OPTIMUM LOCATION OF HINGED JOINTS
IN H-FRAME STRUCTURES

Joseph Gallaccio

A

Dissertation

in

The Faculty

of

Engineering

Presented in Partial Fulfillment of the Requirements
for the Degree of Master of Engineering at
Concordia University
Montréal, Québec, Canada

April 1980

OPTIMUM LOCATION OF HINGED JOINTS IN H-FRAME STRUCTURES

Joseph Gallaccio

ABSTRACT

The application of prefabricated concrete structures has increased over the years. Because of the growing concern for reducing escalating construction costs, and the growing prefabrication industry of concrete components, the need to study the behavioral qualities of moment released prefabricated H-Frames has become of great interest.

Congested reinforcement in precast H-Frames often makes the fabrication of these units difficult to manufacture. However when hinge joints are efficiently located, precast units can be produced such that their shape reflects the least amount of reinforcement required for the strength and serviceability of the structure.

The development of a computer program which is geared especially to the analysis of hinged frames accurately displays the behaviour as joint locations vary. The comparison of rigid frames versus the optimized hinged frames aided by the computer analysis indicates the performance of geometrically identical structures under identical loading conditions.

Acknowledgement

The author wishes to thank Prof. Z. A. Zielinski for his guidance and for allowing the inclusion of material regarding his (Patent Pending) Modular H-Frame system. Also a word of thanks to Prof. M. McC. Douglass for his helpful suggestions.

Table of Contents

Table of Contents (cont.)

List of Tables

List of Illustrations

List of Illustrations (cont.)

List of Illustrations (cont.)

List of Illustrations (cont.)

List of Illustrations (cont.)

List of Symbols

(Q)	Structure joint load matrix.
(S)	Structure stiffness matrix.
(D)	Structure joint displacement matrix.
h	Storey height.
y	Column pin location.
L	Bay width.
b, d	Frame member cross-section dimensions.
E_c	Modulus of elasticity.
A_c, A_b	Cross-sectional area of column and beam.
I_c, I_b	Moment of inertia of column and beam.
ΔL	Differential settlement.
β	Non-dimensional settlement ratio.
N	Segmental divisions in a column.
NST	Number of structure storeys.
X, Y	Global axis coordinates.
i, j	Member nodes.
Fx, Fy, Mxy	Joint Actions (global Axis).
w	Uniformly distributed member load.
p	Concentrated member load.
x	Reference dimension.
Dx, Dy, Dxy	Support settlements.

1. INTRODUCTION:

The objective of this dissertation is to investigate how the performance of H-Frame structures is influenced by the locations of the hinge joints. In particular, a computer program has been developed (Appendices A, B and C) to aid in the optimization process.

Although buildings may consist of any combination of frames, shear walls, cores and other structural systems, the type of structure under consideration is the planar frame. This planar restriction is satisfactory since three-dimensional buildings are usually reduced to an assemblage of two-dimensional sub-structures for ease in the analysis, and also the two-dimensional analysis yields conservative results with respect to the three-dimensional structure.

The comparison of geometrically similar rigid and hinged frames, each under identical loading conditions, are made by comparing:

1) area of the maximum bending moment envelope.

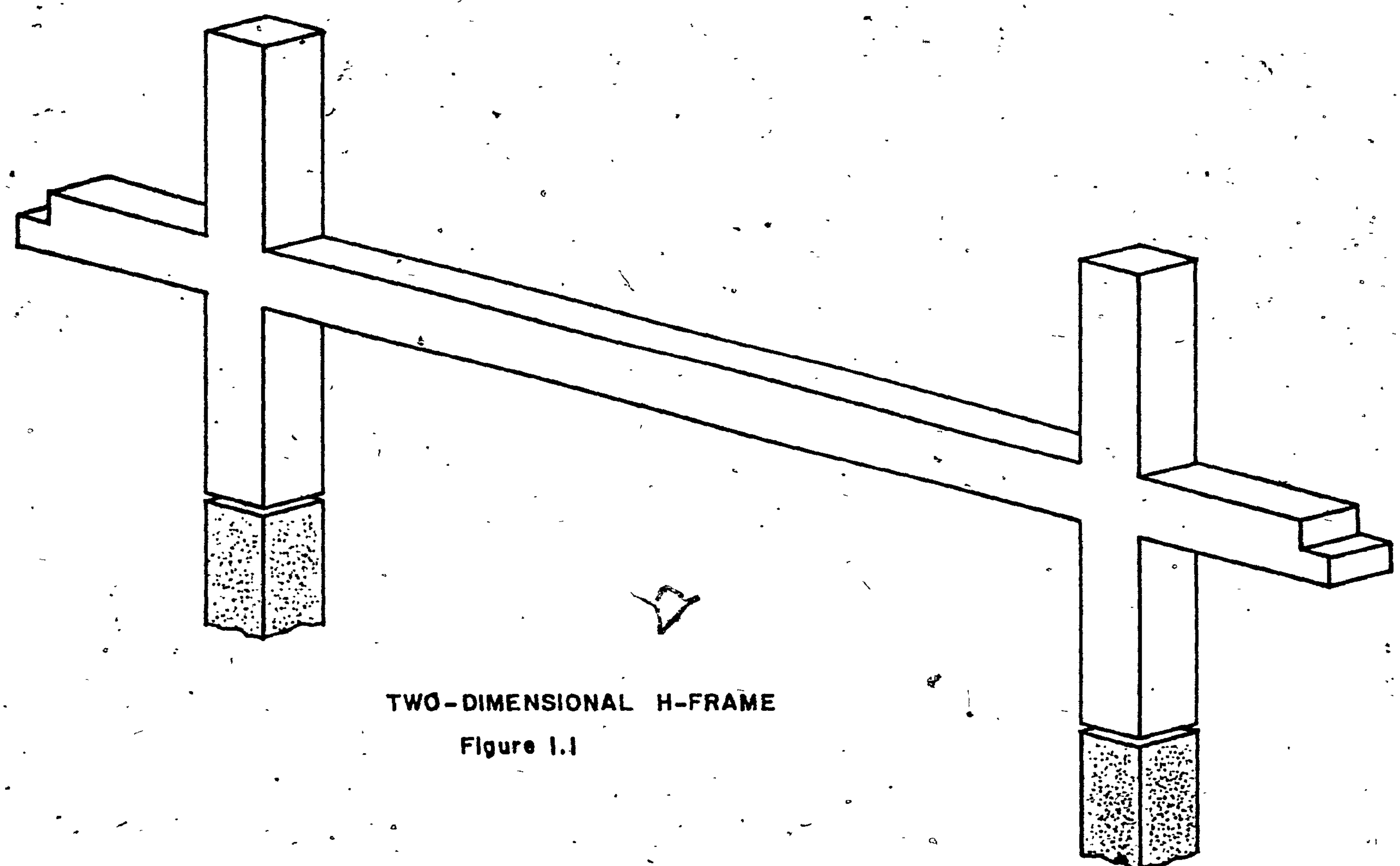
2) the structure's drift.

The types of loading cases considered are gravity and lateral loads (wind or earthquake) as well as differential foundation settlements.

1.1 APPLICATION OF H-FRAMES:

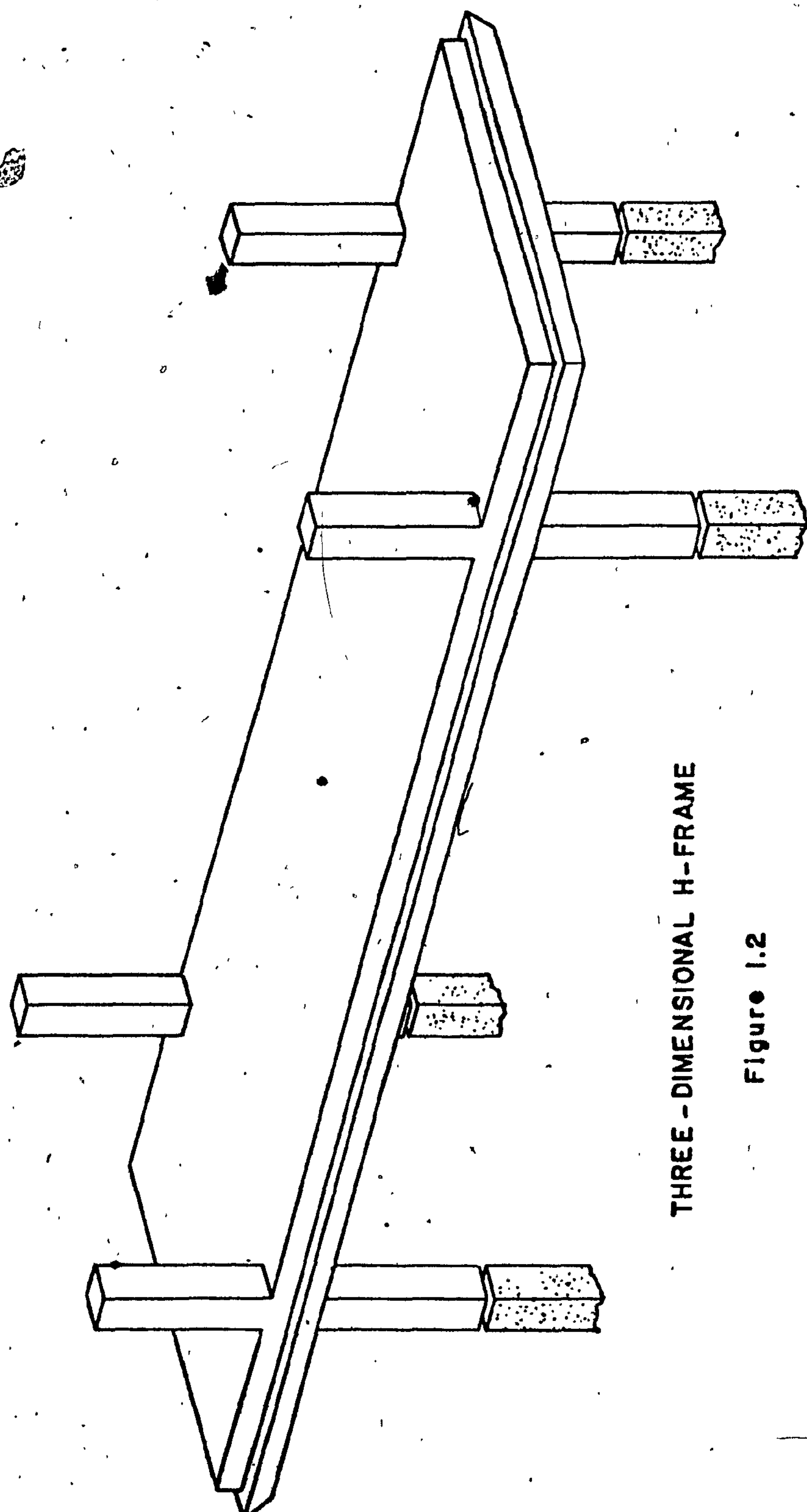
H-Frames can be classified under two different types:

1) plane H-Frame (two-dimensional)



TWO-DIMENSIONAL H-FRAME

Figure I.I



THREE-DIMENSIONAL H-FRAME

Figure 1.2

and 2) spatial H-Frames (three-dimensional)

Perspective views of the plane and spatial (Zielinski) H-Frames are shown in Figure 1.1 and 1.2 respectively.

In practice, both of these systems have been used with great success because of their applicability to prefabrication. Bobrowski (Ref. 1) used H-Frame units for the construction of grandstands (Figure 1.3). In other projects, Zielinski has used H-Frame components as two and three-dimensional units. An office/apartment building in Lloydminster, Alberta (Figure 1.4) was constructed as a modified H-Frame system with hinge joints located at the column base. Three-dimensional H-Frame units were also used for the construction of a motel/office complex (Figure 1.5) in Edmonton, Alberta.

Although the H-Frames of Figures 1.1 and 1.2 differ in geometry, their structural performance in the longitudinal direction is similar. With respect to a multi-bay/multi-storey structure, both frame types can be idealized as a combination of H-Frame components coupled by link-beams. A typical frame model is illustrated in Figure 1.6 where hinge joints are "naturally" located where H-Frames and/or link beams are connected. These joints (pins)* are member discontinuities which transmit axial and shear forces but do not transmit moments.

*The terms hinge and pins are used synonymously.

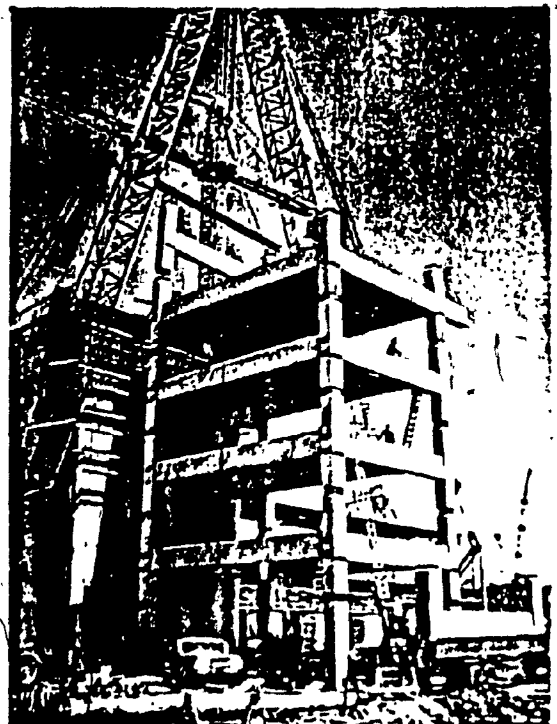


Figure 1.3

Erection of the Calgary Stampede and Exhibition Grandstand.

Figure 1.4

Application of a two-dimensional H-Frame system in Lloydminster, Alberta.
(Designed by Zielinski, Z.A.,1978)

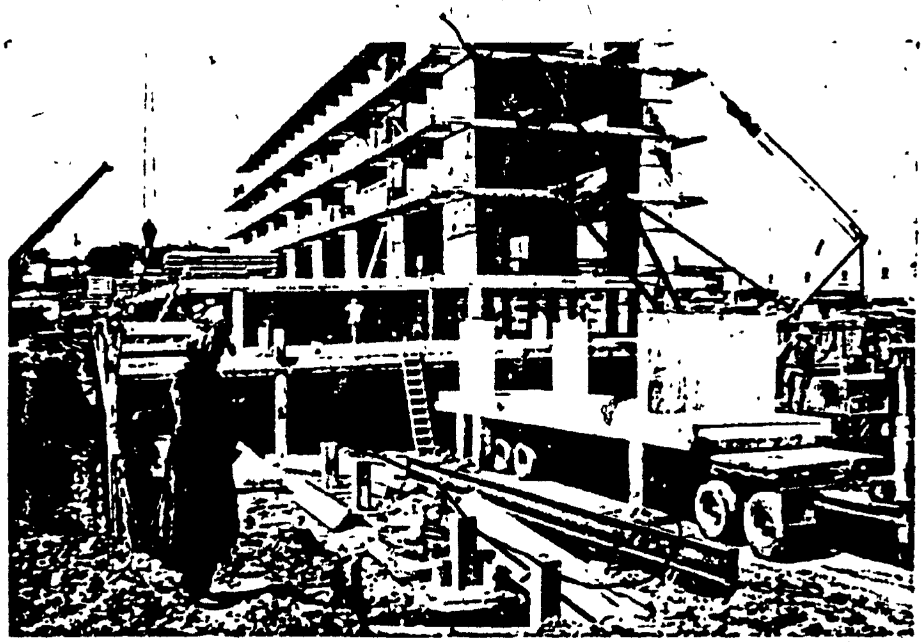
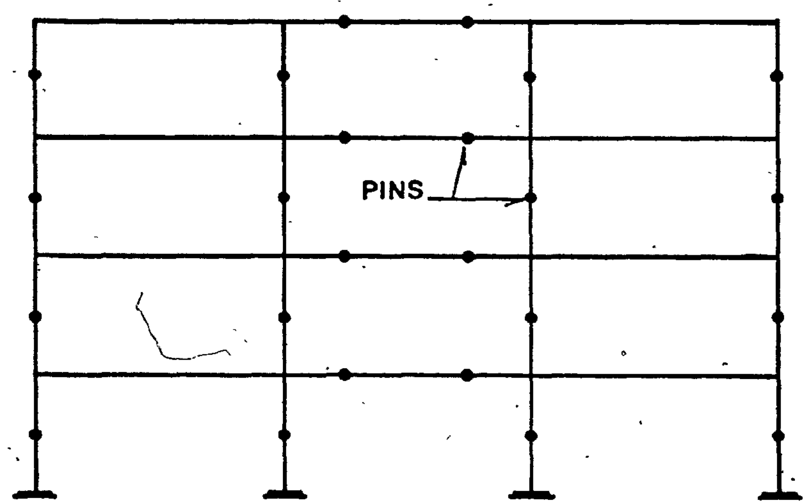


Figure 1.5

Application of a three-dimensional
H-Frame system in Edmonton, Alberta.
(Designed by Zielinski, Z.A.,1978)



H-FRAME MODEL

Figure 1.6
Typical H-Frame model with "natural" pin locations.

2. METHOD OF ANALYSIS:

The most commonly used techniques for the analysis of indeterminate structures are the "Flexibility" and "Stiffness" methods for which numerous computer programs have been developed. The program listed in Appendix C employs the stiffness method of analysis. This method implies that all structural materials used in the analysis behave in a linear-elastic fashion. In practice, concrete is assumed to behave as such. The program takes into account bending and axial deformations but neglects shear deformation of frame members.

2.1 ITERATIVE FRAME ANALYSIS APPROACH:

In general terms, the stiffness method of analysis is governed by the linear-elastic relationship

$$(Q) = (S) \times (D) \quad .....(2.1a)$$

where: (Q) = joint load matrix

(S) = structure stiffness matrix

(D) = joint displacement matrix

therefore $$(D) = (S)^{-1} \times (Q) \quad .....(2.1b)$$

equation 2.1b shows that in order to determine the corresponding joint displacements of the structure, the inverse of the stiffness matrix $(S)^{-1}$ must exist. The stiffness method therefore requires that the structure be explicitly defined with respect to dimensions, material properties, loads and the location of any hinge joints, thus, the trend of analysis must then employ a technique of successive trials. The outcome of these trials would then indicate the positions where hinges create the least overall bending moment envelope and the least structure drift.

2.2 COMPUTER ANALYSIS USING PFRAME:

Multi-bay/multi-storey buildings are highly indeterminate structures thus their solution by hand methods is only approximate and often complicated. The use of a computer greatly increases the accuracy and provides results within a short period of time. The stiffness method as well as the iterative approach outlined in section 2.1 have been used to develop the computer program PFRAME.

The internal organization of the program is outlined in Appendix A. This organization indicates how the program is structured in terms of the types of analyses that are possible, the general flow of the program and the functions of individual subroutines.

A users manual has been included in Appendix B where structures are analyzed according to the four possible Dispose Types (also in Appendix A). In order to explain the required input data, descriptive notes have been added.

Lastly, a listing of PFRAME is presented in Appendix C.

3. SAMPLE FRAME ANALYSIS:

Studies have been undertaken (Ref's 2 & 3) to compare the performance of rigid and hinged frames for various load cases. In particular, typical multi-bay/multi-storey plane frames have been analyzed for 1) gravity and lateral loads and 2) for differential foundation settlements.

3.1 GRAVITY AND LATERAL LOADS:

The most commonly applied loads on a building (residential or commercial) are gravity and lateral loads where gravity loads consist of the stationary dead load present during the structure's life-time, and where live loads (transient) are due to occupancy. The lateral loads are either wind or earthquake forces which (according to code specifications) are usually lumped as equivalent forces applied at the floor levels.

These loads have been applied to the typical frame of Fig. 3.1.1 where the hinged frame has been studied for various column pin locations (Dispose Type 11). The structure of Fig. 3.1.1 is reduced to two sub-structures (Fig's 3.1.2 and 3.1.3). These sub-structures differ in that beam cantilevers are not present along one column line of Fig. 3.1.3.

Table 3.1 summarizes the assumed frame geometry, element cross-sections and different loading cases applied to the sub-structures.

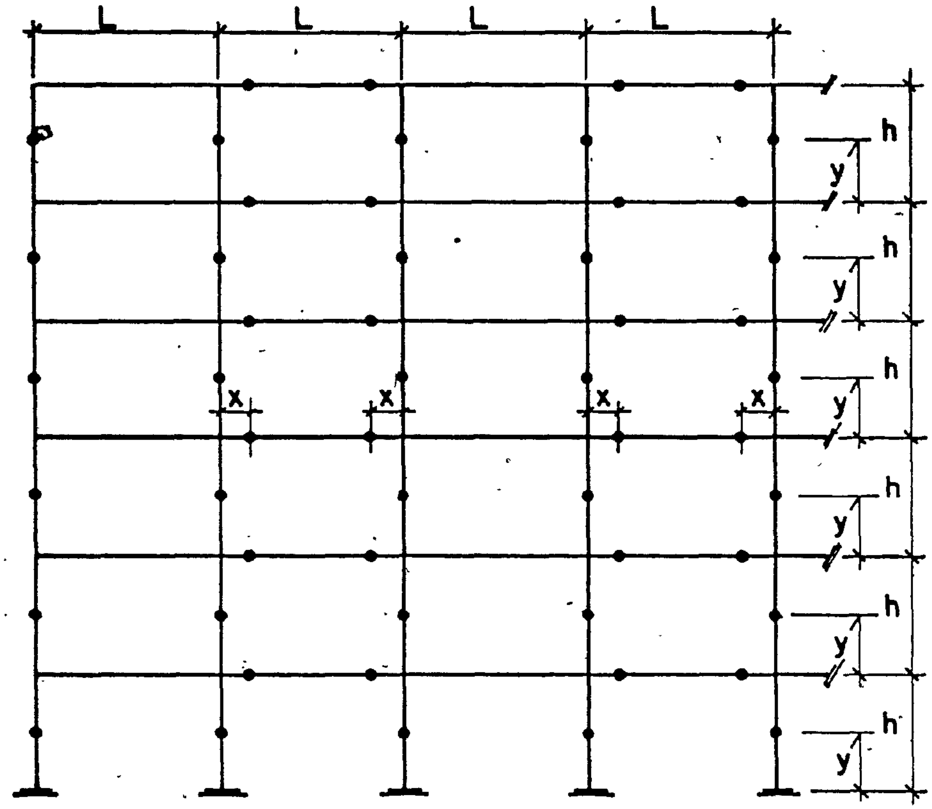


Figure 3.1.1 (x=0.15L)
Multi-bay/multi-storey H-Frame structure.

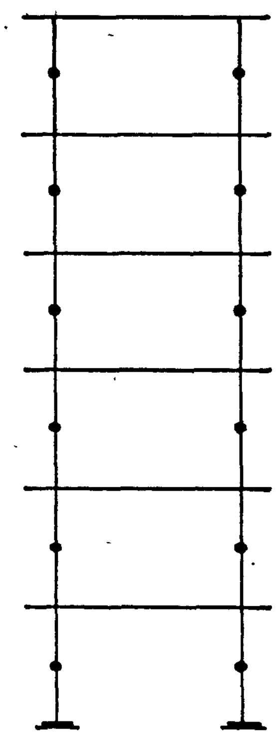


Figure 3.1.2
Single-bay/multi-storey sub-structures with cantilivers on both sides.

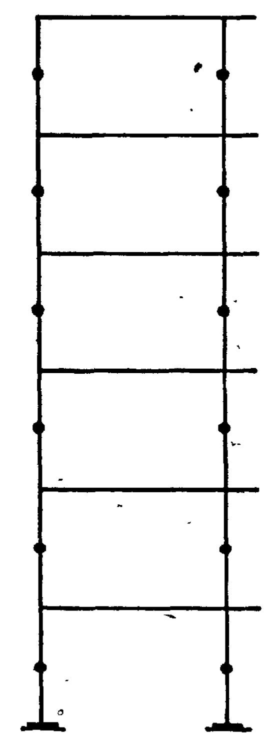


Figure 3.1.3
Single-bay/multi-storey sub-structures with cantilevers on one side.

Table 3.1 Frame Properties and Applied Loading

Study Case Number	Presence of Cantilevers	Storey Height h	Bay Width L	L/h	Column Cross Section b x d	Beam Cross Section b x d	Ec Modulus of Elasticity for Concrete	Type of Loading
		m	m	-	cm x cm	cm x cm	kg/cm²	
1	both sides	3.0	6.0	2	30 x 60	30 x 70	200.000	G.L.: D.L. = 2.5t/m + L.L. = 2.5t/m
2	both sides	3.0	9.0	3	40 x 80	40 x 90	200.000	G.L.: D.L. = 2.5t/m + L.L. = 2.5t/m
3	both sides	3.0	12.0	4	50 x 100	50 x 110	200.000	G.L.: D.L. = 2.5t/m + L.L. = 2.5t/m
4	both sides	3.0	6.0	2	30 x 60	30 x 70	200.000	W.L.: 1.5t in each beam level
5	both sides	3.0	9.0	3	40 x 80	40 x 90	200.000	W.L.: 1.5t in each beam level
6	both sides	3.0	12.0	4	50 x 100	50 x 110	200.000	W.L.: 1.5t in each beam level
7	both sides	3.0	6.0	2	30 x 60	30 x 70	200.000	E.L.: 5.0% of G.L.
8	both sides	3.0	9.0	3	40 x 80	40 x 90	200.000	E.L.: 5.0% of G.L.
9	both sides	3.0	12.0	4	50 x 100	50 x 100	200.000	E.L.: 5.0% of G.L.
10	both sides	3.0	6.0	2	30 x 60	30 x 70	200.000	G.L. & W.L.: 2.5t/m + 2.5t/m + 1.5t in each level
11	both sides	3.0	9.0	3	40 x 80	40 x 90	200.000	G.L. & W.L.: 2.5t/m + 2.5t/m + 1.5t in each level
12	both sides	3.0	12.0	4	50 x 100	50 x 110	200.000	G.L. & W.L.: 2.5t/m + 2.5t/m + 1.5t in each level
13	both sides	3.0	6.0	2	30 x 60	30 x 70	200.000	G.L. & E.L.: 2.5t/m + 2.5t/m + 5.0% of G.L.
14	both sides	3.0	9.0	3	40 x 80	40 x 90	200.000	G.L. & E.L.: 2.5t/m + 2.5t/m + 5.0% of G.L.
15	both sides	3.0	12.0	4	50 x 100	50 x 110	200.000	G.L. & E.L.: 2.5t/m + 2.5t/m + 5.0% of G.L.
16	one side	3.0	6.0	2	30 x 60	30 x 70	200.000	G.L.: 2.5t/m + 2.5t/m
17	one side	3.0	9.0	3	40 x 80	40 x 90	200.000	G.L.: 2.5t/m + 2.5t/m
18	one side	3.0	12.0	4	50 x 100	50 x 110	200.000	G.L.: 2.5t/m + 2.5t/m
19	one side	3.0	6.0	2	30 x 60	30 x 70	200.000	E.L.: 5.0% of G.L.
20	one side	3.0	9.0	3	40 x 80	40 x 90	200.000	E.L.: 5.0% of G.L.
21	one side	3.0	12.0	4	50 x 100	50 x 110	200.000	E.L.: 5.0% of G.L.
22	one side	3.0	6.0	2	30 x 60	30 x 70	200.000	G.L. & E.L.: 2.5t/m + 2.5t/m + 5.0% of G.L.
23	one side	3.0	9.0	3	40 x 80	40 x 90	200.000	G.L. & E.L.: 2.5t/m + 2.5t/m + 5.0% of G.L.
24	one side	3.0	12.0	4	50 x 100	50 x 110	200.000	G.L. & E.L.: 2.5t/m + 2.5t/m + 5.0% of G.L.

Notations: D.L. = Dead Load W.L. = Wind Load G.L. = Gravity Load (G.L. = D.L. + L.L.)
L.L. = Live Load
E.L. = Estimated Equivalent Earthquake Load

The results of the Study Cases No. 1 to No. 24 of Table 3.1 are represented by the graphs of Fig's 3.1.4 to 3.1.14. These graphs relate the area of the maximum bending moment envelopes as a function of the column pin locations (Fig's 3.1.4 to 3.1.11) and also the roof level drift (Fig's 3.1.12 to 3.1.14) as a function of the column pin locations. Similar rigid frame responses have been included and are represented by the dashed lines. (Refer to the list of illustrations for additional explanation concerning these graphs)

3.1.1 REMARKS CONCERNING GRAVITY AND LATERAL LOADS:

The results regarding gravity and lateral loads clearly show the best column pin locations and are summarized as follows:

1) For the maximum bending moment envelope:

For gravity loads (dead and live loads) the best pin location is at y/h = 0.0 (at the column base).

For lateral loads (wind or earthquake) the best pin location is at y/h = 0.6 (from 0.55 to 0.65). This case shows that both frames, hinged or rigid behave almost similarly.

For combined gravity and lateral loads, the best pin location is at y/h = 0.2 (from 0.1 to 0.3) or y/h = 0.9 (from 0.8 to 1.0). For this load case, the performance of

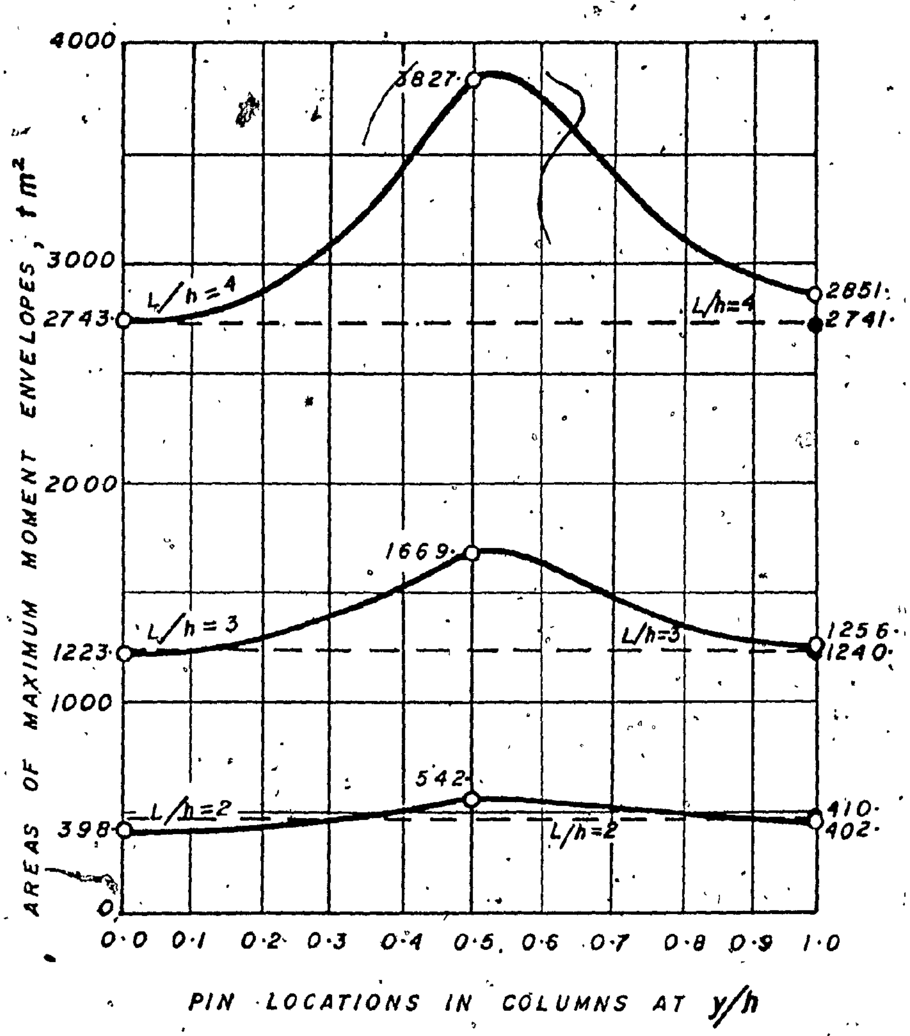


Figure 3.1.4
Study Cases No. 1,2 & 3

Areas of maximum moment envelopes for different pin locations for frames with cantilevers on both sides under gravity loads. Dashed lines indicate rigid frames.

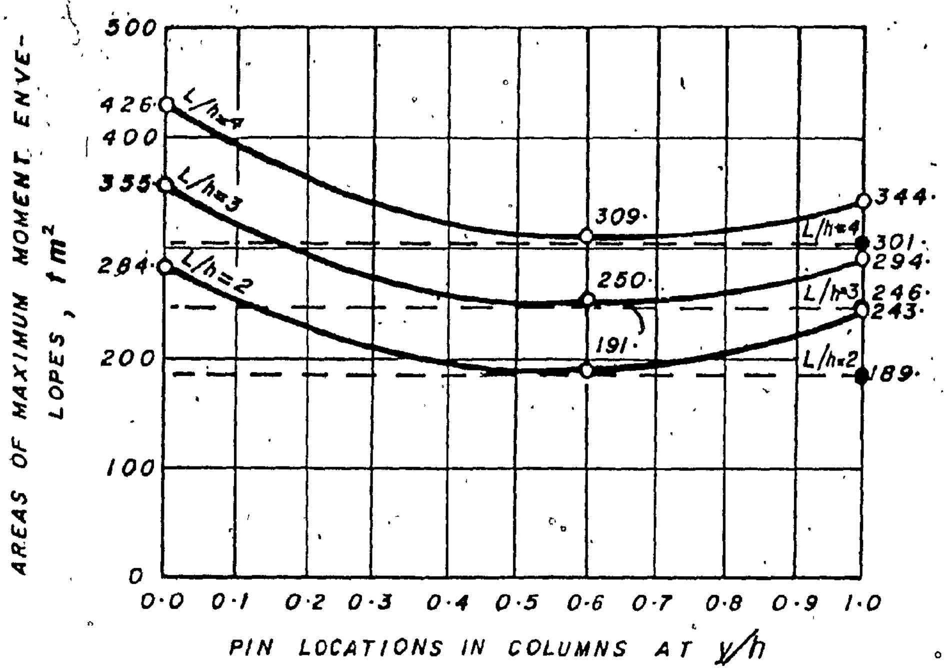


Figure 3.1.5 Study Cases No. 4, 5 & 6
Areas of maximum moment envelopes for different pin locations under wind loading. Dashed lines indicate rigid frames.

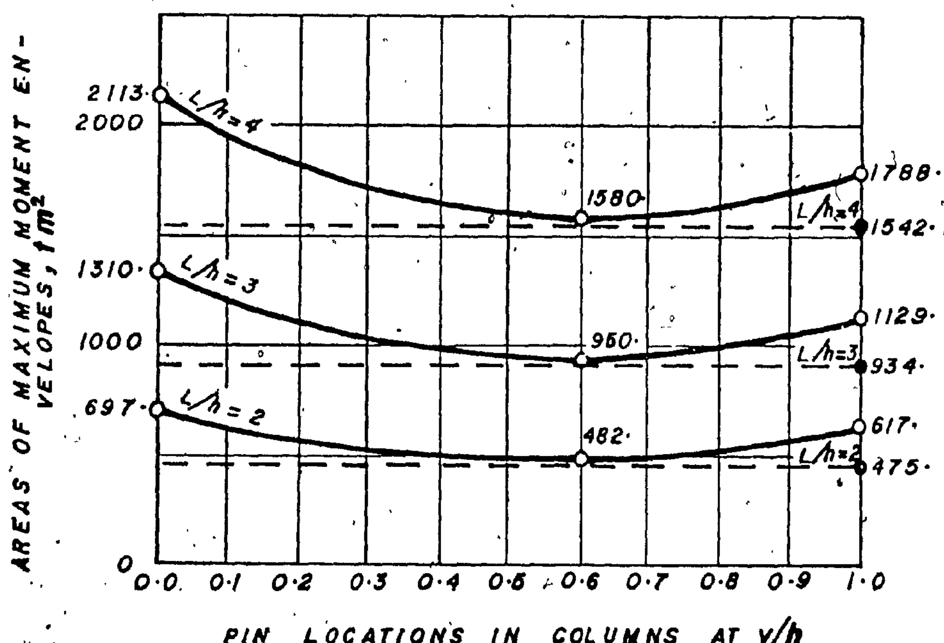


Figure 3.1.6 Study Cases No. 7, 8 & 9
Areas of maximum moment envelopes for different pin locations under earthquake loading. Dashed lines indicate rigid frames.

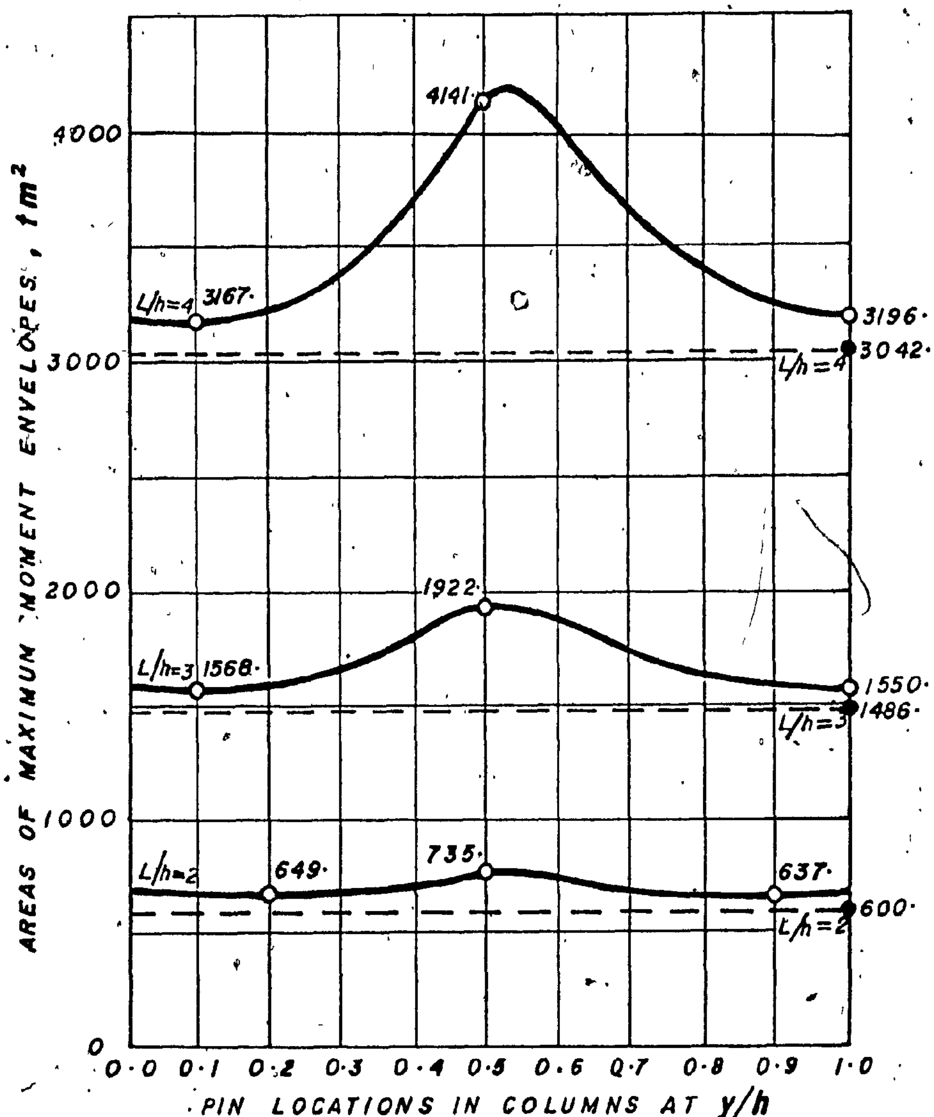


Figure 3.1.7
Study Cases No. 10,11 &12
Areas of maximum moment envelopes for different pin locations under combined gravity and wind loading. Dashed lines indicate rigid frames.

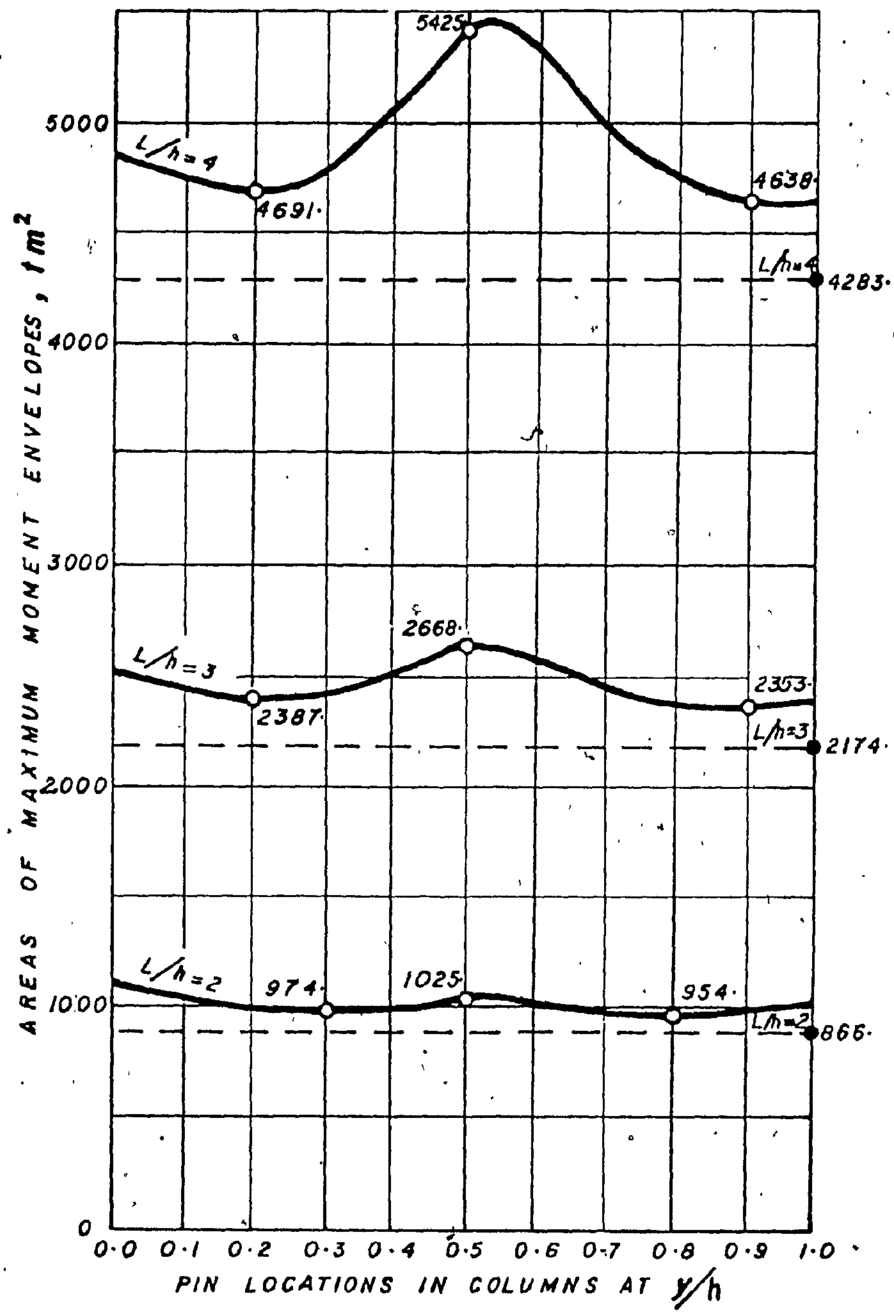


Figure 3.1.8
Study Cases No. 13,14 &15
Areas of maximum moment envelopes for different pin locations under combined gravity and earthquake loading. Dashed lines indicate rigid frames.

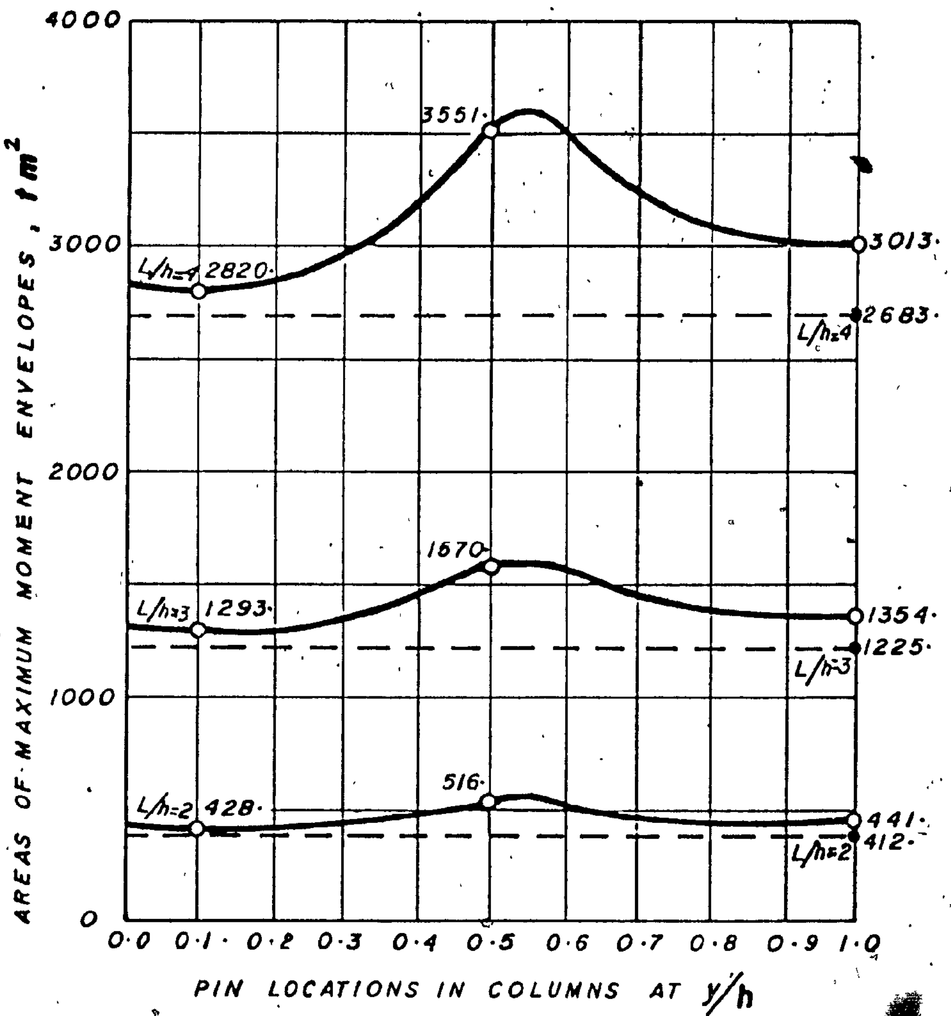


Figure 3.1.9
Study Cases No. 16,17 &18
Areas of maximum moment envelopes for frames with cantilevers on one side under gravity loads. Dashed lines indicate rigid frames.

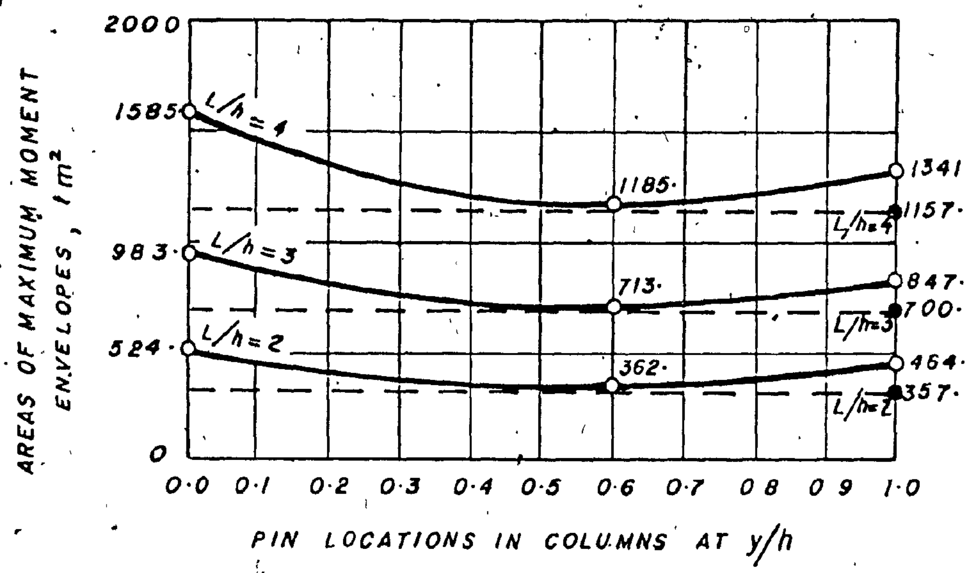


Figure 3.1.10
Study Cases No. 19, 20 &21
Areas of maximum moment envelopes under earthquake loading. Dashed lines indicate rigid frames.

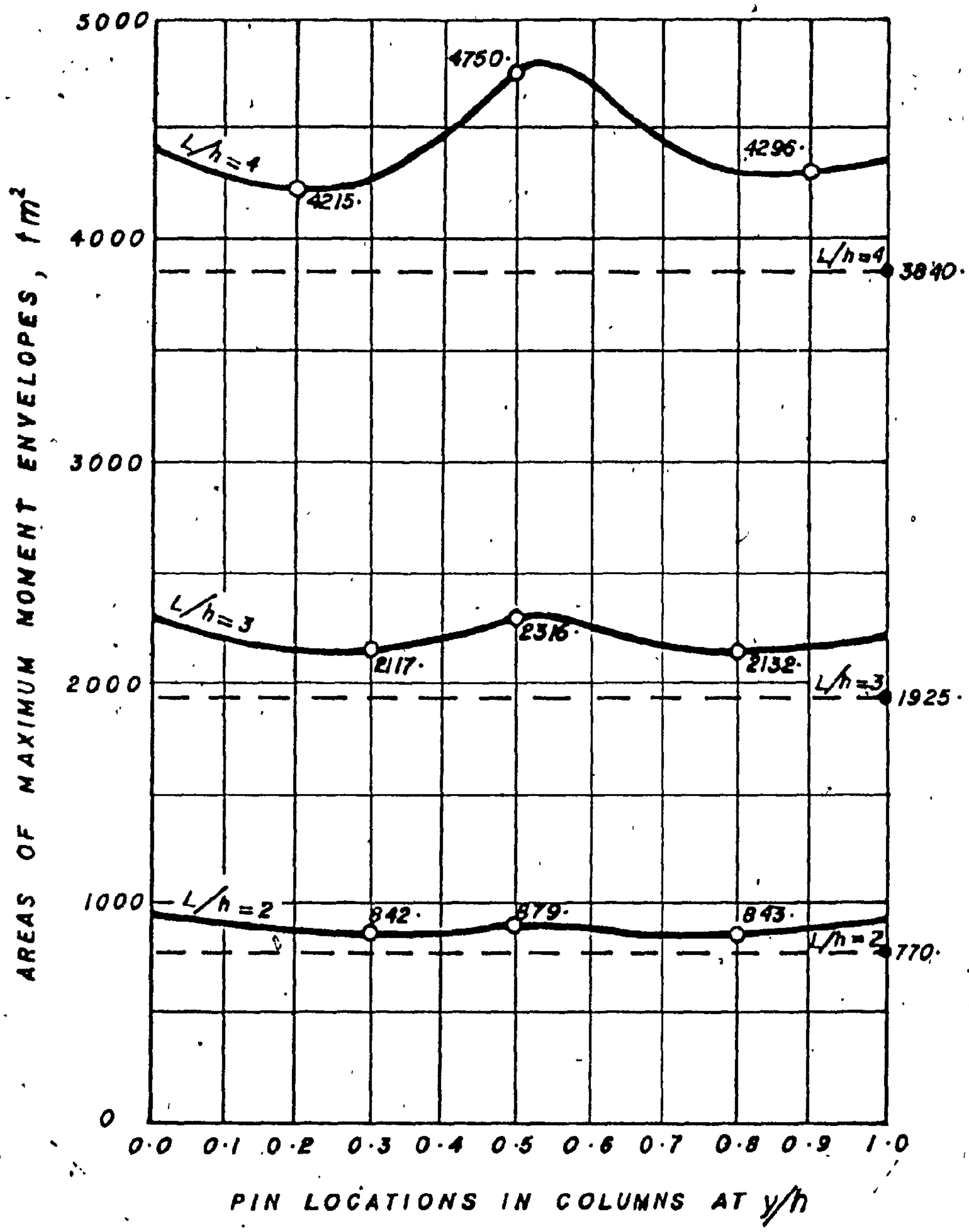


Figure 3.1.11
Study Cases No. 22, 23 &24
Areas of maximum moment envelopes under combined gravity and earthquake loading. Dashed lines indicate rigid frames.

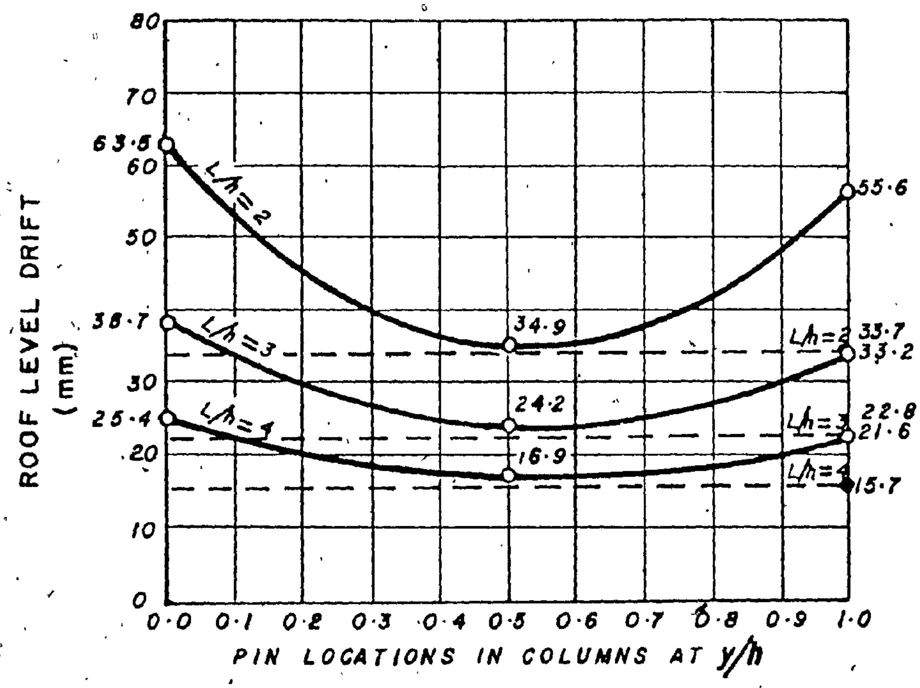


Figure 3.1.12
Study Cases No. 13, 14 & 15
Lateral displacements in frames with cantilevers on both sides under combined gravity and earthquake loading. Dashed lines indicate rigid frames.

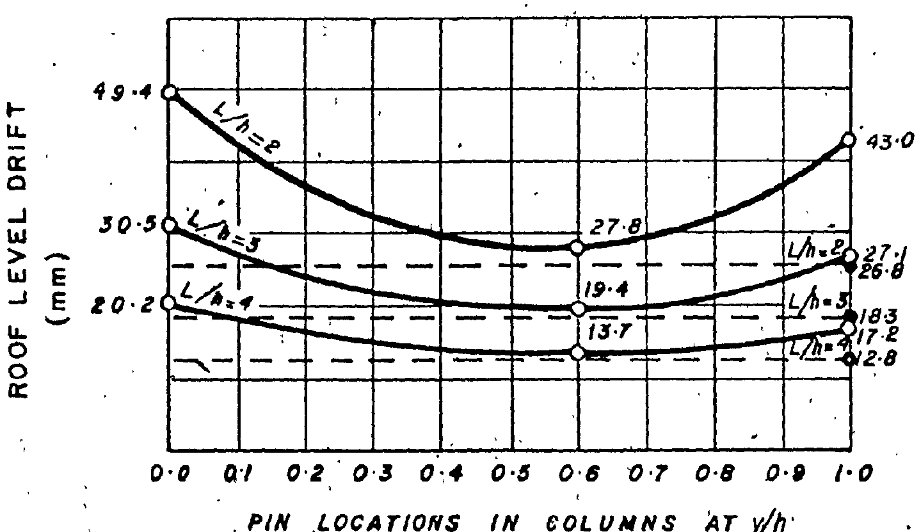


Figure 1.1.13
Study Cases No. 22, 23 & 24
Lateral displacements in frames with cantilevers on one side under combined gravity and earthquake loading. Dashed lines indicate rigid frames.

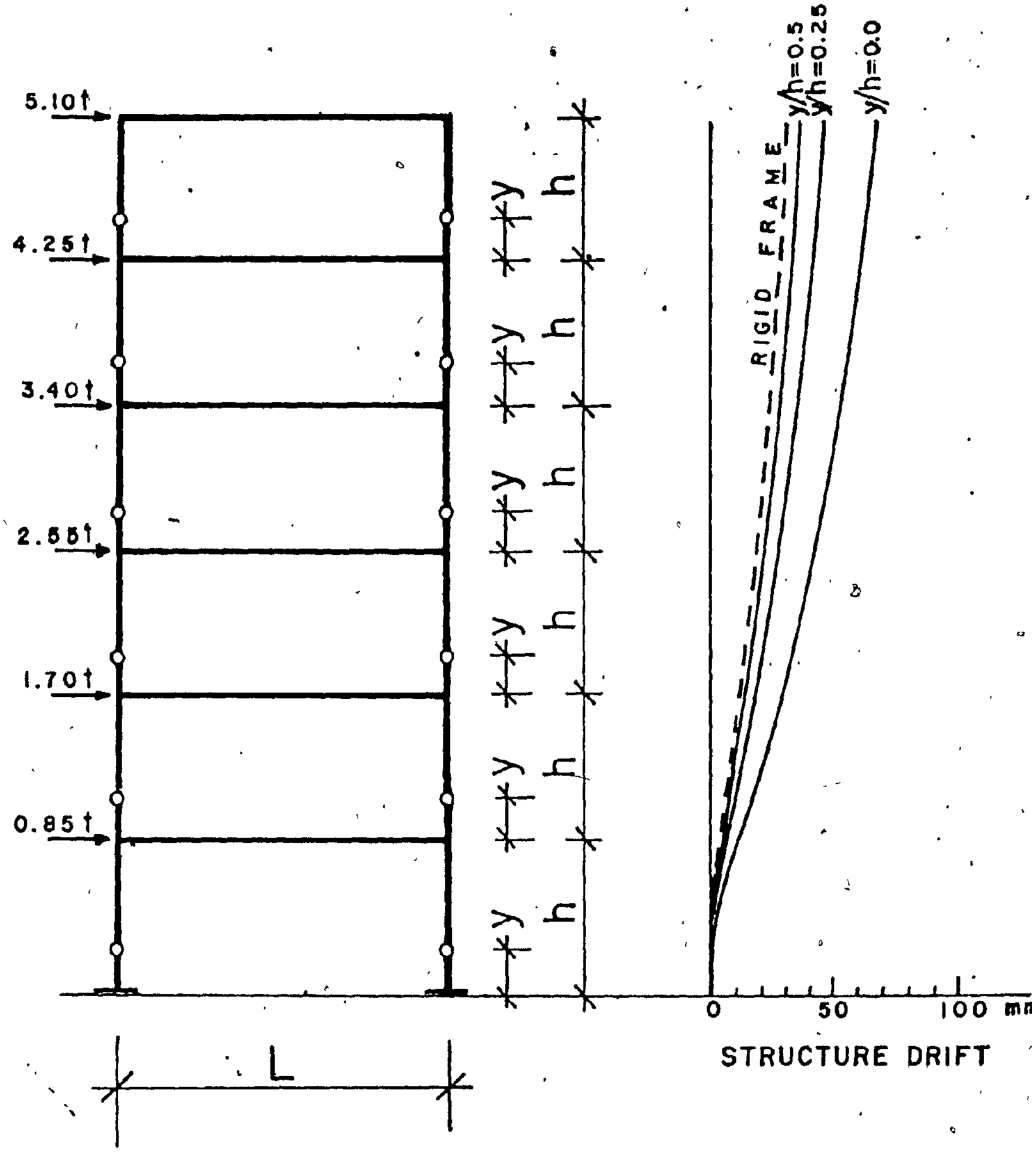


Figure 3.1.14
Study Case No. 7

A comparison of lateral displacements at the floor levels for frames with different pin locations under combined gravity and earthquake loading.

the rigid frame over the hinged frame at the optimum location differs by about 10% (i.e. pinned frame bending moment is larger).

2) Combined bending moment and drift:

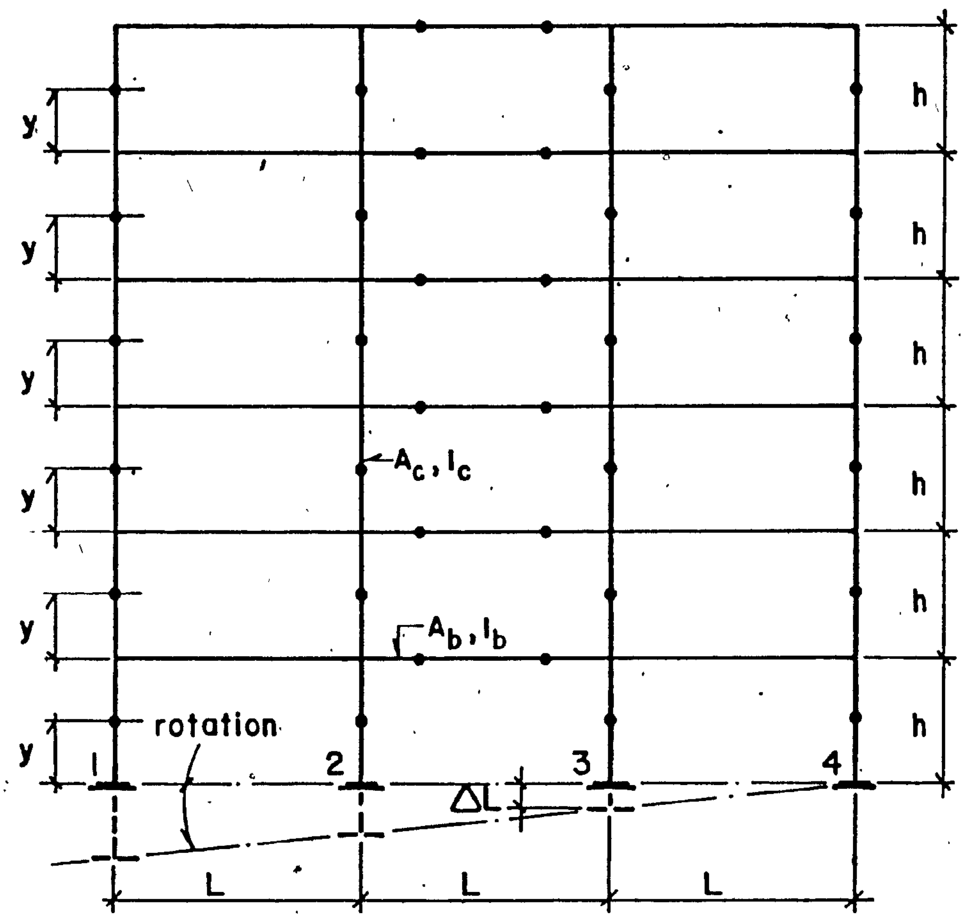
For this combination, the best column pin locations are at y/h = 0.3 (from 0.2 to 0.4) or y/h = 0.8 (from 0.7 to 0.9).

3.2 DIFFERENTIAL FOUNDATION SETTLEMENTS:

Although differential settlement of supports is highly undesirable, this load case may occur and its effect on rigid and hinged frames has been studied. In particular, two types of settlements have been considered 1) overall rotation settlement and 2) localized footing settlement. The additional bending moment area caused by settlements is evaluated as a percentage of the maximum bending moment envelope due to gravity (dead and live) loads. It is important to note that although the best pin location for gravity loads is at y/h = 0.0, the hinged frames for differential settlement analysis will be considered for pin located at y/h = 1.0 with an accompanying minor increase of gravity load bending moment area (less than 5%).

3.2.1 OVERALL ROTATION SETTLEMENT:

For the rigid frame of Fig. 3.2.1 (refer to Table 3.2 for



ROTATION SETTLEMENT

Figure 3.2.1

Multi-bay/multi-storey frame with rotation settlements of supports.

Table 3.2 Frame Properties and Applied Loading

Study Case Number	A_c (m^2)	I_c (m^4)	A_b (m^2)	I_b (m^4)	L (m)	h (m)	y (m)	Type of Loading	ABME[4] (tm^2)
25[1]	0.18	0.0054	0.21	0.0086	6	3	N.A	G.L.: D.L.= 3.0 t/m - L.L. = 3.0 t/m	1423
26[2]	0.32	0.0171	0.36	0.0243	9	3	N.A	G.L.: D.L.= 4.5 t/m - L.L. = 4.5 t/m	6153
27[3]	0.50	0.0416	0.55	0.0610	12	3	N.A	G.L.: D.L.= 6.0 t/m - L.L. = 6.0 t/m	17870
28[1]	0.18	0.0054	0.21	0.0086	6	3	3.0	G.L.: D.L.= 3.0 t/m - L.L. = 3.0 t/m	1372
29[2]	0.32	0.0171	0.36	0.0243	9	3	3.0	G.L.: D.L.= 4.5 t/m - L.L. = 4.5 t/m	6478
30[3]	0.50	0.0416	0.55	0.0610	12	3	3.0	G.L.: D.L.= 6.0 t/m - L.L. = 6.0 t/m	19652

(1) Frame geometry and member properties for frame No. 1
(2) Frame geometry and member properties for frame No. 2
(3) Frame geometry and member properties for frame No. 3
(4) Area of maximum bending moment envelope.

NOTE: Modulus of elasticity for concrete = 2,000,000 t/m^2

for geometric properties) settlement about footing "4" with footings restrained against rotation will develop bending moments throughout the frame with the bulk of the moments present in the members of the lower storeys. The additional bending moment areas for the rigid frames No. 1, 2 and 3 for various β values where

$$\beta = -\log_{10}(\Delta L/L) \quad \text{.........3.2a}$$

are illustrated by the curves of Fig. 3.2.2.

Investigation of these curves indicates that large rotation settlements (small values of β) will cause excessive additional bending moments. In contrast, the comparable hinged frames exhibit very small (of negligible value) percentage increases of bending moments (which are present in the first storey columns) regardless of the column pin location y.

3.2.2 LOCALIZED FOOTING SETTLEMENT:

The frame of Fig. 3.2.3 subjected to localized footing settlements ΔL will produce additional bending moments throughout the rigid and hinged frames.

The localized settlements of footings "3" and "4" are represented by the curves of Fig's 3.2.4 and 3.2.5 respectively. The rigid frame settlement of footing "3" is more critical than the settlement of footing "4".

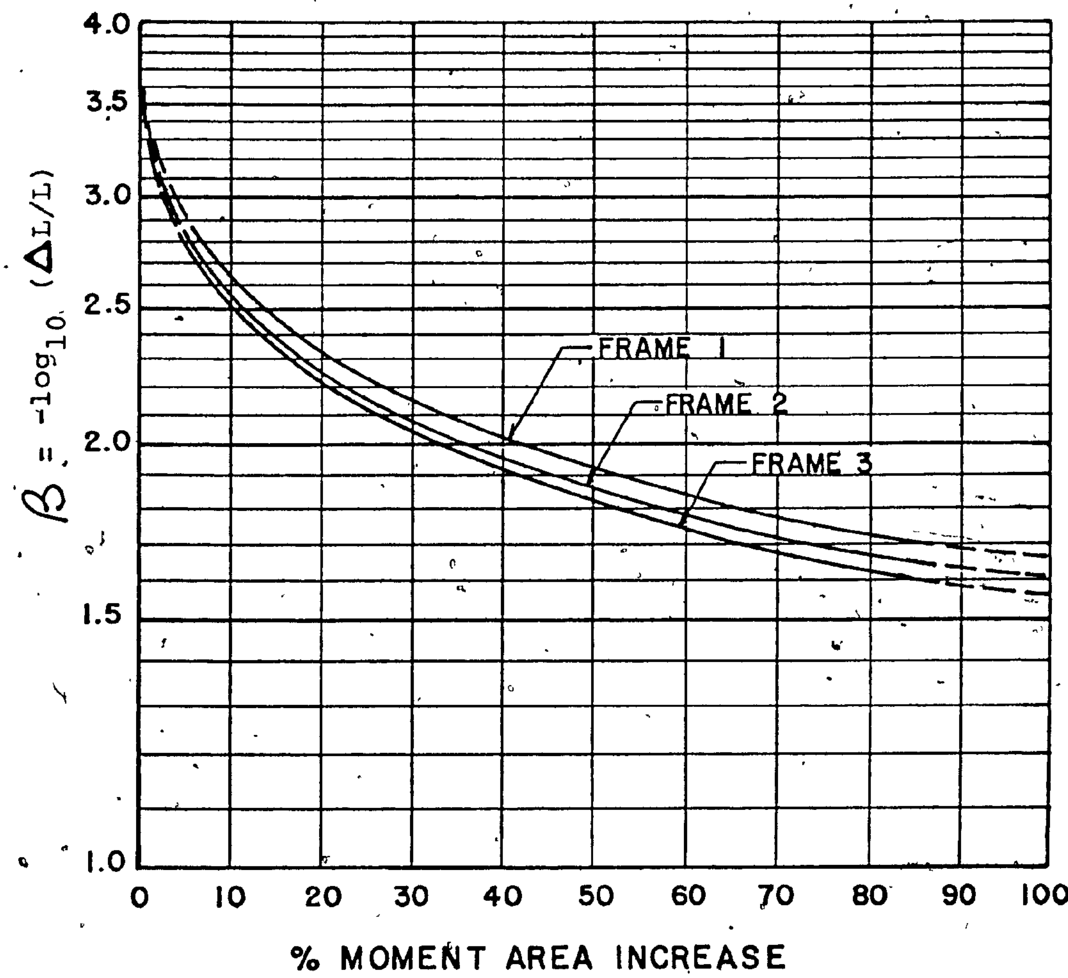


Figure 3.2.2

Plot of moment increase (%) for frame of Fig. 3.2.1 for various rotations.

The comparable hinged frames evaluated once again at y/h = 1.0 are illustrated by the curves of Fig. 3.2.6. Although individual member moments may vary in magnitude and sign (positive or negative), the absolute bending moment areas for the entire structure when settlement of footings "3" or "4" occur, are equal.

3.2.3 REMARKS CONCERNING DIFFERENTIAL SETTLEMENTS:

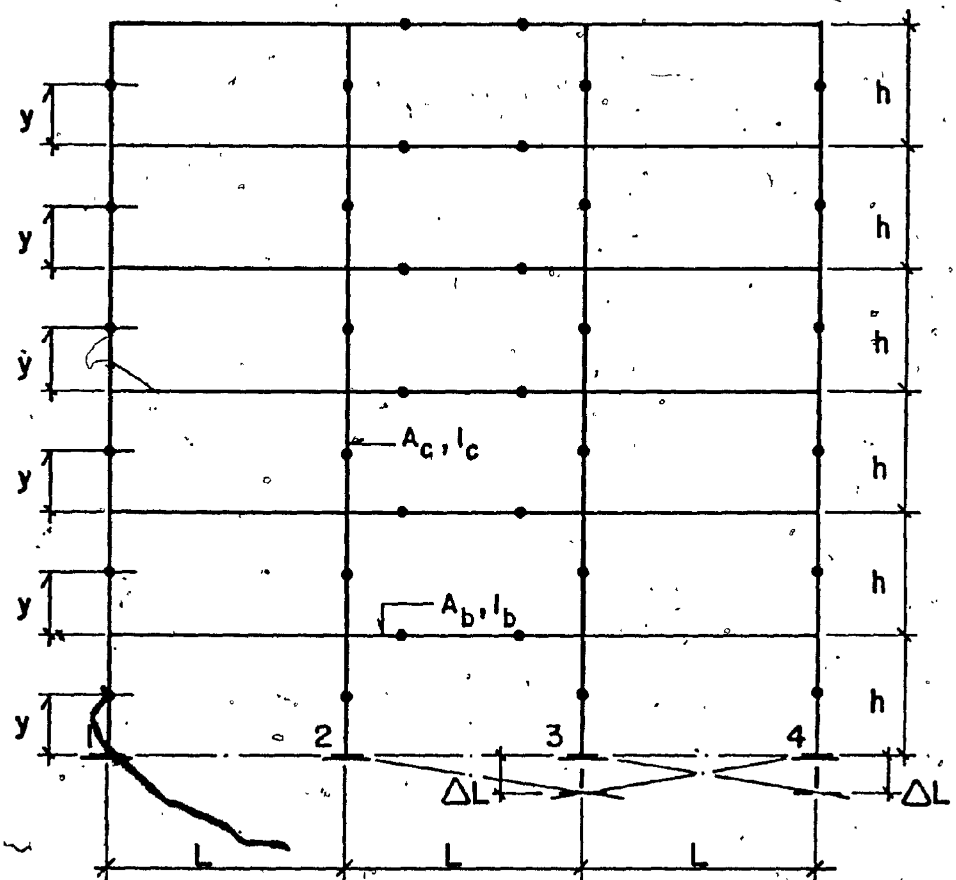
In general, the best location of the column pins is at y/h = 1.0. The hinged frame is better adapted to differential settlements over the rigid frame because of the moment releases (pins) which allows the structure to deform with smaller increases in bending moments.

The column pins located at y/h = 1.0 also produce the least lateral (drift) frame displacements for both rotation settlements and independent footing settlements.

3.3 SUPERPOSITION OF PIN LOCATIONS:

Sections 3.1.1 and 3.2.2 summarize where column pins should be located so as to minimize the bending moment envelope and control drift for gravity and lateral loads as well as differential settlements.

Superimposing the pin location for differential settlements with the pin location for the combined gravity and lateral loads shows that in general, providing column pins



LOCAL SETTLEMENT

Figure 3.2.3

Multi-bay/multi-storey frame with local footing settlements.

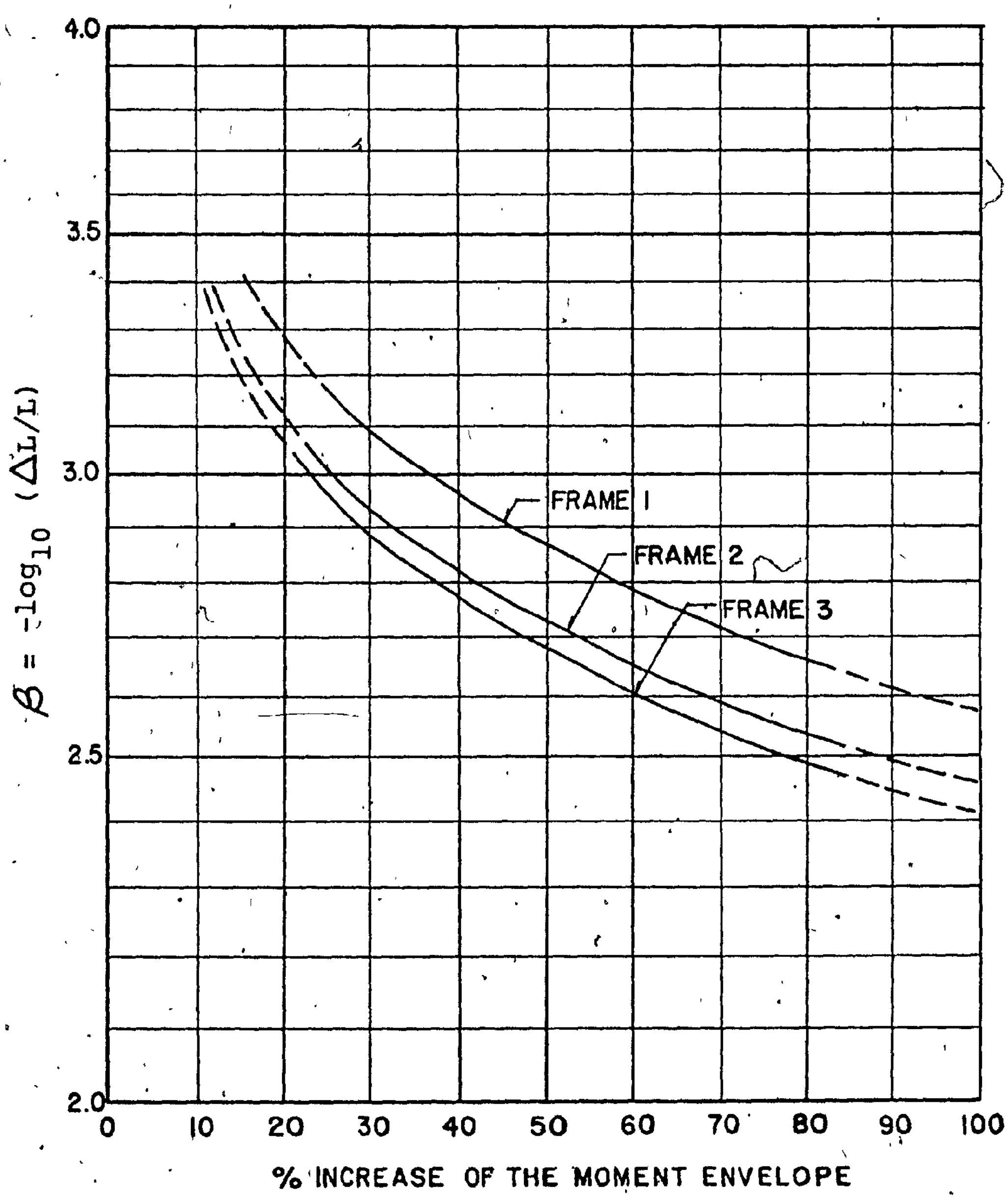


Figure 3.2.4
Plot of rigid frame moment increase (%) for local settlement of footing "3".

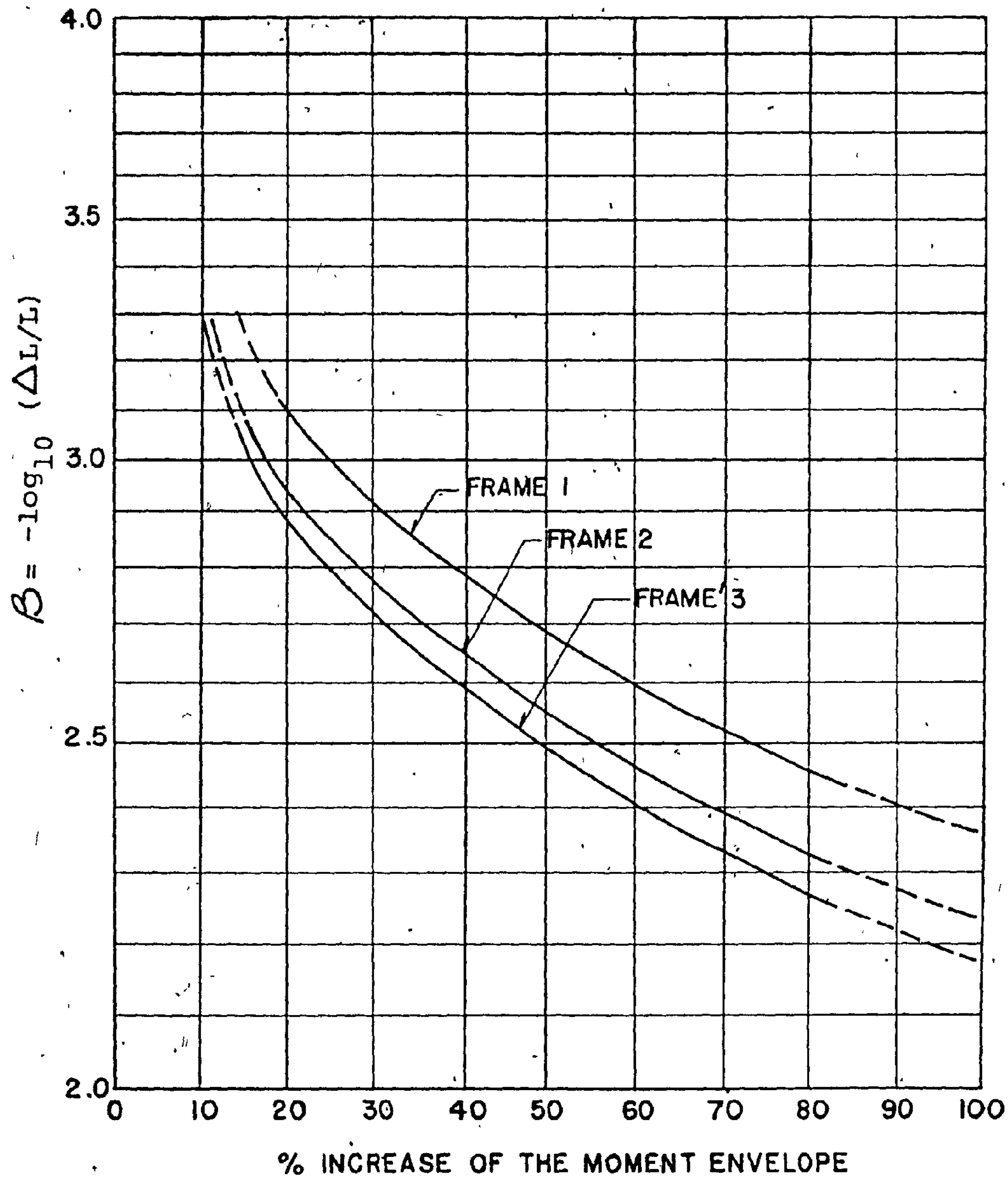


Figure 3.2.5
Plot of ridig frame moment increase (%) for local settlement of footing "4".

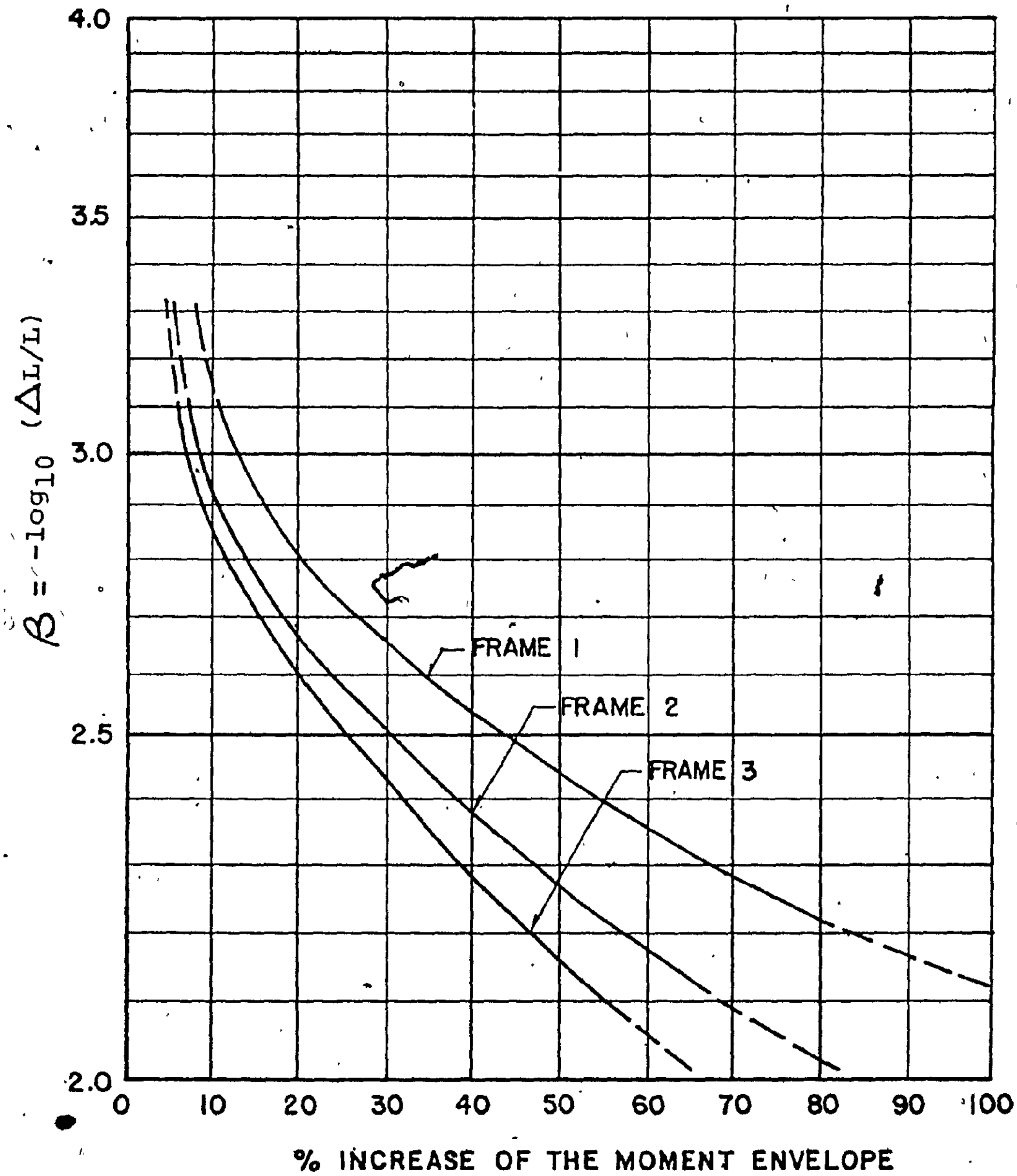


Figure 3.2.6

Plot of hinged frame moment increase (%) for local settlements of footings "3" & "4".

at y/h = 0.8 will produce an optimized structure that would satisfy both gravity and lateral loads as well as differential foundation settlements.

4. CONCLUSIONS:

A computer program has been introduced for the analysis of frame structures which has been developed especially for the optimization of column pin locations for H-Frame structures.

The optimized H-Frame has been evaluated by establishing the best column pin location which will produce the least area of the maximum bending moment envelope and the least structure drift.

Although H-Frames have been traditionally designed with column pins located at column mid-height, this study has indicated that structures influenced by gravity and lateral loads as well as any possible differential foundation settlement exhibit better characteristics when pins are located at y/h = 0.8. This result is not applicable to high-rise structures.

The studies have compared rigid and pinned frames under identical frame geometry, section properties and loading conditions. Although the rigid frame has a lower bending moment area over the optimized pinned frame (10% difference) for the combined gravity and lateral loads, the effect of bending moment area of the pinned frame over the rigid frame for foundation settlements is drastically smaller.

A guideline has been given for establishing the pin location so that the fabrication of H-Frames can be simplified by the reduction of flexural reinforcement which is comparable to the bending moment area.

REFERENCES

(1) Bobrowski, J., Bardhan-Roy, B.K., and Maciag, T., "The Design and Analysis of Grandstand Structures", The Structural Engineer, 2, February 1974.

(2) Zielinski, Z.A., Aydin, H., and Gallaccio, J., "Optimization of Pin-Joint Locations in Multi-Storey/Multi-Bay Precast Concrete H-Frame Systems", Canadian Society for Civil Engineering, May 1980 Annual Conference, Winnipeg, Manitoba.

(3) Zielinski, Z.A., Hanna, A.M., and Gallaccio, J., "Differential Foundation Settlements of Multi-Storey Structures Erected Under Complicated Geological Conditions", CIB Symposium V, June 1980, Madrid, Spain.

(4) "Analysis of Framed Structures", by J.M. Gere and W. Weaver Jr., D. Van Nostrand Company, 1965.

(5) "Matrix Methods of Structural Analysis", by, Chu-Kia Wang, International Textbook Company, 1970 Appendix B.

APPENDIX A

Internal Organization of Program

A.1 Internal Organization of Program:

This program employs the "Stiffness Method" of analysis (Ref. 4) which is based on the assumed linear-elastic behavior of the structural materials. The program is designed as a low capacity in-core storage program which is composed of the six major areas (subroutine organization is shown in Fig. A.1.1).

1) The user has the choice of four possible types of analysis (Dispose Types 1, 11, 111 or 1V) where:

   Type 1.... consists of members that may or may not have pins with a resulting output consisting of member forces and joint displacements for each load case as well as the maximum bending moment envelopes and maximum positive and negative joint displacements and rotations.

   Type 2.... pins are automatically generated and applied to each column starting at the column base and progressively moving towards the top - a total of n positions. The corresponding frame analysis for each pin location yields the maximum bending moment envelope as well as the maximum joint displacements.

   Type 111.... same as Type 11, except that the pins are randomly positioned for the column members.

   Type 1V.... general frame analysis with corresponding member forces as well as the joints displacements and

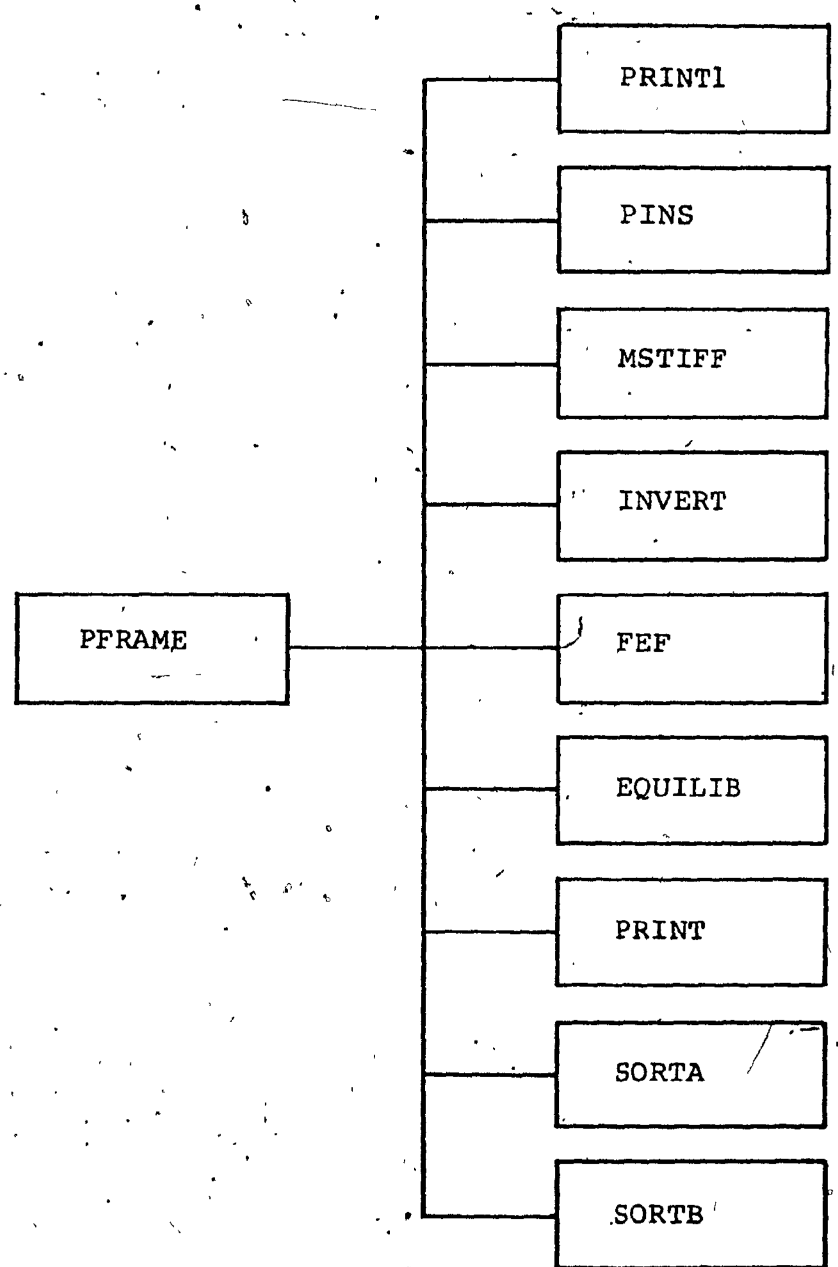


Figure A.1.1
Program Organization

rotations.

2) The next area involves the reading of the input values which define the structure identification, material properties and different load cases applied to the structure.

3) The stiffness matrix is formed and inverted and the structure is analyzed for each load case (member forces and joint displacements).

4) The moment envelope for the entire structure is evaluated for each load case and is accumulated to determine the maximum moment envelope for all the combined load cases. The corresponding joint displacements for each load case are also accumulated to determine the maximum global joint displacements and rotations.

5) The accumulated maximum bending moment envelope is evaluated.

6) Output values are documented via printing subroutines and main-line program printing statements.

A.2 <u>Main-Line Program and Subroutine Description:</u>

The program has nine subroutines all of which may or may not be used during the analysis. Their use depends on the type of output results the user requires. Table A.2.1 indicates when the subroutines are used.

Subroutine Print1: This subroutine prints out all of the input data and also the frame element lengths and their orientation within the planar structure.

SUBROUTINE	DISPOSE TYPE
PRINT1	1, 11, 111, 1V
PINS	111
MSTIFF	1, 11, 111, 1V
INVERT	1, 11, 111, 1V
FEF	1, 11, 111, 1V
EQUILIB	1, 1V
PRINT	1
SORTA	1, 11, 111
SORTB	1, 11, 111

Table A.2.1
Subroutine - Dispose Type
Usage

Subroutine PINS: This subroutine generates all the possible random hinge positions subject to the number of storeys and the number of pin intervals that are to be investigated per storey. The number of intervals divides each column into N equal segments such that the number of pin locations becomes $(N + 1)^{NST}$ times where NST are the number of storeys in the structure. Use of this Dispose Type should be limited since the amount of computation time required may be great.

<u>Subroutine MSTIFF</u>: The member stiffness matrix is formed for three possible types of members A, B or C where each member

A = Fix-Fix member

B = Fix-Pin-Fix member

C = Pin-Pin member

has a defined value for the cross-sectional area, inertia and modulus of elasticity. The program will automatically determine the member lengths. The column pin locations for member type B is automatically generated for Dispose Types 11 and 111 and may be input manually (if used) for Dispose Types 1 and IV.

<u>Subroutine INVERT</u>: This subroutine inverts the accumulated structure stiffness matrix. This inversion technique uses the Gauss-Jordan elimination method (Ref. 5).

<u>Subroutine FEF</u>: Besides global joint forces, the program also accepts member distributed and concentrated loads

which are applied (by the computer) to the structure as equivalent member fixed end forces.

Subroutine EQUILIB: A joint equilibrium check is performed for Dispose Types 1 and IV.

Subroutine PRINT: Prints the member moment diagram and the moment envelope.

Subroutine SORTA: The bending moment effects of each load case for all members is considered for the construction of the maximum bending moment envelope and the maximum joint displacements.

Subroutine SORTB: The area of the maximum moment envelope is determined.

APPENDIX B

Program PFRAME.....Users Manual

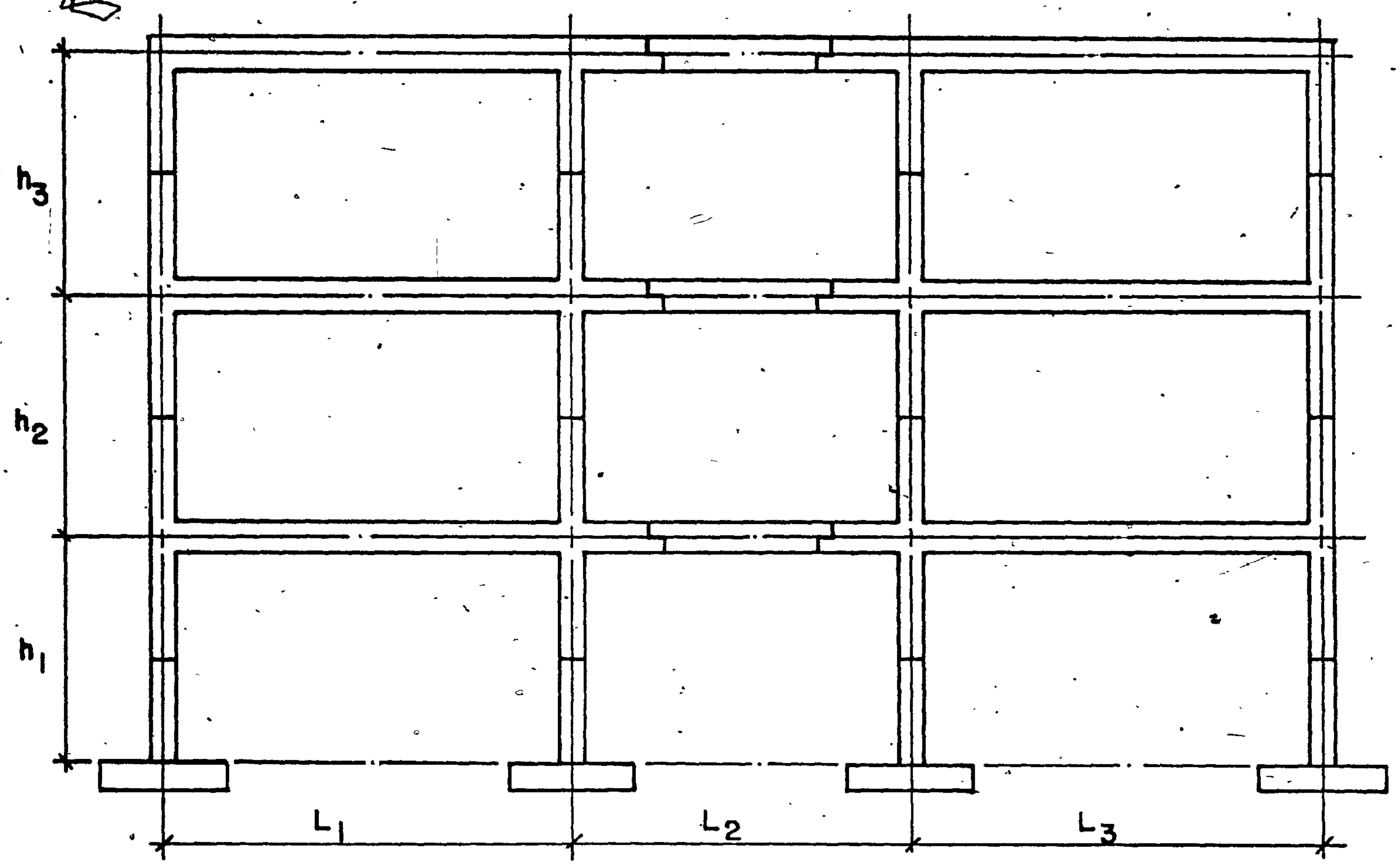
B.1 Numerical Definition of the Building:

For the purpose of preparing the numerical input to the computer program, the building must be separated into planar frames. A typical frame is shown in Fig. B.1.1. Because of the types of structures investigated, shear deformation of members is not considered.

All plane frames consist of members that are long in comparison to their cross-sectional dimension. The joints are points of intersection of members, as well as points of support and free ends. Joints are referenced in a global (*X* - *Y* coordinate) axis system. Individual members have section properties of area, inertia and modulus of elasticity and are oriented within the frame from joint *i* to joint *j*.

Loads are applied either as global joint actions (*Fx*, *Fy* or *Mxy*) or as uniformly distributed member load *w* over span L or concentrated force *p* (normal to the member) at a point x from joint *i*. Displacement of supports (*Dx*, *Dy* or *Dxy*) are accounted for as equivalent member forces which are referenced in the local axis system.

Subsequent numerical input will be classed under the Dispose type required (Appendix A.1) by the user. In general, all input data are given as format-free with individual data being separated by a comma or a blank space. The units of dimension must be consistent.



TYPICAL FRAME

Figure B.1.1

Typical Planar H-Frame structure.

B.2 Numerical Input for Dispose Type 1 & Type IV:

	Argument*	Note
1. Title card:	A	(1)
2. Control card:		
Data check	I	(2)
Dispose type	I	(3)
Modulus of elasticity	R	
Frame ID number	I	(4)
Number of frame elements	I	(5)
" " frame joints	I	(6)
" " support joints	I	(7)
" " load cases	I	(8)
3. Joint data cards:		(9)
Joint ID number	I	
X-coordinate	R	
Y-coordinate	R	
4. Member data cards:		(10)
Member ID number	I	(11)
Member type	I	(12)
Node i	I	(13)
Node j	I	(13)
Cross-section area	R	
Moment of inertia	R	
Pin location	R	(14)

*) A = Alphanumeric, I = Integer, R = Real

	Argument	Note
5. Support joints:		(15)
Joint ID numbers	I	
6. Load cases:		(16)
Load case ID number	I	(17)
Number of loaded joints	I	(18)
Number of loaded members	I	(19)
7. Joint loads:		(20)
Joint ID number	I	(21)
Force in X-direction	R	(21)
Force in Y-direction	R	(21)
Moment	R	(21)
8. Member loads:		(22)
Member ID number	I	(23)
Load type	I	(24)
P1	R	(24)
P2	R	(24)
P3	R	(24)

B.3 Numerical Input for Dispose Type 11:

Refer to Dispose Type 1
for cards 1,3,4,5,6,7 &8

2. Control card:

Data check

Dispose type

	Argument	Note
Modulus of elasticity		
Frame ID number		
Number of frame elements		
" " frame joints		
" " support joints		
" " columns	I	(25)
" " beams	I	(25)
" " pin intervals	I	(26)
" " load cases		

B.4 <u>Numerical Input for Dispose Type 111:</u>

Refer to Dispose Type 1
for cards 1,3,4,5,6,7 &8

2. <u>Control card:</u>

	Argument	Note
Data check		
Dispose type		
Modulus of elasticity		
Frame ID number		
Number of frame elements		
" " frame joints		
" " support joints		
" " columns	I	(25)
" " beams	I	(25)
" " pin intervals	I	(26)

Number of load cases			
" " bays	I	(27)
" " storeys	I	(27)

<u>Notes:</u>

(1) Maximum of 80 characters to be printed with output.

(2) EQ. 0; Data check only.

NE. 0; Execute program.

(3) For dispose Type 1 = 1

Type 11 = 2

Type 111 = 3

Type 1V = 4

(4) Frame identification number to be printed with output.

(5) Total number of elements in the frame.

(6) Total number of joints in the frame.

(7) Total number of support joints.

(8) Total number of independent load cases to be analyzed.

(9) Prepare one card for each joint in note 6 with a joint number and the X - Y coordinate in the global axis system.

(10) Prepare one card for each member in note 5. For Dispose Types 11 & 111, provide all data for column members first, then the data for the beam elements.

(11) All members must have an identification number.

(12)

MEMBER	TYPE
Fix - Fix	1
Fix - Pin - Fix	2
Pin - Pin	3

Note: Omit for column elements of Dispose Types 11 & 111.

(13) The member starts at joint i and ends at joint j.

(14) Distance "x" (see note 12) to pin is measured from joint i.

Note: Omit for column elements of Dispose Types 11 & 111.

(15) List the joints that are fixed (see note 7).

(16) Prepare one set of cards for each load case specified by note 8. For dead plus live load analyses, the first load case must be the dead loads.

(17) Load case ID number to be printed with output.

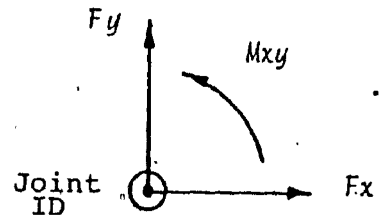
(18) Total number of joints that are loaded. Provide a zero if no joints are loaded.

(19) Total number of members that are loaded. Each concentrated and distributed load must be considered as an independent load. The program allows a single member to have more than one independent load. Provide a zero if no members are loaded.

(20) Prepare one card for each loaded joint of note 18.

(21) Each joint referred to must have 3 corresponding global actions. The order is Fx, Fy and Mxy. Provide a zero for

actions that are not present.

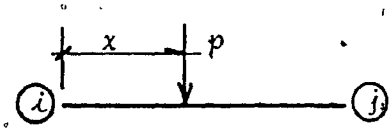
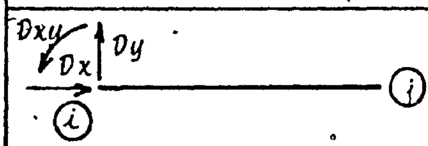


... considered to be positive. (Global Axis System)

(22) Prepare one card for each loaded member of note 19.

(23) Member ID number which has an externally applied load. A differential foundation settlement is considered to be externally applied. It is applicable to any member type of note 12.

(24) Load types and intensities.

LOADING (considered positive)	TYPE	P1	P2	P3
w (uniform loads) 	1	w	*	*
	2	p	x	*
	3**	Dx	Dy	Dxy

---

*) omit for these load cases.

**) provide a zero if displacements do not occur in a particular sense.

(25) Number of columns which will have varying pins. Remaining elements become the number of beams.

(26) Number of pin intervals to be considered along the column.

(27) Number of bays and number of storeys in the frame.

APPENDIX C

Listing of Program PFRAME

```
      PROGRAM PFRAME(INPUT,OUTPUT)
C
C     PROGRAMED BY JOSEPH GALLACCIO....APRIL,1980.
C
C     PROGRAM FOR THE ANALYSIS OF PLANE FRAMES WITH
C     RIGID OR HINGED ELEMENTS. PRESENTED AS PARTIAL
C     FULFILLMENT OF THE REQUIRMENTS FOR THE DEGREE
C     OF MASTER OF ENGINEERING AT CONCORDIA UNIVER-
C     SITY, MONTREAL.
C
      DIMENSION AE(120),AC(120),F(60,6),RCT(60)
     +,DISPL(120),MRL(120),NLCS(30),M(60),IPIN(625,4)
      INTEGER RL(120),DISPOSE
      COMMON/C1/J(40),X(40),Y(40),JJ(60),JK(60),K(60),KK(40,30),A(
     +120,30),NLJS(40),DISPOSE,NCS,NBS,MI,NJ,NRJ,NUM,NINT,FG(60,6)
      COMMON/C2/PP(60,30),MM(60,30),ITYPE(60,30),BB(60,30),NLMS(60),NL
     +C,PBA(10,10)
      COMMON/C3/SMD(6,6)
      COMMON/C4/AX(60),CX(60),CY(60),E(60),AIN(60)
      COMMON/C5/JTYPE(60),AL(60),AB(60),TITLE
      COMMON/C6/AM(60,6)
      COMMON/C7/S(70,70),N,IFLAG,INDEX(70,2)
      COMMON/C8/SORT(53,96),D(120),DISP(40,7),AVT(53,48)
      READ 5,TITLE
    5 FORMAT(5A10)
      READ*,ICHECK,DISPOSE,YOUNGS
      IF(DISPOSE.EQ.2.OR.DISPOSE.EQ.3) GO TO 50
C**** DATA INPUT FOR DISPOSE TYPE...1 OR 4
      READ*,NUM
      READ*,MI,NJ,NRJ,NLC
      DO 10 I=1,NJ
   10 READ*,J(I),X(I),Y(I)
      DO 20 I=1,MI
      READ*,M(I),JTYPE(I),JJ(I),JK(I),AX(I),AIN(I),AB(I)
   20 E(I)=YOUNGS
      GO TO 110
   30 DO 40 I=1,NRJ
   40 READ*,K(I)
      GO TO 130
C**** DATA INPUT FOR DISPOSE TYPE...2
   50 READ*,NUM
      READ*,MI,NJ,NRJ,NCS,NBS,NINT,NLC
      IF(DISPOSE.EQ.3) READ*,NBAYS,NST
      DO 60 I=1,NJ
   60 READ*,J(I),X(I),Y(I)
      DO 70 I=1,NCS
      JTYPE(I)=2
      READ*,M(I),JJ(I),JK(I),AX(I),AIN(I)
   70 E(I)=YOUNGS
      DO 80 I=1,NBS
      READ*,M(I+NCS),JTYPE(I+NCS),JJ(I+NCS),JK(I+NCS),AX(I+NCS),AIN(I+NC
     +S),AB(I+NCS)
   80 E(I+NCS)=YOUNGS
      GO TO 110
   90 DO 100 I=1,NRJ
  100 READ*,K(I)
      GO TO 130
```

```
C**** X AND Y COMPONENTS OF LENGTH OF EACH MEMBER.
  110 DO 120 I=1,MI
      XCL=X(JK(I))-X(JJ(I))
      YCL=Y(JK(I))-Y(JJ(I))
      AL(I)=SQRT(XCL**2+YCL**2)
      CX(I)=XCL/AL(I)
  120 CY(I)=YCL/AL(I)
      IF(DISPOSE.EQ.1.OR.DISPOSE.EQ.4) GO TO 30
      IF(DISPOSE.EQ.2.OR.DISPOSE.EQ.3) GO TO 90
C**** INPUT DATA FOR ALL LOAD CASES.
  130 DO 170 I=1,NLC
      READ*,NLCS(I)
      READ*,NLJS(I),NLMS(I)
      IF(NLJS(I).EQ.0) GO TO 150
      NLJSI=NLJS(I)
      DO 140 INPT=1,NLJSI
  140 READ*,KK(INPT,I),A((3*KK(INPT,I)-2),I),A((3*KK(INPT,I)-1),I),A((3*
     +KK(INPT,I)),I)
  150 IF(NLMS(I).EQ.0) GO TO 170
      NLMSI=NLMS(I)
      DO 160 INPT=1,NLMSI
      READ*,MM(INPT,I),ITYPE(INPT,I)
      IF(ITYPE(INPT,I).EQ.1) READ*,PP(INPT,I)
      IF(ITYPE(INPT,I).EQ.2) READ*,PP(INPT,I),BB(INPT,I)
      IF(ITYPE(INPT,I).EQ.3) READ*,PP(INPT,I),BB(INPT,I),PBA(INPT,I)
  160 CONTINUE
  170 CONTINUE
      CALL PRINTI(M,NBAYS,NST)
      IF(ICHECK.EQ.0) STOP
C**** INITIALIZE ALL JOINT DISPLACEMENTS TO 0
      NJ1=3*NJ
      DO 180 I=1,NJ1
  180 RL(I)=0
C**** ESTABLISH THE RESTRAINED DISPLACEMENTS
      DO 190 I=1,NRJ
  190 RL(3*K(I)-2)=RL(3*K(I)-1)=RL(3*K(I))=1
C**** DETERMINE THE MODIFIED RESTRAINT LIST
      II=0
      DO 210 I=1,NJ1
      IF(RL(I).EQ.1) GO TO 200
      MRL(I)=II=II+1
      GO TO 210
  200 MRL(I)=0
  210 CONTINUE
      N=II
      MI2=2*MI
C****     1) FRAME ANALYSIS FOR DISPOSE TYPE...1
C**** OR 2) PIN GENERATION AND FRAME ANALYSIS FOR DISPOSE TYPE...2 OR 3
C**** OR 3) GENERAL FRAME ANALYSIS FOR DISPOSE TYPE...4
      DO 215 L3=1,MI
      SORT(1,2*L3-1)=L3
  215 SORT(1,2*L3)=L3
      ICTR=0
      IF(DISPOSE.EQ.1.OR.DISPOSE.EQ.4) NCAL=1
      IF(DISPOSE.EQ.2) NCAL=NINT+1
      IF(DISPOSE.EQ.3) GO TO 303
      GO TO 302
```

```
  303 NCAL=(NINT+1)**NST
      CALL PINS(NINT,NST,IPIN)
      GO TO 301
C**** DISPOSE=1,2 OR 4
  302 ICTR=ICTR+1
      IF(ICTR.GT.NCAL) GO TO 890
      IF(DISPOSE.EQ.1) GO TO 211
      IF(DISPOSE.EQ.4) GO TO 221
C**** DISPOSE=2
      ID22=ICTR-1
      NINTT=NCAL-1
      PRINT 400,ID22,NINTT
  400 FORMAT(1H1,10X,*COLUMN PIN LOCATIONS AT*,I3,* OF*,I3,* INTERVALS*,
     +//,14X,*AREA OF BENDING MOMENT DIAGRAM PER MEMBER*,71X,*TOTAL AREA
     +*,/)
  211 DO 214 L1=1,NJ
      DO 214 L2=1,7
  214 DISP(L1,L2)=0.0
      DO 216 L1=2,53
      DO 216 L2=1,M12
  216 SORT(L1,L2)=0.0
      IF(DISPOSE.EQ.1) GO TO 221
      DO 220 ID3=1,NCS
  220 AB(ID3)=(FLOAT(ICTR-1)/FLOAT(NCAL-1))*AL(ID3)
      GO TO 222
C**** DISPOSE TYPE 3
  301 NCL=NBAYS+1
  304 ICTR=ICTR+1
      IF(ICTR.GT.NCAL) GO TO 890
      PRINT 401,(IPIN(ICTR,L1),L1=1,NST)
  401 FORMAT(//,5X,*COLUMN PIN LOCATIONS AT RANDOM INTERVALS*,10(I5))
      PRINT 402
  402 FORMAT(/,5X,*AREA OF BENDING MOMENT DIAGRAM PER MEMBER*,81X,*TOTAL
     + AREA*,/)
      ICT=0
      DO 217 LX=1,NST
      DO 217 LY=1,NCL
      ICT=ICT+1
  217 AB(ICT)=(FLOAT(IPIN(ICTR,LX))/FLOAT(NINT))*AL(ICT)
      DO 218 L1=1,NJ
      DO 218 L2=1,7
  218 DISP(L1,L2)=0.0
      DO 219 L1=2,53
      DO 219 L2=1,M12
  219 SORT(L1,L2)=0.0
  222 ICOUNT=M1
      M2=1
      M3=0
  225 ICOUNT=ICOUNT-10
      IF(ICOUNT.GE.0) JCOUNT=10
      IF(ICOUNT.LT.0) JCOUNT=ICOUNT+10
      M3=M3+JCOUNT
      PRINT 226
  226 FORMAT(5X,*MEMBERS*)
      PRINT 227,(SORT(1,2*L1-1),L1=M2,M3)
  227 FORMAT(1H+,14X,10(F7.0,4X))
      IF(ICOUNT.LT.0) GO TO 228
```

```
      M2=M3+1
      GO TO 225
  228 ICOUNT=M1
      M2=1
      M3=0
  229 ICOUNT=ICOUNT-10
      IF(ICOUNT.GE.0) JCOUNT=10
      IF(ICOUNT.LT.0) JCOUNT=ICOUNT+10
      M3=M3+JCOUNT
      PRINT 230
  230 FORMAT(5X,*AT A=*)
      PRINT 232,(AB(L1),L1=M2,M3)
  232 FORMAT(1H+,14X,10(3X,F6.2,2X))
      IF(ICOUNT.LT.0) GO TO 221
      M2=M3+1
      GO TO 229
C**** GENERATION OF MEMBER STIFFNESS MATRIX AND
C**** ITS ASSEMBLY INTO THE OVERALL STIFFNESS MATRIX
  221 DO 291 L1=1,N
      DO 291 L2=1,N
  291 S(L1,L2)=0.0
      DO 350 I=1,M1
      J1=MRL(3*JJ(I)-2)
      J2=MRL(3*JJ(I)-1)
      J3=MRL(3*JJ(I))
      K1=MRL(3*JK(I)-2)
      K2=MRL(3*JK(I)-1)
      K3=MRL(3*JK(I))
      CALL MSTIFF(I)
      IF(J1.EQ.0) GO TO 310
      S(J1,J1)=S(J1,J1)+SMD(1,1)
      S(J2,J1)=S(J2,J1)+SMD(2,1)
      S(J3,J1)=S(J3,J1)+SMD(3,1)
      IF(K1.EQ.0) GO TO 290
      S(K1,J1)=SMD(4,1)
      S(K2,J1)=SMD(5,1)
      S(K3,J1)=SMD(6,1)
  290 S(J1,J2)=S(J1,J2)+SMD(1,2)
      S(J2,J2)=S(J2,J2)+SMD(2,2)
      S(J3,J2)=S(J3,J2)+SMD(3,2)
      IF(K1.EQ.0) GO TO 300
      S(K1,J2)=SMD(4,2)
      S(K2,J2)=SMD(5,2)
      S(K3,J2)=SMD(6,2)
  300 S(J1,J3)=S(J1,J3)+SMD(1,3)
      S(J2,J3)=S(J2,J3)+SMD(2,3)
      S(J3,J3)=S(J3,J3)+SMD(3,3)
      IF(K1.EQ.0) GO TO 310
      S(K1,J3)=SMD(4,3)
      S(K2,J3)=SMD(5,3)
      S(K3,J3)=SMD(6,3)
  310 IF(K1.EQ.0) GO TO 350
      IF(J1.EQ.0) GO TO 320
      S(J1,K1)=SMD(1,4)
      S(J2,K1)=SMD(2,4)
      S(J3,K1)=SMD(3,4)
  320 S(K1,K1)=S(K1,K1)+SMD(4,4)
```

```
      S(K2,K1)=S(K2,K1)+SMD(5,4)
      S(K3,K1)=S(K3,K1)+SMD(6,4)
      IF(J1.EQ.0) GO TO 330
      S(J1,K2)=SMD(1,5)
      S(J2,K2)=SMD(2,5)
      S(J3,K2)=SMD(3,5)
  330 S(K1,K2)=S(K1,K2)+SMD(4,5)
      S(K2,K2)=S(K2,K2)+SMD(5,5)
      S(K3,K2)=S(K3,K2)+SMD(6,5)
      IF(J1.EQ.0) GO TO 340
      S(J1,K3)=SMD(1,6)
      S(J2,K3)=SMD(2,6)
      S(J3,K3)=SMD(3,6)
  340 S(K1,K3)=S(K1,K3)+SMD(4,6)
      S(K2,K3)=S(K2,K3)+SMD(5,6)
      S(K3,K3)=S(K3,K3)+SMD(6,6)
  350 CONTINUE
      IFLAG=0
C**** INVERT THE STIFFNESS MATRIX.
      CALL INVERT
      IF(IFLAG.EQ.1) PRINT 360
  360 FORMAT(//,5X,*STRUCTURAL STIFFNESS MATRIX ERROR*)
      DO 911 IGG=1,NLC
      CALL FEF(IGG)
C**** ASSEMBLY OF THE LOAD VECTORS.
      DO 231 L1=1,NJ1
  231 AE(L1)=AC(L1)=0.0
      IF(NLMS(IGG).EQ.0) GO TO 250
      DO 240 I=1,M1
      AE(3*JJ(I)-2)=AE(3*JJ(I)-2)-AM(I,1)*CX(I)+AM(I,2)*CY(I)
      AE(3*JJ(I)-1)=AE(3*JJ(I)-1)-AM(I,1)*CY(I)-AM(I,2)*CX(I)
      AE(3*JJ(I))=AE(3*JJ(I))-AM(I,3)
      AE(3*JK(I)-2)=AE(3*JK(I)-2)-AM(I,4)*CX(I)+AM(I,5)*CY(I)
      AE(3*JK(I)-1)=AE(3*JK(I)-1)-AM(I;4)*CY(I)-AM(I,5)*CX(I)
      AE(3*JK(I))=AE(3*JK(I))-AM(I,6)
  240 CONTINUE
C**** COMBINED JOINT LOADS.
  250 DO 260 I=1,NJ1
  260 AC(I)=A(I,IGG)+AE(I)
C**** RE-ARRANGEMENT OF THE COMBINED JOINT VECTORS.
      IA=1
      DO 280 I=1,NJ1
      IF(MRL(I).EQ.0) GO TO 279
      AC(IA)=AC(I)
      IA=IA+1
  279 AC(I)=0.0
  280 CONTINUE
C**** CALCULATION OF THE JOINT DISPLACEMENTS.
      DO 361 L1=1,NJ1
  361 DISPL(L1)=D(L1)=0.0
      DO 370 J4=1,N
      DO 370 K4=1,N
  370 DISPL(J4)=DISPL(J4)+S(J4,K4)*AC(K4)
C**** ORGANIZE "DISPL" INTO THE TRUE "D" VECTOR
      IA=1
      DO 390 I=1,NJ1
      IF(MRL(I).EQ.0) GO TO 390
```

```
      D(I)=DISPL(IA)
      IA=IA+1
  390 CONTINUE
      IF(DISPOSE.EQ.2.OR.DISPOSE.EQ.3) GO TO 495
C**** OUTPUT FOR "NLC" LOAD CONDITIONS
C**** PRINT THE JOINT DISPLACEMENTS
      PRINT 440,ICG
  440 FORMAT(1H1,4X,*JOINT DISPLACEMENTS  LOAD CASE*,I3,/,5X,*JT*,3X,*X-
     +DISP*,6X,*Y-DISP*,6X,*Z-ROTATION*)
      DO 450 I=1,NJ
  450 PRINT 460,I,D(3*I-2),D(3*I-1),D(3*I)
  460 FORMAT(4X,I3,1X,3(G10.4,2X))
C**** REVISED JOINT LOCATIONS
      PRINT 470,ICG
  470 FORMAT(/,5X,*REVISED JOINT COORDINATES FOR  LOAD CASE*,I3,/,5X,*JT
     +*,5X,*X*,11X,*Y*)
      DO 480 I=1,NJ
      XX=X(J(I))+D(3*J(I)-2)
      YY=Y(J(I))+D(3*J(I)-1)
  480 PRINT 490,I,XX,YY
  490 FORMAT(4X,I3,1X,2(G10.4,2X))
C**** CALCULATION OF THE MEMBER END ACTIONS
  495 DO 530 I=1,MI
      J1=3*JJ(I)-2
      J2=3*JJ(I)-1
      J3=3*JJ(I)
      K1=3*JK(I)-2
      K2=3*JK(I)-1
      K3=3*JK(I)
      IF(JTYPE(I).EQ.2) GO TO 500
      IF(JTYPE(I).EQ.3) GO TO 510
C**** FIX-FIX MEMBER END ACTIONS (LOCAL AXIS)
      S1=(E(I)*AX(I))/AL(I)
      S2=(4*E(I)*AIN(I))/AL(I)
      S3=(1.5*S2)/AL(I)
      S4=(2.*S3)/AL(I)
      F(I,1)=AM(I,1)+S1*((D(J1)-D(K1))*CX(I)+(D(J2)-D
     +(K2))*CY(I))
      F(I,2)=AM(I,2)+S4*(-(D(J1)-D(K1))*CY(I)+(D(J2)-
     +D(K2))*CX(I))+S3*(D(J3)+D(K3))
      F(I,3)=AM(I,3)+S3*(-(D(J1)-D(K1))*CY(I)+(D(J2)-
     +D(K2))*CX(I))+S2*(D(J3)+D(K3)/2.)
      F(I,4)=AM(I,4)+S1*(-(D(J1)-D(K1))*CX(I)-(D(J2)-
     +D(K2))*CY(I))
      F(I,5)=AM(I,5)+S4*((D(J1)-D(K1))*CY(I)-(D(J2)-D
     +(K2))*CX(I))-S3*(D(J3)+D(K3))
      F(I,6)=AM(I,6)+S3*(-(D(J1)-D(K1))*CY(I)+(D(J2)-
     +D(K2))*CX(I))+S2*(D(J3)/2.+D(K3))
      GO TO 520
C**** FIX-PIN-FIX MEMBER END ACTIONS (LOCAL AXIS)
  500 B=AL(I)-AB(I)
      S1=(E(I)*AX(I))/AL(I)
      S2=(3*E(I)*AIN(I))/(AB(I)**3+B**3)
      S3=S2*AB(I)
      S4=S3*AB(I)
      S5=S3*B
      S6=S2*B
```

```
      S7=S6*B
      F(I,1)=AM(I,1)+S1*((D(J1)-D(K1))*CX(I)+(D(J2)-D
     +(K2))*CY(I))
      F(I,2)=AM(I,2)+S2*((D(J2)-D(K2))*CX(I)+(D(K1)-D
     +(J1))*CY(I))+S3*D(J3)+S6*D(K3)
      F(I,3)=AM(I,3)+S3*((D(J2)-D(K2))*CX(I)+(D(K1)-D
     +(J1))*CY(I))+S4*D(J3)+S5*D(K3)
      F(I,4)=AM(I,4)+S1*((D(K1)-D(J1))*CX(I)+(D(K2)-D
     +(J2))*CY(I))
      F(I,5)=AM(I,5)+S2*((D(K2)-D(J2))*CX(I)+(D(J1)-D
     +(K1))*CY(I))-S3*D(J3)-S6*D(K3)
      F(I,6)=AM(I,6)+S6*((D(J2)-D(K2))*CX(I)+(D(K1)-D
     +(J1))*CY(I))+S5*D(J3)+S7*D(K3)
      GO TO 520
C**** PIN-PIN MEMBER END ACTIONS (LOCAL AXIS)
  510 S1=(E(I)*AX(I))/AL(I)
      F(I,1)=AM(I,1)+S1*((D(J1)-D(K1))*CX(I)+(D(J2)-D
     +(K2))*CY(I))
      F(I,2)=AM(I,2)
      F(I,3)=AM(I,3)
      F(I,4)=AM(I,4)+S1*((D(K1)-D(J1))*CX(I)+(D(K2)-D
     +(J2))*CY(I))
      F(I,5)=AM(I,5)
      F(I,6)=AM(I,6)
  520 FG(I,1)=CX(I)*F(I,1)-CY(I)*F(I,2)
      FG(I,2)=CY(I)*F(I,1)+CX(I)*F(I,2)
      FG(I,3)=F(I,3)
      FG(I,4)=CX(I)*F(I,4)-CY(I)*F(I,5)
      FG(I,5)=CY(I)*F(I,4)+CX(I)*F(I,5)
      FG(I,6)=F(I,6)
  530 CONTINUE
      IF(DISPOSE.EQ.2.OR.DISPOSE.EQ.3) GO TO 655
C**** CALCULATION OF THE SUPPORT REACTIONS
      DO 531 LI=1,60
  531 RCT(LI)=0.0
      DO 570 IR=1,NRJ
      DO 550 JR=1,MI
      IF(K(IR).EQ.JJ(JR)) GO TO 540
      GO TO 550
  540 RCT(3*IR-2)=RCT(3*IR-2)+FG(JR,1)
      RCT(3*IR-1)=RCT(3*IR-1)+FG(JR,2)
      RCT(3*IR)=RCT(3*IR)+FG(JR,3)
  550 CONTINUE
      DO 570 JR=1,MI
      IF(K(IR).EQ.JK(JR)) GO TO 560
      GO TO 570
  560 RCT(3*IR-2)=RCT(3*IR-2)+FG(JR,4)
      RCT(3*IR-1)=RCT(3*IR-1)+FG(JR,5)
      RCT(3*IR)=RCT(3*IR)+FG(JR,6)
  570 CONTINUE
      PRINT 580,IGG
  580 FORMAT(/,5X,*MEMBER END FORCES (LOCAL AXIS SYSTEM)  LOAD CASE*,I3,
     +/,5X,*M *,2(1X,*JT*,4X,*AXIAL*,6X,*SHEAR*,5X,*MOMENT  *))
      DO 590 I=1,MI
  590 PRINT 600,I,JJ(I),F(I,1),F(I,2),F(I,3),JK(I),F(I,4),F(I,5),F(I,6)
  600 FORMAT(3X,I3,1X,I3,3(1X,G10.4),1X,I3,3(1X,G10.4))
      PRINT 610,IGG
```

```
610 FORMAT(/,5X,*MEMBER END FORCES (GLOBAL AXIS SYSTEM)  LOAD CASE*,I3
   +,/,5X,*M *,2(IX,*JT*,4X,*  X   *,5X,*  Y   *,5X,*  Z      *))
    DO 620 I=1,MI
620 PRINT 600,I,JJ(I),FG(I,1),FG(I,2),FG(I,3),JK(I),FG(I,4),FG(I,5),F
   +G(I,6)
    PRINT 630,IGG
630 FORMAT(/,5X,*SUPPORT REACTIONS (GLOBAL AXIS SYSTEM)  LOAD CASE*,I3
   +,/,5X,*JT*,5X,*X*,11X,*Y*,11X,*Z*)
    DO 640 I=1,NRJ
640 PRINT 650,K(I),RCT(3*I-2),RCT(3*I-1),RCT(3*I)
650 FORMAT(4X,I3,1X,3(G10.1,2X))
    CALL EQUILIB(IGG)
    IF(DISPOSE.EQ.4) GO TO 911
655 DO 740 I=1,MI
    VM=XA=0.0
C**** MOMENT DIAGRAM DUE TO END FORCES.
    DO 660 II6=2,52
    AMT(II6,I)=F(M(I),3)-(F(M(I),2)*XA)
660 XA=XA+AL(M(I))/50.
    IF(NLMS(IGG).EQ.0) GO TO 720
C**** CHECK FOR APPLIED LOADING ON MEMBER.
    NLMSI=NLMS(IGG)
    DO 710 II5=1,NLMSI
    IF(M(I).EQ.MM(II5,IGG)) GO TO 670
    GO TO 710
C**** DETERMINE WHAT TYPE OF LOADING
670 IF(ITYPE(II5,IGG).EQ.2) GO TO 690
C**** UDL  MEMBER.
    XA=0.0
    DO 680 II6=2,52
    AMT(II6,I)=AMT(II6,I)+(PP(II5,IGG)*XA**2/2.)
680 XA=XA+AL(M(I))/50.
    GO TO 710
C**** CONCENTRATED LOAD MEMBER.
690 XA=0.0
    DO 700 II6=2,52
    IF(XA.LT.BB(II5,IGG)) GO TO 700
    XAY=XA-BB(II5,IGG)
    AMT(II6,I)=AMT(II6,I)+(PP(II5,IGG)*XAY)
700 XA=XA+AL(M(I))/50.
710 CONTINUE
C**** AREA OF BENDING MOMENT UNDER B.M.D.
720 XA=AL(M(I))/50.
    DO 730 ID=2,51
    IE=ID+1
730 VM=VM+(ABS(AMT(ID,I))+ABS(AMT(IE,I)))/2.*XA
    AMT(53,I)=VM
740 CONTINUE
    IF(DISPOSE.EQ.2.OR.DISPOSE.EQ.3) GO TO 790
CCCC  LFLAG=1
CCCC  CALL PRINT(MI,IGG,LFLAG)
    GO TO 951
790 ICOUNT=MI
    M2=1
    M3=0
801 ICOUNT=ICOUNT-10
    IF(ICOUNT.GE.0) JCOUNT=10
```

```
      IF(ICOUNT.LT.0) JCOUNT=ICOUNT+10
      M3=M3+JCOUNT
      PRINT 802,IGG
  802 FORMAT(5X,*LC*,I3)
      PRINT 803,(AMT(53,L1),L1=M2,M3)
  803 FORMAT(1H+,14X,10(1X,G10.4))
      IF(ICOUNT.LT.0) GO TO 820
      M2=M3+1
      GO TO 801
  820 TOTAL=0.0
      DO 830 I=1,M1
  830 TOTAL=TOTAL+AMT(53,I)
      PRINT 850,TOTAL
  850 FORMAT(1H+,126X,G10.4)
  851 CALL SORTA(M1,J,NJ,IGG,NLC)
      IF(DISPOSE.EQ.2.OR.DISPOSE.EQ.3) GO TO 911
CCCC  LFLAG=2
CCCC  CALL PRINT(M1,IGG,LFLAG)
  911 CONTINUE
      IF(DISPOSE.EQ.4) GO TO 890
      CALL SORTB(M1,M,TOTAL)
      IF(DISPOSE.EQ.2.OR.DISPOSE.EQ.3) GO TO 915
      LFLAG=4
      CALL PRINT(M1,IGG,LFLAG)
  915 PRINT 950,TOTAL
  950 FORMAT(//,5X,*CONDENSED VOLUME OF MAXIMUM MOMENT ENVELOPE=*,G10.4)
      IF(DISPOSE.LE.2) PRINT 925
      IF(DISPOSE.EQ.3) PRINT 926
  926 FORMAT(//,5X,*MAXIMUM POS. AND NEG. JOINT DISPLACEMENTS*,/)
      PRINT 949
  925 FORMAT(1H1,4X,*MAXIMUM POS. AND NEG. JOINT DISPLACEMENTS*,/)
  949 FORMAT(5X,*JT     X +VE        X -VE        Y +VE        Y -VE        Z +VE
     +      Z -VE*,/)
      DO 1000 I=1,NJ
 1000 PRINT 1100,(DISP(I,L1),L1=1,7)
 1100 FORMAT(5X,F3.0,6(1X,G10.4))
      IF(DISPOSE.EQ.3) GO TO 394
      IF(DISPOSE.NE.3) GO TO 392
  890 STOP
      END
```

```
      SUBROUTINE PINS(NINT,NST,IPIN)
      DIMENSION IPIN(625,4)
      N1=NINT+1
      N2=N1**NST
      M1=N2
      DO 500 L1=1,NST
      M1=M1/N1
      M2=N1**(L1-1)
      ICOUNT=0
      DO 500 L2=1,M2
      DO 500 L3=1,N1
      DO 500 L4=1,M1
      ICOUNT=ICOUNT+1
  500 IPIN(ICOUNT,L1)=(L3-1)
      RETURN
      END
```

```
      SUBROUTINE SORTA(MI,J,NJ,IGG,NLC)
      COMMON/C8/SORT(53,96),D(120),DISP(40,7),AMT(53,48)
      DIMENSION J(40)
C**** ACCUMULATION OF POS. AND NEG. BENDING MOMENTS
      IF(NLC.EQ.1) GO TO 50
      IF(IGG.EQ.1) GO TO 1250
   50 DO 300 N1=1,MI
      DO 300 N2=2,52
      IF(AMT(N2,N1)) 100,300,200
  100 SORT(N2,2*N1)=SORT(N2,2*N1)+AMT(N2,N1)
      GO TO 300
  200 SORT(N2,2*N1-1)=SORT(N2,2*N1-1)+AMT(N2,N1)
  300 CONTINUE
C**** ACCUMULATION OF POS. AND NEG. JOINT DISPLACEMENTS.
      DO 1200 I=1,NJ
      DISP(I,1)=J(I)
      IF(D(3*J(I)-2)) 600,400,700
  400 IF(D(3*J(I)-1)) 800,500,900
  500 IF(D(3*J(I))) 1000,1200,1100
  600 DISP(I,3)=DISP(I,3)+D(3*J(I)-2)
      GO TO 400
  700 DISP(I,2)=DISP(I,2)+D(3*J(I)-2)
      GO TO 400
  800 DISP(I,5)=DISP(I,5)+D(3*J(I)-1)
      GO TO 500
  900 DISP(I,4)=DISP(I,4)+D(3*J(I)-1)
      GO TO 500
 1000 DISP(I,7)=DISP(I,7)+D(3*J(I))
      GO TO 1200
 1100 DISP(I,6)=DISP(I,6)+D(3*J(I))
 1200 CONTINUE
      GO TO 1300
 1250 DO 1275 L1=1,MI
      DO 1275 L2=2,52
 1275 SORT(L2,2*L1-1)=SORT(L2,2*L1)=AMT(L2,L1)
 1300 RETURN
      END
```

```
      SUBROUTINE SORTB(MI,M,TOTAL)
      DIMENSION M(50)
      COMMON/C5/JTYPE(60),AL(60),AB(60),TITLE
      COMMON/C8/SORT(53,96),D(120),DISP(40,7),AMT(53,48)
C**** REARRANGEMENT OF MOMENT ENVELOPE VALUES
      DO 500 L1=1,MI
      DO 500 L2=2,52
      IF(SORT(L2,2*L1).GT.0.AND.SORT(L2,2*L1-1).GT.0) GO TO 50
      IF(SORT(L2,2*L1).LT.0.AND.SORT(L2,2*L1-1).LT.0) GO TO 150
      GO TO 500
   50 IF(SORT(L2,2*L1-1).GT.SORT(L2,2*L1)) GO TO 100
      SORT(L2,2*L1-1)=SORT(L2,2*L1)
      SORT(L2,2*L1)=0.0
      GO TO 500
  100 SORT(L2,2*L1)=0.0
      GO TO 500
  150 IF(ABS(SORT(L2,2*L1-1)).GT.ABS(SORT(L2,2*L1))) GO TO 200
      SORT(L2,2*L1-1)=0.0
      GO TO 500
  200 SORT(L2,2*L1)=SORT(L2,2*L1-1)
      SORT(L2,2*L1-1)=0.0
  500 CONTINUE
C**** AREA OF CONDENSED MAXIMUM BENDING MOMENT ENVELOPE
      TOTAL=0.0
      L2=2*MI
      DO 300 L1=1,L2
      L3=IFIX(SORT(1,L1))
      VM=0.0
      XA=AL(M(L3))/50.
      DO 400 L3=2,51
      L4=L3+1
  400 VM=VM+(ABS(SORT(L3,L1))+ABS(SORT(L4,L1)))/2.*XA
      SORT(53,L1)=VM
      TOTAL=TOTAL+VM
  300 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE INVERT
      COMMON/C7/S(70,70),N,IFLAG,INDEX(70,2)
      DO 108 I=1,N
108   INDEX(I,1)=0
      II=0
109   AMAX=-1.
      DO 110 I=1,N
      IF(INDEX(I,1)) 110,111,110
111   DO 112 J=1,N
      IF(INDEX(J,1)) 112,113,112
113   TEMP=ABS(S(I,J))
      IF(TEMP-AMAX) 112,112,114
114   IROW=I
      ICOL=J
      AMAX=TEMP
112   CONTINUE
110   CONTINUE
      IF(AMAX) 225,115,116
116   INDEX(ICOL,1)=IROW
      IF(IROW-ICOL) 119,118,119
119   DO 120 J=1,N
      TEMP=S(IROW,J)
      S(IROW,J)=S(ICOL,J)
120   S(ICOL,J)=TEMP
      II=II+1
      INDEX(II,2)=ICOL
118   PIVOT=S(ICOL,ICOL)
      S(ICOL,ICOL)=1.
      PIVOT=1./PIVOT
      DO 121 J=1,N
121   S(ICOL,J)=S(ICOL,J)*PIVOT
      DO 122 I=1,N
      IF(I-ICOL) 123,122,123
123   TEMP=S(I,ICOL)
      S(I,ICOL)=0.
      DO 124 J=1,N
124   S(I,J)=S(I,J)-S(ICOL,J)*TEMP
122   CONTINUE
      GO TO 109
125   ICOL=INDEX(II,2)
      IROW=INDEX(ICOL,1)
      DO 126 I=1,N
      TEMP=S(I,IROW)
      S(I,IROW)=S(I,ICOL)
126   S(I,ICOL)=TEMP
      II=II-1
225   IF(II.LT.0.OR.II.GT.0) GO TO 125
      GO TO 134
115   PRINT 133
133   FORMAT(//,1X,23H*** SINGULAR MATRIX ***)
      IFLAG=1
134   RETURN
      END
```

```
      SUBROUTINE MSTIFF(I)
      COMMON/C3/SMD(6,6)
      COMMON/C4/AX(60),CX(60),CY(60),E(60),AIN(60)
      COMMON/C5/JTYPE(60),AL(60),AB(60),TITLE
      DO 100 IL=1,6
      DO 100 JL=1,6
  100 SMD(IL,JL)=0.
      IF(JTYPE(I).EQ.2) GO TO 200
      IF(JTYPE(I).EQ.3) GO TO 300
C**** FIX-FIX MEMBER STIFFNESS MATRIX
      S1=(E(I)*AX(I))/AL(I)
      S2=(4*E(I)*AIN(I))/AL(I)
      S3=(1.5*S2)/AL(I)
      S4=(2.*S3)/AL(I)
      SMD(1,1)=SMD(4,4)=S1*CX(I)**2+S4*CY(I)**2
      SMD(1,4)=SMD(4,1)=-SMD(1,1)
      SMD(1,2)=SMD(2,1)=(S1-S4)*CX(I)*CY(I)
      SMD(4,5)=SMD(5,4)=SMD(1,2)
      SMD(1,5)=SMD(5,1)=SMD(2,4)=SMD(4,2)=-SMD(1,2)
      SMD(1,3)=SMD(3,1)=SMD(1,6)=SMD(6,1)=-S3*CY(I)
      SMD(3,4)=SMD(4,3)=SMD(4,6)=SMD(6,4)=-SMD(1,3)
      SMD(2,2)=SMD(5,5)=S1*CY(I)**2+S4*CX(I)**2
      SMD(2,5)=SMD(5,2)=-SMD(2,2)
      SMD(2,3)=SMD(3,2)=SMD(2,6)=SMD(6,2)=S3*CX(I)
      SMD(3,5)=SMD(5,3)=SMD(5,6)=SMD(6,5)=-SMD(2,3)
      SMD(3,3)=SMD(6,6)=S2
      SMD(3,6)=SMD(6,3)=S2/2.
      GO TO 400
C**** FIX-PIN-FIX MEMBER STIFFNESS MATRIX
  200 A=AB(I)
      B=AL(I)-AB(I)
      S1=(E(I)*AX(I))/AL(I)
      S2=(3*E(I)*AIN(I))/(A**3+B**3)
      S3=S2*A
      S4=S3*A
      S5=S3*B
      S6=S2*B
      S7=S6*B
      SMD(1,1)=SMD(4,4)=S1*CX(I)**2+S2*CY(I)**2
      SMD(1,4)=SMD(4,1)=-SMD(1,1)
      SMD(1,2)=SMD(2,1)=SMD(4,5)=SMD(5,4)
     +=S1*CX(I)*CY(I)-S2*CX(I)*CY(I)
      SMD(1,5)=SMD(2,4)=SMD(4,2)=SMD(5,1)=-SMD(1,2)
      SMD(3,4)=SMD(4,3)=S3*CY(I)
      SMD(1,3)=SMD(3,1)=-SMD(3,4)
      SMD(4,6)=SMD(6,4)=S6*CY(I)
      SMD(1,6)=SMD(6,1)=-SMD(4,6)
      SMD(2,2)=SMD(5,5)=S1*CY(I)**2+S2*CX(I)**2
      SMD(2,5)=SMD(5,2)=-SMD(5,5)
      SMD(2,3)=SMD(3,2)=S3*CX(I)
      SMD(3,5)=SMD(5,3)=-SMD(2,3)
      SMD(2,6)=SMD(6,2)=S6*CX(I)
      SMD(5,6)=SMD(6,5)=-SMD(6,2)
      SMD(3,3)=S4
      SMD(3,6)=SMD(6,3)=S5
      SMD(6,6)=S7
      GO TO 400
```

```
C**** PIN-PIN MEMBER STIFFNESS MATRIX
 300  SI=(E(I)*AX(I))/AL(I)
      SMD(1,1)=SMD(4,4)=SI*CX(I)**2
      SMD(2,2)=SMD(5,5)=SI*CY(I)**2
      SMD(1,2)=SMD(2,1)=SMD(4,5)=SMD(5,4)=SI*CX(I)*CY(I)
      SMD(1,4)=SMD(4,1)=-SMD(1,1)
      SMD(1,5)=SMD(2,4)=SMD(4,2)=SMD(5,1)=-SMD(1,2)
      SMD(2,5)=SMD(5,2)=-SMD(2,2)
 400  RETURN
      END
```

```
C**** SUBROUTINE FEF(FIXED END FORCES) FOR DETERMINING
C**** THE NODE FORCES DUE TO APPLIED MEMBER LOADINGS
      SUBROUTINE FEF(IGG)
      COMMON/C2/PP(60,30),MM(60,30),ITYPE(60,30),BB(60,30),NLMS(60),NL
     +C,PBA(10,10)
      COMMON/C4/AX(60),CX(60),CY(60),E(60),AIN(60)
      COMMON/C5/JTYPE(60),AL(60),AB(60),TITLE
      COMMON/C6/AM(60,6)
      REAL MA,MB
      INTEGER BC
      NLM=NLMS(IGG)
      DO 50 L1=1,60
      DO 50 L2=1,6
   50 AM(L1,L2)=0.0
      IF(NLM.EQ.0) GO TO 1001
      DO 1000 II=1,NLM
      P=PP(II,IGG)
      B=BB(II,IGG)
      D=AL(MM(II,IGG))
      C=D-B
      IF(ITYPE(II,IGG).EQ.2) GO TO 500
      IF(ITYPE(II,IGG).EQ.3) GO TO 800
      IF(JTYPE(MM(II,IGG)).EQ.2) GO TO 200
      IF(JTYPE(MM(II,IGG)).EQ.3) GO TO 300
C**** UDL  MEMBER TYPE 1.
      RA=P*D/2.
      MA=P*D**2/12.
      AM(MM(II,IGG),2)=AM(MM(II,IGG),2)+RA
      AM(MM(II,IGG),5)=AM(MM(II,IGG),5)+RA
      AM(MM(II,IGG),3)=AM(MM(II,IGG),3)+MA
      AM(MM(II,IGG),6)=AM(MM(II,IGG),5)-MA
      GO TO 1000
C**** UDL  MEMBER TYPE 2.
  200 H=AB(MM(II,IGG))/D
      MA=P*D**2*H*(6*H**2-8*H+3)/3*(3*H**2-3*H+1)
      MB=-P*D**2*(1-H)*(6*H**2-4*H+1)/3*(3*H**2-3*H+1)
      AM(MM(II,IGG),3)=AM(MM(II,IGG),3)+MA
      AM(MM(II,IGG),6)=AM(MM(II,IGG),6)+MB
      AM(MM(II,IGG),2)=AM(MM(II,IGG),2)+(MA+MB)/D+P*D/2.
      AM(MM(II,IGG),5)=AM(MM(II,IGG),5)-(MA+MB)/D+P*D/2.
      GO TO 1000
C**** UDL  MEMBER TYPE 3.
  300 RA=P*D/2.
      AM(MM(II,IGG),2)=AM(MM(II,IGG),2)+RA
      AM(MM(II,IGG),5)=AM(MM(II,IGG),5)+RA
      GO TO 1000
  500 IF(JTYPE(MM(II,IGG)).EQ.2) GO TO 600
      IF(JTYPE(MM(II,IGG)).EQ.3) GO TO 700
C**** PT. LOAD  MEMBER TYPE 1.
      MA=P*B*C**2/D**2
      MB=-P*B**2*C/D**2
      RA=P*C**2*(3*B+C)/D**3
      RB=P*B**2*(B+3*C)/D**3
      AM(MM(II,IGG),2)=AM(MM(II,IGG),2)+RA
      AM(MM(II,IGG),3)=AM(MM(II,IGG),3)+MA
      AM(MM(II,IGG),5)=AM(MM(II,IGG),5)+RB
      AM(MM(II,IGG),6)=AM(MM(II,IGG),6)+MB
```

```
      GO TO 1000
C**** PT. LOAD/ MEMBER TYPE 2.
 600  H=AB(MM(II,IGG))/D
      HK=B/D
      IF(HK.LT.H) GO TO 625
      IF(HK.GT.H) GO TO 650
      MA=P*D*H*(1-H)**3/(3*H**2-3*H+1)
      MB=-P*D*H**3*(1-H)/(3*H**2-3*H+1)
      RA=P*D/2.
      AM(MM(II,IGG),2)=AM(MM(II,IGG),2)+RA
      AM(MM(II,IGG),3)=AM(MM(II,IGG),3)+MA
      AM(MM(II,IGG),5)=AM(MM(II,IGG),5)+RA
      AM(MM(II,IGG),6)=AM(MM(II,IGG),6)+MB
      GO TO 1000
 625  MA=P*D*(HK-((HK**2*H*(3*H-HK))/2*(3*H**2-3*H+1)))
      MB=-P*D*HK**2*(1-H)*(3*H-HK)/2*(3*H**2-3*H+1)
      RA=(MA+MB)/D+P*C/D
      RB=-(MA+MB)/D+P*B/D
      AM(MM(II,IGG),2)=AM(MM(II,IGG),2)+RA
      AM(MM(II,IGG),3)=AM(MM(II,IGG),3)+MA
      AM(MM(II,IGG),5)=AM(MM(II,IGG),5)+RB
      AM(MM(II,IGG),6)=AM(MM(II,IGG),6)+MB
      GO TO 1000
 650  MA=P*D*H*(1-HK)**2*(HK+2-3*H)/2*(3*H**2-3*H+1)
      MB=-P*D*(1-HK-((1-HK)**2*(1-H)*(HK+2-3*H)/2*(3*H**2-3*H+1)))
      RA=(MA+MB+P*C)/D
      RB=(-(MA+MB)+P*B)/D
      AM(MM(II,IGG),2)=AM(MM(II,IGG),2)+RA
      AM(MM(II,IGG),3)=AM(MM(II,IGG),3)+MA
      AM(MM(II,IGG),5)=AM(MM(II,IGG),5)+RB
      AM(MM(II,IGG),6)=AM(MM(II,IGG),6)+MB
      GO TO 1000
C**** PT. LOAD  MEMBER TYPE 3
 700  AM(MM(II,IGG),2)=AM(MM(II,IGG),2)+(P*C/D)
      AM(MM(II,IGG),5)=AM(MM(II,IGG),5)+(P*B/D)
      GO TO 1000
C**** SUPPORT SETTLEMENT MEMBER FIX-END FORCES
C**** FOR MEMBERS TYPE 1 AND 2.
C**** LOCAL AXIS SUPP. DISP. AT NODE(S) I OF MEMBER(S) M
C     DISPLACEMENT IN X=PP(II,IGG)
C                  IN Y=PB(II,IGG)
C                  IN Z=PBA(II,IGG)
 800  PB=PBA(II,IGG)
      BC=MM(II,IGG)
      IF(JTYPE(BC).EQ.2) GO TO 900
C**** FEF MEMBER TYPE 1.
      F1=AX(BC)*E(BC)/AL(BC)
      F2=12*E(BC)*AIN(BC)/AL(BC)**3
      F3=F2*AL(BC)/2
      F4=F3*AL(BC)/1.5
      AM(BC,1)=AM(BC,1)+F1*P
      AM(BC,2)=AM(BC,2)+F2*B+F3*PB
      AM(BC,3)=AM(BC,3)+F3*B+F4*PB
      AM(BC,4)=AM(BC,4)-F1*P
      AM(BC,5)=AM(BC,5)-F2*B-F3*PB
      AM(BC,6)=AM(BC,6)+F3*B+F4*PB/2.
      GO TO 1000
```

```
C**** FEF MEMBER TYPE 2.
  200 ABB=AB(BC)
      ABC=D-ABB
      F1=AX(BC)*E(BC)/AL(BC)
      F2=3*E(BC)*RIN(BC)/(ABB**3+ABC**3)
      AM(BC,1)=AM(BC,1)+F1*P
      AM(BC,2)=AM(BC,2)+F2*B+F2*PB*ABB
      AM(BC,3)=AM(BC,3)+F2*B*ABB+F2*PB*ABB**2
      AM(BC,4)=AM(BC,4)-F1*P
      AM(BC,5)=AM(BC,5)-F2*B-F2*PB*ABB
      AM(BC,6)=AM(BC,6)+F2*B*ABC+F2*PB*ABB*ABC
 1000 CONTINUE
 1001 RETURN
      END
```

```
      SUBROUTINE EQUILIB(IGG)
      INTEGER DISPOSE
      COMMON/CI/J(40),X(40),Y(40),JJ(60),JK(60),K(60),KK(40,30),
     +A(120,30),NLJS(40),DISPOSE,NCS,NBS,MI,NJ,NRJ,NUM,NINT,FG(60,6)
      PRINT 100,IGG
      DO 10 L1=1,NJ
      DO 5 L2=1,NRJ
    5 IF(L1.EQ.K(L2)) GO TO 10
      AX=AY=AZ=0.0
      AX=AX-A(3*L1-2,IGG)
      AY=AY-A(3*L1-1,IGG)
      AZ=AZ-A(3*L1,IGG)
      DO 30 L3=1,MI
      IF(L1.NE.JJ(L3)) GO TO 30
      AX=AX+FG(L3,1)
      AY=AY+FG(L3,2)
      AZ=AZ+FG(L3,3)
   30 CONTINUE
      DO 40 L4=1,MI
      IF(L1.NE.JK(L4)) GO TO 40
      AX=AX+FG(L4,4)
      AY=AY+FG(L4,5)
      AZ=AZ+FG(L4,6)
   40 CONTINUE
      PRINT 200,L1,AX,AY,AZ
   10 CONTINUE
  100 FORMAT(//,5X,*JOINT EQUILIBRIUM CHECK (GLOBAL AXIS)  LOAD CASE*,I4
     +,/,5X,*JT     DELTA-X         DELTA-Y        DELTA-Z*,/)
  200 FORMAT(5X,I2,3(1X,G11.4,1X))
      RETURN
      END
```

```
      SUBROUTINE PRINT(M1,IGG,LFLAG)
      COMMON/CB/SORT(53,96),D(120),DISP(40,7),AMT(53,48)
      IF(LFLAG.EQ.1) ICOUNT=M1
      IF(LFLAG.NE.1) ICOUNT=2*M1
      M2=1
      M3=0
   50 ICOUNT=ICOUNT-10
      IF(ICOUNT.GE.0) JCOUNT=10
      IF(ICOUNT.LT.0) JCOUNT=ICOUNT+10
      M3=M3+JCOUNT
      IF(LFLAG.EQ.2.OR.LFLAG.EQ.4) GO TO 500
      PRINT 100,M2,M3,IGG
      PRINT 200,(SORT(1,2*L1),L1=M2,M3)
      DO 300 L1=2,53
      L9=L1-2
  300 PRINT 400,L9,(AMT(L1,L2),L2=M2,M3)
      GO TO 1100
  500 IF(LFLAG.EQ.4) M4=53
      IF(LFLAG.EQ.2) M4=52
      PRINT 600
      PRINT 200,(SORT(1,L1),L1=M2,M3)
      DO 700 L1=2,M4
      L9=L1-2
  700 PRINT 400,L9,(SORT(L1,L2),L2=M2,M3)
 1100 IF(ICOUNT.LE.0) GO TO 1500
      M2=M3+1
      GO TO 50
  100 FORMAT(1H1,4X,*BENDING MOMENT DIAGRAM...MEMBERS*,I3,* TO*,I3,* LOA
     +D CASE*,I3,/)
  200 FORMAT(8X,10(F3.0,9X),/)
  400 FORMAT(1X,I2,*) *,10(1X,G11.4))
  600 FORMAT(1H1,4X,*CUMULATIVELY SORTED MAXIMUM BENDING MOMENT ENVELOPE
     +*,/)
 1500 RETURN
      END
```

```
      SUBROUTINE PRINT1(M,NBAYS,NST)
      COMMON/C1/J(40),X(40),Y(40),JJ(60),JK(60),K(60),KK(40,30)
     +,A(120,30),NLJS(40),DISPOSE,NCS,NBS,MI,NJ,NRJ,NUM,NINT,FG(60,6)
      COMMON/C2/PP(60,30),MM(60,30),ITYPE(60,30),BB(60,30),NLMS(60),NL
     +C,PBA(10,10)
      COMMON/C4/AX(60),CX(60),CY(60),E(60),AIN(60)
      COMMON/C5/JTYPE(60),AL(60),AB(60),TITLE
      DIMENSION M(60)
      INTEGER DISPOSE
      PRINT 140
      PRINT 145,TITLE
      PRINT 150,NUM,DISPOSE
      IF(DISPOSE.EQ.2.OR.DISPOSE.EQ.3) GO TO 10
      PRINT 160,MI,NJ,NRJ,NLC
      GO TO 20
   10 PRINT 160,MI,NJ,NRJ,NLC
      PRINT 280,NCS,NBS,NINT
      IF(DISPOSE.EQ.3) PRINT 285,NBAYS,NST
   20 PRINT 170
      DO 30 I=1,NJ
   30 PRINT 190,J(I),X(I),Y(I)
      PRINT 200
      DO 40 I=1,MI
   40 PRINT 220,M(I),JTYPE(I),JJ(I),JK(I),AX(I),AIN(I),E(I)
      PRINT 210
      DO 60 I=1,MI
   60 PRINT 240,I,AL(I),CX(I),CY(I),AB(I)
      PRINT 230
      DO 100 I=1,NRJ
  100 PRINT 250,K(I)
      DO 320 IG=1,NLC
      IF(NLJS(IG).EQ.0) GO TO 120
      PRINT 270,IG
      IGG=NLJS(IG)
      DO 110 I=1,IGG
  110 PRINT 290,KK(I,IG),A((3*KK(I,IG)-2),IG),A((3*KK(I,IG)-1),IG)
     +,A((3*KK(I,IG)),IG)
  120 IF(NLMS(IG).EQ.0) GO TO 320
      PRINT 300,IG
      IGG=NLMS(IG)
      DO 130 I=1,IGG
      IF(ITYPE(I,IGG).NE.3) GO TO 121
      PRINT 311,MM(I,IG),ITYPE(I,IG),PP(I,IG),BB(I,IG),PBA(I,IG)
      GO TO 130
  121 PRINT 310,MM(I,IG),ITYPE(I,IG),PP(I,IG),BB(I,IG)
  130 CONTINUE
  140 FORMAT(1H ,4X,*PFRAME VERSION 1.1 DEVELOPED BY JOSEPH GALLACCIO*,/
     +,5X,*ANALYSIS OF PLANE STRUCTURES  (STIFFNESS METHOD)*)
  145 FORMAT(//,5X,5A10)
  150 FORMAT(/,5X,*FRAME*,I4,* ...DISPOSE TYPE*,I2)
  160 FORMAT(/,5X,I3,* MEMBERS/*,I3,* JOINTS/*,I3,* RESTRAINED JOINTS/*,
     +I3,* LOAD CASE(S)*)
  170 FORMAT(/,5X,*JOINT COORDINATES*,/,5X,*JT*,5X,*X*,7X,*Y*)
  190 FORMAT(4X,I3,2(2X,F6.2))
  200 FORMAT(/,5X,*MEMBER DESIGNATION AND PROPERTIES*,/,5X,*MEMBER TYPE
     + I  J*,4X,*AREA*,7X,*INERTIA*,5X,*M OF E*)
  210 FORMAT(5X,*MEMBER  LENGTH  COS-X  COS-Y       A*)
```

```
220 FORMAT(5X,I3,6X,I1,2(2X,I2),2X,3(G10.4,1X))
230 FORMAT(/,5X,*RESTRAINED SUPPORTS*,/,5X,*JT*)
240 FORMAT(5X,I3,5X,F6.2,2(2X,F5.2),2X,F6.2)
250 FORMAT(4X,I3)
260 FORMAT(5X,I3,5X,F6.2,2(2X,F5.2))
270 FORMAT(/,5X,*JOINT FORCES (GLOBAL AXIS SYSTEM)  LOAD CASE*,I3,/,5X
   +,*JT*,8X,*X*,11X,*Y*,11X,*Z*)
280 FORMAT(5X,I3,* COLUMNS/*,I3,* BEAMS/ *,I3,* PIN INTERVALS*)
285 FORMAT(1H+,45X,*/*,I4,* BAY(S)/*,I4,* STORY(S)*)
290 FORMAT(4X,I3,1X,F10.2,2(2X,F10.2))
300 FORMAT(/,5X,*MEMBER  LOAD (MEMBER AXIS SYSTEM)  LOAD CASE*,I3,/,5X
   +,*MEMBER TYPE*,7X,*A1*,7X,*A2*,7X,*A3*)
310 FORMAT(6X,I3,2X,I3,3X,2(1X,F8.4))
311 FORMAT(6X,I3,2X,I3,3X,3(1X,F8.4))
320 CONTINUE
    RETURN
    END
```