# OPTIMIZATION OF OSCILLATORY CONVEYERS

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## ABSTRACT

The mathematical model of an object placed on an oscillatory conveyer has been derived, and an optimizing procedure for a conveyer driven by a four-bar mechanism developed.

The performance of a conveyer excited by any form of driving cycle can be quickly evaluated by the methods presented and one obvious extension of the method is the calculation of optimal driving cycles, irrespective of mechanism, and then applying a synthesis procedure to obtain the mechanism to generate this cycle.

The motion of an object on an oscillatory conveyer driven by a four-bar mechanism is simulated digitally. Using this simulation, sets of conveyer system parameters which maximize the rate of travel of the particle can be found. Optimal combinations can be used in the design of oscillatory conveyer systems, and an example of their use is given.

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# NOMENCLATURE

| | |
|---|---|
| $t$ | Time (seconds) |
| $g$ | Acceleration due to gravity (ft/sec$^2$) |
| $\xi, \eta$ | Axes of reference |
| $\ddot{y}$ | Acceleration cycle applied to the trough (ft/sec$^2$) |
| $\dot{y}$ | Velocity of the driving point (ft/sec) |
| $y$ | Displacement of the driving point (ft.) |
| $\ddot{x}$ | Acceleration of conveyed object (ft/sec$^2$) |
| $\dot{x}$ | Velocity of conveyed object (ft/sec) |
| $x$ | Object displacement (ft.) |
| $\ddot{y}_i$ | Acceleration component applied in the plane of the trough |
| $\ddot{y}_n$ | Acceleration component applied normal to the plane of the trough |
| $y_i$ | Velocity component of the trough along the $\zeta$ plane |
| $\dot{y}_n$ | Velocity component of the trough along the $\eta$ plane |
| $\mu$ | Coefficient of friction |
| $B_U$ | Upper bound to object acceleration defined in Chapter 2 (ft/sec$^2$) |
| $B_L$ | Lower bound to object acceleration defined in Chapter 2 (ft/sec$^2$) |
| $m$ | Mass of the object (lb.) |
| $\alpha$ | Trough angle with the horizontal (degrees) |
| $N$ | Force component normal to trough plane (lb.ft/sec$^2$) |
| $\beta$ | Angle of accelerating force with the horizontal (degrees) |
| $B, S, B_O, S_O$ | Terms defined in Chapter 2 |
| $A_m$ | Maximum positive acceleration ordinate (ft/sec$^2$) |

NOMENCLATURE (CONT'D)

| | |
|---|---|
| $A_n$ | Maximum negative acceleration ordinate ($ft/sec^2$) |
| $r_1, r_2$, $r_3, r_4$ | Dimensions of the four bar linkage (ft.) |
| $R_2, R_3$, $R_4$ | Arms ratios of four bar linkage |
| $\phi$ | Free flight factor |
| $A_1, A_2$, LEAD, LAG | Logic signals defined in Chapter 4 |
| $\overline{A_1}, \overline{A_2}, \overline{LEAD}, \overline{LAG}$ | The logical complement signals of $A_1, A_2$, LEAD, and LAG signals |

# CHAPTER I

# INTRODUCTION

# CHAPTER I

# INTRODUCTION

## 1.1 General

Conveyers are used in industry for the transport of large quantities of material and for transporting individual items. They are useful in such different applications as feeding delicate components to automatic assembly lines, transporting mineral ores, controlled mixing of chemicals and other purposes.

The moving belt type of conveyers is in common use. These consist of an endless belt of flexible material, moving over rollers, transporting material placed on the belt. Another type, the subject of the present study, are oscillatory conveyers. These consist of a reciprocating trough which transports material by vibratory motion, and a driving mechanism to generate the motion of the trough. The trough is attached to a carriage which can be freely moved by the driving mechanism along stationary guides. The conveyed objects are placed on the trough, which may be a sheet of metal, glass, wood, or any other suitable material. Oscillatory conveyers can be divided into two major types

a) In-plane oscillatory conveyers

b) In-and-normal to plane oscillatory conveyers.

The driving cycle of oscillatory conveyers is usually produced by a linkage mechanism. The resulting motion of the conveyed objects can be a pure sliding motion for in-plane oscillatory conveyers or a combination of sliding and free flight for in-and-normal-to-plane oscillatory conveyers. The free flight motion always ends with an impact, which is often undesirable. Accordingly, the present study emphasizes conveyers with pure sliding motion,as these are likely to be more useful, and only discusses the other type with some simplifying assumptions.

To achieve an optimum design for a given purpose, the conveyer designer needs to predict the net distance travelled by the conveyed material over a period of time, and it is essential to know the effect of changes in the conveyer system parameters on this distance. Such design and optimization procedures are best carried out on the mathematical model of the system, since the number of parameters affecting the performance of these conveyers is large. However, the differential equations describing the motion of the conveyed material are highly non-linear, hence making analytical solutions unlikely; indicating a simulation approach to the optimization.

## 1.2 Review of Previous Work

Analytical studies of oscillatory conveyers received little attention prior to 1960. Most of the work introduced until that time by Roebuck, Cornegie, and Taylor [1] and Berry [2,3] was descriptive and experimental. These authors discussed the principles of oscillatory feeders subjected to sinusoidal drive. Akhmechet [4] and Moskwitz [5] in 1959, and Zilly [6], in 1960, discussed the design of oscillatory conveyers, deriving approximate design calculations of limited use in practical applications. However, they introduced the use of performance indices for assessment of oscillatory conveyers.

Analysis of sinusoidal acceleration cycles based on experimental work was carried out by Povidaylo [7,8] and Booth [9]. They resorted to experiment to confirm approximate design criteria. Their analysis was based on graphical and iterative approaches, and yielded expressions for the mean conveying velocity. These provide guidelines for the designer which can be used in predicting the performance of conveyers subjected to sinusoidal drive.

The only work which has been done on non-sinusoidal driving cycles was carried out by Morcos and Massoud [10]. A hypothetical rectangular trough acceleration cycle was assumed in order to simplify the analysis; accordingly it was possible to find performance criteria as guidelines for the design. However, the difficulty of generating such acceleration cycles was not considered.

## 1.3 Objectives of the Present Study

It is the purpose of this thesis to study oscillatory conveyers whose trough acceleration is generated by common linkage mechanisms, rather than by assumed analytically tractable acceleration cycles, and to provide optimal design criteria for conveyers driven by a mechanism of the four-bar type.

The design method presented here, is based on simulation and covers most of the design problems normally encountered. This method can be applied with or without constraints on the conveying system parameters, once the conveyer type is specified and the driving mechanism is selected. Application of the method will result in optimum conveyer design for particular problem requirements.

## 1.4 Plan of Study

Chapter II presents the derivation of equations of motion of the transported object for the case of Coulomb friction between the trough and the object. The construction of the trough and the in-plane driving cycle produced by a four-bar mechanism connected to the trough are discussed in Chapter III. Assessment of different types of simulation techniques is made in Chapter IV. Optimization of the conveying system parameters is also carried out in this Chapter. A design example utilizing the computed optimum data, as well as brief consideration of the merits and demerits of simulation on digital and analog computers are given in Chapter IV. Chapter V discusses the results obtained from the digital and the analog computers, and draws conclusions about the effect of various conveyer parameters on conveyer performance.

# CHAPTER II

# DYNAMICS OF TRANSPORTED OBJECT

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# DYNAMICS OF TRANSPORTED OBJECT

## 2.1 Introduction

The conveyed object's motion is studied for the case of an arbitrary driving cycle applied to the trough. The acceleration cycle is assumed to have two components, one in the plane of the trough, and the other normal to it. The object's motion can take several forms. These are sliding, rolling, stationary relative to the trough, and free flight.

The in-plane driving component will produce the sliding and the rolling modes, while the driving cycle component normal to the plane of the trough will produce free flight. For simplicity, only one conveyed object is considered, and rolling is excluded. Further, Coulomb friction between trough and object is assumed.

Strictly speaking, the friction depends on the relative velocities of the object and the trough, but Coulomb friction was justified by Booth [9], and leads to a somewhat simpler model. However, there is no theoretical restriction to this type of friction, and others can be considered at the expense of a somewhat more complex model.

It should be remembered that the dynamics of one object travelling along the trough will differ from the dynamics of many objects travelling at the same time. In that case, layers may form, in which not all objects are in direct contact with the trough, and objects may interact with one another. Hence, the present study is more suited to the case where individual objects not in contact, are to be transported, such as parts on an assembly line.

Two cases are considered in this study. In the first case, it is assumed that the object is continuously in contact with the trough surface. This can be achieved by confining the trough to movement in its own plane.

In the other case considered, free flight may occur as a result of the trough being driven in its plane and normal to it. At the termination of free flight, it is assumed that the object will acquire the velocity and acceleration of the trough at the moment the object touches the trough.

The previous assumptions will only allow the object to be moving in one of the following modes:

a) Both object and trough are moving with the same velocity relative to a stationary reference system; the object is stationary, with respect to the trough.

b) The velocity of the object is less than the velocity of the trough; the object is relatively moving downward.

c) The velocity of the object is greater than the velocity of the trough; the object is relatively moving upward.

d) The object leaves the trough when the normal component of trough acceleration is zero and continues in free flight under the influence of gravity until its trajectory again intersects the trough.

## 2.2 Forces Acting on the Object

Figures 1 and 2, show the axes of reference, assumed fixed in space, and forces on the conveyed object, used in the further development. One axis is parallel to the conveyer trough at an angle $\alpha$ to the horizontal, the second is normal to the first. The conveyer trough has acceleration $\ddot{y}$, and velocity $\dot{y}$, in general, with components $\ddot{y}_i$ and $\dot{y}_i$, in line with the trough, and $\ddot{y}_n$ and $\dot{y}_n$, normal to it.

The forces acting on the conveyed object due to gravity and the motion of the trough, are shown for the case where the coefficient of sliding friction between object and trough is $\mu$. The object's motion is given by acceleration $\ddot{x}$, velocity $\dot{x}$, and displacement $x$, again in general with components in the plane of the trough and normal to it.

## 2.3 Governing Equations for the Object's Motion

Consider the downward direction of motion to be positive and the upward direction negative. Therefore, the equation governing the motion of the object during downward travel in the $\zeta$ direction is

$$m\ddot{x}_i + mg\sin(\alpha) = \mu mg\cos(\alpha) \qquad (1)$$

where $\mu mg\cos(\alpha)$ is the force of friction which is always in opposite direction to the object velocity. Also, the equation that governs the object's upward travel is

$$-m\ddot{x}_i + mg\sin(\alpha) = -\mu mg\cos(\alpha) \qquad (2)$$

Let the net force component normal to the trough plane acting on the object be N. For an object only subjected to gravitational force, one obtains

$$N = mg\cos(\alpha) \qquad (3)$$

while for an object moving in the $\eta$ direction with acceleration $\ddot{x}_n$ the governing equation for object motion is described by

$$m\ddot{x}_n = N - mg\cos(\alpha) \tag{4}$$

From Equations (1) and (2) the following is obtained

$$\ddot{x}_i = g\cos(\alpha) - g\sin(\alpha) \tag{5}$$

$$\ddot{x}_i = g\cos(\alpha) + g\sin(\alpha) \tag{6}$$

Also from Equations (3) and (4) one can get the following

$$\ddot{x}_n = \frac{N}{m} - g\cos(\alpha) \tag{7}$$

It should be noted that if the term $\ddot{x}_i$ is positive, Equation (5) can be applied, and for $\ddot{x}_i$ negative Equation (6) applies.

The first case studied, that of sliding only, is governed by Equations (5) and (6); the other case, sliding and free flight, is governed by Equations (5), (6) and (7). The study of each case is presented in the following.

### 2.3.1 Governing Equations for an In-Plane Driving Cycle

Define $B_U$ the maximum upper bound and $B_L$ is the minimum lower bound on the object's acceleration, where these quantities are given by

$$B_U = g[\mu\cos(\alpha) - \sin(\alpha)] \tag{8}$$

$$B_L = g[\mu\cos(\alpha) + \sin(\alpha)] \tag{9}$$

Equations (8) and (9) show that the maximum and minimum values of $\ddot{x}_i$ are the absolute value of $B_U$ and $B_L$, respectively. When the object is placed on the trough, the velocities of both trough and object will be the same, provided that the following conditions are satisfied

$$B_U > \ddot{x}_i > - B_L \qquad (10)$$

If the object is moving upward, the following applies

$$\dot{x}_i > \dot{y}$$

and the trough acceleration must satisfy the relationship:

$$\ddot{y} < -B_L \qquad (11)$$

Similarly, for an object moving downward, one can obtain the following relationships:

$$\begin{aligned} \dot{x}_i &< \dot{y} \\ \ddot{y} &> B_U \end{aligned} \qquad (12)$$

### 2.3.2 Governing Equations for In-Plane and Normal-to-Plane Driving Cycle

Substituting Equation (3) into (5) and (6), one obtains

$$\ddot{x} = \frac{\mu N}{m} \pm g \sin(\alpha) \qquad (13)$$

Equation (7) can be rearranged as follows:

$$\frac{N}{m} = \ddot{x}_n + g\cos(\alpha) \qquad (14)$$

Let B and S be the maximum and minimum values of the object's acceleration, where

$$B = \frac{\mu N}{m} - g\sin(\alpha) \qquad (15)$$

$$S = \frac{\mu N}{m} + g\sin(\alpha) \qquad (16)$$

Figure 2 shows the acceleration applied to the trough forming an angle $\beta$ with the horizontal. The two acceleration cycle components of the trough in the $\zeta$ plane and normal to it, are denoted by $\ddot{y}_i$ and $\ddot{y}_n$ respectively, as previously defined. Therefore, at any particular instant, the conveyed object will be subjected to all or part of the forces shown in Figure 2.

The object will remain in contact with the trough as long as the normal reaction is equal to or less than zero.

Consider the case where both object and conveyer have zero acceleration and equal velocity; therefore

$$\ddot{x} = \ddot{y}_i = 0 \text{ and } \dot{x} = \dot{y} \neq 0 \qquad (17)$$

Assuming the conveyer starts to accelerate, $\ddot{x}$ will remain equal to $\ddot{y}_i$ providing that $\ddot{y}_i$ is less than B.

For trough deceleration, $\ddot{x}$ will remain equal to $\ddot{y}_i$ as long as $\ddot{y}_i$ is greater than $-S$.

Therefore, the conveyer can accelerate and decelerate with the object remaining stationary on its surface as long as the following relations hold

$$-S < \ddot{y}_i < B \quad \text{and} \quad N > 0 \qquad (18)$$

Referring to Equation (14), it will be noted that for negative $\ddot{x}_n$, N may be both positive or negative. The transition from positive to negative N occurs when $\ddot{x}_n$ equals $-$ mg cos ($\alpha$) and for values of $\ddot{x}_n$ greater than this, N will be positive. Defining the terms $B_o$ and $S_o$, as the upper and lower bounds of the object's acceleration when $\ddot{y}$ is at an angle $\alpha$ with the horizontal. Equations (15) and (16) are rewritten as

$$B_o = \mu g \cos(\alpha) - g \sin(\alpha) \qquad (19)$$

$$S_o = \mu g \cos(\alpha) + g \sin(\alpha) \qquad (20)$$

If $\ddot{x}_n$ is equal to $\ddot{y}_n$ one obtains

$$N = m\ddot{y}_n + mg \cos(\alpha) \qquad (21)$$

and therefore, Equations (15) and (16) can be written as follows:

$$B = B_o + \mu\ddot{y}_n \qquad (22)$$

$$S = S_o + \mu\ddot{y}_n \qquad (23)$$

## 2.4 Effect of Trough Motion on Object's Motion

It is the objective of the conveyer designer to find a driving cycle which moves the object upwards. Therefore, during the upward stroke of the trough, one should not exceed an object acceleration which overcomes the limiting friction force between the object and the trough. On the other hand, one would like the downward acceleration of the trough to be very much larger than the limit set by friction, in order that the object remains nearly stationary.

The object will be in one of four modes of motion at one time. Each mode will start whenever certain conditions are met and will terminate whenever another set of conditions becomes true. These modes will be discussed in a later section in this Chapter.

An in-plane symmetric driving cycle applied to the trough will not provide upward travel of the object. For such a cycle, the object will either remain stationary on the trough, or move downward, since the magnitude of downward acceleration for the object is greater than the magnitude of upward acceleration. This was first noted by Morcos and Massoud [10] .

Therefore, there are two alternatives to obtain upward object travel. One can use

a) A nonsymmetric driving cycle, or

b) Two symmetric driving cycles, one in-plane and the other normal to it.

## 2.5 Object Modes

There are four possible modes of motion for a conveyed object. Two cases are discussed, one for an in-plane drive, and the other for a sinusoidal drive forming an angle $\beta$ with the horizontal, as shown in Figures (3) and (4), respectively.

### 2.5.1 The In-Plane Nonsinusoidal Drive

Only sliding can take place in this case

a) Mode I (the track mode)

In this mode, the trough and the object are moving at the same velocity, therefore $\dot{x}$ is equal to $\dot{y}$. Also, the following conditions must be satisfied, as long as the object is in this mode

$$\ddot{x} - B_U < 0$$
$$\ddot{x} + B_L > 0 \qquad (24)$$

The object will leave this mode whenever one of the conditions in Equation (24) is violated. Let $x(t_o)$ be the displacement coordinate of the object, at its start in this mode, and $t$ the time elapsed while the object was in this mode. $x(t)$ will be the object displacement coordinate after time $t$ is elapsed. The governing equations of the object motion in this mode are then

$$\begin{aligned} \ddot{x} &= \ddot{y} \\ \dot{x} &= \dot{y} \\ x(t) &= x(t_o) + [y(t) - y(t_o)] \end{aligned} \tag{25}$$

where $y_t$ and $y_o$ are the trough displacements at time $t$ and at the start of motion in this mode.

b) <u>Mode II</u> (the lag mode)

In this mode, the object is sliding downward. While in this mode, the object motion satisfies the following relation

$$\ddot{x} = B_U \tag{26}$$

Accordingly, the velocity of the object will be less than that of the trough and the following relations will hold, as long as the object stays in this mode

$$\dot{x} - \dot{y} < 0 \tag{27}$$

The governing equations of the object motion then, are

$$\ddot{x} = B_U$$
$$\dot{x} = \dot{x}(t_o) - B_U(t-t_o) \qquad (28)$$
$$x(t) = x(t_o) - \dot{x}(t_o)(t-t_o) + \tfrac{1}{2} B_U(t-t_o)^2$$

where $t_o$ is the time at which the object starts to slide downward. This mode will terminate when the relation (27) is no longer valid.

c) Mode III (the lead mode)

The object moves upward in this mode. The necessary condition to be in this mode is

$$\ddot{x} = -B_L \qquad (29)$$

In this mode the following relation holds

$$\dot{x} - \dot{y} > 0 \qquad (30)$$

Whenever the relation (30) is not satisfied, this mode will terminate. The object motion during this mode will be governed by the following equations

$$\ddot{x} = -B_L$$
$$\dot{x} = \dot{x}(t_o) + B_L(t-t_o) \qquad (31)$$
$$x(t) = x(t_o) - \dot{x}(t_o)(t-t_o) - \tfrac{1}{2} B_L(t-t_o)^2$$

where $t_o$ is again the time at the beginning of this mode of object's motion.

### 2.5.2 The Sinusoidal In-Plane and Normal To Plane Drives

Let the acceleration cycle $\ddot{y}$ applied to the trough be at an angle $\beta$ to the horizontal. For a given sinusoidal cycle of maximum amplitude $a$, the acceleration can be written in the form

$$\ddot{y} = a \sin(\omega t) \quad (32)$$

Referring to Figure 2, the components of this cycle $\ddot{y}_i$ and $\ddot{y}_n$ in and normal to the trough plane are

$$\begin{aligned} \ddot{y}_i &= \ddot{y} \cos(\beta-\alpha) \\ \ddot{y}_n &= \ddot{y} \sin(\beta-\alpha) \end{aligned} \quad (33)$$

When $\ddot{y}_n$ is equal to zero, Equations (22) and (23) will be reduced to

$$\begin{aligned} B &= B_o = \mu\, g \cos(\alpha) - g \sin(\alpha) \\ S &= S_o = \mu\, g \cos(\alpha) + g \sin(\alpha) \end{aligned} \quad (34)$$

Introducing the following notation

$$\begin{aligned} A &= \ddot{y} \cos(\beta - \alpha) \\ \phi &= \mu \tan(\beta - \alpha) \end{aligned} \quad (35)$$

Equations (22) and (23) can be written in the following form

$$B = B_o + \phi A \sin(\omega t)$$
$$S = S_o + \phi A \sin(\omega t) \qquad (36)$$

Referring to Figure 4 and Equation (36), it can be seen that the normal reaction will not vanish, providing that each of the terms $B_o$ and $S_o$ is greater than $\phi A$. This restriction is illustrated in Figure 4, and can be written in compact form as follows

$$B_o + S_o > 2\phi A \qquad (37)$$

$\phi$ can be called the free flight factor. Thus the object can be in one of the four following modes.

a) Mode I (the track mode)

The object has the same velocity and acceleration as the trough. It stays in this mode as long as the following relations hold

$$\ddot{y}_i - B < 0$$
$$\ddot{y}_i + S > 0 \qquad (38)$$
$$N > 0$$

The equations governing the object dynamics in this mode are

$$\ddot{x} = \ddot{y}_i$$
$$\dot{x} = \dot{y}_i \qquad (39)$$
$$x(t) = x(t_o) + [y_i(t_o) - y_i(t)]$$

b) <u>Mode II</u> (the lag mode)

In this mode, the absolute value of the object's velocity is less than that of the trough. The object will stay in this mode if the following relations hold

$$\ddot{y}_i - B > 0$$
$$\dot{y}_i - \dot{x} > 0 \qquad (40)$$
$$N > 0$$

The governing equations of this mode are

$$\ddot{x} = B$$
$$\dot{x} = \dot{x}(t_o) + \int_{t_o}^{t} B\, dt \qquad (41)$$
$$x(t) = x(t_o) + \int_{t_o}^{t} \dot{x}\, dt$$

c) <u>Mode III</u> (the lead mode)

In this mode, the absolute value of the object's velocity is greater than that of the trough and the following conditions are fulfilled

$$\dot{y}_i + S < 0$$

$$\dot{x} - \dot{y}_i > 0 \qquad (42)$$

$$N > 0$$

The governing equations of the object dynamics in the mode are

$$\ddot{x} = -S$$

$$\dot{x} = \dot{x}(t_o) - \int_{t_o}^{t} S\,dt \qquad (43)$$

$$x(t) = x(t_o) + \int_{t_o}^{t} x\,dt$$

d) <u>Mode IV</u> (the free flight mode)

In this mode, the object leaves the trough surface when the normal reaction is equal to zero. The object velocity will have two components along the reference axes denoted as $\dot{x}_\zeta$ and $\dot{x}_\eta$. At the instant the object leaves the trough, its velocity will be $(\dot{x})_o$ with $\zeta$ and $\eta$ components will be $(\dot{x}_\zeta)_o$ and $(\dot{x}_\eta)_o$. It is obvious that the normal reaction will only become zero or negative during the downward stroke of the trough.

While in free flight, the object is under the influence of gravitational forces only. The governing equations of the object's motion along the $\zeta$ axis are

$$\begin{aligned} \ddot{x}_\zeta &= - g \sin(\alpha) \\ \dot{x}_\zeta &= (\dot{x}_\zeta)_o - g(t-t_o) \sin(\alpha) \qquad (44) \\ x_\zeta &= (x_\zeta)_o - \tfrac{1}{2} g(t-t_o)^2 \sin(\alpha) + \\ &\quad + (\dot{x}_\zeta)_o(t-t_o) \end{aligned}$$

and along the $\eta$ axis are

$$\begin{aligned} \ddot{x}_\eta &= - g \cos(\alpha) \\ \dot{x}_\eta &= (\dot{x}_\eta)_o - g(t-t_o) \cos(\alpha) \qquad (45) \\ x_\eta &= (x_\eta)_o - \tfrac{1}{2} g(t-t_o) \cos(\alpha) + \\ &\quad + (\dot{x}_\eta)_o(t-t_o) \end{aligned}$$

The object will stay in the free flight mode as long as it is not in contact with the trough, which implies the following relation

$$x_\eta - y_\eta > 0 \qquad (46)$$

## 2.6 Transitions Between Object Modes

As previously discussed, several restrictions affect the object motion. These restrictions depend on the current mode of object motion. The object will remain in this mode as long as the conditions governing the mode remain true. If a change in the object and trough states is sufficient to cause violation of the mode conditions, a mode transition will occur.

Figure 5, Tables (1) and (2) show the possible object states and the transitions that can occur between object modes for both pure sliding, and sliding and free flight transport, respectively.

# CHAPTER III

# THE CONVEYER TROUGH AND DRIVING MECHANISM

# CHAPTER III

# THE CONVEYER TROUGH AND DRIVING MECHANISM

## 3.1 The Conveyer Trough

As previously mentioned, the conveyed objects are placed on a trough, constructed of a suitable material. The type of material used will define the coefficient of friction between object and trough $\mu$. Some of the most commonly used materials and their coefficients of friction are given in Table (3).

The trough is placed at an angle $\alpha$, so as to convey objects from one point at a lower level to another, at a higher level. The trough motion is cyclical and can be purely in the plane of the trough, or can have components in the trough plane and normal to it. The net travel of the trough over one cycle is zero, thus only the different velocities imparted by the trough to the object during its forward and backward strokes will provide the object's upward motion.

### 3.1.1 Parameters of the Trough

Mathematically for in-plane trough movement, the two parameters describing the trough's effect on the object's motion are its angle of inclination $\alpha$, and the coefficient of friction $\mu$.

For a given driving cycle, let the maximum positive acceleration ordinate be $A_m$. To efficiently utilize the driving cycle, it is necessary to choose the coefficient of friction $\mu$ in such a way that the maximum possible object acceleration is as close to the value $A_m$. Thus, referring to Figure 6, the energy lost from the driving cycle will be reduced to a minimum for a given upward travel of the conveyed object.

The following relation gives the maximum value of $\mu$ that satisfies the above requirement. It is obtained by substituting $A_m$ into Equation (5) and rearranging

$$\mu_{max} = \frac{A_m/g + \sin(\alpha)}{\cos(\alpha)} \qquad (47)$$

The trough angle $\alpha$ will determine the minimum value of $\mu$. For values of $\mu$ smaller than this minimum, the object will continuously move downward irrespective of the motion of the trough. From Equations (5) and (6) the minimum value is as follows

$$\mu_{min} = \tan(\alpha) \qquad (48)$$

For the second case discussed in Chapter II, with the sinusoidal drive applied to the trough at an angle $\beta$, the trough parameters will be $\mu$, $\alpha$ and $\beta$. The conveyed object can be in the free flight mode,as governed by the condition given by Equation (37).

The occurrence of this mode does not only depend on the coefficient of friction $\mu$, but depends also on $\alpha$, $\beta$ and the amplitude and frequency of the acceleration cycle. To prove this, Equation (37) can be written as follows

$$\frac{\cos(\alpha)}{\sin(\beta-\alpha)} > \frac{\ddot{y}}{2g} \tag{49}$$

The simulation of this case is shown in a later Chapter, and is restricted to the sliding modes only.

## 3.2 The Driving Mechanism

A linkage mechanism is usually used in driving the nonsinusoidal types of oscillatory conveyers. The four bar mechanism of the crank lever type is chosen for the present study. The driving cycle produced by this linkage mechanism has rapid variations of acceleration. Such a driving cycle is capable of producing relative motion between trough and object, because rapid changes in trough velocity occur in a short time.

Figure 7 shows the dimensions of a four bar mechanism. The four links are hinged at 0, a, b and $0_1$. Link $r_2$ is the driver and rotates with a constant angular velocity $\omega$. The movement of point b is transferred to the conveyer in such a way that only the component of motion in the plane of the trough acts on the conveyer. In this case only an in-plane nonsinusoidal acceleration cycle will be acting on the trough. The dimensions $r_1, r_2, r_3$ and $r_4$ can be varied under certain constraints which will be discussed later. When these constraints are not met, locking occurs.

### 3.2.1 The Mathematical Model of the Four Bar Mechanism

Figure 7 shows the mechanism with the driver link at the starting position. The displacement components of the coupling point b at start are derived in Appendix I, with respect to the coordinate axes established in this Figure. These components are designated $(x_b)_o, (y_b)_o$ and are as follows

$$(x_b)_o = (r_1^2 + r_3^2 - r_2^2 - r_4^2)/2(r_1 - r_2) \quad (50)$$

$$(y_b)_o = \sqrt{r_3^2 - (x_b - r_2)^2} \quad (51)$$

A numerical approach is used for incrementing the starting values of these components iteratively to obtain the $x_b$ and $y_b$ components at the various equiangular driving crank stations. These stations will cover one complete turn of the driven link.

The velocity and acceleration components of the coupling point b are derived in Appendix J. The velocity components

$\dot{x}_b$, $\dot{y}_b$ are as follows

$$\dot{x}_b = [\dot{y}_b\, y_b(y_b - y_a) + \dot{x}_a\, y_b(x_b - x_a)/$$

$$[y_b(x_b - x_a) + (y_b - y_a)(r_1 - x_b)] \qquad (52)$$

$$\dot{y}_b = \dot{x}_b(r_1 - x_b)/y_b$$

and the acceleration components $\ddot{x}_b$, $\ddot{y}_b$ are given by

$$\ddot{x}_b = [\ddot{x}_a y_b(x_b - x_a) - y_b(\dot{x}_b - \dot{x}_a)^2 + (y_b - y_a)(\dot{x}_b + \dot{y}_b{}^2 + y_b\ddot{y}_b)$$

$$- y_o(\dot{y}_a - \dot{y}_b)^2]/[y_b(r_1 - x_a) + (y_b - y_a)(r_1 - x_b)] \qquad (53)$$

$$\ddot{y}_b = [\ddot{x}_b(r_1 - x_b) - \dot{x}_b{}^2 - \dot{y}_b{}^2]/y_b$$

### 3.2.2 Different Methods of Solving the Mathematical Model

#### 3.2.2.1 The Numerical Method

The displacement components $x_b, y_b$ can be established from Equations (50) and (51). Thus, the starting values of these components are known. Using a suitable iterative method the displacement components are obtained for the different driving crank locations. There are many iterative methods that can be used such as Newton Raphson, Halley's, Regula Falsi, secant method.

The Newton-Raphson method is recommended. It is a first order iteration scheme and only requires one starting point. Halley's method is a third order iteration scheme, while Regula Falsi and secant methods are multi point iteration schemes [11]. These methods require longer computation time than the Newton Raphson Method.

Eq. (1.4A) is the general equation which describes the locus of the coupling point b. This equation can be written in a compact form as follows

$$\begin{aligned} f_1(x,y) &= 0 \\ f_2(x,y) &= 0 \end{aligned} \qquad (54)$$

Let $x_o$, $y_o$ be an approximate solution to Equation (54) and $\Delta x, \Delta y$ be the unknown corrections, such that the following values of $x$ and $y$ result in a better approximation.

$$\begin{aligned} x &= x_o + \Delta x \\ y &= y_o + \Delta y \end{aligned} \qquad (55)$$

Equations (54) and (55) lead to

$$\begin{aligned} f_1(x_o + \Delta x,\ y_o + \Delta y) &= 0 \\ f_2(x_o + \Delta x,\ y_o + \Delta y) &= 0 \end{aligned} \qquad (56)$$

Expanding Equation (56) in Taylor Series for a function of two variables and dropping higher order terms gives

$$f_1(x_o+\Delta x, y_o+\Delta y) = f_1(x_o,y_o) + \Delta x[\frac{\partial f_1}{\partial x}]_o + \Delta y[\frac{\partial f_1}{\partial y}] + \ldots$$
$$f_2(x_o+\Delta x, y_o+\Delta y) = f_2(x_o,y_o) + \Delta x[\frac{\partial f_2}{\partial x}]_o + y[\frac{\partial f_2}{\partial y}] + \ldots \tag{57}$$

Rearranging Equation (57) in a matrix form gives

$$\begin{bmatrix} (\frac{\partial f_1}{\partial x})_o & (\frac{\partial f_1}{\partial y})_o \\ (\frac{\partial f_2}{\partial x})_o & (\frac{\partial f_2}{\partial y})_o \end{bmatrix} \begin{Bmatrix} \Delta x \\ \Delta y \end{Bmatrix} = \begin{bmatrix} -f_1(x_o,y_o) \\ -f_2(x_o,y_o) \end{bmatrix} \tag{58}$$

Solving for $\Delta x, \Delta y$ as shown in Appendix I, leads to the following

$$\Delta x = (UV_2 - VU_2)/D$$
$$\Delta y = (VU_1 - UV_1)/D \tag{59}$$

where $U$, $U_1$, $U_2$, $V$, $V_1$, $V_2$, and $D$ are as follows

$$U = r_3^2 - (x_b - r_2\cos\omega t)^2 - (y_b - r_2\sin\omega t)^2$$
$$U_1 = 2(x_b - r_2\cos\omega t)$$
$$U_2 = 2(y_b - r_2\sin\omega t)$$
$$V = r_4^{\,2} - (x_b - r_1)^2 - y_b^2$$
$$V_1 = 2(x_b - r_1)$$

$$V_2 = 2y_b$$

$$D = U_1V_2 - U_2V_1$$

$(x_b)_o$ and $(y_b)_o$ of the coupling point b are known. In the next position of $r_2$; $x_b$ and $y_b$ can be obtained by incrementing the previous values by $\Delta x$ and $\Delta y$ from Equation (59). This process is carried on a sufficient number of times until the new values of $x_b$ and $y_b$ satisfy this prescribed equation within a predetermined tolerance. Once the values of $x_b$ and $y_b$ are known at a crank location, the velocity, as well as the acceleration can be directly computed from Equations (52) and (53). The process repeats for each position of the crank $r_2$.

### 3.2.3 The Analog Method

The four bar mechanism is shown in Figure 8, with defined dimensions. Algebraic and differential equations for the displacement, velocity and acceleration components of the four bar mechanism were derived in Appendix II. The analog circuits simulating these equations are shown in Figure 9 for the displacement components, in Figure 10, for the velocity components and in Figure 11 for the acceleration components. The complexity involved in the analog simulation is easily recognized, especially in Figure 10 and Figure 11. Eighteen multipliers are required, in addition to the other components, in order to carry out this simulation.

Due to a shortage of multipliers, this study was restricted to the simulation of displacement components. Results on the analog computer closely resembled the ones obtained digitally.

Analog simulation is recommended when a sinusoidal drive is applied to the trough, either in-plane, or in and normal to it. This case is further studied in Chapter 4.

## 3.3 Generation of Possible Link Combinations

### 3.3.1 Locking Conditions

The locking conditions are illustrated in Figure 12. For the crank lever type [11] these conditions are as follows

$$\begin{aligned} r_1 + r_2 &< r_3 + r_4 \\ r_2 &< r_1, r_3, r_4 \\ r_1 + r_4 &> r_2 + r_3 \end{aligned} \tag{60}$$

Rearranging the inequalities in Equation (60) leads to

$$\begin{aligned} r_1 + r_2 &< r_3 + r_4 \\ r_1 + r_4 &> r_2 + r_3 \end{aligned} \tag{61}$$

which can be written in the following form

$$r_1 + r_2 - r_4 < r_3 < r_1 + r_4 - r_2 \tag{62}$$

If all link lengths are expressed as ratios of $r_1$ this condition becomes

$$1 + R_2 - R_4 < R_3 < 1 + R_4 - R_2 \qquad (63)$$

where $R_2, R_3$ and $R_4$ are the link ratios relative to the distance between the hinges 0 and 01.

### 3.3.2 Generation of Link Combinations for Simulation

Referring to Equation (63) the nondimensional length $R_3$ will have a maximum and a minimum value for given values of the other arm lengths $R_2$ and $R_4$. Therefore, to generate most of the driving cycles from the four bar mechanism, the arms ratios $R_2$ and $R_4$ are incremented from 0.1 to 0.9, each in turn, covering all possible sets of $R_2$, $R_4$. Also, for every combination of $R_2$ and $R_4$ there exists a set of values for the ratio $R_3$ which can be calculated in ascending order in steps of 0.1, starting from the minimum value to the maximum value, as calculated from Equation (63). The parameter sets generated are shown in Appendix I.

### 3.3.3 Effect of Changes in Parameters on the Driving Cycle

The parameters of the driving mechanism are $r_1, r_2, r_3$, $r_4$ and $\omega$. For a given set of parameters, the acceleration cycle of the coupling point b will have a certain form. This form will not be affected by the changes in the driving speed $\omega$. Only a change in the relative amplitudes of the acceleration cycles will be noticed. This change is proportional to $\omega^2$.

Also, for different sets of arm lengths, there exist different forms of driving cycles. But for one set of arm length ratios, there will be only one wave form. If all arm lengths are multiplied by a constant, the amplitude of the acceleration cycle of the coupling point will increase by the same constant amount.

# CHAPTER IV

# SIMULATION AND OPTIMIZATION OF OSCILLATORY CONVEYERS

# CHAPTER IV

# SIMULATION AND OPTIMIZATION OF OSCILLATORY CONVEYERS

## 4.1 Simulation Methods

Several simulation methods are developed, each allowing optimization of conveyer parameters. In the following description the simulation strategies are presented in modules. These modules contain the mathematical equations governing the object during the different modes of transport and the logic of transition between these modes.

### a) Method I (series operations)

It is convenient to think of the simulation as having a base where decisions can be taken. The object leaves one mode for another under control of this base. A series of logical tests are performed in the base, and the simulation enters a particular module, according to the results of these. This method is represented in Figure 13.

### b) Method II (series-parallel operations)

Another concept is to visualize the entire model as switches which provide a path for the system's inputs to its outputs, as illustrated in Figure 13. These switches are controlled by a base. This base continuously monitors the input and output of the simulation and performs logical tests to produce control signals. The control signals will open and close switches according to the object mode selected by the base.

## 4.2 Assessment of Digital, Analog and Hybrid Computation

Method I is most suited to digital computation because of its serial nature. At first sight numerical integration is required, usually a time-consuming procedure, but if an appropriate algorithm is developed, the entire simulation can be reduced to function evaluation. This will be further discussed later. Since analog computers operate in parallel, that is mathematical operations carried out simultaneously, Method II is most suited to analog simulation. The complexity of the describing equations of the system is the deciding factor.

Equations (52) and (53) describing the velocity and acceleration of the four bar mechanism require many analog components for their simulation, as has already been discussed. Complete analog simulation is possible for simple cases, such as the sinusoidal driving function case. However, general simulation of conveyer, object and driver mechanism requires a well equipped analog computer.

Hybrid computation offers a solution by allowing simulation of the dynamics of the analog computer, while using the digital computer to generate the driving functions and to monitor the object modes and carry out the switching functions. Thus, both types of computers are used to their best advantage.

## 4.3 Optimization Problem and Methods Adopted For Simulation

In the sinusoidal driving function case, the system parameters are $A, \omega, \alpha, \beta$. For the case of a conveyer driven by a four-bar mechanism, the parameters are $r_1, r_2, r_3, r_4, \omega$, $\alpha$ and $\mu$ . It is desired to produce a design tool, based on simulation to allow the conveyer designer to select one optimal set of these parameters for a particular application. Design charts covering the most common ranges of parameters would be desirable. The simulation method, itself, can be used for cases falling outside the range of charts.

Methods of obtaining optimal design parameter conditions are now discussed.

### 4.3.1 The Nonsinusoidal Driving Function Case

Digital simulation is recommended in this case. Method I, the series method, is used. The simulation will be discuss-ed later. It is applied to design a system with physical con-straints on some of its parameters. Numerical integration is avoided by developing an algorithm, based on function evaluation.

### 4.3.2 The Sinusoidal Driving Function Case

In this case, optimization of the parameters can be obtained by an analog approach. Method II, the series parallel approach, is used for simulating the system dynamics and is described later. The free flight mode, discussed in Chapter 2, is not considered in this simulation. Optimum parameter values can easily be found by a manual search using potentiometers to vary parameters and observing object travel on an oscilloscope.

## 4.4 The Digital Simulation

The essence of this method is to store one driving cycle in the memory of the computer. The stored information is the acceleration, velocity and displacement of the trough in discrete form. The discrete data are taken at $n$ equally spaced points over the entire driving cycle in such a manner that all of the acceleration positive ordinates appear first. This driving cycle is applied to the trough.

The driving crank is moved in equiangular steps. The crank steps are used in simulating real time. Each step corresponds to an increase in time equal to $\Delta t$ defined by

$$\Delta t = 2\pi/\omega(n-1)$$

Stations number one and $n$ are identical, as these positions correspond to the crank at angles zero and $2\pi$, respectively.

Denoting the crank stations by an index K incremented from one to n, the time t is increased by $\Delta t$ for every incrementation of K. This process can be continued until K reached n because there is only one cycle stored. At this stage, changing K to (K - n + 1) will correspond to a return back to the station whose index is K equal to one. K can then be incremented again on the same stored cycle. The time t is continuously increasing by $\Delta t$. In this manner, the simulation can be run a number of cycles until the object motion reaches an oscillatory steady state. In practice, it is found that only the first cycle is different, for all other cycles the object is moving in an oscillatory steady state.

For a given set of parameters $r_1, r_2, r_3, r_4, \omega$, $\alpha$ and $\mu$ the through acceleration cycle can be generated from the first six parameters. It is available to the decision-making module with the other variables of interest. Closed form mathematical algorithms using equations (25), (28), and (31) are provided to calculate object travel for each mode to avoid numerical integration. Therefore, the object travel can be easily computed.

The simulation provides either partial or complete scanning of the parameter sets, described in Appendix I, for a given crank speed $\omega$. The designer can restrict the scanning process only to the sets that will suit his requirements.

### 4.4.1 Generation of the Driving Cycle from the Four Bar Mechanism

The block diagram given in Appendix I shows that the mechanism parameters are specified in the main program OPT. Subroutine NEWTON is called from the main program. This subroutine computes the $\zeta$ and $\eta$ displacement components of the coupling point b at 200 equiangular crank stations. The velocity and acceleration is also evaluated at these 200 locations. By storing these values, the displacement, velocity and acceleration of the coupling point b over a complete crank revolution will be known. Two hundred points are sufficient to show most of the driving cycle details. The accuracy of the computed ordinates is $10^{-4}$. Linear interpolation can be used for calculating the ordinates of the driving cycle for the in between crank locations, as shown in Fig. 14. This type of interpolation will not provide accurate results, if the number of points is small. Increasing the number of points will obviously increase the computation time. Accordingly, the crank stations adopted are of angular step size slightly less than two degrees. This choice allows linear interpolation to be used and the ordinates evaluated show accurate results.

The driving cycle of the trough is obtained from the stored values of the coupling point b The in-plane driving cycle acting on a trough, placed at an angle $\alpha$, is given by the following equation

$$\begin{aligned} y_1 &= x\cos(\alpha) + y\sin(\alpha) \\ \dot{y}_1 &= \dot{x}\cos(\alpha) + \dot{y}\sin(\alpha) \qquad (64) \\ \ddot{y}_1 &= \ddot{x}\cos(\alpha) + \ddot{y}\sin(\alpha) \end{aligned}$$

The trough driving cycle is computed at the 200 point of the coupling point b of the four bar mechanism. The driving cycle will change when varying the trough angle. The relations in Equation (64) only will be computed again to obtain the new values of the in-plane driving cycle of the trough for different driving angles.

### 4.4.2 Sorting of the Driving Cycle

It is necessary to the algorithm to sequentially re arrange the ordinates of the driving cycle in such a manner that the positive acceleration ordinates appear first. This strategy is adopted to provide simple means for simulating the object modes described in later sections.

Computing the ordinates of one driving cycle in the trough plane, for the 200 points representing this cycle, results in one of the following possible arrangements:

a) Negative ordinates followed by positive, then followed by negative.

b) Positive ordinates followed by negative, then followed by positive.

c) Zero ordinate followed by negative ordinates and then by positive.

d) Zero ordinate followed by positive ordinates and then by negative.

The subroutine SORT arranges the cycle ordinates for cases a), b) and c), so that these ordinates are available in the form described in d).

### 4.4.3 Coefficient of Friction $\mu$

Practical values for $\mu$ range from .1 to .5 [12] The whole range of $\mu$ can be used at small trough angles. Increasing the trough angle $\alpha$ will not allow low values of $\mu$ to be used in association with specific driving cycles applied to the trough, since slipping will occur. In some design problems $\mu$ is constrained due to a limited choice in the trough's material.

The applicable band of $\mu$ is scanned in equal increments. This ultimately leads to the value of $\mu$ that gives optimum performance for the given remaining parameters.

## 4.5 Optimization Criterion

In any optimization, one must define an objective function to be minimized or maximized. In this study, we wish to maximize object travel per cycle by varying conveying system parameters. The driving cycle is changed by varying $r_3$ while keeping $r_1, r_2, r_4$, and $\omega$ unchanged. To find the optimum value of $r_3$, it is necessary to develop a criterion that cuts down the number of trials. Referring to Figure 15, the following rules are applied:

a) The trough driving cycles generated are sorted into two classes, relying on the fact that the positive and negative areas of the trough acceleration cycle are equal. The first class are the useful cycles which can provide upward object travel. The other class of driving cycles gives downward travel. To differentiate between the two types, the acceleration cycle is checked. Let the index $k_o$ shown in Fig. 15, be used for the ordinate, at which this cycle goes from positive to negative. If it has $k_o > 100$, for the 200 crank stations previously selected, it is considered useful. This criterion will guarantee that the average amplitude of the positive portion of the acceleration cycle is less than the average amplitude of its negative portion.

b) The trough angle $\alpha$, ranging from 5 to 25 degrees, covers most of the widely used values in practice. The optimum value of $\mu$ is obtained by scanning its applicable band. It is possible to scan $\mu$ in such a way that increases in object travel occur at the start of the scanning process, then decreases.

To determine the applicable band of $\mu$, the maximum positive peak of the acceleration cycle $A_m$ is detected. Equating the maximum positive acceleration of the object $B_U$ to $A_m$ at a given trough angle $\alpha$, the relation

$$B_U = g[\mu \cos(\alpha) - \sin(\alpha)] = A_m \qquad (65)$$

will specify the maximum $\mu$ that can be used. Values of $\mu$ exceeding this maximum will result in equal movements of $B_U$ and $B_L$ away from the line $-g \sin(\alpha)$, and object upward travel decreases as $B_L$ increases.

The limiting case presented shows that the gain in object travel will decrease if $\mu$ is increased beyond that specified value.

Therefore, for a given $\alpha$, the defined band of $\mu$ can be scanned in a descending order. The scanning process of $\mu$ is ceased as soon as object travel decreases. The remaining values of $\mu$ need not be investigated.

c) Another constraint governs $\mu$, its extreme value in practice is .5. If the maximum calculated value is greater than .5, the upper bound of this parameter is limited to .5.

The decision making in the main program OPT falls within the guidelines prescribed in a, b and c. These guidelines lead to optimum performance after a few trials.

Optimum performance can be achieved by minimizing the duration of the object in Mode II, and maximizing it for Mode III.

## 4.6 Object Travel

Object travel is computed in the subroutine CONV. The ordinates of the acceleration cycle are sequentially compared with the calculated values of $B_U$ and $B_L$ at a given parameter set. Accordingly, one of the simulated modes in this subroutine will be activated. Transitions between modes occur when $B_U$ or $B_L$ is less than the absolute value of the acceleration ordinates. At transition points, the object is in two different modes at two consecutive ordinates of the acceleration cycle. As illustrated in Figure 14, linear interpolation is used to calculate the exact object acceleration, velocity and displacement at these points. The object travel is computed from the displacement results displayed in the second cycle of trough motion, as transients always appear in the results of the first cycle.

In Appendix II, a FORTRAN IV program is described which carries out the simulation and optimization. Its block diagram and flow charts show the steps of computation and display the methods used in implementing the previously mentioned guidelines. Both nonsinusoidal and sinusoidal driving function cases are run on the digital computer. The sample results obtained are plotted and given in Appendix II.

## 4.7 Design Example

It is required to transport the work pieces coming out of a machine placed in a production line to another machine which performs packaging of 10 pieces per box. This machine can produce approximately three boxes per minute. The distance between the two machines is six feet and the difference in elevation between the output of the first to the output of the second is 18 inches. It is required to design a feeder that can arrange and handle 10 work pieces in a time that can match the packing machine output.

Practical values for the speed of a driving mechanism is multiples of 60 as the driving source is usually a motor, connected to A.C. power of a frequency 60 cycles per second. As previously mentioned, low speeds are desirable, let the band of speeds adopted be 60 r.p.m. to 120 r.p.m. As will be seen later, to compensate for the decrease in u, the speed selected is 100 r.p.m., and can be increased to 120 r.p.m. when required.

The constraints on the system parameters are

$$\tan\alpha = \frac{18}{6 \times 12} = 0.249$$

therefore

$$\alpha = 14°$$

Distance travelled by objects $= \frac{6}{.97} = 6.2$ ft.

Angular velocity selected $= \frac{2 \times \pi \times 100}{60}$

$= 10.5$ radians / sec.

Required travel / sec. for one work piece

$$= \frac{6.2 \times 10}{20} = 3.1 \text{ ft./sec.}$$

From Figure 16, it is much more favourable to have the driving mechanism installed under the conveyer which will save floor space.

The design constraints implied that $r_1$, $r_2$ to be taken as 1 ft. and 0.6, respectively, as shown in Fig. 16.

The program OPT is run to evaluate the optimum design for the remaining parameters $r_3$,$r_4$ and $\mu$. The results are obtained and shown in Appendix III.

The maximum travel obtained is .29 ft./sec. From the plotted results, Fig. 17 at $r_3$ and $r_4$ equal .75 and .9, respectively, at a coefficient of friction equal to .4. Fig. 18 shows the effect of the parameter $r_3$ on object travel.

As the gain/cycle is 0.29 ft. the gain per sec. will be 2.9 ft. therefore the time required to cover a distance of 6.2 ft. for one work piece is

$$\frac{6.2}{2.9} = 2.1 \text{ sec.}$$

Therefore, the time required to handle 10 work pieces will be 21 seconds. The packaging machine will have a unit output in $\frac{60}{3} = 20$ sec.

It is obvious that the obtained results give the optimum object travel and the values of its corresponding parameters which meet the constraints of required oscillatory conveyer.

4.8 The Analog Simulation

Method II, previously described is applied to this simulation. The case of sinusoidal drive, in and normal to trough plane, is studied. The simulation only considered the sliding modes by keeping the normal reaction N greater than zero. The trough is subjected to two acceleration components $\ddot{y}_i$ and $\ddot{y}_n$, and for a given parameter of the trough B and S can be evaluated from equations (22) and (23).

The object acceleration cycle can be formed from $\ddot{y}_i$, $\ddot{y}_n$, B and S components. The analog circuit shown in Figure 19 simulates the system model. It is composed of the following major parts.

a) The control logic circuit.

The components used in this circuit are comparators and AND gates whose function is briefly described in Appendix III. Figure 20 shows the logic signals generation. These signals will control the system modules whose logic values appear in Table 4.

b) The system modules.

These are the analog representation of the track, lag and lead modes. Figure 19 shows three controlled electronic switches. Every switch has a control signal which causes it to open whenever the applied logic signal is zero, and causes it to close whenever the control signal changes to one.

4.8.1 <u>Circuit Description</u>

Referring to Figure 20, the comparators designated $A_1$ and $A_2$ continuously check the acceleration components of the trough against the upper and lower bounds of the object acceleration B and S, respectively. The comparators designated $V_1$ and $V_2$ also continuously check the velocity component $\dot{y}_i$ of the trough against the velocity of the object. Their logic outputs are used to latch the $A_1$ and $A_2$ comparators. The two comparators used to check object and trough velocities are required to distinguish three different states. These states are as follows:

a) The velocity of the object is less than that of the trough.

b) The velocity of the object is equal to that of the trough.

c) The velocity of the object is greater than that of the trough.

Accordingly, a small fixed bias has been introduced to the $V_1$ and $V_2$ comparators to produce a narrow dead band. Thus, equal object and trough velocities can be detected. The percentage error in the simulation is $2 \times 10^{-1}$, which is the amount of bias used to produce this dead band.

The AND gates are used to combine the following signals:

a) The $A_1$ and $A_2$ signals are combined in Gate I, to produce the control signal T. Therefore, T is equal to $A_1.A_2$.

b) The $\overline{A_1}$ and $\overline{LEAD}$ signals are combined in Gate II, to produce the control signal LAG. Therefore LAG is equal to

$$\overline{A_1}.\overline{LEAD}$$

c) The $\overline{A_2}$ and $\overline{LAG}$ signals are combined in Gate III to produce the control signal LEAD. Therefore, LEAD is equal to

$$\overline{A_2}.\overline{LAG}$$

Table 4 emphasizes the fact that the control signals are set to logic one, one at a time. Therefore, only one switch among the $SW_1$, $SW_2$ and $SW_3$ switches shown in the block diagram Figure 21, will be closed at any time. This results in forming $\ddot{x}$ from B, $\ddot{y}_i$, and -S components. Two integrators are used to provide $\dot{x}$ and $\dot{y}_i$ required for the generation of the control logic signals.

4.8.2 Circuit Construction

Referring to Figure 19, at given values for $\mu$ and $\alpha$, the settings of the potentiometers designated $K_o$ and $S_o$ can be calculated, using equation (32). Also, at a known value of B, the potentiometer designated $\phi/2$ can be set to the calculated value using equation (33). The potentiometer designated E provides a slight amount of negative bias to the $V_1$ and $V_2$ comparators to overcome the difficulty encountered in detecting analog signals of zero magnitude on the analog computer, as previously discussed. This pot is set to .002, and considering amplitude scaling the percentage error introduced is $2 \times 10^{-1}$.

A sample result is given in Figs. 22,23,for one set of system parameters. The cases of an in-plane sinusoidal trough drive and in-plane and normal to trough plane drives are illustrated for the sliding modes only.

The designer can run his problem in the same way previously described and set the analog computer potentiometers to the values that can meet the required constraints on the system parameters. Optimization can be done to the unconstrained system parameters by varying their corresponding potentiometers until maximum object travel is obtained.

4.8.3 <u>The Sinusoidal Drive</u>

As previously mentioned, one sinusoidal acceleration cycle in plane will never provide upward travel of the conveyed objects. The analog simulation is applied to illustrate an in-plane sinusoidal trough drive. Results are given in Fig. 24 , showing a net downward object travel. A similar case has been proven by Morcos and Massard [10] for a hypothetical cycle.

In the second case, the potentiometer $\phi/2$ is set for a value not greater than $B_o + S_o$. This will not let the object enter the free flight mode. Optimization is obtained in this case, by adjusting the free flight factor to the threshold at which the free flight mode will start to occur.

Gain in object travel can be optimized by varying the potentiometer settings which correspond to a variation in the system parameters. Optimization of this type of conveyors may lead to general trends, as there is always a definite pattern for its acceleration cycles, which is not the case for linkage mechanism driven conveyors. This generalization was not attempted, in the present study.

4.8.4 Example Covering the Upward Travel

The analog simulation sample results obtained with the following parameters

$$B = .36 \text{ ft/sec}^2$$
$$S = .4 \text{ ft/sec}^2$$
$$\phi/2 = .1$$
$$a = 1 \text{ ft/sec}^2$$
$$\omega = 1 \text{ cycle/sec}$$

which correspond to

$$\text{trough angle } \alpha = 2'$$
$$\mu = .01$$

are shown in Figure 24, for the case of in-plane sinusoidal acceleration. The area lost from the acceleration cycle is greater than the area gained, as a result, the object travels downward.

Figures 22 ,23 show the case of two sinusoidal acceleration cycles, one in-plane and the other normal to the plane of the trough. The object acceleration and velocity when superimposed on the trough acceleration and velocity cycles, Figure 23 shows that the object acceleration is bounded by the sinusoidal wave form of amplitude $\phi A$ superimposed on $B_o$ and $S_o$. Further, examination of the area under the velocity curve shows that the object is travelling upward.

# CHAPTER V

# DISCUSSION AND CONCLUSION

# CHAPTER V

## DISCUSSION

Most of the forward travel gained by a conveyed object placed on an oscillatory conveyer is achieved in the lead mode and the free flight mode. The free flight mode is not always desirable, as it may damage the objects on each impact, and reduce the quality of the trough surface.

The optimization tool provided in Chapter 4, considered only the sliding modes and maximized the lead mode for a given driving cycle. This is achieved by changing $\mu$ and $\alpha$ until maximum upward travel over one cycle was obtained. A pure digital approach is applied to provide optimization of the in-plane non-sinusoidal type of trough drive provided by a four-bar mechanism. All possible arm combinations of that mechanism are tried, so as to establish general optimum design trends, but general optimal parameter sets are not found,due to the large number of parameters involved.

For an in-plane and normal-to-trough plane driving cycle, only sinusoidal drives are evaluated. The free flight mode was not studied. Analog simulation was applied in this case, and maximum upward travel was achieved for the case of pure sliding with the system parameters adjusted to the threshold of the free flight mode.

It is difficult to establish general trends for the four bar-driven types of conveyers, whose simulation was discussed in the previous Chapters.

In general, an increase in $\mu$ will result in an increase in the object's upward travel. It is important to note that $\mu$ may change through the normal wear of the trough material, and can result in a downward travel, rather than an upward object travel,after some time.

The changes in $\alpha$ will not only affect the topology of the driving cycle, but also will change the boundaries BU and BL. Changes in $\alpha$, especially at large values of $\mu$, considerably affect conveyor performance.

However, at low values of $\mu$, this effect is decreased.

To achieve optimum design, it is necessary to use the FORTRAN program which has been developed to cover possible combinations of the four-bar mechanism.

A complete or partial scanning of these combinations can be adjusted by the designer, as required. The object travel per cycle is given as one of the program output at different values of $\mu$ and $\alpha$. The designer can easily select the optimum object travel and the associated system parameters. When constraints are imposed on the parameters, the designer can still use the optimization tool previously discussed and adjust the constrained parameters to suit his design problem. A design example was given to illustrate the above concepts.

# CONCLUSION

A method for the design of oscillatory conveyers has now been developed. The method is based on a simulation model of an actual conveying system, and allows optimization of design parameters for particular applications.

The particular cases of conveyers driven by a four bar mechanism has been studied. The same method can be used for other types of driving mechanism by changing the digital simulation of the driving mechanism to the type of mechanism available.

The dynamics of one object placed on the conveyer are considered.

The analog simulation gives another alternative tool to the designer for optimization of sinusoidal and non-sinusoidal types of conveyers.

The four bar mechanism used to drive the trough shows that at small ratios of links $r_1, r_2, r_3, r_4$ the speed has to be increased, in order to keep $A_m$ and $A_n$ greater than $B_U$ and $B_L$ which results in object's relative displacements to the trough.

The effect of increasing the dimension of the link $r_3$ in this mechanism severely affects the object travel as it changes the configuration of the acceleration cycle at a given trough angle.

A useful acceleration cycle is that which has its positive portion last longer than its negative portion, so its average positive value is less than its average negative value. Therefore, the object travel will increase with an increase in $\mu$ up to a turning point decided by the leading intersection points of the trough acceleration cycle to the $B_U$ and $B_L$ lines.

The optimum performance can be obtained at all values of the unconstrained system parameters, but not all of these values are realistic. Optimum design charts for realistic values can be provided for some defined parameter sets, but still it will not fulfil and meet the designer requirements for the unpredicted constraints in his design problem. It is found that there is no consistent pattern in the system performance when variations in the parameters occur. Therefore, the FORTRAN PROGRAM previously established, can be run. This will provide the design parameters at which optimum conveyer performance is achieved. Also these parameters will meet the desired constraints of the design problem.

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FIGURES

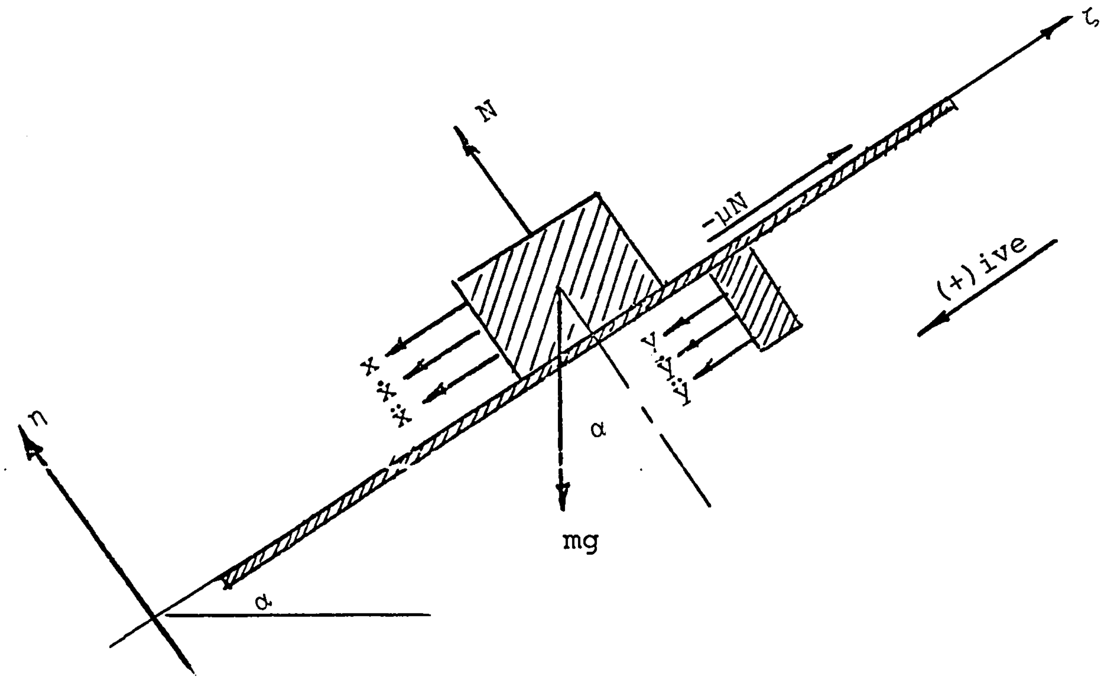



FIG.1 Object dynamics

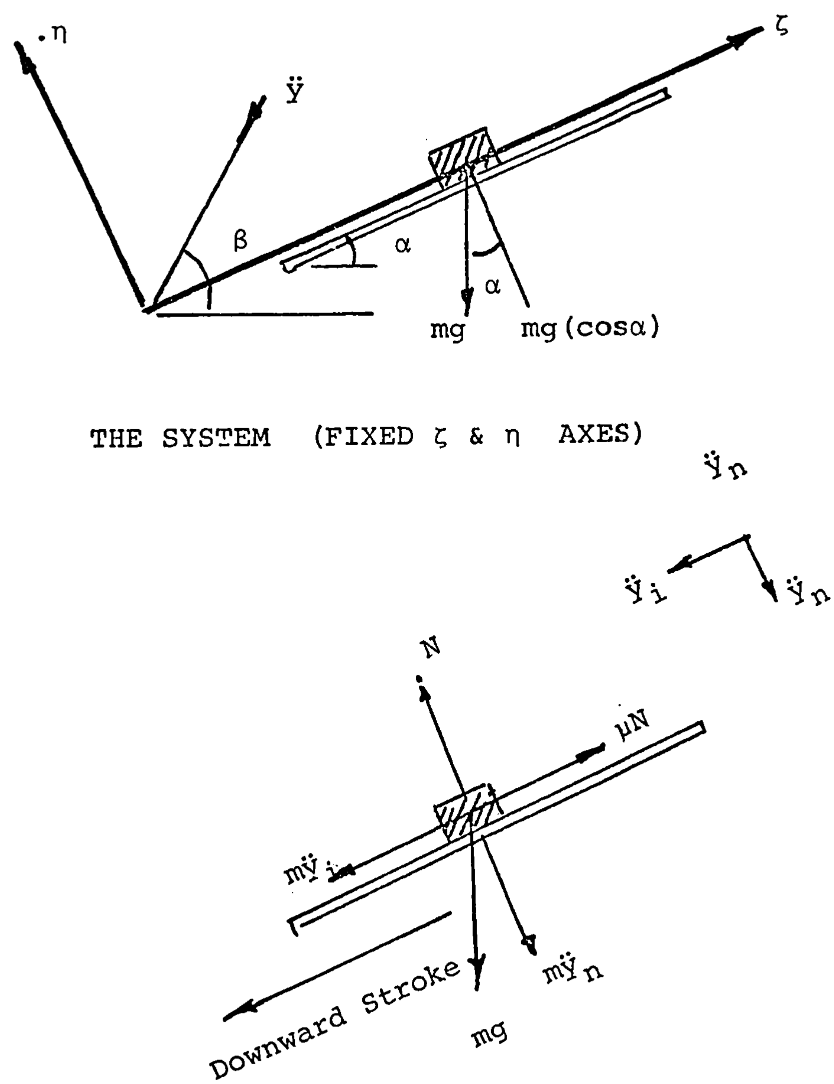



FIG. 2 Object dynamics when subjected to two acceleration cycles in and normal to conveyer plane.

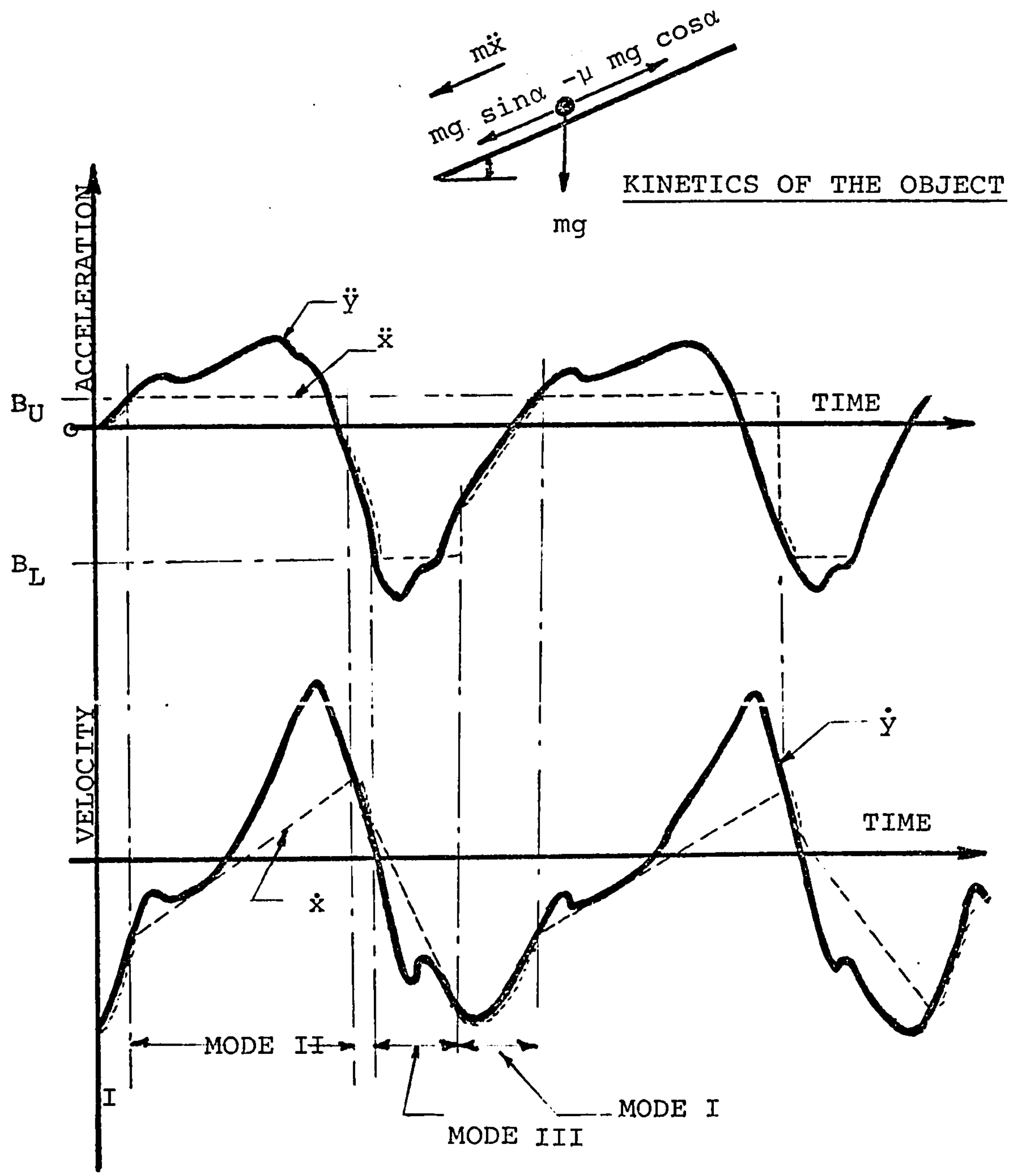



FIG.3 The three modes of an object moving along an oscillatory conveyer driven by an in-plane nonsinusoidal acceleration cycle.

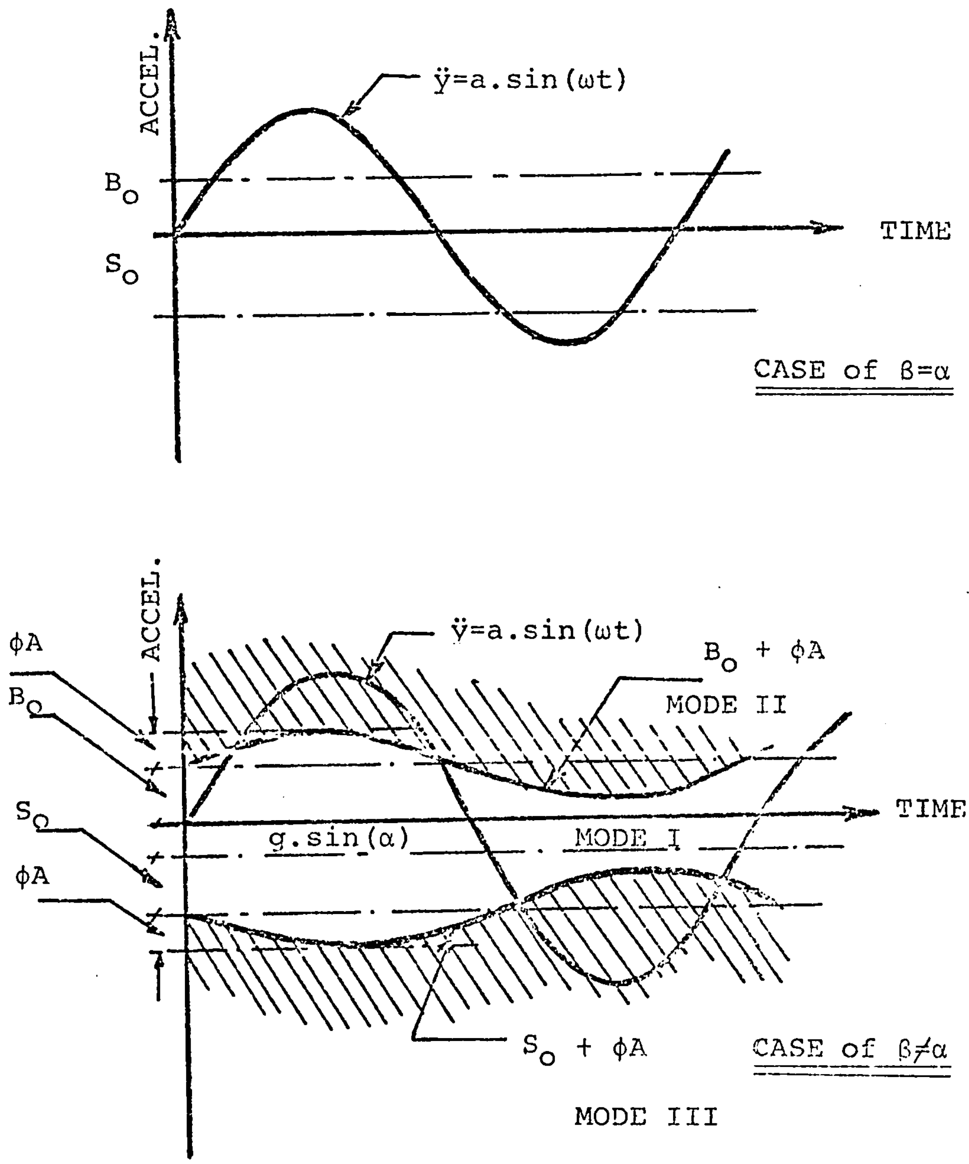



FIG. 4 The upper and lower bounds of the object acceleration when in-plane and normal-to-plane sinusoidal acceleration cycles are applied to the trough.

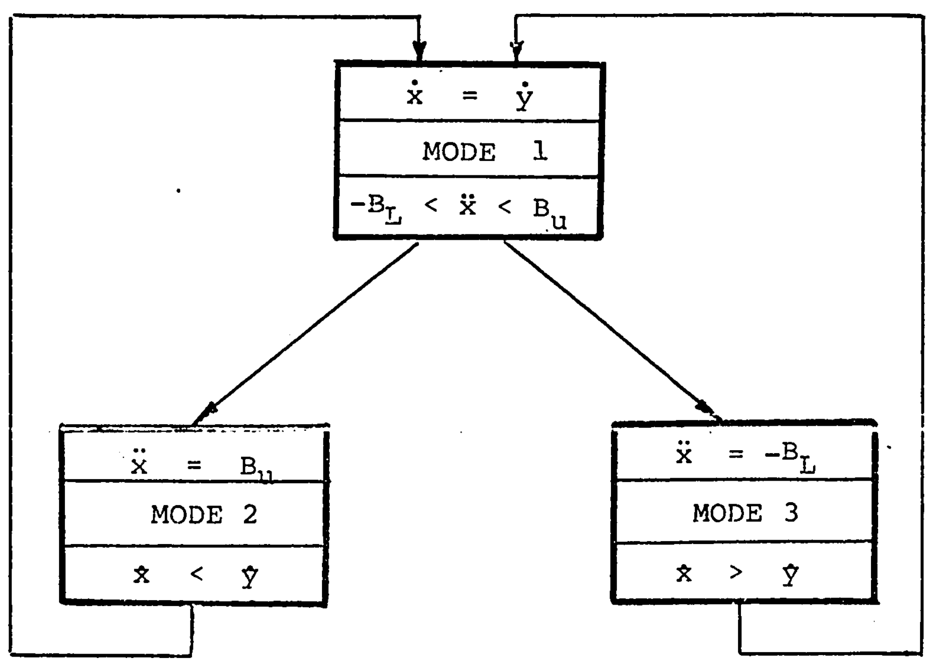



FIG. 5 Transitions between object's modes.

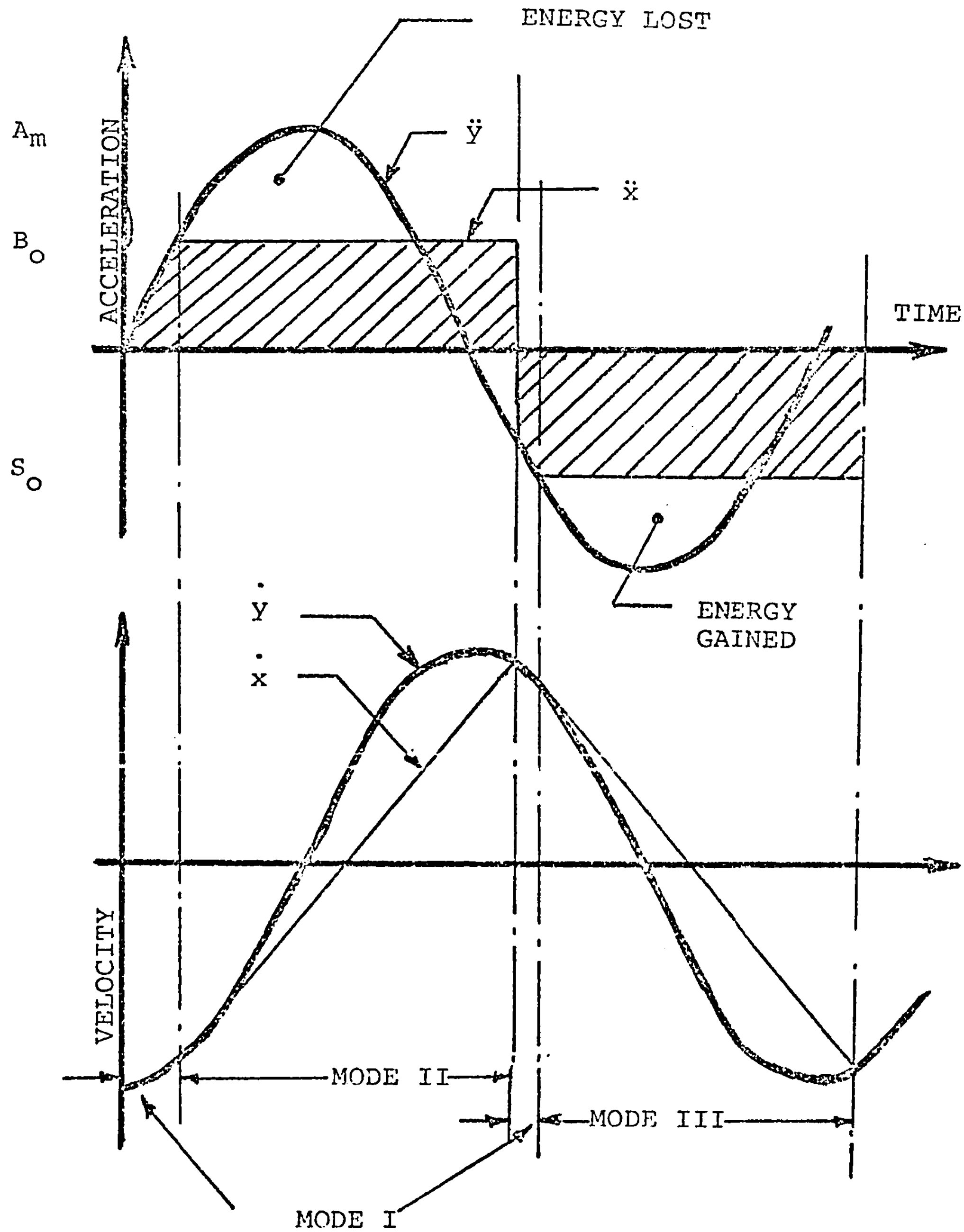



FIG. 6 Energy lost and energy gained for an in-plane sinusoidal driving cycle.

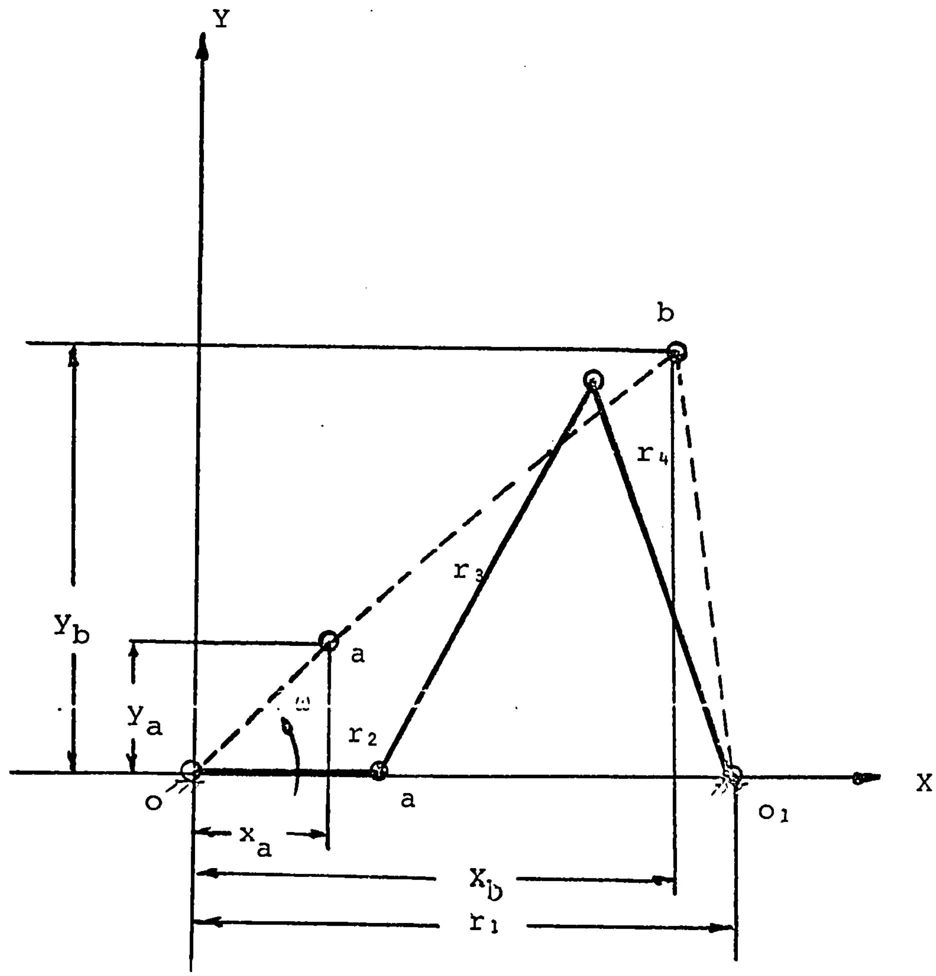



FIG. 7 Displacement components of the coupling point B in the four bar mechanism.

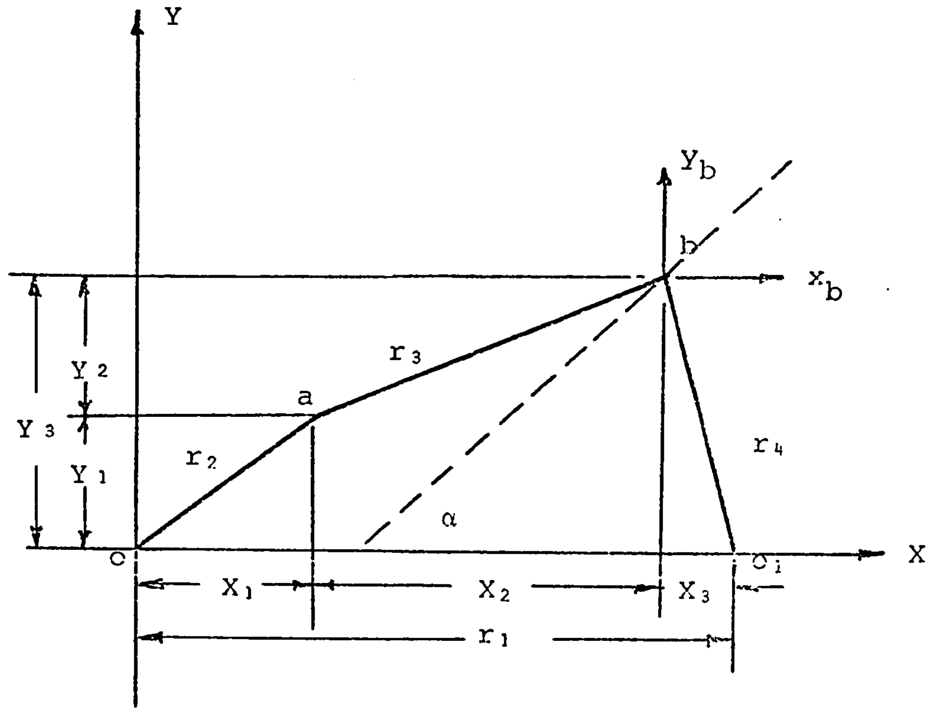



FIG. 8 Coordinates of the driver link and the coupling point (b) of the four bar mechanism used in analog simulation

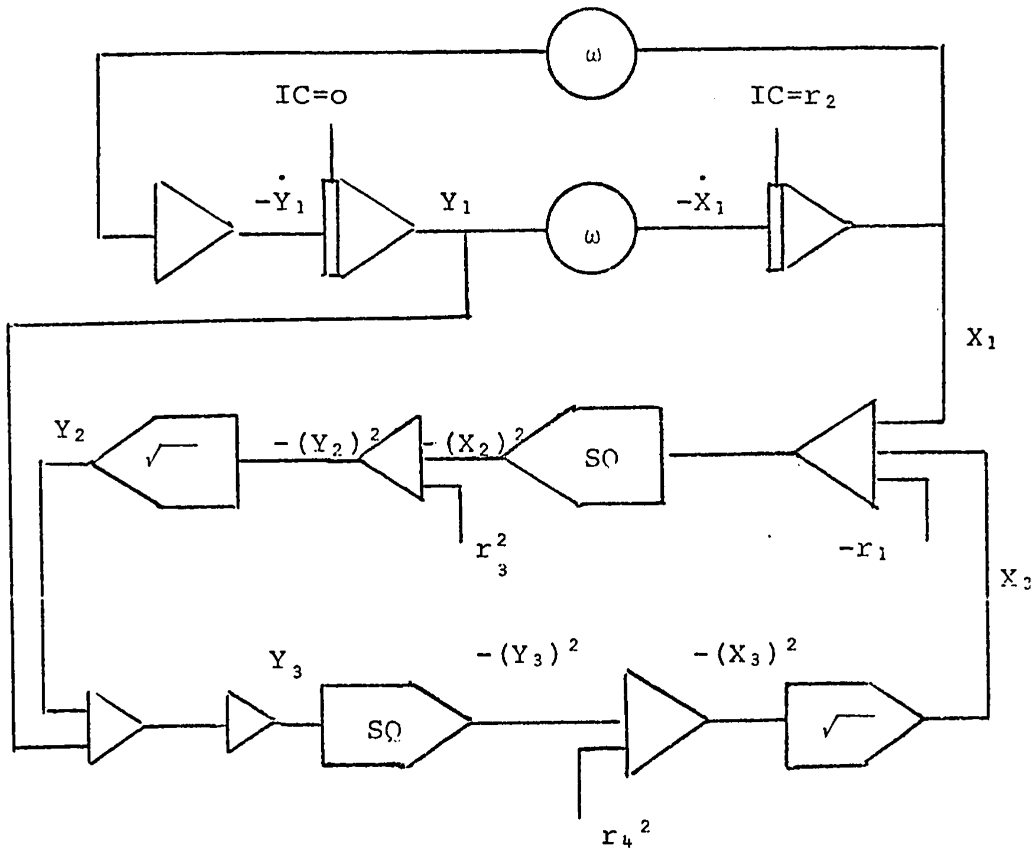



FIG. 9 Analog simulation of the displacement components X and Y of the four bar mechanism.

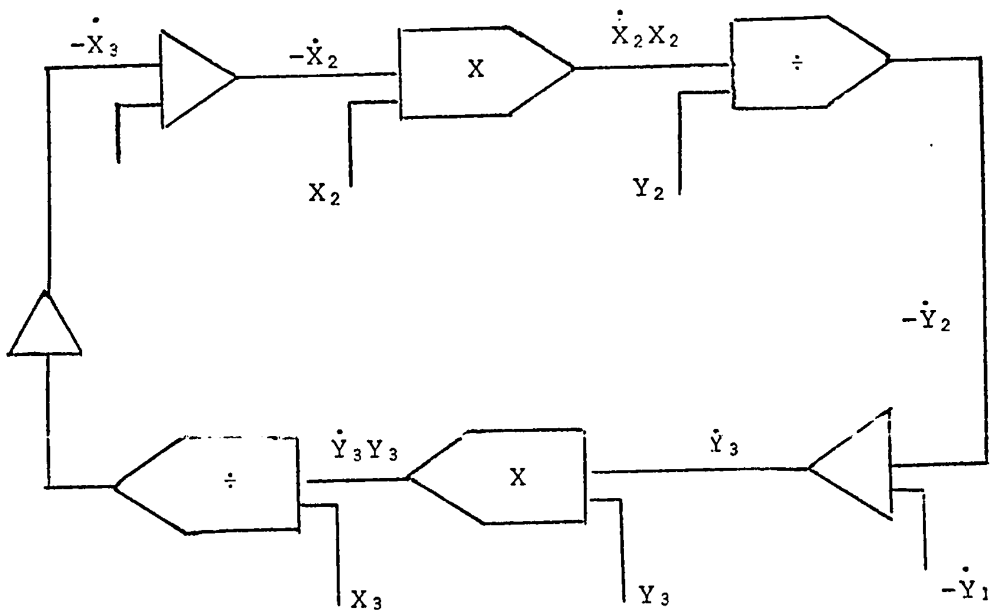



FIG. 10 Analog simulation of the velocity components $\dot{X}$ and $\dot{Y}$ of the four bar mechanism.

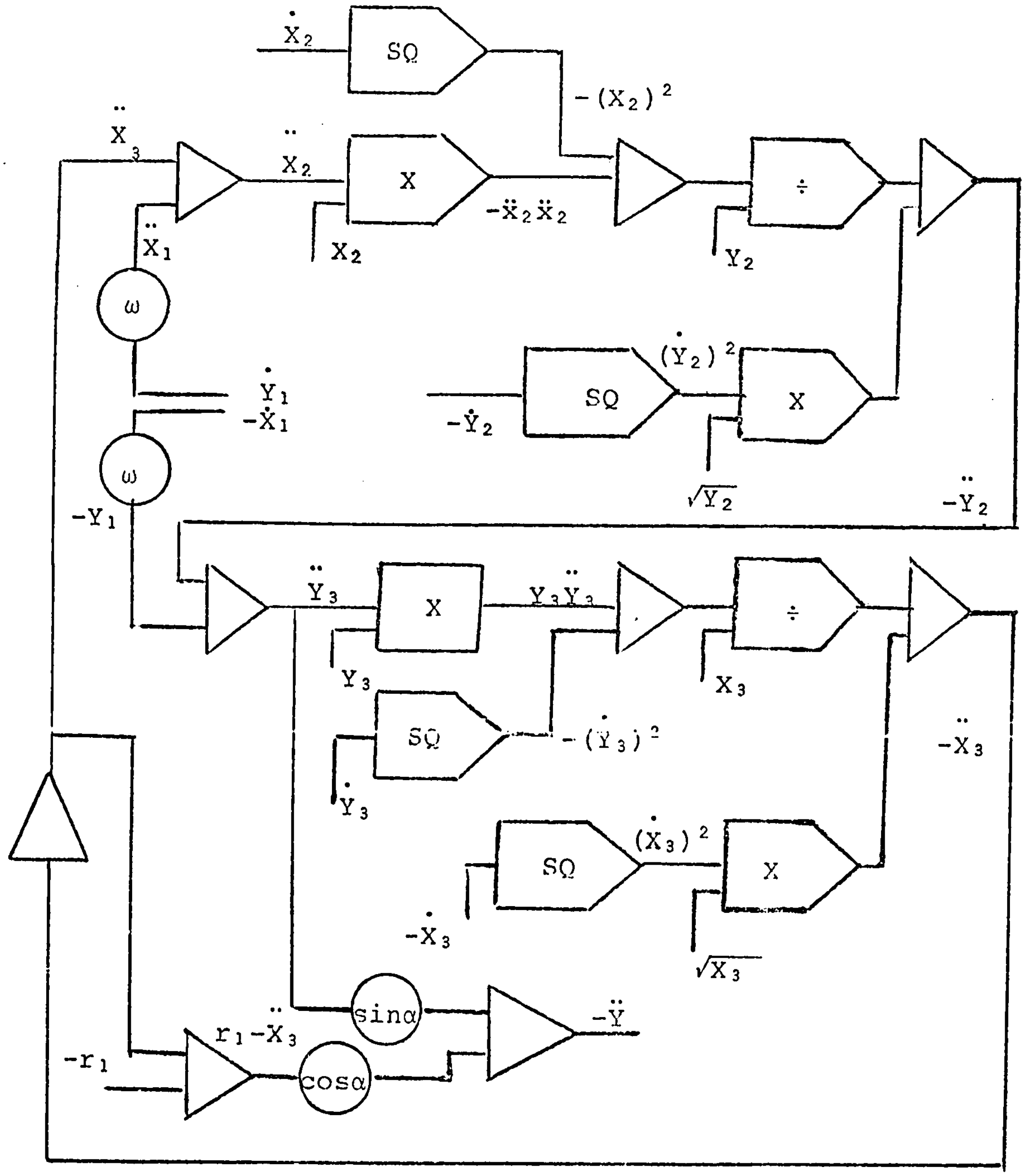
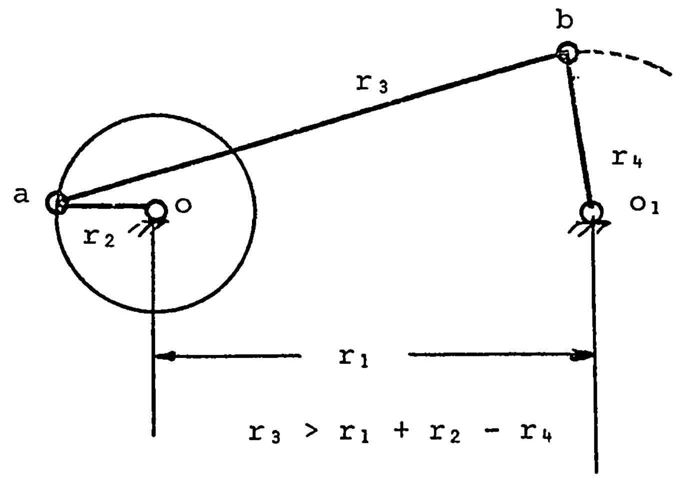



FIG. 11 Analog simulation of acceleration components $\ddot{X}$ and $\ddot{Y}$ that can provide the trough drive acceleration cycle



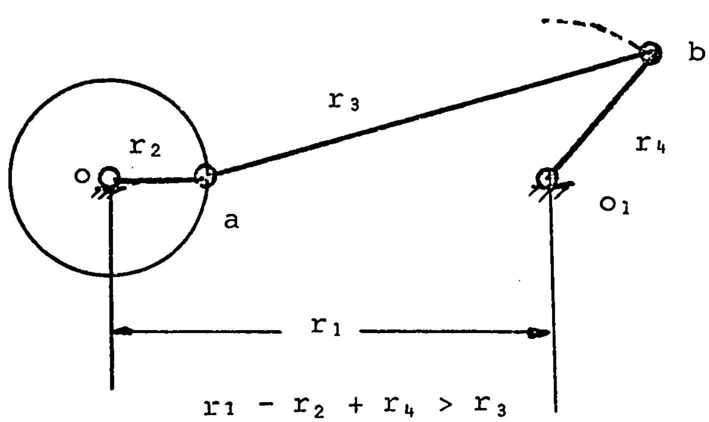
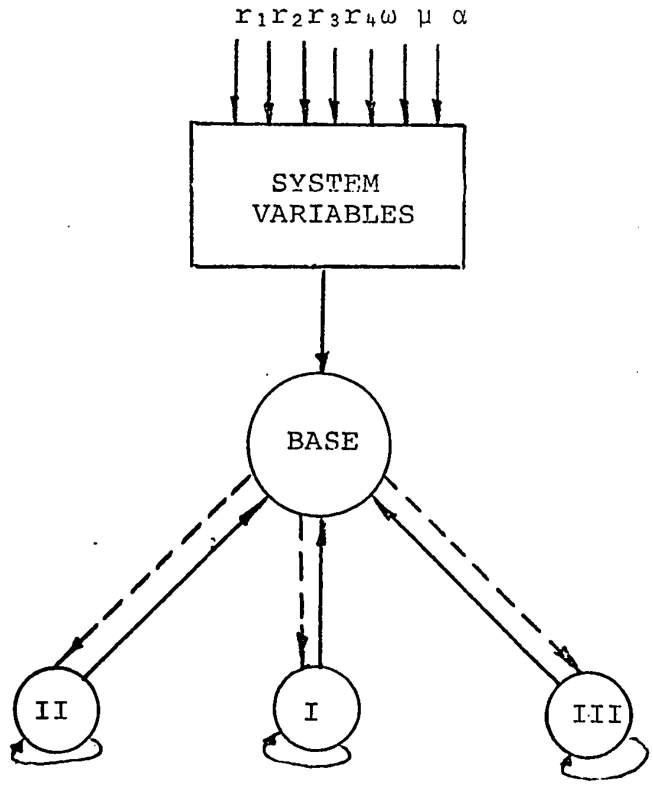
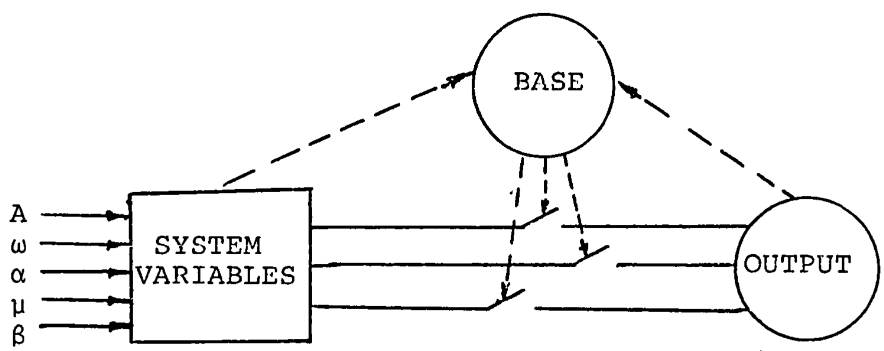




FIG. 12 Locking conditions for the crank lever type of the four bar mechanism.




METHOD I




METHOD II

FIG. 13 Methods of simulation

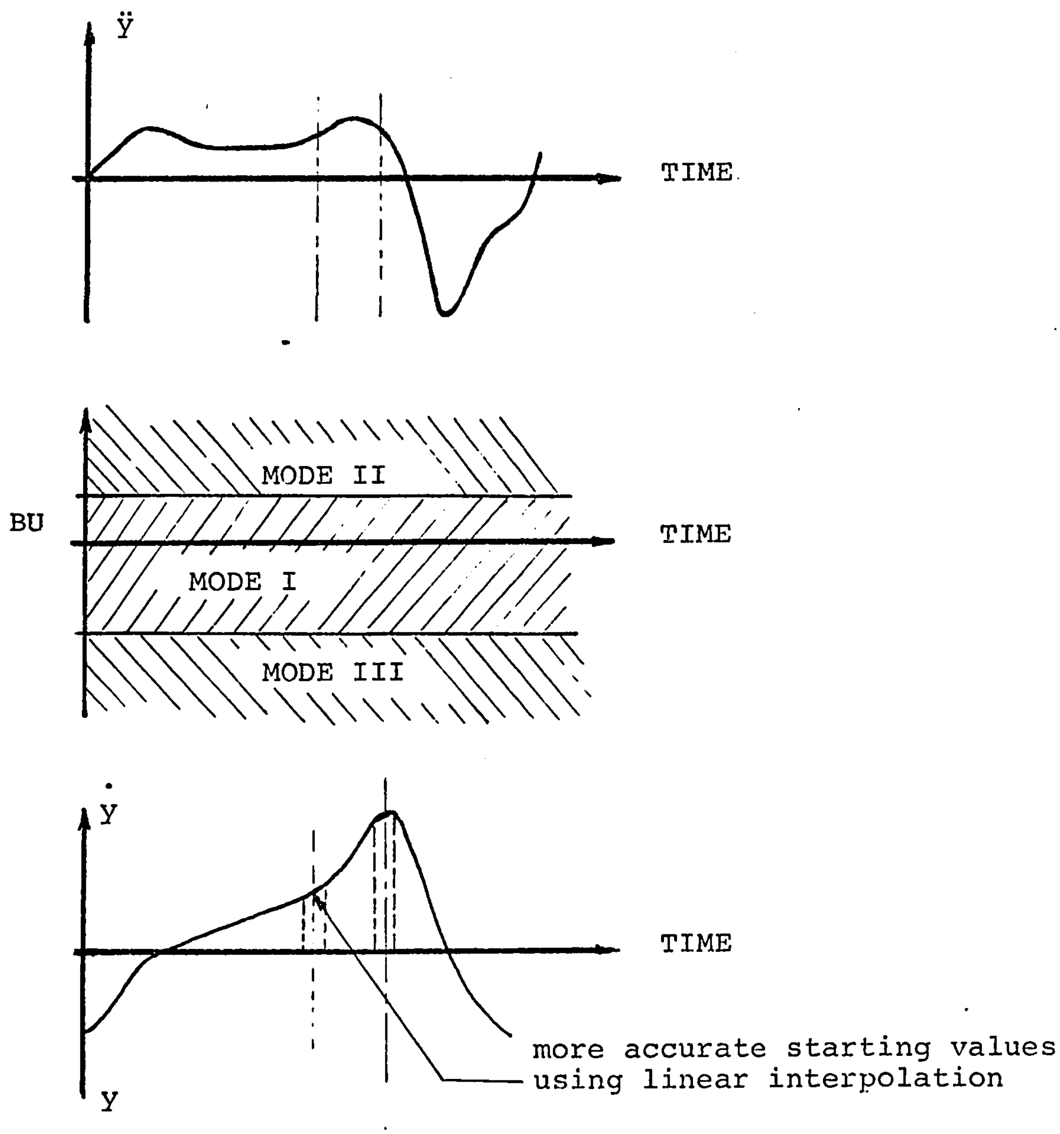



FIG. 14 Detection of object modes and computation of refined values at transition points.

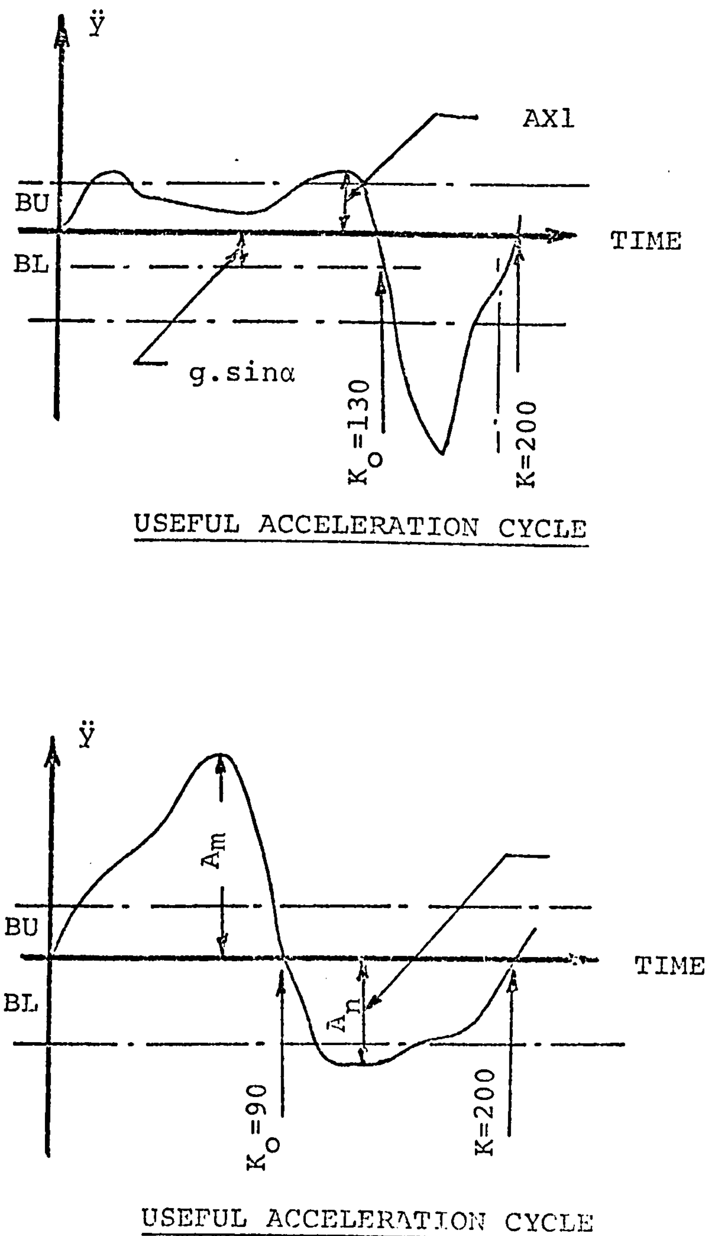



FIG. 15 Optimization criterion for the trough acceleration cycle.

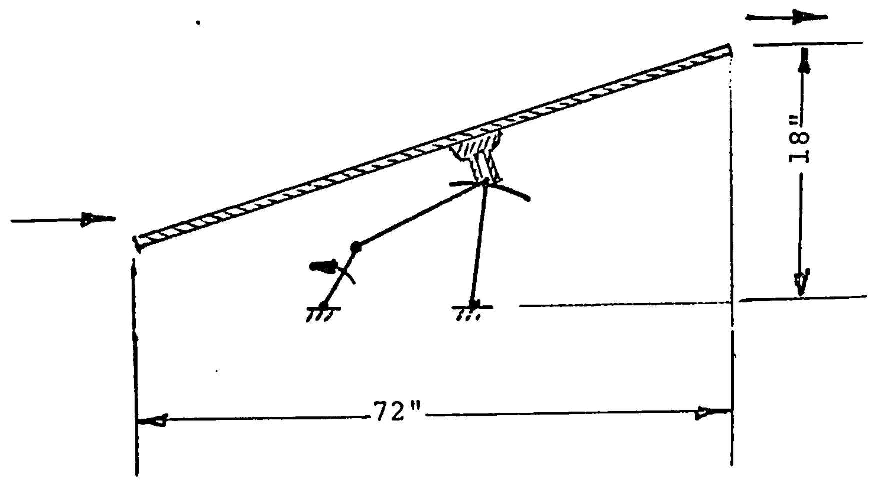



FIG. 16 The design example of an oscillatory conveyer.

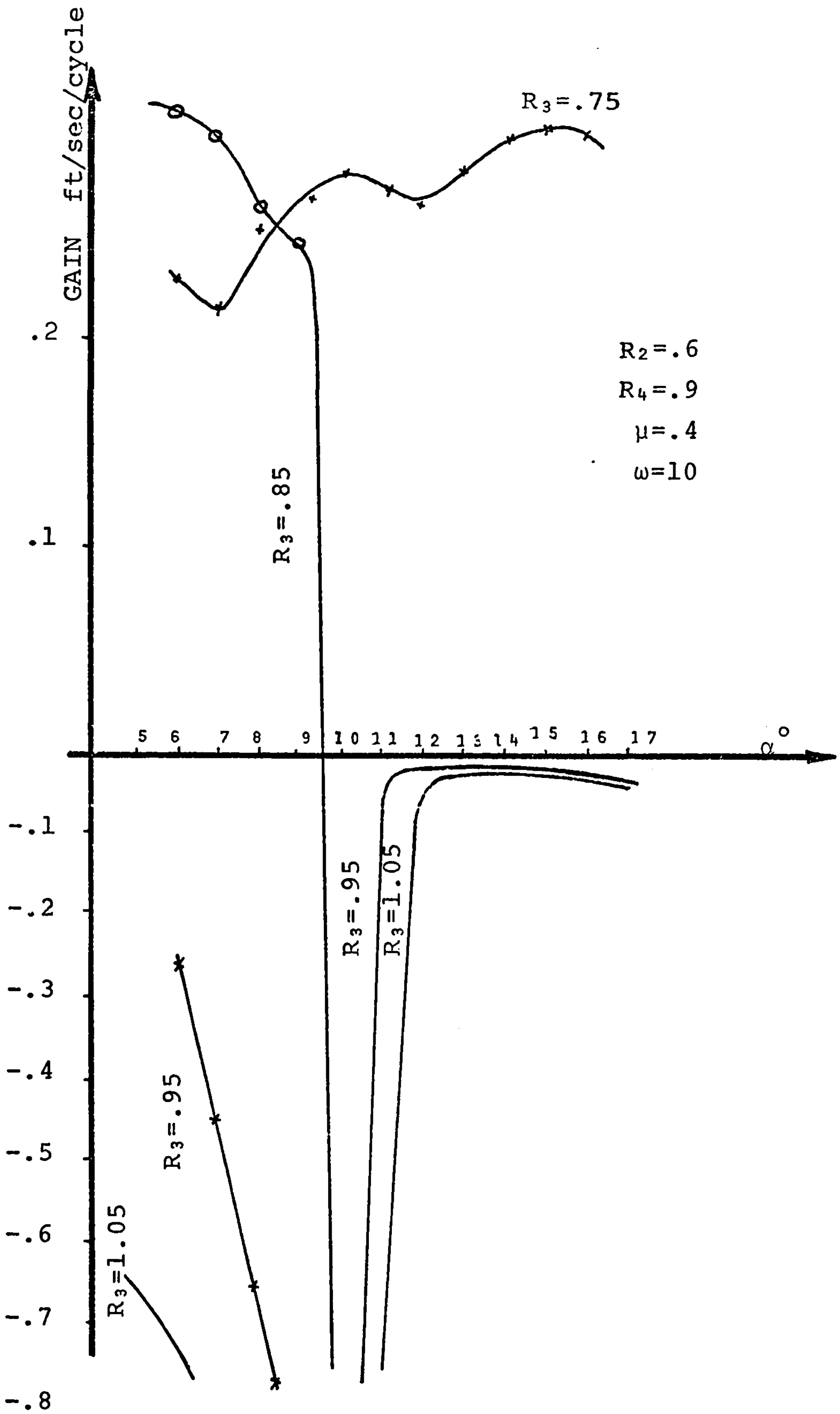



FIG. 17 Effect of the parameter R3 on object travel.

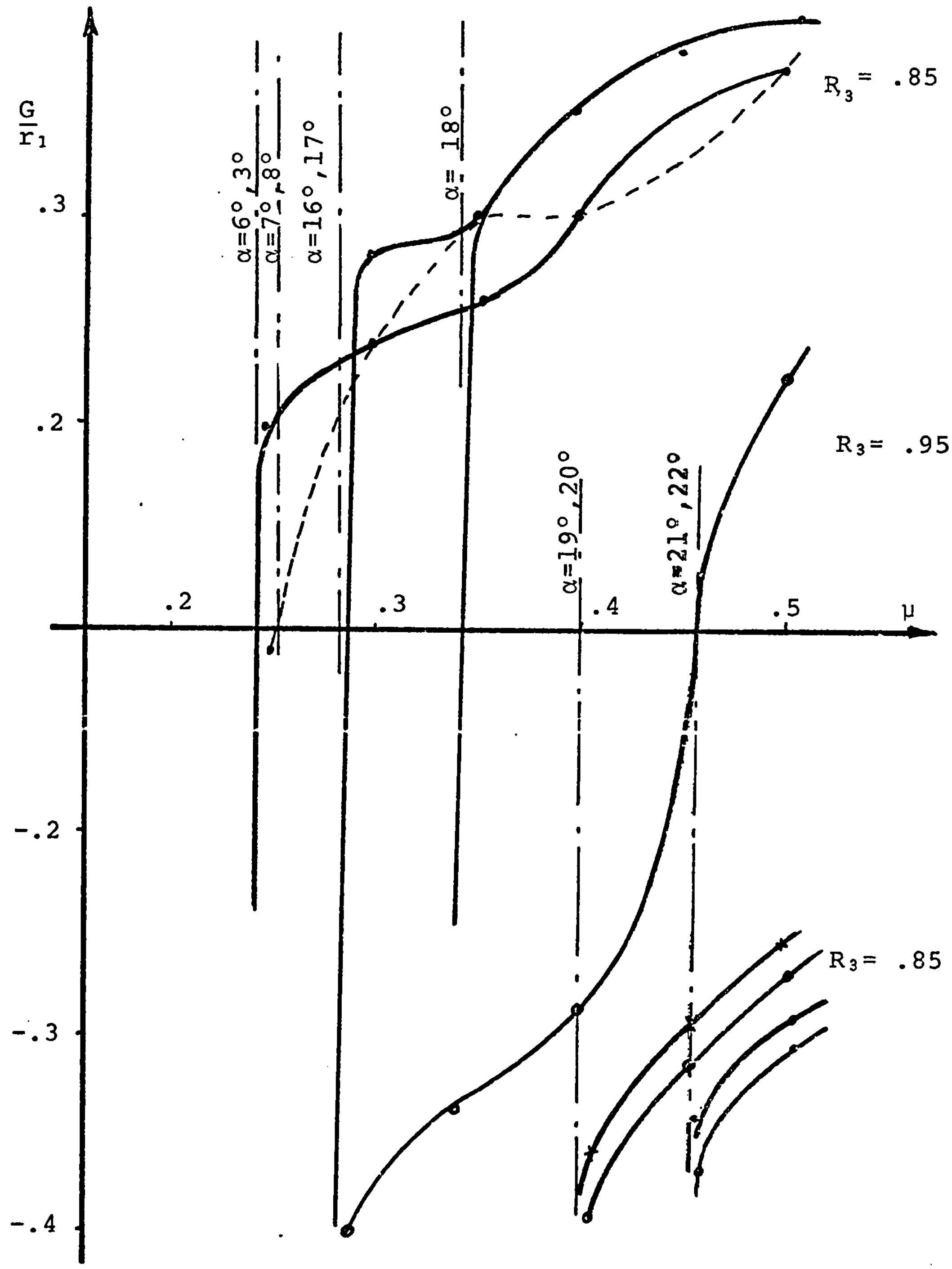



FIG. 18 Effect of μ,α and R3 on object travel, keeping R2, R4, and R1 constant. R2=.7, R4=.9, and R1=10. radians/sec.

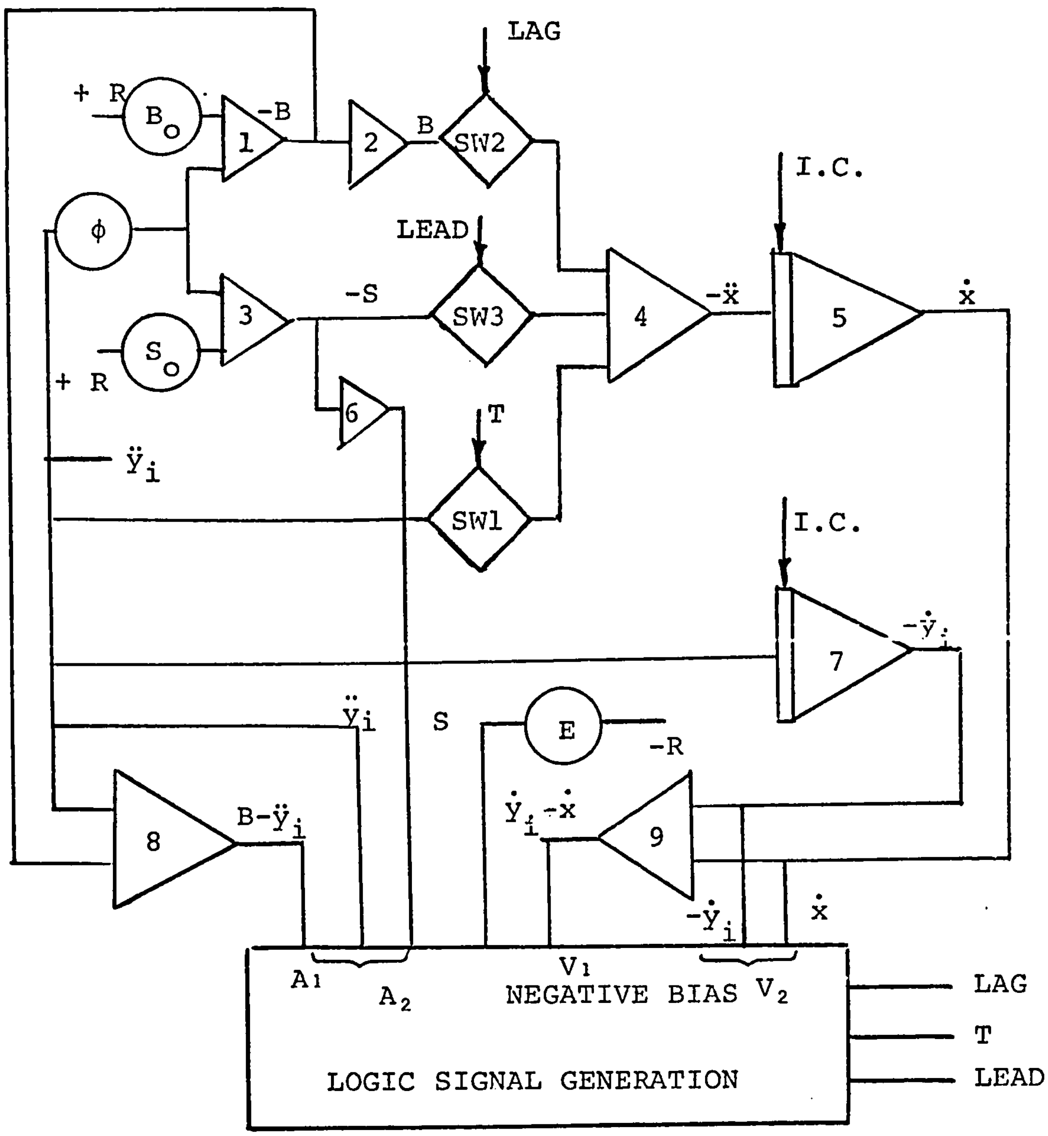



FIG. 19 The analog simulation of the sliding modes.

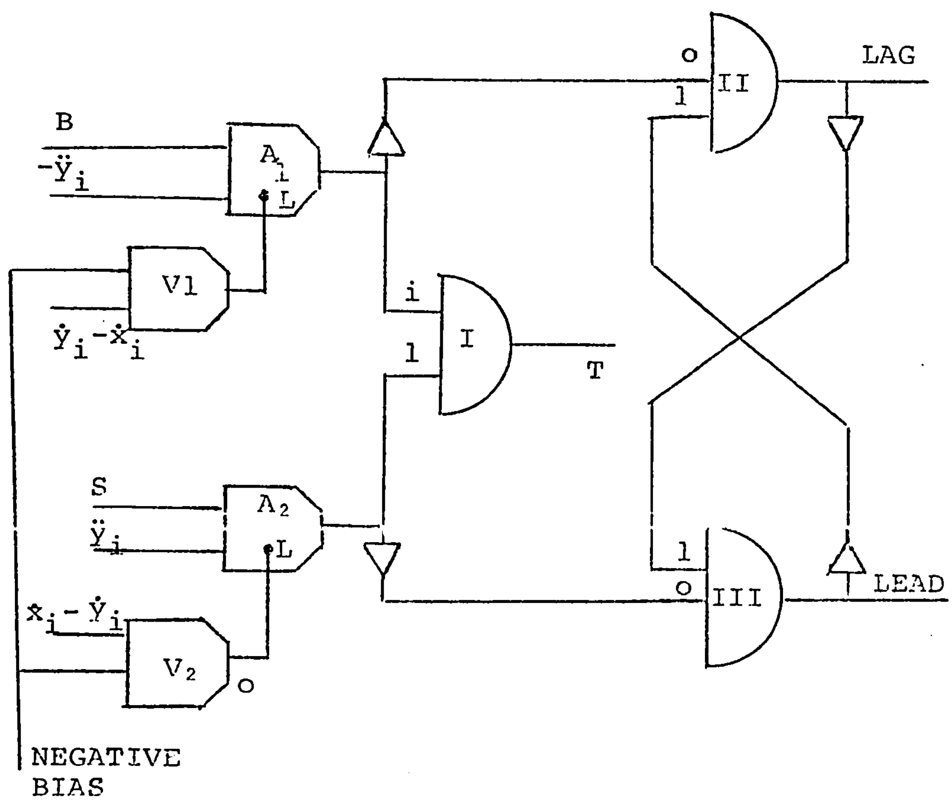



FIG. 20 Logic control signals generation.

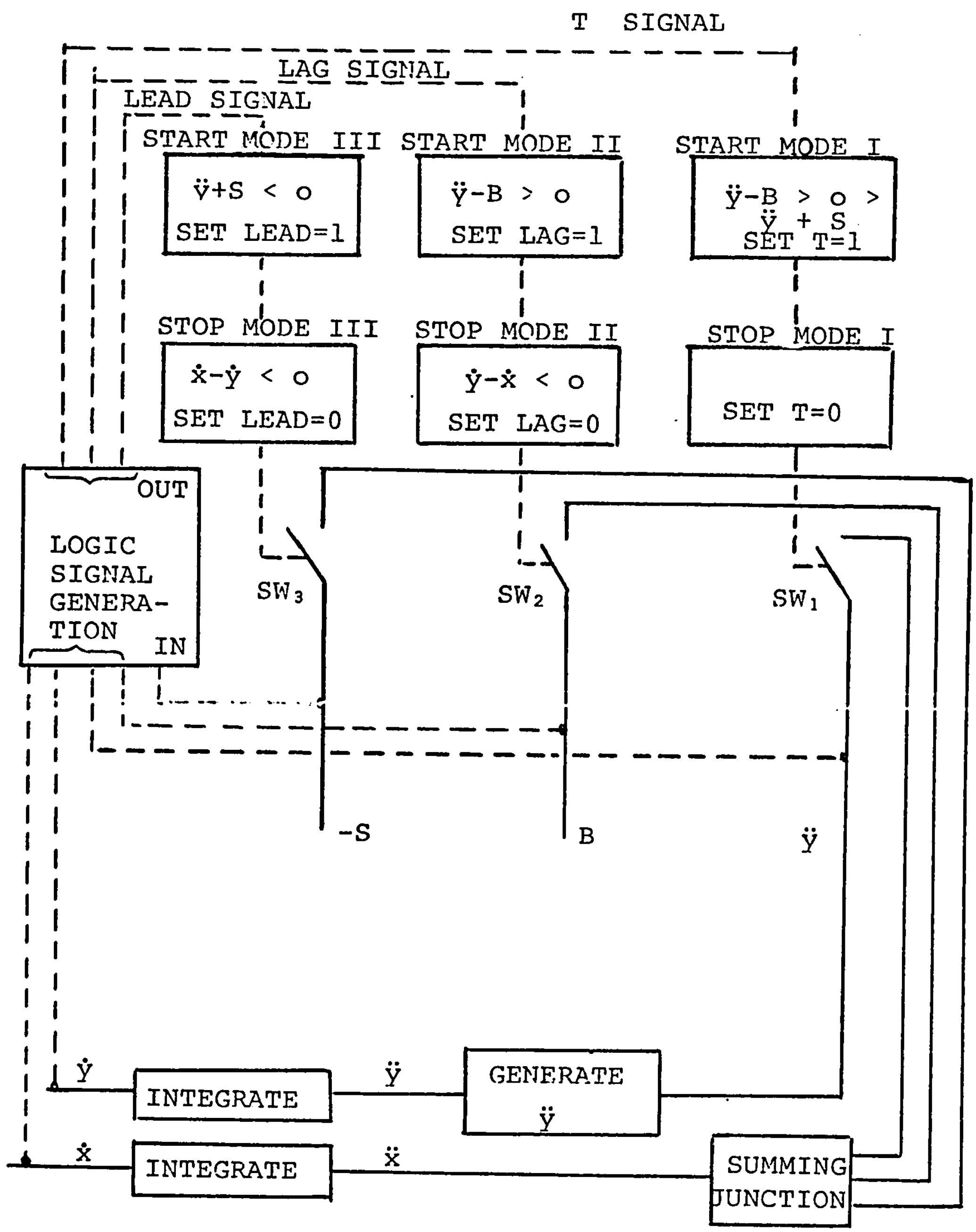



FIG. 21 Analog simulation block diagram

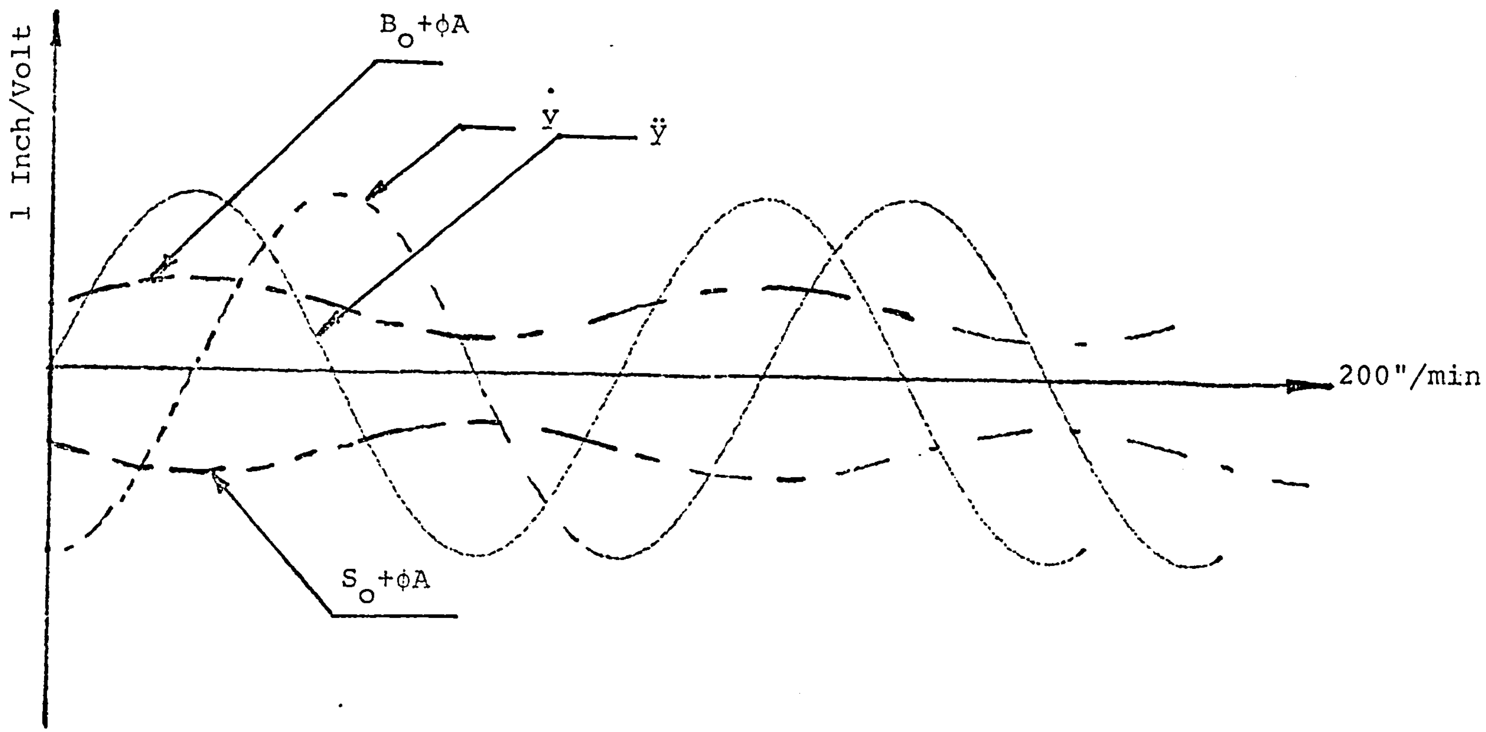



FIG. 22 Analog simulation result showing the boundaries of the object's acceleration cycle for an in-plane and normal-to-plane trough drive.

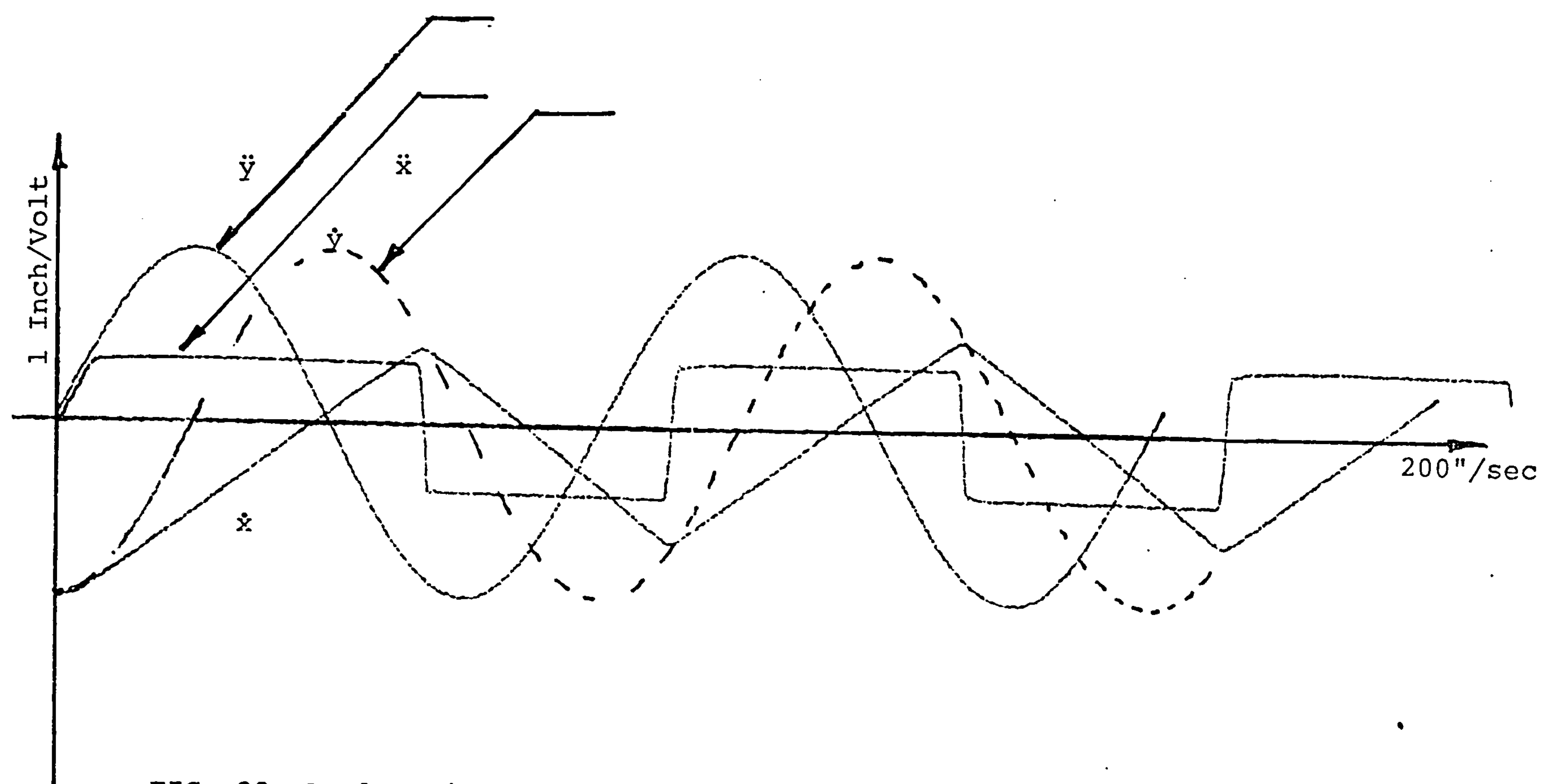



FIG. 23 Analog simulation result of sinusoidal trough drive in-plane and normal to it, showing the object's velocity and acceleration cycles.

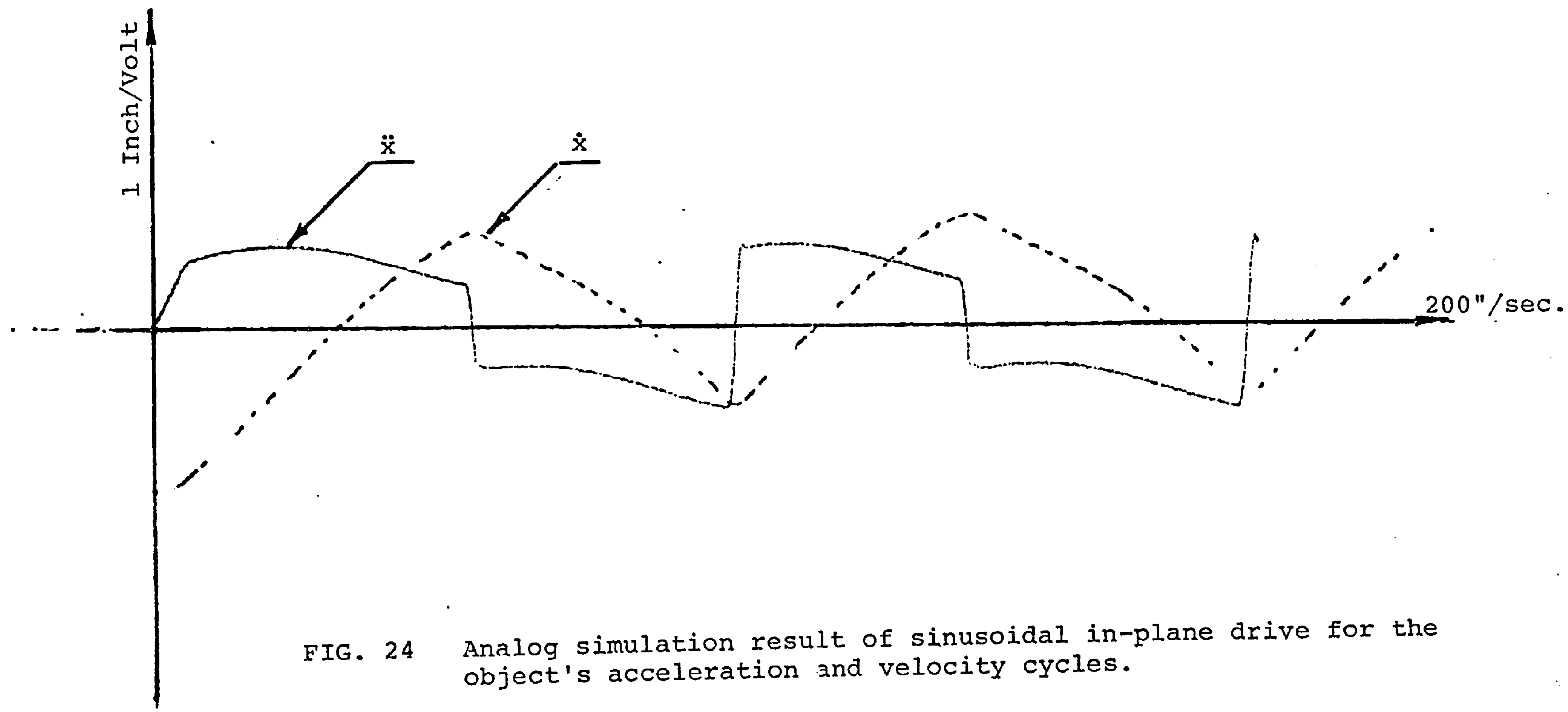



FIG. 24 Analog simulation result of sinusoidal in-plane drive for the object's acceleration and velocity cycles.

TABLES

| STATE | $y-B_{IJ}$ | $y+E_L$ | $\dot{y}-\dot{x}$ | $\dot{x}-\dot{y}$ | Mode |
|---|---|---|---|---|---|
| 1 | - | + | $\dot{y}=\dot{x}$ | $\dot{x}=\dot{y}$ | I |
| 2 | + | + | $\dot{y}=\dot{x}$ | $\dot{x}=\dot{y}$ | II |
| 3 | + | + | + | - | II |
| 4 | - | + | + | - | II |
| 5 | - | - | + | - | II |
| 6 | - | - | $\dot{y}=\dot{x}$ | $\dot{x}=\dot{y}$ | III |
| 7 | - | - | - | + | III |
| 8 | - | + | - | + | III |
| 9 | + | + | - | + | III |

TABLE 1

Possible states for the conveyed object when sliding conditions only are considered.

| STATE | y-B | y+B | $\dot{y}-\dot{x}$ | $\dot{x}-\dot{y}$ | N | $x_\eta - y_\eta$ | Mode |
|---|---|---|---|---|---|---|---|
| 1 | - | + | $\dot{y}=\dot{x}$ | $\dot{x}=\dot{y}$ | + | | I |
| 2 | + | + | $\dot{y}=\dot{x}$ | $\dot{x}=\dot{y}$ | + | | II |
| 3 | + | + | + | - | + | | II |
| 4 | - | + | + | - | + | | II |
| 5 | - | - | + | - | + | | II |
| 6 | - | - | $\dot{y}=\dot{x}$ | $\dot{x}=\dot{y}$ | + | | III |
| 7 | - | - | - | + | + | | III |
| 8 | - | + | - | + | + | | III |
| 9 | + | + | - | + | + | | III |
| 10 | | | | | - | + | IV |

TABLE 2

Possible states of the conveyed object subjected to in-plane and normal-to-trough plane acceleration cycles, to show the free flight mode.

| Trough Material | Object Material | Static Friction | Sliding Friction |
|---|---|---|---|
| Mild Steel | Plastic | - | .35 |
| | Aluminum | .61 | .47 |
| | Magnesium | - | .42 |
| Cast Iron | Steel | - | .23 |
| | Copper | 1.05 | .29 |
| | Brass | - | .30 |
| | Zinc | - | .21 |
| | Lead | - | .43 |
| | Tin | - | .32 |
| | Leather | - | .56 |
| Aluminum | Aluminum | 1.05 | 1.4 |
| Glass | Copper | 1.10 | .15 |
| Oak | Oak | .54 | .32 |
| | Leather | .61 | .52 |

TABLE 3

Coefficients of dry static and dry sliding friction.

| STATE | A1 | A2 | V1 | V2 | T * | LAG ** | LEAD *** | MODE |
|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | I |
| 2 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | II |
| 3 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | II |
| 4 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | II |
| 5 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | II |
| 6 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | III |
| 7 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | III |
| 8 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | III |
| 9 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | III |

* $T = A1.A2$

** $LAG = \overline{A1}.\overline{LEAD}$

*** $LEAD = \overline{A2}.\overline{LAG}$

TAELE 4

Truth table for the logic control signals

APPENDIX I

# APPENDIX I

# THE FOUR BAR MECHANISM

## 1.1A Mathematical Relations

### 1.1.1A Relations Directly Obtained from Fig. (7)

$$x_a = r_2 \cos \omega t$$
$$y_a = r_2 \sin \omega t \qquad (1.1A)$$

$$\dot{x}_a = \omega y_a$$
$$\dot{y}_a = \omega x_a \qquad (1.2A)$$

$$\ddot{x}_a = \omega^2 x_a$$
$$\ddot{y}_a = \omega^2 y_a \qquad (1.3A)$$

$$(x_b - x_a)^2 + (y_b - y_a)^2 = r_3^2$$
$$(r_1 - x_b)^2 + y_b^2 = r_4^2 \qquad (1.4A)$$

### 1.1.2A Initial Conditions for the Driving Link are at $t = o$

$$\theta = o, \; x_a = r_2 \text{ and } y_a = o$$

Substituting in equation (1.4A) gives

$$(x_b - r_2)^2 + y_b^2 = r_3^2$$
$$(r_1 - x_b)^2 + y_b^2 = r_4^2 \qquad (1.5A)$$

Solving equation (1.5A) for $x_b$ and $y_b$ and evaluating their values at $t = o$ leads to

$$x_b = (r_1^2 + r_3^2 - r_2^2 - r_4^2)/2(r_1 - r_2)$$
$$y_b = \pm \sqrt{r_3^2 - (x_b - r_2)^2} \qquad (1.6A)$$

In equation (1.6A), $y_b$ can be positive or negative. The positive sign is chosen as the links of the four bar and are placed above the $x$ axis; the negative sign will provide the image of $y_b$ with respect to the $x$ axis, which is not used in the previous analysis.

1.1.3A <u>Velocity Components</u>

Velocity components are obtained by differentiating, with respect to $(t)$ equation (1.1.4A) which gives

$$(x_b - x_a)(\dot{x}_b - \dot{x}_a) + (y_b - y_a)(\dot{y}_b - \dot{y}_a) = 0$$
$$-(r_1 - x_b)\dot{x}_b + y_b\dot{y}_b = 0 \qquad (1.7A)$$

Solving for $\dot{x}_b$ and $\dot{y}_b$ and rearranging leads to

$$\dot{x}_b = y_b\dot{y}_b(y_b - y_a) + \dot{x}_a y_b(x_b - x_a)/$$
$$[y_b(x_b - x_a) + (y_b - y_a)(r_1 - x_b)] \qquad (1.8A)$$

$$\dot{y}_b = \dot{x}_b(r_1 - x_b)/y_b$$

### 1.1.4A Acceleration Components

Acceleration components are obtained by differentiating with respect to (t) equation (1.7A) which gives

$$(x_b-x_a)(\ddot{x}_b-\ddot{x}_a) + (\dot{x}_b-\dot{x}_a)^2 + (y_b-y_a)(\ddot{y}_b-\ddot{y}_a) + (\dot{y}_b-\dot{y}_a)^2 = 0$$

$$- (r_1-x_b)\ddot{x}_b + \dot{x}_b^2 + y_b\ddot{y}_b + \dot{y}_b^2 = 0 \qquad (1.9A)$$

Solving equation (1.9A) for $\ddot{x}_b$ and $\ddot{y}_b$ and rearranging

$$\ddot{x}_b = [\ddot{x}_a y_b(x_b-x_a) - y_b(\dot{x}_b\ \dot{x}_a)^2 + (y_b-y_a)(\dot{x}_b^2+y_b^2+y_b\ddot{y}_b)$$

$$- y_b(\dot{y}_a-\dot{y}_b)^2]/[y_b(x_b-x_a) + (y_b-y_a)(r_1-x_b)] \qquad (1.10A)$$

$$\ddot{y}_b = [\ddot{x}_b(r_1-x_b) - \dot{x}_b^2 - \dot{y}_b^2]/y_b$$

### 2.1A Application of the Newton Raphson Iteration Method

Equation (1.4A) describes the displacement components $x_b, y_b$ of the coupler point b in the four-bar mechanism. The values $\Delta x$ and $\Delta y$ used in finding the new values of the $x_b, y_b$ relative to the different driver link position are given as follows:

$$\frac{\partial f_1}{\partial x_b} = 2(x_b-r_2\cos\omega t) = U1 \qquad (1.11A)$$

$$\frac{\partial f_1}{\partial y_b} = 2(y_b-r_2\sin\omega t) = U2 \qquad (1.12A)$$

$$\frac{\partial f_2}{\partial x_b} = 2(x_b - r_1) = VI \tag{1.13A}$$

$$\frac{\partial f_2}{\partial y_b} = 2y_b = V2 \tag{1.14A}$$

$$-f_1(x_o,y_o) = r_3^2 - (x_b - r_2 \cos \omega t)^2 - (y_b - r_2 \sin \omega t)^2 =$$

$$= U \tag{1.15A}$$

$$-f_2(x_o,y_o) = r_4^2 - (x_b - r_1)^2 - y_b^2 = V \tag{1.16A}$$

$$\begin{bmatrix} U_1 & U_2 \\ V_1 & V_2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} U \\ V \end{bmatrix}$$

and in compact form

$$[A] \quad \{\delta\} \quad = \quad \{b\}$$

The solution will be in this form

$$\{\delta\} = \text{cofactor}\frac{[A]^T}{|A|}\{b\}$$

$$|A| = U1V2 - U2V1 = D \tag{1.17A}$$

Thus

$$\Delta x = (UV2 - VU2)/D$$

$$\Delta y = (VU1 - UV1)/D \tag{1.18A}$$

### 3.1A The Possible Four-Bar Link Combinations

A FORTRAN program is developed according to the basic ideas described in Chapter III. The following is the possible combination of R2, R3, and R4, with R1, taken as unity.

# APPENDIX II

# BLOCK DIAGRAMS AND

# FLOW CHARTS

# APPENDIX II

# OPTIMIZATION

## 2.1A OPTIMIZATION

Block diagrams and flow charts for the following:

i) Program Optimization

ii) Subroutine Newton

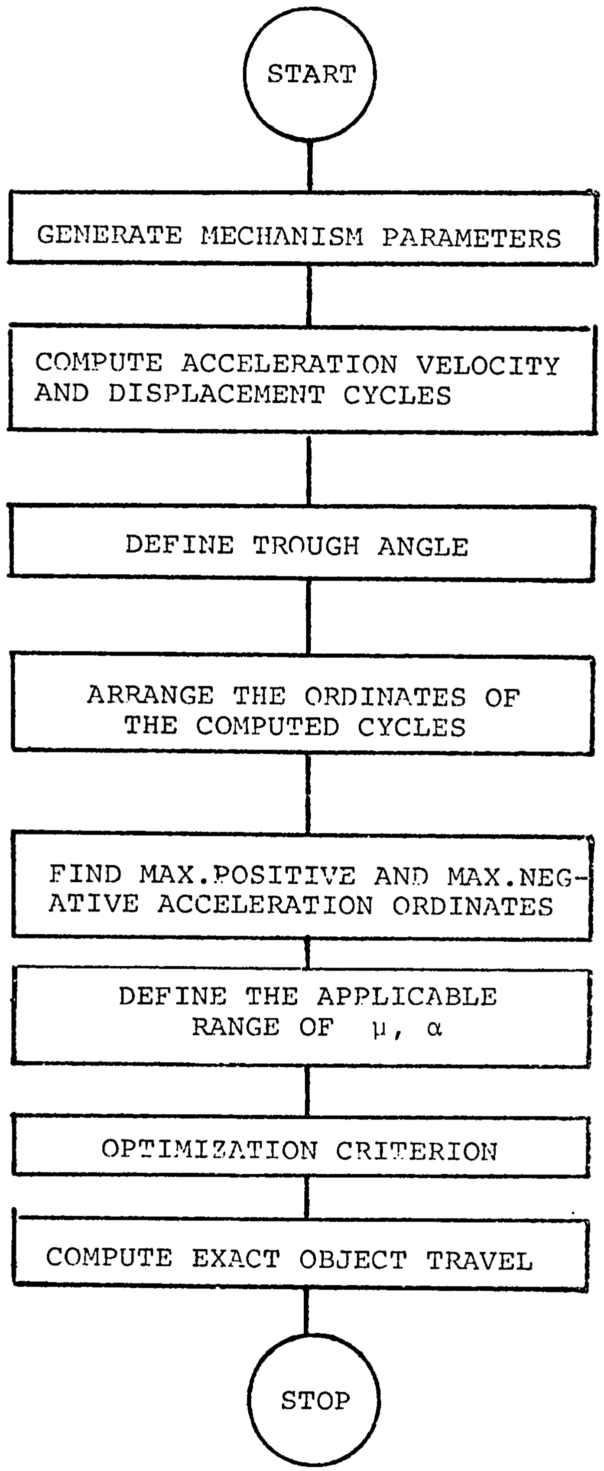
iii) Subroutine Sort

iv) Subroutine Conveyer

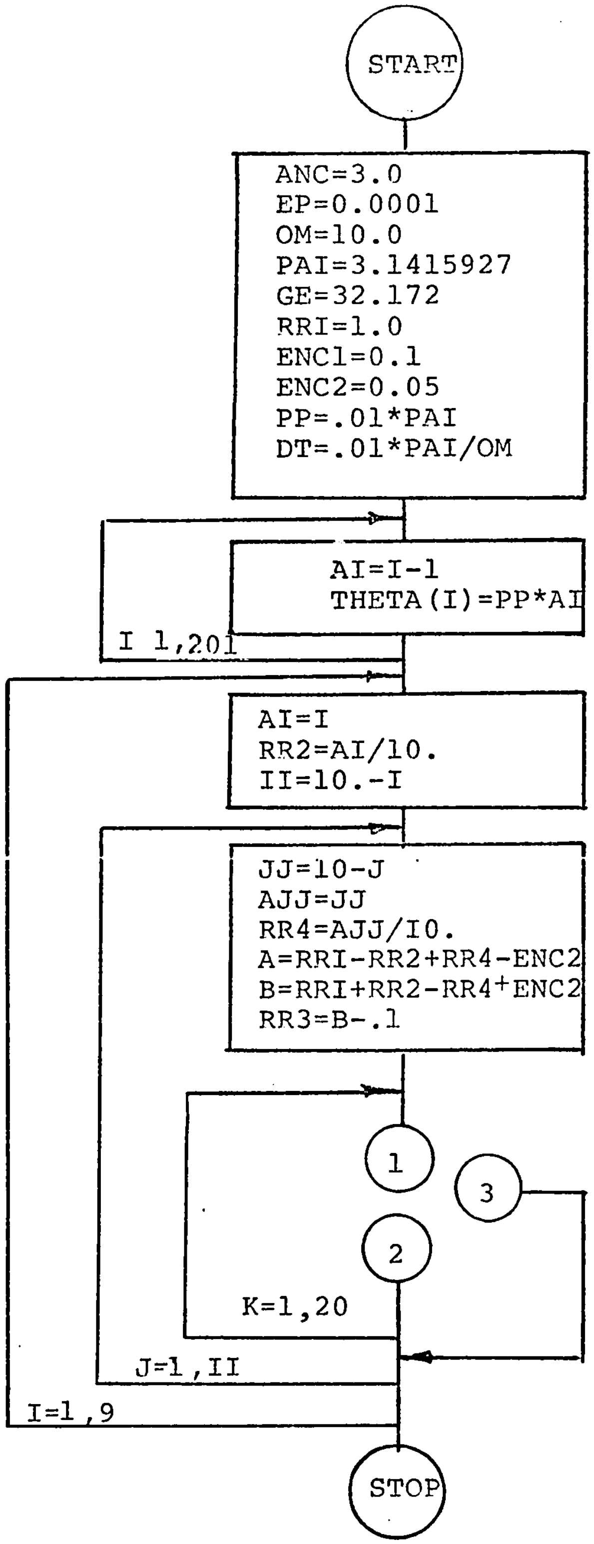
## 2.2A THE FORTRAN PROGRAM

The Fortran Program and plotted results.

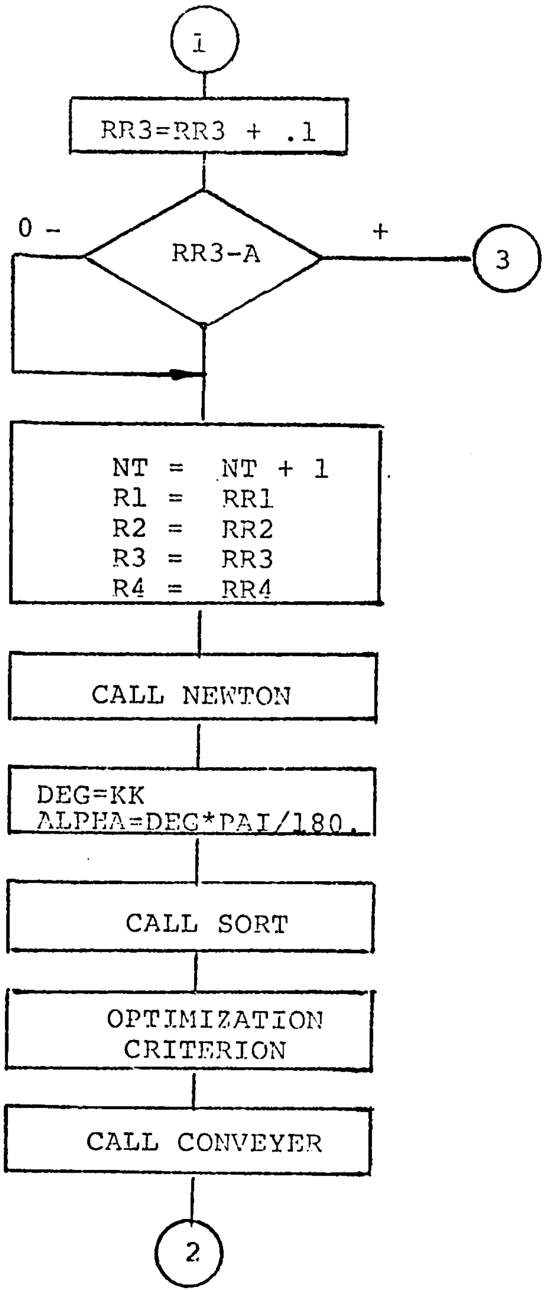
## 2.3A SAMPLE RESULTS




BLOCK DIAGRAM

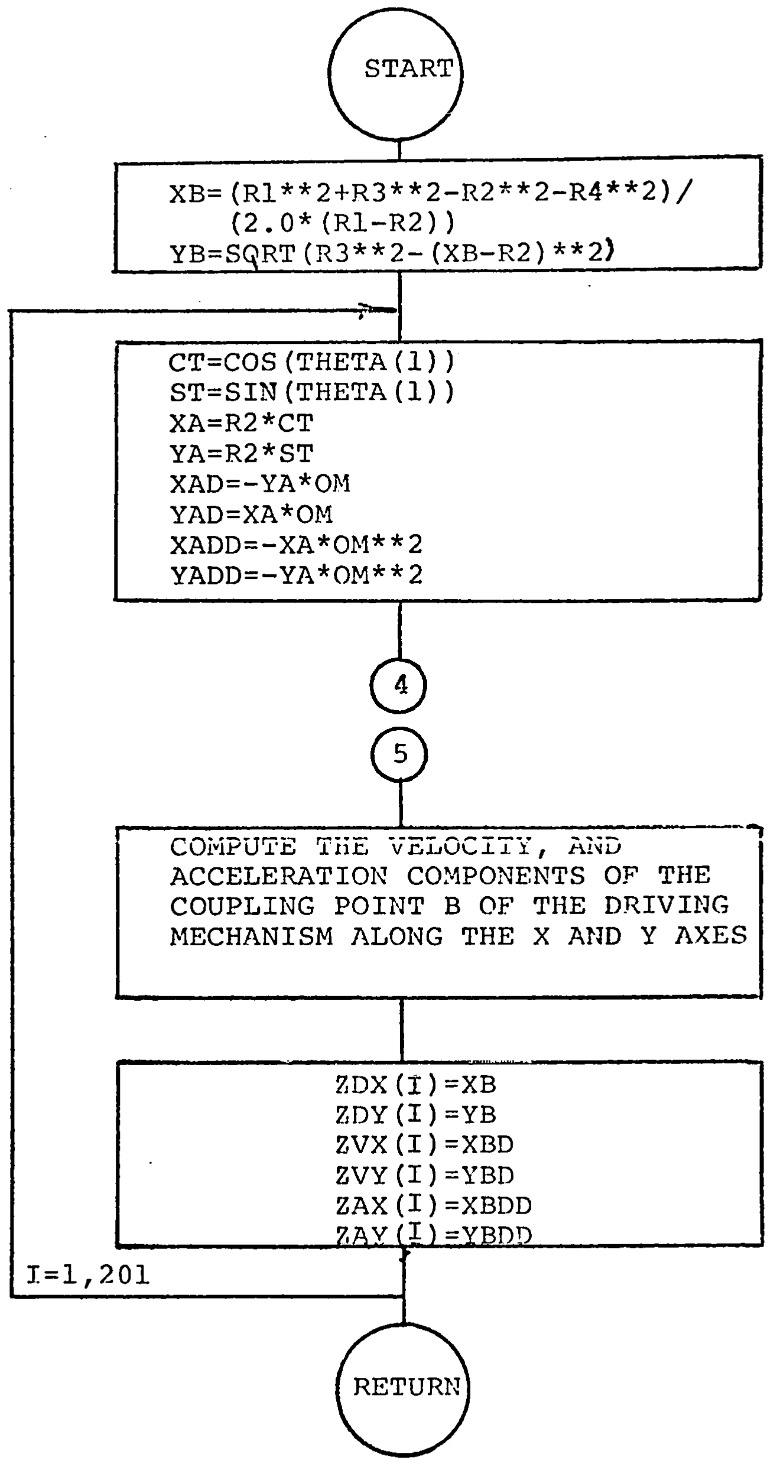



PROGRAM OPT
For generating the crank angles and
the sets of arms of the driving mechanism



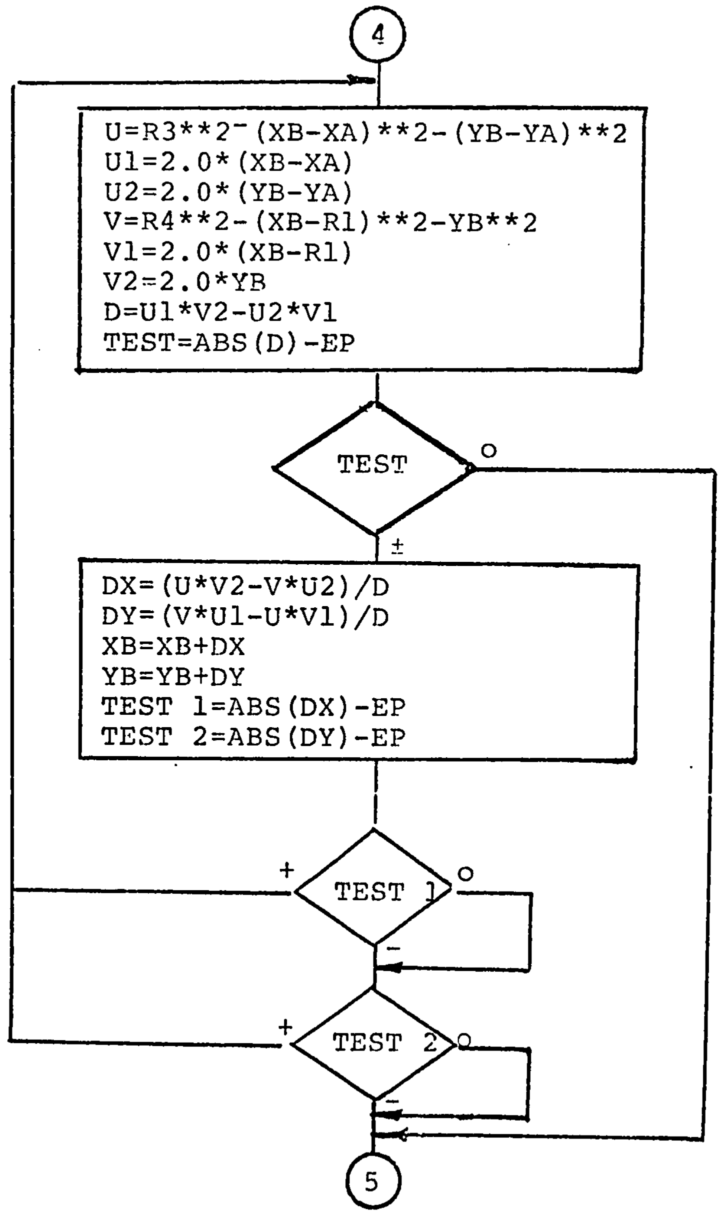

PROGRAM OPT. (CONT'D)

Computing the displacement, velocity, and acceleration cycle of the trough, and selecting the best coefficient of friction and the trough angle that gives maximum forward travel at a specified crank speed and arm's ratio of the driving mechanism.



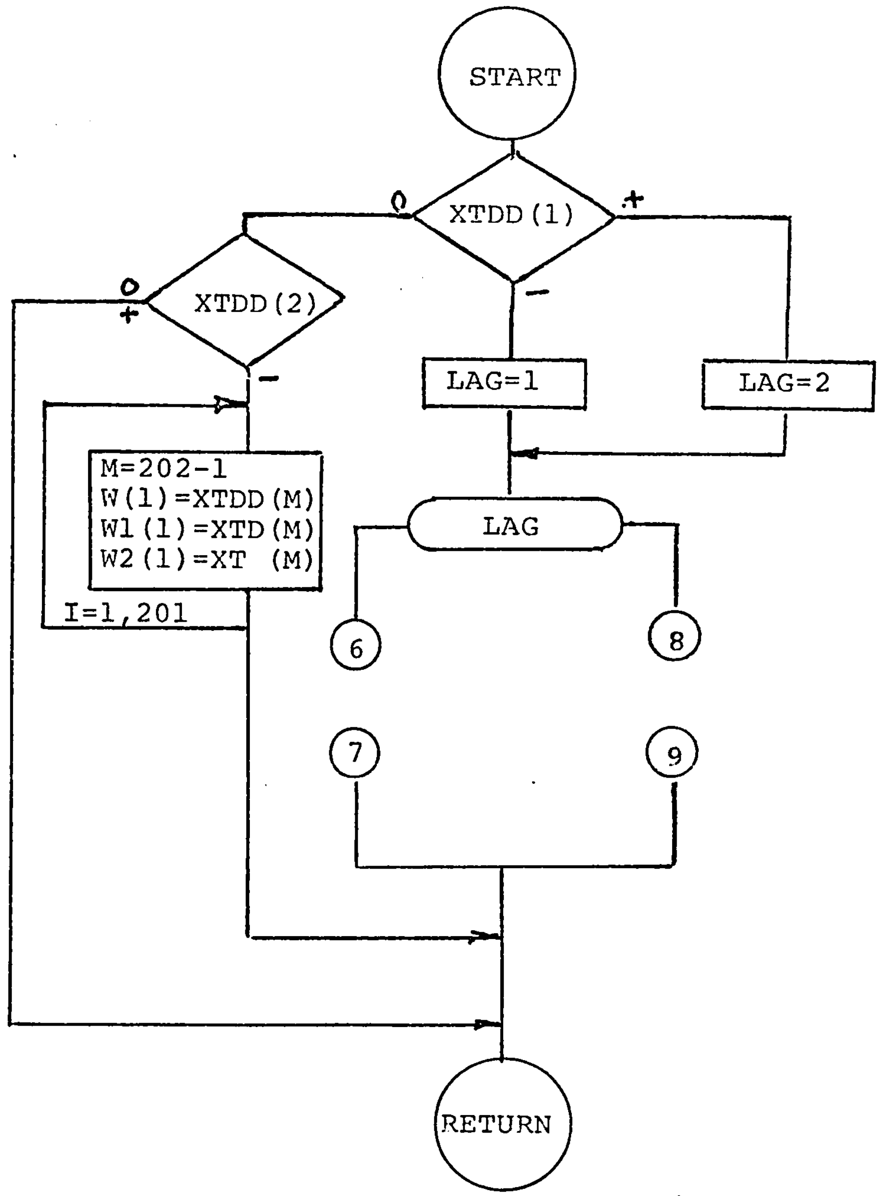

SUBROUTINE NEWTON

For computing displacement, velocity, and acceleration components of point B on the four-bar mechanism.



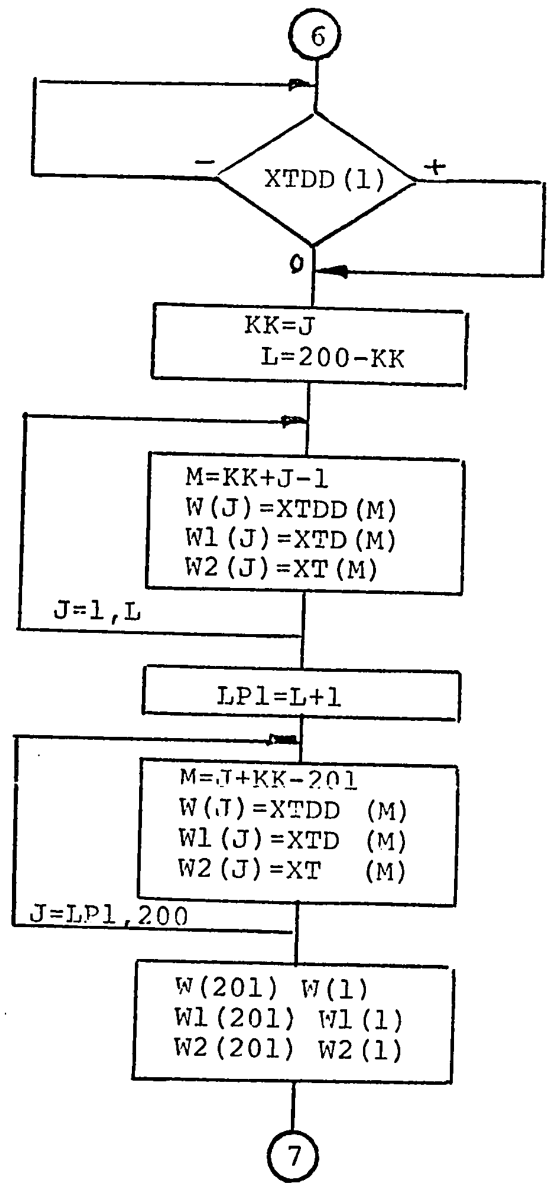

SUBROUTINE NEWTON (CONT'D)

Newton Raphson iteration method used for computing the displacement components along the X and Y axes.



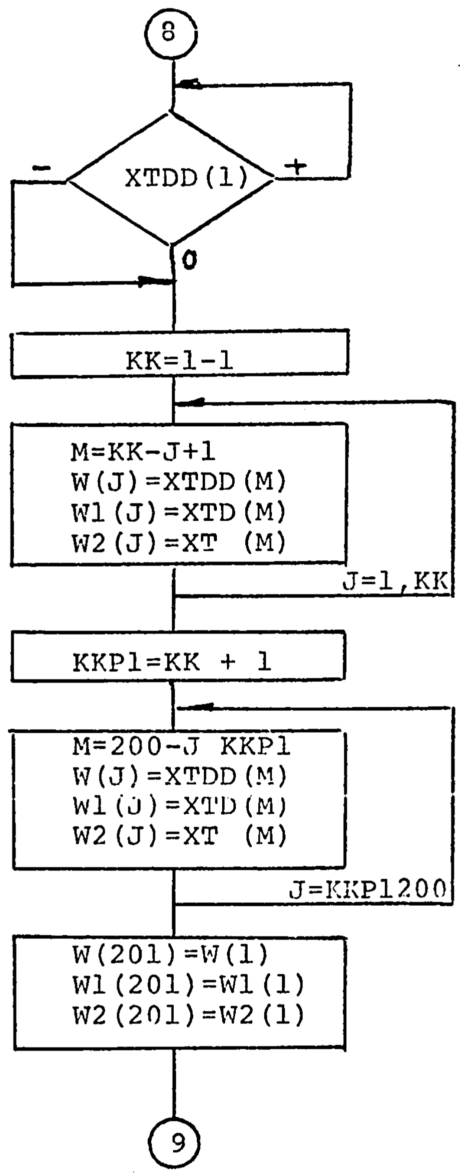

SUBROUTINE SORT

For adjusting the start point of the driving crank



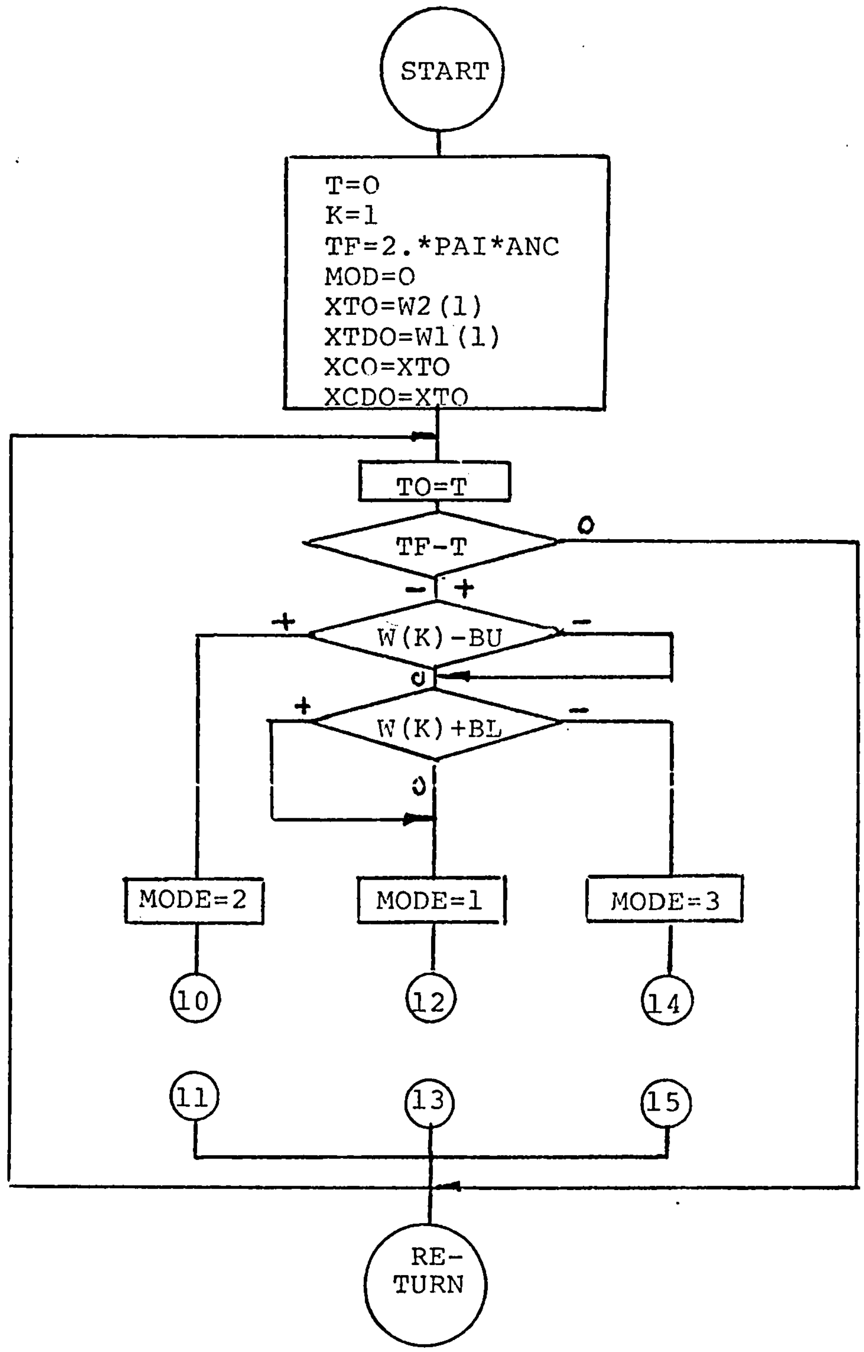

SUBROUTINE SORT (CONT'D)

Looks after acceleration cycle with negative starting ordinates



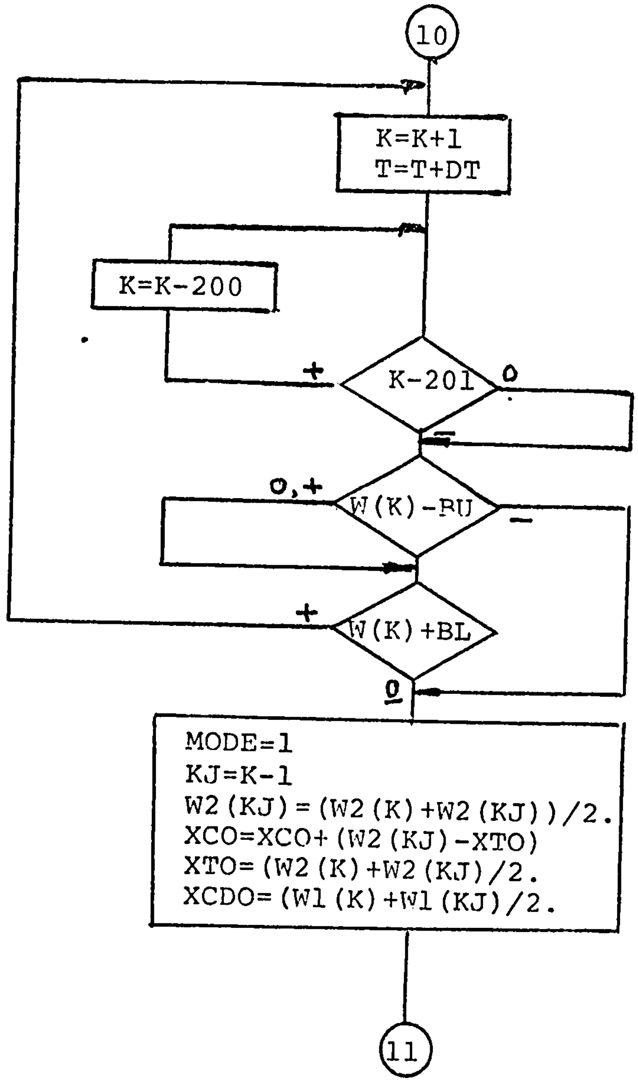

SUBROUTINE SORT (CONT'D)

Looks after acceleration cycle with positive starting ordinates

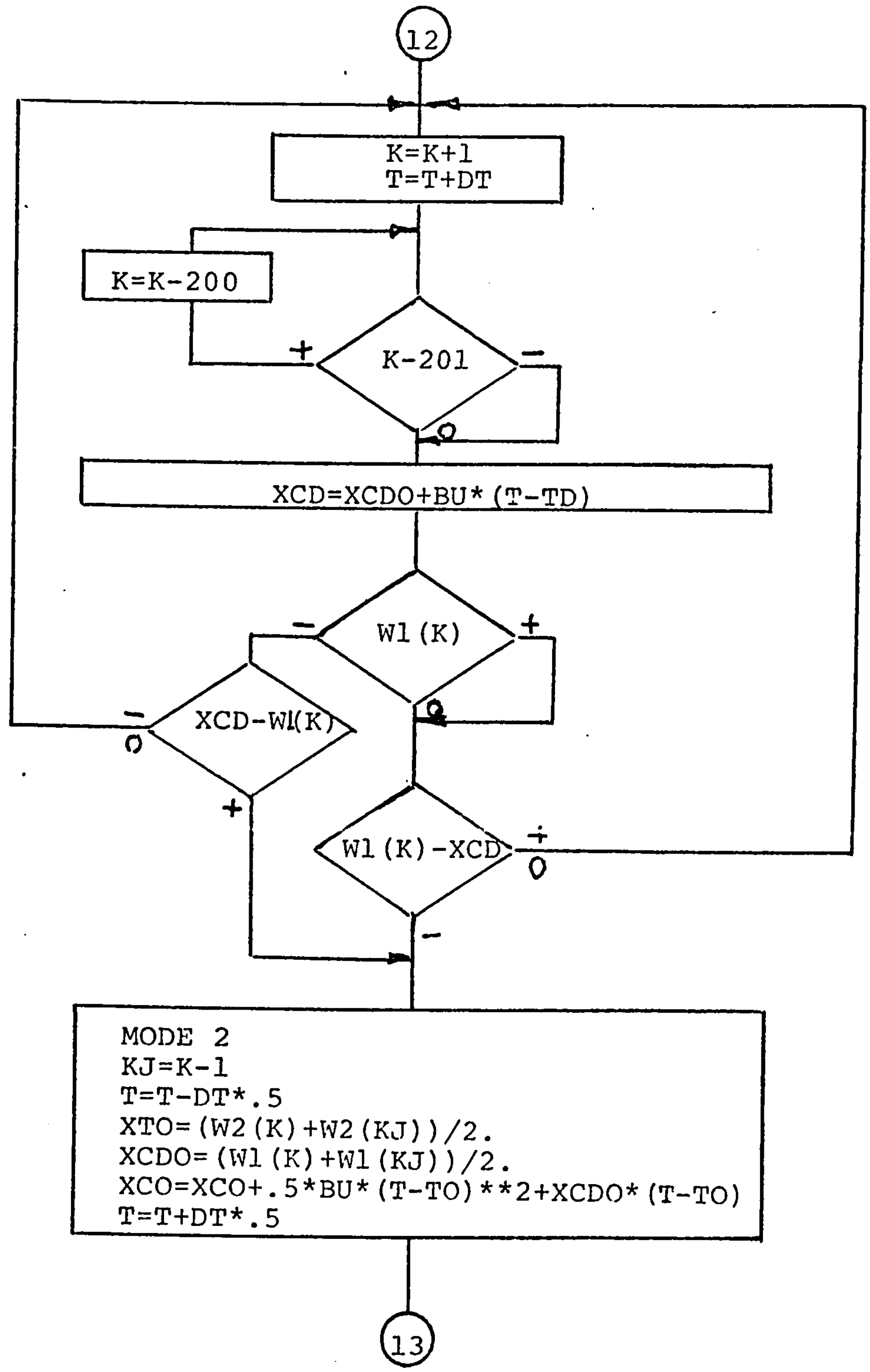



SUBROUTINE CONVEYER

For exact computation of object travel

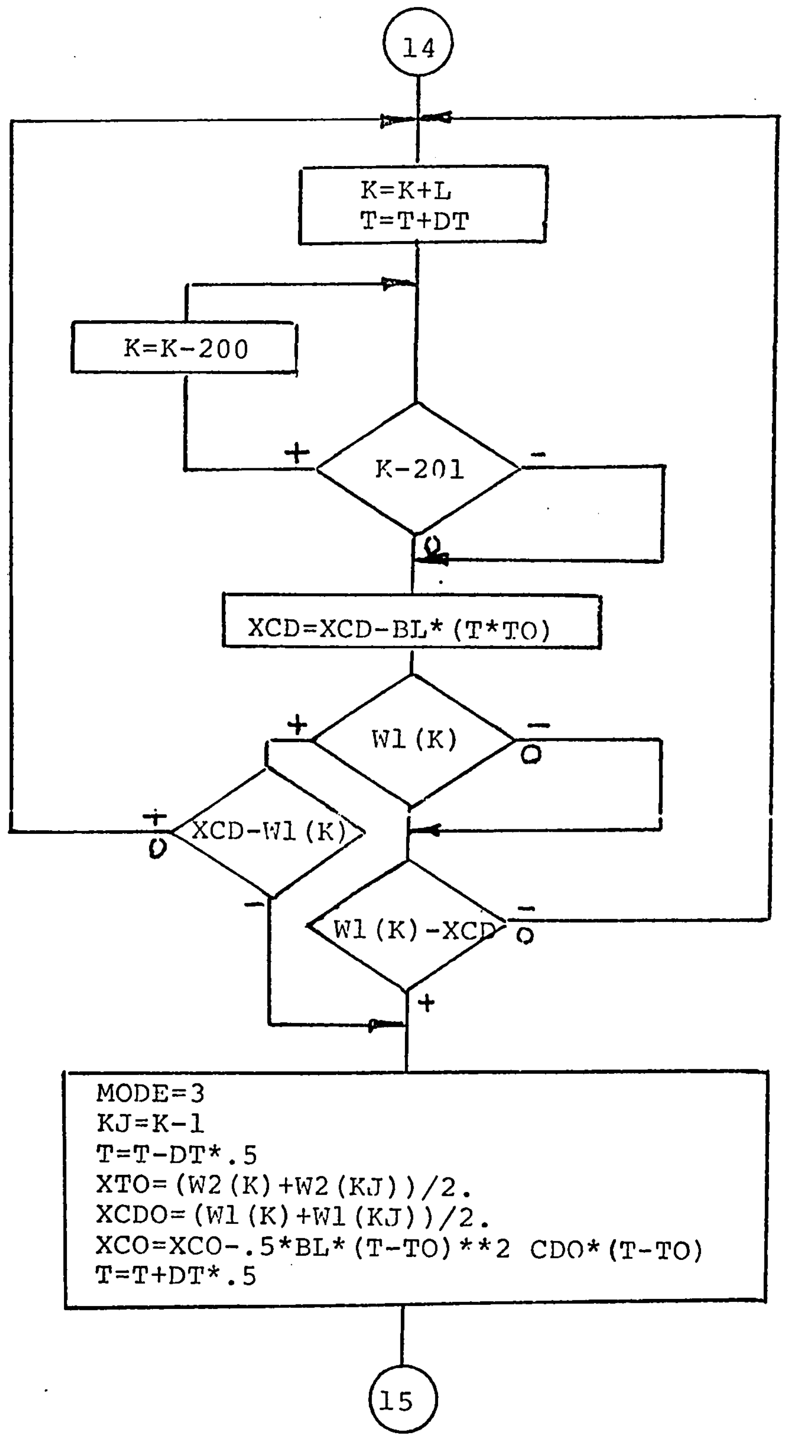



SUBROUTINE CONVEYER (CONT'D)
MODE I




SUBROUTINE CONVEYER (CONT'D)

MODE II




SUBROUTINE CONVEYER (CONT'D)

MODE III

# FORTRAN PROGRAM

```
        [illegible] OPT
        [illegible]
        [illegible]1.0
        FNC1=.1
        FNC2=.05
        DO 35 I=1,9
        A1=I
        X2=A1/10.0
        II=9-I+1
        DO 34 J=1,II
        JJ=9-J+1
        AJJ=JJ
        X4=AJJ/10.0
        SH=FNC2
        IF(X4-X2)29,29,30
 29     X4=X4+FNC[illegible]
        SH=SH-FNC2
 30     CONTINUE
        A=1-X2+[illegible]-SH
        B=1+X2-[illegible]+SH
        FNC=0.0
        DO 32 K=1,20
        XJ=A+FNC
        IF(XJ-[illegible])32,32,31
 31     GO TO 3[illegible]
 32     CONTINUE
        FNC=FNC+FNC1
        [illegible]
        [illegible]
        [illegible]
        [illegible]
        [illegible]
        [illegible]
        [illegible] XT,X2,X4,AA,X3,BB,CC,DD,EE
 [illegible] FORMAT([illegible],F4,8F8.2)
 33     CONTINUE
 34     CONTINUE
 35     CONTINUE
        END
```

FORTRAN DIAGNOSTIC RESULTS FOR OPT

NO ERRORS

COMPILED LENGTHS OF OPT - P = 0[illegible]313 C = 00000 D = 00000

X,LGO

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 1 | .10 | .90 | 1.80 | .25 | .20 | .90 | .15 | .80 |
| 2 | .10 | .90 | 1.80 | .35 | .20 | .90 | .25 | .80 |
| 3 | .10 | .90 | 1.80 | .45 | .20 | .90 | .35 | .80 |
| 4 | .10 | .90 | 1.80 | .55 | .20 | .90 | .45 | .80 |
| 5 | .10 | .90 | 1.80 | .65 | .20 | .90 | .55 | .80 |
| 6 | .10 | .90 | 1.80 | .75 | .20 | .90 | .65 | .80 |
| 7 | .10 | .90 | 1.80 | .85 | .20 | .90 | .75 | .80 |
| 8 | .10 | .90 | 1.80 | .95 | .20 | .90 | .85 | .80 |
| 9 | .10 | .90 | 1.80 | 1.05 | .20 | .90 | .95 | .80 |
| 10 | .10 | .90 | 1.80 | 1.15 | .20 | .90 | 1.05 | .80 |
| 11 | .10 | .90 | 1.80 | 1.25 | .20 | .90 | 1.15 | .80 |
| 12 | .10 | .90 | 1.80 | 1.35 | .20 | .90 | 1.25 | .80 |
| 13 | .10 | .90 | 1.80 | 1.45 | .20 | .90 | 1.35 | .80 |
| 14 | .10 | .90 | 1.80 | 1.55 | .20 | .90 | 1.45 | .80 |
| 15 | .10 | .90 | 1.80 | 1.65 | .20 | .90 | 1.55 | .80 |
| 16 | .10 | .80 | 1.70 | .35 | .30 | .90 | .25 | .70 |
| 17 | .10 | .80 | 1.70 | .45 | .30 | .90 | .35 | .70 |
| 18 | .10 | .80 | 1.70 | .55 | .30 | .90 | .45 | .70 |
| 19 | .10 | .80 | 1.70 | .65 | .30 | .90 | .55 | .70 |
| 20 | .10 | .80 | 1.70 | .75 | .30 | .90 | .65 | .70 |
| 21 | .10 | .80 | 1.70 | .85 | .30 | .90 | .75 | .70 |
| 22 | .10 | .80 | 1.70 | .95 | .30 | .90 | .85 | .70 |
| 23 | .10 | .80 | 1.70 | 1.05 | .30 | .90 | .95 | .70 |
| 24 | .10 | .80 | 1.70 | 1.15 | .30 | .90 | 1.05 | .70 |
| 25 | .10 | .80 | 1.70 | 1.25 | .30 | .90 | 1.15 | .70 |
| 26 | .10 | .80 | 1.70 | 1.35 | .30 | .90 | 1.25 | .70 |
| 27 | .10 | .80 | 1.70 | 1.45 | .30 | .90 | 1.35 | .70 |
| 28 | .10 | .80 | 1.70 | 1.55 | .30 | .90 | 1.45 | .70 |
| 29 | .10 | .70 | 1.60 | .45 | .40 | .90 | .35 | .60 |
| 30 | .10 | .70 | 1.60 | .55 | .40 | .90 | .45 | .60 |
| 31 | .10 | .70 | 1.60 | .65 | .40 | .90 | .55 | .60 |
| 32 | .10 | .70 | 1.60 | .75 | .40 | .90 | .65 | .60 |
| 33 | .10 | .70 | 1.60 | .85 | .40 | .90 | .75 | .60 |
| 34 | .10 | .70 | 1.60 | .95 | .40 | .90 | .85 | .60 |
| 35 | .10 | .70 | 1.60 | 1.05 | .40 | .90 | .95 | .60 |
| 36 | .10 | .70 | 1.60 | 1.15 | .40 | .90 | 1.05 | .60 |
| 37 | .10 | .70 | 1.60 | 1.25 | .40 | .90 | 1.15 | .60 |
| 38 | .10 | .70 | 1.60 | 1.35 | .40 | .90 | 1.25 | .60 |
| 39 | .10 | .70 | 1.60 | 1.45 | .40 | .90 | 1.35 | .60 |
| 40 | .10 | .60 | 1.50 | .55 | .50 | .90 | .45 | .50 |
| 41 | .10 | .60 | 1.50 | .65 | .50 | .90 | .55 | .50 |
| 42 | .10 | .60 | 1.50 | .75 | .50 | .90 | .65 | .50 |
| 43 | .10 | .60 | 1.50 | .85 | .50 | .90 | .75 | .50 |
| 44 | .10 | .60 | 1.50 | .95 | .50 | .90 | .85 | .50 |
| 45 | .10 | .60 | 1.50 | 1.05 | .50 | .90 | .95 | .50 |
| 46 | .10 | .60 | 1.50 | 1.15 | .50 | .90 | 1.05 | .50 |
| 47 | .10 | .60 | 1.50 | 1.25 | .50 | .90 | 1.15 | .50 |
| 48 | .10 | .60 | 1.50 | 1.35 | .50 | .90 | 1.25 | .50 |
| 49 | .10 | .50 | 1.40 | .65 | .60 | .90 | .55 | .40 |
| 50 | .10 | .50 | 1.40 | .75 | .60 | .90 | .65 | .40 |
| 51 | .10 | .50 | 1.40 | .85 | .60 | .90 | .75 | .40 |
| 52 | .10 | .50 | 1.40 | .95 | .60 | .90 | .85 | .40 |
| 53 | .10 | .50 | 1.40 | 1.05 | .60 | .90 | .95 | .40 |
| 54 | .10 | .50 | 1.40 | 1.15 | .60 | .90 | 1.05 | .40 |
| 55 | .10 | .50 | 1.40 | 1.25 | .60 | .90 | 1.15 | .40 |
| 56 | .10 | .40 | 1.30 | .75 | .70 | .90 | .65 | .30 |
| 57 | .10 | .40 | 1.30 | .85 | .70 | .90 | .75 | .30 |
| 58 | .10 | .40 | 1.30 | .95 | .70 | .90 | .85 | .30 |
| 59 | .10 | .40 | 1.30 | 1.05 | .70 | .90 | .95 | .30 |
| 60 | .10 | .40 | 1.30 | 1.15 | .70 | .90 | 1.05 | .30 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 61 | [illegible] | [illegible] | 1.20 | .85 | .60 | .90 | .75 | .20 |
| 62 | .10 | .?0 | 1.20 | .95 | .60 | .90 | .85 | .20 |
| 63 | .10 | [illegible] | 1.20 | 1.05 | .60 | .90 | .95 | .20 |
| 64 | .1 | .20 | 1.10 | .95 | .90 | .90 | .85 | .10 |
| 65 | .10 | .15 | 1.05 | .95 | .95 | .90 | .85 | .05 |
| 66 | .20 | .90 | 1.70 | .35 | .30 | .80 | .15 | .70 |
| 67 | .20 | .90 | 1.70 | .45 | .30 | .80 | .25 | .70 |
| 68 | .20 | .90 | 1.70 | .55 | .30 | .80 | .35 | .70 |
| 69 | .20 | .90 | 1.70 | .65 | .30 | .80 | .45 | .70 |
| 70 | .20 | .90 | 1.70 | .75 | .30 | .80 | .55 | .70 |
| 71 | .20 | .90 | 1.70 | .85 | .30 | .80 | .65 | .70 |
| 72 | .20 | .90 | 1.70 | .95 | .30 | .80 | .75 | .70 |
| 73 | .20 | .90 | 1.70 | 1.05 | .30 | .80 | .85 | .70 |
| 74 | .20 | .90 | 1.70 | 1.15 | .30 | .80 | .95 | .70 |
| 75 | .20 | .90 | 1.70 | 1.25 | .30 | .80 | 1.05 | .70 |
| 76 | .20 | .90 | 1.70 | 1.35 | .30 | .80 | 1.15 | .70 |
| 77 | .20 | .90 | 1.70 | 1.45 | .30 | .80 | 1.25 | .70 |
| 78 | .20 | .90 | 1.70 | 1.55 | .30 | .80 | 1.35 | .70 |
| 79 | .20 | .80 | 1.60 | .45 | .40 | .80 | .25 | .60 |
| 80 | .20 | .80 | 1.60 | .55 | .40 | .80 | .35 | .60 |
| 81 | .20 | .80 | 1.60 | .65 | .40 | .80 | .45 | .60 |
| 82 | .20 | .80 | 1.60 | .75 | .40 | .80 | .55 | .60 |
| 83 | .20 | .80 | 1.60 | .85 | .40 | .80 | .65 | .60 |
| 84 | .20 | .80 | 1.60 | .95 | .40 | .80 | .75 | .60 |
| 85 | .20 | .80 | 1.60 | 1.05 | .40 | .80 | .85 | .60 |
| 86 | .20 | .80 | 1.60 | 1.15 | .40 | .80 | .95 | .60 |
| 87 | .20 | .80 | 1.60 | 1.25 | .40 | .80 | 1.05 | .60 |
| 88 | .20 | .80 | 1.60 | 1.35 | .40 | .80 | 1.15 | .60 |
| 89 | .20 | .80 | 1.60 | 1.45 | .40 | .80 | 1.25 | .60 |
| 90 | .20 | .70 | 1.50 | .55 | .50 | .80 | .35 | .50 |
| 91 | .20 | .70 | 1.50 | .65 | .50 | .80 | .45 | .50 |
| 92 | .20 | .70 | 1.50 | .75 | .50 | .80 | .55 | .50 |
| 93 | .20 | .70 | 1.50 | .85 | .50 | .80 | .65 | .50 |
| 94 | .20 | .70 | 1.50 | .95 | .50 | .80 | .75 | .50 |
| 95 | .20 | .70 | 1.50 | 1.05 | .50 | .80 | .85 | .50 |
| 96 | .20 | .70 | 1.50 | 1.15 | .50 | .80 | .95 | .50 |
| 97 | .20 | .70 | 1.50 | 1.25 | .50 | .80 | 1.05 | .50 |
| 98 | .20 | .70 | 1.50 | 1.35 | .50 | .80 | 1.15 | .50 |
| 99 | .20 | .60 | 1.40 | .65 | .60 | .80 | .45 | .40 |
| 100 | .20 | .60 | 1.40 | .75 | .60 | .80 | .55 | .40 |
| 101 | .20 | .60 | 1.40 | .85 | .60 | .80 | .65 | .40 |
| 102 | .20 | .60 | 1.40 | .95 | .60 | .80 | .75 | .40 |
| 103 | .20 | .60 | 1.40 | 1.05 | .60 | .80 | .85 | .40 |
| 104 | .20 | .60 | 1.40 | 1.15 | .60 | .80 | .95 | .40 |
| 105 | .20 | .60 | 1.40 | 1.25 | .60 | .80 | 1.05 | .40 |
| 106 | .20 | .50 | 1.30 | .75 | .70 | .80 | .55 | .30 |
| 107 | .20 | .50 | 1.30 | .85 | .70 | .80 | .65 | .30 |
| 108 | .20 | .50 | 1.30 | .95 | .70 | .80 | .75 | .30 |
| 109 | .20 | .50 | 1.30 | 1.05 | .70 | .80 | .85 | .30 |
| 110 | .20 | .50 | 1.30 | 1.15 | .70 | .80 | .95 | .30 |
| 111 | .20 | .40 | 1.20 | .85 | .80 | .80 | .65 | .20 |
| 112 | .20 | .40 | 1.20 | .95 | .80 | .80 | .75 | .20 |
| 113 | .20 | .40 | 1.20 | 1.05 | .80 | .80 | .85 | .20 |
| 114 | .20 | .30 | 1.10 | .95 | .90 | .80 | .75 | .10 |
| 115 | .20 | .25 | 1.05 | .95 | .95 | .80 | .75 | .05 |
| 116 | .30 | .90 | 1.60 | .45 | .40 | .70 | .15 | .60 |
| 117 | .30 | .90 | 1.60 | .55 | .40 | .70 | .25 | .60 |
| 118 | .30 | .90 | 1.60 | .65 | .40 | .70 | .35 | .60 |
| 119 | .30 | .90 | 1.60 | .75 | .40 | .70 | .45 | .60 |
| 120 | .30 | .90 | 1.60 | .85 | .40 | .70 | .55 | .60 |
| 121 | .30 | .90 | 1.60 | .95 | .40 | .70 | .65 | .60 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 122 | .30 | .70 | 1.60 | 1.05 | .40 | .70 | .75 | .60 |
| 123 | .30 | [illegible] | 1.60 | 1.15 | .40 | .70 | .85 | .60 |
| 124 | .30 | .70 | 1.60 | 1.25 | .40 | .70 | .95 | .60 |
| 125 | [illegible] | .70 | 1.60 | 1.35 | .40 | .70 | 1.05 | .60 |
| 126 | .30 | .70 | 1.60 | 1.45 | .40 | .70 | 1.15 | .60 |
| 127 | .30 | .80 | 1.50 | .55 | .50 | .70 | .25 | .50 |
| 128 | .30 | .80 | 1.50 | .65 | .50 | .70 | .35 | .50 |
| 129 | .30 | .80 | 1.50 | .75 | .50 | .70 | .45 | .50 |
| 130 | .30 | .80 | 1.50 | .85 | .50 | .70 | .55 | .50 |
| 131 | .30 | .80 | 1.50 | .95 | .50 | .70 | .65 | .50 |
| 132 | .30 | .80 | 1.50 | 1.05 | .50 | .70 | .75 | .50 |
| 133 | .30 | .80 | 1.50 | 1.15 | .50 | .70 | .85 | .50 |
| 134 | .30 | .80 | 1.50 | 1.25 | .50 | .70 | .95 | .50 |
| 135 | .30 | .80 | 1.50 | 1.35 | .50 | .70 | 1.05 | .50 |
| 136 | .30 | .70 | 1.40 | .65 | .60 | .70 | .35 | .40 |
| 137 | .30 | .70 | 1.40 | .75 | .60 | .70 | .45 | .40 |
| 138 | .30 | .70 | 1.40 | .85 | .60 | .70 | .55 | .40 |
| 139 | .30 | .70 | 1.40 | .95 | .60 | .70 | .65 | .40 |
| 140 | .30 | .70 | 1.40 | 1.05 | .60 | .70 | .75 | .40 |
| 141 | .30 | .70 | 1.40 | 1.15 | .60 | .70 | .85 | .40 |
| 142 | .30 | .70 | 1.40 | 1.25 | .60 | .70 | .95 | .40 |
| 143 | .30 | .60 | 1.30 | .75 | .70 | .70 | .45 | .30 |
| 144 | .30 | .60 | 1.30 | .85 | .70 | .70 | .55 | .30 |
| 145 | .30 | .60 | 1.30 | .95 | .70 | .70 | .65 | .30 |
| 146 | .30 | .60 | 1.30 | 1.05 | .70 | .70 | .75 | .30 |
| 147 | .30 | .60 | 1.30 | 1.15 | .70 | .70 | .85 | .30 |
| 148 | .30 | .50 | 1.20 | .85 | .80 | .70 | .55 | .20 |
| 149 | .30 | .50 | 1.20 | .95 | .80 | .70 | .65 | .20 |
| 150 | .30 | .50 | 1.20 | 1.05 | .80 | .70 | .75 | .20 |
| 151 | .30 | .40 | 1.10 | .95 | .90 | .70 | .65 | .10 |
| 152 | .30 | .35 | 1.05 | .95 | .95 | .70 | .65 | .05 |
| 153 | .40 | .90 | 1.50 | .55 | .50 | .60 | .15 | .50 |
| 154 | .40 | .90 | 1.50 | .65 | .50 | .60 | .25 | .50 |
| 155 | .40 | .90 | 1.50 | .75 | .50 | .60 | .35 | .50 |
| 156 | .40 | .90 | 1.50 | .85 | .50 | .60 | .45 | .50 |
| 157 | .40 | .90 | 1.50 | .95 | .50 | .60 | .55 | .50 |
| 158 | .40 | .90 | 1.50 | 1.05 | .50 | .60 | .65 | .50 |
| 159 | .40 | .90 | 1.50 | 1.15 | .50 | .60 | .75 | .50 |
| 160 | .40 | .90 | 1.50 | 1.25 | .50 | .60 | .85 | .50 |
| 161 | .40 | .90 | 1.50 | 1.35 | .50 | .60 | .95 | .50 |
| 162 | .40 | .80 | 1.40 | .65 | .60 | .60 | .25 | .40 |
| 163 | .40 | .80 | 1.40 | .75 | .60 | .60 | .35 | .40 |
| 164 | .40 | .80 | 1.40 | .85 | .60 | .60 | .45 | .40 |
| 165 | .40 | .80 | 1.40 | .95 | .60 | .60 | .55 | .40 |
| 166 | .40 | .80 | 1.40 | 1.05 | .60 | .60 | .65 | .40 |
| 167 | .40 | .80 | 1.40 | 1.15 | .60 | .60 | .75 | .40 |
| 168 | .40 | .80 | 1.40 | 1.25 | .60 | .60 | .85 | .40 |
| 169 | .40 | .70 | 1.30 | .75 | .70 | .60 | .35 | .30 |
| 170 | .40 | .70 | 1.30 | .85 | .70 | .60 | .45 | .30 |
| 171 | .40 | .70 | 1.30 | .95 | .70 | .60 | .55 | .30 |
| 172 | .40 | .70 | 1.30 | 1.05 | .70 | .60 | .65 | .30 |
| 173 | .40 | .70 | 1.30 | 1.15 | .70 | .60 | .75 | .30 |
| 174 | .40 | .60 | 1.20 | .85 | .80 | .60 | .45 | .20 |
| 175 | .40 | .60 | 1.20 | .95 | .80 | .60 | .55 | .20 |
| 176 | .40 | .60 | 1.20 | 1.05 | .80 | .60 | .65 | .20 |
| 177 | .40 | .50 | 1.10 | .95 | .90 | .60 | .55 | .10 |
| 178 | .40 | .45 | 1.05 | .95 | .95 | .60 | .55 | .05 |
| 179 | .50 | .90 | 1.40 | .65 | .60 | .50 | .15 | .40 |
| 180 | .50 | .90 | 1.40 | .75 | .60 | .50 | .25 | .40 |
| 181 | .50 | .90 | 1.40 | .85 | .60 | .50 | .35 | .40 |
| 182 | .50 | .90 | 1.40 | .95 | .60 | .50 | .45 | .40 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 183 | [illegible] | [illegible] | 1.40 | 1.25 | .80 | .50 | .55 | .40 |
| 184 | [illegible] | .9[illegible] | 1.40 | 1.15 | .60 | .50 | .65 | .40 |
| 185 | [illegible] | .9[illegible] | 1.60 | 1.25 | .60 | .50 | .75 | .40 |
| 186 | [illegible] | [illegible] | 1.30 | .75 | .70 | .50 | .25 | .30 |
| 187 | .5[illegible] | [illegible] | 1.30 | .85 | .70 | .50 | .35 | .30 |
| 188 | [illegible] | [illegible] | 1.30 | .95 | .70 | .50 | .45 | .30 |
| 189 | .50 | .80 | 1.30 | 1.05 | .70 | .50 | .55 | .30 |
| 190 | .50 | .80 | 1.30 | 1.15 | .70 | .50 | .65 | .30 |
| 191 | .50 | .70 | 1.20 | .85 | .80 | .50 | .35 | .20 |
| 192 | .50 | .75 | 1.20 | .95 | .80 | .50 | .45 | .20 |
| 193 | .50 | .70 | 1.20 | 1.05 | .80 | .50 | .55 | .20 |
| 194 | .50 | .70 | 1.10 | .95 | .90 | .50 | .45 | .10 |
| 195 | .50 | .65 | 1.05 | .95 | .95 | .50 | .45 | .05 |
| 196 | .60 | .90 | 1.30 | .75 | .70 | .40 | .15 | .30 |
| 197 | .60 | .90 | 1.30 | .85 | .70 | .40 | .25 | .30 |
| 198 | .60 | .95 | 1.30 | .95 | .70 | .40 | .35 | .30 |
| 199 | .60 | .90 | 1.30 | 1.05 | .70 | .40 | .45 | .30 |
| 200 | .60 | .90 | 1.30 | 1.15 | .70 | .40 | .55 | .30 |
| 201 | .60 | .80 | 1.20 | .85 | .80 | .40 | .25 | .20 |
| 202 | .60 | .80 | 1.20 | .95 | .80 | .40 | .35 | .20 |
| 203 | .60 | .70 | 1.20 | 1.05 | .80 | .40 | .45 | .20 |
| 204 | .60 | .70 | 1.10 | .95 | .90 | .40 | .35 | .10 |
| 205 | .60 | .65 | 1.05 | .95 | .95 | .40 | .35 | .05 |
| 206 | .70 | .90 | 1.20 | .85 | .80 | .30 | .15 | .20 |
| 207 | .70 | .90 | 1.20 | .95 | .80 | .30 | .25 | .20 |
| 208 | .70 | .90 | 1.20 | 1.05 | .80 | .30 | .35 | .20 |
| 209 | .70 | .80 | 1.10 | .95 | .90 | .30 | .25 | .10 |
| 210 | .70 | .75 | 1.05 | .95 | .95 | .30 | .25 | .05 |
| 211 | [illegible] | .90 | 1.10 | .95 | .90 | .20 | .15 | .10 |
| 212 | [illegible] | [illegible] | 1.05 | .95 | .95 | .20 | .15 | .05 |
| 213 | .9[illegible] | .9[illegible] | 1.05 | .95 | .95 | .10 | .05 | .05 |

```
      SUBROUTINE NEWTON
      COMMON GE,ALFA,DT,BU,BL,ENC1,ENC2,DEG,AX7,AX2,EMU,
     1W(201),W1(201),W2(201),ZDX(201),ZDY(201),ZVX(201),ZVY(201),
     2R1,R2,R3,R4,OM,EP,PAI,CA,SA,
     3NT,THETA(201),XTDD(201),XTD(201),XT(201),ANC,
     4ZAX(201),ZAY(201)
      WRITE(61,101) R1,R2,R3,R4
C
C         DISPLACEMENT OF COUPLING POINT B
C
      XB=(R1**2+R3**2-R2**2-R4**2)/(2.0*(R1-R2))
      YB=SQRT(R3**2-(XB-R2)**2)
C
C         CRANK ROTATION
      DO 6 I=1,201
      CT=COS(THETA(I))
      ST=SIN(THETA(I))
      XA=R2*CT
      YA=R2*ST
      XAD=-YA*OM
      YAD=XA*OM
      XADD=-XA*OM**2
      YADD=-YA*OM**2
C
C         ITERATION BY NEWTON RAPHSON
C
    2 U=R3**2-(XB-XA)**2-(YB-YA)**2
      U1=2.0*(XB-XA)
      U2=2.0*(YB-YA)
      V=R4**2-(XB-R1)**2-YB**2
      V1=2.0*(XB-R1)
      V2=2.0*YB
      D=U1*V2-U2*V1
      TEST=ABS(D)-EP
      IF(TEST) 3,5,3
    3 DX=(U*V2-V*U2)/D
      DY=(V*U1-U*V1)/D
      XB=XB+DX
      YB=YB+DY
      TEST1=ABS(DX)-EP
      TEST2=ABS(DY)-EP
      IF(TEST1)4,4,2
    4 IF(TEST2) 5,5,2
    5 U1=YB-YA
      U2=XB-XA
      U3=R1-XB
C
C         VELOCITY OF POINT B
      ENM1=YAD*YB*U1+XAD*YB*(XB-XA)
      DEN1=YB*U2+U1*U3
      XBD=ENM1/DEN1
      YBD=U3*XBD/YB
      ENO1=YB*U2*XADD-YB*(XBD-XAD)**2+U1*(XBD**2+YBD**2+YADD*YB)
      ENO2=-YB*(YBD-YAD)**2
```

```
      ENM2=ENO1+ENO2
      DEN2=U2*YB+U1*U3
C
C          ACCELERATION OF POINT B
      XBDD=ENM2/DEN2
      YBDD=(U3*XBDD-XBD**2-YBD**2)/YB
      ZDX(I)=XB
      ZDY(I)=YB
      ZVX(I)=XBD
      ZVY(I)=YBD
      ZAX(I)=XBDD
      ZAY(I)=YBDD
    6 CONTINUE
  101 FORMAT(///5X,4H R1=,F5.2,5X,4H R2=,F5.2,5X,4H R3=,F5.2,
     15X,4H R4=,F5.2)
      RETURN
      END
```

FORTRAN DIAGNOSTIC RESULTS FOR      NEWTON

NO ERRORS

NEWTON    P   00675   C   12225   D   00000

```
      SUBROUTINE SORT
      COMMON GE,ALFA,DT,BU,BL,ENC1,ENC2,DEG,AX1,AX2,EMU,
     1W(201),W1(201),W2(201),ZDX(201),ZDY(201),ZVX(201),ZVY(201),
     2R1,R2,R3,R4,OM,EP,PAT,CA,SA,
     3NT,THETA(201),XTDD(201),XTD(201),XT(201),ANC,
     4ZAX(201),ZAY(201)
      IF(XTDD(1))  7,21,8
    7 LAG=1
      GO TO 9
    8 LAG=2
    9 DO 12 I=2,201
      GO TO (10,11),LAG
   10 IF(XTDD(I)) 12,13,13
   11 IF(XTDD(I)) 17,17,12
   12 CONTINUE
   13 KK=I
      L=200-KK
      DO 14 J=1,L
      M=KK+J-1
      W(J)=XTDD(M)
      W1(J)=XTD(M)
      W2(J)=XT(M)
   14 CONTINUE
      LP1=L+1
      DO 15 J=LP1,200
      M=J+KK-200
      W(J)=XTDD(M)
      W1(J)=XTD(M)
      W2(J)=XT(M)
   15 CONTINUE
      W(201)=W(1)
      W1(201)=W1(1)
      W2(201)=W2(1)
      GO TO 25
   17 KK=I-1
      DO 18 J=1,KK
      M=KK-J+1
      W(J)=XTDD(M)
      W1(J)=XTD(M)
      W2(J)=XT(M)
   18 CONTINUE
      KKP1=KK+1
      DO 19 J=KKP1,200
      M=200-J+KKP1
      W(J)=XTDD(M)
      W1(J)=XTD(M)
      W2(J)=XT(M)
   19 CONTINUE
      W(201)=W(1)
      W1(201)=W1(1)
      W2(201)=W2(1)
      GO TO 25
   21 IF(XTDD(2)) 22,25,25
   22 DO 23 I=1,201
```

FORTRAN (3.2)/MASTER

```
      M=202-I
      W(I)=XTDD(M)
      W1(I)=XTD(M)
      W2(I)=XT(M)
   23 CONTINUE
  25  CONTINUE
      RETURN
      END
```

FORTRAN DIAGNOSTIC RESULTS FOR SORT

NO ERRORS

SORT P 00363 C 12225 D 00000

```
      SUBROUTINE CONV
      COMMON GE,ALFA,DT,BU,BL,ENC1,ENC2,DEG,AX1,AX2,EMU,
     1W(201),W1(201),W2(201),ZDX(201),ZDY(201),ZVX(201),ZVY(201),
     2R1,R2,R3,R4,OM,EP,PAI,CA,SA,
     3NT,THETA(201),XTDD(201),XTD(201),XT(201),ANC,
     4ZAX(201),ZAY(201)
      MT=1
      T=0
      K=1
      TF=(2.0*PAI*ANC)
      TSF=2.*PAI*(ANC-1.)
      MODE=0
      XTO=W2(1)
      XTDO=W1(1)
      XCO=XTO
      XCDO=XTDO
   40 TO=T
      IF(MT-MODE) 14,15,14
   14 MT=MODE
      WRITE(61,105) MODE,T,XCO,XCDO,K
   15 CONTINUE
      IF (TF-T) 70,70,450
  450 IF(TSF-T) 440,10,440
   10 CAP=XCO
  440 IF(W(K)-BU) 41,41,48
   41 IF(W(K)+BL) 61,42,42
   42 K=K+1
      T=T+DT
      IF(TF-T) 70,70,5
    5 CONTINUE
   43 IF(K-201) 45,45,44
   44 K=K-200
      GO TO 43
   45 IF(W(K)-BU) 46,47,47
   46 IF(W(K)+BL) 47,47,42
   47 MODE=1
      KJ=K-1
      W2(KJ)=(W2(K)+W2(KJ))/2.
      XCO=XCO+(W2(KJ)-XTO)
      XTO=(W2(K)+W2(KJ))/2.
      XCDO=(W1(K)+W1(KJ))/2.
      GO TO 40
   48 K=K+1
      T=T+DT
   54 IF(K-201) 56,56,55
   55 K=K-200
      GO TO 54
   56 XCD=XCDO+BU*(T-TO)
      IF(W1(K)) 1,2,2
    1 IF(XCD-W1(K)) 48,48,57
    2 IF(W1(K)-XCD) 57,48,48
   57 MODE=2
      KJ=K-1
      T=T-DT*.5
```

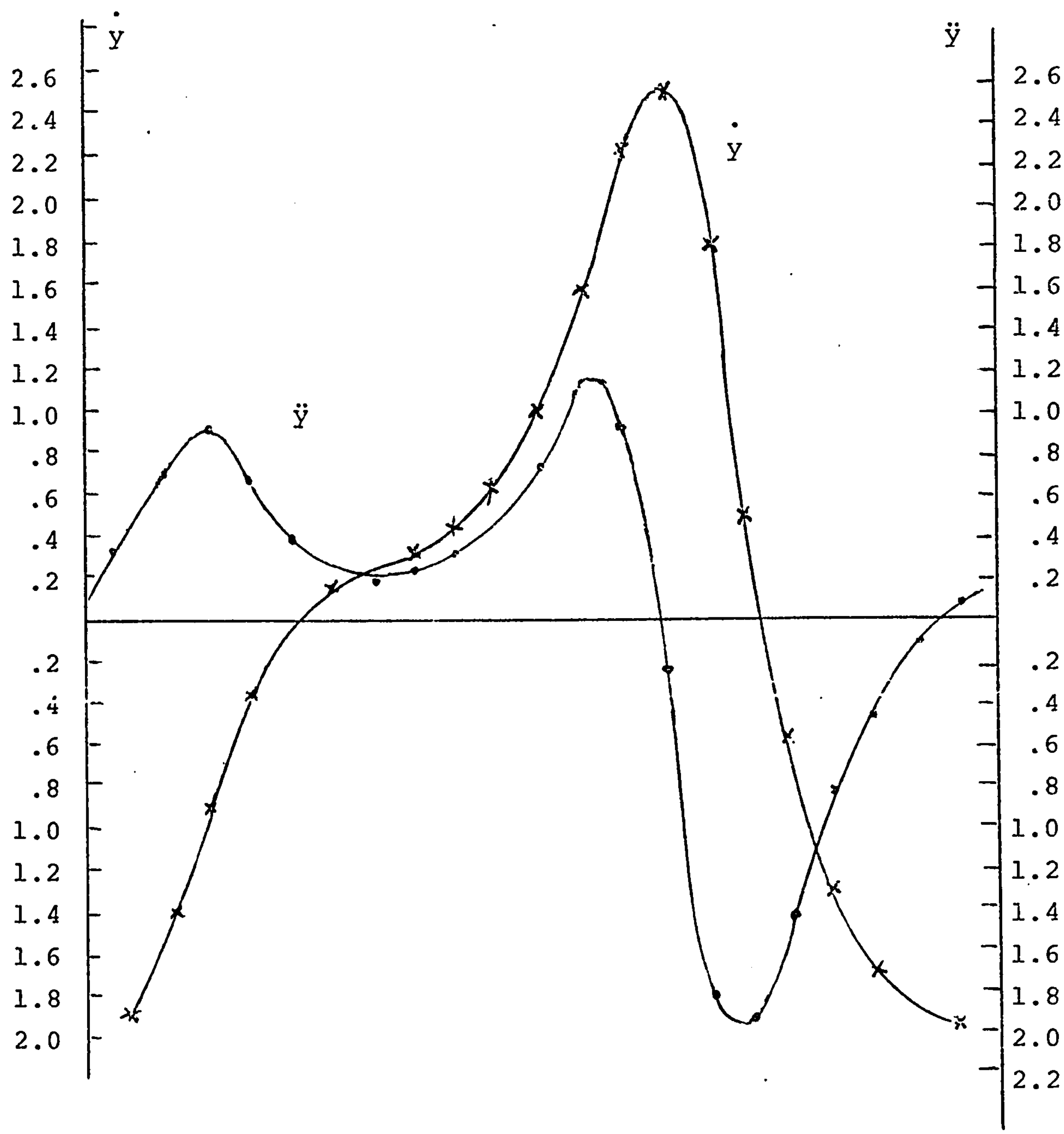
FORTRAN (3.2)/MASTER

```
      XTO=(W2(K)+W2(KJ))/2.
      XCDO=(W1(K)+W1(KJ))/2.
      XCO=XCO+0.5*BU*(T-TO)**2+XCDO*(T-TO)
      T=T+DT*.5
      GO TO 40
   61 K=K+1
      T=T+DT
   62 IF(K-201)64,64,63
   63 K=K-200
      GO TO 62
   64 XCD=XCDO-BL*(T-TO)
      IF(W1(K)) 3,3,4
    3 IF(W1(K)-XCD) 61,61,65
    4 IF(XCD-W1(K)) 65,61,61
   65 MODE=3
      KJ=K-1
      T=T-DT*.5
      XTO=(W2(K)+W2(KJ))/2.
      XCDO=(W1(K)+W1(KJ))/2.
      XCO=XCO-0.5*BL*(T-TO)**2+XCDO*(T-TO)
      T=T+DT*.5
      GO TO 40
  105 FORMAT(5X,6H MODE=,I1,5X,3H T=,F10.5,5X,5H XCO=,F10.5,5X,
     16H XCDO=,F10.5,5X,3H K=,I4)
   70 CONTINUE
      RETURN
      END
```

FORTRAN DIAGNOSTIC RESULTS FOR CONV

NO ERRORS

```
CONV        P    00606    C    12225    D    00000
OBJ,L30
```




The digital computer output plotted for the acceleration and velocity cycles of the coupling point b of the four-bar mechanism.

| $\ddot{y}$ | $\dot{y}$ | $y$ |
|---|---|---|
| .29030 ← | -1.96662 | .60682 ← |
| .67215 | -1.98360 | .59434 |
| 1.05075 | -1.97618 | .58190 |
| 1.42615 | -1.97040 | .56949 |
| 1.79837 | -1.96027 | .55714 |
| 2.16742 | -1.94781 | .54486 |
| 2.53331 | -1.93304 | .53267 |
| 2.89605 | -1.91598 | .52058 |
| 3.25563 | -1.89666 | .50860 |
| 3.61203 | -1.87508 | .49675 |
| 3.96521 | -1.85127 | .48504 |
| 4.31512 ← | -1.82526 | .47349 ← |
| 4.66165 | -1.79706 | .46211 |
| 5.00456 | -1.76669 | .45091 |
| 5.34394 | -1.73418 | .43991 |
| 5.67921 | -1.69955 | .42913 |
| 6.01006 | -1.66282 | .41856 |
| 6.33595 | -1.62403 | .40824 |
| 6.65616 | -1.58322 | .39816 |
| 6.96974 | -1.54041 | .38834 |
| 7.27548 | -1.49565 | .37881 |
| 7.57181 ← | -1.44900 | .36955 ← |
| 7.85679 | -1.40053 | .36060 |
| 8.12802 | -1.35030 | .35196 |
| 8.38261 | -1.29842 | .34364 |
| 8.61715 | -1.24501 | .33565 |
| 8.82770 | -1.19019 | .32800 |
| 9.00985 | -1.13414 | .32069 |
| 9.15883 | -1.07704 | .31375 |
| 9.26965 | -1.01913 | .30716 |
| 9.33743 | -0.96065 | .30094 |
| 9.35765 ← | -0.90189 | .29509 ← |
| 9.32657 | -0.84317 | .28961 |
| 9.24162 | -0.78481 | .28449 |
| 9.10179 | -0.72716 | .27974 |
| 8.90766 | -0.67055 | .27535 |
| 8.66258 | -0.61533 | .27132 |
| 8.37059 | -0.56179 | .26762 |

| | | |
|---|---|---|
| 8.08826 | -0.51023 | .26425 |
| 7.67325 | -0.46085 | .26120 |
| 7.28403 | -0.41385 | .25846 |
| 6.87935 ← | -0.36935 | .25600 ← |
| 6.46773 | -0.32742 | .25381 |
| 6.05700 | -0.28808 | .25188 |
| 5.65399 | -0.25129 | .25018 |
| 5.26437 | -0.21700 | .24871 |
| 4.89253 | -0.18511 | .24745 |
| 4.54168 | -0.15548 | .24638 |
| 4.21390 | -0.12799 | .24549 |
| 3.91033 | -0.10248 | .24477 |
| 3.63132 | -0.07880 | .24420 |
| 3.37660 ← | -0.05679 | .24378 ← |
| 3.14545 | -0.03632 | .24349 |
| 2.93665 | -0.01722 | .24332 |
| 2.74953 | .00063 | .24327 |
| 2.58215 | .01737 | .24332 |
| 2.43332 | .03312 | .24348 |
| 2.30164 | .04798 | .24374 |
| 2.18577 | .06207 | .24408 |
| 2.08446 | .07548 | .24452 |
| 1.99651 | .08829 | .24503 |
| 1.92085 ← | .10059 | .24562 ← |
| 1.85649 | .11246 | .24629 |
| 1.80254 | .12394 | .24704 |
| 1.75821 | .13513 | .24785 |
| 1.72280 | .14606 | .24873 |
| 1.69569 | .15679 | .24969 |
| 1.67636 | .16738 | .25070 |
| 1.66433 | .17787 | .25179 |
| 1.65921 | .18831 | .25294 |
| 1.66067 ← | .19874 | .25416 ← |
| 1.66845 | .20919 | .25544 |
| 1.68233 | .21971 | .25678 |
| 1.70213 | .23034 | .25820 |
| 1.72774 | .24112 | .25968 |
| 1.75908 | .25207 | .26123 |
| 1.79611 | .26323 | .26285 |
| 1.83884 | .27465 | .26454 |
| 1.88731 | .28635 | .26630 |
| 1.94158 | .29838 | .26814 |
| 2.00177 ← | .31076 | .27005 ← |
| 2.06801 | .32354 | .27204 |
| 2.14049 | .33676 | .27412 |
| 2.21940 | .35045 | .27627 |
| 2.30498 | .36466 | .27852 |
| 2.39749 | .37943 | .28086 |
| 2.49723 | .39481 | .28329 |
| 2.60453 | .41083 | .28582 |
| 2.71973 | .42755 | .28845 |
| 2.84321 | .44502 | .29119 |
| 2.97538 ← | .46330 | .29405 ← |
| 3.11667 | .48243 | .29702 |
| 3.26754 | .50248 | .30011 |
| 3.42846 | .52351 | .30333 |
| 3.59992 | .54559 | .30669 |
| 3.78241 | .56877 | .31019 |
| 3.97645 | .59314 | .31384 |
| 4.18254 | .61876 | .31765 |

| | | | | |
|---|---|---|---|---|
| 4.40117 | ← | .64572 | .32162 | ← |
| 4.63281 | | .67410 | .32577 | |
| 4.87788 | | .70397 | .33009 | |
| 5.13675 | | .73542 | .33461 | |
| 5.40972 | | .76855 | .33934 | |
| 5.69696 | | .80343 | .34428 | |
| 5.99853 | | .84016 | .34944 | |
| 6.31429 | | .87884 | .35484 | |
| 6.64390 | | .91954 | .36049 | |
| 6.98673 | | .96235 | .36640 | |
| 7.34185 | ← | 1.00736 | .37258 | ← |
| 7.70791 | | 1.05463 | .37906 | |
| 8.08310 | | 1.10424 | .38584 | |
| 8.46507 | | 1.15622 | .39294 | |
| 8.85082 | | 1.21062 | .40037 | |
| 9.23663 | | 1.26744 | .40816 | |
| 9.61796 | | 1.32667 | .41631 | |
| 9.98934 | | 1.38828 | .42483 | |
| 10.34431 | | 1.45217 | .43376 | |
| 10.67533 | | 1.51821 | .44309 | |
| 10.97376 | ← | 1.58624 | .45284 | ← |
| 11.22981 | | 1.65602 | .46302 | |
| 11.43261 | | 1.72725 | .47365 | |
| 11.57028 | | 1.79955 | .48473 | |
| 11.63009 | | 1.87247 | .49627 | |
| 11.59874 | | 1.94550 | .50826 | |
| 11.46268 | | 2.01801 | .52071 | |
| 11.20856 | | 2.08929 | .53362 | |
| 10.82376 | | 2.15858 | .54696 | |
| 10.29705 | | 2.22501 | .56073 | |
| 9.61921 | ← | 2.28765 | .57491 | ← |
| 8.78380 | | 2.34555 | .58947 | |
| 7.78782 | | 2.39769 | .60437 | |
| 6.63235 | | 2.44308 | .61959 | |
| 5.32300 | | 2.48071 | .63506 | |
| 3.87023 | | 2.50967 | .65074 | |
| 2.28935 | | 2.52908 | .66657 | |
| 2.53645 | | .69845 | 0 | |
| -1.18338 | | 2.53640 | .69854 | |
| -3.01335 | ← | 2.52324 | .71444 | ← |
| -4.87007 | | 2.49848 | .73022 | |
| -6.72273 | | 2.46205 | .74581 | |
| -8.54067 | | 2.41408 | .76114 | |
| -10.29460 | | 2.35486 | .77613 | |
| -11.95775 | | 2.28490 | .79071 | |
| -13.50662 | | 2.20484 | .80482 | |
| -14.92267 | | 2.11546 | .81839 | |
| -16.19072 | | 2.01763 | .83138 | |
| -17.30116 | | 1.91234 | .84373 | |
| -18.24877 | ← | 1.80057 | .85540 | ← |
| -19.03268 | | 1.68336 | .86635 | |
| -19.65580 | | 1.56174 | .87654 | |
| -20.12430 | | 1.43669 | .88596 | |
| -20.44651 | | 1.30917 | .89459 | |
| -20.63427 | | 1.18004 | .90241 | |
| -20.69832 | | 1.05013 | .90942 | |
| -20.65169 | | .92018 | .91561 | |
| -20.50725 | | .79083 | .92098 | |
| -20.27767 | | .66266 | .92555 | |
| -19.97514 | ← | .53617 | .92931 | ← |

| | | |
|---|---|---|
| -19.61109 | .41178 | .93229 |
| -19.19606 | .26985 | .93449 |
| -18.73961 | .17065 | .93594 |
| -18.25625 | .05443 | .93664 |
| -17.73549 | -0.05863 | .93663 |
| -17.20182 | -0.16839 | .93591 |
| -16.65460 | -0.27476 | .93452 |
| -16.09914 | -0.37766 | .93247 |
| -15.53874 | -0.47705 | .92978 |
| -14.97682 ← | -0.57291 | .92648 ← |
| -14.41597 | -0.66525 | .92259 |
| -13.85823 | -0.75407 | .91813 |
| -13.30521 | -0.83940 | .91312 |
| -12.75810 | -0.92128 | .90759 |
| -12.21776 | -0.99974 | .90155 |
| -11.68480 | -1.07482 | .89503 |
| -11.15958 | -1.14658 | .88805 |
| -10.64228 | -1.21507 | .88063 |
| -10.13296 ← | -1.28033 | .87279 |
| -9.63154 | -1.34242 | .86455 ← |
| -9.13787 | -1.40138 | .85593 |
| -8.65174 | -1.45726 | .84695 |
| -8.17269 | -1.51011 | .83762 |
| -7.70102 | -1.55997 | .82798 |
| -7.23583 | -1.60689 | .81803 |
| -6.77701 | -1.65091 | .80779 |
| -6.32425 | -1.69207 | .79729 |
| -5.87724 | -1.73040 | .78653 |
| -5.43571 ← | -1.76593 | .77555 ← |
| -4.99936 | -1.79871 | .76435 |
| -4.56795 | -1.82876 | .75295 |
| -4.14125 | -1.85612 | .74137 |
| -3.71903 | -1.88081 | .72963 |
| -3.30109 | -1.90286 | .71774 |
| -2.88727 | -1.92230 | .70573 |
| -2.47741 | -1.93915 | .69359 |
| -2.07137 | -1.95344 | .68136 |
| -1.66903 | -1.96519 | .66905 |
| -1.27026 | -1.97442 | .65667 |
| -0.87505 | -1.98116 | .64425 |
| -0.48324 | -1.98543 | .63178 |
| -0.09481 | -1.98724 | .61930 |
| .29030 | -1.98662 | .60682 ← |

?00 .00010

.300 .1763 11.8639 18.6361 -25.3473 .4681 .0095

| | | |
|---|---|---|
| = | -0.78760 | K= 1 |
| = | -0.77001 | K= 8 |
| = | .01556 | K= 148 |
| = | -0.77001 | K= 8 |
| = | .01556 | K= 148 |
| = | -0.77001 | K= 8 |
| = | .01556 | K= 148 |

COMMON DATA

| | | | | | |
|---|---|---|---|---|---|
| 1 | 3 | 1 | 3.14159 | 32.17200 | .00010 |

| | | | |
|---|---|---|---|
| R1= 1.00 | R2= .70 | R3= .85 | R4= .90 |
| R1= 1.00 | R2= .70 | R3= .95 | R4= .90 |
| R1= 1.00 | R2= .70 | R3= 1.05 | R4= .90 |
| R1= 1.00 | R2= .70 | R3= 1.15 | R4= .90 |
| R1= 1.00 | R2= .70 | R3= .95 | R4= .80 |
| R1= 1.00 | R2= .70 | R3= 1.05 | R4= .80 |
| R1= 1.00 | R2= .60 | R3= .95 | R4= .90 |
| R1= 1.00 | R2= .60 | R3= 1.05 | R4= .90 |

17 .300 .1763

5.70189

| | | | | |
|---|---|---|---|---|
| MODE=0 | T= 0 | XCO= 1.04263 | XCDO= -0.78760 | K= |
| MODE=1 | T= .21991 | XCO= .86304 | XCDO= -0.77001 | K= |
| MODE=2 | T= 4.61614 | XCO= 2.64438 | XCDO= .01556 | K= |
| MODE=1 | T= 6.50310 | XCO= 1.67734 | XCDO= -0.77001 | K= |
| MODE=2 | T= 10.90133 | XCO= 3.43868 | XCDO= .01556 | K= |
| MODE=1 | T= 12.78628 | XCO= 2.46861 | XCDO= -0.77001 | K= |
| MODE=2 | T= 17.18451 | XCO= 4.22995 | XCDO= .01556 | K= |

TRAVEL .79127

# APPENDIX III

# APPENDIX III

## DERIVATIONS USED IN ANALOG SIMULATION FOR THE NONSINUSOIDAL DRIVE

### 3.1A Acceleration Cycle of the Four-Bar Mechanism

The following relations are directly obtained from Figure(8).

$$\begin{aligned} x_a &= X_1 \\ x_b - x_a &= X_2 \\ y_a &= Y_1 \\ y_b - y_a &= Y_2 \\ r_1 - x_b &= X_3 \\ y_b &= Y_3 \end{aligned} \qquad (3.1A)$$

$$\begin{aligned} x_1 + x_2 + x_3 &= r_1 \\ y_1 + y_2 - y_3 &= 0 \end{aligned} \qquad (3.2A)$$

The mathematical relations for the four bar mechanism (2) and (4) in Appendix I, can be written using the above relations to read as follows:

$$x_1 = r_2 \cos(\omega t)$$
$$Y_1 = r_2 \sin(\omega t)$$
$$X_2{}^2 + Y_2{}^2 = r_3{}^2 \qquad (3.3A)$$
$$X_3{}^2 + Y_3{}^2 = r_4{}^2$$

From (2) and (3) rearranging terms leads to

$$X_1 = r_2 \cos(\omega t)$$
$$Y_1 = r_2 \sin(\omega t)$$
$$X_2 = r_1 - X_1 - X_2$$
$$Y_2 = \sqrt{r_3{}^2 - X_2{}^2} \qquad (3.4A)$$
$$X_3 = \sqrt{r_4{}^2 - Y_3{}^2}$$
$$Y_3 = Y_1 + Y_2$$

which are the displacement components of the four bar mechanism. Differentiating the equations given in (4) provides the velocity components.

$$\dot{X}_1 = -\omega Y_1$$
$$\dot{Y}_1 = \omega X_1$$
$$\dot{X}_2 = -\dot{X}_1 - \dot{X}_3$$
$$\dot{Y}_2 = -(X_2 . \dot{X}_2)/Y_2 \qquad (3.5A)$$
$$\dot{X}_3 = -(Y_3 . \dot{Y}_3)/X_3$$
$$\dot{Y}_3 = \dot{Y}_1 + \dot{Y}_2$$

Differentiating the equations given in (5) provides the acceleration components.

$$\ddot{X}_1 = -\omega\dot{Y}_1$$
$$\ddot{Y}_1 = \omega\,\dot{X}_1$$
$$\ddot{X}_2 = -\ddot{X}_1-\ddot{X}_3$$
$$\ddot{Y}_2 = -\frac{\ddot{X}_2 X_2-(\dot{X}_2)^2}{Y_2} + \frac{X_2{}^2\dot{X}_2{}^2}{Y_2{}^{1.5}} \qquad (3.6A)$$
$$\ddot{X}_3 = -\frac{\ddot{Y}_3 Y_3+(\dot{Y}_3)^2}{X_3} + \frac{Y_3{}^2\ddot{Y}_3{}^2}{X_3{}^{1.5}}$$
$$\ddot{Y}_3 = \ddot{Y}_1+\ddot{Y}_2$$

The acceleration components of the ocupling point b using the relations given in (1) are as follows:

$$\ddot{x}_b = r_1-\ddot{X}_3$$
$$\ddot{y}_b = \ddot{Y}_3 \qquad (3.7A)$$

Using the relations given in (6) and (7) the in-plane acceleration cycle for a trough forming an angle $\alpha$ to the horizontal is as follows:

$$\ddot{y} = (r_1-\ddot{X}_3)\cos(\alpha) + \ddot{Y}_3\sin(\alpha) \qquad (3.8A)$$

### 3.2A Basic Functions of the "Comparator" and the AND Gate

The comparator is a component in the analog computer. Its input is located in the analog field and its output is found at the logic field. It accepts analog signals as inputs and provides logic outputs. Logic one appears at its output for a positive analog input, this logic output changes to logic zero whenever the analog input changes to negative value.

If another logic signal is connected to the comparator at the terminal designated L, latching occurs. This means that the comparator output stays unchanged, irrespective of the changes that occur at its analog input when L is logic one. If L is logic zero, the comparator operates in the usual manner previously described.

The AND gate is a device that receives two logic signals at its input terminals and provides either logic one at its output when both input signals are high or logic zero when either of the input signals is zero.

It should be noted that the logic signal output of the comparators and the AND gates and their compliment signals are directly available in the logic area of the analog computer.