Pair Production and Decay of the Higgs Fields in the Left-Right Symmetric Extension of the Standard Model

Homayoun Hamidian

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ABSTRACT

Pair Production and Decay of the Higgs Fields in the Left-Right Symmetric Extension of the Standard Model

The Higgs sector of the (minimal) left-right symmetric extension of the Standard Model is studied in some detail. Using constraints coming from low energy phenomenology certain vacuum expectation value scenarios have been singled out as the most natural ones. Scattering cross sections for Higgs pair productions through e^+e^- annihilation and the renormalization group improved decay widths for the Higgs fields in the left-right symmetric theory are also calculated.

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INTRODUCTION

One of the major triumphs of theoretical physics in the 20th century is the development of the Standard Model of strong, electromagnetic and weak interactions based on the gauge group $SU(3)_c \times SU(2)_L \times U(1)_Y$. The $SU(3)_c$ gauge theory is now widely believed to describe strong interactions, although some problems such as the proof of the hypothesized confinement still persist. The $SU(2)_L \times U(1)_Y$ is an attempt to unify electromagnetic and weak interactions through the gauge structure of the theory. Among other things, this requires considering quarks and leptons on the same footing as far as their representation properties are concerned. This, however, is not totally realized in this model since right-handed neutrinos are absent. Also, since quarks and some of the leptons have non-zero masses, the $SU(2)_L$ symmetry must break down. This symmetry is broken by introducing certain scalar bosons in the theory which have non-vanishing vacuum expectation values. It is found that this model correctly describes weak interactions and the agreement between experiment and some of its predictions such as the existence of neutral currents and the W and Z particles has been spectacular.

However, the model suffers from various problems. For instance, there are 19 free parameters in the theory whose values are put in by hand and there is no explanation from the gauge structure of the theory to explain the assumed handedness of neutrinos. Moreover the Higgs field remains undetected and elusive, apart from being totally mysterious from a purely theoretical point of view. (Except for the fact that, so far, it is the only known way to give masses to the weak gauge bosons and at the same time maintain the renormalizability of the theory).

Due to these and other problems, and in the light of general support for (quantized) gauge field theories, physicists have been searching for higher and higher gauge symmetries which would partially answer some of the problems at the electroweak energy scale.

One of the most attractive theories beyond the $SU(2)_L \times U(1)_Y$ theory is its left-right symmetric (LRS) extension based on the gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. This model relates the question of handedness to the symmetry-breaking energy scale associated with the $SU(2)_R$ part of the gauge group. Its structure is sufficiently rich to include a mechanism to generate very light left-handed and massive right-handed neutrinos consistent with laboratory and cosmological data.

In this thesis an analysis of the Higgs sector of the minimal version of the LRS theory has been made. In particular, the Higgs potential is studied in Chapter 3 and certain consequences of it are analyzed. The renormalization group improved hadronic decay width of the Higgs field in the LRS theory are calculated in Chapter 4. In Chapter 5 the scattering cross sections for the production of the LRS theory Higgs fields through e^+e^- annihilations have been calculated. The various bounds for the masses of the underlying Higgs fields in the calculations of Chapters 4 and 5 have been chosen by keeping in mind some general constraints discussed in chapter 3.

For the sake of completeness, some background material on the Standard Model and its LRS extension are provided in chapters 1 and 2. Also, some preliminaries on the QCD renormalization group equations, related to the subject of chapter 4, are given in appendix B. The notation and conventions used throughout the thesis are given in Appendix A.

The thesis is concluded with some remarks and future prospects in the LRS theory.

CHAPTER1

THE STANDARD MODEL

1.1 Introduction

In this chapter the unified theory of weak and electromagnetic interactions of Glashow, Weinberg and Salam[1-3](GWS) will be described in some detail. In 1961 Glashow constructed a model of weak and electromagnetic interactions of leptons which was based on the gauge group SU(2) × U(1). This theory is based on the assumption that, together with the photon, there exist also charged W and neutral Z intermediate bosons. The masses of the W and Z bosons were introduced "by hand". As a result, the model was unrenormalizable. In 1967-68 Weinberg[2] and Salam[3] constructed the SU(2) × U(1) model of electroweak interactions of leptons with spontaneous gauge- symmetry breakdown. In 1971-72 't Hooft[4] and others[5] proved that models of this type were renormalizable. The model was subsequently generalized to include quarks[6] using the mechanism proposed by Glashow, Illiopoulos and Maiani[7]. The GWS theory is based on the assumption that there exist charged and intermediate vector bosons and it is constructed so that massless fundamental fermions (leptons and quarks) obey local SU(2) × U(1) gauge invariance. Then the interaction (again locally gauge invariant) of Higgs scalar fields, with both gauge vector bosons and fermions, is introduced. As a consequence of the spontaneous breakdown of the underlying symmetry, leptons, quarks and intermediate bosons all acquire masses. In the following sections these steps will be described. Further details can be found in Refs.9-13.

1.2 Quantum Electrodynamics (QED)

A good starting point for a discussion of GWS theory is Quantum Electrodynamics (QED). QED is the first and simplest gauge theory.

The Lagrangian L for the massless electromagnetic field A_μ interacting with a spin $\frac{1}{2}$ field ψ of bare mass m is[9,10]

$$L = -\frac{1}{4} F_{\mu\nu}F^{\mu\nu} + \bar{\psi}\,(i\gamma^\mu D_\mu - m)\,\psi \qquad (1.1)$$

here, $F_{\mu\nu}$ is the electromagnetic field tensor

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

and D_μ is the *covariant* derivative

$$D_\mu = \partial_\mu + ie\, A_\mu Q$$

where e is the unit electric charge and Q is the charge operator. This Lagrangian is invariant under *local gauge transformations*

$$\psi(x) \longrightarrow U(x)\psi(x) \quad , \quad A_\mu(x) \longrightarrow A_\mu(x) + \partial_\mu\alpha(x) \qquad (1.2)$$

with

$$U(x) = \exp\,(-ie\ Q\ \alpha(x))$$

and arbitrary function $\alpha(x)$. For infinitesimal $\alpha(x)$,

$$\psi(x) \longrightarrow (1 - ie\ Q\ \alpha(x))\psi(x) \qquad (1.3)$$

It is sufficient to discuss infinitesimal transformations since finite transformations can be obtained from them by integration. Retaining only terms of lowest order in $\alpha(x)$, it can be shown that L remains invariant under the gauge transformations (1.2) and (1.3).

Local gauge invariance demands that there be a *gauge field* A_μ which interacts with fermions in a prescribed way. Had one started from non-invariant ψ fields, the Lagrangian would already have been invariant under global transformations (U independent of x) but local gauge invariance would have required the existence of A_μ fields plus the interaction term

$$L_{int} = -e\ J^\mu_{em} A_\mu$$

where $J^\mu_{em} = \bar{\psi}\ \gamma^\mu Q\ \psi$ is the electromagnetic current.

The Euler-Lagrange equation

$$\partial_\mu[\ \partial L\ /\ \partial(\partial_\mu\phi_r)\ ] = \partial L\ /\ \partial\phi_r$$

obtained from the requirement of stationary action for any field ϕ_r, yields the following equations of motion of electromagnetism

$$\partial_\nu F^{\mu\nu} = J^\mu_{em} \quad , \quad (i\gamma^\mu D_\mu - m)\psi = 0 \tag{1.4}$$

As a result of (1.4), $\partial_\mu J^\mu_{em} = 0$, implying that the electromagnetic current and hence the electric charge q is conserved, where

$$q = \int J^0_{em} \, d^3x \tag{1.5}$$

This is an example of the celebrated Nother's theorem, stating that for each continuous symmetry ,

$$\phi_r \longrightarrow \phi_r - i \, \varepsilon \, \lambda_{rs}\phi_s \tag{1.6}$$

with ε an infinitesimal parameter and λ_{rs} constant coefficients, there is a conserved current

$$J_\mu(x) = - i \, \lambda_{rs}\phi_s \, \partial L \, / \, \partial \, (\partial_\mu\phi_r) \tag{1.7}$$

In the language of group theory, the QED gauge transformations with scalar phase $\alpha(x)$ belong to the Abelian unitary group U(1) and the full Lagrangian is said to have $U(1)_Q$ symmetry, with the charge operator Q as the generator.

The QED *minimal coupling* of the photon A_μ to spinors (see (1.4)) is introduced through the covariant derivative $D_\mu\psi$ and is determined purely by the transformation properties of ψ under the $U(1)_Q$ gauge group. Other gauge invariant couplings, however, can be constructed (e.g., $\bar{\psi}\,\sigma_{\mu\nu}F^{\mu\nu}\psi$) but they are not renormalizable, as can be checked by simple dimension counting[9-12].

1.3 Yang-Mills Fields and the Unbroken $SU(2)_L \times U(1)_Y$ Model

Gauge transformations can also involve internal degrees of freedom. For example, consider an internal symmetry group SU(2) such as isospin under which spin $\frac{1}{2}$ fields transform as doublets. Their free field Lagrangian is

$$L = \bar{\psi}\ (\ i\ \gamma^\mu\partial_\mu - m\ )\psi \qquad (1.8)$$

where $\bar{\psi}$ and ψ are row and column vectors in the isospin space, respectively. In analogy with QED, one can now require invariance under the infinitesimal local gauge transformation

$$\psi(x) \longrightarrow [\ 1 - ig\ \vec{\alpha}(x)\cdot\vec{T}\ ]\ \psi(x) \qquad (1.9)$$

where $\vec{\alpha}(x)$ is an arbitrary infinitesimal vector in the isospin space and $\vec{T} = (\ T_1, T_2, T_3\ )$ is the isospin operator whose components T_i are generators of SU(2) group transformations. The T_i form a *Lie Algebra*[11,12] with the commutation relations

$$[\ T_i, T_j\ ] = i\ \varepsilon_{ijk}\ T_k \tag{1.10}$$

When operating on isospin doublets, the matrix representation is $T_i = \frac{1}{2}\ \tau_i$ where τ_i are the Pauli matrices.

In order to make the ψ-field part of the Lagrangian invarint one has to introduce an appropriate covariant derivative D_μ. Thus

$$L_\psi = \bar{\psi}\ (\ i\ \gamma^\mu D_\mu - m\ )\ \psi \quad , \quad D_\mu = \partial_\mu + ig\ \vec{W}_\mu \cdot \vec{T} \tag{1.11}$$

provided that an isospin triplet of *Yang-Mills* gauge fields $W_{i\mu}$ (i = 1, 2, 3) exists and transforms simultaneously as

$$\vec{W}_\mu \longrightarrow \vec{W}_\mu(x) + \partial_\mu \vec{\alpha}(x) + g\ \vec{\alpha}(x) \times \vec{W}_\mu(x) \tag{1.12}$$

This transformation on the gauge fields is more complicated than in the QED case, because of the non-Abelian nature of the group SU(2). The gauge invariance of L_ψ is evident upon inspection.

A gauge-invariant form for the $\vec{W}$ part of the Lagrangian can be written as

where

$$\vec{W}_{\mu\nu} = \partial_\mu \vec{W}_\nu - \partial_\nu \vec{W}_\mu - g\, \vec{W}_\mu \times \vec{W}_\nu \qquad (1.14)$$

In addition to the usual kinetic energy terms, this introduces cubic and quartic self-couplings of the $\vec{W}_\mu$ fields. This is necessary to keep the gauge invariance of the theory and is a result of (1.10) and (1.12). An SU(2) gauge model is a canditate for weak interaction theory, since the isospin triplet $\vec{W}$ could consist of W^+, W^0, W^- bosons to transmit the weak force, with

$$W^\pm_\mu = \frac{1}{\sqrt{2}} \left(W_{1\mu} \mp i\, W_{2\mu} \right) \quad , \quad W^0_\mu = W_{3\mu} \qquad (1.15)$$

where the field operators $W^\pm_\mu$ are defined to annihilate $W^\pm$ bosons. This model, however, is unsatisfactory for a variety of reasons. Most importantly, the effective low-energy form of weak interactions implies that the charged bosons must be very massive, and also implies a left-handed structure for the charged-current couplings[9-12]. Also, it is desirable to unify weak and electromagnetic interactions into single gauge theory. To generate the left-handed structure of charged-current weak interaction, an SU(2) gauge symmetry is applied to left-handed fermion fields ψ_L only, where left- and right-handed fermion fields are defined as

$$\psi_L = \frac{1}{2} (1 - \gamma_5)\psi \quad , \quad \psi_R = \frac{1}{2} (1 + \gamma_5)\psi$$

The fermion mass term $m\bar{\psi}\psi = m(\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L)$ is, however, not invariant under $SU(2)_L$. Therefore, at this stage, one takes the fermions to be massless.

The conserved quantum number corresponding to $SU(2)_L$ is the *weak isospin* $\vec{T}_L$. In addition to $SU(2)_L$, an independent $U(1)_Y$ gauge symmetry is introduced whose conserved quantum number Y is called *weak hypercharge*. The $U(1)_Y$ symmetry is essential to incorporate conservation of the electric charge Q and unify the weak and electromagnetic interactions in a common gauge structure. The weak hypercharges are specified according to the formula

$$Q = T_3 + \frac{1}{2} Y$$

in analogy with the Gell-Mann– Nishijima formula. This formula simply reflects the fact that the gauge group, namely $SU(2)_L \times U(1)_Y$ is a *direct product* of two other gauge groups. Right-handed fermions are assigned to transform under $U(1)_Y$ only; this is to prevent introducing right-handed neutrinos. Left-handed fermions transform non-trivially under both $SU(2)_L$ and $U(1)_Y$. The *electroweak* quantum numbers for the first generation of quarks and leptons are given in Table 1 below. It is a mystery of the Standard Model why Y takes values such that the lepton and quark charges are obtained. With equal numbers of quarks and leptons and three quark colours, these quantum numbers lead to the cancellation of divergent chiral anomalies[14-17] (see Refs. 11 and 13 for details).

Table 1

	T	T_3	$\frac{1}{2}Y$	Q
ν_{eL}	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0
e_L	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	-1
u_L	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{2}{3}$
d_L	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{6}$	$-\frac{1}{3}$
e_R	0	0	-1	-1
u_R	0	0	$-\frac{2}{3}$	$-\frac{2}{3}$
d_R	0	0	$-\frac{1}{3}$	$-\frac{1}{3}$

The massless gauge fields in this model are an isotriplet $\vec{W}_\mu$ for $SU(2)_L$ and a singlet B_μ for $U(1)_Y$. The Lagrangian is

$$L = -\frac{1}{4}\vec{W}^{\mu\nu}\cdot\vec{W}_{\mu\nu} - \frac{1}{4}B^{\mu\nu}B_{\mu\nu} + \bar{\psi}\, i\gamma^{\mu}D_{\mu}\psi \qquad (1.16)$$

with a seperate fermion term for each field ψ_L and ψ_R. The field tensor $\vec{W}_{\mu\nu}$ is defined as in (1.14) and

$$B_{\mu\nu} = \partial_{\mu}B_{\nu} - \partial_{\nu}B_{\mu}$$

The covariant derivative is

$$D_{\mu} = \partial_{\mu} + ig\,\vec{W}_{\mu}\cdot\vec{T} + ig'\frac{1}{2}B_{\mu}Y \qquad (1.17)$$

The Lagrangian is invariant under infinitesimal local gauge transformations $SU(2)_L$ and $U(1)_Y$ independently. Applied to the isodoublet field ψ_L, the weak isospin operator $\vec{T}$ can be represented as $\vec{\tau}/2$ in terms of the Pauli matrices. One defines isospin raising and lowering operators $T^{\pm} = (T_1 \pm iT_2)/\sqrt{2}$ and hence, $\vec{W}\cdot\vec{T} = W^{+}T^{+} + W^{-}T^{-} + W_3T_3$. For the electromagnetic interaction to be unified with the weak interaction in this model, the electromagnetic term ieQ must be contained in the neutral term $i(\,gW_{3\mu}T_3 + g'\frac{1}{2}B_{\mu}Y\,)$ of (1.17). Therefore the W_3 and B fields must be linear combinations of A and another neutral field Z; since all the vector boson terms have the same normalization, one can write this relation as

$$\begin{pmatrix} W_3 \\ B \end{pmatrix} = \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} Z \\ A \end{pmatrix} \tag{1.18}$$

where θ_W is the electroweak mixing angle (Weinberg angle). Hence

$$ig\, W_3 T_3 + ig' \frac{1}{2} B\, Y = iA\, [\, g \sin\theta_W T_3 + \frac{1}{2} g' \cos\theta_W Y \,]$$

$$+ iZ\, [\, g \cos\theta_W T_3 - \frac{1}{2} g' \sin\theta_W Y \,]$$

For the coefficient of A to equal $ieQ = ie\,(T_3 + \frac{1}{2})$, one needs

$$g = e/\sin\theta_W \quad , \quad g' = e/\cos\theta_W \tag{1.19}$$

and hence, $1/g^2 + 1/g'^2 = 1/e^2$. The Z term of the covariant derivative can be written as

$$D^Z_\mu = ig_Z\, Z_\mu\, (\, T_3 - x_W Q \,)$$

where

$$g_Z = \frac{e}{\sin\theta_W \cos\theta_W} \quad \text{and} \quad x_W = \sin^2\theta_W \tag{1.20}$$

The interaction of gauge bosons with any fermion field ψ arises from the term $\bar{\psi} i \gamma^\mu D_\mu \psi$ in L which can be written as

$$-L' = eJ^\mu_{em} A_\mu + \frac{g}{\sqrt{2}}\, (\, J^{+\mu}_L W^+_\mu + J^{-\mu}_L W^-_\mu \,) + g_Z J^\mu_Z\, Z_\mu \tag{1.21}$$

where

$$J_L^{\pm\mu} = \sqrt{2}\ \bar{\psi}\ \gamma^\mu\ T_L^\pm\ \psi \tag{1.22}$$

$$J_Z^\mu = \bar{\psi}\ \gamma^\mu\ [\ T_{3L} - x_W Q\ ]\ \psi \tag{1.23}$$

$$J_{em}^\mu = \bar{\psi}\ \gamma^\mu Q\ \psi \tag{1.24}$$

The angle θ_W is a parameter of the model. For given θ_W all gauge couplings are determined by the electric charge e; the weak and electromagnetic interactions are thereby unified.

The deficiency of this model as it stands is that the $W^\pm$ and Z bosons and the fermions are all massless. The problem is to generate the required masses while preserving the renormalizability of the theory. This is achieved, subsequent to *spontaneous symmetry breaking*, by the Higgs mechanism where the gauge symmetry of the Lagrangian remains but is "hidden" by the appearance of a preferred direction in weak isospin space, as described below.

1.4 The Higgs Mechanism

In the Standard Model an SU(2) doublet of scalar fields Φ is introduced. Its self-interactions provide the (Higgs) mechanism subsequent to spontaneous symmetry breaking (SSB), thus giving masses to gauge and fermion fields. It also gives rise to a new neutral scalar particle, the Higgs boson. The Lagrangian (1.21) is thus modified through the addition of L_Φ and L_Φ^F where

$$L_\Phi = |D_\mu \Phi|^2 - V(|\Phi|^2) \tag{1.25}$$

Here, $|\Phi|^2 = \Phi^\dagger \Phi$ and L_Φ^F is the term representing the Yukawa coupling of Φ to fermions, to be discussed later. The most general renormalizable expression for the scalar potential V is

$$V = \mu^2 |\Phi|^2 + \lambda |\Phi|^4 \tag{1.26}$$

and one specifies the isodoublet as

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \tag{1.27}$$

where ϕ^+ and ϕ^0 are each complex fields with the following quantum numbers

	T	T_3	$\frac{1}{2}Y$	Q
ϕ^+	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1
ϕ^0	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	0

In a classical theory with $\mu^2<0$ the ground state of $|\Phi|^2$ occurs at $|\Phi|^2 = -\frac{1}{2}\mu^2/\lambda$. The quantum analog is a non-vanishing expectation value of $|\Phi|^2$ in the physical vacuum state. The appearance of this non-vanishing vacuum expectation value (VEV) selects a preferred direction in weak isospin space and thereby "spontaneously breaks" the $SU(2)\times U(1)$ symmetry.

Since conventional perturbation theory is formulated for fields with zero VEV, it is appropriate to separate out the VEV and redefine the scalar doublet Φ as

$$\Phi(x) = \exp\left(\frac{i\ \vec{\xi}(x)\cdot\vec{\tau}}{2v}\right)\begin{pmatrix} 0 \\ (v + H(x))/\sqrt{2} \end{pmatrix} \tag{1.28}$$

where $v/\sqrt{2} = (-\mu^2/2\lambda)^{1/2}$ and the real fields $\xi_1(x)$, $\xi_2(x)$, $\xi_3(x)$, and $H(x)$ have zero VEVs. By a finite gauge transformation under $SU(2)_L$ with $\vec{\alpha}(x) = \vec{\xi}(x)/v$, one can remove the phase factor from $\Phi(x)$ in (1.28), eliminating the explicit appearance of $\vec{\xi}(x)$ in the Lagrangian. In this "unitary gauge" the $\vec{\xi}$ degrees of freedom seem to vanish but essentially reappear as the longitudinal components of $W^{\pm}$ and Z when they acquire masses; the $\vec{\xi}$ degrees of freedom are said to have been "eaten" by the gauge fields.

The Goldstone theorem[19] states that massless spin-0 particles appear in a theory when a continuous symmetry is spontaneously broken; physically they embody the zero energy exitations that were previously described by symmetry transformations. For global symmetry breaking such *Goldsone bosons* are unavoidable but in gauge symmetry breaking by the

Higgs mechanism the situation is different. There are three Goldstone bosons in the present case that one can represent by the $\vec{\xi}$ degrees of freedom. These degrees of freedom are gauged away from the scalar sector but essentially reappear (with mass) in the gauge field sector, where they provide longitudinal masses for $W^{\pm}$ and Z^0.

The covariant derivative operation on an isodoublet field expressed in terms of the physical A, W^{+}, and Z fields is

$$D_{\mu} = \partial_{\mu} + ieQ\, A_{\mu} + \frac{i}{\sqrt{2}}\, g\, (\tau^{+}W^{+}_{\mu} + \tau^{-}W^{-}_{\mu})$$

$$+ ig_{z}\, (\frac{1}{2}\, \tau_{3} - x_{W}Q)Z \qquad (1.29)$$

where

$$\tau^{+} = \sqrt{2}\, T^{+} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \quad , \quad \tau^{-} = \sqrt{2}\, T^{-} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

Since in the unitary gauge $\Phi(x)$ has only a neutral component, one obtains

$$D_{\mu}\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{1}{\sqrt{2}}\, igW^{+}(v + H) \\ \partial_{\mu}H - \frac{1}{2}\, ig_{z}Z(v + H) \end{pmatrix}$$

which after substitution in (1.25) gives

$$L_\Phi = \frac{1}{2} (\partial H)^2 + \frac{1}{4} g^2 W^+ W^- (v + H)^2 + \frac{1}{8} g_Z^2 ZZ (v + H)^2$$

$$- V \left[\frac{1}{2} (v + H)^2 \right] \tag{1.30}$$

The v^2 terms provide W and Z boson mass terms

$$M_W^2 W^+ W^- + \frac{1}{2} M_Z^2 ZZ$$

with

$$M_W = \frac{1}{2} g v \qquad \text{and} \qquad M_Z = \frac{1}{2} g_Z v = \frac{M_W}{\cos\theta_W} \tag{1.31}$$

while the photon (A_μ) remains massless.
The kinetic and potential terms in L_Φ give

$$\frac{1}{2} (\partial H)^2 - \frac{1}{2} (-2\mu^2) H^2 + \frac{1}{4} \mu^2 v^2 \left[-1 + \frac{4H^3}{v^3} + \frac{H^4}{v^4} \right] \tag{1.32}$$

describing a physical Higgs scalar boson of mass $m_H = \sqrt{-2\mu^2}$ with cubic and quartic self-interactions.

As one can see from (1.30), the field H has no electromagnetic interaction and its interaction with the other gauge fields are given by the cubic and quartic terms

$$\left(\frac{1}{4} g^2 W^+ W^- + \frac{1}{8} g_Z^2 ZZ \right) \left(H^2 + 2vH \right) \tag{1.33}$$

all of which are completely specified by the gauge couplings.

In physical terms, the final result of the Higgs mechanism is that the vacuum everywhere can emit and absorb a neutral colourless quantum of the Higgs field that carries weak isospin and hypercharge quantum numbers. As a result, the fermions and the W and Z bosons that couple to such a quantum effectively acquire masses, but the photon and the gluons that cannot couple to it remain massless.

1.5 Parameters of the Gauge Sector

As has been described above, the parameters g, g', and v determine the gauge field masses and interactions in the standard model. However, it is customary to work with other more convenient sets of parameters. For low energy electroweak interactions $\alpha = e^2/4\pi$, $G_F = \frac{\sqrt{2}\, g^2}{8\, M_W^2}$ (the Fermi coupling constant) and $\sin^2\theta_W$ are commonly used because the first two are very accurately known, leaving $\sin^2\theta_W$ as the single parameter (characteristic of unification) to be specified. The basic parameters are related by $g = e/\sin\theta_W$, $g' = e/\cos\theta_W$ and

$$v = 2M_W\,/g = (\ \sqrt{2}\ G_F\ )^{-1/2} = 246\ \text{GeV} \tag{1.34}$$

and the weak boson masses are

$$M_W = A\ /\ \sin\theta_W \qquad \text{and} \qquad M_Z = A\ /\ \sin\theta_W\cos\theta_W$$

where

$$A = (\pi \alpha / \sqrt{2} G_F)^{1/2} = 37.2810 \pm 0.0003 \text{ GeV}$$

with G_F determined by the mean lifetime of the W particle within certain radiative corrections.

1.6 Lepton Masses

Spontaneous symmetry breaking will generate an electron mass if one adds a Yukawa interaction term for leptons and the Φ fields to the Lagrangian. This term must, of course, have the property of being both renormalizable and gauge invariant under the $SU(2)_L \times U(1)_Y$ gauge transformations. It is given by

$$L = - G_\ell [\bar{\ell}_R(\Phi^\dagger \ell_L) + (\bar{\ell}_L \Phi)\ell_R] \tag{1.35}$$

where G_ℓ is a new coupling constant (ℓ = e, μ, τ are the lepton-type indices) and

$$\ell_L = \begin{pmatrix} \nu_\ell \\ \ell \end{pmatrix}$$

Substituting for $\Phi(x)$ in the unitary gauge (see paragraph following (1.28)) gives

$$L = - (G_e v / \sqrt{2}) \bar{e} e - (G_e / \sqrt{2}) H \bar{e} e \tag{1.36}$$

Thus the electron acquires a mass

$$m_e = G_e v / \sqrt{2}$$

and also a coupling to the Higgs boson. Replacing G_e by $\sqrt{2}$ m_e/v and using $v = (\sqrt{2}\ G_F)^{-1/2}$, the Higgs boson coupling to the electron can be written as

$$-\frac{m_e}{v} H \bar{e} e = -2^{1/4} \sqrt{G_F}\, m_e H \bar{e} e$$

This coupling is very small, $G_e = 2.9 \cdot 10^{-6}$.

One can apply similar arguments to second and third generations of leptons to obtain the Yukawa interaction terms

$$-2^{1/4} \sqrt{G_F}\ (\ m_e H\, \bar{e}\, e + m_\mu H\, \bar{\mu}\, \mu + m_\tau H\, \bar{\tau}\, \tau\ ) \qquad (1.37)$$

The Higgs Mechanism for generating masses introduces an arbitrary coupling parameter for each fermion mass, and hence provides no fundamental understanding of the various mass values.

The neutrinos cannot acquire masses or couplings to the H field in an analogous way, since ν_R do not exist in the Standard Model. In the scalar interactions with leptons one cannot consider couplings that mix generations because for massless neutrinos mixing among the three neutrino states has no meaning. In this manner, ν_e is defined to be the partner of e, ν_μ of μ, and ν_τ of τ.

1.6 Quark Masses and Mixings

Quark masses are also generated through Yukawa couplings to the scalars. The fundamental quark field eigenstates of the unbroken gauge theory are

$$Q_{iL} = \begin{pmatrix} u_i \\ d_i \end{pmatrix}_L \quad , \quad u_{iR} \quad , \quad d_{iR} \qquad (i = 1,2,3)$$

where Q_{iL} is an $SU(2)_L$ doublet with $Y = 1/3$ and u_{iR} and d_{iR} are SU(2) singlets with $Y = 4/3$ and $-2/3$, respectively, and i is the generation index.

In order to generate quark masses for both u- and d-type quarks one needs not only the doublet Φ with $Y = 1$, but also the conjugate doublet

$$\tilde{\Phi} = i\ \tau_2\ \Phi^* = \begin{pmatrix} \phi^0 \\ -\phi^- \end{pmatrix} \tag{1.38}$$

which transforms as a doublet with $Y = -1$. The most general $SU(2)_L \times U(1)_Y$ invariant renormalizable Yukawa interaction term in the Lagrangian is then

$$L = -\sum_{i=1}^{3} \sum_{j=1}^{3} \left[\tilde{G}_{ij} \bar{u}_{iR} (\tilde{\Phi}^\dagger Q_{jL}) + G_{ij} \bar{d}_{iR} (\Phi^\dagger Q_{jL}) \right] + \text{h.c.} \tag{1.39}$$

where inter-generation couplings are allowed. As before, from the VEVs of the Φ and $\tilde{\Phi}$ one obtains mass terms for the charge 2/3 and charge -1/3 quarks

$$(\bar{u}_1 , \bar{u}_2 , \bar{u}_3)_R M^u \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}_L + \text{h.c.} \tag{1.40}$$

$$(\bar{d}_1 , \bar{d}_2 , \bar{d}_3)_R M^d \begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix}_L + \text{h.c.} \tag{1.41}$$

where

$$M^u_{ij} = \frac{v}{\sqrt{2}} \tilde{G}_{ij} \qquad , \qquad M^d_{ij} = \frac{v}{\sqrt{2}} G_{ij}$$

are quark mass matrices in generation space, each depending on 9 complex parameters. These matrices are in general not Hermitian. Moreover, since these are (non-degenerate) complex matrices they can be diagonalized by bi-unitary transformations. Then one has

$$\begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}_{L,R} = U_{L,R} \begin{pmatrix} u \\ c \\ t \end{pmatrix} \qquad , \qquad \begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix} = D_{L,R} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

which simultaneously diagonalize M^u and M^d

$$U_R^{-1} M^u U_L = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix} \tag{1.42}$$

and similar expressions for $D_R^{-1} M^d D_L$, where U_R, U_L, D_R, and D_L are unitary matrices and the diagonal entries are the (physical) quark masses. The weak eigenstates u_1, u_2, u_3 are linear superpositions of the mass eigenstates u, c, t and likewise d_1, d_2, d_3 are superpositions of d, s, b with separate relations for L and R components.

In the charged-current weak interaction one encounters the bilinear terms $\bar{u}_{1L}\gamma^\mu d_{1L}$, $\bar{u}_{2L}\gamma^\mu d_{2L}$, $\bar{u}_{3L}\gamma^\mu d_{3L}$ (see(1.22)) whose sum can be represented as an inner product of vectors in the generation space

$$(\bar{u}_1 \ , \ \bar{u}_2 \ , \ \bar{u}_3)_L \gamma_\mu \begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix}_L = (\bar{u} \ , \ \bar{c} \ , \ \bar{t})_L \ U_L^\dagger D_L \ \gamma_\mu \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L \tag{1.43}$$

Therefore in general there will be generation mixing of the mass eigenstates, described by the matrix

$$V = U_L^\dagger D_L \tag{1.44}$$

In the neutral-current interaction of the standard model one encounters instead bilinear forms such as

$$(\bar{u}_1, \bar{u}_2, \bar{u}_3)_L \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}_L = (\bar{u}, \bar{c}, \bar{t})_L U_L^\dagger U_L \begin{pmatrix} u \\ c \\ t \end{pmatrix}_L \qquad (1.45)$$

Hence, there is no mixing in this case and the standard model prohibits flavour changing neutral current (FCNC) interactions.

Like the mass matrix itself, the mixing of quark flavours in the charged-current weak interaction has no fundamental explanation, though theoretical attempts to predict the mixing angles have been made in extended gauge models.

In terms of the above general mixing matrix V, the charged weak currents for quarks are

$$J^\dagger_{L\mu} = (\bar{u}, \bar{c}, \bar{t})_L \gamma_\mu V \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L \qquad (1.46)$$

Usually (as a matter of convention), the mixing is ascribed completely to the $T_3 = -1/2$ states by defining

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix}_L = V \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L$$

and therefore the (physical) quark weak eigenstates are

$$\begin{pmatrix} u \\ d' \end{pmatrix}_L , \begin{pmatrix} c \\ s' \end{pmatrix}_L , \begin{pmatrix} t \\ b' \end{pmatrix}_L , u_R , c_R , t_R , d_R , s_R , b_R$$

It can be shown[9] that for N generations the quark mixing matrix contains $(N-1)^2$ physically independent parameters (after absorbing all the irrelevant unphysical phases). Since a general $N \times N$ real unitary matrix (i.e., an orthogonal matrix) has $N(N-1)/2$ independent parameters, the mixing matrix generally contains $(N-1)(N-2)/2$ independent phase angles.

Therefore a complex phase in L which can be realized in the quark sector through V requires the introduction of three or more generations. This complex phase can be taken to be responsible for CP violation in $K^0 - \bar{K}^0$ system.

For three generations, a convenient parametrization widely used is due to Kobayashi and Maskawa[21] (KM)

$$V_{KM} = \begin{pmatrix} c_1 & -s_1 c_3 & -s_1 s_3 \\ s_1 c_2 & c_1 c_2 c_3 - s_2 s_3 e^{i\delta} & c_1 c_2 s_3 + s_2 c_3 e^{i\delta} \\ s_1 s_2 & c_1 s_2 c_3 + c_2 s_3 e^{i\delta} & c_1 s_2 s_3 - c_2 c_3 e^{i\delta} \end{pmatrix} \qquad (1.47)$$

where $s_i = \sin\theta_i$ and $c_i = \cos\theta_i$ $(i = 1, 2, 3)$. By suitably choosing the signs of the quark fields, one can restrict the angles to the ranges

$$0 \le \theta_i \le \pi/2 \qquad , \qquad 0 \le \delta \le 2\pi$$

The phase δ gives rise to CP-violating effects (see Ref.9 for details and further references).

By using experimental data from processes such as $K \to \pi e \nu$, charm production in neutrino collisions, $\nu_\mu N \longrightarrow \mu$ + charm + X, and $B \longrightarrow \ell \nu X$, various bounds on the values of the moduli of some elements of the KM matrix V_{KM} have been found[22]. These can be combined with the unitarity property of V_{KM} to yield the moduli of other elements. The moduli of the matrix elements lying in the 1σ ranges are as follows

$$|V_{KM}| = \begin{pmatrix} 0.974\text{-}0.976 & 0.218\text{-}0.222 & 0.000\text{-}0.012 \\ 0.183\text{-}0.231 & 0.81\text{-}1.0 & 0.035\text{-}0.049 \\ 0.000\text{-}0.022 & 0.032\text{-}0.050 & 0.998\text{-}0.999 \end{pmatrix} \qquad (1.48)$$

Information about the CP-violating phase δ can be inferred from theoretical calculations of the ε parameter in the $K^0 - \bar{K}^0$ system. These studies suggest that the phase δ is in the range $30^\circ \le \delta \le 177^\circ$.

This concludes our brief review of the basic ingredients of the standard model.

CHAPTER 2

BASIC FEATURES OF THE LEFT-RIGHT SYMMETRIC MODELS

2.1. Introduction.

In spite of its many successes, the standard $SU(3)_C \times SU(2)_L \times U(1)_Y$ model of particle physics, which provides a fundamental theory of the non-gravitational interactions of quarks and leptons valid up to energies of order 1 TeV, leaves many problems unanswered. Included in its shortcomings are the following.

- The model is not really unified as there are three coupling constants, g_s, g', g, corresponding to the respective gauge groups.

- Quarks and leptons are not really unified as they appear in different multiplets. Because of this fact, there is no *a priori* reason to expect the magnitudes of the electron and proton charges to be equal, yet they *are* so to many significant figures.

- The model contains at least 21 parameters- quark masses, the ϑ angle of QCD, CKM mixing angles, ϑ_W, etc.- which are not predicted by the theory itself and must be put in by hand.

- There is no explanation for the family structure of three (or more) generations; as I. I. Rabi put it, "who ordered the muon?" Moreover, so far as everyday life goes, the first generation is all that we require. Related is the question of fermion masses; in the standard model the masses are "dialed in" as Yukawa couplings.

- Why is the theory left-handed?

- Where does the weak scale come from, why is it so small compared to the Planck scale, and how is it to be stabilized against radiative corrections?

- Where does gravity fit in?

- Why is the vacuum energy so small today (this is inferred from the very small value for the cosmological constant $\Lambda \approx 0$)?

These shortcomings/questions clearly point to a grander theory that goes beyond the Standard Model. In the search for physics beyond the standard model theorists have explored various possibilities, such as: compositeness, left-right symmetry, grand unification, supersymmetry, superstrings. Each approach addresses a particular aspect of this search and is not necessarily in conflict with the others. Our goal in this chapter is to present some of the basic features of the left-right symmetric extension of the Standard Model[1]. This condensed version will enable us to present a rather detailed analysis of the Higgs sector of the minimal L-R (left-right) symmetric model in the next chapter.

2.2. Why Left-Right Symmetry?

The original motivation for introducing the L-R symmetric models based on the gauge group $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ was to provide an understanding of the origin of parity violation in weak interactions[2]. Accordingly, one starts with a weak interaction Lagrangian which prior to the breaking of gauge symmetry respects all spacetime symmetries, just like the strong, electromagnetic, and gravitational interactions. Afterwards, the observed violation of parity shows up as a result of spontaneous symmetry breaking. In other words, in this model the violation of parity is a result of the non-invariance of the vacuum under parity. The most interesting feature of this model is that it reproduces all the features of the Standard Model of electroweak interactibns at low energies, and as one goes to higher energies, new effects associated with the parity invariance of the original L-R symmetric model are supposed to appear.

There exist several other considerations, related to the weak interaction, which are addressed naturally in a L-R symmetric model. Among these a very important one is the mass of the neutrino. Astrophysical and cosmological considerations are easily understood if neutrino has a non-vanishing mass[3] in the electron volt range. If however, $m_\nu \neq 0$, and is in the electron volt range, a most natural framework to understand it is in the L-R symmetric models.

Also, if weak interaction symmetries are the result of some more fundamental substructure of quarks and leptons, and if the forces at the substructure level are assumed to be similar to those of QCD, then there are favourable arguments[4] indicating that the $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ is the weak interaction symmetry rather than the $SU(2)_L \times U(1)_Y$ of the Standard Model.

Another shortcoming of the Standard Model is the lack of any physical meaning of the $U(1)_Y$ generator of the gauge group. In the L-R symmetric models, this generator corresponds to the B-L quantum number[5], and consequently all the weak interaction symmetry generators will have a physical meaning. In fact, in the $SU(2)_L \times U(1)_Y$ model, the only anomaly free quantum number left ungauged is (B-L) and once (B-L) is included as a gauge generator, the weak gauge group becomes $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ and the electric charge is given by[5]

$$Q = I_{3L} + I_{3R} + \frac{B-L}{2} \tag{2.1}$$

As a final comment, we consider the status of CP-violation in gauge theories. In the Standard Model, as seen in the previous chapter, three generations are required to have *non*-trivial CP-violation and all CP-violations are parametrized by only one phase, δ_{KM}, the Kobayashi-Maskawa phase. But the model does not explain why the observed CP-violation has milliweak strength. The L-R symmetric models provide a more interesting alternative[6,14-16], according to which the smallness of V+A currents can be understood through the smallness of

$$\eta_{+-} \simeq (M_L/M_R)^2 \sin\delta \tag{2.2}$$

where M_L and M_R are the masses of the left- and right-handed gauge bosons (see below), respectively, and δ is the CP-violating KM phase.

If parity and CP-violation both owe their origin to the spontaneous breakdown of gauge symmetry, then (2.2) can be proved[7,8] for three generations or more, and becomes valid regardless of the contribution of the Higgs sector.

Having listed some of the motivations for studying L-R symmetric models, we proceed to a discussion of the minimal L-R symmetric model. This is the model used for the phenomenological studies presented in the forthcoming chapters.

2.3. The Minimal Left-Right Symmetric $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ Model.

In the L-R symmetric models[1] the left- and right-handed fermions transform as doublets under $SU(2)_L$ and $SU(2)_R$, respectively. Defining the U(1) generator as in (2.1), the quark and lepton doublets transform according to the following representations

$$Q_{iL} = \begin{pmatrix} u' \\ d' \end{pmatrix}_{iL} = (1/2\ ,\ 0\ ,\ 1/3) \tag{2.3}$$

$$Q_{iR} = \begin{pmatrix} u' \\ d' \end{pmatrix}_{iR} = (0\ ,\ 1/2\ ,\ 1/3) \tag{2.3'}$$

$$\psi_{iL} = \begin{pmatrix} \nu' \\ \ell' \end{pmatrix}_{iL} = (1/2\ ,\ 0\ ,\ -1) \tag{2.4}$$

$$\psi_{iR} = \begin{pmatrix} \nu' \\ \ell' \end{pmatrix}_{iR} = (0\ ,\ 1/2\ ,\ -1) \tag{2.4'}$$

respectively. In (2.3) to (2.4') the primes indicate that the fermions are gauge group rather than mass eigenstates, and $i = 1,\ldots,N$ is the generation index. In order to generate masses for quarks and charged leptons one requires at least one Higgs bidoublet, Φ, of the form[9]

$$\phi = \begin{pmatrix} \phi_1^0 & \phi_1^+ \\ \phi_2^- & \phi_2^0 \end{pmatrix} = (1/2\ ,\ 1/2^*\ ,\ 0) \tag{2.5}$$

The most general form of the vacuum expectation value (VEV) of ϕ that is invariant under the electromagnetic $U(1)_Q$ is

$$<\phi> = \frac{1}{\sqrt{2}} \begin{pmatrix} k^* & 0 \\ 0 & k' \end{pmatrix} \tag{2.6}$$

where k and k' are, in general, complex. In order to break the symmetry down to $U(1)_Q$ additional Higgs multiplets with $B-L\neq 0$ are needed. A very popular choice[9,10] (see also chapters 3 and 5) is to introduce Higgs triplets Δ_L and Δ_R, transforming under the gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ according to the following representations

$$\Delta_L = \begin{pmatrix} \delta_L^{+} & \delta_L^{++} \\ \delta_L^{0} & \delta_L^{+} \end{pmatrix} = (1\ ,\ 0\ ,\ 2) \tag{2.7}$$

$$\Delta_L = \begin{pmatrix} \delta_R^{+} & \delta_R^{++} \\ \delta_R^{0} & \delta_R^{+} \end{pmatrix} = (0\ ,\ 1\ ,\ 2) \tag{2.8}$$

with VEVs given by

$$<\Delta_{L,R}> = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 \\ v_{L,R} & 0 \end{pmatrix} \tag{2.9}$$

To be more explicit, let us note that under the gauge transformations U_L and U_R of the gauge groups $SU(2)_L$ and $SU(2)_R$, respectively, the Higgs fields ϕ, $\tilde{\phi}$, Δ_L, and Δ_R, where

$$\tilde{\phi} = \tau_2\ \phi^{*}\ \tau_2 \tag{2.10}$$

transform as follows

$$\phi \longrightarrow U_L\ \phi\ U_R^{\dagger} \quad , \quad \tilde{\phi} \longrightarrow U_L\ \tilde{\phi}\ U_R^{\dagger} \tag{2.11}$$

$$\Delta_L \longrightarrow U_L \Delta_L U_L^{\dagger} \quad , \quad \Delta \longrightarrow U_R \Delta_R U_R^{\dagger} \tag{2.12}$$

Then, in order to have parity as a spontaneously broken symmetry, one requires the discrete symmetry

$$\psi_{iL} \longleftrightarrow \psi_{iR} \; , \; \Delta_L \longleftrightarrow \Delta_R \; , \; \phi \longleftrightarrow \phi^{\dagger} \tag{2.13}$$

which also implies that $g_L = g_R = g$. As a consequence of this, the most general Yukawa Lagrangian for the Higgs-Fermion couplings can be written as

$$L_Y = f\, \bar{\psi}_L \phi\, \psi_R + h\, \bar{\psi}_L \tilde{\phi}\, \psi_R + h_1 \bar{Q}_L \phi\, Q_R$$

$$+ h_2 \bar{\psi}_L \phi\, \psi_R + i h_3 (\, \psi_L^T C \tau_2 \Delta_L \psi_L + \psi_R^T C \tau_2 \Delta_R \psi_R \,) + \text{h.c.} \tag{2.14}$$

where C is the charge conjugation matrix and due to left-right symmetry the Yukawa coupling matrices f and h_i are all taken to be hermitian.

The gauge covariant derivatives for the left- and right-handed fermions $f'_{L,R}$ of the theory are given by

$$D^{\mu} f' = \partial^{\mu} f'_{L,R} + \frac{i}{2} (\, g\, \tau^a\, W^{\mu a}_{L,R} + g' Y B^{\mu} \,)\, f'_{L,R} \tag{2.15}$$

where τ^a are the Pauli matrices, g' is the U(1) gauge coupling, and W^a_L , W^a_L , and B are the $SU(2)_L$, $SU(2)_R$, and U(1) gauge bosons, respectively. The covariant derivatives of the fields Δ_L and Δ_R are defined similarly to (2.15), with the τ^a matrices replaced by matrices of appropriate dimension. Similarly, one has

$$D^{\mu}\phi = \partial^{\mu}\phi + \frac{i}{2} \left(g\, \tau^a W_L^{a\mu}\, \phi - g\, \phi\, \tau^a W_R^{a\mu} \right) \qquad (2.16)$$

The charged boson mass matrix can be read off, as usual, from the kinetic term of the Lagrangian

$$L_k = \mathrm{Tr}\left[(D_\mu \phi)^\dagger D^\mu \phi + (D_\mu \Delta_L)^\dagger D^\mu \Delta_L + (D_\mu \Delta_R)^\dagger D^\mu \Delta_R \right] \qquad (2.17)$$

which in the $W_L^+ - W_R^+$ basis, is given by

$$M_W^2 = \begin{pmatrix} M_L^2 & M_{LR}^2 e^{i\alpha} \\ M_{LR}^2 e^{-i\alpha} & M_R^2 \end{pmatrix} \qquad (2.18)$$

where

$$M_L^2 = \frac{1}{4} g^2 (|v_L|^2 + |k|^2 + |k'|^2)$$

$$M_R^2 = \frac{1}{4} g^2 (|v_R|^2 + |k|^2 + |k'|^2)$$

$$M_{LR}^2 e^{i\alpha} = -\frac{1}{2} g^2 k'^* k$$

where α is the (relative) phase of k' , k^*.

Since M_W^2 is a hermitian matrix, it can be diagonalized by a unitary transformation, which can be written in terms of one angle and three phases (cf. the discussion of the KM matrix in ch.1). Since two of these phases can be absorbed in the redefinition of the mass eigenstates $W_{1,2}$, the gauge eigenstates can be written as

$$\begin{pmatrix} W_L^+ \\ W_R^+ \end{pmatrix} = \begin{pmatrix} \cos\zeta & -\sin\zeta \\ e^{i\omega}\sin\zeta & e^{i\omega}\cos\zeta \end{pmatrix} \begin{pmatrix} W_1^+ \\ W_2^+ \end{pmatrix}$$

where ζ is the mixing angle of the left- and right-handed gauge bosons and ω is a phase. The mass eigenvalues are

$$M_{1,2}^2 = \frac{1}{2}\left(M_L^2 + M_R^2 \pm \left[(M_R^2 - M_L^2)^2 + 4|M_{LR}^2|^2 \right]^{1/2} \right) \qquad (2.19)$$

and ζ and ω are given by

$$\tan 2\zeta = \frac{\mp 2\, M_{LR}^2}{M_R^2 - M_L^2} \quad ; \quad e^{i\omega} = \mp e^{i\alpha} \qquad (2.20)$$

where the two different signs in (2.20) represent two physically equivalent phase conventions for W_2 [11].

For $|v_R|^2 \gg |k|^2, |k'|^2, |v_R|^2$, which is the physically relevant case [9,10] , one has

$$M_1^2 \simeq \frac{1}{4} g^2 (|v_L|^2 + |k|^2 + |k'|^2) \quad (2.21)$$

$$M_2^2 \simeq \frac{1}{4} g^2 |v_R|^2 \quad (2.22)$$

$$\zeta \simeq \mp \frac{2|k'k|}{|v_R|^2} \quad (2.23)$$

Next, we consider the quark gauge- and mass-eigenstates. In order to determine the quark mass matrices after spontaneous symmetry breaking we have to use the VEV of the field ϕ. Then, by replacing ϕ with $<\phi>$ in (2.14) and expanding the relevant terms, we obtain

$$\frac{1}{\sqrt{2}} \bar{u}'_L (fk + hk'^*) u'_R + \frac{1}{\sqrt{2}} \bar{d}'_L (fk' + hk^*) d'_R + \text{h.c.} \quad (2.24)$$

from which the equations for the quark mass matrices are readily found

$$M^u = \frac{1}{\sqrt{2}} (fk + hk'^*) \quad (2.25)$$

$$M^d = \frac{1}{\sqrt{2}} (fk' + hk^*) \quad (2.26)$$

As we can see L-R symmetry alone is not sufficient to make M^u and M^d hermitian or symmetric. However, if k and k' are real (which is not natural if there are explicit CP-violating phases in f and h), then $M^{u,d}$ are hermitian (manifest L-R symmetry). Similarly, if f and h are real but k and/or k′ are complex (spontaneous CP-violation) then $M^{u,d}$ are complex symmetric matrices (pseudomanifest L-R symmetry). Later on when we will analyze various possibilities for the Higgs sector of the minimal L-R symmetric theory, it will become clear which choice for the quark mass matrices is the most viable one. But for now, we will proceed with the general case without making any specific choices.

The gauge and mass eigenstates of the quark fields are related to one another through the (bi)unitary transformations

$$u_{L,R} = V^{u}_{L,R}\, u'_{L,R} \tag{2.27}$$

$$d_{L,R} = V^{d}_{L,R}\, d'_{L,R} \tag{2.28}$$

where the matrices $V^{u}_{L,R}$, $V^{d}_{L,R}$ constitute the biunitary transformations required to diagonalize the mass matrices M^{u} and M^{d},

$$V^{u}_{L}\, M^{u}\, V^{u\dagger}_{R} = D^{u} \tag{2.29}$$

$$V^{d}_{L}\, M^{d}\, V^{d\dagger}_{R} = D^{d} \tag{2.30}$$

The charged current interaction terms in the Lagrangian can therefore be written as

$$-L_{CC} = \frac{g}{\sqrt{2}} \bar{u}_{iL} \gamma_\mu U^L_{ij} d_{jL} W^{\mu+}_L$$

$$+ \frac{g}{\sqrt{2}} \bar{u}_{iR} \gamma_\mu U_{ij} d_{jR} W^{\mu+}_R + h.c. \qquad (2.31)$$

where

$$U^L = V^u_L V^{d\dagger}_L \quad , \quad U^R = V^u_R V^{d\dagger}_R \qquad (2.32)$$

are the left- and right-handed Kobayashi-Maskawa matrices.

Even though $M^{u,d}$ in equations (2.25-26) are not the most general complex matrices, there is no simple linear relationship we can find between the phases in U_L and U_R. There are $N^2 - 1/2\ N(N-1) = 1/2\ N(N+1)$ phases each in U_L and U_R. There are 2N-1 relative phases between 2N quarks that can be used to remove some of the phases in U_L and U_R. Therefore, one is left with $2 \cdot 1/2\ N(N+1)-(2N-1)=N^2-N+1$ phases . Note that there is originally a phase in the charged gauge boson mass matrix (2.18), but this phase can be removed by redefining the phase of $W^{\mp}_R$ (or $W^{\mp}_L$) so that this phase will be combined with the overall phase of U_R (or U_L). Alternatively, one may choose to move the overall phase of U_R (or U_L) to the gauge boson mass matrix. If this is done, there will be only N^2-N phases left over in U_L and U_R. For N=2, this number is 2, and for N=3, this number is 6. Some of the interesting ways to place these phases are[13]

$$U_L = \begin{pmatrix} \cos\vartheta_L & \sin\vartheta_L \\ -\sin\vartheta_L & \cos\vartheta_L \end{pmatrix} , \quad U_R = \begin{pmatrix} (e^{-i\delta_2})\cos\vartheta_R & (e^{-i\delta_1})\sin\vartheta_R \\ -(e^{i\delta_1})\sin\vartheta_R & (e^{i\delta_2})\cos\vartheta_R \end{pmatrix}$$

$$U_L = \begin{pmatrix} \cos\vartheta_L & (e^{i\delta_1})\sin\vartheta_L \\ -(e^{-i\delta_1})\sin\vartheta_L & \cos\vartheta_L \end{pmatrix} , \quad U_R = \begin{pmatrix} (e^{-i\delta_2})\cos\vartheta_R & \sin\vartheta_R \\ -\sin\vartheta_R & (e^{i\delta_2})\cos\vartheta_R \end{pmatrix}$$

and it can be shown that these are related by a change of phase convention. It is worthwhile to note that, although the discovery of a third generation of quarks makes it possible to have a Kobayashi-Maskawa (KM) CP-violation without an extension of the standard model, the small value of the mixing of the third generation may raise the question of whether the KM model is qualitatively satisfactory[17]. In a recent work[18] it has been shown that both a relatively large $D_n \simeq 10^{-25} e$ cm [19] for the electric dipole moment of the neutron and $\varepsilon'/\varepsilon \simeq 10^{-3}$ (see Ref.20) can be obtained in a left-right symmetric extension of the two generation version of the standard model. This indicates yet another very attractive feature of the L-R symmetric models, which is totally absent in the Standard Model. Going back to the left- and right-handed KM matrices U^L and U^R , let us note that for the special case of manifest L-R symmetry $M^{u,d}$ are hermitian, so that $U^R=U^L$. For pseudomanifest L-R symmetry $U^R=U^{L*}K$, where K is a diagonal phase matrix[13].

Transforming the gauge bosons into mass eigenstates and using the L-R KM matrices $U^{L,R}$, the charged current Lagrangian in (2.31) can be rewritten as

$$-L_{cc} = \frac{g\cos\zeta}{\sqrt{2}}\, \bar{u}\, \gamma_{\mu}\Big[(U^{L}\gamma_{L} + \tan\zeta\, e^{i\omega}\, U^{R}\gamma_{R})W_{1}^{\mu+}$$

$$+ (-\tan\zeta\, U^{L}\, \gamma_{L} + e^{i\omega}\, U^{R}\gamma_{R})W_{2}^{\mu+}\Big]d + h.c. \qquad (2.33)$$

where $\gamma_{R,L} = (1\mp\gamma_5)/2$. As indicated above, one can absorb the phase $e^{i\omega}$ into the quark mixing matrix U^{R}. Once this is done, the sign of ζ is fixed if one picks a definite convention for the phase of W_2. The leptonic charged current interactions is analogous to (2.33), with $u \longrightarrow \nu$ and $d \longrightarrow e$, and $U^{L,R} \longrightarrow V^{L,R}$, where $V^{L,R}$ are the leptonic analogues of the L-R KM matrices[21].

This concludes our survey of some of the basic features of the L-R symmetric models that will be needed for the purposes of this thesis in the forthcoming chapters. Also, some important matters that have been only pointed at, in this chapter, will be elaborated on as we need them.

CHAPTER 3

THE HIGGS SECTOR OF THE MINIMAL LEFT-RIGHT SYMMETRIC MODEL

3.1. Introduction

As was pointed out in the previous chapter, in the Left-Right symmetric extension of the standard model, parity is an exact symmetry of the Lagrangian, and is only broken spontaneously due to a specific form of the scalar field potential.

The Higgs sector of L-R symmetric theories contains a bidoublet field ϕ (see 2.5) which is used to generate masses for the standard W_L and Z_L through the vacuum expectation values (VEVs) k and k'.
On the other hand, since experimental constraints from K_L-K_S mixing force W_R to be very heavy[1] ($\gtrsim$ 1.6 TeV), an additional Higgs representation with large VEV v_R (v_R>>k,k') for its neutral member that couples primarily to W_R is required.

In order to keep the full L-R symmetry of the unbroken Lagrangian, there must be a corresponding Higgs field coupling to W_L, but the VEV of its neutral member v_L must be small (v_L<<max(k,k')) in order to preserve the standard model relation between the W_L and Z_L masses (see chapter.1). If the additional Higgs fields are members of doublets, then all the above criteria can be met, but the theory will fail

to incorporate a natural explanation of the smallness of neutrino masses. On the other hand, if these extra Higgs fields are triplets, then v_R can induce a large Majorana mass term for the Majorana neutrino N, in addition to the Dirac mass terms induced by the VEV's of the bidoublet fields that mix the N and the ν (see Ref.2 for details). Then, since the ratio of Majorana to Dirac mass terms is of order $v_R/\max(k,k')$, one is led naturally to the standard "see-saw" mechanism which yields a very small Majorana mass for the left-handed neutrinos.

It is due to the attractiveness of the latter alternative that we choose to investigate models containing extra triplet Higgs fields, Δ_R and Δ_L (see §2.3). The resulting Higgs field has many exotic features, and our ability to probe experimentally these features in the not too distant future is an important issue.

The principal source of uncertainty in dealing with the Higgs sector of the L-R symmetric models is the exact form of the scalar potential V. An important question regarding a possible form for V is whether the phenomenologically required hierarchy for the VEVs : $v_R \gg \max(k,k') \gg v_L$ is natural. The most general form of this potential must contain only quadratic or quartic terms[1], in view of renormalizability and gauge invariance of the theory. In this chapter we to study the consequences of this most general scalar potential (with some modifications) by using the results related to very important constraints coming from bounds on the FCNC (Flavour Changing Neutral Currents), K_L- K_S mixings, etc. After these considerations we will find that there are two Higgs scenarios which are most viable. The consequences of choosing these two scenarios (one of which, as we will argue later, is the most natural possibility) to be the physically relevant cases will be discussed in the forthcoming chapters.

3.2. The Scalar Potential

The Higgs fields of the minimal L-R symmetric model, ϕ, Δ_L, Δ_R were given in the previous chapter in their respective representations (see 2.5-8). In what follows, we will use the convenient convention of writing a neutral field ϕ^0 in terms of correctly renormalized real and imaginary components as

$$\phi^0 = (1/\sqrt{2})(\phi^{0r} + i\phi^{0i}) \tag{3.1}$$

We now discuss the form of the scalar field potential. L-R symmetry requires, first of all, that the potential be invariant under

$$\Delta_R \leftrightarrow \Delta_L \quad , \quad \phi \leftrightarrow \phi^\dagger \tag{3.2}$$

Furthermore, the most general scalar field potential cannot have any tri-linear terms, due to gauge invariance. Because of the non-zero B-L quantum numbers of the Δ_L and Δ_R

triplets, these must always appear in the quadratic combinations $\Delta_L^\dagger\Delta_L$, $\Delta_R^\dagger\Delta_R$, $\Delta_L^\dagger\Delta_R$ or $\Delta_R^\dagger\Delta_L$. These combinations, of course, cannot be combined with a single bidoublet ϕ in such a way as to form an $SU(2)_L$ and $SU(2)_R$ singlet. Nor can these bidoublets be combined so as to yield a singlet. However, quartic combinations of the form $Tr(\Delta_L^\dagger\phi\ \Delta_R\phi^\dagger)$ are in general allowed by L-R symmetry. But as discussed later it is physically desirable[3] to eliminate such terms in order that the natural minimum of the potential have $v_L=0$. To accomplish this, one imposes invariance under the additional discrete symmetry

$$\Delta_L \to \Delta_L \quad , \quad \Delta_R \to -\Delta_R \quad , \quad \phi \to i\phi \tag{3.3}$$

This discrete symmetry implies that terms which are linear combinations of $Tr(\tilde{\phi}^\dagger\phi)$ and its conjugate transpose cannot be included in the potential either. This is in fact quite desirable, since it offers a natural potential minimum which avoids FCNC problems most economically.[3,4]

Putting all this together, the most general form of V is then

$$V = -\mu_1^2 Tr\phi^\dagger\phi + \lambda_1(Tr\phi^\dagger\phi)^2 + \lambda_2 Tr(\phi^\dagger\phi\ \phi^\dagger\phi)$$

$$+ \frac{1}{2}\lambda_3(Tr\phi^\dagger\tilde{\phi} + Tr\tilde{\phi}^\dagger\phi)^2 + \frac{1}{2}\lambda_4(Tr\phi^\dagger\tilde{\phi} - Tr\tilde{\phi}^\dagger\phi)^2$$

$$+ \lambda_5 Tr\phi^\dagger\phi\ \tilde{\phi}^\dagger\ \tilde{\phi} + \frac{1}{2}\lambda_6\left[Tr\phi^\dagger\ \tilde{\phi}\ \phi^\dagger\ \tilde{\phi} + Tr\tilde{\phi}^\dagger\ \phi\ \tilde{\phi}^\dagger\ \phi\right]$$

$$- \mu_2^2\left[\mathrm{Tr}\Delta_L^\dagger\Delta_L + \mathrm{Tr}\Delta_R^\dagger\Delta_R\right] + \rho_1\left[(\mathrm{Tr}\Delta_L^\dagger\Delta_L)^2 + (\mathrm{Tr}\Delta_L^\dagger\Delta_L)^2\right]$$

$$+ \rho_2\left[\mathrm{Tr}\Delta_L^\dagger\,\Delta_L\,\Delta_L^\dagger\,\Delta_L\right] + \rho_3(\mathrm{Tr}\Delta_L^\dagger\Delta_L)(\mathrm{Tr}\Delta_R^\dagger\,\Delta_R)$$

$$+ \alpha_1(\mathrm{Tr}\phi^\dagger\phi)(\mathrm{Tr}\Delta_L^\dagger\Delta_L + \mathrm{Tr}\Delta_R^\dagger\Delta_R)$$

$$+ \alpha_2(\mathrm{Tr}\Delta_R^\dagger\,\phi^\dagger\,\phi\,\Delta_R + \mathrm{Tr}\Delta_L^\dagger\,\phi\,\phi\,\Delta_L^\dagger)$$

$$+ \alpha_2'(\mathrm{Tr}\Delta_R^\dagger\,\tilde{\phi}^\dagger\,\tilde{\phi}\,\Delta_R + \mathrm{Tr}\Delta_L^\dagger\,\tilde{\phi}\,\tilde{\phi}^\dagger\,\Delta_L) \qquad (3.4)$$

As we can see, there are many more parameters involved here than in the standard model. Since we are considering the most general possibility, one does not know at first whether these parameters can be taken to be complex or not (in other words, whether the scalar potential is CP-conserving or CP-violating). We will offer a simple argument to justify the generally assumed CP-conserving scalar potential in the minimal L-R symmetric model.

If one takes the Higgs fields of the minimal L-R symmetric model to be dynamical fields, then all the above arguments and restrictions based on the various symmetries of the theory and constraints from low energy phenomenology still apply. The only difference that will arise will be the inclusion of various radiative corrections (up to some arbitrary loop order) which would contribute to the scalar field potential. It would be a lengthy undertaking to wrtite these radiative corrections for the L-R symmetric theory in detail. We do not need, however, to go through such details, and can draw very useful conclusions from arguments analogous to the ones brought forth in the case of the Standard Model. In the Standard Model it has been shown that the spontaneous breaking of symmetry can be understood as having been driven by radiative corrections[5]. Moreover, one of the most important features of this procedure of dynamically driven spontaneous symmetry breaking is that it makes it possible to set a lower bound for the mass of the Higgs field, when applying the theory to the Standard Model.[6] All these calculations are simply the result of considering the effective potential (which involves radiative corrections arising from n-loop contributions, in contrast to the ordinary situation in which the tree level potential is used to the end). Suppose now that we applied the same techniques to the Higgs potential of the L-R symmetric model. We would immediately discover that if the potential parameters were allowed to be complex, it would not be possible, in general, to set a *real* lower bound for the masses of the Higgs fields in this theory. Therefore, apart from an inherent inconsistency, the theory would not allow us to determine the symmetry breaking scale in a natural way.

Based on this argument, we will continue by assuming that the potential is CP-conserving , i.e. we take all the physical parameters in the potential to be real.

3.3. Higgs eigenfields and their corresponding masses

Since the vacuum expectation values of the Higgs field can be taken to be real and non-negative, we have (see 3.1-2)

$$<\delta_R^0> = \frac{1}{\sqrt{2}} <\delta_R^{0r}> = \frac{1}{\sqrt{2}} v_R \;, \; <\delta_L^0> = \frac{1}{\sqrt{2}} <\delta_L^{0r}> = \frac{1}{\sqrt{2}} v_L \qquad (3.5)$$

$$<\phi_1^0> = \frac{1}{\sqrt{2}} <\delta_1^{0r}> = \frac{k}{\sqrt{2}} \;, \; <\phi_2^0> = \frac{1}{\sqrt{2}} <\phi_2^{0r}> = \frac{k'}{\sqrt{2}} \qquad (3.6)$$

From the twenty real degrees of freedom of the Higgs fields, six are absorbed in giving masses to the left- and right-handed gauge bosons, $W_L^{\pm}$, $W_R^{\pm}$, Z_L and Z_R.

In order to minimize the potential when all the neutral fields are evaluated at their respective vacuum expectation values, we must require that

$$\frac{\partial V}{\partial v_L} = \frac{\partial V}{\partial v_R} = \frac{\partial V}{\partial k} = \frac{\partial V}{\partial k'} = 0 \qquad (3.7)$$

which after expanding (3.4) implies that

$$\frac{\partial V}{\partial v_L} = \Big[-2\mu_2^2 + \rho_3 v_R^2 + \alpha_1 (k^2 + k'^2) + (\alpha_2 k'^2 + \alpha_2' k^2)$$

$$+ 2\ (\rho_1 + \rho_2) v_L^2 \Big] v_L = 0 \qquad (3.8)$$

$$\frac{\partial V}{\partial v_R} = \Big[-2\mu_2^2 + \rho_3 v_L^2 + \alpha_1 (k^2 + k'^2) + (\alpha_2 k'^2 + \alpha_2' k^2)$$

$$+ 2 (\rho_1 + \rho_2) v_R^2 \Big] v_R = 0 \qquad (3.9)$$

$$\frac{\partial V}{\partial k} = \Big[-2\mu_1^2 + 2 (\lambda_1 + 4\lambda_3 + \lambda_5 + \lambda_6) k'^2 + (\alpha_1 + \alpha_2') (v_R^2 + v_L^2)$$

$$+ 2 (\lambda_1 + \lambda_2) k^2 \Big] k = 0 \qquad (3.10)$$

$$\frac{\partial V}{\partial k'} = \Big[-2\mu_1^2 + 2 (\lambda_1 + 4\lambda_3 + \lambda_5 + \lambda_6) k^2 + (\alpha_1 + \alpha_2) (v_R^2 + v_L^2)$$

$$+ 2 (\lambda_1 + \lambda_2) k'^2 \Big] k' = 0 \qquad (3.11)$$

Also, we must keep in mind that, at a true local minimum, all physical Higgs bosons must have positive squared masses for a solution of equations (3.8-11). From these equations we have $\partial V/\partial v_i = v_i \, f_i(v_i)$ for $v_i = v_L$, v_R, k, k'. Thus the minimization condition for each v_i can be satisfied either by $v_i = 0$ or $f_i(v_i) = 0$. We already know that $v_R \neq 0$ so that constraints coming from low-energy phenomenology, namely, the standard electroweak theory will be met satisfactorily. Adopting the non-restrictive convention $k > k'$, $k \neq 0$, we will consequently be left with four possible scenarios for the remaining VEVs, v_L and k': (a) $v_L \neq 0$, $k' \neq 0$; (b) $v_L = 0$, $k' \neq 0$; (c) $v_L \neq 0$, $k' = 0$; (d) $v_L = 0$, $k' = 0$. Let us start our considerations with v_L. From equations (3.8-9) we realize that

$$(\rho_3 - 2\rho_2 - 2\rho_1)(v_R^2 - v_L^2) = \bar{\rho}'(v_R^2 - v_L^2) = 0 \qquad (3.12)$$

but since we clearly don't want to impose $v_L^2 = v_R^2$(which would simply render the theory inconsistent), a non-zero v_l would require $\bar{\rho}' = 0$.

On the other hand there is *no* symmetry in the theory which would ensure such a relation among the parameters of the Higgs potential (where, of course, the renormalizability of the theory is automatically taken care of[7], without any reference to a more spcific choice of the parameters). So we conclude that for all but the specific choice of the coupling constant $\bar{\rho}' = 0$, we must have $v_L = 0$. This is because if we take $v_L = 0$, then the constraints (3.12) would simply not arise in the first place. We will take this to be a natural choice and consequently rule out possibilities (a) and (c).

Next we have to consider the remaining two scenarios. From equations (3.10) and (3.11) we obtain (after taking $v_L = 0$)

$$(-\lambda_2 + 4\lambda_3 + \lambda_5 + \lambda_6)(k^2 - k'^2) + \Delta\alpha v_R^2 = 0 \qquad (3.13)$$

where $\Delta\alpha = (\alpha_2 - \alpha'_2)/2$. This equation can be interpreted as a constraint on the V.E.V's k, k' and v_R, which is quite unnatural on the following grounds.

One can argue, from the point of view of dynamically driven spontaneous symmetry breaking, that if such a constraint among the VEVs existed at the tree level (i.e. 3.13), it would be quite hard to imagine just what kind of argument could ensure similar constraints at any desired n-loop order, so that the minimization conditions on the effective potential would all be satisfied. In addition, it can be shown[8] that non-zero values for both k and k' would almost inevitably lead to the impossibility of a relatively light Higgs boson without large FCNC.

On the other hand if we take $v_L = 0$ and $k' = 0$, the equations (3.9) and (3.10) reduce to

$$2\mu_1^2 = (\alpha_1 + \alpha'_2)v_R^2 + 2(\lambda_1 + \lambda_2)k^2 \tag{3.14}$$

$$2\mu_2^2 = 2(\rho_1 + \rho_2)v_R^2 + (\alpha_1 + \alpha'_2)k^2 \tag{3.15}$$

which, as we can see, for a particular choice of potential parameters uniquely determines the values of k and v_R. Also, when $k' = 0$, there is automatically no mixing between the left- and the right-handed gauge bosons. Generally, this mixing is given by the angle ζ where $\tan 2\zeta \simeq 2kk'/v_R^2$. Such mixing might lead to phenomenological difficulties.

Such mixing might lead to phenomenological difficulties. For instance, for $m(W_R) = 1.6\text{TeV}$ one finds $\tan 2\zeta \simeq 0.005$, which comes quite close to violating the present experimental constraint $\zeta \leq 0.0055$ [9]. Substantial improvements in this constraint would require that k' be significantly smaller than k. In addition, it is desirable to have k significantly different from k' in order to easily generate a large mass ratio for m_t/m_b [10,11].
We are therefore led to choose the possibility (d): $v_L = k' = 0$ as the scenario with the most natural expectation value.

In what follows, we shall be using mainly this scenario for phenomenological studies, although some brief studies on the scenario (b) will be discussed in chapter 4.

At this point, the first thing that has to be done is to find the physical Higgs eigenstates and their corresponding masses. This is achieved by the usual diagonalization of the appropriate mass matrices that arise from the Higgs potential. After some lengthy algebra one finds the following mass matrices (see 3.20 for conventions)

(1) In the basis $\phi_1^{0r} - \phi_2^{0r} - \delta_R^{0r} - \delta_L^{0r}$

$$M_{0r}^2 = \begin{pmatrix} 2k^2\bar{\lambda} & 0 & v_R k\bar{\alpha}' & 0 \\ 0 & \bar{\Delta} & 0 & 0 \\ v_R k\bar{\alpha}' & 0 & 2v_R^2\bar{\rho} & 0 \\ 0 & 0 & 0 & 2^{-1}v_R^2\bar{\rho}' \end{pmatrix} \tag{3.16}$$

(2) In the $\delta_R^{++} - \delta_L^{++}$ basis

$$M_{++}^2 = \begin{pmatrix} -\rho_2 v_R^2 + \Delta\alpha k^2 & 0 \\ 0 & 2^{-1}\bar{\rho}' v_R^2 + \Delta\alpha k^2 \end{pmatrix} \qquad (3.17)$$

(3) In the $\phi_1^+ - \phi_2^+ - \delta_R^+ - \delta_L^+$ basis

$$M_+^2 = \begin{pmatrix} \Delta\alpha v_R^2 & 0 & 2^{-1/2} v_R k & 0 \\ 0 & 0 & 0 & 0 \\ 2^{-1/2}\Delta v_R k & 0 & 2^{-1/2}\Delta\alpha\kappa^2 & 0 \\ 0 & 0 & 0 & 2^{-1}(\bar{\rho}' v_R^2 + \Delta\alpha k^2) \end{pmatrix} \qquad (3.18)$$

(4) In the $\phi_1^{0i} - \phi_2^{0i} - \delta_R^{0i} - \delta_L^{0i}$ basis

$$M_{0i}^2 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \Delta\alpha v_R^2 + \bar{\Sigma} k^2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2^{-1}\bar{\rho}' v_R^2 \end{pmatrix} \qquad (3.19)$$

where

$$\bar{\lambda} = \lambda_1 + \lambda_2 \;, \quad \bar{\alpha}' = \alpha_1 + \alpha_2' \;, \quad \Delta c = (\alpha_2 - \alpha_2')/2 \;, \quad \bar{\rho} = \rho_1 + \rho_2$$

$$\bar{\rho}' = \rho_3 - 2(\rho_2 + \rho_1) \ , \ \bar{\Sigma} = \lambda_5 - \lambda_2 - 4\lambda_4 - \lambda_6 \qquad (3.20)$$

$$\bar{\Delta} = \Delta\alpha v_R^2 - k^2[\lambda_2 - (4\lambda_3 + \lambda_5 + \lambda_6)] \ , \ \bar{\Sigma}' = \lambda_2 - (4\lambda_3 + \lambda_5 + +l_6)$$

The mass eigenstates can now be obtained by diagonalizing the matrices (3.16)-(3.19). We find that the doubly charged sector consists of δ_R^{++} and δ_L^{++}, which are unmixed mass eigenstates, with masses

$$m^2(\delta_R^{++}) = -\rho_2 v_R^2 + \Delta\alpha k^2 \qquad (3.21)$$

$$m^2(\delta_L^{++}) + 2^{-1}\bar{\rho}' v_R^2 + \Delta\alpha k^2 \qquad (3.22)$$

The singly charged sector consists of δ_L^{+} and h^{+}, given by

$$h^{+} = (\ \phi_1^{+} + 2^{-1/2}k\delta_R^{+}\ )/(\ 1 + 2^{-1}k^2 v_R^{-2}\ )^{1/2} \qquad (3.23)$$

with masses

$$m^2(\delta_L^{+}) = 2^{-1/2}(\ \bar{\rho}' v_R^2 + \Delta\alpha k^2\ ) \qquad (3.24)$$

$$m^2(h^{+}) = \Delta\alpha\ (\ v_R^2 + 2^{-1}k^2\ ) \qquad (3.25)$$

The neutral *imaginary* sector consists of ϕ_2^{0i} and δ_L^{0i} with masses

$$m^2(\phi_2^{0i}) = \Delta\alpha v_R^2 + k^2\bar{\Sigma} \tag{3.26}$$

$$m^2(\delta_L^{0i}) = 2^{-1}\bar{\rho}' v_R^2 \tag{3.27}$$

The neutral *real* sector consists of δ_L^{0r}, ϕ_2^{0r}, h^0, and H^0, given by

$$h^0 = \cos\alpha\ \phi_1^{0r} - \sin\alpha\ \delta_R^{0r} \tag{3.28}$$

$$H^0 = \cos\alpha\ \delta_R^{0r} + \sin\alpha\ \phi_1^{0r} \tag{3.29}$$

where $\tan 2\alpha = v_R k\bar{\alpha}'/(v_R^2\bar{\rho} - k^2\bar{\lambda})$. The masses of these neutral fields are given by

$$m^2(\delta_L^{0r}) = 2^{-1}\bar{\rho}' v_R^2 \tag{3.30}$$

$$m^2(\phi_2^{0r}) = \Delta\alpha v_R^2 - k\bar{\Sigma}' \tag{3.31}$$

$$m^2(h^0) = k^2 \left(2\bar{\lambda} - \bar{\alpha}'^2/2\bar{\rho} \right) \tag{3.32}$$

$$m^2(H^0) = 2v_R^2\bar{\rho} - 2k^2\bar{\lambda} + k^2\bar{\alpha}'/\bar{\rho} \tag{3.33}$$

Although a detailed knowledge of the many parameters of the Higgs potential simply does not exist, one is nevertheless able to put reasonably realistic bounds on the masses of the various Higgs mass-eigenstates. For example, since FCNC interactions arise from couplings involving

ϕ_2^{0r} and ϕ_2^{0l} fields, one can use the results of Ref.1 to suppress sufficiently the masses of these Higgs fields. Thus, using (3.31) and (3.26) one finds that in order to satisfy the FCNC constraints, one must have $\Delta\alpha v_R^2 > (5\ \mathrm{TeV})^2$, which also implies that for the minimum value of $v_R \approx 3.3$ TeV (coming from $m(W_R) \approx 1.6$ TeV) one has $\Delta\alpha \geq 2.3$. As a consequence of this, we learn that the singly charged h^+ field is then also forced to be heavy and approximately degenerate with ϕ_2^{0r} and ϕ_2^{0l}.

Regarding the field δ_R^{++}, we first note that (see (3.21)) ρ_2 must be negative or very small to avoid having negative mass squared for δ_R^{++}. In the former case it is probably natural for δ_R^{++} to be quite heavy. But on the other hand, $\bar{\rho} = \rho_1 + \rho_2$ must be positive (see (3.32)) in order that both $m^2(H^0)$ and $m^2(h^0)$ be positive (for $\bar{\lambda} > 0$). Since substantial cancellations are possible, $\bar{\rho}$ could be much smaller than either ρ_1 or ρ_2. This implies that H^0 could be lighter than what one would at first anticipate from (3.33). In general there is clearly considerable uncertainty associated with both δ_R^{++} and H^0 masses.

A similar uncertainty arises when we consider the masses of the fields δ_L^{++} , δ_L^{+} , δ_L^{0l} , δ_L^{0r}. The main reason for this is that, as was noted earlier, there is no natural symmetry calling for $\bar{\rho}' = 0$.

As for the mass of h^0, it can be verified from (3.28-29) and (3.32-33) that parameters are chosen in such a way that h^0 and H^0 couplings to $W_L W_L$ and $Z_L Z_L$ (through their

ϕ_1^{0r} components) will not cause any unitarity problems.

These bounds on the masses of the various physical Higgs fields will enable us to estimate the decay widths of the neutral and singly charged Higgs fields into hadrons. This will be the topic of chapter 4, in which, by using renormalization group estimates (taken in conjunction with QCD sum rules), the decay width of the physical Higgs into hadrons will be given up to second-order contributions. The values of the physical Higgs masses can also be used in the formulae for the total cross sections of processes $e^+e^- \longrightarrow$ (Higgs)(Higgs) in the minimal left-right symmetric model. The calculation of the total cross sections for these processes will be discussed in Chapter 5.

CHAPTER 4

THE RENORMALIZATION GROUP ESTIMATE OF THE HADRONIC DECAY WIDTH OF THE HIGGS BOSONS IN THE MINIMAL LEFT-RIGHT SYMMETRIC MODEL

4.1 Introduction

In the previous chapter we learned that the masses of the Higgs fields h^{+}, ϕ_2^{0r} and ϕ_2^{0i} fields lie in the (multi)-TeV region. Since these heavy fields couple to the quarks it is natural to estimate the total hadronic decay widths of the Higgs bosons. This can be achieved most elegantly and to a very good approximation by using QCD operator product expansions and the renormalization group equations. The result should be of some phenomenological interest in the study of future experiments with very high-energy accelerators and colliders.

After bringing the necessary arguments, we will apply the results of ref.1 to the minimal left-right symmetric model under our scrutiny, using the left-right theory of chapters 2 and 3. The authors of Ref.1 use the techniques of QCD operator product expansions and renormalization group equations to estimate the total hadronic decay width of the Weinberg-Salam type (heavy) Higgs bosons with mass $\leq$ 1 TeV. In the next section we review the essential and final results of their work on the perturbative formulae for the total hadronic decay width of the Higgs bosons. A short review of the QCD renormalization group equations is given in appendix B.

4.2 Renormalization Group Method and the perturbative decay width formula

We start with the Lagrangian of the Standard Model involving the quark fields which is given, in renormalized form, as

$$L = L_{QCD} + L_{Higgs} + L_{int} \tag{4.1}$$

where

$$L_{QCD} = Z_2 \bar{\psi} i \gamma_\mu \partial^\mu \psi - Z_{m(q)} Z_2 \bar{\psi} D^q \psi$$

$$+ Z_1 \mu^{-\varepsilon/2} f \bar{\psi} \gamma_\mu A^\mu \psi + L_{gluons} \tag{4.2}$$

$$L_{Higgs} = 1/2\ Z_3' (\partial_\mu \phi)^\dagger (\partial^\mu \phi) - 1/2\ Z_{m(H)} Z_3' m^2(H) \phi^2 \tag{4.3}$$

$$L_{int} = -\mu^{\varepsilon/2} g \phi J_H \tag{4.4}$$

Here Z_i are the renormalization constants and the scalar currents J_H are defined in terms of the renormalized quark field ψ and the left gauge boson mass M_L by

$$J_H = Z_1' \frac{1}{2M_L} \bar{\psi} D^q \psi \tag{4.5}$$

with the diagonal quark mass matrices D^q. In order to handle the ultraviolet divergences, the minimal subtraction scheme[2,3] using the method of dimensional regularization[2] in D-dimensions is employed This gives rise to an arbitrary mass scale μ and a parameter $\varepsilon = 4-D$. We emphasize that in what follows we are concerned with QCD corrections to all orders in the QCD coupling f, while restricting ourselves to the lowest order in the weak coupling g. The total hadronic decay width is then found by using the formula[2]

$$\Gamma_H = \frac{g^2}{m_H} \, \mathrm{Im}\, \Pi_H(q^2 = m_H^2) \qquad (4.6)$$

where the spectral function $\Pi_H(q^2)$ is defined by[2]

$$\Pi_H(q^2) = i \int d^Dx \, e^{iqx} < T\, [J_H(x) J_H(0)] > \qquad (4.7)$$

When one considers the case of a Higgs boson much heavier than the ordinary hadronic mass scale, one has to consider large q^2 behaviour of the spectral function. In other words, one is concerned with the short distance property of the T-product of the two scalar currents in $\Pi_H(q^2)$.

Then one proceeds as follows.[1,2] First, one applies the operator product expansion to the T-product in $\Pi_H(q^2)$. Then, one derives and solves the renormalization group equations satisfied by the coefficient functions in the space-like q^2 region. Finally , one makes an analytic continuation of the obtained solutions from the space-like to the time-like q^2 and takes the absorptive part at $q^2 = m_H^2$.

The operator product expansion at short distance is given by[2]

$$i\int d^Dx\, e^{iqx}\, T[\, J_H(x)\; J_H(0)\,] = \sum_{d,\ell} C_d^\ell(q^2)\, \mathcal{O}_d^\ell \qquad (4.8)$$

where the local operators $\mathcal{O}_d^\ell$ are classified according to their canonical dimensions d and the additional suffix ℓ running over independent operators of the same dimension[2]. Equation (4.8) implies that the large-q^2 behaviour is governed by operators of low dimensionality, e.g.

$$\mathcal{O}_0 = 1\ ,\ \mathcal{O}_4^1 = \bar{\psi} D^q \psi\ ,\ \mathcal{O}_4^2 = G^a_{\mu\nu} G^{\mu\nu}_a \qquad (4.9)$$

where $G^a_{\mu\nu}$ is the gluonic strength tensor. The orders of magnitude of $\mathcal{O}_4^1$ and $\mathcal{O}_4^2$ can be estimated by the standard methods of Ref.4. The q^2 dependence of the coefficient functions is determined by the renormalization group equations. The equation for $C^l_{d\neq 0}$ is of the usual type encountered in QCD [2,4], whereas the derivation of the equations for $C_{d=0}$ requires some care, since the two-point function is not multiplicatively renormalizable[2], i.e.

$$C_0(q^2) = \mu^{-\varepsilon}(\, Sq^2 - Tm_H^2\,) + C_0^f(q^2) \qquad (4.10)$$

where

$$S = (1 - Z'_3)/g^2 \text{ and } T = (1 - Z_{m(H)} Z'_3)/g^2$$

are the subtraction terms corresponding to the wavefunction and mass renormalizations of the Higgs boson, and can be expanded as Laurent series [4,9]

$$S(f, m_i/M_L, \varepsilon) = \sum_{k=1}^{\infty} \frac{1}{\varepsilon^k} S_k(f, m_i/M_L) \qquad (4.11)$$

$$T(f, m_i/M_L, M_L/M_H, \varepsilon) = \sum_{k=1}^{\infty} \frac{1}{\varepsilon^k} T_k(f, m_i/M_L, M_L/M_H) \qquad (4.12)$$

where m_i are the quark masses, and f is the QCD coupling.

Based on (4.11) and (4.12), the authors of Ref.1 carry out a perturbative analysis of the total hadronic decay width of the Higgs boson of the Standard Model and find that, up to the next-to-leading order one obtains

$$\frac{\Gamma_H}{m_H} = \frac{3\sqrt{2}}{8\pi} G_F \sum_i \hat{m}_i^2 \left(\ln \frac{m_H^2}{\Lambda^2} \right)^{-8/7} \left\{ 1 - \frac{208}{343} \frac{\ln\ln(m_H^2/\Lambda^2)}{\ln(m_H^2/\Lambda^2)} + \frac{4976}{1029} \frac{1}{\ln(m_H^2/\Lambda^2)} + \delta_i \right\} \qquad (4.13)$$

where

$$\delta_i = -6 \frac{\hat{m}_i^2}{m_H^2} \left(\ln \frac{m_H^2}{\Lambda^2} \right)^{-8/7} \left\{ 1 - \frac{416}{343} \frac{\ln\ln(m_H^2/\Lambda^2)}{\ln(m_H^2/\Lambda^2)} + \frac{7208}{1029} \frac{1}{\ln(m_H^2/\Lambda^2)} \right\} \quad (4.14)$$

where $G_F = g^2/(4\sqrt{2}M_L^2)$ is the Fermi constant, $\hat{m}_i$ are the constituent quark masses, and Λ is given by[5]

$$\Lambda_{\overline{MS}} = \Lambda \exp \left\{ \frac{1}{2} (\ln 4\pi - \gamma_E) \right\} \quad (4.15)$$

where $\gamma_E \doteq 0.577$ is the Euler-Mascheroni constant, and we take[5]

$$\Lambda_{\overline{MS}} = 0.52 \text{ GeV} \quad (4.16)$$

Since we are dealing with the gauge group $SU(3)_C \cdot SU(2)_L \cdot SU(2)_R \cdot U(1)_{B-L}$ in the minimal left-right symmetric model, and since there are heavy Higgs fields coupling to the quarks, all the above results are applicable to the minimal left-right symmetric model. The difference is that in this case we have both charged and neutral Higgs fields. Once the charged and the neutral current Lagrangians are known, it is straightforward to read the couplings and proceed as above. These tasks will be completed in the next two sections.

4.3 Estimates of the Hadronic Decay Width of the Neutral Higgs bosons in the L-R Symmetric Model

We start with the neutral current Lagrangian involving the quarks and the Higgs fields. We already know (see (2.14)) that only the components of the Higgs bidoublet ϕ couple to the quarks. Now, choosing the neutral (physical) Higgs fields involving the components of ϕ in a convenient form[6] as

$$\Phi_1 = 2^{-1/2}V^{-1} \left[k^*\phi_1^0 + k\phi_1^{0*} + k'^*\phi_2^0 + k'\phi_2^{0*} \right] \quad (4.17)$$

$$\Phi_2 = 2^{-1/2}V^{-1}\left[\left|\frac{k}{k'}\right|(k'^*\phi_2^0 + k'\phi_2^{0*}) - \left|\frac{k'}{k}\right|(k^*\phi_1^0 + k\phi_1^{0*})\right] \quad (4.18)$$

$$\Phi_3 = i2^{-1/2}V^{-1}\left[\left|\frac{k}{k'}\right|(k'^*\phi_2^0 - k'\phi_2^{0*}) + \left|\frac{k'}{k}\right|(k\phi_1^0 - k\phi_1^{0*})\right] \quad (4.19)$$

one can write the neutral current Lagrangian as

$$-L_Y^0 = \frac{\Phi_1}{V} (\bar{u}_L D^u u_R + \bar{d}_L D^d d_R)$$

$$+ \sum_{i=2}^{3} \Phi_i (\bar{u}_L G_i^u u_R + \bar{d}_L G_i^d d_R) + h.c. \quad (4.20)$$

where

$$G_2^u = -iG_3^u = -(|k|^2 - |k'|^2)^{-1}(2|kk'|D^u/V + e^{i\omega}VU^L D^d U^{R\dagger}) \tag{4.21}$$

$$G_2^d = iG_3^d = -(|k|^2 - |k'|^2)^{-1}(2|kk'|D^d/V + e^{-i\omega}VU^{L\dagger} D^u U^R) \tag{4.22}$$

In (4.17-22) V is defined as $V^2 = |k|^2 + |k'|^2$, and D^u, D^d are the diagonal quark mass matrices (2.29) and (2.30). The matrices U^L and U^R have been defined in (2.32). Let us also note that from (2.19) we have

$$M_{1,2}^2 = \frac{1}{4} g^2\left[V^2 + |v_R|^2 + |v_L|^2 \mp \sqrt{(|v_R|^2 - |v_L|^2)^2 + 4|kk'|^2}\,\right] \tag{4.23}$$

which in the physically relevant limit $|v_L|^2 << V^2 << |v_R|^2$ gives

$$M_1^2 \simeq \frac{1}{4} g^2 (|k|^2 + |k'|^2) = \frac{1}{4} g^2 V^2 \tag{4.24}$$

$$M_2^2 \simeq \frac{1}{4} g^2 (2|v_R|^2 + V^2) \tag{4.25}$$

so also $V^2 \simeq (\sqrt{2}G_F)^{-1}$. Using the expressions (4.17-4.19), together with the relations (4.23-25), we find that for two of the VEV scenarios discussed in chapter 2, namely

i) $k' = 0$, v_L, k, $v_R \neq 0$

ii) $v_L = 0$, k, k', $v_R \neq 0$

one will obtain

Case (i):

$$\Phi_1 = 2^{-1/2}V^{-1}k\ (\phi_1^0 + \phi_1^0) = \sqrt{2}\ \mathrm{Re}(\phi_1^0) \tag{4.26}$$

$$\Phi_2 = 2^{-1/2}V^{-1}k\ (\phi_2^0 + \phi_2^{0*}) = \sqrt{2}\ \mathrm{Re}(\phi_2^0) \tag{4.27}$$

$$\Phi_3 = i\ 2^{-1/2}V^{-1}(\phi_2^0 - \phi_2^{0*}) = \sqrt{2}\ i\ \mathrm{Im}(\phi_2^{0*}) \tag{4.28}$$

the coupling matrices in (4.21) and (4.22) will simplify to

$$G_2^u = -i\ G_3^u = -k^{-1}\ (\ U^L D^d U^{R\dagger}\ ) \tag{4.29}$$

$$G_2^d = i\ G_3^d = -k'^{-1}\ (\ U^{L\dagger} D^u U^R\ ) \tag{4.30}$$

Finally (omitting the h.c. part), the neutral current Lagrangian can be rewritten, term by term, as

$$-L_Y^0 = \frac{g}{2M_L}\ \Phi_1\ (\ \bar{u}_L D^u u_R + \bar{d}_L D^d d_R\ )$$

$$-\frac{g}{2M_L}\ \Phi_2\ (\ \bar{u}_L U^L D^d U^{R\dagger} u_R\ ) + \frac{ig}{2M_L}\ \Phi_3\ (\ \bar{u}_L U^L D^d U^{R\dagger} u_R\ )$$

$$-\frac{g}{2M_L}\ \Phi_2\ (\ \bar{d}_L U^{L\dagger} D^u U^R d_R\ ) - \frac{ig}{2M_L}\ \Phi_3\ (\ \bar{d}_L U^{L\dagger} D^u U^R d_R\ ) \tag{4.31}$$

J_H in (4.4) is now to be identified with the current multiplying Φ_1, and its analogs which couple to Φ_2 and Φ_3

fields are, respectively

$$J_2 = \bar{u}_L U^L D^d U^{R\dagger} u_R + \bar{d}_L U^{L\dagger} D^u U^R d_R \qquad (4.32)$$

$$J_3 = \bar{u}_L U^L D^d U^{R\dagger} u_R - \bar{d}_L U^{L\dagger} D^u U^R d_R \qquad (4.33)$$

Since D^u = diag (m_u, m_c, m_t) and D^d = diag (m_d, m_s, m_b), and since m_b and m_t are much heavier than the other quarks, the leading contributions arise through setting $\hat{m}_i = \hat{m}_b, \hat{m}_t$ in (4.13-14).

From (4.31) we see that the couplings are proportional to $g/2M_L$, just like the Standard Model part of the neutral current Lagrangian. So everything in (4.13) and (4.14) will hold for the left-right symmetric theory but different mass ranges of the underlying Higgs fields must be considered. As a final remark, we emphasize that the presence of the KM-type matrices U^L and U^R need not worry us, since we do not intend to address questions of CP-violation and quark mixings here.

Next we have to consider case (ii), i.e. when $k' \neq 0$. Again using the general expressions (4.17-4.19) we find the following:

Case (ii)

$$\Phi_1 = 2^{1/2} V^{-1} [\text{ Re } (\phi_1^0) + \text{ Re } (\phi_2^0)] \qquad (4.34)$$

$$\Phi_2 = 2^{1/2}V^{-1}[\ k\ \mathrm{Re}\ (\phi_2^0) + k'\ \mathrm{Re}\ (\phi_1^0)\ ] \qquad (4.35)$$

$$\Phi_3 = i2^{1/2}V^{-1}[\ k\ \mathrm{Im}\ (\phi_2^0) + k'\ \mathrm{Im}(\phi_1^0)\ ] \qquad (4.36)$$

and the coupling matrices become (see (4.21-4.22))

$$G_2^u = -iG_3^u = -\frac{1}{k^2 - k'^2}\left[\frac{2kk'}{V}D^u + VU^L D^d U^{R\dagger}\right] \qquad (4.37)$$

$$G_2^d = iG_3^d = -\frac{1}{k^2 - k'^2}\left[\frac{2kk'}{V}D^d + VU^{L\dagger} D^u U^R\right] \qquad (4.38)$$

from which we can read the coupling strengths

$$\frac{2kk'}{V(k^2-k'^2)}\quad , \quad \frac{V}{k^2 - k'^2} \qquad (4.39)$$

If we now recall[6] the approximate relation

$$\frac{m_b}{m_t} \simeq \frac{k'}{k} \qquad (4.40)$$

we can write

$$\frac{2kk'}{V(k^2-k'^2)} \simeq \frac{2}{k}\left(\frac{m_b}{m_t}\right)\left(1 + \frac{m_b^2}{m_t^2}\right)^{-1/2}\left(1 - \frac{m_b^2}{m_t^2}\right)^{-1}$$

$$\simeq \frac{2}{k}\frac{m_b}{m_t} \qquad (4.41)$$

$$\frac{V}{k^2 - k'^2} \simeq \frac{1}{k}\left(1 + \frac{m_b^2}{m_t^2}\right)^{1/2}\left(1 - \frac{m_b^2}{m_t^2}\right)^{-1}$$

$$\simeq \frac{1}{k} \qquad (4.42)$$

The couplings (4.41) and (4.42) amount to replacing G_F with

$$4\left(\frac{m_b^2}{m_t^2}\right)G_F \quad \text{and} \quad \left(1 + \frac{m_b^2}{m_t^2}\right)G_F \qquad (4.43)$$

in (4.13), respectively.

4.4 Estimates of the Hadronic Decay Width of the Charged Higgs Boson in the L-R Symmetric Model

In this section we consider charged Higgs bosons of the minimal left-right symmetric model which couple to quarks. All the arguments concerning perturbative QCD expansions etc., presented in the previous section for the neutral-Higgs decay widths, remain valid in the charged-Higgs boson case as well. Thus, all we have to do is to find the coupling strength of the charged-Higgs fields to the quarks and proceed as before.

From the Yukawa Lagrangian of (2.24) it can be readily seen that from all the charged physical Higgs fields, only those that involve the components of the multiplet ϕ couple to the quarks. This leaves us with a single charged field

$$\Phi^+ = N \left(k^* \phi_1^+ + k' \phi_2^+ + 2^{-1/2} \frac{|k|^2 - |k'|^2}{v_R} \delta_R^+ \right) \quad (4.44)$$

where

$$N = \left[V^2 + \left((|k|^2 - |k'|^2)/\sqrt{2}\ v_R \right)^2 \right]^{-1/2} \quad (4.45)$$

and $V^2 = |k|^2 + |k'|^2$.

As we can see, for $k' = 0$, the expression (3.23) is recovered. The corresponding Yukawa Lagrangian is then given by

$$-L_Y^+ = (u_L G_R d_R + u_R G_L d_L) \Phi^+ + h.c. \quad (4.46)$$

where

$$G_R = \frac{\sqrt{2}\ N}{|k|^2 - |k'|^2} (V^2 D^u U^R - 2\ k\ k'^* U^L D^d) \quad (4.47)$$

$$G_L = \frac{\sqrt{2}\ N}{|k|^2 - |k'|^2} (2\ k\ k'^* D^u U^L - V^2 U^R D^d) \quad (4.48)$$

Thus, from (4.36) and (4.38) it follows that the couplings that we have to consider are

$$\frac{\sqrt{2}\ N}{|k|^2 - |k'|^2} V^2 \quad , \quad \frac{2\sqrt{2}\ N k k'^*}{|k|^2 - |k'|^2} \quad (4.49)$$

Now, as before, we can consider the two cases $k' = 0$, and k'

$\neq 0$ and *real.*

i) $k' = 0$

In this case the coupling is $\sqrt{2}N$ which, using (4.45), is given by

$$\sqrt{2}\ N \simeq \frac{\sqrt{2}}{k} \left(1 - \frac{k^2}{4v_R^2} \right) \tag{4.50}$$

which, because of the VEV hierarchy $|v_L|^2 << V^2 << |v_R|^2$ implies that in the estimate for Γ_H/m_H one has to replace $\sqrt{2}$ G_F with $2\sqrt{2}G_F$.

ii) $k' \neq 0$, *(real)*

In this case

$$N \simeq \frac{1}{k} \left(1 - \frac{1}{2} \frac{m_b^2}{m_t^2} \right) \tag{4.51}$$

where, again, we have used the approximate relation $m_b/m_t \simeq k'/k$ (see (4.40)). The couplings in (4.49) are therefore approximately given by

$$\frac{\sqrt{2}\ N}{k^2 - k'^2} V^2 \simeq \frac{1}{k} \left(1 - \frac{1}{2} \frac{m_b^2}{m_t^2}\right) + O(\ m_b^2/m_t^2) \tag{4.52}$$

and

$$\frac{\sqrt{2}\ Nkk'}{k^2 - k'^2} \simeq \frac{2\sqrt{2}}{k} \left(\frac{m_b}{m_t} \right) \tag{4.53}$$

As we can see, if we neglect the m_b/m_t contribution in

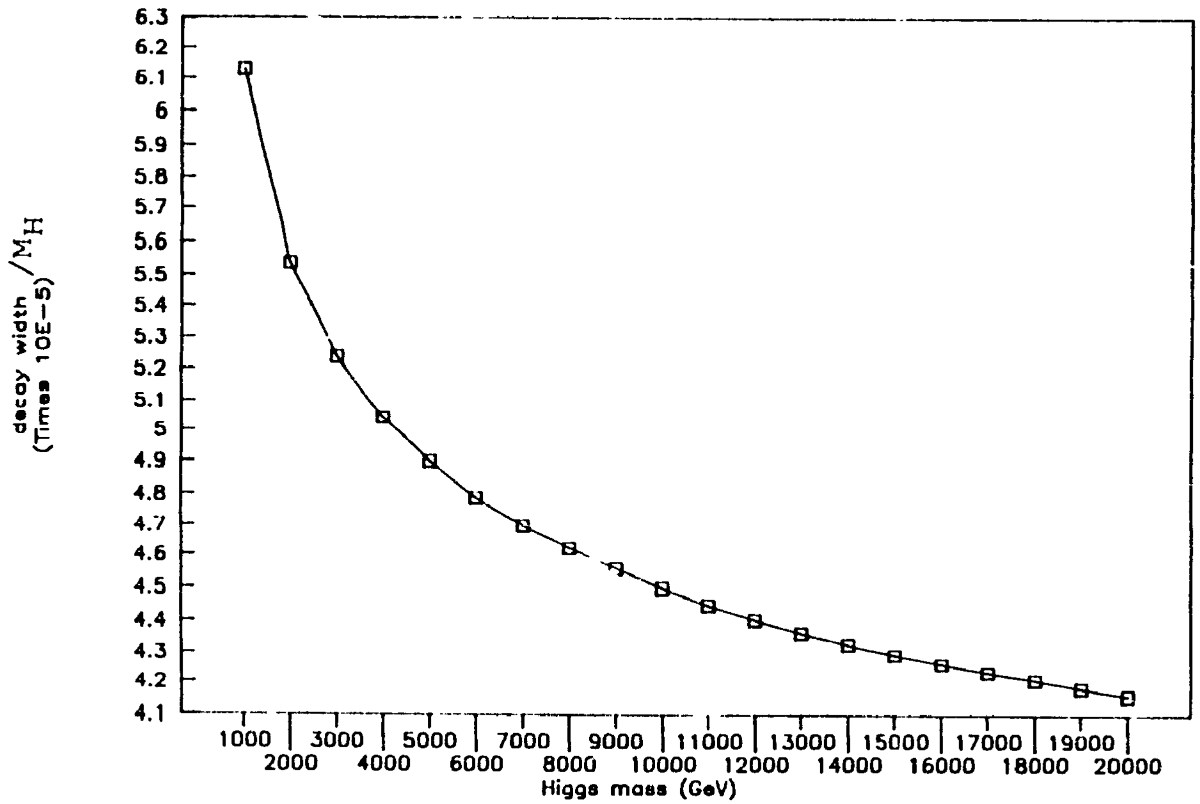
(4.53) in comparison with (4.52), we will be left with 1/k, which means that we have to retain $\sqrt{2}\ G_F$ in (4.13) for the corresponding decay width. This concludes our considerations on the renormalization group estimates of the *total* hadronic decay widths of the neutral and charged Higgs fields of the minimal left-right symmetric model, which couple to the quark fields.

These results enable us to make theoretical estimates of the total hadronic decay widths which differ from the Standard Model estimates. Thus, assuming that the top quark will be discovered in the not too distant, future one can infer the Higgs mass, or vice versa. This will serve as an interesting testing ground for physics beyond the Standard Model. We can now use the couplings in (4.43) namely

$$4\left(\frac{m_b^2}{m_t^2}\right)G_F \quad \text{and} \quad \left(1+\frac{m_b^2}{m_t^2}\right)G_F$$

corresponding to two of the (non-standard) Higgs fields in case (ii), i.e. ϕ_2 and ϕ_3 and calculate the decay width of these fields by using the formulae (4.13-14). The corresponding graphs are given in Figs. 4.1 and 4.2. Similarly, we can find the hadronic decay width of the charged Higgs field ϕ^+ in case(i) is found by replacing G_F with $2G_F$ in (4.13-14). This is graphically presented in Fig. 4.3.

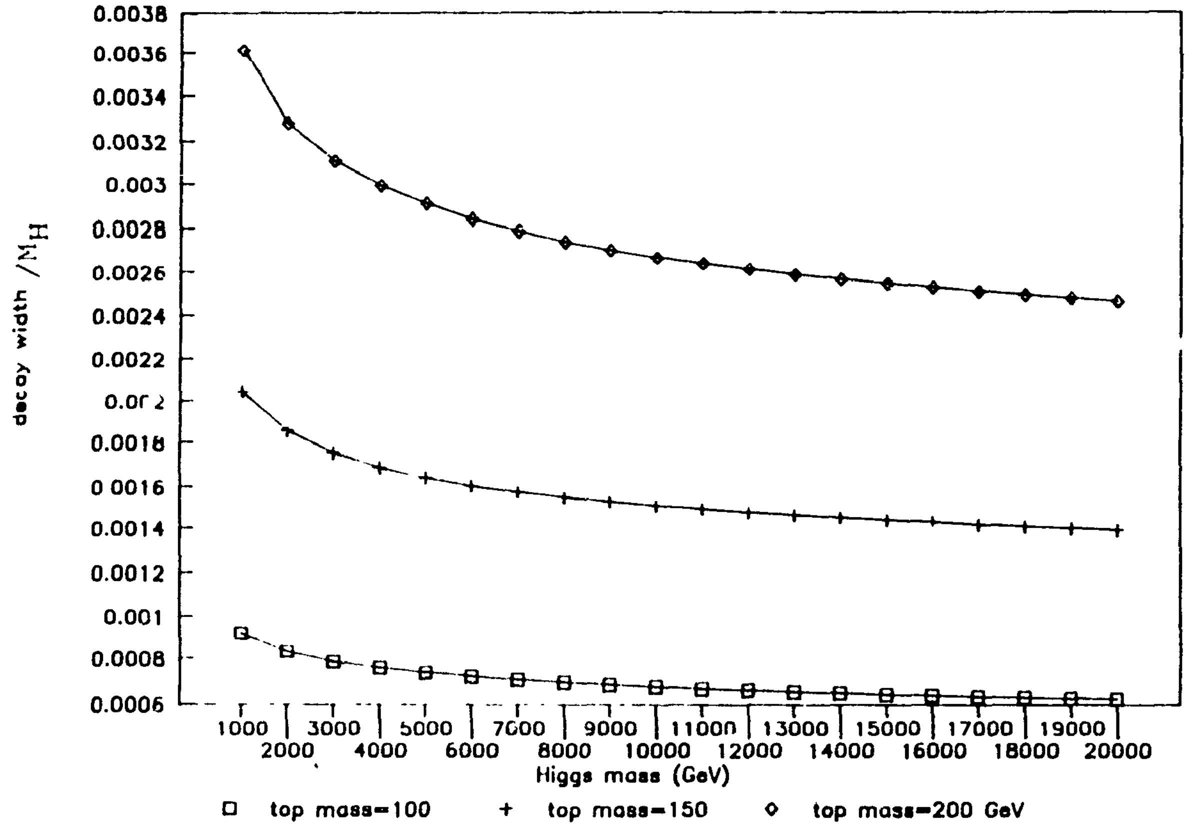
Fig.4.1



□ top mass=100 GeV

Neutral-Higgs decay width vs. the Higgs mass

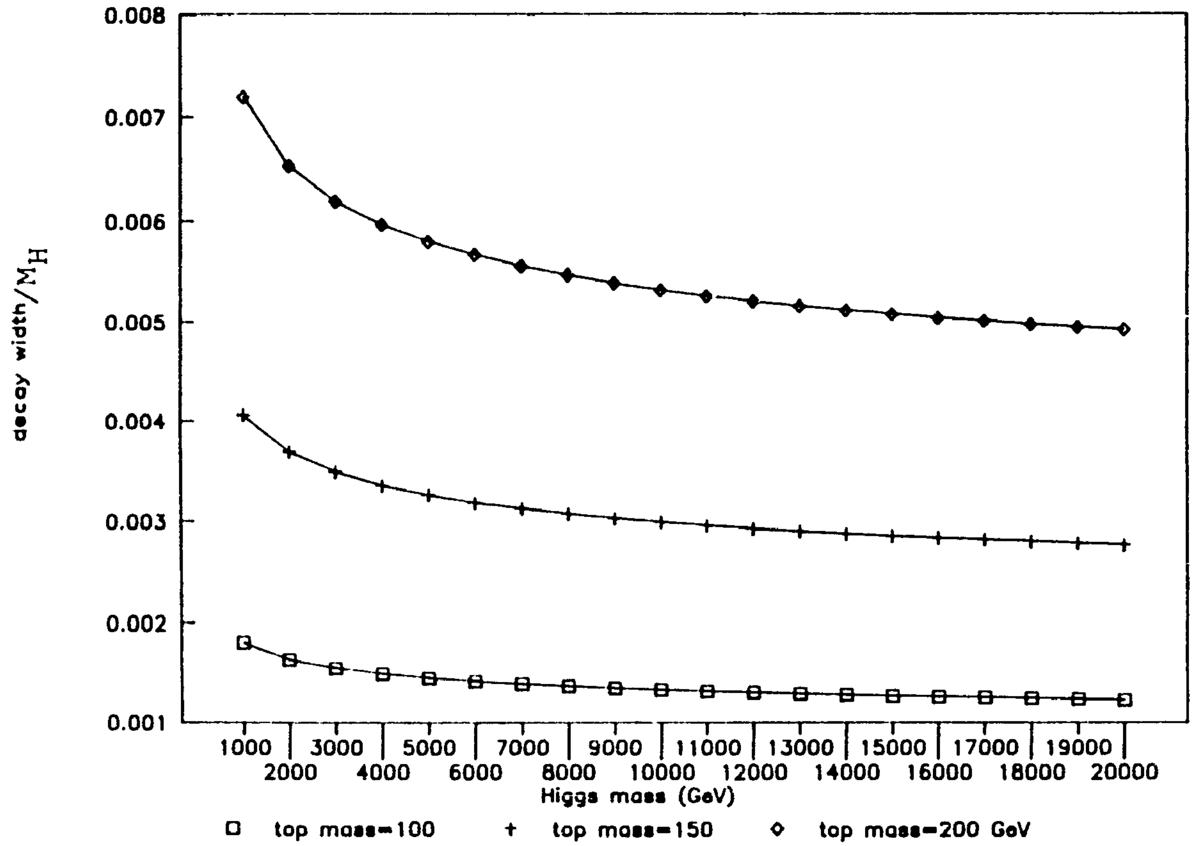
Fig.4.2



Charged-Higgs decay width vs. the Higgs mass

in case (i)

Fig.4.3



Charged-Higgs decay width vs. the Higgs mass in case (ii)

CHAPTER 5

HIGGS PRODUCTION THROUGH THROUGH ELECTRON-POSITRON ANNIHILATION

5.1 Introduction.

Several accelerators have been constructed in different laboratories around the world to study particle collision at higher and higher energies. In quantum theory high energies imply short wavelengths, which are essential for probing small scale phenomena. The latest of these accelerator projects are "colliders", based on the principle of colliding particle beams, which is now the most economical way of achieving high energies. Future plans such as the SSC are centered on colliders as the best way towards new physics.

In the spirit of these future plans, and in pursuing the phenomenological consequences of the minimal left-right symmetric extension of the standard model, total differential cross sections for various processes of the type $e^+e^- \longrightarrow H\bar{H}$, where H denotes the Higgs field, will be calculated in this chapter.

5.2 Vertex factors

The first step for calculating total differential cross sections for the processes $e^+e^- \longrightarrow H\bar{H}$ is the collection of all the vertex factors (couplings) in the Feynman diagrams associated with these processes. The underlying technique for finding the vertex factors is the familiar one from the Standard Model, which will be briefly reviewed below. In the Standard Model, one starts with the neutral current Lagrangian[1]

$$L_{N.C} = e\, J_\mu^{em} A^\mu + (g/\cos\theta_W)\; J_\mu^0 Z^\mu \qquad (5.1)$$

with $J_\mu^0 = J_\mu^3 - \sin^2\theta_W\, J_\mu^{em}$. J_μ^0 can be expressed explicitly in terms of the fermion fields as

$$J_\mu^0 = \sum_f \left[\, g_L^f\, \bar{f}_L\, \gamma_\mu\, \bar{f}_L + g_R^f\, \bar{f}_R\, \gamma_\mu\, f_R \,\right]$$

$$= \frac{1}{2} \sum_f \left[\, g_L^f\, \bar{f}\, \gamma_\mu (1-\gamma_5)\, f + g_R^f\, \bar{f}\, \gamma_\mu (1-\gamma_5)\, f \,\right] \qquad (5.2)$$

where f denotes the quark and lepton fields. The weak neutral couplings are given by

$$g_{L,R}^f = T_3(\, f_{L,R}\,) - Q(\, f\,)\, \sin^2\theta_W \qquad (5.3)$$

where T_3 and Q are the third component of the weak isospin and the electric charge operators respectively (see chapter 1). Thus, for the electron field for instance, one has

$$J_\mu^{0e} = \frac{1}{2}\left(-\frac{1}{2} + \sin^2\theta_W \right) \bar{e}\, \gamma_\mu \,(\, 1 - \gamma_5\,)\, e$$

$$+ \frac{1}{2} (\sin^2\theta_W) \bar{e} \gamma_\mu (1 - \gamma_5) e \qquad (5.4)$$

which, when inserted in (5.1), gives the $e^+ e^- Z_L^\mu$ vertex factor as

$$i e \gamma_\mu \frac{1}{4 \sin\theta_W \cos\theta_W} \left[(-1 + 4 \sin^2\theta_W) + \gamma_5 \right] \qquad (5.5)$$

In the left-right symmetric model the vertex factors can be found in a similar fashion. In this case, the neutral current Lagrangian is[2]

$$L_{N.C} = Q J_\mu^{em} A^\mu + \frac{g}{\cos\theta_W} \left\{ Z_L^\mu \left[J_{L\mu}^Z - \eta \cos\theta_W (\sin^2\theta_W J'_{L\mu} \right.\right.$$

$$\left. + \cos^2\theta_W J_{R\mu}^Z) \right] + \frac{1}{\sqrt{\cos 2\theta_W}} Z_R^\mu$$

$$\left. (\sin^2\theta_W J_{L\mu}^Z + \cos^2\theta_W J_{R\mu}^Z) \right\} \qquad (5.6)$$

where $\sin\theta_W = e/g$, $\eta = (M_L/M_R)^2$, and $J_{L,R}^Z = J_{L,R}^3 - Q \sin^2\theta_W$.

Here, $J_{L,R}^3$ are the diagonal generators of the $SU(2)_{L,R}$ gauge groups respectively, and $M_{L,R}$ are the physical masses of the left- and right-handed charged gauge bosons. Using (5.6), one can find the couplings to the photon A_μ and the neutral intermediate gauge bosons Z_μ^L and Z_μ^R. In terms of the operators $J^3_{L,R}$ and Q, these are given by

$$A^\mu: \quad e\,Q \quad (e \text{ is the positive unit electric charge})$$

$$Z^L_\mu: \quad \frac{e}{\sin\theta_W \cos\theta_W}\Big[J^3_L - Q\sin^2\theta_W - \eta\sin^2\theta_W\cos\theta_W J^3_L$$

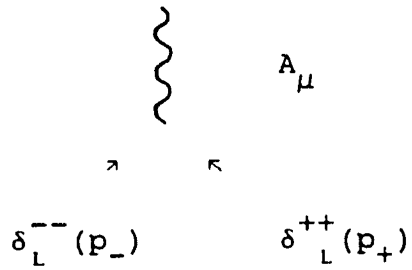
$$+ \eta\, Q\sin^4\theta_W\cos\theta_W - \eta\cos^3\theta_W J^3_R + \eta\, Q\sin^2\theta_W\cos^3\theta_W\Big] \qquad (5.8)$$

$$Z^R_\mu: \quad \frac{e}{\sin\theta_W\cos\theta_W\sqrt{\cos 2\theta_W}}\Big[\cos^2\theta_W J^3_R + \sin^2\theta_W (J^3_L - Q)\Big] \qquad (5.9)$$

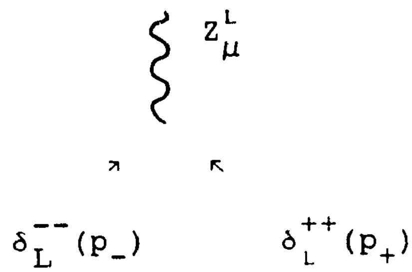
Ignoring terms of order $\eta = (M_L/M_R)^2$ in (5.8), the coupling to Z^L_μ simplifies to

$$\frac{e}{\sin\theta_W\cos\theta_W}\,(J^3_L - Q\sin^2\theta_W) \qquad (5.10)$$

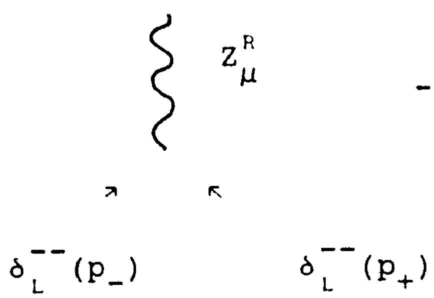
Using the general couplings (5.7) to (5.10), together with (2.5), (2.7) and (2.8), all the required vertex factors for the calculation of the differential cross sections for the processes $e^+e^- \rightarrow H\bar{H}$ can be found. The relevant vertex diagrams and the associated couplings are given below



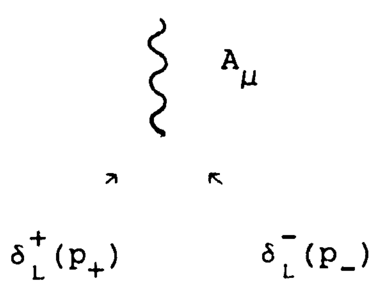
$$-2ie(p_- - p_+)_\mu = -i\lambda_1 (p_- - p_+)_\mu \quad (5.11)$$



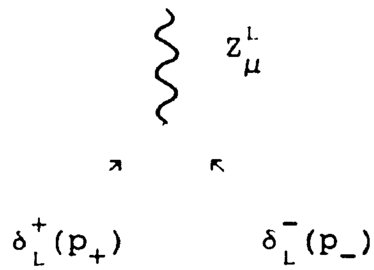
$$-ie\left(\frac{1 - 2\sin^2\theta_W}{\sin\theta_W \cos\theta_W} \right)(p_- - p_+)_\mu = -i\lambda_1^L (p_- - p_+)_\mu \quad (5.12)$$



$$-ie\left(\frac{-\tan\theta_W}{\sqrt{\cos 2\theta_W}} \right)(p_- - p_+)_\mu = -i\lambda_1^R (p_- - p_+)_\mu \quad (5.13)$$

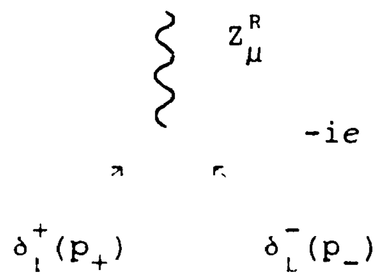


$$-ie(p_- - p_+)_\mu = -i\lambda_2 (p_- - p_+)_\mu \quad (5.14)$$



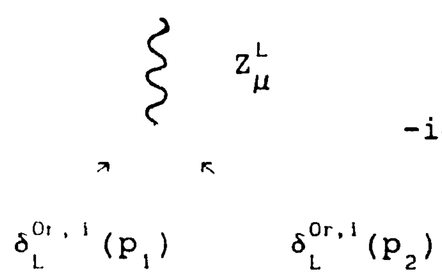
$$-ie\left(\frac{-\sin^2\theta_W}{\sin\theta_W\ \cos\theta_W}\right)(p_- - p_+)_\mu$$

$$= -i\lambda_2^L(\ p_- - p_+)_\mu \tag{5.15}$$



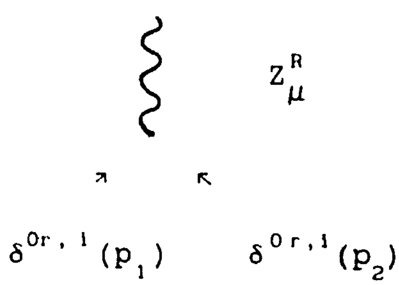
$$-ie\left(\frac{-\sin_W^2\theta}{\sin\theta_W\ \cos\theta_W\ \sqrt{\cos 2\theta_W}}\right)(p_- - p_+)_\mu$$

$$= -i\lambda_2^R(\ p_- - p_+)_\mu \tag{5.16}$$



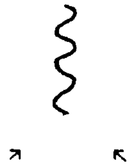
$$-ie\left(\frac{-1}{\sin\theta_W\ \cos\theta_W}\right)(p_2 - p_1)_\mu$$

$$= -i\lambda_3^L(\ p_2 - p_1)_\mu \tag{5.17}$$



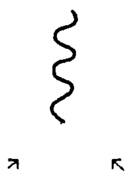
$$-ie\left(\frac{-1}{\sin\theta_W\cos\theta_W}\right)(p_2 - p_1)_\mu$$

$$= -i\lambda_3^R(\ p_2 - p_1)_\mu \tag{5.18}$$



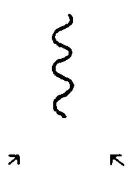
$$-ie\left(\frac{-1}{2\sin\theta_W\cos\theta_W}\right)(p_2-p_1)_\mu$$

$$= -i\lambda^L_4(p_2-p_1)_\mu \qquad (5.19)$$

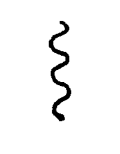


$$-ie\left(\frac{\cos^2_W\theta - \sin^2\theta_W}{2\sin\theta_W\cos\theta_W\sqrt{\cos 2\theta_W}}\right)(p_2-p_1)_\mu$$

$$= -i\lambda^R_4(p_2-p_1)_\mu \qquad (5.20)$$

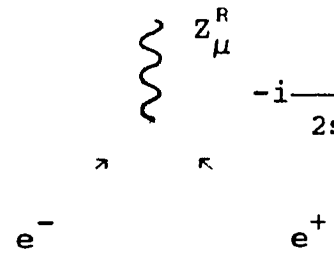


$$-ie\,\gamma_\mu = -i\lambda\gamma_\mu \qquad (5.21)$$



$$-i\frac{e}{4\sin\theta_W\cos\theta_W}\gamma_\mu\left(-1+4\sin^2\theta_W+\gamma_5\right)$$

$$= -i\,\gamma_\mu\,(c^L_1 - c^L_2\,\gamma_5) \qquad (5.22)$$



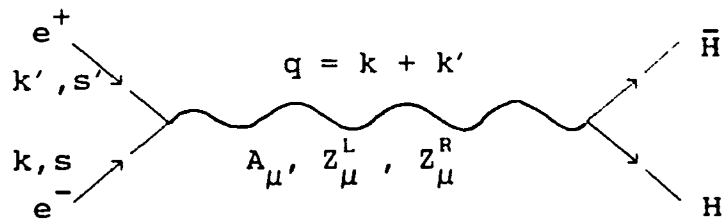
$$-i\frac{e}{2\sin 2\theta_W\sqrt{\cos 2\theta_W}}\,\gamma_\mu\left(-1+4\sin^2\theta_W-\cos(2\theta_W)\gamma_5\right)$$

$$= -i\,\gamma_\mu\,(\,c_1^R + c_2^R\,\gamma_5\,) \tag{5.23}$$

Now that all the vertex couplings are explicitly collected, one can proceed with the calculation of the total differential cross section for general cases and insert any of the desired couplings. This task will be completed in the next section and the corresponding expressions for the total differential cross sections will be given.

5.3 Calculation of the total differential cross section for the processes of the type $e^+e^-\longrightarrow H\bar{H}$

The Feynman diagram corresponding to the process $e^+e^-\longrightarrow H\bar{H}$, where H is any of the Higgs fields δ_L^{++}, δ_L^{+}, $\delta_L^{0r,i}$, $\phi_2^{0r,i}$, is typically of the following form.



(5.24)

where k, k', p, p' are the four-momenta of the incoming and the outgoing particles respectively, and s and s' are the spins of the incoming electron and positron.

The underlying calculation is quite similar to the one of the process $e^+e^- \longrightarrow \pi^+\pi^-$ in the no-structure differential cross section approximation except that, since one is dealing with very heavy Higgs fields (in the multi-TeV region), even at very high energies the contributions coming from the masses of the Higgs fields must always be strictly retained. Using the usual techniques for calculating cross sections[1] one estimates the scattering amplitude for the process involving the exchange of Z^{l}_{μ} which can be written as

$$\mathcal{A} = -i\, \mathcal{N}\, \mathcal{N}'\, N\, N' \lambda^{L}_{1} (2\pi)^4 \delta^4(p'+p-k'-k)\ \bar{v}\,(k',s')\,\gamma_{\mu}(c^{L}_{1} - c^{L}_{2}\gamma_5)$$

$$u\,(k,s) \left(- \frac{g^{\mu\nu} - q^{\mu}q^{\nu}/M_L^2}{q^2 - M_L^2} \right) (p' - p)_{\nu} \qquad (5.25)$$

where $\mathcal{N}$, $\mathcal{N}'$, N, N' are the appropriate normalization constants and λ^{L}_{1} can be any of the numerical factors read from (5.11-21), except (5.13), (5.16) and (5.20). v and u are the positron and electron spinors respectively. One can now read the invariant amplitude from (5.25), which in its expanded form is

$$F_{ss'} = F^1_{ss'} + F^2_{ss'} + F^3_{ss'} + F^4_{ss'} \tag{5.26}$$

where

$$F^1_{ss'} = \lambda' c^L_1 \bar{v}(k',s')\, \gamma_\mu\, u(k,s) \left(\frac{g^{\mu\nu}}{q^2 - M^2_L}\right) (p - p')_\nu \tag{5.27}$$

$$F^2_{ss'} = \lambda' c^L_1 \bar{v}(k',s')\, \gamma_\mu\, u(k,s) \left(- \frac{q^\mu q^\nu}{M^2_L (q^2 - M^2_L)}\right) (p - p')_\nu \tag{5.28}$$

$$F^3_{ss'} = \lambda' c^L_2 \bar{v}(k',s')\, \gamma_\mu \gamma_5\, u(k,s) \left(- \frac{g^{\mu\nu}}{q^2 - M^2_L}\right) (p - p')_\nu \tag{5.29}$$

$$F^4_{ss'} = \lambda' c^L_2 \bar{v}(k',s')\, \gamma_\mu \gamma_5\, u(k,s) \left(\frac{q^\mu q^\nu}{M^2_L (q^2 - M^2_L)}\right) (p - p')_\nu \tag{5.30}$$

Now, in order to calculate the differential cross section, one has to sum the scattering amplitude over initial spins and average over final spins

$$\frac{1}{4} \sum_{s,s'} |F_{ss'}|^2 \tag{5.31}$$

which can be considerably simplified when one uses the center of momentum (CM) reference frame, which is also the most suitable frame for collider calculations. In the CM frame one has

$$q\cdot(p - p') = (p + p')\cdot(p - p') = 0$$

which implies that $F^2_{ss'} = F^4_{ss'} = 0$. Consequently the differential cross section has to be found from

$$\frac{1}{4} \sum_{s,s'} \left(F^{1\dagger}_{ss'} + F^{3\dagger}_{ss'} \right) \left(F^1_{ss'} + F^3_{ss'} \right) = \frac{1}{4} \bar{\mathcal{F}} \tag{5.32}$$

The summation over spin states s, s′ can be carried out by using well known[1] trace formulae. This gives

$$(\lambda^L_1 c^L_1)^2 \mathrm{Tr}\left[(\not{k}' - m)\gamma_\mu(\not{k} + m)\gamma_\nu\right] \frac{1}{(q^2 - M^2_L)^2}(p - p')^\mu(p - p')^\nu$$

$$= (\lambda^L_1 c^L_1)^2 \, 4\left[k'_\mu k_\nu + k'_\nu k_\mu - \frac{q^2}{2} g_{\mu\nu}\right] \frac{(p - p')^\mu(p - p')^\nu}{(q^2 - M^2_L)^2} \tag{5.33}$$

$$(\lambda^L_1 c^L_1 c^L_2) \mathrm{Tr}\left[(\not{k}' - m)\gamma_\mu(\not{k} + m)\gamma_\nu\gamma_5\right] \frac{-1}{(q^2 - M^2_L)^2}(p-p')^\mu(p-p')^\nu$$

$$= \lambda_1^L c_1^L c_2^L \; 4i\varepsilon_{\alpha\mu\beta\nu} \; \frac{k'^{\alpha}k^{\beta}(p - p')^{\mu}(p - p')^{\nu}}{(q^2 - M_L^2)^2} = 0 \tag{5.34}$$

$$(\lambda_1^L c_2^L)^2 \; \mathrm{Tr}\left[(\not{k}' - m)\gamma_\mu\gamma_5(\not{k} + m)\gamma_\nu\gamma_5\right]\frac{(p - p')^{\mu}(p - p')^{\nu}}{(q^2 - M_L^2)^2}$$

$$= (\lambda_1^L c_2^L)^2 4\left[k'_\mu k_\nu + k'_\nu k_\mu - \frac{q^2}{2} g_{\mu\nu} + 2m^2 g_{\mu\nu}\right]\frac{(p - p')^{\mu}(p - p')^{\nu}}{(q^2 - M_L^2)^2} \tag{5.35}$$

for $\sum_{s,s'} F^{1\dagger}_{ss'} F^{1}_{ss'}$, $\sum_{s,s'} F^{1\dagger}_{ss'} F^{1}_{ss'}$, $\sum_{s,s'} F^{3\dagger}_{ss'} F^{3}_{ss'}$, respectively.

In (5.33-35) the following identities have been used

$$q^2 = (k + k')^2 = (p + p')^2 = 2m^2 + 2k\cdot k'$$

Furthermore, since $p - p' = 2p - q$, and since

$$q^{\mu} \; [\; k'_\mu k_\nu + k'_\nu k_\mu - \frac{q^2}{2} g_{\mu\nu} \;] = 0 \tag{5.36}$$

one obtains, after neglecting the m^2 (m is the electron mass) term

$$\bar{\mathcal{F}} = 16 \; [(\lambda_1^L c_1^L)^2 + (\lambda_1^L c_2^L)] \frac{2(p\cdot k)(p\cdot k') - \frac{q^2}{2} M_H^2}{(q^2 - M_L^2)^2} \tag{5.37}$$

Next, using the CM frame kinematical identities

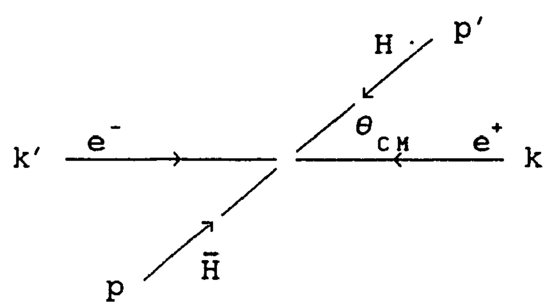
$$s = (k + k')^2 = (p + p')^2 = 4E^2$$

$$t = (k - p')^2 = (p - k')^2$$

$$u = (k - p)^2 = (p' - k')^2$$

$$(p\cdot k)(p\cdot k') = E^4 - |\mathbf{p}|^2|\mathbf{k}|^2 \cos^2\theta_{CM}$$

where (see the following diagram)



$$k = (E,\mathbf{k}) \;,\; k' = (E,-\mathbf{k}) \;,\; p = (E,\mathbf{p}) \;,\; p' = (E,-\mathbf{p})$$

and neglecting m^2 terms, one obtains

$$(p\cdot k)(p\cdot k') \cong \frac{1}{16}\, s^2\, (1 - \cos^2\theta_{CM}) + \frac{1}{4}\, s\, M_H^2 \cos^2\theta_{CM} \qquad (5.38)$$

which, after inserting (5.38) in (5.37), produces the desired expression. The total differential cross section is then found after integrating over the solid angle and dividing by $64\pi^2 s$

$$\sigma_{total} = \frac{1}{64\pi^2 s} \int_0^{2\pi} \int_0^{\pi} \bar{\mathcal{F}} \sin\theta \, d\theta \, d\phi \tag{5.39}$$

So, for individual diagrams involving Z_L and Z_R gauge bosons the total differential cross sections are respectively

$$\sigma_{total,Z_L} = [(\lambda_1^L c_1^L)^2 + (\lambda_1^L c_2^L)^2] \frac{s - 4 M_H^2}{48 \pi (s - M_L^2)^2} \tag{5.40}$$

$$\sigma_{total,Z_R} = [(\lambda_1^L c_1^R)^2 + (\lambda_1^L c_2^R)^2] \frac{s - 4 M_H^2}{48 \pi (s - M_R^2)^2} \tag{5.41}$$

In order to find the total differential cross section when a photon is exchanged one can use either of (5.40) or (5.41) and set $c_1^{L,R} = 2$, $c_2^{L,R} = 0$, and $M_{L,R} = 0$. This yields

$$\sigma_{total,\gamma} = \frac{(\lambda\lambda')^2 (s - 4M_H^2)}{48 \pi s^2} \tag{5.42}$$

To complete the calculation of the total differential cross section, interference terms must also be included. There are three Feynman diagrams contributing to the process $e^+e^- \longrightarrow H\bar{H}$ (see 5.24). Denoting the sum of the invariant amplitudes corresponding to the photon and the left- and right-handed gauge bosons symbolically as

$$\mathcal{M} = \mathcal{M}_\gamma + \mathcal{M}_L + \mathcal{M}_R$$

one can express the interference terms arising from $\mathcal{M}^\dagger \mathcal{M}$ as the sum of the following terms

$$\mathcal{M}_\gamma^\dagger \mathcal{M}_L + \mathcal{M}_L^\dagger \mathcal{M}_\gamma \quad (5.43)$$

$$\mathcal{M}_\gamma^\dagger \mathcal{M}_R + \mathcal{M}_R^\dagger \mathcal{M}_\gamma \quad (5.44)$$

$$\mathcal{M}_R^\dagger \mathcal{M}_L + \mathcal{M}_L^\dagger \mathcal{M}_R \quad (5.45)$$

After some straightforward but lengthy algebra, and upon neglecting contributions coming from the square of the electron mass, expressions (5.43-45) are found to be respectively

$$\frac{8\lambda\lambda_1\lambda_1^L c_1^L}{q^2(q^2 - M_L^2)} [k'_\mu k_\nu + k'_\nu k_\mu - g_{\mu\nu}\frac{q^2}{2}] (p - p')^\mu (p - p')^\nu \quad (5.46)$$

$$\frac{8\lambda\lambda_1\lambda_1^R c_1^R}{q^2(q^2 - M_R^2)} [k'_\mu k_\nu + k'_\nu k_\mu - g_{\mu\nu}\frac{q^2}{2}] (p - p')^\mu (p - p')^\nu \quad (5.47)$$

$$\frac{8\lambda_1^L\lambda_1^R(c_1^L c_1^R + c_2^L c_2^R)}{(q^2 - M_L)\ (q^2 - M_R^2)} [k'k_\nu + k'k_\mu - g_{\mu\nu}\frac{q^2}{2}] (p - p')^\mu (p - p')^\nu \quad (5.48)$$

Next, as before, one has to integrate each of (5.46)-(5.48) over the solid angle and use the previous CM-frame kinematical equations, and finally divide by $64\pi^2 s$.

This will yield for (5.46)-(5.48), respectively

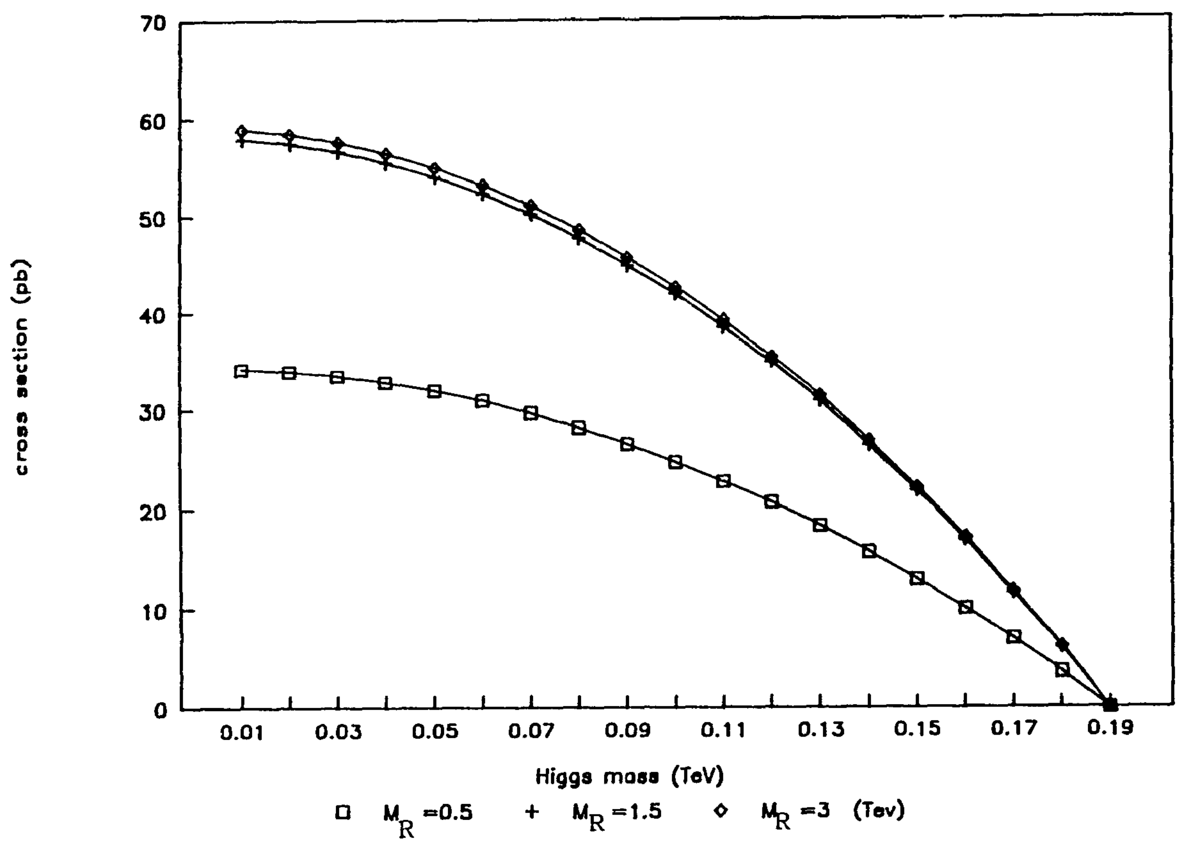
$$\frac{\lambda\lambda_1\lambda_1^L c_1^L}{12\pi\ s\ (s - M_L^2)}\ (s - 4M_H^2) \tag{5.49}$$

$$\frac{\lambda\lambda_1\lambda_1^R c_1^R}{12\pi\ s\ (s - M_R^2)}\ (s - 4M_H^2) \tag{5.50}$$

$$\frac{\lambda_1^L\lambda_1^R (c_1^L c_1^R + c_2^L c_2^R)}{3(s - M_L^2)(s - M_R^2)}\ (s - 4M_H^2) \tag{5.51}$$

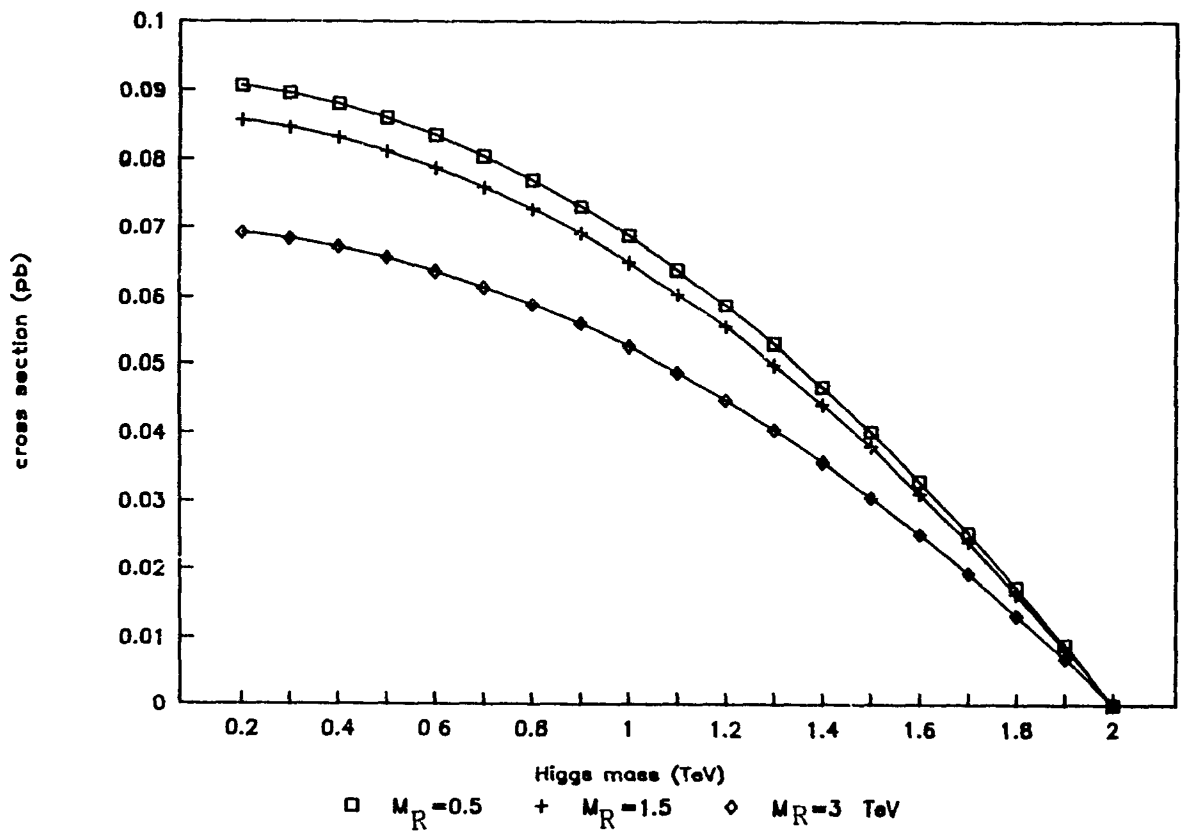
One can now carry out some numerical analysis of the total cross section for the Higgs pair production. However, before proceeding to the numerical study of total cross sections, a few comments are in order. First of all, since the fields ϕ_2^{0r} and ϕ_2^{0i} are responsible for the FCNC interactions, their masses must be sufficiently heavy (≥ 5 TeV) to be consistent with low energy phenomenology. This, together with a minimum value of approximately 3.3 TeV for v_R will make the field ϕ_1^{0r} also very heavy. These fields are therefore not suitable for experimental studies in e^+e^- colliders. But in contrast to the case of ϕ_1^{0r} and $\phi_2^{0r,i}$ fields , there is considerable uncertainty associated with the masses of the triplet fields. The uncertainty comes from the various possibilities for the values of the parameters in the Higgs potential and the VEVs for the Higgs fields. As was discussed earlier (see chapter 3), two most interesting possibilities correspond to the cases $v_L = k' = 0$, $k, v_R \neq 0$ and $v_L, k, v_R \neq 0$, $k' = 0$ (with v_L very small in the second case). When $v_L \neq 0$ but very small, δ_L^0 turns out to be exactly massless (except for radiative corrections which could produce a very small mass) and constraints from the deviation in the electroweak ρ parameter, consistent with $\Delta\rho \leq 0.01$ result in an upper bound of approximately 200 GeV for the masses of Δ_L^+ and Δ_L^{++}. However, if ϕ_2^0 is heavier than the known lower bound of approximately 5 TeV, then no upper limit on the mass of Δ_L^+ can be inferred. In the case of $v_L = 0$ it is possible to have all the scalar bosons except the standard model Higgs-like h^0 to have masses proportional to v_R and hence very heavy. however, this need not necessarily be the case. The reason for this is that the v_R terms in the masses of these fields are proportional to a certain combination of the potential parameters (cf. 3.21-3.33) which can be suitably adjusted to produce smaller masses for these fields. These adjustments can give rise to masses between 200 and 1000 GeV for the fields δ_L^{++}, δ_L^+ and δ_L^0 . In view of these remarks a number graphs for σ vs. M_H and s are given in Figs.1-6. As can be seen in these graphs, wide ranges of masses for the underlying Higgs fields are allowed. Also, a wide range of CM energy makes these results more suitable for experimental studies.

Fig.5.1



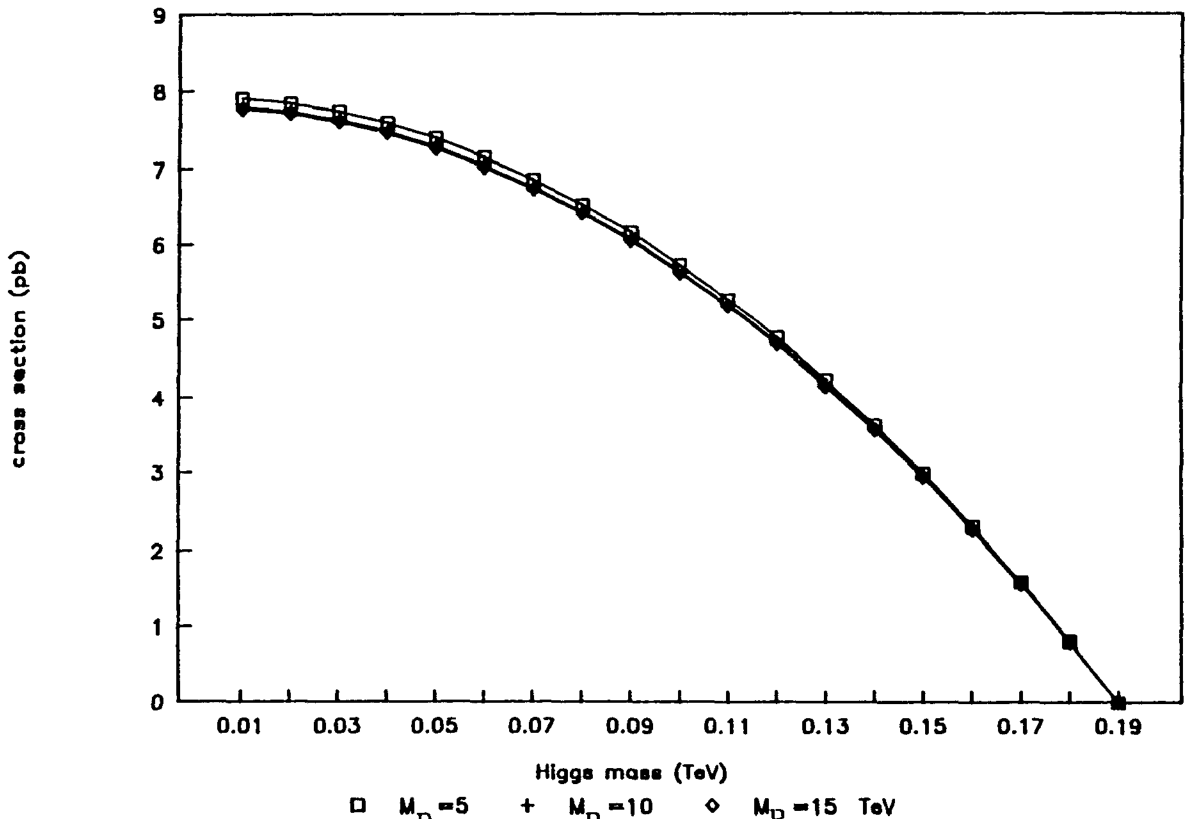
Pair-production cross section vs. the Higgs mass for the δ_L^{++} field

Fig.5.2



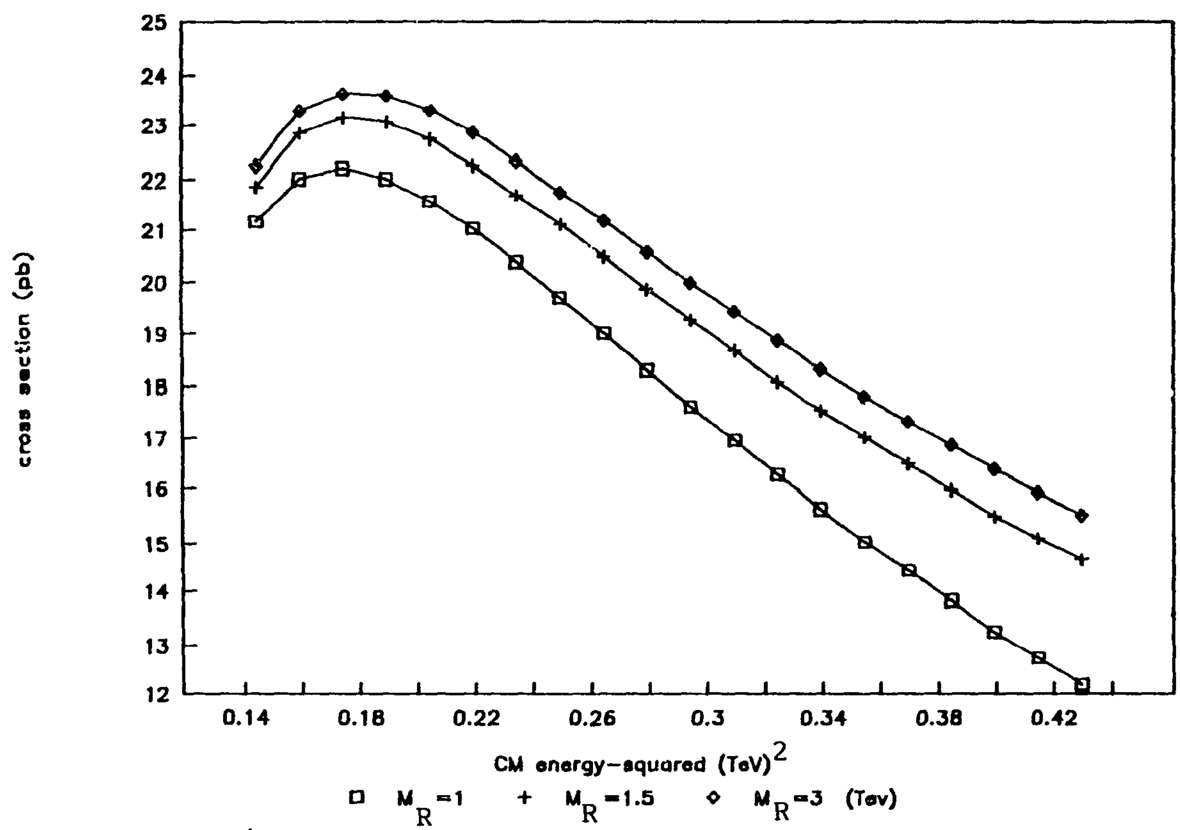
Pair-production cross section vs. the Higgs mass for the δ_L^+ field

Fig.5.3



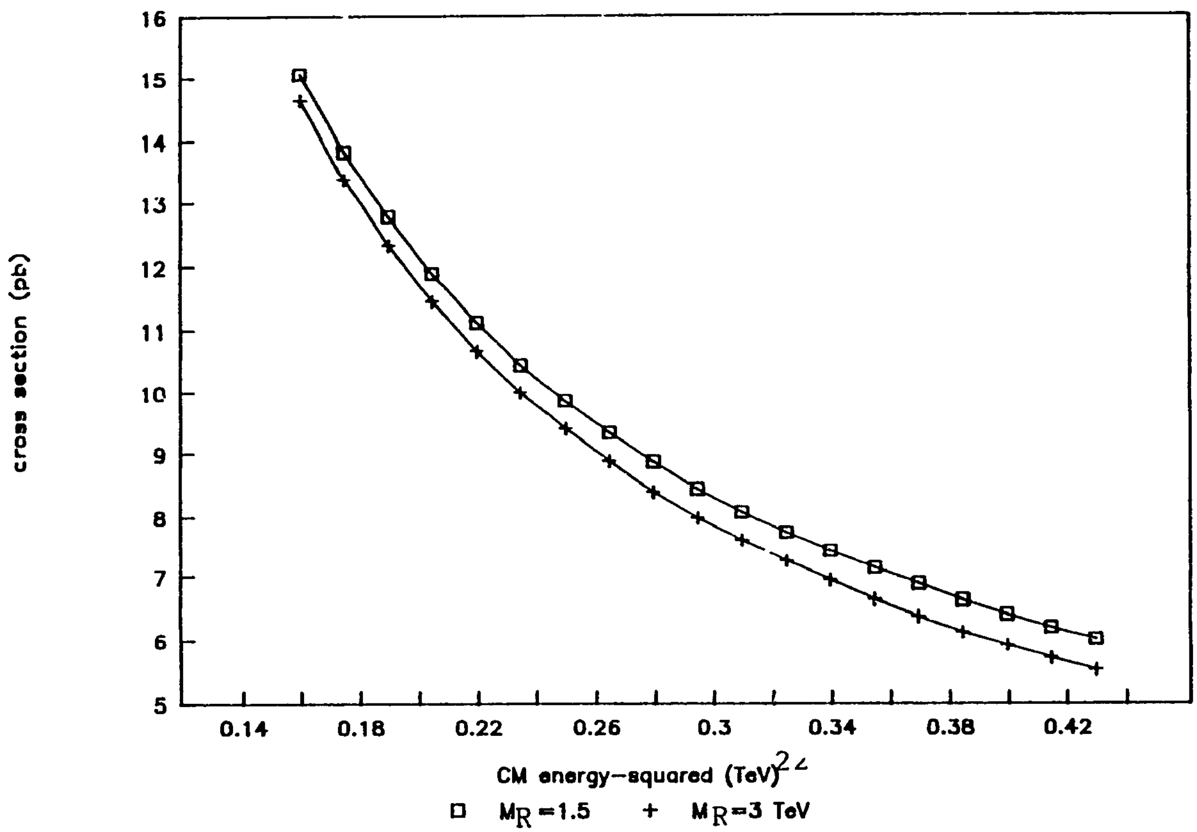
Pair-production cross section vs. the Higgs mass for the δ_L^0 field

Fig.5.4



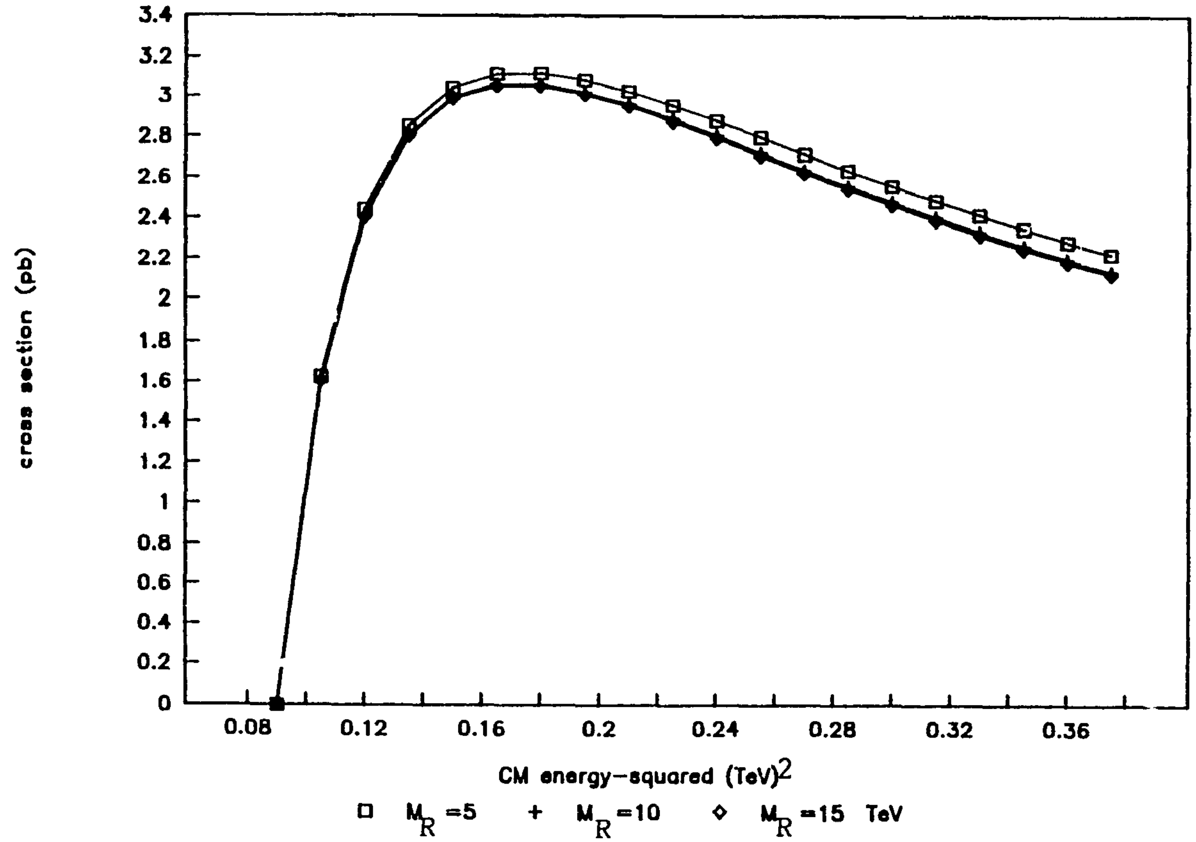
Pair-production cross section vs. CM energy-squared for the δ_L^{++} field

Fig.5.5



Pair-production cross section vs. CM energy-squared for the δ_L^+ field

Fig.5.6



Pair-production cross section vs. CM energy-squared for the δ_L^0 field

CONCLUSION

In this thesis the Higgs sector of the minimal left-right symmetric model (MLRSM) has been studied in some detail. In Chapter 3 it was found that although the Higgs potential in MLRSM contains many more parameters than the one of the Standard Model, it is still possible to discuss mass ranges and vacuum expectation values of these fields with an eye on the low-energy phenomenological data. Due to the many parameters appearing in the Higgs potential, the analysis remains incomplete.
Therefore one must calculate new quantities which can be measured in the forthcoming very high-energy experiments in the TeV range. Thus in Chapter 4 we have produced the (renormalization group improved) hadronic decay widths of the MLRSM Higgs fields. In Chapter 5 the scattering cross section for the production of Higgs fields through e^+e^- annihilation was studied. Wide mass-ranges and center-of-momentum energies have subsequently been used to graph the results for the decay widths and scattering cross sections. These were done in the hope that new data from future very high-energy experiments would help us test the validity of the theory; and if the theory turns out to be valid, we could further restrict the ranges of certain parameters to narrow down the arbitrariness in choosing them.

More studies with respect to the Higgs sector of the MLRSM must be done to widen our understanding of the structure of the theory. For instance, a great deal can be learned through cosmological and astrophysical considerations. In particular, to make contact with the real world, MLRSM must be thoroughly reconsidered within the framework of finite-temperature field theories. Also, new phenomenological and astrophysical data on the phenomenon of CP-violation , both in the hadronic and leptonic sector could provide valuable clues for more realistic investigations.

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APPENDIX A

NOTATION AND CONVENTIONS

A.1 Units

Throughout this thesis, unless otherwise stated, the natural units $\hbar = c = 1$ have been used, where $\hbar = h/2\pi$ with h the Planck constant and c the velocity of light in vacuum. With 1 eV = 1.602×10^{-12} erg, 1 GeV = 10^3 MeV = 10^6 keV = 10^9 eV, and 1 b (barn) = 10^{-24} cm^2 and 1 mb = 10^{-3} b, one has 1 f = 10^{-13} cm, 1 mb = $(\hbar c)^2/0.624\ (GeV)^2$

A.2 Metric and Four-vectors

The metric in Minkowski space { x^μ: μ = 0,1,2,3 } is given by $g^{\mu\nu}$ with

$$g^{00} = +1 \ , \ g^{11} = g^{22} = g^{33} = -1 \ , \ \text{otherwise} = 0$$

The contravariant vectors of the space-time coordinate and energy-momentum are given by

$$x^\mu = (ct \ , \ \mathbf{r}) \quad , \quad p^\mu = (E/c \ , \ \mathbf{p})$$

where t and **r** are the time and space coordinates respectively and E and **p** are the energy and momentum. The bold-faced symbols represent the three dimensional vectors. The covariant vectors are

$$x_\mu = g_{\mu\nu}x^\nu = (ct\ ,\ -\mathbf{r})$$

$$p_\mu = g_{\mu\nu}p^\nu = (E/c\ ,\ -\mathbf{p})$$

and hence

$$p\cdot x \equiv p^\mu x_\mu = g_{\mu\nu}p^\mu p^\nu = Et - \mathbf{p}\cdot\mathbf{r}$$

Here it is understood that repeated indices are summed (Einstein's notation). The contravariant vector of space and time differentiation is defined by

$$\partial^\mu \equiv \frac{\partial}{\partial x_\mu} = \left(\frac{\partial}{\partial t}\ ,\ -\nabla \right)$$

A.3 Dirac Matrices

The Dirac gamma matrices $\gamma^\mu = (\gamma^0\ ,\ \gamma^i)$ satisfy the relation

$$\{\gamma^\mu\ ,\ \gamma^\nu\} = 2g^{\mu\nu}$$

where $\{A,B\} \equiv AB + BA$. The matrix γ_5 is defined by

$$\gamma_5 = i\ \gamma^0\gamma^1\gamma^2\gamma^3$$

and anticommutes with all γ^μ:

$$\{\gamma_5\ ,\ \gamma^\mu\} = 0$$

he hermitian conjugate of γ^μ is taken to be

$$\gamma^{\mu\dagger} = \gamma^0\gamma^\mu\gamma^0$$

so that according to the defining relation for γ_5 one will have

$$\gamma_5^\dagger = \gamma_5$$

From the above definition of the matrices γ^μ it follows that

$$(\gamma^0)^2 = 1 \qquad , \qquad (\gamma^i)^2 = -1 \qquad , \qquad \gamma_5^2 = 1$$

where the Latin index i denotes spatial indices 1,2,3. In the representation where γ^0 is diagonal, the explicit 4 · 4 form of the gamma matrices reads

$$\gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad , \quad \vec{\gamma} = \begin{pmatrix} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{pmatrix} \quad , \quad \gamma_5 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

where 1 and 0 are the 2 · 2 unit and zero matrices respectively and $\vec{\sigma} = (\sigma_1 , \sigma_2 , \sigma_3)$ with σ_i (Pauli matrices) given by

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad , \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad , \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

APPENDIX B

QCD RENORMALIZATION GROUP EQUATIONS (RGE)

B.1 The QCD Lagrangian

Strong interactions are described by a local non-Abelian gauge theory of quarks and gluons. SU(3) is the gauge group and gluons are the gauge bosons. Three colored quarks of each flavor form a triplet in the fundamental representation of SU(3) and eight gluons form an octet in the adjoint representation (defined to have the same dimension as the group). The QCD Lagrangian is then given by

$$L = -\frac{1}{4} F_a^{\mu\nu} F_{a\mu\nu} + \bar{\psi}_j (i\gamma_\mu D_j^\mu - M_{jk}) \psi_k \qquad \text{(B.1)}$$

where the indices a,j and k refer to color and assume the values a = 1,2,3. The covariant derivative acting on a quark field is

$$D_{jk}^\mu = \delta_{jk} \partial^\mu + ig\ (T_a)_{jk}\ G_a^\mu \qquad \text{(B.2)}$$

where G_a^μ are the gluon fields, T_a are the SU(3) generators, and g is the strong coupling constant; M_{jk} is the quark mass matrix. The gluon field tensor is

$$F_a^{\mu\nu} = \partial^\mu G_a^\nu - \partial^\nu G_a^\mu - g\ f_{abc}\ G_b^\mu G_c^\nu \qquad \text{(B.3)}$$

where f_{abc} are the structure constants of SU(3), defined by the commutation relations among the SU(3) generators

$$[T_a, T_b] = if_{abc}T_c \qquad (B.4)$$

The f_{abc} are antisymmetric ler the interchange of any two indices and are given by

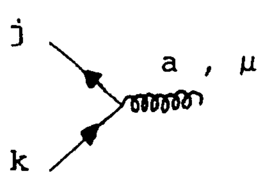
$$f_{123}=1 \quad , \quad f_{147} = f_{246} = f_{257} = f_{345} = f_{516} = f_{637} = \frac{1}{2}$$
$$f_{458} = f_{678} = \frac{\sqrt{3}}{2} \qquad (B.5)$$

The Lagrangian (B.1) is invariant under the infinitesimal local gauge transformations

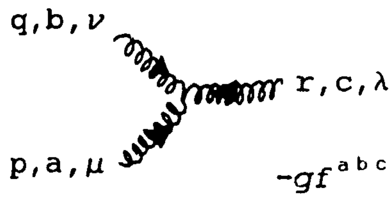
$$\psi(x) \longrightarrow [\ 1 - ig\ \alpha_a(x)\ T_a]\ \psi(x)$$

$$G_a^{\mu}(x) \longrightarrow G_a^{\mu}(x) + \partial^{\mu}\alpha_a(x) + g\ f_{abc}\ \alpha_b(x)\ G_c(x)$$

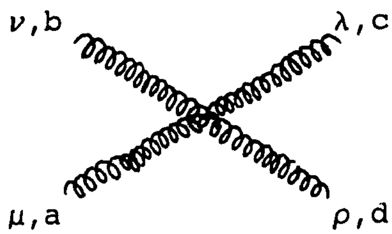
The Feynman rules corresponding to the Lagrangian (B.1) are as follows



$$-ig\gamma^{\mu}(T_a)_{jk} \qquad (B.5)$$



$$-gf^{abc}[(p-q)_\nu g_{\lambda\mu}+(q-r)_\lambda g_{\mu\nu}+(r-p)_\mu g_{\nu\lambda}]$$
$$\text{where } p+q+r=0 \qquad \text{(B.6)}$$



$$\begin{aligned}-ig^2[f^{abc}f^{cde}&(g_{\mu\lambda}g_{\nu\rho}-g_{\mu\rho}g_{\nu\lambda})\\ +f^{ace}f^{dbe}&(g_{\mu\rho}g_{\nu\lambda}-g_{\mu\nu}g_{\lambda\rho})\\ +f^{ade}f^{bce}&(g_{\mu\nu}g_{\lambda\rho}-g_{\mu\lambda}g_{\nu\rho})\end{aligned}$$

(B.7)

In QCD calculations the following identities are frequently used

$$\{T^a,T^b\}=\frac{1}{3}\delta^{ab}+d^{abc}T^c$$

$$T^aT^b=\frac{1}{2}\left(\frac{1}{3}\delta^{ab}+d^{abc}T^c+if^{abc}T^c\right)$$

$$\text{Tr } T^a=0\ ,\ \text{Tr } T^aT^b=0$$

$$\text{Tr } T^aT^bT^c=\frac{1}{4}[d^{abc}+if^{abc}]\ ,\ \text{Tr } T^aT^bT^aT^c=-\frac{1}{12}\delta^{bc}$$

$$f^{abb}=0\ ,\ f^{acd}f^{bcd}=3\delta^{ab} \qquad \text{(B.8)}$$

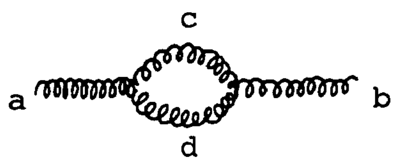
The f^{abc} are antisymmetric under interchanges of any two indices; the d^{abc} are symmetric. The non-vanishing d^{abc} are

$$d^{118} = d^{228} = d^{338} = d^{888} = \frac{1}{\sqrt{3}}$$

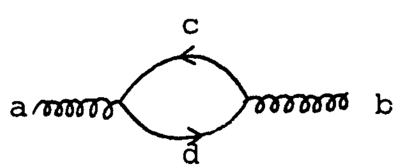
$$d^{146} = d^{157} = d^{256} = d^{344} = d^{355} = -d^{247} = -d^{366} = -d^{377} = \frac{1}{2}$$

$$d^{448} = d^{558} = d^{668} = d^{778} = -\frac{1}{2\sqrt{3}} \qquad \text{(B.9)}$$

From (B.4-5) and (B.8-9) the color (group theoretic) factors corresponding to the following loops are



$$\sum_{c,d=1}^{8} f_{acd} f_{bcd} = C_2(G)\ \delta_{ab}$$



$$\sum_{i,j=1}^{3} (T_a)_{ij} (T_a)_{ij} = \mathrm{Tr}(T_a T_b)$$
$$= T(R)\ \delta_{ab}$$
(B.10)

where R denotes the fundamental representation. For G = SU(3) and for a flavor SU(N_f) group, the color factors are

$$C_2(G) = 3 \quad , \quad C_2(R) = \frac{4}{3} \quad , \quad T(R) = \frac{1}{2} N_f \qquad \text{(B.11)}$$

Additional Feynman rules for ghost contributions (that cancel unphysical degrees of freedom in loop diagrams) are not included here since they will not be needed in what follows.

B.2 The Renormalization Group Equations (RGE)

In evaluating Feynman diagrams that contain loops, divergent integrals over loop momenta occur. To make sense of these quantities, the divergent expressions are first made "temporarily finite" by some *regularization* procedure which introduces additional parameters (e.g., a gluon mass m_g , an ultraviolet cut-off Λ, or a fractional space-time dimension $D=4-2\varepsilon$). In this way the divergences of perturbation theory are re-expressed in a well-defined way (though still with divergent limits). These regularized divergences of perturbation theory are then removed by absorbing them into the definition of physical quantities through a *renormalization* procedure[1]. This is done by some specified (but arbitrary) prescription which introduces a new dimensional scale μ in the theory. Renormalized quantities in the theory, such as mass m and the basic vertex coupling strength g, will now depend explicitly on μ. Different renormalization prescriptions with different μ must all lead to the same observable amplitudes. The μ-dependent transformations of renormalized operators form a Lie group,as first recognized by Stuckelberg and Peterman and named by them the *Renormalization Group*. The equations that express the invariance of the underlying physics under changes of the parameter μ are known as the *Renormalization Group Equations* (RGE).

Renormalization is done on the sum of connected Feynman diagrams with external propagators removed (including their self-energy parts). In more technical terms, one deals with one-particle-irreducible Green's functions Γ which cannot be disconnected by cutting any single internal line. One way to control divergences in Γ is to introduce an ultraviolet cut-off Λ in the loop momentum integrals, thus obtaining unrenormalized Green's functions $\Gamma_U(p_i, g_0, \Lambda)$ where p_i denotes external particle momenta and g_0 is the basic vertex coupling in the Lagrangian. For *renormalizable* field theories, such as QED and QCD, it is possible to define renormalized Green's functions Γ_R by

$$\Gamma_R(p_i, g, \mu) = Z_\mu(g_0, \Lambda/\mu)\ \Gamma_U(p_i, g_0, \Lambda) \tag{B.12}$$

which are finite in the $\Lambda \longrightarrow \infty$ limit but depend on the *subtraction point* (prescription parameter) μ and a renormalized coupling g. Z_μ is a product of factors Z_i, one for each external particle i of the Green's function Γ.

Since Γ_U does not depend on μ, one obtains after differentiation

$$\frac{d\Gamma_U}{d\mu} = \frac{d}{d\mu}\left[Z_\mu^{-1}\ \Gamma_R\right] =$$

$$\frac{\partial Z_\Gamma^{-1}}{\partial \mu}\Gamma_R + Z_\mu^{-1}\left(\frac{\partial}{\partial \mu} + \frac{\partial g}{\partial \mu}\frac{\partial}{\partial g}\right)\Gamma_R = 0 \tag{B.13}$$

which is usually written

$$\left(\mu \frac{\partial}{\partial\mu} + \beta(g) \frac{\partial}{\partial g} + \gamma_\Gamma \right) \Gamma_R(p_i,g,\mu) = 0 \qquad \text{(B.14)}$$

where the *beta function* $\beta(g)$ and the *anomalous dimension* $\gamma(g)$ have been defined by

$$\beta = \mu \frac{\partial g}{\partial\mu} \quad , \quad \gamma_\Gamma = \frac{\mu}{Z_\mu} \frac{\partial Z_\Gamma}{\partial\mu} \qquad \text{(B.15)}$$

Here, Λ is held constant in the differentiation and subsequently the limit $\Lambda \longrightarrow \infty$ is taken. The beta fuction is universal (due to the universality of gauge interactions), but the γ function depends on the Green's functions, that is, on the wave function renormalizations of the external particles. If Z_Γ is expressed as a product of renormalization factors (such as those of the wave function and the vertex coupling), then γ may be expressed as the sum of corresponding contributions.

Now consider the case in which there is a single large momentum scale Q. All momenta p_i can then be expressed as fixed fractions α_i of Q. Introducing the variable

$$t = \frac{1}{2} \ln \left(\frac{Q^2}{\mu^2} \right) \qquad \text{(B.16)}$$

one has

$$d\mu = -Q\, e^{-t} dt \Rightarrow \mu \frac{\partial}{\partial\mu} = - \frac{\partial}{\partial t} \qquad \text{(B.17)}$$

and hence the RGE (B.14) can be re-expressed as

$$\left(- \frac{\partial}{\partial t} + \beta(g)\frac{\partial}{\partial g} + \gamma_\Gamma(g) \right) \Gamma(t,g,x_i) = 0 \qquad \text{(B.18)}$$

Defining a *running coupling* $\bar{g}(t)$ by the integral equation

$$t = \int_{g=\bar{g}(g,0)}^{g(t)=\bar{g}(g,t)} \frac{dg'}{\beta(g')}. \qquad \text{(B.19)}$$

the general solution to RGE (B.18) can be written as

$$\Gamma(t\ ,\ \bar{g}(g,0)\ ,\ x_i) = \Gamma(0\ ,\ \bar{g}(g,t)\ ,\ x_i)\ \exp \int_g^{g(t)} dg' \frac{\gamma_\Gamma(g')}{\beta(g')} \qquad \text{(B.20)}$$

which can be checked directly by noting that differentiating both sides of (B.19) with respect to t and g gives respectively

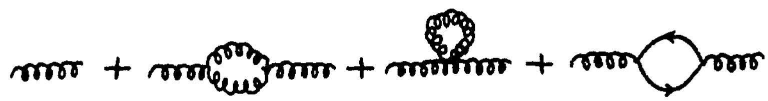
$$\beta(g(t)) = \frac{\partial g(t)}{\partial t} \qquad \text{(B.21)}$$

$$\beta(g(t)) = \beta(g)\ \frac{\partial g(t)}{\partial t} \qquad \text{(B.22)}$$

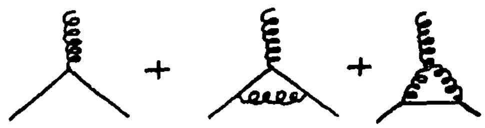
Equation (B.20) is a very important result. It shows that the whole Q^2 dependence of Γ arises through $g(t)$.

B.3 The running coupling

In practice Γ, γ_Γ and β can only be calculated in perturbation series in the coupling $g = \bar{g}\,(g,0) = g(0)$. Let us denote by $\Gamma^{n,m}$ the renormalized truncated QCD Green's functions with n gluon and m quark external legs. The lowest order diagrams contributing to $\Gamma^{2,0}$, $\Gamma^{0,2}$ and $\Gamma^{1,2}$ are

$\Gamma^{2,0}=$ 

$\Gamma^{0,2}=$ 

$\Gamma^{1,2}=$ 

The perturbative expansions to order g^2 are

$$\Gamma^{2,0}= (p_\mu p_\nu - g_{\mu\nu}p^2)\left\{1+\left[\frac{13}{6}\ C_2(G) - \frac{4f}{3}T(R)\right] \frac{g^2}{16\pi^2}\ \ln\left(\frac{-p^2}{\mu^2}\right) \tag{B.23}$$

$$\Gamma^{0,2}= p + O(g^4) \tag{B.24}$$

$$\Gamma^{1,2}= g\gamma_\mu T_1\ \left[1 - \frac{3}{4}\ C_2(G)\ \frac{g^2}{16\pi^2}\ \ln\left(\frac{-p^2}{\mu^2}\right)\right. \tag{B.25}$$

with incoming momentum configurations (p,-p) for the two-leg and (0,-p,p) for the three-leg cases normalized at $p^2 = -\mu^2$, where μ is the arbitrary subtraction point. In (B.23-25) the Landau gauge has been used, in which the gluon propagator is $-i\delta_{ab}(g_{\mu\nu}p^2 - p_\mu p_\nu)/p^4$. The coefficient f is the number of flavors and the group theoretical factors $C_2(G)$ and T(R) were given in (B.11).

Applying the RGE to the (truncated) Green's functions in (B.23-25) gives

$$\left(\mu \frac{\partial}{\partial \mu} + \beta \frac{\partial}{\partial g} + 2\gamma_G \right) \Gamma^{2,0} = 0 \qquad \text{(B.26)}$$

$$\left(\mu \frac{\partial}{\partial \mu} + \beta \frac{\partial}{\partial g} + 2\gamma_Q \right) \Gamma^{0,2} = 0 \qquad \text{(B.27)}$$

$$\left(\mu \frac{\partial}{\partial \mu} + \beta \frac{\partial}{\partial g} + 2\gamma_Q + \gamma_G \right) \Gamma^{1,2} = 0 \qquad \text{(B.28)}$$

where γ_G and γ_Q are the contributions due to each gluon and quark leg.

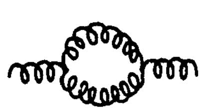
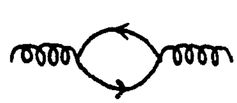
Then by using (B.23-25) in (B.26-28) it can be easily verified that (in the Landau gauge)

$$\gamma_G = \left[\frac{13}{6} C_2(G) - \frac{4f}{3} T(R) \right] \frac{g^2}{16\pi^2} + O(g^4) \qquad \text{(B.29)}$$

$$\gamma_Q = 0 + O(g^4) \qquad \text{(B.30)}$$

$$\beta = - \left[\frac{11}{3} C_2(G) - \frac{4f}{3} T(R) \right] \frac{g^3}{16\pi^2} + O(g^5) \qquad \text{(B.31)}$$

This result shows that $\beta(g)$ arises from the loop diagrams

and its expansion in powers of g begins, in general, as

$$\beta(g) = -bg^3(1 + b'g^2 + \dots) \qquad \text{(B.32)}$$

where, for the QCD case

$$b_{QCD} = \frac{1}{48\pi^2}(33 - 2f) \tag{B.33}$$

In the one-loop approximation $\beta(g) \simeq - bg^3$ which together with (B.20) yields, after integration,

$$g^2(t) = \frac{g^2(0)}{1 + 2bg^2(0)t} \tag{B.34}$$

Similarly, the two-loop approximation (which is the next-to-leading-logarithm approximation) yields

$$\frac{1}{g^2(t)} - \frac{1}{g^2(0)} + b'\ln\left[\frac{b' + 1/g^2(0)}{b' + 1/g^2(t)}\right] = 2bt \tag{B.35}$$

From (B.34) it follows that with $f \le 16$ flavors b_{QCD} is positive and $g^2(t) \longrightarrow 0$ as $t \longrightarrow \infty$. This important result, that the running coupling constant going to zero as $Q^2 \longrightarrow \infty$ is known as *asymptotic freedom* and allows RGE-improved perturbative calculations at large Q^2 (short distances). It is convenient to use

$$\alpha_s(Q^2) = \frac{1}{4\pi b\ \ln(Q^2/\Lambda^2)} = \frac{12\pi}{(33 - 2f)\ \ln(Q^2/\Lambda^2)} \tag{B.37}$$

The parameter Λ (which has nothing to do with the renormalization cut-off!) is a fixed parameter and is to be determined from experiment. The theory is applicable only for $Q^2 >> \Lambda^2$ for which α_s is small. The number f of participating flavors depends on Q^2; in general, a quark i of mass m_i is expected to contribute to the loops only when $|Q^2| \geq 4m_i^2$.

As introduced here in the leading logarithmic approximation context, Λ has a precise operational meaning (it fixes $\alpha_s(Q^2)$ with which we calculate) but its theoretical basis is somewhat subtle. If we added a non-leading logarithmic term, by changing the denominator factor from $\ln(Q^2/\Lambda^2)$ to $\ln(Q^2/\Lambda^2) + X$, it would be equivalent to changing $\Lambda \longrightarrow \Lambda' = \Lambda \exp(-1/2\ X)$. Thus if we determine Λ from a particular experiment, using leading logarithmic formulae, the resulting empirical value of Λ parametrizes, to some extent, the non-leading terms that we have neglected. These non-leading terms arise from the higher-loop contributions to $\beta(g)$ and $\gamma(g)$. Although $\beta(g)$ is universal, the relevant $\gamma(g)$ depends on the experimental measurements in question. Hence the empirical value of Λ found from data fitting with leading logarithmic approximate formulae will depend in general on the class of data being used. To determine whether these different values of Λ are consistent with a common QCD interpretation, one must consistently include non-leading logarithmic terms in the formulae. In QCD the second coefficient b′ is given by

$$b' = (102 - 38f/3)/(16\pi^2)^2 \qquad \text{(B.38)}$$

and the two-loop correction to $\alpha_s(Q^2)$ can be written as

$$\alpha_s(Q^2) = \frac{1}{4\pi b \ln(Q^2/\Lambda^2)} \left\{ 1 - \frac{b' \ln[\ln(Q^2/\Lambda^2)]}{b \ln(Q^2/\Lambda^2)} \right\} \quad \text{(B.39)}$$

In QED there is no analog to the gluon loops, so $C_2(G) = 0$ and $T(R) = \Sigma e_i^2$, summed over all fermions (with charge e_i) appearing in the fermion loops. Hence

$$b_{QED} = -\frac{1}{12\pi^2} \sum e_i^2 \quad \text{(B.40)}$$

This concludes our RGE preliminaries in this appendix. More details can be found in Ref.1.