Production and Evaluation
of a Topdown Structured Text
Some Mathematical Pre-Requisites to Cybernetics

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A Thesis Equivalent

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ABSTRACT

Production and Evaluation
of a Topdown Structured
"Some Mathematical Pre-Requisites to Cybernetics"

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This study deals with the design, development and evaluation of a Structured Text, "programmed" according to the methods and techniques of INFOSTRUCT: A pedagogy using structures instead of isolated facts, concepts, rules and principles to present knowledge (information) to the learner.

The structured text (See Appendix IV) is called:
SOME MATHEMATICAL PRE-REQUISITES TO CYBERNETICS
and is designed for Educational Technology graduate students at Concordia University. The pre-test was administered one week before the subjects took the course, and the posttest and evaluation questionnaire were administered one week after the subjects had completed the course.

Both formative and summative evaluations were performed to assess the strengths and weaknesses of the structured text as well as the methods and techniques of Infostruct and Topdown Conceptual Analysis.

The results were that a statistically significant gain in factual knowledge occurred and that the students expressed satisfaction with their use of this structured study material.

ACKNOWLEDGEMENTS

This Thesis-Equivalent would not have happened without the help of many people.

I am particularly indebted to Prof. David P. Mitchell for having accepted me into the Ed. Tech. program.

Prof. Gary M. Boyd, my supervisor, is to be thanked for his instruction, guidance, support and unfailing good humour.

Profs. Dennis Dicks, Richard Schmid and Robert Bernard are to be thanked for their instruction, help and continual support.

Credit is due to the thirty-five Ed. Tech. students who were the subjects in the evaluation of the structured text.

A larger number of classmates have aided me in my attempt to explain Infostruct and Topdown Conceptual Analysis, and for their patience I am grateful to them all.

Montreal
August, 1986

Xuan Le

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I : INTRODUCTION

INTRODUCTION

Educational Problem

It was found that graduate students in Educational Technology frequently lacked the mathematical background to properly study Educational Cybernetics (G.Boyd, personal communication, 1979). Since there was no time nor resourses availabe for an extra course in this area, it seemed advisable to consider the provision of self-study materials.

The logical and mathematical nature of the problem suggested that textual materials might prove adequate if properly designed.

The author's longstanding interest and experience in the design of instructional materials led to this Thesis-equivalent as a means to prepare students to study Educational Cybernetics.

THE PROBLEM OF DEVELOPING HIGH QUALITY INSTRUCTIONAL MATERIALS

"Education, Professor Jerrold Zacharias of MIT had suggested in a 1956 memorandum which led to the creation of the Physical Sciences Study Committee (PSSC), is like high fidelity. To have a good hi-fi, you need a good performer on a good day, a good recording, a good pressing of it, a good pickup and amplifier; you need a room with good acoustical qualities and a person with the intent to listen. 'But most important of all,' Zacharias argued, 'is the composition itself; without a great composition, everything else is pointless." Silberman(1970)

To have a good learning system you need high quality instructional materials or knowledge base and to have high quality instructional materials and knowledge base you need first class subject matter specialists (SMS) or domain experts (DE) and first class instructional designers (ID).

Cooperation Between SMS and ID

The planning of good learning systems development projects ideally provides for close cooperation between subject matter specialists (SMS) and instructional designers (ID). In this study the term subject matter specialist (SMS) or domain expert (DE) is a person who normally earns, or could earn a living as a worker in a field of specialization, and in a higher level position than the level for which the system is to be bilt. S/he should know the subject matter or domain of her/his field thoroughly and the problems encountered at various levels within her/his specialization.

The term Instructional designer (ID) or applied educational psychologist refers to a person who has learned to apply learning theories to the practical task of learning and teaching. Ideally s/he should have also acquired the methods and techniques of topdown conceptual analysis (TCA). The process of developing learning systems involves close cooperation between SMSs and IDs.

Allocation of Responsibilities

Although a module or course can be developed by one SMS working with one ID, or by one person acting both as SMS and ID, it is often better to have one ID working with several SMS, programmers, editors, and word processor operators. Based on this author's experience it would be a waste of talents for SMS and IDs to perform each of the steps for the development of a system of any length.

Set of Steps

While designing learning systems, the SMS and the ID go through a set of such steps as:

1. Determining the needs
2. Identifying the problem

3. Specifying the target population
4. Specifying the entering behaviour
5. Specifying the terminal behaviour
6. Generating the knowledge base (KB)
7. Converting the KB into teaching sequences
8. Deciding on the type of instruction
9. Testing and revising until the materials meet their specifications
10. Validating the materials
11. Implementing the programs
12. Evaluating the system, and so forth.

Some textbooks explain quite clearly how to perform all the steps except one : Generating the knowledge base. From experience the author believes that the success of a learning system depends on the quality of the knowledge base. It is the raw material with which we build the composition of a system.

SURVEY OF THE LITERATURE

Introduction

The author believes that generating the knowledge base is a fundamental step in the design and development of any system : learning system, expert system, and so forth. Up until now several methods and techniques have been developed which indicate how the instructional designer should accomplish this task. Each method and technique presents interesting ways of performing the task but suffers from a lack of precision and concreteness (See examples below) in the way the procedure should be conducted, as in its capacity to exhaust the capabilities which have to be presented. In developing the procedure which will be described in this study the author has attempted to combine the advantages of all the methods while avoiding their defects. Basically the procedure relies on the methods and techniques of topdown conceptual analysis (TCA) which are explained on page 19.

An ever increasing number of instructional designers have become aware of the challenge of educational technology and have searched for more ways of improving methods and techniques which will assure the most efficient and effective acquisition of capabilities and knowledge on the part of the learners.

Quite often, however, despite the perfectly written objectives and the undoubted recognition that we know where we want to go, we feel unsure of having found methods and techniques to generate - without involuntary omissions or repetitions - the knowledge base to build the "bridge" which will take our learners from where they are to where we want them to go.

Searching the literature on this important step of the design process the author has found interesting methods and techniques but they are usually not easy to follow.

Exemple 1

Robert Ascroft (1981) recommends the following procedure:

1. General objective
2. Terminal objectives
3. Content description
4. Audience characteristics
5. Enabling objectives
6. Posttest
7. Media selection and so forth.

To generate the knowledge base for the development of a learning system, one performs step 3 (Content description) as follows:

1. From the general objective ask:
   What subskills and knowledge will the student need to reach this objective?

2. Having identified the major components of the main task ask:

What subskills and subconcepts must the student possess in order to master the main component?

A complete content description specifies, in order, the sequences, skills and knowledge the learner will be experiencing as s/he is successfully performing the task. You will eventually trace back to skills and knowledge that the student enters the already knowing. This is the point at which to stop the analysis.

<u>Limitations</u>: Though the method and technique is quite effective, it does not provide an automatic check against involuntary omissions or repetitions.

<u>Exemple 2</u>

Paul A. Friessen (1971) of Friessen, Kaye and Associates Ltd., prescribes the following steps:

1. Problem identification
2. Classification of problem
3. Analyze the performance component
4. Population description
5. Entering behaviour
6. Subject analysis
7. Levels of learning (cognition, motor, problem solving)

8. Method of measurement

9. Measurement criteria

10. Instructional objectives

11. Teaching points

12. Sequencing, etc.

According to Friessen, to generate the knowledge base, one performs step 11 (teaching points) as follows:

> Now, it is necessary to list, point by point, what has to be learned in order for the learner to meet the measurement criteria already stated.

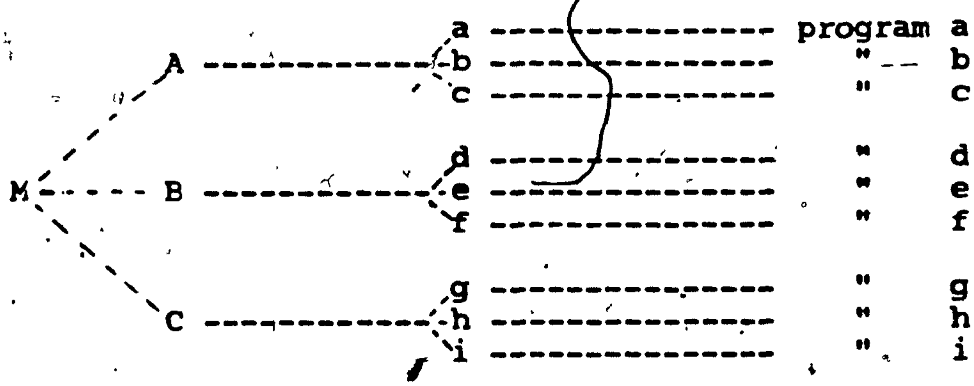
<u>Key Questions</u>: What teaching points must be covered in order that the learner will be able to answer the sub-criterion question?

<u>Comment</u>: The method and technique is purely intuitive because it does not provide an automatic check against involuntary omissions or repetitions.

<u>Example 3</u>

Robert Brien of Université Laval and Silvio Lagana (1978) of Florida State University give the following procedure to generate the knowledge base which they call "learning hierarchies" after Gagné (1965).

"If capabilities can be thought of as a computer program and subprograms, the problem of developing learning hierarchies may be seen as the recursive analysis of a main program into components. In the APL language, as an example, the analysis of the main program in its subprograms A, B and C is represented schematically in the figure below, where the execution of the main program is dependent upon the execution of A, B and C, whose execution is in turn dependent upon the execution of programs a, b, and c for A; d, e, and f for B; and g, h, and i for C.



"By analogy, such a schema may be considered as a 'learning hierarchy' where M is the higher-order concept or rule; A, B, and C are sub-higher-order concepts or rules; and a, b, c, d, e, f, g, h and i are first-order concepts or rules."

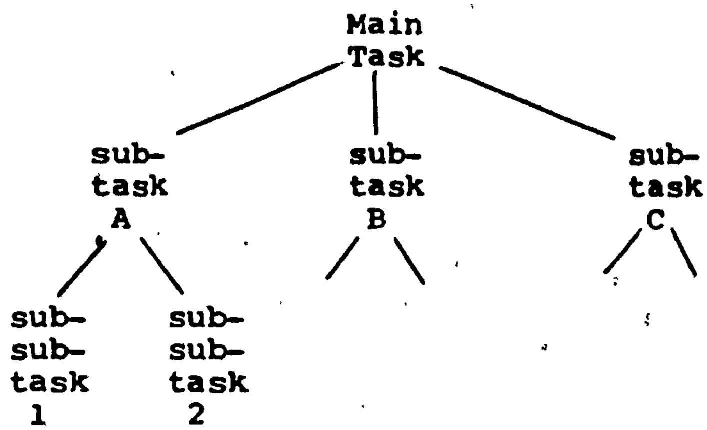
This method and technique is one of the several versions of topdown analysis.

Comment: It is very effective when the learning hierarchy is made up of less than 10 concepts or rules, because a tree of 20 concepts or rules does not fit on a page.

Limitations: When the learning hierarchy contains more than 10 concepts or rules, another model should be used before one can represent it by a tree diagram.

Example 4

Sivasailam Thiagarajan (1971) gives the following procedure to generate the skills and specifications:



This another version of topdown analysis.

Limitations: When the hierarchy contains more 10 sub-tasks, another model should be used.

Example 5

Robert E. Horn (1976) of Information Resources Inc. gives the following procedure:

Task 1. List job procedures.

Examine each of the job responsibilities and specific objectives listed in the description of objectives. For each of the objectives ask yourself:

"Are there any procedures which a student has to learn that are connected with this job responsibility?"

Ex.: Given two binary expressions, students will be able to

* add them
* subtract them them,etc

Task 2. List the knowledge topics found in the objectives, job responsibilities, and procedures.

Examine each of the job responsibilities, specific objectives, and procedures listed in the description of objectives. Make a list of

* the major technical items which will be introduced to the student
* the major pieces of equipment and the major parts of this equipment
* the names of other major physical structures
* the names of any major processes

Ex.: Objectives

1. to be able to tell the difference between positional and non-positional number numeration systems

2. given a binary expression, to be able to

   * convert it to decimal expression
   * convert it to octal expression

List of knowledge topics

1. Positional number systems
2. Non-positional number systems
3. Radix,etc.

Comment: In a way this also a version of topdown analysis.

Limitations: This procedure is less effective than the method using the tree diagram, because it does not provide an automatic check against involuntary omissions or repetions.

Example 6

Walter Dick & Lou Carey (1978) prescribes the following procedure:

1. Identifying an instructional goal
2. Conducting an instructional analysis
3. Identifying entering behaviors
4. Writing performance objectives
5. Developing criterion-referenced tests
6. Developing and selecting instruction
7. Developing an instructional strategy
8. Designing and conducting the formative evaluation
9. Revising instruction

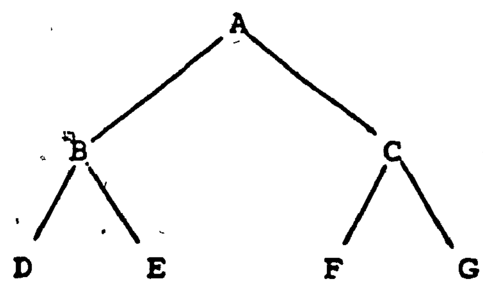
10. Conducting summative evaluation, and so forth

To generate the knowledge base, Dick and Carey suggest two approaches:

1. The procedure approach:

   S ----------→ R · S ---------→ R, and so forth

2. The hierarchical approach:



How does the designer go about identifying the critical subordinate skills a student must learn in order to achieve a higher level intellectual skill? The process suggested by Gagné is one of asking the question, "what does the student have to already know how to do, so that with a minimal amount of instruction this task can be learned? and so on and so forth.

<u>Comment and limitations</u>: The same as above.

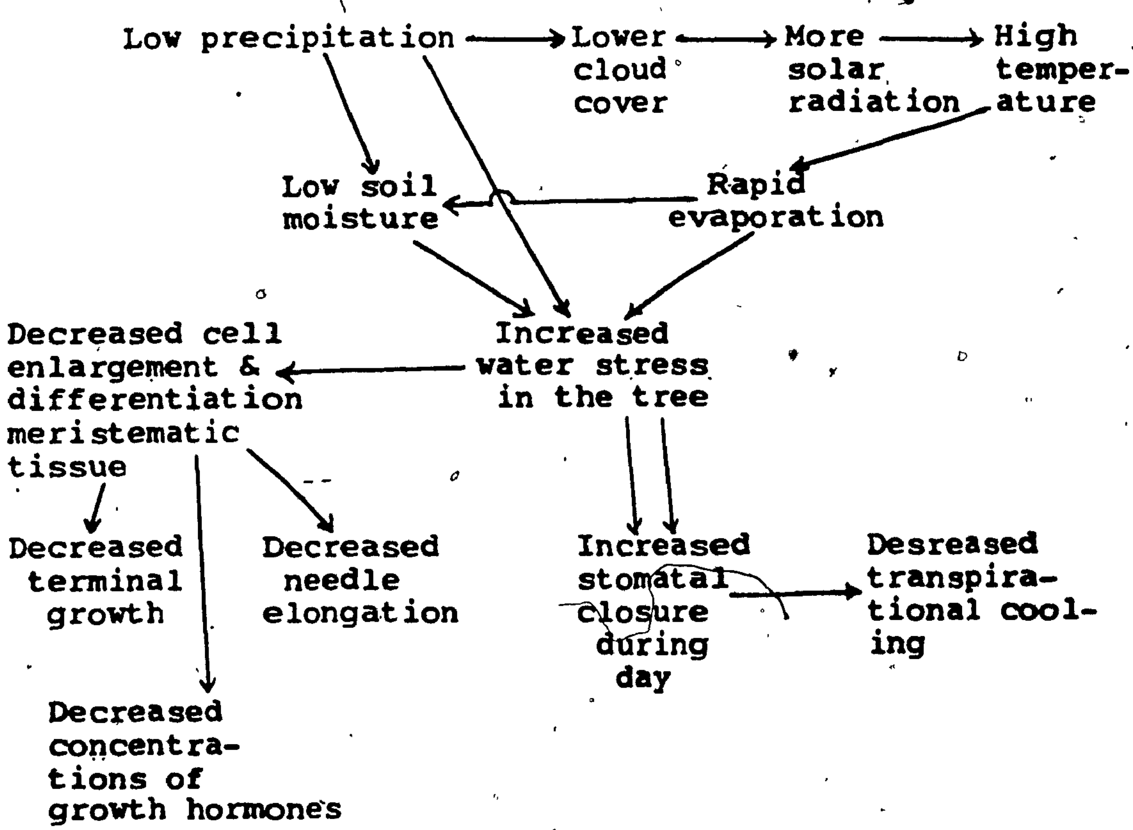
<u>Example 7</u>

The following are some of the approaches suggested by Romiszowski (1986) to generate lesson contents (knowledge base, tasks, skills,etc)for the development of instructional materials:

1. The task analysis approach:

No.	Steps in performing the task
1	Note the plug location relative to the cylinder
2	Remove all spark plugs
3	Identify the type of plags
4	Decide whether to adjust or replace plugs
5	Clean plugs, if necessary
6	Adjust plugs, if appropriate
7	Replace spark plugs in engine
8	Connect ignition wires to appropriate plugs
9	Check for performance

2. The topic analysis approach:



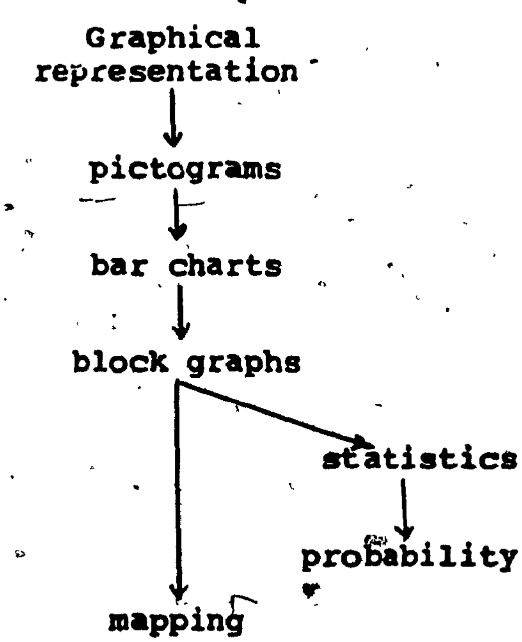
3. The Mathetics analysis approach:

$S_1 \longrightarrow$	R_1	.	$S_2 \longrightarrow$	R_2	.	S_3 ...
Telephone rings	Identify the line		External light is on	Connent the red cord		Connection made

4. The Clarke analysis approach:

Procedure

1. Identify the key concepts/rules/principles that are included in the subject.
2. Write them on a large sheet of paper, well spaced out.
3. Write, round each key topic, the subsidiary topics of which it is composed. Use arrows to indicate sequence/dependency, etc. (Be consistent as regards the meanings of arrows.)
4. Continue until all key concepts are interlinked by a network of subsidiary concepts.
5. Check for corrections/completeness/consistency/usefulness.



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Comment: The approaches suggested by Romiszowski are all very effective. However, the examples we have examined are very short ones - perhaps requiring at most a few hours to develop. This is a very far cry indeed from an operating system or a database management system which may require tens or even hundreds of thousands of instructions. Furthermore, the end-products of these approaches are not collections of propositions but collections of terms and phrases.

Tentative Conclusion

The examples we have examined provide good models for generating knowledge bases for simple subject matter contents. For subject matter contents requiring many learning hierarchies and procedures, they do not fully specify what needs to be done. The author has been using a method and technique called Topdown Conceptual Analysis (Le Xuan, 1981). It is another version of the topdown approach using the tree diagram last, when all concepts, rules and principles have been generated.

According to Ramamoorthy (1974), a pionner in Topdown Analysis, "not only was the software always late and expensive but also the final delivered product was very unreliable. Many software systems were released with thousands of bugs still in them. Each new release of the OS/360 contained roughly 1000 new software errors. Even after the program was considered to be thoroughly tested, there were 18 discrepancies found in the software during the 10-day flight of Apollo-14. This becomes more scary when we consider the complexity of the program for national defence and air traffic control.

The need for better techniques is illustrated in some of the quotations from design authorities:

> We build systems like the Wright brothers built airplanes - build the whole thing, push it off a cliff, let it crash, and start over again.
>
> Professor Graham (1969)

> The attempt to build a discipline of software engineering on such shoddy foundations must surely be doomed like trying to base chemical engineering on the phlogiston theory, or astronomy on the assumption of a flat earth.
>
> Hoare (1975), Professor of Computing

II : THE TCA-STRUCTURING METHODS & TECHNIQUES

TOPDOWN CONCEPTUAL ANALYSIS

What is Topdown Conceptual Analysis?

Topdown Conceptual Analysis (TCA) is a rigorous procedure to generate - without involuntary omissions or repetitions - the knowledge base for the development of learning systems. What is really important in TCA is the idea that a concept, rule, principle, or objective can be broken down into meaning pieces, and that these pieces can be broken into yet finer pieces, and so on, until we finally reach a stage at which the detailed writing of line after line of a "program" or system is appriopriate. Topdown analysis is an important phase in information science, and it implies a progression from the abstract ('top') to the particular ('bottom'). The tree diagrams we have examined are one way to visualize TCA.

An automobile engineer who starts his work by drawing sketches and then blue-prints for a propose car is using the topdown approach, whereas an engineer who starts by consulting a tire catalogue is not using the topdown approach. In the same manner, an instructional designer who starts her/his work by writing a line here and a line there is not using the topdown approach but the bottom-up approach with the consequence that her/his materials are full of errors (bugs), and that is exactly what most instructional designers are now doing.

While it may sound incredible that a topdown analyst can assist a subject matter specialist in analyzing the latter's own areas of knowledge, it is true that, by workin close cooperation and harmony with the subject matter specialist, the analyst can help the subject matter specialist organize the subject matter in such a way that it lends itself readily to any strategy appropriate for the target population or user.

In the course of performing the TCA the analyst may ask the subject matter specialist such questions as:

1. Why is the concept so important?
2. What are some positive examples of the concept?
3. What are some negative examples of the concept?
4. What are some facts that must be presented to support this principle?
5. What are some subordinate concepts of this concept?
6. What are some related rules of the concept? and so forth.

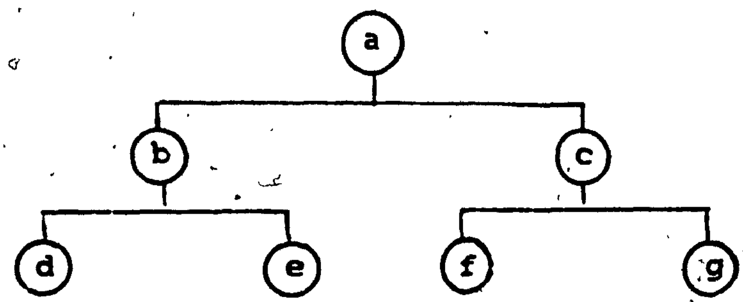
All the material treated in the TCA is contributed by the subject matter specialist, the analyst functioning merely as prober or prompter. S/he neither adds or subtracts anything from the domain of knowledge as it is construed by the subject matter specialist.

The Idea of TCA Is Not New

The term topdown conceptual analysis (TCA) may be new but the idea of TCA is not new. We all have used it in some form or other without realizing that it is called TCA.

1. In his "Discourse on the Method", René Descartes (1637) was certainly using the method and technique of TCA when he wrote about dividing each problem into as many parts as feasible and requisite for the solution of the problem. Unfortunately, Descartes never showed us exactly how to go about such division.

2. Robert Gagné (1968) was certainly referring to TCA when he said with regard to the problem of learning. "Ensure that the learner has acquired the pre-requisites and he will be able to learn. When he is capable of tasks d and e, he is capable of learning task b; when he is capable of tasks f and g, he is ready to learn c; and when he is capable of tasks b and c, he is ready to learn a. This situation can be visualized by the following tree diagram:



3. Henry R. Hatfield (1914) used TCA techniques (without so naming them) to breakdown the concept of proprietorship account into component parts:

- Proprietorship
  - Original
    - Contributed by A
    - " by B
    - " by C
  - Profit & Loss
    - Accumulated
      - Surplus
      - Reserve for A
      - Reserve for B
      - etc
    - Current
      - Accretions (Profits)
        - On Sales
        - Interest
        - Other gains
      - Deductions (Losses)
        - General Expenses
        - Business losses
        - etc.

4. Ken Orr & J.D. Warnier (1981) gave a new name to the tree diagram that we have introduced: Warnier/Orr diagram:

- Subscription System
  - In
    - Transactions
      - In
        - Unedited Transactions
      - Out
        - Edited Transations
    - Subscriber Master File
  - Out
    - Updated Subscriber Master File
    - Bills/Refunds

<u>Some Uses of TCA</u>

The methods and techniques of TCA are one of the most powerful tools for instructional designers, systems designers and students. Here are some of the uses of TCA:

1. Teaching oneself any subject matter with the help of textbooks, manuals and encyclopedias.

2. Measuring the level of difficulty of term, expressions or text.

3. Discovering subordinates of any concept.

4. Generating knowledge bases for the development of learning systems, expert systems, and so forth.

5. Generating raw materials for the construction of tests: pre-requisite tests, terminal tests, performance tests, and so forth.

STRUCTURED TEXT & INFOSTRUCT

The Problem

Technical handbooks, procedure books, technical training manuals, textbooks, and so forth, are usually too wordy or too condensed (abstract). Some authors lose track of what they were talking about in the previous page, because they include too many concepts, examples, rules and principles in a single page, with the consequence that readers spend too much time sorting out and reorganizing the materials to get the essential information they need. Other authors use words without rigorous definitions and/or fail to supply sufficient facts and examples to support their theories. But is it possible to present essential yet understandable information in a form that will communicate more in less time and space?

A Solution: Infostruct

Jean-Claude Chassain and the author (1975) have developed a method and technique which we call INFOSTRUCT (or STRUCTURED TEXT), which abandons the conventional paragraph form of writing and replaces it with functionally designed and labelled Information Structure and Information Elements. The information elements, of which there are an infinite number, define themselves by the kind of information they present: "definition elements," for instance, contain only definition, "example elements" contain only examples, and so forth. Fitted into one of the structures (networks, meshes, and so forth), the elements which may contain diagrams, tables, photographs, and formulas where necessary, allow a reader to absorb the information through scanning.

What Is It For?

Learning and reference work are the primary applications anticipated for Infostruct.

How Is It Used?

Infostruct materials must be produced in book form or organized into knowledge bases for intelligent tutoring systems and expert systems.

In books designed for initial learning and reference, the information is carried in clearly labelled information elements, arranged in an order prescribed for the kind of information involved. Other features of these self-instructional books include feedback questions and answers, special structures to facilitate learning and retention, charts and displays for easy retrival of topics for review and reference purposes.

For multi-purpose computer systems, an information structured database would be composed of separable labelled elements of information together with their interconnections. Only those parts of the elements required for a specific purpose need be called up. This flexible system would permit the user to organize sequences of elements and to display them in the order best serve her/his purpose, whether it be learning, reference, or browing.

Main Product

Most of the research and development work so far has been involved book versions of topics in mathematics, basic science, educational technology, and artificial intelligence. One of the product of this new system for which we have experimental data is a self-instructional text on Relations and related concepts for educational technology graduate students with minimum preparation in mathematics.

How Was It Developed

One of the main tenets of Infostruct is that the most reliable way to obtain material effectiveness is to make impirical testing and revision an integral part of the design and development process. Thus the text on Relations and related concepts was shaped, corrected and improved by tryout-and-revision cycles.

INFORMATION STRUCTURES & INFORMATION ELEMENTS

Introduction

All the materials used in INFOSTRUCT are organized into a grand structure made up of the following:

1st information elements
2nd information structures
3rd chapters or parts
4th modules or course

Definitions

An information structure is a collection of related information elements. In other words, an information structure is a collection of relevant information elements about a topic.

An information element is a component part of an information structure. It consists of

1. One or more sentences - with diagrams, tables, photos, drawings, and so forth - about a logical coherent fragment of subject matter, and
2. A label which describes the function or content of the element.

Types of Elements & Structures

The number of types of information structures and information elements seems to be infinite. Bu the author has used such types of structures as:

1. Conceptual structures which define and provide positive and negative examples of concepts.
2. Procedure structures which explain how to do things and in what order to do them.

3. <u>Overview structures</u>
4. <u>Preview structures</u>
5. <u>Recap structures</u>, and so forth.

The following are some of the types of information elements the author has used: Introduction, definition, positive examples, negative examples, description, procedure, flow charts, flow diagrams, use, notation, symbol, axiom, connective summary, pattern, feedback, input, output, historical antecedent, and so forth.

ORIGINS OF TOPDOWN CONCEPTUAL ANALYSIS & INFOSTRUCT

Introduction

In an effort to develop more efficient materials for learning and reference, I have drawn upon accumulated knowledge in science and technology. Research findings, generalizations, principles, rules and procedures from many domains were considered with a view to their possible practical value for instruction or reference.

Domains Drawn Upon

Gradually I evolve the set of guidelines and rules for generating, organizing and displaying the knowledge that has been referred to as INFOSTRUCT. These guidelines have their origins in such domains as:

1. Information processing theories
2. Applications of cognitive science to instruction
3. Schemata and conceptual networks
4. Content of secondary memory
5. Representation of information in second memory
6. Logical analysis of subject matter
7. Learning research findings
8. Teaching practice
9. Commuication technologies, and so forth.

The implications of the various concepts were translated into practical form and were documented as rules and procedures for developing information structures.

Some Theories Behind The Techniques

Although Infostruct and TCA mathods and techniques have their origins in several domains, there is no doubt that their principal foundations lie in learning theories and education research: cognitive science, information science and related domains.

III : SOME THEORIES BEHIND TCA & INFOSTRUCT

GENERALIZATION AND DISCRIMINATION

Introduction

Generalization and discrimination are the two most important concepts in the field of education. In fact, the aim of education is to create situations which will allow the student to generalize within a class and to discriminate between that class and other classes. In other words, to design materials which will allow the learner to generalize and to discriminate is the goal of instructional design. Yet, not many teachers and instructional designers are capable of explaining in a clear and simple language what these concepts mean.

Definitions

1. To generalize within a class is to make the same response to different stimuli. To behave in the same manner in front of different situations is also to generalize within a class of situations. To go beyond what is given is also to generalize within a class of objects and events.

2. To discriminate between classes is to make diferent responses to different stimuli. To behave in different manner in front of different objects or events is also discriminate between classes of objects and events.

Negative and Positive Examples

1. To say "fruit" in the presence of bananas, oranges, pears, and lemons is to generalize within a class of stimuli (objects). To visualize this we can draw diagrams like these:

Bananas -----> fruits
oranges -----> fruits
pears -------> fruits
lemons ------> fruits

Bananas ------
oranges ------ ---> fruit
pears --------
lemons -------

2. To say "bird" at the sight of herons, pelicans, ducks and eagles is to generalize within a class of stimuli (objects).

3. To say "foxes" in the presence of foxes but not in the presence of dogs is to discriminate between the class of foxes and the class of dogs. To visualize this we can draw a diagram like this:

Foxes -----------> foxes
Dogs ------------> dogs

4. To run away from cobras but not from water snakes is to discriminate between two classes of objects.

WHEN CAN WE SAY THAT AN INDIVIDUAL HAS LEARNED A DESCRIPTIVE CONCEPT?

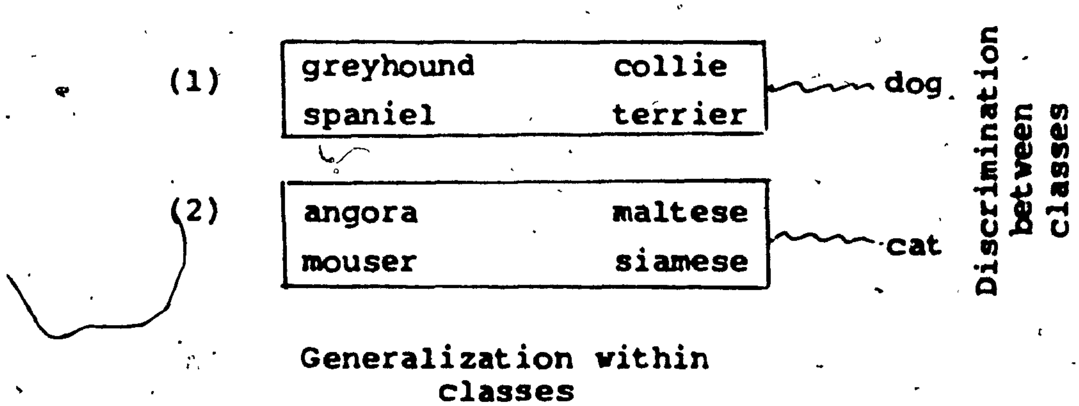
Introduction

When can we say that an individual has learned a concept? In other words, what must an individual do to show that s/he has learned a concept?

1. S/he must be able to generalize within a class of objects, events and relations, and
2. discriminate between that class and other classes.

Example 1

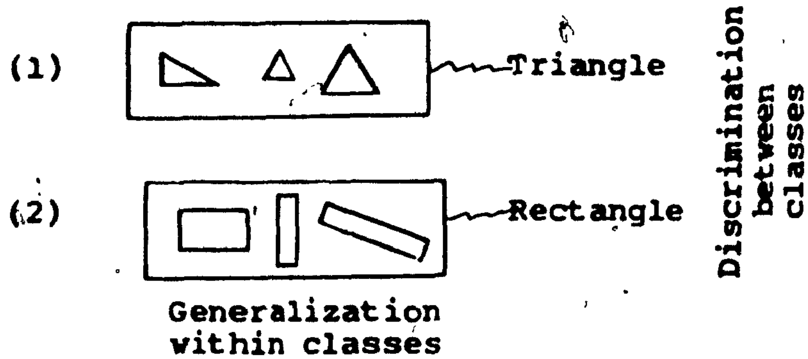
Consider the following class of objects:



When an individual makes the same response "dog" to each of the animals of class (1), and the same response "cat" to each member of class (2), he is generalizing within a class of dogs and a class of cats; s/he is also discriminating between a class of dogs and a class of cats. The psychologist would say that the individual has learned the concept of dog and the concept of cat.

Example 2

Consider the following classes:



When an individual is capable of generalizing within a class of triangles and a class of rectangles, and of discriminating between a class of triangles and a class of rectangles, we say that s/he has learned the concept of triangle and the concept of rectangle.

Negative Example 1

Paul said "square" in the presence of square figures and also "square" in the presence of figures which resemble squares but are not square. Paul has learned to generalize within a class of squares but he has not learned to discriminate between the class of squares and the class of non-squares. We say that Paul has not learned the concept of square.

Negative Example 2

A dog has been trained to detect the enemy. After three months' training the dog was still unable to discriminate between friends and foes. We say that it has not learned the concept of enemy.

Negative Example 3

When a person is capable of giving the definition of a term but is unable to give or recognize an example of the concept represented by the term, s/he has not learned the concept represented by the term.

HOW TO TEACH A DESCRIPTIVE CONCEPT

Introduction

In the preceding structure the behavior of the learner was analyzed to see what he does when he learns a concept: S/he generalizes within a class of objects (stimuli), and s/he discriminates between that class and other classes. Now we are going to analyze the behavior of the instructor to see what s/he does when s/he teaches a concept, that is, how s/he goes about teaching the learner to generalize and to discriminate among things.

Procedure

Suppose we have to teach the concept of organism, here is how we could proceed:

1. Show the learner one positive example of the concept "organism":

   "A dog is aliving thing and it is independent: a dog is an organism."

2. Show h/her a negative example of organism:

   "The heart of a dog is a living thing but it is not independent. It is part of a dog. The heart is not an organism. In fact, it is an organ.

3. Show h/her another positive example:

   "A cat is a living thing that can live by itself. The cat is an organism."

   Give h/her another negative example:

   "The lung of an animal cannot live outside the body of that animal. The lung is not an organism.

5. Give h/her a test to see whether or not s/he has learned the concept; that is, whether or not s/he can generalize and discriminate among organisms and non-organisms.

<u>Summary</u>

In addition to responding to similar things in the same manner in different situations, an individual also learns to make appropriate responses to different stimulus situations. For instance, s/he learns to discriminate between regular verbs and irregular verbs. When s/he calls a strange animal that s/he has never seen before by the right name or when s/he says the word "dog" to the picture of a pekinese or to the barking of a dog, s/he has learned the concept of dog.

POSITIVE AND NEGATIVE EXAMPLES

Introduction

We have seen that:

1. To teach a concept we must teach the learner to generalize within a class of things and to discriminate between that class and other classes, and

2. To teach the learner to generalize and to dis- discriminate, one must present h/her at least two positive examples and two negative examples of the concept. The number ~~of positive~~ and negative examples necessary for these two operations depends on the pre-requisites of the target audience and the complexity of the subject matter.

Now, where do these positive and negative examples come from? Usually, it is the subject matter expert who has to supply them.

Definitions

1. The positive examples (or simply examples) of a class or concept are the members or elements of that class or concept.

2. The negative examples (or non-examples) of a class or concept are the members or elements of a class which is similar to but different from the class or concept under study.

Positive and Negative Examples

1. A foxhound belongs to a class of objects called dog. A foxhound is said to be a positive example (instance,

examplars) of the concept (class) dog. A fox which looks somewhat like a dog but is not a dog, would be a good negative example of the concept dog. An elephant which has very few common characteristics with a pekinese, would not be a good negative example of the concept dog.

2. Chair is an excellent negative example of stool, because chair and stool both possess the same attributes (properties) except one: a chair has a back whereas a stool does not.

3. The following are some negative examples - in descending order of subtlety - of the concept dog:

.1 Jackals or foxes are negative examples of high subtlety.

.2 Cats are negative examples of average subtlety.

.3 Black and white are negative examples of obvious irrelevancy.

ORGANIZATION OF INFORMATION IMPORTANT FOR LEARNING

Introduction

Some important features of Infostruct owe their origins to a topic of current theoretical interest among learning theorists - namely, the logical and psychological structures of knowledge and their impact on learning and retention.

Theoretical Discussions

1. Piaget had long ago speculated that "learning is facilitated by presenting materials in a fashion amenable to organization" (Flavell, 1963), but it is only in recent years that learning theorists have actively taken up the problems of how knowledge structures develop and of the role of organization of learning and retention.

2. In a symposium on "Education and Structure of Knowledge" P.H. Phenix (1964) remarked: "It is difficult to imagine how any effective learning could take place without regard for the inherent patterns of what is to be learned.

3. Bruner (1966) asserted that the teaching/learning process is facilitated by STRUCTURE because it affords systematic means of organizing facts, events, things and relations. Structure fosters understanding, retention, and transfer by enabling the learner to more clearly discern relatioships among otherwise inert facts. "Grasping the structure of a subject is understanding it in a way that permits many other things to be related to it meaningfully. To learn structure,

in short, is to learn how things are related," Bruner concluded.

4. David Ausubel (1960,1963,1964,1968) has developed a logical and psychological case for believing that learning and long-term retention are facilitated by 'organizers' which provide an ideational scaffolding.'

5. The relation of organization of materials to ease learning also finds support in the domain of verbal learning research (Underwood, 1966).

Implications for Infostruct

Infostruct is a method and technique of organizing concepts, rules, principles, facts, and so forth in STRUCTURES. Everything that is interesting and meaningful is coherent or structured. A STRUCTURE is more than the sum of its parts, and therefore can only make sense as a whole. This wholeness derives from structures or systems.

FEATURES TO AID IN ORGANIZATION

Introduction

The following lists of features designed to promote organization of concepts and relationships contains some that I have already adopted in organizing the content of this study. For instance, the guidelines called for practice questions and answers throughout the structured text because learning research suggested their value in several ways; but questions can also be phrased to encourage organization of concepts and rules.

List of Features

1. Reviews and previews to take stock of the concepts developed up to that point and to prepare the ground for relating them to new concepts about to be encountered.
2. Introduction to each structure to relate new concepts to previous concepts or to the learner why s/he has to learn such and such concepts or rules.
3. Recaps to summarize succinctly the essential ideas or rules or principles in nushell form.
4. Diagrams to visualize the concepts, rules or procedures of a topic to show the role of each and its links to others.
5. Summary tables to chart in easy reference form the main concepts of a domain.
6. Review tests to promote the integration of several concepts and to practice using them in problem solving, and so forth.

A MODEL FOR MEASURING DESCRIPTIVE CONCEPT ACQUISITION

Introduction

How do we know that an individual has "acquired" a concept? S/he must be able to generalize and discriminate among things, concepts, facts, and events or relations and this involves at least ten things s/he must do to show that s/he has "acquired" a concept.

List Of Things One Must Do To Show That One Has Acquired A Concept

1. Given the name of an attribute value, s/he must be able to produce an example of the attribute value.
2. Given the name of a concept, s/he must be able to produce an positive example of the concept.
3. Given the name of a concept, s/he must be able to produce a negative example of the concept.
4. Given an example of a concept, s/he must be able to give the name of the concept.
5. Given the definition of a concept, s/he must be able to give the name of the concept.
6. Given the name of a concept, s/he must be able to give the definition of the concept.
7. Given the name of a concept, s/he must be able to produce the name of its subordinate.
8. Given the name of a concept, s/he must be able to give the name of its supraordinate.

9. Given the name of a concept, s/he must be able to produce its coordinates.

10. Given the concepts of a hierarchy, s/he must be able to identify the relations connecting these concepts, and so forth.

PRIMARY, SECONDARY & NTH-ORDER CONCEPTS

Introduction

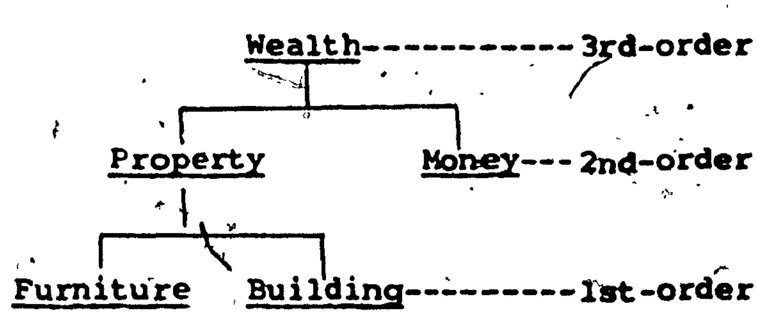
Most instructional designers I have met never heard of such terms as: primary, secondary, nth-order concepts; subordinate, coordinate and superordinate concepts. Yet the concepts represented by these terms are very important in instructional design and knowledge acquisition.

Definitions

1. According to Richard R. Skemp (1971), concepts that are derived from our sensory and motor experiences of the outside world are called primary concepts and concepts that are derived from parimary concepts are called secondary concepts.
2. For the sake of clarity we will use the term "1st-order concept" for "primary concept," and "2nd-order, 3rd-order, ... , nth-order concepts," as the case may require, for "secondary concepts." (There is nothing absolute about this "1st-order". In some other analyses it might be 7th-order).

Example

Below is a diagram of a conceptual structure:



In this example, furniture and building are "1st-order concepts. Furniture would become a "2nd-order concept" if it is broken down into chair, bed, stool. But in general, 2nd-order concepts are those that are derived from 1st-order concepts; 3rd-order concepts are those that are derived from 2nd-order concepts, and so forth.

Supra-ordinate or superordinate concepts are concepts that are of the highest order or level in a conceptual structure.

Coordinate concepts are those that are of the same rank or order in a conceptual structure.

Subordinate concepts are those that are of lover-order or rank in a conceptual structure.

CHAINS OF BEHAVIORS

Introduction

We have seen that a conceptual structure (or conceptual hierarchy, or hierarchical set) is a structure in which the lower-order concepts are prerequsites to the higher-order concepts. In this structure we will show you another type of structure, known as a chain. It is a sequence of ordered actions, that is to say some actions must precede others. For example, the action, "putting on socks," must precede the action, "putting on shoes," and so forth.

Any task that is to be taught effectively should be explicitly described before the instructional material is designed. In describing the task, we are interested first of all in how a really good specialist would perform in a given situation. Based on the description of the "best worker's" behavior, we determine exactly which steps make up the task. (Thomas Gilbert, 1961)

Definitions

1. To psychologists, a chain is a sequence of alternating stimuli and responses where each response produces the stimulus for the next response and so on. (Francis Mechner, 1964)
2. A logical sequence of stimuli and response following the other, is called a chain. (Thomas Gilbert, 1961)

3. A sequence of reflexes in which each R produces the S for the next R and so forth, is called a chain. (Keller & Schoenfeld, 1960)

1st Example

If we were to describe the behavior of a person when s/he buys a metro ticket from an automat, we might come up with something like this:

S_1 Having money in hand and facing the automat,

R_1 S/he checks to see if the machine is owkring.

S_2 S/he presses the button next to the station where s/he wants to go,

R_2 S/he puts in as many coins as necessary until a ticket is ejected,

S_3 S/he sees that a ticket is ejected,

R_3 S/he removes the ticket from the automat.

This sequence of stimuli and responses where each response produces the stimulus for the next responses and so forth, is called a chain.

2nd Example

The following is what one usually does before going to bed.

S_1	Takes off coat	R_1	Takes off shirt
S_2	Takes off shoes	R_2	Takes off socks
S_3	Takes off trousers	R_3	Washes
S_4	Brushes teeth	R_4	Puts on pyjamas, etc.

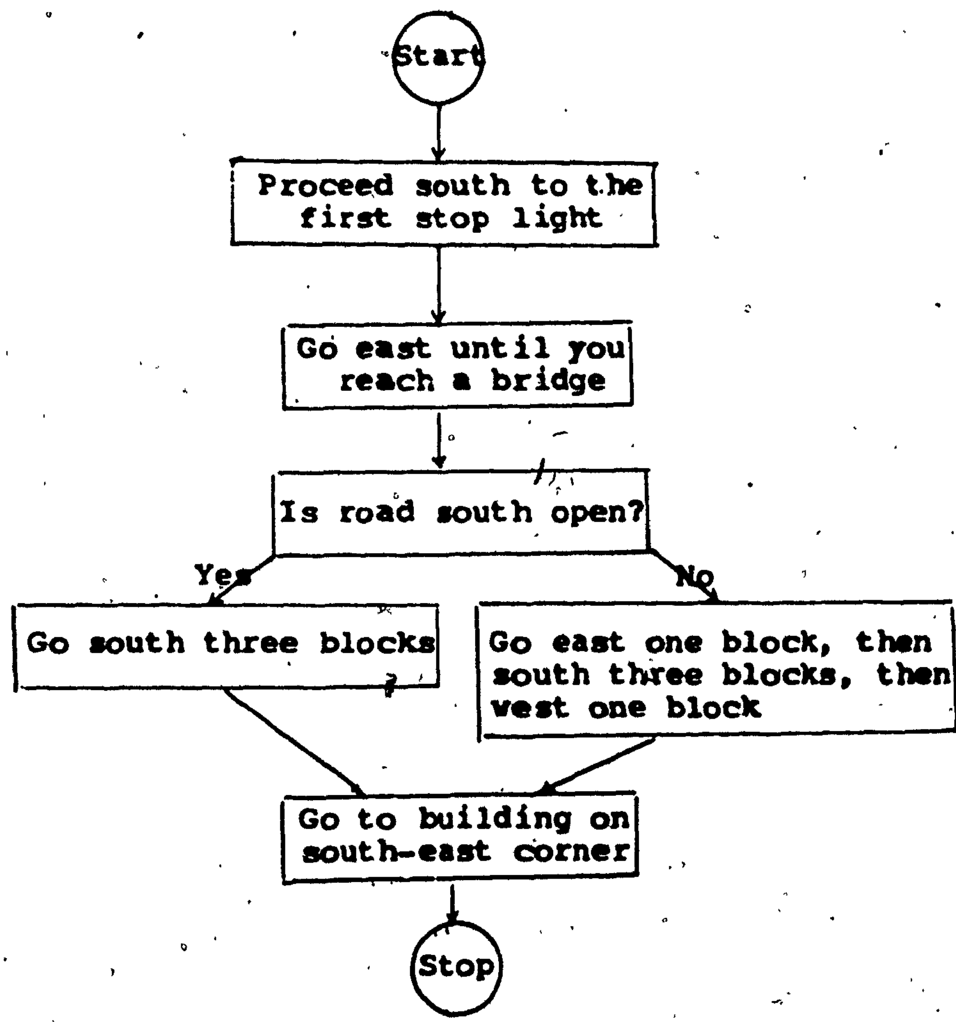
This sequence of stimuli and responses, one following the other, is called a chain.

3rd Example

We have all had the experience of telling a stranger how to find his way to a certain part or address of a city. Our instructions could be the following:

"Go south to the first stop light. Then go east until you reach a bridge. If the roat south from that point is open, go south three blocks, and the place you're looking for is on the south-east corner. But if the roat is blocked off, you will have to proceed one block further east, then three blocks south, then one block west."

These instructions can be described in a more pictorial (graphical) way in the following diagram, which is an example of a chain with decision points, commonly called decision flow chart or algorithm.



ALGORITHMS AND HEURISTICS

Introduction

In the study of problems, plans and operators are divided into two types: algorithms and heuristics. In non-AI programming, the programmer first develops a step-by-step method, called algorithm, for solving a certain class of problems.

There are unfortunately, two difficulties with this approach:

1. There are some types of problems for which there is no algorithm that will solve them.
2. And when there is an algorithm that will solve a certain types of problems, the algorith may be practical only for the smallest examples of the problem, and so inefficient as to be out of the question for practically sized problems.

Definitions

An algorithm is an exhaustive and resultative set of rules which, if followed, will automatically generate the correct solution. A heuristic is any rule of thumb or probabalistic procedure that helps us discover a solution to a problem.

Positive and Negative Examples

1. The rules for multiplication constitute an algorithm: if you use them properly, you always get the right answer.

2. Chess manuals do not give a prescription guaranteed to lead to success. Rather, they contain such rules as:

- ★ A bird in the hand is worth two in the bush.
- ★ A stitch in time saves nine.
- ★ Look before you leap.

Note that each proverb is a good advice. That is, each is certainly worth considering in the appropriate situation. Unfortunately, no proverb is guaranteed to always yield desirable results. Nevertheless, heuristic search is the principal problem-solving technique of Artificial Intelligence.

Measurement of Procedure Acquisition

To show that one has acquired the concepts of algorithm and heuristics, one must be able to:

1. discriminate between algorithms and heuristics.
2. determine if the task is well-defined: If the defining terms are vague, one cannot apply the algorithmic procedure.
3. determine if the task is manageable, that is, one must be aware of combinatorial explosions. For instance, the 10^{120} possible moves of a chess game: the fastest computer operating today could not explore them all in the time that remains before the death of our Sun.
4. know when to use algorithmic procedures and when to use heuristics.

KNOWLEDGE AND SKILLS IN A GIVEN DOMAIN

Introduction

What are knowledge and skills in a given domain or field? Dictionaries give such definitions as: "knowledge in a given domain is what we know in this domain," and "skill is the ability gained by practice," which are quite vague. In this structure we will try to define and explain these two terms in a more precise and concrete way.

Definitions and Explanations

Knowledge in a given domain consists of descriptions, relationships, and procedures in that domain. In short, knowledge in a given domain consists of (1) the symbolic descriptions and relationships in that domain and (2) the symbolic descriptions of procedures for manipulating these descriptions. The descriptions in a knowledge are sentences in some language whose elementary components consist of concepts, rules, principles, and procedures for applying and interpreting descriptions in specific applications. To have skills is to have knowledge and use it correctly (Frederick Hayes-Roth et al, 1983). N.B. Keep in mind descriptions of concepts, rules and procedures are not knowledge, any more than an encyclopedia is knowledge. We can say, metaphorically, that a book is a source of knowledge, but without a reader, the book is just ink on paper. Similarly, data structures in an AI database are knowledge when they represent concepts, rules and procedures when used by a certain program to behave in a knowledgeable way.

Examples of Knowledge

The following are some examples of knowledge:

1. Symbolic descriptions of concepts, rules or procedures used to infer situations from observables.

2. Symbolic descriptions of rules and procedures used infer likely consequences fron given situations.

3. Symbolic descriptions of rules and procedures used to infer system malfunctions from observables.

4. Symbolic descriptions of concepts, rules and procedures used to prescribe remedies for malfunctions.

Examples of Skills

1. A mechanic who can repair an engine in two hours instead of ten is said to have acquired great skill.

2. A person who is capable of applying knowledge to produce solutions, both efficiently and effectively, is said to be very skillful.

IV : THE DESIGN AND PRODUCTION OF SOME MATHEMATICAL PRE-REQUISITES TO CYBERNETICS

USING TCA TO GENERATE THE KNOWLEDGE BASES

The knowledge base is the raw material with which we build learning sytems. It is made up of two components: the declarative knowledge base which contains concepts, rules and principles, and the procedural knowledge base which contains procedures to perform such tasks as doing an experiment to illustrate scientific concepts, starting a car in the morning, or changing a tire.

The following is the TCA the author has performed to generate the knowledge base for the development of the structure, RELATIONS, one of the component parts of the structured text, "Some Mathematical Pre-requisites to Cybernetics". The knowledge bases for the rest of the text have been generated in the same manner. This TCA has been performed from a term (Relations) but a TCA can also be performed from any symbol, phrase, sentence, or text.

TCA : RELATIONS
Part A: Input & Output Inventory

Input	Outputs
1.0 Relations →	2.0 Introduction The concept of relation is very important both in everyday life and in science. Without the concept of relation we will not be able to describe or explain anything.

1.0 Relations →	3.0 **Definition** A <u>relation</u> is something that connects objects. It can be likened to cement or glue which holds things together, but it is not something that we can see, touch or photograph. It can be represented by a verb or verbal phrase. Other names for relation are: link, dependence, association, function, correspondence, and so forth. 4.0 **Positive Examples** 4.1 Consider the sentence: "Paul loves Jane." The verb "loves" which relates Paul to Jane states a <u>relation</u>. 4.2 The following are some other examples of relations: ... is north of ... ... depends on ... ... borrows ... ... borrows ... from ... ... is equal to ... ... is between ... and ... ... has the same address as ...

1.0 Relations →	**5.0 Negative Examples**
	5.1 Consider the following group of words: "Paul Jane." We cannot say anything about Paul and Jane because of the absence of such verbs and verbal phrases as: ... is in love with ... / ... likes ... / ... hates ..., etc. which expresses a certain link between Paul and Jane.
	5.2 Consider the sentence: "John borrows money from the bank." Now, if we strike out the verb "borrows", this group of words would be meaningless. In other words, without the concept of relation we cannot describe or explain anything.
	6.0 Arrow Diagrams
	6.1 Consider the relation ... is the brother of ... in a set of people: "John is the brother of Helen." We can represent (visualize) this relation by drawing an arrow starting from John to Helen like this: John ⟶ Helen or like this Helen ⟵ John.

1.0 Relations →	6.1 The important thing here is that the arrow goes from John to Helen. No arrows go from Helen to John because Helen is not the brother of John.
	6.2 Consider the relation "is the brother of" in a set of two males: "John is the brother of Paul." To represent the relation "is the brother of" in a set of two males, we draw double arrows like this: John ⇄ Paul or like this: John ⟷ Paul.
	7.0 Some Related Rules
	7.1 One cannot be conscious of one thing only. In order to have one thing, we must have at least three: a thing, a relation, and another thing. To explain what "uncle" is, for instance, we must have three things: 1. Uncle (the thing to be explained), 2. Is the brother of (a relation) 3. One's father or mother (another thing).
	7.2 Two unrelated terms cannot exclude each other (Hamelin, 1859-1907). In other words, the excluded thing must also be present before we can

1.0 Relations

7.2 have an exclusion. To suppress one term, the other will disappear just as shadow will disappear when light vanishes.

8.0 Exercises

(1) Underline the relations in the following:

a. Mark is as tall as John.
b. 4 is the square root of 16.
c. 16 is the square of 4.
d. Ottawa is west of Montreal.

(2) Which of the following is/are true?

a. We cannot describe or explain anything without the concept of relation.
b. Relations are something that holds things together.
c. A relation can be expressed by a verbe.
d. A relation can be expressed by a verbal phrase.

9.0 Answers

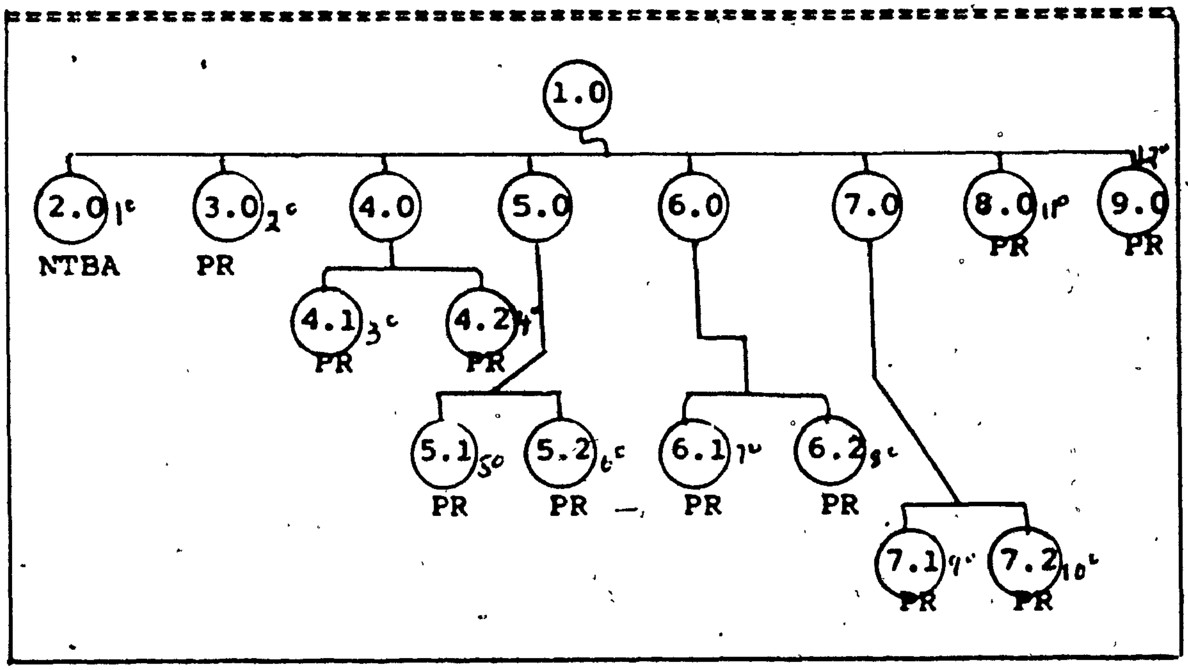
(1) You should have underlined: is as tall as, is the square root of, is the square of, is west of.

(2) All are true.

TCA : RELATIONS
Part A : Input & Output Inventory

2.0 Introduction →	NTBA	Short for Not To Be Analyzed. This means that by convention we will not analyze the content of the introduction.
3.0 Definition →	PR	Short for Pre-Requisite, by which we mean that this item contains no concepts which call for further analysis.
4.0 Positive Ex. 4.1 Consider ... → 4.2 The ... →	 PR PR	
5.0 Negative Ex. 5.1 Consider ... → 5.2 Consider ... →	 PR PR	
6.0 Arrow Diagrams 6.1 Consider ... → 6.2 Consider ... →	 PR PR	
7.0 Some Related Rules 7.1 One ... → 7.2 Two ... →	 PR PR	
8.0 Exercises →	PR	
9.0 Answers →	PR	

TCA : RELATIONS
Part B : Representation of TCA



TCA : RELATIONS
Part C : Sequential Index

1° (2.0 - 1.0) <u>Introduction</u>: The concept of relation is very important both in everyday life and in science. Without the concept of relation we will not be able to describe or explain anything.

2° (3.0 - 1.0) <u>Definition</u>: A <u>relation</u> is something that connects objects. It can be likened to cement or glue which holds things together, but it is not something that we can see, touch or photograph. It can be represented by a verb or verbal phrase. Other names for relations are: link, dependence, association, function, correspondence, and so forth.

3° (4.1 - 4.0 - 1.0) <u>Positive Examples</u>
Consider the sentence: " Paul loves Jane." The verb "loves" which relates Paul to Jane states a <u>relation</u>.

4° (4.2 - 4.0 - 1.0) Positive Example

The following are some other examples of relations: ... is north of ... , ... depends on ... , ... borrows ... , ... borrows ... from ... , ... is equal to ... , ... is between ... and ... , ... has the same address as ...

5° (5.1 -5.0 - 1.0) Negative Example

Consider the following group of words: "Paul Jane." We cannot say anything about Paul and Jane because of the absence of such verbs and verbal phrases as:

... is in love with ...
... likes ...
... hates ... , etc.

which expresses a certain link between Paul and Jane.

6° (5.2 - 5.0 - 1.0) Negative Example

Consider the sentence:

"John borrows money from the bank."

Now, if we strike out the verb "borrows", this group of words would be meaningless. In other words, without the concept of relation we cannot describe or explain anything.

7° (6.1 - 6.0 - 1.0) Arrow Diagrams

Consider the relation ... is the brother of ... in a set of people: "John is the brother of Helen." We can represent (visualize) this relation by drawing an arrow starting from John to Helen like this: John ⟶ Helen, or like this: Helen ⟵ John. The important thing here is that the arrow goes from John to Helen. No arrows go from Helen to John because Helen is not the brother of John.

8° (6.2 - 6.0 - 1.0) Arrow Diagrams

Consider the relation "is the brother of" in a set of two males: "John is the brother of Paul." To represent the relation "is the brother of" in a set of two mles, we draw double arrows like this:

$$\begin{tikzcd} \text{John} \arrow[r, bend left] & \text{Paul} \arrow[l, bend left] \end{tikzcd}$$ or like this: John $\longleftrightarrow$ Paul.

9° (7.1 - 7.0 - 1.0) Some Related Rules

One cannot be conscious of one thing only. In order to have one thing, we must have at least three things: a thing, a relation, and another thing. To explain what "uncle" is, for instance, we must have three things:

1. Uncle (the thing to be explained),
2. Is the brother of (a relation),
3. One father or mother(another thing).

10° (7.2 - 7.0 - 1.0) Some Related Rules

Two unrelated terms cannot exclude each other (Hamelin, 1856-1907). In other words, the excluded thing must also be present before we can have an exclusion. To suppres one, the other will disappear just as shadow will disappear when light vanishes.

11° (8.0 - 1.0) Exercises

(1) Underline the relations in the following:
   a. Mark is as tall as.
   b. 4 is the square root of 16.
   c. 16 is the square of 4.
   d. Ottawa is west of Montreal.

(2) Which of the following is/are true?
   a. We cannot describe or explain anything without the concept of relation.

b. Relations are something that holds things together.

c. A relation can be expressed by a verb.

d. A relation can be expressed by a verbal phrase.

11° (9.0 - 1.0) Answers

(1) You should have underlined: is as tall as, is the square root of, is the square of, is west of.

(2) All are true.

How Was Part A Generated?

Each input is written on the left side of the page. It generates one or more outputs which appear(s) on the right side of the page. Each of these outputs will in turn become new inputs for further outputs and so on, until we reach outputs which call for no further analysis: these are the pre-requisites.

In general, each new concept generates a structure which is composed of such elements as the following:

1. A introduction
2. A definition
3. Some positive examples (or some experiments if it is a science concept)
4. Some neg tive examples
5. A procedure (if necessary)
6. Some related concepts
7. Some related rules
8. An exercise with answers,etc.

How Was Part A Generated?

Rule: Each ouput (except those that are followed by the symbol PR) is transferred to the left to become a new input which will generate one or more outputs.

To generate Part A of the analysis I proceeded as follows:

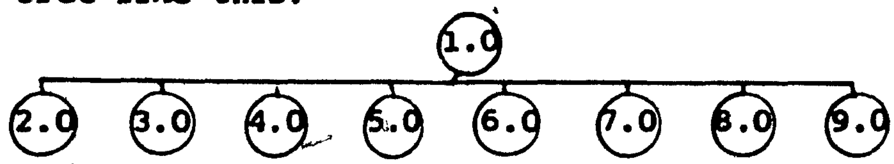
1. I divided all the pages into two parts, forming two columns: one for inputs and the other for outputs.
2. I recorded the material to be analyzed (in this case, the concept of relation) in the input column and numbered it 1.0.
3. Input 1.0 produced eight outputs: 2.0, 3.0, 4.0, 5.0, 6.0, 7.0, 8.0 and 9.0.
4. Input 4.0 generated outputs 4.1 and 4.2.
5. Input 5.0 generated outputs 5.1 and 5.2.
6. Input 6.0 generated outputs 6.1 and 6.2.
7. Input 7.0 generated outputs 7.1 and 7.2.
8. Last, all the outputs were transferred to the left side, where they became inputs for further outputs. As they contained no new concepts which called for further analysis, the symbol PR was written in front of each one of them, in the output column. This completed the analysis.

How Was Part B Generated?

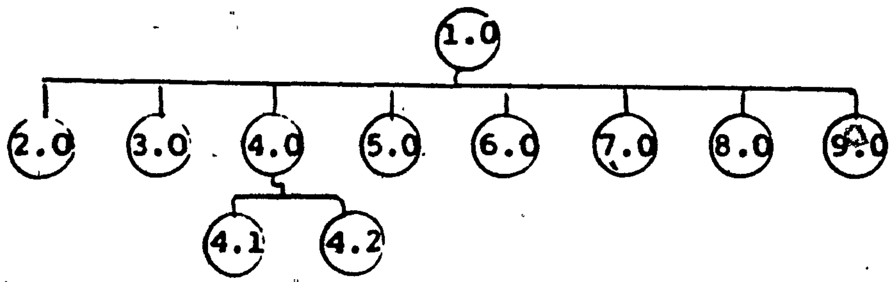
To reorganize the input-output inventory into a conceptual hierarchy I proceeded as follows:

How Was Part B Generated?

1. As input 1.0 produced eight outputs: 2.0, 3.0, 3.0, 4.0, 5.0, 6.0, 7.0, 8.0, and 9.0, I drew a partial tree like this:



2. As input 4.0 produced outputs 4.1 and 4.2, I added 4.1 and 4.2 to the above partial inverted tree which now looks like this:



3. I proceeded in the same manner until I exhausted all the inputs and outputs of the inventory and obtained the final tree.

4. The symbols 1°, 2°, 3°, ... , represent the order in which the items should be presented. The ordering begins with the first branch, from left to right, and the bottom upwards.

5. As shown, the TCA begins at the top to moves downwards. But the concepts and rules (represented by the nodes) are ranked from the bottom upwards. This means that the ana ysis proceeds from the highest order concepts to the lower order concepts, and the presentation of the concepts and rules proceeds from the lower order concepts to the higher order concepts.

How Was Part C Generated?

To generate Part C I proceeded as follows:

1. I picked out Item 1^{o} on the tree diagram, then looked at the input-output inventory, and copied the content of 1^{o} on a separate sheet of paper. This is the first element of the structure.
2. Next, I picked out Item 2^{o} and copied its content. I proceeded in the same manner until the last item had been disposed of. The end result is the sequential index or knowledge base I seek to develop. It is the raw material which I used to develop the structured text for this study.

Sources Used

In developing the structured text for this study, I used both domain experts and printed documents.

USING TCA TO GENERATE
THE KNOWLEDGE BASE FOR "ORDERED PAIR"

The following is the TCA that I performed to generate the knowledge base for the development of the structure, ORDERED PAIRS. This TCA was performed from a terminal test.

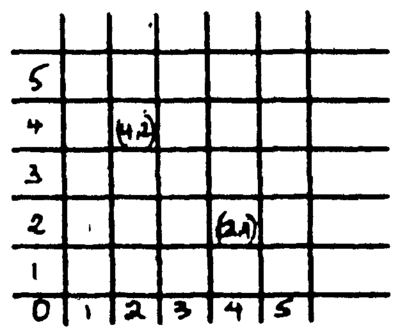
TCA : ORDERED PAIRS
Part A : Input & Output Inventory

1.0 (1) To symbolize ordered pairs we use braces or parentheses? (2) What is a pair? (3) What is an ordered pair?	2.0 Ordered pair.
2.0 Ordered pair	3.0 **Introduction** In many places in mathematics we need to distinguish between the pair (a,b) and the pair (b,a). The usual two-dimensional graph represents these pairs by distinct points unless, of course, a = b. When the order of a pair is of importance the pair is referred to as an ordered pair. 4.0 **Definition** An ordered pair of objects is a set of two objects in which it has been decided which is the first & which is the 2nd.

TCA : ORDERED PAIRS
Part A : Input & Output Inventory

2.0 Ordered pair →	5.0 **Positive example**

On a road map, one is often directed to find a location by scanning the area designated by the pair of numbers 4 and 2, for instance. This means that one can locate the area one is looking for by following the horizontal boundary until one reaches section 4 (see diagram below); then follows the



vertical boundary until one reaches section 2. Thus the direction (4,2) locates a position on a map. Now suppose one is trying to find another position by scanning the area designated by (2,4), what would one do? One would have to follow the horizontal boundary until one reaches section 2; then the vertical boundary until one reaches section 4. This direction (2,4) locates another position on a map.

TCA : ORDERED PAIRS
Part A : Input & Output Inventory

2.0 Ordered pair →	5.0 Positive example These pairs of objects (4,2) & (2,4) in which the first object does not have the same meaning as the second are called ordered pairs. Note that (4,2) & (2,4) represent different squares on the map.
	6.0 Negative Example A set which is composed of two element a and b, for instance, is called a pair and is written {a,b} or {b,a} because the order in which we list the elements of a set is not important.
	7.0 Notation Two objects, a and b, separated by a comma or semi-colon and written within parentheses: (a,b) or (a;b) in which "a" occupies the first place and "b" the second, is called an ordered pair.
	8.0 Exercise (1) To symbolize ordered pairs do we use brances or pa-theses?

TCA : ORDERED PAIRS
Part A : Input & Output Inventory

2.0 Ordered pair → 8.0, 9.0	**8.0 Exercise** (2) What is a pair? (3) What is an ordered pair? **9.0 Answers** (1) To symbolize ordered pairs we use parentheses. (2) A pair is a set containing two elements. (3) An ordered pair is a set consisting of two objects in a special order.

Important Remarks

1. The end-product of a TCA is a non-empty class or collection of structures; that is, a class consisting of at least one structure which contains enough materials for the analyst to say something about "ordered pairs."

2. This structure also contains enough materials for the analyst to measure the level of difficulty of the text; that is, to list the number of subordinate concepts contained in the text: graph, pair, and class. To obtain the list of the subordinate concepts of the structure, one just counts the number of technical terms used to define and explain the supra-ordina e (highest-order) concept: ordered pair.

3. With this structure, we can develop a lesson to teach a person something about "ordered pairs."

4. This structure is the knowledge base for the development of a short intelligent tutoring system.

CHOICE OF INSTRUCTIONAL MODE

Production Activities

To develop the Structured Text I proceeded as follows:

1. Determined the needs
2. Identified the problem
3. Specified the target population
4. Specified the entering behavior
5. Specified the terminal behavior
6. Generated the knowledge base
7. Converted the KB into teaching sequences
8. Decided on the mode of instruction, and so forth.

Why Structured Text?

The following are some of the reasons why I have chosen the paper booklet rather than the CAI method and technique:

1. I wanted to see how Educational Technology graduate students will react to the methods and techniques of Infostruct.
2. And then, Infostruct is a very effective tool for learners and teachers, because it is derived from such learning researches and teaching practices as:
   (a) Active responding generally aids learning. (Lumsdaine and May, 1965; Briggs, 1968; Glaser, 1965)

   (b) The act of writing responses helps some learners. (Edling, 1968)

   (c) Feedback or knowledge of results (or reinforcement often facilitates learning by:
   * confirming or correcting learner's understanding

(c) * providing a motivational effect
* improving scanning behavior
(Lumsdaine & May, 1965; Smith, 1964; Gagné & Rohwer, 1969; Glaser, 1965)

(d) The insertion of questions, "test-like events," after text segments has a positive effect on learning. Giving knowledge of results further increases the effect. (Gagné & Rohwer, 1969; McKeachie, 1963)

(e) Self-tests, pretests facilitate retention. (Glaser, 1965; Briggs, 1968, Lumsdaine, 1963)

(f) In concept learning, a variety of examples promotes learning. (Gagné & Rohwer, 1969; Lumsdaine, 1963)

(g) Instructions are useful in calling learner's attention to important features. (Gagné & Rohwer, 1969; Gagné, 1965)

(h) Judicious use of underlining often helps to focus attention on key elements. (Hershberger & Terry, 1963)

(i) Labelling appears to aid by alerting learner to nature of upcoming information and informing him what his learning task is. (Briggs, 1968)

(j) Pictorial materials often help learning. (Briggs, 1968)

(k) For some kinds of materials, charts of the information are valuable. (Feldman, 1965).

V : FORMATIVE EVALUATION, SUMMATIVE EVALUATION, CONCLUSION & RECOMMENDATIONS

INTRODUCTION TO THE FORMATIVE & SUMMATIVE EVALUATION STUDIES

Introduction

With the advent of any new educational product, questions of its usefulness and of its attractiveness to potential users must be dealt with. To answer such questions for Infostruct my approach has been primarily to seek performance data from subjects using structured text (Infostruct) to tab subjects' reactions to the materials through try-outs and questionnaires.

Background

My approach to the evaluation problem may be summed up by saying that I consider that:

1. "Media comparisons" are to be avoided.
2. Developmental testing with tryout-and-revision cycles is the key method for producing effective learning materials.
3. Pretest-posttest differences can be meaningful evidence for judging whether or not a given instructional system has achieved its objectives.

In actual practice this means that I will evaluate specific Infostruct products against explicitly stated learning objectives (terminal tests).

The spuriousness of "media comparisons" which abound in the literature of programmed instruction has been frequently discussed. In 1962, for instance, Stolurov (1962) detailed his reasons for judging media comparisons inappropriate and he expressed the "prediction and firm hope ... that the comparative study will become extinct."

In their 1965 review of the educational field, Lumsdaine and May (1965) summarize the shortcomings of media comparisons with 'conventional' instruction, and a corresponding increase in the proportion of studies which attempt to manipulate specifiable variables."

My General Approach

The evaluation program addresses itself to two issues:

1. The practical one of validating the strcutured texts, and
2. The more experimental one of investigating certain key parameters of these structured materials and of determining their influences on instructional outcomes.

Attitude of the user toward the new method is recognized as being a very important facet of the evaluation program. It matters little if a program is strikingly effective in teaching, yet repels students from further contact with the subject matter. Many educators are now turning to attitude and user-preference data as being even more informative than performance data in assessing the value of instructional techniques and methods. In my evaluation I use both performance data and attitude questionnaire.

The Evaluation Study

The instructional materials evaluated during this study dealt with modern mathematics : the units were entitled "Some Mathematical Pre-Requisites to Cybernetics". The text was aimed primarily at Educational Technology graduate students of Concordia University and the intention was to make the topic understandable even to students with minimal mathematical background. The materials were to be self-instructional.

THE EDUCATIONAL TECHNOLOGY TRYOUT

Introduction

The first planned formative and summative evaluation study of learning from an Infostruct text took place in September 1981. The instructional trxt was entitled Some Mathematical Pre-requisites to Cybernetics (See Appendix IV), and it consisted of some 40 pages of structured texts and of feedback questions coverning the following topics:

1. Understanding and Learning
2. Some Pre-requisites to Relations, Functions and Transformation
3. Ordered Pairs
4. Relations
5. Cartesian Products
6. Binary relations
7. Functions
8. Mappings
9. Correspondence
10. Transformations
11. Closed Under the Transformation
12. Graph Theory : Undirected Graphs
13. Degree of a Vertex

The first tryout was a simple, short-time one designed mainly to explore the effects of the structured text when used in a "natural" mode of use in home study.

Basic Purpose

The Ed. Tech. formative & summative evaluation was conducted to enable the author to:

1. Estimate student achievement as a result of the learning material under one condition only: in home study over a week's time.
2. Discover students' assessment of the Infostruct text as a communication technique and of its value in self-instruction.
3. Locate areas of the text that requires revision.
4. Obtain estimates of the time required to study the text.

Subjects

The 35 students for this study were members of courses: one in Cybernetics (ET 606) and one in Systems Analysis (ET 654) given in the Ed. Tech. Graduate Program of Concordia University.

General Plan

The tryout of the Infostruct text was planned to consist of a non-supervised home-study of one week. Pretests (See Appendix I) were administered to the subjects. Following that phase of the tryout, the subjects were to take the texts home, work on them during the week and return to the next class period prepared to take a final exam (posttest) (See Appendix II).

The plan to have students study at home worked out quite well. According to their notes and comments, the posttest given after the intervening week was for many a test of their grasp of the new material. The results section will reproduce the relevant data.

Data Collection Form

1. Pretests and posttests are used to measure the subjects' understanding of the topics before and after a home-study period.
2. A post-study attitude questionnaire tapped subjects' reactions to learning from the Infostruct text, to the presentation methods of the structured text, the specific format features and toward the possibility of further study from such materials.
3. The instructions ask subjects to write criticisms and comment throughout the text whenever they wish.

Procedural Details

1. Orientation: General explanation of the plan for the tryout and the range of the material to be covered was given. Students already familiar with the topic were encouraged not to take part in the tryout.
2. Pretest: This was administered under conditions where each subject's time was recorded.
3. Study Instruction: Texts were distributed and instructions were to answer feedback questions and to write criticisms as they study the texts.
4. Home-study: The class was instructed to finish to finish studying the text during the coming week and to record time spent; they were to prepare for the post-test a week later.
5. Post-test (See Appendix II): This was administered under conditions where each subject's time was recorded.
6. Post-study attitude questionnaire (See Appendix III) were filled out.
7. Texts were returned to students who requested them.

Data Analysis Methods

Because this is simply an evaluation of learning outcomes and attitudes toward this kind of instructional material, the data analysis methods consisted of t-tests of the difference of the means of the pre- and post-test scores given a week later after possible home study.

Attitude data were simply summarized to indicate the strength of subjects' opinion about various fatures of the learning material.

Formative evaluation: comments recorded in the texts were compiled as a guide for further revisions of the material.

TRYOUT RESULTS

Achievement Scores (Summative Evaluation)

Scores on the subject matter tests were stated in terms of percentage of items successfully passed. These are given in the table below for the total result of this study.

Posttest was the test given a week later after the possibility of home study.

Text	Pretest (N= 35)	Posttest (N= 35
Whole Text	Mean Score 47% S.D. 23.95	Mean Score 86% S.D. 12.96

From inspection we can see that the mean improvement is quite large.

A t-test of the difference in mean scores between pre-test and posttest is statistically significant with $P < .001$ ($t_d = 8.75$, $df = 33$).

The central point of the achievement score evaluation is simply that the learning material has made a significant impact on the subjects' knowledge of the subject domain.

Time

If the total time spent with the learning material is divided by the total number of pages the subjects covered in the period, we have the first estimate of how long it takes to study the Infostruct pages.

The total number of minutes divided by the total number of pages completed gives 15 minutes as the estimate of mean time for a page.

Attitude Results

Often the most important effects of an instructional system lie in its capacity to attract and sustain students' interest. For this reason the author was particularly concerned to learn the subjects' reactions to the Infostruct material.

Items in the attitude survey consisted mainly of questions which can be answers by YES or NO.

Summary of Attitude Test Result

Rather than reproduce the questions verbatim (for exact text see appendix III), I shall give the gist of them more tersely along with the percentage of subjects expressing given views:

1. 70% of subjects completed the module, because of the section on directed graphs which they found too difficult.

2 & 3. Subjects spent about 3 hours to complete the module, or about 15 minutes for each structure (as reported by students).

4. 97% said that the material is organized to promote learning.

5. 85% said that the text gives a clear picture of the subject matter.

6. 97% said that the language used is clear and easy to understand.

7. 82% said that positive and negative examples are helpful in illustrating the concepts to be learned.

8. 88% said that the format of the structure favors scanning.

9. 82% said that graphic cues enhance learning.

10. 91% said that they prefer the format of this module to that of the traditional textbooks.

11. 94% said that the module is helpful as a self-instructional material.

12. 100% said that the module gave them an idea of what new mathematics is.

13. 91% said that the module is easy to learn.

14. 76% said that the module is well-structured for beginners.

15. 88% said that the module provides a good progression: from simple to complex materials.

16. 79% said that the method of presenting information gives confidence to the learners.

17. 67% said that the module helped them learn Cybernetics.

18. 85% said that they like structured lessons.

19. 61% said they would like to see some of the courses converted into structured lessons like those in the module.

ATTITUDE RESULTS : OTHER COMMENTS & REMARKS

Introduction

Though subjects had very little time while studying Cybernetics (which is considered by most students as very fascinating but difficult) some have taken the trouble of writing a comment.

Types of Comments or Remarks

1. I think it is a very important educational material an am looking forward to subsequent modules. Thank you very much.
2. I believe that I spent too little time on the module, because it was a lot of work to assimilate the material even though it was quite simple.
3. I feel that the material is a good instroduction but there is not enough practice or application to promote long term retention.
4. I was already familiar with most of the material presented, but I found the module helpful and well-structured for reviewing it. It was easier to understand than the text I had used which is a Discrete (mathematical) Structure course given on undergraduate level (not at Concordia).
5. Congratulation!
   Perhaps I will not have to remain an algebraic moron for the rest of my life.
6. I liked the way the material was presented, but in some way it bothered me the negative examples. The last unit I could not understand, I think it was unclear.

7. Did not understand undirected graphs and degrees of a vertex. All in all, I enjoyed doing this module.

8. I feel quite comfortable with this material and I do have an antipathy toward math. The section on <u>Functions</u> was the one which gave me the most trouble, perhaps a little more explanation there would be helpful. I appreciate the opportunity to participate in this. It has defnitely helped me.

<u>Experimenter's Observations</u>

Most of those who did not complete, actually did complete everything except the section on directed graphs.

OVERVIEW OF THE EVALUATION STUDY

Introduction

First of all, two points about the evaluation study in general:

The experimental data bear on the specific product and only indirectly on the method (though we can always infer the quality of the method from the quality of the product). Just as one given textbook cannot be considered as representative of a so broad category as "textbooks in general" so the Infostruct text the author developed and tested is only one sample of the possible products of Infostruct methods and techniques.

Achievement Data

The evaluation study has shown some of the effects of what can be expected from the use of this Infostruct text under conditions of home-study. After some hours of study in a setting free from the usual educational pressures, subjects showed significant evidence of learning. The fact in itself is not surprising in view of the empirical tryouts and revisions that the text underwent through its development.

Attitude Data

The difficulty of trying to establish an objective standard for comparing educational materials has led researchers to turn to attitude data as being highly revealing about the attractive power of a learning system.

The reaction of our subjects to Infostruct texts was generally quite favorable; they rated the materials as effective, said they would recommend them to others, and singled out many of the special features for favorable mention.

CONCLUSION AND RECOMMENDATIONS

The Structured Text

The Structured Text has been revised several times to improve its effectiveness. This experiment showed that subjects did learn what the text was supposed to teach. The time factor appears to be the main one which determines the success of students' performance. Based on the learning outcome of the learners, their observed attitudes and other parts of the formative evaluation, it is concluded that the text has reached a high level of success in terms of its effectiveness.

Cost Effectiveness

The cost effectiveness of an instructional material is the analysis of its cost and achievement (Haller, 1974; Harmon, 1970; Hartley, 1968). It provides a decision tool to select the best choice among the feasible alternatives on the basis of the least cost and the greatest effectiveness. Hence, to determine the cost effectiveness of this structured text, two factors have to be examined:

1. its cost as compared with that of others, and
2. whether it has achieved its behavioral objectives or not.

The estimate cost in producing this structured text is about $9,000.00 (which covers the expenses for the designer, domain expert, research, developing costs and photocopying) while the market price would be about $5.00. It would be fully economic if two thousand copies were sold to recover the cost and make a reasonable profit.

Recommendations

Bruner has shown that the learning and teaching of structure is an effective instructional technique and the method has been implemented to develop a variety of instructional materials. In this present study the technique is also rated favorably because of its capacity for increasing learning, retention and transfer. Nevertheless, further research is recommended to compare Structured Text with other types of programmed texts to determine relative effectiveness. Other types of questions which can be asked in the future studies are: When is Structured Text most appropriately used? Are there any underlying factors that determine the use of Structured Texts? How can Structured Text methods and techniques be further improved?

And on the basis of the observation that most of those subjects who did not complete the text, actually did complete everything except the section on directed graphs, it is clear that the section on directed graphs needs to be revised and expanded.

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APPENDIX I

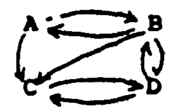
SOME MATHEMATICAL PRE-REQUISITES TO CYBERNETICS

Appendix I Pre-Test Circle the right answer *Your (Code) Name*

1. Which of the following symbolizes (represents) an ordered pair?
   a.]a,b) b. {a,b} c. (4,5) d. (4&5)

2. Which of the following represents a binary relation?
   a. (a,b)R b. a R a c. R(a,b,c) d. {a,b} ∈ R

3. The following is an incomplete arrow diagram of the relation R "is the brother of" in a set of boys:

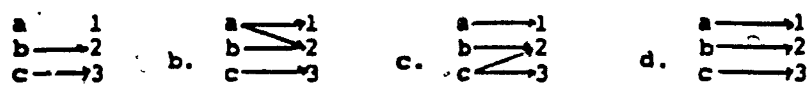


   How many arrows are missing?
   a. 4 b. 5 c. 6 d. 7

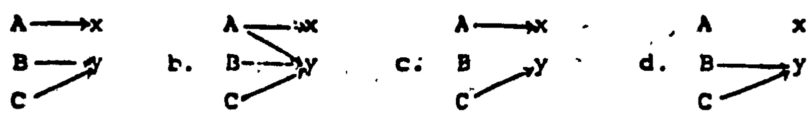
4. The domain of the relation R = {(1,2),(2,3),(3,4)} is
   a. {1,2,3} b. {2,3,4} c. (1,2,3) d. (3,2,4)

5. The co-domain (or range) of the relation R = {(1,2),(2,3),(3,4)} is
   a. {1,2,3} b. {2,3,4} c. (1,2,3) d. (3,2,4)

6. Which of the following arrow diagrams describes a function of A into B. A = {a,b,c} and B = {1,2,3}.



7. Which of the following arrow diagrams describes a mapping from a set of people to a set of subject matters?



8. What is the operator or rule of the following transformation?

   $$T \downarrow \begin{array}{cccc} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \end{array}$$

   a. Add 2 b. Add 4 c. Add 5 d. Subtract 2

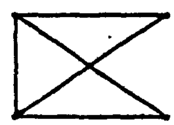
9. Which of the following does not describe a function?
   a. $f : x \longrightarrow x + 5$
   b. $g : x \longrightarrow x^2$
   c. $h : x \longrightarrow x - 5$
   d. $j : x \longrightarrow$ all y less than x

10. Which of the following symbolizes an unordered pair?
   a. {a,b{
   b. (a,b)
   c. (a;b)
   d. (a&b)

11. A graph G consists of three things. What are they?

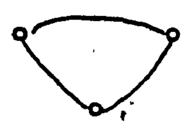
    a. A set of points, a set of nodes, and a set of vertices.
    b. A set of points, a set of unordered pairs, and a function.
    c. A set of nodes, a set of ordered pairs and a relation.
    d. A set of points, a set of rules and a relation.

12. How many vertices has the following graph?



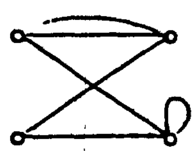
    a. Two
    b. Three
    c. Four
    d. Five

13. How many nodes has the following undirected graph?



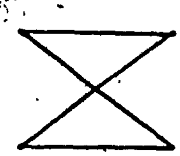
    a. Two
    b. Three
    c. Four
    d. Five

14. How many edges has the following undirected graph?



    a. Four
    b. Five
    c. Six
    d. Seven

15. What is the sum of the degrees of the vertices of the following undirected graph?



a. 8 b. 9 c. 10 d. 11

APPENDIX II

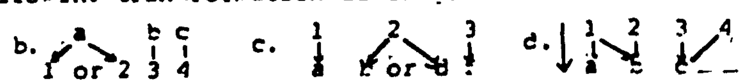
Appendix II

SOME MATHEMATICAL PRE-REQUISITES TO CYBERNETICS

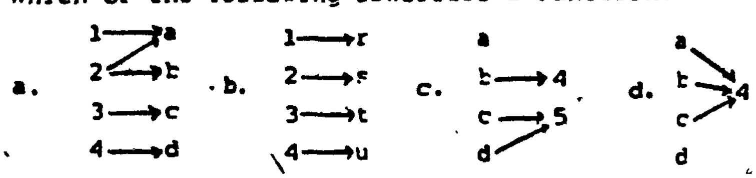
Your (Code) Name ________ Post-Test Circle the right answer

1. How many vertices has the following graph?
   a. 5 b. 6 c. 7 d. 8

2. How many edges?
   a. 5 b. 6 c. 7 d. 8
3. Which of the following transformations is closed?
   a. $T\downarrow\begin{matrix}1 & 2 & 3 & 4 & 5\\ a & b & c & d & e\end{matrix}$ b. $\downarrow\begin{matrix}4 & 5 & 6 & 7\\ 4 & 5 & 6 & 7\end{matrix}$ c. $\downarrow\begin{matrix}A & B & C & \ldots & Z\\ 3 & 6 & 7 & \ldots & 9\end{matrix}$ d $\downarrow\begin{matrix}b & c & d & e & f\\ a & b & c & d & h\end{matrix}$
4. A transformation is a kind of a
   a. Relation b. Function c. mapping d. All of the above
5. Which of the following is an ordered pair?
   a. {4,H} b. [4,H[c. (4,H) d. (4&H)
6. What is the set of operands of this tranformation?
   $T\downarrow\begin{matrix}1 & 2 & 3 & 4\\ r & s & t & u\end{matrix}$
   a. {1, 2, 3, 4} b. {r, s, t, u}
7. A relation can be
   a. One-to-one correspondence b. One-to-many correspondence
   c. Many-to-one correspondence d. All of the above
8. Which of the following transformation is single-valued?
   a. $T\downarrow\begin{matrix}1 & 2 & 3 & 4\\ 5 & 6 & 7 & 8\end{matrix}$

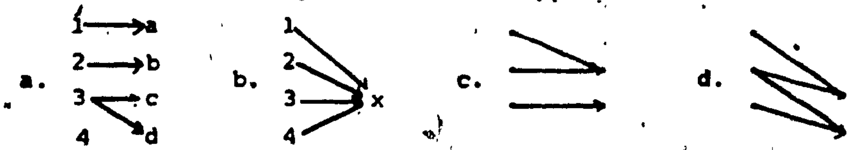


9. The transformation
   $T\downarrow\begin{matrix}1 & 2 & 3 & 4\\ 6 & 7 & 8 & 9\end{matrix}$
   a. $n \longrightarrow n + 6$ b. $n \longrightarrow n + 5$ c. $n \longrightarrow n + 4$ d. $n \longrightarrow n + 3$
10. What is the rule of this function f?
   $1 \xrightarrow{f} 4$
   $2 \longrightarrow 5$
   $3 \longrightarrow 6$
   a. add 2 b. add 3 c. add 4 d. add 5
11. Which of the following describes a function?

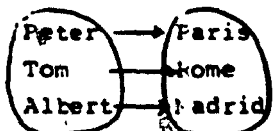


Appendix II

12. Which of the following describes a mapping?



13. The following arrow diagram describes a relation:



What is its domain?

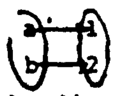
a. {Peter, Tom, Albert}

b. {Paris, Rome, Madrid}

14. What is the range of the above relation?

a. {Peter, Tom, Albert} b. {Paris, Rome, Madrid}

15. What is the co-domain of the relation represented by the following diagram:



a. {a, b} b. {1, 2}

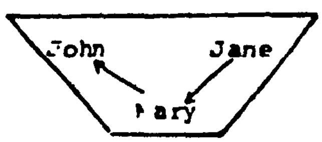
16. What is the difference between a relation and a mapping?

a. In a relation each member of the domain may/zero, one or more than one images whereas in a mapping each member of the first set has exactly one element of the second set paired with it.

have

b. In a relation each member of the first set may not have more than images whereas in a mapping each member of the domain may have zero one or several images.

17. The following is an incomplete arrow diagram of the relation, "is the sister of," in a set of boys and girls.



How many arrows are missing?

a. 1 b. 2 c. 3 d. 4

18. A relation R is described by the following set of ordered pairs.

R = {(Paris,France),(Rome,Italy),(Madrid,Spain)}

Its domain is:

a. {Paris, Rome, Madrid} b. {France, Italy, Spain}

19. A relation is composed of three things. What are they?

a. A set A called the domain, a set B called the co-domain or range, and a rule.

b. A set S called the co-domain, a set T called the range, and a rule.

c. A first set, a domain and a rule.

d. A co-domain, a second set and a rule.

APPENDIX III

Appendix III

QUESTIONNAIRE

Your Name:

1. Did you complete the course? ______
2. How much time did you spend on the course? ______
3. How much time did you spend on each sequence? ______
4. Is the material organized to promote learning? ______
5. Does the text give a clear picture of the subject? ______
6. Is the language used clear & easy to understand? ______
7. Are the positive and negative examples helpful in illustrating the concepts to be learned? ______
8. Does the format of the sequence (the organization of the sequence into elements) favor scanning? ______
9. Do the graphic cues (arrow diagrams and underlinings) enhance learning through emphasis and attention focusing? ______
10. Do you prefer the format of this course to that of the traditional textbooks? ______
11. Is the course helpful as a self-instructional material? ______
12. Does the course give you a clear idea about Instructional Design is? ______
13. Is this course easy to learn? ______
14. Is this course well-structured for beginners? ______
15. Does this course provide a good progression from simple to complex materials? ______
16. Does this method of presenting information give confidence to the learners? ______
17. Does the content of this module give you any help in studying new subjects? ______
18. On the whole do you like this method of presenting information? ______
19. Would you like to see all textbooks converted into structures lessons like those presented in this course? ______
20. Other comments or remarks are welcome!

APPENDIX IV

Appendix IV

SOME MATHEMATICAL PRE-REQUISITES TO CYBERNETICS

By Le Xuan

Table of Contents

Appendix IV : Some Mathematical Pre-requisites to Cybernetics

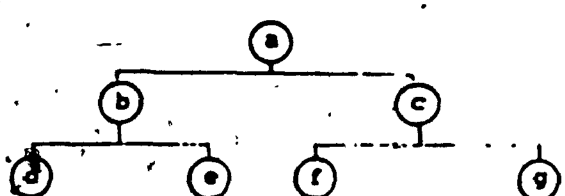
1 : UNDERSTANDING AND LEARNING

The Problem.

There is a close relationship between understanding and learning. When there is no understanding there is no learning, except perhaps rote learning. This view has been clearly expressed by many educators and educational technologists.

According to Skemp /1/, concepts of higher order than those which a person already has cannot be communicated to him by a definition, but only by arranging for him to encounter a suitable collection of examples. And since in mathematics these examples are almost invariably other concepts, it must first be ensured that these concepts are already formed in the mind of the learner. In other words, the lower-order concepts must be present before the next stage of abstraction is possible. In building the hierarchy (or structure) of successive abstractions, if a particular level is imperfecly understood, everything from there on is in peril. This dependency is probably greater in mathematics (and related subjects such cybernetics and information theory) than in other subjects. One can understand the geography of Africa even if one has missed that of Europe; one can understand the history of the nineteenth century even if one has missed that of the eighteenth; in physics one can understand heat and light even if one has missed sound. But to understand algebra without ever having really understood arithmetic is an impossibility. To learn calculus without analytic geometry and trigonometry is not possible.

Gagné /2/maintains that readiness to learn is essentially a function of the presence or absence of pre-requisite learning. "Ensure that the learner has acquired the pre-requisite capabilities and he will be able to learn. When he is capable of task d and e, he is by definition ready to learn task b, and when he is capabl of tasks b and c, he is ready to learn a:



A Solution

What Skemp and Gagné wanted to say is: Before we can understand and learn a new concept, we have to find out what its contributory (or related concepts) are and learn them first. In this book you will find some pre-requisite concepts to Educational Cybernetics. Get acquainted with them and look them up when you meet them again in your textbooks /3/.

Appendix IV

1 : UNDERSTANDING AND LEARNING-2

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Appendix IV

2 : SOME PRE-REQUISITES TO RELATIONS, FUNCTIONS & TRANSFORMATIONS

Set and Element

We start with the concepts of <u>set</u> and <u>element</u> taken as understood. What is essential is that given an object we must be able to tell whether this object belongs or does not belong to the collection under study. This fact will be written as $x \in S$ or as $x \notin S$. If the set is described by the naming of its individual elements, it will written within braces (curly brackets), for example as $\{1,2,3\}$. Repetitions of an element within a set will be ignored. The <u>empty set</u>, that with no elements, will be represented by $\{\ \}$ or ~~by $\emptyset$~~.

If two sets are such that every element of A is also an element of B, we write $A \subset B$ (read, "A is a subset of B). If A and B are composed of the same set of elements we write $A = B$.

Complement

Given two set A and B, the set $A - B$ (read, "A not-B) is defined as consisting of those elements that are in A but <u>not</u> in B. If A is some basic set that can be taken for granted, B' will signify the set of elements not in B ("but in A" is understood). B' is the <u>complement</u> of B.

Union & Intersection

Given two sets A and B, their <u>union</u>, written $A \cup B$ (read,"A union B" or "A cup B"), is the set of elements that belong either to A or to B or to both.

Their <u>intersection</u>, written $A \cap B$ (read,"A inter B" or "A cap B"), is the set of elements that belong to A and also to B.

Implication

If statement p <u>implies</u> statement q, that is, if p's being true implies thatq must be true, we shall write $p \longrightarrow q$ (read,"if p then q"). When both $p \longrightarrow q$ and $q \longrightarrow p$, we shall write $p \longleftrightarrow q$(p if and only if q).

Quantifiers

Phrases that convey the idea of quantity are called <u>quantifiers</u>. For instance, "for all x" ($\forall x$),"there exists an x such that"($\exists x$:) are examples of quantifiers. The quantifier <u>all</u> is called the <u>universal quantifier</u>, the quantifier <u>there exists</u> is called the <u>existential quantifier</u>. An alternative expression for "there exists" is "for some."

Exercise

1. Say in your own words what a set is.
2. Write in symbols, "P cup Q" and "P cap Q."
3. Symbolize, "a is an element of A" and "if p then q."

Answers

1. A set is a collection of distinct objects.
2. $P \cup Q$ and $P \cap Q$.
3. $a \in A$ and $p \longrightarrow q$.

Appendix IV

3: ORDERED PAIRS

Introduction

In many places in mathematics we need to distinguish between the pair (a,b) and the pair (b,a). The usual two-dimensional graph represents these pairs by distinct points unless, of course, a = b. When the order of a pair is of importance the pair is referred to as an ordered pair.

Definition

An <u>ordered pair</u> of objects is a set of two objects for which it has been decided which is first in the pair and which is second.

Notation

Two objects, a and b, separated by a comma or a semi-colon and written within parentheses:

(a,b) or (a;b)

in which a occupies the first place and b the second place is called an ordered pair.

Positive Example

On a road map, one is often directed to find a location by scanning the area designated by the pair of numbers 4,2, for instance. This means that one can locate the area one is looking for by following the horizontal boundary until one reaches section 4; then follows the vertical boundary until one reaches section 2. Thus the direction (4,2) locates a position on a map. Now suppose one is trying to find another position by scanning the area designated by (2.4), what would one do? One would

5					
4		2,4			
3					
2				4,2	
1					
0	1	2	3	4	5

have to follow the horizontal boundary until one reaches section 2; then follow the vertical boundary until one reaches section 4. This direction (2,4) locates another position on the map. These pairs of objects

(4,2) and (2,4)

in which the first object does not have the same meaning as the second are called <u>ordered pairs</u>. Note that (4,2) and (2,4) represent different squares on the map.

Appendix IV

3 : ORDERED PAIRS-2

Negative Example

A set which is composed of two elements a and b for instance, is called a pair and is written

$$\{a,b\} \text{ or } \{b,a\}$$

because the order in which we list the elements of a set is not important.

Exercises

(1) To symbolize ordered pairs we use braces or parentheses?

(2) What is a pair?

(3) What is an ordered pair?

Answers

(1) To symbolize ordered pairs we use parentheses.

(2) A pair is a set containing two elements.

(3) An ordered pair is a set consisting of two objects in a specified order.

Appendix IV

4 : RELATIONS

Introduction

The concept of RELATION is just as important as the concept of SET. Relations are present not only in science and mathematics but also in everyday life. We meet the concept of relation everyday when we use such phrases as:

is a friend of   is a colleague of   was born at
is in love with   live at   went to, etc.

Relations are very much in use in all scientific fields social sciences, physical sciences, and so forth. We are familiar with such phrases as:

cost more than   has the same income as   feed on
has the same I.Q. as   has the same density as
has the same ethnic origin as   belong to the same race as   means the same thing as   etc.

Elementary mathematics makes frequent use of such relations as:

is a subset of   belong to set   is less than
is greater than   is congruent to   is equal to, etc.

The concept of relation is so important in science that if we had to summarize the goal of science we would not be wrong in stating that

Science is the study of relations.

Positive examples

1. Consider the sentence:

   "Paul loves Jane."

   The verb "loves" which relate Paul to Jane is a <u>relation</u>.

2. The following are some other examples of relation:

   ... has the same age as ...
   ... is north of ...
   ... depend on ...
   ... is parallel to ...
   ... is between ... and ...

Negative examples

1. Consider the following groups of words:

   "Paul Jane"

   We cannot say anything about Paul and Jane because of the absence of such verbs or verbal phrases as "... is in love with ...," "likes," " hates", etc., which express a certain relation between Paul and Jane.

Appendix IV

4 : RELATIONS-2

Negative examples (cont.)

2. Consider the sentence:

"John borrows money from the bank for his sister."

Now, if we strike out "borrows", this group of words would be meaningless. In other words, we cannot describe or explain anything without relations.

Arrow diagrams

1. Consider the sentence:

"John is the brother of Helen."

We can represent (visualize) this relation "is the brother of" in a set of two people, a brother and a sister, by drawing an arrow diagram starting from John to Helen like this:

John $\longrightarrow$ Helen or like this Helen $\longleftarrow$ John,

or like this John $\curvearrowright$ Helen

The important thing here is that the arrow starts from John to Helen. The arrow here means "is the brother of". No arrows have been drawn from Helen to John because Helen is not the brother of John. The arrow diagram here is complete with one arrow.

2. Examine the following sentence:

"John is the brother of Peter."

To represent this relation "is the brother of" in a set of two boys, we can draw an arrow from John to Peter and as Peter is also the brother of John, we can draw another arrow from Peter to John and obtain the following diagram:

John $\rightleftarrows$ Peter

Here the diagram is complete with two arrows.

Definition

At the elementary level we can think of a relation as something which relates objects. It is like "Glue" or "cement" which holds things together. It cannot be seen, touched or photographed but it can be represented by a verb or verbal phrase.

Related concepts

Function, correspondence, mapping, transformation, etc.

Some related rules

1. One cannot be conscious of one thing and one thing

Appendix IV

4 : RELATIONS-3

Some related rules (cont.)

only. In order to have one, we must have at least three: a thing, a relation, and another thing. For instance, to explain what "uncle" is, we must have three things: (1) uncle, (2) a relation: is the brother of, and (3) another thing: one's father or mother.

2. Two unrelated terms cannot exclude each other (Hamelin). For instance, we always something from something else. In other words, the excluded thing must also be present before we can have an exclusion. To suppress one term, the other will disappear just as shadow will disappear when light vanishes(Hamelin).

3. Without adding or subtracting any of the factors in the composition of a thing, we may utterly change the characteristics of that thing by changing the relations of the various factors to each other. For instance, consider the three names, "Ronald," "Arnold," and "Roland"; they contain exactly the same letters, but the relative positions of these letters - that is to say, their mutual relations of "before," "after," or "right" and "left" - are different in each case, and utterly different words result(Langer,1937).

Exercises

(1) Underline all the relations in the following:

a. Mark is as tall as John.
b. 4 is the square root of 16.
c. 16 is the square of 4.
d. "Animal" is a subordinate of "Living thing."
e. Ottawa is west of Montreal.

(2) Which of the following statements is/are true?

a. Without relations we cannot explain and describe things and events.
b. We can describe and explain things and events without relations.
c. Relations are something that relates objects and are usually represented by verbs or verbal phrases.
d. Relations can be likened to "cement" or "glue" which hold things together.

Answers to exercises (1) You must have underlined the following phrases:

is as tall as - is the square root of - is the square of - is a subordinate of - is west of.

(2) (a), (c), (d).

Appendix IV

5 : CARTESIAN PRODUCT

Introduction

We must understand the concept of a CARTESIAN PRODUCT to understand the formal definition of a relation. Cartesian products (sometimes called simply product sets) are so named in honor of the famous French mathematician, René Descartes (1596-1650)

Positive and negative examples

1. Consider the following situation:

   Paul is going to eat his lunch. He has the choice of hamburger, sausage, or vegetable. After eating the hamburger, or sausage, or vegetable, he will drink either tea or coffee. What are all the combinations of dishes and beverages he may choose? Remember, he is going to eat one of the dishes first, then drink one of the beverages.

   Here are all the possibilities:

   (hamburger,coffee) (hamburger,tea)
   (sausage, coffee) (sausage, tea)
   (vegetable, coffee) (vegetable, tea)

   We may think of all these possibilities as forming a set. This set has pairs for its elements, and the set has six of these pairs. For example, the pair (hamburger, coffee) is an element of the set; the pair (hamburger, tea) is another element of the set,etc.

   What is the mathematical significance of the above situation?

   We are given two sets: A and B.

   A = {hamburger, sausage, vegetable}
   B = {coffee, tea}

   From these two sets we construct a third set of six pairs whose first components are from A and whose second components are from B. This new set of ordered pairs is called the Cartesian product, or cross product, or product set of A and B, denoted by A x B (Read "A cross B"). A is the domain and B is the co-domain or range of this Cartesian product. Note that the cross symbol x is also used to indicate the multiplication of two numbers. Hence we may also write 4 x 5, read "4 times 5" and not "4 cross 5".

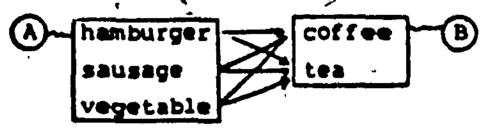
   There are several devices which make it possible to write out - without repetitions or omissions - all the ordered pairs whose first component is an element of A and whose second component is an element of B.

Appendix IV

5 : CARTESIAN PRODUCT-2

Positive & negative examples (cont.)

Below is the first device called arrow diagram:

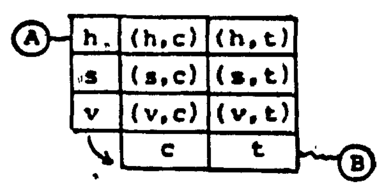


From each element of A leave two arrows: one toward coffee and one toward tea. We see six arrows altogether. To each arrow corresponds an ordered pair whose first component is an element of A and whose second component is an element of B. The totality of these six ordered pairs is what we have found by the intuitive method. We can summarize thus:

$$A \times B = \{(h,c),(h,t),(s,c), (s,t), (v,c), (v,t)\}$$

where h represents hamburger, c represents coffee, etc.

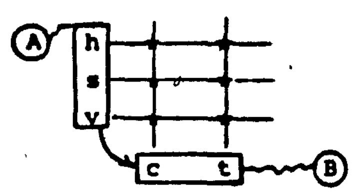
Below is a second device called Cartesian diagram:



We can readily see the six ordered pairs whose 1st component is an element of A and whose second component is an element of B.

Note the little arrow emanating from the domain A and terminating at the co-domain or range B.

Below is a variant of the above diagram:



Each star * is the intersection of a vertical line and a horizontal line. There are as many stars as there are ordered pairs whose 1st component is an element of A and whose 2nd component is an element of B.

Appendix IV

5 : CARTESIAN PRODUCT-3

Positive and negative examples (cont.)

2. Consider the following Cartesian diagram:

R	3	(1,3)	(2,3)
	4	(1,4)	
		1	2	D

The set of ordered pairs {(1,3),(1,4),(2,3)} does not represent (constitute) the totality of the ordered pairs whose 1st component is an element of D and whose second component is an element of R.

This set of ordered pairs is not the Cartesian product of D cross R. Note that the arrow here goes from D to R.

3. Examine the following table:

N	1	1x1	2x1
	2	1x2	2x2
	x	1	2	N

The above four elements (objects) (1x1),(1x2), (2x1), and (2x2) are not ordered pairs. They are the products of two numbers of a multiplication table and do not represent (constitute) the Cartesian product of N cross N. Note that the symbol x here is read "times".

Definition

The Cartesian product or cross product or product set of set A and B, denoted by A x B (Read "A cross B" in that order) is the set of all ordered pairs whose 1st component is an element of A and whose 2nd components is an element of B. A is called the domain and B, the co-domain or range of the Cartesian product. Note that A x B and B x A are equal if and only if A = B.

Exercises

(1) Which of the following statements is/are true?

a. The domain of the Cartesian product R cross S is the set of the first components of the ordered pairs that define it.

b. The co-domain or range of the set product R cross S is the set of the second components of the ordered pairs that define it.

c. An ordered pair is a pair whose first component is from the domain and whose second component is from the co-domain or range.

Exercises (cont.).

(2) Which of the following statements is/are true?

a. Given two sets P and Q, in that order, the totality of the ordered pairs whose 1st components are from P and whose 2nd components are from Q, is called the cross product or Cartesian product of P and Q.

b. The term "component" is used for ordered pairs and the term "element" is used for sets.

(3) Let $A = \{1,2\}$ and $B = \{a,b\}$. Determine

a. $A \times B$, and

b. $B \times A$

(4) In what case are $A \times B$ and $B \times A$ equal?

(5) Let $C = \{m,n\}$ and $D = \{1,2\}$

a. Write the roster symbols for C cross D.

b. Draw an arrow diagram for D cross C.

Answers to exercises

(1) They are all true.

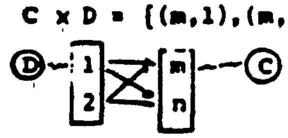
(2) Both are true.

(3) a. $A \times B = \{(1,a),(1,b),(2,a),(2,b)\}$

b. $B \times A = \{(a,1),(a,2),(b,1),(b,2)\}$

(4) When $A = B$.

(5) a. $C \times D = \{(m,1),(m,2),(n,1),(n,2)\}$

b. 

Appendix IV

6 : BINARY RELATIONS

Positive examples

1. A farmer investigating the way in which the growth of young chicks is related to their age weighed a chick every month until it has reached the proper size. The following table records the weights in kilograms.

A	Age	1	2	3	4	5	6	7
B	Weight	.3	1.0	1.5	1.8	2.0	3.0	3.5

This table specifies a set of ordered pairs of numbers, the first number in each pair being the age, and the second number, the weight of the chick.

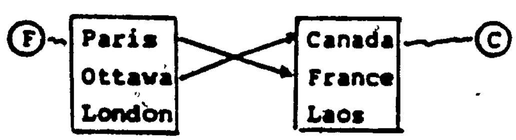
This set of ordered pairs {1;.3),(2;1.0), ... } is called a <u>binary relation</u> from A to B. The set of all the first components of the ordered pairs is the domain of the relation, A in this case; and the set of all the second components or coordinates of the ordered pairs is the co-domain or range of the relation.

2. Consider a set of countries C and a set of capital cities F:

C = {Canada, France, Laos}

F = {Paris, Ottawa, London}

the relation "... is the capital of ...", and its arrow and Cartesian diagrams:



F			
Paris		(Paris,France)	
Ottawa	(Ottawa,Canada)		
London			
C	Canada	France	Laos

To each arrow of the arrow diagram corresponds an ordered pair and to each filled-in block of the Cartesian diagram corresponds an ordered pair. We therefore have a set of two ordered pairs in all, (Paris,France),(Ottawa,Canada).

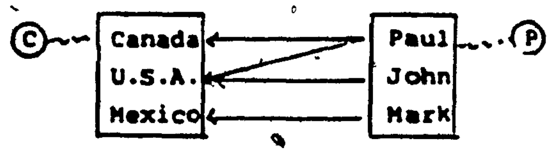
Appendix IV

6 : BINARY RELATIONS-2

Positive examples (cont.)

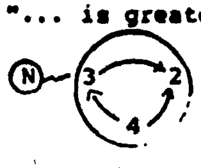
This set of ordered pairs is a subset of the Cartesian product of F cross C (F × C). It is the binary relation ".. is the capital of ..." from F to C.

3. Now consider the relation "... has visited ..." from P to C and the arrow diagram of this relation:



On the diagram we see four arrows which correspond to four ordered pairs (Paul,Canada), (Paul, U.S.A.), (John,U.S.A.), (Mark,Mexico). This set of four ordered pairs is the binary relation "... has visited ..." from P to C.

4. Consider the following diagram of the relation "... is greater than ..." in a set N.



The three arrows correspond to three ordered pairs (4,2),(4,3) and (3,2). This set of ordered pairs is the binary relation "... is greater than ..." in N.

Negative examples

1. Examine the following sentence:

"Anatole teaches philosophy at the University."

The verbal phrase "... teaches ... at ..." relates three objects: Anatole, philosophy and University. It is not a binary relation but a ternary (driadic or three-ary) relation.

2. The verbal phrase "... borrows ... from ... for ... " in the sentence

"John borrows money from the bank for a friend."

relates four objects: John, money, bank and friend. It is a quaternary or four-ary relation.

3. "Paris is noisy."

The verb "is" here relates noisy (which is not an object) and Paris. This type of relation is a unary relation.

Appendix I

6 + BINARY RELATIONS-3

Notation

1. Binary relations may be denoted by upper-case letters such as R, G, T or any other symbols. If two objects x and y are in the relation R, G or T with each other, this will be denoted by

$$x\ R\ y \qquad x\ G\ y \qquad \text{or } x\ T\ y.$$

If the letter R represents the relation "is less than", then the notation

$$x\ R\ y$$

means "x is less than y." Note that instead of writing x R y we can also write

$$R(x,y) \text{ or } (x,y) \in R$$

2. Three-ary, four-ary and n-ary relations may also be denoted by such letters as R, G, T or any other symbols.

If three objects x, y and z are in the relation R, then we can write

$$R(x,y,z) \text{ or } (x,y,z) \in R$$

If four objects v, x, y and z are in the relation R, then we can write

$$R(v,x,y,z) \text{ or } (v,x,y,z) \in R$$

etc.

Definitions

1. At the elementary stage we can think of a binary relation as a relation relating two objects; a ternary or three-ary relation as a relation relating three objects, etc.; and a unary relation as a relation connecting an object with itself.

More sophisticatedly

2. A relation R on a set S is the subset of S x S; that is, a binary relation is a set of ordered pairs. The set of the 1st components of the pairs is called the domain of the relation and the set of the 2nd components of the pairs is called the co-domain or range of the relation R.

3. A relation R between sets A and B (or a relation R from A to B) is a subset of A x B. Set A is the domain and B is the co-domain or range of R.

Appendix IV

6 : BINARY RELATIONS-4

Exercises

(1) What relations are suggested by the following sets of ordered pairs?

a. {(France,Paris),(Japan,Tokyo),(Spain,Madrid)}

b. {(stop,go),(slow,fast),(clever,stupid)}

(2) Which of the following statements is/are true?

a. A binary relation is a relation relating two objects.

b. A binary relation is a set of ordered pairs.

c. A binary relation is a set of objects.

(3) Binary relations require how many objects?

(4) Ternary relations?

(5) N-ary relations?

(6) Find the domain and the range of the relation R:

R = {(1,2),(2,3),(3,4)}

(7) What does the notation

a R b

mean if R represents the relation "... is north of ..."?

Answers to exercises

(1) a. "... has for capital ...".

b. "... is opposite of ...".

(2) They are all true.

(3) Two objects.

(4) Three objects.

(5) N objects.

(6) The domain of R is {1,2,3} and the range of R is {2,3,4}.

(7) "a is north of b".

Appendix IV

7 : FUNCTIONS

Introduction

A function is a special relation. In the classical sense it was a rule that assigns to a number, another number. In the modern sense, it is a rule that assigns to an element of one set an element of another set.

Positive examples

1. Consider the collection of students whom we shall call set S, and a collection of chairs in a classroom which we shall call set C. As each student enters the classroom he is assigned a chair by the teacher.

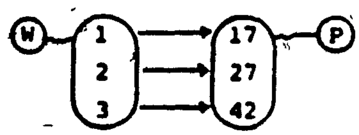
$$\begin{array}{l} S = \{s_1, s_2, s_3, \ \ldots \ , s_n\} \\ \quad\ \ \updownarrow \ \ \updownarrow \ \ \updownarrow \qquad\quad \updownarrow \\ C = \{c_1, c_2, c_3, \ \ldots \ , c_n\} \end{array}$$

We can see that each student is assigned one and one (syn.: a unique) chair.

Here the domain is the set of student S and the co-domain or range is the set of chairs C.

When this information is written as a set of ordered pairs, the result is a functional relation or simply function.

2. The rate of postage on first class mail within Canada is given in the diagram below:



W being the set of weights in ounces,
P being the set of "amounts" in cents.

To each element of W (the domain) there corresponds a unique element of P (the co-domain or range), and we say that the "weight determines postage" and we write

$W \longrightarrow P$ or Weight $\longrightarrow$ Postage

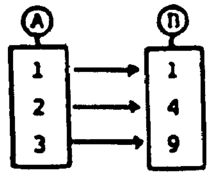
The precise relationship is given if we write down the set of corresponding ordered pairs $\{(1;17),(2;27),(3;42)\}$. This set of ordered pairs describes (defines) a functional relation or function.

3. Consider the relation represented by the following diagram:

Appendix IV

7 : FUNCTIONS-2

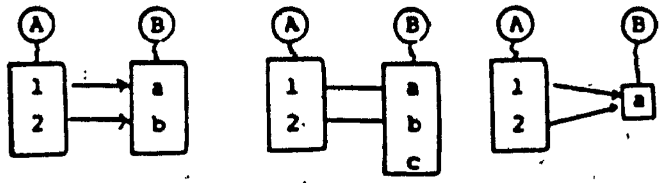
Positive examples (cont.)



'1' is the square root of '1'
'2' is the square root of '4'
'4' is the square root of '9'

To each value of A there corresponds a unique value of B. The set of ordered pairs [(1,1),(2,4),(3,9)] is a <u>functional relation</u> or <u>function</u> from A to B.

4. The following diagrams represent <u>functions</u>:



A unique arrow leaves each element of the domain; that is, every element of A has exactly one element of B paired with it. This can also be translated by the following sentences (statements or propositions):

a. No element of A can be the first component of more than one pair;

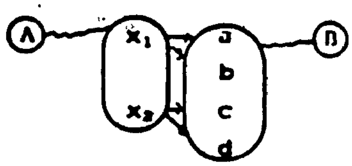
b. Each element of the domain is associated with one and only one element of the range;

c. No two distinct ordered pairs have the same first component.

Note that the above statements describe some of the critical properties of <u>functions.</u>

Negative examples

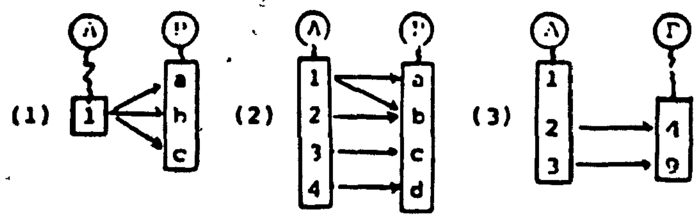
1. The following diagram represents the relation H "... is the husband of ..." from A to B in a polygamous community:



Two arrows leave each point of the domain; that is, each element of the domain is associated with more than one element in the range. The relation H here is <u>not</u> a function.

Negative examples (cont.)

2. Examine the relations represented by the following diagrams:



In (1) three arrows leave the only element of A.

In (2) one element of A is first component of two ordered pairs as shown by two arrows emanating from A.

In (3) no arrows leave point '1' of A; that is, one element of A is associated with no element of B.

They are all ordinary relations and not functions.

Notation

1. It is a common practice to represent a function by the letter f. But any letter or symbol will do and the most popular ones are f, g, h, F, G, and H. If x is an element of the domain of a function f, then the object which f associates with x is denoted by f(x) (read "the value of f at x" or simply "f at x", or "f of x").

Consider the function of x.

$y = 2x + 3$

In functional notation, this would be written as

$f(x) = 2x + 3$

In other words $y = f(x)$.

Functional notation is especially advantageous when we wish to indicate the value of the function y when x takes a specific value, say x = 1. For the above example, we see that when x = 1,

$$\begin{aligned} y &= 2x + 3 \\ &= 2(1) + 3 \\ &= 2 + 3 \\ &= 5 \end{aligned}$$

Rather than write this , we can use the simpler notation

$f(1) = 5$

Notation (cont.)

2. Find g(2) if g(x) is the function defined by the formula

$$g(x) = \frac{2}{x} + 2x$$

Solution. Whenever a formula is given for g(x) and we want to find g(2), we merely replace x by 2 in the formula and compute in accordance with formula. By definition

$$g(x) = \frac{2}{x} + 2x; \quad \text{hence} \quad g(2) = \frac{2}{2} + 2(2) = 5$$

3. The above function

$$g(x) = \frac{2}{x} + 2x$$

may be written in any of the following ways:

a. $y = \frac{2}{x} + 2x$

b. $g: X \longrightarrow Y$ (Read "The set X is mapped, by means of g, onto the set Y). It is the function whose values are given $g(x) = \frac{2}{x} + 2x$

c. $g:(x,y)$ is the function whose ordered pairs are $(x, \frac{2}{x} + 2x)$

d. $\{(x,y)/y = \frac{2}{x} + 2x$

but the most useful notations are

$$y = \frac{2}{x} + 2x \quad \& \quad g(x) = \frac{2}{x} + 2x$$

Some definitions

1. A function f is a relation between two sets:

   a. a set X called the domain, and
   b. a set Y called the co-domain or range, and
   c. a rule which assigns to each element of X a unique element of Y.

2. A function f is a set of ordered pairs (x,y) such that

   a. x is an element of a set X,
   b. y is an element of a set Y, and
   c. no two pairs in f have the same 1st component.

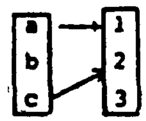
3. If, in a relation, to every element of a set called the domain there is assined exactly one element of a set called the range, then the relation is a function.

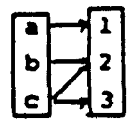
Appendix IV

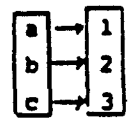
7 : FUNCTIONS-5

Exercises

(1) State whether or not each of the following arrow diagrams describes a function of A = {a,b,c} into B = {1,2,3}.

(a) 

(b) 

(c) 

(2) In each of the following, decide whether or not the given set of ordered pairs describe a function.

a. {(1,0),(2,2),(3,1)}

b. {(1,0),(1,1),(1,2)}

(3) Which of the following does not/do not describe a function/functions, when x R y?

a. $f : x \longrightarrow x + 5$

b. $g : x \longrightarrow x^2$

c. $h : x \longrightarrow x - 5$

d. $f : x \longrightarrow$ all y less than x.

(4) Specify the domain and range of the following functions where $x, f(x) \in R$ (real numbers)

a. $f : x \longrightarrow x$

b. $f : x \longrightarrow x^2$

c. $f : x \longrightarrow \sqrt{x}$

(5) Read the following notation

$f : x \longrightarrow f(x)$

(6) Determine if the following relations are functions. If so, indicate the domain and range.

a. {(1,2),(1,3),(1,4)}

b. {(2,1),(3,1),(4,1)}

(7) If $f(x) = x^2 - 1$, find $f(-1)$, $f(0)$, and $f(1)$ and complete the following table:

x	-1	0	1
f(x)			

(8) What is a function?

7 : FUNCTIONS-6

Answers to exercises

(1) a. No, because to element b of A is assigned no element of B.

b. No, because to element c of A are assigned two elements of B: 2 & 3.

c. Yes, because to each element of A is assigned a unique element of B.

(2) a. Yes, because different ordered pairs have different first components.

b. No, becaause all the three ordered pairs have the same first components.

(3) d. Because it is a multiple-valued relation; that is, to every x of the domain X are assigned many elements of Y.

(4) a. R (real numbers)

b. R

c. Non-negative R

(5) "f takes x into f at x" or "f takes x into f of x".

(6) a. Not a function, because the distinct ordered pairs (1,2),(1,3), and (1,4) have the same first component.

b. Is a function, since no two distinct ordered pairs have the same first component. The domain of the function is {2,3,4} and the range is {1}.

(7) Since $f(x) = x^2 - 1$.

$$f(-1) = (-1)^2 - 1 = 0$$

$$f(0) = 0^2 - 1 = -1$$

$$f(1) = 1^2 - 1 = 0$$

The completed table is

x	-1	0	1
y = f(x)	0	-1	0

(8) A function f is a relation between two sets:

a. a set X called the domain,

b. a set Y called the range, and

c. a rule which assigns to each element of X a unique element of Y.

Appendix IV

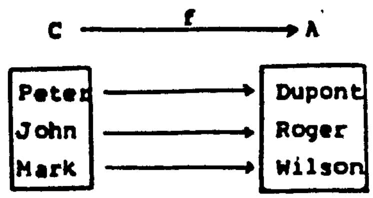
8 : MAPPINGS

Introduction

A mapping is another type of relation and is a synonym of function.

Positive examples

1. Consider a set of children C, a set of adults A, and a relation f "... is the son of ..." from C to A represented by the following arrow diagram:

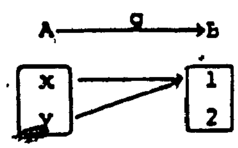


From each element of C leaves one and only one arrow. The relation f here is a mapping of C into A. This can be symbolized as

$f : C \longrightarrow A$ (f maps C into A)

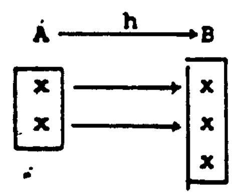
We see that it behaves exactly like a function, except for the vocabulary used to describe it.

2. The relation g represented by the diagram on the right is a mapping. Each element of A has a unique image. It is also a function.



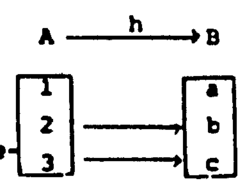
3. The relation h represented by the diagram on the right is a mapping.

One and only one arrow originates from each element of A. It is also a function.



Negative examples

1. Look at the relation h represented by the diagram on the right. The element "1" of the domain A has no image. This is shown by the absence of an arrow going from "1" to an element of the range B.



The relation h here is neither a mapping or a function.

2. Examine the relation R represented by the following set of ordered pairs:

$R = \{(1,2),(1,3),(1,4)\}$

Appendix IV

8 : MAPPINGS-2

Negative examples (cont.)

We notice that the three ordered pairs have the same first component. This is neither a mapping or function.

3. The domain of the relation "... is the mother of" is a set of all people of both sexes. Is this relation a mapping?

No, because there exists at least one element in the domain who is male and a male cannot be the mother of anybody.

Definitions

The following are some of the acceptable definitions of a mapping.

1. A mapping m from a set A into a set B is the relation or rule that associates a unique element of B with each element of A. Such a mapping may be denoted by $m : A \longrightarrow B$ where the relation is written $m(a) = b$. The element b is called the image or correspondent of a under the mapping m. This implies that

   1. If a is an element of A, then there is exactly one element b of B such that $b = m(a)$;
   2. Every element in the image set B does not have to be used under the mapping m; and
   3. It is possible that two or more elements of A will be paired with the same element b of B.

2. A relation f from a set A into a set B is said to be a mapping from A onto B, denoted by

$$f : A \xrightarrow{\text{onto}} B$$

if every element of the set B is the image of some element of A.

3. A mapping g from a set B into a set D is said to be one-to-one, denoted 1:1, if distinct elements of B (here the domain) have distinct images in the range D.

   This third definition implies that:

   1. If g is a one-to-one mapping from B into D, then each element of the domain B is associated with a unique element of the range D.
   2. A one-to-one mapping must be both into & onto.

4. A mapping f is a function if to each element of its domain there corresponds a single (unique) element of the range.

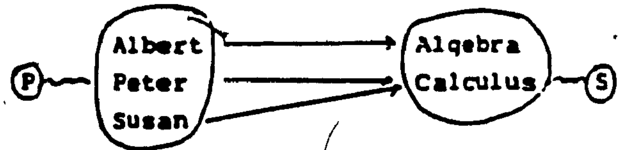
Appendix IV

8 : MAPPINGS-3

Exercises

MAPPINGS-3

(1) The following diagram represents a relation:

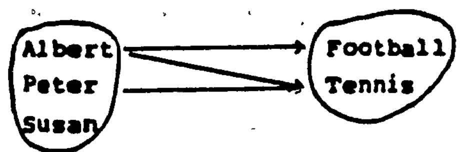


a. Explain why this is a mapping from P to S.

b. What is the domain?

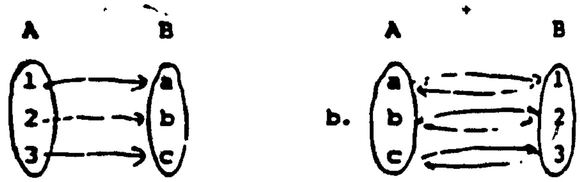
c. What is the co-domain or range?

(2) The following arrow diagram does not represent a mapping because two requirements have not been fulfilled. What are these requirements?



(3) Which of the following diagrams represent a one-to-one mapping? Explain.

a. b.



(4) Which of the following statements is/are true?

a. Function and mapping are synonyms.

b. A mapping is a set of ordered pairs whose first components are elements of the domain and whose second components are elements of the co-domain or range, and no two distinct ordered pairs have the same first components.

(5) What is the difference between a mapping and a simple relation?

(6) Is a many-to-one relation a mapping?

(7) Is a one-to-many relation a mapping?

Appendix IV

B : MAPPINGS-4

MAPPINGS-4

Answers

(1) a. A single arrow leaves each element of P.

b. The domain is the set P.

c. The co-domain or range is the set S.

(2) Two arrows leave the element "Albert" of the domain and the element "Susan" of the domain has no images.

(Or equivalent answers)

(3) The diagram (b) represents a one-to-one mapping, because each element of A is associated with a unique element of B and each element of B is associated with a unique element of A.

(Or equivalent answers)

(4) They are both true.

(5) The difference between a mapping and a simple relation is that in a mapping "each element of the domain has one and only image."

(Or equivelent answers)

(5) Yes.

(6) No.

Appendix IV

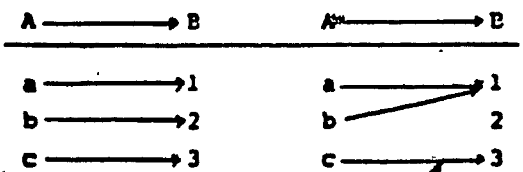
9 : CORRESPONDENCES

Introduction

We have seen that the elements of one set may usually be assigned arbitrarily or by some rule to those of another in various ways. The resulting set of ordered pairs is, of course, called a relation. A correspondence is another kind of special relation. To some authors correspondence, function and mapping are synonyms and others they are not.

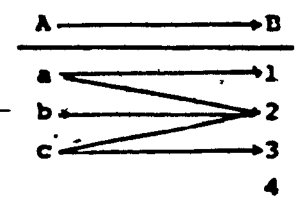
Positive & negative examples

1. The relations represented by the diagrams below are functions or mappings:

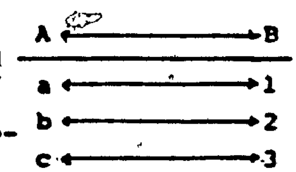


Each element of the domain corresponds to exactly one elements of the co-domain or range. They are also correspondences.

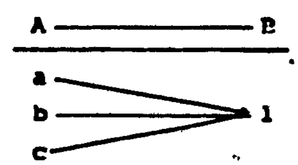
2. The diagram in the figure on the right represents a relation that is not a correspondence, since some of the elements of set A, in this case "a" and "c", are assigned to more than one elements of the range.



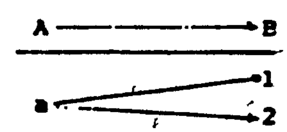
3. In the figure on the right, each element of A is paired with exactly one element of B and each member of B is paired with exactly one element of A, as shown by double-headed arrows. This is a one-to-one correspondence or mapping, denoted 1:1.



4. The diagram on the right represents a many-to-one correspondence or mapping. A is the domain and B is the co-domain or range of the correspondence.



5. The diagram on the right does not represent a correspondence but a one-to-many relation.

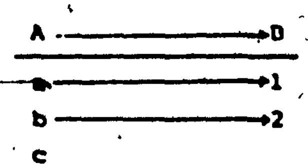


Appendix IV

9 : CORRESPONDENCES-2

Positive & negative examples (cont.)

6. The diagram at the right does not represent a correspondence, since "c" of A is not paired with any element of B. This is a simple relation.



Definitions

1. Let T and U be non-empty sets. A correspondence R in T into U is a rule or law that assigns to each element t of T one and only one element of U. This correspondence R is denoted by

$$t \xrightarrow{R} u, \quad t \in T \text{ and } u \in U$$

If "t" of T corresponds to "u" of U, then "u" is called the image of "t" and "t" is called the pre-image of "u".

2. If each element of A is paired with exactly one element of B and if each element of B is paired with exactly one element of A, then R is said to be a one-to-one correspondence, function or mapping.

3. If more than one element of A is paired with one element of B, then R is said to a many-to-one correspondence, function or mapping.

Exercises

(1) The function f = {(1,2),(2,3),(3,4)} is a correspondence.

a. What is its domain?

b. What is its co-domain or range?

c. Is it a one-to-one correspondence? Why?

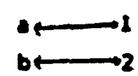
(2) Draw an arrow diagram to represent the following relation R = {(1,2),(2,3),(3,2),(4,3)}

a. Is it a function? Why?

b. Is it a one-to-one correspondence? Why?

(3) Which of the following diagrams represents a one-to-one correspondence?

a. 

b. 

Appendix IV

9 : CORRESPONDENCES-3

Answers to exercises

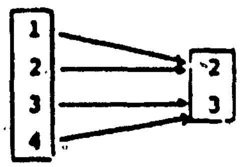
(1) a. {1,2,3}

b. {2,3,4}

c. Yes, since each first component occurs only once in the set of ordered pairs, and each second component occurs once in the set of ordered pairs.

(Or equivalent answers)

(2)



a. It is a function, because one and only one arrow leaves each element of the domain (the first set).

b. It is not a one-to-one correspondence, because some second components occur more than once in the set of ordered pairs.

(or equivalent answers)

(3) (b) represents a one-to-one correspondence, since each first component occurs only once in the set of ordered pairs, and each second component occurs once in the set of ordered pairs.

Appendix IV

10 : TRANSFORMATIONS

Introduction

To have a relation, a function, a mapping, or a correspondence, we need two sets of objects (which may actually be the same), and a rule or law which associates each element of the first set with an element of the second. To have a transformation we need two sets of ob ects and a rule or law. The only difference here is that the domain here shall be called the set of operands, the co-domain or range the set of transforms, and the rule or law shall be called the operator which we borrow from Ashby's* famous book "An Introduction to Cybernetics".

Positive & negative examples

1. Consider a set of operands O = {pale skin, cold soil, unexposed film}, a set of transforms T = {dark skin, warm soil, exposed film}, and the operator R "... is changed to ..." which relates each element of O to exactly one element of T, giving the following set of ordered pairs whose first components are element of O and whose second components are elements of T:

   {(pale skin, dark skin),(cold soil,warm soil (unexposed film, exposed film)}

   Another way of exhibiting the relationship here is to write

Operands ⟶	Transforms
Pale skin ⟶	Dark skin
Cold soil ⟶	Warm soil
Unexposed film ⟶	Exposed film

   The symbol ⟶ can be read in a variety of ways:

   "under the influence of sunshile, pale skin is changed to dark skin", or "under the influence of sunshile, pale skin is transformed into dark skin",etc.

   The above set of ordered pairs is called a <u>transformation</u>, or function, mapping or correspondence.

2. Consider the set of letters of the alphabet and the operator T "... turns to the one that follows it ...", and the set of ordered pairs:

   $$\begin{array}{l} A \longrightarrow B \\ B \longrightarrow C \\ \quad \cdots \\ Z \longrightarrow A \end{array}$$

Appendix IV

10 : TRANSFORMATIONS-2

Positive & negative examples (cont.)

The above set of ordered pairs whose first components and whose second components are elements of the same set of the letters of the alphabets:

{(A,B),(B,C), (C.D), ... }

is called a transformation, function, mapping, or correspondence

For convenience of printing, such a transformation can also be written thus:

$$\downarrow\begin{array}{cccc} A & B & \dots & Z \\ B & C & \dots & A \end{array}$$

3. If the operands are 1,2.3, and 4, the transforms 4,5,6, and 7, and the operator is "add three to it", the transformation is the set of ordered pair

$$\downarrow\begin{array}{cccc} 1 & 2 & 3 & 4 \\ 4 & 5 & 6 & 7 \end{array}$$

4. The transformation

$$\downarrow\begin{array}{cccc} A & B & C & D \\ B & A & A & D \end{array}$$

which converts each operand to only one transform is called a single-valued transformation, or function or mapping.

5. The transformation

$$\downarrow\begin{array}{cccc} A & B & C & D \\ B \text{ or } D & A & B \text{ or } C & D \end{array}$$

which converts some of its operands into more than one transform, is called a multi-valued transformation or relation but not function or mapping.

6. In the following single-valued transformation the transforms are all different from one another:

$$\downarrow\begin{array}{cccccc} A & B & C & D & E & F \\ B & C & D & E & F & G \end{array}$$

It is a one-to-one transformation or function, mapping, or correspondence.

Appendix IV

10 : TRANSFORMATIONS-3

Notation

When it is not convenient to write transformations in full, we use three dots ... to represent the transform that are not given individually just as we do with sets For example, the set of positive integers can be writt {1,2,3, ... }.

Often the description of a transformation is made simpl by some relation or rule that links all the operands to their respective transforms. Thus the transformation

$$\downarrow\begin{array}{cccc} 1 & 2 & 3 & 4 \\ 4 & 5 & 6 & 7 \end{array}$$

can be replaced by the formula

Operand ⟶ Operand plus three

or by

$$n \longrightarrow n + 3$$

where n represents an operand.

We can also use another symbol which resembles the one we use for functions. If n is the operand, and transformation T is applied, then the transform is represented by T(n). For example,

The transformation

$$\downarrow\begin{array}{ccc} 1 & 2 & 3 \\ 7 & 8 & 9 \end{array}$$

can be condensed into

$$n \longrightarrow n + 6$$

The transform f is

$$f : \downarrow\begin{array}{ccc} 1 & 2 & 3 \\ 6 & 12 & 18 \end{array}$$

Note: We must first find the rule which in this case is n + 6.

f can be condensed into

$$f : n \longrightarrow n \times 6$$

Note that the rule here is n × 6.

Appendix IV

10 : TRANSFORMATIONS-4 TRANSFORMATIONS-4

Notation (cont.)

Instead of writing

$n \longrightarrow n + 3$

We can also write

$n' = n + 3$

Similarly

$n \longrightarrow n \times 6$

Can also be written

$n' = n \times 6$, etc.

Definition

Let O be a set of operands and T a set of transforms. A transformation G is a set of ordered pairs (o,t) such that

a. o is an element of set O,

b. t is an element of set T, and

c. an operator which assigns to each element of O at least an element of T.

A one-to-one transformation is a function, mapping or correspondence.

A many-to-one transformation is also a function, mapping or correspondence.

A one-to-many transformation is not a function, mapping or correspondence but a relation.

A transformation can be symbolized by such formulas as

Operand ⟶ Operand plus four

or by $n \longrightarrow n + 4$, where n represents an operand.

or by $n' = n + 4$.

A transformation can also be symbolized by the notation we use for functions:

$G : n \longrightarrow G(n)$

(Read "G takes n into G(n).

Appendix IV

10 : TRANSFORMATIONS-5

Exercises

(1) A function consists of three things:

1. The domain
2. The co-domain or range, and
3. A rule

A transformation also consits of three things. What are they?

(2) What is the operator of each of the following transformation?

a. $$T \ \Big\downarrow \begin{array}{cccc} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \end{array}$$

b. $$T \ \Big\downarrow \begin{array}{cccc} 1 & 2 & 3 & 4 \\ 3 & 6 & 9 & 12 \end{array}$$

c. $$T \ \Big\downarrow \begin{array}{cccc} 1 & 2 & 3 & 4 \\ 1 & 8 & 27 & 64 \end{array}$$

(3) Condense into one line each of the following transformations:

a. $$T \ \Big\downarrow \begin{array}{cccc} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \end{array}$$

b. $$T \ \Big\downarrow \begin{array}{cccc} 1 & 2 & 3 & 4 \\ 3 & 6 & 9 & 12 \end{array}$$

c. $$T \ \Big\downarrow \begin{array}{cccc} 1 & 2 & 3 & 4 \\ 1 & 8 & 27 & 64 \end{array}$$

d. $$T \ \Big\downarrow \begin{array}{cccc} 1 & 2 & 3 & 4 \\ 7 & 14 & 21 & 28 \end{array}$$

(4) Write out in full the transformation in which the operands are 1,2, and 3 and in which $n' = n - 3$.

(5) Given the set of operands $O = \{4,5,6\}$ and the rule (or operator) "subtract 3 from it". Write out the transformation in full.

(6) Which of the following transformations is/are single-valued?

a. $$T \ \Big\downarrow \begin{array}{ccccc} A & B & C & D & E \\ B & C & D & E & F \end{array}$$

Appendix IV

10 : TRANSFORMATIONS-6

Exercises (cont.)

D.

T ↓	A	B	C	D	E
	B or C	A	D	E	A

(7) Construct and describe the transformation in R determined by the following mapping:

$$x \longrightarrow f(x) = x + 3$$

Answers

(1) A transformation consists of three things:

1. The set of operands
2. The set of transforms, and
3. The operators

(2) a. "add 4 to it"
b. "multiply it by 3"
c. "raise it to the 3rd power"

(3) a. $n' = n + 4$
b. $n' = 3n$
c. $n' = n^3$
d. $n' = 7n$

(4)

T ↓	1	2	3
	-2	-1	0

(5)

T ↓	4	5	6
	1	2	3

(6) (a) is single-valued.

(7)

T ↓	1	2	3	4
	4	5	6	7

Appendix IV

11 : CLOSED UNDER THE TRANSFORMATION

Introduction

When an operator acts on a set of operands it may happen that the set of transforms obtained contains no new elements. When this occurs, the set of operands is closed under the transformation.

Definition

If a binary operation (addition, multiplication, transformation, etc) on any two elements in a given set produces a result that is also an element in the set, the set is said to be closed under that operation.

Positve Examples

1. Consider the transformation

$$\begin{array}{cccccc} A & B & C & \ldots & Y & Z \\ \downarrow B & C & D & \ldots & Z & A \end{array}$$

Every element in the lower line occurs also in the upper. When this occurs, the set of operands is closed under the transformation.

2. When we add two natural numbers, the sum we get is also a natural number: $4 + 5 = 9$.

To express the fact that we do not have to go outside of the natural number system to find the sum of two natural numbers, we say that the natural number system is closed under the operation of addition, or simply that "addition is a closed operation".

3. Consider the operation

$$4 \times 5 = 20$$

The product of two natural numbers is also a natural number. To express this fact we say that multiplication is a closed operation.

Negative Examples

1. Consider the transformation

$$\begin{array}{cccc} 1 & 2 & 3 & 4 \\ \downarrow 4 & 5 & 6 & 7 \end{array}$$

Here the set of operands has produced three new elements (5,6, and 7). To express this fact we say that the set of operands is not closed under the transformation.

2. While we can subtract 2 from 5 in the natural number system ($5 - 2 = 3$), we cannot subtract 5 from 2 in that system. The latter subtraction becomes possible only in an expanded number system that contains the negative integer (whole number) -3, a number that is outside the natural number system. To express this fact we say that subtraction is not a closed operation.

Appendix IV

11 : CLOSED UNDER THE TRANSFORMATION-2

Exercise

1. Is the following transformation closed? Explain.

$$S \downarrow \begin{array}{cccc} a & b & c & d \\ 1 & 2 & 3 & 4 \end{array}$$

2. In A below the operands are odd numbers from 1 onwards to 7, the transforms are their squares.

$$A \downarrow \begin{array}{cccc} 1 & 3 & 5 & 7 \\ 1 & 9 & 25 & 49 \end{array}$$

Is A closed? Explain.

3. Is the transformation I below closed? Explain.

$$I \downarrow \begin{array}{cccc} a & b & c & d \\ a & b & c & d \end{array}$$

Answers

1. It is not closed, for the transforms are entirely different from the operands: the transformation has created a set of new elements.

2. No, for the transformation has created a set of new elements.

3. Yes, for the transformation has created no new elements: the set of transforms is the same as the set of operands.

Appendix IV

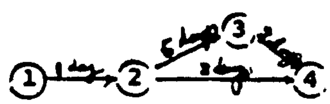
12 : UNDIRECTED GRAPHS

Introduction

The theory of graph deals with the analysis of certain structures called linear graphs. These are not graphs of functions (which are defined as sets of ordered pairs), but rather are structures such as the edges and vertices of a polyhedron (a solid bounded by plane polygons called the faces) that is, structures made up of points joined by line segments.

The study of graphs may be divided into a number of ways:

Undirected versus directed graphs. In a directed graph, each line segment has direction assigned to it. Such graphs are useful for the analysis of structure involving flow such as flowcharts and PERT diagrams. Undirected graphs have no such assignment on their edges. They relate to more static situations, such as the study of data base structures:



PERT diagram

Data base structure

Finite versus Infinite Graphs. It is possible for a graph to have infinitely many vertices and edges, but we shall consider only finite graphs.

We begin with the study of finite, undirected and unlabelled graphs.

Definitions

Let $G = (V,E,f)$, where $V = \{v_1,v_2,v_3, \ldots ,v_n\}$ is the set of vertices of G, $E = \{e_1,e_2,e_3, \ldots ,e_n\}$ is the set of edges of G, and $f : E \longrightarrow V \ominus V$ is a correspondence mapping edges into unordered pairs of vertices. Then G is called an undirected graph. For $e \in E$, if $f(e) = (v,w)$, we say that e is incident to v and to w. For edge $f(e) = (v,w)$, we also use the notation $e = (v,w)$. Note that to each edge e_k is assigned a pair (v_i,v_j) of vertices, but generally there are vertex pair which are assigned to no edge. Vertices v_1 and v_2 are said to be adjacent if there is an edge $\{v_1,v_2\}$.

Description

A graph G consists of three things:

1. A set V whose elements are called vertices, points or nodes.
2. A set E of unordered pairs of vertices called edges.
3. A correspondence, function or mapping f.

Diagrams

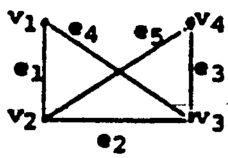
We represent graphs by diagrams in a plane. Each vertex v in V is represented by a dot (or small circle) and each edge $e = \{v_1,v_2\}$ is represented by a curve which

12 : UNDIRECTED GRAPHS-2

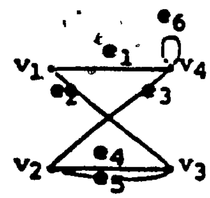
Diagrams (cont.) connects its endpoints v_1 and v_2. For example, Figure (a) below represents the graph with vertices $\{v_1, v_2, v_3, v_4\}$, edges $E = \{e_1, e_2, e_3, e_4, e_5\}$, and f defined by the following table

e	f(e)
e_1	(v_1, v_2)
e_2	(v_2, v_3)
e_3	(v_3, v_4)
e_4	(v_1, v_3)
e_5	(v_2, v_4)

(a) Graph



(b) Multi graph

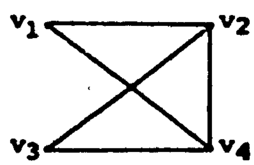


It consists of four vertices and five edges. In drawing graphs, the incidence of edges to vertices is the important consideration. An edge need not be straight, nor of any particular length or direction.

The diagram of Figure (b) above is not a graph but a multigraph. The reason is that e_4 and e_5 are multiple edges, i.e. edges connecting the same endpoints, and e_6 is a loop, i.e. an edge whose endpoints are the same vertex.

Exercise Draw the diagram of a graph whose $V = \{v_1, v_2, v_3, v_4\}$ and whose $E = [\{v_1, v_2\}, \{v_1, v_4\}, \{v_2, v_3\}, \{v_2, v_4\}, \{v_3, v_4\}]$

Answer



Draw a dot for each vertex v in V, and for each edge $\{x, y\}$ in E draw a curve from the vertex x to the vertex y, as shown in the above figure.

Appendix IV

13 : DEGREE OF A VERTEX

Introduction

If v is an endpoint of an edge e, then we say that e is <u>incident on v</u>. The degree of a vertex v, written deg(v), is equal to the number of edged which are incident on v. Since each edge is counted twice the degrees of the vertices of a graph, we have the following simple but important result.

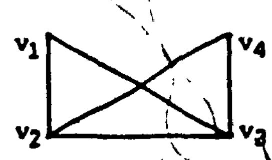
Theorem

The sum of the degrees of the vertices of a graph is equal to twice the number of edges.

Example

In the figure below we have

$$\deg(v_1) = 2 \quad \deg(v_2) = 3 \quad \deg(v_3) = 3 \quad \deg(v_4) = 2$$

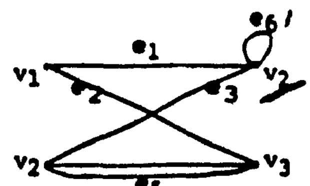


The sum of the degrees equals ten which, as expected, is twice the number of edges. A vertex is said to be <u>even</u> or <u>odd</u> according as its degree is an even or an odd number. Thus v_1 and v_4 are even vertices whereas v_2 and v_3 are odd vertices.

Note: The theorem also holds for multigraphs where a loop is counted twice towards the degree of its endpoint. For example, in the figure below, we have

$$\deg(v_4) = 4$$

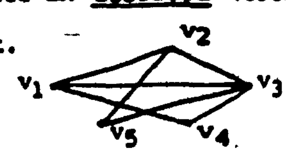
since the edge e_6 is counted twice; hence v_4 is an even vertex.



A vertex of degree zero is called an <u>isolated</u> vertex.

Exercise

Look at the figure on the right. Find the degree of each vertex and verify the Theorem above for this graph.



Answer

$\deg(v_1) = 3$ $\deg(v_2) = 3$ $\deg(v_3) = 4$ $\deg(v_4) = 2$ $\deg(v_5) = 2$. The sum of the degrees of the vertices is $3 + 3 + 4 + 2 + 2 = 14$ which does equal twice the number of edges.

13 : DEGREE OF A VERTEX

Introduction

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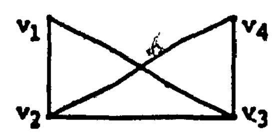

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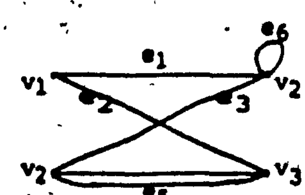

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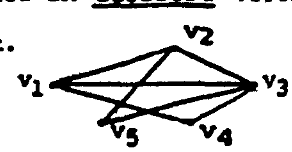

Answer

$\deg(v_1) = 3$ $\deg(v_2) = 3$ $\deg(v_3) = 4$ $\deg(v_4) = 2$ $\deg(v_5) = 2$. The sum of the degrees of the vertices is $3 + 3 + 4 + 2 + 2 = 14$ which does equal twice the number of edges.