OPTIMAL VIBRATION ABSORBER

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ABSTRACT

OPTIMAL VIBRATION ABSORBER

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The investigation presented here concerns with optimization of vibrating systems using simple vibration absorbers. The criteria for design of an optimum absorber depend entirely on the requirements placed on the absorber function. Generally, an optimum absorber has to provide a reduction of amplitude of vibration of the main mass over a required frequency range. Sometimes, an optimum absorber, such as those found in the milling and cutting machines, is also used for increasing the chatter-free machining capability. Such optimum absorbers are discussed in this report. Also the case of a vibration absorber with both viscous and hysteretic damping attached to both damped and undamped main mass system is also investigated. The technique of optimization of a mechanical system with more than one vibration absorber which may be attached at a point remote from the point of action of the disturbing force is also investigated.

ACKNOWLEDGEMENTS

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NOMENCLATURE

A	Designation of coupled system
A^*	Designation of intermediate coupled system
B	Designation of original main system
C, D	Designation of additional system
C	Damping coefficient of main system
c	Damping coefficient of absorber system
c_c	Critical damping, equal $2m\omega_a$
c_{opt}	Optimum damping
F	Force
f	Ratio of the absorber natural frequency to the main mass natural frequency
f_{opt}	Optimum f
h, H	Constants
K	Spring stiffness
K^*	Complex spring stiffness
k	Absorber stiffness
k^*	Complex absorber spring stiffness
M	Main mass
m	Absorber mass
P_o	External force
q	Ratio of the exciting frequency to the main mass natural frequency
R_a	Receptance of system A
R_a^*	Receptance of system A^*

R_b	Receptance of system B
R_c	Receptance of system C
R_d	Receptance of system D
$x_{1,2}$	Displacement vectors
$X, X_{1,2}$	Displacements
X_{st}	Ratio of the exciting force amplitude to the spring constant of the main system
$\bar{x}_1$	Equal $X_1 e^{-i\theta_1}$
$\bar{x}_2$	Equal $X_2 e^{-i\theta_2}$
α	Absorber hysteretic damping ratio
β	Main mass hysteretic damping ratio
δ	Value equal to $C/2M\Omega$
$\theta_{1,2}$	Phase angles
ϕ	Frequency ratio, equal to ω/ω_a
ζ	Damping ratio, equal to $c/(2\sqrt{mk})$
ω	Forcing frequency
ω_a	Natural frequency of absorber
Ω	Natural frequency of main mass system
μ	Mass ratio m/M

CHAPTER 1

INTRODUCTION

All engineering materials, mechanical systems and structures are to some extent elastic and therefore prone to vibrations. Unless specifically needed, vibration of an engineering structure is often detrimental to its function and life. Under certain conditions, vibration can induce very high stress in materials, cause fatigue failure and temperature variations. Assemblies comprising moving parts are subject to wear because of one part impacting on another and due to the break-down of the oil film, both as a result of vibration. Chip-forming machines such as milling machines may not be directly affected by wear in the change wheels, but the finish of the machined part may be severly affected at certain operating speeds. The moving parts of all machines inherently produce vibration, and for this reason the mechanical designer must expect vibration to exist in the product he designs. Vibration may be either free or forced. A machine element is said to have free vibration if the periodic motion continues after the cause or the original disturbance is removed, but if the vibratory motion persists because of the existence of a disturbing force, then it is called forced vibration. Due to the damping, any free vibration of a mechanical system will eventually cease because of loss of energy. A heavily damped system is one in which the vibration decays rapidly. The merit in design lies in anticipating a vibration problem and in minimizing its undesirable effects during the life of the machine. Even after precautions have been taken, unexpected vibratory motions are often found after a machine has been designed and constructed. Amplitude of such vibration can be reduced only by the addition of an absorber. The dynamic vibration absorber has been found to be a very useful component for limiting vibrations with excessive amplitude. An absorber is inexpensive, effective, and has negligble effect on the other functions of the structure.

A vibration absorber, normally consists of a mass coupled to the vibrating structure by spring and damping elements. The addition of damping elements to the dynamic absorber makes the absorber to be more effective over a large range of forcing frequencies. The attachment may be remote from the action of disturbing force. There may be more than one absorber in a complex engineering structure. An optimum absorber is said to be tuned if the three parameters representing mass, spring stiffness and damping of the absorber have been properly chosen. The condition for optimization depends entirely on the imposed requirements. Generally. an optimum absorber is one which reduces the main system responses to the smallest possible value for the entire operating frequency range.

In this report, the optimization of a shock absorber with a single mass system in forced vibration under steady state condition(1)(2)(3)(4)* is discussed. A graphical method for use on idealized system with restricted damping developed by Stone and Simock (5) for optimal vibration absorber is discussed in the following chapter. In Chapter 3, a method developed by Stone and Andrew (6), which can predict the effect of the relative displacement experienced by the absorber when added at a point remote from the existing force for complex structure is also discussed. The experiments performed by Stone and Andrew (6) prove that the predicted results by the analytical method developed agree with the measured results. The major assumptions for the ideal system considered here are that the mass is guided to move only in the vertical direction, the spring and the dashpot are massless, the mass is absolutely rigid and all the damping is concentrated in the dashpot.

* Number in paranthesis refer to references at the end of report

CHAPTER 2

OPTIMAL VIBRATION ABSORBER

2.1 SYSTEM WITH UNRESTRICTED DAMPING

When an absorber with a viscous damper is added to a simple undamped main mass system under harmonic excitation, as represented by the symbolic diagram shown in Fig.1, the equation of motion for the main mass can be obtained as

$$M\ddot{x}_1 + Kx_1 + k(x_1 - x_2) + c(\dot{x}_1 - \dot{x}_2) = P_o(t) \quad \ldots\ldots(1)$$

where a dot representeds differentiation with respect to time. The equation of motion for the absorber mass is tHen

$$m\ddot{x}_2 + k(x_2 - x_1) + c(\dot{x}_2 - \dot{x}_1) = 0 \quad \ldots\ldots\ldots\ldots(2)$$

For steady state response, the displacements x_1 and x_2 can be represented in general as $x_1 = X_1 e^{(i\omega t - \theta_1)}$, $x_2 = X_2 e^{(i\omega t - \theta_2)}$, and $P_o(t) = P_o e^{i\omega t}$. The impressed force and the resulting displacement are having vectors of magnitude P_o and X, and the latter lagging the former by the angle θ.

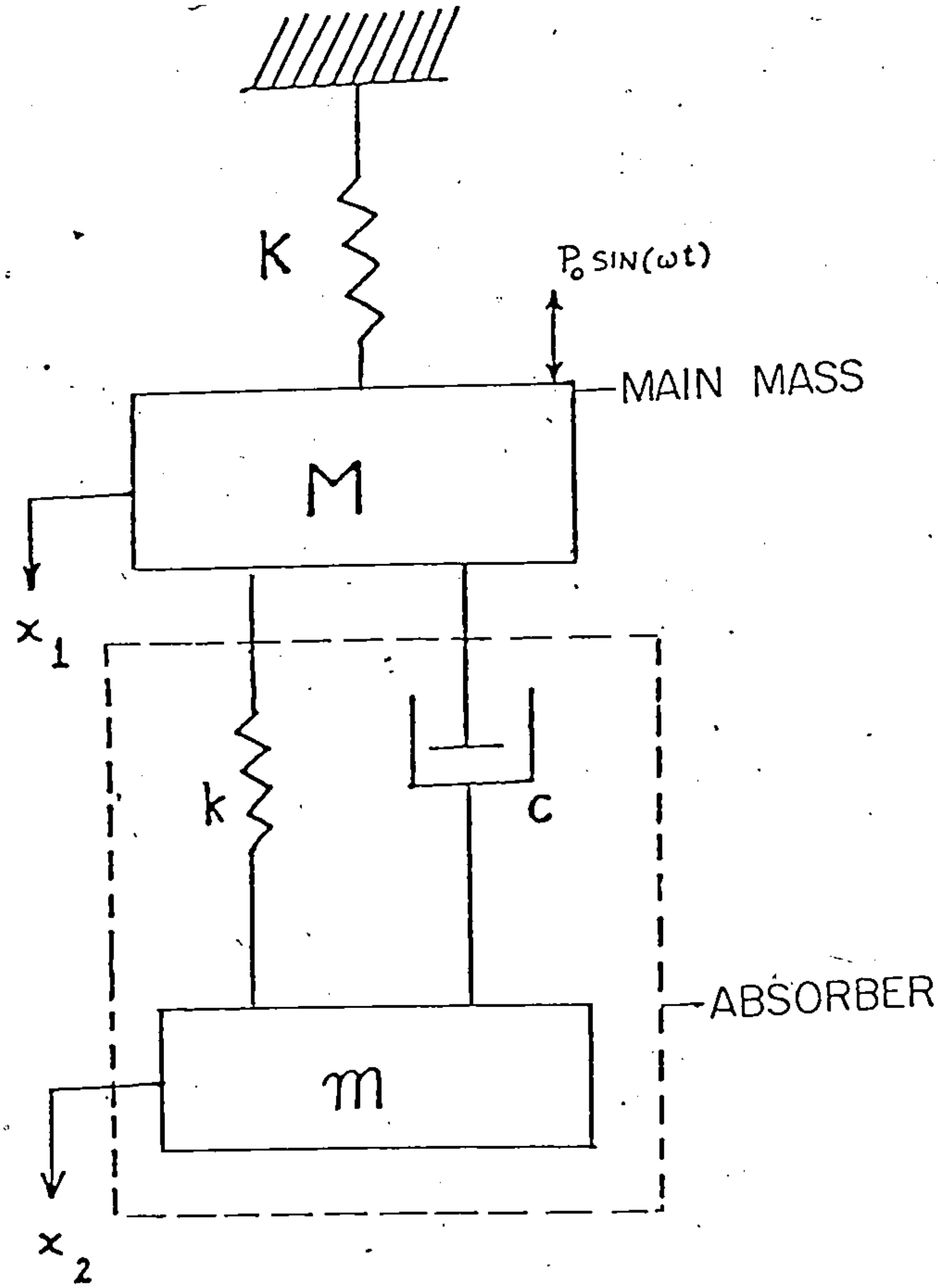


FIG. 1

By letting $\bar{x}_1 = X_1 e^{-i\theta_1}$, $\bar{x}_2 = X_2 e^{-i\theta_2}$, and rewriting equation (1) and (2) in complex form

$$-M\omega^2\bar{x}_1 + K\bar{x}_1 + k(\bar{x}_1 - \bar{x}_2) + i\omega c(\bar{x}_1 - \bar{x}_2) = P_o$$

$$-(k + i\omega c)\bar{x}_1 + (-m\omega^2 + k + i\omega c)\bar{x}_2 = 0$$

Solving these two simultaneous equations, we get

$$\bar{x}_1 = \frac{P_o\left[(k - m\omega^2) + i\omega c\right]}{\left[(-M\omega^2 + K)(-m\omega^2 + k) - m\omega^2 k\right] + i\omega c(-M\omega^2 + K - m\omega^2)}$$

and
$$\left|\frac{X_1}{P_o}\right| = \frac{(k - m\omega^2)^2 + \omega^2 c^2}{\left[(-M\omega^2 + K)(-m\omega^2 + k) - m\omega^2 k\right]^2 + \omega^2 c^2(-M\omega^2 + K - m\omega^2)^2}$$

Transmissibility is the ratio of transmitted force to the main mass and the impressed force. That is,

$$\text{Transmissibility} = \frac{KX_1}{P_o} = \frac{X_1}{X_{st}}$$

$$\text{or} \quad \frac{X_1}{X_{st}} = \frac{K\left[(k - m\omega^2) + i\omega c\right]}{(-M\omega^2+K)(-m\omega^2+k) - m\omega^2 k + i\omega c(-M\omega^2+K-m\omega^2)}$$

Transforming the above equation into a non-dimensional form by defining

$$f = \frac{\omega a}{\Omega} \quad ; \quad q = \frac{\omega}{\Omega} \quad ; \quad \mu = \frac{m}{M}$$

$$\frac{X_1}{X_{st}} = \pm\sqrt{\frac{(2\frac{c}{c_c}q)^2 + (q^2 - f^2)^2}{(2\frac{c}{c_c}q)^2(q^2 - 1 + \mu q^2)^2 + \left[\mu f^2 q^2 - (q^2 - 1)(q^2 - f^2)\right]^2}}$$

....(3)

A graph of the amplitude ratio ($\frac{X_1}{X_{st}}$) as a function of the frequency ratio is shown in Fig.2, and it can be seen that the peak amplitudes occur at infinity when c=0 and ∞. Somewhere in between there must be an optimum damping c_{opt} which brings the resonant peak of the amplitude down to its lowest possible value, and that is the object of adding an absorber to the system.

The work done by the damping force is given by the displacement through which it operates. In the present

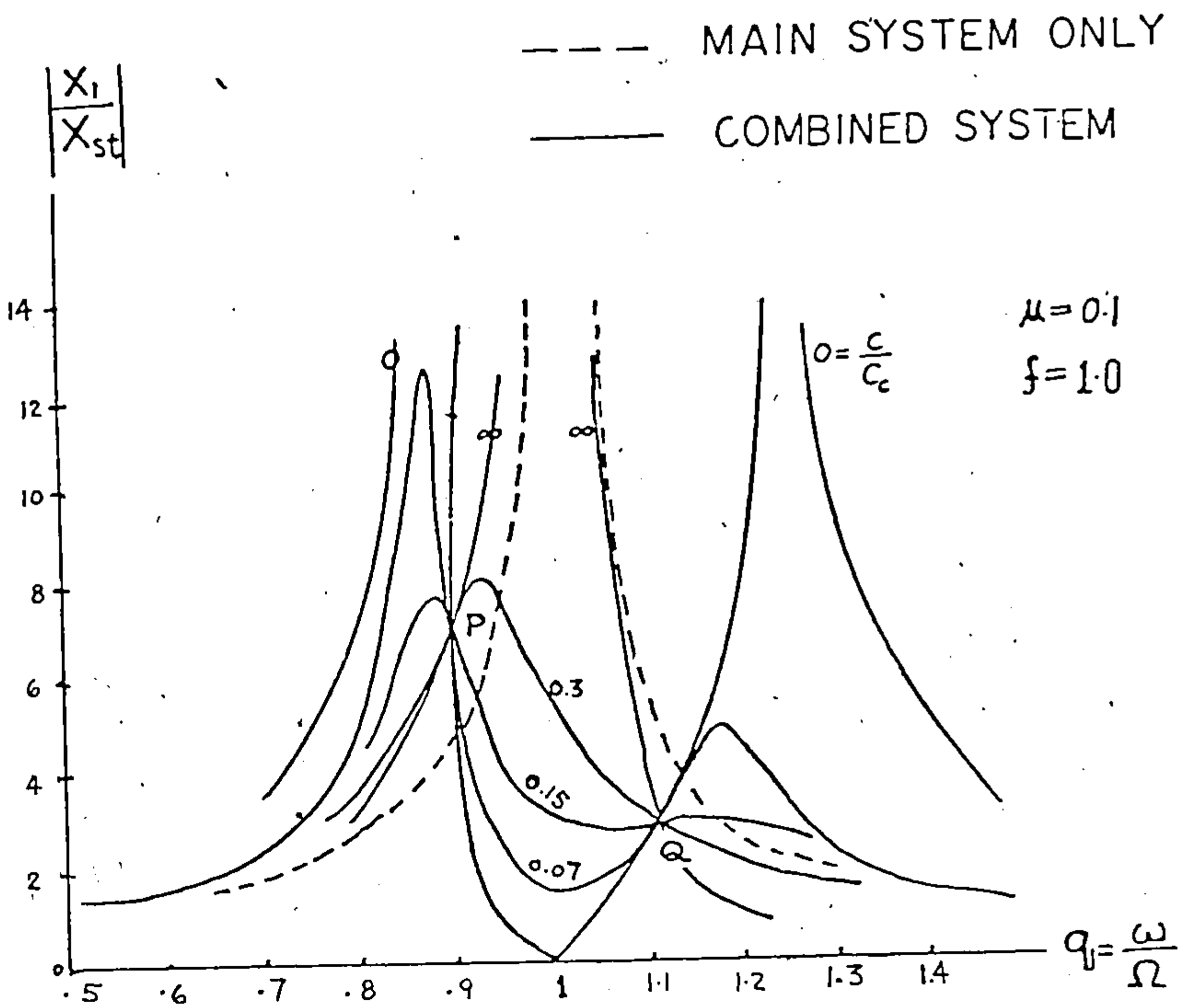


FIG.2 AMPLITUDE OF THE MAIN MASS FOR VARIOUS VALUES OF ABSORBER DAMPING

case, the displacement is the relative motion between the two masses. The reason for the peaks at infinity when c=0 and $c=\infty$, is that there is zero energy dissipation in the system. When c=0, the damping force is zero, no work is done. When $c=\infty$, the two masses are locked to each other so that their relative displacement is zero resulting in a single degree of freedom. only if the damping force does considerable work, much energy would be disspated, consequently, the amplitude will remain small at resonance.

It may be observed that all curves pass through the two points P and Q in Fig.2, and are independent of the damping present in the system. For optimum damping, the curve should have a horizontal tangent through the highest of the two fixed points P or Q. The best obtainable "resonance amplitude" will then be the ordinate of that point. The two fixed points can be shifted up and down the curve for c=0, by changing the relative "tuning" $f=\frac{\omega_a}{\Omega}$ of the damper with respect to the main system. By changing f, one point will go up and the other down. Clearly, the most favourable case will be

(i) through a proper choice of f the two points P and Q are adjusted to remain at equal heights, and

(ii) Through a proper choice of c/c_c the curve is adjusted to pass with a horizontal tangent through them.

The resonance curve for the main mass fitted with the most favourably tuned vibration-absorber system is shown in Fig.3.

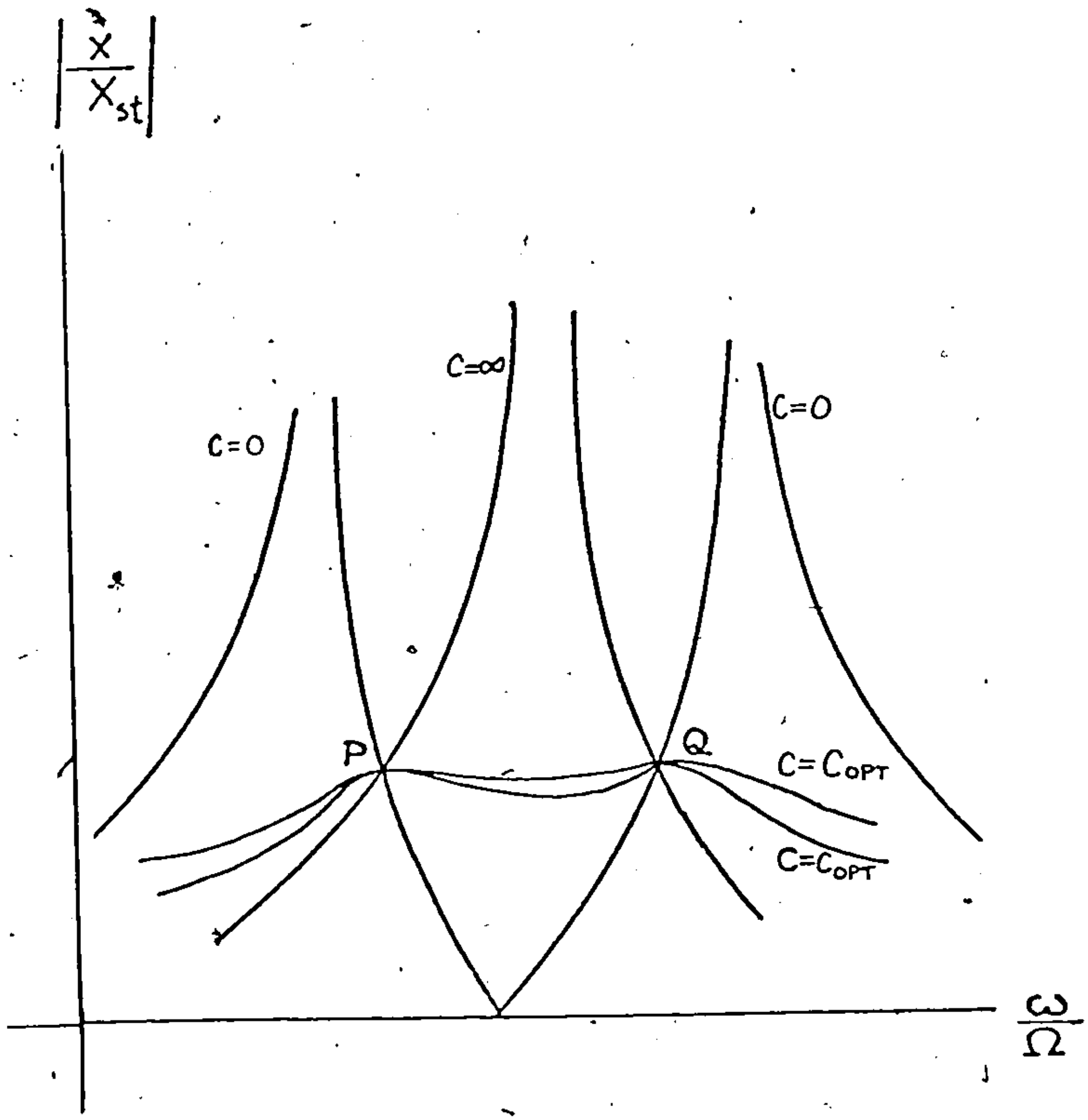


FIG. 3 SCHEMATIC DIAGRAM OF RESONANCE CURVE FOR THE MAIN MASS FITTED WITH THE MOST FAVOURABLY TUNED VIBRATION-ABSORBER SYSTEM

Now consider the expression under radical sign in the denominator of equation (3). This can be rewritten in the form

$$(q^2 - 1 + \mu q^2)^2 \left[(2 \frac{c}{c_c} q)^2 + A \right]$$

where

$$A = \frac{(\mu f^2 q^2 - (q^2 - 1)(q^2 - f^2))^2}{(q^2 - 1 + \mu q^2)^2}$$

Now, if A were equal to $(q^2 - f^2)^2$, and substituting this into equation (3), gives,

$$\frac{X_1}{X_{st}} = \pm \frac{1}{(q^2 - 1 + \mu q^2)}$$

and becomes independent of c/c_c . Equating the expression for A above equal to $(q^2 - f^2)^2$, in order to make the amplitude independent of damping,

$$\frac{\left[\mu f^2 q^2 - (q^2 - 1)(q^2 - f^2)\right]^2}{(q^2 - 1 + \mu q^2)^2} = (q^2 - f^2)^2$$

or

$$(2 + \mu) q^4 - \left[2(1 + \mu)f^2 + 2 \right] q^2 + 2f^2 = 0 \quad \ldots (4)$$

Rearranging, this becomes

$$q^4 - \left(\frac{2(1+\mu)f^2 + 2}{2+\mu} \right) q^2 + \frac{2f^2}{2+\mu} = 0 \quad \ldots.(4.a)$$

Since the negative value of q^2 is not acceptable, there exists two values of q for which the transmissibility is independent of c/c_c .

If we choose $c=\infty$ and substitute this value into equation (3), it becomes the case of ∞/∞ . By employing L'Hopital's Rule, setting limit $c \to \infty$, and differentiating the numerator and denominator of equation (3) with respect to c , the following espression can be obtained.

$$\frac{X_1}{X_{st}} = \pm \frac{1}{1 - q^2(1+\mu)}$$

Let q_1^2 and q_2^2 be the two solutions of equation (4), we can write

$$\frac{X_1}{X_{st}} = \frac{1}{1 - q_1^2(1+\mu)} = \frac{-1}{1 - q_2^2(1+\mu)}$$

Hence, $$q_1^2 + q_2^2 = \frac{2}{1+\mu} \quad \ldots\ldots(5)$$

Writing equation (4) in the form $(q^2 - q_1^2)(q^2 - q_2^2) = 0$, and expanding $q^4 - (q_1^2 + q_2^2)q^2 + q_1^2 q_2^2 = 0$. Comparing coefficients with equation (4.a),

$$q_1^2 + q_2^2 = \frac{2(1 + f^2 + \mu f^2)}{2 + \mu}$$

Combining this with equation (5) results in the relationship

$$f = \frac{1}{1 + \mu} \qquad \ldots\ldots\ldots\ldots \quad (6)$$

This is the optimal or most favourable tuning for each absorber size. Since $f = \omega_a/\Omega$ or $f = \frac{1}{\Omega}\sqrt{\frac{k}{m}}$, the correct tuning of the system can be obtained by adjusting the spring stiffness of the absorber. When the optimum tuning is determined, the optimum damping can be found. For optimum damping, the curve passes horizontally through either P or Q. In other words, the slope of the curve at P or Q is equal to zero, i.e.

$$\frac{d}{dq}\left(\frac{X_1}{X_{st}} \right) = 0$$

Substituting equation (6) into equation (3) and differentiating with respect to q , thus finding the slope, and equate that slope to zero for the point P, the value of c/c_c can be calculated.

$$\left(\frac{c}{c_c}\right)^2 = \frac{\mu\left[3 - \sqrt{\mu/(\mu + 2)}\,\right]}{8(1 + \mu)^3}$$

On the other hand, if the slope is set equal to zero at point Q, we get

$$\left(\frac{c}{c_c}\right)^2 = \frac{\mu\left[3 + \sqrt{\mu/(\mu + 2)}\,\right]}{8(1 + \mu)^3}$$

The average value between the two gives the optimum damping for the most favourable tuning to achieve the optimum amplitude of vibration for the main system.

2.1.1 SYSTEM UNDER RANDOM EXCITATION

For system under random excitation, the optimal absorbers are designed so as to minimize the mean square response of the main mass. The governing equations of motion for the conventional damped dynamic absorber system shown in Fig.4 is as follows:

$$M\ddot{x}_1 + C\dot{x}_1 + Kx_1 + c(\dot{x}_1 - \dot{x}_2) = F(t)$$

$$m\ddot{x}_2 + c(\dot{x}_2 - \dot{x}_1) + k(x_2 - x_1) = 0$$

By taking the Laplace transformation of the above equations and solving simultaneously will yield the transfer function G(s) relating the s-domain response x(s) of the main mass to the applied force F(s).

The transfer function $G(s) = \frac{x(s)}{F(s)}$

If $\xi = \frac{c}{\sqrt{2mk}}$, $f = \frac{\omega a}{\Omega}$, $\delta = \frac{C}{2M\Omega}$,

then

$$G(s) = \frac{1}{M}\left\{ \frac{S^2 + 2\xi\Omega f S + f^2\Omega^2}{S^4 + 2\Omega[\delta + f\xi(1+\mu)]S^3 + (f^2\Omega^2 + 4\xi\delta f\Omega^2 + \Omega^2 + f^2\Omega^2\mu)S^2 + (2\delta f^2\Omega^2 + 2\xi f\Omega^3)S + f^2\Omega^4} \right\}$$

....[see ref.(7)(8)]

For a stationary input, the power spectral density of the response is related to that of the forcing function by

$$S_x(\omega) = \left|G(s)\right|^2_{s=j\omega} S_f(\omega)$$

If the forcing function has a zero mean then so will the response, and thus the mean square response may be obtained by integrating the power spectral density function

$$\sigma_x^2 = \frac{1}{2\pi} \int_{-\infty}^{\infty} |G(j\omega)|^2 \, S_f(\omega) \, d\omega$$

If the power spectral density of the forcing function, F(t), is idealized by a white noise, that is

$$S_f(\omega) = S_o$$

then the mean square response is

$$\sigma_x^2 = \frac{S_o}{2\pi} \int_{-\infty}^{x} | G(j\omega)|^2 \, d\omega$$

$$\sigma_x^2 = \frac{S_o}{2\pi j} \int_{-j\infty}^{j\infty} G(j\omega)G(-j\omega) \, dj\omega$$

$$= \frac{S_o}{M^2\Omega^3 4} \left(\frac{B_1\xi^3 + B_2\xi^2 + B_3\xi + B_4}{A_1\xi^3 + A_2\xi^2 + A_3\xi + A_4} \right) \ldots (7) \quad \text{see ref.(7)}$$

where

$A_1 = 4f^2\delta(1 + \mu)$

$A_2 = f\mu + 4f\delta^2 + 4f^3\delta^2(1 + \mu)$

$A_3 = \delta + \delta f^4(1 + \mu)^2 + 4\delta^3 f^2 - 2f^2\delta$

$A_4 = f^3\delta^2\mu$

$B_1 = 4f^2(1 + \mu)$

$B_2 = 4(f\delta + f^3\delta(1 + \mu))$

$B_3 = 1 - f^2(2+\mu) + f^4(1+\mu)^2 + 4\delta^2 f^2$

$B_4 = f^3\delta\mu$

To minimize the mean square motion of the main mass, equation (7) is differentiated with respect to ξ, and equated to zero. The numerator of the resulting fraction must vanish and thereby resulting in the following polynomial to be solved for the optimal values of ξ

$$\xi^4 + C_1\xi^3 + C_2\xi^2 + C_3\xi + C_4 = 0 \qquad \text{...see ref.(7)}$$

where

$$C_1 = (\delta/f\mu)\left[1 + f^4(1+\mu)^2 + 2\delta^2 f^2 - 2f^2 + f^2(2+\mu)\right]$$

$$C_2 = f^2\delta^2 + \left\{f^2\mu(2+\mu) + 4\delta(1-\delta) - \mu\left[1 + f^4(1+\mu)^2\right]\right\}$$

$$C_3 = -f\mu\delta/2(1+\mu)$$

$$C_4 = -\delta^2 f^2\mu/4(1+\mu)$$

The optimal absorber damping ratio as a function of the main mass ratio, for several values of the tuning ratio f, and zero damping on the main mass are shown in Fig.5. The minimal mean square motions obtained for the optimal absorbers are given in Fig.6 for the same case examined in Fig.5. As can be seen from graphs in Fig.5 and 6, the higher the ratio of absorber frequency to natural frequency of main mass system, higher the optimum absorber ξ_{opt}, and higher the variance of motion of M.

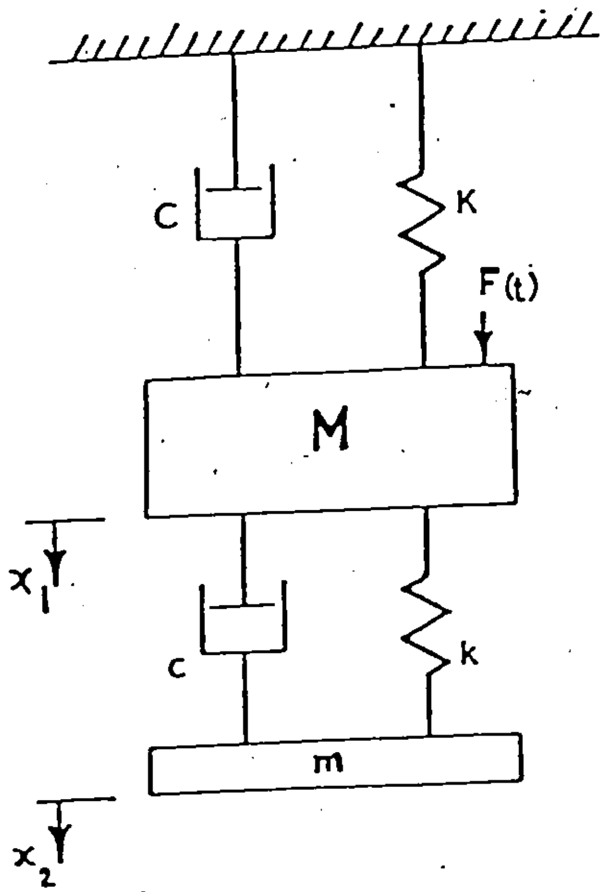


FIG. 4 DAMPED TWO-DEGREE-OF-FREEDOM WITH DYNAMIC VIBRATION ABSORBER

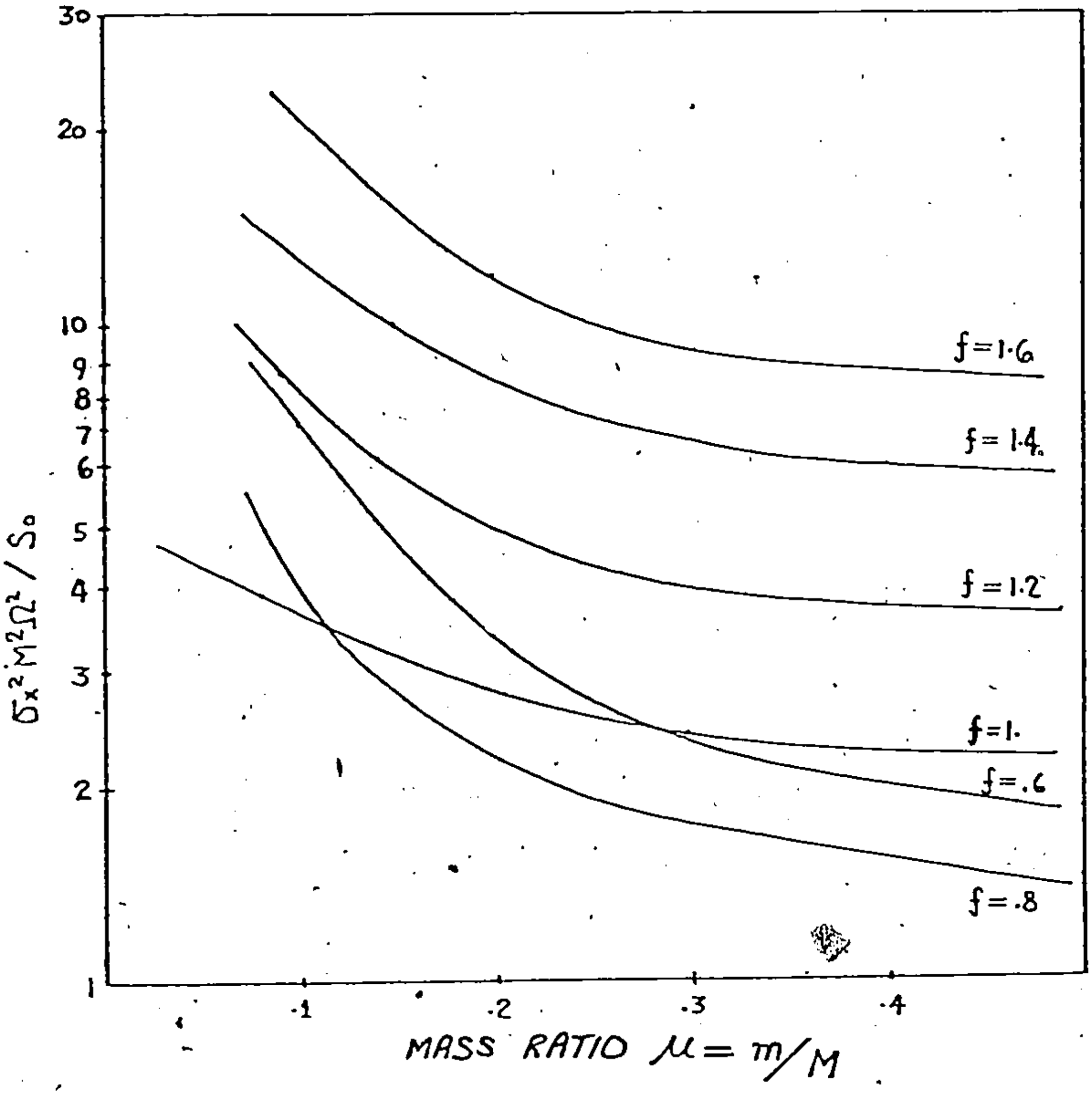


FIG. 6 MINIMAL VALUE OF THE VARIANCE OF MOTION OF M FOR $\delta = 0$

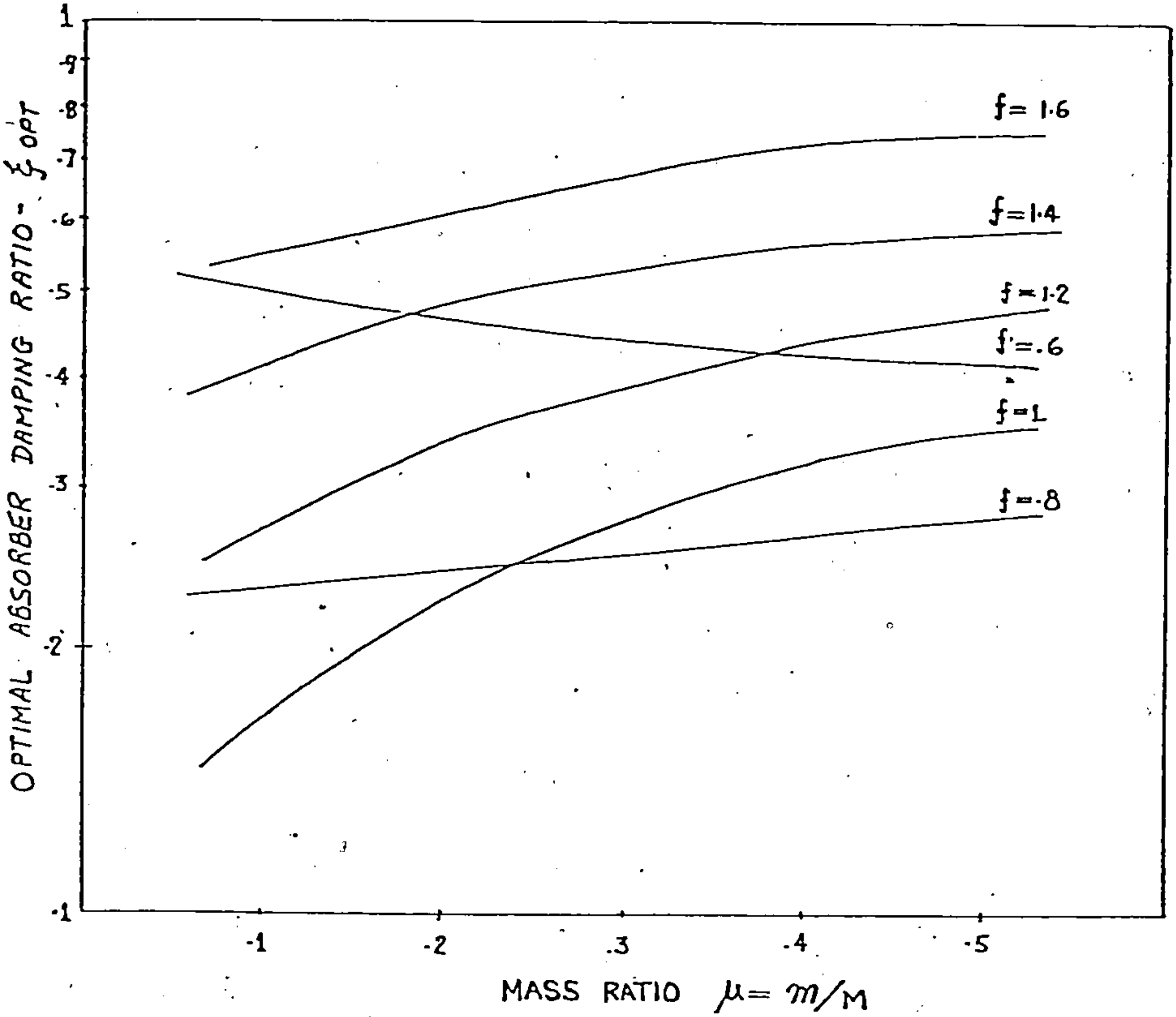


FIG.5 OPTIMAL ABSORBER DAMPING RATIO, ξ AS A FUNCTION OF MASS RATIO FOR VARIOUS TUNING RATIOS f. THE MAIN DAMPING RATIO $\delta=0$

2.2 SYSTEM WITH RESTRICTED DAMPING

Sometimes, the required optimum damping cannot be achieved in practice, and the effect of damping in the main system cannot be neglected. Usually, damping in the absorber system is restricted and we can only vary the spring stiffness to obtain the optimum absorber.

Damping forces in a system can either be proportional to the instantaneous velocity such as those existing between a shaft and bearing with oil in between, or that the damping forces are harmonic and in quadrature with the displacement, usually found in springs due to imperfection, with the magnitude being proportional to the displacement. The first is known as the viscous damping while the second is known as the hysteretic damping.

2.2.1 UNDAMPED MAIN MASS SYSTEM : ABSORBER WITH VISCOUS DAMPING

An elastic system with an undamped main mass and an absorber is shown in Fig.7. It is assumed that all the elements of the system are linear and the response to a harmonic force $Fe^{i\omega t}$ will be a harmonic displacement $Xe^{i\omega t}$. This system can be considered as consisting of two components, the main system and the absorber. If F_1/X_1 and F_2/X_2 are the dynamic stiffness of the main system and absorber respectively when they are uncoupled, then the dynamic stiffness, F/X of the coupled system at the point of coupling is the vectorial sum F_1/X_1 and F_2/X_2.

$$\frac{F_1}{X_1} + \frac{F_2}{X_2} = \frac{F}{X} \qquad \text{........(8)}$$

The detailed procedure for calculating the dynamic stiffness of the main system and the absorber and the coupled system are outlined in Appendix.

Equation (8) holds for the general case when the stiffnesses $K^*(\omega)$ and $k^*(\omega)$ are themselves complex and frequency dependent. In this system k^* has been reduced to a simple spring of stiffness k in parallel with a viscous damper having a damping coefficient c. Then $K^*(\omega) = K$ and $k^*(\omega) = k + ic\omega$.

For the uncoupled system, the response are

$$\frac{F_1}{X_1} = K - M\omega^2 \qquad \text{..................(9)}$$

and

$$\frac{F_2}{X_2} = \frac{(\ k + ic\omega\ )\ m\omega^2}{m\omega^2 - (\ k + ic\omega\ )} \qquad \text{...........(10)}$$

These equations are derived in the Appendix.

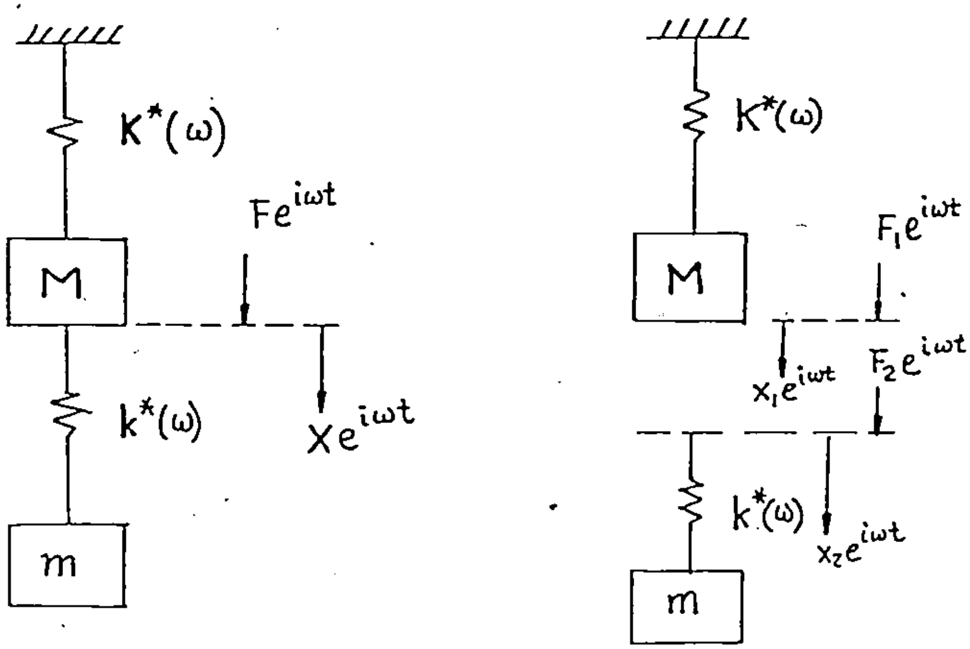


FIG. 7 MAIN SYSTEM WITH ABSORBER

Combining equations (9) and (10)

$$\frac{F}{X} = \underbrace{\frac{(k + ic\omega)(m\omega^2)}{m\omega^2 - (k + ic\omega)}}_{\text{ABSORBER}} + \underbrace{K - M\omega^2}_{\text{MAIN SYSTEM}} \quad \ldots\ldots\ldots(11)$$

Defining $\xi = \frac{c}{2\sqrt{mk}}$, $\phi = \frac{\omega}{\omega_a}$, $f = \frac{\omega_a}{\Omega}$

and dividing equation (11) throughout by k, equation (11) takes the non-dimensional form

$$\frac{F}{kX} = \frac{1}{\mu f^2} - \frac{\phi^2}{\mu} + \frac{(1 + 2\xi i\phi)\phi^2}{\phi^2 - (1 + 2i\xi\phi)} \quad \ldots\ldots(12)$$

This equation describes the coupled system dynamic stiffness as a function of the exciting frequency, and it can be represented by a vector locus plotted as a function of ϕ. In the particular case, when μ=0.2 and ξ=0.1, the locus of

$$\frac{-\phi^2}{\mu} + \frac{(1 + 2i\xi\phi)\,\phi^2}{\phi^2 - (1 + 2i\xi\phi)}$$

is shown in Fig.8. The remaining term $1/\mu f^2$ is independent of ϕ and is always a real quantity. Thus the addition of this term to the curve moves the locus bodily towards the positive real direction by an amount depending on the values of μ and f. The value of μ is a known quantity and f is going to be optimized. In achieving optimum absorber tuning, the magnitude of $|F/kX|$ must be made as large as possible, since the smallest value of it corresponds to a resonance of the non-dimensional system response. When a certain value F on the real axis is chosen so that the origin is moved a distance OF, two minimum values of $|F/kX|$ can be found to be given by FG and FH as in Fig.8. When F is moved to increase FG, FH will be decreased. It thus follows that in the optimum case, FG and FH must be equal and therefore that the point F is at A, the centre of the inscribed circle of curve with centre on the real axis. Optimization is therefore effected by putting $AO=1/\mu f^2$, and the value of f thus obtained is the optimum value, f_{opt}. Since the radius r represents a particular value $|F/kX|$, and $AO=1/\mu f^2=K/k$ the corresponding value of $|F/x|$ is given by $kr=(r/AO)K$. The amplitude and phase of the dynamic stiffness vectors are measured from the centre of the optimization circle, and the amplitude multiplied by the dimensional factor $(AO)K$. Since $f=\frac{\omega a}{\Omega}$, $\omega_a=\sqrt{k/m}$, $\mu=m/M$ where μ, M, and Ω are defined quantities, the

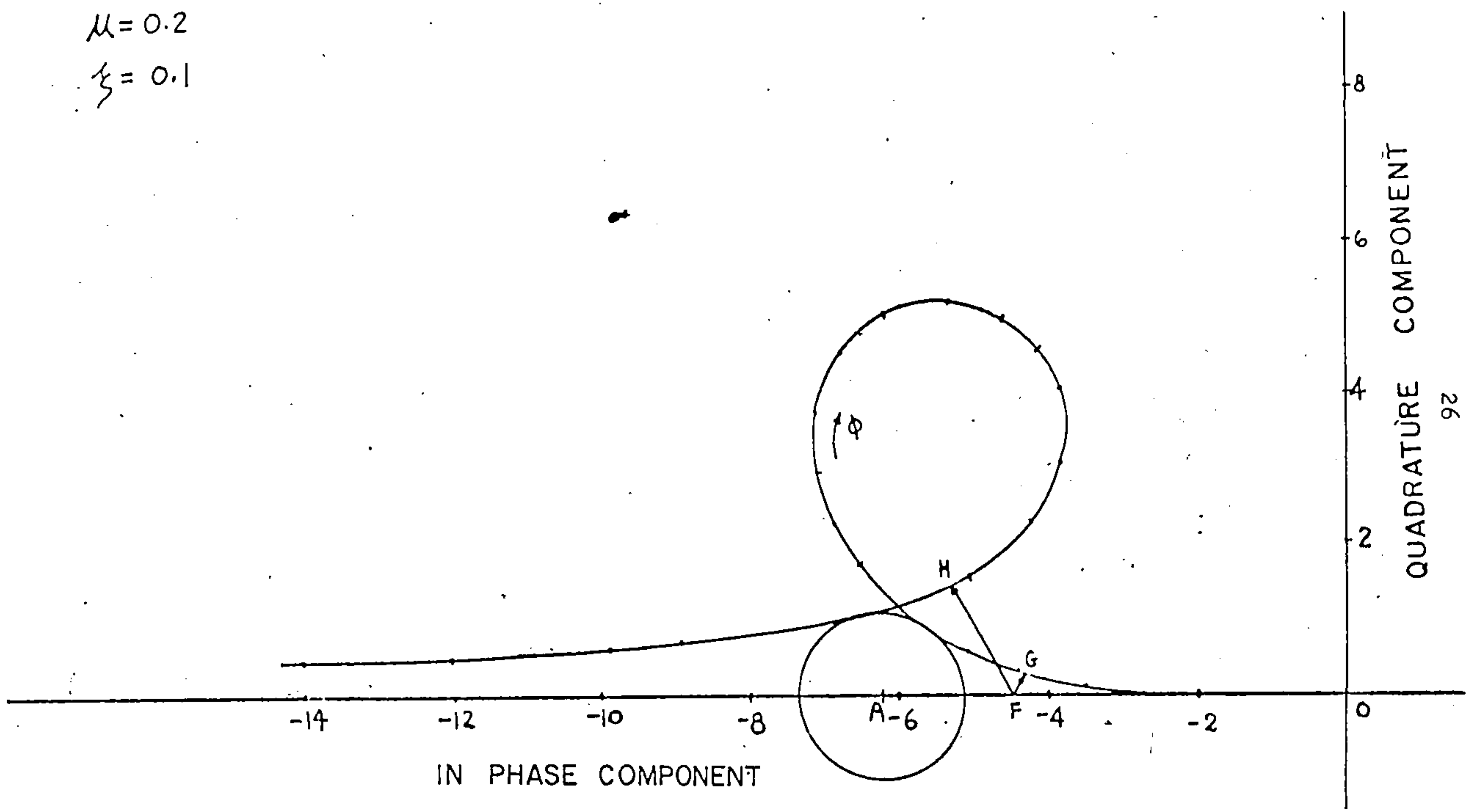


FIG.8 NON-DIMENSIONAL DYNAMIC STIFFNESS LOCI AND OPTIMIZATION PROCEDURE : ABSORBER WITH - VISCOUS DAMPING

value f_{opt} can be obtained by adjusting the spring stiffness k of the absorber.

2.2.2 UNDAMPED MAIN SYSTEM : ABSORBER WITH HYSTERETIC DAMPING

An ideal spring does not exist in reality. A hysteretic damping is often associated with a spring. Hysteretic damping involves a loss of vibration energy per cycle which is proportional to the amplitude of vibration but independent of frequency. According to Bishop and Johnson (1), hysteretically damped spring can be treated as having a complex stiffness $k^*=k+ih$. Then for the uncoupled absorber system (see Appendix for details)

$$\frac{F_2}{X_2} = \frac{(k + ih) m\omega^2}{m\omega^2 - (k + ih)} \qquad \ldots\ldots\ldots(13)$$

and for the complete system, from equation (8), (9) and (13) the dynamic stiffness

$$\frac{F}{kX} = \frac{1}{\mu f^2} - \frac{\phi^2}{\mu} + \frac{(1 + i\alpha)\phi^2}{\phi^2 - (1 + i\alpha)} \qquad \ldots.(14)$$

where $\alpha = \frac{h}{k}$, known as the absorber hysteretic damping ratio. The optimization procedure and the conditions that this must satisfy to obtain an optimal response F/X, are the same as that for a viscously damped absorber discussed in section 2.2.1. For a particular case, when $\mu=0.2$, $\alpha=0.2$, the vector locus plot for equation (14) as a function of ϕ is shown in Fig.9.

2.2.3 MAIN SYSTEM WITH HYSTERETIC DAMPING : ABSORBER WITH VISCOUS DAMPING

The stiffness $K^*(\omega)$ is now reduced to a simple spring in parallel with a hysteretic damper such that $K^*(\omega) = K + iH$. Then for the uncoupled main system, the dynamic stiffness becomes

$$\frac{F_1}{X_1} = K + iH - M\omega^2$$

(see Appendix for details). Considering the main system coupled to absorber with viscous damping, the coupled dynamic stiffness in the dimensionless form can be given as

$$\frac{F}{kX} = \frac{1 + i\beta}{\mu_f^2} - \frac{\phi^2}{\mu} + \frac{(1 + 2i\zeta\phi) \phi^2}{\phi^2 - (1 + 2i\zeta\phi)} \quad ....(15)$$

where $\beta = \frac{H}{k}$

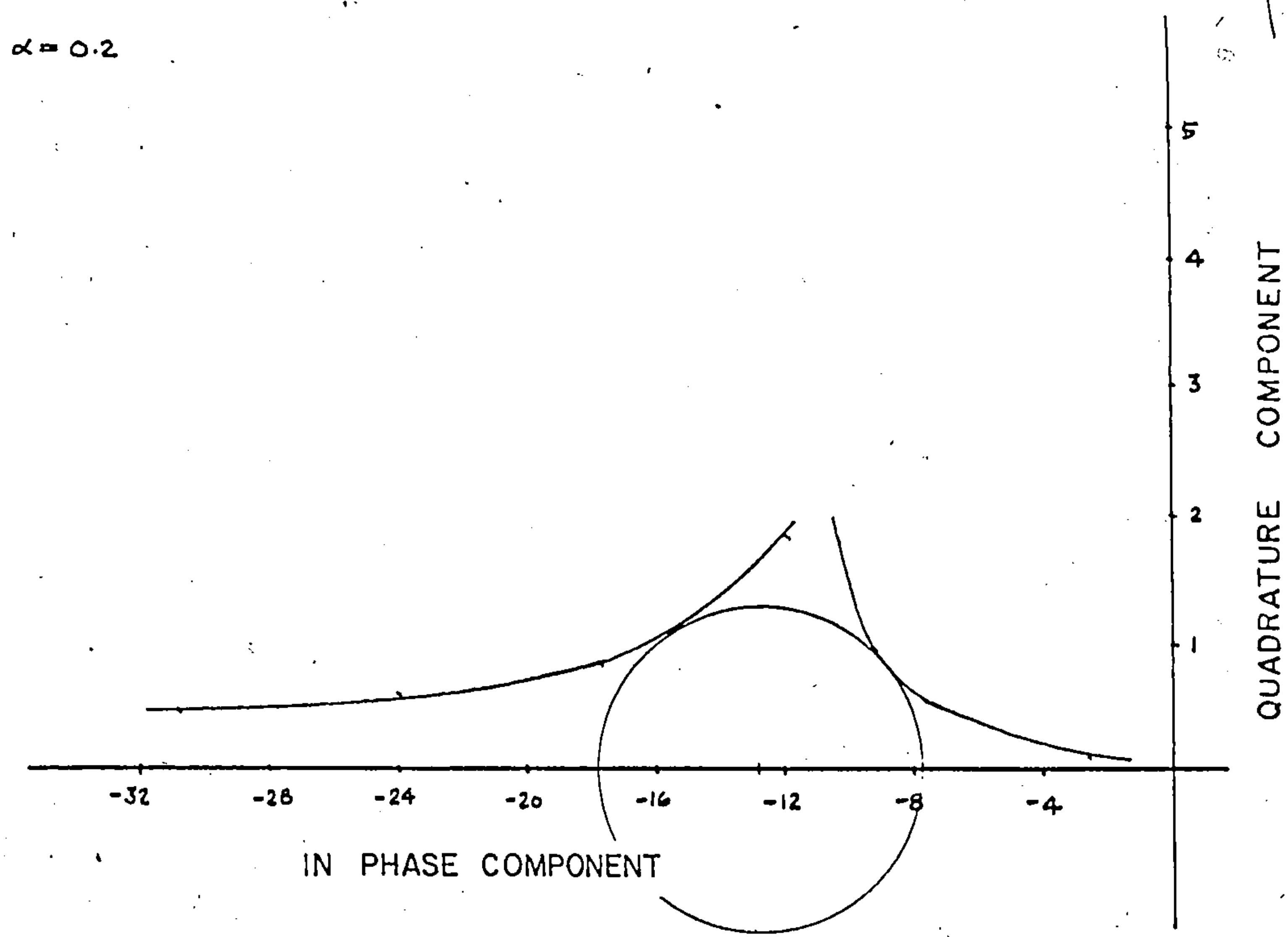


FIG.9 UNDAMPED MAIN SYSTEM ABSORBER WITH HYSTERETIC DAMPING

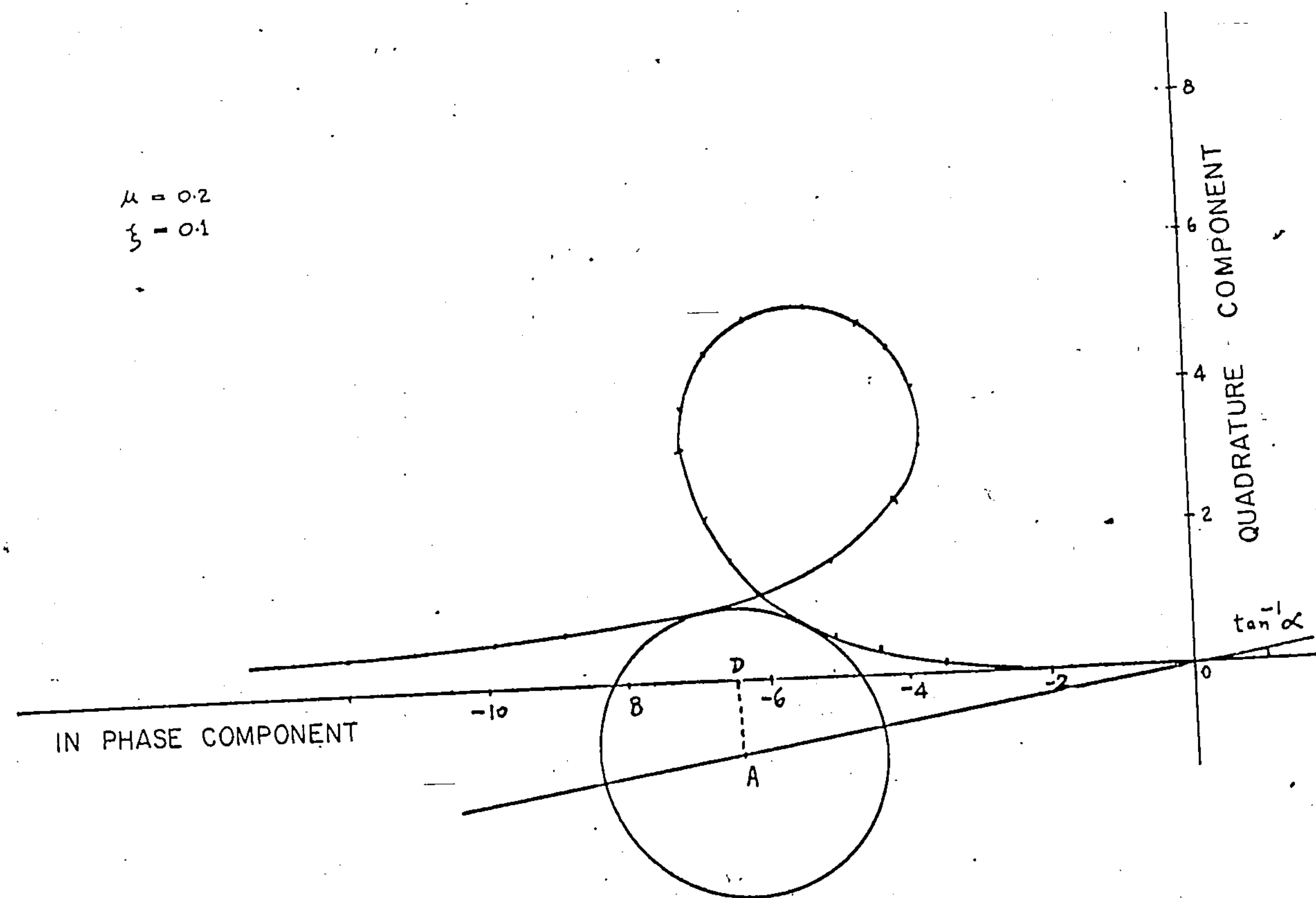


FIG.10. OPTIMIZATION PROCEDURE : MAIN SYSTEM WITH HYSTERETIC DAMPING

The second and the third terms in this expression for F/kX are the same as that for the case considered earlier with the main system undamped and with viscous damping in the absorber. The first term of equation (15) contains not only real quantity but it also contains a quadrature component. If the origin is moved a distance $1/\mu f^2$ in the positive real direction, it has to be moved a distance $\alpha/\mu f^2$ in the positive imaginary direction too. Hence the transformed origin must lie on a line which cuts the origin and making an angle α with the horizontal axis. This system can be optimized by using the same method as the one used in section 2.2.2. When the centre of the optimized circle has been found on the transformed axis as shown in Fig.10, the real component of OA is OD and is equal to $1/\mu f^2$. From this, the optimized spring stiffness can be calculated and hence the optimal vibration absorber can be obtained. For particular case, when μ=0.2 and ζ=0.1, the vector locus plot for equation (15) as a function of ϕ is shown in Fig.10.

2.2.4 MAIN SYSTEM WITH HYSTERETIC DAMPING & ABSORBER WITH VISCOUS AND HYSTERETIC DAMPING

A schematic diagram of this type of system is shown in Fig.11. In this case the expression for the dynamic stiffness of the uncoupled system can be obtained from the F_1/X_1 and F_2/X_2 ratios derived in section 2.2.1 and 2.2.2.

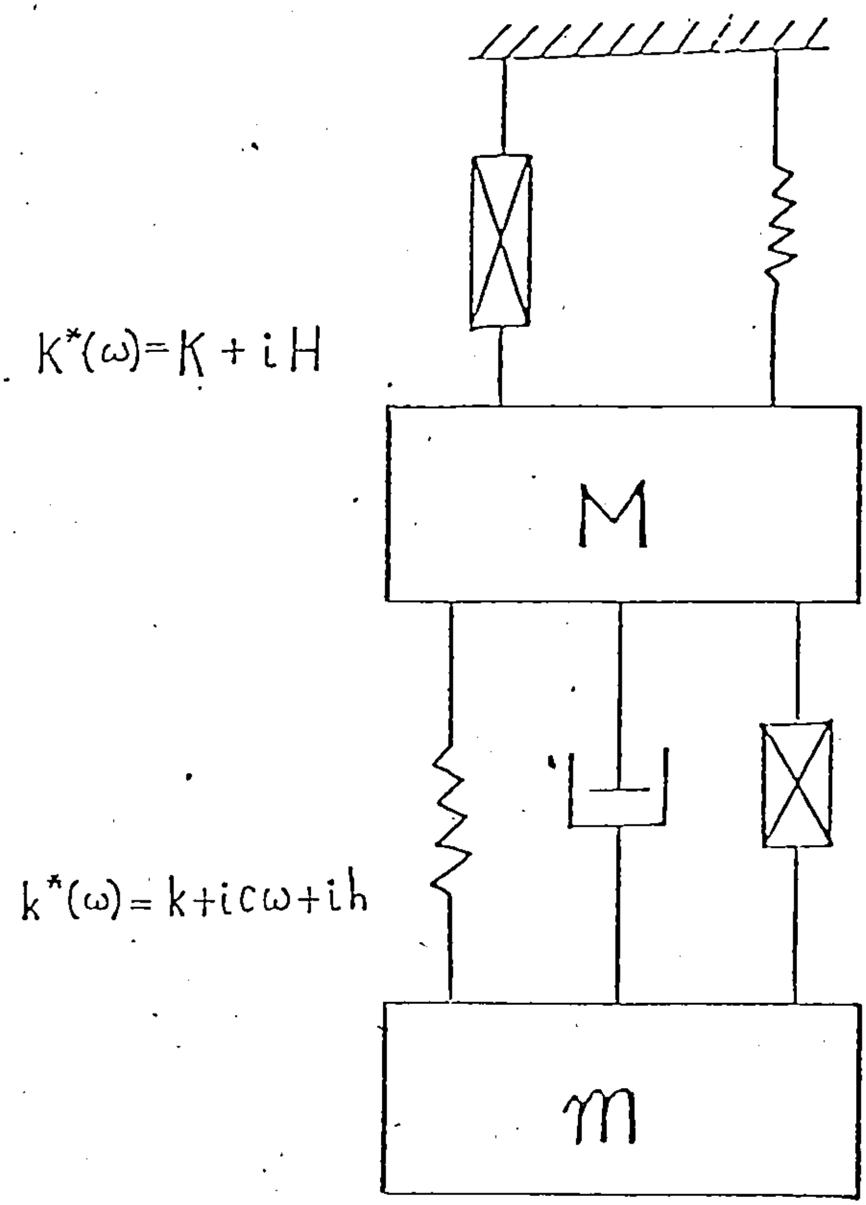


FIG.11 MAIN SYSTEM WITH HYSTERETIC DAMPING ABSORBER WITH VISCOUS AND HYSTERETIC DAMPING

In the previous case for a main system with hysteretic damping uncoupled, the dynamic stiffness is

$$\frac{F_1}{X_1} = K + iH - M\omega^2$$

and for the absorber system uncoupled, the dynamic stiffness is

$$\frac{F_2}{X_2} = \frac{k^*(\omega)\ m\omega^2}{m\omega^2 - k^*(\omega)}$$

In this case $k^*(\omega) = k + ih + ic\omega$. The dynamic stiffness of the coupled system becomes

$$\frac{F}{X} = (\ K + iH - M\omega^2\ ) - M\omega^2 + \frac{m\omega^2\ (\ k + ih + ic\omega\ )}{m\omega^2 - (\ k + ih + ic\omega\ )}$$

Dividing the entire equation by k, we obtain the non-dimensional form

$$\frac{F}{kX} = \underbrace{\frac{1 + i\beta}{\mu f^2} - \frac{\phi^2}{\mu}}_{\text{MAIN SYSTEM}} - \underbrace{\frac{(\ 1 + 2\xi\phi i + i\alpha\ )\ \phi^2}{\phi^2 - (\ 1 + i\alpha + 2\xi\phi i\ )}}_{\text{ABSORBER SYSTEM}}$$

Transform the second and the third terms into a complex (A + iB) form, it becomes

$$\left[\frac{\phi^2}{\mu} - \frac{\phi^2(\phi^2 - 1) - (2\xi\phi + \alpha)\phi^2}{(\phi^2 - 1)^2 + (\alpha + 2\xi\phi)^2}\right] + \left[\frac{\phi^4(\alpha + 2\xi\phi)\, i}{(\phi^2 - 1)^2 + (\alpha + 2\xi\phi)^2}\right]$$

The graph of in-phase component plotted against quadrature component from this equation is similar to the previous cases. We can optimize the absorber by employing the same method that has been used in **section 2.2.1.** The centre of optimization circle lies on the transformed axis and the projection of it on the in phase component axis is equal to $1/(\mu f^2)$. The value f which is the optimum spring stiffness of the absorber can be calculated. The radius of the circle represents a particular value of $F/(kX)$, where X is the maximum amplitude of vibration for the system.

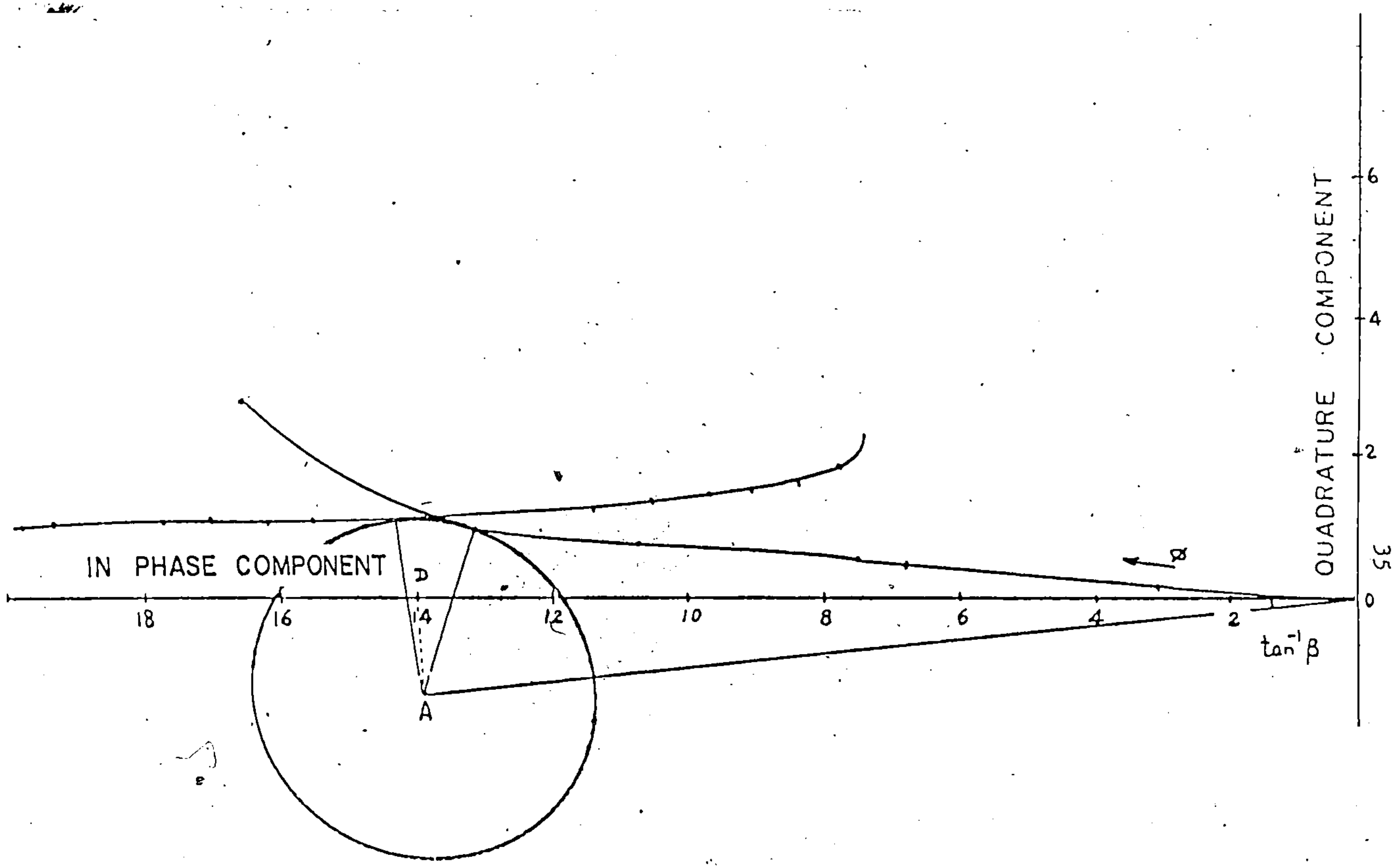


FIG. 11B MAIN SYSTEM WITH HYSTERETIC DAMPING
ABSORBER WITH VISCOUS AND
HYSTERETIC DAMPING

CHAPTER 3

VIBRATION ABSORBER FOR COMPLEX STRUCTURES

The techniques that are discussed for the optimization of vibration absorbers are limited to situations where the disturbing force act on the main system at the same point and in the same direction as the relevant resulting displacement and the attachment of absorber. However, in practice the absorber can rarely be attached at the point of action of the disturbing force and the relevant displacement of the main system is not necessarily at the same position or in the same direction as this force. In particular if the absorber is to be applied to a machine tool, certain other requirements are to be imposed, namely, the force is an internal force between the two points on the main system (e.g. the cutting edge and work piece) and the relevant displacement of the main system is the relative displacement between the same two points but generally in a different direction from the force. Further in machining operations, the increase of the chatter-free machining capacity is often the criterion for optimization; in this case the minimization of the amplitude of the response is not necessarily the condition for the absorber to have been optimized. In this chapter a method which can predict the effect of relative displacement of a vibration absorber at a point remote from an internal force, due to the addition of a vibration absorber at a point away from the point of action of the force is discussed . The method outlined in chapter 2 for the optimization of the absorber can also be extended to two-dimensional absorbers, i.e., two degree of freedom absorber.

Dynamic receptance R_{b12} is defined as X_1/F_2, where X_1 is the displacement vector of the harmonic displacement $X_1e^{i\omega t}$, resulting from the harmonic force $F_2e^{i\omega t}$. When a force applied to the main system B in the direction and at the position denoted by 2 as shown in diagram 12(a), the relevant displacement is in the direction and at the position denoted by 1. The additional system C is added to the main system B in the direction and at the position denoted by 3. Each of this subscripts may refer to either absolute or relative quantities. For example, if 3 denotes an absolute displacement then system C may be an absorber, or if 3 denotes a relative displacement then system C may be insert of some form, such that equal and opposite forces are applied to the main system. The subscript 1 denotes a given direction and position of displacement and may refer to either an absolute or a relative displacement. The subscript 2 denotes a similar direction and position for the force, which may also be absolute or relative.

The forces and displacements relationship shown in Fig.12 are

$$X_1 = R_{b12} F_2 + R_{b13} F_{b3} \quad \text{............(16)}$$

$$X_{b3} = R_{b32} F_2 + R_{b33} F_{b3} \quad \text{............(17)}$$

The motions resulting from F_2 and F_3 may be superimposed, since the system is assumed to be linear.

From the sub-system C

$$X_{c3} = R_{c33} F_{c3} \quad \text{.............(18)}$$

There is no external force at 3, so

$$F_{b3} + F_{c3} = 0 \quad \text{..............(19)}$$

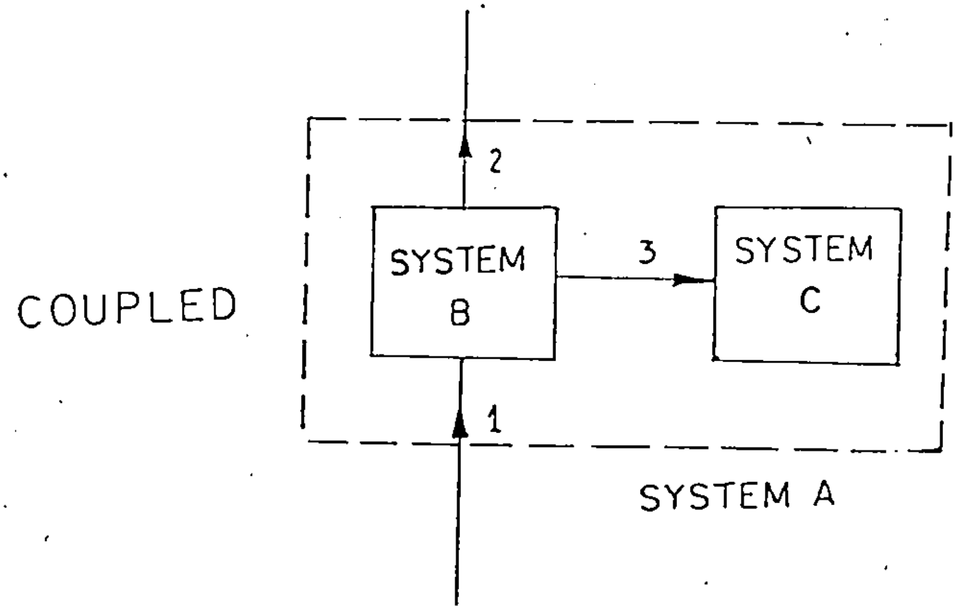
and for compatibility

$$X_{b3} = X_{c3} \quad \text{.............(20)}$$

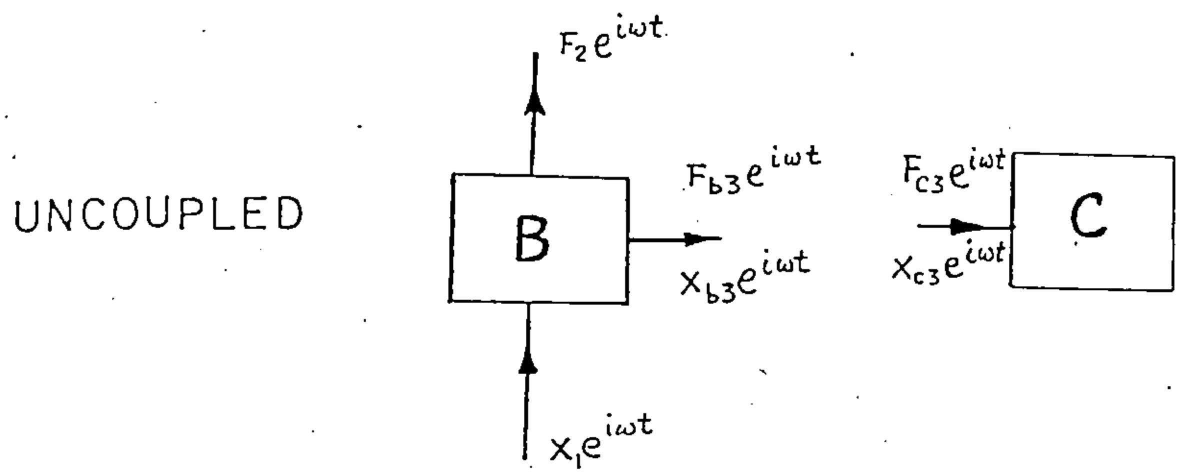
Substitute for equation (18), (19) and (20) in equation (16) and (17) gives

$$R_{a12} = R_{b12} - \frac{R_{b13} R_{b32}}{R_{b33} + R_{c33}} \quad \text{......(21)}$$

If more systems are to be added to the main system, it can be considered firstly as the addition of one system



(A)



(B)

FIG. 12. GENERAL CASE OF ADDITIONAL SYSTEM ADDED REMOTELY FROM MAIN SYSTEM FORCE AND DISPLACEMENT POSITION

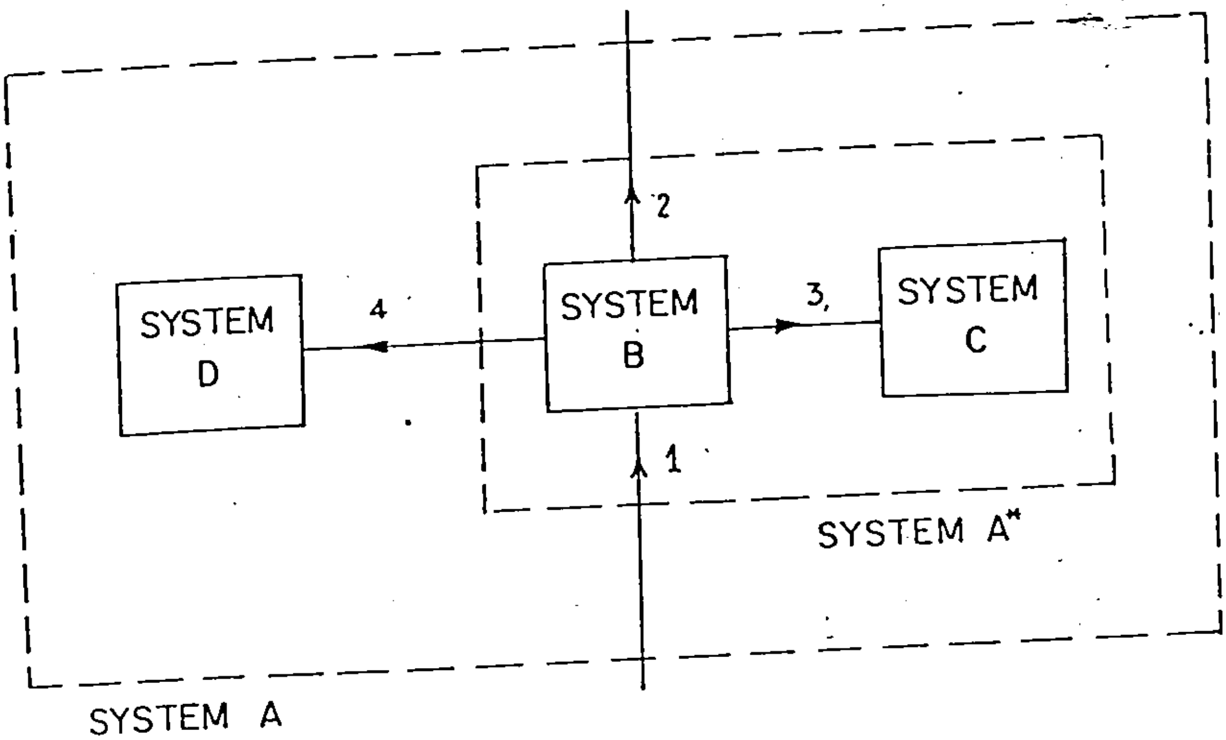


FIG.13 GENERAL CASE OF TWO ADDITIONAL SYSTEM ADDED REMOTELY FROM MAIN SYSTEM FORCE AND DISPLACEMENT POSITION

to form a new main system A^*, and then the addition of the other system to A^*, as shown in Fig. 13 for the two dimensional absorber. From equation (21)

$$R^*_{a12} = R_{b12} - \frac{R_{b13}\, R_{b32}}{R_{b33} + R_{c33}}$$

Similarly

$$R^*_{a14} = R_{b14} - \frac{R_{b13}\, R_{b34}}{R_{b33} + R_{c33}}$$

$$R^*_{a42} = R_{b42} - \frac{R_{b43}\, R_{b32}}{R_{b33} + R_{c33}}$$

and $$R^*_{a44} = R_{b44} - \frac{R_{b34}^{\ 2}}{R_{b33} - R_{c33}}$$

In the last expression, Maxwell's reciprocal theorem is assumed, that is $R_{43} = R_{34}$. The system D is therefore added to the system A^* having the receptances defined above. Thus the receptance of the combined system is given by

$$R_{a12} = R^*_{a12} - \frac{R^*_{a14}\, R^*_{a42}}{R^*_{a44} + R_{d44}} \qquad \text{........(22)}$$

For the chatter-free machine, the receptance to be optimized is the chatter receptance. That is, the forced vibration receptance modulus requires to be minimized. We can employ the graphical method that we have been used in the previous cases to obtain the optimum absorber, but an iterative method using a computer is preferred due to the complexity of the receptance equation. It is assumed that all terms in the main system are defined, and only the receptance of the absorber (R_c , R_d) can be varied to obtain the optimum absorber.

CHAPTER 4

CONCLUSION

It is shown that different methods can be used to optimize an absorber. The mass of an absorber is usually restricted, and only the spring stiffness and the damping coefficient can be varied in order to achieve the optimal absorber.

The first method that has been discussed in this report is to optimize an absorber which has restricted damping. From Fig.3 and by calculation, the most favourable tuning for the absorber can be obtained. The most favourable tuning of the absorber can be obtained by adjusting the stiffness of the absorber. After determining the correct tuning, the optimum damping of this system can be calculated. When the absorber has the most favourable tuning and optimum damping, the maximum amplitude of vibration for the whole frequency range will be reduced to the minimum. This kind of absorber is usually found in the landing device of an aeroplane.

When the absorber has only restricted damping in the spring, a different method is used to obtain the optimum absorber. An expression is obtained with $F/(kX)$ in terms of the other variables. $F/(kX)$ corresponds to a resonance of the non-dimensional response, and the absorber is optimized if it is made as large as possible. A graphical method is used in achieving the correct spring stiffness for the optimum absorber. The same technique is being used to optimize the absorber with damped and undamped main mass system, and the absorber with viscous, hysteretic or these dampers.

Sometimes, the reduction of amplitude of vibration of the main mass for the whole frequency range is not the sole purpose of an optimum absorber. In cutting and milling machines, the increase of the chatter-free machining capacity is often the criterion for optimization. An absorber is not necessarily connected to the point of the disturbing force. The attachment can be remote from the position of interest in the main system. The optimization of one-dimensional absorber and two-dimensional absorber attached to a complex system are discussed in the last part of this report. For the chatter-free machine, the object of an optimum absorber is to minimize the forced vibration receptance. Since the receptance equation for the complex system is so complicated, an iterative method using a computer, is to be employed to obtain the correct stiffness of the optimum absorber.

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APPENDIX

DYNAMIC STIFFNESS OF UNCOUPLED MAIN SYSTEM

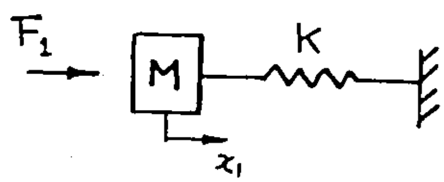
When a force $F_1e^{i\omega t}$ applied to the mass M, the response of the main mass system is

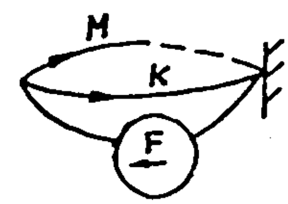
$$M\ddot{x}_1 + K^*(\omega)x_1 = F_1e^{i\omega t} \quad \text{........(a)}$$

Put $X_1 = x_1e^{i\omega t}$.

then $-MX_1\omega^2e^{i\omega t} + K^*(\omega)X_1e^{i\omega t} = F_1e^{i\omega t}$ .....(b)

Hence $F_1/X_1 = K^*(\omega) - M\omega^2$ ....(c)



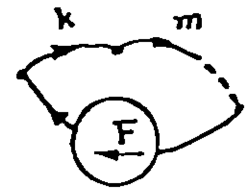


DYNAMIC STIFFNESS OF UNCOUPLED ABSORBER

The response of the absorber system to a force $F_2e^{i\omega t}$ applied to the point of coupling is given by the following simultaneous equations

$$m\ddot{x}_3 + k^*(\omega)(x_3 - x_2) = 0 \quad \text{......(d)}$$

$$m\ddot{x}_3 = F_2 e^{i\omega t} \quad \ldots\ldots\ldots\ldots(e)$$



Put $x_2 = X_2 e^{i\omega t}$

$$x_3 = X_3 e^{i\omega t}$$

then $$-MX_3\omega^2 + k^*(\omega)(X_3 - X_2) = 0 \quad \ldots\ldots(f)$$

$$-mX_3\omega^2 = F_2 \quad \ldots\ldots\ldots(g)$$

From equation (f) and (g), we obtain

$$\frac{F_2}{X_2} = \frac{k^*(\omega)\, m\omega^2}{m\omega^2 - k^*(\omega)} \quad \ldots\ldots\ldots(h)$$

DYNAMIC STIFFNESS OF COUPLED SYSTEM

The dynamic stiffness is obtained by adding equations (c) and (h), gives

$$\frac{F_1}{X_1} + \frac{F_2}{X_2} = \frac{F}{X}$$

Thus

$$\frac{F}{X} = K^*(\omega) - M\omega^2 + \frac{m\omega^2 k^*(\omega)}{m\omega^2 - k^*(\omega)}$$