Monetary-Fiscal Policy Performance: Some Optimal Control Experiments with A Small Econometric Model of Canada

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ABSTRACT

Monetary-Fiscal Policy Performance: Some Optimal Control Experiments With A Small Econometric Model of Canada

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In this thesis we conducted a number of experiments in monetary-fiscal policies, using a small econometric model of Canada and the technique of optimal control. The planning horizon covers fourteen quarters, from 1975 III to 1978 IV. In this period, the Bank of Canada (BOC) pursued the monetarist policy, also known as monetary gradualism, to combat inflation.

The experiments deal with three issues:

1. How does an optimal monetary-fiscal coordination perform with respect to the historical policy? This question is the focus of the thesis.

2. How is the relative effectiveness among various optimal policies: monetary, fiscal and joint? How is the impact of constant growth rate rule (CGRR)?

3. How important is the effect of inside lag?

We first constructed our econometric model. Several steps were taken: model specification, estimation, and validation. Bounded control is then applied. There are two controls: money supply and government spending.

Results indicate there is no evidence of a substantial gain over the historical policy from the optimal

joint policies. Several implications emerge. For example, alternative solutions suggested by the BOC critics appear unsupported. Monetary-fiscal policies as demand management tools do not appear effective in providing a solution to the Canadian unemployment-inflation problem in the short-run. It results that a gradual approach to end inflation, even announced, will inevitably involve high unemployment and low economic growth; the necessary time can be long, particularly for unemployment...

Regarding the issue of relative effectiveness, fiscal policy is generally better than monetary policy; but the degree of superiority is rather negligible and does not reflect multiplier analysis calculations. In one experiment the outcome is reversed when inside lag is incorporated. These suggest in a multi-target context, multiplier analysis could be a misleading tool in evaluating relative effectiveness.

Regarding CGRR, the controls' performance diminishes when they are subject to this rule. The loss in effectiveness, however, is negligible for money supply.

We have to say that several factors can affect the results: model specification, control bounds, welfare weights, etc... In spite of sensitivity analysis experiments, the results should be viewed as suggestive.

To Thu Thi Luu and

My Father

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CHAPTER 1

INTRODUCTION

This thesis is an empirical study of the Canadian econonomy. In the thesis we perform some deterministic optimal control experiments in monetary-fiscal policies,[1] using a small-size non-linear econometric model of Canada. There are two control variables: nominal narrow money M1 as the monetary instrument and deflated government spending as the fiscal instrument.[2] The planning horizon covers fourteen quarters and extends from 1975 III to 1978 IV. It fits with the period under which the Bank of Canada (BOC) adopted the policy of announcing the target range for the rate of growth of narrow money M1. That policy stance is also known as monetary gradualism since the BOC, to combat inflation, has committed itself to bring down the rate of growth of M1 gradually in the belief that a rapid transition would be too disruptive in economic and social terms (see, for example, Courchene, 1977, 1981b). The policy is interesting in two aspects: i) it allows us to treat M1 as a control variable, ii) it is a monetarist experiment in the face of world stagflation as Lipsey (1981, p. 567) wrote:

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1 For terminology clarification, a hyphen in 'monetary-fiscal' implies monetary, fiscal, and joint policies while 'monetary and fiscal' excludes the joint policy. The joint policy is the policy with both monetary and fiscal control variables.

2 To evaluate the robustness of the results, a tax rate is also used as a control variable in a few experiments.

'We have in Canada today a purely monetarist experiment.'

Results from the optimal monetary-fiscal experiments aim at answering three questions which are somewhat interrelated.

i) How does an optimal monetary-fiscal coordination perform with respect to the historical policy?

ii) How is the relative effectiveness among optimal monetary, fiscal, joint policies, and inside optimal monetary and fiscal policies when the control variable is subject to a constant growth rate?

iii) How important is the role of inside lag in affecting the policy effectiveness outcome?

Before we go any further, some clarification is needed.

First, the word 'optimal' has to be understood in the sense that the policy solution is obtained from an optimal control problem which is the optimization of a welfare function, subject to economic contraints, among other assumptions. It should not be understood in the sense of 'best' for the simple reason that the true welfare function might not be known or necessarily unique, and therefore different hypothetical welfare functions would yield different optimal solutions (of course the welfare function cannot be arbitrary either; it is supposed to reflect the decision-makers' preferences or alternative policies which are economically acceptable, politically acceptable (i.e., institutionally feasible)); only in that case, optimal solutions can meaningfully serve as a standard of reference for comparisons of policies).

Second, in the literature there is a branch of thought in which the government agents pursue self-interest motives in the sense of Nordhaus (1975)'s 'The Political Business Cycle'. Instead of optimizing a purely economic welfare function, they optimize a welfare function to achieve a political objective of winning the election: e.g., maximize an aggregate voting function subject to economic constraints as in Nordhaus (1975), Ginsburgh and Michel (1983), or minimize a vote-loss function as in MacRae (1977). We like to mention that we do not pursue this line of thought here.

Let us now return to our three questions.

Gain over the historical policy: The first question will be the focus of the thesis. It concerns the BOC monetarist monetary policy. It can be put in a more precise way: Is there a substantial gain over the historical policy from an optimal monetary and fiscal mix? If such a gain exists, one may attribute undesirable results of the Canadian economy from 1975 III to 1978 IV to the BOC restrained monetary policy. On the other hand, if it does not exist, one could not draw that conclusion.

Critics, monetarists or Keynesians, have raised a lot of criticism against the BOC policy. An important circumstance was the symposium on 'Has Monetarism Failed?'[3,4] organized in 1980 by the well-known monetarist D. Laidler for the Canadian Economics Association. In the meeting, a group of economists attempted to evaluate the BOC monetary policy. The impression from the majority of participants was that the BOC has brought a modest rate of inflation at an expensive cost of lost output and higher unemployment, that its policy had been a failure. Unfortunately, critics also raised a lot of confusion. For example, as a recognized Canadian monetarist, Courchene (1981a, also 1976, 1977, 1981b) did not rest the blame on monetarism. Critical of the facts that a narrow rather than broad money supply growth target has been chosen, that gradualism has not been pursued with any degree of firmness and determination but with such emphasis on short-run exchange rate concerns, he was inclined to argue that the BOC policy did not represent monetarism, and therefore monetarism has not yet been tried. Another participant, Wirick (1981), had a considerable sympathy with the underlying economic rationale for monetarism but

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3 The papers appeared in the Canadian Public Policy, April 1981 (Supplement): Laidler (1981a, 1981b), Barber and McCallum (1981), Peters and Donner (1981), Courchene (1981a), Wirick (1981), Freedman (1981).

4 'Monetarism' is a word which means different things to different people. Laidler (1981b, p. 215) argued that it should be understood in the Canadian context as a practical policy doctrine which suggests a set of principles for the conduct of monetary policy such as the rate of growth of some monetary aggregate put at the center of the monetary policy, controlled in accordance with some pre-stated rules or targets rather than manipulated for fine-tuning purposes.

suggested that the BOC gradualism has been too gradual to have effect on inflation expectations; as a result, the policy was not effective in reducing inflation, the slow progress painful. On the other side, Barber and McCallum (1981, also 1980), Peters and Donner (1981, also Donner and Peters, 1979) who may loosely be described as post-Keynesians were very concerned about output and employment; they thought that money had been too tight and the cause for the BOC policy failure. Peters and Donner (p. 226) particularly wrote: 'Thus, the appropriate policy mix for a floating rate country such as Canada is surely the opposite one, a relatively easy monetary policy combined with a tight fiscal policy.' How conflicting these ideas are! One should wonder whether critics have truly perceived deep problems of stagflation.

To answer the question meaningfully, attention will be paid to the two positions taken by Barber and McCallum, Peters and Donner on the one hand, and by Wirick on the other.[5] The first advocated that more concern should be put on unemployment and GNP, and as prescription, money should not have been too tight. The second recommended a less gradual monetary policy to permit an effective reduction in

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[5] We will not touch on Courchene's point of view that monetarism has not been tried.

inflation.[6] Two reasons compel us to look closely at these views. First, as will be clear in Chapter 9, raised as alternatives to the BOC policy, they perfectly fit the purposes of the sensitivity analysis experiments. Second, we personally believe that they had been shared by a fair number of economists and therefore are worth being clarified.[7]

What are the criteria to determine if the gain over the historical policy is meaningful, substantial? As the Canadian economy was facing stagflation under the optimization period, the fulfillment of one of the following conditions would allow to conclude that the gain is substantial:-

i) the optimal policy brings a substantial and simultaneous improvement in GNP, unemployment and inflation rates relative to the historical policy.

ii) The optimal policy yields much better results in GNP and unemployment without much deteriorating the rate of inflation relative to the past policy.

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6 A belief that a less gradual monetary restraint will act upon inflation expectations and makes them adjust more rapidly has to presume that the model structure will accordingly change. To such a belief, there is no hope for our model to meaningfully conduct a less gradual monetary experiment. It is not clear to us how strong Wirick's belief is. On the other hand, there are economists who recognize some inertia in inflation expectations but believe a less gradual monetary policy can bring a sharper reduction in inflation. It is this belief we are looking at.

7 In the text we sometimes mention the critics' names explicitly or just use the word 'critics'. Since their views, we believe, had been shared by a fair number of economists, we truly refer to a larger group.

iii) the optimal policy substantially reduces the rate of inflation without much deteriorating GNP and unemployment relative to the past policy.[8]

Relative effectiveness: Turning to the second question of relative effectiveness among monetary, fiscal, joint policies, and inside monetary and fiscal policies, we do not intend to restrict the exercises solely to the issue of relative effectiveness. The scope is somewhat larger. We like to draw from results two implications:

i) regarding the traditional multiplier analysis as a tool of evaluating relative effectiveness

ii) regarding some desirable conduct of monetary policy. For example, does it make much difference if M1 is discretionary or follows a constant growth rate rule?

Inside lag: Some aspects of relative effectiveness would be evaluated more meaningfully if the role of inside lag was taken into account. Of course if inside lag was important, it could affect the outcome of the result from the first question. Economists have recognized the importance of inside and outside lags; but there is little effort in quantitatively examining the impact of inside lag. It is no surprise that one can come across a statement such as:

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8. The gain may also be regarded as meaningful if the optimal policy substantially improves GNP without much changing the inflation and unemployment rates relative to the past policy. This condition is weaker than conditions (i) and (ii).

> Because of the legislative and bureaucratic lags in the enactment and implementation of fiscal policy changes, many Keynesians are now coming to the view that fiscal policy is not a useful stabilization weapon and are placing more emphasis on active variations in the money supply as the appropriate way of stabilizing the economy [Parkin, 1982, p. 495].

One may find exaggeration or bias in the statement; but is there a little truth? We will tentatively try to quantify the impact of inside lag.

Summary of the results

Several interesting results emerge from the experiments. Below they are briefly summarized.

First, it could be said that there is no evidence of a substantial gain over the historical policy from an optimal monetary and fiscal coordination. Embodied in this result are several important implications:

i) Alternative solutions suggested by critics appear unsupported by empirical evidence.

ii) Unless we believe in a quick disinflationary policy, monetary-fiscal policies as demand management tools do not seem effective in providing a satisfactory solution to the Canadian unemployment-inflation problem in the short-run;

iii) it results that a gradual approach to end inflation, even announced, will inevitably involve high unemployment and low economic growth. The necessary time can be long, particularly for unemployment.

iv) In view of these points, attributing bad results of the economy to the BOC monetarist policy would not have a valid foundation.

At the present time, economists have learned more about inflation. They are now less enthusiastic about the belief that there is anything to be gained from increasing demand through stimulative monetary policies because such policies would soon bring higher inflation, higher interest rates, less investment and more unemployment. In retrospect, is the rationale of the BOC monetary restraint fundamentally different from the present attitude?

Next, with respect to the issue of relative effectiveness, the joint policy is naturally the best. The fiscal policy through government spending is better than the monetary policy; but the degree of superiority is rather negligible. Under the assumption that the policy-makers' past preferences were preserved, incorporating the effect of inside lag would reverse the outcome. These findings are in contrast with multiplier analysis calculations which show a very high degree of effectiveness for the fiscal policy. The implication is that in a multi-target framework, multiplier analysis would no longer be adequate and could be a misleading tool in assessing relative effectiveness.

With respect to the effect of rule, imposing a constant growth rate on the controls would diminish their performance. But the loss in effectiveness is found to be negligible for M1; it appears that in the conduct of monetary policy, when a target range has been set for a certain planning period, on a quarterly basis it would matter little whether M1 (seasonally adjusted) is discretionary or follows the rule.

Plan

The remainder of the thesis is organized as follows.

Besides Chapter 1 (Introduction) and Chapter 10 (Conclusion) there are three parts.

Part I contains two chapters. Chapter 2 is a survey of the literature. Our research takes place at a time when there is less enthusiasm in the government intervention. For years, monetarists have objected to activist stabilization policy. That position is presently consolidated by the rational expectations school which, furthermore, questions the usefulness of econometric models in policy evaluation, the validity of optimal control techniques in economic planning. To some degree, our thesis falls under similar criticisms. Therefore, it is indispensable to review the rational expectations controversy. Although our topic is also related to the monetarist debate, here we will not go over it as it is covered in almost every macroeconomics textbook. Chapter 3 discusses methodology and framework of study. First, some arguments are advanced to support the use of optimal control; then, a short description of optimal control techniques is provided; finally, the optimization experiments are listed. It is worth mentioning that this thesis presents an application of bounded control. In the empirical literature, for realistic and technical reasons, the common practice is to attach costs to the control variables in the welfare loss function. Some drawbacks might result; particularly, the procedure may alter the relative balance of attention accorded to the legitimate welfare variables. An alternative is to apply appropriate bounds on the control variables. To our knowledge, only a few studies have experimented with bounded control (McCarthy and Palash, 1977; Craine, 1978; Sandblom, Banasik, and Parlar, 1981; Sandblom and Banasik, 1984, 1985).

Part II focuses on the model which was built with the explicit purpose of evaluating monetary-fiscal policies among themselves and relative to the historical performance in a particular period of time which witnesses the monetarist experience. Chapter 4 provides an outline of the model. Chapter 5 discusses the model specification and estimation by ordinary least squares. Chapter 6 treats the estimation by two-stage least squares, and Chapter 7 deals with the model validation.

Part III reports the optimization results (Chapter 9).But before that, the specification of the welfare loss function is carefully described in Chapter 8.

Chapter 10 is the conclusion. Main findings of the research are summarized, and implications are drawn.

PART I

REVIEW OF THE LITERATURE - FRAMEWORK OF STUDY

CHAPTER 2

RATIONAL EXPECTATIONS CONTROVERSY IN MACROECONOMICS: REVIEW OF THE LITERATURE

The purpose of this chapter is to provide a review of the literature on the rational expectations (RE) controversy with a limited scope on principal issues and a focus on important studies. Presently, there have been several publications discussing the subject, e.g., Shiller (1978), McCallum (1980), Buiter (1980), Gordon (1981), Begg (1982), Maddock and Carter (1982), Sheffrin (1983)...To an extent, our review is based on Sheffrin (1983) which appears to provide the most complete survey.

The RE concept was introduced in macroeconomics by Muth in 1961. But its profound impact was not seen until the contributions of Lucas, Sargent, Wallace, and Barro in the 1970s. Essentially it postulates that the unobservable subjective expectations of individuals are exactly the true mathematical conditional expectations implied by the model itself, given the available information. Economic agents act as if they are fully informed of the process which ultimately generates the outcome of the variable concerned; hence, they do not waste information and behave in the most efficient and economical manner that they can.[9]

The adoption of the RE hypothesis leads to several propositions which have raised a lot of controversy between

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9 On testing and criticizing the RE hypothesis itself, see Sheffrin (1983, pp. 11-17) for a short review.

Keynesians and the RE theorists who call themselves new classical macroeconomists. Debate has been centered around the three following points initiated by the RE proponents:

i) Stabilization policy ineffectiveness: any systematic action on the part of the government would have no effect on output or other real variables (Lucas, 1972; Sargent and Wallace, 1975, 1976; etc...).

ii) Econometric policy evaluation: the structure of existing macroeconometric models is not invariant to policy change; therefore, policy evaluation is meaningless (Lucas, 1976).

iii) Optimization policy: optimal control techniques are not appropriate for economic planning; optimal plans are inconsistent (Kydland and Prescott, 1977).

The propositions seriously challenge orthodox macroeconomics, rejecting the intervention doctrine, suggesting instead predictable fixed rules, and accordingly vindicating monetarists' constant growth rate rule.

These three issues will successively be discussed. But before that, it seems necessary to briefly consider Lucas's theory of business cycle at least for two reasons:

i) The theory will provide the background for the so-called Lucas supply equation which appears in the policy ineffectiveness proof.

ii) The theory has been claimed as an alternative to Keynesian theory in explaining the business cycle.

2.1 LUCAS'S THEORY OF BUSINESS CYCLE[10]

Lucas perceives the economy as a world of competitive markets in which tastes and technology are fluctuating, implying continually changing relative prices. To these price movements respond movements in quantities (employment, consumption, investment...) which are the outcome of free decisions taken by self-interested (utility and profit) maximizers who utilize their incomplete information efficiently. There is no reason, however, to believe that business cycles will occur in that world because, although tastes and technology affect individual markets, the effects would tend to cancel over markets. Nevertheless, if an additional source of 'noise', unsystematic movements in money is introduced, the general price movements will be generated, and the economic system will generate observed facts in prices and quantities (i.e., observed business cycles), with the system always in general equilibrium, markets always clearing.

How different is Lucas's account of business cycle from orthodox macroeconomics in which output fluctuations take place, not because of wage and price fluctuations, but instead of them (Laidler, 1982, p. 86).

To clarify Lucas's point of view, let us consider how a representative worker-producer will respond to an

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10 Based largely on Lucas (1977).

unexpected monetary change, e.g., an increase. As money increases, it will induce an increase in the general price, affecting the selling price of our agent who has to make two distinctions.

i) How much is an increase in his selling price due to an increase in relative price, how much is it due to general inflation?

Our agent, of course, will not respond if the increase is purely inflationary. The question arises since he has imperfect information in aggregate price. Although Lucas suggested a strong link between money and general price in the long-run, he regarded it loose in the short-run, which prevents the agent from making accurate forecast.

ii) Is the increase in relative price temporary or permanent?

To an extent that the agent perceives the increase in his selling price as a temporary relative increase, he will supply more employment. On the aggregate, employment and output will increase with price increases, as the aggregate price level turns out to be higher than anticipated. In the case of a sustained inflation, agents will be less likely to respond and distrust their price changes as signals of relative price change.[11]

Lucas's explanation of the business cycle relies particularly on two assumptions: imperfect information and intertemporal substitution.

Critics question the validity of the imperfect information since a wealth of information about prices, wages, employment...is available virtually instantaneously and essentially for free (Hall, 1980b, p. 23).

The intertemporal substitution hypothesis explains fluctuations in employment and unemployment as the response of workers to perceived temporary movements in the real wage. According to it, workers treat current leisure as highly substitutable with leisure and goods in future periods; therefore, movements in current real wage relative to expected future real wage will induce a large labour supply response (Altonji, 1982, p. 783). To Lucas, the hypothesis is important:

i) To explain observed facts (that producers respond to small price fluctuations with large fluctuations in output

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11 Lucas (1973) claimed evidence of a relationship between higher variance in demand management (i.e., fiscal and monetary policy) and the terms of inflation-output trade-off. But as noted by Sheffrin (1983, p. 60), the relationship holds only in extreme cases (country with high volatility of inflation as Argentina, and country with low inflation as the U.S.). Another study by Froyen and Waud (1980), while econometrically supporting Lucas's hypothesis that agents react to relative prices and form expectations optimally but with imperfect information, found a very weak connection between the volatility of demand management policy and the terms of inflation-output trade-off.

and employment), based on substitution effects and not on 'disequilibria' notions (Lucas, 1977, p. 17).

ii) To show that it is a free choice which affects unemployment. Hence, there is no such 'involuntary unemployment' (if unemployment is involuntary, his hypothesis of market-clearing is no longer tenable).

Lucas accepted the intertemporal substitution, based on empirical evidence obtained from his study with Rapping (Lucas and Rapping, 1969). However, in a very recent paper, using the Lucas-Rapping model and replacing their assumption of adaptive expectations by RE, Altonji (1982) found that his results do not support the intertemporal substitution but remarked that evidence apart from his study indicates a very low elasticity of labour supply with respect to temporary wage changes in the range of .10 to .60 as compared to 4.6 in Lucas and Rapping's work (see Altonji, p. 783 and p. 816). He also attributed many poor results partly to the false assumption of market-clearing. One can see how important the concept of market-clearing is. It plays an important role in Lucas's vision of the business cycle and in the new classical macroeconomics.

2.2 STABILIZATION POLICY INEFFECTIVENESS

The proposition asserts that systematic government policy is impotent, only predictable changes can affect output and other real variables. Consequently, it does not matter if the monetary authority pursues an activist policy or follows a constant growth rate rule. Note, however, that the price behaviour is still affected by systematic or unsystematic policies.

The proposition is easily seen through a simple equilibrium model.

2.2.1 Model[12]

Supply equation[13]

$$y_t = y_t^n + a(p_t - p_t^e) + u_t \quad (2.2.1)$$

Demand equation

$$y_t = - bp_t + cx_t \quad (2.2.2)$$

Expectations

$$p_t^e = E_{t-1} \; p_t \quad (2.2.3)$$

where y_t = real GNP

y_t^n = real GNP at the natural level (exogenous)

p_t = price

x_t = government policy instrument, e.g., money supply

u_t = random variable with zero mean

E_{t-1} p_t = expectation of p_t as of time t-1.

Following the RE literature practice, we express all variables in Section 2.2.1 in log form. Equating demand and supply yields:

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12 Taken directly from Maddock and Carter (1982, p. 50)

13 For simplicity, we ignore the persistence factor. On the issue of persistence in output..., see Sheffrin (1983, pp. 54-58).

$$p_t = \frac{1}{a+b} \left(ap_t^e + cx_t - y_t^n - u_t\right) \qquad (2.2.4)$$

Take expectation of p_t and by virtue of (2.2.3):

$$p_t^e = E_{t-1}\, p_t = \frac{1}{a+b} \left(ap_t^e + cE_{t-1}\, x_t - y_t^n\right) \qquad (2.2.5)$$

Subtract (2.2.5) from (2.2.4):

$$p_t - p_t^e = \frac{1}{a+b} \left[c(x_t - E_{t-1}\, x_t) - u_t\right] \qquad (2.2.6)$$

Substitute (2.2.6) in (2.2.1):

$$y_t - y_t^n = \frac{ac}{a+b} (x_t - E_{t-1}\, x_t) + \frac{b}{a+b}\, u_t \qquad (2.2.7)$$

Examining (2.2.7), we see that only the unsystematic part of the government policy $(x_t - E_{t-1}\, x_t)$, can cause output to deviate from its full employment level.

2.2.2 Theoretical Criticisms

The ineffectiveness results depend on three characteristics:

i) RE

ii) Lucas supply equation

iii) All individuals have access to the same information.

Relaxing the RE hypothesis naturally reestablishes the policy effectiveness. But alternative kinds of expectations are ad hoc. It seems interesting to examine the remaining characteristics.

a. Lucas's supply equation

The supply equation (2.2.1) is usually attributed to Lucas.[14] It is based on Lucas (1973) work: prices are fully flexible, and output can deviate from its full-employment level only when actual price deviates from its level which was expected to prevail on the basis of past information. In the absence of any surprise in the price level, output will always be at the full-employment level.

Implicit in Lucas's supply equation is the hypothesis of market-clearing prices which is strongly criticized by Keynesians (see Tobin, 1980a). Although realizing that prices can be quite flexible in markets with specialized traders, they believe there is ample evidence as to the existence and persistence of disequilibrium in factor and product markets (Buiter, 1980, p. 41). They cannot think of a 12% rate of unemployment as an indication of a labour market in equilibrium.

Early Keynesian studies by Fischer (1977), Phelps and Taylor (1977) maintained RE without rejecting the assumption of market clearing, introduced sticky prices and wages and

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14 The supply equation can be also derived from Friedman (1968) work (see McCallum, 1980, pp. 720-721; Gordon, 1981, pp. 504-505). In a RE framework, Lucas's interpretation is preferable. Indeed, in Friedman's, firms calculate real wage by using the current product price (which, on the aggregate, is equal to the aggregate price) while workers use the expected aggregate price. This asymmetry is avoided in Lucas (1973) because workers take into account the information of the current price of their market's product in forming the expected aggregate price. Wasting this kind of information is not consistent with RE hypothesis (Parkin, 1982, p. 349).

found no theoretical support for the ineffectiveness proposition which is also invalidated by disequilibrium models (e.g., Buiter, 1980; also, see references in Begg, 1982, p. 170).

Fischer's paper is quite interesting as one of the pioneer studies from the Keynesian camp and also in providing an example of unequal information to be discussed in the next section. It is worthwhile to review it.

Fischer injected an element of short-run wage stickiness by setting over-lapping labour contracts, with each being made for two periods. The monetary policy is fully anticipated; but there is still room for it to maneuver, because between the time the two-year contract is drawn up and the last year of operation of that contract, there is time for the monetary authority to react to new information about recent economic disturbances while workers must stick to their nominal contracts. Given the negotiated second-period nominal wage, the monetary authority by reacting to disturbances can affect the real wage of the second period of the contract and thus output.

Fischer did not provide any microeconomic foundation for the existence of long-term nominal contracts but noted that transaction costs of frequent price setting and wage negotiations must be part of the explanation. There have also been some attempts to restore the ineffectiveness proposition in front of sticky prices. McCallum, in a series of papers, argued that the recognition of price stickiness does not, in and of itself, negate the proposition. Frydman (1981) proved that the McCallum (1977) model is flawed and implicitly embraces an unacceptable assumption regarding the producers' supply behaviour. McCallum (1980) reconstructed the argument supporting the ineffectiveness proposition for prices which are rigid for one period (i.e., prices independent of current demand and supply) but are free to jump by any amount in the next period. Gordon (1981, p. 511, footnote 12) remarked that this type of one-period rigidity does not seem to have any empirical evidence to the aggregate U.S. economy.

b. Information structure

The ineffectiveness proposition depends closely on the information structure in the model. Early models restrict individuals' access to contemporaneous information to prices in local markets; nevertheless, it is plausible to make some global information available to agents: for example, McCallum (1980) showed that including an aggregate interest rate can invalidate the neutrality of money. Begg (1982, Chapter 6) demonstrated that if the government gains an informational advantage over the private sector, the proposition does not hold. The Fischer (1977) study as just discussed is another example. In short, the message is simple: the proposition is very sensitive to the information structure built into the model.

2.2.3 Empirical Evidence and Criticisms

Barro (1977) provided the first empirical evidence that only unanticipated movement in money would affect real economic variables like unemployment. Essentially he tested the following equation:[15]

$$UR = UR^n + f(MG - MG^e) \qquad (2.2.8)$$

where UR = unemployment rate

UR^n = natural level of UR, determined by minimum wage and military draft variables

MG = money growth (actual)

MG^e = money growth (anticipated)

$(MG - MG^e)$ represents unanticipated movement in money. To measure it, Barro specified an equation for MG which depends on: a measure of federal expenditure relative to normal, a lagged value of unemployment variable and two lagged values of MG. He then equated $(MG - MG^e)$ to the equation's prediction error. Results show that the coefficient of $(MG - MG^e)$ is significant. Further tests reject the relevance of actual money growth for unemployment.

Barro's study was criticized by Small (1979) on three respects:

i) The money equation left, as prediction error, monetary movements in the war periods whose information were readily available to rational agents. Hence, the use

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[15] The equation was taken from Small (1979).

of dummies to capture federal expenditure in the war years is necessary.

ii) A distinction should be made between anticipated and unanticipated increase in federal expenditure; only the latter should enter the money equation.

iii) The military draft variable as a determinant of the natural unemployment rate had an implausible effect on the labour market.

Taking into account the arguments in (i) and (ii), Small concluded that Barro's evidence on the ineffectiveness was weak.

Barro (1979), rejecting argument (ii) completely, introduced dummies as suggested in (i), but found that his results were still robust. He acknowledged, however, that the estimated effects of the determinants of the natural unemployment rate (i.e., minimum wage and military draft) were doubtful.

The works by Barro and Small assume the RE hypothesis without testing it. A very recent study by Mishkin (1982) attempted to test RE and ineffectiveness hypotheses jointly and separately.

First, Mishkin specified an equation in which the deviation of a real economic variable (output or unemployment rate) from its natural level, to be denoted by YD_t, is governed by unanticipated and anticipated money growth:

$$YD_t = \sum_{i=0}^{n} \alpha_i \, (MG_{t-i} - MG^e_{t-i}) + \sum_{i=0}^{n} \beta_i \, MG^e_{t-i} + u_t \qquad (2.2.9)$$

Neutrality of money will imply $\beta_i = 0$ for all i. Mishkin, then, postulated a function for MG such that:

$$MG_t = Z_t \gamma + v_t \qquad (2.2.10)$$

where Z_t is a vector of variables known at time t-1. Hence:

$$MG_t^e = Z_t \gamma \qquad (2.2.11)$$

If RE are imposed (2.2.9) can be written as:

$$YD_t = \Sigma \alpha_i (MG_{t-i} - Z_{t-i} \gamma^*) + \Sigma \beta_i Z_{t-i} \gamma^* + u_t \qquad (2.2.12)$$

where $\gamma = \gamma^*$. If neutrality is imposed as well, (2.2.12) becomes:

$$YD^t = \Sigma \alpha_i (MG_{t-i} - Z_{t-i} \gamma^*) + u_t \qquad (2.2.13)$$

and $\gamma = \gamma^*$.

The joint test for RE and neutrality can now be carried out. This involves a likelihood ratio test based on two systems: constrained (2.2.10) and (2.2.13) with $\gamma = \gamma^*$ and unconstrained (2.2.10) and (2.2.12) without imposing $\gamma = \gamma^*$. If the joint test is rejected, RE and neutrality can be tested separately. Note that Mishkin's procedure is joint estimation of two equations at a time. This can be done since there is no restriction that the covariances of α and γ are zero as in Barro and Small. Parameters will therefore be more efficient.

Mishkin's own empirical results somewhat support the RE hypothesis but generally cast doubt on the money neutrality proposition; particularly for output, results completely refute the neutrality proposition. Mishkin attributed favorable results from Barro to the misspecification of the lag structure (too short in Barro's case) which yields incorrect statistics. However, as his study (as well as Barro and Small's) adopted the reduced-form approach with the implicit assumption that the reduced-form is true, he cautiously expressed: 'it is not clear whether the invalidity of this assumption might lead to the rejection of the macro RE hypothesis [the policy ineffectiveness proposition in Mishkin's terminology] even if it is true'.

To conclude this section, as Sheffrin (1983, p. 66) has summarized, there now exist several studies which purport to show that only unanticipated variables affect real variables such as GNP or unemployment; however, recent studies cast doubt on these results, particularly when they are examined in the context of more general models.

2.3 ECONOMETRIC POLICY EVALUATION

2.3.1 Lucas's Critique

In a penetrating article Lucas (1976) questioned the assumption of model structure invariance in policy evaluation and reached the conclusion that existing macroeconometric models are useless for policy analysis. His critique has been too influential.

A typical econometric model can be written as:

$$y_{t+1} = f(y_t, x_t, \theta, \varepsilon_t) \qquad (2.3$$

where y_t is a vector of endogenous variables, x_t a vec[illegible]f exogenous variables (including policy variables), ε_t a vector of serially uncorrelated, identically distributed disturbances, and θ a vector of parameters.

The model can be simulated under alternative policies on the assumption that θ is stable, invariant to policy changes. Lucas's theme is the opposite:

> given that the structure of an econometric model consists of optimal decision rules of economic agents, and that optimal decision rules vary systematically with changes in the structure of series relevant to the decision-maker, it follows that any change in policy will systematically alter the structure of econometric models [Lucas, 1976, p. 41].

The argument is easily understood in a RE context. In an econometric model expectations enter through many equations. With RE, agents will form their expectations consistent with their environment; as the environment changes due to policy changes, their expectations will adjust to the new environment, affecting the structure of the model. The example below illustrates the point.

Suppose the government attempts to evaluate certain monetary policies, based on the following equation which has been estimated over past sample period:[16]

$$YD_t = \alpha + \beta MG_t + \delta YD_{t-1} + u_t \qquad (2.3.2)$$

YD_t and MG_t have the same meaning as in the last section. (2.3.2) can be thought as a reduced-form of a model which can be described by the three following equations:

$$YD_t = a_0 + a_1\ (MG_t - E_{t-1}\ MG_t) + a_2 YD_{t-1} + u_t \qquad (2.3.3)$$

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16 Taken from Sargent and Wallace (1976).

$$MG_t = g_0 + g_1 YD_{t-1} + v_t \tag{2.3.4}$$

$$E_{t-1}\, MG_t = g_0 + g_1 YD_{t-1} \tag{2.3.5}$$

Equation (2.3.4) governs the money supply, and (2.3.5) is the public's expectations of the money rate of growth MG. According to (2.3.5), the public knows the government's rule and takes it into account in forming its expectations.

Equations (2.3.3), (2.3.4), and (2.3.5) can reduce to

$$YD_t = (a_0 - a_1 g_0) + a_1 MG_t + (a_2 - a_1 g_1) YD_{t-1} + u_t \tag{2.3.6}$$

which is in the form of (2.3.2) with $\alpha = (a_0 - a_1 g_0)$, $\beta = a_1$, and $\delta = (a_2 - a_1 g_1)$. While (2.3.2) and (2.3.6) are identical in form, the coefficients of (2.3.6) depend on control parameters, g_0 and g_1. The implication is clear: if the government desires to change the policy rule (2.3.4), e.g., changing g_0 or g_1 or both, and under the assumption that the public knows the rule, α and δ will alter; since the system is no longer structurally invariant, it is not valid to use (2.3.2) for policy evaluation.

As an alternative, Lucas cautiously proposed that policy analysis should be formulated as choice among rules because optimal decisions of agents cannot be discussed under arbitrary sequence (x_t) of future shocks.

Let policies be characterized by:

$$x_t = h(y_t, \lambda, \eta_t) \tag{2.3.7}$$

where h is known, λ a fixed parameter vector, and η_t a vector of disturbances. The econometric problem is to estimate $\theta(\lambda)$ in:

$$y_{t+1} = F[y_t, x_t, \theta(\lambda), \varepsilon_t] \quad (2.3.8)$$

A change in λ implies a change in policy. Lucas believed that only if policy changes occur as fully discussed and understood changes in rules, there is some chance that the resulting structural changes can be forecast on the basis of estimation from past data of $\theta(\lambda)$ (Lucas, 1976, p. 41).[17]

2.3.2 Criticisms

Lucas's attack is devastating. So far there is not much reply from leading Keynesians, which does not mean that they fully accept Lucas's argument.[18]

The strongest response came from Sims (1980, 1982). On the one hand, Sims criticized conventional large-scale macromodels for their false identification; nevertheless, he argued that their forecasts are useful because their identifying restrictions which are more convenient simplifications than a priori knowledge imposed on the data are pragmatically adjusted to avoid obvious conflict with data. In spite of his criticism and proposal for an alternative of macroeconometrics (vector autoregressive approach), he rejected, on the other hand, the Lucas-Sargent statement that 'modern macro-economic models are of no value

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17 Presently there have been some research in developing new methods for estimating RE models (see Wallis, 1980; Hansen and Sargent, 1980; Chow, 1983, Chapter 12; Fair and Taylor, 1983). Works are still in an experimental stage.

18 See 'Comments and Discussion' following Sims (1982) paper.

in guiding policy and this condition will not be remedied by modifications along any line which is currently pursued' (Lucas and Sargent, 1979, p. 2). He regarded the RE critique of econometric analysis as a cautionary footnote of a general principle (that extrapolation far beyond the range of history to which the model was fitted will generate unreliable results) rather than a deep objection to its foundation (Sims, 1981, pp. 108 and 122). On this respect, he believed that one of the main contributions of the RE assumption has been to provide examples showing how policy action whose size is within the historically normal range could be far outside the range of past experience if its time patterns were historically unusual.

Sim's arguments advanced to support the invariance of the structure model can be summarized as follows.

Policy analysis, in practice, is more often projecting the effect of a change in policy variables on the evolution of the economy rather than projecting the effect of changing the parameters of a model equation; therefore, it can maintain the exogeneity (in reality an approximate exogeneity) of the policy variables if they have varied exogenously: in that case, the reduced-form is structurally invariant. It is a mistake to think that decisions about policy can only be described as choice among permanent rules of behaviour; on the opposite, given the political system, policy shifts, if occurred, are short-lived, and many policy announcements are never carried out or believed. Changes as envisaged by Lucas are rare, resulting in a sequence of policy actions without precedent; so, there is no hope of looking at historical data for similar actions to determine the likely consequences. In short, to Sims, policy analysis and projections should be portrayed as attempts to effectively implement a stable optimal or existing policy regime or a slowly changing rule.

Sims's view is shared by Goldfeld[19] who made the remark: if changes in policy rules hardly occur at random, they should be visualized as brought about by the model, i.e., the real world and not the other way around; in that case, Lucas fails to consider the nature of choices embodied in historical data. Should there be a need for rethinking the RE critique?

Apart from Sims's, Duesenberry[20] advanced the arguments suggesting that the domain in which the critique applies is limited: i) only a fraction of policy developments could be interpreted as changes which should actually cause parameters to vary, ii) there are many economic relations in which expectations about macroeconomic policy are not that important, and this situation characterizes most individual equations in large models.

Recently, Engel, Hendry, and Richard (1983) related the concept of exogeneity to the issue of policy

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19 In 'Comments and Discussion' following Sims (1982) paper, p.153.

20 In 'Comments and Discussion' following Sims (1982) paper, p.163.

evaluation. They defined new concepts of exogeneity. Interestingly, if a variable is 'super exogenous', policy simulations or control exercises are appropriate. A variable is super exogenous if:

i) it is weakly exogenous (essentially a variable is weakly exogenous for estimating a set of parameters if inference about those parameters conditional on the variable involves no loss of information).

ii) The conditional model is structural invariant to any change in the distribution of the conditioning variable.

The authors wrote:

> The concept of structurally invariant conditional models characterizes the conditions which guarantee the appropriateness of "policy simulations" or other control exercises, since any change in the distribution of the conditioning variables has no effect on the conditional submodel and therefore on the conditional forecasts of the endogenous variables [p. 284].

The assumption of 'super exogeneity' is testable by examining the behaviour of the conditional model for invariance when the parameters of the exogenous variable changed. However, Engel, Hendry, and Richard noted that while weak exogeneity and parameter constancy are conjectural features in a conditional modelling exercise, if the data generating process of the exogenous variable has changed, but the conditional model has not, then some credibility must attach to the latter until refuted (p. 302). And that is the standard scientific practice.

2.4 POLICY OPTIMIZATION

Issues also arise when policy optimization is considered in RE models. RE imply economic agents behave optimally, given all information available to them. In consequence, both the private sector and the government know each other's aims and anticipate each other's actions; that has been manifested in Lucas (1976) critique: as the decision rules of the government form a part of the environment facing private economic agents, the decision rules of the latter will change in response to changes in government policy.

In such a game between rational agents, Kydland and Prescott (1977) arrived at a paradox: optimal plans are either consistent and suboptimal or optimal but inconsistent. They went further to draw the conclusion: unless economic agents' expectations are invariant to the future policy plan selected, there is no way optimal control can be made applicable to economic planning when expectations are rational; therefore, rules (with good operating characteristics, e.g., constant money growth) should be preferred over discretion.

The phenomenon of optimal plan inconsistency occurs in multiperiod decision rules because there may be a gain to reverse a rule adopted earlier. A two-period example taken from Buiter (1980) would illustrate the point.

There are two states of the world in period two: good (no flood), bad (flood). In period one, the optimal policy of the government is to discourage people to settle in the potential flood area by promising not to send relief if flood occurs. If people have disregarded the warning and settled, in period two, the optimal policy will require the government to send relief in case of disaster. Rational agents anticipating the government's behaviour will be inclined to settle in the area. The picture is well clear: if the flood occurs, either plans are optimal but inconsistent (the government breaks its promise) or consistent (promise kept) but suboptimal.

As a matter of fact, not all models in which agents are rational will have inconsistent optimal policies. For example, in the field of taxation, the general message that seems to emerge from the literature is that if taxes are not distorting, optimal policies will not be inconsistent (see Sheffrin, 1983, p. 97).

Regarding the usefulness of optimal control, Kydland and Prescott's paper have raised some confusion at the beginning. There is now a consensus that traditional optimal control techniques which fail to take into account the expectation formation mechanism will be inadequate and suboptimal if expectations are rational, but optimal control can still be used for policy analysis if the expectation endogeneity is incorporated (Taylor, 1979; Buiter, 1980; Chow, 1981, Chapter 15).

2.5 CONCLUSION

As some issues are still controversial, it is hard to claim that our review has been balanced. However, no matter how controversial they are, RE economists force us to confront these issues. That is a very important contribution on the part of the RE exponents.

As far as optimal control is concerned, the general agreement is that it is an appropriate tool as long as expectations, if rational, are endogenously incorporated.

The policy ineffectiveness proposition seems extreme. It goes beyond monetarists' allegation of no trade-off between inflation and unemployment in the long-run. Even, McCallum (1980, p. 738), highly sympathetic to the RE school, stated: 'it seems difficult to sustain the policy ineffectiveness propostion is applicable to the U.S. economy'.

Lucas's critique is less controversial. The dependence of the model structure on policy changes when RE are incorporated is a meaningful theoretical insight which could easily be accepted by the economics profession. It is not well clear how the profession will react to his conclusion that existing econometric models are of no use; so far it is somewhat quiet. Is it an expression of acceptance or an attitude of reservation due to uncertainty? Deeply, that attitude might reflect Buiter's (1980, p. 46) view that the practical importance of Lucas's critique on policy evaluation exercises using econometric models still remains to be established on a case-by-case basis.

Turning to his remedy, Lucas is optimistic that it is feasible to model business cycles in a dynamic optimal framework consistent with general equilibrium. If we ignore problems in developing new econometric techniques, new optimization methods which will require large amount of computer resources, his approach surely poses serious difficulties in the model identification. One truly doubts about this. In the words of Sims (1982, p. 151):

> The ambitious, probably unattainable goal of the rational expectations school -- to identify parameters of behaviour that would be invariant to unprecedented permanent changes in rule -- should not condition an entire research plan.

In any case, new tools for policy analysis, free of critique, do not seem to appear in a near future.

With respect to policy recommendation, it is obvious that RE theorists favour predictable policy rules. To some extent, they vindicate monetarists' proposal of non-intervention. But it is misleading to think that all (or the majority of?) monetarists embrace their approach.[21] As the monetarist Laidler (1982, p. 104) expressed, the proposition that the operations of market (always clearing) leave no room for government improvement cannot be sustained; of course the expression does not suggest that intervention be desirable,

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21 Laidler (1982, p. 21) noted that it is a mistake to treat the new classical view of price-output interaction as synonymous with monetarism (see, e.g., Hahn, 1980). He preferred to call it the neo-Austrian doctrine while Tobin (1980a) labelled the new classical economics monetarism - Mark II.

given the state of knowledge. Laidler went further to claim it is almost impossible to reconcile the RE approach to the analysis of business cycle with evidence (Laidler, 1982, p. 96). To us, Laidler put forward the most original argument against the hypothesis of market-clearing in particular and the RE approach to business cycle in general. As the market-clearing notion is one of two pillars (apart from RE hypothesis) of the new classical macroeconomics (see Tobin, 1980a, p. 22), it is important to reserve the remainder of this chapter for a look at Laidler's argument.

His premise was: in a general equilibrium theory, if something is going wrong in the labour market, it will manifest itself in some other market or markets as well. What Laidler wanted to show is that it manifests in the market for real money balances.

Take the demand for money function: a lagged dependent variable seems to be needed. In a world where money supply is exogenous (as assumed by RE proponents), it is not logical to justify its presence on the ground that the nominal money supply adjusts with a distributed lag to variations in the arguments of the demand for money function. Nominal money is either exogenous or endogenous; it cannot be both. That leaves two ways to explain the presence of the lagged dependent variable. If one ignores the learning error hypothesis in modelling permanent income (which is not consistent with RE), the only explanation is: it reflects the slow adjustment of real balances, and therefore of the price level, towards their long-run equilibrium value, rather than a slow adjustment of nominal balances. If one accepts the explanation, the short-run demand for money function is a hybrid which is part long-run structural demand for money function and part price level adjustment equation; the coefficient which measures the speed of adjustment does not reflect the transaction costs of agents trading in asset markets but the slow real balance effect operating on prices. This interpretation, if accepted, is consistent with the literature that money wages and therefore perhaps prices might display considerable sluggishness in moving toward market-clearing levels after a monetary shock.

As long as there is no satisfactory theory of why prices are sticky, Laidler added, it is not surprising that such assumption of market-clearing is acceptable, just as before, the assumption of fixed prices.

CHAPTER 3

METHODOLOGY AND FRAMEWORK OF STUDY

This chapter is the background of the optimization experiments. In Section 3.1 on methodology, we advance some arguments supporting the use of optimal control, followed by a brief discussion on optimal control techniques with emphasis on some particular features of the algorithm we use. In Section 3.2 on framework of study, we describe the set of optimization experiments necessary to answer the questions raised in the Introduction Chapter; finally, we touch again on Lucas's critique as far as our thesis is concerned.

3.1 METHODOLOGY

3.1.1 Justification for the Use of Optimal Control

Our Introduction Chapter has set out three questions for study. Among them are judging history and evaluating relative effectiveness.

One way of judging historical performance is to compare it with an optimal solution. On the other hand, optimization is technically desirable as Holbrook (1974, p. 155) explained:

> ...a policy-maker typically is concerned with a multidimensional policy decision which takes into account both current and future goals, a decision requiring some form of optimal control technique rather than the standard fare of simulation results commonly provided.

On the issue of evaluating relative effectiveness, traditionally it is done by means of multipliers with comparisons conducted in terms of single targets. The usual tool is policy simulation. For example, two simulations are run over the same period of time: one uses actual values of all exogenous variables, and the other uses the same set of exogenous values except that, at some chosen moment, the time path of one of the policy variables is given a step-wise increase or decrease and thereafter follows a path that is at a fixed distance above or below the actual path. Results give two different time paths for the endogenous variables. Multipliers are then derived, showing the effect on an endogenous variable divided by the step change in the policy variable after one quarter, two quarters... (Christ, 1975, p. 57). The target commonly used is GNP. But inflation recently became one main objective although GNP and unemployment are still policy-makers' concerns. Clearly, in a multi-target context, particularly under a period of high inflation, multipliers solely in terms of real GNP would not be adequate to evaluate relative effectiveness. Although multipliers can be calculated for each target, it is not clear how the individual multipliers are weighted to yield a total impact on all targets. A few authors noted the weakness of multiplier analysis. Brainard (1967, p. 411) argued that the question of relative effectiveness should be related to how the policy instruments affect the policy-makers' performance in meeting their objectives rather than be based on the examination of the multipliers of these instruments. Chow (1975, p. 220) was more precise:

> Comparison of the various multipliers, however, does not provide a precise answer to question at hand. The interactions of the current and delayed effects of the instruments through the elements of the matrices A and C have to be properly incorporated, the dependent variables affected by them have to be properly weighted, and the instruments must be given any opportunity to perform their best in terms of stated objectives of policy. All these considerations are systematically taken into account in the optimal control calculations for the comparison of relative effectiveness of subsets of policy instruments. 22

If so, as Abel (1975, p. 243) wrote, multiplier analysis is unnecessary, if not misleading, since any relevant features of the multipliers will be reflected in the value of the welfare loss function. Part of this thesis is to present evidence that multiplier analysis could be truly misleading in assessing relative effectiveness (we are not aware of any empirical evidence in the literature).

3.1.2 Deterministic Optimal Control with Bounded Controls

As we move to the computational techniques, there now exist several algorithms capable of solving control problems in economics. The simplest one is to solve a deterministic model whose parameters are known with certainty, and disturbances are assumed to be zero. Control becomes stochastic when introducing uncertainty in the form of an additive noise to the system equations, and in the form of uncertainty about parameters. If parameters are allowed to be learned, it is adaptive or dual control since the control variable is chosen with the two purposes of achieving desired

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22 The matrices A and C are the coefficient matrices of the econometric model. See Equation (3.2.1) in this chapter.

targets and learning the parameters (for more on the classification, see Kendrick, 1981a, who also provided brief discussions on various existing algorithms). Kendrick (1981a, p. 5) also noted that most economic control models which have been solved to date are of the deterministic variety, that as one progresses from deterministic to stochastic control without learning, to dual control, the size of the numerical models rapidly declines due to the computational complexity in the uncertainty treatment. For example, deterministic control models can contain hundreds of equations, stochastic without learning control models usually have tens of equations while dual control models contain fewer than ten equations.

The computer code we have used for this research is the CONOPT,[23] written by Drud (1978) at the Technical University of Denmark and improved by Drud and Meeraus (1980). The CONOPT is apparently the most efficient and up-to-date optimization program for large-scale econometric models (Sandblom and Banasik, 1980, p. 72). It is designed for deterministic discrete control by implementing a generalized reduced gradient method and using sparse matrix techniques (see Drud, 1978). It has very interesting features;

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23 The CONOPT is available at Concordia University by courtesy of Professor Sandblom. Also available from Professor Sandblom are programs developed by Chow (1975, 1981) which, however, do not suit our purposes, e.g., it is not feasible to impose bounds on control variables and in non-linear models to constrain them to constant growth rates etc...

among them are: i) any objective function that can be written in a FORTRAN subroutine can be used, ii) inequality constraints on endogenous or control variables are allowed.[24] The first feature avoids the problem of symmetrical penalty, i.e., deviations of variables from their targets are equally undesirable regardless of their signs. With the second feature there is no need to attach costs to control variables in the objective function to ensure the model convergence and the practical feasibility (from a political as well as an economic point of view) of the resulting optimal solutions;[25] in that case the objective can solely express the decision-makers' preferences. Sandblom, Banasik, and Parlar (1981) advanced strong arguments suggesting the elimination of the habit of attaching costs to control variables as found in the economic optimal control literature. First, including a policy variable in the objective function would arbitrarily detract from the welfare aspect of optimization and may well alter the relative balance of attention accorded to the legitimate welfare variables. Second, mathematically in linear-quadratic models with continuous time as in control

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24 The CONOPT can also be used as a simulation program when the objective function is not specified. Indeed, it has been used in our simulation exercises to be reported in Chapter 7.

25 Klein and Su (1980, pp. 228-229) noted that without any constraints on the control variables (e.g., penalties in the welfare function or lower and upper bounds) their optimal solutions may become unrealistic and unreasonable, which can disturb the model convergence.

engineering literature, it is necessary to attach costs to control variables (as long as bounds are not imposed on controls) to ensure the existence of a unique solution; for discrete time models, it is not necessary. Economic models are in general of the discrete nature. In view of these problems, they advocated the imposition of bounds on the control variables for realistic reason. Furthermore, one advantage of bounded control is to reduce the number of variables entering the objective function. Thus, the task of searching for the appropriate weights, of varying the weights for sensitivity analysis purposes is easier. Surely, difficulty in generating the control bounds arises; but that should not be a problem as long as the number of control variables to be used is small.

In short, thanks to these convenient features of the CONOPT, we can allow asymmetrical penalty and follow the bounded control approach as suggested by Sandblom et al. This will lessen some criticisms against the linear-quadratic formulation (see, e.g., Bock v. Wulfingen and Pauly, 1978).

3.2 FRAMEWORK OF STUDY

3.2.1 Optimization Experiments

All experiments can be grouped into three categories. The difference between the first two mainly lies in the exclusion (First Category) or inclusion (Second Category) of inside lag.

An important note is that in all optimization experiments, following Chow (1981, p. 55), Klein and Su (1980, p. 232), we add residuals back to all equations. This is feasible for the type of analysis in the evaluation of past policies and seems meaningful; otherwise, the comparison has to be directed at the historically simulated policy which is not our interest.

Our econometric model (to be discussed in Part II) provides two control variables: M1 for monetary policy and G1 for fiscal policy. M1 is the nominal quantity of narrow money; G1 is real expenditure of federal, provincial, and local governments (total government spending consists of G1 and G2 which is real expenditure of Hospitals, Quebec and Canada Pension Plans and is very small compared to G1).

Below we will present a listing of all optimization experiments; explanation will be provided later. But before that, a clarification of terminology is needed: a control variable, if subject to constant growth rate rule (CGRR), is defined as passive; otherwise, it is called discretionary.

CATEGORY I: Experiments without lag consideration

i) Basic experiments

(1) Monetary policy: M1 discretionary, G1 exogenous

(2) Fiscal policy: G1 discretionary, M1 exogenous

(3) Joint policy: M1, G1 discretionary[26]

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[26] An extra experiment is performed for the joint policy in which three instruments are used: M1, G1, and tax rate. In a few other experiments, the tax rate is also treated as a control variable in conjunction with M1 and G1.

ii) Extension

(4) Monetary policy: M1 discretionary, G1 passive

(5) Fiscal policy: G1 discretionary, M1 passive

(6) Monetary policy: M1 passive, G1 exogenous

(7) Fiscal policy: G1 passive, M1 exogenous

CATEGORY ~~II~~: Experiments with lag consideration

(8) (1-quarter lagged) Monetary policy: from (1), obtain optimal M1. Denote it by $M1^*$. Now set G1 and M1 exogenous with $M1 = M1^*(-1)$

(9) (2-quarter lagged) Fiscal policy: from (2), obtain optimal G1. Denote it by $G1^*$. Now set M1 and G1 exogenous with $G1 = G1^*(-2)$

CATEGORY III: Sensitivity analysis experiments

i) Sensitivity with respect to a change in the model coefficients

(10) Joint policy: as in (3) with some new parameters

ii) Sensitivity with respect to a change in the welfare function weights

(11) Joint policy: as in (3)

(12) Monetary policy: as in (1)

(13) Monetary policy (with CGRR): as in (6)

(14) Fiscal policy: as in (2)

(15) Fiscal policy (with CGRR): as in (7)

iii) Sensitivity with respect to a change in the control variable (M1) bounds

(16) Joint policy: as in (3)

Let us examine these experiments more closely.

a. Experiments without lag consideration

In the basic experiments the monetary policy is defined as the one in which M1 is the control variable, and G1 is set at its historical path. The reverse holds for the fiscal policy. In the joint policy, both M1 and G1 are control variables. This kind of definition was found in Craine, Havenner, and Tinsley (1976). For the monetary and fiscal policies alone, it allows M1 and G1 to play the role of control variable alternatively; if they are compared to the joint policy, it allows to determine how the contribution will increase with one more instrument permitted. Moreover, the joint policy will be used to judge the historical performance.

The above definition, however, does not isolate monetary effects from fiscal effects and vice versa. The separation requires in the monetary policy G1 be held constant while M1 is a discretionary instrument, in the fiscal policy M1 be held constant while G1 is a discretionary instrument (see, e.g., Friedman, 1970a, p. 18). Practically, holding an instrument constant for a long period of 14 quarters hardly maintains the invariance of the model structure (recall Lucas's critique on policy evaluation). This demands a less stringent definition. Abel (1975, p. 239), in a study of the monetarist-fiscalist debate, to evaluate the effectiveness of an instrument designated that instrument as a discretionary instrument and the other instrument as a passive instrument and solved an optimal control problem. The values of the discretionary instrument are determined subject to feedback control, but the passive instrument is constrained to change at a constant growth rate (also determined in the optimization process). Constrained to a constant growth rate, the passive instrument can be considered as following a neutral policy rule (Hallet and Rees, 1983, p. 325). Adopting Abel's definition generates Experiments Nos. 4 and 5 for monetary and fiscal policies. We also carry out a monetary policy, Experiment No. 6, in which G1 is exogenous and M1 a control variable subject to constant growth rate. This is based on monetarists' recommendation for the conduct of monetary policy. Experiment No. 6 can be compared to No. 2 to determine the loss due to the adoption of the rule (i.e., CGRR). Experiment No. 7 is in the same spirit, but applied to fiscal policy.

b. Experiments with lag consideration

To many economists, lags and uncertainty are the two most important factors which make any government intervention undesirable. In the literature it is customary to divide lags into inside lag and outside lag.[27] The former is the time it takes to undertake an action while the latter describes the timing of the effects of the policy action on the economy. Here we are concerned only with inside lag

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[27] The description on lags was taken from Dornbusch, Fischer, and Sparks (1982, Chapter 9).

while outside lag, in principle, is captured by the model structure.

Inside lag is again divided into recognition, decision, and action lags. Recognition lag is due to the delay in recognizing that there is a stabilization problem that requires government action. Recognition lag should be the same for monetary and fiscal policies while decision and action lags are much longer for fiscal policy because of the legislative and bureaucratic lags in the enactment and implementation of fiscal changes. Moreover, inside lag for government spending is longer than that for tax because when the government purchases goods and services, a long process evolves: determining what goods to buy, bidding from the private sector, awarding contracts etc...

How long is inside lag for monetary and fiscal policies? This is not said in the literature. For that reason we tentatively assume that there is one-quarter lag for monetary policy and two-quarter lag for fiscal policy. The attempt to evaluate the influence of inside lag on relative effectiveness is restricted to the basic monetary and fiscal policies. Of course the study of inside lag can be extended to the joint policy. But in this research we are content with a limited experimentation.

The procedure of introducing inside lag is rather simple.[28] Take the case of monetary policy. First carry out

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[28] I owe to Professor Sandblom the idea and procedure of 'lagged' experiments.

Experiment No. 1. Optimal value of M1 is obtained. Denote it by M1*. Now lag it one quarter, and set M1 = M1*(-1) exogenous in a straightforward simulation. The value of the welfare loss function can then be computed. The same procedure is applied to fiscal policy. The rationale is as follows. For the monetary policy, the monetary authority, assuming that the amount of government spending is given, attempts to optimize the economy and obtains an optimal path for M1 which, in practice, can be implemented one quarter later due to recognition, decision, and action lags; for the fiscal policy, the fiscal authority, assuming M1 is known, attempts to optimize the economy and obtains an optimal path for G1 to be implemented two quarters later due to lags.

c. Sensitivity analysis experiments

They are in three forms. The first deals with a change in the model coefficients; the second considers a change in the specification of the welfare function weights; and the last examines the change in the control variable bounds.

With respect to a change in the model coefficients

The problem consists of breaking data series into two subsets due to an important change in policy. A subset is then used to reestimate some key equations (consumption, investment...). With new parameters, an already executed experiment (No. 3) is repeated, and comparison between the two versions can be made. The above exercise is more or less in the same spirit as the Chow stability tests, but deals with the whole model. It cannot directly answer Lucas's critique for the simple reason that we are conducting a change in past policy while Lucas's critique concerns the adoption of a new policy which, as Lucas claimed, will no longer maintain past relationships. Nevertheless, the exercise can show that, if it is found that there is no significant difference between the two optimal versions, then important changes in policy might not be those envisaged by Lucas (recall Sims's arguments).

With respect to a change in the welfare functions weights

In general, optimization exercises require a proper specification of the objective function. One aspect of our thesis topic is to evaluate the historical performance. This necessitates that the objective function reflects the preferences of policy-makers in the period under consideration. Nevertheless, those preferences might not reflect a best or right choice; hence, there might be a need to respecify the objective function. That is the purpose of the Experiment No. 11 which repeats Experiment No. 3 for the joint policy to determine if there is a better performance for the economy with other welfare functions which allow more room for optimization, lay more emphasis on the unemployment and GNP targets. Experiments Nos. 12, 13, 14, and 15, as will be clear in Chapter 9, are to investigate if a modified objective function can affect the results on optimality and suboptimality due to the adoption of the rule.

With respect to a change in the control variable (M1) bounds

In Experiment No. 16, the joint policy is conducted with some alternative, larger bounds of M1. The purpose is to examine first the robustness of the optimal results, and secondly the performance of a less gradual monetary policy.

3.2.2 Lucas's critique on policy evaluation again

Before we leave Part I, it is worthwhile to examine to what extent Lucas's critique can be satisfied. In the previous chapter we have presented Sims's arguments which regard Lucas's critique as a cautionary footnote of a general principle that extrapolation outside normal historical range to which the model was fit yields unreliable results. One might or might not agree with the remark; but as long as a change in policy is in normal range, there is more chance that the model structure is stable. Two conditions appear essential for keeping the policy effects inside historically normal range.

i) Capture the policy-makers' preferences as reflected in observed data, past econometric relationships (the same objective function will then be used in most experiments).

ii) Impose realistic bounds on control variables.

How these are done will be discussed in Chapter 8. But let us examine the logic of these two conditions.

Assume for simplicity the econometric model is represented by

$$y_t = A_t y_{t-1} + C_t x_t + b_t \quad (3.2.1)$$

where y_t is a vector of state variables, x_t a vector of control variables, b_t a vector of exogenous variables and A_t, C_t matrices of known coefficients. (3.2.1) has been converted into state-space form with y_t incorporating x_t as a subvector (see Chow, 1975, pp. 153-154).

Suppose now that x_t is the optimal decision of the government which minimizes:

$$J = \sum_{t=1}^{n} (y_t - a_t)' K_t (y_t - a_t) \quad (3.2.2)$$

subject to (3.2.1), where a_t is a vector of targets, and K_t a symmetric positive semi-definite weighting matrix.

It is well-known that a feedback rule can be derived (see, e.g., Chow, 1975, Chapter 7) such that:

$$x_t = G_t y_{t-1} + g_t \quad (3.2.3)$$

where G_t and g_t are a matrix and a vector of coefficients.

Note that both the weighting parameters (K_t) and the model coefficients (A_t, C_t) enter G_t and g_t (a_t and b_t also enter g_t).

Substitute (3.2.3) in (3.2.1):

$$y_t = (A_t + C_t G_t)\, y_{t-1} + (b_t + C_t g_t) \quad (3.2.4)$$

If past and observed data of y_t and x_t have been generated by (3.2.4) and (3.2.3), a weighting scheme which does not reflect policy-makers' preferences could change greatly G_t, g_t and could generate y_t and x_t outside their historical range.

In the economic literature, it is customary to attach costs to control variables, which is captured in K_t. As this tradition is not respected here, the obvious requirement is that realistic bounds should be imposed on control variables. On the other hand, various policies can have the same objective function truly reflecting policy-makers' preferences for target variables but use policy instruments in different ways. That should also require the imposition of realistic bounds.

In short, if the above two conditions are met, the model structure is likely to be maintained. In our context, the first condition of capturing the decision-makers' past preferences is also necessary for two other reasons. The first is that a great number of optimization experiments are involved; for a majority of them it is not feasible to conduct them with several sets of welfare weights. Practicality would restrict to one or two sets. Instead of an arbitrary one, a natural candidate is the one which most reflects the policy-makers' past preferences. The second reason is that the objective function is to represent the preferences of the policy-makers who are presumed to act in the best interest of the whole nation. To rely on an objective function which does not reflect their preferences in order to evaluate past policy might not be well meaningful and fair. Of course, there is no guarantee that these preferences constitute a best or right choice; hence there is still a need to consider alternatives. But the point is: the objective function should capture the policy-makers' preferences; sensitivity analysis is an important but complementary issue.

In closing, for all reasons which have been discussed, in general realistic restrictions will be imposed on the control variables, and the same welfare function which hopefully can preserve more or less the policy-makers' preferences will be used; thus, the model structure would have a good chance to be maintained. There is, however, some reservation with respect to sensitivity analysis exercises which experiment with large control bounds, and with preferences diverging much from those of the past.

PART II

SPECIFICATION, ESTIMATION AND VALIDATION OF THE MODEL

CHAPTER 4

OUTLINE OF THE MODEL[29]

The model was built with the main purpose of evaluating, through optimal control technique, certain policies related to the Canadian economy for a special period of time from 1975 III to 1978 IV in which the Bank of Canada adopted the monetarist approach. Furthermore, it is for control studies and not for forecasting.

The model is concerned with the short-run which is reflected by the absence of capital growth, the presence of sluggish wage, the inertia in inflationary expectations etc...The analysis is restricted to the short-run since it seems that in the long-run, monetarists and non-monetarists' views would not be different. Although short-run, the model is dynamic by the dynamic structure of the government budget constraint, by lagged adjustment functions.

The model mainly contains three sectors (product, money, and labour), an equation for the foreign exchange rate, and the government budget constraint which links the government to the rest of the economy.

Below we present a simplified version of it. We hope it will facilitate the understanding of the full model as discussed in the next chapter. However, we will not attempt here to justify the forms of the equations as explanations

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29 This chapter is based on the description of the TRACE model in Choudhry et al. (1972, Chapter 1).

will be provided in the next chapter. No importance should be given to the equations' lag structures as they are too much simplified for the sake of discussion.

4.1 DEFINITIONS OF VARIABLES

Endogenous variables

BOND = government debt (stock) held by the private sector

CON = real consumption

CUR = currency

DD = demand deposits

DSN = time deposits

EX = real export of goods and services

I = real investment

L = employment

LF = labour force

IM = real import of goods and services

M2 = quantity of broad money

MB = monetary base

P = price level

PE = price of export

PFX = foreign exchange rate

R = domestic interest rate

RR = chartered bank reserves

TAX = nominal tax revenues

UR = employment rate

V = real wealth

W = wage rate

Y, Y^s	=	demanded and supplied outputs (real GNP)

Exogenous variables

G	=	real government expenditure
M^f	=	foreign money supply
M1	=	quantity of narrow money
P^f	=	foreign price level
PIMA	=	price of import
R^f	=	foreign interest rate
TIME	=	time trend
Y^f	=	foreign income

4.2 EQUATIONS

i. Aggregate demand

Product market

Total aggregate demand

$$Y = CON + I + G + EX - IM \qquad (4.2.1)$$

Consumption

$$CON = f[Y - \frac{TAX}{P}, V, CON(-1)] \qquad (4.2.2)$$

$$\text{where } TAX = f(P.Y) \qquad (4.2.3)$$

Investment

$$I = f[Y, R, I(-1)] \qquad (4.2.4)$$

Export

$$EX = f(Y^f, PFX, P^f, PE) \qquad (4.2.5)$$

Import

$$IM = f(Y, PFX, PIMA, P) \qquad (4.2.6)$$

Money Market

Demand for currency

$$CUR/P = f(Y, R) \qquad (4.2.7)$$

Demand for demand deposits

$$DD/P = \frac{M1 - CUR}{P} \qquad (4.2.8)$$

Demand for time deposits

$$DSN/P = f(Y, R, V) \qquad (4.2.9)$$

Interest rate

$$R = f(R^f, \frac{M1}{P}, Y) \qquad (4.2.10)$$

Chartered bank reserves

$$RR = f(DD, DSN) \qquad (4.2.11)$$

Monetary base

$$MB = CUR + RR \qquad (4.2.12)$$

Broad money

$$M2 = M1 + DSN \qquad (4.2.13)$$

Real wealth (liquid assets)

$$V = (MB + BOND)/P \qquad (4.2.14)$$

Government budget constraint

$$MB - MB(-1) + BOND - BOND(-1) = P\,G - TAX \qquad (4.2.15)$$

Foreign exchange rate

$$PFX = f[M2, M^f, Y, Y^f, (R - R^f)] \qquad (4.2.16)$$

ii. Aggregate supply

Labour market, wage and prices

Employment

$$L = f[Y^s, P, W, L(-1)] \qquad (4.2.17)$$

Labour force

$$LF = f(L, W, TIME) \qquad (4.2.18)$$

Unemployment rate

$$UR = \frac{(LF - L)}{LF} \cdot 100 \qquad (4.2.19)$$

Wage

$$\frac{W - W(-1)}{W(-1)} = f(UR, \pi) \qquad (4.2.20)$$

where π = expected rate of inflation as a distributed lag of current and past rates of inflation

Price level

$$P = f(W, UR) \qquad (4.2.21)$$

Price of export

$$PE = f(P) \qquad (4.2.22)$$

iii. Equality of demanded and supplied outputs

$$Y = Y^s \qquad (4.2.23)$$

Several things need to be said.

i) The simplified model is complete with 23 equations and 23 endogenous variables.

ii) In the product sector wealth enters the consumption function, and foreign exchange rate enters both export and import.

iii) In the monetary sector the supply of (narrow) money M1 is exogenous. If we impose the equilibrium condition that demand for M1 equals its supply, the interest rate can be obtained. Therefore, Equation (4.2.10) for the interest rate can be considered as a normalized demand for money. As M1 is exogenous, demand deposits are derived residually after currency has been specified. Demand and time deposits then determine chartered bank reserves, a component of monetary base.

iv) With monetary base and tax endogenous, government spending exogenous, the government budget constraint determines bonds residually.

v) There is no equation for the balance of payments (BOP). Theoretically, in a flexible exchange rate BOP is zero. Looking carefully at (4.2.16), we can see that the foreign exchange formulation rate is based on the monetary view. With the monetary approach, there is no need for a BOP equation. However, as our estimation sample extends from 1967 I to 1978 IV, it covers both regimes of exchange rate (the exchange rate started to be floating again in June 1970). The BOP must be present in our model. Nevertheless, we treat the foreign exchange rate as exogenous from 1967 I to May 1970 and decide not to include the BOP for the following reasons:

- the period of fixed exchange rate is rather short
- for optimal control exercises we have chosen the period of flexible exchange since under this regime the monetary authority is believed to have some degree of control in pursuing its monetary policy. Therefore, if a BOP equation is constructed, it will be thereafter deleted for optimal control exercises.
- finally it might take a considerable time to obtain good statistical results for BOP.

In short, we think the treatment should not cause any problem as the model is built for policy evaluation.

vi) The output side is seen to depend soley on the labour factor. There is no presence of capital stock. In the labour market money wage is assumed to be sluggish in the short-run, responding slowly to a non-zero excess supply. The wage equation is of the expectations augmented Phillips curve.

vii) Let us see how the model is working. Substitute Equations (4.2.2) to (4.2.6) in (4.2.1′):

$$Y = f[P, R, V, PFX, PE; CON(-1), I(-1), G, PIMA, Y^f, P^f] \qquad (4.2.24)$$

The monetary sector, the government budget constraint, and the foreign exchange rate can be condensed into three equations after TAX is replaced by (4.2.3):

$$R = f(Y, P; M1, R^f) \quad (4.2.25)$$

$$V = f[Y, P, R; MB(-1), BOND(-1), M1, G] \quad (4.2.26)$$

$$PFX = f(Y, R, V; M1, M^f, Y^f, R^f) \quad (4.2.27)$$

Substitute (4.2.27), (4.2.26), (4.2.25), and (4.2.22) for PFX, V, R, and PE in (4.2.24):

$$Y = f[P; CON(-1), I(-1), MB(-1), BOND(-1), G, PIMA, Y^f, P^f, M1, M^f, R^f] \quad (4.2.28)$$

(4.2.28) is the aggregate demand function.

Except (4.2.22) for the export price, the labour market has five equations and six endogenous variables. After substitution:

$$Y^s = f[P; P(-1), P(-2)..., W(-1), L(-1), TIME] \quad (4.2.29)$$

This is the aggregate supply function.

The equality condition (4.2.23) combined with (4.2.28) and (4.2.29) determines output and its price.[30]

Any changes in the policy variables (M1, G), in the predetermined variables (CON(-1), I(-1), PIMA, P^f, Y^f...) will shift the aggregate demand function; on the other hand, lagged employment, the exogenous growth of labour force (captured by the time trend) etc... will shift the aggregate supply. In short, the existence of policy variables, of exogenous and lagged variables keeps shifting the aggregate

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[30] In practice the algorithm solves the whole system of equations, (4.2.1) to (4.2.23), simultaneously.

demand and supply, generating new values of output and price; moreover, given initial conditions and values of exogenous variables, the model can generate values of endogenous variables for future periods.

CHAPTER 5

SPECIFICATION AND ESTIMATION OF THE MODEL BY ORDINARY LEAST SQUARES

In this chapter we will specify the equations of the model. In the tradition of applied macroeconomics, the model is first explored by ordinary least squares (OLS). Although it is not the right way, it is most practical; especially, OLS estimates seldom differ much from estimates produced by consistent estimation methods (Kuh and Schmalensee, 1973, p. 8). The specification, however, does not rely completely on the estimation; results from multiplier and simulation analysis sometimes play a decisive role.

Our model is quarterly with non-seasonally adjusted (N.S.A.) data except a few U.S. data. The use of N.S.A. data is based on a study by Wallis (1974). He showed that if seasonally adjusted (S.A.) data are used and if seasonal adjustment procedures are inappropriately applied, the relationship between the variables can be distorted; this will cause autocorrelation and dynamic specification problems. The period of estimation extends from 1967 I to 1978 IV.[31] The only exception is the foreign exchange rate estimated from 1970 III to 1978 IV (it began to float in June 1970); before 1970 III, the exchange rate is exogenous.

The model consists of 28 equations, of which 19 are stochastic, and 9 are identities. It contains 52

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[31] The estimation was carried out on the Concordia University computer, using the TSP (Time Series Processor) package.

variables, of which 24 are exogenous. It includes three sectors: product, money, and labour; besides, there is one equation for the foreign exchange rate and one equation for the government budget constraint.

Sections 5.1 and 5.2 will deal with the product and money sectors. Sections 5.3, 5.4 will be concerned with the formulation of the government budget constraint and the foreign exchange rate. Section 5.5 will discuss the labour market with wage and price equations included. And finally, in Section 5.6 and 5.7 we will provide a listing of the final equations, and variables with their definitions.

Before starting the model specification, we would like to make two remarks.

i) The first is related to a point raised by Hendry and Mizon (1978) on the treatment of autocorrelation. When an equation suffers from autocorrelation, the common practice is to correct it by some procedure, for example, CORC (Cochrane-Orcutt). According to Hendry and Mizon, the residual autocorrelation might reflect little more than dynamic misspecification, and therefore, the CORC procedure might be a poor approximation to a true dynamic model. The appropriate strategy should start with a general model allowing freer lags than the CORC which imposes a 'common factor'. We adopt that suggestion and will have recourse to the CORC only when all attempts seem to fail.

ii) The second remark concerns the dynamic specification of the equations. Recently, a class of models have been analyzed, namely the class of AD ($m_0, m_1, \ldots, m_n$) - models where AD stands for Autoregressive-Distributed (lag) [see, e.g., Davidson et al., 1978; Hendry (1980); Hendry and Richard (1983); Hendry, Sargan, and Pigan (1984)]. It can be shown that the models we are going to analyze are nested in this class. Further specification searches along the lines of these papers, however, are clearly beyond the scope of this thesis. We are, therefore, restricted to the more traditional specification.

5.1 PRODUCT SECTOR

The product sector contains seven stochastic equations and three identities. Apart from describing the components of GNP, for convenience we also place tax and revenue equations in this sector.

The first identity is real GNP.

Total aggregate demand (Y)

$$Y = CON + (IIN + IRCNR) + (G1 + G2) + EX - IM - RES \quad (5.1.1)$$

where Y = real GNP or GNE (gross national expenditure)

CON = real consumption (total personal expenditure on goods and services)

IIN = real private investment in inventory

IRCNR = real gross private fixed investment

G1 = real government expenditure (current expenditure + gross capital formation) from federal, provincial, and local governments

G2 = real government expenditure from Hospitals, Quebec Pension Plan, Canada Pension Plan

EX = real export of goods and services

IM = real import of goods and services

RES = residual error of estimate

There are a few remarks.

i) IRCNR will be broken down into two more components. In short, total private investment is disaggregated into three components as this would give better statistical results than the estimation based on a single aggregate.

ii) There are also two components for government expenditure. This is, as will be seen, because the government budget constraint is restricted only to the three levels of governments.

iii) G1, G2, and RES are exogenous in the model.

5.1.1 Real Consumption (CON)

We start with the basic form:

$$CON = f[Y - \frac{TAX}{P}, CON(-1)] \qquad (5.1.2)$$

where TAX = nominal tax revenues from federal, provincial, and local governments

(5.1.2) can be derived as a consequence of a version of Friedman's permanent income hypothesis, or from the partial adjustment model, or from Brown (1952) habit formation hypothesis (according to Brown there is a lagged effect in consumer demand which was produced by consumption habit people formed as a result of past consumption;

therefore, the appropriate lagged variable is previous real consumption and not previous income). For more on the discussion of this functional form, see Kuh and Schmalensee (1973, Chapter 3), Wallis (1973, Chapter 1), and also Choudhry et al. (1972, p. 16) with respect to the TRACE model of the Canadian economy.

To (5.1.2) is added the wealth variable V. As capital stock does not exist in our model, V is approximated by real financial wealth (= sum of the stocks of monetary base and government debt held by the private sector).

Equation (5.1.2) with V yields:

$$\begin{aligned} CON = &\ \underset{(1.70)}{711.180} + \underset{(7.10)}{0.527}\left(Y - \frac{TAX}{P}\right) + \underset{(4.74)}{0.395}\ CON(-1) \\ &- \underset{(-.64)}{.008}\ V - \underset{(-7.69)}{2111.61}\ Q1 - \underset{(-9.69)}{1112.62}\ Q2 \\ &- \underset{(-20.50)}{2733.49}\ Q3 \end{aligned} \qquad (5.1.3)$$

$R^2 = .994 \quad \bar{R}^2 = .993 \quad F(6,41) = 1105.58$

$SER = 257.636 \quad DW = 2.03 \quad D's\ H = -.14$

where Q1 = seasonal dummy, 1 in quarter I, 0 elsewhere

Q2 = seasonal dummy, 1 in quarter II, 0 elsewhere

Q3 = seasonal dummy, 1 in quarter III, 0 elsewhere

Note that values reported under the coefficients are t-statistics.

As can be seen, V is not significant at all. Various lag combinations of V are tried, e.g., i) V, V(-1); ii) V, V(-1), V(-2); iii) V(-1), V(-2). The disturbing finding is that with these lag structures, the wealth coefficients are significant, but the value of their sum is always negative. For example, with the inclusion of V and V(-1):

$$CON = \underset{(2.29)}{914.199} + \underset{(7.52)}{0.523} \frac{(Y - TAX)}{P} + \underset{(5.31)}{0.417} CON(-1)$$

$$+ \underset{(1.96)}{0.048} V - \underset{(-2.60)}{0.067} V(-1) - \underset{(-7.50)}{1971.86} Q1$$

$$- \underset{(-6.77)}{905.829} Q2 - \underset{(-15.94)}{2489.73} Q3 \qquad (5.1.4)$$

$R^2 = .995$ $\bar{R}^2 = .994$ $F(7,40) = 1081.44$

SER = 241.283 DW = 1.94 D's H = .38

The signs of V and V(-1) are acceptable, but V(-1) has a higher coefficient. As this seems doubtful, we try other models:

- the Houthakker-Taylor demand model[32]
- the Zellner-Huang-Chau (1965) consumption model with real liquid assets
- Pindyck-Rubinfeld consumption equation (1976, p. 378)

<u>The Houthakker-Taylor demand model</u>

Ignoring price variables, it can be written in discrete form as:

$$CON = f[\Delta(Y - \frac{TAX}{P}), (Y - \frac{TAX}{P}), CON(-1)] \qquad (5.1.5)$$

To (5.1.5) the same types of lag structures for V as mentioned above are included. The same result for V is

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[32] A brief discussion on the well-known Houthakker-Taylor model can be found in Kuh-Schmalensee (1973, pp. 37-40).

repeated. Indeed, when V and V(-1) enter (5.1.5):

$$CON = \underset{(.61)}{313.515} - \underset{(-1.80)}{0.228}\ \Delta\left(Y - \frac{TAX}{P}\right) + \underset{(6.28)}{0.672}\left(Y - \frac{TAX}{P}\right) + \underset{(2.07)}{0.249}\ CON(-1) + \underset{(2.57)}{0.067}\ V$$

$$- \underset{(-3.20)}{0.091}\ V(-1) - \underset{(-3.64)}{1433.38}\ Q1 - \underset{(-.64)}{248.921}\ Q2 - \underset{(-5.38)}{1912.27}\ Q3 \qquad (5.1.6)$$

$R^2 = .995$ $\bar{R}^2 = .994$ $F(8,39) = 999.29$

SER = 234.838 DW = 1.91 D's H = .59

<u>The Zellner-Huang-Chau model</u>

In their consumption study, Zellner *et al.* introduced real liquid assets whose definition is quite close to ours. The model starts with:

$$CON_t = a\ Y_t^e + b\ (LA_{t-1} - LA_t^*)$$

where Y_t^e = expected income (estimated permanent income)

LA_{t-1} = real liquid asset holdings at the start of the period

LA_t^* = desired level of LA for the current period and $b > 0$

Y_t^e is approximated by:

$$Y_t^e = (1 - c) \sum_{j=0} c^j\ Y_{t-j}$$

and LA_t^* is assumed to be proportional to Y_t^e:

$$LA_t^* = d\ Y_t^e$$

After substitution and using Koyck transformation:

$$CON_t = (a - bd)(1 - c)\,Y_t + c\,CON_{t-1} + b\,LA_{t-1} - bc\,LA_{t-2} \qquad (5.1.7)$$

Non-linear estimation applied to (5.1.7) yields, after replacing Y by $(Y - \frac{TAX}{P})$ and LA by V:

$$\begin{aligned} CON = {} & \underset{(2.19)}{931.589} + \underset{(7.62)}{0.551}\,(Y - \tfrac{TAX}{P}) + \underset{(4.77)}{0.386} \\ & CON(-1) - \underset{(-1.37)}{0.014}\,V(-1) + (0.386)(0.014)\,V(-2) \\ & - \underset{(-7.43)}{2034.14}\,Q1 - \underset{(-9.66)}{1080.40}\,Q2 - \underset{(-23.65)}{2717.72}\,Q3 \end{aligned} \qquad (5.1.8)$$

$R^2 = .994 \quad \bar{R}^2 = .993 \quad F(6,41) = 1145$

$SER = 253.188 \quad DW = 1.97$

The result still shows that total impact of V is negative.

<u>Pindyck-Rubinfeld consumption equation</u>

Pindyck-Rubinfeld (1976, p. 378) included wealth in differenced form:

$$CON = f[Y - \frac{TAX}{P}, CON(-1), V - V(-1)]$$

Ironically, this simple form is working well.

$$\begin{aligned} CON = {} & \underset{(1.66)}{470.430} + \underset{(7.28)}{0.491}\,(Y - \tfrac{TAX}{P}) + \underset{(5.24)}{0.418}\,CON(-1) \\ & + \underset{(2.16)}{0.054}\,(V - V(-1)) - \underset{(-7.78)}{2044.82}\,Q1 \\ & - \underset{(-6.79)}{921.389}\,Q2 - \underset{(-15.63)}{2451.97}\,Q3 \end{aligned} \qquad (5.1.9)$$

$R^2 = .994 \quad \bar{R}^2 = .994 \quad F(6,41) = 1219.99$

$SER = 245.329 \quad DW = 1.88 \quad D's\ H = .50$

All coefficients including wealth are significant. The long-run marginal propensity to consume is reasonable with a value of .84. As the above form imposes the constraint that V and V(-1) have the same coefficient, the null hypothesis is checked by a F-test:

$$F = \frac{\dfrac{SSR^R - SSR^U}{1}}{\dfrac{SSR^U}{41}}$$

where SSR^R = sum of squared residuals of the restricted model (5.1.9)

SSR^U = sum of squared residuals of the unrestricted model (5.1.4)

and 1 is the number of restriction, 41 the degrees of freedom for (5.1.9).

The computed F-statistic in our case is 2.39 and $F_{.01}$ (1,40) = 7.31 and $F_{.01}$ (1,60) = 7.08. Therefore, we accept the null hypothesis of identical coefficients for V and V(-1).

(5.1.9) is our chosen equation. As it is quite good, we do not want to try other models with wealth in differenced form.

Despite its significance as widely suggested by theory, the impact of wealth on consumption is quite small. The coefficient value of (V - V(-1)) is only 0.054, and moreover, wealth is in differenced form. An equation estimated without wealth effect proves $\bar{R}^2$ hardly improves.

$$CON = \underset{(1.77)}{521.052} + \underset{(7.34)}{0.511}\left(Y - \frac{TAX}{P}\right) + \underset{(4.82)}{0.398}\, CON(-1)$$

$$- \underset{(-7.87)}{2131.47}\, Q1 - \underset{(-9.80)}{1099.77}\, Q2 - \underset{(-23.28)}{2691.80}\, Q3 \qquad (5.1.10)$$

$R^2 = .994$ $\bar{R}^2 = .993$ $F(5,42) = 1345.44$

SER = 255.826   DW = 2.01   D's H = -.03

5.1.2 Real Private Investment

Real private investment consists of: i) investment in inventory, ii) gross investment in non-residential construction, and machinery & equipment, and iii) gross investment in residential construction. Each component will be specified by a stochastic equation; besides, an identity defining gross private fixed investment links the last two components together.

a. Real private investment in inventory (IIN)

The formulation adopts the stock adjustment model. Most inventory investment models start with the assumption that:

$KIN^* = a\, S^*$

where KIN = stock of inventory

KIN^* = desired stock of inventory

S = sales or output

S^* = permanent sales or output

Assuming S^* is a distributed lag function of S with geometrically declining weights:

$KIN^* = b\, S + c\, KIN(-1)^*$

The common assumption is that $KIN^* = KIN$ (Kuh and Schmalensee, 1973, p. 90). It results that:

$$IIN = KIN - KIN(-1) = b\,[S - S(-1)] + c\,IIN(-1) \qquad (5.1.11)$$

With real income as a proxy for S, (5.1.11) yields:

$$IIN = \underset{(-5.49)}{-\,975.148} + \underset{(1.72)}{0.178}\,[Y - Y(-1)] + \underset{(3.91)}{0.541}\,IIN(-1)$$

$$+ \underset{(5.16)}{1429.340}\,Q1 + \underset{(2.65)}{722.814}\,Q2 + \underset{(6.55)}{1880.050}\,Q3 \qquad (5.1.12)$$

$R^2 = .897$ $\bar{R}^2 = .884$ $F(5,42) = 72.86$

SER = 233.670 DW = 2.14 D's H = -1.66

As we expect the desired stock to increase with permanent sales, the positive sign of Y - Y(-1) is meaningful. The sigh of IIN(-1) is all right. The problem is that the H-statistic reveals autocorrelation.

We follow Pindyck (1973, pp. 48-49) and add a new variable: change in consumption.

$$IIN = \underset{(-.58)}{-\,143.348} + \underset{(4.31)}{0.501}\,[Y - Y(-1)] - \underset{(-4.21)}{0.451}$$

$$[CON - CON(-1)] + \underset{(6.06)}{0.805}\,IIN(-1) + \underset{(.81)}{288.877}\,Q1$$

$$- \underset{(-.27)}{79.937}\,Q2 + \underset{(.69)}{306.546}\,Q3 \qquad (5.1.13)$$

$R^2 = .928$ $\bar{R}^2 = .917$ $F(6,41) = 87.82$

SER = 197.650 DW = 2.26 D's H = -2.31

Compared to (5.1.12), (5.1.13) improves rather well, statistically. The coefficient of [CON - CON(-1)] is negative as increase in consumption is expected to deplete the stock of inventory. The coefficients of the seasonal dummies are not significant; this might be explained by the fact that [CON - CON(-1)] plays the role of a 'shock absorber' and captures all seasonal effects. Autocorrelation is still present. After a search for the appropriate lag structure, we end up with:

$$IIN = -\underset{(-.30)}{9.684} + \underset{(17.66)}{0.514}[Y - Y(-1)] + \underset{(2.40)}{0.070}[Y(-1) - Y(-2)] - \underset{(-12.73)}{0.554}[CON - CON(-1)] + \underset{(6.47)}{0.773}\, IIN(-1) \qquad (5.1.14)$$

$R^2 = .926 \quad \bar{R}^2 = .920 \quad F(4,43) = 135.24$

$SER = 194.916 \quad DW = 2.25 \quad D's\ H = 1.53$

(5.1.14) is free of autocorrelation. Seasonal dummies, still insignificant, are left out of the equation.

b. <u>Real gross private fixed investment (IRCNR)</u>

$$IRCNR = INR + IRC \qquad (5.1.15)$$

where INR = real gross private investment in non-residential construction, and machinery & equipment

IRC = real gross private investment in residential construction

c. <u>Real gross private investment in non-residential construction, and machinery & equipment (INR)</u>

Junankar (1972, p. 62) noted that in general most of the empirical works on fixed investment in machinery and equipment are ad hoc or eclectic.

Jorgenson's model, despite its shortcomings, is commonly used. Applying Jorgenson's model requires the inclusion of capital stock. Since the latter does not figure in our model, the two main determinants for investment one could imagine of are long-term interest rate RL (measured by the government of Canada bond yield, 10 years over), and income, i.e.,

INR = f(RL, Y)

It is well known that investment in machinery and equipment responds very slowly to its determinants. An intensive search is made for the lag structure of INR; particularly, Almon lag is tried for both Y and RL, and for each separately. Our problem is that with Almon lag the equation suffers from very high autocorrelation, and the TSP package does not contain a procedure of correcting it when Almon lag is used. High autocorrelation in investment studies is commonly found in many Canadian studies (for example, Scotland, 1981; Banasik and Sandblom, 1981, pp. 68-69). Without Almon lag, the lagged terms of RL turn out to be generally insignificant or have the wrong sign. We finally obtain the following equation, statistically satisfactory but ad hoc in terms of lag structure.

$$\begin{aligned} \overline{INR} = &- \underset{(-3.13)}{288.612} + \underset{(5.47)}{0.082}\, Y - \underset{(-3.59)}{43.591}\, [RL(-8) + RL(-9)] \\ &+ \underset{(9.31)}{0.650}\, INR(-1) - \underset{(-.25)}{10.566}\, Q1 + \underset{(12.20)}{466.895}\, Q2 \\ &- \underset{(-3.69)}{135.484}\, Q3 \end{aligned} \qquad (5.1.16)$$

$R^2 = .985$ $\bar{R}^2 = .982$ $F(6,41) = 438.86$

SER = 88.632   DW = 2.18   D's H = -.72

Both RL(-8) and RL(-9) are constrained to have the same coefficient. A F-test, as described in the consumption section, is performed and shows that the constraint is valid.

As we are not satisfied with the lag structure of (5.1.16), we attempt to estimate INR with real interest rate. The following equation is found most satisfactory with real interest rate included.

$$INR = -\underset{(-1.34)}{142.188} + \underset{(4.30)}{0.048}\, Y - \underset{(-2.46)}{12.843}\left[RL - 100\left(\frac{P}{P(-4)} - 1\right)\right] + \underset{(9.21)}{0.681}\, INR(-1) - \underset{(-1.48)}{63.621}\, Q1$$

$$+ \underset{(11.28)}{465.357}\, Q2 - \underset{(-2.90)}{113.072}\, Q3 \qquad (5.1.17)$$

$R^2 = .982$ $\bar{R}^2 = .980$ $F(6,41) = 382.08$

SER = 0.095   DW = 2.08   D's H = -.30

In terms of $\bar{R}^2$, (5.1.16) and (5.1.17) perform equally well. In terms of t-statistics of main variables, (5.1.16) is slightly better. On the contrary, the lag structure of (5.1.16) is much more ad hoc than that of (5.1.17).

Statistically, we prefer (5.1.17). Unfortunately, (5.1.17) exhibits a suspicious long-run dynamic property. In a seven year shock response experiment when (5.1.17) is used, the real GNP multiplier (from an increase in government spending) peaks at a value of 2.03 after three quarters, then declines until it reaches a lowest value of .61 after 13 quarters, then keeps moving up in the last 13 quarters with a final value of 1.42. As the multiplier series is on an increasing path in the last portion of the simulation period, and since the final value of 1.42 is quite high relatively to the peak value of 2.03, we have some reservation with respect to the long-run property of crowding-out effect. We believe the investment equation (5.1.17) to some extent is the cause of that behaviour. This might be explained as follows. When government spending increased, price would increase due to excess demand in the product sector. As nominal money stock remains constant, real money stock decreases, pushing up nominal interest rate. If INR depends on nominal interest, there will be an adverse effect on it, contributing to the weakening of real income in the long-run. But, if it depends on real interest rate the impact of the latter is subject to the interaction of the increase of nominal interest rate and the increase of the inflation rate which, if stronger, might not cause an adverse effect on investment. It results that real GNP would decline less and more slowly than in the case of nominal interest rate.

When Equation (5.1.16) is used, the dynamic multiplier seems to better satisfy the long-run crowding-out effect (Chapter 7).[33] In view of all this, we are reluctant to accept (5.1.17); we choose (5.1.16) instead.

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[33] With respect to monetary effectiveness as expressed by GNP multiplier, it does not matter whether (5.1.16) or (5.1.17) is used.

d. Real gross private investment in residential construction (IRC)

As Pindyck and Rubinfeld (1976, p. 375) noticed, in a small macroeconometric model it is very difficult to accurately explain investment in residential construction. In fact, IRC depends on a lot of factors: mortgage rate, credit availability, constructon costs, number of families...None of these is included in our model.

We start with the basic form:

$$IRC = f[Y, RS, (RL - RS)]$$

where RS = short-term interest rate measured by the 90-day finance company paper rate.[34]

RS is used as a proxy for the mortgage rate, while the interest differential (RL-RS) is a crude measure of credit availability (credit is tighter when the short-term rate approaches the long-term rate). In our specification (RL - RS) is never significant and therefore deleted. Long lags for RS up to the eighth quarter have been tried, but generally their coefficients are not significant. The following equation appears to behave better:

$$\begin{aligned} IRC = & - \underset{(-.69)}{69.622} + \underset{(2.66)}{0.024}\, Y - \underset{(-2.50)}{10.992}\, [RS(-1) + RS(-2)] \\ & + \underset{(5.35)}{0.652}\, IRC(-1) - \underset{(-4.89)}{188.915}\, Q1 + \underset{(3.38)}{204.914}\, Q2 \\ & + \underset{(4.29)}{215.204}\, Q3 \qquad (5.1.18) \end{aligned}$$

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34 The 90-day finance company rate is chosen for the purpose of the demand of money study. In the Canadian context, it is most widely used to reflect the short-term interest rate.

$R^2 = .934 \quad \bar{R}^2 = .924 \quad F(6,41) = 96.52$

SER = 91.523 DW = 1.54 D's H = 2.95

Coefficients look meaningful, but H-statistic reveals autocorrelation. Instead of CORC, the alternative of adding more lagged explanatory variables is tried, yielding:

$$\begin{aligned} IRC = &- \underset{(-1.48)}{173.253} + \underset{(3.12)}{0.030}\, Y - \underset{(-2.06)}{9.169} [RS(-1) + RS(-2)] \\ &+ \underset{(5.24)}{0.817}\, IRC(-1) - \underset{(-1.65)}{0.250}\, IRC(-2) - \underset{(-1.91)}{113.353}\, Q1 \\ &- \underset{(-3.62)}{300.822}\, Q2 + \underset{(3.57)}{186.537}\, Q3 \end{aligned} \qquad (5.1.19)$$

$R^2 = .938 \quad \bar{R}^2 = .927 \quad F(7,40) = 86.58$

SER = 89.667 DW = 1.61 D's H = (test breaks down)

(5.1.19) improves relatively to (5.1.18). The H-test breaks down due to the negative value of the terms under the square root sign (see Johnston, 1972, p. 313). We then rely on the residual plot and the Geary sign test[35] which do not indicate any autocorrelation.

(5.1.19) is also tried with the addition of the interest rate term lagged once (i.e., RS(-2) + RS(-3)), but the latter turns out to be insignificant. It seems (5.1.19) exhibits the appropriate lag structure.

Examining (5.1.19), we can see that IRC depends on RS(-1) + RS(-2). What it means is that IRC depends on the average value of the interest rate in two successive periods;

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[35] For Geary sign test, see Habibagahi and Pratschke (1972).

this implies RS(-1) and RS(-2) are imposed to have identical coefficients. A F-test proves that the restricted model is acceptable.

5.1.3 Real Exports of Goods and Services (EX)

Real export is assumed to depend on the activity in the rest of the world, on the relative price and the foreign exchange rate.

Two proxies are needed for estimation: one to represent the activity in the rest of the world and the other to measure the foreign price. The former is approximated by the index of world industrial production (see Rhomberg, 1964, pp. 8-9; Choudhry et al., 1972). The latter is measured by the U.S. business product price deflator.

$$EX = f(WIP, \frac{PE}{PYBU.PFX})$$

where WIP = index of world industrial production (exogenous)

PE = implicit price deflator, export of goods and services

PYBU = implicit price deflator, U.S. business product (exogenous)[36]

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[36] All data used in the model are seasonally unadjusted except PYBU and U.S. GNP which will enter the foreign exchange equation. In the case of the export equation, a check has been performed to see how the results change if both price data are seasonally adjusted (PE is adjusted, using the seasonality adjustment procedure in the TSP): almost no change for the coefficients and minor changes for the constant and the seasonal dummies. Repeating the same for the exchange rate equation, we find no change for the coefficients, and small changes with respect to the constant and the dummies.

PFX = spot foreign exchange rate: number of Can[illegible] dollars per U.S. dollar

The following form is obtained

$$EX = \underset{(4.04)}{1092.72} - \underset{(-4.17)}{1536.17} \left[\frac{PE(-3)}{PYBU(-3) \cdot PFX(-3)} \right]$$

$$+ \underset{(5.99)}{4289.63} \, WIP + \underset{(2.57)}{0.300836} \, EX(-1)$$

$$- \underset{(-7.52)}{582.704} \, Q1 + \underset{(4.13)}{381.048} \, Q2 - \underset{(-1.03)}{105.445} \, Q3 \qquad (5.1.20)$$

$R^2 = .977$ $\bar{R}^2 = .973$ $F(6,41) = 285.19$

$SER = 183.322$ $DW = 2.11$ $D's\ H \doteq -.68$

All coefficients have the correct sign. The relative price is lagged three quarters, which is a bit shorter as compared to results seen in the international trade literature (see Artus and Young, 1979). The long-run price elasticity calculated at the sample mean is found to be -0.4 compared to the range of -0.5 to -1.5 reported in the literature (Artus and Young, 1979, p. 666).

In the import equation (as will be seen) the unemployment rate used to enter to capture excess capacity. We also try it with export, but truly we do know if it can play the same role. It is found to be highly significant but with the wrong sign (positive). One explanation is: as unemployment rate increases, final demand declines, but export might increase due to the reduction of the domestic market. It is not sure if the above explanation can be applied to a developed country like Canada. It is decided to leave it out.

5.1.4 Real Import of Goods and Services (IM)

IM is a function of income, relative price (import over domestic prices), and foreign exchange rate. That is the formulation found in the TRACE model and in Rhomberg (1964, p. 9).

Since both prices are expressed in domestic currency, there is no room for the exchange rate PFX if it enters the relative price multiplicatively. To circumvent this drawback, we follow Rhomberg and include the adjusted import price expressed in foreign currency (i.e., the adjusted import price is equal to the original import price divided by PFX).

In many empirical studies (see, for example, Norton and Henderson, 1973, p. 69), a variable expressing excess capacity is added. In our model it is approximated by the unemployment rate. so the basic form is:

$$IM = f(Y, \frac{PIMA \cdot PFX}{P}, UR) \qquad (5.1.21)$$

where PIMA = import price in foreign currency

UR = unemployment rate

As import responds slowly to the relative price, we try various lag structures. What we find is: when both UR and relative price (in ratio form) appear simultaneously, the latter is not significant; when the former is removed, prices are significant but usually not in a ratio form. This leads us to include UR and besides relative price in ratio form, either i) individual import price or ii) individual domestic price. This makes both UR and relative price significant; but with (i) the individual import price has the positive sign, which does not make sense. We choose the (ii) alternative and obtain the two following equations:

$$\begin{aligned} IM = &- \underset{(-0.63)}{766.239} + \underset{(14.71)}{0.362}\, Y - \underset{(-1.82)}{1752.050} \\ &\left[\frac{PIMA(-2)\ .\ PFX(-2)}{P(-2)}\right] + \underset{(2.50)}{867.579}\, P(-2) \\ &- \underset{(-4.57)}{218.34}\, UR + \underset{(4.28)}{495.509}\, Q1 \\ &+ \underset{(6.56)}{537.14}\, Q2 - \underset{(-10.10)}{846.765}\, Q3 \end{aligned} \qquad (5.1.22)$$

$R^2 = .987 \quad \bar{R}^2 = .985 \quad F(7,40) = 425.36$

$SER = 190.183 \quad DW = 1.77$

and

$$\begin{aligned} IM = &- \underset{(-.24)}{314.515} + \underset{(13.57)}{0.354}\, Y - \underset{(-2.01)}{2057.71} \\ &\left[\frac{PIMA(-3)\ .\ PFX(-3)}{P(-3)}\right] + \underset{(2.64)}{942.152}\, P(-2) \\ &- \underset{(-4.64)}{219.716}\, UR + \underset{(4.46)}{503.237}\, Q1 \\ &+ \underset{(5.74)}{492.552}\, Q2 - \underset{(-10.79)}{871.970}\, Q3 \end{aligned} \qquad (5.1.23)$$

$R^2 = .987 \quad \bar{R}^2 = .985 \quad F(7,40) = 432.57$

$SER = 188.611 \quad DW = 1.63$

As there is no reason to prefer (5.1.22) or (5.1.23), the latter is arbitrarily chosen. The income elasticity (evaluated at the sample mean) is 1.40, which is meaningful, while the relative price elasticity is -0.35 compared to the range of -0.50 to -1.00 in international trade as reported in Artus and Young (1979, p. 666). Note that in the price elasticity computation the term P(-2) is not taken into account.

5.1.5 Nominal Taxes, Net of Subsidies and Transfers from Federal, Provincial, and Local Government (TAX)

The equation for TAX is very simple. TAX is assumed to be related to nominal GNP.

$$TAX = f(P.Y)$$

For estimation, apart from the linear form, a log-form and the following non-linear form are tried.

$$TAX = f[P.Y, (P.Y)^2] \tag{5.1.24}$$

Only the non-linear form (5.1.24) does not suffer from autocorrelation.

$$\begin{aligned} TAX = & - \underset{(-1.63)}{822.993} + \underset{(10.13)}{0.305}\ (P.Y) - \underset{(-4.36)}{0.00002}\ (P.Y)^2 \\ & + \underset{(.51)}{93.125}\ Q1 - \underset{(-3.64)}{662.944}\ Q2 - \underset{(-2.85)}{517.469}\ Q3 \end{aligned} \tag{5.1.25}$$

$R^2 = .972$ $\bar{R}^2 = .969$ $F(5,42) = 289.605$

$SER = 444.717$ $DW = 1.82$

But the coefficient of $(P.Y)^2$ has a negative sign, which is not logical in the Canadian context.

Between the log and linear forms, there is a significant decline of the DW value from the second to the first, indicating there should be higher misspecification with the log-form. This leads us to select the linear form estimated by CORC.

$$TAX = \underset{(3.99)}{1247.46} + \underset{(21.81)}{0.174}\ P.Y + \underset{(.98)}{169.868}\ Q1$$

$$- \underset{(-3.33)}{622.143}\ Q2 - \underset{(-3.26)}{545.225}\ Q3 \qquad (5.1.26)$$

$R^2 = .923 \quad \bar{R}^2 = .915 \quad F(4,42) = 119.37$

SER = 484.313   DW = 2.06   RHO = .36 (2.68)

(The number in brackets next to the RHO value is its t-statistic).

A small note worthy of mentioning is that when (5.1.26) is estimated by two-stage least squares, $\bar{R}^2$ increases from .915 to .989.

<u>Nominal total government revenues (TR)</u>

TR is defined as:

TR = TAX + OREV (5.1.27)

where OREV = other revenues from federal, provincial, and local governments (exogenous).

5.2 MONEY SECTOR

5.2.1 <u>Introduction to the Basic Structure</u>

In the specification of the monetary sector, one has to answer one important question: which monetary aggregate is directly controlled by the central bank?

In the American literature and textbooks, there has been an emphasis on the monetary base as the policy instrument while the money supply is considered to be controlled indirectly (endogenous, therefore, in the model). In our first attempt, the above line was followed: broad money supply (M2) is endogenous, and the supply of monetary base ($\overline{MB}^s$) exogenous.[37] Since the central bank cannot simultaneously set the monetary base (and indirectly the monetary supply) and interest rate at whatever levels it wants, once it has decided to set the monetary base, the interest rate will play a role of clearing the demand and supply of monetary base. In that framework, a few ways of constructing the monetary sector are conceivable. A simple example will elucidate that.

Define: CUR = currency

DD = demand deposits

DSN = time deposits

M2 = broad money

RS = short-term interest rate

RR = chartered bank reserves

1. Demand for demand deposits

DD / P = f(Y, RS)

2. Demand for time deposits

DSN / P = f(Y, RS)

3. Money demand and supply equilibrium

M2 / P = ($\overline{CUR}$ + DD + DSN) / P

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37 For convenience of discussion a bar is sometimes written over a variable to indicate that it is exogenous.

4. Monetary base demand and supply equilibrium

$$\overline{CUR} + RR = \overline{MB}^{s}$$

5. Demand for chartered bank reserves

RR = f(RS, DD, DSN)

There is no need to explain equations 1 to 4. As to 5, RR consists of required reserves and excess reserves held by chartered banks. The latter varies inversely to RS,[38] while the former is determined by fixed ratios applied to demand and time deposits.

For the sector as a whole, if we make substitution, we will obtain a relationship between the money stock and the monetary base through the money multiplier.

If we ignore Y and P which are determined elsewhere in the rest of the model, the monetary sector is complete with five endogenous variables (DD, DSN, M2, RS, RR) and five equations. One among other alternatives to the above formulation is as follows. Treat CUR endogenous (i.e., adding an equation for CUR which will depend on Y and RS), and replace Equation 5 by the equation for (CUR + RR), i.e., the equation of the demand for monetary base whose explanatory variables include those which have entered RR and CUR equations.

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38 In estimation, RS is never significant in 5. This might be due to the correlation between RS and DD, DSN. Thereafter, RS will be deleted from the RR equation.

Both ways have been tried. The astonishing result was that the model could not converge when simulated, unless nominal interest rate (RS) was allowed to have negative values. And its root-mean square percent error (to be defined in chapter 7) was over 20%. Those failures force us to abandon the monetary base as the policy instrument. As a matter of fact, that treatment contradicted the Canadian reality. In the history of Canadian monetary policy, there has been no attempt from the Bank of Canada to control the monetary base. Of course, it is inaccurate to say that the money supply is completely exogenous. To a large extent, it
7 is partly endogenous, partly exogenous. For the period after May 1975, in which the Bank of Canada announced the M1 target range, the exogeneity assumption is more acceptable.

In the official 1980 version of the RDXF, a model of the Bank of Canada,[39] its builders offer two alternatives for the treatment of the short-term interest rate: it can be exogenous or one may want to invert the money demand equation to yield short-term paper rate endogenously, given an exogenously determined money supply (Bank of Canada, 1980b, p. 200). Obviously, the most correct way is to treat M1 endogenous before 1975 II and exogenous after that. In practice, it is complex to deal simultaneously with both regimes; we decide to treat M1 exogenous for the full period. This is not serious since our optimization experiments occur

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[39] RDXF is a successor of the R[illegible] models and developed for medium-term forecast. It has around 400 equations.

after 1975 II. With M1 exogenous, the interest rate will be endogenous and plays the role of clearing the demand and supply of narrow money. This can be done in two ways:

- the demand for M1 is estimated directly, and no explicit equation for interest rate (RS) or
- RS is estimated directly by essentially normalizing the demand for money.

The second alternative is selected on the main ground that it will generate a better performance for RS in simulation. This has also been remarked by Clinton and Masson (1975, p. 3) in their monthly model of the Canadian financial system:

> Interest rates themselves are determined within the model by explicit equations rather than through the interaction of supply and demand...interest rates solved implicitly from supply-demand relationships are apt to track historical data poorly and to behave impredictably in simulation. 40

Apart from the justification of good tracking, estimating RS directly fits the Bank of Canada monetary conduct after May 1975, which aims at achieving the level of RS (through the monetary base) in such a way that the supply of money will equal its target, given forecasted price and income (see Parkin, 1982, pp. 453-456).

We can now move to the specification of the monetary sector. As before, for ease of understanding, the basic structure is explained first.

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40 In our simulation with the demand for money estimated explicitly, the root-mean square percent error of RS is around 3% higher, which yields worse results for GNP.

Basic structure

1. Demand for currency

$CUR/P = f(Y, RS, V)$

2. Demand for demand deposits

$DD/P = f(Y, RS, V)$

3. Narrow money demand and supply in equilibrium

$(CUR + DD - \overline{M1})/P = 0$

4. Demand for time deposits

$DSN/P = f(Y, RS, V)$

5. Short-term interest rate

$RS = f(Y, \frac{\overline{M1}}{P})$

6. Demand for chartered bank reserves

$RR = f(DD, DSN)$

7. Broad money

$M2 = \overline{M1} + DSN$

8. Demand for monetary base

$MB = CUR + RR$

9. Real financial wealth

$V = (MB + BOND)/P$

Again, if we ignore Y and P which are determined elsewhere, and BOND (government debt held by the private sector) which is determined from the government budget constraint, there will be nine equations but only eight endogenous variables: CUR, DD, DSN, RS, RR, MB, M2, V. One equation has to be deleted. Note that by chance 1,2, and 3 can be satisfied simultaneously. In other words, either CUR or DD must be derived residually from three, and its equation deleted. We choose to delete 2 and derive DD residually for good tracking purpose. This is understandable as the currency equation is much more stable than the demand deposit equation (this will be clearer in Section 5.2.2). The choice affects, however, a few financial variables such as CUR, DD (seriously), and RR, MB (slightly) and leaves unaltered the remaining variables of the model. The reason is simple. Any choice of either CUR or DD as a residual will affect MB which leads to a change of BOND in the opposite direction, leaving financial wealth V intact. As V remains constant, and interest rate is determined by an exogenous M1, the product sector is insulated.

So is the basic structure. In reality, the money sector is a little more complex, for example, time deposit rate enters DSN rather than the short-term rate RS etc...It has two more equations for the term structure of interest rates, in total, ten equations, (eight from the basic model -- note that either 1 or 2 has to be left out from the basic model).

Let us move to the full specification.

5.2.2 Demand for assets

In Chapter 7 we will present simulation results of two alternatives (first DD is derived residually, then CUR is derived residually); therefore, here we will discuss the formulation of both DD and CUR.

In theory, demands for assets depend on real income, positively on their own interest rates and negatively on competitive rates, and finally on wealth. In estimation, one usually encounters some problems regarding interest rates and wealth.

In our model there are five types of interest rates:

i) short-term interest rate (RS) measured by the 90-day finance company paper rate[41]

ii) time deposit rate (RTD)[42]

iii) long-term interest rate (RL)

iv) U.S. short-term interest rate (RSU) measured by the U.S. 90-day short-term paper rate

v) U.S. long-term interest rate (RLU)

Through the term structure of interest rates, other Canadian rates are related to the short-term rate. And the latter generally moves along the U.S. rate. So, collinearity among different interest rates makes their co-existence hardly significant in estimation.

As to wealth its significance in the demand for money studies is not widely found. There should be two reasons:

i) multicollinearity between Y and V

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41 There is now a tendency to use the 90-day finance company paper rate rather than the 90-day treasury bill; see, for example, Cameron (1979), Clinton and Lynch (1979).

42 Time deposits consist of: i) personal deposits at chartered banks and ii) non-personal term and notice deposits at chartered banks. The rate on time deposits is the weighted average of the rates of (i) and (ii) (data obtained from the Bank of Canada).

ii) aggregation: while there is reason to speculate its influence in time deposits, one is less certain of its role in money for transaction purpose.

Due to these problems we start with the following basic forms for asset demands:

$$\frac{DD}{P} = f(Y, RS)$$

$$\frac{CUR}{P} = f(Y, RS)$$

$$\frac{DSN}{P} = f(Y, RS, V)$$

More complexity will be introduced at later stages.

a. <u>Demand deposits (DD)</u>

The basic form

$$\frac{DD}{P} = f(Y, RS) \qquad (5.2.1)$$

is estimated in three versions:

i) linear form

ii) log-form for every term

iii) log-form for all except RS

(All logs are natural)

In log-form equations, certain studies prefer to let RS enter linearly to avoid one theoretical shortcoming: for example, if RS enters in log-form, raising it from 3% to 6% would imply the same proportionate effect on the asset demand as an increase from 6% to 12% even though the absolute profit incentive in the second case is double that in the first (White, 1976, p. 123).

The first result obtained from the basic form shows all versions suffer from serious autocorrelation. For example, in linear form:

$$\frac{DD}{P} = \underset{(12.60)}{4458.60} + \underset{(8.23)}{0.122}\ Y - \underset{(-2.09)}{68.432}\ RS - \underset{(-.86)}{126.385}\ Q1$$

$$+ \underset{(-1.60)}{230.163}\ Q2 - \underset{(-2.66)}{382.678}\ Q3 \qquad (5.2.2)$$

$R^2 = .661 \quad \bar{R}^2 = .621 \quad F(5,42) = 16.39$

$SER = 350.821 \quad DW = .40$

$\bar{R}^2$ is also very low; this leads us to add two postal strike dummies: QMAIL74 (1 in 1974 II, 0 elsewhere) and QMAIL75 (1 in 1975 IV, 0 elsewhere), based on a study by Clinton and Lynch (1979, pp. 34-36). One observes that postal strikes had the effect of increasing real cash balances.[43]

$$\frac{DD}{P} = \underset{(13.67)}{4593.04} + \underset{(2.33)}{856.535}\ QMAIL74 + \underset{(1.63)}{566.239}\ QMAIL75$$

$$+ \underset{(8.87)}{0.124}\ Y - \underset{(-3.01)}{98.787}\ RS - \underset{(-.70)}{98.061}\ Q1$$

$$- \underset{(-1.88)}{265.956}\ Q2 - \underset{(-2.47)}{340.566}\ Q3 \qquad (5.2.3)$$

$R^2 = .716 \quad \bar{R}^2 = .667 \quad F(7,40) = 14.43$

$SER = 328.910 \quad DW = .28$

The dummies are found to be significant, improve $\bar{R}^2$, but autocorrelation still exists.

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43 Mackinnon (1980) offered a plausible explanation: as postal strike tends to delay customers' payments to firms which cannot postpone their financial obligation (e.g., payroll) and have to borrow, the Bank of Canada has to accommodate this additional and temporary demand for liquidity.

From the above results it seems some dynamic mechanism is necessary. As usual, a search for the appropriate lag structure is undertaken by trying various combinations of lagged and unlagged explanatory variables, jointly with a variety of interest rates (except RLU). A few interesting things emerge:

i) RS(-1) is never significant beside RS

ii) multicollinearity arises when different interest rates are together

iii) the log-form is generally better than the linear form

iv) wealth is not significant

v) when lagged dependent appears, the combination of log (Y) and log (Y(-1)) prevail over log (Y) alone. Indeed:

$$\text{Log}\,\frac{DD}{P} = \underset{(2.10)}{0.828} + \underset{(2.49)}{0.062}\,\text{QMAIL74} + \underset{(3.44)}{0.082}\,\text{QMAIL75}$$

$$+ \underset{(3.82)}{0.140}\,\text{Log Y} - \underset{(-5.66)}{0.087}\,\text{Log RS} + \underset{(11.13)}{0.768}$$

$$\text{Log}\,\frac{DD(-1)}{P(-1)} - \underset{(-5.46)}{0.057}\,\text{Q1} - \underset{(-3.16)}{0.031}\,\text{Q2}$$

$$- \underset{(-3.24)}{0.031}\,\text{Q3} \qquad (5.2.4)$$

$R^2 = .939 \quad \overline{R}^2 = .926 \quad F(8,39) = 74.58$

SER = 0.023 DW = 1.70 D's H = 1.18

$$\text{Log}\,\frac{DD}{P} = \underset{(2.24)}{0.817} + \underset{(2.88)}{0.067}\,\text{QMAIL74} + \underset{(3.40)}{0.076}\,\text{QMAIL75}$$

$$+ \underset{(3.3)}{0.724}\,\text{Log Y} - \underset{(-2.70)}{0.587}\,\text{Log Y(-1)} - \underset{(-5.46)}{0.079}$$

$$\text{Log RS} \underset{(12.06)}{+ 0.773} \text{ Log } \frac{DD(-1)}{P(-1)} \underset{(-3.71)}{- 0.042 \text{ Q1}}$$

$$\underset{(-3.81)}{- 0.089 \text{ Q2}} \underset{(-3.79)}{- 0.092 \text{ Q3}} \qquad (5.2.5)$$

$R^2 = .949$ $\bar{R}^2 = .936$ $F(9,38) = 77.75$

SER = 0.021 DW = 1.62 D's H = 1.47

From (5.2.2) to (5.2.5), $\bar{R}^2$ increases tremendously from .667 to .936. Autocorrelation disappears. The long-term elasticities of real demand deposits with respect to income and interest rate are .61 and -.35. In the literature, there is more evidence on the demand for money elasticities but less report on the demand deposit elasticities. We have to rely on the Bank of Canada study by Clinton and Lynch (1979) which ran the following equation for demand deposits:[44]

$$\text{Log } \frac{DD}{P} = f\ [\text{QMAIL74, QMAIL75, Log Y, RS,}$$

$$\text{Log } \frac{DD(-1)}{P(-1)}\ ] \qquad (5.2.6)$$

Their data seem to be the same as ours. Based on their reported coefficients, the elasticities can be calculated and are .79 for income and -.35 for interest rate. Our income elasticity is lower for two reasons: differences in period estimation and equation form. To pinpoint the exact cause, we run Clinton-Lynch equation from

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44 Also see Wirick (1981, p. 254 and p. 258) for income elasticity comparison.

1967 I to 1978 IV (i.e., our estimation period). The result is a little worse compared to (5.2.5), and income elasticity does not change much (.65). So, the main cause is difference in period estimation.

Although (5.2.5) is satisfactory, we also run it with log (RS) replaced by RS. The new equation is better and most preferred.

$$\begin{aligned} \text{Log}\,\frac{DD}{P} = {} & \underset{(1.80)}{0.638} + \underset{(3.46)}{0.079}\ QMAIL74 + \underset{(3.58)}{0.077}\ QMAIL75 \\ & + \underset{(3.50)}{0.733}\ \text{Log}\,Y - \underset{(-2.85)}{0.592}\ \text{Log}\,Y(-1) - \underset{(-5.99)}{0.012}\ RS \\ & + 0.782\ \text{Log}\,\frac{DD(-1)}{P(-1)} - \underset{(-3.99)}{0.043}\ Q1 - \underset{(-4.07)}{0.091}\ Q2 \\ & - \underset{(-4.0)}{0.093}\ Q3 \end{aligned} \tag{5.2.7}$$

$R^2 = .953 \quad \bar{R}^2 = .942 \quad F(9,38) = 85.18$

$SER = 0.020 \quad DW = 1.61 \quad D's\ H = 1.50$

The long-term elasticities with respect to income and interest rate are .65 and -.41.

b. Currency (CUR)

The demand for currency in principle should have the same form as DD. We follow the same steps taken for DD. One small difference is that the U.S. short-term interest rate is significant along the Canadian short rate. Rather than repeating the whole procedure, we report only the best equation for CUR.

$$\text{Log}\ \frac{CUR}{P} = -\underset{(-.84)}{0.120} + \underset{(.93)}{0.014}\ QMAIL74 + \underset{(2.60)}{0.037}\ QMAIL75$$
$$+ \underset{(6.38)}{0.867}\ \text{Log}\ Y - \underset{(-3.16)}{0.481}\ \text{Log}\ Y(-1) - \underset{(-3.69)}{0.007}\ RS$$
$$+ \underset{(2.17)}{0.003}\ RSU + \underset{(5.22)}{0.547}\ \text{Log}\ \frac{CUR(-1)}{P(-1)} - \underset{(-.32)}{0.003}\ Q1$$
$$- \underset{(-3.97)}{0.064}\ Q2 - \underset{(-5.66)}{0.080}\ Q3 \qquad (5.2.8)$$

$R^2 = .992$ $\bar{R}^2 = .990$ $F(10,37) = 480.49$

SER = 0.013 DW = 2.18 D's H = -.90

The positive sign of RSU is consistent with the view that real cash balances are negatively related to the Canadian rate and positively related to the U.S. rate (see Choudhry _et_ _al_., p. 47, for the TRACE model). Long-term elasticities are .85 for income and -.12 for interest rate.

c. Time deposits (DSN)

From the results of demand deposits and currency, we might figure out that without lagged dependent variable, autocorrelation will plague the DSN equation; furthermore, we might guess better results would be obtained not from the inclusion of the short-term rate RS but from the own rate of time deposits (RTD). Finally, without surprise wealth turns out to be significant.

For the estimation period there was an important event, the introduction of the Bank Act Revision in 1967 III which removed interest ceiling. A dummy will be included to take into account of this structural change. As an initial step we can examine the following equation:

$$\frac{DSN}{P} = \underset{(-3.82)}{-8530.11} + \underset{(1.89)}{1279.69}\ QBANK + \underset{(2.02)}{0.252}\ Y$$
$$+ \underset{(2.06)}{194.422}\ RTD + \underset{(3.70)}{0.152}\ V + \underset{(9.46)}{0.734}\ \frac{DSN(-1)}{P(-1)}$$
$$+ \underset{(.14)}{49.726}\ Q1 + \underset{(.61)}{170.380}\ Q2 - \underset{(-3.10)}{978.168}\ Q3 \qquad (5.2.9)$$

$R^2 = .994 \quad \bar{R}^2 = .993 \quad F(8,39) = 812.88$

SER = 629.235 DW = 1.67 D's H = 1.34

where QBANK = dummy, 1 in 1967 III and 0 elsewhere, to capture the effect of the Bank Act Revision

RTD = interest rate on time deposits

V = real wealth (liquid assets)

The equation performs extremely well. The dummy is significant. RTD has the right sign as expected. Also is wealth. There is no symptom of autocorrelation.

We also find the equation can be improved on many respects ($\bar{R}^2$, t-statistics, DW) when RTD(-1) replaces RTD.

$$\frac{DSN}{P} = \underset{(-4.51)}{-\ 10168.6} + \underset{(2.43)}{1581.50}\ QBANK + \underset{(2.46)}{0.280}\ Y$$
$$+ \underset{(3.00)}{275.057}\ RTD(-1) + \underset{(4.39)}{0.181}\ V + \underset{(8.98)}{0.686}\ \frac{DSN(-1)}{P(-1)}$$
$$+ \underset{(.26)}{87.909}\ Q1 + \underset{(1.07)}{288.459}\ Q2 - \underset{(-2.86)}{865.000}\ Q3 \qquad (5.2.10)$$

$R^2 = .995 \quad \bar{R}^2 = .994 \quad F(8,39) = 902.63$

SER = 597.317 DW = 1.70 D's H = 1.21

Finally the best equation is obtained when V(-1) is also included.

$$\frac{DSN}{P} = -\underset{(-3.32)}{6448.18} + \underset{(1.67)}{892.036}\ QBANK + \underset{(3.00)}{0.271}\ Y$$

$$+ \underset{(1.73)}{135.015}\ RTD(-1) + \underset{(7.36)}{0.361}\ V - \underset{(-4.92)}{0.282}\ V(-1)$$

$$+ \underset{(12.30)}{0.781}\ \frac{DSN(-1)}{P(-1)} + \underset{(2.44)}{729.339}\ Q1$$

$$+ \underset{(3.76)}{950.036}\ Q2 + \underset{(.01)}{3.906}\ Q3 \qquad (5.2.11)$$

$R^2 = .997$ $\bar{R}^2 = .996$ $F(9,38) = 1281.96$

$SER = 473.039$ $DW = 1.82$ $D's\ H = .70$

We also try (5.2.11) in log-form, but the t-statistic of the time deposit rate (either in linear or log-form) declines tremendously with a value barely exceeding 1.

5.2.3 Short-term interest rate (RS)

Essentially it is the normalized demand for money. This can be seen as follows.

Let: $\frac{M1^d}{P} = f(Y, RS)$

$\overline{M1} = M1^d$

where $M1^d$ is the demand for money. Solving these two equations:

$RS = f(\frac{\overline{M1}}{P}, Y)$

In the TRACE model (see Choudhry et al., 1972, p. 691), $\frac{M1}{P}$ is scaled down by income. There are a few reasons which compel us to follow the same line:

- collinearity betwen Y and $\frac{M1}{P}$.

- the search for an appropriate dynamic structure will be difficult if both $\frac{M1}{P}$ and Y are introduced in the equation
- as there are a lot of variables (including four extra dummies besides seasonal dummies) in the RS equation, this would save degrees of freedom
- and finally, the goodness of fit and statistical significance are better with the scaling.

So the main explanatory variable is $\frac{M1}{P.Y}$. A second one is the U.S. short rate RSU which plays an important role since the Canadian rate usually moves alongside. Of course there are also postal strike dummies which affect real cash balances. Apart from these there are two extra dummies:

- QPFX: 1 in 1971 II, 0 elsewhere, to capture the effect of the Bank of Canada measures in moderating the exchange rate appreciation right after its floating (for example, intervention in the exchange market, reduction in the bank rate, exhortation to borrowers to obtain their funds in the domestic market).
- QMR: 1 in 1973 IV, 0 elsewhere, to capture the lagged effect of the Bank of Canada measures in restraining the monetary growth.

In short, the basic form is:

$$RS = f(QMAIL74,\ QMAIL75,\ QPFX,\ QMR,\ RSU,\ \frac{M1}{P.Y},\ Q1,\ Q2,\ Q3) \qquad (5.2.12)$$

The first thing we can predict is that (5.2.12) suffers from serious correlation.

$$RS = \underset{(6.69)}{13.850} + \underset{(1.48)}{1.649}\ QMAIL74 + \underset{(2.01)}{2.095}\ QMAIL75$$
$$- \underset{(-2.36)}{2.456}\ QPFX + \underset{(.96)}{1.010}\ QMR + \underset{(5.97)}{0.512}\ RSU$$
$$- \underset{(-5.45)}{23.394}\ \frac{M1}{P.Y} + \underset{(.82)}{0.355}\ Q1 - \underset{(-.16)}{0.072}\ Q2$$
$$- \underset{(-1.51)}{0.663}\ Q3 \qquad (5.2.13)$$

$R^2 = .767 \quad \bar{R}^2 = .712 \quad F(9,38) = 13.89$

$SER = 0.990 \quad DW = .56$

When the lagged dependent variable is added, autocorrelation does not disappear either, although $\bar{R}^2$ improves substantially.

$$RS = \underset{(2.07)}{4.356} + \underset{(2.10)}{1.648}\ QMAIL74 + \underset{(1.96)}{1.458}\ QMAIL75$$
$$- \underset{(-2.11)}{1.583}\ QPFX + \underset{(1.33)}{0.995}\ QMR + \underset{(3.99)}{0.283}\ RSU$$
$$- \underset{(-1.90)}{7.495}\ \frac{M1}{P.Y} + \underset{(6.26)}{0.576}\ RS(-1) - \underset{(-.27)}{0.084}\ Q1$$
$$+ \underset{(.60)}{0.189}\ Q2 - \underset{(-.03)}{0.009}\ Q3 \qquad (5.2.14)$$

$R^2 = .887 \quad \bar{R}^2 = .856 \quad F(10,37) = 28.98$

$SER = .699 \quad DW = 1.13 \quad D's\ H = 3.90$

After a long search for a more complicated lag structure, our best equation is obtained with the following form:

$$RS = f\left[QMAIL74,\ QMAIL75,\ QPFX,\ QMR,\ RSU,\ RSU(-1),\ \frac{M1}{P.Y},\ \frac{M1(-1)}{P(-1).Y(-1)},\ RS(-1)\right] \qquad (5.2.15)$$

(5.2.15) is also tried in log form, but the linear form has a better fit. Below we present (5.2.15) in both forms:

$$\text{Log RS} = \underset{(1.69)}{0.235} + \underset{(1.91)}{0.154}\ \text{QMAIL74} + \underset{(3.45)}{0.273}\ \text{QMAIL75}$$

$$- \underset{(-4.22)}{0.320}\ \text{QPFX} + \underset{(1.57)}{0.125}\ \text{QMR} + \underset{(6.37)}{0.559}\ \text{Log RSU}$$

$$- \underset{(-4.50)}{0.454}\ \text{Log RSU}(-1) - \underset{(-3.01)}{1.785}\ \text{Log}\ \frac{\text{M1}}{\text{P.Y}} + \underset{(2.81)}{1.541}$$

$$\text{Log}\ \frac{\text{M1}(-1)}{\text{P}(-1).\text{Y}(-1)} + \underset{(9.57)}{0.729}\ \text{Log RS}(-1)$$

$$- \underset{(-1.84)}{0.071}\ \text{Q1} - \underset{(-2.52)}{0.189}\ \text{Q2} - \underset{(-2.80)}{0.217}\ \text{Q3} \qquad (5.2.16)$$

$R^2 = .945 \quad \bar{R}^2 = .926 \quad F(12,35) = 50.19$

SER = 0.070 DW = 1.60 D's H = 1.64

$$\text{RS} = \underset{(2.09)}{3.278} + \underset{(2.50)}{1.374}\ \text{QMAIL74} + \underset{(3.76)}{1.905}\ \text{QMAIL75}$$

$$- \underset{(-3.08)}{1.501}\ \text{QPFX} + \underset{(2.64)}{1.389}\ \text{QMR} + \underset{(7.57)}{0.614}\ \text{RSU}$$

$$- \underset{(-5.52)}{0.533}\ \text{RSU}(-1) - \underset{(-2.78)}{26.873}\ \frac{\text{M1}}{\text{P.Y}}$$

$$+ \underset{(2.67)}{23.443}\ \frac{\text{M1}(-1)}{\text{P}(-1).\text{Y}(-1)} + \underset{(10.12)}{0.773}\ \text{RS}(-1)$$

$$- \underset{(-2.07)}{0.511}\ \text{Q1} - \underset{(-2.55)}{1.297}\ \text{Q2} - \underset{(-2.72)}{1.389}\ \text{Q3} \qquad (5.2.17)$$

$R^2 = .955 \quad \bar{R}^2 = .939 \quad F(12,35) = 61.62$

SER = 0.454 DW = 1.65 D's H = 1.42

All coefficients in (5.2.17) are well significant and have the expected sign. To (5.2.17) the rate of inflation in various lagged forms is included; but it is not significant, and $\bar{R}^2$ declines.

5.2.4 Demand for Chartered Bank Reserves (RR)

RR consists of required reserves and excess reserves. Chartered banks were required to hold reserves of 12% against demand deposits and 4% against time deposits.[45] The Government of Canada deposits at chartered banks are counted as demand deposits. Excess reserves theoretically vary inversely with interest rate. As mentioned in footnote 38 of this chapter, the latter is never significant, maybe due to its correlation with demand and time deposits. This leads us to formulate RR in a very simple way.

$$RR = f[.12\ (DD + DG),\ .04\ (DSN)] \qquad (5.2.18)$$

where RR, DD and DSN have been defined as before and

DG = Government of Canada deposits at chartered banks

The equation performs well in this form:

$$\begin{aligned} RR \;=\; & -\underset{(-1.64)}{60.305} + \underset{(8.26)}{0.077}\ (DG + DD) + \underset{(4.14)}{0.015}\ DSN \\ & + \underset{(7.91)}{0.552}\ RR(-1) + \underset{(3.87)}{68.054}\ Q1 + \underset{(1.30)}{24.717}\ Q2 \\ & + \underset{(1.95)}{35.900}\ Q3 \end{aligned} \qquad (5.2.19)$$

$R^2 = .999 \quad \bar{R}^2 = .999 \quad F(6,41) = 6692.92$

$SER = 42.665 \quad DW = 1.95 \quad D's\ H = .18$

As $\bar{R}^2$ is very high, there should be no worry for the exclusion of the short-term interest rate.

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[45] In practice, these ratios are not applied to the current level of those deposits but to their lagged values.

Before we start the discussion on the term structure of interest rates, we like to remind that there still exist in the money sector four identities defining the equilibrium condition for money, the demand for monetary base, broad money and real financial wealth (see section on the basic structure).

Narrow money demand and supply in equilibrium

$$\frac{CUR + DD}{P} - M1 = 0 \qquad (5.2.20)$$

Depending on whether CUR or DD is explicitly specified, (5.2.20) will determine the other residually.

Broad money (M2)

$$M2 = M1 + DSN \qquad (5.2.21)$$

The presence of broad money (M2) is necessary since it will enter the foreign exchange rate equation.

Monetary base (MB)

$$MB = CUR + RR \qquad (5.2.22)$$

MB will enter the government budget constraint

Real financial wealth (V)

$$V = (MB + BOND)/P \qquad (5.2.23)$$

5.2.5 Term Structure of Interest Rates

As the short-term interest rate RS plays the main role of determining the equilibrium condition of the money market, the other interest rates (long-term and time deposit) have to be linked to it.

a. <u>Long-term interest rate (RL)</u>

Its function is derived from expectation models of the term structure of interest rates (for a summary on the term structure theories, see Masera, 1972, Chapter 1). According to the expectation theory, the return from holding a n-period security is equal to the expected return from holding a series of one-period securities over n periods. Then:

$$(1 + RL_t) = (1 + {}_tRS^e_t)\ (1 + {}_tRS^e_{t+1}) \ldots (1 + {}_tRS^e_{t+n-1})$$

where ${}_tRS^e_{t+i}$ denotes the expected one-period of return for period t+i, conditional on information available at time t.

As ${}_tRS^e_{t+i}$ is unobserved, the usual assumption is that it is a distributed lag function of actual RS with geometrically declining weights. This yields:

$$RL = f[RS,\ RL(-1)]$$

or in estimation form:

$$\begin{aligned} RL = {} & \underset{(1.57)}{0.547} + \underset{(3.83)}{0.129}\ RS + \underset{(14.52)}{0.800}\ RL(-1) + \underset{(.50)}{0.066}\ Q1 \\ & + \underset{(2.63)}{0.343}\ Q2 + \underset{(1.28)}{0.165}\ Q3 \end{aligned} \qquad (5.2.24)$$

$R^2 = .927 \quad \bar{R}^2 = .919 \quad F(5,42) = 107.105$

$SER = 0.316 \quad DW = 1.59 \quad D's\ H = 1.52$

Looking at $\bar{R}^2$, we believe the equation can improve. We can imagine of adding U.S. interest rates. Both U.S. short-term and long-term rates are tried with various lag structures. The long-term rate is found to perform better. The following equation is our most satisfactory one.

$$RL = \underset{(.57)}{0.147} + \underset{(3.37)}{0.148}\ RS - \underset{(-2.14)}{0.099}\ RS(-1)$$
$$+ \underset{(4.90)}{0.729}\ RLU - \underset{(-3.14)}{0.575}\ RLU(-1) + \underset{(7.96)}{0.801}\ RL(-1)$$
$$+ \underset{(.65)}{0.062}\ Q1 + \underset{(2.36)}{0.219}\ Q2 + \underset{(1.47)}{0.133}\ Q3 \qquad (5.2.25)$$

$R^2 = .967$ $\bar{R}^2 = .961$ $F(8,39) = 143.99$

SER = 0.220 DW = 1.94 D's H = .26

$\bar{R}^2$ increases from .919 to .961. DW or D's H-statistic is also better, and coefficients have the correct sign.

b. Interest rate on time deposits (RTD)

The formulation is straightforward and ad hoc, based on a similar treatment found in Kuh and Schmalensee (1973, p. 164 and p. 168). RTD is related to the short-term rate with the dummy QBANK included to capture the effect of removing interest ceiling by the Bank Act Revision in 1967 III.

The following form is found not to suffer from autocorrelation.

$$RTD = -\underset{(-1.85)}{0.312} + \underset{(2.84)}{0.776}\ QBANK + \underset{(13.85)}{0.558}\ RS$$
$$- \underset{(-4.09)}{0.259}\ RS(-1) + \underset{(11.96)}{0.705}\ RTD(-1) \qquad (5.2.26)$$

$R^2 = .976$ $\bar{R}^2 = .973$ $F(4,43) = 428.98$

SER = 0.259 DW = 2.08 D's H = -.30

Seasonal dummies are not significant and left out of the equation.

U.S. short-term rate RSU is also tried, but its influence is too small and is never significant. One reason might be that RSU cannot approximate U.S. time deposit rate. Below we present one version with U.S. rate included.

$$\begin{aligned} RTD = & -\underset{(-2.13)}{0.383} + \underset{(2.98)}{0.833}\ QBANK + \underset{(7.89)}{0.503}\ RS \\ & - \underset{(-2.96)}{0.237}\ RS(-1) + \underset{(1.05)}{0.060}\ RSU - \underset{(-.55)}{0.031}\ RSU(-1) \\ & + \underset{(11.73)}{0.723}\ RTD(-1) \qquad (5.2.27) \end{aligned}$$

$R^2 = .976$ $\bar{R}^2 = .973$ $F(6,41) = 283.21$

SER = 0.260 DW = 2.05 D's H = -.20

5.3 GOVERNMENT BUDGET CONSTRAINT

It specifies that the total flow of government expenditure must equal the total flow of financing of all sources (taxes, government bonds, high-powered money), restricting therefore the freedom of the government in assigning arbitrary values to policy variables. Given n policy variables, at most n-1 can be assigned arbitrarily; the remaining one must be determined residually. Another important aspect is that, to the extent that the crowding-out effect can take place following a fiscal action, the government budget constraint (GBC) provides the necessary link between the fiscal sector and the rest of the economy (Choudhry, 1976).

In theoretial studies, the GBC is usually expressed simply as:

$$MB - MB(-1) + BOND - BOND(-1) = P(G - \frac{TAX}{P}) \qquad (5.3.1)$$

where G = total real government spending

Tax = nominal tax revenue

In reality, the relationship is highly complex.

To have an idea about such complexity, we present below the components which constitute the left hand side of the federal GBC as formulated in the most disaggregated econometric model of Canada, the RDX2 (see Bank of Canada, 1976).

In the RDX2's formulation, the right hand side is defined as the federal government deficit (surplus) on a national account basis and equal to the left hand side which consists of the following elements:

i) + change in the monetary base

ii) + change in the stock of federal debt held by the private sector

iii) - change in the federal government cash balances (exogenous in the RDX2)

iv) + difference between corporation tax collected and accrued

v) - change in foreign reserve

vi) - changes in miscellaneous assets and liabilities accounts of the government of Canada (exogenous in the RDX2 and derived residually)

The right hand side is much more complicated with a high number of exogenous and endogenous variables reflecting different types of taxes or revenues (for example, federal personal income tax, other federal personal direct tax, federal corporation income tax, excise tax and duties etc...), different types of expenditures (federal wages, federal current non wage expenditure, unemployment insurance benefits etc...).

It is well clear, in the context of a small model, that an over-simplification is needed if one wishes to incorporate the GBC into the model.

Let us look first at the federal GBC. Its deficit (surplus), on a national account basis, includes the following items in nominal value (please refer to any issue of the System of National Accounts):

i) - TRF = federal total revenues (= Tax + Other revenue)

ii) + GF = total federal current expenditure (= Current expenditure + Other current expenditure)

iii) + GFCF = federal gross capital formation

iv) - GFDEP = federal capital consumption allowance (i.e., depreciation)

Note that 'Other revenue' is just the sum of all components which do not belong to the category 'Tax' (for the time being, let's assume we know well what constitutes 'Tax'). In the same way, 'Other current expenditure' is the sum of all items which do not belong to 'Current expenditure'. Let's denote 'Current expenditure' by G1OF and 'Other current expenditure' by G2F. Because, in our income equilibrium equation (see Equation (5.1.1)), both G1 and G2 include government current expenditure, (not total current expenditure) and government investment, it is desirable to combine G1OF and GFCF into one component (call it G1F).

Then, the right hand side of the federal government budget constraint is:

- TRF + (G1F + G2F) - GFDEP

Let's now go back to the left hand side of the federal GBC as formulated in the RDX2. We now lump all items (iii, iv, v, vi) together into a single component. Call it by the name 'OTHER' (it is treated as exogenous). The left hand side is now:

MB - MB(-1) + BF - BF(-1) + OTHER

(BF = federal debt held by the private sector)

Combine both sides:

MB - MB(-1) + BF - BF(-1) + OTHER = (G1F + G2F)
- TRF - GFDEP (5.3.2)

Since there is no capital stock in our model, depreciation does not exist either. Let's include it in 'OTHER'.

(5.3.2) becomes:

MB - MB(-1) + BF - BF(-1) + OTHER = (G1F + G2F)
- TRF (5.3.3)

It is obvious that the item 'OTHER' is just the balancing item to make both sides of the GBC as defined above balance. It will be derived as a residual and considered as exogenous (we treat it in the same way as the item 'changes in miscellaneous asset and liabilities account of the Government of Canada' in the RDX2 equation).

First of all, regarding (5.3.3), we have to say immediately it reflects the budget constraint of the federal government.

In Canada, there are three levels of governments (ignoring Hospitals, Quebec Pension Plan, Canada Pension Plan). Only the federal government has the right to print money. But the provincial and local governments can issue bonds. In other words, if we generalize (5.3.3) to other levels of government, the terms MB - MB(-1) will disappear in the provincial and local governments' budget constraints.

So, if (5.3.3) can be accepted for the federal government, a generalization can be made to consolidate all three levels of governments such that we now have:

$$[MB - MB(-1)] + [BOND - BOND(-1)] + OTHER = P(G1 + G1R) - TR \qquad (5.3.4)$$

where TR = nominal total revenues from three levels of governments[46]

G1 = real government expenditures (= current expenditure + investment) from three levels of governments

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46 The variables TR and G1 have appeared earlier in Equations (5.1.1) and (5.1.27) respectively.

G1R = other real current government expenditure from three levels of governments (exogenous)

MB = monetary base

BOND = stock of government (three levels) debt held by the private sector[47]

OTHER= balancing item (exogenous)

Two remarks should be made:

i) Since TR and MB are endogenous elsewhere in the model, and G1, G2R are exogenous by assumption, BOND is the only remaining instrument which must be determined endogenously and residually from the budget contraint.

ii) The term 'OTHER' is the balancing item. It is treated as exogenous and includes a variety of variables, one of them is the change in the foreign reserves. Under a system of flexible exchange rate, the change in foreign reserves, by definition equal to total balance of payments, is theoretically zero. There is no problem in treating it as exogenous and lumped into 'OTHER'. But under a fixed exchange rate regime, 'foreign reserves' is endogenous. Our treatment is surely not appropriate. But as we argued earlier in the previous chapter, the exclusion of the foreign reserves should not cause any serious problem for our optimal control exercises which will take place in a flexible

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47 Data for provincial and local bonds are obtained from the Bank of Canada. Data for federal debt are taken from CANSIM.The types of federal debt entering the federal GBC were described in the RDX2 manual (see Bank of Canada, 1976, pp. 214-215).

exchange regime. Furthermore, the GNP price is used as a proxy for total government expenditure price deflator; any deviation in the amount of nominal government expenditure caused by the approximation is also captured by the term 'OTHER'.

5.4 FOREIGN EXCHANGE RATE (PFX)[48]

The Canadian foreign exchange rate was fixed before June 1970. After that it has been floating again. Our treatment is that from 1967 I to 1970 II, PFX is assumed to be exogenous, and from 1970 III on it is endogenous.

Much of the recent work on floating exchange rates goes under the name of the 'monetary' or 'asset' view: the exchange rate is viewed as moving to equilibrate the international demands for stocks of assets (see Frankel, 1979, and particularly, Bilson, 1979, for a survey of the asset view). This view emphasizes the role of the supply and demand for monies (or assets) as stocks, in contrast to the traditional flow view whose analysis is based on the components of the balance of payments (BOP). Of course, as Bilson noted, in a fully specified general equilibrium system any excess supply of money is balanced by an offsetting excess demand for goods and non-monetary assets, so that the balance of payment components view and the monetary view are equivalent in a fully specified model.

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48 The formulation of the foreign exchange rate and the presentation of this section are drawn heavily from a study by Frankel (1979).

With respect to the traditional flow view, while the theoretical aspects of the current account are comparatively well developed, and the import and export demand relations are highly standardized, the theory of capital account relationship is not well formalized. Most of the empirical studies provide poor statistical results (see Leamer and Stern, 1970, Chapter 4). A typical example is the Canadian RDX2 model (Bank of Canada, 1976): while the short-term capital flow is determined residually from the BOP identity, the long-term capital flow equations (12 in total) exhibit very low R^2 which are, in most cases, below .50.

We have tried to build our foreign exchange sector, based on the traditional view, but our results are not better at all. It is then decided to pursue the asset view which provides us with encouraging results.

Regarding the asset view, within it there are two different approaches which lead to conflicting implications for the relationship between the exchange rate and the interest rate. The first approach might be called the 'Chicago' theory. It assumes prices are perfectly flexible. In consequence, changes in nominal interest rate reflect changes in the expected inflation rate. An increase in the domestic interest rate relative to the foreign rate is due to higher domestic inflation, to the fall in the demand for domestic currency relative to the foreign currency which causes a depreciation of the domestic currency and a rise in the exchange rate, defined as the price of foreign currency.

The relationship between the exchange rate and the nominal interest differential is positive.

The second approach might be called 'Keynesian' because it assumes prices are sticky, at least in the short-run (see Dornbusch, 1976). Consequently, changes in nominal interest rate reflect changes in the tightness of the monetary policy. A domestic contraction in the domestic money supply without a matching fall in prices will yield a higher domestic interest rate relative to the foreign rate, which attracts a capital inflow. The exchange rate will depreciate (appreciation of the domestic currency). The relationship between the exchange and the nominal interest differential is negative.

The Chicago approach would be appropriate when the inflation differential is large as in the German inflation of 1920's, while the Keynesian theory would be more realistic for the case of small inflation differential. For a situation of moderate variation in the inflation differential, both effects might be present, that is the theme proposed by Frankel (1979). He developed a more general model which is still a version of the asset view, in that it emphasizes the role of expectations and rapid adjustments in the capital markets, and which combines the assumption of sticky prices of the Keynesian theory with the Chicago assumption that there are secular rates of inflation. The model predicts that the exchange rate is negatively related to the nominal interest rate differential but positively related to the expected long-run inflation differential.

Frankel's equation proposed for empirical testing is (for the development of his model, please refer to his paper):

$$\text{Log PFX} = \text{Log}\,\frac{M}{M^f} + \text{a1 Log}\,\frac{Y}{Y^f} + \text{a2}\,(RS - RS^f)$$

$$+\ \text{a3}\ (\pi - \pi^f) \qquad (5.4.1)$$

where PFX = foreign exchange rate

M = domestic money supply (= domestic money demand) to be defined later

M^f = foreign money supply (= domestic money demand) to be defined later

Y = domestic income

Y^f = foreign income

R = domestic interest rate (short rate)

R^f = foreign interest rate (short rate)

π = domestic expected rate of inflation

π^f = foreign expected rate of inflation

Examining the equation, we see the presence of money supplies which come from the monetary market equilibrium condition in each country. That is the reason why the view to the flexible exchange rate is labelled 'monetary'. The presence of incomes links the money and product markets together. Therefore, the monetary view does not focus solely on monetary factors; it just emphasizes the role of assets (monies).

Let us turn to the signs of the coefficients. $\frac{M}{M^f}$ is expected to have a positive sign and to equal unity. As the domestic money supply increases relative to the foreign money supply, the value of domestic currency is expected to depreciate, in other words, a rise in the exchange rate. (a1) is expected to be negative: an increase in the domestic income relative to the foreign income will increase the domestic demand for money which will appreciate the value of domestic currency (or a decrease in the foreign exchange rate). If Frankel model was valid, as discussed earlier (a2) will be negative and (a3) positive.

Let us turn to the data side. Because PFX is defined as the number of Canadian dollars per U.S. dollar, the choice for the foreign country is simple: it is the U.S. With respect to the stocks of monies, we try both narrow and broad monies. No wonder broad money is more meaningful. The only problem is to measure $(\pi - \pi^f)$. As π and π^f are unobserved, Frankel tried two proxies for the inflation differential: i) long-term interest rate differential (under the rationale that the long-term real rates are equal) and ii) past inflation differentials (based on quarterly data and averaged over the preceding year). We use three proxies for $(\pi - \pi^f)$: i) long-term interest rate differential, ii) past inflation differentials in the last three quarters with weights 1/2, 1/3, and 1/6 (suggested by Fair, 1979, p. 543), and iii) past inflation differentials in the last three quarters with equal weights.

Equation (5.4.1) is first run in the original form suggested by Frankel. The inflation differential has the wrong sign, but insignificant. The interest differential has the negative sign, but also insignificant. The DW value is in general less than 1; even with CORC, autocorrelation persists. Frankel's equation, when derived, was based on the demand for money without a lagged dependent variable. This form of money demand might be considered as long-run. Our equations on currency, demand, and time deposits all incorporated lagged dependent variables. Certain authors think the lagged dependent variable should enter the demand for money and, therefore, suggest the inclusion of lagged PFX in the PFX equation. They find improvement in their equation (for example, Bilson, 1978). We decide to include the lagged dependent variable. There is some improvement, but $(\pi - \pi^f)$ is still insignificant. For example, with $(\pi - \pi^f)$ proxied by the long-term interest differential:

$$\begin{aligned}
\text{Log PFX} = & -\underset{(-.58)}{0.227} + \underset{(4.37)}{0.187}\ \text{Log}\ \frac{M2}{MU} - \underset{(-1.63)}{0.190}\ \text{Log}\ \frac{Y}{YU} \\
& - \underset{(-3.13)}{0.008}\ (RS - RSU) - \underset{(-.27)}{0.003}\ (RL - RLU) \\
& + \underset{(7.14)}{0.736}\ \text{Log PFX}(-1) - \underset{(-1.69)}{0.015}\ Q1 - \underset{(-1.75)}{0.009}\ Q2 \\
& - \underset{(.01)}{0.00006}\ Q3 \qquad (5.4.2)
\end{aligned}$$

$R^2 = .941 \quad \bar{R}^2 = .921 \quad F(8,24) = 47.494$

$SER = 0.010 \quad DW = 1.79 \quad D's\ H = .77$

$RHO = .325\ (1.97)$

where MU = U.S. broad money

YU = U.S. real GNP

Since the estimation requires good proxies for $(\pi - \pi^f)$, and since the contention between the two approaches focuses on the short-term interest differential, we decide to exclude $(\pi - \pi^f)$, which yields the following result.

$$\begin{aligned} \text{Log PFX} = &- \underset{(-.77)}{0.274133} + \underset{(4.43)}{0.186332} \text{ Log } \frac{M2}{MU} - \underset{(-1.82)}{0.201328} \\ &\text{Log } \frac{Y}{YU} - \underset{(-3.59)}{0.0083396} (RS - RSU) + \underset{(7.80)}{0.743879} \\ &\text{Log PFX}(-1) - \underset{(-1.76)}{0.0152353}\, Q1 - \underset{(-1.92)}{0.00977621}\, Q2 \\ &- \underset{(-.02)}{0.000112176}\, Q3 \end{aligned} \qquad (5.4.3)$$

$R^2 = .939$ $\bar{R}^2 = .922$ $F(7,25) = 46.42$

SER = 0.010 DW = 1.77 D's H = .78

RHO = .333225 (2.03)

As can be seen, the equation is estimated by CORC. This is our last resort after many attempts have failed in eliminating autocorrelation. The terms $\frac{M2}{MU}$ and $\frac{Y}{YU}$ have the right signs with the former having a long-run elasticity equal to .73. The coefficient of (RS - RSU) is negative, which seems to support the Keynesian view.

In a very recent book Macesich (1983, pp. 208-209) reports the results on the Canadian foreign exchange rate equation from his joint study with Tsai.[49] [illegible]esich and

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[49] For reference, see Macesich (1983, footno[illegible] p. 215)

Tsai, using annual data from 1973 to 1980, estimated a foreign exchange rate equation, quite similar to ours with two minor differences: interest differential in log-form, no lagged dependent variable (there is no indication that CORC or OLS is used). They obtained a negative sign for the interest differential. However, as their model was of the Chicago approach, Macesich concluded the sign was wrong.

To end the foreign exchange section, we like to report the results based on the use of narrow money.

$$\begin{aligned}\text{Log PFX} = &- \underset{(-.47)}{0.248} + \underset{(2.06)}{0.153} \text{ Log } \frac{M1}{M1U} - \underset{(-1.13)}{0.189} \\ &\text{Log } \frac{Y}{YU} - \underset{(-1.91)}{0.006} (RS - RSU) + \underset{(7.03)}{0.845} \\ &\text{Log PFX}(-1) - \underset{(-1.11)}{0.013} Q1 - \underset{(-1.28)}{0.007} Q2 \\ &+ \underset{(.08)}{0.0006} Q3 \end{aligned} \qquad (5.4.4)$$

$R^2 = .850$ $\bar{R}^2 = .817$ $F(7,25) = 16.98$

SER = 0.012 DW = 1.74 D's H = 1.03

RHO = .555 (3.84)

where M1U = U.S. narrow money

Compared to (5.4.3), (5.4.4) is much worse in many respects: $\bar{R}^2$, DW value etc... Besides, t-statistic of $\frac{Y}{YU}$ declines tremendously. In view of these, (5.4.3) is our choice.

5.5 LABOUR SECTOR AND PRICES

5.5.1 Labour and Wage

Money wage, in the short-run, is assumed to be sluggish in the labour market, responding slowly to a non-zero excess supply. The labour market is the interaction between the civilian labour force and its demand. The gap determines the unemployment rate which causes the wage rate to move according to the expectations augmented Phillips curve. Wage in turn influences the general price level.

a. Employment (L)

The desired demand for labour is derived from profit maximization assumption which allows some degree of monopoly. If we assume a Cobb Douglass production function, the marginal product condition yields:

$$L^* = (1 + \frac{1}{\varepsilon})\, \alpha Y . \frac{P}{W} \qquad (5.5.1)$$

where L^* = desired demand for labour

ε = elasticity of demand with respect to price

α = elasticity of output with respect to labour

Y = real income, used as a proxy for business output

P = price

W = money wage (= total manufacturing average hourly earning)

If we assume a partial adjustment, the actual employment (L) equation will be of the form:

$$L - L(-1) = \gamma [L^* - L(-1)]$$

or

$$L = \gamma L^* + (1 - \gamma)\ L(-1)$$

where γ = coefficient of adjustment.

Substitute for L^*:

$$L = \gamma\left(1 + \frac{1}{\varepsilon}\right) \alpha\, Y.\frac{P}{W} + (1 - \gamma)\ L(-1)$$

or

$$L = f[Y.\frac{P}{W},\ L(-1)] \qquad (5.5.2)$$

The estimated equation is

$$L = \underset{(-4.21)}{-\ 475.171} + \underset{(2.17)}{0.090}\ \frac{Y.P}{W} + \underset{(32.06)}{0.947}\ L(-1) + \underset{(.23)}{6.171}\ Q1$$

$$+ \underset{(23.65)}{653.174}\ Q2 + \underset{(13.18)}{520.741}\ Q3 \qquad (5.5.3)$$

R^2 = .997 $\bar{R}^2$ = .996 $F(5,42)$ = 2374.19

SER = 55.367 DW = 1.89 D's H = .38

b. <u>Labour force (LF)</u>

On the supply side of the labour market, economists' investigations are sparse as compared to the amount of work expended on other parts of standard macroeconomic models (for a brief survey of the labour supply, see Byers, 1976). On the theory side, the labour supply function is commonly based on the theory of income-leisure choice which postulates a positive relationship between labour supply and wage rate. On the empirical side, the 'discouraged worker' hypothesis is commonly incorporated in equations of aggregate labour force (or of labour force participation). According to this hypothesis, workers are encouraged to enter the market in large numbers when the market is tight, and vice versa. A measure of market tightness then enters the equations of labour force (or labour force participation). This approach is found in numerous econometric models, for example, in:

- the Wharton Quarterly Econometric Forecasting Model: Mark III (McCarthy, 1972)
- the quarterly econometric model of the U.S. economy by Kuh and Schmalensee (1973)
- Pindyck's model (Pindyck, 1973)
- the RDX2 model for Canada (Bank of Canada, 1977)
- the Australian econometric model (Norton and Henderson, 1973) etc...

Here, both aspects will be taken into consideration, i.e., the wage rate and a measure of the degree of market tightness; furthermore, a linear time trend variable (TIME) is also included to capture the secular influence on the labour force. Two points need to be discussed.

i) Should money wage or real wage rate be used?

On this point, both types of wage rate are tried. Results show that the coefficient for the real wage rate is not significant or has the wrong sign; therefore, real wage is discarded.

ii) How do we measure market tightness?

Usually, it is proxied by the unemployment rate or the ratio of employment (L) over population (POP). A last candidate is employment (L) as found in the Wharton and Australian models. Results point out that the unemployment rate performs poorly (the same conclusion was reached by Kuh and Schmalensee for their model). But L/POP and L alone perform equally well.

Below the results using the last two proxies are exhibited.

<u>Using the ratio L/POP</u>

$$\begin{aligned} LF = {} & \underset{(4.79)}{2280.94} + \underset{(7.26)}{152.537}\ W + \underset{(10.01)}{13485.4}\ (L/POP) \\ & + \underset{(12.57)}{37.152}\ TIME - \underset{(-.76)}{17.609}\ Q1 + \underset{(4.85)}{56.421}\ Q2 \\ & + \underset{(6.16)}{134.06}\ Q3 \end{aligned} \qquad (5.5.4)$$

$R^2 = .999 \quad \bar{R}^2 = .999 \quad F(6,40) = 9076.01$

SER = 29.676 DW = 1.87 RHO = .38 (2.80)

<u>Using L</u>

$$\begin{aligned} LF = {} & \underset{(6.35)}{2581.96} + \underset{(5.98)}{120.027}\ W + \underset{(10.99)}{0.638}\ L \\ & + \underset{(5.86)}{22.029}\ TIME - \underset{(-.42)}{9.223}\ Q1 + \underset{(5.47)}{58.935}\ Q2 \\ & + \underset{(6.18)}{126.560}\ Q3 \end{aligned} \qquad (5.5.5)$$

$R^2 = .999 \quad \bar{R}^2 = .998 \quad F(6,40) = 3887.32$

SER = 27.665 DW = 1.95 RHO = 0.38 (2.81)

Both versions are estimated by CORC; we have not succeeded with the usual approach of eliminating autocorrelation. Both exhibit highly significant coefficients. Statistically, both perform equally well. For simplicity reason we chose (5.5.5).

As said earlier, real wage is also tried. Below is a version of (5.5.5) with money wage replaced by real wage.

$$LF = \underset{(2.89)}{2155.79} - \underset{(-.64)}{51.053}\ \frac{W}{P} + \underset{(8.11)}{0.739}\ L + \underset{(4.57)}{30.407}$$

$$TIME + \underset{(.80)}{24.457}\ Q1 + \underset{(5.47)}{55.649}\ Q2 + \underset{(3.10)}{86.840}\ Q3 \qquad (5.5.6)$$

$R^2 = .999 \quad \bar{R}^2 = .999 \quad F(6,40) = 8582.01$

$SER = 30.518 \quad DW = 2.04 \quad RHO = .81\ (9.45)$

The result is not satisfactory either when W and P enter the equation separately.

c. Unemployment rate (UR)

It is defined as:

$$UR = \frac{(LF - L)}{LF}\ 100 \qquad (5.5.7)$$

d. Wage rate (W)

The equation for nominal wage is:

$$\frac{W - W(-1)}{W(-1)} = f(UR, \pi\ ) \qquad (5.5.8)$$

where π is the expected rate of inflation.

Since π is unobserved, some form of expectation has to be assumed. Three expectation hypotheses are tried.

i) Adaptive expectation: in this case (5.5.8) becomes

$$\frac{W - W(-1)}{W(-1)} = f\Big[UR,\ UR(-1),\ \frac{W(-1) - W(-2)}{W(-2)},$$

$$\frac{P - P(-1)}{P(-1)}\Big] \qquad (5.5.9)$$

The coefficient of π can be recuperated and equal to:

$$\frac{\text{coefficient of } [P - P(-1)]/P(-1)}{1 - \text{coefficient of } [W(-1) - W(-2)]/W(-2)}$$

(Johnston, 1972, pp. 301-302).

ii) π is an Almon distributed lag or iii) general distributed lag of current and past rates of inflation.

A point deserves comment:

Either with Almon or distributed lag,

$$\pi = \sum_{i=0}^{n} w_i (\Delta P_{t-i})/P_{t-i-1} \qquad (5.5.10)$$

To recuperate the coefficient of π, it is necessary to impose the restriction that the sum of w_i equals 1. Such a restriction, as pointed out by Sargent (1971), has no foundation and is utterly misleading. Here it is disregarded.

Adaptive expectation

We start with the basic form (5.5.9):

$$\frac{W - W(-1)}{W(-1)} = \underset{(4.68)}{0.023} + \underset{(.05)}{0.0002}\, UR - \underset{(-.10)}{0.0003}\, UR(-1)$$

$$+ \underset{(4.71)}{0.583} \frac{W(-1) - W(-2)}{W(-2)} + \underset{(2.10)}{0.105} \frac{P - P(-1)}{P(-1)}$$

$$- \underset{(-3.16)}{0.017}\, Q1 - \underset{(-2.70)}{0.013}\, Q2 - \underset{(-5.80)}{0.025}\, Q3 \qquad (5.5.11)$$

$R^2 = .609 \quad \bar{R}^2 = .541 \quad F(7,40) = 8.91$

SER = 0.007 DW = 2.14 D's H = -.92

UR and UR(-1) have the right sign but are not significant. In principle, with the adaptive expectation first-order autocorrelation should appear, but it does not show up here. The coefficient of π is recuperated and equal to .25, which is very low. We then try (5.5.9) with different lags of UR, but the coefficient of π is never greater than .25. In view of these results, we discard the adaptive expectation hypothesis.

Almon lag

As to the Almon lag, an intensive search is made for the appropriate degree of polynomial and the length of lag. The best equation is:

$$\frac{W - W(-1)}{W(-1)} = \underset{(5.46)}{0.023} - \underset{(-1.48)}{0.001}\ UR + \underset{(1.76)}{0.084}\ \frac{P - P(-1)}{P(-1)}$$
$$+ \underset{(5.19)}{0.166}\ \frac{P(-1) - P(-2)}{P(-2)} + \underset{(6.45)}{0.206}\ \frac{P(-2) - P(-3)}{P(-3)}$$
$$+ \underset{(6.06)}{0.203}\ \frac{P(-3) - P(-4)}{P(-4)} + \underset{(3.22)}{0.157}\ \frac{P(-4) - P(-5)}{P(-5)}$$
$$- \underset{(-2.21)}{0.006}\ Q1 - \underset{(-1.99)}{0.006}\ Q2 - \underset{(-5.45)}{0.017}\ Q3 \qquad (5.5.12)$$

$R^2 = .736 \quad \bar{R}^2 = .689 \quad F(7,40) = 15.89$

SER = 0.006 DW = 1.73 Degree = 2 Lag = 4

The inflation coefficients are highly significant with their sum value equal to .81.

The coefficient of UR has the right sign, but its t-statistic is significant only at the 10% level. The low significance can be partly explained by the fact that during the estimation span there were certain periods in which the trade-off between inflation and unemployment did not exist. The value of the UR coefficient should not be taken seriously. The reason is: because data are seasonally unadjusted, the rate of inflation as defined above is not a suitable indicator of a quarter-to-quarter change in price. For that reason we also try (5.5.12) instead with $\frac{W - W(-4)}{W(-4)}$, $\frac{P - P(-4)}{P(-4)}$ etc... It results that the t-statistic of UR increases to 1.66 and its value is .0023 or .23 if wage and price inflation terms are measured in percent. The serious problem is that the equation suffers from high autocorrelation (DW = .56), and there is no procedure to correct autocorrelation in presence of Almon lag with the TSP computer program available at Concordia University.

Equation (5.5.12) is also tried with UR replaced by the non-linear form (1/UR). The non-linear form is more suggested in the literature as the behaviour in the money wage equation should be asymmetrical, i.e., wages rise with excess demand for labour or expected inflation but do not fall at the same rate in the circumstance of even heavy unemployment or expected deflation (see, e.g., Dornbusch, Fischer, and Sparks, 1982, pp. 407-408, also Lipsey, 1981). It is found that the non-linear form performs less satisfactorily than the linear form on all respects ($\bar{R}^2$, DW, t-statistics). For example, the t-statistic of (1/UR) is only 1.25. Theoretically, the non-linear form should be taken in spite of a small decrease in performance; but we have chosen instead the linear form without paying much attention to the asymmetry in the Phillips curve. The importance of the asymmetry feature in a stagflation world came to our mind only when we started to analyze the optimization results.

Unconstrained distributed lag

As the length of the lag in (5.5.12) is short compared to the polynomial degree, there is not much gain from the Almon lag versus the unconstrained distributed lag.

$$\frac{W - W(-1)}{W(-1)} = \underset{(5.40)}{0.023} - \underset{(-1.41)}{0.00092}\, UR + \underset{(1.84)}{0.113}$$

$$\frac{P - P(-1)}{P(-1)} + \underset{(3.75)}{0.161} \frac{P(-1) - P(-2)}{P(-2)}$$

$$+ \underset{(4.30)}{0.185} \frac{P(-2) - P(-3)}{P(-3)} + \underset{(5.31)}{0.229}$$

$$\frac{P(-3) - P(-4)}{P(-4)} + \underset{(2.15)}{0.133} \frac{P(-4) - P(-5)}{P(-5)}$$

$$- \underset{(-2.06)}{0.006}\, Q1 - \underset{(-2.20)}{0.007}\, Q2 - \underset{(5.40)}{0.017}\, Q3 \qquad (5.5.13)$$

$R^2 = .742 \quad \bar{R}^2 = .681 \quad F(9,38) = 12.16$

$SER = 0.0058 \quad DW = 1.68$

Although the gain is small, we pick up the Almon lag equation (5.5.12).

5.5.2 Prices

The price block contains two equations: one for the general price level and one for the export price.

a. Price level (P)

Many studies on price equations have adopted the mark-up theory (Eckstein and Fromm, 1968, Nordhaus, 1972). The theory is particularly designed for pricing in non-competitive markets: it is assumed that price is basically determined by a mark-up on unit normal labour cost (UNLC). In competitive markets, price responds to variations in current unit labour cost (ULC) (see Eckstein and Fromm, ibid, pp. 1162-1163). So, we would expect that in a mixed market both UNLC and ULC affect price. ULC is related to wage rate. UNLC should be too. Indeed, to measure UNLC many empirical works use moving average of past ULC, or it is assumed that UNLC is a function of wage which declines exponentially over time, reflecting the growth in average labour productivity (Eckstein and Fromm, ibid, p. 1169). The immediate implication is that price depends on wage rate (W). Apart from the effect of wage rate, price depends on demand pressure. Most studies use some measure of capacity utilization to capture the imbalance in the product market. Here the unemployment rate is used as a proxy.

So the basic form of the price equation is

$$P = f(W, UR) \qquad (5.5.14)$$

Since UR also appears in the wage equation in which the wage is in rate of change form [i.e., $\frac{W - W(-1)}{W(-1)}$], for consistency, we might be tempted into running the price equation in the same form. In case we want to avoid the non-linear form, the linear form should be in principle derived from the mathematical transformation of the non-linear form if the right functional form is non-linear. The point is whether the non-linear form is the appropriate one.

We begin with the non-linear form in various lag structures. The best equation we can obtain is:

$$\underset{}{\frac{P - P(-1)}{P(-1)}} = \underset{(-3.06)}{-\,0.038} + \underset{(3.43)}{0.880}\,\frac{W - W(-1)}{W(-1)}$$

$$+ \underset{(.55)}{0.0008}\ UR(-1) + \underset{(2.64)}{0.027}\ Q1$$

$$+ \underset{(4.25)}{0.033}\ Q2 + \underset{(5.16)}{0.056}\ Q3 \qquad (5.5.15)$$

$R^2 = .530 \quad \bar{R}^2 = .473 \quad F(5,41) = 9.245$

$SER = 0.019 \quad DW = 2.04 \quad RHO = -.36\ (-2.67)$

As can be seen, the coefficient of UR(-1) is not significant and has the wrong sign. This behaviour is present in all runs with price expressed as percentage change. In terms of R^2 and F-statistic, the above non-linear form performs poorly. So, there is no reason to derive a linear approximation from it. With this in mind we estimate the price equation in level form and obtain:

$$P = \underset{(22.94)}{0.345} + \underset{(49.46)}{0.256}\ W - \underset{(-6.51)}{0.035}\ UR(-1) + \underset{(.37)}{0.003}\ Q1$$

$$+ \underset{(4.74)}{0.057}\ Q2 + \underset{(6.47)}{0.062}\ Q3 \qquad (5.5.16)$$

$R^2 = .996 \quad \bar{R}^2 = .996 \quad F(5,42) = 2139.92$

$SER = 0.022 \quad DW = 1.65$

The coefficients of W and UR(-1) are highly significant with signs correct as expected. Although, $\bar{R}^2$ is extremely high, we are tempted into including the import price which is not found to be significant, and the world inflation rate in the hope that it might capture to some extent the transmission of world inflation. In Turnovsky (1977, Chapter 12) the imported inflation is measured as the sum of the rate of inflation of foreign goods and the rate of depreciation of the foreign exchange PFX. As our price is not in percent, we use (PFX.PWE), and PWE = world export price, proxied by the implicit deflator of U.S. business product. The variable which represents the imported inflation is not significant. We then run the equation with all variables in percentage change except the unemployment rate. The results do not improve at all.

In short, (5.5.17) is our price equation.

b. Price of export (PE)

In general, the empirical formulations as found in the literature for the disaggregated prices are rather ad hoc. The main variable which commonly enters these equations is a general price level. This is a measure of general influences on disaggregated prices, influences such as overall prices, wage rates, prices of imports...here this tradition is followed: the price level (P) will be the principal determinant. Besides, the unemployment rate is included to reflect demand pressure. In certain econometric models, for example, the Wharton (McCarthy, 1972), the TRACE etc...a time trend is added. No explanation is given; it might be that the trend is to capture the increasing trend of the export price.

With these determinants the export price is estimated as follows:

$$\begin{aligned} PE &= \underset{(1.47)}{0.062} + \underset{(1.97)}{0.198}\, P - \underset{(-4.10)}{0.031}\, UR + \underset{(2.78)}{0.004} \\ &\quad TIME + \underset{(12.01)}{0.833}\, PE(-1) + \underset{(5.05)}{0.067}\, Q1 + \underset{(2.65)}{0.025}\, Q2 \\ &\quad + \underset{(1.93)}{0.014}\, Q3 \end{aligned} \qquad (5.5.17)$$

$R^2 = .995 \quad \overline{R}^2 = .994 \quad F(7,39) = 896.05$

$SER = 0.020 \quad DW = 2.01 \quad D's\ H = .04$

$RHO = .41\ (3.08)$

The export price terminates the discussion on the model specification. In the next section we present first a summary of the final OLS equations and then the listing of variables with their definitions.

5.6 LISTING OF MODEL EQUATIONS (OLS)

5.6.1 Product Sector

Total aggregate demand (Y)

$$Y = CON + (IIN + IRCNR) + (G1 + G2) + EX - IM + RES \quad (5.6.1)$$

Real consumption (CON)

$$CON = \underset{(1.66)}{470.430} + \underset{(7.28)}{0.491} \left(Y - \frac{TAX}{P}\right) + \underset{(2.16)}{0.054}$$

$$[V - V(-1)] + \underset{(5.24)}{0.418}\, CON(-1) - \underset{(-7.78)}{2044.82}\, Q1$$

$$- \underset{(-6.79)}{921.389}\, Q2 - \underset{(-15.63)}{2451.97}\, Q3 \quad (5.6.2)$$

$R^2 = .994$ $\bar{R}^2 = .994$ $F(6,41) = 1219.99$

SER = 245.329 DW = 1.88 D's H = .50

Real private investment in inventory (IIN)

$$IIN = - \underset{(-.30)}{9.684} + \underset{(17.66)}{0.514}\, [Y - Y(-1)] + \underset{(2.40)}{0.070}$$

$$[Y(-1) - Y(-2)] - \underset{(-12.73)}{0.554}\, [CON - CON(-1)]$$

$$+ \underset{(6.47)}{0.773}\, IIN(-1) \quad (5.6.3)$$

$R^2 = .926$ $\bar{R}^2 = .920$ $F(4,43) = 135.24$

SER = 194.916 DW = 2.25 D's H = 1.53

Real gross private fixed investment (IRCNR)

$$IRCNR = INR + IRC \quad (5.6.4)$$

Real gross private investment in non-residential construction, and machinery & equipment (INR)

$$INR = -288.612 + 0.082\,Y - 43.591\,[RL(-8) + RL(-9)] + 0.650\,INR(-1) - 10.566\,Q1 + 466.895\,Q2 - 135.484\,Q3 \qquad (5.6.5)$$

t-statistics: (−3.13) (5.47) (−3.59) (9.31) (−.25) (12.20) (−3.69)

$R^2 = .985 \quad \bar{R}^2 = .982 \quad F(6,41) = 438.86$

$SER = 88.632 \quad DW = 2.18 \quad D's\ H = -.72$

Real gross private investment in residential construction (IRC)

$$IRC = -173.253 + 0.030\,Y - 9.169\,[RS(-1) + RS(-2)] + 0.817\,IRC(-1) - 0.250\,IRC(-2) - 113.353\,Q1 - 300.822\,Q2 + 186.537\,Q3 \qquad (5.6.6)$$

t-statistics: (−1.48) (3.12) (−2.06) (5.24) (−1.65) (−1.91) (−3.62) (3.57)

$R^2 = .938 \quad \bar{R}^2 = .927 \quad F(7,40) = 86.58$

SER = 89.667 DW = 1.61 D's H = (test breaks down)

Real export of goods and services (EX)

$$EX = 1092.72 - 1536.17\left[\frac{PE(-3)}{PYBU(-3)\cdot PFX(-3)}\right] + 4289.63\,WIP + 0.300836\,EX(-1) - 582.704\,Q1 + 381.048\,Q2 - 105.445\,Q3 \qquad (5.6.7)$$

t-statistics: (4.04) (−4.17) (5.99) (2.57) (−7.52) (4.13) (−1.03)

$R^2 = .977 \quad \bar{R}^2 = .973 \quad F(6,41) = 285.19$

$SER = 183.322 \quad DW = 2.11 \quad D's\ H = -.68$

Real import of goods and services (IM)

$$IM = -314.515 + 0.354\,Y - 2057.71 \left[\frac{PIMA(-3)\,.\,PFX(-3)}{P(-3)}\right] + 942.152\,P(-2) - 219.716\,UR + 503.237\,Q1 + 492.552\,Q2 - 871.970\,Q3 \qquad (5.6.8)$$

t-statistics: (−.24) (13.57) (−2.01) (2.64) (−4.64) (4.46) (5.74) (−10.79)

$R^2 = .987$ $\bar{R}^2 = .985$ $F(7,40) = 432.57$

$SER = 188.611$ $DW = 1.63$

Nominal taxes, net of subsidies and transfers, from federal, provincial, and local governments (TAX)

$$TAX = 1247.46 + 0.174\,P\,.\,Y + 169.868\,Q1 - 622.143\,Q2 - 545.225\,Q3 \qquad (5.6.9)$$

t-statistics: (3.99) (21.81) (.98) (−3.33) (−3.26)

$R^2 = .923$ $\bar{R}^2 = .915$ $F(4,42) = 119.37$

$SER = 484.313$ $DW = 2.06$ $RHO = .36\ (2.68)$

Nominal total government revenues (TR)

$$TR = TAX + OREV \qquad (5.6.10)$$

5.6.2 Money Sector

Currency (CUR)

$$\text{Log}\,\frac{CUR}{P} = -0.120 + 0.014\,QMAIL74 + 0.037\,QMAIL75 + 0.867\,\text{Log}\,Y - 0.481\,\text{Log}\,Y(-1) - 0.007\,RS$$

t-statistics: (−.84) (.93) (2.60) (6.38) (−3.16) (−3.69)

$$+ \underset{(2.17)}{0.003}\ RSU + \underset{(5.22)}{0.547}\ Log\ \frac{CUR(-1)}{P(-1)} - \underset{(-.32)}{0.003}\ Q1$$

$$- \underset{(-3.97)}{0.064}\ Q2 - \underset{(-5.66)}{0.086}\ Q3 \qquad (5.6.11)$$

$R^2 = .992 \quad \bar{R}^2 = .990 \quad F(10,37) = 480.49$

$SER = 0.013 \quad DW = 2.18 \quad D's\ H = .85$

Demand deposits (DD)

$$\frac{CUR + DD - M1}{P} = 0 \qquad (5.6.12)$$

Time deposits (DSN)

$$\frac{DSN}{P} = - \underset{(-3.32)}{6448.18} + \underset{(1.67)}{892.036}\ QBANK + \underset{(3.00)}{0.271}\ Y$$

$$+ \underset{(1.73)}{135.015}\ RTD(-1) + \underset{(7.36)}{0.361}\ V - \underset{(-4.92)}{0.282}\ V(-1)$$

$$+ \underset{(12.30)}{0.781}\ \frac{DSN(-1)}{P(-1)} + \underset{(2.44)}{729.339}\ Q1 + \underset{(3.76)}{950.036}\ Q2$$

$$+ \underset{(.01)}{3.906}\ Q3 \qquad (5.6.13)$$

$R^2 = .997 \quad \bar{R}^2 = .996 \quad F(9,38) = 1281.96$

$SER = 473.039 \quad DW = 1.82 \quad D's\ H = .70$

Short-term interest rate (RS)

$$RS = \underset{(2.09)}{3.278} + \underset{(2.50)}{1.374}\ QMAIL74 + \underset{(3.76)}{1.905}\ QMAIL75$$

$$- \underset{(-3.08)}{1.501}\ QPFX + \underset{(2.64)}{1.390}\ QMR + \underset{(7.57)}{0.614}\ RSU$$

$$- \underset{(-5.52)}{0.533}\ RSU(-1) - \underset{(-2.78)}{26.873}\ \frac{M1}{P.Y} + \underset{(2.67)}{23.443}$$

$$\frac{M1(-1)}{P(-1)\ .\ Y(-1)} + \underset{(10.12)}{0.773}\ RS(-1) - \underset{(-2.07)}{0.511}\ Q1$$

$$- \underset{(-2.55)}{1.296}\ Q2 - \underset{(-2.72)}{1.389}\ Q3 \qquad (5.6.14)$$

$R^2 = .955 \quad \bar{R}^2 = .939 \quad F(12,35) = 61.62$

$SER = 0.454 \quad DW = 1.65 \quad D's\ H = 1.42$

Demand for chartered bank reserves (RR)

$$RR = -\underset{(-1.64)}{60.305} + \underset{(8.26)}{0.077}\,(DG + DD) + \underset{(4.14)}{0.015}\,DSN$$
$$+ \underset{(7.91)}{0.552}\,RR(-1) + \underset{(3.87)}{68.054}\,Q1 + \underset{(1.30)}{24.717}\,Q2$$
$$+ \underset{(1.95)}{35.900}\,Q3 \qquad (5.6.15)$$

$R^2 = .999 \quad \bar{R}^2 = .999 \quad F(6,41) = 6692.92$

$SER = 42.665 \quad DW = 1.95 \quad D's\ H = .18$

Broad money (M2)

$$M2 = M1 + DSN \qquad (5.6.16)$$

Monetary base (MB)

$$MB = CUR + RR \qquad (5.6.17)$$

Real financial wealth (V)

$$V = (MB + BOND) / P \qquad (5.6.18)$$

Long-term interest rate (RL)

$$RL = \underset{(.57)}{0.147} + \underset{(3.37)}{0.148}\,RS - \underset{(-2.14)}{0.099}\,RS(-1)$$
$$+ \underset{(4.90)}{0.729}\,RLU - \underset{(-3.14)}{0.575}\,RLU(-1) + \underset{(7.96)}{0.801}$$
$$RL(-1) + \underset{(.65)}{0.062}\,Q1 + \underset{(2.36)}{0.219}\,Q2 + \underset{(1.47)}{0.133}\,Q3 \qquad (5.6.19)$$

$R^2 = .967 \quad \bar{R}^2 = .961 \quad F(8,39) = 143.99$

$SER = 0.220 \quad DW = 1.94 \quad D's\ H = .26$

Interest rate on time deposits (RTD)

$$RTD = -\underset{(-1.85)}{0.312} + \underset{(2.84)}{0.776}\ QBANK + \underset{(13.85)}{0.558}\ RS$$

$$-\underset{(-4.09)}{0.259}\ RS(-1) + \underset{(11.96)}{0.705}\ RTD(-1) \qquad (5.6.20)$$

$R^2 = .976 \quad \bar{R}^2 = .973 \quad F(4,43) = 428.98$

$SER = 0.259 \quad DW = 2.08 \quad D's\ H = -.30$

5.6.3 Government Budget Constraint

$$MB - MB(-1) + BOND - BOND(-1) + OTHER = P(G1 + G1R) - TR \qquad (5.6.21)$$

5.6.4 Foreign Exchange Rate (PFX)

$$Log\ PFX = -\underset{(-.77)}{0.274133} + \underset{(4.43)}{0.186332}\ Log\ \frac{M2}{MU} - \underset{(-1.82)}{0.201328}$$

$$Log\ \frac{Y}{YU} - \underset{(-3.59)}{0.0083396}\ (RS - RSU) + \underset{(7.80)}{0.743879}$$

$$Log\ PFX(-1) - \underset{(-1.76)}{0.0152353}\ Q1 - \underset{(-1.92)}{0.00977621}\ Q2$$

$$-\underset{(-.02)}{0.000112176}\ Q3 \qquad (5.6.22)$$

$R^2 = .939 \quad \bar{R}^2 = .922 \quad F(7,25) = 46.42$

$SER = 0.010 \quad DW = 1.77 \quad D's\ H = .78$

$RHO = .333225\ (2.03)$

5.6.5 Labour Sector and Prices

Employment (L)

$$L = -\underset{(-4.21)}{475.171} + \underset{(2.17)}{0.090}\ \frac{Y\ .\ P}{W} + \underset{(32.06)}{0.947}\ L(-1)$$

$$+ \underset{(.23)}{6.171}\ Q1 + \underset{(23.65)}{653.174}\ Q2 + \underset{(13.18)}{520.741}\ Q3 \qquad (5.6.23)$$

$R^2 = .997 \quad \bar{R}^2 = .996 \quad F(5,42) = 2374.19$

$SER = 55.367 \quad DW = 1.89 \quad D's\ H = .38$

Labour force (LF)

$$LF = \underset{(6.35)}{2581.96} + \underset{(5.98)}{120.027}\ W + \underset{(10.99)}{0.638}\ L + \underset{(5.86)}{22.029}\ TIME$$

$$- \underset{(-.42)}{9.223}\ Q1 + \underset{(5.47)}{58.935}\ Q2 + \underset{(6.18)}{126.560}\ Q3 \qquad (5.6.24)$$

$R^2 = .999 \quad \bar{R}^2 = .998 \quad F(6,40) = 3887.32$

$SER = 27.665 \quad DW = 1.95 \quad D's\ H = .20$

$RHO = 0.38\ (2.81)$

Unemployment rate (UR)

$$UR = \frac{(LF - L)}{LF} \cdot 100 \qquad (5.6.25)$$

Wage (W)

$$\frac{W - W(-1)}{W(-1)} = \underset{(5.46)}{0.023} - \underset{(-1.48)}{0.001}\ UR + \underset{(1.76)}{0.084}\ \frac{P - P(-1)}{P(-1)}$$

$$+ \underset{(5.19)}{0.166}\ \frac{P(-1) - P(-2)}{P(-2)} + \underset{(6.45)}{0.206}\ \frac{P(-2) - P(-3)}{P(-3)}$$

$$+ \underset{(6.06)}{0.203}\ \frac{P(-3) - P(-4)}{P(-4)} + \underset{(3.22)}{0.157}\ \frac{P(-4) - P(-5)}{P(-5)}$$

$$- \underset{(-2.21)}{0.006}\ Q1 - \underset{(-1.99)}{0.006}\ Q2 - \underset{(-5.45)}{0.017}\ Q3 \qquad (5.6.26)$$

$R^2 = .736 \quad \bar{R}^2 = .689 \quad F(7,40) = 15.89$

$SER = 0.006 \quad DW = 1.73$

Price level (P)

$$P = \underset{(22.94)}{0.345} + \underset{(49.46)}{0.256}\ W - \underset{(-6.51)}{0.035}\ UR(-1) + \underset{(.37)}{0.003}\ Q1$$

$$+ \underset{(4.74)}{0.057}\ Q2 + \underset{(6.47)}{0.062}\ Q3 \qquad (5.6.27)$$

R^2 = .996 $\bar{R}^2$ = .996 F(5,42) = 2139.92

SER = 0.022 DW = 1.65

Price of export (PE)

$$PE = \underset{(1.47)}{0.062} + \underset{(1.97)}{0.198}\ P - \underset{(-4.10)}{0.031}\ UR + \underset{(2.78)}{0.004}\ TIME$$

$$+ \underset{(12.01)}{0.833}\ PE(-1) + \underset{(5.05)}{0.067}\ Q1 + \underset{(2.65)}{0.025}\ Q2$$

$$+ \underset{(1.93)}{0.014}\ Q3 \qquad (5.6.28)$$

R^2 = .995 $\bar{R}^2$ = .994 F(7,39) = 896.05

SER = 0.020 DW = 2.01 D's H = .04

RHO = .41 (3.08)

5.7 LISTING OF MODEL VARIABLES WITH DEFINITIONS

5.7.1 Endogenous Variables

BOND = federal, provincial, and local government debt (stock) held by the private sector, millions of current dollars

CON = real consumption (total personal expenditure on goods and services), millions of 1971 dollars

CUR = currency, millions of current dollars

DD = demand deposits, millions of current dollars

DSN = time deposits, millions of current dollars

EX = real export of goods and services, millions of 1971 dollars

IM = real import of goods and services, millions of 1971 dollars

IIN = change in business inventories, millions of 1971 dollars

INR = real gross private investment in non-residential construction, and machinery & equipment, millions of 1971 dollars

IRC = real gross private investment in residential construction, millions of 1971 dollars

IRCNR = real gross private fixed investment, millions of 1971 dollars

L = employment, thousands of persons

LF = labour force, thousands of persons

M2 = broad money quantity, millions of current dollars

MB = monetary base, millions of current dollars

P = price level (implicit GNE price deflator), 1971 = 1

PE = implicit price deflator, export of goods and services, 1971 = 1

PFX = spot exchange rate: number of Canadian dollars per U.S. dollar[50]

RL = long-term interest rate (Government of Canada bond yield averages, 10 years and over), in percent

RR = chartered bank reserves, millions of current dollars

RS = short-term interest rate (90-day finance company paper rate), in percent

RTD = interest rate on time deposits, in percent

TAX = taxes, net of subsidies and transfers, from federal, provincial, and local governments, millions of current dollars

TR = total revenues of federal, provincial, and local governments, millions of current dollars

UR = unemployment rate, in percent

V = real financial wealth, millions of 1971 dollars

W = wage rate (= total manufacturing average hourly earning)

Y = real gross national expenditure (GNE) or GNP, millions of 1971 dollars.

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[50] Exogenous under the fixed exchange regime

5.7.2 Exogenous Variables[51]

DG = Government of Canada deposits at chartered banks, millions of current dollars

G1 = real expenditure of federal, provincial, and local governments (current expenditure on goods and services + gross capital formation), millions of 1971 dollars.

G1R = real other expenditure of federal, provincial, and local governments, millions of 1971 dollars

G2 = real expenditure of Hospitals, Quebec Pension Plan, and Canada Pension Plan (current expenditure on goods and services + gross capital formation), millions of 1971 dollars

M1 = narrow money quantity, millions of current dollars

MU = U.S. broad money quantity, millions of current dollars

OREV = other revenues of federal, provincial, and local governments, millions of current dollars

OTHER = balancing item in the government budget constraint, millions of current dollars

PIMA = implicit price deflator, import of goods and services (expressed in U.S. currency)

PYBU = implicit price deflator, U.S. business product, 1971 = 1 (rebased)

Q1 = seasonal dummy, 1 in quarter I, 0 elsewhere

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51 First letter 'Q' denotes dummy variables

Q2 = seasonal dummy, 1 in quarter II, 0 elsewhere

Q3 = seasonal dummy, 1 in quarter III, 0 elsewhere

QBANK = dummy variable to reflect the introduction of the Bank Act Revision in 1967 III, removing interest rate ceiling, 1 in 1967 III, 0 elsewhere

QMAIL74 = dummy variable to capture the postal strike in 1974 II, 1 in 1974 II, 0 elsewhere

QMAIL75 = dummy variable to capture the postal strike in 1975 IV, 1 in 1975 IV, 0 elsewhere

QMR = dummy variable to capture the lagged effect of the Bank of Canada measures in restraining the monetary growth (increase of the bank rate in August 6 and September 12, 1973), 1 in 1973 IV, 0 elsewhere

QPFX = dummy variable to capture the effect of the Bank of Canada measures in moderating the exchange rate appreciation right after its floating (for example, intervention in the exchange market, reduction of the bank rate, exhortations to borrowers to obtain their funds in the domestic market), 1 in 1971 II, 0 elsewhere

RES = residual error of estimate in the income equation, millions of 1971 dollars

RLU = U.S. Government long-term bond yield, in percent

RUS = U.S. short-term interest rate (90-day short-term paper), in percent

TIME = time trend (=1, 2, 3...first period 1967 I)

WIP = index of world industrial production, 1971 = 1

YU = real U.S. GNP, millions of 1971 dollars (rebased)

CHAPTER 6

ESTIMATION OF THE MODEL BY TWO-STAGE LEAST SQUARES

6.1 Introduction

The model has more predetermined variables (58) than observations (48). That is the case of undersized samples. To circumvent this the model is divided into small modules, and two-stage least squares (2SLS) limited to them.[52]

Originally, the model was partitioned into six modules.

The product sector was broken down into the product and foreign trade modules. The product module was constituted by all equations in the product sector except export and import which formed the foreign trade module. The idea of a separate module for export and import stems from the belief that it would more truly reflect the traditional Keynesian view on the product sector which leaves out export and import.

The monetary module consisted of all money sector equations and the government budget constraint identity.

The foreign exchange rate was formed by the unique equation of foreign exchange rate.

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52 This amounts to a regular instrumental variables technique. One alternative of selecting a subset of predetermined variables from the original set is the structural ordering method suggested by Fisher (1965).

The labour and price block was divided into two modules: the labour module with equations on employment, labour supply, unemployment, wage and the price module with two price equations (general price level and export price).

What we have found is that inside the labour module, the employment equation was unsuccessfully estimated. This forced us to combine the labour module with the product module. As the labour module becomes part of the product module, there remains no ground to have a separate module for import and export equations which are more related to the product sector than labour equations. In other words, the product module will now include import and export.

With respect to the foreign exchange module, the only equation which exists is the foreign exchange rate equation, and there won't be any attempt to estimate it by 2SLS. The reasons are:

i) lack of data (the starting date is 1970 III instead of 1967 I as for other variables)

ii) autocorrelation appeared in the OLS run.

If it is part of another module (in this case, the proper one is the money module), 2SLS with correction for autocorrelation (Fair, 1970) will leave just a few degrees of freedom. Even when the foreign exchange rate equation is the only one in the module, there are difficulties in dealing with consistency, autocorrelation in the whole context of simultaneous equations. The lack of data is also observed in the TRACE model in which the foreign exchange was estimated by OLS rather than by 2SLS.

In short, the final partition is as follows:

i) The product module: all equations from the product sector and equations from labour and prices block except the two price equations.

Inside this module the export equation is not estimated by 2SLS since all explanatory variables in it are predetermined.

ii) The money module: all equations from the money sector and the government budget constraint identity.

iii) The foreign exchange module: the foreign exchange rate equation which is estimated by OLS.

iv) The price module: general price and export price equations.

Before we determine the sets of predetermined variables entering each module (except the foreign exchange), some treatments require particular attention.

i) Any function of endogenous and predetermined variables which appear in the right hand side of the equation is treated as an endogenous variable. This is equivalent to adding a new (latent) equation in the model for this variable; further, those predetermined variables will be also among the set of predetermined variables used at the first stage of 2SLS.

Predetermined variables which appear in non-linear form will be treated as a single predetermined variable; therefore, individual predetermined variables are not among the set of predetermined variables unless they appear elsewhere in the module.

These treatments are in the same line as those found in a study for Canada by Nehlawi (1975, pp. 10 and 16).

ii) The wage equation used Almon lag. A procedure to combine distributed lag and 2SLS, and at the same time to save the degrees of freedom was suggested by Klein (1973).

Suppose the distributed lag function is:

$$y_{jt} = a + \sum_{i=0}^{n} b_i z_{j,t-i}$$

where y_{jt} is any endogenous variable, and z_{jt} can be endogenous or exogenous.

Treating each lagged z_j as an individual predetermined variable will reduce the number of degrees of freedom. It is a problem in an undersized sample. Note that:

$$\sum_{i=0}^{n} b_i z_{j,t-i} = b_0 \, z_{jt} + b_0 \sum_{i=1}^{n} \frac{b_i}{b_0} z_{j,t-i}$$

Form a single predetermined variable for

$\sum_{i=1}^{n} \frac{b_i}{b_0} z_{j,t-i}$. Call it zz_{jt}.

Assume the relative size of the lag coefficients, as established by the Almon distributed lag, is correct. The 2SLS consists of running the following equation:

$$y_{jt} = a^{2SLS} + b_0^{2SLS} z_{jt} + b_0^{2SLS} zz_{jt}$$

Note that z_{jt} and zz_{jt}' are constrained to have the same coefficient. The b_i^{2SLS} (i=1,...,n) can be recuperated as follows:

$$b_i^{2SLS} = b_0^{2SLS} \cdot \frac{b_i^{OLS}}{b_0^{OLS}}$$

The procedure described above will apply to the wage equation (5.6.26) where zz is now defined as:

$$zz = b_1 \frac{P(-1) - P(-2)}{P(-2)} + b_2 \frac{P(-2) - P(-3)}{P(-3)} + b_3 \frac{P(-3) - P(-4)}{P(-4)}$$

$$+ b_4 \frac{P(-4) - P(-5)}{P(-5)}$$

b_1, b_2, b_3 and b_4 are OLS estimates. With this procedure the 2SLS t-statistics for the b_i (i=1, 2, 3, 4) will not be provided.

The following are the sets of predetermined variables to be used in the first stage of 2SLS.

i) Product module:

constant, Q1, Q2, Q3, G1, G2, RES, V(-1), CON(-1), Y(-1), Y(-2), IIN(-1), RS(-1), RS(-2), IRC(-1), IRC(-2), RL(-8), RL(-9), INR(-1), OREV, W(-1), L(-1), TIME, P(-1), zz, WIP, $\frac{PE(-3)}{PYBU(-3).PFX(-3)}$, EX(-1), $\frac{PIMA(-3).PFX(-3)}{P(-3)}$, P(-2)

ii) Money module:

constant, Q1, Q2, Q3, QMAIL74, QMAIL75, QPFX, QMR, QBANK, M1, Log $\frac{CUR(-1)}{P(-1)}$, RTD(-1), V(-1), $\frac{DSN(-1)}{P(-1)}$, Log Y(-1), DG, RR(-1), MB(-1), BOND(-1), OTHER, G1, G1R, RSU, RSU(-1),

$\frac{M1(-1)}{P(-1).Y(-1)}$, RS(-1), RL(-1), RLU, RLU(-1)

iii) Price module:

constant, Q1, Q2, Q3, P(-1), TIME, PE(-1), UR(-1).

The next section gives a summary of all equations estimated by 2SLS. Please note that in 2SLS neither DW or H-statistic is reliable; it is therefore decided to report only the DW value to allow a comparison with its OLS value and supplement the autocorrelation checking by Geary sign test and residual plot examination.

At a quick glance, we can recognize in general the 2SLS estimates are quite close to the OLS estimates.

6.2 LISTING OF MODEL EQUATIONS (2SLS)

6.2.1 Product Sector

Total aggregate demand (Y)

$$Y = CON + (IIN + IRCNR) + (G1 + G2) + EX - IM + RES \quad (6.2.1)$$

Real consumption (CON)

$$CON = \underset{(1.52)}{438.718} + \underset{(7.13)}{0.503211}\left(\frac{Y-TAX}{P}\right) + \underset{(1.95)}{0.0513596}[V - V(-1)] + \underset{(4.84)}{0.40375}\,CON(-1) - \underset{(-7.33)}{2007.42}\,Q1 - \underset{(-6.71)}{935.106}\,Q2 - \underset{(-15.20)}{2469.77}\,Q3 \quad (6.2.2)$$

$R^2 = .994$ $\bar{R}^2 = .994$ $F(6,41) = 1218.88$

SER = 245.439 DW = 1.85

Real private investment in inventory (IIN)

$$IIN = -\underset{(-.22)}{7.11855} + \underset{(17.38)}{0.509377}[Y - Y(-1)] + \underset{(2.50)}{0.073485}[Y(-1) - Y(-2)] - \underset{(-12.45)}{0.546111}\,CON(-1) + \underset{(6.26)}{0.753653}\,IIN(-1) \quad (6.2.3)$$

$R^2 = .926$ $\bar{R}^2 = .919$ $F(4,43) = 135.12$

SER = 194.993 DW = 2.22

Real gross private fixed investment (IRCNR)

$$IRCNR = INR + IRC \quad (6.2.4)$$

Real gross private investment in non-residential construction and machinery & equipment (INR)

$$INR = -\underset{(-3.01)}{278.494} + \underset{(5.17)}{0.0783787}\ Y - \underset{(-3.40)}{41.4999}\ [RL(-8)$$

$$+ RL(-9)] + \underset{(9.43)}{0.664609}\ INR(-1) - \underset{(-.37)}{15.8799}\ Q1$$

$$+ \underset{(12.24)}{469.150}\ Q2 - \underset{(-3.67)}{134.799}\ Q3 \qquad (6.2.5)$$

$R^2 = .985 \quad \bar{R}^2 = .982 \quad F(6,41) = 438.25$

$SER = 88.692 \quad DW = 2.22$

Real gross private investment in residential construction (IRC)

$$IRC = -\underset{(-1.43)}{167.947} + \underset{(3.04)}{0.0293904}\ Y - \underset{(-2.05)}{9.12731}\ [RS(-1)$$

$$+ RS(-2)] + \underset{(5.26)}{0.82157}\ IRC(-1) - \underset{(-1.62)}{0.24619}\ IRC(-2)$$

$$- \underset{(-1.93)}{114.799}\ Q1 + \underset{(3.63)}{302.066}\ Q2 + \underset{(3.61)}{188.932}\ Q3 \qquad (6.2.6)$$

$R^2 = .938 \quad \bar{R}^2 = .927 \quad F(7,40) = 86.57$

$SER = 89.673 \quad DW = 1.63$

Real export of goods and services (EX)

See OLS equation (5.6.7) (6.2.7)

Real import of goods and services (IM)

$$IM = -\underset{(-.32)}{421.762} + \underset{(13.59)}{0.357}\ Y - \underset{(-1.93)}{1990.46}\ \left[\frac{PIMA(-3)\ .\ PFX(-3)}{P(-3)}\right]$$

$$+ \underset{(2.56)}{918.216}\ P(-2) - \underset{(-4.61)}{222.897}\ UR + \underset{(4.52)}{514.521}\ Q1$$

$$+ \underset{(5.78)}{497.151}\ Q2 - \underset{(-10.82)}{875.104}\ Q3 \qquad (6.2.8)$$

$R^2 = .987 \quad \bar{R}^2 = .985 \quad F(7,40) = 432.35$

$SER = 188.659 \quad DW = 1.64$

Nominal taxes, net of subsidies and transfers, from federal, provincial, and local governments (TAX)

$$TAX = \underset{(3.59)}{1120.01} + \underset{(22.08)}{0.176725}\ Y \cdot P + \underset{(.78)}{135.678}\ Q1$$

$$- \underset{(-3.27)}{618.690}\ Q2 - \underset{(-3.19)}{541.094}\ Q3 \qquad (6.2.9)$$

$R^2 = .990 \quad \bar{R}^2 = .989 \quad F(4,43) = 954.64$

$SER = 491.958 \quad DW = 2.04 \quad RHO = .370597\ (2.59)$

Nominal total government revenues (TR)

$$TR = TAX + OREV \qquad (6.2.10)$$

6.2.2 Money Sector

Currency (CUR)

$$\text{Log}\frac{CUR}{P} = - \underset{(-.64)}{0.0920447} + \underset{(.85)}{0.0127169}\ QMAIL74 + \underset{(2.54)}{0.0363002}$$

$$QMAIL75 + \underset{(5.26)}{0.835556}\ \text{Log}\ Y - \underset{(-2.71)}{0.466514}\ \text{Log}\ Y(-1)$$

$$- \underset{(-3.21)}{0.00655085}\ RS + \underset{(1.86)}{0.00306592}\ RSU(-1) + \underset{(5.33)}{0.564712}$$

$$\text{Log}\ \frac{CUR(-1)}{P(-1)} - \underset{(-.51)}{0.00489635}\ Q1 - \underset{(-3.49)}{0.0626798}\ Q2$$

$$- \underset{(-4.82)}{0.0839932}\ Q3 \qquad (6.2.11)$$

$R^2 = .992 \quad \bar{R}^2 = .990 \quad F(10,37) = 478.33$

$SER = 0.013 \quad DW = 2.23$

Demand Deposits (DD)

$$\frac{CUR + DD - M1}{P} = 0 \qquad (6.2.12)$$

Time Deposits (DSN)

$$\frac{DSN}{P} = \underset{(-3.05)}{-\,6019.61} + \underset{(1.75)}{941.564}\ QBANK + \underset{(2.73)}{0.252526}\ Y$$

$$+ \underset{(1.69)}{132.510}\ RTD(-1) + \underset{(6.31)}{0.332169}\ V - \underset{(-4.31)}{0.25784}\ V(-1)$$

$$+ \underset{(12.26)}{0.79553}\ \frac{DSN(-1)}{P(-1)} + \underset{(2.00)}{617.965}\ Q1 + \underset{(3.28)}{855.739}\ Q2$$

$$- \underset{(-.33)}{101.545}\ Q3 \qquad (6.2.13)$$

$R^2 = .997 \quad \bar{R}^2 = .996 \quad F(9,38) = 1269.69$

$SER = 475.312 \quad DW = 1.83$

Short-term interest rate (RS)

$$RS = \underset{(1.68)}{2.86784} + \underset{(2.31)}{1.30099}\ QMAIL74 + \underset{(3.55)}{1.84103}\ QMAIL75$$

$$- \underset{(-3.06)}{1.49347}\ QPFX + \underset{(2.71)}{1.45527}\ QMR + \underset{(7.47)}{0.626308}\ RSU$$

$$- \underset{(-5.49)}{0.548094}\ RSU(-1) - \underset{(-1.96)}{22.9064}\ \frac{M1}{P \cdot Y} + \underset{(1.91)}{19.9871}\ \frac{M1(-1)}{P(-1) \cdot Y(-1)}$$

$$+ \underset{(9.75)}{0.788794}\ RS(-1) - \underset{(-1.78)}{0.463149}\ Q1 - \underset{(-1.88)}{1.11214}\ Q2$$

$$- \underset{(-2.00)}{1.19814}\ Q3 \qquad (6.2.14)$$

$R^2 = .955 \quad \bar{R}^2 = .939 \quad F(12,35) = 61.31$

$SER = 0.455 \quad DW = 1.67$

Demand for chartered bank reserves (RR)

$$RR = -\underset{(-1.77)}{66.0438} + \underset{(8.18)}{0.0763737}\,(DG + DD) + \underset{(3.85)}{0.0140307}\,DSN$$

$$+ \underset{(8.02)}{0.567436}\,RR(-1) + \underset{(3.86)}{67.9104}\,Q1 + \underset{(1.25)}{23.6396}\,Q2$$

$$+ \underset{(1.93)}{35.4816}\,Q3 \qquad (6.2.15)$$

$R^2 = .999$ $\bar{R}^2 = .999$ $F(6,41) = 6684.15$

$SER = 42.693$ $DW = 1.99$

Broad money (M2)

$$M2 = M1 + DSN \qquad (6.2.16)$$

Monetary base (MB)

$$MB = CUR + RR \qquad (6.2.17)$$

Real financial wealth (V)

$$V = (MB + BOND) / P \qquad (6.2.18)$$

Long-term interest rate (RL)

$$RL = \underset{(.57)}{0.145221} + \underset{(3.05)}{0.143122}\,RS - \underset{(-1.94)}{0.0947432}\,RS(-1)$$

$$+ \underset{(4.85)}{0.737025}\,RLU - \underset{(-3.15)}{0.57949}\,RLU(-1) + \underset{(7.91)}{0.798745}\,RL(-1)$$

$$+ \underset{(.61)}{0.0585409}\,Q1 + \underset{(2.36)}{0.218796}\,Q2 + \underset{(1.48)}{0.133442}\,Q3 \qquad (6.2.19)$$

$R^2 = .967$ $\bar{R}^2 = .961$ $F(8,39) = 143.95$

$SER = 0.220$ $DW = 1.94$

Interest rate on time deposits (RTD)

$$RTD = -\underset{(-1.87)}{0.315097} + \underset{(2.83)}{0.775986}\ QBANK + \underset{(13.32)}{0.561479}\ RS$$

$$- \underset{(-4.08)}{0.262824}\ RS(-1) + \underset{(11.96)}{0.70516}\ RTD(-1) \qquad (6.2.20)$$

$R^2 = .976 \quad \bar{R}^2 = .973 \quad F(4,43) = 428.88$

$SER = 0.259 \quad DW = 2.09$

6.2.3 Government Budget Constraint

$$MB - MB(-1) + BOND - BOND(-1) + OTHER$$

$$= P(G1 + G1R) - TR \qquad (6.2.21)$$

6.2.4 Foreign Exchange Rate (PFX)

See OLS Equation (5.6.22) (6.2.22)

6.2.5 Labour Sector and prices

Employment (L)

$$L = -\underset{(-4.01)}{462.762} + \underset{(1.88)}{0.0822046}\ \frac{Y}{W} \cdot P + \underset{(30.60)}{0.952226}\ L(-1)$$

$$+ \underset{(.14)}{3.8660}\ Q1 + \underset{(23.36)}{655.680}\ Q2 + \underset{(12.81)}{526.496}\ Q3 \qquad (6.2.23)$$

$R^2 = .997 \quad \bar{R}^2 = .996 \quad F(5,42) = 2372.38$

$SER = 55.388 \quad DW = 1.90$

Labour force (LF)

$$LF = \underset{(6.91)}{2709.73} + \underset{(6.77)}{122.926}\ W + \underset{(11.03)}{0.619568}\ L + \underset{(6.92)}{22.8235}\ TIME$$

$$- \underset{(-.71)}{14.9126}\ Q1 + \underset{(5.55)}{59.741}\ Q2 + \underset{(6.63)}{132.469}\ Q3 \qquad (6.2.24)$$

$R^2 = 1 \quad \bar{R}^2 = 1 \quad F(6,41) = 349489$

$SER = 27.376 \quad DW = 1.94 \quad RHO = .347387\ (2.38)$

Unemployment rate (UR)

$$UR = \frac{(LF - L)}{LF} \cdot 100 \qquad (6.2.25)$$

Wage (W)

$$\frac{W - W(-1)}{W(-1)} = \underset{(5.97)}{0.022385} - \underset{(-1.42)}{0.00088038}\ UR + \underset{(8.35)}{0.0834713}$$

$$\frac{P - P(-1)}{P(-1)} + 0.164358803\ \frac{P(-1) - P(-2)}{P(-2)}$$

$$+ 0.20334729\ \frac{P(-2) - P(-3)}{P(-3)} + 0.200435176\ \frac{P(-3) - P(-4)}{P(-4)}$$

$$+ 0.155623451\ \frac{P(-4) - P(-5)}{P(-5)} - \underset{(-2.54)}{0.00602508}\ Q1$$

$$- \underset{(-2.53)}{0.00581121}\ Q2 - \underset{(-7.24)}{0.0166822}\ Q3 \qquad (6.2.26)$$

$R^2 = .735 \quad \bar{R}^2 = .704 \quad F(5,42) = 23.35$

$SER = 0.006 \quad DW = 1.73$

Price Level (P)

$$P = \underset{(22.95)}{0.345404} + \underset{(49.22)}{0.256395}\ W - \underset{(-6.58)}{0.035748}\ UR(-1)$$

$$+ \underset{(.38)}{0.00340466}\ Q1 + \underset{(4.80)}{0.0575959}\ Q2 + \underset{(6.50)}{0.0620362}\ Q3 \qquad (6.2.27)$$

$R^2 = .996 \quad \bar{R}^2 = .996 \quad F(5,42) = 2139.19$

$SER = 0.022 \quad DW = 1.66$

Price of export (PE)

$$PE = \underset{(2.01)}{0.087884} + \underset{(2.14)}{0.217501}\ P(-1) - \underset{(-4.03)}{0.0401839}\ UR$$

$$+ \underset{(3.42)}{0.00456144}\ TIME + \underset{(11.82)}{0.822845}\ PE(-1) + \underset{(4.93)}{0.0805271}\ Q1$$

$$+ \underset{(2.91)}{0.029631}\ Q2 + \underset{(1.83)}{0.0134186}\ Q3 \qquad (6.2.28)$$

$R^2 = .999$ $\quad \bar{R}^2 = .999$ $\quad F(7,40) = 12052.5$

SER = 0.020 $\quad$ DW = 1.99 $\quad$ RHO = .366096 (2.48)

CHAPTER 7

VALIDATION OF THE MODEL

The purpose of the model validation is to evaluate the performance so that optimal control experiments can be conducted with some confidence. Since the main objective of the model is for policy evaluation which also involves comparison with past economic policy, the model should be validated for certain characteristics: first, we perform historical simulation to see how it represents the real economy; second, we take a look at some dynamic properties concerning the long-run crowding-out and neutrality of money. To analyze dynamic responses, we will compute multipliers for fiscal and monetary policies, a tool commonly used to evaluate policy effectiveness.

This chapter has two sections. Section 7.1 deals with the historical simulation analysis. Section 7.2 studies some dynamic properties of the model.

7.1 HISTORICAL SIMULATION ANALYSIS

As a step in validating a model, dynamic simulations[53] over the sample period are necessary to forecheck its ability to reproduce the historical path of the economy. Due to data limitations no simulation outside the sample is experimented;

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53 By 'dynamic', we mean a process in which the values of lagged endogenous variables are generated by the simulation itself.

this would be acceptable as forecasting is not the purpose of the model design.

Two simulation periods are chosen:

i) from 1972 I to 1978 IV

ii) from 1975 III to 1978 IV

The first one spans seven years. A longer interval will accumulate errors, and we believe the results might not generate a meaningful picture.[54]

The second corresponds to the optimization period. The 3 1/2 year span is long enough for our short-run analysis. If the model was able to simulate the historical path in this period with some degree of accuracy, we would have more confidence in our optimization results.

Regarding evaluation criteria, in the literature a variety of statistics are relied upon: root-mean square error (RMSE), root-mean square error in percent or root-mean square percent error (RMSPE), number of turning-point errors, Theil's inequality coefficients...

As Klein and Young (1980, p. 66) noted, for a quarterly macroeconomic model, perhaps the most important dynamic statistics are those relating to turning-point errors and measures of deviations of major economic variables from their historical paths measured by percent in the case of dollar-value variables and levels with

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[54] In a NBER/NSF model comparison seminar, participating major U.S. models were asked to perform 7-year simulations (see Fromm and Klein (1976)). Another reason for limiting the time span is due to the problem caused by the existence of two regimes of exchange rate.

respect to growth, inflation and unemployment rates.

Here we report RMSE and RMSPE for all variables except UR (unemployment rate) and IIN (investment in inventory). As to IIN, since it fluctuates widely from negative to positive values, those statistics are not meaningful and therefore are not calculated.[55] On the other hand, extra computation is done for financial assets ($\frac{CUR}{P}$, $\frac{DD}{P}$, and $\frac{DSN}{P}$).

RMSE is defined as

$$\left[\frac{1}{n} \sum_{t=1}^{n} (X^s_t - X^a_t)^2 \right]^{1/2}$$

where X^s and X^a are respectively simulated and actual values of X. RMSPE is RMSE divided by the variable mean. Tables 7.1 and 7.2 (columns 3 and 4) exhibit RMSE and RMSPE. In the last column of Table 7.1 are also reported the RMSPE(s) of the simulation in which (DD/P) enters as an equation, and currency derived as a residual.

The turning points are illustrated by graphs exhibited at the end of this section for major economic variables (Figs. 7.1 to 7.17). Since no statistic is computed for IIN, it will be plotted. As variables are of different scales, plots might be misleading indicators of deviations.

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55 The same treatment is found in the RDX2 manual (see Bank of Canada, 1976, p. 262). A similar remark was expressed by Kuh and Schmalensee (1973, p. 190).

TABLE 7.1: 1972 I – 1978 IV Simulation

VARIABLE	MEAN	RMSE	RMSPE (%)	RMSPE (%)
BOND	78835.3	3297.	4.18	4.31
CON	17623.3	601.9	3.42	3.42
CUR	6062.98	129.1	2.13	7.03
(CUR/P)	4165.3	67.83	1.63	7.64
DD	10493.4	129.1	1.23	4.06
(DD/P)	7277.5	188.1	2.58	3.69
DSN	48003.2	1537.	3.92	3.92
(DSN/P)	32316.1	1247.	3.86	3.86
EX	6584.79	198.8	3.02	3.02
IM	7563.93	243.9	3.22	3.22
INR	3989.96	175.6	4.40	4.40
IRC	1484.61	117.8	7.93	7.93
IRCNR	5474.57	231.	4.22	4.22
L	9230.52	85.85	.93	.93
LF	9910.47	56.31	.57	.57
M2	64559.6	1537.	2.38	2.38
MB	9715.98	133.1	1.37	3.98
P	1.44489	.02731	1.89	1.89
PE	1.57947	.08331	5.27	5.27
PFX	1.02515	.02465	2.40	2.40
RL	8.5538	.4255	4.97	4.97
RR	3653.	59.16	1.62	1.97
RS	8.06809	.8486	10.52	10.52
RTD	6.9621	.6761	9.71	9.71
TAX	8356.39	633.7	7.58	7.19
TR	18270.5	633.7	3.47	3.28
UR	6.81377	.4939		
V	60555.1	2386.	3.94	3.94
W	5.11333	.07168	1.40	1.40
Y	28569.1	607.	2.12	2.12

Note: Last column is obtained from the model in which the equation $\left(\frac{CUR}{P}\right)$ is replaced by the equation $\left(\frac{DD}{P}\right)$, and $\left(\frac{CUR}{P}\right)$ is derived residually.

TABLE 7.2: 1975 III - 1978 IV Simulation

VARIABLE	MEAN	RMSE	RMSPE (%)
BOND	97251.7	2068.	2.13
CON	19212.1	487.7	2.54
CUR	7274.64	140.7	1.93
(CUR/P)	4306.42	70.65	1.64
DD	12185.3	140.7	1.15
(DD/P)	7222.77	225.7	3.12
DSN	61969.8	1775.	2.86
(DSN/P)	36544.3	938.6	2.57
EX	6932.86	149.4	2.15
IM	8138.14	228.7	2.81
INR	4292.79	120.3	2.80
IRC	1569.	132.	8.41
IRCNR	5861.79	225.1	3.84
L	9672.26	69.22	.72
LF	10478.5	56.5	.54
M2	81429.7	1775.	2.18
MB	11780.9	146.2	1.24
P	1.68664	.03526	2.09
PE	1.8567	.06761	3.64
PFX	1.05777	.02055	1.94
RL	9.10952	.3845	4.22
RR	4506.22	61.61	1.38
RS	8.53309	.7922	9.28
RTD	7.4975	.7874	10.52
TAX	9913.14	707.8	7.23
TR	22484.1	707.8	3.16
UR	7.6883	.3641	
V	64327.4	1544.	2.40
W	6.16143	.1303	2.11
Y	30427.6	529.9	1.74

Note: For measurement units of variables presented in Tables 7.1 and 7.2, see Section 5.7.

Before we go into the evaluation, one thing needs to be said. In our analysis other U.S. and Canada models are sometimes referred to. This is just for illustrative purposes; due to differences in sample size, sample period and nation, interpretation should be careful.

7.1.1 Simulation: Period 1972 I - 1978 IV

In general, the results are encouraging. The most important variable Y (= real GNP) has a RMSPE of 2.12 for a seven-year period.

The labour market performs best. RMSPE of employment (L) and labour force (LF) are below 1%. Their plots (Figs. 7.11 and 7.12) show that the historical paths are almost perfectly reproduced.[56] In the literature there is still a tendency to report the unemployment rate (UR) by its RMSPE. If calculated, it is 7.25% (and 4.74% for simulation from 1975 III to 1978 IV). The RDX2 in a 7-year simulation of different sample generated 17.85% for UR (see Bank of Canada, 1976, p. 263).

In terms of RMSPE price level (P) and wage (W) are quite good. However, for P the seasonality feature is not completely satisfactory; indeed, at several 4th quarters, while the actual price declines, the simulated P remains constant (Fig. 7.13).

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56 Rather than plotting the unemployment rate, we prefer to plot employment and labour force.

The monetary sector performs in general well except the interest rates (short-term rate and time deposit rate).

The plots for currency (CUR), demand deposits (DD), and time deposits (DSN) indicate a trendy path with more fluctuations for DD; however, simulated DD reproduces turning-points accurately. To a large extent this is explained by the way it is derived residually. CUR misses turning-points in three instances, but the degree is quite small. The last column of Table 7.1 exhibits the RMSPE results from the simulation in which the equation $\frac{CUR}{P}$ is replaced by $\frac{DD}{P}$ (estimated by 2SLS), in other words the former is now derived residually. As can be seen, results are unchanged except for nominal, real CUR and DD, monetary base (MB), chartered bank reserves (RR), and government bonds (BOND). RMSPE for nominal, real CUR, and DD become worse; that is why we have preferred to derive DD residually.

The worst variable in the monetary sector is the short-term interest rate (RS) with a RMSPE of 10.52%. It is hard to obtain a good estimation for RS. A typical example is Kuh-Schmalensee medium-sized quarterly model (1973, p. 189) in which the endogenous short-term rate was not estimated directly: over a 16-year simulation its RMSPE was 50.37%. This result validates our argument that for good tracking it is essential to estimate RS directly. A second example is Pindyck's small model (1973): over a 13-year simulation, the RMSPE of RS was 18.63%.[57] In short, although RS has the worst RMSPE, it should be considered relatively satisfactory, taking into account all difficulties in obtaining a goodness of fit for it. We believe that if the RS equation could be improved, the effect would spread favorably over the estimation of the investment equations. As to its plot (Fig. 7.15), historical and simulated paths have almost the same form, though they are somewhat out of phase from 1976 I to 1978 II.

Next to RS in terms of high RMSPE is RTD. This is understandable as RTD is solely related to RS and its lagged value. We have tried to diminish its dependence on RS by including the U.S. short-term interest rate. But the U.S. rate has a t-statistic barely exceeding 1, and the simulation does not show any improvement.

A variable of interest is the foreign exchange rate (PFX). A lot of controversies have arisen with the recent development of the monetary approach to the foreign exchange rate which is adopted here. Our simulation generates a RMSPE of 2.40%, which is an encouraging result for our PFX formulation. Carefully examining its path (Fig. 7.14), we can observe from 1976 III that the simulated PFX follows its counterpart quite well as this period witnessed a continued depreciation of the Canadian dollar. But before that the Canadian dollar was much unstable, and the simulated PFX

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[57] Calculation is based on the RMSE statistic reported in Pindyck (1973, p. 65) and on RS data found in Pindyck and Rubinfeld (1976, p. 415).

misses two turning-points in 1974 II and 1976 II.

The last sector to be discussed is the product sector. In general turning-points as well as seasonality are well captured.

Consumption, import, and export perform almost equally well in terms of RMSPE, yielding a very good RMSPE for GNP (2.12%). With respect to import, the simulated path follows quite closely the historical path (Fig. 7.6). It fails, however, to reproduce a minor downturn in 1978 I.

The investment variables, particularly INR and IRC, are comparatively less desirable.

The plot of inventory investment (Fig. 7.7) exhibits wide swings from negative to positive values. The simulated IIN fails to pick-up some turning-points. Considering those extreme oscillations, the performance of IIN should be regarded as satisfactory; moreover, note that its size is too small.

Investment in non-residential construction and machinery & equipment (INR) is also characterized by a lot of oscillations (Fig. 7.8). A minor turning-point is missed in 1976 III. We are not truly happy with the INR equation which depends mainly on income, long-term interest rate RL (lagged 8 and 9 quarters), and its lagged value (see Chapter 5). We prefer a distributed lag to enter the equation rather than ours which is quite ad hoc, but we never succeed.

Investment in residential construction, as expected, is the worst in the product sector with a RMSPE of 7.93%. As noted by Pindyck and Rubinfeld (1976, p. 375), IRC is difficult to explain in a small macroeconomic model: theoretically, it should also depend on mortgage rate, on mortgage availability, and construction costs, and none of these factors is included in our model. The plot (Fig. 7.9) indicates large deviations. A consoling point is that the simulated path follows the direction of the actual path, and IRCNR as the sum of IRC and INR behaves better (its RMSPE is 4.22%).

7.1.2 Simulation: Period 1975 III - 1978 IV

The striking fact is that substantial improvement emerges as the simulation period shortens. The RMSPE decreases for almost every variable.

Residential construction investment (IRC) is one exception: its RMSPE increases from 7.93% to 8.41%; however, investment in non-residential construction, and machinery & equipment (INR) records a sharp decline from 4.40% to 2.80%, making private fixed investment (IRCNR) falling from 4.15% to 3.84%.

Other components of GNP such as consumption, export, and import greatly improve. The total impact yields a very low GNP RMSPE (1.74%) for a 3 1/2 year simulation.

Considering the difference in time span, it seems both simulations perform almost equally well. In that case one might think there is some stability in the economic relationships for the period from 1972 I to 1978 IV. This remark, if true, coupled with quite good simulation results from 1975 III to 1978 IV, should give us a lot of confidence in our optimization analysis.

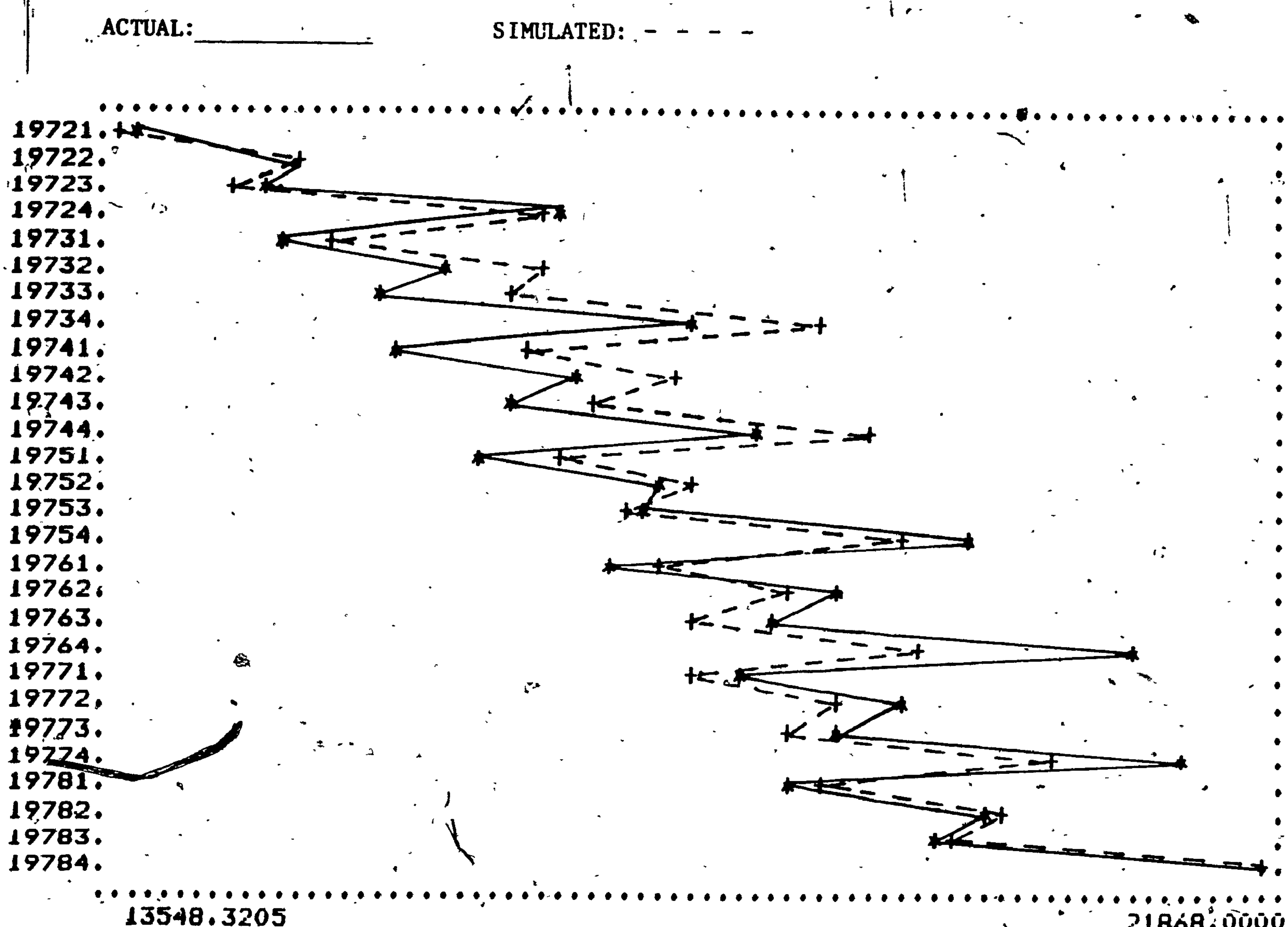


FIGURE 7.1: Actual and Simulated CON (Real Consumption). Millions of 1971 Dollars.

(Note: Under every graph are minimum and maximum values).

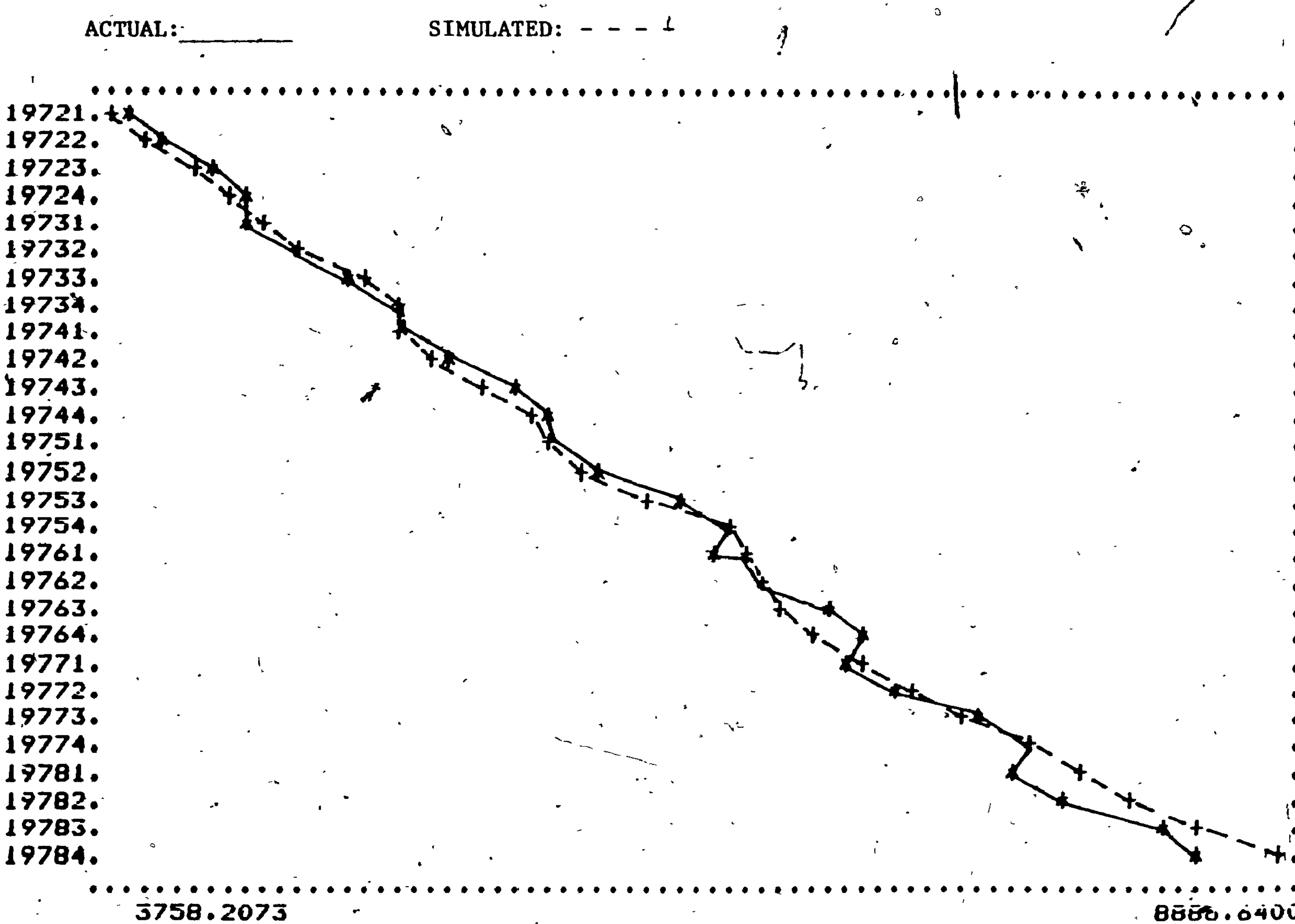


FIGURE 7.2: Actual and Simulated CUR (Currency). Millions of Current Dollars.

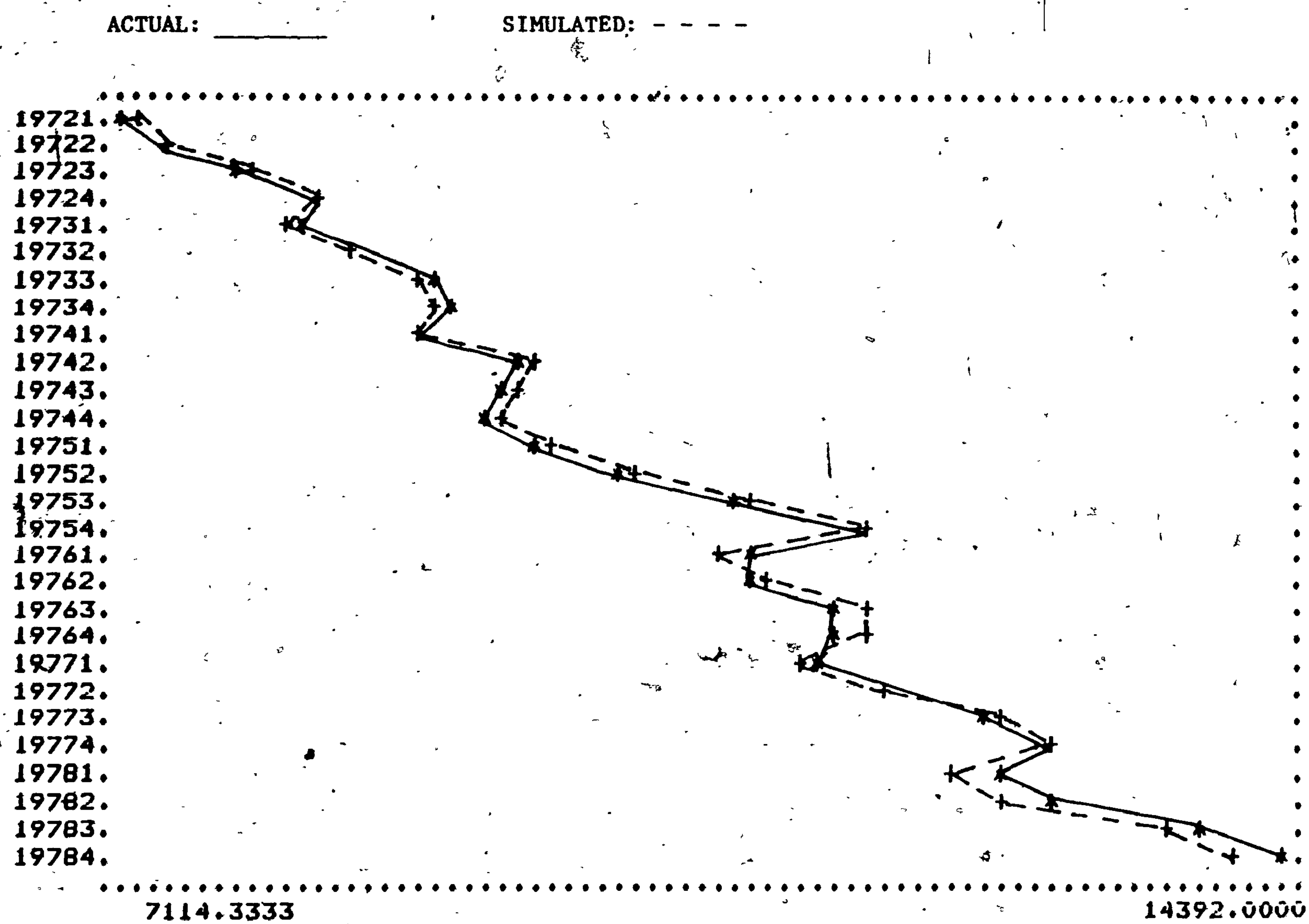


FIGURE 7.3: Actual and Simulated DD (Demand Deposits). Millions of Current Dollars

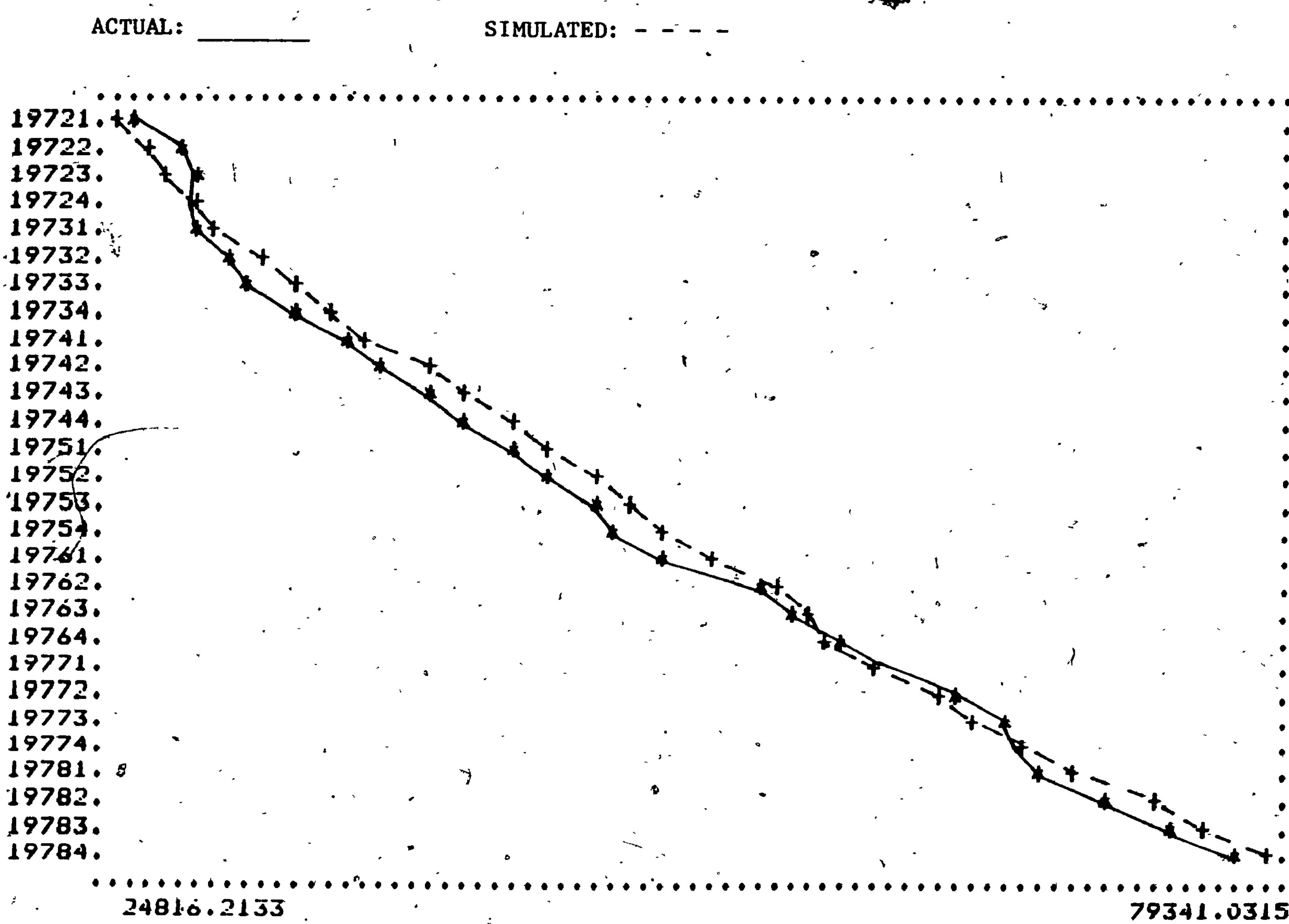


FIGURE 7.4: Actual and Simulated DSN (Time Deposits). Millions of Current Dollars.

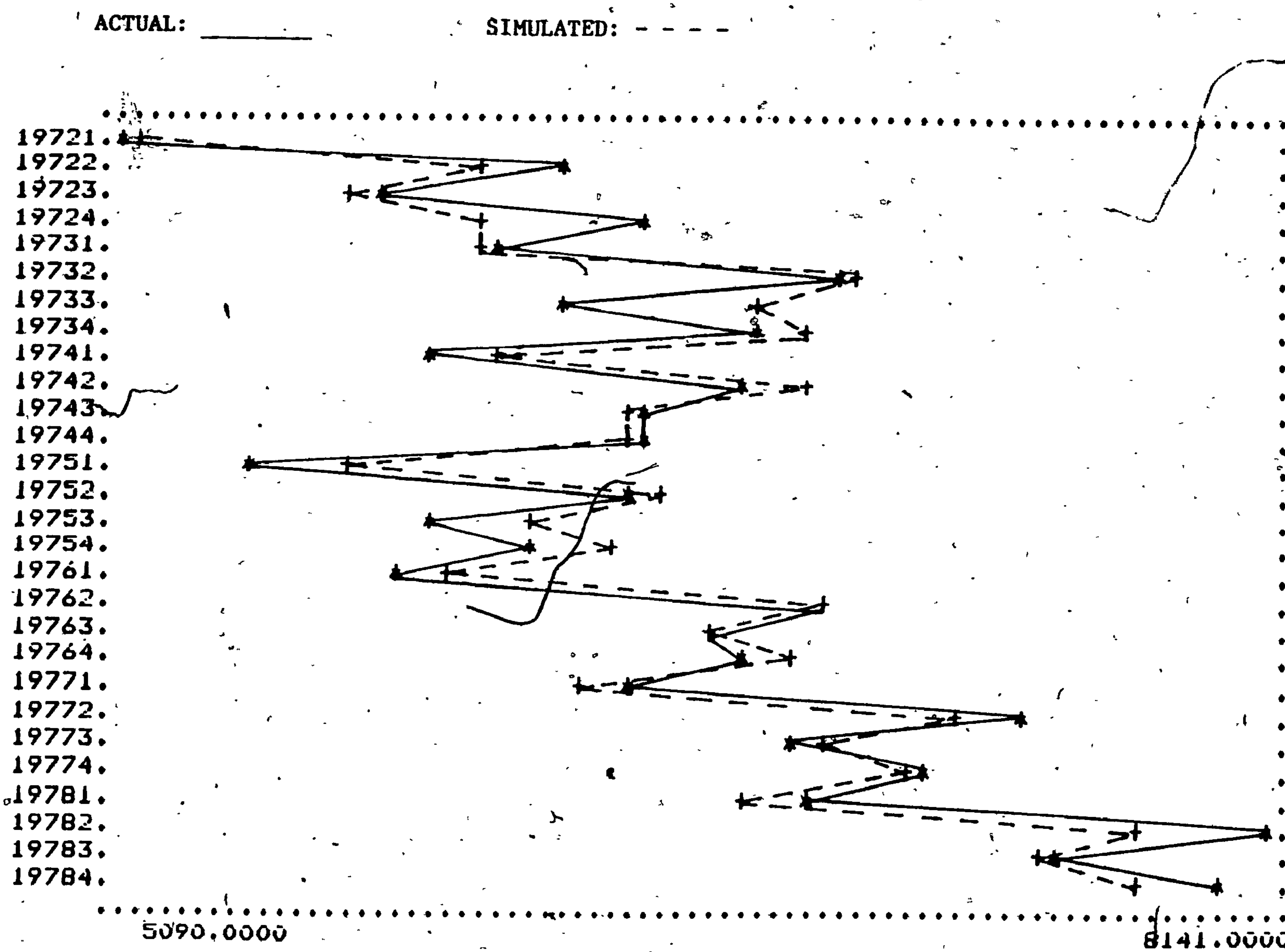


FIGURE 7.5: Actual and Simulated EX (Real Export of Goods and Services). Millions of 1971 Dollars.

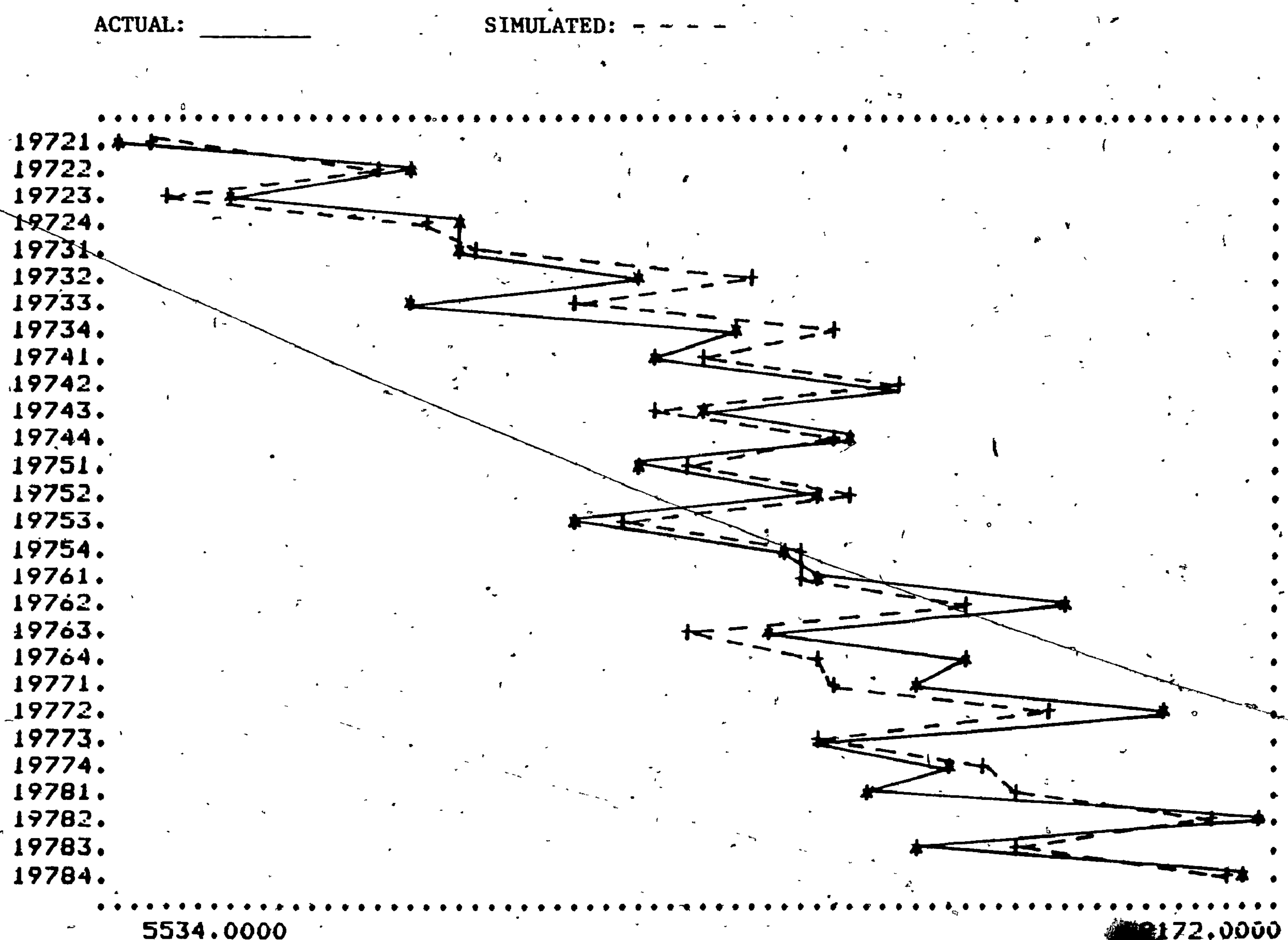


FIGURE 7.6: Actual and Simulated IM (Real Import of Goods and Services). Mi[illegible] of 1971 Dollars.

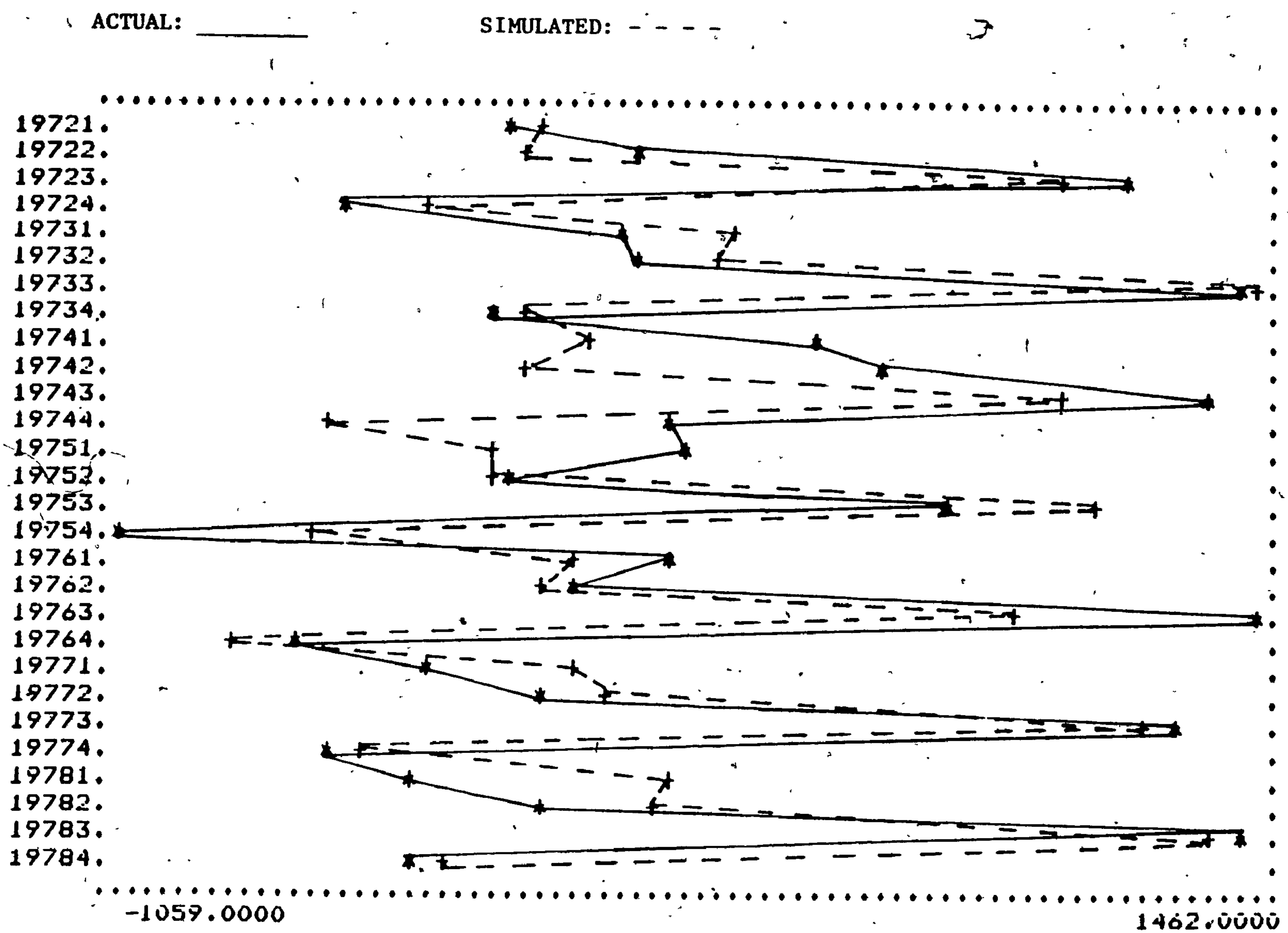


FIGURE 7.7: Actual and Simulated IIN (Real Investment in Inventory). Millions of 1971 Dollars.

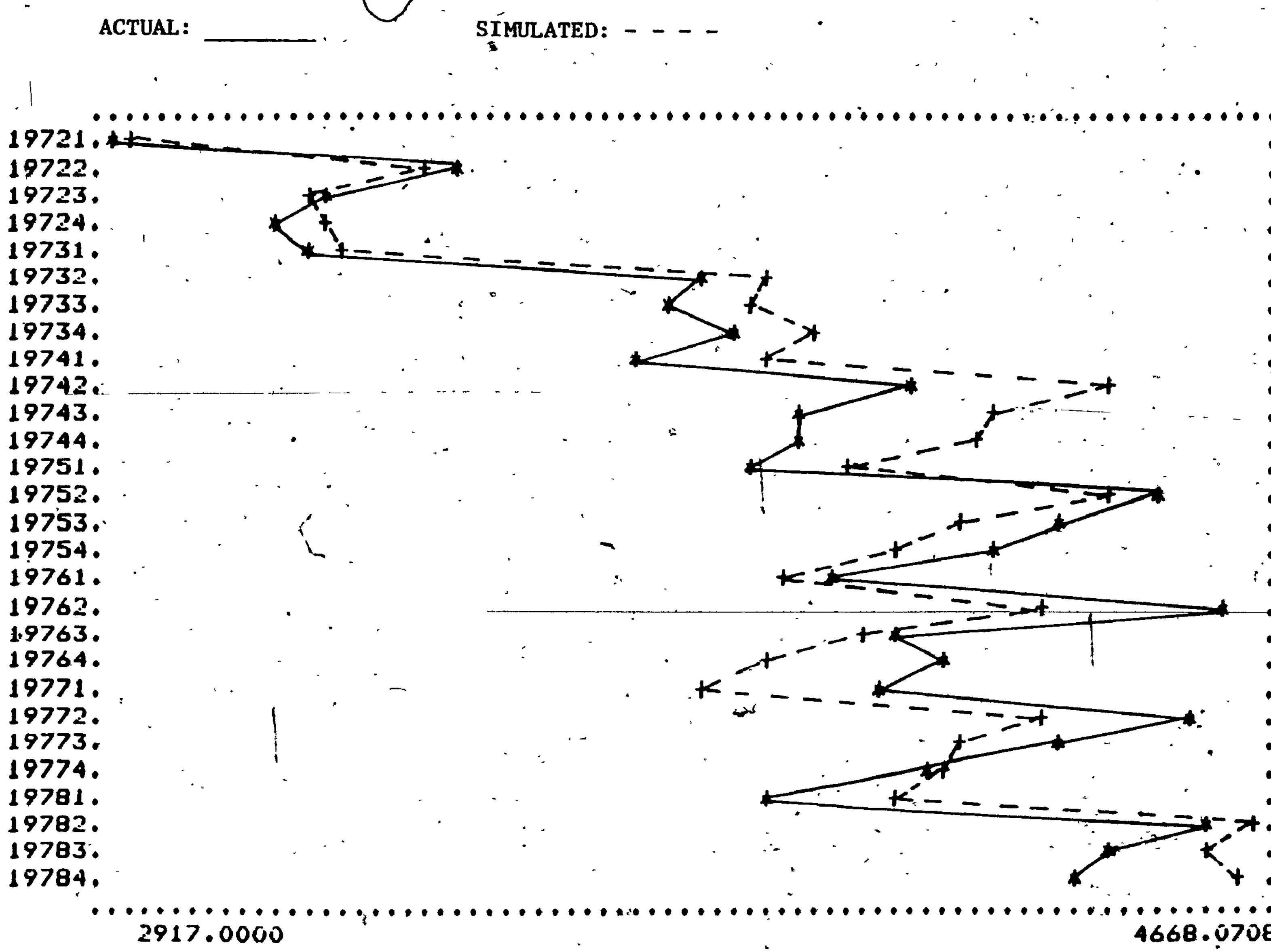


FIGURE 7.8: Actual and Simulated INR (Real Gross Private Investment in Non-Residential Construction, and Machinery & Equipment). Millions of 1971 Dollars.

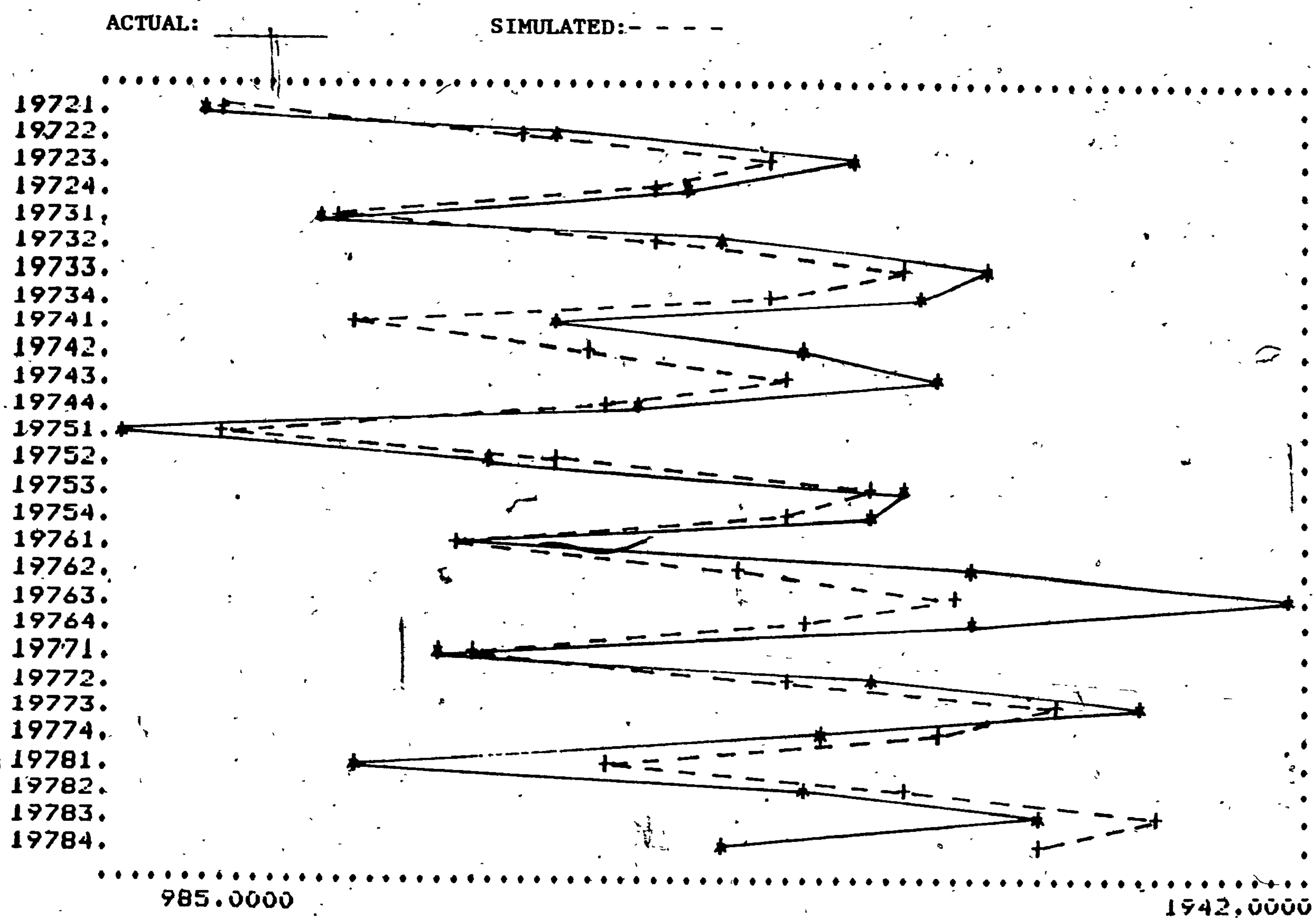


FIGURE 7.9: Actual and Simulated IRC (Real Gross Private Investment in Residential Construction). Millions of 1971 Dollars.

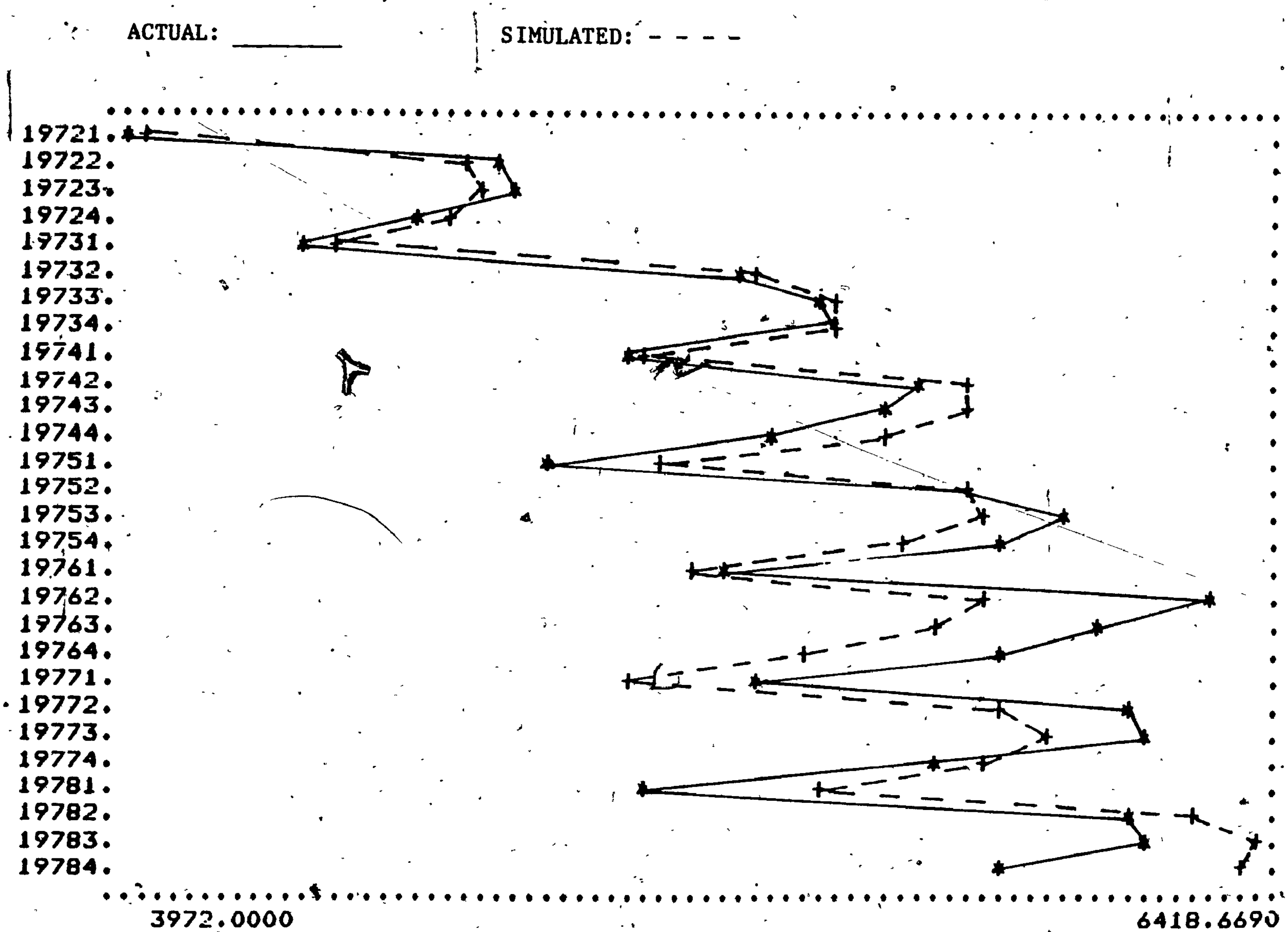
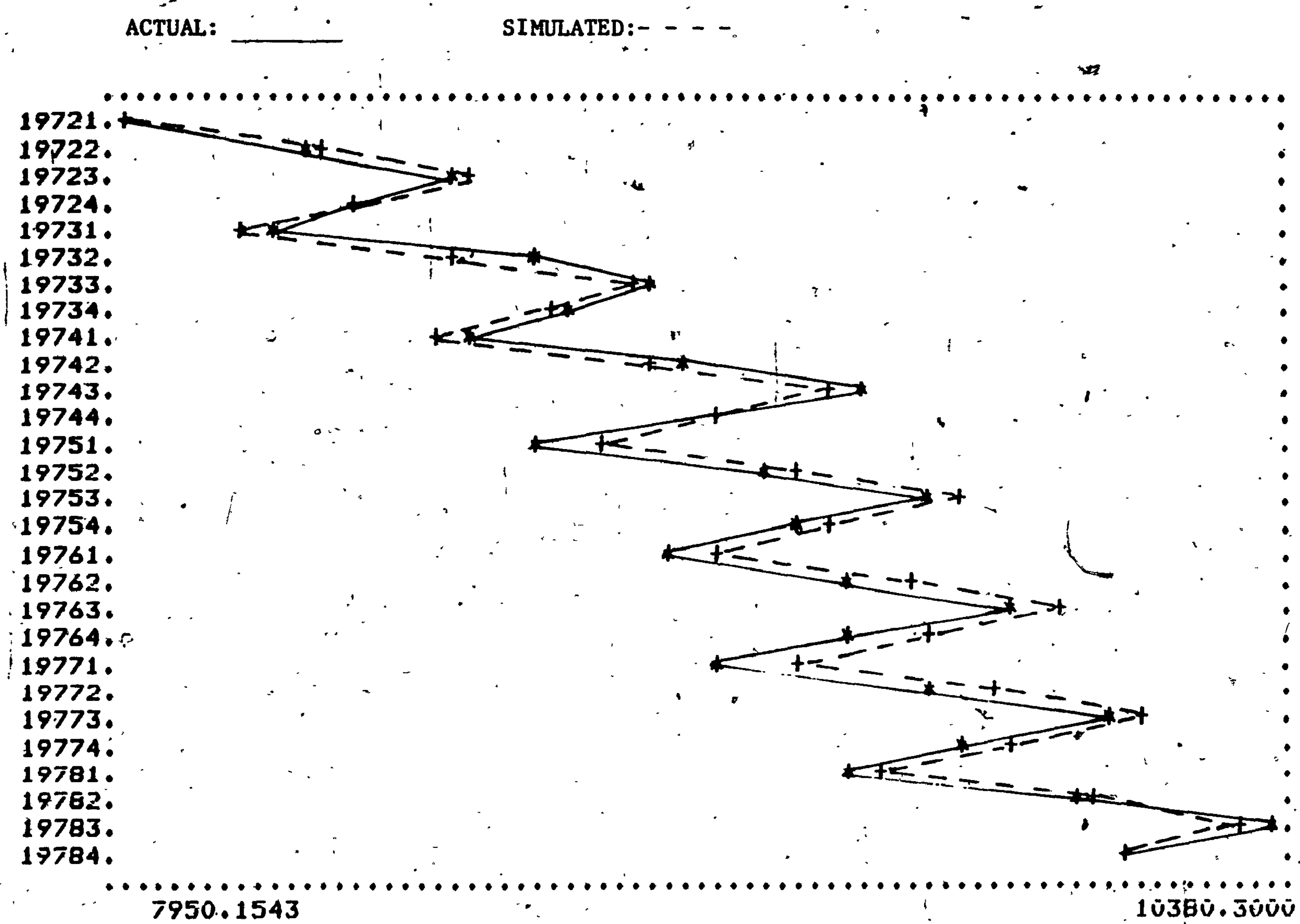


FIGURE 7.10: Actual and Simulated IRCNR (Real Gross Private Fixed Investment). Millions of 1971 Dollars.



<u>FIGURE 7.11</u>: Actual and Simulated L (Employment). Thousands of Persons.

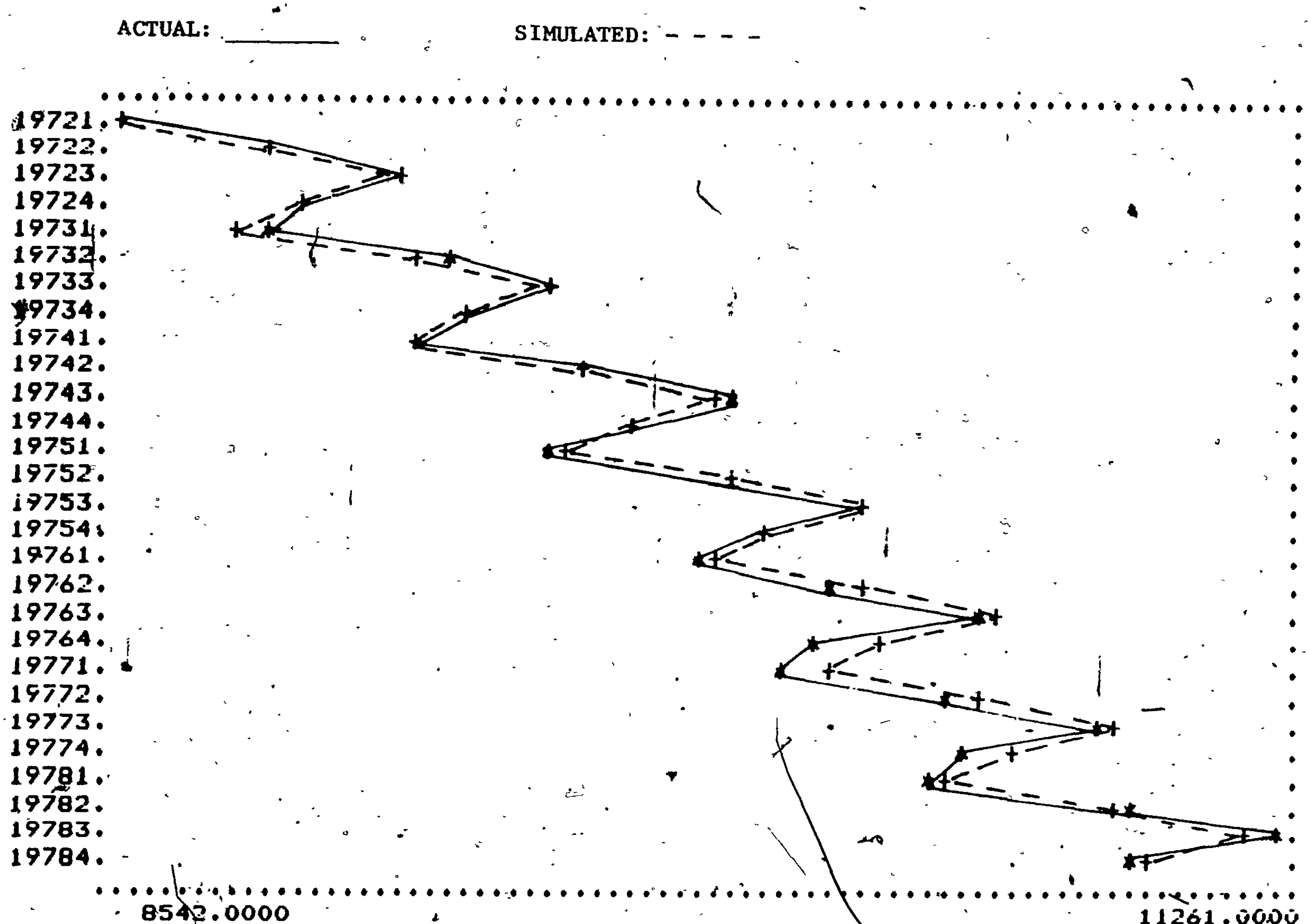


FIGURE 7.12: Actual and Simulated LF (Labour Force). Thousands of Persons.

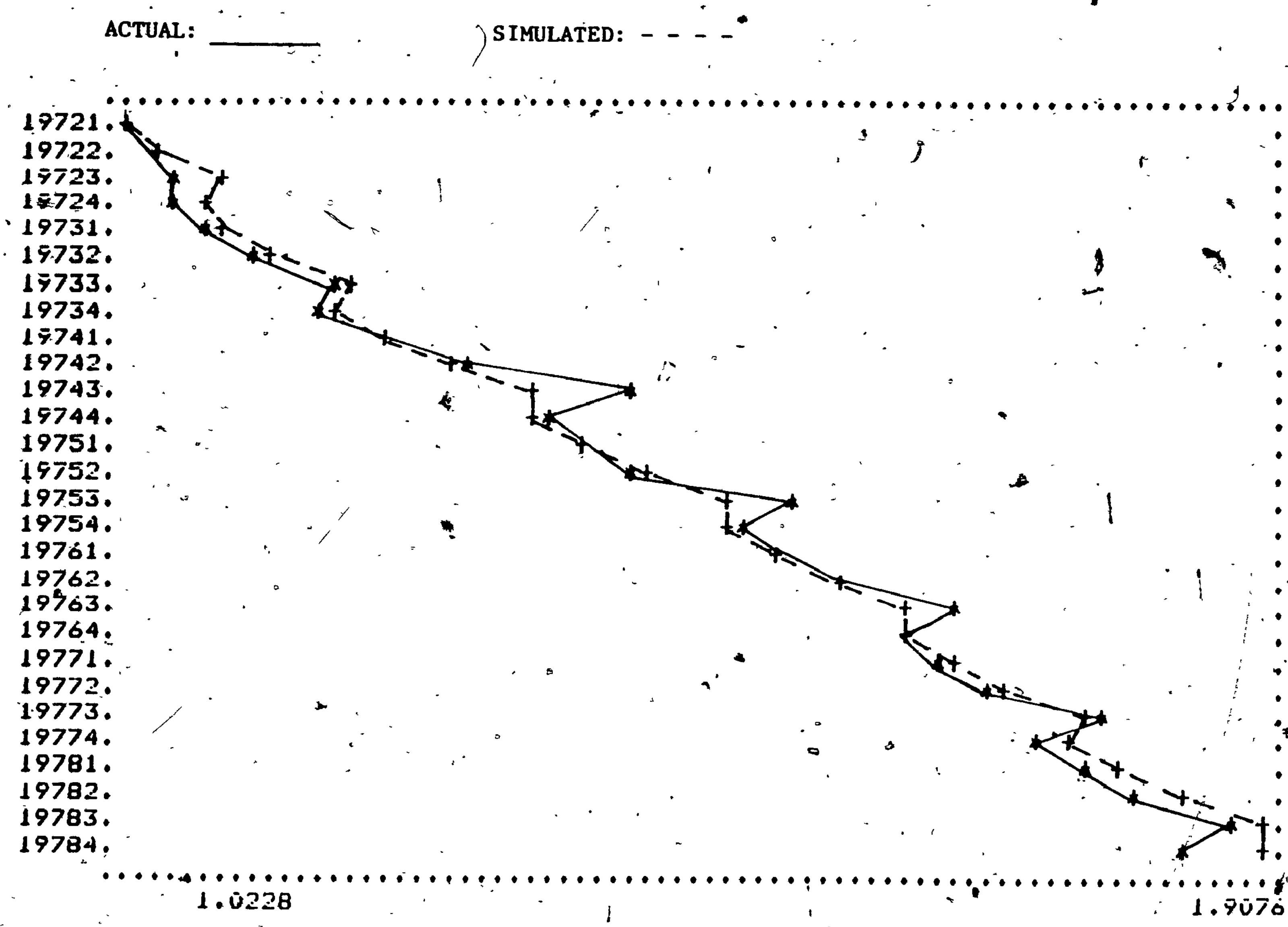


FIGURE 7.13: Actual and Simulated P^s (Price Level). 1971 = 1

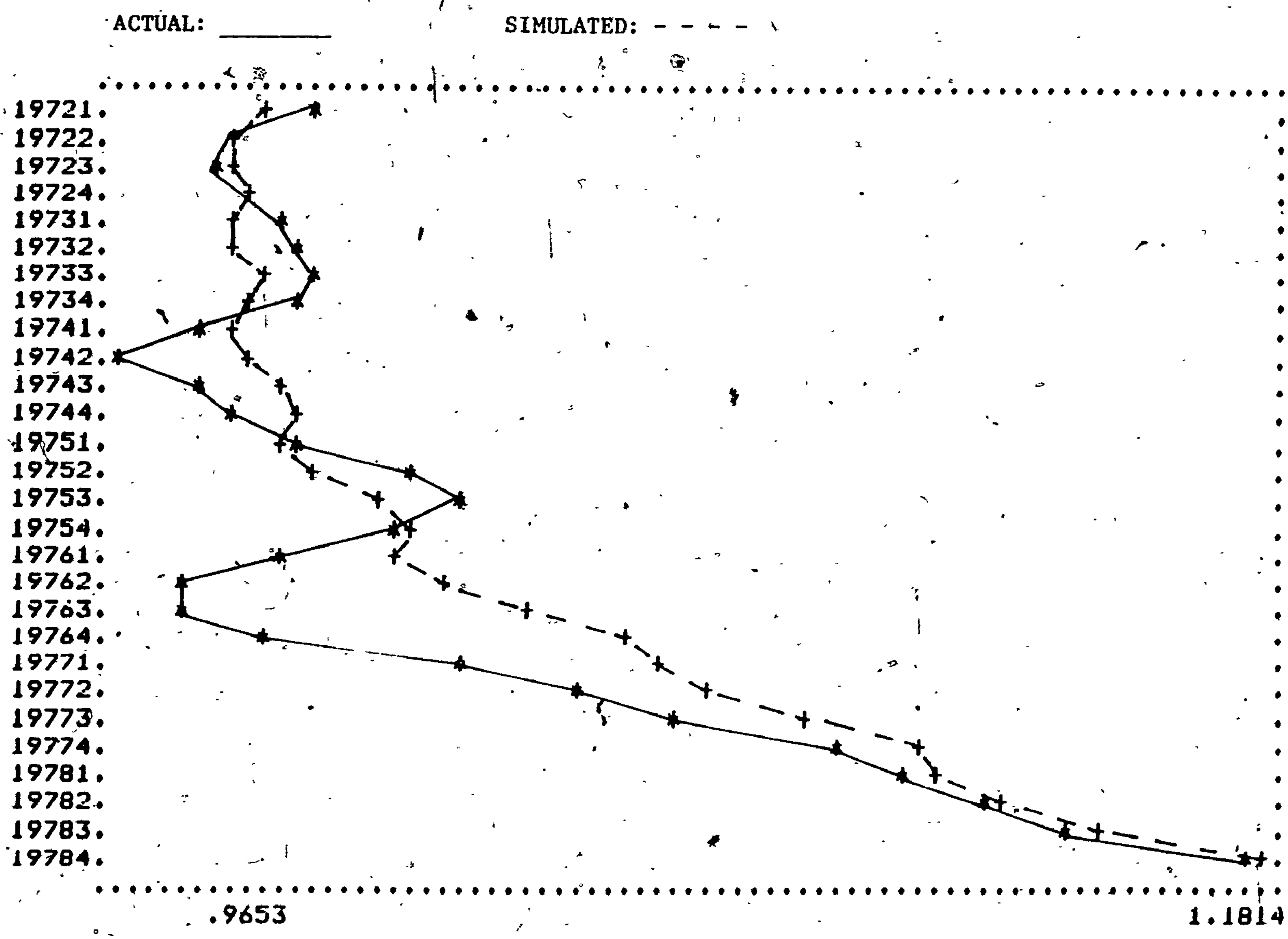


FIGURE 7.14: Actual and Simulated PFX (Foreign Exchange Rate).

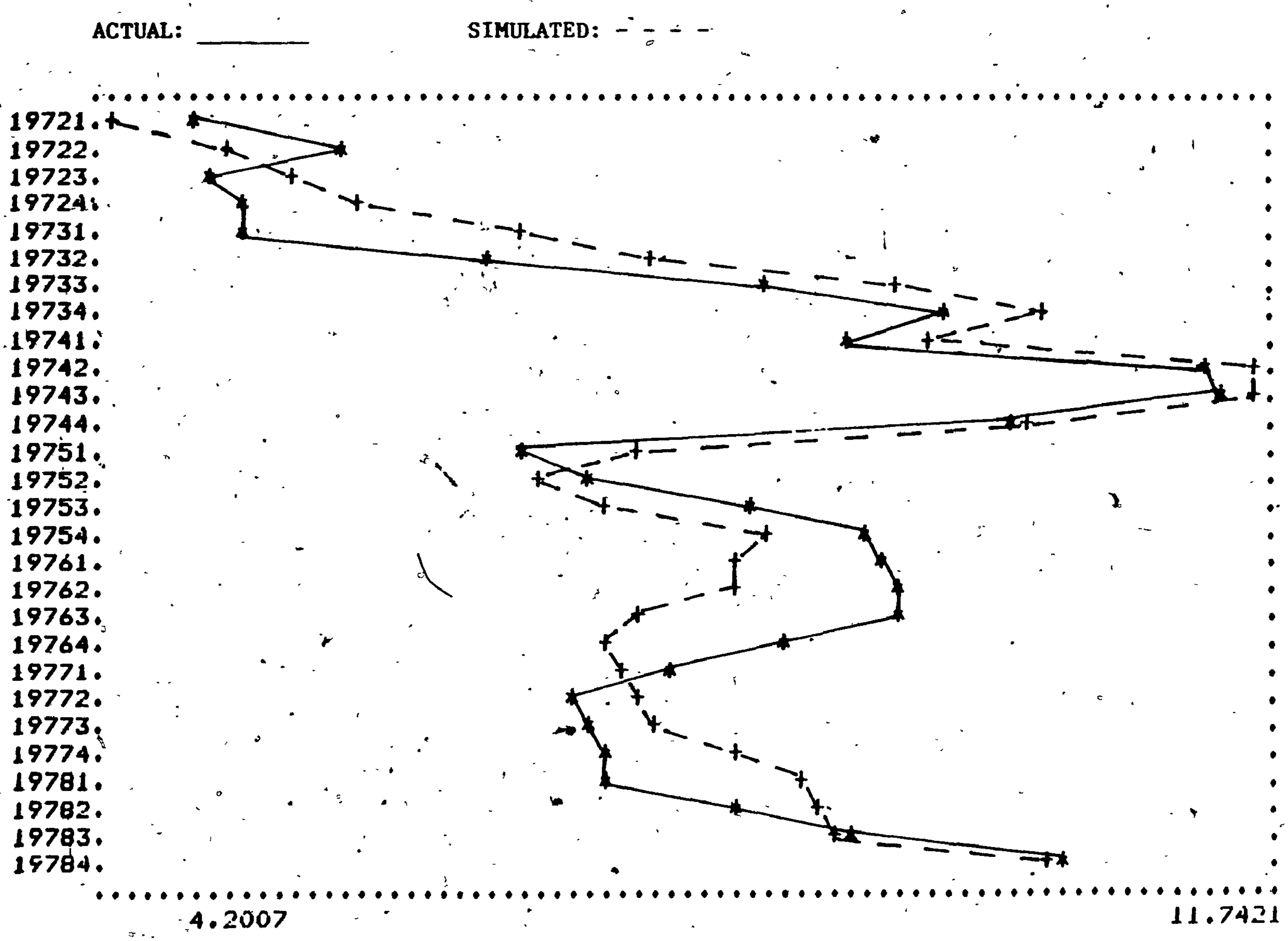


FIGURE 7.15: Actual and Simulated RS (Short-term Interest Rate), In Percent.

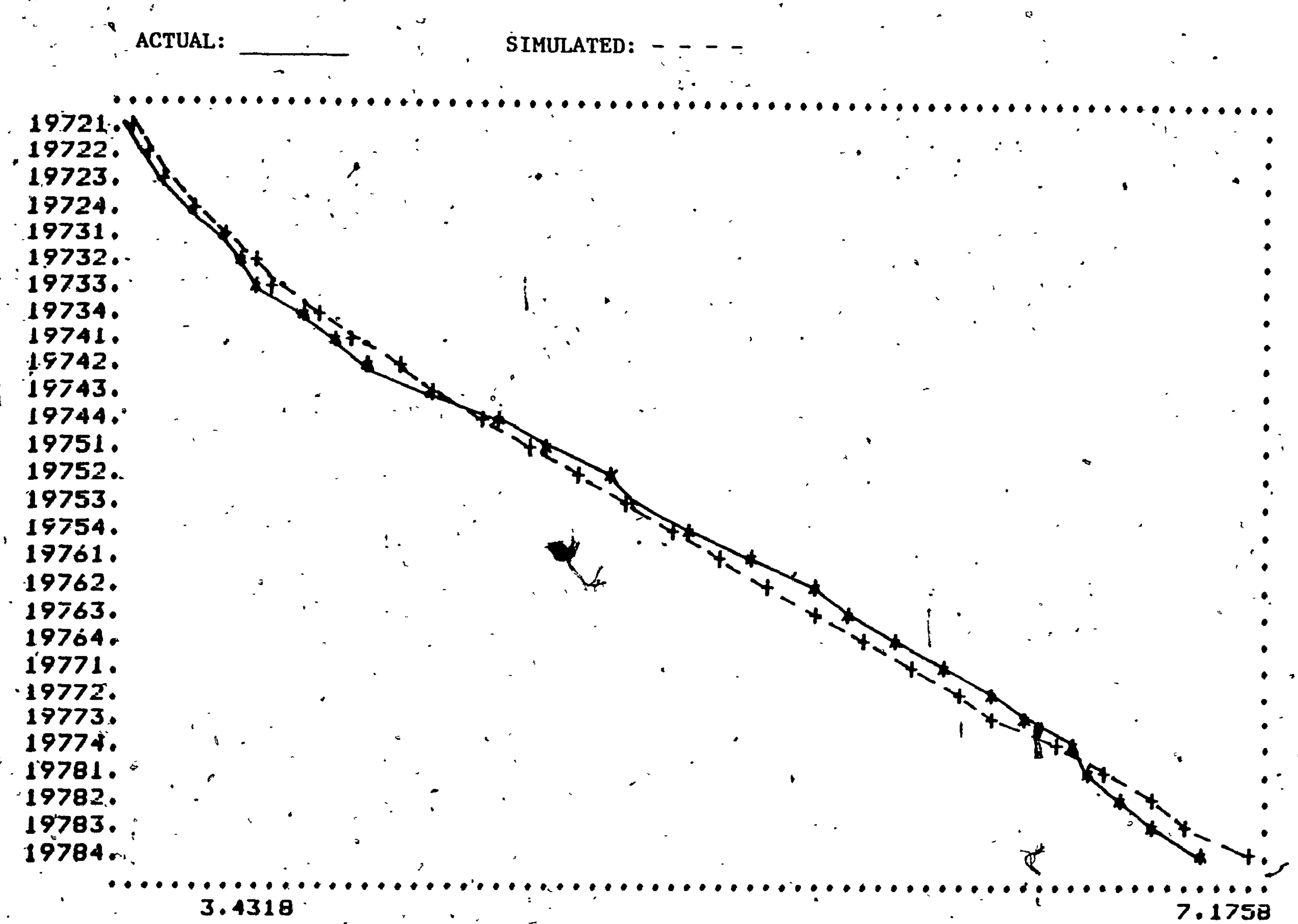


FIGURE 7.16: Actual and Simulated W (Wage Rate).

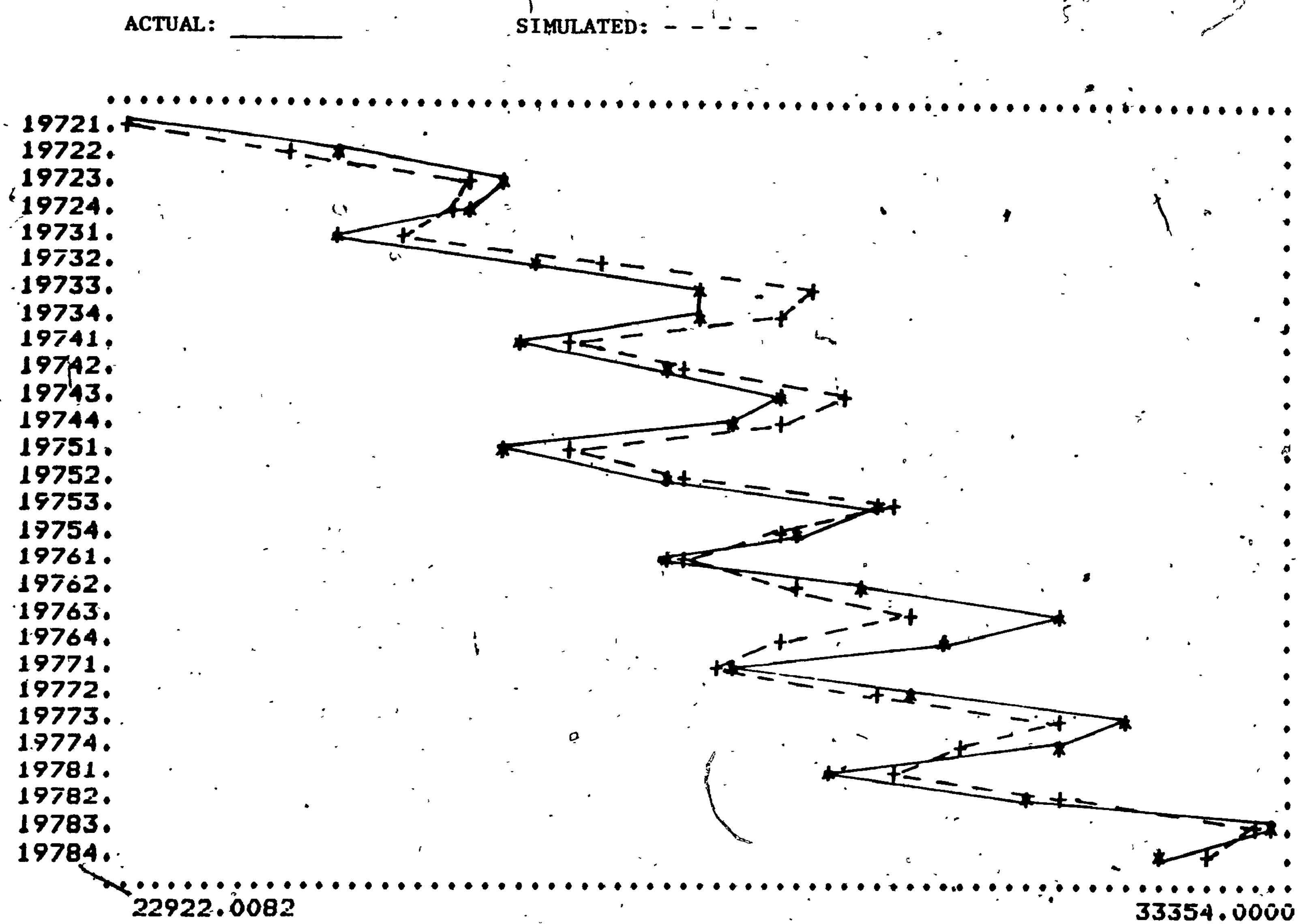


FIGURE 7.17: Actual and Simulated Y (Real GNP). Millions of 1971 Dollars.

7.2 SOME DYNAMIC PROPERTIES: LONG-RUN CROWDING-OUT AND NEUTRALITY OF MONEY

Although the model was constructed with a short-run objective in mind, it is interesting to know if in the long-run, it can meet the two conditions of crowding-out and neutrality of money.

The crowding-out is a phenomenon in which increase in government spending has the only effect of displacing private sector spending with no impact on real income. On the other hand, if money is neutral in the long-run, a money stock increase results only in an equiproportionate increase in price, leaving real income unaffected.

The issue with these effects is that how long is the 'long-run'? For instance, in discussing about fiscal action, Ando and Modigliani (1976, p. 18) stated that when a new long-run equilibrium is reached, a macro fiscal action will have no significant effect, at least, on real income but added that the long-run equilibrium might require a very long time if not forever. Bearing this in mind, we turn to empirical evidence.

To study the above dynamic properties, shock response experiments were conducted. All involved comparisons of a simulation based on non-historical values of a policy variable with a simulation based on historical values of the policy variable. The simulation period extends from 1972 I to 1978 IV (same period as our longest simulation described in Section 7.1.1). In fiscal experiment real government spending (G1) was raised by 100 million in 1971 dollars over historical data for every quarter. In monetary experiment nominal money supply (M1) was raised by 100 million current dollars for every quarter. From simulation results, changes in GNP can be calculated; if they are divided by 100, multipliers are obtained. Here multipliers are reported, as traditionally done. A problem with multipliers is that in non-linear models they are not invariant to the time origin, the level of economic activity, and the size of exogenous change (see, for example, Hallet and Rees, 1983, p. 56). This suggest us to start the shocks at two different dates: 1972 I and 1973 I; the second date is to allow time for lagged terms to have some effect.

Table 7.3 provides the values of fiscal and monetary multipliers obtained from shocks starting 1 year later (i.e., in 1973 I). As the period is not long, the fiscal multiplier value does not approach zero, but its graph (Fig. 7.18) shows that it is decreasing in the last 4 quarters. As the path is cyclical,[58] and the period is short, we cannot be certain to tell whether the multiplier will approach the zero value or not, given enough time.

Regarding the monetary multiplier, although Figure 7.19 shows that it augments in the last quarter, Table 7.3

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58 The cyclical feature is also found in certain Canadian models. For example, Sandblom, Banasik, and Parlar (1981, p. 20) compared fiscal shock response of real GNP from their model (a linearized condensed model based on the RDX2) with that from a version of RDX2: both models exhibited a similar cyclical pattern although with rather large divergence.

indicates its values are negative for the last 8 quarters.

To some extent we might say the conditions of crowding-out and neutrality of money have been met. To lend some support, we performed another set of fiscal and monetary experiments with a very much longer period: 25 years and 1 quarter.[59] For these, we generated artificial data for exogenous variables, assuming that they grow from 1978 IV onwards, at a constant rate (obtained from regression on time). Of course data are not correct, but we feel that this should not be serious as long as data errors appear in both historical and shock simulations. Policy variables were raised by the same amount as earlier, but only in one quarter (1981 I); simulation starts in 1978 IV to allow lagged terms to take full effect.

What we find is: 11 years after the shock dynamic fiscal multiplier becomes stable around a value of .16, while 8 years after the shock money multiplier tends to stabilize around .001. These results would give more credence to the model as far as the long-run properties of crowding-out and neutrality of money are concerned.

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59 Experiment idea was taken from Professor Anastasopoulos.

TABLE 7.3: Fiscal and Monetary Multipliers

PERIOD	FISCAL	MONETARY
	ΔY/ΔG1 (constant $)	ΔY/ΔM1 (constant $/current $)
1973 I	1.761	.00000003
II	2.154	.013
III	2.245	.037
IV	2.065	.122
1974 I	1.807	.190
II	1.532	.227
III	1.267	.243
IV	.975	.236
1975 I	.742	.217
II	.591	.195
III	.479	.171
IV	.381	.128
1976 I	.350	.099
II	.391	.067
III	.416	.048
IV	.472	.023
1977 I	.614	-.0009
II	.717	-.017
III	.917	-.044
IV	1.044	-.046
1978 I	1.083	-.046
II	1.031	-.066
III	.979	-.067
IV	.932	-.056

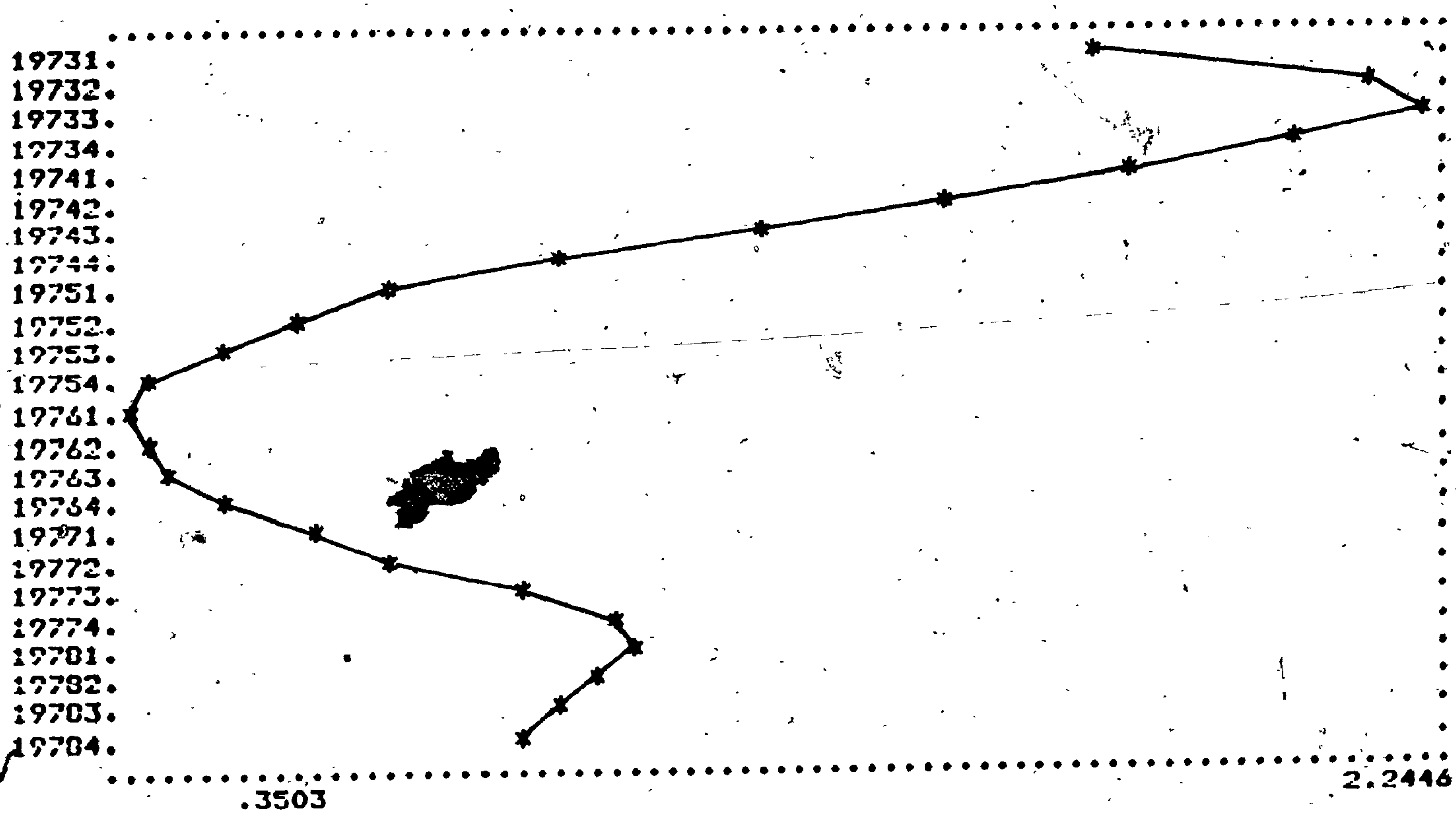

FIGURE 7.18: Fiscal Multiplier.

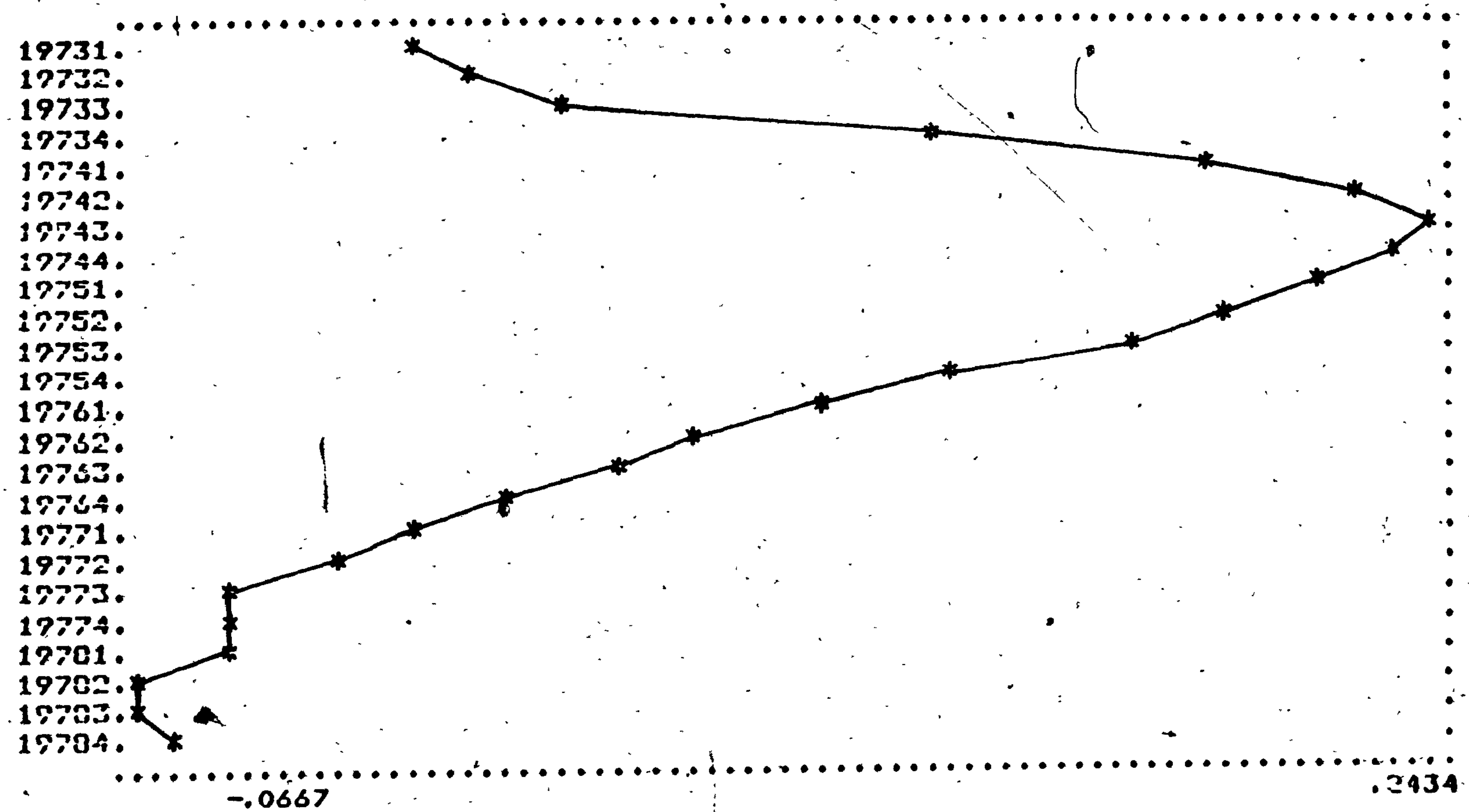

FIGURE 7.19: Monetary Multiplier.

Before leaving this chapter, it is necessary to say how effective, in terms of impact on Y, fiscal and monetary policies are. Table 7.3 clearly indicates that monetary effectiveness is very weak while fiscal policy enjoys a high degree of effectiveness which is commonly found in the empirical macroeconomic literature. The only external Canadian evidence about monetary effectiveness available to us comes from the TRACE model (Choudhry et al., 1972, Chapter 4). In the TRACE manual, a few experiments were performed, based on different starting dates with a once-and-for all raise in M1. Multipliers were not calculated, but enough information were provided to permit the computation roughly. Money multipliers are found to be very small. For example, in the 1957 experiment the impact and interim money multipliers are found to be .07 for the first year (annual data), .02 for the second year, .006 for the third year, and become negative after that. In experiments with other starting dates, money multipliers are smaller.

Note that as the optimization period covers 14 quarters and extends from 1975 III to 1978 IV, we also repeated the multiplier analysis runs for the same period. G1 and M1 were raised by 100 million over historical data for every quarter. In general, results are quite similar to those reported in Table 7.3 for the first 14 quarters although the new values of fiscal multiplier are a bit higher: e.g., at the end of the 14-quarter simulation period, the value is 0.6 (0.39 in Table 7.3).

PART III

OPTIMAL CONTROL EXPERIMENTS

CHAPTER 8

SPECIFICATION OF THE WELFARE LOSS FUNCTION

To calculate an optimal control solution for a model, the specification of the objective or welfare loss function is required. Ours is assumed to be a piecewise quadratic function. This will allow asymmetric penalty and avoid the problem that deviations of variables from their targets are equally undesirable regardless of their signs. Traditionally, the next two steps are to determine the set of targets and instruments, and the weights. For reasons discussed in Chapter 3, the common practice of including control costs in the objective is not followed here; instead, we will establish lower and upper bounds on control variables. Using a piecewise quadratic function and imposing bounds on control variables help to remove some of the criticisms voiced against the quadratic formulation (Bock v. Wulfingen and Pauly, 1978, for example).

In Section 8.1 we treat the subject of targets. In Section 8.2 we deal with the issue of control bounds. And finally, in Section 8.3 we provide a detailed description on how the weights are determined.

8.1 TARGETS

In a NBER/NSF Econometric Model Comparison seminar chaired by L. Klein, major U.S. models were asked to perform deterministic optimal control experiments by essentially using the same quadratic loss function with four targets: GNP gap, unemployment rate, inflation rate, and (nominal) trade balance as a percentage of (nominal) GNP (see Chow, 1981, Chapter 4; Klein and Su, 1980).

We follow the same line. Note that the GNP gap is treated as a target instead of the traditional GNP target to emphasize the idle capacity utilization in the period under consideration.

In short, the objective function (J) to be minimized is:

$$J = \sum_{1975\ III}^{1978\ IV} \left[w1\ \left(100 \left(\frac{Y}{YP} - 1\right) - 0\right)^2 + w2\ (UR - UR^*)^2 + w3\ (INFL - INFL^*)^2 + w4\ (TB - 0)^2 \right]$$

where Y = real GNP

YP = real potential GNP

UR = unemployment rate

UR^* = desired value or target of UR

INFL = inflation rate (to be defined later)

$INFL^*$ = desired value or target of INFL

TB = trade balance as a percentage of GNP (to be defined later

and w1, w2, w3, w4 are the weights of the four targets.

Several explanations are in order.

i) The optimization period covers 14 quarters, starting in 1975 III and ending in 1978 IV. It is particularly chosen to give more support in treating M1 as a control and to provide us an opportunity of studying the Bank of Canada's monetarist experiment.

ii) Penalty will be zero if the GNP gap exceeds its desired value, and if the unemployment and inflation rates are smaller than their desired values.

iii) The GNP gap is defined as $(\frac{Y}{Y^p} - 1)100$ with a desired value of zero. The potential GNP series is generated by assuming the quarterly seasonally adjusted GNP in 1973 III to grow at a constant annual rate of 4.9% (it is mainly a 4.9% trend line).[60] That rate was taken from a study of Crozier (1977) from the Conference Board of Canada. Crozier estimated the full-employment GNP rate of growth for the period 1973-1980 to be 4.9%, defined at the 5% rate of unemployment.[61] The series is then transformed into unadjusted data by multiplying it by a ratio of actual non-seasonally adjusted (N.S.A.) to actual seasonally adjusted (S.A.) GNP. Note that as GNP gap is in ratio form, it does not matter whether we seasonalize Y or deseasonalize Y^p.

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60 Studies by Crozier (1977), Brox and Cluf (1979) showed that GNP was equal to its potential level in 1973 III.

61 The value of 4.9% is very close to our estimate (=5.1%) obtained by running a regression of log(Y) over TIME during the period 1958 I - 1978 IV.

iv) The desired value of the unemployment rate, UR*, is a constant 5%. Please note that the desired values of targets policy-makers talk about are for S.A. data. Therefore, UR* is transformed into N.S.A. by using the ratio of N.S.A. to S.A. data. This procedure had been used by Pindyck and Roberts (1974, p. 222).

v) To be realistic, the desired value of the inflation rate, INFL*, is set equal to 6% from 1975 III to 1977 I and 5% from 1977 II on. The rate of inflation is defined as $100[\frac{P}{P(-4)} - 1]$ rather than $400[\frac{P}{P(-1)} - 1]$. The former has removed to some extent seasonality while the latter, N.S.A., is not a suitable indicator of quarter-to-quarter change in price. Holbrook (1973, p. 61) defined the rate of inflation as $[\frac{P}{P(-1)} - 1]$ and transformed it into S.A. data by using the dummy approach. He remarked that even though inflation data had been seasonally adjusted, the presence of seasonality was still strong, suggesting that the seasonality of the model differs to some degree from that of historical data. We have encountered the same problem when defining the inflation rate as $400[\frac{P}{P(-1)} - 1]$ and seasonalizing it by the ratio of N.S.A. to S.A. data. This urges us to use $100[\frac{P}{P(-4)} - 1]$.

vi) The last target TB is the ratio of nominal trade balance over nominal GNP. It will be referred to as trade balance for short. It is defined as:

$$TB = 100\left(\frac{EX.PE - IM.PIMA.PFX}{Y.P}\right)$$

(all variables keep the same meaning as in Chapter 5).

8.2 CONTROL VARIABLES

There are two candidates for control variables. For monetary policy it is the nominal stock of narrow money M1. For fiscal policy it is G1. Notice that total real government spending consists of G1 (real government expenditure of federal, provincial, and local governments) and G2 (real expenditure of Hospitals, Quebec, and Canada Pension Plan). The latter is a very small amount of total government spending.

For the resulting optimal trajectories to be acceptable from a political as well as an economic point of view, and for technical reasons, we will impose lower and upper bounds on the control variables. To our knowledge a few studies have pursued that direction. In the two works by McCarthy and Palash (1977), and Craine (1978) the control variable is restricted to be a polynomial function of time, à la Almon. Apart from the difficulty in determining the desirable degree of the polynomial, their procedure calls for the development of appropriate optimal control algorithms; as a matter of fact, the authors were more interested in the aspect of computational efficiency which is gained from imposing the control restrictions. In Sandblom, Banasik, and Parlar (1981), and Sanblom and Banasik (1984), the authors set up bounds as some multiples of historical range. In the first study time independent and time dependent bounds are both considered. With time independent bounds the control variable is restricted to stay within a band which is centered around the midpoint of its historical range and constant over time. With time dependent bounds, the band which is centered around the individual historical values moves over time according to the historical values of the control variable, but the bandwidth remains the same. The Sandblom-Banasik-Parlar procedure is simple and easy to implement. However, if the historical range defined as the difference between the smallest and largest values of the control variable under a certain period is quite large, the resulting bounds, particularly time dependent bounds, will be large. Here a different procedure is used, relying mainly on past behaviour of the control variables. This is feasible due to the availability of past information and the existence of only two policy instruments. Another justification for our choice is that as a great number of optimization experiments are to be conducted, we are inclined to generate bounds which are wider than the historical values but still reasonable such that the model structure has a higher probability to be maintained. For sensitivity analysis, the bounds need to be widened; in that case, the Sandblom-Banasik-Parlar procedure will be adopted. Below a detailed discussion on how our control bounds are generated is provided; this seems useful as the bounded control approach is rarely seen in the empirical literature.

8.2.1 Bounds for the Monetary Control (M1)

The point of reference is the Bank of Canada (BOC) data on M1 target range. Table 8.1 exhibits M1 targets as announced by the BOC.

TABLE 8.1: Bank of Canada Target Range for the Rate of Growth of M1 (S.A.)

PERIOD	TARGET RANGE (%)
May 1975 - February 1976	10 - 15
March 1976 - May 1977	8 - 12
June 1977 - May 1978	7 - 11
June 1978 - December 1978	6 - 10

Source: Bank of Canada (data requested)

We have two choices: either accept the BOC target range as it is or expand it within reasonable limits. The latter is taken for two reasons:

i) Past monetary policy might not be optimal. The range could be expanded to allow some more room for optimization.

ii) Theoretically when the range has been established, actual data should fall inside it. Broadly speaking, M1 growth stayed inside the target band. But on a quarterly basis, actual M1 (S.A.) fell outside on several occasions (four times M1 was small than its lower bound, and three times it was higher than its upper bound).

One of the reasons why M1 strayed from its target band would stem from the original procedure used by the BOC for the period from May 1975 to May 1977. The method was as follows: first a base value was chosen, then to it applied the lower and upper bound rates (on a monthly basis) to generate monthly data. It results that for the first month, lower and upper bounds have a common value, but they increasingly diverge over time until a new target is set up; there is consequently a high chance that actual data fell outside bounds during initial periods. It might be due to this drawback that the BOC switched to another procedure from June 1977 on. The second is a little bit complicated and needs an example. Suppose the target range was announced to be 6 - 10% with a base value BV. For the first month the lower bound is (.98).BV, and the upper bound (1.02).BV because the range is equal to 4% (=10-6). A common rate of (6 + 10)/2 = 8% (on a monthly basis) is then applied to initial lower and upper bounds to generate the series.

In view of the problem arisen from the first method, we adopt the second uniformly for the whole period 1975 III - 1978 IV. We begin by attempting to extend the BOC band by ±1%, i.e.,:

May 1975 - February 1976: 9% - 16%

March 1976 - May 1977: 7% - 13% etc...

We find that actual S.A. M1 is still higher than our upper bound in 1975 III by 45 million (due to the postal strike), smaller than our lower bound in 1976 IV by six million and in 1977 I by one million. The gaps in 1976 IV and 1977 I are negligible, but the one in 1975 III is significant. We might choose that target band (i.e., BOC range ±1%) except the M1 value in 1975 III equals the actual level. The problem is that the correction in 1975 III will cause some inconsistency in the data series. We decide to expand the BOC target band a little more.

What we have observed is that it is necessary to reduce the BOC lower bound by at least 1% to prevent actual M1 to fall outside bounds. From that observation we arrive at two alternatives for M1 target range (exactly target range for the rate of growth of S.A. M1).

<u>First alternative:</u> BOC Lower Bound -1%,
BOC Upper Bound +2%, i.e.:

May 1975 - February 1976: 9% - 17%

March 1976 - May 1977 : 7% - 14% etc.

The range is quite realistic since if M1 is always at the upper bound, on the average it will be 1.15% higher than the actual value.

<u>Second alternative:</u> BOC target range ±2%.

An intermediate case would be: BOC Lower Bound -1%, BOC Upper Bound +3%. But we find the results fall between those of the first two alternatives, which renders this case in fact redundant.

We believe expanding the BOC target range more than that of the second alternative would make the announced policy lose its significance as the band is now too large.

We also think the first alternative is most realistic. Once target range has been chosen, we can generate lower and upper bounds for S.A. M1 which are then transformed into N.S.A. data by using the ratio of N.S.A. to S.A. data of M1.

As an abbreviation, we will refer to the two above alternatives of bounds for M1 respectively as BND1(M1) and BND2(M1). Note that BND1(M1) is applied to the first alternative.

8.2.2 Bounds for the Fiscal Control (G1)

In the empirical optimal control literature, the desired value of (real) government spending is usually assumed to grow exponentially at the same rate as (real) potential GNP (Holbrook, 1973, p. 45; Pindyck, 1973, p. 107). Unfortunately, no explanation was provided. Crozier (1977) from the Conference Board estimated the growth rate of full-employment GNP to be 5.4% during the period 1966-1973. In the same period actual GNP and total government spending respectively grew at 5.54% and 5.61% (5.38% for G1). Considering this evidence, a plausible explanation is that over a fairly long period the ratio of government spending over GNP changes little. If we assume it constant, the rates of growth of GNP and government spending would be the same. As actual GNP is closer to potential GNP over a long period than over a yearly basis, the assumption that the desired value of government spending grows at the same rate as potential GNP would be meaningful.

Given the significance of the assumption, for our first upper bound we assume G1 grows at 5% (recall the rate of growth of potential GNP has been assumed to be 4.9%). Examining historical data, we find from 1974 to 1978, the average rates of growth of GNP, total government spending, and G1 were respectively 3.20%, 2.49%, and 2.47%. From 1975 to 1978, they were 3.10%, 2.02%, and 1.94%. Clearly, for the optimization period the rate of 5% applied to G1 upper bound is much higher than its actual value. However, it should not be discarded a priori for this reason because in that period, due to the authority's concern about inflation, government spending was reduced below its trend, and an optimal policy might reset government spending at the normal level.

As a second alternative for G1 upper bound, we choose a 4% rate, i.e., G1 is assumed to grow at 4% annually. This alternative is more restricted but more realistic for the period under consideration. We cannot go below 4% because in 1975 total government spending and G1 grew at 4% and 3.86% respectively (below 4%, actual G1 can fall outside bounds).

In short, two rates are chosen (4% and 5%) and applied to quarterly S.A. G1 (the initial period was 1975 II). The S.A. series are then transformed into N.S.A. data by using the ratio of N.S.A. to S.A. G1.

b. <u>Lower bound</u>

For lower bound we might think that a zero growth rate for G1 would be desirable. In reality, there are two minor problems: i) actual S.A. G1 in 1975 II (initial period to generate bound series) was higher than its level in 1975 III, ii) there was a negative growth in 1976. These lead us to set the lower bound as follows:

i) From 1975 III to 1976 IV: S.A. G1 is assumed to grow annually at -2%.

ii) From 1977 I to 1978 IV: G1 will be constant and equal to its value in 1976 IV (i.e., zero growth rate).

Note that the same lower bound is valid for both alternatives chosen for G1 upper bound. In short, as an abbreviation we will refer to the two kinds of bounds for G1 as BND1(G1) and BND2(G1). Note that BND1(G1) denotes bounds using a 4% rate for G1 upper bound.

8.3 DETERMINATION OF WEIGHTS

8.3.1 Search Method

What are the numerical values to be assigned to the weights w1, w2, w3, and w4? This is an important question. Sadly, it is often neglected. Sandblom (1979)[62] provided an interesting numerical example in searching for the appropriate weights. He took the idea from Rustem, Velupillai, and Westcott (1976). Let us suppose the objective function contains only two targets with the values of w1 and w2 to be assigned. w1 can be arbitrarily set equal to 1. The idea from Rustem, Velupillai, and Westcott is that

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62 See also Sandblom (1980).

we can start with any w2 and find the corresponding optimal path. The result should be shown to the policy-makers whose comments would be used to update w2. The model would be optimized again, with result shown to policy-makers and so on. If the policy-makers are consistent, the procedure can be made to converge.

Based on Rustem et al. idea, Sandblom's approach starts with any arbitrary value of w2. The model is optimized; optimal trajectories of targets are plotted against their historical counterparts. Depending on the nature of deviations, w2 would increase or decrease. After many attempts, hopefully the historical trajectories can reveal the policy-makers' preferences judged by the actual historical performance.[63]

It must be emphasized that in Sandblom's approach, the historical performance is an important guide to reveal policy-makers' preferences. It seems worthwhile to take a short pause and say a few words about the main features of the Canadian macroeconomic policy from 1975 to 1978.

8.3.2 Digression: A Few Words on the Canadian Macroeconomic Policy from 1975 to 1978[64]

From 1971 to 1974, the Canadian money supply (M1) grew at an annual rate of 12.8% as compared to 5.9% in the

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63 For an empirical econometric approach to revealed preferences, see Friedlaender (1973). Within a rational expectations model see Chow (1981, Chapter 16).

64 Parkin (1982, Chapter 37) provided a very brief summary. Detailed discussions can be found in Courchene (1981b), Donners and Peters (1979), Barber and McCallum (1980).

1967-1970 period. In 1974, the rate of inflation based on the GNP implicit price index was 15.3%. These led to an important change in the Canadian monetary policy in 1975. In the direction of combatting inflation, the Bank of Canada embarked upon a new monetary policy: from May 1975, it announced targets for M1 which were revised downward several time during our optimization period. This resulted in an annual rate of growth of 10.03% for M1 in the period 1975-1978. To many economists, by adopting a target for M1 and for its concern more with inflation than unemployment, the Bank of Canada has followed a monetarist monetary approach. Whether it has pursued it strictly and consistently is another issue. Furthermore, the Bank of Canada also called for a tighter fiscal policy and a move to a more balanced federal budget. In the fall of 1975, the Government of Canada launched a wage and price control program (which ended in December 1978). At the same time, the government spoke of a need for fiscal restraint. As far as government spending was concerned, the actual course of fiscal policy in the four years of 1975 to 1978 was in the direction of restraint: from an annual rate of growth of 4.91% for the period 1970-1974, total government spending (real) reduced to 2.02% for the period 1975-1978. In terms of the share of government spending in real GNP, it is also true that fiscal policy was tighter. However, in this period government deficits soared; lower revenues were due to many reasons: high unemployment, indexing of personal income tax exemptions, indexing of social assistance benefits etc...(see Donner and Peters, 1979, p. 75). On that respect, certain economists would not accept that fiscal policy was restrained (Wirick, 1981, e.g.).

Table 8.2 below summarizes some statistics pertaining to GNP growth, the rates of inflation and unemployment. It shows that, for the period 1975-1978 the inflation was reduced at the expense of high unemployment. It is obvious that policy-makers paid more concern to inflation than unemployment. This indicator is important in the task of determining the weights of the objective function to which we now turn.

TABLE 8.2: Output Growth,Unemployment and Inflation Rates (%)

YEAR	GNP	UNEMPLOYMENT	INFLATION (GNP Deflator)
1973	7.5	5.5	9.2
74	3.6	5.3	15.3
75	1.2	6.9	10.8
76	5.5	7.1	9.5
77	2.2	8.1	7.0
78	3.4	8.4	6.4
79	2.7	7.5	10.3

Source: Economic Review, Department of Finance, Canada, 1980, Reference Tables 4, 28, and 43.

8.3.3 Findings

In the experiments below, both fiscal and monetary controls are used in combination since this best reflects the reality; moreover, the most realistic and restricted bounds are selected, i.e., BND(M1) for M1 and BND(G1) for G1.

When values are assigned to the weights, one of them can be treated as the base and be arbitrarily given a value of 1. We choose w3 (inflation rate) as the base.

As a starting point, we arbitrarily pick up the weights[65] used by U.S. models participating in the NBER/NSF seminar mentioned earlier:

w1 = .75 (GNP gap)

w2 = .75 (unemployment rate)

w3 = 1 (inflation rate)

w4 = 1 (trade balance)

Instead of plots, results from historical and optimal runs are illustrated in Table 8.3. Average values of targets in the entire period are reported in conjunction with those of controls. As GNP is an important statistic and much easier to evaluate, we also include it. For our present task, more attention should be paid to the four targets. Note that the GNP gap is denoted by YGAP.

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65 Klein and Su (1980, p. 242) explained why the weights were given these values.

TABLE 8.3: Historical and Optimal Solutions with U.S. Weights w1 = w2 = .75 and w3 = w4 = 1: Average Values of Real GNP, Target and Control Variables.

ITEM	HISTORICAL	OPTIMAL
Y	30428	31026
YGAP	-6.748	-4.979
UR	7.688	7.463
INFL	8.084	8.677
TB	-2.339	-2.769
G1	5715	6007
M1	19460	19614

Note: In this table and elsewhere, YGAP, UR, INFL, and TB are expressed in percent, Y and G in millions of 1971 dollars, and M1 in millions of current dollars.

With the weights as assigned above, the total values of the welfare loss function (LOSS) are respectively 799.29 and 608.234 for historical and optimal performances. If the optimal policy was implemented, the economy could be improved by 23.9% [= 1 - (608.234/799.29)]. Unfortunately, the above optimal solution does not seem realistic since large divergences exist between actual and optimal targets, especially for YGAP and INFL. Let us examine the variable INFL. The optimal value (average) increases by 7.34% [= 1 - (8.677/8.084)] compared to the historical level.[66] On

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66 Relative comparison between two values of a variable measured in percent (inflation or unemployment rate for example) is calculated as 1 minus the ratio of the two values, and not as difference between the two values, unless indicated otherwise.

the other hand, Table 8.3 (last Column) shows that in 1978 the actual rate of inflation was reduced only by 8.57% [= 1 - (6.4/7)] relative to the previous year's level. Clearly, the optimal solution yields too much higher inflation to be accepted by the policy-makers whose main objective was to combat inflation. The obvious remedy is to impose lower weight on YGAP so that its optimal value will be closer to its actual one, and higher weight on INFL (giving lower penalties to YGAP, UR...is equivalent to attributing higher weight to INFL).

After many trials in the direction just described, a 'convergent' solution is found with the following weights w1 = .15, w2 = .50, and w3 = w4 = 1. To prove that it is indeed convergent, in Table 8.4 we report it together with its two adjacent solutions (immediately before and after it). The difference among them lies in the value of w1. It should be understood that the 'convergent' solution can be obtained in principle by a multitude of different weight combinations.

Two evidences show that the 'convergent' solution is truly converging.

i) If we compare the 'convergent' and 'before-convergent' solutions to the historical one (shown in Table 8.3, 2nd column), we can observe that the optimal values of the four targets from the 'convergent' are closer to historical values than those from the 'before-convergent'. Compared to the 'after-convergent', the 'convergent' is still closer to the historical solution in three targets except YGAP.

TABLE 8.4: Comparison of the 'Convergent' Solution and its Neighbours: Average Values of Real GNP, Target and Control Variables.

ITEM	BEFORE	CONVERGENT	AFTER
	w1=.20 w2=.50 w3=w4=1	w1=.15 w2=.50 w3=w4=1	w1=.10 w2=.50 w3=w4=1
Y	30781	30655	30452
YGAP	-5.752	-6.131	-6.731
UR	7.626	7.675	7.742
INFL	8.188	8.055	7.884
TB	-2.493	-2.400	-2.256
G1	5898	5831	5722
M1	18889	18841	18841

ii) On the other hand when we examine the relationship between unemployment and inflation among all three optimal solutions, the 'convergent' is the only solution which is characterized by the simultaneous improvement of UR and INFL relative to historical levels. Beyond the 'convergent' on either side, INFL or UR can be improved only at the expense of the other.

From these facts the solution with w1 = .15, w2 = .50, and w3 = w4 = 1 can be considered to converge. The question is: should we accept the 'convergent' solution as a representative of the policy-makers' preferences, or should we adopt another which allows more room for optimization? The reason why it was raised is this. Recall that in the Introduction Chapter, we mentioned about the Canadian symposium on the topic 'Has Monetarism Failed?' in which the majority of the papers concluded that in the five years following the Bank of Canada new monetary policy, inflation has been bought at a higher price in terms of unemployment and lost output. Examining Tables 8.4 and 8.3 closely, we find the 'convergent' solution has improved both INFL and UR, but the improvement is quite small compared to historical policy: for example UR is just 0.2% [=1 - (7.675/7.688)] lower, and INFL 0.4% [=1 - (8.055/8.084)] lower. In other words, is there any reason to believe that the past policy has been near the most desirable performance? To answer the question requires us to find out how the economy would be improved with respect to each of the three above optimal policies. This necessitates computing the values of the loss function from all three runs in conjunction with their historical counterparts. What we have obtained is that the economy would be 8.74% better (relative to the historical performance) if the 'before-convergent' was adopted, 7.74% better with the 'convergent', and 6.77% better with the 'after-convergent'. The implication is obvious: to move to a better optimal solution in terms of welfare loss, high penalty should be imposed on the target YGAP; put it in another way, policy-makers should be more concerned about output and employment. If that was the case, would realism allow us to chose any policy with w1 > .20 (the remaining weights remain unchanged)? Attempting the experiment with w1 = .25, we find that the economy would be improved by 10.57% relative to the historical policy; but careful consideration reveals that the optimal solution would no longer reflect past reality. This will be discussed in detail in the sensitivity analysis experiment. For the time being, we just like to make a remark that with w1 = .25 (and the remaining weights unchanged) the optimal solution would imply a set of relative weights diverging much from that of the past policy. To see the point, assume there exist, in the criterion function, only two targets: $y_{1,t}$ and $y_{2,t}$ with $y^{*}_{1,t}$ and $y^{*}_{2,t}$ as their desired values. The individual costs of deviation are then i) $q\Sigma(y_{1,t} - y^{*}_{1,t})^2$ and ii) $v\Sigma(y_{2,t} - y^{*}_{2,t})^2$ where q and v are the weights. Suppose now that policy-makers attach an equal (total) cost to each target. That implies

$$q\Sigma(y_{1,t} - y^{*}_{1,t})^2 = v\Sigma(y_{2,t} - y^{*}_{2,t})^2$$

We can arbitrarily set q = 1, and the relative weight v can be derived from the equality,[67] given the values of $y_{1,t}$, $y^{*}_{1,t}$, $y_{2,t}$, and $y^{*}_{2,t}$. Let us extend the analysis to our four targets, and let $y_{1,t}$ stand for INFL and $y_{2,t}$ for YGAP, UR, and TB in turn. As in the two-target case, when q (temporarily the weight of INFL) is set equal to 1, the above equality can be used to determine for historical and various optimal solutions, the relative weights of YGAP, UR, and TB

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67 For a somewhat similar weighting scheme, see Stanhouse and Fackler (1980, p. 385).

respectively.[68] Denote those weights by v_{YGAP}, v_{UR}, and v_{TB}. Table 8.5 reports the results of the calculations.

TABLE 8.5: Values of Relative Weights among Historical and Various Optimal Solutions

RELATIVE WEIGHTS	HISTORICAL	OPTIMAL		
		w2 = .50, w3 = w4 = 1		
		w1 = .15	w1 = .20	w1 = .25
v_{YGAP}	0.18	0.22	0.26	0.29
v_{UR}	1.10	1.08	1.18	1.26
v_{TB}	1.31	1.29	1.26	1.28

Note: All three optimal solutions differ in the value of w1 (= weight of YGAP). Recall when w1 = .15, it is the 'convergent' solution and when w1 = .20, it is the 'before-convergent' solution.

Looking at Table 8.5, one can observe that the set of relative weights generated by the 'convergent' solution (in which w1 = .15) is close to that of the historical solution. As one moves away from it, the divergence becomes larger and largest with the solution in which w1 = .25.

In short, if we like to achieve a better optimal solution beyond the 'convergent', it seems that the most we can accomplish without much distorting the policy makers' past preferences is the 'before-convergent'. Unfortunately, the later registered almost no improvement with respect to

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68 That means, equating the cost (total) of INFL with that of YGAP, then with that of UR etc...One should be aware in the case of more than two targets, the above procedure neglects to take into account the relative balance among all the weights.

the former (in terms of welfare loss) to prefer it, to believe that its adoption could affect the conclusions drawn from the 'convergent' solution.

There are other reasons to select the 'convergent' as explained carefully in Section 3.2.2: namely, concern with Lucas's critique on the model structure invariance, choice of a least arbitrary set of weights to be used for a high number of experiments etc...

Being satisfied with our 'convergent' solution, in the remainder of this chapter, we will present graphs (Figs. 8.1 to 8.6) displaying the trajectories of historical versus optimal values of targets and controls.[69] The plots, we believe, will give us some feeling about the search methodology. Note that we plot GNP rather than GNP gap as both can be considered as two sides of a same coin. A quick glimpse shows that between historical and optimal paths, fluctuations are small for targets but larger for controls. On the average, G1 is 2.03% higher than its historical value, while M1 is 3.18% lower. From our results, G1 is found to fall sometimes at lower bounds, sometimes at upper bounds, and sometimes inside. M1 is always at lower bounds: that behaviour is understandable and consistent with a policy which puts high weight in fighting inflation (more on this in the next chapter).

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[69] Appendix B (Tables B.1 to B.6) provides the data.

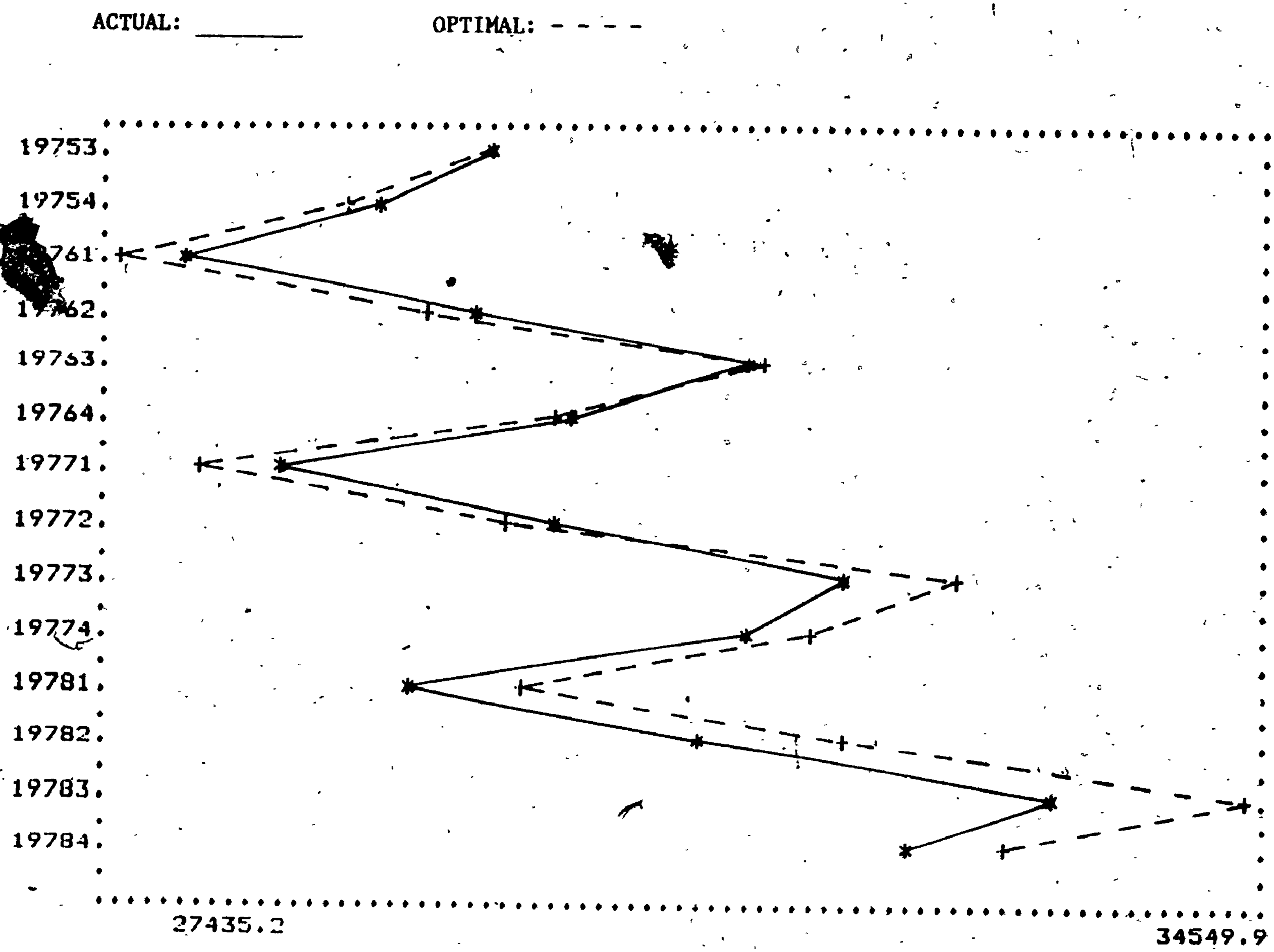

FIGURE 8.1: Actual and Optimal Y (Real GNP). Millions of 1971 Dollars.

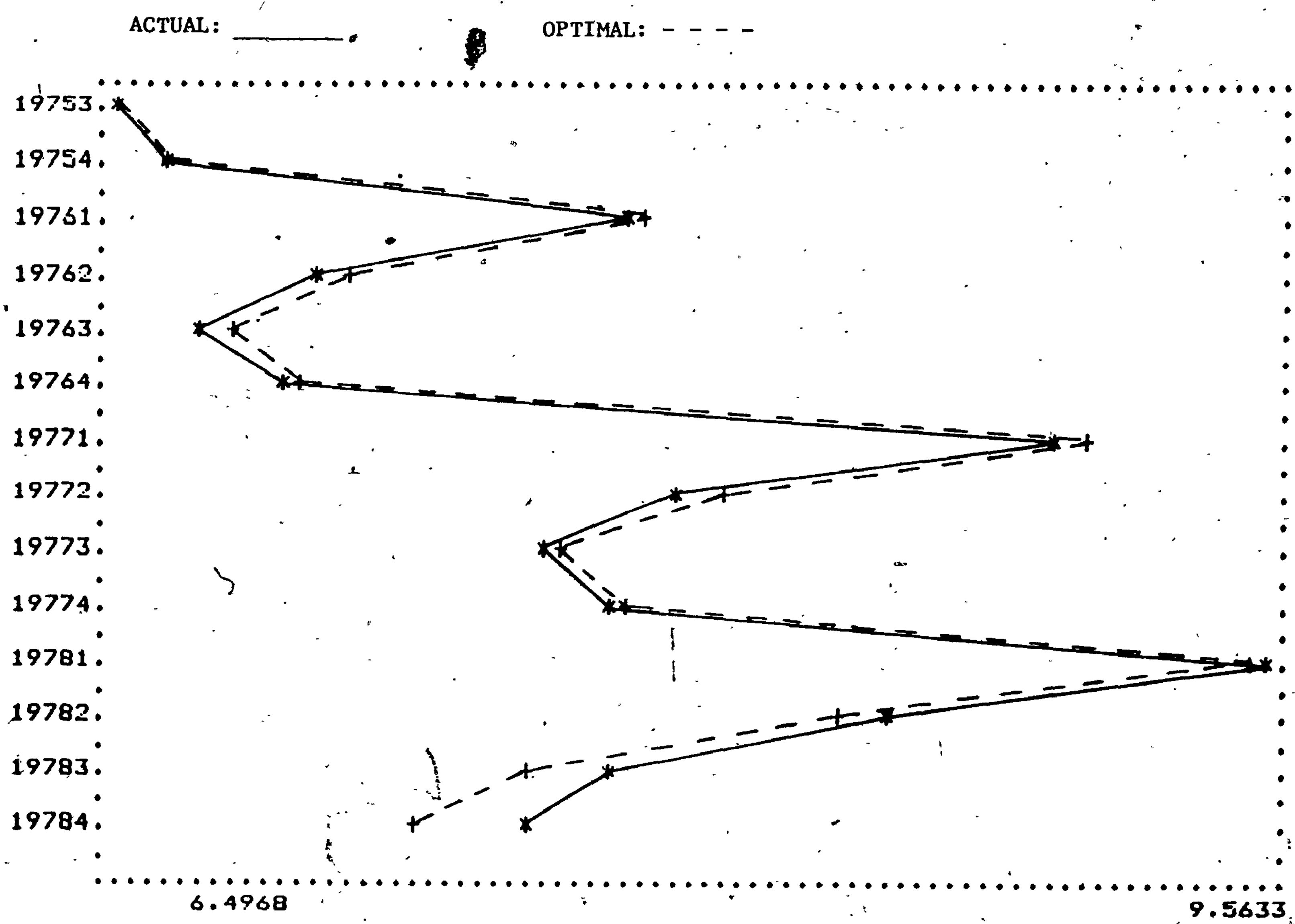

FIGURE 8.2: Actual and Optimal UR (Unemployment Rate). In Percent.

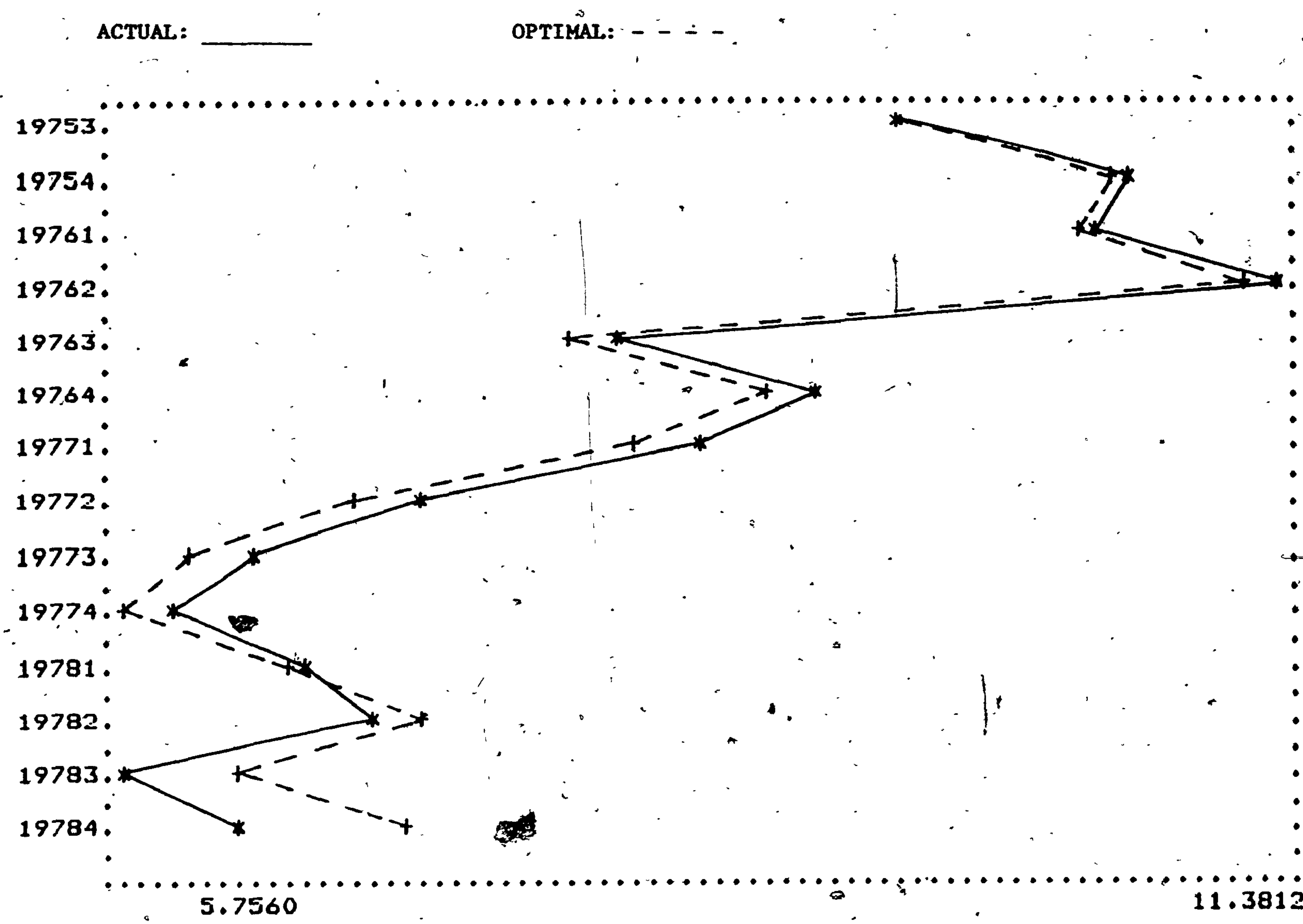

FIGURE 8.3: Actual and Optimal INFL (Inflation Rate). In Percent.

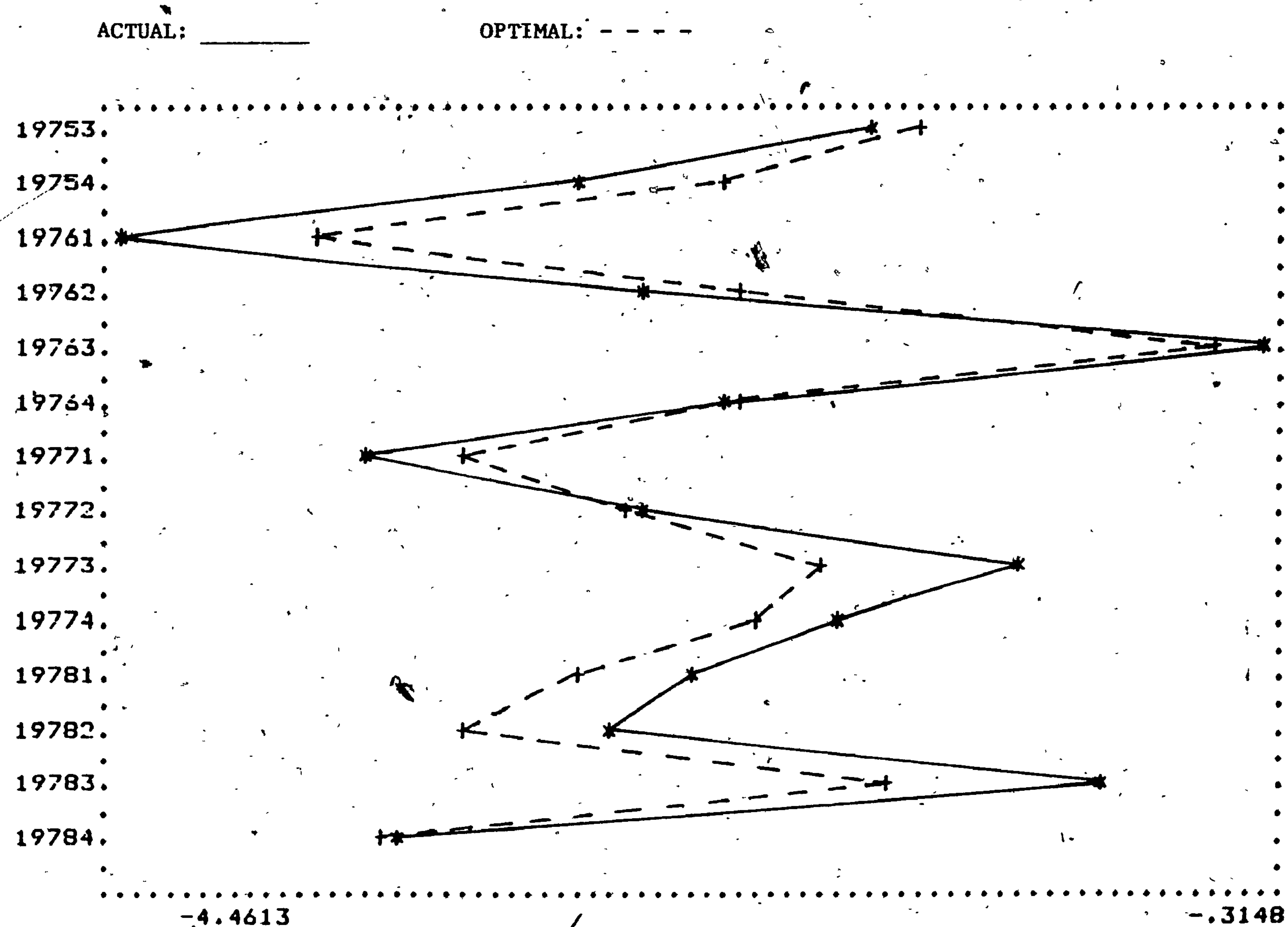

FIGURE 8.4: Actual and Optimal TB (Nominal Trade Balance as a Percentage of Nominal GNP). In Percent.

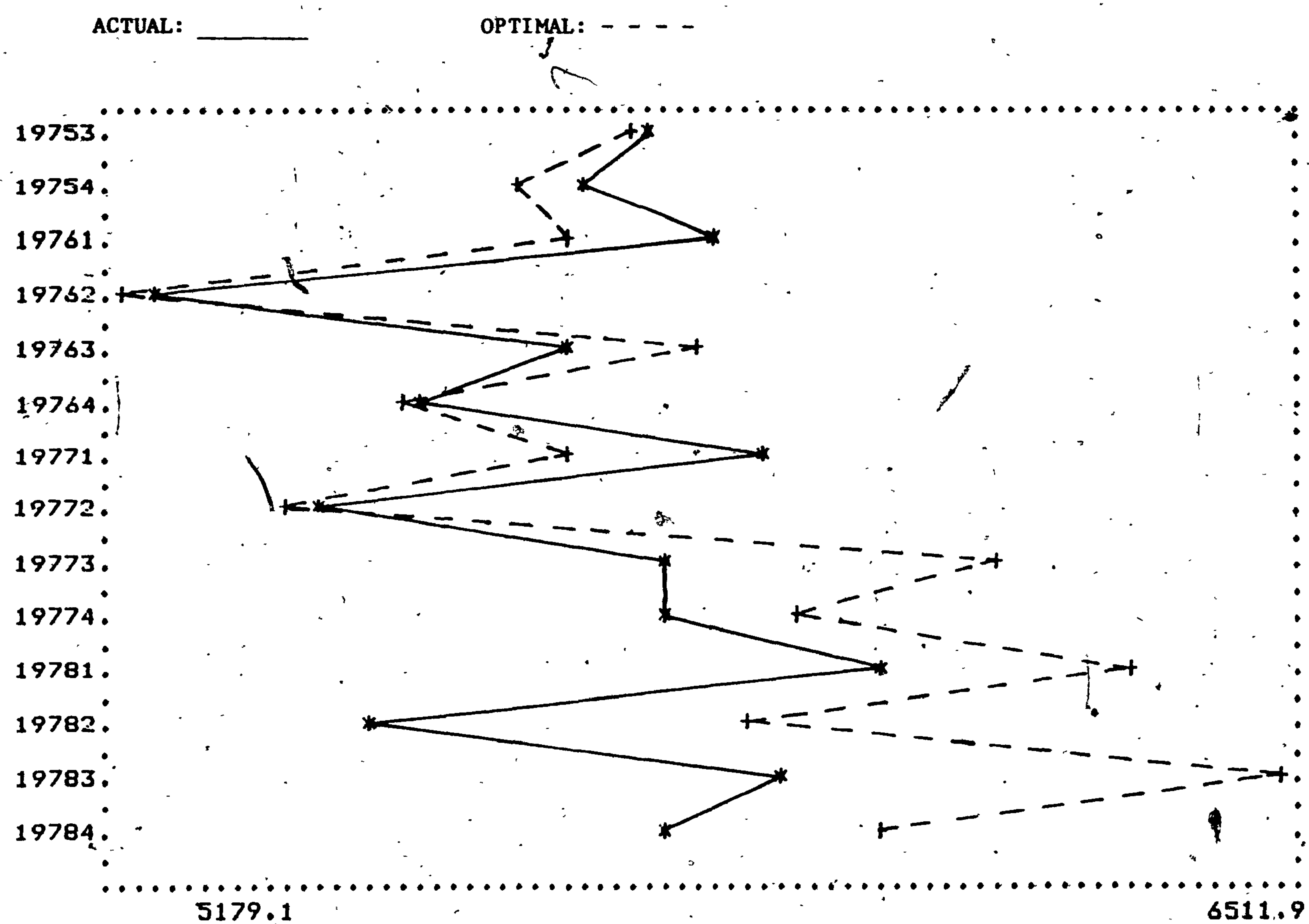

FIGURE 8.5: Actual and Optimal G1 (Real Expenditure of Federal, Provincial, and Local Governments). Millions of 1971 Dollars.

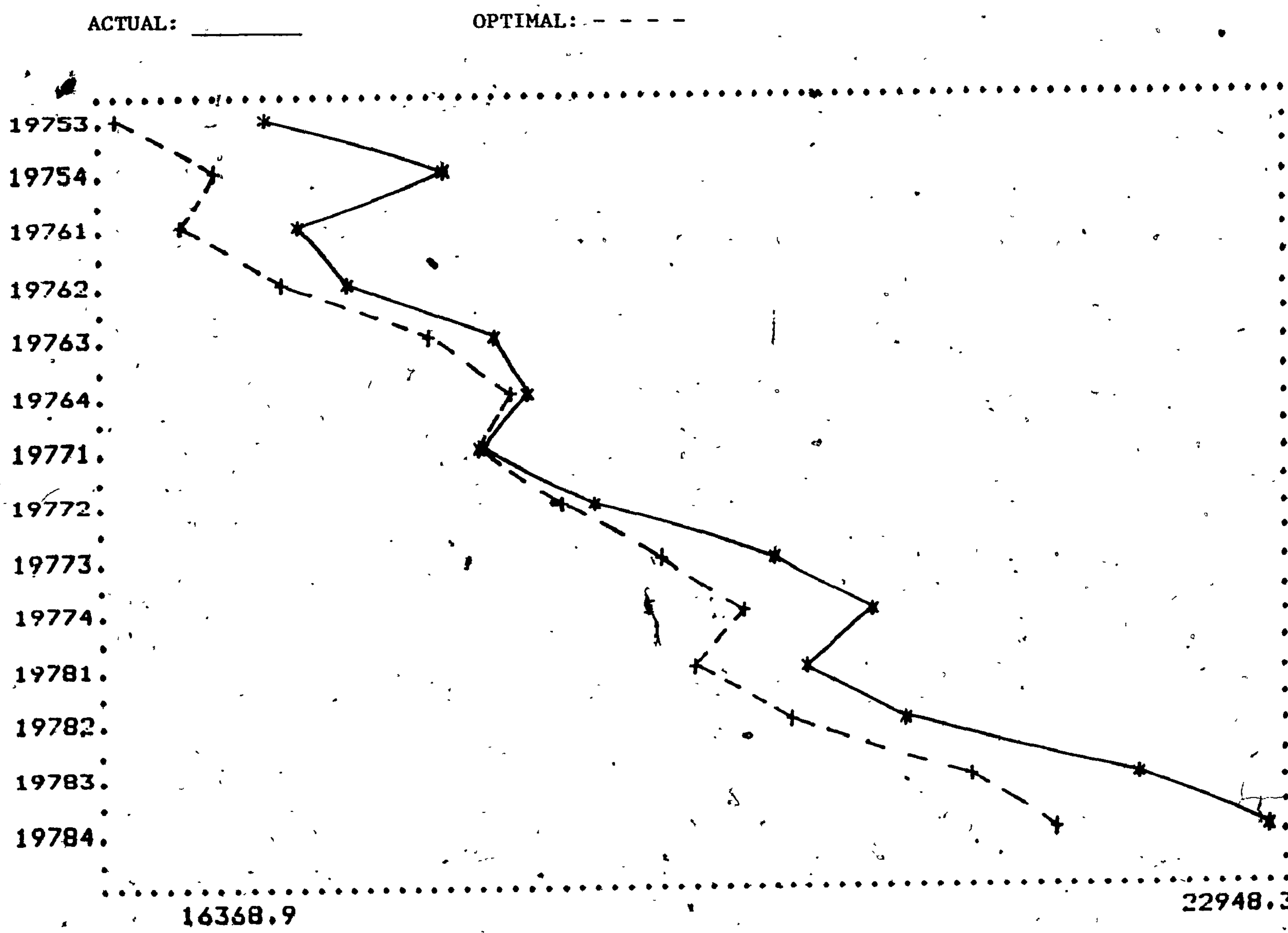

FIGURE 8.6: Actual and Optimal M1 (Narrow Money Quantity). Millions of Current Dollars.

CHAPTER 9

RESULTS OF OPTIMAL CONTROL EXPERIMENTS

This chapter is the condensation of results from all optimization experiments which have been described rather carefully in Chapter 3. To recapitulate, the experiments have been classified by three categories. The first includes monetary-fiscal policies without the element of inside lag which is injected later in the second. The third concerns sensitivity analysis.

As a number of experiments are to be performed, it is impossible and impractical to give each equal treatment as far as analysis is concerned. More focus is devoted to the basic experiments (Category I) because they serve as a vehicle of comparison for the rest, and they are apparently most realistic.

Before we discuss the results, three things need to be said.

First, the results are dependent on many factors: e.g., the formulation of the model which is an approximation of the real world, the period chosen for study, the short or long-term nature of analysis. Estimation can also be an influencing factor; particularly, fitting the historical data as close as possible could build into the model some bias which makes optimal solution close to the historical counterpart.[70] Bounds on the control vector, the welfare loss function are other influencing factors. To evaluate the result robustness, efforts will be put to experiment, when necessary, with alternatives of bounds, weights. But the results in no way presume to be definitive; they should be regarded as suggestive.

Second, Lucas's critique on the model structure stability deserves attention. One has to acknowledge the limitations of an economic model and refrain from considering policies that are too far beyond the range of historical experience. In that spirit, our model will not be asked to perform policies such as quick disinflation. Consequently, as far as monetary policy is concerned, the exercises still fall under the gradualism framework,

Third, the exercises are confined to the traditional monetary-fiscal policies as demand management tools. There will be no attempt to examine proposals of wage-price control, tax-based income policy etc...Most economists would believe that such proposals are complementary rather than substitute prescriptions for the stagflation problem.

9.1 EXPERIMENTS WITHOUT LAG CONSIDERATION

The discussion begins with the basic experiment for monetary-fiscal policies. In the monetary policy, M1 is a control variable (i.e., a discretionary instrument in Abel's

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70 The bias problem was pointed out to me by Professor Jackson.

terminology) while G1 is set exogenous. The reverse holds in the fiscal policy. In the joint policy both M1 and G1 are discretionary.

The discussion then extends to experiments which let the policy variables follow a passive rule, i.e., the policy variables are control variables subject to constant growth rate.

9.1.1 <u>Basic</u> <u>Experiments</u>

We might recall for each control variable there are two bounds which have been referred to as BND1(M1), BND2(M1) for M1 and BND1(G1), BND2(G1) for G1. These bounds generate two experiments for the monetary policy, two for the fiscal policy, and four for the joint policy. Tables 9.1, 9.2, 9.3 and 9.4 present results with respect to the values of the welfare loss function (LOSS), of targets, and controls. The value of GNP is also reported along the value of GNP gap (YGAP). Please note that total value is measured for LOSS while the average values are calculated for the remaining variables.

Carefully examining Tables 9.1 to 9.3, one can realize in terms of welfare loss, only a small improvement is gained from using the large bounds BND2(M1) and BND2(G1). For the sake of analysis, the general discussion can be restricted from now on to the experiments with narrow bounds without causing distortion.

The next tables (Tables 9.4 to 9.7) summarize essential information for an understanding of the monetary-fiscal policy performance in the period 1975 III - 1978 IV.

Several interesting things can be observed.

i) By the criterion of welfare loss all optimal policies perform better than the historical one (Table 9.4). Although the fiscal policy is most successful in terms of real GNP, the joint policy is found to be the best in terms of welfare loss. This is true mathematically since any optimal solution to the above monetary or fiscal model is a feasible solution to the joint policy model, and not vice versa.[71] If the joint policy was implemented, in terms of welfare loss the economy would be improved relative to the historical policy by 7.25% (= 100% - 92.752% from Table 9.5). Among the two instruments, the fiscal tool contributes most. Its policy would be 4.5% better than the historical policy while the monetary policy would register a gain of 2.54% (Table 9.5, Row 1). These statistics, however, do not indicate a great effectiveness of the fiscal policy relative to the monetary policy as multiplier analysis has demonstrated in Chapter 7. As we progress with more experiments, results will give more indication that in a multi-target framework and under a certain circumstance, multiplier analysis could be a misleading tool in evaluating relative effectiveness.

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71 This was pointed out to me by Dr. Drud (author of the CONOPT).

TABLE 9.1: Monetary Policy (Basic Experiments): Total Value of Welfare Loss and Average Values of Real GNP, Target and Control Variables.

ITEM	BND1(M1)	BND2(M1)
LOSS	359.402	358.018
Y	30377	30368
YGAP	-6.905	-6.932
UR	7.720	7.725
INFL	7.989	7.977
TB	-2.191	-2.165
G1	5715	5715
M1	18855	18763

TABLE 9.2: Fiscal Policy (Basic Experiments): Total Value of Welfare Loss and Average Values of Real GNP, Target and Control Variables.

ITEM	BND1(G1)	BND2(G1)
LOSS	352.181	351.863
Y	30665	30689
YGAP	-6.097	-6.030
UR	7.664	7.659
INFL	8.090	8.099
TB	-2.522	-2.540
G1	5803	5815
M1	19460	19460

TABLE 9.3: Joint Policy (Basic Experiments): Total Value of Welfare Loss and Average Values of Real GNP, Target and Control Variables.

ITEM	BND1(G1)	
	BND1(M1)	BND2(M1)
LOSS	342.048	340.227
Y	30655	30653
YGAP	-6.131	-6.139
UR	7.675	7.676
INFL	8.055	8.052
TB	-2.400	-2.375
G1	5831	5837
M1	18841	18721

TABLE 9.3. (Cont'd): Joint Policy (Basic Experiments): Total Value of Welfare Loss and Average Values of Real GNP, Target and Control Variables.

ITEM	BND2(G1)	
	BND1(M1)	BND2(M1)
LOSS	341.588	339.729
Y	30687	30686
YGAP	-6.044	-6.045
UR	7.668	7.668
INFL	8.068	8.066
TB	-2.423	-2.398
G1	5846	5853
M1	18841	18718

TABLE 9.4: Historical and Optimal Policies (Basic Experiments): Total Value of Welfare Loss and Average Values of Some Important Variables.

ITEM	HISTORICAL	MONETARY	FISCAL	JOINT
LOSS	368.776	359.402	352.181	342.048
Y	30428	30377	30665	30655
YGAP	-6.748	-6.905	-6.097	-6.131
UR	7.688	7.720	7.664	7.675
INFL	8.084	7.989	8.090	8.055
TB	-2.339	-2.191	-2.522	-2.400
G1	5715	5715	5803	5831
M1	19460	18855	19460	18841
BOND	97252	97411	97069	97450
CON	19212	19179	19353	19346
EX	6933	6923	6941	6927
IM	8138	8120	8223	8228
IRCNR	5862	5838	5907	5893
L	9672	9663	9676	9673
LF	10479	10472	10480	10478
M2	81430	80823	81544	81100
PFX	1.058	1.050	1.053	1.045
RS	8.533	8.715	8.578	8.777

Note: In this table and elsewhere YGAP, UR, INFL, TB, and RS are in percent, Y, G1, CON, EX, IM, IRCNR in millions of 1971 dollars, M1, BOND, M2 in millions of current dollars, and L, LF in thousands of persons.

TABLE 9.5: Selected (Optimal/Historical) Ratios in Percent from the Basic Experiments.

ITEM	MONETARY	FISCAL	JOINT
LOSS	97.458	95.500	92.752
Y	99.832	100.779	100.746
UR	100.416	99.688	99.831
INFL	98.825	100.074	99.641

Note: Ratios are obtained from Table 9.4 by dividing the optimal levels of welfare loss (LOSS), real GNP (Y), unemployment rate (UR), and inflation rate (INFL) by their historical levels.

TABLE 9.6: Historical and Optimal policies (Basic Experiments): Total Values of Absolute Losses of Target Variables.

ITEM	HISTORICAL	MONETARY	FISCAL	JOINT
YGAP	100.752	104.966	79.854	80.737
UR	55.001	56.391	53.706	54.161
INFL	120.817	116.399	118.140	116.495
TB	92.203	81.646	100.481	90.654
TOTAL	368.773	359.402	352.181	342.048

TABLE 9.7: Historical and Optimal Policies (Basic Experiments): Percentage (%) Losses of Target Variables.

ITEM	HISTORICAL	MONETARY	FISCAL	JOINT
YGAP	27.32	29.21	22.67	23.60
UR	14.91	15.69	15.25	15.83
INFL	32.76	32.39	33.55	34.06
TB	25.00	22.72	28.53	26.50
TOTAL	100.00	100.00	100.00	100.00

ii) With the monetary policy the rate of inflation declines relative to its historical level, but the rate of unemployment turns out to be higher; the opposite occurs to the fiscal policy (see Table 9.4). This feature displays the particular strength of each instrument. A coordination is desirable since the joint policy has brought down inflation and unemployment simultaneously.

iii) Unfortunately, the gain of the best optimal policy (i.e., the joint policy) relative to the historical policy is modest, if not negligible in terms of UR, INFL, and welfare loss. The optimal values of INFL and UR are respectively 8.055% and 7.675% versus 8.084% and 7.688% from the past (Table 9.4). Expressed in optimal/historical ratios, the results amount to 99.641% and 99.831% for INFL and UR (Table 9.5).[72] In terms of average annual rate of growth, optimal real GNP grows at 3.8% for the full period 1975 - 1978,[73] compared to an actual 3.1%. The optimal rate is still much lower than those achieved right before the monetary restraint: for example, the actual rate was 6.03%

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72 In simulation exercises the RMSPE (root-mean square percent error) statistic is commonly reported to evaluate how the model fits historical data. That is not the practice in reporting optimal results. There should be two reasons. First reproducing the historical path of the economy is no longer the objective. Secondly, in the RMSPE computation the sign of the deviation is of no concern; in optimization exercises, one is much concerned with whether the optimal values of certain important variables are higher or lower than the historical ones. Based on average values, we have calculated optimal/historical ratios (Table 9.5) which show that the optimal values of INFL and UR in the joint policy are very close to their actual ones. It might be useful to compute another summary statistic to confirm such closeness. Due to the second reason mentioned above, the RMSPE is not appropriate. It is decided to calculate the mean percent error (MPE) defined as

$$\frac{1}{n} \Sigma \frac{(X_t^o - X_t^a)}{X_t^a}$$

where X^o and X^a are optimal and actual values of X and n is the number of observations (see Pindyck and Rubinfeld, 1976, p. 317). From our calculations the MPE(s) of INFL, UR, and Y are respectively -0.025%, -0.148%, 0.684%.

73 To compute the optimal annual rate of growth in 1976, optimal data for 1975 I, II are required. They are not, however, available since the optimization period starts from 1975 III on. This forces us to use actual data for 1975 I, II and calculate the average annual rate for the full period 1975 I - 1978 IV rather than restrict to the optimization period 1975 III - 1978 IV. Furthermore, without using actual data for 1975 I, II, there is no clear-cut and consistent way of calculating the annual rate of growth for the two quarters 1975 III and IV.

from 1971 to 1974 and 5.32% from 1970 to 1974. In spite of this, there is some improvement in GNP which is seen to perform better than UR and INFL. It must be emphasized, however, that the average rate of growth statistic can be a bad, if not misleading indicator of performance, because when two data series from two solutions are compared, the rates which purport to measure year-over-year variations of each series, may not capture the relative overall performance, and the result may apparently look better or worse than it truly is. A plausible symptom is that a solution may have a higher average rate of growth but a lower average value of GNP, or vice versa. Indeed, when the historical and optimal GNP data are examined from 1975 to 1977 only, the optimal solution is found to have a higher average annual rate of growth but a lower average value. This inconsistency will also show up in the sensitivity analysis experiments (Section 9.3.2). In short, apart from the GNP average rate of growth, other statistics which may be of much more importance have to be relied upon for a proper evaluation. Let's reexamine the performance of optimal GNP. Other information would indicate that the gain is truly modest: on the average, the optimal value of GNP is only 0.75% higher than the historical level (Table 9.5). When all the improvements in GNP, UR, and INFL are taken into account, the joint policy would improve the economy by a meagre reduction of 7.25% in welfare loss relative to the historical policy.

One might wonder whether results could be improved in case of a more active role from taxation. To answer it we perform an extra experiment in which three control variables are used in combination: M1, G1, and the tax rate. The previous value of the tax rate coefficient was .1767 (see Equation 6.2.9); now it will still be kept fixed but determined optimally. Results show that in terms of welfare loss the revised joint policy still records a negligible improvement of 9.39% relative to the historical performance with the tax rate increasing to .1908 (one would question the implementation of this policy for its unpopularity in the presence of high unemployment and low economic growth). The inflation rate (=7.732) is a little lower than the actual level while the unemployment rate (=7.794) is higher. Real GNP (=30427 million) is the same as its historical counterpart. The values of M1 (nominal) and G1 (real) are respectively 18930 and 5972 million . In short, even with one more control variable the optimal mix could not substantially change past results.

One remark deserves to be mentioned: Certain critics urged a loosening of monetary control (Barber and McCallum, 1981; Peters and Donner, 1981) and tight fiscal policy (Barber and McCallum). Optimal results of the joint policy call for more monetary restraint and less tighter government spending with respect to the historical policy: on the average, the optimal value of M1 is 3.18% lower than the historical level while improving just slightly Y and UR necessitates a relative increase of 2.03% in G1 (Table 9.4). This is in the opposite direction to what Barber and McCallum (1981, p. 226) recommended:

> Thus, the appropriate mix for a floating country such as Canada is surely the opposite one, a relatively easy monetary policy combined with tight fiscal policy.

iv) Tables 9.6 and 9.7 give a breakdown of the total welfare loss into various components. The loss due to high UR looks smallest. The importance of UR, as noted by Klein and Su (1980, p. 245) in a U.S. study, is understated because a part of the loss due to idle capacity (i.e., GNP gap) is contributed by the low level of employment. If one merges together the losses due to high UR and large GNP gap, one can realize that the fiscal and joint policies will enjoy an improvement in GNP and UR at the expense of INFL and trade balance TB: e.g., the combined percentage loss due to UR and GNP gap is reduced from 42.23% (historical) to 37.92% (fiscal), to 39.43% (joint). Part of the worsening of the trade balance can be attributed to higher import originating from higher GNP. On the other hand, there is little difference between the historical and monetary policies.

The above discussion, hopefully, has given us some insight into the performance of the historical and various optimal policies. In the remainder of this section, just to have some flavour we will take a look at the behaviour of certain key variables behind the three optimal policies with more attention paid to the control variables. Table 9.4 in conjunction with Figs. 9.1 to 9.12 form the background of the analysis. For the sake of examination, data in relation to these figures are supplied in Appendix B (Tables B.1 to B.12). Figs. 9.1 to 9.4 display the paths of the four targets (Y instead of YGAP is plotted); Figs. 9.5, 9.6 exhibit the trajectories of the control variables while the rest concerns some important endogenous variables.[74]

Let us first examine the control variables (recall that M1 is exogenous in the fiscal policy and G1 exogenous in the monetary policy). Without surprise, the monetary policy has the lowest level of M1 among the three optimal policies; but the policy which maintains the highest G1 turns out to be the joint policy. This may not be much in agreement with some belief that a combined policy would take advantage of the coordination of both control variables by exhibiting less fluctuations in both controls or utilizing less amount (one has to say immediately that G1 in the joint policy is just a bit higher than that in the fiscal policy). Fig. 9.6 shows that the monetary and joint policies almost

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74 Figs. 9.1 to 9.12 display only the three optimal trajectories (basic experiments). Although it is desirable to plot the historical path alongside, there have been two severe technical obstacles. First, due to the extreme closeness among the four policies in various places, it is almost impossible to draw visible trajectories. Secondly, also due to the closeness, the graphs produced by the TSP program do not generate, for each quarter, four different symbols corresponding to the four data series; in general, only one or two symbols are generated, which forces us to inspect individual observations for every quarter even in drawing the three optimal trajectories.

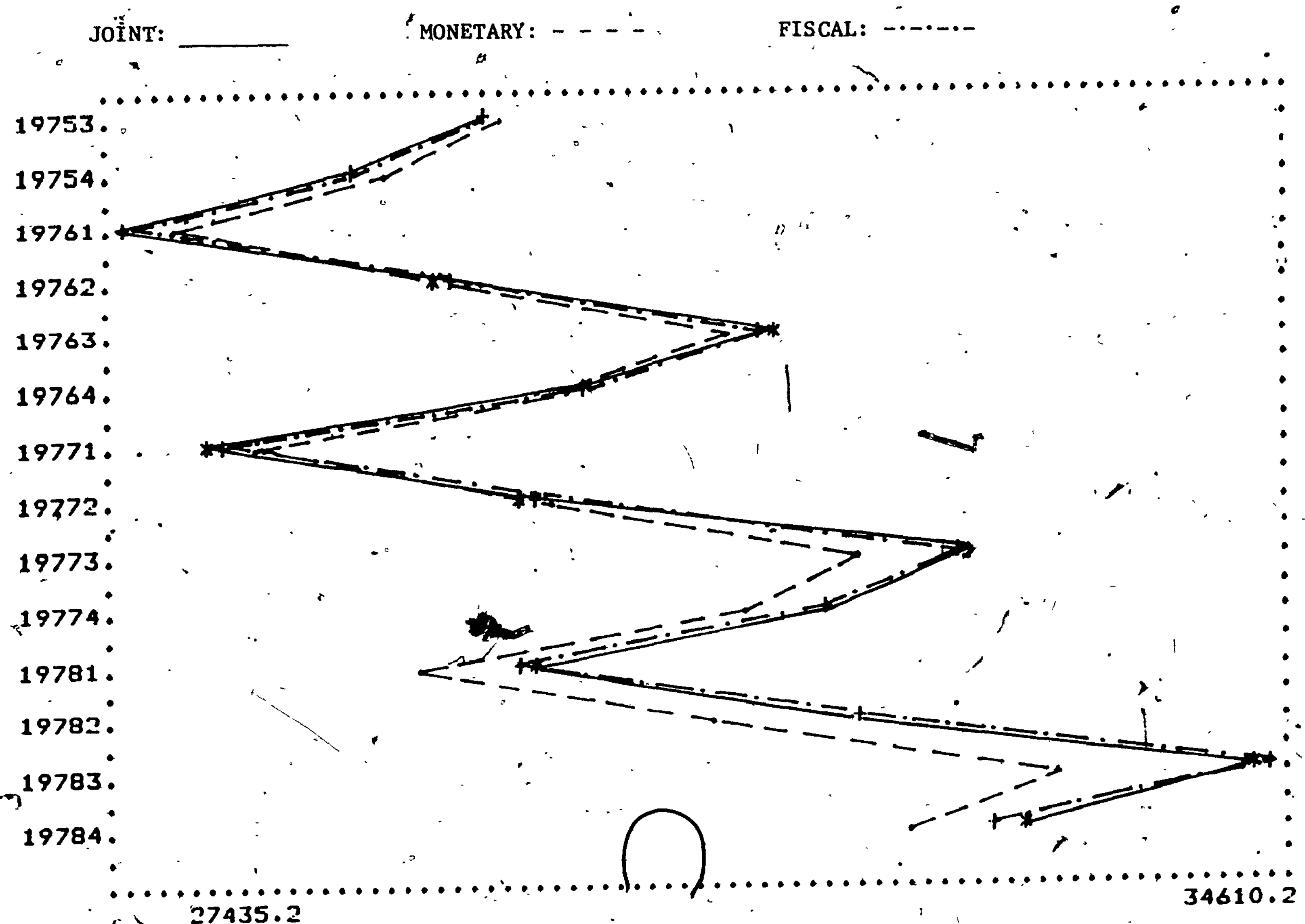


FIGURE 9.1: Optimal Time-Paths of Y (Real GNP) in the Basic Experiments. Millions of 1971 Dollars.

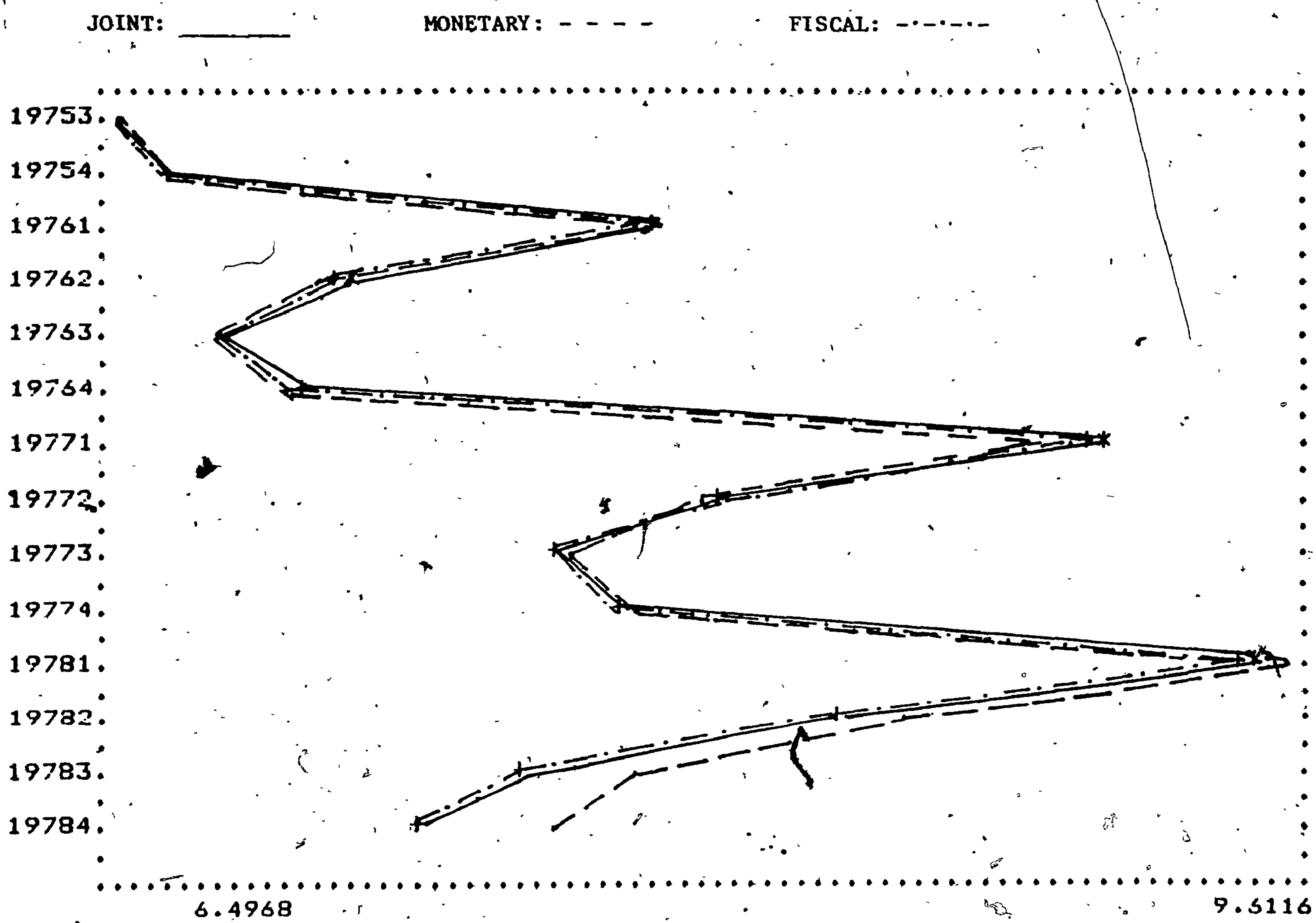


FIGURE 9.2: Optimal Time-Paths of UR (Unemployment Rate) in the Basic Experiments. In Percent.

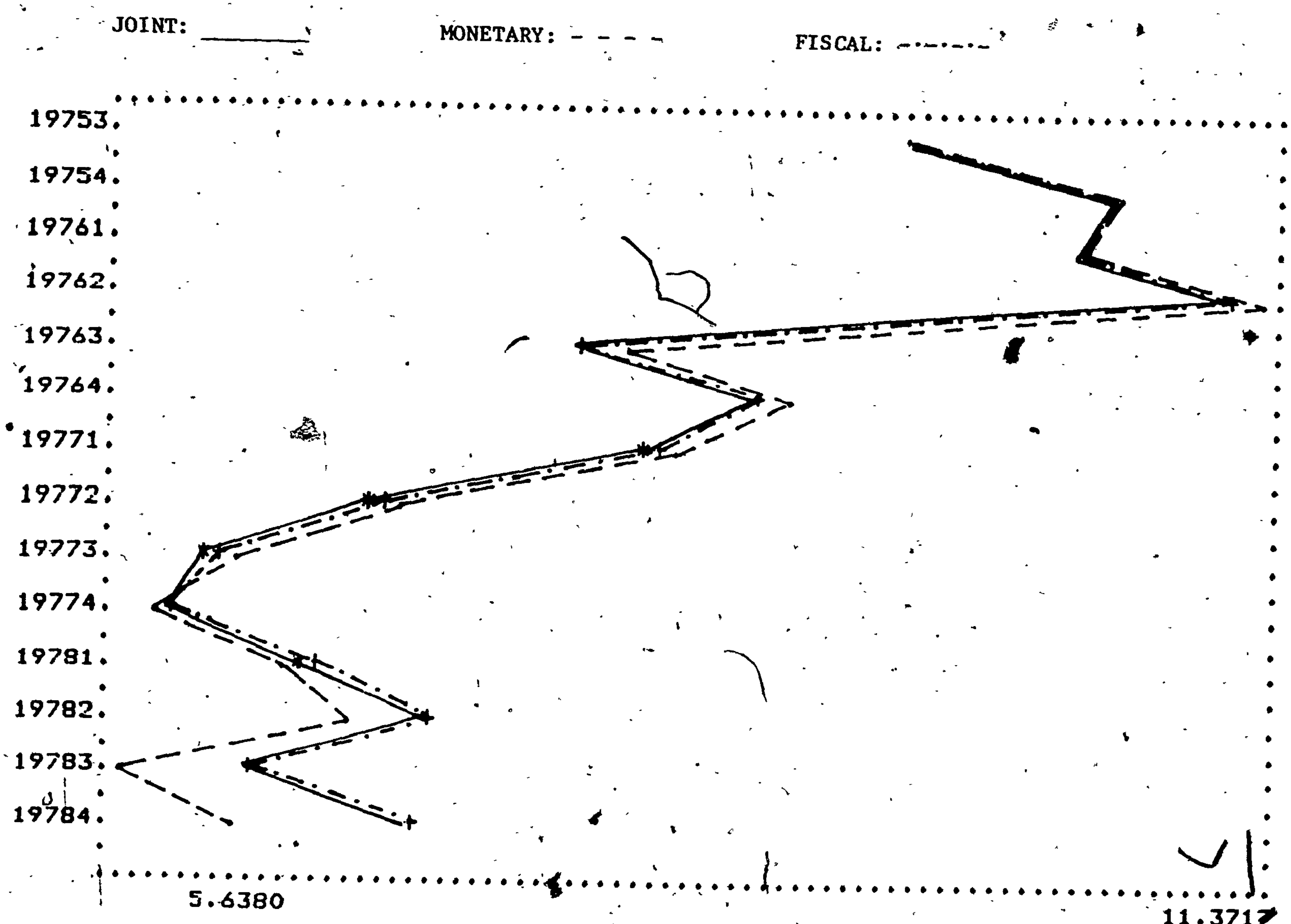


FIGURE 9.3: Optimal Time-Paths of INFL (Inflation Rate) in the Basic Experiments. In Percent.

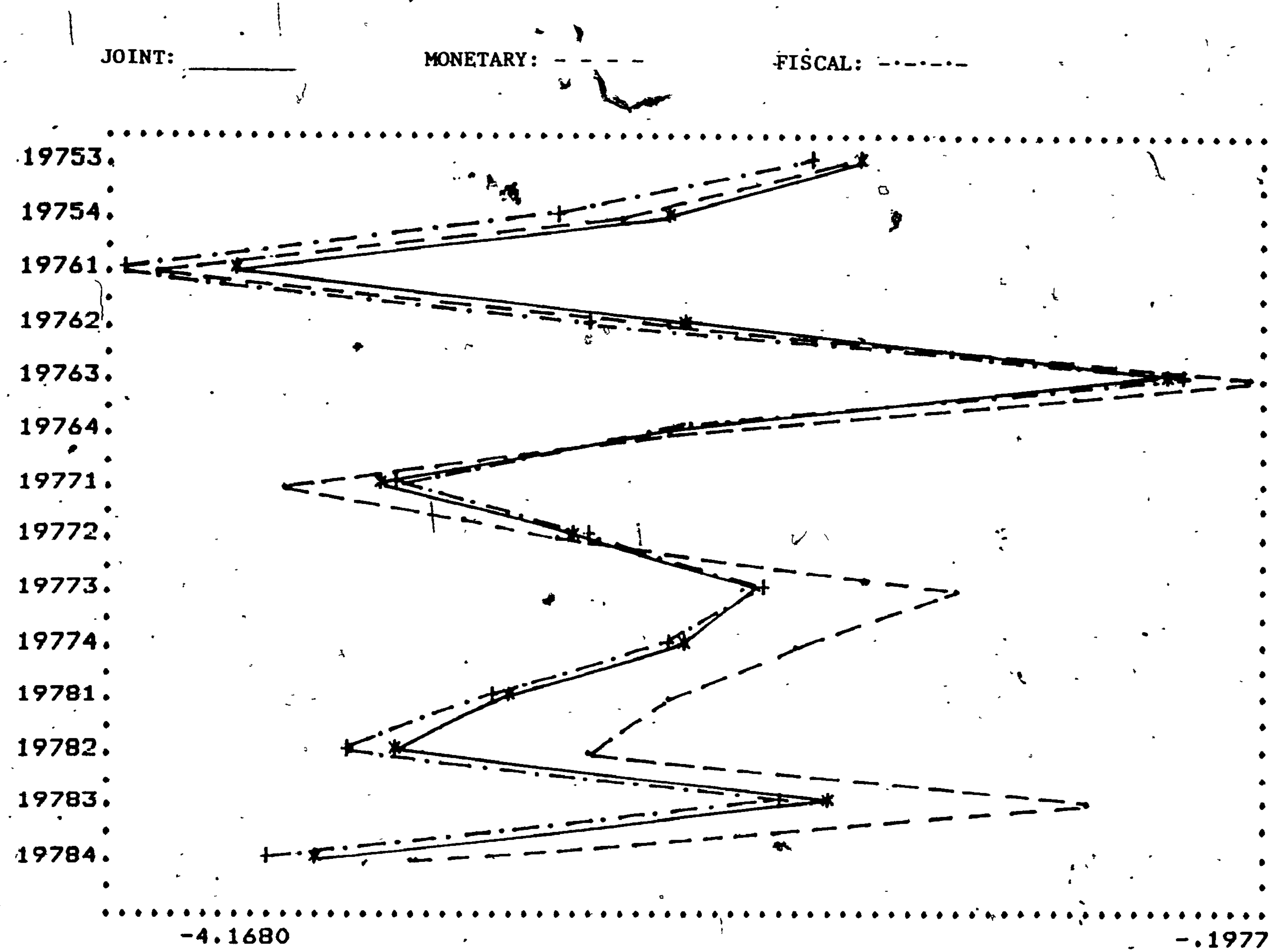


FIGURE 9.4: Optimal Time-Paths of TB (Nominal Trade Balance as a Percentage of Nominal GNP) in the Basic Experiments. In Percent.

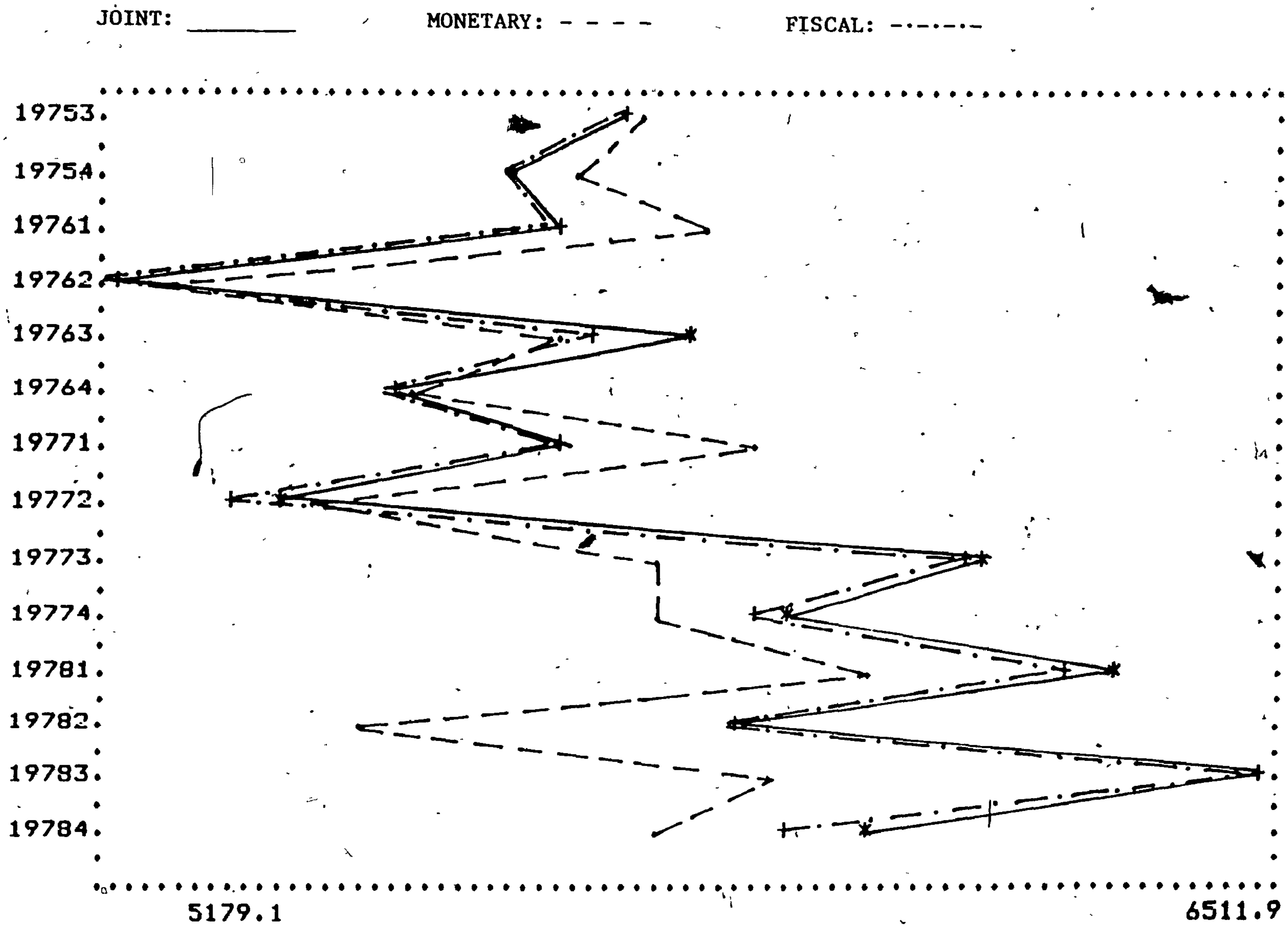


FIGURE 9.5: Optimal Time-Paths of GI (Real Expenditure of Federal, Provincial, and Local Governments) in the Basic Experiments. Millions of 1971 Dollars.

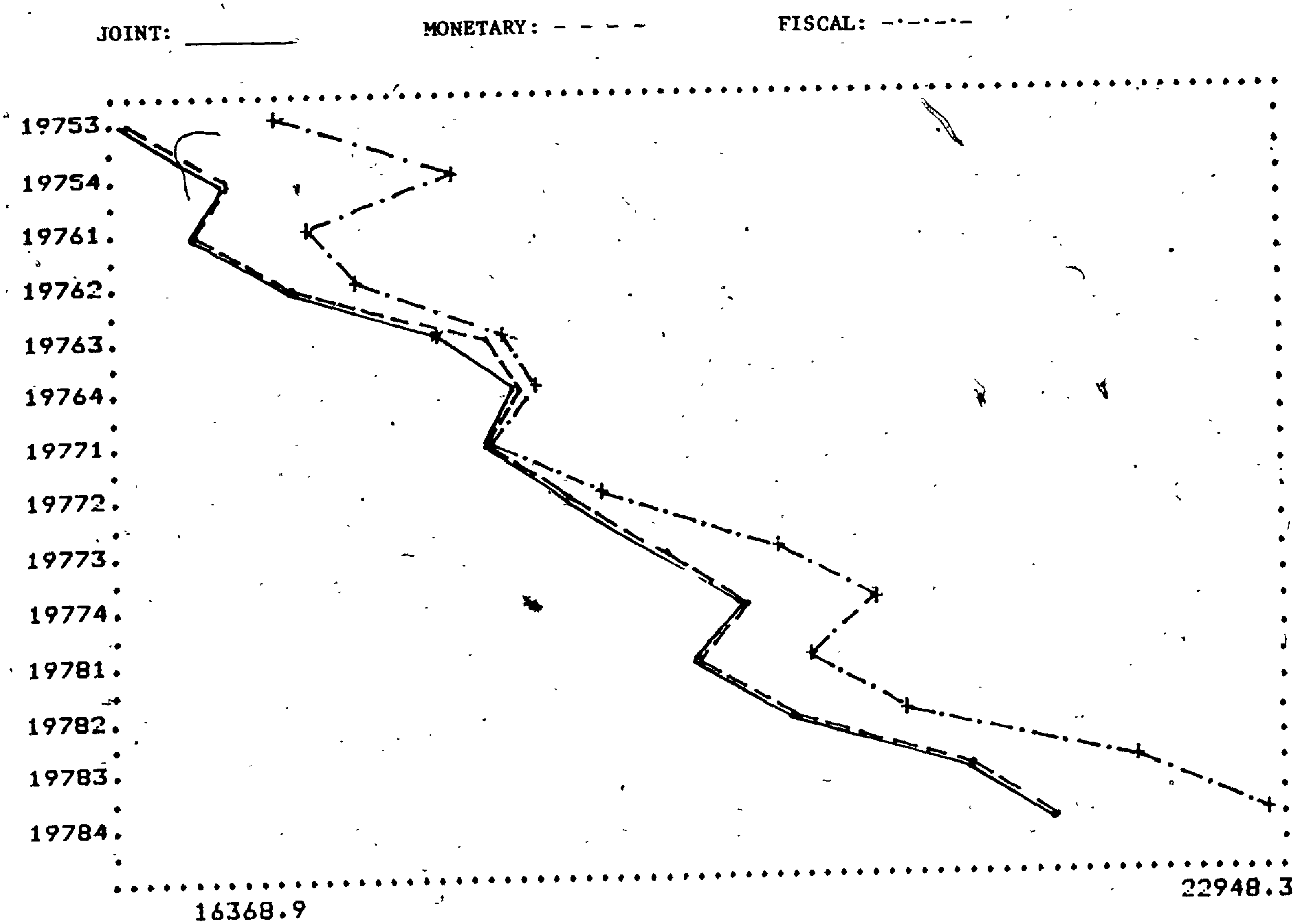
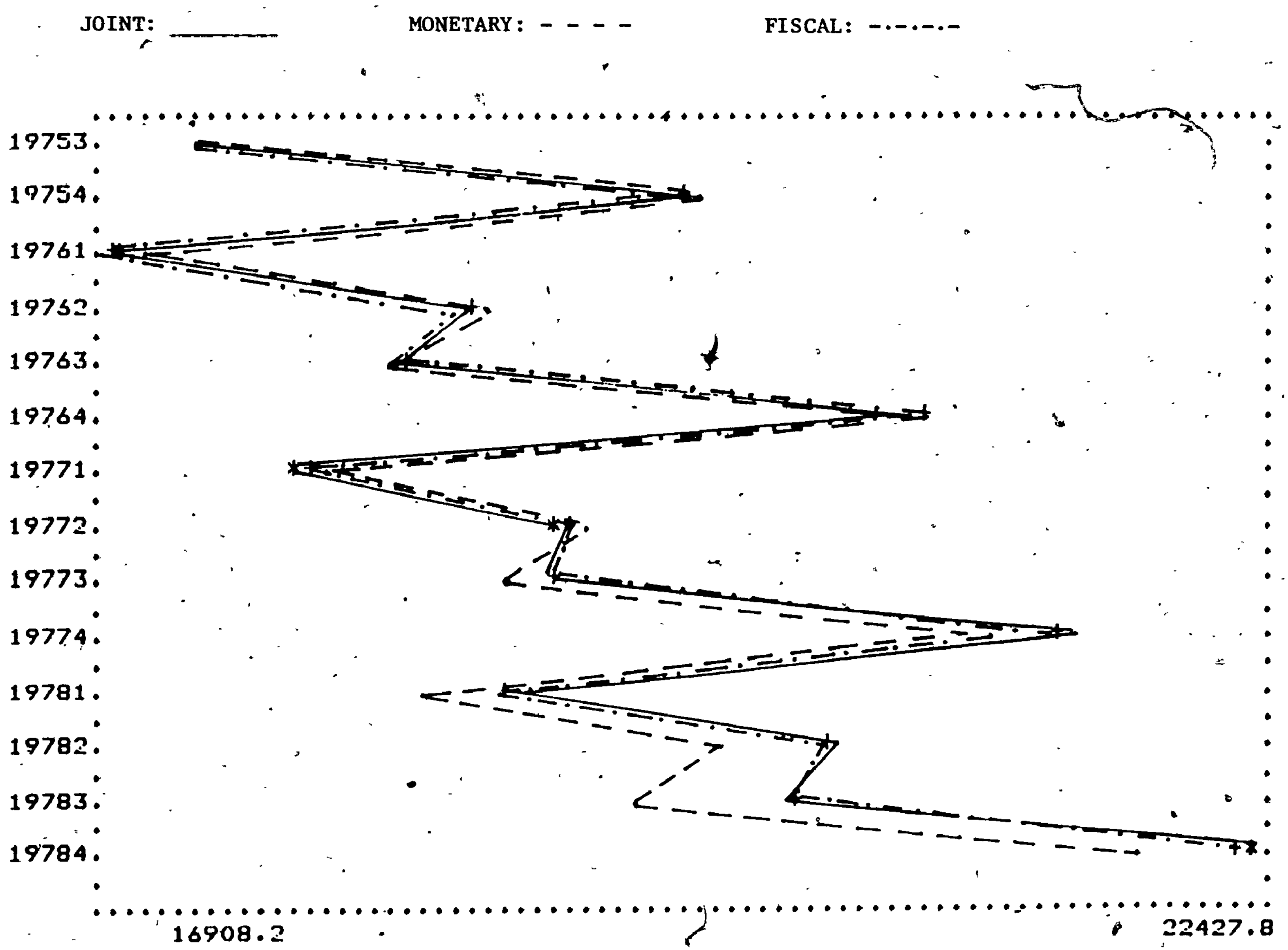


FIGURE 9.6: Optimal Time-Paths of M1 (Narrow Money Quantity) in the Basic Experiments. Millions of Current Dollars.



<u>FIGURE 9.7</u>: Optimal Time-Paths of CON (Real Consumption) in the Basic Experiments. Millions of 1971 Dollars.

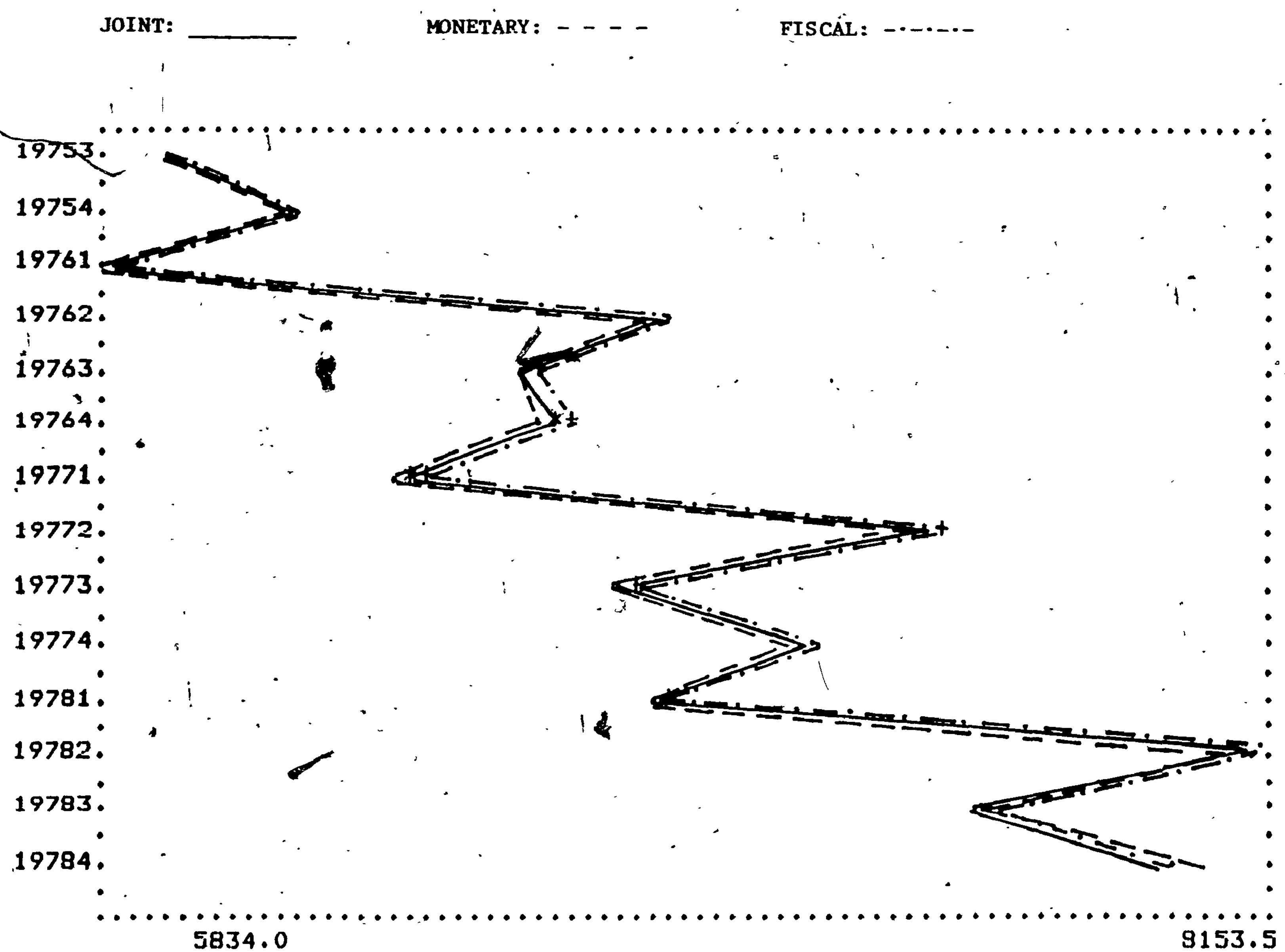


FIGURE 9.8: Optimal Time-Paths of EX (Real Export of Goods and Services) in the Basic Experiments. Millions of 1971 Dollars.

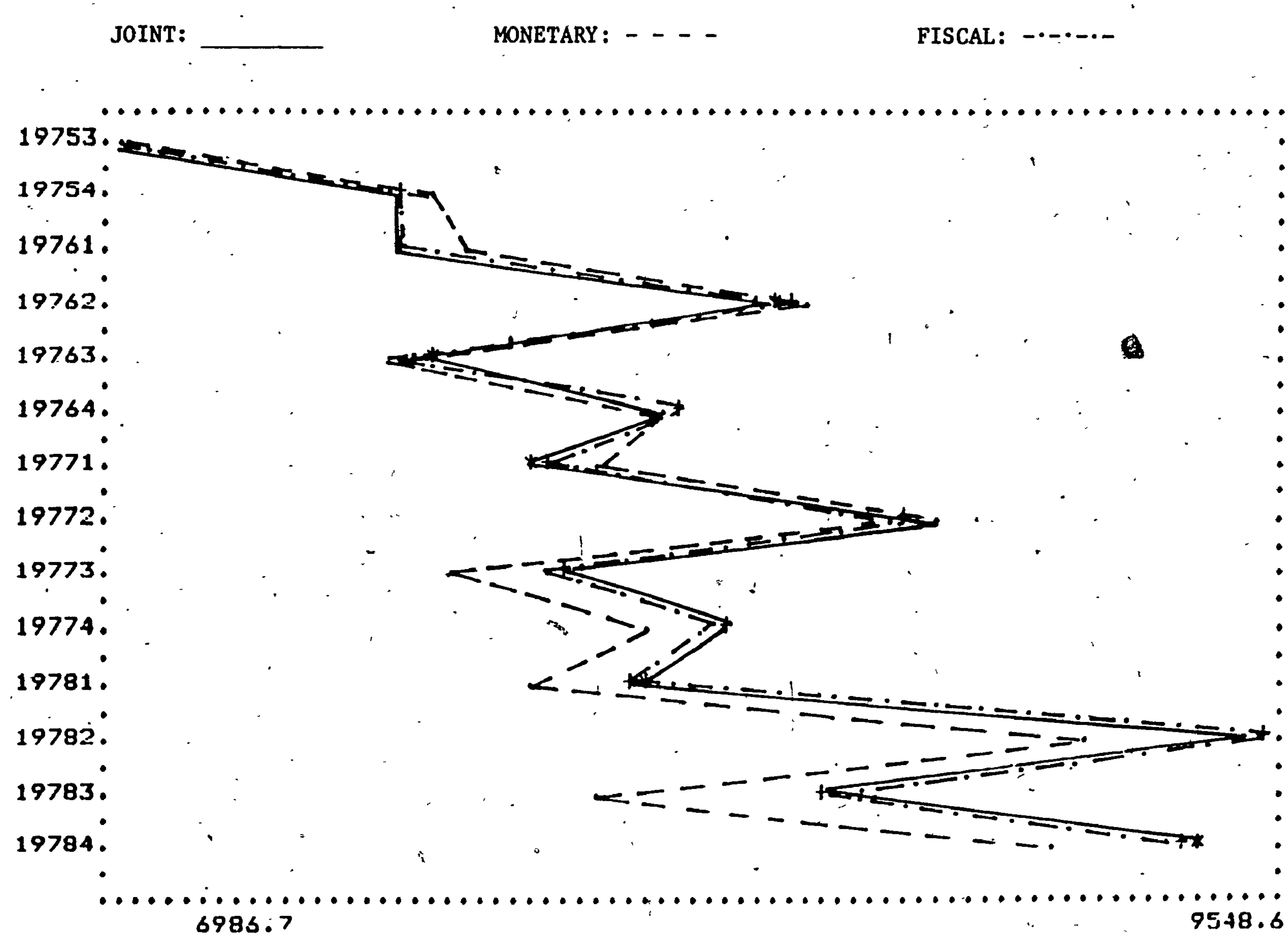


FIGURE 9.9: Optimal Time-Paths of IM (Real Import of Goods and Services) in the Basic Experiments. Millions of 1971 Dollars.

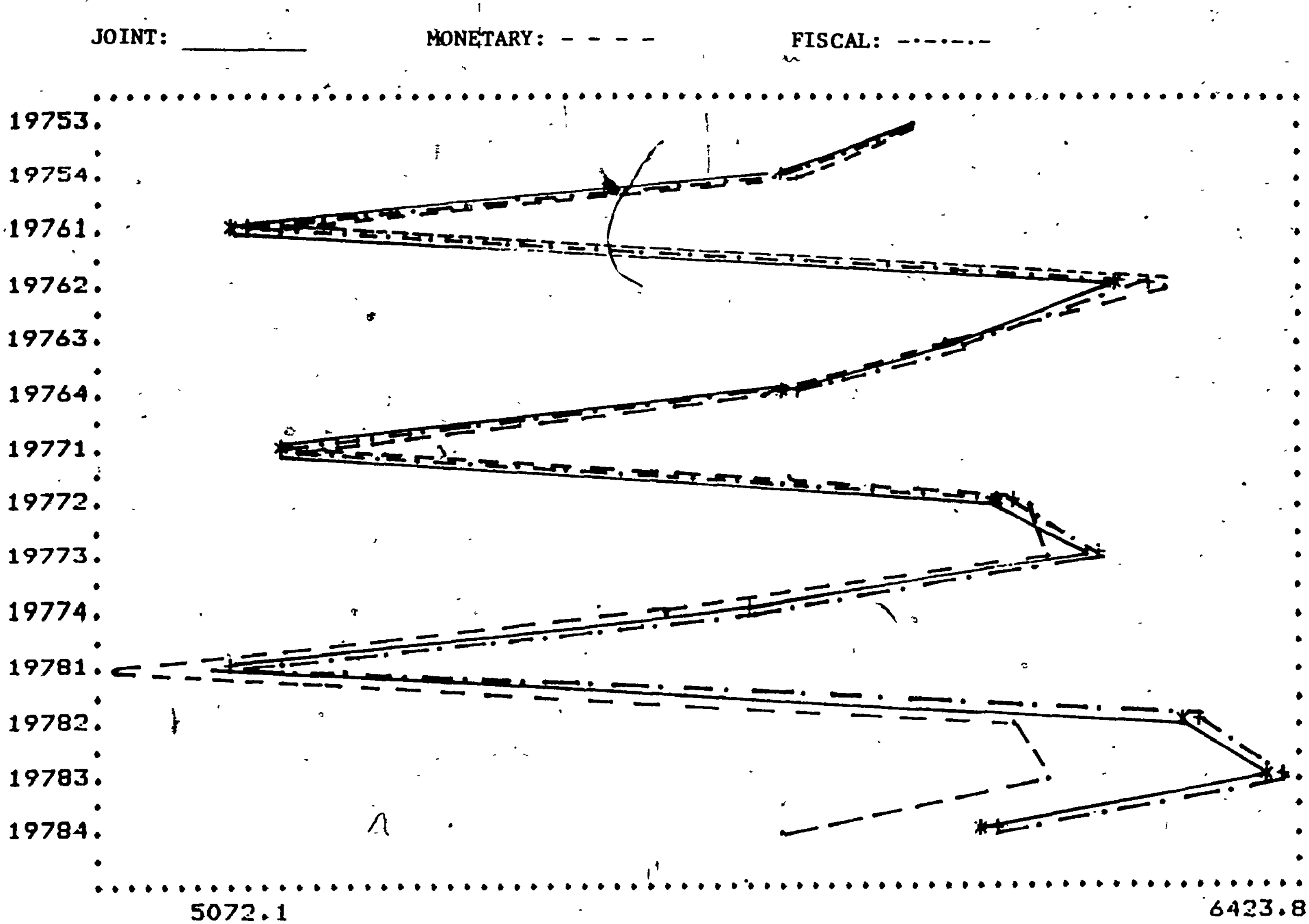


FIGURE 9.10: Optimal Time-Paths of IRCNR (Real Gross Private Fixed Investment) in the Basic Experiments. Millions of 1971 Dollars.

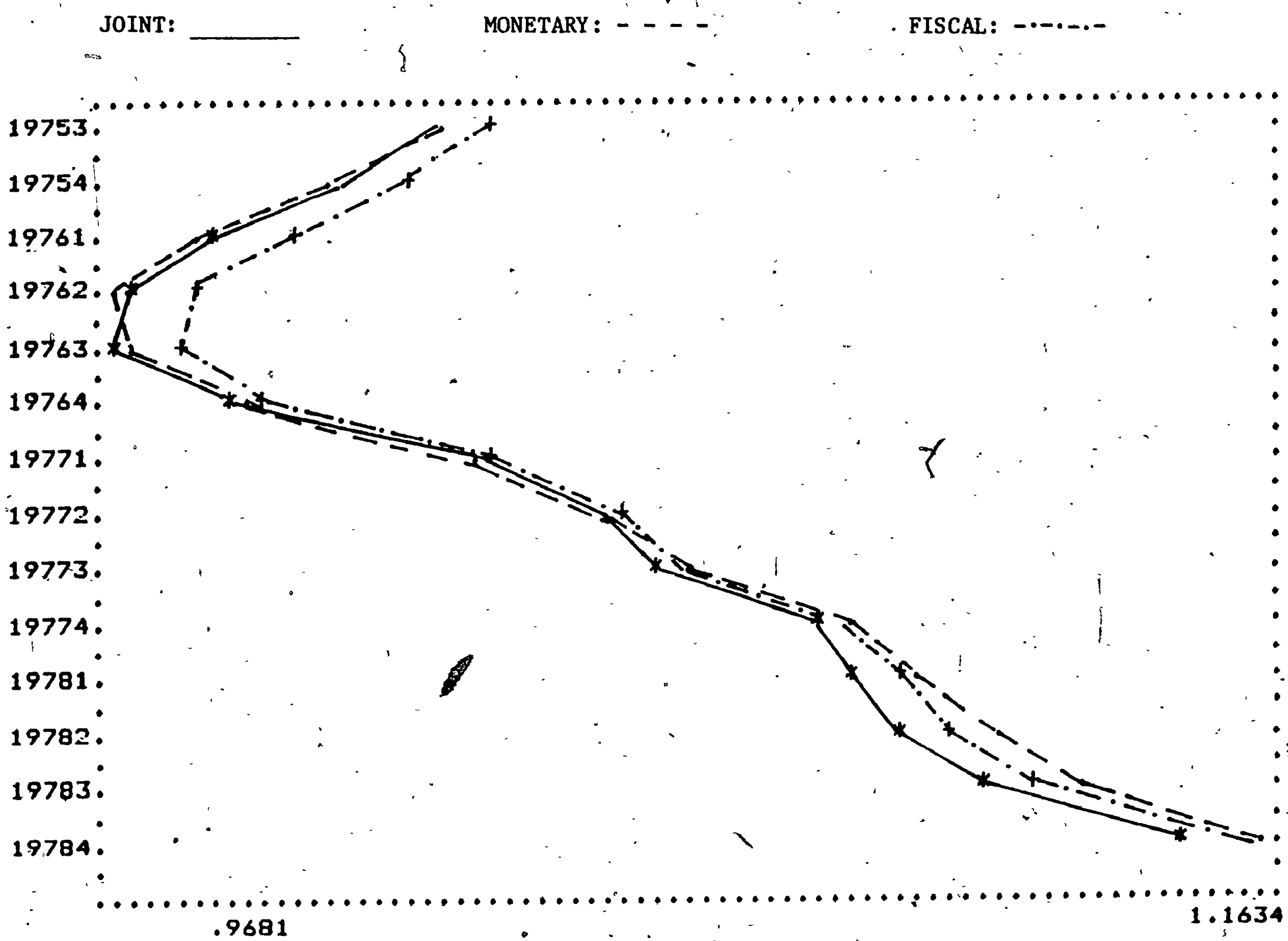


FIGURE 9.11: Optimal Time-Paths of PFX (Foreign Exchange Rate) in the Basic Experiments.

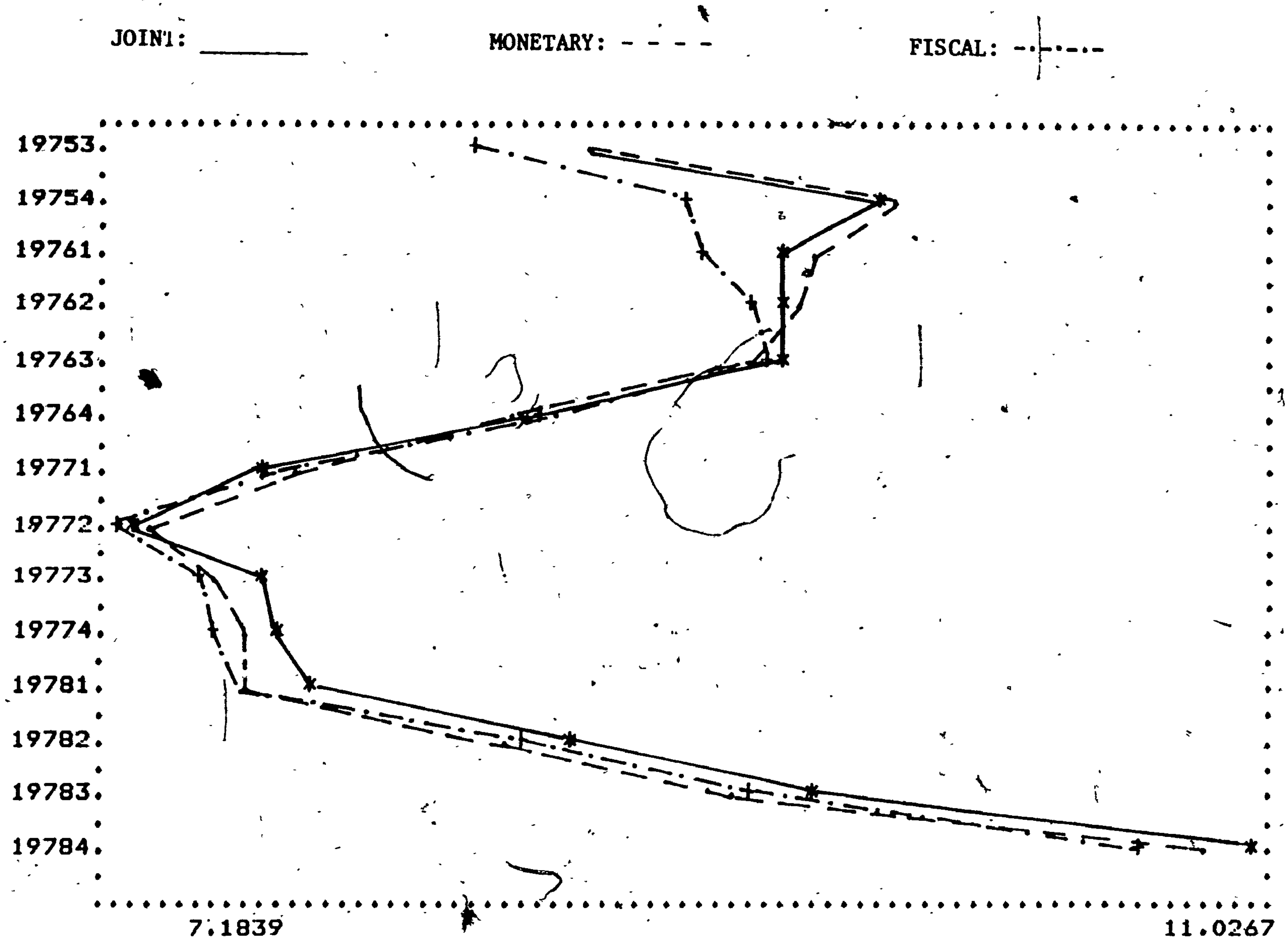


FIGURE 9.12: Optimal Time-Paths of RS (Short-term Interest Rate) in the Basic Experiments. In Percent.

have the same M1 path. On the other hand, in Fig. 9.5 the paths of G1 from the fiscal and joint policies are very close; they abruptly diverge from the historical (= monetary) path in the last seven quarters, which explains pronounced variations among the three policies in certain variables (e.g., GNP, TB) during that interval. How do the control variables behave with respect to their bounds? Examination of the CONOPT output reveals that in the joint policy, M1 is always at lower bounds while G1 is six times at lower bounds, two times at upper bounds, and six times inside bounds in a 14-quarter period. In the monetary policy, M1 is always at lower bounds except in one case it falls inside bounds. In the fiscal policy, G1 is six times at lower bounds, one time at upper bound, and seven times inside bounds. One also remarks a close similarity in timing between the joint and fiscal policies on the one hand, and the joint and monetary policies on the other hand: e.g., when G1 in the joint policy falls inside bounds, G1 in the fiscal policy also falls inside bounds; when M1 in the joint policy hits the lower bound, the same behaviour is observed for M1 in the monetary policy. This explains a point raised earlier concerning the closeness among the paths of G1 between the joint and fiscal policies, and the closeness among the paths of M1 between the joint and monetary policies.

Regarding the behaviour of M1, one could question why its optimal level is always at lower bounds. The phenomenon is interesting to be explored. To explain it, some evidence from Sandblom-Banasik-Parlar research on bounded control will be relied upon. Using a small Canadian econometric model, the authors carried out optimization experiments involving two targets (inflation and unemployment) and one control variable, GCNW (government non-wage expenditure). Time independent and time dependent bounds were both considered. Here the second kind of bounds will not be discussed due to lack of information. With time independent bounds, the control variable is restricted to stay within a band which is constant over time. The range of GCNW is centered around the mid point of its historical range during a 12-quarter control beginning in 1970 I and is some integer multiple of the historical range.[75] The authors observed when GCNW is allowed to vary within a relatively narrow range (one, two, and three times the historical range), its values oscillate less and have more frequent contacts with the bounds (i.e., the values are inside bounds less often). For example, Figures 8 and 9 (pp. 33-34) in their paper reveal the following behaviour of GCNW.

- For a range which is twice the historical one, optimal GCNW is:

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[75] The historical range of GCNW is 437 million in 1961 dollars (p. 27). As to its relative magnitude, the ratio of (actual) GCNW over (simulated GNP is 8.54% (=1463/17125) on the average (p. 42). In our case the ratio of G1 over GNP is 18.78% on the average.

9 times at upper bounds

2 times at lower bounds

1 time inside bounds

- For a range which is three times the historical one, optimal GCNW is:

5 times at upper bounds

4 times at lower bounds

3 times inside bounds

- For the maximum range of six times the historical one, optimal GCNW is:

4 times at upper bounds

4 times at lower bounds

4 times inside bounds

The interesting evidence is: when the ranges are narrow and therefore more realistic,[76] not only that GCNW has more frequent contacts with the bounds, it has the tendency to hit upper bounds more often because it is geared toward the most concerned target, UR (p. 29). If this was true, the behaviour of M1 which is the instrument geared toward the INFL target would be logical and consistent with a monetary restraint policy which puts heavy weight on INFL but allows relatively very narrow bounds (please see footnote 79 later). The implication is obvious: if less heavy weight is assigned to INFL, and wider bounds are imposed on M1, the optimal

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76 Sandblom-Banasik-Parlar study found that a bandwidth of twice the historical range appears the most attractive because larger bounds result only in modest reductions in the welfare loss (p. 45), and model solutions do not appear to improve (p. 40).

values of M1 should not always hit lower bounds. This is indeed confirmed by our experiments.

First, consider the case where less heavy weight is attached to INFL. The typical experiment is the one which makes use of the U.S. weights; it produces the following result.

- With our narrow bounds BND1(M1) and BND1(G1), optimal M1 is:

  6 times at upper bounds

  8 times at lower bounds

- With our larger bounds BND2(M1) and BND2(G1), optimal M1 is:

  6 times at upper bounds

  6 times at lower bounds

  2 times inside bounds

Next, consider the case where the weights of our convergent solution are kept intact, but much widened bounds are applied on M1. For this purpose, Sandblom-Banasik-Parlar procedure of generating control bounds is followed. To be consistent with our original time dependent M1 bounds, to encourage the control variable to retain some of its historical pattern (e.g., trend, seasonality etc...) and so enhance the realism of solution values,[77] time dependent bounds are generated. Three different sets allowing M1 to vary within one, two, and three times its historical range

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77 See Sandblom, Banasik, Parlar, p. 41.

yield the following result.[78]

i) For a range which is of the same order of magnitude as the historical one, optimal M1 is:

12 times at lower bounds

2 times inside bounds

ii) For a range which is twice the historical one, optimal M1 is:

10 times at lower bounds

4 times inside bounds

iii) For a range which is three times the historical one, optimal M1 is:

7 times at lower bounds

7 times inside bounds[79]

In short, we can conclude that the behaviour of optimal M1 is not unusual. Let us now turn quickly to some endogenous variables (including targets).

In terms of Y, the fiscal policy performs best among the optimal ones. Even the joint policy has a smaller Y. One

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78 Beyond the range of three times the historical one, the bounds of M1 look unrealistic: from 1975 III to 1978 IV, the historical range of M1 is 5679 million current dollars; when M1 is restricted to stay within a band of three times the historical range, it is observed that the lowest value of M1 is lower than the actual value of M1 in 1970 I. And the 1970 year was characterized by very low rates of growth of M1 (2.2%) and of inflation (3.2%). Thus, we do not see any realism to widen the bounds beyond the three-time historical range.

79 In case (i), the optimal average value of M1 is 13% lower than its historical counterpart. In case (ii) and (iii), the percentages are respectively 23.11% and 28.92% as compared to 3.18% from the convergent solution.

explanation is: tighter money in the joint policy results in higher interest rate RS which has adverse effects on private investment IRCNR. Tight money also explains why the monetary policy with its M1 3.11% below the historical level registers a level of real GNP even lower than the actual one.

Results on Y have impact on UR and INFL. In terms of low UR, the ranking is: fiscal, joint, monetary. The reverse order is for INFL.

Higher Y will also lead to higher consumption CON and higher investment IRCNR in the fiscal and joint policies. The behaviours of import IM and export EX are more difficult to be determined a priori for the reason that they depend on several factors such as relative price, foreign exchange rate etc. In spite of the complexity, Fig. 9.8 reveals there is not much difference in the amounts of export among the three policies while Fig. 9.9 shows that import is worse in the fiscal and joint policies due to their higher GNP. These translate into a worse trade balance in the fiscal and joint policies.

The behaviour of the foreign exchange rate PFX is also complex. It depends positively on broad money M2, negatively on domestic Y and RS. The interaction of these factors yields the lowest value of PFX (i.e., the highest value of the Canadian dollar) for the joint policy.

The last variable of interest is RS whose value is determined mainly by the behaviour of M1. For example, the fiscal policy has the highest amount of M1 and the lowest value of RS consequently.

9.1.2 Extension

Section 9.1.1 has shed some light on the relative effectiveness: the fiscal policy is found to be superior to the monetary policy but on a very small scale. A shortcoming in the definition of the monetary and fiscal policies from the basic experiments is that monetary effects cannot be isolated from fiscal effects or vice versa. As discussed in Chapter 3, Abel (1975) proposed a different definition: for each policy its own instrument is discretionary (i.e., variable subject to feedback control or 'free' control variable in simple language) while the other instrument is passive (i.e., control variable subject to constant growth rate rule). It results that in Abel's sense, the fiscal policy involves a discretionary fiscal control (variable) and a passive monetary control while the monetary policy is characterized by a discretionary monetary control and a passive fiscal control. Abel's definition would put the comparison on a fairer basis. Note that either discretionary or passive the control remains under its bounds.

The first part of this section will reexamine the relative effectiveness of monetary and fiscal policies, based on the new definition. The second part will specially deal with the optimality and suboptimality due to the adoption of the rule. Monetary and fiscal policies are defined as in the basic experiments; furthermore, the control variable is subject to (constant growth rate) rule. For example, in the monetary policy M1 is a control variable subject to rule while G1 is set exogenous.

a. Monetary and fiscal experiments: Monetary policy (M1 discretionary, G1 passive – Fiscal policy (G1 discretionary, M1 passive.)[80]

Table 9.8 reports some statistics pertinent to the experiments.

TABLE 9.8: Monetary Policy with M1 Discretionary, G1 Passive and Fiscal Policy with G1 Discretionary, M1 Passive: Total Value of Welfare Loss and Average Values of Real GNP, Target and Control Variables.

ITEM	MONETARY	FISCAL
LOSS	363.463	346.652
Y	30519	30660
YGAP	-6.463	-6.113
UR	7.626	7.670
INFL	8.268	8.070
TB	-2.272	-2.461
G1	5817	5818
M1	18841	19135

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80 The constant growth rates are applied to S.A. data of M1 and G1. A following equation is added to the model: (X/a)=h [X(-1)/a(-1)] where X is either M1 or G1, a the actual ratio of N.S.A. to S.A. data of X, and h = (1 + rate of growth) to be determined optimally in the optimization process.

Two important results emerge from Table 9.8 (see also Table 9.4).

i) Compared to the historical policy, if the fiscal policy was implemented, the economy could be improved by 6% in terms of welfare loss. The monetary policy would improve the economy by 1.44%.

Recall in the basic experiments, the economy would be improved by 4.5% with the fiscal policy and by 2.54% with the monetary policy. Therefore, by moving to Abel's definition, more advantage shifts to fiscal policy. But in terms of welfare loss and especially in terms of GNP and UR, the advantage is still small.

ii) On second thought, one can realize the sole difference between the joint policy (basic experiments) and Abel's monetary policy (in which M1 is discretionary and G1 passive) lies in the nature of G1 which can be either discretionary or passive. For example, if G1 is passive rather than discretionary, the joint policy will become Abel's monetary policy, and it results that the improvement will greatly decline from 7.25% (joint) to 1.44% (monetary). By the same token, if M1 is passive rather than discretionary, the joint policy will transform into Abel's fiscal policy (in which G1 is discretionary and M1 passive), but the improvement will slightly decline from 7.25% to 6%. What one learns is: if the rule is imposed, the effectiveness or performance of the monetary instrument is hardly affected, but the loss in performance is high for the fiscal instrument.

b. Monetary and fiscal experiments: Monetary policy (M1 passive, G1 Exogenous) - Fiscal policy (G1 passive, M1 exogenous).

Fig. 9.13 displays the trajectories of S.A. M1 among the three policies: historical policy, monetary policies without and with rule. S.A. data are deliberately plotted because the constant growth rate is applied to S.A. data of M1 (see footnote 80) and for consistent comparison. As can be seen, the path of the monetary policy with rule follows a straight line. One also observes a quite smooth path in the monetary policy without rule.

Table 9.9 below in comparison with Table 9.4 gives a few results.

i) Although it has M1 1% lower than the historical level, the monetary policy with rule is still a bit better than the historical policy in terms of welfare loss.

ii) In terms of welfare loss, both monetary and fiscal policies with rule are dominated by their counterparts without rule. Cooper and Fischer (1972; also Fischer and Cooper, 1973) obtained the same result. They found that there is room for improvement by use of some policy other than the constant growth rate rule and nowhere that rule is optimal.

iii) There is a slight decrease of performance from the monetary policy without rule to the monetary policy with rule. For example, as compared to the historical policy, the improvement in welfare declines from 2.54% to 1.49%

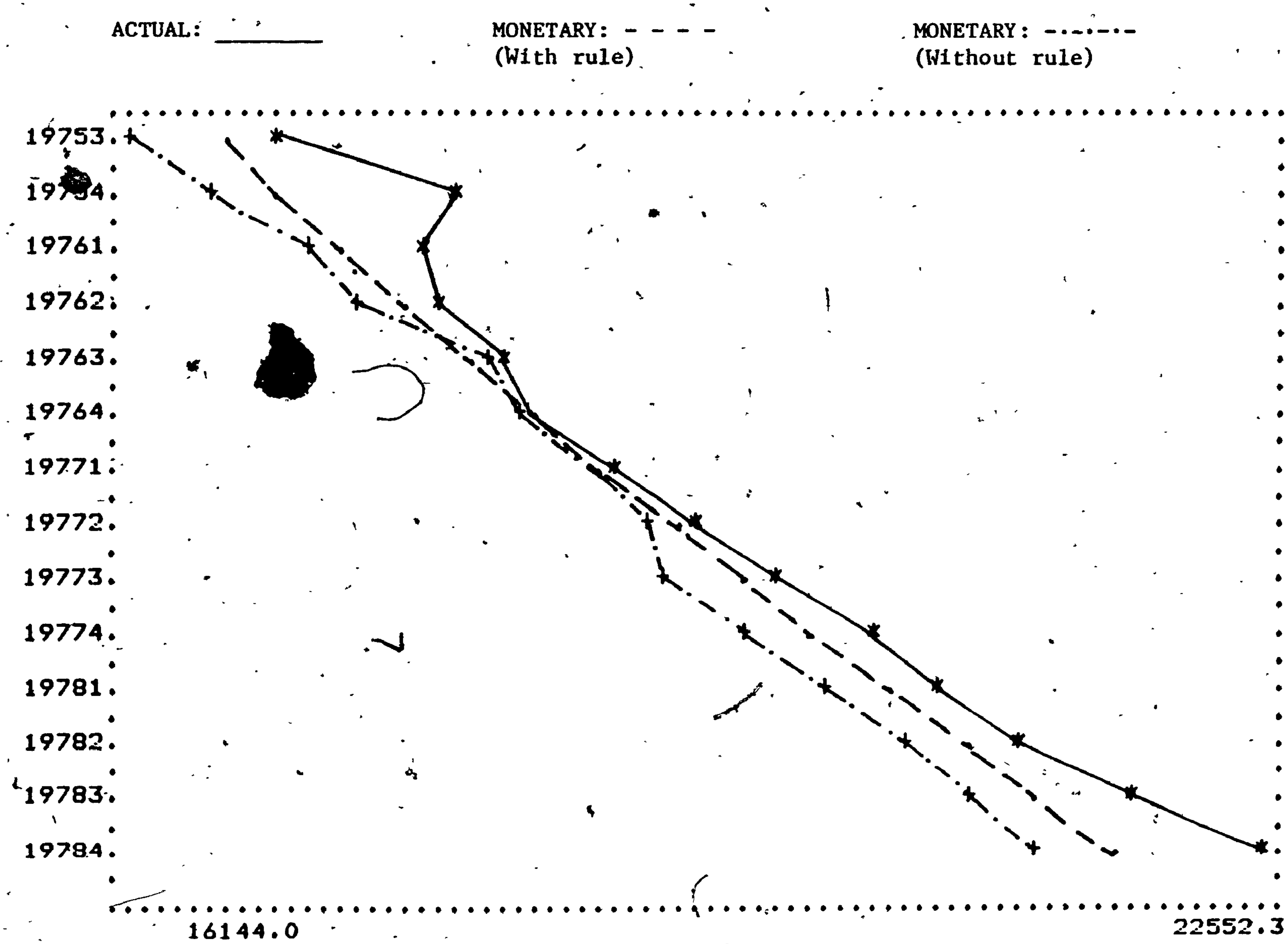

FIGURE 9.13: Actual and Optimal Time-Paths of S.A. M1 (Narrow Money Quantity) in the Monetary Policies with and without Rule. Millions of Current Dollars.

TABLE 9.9: Monetary Policy with M1 Passive, G1 Exogenous and Fiscal Policy with G1 Passive, M1 Exogenous: Total Value of Welfare Loss and Average Values of Real GNP, Target and Control Variables.

ITEM	MONETARY	FISCAL
LOSS	363.277	374.611
Y	30401	30495
YGAP	-6.832	-6.532
UR	7.707	7.627
INFL	8.028	8.272
TB	-2.263	-2.370
G1	5715	5772
M1	19135	19460

Note: Monetary and fiscal results are reported together for convenience reason; they cannot be compared with each other. For the issue of optimality and suboptimality due to rule, the monetary policy in Table 9.9 must be compared with that in Table 9.4 (the same remark applies to the fiscal policy).

iv) The decrease of performance is severe to the fiscal policy. From a value of 4.50% (no rule) the improvement turns out to be a negative (-) 1.58% (with rule).

Points (iii) and (iv) consolidate the previous finding concerning the effect of rule on the performance of the controls. The effect on G1 should be due to some of its advantages which must prevent it from favourably taking a steady course. One could conceive of at least two advantages.

- First, it is an advantage of shorter outside lag. The economy is understood to adjust slowly to changes in monetary and fiscal policies. Nevertheless, government spending acts directly on aggregate demand. On the other hand, a change in M1 will affect short-term interest rate RS with a lag. RS, in turn, will affect residential investment IRC and long-term interest rate RL also with a lag. It will take quite a long time for RL to have impact on investment in non-residential construction and machinery & equipment INR...Briefly, shorter outside lag allows government spending to respond more quickly and strongly to erratic changes in exogenous variables; and this should be reflected in wider fluctuations of G1. Indeed, examining Figs. 9.5, 9.6 (and also Fig. 9.13), one can find that the paths of G1 display much larger fluctuations than those of M1. The same pattern is also found for a longer period from 1967 I to 1978 IV (our estimation period) and even with seasonally adjusted data. Therefore, keeping G1 grow steadily will lead it to a direction opposite to its natural course and will certainly lessen its effectiveness.

Athans <u>et al</u>. (1975, p. 294) in their studying of the control of the U.S. economy provided a similar explanation with respect to the tax rate behaviour:

> As we would expect, there is slightly more fluctuation in the tax rate than in unborrowed reserves due to the much shorter lags with which the former operates, thus permitting tax rates to react more quickly and vigorously to any erratic behaviour in the exogenous variables.

- Second, as a control variable G1 powerfully enters the income equation with a coefficient value of 1; that is not the case for M1.[81] One must, however, realize in a dynamic framework the immediate strength particularly matters because the policy is restricted to the short-run and because the control variable (i.e., G1) exhibits a shorter outside lag.

To sum up the experiments without lag consideration, three main results have emerged.

i) Although the controls have been allowed larger bounds relative to the past, an optimal monetary and fiscal mix would bring a modest improvement over the historical policy. The improvement is negligible in terms of UR, INFL, and welfare loss.

ii) In a multi-target framework, the fiscal policy is consistently found superior to the monetary policy; but the degree of relative effectiveness mainly through the comparison of the welfare loss values is small and does not match the one measured by multipliers.

iii) Rule very slightly affects the performance of the monetary instrument. This does not apply to the fiscal instrument.

9.2 EXPERIMENTS WITH LAG CONSIDERATION

We now incorporate inside lag in the experiments. There will be one-quarter lag for monetary policy and two-

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81 The point can easily be verified by comparing the impact multipliers of G1 and M1 or their multiplier values in the first four quarters (see Table 7.3).

quarter lag for fiscal policy. The principle consists of two steps.[82] First, derive optimal values of M1 and G1 from the basic monetary and fiscal policies; denote them by $M1^*$ and $G1^*$. Then set M1 = $M1^*(-1)$ and G1 = $G1^*(-2)$ exogenous in straight simulations. With 'lagged' experiments, two observations will be lost.

Table 9.10 reports the results of the experiments.

TABLE 9.10: Lagged Monetary and Fiscal Policies (Basic Experiments), Period 1976 I - 1978 IV: Total Value of Welfare Loss and Average Values of Real GNP, Target and Control Variables.

ITEM	HISTORICAL	MONETARY	FISCAL
LOSS	362.811	350.975	360.134
Y	30599	30499	30664
YGAP	-6.929	-7.235	-6.766
UR	7.876	7.929	7.919
INFL	7.752	7.599	7.573
TB	-2.352	-2.098	-2.424
G1	5710	5710	5725
M1	19751	18835	19751

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82 In reality, the procedure is slightly modified due to seasonality. For example, after obtaining $M1^*$, we transform it into S.A. data by the usual way, then lag S.A. $M1^*$; the lagged S.A. $M1^*$ is then transformed back into N.S.A. data. The same principle applies to G1.

So far, the fiscal policy is found to be more efficient in reducing unemployment relative to the historical level at the detriment of higher inflation. When lag is introduced, the reverse occurs. Again, this indicates the importance of the timing of government spending.

For the first time the monetary policy performs better than the fiscal policy. In terms of welfare loss and relative to the historical policy, the lagged monetary policy would improve the economy by 3.26%; the lagged fiscal policy would improve the economy by only .7%. The result validates economists' concern about the effect of inside lag. Also importantly, the result has demonstrated that multiplier analysis as a device to evaluate relative effectiveness could be misleading under a certain economic context with multiple objectives.

9.3 SENSITIVITY ANALYSIS EXPERIMENTS

The objective of the section is to try to understand if many previous optimal results would be sensitive to a change in i) the model coefficients, ii) the specification of the welfare weights, and iii) the control variable bounds. Interestingly, experiments will also provide opportunities to look at some alternatives to the BOC policy. The first kind of experiments is a reflection of our concern with Lucas's critique on policy evaluation: the joint policy will be reexecuted with some reestimated coefficients. Since the experiment deals with a past event, it cannot directly answer

Lucas's critique; however, it should be able to point out that if the results were insignificantly altered, not every important change in policy would reflect those envisaged by Lucas. The last two kinds of experiments take further steps in assessing the validity of certain criticisms voiced against the BOC policy. In the second, two new sets of weights will alternatively be tried, with more attention paid to the GNP and UR targets; the motive is to explore whether the economy could be better off if policy-makers showed more concern about these targets. Finally, in the third kind much larger bounds will be imposed on M1, giving the monetary authority ample room to fight inflation; the purpose of the experiments is to gain some insight into the effect of a less gradual monetary policy at least on the inflation side.

9.3.1 Sensitivity with Respect to a change in the Model Coefficients

Would an important policy change in the past affect optimal results through its impact on the model structure? To deal with some aspect of model structure stability, instead of the Chow tests, here we take another road. First, we will break the data series into two subsets depending upon the date of the event, and one subset is selected for the reestimation. Then, three key equations, believed to be the most important and traditionally economists' concern for stability, will be reestimated by OLS as it is unthinkable to reestimate the whole model. Finally, an already executed optimization exercise, the joint policy, will be repeated with the new coefficients.

These three equations are CON (consumption), INR (real gross private investment in non-residential construction, and machinery & equipment), and RS (short-term interest rate). Inside the product sector economists pay particular attention to consumption and investment; inside the monetary sector they give attention to the demand for money. Our model contains three investment equations with INR the most important one. In the monetary sector, there exist two stochastic equations: currency and time deposits; the latter plays a very weak rôle. Demand deposits are derived residually. Clearly, RS is the right choice; especially it is in reality the normalized demand for narrow money.

The next question is where we can break the data series. During the estimation period 1967 I - 1978 IV, two events stand out: return to the flexible exchange regime in June 1970, and adoption of the monetarist monetary policy in May 1975.

If we are interested in the monetary event, the data series can be broken into two subsets: 1967 I - 1975 II and 1975 III - 1978 IV. The second subset with only 14 observations is too short for estimation (the RS equation has 13 parameters). The first also presents a special problem: estimation under the period 1967 I - 1975 II does not supply for the optimization period 1975 III - 1978 IV necessary residuals to be added back to the model equations (this procedure has been followed so far).

For these problems we will not choose the monetary policy change. Nevertheless, the opportunity of examining the effect of the monetarist event should not be considered as lost if we remember a remark made earlier in Section 7.1.2 related to the simulation exercises: both simulations 1972 I - 1978 IV and 1975 III - 1978 IV perform almost equally well; this might suggest some stability in the economic relationships as a whole for the period 1972 I - 1978 IV.

Let's turn to the foreign exchange rate event. The data series can be broken into two subsets: 1967 I - 1970 II and 1970 III - 1978 IV. The first subset suffers from the two problems as said earlier, leaving the second subset as the period of reestimation.

Table 9.11 presents results of the new optimization. To facilitate comparison, historical and original optimization results are also reproduced.

If we take the historical solution as the base, in terms of welfare loss, the new optimal policy improves a bit relative to the initially optimal solution: 8.34% versus 7.25%. But the discrepancy does not hide the fact that both optimal solutions are virtually the same, item by item; e.g., the revised version still registers an improvement in both UR and INFL etc...

In view of the results, the shift from a fixed exchange regime to a flexible regime would not alter the structure of the model as a whole. And if we recall a point repeated earlier that the simulation results would suggest

TABLE 9.11 Joint Policy (Sensitivity Experiment with Some Reestimated Coefficients): Total Value of Welfare Loss and Average Values of Real GNP, Target and Control Variables.

ITEM	HISTORICAL	OPTIMAL (1)	OPTIMAL (2)
LOSS	368.780	342.048	338.202
Y	30428	30655	30668
YGAP	-6.748	-6.131	-6.089
UR	7.688	7.675	7.670
INFL	8.084	8.055	8.068
TB	-2.339	-2.400	-2.355
G1	5715	5831	5865
M1	19460	18841	18862

Notes: (1) initial coefficients

(2) reestimated coefficients

some degree of stability in the economic relationships as a whole for the period 1972 I – 1978 IV in which the important monetarist event occured, there is some weak evidence that not any important change in policy would change the model structure or change it substantially.

9.3.2 Sensitivity with Respect to a Change in the Welfare Function Weights

Two new sets of weights will successively be tried with increasing importance given to GNP and UR. The first consists of w1 = .25, w2 = .50, w3 = w4 = 1. The second which may not have any connection with the Canadian reality borrows the U.S. weights: w1 = w2 = .75, w3 = w4 = 1. With new weights, much more emphasis than in the past is put on the GNP and UR targets. Would the results be more satisfactory to the economy? That will be the topic of part (a). Hopefully, the answer could clarify the validity of criticisms and stances taken by Barber and McCallum, Peters and Donner.

The issue of optimality and suboptimality due to rule will also be reexamined, but only the first set of weights will be considered as it has higher probability of maintaining the stability of the model structure. The discussion is very short and presented in part (b).

a) Is there a substantial gain over the historical performance?

Weights: w1 = .25, w2 = .50, w3 = w4 = 1

Chapter 8 has quite carefully described how the objective function is specified. The four targets of the 'convergent' solution were given the values:

w1 = .15 (YGAP)

w2 = .50 (UR)

w3 = 1 (INFL)

w4 = 1 (TB)

We have remarked that in terms of welfare loss, to move to a better solution higher weight should be attached to YGAP; in other words, more attention should be given to GNP and unemployment. That requires the values of w1 be increased. We have also explained why the set of weights with w1 = .25 and the rest unchanged was not, however, chosen. One of the justifications is that these weights no longer reflect past preferences.

What we like to do now is to reconsider these weights, to see how the economy would perform if w1 was given a value of .25. The new weights will naturally allow more room for optimization and would more satisfy critics who were frustrated in the bad performance of GNP and unemployment caused by a monetary restraint. For our purpose, the joint policy (Experiment No. 3) will be optimized again with a modified welfare loss function.

Table 9.12 presents the results of the new joint policy along those of the historical and 'convergent' solutions.

In terms of welfare loss, the alternative optimal solution would improve the economy by 10.56% relative to the historical performance, compared to the initial result of 7.25%. It would also yield a better GNP which was before 0.75% higher than its historical level and is now 1.33% higher. From our calculations, optimal GNP would grow at an average annual rate of 3.96% for the four-year period from 1975 to 1978 (the actual rate was 3.1% for the same period). Unemployment would improve little from 7.675 (convergent) to 7.595 (modified); but the improvement relative to the historical level (=7.688) is still very weak and materializes to the detriment of inflation. Optimal INFL would be rather

TABLE 9.12: Joint Policy (Sensitivity Experiment with Weights w1 = .25, w2 = .50, w3 = w4 = 1): Total Value of Welfare Loss and Average Values of Real GNP, Target and Control Variables.

ITEM	HISTORICAL	OPTIMAL (1)	OPTIMAL (2)
LOSS	(a)	342.048	389.881
Y	30428	30655	30834
YGAP	-6.748	-6.131	-5.586
UR	7.688	7.675	7.595
INFL	8.084	8.055	8.280
TB	-2.339	-2.400	-2.529
G1	5715	5831	5927
M1	19460	18841	18926

Notes: (1) 'convergent' welfare function

(2) modified welfare function with weights w1 = .25, w2 = .50, w3 = w4 = 1

(a) historical loss = 367.78 with the 'convergent' welfare function

= 435.944 with the modified welfare function

worse than its historical counterpart: 8.084 (historical), 8.055 (convergent), 8.280 (modified); this has led us to say that the modified objective function would no longer reflect decision-makers' past preferences. Results also confirm our earlier statement (Section 8.3.3) that beyond the 'convergent' solution, UR or INFL could be improved relative to the historical values only at the expense of the other. To recapitulate, statistics on UR and INFL are still disappointing; there is, however, some relief in low economic growth although it looks somewhat modest.[83]

Consider the control variables. The optimal M1 would be 2.74% lower than its actual level; in other words, tighter monetary restraint relative to the BOC's past monetary policy is still called for. The variable G1 needs detailed discussion. The modified objective function would require higher government spending and much larger deficit as a consequence. From our calculations, the 'convergent' solution necessitates an increase of government deficit[84] of 456 million current dollars on the annual average, compared to the historical policy. The alternative solution requires 1079 million . To many critics of the past monetary and fiscal policies, large deficits are not welcome or acceptable. Indeed, in Chapter 8 on the description of the history of Canadian macroeconomic policy, we have said at least on the expenditure side there has been some restraint

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83 For a proper evaluation of the GNP performance, we might have to wait for the results on GNP and welfare loss from the next experiment using the U.S. weights and three controls.

84 Government deficit is measured approximately by the sum of change in government bonds and monetary base. It should be clear from our discussion of the government budget constraint that this measure does not correspond to the actual and published data of the government deficit on the national accounts basis; however, the comparison among various solutions is completely meaningful.

in fiscal policy; nevertheless, many participants[85] at the symposium on 'Has Monetarism Failed?' were critical of the fiscal stimulus. Of course what they had in mind is large amounts of budget deficits which were on the average almost 8 billion current dollars per year (for three levels of governments) from 1975 to 1978, compared to a deficit of 942 million in 1974 and a surplus in 1973.[86] The general impression is that regardless of the antipathy to or the sympathy with the monetarist monetary policy, large government budget deficits were not supported.[87] In the literature, many arguments have been advanced against big deficits. E.g., large deficits will push up the interest rate through the government competition with the private sector for funds. Also, deficits, if continue, will accumulate; a time will come that a large part of government revenues is just to finance interest payments;[88] that

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See e.g., Wirick (1981), Freedman (1981).

86
These are actual and published data of the government deficit (surplus) on the national account basis. Sources: System of National Accounts, National Income and Expenditure Accounts, Second Quarter 1979, Statistics Canada.

87
If we ignore its policy's sympathizers, the Bank of Canada itself called for a tighter fiscal policy and a move to a more balanced federal budget. Its critics such as Barber and McCallum (1981, p. 226) favoured tight fiscal policy and relatively easy monetary policy.

88
In 1983, interest charges on the federal debt were 31.2% of the federal government revenues (J.S. McCallum, 1984, p. 56).More on the deficit issue, see the same reference.

situation will restrict the government's capacity to achieve macroeconomic objectives and makes difficult to manage the government's finances on a day-to-day basis. Also, deficits will enhance inflationary expectations because the public believes that they will eventually be financed by printed money (Parkin, 1982, Chapter 37); to Tobin (1980b, p. 57), there is scant evidence on this effect. But Lipsey (1981, p. 569) in his presidential address to the Canadian Economics Association and in a seminal paper 'The understanding and control of inflation: is there a crisis in macro-economics?' proposed more restrained fiscal policy along tight monetary policy in an anti-inflation package. He wrote:

> Second, there is increased fiscal restraint. This policy would be partly psychological, since there is not much evidence that current budget deficits are fuelling current inflationary fires. But psychology is an important element of inflationary inertia. The government must be willing to bear its share of the burden, and some kind of Reaganesque budget -- slashing (although without the bias against the poor of Reagan's measure) would be an important part of the package.

If large deficits are not favoured, it is difficult to see how the above modified welfare function can be viewed as representing the policy-makers' preferences. One may now feel that our choice of the weights of the 'convergent' as reflecting the authorities' preferences is not arbitrary or unreasonable. Unfortunately, the matter is not so simple. There is a fair number of economists who are willing to tolerate substantial budget deficits and believe that unemployment can be much improved. They would argue for attaching heavier weights to GNP gap and UR beyond the value of w1 = .25. To find where the truth lies, it is interesting to go back to the experiment in which the U.S. weights are used.

U.S. Weights: w1 = w2 = .75, w3 = w4 = 1

Recall that in the process of searching for the weights, we started by arbitrarily borrowing those applied to U.S. models participating in the NBER/NSF Econometric Model Comparison seminar. They were: w1 = w2 = .75 for GNP gap and UR, w3 = w4 = 1 for INFL and TB. Part of the optimal results obtained from those weights were reported in Table 8.3 and discussed in Section 8.3.3. For the sake of presentation, optimal results from Table 8.3 are reproduced in Column three of Table 9.13.

Two remarks are in order.

i) With respect to the U.S. economy, those U.S. weights would put GNP, UR at the same level of concern as INFL for the period 1971 I - 1975 IV (see Klein and Su, 1980). Therefore, there seems to be no ground to suggest that these weights would bear any relationship to the Canadian economy under the optimization period 1975 III - 1978 IV. Moreover, one has to admit that by using the U.S. weights, the decision-makers' preferences are substantially altered: compared to the convergent solution, the GNP and UR targets are assigned very heavier costs; as an indication, contrast the optimal values of targets with their historical levels (Table 9.13, Columns one and two).

TABLE 9.13: Joint Policy (Sensitivity Experiment with U.S. Weights): Total Value of Welfare Loss and Average Values of Real GNP, Target and Control Variables.

ITEM	HISTORICAL	OPTIMAL	
		CONTROL VARIABLES	
		G1, M1	G1, M1, Tax Rate
LOSS	799.29	608.234	539.226
Y	30428	31026	31645
YGAP	-6.748	-4.979	-3.107
UR	7.688	7.463	7.180
INFL	8.084	8.677	9.430
TB	-2.339	-2.769	-3.145
G1	5715	6007	5878
M1	19460	19614	18845

ii) While changing the weights, we have no intention of simultaneously varying the control bounds. There are two reasons. Those who are greatly concerned about UR and GNP surely want to alleviate the adverse effects of the monetary restraint; they would not like to reduce M1 below the historical level. Hence, original bounds on M1 would suffice. As to G1, the optimal level can be expected to increase relative to the historical level. In constructing its bounds, the upper limit has been assumed to grow at a steady rate of 4.0% in lieu of 1.94% as in the past;[89] that seems to give G1 enough room.

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[89] The growth rate is 4% for BND1(G1) and 5% for BND2(G1). Experimenting with larger bounds of M1 and G1 [i.e., BND2(M1) and BND2(G1)] does not alter the UR result.

Let's look at Table 9.13. The optimal policy would generate an inflation rate 7.34% [= 1 - (8.677/8.084)] higher than the historical level. Considering the fact that in 1978 the monetary authorities can reduce the rate of inflation only by 8.57% [= 1 - (6.4/7)] relative to the previous year's level,[90] that outcome is not easy for them to swallow. With respect to GNP, there is a meaningful improvement in terms of average value: optimal GNP is now 1.97% higher than the historical level. In terms of average annual rate of growth, the result is, however, less satisfactory, if not disappointing: from 1975 to 1978, optimal GNP grows at 3.92% as compared to 3.1% from the past policy and 3.96% from the previous experiment with the modified welfare function: w1 = .25, w2 = .50, w3 = w4 = 1. Notice that the policy with the U.S. weights results in a lower average annual rate but a higher average value of GNP relative to the optimal policy of the previous experiment. The conflicting signals (a point we have raised in Section 9.1.1) make the interpretation of the GNP performance difficult. Turning to UR, we see that it improves. But the decline is very small from 7.688 (historical) to 7.463 (optimal); this is transformed into an average reduction of 21 thousand unemployed workers relative to the historical performance. As a matter of fact, the gain in job creations is higher. Compared to the historical policy, the optimal one would, on the average,

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See Table 8.2.

increase employment (L) by more than 63 thousand but at the same time would augment labour force (LF) by more than 42 thousand , resulting in a net reduction of 21 thousand with respect to an actual, average level of 807 thousand unemployed persons.[91] One should realize how small the improvement in unemployment is.

Let us quickly consider the performance of the control variables. Optimal G1 much increases relative to the historical level by 5.11%: Optimal M1 also increases but insubstantially by 0.80%. When larger bounds, BND2(M1) and BND2(G1), are used, M1 declines relative to the historical level from 19460 (historical) to 19441 (optimal). The decline in optimal M1 is also observed in the previous experiment with weights w1 = .25, w2 = .50, w3 = w4 = 1 (see Table 9.12). We could conclude that even with more attention given to the GNP and UR targets, loose rather than tight government spending is called for, and generally monetary restraint is still necessary. This is at variance with some recommendation of easy money (Barber and McCallum, Peters and Donner), or easy money combined with tight fiscal policy (Barber and McCallum).

In short, although much attention has been accorded to UR and GNP targets, the improvement in UR is small. In view of the bad performance in UR, we are wondering whether

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91 The actual value of unemployment (= 807 thousand) was obtained from Table 9.4 as the difference between LF and L.

the result can improve in case that the fiscal authorities make use of two instruments: G1 and tax rate. As before, the tax rate will be a control, fixed but optimally determined in the optimization process. Optimal results are reported in the last column of Table 9.13. Now there is a tremendous reduction in welfare loss relative to the past policy: 32.54%. As preferences shift more toward GNP and UR, the tax rate declined from a historical value of .1767 (Equation 6.2.9) to .1507.[92] This combined with a higher government spending relative to the past level yields an impressive gain in GNP which is 4% higher than its historical level. From 1975 to 1978, optimal GNP grows at 4.75% on the annual average against an actual 3.1%. It is the first time we can say without ambiguity that the improvement in GNP is significant, substantial. As to UR, it is reduced from 7.688 (historical) to 7.180 (optimal). This translates into a reduction of 48000 unemployed workers; again the number of job creations is much higher: 149000 new jobs.[93] This is so far the best performance in UR, but objectively one has to

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92 In the case of the experiment with weights w1 = .25, w2 = .50, w3 = w4 = 1, introducing tax does not make any difference to the results. First, the values of the welfare loss function are almost identical regardless of tax or no-tax. Second, the historical tax rate hardly changes (optimal value of the tax rate is .1790).

93 Relative to the historical policy, employment L increases by 149000 persons, but at the same time labour force LF also increases by 101000 due to improved economic conditions.

say that the improvement is not great, if not modest.[94] On the inflation side, the gains in GNP and UR lead to a substantial increase in INFL relative to the past policy: INFL augments from 8.084 (historical) to 9.430 (optimal). It is quite clear that the battle against inflation has been lost.

Let us recapitulate. Essentially three experiments are conducted. The first is based on a welfare function with the target weights: w1 = .25, w2 = .50, w3 = w4 = 1. The second tries the U.S. weights: w1 = w2 = .75, w3 = w4 = 1. The last differs from the second in having one extra control, the tax rate. The aim of the experiments is to study the effects of a policy which shows more or much more concern about economic growth and employment. First, relative to the past performance GNP and UR can improve only at the expense of inflation. Second, the reduction in UR is modest or negligible. Third, GNP performs better. In general, to say whether its improvement is modest or fairly satisfactory is a matter of subjective judgement. In the case where the U.S.

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94 Throughout the discussion of the optimization experiments, we can observe that in the process of varying the weights of the objective function, focus has been laid almost solely on that of GNP gap. In reality, from the starting point which makes use of the U.S. weights to the arrival of the 'convergent' solution, both the weights of GNP gap and UR are tried extensively; due to its small impact, the weight of TB is seldom changed. Besides, based on the relative weights implied by the historical performance (see Table 8.5, Column 2) we also set w1 = .2, w2 = 1.1, w3 = 1 and w4 = 1.3. In general, we could say the optimal solution using the U.S. weights would generate the best performance for the unemployment rate.

weights are used in combination with three controls (G1, M1, and tax rate), we can definitively conclude that the improvement is substantial; but the sacrifice is very high: the battle of controlling inflation which has openly been supported throughout Western nations is lost. Fourth, the realization of these outcomes generates higher government spending and more reduction in money relative to the historical policy. This result must disappoint critics who favour more growth and employment but are not prepared to tolerate large government deficits.

b. Optimality and Suboptimality Due to Rule

The monetary policies without and with rule (Experiments Nos. 1, 6) and the fiscal policies without and with rule (Experiments Nos. 2, 7) will be executed again, based on the welfare function whose weights are w1 = .25, w2 = .50, w3 = w4 = 1.

To mention very briefly, in terms of welfare loss relative to that of the historical performance, the improvement declines from 2.10% to 1% depending on whether the monetary policy follows no rule or rule. In the case of the fiscal policies, the improvement is tremendously reduced from 8.42% (no rule) to 2.08% (with rule).

These findings confirm previous results concerning the effectiveness of monetary and fiscal instruments when subjected to rule.

9.3.3 Sensitivity with Respect to a Change in the Control Variable (M1) Bounds.

To many critics of the BOC policy (e.g., Wirick, 1981), gradualism has brought only modest results on the inflation rate. They accepted inflation as the main concern and fully agreed with the BOC's strategy of achieving the target by reducing the growth of the money supply. They would believe that the BOC's gradualism has been too gradual and therefore was a bad tactic in permitting an effective reduction in inflation. The last experiments will focus on the impact of a less gradual monetary restraint. The bounds of G1 will be kept intact, but those of M1 will be much widened so that larger reduction in M1 relative to those in previous experiments are possible.[95] Sandblom-Banasik-Parlar procedure is used (actually, has been used). Recall in Section 9.1.1 three sets of time dependent bounds were generated, allowing M1 to vary within one, two, and three times its historical range. For expository purposes, denote them successively by B1(M1), B2(M1), and B3(M1). In Section 9.1.1, we have also noted that beyond B3(M1), the bounds values are no longer realistic. Even B3(M1) produces unreasonable values for M1: from 1975 III to 1978 IV, the average value of optimal M1 is 13832 million current dollars. Contrast it with actual, annual values of M1 from 1973 to 1978: 13444, 14729, 16729, 18082, 19577, and 21557. In the light of this, it is decided to report only the

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95 Introducing the tax rate does not fundamentally affect the results.

results obtained from more restrictive bounds sets, B1(M1) and B2(M1).

Table 9.14 displays the results along with those of the historical and convergent solutions.

TABLE 9.14: Joint Policy (Sensitivity Experiment with Widened Bounds of M1): Total Value of Welfare Loss and Average Values of Real GNP, Target and Control Variables.

ITEM	HISTORICAL	CONVERGENT	OPTIMAL (1)	OPTIMAL (2)
LOSS	368.776	342.048	315.712	294.949
Y	30428	30655	30603	30479
YGAP	-6.748	-6.131	-6.282	-6.649
UR	7.688	7.675	7.696	7.752
INFL	8.084	8.055	8.001	7.853
TB	-2.339	-2.400	-1.974	-1.491
G1	5715	5831	5922	5966
M1	19460	18841	16933	14963

Notes: 1) Based on B1(M1): bounds range equal to the historical range

2) Based on B2(M1): bounds range equal to twice the historical range

In terms of welfare loss, the optimal policies based on the new sets of M1 bounds perform much better than the convergent solution. This is well understandable: as the criterion function is mainly geared toward the inflation target, allowing M1 more room should naturally bring a larger reduction in welfare loss. That statistic, however, does not unfold the whole story.

Take the historical values as benchmarks: contrary to the convergent solution, the new optimal policies exhibit a trade-off between UR, GNP, and INFL. The message is: manipulating M1 in the direction of lowering the rate of inflation would require higher UR and heavier loss in output. Unfortunately, the improvement in INFL is insubstantial. With B1(M1), the inflation rate goes down from 8.084 (historical), 8.055 (convergent) to 8.001 and calls for a larger reduction in M1: optimal M1 is 12.99%[= 1 - (16933/19460)] lower than the actual level against 3.18% from the convergent solution. With B2(M1), the inflation goes down a bit further to 7.853 but demands a tremendous reduction in M1: optimal M1 is 23.11% lower than the actual level. We like to remark that with B2(M1), the average value of optimal M1 is 14963 while the actual value of M1 in 1974 was 14729. To a large extent, if we take 1974 as the benchmark year, the policy using B2(M1) could be regarded as attempting to practically impose a freeze[96] on the rate of growth of M1 for the whole optimization period from 1975 III to 1978 IV. In view of the large dosage, that policy might not be conceived of as gradual.

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96 Indeed, when examining the optimal plot of M1, we observe that except 1976 II, in any quarter the value of optimal M1 is always lower than the actual value of M1 in 1975 III which is the starting quarter of the planning period.

In summary, with some reservation about the model coefficients stability, the last experiments indicate that as long as some form of gradualism is pursued, allowing more room on M1 could not bring better results relative to the historical policy; even on the inflation side, the improvement is small while UR, GNP are worse, and higher government spending, which is not welcomed by monetarists, is still required.

CHAPTER 10

CONCLUSION

This has been a long discussion. We started by stating a few macroeconomic problems we like to study for the Canadian economy. The vehicle proposed for studying is a small-sized quarterly econometric model, and the technique is optimal control. The task proceeds with the review of the literature on the rational expectations controversy. This is indispensable because, to an extent the discussion of present macroeconomic problems will lose its full significance if there is no mention of the rational expectations school whose ideas go to the heart of every stabilization attempt, of policy evaluation...Our task continues with the construction of the econometric model which requires several steps: specification and estimation by OLS, estimation by 2SLS, validation. We then conduct a number of optimization experiments, using the bounded control approach. This approach has certain advantages which are not well recognized or appreciated in the empirical literature. Hopefully the description on techniques of imposing bounds, on the controls behaviour under bounds proves to be useful.

In closing we like to summarize the results of the optimization experiments and derive some implications (old and new); but before that, it seems meaningful to draw a few interesting results from the estimation and simulation chapters. We personally believe part of the thesis contribution comes from the efforts of building the model.

Econometric results

i) The role of wealth is found to be very minor in the consumption equation. This is in contrast with its emphasis or the importance attached to a related concept (real balances effect in consumption) as found in textbooks and the literature. An example we came across is Begg (1982, pp. 147-149). In his book, Begg proved that if real balances effect is incorporated in consumption, the strong neutrality proposition fails. In the context of rational expectations, we have discussed the policy ineffectiveness proposition which asserts that any systematic action from the government will not affect the deviation of output from its natural rate. The strong proposition says that even in the long-run systematic monetary action cannot affect the natural rate of output.

Our result would suggest at least for Canada some caution in attaching an importance to the role of wealth in consumption.

ii) Again, wealth is not found significant in the currency and demand deposits equations; it is significant only in the time deposits equation. In the literature (see, e.g., Laidler, 1969, p. 90 and for Canada, White, 1976, pp. 78-80), it is often reported insignificant when GNP and wealth enter the demand for money simultaneously. The usual explanation is multicollinearity. Our result points out an aggregation problem.

In the macroeconomic literature, the integration of the government budget constraint in an economic model is highly recommended. Apart from the reason that one among the set of instruments has to be determined residually, its inclusion will permit to trace out the monetary and debt effects of government deficit financing: impact on the IS curve via wealth effect in consumption, crowding-out due to bond financing etc...The above results (very minor role of wealth in consumption and its insignificance in currency and demand deposits) would diminish this important aspect of the government budget contraint in a short-run model.

iii) The last result we like to mention concerns the foreign exchange rate equation which performs nicely with the monetary view rather than with the traditional view whose analysis is based on the components of the balance of payments. And inside the monetary view the Keynesian approach of sticky prices would be more appropriate than the Chicago approach of flexible prices.

Recall how important the assumption of flexible prices is for the rational expectations theorists.

Simulation results

One important finding is that the model does not converge in simulation if we follow American textbooks and assume that the Bank of Canada controls the monetary base rather than the money supply M1. This is not a place to criticize or defend the Bank of Canada for its emphasis on M1 as a policy instrument. It appears that in the Canadian context, treating the monetary base as the policy instrument might not reproduce the historical path of the economy accurately.

Optimization results

All optimization experiments have been a process undertaken to answer three specific problems, namely:

i) Is there any substantial gain over the historical performance from an optimal coordination of monetary and fiscal policies in order to attribute disappointing results of the Canadian economy to the monetarist policy of the Bank of Canada?

ii) Relative effectiveness among monetary-fiscal policies, and inside each monetary or fiscal policy due to the adoption of the (constant growth rate) rule.

iii) Impact of inside lag on relative effectiveness.

The main findings can be summarized as follows.

Based on the criteria[97] stated in Chapter 1 it could be said that there is no evidence of a substantial

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[97] The fulfillment of one of the following conditions would allow to conclude that the gain is substantial:

i) the optimal policy brings a substantial and simultaneous improvement in GNP, UR, and INFL relative to the past policy

ii) the optimal policy yields much better results in GNP and UR without much deteriorating INFL relative to the past policy

iii) the optimal policy substantially reduces INFL without much deteriorating GNP and UR relative to the past policy.

gain over the historical policy. The 'convergent' solution generates little gain in spite of a simultaneous improvement in GNP, UR, and INFL. Optimal policies which, relative to the past policy, exhibit trade-offs between GNP, UR on the one hand and INFL on the other, appear ineffective in reducing high unemployment and inflation. In case more attention was paid to GNP and UR without much sacrificing INFL, some relief in low economic growth could be felt, although it looks somewhat modest.[98] In case the decision-makers were willing to tolerate much higher inflation relative to the past level, optimal policy could deliver a substantial improvement in GNP;[99] considering, however, the efforts industrialized nations have put to control inflation, one would think that this solution is out of tune with the international concern at that time.

Among the three policies, in terms of welfare loss the joint policy is the best as mathematically expected. The fiscal policy is next; but its superiority to the monetary policy is negligible or rather negligible[100] depending on

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98 i.e., the solution with weights: w1 = .25, w2 = .50, w3 = w4 = 1.

99 i.e., the solution with the U.S. weights and with the use of three controls.

100 based largely on the evaluation of the welfare loss function values. Other statistics (GNP, UR...) are also considered explicitly or implicitly.

the welfare weights,[101] and does not reflect the degree of effectiveness it enjoys from the multiplier calculations.[102] Under the 'convergent' weights, when the effect of inside lag is taken into account, the superiority ceases, and the monetary policy turns out to be slightly more effective; this outcome would affect the already small gain of the (convergent) joint policy over the historical policy because without lag consideration, the fiscal instrument contributes most in the joint policy.

Inside monetary or fiscal policy, the imposition of the rule will diminish the controls' performance. The loss in effectiveness would be small for monetary policy but high for fiscal policy. The word 'high' should be understood in a relative sense, first relative to the loss due to the adoption of the monetary rule, second relative to the context. In an anti-inflation period, the loss would not be high. But that would not be the case in a period under which the focus of attention ~~is~~ employment and economic growth.

What kinds of implications can we draw from these results?

Results on relative effectiveness between monetary and fiscal policies imply at least one thing: in the stagflation world, it is obvious that expressing relative effectiveness in terms of a single target GNP has little meaning:

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[101] They belong to the 'convergent' solution or to the one with w1 = .25, w2 = .50, w3 = w4 = 1.

[102] The results were reported in Chapter 7, Section 7.2.

in presence of multiple targets, not only it would be no longer adequate, multiplier analysis could be a misleading tool in assessing relative effectiveness.

Results on optimality and suboptimality due to rule lead to an interesting phenomenon: in the conduct of monetary policy, after a target range has been set for a certain planning period, on a quarterly basis it would matter little whether M1 is discretionary or takes a steady growth path.[103] The result, however, begs the caution since it might well depend on the period of study; further empirical evidence is required for it to be substantiated.

Results on the evaluation between the historical and optimal policies yield several implications.

1. The first is that some recommendations and prescriptions given by critics in the symposium whose views might have been shared by a number of economists appear unsupported by empirical evidence.

i) A policy, even geared more heavily toward GNP and UR, could not bring a substantial relief in high unemployment. GNP could substantially improve, but the battle against inflation would be lost.

ii) A less gradual monetary policy could not do better. Particularly no sharp decline in INFL was obtained.

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[103] The observation applies to M1 seasonally adjusted.

iii) Many Keynesians may have believed that the bad performance of the world economy was due to the monetary restraint pursued in many industrialized countries. To improve economic growth and employment, they would have preferred a policy mix of loose monetary and tight fiscal policy. As long as inflation is an important economic objective, in our Canadian context optimal results generally call for tighter monetary restraint relative to the past. On the other hand, attempting to improve GNP or UR requires looser government spending than the historical level which had been much below the normal trend.

2. Unless we advocate a quick disinflationary policy with the belief that there are no real side-effects or they are short-lived (more on this later), monetary and fiscal policies as demand management tools, even in coordination, do not seem effective in providing a satisfactory solution to the Canadian unemployment-inflation problem in the short-run. With respect to inflation, Laidler (1982, p. 175) remarked that we have taken fifteen years to get into our current solution, so there is no reason why we should not expect to take close to a decade to unwind the inflation rate. Tobin, a long time ago, particularly wrote:

> As I have already noted, wage equations that assign high coefficients to past price experience do not assign a strong influence to unemployment. The short-run Phillips curve is flat at high rates of unemployment. Since it is steep at low rates, a much longer time is required to unwind an inflation than to generate one.
>
> In the circumstances, neither monetary policy nor aggregate-demand policy is in general a useful tool [Tobin, 1974, pp. 231-232].

3. In view of (1) and (2), a gradual approach to end inflation, even announced, will inevitably involve high unemployment and low economic growth. And the necessary time can be long, and longer for unemployment as optimal results would imply.

4. In view of the above points, attributing undesirable results of the economy to the Bank of Canada monetarist policy of controlling inflation by participants at the Canadian symposium on 'Has Monetarism Failed?' in particular would not have a valid foundation; especially one must recall that monetary policy is but one tool of macro-economic management, and other tools are quite beyond the control of the Bank of Canada.

It should be added that certain economists (e.g., Tobin, 1980b; Lipsey, 1981) who are well aware of the bad performance of the traditional monetary and fiscal policies in the present situation have advocated controls as a supplementary tool to lessen the real effects of monetary restraint. On the effectiveness or ineffectiveness of controls, our thesis has very little to say but mentions that in the period of study Canada was under a wage and price control program (for a view against controls see, e.g., Laidler, 1982, pp. 181-187).

As points (3) and (4) are quite important in the Canadian context, are there any studies which would support them?

If one restricts macroeconomic tools to the traditional fiscal and monetary policies, the previous quotation from Tobin, a leading Keynesian, would be an evidence of support. However one has to say immediately that Tobin (1980b), concerned with high costs of unemployment and output due to a monetary disinflation, did not think that the traditional demand management is a sufficient package.

Switching to the monetarist camp, Friedman on several occasions stated that there is no way to end inflation which does not involve a temporary, though perhaps fairly protracted, period of low economic growth and high unemployment (see, e.g., Friedman, 1974, p. 23). He also expressed that present rates of growth of monetary aggregates should be modified to achieve a long-run target in a gradual, systematic, and preannounced fashion (Friedman, 1982a, p. 101). But to our knowledge, the true sympathizer of the Bank of Canada policy was Laidler. As an organizer of the symposium on 'Has Monetarism Failed?', he stood alone among participants for his advocacy of gradualism (Laidler, 1981a and b). This theme is repeated at length in his recently published book (Laidler, 1982, Chapter 5). As our results are empirical, it is worthwhile to examine them from another angle, theoretical. For this let us take a brief look at Laidler's theme.

Gradualism is a program of coping with inflation by slowly reducing the rate of growth of money by one or two percentage points a year until long-run price stability is reached. To Laidler, the program is not painless or short-lived; but it is the only policy that is likely to be found tolerable compared to alternatives which are worse in the current state of knowledge. The main short-run, not necessarily short-lived, effects of reducing the monetary supply in an economy where inflation is well entrenched is not the reduction in the inflation rate, but a down-turn in real activity and an increase in unemployment. This is due to the inertia in inflation expectations, the long-term contracts which embody those expectations, and the difficulty that firms and other economic agents encounter in distinguishing random fluctuations in demand from long-term changes. The effect on inflation will come later. And in due course, the falling inflation rate will catch up with the rate of monetary expansion and also can overtake it. Under a gradualist policy, the inflation rate (as well as unemployment and output) is likely to follow a cyclical path. This implies that there is no reason to expect a close relationship between the rate of inflation and the rate of monetary expansion in the long transitional period; consequently, any absence of such a correlation is not a sign of failure, and a satisfactory rate of inflation does not suggest it will be sustained.

Is a quick cure better? One justification is based on the rational expectations hypothesis and grounded on the belief that much inflationary inertia comes from expectations, and since economic agents form their expectations rationally, an announced and clear-cut change would affect those expectations instantaneously, without real side effects. Laidler questioned first if the announcement is believed, secondly if agents expect it to be effective, and thirdly if they are in a position to act upon those expectations (e.g., agents can be bound by long-term contractual arrangements). Monetarists believe a reduction of money will reduce inflation; but the general public does not understand the monetary theory of inflation or believe in it. So, it is inconsistent to argue that such a policy will change the expectations of the people who do not believe in it.

Others (e.g., Lipsey, 1979) would advocate a quick cure on the belief that its effects, although more painful, will be relatively quick. A quick cure could bring a sharp decrease in the inflation rate, but Laidler asked if the subsequent convergence of the economy to its natural unemployment rate would be any quicker than under a gradualist policy. Furthermore, is there reason to believe cycles generated by a quick cure will be shorter? It is known that in many economic models, factors which determine the period of any inherent cycle do not depend on the size of the shock; therefore, it is too bold to speculate about the dynamic process underlying the interaction of monetary expansion, unemployment and inflation.

Briefly, to Laidler a quick cure could be less costly, but its success would be the result of expectation effects, and of certain dynamic properties of market processes which one cannot a priori rely upon due to the lack of evidence. Therefore, a quick cure is risky, and it is on this that the gradualist alternative rests.

One may feel that our results are consistent with Laidler's theme. It is an encouragement that part of our study has some theoretical support. Just as now, it appears that the lessons of inflation have been well learned. Less enthusiasm is given to the belief that there is anything to be gained from increasing demand through stimulative monetary policies because such policies would soon bring higher inflation, higher interest rates, less investment and more unemployment.[104]

In closing, we like to remind again that our results should be viewed as suggestive. Efforts have been made to assess their robustness, but several factors may play an influencing role: formulation of the model, short-run analysis, specification of the welfare loss, control bounds, limited number of experiments...

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104 From Emile Van Lennep (1984, p. 3), OECD's Secretary General.

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APPENDIX A

DATA SOURCES

The first part of the Appendix A provides the sources of the endogenous and exogenous variables entering the econometric model. Two exceptions are dummies which will be left out completely, and definition variables. Apart from a few series which were obtained directly from the Bank of Canada by request, the Canadian and American series were retrieved from Statistics Canada's computerized databank, CANSIM Mini Base,[105] which is reviewed every six months.[106] The series will be identified respectively by the CANSIM matrix and databank identification numbers. Data are N.S.A. (non-seasonally adjusted) and quarterly unless indicated.

The second part is concerned with the sources of the data related to the seasonal factor ratios which are used implicitly or explicitly in some optimization experiments, in the welfare loss function, and in the calculation of control bounds. As their period which corresponds approximately to the optimization period (1975 III - 1978 IV) is quite short, the series are mainly loaded manually.

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105 The title of the directory is "CANSIM Mini Base Series Directory", Statistics Canada.

106 The majority of our data were retrieved in July 1979, some around December 1980, March 1981, and October 1982.

1. VARIABLES OF THE ECONOMETRIC MODEL

1.1 Endogenous Variables

BOND	=	federal, provincial, and local government debt (stock) held by the private sector, millions of current dollars.		
		. End-of-quarter stock of Government of Canada (i.e., federal) direct and guaranteed securities (excluding Bank of Canada holdings) is the sum of:		
		+ Treasury bills held by the chartered	923	B2473
		banks and the general public	923	B2477
		+ Bonds held by the chartered banks	923	B2474
		+ Marketable bonds held by the public	923	B2478
		+ Canada Savings bonds.	923	B2406
		. End-of-quarter stock of provincial and municipal bonds, direct and guaranteed: data obtained from the Bank of Canada.		
CON	=	real consumption (personal expenditure on real consumer goods and services), millions of 1971 dollars.	525	D40562
CUR	=	currency outside banks, millions of current dollars. Average of monthly data.	921	B2001
DD	=	demand deposits at the chartered banks, millions of current dollars. Average of monthly data.	921	B2008

DSN = time deposits at the chartered banks, millions of current dollars. Average of monthly data.

. Sum of:

+ Personal savings deposits 921 B2006

+ Non-personal term and notice deposits. 921 B2007

EX = real export of goods and services, millions of 1971 dollars. 525 D40586

IM = real import of goods and services, millions of 1971 dollars. 525 D40588

IIN = change in business inventories, millions of 1971 dollars.

. Sum of:

+ Business: non-farm 525 D40583

+ Business: farm and grain in commercial channels. 525 D40584

INR = real gross private investment in non-residential construction, and machinery & equipment, millions of 1971 dollars.

. Sum of:

+ Business gross fixed capital formation in non-residential construction 525 D40577

+ Business gross fixed capital formation in machinery & equipment 525 D40578

IRC	= real gross private investment in residential construction, millions of 1971 dollars.	525	D40576
IRCNR	= real gross private fixed investment, millions of 1971 dollars.		
	. Sum of INR and IRC.		
L	= employment (both sexes, 15 years and over), thousands of persons. Average of monthly data.	2074	D767286
LF	= labour force (both sexes, 15 years and over), thousands of persons.	2074	D767285
M2	= broad money quantity, millions of current dollars.		
	. Sum of narrow money M1 and time deposits DSN.		
MB	= monetary base, millions of current dollars.		
	. Sum of currency CUR and chartered bank reserves RR.		
P	= implicit price deflator, gross national expenditure (GNE), 1971 = 1.		
	. Ratio of:		
	GNE in current dollars to	1001	D40548
	GNE in 1971 dollars.	525	D40561

PE	= implicit price deflator, export of goods and services, 1971 = 1.		
	. Ratio of:		
	Export in current dollars to	1001	D40029
	Export in 1971 dollars	525	D40586
PFX	= spot exchange rate, number of Canadian dollars per U.S. dollars. Average of monthly data.	926	B3400
RL	= long-term interest rate (Government of Canada bond yield, 10 years and over). Average of monthly data.	2560	B14013
RR	= chartered bank reserves, millions of current dollars. Average of monthly data.		
	. Sum of:		
	+ Notes in circulation held by the chartered banks	911	B52
	+ Chartered bank deposits at the Bank of Canada	911	B55
RS	= short-term interest rate (90-day finance company paper rate). Average of monthly data.	2560	B14017
RTD	= interest rate on time deposits.		
	. Weighted average of interest rates on (chartered bank) personal savings deposits and on non-personal term and		

notice deposits: series on individual interest rates are obtained from the Bank of Canada; weights are computed and equal to the shares of each type of deposit over their sum.

TAX = taxes, net of subsidies and transfers, from federal, provincial, and local governments, millions of current dollars.

Net sum of:

+	Direct taxes - Persons: Federal	1005	D40059
+	As above: Provincial	1005	D40060
+	As above: Local	1005	D40061
+	Direct taxes - Corporate and Government business enterprises: Federal	1005	D40065
+	As above: Provincial	1005	D40067
+	Indirect Taxes	1005	D40070
+	Other current transfers from persons: Federal	1005	D40075
+	As above: Provincial	1005	D40076
+	As above: Local	1005	D40077
-	Transfer payments to persons: Federal	1005	D40118
-	As above: Provincial	1005	D40119
-	As above: Local	1005	D40120
-	Subsidies (Federal and Provincial)	1005	D40124

TR	= total revenues of federal, provincial, and local governments, millions of current dollars.		
	. Sum of:		
	+ Total revenues: Federal	1005	D40093
	+ Total revenues: Provincial	1005	D40094
	+ Total revenues: Local	1005	D40095
UR	= unemployment rate.		
	. Defined as $\frac{(LF - L)}{LF} \cdot 100$		
V	= real financial wealth, millions of 1971 dollars.		
	. Defined as (MB + BOND)/P.		
W	= wage rate (total manufacturing average hourly earnings). Average of monthly data.	75	D1518
Y	= real gross national expenditure (GNE) or GNP, millions of 1971 dollars.	525	D40561

1.2 Exogenous Variables

DG	= Government of Canada deposits at the chartered banks, millions of current dollars. Average of monthly data.	921	B2005
G1	= real expenditure of federal, provincial, and local governments (current expenditure on goods and services + gross capital formation), millions of 1971 dollars.		

- G1 is derived in two steps. First, its nominal value is calculated. Then it is deflated by its price deflator.
- The nominal value of G1 is the sum of:

+	Current expenditure on goods and services: Federal	1005	D40112
+	As above: Provincial	1005	D40114
+	As above: Local	1005	D40115
+	Gross capital formation: Federal	1005	D40157
+	As above: Provincial	1005	D40158
+	As above: Local	1005	D40159

- As the implicit price deflator of G1 is not available, it is proxied by the price deflator of total government expenditure (= sum of G1 and G2 in the income equation).

  Let us denote it by 'PG'.
- PG is equal to the nominal value of total government expenditure divided by its counterpart in 1971 dollars.

  The former is the sum of:

+	Government current expenditure on goods and services (current dollars)	1001	D40015
+	Government gross fixed capital formation (current dollars)	1001	D40017

+ Government value of physical change in inventory (current dollars). 1001 D40026

The latter (let us denote by 'G') is the sum of:

+ Government current expenditure on goods and services (1971 dollars) 525 D40568
+ Government gross fixed capital formation (1971 dollars) 525 D40570
+ Government value of physical change in inventory (1971 dollars). 525 D40582

G1R = real other expenditure of federal, provincial, and local governments, millions of 1971 dollars.

. Net sum of:

+ (Total current expenditure in current dollars/PG)
+ (Gross capital formation in current dollars/PG)
- G1

. Total current expenditure in current dollars is the sum of:

+ Total current expenditure: Federal 1005 D40143
+ As above: Provincial 1005 D40144
+ As above: Local 1005 D40145

. Gross capital formation is the sum of:

+	Gross capital formation: Federal	1005	D40157
+	As above: Provincial	1005	D40158
+	As above: Local	1005	D40159

G2 = real expenditure of Hospitals, Quebec Pension Plan, and Canada Pension Plan (current expenditure on goods and services + gross capital formation), millions of 1971 dollars.

. G - G1

.. For the derivation of G, see explanation on G1.

M1 = narrow money quantity, millions of current dollars.

. Sum of currency CUR and demand deposits DD.

MU = U.S. broad money quantity, millions of current dollars.

. Sum of:

+	U.S. currency and demand deposits	2544	B54307
+	U.S. time and savings deposits.	2544	B54308

OREV = other revenues of federal, provincial, and local governments, millions of current dollars.

. TR - TAX

OTHER = balancing item in the government budget constraint, millions of current dollars.

. Residually derived from the government budget constraint.

PIMA = implicit price deflator (expressed in foreign currency), import of goods and services.

. Ratio of implicit price deflator (expressed in domestic currency), import of goods and services, to the foreign exchange rate. Let us denote this import price by 'PIM'.

. PIM is the ratio of:

Import of goods and services, in current dollars to	1001	D40030
Import of goods and services, in 1971 dollars.	525	D40588

PYBU = implicit price deflator, U.S. business product, 1971 = 1 (rebased).

. Ratio of:

U.S. business gross domestic product (in current dollars, S.A.) to	2548	B51302
U.S. business gross domestic product (in 1972 dollars, S.A.).	2549	B51402

RES = residual error of estimate in the income equation, millions of 1971 dollars.

. Derived as the difference between the

	right-hand and left-hand sides of the income equation.		
RLU	= U.S. long-term government bond yield. Average of monthly data.	2545	B54403
RSU	= U.S. short-term interest rate (90-day short-term paper). Average of monthly data.	2545	B54412
TIME	= time trend (= 1, 2, 3... first period 1967 I).		
WIP	= index of world industrial production. . Source: United Nations, Monthly Bulletin of Statistics (various issues). . Series do not have the same base. A linking is necessary; the series is then rebased.		
YU	= real U.S. GNP, millions of 1971 dollars, S.A. (rebased). . First, the GNP price deflator is derived. It is the ratio of:		
	U.S. GNP in current dollars to	2533	B50201
	U.S. GNP in 1972 dollars.	2534	B50301
	. The price deflator is then rebased. . Finally, GNP in 1971 dollars is the ratio of nominal GNP to the rebased price deflator.		

2. SEASONAL FACTOR RATIOS IN THE OPTIMIZATION EXPERIMENTS

In the welfare loss function as well as in some optimization experiments, we have to transform certain N.S.A. data into S.A. data or vice versa by using the seasonal factor ratios defined as ratios of N.S.A. to S.A. data. There exist four ratios for GNP, G1, UR, and M1. As N.S.A. series have been discussed earlier, below we will almost mention the sources of S.A. data.

i) For GNP

S.A. data was taken directly from Table 20 of 'System of National Accounts, National Income and Expenditure Accounts', Statistics Canada (Second Quarter 1979).

ii) For G1

S.A. data was taken from the same publication as for GNP. On how it is defined, see the previous section for N.S.A. G1.

iii) For UR

The source is Section 4, Table 3 of 'Canadian Statistical Review', Statistics Canada (various issues). First, quarterly data are formed for N.S.A. and S.A. UR by taking the average of three suitable months. Then, the seasonal factor ratio is calculated.

iv) For M1

The source for S.A. data of M1 is CANSIM Mini Base, CANSIM matrix No. 919, identification No. B1609.

APPENDIX B

DATA SERIES

Appendix B provides a number of data series which are intended to facilitate the examination of certain displayed figures. It contains 14 tables. The first twelve (Table B.1 to B.12) show actual and optimal data from the basic experiments of the monetary-fiscal policies; data correspond to Figs. 8.1 - 8.6 of Chapter 8, and to Figs. 9.1 - 9.12 of Chapter 12. The last two (Tables B.13 and B.14) show actual and simulated data. As suggested by the thesis committee members, data for only two endogenous variables (Y and RS) are supplied; they correspond respectively to Figs. 7.17 and 7.15 of Chapter 7. The variable Y (real GNP) is the key variable of the simulation exercises. As to RS (short-term interest rate), recall that it is the worst variable in terms of root-mean square percent error.

TABLE B.1: Actual and Optimal Values (Basic Experiments) of Y (Real GNP). Millions of 1971 Dollars.

19753	29760.0	29735.5	29760.0	29735.5
19754	29034.0	28873.6	29027.6	28880.0
19761	27801.0	27435.2	27777.5	27458.8
19762	29640.0	29414.3	29564.6	29489.9
19763	31361.0	31484.9	31219.3	31407.3
19764	30314.0	30216.3	30160.5	30320.0
19771	28414.0	27957.2	28286.0	28074.4
19772	30130.0	29921.2	30053.8	29947.6
19773	32053.0	32689.6	32022.4	32689.1
19774	31352.0	31830.4	31350.2	31792.9
19781	29306.0	29993.9	29308.2	29923.4
19782	31085.0	32046.5	31074.4	32058.7
19783	33354.0	34549.9	33326.2	34610.2
19784	32382.0	33021.9	32345.4	32911.4

TABLE B.2: Actual and Optimal Values (Basic Experiments) of UR (Unemployment Rate). In Percent.

19753	6.49675	6.49925	6.49677	6.49925
19754	6.62638	6.64455	6.62703	6.64393
19761	7.85134	7.90605	7.65433	7.90309
19762	7.02725	7.10016	7.03715	7.09027
19763	6.72874	6.78566	6.75148	6.78353
19764	6.91802	6.98313	6.95546	6.97155
19771	8.99506	9.10235	9.04501	9.07957
19772	7.99276	8.10861	8.04529	8.08528
19773	7.63166	7.68143	7.68257	7.65996
19774	7.83143	7.83550	7.88114	7.81762
19781	9.56333	9.50128	9.61156	9.49033
19782	8.55515	8.41004	8.59837	8.40000
19783	7.82080	7.58135	7.86131	7.56770
19784	7.59744	7.30485	7.63899	7.30106

TABLE B.3: Actual and Optimal Values (Basic Experiments) of INFL (Inflation Rate). In Percent.

19753	9.52881	9.52878	9.52899	9.52878
19754	10.6257	10.6169	10.6258	10.6169
19761	10.5101	10.4511	10.5082	10.4532
19762	11.3812	11.2063	11.3717	11.2160
19763	8.17097	7.93084	8.14115	7.75892
19764	9.10729	8.85402	9.03424	8.86941
19771	8.56279	8.27916	8.43731	8.32263
19772	7.23980	6.90585	7.06533	6.97866
19773	6.36531	6.04254	6.17816	6.10868
19774	6.01952	5.78564	5.82934	5.87918
19781	6.64368	6.55660	6.47151	6.62973
19782	6.95872	7.19208	6.81452	7.22542
19783	5.75597	6.30082	5.63796	6.32366
19784	6.30303	7.11768	6.19604	7.14262

TABLE B.4: Actual and Optimal Values (Basic Experiments) of TB (Nominal Trade Balance as a Percentage of Nominal GNP). In Percent.

19753	-1.73793	-1.57837	-1.60282	-1.71348
19754	-2.77632	-2.25600	-3.40464	-2.62845
19761	-4.46128	-3.76169	-4.05726	-4.16800
19762	-2.56025	-2.19100	-2.24225	-2.51163
19763	-.314780	-.467011	-.197716	-.416665
19764	-2.25138	-2.23844	-2.22963	-2.26753
19771	-3.56033	-3.24461	-3.62001	-3.22562
19772	-2.59418	-2.59893	-2.64851	-2.50547
19773	-1.23041	-1.88942	-1.21256	-1.87614
19774	-1.86937	-2.17140	-1.77198	-2.23541
19781	-2.39582	-2.78911	-2.25133	-2.87324
19782	-2.67478	-3.22318	-2.52671	-3.39538
19783	-.883720	-1.67855	-.736592	-1.86174
19784	-3.42858	-3.51307	-3.17339	-3.63196

TABLE B.5: Actual and Optimal Values (Basic Experiments) of G1 (Real Expenditure of Federal, Provincial, and Local Governments). Millions of 1971 Dollars.

19753	5781.34	5767.49	5781.34	5767.49
19754	5711.20	5627.45	5711.20	5627.45
19761	5873.43	5700.39	5873.43	5700.39
19762	5219.26	5179.05	5219.26	5179.05
19763	5686.08	5848.68	5686.08	5724.76
19764	5523.22	5493.65	5523.22	5493.65
19771	5919.76	5691.35	5919.76	5671.35
19772	5411.63	5363.10	5411.63	5321.63
19773	5806.37	6188.65	5806.37	6160.65
19774	5806.80	[illegible]58.74	5806.80	5924.60
19781	6056.23	6335.70	6056.23	6288.39
19782	5469.62	5910.29	5469.62	5902.67
19783	5944.97	6511.94	5944.97	6511.94
19784	5802.40	6060.39	5802.40	5952.33

TABLE B.6: Actual and Optimal Values (Basic Experiments) of M1 (Narrow Money Quantity). Millions of Current Dollars.

19753	17179.0	16368.9	16368.9	17179.0
19754	18250.0	16922.2	16922.2	18250.0
19761	17431.3	16772.6	16772.6	17431.3
19762	17714.3	17271.7	17271.7	17714.3
19763	18500.3	18189.9	18396.7	18500.3
19764	18691.3	18646.0	18646.0	18691.3
19771	18477.7	18458.1	18458.1	18477.7
19772	19119.7	18871.8	18871.8	19119.7
19773	20095.7	19490.0	19490.0	20095.7
19774	20670.3	19955.6	19955.6	20670.3
19781	20327.3	19699.1	19699.1	20327.3
19782	20864.7	20207.5	20207.5	20864.7
19783	22169.3	21231.1	21231.1	22169.3
19784	22948.3	21682.2	21682.2	22948.3

TABLE B.7: Actual and Optimal Values (Basic Experiments) of CON (Real Consumption). Millions of 1971 Dollars.

19753	17322.0	17311.4	17322.0	17311.4
19754	19730.0	19646.6	19717.4	19649.1
19761	17094.0	16908.2	17083.5	16918.7
19762	18777.0	18611.3	18742.3	18646.0
19763	18322.0	18314.6	18250.9	18290.5
19764	20900.0	20854.9	20809.9	20687.5
19771	18017.0	17801.0	17929.0	17861.3
19772	19232.0	19061.6	19166.8	19093.7
19773	18806.0	19014.5	18767.0	19026.4
19774	21217.0	21498.9	21201.1	21486.4
19781	18398.0	18799.4	18391.3	18766.3
19782	19838.0	20407.1	19832.0	20378.6
19783	19459.0	20193.0	19444.1	20215.0
19784	21868.0	22427.8	21846.5	22386.2

TABLE B.8: Actual and Optimal Values (Basic Experiments) of EX (Real Export of Goods and Services). Millions of 1971 Dollars.

19753	5927.00	5926.99	5926.99	5926.99
19754	6183.00	6182.99	6182.99	6182.99
19761	5834.00	5833.99	5833.99	5833.99
19762	6968.00	6957.48	6957.03	6968.45
19763	6679.00	6653.15	6649.53	6682.64
19764	6739.00	6715.02	6703.60	6750.42
19771	6441.00	6424.10	6408.74	6456.33
19772	7506.00	7492.59	7484.55	7520.53
19773	6868.00	6864.42	6858.48	6881.79
19774	7242.00	7254.91	7242.44	7261.93
19781	6931.00	6951.18	6935.44	6954.30
19782	8141.00	8148.35	8143.26	8153.52
19783	7594.00	7587.51	7592.00	7587.04
19784	8007.00	7984.35	8002.07	7997.64

TABLE B.9: Actual and Optimal Values (Basic Experiments) of IM (Real Import of Goods and Services). Millions of 1971 Dollars.

19753	6996.00	6986.69	6995.99	6986.69
19754	7683.00	7621.67	7680.57	7624.09
19761	7748.00	7605.20	7738.94	7614.28
19762	8541.00	8455.14	8523.31	8472.89
19763	7626.00	7681.35	7597.70	7626.64
19764	8234.00	8201.04	8197.63	8213.65
19771	8090.00	7912.25	8054.26	7937.83
19772	8851.00	8758.38	8822.86	8756.15
19773	7747.00	7963.65	7727.65	7959.19
19774	8184.00	8339.93	8168.73	8328.65
19781	7902.00	8143.64	7887.37	8121.77
19782	9172.00	9546.57	9157.78	9548.57
19783	8062.00	8554.15	8046.49	8569.37
19784	9098.00	9416.79	9080.99	9366.12

TABLE B.10: Actual and Optimal Values (Basic Experiments) of IRCNR (Real Gross Private Fixed Investment). Millions of 1971 Dollars.

19753	.	5996.00	5993.36	5996.00	5993.36
19754	.	5868.00	5845.22	5863.63	5849.60
19761	.	5279.00	5213.98	5263.22	5229.81
19762	.	6308.00	6231.42	6279.76	6259.60
19763	.	6069.00	6030.98	6031.74	6044.75
19764	.	5874.00	5842.32	5834.39	5861.28
19771	.	5327.00	5259.91	5291.16	5282.96
19772	.	6155.00	6090.88	6127.27	6105.81
19773	.	6181.00	6[illegible]08.85	6159.29	6219.00
19774	.	5731.00	5797.61	5708.69	5608.31
19781	.	5097.00	5204.48	5072.10	5215.12
19782	.	6138.00	6302.58	6112.40	6319.19
19783	.	6172.00	6399.78	6147.03	6423.60
19784	.	5870.00	6079.44	5846.04	6086.33

TABLE B .11: Actual and Optimal Values (Basic Experiments) of PFX (Foreign Exchange Rate).

19753	.	1.03074	1.02504	1.02485	1.03093
19754	.	1.01751	1.00530	1.00398	1.01869
19761	.	.995293	.985516	.981975	.993948
19762	.	.978981	.971132	.968132	.982065
19763	.	.977480	.969307	.970969	.978572
19764	.	.992357	.987238	.989247	.992403
19771	.	1.02995	1.03016	1.02919	1.03216
19772	.	1.05237	1.05222	1.05125	1.05427
19773	.	1.06971	1.06073	1.06585	1.06499
19774	.	1.10174	1.08712	1.09485	1.09405
19781	.	1.11329	1.09355	1.10474	1.10202
19782	.	1.12735	1.10299	1.11732	1.11169
19783	.	1.14369	1.11477	1.13228	1.12448
19784	.	1.17830	1.14841	1.16342	1.16240

TABLE B.12: Actual and Optimal Values (Basic Experiments) of RS (Short-term Interest Rate). In Percent.

19753	8.37666	8.77358	8.78031	8.36961
19754	9.16333	9.77884	9.82654	9.11192
19761	9.28999	9.43917	9.55237	9.17275
19762	9.37666	9.43249	9.47518	9.33352
19763	9.38333	9.45576	9.29773	9.39525
19764	8.63666	8.53579	8.53473	8.63167
19771	7.90000	7.67880	7.80296	7.77316
19772	7.24999	7.23222	7.28514	7.18394
19773	7.33666	7.66772	7.51871	7.48240
19774	7.41666	7.75687	7.64823	7.51474
19781	7.45333	7.82539	7.64561	7.61381
19782	8.33666	8.73818	8.50656	8.56873
19783	9.04666	9.54187	9.28327	9.31621
19784	10.4967	11.0267	10.8589	10.6249

TABLE B.13: Actual and Simulated Values of Y (Real GNP). Millions of 1971 Dollars.

19721	.	22946.0	22922.0
19722	.	24907.0	24399.3
19723	.	26323.0	26013.4
19724	.	26072.0	25935.3
19731	.	24917.0	25482.7
19732	.	26581.0	27295.4
19733	.	28170.0	29221.1
19734	.	28144.0	28945.8
19741	.	26430.0	26925.4
19742	.	27831.0	28050.7
19743	.	28926.0	29446.1
19744	.	28491.0	28813.1
19751	.	26298.0	26965.3
19752	.	27913.0	28056.7
19753	.	29760.0	29912.4
19754	.	29034.0	28928.2
19761	.	27801.0	27971.0
19762	.	29640.0	28990.2
19763	.	31361.0	30102.6
19764	.	30314.0	28844.0
19771	.	28414.0	28296.8
19772	.	30130.0	29716.5
19773	.	32053.0	31397.6
19774	.	31352.0	30479.7
19781	.	29306.0	29919.9
19782	.	31085.0	31422.4
19783	.	33354.0	33260.0
19784	.	32382.0	32737.5

TABLE B.14: Actual and Simulated Values of RS (Short-Term Interest Rate). In Percent.

19721	4.79000	4.20074
19722	5.71333	4.92443
19723	4.84667	5.34748
19724	5.05667	5.83841
19731	5.08000	6.92180
19732	6.62666	7.75400
19733	8.45667	9.37285
19734	9.65000	10.3502
19741	9.02666	9.55412
19742	11.4367	11.6889
19743	11.4833	11.7421
19744	10.0767	10.2171
19751	6.86333	7.62510
19752	7.33666	7.03577
19753	8.37666	7.41819
19754	9.16333	8.54592
19761	9.28999	8.29183
19762	9.37666	8.25168
19763	9.38333	7.61451
19764	8.63666	7.39342
19771	7.90000	7.54456
19772	7.24999	7.61060
19773	7.33666	7.71134
19774	7.41666	8.34458
19781	7.45333	8.67348
19782	8.33666	8.80607
19783	9.04666	8.92402
19784	10.4967	10.3515