Finally a technique for the stabilization of an unstable nonlinear system is indicated.

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CHAPTER I

INTRODUCTION

1.1 GENERAL

The birth of alternating current power transmission can be traced back to the winter of 1885-1886, when William Stanley supplied electric energy to 150 lamps in the town of Barrington, Massachusetts [30].

From 1920 onwards, the installed generating capacity and energy production has doubled every ten years, so that, by 1970, the total annual production figure for the United States was two trillion killowatt hours, and total plant and equipment investment was ninety billion dollars, thus making it the single largest industry in the world.

The North American electric energy generation and distribution facilities have tended to be under the control of the private sector. For example, in the United States, seventy-six percent of the industry is privately owned. While the situation is somewhat different in Canada because of provincial ownership of the utilities , these public utilities have nevertheless followed the operational philosophy of the private sector.

Being either privately owned or under provincial authority, the American and Canadian power system structures have moved towards total regional control and self-sufficiency and this has resulted in a continent-wide network with very weak interconnections and hence one which operates with less than maximum national efficiency, economy and safety[9]. The continued growth and complexification of the electric power industry has made the twin problems of efficiency and stability of paramount importance and this in turn has given rise to a corresponding growth in the research effort and number of publications devoted to solutions of these two questions[28].

1.2 POWER SYSTEM STABILITY PROBLEM

In non-mathematical terms, the stability problem is concerned with the behaviour of a system of interconnected synchoonous generators after some disturbance. A complex power system can sustain many types of perturbations ranging from the pathological three phase short circuit to ground through to mild fluctuations in generation power output demand or control variable. For the purpose of this thesis, only stability problems arising from variations in generator demand or control values will be considered, and unstable operation due to changes in network load structure will not be studied.

The first stability problem of interest is one where a power system with a constant load is mildly perturbed from its steady state operating condition, causing the readjustment of all machine voltage angles. If the period of time required to reach the new operating point is called " transient period ", then the desired stable behaviour is that the generators maintain synchronism throughout the transient period[2]. During the transient period the power network undergoes what is referred to as " dynamic system performance ".

The second area of concern is the tie line stability problem. Tie lines interconnect groups of machines, whose angles are in continuous oscillation with respect to each other because of minor operational changes, and these angular variations give rise to power flow fluctuations over the tie lines. In a stable system the power swings will eventually become damped out and a new stable operating point attained. However, if the oscillations are too large, the tie line protection circuitry will trip, thereby disconnecting one machine group from another. The effect of such an action on overall system stability is referred to as the " tie line-stability problem ".

Due to the structure of the North American power system, that is, a large power network consisting of many machine groups with relatively weak interconnection, transient periods following a distrubance tend to be large and tie line oscillations underdamped. This in turn can lead to a chain reaction in which perturbations, propagating from group to group with ever increading magnitude, trip tie line protection circuitry and result, eventually, in total system shutdown[2].

1.3 SOME METHODS FOR POWER SYSTEM STABILIZATION

With the increased size of power systems, the need has arisen for a greater degree of control in order to ensure stable and uninterrupted operation. Three different methods for stabilization will be considered and are as follows :

1) Adaptive Control
2) Proportional Feedback
3) Linearized Pole Placement Design.

A detailed analysis of each will not be carried out , rather the techniques will merely be outlined in the following sections and some drawbacks of each method will be presented.

1.3.1 Stabilization By Adaptive Control

An adaptive control scheme for generators is proposed by Irving et al [13-14]. The basic parallel reference model adaptive control is shown in Fig. 1.1 and consists of ;

1) A reference model which embodies the desired closed-loop system behaviour.
2) A controlled system.
3) An adaptation mechanism which can adjust the regulator characteristics.
4) An adjustable regulator.

While in general, adaptation techniques are conditionally stable, Landau[20], using hyperstability concepts, has demonsrated the possibility of designing unconditionally stable adaptive controllers. The requirements of such a design procedure are :

1) Linearized or linear model of the controlled system.
2) All state variables of the model must be available at the output.

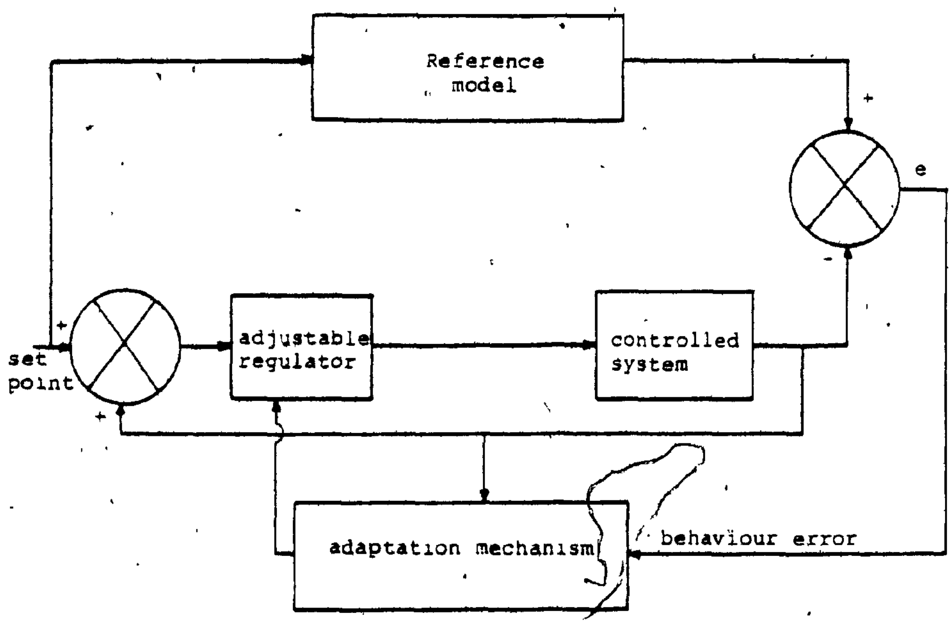


Figure 1.1 : Parallel Reference model Adaptive System

If the power system is described by a nonlinear set of differential equations or if not all the state variables are directly measurable, then the known adaptive control methodology cannot be implemented.

1.3.2 Stabilization Through Proportional Feedback

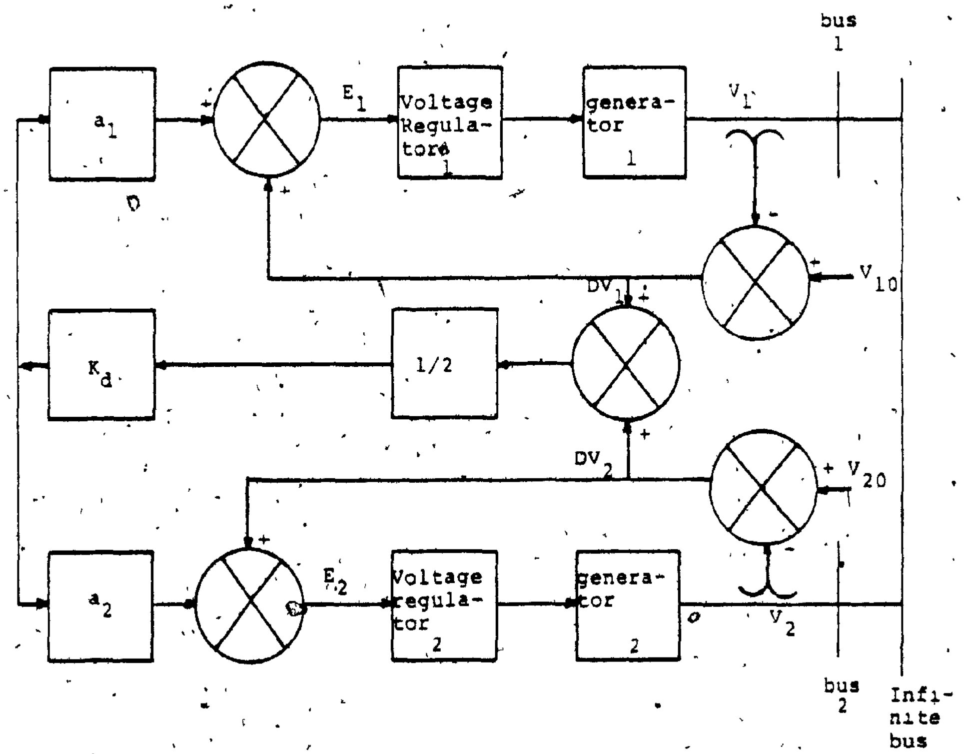
The method is proposed by Dubé [7] in order to improve the dynamic system performance of the Hydro Quebec electric power network (Figure 1.2).

This diagram represents the inter-installation proportional feedback technique for a two machine system feeding an infinite bus. Mathematically the regulator input voltages can be written as

$$E_1 = [1+\frac{kd}{2} a_1] [V_{10}-V_1] + \frac{kd}{2} a_1 [V_{10}-V_1+V_{20}-V_2] \quad (1.1a)$$

$$E_2 = [1+\frac{kd}{2} a_2] [V_{20}-V_2] + \frac{kd}{2} a_2 [V_{10}-V_1+V_{20}-V_2] \quad (1.1b)$$

where kd, a_1, a_2 are constant gains and V_{10}, V_{20} are fixed reference voltages. The coefficients a_1 and a_2 are weighting factors which determine the degree of influence each generator has upon the system as a whole. It is apparent from Equations (1.1) that any deviation from the nominal values V_{10} and V_{20} will produce error signals which will tend eventually to disappear. However since no stability analysis whatsoever, is presented for this type of controller, its usefulness for the power system engineer is only marginal at best.



<u>Figure 1.2 : Control of a two machine system by proportional feedback</u>

1.3.3 Stabilization with the aid of quasi-linearization and pole shifting

For the classical model of an n generator network the machine equations are of form

$$\frac{2H_i}{\omega_R}\frac{d\omega_i}{dt} + D_i\omega_i = P_{mi} - [E_i^2 G_{ii} + \sum_{j=1}^{n} E_i E_j Y_{ij} \cos(\theta_{ij} - \delta_i + \delta_j)] \qquad (1.2a)$$

$$\frac{d\delta_i}{dt} = \omega_i - \omega_R \qquad i = 1,2 \; .... \; n \qquad (1.2b)$$

where ω_i, δ_i, H_i and D_i are the speed, voltage angle, time constant and damping coefficient values respectively for generator i. From Equation(1.2) it appears that 2n differential euqations are needed in order to describe the complete system. It is also apparent that the equations are nonlinear and therefore standard techniques [2,28] can be applied to obtain a quasi-linear state space system of equations of the type

$$\begin{bmatrix} \Delta\dot{\omega} \\ \Delta\dot{\delta} \end{bmatrix} = \begin{bmatrix} 0 & A \\ I & 0 \end{bmatrix} \begin{bmatrix} \Delta\omega \\ \Delta\delta \end{bmatrix} + \begin{bmatrix} F \\ 0 \end{bmatrix} \Delta V + \begin{bmatrix} B \\ 0 \end{bmatrix} \Delta P_m \qquad (1.3)$$

If feedback of the form

$$\begin{bmatrix} F \\ 0 \end{bmatrix} \Delta V + \begin{bmatrix} B \\ 0 \end{bmatrix} \Delta P_m = \begin{bmatrix} -k_1 & 0 \\ 0 & -k_2 \end{bmatrix} \begin{bmatrix} \Delta\omega \\ \Delta\delta \end{bmatrix} \qquad (1.4)$$

is applied to the system described by (1.3) the resulting model becomes

$$\begin{bmatrix} \Delta\dot{\omega} \\ \Delta\dot{\delta} \end{bmatrix} = \begin{bmatrix} -k_1 & A \\ I & -k_2 \end{bmatrix} \begin{bmatrix} \Delta\omega \\ \Delta\delta \end{bmatrix} \qquad (1.5)$$

and as can easily be verified, its poles may be positioned in the left half plane by suitable choice of the gain vectors

k_1 and k_2.

While this design technique is valid, the model representation, especially for a fault located close to a generator i tends to be inaccurate[2]. In order to circumvent this difficulty Okongwu et al [25] design a feedback stabilizing control based on a third order linearization. It is assumed, however, that some states are unavailable for direct feedback and therefore must be reconstructed by a Luenberger observer[3].

In an alternative approach taken by Takata et al [31-32,37-38], the states of a nonlinear electric power system are estimated from a discretized set of previous output measurements. The observer described in [37], is constructed by expanding and linearizing a five state variable network in a second order Taylor series.

1.4 SHORTCOMINGS OF CLASSICAL MODEL

The classical model given by equation(1.2) tacitly assumes a constant generator main field-winding flux linkage. However, with relatively weak interconnections, the power network inertias and transient periods have increased from under one second to at least five or six seconds[2]. For short transient periods the classical model is adequate, but for longer time intervals the change in field winding flux linkage must be accounted for. In addition the devices used for field voltage excitation control are fast acting and influence the field winding flux as well as output power.

Therefore in a subsequent chapter, the seventh order differential equation representation of the synchronous machine, based on the work of Concordia[5] will be developed.

1.5 VALIDITY OF THE HIGH ORDER SYSTEM

In order to better understand the qualitative behaviour of power networks from the control engineering point of view, the usual approach, that is, simulation of a power system either by a set of second order models or a single seventh order generator connected to an infinite bus, is abandoned. The classical model, although accurate in representing machines far removed from an impact centre, fails to faithfully reproduce the transient behaviour of a real generating system at the disturbance point. The isolated seventh order model, on the other hand, is accurate in characterising synchronous machine transient behaviour following disturbances, but cannot provide any realistic picture of the system as a whole. Therefore the approach taken is to describe the power system (in the form of two seventh order generators with interconnections) as a thirteenth order set of nonlinear differential equations. This type of model is sufficiently realistic in its reproduction of transients at a point of impact, as well as in its provision of a good overview of system behaviour.

1.6 OBJECTIVES OF THE THESIS

The objectives of the thesis can be summarized as follows :

1) To show that, under certain conditions, the state variables of an observer for a nonlinear system can be used to stabilize the given system.
2) Develop the nonlinear state space model of a two synchronous machine interconnected power system.
3) Design and implement in software a nonlinear observer for the aforementioned system, prove its convergence analytically as well as computationally.
4) Discuss drawbacks and limitations of the observer design and point out areas for further investigation.

1.7 SCOPE OF THE THESIS

This thesis is concerned with the design and software implementation of an observer for a two generator interconnected power system in which each machine is described by seven first order nonlinear differential equations.

A model of the synchronous machine under investigation is developed in chapter 2. Since the state variables are currents, it is tacitly assumed that no saturation effects are present. Although this somewhat detracts from the accuracy of the representation, it is felf that since the objective of the thesis is the implementation of a nonlinear observer, the

the exclusion of saturation effects does not cause the thesis to deviate from the stated aims.

In Chapter 3 the interconnection equations are developed and the complete power system is presented.

Relevant definitions and theorems from Lyapunov stability theory are presented in Chapter 4. The nonlinear observer design method is demonstrated and convergence of the observer states to the system states is proved analytically. Resuslts of a computer simulation of both the system and observer are tabulated and plots of the state variable trajectories given. In addition, it is demonstrated that, under certain conditions, a system can be stabilized with observer state feedback.

The final chapter contains conclusions and areas for further research.

CHAPTER 2

SYNCHRONOUS MACHINE EQUATIONS

2.1 INTRODUCTION

The synchronous generator constitutes one of the basic components of an electric energy system and a detailed understanding of its dynamic behaviour is essential for the specialist or engineer involved in power system design[9]. While there are several excellent texts, each of which provide detailed mathematical models of the synchronous machine,[1,5,17] the development followed in this thesis is essentially due to Anderson and Fouad[2], Meisel[23], and Dubé[8].

The synchronous machine under consideration is a three-phase, two-pole device with three stator windings a,b,c, each located 120° from the other, one field winding F and two amortisseur or damper windings D and Q, all of which are magnetically coupled. Note that the F,D and Q coils are on the rotor. A cross sectional representation of this machine is given in Figure 2.1.

2.2 MACHINE INDUCTANCES

A co-ordinate system can be introduced in Fig.2.1 by defining the d-axis of the rotor, at some instant of time, to be at an angle θ with respect to a fixed reference position, which, for the sake of convenience, can be the a- winding axis. Electrically the generator can be regarded as a network consisting of six mutually coupled coils a-a', b-b', c-c', F-F',

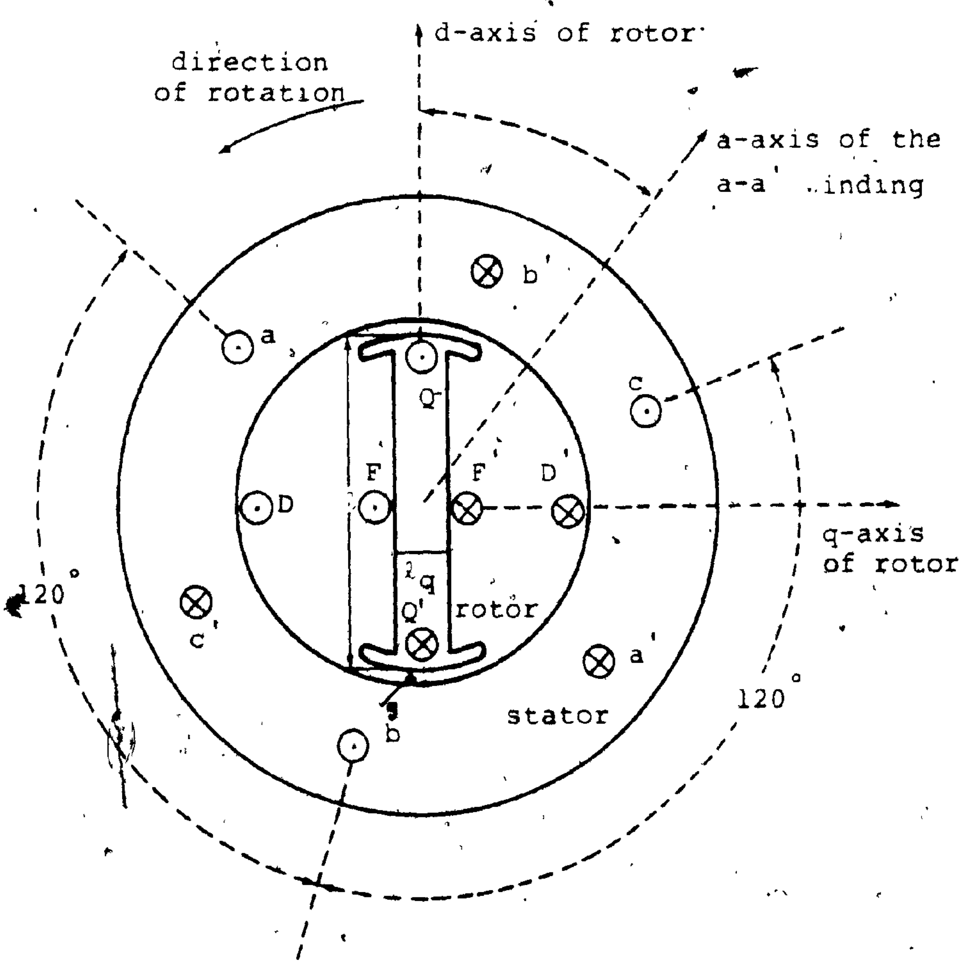


Figure 2.1 : Cross Sectional Representation of 3-phase, 2-pole Synchronous Machine

D-D and Q-Q. The flux linkage equations describing these six circuits are written as

$$\begin{bmatrix} \lambda_a \\ \lambda_b \\ \lambda_c \\ \lambda_F \\ \lambda_D \\ \lambda_Q \end{bmatrix} = \begin{bmatrix} L_{aa} & L_{ab} & L_{ac} & L_{aF} & L_{aD} & L_{aQ} \\ L_{ba} & L_{bb} & L_{bc} & L_{bF} & L_{bD} & L_{bQ} \\ L_{ca} & L_{cb} & L_{cc} & L_{cF} & L_{cD} & L_{cQ} \\ L_{Fa} & L_{Fb} & L_{Fc} & L_{FF} & L_{FD} & L_{FQ} \\ L_{Da} & L_{Db} & L_{Dc} & L_{DF} & L_{DD} & L_{DQ} \\ L_{Qa} & L_{Qb} & L_{Qc} & L_{QF} & L_{QD} & L_{QQ} \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \\ i_F \\ i_D \\ i_Q \end{bmatrix} \qquad (2.1)$$

where L_{uu} indicate self and L_{uv} mutual inductance parameters. The actual values will be determined according to the method of Thaler[33].

2.2.1. Rotor Inductances

If slot effects and saturation are neglected, the rotor self-inductances can be shown to be constant. The inductance equation of an N - turn coil is of the form[19]

$$L = \frac{N^2 \mu_r \mu_o A}{d} \qquad (2.2)$$

where L = coil inductance

N = number of coil turns

d = length of path through which magnetic flux traverses

μ_r = relative permeability of material medium in flux path

μ_o = absolute permeability of free space

A = effective cross sectional area through which flux passes.

From Figure 2.1 it is apparent that the flux produced by coil F-F' traverses through both the iron rotor as well as the air gap. The field winding self inductance is then deduced from Equation (2.2) to be

$$L_{FF} = \frac{N_{FF}^2}{\frac{\ell}{\mu_o \mu_r A} + \frac{2g_{da}}{\mu_o A}} \simeq \frac{N_{FF}^2 \mu_o A}{2g_{da}} \quad (2.3)$$

where L_{FF} = Field coil self inductance

N_{FF} = number of turns of field winding

A = effective cross sectional area in flux direction.

$2g_{da}$ = effective flux path length through air gap

and where note is taken of the fact that $\mu_r >> \mu_o$.

A similar set of equations can be derived for the coils viz.

$$L_{DD} = \frac{N_{DD}^2 \mu_o A}{2g_{da}} \quad (2.4a)$$

$$L_{QQ} = \frac{N_{QQ}^2 \mu_o A'}{2g_{da}} \quad (2.4b)$$

where subscripts D and Q apply to the D and Q coils respectively and $2g_{qa}$ is the air gap length along the direction of the q - axis in Figure 2.1.

From inspection of Figure 2.1 and with the aid of Equation (2.2), the rotor mutual inductances are seen to be of the form ;

$$L_{DQ} = L_{QD} = 0 \quad (2.5a)$$

$$L_{FQ} = L_{QF} = 0 \quad (2.5b)$$

$$L_{DF} = L_{FD} = \frac{N_{DD} \, N_{FF} \, \mu_o \, A}{2g_{da}} \quad (2.5c)$$

where subscripts refer to the appropriate windings.

2.2.1 Stator Inductances

Stator self and mutual inductances, on the other hand, are dependent on rotor angle θ. The flux generated by current i flowing in an N - turn coil is given by ;

$$\phi = \frac{N_i \, \mu_r \, \mu_o \, A}{d} \quad (2.6)$$

where A = effective cross sectional area of flux path

d = effective path length that flux traverses

μ_r = relative permeability of medium along flux path

μ_o = absolute permeability of free space.

Therefore the d and q components of ϕ, produced by a current i flowing through the N_a - turn stator winding a-a', are calculated respectively as

$$\phi_{ad} = \frac{N_a i \mu_r \mu_o A \cos\theta}{\ell_{da} + 2g_{da} \mu_r} \simeq \frac{N_a i \mu_o A \cos\theta}{2g_{da}} \quad (2.7a)$$

$$\phi_{aq} = \frac{N_a i \mu_r \mu_o A \sin\theta}{\ell_{qa} + 2g_{qa} \mu_r} \simeq \frac{N_a i \mu_o A \sin\theta}{2g_{qa}} \quad (2.7b)$$

where ℓ_{da} = effective flux path length along the d-axis through the rotor

ℓ_{qa} = effective flux path length along the q-axis through the rotor

$2g_{da}$ = effective air gap flux path length along the d-axis

$2g_{qa}$ = effective air gap flux path length along the q-axis.

Equation (2.7) is consistent with the observation that flux in a magnetic circuit consisting of an iron medium and an air-gap appears mostly in the gap region. Since the inductance of the stator coil a-a' is desired, the total flux impinging on this winding is given by

$$\phi_a = \phi_{ad} \cos\theta + \phi_{aq} \sin\theta \quad (2.8)$$

The general inductance equation (2.2) can be rewritten in the form

$$L = \frac{N\phi}{i} \quad (2.9)$$

Then L_{aa} can be computed from (2.8) as

$$L_{aa} = N_a^2 \mu_o A \left[\frac{\cos^2\theta}{2g_{da}} + \frac{\sin^2\theta}{2g_{qa}}\right] \quad (2.10)$$

However, substituting the trigonometric identities

$$\cos^2\theta = \frac{\cos 2\theta + 1}{2} \qquad (2.11a)$$

$$\sin^2\theta = \frac{1 - \cos 2\theta}{2} \qquad (2.11b)$$

into Equation (2.10), the a-a' winding self-inductance then becomes

$$L_{aa} = L_s + L_m \cos 2\theta \qquad (2.12)$$

where

$$L_s \triangleq \frac{N_a^2 \mu_o A}{2} \left[\frac{1}{2g_{qa}} + \frac{1}{2g_{da}} \right]$$

$$L_m \triangleq \frac{N_a^2 \mu_o A}{2} \left[\frac{1}{2g_{da}} - \frac{1}{2g_{qa}} \right]$$

Since coil b-b' (c-c') is displaced by -120° ($+120^\circ$) from winding a-a', its self inductance can be calculated by simply replacing θ in (2.12) with $\theta - \frac{2\pi}{3}$ ($\theta + \frac{2\pi}{3}$). The values for L_{bb} and L_{cc} can then be written as

$$L_{bb} = L_s + L_m \cos 2 \left(\theta - \frac{2\pi}{3}\right) \qquad (2.13a)$$

$$L_{cc} = L_s + L_m \cos 2 \left(\theta + \frac{2\pi}{3}\right) \qquad (2.13b)$$

where L_s and L_m have been previously defined.

The mutual inductance between windings a-a' and b-b' can be calculated as follows ;

1) compute the components of flux due to current i_1 in coil a-a' along the d and q axis respectively

2) project the two flux components onto the b-b' coil and form the vector sum.

After implementing the above procedure and assuming equal turns for each winding, the resulting equation is of the form

$$L_{ab} = -M_s - L_m \cos 2\left(\theta + \frac{\pi}{6}\right) \qquad (2.14a)$$

where $M_s \triangleq L_s \cos\frac{\pi}{3}$.

The remaining stator mutual inductances, which are derived by the aforementioned procefure, can be written as ;

$$L_{ac} = -Ms - Lm \cos 2\left(\theta + \frac{5\pi}{6}\right) \qquad (2.14b)$$

$$L_{bc} = -Ms - Lm \cos 2\left(\theta - \frac{\pi}{2}\right) \qquad (2.14c)$$

where M_s and L_m have been previously defined.

2.2.3 ROTOR - STATOR MUTUAL INDUCTANCES

The stator-rotor mutual inductances are easily calculated with the aid of equations of the form of (2.2) and and (2.7), that is

$$L_{aF} = L_{Fa} = \frac{N_a N_F \mu_o A \cos\theta}{2g_{da}} \qquad (2.15a)$$

or

$$L_{aF} = M_F \cos\theta \qquad (2.15b)$$

where $M_F \triangleq \dfrac{N_a N_F \mu_o A}{2g_{da}}$

Mutual inductances between field coil and phase b(c) can be calculated by substituting $\theta - \frac{2\pi}{3}$ $(\theta + \frac{2\pi}{3})$ in place of θ in Equation (2.15). The damper-stator inductances are similar in form to the field-stator quantities. Table 2.1 contains the thirty six self and mutual inductances of the synchronous machine. The constants in Table 2.1 are defined below :

1) $L_s \triangleq \dfrac{N_a^2 \mu_o A}{2} \left[\dfrac{1}{2g_{qa}} + \dfrac{1}{2g_{da}}\right]$

2) $L_m \triangleq \dfrac{N_a^2 \mu_o A}{2} \left[\dfrac{1}{2g_{qa}} - \dfrac{1}{2g_{da}}\right]$

3) $M_s \triangleq L_s \cos \dfrac{\pi}{3}$

4) $M_R \triangleq \dfrac{N_{DD} N_{FF} \mu_o A}{2g_{da}}$

5) $M_D \triangleq \dfrac{N_a N_{DD} \mu_o A}{2g_{da}}$

6) $M_Q \triangleq \dfrac{N_a N_{QQ} \mu_o A}{2g_{qa}}$

7) $M_F \triangleq \dfrac{N_a N_{FF} \mu_o A}{2g_{da}}$

Winding type	Field coil	D-coil	Q-coil	a-ccil	b-coil	c-coil
Field coil	$L_{FF} = L_F = \frac{N_{FF}^2 \mu_o A}{2g_{da}}$	$L_{FD}=M_R$		$L_{Fa} = M_F \cos\theta$	$L_{Fb} = M_F \cos(\theta-120^\circ)$	$L_{Fc} = M_F \cos(\theta+120^\circ)$
D-coil	$L_{DF}=M_R$	$L_{DD}=L_D \frac{N_{DD}^2 \mu_o A}{2g_{da}}$		$L_{Da} = M_D \cos\theta$	$L_{Db} = M_D \cos(\theta-120^\circ)$	$L_{Dc} = M_D \cos(\theta+120^\circ)$
Q-coil			$L_{QQ}=L_Q = \frac{N_Q^2 \mu_o A}{2g_{qa}}$	$L_{Qa} = M_Q \sin\theta$	$L_{Qb} = M_Q \sin(\theta-120^\circ)$	$L_{Qc} = M_Q \sin(\theta+120^\circ)$
a-coil	$L_{aF} = M_{Fccs}\ \theta$	$L_{aD}=L_{Da}$	$L_{aQ}=L_{Qa}$	$L_{aa} = L_s+L_M \cos\theta$	$L_{ab} = -M_s -L_M \cos 2(\theta+\frac{\pi}{6})$	$L_{ac} = -M_s -L_M \cos 2(\theta+\frac{5\pi}{6})$
b-coil	$L_{bF}=L_{Fb}$	$L_{bD}=L_{Db}$	$L_{bQ}=L_{Qb}$	$L_{ba}=L_{ab}$	$L_{bb} = L_s +b_M \cos 2(\theta-\frac{2\pi}{3})$	$L_{bc} = M_s -L_M \cos 2(\theta-\frac{\pi}{2})$
c-ccil	$L_{cF}=L_{Fc}$	$L_{cD}=L_{Dc}$	$L_{cQ}=L_{Qc}$	$L_{ca}=L_{ac}$	$L_{cb}=L_{bc}$	$L_{cc} = L_s +L_M \cos 2(\theta+\frac{2\pi}{3})$

Table 2.1 Synchronous Machine Inductances

2.3 PARK'S TRANSFORMATION OF MACHINE INDUCTANCES

From Table 2.1 it is apparent that the stator rotor and stator-stator inductances (self and mutual), are time dependent functions of rotor angle θ. In order to simplify the equations and render the inductors time independent, a co-ordinate transformation matrix P, usually referred to as Park's Transformation, is applied to (2.1). Among the unique features of the particular P matrix [2] to be used, as opposed to earlier ones commonly found [26-27], are the following:

(1) Orthogonality; this implies that $P^{-1} \equiv P^T$

(2) Power invariance for voltage, current and flux tranformations from one reference frame to another [24].

The P matrix projects a, b and c phase currents, voltages or flux along three new directions, d, q and o which are the rotor d and q axis as shown in Figure 2.1 and a fictitious stationary axis labelled o.

2.3.1 Flux Equation Transformation

The P matrix is of the form

$$P = \sqrt{\frac{2}{3}} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \cos\theta & \cos(\theta-120^\circ) & \cos(\theta+120^\circ) \\ \sin\theta & \sin(\theta-120^\circ) & \sin(\theta+120^\circ) \end{bmatrix} \quad (2.16)$$

For ease of analysis, Equation (2.1) can be written as

$$\begin{bmatrix} \lambda_{abc} \\ \lambda_{FDQ} \end{bmatrix} = \begin{bmatrix} L_{SS} & L_{SR} \\ L_{RS} & L_{RR} \end{bmatrix} \begin{bmatrix} i_{abc} \\ i_{FDQ} \end{bmatrix} \quad (2.17)$$

where

$$\lambda_{abc} \triangleq \begin{bmatrix} \lambda_a & \lambda_b & \lambda_c \end{bmatrix}'$$

$$i_{abc} \triangleq \begin{bmatrix} i_a & i_b & i_c \end{bmatrix}'$$

$$\lambda_{FDQ} \triangleq \begin{bmatrix} \lambda_F & \lambda_D & \lambda_Q \end{bmatrix}'$$

$$i_{FDQ} \triangleq \begin{bmatrix} i_F & i_D & i_Q \end{bmatrix}'$$

where $[\cdot]'$ denotes the transpose of $[\cdot]$

and where L_{SS} = stator self and mutual inductances

$L_{SR} = L_{RS}$ = stator-rotor mutual inductances

L_{RR} = rotor self inductances

Since L_{RR} is independent of time, only L_{SS} and L_{SR}, L_{RS} need to be modified. Accordingly the transformation used is of the type

$$\begin{bmatrix} P & O \\ O & I_3 \end{bmatrix} \begin{bmatrix} \lambda_{abc} \\ \lambda_{FDQ} \end{bmatrix} = \begin{bmatrix} P & O \\ O & I_3 \end{bmatrix} \begin{bmatrix} L_{SS} & L_{SR} \\ L_{RS} & L_{RR} \end{bmatrix} \begin{bmatrix} P^{-1} & O \\ O & I_3 \end{bmatrix} \begin{bmatrix} P & O \\ O & I_3 \end{bmatrix} \begin{bmatrix} i_{abc} \\ i_{FDQ} \end{bmatrix} \quad (2.18)$$

where I_3 = 3 x 3 identity matrix

P^{-1} = matrix inverse of P

Flux linkage λ_a, λ_b, λ_c are then transformed to λ_o, λ_d, λ_q while currents i_a, i_b, i_c undergo a similar change, so that Equation (2.18) can be simplified and rewritten as

$$\begin{bmatrix} \dot{\lambda}_{odq} \\ \dot{\lambda}_{FDQ} \end{bmatrix} = \begin{bmatrix} PL_{SS}P^{-1} & PL_{SR} \\ L_{RS}P^{-1} & L_{RR} \end{bmatrix} \begin{bmatrix} i_{odq} \\ i_{FDQ} \end{bmatrix} \qquad (2.18)$$

By combining (2.16) and stator-rotor inductances from Table 2.1, PL_{SR} becomes

$$PL_{SR} = \frac{1}{\sqrt{\frac{2}{3}}} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \cos\theta & \cos(\theta-120°) & \cos(\theta+120°) \\ \sin\theta & \sin(\theta-120°) & \sin(\theta-120°) \end{bmatrix} \begin{bmatrix} M_F\cos\theta & M_D\cos\theta & M_Q\sin\theta \\ M_F\cos(\theta-120°) & M_D\cos(\theta-120°) & M_Q\sin(\theta-120°) \\ M_F\cos(\theta+120°) & M_D\cos(\theta+120°) & M_Q\sin(\theta+120°) \end{bmatrix} \qquad (2.20)$$

After computation, the result is of the form

$$PL_{SR} = \begin{bmatrix} 0 & 0 & 0 \\ kM_F & kM_D & 0 \\ 0 & 0 & kM_Q \end{bmatrix} \qquad (2.21a)$$

Since $(PL_{SR})^T$ and $L_{RS}P^{-1}$ are identical

$$L_{RS}P^{-1} = \begin{bmatrix} 0 & kM_F & 0 \\ 0 & kM_D & 0 \\ 0 & 0 & kM_Q \end{bmatrix} \qquad (2.21b)$$

where $k \triangleq \sqrt{\frac{3}{2}}$.

Following a similar procedure the matrix $PL_{SS}P^{-1}$ becomes

$$PL_{SS}P^{-1} = \begin{bmatrix} L_0 & 0 & 0 \\ 0 & L_d & 0 \\ 0 & 0 & L_q \end{bmatrix}$$

where

$$L_0 \triangleq L_S - 2M_S$$

$$L_d \triangleq L_S + M_S + 3/2\ L_M$$

$$L_q \triangleq L_S + M_S - 3/2\ L_M$$

In summary, (2.19) can now be written as

$$\begin{bmatrix} \lambda_o \\ \lambda_d \\ \lambda_q \\ \lambda_F \\ \lambda_D \\ \lambda_Q \end{bmatrix} = \begin{bmatrix} L_o & 0 & 0 & 0 & 0 & 0 \\ 0 & L_d & 0 & kM_F & kM_D & 0 \\ 0 & 0 & L_q & 0 & 0 & kM_Q \\ 0 & kM_F & 0 & L_F & M_R & 0 \\ 0 & kM_D & 0 & 0 & L_D & 0 \\ 0 & 0 & kM_Q & 0 & 0 & L_Q \end{bmatrix} \begin{bmatrix} i_o \\ i_d \\ i_q \\ i_F \\ i_D \\ i_Q \end{bmatrix} \quad (2.23)$$

2.4 GENERATOR VOLTAGE EQUATIONS

A three-phase synchronous generator, with mutual inductances omitted for clarity, is represented schematically in Figure 2.2. The voltage equation for a single phase can be obtained from Kirchoff's voltage and current laws as

$$V_a = ri_a - \dot{\lambda}_a + v_n \qquad (2.24)$$

where $v_n \triangleq -r_n [\, i_a+i_b+i_c \,] - \ell_n [\, \dot{i}_a+\dot{i}_b+\dot{i}_c \,]$,

and n defines a netural phase. For the total set of machine windings the voltage are of the form

$$\begin{bmatrix} V_{abc} \\ V_{FDQ} \end{bmatrix} = -\begin{bmatrix} R_{abc} & 0 \\ 0 & R_{FDQ} \end{bmatrix}\begin{bmatrix} i_{abc} \\ i_{FDQ} \end{bmatrix} - \begin{bmatrix} \dot{\lambda}_{abc} \\ \dot{\lambda}_{FDQ} \end{bmatrix} + \begin{bmatrix} V_n \\ 0 \end{bmatrix} \qquad (2.25)$$

where the subscripts refer to the appropriate phase or winding values and where V_n is defined as

$$V_n \triangleq -r_n \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} - \ell_n \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} \dot{i}_a \\ \dot{i}_b \\ \dot{i}_c \end{bmatrix}$$

or equivalently

$$V_n \triangleq -R_n \, i_{abc} - L_n \, \dot{i}_{abc} \, .$$

2.4.1 Park's Transformation of Voltage Equations

From (2.25) it can be ascertained that the three-phase voltages are dependent on rotor angle θ. Therefore,

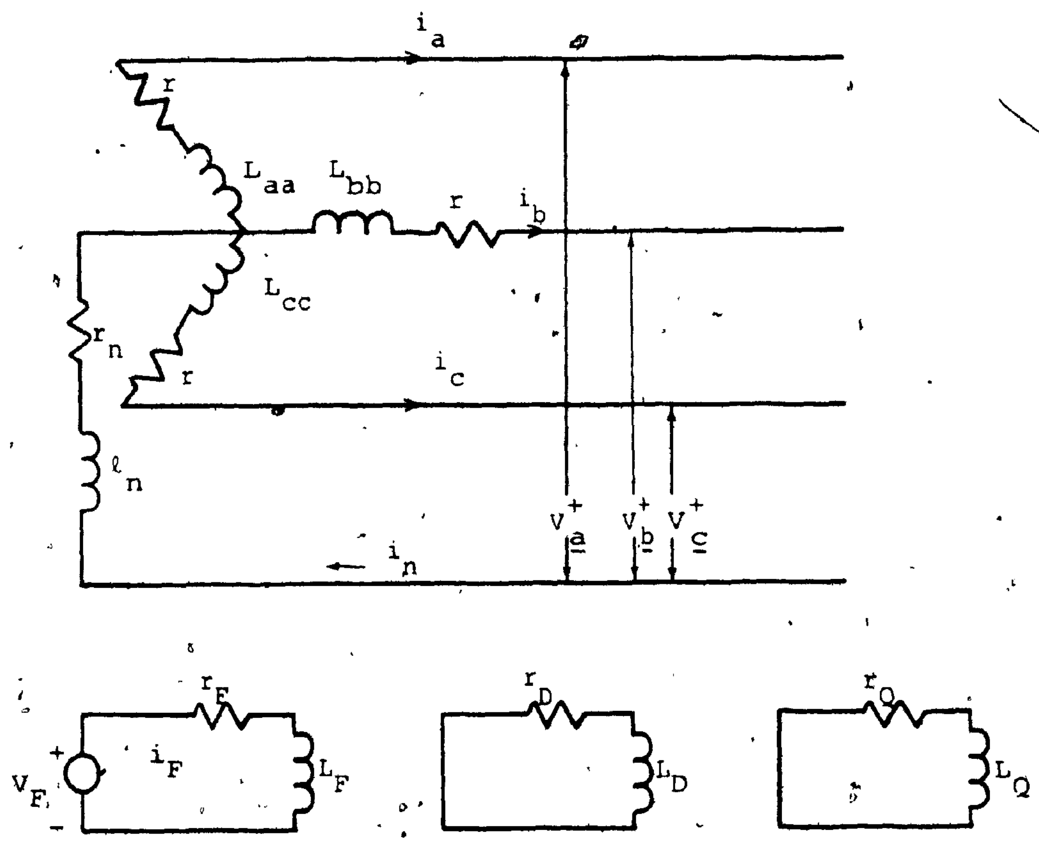


Figure 2.2 :Schematic Diagram of three phase Synchronous Generator

as in Section 2.3, a co-ordinate transformation is applied in order to convert voltages, currents, fluxes and their derivatives with respect to time, into constant quantities. By applying the previously defined P matrix to Equation (2.25) the resulting product is of the form

$$\begin{bmatrix} P & O \\ O & I_3 \end{bmatrix}\begin{bmatrix} V_{abc} \\ V_{FDQ} \end{bmatrix} = -\begin{bmatrix} P & O \\ O & I_3 \end{bmatrix}\begin{bmatrix} R_{abc} & O \\ O & R_{FDQ} \end{bmatrix}\begin{bmatrix} P^{-1} & O \\ O & I_3 \end{bmatrix}\begin{bmatrix} P & O \\ O & I_3 \end{bmatrix}\begin{bmatrix} i_{abc} \\ i_{FDQ} \end{bmatrix}$$

$$- \begin{bmatrix} P & O \\ O & I_3 \end{bmatrix}\begin{bmatrix} \dot{\lambda}_{abc} \\ \dot{\lambda}_{FDQ} \end{bmatrix} + \begin{bmatrix} P & O \\ O & I_3 \end{bmatrix}\begin{bmatrix} V_n \\ O \end{bmatrix} \qquad (2.26)$$

where

$$\begin{bmatrix} V_{odq} \\ V_{FDQ} \end{bmatrix} \triangleq \begin{bmatrix} P & O \\ O & I_3 \end{bmatrix}\begin{bmatrix} V_{abc} \\ V_{FDQ} \end{bmatrix}$$

$$\begin{bmatrix} i_{odq} \\ i_{FDQ} \end{bmatrix} \triangleq \begin{bmatrix} P & O \\ O & I_3 \end{bmatrix}\begin{bmatrix} i_{abc} \\ i_{FDQ} \end{bmatrix}$$

Since the flux vector, λ_{odq}, is given by

$$\lambda_{odq} \triangleq P\,\lambda_{abc}$$

it can easily be demonstrated that

$$P\dot{\lambda}_{abc} = \dot{\lambda}_{odq} - \dot{P}P^T\lambda_{odq} \qquad (2.27)$$

where $\dot{x}$ represents the time derivative of the variable x.

Expansion of $P\,V_n$ yields

$$P\,V_n = -PR_nP^{-1}\,i_{odq} - PL_nP^{-1}\,\dot{i}_{odq} \qquad (2.28)$$

where

$$PR_nP^{-1} \triangleq \begin{bmatrix} 3\,r_n & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$PL_nP^{-1} \triangleq \begin{bmatrix} 3\,\ell_n & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

In addition $\dot{P}P^T$ is represented as

$$\dot{P}P^T = \omega \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \qquad (2.29)$$

where $\omega \triangleq \frac{\theta}{t}$.

Taking the derivatives with respect to time, of currents and fluxes in Equation (2.23), the resulting vector is of the form

$$\begin{bmatrix} \dot{\lambda}_0 \\ \dot{\lambda}_d \\ \dot{\lambda}_q \\ \dot{\lambda}_F \\ \dot{\lambda}_D \\ \dot{\lambda}_Q \end{bmatrix} = \begin{bmatrix} L_0 & 0 & 0 & 0 & 0 & 0 \\ 0 & L_d & 0 & kM_F & kM_D & 0 \\ 0 & 0 & L_q & 0 & 0 & kM_Q \\ 0 & kM_F & 0 & L_F & M_R & 0 \\ 0 & kM_D & 0 & 0 & L_D & 0 \\ 0 & 0 & kM_Q & 0 & 0 & L_Q \end{bmatrix} \begin{bmatrix} \dot{i}_0 \\ \dot{i}_d \\ \dot{i}_q \\ \dot{i}_F \\ \dot{i}_D \\ \dot{i}_Q \end{bmatrix} \qquad (2.30)$$

Finally, with a substitution of (2.27), (2.28), (2.29) and (2.30) into (2.26), the required set of voltages is obtained, viz.

$$\begin{bmatrix} v_0 \\ v_d \\ v_q \\ -v_F \\ 0 \\ 0 \end{bmatrix} = - \begin{bmatrix} r+3r_n & 0 & 0 & 0 & 0 & 0 \\ 0 & r & \omega L_q & 0 & 0 & \omega KM_Q \\ 0 & -\omega L_d & r & -\omega kM_F & -\omega kM_D & 0 \\ 0 & 0 & 0 & r_F & 0 & 0 \\ 0 & 0 & 0 & 0 & r_D & 0 \\ 0 & 0 & 0 & 0 & 0 & r_Q \end{bmatrix} \begin{bmatrix} i_0 \\ i_d \\ i_q \\ i_F \\ i_D \\ i_Q \end{bmatrix}$$

$$- \begin{bmatrix} L_0+3\ell_n & 0 & 0 & 0 & 0 & 0 \\ 0 & L_d & 0 & kM_F & kM_D & 0 \\ 0 & 0 & L_q & 0 & 0 & kM_Q \\ 0 & kM_F & 0 & L_F & M_R & 0 \\ 0 & kM_D & 0 & M_R & L_D & 0 \\ 0 & 0 & kM_Q & 0 & 0 & L_Q \end{bmatrix} \begin{bmatrix} \dot{i}_0 \\ \dot{i}_d \\ \dot{i}_q \\ \dot{i}_F \\ \dot{i}_D \\ \dot{i}_Q \end{bmatrix} \quad (2.31)$$

2.5 VOLTAGE NORMALIZATION

Since stator voltage magnitudes are of the order of several hundred thousands, while corresponding field quantities are much smaller, the matrix equation (2.31) tends to be numerically ill-conditioned. It is therefore of some convenience, computationally, to normalize all generator currents and voltages.

Several authors have considered this problem in detail [11,21,29] and the technique employed here is essentially due to Lewis[21].

Per unit normalization consists in choosing base quantities that together involve all the independent dimensions in a system. For example, in any given circuit the independent dimensions can be volts, amperes and time, so that the base quantities are the following :

1) $S_B \triangleq$ stator rated VA / phase
2) $V_B \triangleq$ stator rated line-neutral voltage
3) $\omega_B \triangleq$ generator rated speed.

In order to scale any quantity, it is simply divided by the base value of the same dimension, that is for current i,

$$i_u \triangleq \frac{i}{I_B}$$

where i_u is the per unit current

I_B is the base current and is defined to be

$$I_B \triangleq \frac{S_B}{V_B} .$$

The other base entities are given as :

4) base flux, $\lambda_B \triangleq V_B' \; t_B = L_B I_B$

5) base time, $t_B \triangleq \frac{1}{\omega_B}$

6) base resistance, $R_B \triangleq \frac{V_B}{I_B}$

2.5.1 Per Unit Voltages

Any voltage v or current i may be expressed on a per unit basis, for example, V is of the following form

$$V = V_u V_B$$

where V is the given voltage

V_u is the corresponding per unit quantity

V_B is the base voltage.

Therefore it is possible to express the dynamical equations (2.31) as

$$\begin{bmatrix} V_{du}\,V_B \\ V_{qu}\,V_B \\ -V_{Fu}\,V_{FB} \\ 0 \\ 0 \end{bmatrix} = - \begin{bmatrix} r & \omega L_q & 0 & 0 & \omega k M_Q \\ -\omega L_d & r & -\omega k M_F & -\omega k M_D & 0 \\ 0 & 0 & r_F & 0 & 0 \\ 0 & 0 & 0 & r_D & 0 \\ 0 & 0 & 0 & 0 & r_Q \end{bmatrix} \begin{bmatrix} i_{du}\,I_B \\ i_{qu}\,I_B \\ i_{Fu}\,I_{FB} \\ i_{Du}\,I_{DB} \\ i_{Qu}\,I_{QB} \end{bmatrix}$$

$$- \begin{bmatrix} L_d & 0 & kM_F & KM_D & 0 \\ 0 & L_q & 0 & 0 & kM_D \\ kM_F & 0 & L_F & M_R & 0 \\ kM_D & 0 & M_R & L_D & 0 \\ 0 & kM_Q & 0 & 0 & L_Q \end{bmatrix} \begin{bmatrix} \dot{i}_d\,I_B \\ \dot{i}_q\,I_B \\ \dot{i}_F\,I_{FB} \\ \dot{i}_D\,I_{DB} \\ \dot{i}_Q\,I_{QB} \end{bmatrix} \qquad (2.32)$$

where voltage V_o and current i_o are deleted since, under balanced three phase conditions, they are zero. Because I_B is much larger than the rotor values, the base quantities for rotor currents and voltages are chosen to be I_{FB}, I_{DB}, I_{QB}, V_{FB}, V_{DB}, and V_{QB} respectively. Similarly ω is defined, on a per unit basis, to be

$$\omega \triangleq \omega_B\,\omega_u.$$

If both sides of (2.32) are divided through by the appropriate base voltages, the resulting terms are of the form

$$\begin{bmatrix} V_{du} \\ V_{qu} \\ -V_{Fu} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} r_u & \omega_u L_{qu} & 0 & 0 & \omega_u kM_{Qu} \\ -\omega_u L_{du} & r_u & -\omega_u kM_{Fu} & -\omega_u kM_{Du} & 0 \\ 0 & 0 & r_{Fu} & 0 & 0 \\ 0 & 0 & 0 & r_{Du} & 0 \\ 0 & 0 & 0 & 0 & r_{Qu} \end{bmatrix} \begin{bmatrix} i_{du} \\ i_{qu} \\ i_{Fu} \\ i_{Du} \\ i_{Qu} \end{bmatrix}$$

$$- \begin{bmatrix} \frac{L_{du}}{\omega_B} & 0 & \frac{kM_{Fu}}{\omega_B} & \frac{kM_{Du}}{\omega_B} & 0 \\ 0 & L_{qu} & 0 & 0 & kM_{Qu} \\ \frac{km_{Fu}}{\omega_B} & 0 & \frac{L_{Fu}}{\omega_B} & \frac{M_{Ru}}{\omega_B} & 0 \\ \frac{km_{Du}}{\omega_B} & 0 & \frac{M_{Ru}}{\omega_B} & \frac{L_{Du}}{\omega_B} & 0 \\ 0 & \frac{kM_{Qu}}{\omega_B} & 0 & 0 & \frac{L_{Qu}}{\omega_B} \end{bmatrix} \begin{bmatrix} \dot{i}_{du} \\ \dot{i}_{qu} \\ \dot{i}_{Fu} \\ \dot{i}_{Du} \\ \dot{i}_{Qu} \end{bmatrix} \quad (2.34)$$

where the per unit quantities are defined as follows :

1) $r_u \triangleq \frac{r}{R_B}$

2) $L_{qu} \triangleq \frac{L_q I_B \omega_B}{V_B} = \frac{L_q}{L_B}$

3) $M_{Qu} \triangleq \frac{M_Q I_{QB} \omega_B}{V_B} = \frac{M_Q}{M_{QB}}$.

4) $L_{du} \triangleq \dfrac{L_d \omega_B I_B}{V_B} = \dfrac{L_d}{L_B}$

5) $M_{Fu} \triangleq \dfrac{M_F I_{FB} \omega_B}{V_B} = \dfrac{M_F}{M_{FB}}$

6) $M_{Du} \triangleq \dfrac{M_D I_{DB} \omega_B}{V_B} = \dfrac{M_D}{M_{DB}}$

7) $r_{Fu} \triangleq \dfrac{r_F I_{FB}}{V_{FB}}$

8) $M_{Ru} \triangleq \dfrac{M_R I_{DB} \omega_B}{V_{FB}}$

9) $L_{Fu} \triangleq \dfrac{L_F I_{FB} \omega_B}{V_{FB}}$

10) $L_{Du} \triangleq \dfrac{L_D I_{DB} \omega_B}{V_{DB}}$

11) $L_{Qu} \triangleq \dfrac{L_Q I_{QB} \omega_B}{V_{QB}}$

In order to eliminate terms with $1/\omega_B$, the time t may be scaled according to the formula

$$\tau = \omega_B t$$

so that derivative terms $\dfrac{1}{\omega_B} \dfrac{d}{dt}(.)$ become $\dfrac{d(.)}{d\tau}$. Substituting the normalized time into the voltage matrix and dropping the subscript u, (2.34) can represented in the form of (2.31) with the first row and column deleted.

2.6 PER UNIT TORQUE AND ANGLE EQUATIONS

In order to complete the physical description of the synchronous machine, normalized torque equations, describing the generators' mechanical behaviour, must be added. The usual torque model is of the form

$$\dot{\omega} = \frac{T_{Mu}}{\tau_j} - \frac{T_{e\phi u}}{3\tau_j} - \frac{D\omega_u}{\tau_j} \qquad (2.35)$$

where T_{Mu} is the per unit mechanical torque

$\frac{T_{e\phi u}}{3}$ is the per unit electrical torque

$D\omega_u$ is the per unit damping torque

τ_j is the machine time constant

If the time and angular velocity, t, ω, are scaled as in Section 2.5.1, (2.35) becomes, after dropping the u subscripts,

$$\frac{d\omega}{d\tau} = \frac{T_m}{\tau_j} - \frac{T_{e\phi}}{3\tau_j} - \frac{D\omega}{\tau_j} \qquad (2.36)$$

2.6.1 Electrical Torque

The per unit electrical torque equation for a magnetically coupled system is defined to be.[33]

$$T_{e\phi} \triangleq \frac{\partial P_{f\ell d}}{\partial \omega}$$

where $P_{f\ell d}$ is the power stored in the magnetic field. In a synchronous machine $P_{f\ell d}$ and output power P_O are equivalent and furthermore P_O remains invariant under a transformation of co-ordinates. The formula for electrical torque can then be rewritten as

$$T_{e\phi} = \frac{\partial}{\partial \omega} (v_d i_d + v_q i_q) \qquad (2.37)$$

where v_d, v_q, i_d, i_q have been previously defined. However from (2.34), the d and q voltages are related to currents by

$$v_d = -(r i_d + \omega [L_q i_q + kM_Q i_Q])$$

$$- (L_d i_d + kM_F \dot{i}_F + kM_D \dot{i}_D) \qquad (2.38a)$$

$$v_q = \omega (L_d i_d + kM_F i_F + kM_D i_D)$$

$$- (r i_q + L_q \dot{i}_q + kM_Q \dot{i}_Q) \qquad (2.38b)$$

Electrical torque can then be reformulated as

$$T_{e\phi} = - (L_q i_q + kM_Q \, i_Q) \, i_d$$

$$+ (L_d i_d + kM_F \, i_F + kM_D \, i_D) \, i_q \quad (2.39)$$

By combining Equations (2.36) and (2.39), the per unit torque equation becomes

$$\dot{\omega} = \frac{T_m}{\tau_j} + \left[\frac{-L_d i_q}{3\tau_j} \quad \frac{L_q i_d}{3\tau_j} \quad \frac{-kM_F i_q}{3\tau_j} \quad \frac{-kM_D i_q}{3\tau_j} \quad \frac{kM_Q i_d}{3\tau_j} \quad \frac{-D}{3\tau_j} \right] \begin{bmatrix} i_d \\ i_q \\ i_F \\ i_D \\ i_Q \\ \omega \end{bmatrix} \tag{2.40}$$

2.6.2 Synchronous Torque Angle Equations

At time t=0, the angular displacement between the a-phase axes and q axes is, from Figure 2,3, δ. Similarly the angle between a and d axes is seen to be $\delta+\frac{\pi}{2}$ radians. At time t>0, d and q axes have rotated an additional $\omega_B t$ and the resultant displacement, θ, between the direct axis and a-winding is given by

$$\theta = \omega_B t + \delta + \frac{\pi}{2} .$$

Angular speed is therefore of the form

$$\frac{d\theta}{dt} = \omega_B + \frac{d\delta}{dt} .$$

and in per unit values the equation becomes

$$\omega_B \frac{d\theta}{d\tau} = \omega_B + \omega_B \frac{d\delta}{d\tau} .$$

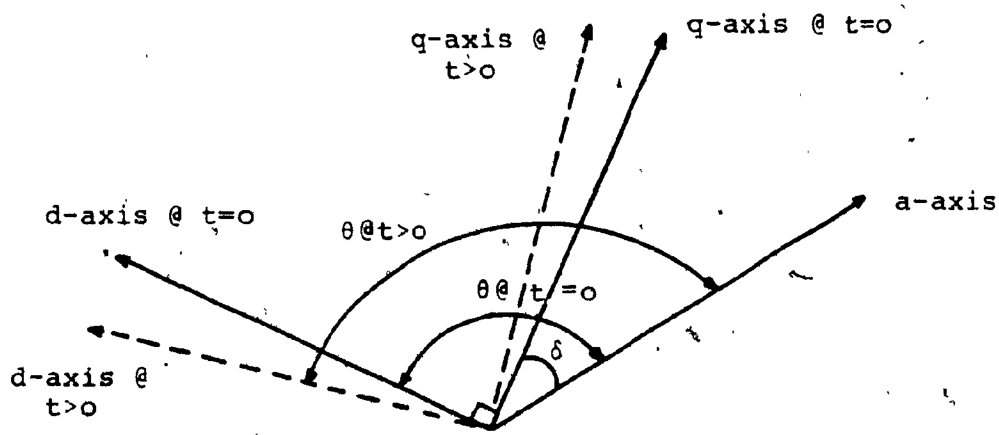


Figure 2.3 : Phasor Representation of angular displacement

After dividing by ω_B and rearranging, the angular velocity equations can be written as

$$\dot{\delta} = \omega - 1 \qquad (2.41)$$

where both ω and $\dot{\delta}$ are expressed in per unit quantities.

2.7 STATE SPACE GENERATOR MODEL

The generator voltage equation (2.34) can be written symbolically as

$$V = Ri - A\dot{i} \qquad (2.42)$$

To obtain the desired state space formulation, (2.42) is permultiplied by A^{-1} and rewritten in the form

$$\dot{i} = A^{-1}Ri - A^{-1}V \qquad (2.43)$$

From linear algebra [24] it is well known that the inverse of a matrix A is defined to be

$$A^{-1} \triangleq \frac{\text{adj}A}{|A|} \triangleq \frac{1}{|A|}\begin{bmatrix} A_{11} & -A_{21} & A_{31} & -A_{41} & A_{51} \\ -A_{12} & A_{22} & -A_{32} & A_{42} & -A_{52} \\ A_{13} & -A_{23} & A_{33} & -A_{43} & A_{53} \\ -A_{14} & A_{24} & -A_{34} & A_{44} & -A_{54} \\ A_{15} & -A_{25} & A_{35} & -A_{45} & A_{55} \end{bmatrix} \qquad (2.44)$$

where A_{ij} is the determinant of A with the ith row and jth column deleted.

Therefore, for A as given by Equation (2.34), A^{-1} is clearly of the form

$$A^{-1} = \frac{1}{|A|}\begin{bmatrix} A_{11} & 0 & A_{31} & -A_{41} & 0 \\ 0 & A_{22} & 0 & 0 & -A_{52} \\ A_{13} & 0 & A_{33} & -A_{43} & 0 \\ -A_{14} & 0 & -A_{34} & A_{44} & 0 \\ 0 & -A_{25} & 0 & 0 & A_{55} \end{bmatrix} \quad (2.45)$$

Finally, by using this expansion, and adding the variables $\dot{\omega}$ and $\dot{\delta}$ as previously defined, the state variable generator description turns out to be

$$
\begin{bmatrix} \dot{i}_d \\ \dot{i}_q \\ \dot{i}_F \\ \dot{i}_D \\ \dot{i}_Q \\ \dot{\omega} \\ \dot{\delta} \end{bmatrix}
=
\begin{bmatrix}
\frac{-rA_{22}}{|A|} & \frac{-\omega L_q A_{22}}{|A|} & \frac{-r_F A_{42}}{|A|} & \frac{r_D A_{52}}{|A|} & \frac{-\omega k M_Q A_{22}}{|A|} & 0 & 0 \\
\frac{\omega L_d A_{33}}{|A|} & \frac{-rA_{33}}{|A|} & \frac{\omega k M_F A_{33}}{|A|} & \frac{\omega k M_D A_{33}}{|A|} & \frac{r_Q A_{63}}{|A|} & 0 & 0 \\
\frac{rA_{24}}{|A|} & \frac{-\omega L_q A_{24}}{|A|} & \frac{-r_F A_{44}}{|A|} & \frac{r_D A_{54}}{|A|} & \frac{-\omega k M_Q A_{24}}{|A|} & 0 & 0 \\
\frac{rA_{25}}{|A|} & \frac{\omega L_q A_{25}}{|A|} & \frac{r_F A_{45}}{|A|} & \frac{-r_D A_{55}}{|A|} & \frac{\omega k M_Q A_{25}}{A} & 0 & 0 \\
\frac{-\omega L_d A_{36}}{|A|} & \frac{rA_{36}}{|A|} & \frac{-\omega k M_F A_{36}}{|A|} & \frac{-\omega k M_D A_{36}}{|A|} & \frac{-A_{66} r_Q}{|A|} & 0 & 0 \\
\frac{-L_d i_q}{3\tau_j} & \frac{L_q i_d}{3\tau_j} & \frac{-k M_F i_q}{3\tau_j} & \frac{-k M_D i_q}{3\tau_j} & \frac{k M_Q i_d}{3\tau_j} & \frac{-D}{3\tau_j} & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0
\end{bmatrix}
\begin{bmatrix} i_d \\ i_q \\ i_F \\ i_D \\ i_Q \\ \omega \\ \delta \end{bmatrix}
$$

Correction ; all subscripts of the form A_{ij}, $_{ii}$, should be $A_{i-1,\,j-1}$, $A_{i-1,i-1}$ on this page only.

$$
+\frac{1}{A}\begin{bmatrix}
-A_{11} & 0 & -A_{31} & A_{41} & 0 & 0 & 0 \\
0 & -A_{22} & 0 & 0 & A_{52} & 0 & 0 \\
-A_{13} & 0 & -A_{33} & A_{43} & 0 & 0 & 0 \\
A_{14} & 0 & A_{34} & -A_{44} & 0 & 0 & 0 \\
0 & A_{24} & 0 & 0 & -A_{55} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & |A| & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & |A|
\end{bmatrix}
\begin{bmatrix}
v_d \\ v_q \\ v_F \\ 0 \\ 0 \\ -\frac{T_M}{\tau_j} \\ -1
\end{bmatrix}
\qquad (2.46)
$$

2.8 CONCLUSIONS

In this chapter the synchronous generator differential equations are developed. The analysis begins with a derivation of the machine rotor and stator inductances in a manner similar to that of Thaler(33) and Dubé(8) but not found in Anderson(2). Park's transformation, P, is applied to the generator inductances and the resulting simplified machine inductances represented by (2.23). Since the differential model is derived from the product PV, where V represents generator voltages, some of the details of the matrix multiplication not shown in Anderson's derivation(2) are included for the sake of clarity. The per unit normalization scheme used both by Lewis(21) and Anderson(2) is explained, the appropriate p.u. constants defined and equations stated as in (2.34). Finally the model is presented by Anderson in the form

$$V = Ri - A\dot{i}$$

whereas, in this thesis, the set of generator differential equations is written as

$$\dot{i} = A^{-1}Ri - A^{-1}V$$

with the specifix matrix entries shown in (2.46).

CHAPTER 3

INTERCONNECTED POWER SYSTEM EQUATIONS

3.1 INTRODUCTION

The previous chapter contains the development of a seventh order single generator model. By extension from Chapter 2, an n machine system can be described by a set of 7 n differential equations. However, careful examination of machine formulae reveals that there are an additional two unknowns, v_d and v_q. For a multimachine system, this would result in 9 n unknowns, thereby requiring 2 n extra equations in order to completely specify the power network. The objective, therefore, is to derive a relationship between the generator voltages v_d, v_q and corresponding currents i_d, i_q. Once that is done, the voltages in Equation (2.46) may be replaced by the currents and the state variable description appropriately modified[39].

It is apparent that there exists yet another technique for solving the unknowns v_d and v_q. Currents i_d, i_q can be deduced by applying any number of differential equation solution methods to (2.46) with v_d, v_q fixed at some arbitrary level. Values of i_d and i_q so obtained can then be used to algebrically solve for voltages v_d, v_q. The solution procedure alternates between these two phases until some desired time t is reached. However, this method tends to be highly inaccurate in simulating a power system, and is therefore not used in this thesis. The one advantage of the two stage solution procedure is its ability to to easily incorporate any change in the loads, while the state space formulation must undergo a change in model structure to adequately account for any load variations. If the system loads are assumed constant, however, the state variable model is valid.

3.1.1 Network under Transient Conditions

From Equations (2.27) and (2.29) it can be shown that Park's transformation of $\dot{i}_{abc}$ is given by

$$P\,\dot{i}_{abc} = \dot{i}_{odq} - \omega \begin{bmatrix} 0 \\ -i_q \\ i_d \end{bmatrix} \tag{3.1}$$

Suppose a network consisting of inductors, resistors and a single generator is assumed ; then, for any branch k, with resistance r_k, inductance ℓ_k and current i_k , the branch voltage can be written as

$$v_k = \ell_k\,\dot{i}_k + r_k\,i_k \tag{3.2}$$

From (3.1), the branch voltage for three phase balanced conditions is then of the form

$$\begin{bmatrix} v_{dk} \\ v_{qk} \end{bmatrix} = \ell_k \begin{bmatrix} \dot{i}_{dk} \\ \dot{i}_{qk} \end{bmatrix} + \omega\ell_k \begin{bmatrix} i_{qk} \\ -i_{dk} \end{bmatrix} + r_k \begin{bmatrix} i_{dk} \\ i_{qk} \end{bmatrix} \tag{3.3}$$

where $v_{dk} \triangleq$ the d- axis voltage in branch k

$v_{qk} \triangleq$ the q- axis voltage in branch k.

The following assumptions are made in order to simplify the analysis :

1) $\omega \simeq \omega_B$

2) $\ell_k \dot{i} \ll \omega \ell_k i$

Taking the two assumptions into account and letting $\omega \ell_k$ be represented as x_k, (3.3) becomes

$$\begin{bmatrix} v_{dk} \\ v_{qk} \end{bmatrix} = r_k \begin{bmatrix} i_{dk} \\ i_{qk} \end{bmatrix} + x_k \begin{bmatrix} i_{qk} \\ i_{dk} \end{bmatrix} \qquad (3.4)$$

If v_k and i_k are defined as

$$v_k \triangleq v_{qk} + j\, v_{dk}$$

$$i_k \triangleq i_{qk} + j\, i_{dk}$$

where $j \triangleq \sqrt{-1}$

then, in terms of v_k and i_k, (3.4) is of the form

$$v_k = (r_k + j x_k) (i_{qk} + j\, i_{dk}) \qquad (3.5a)$$

$$= z_k\, i_k \qquad (3.5b)$$

3.1.2 Converting to a Common Reference Frame

Figure 3.1 represents a voltage v_k embedded within two different co-ordinate systems, one of which (the D-Q axis) is moving at synchronous speed. Voltage v_k is therefore expressable in both reference frames as

$$\hat{v}_k = v_{Qk} + j v_{Dk} \quad (3.6a)$$

$$v_k = v_{qk} + j v_{dk} \quad (3.6b)$$

From Figure 3.1 the relationship between the different voltages can be obtained, that is

$$v_{Qk} = v_{qk} \cos \delta - v_{dk} \sin \delta \quad (3.7a)$$

$$v_{DK} = v_{qk} \sin \delta + v_{dk} \cos \delta \quad (3.7b)$$

If the complex number $v_{Qk} + j\, v_{Dk}$ is written symbolically as $\hat{v}_k$, then

$$\hat{v}_k = v_k e^{j\delta} \quad (3.8)$$

Since v_k is known from (3.5) and $\hat{i}_k$ is of the same form as v_k, it follows that

$$\hat{v}_k = z_k \hat{i}_k \quad (3.9)$$

Letting $\hat{v}_b$, $\hat{i}_b$ represent the vectors of all branch voltages and currents and z_b the matrix of all branch impedances, (3.9) can be rewritten for the whole network as

$$\hat{v}_b = z_b \hat{i}_b \quad (3.10)$$

Alternatively, the node voltage relationship may be derived from branch values [16] by the use of a nodal incidence matrix u and the desired equation expressed as

$$\hat{i} = u^T y_b u \hat{v} \triangleq \bar{Y} \hat{v} \quad (3.11)$$

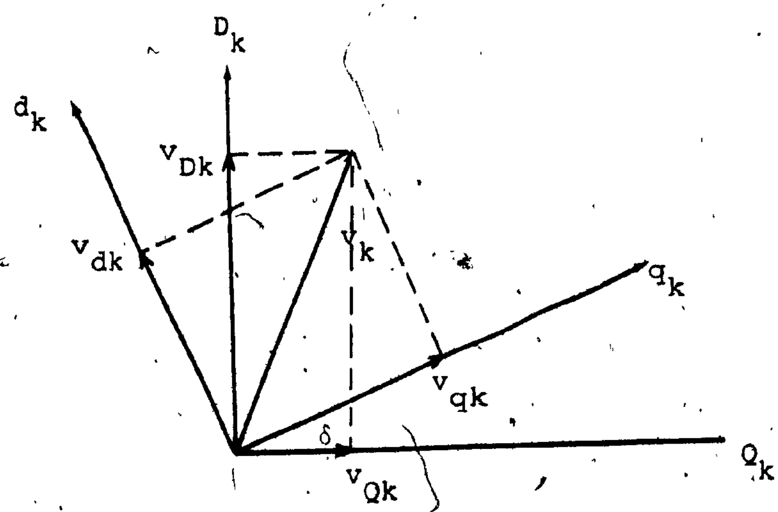


Figure 3.1 Two frames of reference for a voltage v_k

where $u \triangleq (u_{pq}) = 1$ if current in branch p enters node q

$= -1$ if current in branch p leaves node q

$= 0$ if node q is not connected to branch p

$$y_b \triangleq z_b^{-1}$$

$$\bar{Y} \triangleq \begin{bmatrix} Y_{11}\, e^{j\theta_{11}} & Y_{12}\, e^{j\theta_{12}} \cdots\cdots & Y_{1n} e^{j\theta_{1n}} \\ \vdots & & \vdots \\ Y_{n1} e^{j\theta n1} \cdots\cdots & \cdots\cdots & Y_{nn}\; e^{j\theta nn} \end{bmatrix}$$

for an n - node network.

3.1.3 Relations Governing machine currents and Voltages

A voltage $v_{abc\,i}$ at node i of an n machine and n node system is converted to $v_{dq\,i}$ by the application of Park's transformation. If voltage and current vectors for all nodes are defined as

$$\bar{v} \triangleq v_q + j\, v_d$$

$$\bar{i} \triangleq i_q + j\, i_d$$

where $v_q \triangleq \begin{bmatrix} v_{q1} \\ v_{q2} \\ \vdots \\ v_{qn} \end{bmatrix}$, $i_q \triangleq \begin{bmatrix} i_{q1} \\ i_{q2} \\ \vdots \\ i_{qn} \end{bmatrix}$

then, with respect to common reference frames, the voltages can be written as

$$\begin{bmatrix} v_{Q1} & + & jv_{D1} \\ \vdots & & \vdots \\ v_{Qn} & + & jv_{Dn} \end{bmatrix} = \begin{bmatrix} e^{j\delta 1} & & O \\ & \ddots & \\ O & & e^{j\delta n} \end{bmatrix} \begin{bmatrix} v_{q1} & + & jv_{d1} \\ \vdots & & \vdots \\ v_{qn} & + & jv_{dn} \end{bmatrix} \qquad (3.12)$$

Equation (3.12) is stated symbolically in the form

$$\tilde{v} = T\,\bar{v} \qquad (3.13a)$$

A similar relationship can be shown to exist for currents, so that

$$\tilde{i} = T\,\bar{i} \qquad (3.13b)$$

Since $\tilde{v}$ is assumed to be a vector of nodal voltages then the formula

$$\tilde{i} = \bar{Y}\,\tilde{v}$$

is seen to be valid. Therefore $\bar{i}$ can be obtained directly in terms of $\tilde{v}$ as

$$\bar{i} = T^{-1}\,\tilde{i} = T^{-1}\,\bar{Y}\,\tilde{v} = T^{-1}\,\bar{Y}\,T\,\bar{v} \qquad (3.14)$$

Equation (3.14) can be written in the form

$$\begin{bmatrix} i_{q1}+ji_{d1} \\ i_{q2}+ji_{d2} \\ \vdots \\ i_{qn}+ji_{dn} \end{bmatrix} = \begin{bmatrix} Y_{11}e^{j\theta_{11}} & Y_{12}e^{j(\theta_{12}-\delta_{12})} & \cdots & Y_{1n}e^{j(\theta_{1n}-\delta_{1n})} \\ \vdots & & & \vdots \\ Y_{n1}e^{j(\theta_{n1}-\delta_{n1})} & \cdots & \cdots & Y_{nn}e^{j\theta_{nn}} \end{bmatrix} \begin{bmatrix} v_{q1}+jv_{d1} \\ v_{q2}+jv_{d2} \\ \vdots \\ v_{qn}+jv_{dn} \end{bmatrix} \tag{3.15}$$

where $\delta_{jk} \triangleq \delta_j - \delta_k$

It is apparent that the voltages and currents form the desired set of 2n additional equations which can be used to solve for unknowns in the dynamical system.

3.1.4 Voltage-current Equations for Two machine system

For the two machine power system model the following assumptions are made :

1) the full system with n+r nodes is reduced to an n-node equivalent system by circuit reduction techniques[2]
2) loads consist only of constant impedance types
3) all parameters are identical for both machines
4) the number of differential equations may be

reduced from 14 to 13 by noting that $\dot{\delta}_1 = \omega_1 - 1$, $\dot{\delta}_2 = \omega_2 - 1$, $\dot{\delta}_1 - \dot{\delta}_2 = \omega_1 - \omega_2$, and $\delta_1 - \delta_2$ is defined to be a state variable.

Equation (3.15), specialized to the two machine case, may be rewritten as

$$\begin{bmatrix} i_{q1} + j i_{d_1} \\ i_{q2} + j i_{d_2} \end{bmatrix} = \begin{bmatrix} Y_{11} e^{j\theta_{11}} & Y_{12} e^{j(\theta_{12} - \delta_{12})} \\ Y_{21} e^{j(\theta_{21} - \delta_{21})} & Y_{22} e^{j\theta_{22}} \end{bmatrix} \begin{bmatrix} v_{q1} + j v_{d_1} \\ v_{q2} + j v_{d_2} \end{bmatrix} \tag{3.16}$$

where $\delta_{kj} \triangleq \delta_k - \delta_j$

and voltage or current subscripts represent the corresponding synchronous machines.

However, since the currents are defined as state variables, the voltages can be expressed in the form

$$\begin{bmatrix} v_{q1} + j v_{d1} \\ v_{q2} + j v_{d2} \end{bmatrix} = \frac{1}{|D'|} \begin{bmatrix} Y_{22} e^{j\theta_{22}} & Y_{12} e^{j(\theta_{12} - \delta_{12})} \\ -Y_{21} e^{j(\theta_{21} - \delta_{21})} & Y_{11} e^{j\theta_{11}} \end{bmatrix} \begin{bmatrix} i_q + j i_{d_1} \\ i_{q2} + j i_{d_2} \end{bmatrix} \tag{3.17}$$

where $|D'| \triangleq Y_{11} Y_{22} e^{j(\theta_{11} + \theta_{22})} - Y_{12} Y_{21} e^{j(\theta_{21} + \theta_{12})}$

After trigonometric expansion and algebraic manipulation, (3.17) may be rewritten as

$$
\begin{bmatrix} v_{d1} \\ v_{q1} \\ 0 \\ 0 \\ 0 \\ v_{d2} \\ v_{q2} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}
= \frac{1}{|D'|}
\begin{bmatrix}
Y_{22}\cos\theta_{22} & Y_{22}\cos\theta_{22} & \overset{3col}{\leftrightarrow} & -Y_{12}\cos(\theta_{12}-\delta_{12}) & -Y_{12}\sin(\theta_{12}-\delta_{12}) & \overset{6col}{\leftrightarrow} \\
-Y_{22}\sin\theta_{22} & Y_{22}\cos\theta_{22} & & Y_{12}\sin(\theta_{12}-\delta_{12}) & -Y_{12}\cos(\theta_{12}-\delta_{12}) & \\
0 & 0 & & 0 & 0 & \\
0 & 0 & & 0 & 0 & \\
0 & 0 & & 0 & 0 & \\
-Y_{21}\cos(\theta_{21}-\delta_{21}) & -Y_{21}\sin(\theta_{21}-\delta_{21}) & 0 & Y_{11}\cos\theta_{11} & Y_{11}\sin\theta_{11} & 0 \\
Y_{21}\sin(\theta_{21}-\delta_{21}) & -Y_{21}\cos(\theta_{21}-\delta_{21}) & & -Y_{11}\sin\theta_{11} & Y_{11}\cos\theta_{11} & \\
0 & 0 & & 0 & 0 & \\
0 & 0 & & 0 & 0 & \\
0 & 0 & & 0 & 0 & \\
0 & 0 & & 0 & 0 & \\
0 & 0 & & 0 & 0 & \\
0 & 0 & & 0 & 0 &
\end{bmatrix}
\begin{bmatrix} i_{d1} \\ i_{q1} \\ 0 \\ 0 \\ 0 \\ i_{d2} \\ i_{q2} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}
\tag{3.18}
$$

The resulting differential system, for the power network described, are, from (3.18) and (2.46), of the form

$$
\begin{bmatrix} i_{d1} \\ i_{q1} \\ i_{F1} \\ i_{D1} \\ i_{Q1} \\ i_{d2} \\ i_{q2} \\ i_{F2} \\ i_{D2} \\ i_{Q2} \\ \omega_1 \\ \omega_2 \\ \delta_1-\delta_2 \end{bmatrix}
=
\left[\begin{array}{ccccccccccccc}
-r\frac{A_{22}}{|A|} & -\omega_1 L_q\frac{A_{22}}{|A|} & -r_F\frac{A_{42}}{|A|} & -r_F\frac{A_{42}}{|A|} & -\omega_1 kM_A\frac{A_{22}}{|A|} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\omega_1 L_d\frac{A_{33}}{|A|} & -r\frac{A_{33}}{|A|} & \omega_1 kM_F\frac{A_{33}}{|A|} & \omega_1 kM_D\frac{A_{33}}{|A|} & r_Q\frac{A_{63}}{|A|} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-r\frac{A_{24}}{|A|} & -\omega_1 L_q\frac{A_{24}}{|A|} & -r_F\frac{A_{44}}{|A|} & r_D\frac{A_{54}}{|A|} & -\omega_1 kM_Q\frac{A_{24}}{|A|} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
r\frac{A_{25}}{|A|} & \omega_1 L_q\frac{A_{25}}{|A|} & r_F\frac{A_{45}}{|A|} & -r_D\frac{A_{55}}{|A|} & \omega_1 kM_Q\frac{A_{25}}{|A|} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\omega_1 L_d\frac{A_{36}}{|A|} & r\frac{A_{36}}{|A|} & -\omega_1 kM_F\frac{A_{36}}{|A|} & -\omega_1 kM\frac{A_{36}}{|A|} & -\frac{A_{66}}{|A|}r_Q & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -r\frac{A_{22}}{|A|} & -\omega_2 L_q\frac{A_{22}}{|A|} & -r_F\frac{A_{42}}{|A|} & -r_F\frac{A_{42}}{|A|} & -\omega_2 kM_Q\frac{A_{22}}{|A|} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & +\omega_2 L_d\frac{A_{33}}{|A|} & -r\frac{A_{33}}{|A|} & \omega_2 kM_F\frac{A_{33}}{|A|} & \omega_2 kM_D\frac{A_{33}}{|A|} & r_Q\frac{A_{63}}{|A|} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -r\frac{A_{24}}{|A|} & \omega_2 L_q\frac{A_{24}}{|A|} & -r_F\frac{A_{44}}{|A|} & r_D\frac{A_{54}}{|A|} & -\omega_2 kM_Q\frac{A_{24}}{|A|} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & r\frac{A_{25}}{|A|} & \omega_2 L_q\frac{A_{25}}{|A|} & r_F\frac{A_{45}}{|A|} & -r_D\frac{A_{55}}{|A|} & \omega_2 kM_Q\frac{A_{25}}{|A|} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -\omega_2 L_d\frac{A_{36}}{|A|} & r\frac{A_{36}}{|A|} & -\omega_2 kM_F\frac{A_{36}}{|A|} & -\omega_1 kM_F\frac{A_{36}}{|A|} & -\frac{A_{66}r_Q}{|A|} & 0 & 0 & 0 \\
-\frac{L_d i_{q1}}{3\tau_j} & \frac{L_q i_{d1}}{3\tau_j} & -\frac{kM_F i_{q1}}{3\tau_j} & -\frac{kM_D i_{q1}}{3\tau_j} & \frac{kM_Q i_{d1}}{3\tau_j} & 0 & 0 & 0 & 0 & 0 & -\frac{D}{\tau_j} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{L_d i_{q2}}{3\tau_j} & \frac{L_q i_{d2}}{3\tau_j} & -\frac{kM_F i_{q2}}{3\tau_j} & -\frac{kM_D i_{q2}}{3\tau_j} & \frac{kM_Q i_{d2}}{\tau_j} & 0 & -\frac{D}{\tau_j} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0
\end{array}\right]
\begin{bmatrix} i_{d1} \\ i_{q1} \\ i_{F1} \\ i_{D1} \\ i_{Q1} \\ i_{d2} \\ i_{q2} \\ i_{F2} \\ i_{D2} \\ i_{Q2} \\ \omega_1 \\ \omega_2 \\ \text{[illegible]} \end{bmatrix}
$$

Correction : all subscripts of the form A_{ij}, A_{ii}, should be $A_{i-1,\ j-1}$, $A_{i-1,\ j-1}$ on this page only

$$
+\frac{1}{|A|}
\left[\begin{array}{ccccccccccccc}
-A_{11} & 0 & -A_{31} & A_{41} & 0 & & & & & & & & \\
0 & -A_{22} & 0 & 0 & A_{52} & & & \text{O} & & & & \text{O} & \\
-A_{13} & 0 & -A_{33} & A_{43} & 0 & & & & & & & & \\
A_{14} & 0 & A_{34} & -A_{44} & 0 & & & & & & & & \\
0 & A_{24} & 0 & 0 & -A_{55} & & & & & & & & \\
 & & & & & -A_{11} & 0 & -A_{31} & A_{41} & 0 & & & \\
 & & & & & 0 & -A_{22} & 0 & 0 & A_{52} & & & \\
 & & \text{O} & & & -A_{13} & 0 & -A_{33} & A_{43} & 0 & & & \\
 & & & & & A_{14} & 0 & A_{34} & -A_{44} & 0 & & & \\
 & & & & & 0 & A_{24} & 0 & 0 & -A_{55} & & & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & |A| & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & |A| & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right]
\left[\begin{array}{c}
0 \\ 0 \\ -v_{F1} \\ 0 \\ 0 \\ 0 \\ 0 \\ -v_{F2} \\ 0 \\ 0 \\ \frac{T_{M1}}{\tau_j} \\ \frac{T_{M2}}{\tau_j} \\ 0
\end{array}\right]
$$

$$
+\frac{1}{|D|}\cdot\frac{1}{|A|}
\left[\begin{array}{ccccc|ccccc|ccc}
-A_{11} & 0 & -A_{31} & A_{41} & 0 & & & & & & & & \\
0 & -A_{22} & 0 & 0 & A_{52} & & & \text{O} & & & & \text{O} & \\
-A_{13} & 0 & -A_{33} & A_{43} & 0 & & & & & & & & \\
A_{14} & 0 & A_{34} & -A_{44} & 0 & & & & & & & & \\
0 & A_{24} & 0 & 0 & -A_{55} & & & & & & & & \\
\hline
 & & & & & -A_{11} & 0 & -A_{31} & A_{41} & 0 & & & \\
 & & & & & 0 & -A_{22} & 0 & 0 & A_{52} & & & \\
 & & \text{O} & & & -A_{13} & 0 & -A_{33} & A_{43} & 0 & & & \\
 & & & & & A_{14} & 0 & A_{34} & -A_{44} & 0 & & & \\
 & & & & & 0 & A_{24} & 0 & 0 & -A_{55} & & & \\
\hline
 & & & & & & & & & & |A| & 0 & 0 \\
 & & \text{O} & & & & & & & & 0 & |A| & 0 \\
 & & & & & & & & & & 0 & 0 & 0
\end{array}\right]
$$

Any further detailed expansion is not attempted , rather the model is given in terms of machine parameters in the following section.

3.2 SYSTEM MODEL FROM MACHINE PARAMETERS

In order to simulate the two machine system and interconnections, it is necessary to assign typical values to the inductances, damping coefficents and resistances. Without loss of generality, it is assumed that both generators have the same per unit parameters[2] listed below.

1) $L_d = 1.70$

2) $L_F = 1.65$

3) $L_D = 1.605$

4) $L_q = 1.64$

5) $L_Q = 1.526$

6) $kM_D = kM_F = M_R = 1.55$

7) $kM_Q = 1.44$

8) $r = 0.001096$

9) $r_F = 0.00742$

10) r_D 0.0131

11) $r_Q = 0.054$

12) $k = \sqrt{\frac{3}{2}}$

13) $\tau_j = 1786.94$

14) $D = 2.0$

15) $\theta_{11} = -\theta_{22} = 30^\circ$

16) $\theta_{12} = -\theta_{21} = 60^\circ$

17) $Y_{12} = Y_{21} = 1$

18) $Y_{11} = Y_{22} = \sqrt{2}$

The state variables are defined to be :

$$x_1 \stackrel{\Delta}{=} i_{d1}$$
$$x_2 \stackrel{\Delta}{=} i_{q1}$$
$$x_3 \stackrel{\Delta}{=} i_{F1}$$
$$x_4 \stackrel{\Delta}{=} i_{D1}$$
$$x_5 \stackrel{\Delta}{=} i_{Q1}$$
$$x_6 \stackrel{\Delta}{=} i_{d2}$$
$$x_7 \stackrel{\Delta}{=} i_{q2}$$
$$x_8 \stackrel{\Delta}{=} i_{F2}$$
$$x_9 \stackrel{\Delta}{=} i_{D2}$$
$$x_{10} \stackrel{\Delta}{=} i_{Q2}$$
$$x_{11} \stackrel{\Delta}{=} \omega_1$$
$$x_{12} \stackrel{\Delta}{=} \omega_2$$
$$x_{13} \stackrel{\Delta}{=} \delta_1 - \delta_2 .$$

For the given machine parameters, the two generator system takes the form

$$\dot{x} = V(x)\, x + Bu \qquad (3.20)$$

where V(x), obtained from expansion of (3.19), contains elements which are functions of the state variables, and B is a constant matrix. The entries of V(x) as well as B and u are listed as follows :

$$V(x) \triangleq [v_{ij}(x)]$$

$$B \triangleq [b_{ij}]$$

$v_{11} = -6.63300$	$v_{16} = 2.70549\cos x_{13} + 4.68605\sin x_{13}$
$v_{12} = 3.82614 - 8.87401\,x_{11}$	$v_{17} = 4.68605\cos x_{13} - 2.70549\sin x_{13}$
$v_{13} = 0.01383$	$v_{18} = v_{111} = 0$
$v_{14} = 0.04486$	$v_{19} = v_{112} = 0$
$v_{15} = -8.06237\,x_{11}$	$v_{110} = v_{113} = 0$
$v_{21} = 3.81908 + 9.18107x_{11}$	$v_{26} = -4.67704\cos x_{13} + 2.70050\sin x_{13}$
$v_{22} = -6.62977$	$v_{27} = 2.70050\cos x_{13} + 4.67740\sin x_{13}$
$v_{23} = 8.37155\,x_{11}$	$v_{28} = v_{211} = 0$
$v_{24} = 8.37155\,x_{11}$	$v_{29} = v_{212} = 0$
$v_{25} = 0.28477$	$v_{210} = v_{213} = 0$
$v_{31} = 2.28604$	$v_{36} = -0.93244\cos x_{13} - 1.61503\sin x_{13}$
$v_{32} = -1.31866 + 3.05839x_{11}$	$v_{37} = -1.61503\cos x_{13} + 0.93244\sin x_{13}$
$v_{33} = -0.05291$	$v_{38} = v_{311} = 0$
$v_{34} = 0.06662$	$v_{39} = v_{312} = 0$
$v_{35} = 2.77866\,x_{11}$	$v_{310} = v_{313} = 0$
$v_{41} = 4.19800$	$v_{46} = -1.71229\cos x_{13} - 2.96578\sin x_{13}$
$v_{42} = -2.42155 + 5.61633x_{11}$	$v_{47} = -2.96578\cos x_{13} + 1.71229\cos x_{13}$
$v_{43} = 0.03773$	$v_{48} = v_{411} = 0$
$v_{44} = -0.11583$	$v_{49} = v_{412} = 0$
$v_{45} = 5.10263\,x_{11}$	$v_{410} = v_{413} = 0$
$v_{51} = 3.72899 - 8.96510x_{11}$	$v_{56} = 4.56706\cos x_{13} - 2.63679\sin x_{13}$
$v_{52} = 6.46458$	$v_{57} = -2.63679\cos x_{13} - 4.56706\sin x_{13}$
$v_{53} = -8.17406\,x_{11}$	$v_{58} = v_{511} = 0$

$v_{54} = -8.17406x_{11}$ $\quad$ $v_{59} = v_{512} = 0$

$v_{55} = -0.31344$ $\quad$ $v_{510} = v_{513} = 0$

$v_{61} = 2.70549\cos x_{13}+4.68605\sin x_{13}$ $\quad$ $v_{66} = -6.63300$

$v_{62} = 2.70549\sin x_{13}-4.68605\cos x_{13}$ $\quad$ $v_{67} = -3.82614-8.87401x_{12}$

$v_{63} = v_{611} = 0$ $\quad$ $v_{68} = 0.01384$

$v_{64} = v_{612} = 0$ $\quad$ $v_{69} = 0.04486$

$v_{65} = v_{613} = 0$ $\quad$ $v_{610} = -8.06237x_{12}$

$v_{71} = 4.67741\cos x_{13}-2.70050\sin x_{13}$ $\quad$ $v_{76} = 3.81909+9.18171x_{12}$

$v_{72} = 2.70050\cos x_{13}+4.67741\sin x_{13}$ $\quad$ $v_{77} = -6.62077$

$v_{73} = v_{711} = 0$ $\quad$ $v_{78} = 8.37156x_{12}$

$v_{74} = v_{712} = 0$ $\quad$ $v_{79} = 8.37156x_{12}$

$v_{75} = v_{713} = 0$ $\quad$ $v_{710} = 0.28477$

$v_{81} = -0.93244\cos x_{13}-1.61503\sin x_{13}$ $\quad$ $v_{86} = 2.28604$

$v_{82} = 1.61503\cos x_{13}-0.93244\sin x_{13}$ $\quad$ $v_{87} = 1.31867+3.05839x_{12}$

$v_{83} = v_{811} = 0$ $\quad$ $v_{88} = -0.05291$

$v_{84} = v_{812} = 0$ $\quad$ $v_{89} = 0.06663$

$v_{85} = v_{813} = 0$ $\quad$ $v_{810} = 2.77866x_{12}$

$v_{91} = -1.71229\cos x_{13}-2.96579\sin x_{13}$ $\quad$ $v_{96} = 4.19800$

$v_{92} = -2.96578\cos x_{13}+1.71229\sin x_{13}$ $\quad$ $v_{97} = 5.61633x_{12}+2.42155$

$v_{93} = v_{911} = 0$ $\quad$ $v_{98} = 0.03774$

$v_{94} = v_{912} = 0$ $\quad$ $v_{99} = -0.11583$

$v_{95} = v_{913} = 0$ $\quad$ $v_{910} = 5.10264x_{12}$

$v_{101} = -4.56706\cos x_{13}+2.63679\sin x_{13}$ $\quad$ $v_{106} = -3.72899-8.96510x_{13}$

$v_{102} = -2.63679\cos x_{13}-4.56706\sin x_{13}$ $\quad$ $v_{107} = 6.46458$

$v_{103} = v_{1011} = 0$ $\quad$ $v_{108} = -8.17406x_{12}$

$v_{104} = v_{1012} = 0$ $\quad$ $v_{109} = -8.17406x_{12}$

$v_{105} = v_{1013} = 0$ $\quad$ $v_{1010} = -0.31344$

$v_{111} = -0.00317\ x_2$ $\quad$ $v_{126} = -0.00317\ x_7$

$v_{112} = 0.00031\ x_1$ $\quad$ $v_{127} = 0.0031\ x_6$

$v_{113} = -0.00029\ x_2$ $\quad$ $v_{128} = -0.00029\ x_7$

$v_{114} = 0.00028\ x_2$ $\quad$ $v_{129} = 0.00028\ x_7$

$v_{115} = -0.00029\ x_2$ $\quad$ $v_{1210} = -0.00029\ x_7$

$v_{1111} = -0.00037$ $\quad$ $v_{1212} = -0.00037$

$v_{1311} = 1$ $\quad$ $v_{1312} = -1$

$v_{116} = v_{121} = 0$ $\quad$ $v_{131} = v_{136} = 0$

$v_{117} = v_{122} = 0$ $\quad$ $v_{132} = v_{137} = 0$

$v_{118} = v_{123} = 0$ $\quad$ $v_{133} = v_{138} = 0$

$v_{119} = v_{124} = 0$ $\quad$ $v_{134} = v_{139} = 0$

$v_{1110} = v_{125} = 0$ $\quad$ $v_{135} = v_{1310} = v_{1313} = 0$

$b_{11} = b_{66} = -5.41098$ $\quad$ $b_{21} = b_{76} = 0$

$b_{12} = b_{67} = 0$ $\quad$ $b_{22} = b_{77} = -5.40101$

$b_{13} = b_{68} = 1.86488$ $\quad$ $b_{23} = b_{78} = 0$

$b_{14} = b_{69} = 3.42459$ $\quad$ $b_{24} = b_{79} = 0$

$b_{19} = b_{610} = 0$ $\quad$ $b_{25} = b_{710} = 5.27359$

$b_{31} = b_{86} = 1.86488$ $\quad$ $b_{41} = b_{96} = 3.42459$

$b_{32} = b_{87} = 0$ $\quad$ $b_{42} = b_{97} = 0$

$b_{33} = b_{88} = -7.13137$ $\quad$ $b_{43} = b_{98} = 5.08602$

$b_{34} = b_{89} = 5.08602$ $\quad$ $b_{44} = b_{99} = -8.84203$

$b_{35} = b_{810} = 0$ $\quad$ $b_{45} = b_{910} = 0$

$b_{51} = b_{106} = 0$ $\quad$ $b_{1112} = 1$

$b_{52} = b_{107} = 5.27359$ $\quad$ $b_{1212} = 1$

$b_{53} = b_{108} = 0$ $\quad$ all remaining elements

$b_{54} = b_{109} = 0$ $\quad$ are zero

$b_{55} = b_{1010} = -5.80449$ $\quad$ $u_1 = u_2 = u_4 = u_5 = 0$

$u_6 = u_7 = u_9 = u_{10} = 0$ $\quad$ $u_3 = -v_{F1}$, $u_8 = -v_{F2}$

$u_{11} = \frac{T_{M1}}{\tau_j}$, $u_{12} = \frac{T_{M2}}{\tau_j}$, $u_{13} = 0$

From the entries in V(x), it is apparent that the matrix can be separated into a linear and nonlinear part. Then (3.20) becomes

$$\dot{x} = D(x)\, x + Ex + Bu \tag{3.21}$$

where D(x) is a nonlinear matrix

E is a constant matrix.

In order to complete the system description, the outputs must be specified.

Outputs of a power network are defined to be :

1) $i_{d1} + j i_{q1} = x_1 + j x_2$

2) $i_{F1} = x_3$

3) $i_{d2} + j i_{q2} = x_6 + j x_7$

4) $i_{F2} = x_8$

5) $\omega_1 = x_{11}$

6) $\omega_2 = x_{12}$

7) $\delta_1 - \delta_2 = x_{13}$

In vector form, the outputs can be restated as $y = cx$, or

$$\begin{bmatrix} x_1 + jx_2 \\ x_3 \\ x_6 + jx_7 \\ x_8 \\ x_{11} \\ x_{12} \\ x_{13} \end{bmatrix} = \begin{bmatrix} 1 & j & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & j & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \\ x_{10} \\ x_{11} \\ x_{12} \\ x_{13} \end{bmatrix} \qquad (3.22)$$

However it is undesirable to have complex quantities in the c matrix, therefore, in accordance with accepted practice[6], the available outputs are assumed to be

1) $i_{F1} = x_3$

2) $i_{F2} = x_8$

3) $\omega_1 = x_{11}$

4) $\omega_2 = x_{12}$

5) $\delta_1 - \delta_2 = x_{13}$.

The c matrix is then of the form

$$\begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Symbolically, the complete two machine system can be represented as

$$\dot{x} = Ex + D(x)x + Bu \tag{3.23}$$

$$y = Cx$$

where all matrix and vectors have been previously defined.

3.3 CONCLUSIONS

Chapter 3 begins with the study of an interconnected generator system subjected to some perturbation in the 3 - phase currents. After suitable transformation to a common reference frame the state variable currents are shown to be related to unknown system voltages in the form of Equation (3.15). The mathematical treatment follows closely that of Anderson[2] and Undrill[39]. However for the two machine case the equations relating complex voltages $v_q + jv_d$ to currents $i_q + ji_d$ are replaced by the real and imaginary components v_q, v_d, i_q, i_d and the result written as in (3.18). This allows unknown voltages v_d, v_q, to be expressed directly in terms of the state variable currents i_d, i_q, thereby completing the description of the two machine interconnected system. Finally, after the assignment of typical machine parameters to Equation (3.19), the system is represented by the values V(x) and B of (3.20). One detail, absent in both Anderson[2] and Undrill[39], is the specification of suitable machine outputs $y = Cx$. The C matrix used, is shown in detail following (3.22), and is in agreement with that of Davison[6].

CHAPTER 4

OBSERVER DESIGN FOR POWER SYSTEMS

4.1 INTRODUCTION

One method frequently used[40] to stabilize nonlinear autonomous systems described by

$$\dot{x} = f(x,u) \qquad (4.1)$$

is to provide an appropriate linear feedback control law of the form

$$u = -kx$$

where $k \in R^{m x n}$, $x \in R^n$, $f \in R^n \times R^m \to R^n$.
In addition, nonlinear feedback control of the type

$$u = kx + u_p$$

where $u_p = 1$, if $\beta < ||x|| < \alpha$

$u_p = 0$ if $||x|| \geq \alpha$ or $||x|| \leq \beta$

and where $||.||$ denotes the Euclidian norm, is applied to increase the region of stability of nonlinear, autonomous biped motional systems [4,12] modelled by (4.1).

In the aforementioned design procedures, it is tacitly assumed that all the states are available at the output. As is apparent, however, from Chapter 3, it is not always possible to measure the full state vector.

To overcome this problem for the linear case, a Luenberger observer[22] can be formulated to reconstruct the unavailable state variables of the desired system, if the system is observable.

For the nonlinear autonomous case, it is feasible, as well, to design observers[18,34], although additional constraints are imposed..

One final method, frequently used by control engineers, is to linearize a nonlinear system around an operating point x_o, u_o so that a system of the form (4.1) becomes

$$\dot{x} = Ax + Bu \tag{4.2}$$

where $A \triangleq$ Jacobian matrix of $f(x,u)$ with respect to x @ x_o, u_o

$B \triangleq$ Jacobian matrix of $f(x,u)$ with respect to u @ x_o, u_o

and then design a standard Luenberger observer for this linearization.

The approach taken in this thesis is to design a nonlinear observer according to the method of Thau[34] and compare its convergence properties to that of the linearized power system Luenberger observer.

4.2 Lyapunov Stability Theorems

Since Lyapunov stability theory is the analytical tool used in proving the convergence of the nonlinear observers' states to those of the system, it is necessary to provide a theoretical framework within which the observer design can be understood.

There are three basic types of stability commonly used by power system engineers[10] :

1) bounded input-bounded output stability
2) Lagrange stability
3) stability in the sense of Lyapunov.

The definition of Lyapunov stability is as follows[15,40]:

The equilibrium point O at time t_o for a system $x = f(x)$, is stable at time t_o if, for each $\varepsilon \geq$ there exists a $\delta\ (t_o,\varepsilon)>o$ such that

$$||x(t_o)||<\delta\ (t_o,\varepsilon) \Rightarrow ||x(t)|| < \varepsilon\ \forall t \geq t_o$$

where $||\ .\ ||$ denotes euclidean norm

The intent of the definition can be seen from Figure 4.1. If the norm $||x(t)||$ is not to exceed a pre-specified ε and it is possible to find a bound $\delta(t_o,\varepsilon)$ on the norm $||x(t_o)||$ of the initial conditions such that any solution trajectory starting with $||x(t_o)|| < \delta$ always

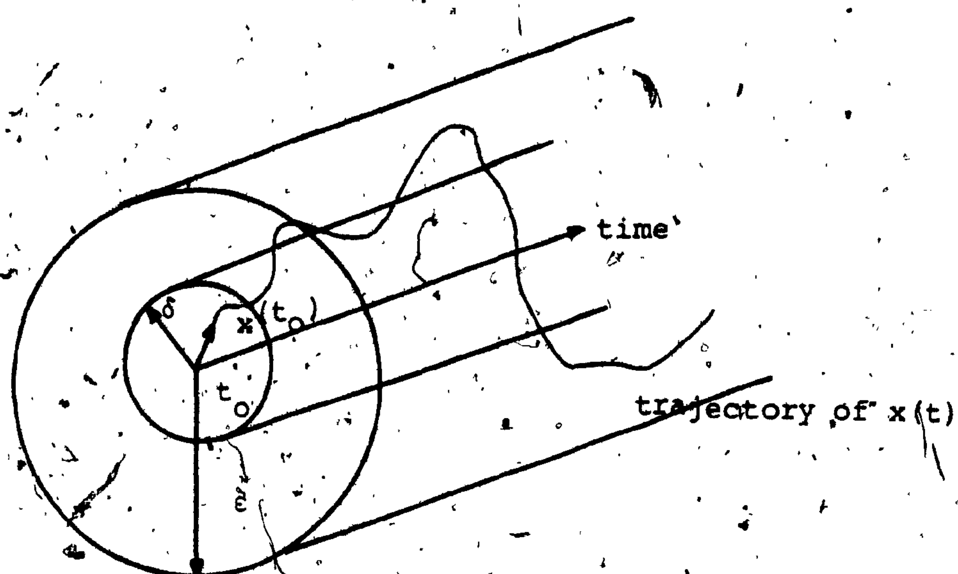


Figure 4.1 : Illustration of Lyapunov Stability

remains within $||x(t)|| < \varepsilon$, then the equilibrium pt. O is Lyapunov stable. An equilibrium point O at time t_o is asymptotically stable if

1) it is stable at time t_o and
2) there exists a number $\delta(t_o)$ such that
$||x(t_o)|| < \delta_1(t_o) \Rightarrow ||x(t)|| \to o$ as $t \to \infty$.

4.2.1 Stability of Linear Autonomous System

The fundamental theorem of Lyapunov's direct method (40 p.148) is stated as follows :

Theorem 4.1 The equilibrium point O of $\dot{x} = f(x)$ is stable if there exists a continuously differentiable locally positive definite function V such that

$$\dot{V}(x) \le o \qquad \forall x \in B_r$$

where $B_r = \{ x \in R^n ; ||\bar{x}|| < r \}$.

One type of dynamical system commonly used in observer design[22] is given as

$$\dot{z} = (A-Kc)\, z \equiv \bar{A}\, z \qquad (4.3)$$

and which is, by inspection, both linear and autonomous. For autonomous differential equations the appropriate theorems are stated in the form (40 pp.171 and 175) :

Theorem 4.2 The equilibrium point O of $\dot{z} = \bar{A}z$ is asymptotically stable if and only if all eigenvalues of

$\bar{A}$ have negative real parts .

Theorem 4.3 Given a matrix $\bar{A} \varepsilon R^{nxn}$, the following three statements are equivalent :

(i) All eignvalues of $\bar{A}$ have negative real parts

(ii) There exists some positive definite matrix $Q\varepsilon R^{nxn}$ such that $\bar{A}^T P + P\bar{A} = -Q$ has a unique solution for P, and this solution is positive definite.

(iii) For every positive definite matrix $Q\varepsilon R^{nxn}$, $\bar{A}^T P + P\bar{A} = -Q$ has a unique solution for P and this solution is positive definite.

4.2.2 Lyapunov's Indirect method

Under certain conditions, the stability of a nonlinear system can be determined by studying the behaviour of an associated linear system. For autonomous differential equations of the form

$$\dot{x} = f(x) \tag{4.4}$$

the theorem of interest is [40,p.189] given as follows :

Theorem 4.4 Consider (4.4). Suppose that $f(o) = o$ and that f is continuously differentiable. Define $A = \frac{\partial f}{\partial x}\Big|_{x=o}$, where A is the Jacobian matrix of f with respect to x. Under these conditions, the equilibrium point O of (4.4) is asymptotically stable if all eigenvalues of A have negative real parts,

4.3 Nonlinear Observer Structure

If the given power network is modeled as

$$\dot{x} = Ex + D(x)\,x + Bu \equiv Ex + F(x) + Bu \qquad (4.5)$$

$$y = cx$$

where $F(x) \equiv D(x)\,x$ and where $F(x)$ contains only terms of second order or higher, then it is possible to construct an observer of the type [34]

$$\dot{z} = [\,E-Kc\,]\,z + F(z) + Kcx + Bu \qquad (4.6a)$$

where K is a gain matrix, in R^{nxm}, chosen to ensure that all eigenvalues of E-Kc have negative real parts.

However if F(x) in (4.5) contains first order nonlinearities, the design procedure must be modified in order to satisfy the requirements of Theorem 4.4.

It is apparent that F(x) can be rewritten in the form

$$F(x) = Gx + R(x) \qquad (4.6b)$$

where $G = \left.\frac{\partial F}{\partial x}\right|_{x=x_o}$

and $R(x)$ contains only second and higher order terms.

Then the system described by (4.5) must be reformulated as

$$\dot{x} = (E+G)\ x + R(x) + Bu \qquad (4.7)$$
$$y = cx$$

and the corresponding nonlinear state reconstructor therefore becomes

$$\dot{z} = (E+G-Kc)\ z + Kcx + R(z) + Bu \qquad (4.8)$$

where K is chosen to make E+G-Kc a stable matrix.

4.3.1 Analytical Convergence of Observer States

The system and observer equations are respectively

$$\dot{x} = (E+G)\ x + R(x) + Bu$$
$$y = cx$$
$$\dot{z} = Hz + R(z) + Kcx + Bu$$

where x_o is the equilibrium point of the system and where $H \triangleq E+G-Kc$.

If the matrix H has all its eigenvalues negative real, then it is clear from Theorem 4.3 that a matrix P can be found such that

$$H^TP + PH = -I_n \qquad (4.9)$$

where I_n is the identity matrix.

Since an observer must reproduce the states of the original system, it is sufficient to study the convergence of the vector e, defined as

$$e \triangleq z - x \tag{4.10a}$$

and $$\dot{e} = \dot{z} - \dot{x} \tag{4.10b}$$

After combining (4.6) and (4.10), the resulting dynamical system is of the form

$$\dot{e} = He + F(z) - F(x) + Gx - Gz \tag{4.11}$$

One possible Lyapunov function candidate V is written as

$$V = e^T Pe \tag{4.12}$$

where P is the solution matrix of (4.9).

The time derivative of V then becomes

$$\dot{V} = -e^T I_n e + 2e^T P [F(z)-F(x) +Gx - Gz] \tag{4.13}$$

Is is obvious that

$$-e^T I_n e = -||e||^2 \tag{4.14a}$$

and therefore any $c_o < 1$ results in an inequality of the form

$$-||e||^2 \leq -c_o||e||^2 \tag{4.14b}$$

Since the system being modelled is a power network, it is reasonable to assume that (4.5) has a unique solution around the equilibirum point x_o, and that a constant k can be found such that

$$||F(z) - F(x)|| \leq k \, || e|| \tag{4.14c}$$

where $||.||$ denotes the euclidean norm.

From the trianular inequality, it is apparent that

$$||F(z)-F(x)+ Gx-Gz|| \leq ||F(z)-F(x)||+ ||Gx-Gz||. \tag{4.15}$$

But from (4.14) and the fact that (40,p.65)

$$||Gx|| \leq ||G||_i \, ||x||$$

where $|| G||_i$ represents the induced matrix norm, Equation (4.15) can be rewritten as

$$||F(z)-F(x)+Gx-Gz|| \leq k \, ||e|| + ||G||_i \, ||e|| \tag{4.16}$$

combining (4.14) and (4.16) and evaluating the norms on the right side of (4.13), $\dot{V}$ can be bounded via an inequality of the form

$$\dot{V} \leq ||e||^2 \, [-C_o+2k||P||_i +2|| P ||_i||G||_i]. \tag{4.17}$$

Therefore $\dot{V}$ is negative definite if

$$c_o > [\ 2k\ ||P||_i + 2\ ||P||_i\ ||G||_i]\qquad (4.18)$$

If the above inequality is valid, then e=o is a stable equilibrium point since V satisfies the conditions of Theorem 4.1. In addition, if (4.11) is rewritten as

$$\dot{e} = He + R(z) - R(x)$$

then from Theorem 4.4, it is apparent that e = o is an asymptotically stable equilibrium point. Kou et al [18] prove that e=o is in fact an exponentially stable equilibrium point, that is, that

$$||e(t)|| \leq K_1\ ||e(o)||\ \exp[-K_2 t],$$ for some

constants K_1, K_2.

4.3.2 Linearized Observer

The linearized observer is described by the equation

$$\dot{z} = (\ E+G-Kc\ )\ z + Kcx + Bu\qquad (4.19)$$

where G is a Jacobian matrix defined previously. A comparison of (4.8) and (4.19) reveals that the term R(z) is excluded from the linearized observer model. Intuitively, therefore, it is clear that the nonlinear observer is expected to have better convergence properties than its linearized counterpart.

4.3.3 Comparison of Linearized and Nonlinear Observers

A two generator power system and interconnections, modeled in the form of (3.23) is solved by a fourth order Runge Kutta technique on a 64 bit CDC computer along with its nonlinear observer of the type

$$\dot{z} = (E+G-Kc)\, z + Kcx + R(z) + Bu.$$

The equilibrium point chosen, is given by

$$x_o \triangleq \begin{bmatrix} x_{10} \\ x_{20} \\ x_{30} \\ x_{40} \\ x_{50} \\ x_{60} \\ x_{70} \\ x_{80} \\ x_{90} \\ x_{100} \\ x_{110} \\ x_{120} \\ x_{130} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \qquad u_o \triangleq \begin{bmatrix} -v_{F10} \\ -v_{F20} \\ \dfrac{-Tm_{10}}{\tau_j} \\ \dfrac{-Tm_{20}}{\tau_j} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

and initial conditions of the system and observer are respectively

$$x_i = 0, \quad z_i = 0 \qquad \forall_i = 1, \ldots, 7$$
$$x_8 = 1, \quad z_8 = 0$$
$$x_{13} = z_{13} = 1$$
$$x_i = 0, \quad z_i = 0 \qquad \forall\ i = 9, \ldots 12.$$

The power network and its linearized observer of the form

$$\dot{z} = (E + G - Kc)\ z + Kcx + Bu$$

are then simulated with the same initial conditions and around the some equilibrium point. Plots of the state variable trajectories x_1, x_7 and x_{13} are given in Figures 4.2 to 4.4 to illustrate the behaviour of the two observers.

The new design methodology should allow the observer to faithfully reproduce the system states over an extended region around the operating point x_o and should therefore be more effective than its linearized counterpart. The simulation results of Figures 4.2, and 4.3 show that the performance of the nonlinear is better than that of the linearized observer. States of the system and two observers at time $t = 3$ are tabulated in Table 4.1 and it is clear, from the table as well as Figures 4.2, 4.3, that both estimators reproduce the system states with approximately equal magnitude, while the nonlinear observer provides the correct sign for twelve

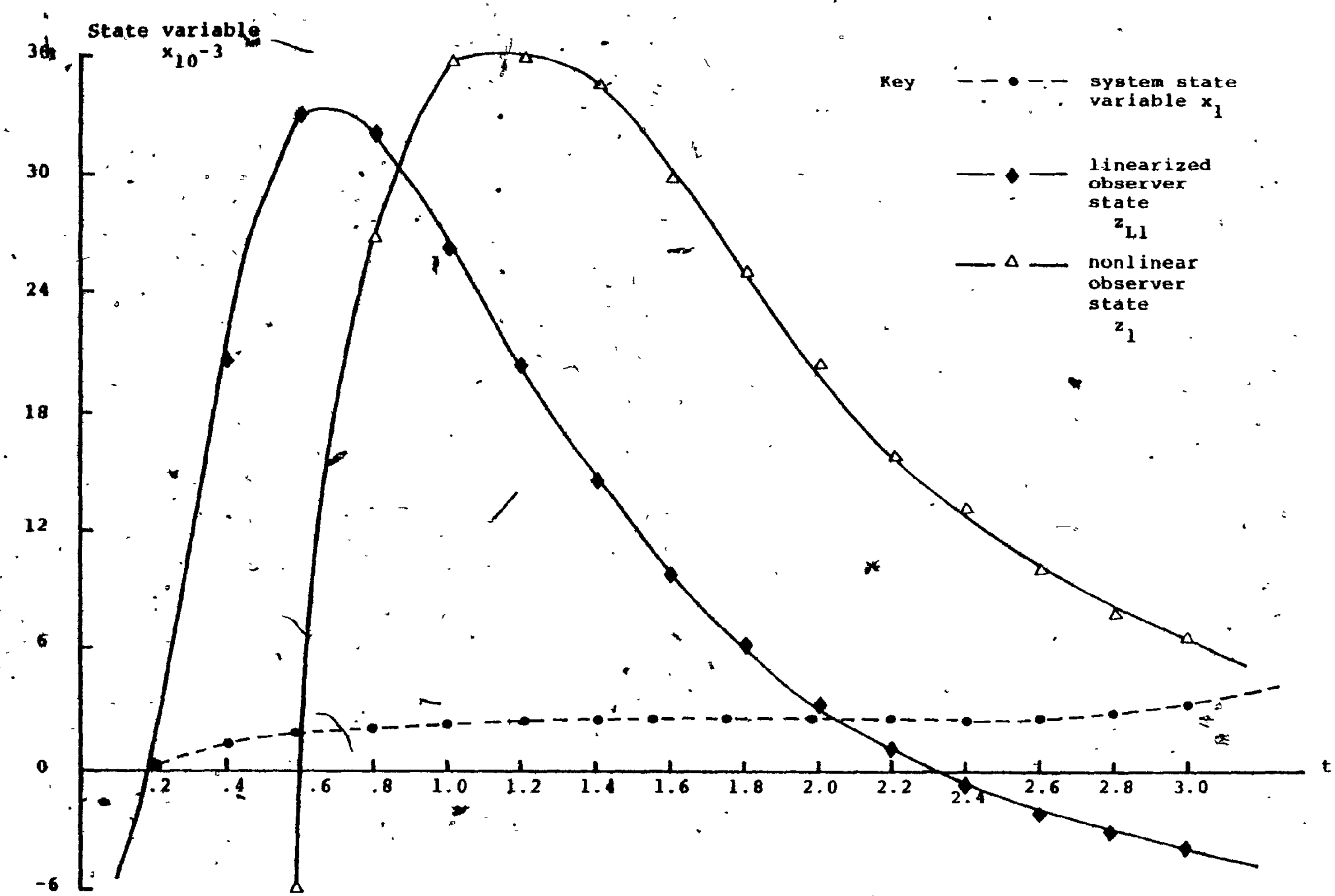


Figure 4.2 : Plot of system, linearized observer and nonlinear observer state variables

x_1, z_{L1}, z_1 trajectories

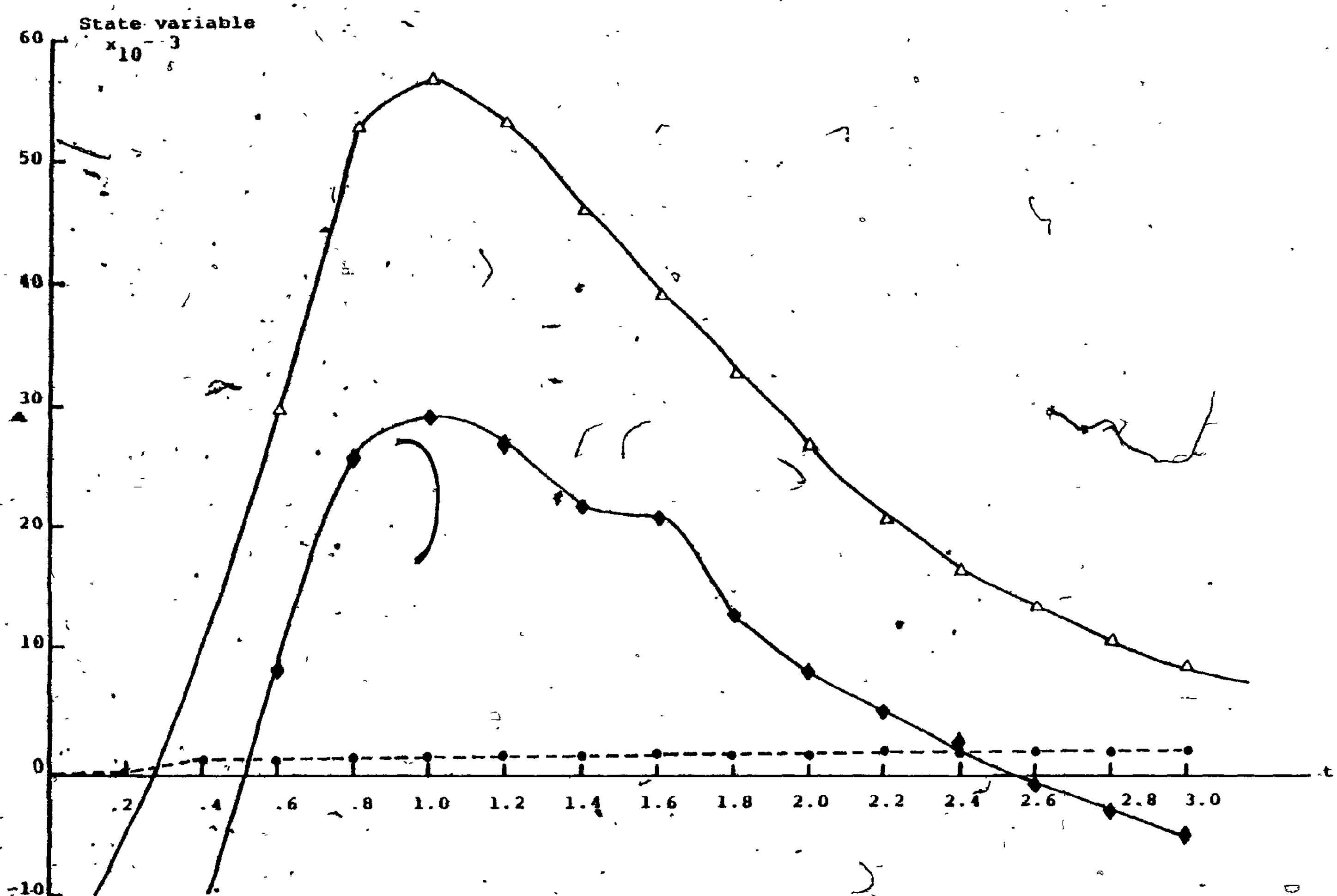


Figure 4.3 : Plot of State variable trajectories

x_7, z_{L7}, z_7

CHAPTER 5

CONCLUSIONS

5.1 SUMMARY

The problem of power system stability and stabilization mentioned in the first chapter, is the unifying theme of this thesis. In order to study stabilization methods in as realistic a way as possible, nontrivial dynamic models must be available. This is the intent of Chapter 2 and 3. The second chapter contains the complete development of a single generator model beginning with the derivation of machine inductances and ending with the per unit torque and angular differential equations. Chapter 3 provides details about the interconnections and the relation between direct and quadrature axis voltages to currents. The full two generator interconnected power model with machine parameters assigned, is presented in this chapter and the preparatory work for a system simulation is completed. In Chapter 4 the nonlinear and linearized observers are designed, their performance compared by simulation on a 64 bit computer where the nonlinear observers is shown to be better than that of its linearized counterpart. Analytical proofs for the convergence of the nonlinear observers state to that of the observed system are provided and it is indicated that an unstable nonliner system with state feedback obtained via the nonlinear observer, can be stablized. The main aims and objectives of the thesis, as stated in the first Chapter, are therefore met.

It should be noted that, in the design procedure for the nonlinear observer, the nonlinear dynamical equations are linearized around some equilibrium point x_o. Although this method might be viewed as being overly restrictive, it is not however so since, any power system functioning under steady state conditions, operates at an equilibrium point. When a distrubance hits the system it pushes the state variables into a region around this point and it is the behaviour of these variables, perturbed from their quiescent state, which ultimately determines system stability.

5.2 AREAS FOR FURTHER RESEARCH

There are several areas into which the existing work can be extended. The first modification would be the simulation of the two machine system given different parameters for each generator. A further extension might be the design of the nonlinear observer and its inclusion in a real on-line power network composed of interconnected micro-alternators.

Other problems, of a theoretical nature which must be solved, are the selection of good eignvalues [22] and the provision of a reasonable gain matrix K for the desired pole locations.

When the loads change, the model cannot adequately handle these variations. It is therefore reasonable to use the developed differential system over a short time frame (on the order of several minutes) however, the model would be inaccurate if used for a longer period of time. Therefore adaptive identification of the nonlinear system with changing loads would seem to be a useful and interesting area in which to concentrate some research effort.

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