Optimum Building Design Decision Making

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ABSTRACT

OPTIMUM BUILDING DESIGN DECISION MAKING

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Construction projects are large, complex and technologically advanced and the number of design decision to be made are growing. The nature of the design decisions changes as a project progresses and also varies for different types of investors. These decisions should satisfy the objectives of the investor, the designer, as well as the regulatory constraints.

Major economic decisions are made at the front end of the project, while most of the technical design decisions are made at the latter stages (pre-design, design, construction). Vital economic decisions must be made at early stages of the project when minimum information is available and when the cost resulting from them cannot be accurately determined. The decisions made by individuals based on their personal experience cannot be accurate for large and complex engineering projects.

The goal of this study is to contribute towards the development of a methodology for the building design problem which reflects the investor's views and provides the designer (architect/ engineer) with enough information to

make better decisions throughout the design phase by selecting design alternatives which will optimize the performance of the system as measured by some combination of financial, technical and user criteria subject to specified constraints.

This thesis presents a general framework for systematic analysis of the building design decision-making problem. In the design practice, first, the building system is decomposed into a series of subsystems/ components. They are designed by different design disciplines and then integrated to constitute the building. Similarly, the performance measures can be decomposed into different components and integrated to measure the performance of the overall building. In this study, the main emphasis is placed on the financial performance measures. The deterministic and probabilistic cost models for capital cost and future costs components are developed, incorporating uncertainties involved in forecasting the future. The time variation of the uncertainty level in the future costs component as well as the effect of maintenance policies on the future performance of the building system are studied and their mathematical formulations are presented.

A four stage design alternative selection process is developed that employs the preference technique and the concept of multiplicative utility function. The use of this methodology has been investigated through a case study.

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TABLE OF CONTENTS

page

page

7

page

LIST OF FIGURES

page

LIST OF TABLES

page

CHAPTER I

1.1-Introduction

The construction industry is large, complex, highly competitive and it is unique when compared to all other industry sectors, in that it involves a group of experts (in conceptual phase, design/ construction phases, and operation phase) brought together for one project and disbanded after completion of that project. Building projects are rarely similar in detail, vary in magnitude, and are diverse in configuration. The decisions made in each stage of project development influence future decisions and affect the overall performance of the building.

Decision-making can be viewed as a sequence of steps: i) defining explicitly the needs of the investor/user; ii) expressing their needs in terms of the decisions to be made; iii) determining alternate sets of solutions, and iv) selecting the best alternative based on their projected performance. The design process is a dynamic one and it has rather unique characteristics. As the project progresses the nature of decisions changes. In making design decisions, a building designer faces several difficulties including :

i) prediction of the future economic environment;

ii) assessment of technological advancements, many of which are unproven in terms of economic and technical performance;

iii) uncertainty involved in the estimation of initial and future costs, reliability of system, etc;

iv) an increasing number of design alternatives to consider;

v) difficulty in coordinating design activities of the various professional disciplines involved in the design process;

vi) existence of multiple, conflicting design objectives; and

vii) existenceof more stringent rules and constraints.

One of the biggest problems identified in the architectural literature (1), is the structure of the design team, the lack of proper communication between design disciplines, and the resulting low design efficiency. Each member of the design team performs a particular task and is generally concerned with a discrete functional area in the building design process. The design team ,therefore, is an association of specialists from different disciplines with an inherent problem of coordination and communication. (2)

1.2-Need for a Decision-Making Framework

There is a need for a coherent framework for design decision-making. Such a framework should consider the hierarchy of the design decisions, the different design disciplines involved and interrelationships between these disciplines, and interaction between different design

* Numbers in paranthesis represent references at the end of the thesis.

desires, and it should be useful in the selection of the best design alternatives (however best is measured and sought).

In developing this framework the building project is considered as a composition of four basic elements. These include : land assembly, design/ construction, operation and disposal (Fig. 1-1).

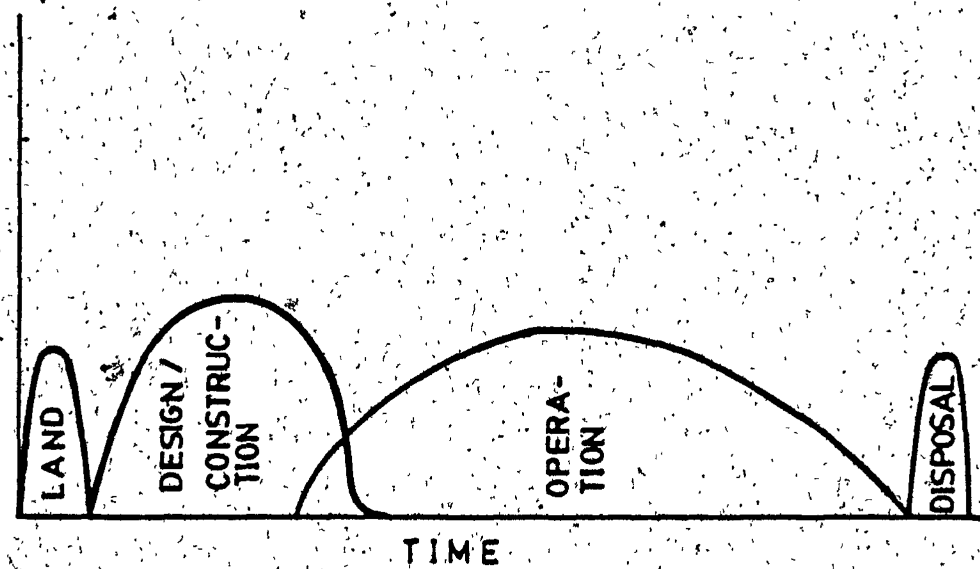


FIG. (1-1) - Basic Elements of a Building Project.

Different types of investors are interested in one or more of the above elements. The different types of investors are:

i) Speculative builder, who is mainly interested in the land, design/ construction, and disposal elements of the building;

ii) Owner, who is interested in all four elements of the building; and

iii) public investor, who is concerned with most of the building elements, plus some external elements such as social/ economic/ political/ environmental impacts of the project.

In order to effectively measure the performance of a building, one needs to define an appropriate set of criteria that reflects the objectives of the investor/ designer at a particular phase of project development. The design evaluation criteria can be classified in four categories (Fig 1-2);

i) single criterion/ deterministic
ii) single criterion/ probabilistic
iii) multiple criteria/ deterministic
iv) multiple criteria/ probabilistic

The use and accuracy of a performance measure is directly influenced by the availability and accuracy of data. They improve as a project progresses from one phase to the next. For example, in the pre-feasibility phase, where the key decisions about investment affecting the construction and lifetime performance of the project are made, the information about size of the building, its location and some historical cost data or cost estimation relationships is available. In the design phase, where technical decisions are made, detailed information about each building subsystem, quantities, unit rates, and

OBJECTIVE FUNCTION / VARIABLE TREATMENT	SINGLE CRITERION 1	MULTIPLE CRITERIA 2
DETERMINISTIC 1	Optimize one of 1) Maximize NPV 2) Minimize LCC 3) Minimize ΣC_{id} 4) Minimize O_{id} for Energy Subject to: i) Equity constraints ii) Code constraints iii) Future expenditure constraints	Optimize two or more of 1) Maximize NPV 2) Minimize LCC 3) Minimize ΣC_{id} 4) Minimize O_{id} for Energy Subject to: i) Equity constraints ii) Code constraints iii) Future expenditure constraints
PROBABILISTIC 2	Optimize one of 1) Maximize $\overline{U}$[NPV] 2) Maximize $\overline{U}$[LCC] 3) Minimize Prob.[LCC $\leq$ budget] 4) Maximize Prob.[O_{id} $\leq$ energy consumption budget] Subject to: i) Equity constraints ii) Code constraints iii) Future expenditure constraints iv) Risk exposure constraints	Optimize two or more of 1) Maximize $\overline{U}$[NPV] 2) Maximize $\overline{U}$[LCC] 3) Minimize Prob.[LCC $\leq$ budget] 4) Maximize Prob.[O_{id} $\leq$ energy consumption budget] Subject to: i) Equity constraints ii) Code constraints iii) Future expenditure constraints iv) Risk exposure constraints

FIG. (1-2) - Treatment of Optimization Problems.(3)

construction schedule becomes available.

1.3-Literature Review

The existing literature is grouped, to the extent possible, under three main headings, i) building design framework, ii) cost modeling, and iii) decision-making methodologies. Some papers relate to more than one topic area.

i) Building design framework

Volker Hartkopf (4) has done a case study in which he demonstrates the usefulness of economic evaluation in selecting a design alternative. The importance of investors' needs and also the consideration of the design objectives in their order of importance have been noted. The effect of the economic environment on the economic ranking of alternatives is demonstrated through a sensitivity analysis.

M.P.T. Linzey, J.F. Brotchie and J.F. Nicholas (5), suggested that, since the building design problem is a multidimensional one and different parties (designer, investor, user and public) have different objectives, the design problem should be treated systematically to accommodate all. The systems approach helps the designer to rationalize the process of the design. The key to a rationalized process is the coordination and the integration of the goals of all concerned and the restatement of these goals as a single objective function. To assess a complex problem, it is important to study the subsystems and the components of the design problem. The system should be capable of manipulation by the user and should reflect the building performance with regard to the decisions made. The decision-making model should not only measure the goal achievement level but should also demonstrate the sensitivity of the design parameters to the design decisions. The authors have also explained different sets of design goals including economical, environmental, and user goals.

ii) Cost modeling

J. E. Tumarkin (6) has suggested the use of statistical cost estimation relationships by decision makers prior to the involvement of architect/ engineer. In order to develop the cost estimation relationship, the project is expressed in terms of its physical characteristics (floor area, volume, total surface area, number of floors, etc.), performance characteristics (type of user and operation phase characteristics), and economic characteristics (investor type, time and location of construction, etc.).

Cost data from three types of building projects (college housing, hospital facilities and educational facilities) are used for the purpose of regression analysis. The floor area and total surface area are determined as principal variables for cost estimation relationship for hospital buildings. In his analysis, Tumarkin found a linear relationship for cost as a function of floor area. In this paper, the sensitivity analysis is performed for capital cost with respect to some of the physical, performance, and economic variables.

V. Kouskoulas and E. Koehn (7), have developed a mathematical expression for the building capital cost estimation at the pre-design stage. The cost of a building is expressed as a function of location, building type, building height, quality of workmanship, technology used and time factor. The authors suggest the use of indexes for each of these variables and express the total cost as;

$$C = \sum_{k} A_k \cdot V_k + A_o$$

Where C is the estimate of the cost /ft^2; A_o and A_k, k=0,....6 are constants determined using historical data and V_k, k=1,....6 are the indexes for the stated variables. For a sample of historical data, the regression analysis has been performed and the results are presented. The percentage of errors for the model varies from 3.3% to -7.4%. It is shown that by eliminating one or more of the variables the coefficient of determination decreases. It has been suggested that the data records be handled with care and updated regularly. Other simplified forms of the pre-design cost estimation relationships have also been reported in the literature (8).

M. Misurak (9), stressed the importance of financial decisions made during the early stages of the project, especially for large engineering projects. He has investigated the use of macro- and micro-economic models at different levels of a project. Sensitivity analysis provides a great deal of understanding about variables and their effect on project performance. He has also examined different capital expenditure models (Beta and Uniform models). Use of all models has been demonstrated through a computer program.

R. Flanagan and G. Norman (10) have described a technique which identifies the probability distribution to be used in capital cost prediction. The general methodology is applicable at any stage of the project construction. They postulate the required characteristics for a probability distribution for cost prediction. They include limited end points, flexibility, and the ability to derive from a finite sample size. They suggest that Beta distribution fulfills these requirements. The process of determining the distribution model has been explained by means of an example using a sample of 10 unit rates for structure and superstructure. However, in this paper, there is no statistical justification for the selection of Beta distribution.

L.C. Bacher (11), has explained the nature of the capital cost estimation problem and the sources of uncertainty. Capital cost estimation is a process in which the estimator is called upon to exercise his skill and judgement in determining quantities, the unit costs for labour, material and equipment, the labour and equipment productivity and the material waste factor. The expression given for estimating is:

$$E = q_i . (r_i (w_i + t_i) + u_i . p_i)$$

where:

q_i = quantity of item i

r_i = productivity factor for labour and equipment for item i

w_i = the composite wage of the crew performing the item i

t_i = the rental rate for required equipment for item i

u_i = waste factor for item i

p_i = unit price of the material for item i

Similar expressions in more detailed form are also suggested in the literature (12). Since there are uncertainties involved in each of the above factors, the cost estimation problem should be treated probabilistically. The determination of the contingency then becomes a matter of establishing an acceptable probability level to accomodate the cost overruns. In Bacher's paper (11), it is suggests that the distribution of the overall cost is Normal and that the mean and the variance explain the nature of the distribution. However, this assumption has not been supported in the literature (10).

C.D. Zinn, W.G. Lesso and B. Motazed (13) have also studied capital investment under uncertainty. They have used Laplace transforms in computing the mean and variance of the net present value "NPV" distribution. They have also recommended the use of semi-variance to detect the skewness of the distribution. Semi-variance is the variance of the distribution to the left of the expected value, measuring the downside risk. The expressions for different types of cash flow under uncertain project life are presented in this work.

iii) Decision-making methodologies

Wilson and Templeman (14), in their study, have gone one step further and have given an almost complete picture of the optimum thermal design of an office building. They have assumed that the structure of the building, including internal and external configuration, ceiling, walls, floor, partition size and material is known. Also, some high level decisions, such as the type of heating system, energy source (e.g. fuel oil) have already been made. The purpose of this paper is to find the balance point in the tradeoff between thermal subsystem and other subsystems. The objective function selected is to minimize life-cycle cost (tradeoff between insulation thickness, fuel consumption, system size and its efficiency). Geometric programming was used for the optimization of the design.

S.O.D. Russell and W. F. Caselton (15), have investigated the use of two surrogate utility functions in the engineering design decision-making problems. The first, is the Kelly criterion, which is based on the maximization of the expected value of wealth. The strategy based on this criterion also maximizes the rate of asset growth. The information required for application of this criterion is the total wealth level and the probability of success. Regardless of the individual characteristics and of the wealth level, this criterion suggests the same strategy. Use of this function has been illustrated through several examples (betting strategy, bidding, investment selection). The second, Shannon's measure of information, is designed for dynamic problems and is built on the use of the Bayesian approach to treat individual cases. The utility function has two distinct characteristics: i) it attains its maximum value with respect to posterior information and ii) it depends on the information gathered for each case while independent of the other cases. These two characteristics make the utility function "proper and local". The authors cite the example of an environmental network design problem with dynamic characteristics and demonstrate the application of this utility function.

E. Bussey and G.T. Stevens Jr. (16), have presented a theoretical approach to the probabilistic capital cost budgeting problem. They introduce the use of cubic and quadratic polynomial functions as an approximation of utility function of a decision maker over a limited range of return. They have noted that cubic polynomial implies that the behavior of a decision-maker is risk-averse up to a certain level of payoff and then becomes risk-prone. The quadratic polynomial, however, allows for a situation where the decision-maker can be risk- prone even when there is no positive payoff. Equations have been derived to compute the expected value of the utility functions as a function of the first four moments (mean, variance, skewness, kurtosis). Since the higher order moments have no physical explanation, the authors have suggested that the use of higher order polynomials is not practical.

A.D. Radford and J.S. Gero (17), have studied a multi-criteria model to be used in decision-making. They have noted the importance of studying trade-offs between performance levels of different criteria. The multiple criteria Pareto optimization technique has been used to select the best design alternative. However, due to the existence of conflicting criteria in multiple-criteria models, the architect/engineer is often the person making the final decisions based on trade-off curves. This is particularly difficult where the number of conflicting criteria is more than two.

A.D. Radford, J.S. Gero and N.S. Murthy (18), have studied the architectural design decision-making problem and suggested a multi-stage design alternative selection process. There are two main steps in this process, i) determination of the Pareto optimal sets and ii) selection of the best design alternative, using the inverse goal programming approach in several stages. In this research work they have tried to overcome the problem of trade-off analysis in the multidimensional (more than three variables) problems. Graphs are developed for pairs of attributes and the Pareto set is determined for each. Then, for each pair, the constraints' boundaries are tightened until an acceptable number of alternatives remains to be considered. The example of surface material selection is presented to illustrate the above process. Details of the methodology are documented by N.S. Murthy (19).

S. Mattar, W. Bitterlich, P. Manning and P. Fazio (20), introduced the concept of decision by exclusion. Here, the authors treat the design decision-making problem as a multi-attribute one. Using an implicit assumption about criteria set (set of independent criteria), they suggest that the additive utility function be used in assessing the overall performance of each design alternative. It has been assumed that the utility values are known, that the attributes are mutually exclusive and that they fully define the design objectives. Using linear algebra techniques, a range is defined for weightings in which a particular alternative is found to be the best. It has been proven that these ranges are mutually exclusive and, only with respect to the boundary values, can two alternatives perform equally where the designer is indifferent to them. The application of this methodology has been illustrated through a building enclosure design example (21).

C.L. Gupta and M. Anson (22), noted in their paper that because of the lack of suitable design decision-making models, the design often tends to be inadequately tested. For optimum thermal design of the building envelope, they have recognized the relationship between capital cost and future operating and maintenance costs. In order to satisfy the environmental standards, the building is considered to be a techno-economic system. So as to optimize such a system, more than one criterion should be considered and it may include the minimization of the initial cost, degree of discomfort, and the air-conditioning plant capacity.

1.4- Focus and Objectives of the Research

Examination of the literature indicates that there is a need and a move towards improving existing design decision-making methodologies. In addition to the foregoing literature reviews, there have been several approaches to improve the decision-making ability of engineers and architects: 1) to develop criteria which can evaluate the consequences of different design decisions e.g. economic consequences, technical consequences and user performance consequences (23); 2) to study the nature of the different variables in a criterion, identify the type (randomness) and investigate ways of incorporating this information into the design evaluation process (23,24,25); 3) to approach the design decision making problem systematically, identifying the hierarchy of the design decisions and selecting the design alternative using operation research techniques (26); and 4) to treat the design decision-making problem as a multi-attribute one (27,28).

Three major factors have been suggested as the basis of design decisions, i) user objectives/constraints ; ii) the building codes and guidelines; and iii) financing and risk. However, recently other factors such as building efficiency at the operation stage (29), user cost (wages and workers productivity) (30); and allowance for technological changes (building flexibility) (1,31) , are being considered.

A realistic design decision framework should include these factors and should reflect the existing economic environment. Tight competition within the construction industry and an unstable economic environment have made urgent the study and the development of such a model to assist the designer/engineer in making design decisions at different phases of the project.

In this study, emphasis is placed on the development of a framework for the building design decision-making problem, on cost modeling, and on the development of a multiple-criteria design decision making methodology. The explicit goal is to study fundamentals of the building design decision -making problem. However, this research work contributes to the foundation for development of a computer-aided design decision-making tool that can help building designers guide their perception of the design related problems.

1.4.1- Building Design Framework

In this component, a framework for building design decision problems will be developed that considers the hierarchy of design decisions and provides a consistent structure to be used in different phases of project development. Use of such a structure will improve the design decision making environment by providing common information to all design disciplines, thus minimizing conflicts and overlaps.

1.4.2- Cost Modeling

Development of a financial performance criterion requires the development of cost models (including cash flow models) for various phases of the project's life cycle , the formulation of financial performance measures in which the time value of money is incorporated, and the study of the uncertainty involved and its effects on the evaluation of performance measures. Here, different models for capital expenditure profile will be examined, mathematical models for some future cost components will be developed, and application of different probabilistic analysis methods in probabilistic cost modeling will be investigated.

1.4.3- Decision-Making Methodology

Under this heading, work will be focused on developing a methodology which reflects the design objectives (investor views). There is more than one objective and they often conflict with each other. A model is developed that permits all of these objectives (or at least prime objectives) to be included and also reflect the uncertain economic environment which affects the outcome of design decisions.

1.4.4-Objectives

In order to contribute towards the development of a framework that includes the above stated components and solves some of the stated problems, the research objectives are set to be:

1- To develop a framework for the building design decision-making problem .

2- To investigate/ develop financial or cost models in the construction and operating phases of building projects;

3- To develop probabilistic cost models, and demonstrate how they can be incorporated into the design decision-making process;

4- To develop a multiple-objective design decision-making model capable of incorporating both quantitative and qualitative objectives; and

5- To examine models and the methodology through a case study.

The basic hypotheses of this research work are that:

1- A realistic design decision- making tool helps the

1- A realistic design decision- making tool helps the designer to examine design alternatives and to select the most appropriate one.

2- The multiple-criteria decision making model is the most realistic for design decision problems as it reflects both qualitative and quantitative objectives.

3- In the present uncertain economic environment there is a pressing need to identify risk sources, to quantify the magnitude of risk and to incorporate risk into the decision making process.

1.5-Thesis Overview

The development of a framework for the building design problem and deterministic cost models for capital cost, future operating, maintenance, repair and renewal, etc. are presented in Chapter II. Chapter III includes the probabilistic treatment of cost models presented in Chapter II. Development of a multiple criteria/objective design decision- making model is treated in Chapter IV. An actual case study, presented in Chapter V, tests the models and methodologies put forth in previous chapters. The conclusion and recommendations for future studies are found in Chapter VI.

CHAPTER II

DETERMINISTIC MODELING

2.0-Introduction

The objectives of this chapter are: i) to develop an overall framework for the building design problem and ii) to investigate and develop cost models for different elements of the building applicable to different phases of project development.

2.1- Design Decision-Making Criteria

In order to measure the performance of different design solutions one needs to identify a proper set of criteria (performance measures) applicable in different phases of project development. Many performance measures are developed to evaluate the design alternatives. Some of these measures are capable of evaluating only one design objective(e.g. capital cost, payback period,...), while others are more comprehensive (e.g. NPV, LCC,....). In fig. (2-1), some of these measures are shown (note that the measures are not exhaustive nor mutually exclusive). They have been categorized in three sections, i) financial performance measures such as Internal Rate of Return (IRR), Net Present Value (NPV), and Life Cycle Cost (LCC); ii) technical performance measures such as structural feasibility, energy conservation,.... ; and iii) user performance measures, for example, thermal comfort, etc.

Generally, the technical/—user requirements of a

CRITERIA / MOTIVATIONS		PERFORMANCE MEASURES																									
		FINANCIAL														TECHNICAL					USER						
		PAYBACK PERIOD	EQUITY DIVIDEND RATE	TAX SHELTER	VECTOR OF CASH FLOWS	NET SALE CASH REVERSION	INTERNAL RATE OF RETURN	NET PRESENT VALUE	DISCOUNTED BENEFIT COST RATIO	LEVERAGE	INITIAL EQUITY	CAPITAL COST	LOAN COVERAGE RATIO	RISK	LIFE CYCLE COST	ENERGY CONSUMPTION	BUILDING HEIGHT	FLOOR EFFICIENCY	ADAPTABILITY	MAINTAINABILITY	WORKER PRODUCTIVITY	AURAL COMFORT	ILLUMINATED COMFORT	THERMAL COMFORT	ENVIRONMENTAL COMFORT	COMMUNICATION EFFICIENCY	USER COST
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	1	2	3	4	5	1	2	3	4	5	6	7
1	HEDGE AGAINST INFLATION				●	●	●	●	●					●	●	○			○								○
2	QUALITY OF CASH FLOW	○	○	●	●		○	○	○			○	●	●	○	○		○	○	○	●					○	
3	RAPID RECOVERY OF EQUITY	●	○	○	○	○	○	○	○					○	○												○
4	LEVERAGE POTENTIAL	○			○		●	●	●	●	○	○				○											
5	INCOME TAX SHELTER			●	○	●	○	○	○	○	○				○	○											
6	CAPITAL APPREC. POTENTIAL	○		○	○	●	○	○	○			○		○	○	●		○	●	○	○					○	○
7	HIGH RATE OF RETURN		○	○	○	○	●	●	●	○	○	○		●	●	●		○	○	○							
8	PORTFOLIO DIVERSIFICATION	○		○	○		○	○	○		○			●													
9	LIQUIDITY	○				●						●		○		○		○	○	○							
10	PRIDE OF OWNERSHIP															○	●			○	○	○	○	○	○	○	○
11	DEMONSTRABLE NEED															○		●	●	●	●	○	○	○	○	●	●
12	PRESENCE															●	●					○	○	○	○		
13	ECONOMIC NEED																										

● THE CRITERION CAN PORTRAY EXPLICITLY THE MOTIVATION.

○ THE CRITERION HAS SOME CONSIDERATION OF THE MOTIVATION.

FIG. (2-1)- Wedding Investor Motivations with Performance Measures. (3)

building are specified in building codes and the investors design specification, and are mostly implemented in the design phase. The financial criteria are used in the front end of the project to measure the feasibility and attractiveness of the project (e.g. IRR,NPV) and in the design phase as a design decision making tool (e.g. LCC).

2.2- Criteria Formulation

The process of building design may be summarized in two words: decomposition and integration. In order to design a building, it is decomposed into a series of systems (e.g. foundation, structure,mechanical system,.....), subsystems (e.g. for mechanical system, HVAC, Plumbing,.....), and components (e.g. chiller, heat generating system,.....). Different systems/ subsystems/ components are designed by experts from different design disciplines . Then, they are integrated to constitute a building (fig. 2-2).

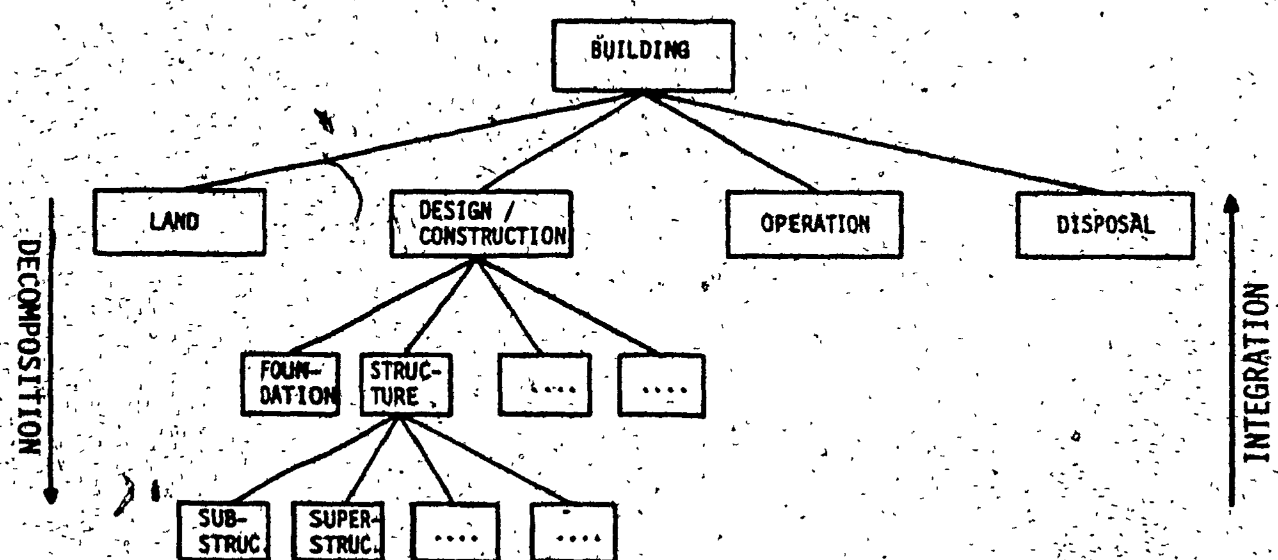


FIG. (2-2) - Building Decomposition and Integration

Similarly, the building design evaluation criteria can be decomposed and integrated to measure the overall performance of a building project. For example, consider the Net Present Value criterion;

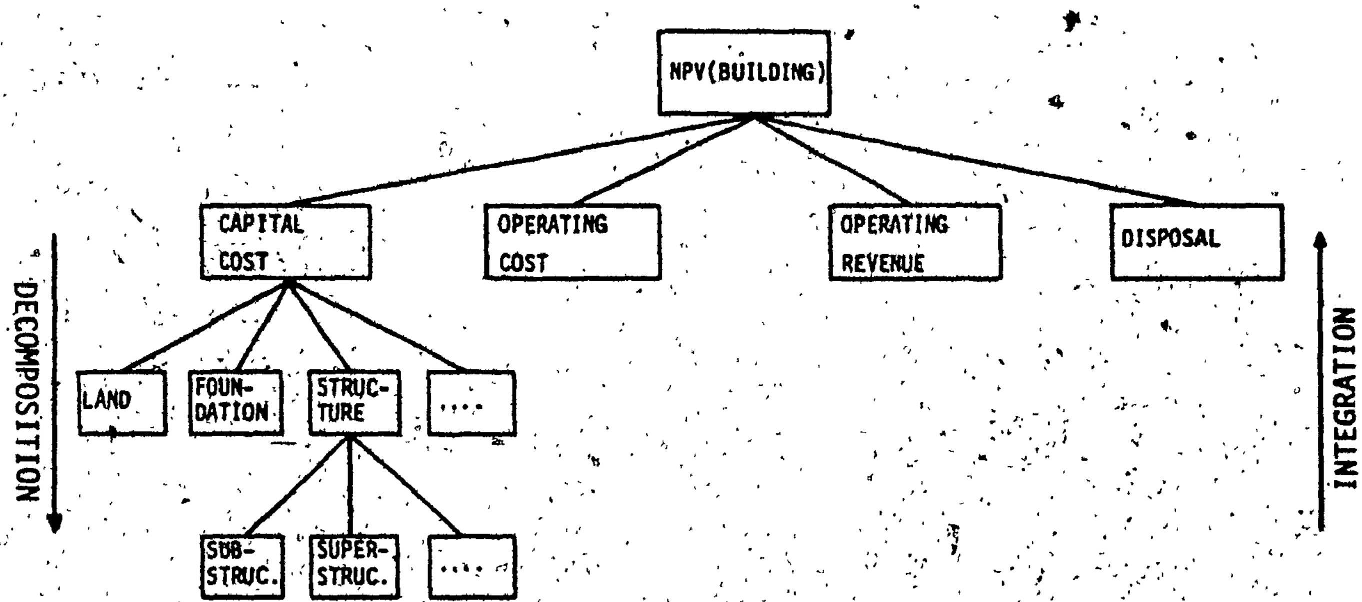


FIG. (2-3)- Building Performance Evaluation

where:

$$NPV(building) = NPV(land) + NPV(design/construction) + \ldots$$

$$NPV(operation) = \sum_j NPV(operation\ of\ system_j)$$

A simple observation in fig. (2-3) is that the NPV criterion has an additive structure (e.g. IRR does not have an additive structure). This characteristic makes the NPV more attractive as a performance measure.

A typical design evaluation criterion "Y" is a function of a set of design decisions or control variables ($\underline{Z}_i$) and a set of cost coefficients ($\underline{X}_i$). In general terms it can be written as;

$$Y = \sum_i f_i(\underline{X}_i, \underline{Z}_i) \tag{2-1}$$

where:

$\underline{X}_i$ = vector of cost related coefficients

$\underline{Z}_i$ = vector of design decision variables

i = system i

In the above expression, $f_i(\underline{X}_i, \underline{Z}_i)$ consists of two main functions;

- design related function
- time related function

$$f_i(\underline{X}_i, \underline{Z}_i) = \sum_j g_{ij}(\underline{X}_{ij}, \underline{Z}_{ij}) * h_{ij}(\underline{Z}_{ij}, t) \tag{2-2}$$

where;

$g_{ij}(\underline{X}_{ij}, \underline{Z}_{ij})$ = design related function for subsystem j

$h_{ij}(\underline{Z}_{ij}, t)$ = time related function for subsystem j (it can be used to describe cost variations either in constant dollar or current dollar terms. In this thesis, it is used to describe constant dollar variation.)

$\underline{X}_{ij}$ = vector of cost related coefficient for subsystem j

$\underline{Z}_{ij}$ = vector of design decision variables for subsystem j

t = time

In the above equation function g_{ij} (for financial performance measures) provides base year cost estimates of construction, operation and disposal elements of a building and function h_{ij} adjusts the constant dollar estimates to reflect the building performance with time. The design decisions vector "$\underline{Z}_{ij}$", contains all design decision variables. Some of these variables are used in function g_{ij} (e.g. total floor area, number of stories, quantities of material required) while others (e.g. aging factor, operating and maintenance policies) are used in computation of h_{ij}. Variables used in g_{ij} are largely controlled by the investor/ designer/ planner. Variables in h_{ij}, on the other hand, are largely beyond the decision makers control. It should be noted that some control may be exerted over the time related variables in h_{ij} through design decisons and operating and maintenance policies.

The design related function may either be continuous or discrete and may be in one of the following forms:

- explicit functional form, in this case a closed form expression is available for g_{ij};
- implicit functional form, in this case a series of functions, a piece of software or other tools are used in design; and
- data base, in this case the information is available in the form of a data base. The information from the data base can be used to develop functional relationships.

In following sections $g_{ij}(\underline{X}_{ij},\underline{Z}_{ij})$ and $h_{ij}(\underline{Z}_{ij},t)$ will be refered to as g_{ij} and h_{ij} respectively.

For example, if an explicit functional form for g_{ij} is available, considering the time varying discount and inflation rates, the expression for current dollar present value becomes:

$$f_i(\underline{X}_i,\underline{Z}_i) = \sum_j \int g_{ij}\, h_{ij}\, e^{(\int_0^t \theta_j(\tau)\, d\tau)}\, e^{(-\int_0^t r(\tau)\, d\tau)}\, dt \qquad (2\text{-}3)$$

where

$\theta_j(\tau)$ = time varying inflation rate for subsystem j

$r(\tau)$ = time varying discount rate

In the above expression h_{ij} represents the constant dollar variation of cost (e.g. variation of cost due to technological changes (construction processes), aging, operation and maintenance policies) with time (Fig 2-4).

In the following sections, some of the more comprehensive financial performance measures are presented.

2.2.1- Net Present Value (NPV)

Net Present Value is a classical financial technique, long taught and successfully used for evaluation of investment proposals. It has been used in practice with some simplification and is well documented in the technical

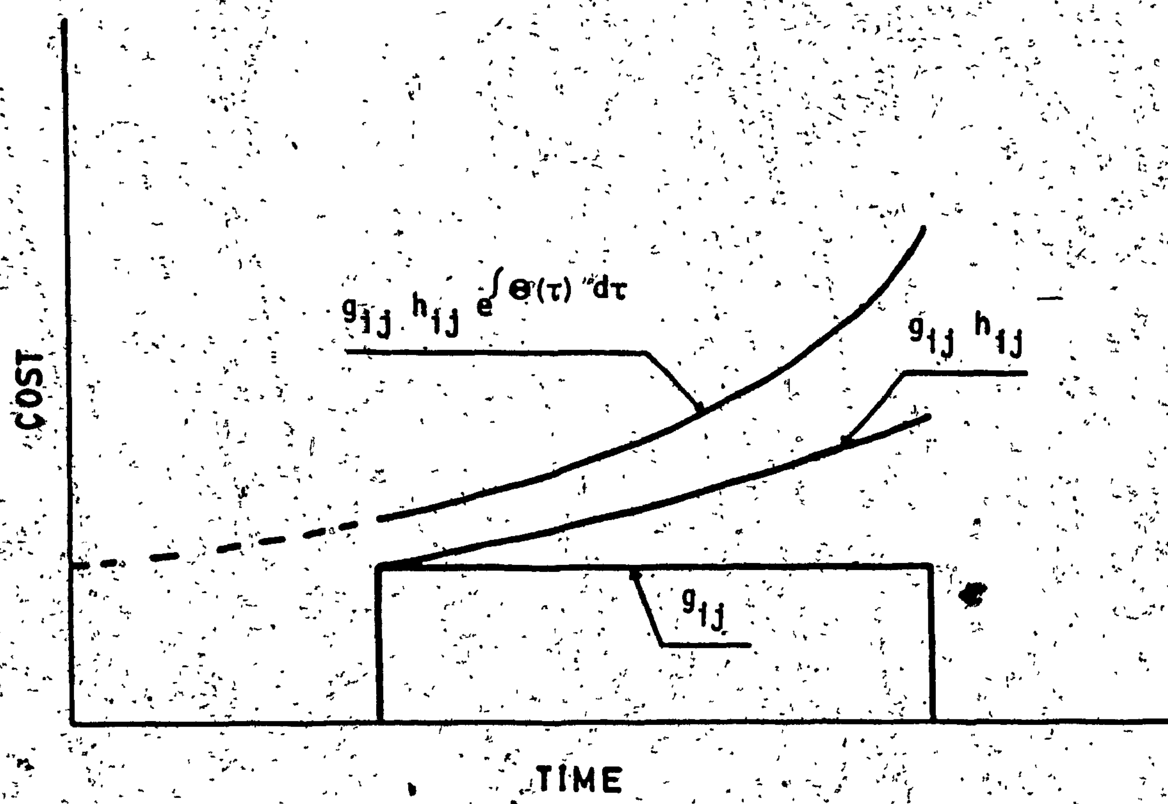


FIG. (2-4) - Graphical Representation of the General Framework.

literature (32).

Mathematically speaking, the net present value can be defined as the present value of all future, after tax, cash flows.

$$NPV = PW\ (\text{ revenue }) - PW\ (\text{ disbursements }) \qquad (2\text{-}4)$$

Assuming continuous cash flow models for revenue and disbursements, the above relationship can be written as;

$$NPV = \int_{T_s}^{T_e} R(t)\, e^{\int_0^t \theta(\tau)\, d\tau}\, e^{-\int_0^t r(\tau)\, d\tau}\, dt - \int_{T_s}^{T_e} C(t)\, e^{\int_0^t \theta(\tau)\, d\tau}\, e^{-\int_0^t r(\tau)\, d\tau}\, dt \qquad (2\text{-}5)$$

where

$R(t)$ = revenue as a function of time

$C(t)$ = disbursements as a function of time including initial equity and interest payments

$\theta(\tau)$ = time varying inflation rate

$r(\tau)$ = time varying discount rate

T_s = start of construction

T_e = study period

In equation (2-5), "r" is the discount rate which represents the interest earned on the unrecovered balance. Whenever taxes are applicable, the discount rate "r" is reduced to an after tax rate.

$$BTCF(t) = R(t) - C(t) - A(t) \qquad (2\text{-}6)$$

$$ATCF(t) = (1-T) * BTCF(t) + T*CCA - T*A(t) \qquad (2\text{-}7)$$

where

BTCF(t) = before tax cash flow at time t

ATCF(t) = after tax cash flow at time t

A(t) = amortization at time t (repayment of principal only)

CCA(t) = capital cost allowance at time t

T = tax rate

It is apparent that the effect of taxation should be considered and that the analysis should be performed based on after-tax cash flow profile. The discount rate should also be an after-tax and inflation adjusted rate. For example, for the case of continuous compounding;

$$r = r^{*} + \theta \qquad (2\text{-}8)$$

where

r^{*} = real rate of return

θ = inflation rate

To include both taxation and inflation in NPV analysis, all cash flow should be measured in current dollars. This is true because of the way taxes are calculated (capital cost allowance is not indexed) and because of the varying nature of inflation with time.

A general expression of the Net Present Value relationship suitable for the building design problem can be written as;

$$NPV = \int_{T_f}^{T_e} R^* e^{\int_0^t \theta_R(\tau)d\tau} e^{-\int_0^t r(\tau)d\tau} dt - \int_{T_s}^{T_f} C_c^* e^{\int_0^t \theta_{C_C}(\tau)d\tau} e^{-\int_0^t r(\tau)d\tau} dt$$

$$- \int_{T_f}^{T_e} C_f^* e^{\int_0^t \theta_{C_f}(\tau)d\tau} e^{-\int_0^t r(\tau)d\tau} dt. \qquad (2\text{-}9)$$

where

R^* = revenue (function of design and time)

C_c^* = initial cost of the project

C_f^* = future cost (function of design and time)

θ_{C_C} = inflation model for initial cost

θ_{C_f} = inflation model for future costs

θ_R = inflation model for revenues

T_f = finish time of construction

In the above expression, R^*, C_c^* and C_f^* represent the constant dollar profile of costs/ revenues ($g_{ij} * h_{ij}$).

2.2.2 - Internal Rate of Return (IRR) (33)

The Internal Rate of Return is defined as the discount rate that equates the net present value of a cash flow to zero. In simple terms, the IRR is the discount rate at which;

$$NPV = 0 \qquad (2\text{-}10)$$

The value of IRR is usually determined by trial and error.

2.2.3 - Life Cycle Costing (LCC) (3)

Life Cycle Cost analysis is a particular application of NPV through which the effects of the design decisions on the overall economic performance of the building can be directly measured. During the past few years, life cycle costing has received a great deal of attention. It is becoming one of the requirements for public projects and an integral part of the codes and design guidelines (e.g. 1980, Systems, ASHRAE Handbook)(34). One of the major reasons for such attention is the rapid escalation of operating costs (consider, for example, the drastic changes in fuel price structure during the early 70's). Although the capital cost of projects can also escalate rapidly, such escalation does not affect the project after the date of completion. Another factor is rapid technological advancement which can increase future maintenance, replacement, and renewal costs of a building (for example, hospitals). Finally, LCC, as a design decision-making tool, incorporates several important variables such as capital cost, future operating, maintenance, repair and renewal expenses, taxes, inflation, etc. .

The Life Cycle Cost relationship can be defined as;

$$\text{LCC} = \text{present worth of initial cost} + \text{present worth of the future costs} \quad (2\text{-}11)$$

Using eq(2-9), the LCC relationship can be expressed as;

$$LCC = \int_{T_s}^{T_f} C_c^* \, e^{\int_0^t \theta_{C_c}(\tau)d\tau} \, e^{-\int_0^t r(\tau)d\tau} \, dt + \int_{T_f}^{T_e} C_f^* \, e^{\int_0^t \theta_{C_f}(\tau)d\tau} \, e^{-\int_0^t r(\tau)d\tau} \, dt. \quad (2\text{-}12)$$

In equation (2-12), the inflation function is assumed to be the same for all subsystems. However, by performing LCC analysis at subsystem level, different inflation functions (in construction and operating phases) can be used for each subsystem. Also, for design purposes, since the characteristics of each building subsystem are unique, it is necessary, i) to perform LCC analysis for each subsystem (e.g. structural , HVAC, electrical,...) and ii) to combine these values to get the total life cycle cost of the building.

In equation (2-12), the constant dollar cost profiles can be written as;

$$C_c^* = \sum_j C_{cj}^* \quad (2\text{-}13)$$

$$C_f^* = \sum_j C_{fj}^* \quad (2\text{-}14)$$

Where

C_{cj}^* = constant dollar profile of capital cost for subsystem j

C_{fj}^* = constant dollar profile of future costs for subsystem j

Furthermore

$$C^*_{fj} = C^*_{oj}+C^*_{mj}+\Delta C^*_{uj}+C^*_{rj}+C^*_{pj}+\Delta F^*_j \qquad (2\text{-}15)$$

Where

C^*_{oj} = constant dollar profile of operating cost for subsystem j

C^*_{mj} = constant dollar profile of maintenance cost for subsystem j

ΔC^*_{uj} = constant dollar profile of user cost increment (decrement) for subsystem j

C^*_{rj} = constant dollar profile of repair/ renewal costs for subsystem j

C^*_{pj} = constant dollar profile of periodical costs for subsystem j

ΔF^*_j = constant dollar profile of savings (or increase) in financing cost due to the design of subsystem j

Now, expressing the constant dollar profiles in terms of design related (g_{ij}) and time related (h_{ij}) functions,

$$C^*_{xj} = C_{xj} * h_{xj} \qquad (2\text{-}16)$$

and, defining an inflation adjusted time factor, B_{xj}, as:

$$B_{xj} = \int h_{xj}\, e^{\int_0^t \theta(\tau)\, d\tau}\ \_e^{-\int_0^t r(\tau) d\tau}\, dt. \qquad (2\text{-}17)$$

Where

C_{xj} = base year cost estimate of future cost component "x", for subsystem j

B_{xj} = present worth of inflation adjusted time factor ,for future cost component "x", for subsystem j

Then, the LCC relationship for building subsystem j, may be expressed as;

$$LCC_j = C_{cj} B_{ij} + (1-T) * (C_{oj} B_{oj} + C_{mj} B_{mj} + \Delta C_{uj} B_{uj}) + C_{rj} B_{rj} +$$

$$C_{pj} B_{pj} - S_j B_{sj} - \Delta F_j B_{fj} - T * C_{cj} B_{ccaj} \qquad (2\text{-}18)$$

where

C_{cj} =base year estimate of capital cost for subsystem j

B_{ij} =present worth of time factor for capital cost of subsystem j

C_{oj} =base year estimate of operating cost for subsystem j

B_{oj} =present worth of time factor for operating cost of subsystem j

C_{mj} =Base year estimate of maintenance cost for subsystem j

B_{mj} =present worth of the time factor of maintenance cost of subsystem j

ΔC_{uj} =increment or decrement in user cost due to adoption of design alternative for subsystem j

B_{uj} =present worth of time factor for user cost of subsystem j

C_{rj} =base year estimate of repair/renewal cost for subsystem j

B_{rj} =present worth of time factor for repair/renewal cost of subsystem j

C_{pj} =base year estimate of periodical repair cost for subsystem j

B_{pj} =present worth of time factor for periodical cost of subsystem j

S_j =base year estimate of net (after tax, expenses), selling price of subsystem j

B_{sj} =present worth of the time factor for selling of subsystem j

ΔF_j =base year estimate of saving (or increase) in financing cost due to the design of subsystem j

B_{fj} =present worth of the financing cost for subsystem j

B_{ccaj} =present worth of the time factor for capital cost of subsystem j

Then the total building life cycle cost;

$$TLCC = \sum_j LCC_j \qquad (2\text{-}19)$$

If the time factors are defined as after tax factors, and time factors for capital cost and capital cost allowance are combined, then equation (2-18) becomes;

$$TLCC = \sum_j \left[C_{cj} B_{ij} + C_{oj} B_{oj} + C_{mj} B_{mj} + C_{rj} B_{rj} + C_{pj} B_{pj} + \Delta C_{uj} B_{uj} - S_j B_{sj} - \Delta F_j B_{fj}\right] \qquad (2\text{-}20)$$

In this study, the main task is to investigate the kinds of models appropriate for design and time related functions, g_{ij}, h_{ij}, at project and subsystem levels that are applicable in different phases of the project development.

2.3 - Deterministic Modeling

The cash flow profile of a building project consists of four main elements, namely, land, design/ construction, operation, and disposal (Fig. 2-5).

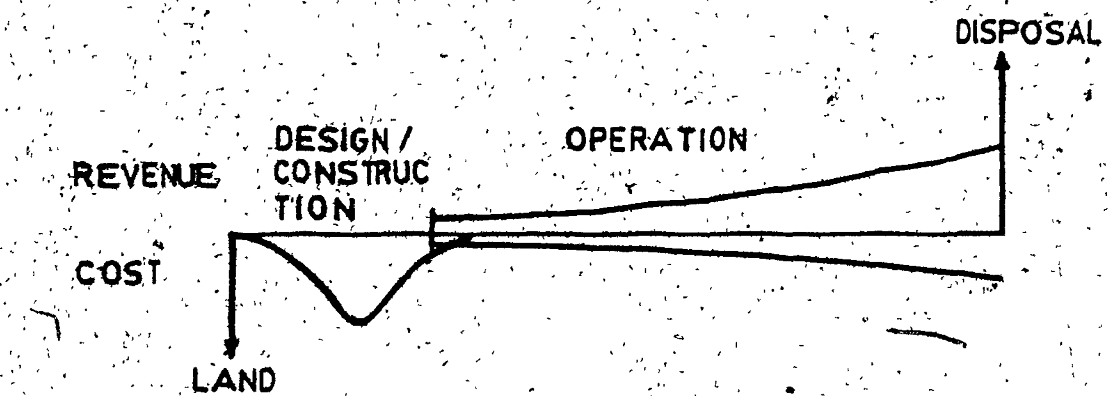


FIG (2-5)- Typical Cash Flow Profile for a Building Project

Newton (35) has suggested that a meaningful cost model should have the following characteristics:

- Initial formulation of the model should not require a detailed knowledge of cost.(Since at the early stages of a project we do not have access to detailed information about building subsystems and their associated costs, the initial formulation of the cost model should not require detailed knowledge of cost (e.g cost information at subsystem and component levels).)

-Cost information is to be consistent throughout the design process.

-Information is to be presented in a familiar form.

-Assistance is to be given in solution generation.

-The model should relate a proposed design to previous, comparable projects and indicate the variability of the relationship.

-The model must link the user to a central data base.

-Cost information is not to interrupt the flow of design. (The design and costing processes should be integrated, but lack of cost information should not interrupt the flow of design).

-The costing mechanism is to be highly flexible and adaptable to the most innovative design solution.

-The model should facilitate a dynamic decision-making process.

-The user must be allowed to interrogate the results in great detail.

-The mechanics of cost generation are to be 'transparent', so that cost consequences can be traced through all inter-relationships. (The mechanism by which data banks are used in cost generation should be clear to the user).

NOTE: The comments in brackets are our interpretation of Newton's specifications for cost models.

Table (2-1) provides a classification of existing research work on cost modeling applicable to different elements of a building project.

ELEMENT	PROJECT LEVEL	SUBSYSTEM LEVEL
CAPITAL COST	Kouskoulas, et al. (7) Tumarkin (6) Russell (8) Karshenas (36)	Bacher (11) Russell, et al. (12) Hardcastle (37)
CAPITAL EXPENDITURE	Sidwell (38) Misurak (9) de La Mare (39) Shlomo Peer (40)	Misurak (9) Russell (41)
FUTURE COSTS	Tucker (42) Holmes, et al. (43)	Arlani (23) Russell (41)
USER COST	Fuller (44) Baron, et al. (45)	

TABLE (2-1) - Sample of Existing Cost Models

2.3.1 - Capital Cost Modeling

Capital cost modeling has been the focus of most of the research described in the literature. Several cost estimation relationships have been suggested (Table (2-1)). In these relationships, mathematical expressions of a system cost as a function of the design decision variables that characterize the future performance of the system have been derived.

The nature of the capital cost estimation problem changes as a project progresses and the accuracy increases as more information becomes available. The Work Breakdown Structure (WBS), is a useful tool for categorizing the building project cost estimation problem. In the WBS the project has been decomposed into a series of systems (e.g. foundations, structure,...); subsystems (e.g. different components of foundation such as standard foundation); and components (e.g. wall foundation,column foundation,...). One possible WBS is given in Table (2-2) (41). It consists of four levels: the project level (level 1); the system level (level 2); the subsystem level (level 3); and the component level (level 4).

The accuracy of the estimate increases as information at a more detailed level (e.g. level 4), becomes available. For example, at the initiation phase, when the feasibility study is being done, only information at the project level (cost estimation relationships or a unit rate based on

LEVEL 2	LEVEL 3	LEVEL 4
01 FOUNDATIONS	01 STANDARD FOUNDATIONS	01 Wall Foundations
		02 Col. Foundations & Pile Caps
	02 SPEC FOUNDATION COND	01 Pile Foundations
		02 Caissons
		03 Underpinning
		04 Dewatering
		05 Raft Foundations
		06 Other Spec Foundation Cond.
02 SUBSTRUCTURE	01 SLAB ON GRADE	01 Standard Slab on Grade
		02 Structural Slab on Grade
		03 Inclined Slab on Grade
		04 Trenches, Pits & Bases
		05 Foundation Drainage
	02 BASEMENT EXCAVATION	01 Excavation for Basements
		02 Structure Fill & Compact
		03 Shoring
	03 BASEMENT WALLS	01 Basement Wall Construction
		02 Moisture Protection
		03 Basement Wall Insulation
	01 FLOOR CONSTRUCTION	01 Susp Basement Floor Const
		02 Upper Floors Construction
		03 Balcony Construction
		04 Ramps
		05 Special Floor Construction
	02 ROOF CONSTRUCTION	01 Flat Roof Construction
		02 Pitched Roof Construction
		03 Canopies
		04 Special Roof Systems
	03 STAIR CONSTRUCTION	01 Stair Structure
04 EXT CLOSURE	01 EXTERIOR WALLS	01 Exterior Wall Construction
		02 Exterior Louvers & Screens
		03 Sun Control Devices (Ext)
		04 Balcony Walls & Handrails
		05 Exterior Soffits
	02 EXT DOORS & WINDOWS	01 Windows
		02 Curtain Walls
		03 Exterior Doors
		04 Storefronts
05 ROOFING		01 Roof Coverings
		02 Traf Topng & Paving Membr
		03 Roof Insulation & Fill
		04 Flashing & Trim
		05 Roof Openings
06 INT CONST	01 PARTITIONS	01 Fixed Partitions
		02 Demountable Partitions
		03 Retractable Partitions
		04 Compartments & Cubicles
		05 Int Balustrades & Screens
		06 Interior Doors & Frames
		07 Interior Storefronts
	02 INTERIOR FINISHES	01 Wall Finishes
		02 Floor
		03 Ceiling Finishes
	03 SPECIALTIES	01 General Specialties
		02 Built-in Fittings
07 CONVEYING SYS		01 Elevators
		02 Moving Stair & Walks
		03 Dumbwaiters
		04 Pneumatic Tube Systems
		05 Other Conveying Systems
		06 General Construction Items

LEVEL 2	LEVEL 3	LEVEL 4
08 MECHANICAL	01 PLUMBING	01 Domestic Water Supply Sys
		02 Sanitary Waste & Vent Sys
		03 Rainwater Drainage System
		04 Plumbing Fixtures
	02 H.V.A.C.	01 Energy Supply
		02 Heat Generating System
		03 Cooling Generating System
		04 Distribution Systems
		05 Terminal & Package Units
		06 Controls & Instrumentation
		07 Systems Testing & Balancing
	03 FIRE PROTECTION	01 Water Supply (Fire Protect)
		02 Sprinklers
		03 Standpipe Systems
		04 Fire Extinguishers
	04 SPEC. MECHANICAL SYS.	01 Special Plumbing Systems
		02 Special Fire Protection Sys.
		03 Misc. Spec. Sys. and Devices
		04 Gen. Const. Items (Mech.)
09 ELECTRICAL	01 SERVICE & DISTRIBUTION	01 High Tension Service & Dist.
		02 Low Tension Service & Dist.
	02 LIGHTING & POWER	01 Branch Wiring
		02 Lighting Equipment
	03 SPEC. ELECTRICAL SYS.	01 Communication & Alarm Sys.
		02 Grounding Systems
		03 Emergency Light & Power
		04 Electric Heating
		05 Floor Raceway Systems
		06 Other Spec. Sys. & Devices
		07 General Construction Items
10 GEN. COND. OH&P		01 Mobilization & Init. Expenses
		02 Site Overheads
		03 Demobilization
		04 Main Off Expense & Profit
11 EQUIPMENT	01 FIXED & MOVABLE EQUIP.	01 Built-in Maintenance Equip.
		02 Checkroom Equipment
		03 Food Service Equipment
		04 Vending Equipment
		05 Waste Handling Equipment
		06 Loading Dock Equipment
		07 Parking Equipment
		08 Detention Equipment
		09 Postal Equipment
		10 Other specialized Equipment
	02 FURNISHINGS	01 Artwork
		02 Window Treatment
		03 Seating
		04 Furniture
		05 Rugs Mats & Furn Acces.
	03 SPECIAL CONSTRUCTION	01 Vaults
		02 Interior Swimming Pools
		03 Modular Prefab Assemblies
		04 Special Purpose Rooms
		05 Other Special Construction
12 SITE WORK	01 SITE PREPARATION	01 Clearing
		02 Demolition
		03 Site Earthwork
	02 SITE IMPROVEMENTS	01 Parking Lots
		02 Roads, Walks, Terraces
		03 Site Development
		04 Landscaping
	03 SITE UTILITIES	01 Water Supply & Dist. Sys.
		02 Drainage and Sewage Systems
		03 Heating and Cooling Dist. Sys.
		04 Elec. Dist. & Lighting Sys.
		05 Snow Melting Systems
		06 Service Tunnels
	04 OFF-SITE WORK	01 Railroad Work
		02 Marine Work
		03 Tunneling
		04 Other Off-Site Work

historical data) is available. The amount of available information is minimal and there is a high degree of uncertainty. When the preliminary design and schedule are completed, a more accurate estimate can be done, thus lowering the uncertainty level. After the completion of the final design, where the quantities and current unit costs are known, the accuracy of the estimate increases.

2.3.1.1 - Capital Cost Modeling at Project Level

At the overall project level, several cost estimation relationships have been developed (Table (2-1)), some of which are more comprehensive than the others. Tumarkin (6) in his study identified sets of physical parameters (area, volume, number of stories,.......), economic parameters (labour contracts, type of investor/ owner, type of contract,...), and performance parameters (type of user and operation phase characteristics). Karshenas (36) presented a simplified relationship to accomodate only the area and height of the building.

The aim of developing cost estimation relationships is: first, to provide a methodology for development of cost estimation relationship and second, to identify some dominant variables and demonstrate their significance using cost data from one or a set of projects (6). Such relationships should continually be updated to account for technological and economical changes.

A cost data bank is a necessity for development of new estimating relationships and for testing the accuracy of existing ones. To date, attention has been directed towards the development of data banks (26).

Cost estimation relationships at the project level should be:

- built based on minimum requirements of costs (e.g. historical information about capital cost of a particular type of building projects).
- adaptable to different building design solutions.
- flexible to facilitate the future application where the cost estimation relationship may vary.
- transparent to allow the user to examine the consequences of his decisions (the mechanism by which the information is being processed should be clear).

2.3.1.2 - Capital Cost Modeling at Subsystem Level

At this level, as more information becomes available, a more accurate, constant dollar estimation of building cost becomes possible. The expression for constant dollar capital cost of component j can be written as (12);

$$C_{1j} = \left(M_j + \frac{C_{1j} Q_j}{P_{1j}} + C_{mj} Q_j + \frac{C_{ej} Q_j}{P_{ej}} + S_j \right) * (1+P)\,(T+0)\,(1+m) \qquad (2-21)$$

where

M_j =mobilization cost for subsystem j

C_{1j} =unit cost of labour for subsystem j

C_{mj} =unit cost of material for subsystem j

C_{ej} =unit cost of equipment for subsystem j

Q_j =quantity needed for subsystem j

P_{lj} =labour productivity for subsystem j

P_{ej} =equipment productivity for subsystem j

P =profit allowance

O =overhead allowance

m =allowance for market condition

S_j = subcontractor cost allowance for subsystem j

2.3.2 - Capital Expenditure Modeling

For a building construction project, the cumulative distribution of the total capital cost is approximated by a "S" curve (Fig (2-6)). It has been stated that the mathematical expression for capital expenditure should have the following characteristics(10);

i. be simple in application and easily understood;

ii. provide versatility, model actual projects, and realistically treat the end points;

iii. have parameters which can be easily computed; and

iv. be able to withstand rigorous statistical operations.

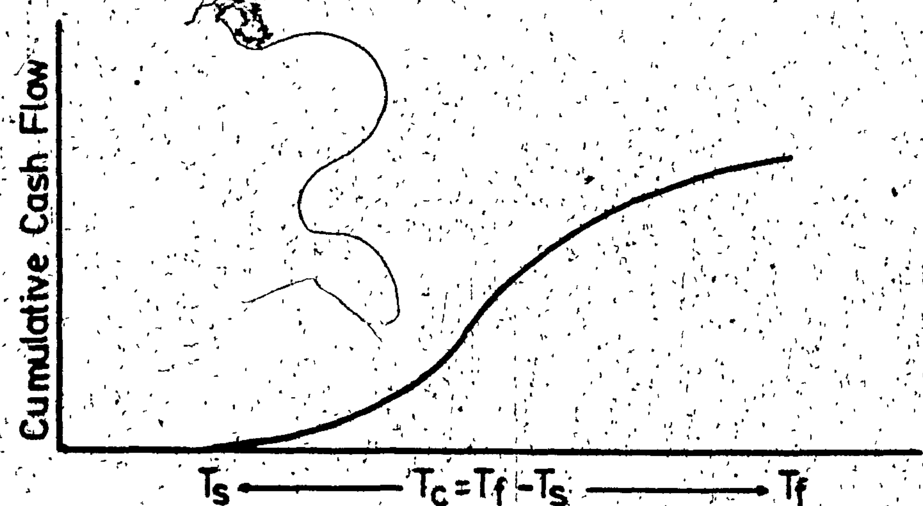


FIG. (2-6)- Capital Expenditure Profile (S curve)

A few expressions have been suggested for capital expenditure modeling. These include the Beta model (9), de la Mare model (39), and linear model (41).

In the following sections, use of a constant discount rate and continuous compounding are assumed in developing expenditure models. These assumptions are made to facilitate the presentation of the model and they do not limit the generality of the models. Any change in these assumptions (e.g. replacing constant discount rate by a time varying discount rate) can easily be incorporated into the models. The goal of the following sections is to demonstrate the application of different capital expenditure models at different levels of Work Breakdown Structure and to measure the magnitude of error that may be induced by selecting a particular model.

2.3.2.1- Capital Expenditure Models at Project Level

1- Beta model

The Beta function represents a flexible distribution which takes different shapes with respect to the parameter values selected. The expression for the Beta distribution, in its general form, can be written as (10):

$$h(x) = \frac{1}{B} \cdot \frac{1}{(b-a)^{p+q-1}} \cdot (x-a)^{p-1} \cdot (b-x)^{q-1} \qquad (2\text{-}22)$$

in which p and q are the parameters of the Beta function, a and b are the lower and upper limits of the distribution. B is the Beta function coefficient which is given by:

$$B = \frac{\Gamma(p) \cdot \Gamma(q)}{\Gamma(p+q)} \tag{2-23}$$

where p and q are integer variables and,

$$\Gamma(p) = (p-1)! \quad , \quad \Gamma(q) = (q-1)! \tag{2-24}$$

If p and q have real values, then;

$$B = \int_0^1 x^{p-1} (1-x)^{q-1} \, dx \tag{2-25}$$

The Beta function parameters, both have positive values (Fig (2-7)).

Although the Beta distribution could be used for both constant and current dollar expenditure profiles, one prefers, however, to treat inflation explicitly. Here, the Beta distribution is used for constant dollar capital expenditure profile.

The expression for the constant dollar cash flow distribution (base year estimate), may be written as;

$$cf(t) = C_{io} \cdot \frac{1}{B} \cdot \frac{1}{(T_c)^{p+q-1}} \cdot (t-T_s)^{p-1} \cdot (T_f-t)^{q-1} \tag{2-26}$$

Where cf(t) is the cost in constant dollars measured at some time t=0, T_c is the total construction duration (T_f-T_s), T_s is the start time of the construction, T_f is the finish time of the construction, and C_{io} is the base year estimate of the construction cost. In the Beta model, the S

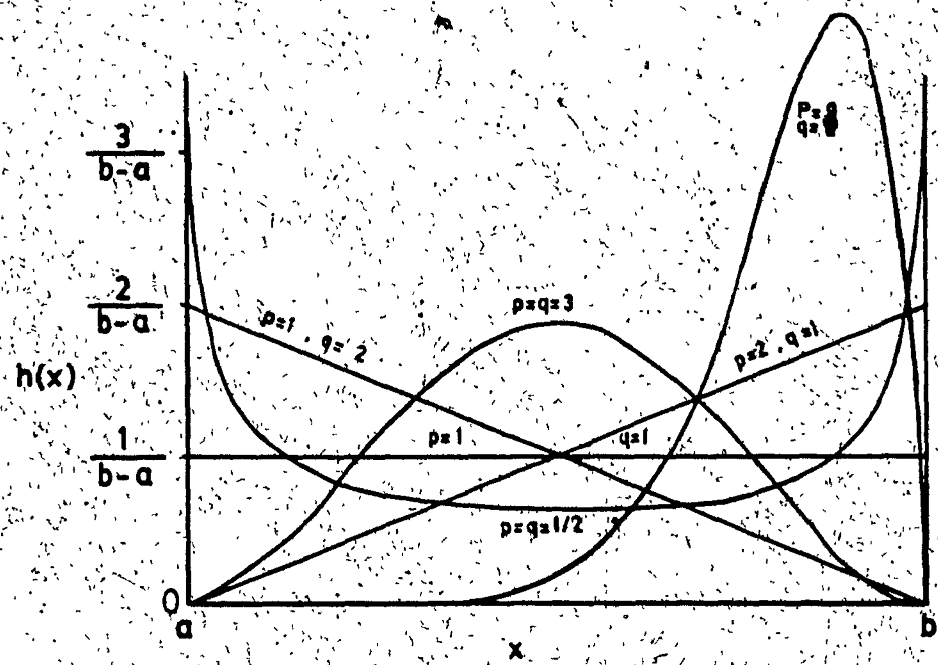


FIG. (2-7) - Beta Density Function.

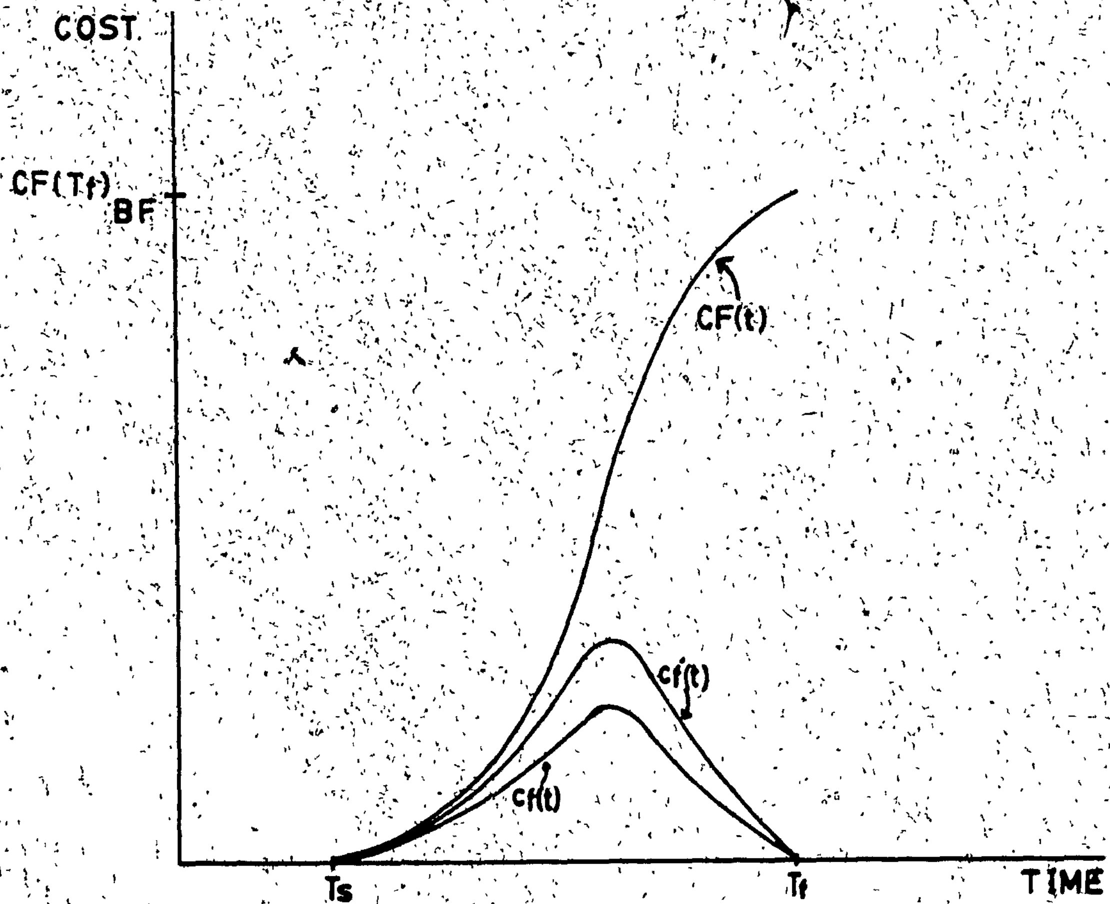


FIG. (2-8)- Cash Flow Diagrams for Total Capital Cost of a Building Project.

curve can be obtained when $p>1$ and $q>1$.

For a realistic model we need to incorporate the project escalation and the financing charges. The before finance capital cost is computed using;

$$cf'(t) = cf(t) \cdot e^{\int_0^t \theta(\tau)\, d\tau} \tag{2-27}$$

$$CF(t)_{BF} = \int_{T_s}^{t} cf'(\tau)\, d\tau \tag{2-28}$$

where

$cf'(\tau)$=inflation adjusted cash flow at time

$CF(t)_{BF}$=cumulative before finance capital cost

Figure (2-8), illustrates the above relations.

If we use α% financing with uniform interest rate of I%, assuming that no payment is made until completion of the entire project, the total financing charges become:

$$Fc = \int_{T_s}^{T_f} cf'(t) \cdot \alpha \cdot (e^{(T_f - t)\cdot I} - 1) \cdot dt \tag{2-29}$$

Consequently;

$$PW(CF(T_f))_{AF} = \int_{T_s}^{T_f} cf'(t) \cdot e^{-rt} \cdot dt + Fc \cdot e^{-r \cdot T_f} \tag{2-30}$$

In which

Fc=total financing charges

$PW(CF(T_f))_{AF}$=present worth of the total after finance capital cost

2- De la Mare model (39)

Based on the same requirements, de la Mare suggested the following one parameter Weibull distribution for the cumulative capital cost (S curve);

$$CF(t) = C_{io} \cdot (1-e^{-(\frac{t}{\eta})^{\beta}}) \qquad (2-31)$$

In which

$$\eta = \frac{Tc}{(6.90776)^{1/\beta}} \qquad (2-32)$$

The β parameter represents the project type/schedule. Figure (2-9), shows the capital cost profiles for different values of β.

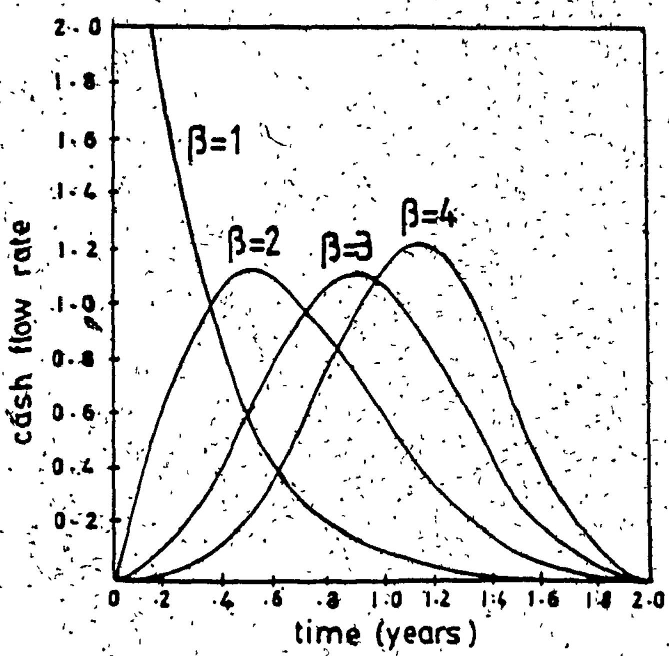


FIG. (2-9)- Capital Expenditure Profiles for Different Values of β Parameter.

If the project starts at time T_s, then equation (2-31) becomes :

$$CF(t) = C_{io} \cdot (1-e^{-(\frac{t-T_s}{\eta})^{\beta}}) \qquad (2-33)$$

In order to include financing expenses and escalation in the analysis, the current dollar cash flow at time t is needed. Using equation (2-33), the expression for constant dollar cash flow profile, cf(t), is:

$$cf(t) = C_{io} \cdot (\beta \cdot (\frac{1}{\eta})^{\beta} \cdot (t-T_s)^{\beta-1} \cdot e^{-(\frac{t-T_s}{\eta})^{\beta}}) \qquad 2-34)$$

Then,

$$PW(CF(T_f))_{AF} = \int_{T_s}^{T_f} cf'(t) \cdot e^{-rt} \cdot dt + Fc \cdot e^{-r \cdot T_f} \qquad (2-35)$$

Although the Beta and de la Mare models are capable of representing the distribution of capital cost (S curve) and they satisfy some of the requirements, in both cases, however, they do not provide a closed form expression that can be used for further theoretical developments. For example, in order to analytically perform statistical analysis, one needs a closed form expression for the project capital cost. In this case a function that, after consideration of inflation and discount rate, can be integrated with respect to time is needed. In both cases, such a closed form does not exist. To overcome this problem, a linear approximation may be used, provided that one is prepared to work at the subsystem level.

2.3.2.2- Capital Expenditure Model at Subsystem Level

1 - The Linear Model

The linear model is a special case of the Beta model when p=q=1. In this model the project capital cost profile is approximated by a combination of linear functions. Using the Work Breakdown Structure (WBS table (2-2)), the total capital cost of the project is the sum of the subsystems' costs. Here, the assumption is that the subsystem's cost occurs from T_{sj} to T_{fj} at a uniform rate. To compute the total capital cost each subsystem is treated separately. For subsystem j, the cash flow at time t is approximated by:

$$cf(t)_j = \frac{C_{cj}}{T_{fj}-T_{sj}} \tag{2-36}$$

For a constant inflation rate θ, using α% financing with I%, interest rate, the expression for after finance capital cost for subsystem j, can be computed.

$$cf'(t)_j = cf(t)_j \, e^{\theta_j t} \tag{2-37}$$

$$CF(T_f)_j = \int_{T_{sj}}^{T_{fj}} cf'(t)_j \cdot dt \tag{2-38}$$

$$Fc_j = \int_{T_{sj}}^{T_{fj}} cf'(t)_j \cdot \alpha \cdot (e^{(T_f - t) I} - 1) \cdot dt \tag{2-39}$$

$$PW(CF(Tf)_j)_{AF} = \frac{C_{ij}}{(Tf_j - Ts_j)} \cdot \left\{ \left(\frac{1}{(\theta_j - r)} \cdot \left(e^{(\theta_j - r)\cdot Tf_j} - e^{(\theta_j - r)\cdot Ts_j} \right) \right) - \right.$$

$$\left. \alpha \cdot e^{-r\cdot Tf} \left[\frac{e^{I\cdot Tf}}{(\theta_j - I)} \left(e^{(\theta_j - I)\cdot Tf_j} - e^{(\theta_j - I)\cdot Ts_j} \right) - \frac{1}{\theta_j} \left(e^{\theta_j \cdot Tf_j} - e^{\theta_j \cdot Ts_j} \right) \right] \right\} \tag{2-40}$$

Finally the total discounted building capital cost becomes:

$$PW(CF(T_f))_{AF} = \sum_{j=1}^{k} PW(CF(T_{fj})_{AF})_j \tag{2-41}$$

2.3.3- Sensitivity Analysis

Sensitivity analysis is applied here in order to study the importance of selecting accurate expenditure profiles for capital cost. For this purpose, seven expenditure profiles that are representative of capital cost distribution in construction projects have been selected, three profiles from the de La Mare model (β=2, 3,4), three profiles from the Beta model (p=2,q=3 ; p=3,q=3 ; p=3,q=2), and the Uniform model (see Fig. (2-10)) (although the uniform model is applicable at subsystem level, it is included in the analysis for comparison purposes). Assuming a project construction duration of 2 years, the escalation cost, as a percentage of the constant dollar estimate, for different inflation rates are calculated and are presented in Table (2-3) and Figure (2-11).

	inflation rate		
	θ = 2%	θ = 4%	θ = 6%
escalation cost (%)	1.20%- 2.42%	2.62%- 4.91%	4% - 7.48%

TABLE (2-3)- Sensitivity of Project Escalation Cost to Capital Expenditure Profile and Escalation Rate (Tc=2 years).

NOTE: The lower limit is found for de La Mare model (B=2) and the upper limit is found for the Beta model (p=3, q=2).

Table (2-3) demonstrates that, for a given construction duration, the maximum error caused by selecting two extreme profiles, for different inflation rates, varies from 1.22% to 3.48%.

Similar analysis is performed to measure the magnitude of the error for different construction duration (inflation rate of 6% is selected for analysis purposes). Table (2-4) demonstrates that the error margin increases with increase in the construction duration.

	construction duration (year)			
	T_c=1	T_c=2	T_c=3	T_c=4
escalation cost (%)	1.91%-3.6%	4%-7.45%	6.2%-11.5%	8.4%-15.6%

TABLE (2-4)- Sensitivity of Project Escalation Cost to Various Capital Expenditure Profiles and Construction Duration (θ=6%).

NOTE: The lower limit is found for de La Mare model (B=2) and the upper limit is found for the Beta model (p=3, q=2).

In the above analysis, selection of the uniform model may lead up to 4.6% error (T_c=4 years, θ=6%). For small projects (e.g. T_c=1 year) the use of uniform model may lead to 1.2% error (θ=6%) (see Fig. 2-12). This study demonstrates that in the subsystem level, the contribution of the error in the selection of expenditure profile to the overall capital cost error is minimal (e.g. <1%), therefore, the use of the uniform model in the subsystem level is recommended. Similar results have been found for the present worth of the capital cost (r-θ=2%,4%,6%). Nevertheless, anaccurate estimate is needed for proper financial management of the project.

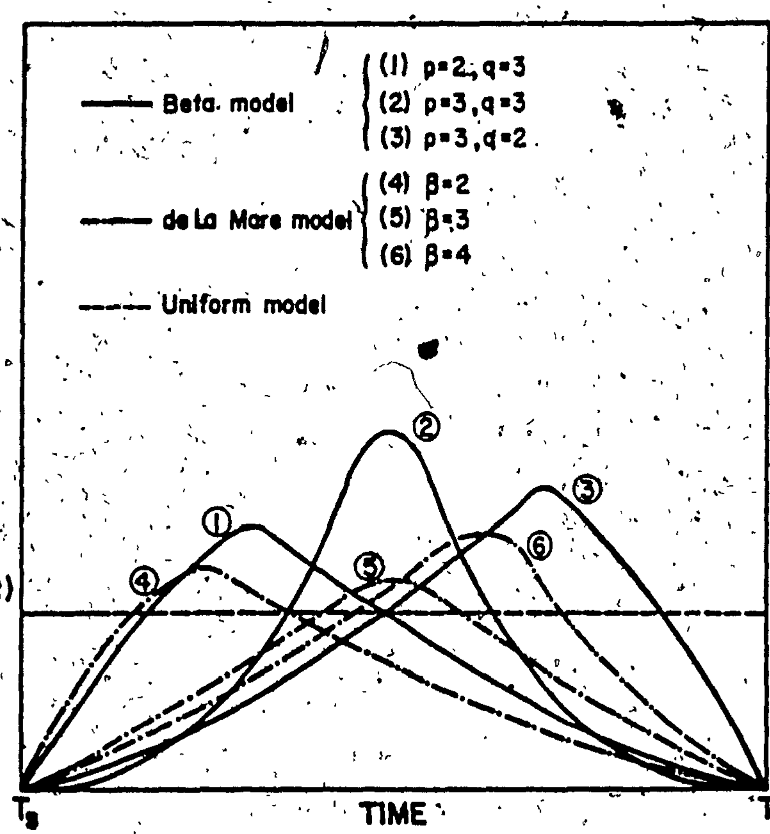


FIG. (2-10) - Capital Expenditure Profiles Used in the Sensitivity Analysis.

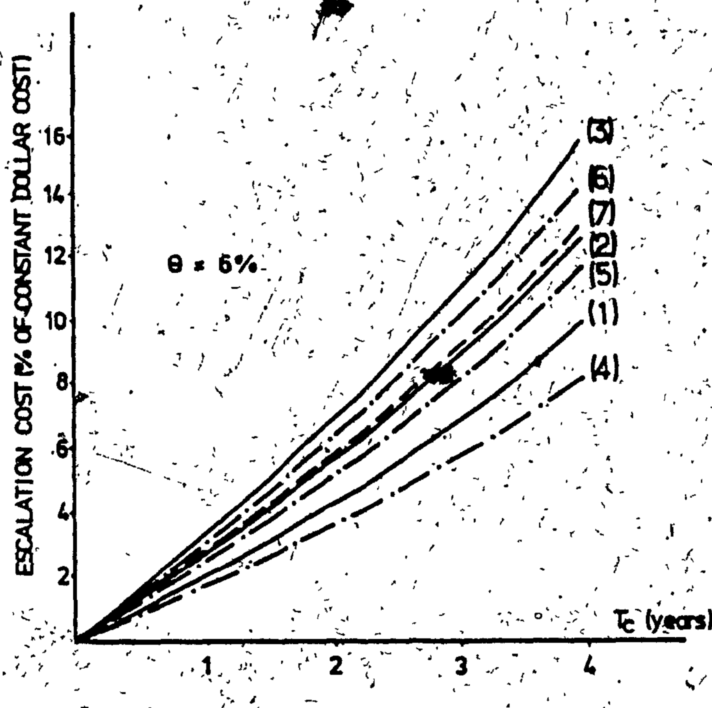


FIG. (2-11) - Escalation Cost vs. Project Duration for Different Capital Expenditure Profiles.

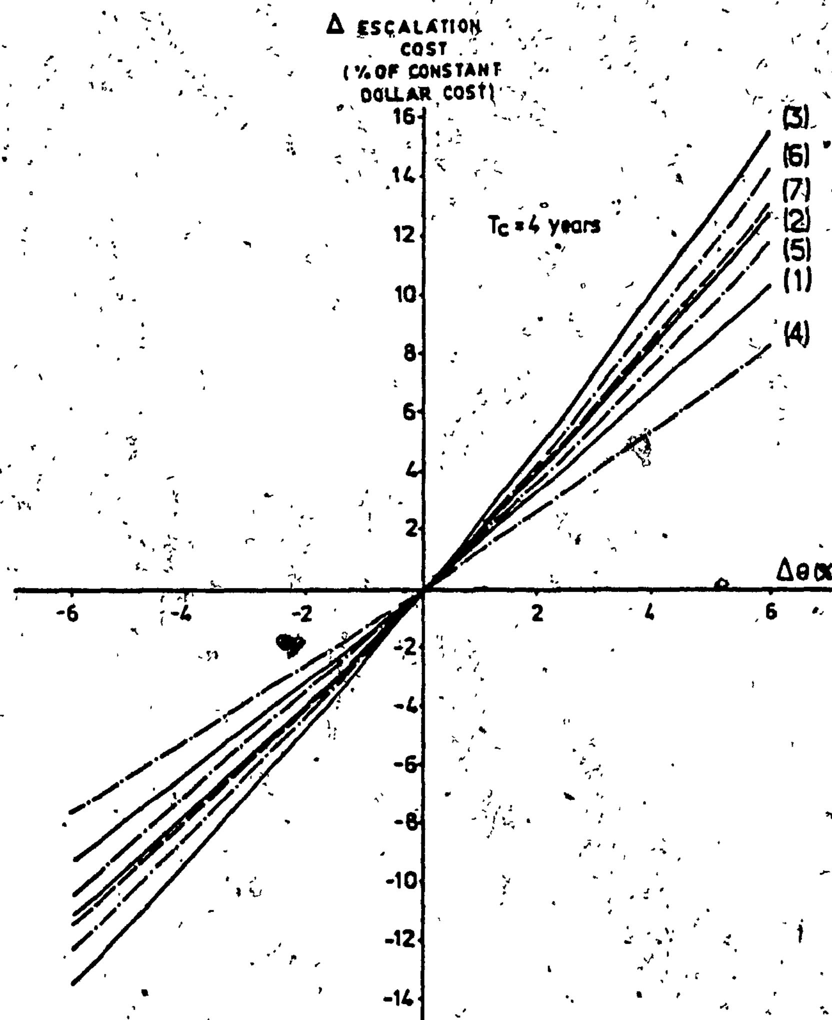


FIG. (2-12) - Escalation Cost vs. Escalation Rate for Different Capital Expenditure Profiles.

2.4- Future Cost Models

The overall building cost during its useful life (initial and the future costs) is composed of several components (Fig. (2-13) (44).

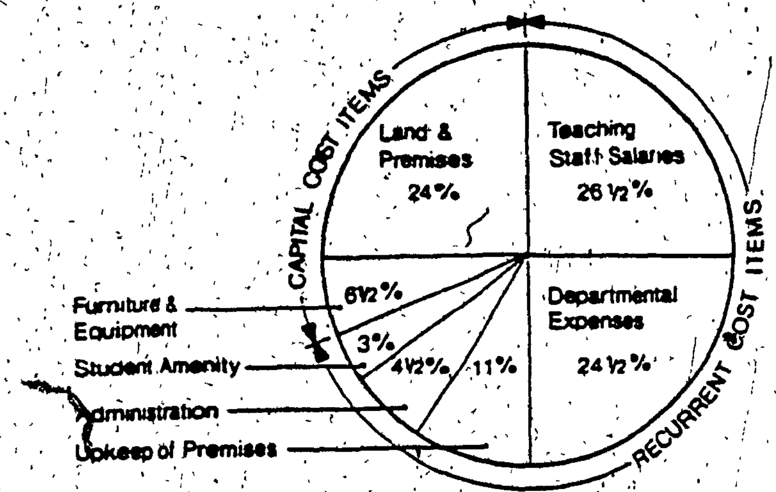


FIG. (2-13) - Cost Breakdown During Life of an Educational Institution (including user cost). (44)

The relative importance of each component varies with the building type. For example, office buildings have high future operating costs (operating, maintenance, etc.) while warehouses have relatively low future operating costs. The relative importance also depends on the level of high technology used in the building. In the case of hospitals and health centers, equipment is constantly being updated. The cost of these changes, whether allowances for the future advancements are made or not, varies. Consideration of future technological changes in the design stage may increase the capital cost substantially and may not be feasible when there are budgetary constraints. However, it should be considered as an important factor in selecting the design alternatives.

The behaviour of several future cost components has been studied in the past (46). Here, we will develop models, for future cost components, that can be integrated into the general framework. The present value of future cost components in their most general form can be written as:

$$C_f = \sum_i \sum_j \int g_{ij} \, h_{ij} \, e^{\int_0^t \theta(\tau)\,d\tau} \, e^{-\int_0^t r(\tau)\,d\tau} \, dt \qquad (2\text{-}42)$$

In the above expression, g_{ij} is the base year cost of operation / maintenance / repair and renewal that are generally determined at the design stage (e.g. the base year cost of heating energy is determined based on the calculated load, the control system, and the operation policy). h_{ij} represents the effect of aging, technological changes, and the selected operating and maintenance policies on constant dollar cost.

In this study g_{ij} and h_{ij} functions are developed for operating cost and some of the maintenance cost components using data taken from the 1982 BOMA report (47). The design related variables selected are total floor area and building height and the time related variable selected is the aging factor (effect of building age). Five functions are selected for the purpose of regression analysis. These include; linear, quadratic, power, asymptotic power and exponential functions. The most suitable functions are selected based on the maximum value of the coefficient of determination (R^2). Functions , for g_{ij} and h_{ij}, are developed for overall operating cost (the overall operating cost includes all maintenance costs exclusive of alteration costs, energy cost, and general administration costs) and three components of the maintenance cost (cleaning, electrical system and combined HVAC) . These models are tabulated in Tables (2-5,6,7).

Expressions in Tables (2-5,6,7) are developed from a small and limited set of data and for a particular type of building (office building). In order to develop meaningful relationships, one requires cost data banks that are continually being updated. The main purpose here is not to develop specific equations for future cost components, but rather is an attempt to develop different components of cost models that can be implemented within the general expression (equation (2-2)).

Profiles of g_{ij} (base year cost of operation/ ft^2) with respect to building height and building total floor area are shown in Figs. (2-14,15). In both cases, the operating cost per square foot decreases as the scale gets bigger (floor area, number of floors). The decreases continues up to a point (670,000. sq. ft.; 15 floors), then the trend changes and cost increases with both parameters.

FUTURE COST COMPONENT		TYPE OF FUNCTION	g_{ij} (\$ / sq. ft)	R^2
OPERATION		QUADRATIC	$g_{ij} = 4.87 - .148\ A + .0058\ A^2$	.983
MAINTENANCE	cleaning	QUADRATIC	$g_{ij} = 1.05 - .026\ A + .0013\ A^2$	.977
	electrical system	QUADRATIC	$g_{ij} = .162 - .005\ A + .0017\ A^2$	.893
	combined HVAC	QUADRATIC	$g_{ij} = .311 - .004\ A + .0003\ A^2$	.930

A = total floor area (sq. ft.) / 50,000.

TABLE (2-5) - Cost Estimation Relationships for Operating and Maintenance Costs Components (g_{ij} as a function of total floor area).

FUTURE COST COMPONET		TYPE OF FUNCTION	g_{ij} (\$ / sq. ft.)	R^2
OPERATION		QUADRATIC	$g_{ij} = 4.33 - .012\,h + .0004\,h^2$	.992
MAINTENANCE	cleaning	LINEAR	$g_{ij} = .871 + .544\,h$	.948
	electrical system	POWER FUNCTION	$g_{ij} = .258\,h^{-.21}$	.936
	combined HVAC	POWER FUNCTION	$g_{ij} = .314\,h^{.032}$	.928

h = number of floors

TABLE (2-6) - Cost Estimation Relationships for Operating and Maintenance costs components (g_{ij} as a function of building height).

FUTURE COST COMPONENT		TYPE OF FUNCTION	h_{ij}	R^2
OPERATION		QUADRATIC	$h_{ij} = 1 + .0269\ t + .000296\ t^2$	.951
MAINTENANCE	cleaning	QUADRATIC	$h_{ij} = 1 + .0248\ t - .000138\ t^2$	.914
	electrical system	ASYMPTOTIC POWER	$h_{ij} = 1 - 5.820\ t^{-1.34}$	.962
	combined HVAC	QUADRATIC	$h_{ij} = 1 + .1720\ t - .000218\ t^2$	.993

t = building age

TABLE (2-7) - Cost Estimation Relationships for Operating and Maintenance Costs Components (h_{ij} as a function of building age).

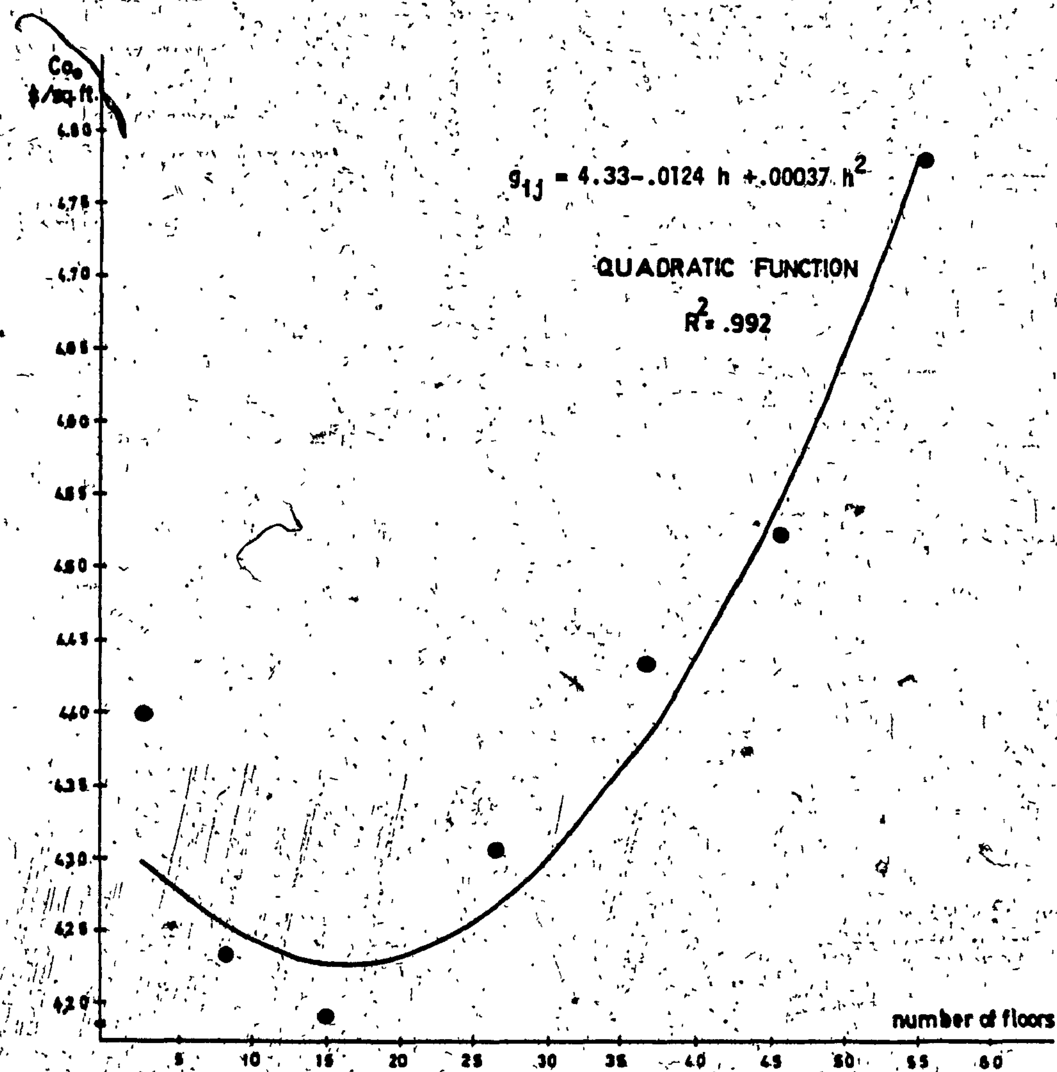


FIG (2-14) - Operating Cost vs. Building height for Office Building Projects.

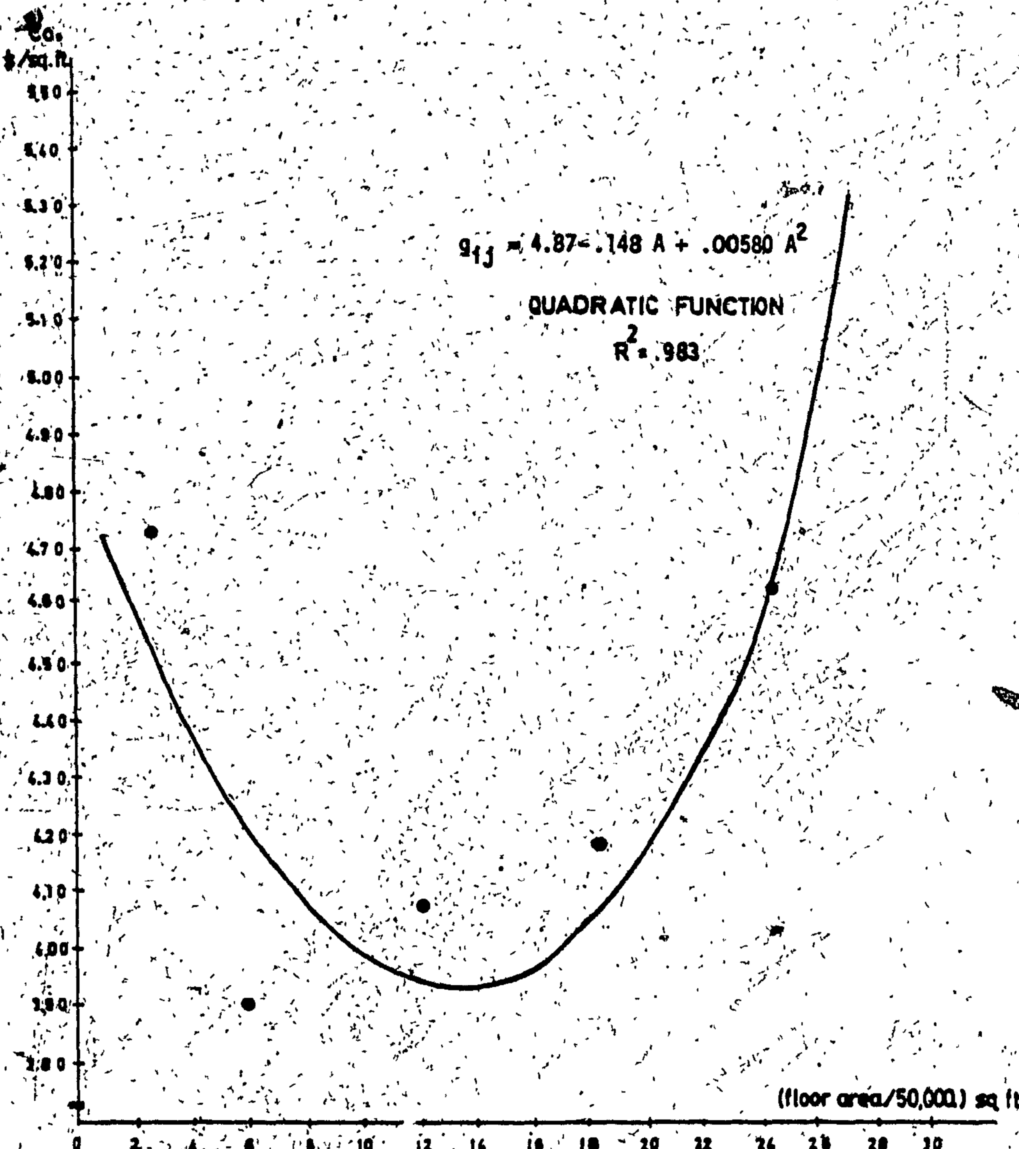


FIG. (2-15) - Operating Cost vs. Total Floor Area (gross floor area is considered).

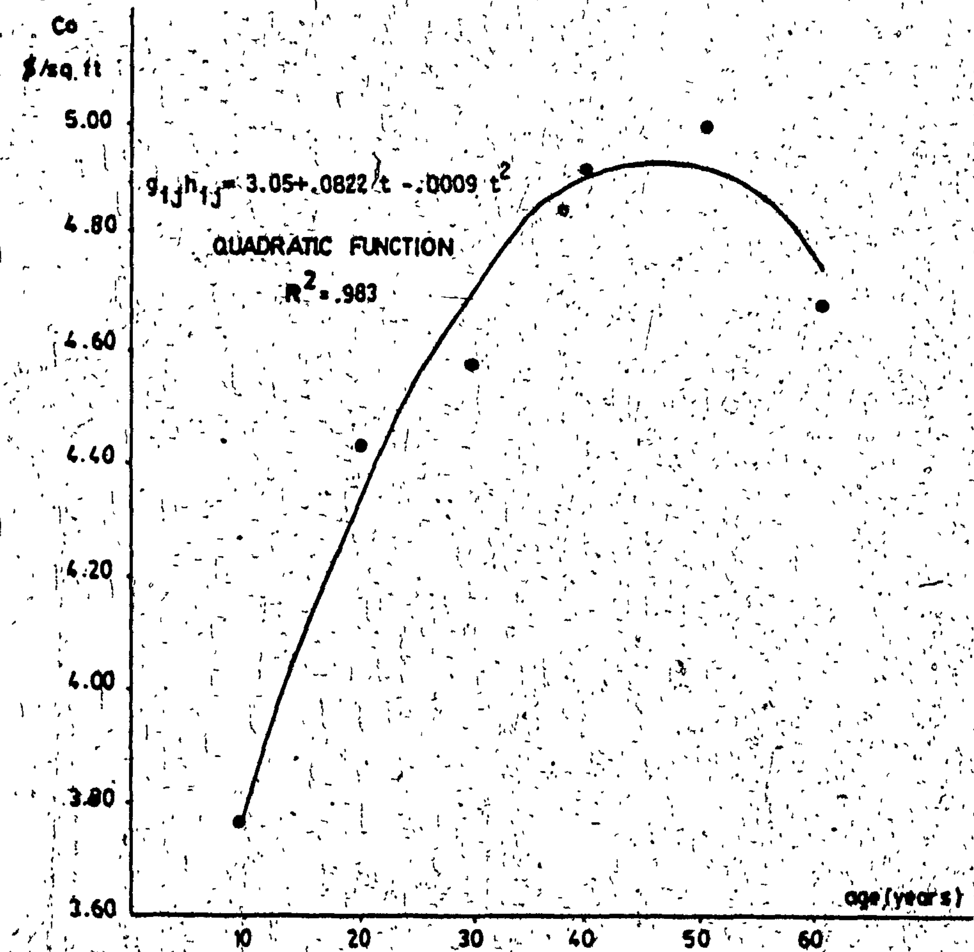


FIG. (2-16) - Constant Dollar Cost Profile for Office Building Projects (building age is considered as time varying factor).

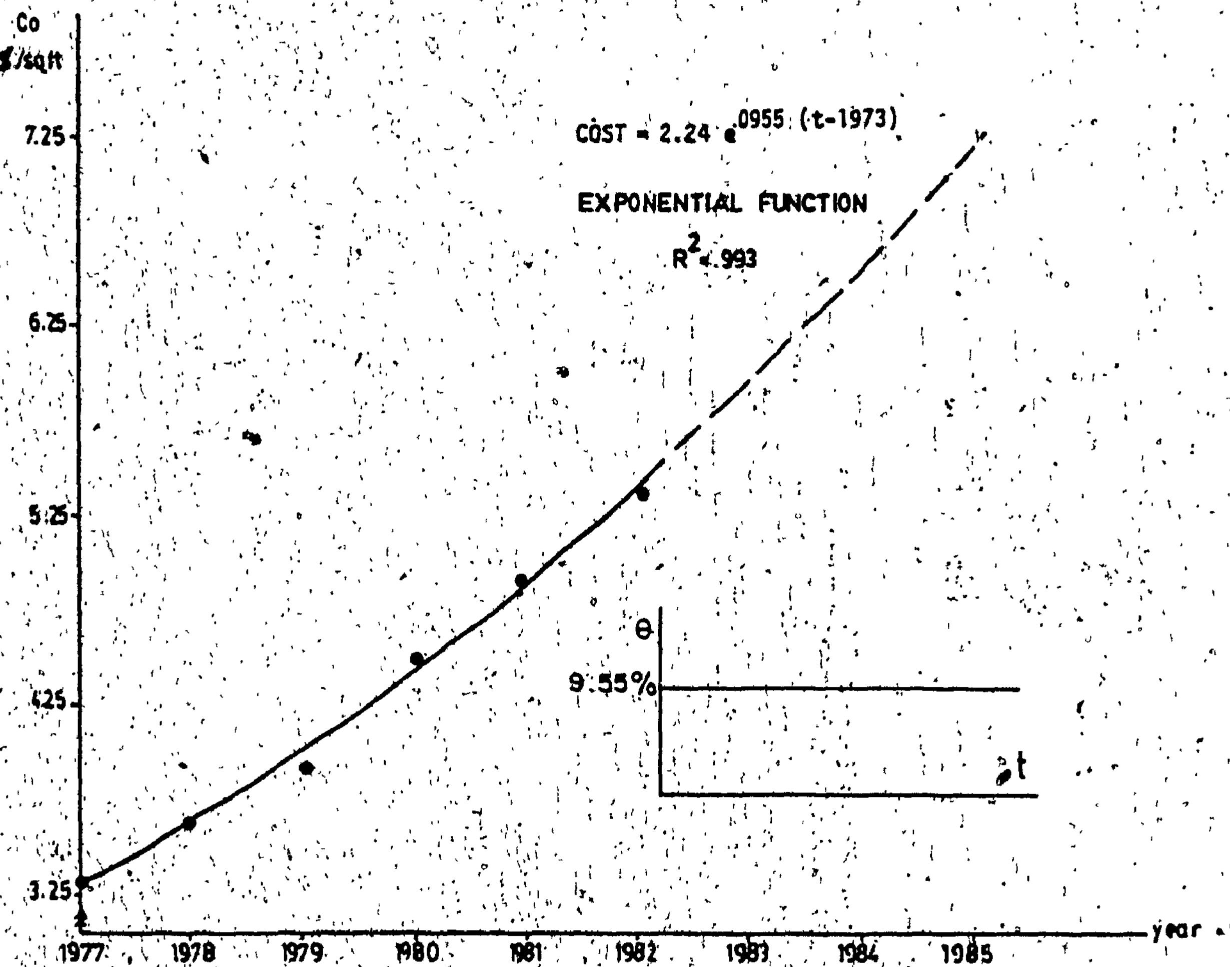


FIG. (2-17) - Current Dollar Operating Cost vs. Time
(office building projects).

In Figure (2-16), the constant dollar profile of operating cost versus aging factor is shown. The operating cost increases as the building ages. The decrease at the end of the curve may be due to the reclassification (degrading) of the building.

In order to determine the inflation rate profile, the variation of operating cost with respect to time has been studied. The exponential function is selected based on the maximum value of the coefficient of determination (Fig. (2-17)). The exponential function represents the uniform inflation rate (e.g. for these set of data, the inflation rate is 9.54%).

EXAMPLE

To demonstrate the importance of time related variables in calculation of the future cost components, the effect of aging factor on the present worth of the overall operating cost is examined. In this example we will consider the first 40 years of the building operation (see fig. 2-16).

Assumptions: $r-\theta = 2\%$, uniform rate with continuous compounding.

The constant dollar profile for the operating cost is shown for two cases: i) aging factor is ignored (uniform cost/ sq. ft.) and ii) the aging factor is considered (quadratic function)

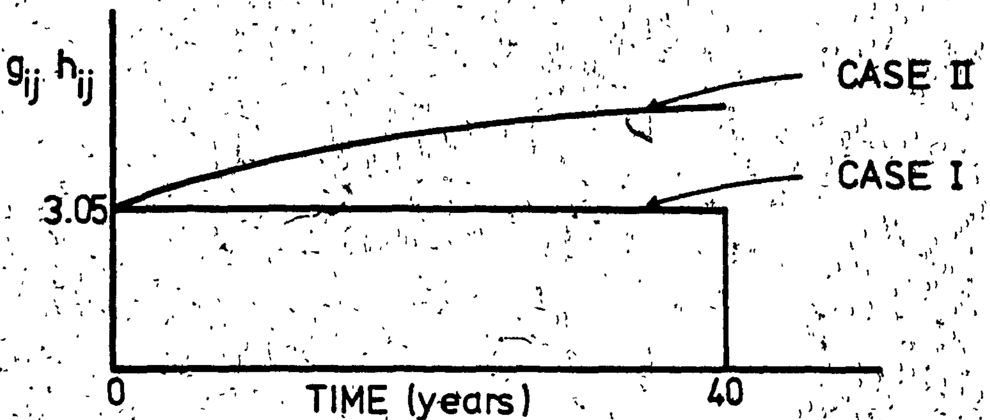


FIG. (2-18)- Constant Dollar Cost Profiles for Operating cost.

Case i- the present worth of the operating cost/ ft^2 is (aging affect is ignored):

$$PW(\text{ operating cost}) = \int_0^{40} 3.05\ e^{-.02\,t}\, dt.$$

$$= 84$$

case ii- the present worth of the operating cost/ ft^2 is (aging effect is considered):

$$PW(\text{operating cost}) = \int_0^{40} (3.05+.0822t-.00091\ t^2)\ e^{-.02\,t}\, dt.$$

$$= 112.5$$

$$\%\ \text{change} = \frac{112.5 - 84}{84} = .34 = 34\%$$

34% underestimation of the operating cost coupled with a decrease in the rate of increase of the rental income can contribute significantly to the inaccuracy of the future cost estimates.

2.4.1- User Cost

Awareness of user productivity and user cost has grown considerably since the late 70's. Following a trend of energy conservation strategies employed in the design and operation of both public and private buildings, there were an increasing number of health related problems in mostly airtight, energy efficient, office buildings. The National Institute for Occupational Safety and Health (NIOSH) alone have conducted over 115 investigations between 1974 and 1981. Sterling, et al. (48), have developed an information system containing information about the internal environment condition and the health history of users. They have surveyed 1100 office workers in nine different buildings in New York and have studied all internal environmental variables (air properties, temperature, air movement,...). However, they fall short of relating this information to the user productivity in a quantitative manner. At present, many studies have been done on environmental conditions (49), including building layout, floor area, lighting conditions, acoustic conditions, air properties, and worker related variables. There is an implicit conclusion that violating the limits of these conditions in the office buildings will decrease employee productivity (sleepiness, temporary exhaustion, reduced attendance at work,....), especially for workers performing complex tasks.

The consideration of user cost as a design decision-making parameter is a concept, the importance of which is starting to be recognized by researchers (3,50). In Fig. (2-13), it was shown that a large portion of the future costs are the user oriented ones (e.g. for an educational institution, it is about 84% of the total future costs). The relative importance of the user cost varies from one building to another. It has three main components; i) labour cost; ii) material cost and iii) equipment cost. The composition of these three components also vary with building type (e.g. in the office buildings, the labour cost is the major component while in the other commercial buildings (e.g. shopping centers), since the material is the main component, the material cost is very high).

The user cost relationship may be written as;

$$C_u = (C_{u,l}/P_{u,l}) + (C_{u,m}/\text{eff.}_{(u,m)}) + (C_{u,e}/P_{u,e}) \qquad (2\text{-}43)$$

Where;

C_u = user cost

$C_{u,l}$ = labour cost (user related)

$C_{U,m}$ = material cost (user related)

$C_{u,e}$ = equipment cost (user related)

$P_{u,l}$ = labour productivity (user related)

$P_{u,e}$ = equipment productivity (user related)

$\text{eff.}_{(u,m)}$ = efficiency of the material used

$$PW(Cu) = \int_{T_f}^{T_e} Cu(t)\, e^{-rt}\, dt \qquad (2\text{-}44)$$

By increasing the labour/equipment productivity or the efficiency of the material used, the total user cost will decrease. An increase in productivity may be achieved through better design (better communication system, transportation, and the allowance for future technological changes), and proper operating and maintenance policies. As shown in Fig. (2-13), if a 25% increase in the cost of upkeeping premises results only in 5% saving in the user costs, the move is justified. In equation (2-43), the productivity terms (especially labour productivity) are directly influenced by environmental conditions. A recent study (50), provides quantitative information about effects of several environmental variables (e.g. noise, temperature fluctuations, etc.) on job performance and job satisfaction. These effects are measured for three types of jobs (managers, professionals, clerical) in terms of absentieesm and turnover related to improvement/ decline in each variable. It has been reported that improvements in work environment can save up to 15% of the user costs (salary). Tables (2-8,9) (50), show the extent of effects of improvements/ decline in environmental variables in office buildings. However, in the present economic situation, because of budgetary constraints, limitations in capital expenditures and cuts in operating budgets are causing a reverse effect.

In the absence of user cost considerations, design decisions are largely influenced by the initial cost of the

FACETS	JOB TYPES					
	Managers (Avg. Salary: $41,500)		Professional/ Technical (Avg. Salary: $31,600)		Clerical (Avg. Salary: $17,400)	
Facets that Relate to Job Satisfaction	**Annual Value**	**NPV 5 Yrs.**	**Annual Value**	**NPV 5 Yrs.**	**Annual Value**	**NPV 5 Yrs.**
Noise	472	1,789	282	1,068	148	560
Temperature Fluctuation	270	1,023	162	613	85	322
Glare	275	1,023	165	625	87	329
Comfort[1]	—	—	234	886	—	—
Floor Area[2]	—	—	—	—	—	—
Relocation Frequency	450	1,705	271	1,026	142	538
Work Surface Width[3]	—	—	—	—	—	—
Storage (Personal)[4]	—	—	—	—	630	2,387
Ease of Communication	75	284	45	170	23	87
Participation[5]	—	—	—	—	—	—
Facets that Relate to Job Performance	**Annual Value**	**NPV 5 Yrs.**	**Annual Value**	**NPV 5 Yrs.**	**Annual Value**	**NPV 5 Yrs.**
Enclosure	3,423	12,971	2,608	9,873	1,438	5,447
Layout	2,491	9,438	1,646	6,236	1,046	3,964

[1]COMFORT: Relates only to professional/technical workers
[2]FLOOR AREA: Only *decreases* in floor area relate to job satisfaction
[3]WORK SURFACE WIDTH: Only *decreases* in work surface width relate to job satisfaction.
[4]STORAGE (PERSONAL): Relates only to clerical workers.
[5]PARTICIPATION: No dollar value estimate can be developed for improvements in participation, given the numbers we have.

TABLE (2-8) - Value of Benefits from Improved Facets of Environment.(50)

FACETS	JOB TYPES					
	Managers (Avg. Salary: $41,500)		Professional/ Technical (Avg. Salary: $31,600)		Clerical (Avg. Salary: $17,400)	
Facets that Relate to Job Satisfaction	**Annual Value**	**NPV 5 Yrs.**	**Annual Value**	**NPV 5 Yrs.**	**Annual Value**	**NPV 5 Yrs.**
Noise	850	3,221	509	1,928	267	1,011
Temperature Fluctuation	692	2,281	361	1,367	189	716
Glare	194	735	116	439	61	231
Comfort[1]	—	—	701	2,656	—	—
Floor Area	1,148	4,354	688	2,607	361	1,367
Relocation Frequency	1,471	5,574	880	3,334	461	1,746
Work Surface Width[3]	609	2,307	364	1,379	191	723
Storage (Personal)[2]	—	—	—	—	233	882
Ease of Communication	673	2,550	403	1,527	211	799
Participation[3]	—	—	—	—	—	—
Facets that Relate to Job Performance	**Annual Value**	**NPV 5 Yrs.**	**Annual Value**	**NPV 5 Yrs.**	**Annual Value**	**NPV 5 Yrs.**
Enclosure	2,568	9,729	1,954	7,405	1,079	4,087
Layout[4]	—	—	—	—	—	—

[1]COMFORT: Relates only to professional/technical workers
[2]STORAGE (PERSONAL): Relates only to clerical workers
[3]PARTICIPATION: No dollar value estimate can be developed for declines in participation, given the numbers we have.
[4]LAYOUT: Only increases in suitability of layout relate to job performance.

TABLE (2-9) - Cost of Declines in Facets of Environment.(50)

building. Fig (2-19), indicates that in the absence of the user costs, the initial costs constitute about 74% of the overall building cost compared to the previous case (Fig. (2-13)) of 30%.

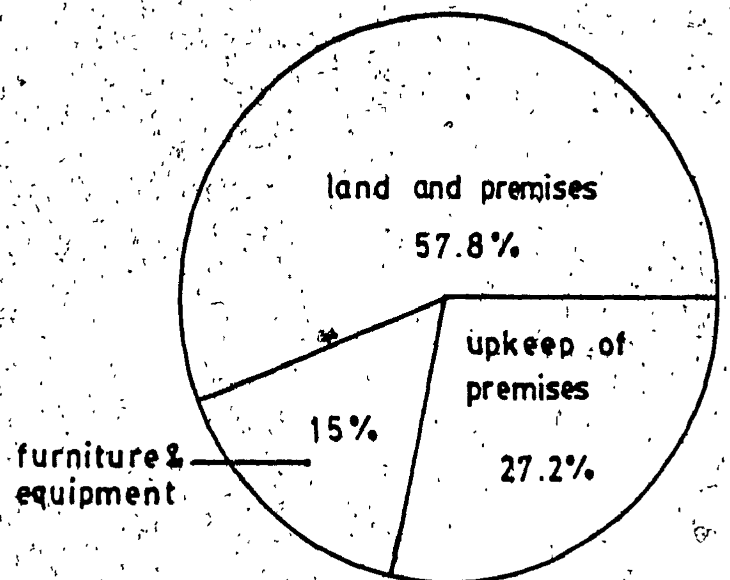


FIG (2-19) - Cost Breakdown During Life of a Building Project (excluding user cost). (educational institution)(50)

CHAPTER III

3.0 - Introduction

In the previous chapter, the underlying assumption was that all variables used in the cost modeling are deterministic in nature. Such an assumption is unrealistic in that one is required to predict future events. In the past two decades, the increasing rate of change of economic condition contributed to the uncertainty in forecasting the future. This is especially true for the time related variables. In order to treat existing uncertainties in a meaningful manner, a probabilistic analysis is required.

The objective of this chapter is to explore some of the available tools that can be used for probabilistic analysis and to develop probabilistic cost models. In the following sections, use of different probabilistic analysis methods, their data requirements, and their application to different classes of building design related problems will be discussed. Application of the moments approach as a useful tool that is applicable within the general framework will be demonstrated. Finally, a formulation of the maintenance/replacement costs problem will be presented and used to examine the effects of deferred maintenance policies that are practiced in the present restrained economic environment.

3.1 - Probabilistic Formulation

Consider the general model, for a constant dollar cost profile;

$$Y = \sum_i \sum_j \int g_{ij}(\underline{X}_{ij}, \underline{Z}_{ij}) \; h_{ij}(\underline{Z}_{ij}, t) \, dt. \tag{3-1}$$

The aim here is to provide information about Y, where variables $\underline{X}_{ij}$ and $\underline{Z}_{ij}$ are random (there can also be uncertainty in the functions g_{ij} and h_{ij} themselves). The random behavior of function Y can be represented by its probability distribution (through numerical simulation), frequency distribution/ density function (through mathematical approaches), or its main descriptors (mean, variance , skewness,.....) through a moments approach. The applicability of any one of these approaches depends on: i) input information available, ii) the output information required, and iii) the complexity of the function Y.

3.1.1- Numerical Simulation (51)

One approach to find the probability distribution of a function is through numerical simulation (e.g. Monte Carlo simulation techniques). In this approach, we need probability distributions of all independent variables. For each variable, values for its probability distribution are generated using a random number generator. These values are used to calculate values of function Y. Calculating numerous experimental values for Y and plotting these values in the form of a histogram will yield an approximate probability density function for Y. If variables are correlated, the simulation technique must account for correlations. In this case, the random numbers are also correlated and the process of generating these numbers should be modified (52,53).

Although simulation is a very versatile tool and invaluable where analytical approaches are inadequate, it requires a large amount of information and computation time. The effectiveness of simulation and the level of accuracy that can be reached depends on the amount of information available, the initial design of the simulation system (simulation procedure), and the method of generating random numbers (e.g. using variance reducing techniques).

The application of numerical simulation for the general model in the subsystem level (eq. 3-1), requires probability distributions of $\underline{X}_{ij}$ and $\underline{Z}_{ij}$. The amount of information required at this level makes the analysis impractical and in most cases impossible. However, numerical simulation can be a useful tool at the overall project level, where historical information about similar projects is available. (10)

3.1.2 - Frequency Distribution/ Density Function

The frequency distribution/ density function of a multiple variable function can also be determined using analytical approaches. The requirements are the frequency distributions of all independent random variables or their joint frequency distribution (which is the same as in simulation). For the calculation of the frequency distribution of function Y (eq. 3-1), one needs frequency distribution functions for the $\underline{X}_{ij}$ and $\underline{Z}_{ij}$ variables. However, in practice, due to i) lack of data availability (or difficulties in gathering such information) about different components of the building design problem (design related and cost related components); and ii) difficulties in finding analytical solutions (integrating) for a combination of several complex functions (frequency distribution functions of different variables), application of the frequency distribution method for problems related to building design becomes impossible. In this research work the frequency distribution method will not be used for the purpose of probabilistic analysis.

3.1.3 - The Moments Approach

The moments of a random variable are defined as expectations of the powers of the random variable having a given distribution. If x is a random variable, the rth central moment of x (r=2,3,....), usually denoted by $u'_r(x)$, is defined as:

$$\mu'_r(x) = E\,[(x-\bar{x})^r] \qquad (3-2)$$

Throughout this study, the first three moments (mean, variance, and skewness) of the variables are used to explain their random behavior. Higher order moments require information that is not available and do not have physical meanings in the kind of problems under study.

Consider the following multiple variable function:

$$y = g(X_1, X_2, \ldots\ldots X_n) \tag{3-3}$$
$$= g(\underline{X})$$

Here the first three terms of the Taylor series expansion about X are used. The higher order terms require higher order moments (4th, 5th, ...) of variables that are not available and do not have a physical meaning. The first three terms, expanded about mean, are:

$$Y = g(\bar{\underline{X}}) + \sum_{i=1}^{n} \left\{ \frac{\delta\, g(\underline{X})}{\delta\, X_i} \bigg|_{\bar{X}} \cdot (X_i - \bar{X}_i) \right\} +$$

$$\frac{1}{2} \left[\sum_{i=1}^{n} \left\{ \frac{\delta^2 g(\underline{X})}{\delta\, X_i^{\,2}} \bigg|_{\bar{X}} \cdot (X_i - \bar{X}_i)^2 \right\} + \right.$$

$$\left. 2 \sum_{i<j}^{n} \sum_{j}^{n} \left\{ \frac{\delta^2 g(\underline{X})}{\delta X_i \;\; \delta X_j} \bigg|_{\bar{X}} \cdot (X_i - \bar{X}_i)(X_j - \bar{X}_j) \right\} \right] + \ldots\ldots \tag{3-4}$$

The moments of this function are (54):

$$\bar{Y} = g(\bar{\underline{X}}) + \frac{1}{2} \left[\sum_{i=1}^{n} \left\{ \frac{\delta^2 g(\underline{X})}{\delta\, X_i^{\,2}} \bigg|_{\bar{X}} \cdot \sigma^2_{X_i} \right\} + \right.$$

$$\left. 2 \sum_{i}^{n} \sum_{j}^{n} \left\{ \frac{\delta^2\, g(\underline{X})}{\delta X_i \;\; \delta X_j} \bigg|_{\bar{X}} \cdot E\,[(X_i - \bar{X}_i)(X_j - \bar{X}_j)] \right\} \right] + \ldots\ldots \tag{3-5}$$

$$\sigma_Y^2 = \sum_{i=1}^{n} \left\{ \left(\frac{\delta g(\underline{X})}{\delta X_i} \bigg|_{\bar{X}} \right)^2 \cdot \sigma_{X_i}^2 \right\} +$$

$$2 \sum_{\substack{i \\ i<j}}^{n} \sum_{j}^{n} \left\{ \frac{\delta g(\underline{X})}{\delta X_i} \bigg|_{\bar{X}} \frac{\delta g(\underline{X})}{\delta X_j} \bigg|_{\bar{X}} \cdot E[(X_i - \bar{X}_i)(X_j - \bar{X}_j)] \right\} +$$

$$\sum_{i=j}^{n} \left\{ \frac{\delta g(\underline{X})}{\delta X_i} \bigg|_{\bar{X}} \cdot \frac{\delta^2 g(\underline{X})}{\delta X_i^2} \bigg|_{\bar{X}} \cdot \gamma_3(X_i) \cdot \sigma_{X_i}^3 \right\} +$$

$$\sum_{\substack{i \\ i \neq j}}^{n} \sum_{j}^{n} \left\{ \frac{\delta g(\underline{X})}{\delta X_i} \bigg|_{\bar{X}} \cdot \frac{\delta^2 g(\underline{X})}{\delta X_j^2} \bigg|_{\bar{X}} \cdot E[(X_i - \bar{X}_i)(X_j - \bar{X}_j)^2] \right\} +$$

$$2 \sum_{\substack{i \\ i \neq j}}^{n} \sum_{j}^{n} \left\{ \frac{\delta g(\underline{X})}{\delta X_i} \bigg|_{\bar{X}} \cdot \frac{\delta^2 g(\underline{X})}{\delta X_i \, \delta X_j} \bigg|_{\bar{X}} \cdot E[(X_i - \bar{X}_i)^2 (X_j - \bar{X}_j)] \right\} +$$

$$2 \sum_{\substack{i \\ i \neq j \neq k}}^{n} \sum_{j}^{n} \sum_{k}^{n} \left\{ \frac{\delta g(\underline{X})}{\delta X_i} \bigg|_{\bar{X}} \cdot \frac{\delta^2 g(\underline{X})}{\delta X_j \, \delta X_k} \bigg|_{\bar{X}} \cdot E[(X_i - \bar{X}_i)(X_j - \bar{X}_j)(X_k - \bar{X}_k)] \right\} + \ldots \tag{3-6}$$

$$\mu_3(Y) = \sum_{i=1}^{n} \left\{ \left(\frac{\delta g(\underline{X})}{\delta X_i} \bigg|_{\bar{X}} \right)^3 \cdot \gamma_3(X_i) \cdot \sigma_{X_i}^3 \right\} +$$

$$3 \sum_{\substack{i \\ i \neq j}}^{n} \sum_{j}^{n} \left\{ \left(\frac{\delta g(\underline{X})}{X_i} \bigg|_{\bar{X}} \right)^2 \cdot \frac{\delta g(\underline{X})}{X_j} \bigg|_{\bar{X}} \cdot E[(X_i - \bar{X}_i)^2 (X_j - \bar{X}_j)] \right\} +$$

$$6 \sum_{\substack{i \\ i<j<k}}^{n} \sum_{j}^{n} \sum_{k}^{n} \left\{ \frac{\delta g(\underline{X})}{\delta X_i} \bigg|_{\bar{X}} \cdot \frac{\delta g(\underline{X})}{\delta X_j} \bigg|_{\bar{X}} \cdot \frac{\delta g(\underline{X})}{\delta X_k} \bigg|_{\bar{X}} \right.$$

$$\left. E[(X_i - \bar{X}_i)(X_j - \bar{X}_j)(X_k - \bar{X}_k)] \right\} \tag{3-7}$$

In the above expressions the following terms represent the correlation between different variables. If the variables are independent, then they are eliminated.

$$E[(X_i-\bar{X}_i)(X_j-\bar{X}_j)] = \rho\, \sigma_{X_i}\, \sigma_{X_j} \tag{3-8}$$

$$E[(X_i-\bar{X}_i)(X_j-\bar{X}_j)^2] = E[X_i X_j^2]-E[X_i]\, E[X_j^2]-2\, E[X_i X_j]\, E[X_j]+ 2E[X_i]\,(E[X_j])^2 \tag{3-9}$$

$$E\left[(X_i-\bar{X}_i)(X_j-\bar{X}_j)(X_k-\bar{X}_k)\right]= E\left[X_i X_j X_k\right]-E\left[X_i\right]E\left[X_j X_k\right]-E\left[X_j\right]E\left[X_i X_k\right]- E\left[X_k\right]E\left[X_i X_j\right]+2\,E\left[X_i\right]E\left[X_j\right]E\left[X_k\right] \tag{3-10}$$

The importance of including correlations in the analysis depends on the magnitudes of coefficients of variations and the function itself. If the coefficients of variations are high, the effect of correlation could be substantial. In deciding whether or not to consider the correlations, one has to make the trade-off between the amount of information required (in this case the correlation coefficients between all variables), the amount of information available, the level of accuracy needed, the complexity of the analysis, and the cost of such accuracy.

The information about correlations should either be supplied by investors/ designers/ contractors or predicted from data that is available in the market. In the case of the building design problem, investors/ designers/ contractors are not familiar with the concept and available data in the market is weak and inappropriate for deriving correlations. This makes it difficult and in most cases impossible to explicitly incorporate the correlations in the analysis.

Since correlations are usually generated in the decomposition process, some suggest that the decomposition process should be limited (55). Such a solution is not realistic as the building design process is one of decomposition and integration. Another solution is the pessimistic and optimistic approach (56). This method suggests that analysis be performed with the most pessimistic view (full correlation) and compared to the most optimistic view (no correlation). If the level of error is acceptable, correlations are ignored, otherwise further study to determine the magnitudes of the correlations should be undertaken (in this method, instead of considering full coorrelation or no correlation, upper and lower limits may be considered. This method is useful in that the maximum level of error that is induced by ignoring correlations can be determined. The method that may be helpful in future research is to isolate the sources of uncertainty and the sources of correlations. For example, there is a high correlation between inflation rates for different cost components. If inflation rates are isolated and further studied, it can be seen that inflation rates for different components are composed of a general inflation rate and an incremental inflation rate. The general inflation rate is the main source of correlation. With this method we may be able to modify cost models and treat the main sources of uncertainty. In a case where a reliable data base becomes available, techniques that are used in social sciences (such as Factor Analysis (72)) should be examined.

The inclusion of the third moment (skewness) in equations (3-5,6,7) has been investigated (23). It was learned that, except in extraordinary situations (very high coefficient of variation), if the coefficient of skewness is less than one, the inclusion of the third moment has very little effect on the expected value of the function. However, its effect on variance largely depends on the magnitude of the coefficients of variation. If the coefficient of variation is high, the effect of including a third moment can be substantial.

To demonstrate the application and the accuracy of the moments approach (in comparing with the frequency method), the following example has been solved using both the frequency and moments approaches.

EXAMPLE:

Consider the following multiple variable function;

$$P = X\ Y\ Z \tag{3-11}$$

In which, variables X,Y,Z are independent random variables having the following distributions;

X Normal distribution

Y Log-Normal distribution

Z Beta distribution (p=3 , q=2)

i) using the frequency distribution method, the moments of function P are calculated as;

$$E[P] = \int\int\int X\ Y\ Z\ f_x f_y f_z\ dx\ dy\ dz \qquad (3\text{-}12)$$

or

$$E[P] = \int X\ f_x\ dx. * \int Y\ f_y\ dy. * \int Z\ f_z\ dz. \qquad (3\text{-}13)$$

and

$$\sigma^2(P) = \int\int\int (X\ Y\ Z - E[P])^2\ f_x f_y f_z\ dx\ dy\ dz \qquad (3\text{-}14)$$

For the given distributions and considering the independence condition;

$$E[P] = E[X] * E[Y] * E[Z] \qquad (3\text{-}15)$$

$$\sigma^2(P) = E[X^2] * E[Y^2] * E[Z^2] - (E[P])^2 \qquad (3\text{-}16)$$

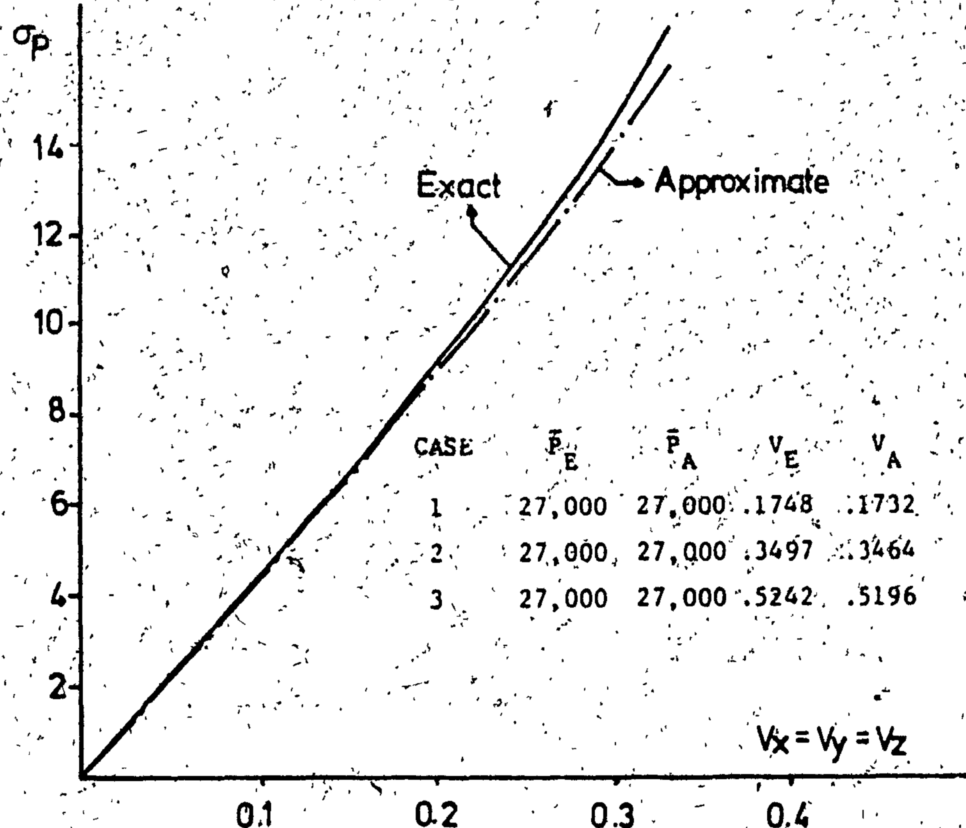
$$E[W^2] = \sigma^2_W + (E[W])^2 \quad \text{for } W=X,Y,Z \qquad (3\text{-}17)$$

ii) Using the moments approach;

$$E[P] = E[X] * E[Y] * E[Z] \qquad (3\text{-}18)$$

$$\sigma^2(P) = (E[Y]*E[Z])^2 * \sigma^2(X) + (E[X]*E[Z])^2 * \sigma^2(Y) + (E[X]*E[Y])^2 * \sigma^2(Z) \qquad (3\text{-}19)$$

In this example three cases have been considered. In all cases the mean values are kept constant ($E(X)=E(Y)=E(Z)=30.$). The coefficient of variation for case 1 is 0.1 ($V_x=V_y=V_z=.1$), for case 2 is 0.2, and for case 3 is 0.3 . Using this information, parameters of each distribution are determined (μ and σ for Normal distribution, λ and ξ for Log-Normal distribution, and upper and lower limits for Beta distribution). Values for mean, standard deviation, and coefficient of variation for each case (using both frequency and moments methods) are computed and are shown below.



CASE	$\bar{P}_E$	$\bar{P}_A$	V_E	V_A
1	27,000	27,000	.1748	.1732
2	27,000	27,000	.3497	.3464
3	27,000	27,000	.5242	.5196

As it is shown, the mean values in both approaches are the same. In all three cases, standard deviations (or coefficients of variations) in the moments approach are underestimated by less than 1% . This example suggests that the moments approach provides an accurate estimate of the major descriptors (in this example mean and variance) of function P. The level of accuracy does not change (substantial change) with changes in the coefficients of variation. However, for higher values of coefficient of variation the effect of inclusion of the coefficient of variation can be substantial. In examining the literature, the author has found no detailed analysis pertaining to limits in accuracy of the moments approach.

3.2 - The Application of Moments Approach

The moments approach will assist us in analyzing some of the problems, at different levels of cost modeling, that are probabilistic in nature. This approach can also be integrated into the main framework specified in Chapter II. This is depicted in Figure (3-1).

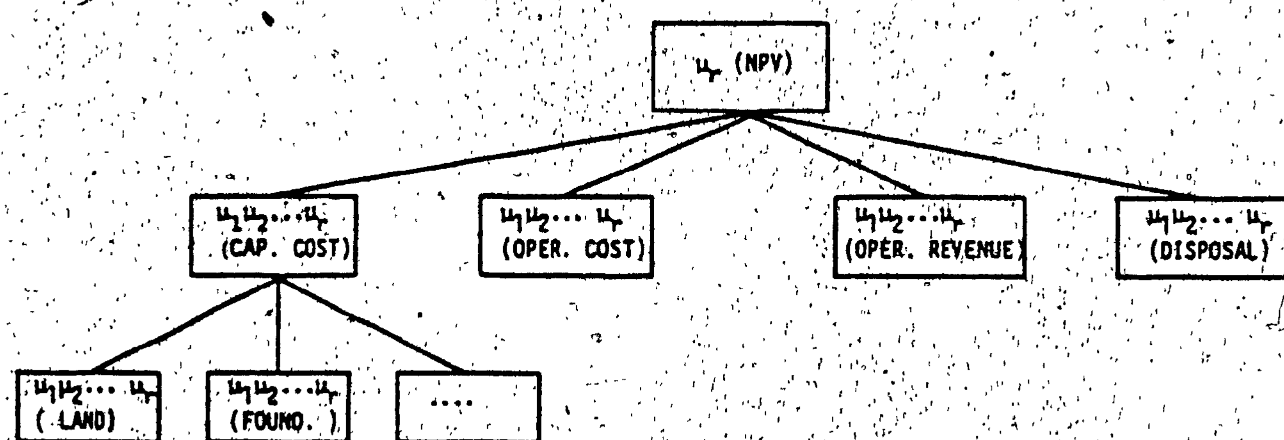


FIG. (3-1) - Application of Moments Approach in the General Framework.

Where μ_r is the rth moment .

3.2.1 - Application of Moments Approach at Subsystem Level

3.2.1.1- Moments of Capital Cost

The capital cost relationship, as stated in the Linear model, is a function of several variables;

$$PW(C_{ij}) = g\ (C_{ij}, \Theta_{ij}, r, T_{sj}, T_{fj}) \tag{3-20}$$

Here, all variables are considered to be random variables. In Equation (3-20) function "g" can be written in a general form.

$$\begin{aligned} g\ (C_{ij}, \Theta_{ij}, r, T_{sj}, T_{fj}) &= g\ (X_1, X_2, \ldots X_n) \\ &= g\ (\underline{X}) \end{aligned} \tag{3-21}$$

Function $g(\underline{X})$ is a multiple variable function similar to Equation (3-3). Moments equations (3-5,6,7) can be used for calculation of the first three moments for the present worth capital cost expression. Moments of capital cost will be used in calculating moments of evaluation criteria (e.g. NPV, LCC).

3.2.1.2 - Application of Moments Approach for Future Cost Components

In forecasting the future, one is dealing with events which are largely beyond control and one's ability to predict accurately. The existing unstable economic environment makes difficult the prediction of the future behavior of the time related variables in the cost models.

The application of the moments approach in the probabilistic analysis of the future cost components are explored on two levels ; I) to show how the expression(s) for moments of different future cost components (e.g. operating, maintenance,....) can be calculated, and II) to investigate and formulate the varying nature of the uncertainty involved in future cost components.

I- Moments Expressions for Future Cost Components

The future cost relationships, as provided in Chapter II, are functions of several variables;

$$PW(c_{fj}) = g\ (C_{fjo}, \theta_{oj}, n, T_f, T_e, r, \ldots) \qquad (3\text{-}22)$$

where;

C_{fjo} = base year cost of future cost component j (e.g. C_{Ojo} for operating cost)

θ_{oj} = inflation rate for future cost of component j

n = alteration period

T_f = finish time of construction

T_e = study period

r = discount rate

Here, all variables are considered to be random. In Equation (3-22) function "g" can be written in a general form.

$$g\ (C_{fjo}, \theta_{oj}, n, T_f, T_e, r, \ldots) = g\ (X_1, X_2, \ldots, X_n) = g\ (\underline{X}) \qquad (3\text{-}23)$$

Function g ($\underline{X}$) is a multiple variable function similar to Equation (3-3). The moments expressions (3-5,6,7) can be used for calculation of the first three moments of the future cost components.

II - Variation of Uncertainty With Time

In formulating the future cost models, one has to predict future trends that are beyond his control. Forecasts of these trends are subject to a high level of uncertainty that varies with time. In this section, the variability of the uncertainty level for a time related variable "inflation rate" will be studied and its treatment using the moments approach will be presented.

Consider the following general expression for present worth of a future cost component.

$$PW\ (CF\ (Tf)) = \int Cf_0\, e^{\int \theta(\tau)\, d\tau}\, e^{-\int r(\tau)\, d\tau}\, dt \qquad (3\text{-}24)$$

Assume that the base year cost, "c_{fo}", and the discount rate (uniform rate or as a function of time), $r(\tau)$, are deterministic (although the discount rate is a function of the inflation rate, for illustration purposes, it is assumed to be deterministic), θ is a function of random variables whose future behavior is unknown. Figure (3-2) illustrates a simple case where boundaries of the inflation rate fluctuations are linear functions of time (in this figure the upper and lower limits of the inflation rate fluctuations are stretched to show the variation of uncertainty with time). The above assumptions do not effect the generality of the procedure and they are only made for illustrative purposes.

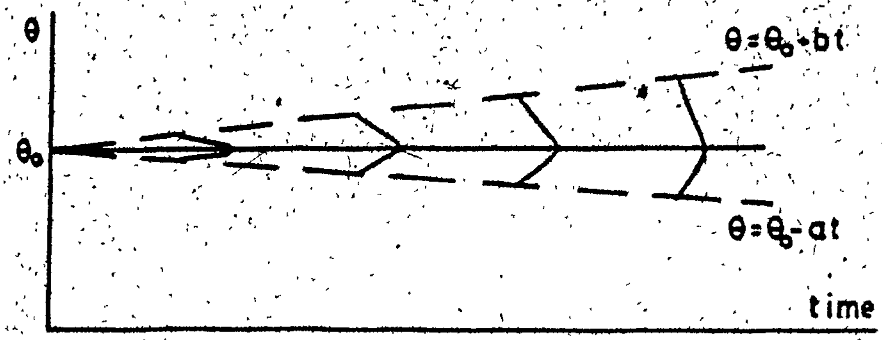


FIG. (3-2) - Time Variations of Inflation Rate Distribution Model

The expected value of the present worth function in Equation (3-24) is:

$$E[PW\ (CF(If\ ))] = \int E[Cf_0\ e^{\int \theta(\tau)\,d\tau}\ e^{-\int r(\tau)\,d\tau}\ dt\ ]$$

$$= \int E[Cf_0\ e^{\int \theta(\tau)\,d\tau}\ ]\,e^{-\int r(\tau)\,d\tau}\ dt \qquad (3\text{-}25)$$

Then;

$$E\,[Cf_0\ e^{\int \theta(\tau)\,d\tau}\ ] = Cf_0\ \ E[\ e^{\int \theta(t)\,d\tau}\ ] \qquad (3\text{-}26)$$

Function $\theta(\tau)$ can be represented as:

$$\theta(\tau) = \theta(\theta_1, \theta_2, \theta_3, \ldots\ldots, \theta_n, \tau) \quad (3\text{-}27)$$

and;

$$E[e^{\int \theta(\tau)\, d\tau}] = E[e^{\theta(\theta_1, \theta_2, \theta_3, \ldots\ldots, \theta_n, t)}] \quad (3\text{-}28)$$

In the above equation, if variables $\theta_1, \theta_2, \ldots.$ are independent random variables which correspond to parameters in the inflation function, using the first two moments, the expected value becomes;

$$E[e^{\theta(\theta_1, \theta_2, \ldots., \theta_n, t)}] = e^{\theta(\bar{\theta}_1, \bar{\theta}_2, \ldots, \bar{\theta}_n, t)} + (1/2)\sum_k \left(\frac{\delta^2 e^{\theta(\theta_1, \theta_2, \ldots, \theta_n, t)}}{\delta \theta_k^2}\right)\sigma^2_{\theta_k} \quad (3\text{-}29)$$

Then;

$$E[PW(CF(Tf))] = \int Cf_0\, e^{-\int r(\tau)\, d\tau}\left[e^{\theta(\bar{\theta}_1, \bar{\theta}_2, \ldots, \bar{\theta}_n, t)} + (1/2)\sum_k \left(\frac{\delta^2 e^{\theta(\theta_1, \theta_2, \ldots, \theta_n, t)}}{\delta \theta_k^2}\right)\sigma^2_{\theta_k}\right] dt \quad (3\text{-}30)$$

Similarly, the variance of the present worth function can be developed.

$$\sigma^2(PW(CF(Tf))) = \int \sigma^2 \left(Cf_0 \, e^{\int \theta(\tau)\, d\tau} \, e^{-\int r(\tau)\, d\tau} \, dt \right)$$

$$= \int Cf_0^2 \, e^{-2\int r(\tau)\, d\tau} \, \sigma^2\left(e^{\int \theta(\tau)\, d\tau}\right) dt \qquad (3\text{-}31)$$

Then;

$$\sigma^2\left(e^{\int\theta(\tau)\,d\tau}\right) = \sigma^2\left(e^{\theta(\theta_1,\theta_2,\ldots,\theta_n,t)}\right)$$

$$= \sum_k \left(\frac{\delta\, e^{\theta(\theta_1,\theta_2,\ldots,\theta_n,t)}}{\delta\,\theta_k}\right)^2 \cdot \sigma^2_{\theta_k} \qquad (3\text{-}32)$$

The expression for the variance of the present worth function is computed using the above equations.

$$\sigma^2(PW(CF(Tf))) = \int Cf_0^2 \, e^{-2\int r(\tau)\,dt}$$

$$\sum_k \left(\frac{\delta\, e^{\theta(\theta_1,\theta_2,\ldots,\theta_n,t)}}{\delta\,\theta_k}\right)^2 \cdot \sigma^2_{\theta_k} \cdot dt \qquad (3\text{-}33)$$

Since the expressions for expected value and variance are complex and their analytical solutions are not necessary, numerical integration is suggested for purposes of evaluating equations (3-30) and (3-33).

The following example is constructed to demonstrate the application of the above procedure and to show the importance of recognizing the time varying nature of the uncertainty, for the time related variables.

Example:

Consider the following, time dependent, inflation model:

$$\theta(t) = \theta_o + \theta_1 t \tag{3-34}$$

In the above expression, θ_o is the base year inflation rate and θ_1 is a random variable, having a uniform distribution (Beta distribution with p=q=1) with upper and lower limits of $\pm a$ (in numerical calculations a=.002 is used).

The mean value of θ_1, using a uniform distribution, is zero and the variance of θ_1 is calculated as:

$$(\theta_1) = a^2/3 \tag{3-35}$$

Then, the mean value of the inflation becomes θ_o, and its variance is the same as the variance of θ_1. The variation of mean and standard deviation, with time, for different values of p,q are also studied and the results are shown in Figs. (3-3,4). These variations are all linear functions of time.

In a continuation of this example, the formulation of the current dollar cost model (Cf'(t)), for base year cost of Cfo is shown.

$$\begin{aligned} E[Cf'(t)] &= Cf_o \cdot E[e^{(\theta_o \cdot t + (1/2) \cdot \theta_1 \cdot t^2)}] \\ &= Cf_o \cdot (e^{\theta_o \cdot t} + (1/8) \cdot t^4 \cdot e^{\theta_o \cdot t} \cdot \sigma^2_{\theta_1}) \\ &= Cf_o \cdot (e^{\theta_o \cdot t} + (1/24) \cdot t^4 \cdot a^2 \cdot e^{\theta_o \cdot t}) \end{aligned} \tag{3-36}$$

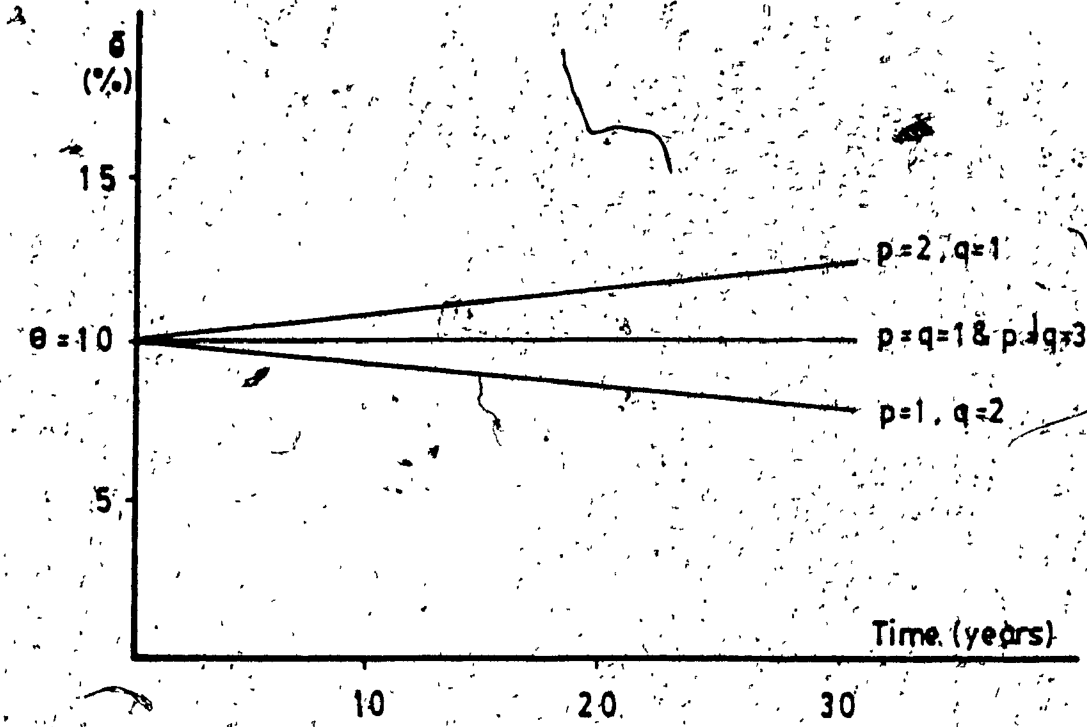


FIG. (3-3) - Time variation of the mean inflation rate.

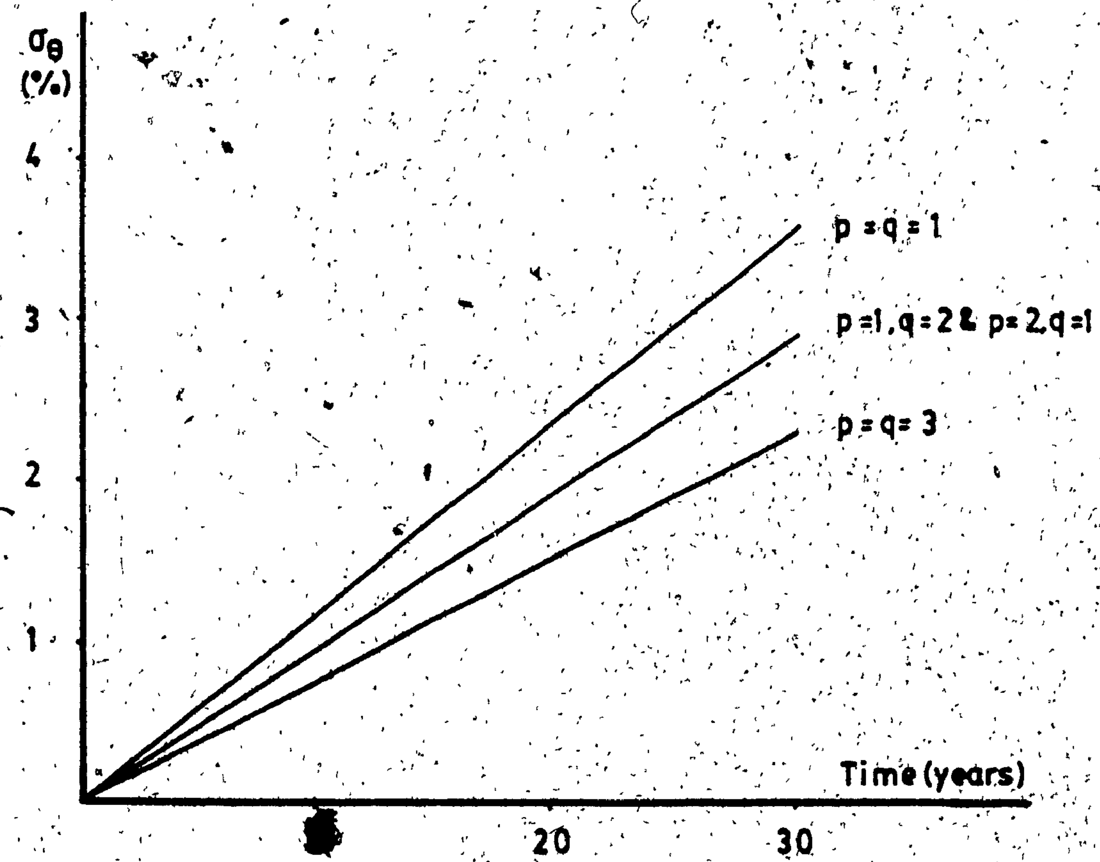


FIG. (3-4) - Time variation of standard deviation of inflation rate.

The variations of the mean value of Cf'(t) are shown, for different values of p and q (p and q are used in calculation of mean and variance of θ_1), in Fig. (3-5).

Similarly, the variance of the current dollar cost is calculated as;

$$\sigma^2 (Cf'(t)) = Cf_0^2 \cdot ((1/12) t^4 a^2 e^{2 \cdot \theta_0 \cdot t}) \qquad (3\text{-}37)$$

Fig. (3-6), illustrates the variations of the standard deviation of Cf'(t) for different values of Beta parameters (p and q).

3.2.2- Moments of Cost Components at System Level

To evaluate moments of cost components at system level, the structure of the general framework is used. The task here is to compute the moments at system level by integrating information that is generated at the subsystem level. For example, for subsystems 1 and 2, at subsystem level we have:

$$Y_1 = g_1 (\underline{X}_1) \qquad (3\text{-}38)$$

$$Y_2 = g_2 (\underline{X}_2) \qquad (3\text{-}39)$$

Then, at system level we will have;

$$Y = g (Y_1, Y_2) \qquad (3\text{-}40)$$

In the above expression function Y is a multiple-

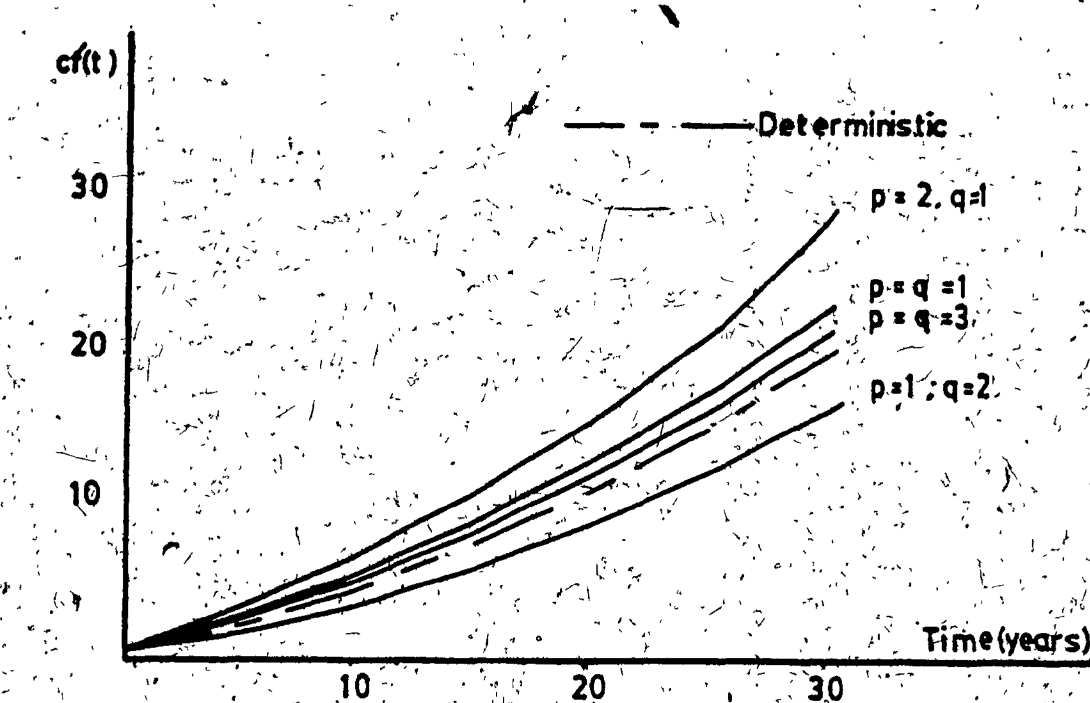


FIG. (3-5) - Time variation of cumulative cash flow using different Beta parameters.

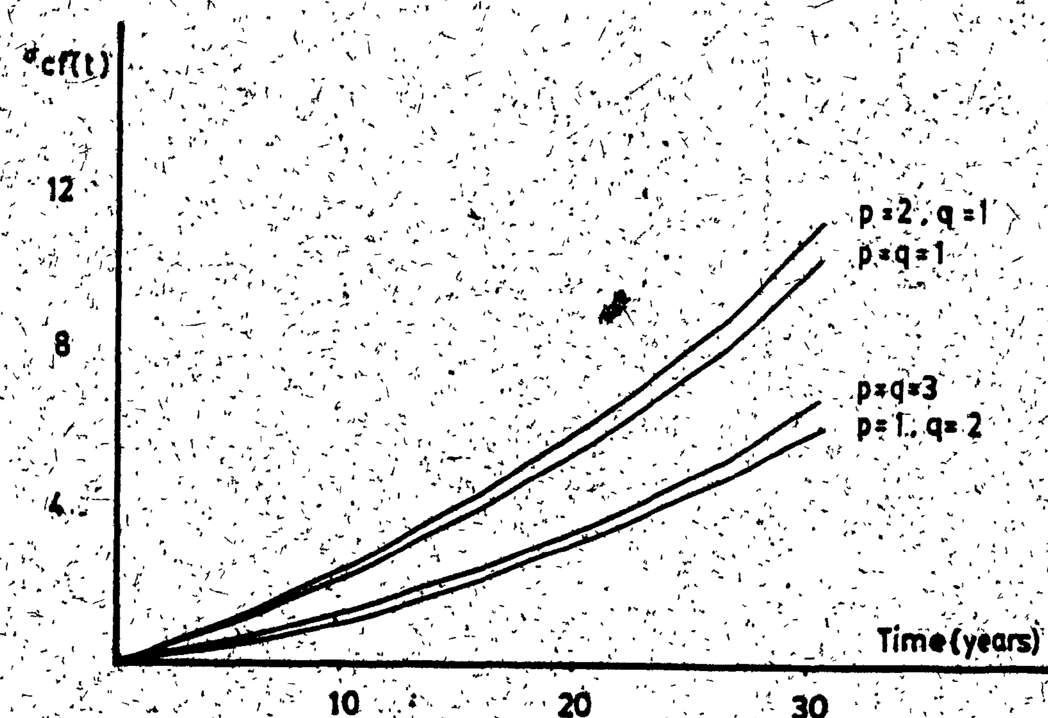


FIG. (3-6) - Time variation of standard deviation of cumulative cash flow using different Beta parameters.

variable function similar to equation (3-3). The moments expressions (eq. 3-5,6,7) can be used for calculation of the first three moments of the cost components at system level. For example, if we have an additive structure (e.g. capital cost component), then

$$Y = Y_1 + Y_2 \tag{3-41}$$

The moments of this function are:

$$E[Y] = E[Y_1] + E[Y_2] \tag{3-42}$$

$$\sigma^2(Y_1+Y_2) = \sigma^2(Y_1) + \sigma^2(Y_2) + COV(Y_1,Y_2) \tag{3-43}$$

$$\mu_3(Y_1,Y_2) = \mu_3(Y_1) + \mu_3(Y_2) + E[(Y_1-\bar{Y}_1)(Y_2-\bar{Y}_2)^2] \tag{3-44}$$

The covariance term represents the cross correlations between variables of Y_1 and Y_2 (e.g. inflation rates for mechanical and electrical systems). In this case, the covariance term could be expressed as:

$$COV(Y_1,Y_2) = \sum_i \sum_j \frac{\delta g_1}{\delta X_{1i}} \frac{\delta g_2}{\delta X_{2j}} COV(X_{1i},X_{2j}) \tag{3-45}$$

$$COV(X_{1i},X_{2j}) = \rho_{(X_{1i},X_{2j})}\, \sigma(X_{1i})\, \sigma(X_{2j}) \tag{3-46}$$

3.2.3 - Moments of Design Evaluation Criteria

Once the moments of different cost components (initial and future) are formulated, they can be used to evaluate the moments of the project evaluation criteria. Here, we will consider three of these, NPV, IRR, and LCC.

3.2.3.1 - Moments of Net Present Value (NPV)

The general form of NPV may be written as;

$$NPV = \sum_j [-PW(C_{cj}) - PW(C_{fj}) + PW(R_{fj}) + PW(S_j)] \qquad (3-47)$$

Where;

C_{cj} = capital cost of subsystem j

C_{fj} = future costs of subsystem j

R_{fj} = future revenues from subsystem j

S_j = salvage value of susbsystem j

Having the moments of different components, the process is a simple one of integrating this information together.

$$E\left[NPV\right] = \sum_j (-E\left[PW(C_{cj})\right] - E\left[PW(C_{fj})\right] + E\left[PW(R_{fj})\right] + E\left[(PW(S_j)\right]) \qquad (3-48)$$

$$\sigma^2(NPV) = \sum_j (\sigma^2(PW(C_{cj})) + \sigma^2(PW(C_{fj})) + \sigma^2(PW(R_{fj})) + \sigma^2(PW(S_j))) \qquad (3-49)$$

$$\mu_3(NPV) = \sum_j (-\mu_3(PW(C_{cj})) - \mu_3(PW(C_{fj})) + \mu_3(PW(R_{fj})) + \mu_3(PW(S_j))) \qquad (3-50)$$

In equations (3-48,49,50), it has been assumed that there are no correlations between different components.

3.2.3.2 - Moments of Internal Rate of Return (IRR)

The Internal Rate of Return is defined as the discount rate that equates the NPV to zero.

$$IRR = f(C_i, C_f, R_f, S, \theta_i, \theta_f, \ldots.) \qquad (3\text{-}51)$$

Since there is no single function for IRR that we can differentiate or integrate, the NPV function will be used in the process of calculating moments of IRR.

$$NPV = \sum_j [-PW(C_{ij}) - PW(C_{fj}) + PW(R_{fj}) + PW(Sj)] \qquad (3\text{-}52)$$

For illustration purposes, let us assume that only the inflation rates are random variables (θ_i, θ_f). In this case, the expected value of IRR is calculated as;

$$E[IRR] = \bar{r}\Big|_{\bar{\theta}} + (1/2) \sum_k \left(\frac{\delta^2 r}{\delta \theta_k^2} \sigma^2_{\theta_k} \right) \qquad (3\text{-}53)$$

To calculate moments of IRR we require first and second derivatives of IRR with respect to random variables (e.g. θ_i). To evaluate the derivatives of "r", the ratio of partial derivatives of NPV are used. The partial derivatives of NPV are obtained from NPV relationship in Chapter II (eq. (2-9).

$$E[IRR] = \bar{r}\Big|_{\bar{\theta}} + (1/2) \sum_k \left(\frac{\frac{\delta}{\delta\theta_k}\left(\frac{\delta NPV}{\delta r}\right) \frac{\delta NPV}{\delta\theta_k} - \frac{\delta^2 NPV}{\delta\theta_k} \frac{\delta NPV}{\delta r}}{\left(\frac{\delta NPV}{\delta r}\right)^2} \sigma^2_{\theta_k} \right) \qquad (3\text{-}54)$$

$$\sigma^2(IRR) = \sum_k \left(\left(\frac{\delta NPV / \delta \theta_k}{\delta NPV / \delta r} \right)^2 \sigma^2_{\theta_k} \right) \qquad (3\text{-}55)$$

$$\mu_3(IRR) = \sum_k \left(\left(\frac{\delta\, NPV / \delta\, \theta_k}{\delta\, NPV / \delta\, r} \right)^3 \gamma_3(\theta_k)\, \sigma^3_{\theta_k} \right) \tag{3-56}$$

For example, if;

$$NPV = \int_{T_f}^{T_e} R_o\, e^{(\theta_R - r)\, t}\, dt - \int_{T_s}^{T_f} C_{to}\, e^{(\theta_t - r)\, t}\, dt \tag{3-57}$$

Then;

$$\frac{\delta r}{\delta \theta_R} = -\frac{\delta NPV/\delta \theta_R}{\delta NPV/\delta r} = -\frac{\int_{T_f}^{T_e} R_o\, t\, e^{(\theta_R - r)\, t}\, dt}{\int_{T_f}^{T_e} R_o (-t) e^{(\theta_R - r) t}\, dt - \int_{T_s}^{T_f} C_{to} (-t) e^{(\theta_t - r) t}\, dt} \tag{3-58}$$

3.2.3.3 - Moments of Life Cycle Cost (LCC)

The moments of LCC can be computed with the information provided in the NPV analysis (sec. 3.2.3.1). First, mean, variance, and skewness of LCC are calculated, using moments of initial and future costs, for different subsystems. Then, this information can be combined to find the moments of LCC for the overall project.

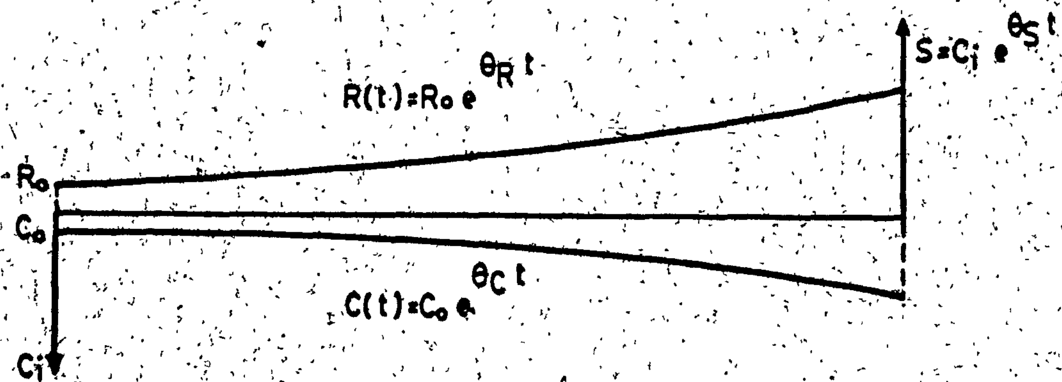
$$E[LCC] = \sum_j E[LCC_j] \tag{3-59}$$

$$\sigma^2(LCC) = \sum_j \sigma^2(LCC_j) \tag{3-60}$$

$$\mu_3(LCC) = \sum_j \mu_3(LCC_j) \tag{3-61}$$

3.2.3.4 - Example

To demonstrate the application of the moments approach in evaluation of decision making criteria, NPV, IRR and LCC, the following cash flow model is considered. In this analysis first two moments of these criteria are calculated.



Where;

C_I = \$3,000,000.

C_o = \$30,000.

R_o = \$150,000.

θ_C = 9%

θ_R = 10%

θ_S = 5%

σ_{θ_C} = 5%

σ_{θ_R} = 2%

σ_{θ_S} = 2%

r = 12% & Te = 30 years

In this hypothetical example inflation rates $(\theta_R, \theta_C, \theta_S)$ are assumed to be random while base year costs and the discount rate are assumed to be deterministic.

1) Net Present Value (NPV)

$$NPV = -3{,}000{,}000. + \int_0^{30} R_o\, e^{(\theta_R - r)t}\, dt - \int_0^{30} C_o e^{(\theta_C - r)t}\, dt + 3{,}000{,}000.\ e^{(\theta_S - r)\ 30}$$

i) Deterministic Analysis

$$NPV = -3{,}000{,}000. + \frac{150{,}000}{(\theta_R - r)}\left(e^{(\theta_R - r)\ 30} - 1\right) - \frac{30{,}000}{(\theta_C - 1)}\left(e^{(\theta_C - r)\ 30} - 1\right) + 3{,}000{,}000.\ e^{(\theta_S - r)\ 30}$$

Using the mean inflation rates;

$$NPV = 157{,}851.$$

ii) Probabilistic Analysis;

$$\overline{NPV} = \tilde{NPV}\Big|_{\bar{\theta}} + 1/2\left(\frac{\delta^2 NPV}{\delta\, \theta_R^2}\sigma_{\theta_R}^2 + \frac{\delta^2 NPV}{\delta\, \theta_C^2}\sigma_{\theta_C}^2 + \frac{\delta^2 NPV}{\delta\, \theta_S^2}\sigma_{\theta_S}^2 \right)\Big|_{\bar{\theta}}$$

$$= 157{,}851 + 1/2\ (374{,}040 - 342{,}000 + 132{,}253)$$

$$= 239{,}997.$$

The variance of NPV is;

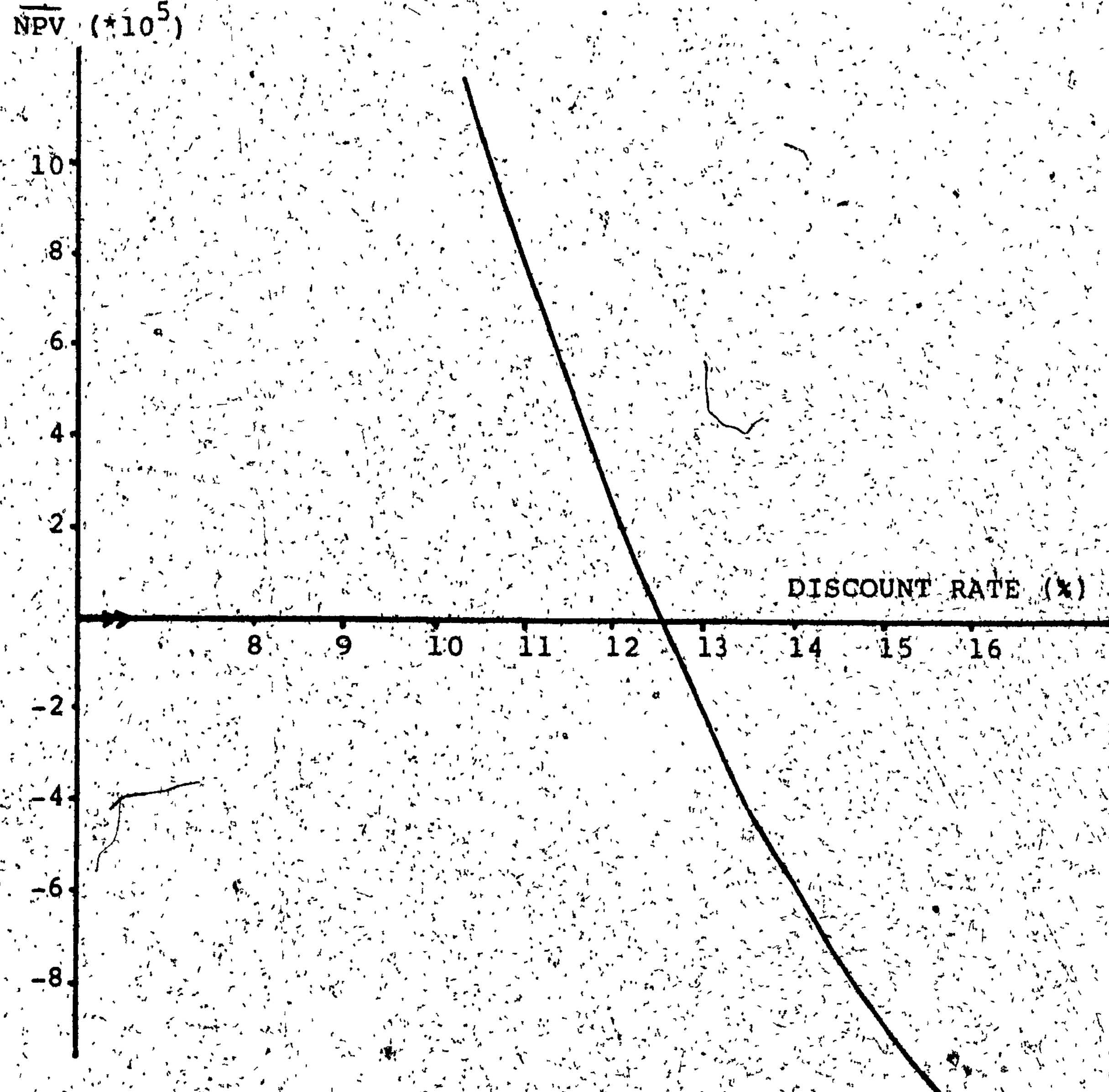


FIG. (3-7) - NPV vs. Discount Rate Diagram for the Example Problem.

$$\sigma^2_{NPV} = \left(\frac{\delta\ NPV}{\delta\ \theta_R}\right)^2 \sigma^2_{\theta_R} + \left(\frac{\delta\ NPV}{\delta\ \theta_C}\right)^2 \sigma^2_{\theta_C} + \left(\frac{\delta\ NPV}{\delta\ \theta_S}\right)^2 \sigma^2_{\theta_S} +$$

$$2\left(\frac{\delta NPV}{\delta\theta_R}\frac{\delta NPV}{\delta\theta_C}\ \rho_{\theta_R,\theta_C}\sigma_{\theta_R}\sigma_{\theta_C} + \frac{\delta NPV}{\delta\theta_R}\frac{\delta NPV}{\delta\theta_S}\ \rho_{\theta_R,\theta_S}\sigma_{\theta_R}\sigma_{\theta_S} +\right.$$

$$\left.\frac{\delta NPV}{\delta\theta_C}\frac{\delta NPV}{\delta\theta_S}\ \rho_{\theta_C,\theta_S}\sigma_{\theta_C}\sigma_{\theta_S}\right) + \ldots\ldots\ldots\ldots$$

$\sigma_{NPV} = 1,199,508$

If there is no coorelation between different inflation rates, the standard deviation of NPV becomes:

$\sigma_{NPV} = 1,121,500.$

2) Internal Rate of Return (IRR)

i) Deterministic Analysis;

By trial and error we can determine the IRR value.

$r_1 = 12\%$ $\quad NPV_1 = 239,997.$

$r_2 = 13\%$ $\quad NPV_2 = -72,566..$

$$IRR = .12 + .01\left(\frac{239,997}{239,997+72,566}\right) = .1277 = 12.77\%$$

ii) Probabilistic analysis;

$$\overline{IRR} = \bar{r}\Big|_{\bar{\theta}} + 1/2\left(\frac{\delta^2 r}{\delta\ \theta_R^2}\sigma^2_{\theta_R} + \frac{\delta^2 r}{\delta\ \theta_C^2}\sigma^2_{\theta_C} + \frac{\delta^2 r}{\delta\ \theta_S^2}\sigma^2_{\theta_S}\right)\Big|_{\bar{\theta}} +$$

$$+ 2\left(\left(\frac{\delta^2 r}{\delta\theta_R\ \delta\theta_C}\right)\rho_{\theta_R,\theta_C}\ \sigma_{\theta_R}\sigma_{\theta_C} + \left(\frac{\delta^2 r}{\delta\theta_R\ \delta\theta_S}\right)\rho_{\theta_R,\theta_S}\ \sigma_{\theta_R}\sigma_{\theta_S} +\right.$$

$$\left.\left(\frac{\delta^2 r}{\delta\theta_C\ \delta\theta_S}\right)\rho_{\theta_C,\theta_S}\ \sigma_{\theta_C}\sigma_{\theta_S}\right) + \ldots\ldots\ldots\ldots$$

$$\overline{IRR} = .1277 + 1/2 * (.00706 - .00659 + .00242)$$
$$= .1292 = 12.92\%$$

If there is no correlations between different inflation rates, the mean value of IRR will reduce by "$1.5 * 10^7$".

$$\sigma^2_{IRR} = \left(\frac{\delta r}{\delta \theta_R}\right)^2 \sigma^2_{\theta_R} + \left(\frac{\delta r}{\delta \theta_C}\right)^2 \sigma^2_{\theta_C} + \left(\frac{\delta r}{\delta \theta_S}\right)^2 \sigma^2_{\theta_S} +$$

$$2\left(\frac{\delta r}{\delta\theta_R}\frac{\delta r}{\delta\theta_C}\rho_{\theta_R,\theta_C}\ \sigma_{\theta_R}\sigma_{\theta_C} + \frac{\delta r}{\delta\theta_R}\frac{\delta r}{\delta\theta_S}\rho_{\theta_R,\theta_S}\ \sigma_{\theta_R}\sigma_{\theta_S} +\right.$$

$$\left.\frac{\delta r}{\delta\theta_C}\frac{\delta r}{\delta\theta_S}\rho_{\theta_C,\theta_S}\ \sigma_{\theta_C}\sigma_{\theta_S}\right) + \ldots\ldots\ldots$$

$$= .000520$$

$$\sigma_{IRR} = .0228 = 2.28\%$$

If there is no correlation between different inflation rates, the standard deviation of IRR becomes:

$$\sigma_{IRR} = .0208 = 2.08\%$$

3) Life Cycle Cost (LCC)

i) Deterministic Analysis

$$LCC = 3,000,000. + \int_0^{30} C_o\, e^{(\theta_C - 1)\, t}\, dt$$
$$= 3,000,000 + 593,430 = 3,593,430.$$

ii) Probabilistic analysis

$$\overline{LCC} = LCC\Big|_{\bar{\theta}} + 1/2 \left(\frac{\delta^2 LCC}{\delta\, \theta_C^2} \sigma^2_{\theta_C} \right)\Big|_{\bar{\theta}}$$

$$= 3,593,430 + 1/2 * 326,850$$

$$= 3,756,855.$$

$$\sigma^2_{LCC} = \left(\frac{\delta\, LCC}{\delta\, \theta_C} \right)^2 \sigma^2_{\theta_C}$$

$$= 12.659 * 10^{10}$$

$$\sigma_{LCC} = 355,800.$$

Observations made from the foregoing example are:

i- The moments approach can effectively be applied to different types of criteria having either additive or multiplicative structure (e.g. NPV has an additive structure and IRR has a multiplicative structure).

ii- The deterministic analysis can substantially reduce the level of accuracy (e.g. the expected value of NPV is 52% higher than its deterministic value). Furthermore, the deterministic analysis provides no information about the probability of failure which in this case is about 90%.

iii- Although IRR criterion is derived from NPV, the magnitude of the coefficient of variation for IRR does not reflect the magnitude of the coefficient of variation for NPV (e.g. coefficient of variation of NPV is 4.67 while the coefficient of variation of IRR is .168). This difference is due to different types of structures that these two criteria have (e.g. NPV has additive structure and IRR has multiplicative structure).

iv- The inclusion of correlations between inflation rates have little effect on mean values of the NPV and the IRR. However, the effect on the standard deviations depends on the variances of the correlated variables. Furthermore, the percentage of error that may be introduced by ignoring correlations is higher for IRR than NPV. This difference is also due to the structures of NPV anf IRR.

3.3 - Distribution of Capital Cost at Project Level

The probabilistic characteristics of cost components, where historical information is available, may be represented by their distributions. A goodness of fit testing procedure can be used to derive distributions of cost components. First, a set of distributions that are applicable to the problem is selected (distributions are selected based on problem characteristics), then, their goodness of fit to the data set are measured based on an evaluation criterion, and finally, the distribution with best performance is selected. The evaluation criteria include the Chi-square, the one -sided Kolmogorov-Smirnov, the two-sided Kolmogorov-Smirnov, the Least square, the sum of square deviations, etc. . The evaluation criterion is selected based on sample size and the frequency distribution of the data set.

The purpose of this section is to illustrate the procedure by investigating and proposing a statistical distribution model which describes the fluctuating nature of constant dollar building cost. The successful model should have the following characteristics (10). It

a) should be flexible (different shapes with different parameter values);
b) should be easy to derive from a finite set of data;
c) should be capable of being updated as new information becomes available; and
d) should preferably have finite end points.

Based on the above characteristics, Flanagan and Norman (10) selected the Beta distribution as a model which satisfies these conditions. However, it was felt that the omission of a number of distributions from further analysis, based only on the fourth condition by these authors and without further justification, was improper.

In order to further study the issue, a set of 32 office buildings was selected from a set of 140 high-rise buildings built by a major Canadian construction company. Each of these buildings cost (contractor values)between 4 to 8 million dollars. All cost items have been indexed using a composite (time, location) index. Furthermore, one building at each extreme, because of the nature of the projects, was screened off. The distribution of the remaining 30 buildings is shown in Fig. (3-7). In this histogram, the upper and lower limits are wide apart. The cost data bank ,used in this analysis, does not provide any additional information that enables us to further reduce this range or to explain the reasons behind such behavior. We did not have any access to information such as the estimated and actual construction duration, the number of design changes, and estimated and actual construction costs. It would be helpful to include such information in the cost data bank.

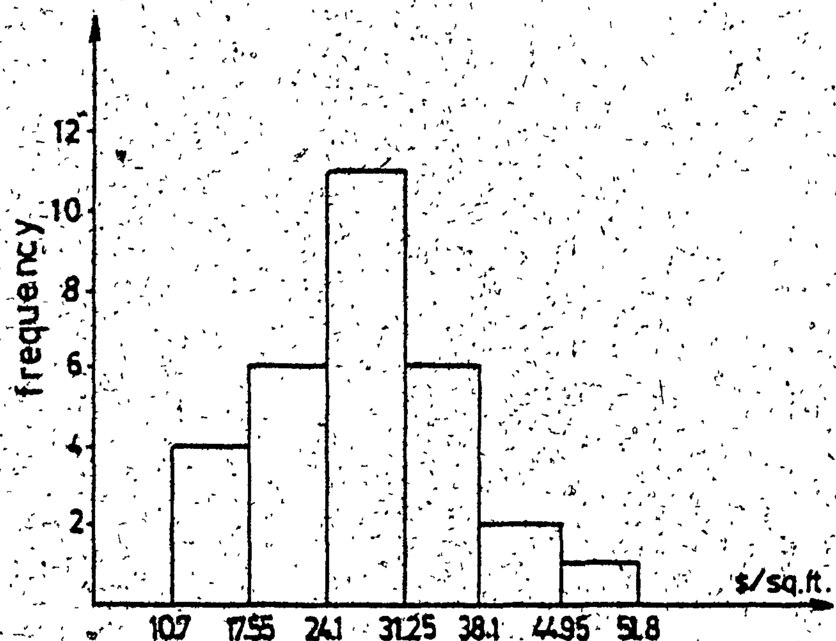


FIG. (3-8) - Capital Cost Data Histogram

Five distributions were selected for preliminary analysis. These include Normal, Beta, Gamma, Log-Normal and Weibull models. The selected evaluation criteria based on the frequency distribution and the sample size are the two-sided Kolmogorov-Smirnov test (57,58), and the sum of the square deviations between the sample data and the selected distribution (the Chi Square test is not appropriate because of the small sample size).

In the Kolmogorov-Smirnov test;

$$P\ (-C < \text{error} < +C) = 1-\alpha \qquad (3\text{-}62)$$

where α is the significance level and $\pm C$ are the upper and lower error limits. As the value of α increases, C will decrease as well as the probability of staying within the limits. The preliminary analysis was performed for the total capital cost as well as the initial cost of mechanical, electrical, exterior closure, and overall energy related systems. For all cost categories, the Weibull distribution either failed the significance test or was far behind the others. Hence, it was eliminated at this stage. Instead the Triangular distribution was included.

The parameters of each distribution have been computed using an applied goodness of fit testing program and are depicted in Fig. (3-9). In order to test the strength of the selected distributions, random sets with smaller sample sizes should be tested (59). In this analysis , ten sets, each of them having 10 elements, were randomly selected from the sample. The tests are repeated for these sets. Based on the Kolmogorov-Smirnov test, for a significance level of

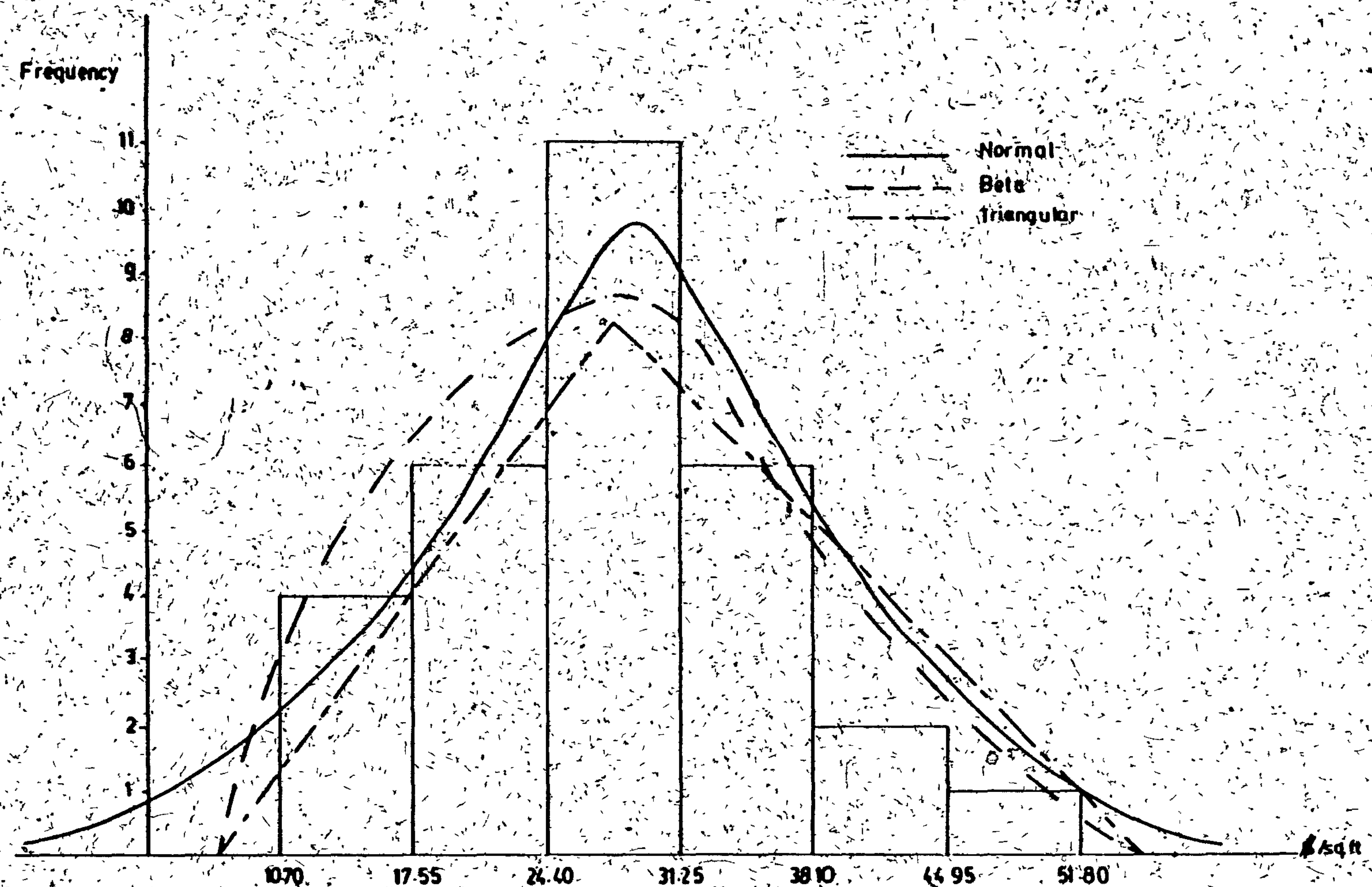


FIG. (3-9) - Probability Distribution Models for Capital Cost.

10%, the Beta, Gamma, and Normal distributions passed the test in 9 out of 10 trials; The Log-Normal distribution in 7 out of 10 trials; and the Triangular distribution in only 4 out of 10 trials. In Table (3-1), it is shown that, in 6 out of 10 times, the Beta distribution provides the best possible approximation. It appears that the Beta model is a suitable distribution for the capital cost. It should also be noted that i) although, the Normal distribution is acceptable for the total project cost with a 10% significance level, it does not represent the skewed nature of the capital cost distribution, and furthermore, in dealing with different building systems (e.g. electrical system), the Normal distribution does not provide an acceptable estimate; ii) the Gamma distribution is acceptable, in most cases, with a good level of significance and iii) the Triangular distribution is not recommended as a capital cost distribution model.

Determining the capital cost distribution is important for capital cost budgeting and control. By using numerical analysis methods one can develop probability distributions, for capital cost, for different types of building projects to measure the probability of staying within the forecasted budget. G.M. Hoffman (60), used this concept in computing the probability of exceeding the budget or amounts near the budget.

In the absence of reliable data bank, the distributions may be estimated by experiments (e.g. acuiring estimates from several engineers/ architects for a set of plans). Once the distribution is derived, it can be used for forecasting purposes and it requires minimum amount of information.

model / test	Normal	Beta	Gamma	Log-Normal	Triangular
Kolmogorov-Smirnov	2/10 *	6/10	2/10	--------	--------
Least sum of squares	3/10	6/10	1/10	--------	--------

* x/n = A particular distribution provides the best fit to x sets out of n sets.

TABLE (3-1) - Comparison of Probability Distribution Models for 10 Random Samples.

3.4- Maintenance Policies and Replacement Costs

The useful life and physical performance of a building depends on decisions made in the construction (design decisions) and the operating (operation/ maintenance policies) phases of the project. The research on optimization of useful life of building projects started in the early 70's (61,62,63), in European countries, where operation/ maintenance costs are very high (62). In "CIB83" conference in Stockholm (see ref. 62), emphasis was put on the development of systematic approaches for maintenance and renewal of buildings and on creating data banks that are useful in predicting the technical performance and developing maintenance policies for different building subsystems. Estimation of maintenance / renewal costs of a building is not such an easy task. There are numerous parameters that should be predicted and many of them are uncertain. These include (61);

- maintenance standard required;
- maintenance policies implemented;
- degree of wear and tear experienced;
- function and use of building;
- original standard of construction; and
- the "knock on" effect of delayed maintenance (knock on effect is the effect of performance of one system on performance of other systems).

In recent years, economic conditions and budgetary constraints have forced public institutions to reduce their operating costs. Maintenance expenditures are kept at a minimum level in the hope that there will be an economic upturn in the near future. Such an approach can have significant negative effects in the form of (61):

1. additional maintenance costs required in the future;
2. additional operating costs required in the future;
3. further damage to the system including system failure;
4. loss of productivity in the building that may result in the loss of rental income; and
5. higher management fees due to conflict with users.

In this section, the problem of selecting an optimal maintenance policy is examined. Our objectives are several fold:

i) To structure the problem and state it in a quantitative framework;

ii) To demonstrate the usefulness of Markov chains as an analytical tool in the treatment of uncertainty for this type of problem; and

iii) To identify the information requirements for treating problems of this type.

Several simplifying assumptions are made. They include:

- the same maintenance policy is kept throughout the study period;
- deterioration of system condition entirely depends on the system condition at the beginning of the year; and
- transition probabilities of going from one state to the next is constant, regardless of system's state in previous years.

Maintenance policies can be divided into four basic groups:

Policy 1. Maintenance of items related to operational safety (in the mechanical , electrical systems, and elevators), as specified in codes and regulations.

Policy 2. Maintenance of safety related items and maintenance of engineering equipment as specified in the guidelines (ducting, controls,..) (e.g ASHRAE guidelines) (34);

Policy 3. Periodical inspections, replacement after failure; and

Policy 4. Full maintenance, regular inspections, and replacements.

Maintenance cost is a function of several factors;

$$Cm = f\ (\text{system design}, \theta, \text{age}, \text{system state}, \text{maintenance policy}) \tag{3-63}$$

The average maintenance cost (constant dollar), during year "i", of system operation may be calculated as;

$$PW(Cm_{ik}) = Cm_{ok}\ e^{(\theta m + a_k - r)\ i} \tag{3-64}$$

where

a_k=combined effect of aging and the maintenance policy k

Cm_{ok}=base year maintenance cost estimate of policy k (Maintenance cost includes maintenance, repair and alteration costs of all those components that are specified in the contract. For example, policy 1 does not include any repair or alteration costs, while policy 4 includes all costs).

$PW(Cm)_{ik}$=present worth of maintenance cost during year i for policy k

θm=inflation rate for maintenance cost

The maintenance cost matrix is constructed as;

$$
\begin{array}{cc}
 & \text{Maintenance Policy} \\
 & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\
\text{year} & \\
Cm= & \begin{array}{c} 1 \\ 2 \\ 3 \\ \cdot \\ i \\ \cdot \\ n \end{array}
\left|\begin{array}{cccc}
PW(Cm)_{11} & PW(Cm)_{12} & PW(Cm)_{13} & PW(Cm)_{14} \\
PW(Cm)_{21} & \cdots & \cdots & PW(Cm)_{24} \\
\cdots & \cdots & \cdots & \cdots \\
\cdots & \cdots & \cdots & \cdots \\
PW(Cm)_{i1} & \cdots & \cdots & PW(Cm)_{i4} \\
\cdots & \cdots & \cdots & \cdots \\
PW(Cm)_{n1} & \cdots & \cdots & PW(Cm)_{n4}
\end{array}\right|
\end{array}
$$

FIG (3-10)- Maintenance Policies Cost Matrix.

With respect to each policy the system may be in one of the following conditions (states):

State 1) need no repair/replacement

State 2) need minor repair/replacements

State 3) need intermediate level of repair/replacements

State 4) need major repair/replacements

Where in the first three categories, the system is operational, in the case of last category, the system fails and requires the replacement of a major component.

The characteristics of the problem are similar to problems in Markov Processes (66), where there is an initial state and a transition matrix that links different states together. For the purpose of modeling the effect of deferred maintenance policies, the application of Markov's process has been investigated , and some of the concepts are used in developing this model.

With respect to the design characteristics (type of system, its performance statistics, and the selected maintenance policy) we assign transition probabilities (conditional probabilities) of going from one state to another within one year (see Fig. (3-10)). The transition probabilities, for a particular maintenance policy (e.g. policy P_k in Fig.(3-10), are p_{11}, p_{12}, p_{13} and p_{14} (probabilities of going from state 1 to states 1,2,3,4), p_{21}, p_{22}, p_{23} and p_{24} (probabilities of going from state 2 to states 1,2,3,4) , and p_{31}, p_{32}, p_{33} and p_{34} (probabilities of going from state 3 to states 1,2,3 ,4), (minor repaires in states 2 and 3 brings the system to state 1 with probabilities that are less than 1). If the system is in state 4, a major overhaul is needed, this brings the system to state 1 (notation "4/1" in Fig. (3-10)). The calculated probabilities for each policy are shown in the probability matrix (Fig. (3-11)). Values in this table are calculated by

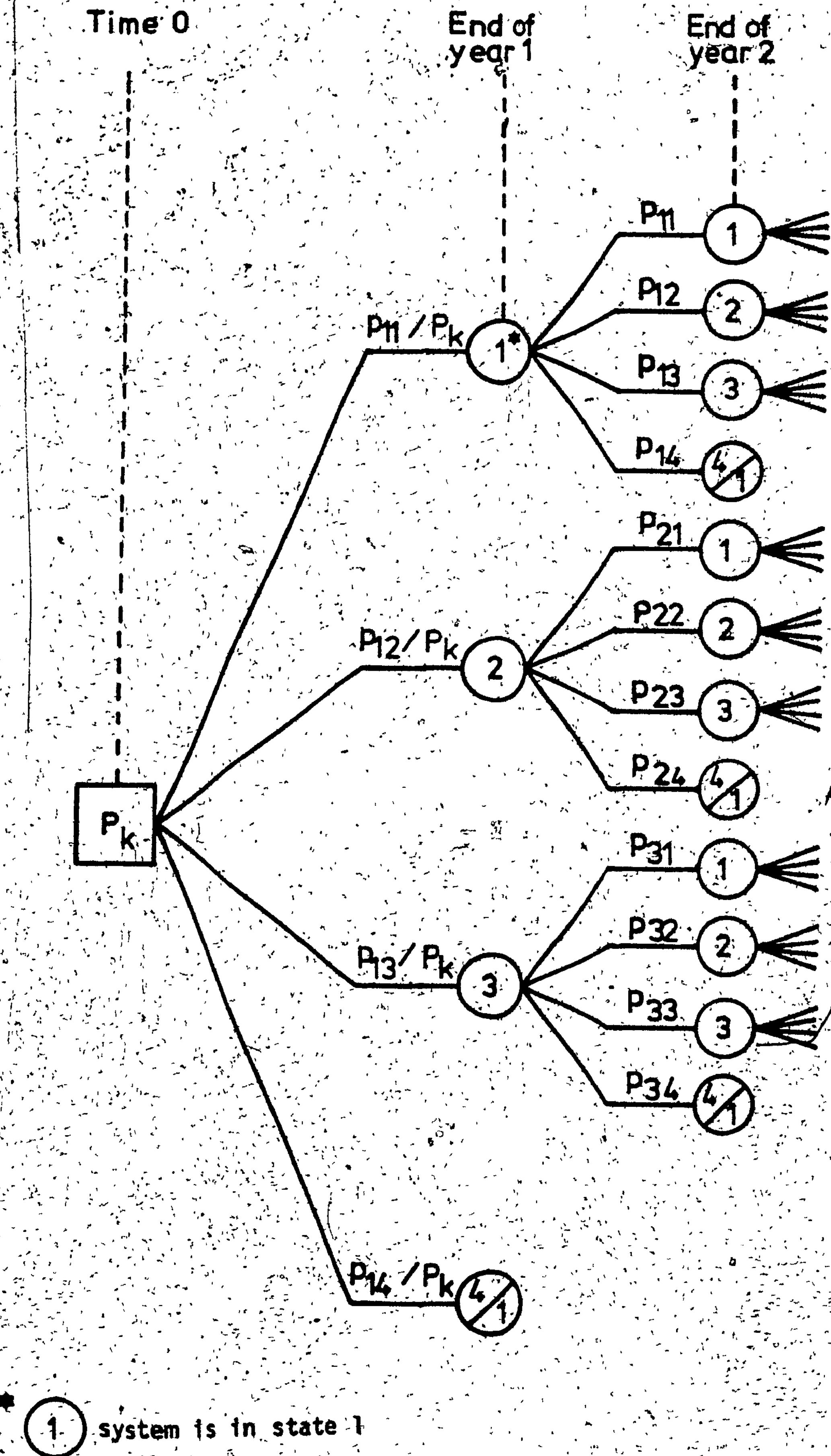


FIG. (3-11) - Transition Probabilities Tree.

adding probabilities of being in a particular state in a reference year, using the probability tree (Fig. (3-10). For example, The probability of being in state 3, at the end of year 2 is calculated as:

$$P_{23} = (p_{11}/P_k)*(p_{13}/P_k)+(p_{12}/P_k)*(p_{23}/P_k)+ (p_{13}/P_k)*(p_{33}/P_k)+(p_{14}/P_k)*(p_{13}/P_k) \qquad (3-65)$$

where;

P_{xj} = probability of going from state x to state j, within one year, for a particular maintenance policy. (transition probabilities) (Fig. (3-11)).

P_{ij} = probability of being in state j, in year i (calculated probabilities) (Fig. (3-12)).

system condition

	year	1	2	3	4
	1	P_{11}	P_{12}	P_{13}	P_{14}
	2	P_{21}	....	....	P_{24}
	3	P_{31}	....	....	....
	.				
P_k=	.				
	i	....	P_{ij}	....	....
	.				
	n	P_{n1}	....	....	P_{n4}

FIG. (3-12) - Probability matrix of system condition for maintenance policy k.

The probability matrix will be used to calculate the expected replacement cost matrix.

The repair/replacement costs (matrix), of retaining the system in operational condition is calculated using;

$$PW(Cr_{ji}) = Cr_{jo} * e^{(\theta rep - r)\, i} \qquad (3-66)$$

where

$PW(Cr_{ji})$ = present worth of replacement cost in year i, if the system is in condition j

θrep = inflation rate for replacement cost

Cr_{jo} = base year estimate of replacement cost, if the system is in condition j

Then

$$\begin{array}{ccc} & \text{system condition} & \begin{array}{cccc} & \text{YEAR} & & \\ 1 & 2 & \cdots\cdots & n \end{array} \\ Cr_k = & \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \left| \begin{array}{cccc} PW(Cr_{11}) & PW(Cr_{12}) & \cdots\cdots & PW(Cr_{1n}) \\ PW(Cr_{21}) & \cdots & \cdots & PW(Cr_{2n}) \\ \cdots & \cdots & \cdots & \cdots \\ PW(Cr_{41}) & \cdots & \cdots & PW(Cr_{4n}) \end{array} \right| \end{array}$$

FIG. (3-13) - Replacement Cost Matrix.

At this stage the expected replacement cost matrix, for maintenance policy k, is calculated by multiplying matrix P_k and matrix Cr_k.

$$E[Cr_k]=P_k*Cr_k=\begin{vmatrix} E[Cr]_{1k} & \ldots\ldots & \ldots\ldots & \ldots\ldots \\ \ldots\ldots & E[Cr]_{2k} & \ldots\ldots & \ldots\ldots \\ & & \ldots\ldots & \\ & & & \\ \ldots\ldots & \ldots\ldots & \ldots\ldots\quad\ldots\ldots & E[Cr]_{nk} \end{vmatrix}$$

FIG (3-14) - Expected replacement cost matrix for maintenance policy k.

Where

$E[Cr]_{ik}$=expected replacement cost in year i, if policy k is selected.

In the above matrix, we are concerned with the diagonal elements only. These elements represent the expected replacement costs, during year 1,2,......n, if policy k is selected.

When the expected cost matrices are obtained for all policies, this information is usefully re-arranged as;

$$
\begin{array}{cc|cccc|}
 & & & \text{maintenance policy} & & \\
 & & 1 & 2 & 3 & 4 \\
 & \text{year} & & & & \\
 & 1 & E[Cr]_{11} & E[Cr]_{12} & E[Cr]_{13} & E[Cr]_{14} \\
 & 2 & E[Cr]_{21} & \cdots & \cdots & E[Cr]_{24} \\
 & 3 & \cdots & \cdots & \cdots & \cdots \\
 & \cdot & & & & \\
E(Cr) = & j & \cdots & E[Cr]_{ik} & \cdots & \cdots \\
 & \cdot & & & & \\
 & \cdot & & & & \\
 & n & E[Cr]_{n1} & \cdots & \cdots & E[Cr]_{n4}
\end{array}
$$

FIG (3-15)- Expected Replacement Cost Matrix for All Maintenance Policies.

Finally, the overall maintenance and replacement costs matrix becomes:

$$E(Cr,m) = Cm + E(Cr) \tag{3-67}$$

and;

year \ policy	1	2	3	4
1	$E[Cr,m]_{11}$ / $P[1,2,3]_{11}$	$E[Cr,m]_{12}$ / $P[1,2,3]_{12}$	$E[Cr,m]_{13}$ / $P[1,2,3]_{13}$	$E[Cr,m]_{14}$ / $P[1,2,3]_{14}$
2	$E[Cr,m]_{21}$ / $P[1,2,3]_{21}$	- / -	- / -	$E[Cr,m]_{24}$ / $P[1,2,3]_{24}$
⋮	⋮	⋮	⋮	⋮
i	-	$E[Cr,m]_{i2}$ / $P[1,2,3]_{i2}$	-	-
⋮	⋮	⋮	⋮	⋮
n	$E[Cr,m]_{n1}$ / $P[1,2,3]_{n1}$	$E[Cr,m]_{n2}$ / $P[1,2,3]_{n2}$	$E[Cr,m]_{n3}$ / $P[1,2,3]_{n3}$	$E[Cr,m]_{n4}$ / $P[1,2,3]_{n4}$

FIG. (3-16) - Expected Cost of Maintenance & Replacement and Probabilities.

In the above matrix, the P(1,2,3) values represent the chance of being operational at any particular year with respect to the maintenance policy selected (P(1,2,3) is the probability of being in states 1,2, and 3). These values are calculated from the probability matrix "P_k" (Fig. 3-12).

For example;

$$P(1,2,3)_{ik} = P_{i1}+P_{i2}+P_{i3} \tag{3-68}$$

To demonstrate the application of the above formulation and its significance, the following example problem is constructed.

Example

Consider the mechanical subsystem of an office building project (approximately 300,000. sq. ft.). The initial capital outlay of this system is \$8.5 / sq ft. For the purpose of this example two maintenance policies (policies 2 and 4, as described previously) are considered. The base year cost estimates of these two policies are:

$$Cm_{o2} = \$23,000. \text{ / year}$$
$$Cm_{o4} = \$30,000. \text{ / year}$$

With respect to four different system's states, the repair/ replacement costs (base year estimates) are:

$Cr_{1o} = \$\ 0.$

$Cr_{2o} = \$\ 2,000.$

$Cr_{3o} = \$10,000.$

$Cr_{4o} = \$30,000.$

Assuming $\theta_m=8\%$, $\theta_r=9\%$, and $r=10\%$, The overall maintenance and replacement costs are calculated for a period of 10 years.

First, using the above information, the present worth, maintenance and replacement cost matrices (Cm,Cr) are;

Maintenance Policy

	Year	2	[illegible]
	1	\$22,711.	\$29,702.
	2	22,320.	29,113.
	3	21,878.	28,537.
	4	21,445.	27,972.
Cm =	5	21,020.	27,418.
	6	20,604.	26,875.
	7	20,296.	26,343.
	8	19,796.	25,821.
	9	19,404.	25,310.
	10	19,020.	24,809.

And,

	Year						
	Sys. Cond.	1	2	3	....	9	10
	1	0	0	0	....	0	0
	2	1,990	1,970	1,950	....	1,837	1,818
Cr =	3	9,950	9,851	9,735	....	9,185	9,093
	4	29,850	29,553	29,159	....	27,555	27,281

At this stage, we require transition probabilities for both maintenance policies. This information can be obtained from past records (records of system maintenance, repair and renewal, and history of system failure). At present, such an information source does not exist and it is necessary to develop. For this example, the transition probabilities are assumed to be:

Transition Probabilities

Policy	P_{11}	P_{12}	P_{13}	P_{14}	P_{22}	P_{23}	P_{24}	P_{33}	P_{34}
2	.25	.20	.30	.25	.20	.30	.50	.30	.70
4	.80	.10	.05	.05	.70	.20	.10	.70	.30

Using the process, shown in Figure (3-11), the probability matrix becomes:

$$P_2 = \begin{array}{c|cccc|} & \multicolumn{4}{c}{\text{System State}} \\ \text{Year} & 1 & 2 & 3 & 4 \\ 1 & .250 & .200 & .300 & .250 \\ 2 & .125 & .140 & .300 & .435 \\ 3 & .140 & .140 & .300 & .420 \\ 4 & .... & .... & .... & .... \\ \cdot & .... & .... & .... & .... \\ \cdot & .... & .... & .... & .... \\ 10 & .140 & .140 & .300 & .420 \end{array}$$

and,

$$
\begin{array}{ll}
 & \text{System State} \\
\text{Year} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\
P_4 = \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ \cdot \\ \cdot \\ 10 \end{array} & \left|\begin{array}{cccc}
.800 & .100 & .050 & .050 \\
.680 & .155 & .098 & .067 \\
.598 & .183 & .137 & .082 \\
\cdots & \cdots & \cdots & \cdots \\
\cdots & \cdots & \cdots & \cdots \\
\cdots & \cdots & \cdots & \cdots \\
.465 & .197 & .223 & .115
\end{array}\right|
\end{array}
$$

Using matrices C_r, P_2, P_4, the expected replacement cost matrix becomes:

$$
\begin{array}{ll}
 & \text{Policy} \\
\text{Year} & \begin{array}{cc} 2 & \quad 4 \end{array} \\
E[C_r] = \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \end{array} & \left|\begin{array}{cc}
10,845 & 0 \\
16,087 & 0 \\
15,488 & 0 \\
15,334 & 0 \\
15,181 & 0 \\
15,030 & 0 \\
14,880 & 0 \\
14,732 & 0 \\
14,586 & 0 \\
14,440 & 0
\end{array}\right|
\end{array}
$$

Finally, The total maintenance and replacement cost matrix is calculated as;

$$E\,[C_{r,m}] = E\,[\,C_r\,] + C_m$$

Maintenance Policy

$E\,[C_{r,m}] =$

Year	2	4
1	33,166 / .750	29,702 / .950
2	38,407 / .565	29,113 / .933
3	37,366 / .580	28,537 / .918
.	.... /	.... / ....
.	.... /	.... / ....
10	33,460 / .500	24,809 / .880

Although values of different parameters, in this example, are not taken from an actual project, some overall conclusions can be derived;

- Deferred maintenance policies (e.g. policy 2) may not save money;
- Probability of failure of the system can substantially be higher for the case of deferred maintenance policy (e.g. for policy 2 is about 50% while for policy 4 is below 20%); and
- Type and amount of information required in this analysis cannot be obtained from the investor/

decision maker. It requires a cost data bank (cost of each maintenance policy, cost of replacing major parts, and if possible the cost of knock-on effect of the system failure), a system performance data bank (history of system performance under different maintenance policies, history of system failure), and clear descriptions of different maintenance policies.

Information required for this analysis include:

- The base year cost of different maintenance policies;
- The base year cost of replacements for different system conditions (e.g. minor repair, major repair,...);
- Inflation rates for maintenance and replacement costs; and
- The transition probabilities for the system (e.g. p_{11}, p_{12}, p_{13},........).

The information about base year costs can directly be obtained from maintenance contractors. Maintenance cost can also be estimated from different building operation cost reports such as BOMA (47) and IREM (67) (these reports do not provide information about different maintenance policies or detailed information about repair costs). In Chapter 2 some basic relationships were developed for estimation of base year maintenance cost as a function of decision making variables.

Inflation rates or effect of time related variables on maintenance/ replacement costs can be estimated from historical data (47,67). In Table (2-7), some simple relationships are given for different maintenance cost components as functions of aging factor and time.

Information about transition probabilities cannot be obtained from one particular source. To estimate these probabilities, for a particular building subsystem, one requires information about subsystem performance, its failure statistics, and its reliability. This information is also needed for estimation of repair costs.

In order to select an effective maintenance policy, a building cost data bank is required. This data bank should include both cost and system performance data for different buliding subsystems and for different types of building. The creation of a building maintenance cost data bank has already been started in European countries where the maintenance cost is a major future cost component (e.g. in Finland the building maintenance cost is as high as 8% of Finland's GNP (61,62)).

CHAPTER IV

4.0- Introduction

The objective of this chapter is to present a rationalized method of decision-making for the types of decision problems found in the building design process. Key elements present in the decision-making process include: i) explicit definition of the needs of the investor/user, ii) expression of these needs in terms of decisions to be made, iii) determination of alternate sets of solutions, iv) selection of evaluation criteria to measure the performance of different design solutions, and v) determination of the best alternative based on the projected performance levels. The design goal is decomposed into a series of objectives that are easy to measure and categorize (e.g. technical, financial and user). A multiple-criteria decision-making methodology is presented that employs preference and utility theory. In this methodology, a multiplicative utility function is developed that treats the problem of criteria overlappings. Basic information (objectives motivations) and their order of importance, weightings for each objective (if necessary), and information about utility function if required) is supplied by the investor. This information is used by the decision maker (architect/engineer) to identify an appropriate set of evaluation criteria and to select design solutions.

The class of problems that can be treated with this methodology is limited to higher level decisions, such as selecting building type at the project level and selecting the energy system that consist of electrical, mechanical and enclosure subsystems at the system level.

In this chapter, we start with examining investor needs/ objectives, identifying different components, and illustrating the objective decomposition process. This is followed by an explanation of different classes of design optimization problems and existing literature on design optimization models. Finally, the design decision-making methodology is presented.

4.1- Investor Needs / Objectives

The investor needs/objectives are mainly determined by considering investor type, required input and output information, and through use of an objective decomposition process.

4.1.1- Investor Type:

Different investor types can be categorized according to their motives. The basic types include: i) speculative builder, ii) private owner, and iii) public investor.

Table (4-1) demonstrates the variation of objectives for different investor types having the same initial goal (for example, design/construction of an economical building).

Investor type	Objectives level 1	Objectives level 2
speculative builder	financial feasibility, technical feasibility, and user satisfaction.	initial cost, marketability, and consideration of code.
private builder	financial feasibility, technical feasibility, and user satisfaction.	initial cost, future cost , reliability , user comfort, and consideration of code.
public	financial feasibility, technical feasibility, and user satisfaction.	initial budget, future operating budget, reliability, social impact, economic need, worker productivity, and user cost.

Table (4-1) - Investor/User Objectives.

In the building design process, the owner/ investor, the architect/ designer, and the ultimate user of the building influence the design decisions. Linzy and Brotchie (5) studied the design objectives and stated that more than one objective should be reflected in the design decision making process. These include performance, cost, reliability, maintainability, risk, etc.. Other research work (18,19,21) also emphasizes the multi-attribute treatment of the building design problem.

4.1.2- Required Input and Output Variables:

Accuracy of an analysis is directly influenced by availability and reliability of the information to measure the objectives. One of the problems, both in the industry and in the academic environment, is the lack of data (cost/performance data for building subsystems).

Basically there are two types of information, i) information about system cost, performance, etc., that is independent of the investor/ decision maker (decision maker may exert little control on this information) and ii) information that is elicited from the investor or determined by the decision maker (information about investor's objectives, weightings, investor's utility function, evaluation criteria set, etc.). For the first type (type one information), regardless of decision-making methodology, we need cost and performance data banks for different building systems/ subsystems/ components. For the second type (type two information), the emphasis is on minimizing the amount of basic information to be supplied by the investor and also, minimizing the workload of the decision-maker (designer) by rationalizing the design decision-making process.

In selecting a design objective, the amount of information required, its availability, and the usefulness of the outcome should be studied.

4.1.3- Objective Decomposition:

To study a set of selected objectives, we need to decompose each of them to a level at which it is more clear and easy to quantify. In many instances, we are dealing with qualitative objectives such as aural comfort, pride of ownership, etc.. These objectives are not easy to measure and there is no existing engineering index to specify them. By decomposing these objectives, we might reach a level at which the sub-objectives are easily measurable (Fig.(4-1)).

The objectives should also be comprehensive, measurable and, where possible, quantitative. In order to have a meaningful decision-making system, the selected objectives should not overlap. The overlapping will cause the reappearance of one or more objectives in different forms, thus creating a false objectives set (a set in which the objectives are not independent).

The objective decomposition process eliminates most of the overlapping and substantially reduces the number of qualitative objectives/sub-objectives by incorporating them into design. For example, user comfort, as an objective, is very difficult to assess. By decomposing, however, we are able to incorporate it into the design process (e.g. user comfort is decomposed into thermal comfort, acoustical comfort, illumination comfort, and environmental comfort (Fig.(4-5)). The thermal comfort component is further decomposed into summer and winter conditions (Fig. (4-6)), that can be incorporated into the design process. In a project, the level of detail which can be reached increases as the project progresses. New objectives are introduced to the system and, at the same time, some of the objectives leave the system. This process is depicted in Figures (4-2,3,4,5,6) (in Figs.(4-4,5,6), only one component is expanded). It has been observed that some of the sub-objectives can be re-arranged under other objectives, for example, code can be implemented in design feasibility, user cost and productivity can be included in the economic feasibility, and the thermal comfort may be included in the energy system design process.

An Objective Breakdown Structure (OBS) can be developed and along with Work Breakdown Structure (WBS) implemented in a computerized package. The process of objective decomposition can be standardized for different types of buildings (residential, commercial, industrial, etc.). The

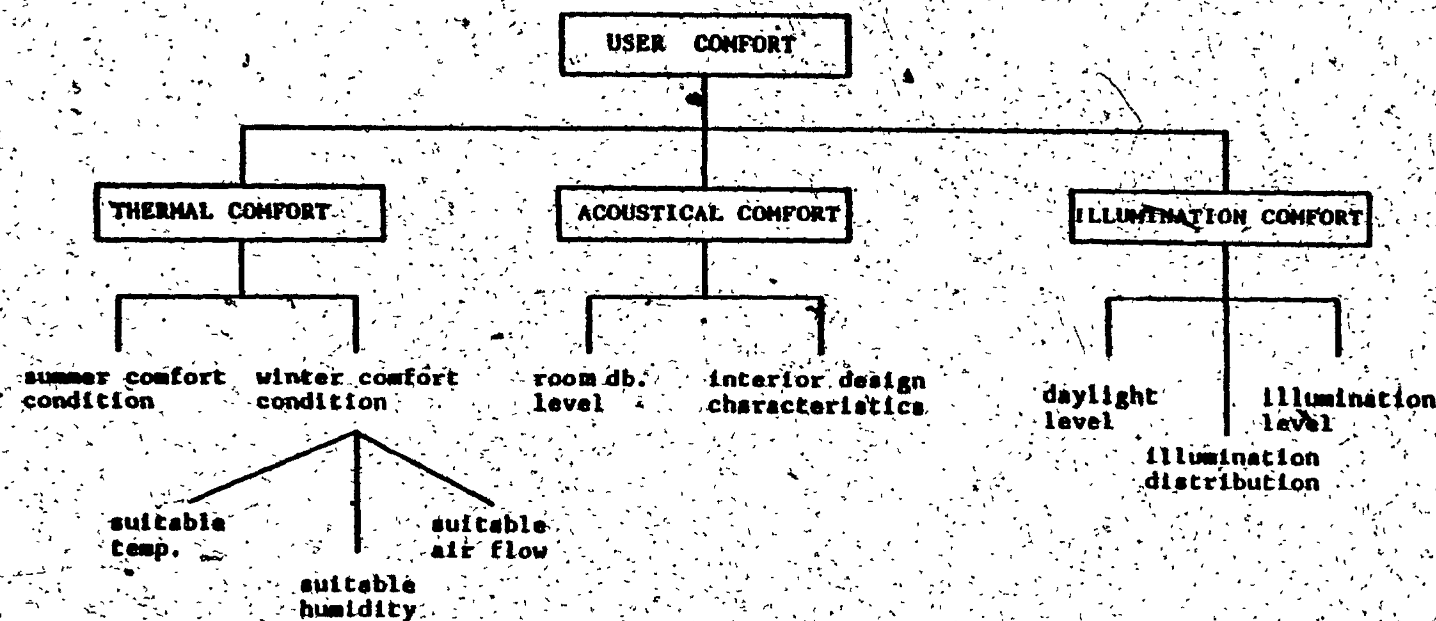


FIG. (4-1) - Decomposition of user objectives (user comfort).

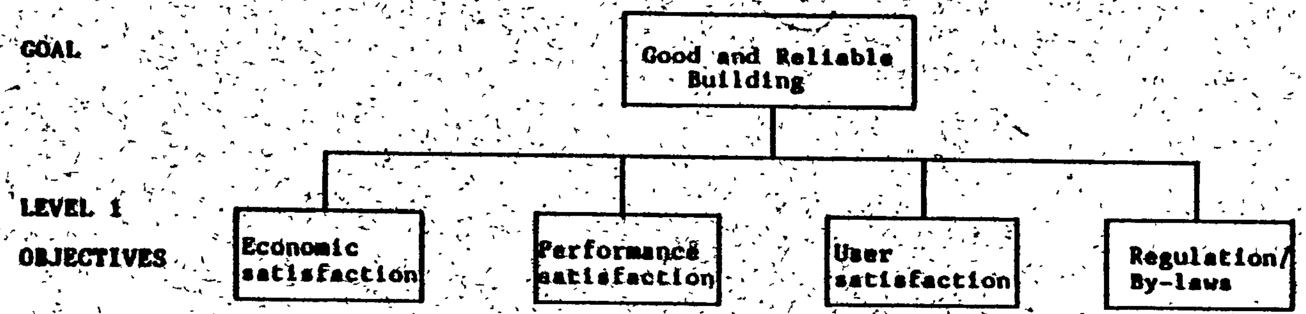


FIG. (4-2) - Design goal and level 1 objectives.

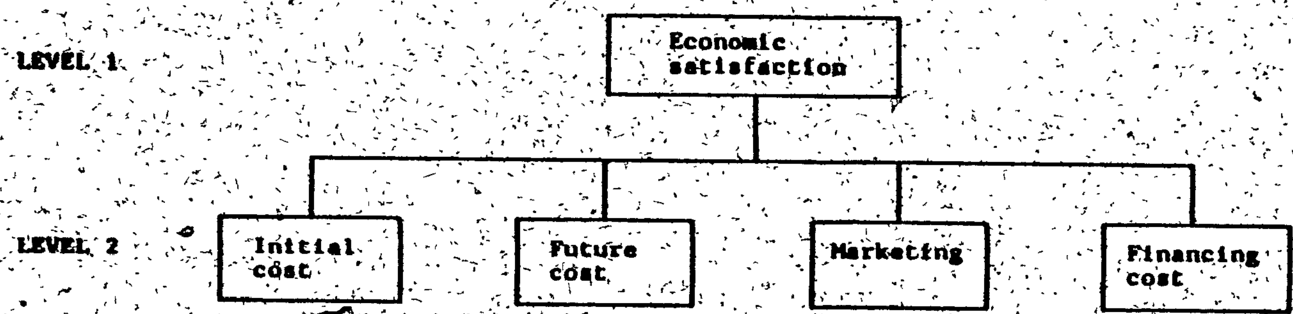


FIG. (4-3) - Level 1 and level 2 objectives.

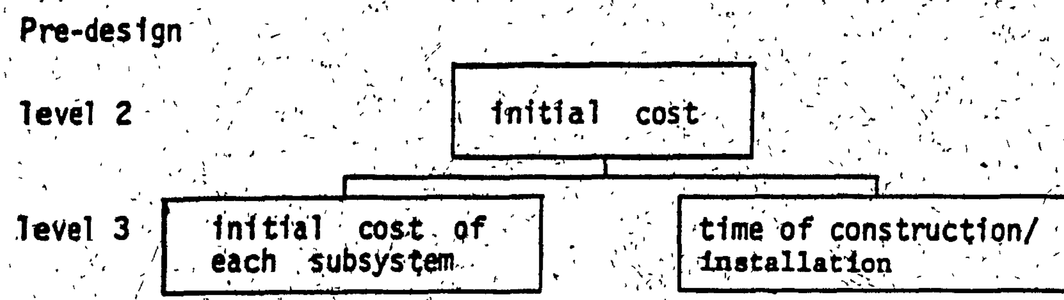


FIG. (4-4) - Level 2 and level 3 objectives for initial cost.

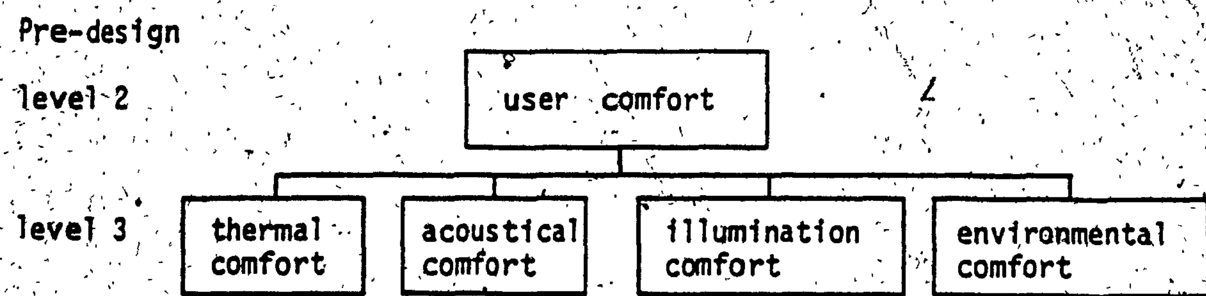


FIG. (4-5) - Level 2 and level 3 objectives for user comfort.

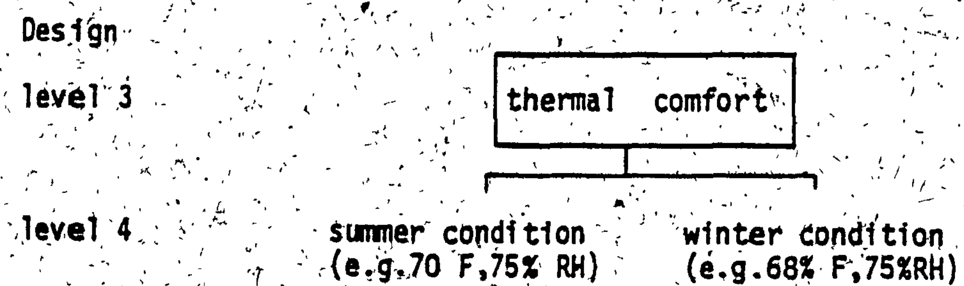


FIG. (4-6) - Level 3 and level 4 objectives for thermal comfort.

investor (or decision maker) should be allowed to refine the OBS for a particular investment (the same process is required for the WBS, where the selection of system/ subsystem/ component is required). Here, WBS requires type one information and OBS requires type two information.

4.1.4 - Performance Measures (Criteria)

Suitability of an objective depends not only on the above mentioned factors but also on the criterion which measures this objective. In Figure (2-1), an objectives (motivations)/ criteria matrix was presented, in which relationships between different objectives and criteria are explored. This matrix can be a helpful tool in proper assessment of an objective by selecting an appropriate criterion.

4.2- Design Decision Optimization Model

Design decision optimization models were classified in Chapter I into four categories (Fig. (1-2)), i) single criterion/deterministic, ii) single criterion/probabilistic, iii) multiple criteria/deterministic, and iv) multiple criteria/probabilistic. In the 1970's, attention was directed primarily to the category "i" models, criteria such as capital cost (32), payback period (32), IRR (33) and NPV (33). The growing uncertainty in the prediction of the economic environment and the sharp changes in energy price trends drew attention to probabilistic analysis and several methodologies were introduced (23,68).

Today, decision makers in the construction industry are confronted with complex design solutions, the number of which have sharply increased. These solutions have their strengths and weaknesses and design optimization based on only one characteristic is not representative of the building design decision making process utilized by practicing professionals. Design decisions are influenced by several individuals (investor, designer, builder) with their respective objectives, These objectives cannot be represented by a single evaluation criteria. The application of multiple-criteria decision making models for building design problems has been the subject of several research works during the past few years (18 ,20 ,69). In these studies,emphasis has been placed more on the technical aspects of the building design problem (e.g. design of the exterior walls), where all variables are assumed to be deterministic (in some cases the capital cost as a deterministic variable is also included). We believe that a realistic decision-making model should be capable of treating several objectives/criteria where the uncertainty in estimation of parameters is incorporated within this methodology. The model should provide the designer/ architect with enough information about the performance of design alternatives with respect to each criterion and the overall criteria set and should suggest a superior solution at each stage of the analysis (instead of leaving the architect with a multi-dimensional trade-off analysis (18)).

The decision maker should be allowed, at each stage, to examine the information (e.g. to perform trade-off analysis) and influence input to further stages.

4.2.1- Multiple Criteria Design Decision-Making Methodology (MCDDMM)

MCDDMM refers to the making of decisions in the presence of more than one criterion, usually conflicting with each other. It is the most realistic (if not, in certain cases, more practical), way of treating a problem. For example, in making the decision about enclosure design of a building, although capital cost minimization, energy efficiency, maximum comfort, and the building appearance may be considered, capital cost minimization may conflict with the other three objectives.

The multiple-criteria design decision making approach has received a great deal of attention in the past two decades (69). Researchers in different fields have developed methodologies for treatment of multiple-criteria problems. For example, in the fields of operations research we have maximin (70) and utility theory (71)); in economics, pareto optimality (28) and welfare program design; and in statistics, factor analysis (72) and multivariate regression analysis (73). Basically, efforts have been made in rationalizing the problem, predicting the behavior of the decision-maker, and suggesting a systematic approach to the problem. In the case of building design, the problem has unique characteristics such as the diversity of objectives, the existence of more than one decision maker (investor, designer, user), difficulties in extracting information, and the nature of performance measures (mostly overlap each other) used for evaluation of the objectives. With regards to these characteristics, some of the above mentioned methods can be applied to the problems related to building design. For example, the Pareto optimality approach is used to determine the non-inferior design alternatives; the additive/ multiplicative utility function is used to treat multiple-criteria design problems; factor analysis may be helpful in treating the correlation problem in cost modeling (the theoretical developments are well documented (72), but its application in cost modeling has not been explored); and multivariate regression analysis can be used in developing constant dollar capital cost models. In the past, attempts have been made in developing multiple-criteria design decision-making models (20,74). Assumptions have been made (implicitly or explicitly), about the decision maker and the nature of the objectives/ criteria set (independent objectives / criteria) (18, 20, 75). In reality, the design decision-making problem includes diverse, often conflicting objectives and/or criteria (financial, technical and user). In this research work, an attempt is made to develop a methodology that takes into account the above mentioned characteristics.

Different steps envisaged in this methodology are:

1. to specify the overall design goal (the design goal

is established by architect/ engineer, based on information provided by the investor);

2. to define the design goal in more detail using different objectives (a detailed discussion of this step was presented in section 4.1.3);

3. to separate the objectives of prime importance from other objectives (selection of prime objectives will reduce the size of the problem. At this step the input of the investor is necessary);

4. to decompose the objectives into a level which is more specific, easy to define and measure (sec. 4.1.3);

5. to study the interrelationships between objectives, determine common grounds, and rank them in their order of preference (this has not been studied ingreat detail, however, it will be applied in the case study);

6. to identify possible criteria that can measure these objectives (criteria selection process is presented in chapter II, (Fig. (2-1));

7. to study the interrelationships between selected criteria and eliminate those that are included within the others (a criterion can be eliminated if and only if it is explicitly represented by another criterion (e.g. NPV and IRR), in the case of implicit representation (e.g. LCC, capital cost), the criterion should not be eliminated);

8. to identify available design alternatives;
9. to assign weightings to each objective, and using objectives (motivations)/ criteria matrix (Fig. (2-1)), compute weightings for each criterion (the procedure of calculating weightings will be presented in section 4.3.2.2); and
10. based on the projected performance, and using a design alternative selection methodology, select the best design alternative (sec. 4.4).

In continuation of this chapter, the main focus will be on the steps 9 and 10.

4.3- Optimization Approaches

There are two overall approaches;

1) examining all available alternatives using one criterion at each stage, and
2) examining all available alternatives and selected criteria within a system.

4.3.1- Examining All Available Alternatives Using One Criterion at Each Stage.

This approach is frequently used in design decision making. It simplifies the multiple criteria model to a multiple-stage/single-criterion model (e.g. first, all alternatives are examined ,using the most important criterion (stage 1). If the best solution is not found, the process continues in the second stage). These methods may be useful in the preliminary screening of the alternatives by considering the existing set of constraints.

Different methods in this approach are;

i) Lexico Graphic Method (76):

This method requires the ranking of criteria in their order of importance. The best alternative is the one with the highest performance with respect to the most important criterion. For instance, take alternatives A1, A2,...,An, criteria set C1,C2,...,Cm, and performance levels of X11,X12,...Xij,...Xnm (Xij is the performance of alternative i, with respect to criteria j).

Stage 1

Consider the most important criterion, C1;

$$A^1 = (Ai \mid \max \ Xi1 \ , i=1,....n) \qquad (4-1)$$

A^1 is the set of alternatives with the maximum level of performance. If the set A^1 has only one element, then it would be the most preferred alternative. Otherwise, the process is continued in stage 2.

Stage 2

Consider the second most important criterion, C2;

$$A^2 = (A^1 \mid \max \ Xi2 \ , i=1,....n) \qquad (4-2)$$

Stage 3 $\longrightarrow$ m

The above process is continued until some A^k

(k=1,...m), containing only one element is found, or all criteria have been considered, in which case, the remaining alternatives are considered to be equally desirable.

ii) Lexico Graphic Semi-Order Method (76):

This method is quite similar to the Lexico Graphic method. The only difference is in assuming some significance range, so as not to judge the alternatives based on slight differences. Here, for those alternatives which have close performance levels, a second criterion is also considered.

iii) Elimination by Aspect (77):

The basic notion in Elimination by Aspect is to eliminate all those alternatives whose performance levels are below our expectations (cut-off point). This method may be used in the preliminary screening stage where constraints such as the maximum amount of equity available, code constraints, and environmental conditions are to be considered. The selection process is as follows;

Stage 1

First, the criteria set should be arranged in their order of importance. Then, considering the most important criterion, all alternatives with levels of performance below the cut-off point are eliminated.

$$A^1 = (Ai | Xi1 >= X1\ min, i=1,....n) \quad (4-3)$$

Where "X1 min" is the cut off point for criterion C1.

A^1 is the set of feasible alternatives remaining after the consideration of the first criterion. In this analysis, there are three possible outcomes:

a) after considering all criteria, only one element is left in set A^m. This alternative is considered to be the optimal and feasible alternative.

b) after considering all criteria, there remains more than one element in set A^m. These alternatives are considered to be feasible and equal.

c) before having considered all the criteria, A^j ($j<m$), is left with only one element. This alternative is therefore considered to be the most preferred alternative, although ~~not~~ necessarily feasible.

Problems:

i) These methods consider the weightings as the primary (most important) variable in the selection process and ignore the magnitude of the criteria performance of different alternatives.

ii) Trade-off analyses are not performed. Thus, there is a possibility of eliminating the most attractive alternative based on its performance with respect to the first two or three criteria.

Positive Points:

Easily applicable; weighting parameters are not required; and the transformation of the units is unnecessary.

Example:

To illustrate the application of the above methods, four design alternatives are considered. The selected evaluation criteria are: capital cost, operating cost, and the comfort condition in the building. The performance of each alternative with respect to three criteria is shown in the tableau below.

Alt.	capital cost	operating cost	comfort condition
1	\$100,000.	\$5,000./ year	medium
2	\$100,000.	\$5,500./ year	good
3	\$110,000.	\$4,000./ year	bad
4	\$120,000.	\$4,000./ year	very good

Note: In the above tableau, criteria are arranged in their order of importance.

i) Lexico Graphic method

Based on the most important criterion (capital cost) the set of optimum solutions is:

$A^1 = (1,2)$

Using set A^1 and based on the operating cost;

$A^2 = (1)$

Alternative 1 is the optimum solution.

ii) Lexico Graphic Semi-Order method

If the following significance ranges are specified for capital cost and operating cost (basic information is supplied by investor, this information is translated into evaluation criteria);

capital cost $\pm$ \$10,000.

operating cost $\pm$ \$500.

Then, based on the most important criterion(capital cost) the set of optimum solutions is:

$A^1 = (1,2,3)$

Using set A^1 and based on operating cost (significance range of $\pm$\$500), the optimum set becomes;

$A^2 = (3)$

Alternative 3, which is more expensive and the comfort condition is bad, is selected as the optimum solution.

iii) Elimination by Aspect.

If the following cut-off points are specified for this analysis;

$$\text{Capital cost} \leqslant \$120{,}000.$$
$$\text{operating cost} \leqslant \$\ 5{,}500.$$
$$\text{comfort condition} \geqslant \text{medium}$$

Then the first set of optimum solutions with respect to the most important criterion, capital cost, is:

$$A^1 = (1,2,3,4)$$

Based on the operating cost, the optimum set becomes;

$$A^2 = (1,3,4)$$

And based on the comfort condition we have;

$$A^3 = (1,4)$$

Using the Elimination by Aspect method, alternatives 1 and 4 are both considered as optimum solutions.

4.3.2- Examining All Available Alternatives and Selected Criteria Within a System.

In this approach, all design alternatives and the selected criteria set are simultaneously considered (76). Here, in order to be effective, a multi-stage analysis is suggested. It is possible to find an optimal/ feasible alternative at any of these stages. In this approach, utility theory is used i) to reflect the attitude of the probabilistic characteristics of the performance measures in the analysis (using expected utility), and iii) to express the multiple-attribute building design decision-making problem as a single utility function (multiplicative utility function). Prior to stating the different stages in this approach, several topics should be studied.

4.3.2.1- Utility Theory

The basis of the utility theory is built on the preference function. This function defines the attitudes of the decision-maker in different situations (financial, economic environment, uncertainty, etc.). The utility function may be defined with respect to parameters such as age, psychological make-up, attitude towards risk, financial status, etc., through suitable questioning (78). According to the theory, each individual has a measurable preference among various choices in a risk situation. The fundamental task in the utility approach is to get a mapping between criteria value and utility. For example, if a decision maker has two options and if there are "n" consequences "X1,X2,....Xi,.....Xn" (in sequence of preference) associated with each option with probabilities of occurrences P1i and P2i (i=1,...,n) ,then in a risky situation, consequences are replaced by their certainty equivalent, in terms of the best consequence (Xn), the worst cosequence (X1), and a probability value (πi) which are asigned by the decision maker.

$$\begin{tikzcd} & X1 \\ E(Xi) \arrow[ur, dash, "(1-\pi i)"] \arrow[dr, dash, "\pi i"'] & \\ & Xn \end{tikzcd}$$

$$E(Xi) = \pi i * Xn + (1-\pi i) * X1 \qquad (4\text{-}4)$$

Preference of a rational decision maker, also indicates that $\pi 1 < \pi 2 < \pi 3 \ldots\ldots < \pi n$;

In utility theory, consequences of a decision $(X1, X2, \ldots, Xn)$ are replaced by " $\pi 1, \pi 2, \pi 3, \ldots \pi n$ " (scaled values of $X1, X2, \ldots Xn$). The utility function is a positive transformation of πi $(i=1,2,\ldots n)$ values. For example, the utility function for Xi can be expressed as:

$$U(Xi) = a + b\ \pi i \qquad (4\text{-}5)$$

Then, utility function for option 1 becomes:

$$\bar{U}(\text{option } 1) = \sum_i P1i \cdot U(Xi) = \sum_i P1i\ ((1-\pi i) \cdot U(X1) + \pi i \cdot U(Xn))$$

and for option 2, it is:

$$\bar{U}(\text{option } 2) = \sum_i P2i \cdot U(Xi) = \sum_i P2i\ ((1-\pi i) \cdot U(X1) + \pi i \cdot U(Xn))$$

The main question here is whether there is an appropriate procedure for deriving a utility curve (function); what is the information required and who provides this information; and whether it is applicable to decision-making problems related to building design.

In practice, determining a utility function relating evaluation criteria to utilities is not easy. The problem is the measurement of the utility of investor. In most cases investors are not familiar with evaluation criteria and are unable to state their preferences appropriately. The only remaining source of information is the decision maker who may be able to translate investor's objectives (or his preferences), but he does not have the same attitude towards risk. Difficulties associated with determining the utility function and ways of overcoming some of these difficulties are documented in the literature (56,71,78).

In the building design problem, utility functions are developed for each criterion and, accordingly, the performance levels are transformed into a single unit (utiles). The determination of the utility function largely depends on; i)the objectives set (how explicit are objectives) and ii) the criteria set (how well a selected criterion measures the objective). The decomposition process, suggested in this chapter, enables us to identify the objectives in a more explicit manner ,and to assign a comprehensive and measurable set of criteria.

The utility function can take almost any form. The amount and level of detailed information that is required for different utility functions vary. Selection of a utility function depends on the type of investment, information required and information available, and required level of accuracy. Here, for illustrative purposes, we define two types that suit our purpose, which is to be able to incorporate the developments of previous chapters in the expected utility function.

1) Polynomial utility function.

The polynomial utility function, in its most general form, can be written as;

$$U(x) = Ao + A_1 X + A_2 X^2 + A_3 X^3 + \ldots\ldots \qquad (4-6)$$

To incorporate both the uncertainty involved in the estimation of the variable X, and the attitude of the decision-maker towards risk, we calculate the expected value of the utility function. The expected utility function of variable X, in general terms, is:

$$E[U(X)] = \int U(X) \ f_x(X)\, dx \qquad (4-7)$$

where

$f_x(X)$ = frequency distribution of variable X

Here, the moments approach is used for the evaluation of the expected value of X. Applying the expectation operator directly to equation (4-6), we get;

$$E(U(X)) = A_0 + A_1\bar{X} + A_2(\bar{X}^2 + \sigma_x^2) + 3A_3(\bar{X}\ \sigma_x^2 + \bar{X}^3\ C_3(X)) \ldots \qquad (4-8)$$

where

X =mean value of variable X

σ_x^2 =variance of variable X

$C_3(X)$ =3rd moment of variable X

Although inclusion of the higher order components contributes to the accuracy of the function, it is neither practical nor realistic (79) because higher order moments do not have a physical meaning and cannot be estimated in practice.

Another way of studying the investor's attitude is to derive the risk coefficient function (71).

$$r(x) = -U''(X) / U'(X) \qquad (4\text{-}9)$$

where

$r(x)$=risk coeff. function

$U'(X)$=first derivative of utility function

$U''(X)$=second derivative of utility function

In equation (4-9), the second derivative (U'') is the main indicator of the shape of the utility curve and consequently of the attitude of investor. If U'' is +ve (within a specific range), the utility curve is convex and the investor is risk-prone. If U'' is -ve, the utility curve is concave and investor is risk-averse.

For the polynomial utility function, the risk coefficient function can be written as;

$$r(x) = -(2\,A_2 + 6\,A_3 X + \ldots) / (A_1 + 2\,A_2 X + 3\,A_3 X^2 + \ldots) \qquad (4\text{-}10)$$

If the utility function is a quadratic function;

$$U(x) = X + A_2\,X^2 \qquad (4\text{-}11)$$

Then,

$$E(U(x)) = \bar{X} + A_2\ (\sigma_x^2 + \bar{X}^2) \tag{4-12}$$

and

$$r(x) = -2\,A_2\ /(1+2\,A_2\,X\,) \tag{4-13}$$

Using equation (4-13), the attitude of the decision maker towards risk can be summarized as:

	$X>=0$	$X=<0$
risk-averse	$r(x)>0$ or $A_2>0$	$r(x)<0$ or $A_2<0$
indifferent	$r(x)=0$ or $A_2=0$	$r(x)=0$ or $A_2=0$
risk-prone	$r(x)<0$ or $A_2<0$	$r(x)>0$ or $A_2>0$

These results are depicted in Figure(4-7), assuming that the coefficient of variation is .2. The relationship between the values of X and A_2 have been studied. It has been shown that the characteristics of the risk coefficient function change as the value of X approaches "$1/(2A_2)$" (Fig. (4-8)).

The utility and the risk coefficient profiles demonstrate the necessity for defining the limits within which a particular utility function can be applied. For example, for a rational decision maker, a second order polynomial utility function is applicable where "$-1/(2A) < X < 1/(2A)$". In fact, the polynomial utility functions are local utility functions that best describe the attitude of the decision-maker within a limited range. Using the risk coefficient, the attitude

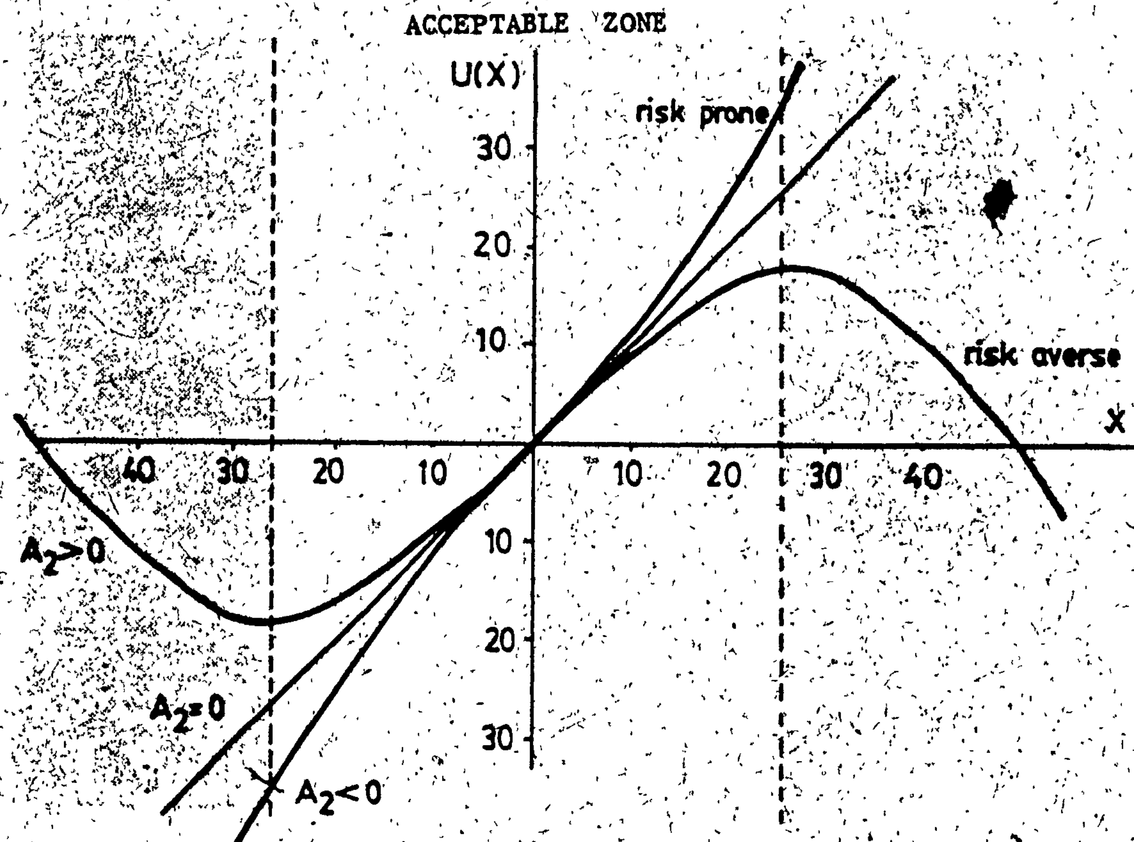


FIG. (4-7) - Utility curves for various attitudes, (polynomial utility function).

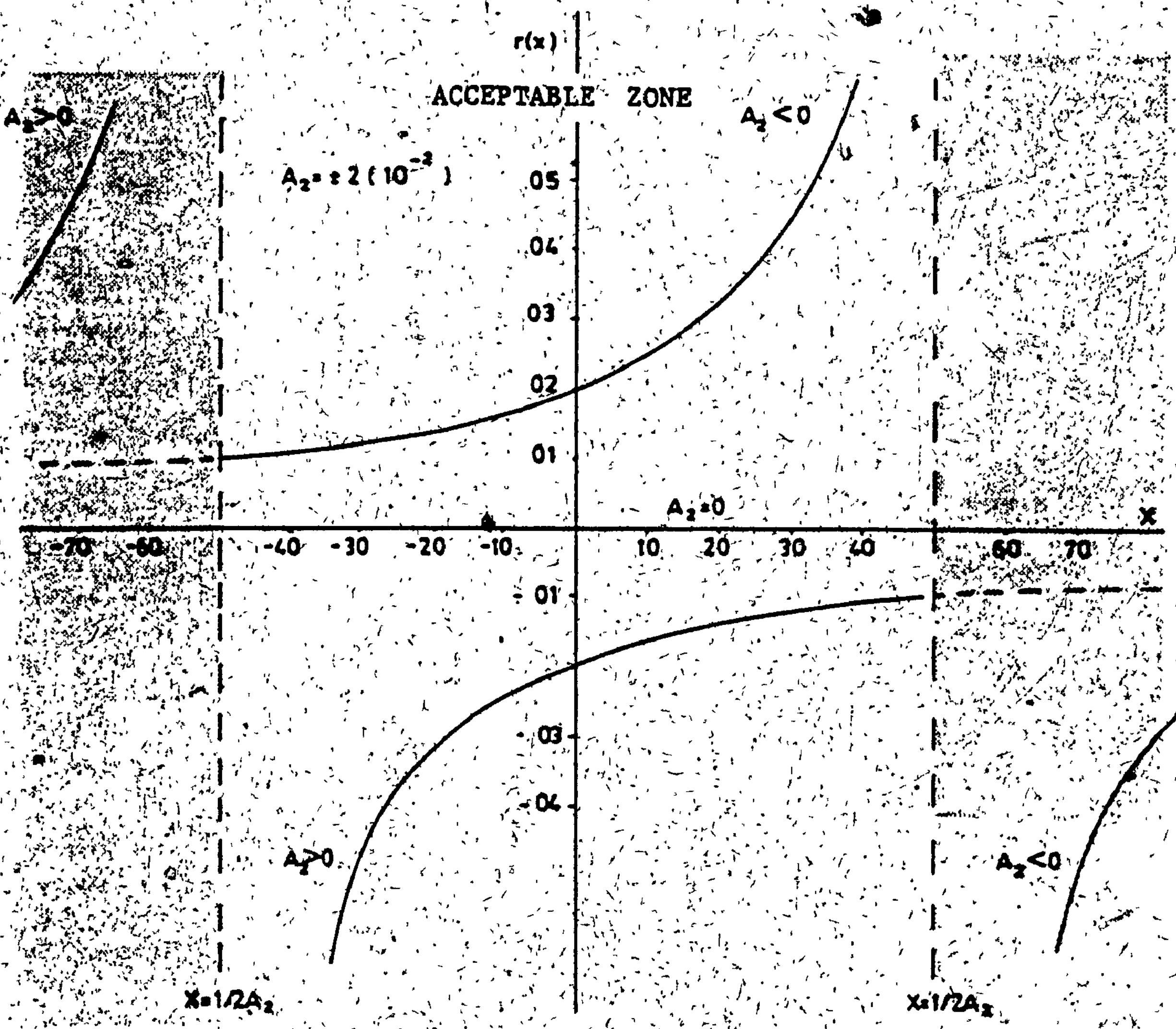


FIG (4-8) - Characteristics of risk coefficient function for polynomial utility function.

of a <u>rational</u> decision-maker towards risk is specified as;

risk-averse	$r(x) > 0$
indifferent	$r(x) = 0$
risk-prone	$r(x) < 0$

2- Exponential utility function.

The exponential utility function may be written as;

$$U(x) = a + b\, e^{CX} \qquad (4\text{-}14)$$

where

$$a, b, C = \text{constants}$$

For this function, the risk coefficient function is

$$r(x) = -(b\, C^2 e^{CX}) \,/\, (b\, C\, e^{CX}) = -C \qquad (4\text{-}15)$$

Where C is the risk coefficient. Attitudes of a decision-maker towards risk can be summarized as;

if	$C<0$	$r(x)>0$	risk-averse
if	$C=0$	$r(x)=0$	indifferent
if	$C>0$	$r(x)<0$	risk-prone

Here, r(x) is constant and represents a uniform attitude towards risk, while the use of polynomial functions as local utility functions implys the changes in the decision maker's attitude.

To derive the expected utility function, equation (4-14) is expanded using Maclaurin's series;

$$U(x)=a+b\ (1+CX+(C^2\ X^2/2)+........) \qquad (4\text{-}16)$$

The expected utility function then becomes,

$$E(U(x))=a+b(1+C\overline{X}\ +(C^2/2)*(\overline{X}^2+\sigma_x^2\ )+.....) \qquad (4\text{-}17)$$

For problems related to finance (80), the exponential utility function has been used in several formats. One of these functions is;

$$U(x) = 1/s * (1-e^{-s\ X}\ ) \qquad (4\text{-}18)$$

The expected utility function for the above equation becomes,

$$E(U(x))= \overline{X} - 1/2 * s * (\overline{X}^2 + \sigma_x^2\ ) \qquad (4\text{-}19)$$

or

$$E(U(X))= \overline{X} - 1/2 * s * \overline{X}^2 * (1+Vx^2) \qquad (4\text{-}20)$$

where

Vx = coefficient of variation
s = coefficient of risk aversion

In deriving the expected utility functions, we have only used the first two moments. These moments may be obtained from the underlying distribution of the variable "X" (e.g. Normal, Beta, ...), or calculated by the moments approach (Chapter II). For the case where information about higher order moments are available and a higher degree of accuracy is needed, the additional terms can be included in both polynomial and exponential utility functions (eqs. 4-6,17).

4.3.2.2- Criteria Weighting Parameter

Determining the weightings in multiple-criteria decision making is a subject that has been touched upon by almost every researcher in this field. It is imperative to consider the different roles of the investor and of the design decision maker as well as the weightings assigned to the objectives by the investor and to the criteria by the designer. This is especially true in the building construction industry. It is obvious that not all investors are design decision makers or familiar with the decision-making process. They are, nonetheless, able to state their preferences (explicitly or implicitly) and/or assign weightings to their objectives. Thus, it is proposed that the investor be the prime source of determining the weighting for each criterion. Then, using the objectives (motivations)/criteria matrix (Fig.(2-1)) the designer identifies an appropriate set of criteria and based on the objective/ criteria relationships he calculates weightings for each criterion. By this method, the subjectivity of the weightings is substantially reduced.

The weightings are computed by:

$$W_j = \sum_{k=1}^{nn} O_k * V_{Kj} \qquad (4\text{-}21)$$

where

W_j =computed weighting for criterion j

O_k =weighting assigned to objective k

V_{kj} =weighting which can be obtained from objectives (motivations)/criteria matrix (see Fig. (2-1))

nn = number of objectives

In the above relationship, O_k's are weightings that are assigned to different objectives by the investor, and V_{kj}'s are weightings that can be obtained from the objective/ criteria matrix (Fig. (2-1)). In Figure (2-1), only two notations are used; "●" for explicit relationship between an objective and a criterion and "Θ" for implicit relationship between an objective and a criterion. These notations can be refined to demonstrate different relationships. For example, "●" where a criterion explicitly defines an objective (e.g. relationship between initial cost objective and capital cost criterion); "Θ" where a criterion implicitly defines an objective (e.g. relationship between initial cost objective and LCC criterion);and "O" where there is weak link between an objective and a criterion (e.g. the relationship between building flexibility and capital cost). We can also develop a more objective objective/criteria matrix in which the objective/ criteria relationships are numerically evaluated. Here, the system of three notations (as specified above) is used.

4.4- Optimization Process (dominance theory)

The design optimization (alternative selection) methodology developed in this research work employs concepts from both approaches in section 4.3 (dominance and utility theories). It consists of four stages, the first two stages require type one information (some basic information about budgetary constraints and technical performance such as comfort conditions that are supplied by the investor), and the last two stages require type two information (objectives' weightings, utility functions). In this methodology, the decision-maker can terminate the process at any stage and use the information for selection of design alternative(s). It is up to the decision-maker to decide based on the information available how far the process should continue. Another feature of this methodology is the use of a multiplicative utility function that treats the overlappings in the criteria set.

Dominance theory was originally developed (81) to assist the decision maker when encountering the possibility of several outcomes (states of nature). Probabilities are assigned to each possible outcome and the best solution is selected based on its expected performance value. Using this concept, we propose the following methodology.

Consider the following alternative / criteria relationship matrix:

criteria

alternative	C1 W1	C2 W2	C3 W3	....	Cm Wm
A1	X11	X12	....		X1m
A2	X21	X22	....		X2m
.	....				
.	....				
.					
An	Xn1	Xn2	....		Xnm

where

A1,.......,An=available alternatives

C1,.......,Cm=selected set of criteria (in descending order)

W1,.......,Wm=weightings computed for criteria 1,...,m

Xij=performance of alternative i using criterion j (original units)

a) Preliminary screening process.

At this stage, one performs a constraint analysis using the Elimination by Aspect method. First, the upper and lower limits for each criterion are established (these limits include code constraints, financial constraints, and user requirements). Then, all alternatives whose performance level lies outside the established limits are eliminated from the alternatives set. In all stages of the decision making process the risk adjusted values of the performance levels will be used.

$$A^1 = (Ai's \,/ Xmin < xij < Xmax, i=1,..n), j=1,..m \qquad (4\text{-}22)$$

where A^1 is the set of feasible alternatives after preliminary analysis.

b) First degree dominance (domination by all aspects)

In first degree dominance, neither weightings nor the transformation of units is required. An alternative is eliminated if, and only if, it is dominated in all criteria (using risk adjusted values) by another alternative.

$$A_k \text{ is preferred to } A_l \text{ if } X_{kj} \geqslant X_{lj}, \text{ for } j=1,...m \qquad (4\text{-}23)$$

The concepts of Efficient Frontier (23), Decision by Exclusion (20), or Pareto Optimality (69) can be used at this stage to determine the non-inferior alternatives set.

$$A^2 = (\text{ set of non dominated alternatives }) \qquad (4\text{-}24)$$

c) Second degree dominance

The aim here is to study the overall performance profile of each alternative and to determine the cumulative weighting level, up to which one alternative dominates another alternative (see Fig. (4-12,13)). At this stage, one must use consistent units (utiles, probability values, dollars, ...). Steps followed at this stage are:

1. Determine weightings for all criteria as proposed in section (4.3.2.2);
2. Normalize weightings using the objectives (motivations)/ criteria matrix. For example, consider the following objectives/ criteria relationships:

Objectives	O_k	Criteria 1	2	3	4
1	.3	●	⊖	⊖	
2	.2		⊖	⊖	⊖
3	.25		O		⊖
4	.25			●	
Wj		.2997	.3125	.500	.225

* V(●)=.9999, V(⊖)=.5, V(O)=.25, V( )=0

The normalized weightings are calculated by normalizing values of V (e.g. V(●),V(⊖),...) for each objective. The numerical representation of the above tableau and the normalized values for objective one are depicted in the following tableau:

	Criteria 1	2	3	4
original	.9999	.5000	.5000	0
normalized	.4999	.2500	.2500	0

This process is repeated for each objective, then using equation (4-21), the normalized weightings are calculated for each criteria.

3. Re-arrange the criteria set in their order of importance;
4. Transform all performance values into a consistent unit (using appropriate expected utility functions and normalizing the expected utility values);
5. Compute expected cumulative utilities (probabilities) ($\bar{U}(X1)$, $\bar{U}(X1)+\bar{U}(X2)$, $\bar{U}(X1)+\bar{U}(X2)+\bar{U}(X3)$,.....).
6. An alternative dominates the others if all its partial cumulative utilities are higher than those of others. This can be better observed by drawing the cumulative utility diagram (Figs. (4-9,10)).

Studying these graphs also results in the selection of an alternative which dominates the others up to a certain point that is satisfactory to the decision maker (e.g. 95% ($\Sigma Wj=.95$)). In this case, one alternative satisfies up to 95% of investor's objectives better than other alternatives).

The optimization process ends if an alternative is found to dominate the others (set A has only one element), otherwise the next step is followed.

d) Third degree dominance

The third degree dominance is used where there are

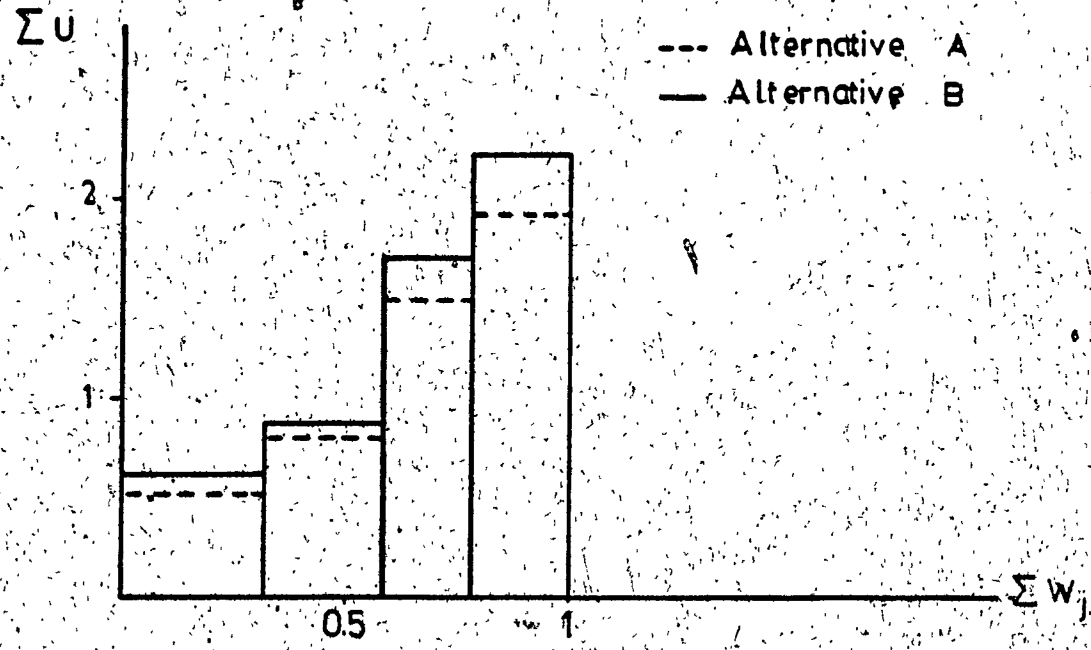


FIG. (4- 9) - Cumulative utility diagrams for two alternatives (B dominates A).

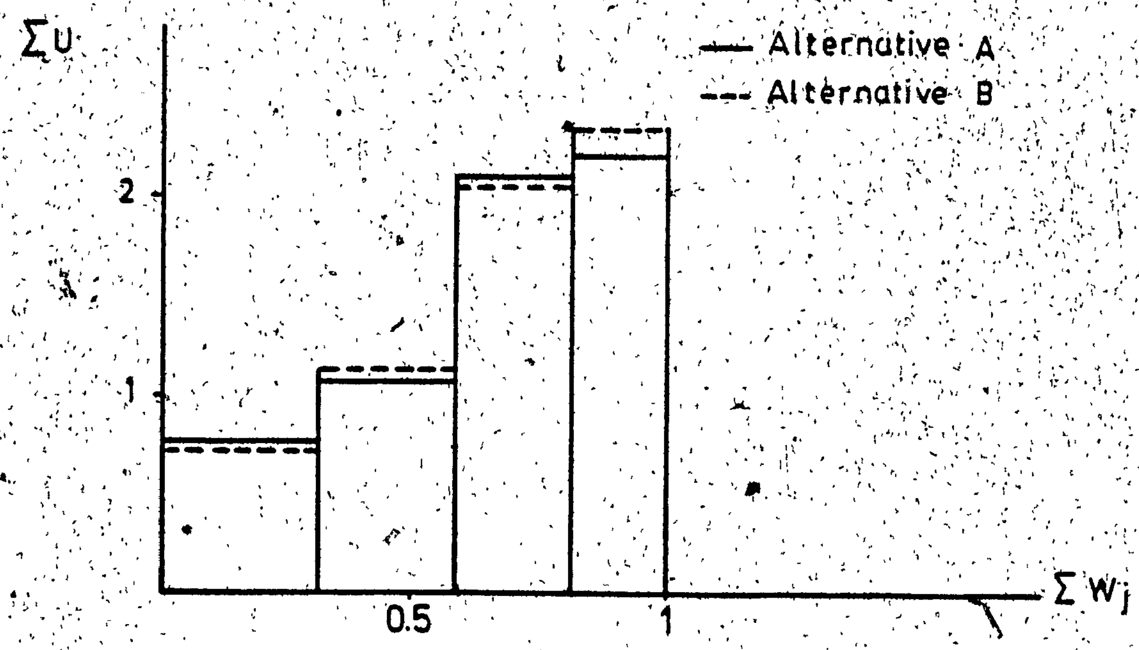


FIG. (4-10) - Cumulative utility diagrams for two alternatives (B does not dominate A).

cross-overs in the cumulative utility diagrams Fig (4-14). In this case, an overall utility function is used for the selection purpose. Basically, the overall utility function is a multi-attribute utility function;

$$U(X_1, X_2, \ldots X_n) = f(U(X_1), U(X_2) \ldots, U(X_n), W1, W2, \ldots Wn) \quad (4\text{-}25)$$

where;

$U(X_1, X_2, \ldots X_n)$ = overall utility function

$U(X_j)$ = utility function for variable X_j

f = a function (additive, multiplicative)

Wj = weighting parameter for variable j (W_j values are not normalized)

The fundamental assumption in constructing a multi-attribute utility function is the "independence condition" (71). The strongest independence condition is that the variables should be mutually utility independent. This implies that the change in the utility function is directly proportional to changes in the variable, with all other variable values kept constant . This condition may not hold for problems related to building design. However, to construct a multiplicative utility function that takes into account the overlappings between different attributes, one needs a combination of preferential independence and utility independence (71). A set of attributes (criteria) are preferentially independent if and only if a subset of attributes can be found such that preferences in this subset are independent of those in the other subsets. The problems related to building design have such characteristics (e.g. consider capital cost and payback period, for two alternatives, the alternative with lower capital cost and constant payback period is preferred regardless of the magnitude of the payback period) . In the past, the additive utility function has been used for computation of the overall utility function, which is not capable of treating the overlaps in the criteria set (20). Here, a multiplicative utility function is developed that treats overlaps for each design objective. This function consists of two main parts: the first part is an additive utility function, and the second part which consists of several terms, measures effects of overlappings. For calculation of overlappings a scaling factor(K_k), which represents the objective/criteria relationships, is used. In this research work, scaling factors are calculated for each objective (in multiplicative utility function suggested by Keeny and Raiffa (71), the scaling factor is calculated at criteria level). If there are positive overlaps, terms in the second part become negative, and if there are negative overlaps (criteria set cannot properly define an objective), these terms become positive. If no overlaps exist, the utility function reduces to an additive one.

$$U_i(C_1,C_2,C_3,\ldots C_m) = \sum_{j=1}^{m}[W_j\, U(X_{ij})] +$$

$$\sum_{k=1}^{nn}[K_k\, O_k \sum_{j}^{m}\sum_{l>j}^{m}(V_{kj}V_{kl}\, U(X_{ij})U(X_{il}))] +$$

$$\sum_{k=1}^{nn}[K_k^2\, O_k \sum_{j}^{m}\sum_{l>j}^{m}\sum_{n>l}^{m}(V_{kj}V_{kl}V_{kn}U(X_{ij})$$

$$U(X_{il})\; U(X_{in}))] + \ldots\ldots \qquad (4\text{-}26)$$

where

$Ui(C1,C2,\ldots\ldots)$=overall utility value for alternative i

Wj=weighting computed for criterion j (Wj values are not normalized)

U(Xij)=utility equivalent of performance level Xij

O_k=weighting assigned to objective k

nn=number of investor objectives

V_{kj}=weighting obtained from objective/criteria matrix for criterion j, objective k

K_k=scaling factor for objective k

$$K_k = \prod_j (1+K_k\, V_{kj}) -1 \qquad (-1<K_k<\infty) \qquad (4\text{-}27)$$

In order to satisfy the independence condition, the limits of K_k are set to be "-1 and OO".This can also be observed by examining equation (4-27), where $0<V_{kj}<1$. This condition establishes a lower bound for K_k (the proof of selecting such limits is documented in Chapter 6 and appendix 6B of Ref. (71)).

The expression for K_k (eq. 4-27) may be written as;

$$f(K_k) = K_k + 1 - \prod_{j} (1+K_k V_{kj}) = 0 \qquad (4\text{-}28)$$

The derivative of the function f with respect to K_k becomes;

$$f'(K_k) = 1 - \sum_{jj} V_{kjj} \prod_{j \neq jj} (1+K_k V_{kj}) \qquad (4\text{-}29)$$

The derivative of function f, <u>within the stated limits</u>, is negative. The negative value indicates that the function f is a decreasing function and with respect to different values of " ΣV_{kj}" has only one root.

In all of the above expressions, it has been assumed that;

$$V_{kj} < 1 \quad \text{for} \quad j=1,2,\ldots\ldots m \qquad (4\text{-}30)$$

The value of $V_{kj}=1$ would indicate that a particular criterion defines an objective in an absolute manner. Such a case in reality (and theoretical sense) may not exist. For developments in this chapter the fundamental assumption is that V_{kj} is always less than one (for cases where there is an explicit relationship between an objective and a criterion, the value of V_{kj} is assumed to be .9999).

Since the polynomial (eq. 4-29) has many roots, The following guideline is used in selecting the K's ;

$$\text{if } \sum_j V_{kj} > 1 \text{ then } -1 < K_k < 0$$
$$\text{if } \sum_j V_{kj} = 1 \text{ then } K_k = 0$$
$$\text{if } \sum_j V_{kj} < 1 \text{ then } 0 < K_k < \infty$$

The value of K_k is negative if there are overlaps between the different criteria ($\sum_j V_{kj} > 1$). For the case where $\sum_j V_{kj} = 1$, the value of K_k becomes zero and the utility function reduces to an additive utility function. $\sum_j V_{kj} < 1$, demonstrates that the selected criteria are unable to properly measure the stated objectives ($K_k > 0$). Following, the proof of selecting limits for K_k is presented.

Proof:

Consider two criteria that evaluate the same objective, with objective/ criteria weightings of V_1 and V_2.

$$K = \prod_{j=1}^{2} (1+K\,V_j) - 1 \qquad (4\text{-}31)$$

$$1 = V_1 + V_2 + K\,(V_1 V_2) \qquad (4\text{-}32)$$

(since there is only one objective, the subscript "k" for variables "K" and "V" ia not shown)

i) If $V_1 + V_2 > 1$, then;

$$K\,\underbrace{(V_1 V_2)}_{+ve} < 0 \qquad (4\text{-}33)$$

$$K < 0 \tag{4-34}$$

and -1 is the lower limit.

ii) If $V_1 + V_2 = 1$; then;

$$K \underbrace{(V_1 \; V_2)}_{+ve} = 0 \tag{4-35}$$

$$k = 0 \tag{4-36}$$

iii) If $V_1 + V_2 < 1$, then;

$$K \underbrace{(V_1 \; V_2)}_{+ve} > 0 \tag{4-37}$$

$$K > 0 \tag{4-38}$$

and the upper limit is OO

The proof is extended to a case where there are 3 criteria evaluating the same objective,

$$K = \prod_{j=1}^{3} (1+K \; V_j) -1 \tag{4-39}$$

$$1 = (V_1+V_2+V_3) + K \; (V_1V_2+V_1V_2+V_2V_3 \;) + K^2 \; (V_1V_2V_3) \tag{4-40}$$

i) If $V_1+V_2+V_3 > 1$, then;

$$K \underbrace{(V_1V_2+V_1V_3+V_2V_3)}_{+ve} + K^2 \underbrace{(V_1V_2V_3)}_{+ve} < 0 \tag{4-41}$$

$$K < 0 \tag{4-42}$$

and the lower limit is -1.

ii) If $V_1+V_2+V_3 = 1$, then;

$$K(V_1V_2+V_1V_3+V_2V_3)+K^2\ (V_1V_2V_3) = 0 \qquad (4-43)$$

$$K = 0 \quad \& \quad K < -1 \qquad (4-44)$$

With regards to the stated limits, only $K = 0$ is acceptable.

iii) If $V_1+V_2+V_3 < 1$, then;

$$K(V_1V_2+V_1V_3+V_2V_3) + K^2\ (V_1V_2V_3) > 0 \qquad (4-45)$$

For the values of " $-1 < K < 0$", the above relationship does not hold. The only feasible region is " $0 < K < \infty$ ".

This proof can be extended for more than 3 criteria.

Example:

The aim of this example is to show the procedure of calculating values of scaling factors K_k (k=1,2,3,4). For illustrative purposes consider the following objective/ criteria matrix;

Objectives	O_k	Criteria 1	2	3	4	K_k
1	.3	●	○	○		-.999
2	.2		○	○	○	-.763
3	.25		O		○	2.0
4	.25			●		0.0
Wj		.2997	.3125	.500	.225	

where $V(\bullet)=.9999$, $V(\ominus)=.5$, $V(\circ)=.25$, $V(\ )=0$

In the above tableau, first, the objective set and related weightings (O_k) are elicited from the investor. Then, a set of criteria (1,2,3,4) that measure these objectives and objectives/ criteria relationships (V_k) are determined by the decision-maker (designer). Finally, values of "K_k" are calculated using equation (4-27) and limitations set forth in part d of section 4.4 .

For example;

$$K_1=(1+.9999K1)*(1+.5\ K_1)*(1+.5\ K_1)\ -1$$

$$K_1=\ 0, \underline{-.9997}, -2.2507$$

similarly;

$$K_2=\ 0, \underline{-.7639}, -5.236$$

$$K_3=\ 0, \underline{2.0}$$

$$K_4=\ 0$$

CHAPTER V

5.0- Introduction

The objective of this chapter is to apply the concepts developed in Chapters II, III, and IV to a case study of energy conservation measures for a major commercial building in order to demonstrate the steps required for their application and to explore the sensitivity of the ranking of design alternatives to different decision criteria.

5.1- Building and Mechanical System Description

5.1.1- General Description of The Building

The building analysed is a 30-storey office building situated in Western Canada, and it is part of a larger complex including a 3 storey podium, (e.g. basement, lobby, and mezzanine), commercial area, and an adjacent enclosed parking garage. The remaining 27 stories (13th floor does not exist) include 2 mechanical floors on the 3rd and 29th level. The building has a gross floor area of 498,000 sq.ft. (office and retail area). Within this total, approximately 347,500 sqft of net leasable office space is available while net retail area consists of 73,000 sq.ft.

The typical floor plan is shown in Figure (5-1), with the sub-divisions of the floor area into North, South, East, West, and core zones, for purposes of energy requirement estimation.

The core area covers the rentable space in the interior

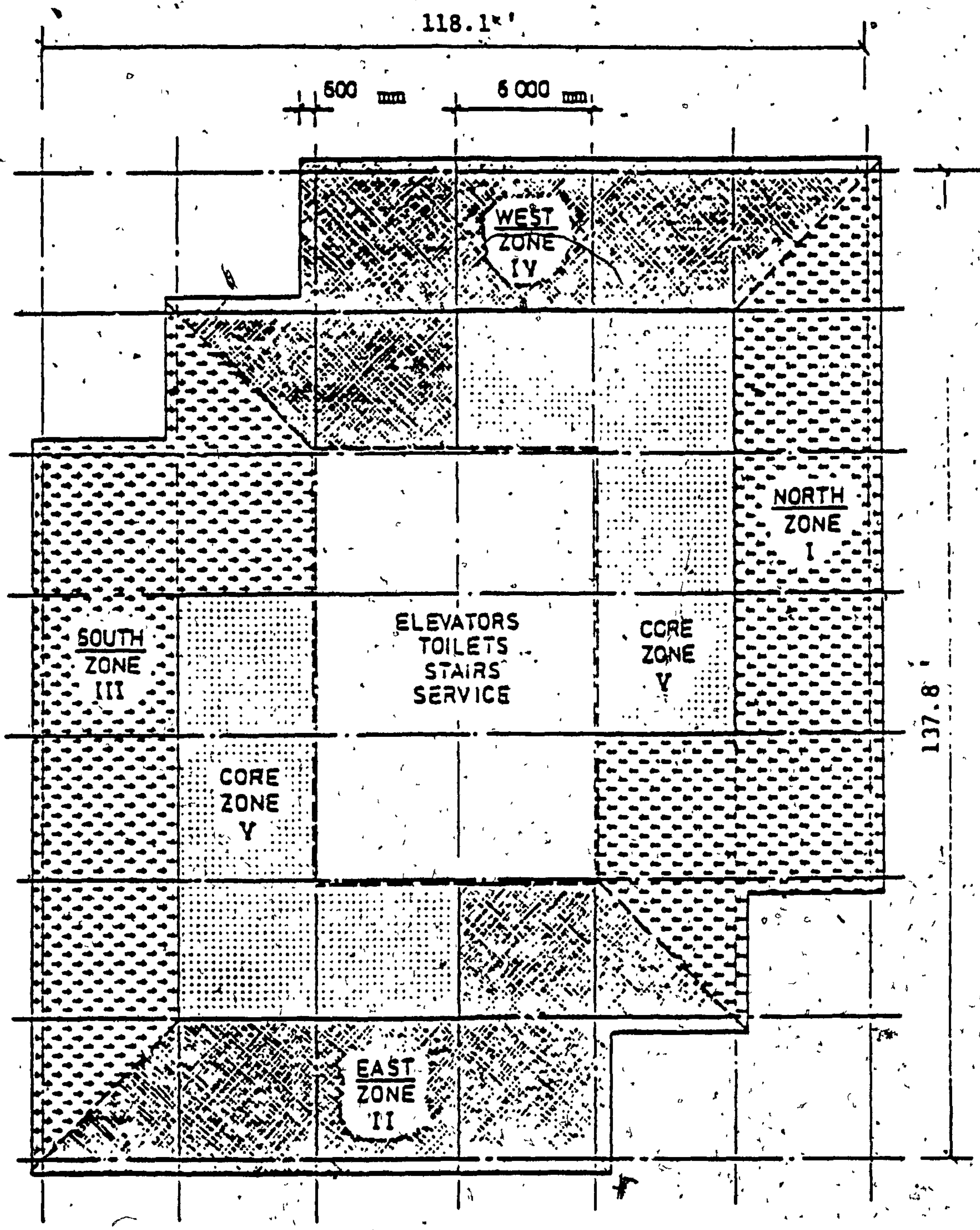


FIG (5-1) - Typical floor plan. (82)

zone, but does not include elevator area, washrooms, duct shafts, and staircases as indicated.

This core area, as such, has no outside walls, hence, no envelope losses or gains.

The building skin or vertical envelope is metal curtain wall with 4 inches of compressed glass fiber insulation in the non-glazed area. The glazed area consists of double-glazed sealed units with bronze solar glass outside and clear glass inside (base case). A typical cross-section of the wall is shown in Figure (5-2).

The perimeter areas are conventionally one bay (15 to 20 ft wide), to account for the effects of outdoor weather conditions and solar radiation.

5.1.2- HVAC System Description

The cooling and ventilation is by means of a VAV (variable air volume) system with sprayed coil cooling and economizer cycle as shown in Figure (5-3). Secondary heating is by means of hot water heated baseboard radiators located on the perimeter of the outer zones. The perimeter heating is controlled by an indoor-outdoor scheduled temperature controller.

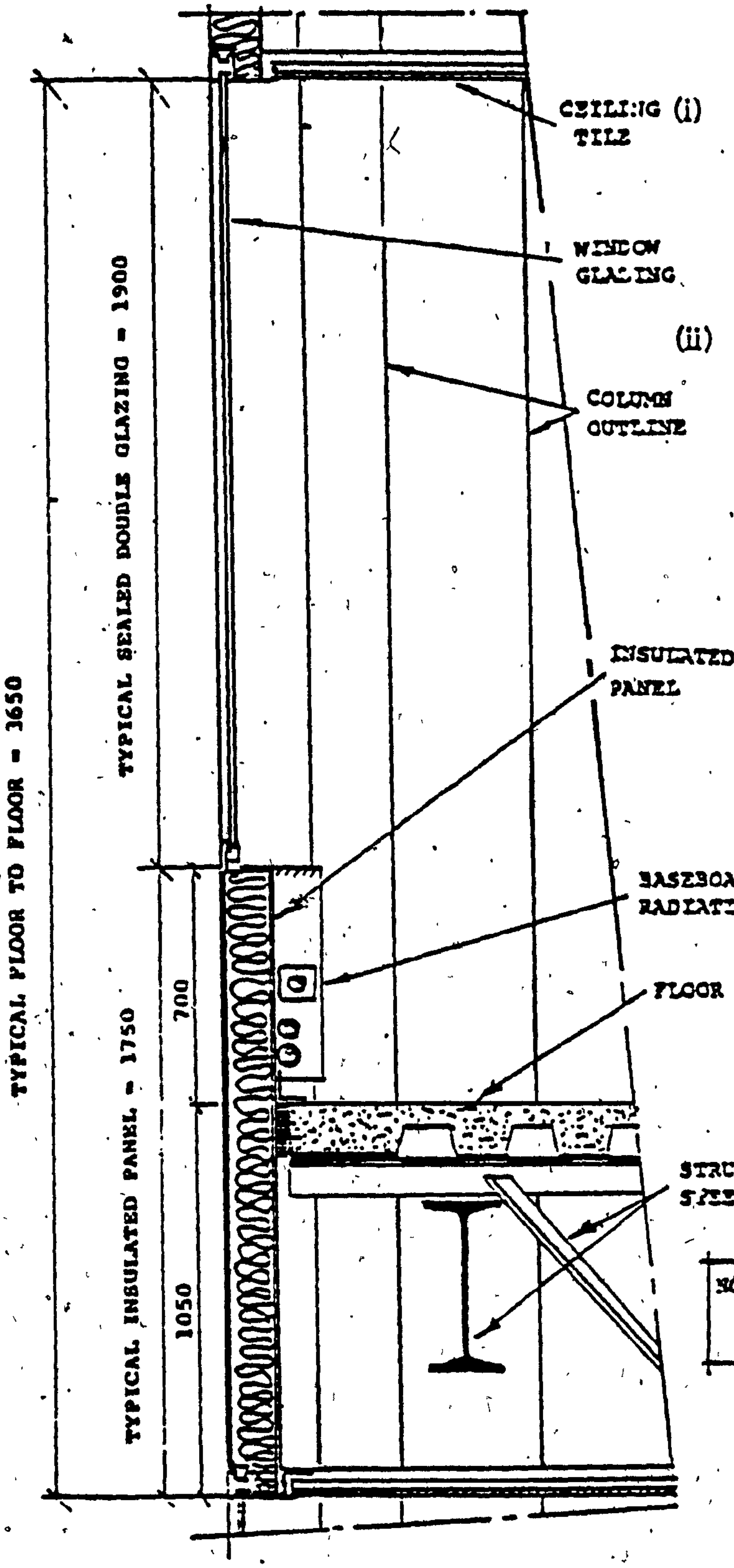
The VAV diffusers are controlled by thermostats in the core zone with volume varying from 40 to 100% of design. In the perimeter zones, the VAV diffusers are used to control the air volume variation. The thermostat controlling the

(i) Windows:

- double glazing
- green solar tint
- U = 0.56 BTU/hr/ft^2/°F (15MPH)

(ii) Walls:

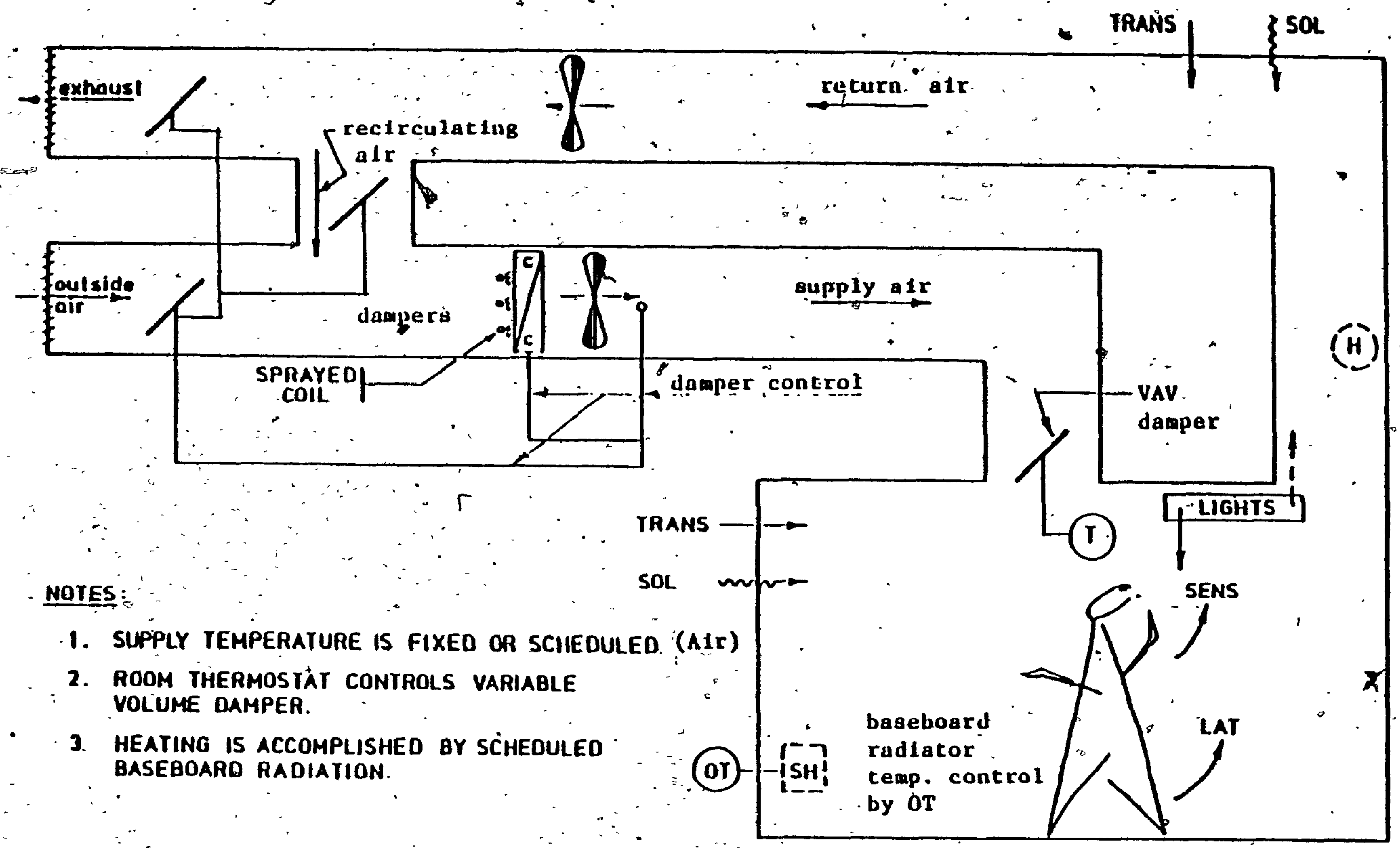
- prefabricated insulated panels
- aluminized reflective exterior
- 4" AS 530 fibreglass insulation
- U = 0.06 BTU/hr/ft^2/°F (15 MPH)



NOTE:
ALL DIMENSIONS IN MM.

FIG (5-2) - Typical wall section

H Humidistat
T Room Thermostat
OT outdoor temperature sensing stat



NOTES:

1. SUPPLY TEMPERATURE IS FIXED OR SCHEDULED (Air)
2. ROOM THERMOSTAT CONTROLS VARIABLE VOLUME DAMPER.
3. HEATING IS ACCOMPLISHED BY SCHEDULED BASEBOARD RADIATION.

FIG (5-3) - VAV air-conditioning system schematic.(82)

variable air volume also controls the perimeter baseboard radiation in sequence when required.

Supply and return fans, cooling and spray coils are located at each of the mechanical rooms.

The two electrically driven centrifugal water chillers having a total capacity of 4578 kw (1302 tons) of refrigerating effect, one for 44% and the other for 56% of the total load are located in the upper mechanical room. The cooling towers and the 4 gas fired boilers each, with a capacity of 1406 kw (6000 MBH) output, are also located in the upper mechanical room. Here, as well, can be found the electrical substations and motor control centers.

Service water is heated electrically on each floor in the service core adjacent to the washrooms.

The control system and its operation may be described as follows.

a) The cooling coils with chillers and cooling towers come into operation when outside air temperature is above 10 °C (50 °F). Below 10 °C (50 °F), outside air is mixed with return air to obtain "free cooling".

b) The supply fan is a draw-through type, i.e. the supply air is cooled and then drawn through the supply fan. As the fan and motor are in the air stream, the air picks up the heat from the motor horsepower input and the supply air is reheated from 2 to 5 F, depending on the mechanical heat available from the fan's electrical motors.

c) The space thermostats control the linear VAV supply diffusers and the perimeter heating baseboard radiation in sequence. When the building is unoccupied, the air system can be shut off to conserve energy. In winter, the perimeter baseboard heating compensates for skin transmission losses. The heating water temperature is controlled by an indoor-outdoor temperature control.

d) Air static pressure controllers, located at various levels, in the supply and return air ducts, at various levels signal to the supply and return air fans to vary the volume of air circulated as a function of the variation in air quantities demanded by the zones, (Figure (5-4)). The air demand varies with variations in the solar load, skin transmission losses, or gains and internal loads (people, lights, etc).

5.1.3- Lighting System Description

A common five foot by five foot ceiling module, with recessed fluorescent light fixtures sufficient to maintain an illumination of 75 foot-candles at desk level, is thought possible with a "checkerboard" light fixture lay-out. This amounts to an average 2.3 w/sqft energy consumption for lighting, including standard ballasts. Light switching for

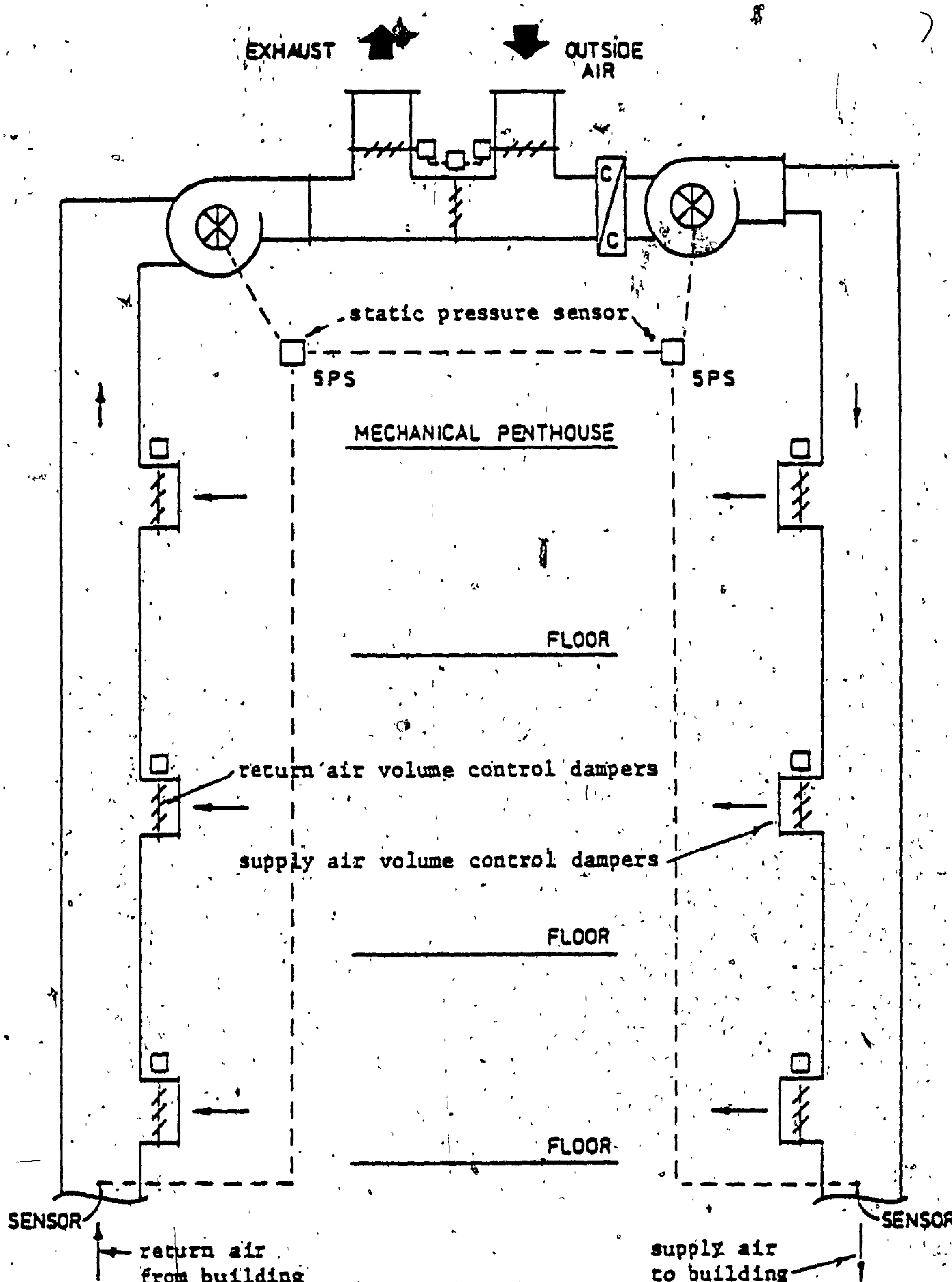


FIG (5-4) - Typical ventilation riser diagram. (82)

each floor, with tenant switching override, forms part of the design. Special light fixtures with aluminized parabolic reflector design and "warm" light fluorescent tubes are used to obtain maximum lighting efficiency and minimum energy consumption. It has been assumed that during the weekends and holidays this level drops to .15 kw/sqft.

5.2- Design Alternatives and Their Technical performance

Eleven enclosure treatments have been studied. The basic variables are the wall insulation thickness and the glazing type. The characteristics of the alternatives are given in Table (5-1).

The technical performance of design alternatives are measured in terms of building energy requirements. The energy analysis of all eleven alternatives were conducted using the Meriwether program (82,83). The energy requirements are presented in Table (5-2). For comparison purposes, the energy consumption levels for heating, cooling, and lighting and power are graphically presented in Fig. (5-5).

5.3- Economic Analysis

For the purpose of economic analysis, the performance of the design alternatives with respect to three criteria have been examined. These include capital cost, payback period and life cycle cost. In this analysis, the following information is used;

Glazing		Wall panel insulation thickness		
		2"	4"	6"
Clear double	U Glass	.490	.490	.490
	U Panel	.114	.060	.040
	[1]U Overall	.310	.284	.274
	[2]S C	.880	.880	.880
Tint double	U Glass	.430	* .430	.430
	U Panel	.114	.060	.040
	U Overall	.278	.252	.243
	S C	.580	.580	.580
Reflective Double	U Glass	NOT RUN	.430	.430
	U Panel		.060	.040
	U Overall		.252	.243
	S C		.380	.380
Clear triple	U Glass	.310	.310	.310
	U Panel	.114	.060	.040
	U Overall	.216	.190	.180
	S C	.710	.710	.710

1 U overall=.52* U Glass+ .48* U Panel
2 Shading Coeffecient (SC)
* Base case

TABLE (5-1) - Characteristics of 11 enclosure alternatives.

Alt.	HEATING		COOLING		LIGHTING	TOTAL ENERGY	
	MBH	kwh	Ton-Hr	kwh COP=3.75	kwh	kwh	kwh/sq.ft
1	12,961,822	3,797,780	398,553	373,679	6,078,194	10,249,653	31.75
2	11,876,646	3,479,825	399,525	374,591	6,082,499	9,936,915	30.78
3	11,476,468	3,362,575	391,074	366,671	6,060,764	9,790,010	30.33
4	11,683,571	3,423,255	372,453	349,212	6,022,211	9,794,678	30.34
5	10,576,216	3,098,803	372,916	349,646	5,980,829	9,429,278	29.21
6	10,165,810	2,978,555	375,452	352,024	5,998,737	9,329,316	28.90
7	--- NOT RUN ---						
8	10,581,840	3,100,451	323,718	303,518	5,838,550	9,242,519	28,63
9	10,572,724	3,097,780	324,851	304,580	5,841,845	9,244,205	28.64
10	9,071,908	2,658,045	393,361	368,815	6,057,884	9,084,744	28.14
11	7,956,746	2,331,305	394,430	369,818	6,062,181	8,763,304	27.15
12	7,552,528	2,212,871	394,860	370,221	6,064,173	8,647,265	26.79

* Floor area= 322,787 ft^2

TABLE (5-2) - Energy consumption levels for 11 enclosure alternatives.

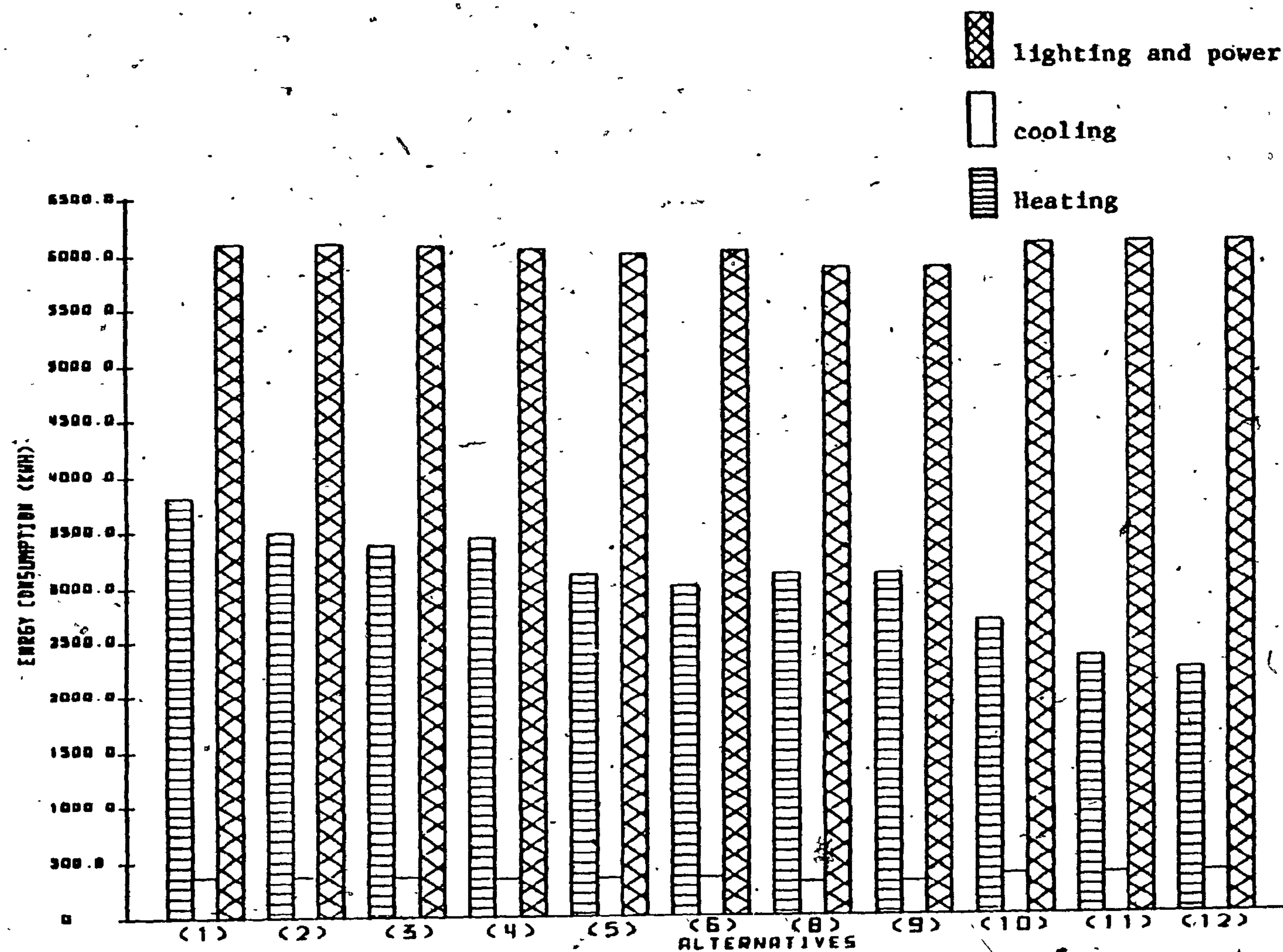


FIG (5-5) - Comparative energy consumption for various wall assemblies.

construction duration; $T_c=2$ years
study period $T_e=30$ years
discount rate $r=15\%$

5.3.1- Capital Cost Analysis

i- Deterministic analysis

The initial cost estimates (expected values) for eleven enclosure alternatives are given in Table (5-3). Since the energy requirements for different alternatives vary, the heating and cooling plants are sized for each case and details are presented in Tables (5-4,5,6) (82). The initial cost estimate for the electrical system was $1,626,900 ($5.04/sqft).

The expected construction schedule and related inflation rates are:

	start-finish time	inflation rate θ *
enclosure	12-18	10%
mechanical system	15-18	12%
electrical system	12-22	14%

* constant inflation rate

* the inflation rates are calculated based on capital cost data (1977-1981).

In capital cost calculation, the uniform model is used. For example, for alternative 1, the present worth of capital cost (including inflation, excluding financing) is calculated as;

$$C_i = \sum_{j=1}^{3} \frac{C_{ij}}{T_{fj}-T_{sj}} \int_{T_{sj}}^{T_{fj}} e^{(\theta_{ij}-r)\,t}\, dt \qquad (5\text{-}1)$$

Panel 48% \ Glass 52%	Clear Double	Tinted Double (base case)	Reflective Double	Clear Triple
2" Insul. glass fibre	11.10 / 22.02	11.94 / 23.70	12.55 / 24.90	12.55 / 24.90
4" Insul. (base case)	11.25 / 22.32	12.09 / 24.00	12.70 / 25.20	12.70 / 25.20
6" Insul.	11.40 / 22.62	12.24 / 24.30	12.85 / 25.50	12.85 / 25.50

Floor area: 322,787

Wall area : 162,654

Cost $ / sq. ft — floor area / wall area

TABLE (5-3) - Capital cost of exterior cladding for 11 enclosure alternatives. (82) (1981 prices)

Alt.	Peak heating MBH	15% pipe loss	Input required MBH using 80% efficiency	Unit required 60%	Unit required 40%	Unit 60%	Unit 40%	Installed cost*	Cost/ sq.ft
1	4,682	702	6,730	4,308	2,962	4,000	3,000	61,760	.191
2	4,290	643	6,166	3,700	3,488	4,000	3,000	61,760	.191
3	4,775	716	6,864	4,118	2,746	4,000	3,000	61,760	.191
4	4,601	690	6,613	3,968	2,645	4,000	3,000	61,760	.191
5	4,435	665	6,375	3,825	2,550	4,000	3,000	61,760	.191
6	3,672	550	5,278	3,167	2,111	3,000	3,000	59,175	.183
7	----------	----------	NOT RUN	----------	----------	----------	----------	----------	----------
8	5,392	808	7,750	4,650	3,100	5,000	3,000	64,345	.199
9	5,246	787	7,541	4,524	3,017	5,000	3,000	64,345	.199
10	3,266	490	4,695	2,817	1,878	3,000	2,000	55,687	.173
11	2,874	431	4,131	2,479	1,652	3,000	2,000	55,678	.173
12	2,728	409	3,921	2,352	1,569	3,000	2,000	55,678	.173

* 1981 prices are used.

TABLE (5-4) – Heating plant cost for 11 enclosure alternatives. (82)

GLASS TYPE	DOUBLE CLEAR	TRIPLE CLEAR	SOLAR BRONZE	REFLECTIVE FILM
Tons Cooling	1660	1466	1288	1034
<u>One Machine</u>	Poor engineering practise to use one only machine for these two selections.			
$ Capital Cost			$208,000	$180,000
$/ ton			$161.50	$174.08
$/ sq. ft			$ 0,64	$ 0,56
<u>Two Machines (Equal)</u>				
$ Capital Cost	$312,000	$304,000	$244,000	$224,000
$/ ton	$187.95	$207.37	$184.49	$216.63
$/ sq. ft	$ 0.97	$ 0.94	$ 0.76	$ 0.69

TABLE (5-5) - Cooling plant cost for 11 enclosure alternatives. (1981 prices) (82)

Alternative	1	2	3	4	5	6	7	8	9	10	11	12
Cost, $/ sq. ft	8.52	8.52	8.52	8.31	8.31	8.30	----	8.26	8.26	8,48	8.48	8.48

Table (5-6) - Overall HVAC system capital cost for 11 enclosure alternatives (1981 cost data) where:

C_{cj} = the base year estimate of capital cost for subsystem j

T_{sj} = the start time of construction for subsystem j

T_{fj} = the finish time of construction for subsystem j

θ_{ij} = cost escalation for subsystem j

r = discount rate

Now

C_{c1}=\$11.1/ sq. ft. * 322,787=\$3,582,936 (Table 5-3)

C_{c2}=\$8.52/ sq. ft. * 322,787=\$2,750,145. (Table 5-6)

C_{c3}=\$1,626,900.

Using values of T_{fj}, T_{sj} and θ_{ij}, as given in the above, the total discounted capital cost becomes:

C_c = \$ 7,608,873.

The discounted capital costs, for all design alternatives, are given in table (5-7).

ii- Probabilistic analysis

Considering the uncertainty involved in initial cost estimations (C_{ij}), future inflation rate (θ_{ij}), start time (T_{sj}) and finish time (T_{fj}), these variables are assumed to be random with the following characteristics:

	Enclosure		HVAC		Electrical	
Variable	C.V.	γ_3	C.V	γ_3	C.V.	γ_3
C_i	.10	.687	.10	.585	.10	1.88
θ_i	.20	1	.167	1	.143	1
T_s	.20	1	.08	1	.10	1
T_f	.10	1	.10	1	.10	1

Alternative	Capital Cost			
	CC	CC	σ_{CC}	$\gamma_3(CC)$
1	7,608,873	7,611,687	473,148	.432
2	7,654,359	7,657,188	476,611	.433
3	7,699,845	7,702,689	480,096	.434
4	7,798,547	7,801,422	489,083	.440
5	7,844,033	7,846,923	492,685	.441
6	7,886,421	7,889,321	496,133	.443
7	NOT RUN			
8	8,013,521	8,016,467	506,675	.447
9	8,059,007	8,061,968	510,373	.448
10	8,036,179	8,039,136	506,768	.442
11	8,081,665	8,084,637	510,421	.443
12	8,127,150	8,130,138	514,092	.444

Note: The coeff. of skewness for actual capital cost data for energy system has been calculated as .49 .

TABLE (5-7) - Present worth total capital cost of energy system for enclosure alternatives (inclusive of inflation, exclusive of financing).

Values in Table (5-7) can be estimated from past experiences (cost data about past projects), by decision maker (architect/ engineer).

Using the moments approach, the first three moments are calculated (Table(5-7)). It is observed that the difference between deterministic and probabilistic mean values are minimal. This trend is the result of two factors: i) the -ve values of "θ-r" and ii) the low value of the coefficient of skewness (γ_3), used in this analysis. The effect of these variables have been investigated in past research work (24) and is shown in Fig. (5-6). As it is shown, for a 0.1 coefficient of variation, the effect of coefficient of skewness is minimal and may be ignored in the future analysis.

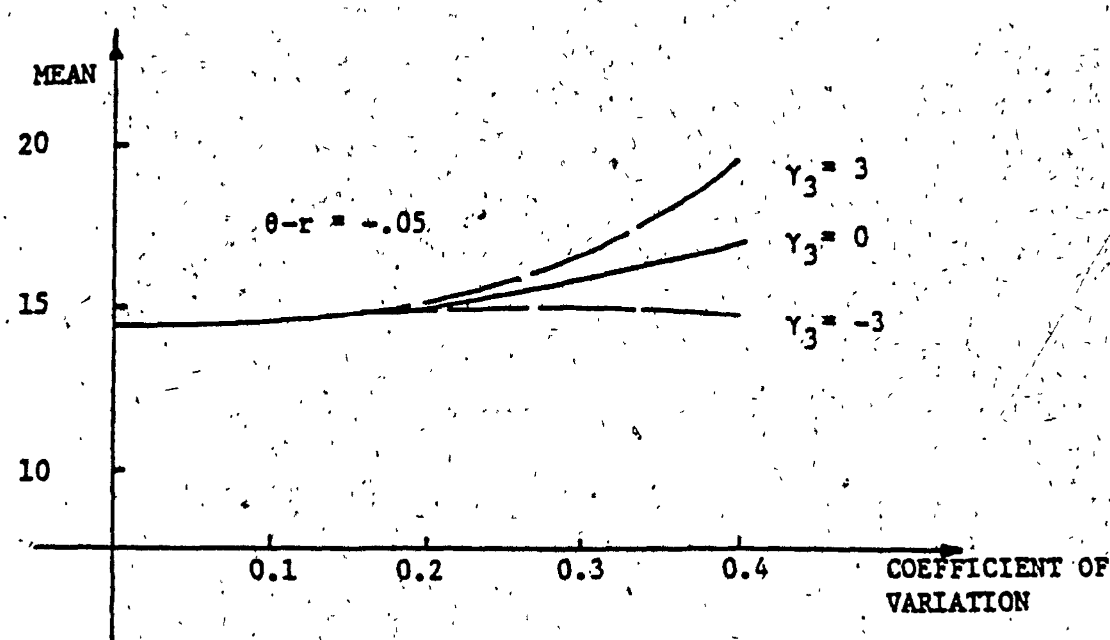


FIG. (5-6)- Variations of Mean as a Function of Coefficient of Variation and γ_3.

5.3.2- Payback Period Analysis

For the purpose of this study, the simple payback method (inflation adjusted cash flows are used), Table (5-9); the discounted payback method (inflation adjusted cash flows and discount rate of 15% are used), Table (5-10); and the expected payback method (based on the discounted expected values of capital costs and operating cost increments), Table (5-11), are used. In order to include all alternatives in the analysis, alternative one is assumed to be the base case for payback analysis (originally alternative number 5 had been used in design).

i- Deterministic analysis

In deterministic analysis, simple and discounted payback periods are calculated from the information provided in Tables(5-3,6,8). For example, considering alternatives 1 and 2, the simple payback period is calculated as;

$$\sum_{j=1}^{3} ES_j \int_{T_c}^{T_c+P} e^{\theta_{oj} t}\, dt = \sum_{j=1}^{3} \Delta C_{cj} \int_{T_{sj}}^{T_{fj}} e^{\theta_{cj} t}\, dt \qquad (5\text{-}2)$$

Where:

ES_j=energy savings for subsystem j

P=payback period

And;

C_{c1}= \$ .15/ sq. ft * 322,787=48418 (Table 5-3)

C_{c2}= 0 (Table 5-6)

C_{c3}= 0

θ_{c1}=.1 T_{s1}=1 T_{f1}=1.5

θ_{c2}=.12 T_{s2}=1.25 T_{f2}=1.5

Alternatives	Heating		Cooling		Lighting	
	base year cost	θ_h	base year cost	θ_c	base year cost	θ_e
1	47,959	.1	3,986	.17	81,567	.17
2	43,944	"	3,995	"	81,610	"
3	42,463	"	3,911	"	81,392	"
4	43,229	"	3,725	"	81,007	"
5	39,132	"	3,729	"	80,593	"
6	37,613	"	3,755	"	80,772	"
7	-------------	----	NOT RUN	----	-------------	----
8	39,153	"	3,237	"	79,170	"
9	39,119	"	3,249	"	79,203	"
10	33,566	"	3,934	"	81,364	"
11	29,440	"	3,944	"	81,407	"
12	27,944	"	3,949	"	81,427	"

TABLE (5-8) - Base year operating costs and ralated inflation rates for HVAC and lighting systems.

$\theta_{c3}=.14 \quad T_{s3}=1 \quad T_{f3}=1.84$

Energy savings data are;

$ES_1=0$

$ES_2=\$\ 4015$ with $\theta_{o2}=.16$ and
$\$\ -9.$ with $\theta_{o2}=.17$

$ES_3=\$\ -43.$ with $\theta_{o3}=.17$ (Table 5-8)

$$48{,}418 * (1.132) = 4{,}015 \left(\frac{1}{.16}*\left(e^{.16(2+P)} - e^{.16(2)}\right)\right) - 52 \left(\frac{1}{.17}*\left(e^{.17(2+P)} - e^{.17(2)}\right)\right)$$

From the above equation p=6 years. Values of simple payback periods for all design alternatives are given in Table (5-9).

For discounted payback periods, the factor θ is replaced by "θ-r". These values are given in Table (5-10).

ii- Probabilistic analysis

For the purpose of probabilistic analysis, some of the variables (ΔC_{cj}, θ_{cj}, T_{sj}, T_{fj},θ_{oj}) in equation (5-2) are assumed to be random. Characteristics of capital cost variables were discussed in section (5.3.1), moments of future inflation rates for different subsystems are:

	θ_{oj}	C.V.	γ_3
Heating	.16	.10	1
Cooling	.17	.20	1
Electricity	.17	.20	1

Since there is not enough information to estimate the uncertainty of energy consumption levels, the energy consumption levels and consequently base-year energy costs are assumed to de deterministic (this assumption is not a realistic assumption and there is uncertainty involved in estimation of building energy requirements, however, because of the lack of data availability they are assumed to be deterministic).

Since the payback period does not have a mathematical expression to be used directly in the moments approach, the probabilistic analysis can only be done by treating the variables in both sides of the equation (5-2), randomly. Using partial derivatives in equation (5-2), expected values of discounted payback periods are calculated and are given in Table (5-11). The standard deviations of payback periods are also calculated, these values will be used for calculation of the expected utilities (sec. 5.4.2).

As it is demonstrated in Tables (5-9,10), by ignoring the time value of money, the payback periods become lower. For the expected payback periods (Table 5-11), since we are dealing with uncertain future operating costs, the values are calculated to be less than those in discounted payback method, and higher than those in simple payback method.

To study the sensitivity of the payback period to energy price/consumption changes, the values are calculated for one, two, three, and four multiplier of energy

Alternatives	1* energy cost	2 * energy cost	3* energy cost	4* energy cost
1	-----	-----	-----	-----
2	6	4	3	3
3	8	8	4	3
4	11	8	6	5
5	10	7	5	4
6	10	7	5	4
7	NOT RUN			
8	11	8	6	5
9	12	8	7	6
10	11	7	6	5
11	10	7	5	4
12	10	7	5	5

TABLE (5-9) - Simple payback period for one, two, three and four multipliers of energy cost/ consumption level.

Alternatives	1 * energy cost	2 * energy cost	3* energy cost	4* energy cost
1	-----	-----	-----	-----
2	11	6	4	3
3	15	6	6	4
4	30	16	11	8
5	21	11	8	6
6	22	12	8	6
7	NOT RUN			
8	28	15	11	8
9	30	17	12	9
10	26	14	10	7
11	23	12	8	6
12	23	12	9	7

TABLE (5-10)– Discounted payback period for one, two, three and four multipliers of energy cost/ consumption level.

Alternatives	1*energy cost	2*energy cost	3*energy cost	4*energy cost
1	--	--	--	--
2	11	6	4	3
3	16	7	7	5
4	30	18	12	9
5	23	12	9	7
6	24	13	7	7
7	NOT RUN			
8	30	18	13	9
9	30	20	14	10
10	26	14	11	8
11	23	12	8	7
12	23	12	9	7

TABLE (5-11) - Expected discounted payback period for one, two, three, and four multipliers of energy cost/ consumption.

cost/consumption. This study reflects the performance of one design alternative in different locations (e.g. Montreal, Edmonton and New York), where the energy costs are different. It also demonstrates how the rate of recovery may change with changes in the economic environment. As shown in Fig. (5-7), the payback period shortens as energy cost/consumption increases (energy cost savings also increases). A similar trend is true for changes in inflation rate; as the inflation rate increases, the payback period decreases (a mild decreasing exponential function) (Fig. (5-8)).

5.3.3- Life Cycle Costing

For the life cycle costing analysis, the following components have been studied:

a) Capital cost: capital cost analysis is documented in section (5.3.1) and the computed values will be used in LCC computation.

b) Operating cost: operating cost is referred to as the cost of energy used for operating the energy system (heating, cooling and electrical system). The base year operating costs and overall energy price escalation rates are provided in Table(5-8). The escalation rates are selected according to the existing price structure of Hydro Quebec (1980,81) (84), and Gas Metropolitan (1980) (85). It has been assumed that there are no major alteration costs and

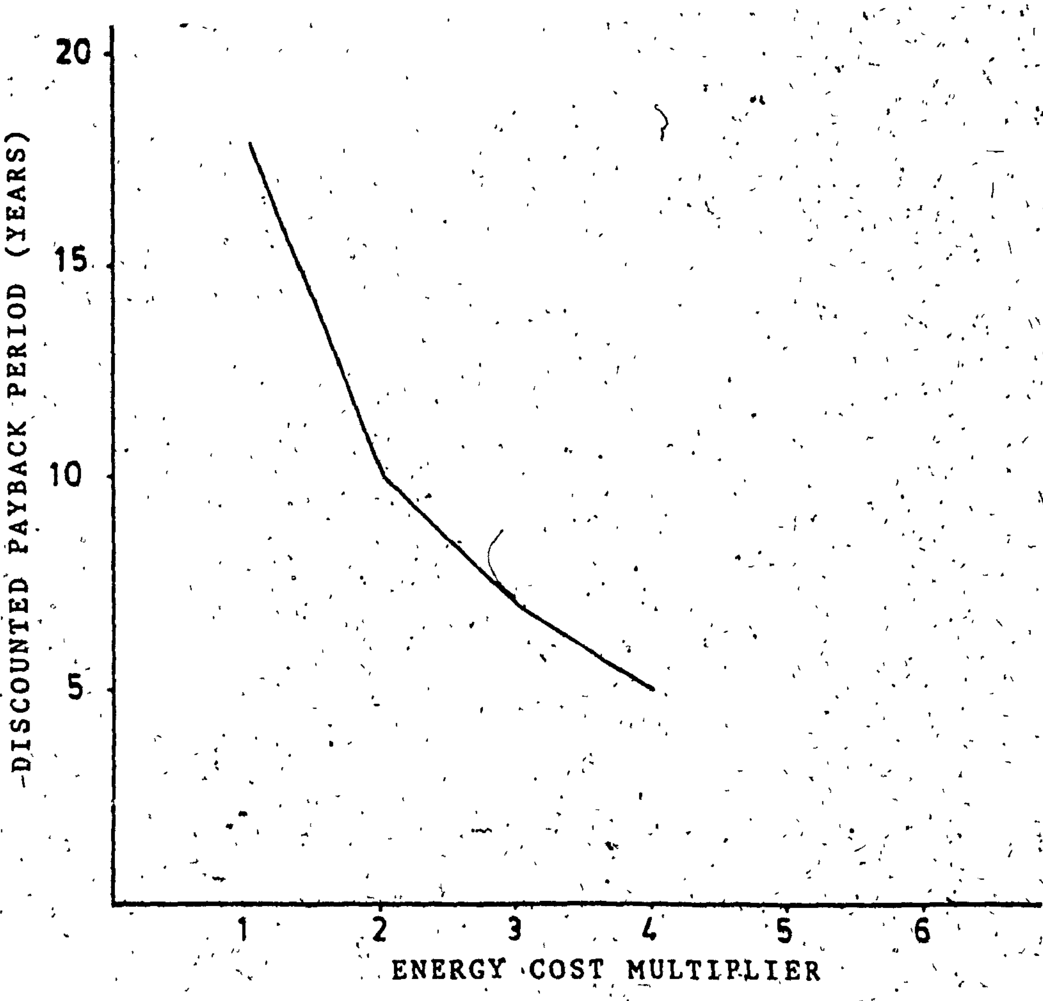


FIG. (5-7) - Payback perion vs. energy cost / consumption.

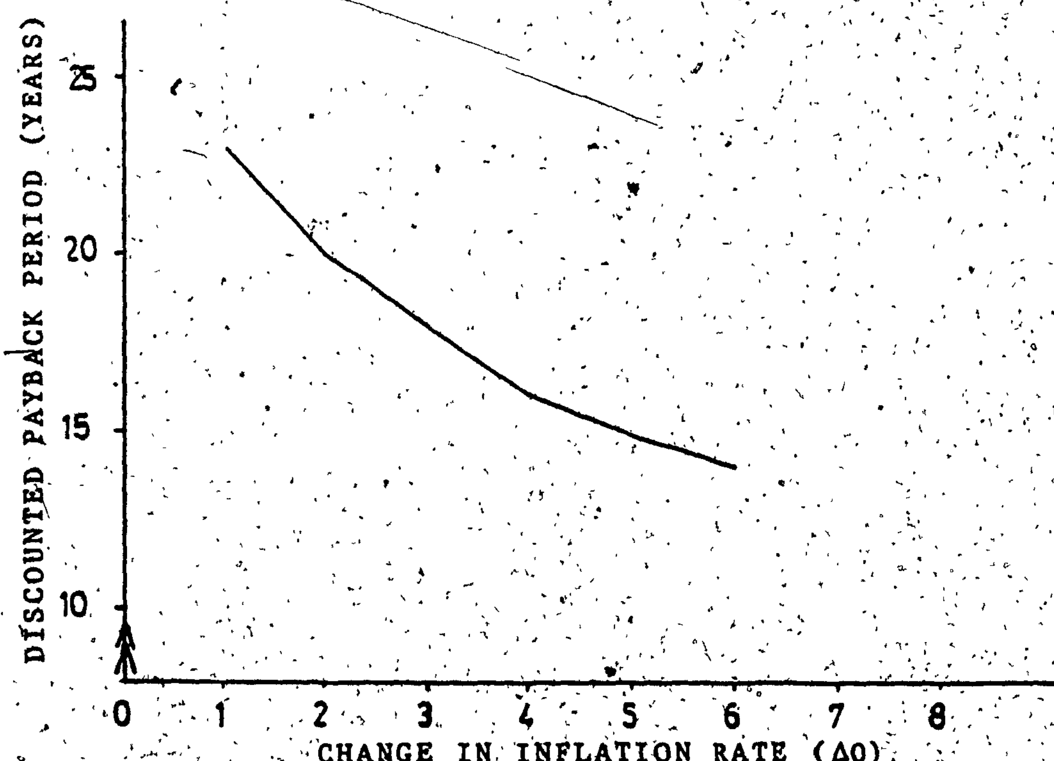


FIG. (5-8) - Payback period vs. inflation rate.

that the maintenance policy includes all repair/renewal and alterations for the subsystems.

c) Maintenance cost: maintenance cost of the energy system is assumed to be constant for all alternatives; \$.07/sq.ft./year for HVAC and \$.03/sq.ft./year for the electrical system (47). The inflation rates are 10% and 11% for HVAC and the electrical system respectively.

i- Deterministic analysis

The life cycle cost is calculated as:

$$LCC = \sum_j C_{cj} + (C_{oj} + C_{mj})$$

where:

C_{cj}=present worth of capital cost for system j

C_{oj}=present worth of operating cost for system j

C_{mj}=present worth of maintenance cost for system j

For example, consider alternative 1;

$$C_c = C_{c1}+C_{c2}+C_{c3} = \$\ 7,608,873 \qquad \text{(Table 5-7)}$$

$$C_o = \int_2^{30} (C_{oj})\, e^{(\theta_{oj} - r)t}\, dt)$$

and

$C_{o1}=0$

$C_{o2}=\$\ 47,959$ with $\theta_{o2}=16\%$ and
$\$\ 3,986$ with $\theta_{o2}=17\%$ (Table 5-8)

$C_{o3}=\$\ 81,567$ with $\theta_{o3}=17\%$ (Table 5-8)

Then, the operating cost becomes;

$$C_o = \$\ 4,922,432.$$

For maintenance;

$$C_m = \int_2^{30} (C_{mj} * e^{(\theta_{mj} - r)\ t}\ dt)$$

Where

$$C_{m1} = 0$$

$$C_{m2} = \$\ .07/\ sq.\ ft * 322,787 = \$\ 22,959.$$

$$C_{m3} = \$\ .03/\ sq.\ ft * 322,787 = \$\ 9,684.$$

and

$$\theta_{m2} = 10\% \quad , \quad \theta_{m3} = 11\%$$

Then, the maintenance cost becomes;

$$C_m = \$\ 459,470.$$

Finally

$$LCC = C_c + C_o + C_m = \$\ 12,990,665.$$

Values of life cycle costs for all design alternatives are given in Table (5-12).

ii- Probabilistic analysis

Probabilistic analysis has been performed, the results of which are presented in Table (5-12). In the probabilistic analysis, the expected values are about 7% higher than those in deterministic analysis. This is due to the exponential effect of the uncertainty involved in estimation of future inflation rates. The coefficients of variation for different alternatives are in the order of .18 . The difference between the life cycle cost of different alternatives are small (about 2.5%), because of the building design

Alternative	Life Cycle Cost (LCC)			
	LCC	$\overline{LCC}$	σ_{LCC}	$\gamma_3(LCC)$
1	12,990,665	13,844,922	2,612,582	.377
2	12,905,825	13,739,622	2,581,979	.384
3	12,890,691	13,715,070	2,564,396	.387
4	12,992,338	13,818,203	2,561,157	.383
5	12,886,747	13,688,689	2,521,561	.391
6	12,887,068	13,682,490	2,516,959	.395
7	NOT RUN			
8	12,982,117	13,772,744	2,480,847	.387
9	13,028,240	13,819,025	2,482,943	.387
10	12,933,533	13,713,590	2,513,641	.403
11	12,845,073	13,604,000	2,492,367	.411
12	12,842,218	13,561,625	2,480,200	.414

TABLE (5-12) - Total life cycle cost and its three moments for energy system.

characteristics. As far as the coefficient of skewness is concerned, since the information about different components (e.g. inflation, project duration, operating expenses) is not known, a clear conclusion cannot be derived.

At this stage, it is useful to present rankings of alternatives for each criteria (technical performance (energy consumption level), capital cost, payback period, and life cycle cost) for deterministic analysis (or expected values). It is clearly shown that the rankings drastically change with respect to each of the stated criteria (objectives) (Table 5-13). Each individual criterion does not explain the desirability of the project. These results should be studied within the context of the system design and performance such as capital and operating budgeting and the yearly performance of these criteria which helps in monitoring the performance of the system.

Thus, considering all objectives of the investor, there is no optimum design solution at this stage.

5.4- Design Decision Making

5.4.1.- Goal, Objectives and Criteria Selection.

For a hypothetical public investor, the goal and related objectives are :

Goal Economical/reliable energy system

objectives: 1. low initial cost

2. energy efficiency

ALTERNATIVES	C_C	LCC	PP	EC
1	1	11	—	11
2	2	7	1	10
3	3	6	2	8
4	4	9	8	9
5	5	4	3	7
6	6	3	4	6
7	NOT RUN			
8	7	8	9	4
9	9	10	10	5
10	8	5	7	3
11	10	2	5	2
12	11	1	6	1

TABLE (5-13) - Rankings of Enclosure Alternatives With Respect to Four Criteria.

3. fast capital recovery on increment of investment

4. reliable system

These objectives are subjected to the following set of constraints:

1. budget constraint

2. technical/ code constraint

3. max. energy consumption level

Different objectives may be evaluated using one or more of the following criteria:

1. capital cost (C.C.)

2. energy consumption (E.C.)

3. life cycle cost (LCC)

4. payback period (P.P.)

The stated objectives/constraints and criteria may vary for different types of investors in achieving the same goal.

In order to reduce size of the criteria set, those criteria that are included in other criteria should be eliminated. This can be done by using a criteria interrelationship matrix (for example, Fig. (5-9) gives a simple matrix that can be used for this purpose. In this case study all four criteria will be used in the decision making process.

			FINANCIAL														TECHNICAL					USER						
			1 PAYBACK PERIOD	2 EQUITY DIVIDEND RATE	3 TAX SHELTER	4 VECTOR OF CASH FLOWS	5 NET SALE CASH REVERSION	6 INTERNAL RATE OF RETURN	7 NET PRESENT VALUE	8 DISCOUNTED BENEFIT COST RATIO	9 LEVERAGE	10 INITIAL EQUITY	11 CAPITAL COST	12 LOAN COVERAGE RATIO	13 RISK	14 LIFE CYCLE COST	1 ENERGY CONSUMPTION	2 BUILDING HEIGHT	3 FLOOR EFFICIENCY	4 ADAPTABILITY	5 MAINTAINABILITY	1 WORKER PRODUCTIVITY	2 AURAL COMFORT	3 ILLUMINATED COMFORT	4 THERMAL COMFORT	5 ENVIRONMENTAL COMFORT	6 COMMUNICATION EFFICIENCY	7 USER COST
FINANCIAL	1	PAYBACK PERIOD	●																									
	2	EQUITY DIVIDEND RATE		●																								
	3	TAX SHELTER			●																							
	4	VECTOR OF CASH FLOWS				●										●												
	5	NET SALE CASH REVERSION					●																					
	6	INTERNAL RATE OF RETURN						●	●	●			●			●												
	7	NET PRESENT VALUE						●	●	●			●			●												
	8	DISCOUNTED BENEFIT COST RATIO						●	●	●			●			●												
	9	LEVERAGE									●																	
	10	INITIAL EQUITY										●																
	11	CAPITAL COST						●	●	●			●			●												
	12	LOAN COVERAGE RATIO												●														
	13	RISK													●	●												
	14	LIFE CYCLE COST				●		●	●	●			●		●	●	●		●			●					●	●
TECHNICAL	1	ENERGY CONSUMPTION														●	●											
	2	BUILDING HEIGHT																●										
	3	FLOOR EFFICIENCY														●			●									
	4	ADAPTABILITY																		●								
	5	MAINTAINABILITY																			●							
USER	1	WORKER PRODUCTIVITY														●						●						
	2	AURAL COMFORT																					●					
	3	ILLUMINATED COMFORT																						●				
	4	THERMAL COMFORT																							●			
	5	ENVIRONMENTAL COMFORT																								●		
	6	COMMUNICATION EFFICIENCY														●											●	
	7	USER COST														●												●

FIG. (5-9) - Criteria interaction matrix.

5.4.2- Multiple-Criteria Design Decision-Making (MCDDM)

The objective in MCDDM is to select the optimum solution with respect to the selected criteria. Here, there are two different approaches:

a) To introduce/select a criterion which includes the others. Life cycle costing, in this case, is the criterion which defines most of the other criteria (except payback period). If this criterion is acceptable, the expected values and the respective ranking can be used as the design decision making tool.

b) To consider all four criteria and the existing constraints within a decision making process. The criteria and constraints are :

1. capital cost (C.C.)
2. energy consumption (E.C.)
3. payback period (P.P.)
4. life cycle cost (LCC)

subject to:

$$C.C. < \$\ 9 * 10^6$$

$$E.C. < 11 * 10^6 \quad \text{Kwh}$$

$$P.P. < Te = 30 \quad \text{years}$$

$$L.C.C. < \$\ 17 * 10^6$$

NOTE: This information is determined by decision maker based on basic information supplied by the investor.

For illustrative purposes, 3 sets of weightings are assigned to the investor's objectives (to illustrate the difference between decisions made by three different types of investors). This table will be referred to as weightings data matrix.

Investor weighting set	OBJECTIVES			
	Low capital cost	Energy efficiency	Fast recovery	Reliable system
set 1	.30	.30	.20	.20
set 2	.50	.15	.30	.05
set 3	.20	.60	.10	.10

Using the objective/criteria matrix (shown below) :

OBJECTIVES	CRITERIA				K_k
	C.C.	E.C.	P.P.	LCC	
Low capital cost	●			◒	-.999
Energy efficiency		●		◒	-.999
Fast recovery			●	○	-.996
Reliable system				●	0

where :

V(●)=.9999, V(◒)=.5, V(○)=.25, V( )=0

Values of K_k are calculated using equation (4-27) and the procedure shown in part "d" of section (4.4).

$$K_k = \prod_j (1+K_k V_{kj}) - 1$$

The weightings for different criteria, using equation (4-21) are calculated to be;

Investor weighting set	CRITERIA			
	C.C.	E.C.	P.P.	LCC
set 1	.30	.30	.20	.537
set 2	.50	.20	.30	.450
set 3	.20	.60	.10	.463

In order to perform analysis at stage 3, we require normalized weighting sets that are calculated from objective weightings data matrix and objective/ criteria relationships. Normalized weightings for different criteria are calculated to be:

Investor weighting set	CRITERIA			
	C.C.	E.C.	P.P.	LCC
set 1	.200	.200	.166	.433
set 2	.333	.100	.240	.326
set 3	.133	.400	.080	.386

Stage 1 : Constraint Analysis

At this stage the performance of each criterion, in its original unit (expected values), is tabulated for all alternatives (Table(5-14)). Since all the alternatives' performance levels are within the stated limits, all are feasible at this stage.

Stage 2 : Domination by All Aspects

Here, all alternatives are compared and those which are dominated by all aspects are eliminated.

i) Alternative 4 is dominated in all aspects by alternative 3

ii) Alternative 9 is dominated in all aspects by alternative 8

Stage 3. (weighting set # 1)

At this stage, only 9 alternatives are left to consider. For calculation of utility values an exponential utility function is used. This function is depicted in Fig. (5-10) and its expression is:

$$U(X) = 1/s * (1 - e^{-sX}) \tag{5-3}$$

The attitude of decision-maker is measured by a risk coefficient which is :

$$r(X) = -U''(X) / U'(X) = s \tag{5-4}$$

ALT. \ CRITERIA	$\overline{LCC}$ ($)	$\overline{CC}$ ($)	$\overline{EC}$ (kwh)	$\overline{PP}$ years
1	13,844,922	7,608,873	10,249,653	---
2	13,739,622	7,654,359	9,936,915	10
3	13,715,070	7,699,845	9,790,010	13
4	13,818,203	7,798,547	9,794,678	24
5	13,688,689	7,844,033	9,429,278	17
6	13,682,490	7,886,421	9,329,316	18
7	NOT RUN			
8	13,772,774	8,013,521	9,242,519	24
9	13,819,025	8,059,007	9,244,205	26
10	13,713,590	8,036,179	9,084,744	22
11	13,604,000	8,081,665	8,763,304	19
12	13,561,625	8,127,150	8,645,265	20

TABLE (5-14) - Expected values of the performance levels for enclosure alternatives with respect to selected criteria

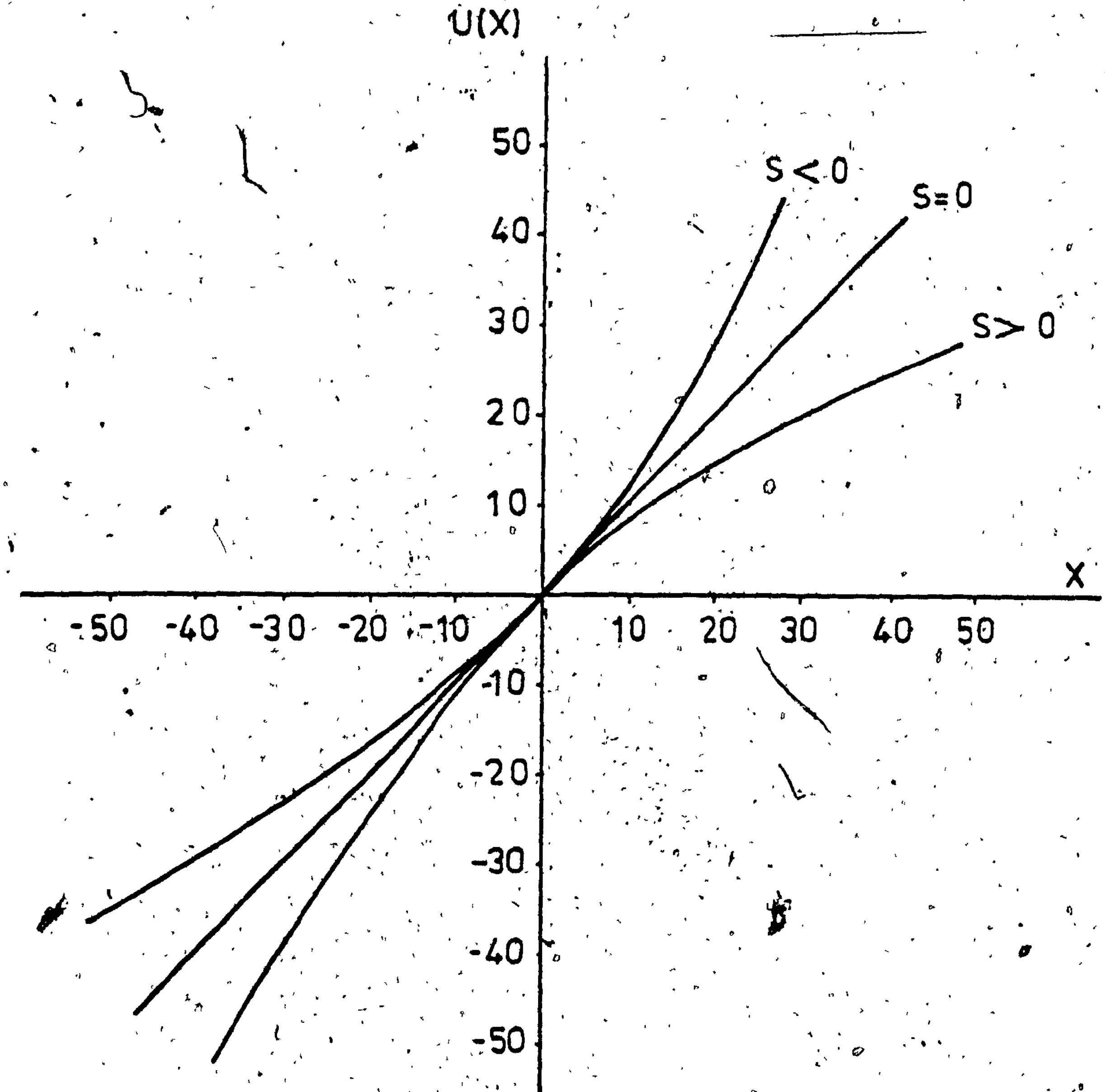


FIG. (5-10) - Utility curves for various attitudes, (exponential function).

Positive values of s represent the attitude of a "risk-averse" decision-maker and negative values of s represent the attitude of a "risk-prone" decision-maker.

In this case study, following values of s are used for different criteria.

$$s = .02 * 10^{-6} \quad \text{(for C.C. and LCC)}$$

$$s = .02 \quad \text{(for P.P.} < 10 \text{ years)}$$

$$s = .03 \quad \text{(for } 10 < \text{P.P.} < 20 \text{ years)}$$

$$s = .05 \quad \text{(for P.P.} > 20 \text{ years)}$$

** Values of risk coefficient "s" represent the attitude of a "low risk-averse" investor.

This utility function with respect to positive values of X is positive (e.g. for payback period and energy consumption criteria), and is negative for negative values of X (e.g. for capital cost and life cycle cost). For the case of negative values of X, the utility decreases (becomes more negative) as the magnitude of X increases (e.g. higher capital cost or life cycle cost). In order to have a consistent scale, all utility values (or expected utility values) will be transformed into a single scale (0-1). A linear transformation will be used to transform both positive and negative utility values into the "0-1" scale.

The utility function for different criteria can be different. In this analysis, the function is the same (for payback period analysis, risk coefficient varies for different values of payback period).

The expected utility values are calculated using the expected utility function;

$$E[U(X)] = \bar{X} - 1/2 * s * (\bar{X}_2 + \sigma^2_X) \qquad (5\text{-}5)$$

The expected utility values are normalized (0-1), using the upper and lower limits for a particular criteria. For example, for alternative 1, considering the capital cost;

$$\begin{aligned} E[U(C.C.)] &= -7.609 - 1/2 * .02 * (7.609^2 + .473^2) \\ &= -8.19 \end{aligned}$$

The upper and lower limits are;

$$\text{Max } C_C = -9 * 10^6$$

$$\text{Min } C_C = -7.5 * 10^6$$

Then, the expected utility in "0-1" scale will be;

$$\begin{aligned} E[U(C.C.)] &= (-9-(-8.19)) / (-9-(-7.5)) \\ &= .538 \end{aligned}$$

The normalized expected utility values for each criterion is shown in Table (5-15).

The partial cumulative utilities are calculated for the remaining design alternatives (Table 5-16). At this stage;

i) alternatives 8, 10 are dominated by alternative 6

Alternative	$\bar{U}$(LCC)	$\bar{U}$(CC)	$\bar{U}$(EC)	$\bar{U}$(PP)
1	.371	.538	.256	1
2	.401	.503	.361	.858
3	.408	.468	.410	.746
4	.379	.392	.409	.149
5	.417	.357	.531	.542
6	.418	.324	.564	.491
7		NOT RUN		
8	.407	.226	.593	.157
9	.380	.191	.593	0
10	.409	.208	.645	.265
11	.441	.173	.750	.435
12	.453	.137	.789	.384

TABLE (5-15) - Normalized Utility Equivalent Values of Enclosure Alternatives' Performance Levels. (exponential expected utility function is used)

CUMULATIVE WEIGHTING	.4332	.6333	.8334	1
CRITERIA / ALT.	$\bar{U}(LCC)$	$\bar{U}(LCC) + \bar{U}(CC)$	$\bar{U}(LCC) + \bar{U}(CC) + \bar{U}(EC)$	$\bar{U}(LCC) + \bar{U}(CC) + \bar{U}(EC) + \bar{U}(PP)$
1	.332	.909	1.165	2.265
2	.401	.904	1.265	2.150
3	.408	.876	1.286	2.032
4	DOMINATED AT STAGE 2			
5	.417	.774	1.305	1.847
6	.418	.742	1.306	1.797
7	NOT RUN			
8	.407	.633	1.226	1.383
9	DOMINATED AT STAGE 2			
10	.409	.617	1.262	1.527
11	.411	.614	1.365	1.800
12	.453	.590	1.379	1.763

TABLE (5-16) - Stage 3 Analysis for Weighting set 1.

Stage 4. (weighting set # 1)

At this stage, the weightings are explicitly used for calculating the overall utility value for different alternatives.

The values of K for the objectives/criteria system are less than zero. The value of K = 0 indicates that the selected criteria are interrelated and this interrelationship should be treated. The overall utility values are calculated (using multiplicative utility function) and alternative 1 is selected as the most desirable one. The ranking of alternatives at this stage is presented in Table (5-19).

5.4.3- Alternatives Ranking for Alternative Weightings

Stage 3. (weighting set # 2)

Using the same procedure, the criteria set is arranged in order of importance and the partial cumulative utilities are calculated (Table (5-17)). At this stage;

i) Alternatives 3, 5, 6, 8, 10, 11, 12 are dominated by alternative 2

Thus, only two alternatives remain to be considered.

Stage 4. (weighting set # 2)

The overall utility values are computed and, alternative 1 is selected as the most desirable one (Table (5-19)).

CUMULATIVE WEIGHTING / CRITERIA / ALT.	.3354	.6598	.9000	1
	$\bar{U}(CC)$	$\bar{U}(CC) + \bar{U}(LCC)$	$\bar{U}(CC) + \bar{U}(LCC) + \bar{U}(PP)$	$\bar{U}(CC) + \bar{U}(LCC) + \bar{U}(PP) + \bar{U}(EC)$
1	.538	.909	1.909	2.165
2	.503	.904	1.762	2.123
3	.468	.876	1.622	2.032
4	DOMINATED AT STAGE 2			
5	.357	.774	1.316	1.847
6	.324	.742	1.233	1.797
7	NOT RUN			
8	.226	.633	.790	1.383
9	DOMINATED AT STAGE 2			
10	.208	.617	.882	1.527
11	.173	.614	1.049	1.800
12	.137	.590	.974	1.763

TABLE (5-17) — Stage 3 Analysis for Weighting set 2.

Stage 3. (weighting set # 3)

Table (5-18), lists all cumulative utility values and respected weightings. At this stage;

i) Alternatives 6, 8, 10 are dominated by alternative 11.

Stage 4. (weighting set # 3)

The overall utility values are computed for four remaining alternatives and alternative 12 is selected as the most preferred alternative. The ranking of alternatives at this stage is provided in table (5-19).

5.5- Summary

In this case study, it has been demonstrated that how the uncertainty effects the performance of a design alternative. Also, the roles of the decision maker in identifying the objectives and indicating his preferences, and of the architect/ engineer in transforming this information into design criteria, have been explored. It has been shown that a single criterion cannot optimize the investor's objectives. In this case study, both deterministic and probabilistic analysis have been performed. Due to the lack of data availability certain assumptions are made. Some of these assumptions may not be realistic (e.g. technical performance is assumed to be deterministic). This points out to the necessity of creating cost/ performance data banks. Variations, in the performance levels, in deterministic and probabilistic analysis

CUMULATIVE WEIGHTING	.4002	.7864	.9198	1
CRITERIA / ALT.	$\bar{U}(EC)$	$\bar{U}(EC) + \bar{U}(LCC)$	$\bar{U}(EC) + \bar{U}(LCC) + \bar{U}(CC)$	$\bar{U}(EC) + \bar{U}(LCC) + \bar{U}(CC) + \bar{U}(PP)$
1	.256	.627	1.165	2.165
2	.361	.762	1.265	2.123
3	.410	.818	1.286	2.032
4	DOMINATED AT STAGE 2			
5	.531	.948	1.305	1.847
6	.564	.982	1.306	1.797
7	NOT RUN			
8	.593	1.000	1.226	1.527
9	DOMINATED AT STAGE 2			
10	.645	1.054	1.262	1.527
11	.751	1.192	1.365	1.800
12	.789	1.242	1.379	1.763

TABLE (5-18) - Stage 3 Analysis for Weighting Set 3.

ALTERNATIVES	WEIGHTINGS		
	set 1	set 2	set 3
1	1	1	6
2	2	2	5
3	3	——	4
4	——	——	——
5	5	——	3
6	7	——	——
7	NOT RUN		
8	——	——	——
9	——	——	——
10	——	——	——
11	4	——	2
12	6	——	1

TABLE (5-19) - Rankings of Alternatives at Stage 4 for Weighting Sets 1,2, and 3 demonstrate the importance of considering uncertainty in the analysis.

The use of the four stage, design alternative selection methodology has been demonstrated. The effect of weighting parameter on rankings of design alternatives shows the sensitivity of the design decisions with respect to this parameter. Weightings are key variables in development of the multiplicative utility function. This is clearly demonstrated in Table (5-19), where, alternatives rankings drastically change from one weighting set to another.

CHAPTER VI

6.1- CONCLUSION

An attempt has been made to develop a rationalized design decision making tool that assists designers during the life cycle of a building project's development.

The objectives set forth for this research work were:

1- To develop a framework for the building design decision-making problem ;

2- To investigate/ develop financial or cost models in the construction and operating phases of building projects;

3- To identify the sources of uncertainty, develop probabilistic cost models, and demonstrate how they can be incorporated into the design decision-making process;

4- To develop a multiple-objective design decision-making model capable of incorporating both quantitative and qualitative objectives; and

5- To examine models and the methodology through a case study.

A general framework has been constructed and used to develop deterministic and probabilistic performance measures. In this framework, evaluation criteria are decomposed into series of components (at system/ subsystem levels). These components are evaluated and integrated to measure the overall performance of a building project. Here, the design evaluation criteria are expressed as a function of a set of design decisions or control variables, a set of cost coefficients, and time. This framework provides a uniform structure that is applicable in different phases of the project development, and also provides consistent information to different design disciplines and the building operators.

Emphasis has been placed on financial performance measures. Cost models and expenditure models are developed for different elements of a building (mainly design/ construction and operation). The sensitivity of total capital cost with respect to expenditure models and different variables in the capital cost has been studied. It has been demonstrated that the capital expenditure model makes little contribution to the inaccuracy of the capital cost estimate and the uniform model provides a good approximation to the capital expenditure model.

Different methods have been examined for treatment of the uncertain nature of some of the variables in the cost models. Based on the output information needed and the input information required, the moments approach was selected. It has been shown that the moments approach can be used for probabilistic analysis of different design decision-making criteria having either additive or multiplicative structures.

It was demonstrated that the building design problem is not a single criterion problem, but a multiple criteria one, where the criteria (or the nature of its variables) change from one stage of the project development to the next. Ignoring one or more of these criteria (especially the important ones) may result in a different design decision. Different decision criteria are cited, in particular the Life Cycle Cost relationship which treats all phases of a building project and includes the objectives of the investor (financial), of the designer (technical), and of the ultimate user of the building.

Multiple criteria decision-making is suggested as the most comprehensive approach to the formulation of the building design decision-making problem. A four-stage design decision-making methodology has been suggested that employs the utility and preference theories. A multiplicative utility function has been developed that treats the problem of criteria overlappings. The weighting parameters, used in this function, are calculated based on the basic information that is supplied by investor. This methodology incorporates both quantitative and qualitative objectives and has the capability of treating the random variables (design, time, cost, etc.). It is recommended that the explicit use of the subjective variables such as weightings be delayed until the last stage of the selection process where it becomes necessary. Through the case study, it has been demonstrated that the choice of a design alternative will vary depending on the type of investor, the stated objectives, and the criteria selected.

6.2- RECOMMENDATIONS

Topics which should be treated in future studies include:

1) Development of performance and cost (capital and future costs) data banks for different building subsystems;
2) Investigating the feasibility of estimating the correlation coefficients between different variables in evaluation criteria (from historical data) and the possibility of using the methods applied in social sciences for treatment of correlations;
3) Treatment of the correlations by isolating the sources of correlations in the cost models and in the evaluation criteria;
4) Development of cost estimation and performance prediction methodologies using cost/ performance data banks, cost models, and statistical methods;
5) Exploring ways and means of gathering information that is required for probabilistic analysis;
6) Future investigation of relationships between design decisions and user performance/ costs and documentation of the results for use by designers and investors;
7) Incorporating the fuzzy set theory in the multiple-criteria decision-making methodology;

8) Investigating the practical application of the methodology presented through interviews with designers, and comparing the actual decisions made and the suggested solution by this methodology; and

9) Implementing the decision-making methodology in a computerized environment and studying meaningful user interfaces to allow or facilitate the use of complex decision making methodologies.

REFERENCES

[1] "Conserving Energy in Rehabilitated or Retrofit Buildings", Round Table, Architectural Records, Dec. 1983.

[2] Sidwell, A.C. , "A Model For The Design of Project Organization", The Department of Civil Engineering and Construction, The University of Astin in Birmingham.

[3] Russell, A.D., Arlani, G.A., " Objective Functions for Optimum Building Design", Computer Aided Design Journal, 1982, pp. 327-337.

[4] Hartkopf, V., "An Economy Model for Generating, Evaluating, and Selecting Architectural Design Alternatives", Institute of Physical/Planning, Carnegie Mellon University, Report No. 38, 1973.

[5] Linzey, M.P.T., Brotchie, J.F., Nicholas, J.F., "Economic Analysis of Tall Buildings - Total Building Design Using Digital Computer", Build International, Vol. 7, 1974.

[6] Tumarkin,J. E., "Parametric Estimating of Construction Cost Using Statistical Methods", Proceedings of AACE Conference, 1971, pp. 198-206.

[7] Kouskoulas, V., Koehn, E., "Predesign Cost-Estimation Function for Buildings", Journal of Construction Division, ASCE, Dec. 1974.

[8] Russell, A.D., "Information Models in Building Design Process", Engineering Issue - Journal of Professional Activities, ASCE, Vol. 101, No. E12, Proc. Paper 11217, April, 1975, pp 133-148.

[9] Misurak, M.M., " Preconstruction Financial Planning of Large Projects" , Thesis submitted in partial fulfillment of the requirements for the degree of M.Eng., Centre for Building Studies, Concordia University, 1983.

[10] Flanagan, R., Norman, G., "Risk Analysis- An Extension of Price Prediction Techniques for Building Work", Construction Papers, Vol. 1, No. 3, 1982, pp. 27-34.

[11] Bacher, L.C., "Contingency Evaluation in Capital Cost", AACE Transactions, 1984, pp. B.6.1-B.6.2.

[12] Russell, A.D., Morrison, R. and Tong, T., "Computer Assisted Economic Evaluation of Design Alternatives: LIFE", Third International Symposium on Building Economics, July 1984, Ottawa, Canada, pp. 20-34.

[13] Zinn, C.D., Lesso, W.G., Motazed, B., "A Probabilistic Approach to Risk Analysis in Capital Investment Projects", The Engineering Economist, Vol. 22, No. 4, 1976, pp. 239-260.

[14] Wilson, A.J., Templeman, A.B., "An Approach to the Optimum Thermal Design of Office Buildings", Building and Environment, Vol. 11, No. 2, 1976, pp. 39-50.

[15] Russell, S.O.D., Caselton, W.F., "Surrogate Utilities in Civil Engineering- The Kelly Criterion and Shannon's Information", Dep. of Civil Engineering, The University of British Columbia, 1984.

[16] Bussey, L.E., Stevens, G.T., "A Solution Methodology for Probabilistic Capital Budgeting Problems Using Complex Utility Function", The Engineering Economist, Vol. 21, No. 2, 1975, pp. 89-109.

[17] Radford, A.D., Gero, J.S., "Tradeoff Diagrams for the Integrated Design of the Physical Environment in Buildings", Building and Environment, Vol. 10, 1982. pp. 3-15.

[18] Radford, A.D., Gero, J.S., Murthy, N.S., "Designing Optimal Multi-Functional Surface Materials", Dep. of Architectural Science, Faculty of Architecture, University of Sydney, Presented at the 8th Australian Conference on Mech. Struct. Mats., Newcastle University, Newcastle, 1982.

[19] Murthy, N.S.H., "Multifunctional Building Enclosure Design", Dep. of Architectural Science, Faculty of Architecture, University of Sydney, 1983.

[20] Mattar, S., Bitterlich, W., Manning, P. and Fazio, P., "A Decision Model for the Design of Building Enclosures", Building and Environment, Vol. 13, 1978, pp 201-216.

[21] Mattar, S., Manning, P., Fazio, P. and Bitterlich, W., "Application of the Decision by Exclusion Rule: The Design of External Walls", Building and Environment, Vol. 13, 1978, pp 217-232.

[22] Gupta, C.L., Anson, M., "Thermal Design of Building Envelopes for Minimum Total Cost", Build International, Nov./Dec. , 1972, pp. 363-371.

[23] Arlani, G.A., "Life Cycle Cost and Risk Analysis", Thesis submitted in partial fulfillment of the requirements for the degree of M.Eng., Centre for Building Studies, Concordia University, 1981.

[24] Wilson, A., "Experiments in Probabilistic Cost Modeling", Building Cost Techniques: New Directions, E. & F.N. Spon, London, 1982, pp. 169-180.

[25] Diekmann, J.E., "Probabilistic Estimating: Mathematics and Application", Journal of Construction Engineering and Management, Vol. 109, No. 3, Sep. 1983, pp. 297-308.

[26] Silver, A., "A Computerized Costing Methodology Applied to Military Systems", Martin Marietta Corporation, Denver, Colorado.

[27] Stadler, W., "A Survey of Multicriteria Optimization or the Vector Maximum Problem. Part I: 1776-1960", Journal of Optimization Techniques and Application, Vol. 29, No. 1, Sep. 1979, pp. 1-52.

[28] Hwang, C.L., Paidy, S.R., Yoon, K. and Masud, A.S.M., "Mathematical Programming with Multiple Objectives: A Tutorial", Computer and Operations Research, Vol. 7, 1980, pp. 5-31.

[29] Thorn, R., Turnbull, J.B.A., "Architectural Form, Function and User Satisfaction for Two High Rise Flats for the Elderly", Architectural Science Review, Vol. 26.3, Dec. 1983, pp. 76-80.

[30] "Documentation and Assessment of GSA/PBS Building Systems Program", National Bureau of Standards/ Centre for Building Technology, NBSIR 83-2662, Feb. 1983.

[31] " The Special Design and Specification Problem in Rehabilitation and Re-use", Round Table, Architectural Record, Dec. 1982.

[32] Newnan, D.G., "Engineering Economic Analysis", 2nd edition, Engineering press Inc. , 1983.

[33] Russell, A.D., "Building Economics- Course Notes", Centre for Building Studies, Concordia University, 1979.

[34] "1980 Fundamentals", American Society of Heating, Refrigerating, and Air-Conditioning Engineers, 1980.

[35] Newton, S. , "Cost Modeling: A Tentative Specification", Building Cost Techniques: New Direction, E. & F.N. Spon, London, 1982, pp. 193-209.

[36] Karshenas, S., " Predesign Cost Estimating Method for Multistory Buildings", ASCE, Journal of Construction Engineering and Management, Vol. 110, No. 1, 1984, pp. 79-86.

[37] Hardcastle, C. , " Capital Cost Estimating and The Method of Presenting Information for Pricing of Construction Work", Building Cost Techniques: New Directions, E & F.N. Spon, London, 1982, pp. 373-377.

[38] Sidwell, A.C. and Rumball, M.A., " The Prediction of Expenditure Profiles for Building Project", Building Cost Techniques: New Directions, E. & F.N. Spon, London, 1982, pp. 324-340.

[39] de La Mare, R.F., " Modeling Capital Expenditure", Engineering and Process Economics, Vol. 4, 1979, pp. 467-477.

[40] Shlomo Peer, "Application of Cash Flow Forecasting Models", ASCE , Vol. 108, June 1982, pp. 226-260.

[41] Russell, A.D., "Life Cycle Costing", Energy, Mines and Resources Canada, Task No. CB81-L1, 1983.

[42] Tucker, S.N., " Modelling Building Life Cycle Cash Flows", Proceedings, Vol 1., Third International Symposium on Building Economics, Ottawa, 1984, pp. 38-48.

[43] Holmes, R. and Droop, C., " Factors Effecting Maintenance Costs in Local Authority Housing", Building Cost Techniques: New Directions, E. & F.N. Spon, London, 1982, pp. 398-409.

[44] Fuller, R., " Life Cycle Costing - A Comprehensive Approach", Building Cost Techniques: New Directions, E.&F.N. Spon, London, 1982, pp. 439-446.

[45] Baron, L.I. and Sterling, E.M., "Does Indoor Air Pollution Affect Office Productivity", Proceedings, Third International Symposium on Building Economics, Vol. 5 , Ottawa, 1984, pp. 21-28.

[46] Reisman, A., Rao, A.K., " Discounted Cash Flow Analysis: Stochastic Approach", Publication No. 1 in the Monigraph Series, Engineering Economy Division, American Institute of Industrial Engineers Inc., 1973.

[47] "Downtown and Suburban Office Building Experience Exchange Report", Buildings Owners and Managers Association International (BOMA).

[48] Sterling, T.D., Sterling, E., and Dimich-Ward H.D., "Air Quality in Public Buildings With Health Related Complaints", ASHRAE Transactions, No. 2773, Vol. 89, 1983.

[49] Poulton, E.C., " The Environment at Work", Applied Ergonomics, Vol. 3, No. 1, 1972, pp. 24-29.

[50] Micheal, B., Margulis, S.T., Konar, E., and BOSTI, " Using Office Design to Increase Productivity", Workplace Design & Productivity, 1984.

[51] Schlaifer,R. " Probability and Statistics for Business Decisions", McGraw-Hill, 1959.

[52] Hiller, F.S. and Lieberman, G.J., "Operations Research", Holden-Day, Inc. 1974.

[53] Crane, M.A. and Iglehart, D.L., "Statistical Analysis of Discrete-Event Simulation", Proceedings of Simulation Conference, Washington D.C., 1974.

[54] Siddall, J.N., "Analytical Decision Making in Engineering Design", Prentice Hall, 1972, pp. 12-79.

[55] Hull, J.C., "The Evaluation of Risk in Business Investment", Pergamon Press, 1980.

[56] Pouliquen, L.Y., "Risk Analysis in Project Appraisal", John Hopkins Press, 1970.

[57] Birnbaum , Z.W., " Numerical Tabulation of the Distribution of Kolmogorov's Statistics for Finite Sample Size", American Statistical Association Journal, 1952, pp. 425-441.

[58] Massey , F.J. Jr., "The Kolmogorov-Smirnov Test for Goodness of Fit", American Statistical Association Journal, 1951, pp. 68-77.

[59] Duran, S.B., "A Survey of Nonparametric tests for scale", Communications in Statistics: Theory and Methods, 1976, pp. 1287,1312.

[60] Hoffman, G.M., " Project Cost Control: Dynamic Risk Analysis of Randomly Ordered Sequential Decisions Under Uncertainty", The Institute of Management Science, 1982, pp. 45-50.

[61] Slater, K., " An Investigation into Hospital Maintenance Expenditure in the North West Regional Health Authcrity", Building Cost Techniques: New Directions, E. & F.N. Spon, London, 1982, pp. 410-420.

[62] Aho, T., "The Building Maintenance Data- A Tool for Energy and Maintenance Planning", CIB83, Vol. 1b, 1983, pp. 153-161.

[63] Janicki, S., Zaleski, S., "Bases for Maintenance, Repair, and Modernization of Building", CIB83, Vol. 1b, 1983, pp. 185-192.

[64] Grover, C., Grover, R., " Exploration Towards An Optimal Inspection Policy for the Failure of Building Components", Building Cost Techniques: New Direction, E.&F.N. Spon, 1982, pp. 421-435.

[65] Meredith, D.D., Wong, K.W., Woodhead, R.W., and Wortman, R.H., "Design and Planning of Engineering Systems", Prentice-Hall, 1973,

[66] Hordijk, A. , " Dynamic Programming and Markov Potential Theory", Amsterdam, Mathematisch Centrum, 1974.

[67] Experience Exchange Committee, "Income/ Expense Analysis: Apartments", Institute of Real Estate Management (IREM).

[68] Kangari, R., Boyer, L.T., "Project Selection Under Risk",Journal of the Construction Division, Proceedings of the American Society of Civil Engineers, ASCE, Vol. 107, No. CO4, Dec. 1981, pp. 597-608.

[69] DeWisepelare, A.R., Sage, A.P., "On Combined Multiple Objective Optimization Theory and Multiple Attribute Utility Theory for Evaluation and Choice-making",Dep. of Engineering Science and Systems, University of Virginia, 1980.

[70] Baired,B.F., "Introduction to Decision Analysis", Duxbury Press, 1982.

[71] Keeny, R.L., Raiffa,H., "Decisions With Multiple Objectives: Preference and Value Tradeoff", Weily, 1976.

[72] Herman, H.H., "Modern Factor Analysis", Chicago Press, 1973.

[73] Limmelblau, D.M., "Process Analysis By Statistical Methods", Sterling Swift Publishing Company, 1970, pp. 143-170.

[74] Roseman, M.A., "Computer Aided Decision-Making in the Buildings", a thesis submitted for the Ph.D. degree in the Dep. of Architectural Science, Faculty of Architecture, University of Sidney, 1981.

[75] Siskos, J., "A way to Deal With Fuzzy Preferences in Multiple Criteria Decision Problems", Europian Journal of Operation Research, Vol. 10, 1982.

[76] Hwang, C., Yoon, K., "Multiple Attribute Decision - Making, A State of Art Survey", No. 186, Springer-Verlag, 1981.

[77] Withmore, G.A., Findlay, M.C., " Stochastic Dominance - An Approach to Decision Making Under Risk", Lexington Books, 1978.

[78] Swalm, R.O., "Utility Theory - Insights into Risk Taking", Harvard Business Review, Nov./Dec. 1966.

[79] Kottas, J.F., Lau, H.S., " A Four-Moments Alternative to Simulation for a Class of Stochastic Management Problem", Management Science, Vol. 28, No. 7, July 1982.

[80] Gupta, S.K. and Cozzolino, J.M., " Fundamentals of Operation Research for Management", Holden- Day, 1975.

[81] Withmore, G.A. " Applied Decision Analysis - Course Notes", Dep. of Management Science, McGill University, 1982.

[82] Hanley, A.J., " A Case Study of the Effect of Enclosure Elements on Energy Consumption In a High Rise Office Building", A major Technical Report, Centre for Building Studies, Concordia University, 1983.

[83] "Computer Aided Building Energy Systems Analysis Using Meriwether Energy System Analysis Program", Reference Manual, Dep. of Public Works, Canada, 1976.

[84] "Electricity Rates", Hydro Quebec, 1980-1981.

[85] " Gas Metropolitan Rate Structure", June 4, 1980.

APPENDIX A

VARIABLE DEFINITIONS

LIST OF VARIABLES

A = Area.

a_k = Combined effect of aging and maintenance policy.

$A(t)$ = Amortization at time t (repayment of principal only).

ATCF = After Tax Cash Flow.

BTCF = Before Tax Cash Flow.

B_c = Present worth of time factor for capital cost.

B_{cca} = Present worth of time factor for capital cost allowance.

B_f = Present worth of time factor of the financing cost.

B_m = present worth of time factor for maintenance cost.

B_o = present worth of time factor for operating cost.

B_p = present worth of time factor for periodical repair cost.

B_r = Present worth of time factor for renewal cost.

B_s = Present worth of time factor for selling cost.

B_u = Present worth of time factor for user cost.

C = Disbursements including initial equity and interest payments.

C_c = Initial cost of the project (C_i in section 2.3.2).

C_f = Future cost of the project.

C_l = Labour cost.

C_m = Maintenance cost.

C_o = Operating cost.

C_p = Periodical repair costs.

C_r = Replacement costs.

$C_{r,m}$ = Total maintenance and replacement costs.

C_u = User costs.

CCA = Capital Cost Allowance.

CF = Cumulative cash flow.

cf = cash flow.

cf' = Inflation adjusted cash flow.

COV = Covariance.

e = Exponent (e=2.71).

E = Expected value.

E.C. = Energy Consumption.

eff = Efficiency.

F = Financing cost (Fc).

g_{ij} = Design related function.

h_{ij} = Time related function.

i = System i.

I = Interest rate.

j = Subsystem j.

K = Scaling factor.

LCC = Life Cycle Cost.

M = Mobilization cost.

NPV = Net Present Value.

O = Overhead allowance.

O_k = Weighting assigned to objective k.

p = Beta model parameter.

P = Profit allowance.

P_e = Equipment productivity.

P_l = Labour productivity.

P_{ij} = Probability of being in state j, in year i.

p_{xj} = Probability of going from state x to state j, within one year.

$P_{u,e}$ = Equipment productivity (user related).

$P_{u,l}$ = Labour productivity (user related)

pp = Payback period.

PW = Present worth

q = Beta model parameter.

Q = Quantity.

r = Discount rate.

r(x) = Risk coefficient as a function of variable x.

R = Revenue.

s = Coefficient of risk aversion.

S = Selling price.

t = Time.

T = Tax rate.

T_c = Construction duration.

T_e = Study Period.

T_f = Finish time of construction.

T_s = Start of construction

Vx = Coefficient of variation.

V_{kj} = Weighting which can be obtained from objective/ Criteria matrix.

W_j = Computed weighting for criteria j.

X_i = Vector of cost related coefficients.

$\bar{X}$ = Mean value of X.

Y = Evaluation criterion Y.

Z_i = Vector of design decision variables.

U(X) = Utility of variable X.

U' = First derivative of utility function.

U'' = Second derivative of utility function.

θ_j = Inflation rate.

τ = Time.

α = % of cost to be financed.

β = de La Mare model parameter.

η = de La Mare model parameter.

μ_r = rth central moment.

σ_X^2 = Variance of variable X.

γ_3 = Coefficient of skewness.

ρ = Correlation.

Π = Product notation.