Power System Generation Expansion Optimization Using a Probabilistic Screening Curves-Branch and Bound Hybrid

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ABSTRACT

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Two new methods of determining long term power system generation expansion scenarios using the limited capabilities of the personal (micro-) computer are presented. Both methods analyze hydro and thermal expansion candidates and efficiently determine near-optimal yearly expansion scenarios. They are based on conventional Screening Curves techniques which are modified to incorporate probabilistic production costing and integer unit additions. The first method, Probabilistic Screening Curves, is used for screening analysis as it is capable of analyzing large numbers of expansion candidates. The second method, Probabilistic Screening Curves - Branch and Bound hybrid, refines the Probabilistic Screening Curves' final scenario using Branch and Bound optimization techniques. The Probabilistic Screening Curves - Branch and Bound hybrid also provides suboptimal plans for sensitivity analysis aiding in the determination of globally optimal expansion sequences. These optimization methods are fast, efficient and well suited to the analysis of large systems within the limitations of the

personal computer. The capabilities of the methods are demonstrated by analysis of actual power system study results.

An extensive survey of optimization methods currently in use and the classification of system modelling, costing and expansion papers are also included.

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TABLE OF CONTENTS

Chapter	Page
1. INTRODUCTION	1
1.1 Motivation	1
1.2 Objectives	2
1.3 Outline	3
2. POWER SYSTEM GENERATION EXPANSION PROBLEM	5
2.1 Introduction	5
2.2 Booth-Baleriaux Formulation	7
2.3 Load Demand Model	8
2.4 Equivalent Load Duration Curve	11
2.5 Power System Reliability Measurements	17
2.6 Costing	20
2.7 Computer Implementation	25
2.8 Generation Expansion Program Capabilities	29
3. GENERATION EXPANSION TECHNIQUES	36
3.1 Introduction	36
3.2 Screening Curves	38
3.3 Linear Programming	44
3.4 Generalized Bender's Decomposition	49
3.5 Generalized Network	54
3.6 Dynamic Programming	60
3.7 Gradient Projection	66
3.8 Heuristics	69
3.9 Optimization Technique Comparison	71
4. PROBABILISTIC SCREENING CURVES OPTIMIZATION TECHNIQUE	78
4.1 Introduction	78
4.2 Screening Curves Optimization Basis Selection	79
4.3 Deterministic Screening Curves Optimization	81
4.4 Probabilistic Screening Curves Optimization	98
4.5 Branch and Bound Optimization	105
4.6 Test Cases and Results	108
4.7 Hydro Units	120
4.8 Hydro Test Cases	123
4.9 Global Expansion Sequence Determination	132
4.10 Summary	135
5. CONCLUDING REMARKS	137
REFERENCES	139
Appendix	
A. Maintenance Modelling Comparison	144
B. Optimization Software Package Comparison	148

Appendix	Page

C. Power System Generation Expansion Planning Bibliography 162
Bibliography 166

LIST OF FIGURES

Figure		Page
2.1	Load Demand Model	10
2.2	Graphic Illustration of Convolution	13
2.3	Probabilistic Method Production Costing	14
2.4	Probabilistic Versus Simplified Analysis	16
2.5	Loss of Load Probability	19
3.1	Optimal Mix - New Units Only	40
3.2	Optimal Mix - 3 Existing Units	42
3.3	Network Representation of Production Costing	58
3.4	Forward Dynamic Programming	63
4.1	Optimal Mix Algorithm - New Units Only	86
4.2	Cases in Which Some Units do Not Participate in Optimal Mix Solution	90
4.3	Positioning Existing Units In Optimal New Unit Mix	92
4.4	Existing Multi-Unit in Optimal New Unit Mix	94
4.5	Inefficient Existing Multi-Unit - Positioning	95
4.6	Probabilistic Screening Curves - Determining New Unit Capacity	101
4.7	Thermal Comparison - Screening Curves and System LDC	112
4.8	Thermal Comparison - Total System Cost	116
4.9	Thermal Comparison - Total System Capacity	117
4.10	Thermal Comparison - Installed Coal	118
4.11	Thermal Comparison - Installed Gas Turbines	119
4.12	Hydro Comparison - Screening Curves	124
4.13	Hydro Comparison - Total System Cost	126
4.14	Hydro Comparison - Total System Capacity	127
4.15	Hydro Comparison - Installed Hydro	128
4.16	Hydro Comparison - Installed Coal	129
4.17	Hydro Comparison - Installed Gas Turbines	130
4.18	Hydro Comparison - New Unit Installations	131
4.19	Globally Optimized Hydro Scenario	134

LIST OF TABLES

Table		Page
2.1	Power System Generation Expansion Criteria	35
3.1	Review of Optimization Methods - Production Costing	72
3.2	Review of Optimization Methods - Investment Costing	73
4.1	Power System Used For Expansion Testing	111
A.1	Comparison of Maintenance Methods	146
B.1	Comparison of Optimizing Expansion Programs	149
C.1	Classification of References Listed in Bibliography	164

SYMBOL DEFINITIONS

α_i	- Dual variable of the existing units (4.3.6).
τ	- Normalized time [per unit].
τ_i	- Breakeven time [per unit].
A	- Capacity availability and system reliability constraint matrix.
B	- Production costing constraint matrix.
b_{it}	- variable cost of unit i in year t [$/MW-year].
BQ	- Variable costs [$].
bq_t	- Variable cost of year t [$].
$\underline{b}_t$	- Variable costs for year t [$/MW year].
C	- Fixed costs of units [$/MW].
cf_{it}	- Capacity factor of unit i in year t [p.u.].
cp_i	- Plant costs of unit i [$/MW] (=0 $i \epsilon I$).
$\underline{c}_t$	- Fixed costs for year t [$/MW year] (=0 $i \epsilon I$).
$\bar{D}$	- Total capacity of EMU [MW].
$\hat{D}$	- Loading point of EMU [MW].
$\bar{D}_i$	- Total capacity of EMU up to existing unit i [MW].
D_i	- Loading level of unit i [MW].
$D_{m,n}$	- Total system capacity in interval m [MW].
E	- Constraint equalities/inequalities.
ELDC	- Equivalent load duration curve.
$ELDC_i(x)$	- Equivalent Load Duration Curve at capacity x after unit i has been loaded [probability].
$ELDC_m$	- Final Equivalent Load Duration Curve in interval m.
EUE_{ann}	- Annual EUE [GWh].

EUE_m - Expected Unserved Energy in interval m [GWh].

EUE_t - Expected unserved energy for year t [MW - year].

F - Capacity availability and system reliability constraints.

G - Production costing constraint matrix.

H - Production costing constraint matrix.

I - Set of existing units.

$ILDC(x)$ - Inverted load duration curve.

i_{rate} - Interest rate [per unit] ie. 10%/100.

$LDC(\tau)$ - Load duration curve.

L_t^m - System peak demand in year t [MW].

$LOLP_{ann}$ - Annual LOLP.

$LOLP_m$ - Loss of Load Probability in interval m [days/interval or hours/interval].

n - Number of units in the system (new and existing).

n_{int} - Number of intervals per year.

npl_i - Plant life [intervals or years].

OTC - Objective function of complete simulation period [$].

p_i - $(1 - q_i)$ [per unit value].

Ω_i - Energy generated by unit i [MW].

q_i - Forced Outage Rate (FOR) [per unit value].

$\bar{r}$ - Index of the first existing unit in the EMU.

$\hat{r}$ - Index of the new unit directly before the first unit of the EMU.

$\bar{s}$ - Index of the last existing unit in the EMU.

$\hat{s}$ - Index of the new unit directly following the last unit of the EMU.

T - Number of years of simulation.

TC_t - Total cost in year t [$].

T_m - Time represented by interval m [days or hours].

u - Dual variable of the peak demand constraint of (4.3.2).

u_i - Dual variable of the non-negativity constraint of (4.3.5).

X - New capacity (Investments) [MW].

x_i - Capacity of new unit i [MW].

$x_{i,t}$ - Capacity of unit i in year t [MW].

Y - Maximum capacity of all units [MW].

y_i - Maximum capacity of unit i [MW].

Y_i - Generating unit's capacity [MW].

$Y_{i,t}$ - Capacity of existing unit i (MW).

$Y_{m,j}$ - Available capacity of unit j in interval m [MW].

Chapter 1.

INTRODUCTION

1.1 Motivation

With the ever increasing demand for electricity and escalating supply costs, long term planning of electrical power system generation is becoming more and more important. Power system generation planning typically considers a time frame of 20-30 years due to the long lead time required for the installation of new generation units and the high cost of not meeting the system load. Thus, there must be careful planning for projected electrical loads and improving and diversifying generation technologies. Ever increasing demands are being placed on power utilities to perform efficiently and to meet their specific needs.

Long term power system generation planning considers a wide variety of phenomena including; the availability of different types of generation and their associated reliability, the availability of fuel and projected cost increases, natural resources and political considerations. They must all be studied in the context of the existing power system generation [24,32].

Many differing computer simulation tools are available to perform this analysis, but tend to require large amounts of computer time and storage. These computer resources are expensive and with the widespread usage and increased capabilities of personal computers are not always warranted. Due to the uncertainty of long term generation expansion planning and the profound effect of social and political phenomena, such sophisticated and accurate techniques are not always required. Interaction with the analyst, near optimal solutions and ease of usage are far more practical. These can be provided for a fraction of the cost on a personal computer.

The personal computer, using hybrids of existing methodology, can be used to calculate realistic, near-optimal expansion scenarios. Thus expensive mainframe computers can be avoided enabling the analysis of projected system needs and sensitivity analysis to be performed quickly and inexpensively.

This research outlines ~~the~~ development of an optimization tool capable of providing a near-optimal yearly integer hydro-thermal expansion mix using probabilistic system analysis. Suboptimal plans are also provided to aid the analyst in developing realizable global expansion scenarios.

1.2 Objectives

There are three primary objectives of this work. The first is to provide a comprehensive discussion of the criteria that are of importance in power system generation modelling and costing. The second is to compare and contrast existing optimization techniques for long-term generation expansion planning. The merits and capabilities of existing optimization software packages utilizing these techniques are also discussed. The third objective is to present an efficient, near-optimal method of generation expansion planning implemented on a personal computer. The capabilities of this method are then demonstrated by comparing it to existing methods using an actual power system.

1.3 Outline

A description of the electrical system generation expansion problem is outlined in Chapter 2. Differing costing techniques and the assumptions in their formulations are discussed.

Chapter 3 outlines techniques to solve the long-term generation expansion planning problem. Various solution techniques are compared and contrasted.

Finally Chapter 4 presents a near-optimal hybrid of long-term optimization techniques outlined in Chapter 3 suitable for personal computer usage. The method's capabilities are demonstrated by comparison with actual power system study results.

Appendix A compares the results of two techniques used to model generation maintenance.

Appendix B summarizes and compares the capabilities of existing optimization generation expansion planning programs.

Appendix C contains a listing and classifications of papers that deal with the subject of system planning and generation expansion providing details beyond the scope of this work.

Chapter 2.

POWER SYSTEM GENERATION EXPANSION PROBLEM

2.1 Introduction

System planners began utilizing computers to solve power system generation expansion planning problems about twenty five years ago. The computers were used to facilitate the simulation of generation strategies by providing estimates of the production costs and system reliability. The planner could easily postulate various expansion strategies and develop a best plan scenario.

Monte Carlo techniques were first used to model random events inherent in power system generation. More recently explicit probabilistic mathematical techniques have replaced Monte Carlo simulation resulting in large reductions in required computer resources. As larger systems were analyzed and the number of possible alternatives increased, the number of feasible expansion strategies expanded beyond existing practical simulation times. Furthermore, due also to increased utility financial constraints, too much reliance was placed on the analyst. Even the optimality of final scenarios could not be assured. There was an obvious need for better planning tools [15].

Why postulate an expansion plan and then simulate it? Why not have the computer directly arrive at an optimal expansion strategy? Methods of optimization were implemented, but restricted computer capabilities forced unacceptable tradeoffs between retaining reasonable simulation accuracy and ensuring optimal expansion selection. Initially, many formulations were not incorporated due to insufficient computer resources. With increased size and speed of modern computers and advances in mathematical optimization, both efficient formulation and problem solutions are now feasible [15]. (For a more detailed historical development see [5].)

Recently, however, with the widespread usage of personal computers, expensive mainframe time appears less attractive resulting in a reverse evolution. Existing methods are being simplified to operate on smaller personal computers, and still give results that will assist the planner in developing a satisfactory generation expansion plan.

This chapter outlines the different aspects of the Power System Generation Expansion Problem and system modelling used in simulation or expansion computer programs. The first three sections, 2.1 - 2.3, discuss system load modelling and the resultant effect of non-ideal generating units. The following two sections, 2.4 - 2.5, discuss reliability and costing criteria used to assess different expansion scenarios. The implementation of the concepts into a computer program is discussed in the following section 2.6. The final section, 2.7, briefly outlines the capabilities, input criteria and topics handled by generation expansion programs.

2.2 Booth-Baleriaux Formulation

Power System Generation Planning is not analyzed in the same way as day to day scheduling of power system operation. This is primarily due to the long time frame in which a system is studied and the inherent uncertainty of input data. The amount of analysis involved for day to day scheduling would be too great over the 20-30 years normal planning horizon. Thus certain approximations were made and, through testing, have been found to correspond closely with actual system operation. One such method is the Booth-Baleriaux formulation.

In the Booth-Baleriaux formulation the system load is modeled as a random variable and generation units are modeled as power sources with available capacity represented by random variables. Both are considered to be independent variables [5,18].

The Booth-Baleriaux framework provides reasonably accurate estimates of generating costs and system reliability. The time dependence of events is not considered important and therefore, is simplified by subdividing each study year into intervals. The system cost and reliability are affected only by the relative duration or probability of load levels and plant outages.

Although the simulation has the form of a difficult nonlinear optimization, it can be solved using this formulation without resorting to nonlinear programming. The efficiency of the probabilistic simulation algorithm stems from its 'forward' nature. Each plant is considered individually in succession and all relevant calculations are performed as it is loaded. There is no need to reconsider the plant once the unit has been loaded. The algorithm is not a search process, but an efficient step by step procedure [4].

2.3 Load Demand Model

A system load fluctuates with time, and hourly predictions are difficult to make due to their random nature. This is especially true in long term analysis. The time aspect of the chronological load demand curve can be collapsed into a time independent probability distribution that models the system load as a random variable and is called the Inverted Load Duration Curve (ILDC). Load forecasts can be obtained through other existing software packages. (Ex. See [15])

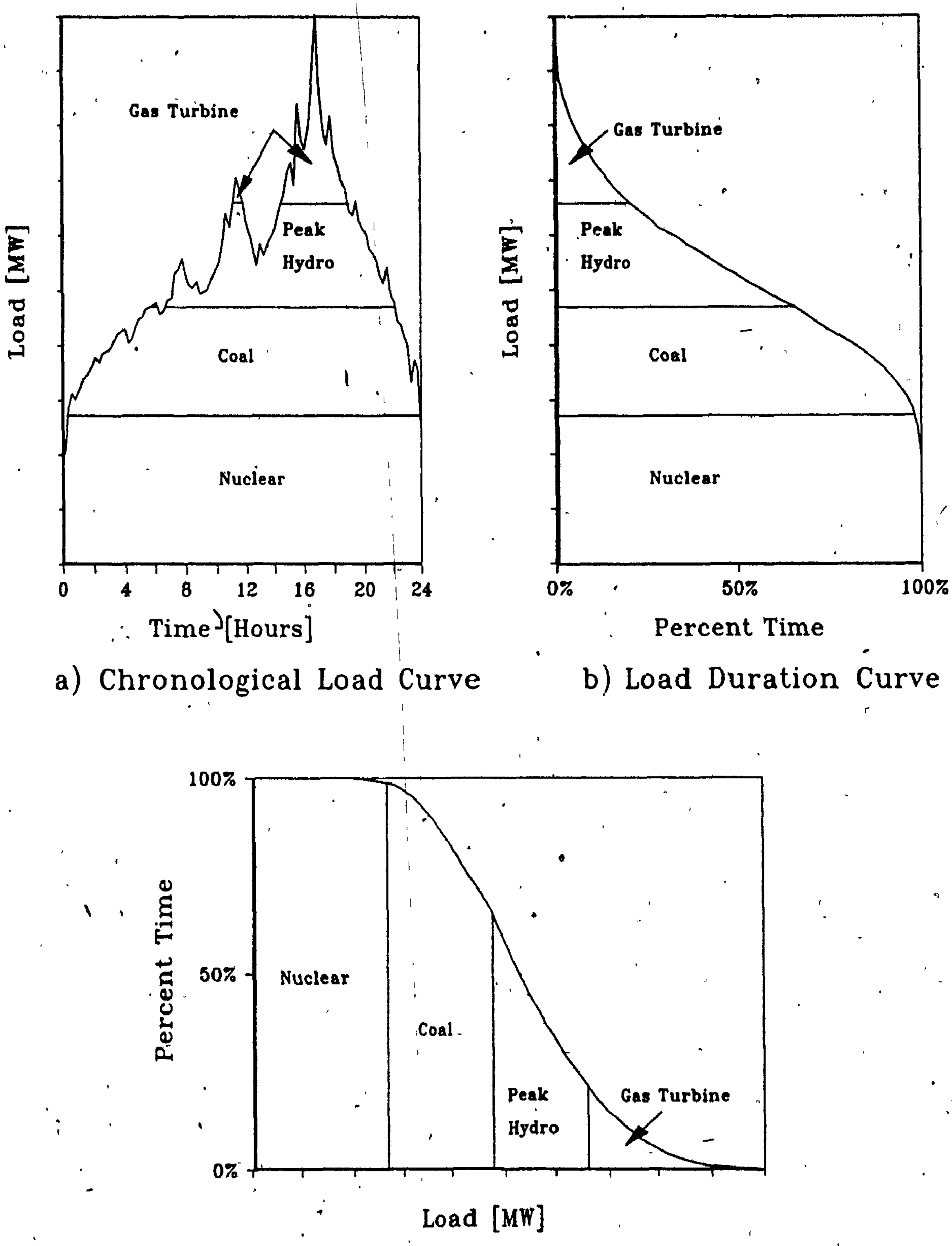
Once the system load forecasts have been derived, the electrical system is modelled as a closed economic market. No consideration is given for potential substitution in other economic sectors. This assumes that the price of electricity has an inelastic relationship with the projected system demand. Analysis has shown this to be a realistic assumption [38].

The Load Duration Curve (LDC) is constructed by sorting the load into increasing hourly values. The LDC is the proportion of time or duration that the load is expected to exceed a certain level. The ILDC is the same as the LDC but with its axis inverted [18,20]. (See Figure 2.1).

If the generating units were 100% reliable, they could meet the load as shown in Figure 2.1. The area of energy represented under each portion of the LDC and ILDC curves is of the same proportion as the original chronological load demand curve. Thus the energy demand is constant despite its change in form.

As generating units are not completely reliable and not all time dependant properties can be ignored, the yearly load is divided into intervals, each with its own representative load duration curve. The required maintenance of generating plants can be represented by having the unit not produce any energy for one or two intervals. If intervals are not used, maintenance outages are approximated by reducing the unit's capacity or increasing the unit's specified rate of random outages (Forced Outage Rate). (See Appendix A for test case comparisons.) Limited energy plants or seasonal dependant generation such as hydro units are modeled by changing the amounts of generating capacity and energy available per interval [18,20,42].

Planned outages are accounted for through the use of intervals, but the model must also account for the random or unexpected outages. These are modelled by the use of the Equivalent Load Duration Curve.



a) Chronological Load Curve

b) Load Duration Curve

c) Inverted Load Duration Curve

Figure 2.1 Load Demand Model.

2.4 Equivalent Load Duration Curve

The random outages of generating units are called Forced Outage Rates (FOR). A more formal definition is the expected fraction of time that the plant is not available for operation. Typical values of FOR are of the order of 15-20% for steam turbine, 15-20% for nuclear, 10-15% for gas turbine and diesel, and 1-5% for hydro-electric generating units. The inability of a generating unit to meet a load 100% of the time results in an increased load for the rest of the operating units. The remaining equivalent demand is the sum of the customer demand and the forced outage rate of the previously loaded units [28].

The outage states for different plants in operation are not treated explicitly since the number of states grows exponentially with the number of plants [5]. Instead the unit's forced outages are modelled probabilistically through convolution and are simplified for digital processing to the following formula [20,42]:

$$ELDC_i(x) = p_i \cdot ELDC_{i-1}(x) + q_i \cdot ELDC_{i-1}(x-Y_i) \qquad i = 1, \ldots, n$$

where

$$ELDC_0(x) = ILDC(x) \tag{2.4.1}$$

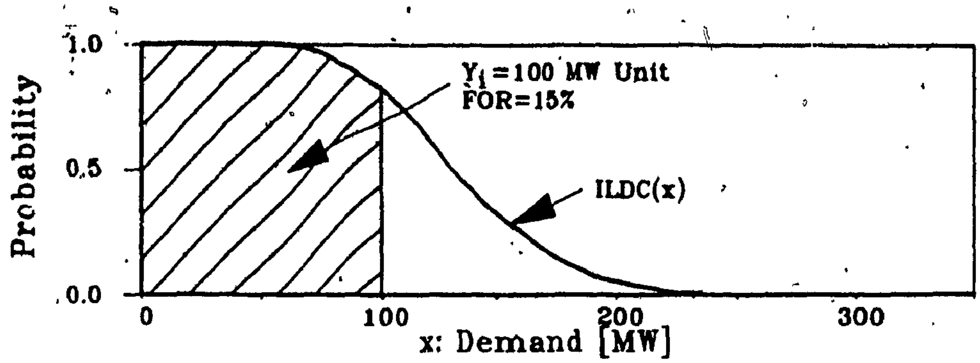
Variables:

$ELDC_i(x)$ - Equivalent Load Duration Curve at capacity x after unit i has been loaded [probability].
x - [MW].
q_i - Forced Outage Rate (FOR) $0 \le q_i \le 1$ [per unit value].
p_i - $(1 - q_i)$ [per unit value].
Y_i - Generating unit's capacity [MW].
n - Number of generating units.

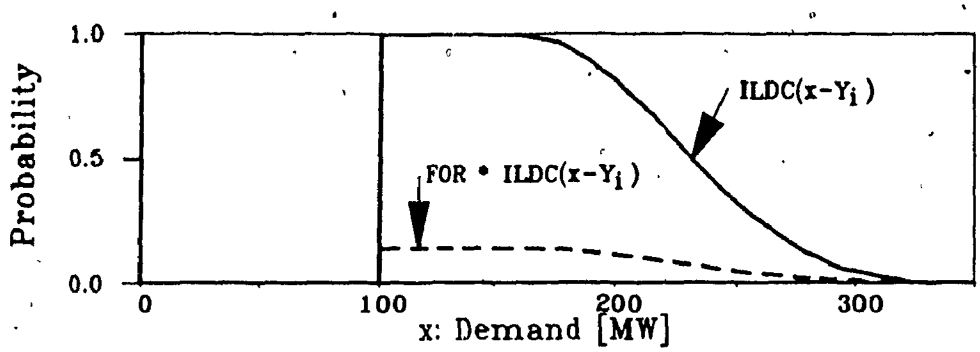
This process is graphically demonstrated in Figures 2.2 and 2.3. (It should be noted that the probability, or percent time axis represents the probability, or the time of this load occurring. A probability of 1.0, or a time of 100% corresponds to the total number of days or hours in that particular interval.)

After a unit has been stacked, the remaining area under the equivalent load duration curve is the energy to be provided by the remaining units. Consecutive application of a number of generating units of this probabilistic simulation results in a sequence similar to that of Figure 2.3. A new curve is computed each time a unit is brought into service. This formulation is for a two state unit model. Multi-state unit modelling is handled in a slightly different fashion, but is beyond the scope of this work. (See [18,20,22] for multi-state methodology).

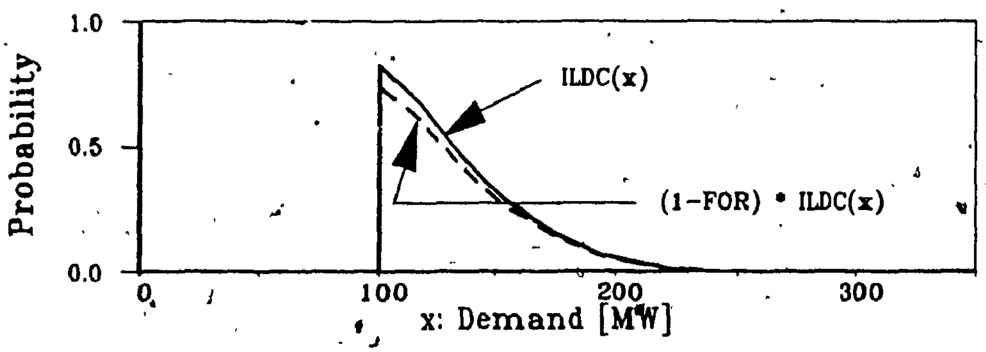
As the number of generating units can be very large, the probabilistic simulation is often avoided in the context of long-term capacity expansion models because of its considerable computational burden. A simplified deterministic approach, similar to Figure 2.1 with the unit's available capacity derated by its forced outage rate, is sometimes used instead. This method is not desirable as it does not represent the random nature of the forced outage rates as in the more realistic equivalent load duration curve method. The difference between a chronological and a load duration curve



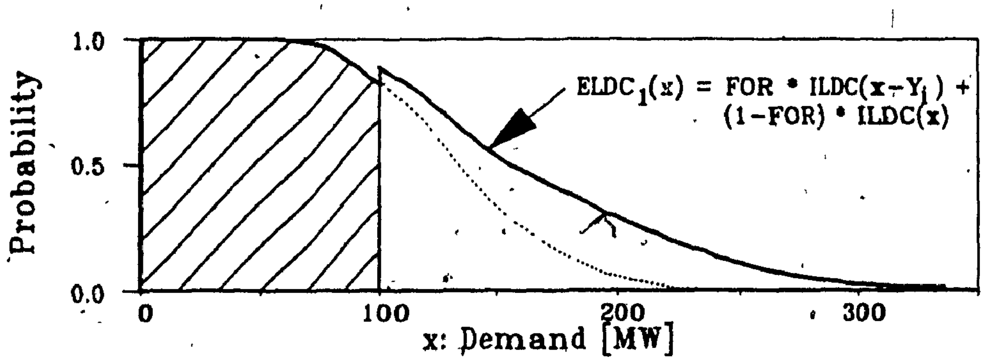
a) Original Inverted Load Duration Curve (ILDC).



b) Load if Unit Fails (15% Probability).

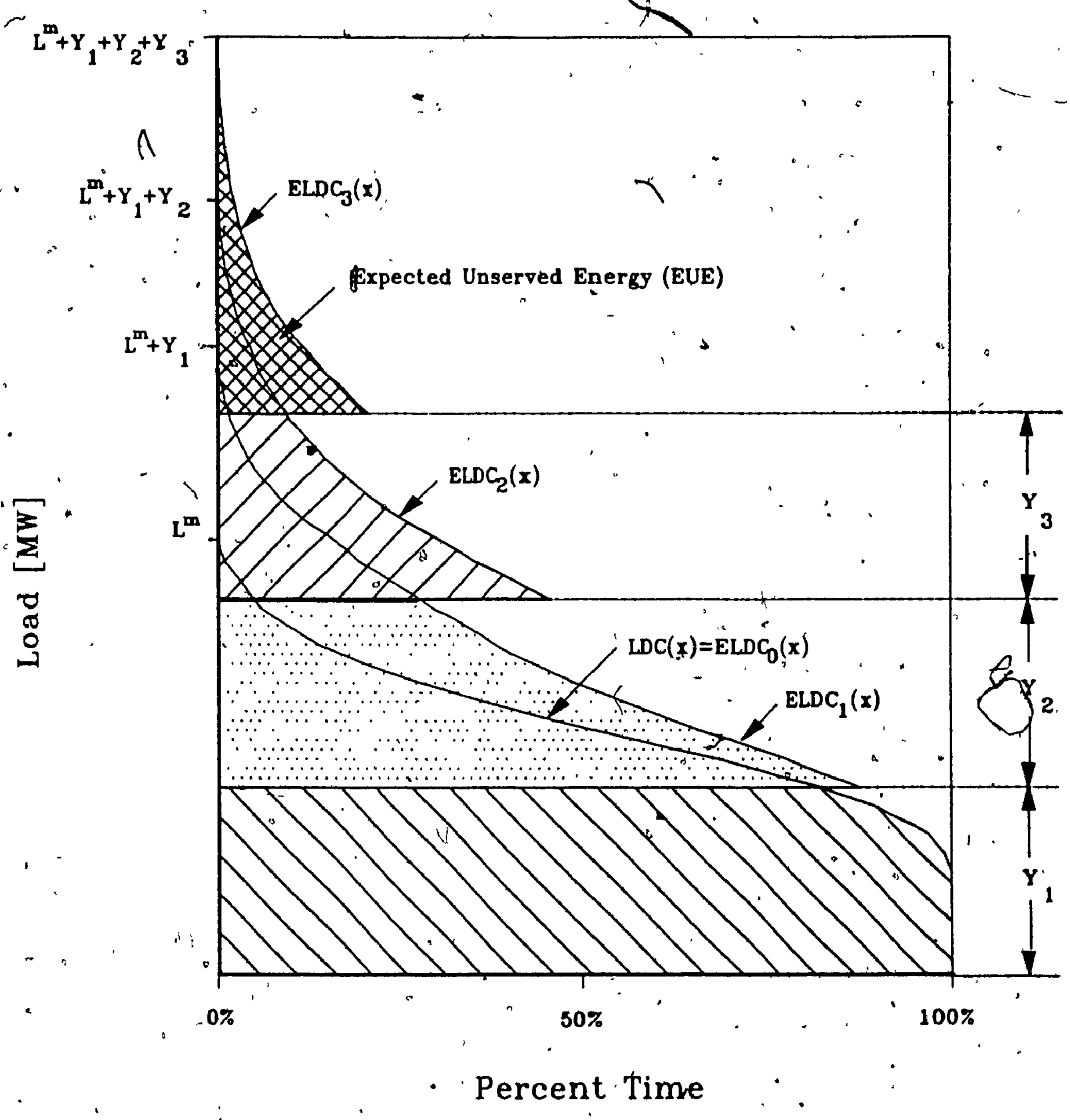


c) Load if Unit Operates (85% Probability).



d) Equivalent Load Duration Curve (ELDC) After Unit Loaded.

Figure 2.2 Graphical Illustration of Convolution.



L^m – Peak Load [MW]. LDC(x) – Load Duration Curve.

Y_i – Capacity of Unit i [MW]. $ELDC_i(x)$ – Equivalent Load Duration Curve.

Figure 2.3 Probabilistic Production Costing Method.

system representation is substantially reduced with the use of probabilistic simulation and interval load divisions [18].

The differences between deterministic and probabilistic simulation can be demonstrated by an example shown in Figure 2.4. Seventy similar typed thermal units (600 MW coal, FOR = 16%) are loaded. The figure shows four load curves. The first curve is the original system load duration curve. The second is the same as the first curve except that it has been divided by the unit's availability (0.84). The third curve is the outer tips of the probabilistic equivalent load duration curve after each unit is loaded. The fourth curve is the actual probabilistic operating range of the thermal units.

The equivalent load duration curve is seen to be a more spiked curve than the original load duration curve. Thus base loaded units operate for shorter periods of time and the peaking units run for longer periods of time. The probabilistic simulation properly determines the unit's operating length, whereas the deterministic production costing reduces the unit's capacity to approximate its forced outage rate. This is important in determining the optimal operating range of a unit when optimizing new unit additions. Improper unit mixes can result if probabilistic production costing is not used.

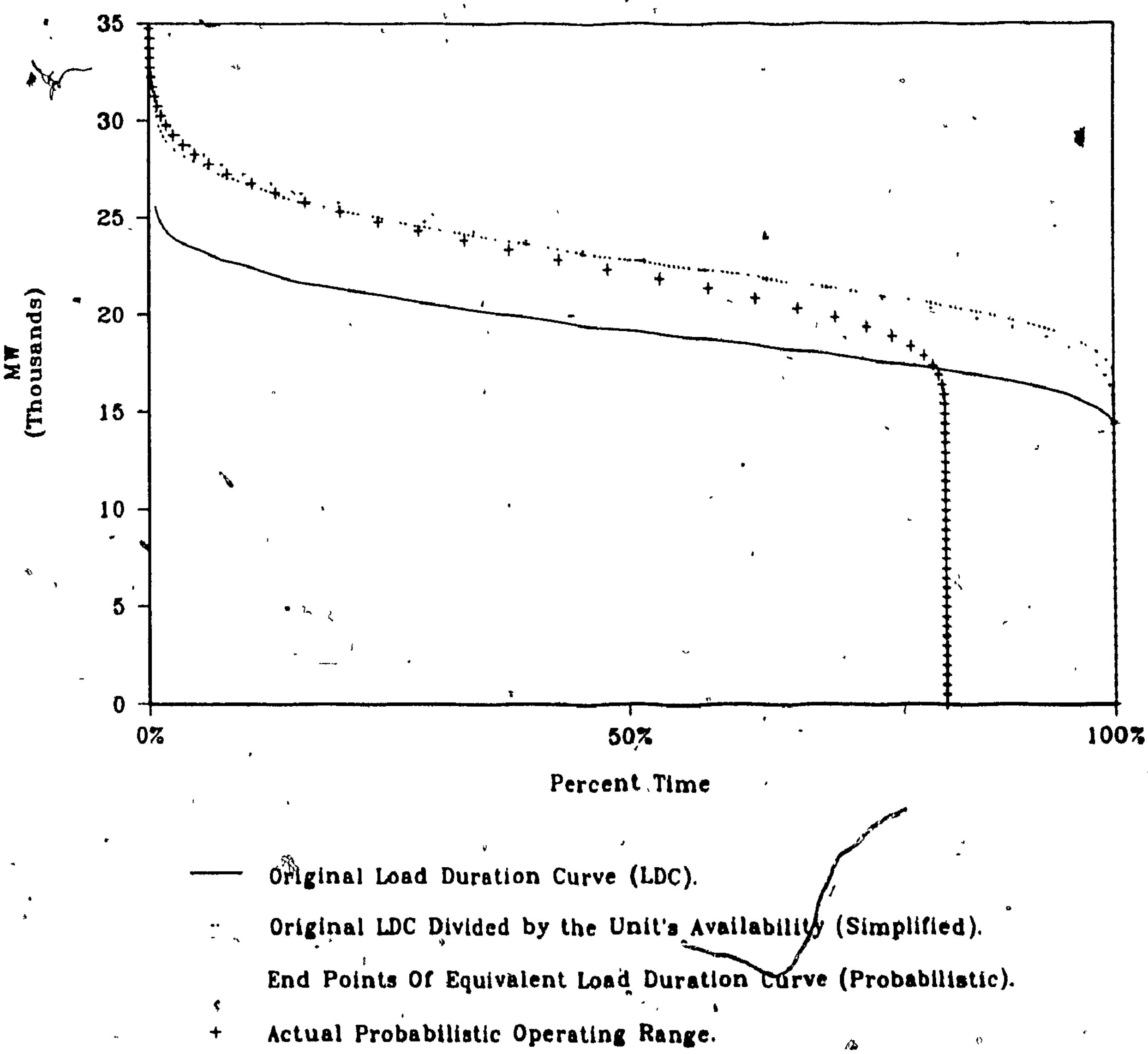


Figure 2.4 Probabilistic Versus Simplified Analysis.

2.5 Power System Reliability Measurements

Power system reliability is a probabilistic measure of the capability of the generation to always meet the system load. The reliability index is dependant on individual generating unit's reliability, maintenance scheduling, load demands, and operating policies.

The reliability of each individual generation unit is determined by modifying the equivalent load duration curve by the unit's forced outage rate. The reliability of the complete system, after all the generation has been loaded, is determined by two criteria; Loss of Load Probability (LOLP) and Expected Unserviced Energy (EUE).

2.5.1 Loss of Load Probability (LOLP)

Loss of load probability is the risk associated with having insufficient generation to meet the system load. It is usually expressed in days per year or hours per year. After all the generation has been loaded for that interval, the loss of load probability value is calculated as follows: (See Figure 2.5).

$$D_{m,n} = \sum_{j=1}^{n} Y_{m,j} \tag{2.5.1}$$

$$LOLP_m = ELDC_{m,n}(D_{m,n}) \cdot T_m \tag{2.5.2}$$

$$LOLP_{ann} = \sum_{m=1}^{n_{int}} LOLP_m \tag{2.5.3}$$

Variables:

$Y_{m,j}$ - Available capacity of unit j in interval m [MW].
$D_{m,n}$ - Total system capacity in interval m [MW].
n - Number of generating units in interval m.
$ELDC_{m,n}$ - Final Equivalent Load Duration Curve in interval m.
LOLPm - Loss of Load Probability in interval m [days/interval or hours/interval].
T_m - Time represented by interval m [days or hours].
$LOLP_{ann}$ - Annual LOLP.
n_{int} - Number of intervals per year.

If the simulation is performed on an interval basis, the loss of load probability for each interval is summed together to obtain the annual LOLP value (2.5.3). The loss of load probability criteria is only accurate to the second decimal place due to the assumptions of the Booth-Baleriaux formulation [42].

2.5.2 Expected Unserved Energy (EUE)

The expected unserved energy is a statistical indication of the energy shortfall in a given year and calculated as follows: (See Figure 2.3).

$$EUE_m = \int_{D_{m,n}}^{\infty} ELDC_{m,n}(x)\,dx \qquad (2.5.4)$$

$$EUE_{ann} = \sum_{m=1}^{n_{int}} EUE_m \qquad (2.5.5)$$

Variables:

EUE_m - Expected Unserved Energy in interval m [GWh]
EUE_{ann} - Annual EUE [GWh]

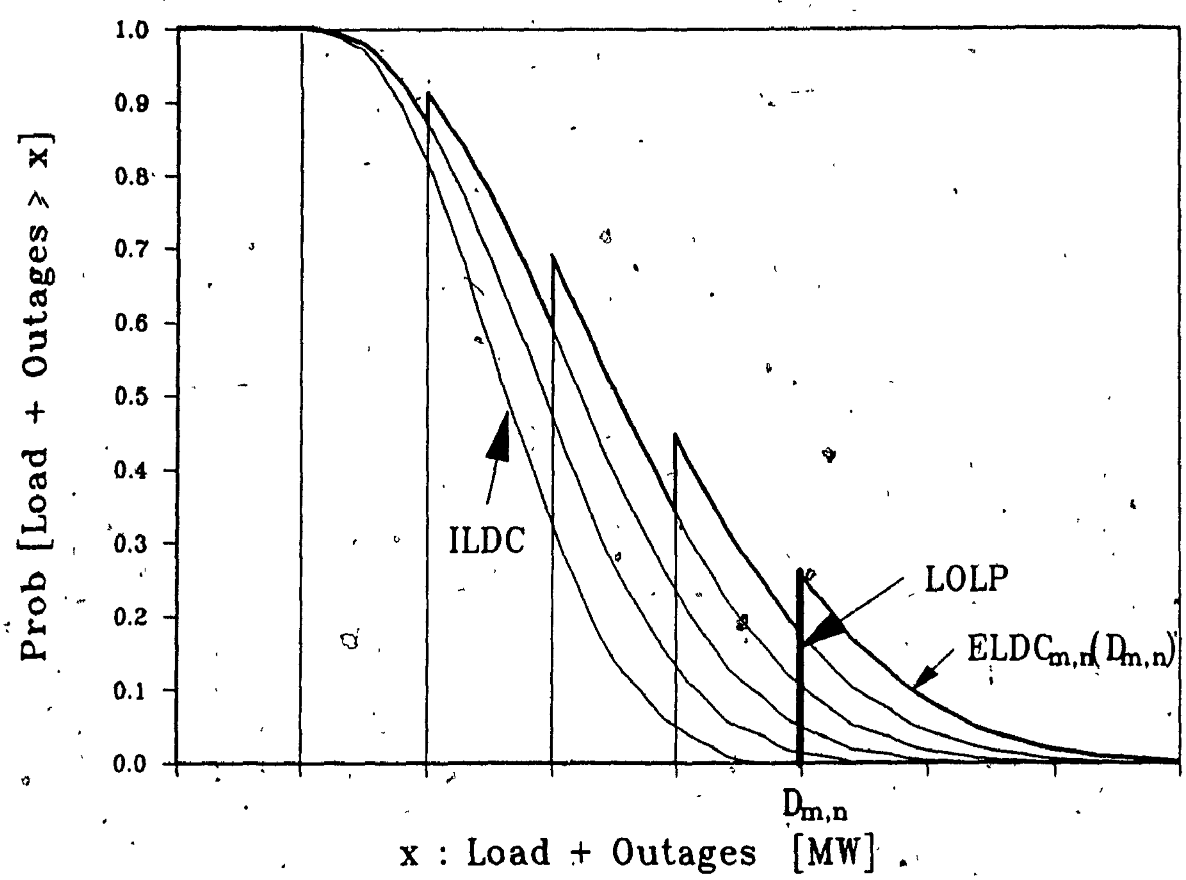


Figure 2.5 Loss of Load Probability.

The expected unserved energy is computed by determining the area remaining under the equivalent load duration curve after all the generation has been loaded. (See Figures 2.3, 2.5). There will always be some unserved energy in probabilistic simulation as a power system's generation cannot meet all of its load 100% of the time. Likewise, there will always be some small probability of insufficient generation available due to the units' forced outage rates. Like the LOLP criteria, if intervals are used, the annual expected unserved energy is the sum of each interval's EUE [42].

These two criteria give a good indication of the expected reliability of the system's generation scenario. No indication of the length of the outages is given, but this is not necessary for long range planning.

2.6 Costing

Production Costing simulates the operation of a power system and determines the cost of the energy produced to meet the system specified load. Many different generation scenarios can be found to meet a given reliability criteria. Production costing is very closely tied to reliability analysis. The desired system reliability affects the cost of producing electrical energy, thus resulting in a tradeoff between system reliability and system costs. As energy and finances are both limited resources, the most cost effective scenario is desired assuming all other factors are equal.

The cost of the energy production is determined by performing a simplified simulation of the system's actual operation. The system load demand is represented by the load duration curve. This energy demand is met by loading (stacking) generating units under the load duration curve in a predetermined 'merit order' [5,18]. The 'merit order' is determined by sorting the variable costs of each unit such that the units with the lowest variable cost are loaded first. The variable costs are proportional to the time the unit is in operation and are expressed per unit of energy. Variable costs consist of fuel, and variable operation and maintenance charges. Units operating in the lower portion of the load duration curve are first in the merit order as they operate more cheaply at a higher capacity factor. (See Figure 2.1).

The scheduling of the operation of generating units is preplanned and deterministic [20,42]. The optimality of the 'merit order' depends on the following simplifying assumptions: absence of generator start-up costs, transmission losses and linearity of operating costs with power output. These assumptions, when compared with the accuracy of the projected data, are acceptable in long-range generation planning models [5,18].

The scheduling of units is affected by the division of the annual load into interval load duration curves. Units scheduled for maintenance in particular interval(s) are removed from the merit order and will not operate during these interval(s). The scheduling of the maintenance of each unit is optimized over a year to achieve generation that corresponds to load fluctuations throughout the year. Unit maintenance is scheduled to minimize interval loss of load probability fluctuations throughout the year. Thus constant system reliability and the lowest production costs due to maintenance scheduling is assured [20]. If only one interval is used, the effect of the maintenance outages can be approximated by capacity reduction or increased forced outage rate. (See Appendix A. for a comparison of various maintenance techniques.)

Proper modelling of certain system phenomena require minimum interval base lengths. Weekly or monthly interval lengths can effectively model generation maintenance. This time frame can also model fluctuations in hydrological conditions affecting hydro unit scheduling. Long term expansion cannot model specific unit commitment and economic dispatch as they require base times of an hour or minute respectively. The required interval base times to model these system phenomena are summarized as follows;

Base Time	**Modelling Capabilities**
Month/Week	Generation maintenance
Month/Week	Hydro scheduling
Hourly	Unit commitment
Minute	Economic dispatch

Long term system planning considers typical base time interval ranges from monthly to a couple of years. Operations that cannot be represented in this time frame must be lumped together in some reasonable way such as modifying a unit's forced outage rate or adjusting the load duration curve [18].

Once the base time has been chosen, representative data for each interval must be obtained for the system load (peak capacity and energy), maintenance scheduling, and available capacity and energy of limited energy units. The operation of the power system is simulated for each interval by loading one unit at a time under the load duration curve and representing the effect of its forced outage rate by modifying the equivalent load duration curve. The area taken by the unit under the ELDC multiplied by its availability represents the energy produced by the unit for that particular interval. The stacking position of the hydro units are not determined using a 'merit order' as with thermal units. Since the operating costs of hydroelectric plants are cheaper than other types of generating units, all their capacity and energy should be used. Thus their loading position is determined to maximize their output of capacity and energy.

Once the energy production of each unit is determined for an interval, the cost of generating this electricity is calculated. The energy costs of each unit consist of fixed and variable operation and maintenance (O&M) costs, and fuel costs. Penalties for unserviced energy are then added to give the total interval production cost. The total yearly production cost is the sum of all the interval costs.

The long term yearly capital costs are calculated using the Uniform Annual Payment method. These costs include capital payments, interim replacement costs, property tax, and insurance costs, and are determined as follows:

$$c_i = \frac{cp_i \cdot i_{rate}}{1 - (1+i_{rate})^{**}npl_i} + \text{(interim replacement, property tax, insurance [\$/MW])} \tag{2.6.1}$$

Variables:

cp_i - Capital plant costs of unit i [$/MW].
c_i - Fixed plant costs for unit i [$/MW year].
i_{rate} - Interest rate [per unit] eg. 10%/100 .
npl_i - Plant life [intervals or years].

These capital costs are added to the annual production costs giving a complete estimate of the system cost per year. Over a 20-30 year simulation annual costs are present worthed back to the first year of simulation. Thus the total cost of differing power systems scenarios can be compared. From this criteria, an optimal least cost generation system can be designed to satisfy the desired reliability standards [20].

The Booth-Baleriaux probabilistic formulation provides a good estimation of the actual costs and operation of a power system. It is, however, limited by the assumptions inherent in the method. The operational costs are based on individual unit energy production which can only be estimated to an accuracy of three to four significant figures. The cost estimates are sufficiently accurate for long-term expansion analysis given the uncertainty of the simulation data. This method of estimating power system costs is widely used as it provides a good balance between long term data estimation, production cost estimates and the required computer resources [18].

2.7 Computer Implementation

The previous section only considered a mathematical approach representing actual generation costs of a particular power system. However, certain problems are encountered when production costing is implemented on the computer. The main problem is with the equivalent load duration curve convolution calculations. When this is represented by a computer algorithm, tradeoffs between computational accuracy, flexibility and efficiency result. The computation of the equivalent load duration curve and the energy produced, which are the basis of production costing and reliability analysis, represent the largest computational burden.

There are two types of algorithms used to calculate the equivalent load duration curve. Each has various advantages with respect to computational and storage requirements, and accuracy. The first type is the conventional Booth-Baleriaux algorithm in which the convolution is the major part of the computational burden. The second type is the analytic algorithm. In this method the calculation of each unit's expected energy generation requires the most computation, whereas the convolution is almost free.

2.7.1 Conventional Booth-Baleriaux Convolution

The conventional Booth-Baleriaux model uses a piecewise linear method consisting of several hundred uniformly spaced rectangular areas to represent the equivalent load duration curve. Convolution is performed using equations (2.4.1) for each rectangular area. This method is considered a benchmark algorithm to which alternative algorithms are compared.

The convolution or deconvolution required to load or unload a generation unit requires the recalculation of each value of the array of rectangular areas representing the equivalent load duration curve. The number of convolutions performed is proportional to the number of units in the system. The computational burden of each convolution process is dependant on the number of areas representing the ELDC which is in turn proportional to the size of the load. These factors combined result in the computational burden increasing exponentially with respect to the power system growth. The large number of calculations also produces numerical accuracy problems due to digital truncation errors inherent in computer computation. As the power system size increases, this problem can result in significant errors. The problem can be partially solved by using double precision calculations, but this doubles the computational burden, and does not totally eliminate the problem [3,18].

If the number of rectangular steps used to represent the equivalent load duration curve is reduced to save on computer resources, additional errors are produced from the poorer fit of the ELDC. The fewer number of steps also increases the dependency on interpolated values if generation unit sizes are not integral multipliers of step sizes. The interpolation is a linear approximation of a nonlinear curve, and thus is a source of error.

These efficiency-accuracy tradeoffs are most pronounced when the power systems are very large and have limited energy generation units. Depending on the method used to represent the limited energy units, they can require repeated loading and unloading to ensure optimal placement in the equivalent load duration curve. The resulting numerical stability problems can be tolerated for most systems, but the enormous computer resources required to overcome them have encouraged the development of alternative algorithms [18].

2.7.2 Analytic Convolution

The basic requirement of probabilistic simulation is to be able to convolve the equivalent load duration curve quickly, and accurately represent the original load duration curve. The conventional piecewise linear method is most widely used but has two main drawbacks. They are its exorbitant computational burden and numerical stability problems in large systems. Analytic algorithms, however, are much faster and do not have numerical stability problems, but tend not to represent the original load duration curve as accurately. Some examples of analytic methods are 50-term Fourier series, Gram-Charlier moments, Edgeworth moments, and Mixture of Normals Approximation (MONA) [18,22]. The last three methods take advantage of the statistical properties of the equivalent load duration curve where the ELDC is one minus the Cumulative Distribution Function (CDF) of the equivalent load. The calculations required to represent the unit's forced outage rate may be performed analytically. This operation's computational process merely consists of a few simple additions [18].

The cause of the analytical methods' poor load duration curve representation can be illustrated by the Mixture of Normals Approximation (MONA) technique. In this method the load duration curve is approximated by a sum of Cumulative Distribution Function (CDF) Normal (or Bell-curve) distribution curves. This method approximates the load duration curve by varying the means and variances of the CDF Normal curves making up the MONA of the load duration curve. The shape is still limited in the different forms it can represent. As the Normal curve is a continuous curve, it tends to smooth out sharp fluctuations in the load duration curve. Due to the length of the simulation horizon and the uncertainty in the load shape forecasts, this is not a serious problem.

Another problem with the MONA representation of the load duration curve is that there will always be a non-zero value for a probability distribution curve no matter how far one is from the mean. The actual load duration curve has a cut-off zero value at the peak load of the system. Problems like these can normally be overcome, through programming, or they can be neglected all together and the computational benefits be realized [18,22].

The original fit of the load duration curve is the most important source of errors in the analytic techniques. Thus it is better to use the piecewise linear method for small systems, or for systems with highly reliable generating units. Analytical algorithms tend to be less accurate in these cases [17]. Despite these limitations the small differences observed between most analytic models and the piecewise linear model can be ignored due to the assumptions on which the methods are based. (For a comparison of several methods see [17,18,22] and other works listed only under probabilistic Production Costing in Appendix C).

2.8 Generation Expansion Program Capabilities

The purpose of the optimization capability in generation expansion is to select from a given set of generating units, the additions that will result in a least present worth cost scenario over the whole production horizon while meeting a specified reliability criteria. The methodology must be compatible with economic long term expansion strategies and properly model generating units having different economic and operating characteristics. Various criteria affecting unit selection are outlined below.

2.8.1 Size of Power System

The size of the power system greatly affects the requirements of computer storage and execution time as they increase exponentially with system size. Thus, only efficient solution techniques can be used and modelling simplifications must be made. There is, therefore, a tradeoff between solution accuracy, power system size and available computer resources.

Due to the limited scope of the problem solution, global phenomena of different scenarios cannot be modelled. Other phenomena such as political and secondary benefits that cannot be approximated by fixed or variable costs are ignored. Their incorporation in the final scenario is left up to the discretion of the analyst. Only a small portion of the complete energy sector is modelled. Thus other sectors are presumed to be inelastic to electrical power sector changes [2].

2.8.2 Simulation Period

The length of the simulation period has similar tradeoffs to system size. Longer, more detailed simulation periods result in larger problem sizes. This in turn requires greater computer resources to solve generation expansion problems.

Due to the long lead time of new unit installation, a simulation period of twenty to thirty years is required. System data projection for this lengthy simulation is difficult and uncertain. The modelling accuracy should be comparable to the accuracy of the simulation data. Thus the lengths of the simulation period, the intervals and the extension period should all reflect the accuracy of the input data. The evaluation period must also be long enough to realize the full benefits of new generation over their lifespan.

2.8.3 Load Forecasts

The accuracy of the load forecasts is crucial in determining final system costs and new generation installations. Longer simulation periods result in more uncertainty in the load projections, thus the load is simplified using load duration curves. However, this simplification cannot model load following units and other time dependant phenomena. Appropriate time frames must be determined bearing in mind these tradeoffs.

2.8.4 Costs

System production costs must be representative of the actual operating costs enabling proper unit cost comparisons. Thus for new unit selection capital costs are determined using uniform annual payments irrespective of the utility's actual financial policies. This costing method weighs the capital costs over the unit's lifespan ensuring proper unit cost comparison. Using consistent costing methods, capital versus production cost tradeoffs can be properly modelled over the unit's operating life. Variations in system costs (variable or fixed) over time must also be reflected in unit selection criteria.

Other costs, such as transmission costs, affect unit selection and are modelled with various degrees of accuracy. When large capital expenditures are involved, power utilities have limitations to the amount of capital they can raise in any particular year. This must also be incorporated into the new unit selection process.

2.8.5 Generating Units

Thermal units are loaded in a 'merit' order of increasing energy costs. Limited energy plants, however, are treated separately as the 'merit order' assumes no energy restrictions. Capacity limitations of all generating units must also be modelled correctly.

Special techniques comparing hydro units on an equal footing with thermal plants must be used. This is due to hydro units having no fuel costs, and seasonal fluctuations in capacity and energy. Likewise, storage plants require special treatment. Not only do they have limited output, but require simulation of the increased load for energy storage.

Existing units are handled differently from new units. Their capital costs are assumed to be sunken and are not considered in unit comparisons. Some optimization models also consider retiring existing units if they are uneconomical when compared to the newer technologically advanced units.

Different sized units do not affect the power system in the same manner. They not only differ in cost, but also in the contribution to system reliability. Thus, the economies of scale must also be considered in unit selection.

Allowances for construction times must also be made when determining if a unit is available for selection.

2.8.6 Reliability

The desired reliability criteria affect the optimal final scenario with respect to new unit size, total system cost, and the number of new units selected. Reliability can either be expressed deterministically or probabilistically.

2.8.7 Summary

These criteria are summarized in Table 2.1. They affect the selection process and the resultant generation mix by varying degrees, and must be treated accordingly in the optimization algorithm. As a general rule, the more detailed the power system model, the more complicated the problem becomes, resulting in a tradeoff due to required computer resources.

Table 2.1 Power System Generation Expansion Criteria

CONSIDERATION	EFFECT
SIZE OF POWER SYSTEM	-Increase computer resources (Nonlinearly) -Restricts methods that can be used -Strategies, benefits and costs constrained (Not global picture.)
SIMULATION HORIZON	-Long term cost of units -Size of Problem -Uncertainty of inputs
-# of Intervals	-Proportional to accuracy of modelling
-Extension Period (Terminal Effects)	-May not realize full value of installed capacity
LOAD FORECASTS	-Uncertainty due to long term predictions
-LDC Representation	-No load following -Energy production
COSTS	
-Uniform Annual Payments	-Not represent other types of costing
-Capital/Production Costing Tradeoffs	-Change mix of units
-Inflation & Escalation Rates	-Change mix of units -Variations in capital, fuel, O & M (fixed and variable) costs
-Transmission	-Generation unit siting -Effective unit capacity
-Limits on Capital	-Constrained optimization problem
GENERATING UNITS	
-Limitations (Capacity, Energy)	-Placement in merit order
-Hydro	-No fuel costs -Different comparison -Special modelling required
-Storage Plants	-Different comparison -Special modelling required
-Construction Lead Time	-Delay first year available
-Existing	-No capital costs in unit comparison -Retired if inefficient
-Economy of Scale	-Different sized units impact cost and reliability differently
-Forced Outage Rate	-Impact cost and reliability differently
RELIABILITY	-Affects optimal mix -Probabilistic vs Deterministic

Chapter 3.

GENERATION EXPANSION TECHNIQUES

3.1 Introduction

There are a number of different optimization formulations used to determine an 'optimal' generation expansion scenario. Each method has different strengths and weaknesses. As computer time and storage are limited resources, tradeoffs must be made to provide the analyst with as much information as possible and in as efficient manner as possible. Certain optimization methods are better suited to different systems and needs. To properly select the appropriate method a basic understanding of the optimization processes is required.

Seven major groupings of optimization methods are discussed and their features compared. They represent the basis of the most widely used methods. A couple of non-standardized techniques are also discussed to give a broader scope of possible solution techniques. Details of specific software packages using these optimization techniques are outlined in Appendix B.

The first method discussed, Screening Curves, gives a graphical interpretation of the basic concepts of the yearly optimization process. These basic principles are then expanded by other techniques incorporating more accurate system modelling and optimization over the whole simulation time period.

The second method, Linear Programming, linearizes the power system and uses standard linear programming packages to determine an optimal expansion scenario. It is a fast and flexible method, but has a poor system representation.

The next four methods use separate techniques for new unit selection and production costing optimization. This enables improved system production costing modelling without solving very large, complex problems. They are iterative techniques and use feedback criteria to transfer information between the separate techniques.

The third algorithm is the Generalized Bender's decomposition which uses a linear programming new unit selection algorithm coupled with a complex production costing process. The next method, Generalized Network formulation, is similar to Generalized Bender's, but uses an integer heuristic method for new unit selection and a less accurate production costing formulation. These methods use similar optimization techniques but have different modelling capabilities and strengths.

The fifth method is Dynamic Programming. This method, although it can be loosely described as a feedback method, is an industrial standard. It relies less on mathematical elegance as it performs an exhaustive search in a prespecified region.

The sixth method, Gradient Projection Method, is a different type of feedback method. Its production costing method is dissimilar to the previously described methods. It gives an example of other, not so well known, optimization techniques being developed.

The final method, Heuristics, represents a group of methods that use 'rule of thumb' to obtain an 'optimal' scenario. They tend to be specific to individual companies and non-standardized, but are none the less used in industry.

Several of these techniques perform global optimizations but in turn sacrifice simulation accuracy, integer (realizable) scenarios, or computational efficiency. On the other hand, the non-global optimization techniques generally do not provide sufficient information to determine a realizable, properly simulated global sequence. The capabilities of each technique are discussed and contrasted with the other optimization techniques. This knowledge can then be used to develop a new generation expansion technique to operate within the limitations of the personal computer (Chapter 4).

3.2 Screening Curves

The Screening Curves (SC) generation expansion optimization method is conceptually the easiest one to understand. As its name implies, curves or graphs are used in the selection process of new units. The basis of the method lies in determining the optimal operating range or time of new units. These operating times are then transformed into the optimal new unit capacities using the load duration curve. Initially only new units are considered. The boundaries of the optimal operating range or breakeven times are determined using their fixed and variable costs.

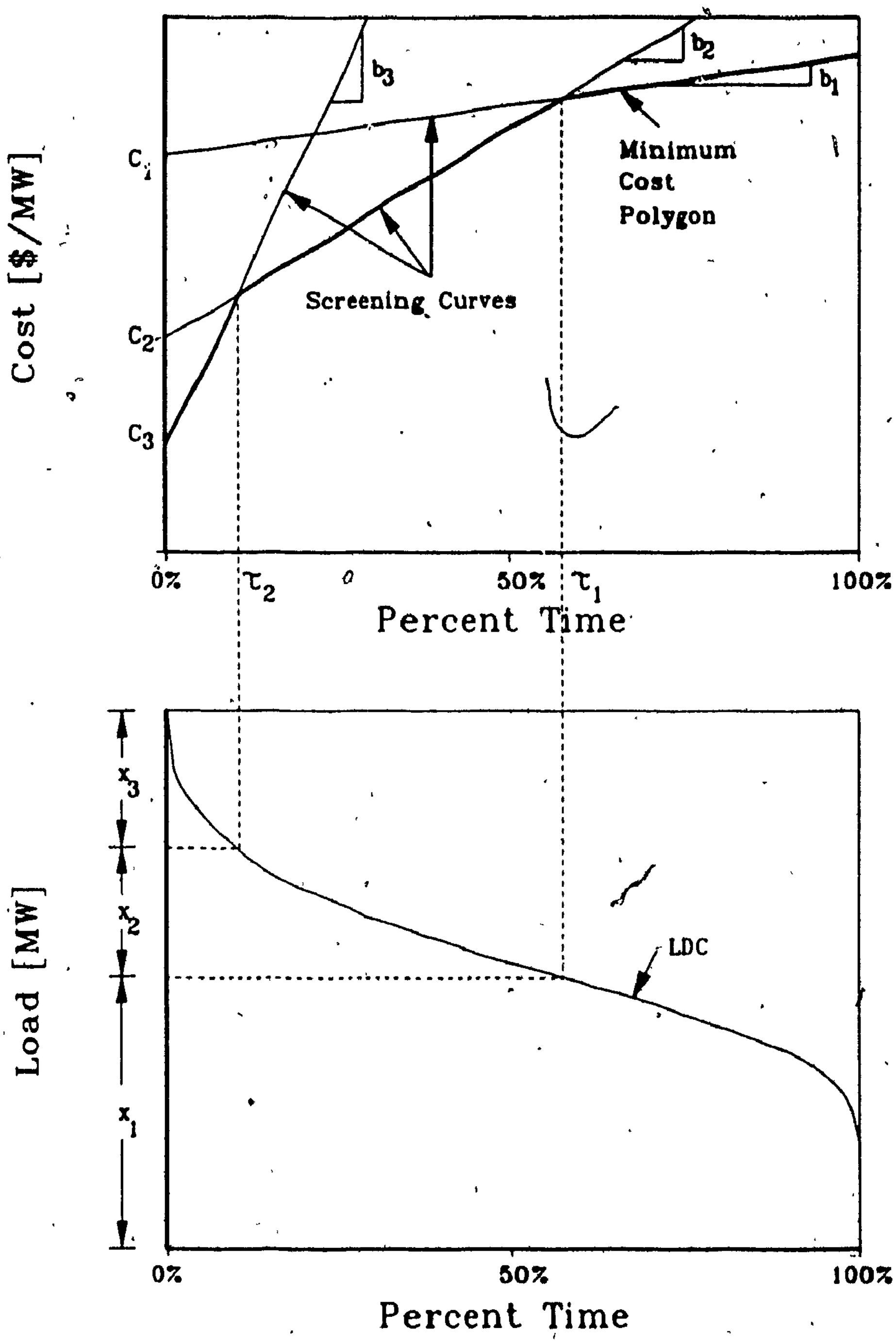
3.2.1 Problem Description

The method can easily be understood graphically where the straight lines, called screening curves, represent each new unit (See Figure 3.1). The y-axis intercept of the screening curve represents the fixed cost, and the x-axis, the length of operation. The slope of the straight line is the variable cost of the unit. The intersections of the screening curves are called breakeven times indicating the operating time where two new units cost the same to operate [11].

The least-cost all new unit mix is shown graphically by the minimum polygon (See Figure 3.1). Other units not included in this polygon are inefficient and are not considered for the optimal mix. This method of unit selection can be easily verified mathematically [34].

Once the breakeven times have been determined, they are transposed onto the system load duration curve. Thus the optimal new unit capacities can be determined. (See Figure 3.1). New unit's optimal capacities are usually non-integer values [21].

Existing units can also be incorporated into the new unit optimal mix. They are represented by screening curves without a known fixed charge or y-intercept. As existing unit's fixed



τ_i Breakeven Time i [per unit time].
b_i Variable cost of unit i [$/MW-time].
c_i Capital Cost of Unit i [$/MW].
x_i Optimal Capacity of New Unit i [MW].

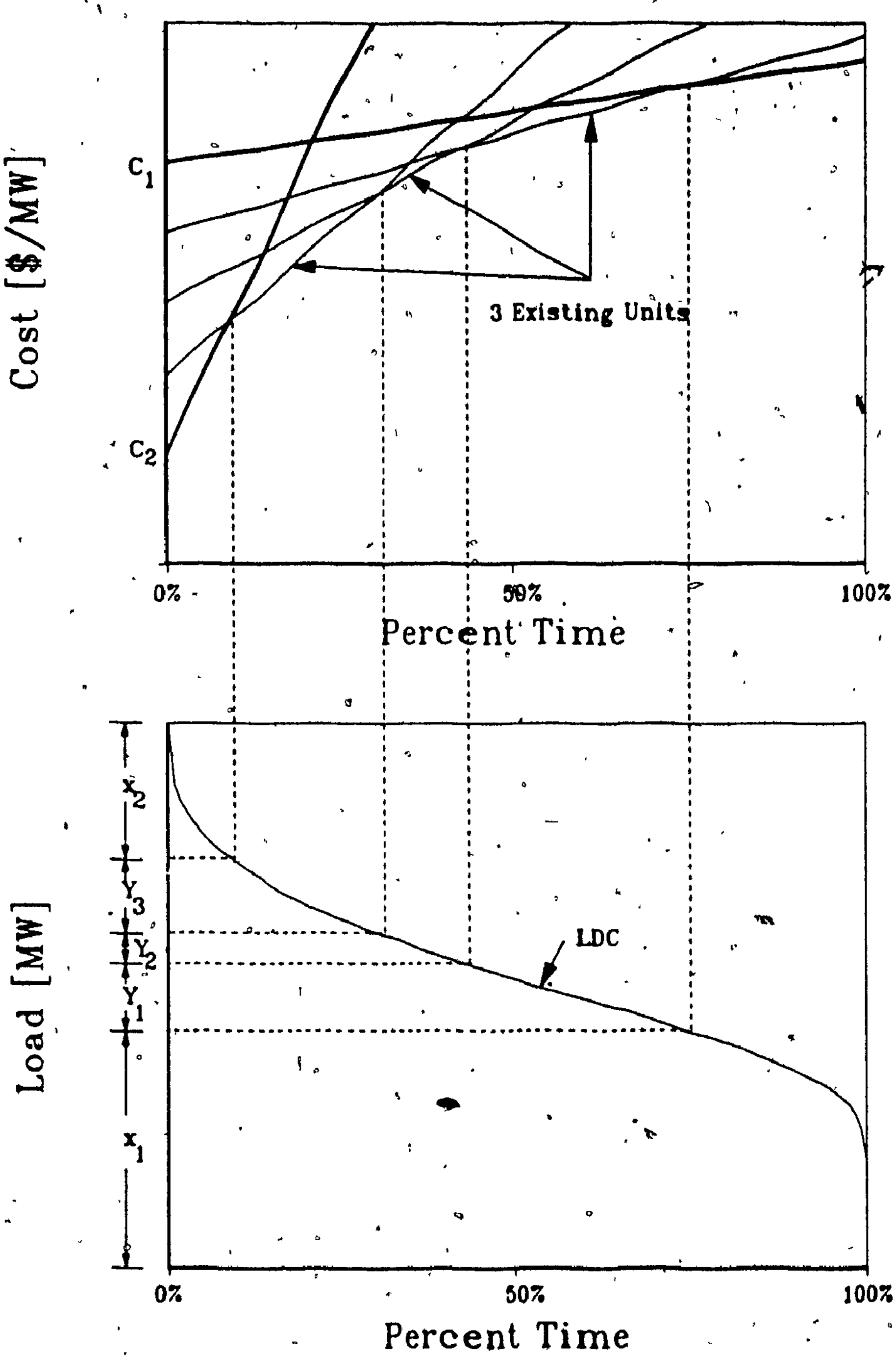
Figure 3.1 Optimal Mix – New Units Only.

charges are assumed to be sunken costs and are not considered. This concept is consistent with the expansion planning formulation. The existing unit's screening curve is placed so that its intersection with other screening curves correspond to its fixed capacity. (See Figure 3.2) [40].

Similarly new limited energy plants can be incorporated. Their fixed costs are known and a variable cost is determined so that it is optimally placed respecting its limited energy constraint [34].

The screening curves problem is generally solved mathematically. Details of their formulations are outlined further in Chapter 4 or can be referenced in [21,33,34,40]. The breakeven points of the screening curve intersections are readily determined as they are the intersections of simple linear equations. Existing units and new limited energy plants are incorporated by solving simple one-dimensional optimization problems (See [33]).

Once the optimal expansion capacities have been determined, operational costs can be solved using deterministic production costing. In deterministic production costing, the load is represented by a numeric load duration curve and forced outages and maintenance are represented by unit capacity deration. Since the reliability index is also deterministic, a percent reserve criteria is used.



C_i Capital Cost of New Unit i.
x_i Optimal Capacity of New Unit i.
Y_i Fixed Capacity of Existing Unit i.

Figure 3.2 Optimal Mix - 3 Existing Units.

3.2.2 Advantages

The advantages of the Screening Curves technique are apparent in its name. The method is excellent in screening different new unit candidates. It can handle many different types of new units with ease. It accurately represents the load duration curve and provides a good estimate of deterministic production system costs. The Screening Curves technique is conceptually easy to understand due to its graphical representation. It is a very fast method and requires little computer storage [33].

3.2.3 Disadvantages

The main disadvantages of the Screening Curves formulation are that the optimal scenario is non-integer and non-globally (yearly) optimized. Another disadvantage is that its production costing estimates are deterministic. It cannot properly model economies of scale due to its non-integer final scenario and thus, portions of new units are required to complete the final system scenario. If the large units are modelled by smaller, similar types of units, their costs are different and the final mix is no longer optimal [21]. As probabilistic phenomena are not modelled, proper unit interactions are not realized. The deterministic reliability criteria of Screening Curves is also less realistic than probabilistic criteria.

The Screening Curves algorithm, although conceptually, straight forward, requires a lot of specific logic programming. The ease of handling new units is somewhat negated by the execution time required in handling large numbers of existing units. Thus Screening Curves is better suited to systems with large numbers of new unit possibilities.

In summary, the Screening Curves method is a fast screening tool useful for analyzing large numbers of different types of new units. It produces non-integer yearly scenarios and is based on deterministic production costing.

3.3 Linear Programming

Linear Programming (LP) generation expansion optimization consists of a linear objective function which is subject to linear constraints. The solution technique seeks to minimize or maximize the objective function within the solution space defined by the linear constraints. Many well established efficient software packages exist for solving general LP formulations.

3.3.1 Problem Formulation

The problem formulation is of the following form:

$$\min_{X} OTC(X) = (C + BCF) \cdot X \tag{3.3.1}$$

$$bcf_{i,t} = b_{i,t} \cdot cf_{i,t} \tag{3.3.2}$$

subject to

$$A \cdot X =, \geq, \leq E \tag{3.3.3}$$

Variables:

OTC - Objective function of whole simulation period [$].
C - Fixed costs of units [$/MW].
X - Sizes of new capacities [MW].
$b_{i,t}$ - Variable cost of unit i in year t [$/MW-year].
$cf_{i,t}$ - Capacity factor of unit i in year t [p.u.].
A - Constraint matrix.
E - Constraint equalities / inequalities.

A piecewise linear approximation of the load duration curve is used to calculate production costs. The LDC is represented by a few (≤ 10) non-equally sized blocks, modelled through the use of constraints. The energy generated by the loaded units is not dictated by the load duration curve in the same way as conventional production costing. The Linear Programming specification requires that the operating capacity factors (CF) of all generating units must be specified as input. Thus the energy produced by the new units under the block load duration curve is set by the capacity factors and is not dictated by a merit order or the shape of the load. Units supply portions of the system load as dictated by the input capacity factor. The energy provided by the generating unit is subtracted from the block load duration curve starting from the first block. As additional generating units are loaded, the resultant energy production is subtracted from the number of load blocks as dictated by the unit's prespecified operating capacity factor.

The accuracy of the system production cost estimate depends on the number of time segments used to represent the load duration curve. Thus there is a tradeoff between the required computer resources (storage and execution time) versus system modelling accuracy. The number of segments representing the load duration curve is directly proportional to the number of variables and constraints used in the Linear Programming formulation [16].

New generating capacity is selected to minimize the present worth investment and operational costs subject to reserve margins constraints. Existing units are not included in the objective function due to their prespecified operation dictated by their capacity factors. Thus the substitution of new cheaper generation replacing expensive existing units cannot be modeled. These new unit selections, due to their prespecified capacity factors can result in inconsistent production costing. These inconsistencies arise when the system load energy, represented by the load duration curve blocks, can be exceeded if the specified capacity factors are not consistent with the Linear Programming optimal expansion plan. This results in unrealistic and suboptimal generation expansion plans [18].

Existing Linear Programming formulations are capable of solving problems with a few thousand variables, subject to several hundred constraints [15]. Most are based on the Simplex or Revised Simplex algorithm. Either the primal or dual problem formulation is solved, depending on which minimizes the number of constraints, computer storage and execution time. [16]

3.3.2 Linear Programming Variations

The Linear Programming technique can be modified to optimize more than one objective function. This technique is called Sequential Objectives. Examples of possible multiple objectives are maximizing revenue or reliability, and minimizing new plant construction, health hazards (emission control) or environmental hazards (heating natural water resources). The objective functions are met sequentially, and are then relaxed to meet subsequent objective functions. To reduce the variance from optimized solutions, previous objective functions are transformed into constraints. With small changes in total system costs, different criteria can be optimized by changing the system mix [38].

The Linear Programming method can also incorporate the following system phenomena: a piecewise linear approximation for nonlinear unit cost versus output relationships; spinning reserve; and hot and cold starts [16]. Hydro and storage unit modelling can also be incorporated, but much data needs to be prespecified and the unit modelling accuracy is poor. (See [18] for implementation). The level of detail modelled depends on available computer resources and the intended usage of the results.

3.3.3 Advantages

The main advantage of the Linear Programming formulation is that it is a fast, flexible global optimization method with a capacity to analyze a large number of expansion alternatives. It can easily be implemented using existing standard Linear Programming processing programs, eliminating the need for expensive, specialized program development. The accuracy of Linear Programming's production costing is dependant on the number of segments used to represent the load duration curve. It is, however, sufficiently accurate for prescreening analysis [18]. Sensitivity analysis is readily available due to the shadow prices inherent in Linear Programming calculations. Mathematically it is a simple tool and is computationally efficient.

3.3.4 Disadvantages

Linear Programming's main disadvantage is its poor estimation of production costs due to the Linear Programming's linearity requirement. It cannot represent probabilistic phenomena pertaining to reliability and unit production. Production cost estimates are further hampered by poor load duration curve representation. The required prespecification of generating unit's operating capacity factors can result in unrealistic and possibly inconsistent costing. The production costing method is also restricted by its inability to enable new generation to affect the operation of existing generation.

Attempts to improve unrealistic non-integer generating unit additions have been realized by modifying the Simplex technique or by using a Branch and Bound post processor. Both methods reduce Linear Programming's efficiency and increase the required size of the computer resources [15].

3.4 Generalized Bender's Decomposition

Generalized Bender's (GB) Decomposition determines a least cost generation expansion sequence using probabilistic reliability criteria and probabilistic production costing methodology. GB performs a global search to derive an optimal investment plan, choosing from a relatively large number of alternatives. This complex nonlinear problem naturally decomposes into two parts: determining optimal new unit investments, and determining system operating costs and reliability. This formulation can be exploited using decomposition techniques which subdivide the optimization problem into a master problem and a subproblem.

3.4.1 Problem Formulation

In this formulation the master problem is a linear program which generates trial solutions to determine a non-integer optimal capacity expansion plan. The subproblem determines the minimum cost of operation (production costing) and reliability of the trial system for each year of the planning horizon.

While performing production costing, dual Langrangian multipliers are computed by measuring changes in system operating costs and reliability resulting from marginal changes in the trial scenarios (See [3,6] for methodology). The dual multipliers provide a first order linear Taylor expansion approximation of operating costs and reliability constraints. The dual multipliers are then fed back to the master problem. Using this updated data, new constraints are formed and a new trial system is generated. This process is continued iteratively until the least cost sequence is obtained [3-6,10]. Constraints, determined by the subproblem, provide new upper and lower bounds, reducing the solution space with each iteration. The algorithm can thus be prematurely terminated with known error bounds.

The problem is formulated as follows [5]:

Master Problem:

$$\min_{X,Y} \; OTC(X,Y) = C \cdot X + \sum_{t=1}^{T} bq_t(\underline{Y}_t) \tag{3.4.1}$$

subject to

EUE $\leq$ reliability for all intervals

$$x_{i,t} \geq 0 \quad i=1,2,\ldots,n; \; t=1,2,\ldots,T \tag{3.4.2}$$

Subproblem:

$$\min_{\underline{Y}_t} \; bq_t = \sum_{i=1}^{n} b_i \int_{D_{i-1}}^{D_i} ELDC_i(x)\, dx \quad t=1,2,\ldots,T \tag{3.4.2}$$

where

$$\begin{cases} D_1 = 0 \\ D_i = \sum_{j=1}^{i-1} x_j \quad i=2, \ldots, n+1 \end{cases} \qquad (3.3.3)$$

$$0 \le x_{i,t} \le Y_{i,t} \quad i=1,2,\ldots,n;\ t=1,2,\ldots,T \qquad (3.4.3)$$

$$EUE_t = \int_{D_{n+1}}^{\infty} ELDC_n(x)\,dx \quad t=1,2,\ldots,T \qquad (3.4.5)$$

Variables:

OTC - Objective function of whole simulation period [\$].
C - Fixed costs of units [\$/MW].
x_i - Capacity of new unit i [MW].
bq_t - Variable cost of year t [\$].
Y_i - Maximum capacity of unit i [MW].
EUE_t - Expected unserved energy for year t [MW-year].
D_i - Loading level of unit i [MW].
$ELDC_i$ - Equivalent Load Duration Curve at capacity x after unit i has been loaded [probability].
n - Number of units.
T - Number of years of simulation.

In Generalized Bender's there is one subproblem for each planning period. The reliability measure of the trial scenario is determined by the expected unserviced energy, ensuring the convexity of the problem formulation [3].

As the master linear programming problem is a non-integer formulation, the equivalent production costs of portions of units are required. This is possible using analytical production costing techniques. Fractional portions of units are represented so that they affect system costs and reliability criteria in a manner consistent with complete generating units. An example of this formulation is outlined in [18] using the Gram-Charlier series load duration curve representation. Thus despite the use of non-integer new unit additions, the effect of individual units is correctly modelled.

3.4.2 Generalized Bender's Variations

There are many variations of the Generalized Bender's formulation. J. A. Bloom, through the use of Lagrangian relaxation and Kuhn-Tucker optimality, transforms the constrained master problem to a larger unconstrained problem [5]. Noonan and Giglio's formulation solves the generation expansion problem using deterministic production costing coupled with a nonlinear Gaussian loss of load probability constraint [5]. Scheweppe et al. and Beglari and Laughton use the Generalized Bender's formulation but heuristics replace the dual multipliers linking the master and subproblem [5]. This method, however, cannot assure convergence. Another is the Generalized Network formulation described in section 3.5 [13].

3.4.3 Advantages

The main advantages of the Generalized Bender's decomposition lie in its ability to probabilistically model system production costing and reliability criteria. Production costing estimates having an accuracy comparable to power system costing programs can be realized, thus basing the optimal scenario on realistic system costs. New capacity additions affect the rest of the system energy generation, resulting in proper unit interactions. Thermal, hydro, limited energy plants (LEP), and storage plants (SP) can all be optimized using the Generalized Bender's formulation. Due to the resultant increased size of the master problem, interval analysis is not recommended, therefore, limited energy plants, storage plants and unit maintenance are not modelled very accurately. Generalized Bender's can also properly analyze the trade-offs between investment costs, production costs and reliability benefits [18].

As the master problem is a linear optimization process, prespecified constraints, such as financial or reserve margins (minimal or maximum), can be implemented. Sensitivity analysis is also inherent in the linear programming formulation through marginal costs of reliability and shadow prices. Since the size of the master problem increases through iterations, the last few iterations use the most computer resources. If these small refinements are not necessary, large amounts of computer time can be saved by early termination. The program terminates with a known error margin since the solution is always bounded. The accuracy of Generalized Bender's has been verified by comparisons to an accepted standard (Dynamic Programming). The solutions were found to match closely both in the time scheduling of new units and total capacity installations [18].

3.4.4 Disadvantages

The main disadvantage of the Generalized Bender's method is its inability to generate a realizable integer optimal scenario. A non-integer solution cannot be rounded or truncated to form an integer solution and still maintain its optimality. Furthermore analytic production costing must be used due to the non-integer new unit additions. This imposes limitations when analyzing small systems. Another disadvantage is the large linear programming memory requirements and computer code size due to the separate master, subproblem formulation. Generalized Bender's also cannot model economies of scale due to non-discrete unit additions.

In summary, Generalized Bender's is a sophisticated screening tool which poses less of a computational burden than exhaustive search methods, yet retains a high degree of production costing accuracy.

3.5 Generalized Network

The Generalized Network formulation of power system generation expansion is very similar to the Generalized Bender's formulation. Both utilize a master, subproblem decomposition to reduce a large problem into two smaller problems. The master problem selects investment alternatives, whereas the subproblem analyzes the operational costs of the proposed master problem scenario. The techniques used in this formulation are an integer programming heuristic master problem and a network flow optimization subproblem.

Generalized Network's main focus is the long term optimization of systems with a high percentage of hydraulic production. The subproblem is able to model water flows and reservoir management. Its production cost model has an accuracy comparable to the Linear Programming formulation. If the storage capabilities of hydro plants are not properly modeled, difficulties can arise and erroneous results occur. The proper handling of hydro units affects the utilization of thermal power plants, peaking requirements, overall system security and energy production [13].

3.5.1 Problem Formulation

The decomposition deals with two decisions, plant mix and equipment operation. They are not decoupled as there are limitations on plant capacity and available energy. The general problem formulation is as follows [13]:

$$\min_{X,Y} \; OTC(X,Y) = C \cdot X + BQ(Y) \qquad (3.5.1)$$

subject to

$$A \cdot X \geq F \qquad (3.5.2)$$
$$G \cdot X + H \cdot Y \geq B \qquad (3.5.3)$$
$$0 \leq X \leq n \cdot Y \quad \text{(no partial units)} \qquad (3.5.4)$$
$$Y \geq 0 \qquad (3.5.5)$$

Variables:

OTC - Objective function of complete simulation period [$].
X - New capacity (Investments) [MW].
Y - Maximum capacity of all units [MW].
C - Fixed costs of units [$/MW].

BQ - Variable costs [\$].
A,F - Capacity availability and system reliability constraints.
G,H,B - Production costing constraints.
n - Total number of units.

Bender's Decomposition uses the block triangular structure of this formulation by fixing the variable x thus reducing the problem to a production costing problem with continuous variables expressed as follows [13].

$$\min_{Y} \ BQ(Y) \tag{3.5.6}$$

subject to

$$H \cdot Y \geq B - G \cdot \bar{x} \qquad (\bar{x} \text{ is fixed}) \tag{3.5.7}$$
$$Y \geq 0 \tag{3.5.8}$$

This method is called Bender's Partitioning Method (BPM). It is an iterative procedure using successive plans based on a relaxed version of the master problem (subproblem ignored). The master problem serves as a filter to eliminate candidates from future considerations. The operating costs of the new investment plan are obtained. These results are then added to the master problem and serve to further constrain the solution space. As the solution space is reduced with each iteration, the method converges to the optimal solution. The efficiency of the method depends on the speed of the subproblem and on the convergence of the master problem [12].

3.5.2 Subproblem

The production costing subproblem is modelled as a network flow optimization problem for which efficient solution techniques exist. A specialization of the Simplex algorithm is one technique that is used. The production costing is deterministic and considers not only the system load and plant availability but also hydraulic inflows. The system load is met when production exceeds demand for each step representing the load duration curve. The system load and generation are modelled by nodes whereby the conservation of energy ensures that the energy entering a node equals the energy leaving the node. (See Figure 3.3). Each arc joining a node is characterized by a capacity and a cost for the energy that flows through it. Unused hydro energy can be passed from one interval to the next by nodal connections. There is one node for all thermal units as they do not have energy limitations. Each arc represents a different thermal unit providing energy to a step of the load duration curve for each production interval [13].

3.5.3 Master Problem

The master problem is solved using an integer optimization process using heuristic methodology. Existing integer programming algorithms are complicated and require too much computer time to be useful in this iterative procedure.

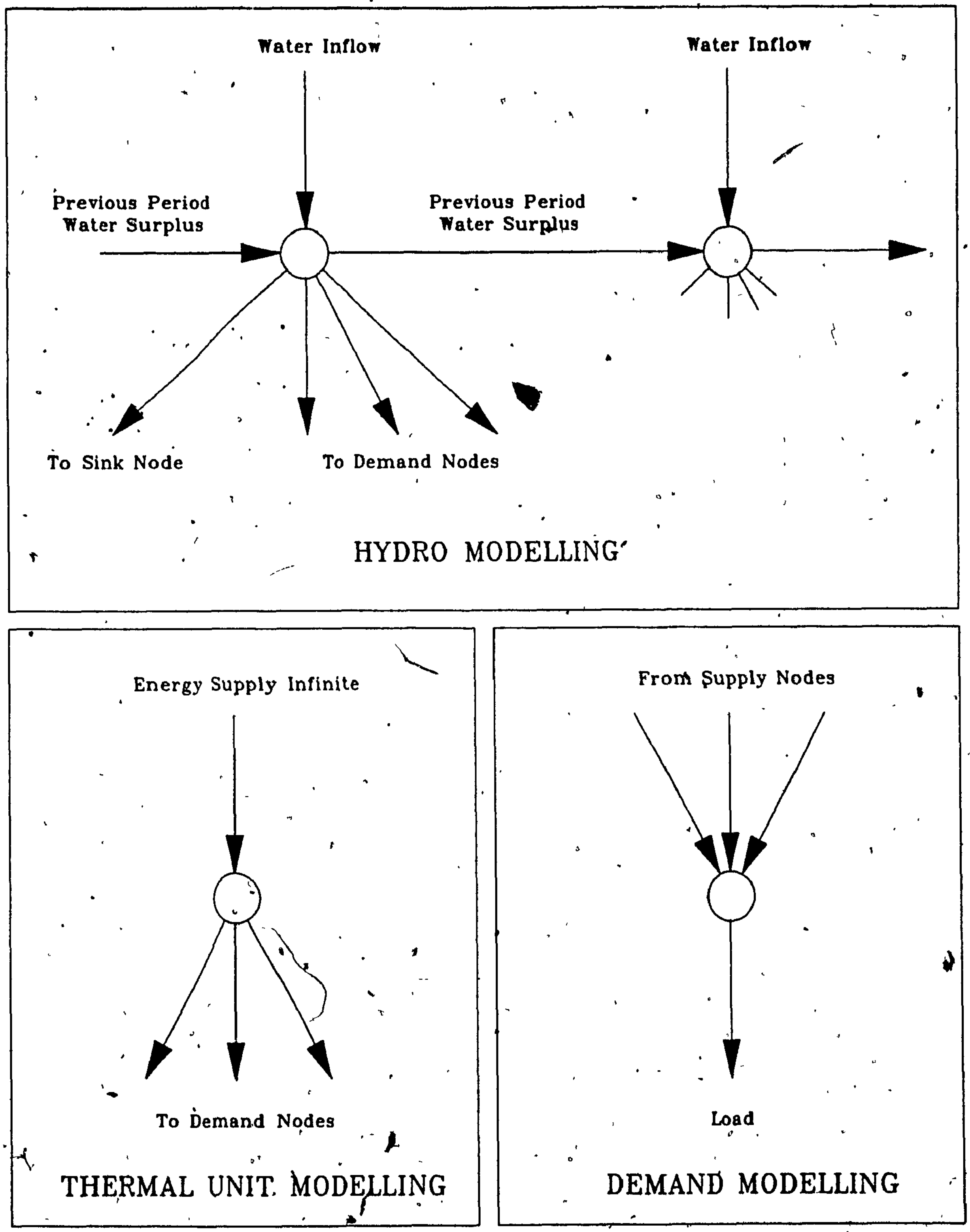


Figure 3.3 Network Representation of Production Costing.
(Adapted from [13]).

The heuristic has a search phase and an improvement phase. The search phase obtains a feasible integer solution. The integer constraint is then relaxed and an upper bounded linear programming algorithm attempts to improve the integer solution. At regular intervals during the search, attempts are made to truncate or round off 'x' to obtain a feasible integer solution. The linear programming problem is small and fast, therefore the heuristic method is efficient [13].

3.5.4 Advantages

This method is useful for systems with large hydraulic facilities where the operation of reservoirs has a significant impact on production costs. The decomposition allows Generalized Network to capitalize on the master/ subproblem decomposition thus reducing the size of the problem treated. It also obtains a realizable integer final scenario which is desirable.

3.5.5 Disadvantages

Generalized Network's main disadvantage is the deterministic approximation of production costing which is of similar accuracy to the Linear Programming formulation. The heuristic method used for the master problem cannot guarantee an optimal solution and its accuracy can only be verified through extensive testing.

It is interesting to note that despite the usage of Bender's decomposition, the results of Generalized Bender's and Generalized Network have very different properties. Generalized Bender's uses probabilistic production costing and ensures mathematical optimality, whereas Generalized Network uses a simplified deterministic production costing method incorporating hydro reservoirs, yet obtains an integer solution with uncertain optimality.

3.6 Dynamic Programming

The simplest and most assured way of determining an optimal solution of a complex nonlinear optimization problem is by 'brute force' or performing a comprehensive search. Dynamic programming (DP) is a refined version of this technique which ensures consistency in new unit additions. DP ensures that once a new unit is added, it will continue to operate throughout its lifespan.

In practical situations, however, the dimensionality of such a power planning problem is so large that such a straight forward approach is computationally infeasible. To limit the solution space of the problem, only combinations of expansion candidates meeting a set of user defined bounds are used. These restrictions, while reducing computational burdens, also sacrifice assured final scenario optimality [39].

Dynamic Programming is widely used in Power System Generation Expansion Analysis. An industrial standard used throughout the world is the Wein Automatic System Planning Package (WASP). WASP uses Dynamic Programming optimization [43]. WASP was used to accredit the methods used to develop the reference sequence for the testing of this work.

3.6.1 Problem Description

The Dynamic Programming formulation applied to power system generation planning seeks to determine the lowest total system cost expansion scenario. The total system cost of operating the system is represented by the objective function consisting of fixed and variable costs over the complete study horizon. The DP formulation does not require the use of any particular production costing method, and normally sophisticated probabilistic methods are used.

Each feasible solution of new unit additions, dictated by use defined bounds, is called a state. A state is defined for a particular period of time depending on the number of intervals used in the production costing simulation. (The number of intervals is limited by the size and simulation length of the power system).

The first state is defined as the first year of simulation in which all units are existing or committed. Subsequent states are restricted by user specified minimum and/or maximum reserve criteria called tunnelling. (See Figure 3.4) These bounds are necessary to limit the number of considered states, thereby reducing the problem size. The states must also be physically realizable ensuring a continuum of new unit additions over the simulation period. System reliability criteria, loss of load probability or expected unserved energy, must also be met for a state to be considered [18,27]. For large power systems it is necessary to reduce the number of states even further using heuristic criteria. (See [36])

The first step of the Dynamic Programming optimization process is determining all feasible states of that particular time period. These system generation mixes represented by states are individually costed. Production and capital costs are included in the state cost. The next step in the Dynamic Programming process is determining least cost routes from the previous time period's states. (See Figure 3.4). As there can be many routes to a particular state, the least cost route is selected and pointers, indicating previous states, are stored. Using these selected least cost routes, the least present worthed total costs (costs to date) of the states are determined. In large systems only the cheapest states are retained due to programming restrictions.

This process is continued for every interval through to the end of the simulation horizon. The cheapest cost to date is the 'optimal' expansion scenario. The optimality is dependant on the effect of the tunnelling boundary conditions. These restrictions are flagged if they affect the optimality of the final scenario. New unit installation dates are determined by tracing the routes of the state pointers back

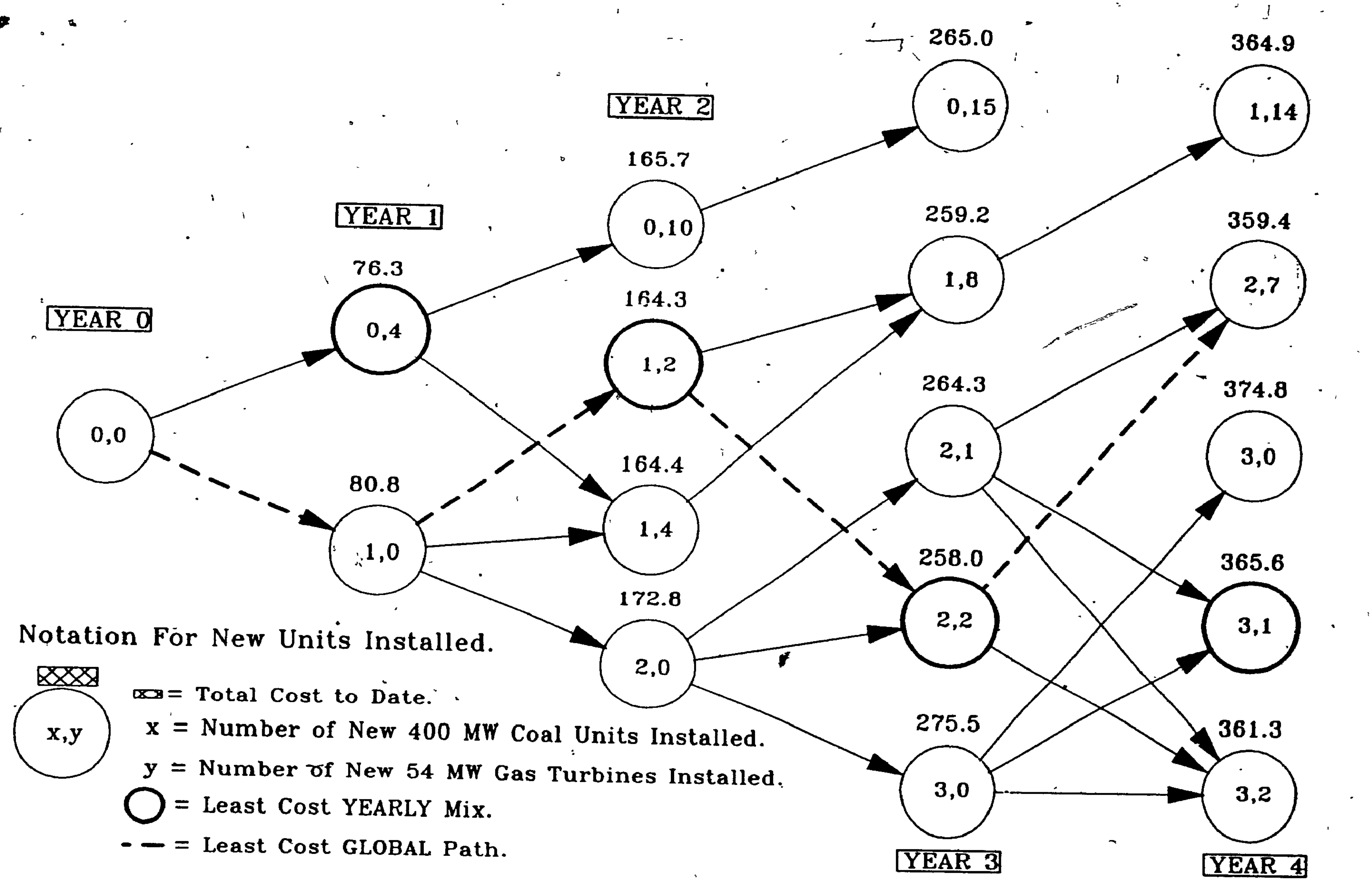


Figure 3.4 Forward Dynamic Programming. (Adapted from [18]).

to the initial state. This technique is called Forward Dynamic Programming [43]. (See [25-27] for the precise mathematical formulation.)

If sensitivity analysis consisting of suboptimal scenarios is desired, Backwards Dynamic Programming is used. The Dynamic Programming cost to date routing is repeated only starting from the final year going toward the first. It is not necessary to repeat the costing process as the results already exist from the Forward Dynamic Programming optimization. The number of suboptimal scenarios available is limited only by computer storage restrictions [8].

Further constraints of the power system can be realized through the selection process of acceptable states. States not meeting constraints including maximum yearly unit installations and yearly capital restrictions are not considered for analysis [43].

The efficiency of the production costing algorithm is very important to Dynamic Programming due to the large number of states that are analyzed. Since almost 80% of the processing time of the Dynamic Programming formulation is spent calculating production costs, analytic methods are widely used. This enables substantial reductions of up to 75% for large systems in the total processing time [39]. There is no restriction, however, on the type of method used for production costing estimates. Many versions of Dynamic Programming use production costing techniques of comparable quality to nonoptimizing generation planning tools. Any type of unit can be modelled using this method as long as the interval time used for the production costing is small enough to represent its operation accurately. These methods are very accurate, but tend to have substantial execution times [39].

3.6.2 Advantages

The main advantage of Dynamic Programming is its determination of a realizable globally optimal (whole unit additions) expansion scenario. Dynamic Programming can be used with any type of production costing and can explicitly handle other constraints including financial and unit installations. Due to the flexibility of the production costing algorithm, many different types of units can be represented including hydro and storage plants [41]. Probabilistic production costing methodology also ensures proper unit interaction. Dynamic Programming also enables the usage of probabilistic reliability criteria, expected unserved energy or loss of load probability. Suboptimal plans are also available giving the analyst a sensitivity analysis through realizable scenarios [18,36].

3.6.3 Disadvantages

The main disadvantage of Dynamic Programming is the 'curse of dimensionality'. The method requires large amounts of computer storage and has a very high computational burden for large systems. The method also relies on the analyst to specify tunnel constraints which, while reducing the size of the problem, can result in a suboptimal solution. For example the addition of a large base load unit which exceeds the maximum reserve margin, might not be considered as a candidate plant [8].

It is ironic that the appealing nature of exhaustive search of the Dynamic Programming formulation in selecting the optimal scenario is the actual factor that forces the restrictions on the method which cause the optimality of the method to be questioned.

3.7 Gradient Projection

The Gradient Projection technique of Youn, Lee and Park [37,44] is fundamentally different from the previously described optimization techniques. The other optimization techniques use a load duration curve to represent the system load whereas this method uses a chronological load curve for production cost estimates. Another fundamental difference is that the probabilistic effects of the unit's forced outage rate are not modelled using the Booth-Baleriaux formulation. Instead the production costing and reliability criteria assume load fluctuations and plant outages follow a Gaussian probability distribution. This is viewed by some critics as inadequate (see discussions in [37,44]) and require further testing for proper validation. It is presented, none the less, to give insight into other possible methods being used to solve the power system generation expansion problem. It is only briefly summarized due to the questions concerning its modelling capabilities. Further details can be found by consulting the following references [30,37,44].

The Gradient Projection technique models load fluctuations using analytic techniques of Normal distributions for each load curve segment. Each segment is represented by normal mean and variances. Generating units' forced outages are similarly modelled, thus enabling their combined effect to be realized by simple additions and subtractions of their respective means and variances. This method is conceptually similar to analytical production costing techniques except that a chronological load curve replaces the traditional load duration curve. Analytic formulas are used not only to determine operating costs and reliability measures but also to calculate their marginal values [44].

The long-range optimization process consists of a master problem to determine the annual investments of new units and subproblems to determine minimum operation capacity costs and reliability [37]. The master problem is formulated as an optimal control problem. The Hamiltonian minimization is performed using a specialized version of the Gradient Projection method [37].

The subproblem analytically determines the operating costs, reliability and their marginal changes. The marginal values are then returned to the master problem which is modified accordingly, and resolved to determine a new investment plan. The optimization is performed iteratively until the optimal scenario is found [30].

3.7.1 Advantages

The main advantage of this optimization technique is the probabilistic handling of production costs and reliability measures. Due to the master-subproblem formulation, the problem size is reduced into two smaller problems enabling a faster solution and more detailed production costing. This method has interval analysis, but insufficient data is available to determine its affect on total simulation period storage and execution requirements.

3.7.2 Disadvantages

The main disadvantage of this method is the unproven accuracy of the representation of the system loads and generation forced outages by the Gaussian approximation. This problem can be rectified through extensive testing and analysis to determine the merits of the method [37,44]. The method also provides non-integer final scenarios, thus requiring the analyst to discretize the final solution into a realizable sequence.

3.8 Heuristics

The boundary that divides mathematical and Heuristic optimization methods is at best ambiguous. Mathematicians resolve this problem through proofs and theories, but in the actual implementation this categorization weakens. It has been shown in Section 2.7, Computer Implementation, that there exists a wide gap between theoretical formulations and working computer programs. Due to the computer's limitations, approximations in the methodology must be made. The approximations are based on the programmer's understanding of the problem and expected system data accuracy. Heuristics or 'rules of thumb' are necessary and are used throughout the implementation of a method; thus the cause for the classification ambiguity.

There are, however, programs that do not have a solid basis in a particular mathematical optimization formulation and are almost completely based on Heuristics. These methods generally cannot be proven to be optimal and are modified or adjusted when the analyst detects conflicting or unexpected results. These programs tend to be developed 'in house' and are based on methods developed through experience or a 'feel' for the system.

An example of a power system generation expansion optimization Heuristic is as follows. Initially the system's reliability criteria is met with new peaking generation. New units of the type corresponding to the 'merit order' are added one at a time until the least cost number of this type is reached. The comparison is based on the cost effect of the new unit on the new peaking generation. The least cost number of new units of this type is added. This process continues until all new unit types have been tried.

The problem with this Heuristic method is that the new generation is only compared to the cost effectiveness of peaking new units candidates. The method provides a realizable final scenario, but its closeness to optimality cannot be asserted.

Such techniques can be sufficiently accurate for some applications, but should only be used by a highly experienced analyst who can determine if the results are reasonable. To ensure the solution's optimality, a lot of hand testing is required which relies exclusively on the intuition of the analyst.

3.8.1 Advantages

Heuristic optimization methods are usually based on years of experience, and if properly incorporated, can provide quick approximations to assist a highly experienced analyst. They tend to have small overhead costs as they are developed over a long period of time and are usually 'in house' programs.

3.8.2 Disadvantages

The main disadvantage is the unknown quantities of a Heuristic method. They may perform as well, if not better than methods based on standard techniques, but this can only be asserted through extensive testing. There is also a lack of standardization, which does not instil confidence in the method. As Heuristic methods tend to be quick approximations, they can be inefficient, and unreliable.

3.9 Optimization Technique Comparison

Various optimization methods exist and implementations of each vary in complexity and versatility. There is considerable overlap in the models described, but each is better suited to a specific purpose. Each requires different input data, and have varying computational requirements. Key similarities and differences are listed below and are summarized in Tables 3.1 and 3.2. Heuristic methods are so varied and nonstandardized that they are not included in the following discussion.

3.9.1 Production Costing

The most accurate production costing estimates are available using Dynamic Programming. The method uses probabilistic production costing and can adequately model limited energy and storage plants for long term optimization. Probabilistic reliability and interval analysis also add to

Table. 3.1 Review of Optimization Methods - Production Costing

Production Costing Attributes	SCREENING CURVES	LINEAR PROGRAMMING	BENDER'S DECOMPOSITION	DYNAMIC PROGRAMMING	GRADIENT PROJECTION
Load Model	Numeric LDC	Block LDC	1) Numeric LDC 2) Analytic LDC	1) Numeric LDC 2) Analytic LDC	Analytic Load curve
Subperiods	No	No	No	Yes	Yes
Production Costing	Deterministic	Deterministic	1) Deterministic 2) Probabilistic	1) Deterministic 2) Probabilistic	Probabilistic
FOR Methodology	Derate Cap.	Derate Cap.	1) Derate Cap. 2) Convolution*	1) Derate Cap. 2) Convolution**	Gaussian Approximation
Reliability	% Reserve	% Reserve	1) % Reserve 2) EUE	1) % Reserve 2) LOLP, EUE	EUE
Maintenance	Derate Cap.	Derate Cap.	1) Derate Cap. 2) Add to FOR	Subperiod Unit Removal	Derate Capacity
LEP Handling	Cut LDC	Cut LDC	1) Cut LDC 2) Merit Order Placement	1) Cut LDC 2) Merit Order Placement	Cut Chronological Load Curve
Unit Costing	Non-Integer	Non-Integer	Non-Int/Integer	Integer	Non-Integer
New Units Affect Existing Ones	Yes	No	Yes	Yes	Yes

* Must use analytic method due to non-integer costing

** Any method can be used

Table. 3.2 Review of Optimization Methods - Investment Costing

Production Costing Attributes	SCREENING CURVES	LINEAR PROGRAMMING	BENDER'S DECOMPOSITION	DYNAMIC PROGRAMMING	GRADIENT PROJECTION*
New Unit Additions	Non-Integer Large #/yr	Non-Integer -20/yr	Non-Integer -5/yr	Integer -5/yr	Non-Integer
New Unit Comparison	Good	Poor	Very Good	Very Good	Very Good
Unit Modelling -Thermal -LEP -SP	 Good Fair Fair	 Fair Fair - Good Fair	 Good-Very Good Very Good Very Good	 Excellent Excellent Excellent	 Very Good Very Good Very Good
Sensitivity Analysis	Dual Variables	Shadow Prices	Shadow Prices	Sub-optimal Plans	Dual Variables
Search Area	Excellent	Very Good	Good	Poor	Good*
Computational Burden	Low	Low - Moderate	Moderate	Moderate-High	Moderate*
Storage Requirements	Low	Moderate	High	High	High*
Power Tool Usage	Screening	Screening	Large Systems	Whole Unit Additions	Large Systems

* Due to the recent development the Gradient Projection Method standardized test results that would allow proper method comparison are unavailable. Thus assumptions are based on mathematical formulations and available test results.

its accurate representation. The Gradient Projection production costing method is not as sophisticated but is probabilistic and has the capabilities of interval and hydro analysis. Generalized Bender's has much the same detail as Dynamic Programming production costing, without interval analysis. Thus the modelling accuracy of maintenance, limited energy and storage plants is reduced. Screening Curves' production costing is as detailed as Generalized Bender's but is not as accurate as deterministic methodology and reliability criteria are used. Generalized Bender's, Screening Curves, Generalized Network and Linear Programming are all unable to model economic interchange due to their non-integer new unit representation.

Generalized Network and Linear Programming have the poorest production costing estimates. Both methods use block load duration curve representations which results in poor modelling accuracy. This is especially true for Linear Programming as the unit's capacity factors are determined exogenously. Generalized Network, however, is the only one of these methods that incorporates hydro reservoir operation into the production cost estimates.

3.9.2 Discrete Unit Additions

Only Dynamic Programming and Generalized Network have the capabilities to determine realizable discrete unit additions. This results in decreased capabilities in system analysis size and increased computational burdens [10]. If the new unit additions are real variables, the accuracy of the final solution is affected. Difficulties arise in the interpretation of the results as it is not always clear how to discretize variables for the final scenario. Blind rounding or truncation can result in a poor, suboptimal final scenarios [10,13]. Generalized Bender's and Gradient Projection, although they provide non-integer expansion scenarios, have the ability to approximate economies of scale and the impact of unit size on cost and reliability. The final expansion scenario, however, dictates the installation of portions of units which contradicts the economics associated with that type of unit.

3.9.3 Computer Resources

Discrete scenario methods require the most execution time and computer storage. They also tend to constrain the search area to avoid the 'dimensionality curse' and large processing times. The more complex production costing methods also require increased computer time and storage. Screening Curves and Linear Programming require the least amount of processing time. Linear Programming due to the Simplex formulation requires more computer storage than Screening Curves. Generalized Bender's and Generalized Network require large amounts of computer storage also due to their Simplex formulations and the detailed production costing. Dynamic Programming has the greatest execution times, which is highly dependant on the production costing methodology.

To give relative execution times, a software package called the Electric Generation Expansion Analysis System (EGEAS) [18] has been used to compare various methods using three standard test systems. Each optimizing method differs in production costing accuracy and in the final expansion scenarios. This is expected as the methods with longer execution times should provide better final scenarios. Using the Linear Programming method's execution time as a base for comparison the following results were obtained. Generalized Bender's required two to six times and Dynamic Programming eleven times the base execution time for the three different sized systems [18]. It can be seen from these results that the more accurate simulation and optimization techniques require substantial more processing times.

There is insufficient data to comment on the execution requirements of the Gradient Projection or Generalized Network methods, but they are expected to be comparable to those of Generalized Bender's and Linear Programming respectively.

3.9.4 Sensitivity Analysis

Dynamic Programming's sensitivity is available through realizable suboptimal scenarios. Generalized Bender's, Generalized Network and Linear Programming all have shadow prices inherent in the Simplex optimization method. Screening

Curves and Gradient Projection methods provide sensitivity analysis through the availability of dual variables.

3.9.5 Power Tool Uses

Screening Curves and Linear Programming can examine large numbers of expansion alternatives, and are therefore good for screening analysis. Due to their low computational burdens, many runs can be made to test the sensitivity of different system components. Generalized Bender's and Gradient Projection are more sophisticated screen tools as they have much better production costing models, but are limited by the size of problem they can handle. Generalized Network is specifically formulated for hydro reservoirs and thus makes its poor thermal production costing model tolerable. Dynamic Programming, due to its very high computational burden, should only be used for small systems or final discrete unit scenarios with limited number of expansion alternatives.

For a comparison of existing software packages using these optimization technique see Appendix B. More detailed comparisons can be obtained through the references outlined in Appendix C.

Chapter 4.

PROBABILISTIC SCREENING CURVES OPTIMIZATION TECHNIQUE

4.1 Introduction

There are few generation expansion optimizing programs available for use on the personal computer. Most existing personal computer software packages are limited in their scope and are not readily available. In a recent Canadian Electrical Association (CEA) survey [19] only six of twenty four generation expansion programs could optimize new unit additions. Of these six, none were implemented on a personal (micro-) computer. It is the objective of this work to develop an optimization software package for the personal computer.

This chapter outlines the development of a hybrid of existing optimization methods and its implementation on the personal computer. This new method is capable of determining a realizable yearly near-optimal hydro-thermal mixes using probabilistic costing analysis.

The optimization hybrid is based on conventional deterministic thermal optimization Screening Curves (SC). Included in the formulation are recent developments incorporating existing thermal units. This technique is then extended to a probabilistic framework called Probabilistic Screening Curves (PSC). As Probabilistic Screening Curves cannot model all probabilistic phenomena, it is coupled with a Branch and

Bound (B&B) technique to form a Probabilistic Screening Curves - Branch and Bound (PSC-B&B) hybrid. The optimization capabilities of the original Screening Curves, Probabilistic Screening Curves, and the Probabilistic Screening Curves-Branch and Bound hybrid are then compared by determining an expansion mix of a test system.

Probabilistic Screening Curves and the Branch and Bound hybrid methods are then extended to include the analysis of hydro expansion candidates. The full hydro-thermal capabilities of the optimization methods are then analyzed using the same test system with hydro candidates.

4.2 Screening Curves Optimization Basis Selection

The Screening Curves formulation was chosen to be the basis of this optimization technique because it is simple, versatile and yet efficient. It is ideally suited for the limitations of the personal computer as it does not require large amounts of computer storage, and the execution code is short and easily implemented and modified. It is a simple step-by-step method, building towards the optimal solution. The technique provides a good first pass non-integer solution approximation. It is also robust and convergence is assured.

The Screening Curves formulation is also very flexible, capable of handling large numbers of new generation alternatives with ease. It is well suited, as implied by its name, for screening analysis. Screening analysis can be loosely defined as the removal of inefficient generation alternatives. Other methods, such as Generalized Bender's, Generalized Network, Dynamic Programming and, to a lesser extent, Linear Programming, require much more storage and computer processing time for the analysis of many new unit alternatives. The Screening Curves method is even used as a pre-screening selection process for these methods to reduce the number of alternatives to be analyzed.

The Screening Curves method can be used to obtain an optimal, yearly non-integer solution efficiently, using deterministic production costing. The analyst is then left to discretize and develop a global optimal final scenario by trial and error testing.

The basic concepts of Screening Curves are easily understood and provide the analyst with a 'feel' of the system. The new unit selection process can be shown graphically and provide visually unit selection criteria and the final system mix. The visual capability of this method enables user-interaction and provides the analyst with a better understanding of the system. Since its optimality can be proven for yearly simulation, the analyst is thus assured of the optimality of the results.

4.3 Deterministic Screening Curves Optimization

4.3.1 Introduction

Conventional Screening Curves techniques for power system generation expansion optimization have been considered since the early 1940's. They were not widely used due to their inability to model existing generation. However, since the 1960's, with the improved mathematical techniques, existing generation has been incorporated in the optimization process.

The following sections outline conventional Screening Curves techniques and Levin's [34] recent nonlinear approach incorporating existing thermal generation.

4.3.2 Thermal Unit Problem Definition

The Screening Curves formulation determines the minimum total yearly costs of a power system expansion scenario using deterministic production costing. The system load is represented by a load duration curve. Both variable and fixed costs for new generating units are considered, whereas only variable costs for existing units are used as their capital costs are assumed to be sunken costs. Capital costs are determined using the Uniform Annual Payments costing method (2.6.1) accrued over the life of the generating unit. The capital costs are assumed to be constant regardless of the actual capacity used. (ie. There are no economies of scale). Variable costs are also assumed to be constant irrespective of the unit's production. Set-up costs, to bring the unit into working condition, are not considered either.

The thermal unit Screening Curves optimization problem is mathematically formulated as follows [34]:

$$\min_{\underline{X}_t} TC_t(\underline{X}_t) = \sum_{i=1}^{n} (\underline{C}_t \cdot \underline{X}_t + \underline{b}_t \cdot \underline{Q}_t) \qquad (4.3.1)$$

Such that

$$\sum_{i=1}^{n} x_{i,t} \geq L_t^m \qquad (4.3.2)$$

$$\left[\begin{array}{l} D_1 = 0 \\ D_i = \sum_{j=1}^{i-1} x_{j,t} \quad i=2, \ldots, n+1 \end{array}\right. \qquad (4.3.3)$$

$$Q_{i,t} = \int_{D_i}^{D_{i+1}} ILDC_t(x)\, dx \quad i=1, \ldots, n \qquad (4.3.4)$$

$$x_{i,t} \geq 0 \qquad i=1, \ldots, n \qquad (4.3.5)$$

$$x_{i,t} \leq Y_{i,t} \qquad , i \in I \qquad (4.3.6)$$

Variables:

- n - Number of units in the system (new and existing).
- I - Set of existing units.
- $x_{i,t}$ - Capacity of unit i in year t [MW].
- $Y_{i,t}$ - Capacity of existing unit i in year t [MW].
- TC_t - Total cost in year t [$].
- $\underline{C}_t$ - Array of fixed costs for year t [$/MW year] (=0 $i \in I$).
- $\underline{b}_t$ - Array of variable costs in year t [$/MW year].
- $\underline{Q}_t$ - Array of energy generated in year t [MW].
- $LDC(\tau)$ - Load duration curve.
- $ILDC(x)$ - Inverted load duration curve.
- L_t^m - System peak demand in year t [MW].
- τ - Normalized time [per unit].

The objective function (4.3.1) minimizes the total cost consisting of all variable costs and capital costs of only new units. It is a nonlinear formulation as the energy term is calculated from the nonlinear inverted load duration curve ILDC(x) in equation (4.3.4). The optimization is constrained to meet the system peak load equation (4.3.2), and to operate the generating units within their peak capacity (4.3.5) - (4.3.6). The total capacity of the loaded units is represented by equation (4.3.3) and is used for the energy production calculation of equation (4.3.4).

This formulation is a convex nonlinear optimization problem. Therefore, the necessary conditions for optimality, given by the Kuhn-Tucker conditions, are also sufficient [1]. Any solution which satisfies these conditions is also the global optimal [34]. Thus an optimization procedure using the Kuhn-Tucker conditions ensures a least cost mix for that year's operation.

Using Lagrangian relaxation [1], the Kuhn-Tucker conditions for optimality of the Screening Curves formulation, in equations (4.3.1)-(4.3.6), are as follows [34]:

$$c_i + R_i - u_i - u = 0 \qquad i \notin I \tag{4.3.7}$$

Where

$$R_i \equiv \sum_{j=1}^{n} (b_j - b_{j+1}) \cdot g(D_{j+1}) \tag{4.3.8}$$

$$\alpha_i + R_i - u_i - u = 0 \qquad i \in I \tag{4.3.9}$$

$$u_i \cdot x_i = 0 \qquad i=1, \ldots, n \qquad (4.3.10)$$

$$u \cdot \left(\sum_{i=1}^{n} x_i - L_m \right) = 0 \qquad (4.3.11)$$

$$\alpha_i \cdot (y_i - x_i) = 0 \qquad i \in I \qquad (4.3.12)$$

$$u_i \geq 0 \qquad i=1, \ldots, n \qquad (4.3.13)$$

$$\alpha_i \geq 0 \qquad i \in I \qquad (4.3.14)$$

$$u \geq 0 \qquad (4.3.15)$$

$$b_{n+1} \equiv 0 \qquad (4.3.16)$$

Variables:

u -Dual variable of the peak demand constraint of (4.3.2).

u_i -Dual variable of the non-negativity constraint of (4.3.5).

α_i -Dual variable of the existing units (4.3.6).

The units in this formulation are indexed by the subscript 'i'. This assigned order is the merit order which sorts the thermal units, both new and existing, into increasing variable costs. For the new units to be efficient, the corresponding fixed costs of these units must be in decreasing order.

$$b_1 < b_2 < \ldots < b_i < \ldots < b_n \quad \text{(merit order)} \qquad (4.3.17)$$

$$c_1 > c_2 > \ldots > c_i > \ldots > c_n \qquad (4.3.18)$$

This can be verified intuitively as a unit with higher fixed and variable cost, irrespective of its loading position, will always be more expensive than a unit with either a lower fixed or variable cost. If (4.3.17), (4.3.18) are not met, the unit violating these conditions is inefficient and is not considered for the optimal mix.

4.3.3 Conventional Screening Curves Methodology:

Conventional Screening Curves determine an optimal non-integer yearly mix consisting only of new thermal generation. The method does not have the capability of including existing generation in the optimization process. This formulation is of little practical value, but is the basis of Levin's [34] formulation. It also gives insight into the basic concepts of power system generation expansion. It is a special case of the more generalized formulation of equations (4.3.1)-(4.3.16). This method has been widely discussed in literature [11,21,29,34,40].

The method is based on a graphical procedure using a breakeven point approach. As new units have different fixed and variable costs, certain units operate more cheaply for specific lengths of operation. Higher fixed, lower variable cost units run efficiently for long periods of time and are base loaded. Likewise, low fixed, high variable cost units are peak loaded. The breakeven point is the operating time where two different units cost the same to operate on a per MW basis.

The breakeven time boundaries determine optimal unit operation for a particular load duration curve. This method can be seen graphically in Figure 4.1. The y-axis of the first graph is the capital costs (c_i), and the normalized time (τ) of the unit's operation is on the x-axis. The slope of the linear screening curve is the variable cost (b_i).

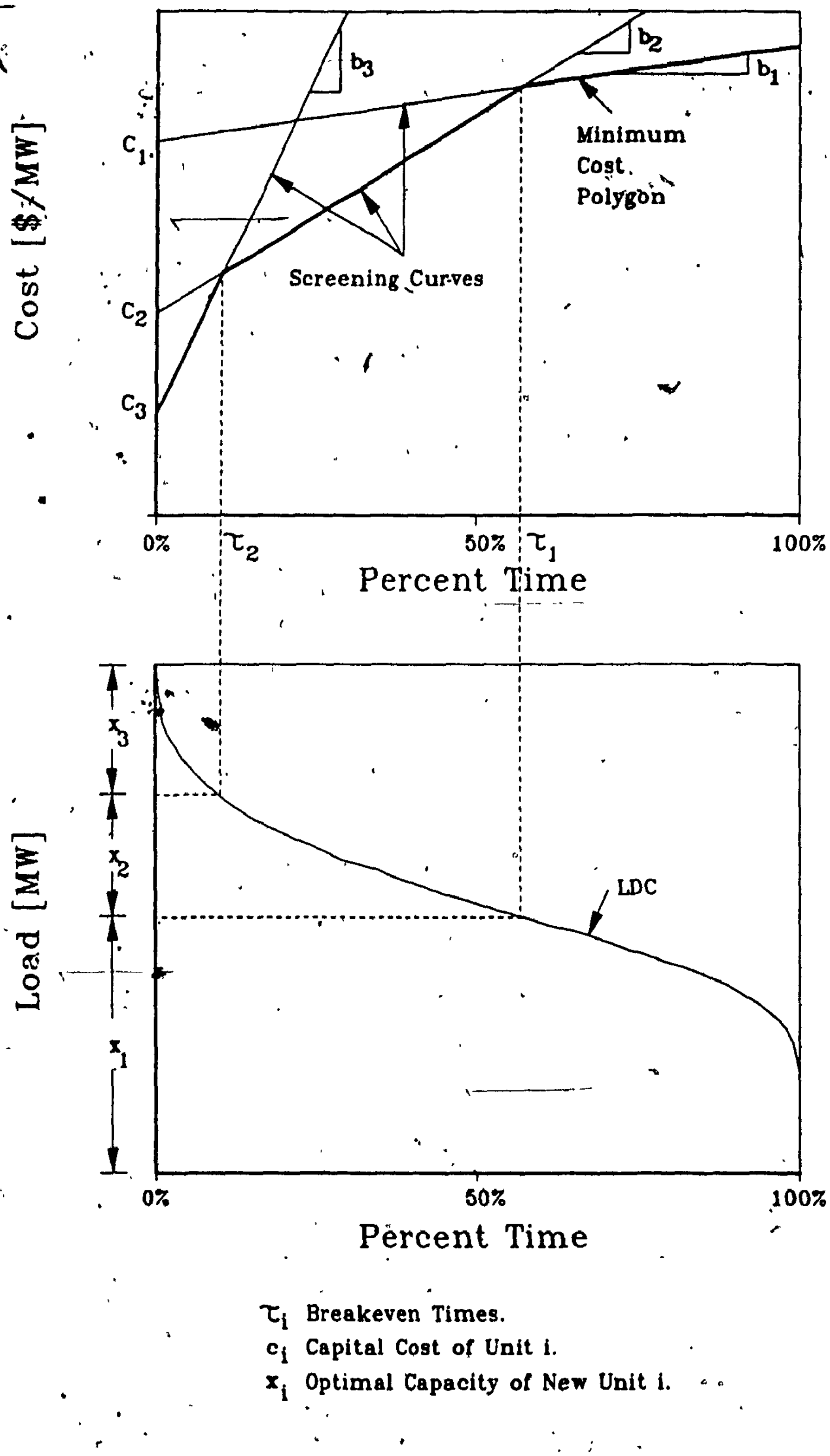


Figure 4.1 Optimal Mix Algorithm – New Units Only

Intersections of the screening curves indicate the breakeven times separating regions where one unit is more efficient to operate than another. τ_1 indicates the intersection of the screening curves for units $i=1$ and $i=2$. If operation times are greater than τ_1, it is more efficient to operate unit 1 and at lower times, unit 2. The optimal unit capacity is obtained by transposing the breakeven times (τ_i) onto the system load duration curve (LDC), resulting in their corresponding capacity (x_i) on the second graph's y-axis (MW). (See Figure 4.1). Thus optimal, non-integer deterministic new unit capacities can be determined.

Breakeven times can also be mathematically derived from the Kuhn-Tucker formulation [40]. They are calculated as follows:

$$\tau_i = \frac{c_i/p_i - c_{i+1}/p_{i+1}}{b_{i+1} - b_i} \tag{4.3.19}$$

Strict inequality signs must be used in equation (4.3.17) to avoid division by zero in the breakeven point equation (4.3.19). Screening curves cannot compare units with the same fixed costs, but intuitively the unit with the higher variable cost is inefficient. Similarly, the unit with the higher fixed cost is inefficient for equal variable costs.

The breakeven times provide boundaries inside which the units should operate to assure an optimal yearly generation mix. The specific units to be used are indicated by the minimum cost polygon. The minimum cost polygon consists of the lower envelope formed by the lowest cost screening curves [34]. (See Figure 4.1)

Screening curves not appearing in the minimum cost polygon are inefficient units. The constraints (4.3.17)-(4.3.18) do not ensure all the units will be efficient or economical. Examples of units which are uneconomical are shown in Figure 4.2. Conditions (4.3.17)-(4.3.18) are not met in (a) and it can be seen that unit 1 is more expensive to operate than unit 2. In (b) conditions (4.3.17), (4.3.18) are met, but unit 2 is still not part of the minimum cost polygon and is inefficient. Case (c) indicates a breakeven time τ_1 greater than 100% time which is physically impossible to realize, therefore the unit is inefficient. Thus for units meeting conditions (4.3.17)-(4.3.18), the following criteria will ensure the inclusion of only efficient new units.

1) $0 < \tau_i < 1$ $\quad i \in I$ (New units only.) (4.3.20)

2) $\tau_i > \tau_{i+1}$ $\quad i \in I$ (New units only.) (4.3.21)

In determining the set of efficient units the following forward recursive methodology avoids calculating all screening curve intersections, as with many new units the large number of intersections is prohibitive. Thus efficient generating units are determined as follows:

1) Calculate new unit fixed and variable costs.

2) Sort new units into merit order ($b_i < b_{i+1}$).

3) Remove uneconomical units ($c_i \nmid c_{i+1}$).

4) Next unit i (Initially i=1).

5) Next unit j (Initially j=2).

6) Calculate $\tau_k = (c_i - c_j) / (b_j - b_i)$ (4.3.19).

7) If $\tau_k \geq 1$ Then remove uneconomical unit i and go to step 4.

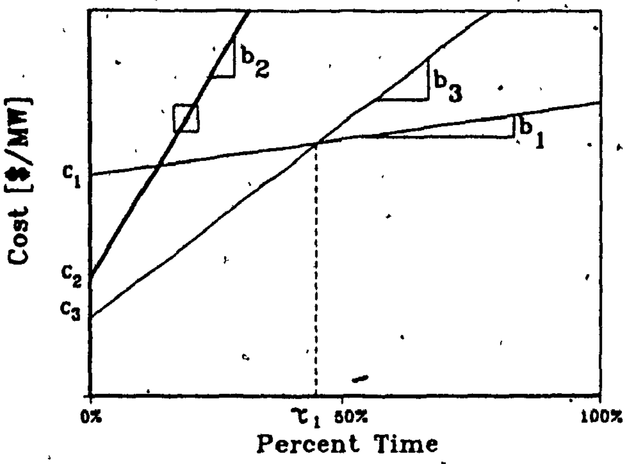
8) If $\tau_k \geq \tau_{k-1}$ Then remove uneconomical unit j and go to step 5.

9) Next k and go to step 4.

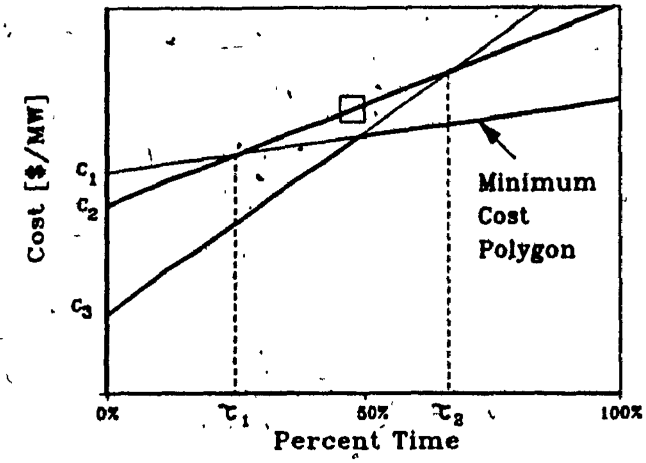
An array of breakeven points τ_k's and the economical units used in the formulation of the minimum cost polygon are thus determined. Uneconomical new units are also identified. This process is very fast and easily implemented.

The breakeven times can then be transformed into new unit capacities using the load duration curve (LDC). (See Figure 4.1.)

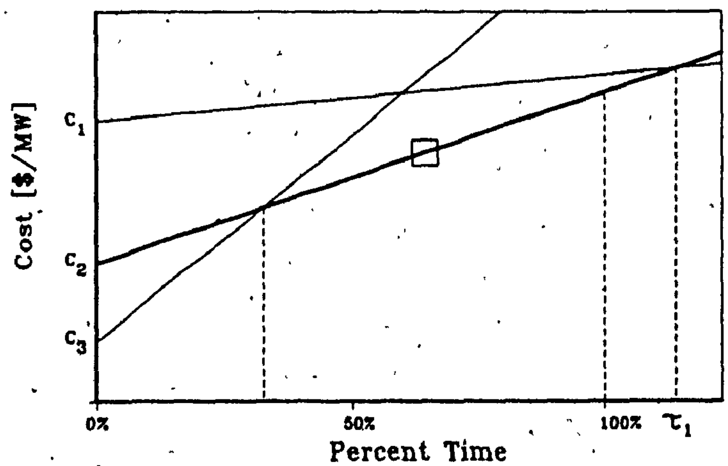
This is the conventional Screening Curves method for determining the optimal yearly, non-integer new thermal unit mix. The total cost of the optimal solution is obtained using deterministic production costing techniques.



a) $b_1 < b_2 \nless b_3$



b) Unit 2 Not Part of Minimum Cost Polygon.



c) $\tau_1 > 1.0$.

τ_i – Breakeven Points

□ – Inefficient Units

Figure 4.2 Cases in Which Some Units do Not Participate in Optimal Mix Solution.

4.3.4 Screening Curves Optimization With Existing Thermal Units

Optimization of a new unit mix is only useful for systems without any existing generation. Most expansion analysis must be performed with a mix of fixed capacity existing units. The fixed capacities of existing units cannot be modelled using conventional Screening Curves techniques. Several extensions to the conventional technique exist to determine the optimal new unit mix with existing generation [21,34]. The method outlined by Levin [34] has been selected for its simple, flexible and computationally efficient formulation.

Conceptually, the existing units are handled similarly to new units with the exception that their fixed costs are ignored and their capacities are fixed. The existing unit's screening curve's slope is fixed by their variable cost b_i as with new units. No capital cost is explicitly assigned to new units, but they do have some unknown capital worth in the optimal mix. This capital worth represents the capital 'earned' by the unit's presence in the final mix.

Graphically, the existing unit's screening curve's y-axis intercept (fixed cost) is shifted up or down until the unit's capacity, dictated by the intersections with the new unit's screening curves and the load duration curve, matches the unit's actual capacity. (See Figure 4.3.) The final y-axis intercept is the capital cost 'earned' for the existing unit being in the optimal mix and is denoted by α_i.

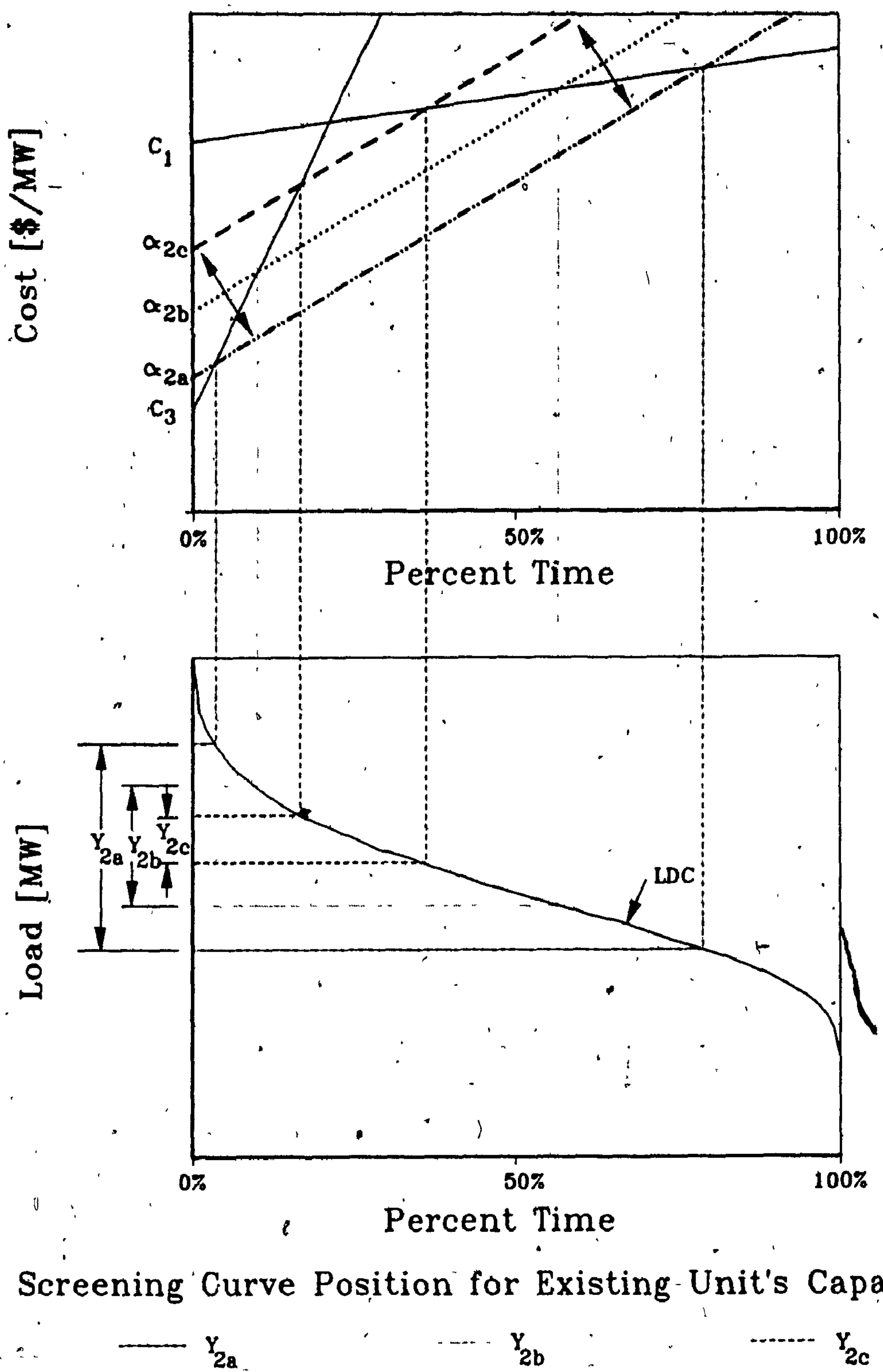


Figure 4.3 Positioning Existing Units In Optimal New Unit Mix.

When many existing units are in sequence in the merit order, they are combined together in the screening curve placement technique to form an existing-multi-unit (EMU) [34]. The loading position of the EMU is determined in a similar fashion to individual existing units. An example can be seen in Figure 4.4 where it is shown that the new units' capacities have been reduced due to the presence of existing units ($\hat{x}_i$ -> x_i).

The existing unit's y-axis intercept is denoted by the variable α_i. If α_i is less than or equal to zero, retaining the complete capacity of the unit in the final mix results in a suboptimal solution, as the unit is inefficient. The reduced capacity of the unit to remain in the solution is determined by forcing the negative α_i to be equal to zero. Thus the unit will operate efficiently at a reduced capacity. (See Figure 4.5).

The existing unit methodology can be confirmed mathematically using Kuhn-Tucker optimality conditions where α_i is a Lagrangian multiplier for existing unit's capacity. The derivation uses the boundary conditions (4.3.2), (4.3.5), (4.3.6) and the Kuhn-Tucker conditions (4.3.7)-(4.3.16) to arrive at the following formula indicating the existing unit's optimal loading point. (See [34,35] for derivation.) A one-dimensional search, using this formula, locates the optimal loading points of the existing units.

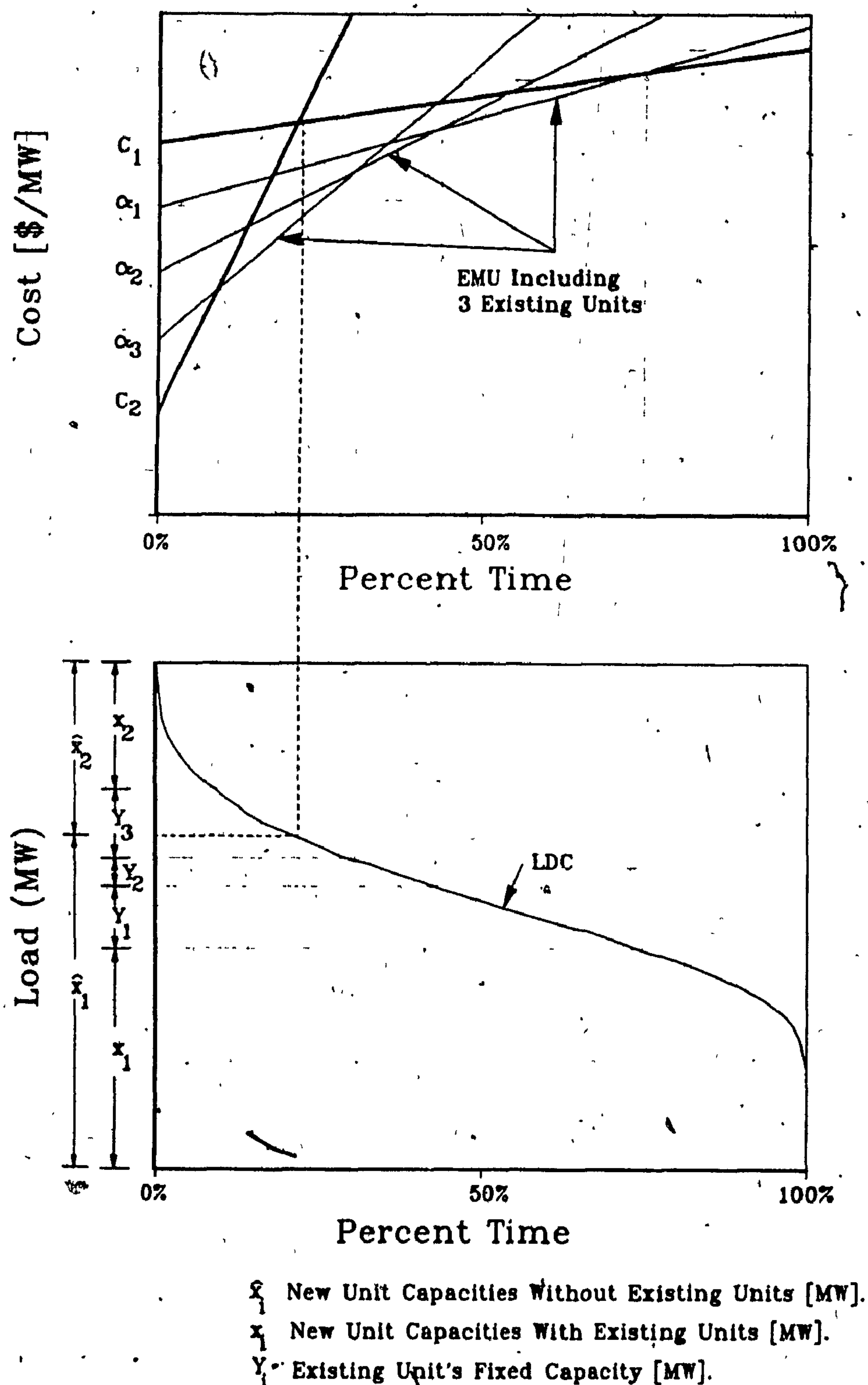
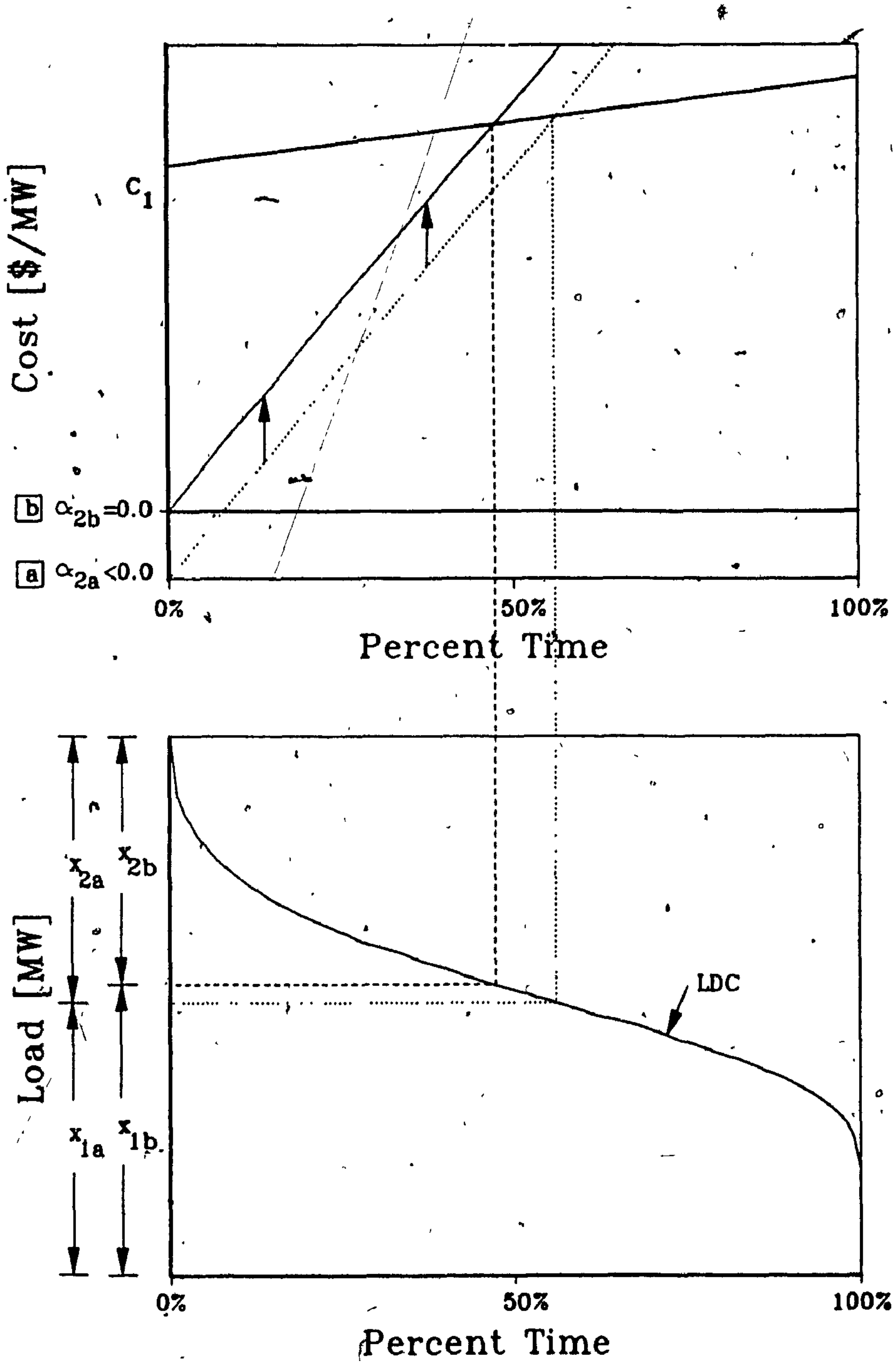


Figure 4.4 Existing Multi–Unit in Optimal New Unit Mix.



a The new unit is inefficient as α_{2a} is less than zero.

b The inefficient EMU x_{2a} is reduced to x_{2b} by forcing α_{2a} to be equal zero. The EMU operating at this capacity is efficient.

Figure 4.5 Inefficient Existing Multi–Unit – Positioning.

This formulation assumes all existing units are efficient and is as follows [34]:

$$q(\hat{r},\hat{s},\hat{D}) = \sum_{j=r}^{s-1} (b_{j+1} - b_j) \cdot \mathrm{ILDC}(\hat{D} + \bar{D}_{j+1}) - c_{\hat{r}} + c_{\hat{s}}$$

$$+ (b_r - b_{\hat{r}}) \cdot \mathrm{ILDC}(\hat{D}) + (b_{\hat{s}} - b_s) \cdot \mathrm{ILDC}(\hat{D} + \bar{D}) \qquad (4.3.22)$$

Subject to:

a) The loading points of the new units before and after the EMU:

$$\hat{D}_{\hat{r}} \leq \hat{D} \leq \hat{D}_{\hat{r}+1} \qquad (4.3.23)$$

$$\hat{D}_{\hat{s}} \leq \hat{D} + \bar{D} \leq \hat{D}_{\hat{s}+1} \qquad (4.3.24)$$

b) $\hat{s}$ is a new unit loaded after the EMU:

$$s < \hat{s} \leq n+1 \qquad (4.3.25)$$

c) $\hat{r}$ is a new unit load before the EMU:

$$1 \leq \hat{r} < r \qquad (4.3.26)$$

Variables:

- $\bar{D}$ - Total capacity of EMU [MW].
- D - Loading point of EMU [MW].
- $\bar{D}_i$ - Total capacity of EMU up to existing unit i [MW].
- $\hat{r}$ - Index of the new unit directly before the first unit of the EMU.
- $\hat{s}$ - Index of the new unit directly following the last unit of the EMU.
- r - Index of the first existing unit in the EMU.
- s - Index of the last existing unit in the EMU.

(Incorporating inefficient units requires only slight modifications as the basic reasoning is similar.)

The optimal loading point for the EMU is determined when $q(\hat{r},\hat{s},\hat{D}) = 0$ in the domain bounded by (4.3.23), (4.3.24). As $q(\hat{r},\hat{s},\hat{D})$ is a monotonically decreasing function and the optimal existing unit placement can be uniquely determined [34]. The optimality assumes that the bounds of the search variable $\hat{D}$ point the search procedure inward. If they do not, the bounds are incorrect and need to be modified.

The bisection search method was chosen to solve this one-dimensional problem. It is a simple, efficient technique which is easily constrained to specified search areas. It converges quickly and can be terminated with known limits. (See [9] for the bisection formulation.)

If the EMU is loaded in the first or last position in the merit order, the search process is not required as its positions and capacities are known. Formulations where all existing unit's capacities must be used, or units are inefficient are similar to the above formulation. Further details can be obtained in [35].

The method for determining the optimal loading point of an EMU can be summarized as follows:

1) Determine efficient new unit τ_i's.
2) Determine EMU bounding new units and boundaries for the search area such that $q(,,D_{lower}) > 0$, and $q(,,D_{higher}) < 0$ and meet equations (4.3.23) - (4.3.24).
3) If EMU is the first or last unit in the merit order, STOP.

4) Determine optimal EMU loading point such that $|q(,,\hat{D})| <$ tolerance.

4.3.5 Optimization With Multiple Existing-Multi-Units

This methodology can easily be generalized to multiple existing-multi-units. Each EMU is determined and processed independently as indicated above. If any EMU's overlap, they are combined into one larger EMU and its optimal loading point is determined. (See [35] for further details.)

Screening Curves enables the annual non-integer optimal thermal unit mix to be determined simply and efficiently. The analyst can be assured of the optimality of the final mix as the method is mathematically optimal. Screening Curves incorporates existing thermal generation and also has the ability to identify inefficient new units. The capacity of existing units is properly modelled and an optimal non-integer solution is obtained.

4.4 Probabilistic Screening Curves Optimization

The previously described Screening Curves technique provides an optimal non-integer yearly scenario. It is limited by its inability to handle discrete unit sizes, economies of scale and probabilistic modelling of forced outages. A more realistic probabilistic production costing method must be used to account for these effects in the overall system cost and in individual unit operation [18]. Thus, while retaining the simplicity of the Screening Curves formulation but using the superior probabilistic costing method, a new optimization method is developed call Probabilistic Screening Curves (PSC).

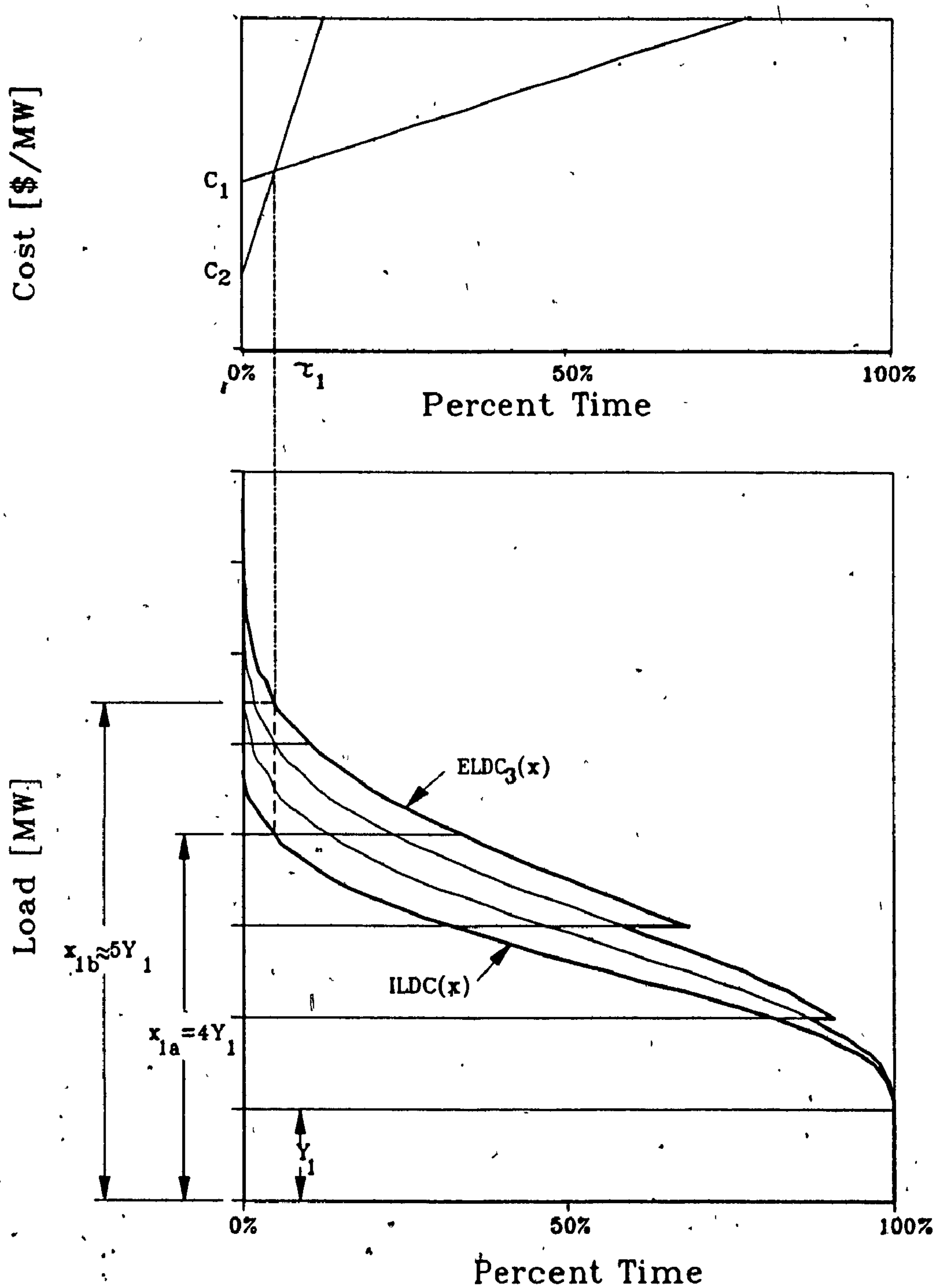
The Screening Curves method uses a variable versus fixed cost breakeven point approach to determine the optimal capacities of new units. The load duration curve transforms the breakeven times to optimal new unit capacities. Transferring this principle to a probabilistic setting, the optimal new unit capacity is taken from the breakeven time intersection of the equivalent load duration curve (ELDC) rather than the load duration curve (LDC). As the equivalent load duration curve changes with the loading of each unit, a solution similar to the one step conventional Screening Curves problem is no longer possible. The optimal screening curves mix must also change to reflect each new equivalent load duration curve. Thus optimal new unit capacities must be determined iteratively. (See Figure 4.6).

The complete recalculation of the screening curve mix at the loading of each unit is too slow and computationally expensive. Examining the unit loading convolution process, Equation (2.4.1) and Figure 2.3, it can be seen that the equivalent load duration curve's values will never decrease. Even if a hypothetical unit's forced outage rate equals zero percent, the equivalent load duration curve will not decrease as it is not modified. For non-zero forced outage rates the load increases due to the loading of each unit.

Using this property of unit convolution, the optimal capacity divided by the new unit's capacity determines the number of new units to be loaded. All these units can be loaded before the screening curve calculations are redone. (See Figure 4.6). The same is true with existing units. Once an existing multi-unit (EMU) is encountered in the loading order, by definition of the EMU, no new units are loaded in between individual existing units. The new units are optimally placed before or after the existing multi-unit.

The original deterministic definition of the breakeven point presents problems due to probabilistic handling of the forced outage rate. The actual operating length of the generating unit is not defined by the equivalent load duration curve due to the unmet energy of the unit's forced outage rate. The unit's actual length of operation is the value of the equivalent load duration curve divided by the unit's availability. Thus the breakeven point is divided by the availability of the new unit.

The breakeven point, however, is determined by the comparison of two units. It must also be modified for the new unit above the breakeven point. It will have the opposite affect on the breakeven point, and thus this unit's availability is multiplied with the breakeven point.



Conventional Deterministic Screening Curves would add four new units. Probabilistic Screening Curves, due to the Probabilistic effect of the Forced Outage Rates, would add five new units of type 1 as the load has shifted.

Figure 4.6 Probabilistic Screening Curves–Determining New Unit Capacity.

The breakeven point is modified to the following form

$$\tau_i = \frac{c_i/p_i - c_{i+1}/p_{i+1}}{b_{i+1} - b_i} \cdot \frac{p_{i+1}}{p_i} \qquad (4.4.1)$$

where: $p_i = (1-q_i)$

This breakeven point formula was confirmed experimentally to give better results than the conventional screening curve breakeven point formula.

The Probabilistic Screening Curves analysis is summarized as follows:

1) Calculate new unit breakeven points (4.4.1) and identify inefficient new units.
2) Group existing units into existing multi-units (EMU's).
3) Determine optimal non-integer mix with most recent ELDC.
4) Load similar typed units first in the merit order and update ELDC.
5) Have reliability criteria (LOLP or EUE) been met? If not go to step 3.

Thus the final yearly probabilistic screening curve integer near-optimal mix is determined.

4.4.1 Advantages

The main advantage of this methodology is its ability to enable Screening Curves to use the more realistic probabilistic production costing technique. The unit mix can now reflect in the system costs discrete unit additions, economies of scale and model the cost of the unit's forced outage rate. Differences between probabilistic and deterministic Screening Curves are especially noticeable with the final costs and mix of the peaking units as deterministic production costing is not as accurate. The probabilistic production costing also enables the use of probabilistic reliability criteria.

4.4.2 Disadvantages

The disadvantages of this modification to the Screening Curves method are the new problems that are presented by the probabilistic nature of modelling units. The method now has the ability to model these many desirable criteria, but the Screening Curves' unit selection process is not designed for them. The unit selection is based only on fixed and variable costs which does not reflect the unit's capacity. Screening Curves cannot model the effect of two similar non-equally sized generating units on the total system generation capacity required to meet a probabilistic reliability criteria.

Thus the optimality of the Probabilistic Screening Curves methodology cannot be asserted. It is, however, based in an optimal non-integer deterministic optimizing method. Thus, confirmed by testing, the Probabilistic Screening Curves method provides a near-optimal yearly mix. In this method the probabilistic production costing is more accurate, but the new unit selection cannot represent these phenomena.

This method in its attempt to represent the effect of whole unit additions, cannot of itself, determine how partial units are to be handled. Heuristics must be used to either round or truncate the final scenario.

4.4.3 Implementation

Probabilistic Screening Curves is computationally less efficient than the deterministic Screening Curves techniques. As the screening curves mix must be recalculated each time a new type of unit is loaded, the following methods can be incorporated to speed up this process.

The values of the new unit's breakeven points (τ_i) need not be recalculated as they are dependant only on the new unit data and not the system load. The original grouping of the existing multi-units (EMU) will not change, but combined EMU's may need to be separated. Therefore the original EMU grouping is retained and need only be regrouped if necessary.

The results of the previous solution is a good starting point for the following Screening Curves optimization, and can greatly speed up the solution process through reduced search areas. Further computational savings can also be realized using the properties of the equivalent load duration curve. Search directions can be predicted due to unit additions.

4.5 Branch and Bound Optimization

Probabilistic Screening Curves incorporated probabilistic production costing into the deterministic Screening Curves methodology. However it cannot ensure an optimal probabilistic scenario due to limits inherent in its formulation. It does, however, provide a good first approximation to the optimal solution.

Probabilistic Screening Curves can efficiently compare many different types of units. As this methodology provides a discrete solution and is near to an optimal mix, it was decided to vary the Probabilistic Screening Curves mix and improve the final mix. The method chosen to perform this integer search is the Branch and Bound search technique.

The Branch and Bound search technique successively tests the addition of a type of new unit in an attempt to decrease the total system cost. It continues to do so until the total cost starts increasing. Thus this branch (or new unit type) of the search is bounded and another branch (generating type) is tried [14].

This process if incorporated for all the different combinations of new units would be too slow due to the immense search area. If indeed it were incorporated, the search area would resemble that of the Dynamic Programming method. This would require significant amounts of computer resources which are unavailable on the personal computer. To limit the search area, the near optimal mix of the Probabilistic Screening Curves method is used.

As base loaded units affect the rest of the system mix, new units are optimized according the 'merit' order. The Branch and Bound technique is used to add or subtract a new unit and the rest of the system mix is determined with the Probabilistic Screening Curves. The number of new units of this type is varied until the total system cost increases, and thus the lowest cost mix of this type is determined. The Branch and Bound technique is then applied to the next new unit in the 'merit' order. Once this process is complete, a new optimal yearly solution is obtained. Even though this is a binary search technique, global yearly optimality cannot be assumed as the use of Probabilistic Screening Curves method does not assure optimality.

The cost function, due to the probabilistic production costing and whole unit additions, does not provide a monotonically decreasing objective cost function. Thus the termination of the new unit additions or subtractions of a particular type must be based on heuristics with an appropriate efficiency versus accuracy tradeoff.

Each iteration of the Branch and Bound technique is computationally more expensive than just the probabilistic production costing due to the Probabilistic Screening Curves calculations. However, the computational savings outline in the Probabilistic Screening Curves technique can be used.

When a branch is bounded for one unit type, the equivalent load duration curve can be modified up to the new unit's loading level and be used as the starting point for all new unit tests. Thus, as the yearly optimization proceeds, the probabilistic production costing and screening curve calculations are reduced. Analytical production costing can also speed up this process due to its faster methodology.

The main advantage of the Probabilistic Screening Curves - Branch and Bound hybrid is its ability to quickly select a yearly near-optimal power generation expansion scenario using probabilistic production costing. Probabilistic Screening Curves - Branch and Bound hybrid will provide a lower cost scenario than Probabilistic Screening Curves as the hybrid can properly cost and model the effects of units size, economies of scale and forced outages. The yearly expansion scenario is realizable as it is an integer solution. The method is efficient, can be incorporated within the limited resources of a personal computer and yet can still handle large numbers of new unit alternatives.

The Branch and Bound technique provides alternative scenarios which also give an indication as to the final solution's sensitivity. This enables the selection of other more expensive expansion scenarios which are better suited to other external considerations. It also aids the analyst in developing a global expansion sequence from the alternative yearly mixes. The unit selection is conceptually easily understood and cost tradeoffs are readily available to the analyst.

The main disadvantage of this method is that the final solution is only a near-optimal yearly optimization. Due to the complexity and size of the problem, in the context of power system generation expansion tools, this is quite acceptable and common. Even 'optimal' methods such as Dynamic Programming and Generalized Bender's cannot guarantee optimality due to restricted search areas and non-integer solutions respectively. Few methods, due to the nonlinear nature of the probabilistic production costing, can guarantee optimality, and even those that do, cannot necessarily be implemented due to other practical considerations.

This method is thus, a compromise. It sacrifices known optimality for decreased computer resources, an integer solution, probabilistic costing and the availability of suboptimal plans. As generation expansion involves much more than just straight financial considerations, this results in this tool being practical but not mathematically optimal.

4.6 Test Cases and Results

The objective of this work was to develop a fast, efficient method of determining a near optimal generation sequence for use on the personal computer. The Probabilistic Screening Curves (PSC) and Probabilistic Screening Curves - Branch and Bound (PSC-B&B) hybrid methods were implemented in FORTRAN. They were combined with an existing Power System Generation Expansion Analysis program kindly made available by Shawinigan Consultants Inc. The system's analysis package is called the Advanced Power System Planning and Production Costing Program (SYPCO) which is available on the personal computer. SYPCO has been used on many international projects and its analysis capabilities have been verified with the Wein, Automatic System Planning Package, an industrial standard. All analysis was performed on a personal (micro-) computer as this was the intent of the research.

Excessive computational requirements are a problem when analyzing large systems with hydro projects. Thus to test the capabilities of the Probabilistic Screening Curves, and Probabilistic Screening Curves - Branch and Bound hybrid methods, such a system was chosen. The data and results used in the testing were also made available by Shawinigan Consultants Inc. The system consisted of a twelve year optimization initially of only thermal units candidates, and was then extended to both hydro and thermal candidates. In this section, the overall system will be described and the thermal candidates available for installation. Hydro optimization is discussed in the subsequent sections.

The test system is based on realistic data, but is not available for full public scrutiny. The system has a capacity of 30,000 MW in the first year of simulation increasing to 52,000 MW in the twelfth year. The load energy increases from 193,000 GWh to 325,000 GWh in the same time frame. This corresponds to about five percent load growth per year and a load factor decreasing from 73% to 71.3%. The discount rate was set at 10% per annum and no escalation was assigned for relative fuel cost changes. The target reliability criteria was a Loss of Load Probability (LOLP) of fifty hours per year based on annual system analysis and deterministic treatment of hydro plants. Production costing was determined using annual probabilistic Booth-Baleriaux annual methodology and the maintenance was modelled using the derated capacity method. (See Table 4.1 for a summary of the system).

The system Load Duration Curve varied in shape as seen in Figure 4.7 (b) throughout the simulation period. The hydro plants for the thermal testing of this section were treated as existing plants and installations dates were taken from the optimized scheme of the project. The hydro plants were inserted under the annual Load Duration Curve based on their average energy production. The resulting Load Duration Curve was used for the thermal optimization.

The existing generation at the first year of simulation consisted of 40 % hydro and 60 % thermal units. The existing thermal ranged in size from 50 MW to 1,000 MW. These thermal

Table 4.1 Power System Used For Expansion Testing

SYSTEM LOAD

Target LOLP = 50 hours/yr   Discount Rate = 10%

	Year 1	Year 6	Year 12
Capacity (MW)	26,468	36,420	49,907
Energy (GWh)	169,813	229,757	310,086

EXISTING GENERATION

THERMAL (Coal)

Capacity Range	Number of Units	FOR [%]	Maintenance [Days]
0 < Y < 100 MW	7	11.0	56
100 < Y < 200 MW	6	11.0	56
200 < Y < 300 MW	13	11.0	56
Y > 400 MW	9	11.0	56

THERMAL (Nuclear)

Capacity Range	Number of Units	FOR [%]	Maintenance [Days]
Y = 1,000 MW	3	11.0	28

HYDRO

Capacity Range	Number of Units Year 1	Number of Units Year 6	Number of Units Year 12	FOR [%]
0 < Y < 100 MW	6	6	6	3.0
100 < Y < 200 MW	13	13	13	3.0
200 < Y < 300 MW	11	11	11	3.0
300 < Y < 400 MW	14	14	16	3.0
Y > 400 MW	4	7	10	3.0

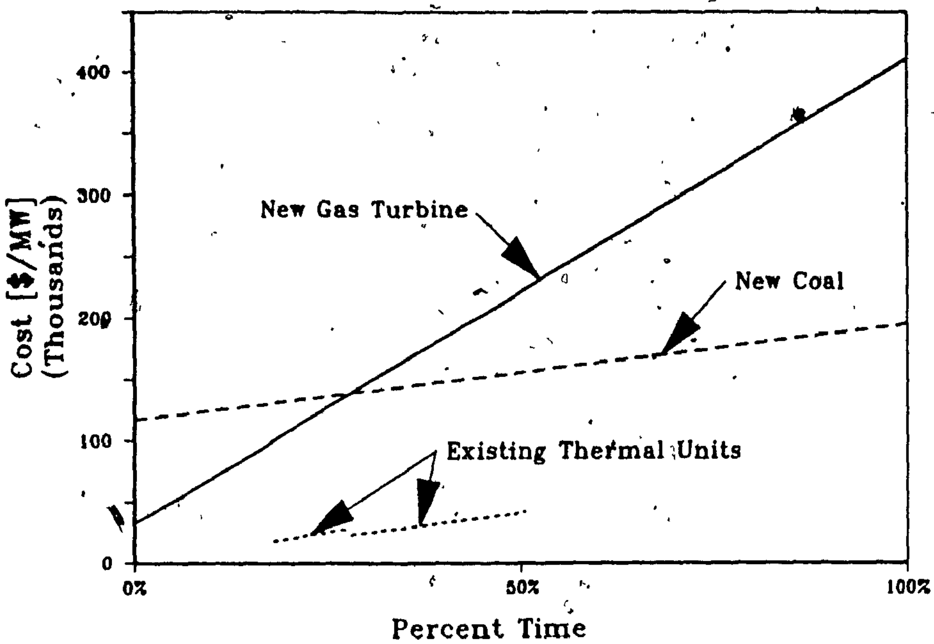
NEW GENERATION

THERMAL (Coal)

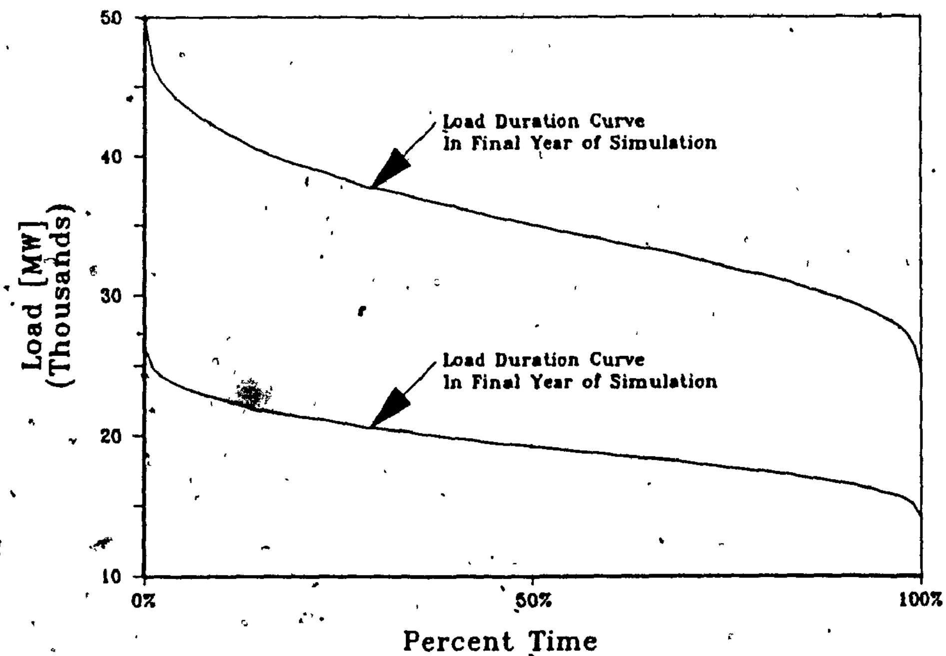
600 MW, FOR = 16 %, Maintenance = 42 days, Life = 30 years

THERMAL (Gas Turbine)

100 MW, FOR = 11 %, Maintenance = 14 days, Life = 20 years



a) New Thermal Unit Screening Curves.



b) System Load Duration Curves.

Figure 4.7 Thermal Comparison - Screeing Curves and System LDC.

plants consisted of Coal and Gas Turbine units with three nuclear plants scheduled for installation during the simulation period. See Table 4.1 for a summary of the existing generation. Figure 4.7 (a) shows graphically the variable costs of the existing thermal units and their approximate operating ranges.

To enable consistent comparisons, the capital costs of the hydro units optimized in the next section's analysis are included in the total costs indicated in the results. Existing unit's capital costs are normally assumed to be sunken costs and are not included.

The system's expansion candidates are 600 MW coal and 100 MW gas turbine units. Nuclear units were also originally included in the candidates for optimization but were determined to be uneconomical by the Screening Curves. Thus, they were forced into the unit sequencing as existing units for political rather than economical reasons. There were no limitations on the number of either unit type installed; thus the unit additions were purely based on present worthed uniform annual payment economic comparisons. The new thermal units' characteristics are summarized in Table 4.1 and in Figure 4.7 (a).

The optimization method used to determine the reference generation sequence was obtained using a heuristic deterministic optimization program called the Generation Sequencing Program (GSP) [23]. (See Appendix B for details of the program). GSP was calibrated to the probabilistic methodology of SYPCO such that an approximate reserve criteria would correlate to the desired Loss of Load Probability (LOLP) criteria. The optimal yearly sequence was then expanded to a realizable global optimal expansion sequence. (As the reference optimization was performed to meet a deterministic reliability criteria, the reference number of peaking units were not used, but were determined using an annual loss of load probability (LOLP) criteria). Thus, the reference sequence has peaking units added or subtracted to the actual 'optimal' sequence ensuring consistent comparisons. This expansion sequence is referred to as the reference sequence.

4.6.1 Test Methodology

Three thermal optimizations were performed and then compared to the reference sequence described above. The first two were the Probabilistic Screening Curves and Probabilistic Screening Curves - Branch and Bound hybrid outlined in the previous sections.

For comparison purposes, the Screening Curves optimization outline by Levin [34] and introduced in section 4.3 was also included. The deterministic Screening Curves mix was obtained and then discretized by rounding the non-integer mix to the nearest integer value. This discrete mix was then probabilistically production costed and new peaking units added until the desired LOLP criteria was met.

Each optimization was performed on an annual basis using similar data. The results are shown graphically in Figures 4.8 - 4.11. The total cost comparison (Figure 4.8) is the yearly cost consisting of the capital costs of new thermal units (and 'new' hydro) plus the variable costs of all units. As the sequence has not been globally optimized the costs are not present worthed and only total yearly costs are compared.

It can be seen from Figure 4.8 that there are no substantial differences in the yearly total costs between the optimization methods. It should be noted, however, that deterministic Screening Curves has consistently higher costs. These, over longer simulation periods, could amount to substantial costs. Thus the merit of the Probabilistic Screening Curves is demonstrated as the costs are lower than the deterministic Screening Curves. The Probabilistic Screening Curves - Branch and Bound (PSC-B&B) hybrid can be seen to be equal to or slightly less than the Probabilistic Screening Curves (PSC) as is expected. As PSC and PSC-B&B are yearly optimizations, the cost is slightly lower in year seven than the reference mix.

The mix of the new unit additions (Figures 4.10, 4.11) demonstrate a close similarity between the number of coal units added for PSC-B&B and PSC compared to the reference sequence. The reference sequence tends to install slightly more coal units replacing gas turbines. This is to be expected due to shortsightedness of the yearly optimization.

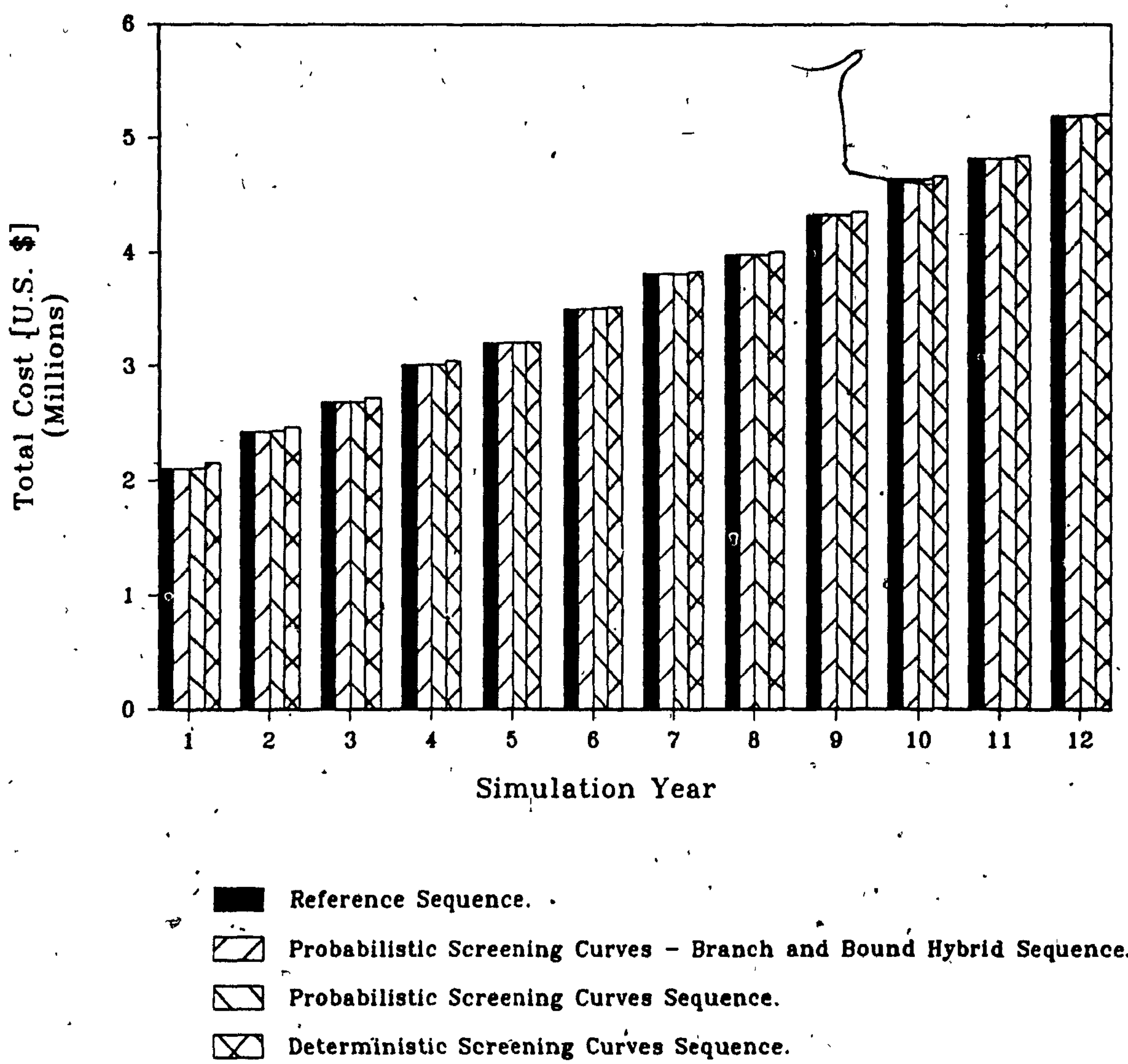


Figure 4.8 Thermal Comparison - Total System Cost.

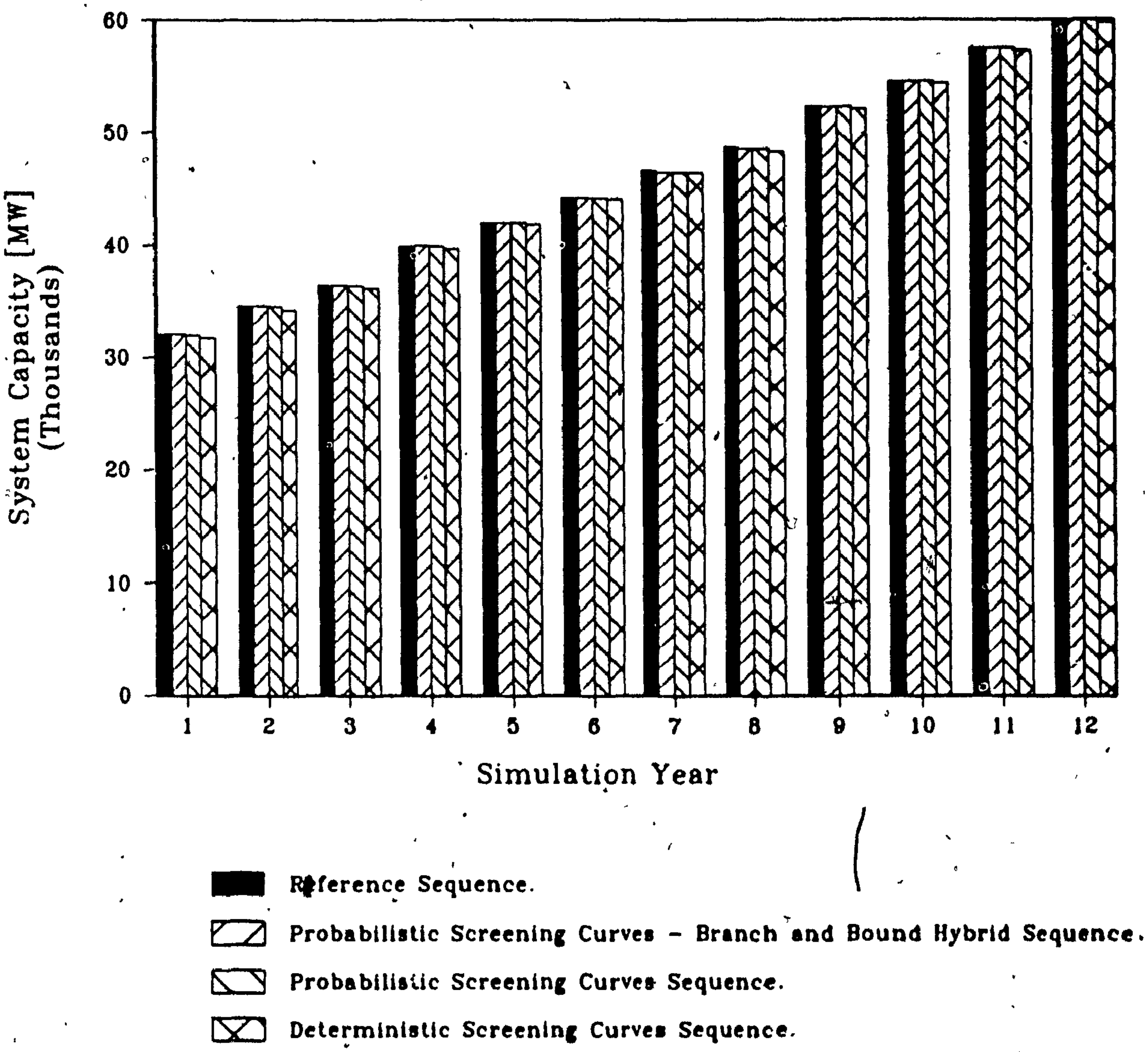


Figure 4.9 Thermal Comparison - Total System Capacity.

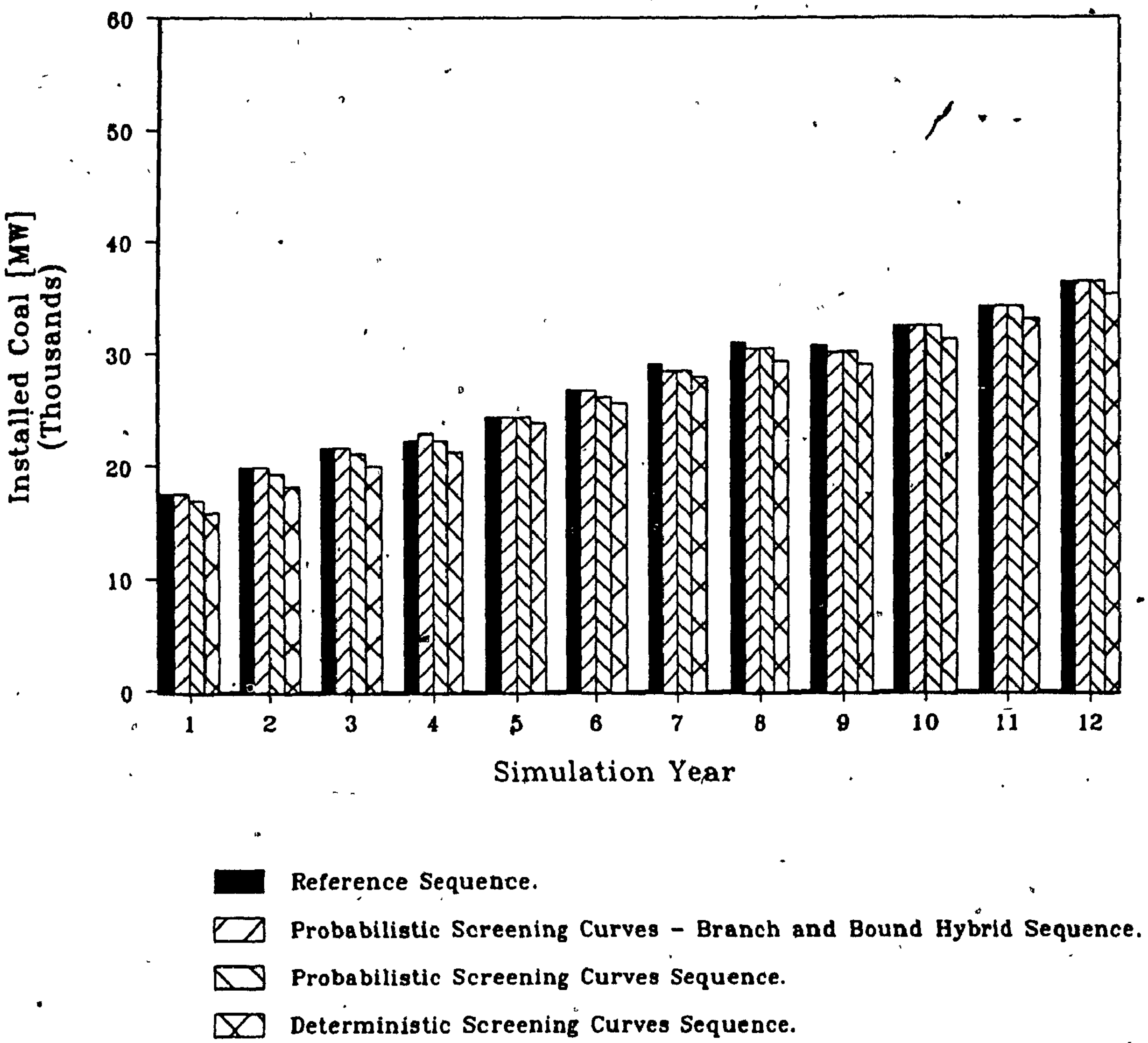


Figure 4.10 Thermal Comparison – Installed Coal.

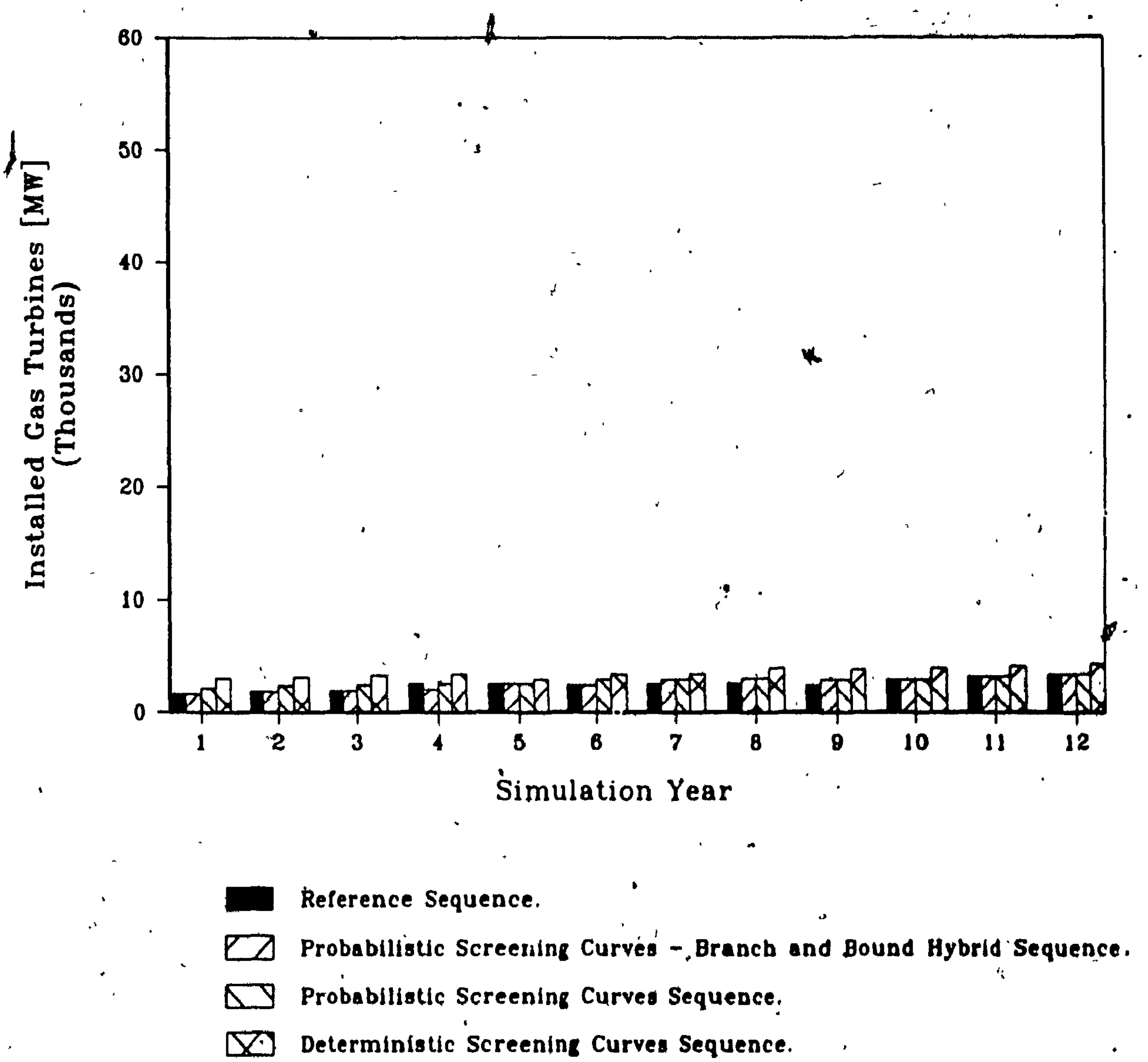


Figure 4.11 Thermal Comparison - Installed Gas Turbines.

The deterministic Screening Curves method consistently adds more gas turbines to the mix. This is due to improper deterministic modelling of the unit's forced outages. The peak of the load duration curve is not modified appropriately with deterministic production costing.

Thus Probabilistic Screening Curves and Probabilistic Screening Curves - Branch and Bound were able to determine a near optimal yearly optimization for this system and were shown to be superior to deterministic Screening Curves.

4.7 Hydro Units

Thus far only thermal units have been considered in the optimizing process. New hydro units must be treated separately from new thermal unit optimization, due to their lack of variable costs and limited energy generation. Hydro projects have fixed capacities and energies, and it is most economical to use both of these to their full capabilities [18]. Thus they have only one loading point on the load duration curve. (In the ensuing discussion, hydro units are treated deterministically due to the small probabilistic impact of these high reliability plants. Large processing times are required for their probabilistic treatment, it was deemed not worth the extra computational effort. The methodology, however, is easily transferred to a probabilistic treatment.)

A new hydro project candidate is either installed in a given year (at its prespecified capacity and energy rating) or it is ignored. This is contrasted to new thermal candidates where many units of the same type can be installed over the simulation period. As hydro plants are site specific, they must be treated individually.

The methodology incorporating hydro unit selections into the Screening Curve methodology as proposed by Levin [33] was not considered practical due to its complex nature and limitation of considering only one new project a year. Therefore the following method was used.

New hydro projects were evaluated by determining the optimal total cost before and after the project was installed. These costs were compared and if the project was economical, it is kept and the next new hydro project was considered. The hydro projects were sorted such that the least expensive projects were considered first. Thus the first unit is considered and if it reduces the total system cost, it is kept and the next unit is considered.

An optimal solution would require all combinations of new hydro projects to be tried. This type of exhaustive search is unnecessary and for large systems, beyond the capabilities of the personal computer. Thus a near-optimal solution is obtained through this methodology.

Determining whether to include a new hydro unit in the final mix can be determined in two ways. The desired accuracy and the computer resources available will determine which method is used. The first method compares the total system costs using the Probabilistic Screening Curves and the second uses the Probabilistic Screening Curve - Branch and Bound hybrid optimization.

The Probabilistic Screening Curves method is summarized as follows:

1) Determine the system cost of all the thermal system using Probabilistic Screening Curves (PSC).
2) Add a new hydro unit.
3) Determine the new system cost using PSC.
4) Compare total system costs.
   If cheaper, keep hydro unit.
5) If more new hydro candidate plants exist, go to step 2.
6) Determine the near optimal Branch and Bound (B&B) thermal solution if desired.

The second method, using Branch and Bound optimization for all hydro tests, is summarized as follows :

1) Determine system cost of all thermal system using PSC.
2) Determine near optimal all thermal B&B solution.
3) Add a new hydro unit.
4) Determine the new system cost using PSC.

5) Determine the near optimal system cost using B&B.

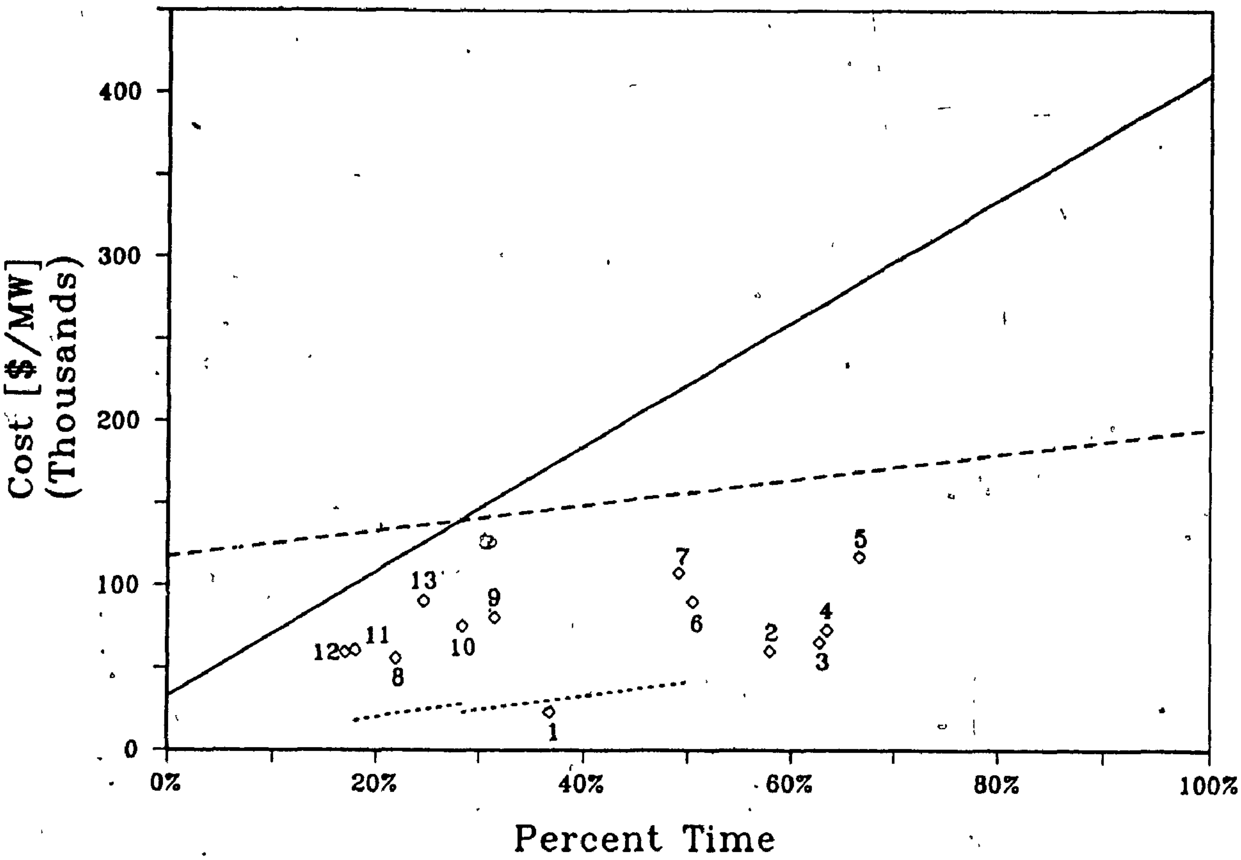
6) Compare total system costs.
   If cheaper, keep hydro unit.

7) If more new hydro candidate plants exist, go to step 2.

The first method (PSC) is very dependent on the uncertain optimality of the Probabilistic Screening Curve methodology and is not as accurate as the second method (PSC-B&B). The Branch and Bound comparison should be used if the computer resources are available and for determining final scenarios. Differences between the two methods can be seen in the following test cases minimum cost scenarios.

4.8 Hydro Test Cases

In this section the outlined hydro sequencing methodology is tested. The power system used to test the hydro optimization capabilities is the same system outlined in section 4.6 except not all the hydro generation is prespecified. Thirteen candidate hydro plants are available for installation the first year of simulation. Each has different capital costs, capacities and energy availabilities. Their cost and operating range relationships to the thermal expansion candidates are shown graphically in Figure 4.12. In this figure the hydro expansion candidates are placed at the intersection of their fixed cost and operating capacity



······ Existing Thermal Units (Approximate Operating Range).
--- New Coal Unit's Screening Curve.
—— New Gas Turbine Unit's Screening Curve
◇ New Hydro Unit i.

Figure 4.12 Hydro Comparison – Screening Curves.

factor. The hydro units are ranked according to increasing cost per capacity factor for the optimization selection.

The same test procedure for thermal only units was used to test the hydro optimization. Screening Curves, Probabilistic Screening Curves, and Probabilistic Screening Curves-Branch and Bound (PSC-B&B) hybrid were all compared to the reference case. (The reference case was determined in the same manner as the thermal sequence, where GSP was used to obtain a hydro-thermal expansion sequence. This sequence corresponded closely to the sequence obtained from WASP). These test results are summarized graphically in Figures 4.13 to 4.18.

The Probabilistic Screening Curves, and Probabilistic Screening Curves - Branch and Bound hybrid test results are very similar in total cost and mixes. They differ in the particular hydro units which are installed in a few simulation years, but do not result in significant cost or mix differences. The PSC-B&B hybrid new unit addition costs are less than or equal to that of the Probabilistic Screening Curves. This is expected as the PSC-B&B is an extension of the Probabilistic Screening Curves methodology. Both methods, as can be seen in Figure 4.18, do not result in realizable sequences as units are added and then removed the following year. This is caused by the non-global optimization, and is expected. It is left up to the analyst to determine a realizable sequence

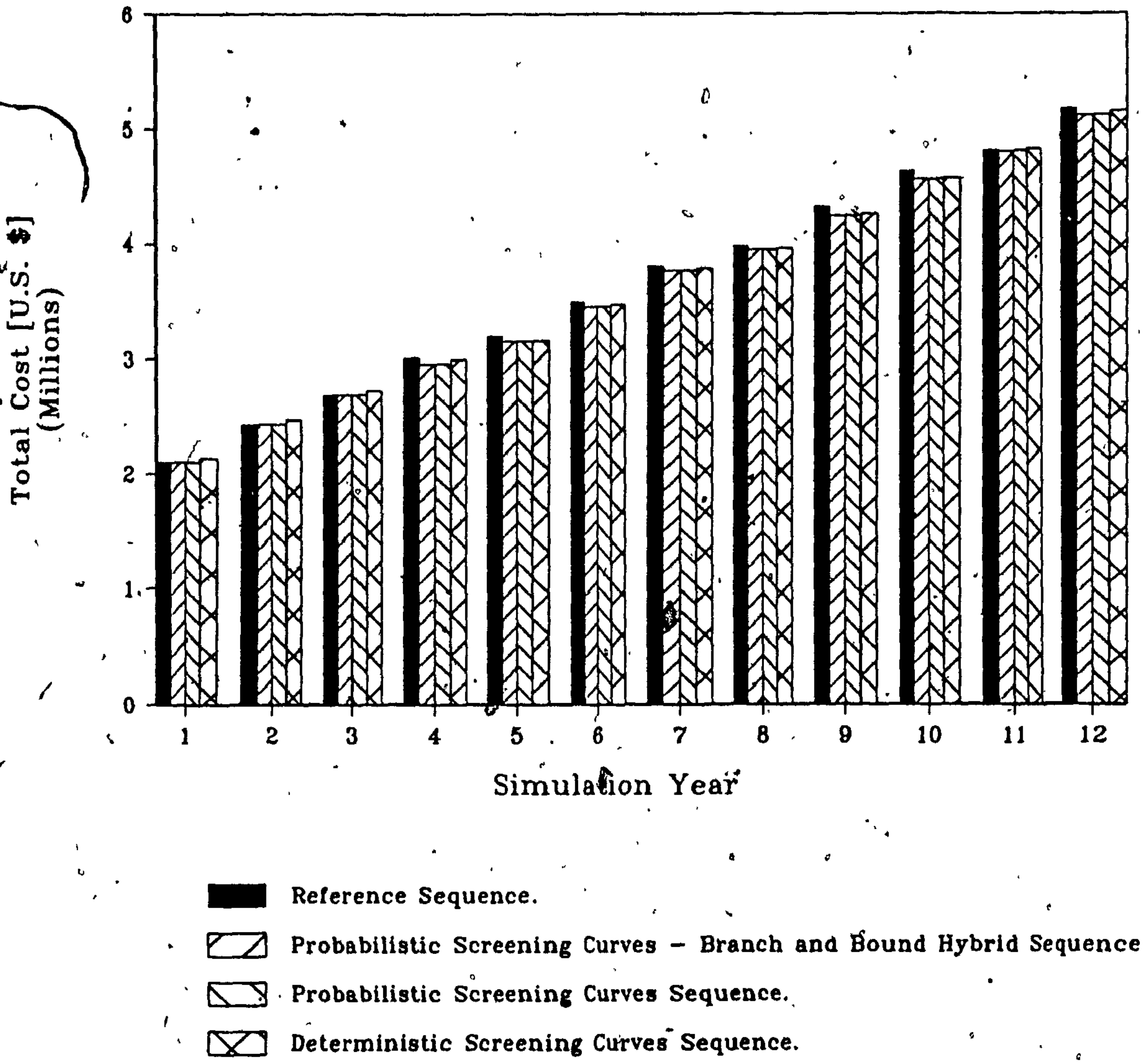


Figure 4.13 Hydro Comparison - Total System Cost.

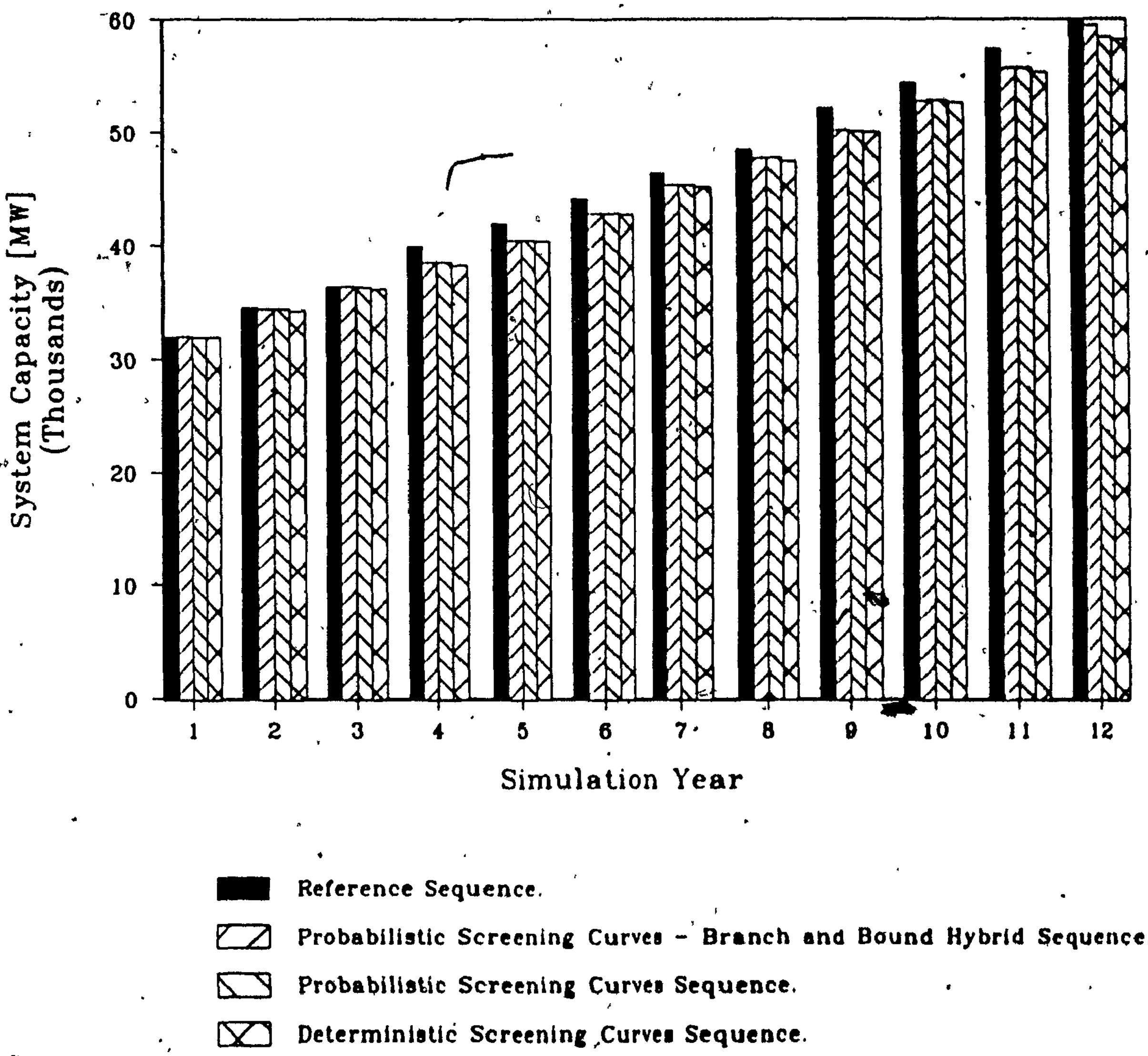


Figure 4.14 Hydro Comparison - Total System Capacity.

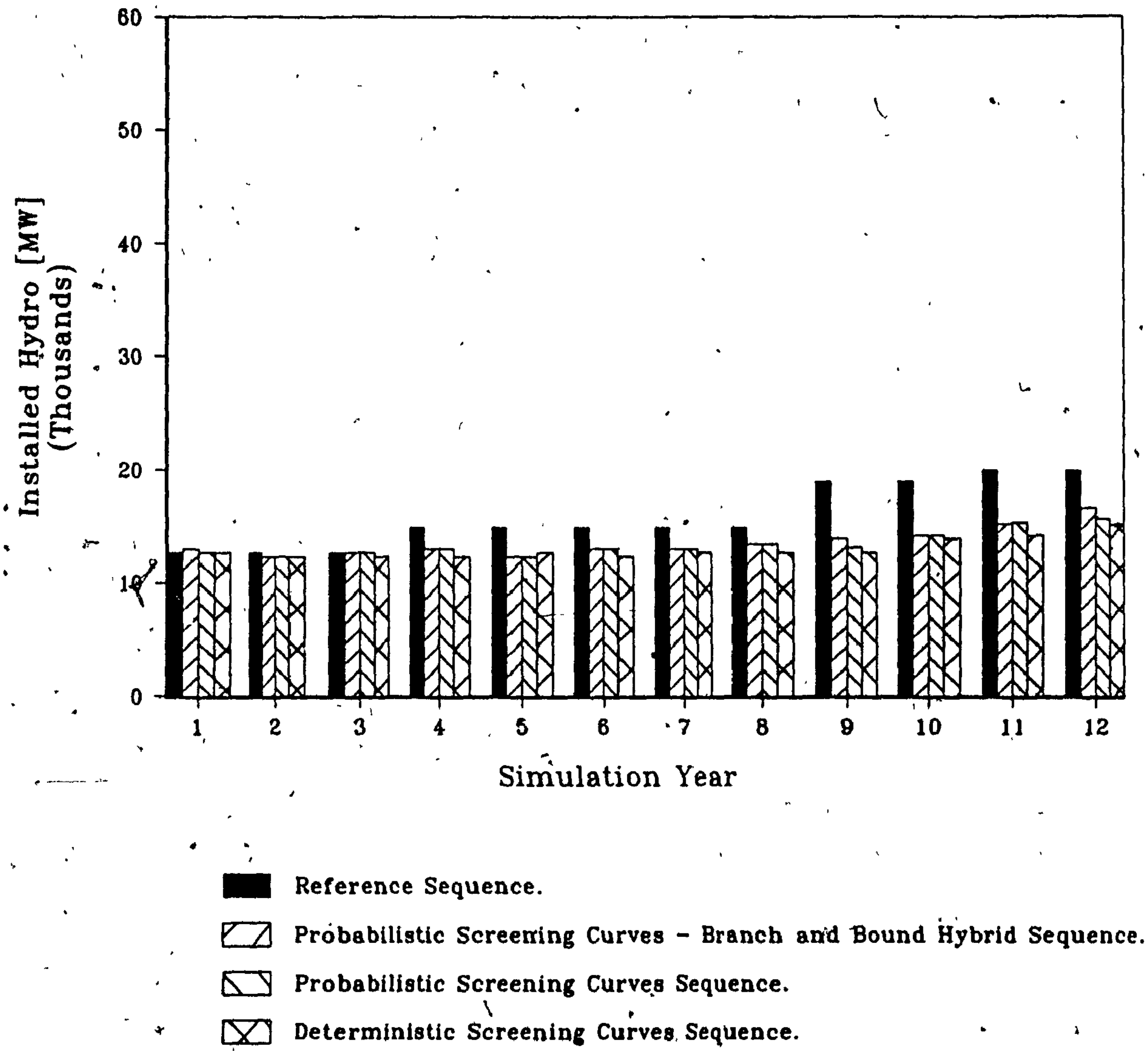


Figure 4.15 Hydro Comparison - Installed Hydro.

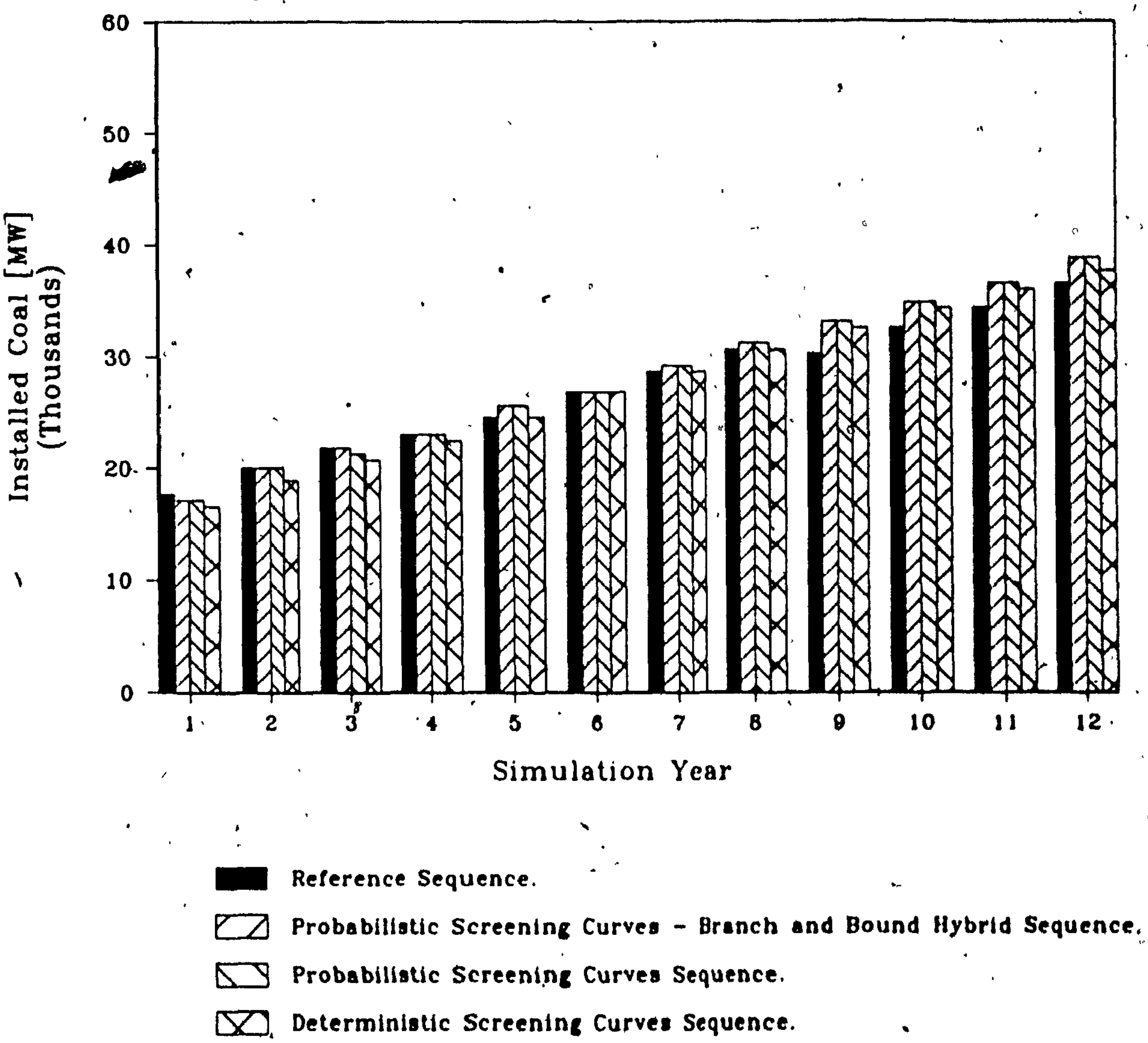


Figure 4.16 Hydro Comparison - Installed Coal.

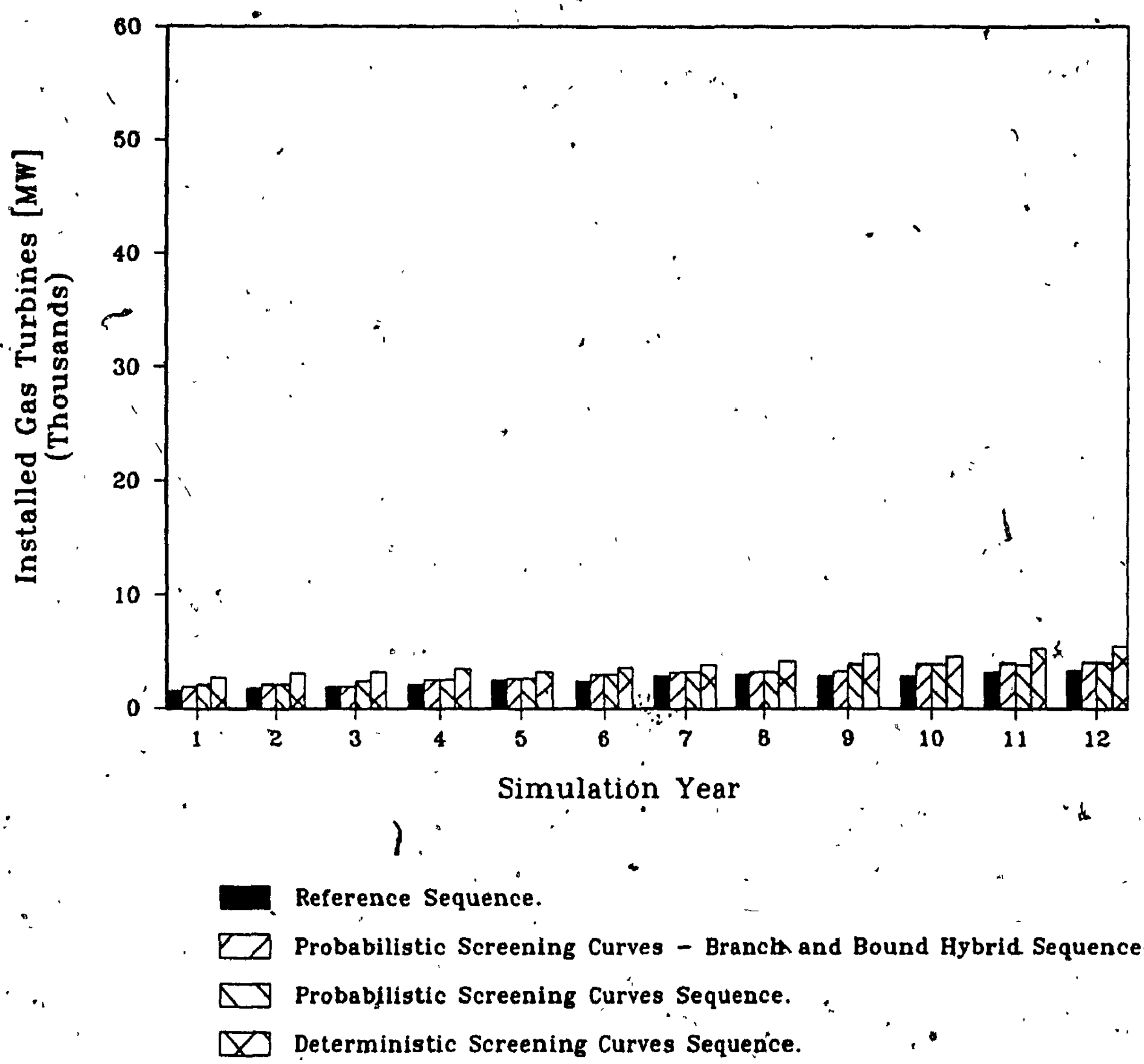


Figure 4.17 Hydro Comparison - Installed Gas Turbines.

REFERENCE SEQUENCE

YEAR	Hydro Unit 1	2	3	4	5	6	7	8	9	10	11	12	13
1	*	*	*	*	*								
2	*	*	*	*	*								
3	*	*	*	*	*								
4	*	*	*	*	*		*			*		*	
5	*	*	*	*	*		*			*		*	
6	*	*	*	*	*		*			*		*	
7	*	*	*	*	*		*			*		*	
8	*	*	*	*	*		*			*		*	
9	*	*	*	*	*	*	*	*	*	*		*	*
10	*	*	*	*	*	*	*	*	*	*		*	*
11	*	*	*	*	*	*	*	*	*	*	*	*	*
12	*	*	*	*	*	*	*	*	*	*	*	*	*

BRANCH AND BOUND - PROBABILISTIC SCREENING CURVES

YEAR	Hydro Unit 1	2	3	4	5	6	7	8	9	10	11	12	13
1	*	*	*	*	*			*					
2	*	*	*	*									
3	*	*	*	*	*								
4	*	*	*	*	*			*					
5	*	*	*	*									
6	*	*	*	*	*			*					
7	*	*	*	*	*			*					
8	*	*	*	*	*					*			
9	*	*	*	*	*					*		*	
10	*	*	*	*	*			*		*		*	
11	*	*	*	*	*	*		*		*			
12	*	*	*	*	*	*		*		*	*	*	

PROBABILISTIC SCREENING CURVES

YEAR	Hydro Unit 1	2	3	4	5	6	7	8	9	10	11	12	13
1	*	*	*	*	*								
2	*	*	*	*									
3	*	*	*	*	*								
4	*	*	*	*	*			*					
5	*	*	*	*									
6	*	*	*	*	*			*					
7	*	*	*	*	*			*					
8	*	*	*	*	*					*			
9	*	*	*	*	*							*	
10	*	*	*	*	*			*		*		*	
11	*	*	*	*	*	*				*		*	
12	*	*	*	*	*	*		*		*		*	

DETERMINISTIC SCREENING CURVES

YEAR	Hydro Unit 1	2	3	4	5	6	7	8	9	10	11	12	13
1	*	*	*	*								*	
2	*	*	*	*									
3	*	*	*	*									
4	*	*	*	*									
5	*	*	*	*	*								
6	*	*	*	*									
7	*	*	*	*	*								
8	*	*	*	*	*								
9	*	*	*	*	*								
10	*	*	*	*	*					*		*	
11	*	*	*	*	*	*							
12	*	*	*	*	*	*					*		

Figure 4.18 Hydro Comparison - New Unit Installations.

from the data of Figure 4.18 and the suboptimal plans of the PSC-B&B method. (See Section 4.9)

The Screening Curves scenario is consistently more costly than the probabilistic optimization methods developed in this work. This deterministic method tends to install less hydro and coal, and replaces it with peaking gas turbine generation. This is caused by the difference between deterministic and probabilistic production costing. However, due to the higher percentage of smaller more reliable Gas Turbines, less total system generation is required. Similarly, due to the smaller percentage of Gas Turbines in the reference, the overall system capacity tends to be higher.

4.9 Global Expansion Sequence Determination

The yearly optimal sequences provided by Probabilistic Screening Curves and Probabilistic Screening Curves-Branch and Bound hybrid do not provide realizable global sequences. The problem with yearly optimizations is that there is no consistency in unit installations. A unit that is installed one year, if mathematically optimal, can be decommissioned the following year. This is, of course, not physically realizable, but occurs in the hydro sequences of Figure 4.18. Thus, the yearly optimization must be modified to determine a least cost expansion sequence.

An example of global optimization is provided for the hydro-thermal test case. (See Figure 4.18). A globally optimized generation sequence was determined based on the results of the Probabilistic Screening Curves-Branch and Bound hybrid's output. (See Figures 4.19). Comparing the global and the yearly sequences, it can be seen that the unit's commissioning dates can sometimes be changed when the system is globally optimized. (Compare commissioning dates of hydro units ten and eleven).

The methodology used to develop globally optimized sequences from yearly optimizations is based on heuristics, the analyst's experience and external considerations. The method used to develop this global sequence involved developing a realizable hydro sequence and then subsequently, a thermal sequence. The hydro methodology consists of selecting one hydro unit and comparing the costs of different installation dates to determine its best commissioning date. The comparisons can usually be performed using the suboptimal plans of the Probabilistic Screening Curves-Branch and Bound hybrid. Once the commissioning date of a particular unit is determined, it is treated as an existing unit and the next hydro unit is analyzed. Three system simulations were required for the global sequence determination. Each subsequent simulations require less processing time as the number of hydro candidates is reduced with each simulation. Once the optimal hydro sequence was developed, a realizable thermal sequence was then determined by advancing or retiring coal commissioning dates.

REFERENCE SEQUENCE

YEAR	Hydro Unit													Thermal Units	
	1	2	3	4	5	6	7	8	9	10	11	12	13	Coal	Gas Turbine
1	*	*	*	*	*									12,000 MW	1,600 MW
2	*	*	*	*	*									15,000 MW	1,800 MW
3	*	*	*	*	*									17,400 MW	1,900 MW
4	*	*	*	*	*		*			*		*		18,600 MW	2,500 MW
5	*	*	*	*	*		*			*		*		20,400 MW	2,500 MW
6	*	*	*	*	*		*			*		*		23,400 MW	2,500 MW
7	*	*	*	*	*		*			*		*		26,400 MW	2,500 MW
8	*	*	*	*	*		*			*		*		27,600 MW	2,600 MW
9	*	*	*	*	*	*	*	*	*	*		*	*	27,600 MW	2,600 MW
10	*	*	*	*	*	*	*	*	*	*		*	*	30,000 MW	2,900 MW
11	*	*	*	*	*	*	*	*	*	*	*	*	*	31,200 MW	3,200 MW
12	*	*	*	*	*	*	*	*	*	*	*	*	*	32,600 MW	3,400 MW

BRANCH AND BOUND - PROBABILISTIC SCREENING CURVES

YEAR	Hydro Unit													Thermal Units	
	1	2	3	4	5	6	7	8	9	10	11	12	13	Coal	Gas Turbine
1	*	*	*	*	*			*						11,400 MW	1,900 MW
2	*	*	*	*	*			*						14,400 MW	2,100 MW
3	*	*	*	*	*			*						16,800 MW	2,200 MW
4	*	*	*	*	*			*						19,200 MW	2,500 MW
5	*	*	*	*	*			*						21,000 MW	2,500 MW
6	*	*	*	*	*			*						23,400 MW	3,000 MW
7	*	*	*	*	*			*						26,400 MW	3,200 MW
8	*	*	*	*	*			*						27,600 MW	3,500 MW
9	*	*	*	*	*			*				*		30,000 MW	3,600 MW
10	*	*	*	*	*			*		*		*		32,400 MW	3,900 MW
11	*	*	*	*	*	*		*		*	*	*		33,600 MW	4,100 MW
12	*	*	*	*	*	*		*		*	*	*		36,000 MW	4,100 MW

Figure 4.19 Globally Optimized Hydro Scenario.

The total present worthed costs for the Probabilistic Screening Curves-Branch and Bound based and reference globally optimized sequences are $ 7,338 million and $ 7,412 million respectively. The merits of the Probabilistic Screening Curves-Branch and Bound is once again demonstrated as it enabled the development of a lower cost global sequence with a few yearly optimization simulations.

4.10 Summary

Probabilistic Screening Curves and Probabilistic Screening Curves - Branch and Bound hybrid are seen to be capable of determining near-optimal yearly optimizations of hydro and thermal alternatives quickly and efficiently in a probabilistic setting. The Probabilistic Screening Curves - Branch and Bound hybrid results in less costly scenarios as its search procedure can account for certain probabilistic phenomena that Probabilistic Screening Curves cannot. The PSC-B&B hybrid also provides suboptimal plans that can aid in determining a global generation sequence. There is, of course, a tradeoff of computational time as the PSC-B&B requires longer execution times.

Probabilistic Screening Curves is ideal for a first pass optimization performed on a personal (micro-) computer in a probabilistic setting. The Probabilistic Screening Curves - Branch and Bound hybrid can then be used to provide lower cost yearly mixes and suboptimal plans that aid the analyst in developing an optimal global sequence.

Chapter 5.

CONCLUDING REMARKS

The purpose of this thesis was twofold. The first was to outline concepts and techniques used in power system generation expansion analysis. The second was to develop a near-optimal analysis tool that had computational requirements suited to the limitations of a personal (micro-) computer.

The concepts and methodology discussed are representative of the probabilistic techniques presently in use in long term generation expansion. Test data stressed the value and importance of the probabilistic treatment of the system's generation.

Various implementations and available software packages demonstrated the capabilities of present day analysis tools. The majority, however, proved to be prohibitive in excesses either in computer resource requirements or modelling over-simplification for long term expansion analysis. Thus, sacrificing global 'optimality' for yearly near-optimality, an efficient yearly hydro-thermal hybrid was developed called Probabilistic Screening Curves and Probabilistic Screening Curves - Branch and Bound hybrid. These methods determined realizable scenarios in a probabilistic setting. They were implemented and tested on a personal computer showing very favourable results. They are capable of handling large systems with many new candidates without demanding exorbitant computer resources. Due to the widespread usage and low cost of personal computers, the power system analysts now have at their disposal a fast, efficient tool to aid them in determining optimal power system generation expansion scenarios.

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APPENDIX A

Maintenance Modelling Comparison

Every generating unit of a power system must be shut down at regular intervals for maintenance. Maintenance is scheduled to maximize system reliability. Detailed production costing programs model maintenance by subdividing each year into intervals and removing units from operation for the number of intervals corresponding to its maintenance requirements. The chosen interval length depends on the desired modelling accuracy and available computer resources. Unit outages are scheduled to either maximize system reserve or reliability (LOLP or EUE) for deterministic or probabilistic production costing respectively. (Maintenance scheduling solution techniques are outlined in [7,14]).

Long-term generation expansion planning do not require such accuracy nor are the computational resources available. The larger interval lengths are incapable of representing individual unit maintenance periods. Maintenance outages can be represented by grouping similar sized units for a combined outage length corresponding to the interval. Unit maintenance grouping is a complex problem and approximations must be made for non-similar units.

Maintenance can also be approximated in a long-term simulation by either reducing the unit's capacity or adding to its forced outage rate. These methods are attractive due to their simplicity and ease of implementation.

They are not as accurate as the more complex unit removal techniques, but approximations must be made to reduce the computational burden. To determine the accuracy of these methods, they were compared through test runs to the realistic interval unit removal method. The testing was performed using two different power systems, a large system (16,000 MW) and a small system (39 MW). The detailed simulation using thirteen intervals per year was the basis for comparison purposes. Total cost, fuel cost and unserviced energy (EUE) were the criteria used in the comparison. The results are summarized in Table A.1. The derated capacity maintenance technique for the large system was more accurate. This is especially evident in the expected unserved energy criteria. Both methods have similar results for the smaller test system.

Maintenance scheduling is far more critical for small systems as each maintenance outage has a profound impact due to the fewer number of reserve units available to meet the increased load. Thus more accurate techniques should be used to simulate maintenance.

As maintenance is a deterministic phenomena, it seems conceptually superior to derated the units' capacity rather than add it to the forced outage rate which is modelled probabilistically. As this reasoning concurred with test

Table. A.1 Comparison of Maintenance Methods

LARGE SYSTEM (16,000 MW, 105,000 GWh)							
Prod. Costing	Mainten. Method	Total Cost (K$)	Error	Fuel Cost (K$)	Error	EUE (GWh)	Error
Interval	Removal	4,260,082		4,064,236		16.5	
Annual	-	4,112,512	-3.46%	3,922,273	-3.49%	0.0	-100.0%
Annual	Maint.+FOR	4,316,809	1.33%	4,120,959	1.40%	89.8	444.2%
Annual	Derate Cap	4,255,435	-0.11%	4,059,661	-0.11%	13.4	-18.8%

SMALL SYSTEM (37.3 MW, 153.5 GWh)							
Prod. Costing	Mainten. Method	Total Cost (K$)	Error	Fuel Cost (K$)	Error	EUE (GWh)	Error
Interval	Removal	14,357.5		12,795.7		1.3	
Interval	-	14,062.2	-2.06%	12,531.2	-2.07%	0.7	-46.2%
Annual	Maint.+FOR	14,284.9	-0.51%	12,731.6	-0.50%	1.6	23.1%
Annual	Derate Cap	14,211.5	-1.01%	12,666.5	-1.01%	1.0	-23.1%

Removal -When scheduled for maintenance the unit is removed for that interval.
Maint.+FOR -The unit's maintenance is added to its <u>F</u>orced <u>O</u>utage <u>R</u>ate in the probabilistic production costing.
Derate Cap -The unit's capacity is derated by the maintenance.

These comparisons were performed using probabilistic production costing. The interval method is able to model unit maintenance more accurately and is used as the basis for comparison. The total affect of maintenance on the system cost is shown by performing a simulation with no maintenance (-). The two methods of annual methods of maintenance are then compared to the interval unit removal method.

results, the maintenance was implemented using the capacity deration technique. For smaller systems, the computer resource limitations are less restrictive and a more detailed analysis can be used.

APPENDIX B

Optimization Software Package Comparison

This appendix contains summaries of seventeen existing software packages used to develop long term generation expansion scenarios. As this work concentrates on automatic unit selection, only such packages are listed. Each method is briefly described including a reference for further information. The software packages are compared by referring to Table B.1. (Note. Sections in Table B.1 are left blank if information is unavailable).

Most methods used in Long Term Power System Generation Expansion Planning are non-optimizing. In a 1985 Canadian Electrical Association (CEA) survey of Canadian Companies [19] only six out of twenty-four long term generation analysis programs could automatically develop an expansion scenario. Much of the power planning industry is still not using existing programs to suggest expansion scenarios but, instead, rely heavily on the experience and intuition of system planners.

TABLE B.1 Comparison of Optimising Expansion Programs

Model / Attribute	AGP	EGEAS			
Expansion Method	Dynamic Programming	Screening Curves	Linear Programming	Generalized Bender's	Dynamic Programming
Variable Cost	Probabilistic Prod. Cost'n	Deterministic LDC	Exogenously Specified	Probabilistic Prod. Cost'n	Probabilistic Prod. Cost'n
Reliability	-LOLP -EUE	Deterministic LDC	% Reserve	EUE	-LOLP -EUE
Optimization Unit Types	-Thermal -Hydro	-Thermal	-Thermal -Hydro -Storage	-Thermal -Hydro -Storage	-Thermal -Hydro -Storage
Constraints	-Min./Max. Reserve		-Financial -Environmental -Siting	-Financial	-Min./Max. Reserve
Computational Burden	High	Low	Low-Moderate	Moderate	Moderate-High
Other	-Good Hydro Simulation -Cumulants	-Suboptimal Plans -Yearly Optimization (Non-Global)		-Suboptimal Plans -Cumulants	-Suboptimal Plans -Cumulants

TABLE B.1 Comparison of Optimising Expansion Programs (Con't)

Model / Attribute	EDF	GENOPT	GMP	GSP	Hydro Quebec
Expansion Method	Optimal Control	Dynamic Programming	Dynamic Programming	Heuristic	Integer Heuristic
Variable Cost	Deterministic LDC	Probabilistic Prod. Cost'n	Probabilistic Prod. Cost'n	Deterministic LDC	Generalized Networks
Reliability	Deterministic	Probabilistic	LOLP	% Reserve	% Reserve
Optimization Unit Types	-Thermal	-Thermal -Hydro -Storage	-Thermal -Storage	-Thermal	-Thermal -Hydro
Constraints		-Fuel Availability	-Min./Max. Reserve -Financial		-Water Availability
Computational Burden	High	High	Moderate-High	Low	Moderate
Other		-Suboptimal Plans -Cumulants	-Suboptimal Plans	-Yearly Optimization (Non-Global)	-Models Hydro Reservoirs and Valleys

TABLE B.1 Comparison of Optimising Expansion Programs (Con't)

Model / Attribute	LEVIN	MIT	MOSES	OGP	OPTGEN
Expansion Method	Screening Curves	Linear Programming	Dynamic Programming	Myopic One Future Period	Dynamic Programming
Variable Cost	Deterministic LDC	Exogenously Specified	Linear Programming	System Operation Simulation	Deterministic LDC
Reliability	Deterministic LDC	% Reserve	% Reserve	LOLP	Approximate Distribution
Optimization Unit Types	-Thermal -Hydro	-Thermal -Hydro -Storage	-Thermal -Hydro -Storage	-Thermal -Hydro -Storage	-Thermal
Constraints					-Min/Max Reserve
Computational Burden	Low	Moderate	Moderate-High	Low-Moderate	Moderate
Other	-Yearly Optimization (Non-Global)		-Incremental Average Prod. Cost Unit Loading Order	-Yearly Optimization (Non-Global)	-Suboptimal Plans

TABLE B.1 Comparison of Optimising Expansion Programs (Con't)

Model / Attribute	PUPS	RPI	University of Massachusetts	WAGP	WASP
Expansion Method	Screening Curves	Sequential Objectives	Integer-Linear Prog.	Branch and Bound	Dynamic Programming
Variable Cost	Chronological Simulation	Linear Programming	Deterministic	Deterministic or Prob.	Probabilistic Prod. Cost'n
Reliability	Chronological Simulation	% Reserve	Approximate Distribution	Deterministic or Prob.	LOLP
Optimization Unit Types	-Thermal	-Thermal -Hydro	-Thermal -Storage	-Thermal -Hydro	-Thermal
Constraints		-Environmental -Fuel Consumption		-Financial -Environmental -Mix Constraints	-Min/Max Reserve
Computational Burden	Low	Moderate	Moderate	Moderate to High	Moderate to High
Other	-Tidal Power evaluation -Yearly Optimization (Non-Global)				-Suboptimal Plans -Cumulants

AGP

The Automatic Generation Planning (AGP) optimization model used Dynamic Programming to optimize thermal and hydro alternatives. It uses analytical probabilistic methods to estimate system variable costs and reliability. AGP has a very good long-term expansion hydro representation. It has a high computational burden due to its Dynamic Programming formulation. AGP was developed by ACRES International [41].

EGEAS

The Electrical Generation Expansion Analysis System (EGEAS) optimization package has four separate optimization options each using a separate optimization technique. They will be discussed individually below.

SC

The first method performs a yearly optimization using Screening Curves (SC). It can only analyze thermal units and uses deterministic variable costing and reliability methods. It has a low computational burden yet still provides suboptimal plans.

LP

The second method analyzes thermal, hydro and storage alternatives using a Linear Programming (LP) formulation. The capacity factors of the generating units must be prespecified for the calculation of system variable costs and reliability. The method is flexible as it models financial, environmental and siting constraints. It has a low to moderate computational burden.

GB

The third method uses Generalized Bender's Decomposition (GB) optimization to determine a least cost thermal, hydro and storage unit sequence. It performs a detailed analytical probabilistic variable cost determination method and uses a probabilistic reliability criteria. It is capable of financial constraint analysis and determining suboptimal plans. It has a moderate computational burden.

DP

The last optimization method of EGEAS is Dynamic Programming (DP) which also analyzes new thermal, hydro and storage units. The very detailed method of system variable costs and reliability criteria determination uses either Booth-Baleriaux or analytical probabilistic simulation. Suboptimal plans are also available. It has a moderate to high computational burden.

The EGEAS package is the most complete and versatile of all the packages discussed. It was developed by the Electric Power Research Institute (EPRI) [18].

EDF

The Électricité de France (EDF) optimization model uses an optimal control and decomposition method to determine thermal expansion sequences. The system simulation uses deterministic methods yet has a high computational burden. The method is used by EDF [18].

GENOPT

The Generation Optimization (GENOPT) optimization algorithm uses Dynamic Programming to optimize thermal, hydro and storage alternatives. It uses analytical probabilistic methods to estimate system variable costs and reliability. It is able to provide suboptimal plans, but has the high computational requirements inherent in Dynamic Programming formulations. GENOPT is also able to model financial limitations. It is used by ACRES International [19].

GMP

The Generation Mix Planning Package (GMP) is an adaptation of the Wien Automatic System Planning Package. It performs a global optimization based on Dynamic Programming (DP) techniques but can only optimize thermal alternatives. GMP uses probabilistic reliability and variable cost estimates to analyze each possible scenario. The search area is limited by minimum and maximum reserve criteria, maximum number of new unit additions and financial considerations. The computational burden is moderate to high. Further information is available through Southern Services Company Inc. [18].

GSP

The Generation Sequencing Program (GSP) determines an optimal thermal yearly sequence using Heuristic methodology. It uses deterministic production costing and reliability criteria. GSP has a low computational burden. It was developed by Canadian International Project Managers (CIPM), Yangtze Joint Venture [22].

Hydro Quebec - GN

Hydro Quebec's Generalized Network formulation is specifically designed for detailed hydro simulation. It optimizes thermal and hydro alternatives based on deterministic variable cost and reliability analysis. It has a moderate computational burden. It was developed in Hydro Quebec's Research Institute IREQ [12,13].

Levin - SC

Levin's Screening Curves (SC) formulation is able to determine an optimal yearly thermal and hydro generation capacities. It uses deterministic production costing techniques and percent reserve reliability criteria. It has a low computational burden. It was developed by Levin [33,34,35].

MIT - LP

The MIT optimization method developed at MIT uses Linear Programming (LP) to determine a least cost scenario. Inherent in its formulation, the user is required to specify the unit's capacity factors to determine the system costs. The reliability criteria is a deterministic reserve margin. It is able to optimize thermal, hydro, and storage alternatives yet only has a moderate computational burden. Its Linear Programming formulation also enables environmental considerations. The system planning package was developed by the MIT Laboratory [18].

MOSES

MOSES uses Dynamic Programming (DP) to determine an optimal sequence of thermal, hydro and storage units. It determines variable costs and reliability using a deterministic Linear Programming method. A major advantage of the method is its ability to model thermal unit's incremental average costs. This is much more realistic but the overall accuracy is still limited by the Linear Programming deterministic methodology. The method has a moderate to high computational burden. MOSES is used by Manitoba Hydro [19].

OGP

The Optimized Generation Planning Model (OGP) performs a yearly optimization based on an iterative procedure using cost and reliability criteria. It has detailed production costing simulation capabilities and thus is useful for determining the minimum cost of generation additions for each year. It analyzes thermal, hydro and storage generation additions. It was developed at General Electric [18].

OPTGEN

OPTGEN uses Dynamic Programming to determine a least cost scenario. The variable cost and reliability estimates are approximate and do not have the accuracy of probabilistic production costing. OPTGEN is only able to optimize thermal unit additions but can determine suboptimal plans. Due to its deterministic system modelling, its computational burden is moderated despite its Dynamic Programming formulation. OPTGEN was developed by Lee [18].

PUPS

The PUPS optimization model uses Screening Curves to optimize thermal sequences. It is specifically designed to analyze tidal power, an unconventional alternative. PUPS models power systems by chronological simulation. Its computational burden is low and was developed by Lee and Dechamps [18,31].

RPI - LP

The Energy Appraisal Model Developed at Rensselaer Polytechnic Institute (RPI) performs a global optimization using a Linear Programming (LP) formulation. Using a sequential objective technique it is able to optimize several criteria while constraining those which were previously optimized. These objectives include minimum present worth power production costs, environmental, fuel restrictions and number of new unit additions. The program is capable of optimizing thermal and hydro unit additions. The computational burden is moderate. The program was developed at RPI [18,38].

University of Massachusetts

The University of Massachusetts model determines an optimal system plan using a mixed Integer-Linear Programming method. Both variable costs and reliability calculations are approximate and do not have the accuracy of probabilistic calculations. It is capable of analyzing thermal and storage unit additions. The program has a moderate computational burden. It was developed by Noonan and Giglio [18].

WAGP

The Automatic Expansion Program (WAGP) performs either an annual or global optimization using Branch and Bound techniques. To reduce the problem size, the new units are preprocessed through a screening curve selection process. Either deterministic or probabilistic costing and reliability criteria may be used. The solution region is defined through the use of financial, environmental and mix constraints. Both thermal and hydro expansion alternatives can be analyzed. The computational burden is moderate to high. WAGP was developed by Westinghouse [18,29].

WASP

The Wien Automatic System Planning Package (WASP) is an industrial standard. It uses Dynamic Programming to optimize new unit additions and is only capable of determining new thermal and hydro unit additions. It uses probabilistic methods to determine variable costs and system reliability. WASP is able to provide up to ten suboptimal plans. It has a high computational burden. WASP was developed by Jenkins and Joy [18,43].

Summary

Each of these optimization methods has its own strengths and weaknesses. There is a tradeoff between modelling accuracy, program capability and computational burden. Modelling accuracy is often sacrificed to reduce the computational burden, but results in reduced confidence levels in final scenarios. Other methods sacrifice modelling accuracy to analyze financial, siting and environmental phenomena.

Still others include all these phenomena with excellent accuracy but have extensive computational burdens.

There is no perfect or best optimization package. Each has a particular usage and provides the appropriate accuracy, capability and computational mix. Most methods with probabilistic modelling accuracy have very high computational burdens. Thus it was the objective of this work to develop a method which used the more accurate simulation method of probabilistic production costing yet keep the computational burden at reasonable levels. The Probabilistic Screening Curve technique is able to provide a yearly optimization using probabilistic simulation with near optimal solutions with moderate computational burdens suitable for personal computer usage.

APPENDIX C

Power System Generation Expansion Planning Bibliography

This appendix contains a bibliography that describes various aspects of Power System Generation Expansion Planning. They have been classified into the following categories:

OPTIMIZATION METHODS:

- SC - Screening Curves
- LP - Linear Programming
- Decomposition
  - GB - Generalized Bender's
  - GN - Generalized Network
- Enumerative
  - DP - Dynamic Programming
  - B&B - Branch and Bound
- Other
  - GP - Gradient Projection
  - H - Heuristic
  - O - Other

UNIT ADDITIONS:

- R - Real (non-integer) unit additions. (Non-realizable scenarios).
- I - Integer unit additions. (Realizable scenarios).

PRODUCTION COSTING METHODS:

Det - Deterministic production costing

Prob - Probabilistic production costing

SURVEY:

S - A comparison or survey of some of the optimization techniques being used.

The following table indicates categories into which each of the listed papers fall. If more than one method is discussed or critiqued, the appropriate methods are indicated. Several papers concentrate on either an optimization formulation or an production costing technique. Thus a production costing method may be indicated without an optimization technique or vice versa. The "SURVEY" category indicates papers that give comparisons of techniques or contain good literature reviews. Papers of a general nature have no specified categorization.

TABLE C.1 Classification of References Listed in Bibliography

PAPER	SC	LP	DECOMPO- SITION	ENUMER- ATIVE	OTHER	UNIT ADDNS	PRODUCTION COSTING	SURVEY
1.							PROB	
2.								S
3.							PROB	S
4.							PROB	
5.							PROB	
6.			GB			R	PROB	
7.			GB			R	PROB	
8.			GB			R	PROB	
9.			GB			R	PROB	
10.				DP		I	PROB	
11.	SC			DP		I	PROB	
12.					O			
13.			GB			R	PROB	
14.	SC					R	DET	
15.			GN			I	DET	
16.			GN			I	DET	
17.		LP				R	DET	
18.		LP		B&B		I	DET	
19.		LP				R	DET	
20.								
21.							PROB	
22.								S
23.	SC					R	DET	S
		LP				R	DET	S
			GB			R	PROB	S
				DP		I	PROB	S
24.								S
25.				DP		I		
26.							PROB	
27.	SC					I	DET	
28.							PROB	
29.							PROB	
30.								S
31.							PROB	
32.					O	I	PROB	
33.								
34.				DP		I		
35.				DP		I		

TABLE C.1 Classification of References Listed in Bibliography (Con't)

PAPER	SC	LP	DECOMPO-SITION	ENUMER-ATIVE	OTHER	UNIT ADDNS	PRODUCTION COSTING	SURVEY
36.				DP		I		S
37.				B&B		I	DET	
38.					GP	R	PROB	
39.							PROB	
40.	SC					R	DET	
41.								
42.	SC					R	DET	
43.	SC					R	DET	
44.								
45.	SC					R	DET	S
46.								S
47.							PROB	
48.								S
49.				B&B		I		
50.		LP				R	DET	
51.				DP		I	DET	
52.								
53.					GP	R	PROB	
54.								S
55.							PROB	
56.								
57.		LP				R	DET	
58.								S
59.							PROB	
60.				DP		I	PROB	
61.							PROB	
62.	SC					R	DET	
63.							PROB	
64.				DP		I	PROB	
65.							PROB	
66.				DP		I		
67.				DP		I	PROB	
68.							PROB	
69.					GP	R	PROB	

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