REALIZATION OF SINGLE AMPLIFIER NETWORKS WITH SPECIFIED POLE-Q OR POLE-FREQUENCY SENSITIVITY

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LIST OF IMPORTANT ABBREVIATIONS AND SYMBOLS

AP	All-pass
BP	Band-pass
C	Capacitance of a Capacitor
D(s)	Denominator Polynomial of T(s)
$D_3(s)$	Third Degree Denominator Polynomial of T(s)
G	Conductance of a Resistor
HP	High-pass
K	Amplifier gain
LP	Low-pass
μ	Open loop gain of OA
μ_o	d.c. gain of OA
N(s)	Numerator polynomial of T(s)
OA	Operational amplifier
Q	Designed pole Q-factor
$\hat{Q}$	Realized Pole Q-factor
ΔQ	Small variation in pole Q factor
Q_z	Zero Q-factor
R	Resistance of a Resistor
s	Complex frequency variable
S_x^a	Sensitivity of 'a' with respect to 'x'
T(s)	Open Circuit Voltage Transfer Function

ω	Variable frequency in radians/sec. or cycles/sec.
ω_o	Designed pole-frequency in radians/sec. or cycles/sec.
$\hat{\omega}_o$	Realised pole frequency in radians/sec. or cycles/sec.
$\Delta\omega_o$	Small variation in Designed pole frequency
ω_c	Pole of the OA
$\omega_c A_o$	Gain Band Width Product of OA
ω_z	Zero Resonant Frequency

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REALIZATIONS OF SINGLE AMPLIFIER NETWORKS

WITH SPECIFIED POLE Q- OR POLE

FREQUENCY SENSITIVITY

NARENDRA PRATAP SINGH

ABSTRACT

The thesis discusses realizations of second order open-circuit voltage transfer functions by single amplifier active RC networks. These realizations are obtained with prescribed sensitivities of Q or ω_o with respect to the amplifier gain while minimizing the gain sensitivity product (GSP) of Q or ω_o. In addition, the element spread in the resulting networks is constrained to lie within acceptable limits.

For this purpose, the bounds on the sensitivities and other properties of the possible decompositions of the second degree denominator polynomial of the transfer function are first studied. Since the amplifier gain has a much larger variation compared to that of any passive element, the problem of prescribing Q- or ω_o- sensitivity with respect to the amplifier gain only is considered.

Realizations for every polynomial decomposition are shown to exist. These realizations may be either of minimal capacitor type or of the non-minimal capacitor type. Only two decompositions (designated IIa and IIb) are found to be

suitable for zero Q-sensitivity realizations. However, these networks are shown to be useful for low Q and low frequency applications only.

Three decompositions (designated as Ic, IIc, and III) provide zero ω_o-sensitivities. The corresponding realizations are always non-minimal. The realizations are found to be suitable for hybrid IC technology. Further, for the decompositions Ic (with positive feedback) and IIc (with negative feedback) either the Q-sensitivity or the gain Q-sensitivity product (GQ-SP) has also been minimized while ω_o-sensitivity has been made either zero or low, of the order of $\frac{1}{20}$.

Design Curves for the high Q realizations for various filters belonging to the decompositions Ic and IIc have been given. For the decomposition IIc, the Band Pass, All Pass and Notch filter realizations can be obtained readily from these curves when Q-sensitivity or GQ-SP is either prescribed or required to be minimum with zero ω_o- sensitivity. For Low Pass and High Pass Filters, while ω_o-sensitivity can not be made zero, it is of the order $\frac{1}{20}$ and the realizations can again be obtained easily for a prescribed or minimized Q-sensitivity or GQ-SP. The GQ-SP obtained ($Q\sqrt{6}$ for Band Pass, Notch and All Pass, 3.1Q for Low Pass and 3.86Q for High Pass) for these realizations is believed to be lower than those of existing networks reported in the literature. For decomposition Ic Band Pass, Low Pass, High Pass and Notch filter realisations can be obtained with ease from the design curves when Q-sensitivity or GQ-SP is prescribed or required to be minimum while ω_o-sensitivity is zero. The values for GQ-SP (lower than 3.52Q) obtained for the Band Pass, Low Pass and Notch filters are once again believed to be lower than those of the existing networks in the literature. In all these high-Q realisations, obtained for the decompositions IIc and Ic the capacitive element spread is constrained to lie within at most five.

One Band Pass filter network for each of the decompositions IIa, IIc and Ic was built with discrete elements and tested. The experimental results agree closely with the theoretical predictions.

CHAPTER I

INTRODUCTION

CHAPTER I

INTRODUCTION

1.1 GENERAL

Considerable amount of literature exists on the design of the resistively terminated passive LC filters [1], [2]. These filters are attractive, since they are absolutely stable and have no serious sensitivity problems associated with them. However, they suffer from some serious limitations, because of the following reasons:

i) Nonlinear frequency dependence of the quality factor Q of the inductors and the variation of Q from one inductor to another make an accurate filter design complicated.

ii) Magnetic coupling between inductive elements may create problems, such as in satellites carrying instruments to measure very weak signals.

iii) Large size and greatly increased cost of inductors for low frequency applications such as in Analog computers, control systems etc.

Recent trend for micro-miniaturization and the increasing interest in Integrated Circuit (IC) technology has confronted the designer with perhaps the most serious drawback in the use of inductors, as they cannot be manufactured with reasonable values and quality factors within acceptable tolerances at low frequencies.

These difficulties may be overcome by employing active RC-filters (3), (4). Their design requires only resistors (R's) and capacitors (C's) along with active elements. The active RC-filters offer additional advantages over RLC-filters. Some of these are:

i) Active RC-Networks are not bound by the two restrictions, passivity and reciprocity, inherent in RLC-Networks and thus they can realize not only what passive RLC-Networks can, but also may be used to realize characteristics not realizable with passive networks.

ii) Input Impedance may be made in general high compared to sourte impedance and thus active RC filters will draw little power from the signal.

iii) Output impedance can be designed to be low compared to that of load, thereby rendering the voltage response independent of the load impedence.

iv) Frequently, these filters provide insertion gain which may be desirable in many applications.

However, active RC filters, if improperly designed, have two major drawbacks, namely:

i) They may become unstable.

ii) They may be sensitive to network parameter variations.

Thus, proper care should be exercised in their design.

1.2 METHODS OF REALIZING RC-ACTIVE NETWORKS:

Various realization procedures using a variety of active elements have been reported in the literature [3], [4]. These may be broadly classified as follows:

i) Polynomial decomposition approach.

ii) Coefficient matching approach.

Both of these methods assume a network configuration with a passive RC sub-network and one or more active elements. In the first approach, proper decomposition and partioning of the denominator polynomial of the transfer function, T(s), to be realized characterises the passive RC network which may be obtained by passive synthesis. The choice of the injection node provides different filter networks without affecting the denominator.

In the second approach, the selected network configuration is analysed for its T(s) and the coefficients in the realised and the desired transfer function are compared to determine the elemental values. Usually the number of variables, that is, the number of elements, exceeds the number of equations and thus gives the designer several realizations or degrees of freedom to optimise a given design.

Realization procedures can also be classified as:

i) Direct approach, where the T(s) is realised as a single section.

ii) Cascade approach, where the desired transfer function is expressed as a product of first and second order transfer functions.

Each of these functions is realised independently and the complete network is obtained by cascading the individual networks correspodning to the individual realizations. The cascade approach is frequently preferred over the direct one because of simpler tuning properties and because of the fact that a small number of universal sections can be designed which can be used to realize most of the practical filter specifications.

1.3 THE OPERATIONAL AMPLIFIER:

Several active devices such as Operational amplifiers (OA), Impedance converters and Gyrators etc. have been used in the realization of active RC filters (3), (4). In this thesis, the operational amplifier (OA) is considered to be the basic element. In fact, all the other active

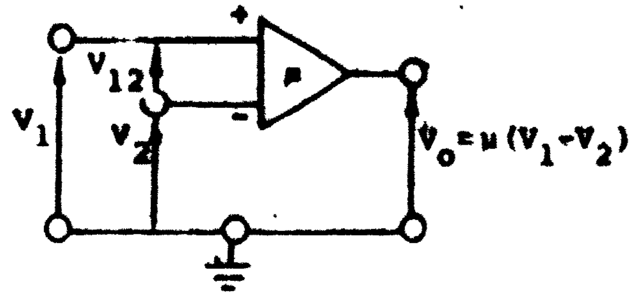

FIG. 1.1 REPRESENTATION OF DIFFERENTIAL INPUT OPERATIONAL AMPLIFIER (OA).

μ IS THE OPEN LOOP GAIN OF THE OA

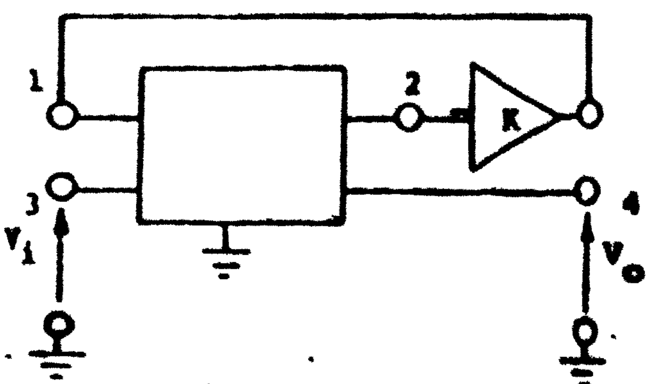

FIG. 1.2 GENERAL SINGLE AMPLIFIER NETWORK CONFIGURATION

$$T(s) = V_o/V_i = -T_{43}/(T_{21} + 1/K)$$

WHERE $T_{43} = V_o/V_3$

$T_{21} = V_2/V_1$

AND [illegible]

devices can be obtained by using OAs. The OA is a non reciprocal two port device, ideally characterised by an infinite gain, infinite input impedance and zero output impedance. In practice, however, the OA has frequency dependent finite gain, μ, a finite input impendance and a non zero output impedance. For example: μA741 has a d.c. gain, μ_o, of 200,000, input impedance of 2MΩ and an output impendance of 75Ω. Besides the OAs are now commercially available as cheap (costing $1.00 or less) off the shelf components. Its symbolic representation is shown in Figure 1.1 and its single pole model is given by:

$$\mu = \mu_o \omega_c / (s + \omega_c) \qquad (1.1)$$

where ω_c is the pole frequency of the OA.

1.4 SINGLE AMPLIFIER NETWORKS:

Second order RC active filters can be realised using one or more amplifiers. However, the cost of supplying power to the filters may become appreciable when more than one amplifier is used. At the present time dc power costs $4.00/watt [5]. Further the problem of cooling the filters due to the heat generated by the dissipated power becomes compounded with additional amplifiers. As a result, the design of second order filter networks with only one amplifier is of great practical interest [6].

Various single amplifier network configurations have been reported in the literature *[6 - 14]. Most of these networks can be generated from the general configurations shown in Figures 1.2, 1.3, and 1.4.

1.5 POLE-Q-AND POLE-FREQUENCY-SENSITIVITIES:

A second order T(s) given by:

$$T(s) = \frac{N(s)}{D(s)} = \frac{N(s)}{a_2 s^2 + a_1 s + a_o} \qquad (1.2a)$$

is usually characterized by the two system parameters, namely:

$$Q = \sqrt{a_o a_2}/a_1 \qquad (1.2b)$$

$$\omega_o = \sqrt{a_o/a_2} \qquad (1.2c)$$

where Q is known as the pole-Q

ω_o is known as the pole frequency

and N(s) is a polynomial of second order or less determining the filter characteristics.

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*Only some of those networks which have appeared in the recent past have been quoted here and these contain many other references.

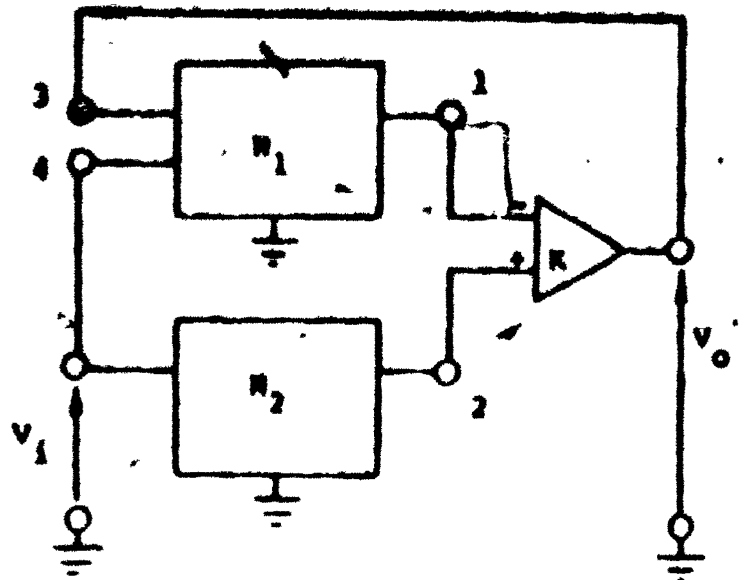

FIG. 1.3 GENERAL SINGLE DIFFERENTIAL AMPLIFIER NETWORK CONFIGURATION

$$T(s) = V_o/V_i = (T_{24} - T_{41})/(T_{31} + 1/K)$$

WHERE $T_{24} = V_2/V_4$

$T_{13} = V_1/V_3$, 4 BEING GROUNDED

$T_{14} = V_1/V_4$, 3 BEING GROUNDED

AND K = AMPLIFIER GAIN.

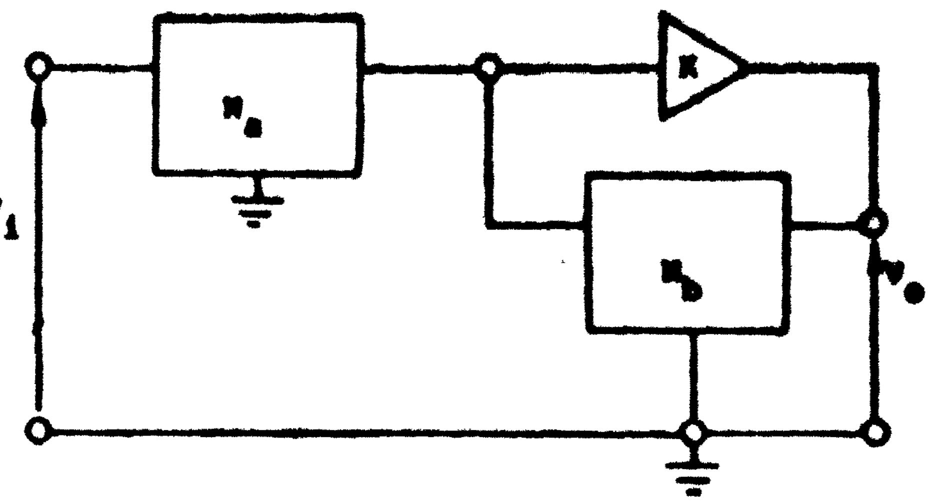

FIG. 1.4 GENERAL SINGLE AMPLIFIER GROUNDED NETWORK CONFIGURATION

$$T(s) = -Ky_{21_a}/(Y_{22_a}+Y_{11_b}+Ky_{12_b})$$

WHERE y_{ij} $i = 1,2,\ j = 1,2$ REFER TO THE ADMITTANCE PARAMETERS CORRESPONDING TO THE PASSIVE RC NETWORKS N_a AND N_b

AND K = AMPLIFIER GAIN

Sensitivity properties of Q and ω_o with respect to the various network elements are of interest. These sensitivities are defined in eq. (1.3)

$$S_x^Q = \frac{x}{Q}\frac{dQ}{dx} \qquad (1.3a)$$

$$S_x^{\omega_o} = \frac{x}{\omega_o}\frac{d\omega_o}{dx} \qquad (1.3b)$$

where 'x' is one of the network elements.

The fractional changes in Q and ω_o due to variations in the network elements are:

$$\frac{\Delta Q}{Q} = \Sigma_\ell \frac{\Delta K_\ell}{K_\ell} S_{K_\ell}^Q + \Sigma_i \frac{\Delta G_i}{G_i} S_{G_i}^Q + \Sigma_j \frac{\Delta C_j}{C_j} S_{C_j}^Q \qquad (1.4a)$$

$$\frac{\Delta\omega_o}{\omega_o} = \Sigma_\ell \frac{\Delta K_\ell}{K_\ell} S_{K_\ell}^{\omega_o} + \Sigma_i \frac{\Delta G_i}{G_i} S_{G_i}^{\omega_o} + \Sigma_j \frac{\Delta C_j}{C_j} S_{C_j}^{\omega_o} \qquad (1.4b)$$

where K_ℓ represents the closed loop gain of an amplifier in the network,

G_i represents the conductance of a resistive element in the network and

C_j represents the capacitance of a capacitive element in the network.

For single amplifier networks, equations (1.4) become:

$$\frac{\Delta Q}{Q} = \frac{\Delta K}{K} S_K^Q + \Sigma_i \frac{\Delta G_i}{G_i} S_{G_i}^Q + \Sigma_j \frac{\Delta C_j}{C_j} S_{C_j}^Q \qquad (1.5a)$$

and $$\frac{\Delta\omega_o}{\omega_o} = \frac{\Delta K}{K} S_K^{\omega_o} + \Sigma_i \frac{\Delta G_i}{G_i} S_{G_i}^{\omega_o} + \Sigma_j \frac{\Delta C_j}{C_j} S_{C_j}^{\omega_o} \qquad (1.5b)$$

where K is the closed loop gain of the amplifier in the network.

1.6 GAIN Q- AND GAIN ω_o-SENSITIVITY PRODUCTS:

An examination of equations (1.3) shows that, not only the variation in Q and ω_o due to K should be designed to be low but also the variation due to the passive elements should be made low. Hybrid IC technology is of interest in this context. In this technology, an active RC filter consists of tantalum thin film RC components and monolithic integrated OAs. This provides almost perfect tracking of passive elements [15], that is, the variations in similar passive elements, due to changes in temperature may be made equal. Further, it is possible to control the process such that:

$$\frac{\Delta G_i}{G_i} = \frac{\Delta C_j}{C_j} = \frac{\Delta G}{G} = \text{Constant for all } i \text{ and } j \qquad (1.6)$$

From equations (1.5) and (1.6), we get:

$$\frac{\Delta Q}{Q} = \frac{\Delta K}{K} S_K^Q + \frac{\Delta G}{G} (\Sigma_i S_{G_i}^Q + \Sigma_j S_{C_j}^Q) \qquad (1.7a)$$

$$\frac{\Delta \omega_o}{\omega_o} = \frac{\Delta K}{K} S_K^{\omega_o} + \frac{\Delta G}{G} (\Sigma_i S_{G_i}^{\omega_o} + \Sigma_j S_{C_j}^{\omega_o}) \qquad (1.7b)$$

Since, from [16]

$$\Sigma_i S_{G_i}^Q + \Sigma_j S_{C_j}^Q = 0$$

and

$$\Sigma_i S_{G_i}^{\omega_o} + \Sigma_j S_{C_j}^{\omega_o} = 0$$

equations (1.7) reduce to:

$$\frac{\Delta Q}{Q} = \frac{\Delta K}{K} S_K^Q \tag{1.8a}$$

and

$$\frac{\Delta \omega_o}{\omega_o} = \frac{\Delta K}{K} S_K^{\omega_o} \tag{1.8b}$$

Thus the effect of variation due to the passive sensitivities is eliminated in thin film technology. Consequently, not only the networks with low passive sensitivities, but also those with high passive sensitivities become attractive. In fact, assuming identical behavior with respect to the active elements embedded in the filter, networks with high passive sensitivities will profit more from the implementation by thin film technology.

The closed loop amplifier gain K is related to the open loop gain μ in the following fashion:

$$K = \frac{K_o}{1 + \frac{K_o}{\mu}} = \frac{K_o \mu}{\mu + K_o} \qquad \text{where } \mu >> K_o \tag{1.9}$$

where K_o is determined by a ratio of resistors used to realize K, then,

$$S_\mu^K = \frac{\mu}{K} \frac{dK}{d\mu} \approx \frac{K}{\mu} \qquad \text{for } \mu >> K,\ K \approx K_o$$

and

$$\frac{\Delta K}{K} = \frac{\Delta\mu}{\mu} S_\mu^K + \frac{\Delta K_o}{K_o} S_{K_o}^K = \frac{K \Delta\mu}{\mu^2} \tag{1.10}$$

where the second term in eq. (1.10) vanishes because of almost perfect tracking and from (16). From equations (1.8) and

(1.10) we get:

$$\frac{\Delta Q}{Q} \approx (KS_K^Q)\ \frac{\Delta \mu}{\mu^2} = (GQ\text{-}SP) \times \frac{\Delta \mu}{\mu^2} \qquad (1.11a)$$

$$\frac{\Delta \omega_o}{\omega_o} \approx (KS_K^{\omega_o})\ \frac{\Delta \mu}{\mu^2} = (G\omega_o\text{-}SP) \times \frac{\Delta \mu}{\mu^2} \qquad (1.11b)$$

where GQ-SP = KS_K^Q and $G\omega_o$-SP = $KS_K^{\omega_o}$ are defined to be the gain Q- and gain ω_o- sensitivity products.

The quantity, $\frac{\Delta \mu}{\mu^2}$ is completely dependent on the OA used and is chosen by the designer from the many OA's available, while the gain-sensitivity products are functions of the properties of the realized network and hence can be controlled by the designer. Thus these two products are of considerable interest [15] particularly so for networks using hybrid tantalum thin film IC technology.

It may be noted that the variation in the transfer function T(s) of a second order network, whose zeros are located far from the high Q poles is given [17] as

$$\left.\frac{\Delta T(s)}{T(s)}\right|_{s = j\omega_o} \doteq \frac{\Delta Q}{Q} + j2Q\ \frac{\Delta \omega_o}{\omega_o} \qquad (1.12)$$

Equation (1.12) shows that for high-Q filters, $G\omega_o$-SP may be more significant than GQ-SP while for low Q filters, GQ-SP appears to be more important. It is also evident that minimization of both $\frac{\Delta Q}{Q}$ and $\frac{\Delta \omega_o}{\omega_o}$ minimizes $\left.\frac{\Delta T(s)}{T(s)}\right|_{s=j\omega_o}$ as well.

1.7 REALIZATIONS WITH PRESCRIBED SENSITIVITY:

Recently attempts have been made to classify active RC filters on the basis of the decompositions of their denominator polynominials [18], [19], and the corresponding realizations have been given using one or more amplifiers. Realizations with some sensitivities prescribed have also been reported in the literature [20 - 26]. Active network realizations with prescribed pole sensitivity have been discussed in [20 - 22]. Realizations corresponding to zero Q-sensitivity with respect to the active element, that is: $S_K^Q = 0$ have been studied in [23 - 24]. Simultaneous realization of T(s) and prescribed sensitivity functions has been discussed in [25]. Also realization of active networks, when the variation of the T(s) is constrained to be within given limits with the additional stipulation that the phase sensitivity remains unchanged, has been discussed in [26]. However, to the best of the author's knowledge any study of second order single amplifier RC networks on the basis of the decompositions of their denominator polynomial along with the prescribed sensitivities of Q or ω_o, and at the same time minimization of the other sensitivity or the gain sensitivity product does not appear to be available in the literature. Such a study appears to be desirable in case of hybrid IC implementation and an attempt is made in this thesis to contribute towards the solution of this problem.

1.8 SCOPE OF THE THESIS:

A detailed study of second order, single amplifier active RC filter networks is presented in this thesis on the basis of their possible denominator polynomial decompositions. The properties of each such decomposition and corresponding realizations are considered when either S_K^Q or $S_K^{\omega_o}$ is prescribed while the other sensitivity or the gain sensitivity product is minimized at the same time.

Chapter II discusses negative feedback filter networks. The bounds on Q- and ω_o- sensitivities for various decompositions of D(s) are first obtained. Realization for each polynomial decomposition is given. These realizations are, in general, of two types:

i) Minimal capacitor type

ii) Non-minimal capacitors type

The minimal capacitor type realization has low passive sensitivities and thus an attempt has been made to control Q and ω_o variations with respect to the active parameter K by considering the corresponding sensitivities as the performance measure. For non-minimal capacitors type realization the passive sensitivities are, in general, high, thus making them suitable only for implementation by hybrid IC technology. In these cases, the gain sensitivity products have been chosen as the performance measure and these are minimized when either S_K^Q or $S_K^{\omega_o}$ is prescribed.

Chapter III deals with, single amplifier active RC filters with positive feedback. The Q- and ω_o-sensitivity bounds are obtained. The Q-sensitivity in these networks is always high and therefore these are more suitable for implementation by hybrid IC technology. Here the gain Q-sensitivity product is minimised while $S_K^{\omega_o}$ is always zero.

Design curves for proposed filter networks, namely Low Pass, High Pass, Band Pass, Notch and All Pass, have been plotted from which it is possible to obtain realisations readily, when the sensitivity is prescribed. It is shown, that, it is possible to prescribe the sensitivity and minimise the gain sensitivity product (GSP) while constraining the element spread at the same time to be within an acceptable value.

For both the negative and positive feedback filter networks proposed, sensitivity properties, stability during activation and the effect of amplifier poles on Q and ω_o have been considered.

Finally, band pass filters for three decompositions (one for $S_K^Q = 0$ and two for $S_K^{\omega_o} = 0$) were built with discrete elements and tested. The experimental results are given.

Chapter IV summarises the results of the present investigation and proposes possible extensions of this work.

CHAPTER II

NEGATIVE FEEDBACK SINGLE AMPLIFIER NETWORKS

CHAPTER II

NEGATIVE FEEDBACK SINGLE AMPLIFIER NETWORKS

2.1 INTRODUCTION:

In this chapter, only networks with negative feedback are considered. Bounds on Q- and ω_o- sensitivities with respect to a network parameter are obtained for various decompositions of D(s). In addition it is shown that a realization exists for each of these decompositions. It is shown that, when either Q- or ω_o- sensitivity is prescribed with respect to amplifier gain, realizations exist only for four decompositions.

2.2 POLYNOMIAL DECOMPOSITIONS:

It is shown that (19) D(s) can be written as:

$$D(s) = s^2 + \frac{\omega_o}{Q} s + \omega_o^2 = D_p(s) + KD_a(s) \quad (2.1)$$

where $D_a(s)$ = That portion of D(s) where K is associated

and $D_p(s)$ = That portion of D(s) where K is not associated

Only the following decompositions are possible depending upon the presence of the parameter of interest 'K' in D(s).

I 'K' appears in only one term out of s^2, [illegible] terms giving rise to the following:

a) 'x' appears in s^2 term only

b) 'x' appears in s^0 term only

c) 'x' appears in s term only

II 'x' appears in two terms out of s^2, s and s^0 terms giving rise to the following:

a) 'x' appears in s^2 and s terms only

b) 'x' appears in s and s^0 terms only

c) 'x' appears in s^2 and s^0 terms only

III 'x' appears in all the three terms.

Polynomial decompositions Ib and IIb may also be obtained from Ia and IIa respectively by RC-CR transformation. But this may lead to a higher capacitive element spread realization from a lower capacitive element spread realization and the number of capacitors will be equal to the number of resistors used in the previous circuit. Therefore, these decompositions are also considered separately. Decompositions I_a and I_b correspond to group A, I_c to group B, II_a and II_b to group C. II_c to group D and III to group E in (19). Also types A, B, C, and D in (18) correspond to decompositions I_b, I_a, III and I_c respectively.

2.3 SENSITIVITIES AND THEIR BOUNDS FOR VARIOUS REALIZATIONS

Bounds on the Q- and ω_o-sensitivities for the case when $D(s)$ is a second degree polynominal shall be derived in this section. For convenience, the sensitivities with respect to the capacitors shall be considered first, while the sensitivities with respect to other network parameters shall be discussed subsequently.

In all the minimal capacitor realizations, eq[n]. (2.1) becomes:

$$D(s) = s^2 C_1 C_2 \alpha + s(C_1\beta_1 + C_2\beta_2) + \gamma \qquad (2.2)$$

where α, β_1, β_2 and γ are functions of the network resistors and amplifier gain only. Therefore:

$$C_1 C_2 \alpha = 1 \qquad (2.3a)$$

$$C_1\beta_1 + C_2\beta_2 = \omega_o/Q \qquad (2.3b)$$

$$\gamma = \omega_o^2 \qquad (2.3c)$$

$$Q = \frac{\sqrt{C_1 C_2 \alpha\gamma}}{C_1\beta_1 + C_2\beta_2} \qquad (2.3d)$$

$$\omega_o = \sqrt{\frac{\gamma}{C_1 C_2 \alpha}} \qquad (2.3e)$$

2.3.1 Q- AND ω_o- SENSITIVITIES WITH RESPECT TO THE CAPACITORS:

From eq. (1.3) and (2.3)

$$S^{Q}_{C_1} = \frac{1}{2} - QC_1\beta_1/\omega_o \qquad (2.4a)$$

and

$$S^{Q}_{C_2} = \frac{1}{2} - QC_2\beta_2/\omega_o \qquad (2.4b)$$

However both $S^{Q}_{C_1}$ and $S^{Q}_{C_2}$ shall be zero if:

$$C_1\beta_1 = C_2\beta_2 = \frac{\omega_o}{2Q} \qquad (2.5)$$

It has been shown [16] that:

$$S^{Q}_{C_1} + S^{Q}_{C_2} = 0 \qquad (2.6)$$

from (2.4) and (2.6) bounds on $S^{Q}_{C_1}$ and $S^{Q}_{C_2}$ are:

$$-\frac{1}{2} < S^{Q}_{C_1} < \frac{1}{2} \qquad (2.7a)$$

$$-\frac{1}{2} < S^{Q}_{C_2} < \frac{1}{2} \qquad (2.7b)$$

Once one of these sensitivities is prescribed the other one is fixed from (2.6)

Irrespective of the values of the capacitors used:

$$S^{\omega_o}_{C_1} = S^{\omega_o}_{C_2} = -\frac{1}{2} \qquad (2.8)$$

Hence it can be seen that only [illegible] can be [illegible]

2.3.2 SENSITIVITIES AND THEIR BOUNDS WITH RESPECT TO OTHER NETWORK PARAMETERS:

For all parameters other than the capacitors, α, β_1 β_2, and γ can be expressed in the form 'ax+b' where 'x' is the parameter of interest and it is non negative for networks considered in this chapter.

from (1.3) and (2.3):

$$S_x^Q = \frac{x}{2}\left(\frac{\alpha'}{\alpha} + \frac{\gamma'}{\gamma} - \frac{2(C_1\beta_1' + C_2\beta_2')}{C_1\beta_1 + C_2\beta_2}\right) \tag{2.9a}$$

and

$$S_x^{\omega_o} = \frac{x}{2}\left(\frac{\gamma'}{\gamma} - \frac{\alpha'}{\alpha}\right) \tag{2.9b}$$

where primes denote differentiation with respect to 'x'.

The derivatives α', β_1', β_2', and γ' can be zero as a minimum and can equal, at the most, $\frac{\alpha}{x}$, $\frac{\beta_1}{x}$, $\frac{\beta_2}{x}$, and $\frac{\gamma}{x}$ respectively and as such the sensitivity bounds are given as follows:

$$-1 < S_x^Q < 1 \tag{2.10a}$$

$$-1/2 < S_x^{\omega_o} < 1/2 \tag{2.10b}$$

Table 2.1 lists these sensitivity bounds for various polynomial decompositions and their special cases. It also gives the additional constraints on Q- and ω_o-sensitivities.

In all the discussions so far 'x' is a general parameter other than capacitors. But the fact, that the amplifier gain has a much larger variation compared to any passive elements, makes the study of Q- and ω_o-sensitivities with respect to the amplifier gain more important than all the passive sensitivities in all the minimal capacitor realizations. Therefore, if 'K' is replaced by 'K' the closed loop gain of the amplifier or the differential amplifier used, Table 2.1 has to be modified, in view of Fialkow Gerst conditions which have to be satisfied by the passive RC transfer function $D_a(s)/D_p(s)$. Table 2.2 gives the Q- and ω_o-sensitivity bounds for various polynomial decompositions, when the amplifier gain 'K' is the parameter of interest.

2.4 REALISABILITY CONDITIONS:

The realisability conditions are derived for the following cases:

i) Sensitivities not prescribed

ii) $S_{C_1}^{Q}$ prescribed

iii) S_K^{Q} prescribed

iv) $S_K^{\omega_o}$ prescribed

v) S_K^{Q} and $S_K^{\omega_o}$ prescribed

More restricted bounds are possible corresponding to each individual realization. In all cases, two distinct classes of realizations exist, one when $C_1 = C_2$ and the other when $C_1 \neq C_2$. These shall be discussed separately.

TABLE [illegible]

SENSITIVITY BOUNDS FOR VARIOUS DECOMPOSITIONS WITH RESPECT TO [illegible] USING A [illegible]

Case		Assumed Description of the Process	Constraints	Sensitivity Bound on [illegible]	Sensitivity Bound on [illegible]	Additional Constraints	Remarks if any
I	a	[illegible]	[illegible] = 0	[illegible] = 1/2	[illegible] = 1/2	[illegible]	Not suitable for sensitivity prescription
			[illegible] ≠ 0	0 < [illegible] < 1/2	-1/2 < [illegible] < 0		
	b	[illegible]	[illegible] = 0	[illegible] = 1/2	[illegible] = 1/2	[illegible]	
			[illegible] ≠ 0	0 < [illegible] < 1/2	0 < [illegible] < 1/2		
	c	[illegible]	[illegible] = 0	[illegible] = -1	[illegible] = 0		[illegible] < 1/2 only possible [illegible]
			[illegible]	-1 < [illegible] < 0	[illegible] = 0		
II	a	[illegible]	[illegible] = 0	-1/2 < [illegible] < 1/2	[illegible] = 1/2	[illegible]	Not suitable for [illegible] prescription.
		[illegible]	[illegible] = 0	1 < [illegible] < 1/2	-1/2 < [illegible] < 0		
			[illegible]	[illegible] = -1/2	[illegible] = -1/2	[illegible]	[illegible] = 0 may be prescribed.
			[illegible]	-1 < [illegible] < 1/2	-1/2 < [illegible] < 0		
	b	[illegible]	[illegible] = 0	-1/2 < [illegible] < 1/2	[illegible] = 1/2	[illegible]	
		[illegible]	[illegible] = 0	-1 < [illegible] < -1/2	0 < [illegible] < 1/2		
			[illegible] = 0	[illegible] = -1/2	[illegible] = 1/2	[illegible]	
			[illegible]	-1 < [illegible] < 1/2	0 < [illegible] < 1/2		

TABLE 2.1 CONTINUED

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

TABLE [illegible]

[illegible]

[illegible]	[illegible]	[illegible]	Sensitivity Bound on [illegible]		Additional Constraints	Remarks if any
			[illegible]	[illegible]		
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]		[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

2.4.1 [illegible] FUNCTIONS:

a) Equal Capacitors Case:

Let: $C_1 = C_2 = C$ (say)

From eq^n (2.3) the realizability conditions are:

$$\frac{1}{C} = \sqrt{\alpha} = Q(\beta_1 + \beta_2)/\omega_o \qquad (2.11a)$$

$$\gamma = \omega_o^2 \qquad (2.11b)$$

b) Unequal Capacitors Case:

$C_1 \neq C_2$

From (2.3) the realizability conditions become:

$$\alpha = 4p_1^2 Q^2 \beta_1 \beta_2/\omega_o^2 \qquad (2.12a)$$

$$\gamma = \omega_o^2 \qquad (2.12b)$$

$$C_1 = \frac{p_1 \pm \sqrt{p_1^2 - 1}}{2 p_1 Q \beta_1} \omega_o \qquad (2.12c)$$

$$C_2 = \frac{p_1 \mp \sqrt{p_1^2 - 1}}{2 p_1 Q \beta_2} \omega_o \qquad (2.12d)$$

Where $p_1 \geq 1$ may be chosen arbitrarily.

As a special case, if $p_1 = 1$, the realizability conditions become:

$$\alpha = 4Q^2\beta_1\beta_2/\omega_o^2 \qquad (2.13a)$$

$$\gamma = \omega_o^2 \qquad (2.13b)$$

$$C_1 = \omega_o/(2Q\beta_1) \qquad (2.13c)$$

$$C_2 = \omega_o/(2Q\beta_2) \qquad (2.13d)$$

In which case $S^Q_{C_1} = S^Q_{C_2} = 0$

2.4.2 $S^Q_{C_1}$ IS PRESCRIBED:

Once $S^Q_{C_1}$ is prescribed, $S^Q_{C_2}$ gets fixed from eq^n (2.6)

From eq^{n}s (2.4) and (2.6)

$$C_1 C_2 = \frac{\omega_o^2}{Q^2 \beta_1 \beta_2}\left[1/4 - (S^Q_{C_1})^2\right] = \frac{\omega_o^2}{Q^2 \beta_1 \beta_2}\left[1/4 - (S^Q_{C_2})^2\right] \tag{2.14}$$

a) Equal Capacitors Case:

Let: $C_1 = C_2 = C$ (say)

From eq^{n}s (1.3), (2.3), and (2.14), the realizability conditions become:

$$\frac{1}{C} = \sqrt{\alpha} = \frac{Q}{\omega_o}(\beta_1 + \beta_2) = \frac{Q}{\omega_o}\sqrt{\beta_1 \beta_2 \Big/ \left[\frac{1}{4} - (S^Q_{C_1})^2\right]} \tag{2.15a}$$

$$\gamma = \omega_o^2 \tag{2.15b}$$

b) Unequal Capacitors Case:

$C_1 \neq C_2$

From eq^{n}s (1.3), (2.12), and (2.14), the realizability conditions become:

$$\alpha = 4 p_1^2 Q^2 \beta_1 \beta_2 / \omega_o^2 = \frac{Q^2 \beta_1 \beta_2 / \omega_o^2}{\frac{1}{4} - (S^Q_{C_1})^2} \tag{2.16a}$$

$$\gamma = \omega_o^2 \tag{2.16b}$$

$$C_1 = \omega_o \left(p_1 \pm \sqrt{p_1^2 - 1}\right) / (2 p_1 Q \beta_1) \tag{2.16c}$$

$$C_2 = \omega_o \left(p_1 \mp \sqrt{p_1^2 - 1}\right) / (2 p_1 Q \beta_2) \tag{2.16d}$$

where $p_1 \geq 1$ may be chosen arbitrarily.

for $p_1 = 1$ it can be shown that $S^Q_{C_1} = S^Q_{C_2} = 0$

2.4.3 S_x^Q IS PRESCRIBED:

a) Equal Capacitors Case:

Let:

$$C_1 = C_2 = C \text{ (say)}$$

From equations (1.3), (2.3), and (2.9) the realizability conditions become:

$$\frac{1}{C}\sqrt{\alpha} = \frac{Q}{\omega_o}(\beta_1+\beta_2) = \frac{2Q(\beta_1'+\beta_2')/\omega_o}{\frac{\alpha'}{\alpha}+\frac{\gamma'}{\gamma}-\frac{2S_x^Q}{x}} \qquad (2.17a)$$

$$\gamma = \omega_o^2 \qquad (2.17b)$$

b) Unequal Capacitors Case:

$$C_1 \neq C_2$$

From equations (2.3), (2.5), and (2.12) the realizability conditions become:

$$\frac{\alpha'}{\alpha}+\frac{\gamma'}{\gamma}-\frac{2S_x^Q}{x} = 4p_1Q\sqrt{\beta_1'\beta_2'/\alpha} > 0 \qquad (2.18a)$$

$$C_1 = \sqrt{\beta_2'/(\alpha\beta_1')}\left[p_1\omega_o \mp \sqrt{p_1^2\omega_o^2-1}\right] \qquad (2.18b)$$

$$C_2 = \sqrt{\beta_1'/(\alpha\beta_2')}\left[p_1\omega_o \pm \sqrt{p_1^2\omega_o^2-1}\right] \qquad (2.18c)$$

$$\omega_o\sqrt{\alpha\beta_1'\beta_2'} = Q\left[\beta_1\beta_2'(\omega_o p_1 \mp \sqrt{p_1^2\omega_o^2-1}) + \beta_2\beta_1'(\omega_o p_1 \pm \sqrt{p_1^2\omega_o^2-1})\right] \qquad (2.18d)$$

where $p_1 \geq 1$ may be chosen arbitrarily

As a special case, if $p_1 = 1$, the realizability conditions become:

$$\frac{\alpha'}{\alpha}+\frac{\gamma'}{\gamma}-\frac{2S_x^Q}{x} = 4Q\sqrt{\beta_1'\beta_2'/\alpha} > 0 \qquad \text{[illegible]}$$

$$C_1 = \sqrt{\beta_2'/(\alpha\beta_1')}\left[\omega_o \mp \sqrt{\omega_o^2 - 1}\right] \qquad (2.19b)$$

$$C_2 = \sqrt{\beta_1'/(\alpha\beta_2')}\left[\omega_o \mp \sqrt{\omega_o^2 - 1}\right] \qquad (2.19c)$$

$$\sqrt{\alpha\beta_1'\beta_2'} = Q\left[\beta_1\beta_2'(\omega_o \mp \sqrt{\omega_o^2 - 1}) + \beta_1'\beta_2(\omega_o \pm \sqrt{\omega_o^2 - 1})\right] \qquad (2.19d)$$

in which case: $S^Q_{C_1} = S^Q_{C_2} = 0$

2.4.4 $S_X^{\omega_o}$ IS PRESCRIBED:

a) **Equal Capacitors Case:**

Let $C_1 = C_2 = C$ (say)

From equations (1.3), (2.3) and (2.9) the realisability conditions become:

$$\frac{1}{C}\sqrt{\alpha} = \frac{Q}{\omega_o}(\beta_1 + \beta_2) \qquad (2.20a)$$

$$\gamma = \omega_o^2 \qquad (2.20b)$$

$$\frac{\gamma'}{\gamma} = \frac{\alpha'}{\alpha} + \frac{2S_X^{\omega_o}}{X} \qquad (2.20c)$$

b) **Unequal Capacitors Case:**

$$C_1 \neq C_2$$

From equations (1.3), (2.3) and (2.12), the realisability conditions become:

$$\alpha = 4p_1^2 Q^2 \beta_1 \beta_2/\omega_o^2 \qquad (2.21a)$$

$$\gamma = \omega_o^2 \qquad (2.21b)$$

$$C_1 = \omega_o(p_1 \pm \sqrt{p_1^2-1})/(2p_1 Q\beta_1) \qquad (2.21c)$$

$$C_2 = \omega_o(p_1 \mp \sqrt{p_1^2-1})/(2p_1 Q\beta_2) \qquad (2.21d)$$

$$\frac{\gamma'}{\gamma} = \frac{\alpha'}{\alpha} + \frac{2S_x^{\omega_o}}{x} \qquad (2.21e)$$

Where $p_1 \geq 1$ may be chosen arbitrarily

As a special case, if $p_1 = 1$, the realizability conditions become:

$$\alpha = 4Q^2 \beta_1 \beta_2/\omega_o^2 \qquad (2.22a)$$

$$\gamma = \omega_o^2 \qquad (2.22b)$$

$$\frac{\gamma'}{\gamma} = \frac{\alpha'}{\alpha} + \frac{2S_x^{\omega_o}}{x} \qquad (2.22c)$$

$$C_1 = \omega_o/(2Q\beta_1) \qquad (2.22d)$$

$$C_2 = \omega_o/(2Q\beta_2) \qquad (2.22e)$$

in which case, $S^Q_{C_1} = S^Q_{C_1} = 0$

2.4.3 <u>S_x^Q AND $S_x^{\omega_o}$ BOTH ARE PRESCRIBED:</u>

a) <u>Equal Capacitors Case</u>

Let $C_1 = C_2 = C$ (say)

From equations (1.3), (2.3), and (2.9), the realizability conditions become:

$$\frac{1}{C} = \sqrt{\alpha} = \frac{Q}{\omega_o}(\beta_1 + \beta_2) = \frac{2Q(\beta_1' + \beta_2')/\omega_o}{\frac{\alpha'}{\alpha} + \frac{\gamma'}{\gamma} - \frac{2S_x^Q}{x}} \qquad (2.23a)$$

$$\gamma = \omega_o^2 \qquad (2.23b)$$

$$\frac{\gamma'}{\gamma} = \frac{\alpha'}{\alpha} + \frac{2S_x^{\omega_o}}{x} \qquad (2.23c)$$

b) <u>Unequal Capacitors Case:</u>

$$C_1 \neq C_2$$

From equations (1.3). (2.3), and (2.12), the realizability conditions become:

$$\frac{\alpha'}{\alpha} + \frac{\gamma'}{\gamma} - \frac{2S_x^Q}{x} = 4p_1 Q\sqrt{\beta_1'\beta_2'/\alpha} > 0 \qquad (2.24a)$$

$$C_1 = (\omega_o p_1 \mp \sqrt{\omega_o^2 p_1^2 - 1})\sqrt{\beta_2'/(\alpha\beta_1')} \qquad (2.24b)$$

$$C_2 = (\omega_o p_1 \pm \sqrt{\omega_o^2 p_1^2 - 1})\sqrt{\beta_1'/(\alpha\beta_2')} \qquad (2.24c)$$

$$\omega_o\sqrt{\alpha\beta_1'\beta_2'} = Q\Big[(\omega_o p_1 \mp \sqrt{\omega_o^2 p_1^2 - 1})\beta_1\beta_2' + \beta_2\beta_1'(\omega_o p_1 \pm \sqrt{\omega_o^2 p_1^2 - 1})\Big] \qquad (2.24d)$$

$$\frac{\gamma'}{\gamma} = \frac{\alpha'}{\alpha} + \frac{2S_x^{\omega_o}}{x} \qquad (2.24e)$$

Where $p_1 \geq 1$ may be chosen arbitrarily.

As a special case, if $p_1 = 1$, the realizability [illegible] become:

$$\frac{\alpha'}{\alpha} + \frac{\gamma'}{\gamma} - \frac{2S_x^Q}{x} = 4Q\sqrt{\beta_1'\beta_2'/\alpha} > 0 \qquad (2.25a)$$

$$C_1 = \sqrt{\beta_2'/(\alpha\beta_1')}\left[\omega_o \mp \sqrt{\omega_o^2 - 1}\right] \qquad (2.25b)$$

$$C_2 = \sqrt{\beta_1'/(\alpha\beta_2')}\left[\omega_o \pm \sqrt{\omega_o^2 - 1}\right] \qquad (2.25c)$$

$$\omega_o\sqrt{\alpha\beta_1'\beta_2'} = Q\left[(\omega_o \mp \sqrt{\omega_o^2 - 1})\beta_1\beta_2' + \beta_2\beta_1'(\omega_o \pm \sqrt{\omega_o^2 - 1})\right] \qquad (2.25d)$$

$$\frac{\gamma'}{\gamma} = \frac{\alpha'}{\alpha} + \frac{2S_x^{\omega_o}}{x} \qquad (2.25e)$$

in which case $S_{C_1}^Q = S_{C_2}^Q = 0$

It may be mentioned that even though either $S_x^Q = 0$ or $S_x^{\omega_o} = 0$ may be prescribed in various cases yet it is not possible to prescribe both $S_x^Q = 0$ and $S_x^{\omega_o} = 0$ as this leads to the condition:

$$\frac{\alpha'}{\alpha} = \frac{C_1\beta_1' + C_2\beta_2'}{C_1\beta_1 + C_2\beta_2} = \frac{\gamma'}{\gamma} \qquad (2.26)$$

which indicates that 'x' is a common factor in α, β_1, β_2, and γ and it may be taken out and may not be associated with the denominator polynomial.

2.5 Q- AND ω_o- SENSITIVITIES WHEN D(s) IS OF THIRD DEGREE:

Active RC-networks with at the most third-degree denominator polynomial in their transfer functions have been considered in this thesis. Third degree denominator polynomial may be written as:

$$D_3(s) = (s + \epsilon_1)(s^2 + \frac{\omega_o}{Q} s + \omega_o^2) = s^3 + s^2(\epsilon_1 + \frac{\omega_o}{Q})$$

$$+ s(\omega_o^2 + \epsilon_1 \frac{\omega_o}{Q}) + \epsilon_1 \omega_o^2 = s^3 + \delta_2 s^2 + \delta_1 s + \delta_o \qquad (2.27)$$

Two cases arise:

a) <u>ϵ_1 is a function of network parameter 'x':</u>

In this case it is not possible to relate Q and ω_o to the network parameter 'x' (which can be either a resistor, capacitor, or amplifier gain) in a simple fashion as for D(s). The Q- and ω_o- sensitivities with respect to the network parameter 'x' can be computed from their relations with the coefficient sensitivities. There is no simple and direct method to prescribe these sensitivities and to obtain the realizability conditions.

From eq^n (2.27) we have:

$$\delta_2 = \epsilon_1 + \frac{\omega_o}{Q} \qquad (2.28a)$$

$$\delta_1 = \omega_o^2 + \epsilon_1 \frac{\omega_o}{Q} \qquad (2.28b)$$

$$\delta_o = \epsilon_1 \omega_o^2 \qquad (2.28c)$$

From equations (2.3) and (2.30) it can be shown that:

$$\begin{bmatrix} \frac{\omega_o}{\delta_2 Q} & -\frac{\omega_o}{\delta_2 Q} & \frac{\epsilon_1}{\delta_2} \\ \frac{2\omega_o^2 + \epsilon_1 \omega_o / Q}{\delta_1} & -\frac{\epsilon_1 \omega_o}{\delta_2 Q} & \frac{\epsilon_1 \omega_o}{Q \delta_1} \\ \frac{2\epsilon_1 \omega_o^2}{\delta_o} & 0 & \frac{\epsilon_1 \omega_o^2}{\delta_o} \end{bmatrix} \begin{bmatrix} S_x^{\omega_o} \\ S_x^{Q} \\ S_x^{\epsilon_1} \end{bmatrix} = \begin{bmatrix} S_x^{\delta_2} \\ S_x^{\delta_1} \\ S_x^{\delta_o} \end{bmatrix}$$

or

$$\begin{bmatrix} S_x^{\omega_o} \\ S_x^{Q} \\ S_x^{\epsilon_1} \end{bmatrix} = \begin{bmatrix} \frac{\omega_o}{\delta_2 Q} & -\frac{\omega_o}{\delta_2 Q} & \frac{\epsilon_1}{\delta_2} \\ \frac{2\omega_o^2+\epsilon_1\omega_o/Q}{\delta_1} & -\frac{\epsilon_1\omega_o}{\delta_1 Q} & \frac{\epsilon_1\omega_o}{\delta_1 Q} \\ 2 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} S_x^{\delta_2} \\ S_x^{\delta_1} \\ S_x^{\delta_o} \end{bmatrix} \tag{2.29}$$

In normalized case, for $\omega_o = 1$ and therefore for $\delta_o = \epsilon_1$, equation (2.29) may be simplified to give:

$$S_x^{\omega_o} = \frac{-\epsilon_1 Q\delta_2 S_x^{\delta_2} + Q\delta_1 S_x^{\delta_1} + \epsilon_1(\epsilon_1 Q-1)S_x^{\delta_o}}{2Q(1+\epsilon_1^2) - 2\epsilon_1} \tag{2.30a}$$

$$S_x^{Q} = \frac{-Q(2Q-\epsilon_1)\delta_2 S_x^{\delta_2} - Q(2\epsilon_1 Q - 1)\delta_1 S_x^{\delta_1} + \epsilon_1(2Q^2+\epsilon_1 Q-1)S_x^{\delta_o}}{2Q(1+\epsilon_1^2) - 2\epsilon_1} \tag{2.30b}$$

$$S_x^{\epsilon_1} = \frac{2Q\epsilon_1\delta_2 S_x^{\delta_2} - 2Q\delta_1 S_x^{\delta_1} + 2Q S_x^{\delta_o}}{2Q(1+\epsilon_1^2) - 2\epsilon_1} \tag{2.30c}$$

b) <u>ϵ_1 is not a function of Network Parameter 'x':</u>

From eqn (2.30) we get, for $S_x^{\epsilon_1} = 0$

$$S_x^{\omega_o} = \frac{1}{2}S_x^{\delta_o} \tag{2.31a}$$

$$S_x^{Q} = \frac{1}{2}S_x^{\delta_o} - Q\delta_2 S_x^{\delta_2} \tag{2.31b}$$

$$\delta_1 S_x^{\delta_1} = S_x^{\delta_o} + \epsilon_1\delta_2 S_x^{\delta_2} \tag{2.31c}$$

In this case Q and ω_o can be related to 'x' in the same fashion as for D(s). $S_x^{\omega_o}$ can be prescribed and a realization can be obtained, S_x^{Q} can be prescribed for D(s) and a realization may be obtained and later on S_x^{Q} can be computed for $D_3(s)$. In general, S_x^{Q} for $D_3(s)$, which is difficult to prescribe, shall not be the same as S_x^{Q} for D(s).

So far for various possible decompositions of the denominator polynomial D(s), the bounds on Q- and ω_o- sensitivities with respect to all the network parameters, namely capacitors, resistors and the amplifier gain have been derived. Further additional constraints on the sum and the difference of Q- and ω_o-sensitivities which have to be satisfied, are also obtained. It is to be pointed out that only the upper bounds on S_K^Q and $S_K^{\omega_o}$ have been obtained in [19] for the denominator polynomial decompositions of negative feedback networks.

In the subsequent sections of this chapter, realizations of negative feedback, single amplifier active networks for each polynomial decomposition shall be discussed.

2.6 REALIZATIONS OF ACTIVE NETWORKS FOR VARIOUS DECOMPOSITIONS:

In this section, we shall present realizations corresponding to each denominator polynomial decomposition given in section 2.3. The specifications for these realizations are Q, ω_o, and either Q- or ω_o sensitivity with respect to the amplifier gain. In a few desired cases, we shall consider the stability of the circuit during activation and also the effect of the pole of the amplifier.

2.6.1 DECOMPOSITION Ia:

The amplifier gain 'K' appears in only s^2 term

D(s) shall be of the type

$$D(s) = s^2 C_1 C_2 (\alpha_1 + K\alpha_2) + s(C_1\beta_1 + C_2\beta_2) + \gamma \quad (2.32a)$$

and

$$Q = \frac{\sqrt{\gamma C_1 C_2 (\alpha_1 + K\alpha_2)}}{C_1\beta_1 + C_2\beta_2} \quad (2.32b)$$

$$\omega_o = \sqrt{\frac{\gamma}{C_1 C_2 (\alpha_1 + K\alpha_2)}} \quad (2.32c)$$

also

$$S_K^Q = \frac{1}{2}\,\frac{K\alpha_2}{\alpha_1 + K\alpha_2} = -S_K^{\omega_o} \quad (2.32d)$$

with the sensitivity bounds being:

$$o < S_K^Q < \frac{1}{2} \quad (2.32e)$$

and

$$-\frac{1}{2} < S_K^{\omega_o} < 0 \quad (2.32f)$$

For all the passive elements, magnitudes of Q- and ω_o-sensitivities are less than unity.

From Fialkow Gerst conditions $\alpha_2 \leq \alpha_1$

solving for K, we get:

$$K = \left[\frac{Q^2 (C_1\beta_1 + C_2\beta_2)^2}{C_1 C_2 \gamma} - \alpha_1\right] \frac{1}{\alpha_2} \quad (2.33)$$

But we should have:

$$(C_1\beta_1 + C_2\beta_2)^2 > 4\gamma C_1 C_2 \alpha_1$$

since poles of the passive portion have to be simple and negative real. Therefore,

$$K > (4Q^2 - 1)\frac{\alpha_1}{\alpha_2} > 4Q^2 - 1 \quad (2.34)$$

In this decomposition the magnitudes of S_K^Q and $S_K^{\omega_o}$ shall be very nearly equal to 1/2, if the [illegible] is a not prescribed. If either S_K^Q or $S_K^{\omega_o}$ is prescribed, K gets specified and the realisable value of Q gets fixed. This means that this decomposition is not suitable for the realization of active networks, when Q and S_K^Q both are specified independently. Nevertheless, a realization is given for the sake of completeness as shown in Figure 2.1.

Analysis yields:

$$\frac{V_o}{V_i} = \frac{-KsC_{f_2}G_{a_1}}{s^2C_{f_1}C_{f_2}(K+1) + s\left[C_{f_1}G_{34} + C_{f_2}(G_{a_1} + G_{34})\right] + G_{a_1}G_{34}} \quad (2.35a)$$

$$\frac{V_{o_1}}{V_i} = \frac{G_{a_1}G_{34}}{s^2C_{f_1}C_{f_2}(K+1) + s\left[C_{f_1}G_{34} + C_{f_2}(G_{a_1} + G_{34})\right] + G_{a_1}G_{34}} \quad (2.35b)$$

a) Equal Capacitors Realization:

Let $C_{f_1} = C_{f_2} = C$ (say)

From eq. (2.11) the realization is:

$$C_{f_1} = C_{f_2} = C = 1/3Q,\ G_{a_1} = G_{34} = 1$$

$$K = 9Q^2 - 1$$

The element spread is: Capacitors: 1:1, Resistors: 1:1

The Transfer function Gain for

- Band Pass: at the centre frequency is $-3Q^2 + 1/3$

Low Pass: is unity at $\omega = 0$.

The Sensitivities are:

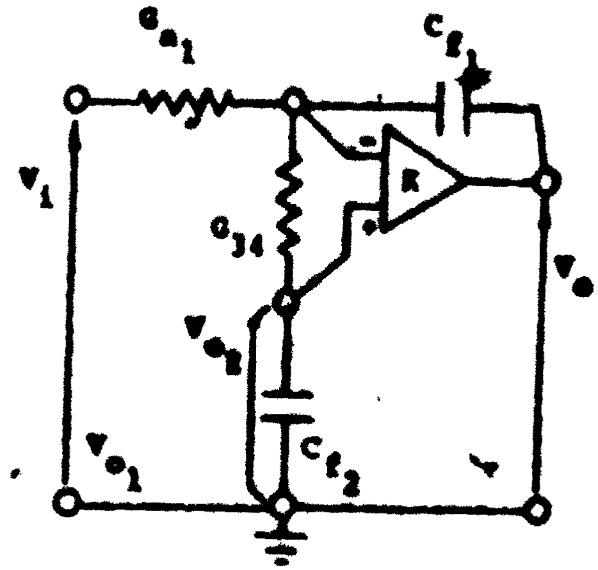

FIG. 2.1 REALIZATION OF AN

ACTIVE NETWORK FOR

DECOMPOSITION Ia

$$S^{Q}_{K} = -S^{\omega_o}_{K} = \frac{1}{2} - \frac{1}{18Q^2}\,.$$

$$S^{Q}_{C_{f_1}} = -S^{Q}_{C_{f_2}} = 1/6$$

$$S^{Q}_{G_{a_1}} = -S^{Q}_{G_{34}} = 1/6$$

$$S^{\omega_o}_{C_{f_1}} = S^{\omega_o}_{C_{f_2}} = -S^{\omega_o}_{G_{a_1}} = -S^{\omega_o}_{G_{34}} = -1/2$$

b) <u>Unequal Capacitor Realization:</u>

$$C_{f_1} \neq C_{f_2}$$

From equations (2.12) and (2.35) the realization is:

$$G_{a_1} = \frac{1}{\sqrt{p_2-1}},\quad G_{34} = \sqrt{p_2-1},\quad C_{f_1} = \frac{1}{2Q\sqrt{p_2-1}},\quad C_{f_2} = \frac{\sqrt{p_2-1}}{2p_2Q},\quad K = 4p_2Q^2-1$$

where p_2 is an arbitrary constant greater than unity.

The element spread is: Capacitors 1: $\frac{p_2}{p_2-1}$,

Resistors 1: p_2-1

The transfer function gain:

for Band Pass at the centre frequency is $-2Q^2 + \frac{1}{2p_2}$

while for Low Pass is unity at $\omega = 0$.

The sensitivities are:

$$S^{Q}_{K} = \frac{1}{2} - \frac{1}{8Q^2p_2} \doteq -S^{\omega_o}_{K}$$

$$S^{Q}_{C_{f_1}} = S^{Q}_{C_{f_2}} = 0$$

$$S^{Q}_{G_{a_1}} = -S^{Q}_{G_{34}} = \frac{1}{2} - \frac{1}{2p_2}$$

$$S^{\omega_o}_{C_{f_1}} = S^{\omega_o}_{C_{f_2}} = -S^{\omega_o}_{G_{a_1}} = -S^{\omega_o}_{G_{34}} = -\frac{1}{2}$$

for a very good capacitive spread, low resistive spread and a reasonable value of K, p_2 may be chosen as 2.

2.6.2 DECOMPOSITION Ib

The amplifier gain K appears in s^o term only: It has been reported in the literature [18], [19], that a realization for this decomposition is obtained from that of decomposition Ia using RC-CR transformation. However, it has to be pointed out that the realization of the network using the above transformation may not result in (i) the same number of capacitors as before and (ii) the same element spread. Also as will be seen presently, the $S_K^{\omega_o}$ changes its sign. Hence, this case is also discussed.

D(s) shall be of the type

$$D(s) = s^2 C_1 C_2 \alpha + s(C_1 \beta_1 + C_2 \beta_2) + \gamma_1 + K\gamma_2 \qquad (2.36a)$$

and

$$Q = \frac{\sqrt{\alpha C_1 C_2 (\gamma_1 + K\gamma_2)}}{C_1 \beta_1 + C_2 \beta_2} \qquad (2.36b)$$

$$\omega_o = \sqrt{\frac{\gamma_1 + K\gamma_2}{C_1 C_2 \alpha}} \qquad (2.36c)$$

also

$$S_K^Q = S_K^{\omega_o} = \frac{1}{2}\,\frac{K\gamma_2}{\gamma_1 + K\gamma_2} \qquad (2.36d)$$

The sensitivity bounds being:

$$0 < S_K^Q < 1/2 \qquad (2.36e)$$

$$0 < S_K^{\omega_o} < 1/2 \qquad (2.36f)$$

For all the passive elements, the magnitudes of Q- and ω_o- sensitivities are less than unity.

From Fialkow Gerst conditions $\gamma_2 \leq \gamma_1$

solving for K from (2.38) gives:

$$K = \left[\frac{Q^2 (C_1\beta_1 - C_2\beta_2)^2}{C_1 C_2 \alpha} - \gamma_1\right]\frac{1}{\gamma_2} \qquad (2.37)$$

But we should have $(C_1\beta_1 + C_2\beta_2)^2 > 4C_1C_2\alpha\gamma_1$,

since the poles of the passive network shall be simple, negative real. Therefore $K > (4Q^2-1)\frac{\gamma_1}{\gamma_2} > 4Q^2-1$

If not prescribed, the magnitudes of S_K^Q and $S_K^{\omega_o}$ shall be very nearly equal to 1/2. In case one of them is prescribed, K gets specified and this in turn fixes the realizable value of Q.

Once again, active networks of this type of decomposition are not suitable for the prescription of S_K^Q or $S_K^{\omega_o}$. Nevertheless, a realization as shown in Figure 2.2 is presented for the sake of completeness.

Analysis yields:

$$\frac{V_o}{V_i} = \frac{-KsC_{a_1}G_{f_2}}{s^2C_{a_1}C_{34} + s\left(C_{34}(G_{f_1} + G_{f_2}) + C_{a_1}G_{f_2}\right) + G_{f_1}G_{f_2}(K+1)} \qquad (2.38a)$$

$$\frac{V_{o_1}}{V_i} = \frac{s^2C_{a_1}C_{34}}{s^2C_{a_1}C_{34} + s(C_{34}(G_{f_1}+G_{f_2}) + C_{a_1}G_{f_2}) + G_{f_1}G_{f_2}(K+1)} \qquad (2.38b)$$

a) Equal Capacitor Realization:

Let: $C_{a_1} = C_{34} = C$(say)

From eq. (2.11) and (2.38) the realization is:

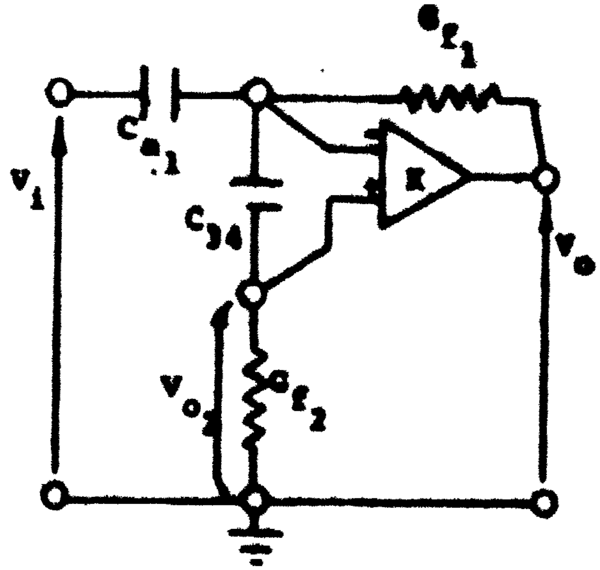

FIG. 2.2 REALIZATION OF AN ACTIVE

NETWORK FOR

DECOMPOSITION Ib

$$C = C_{a_1} = C_{34} = 1$$

$$G_{f_1} = G_{f_2} = 1/3Q$$

$$K = 9Q^2-1$$

The element spread is Capacitors: 1:1, Resistors: 1:1

The Transfer function Gain for:

Band Pass at centre frequency $= -3Q^2 + 1/3$ while for

High Pass at very high frequency $= 1$

The sensitivities are:

$$S_K^Q = S_K^{\omega_o} = \frac{1}{2} - \frac{1}{18Q^2}$$

$$S_{G_{f_1}}^Q = -S_{G_{f_2}}^Q = 1/6$$

$$S_{C_{a_1}}^Q = -S_{C_{34}}^Q = 1/6$$

$$S_{C_{a_1}}^{\omega_o} = S_{C_{34}}^{\omega_o} = -S_{G_{f_1}}^{\omega_o} = -S_{G_{f_2}}^{\omega_o} = -1/2$$

b) <u>Unequal Capacitors Realization:</u>

$$C_{a_1} \neq C_{34}$$

From equations (2.12) and (2.38) the realization is:

$$C_{a_1} = \frac{1}{\sqrt{P_2-1}}, \quad C_{34} = \sqrt{P_2-1}, \quad G_{f_1} = \frac{1}{2Q\sqrt{P_2-1}}, \quad G_{f_2} = \frac{\sqrt{P_2-1}}{2P_2Q},$$

$$K = 4P_2Q^2-1$$

where P_2 is an arbitrary constant greater than unity.

The element spread is:

Capacitors: 1: $\sqrt{P_2-1}$, Resistors: 1: $\dfrac{P_2}{\sqrt{P_2-1}}$

The Transfer function gain for

Band Pass at centre frequency is $+2Q^2 - \dfrac{1}{2P_2}$

High Pass at very high frequency is unity.

The Sensitivities are:

$$S_K^Q = S_K^{\omega_o} = \frac{1}{2} - \frac{1}{4p_2Q^2}$$

$$S_{C_{a_1}}^Q = -S_{C_{34}}^Q = \frac{1}{2} - \frac{1}{2p_2}$$

$$S_{G_{f_1}}^Q = S_{G_{f_2}}^Q = 0$$

$$S_{C_{a_1}}^{\omega_o} = S_{C_{34}}^{\omega_o} = -S_{G_{f_1}}^{\omega_o} = -S_{G_{f_2}}^{\omega_o} = -1/2$$

Once again $p_2 = 2$ may be chosen to provide, a very good element spread for a reasonable value of K.

2.6.3 DECOMPOSITION Ic:

K appears in s term only,

D(s) shall be of the type

$$D(s) = s^2C_1C_2\alpha + s\{C_1(\beta_{11} + K\beta_{12}) + C_2(\beta_{21} + K\beta_{22})\} + \gamma \qquad (2.39a)$$

$$Q = \frac{\sqrt{\gamma C_1 C_2 \alpha}}{C_1(\beta_{11} + K\beta_{12}) + C_2(\beta_{21} + K\beta_{22})} \qquad (2.39b)$$

$$\text{Passive } Q = \frac{\sqrt{\gamma C_1 C_2 \alpha}}{C_1\beta_{11} + C_2\beta_{21}} \qquad (2.39c)$$

$$\omega_o = \sqrt{\frac{\gamma}{C_1C_2\alpha}} \qquad (2.39d)$$

Since K is positive, Q reduces below the passive Q and hence, this decomposition is of no further interest.

2.6.4 DECOMPOSITION IIa:

The amplifier gain appears in s^2 and s terms only.

D(s) shall be of the type

$$D(s) = s^2 C_1 C_2 (\alpha_1 + K\alpha_2) + sC_1(\beta_{11} + K\beta_{12}) + sC_2(\beta_{21} + K\beta_{22}) + \gamma \tag{2.40a}$$

and
$$Q = \frac{\sqrt{\gamma C_1 C_2(\alpha_1 + K\alpha_2)}}{C_1(\beta_{11} + K\beta_{12}) + C_2(\beta_{21} + K\beta_{22})} \tag{2.40b}$$

$$\omega_o = \sqrt{\frac{\gamma}{C_1 C_2(\alpha_1 + K\alpha_2)}} \tag{2.40c}$$

$$S_K^Q = \frac{1}{2}\frac{K\alpha_2}{\alpha_1 + K\alpha_2} - \frac{K(C_1\beta_{12} + C_2\beta_{22})}{C_1(\beta_{11} + K\beta_{12}) + C_2(\beta_{21} + K\beta_{22})} \tag{2.40d}$$

$$S_K^{\omega_o} = -\frac{1}{2}\frac{K\alpha_2}{\alpha_1 + K\alpha_2} \tag{2.40e}$$

$$S_K^Q + S_K^{\omega_o} = -\frac{K(C_1\beta_{12} + C_2\beta_{22})}{C_1(\beta_{11} + K\beta_{12}) + C_2(\beta_{21} + K\beta_{22})} \tag{2.40f}$$

and
$$S_K^Q - S_K^{\omega_o} = \frac{K\alpha_2}{\alpha_1 + K\alpha_2} - \frac{K(C_1\beta_{12} + KC_2\beta_{22})}{C_1(\beta_{11} + K\beta_{12}) + C_2(\beta_{21} + K\beta_{22})} \tag{2.40g}$$

The Sensitivity bounds are:

$$-1 < S_K^Q < 1/2 \tag{2.40h}$$

$$-1/2 < S_K^{\omega_o} < 0 \tag{2.40i}$$

$$-1 < S_K^Q + S_K^{\omega_o} < 0 \qquad (2.40j)$$

$$-1 < S_K^Q - S_K^{\omega_o} < 1 \qquad (2.40k)$$

For all the passive elements, the magnitudes of Q- and ω_o- sensitivities are less than unity. $S_K^{\omega_o}$, in this case, is exactly the same as in decomposition Ia and as such its magnitude shall be very nearly equal to 1/2, if not prescribed. If prescribed, it would fix K and this in turn would fix realizable Q. As stated in Ia, networks obtained from this decomposition are not suitable for the prescription of $S_K^{\omega_o}$.

Once either S_K^Q or $S_K^{\omega_o}$ is prescribed, the other sensitivity gets more restricted according to the bounds on their sum and difference.

If S_K^Q is not prescribed, it is quite low, usually its magnitude being lower than 1/2. However, in this decomposition it is possible to prescribe $S_K^Q = 0$. Networks with $S_K^Q = 0$ with the decomposition of third degree polynomials have been reported in the literature (24). Such decompositions are not discussed in this thesis.

Realizations with $S_K^Q = 0$ shall be presented for the cases with equal and unequal capacitors. Further, stability during activation, Q- and ω_o- sensitivities with respect to open loop OA gains and the resistors employed to realise differential amplifier with closed loop gain K and the effect of OA poles on Q and ω_o shall be considered in this case. Finally, the experimental results shall be given. The network used in these realizations is given in Figure 2.3.

Analysis yields:

$$T(s) = \frac{N(s)}{D(s)} = \frac{-KsG_a(-C_{f_1}G_2 + C_{f_2}G_1)}{s^2C_{f_1}C_{f_2}(K+1)(G_a+G_1+G_2)+s(C_{f_1}\left[(K+1)G_aG_2+G_1G_2\right]+C_{f_2}G_1(G_a+G_2))+G_aG_1G_2}$$

(2.41)

a) <u>Equal Capacitor Realization:</u>

Let $C_{f_1} = C_{f_2} = C$ (say)

From equations (2.17) and (2.41) the design equations are:

$$G_a = G_2 = (12Q^2 - 2)^{-1/3},$$

$$G_1 = (12Q^2 - 2)^{2/3}$$

$$C_{f_1} = C_{f_2} = C = 1/6Q(12Q^2 - 2)^{1/3}),$$

$$K = 36Q^2 - 7,$$

$$\omega_o = 1$$

(2.42)

The element spread is: capacitors 1:1, Resistors: $1:12Q^2-2$. The Transfer Function gain at centre frequency is approximately $-6Q^2 + 3/2$

b) <u>Unequal Capacitor Realization:</u>

$(C_{f_1} \neq C_{f_2})$

From equations (2.21) and (2.41), one set of design equations is:

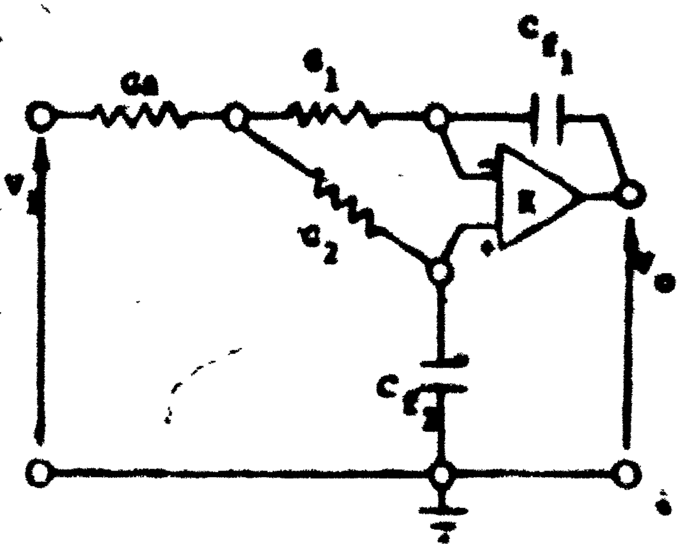

Fig. 2.3 BAND PASS FILTER REALIZATION FOR DECOMPOSITION IIa

$$G_a = G_2 = (2p_2(4Q^2-1))^{1/3}, \quad (2.43a)$$

$$G_1 = (2p_2(4Q^2-1))^{2/3} \quad (2.43b)$$

$$C_{f_1} = \frac{(p_2-1)p_2^{-1/3}(8Q^2-2)^{2/3}}{4Q(p_2(4Q^2-1)+1)}, \quad (2.43c)$$

$$C_{f_2} = \frac{Q(2p_2(4Q^2-1))^{-1/3}}{p_2(4Q^2-1)+1} \quad (2.43d)$$

$$K = \frac{2p_2((4Q^2-1)p_2+1)}{p_2-1} - 1, \quad (2.43e)$$

$$\omega_o = 1 \quad (2.43f)$$

Where p_2 is an arbitrary constant greater than unity.

The element spread is:

Resistors: 1: $2p_2(4Q^2-1)$,

capacitors: 1: $(p_2-1)\ (2-\frac{1}{2Q^2})$

The transfer function gain at centre frequency is:

$$-\frac{2p_2^2(4Q^2-1)+p_2+1}{2p_2(p_2-1)}$$

Before obtaining the different sensitivities of these designs, we shall discuss the differential amplifier realization analysis for stability during activation and the effect of OA poles on Q and ω_o.

2.6.4.1 DIFFERENTIAL AMPLIFIER [illegible]

One of the networks reported in the literature [illegible] for the differential amplifier realisation is given in Figure 2.4.

This upon analysis gives:

$$V_3 = V_2 \frac{\frac{G_{31}}{G_{41}} + 1}{1 + \frac{1}{\mu_2}\left(1 + \frac{G_{31}}{G_{41}}\right)} - V_1 \frac{\left(\frac{G_{11}}{G_{21}} + 1\right)\frac{G_{31}}{G_{41}}}{\left(1 + \frac{1+\frac{G_{11}}{G_{21}}}{\mu_1}\right)\left(1 + \frac{1+\frac{G_{31}}{G_{41}}}{\mu_2}\right)} \qquad (2.44)$$

If: $G_{21}G_{41} = G_{11}G_{31}$

and $$\mu_1 >> 1 + \frac{G_{11}}{G_{21}} = \frac{K}{K-1}, \quad \mu_2 >> 1 + \frac{G_{31}}{G_{41}} = K \qquad (2.45)$$

then: $$\frac{V_3}{V_1 - V_2} = -K = -\left(1 + \frac{G_{31}}{G_{41}}\right) = -\left(1 + \frac{G_{21}}{G_{11}}\right) \qquad (2.46)$$

2.6.4.2 STABILITY ANALYSIS:

Replacing the differential amplifier in Fig. 2.3 by the network of Fig. 2.4 and then analysing it, we get the numerator polynomial $N(s)$ and the denominator polynomial $D(s)$ of the voltage transfer function as:

$$N(s) = -KG_a\left(sC_{f_2}G_1 - sC_{f_1}G_2 - \frac{\frac{K}{K-1}}{\mu_1} G_2(sC_{f_1} + G_1)\right) \qquad (2.47a)$$

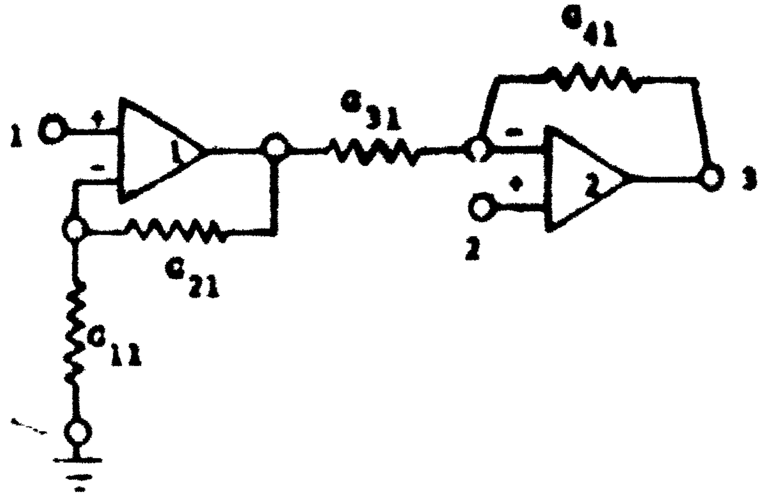

FIG. 2.4 DIFFERENTIAL AMPLIFIER REALIZATION

$$D(s) = s^2 C_{f_1} C_{f_2} (G_a + G_1 + G_2)\left(K + 1 + \frac{K/(K-1)}{\mu_1} + \frac{K}{\mu_2} + \frac{K^2/(K-1)}{\mu_1 \mu_2}\right) +$$

$$s C_{f_1}\left(G_a G_2 \left(K + 1 + \frac{K/(K-1)}{\mu_1} + \frac{K}{\mu_2} + \frac{K^2/(K-1)}{\mu_1\mu_2}\right) + G_1 G_2 \left(1 - \frac{K}{\mu_1} +\right.\right.$$

$$\left.\left.\frac{K}{\mu_2} + \frac{K^2/(K-1)}{\mu_1\mu_2}\right)\right) + s C_{f_2} (G_a G_1 + G_1 G_2)\ \left(1 + \frac{K/(K-1)}{\mu_1} + \frac{K}{\mu_2} +\right.$$

$$\left.\frac{K^2/(K-1)}{\mu_1\mu_2}\right) + G_a G_1 G_2 \left(1 + \frac{K/(K-1)}{\mu_1} + \frac{K}{\mu_2} + \frac{K^2/(K-1)}{\mu_1\mu_2}\right) \quad (2.47b)$$

From the expression of D(s), it is clear that one of the terms is negative. In the odd possibility that during the activation of the circuit μ_1 rises very slowly while μ_2 rises fast, then the possibility of network becoming unstable cannot be discounted [29]. The Q- and ω_o- sensitivities are now worked out.

From (2.47) it can be shown that:

$$Q = \frac{\sqrt{C_{f_1} C_{f_2} (G_a + G_1 + G_2)\left(K + 1 + \frac{K/(K-1)}{\mu_1} + \frac{K}{\mu_2} + \frac{K^2/(K-1)}{\mu_1\mu_2}\right) \times G_a G_1 G_2 \left(1 + \frac{K/(K-1)}{\mu_1} + \frac{K}{\mu_2} + \frac{K^2/(K-1)}{\mu_1\mu_2}\right)}}{C_{f_1} G_2 \left(G_a \left(K + 1 + \frac{K/(K-1)}{\mu_1} + \frac{K}{\mu_2} + \frac{K^2/(K-1)}{\mu_1\mu_2}\right) + C_{f_1} G_1 G_2 \left(1 - \frac{K}{\mu_1} + \frac{K}{\mu_2} + \frac{K^2/(K-1)}{\mu_1\mu_2}\right)\right) + C_{f_2} G_1 (G_a + G_2)\left(1 + \frac{K/(K-1)}{\mu_1} + \frac{K}{\mu_2} + \frac{K^2/(K-1)}{\mu_1\mu_2}\right)}$$

(2.48a)

$$\text{and } \omega_o = \sqrt{\frac{\dfrac{G_a G_1 G_2}{C_{f_1} C_{f_2} (G_a + G_1 + G_2)}}{1 + \dfrac{K}{1 + \frac{K/(K-1)}{\mu_1} + \frac{K}{\mu_2} + \frac{K^2/(K-1)}{\mu_1\mu_2}}}} \quad (2.48b)$$

From (1.3) and (2.48) it can be shown that:

$$g^0_{\mu_1} = \frac{1}{2}\,\frac{\mu_1\left(K + 1 + \frac{K}{\mu_2}\right)}{\mu_1\left(K + 1 + \frac{K}{\mu_2}\right) + \frac{K}{K-1} + \frac{K^2/(K-1)}{\mu_2}} + \frac{1}{2}\,\frac{\mu_1\left(1 + \frac{K}{\mu_2}\right)}{\mu_1\left(1 + \frac{K}{\mu_2}\right) + \frac{K}{K-1} + \frac{K^2/(K-1)}{\mu_2}} - \frac{C_{f_1}\mu_1\left(G_aG_2\left(K + 1 + \frac{K}{\mu_2}\right) + G_1G_2\left(1 + \frac{K}{\mu_2}\right)\right) + C_{f_2}\mu_1\left(1 + \frac{K}{\mu_2}\right)\left(G_aG_1 + G_1G_2\right)}{C_{f_1}\left\{\left(\mu_1\left(K + 1 + \frac{K}{\mu_2}\right)G_aG_2 + \left(\frac{K}{K-1} + \frac{K^2/(K-1)}{\mu_2}\right)G_aG_2 + G_1G_2\left(\mu_1\left(1 + \frac{K}{\mu_2}\right) - K + \frac{K^2/(K-1)}{\mu_2}\right)\right\} + C_{f_2}\left(G_aG_1 + G_1G_2\right)\left\{\left(1 + \frac{K}{\mu_2}\right)\mu_1 + \frac{K}{K-1} + \frac{K^2/(K-1)}{\mu_2}\right\}} \tag{2.49a}$$

$$g^0_{\mu_2} = \frac{1}{2}\,\frac{\mu_2\left(K + 1 + \frac{K/(K-1)}{\mu_1}\right)}{\mu_2\left(K + 1 + \frac{K}{K-1} \times \frac{K}{\mu_1}\right) + K + \frac{K^2/(K-1)}{\mu_1}} + \frac{1}{2}\,\frac{\mu_2\left(1 + \frac{K/(K-1)}{\mu_1}\right)}{\mu_2\left(1 + \frac{K/(K-1)}{\mu_1}\right) + K + \frac{K^2/(K-1)}{\mu_1}} - \frac{C_{f_1}\mu_2G_2\left(G_a\left(K + 1 + \frac{K/(K-1)}{\mu_1}\right) + G_1\left(1 - \frac{K}{\mu_1}\right)\right) + C_{f_2}\mu_2G_1\left(G_a + G_2\right)\left(1 + \frac{K/(K-1)}{\mu_1}\right)}{C_{f_1}G_2\left\{G_a\left(\mu_2\left(K + 1 + \frac{K/(K-1)}{\mu_1}\right) + K + \frac{K^2/(K-1)}{\mu_1}\right) + G_1\left(\mu_2\left(1 - \frac{K}{\mu_1}\right) + K + \frac{K^2/(K-1)}{\mu_1}\right)\right\} + C_{f_2}G_1\left(G_a + G_2\right)\left(\mu_2\left(1 + \frac{K/(K-1)}{\mu_1}\right) + K + \frac{K^2/(K-1)}{\mu_1}\right)} \tag{2.49b}$$

$$S^{\omega_o}_{\mu_1} = \frac{1}{2}\,\frac{\mu_1\left(1+\frac{K}{\mu_2}\right)}{\mu_1\left(1+\frac{K}{\mu_2}\right)+\frac{K}{K-1}+\frac{K^2/(K-1)}{\mu_2}} - \frac{1}{2}\,\frac{\mu_1\left(K+1+\frac{K}{\mu_2}\right)}{\mu_1\left(K+1+\frac{K}{\mu_2}\right)+\frac{K}{K-1}+\frac{K^2/(K-1)}{\mu_2}} \tag{2.49c}$$

$$S^{\omega_o}_{\mu_2} = \frac{1}{2}\,\frac{\mu_2\left(1+\frac{K/(K-1)}{\mu_1}\right)}{\mu_2\left(1+\frac{K/(K-1)}{\mu_1}\right)+K+\frac{K^2/(K-1)}{\mu_1}} - \frac{1}{2}\,\frac{\mu_2\left(K+1+\frac{K/(K-1)}{\mu_1}\right)}{\mu_2\left(K+1+\frac{K/(K-1)}{\mu_1}\right)+K+\frac{K^2/(K-1)}{\mu_1}} \tag{2.49d}$$

All the terms with open loop OA gain in the denominator shall be considered negligible for the remaining Q- and ω_o-sensitivities and they are as follows:

$$S^Q_K = \frac{1}{2}\,\frac{K}{K+1} - \frac{C_{f_1} K G_a G_2}{C_{f_1}\left((K+1)G_aG_2+G_1G_2\right)+C_{f_2}\left(G_aG_1+G_1G_2\right)} \tag{2.50a}$$

$$S^Q_{C_{f_1}} = \frac{1}{2} - \frac{C_{f_1}\left((K+1)G_aG_2+G_1G_2\right)}{C_{f_1}\left((K+1)G_aG_2+G_1G_2\right)+C_{f_2}\left(G_aG_1+G_1G_2\right)} = -S^Q_{C_{f_2}} \tag{2.50b}$$

$$S^Q_{G_a} = \frac{1}{2}\,\frac{G_a}{G_a+G_1+G_2}+\frac{1}{2} - \frac{C_{f_1}G_aG_2(K+1)+C_{f_2}G_aG_1}{C_{f_1}\left((K+1)G_aG_2+G_1G_2\right)+C_{f_2}\left(G_aG_1+G_1G_2\right)} \tag{2.50c}$$

$$S^Q_{G_1} = \frac{1}{2}+\frac{1}{2}\,\frac{G_1}{G_a+G_1+G_2} - \frac{C_{f_1}G_1G_2+C_{f_2}G_1(G_a+G_2)}{C_{f_1}\left((K+1)G_aG_2+G_1G_2\right)+C_{f_2}G_1(G_a+G_2)} \tag{2.50d}$$

$$S^{Q}_{G_2} = \frac{1}{2} + \frac{1}{2}\,\frac{G_2}{G_a+G_1+G_2} - \frac{C_{f_2}G_2\left(G_a(K+1)+G_1\right)+C_{f_2}G_1G_2}{C_{f_1}G_2(K+1)G_a+G_2)+C_{f_2}G_1(G_a+G_2)}$$

(2.50e)

$$S^{\omega_o}_{K} = -\frac{1}{2}\,\frac{K}{K+1} \qquad (2.50f)$$

$$S^{\omega_o}_{C_{f_1}} = S^{\omega_o}_{C_{f_2}} = -\frac{1}{2} \qquad (2.50g)$$

$$S^{\omega_o}_{G_a} = \frac{1}{2} - \frac{1}{2}\;\frac{G_a}{G_a+G_1+G_2} \qquad (2.50h)$$

$$S^{\omega_o}_{G_1} = \frac{1}{2} - \frac{1}{2}\;\frac{G_a}{G_a+G_1+G_2} \qquad (2.50i)$$

$$S^{\omega_o}_{G_2} = \frac{1}{2} - \frac{1}{2}\;\frac{G_1}{G_a+G_1+G_2} \qquad (2.50j)$$

Putting equations (2.47) in terms of G_{11}, G_{21}, G_{31} and G_{41} instead of in terms of K, but this time neglecting all the terms with any open-loop OA gain in the denominator, gives:

$$N(s) = -G_a\left(sC_{f_2}G_1G_{31}(G_{11}+G_{21}) - sC_{f_1}G_2G_{21}(G_{31}+G_{41}) + G_1G_2G_{31}(G_{11}+G_{21}) - G_1G_2(G_{31}+G_{41})G_{21}\right) \qquad (2.51a)$$

$$D(s) = s^2C_{f_1}C_{f_2}(G_a+G_1+G_2)\left(G_{21}G_{41}+G_{31}(G_{11}+G_{21})\right) + sC_{f_2}G_1(G_a+G_2)G_{21}G_{41} + sC_{f_1}G_2\left(G_{11}G_{31}(G_a+G_1)+G_aG_{21}(G_{31}+G_{41})\right) + G_aG_1G_2G_{21}G_{41} \qquad (2.51b)$$

From Eq^n (2.51) it can be shown that:

$$Q = \frac{\sqrt{G_aG_1G_2G_{21}G_{41}}\;\sqrt{C_{f_1}C_{f_2}(G_a+G_1+G_2)(G_{21}G_{41}+G_{31}G_{11}+G_{31}G_{21})}}{C_{f_1}G_2\left(G_{11}G_{31}(G_a+G_1)+G_aG_{21}(G_{31}+G_{41})\right)+C_{f_2}G_1(G_a+G_2)G_{21}G_{41}}$$

(2.52a)

$$\omega_0 = \sqrt{\frac{G_a G_1 G_2 G_{21} G_{41}}{C_{f_1} C_{f_2} (G_a + G_1 + G_2)\,(G_{21}G_{41} + G_{31}G_{11} + G_{31}G_{21})}} \qquad (2.52b)$$

From Eq^{ns} (1.3) and (2.52) it can be shown that:

$$S^{Q}_{G_{11}} = \frac{1}{2}\,\frac{G_{11}G_{31}}{G_{21}G_{41}+G_{11}G_{31}+G_{31}G_{21}} - \frac{G_{11}G_{31}(G_a+G_1)C_{f_1}G_2}{C_{f_1}G_2\left(G_{11}G_{31}(G_a+G_1)+G_aG_{21}(G_{31}+G_{41})\right)+C_{f_2}G_1(G_a+G_2)G_{21}G_{41}}$$

$$= \frac{1}{2}\,\frac{1}{K+1} - \frac{C_{f_1}G_2(G_a+G_1)}{C_{f_1}G_2\left(G_a(K+1)+G_1\right)+C_{f_2}G_1(G_a+G_2)} \qquad (2.53a)$$

$$S^{Q}_{G_{21}} = \frac{1}{2} + \frac{1}{2}\,\frac{G_{21}G_{41}+G_{21}G_{31}}{G_{21}G_{41}+G_{11}G_{31}+G_{31}G_{21}} - \frac{C_{f_1}G_2G_aG_{21}(G_{31}+G_{41})+C_{f_2}G_1(G_a+G_2)\,G_{21}G_{41}}{C_{f_1}G_2\left(G_{11}G_{31}(G_a+G_1)+G_aG_{21}(G_{31}+G_{41})\right)+C_{f_2}G_1G_{21}G_{41}(G_a+G_2)}$$

$$= \frac{1}{2} + \frac{1}{2}\,\frac{K}{K+1} - \frac{KC_{f_1}G_aG_2 + C_{f_2}G_1(G_a+G_2)}{C_{f_1}G_2\left(G_a(K+1)+G_1\right)+C_{f_2}G_1(G_a+G_2)} \qquad (2.53b)$$

$$S^{Q}_{G_{31}} = \frac{1}{2}\,\frac{G_{31}(G_{11}+G_{21})}{G_{21}G_{41}+G_{11}G_{31}+G_{31}G_{21}} - \frac{C_{f_1}G_2\left(G_{11}G_{31}(G_a+G_1)+G_aG_{21}G_{31}\right)}{C_{f_1}G_2\left(G_{11}G_{31}(G_a+G_1)+G_aG_{21}(G_{31}+G_{41})\right)+C_{f_2}G_1G_{21}G_{41}(G_a+G_2)}$$

$$= \frac{1}{2}\,\frac{K}{K+1} - \frac{C_{f_1}G_2(KG_a+G_1)}{C_{f_1}G_2\left((K+1)G_a+G_1\right)+C_{f_2}G_1(G_a+G_2)} \qquad (2.53c)$$

$$S^{Q}_{G_{41}} = \frac{1}{2} + \frac{1}{2}\,\frac{G_{21}G_{41}}{G_{21}G_{41}+G_{11}G_{31}+G_{31}G_{21}} - \frac{C_{f_1}G_2G_aG_{21}G_{41}+C_{f_2}G_1G_{21}G_{41}(G_a+G_2)}{C_{f_1}G_2(G_{11}G_{31}(G_a+G_1)+G_aG_{21}(G_{31}+G_{41}))+C_{f_2}G_1G_{21}G_{41}(G_a+G_2)}$$

$$= \frac{1}{2} + \frac{1}{2}\,\frac{1}{K+1} - \frac{C_{f_1}G_aG_2+C_{f_2}G_1(G_a+G_2)}{C_{f_1}G_2(G_a(K+1)+G_1)+C_{f_2}G_1(G_a+G_2)} \qquad (2.53b)$$

It may be observed that the magnitudes of Q- and ω_o-sensitivities with respect to these passive elements are less than unity. Q- and ω_o- sensitivities with respect to open-loop OA gains are very low thereby showing that for low Q and low ω_o that is, for $\mu \gg Q$, even for a large variation in μ, the variation in Q and ω_o shall be very small.

2.6.4.3 EFFECT OF THE POLE OF THE OA ON Q AND ω_o:

Various procedures have been reported in the literature [30-32] to consider this effect. The method given in [31-32] is mathematically more rigorous than others. However, in the frequency range of interest in this thesis, namely, for $\omega \ll GB$, the approximate method as outlined in [30] is more appropriate and hence this is used for subsequent derivations. From eq. (2.47), assuming $\mu_1 = \mu_2 = \mu$, neglecting terms with μ^2 in the denominator, and replacing μ by

$$\mu = \frac{\mu_o \omega_c / \omega_o}{s + \omega_c / \omega_o}$$

where: ω_c is the pole of the OA.

μ_o is the d.c. gain of the OA.

and ω_o is the designed pole frequency.

We get:

$$D(s) = s^3 C_{f_1} C_{f_2} (K+1)(G_a + G_1 + G_2) \frac{\omega_o}{\mu_o \omega_c} +$$

$$s^2 \left(C_{f_1} C_{f_2} (K+1)(G_a + G_1 + G_2)(1 + \frac{1}{\mu_o}) + C_{f_1} G_a G_2 (K+1) \frac{\omega_o}{\mu_o \omega_c} + C_{f_2} G_1 (G_a + G_2)(K+1) \frac{\omega_o}{\mu_o \omega_c}\right) +$$

$$s \left(C_{f_1} G_a G_2 (K+1)(1 + \frac{1}{\mu_o}) + C_{f_1} G_1 G_2 + C_{f_2} G_1 (G_a + G_2)(1 + \frac{K+1}{\mu_o}) + G_a G_1 G_2 \frac{(K+1)\omega_o}{\mu_o \omega_c}\right) + G_a G_1 G_2 (1 + \frac{K+1}{\mu_o}) \quad (2.54)$$

Comparing this equation with:

$$D(s) = (\frac{s}{\epsilon_2} + 1)(\frac{s^2}{\hat{\omega}_o^2} + \frac{s}{\hat{Q}\hat{\omega}_o} + 1) = \frac{s^3}{\epsilon_2 \hat{\omega}_o^2} + s^2 (\frac{1}{\hat{\omega}_o^2} + \frac{1}{\epsilon_2 \hat{Q}\hat{\omega}_o}) + s(\frac{1}{\epsilon_2} + \frac{1}{\hat{Q}\hat{\omega}_o}) + 1,$$

$$(2.55)$$

Where $\hat{\omega}_o$ is the realized value of the pole frequency

$\hat{Q}$ is the realized value of pole Q

and ϵ_2 is the distant negative real axis pole

gives:

$$\text{i)} \quad \frac{1}{\epsilon_2} \cdot \frac{\omega_o^2}{\hat{\omega}_o^2} = \frac{C_{f_1} C_{f_2} (K+1)(G_a + G_1 + G_2)\omega_o / (\mu_o \omega_c)}{1 + \frac{K+1}{\mu_o}}$$

Since $\hat{\omega}_o$ is approximately equal to ω_o

$$\frac{1}{\epsilon_2} = \frac{C_{f_1}C_{f_2}(K+1)(G_a+G_1+G_2)\omega_o}{\mu_o\omega_c(1+\frac{K+1}{\mu_o})} \qquad (2.56a)$$

ii)

$$\frac{1}{\omega_o^2}+\frac{1}{\epsilon_2 Q\omega_o} = \frac{C_{f_1}C_{f_2}(K+1)(G_a+G_1+G_2)(1+\frac{1}{\mu_o})+(C_{f_1}G_aG_2+C_{f_2}G_1(G_a+G_2))\times\frac{(K+1)\omega_o}{\mu_o\omega_c}}{G_aG_1G_2(1+\frac{K+1}{\mu})}$$

For $\epsilon_2 >> \omega_o$ and $Q > 1$, the second term on the LHS can be neglected compared to the first and this gives:

$$\frac{\hat{\omega}_o}{\omega_o} = \frac{\sqrt{1+\frac{K+1}{\mu_o}}}{\sqrt{(C_{f_1}C_{f_2}(K+1)(G_a+G_1+G_2)(1+\frac{1}{\mu_o})+\frac{K+1}{\mu_o\omega_c}\omega_o(C_{f_1}G_aG_2+C_{f_2}G_1(G_a+G_2)))}}$$

(2.56b)

iii)

$$\frac{1}{\epsilon_2}+\frac{1}{Q\omega_o} = \frac{C_{f_1}G_aG_2(K+1)(1+\frac{1}{\mu_o})+C_{f_1}G_1G_2+C_{f_2}G_1(G_a+G_2)(1+\frac{K+1}{\mu_o})+G_aG_1G_2\frac{(K+1)\omega_o}{\mu_o\omega_c}}{G_aG_1G_2(1+\frac{K+1}{\mu_o})}$$

or

$$\frac{\hat{Q}\hat{\omega}_o}{Q\omega_o} = \frac{G_aG_1G_2(1+\frac{K+1}{\mu_o})}{(C_{f_1}G_aG_2(K+1)(1+\frac{1}{\mu_o})+C_{f_1}G_1G_2+C_{f_2}G_1(G_a+G_2)(1+\frac{K+1}{\mu_o})+G_aG_1G_2\frac{(K+1)\omega_o}{\mu_o\omega_c}-G_aG_1G_2C_{f_1}C_{f_2}(K+1)(G_a+G_1+G_2)\frac{\omega_o}{\mu_o\omega_c})}$$

$$\approx \frac{(C_{f_1}G_aG_2(K+1)+C_{f_1}G_1G_2+C_{f_2}G_1(G_a+G_2))}{G_aG_1G_2} \qquad (2.56c)$$

Eq^{ns} (2.96), when solved, give a good estimate of the effect of the pole of the OA on Q and ω_o. Variations for each realisation using μA741 type OA have been given later.

Now all the sensitivities and other relevant details shall be given for both the realisations presented earlier.

a) <u>Equal Capacitor Realisation</u>:

The design equations are as given in Eq^n (2.42). The sensitivities are:

$$S^Q_K = 0 \qquad S^{\omega_o}_K = -\frac{1}{2} + \frac{1}{12(6Q^2-1)}$$

$$S^Q_{C_{f_1}} = -S^Q_{C_{f_2}} = -1/6 \qquad S^{\omega_o}_{C_{f_1}} = S^{\omega_o}_{C_{f_2}} = -1/2$$

$$S^Q_{G_a} = \frac{-1}{6} + \frac{1}{24Q^2} \qquad S^{\omega_o}_{G_a} = \frac{1}{2} - \frac{1}{24Q^2}$$

$$S^Q_{G_1} = \frac{1}{2} - \frac{1}{12Q^2} \qquad S^{\omega_o}_{G_1} = \frac{1}{12Q^2}$$

$$S^Q_{G_2} = -\frac{1}{3} + \frac{1}{24Q^2} \qquad S^{\omega_o}_{G_2} = \frac{1}{2} - \frac{1}{24Q^2}$$

$$\left.\begin{array}{l} |S^Q_{\mu_1}| < 0.2 \times 10^{-3} \\ |S^Q_{\mu 2}| < 0.1 \times 10^{-5} \end{array}\right| \text{for } Q = 2 \qquad \left.\begin{array}{l} |S^{\omega_o}_{\mu}| < 0.5 \times 10^{-5} \\ |S^{\omega_o}_{\mu_2}| < 0.4 \times 10^{-3} \end{array}\right| \text{for } Q = 2$$

Magnitude and Magnitude squared sums of passive sensitivities are:

$$\Sigma|S_x^Q| = \frac{4}{3} - \frac{1}{6Q^2} \qquad (2.57a)$$

$$\Sigma|S_x^Q|^2 = \frac{1}{36} + \frac{1}{36} + \left(\frac{1}{6} - \frac{1}{24Q^2}\right)^2 + \left(\frac{1}{2} - \frac{1}{12Q^2}\right)^2 + \left(\frac{1}{3} - \frac{1}{24Q^2}\right)^2$$

$$\approx \frac{1}{12} + \frac{1}{4} + \frac{1}{9} = \frac{4}{9} \qquad (2.57b)$$

$$\Sigma|S_x^{\omega_o}| = 2 \qquad (2.57c)$$

$$\Sigma|S_x^{\omega_o}|^2 = \frac{1}{4} + \frac{1}{4} + 2\left(\frac{1}{2} - \frac{1}{24Q^2}\right)^2 + \left(\frac{1}{12Q^2}\right)^2 \approx 1 \qquad (2.57d)$$

Where $12Q^2 >> 1$ has been assumed

Magnitude sum and Magnitude squared sums of the Q-sensitivity with respect to passive elements have been compared with those of the existing networks [23] and found to be almost half of [23]. Thus this realization is superior in this respect.

For this realization, from eq. (2.56), we get:

$$\frac{\hat{\omega}_o}{\omega_o} = \frac{\sqrt{1 + \frac{36Q^2-6}{\mu_o}}}{\sqrt{1 + \frac{1}{\mu_o} + \frac{36Q^2-6}{\mu_o \omega_c}\,\omega_o\left(\frac{1}{6Q(12Q^2-2)} + \frac{1}{36}\right)}}$$

$$\text{and } \frac{\hat{Q}}{Q} = \frac{\sqrt{1 + \frac{36Q^2-6}{\mu_o}}\ \sqrt{1 + \frac{1}{\mu_o} + \frac{36Q^2-6}{\mu_o \omega_c}\,\omega_o\left(\frac{1}{20(36Q^2-6)} + \frac{1}{30}\right)}}{\frac{1}{2}\left(1+\frac{1}{\mu_o}\right) + \frac{1}{6} + \frac{1}{3}\left(1 + \frac{36Q^2-6}{\mu_o}\right) + \frac{(36Q^2-7)}{\mu_o \omega_c}\,\omega_o},$$

Expanding the right hand side gives:

$$\frac{\bar{\omega}_o}{\omega_o} = 1 + \frac{36Q^2-7}{2u_o} - \frac{\omega_o}{2u_o\omega_c}\left(\frac{1}{2Q} + 12Q - \frac{2}{Q}\right) \qquad (2.58a)$$

$$\text{and } \frac{\bar{Q}}{Q} = 1 + \frac{6Q^2-1}{u_o} - \frac{\omega_o}{u_o\omega_c}\left(36Q^2-7-\frac{1}{4Q} - 6Q + \frac{1}{Q}\right) \qquad (2.58b)$$

Variations in ω_o and Q due to the effect of the pole of the OA computed from eq (2.58) for various frequencies are given in Figure (2.5). These approximate variations are within 4% of the exact ones at higher frequencies, which is also shown in Figure 2.5.

Tuning of this circuit is simple. ω_o can be independently controlled by adjusting the amplifier gain K and Q can be controlled by adjusting G_1.

This circuit can be used to obtain realizations with S_K^Q being prescribed between $-\frac{1}{2}$ and $\frac{1}{2}$. The variations in the values of K and the element spread have been given in Figure 2.6. This network was tested for Q = 2.0, with

1% resistors using R_1 = 78.3Ω, R_2 = 3.65KΩ, R_a = 3.65KΩ, K=143.
2% Capacitors C_{f_1} = 39 KPF, C_{f_2} = 39 KPF.

at a frequency of 90 Hz, the results obtained agreed with the theoretical ones within about 3%. Power supply was changed from ±15V to ±10V and no variation in Q or ω_o was observed as shown in Figure 2.7.

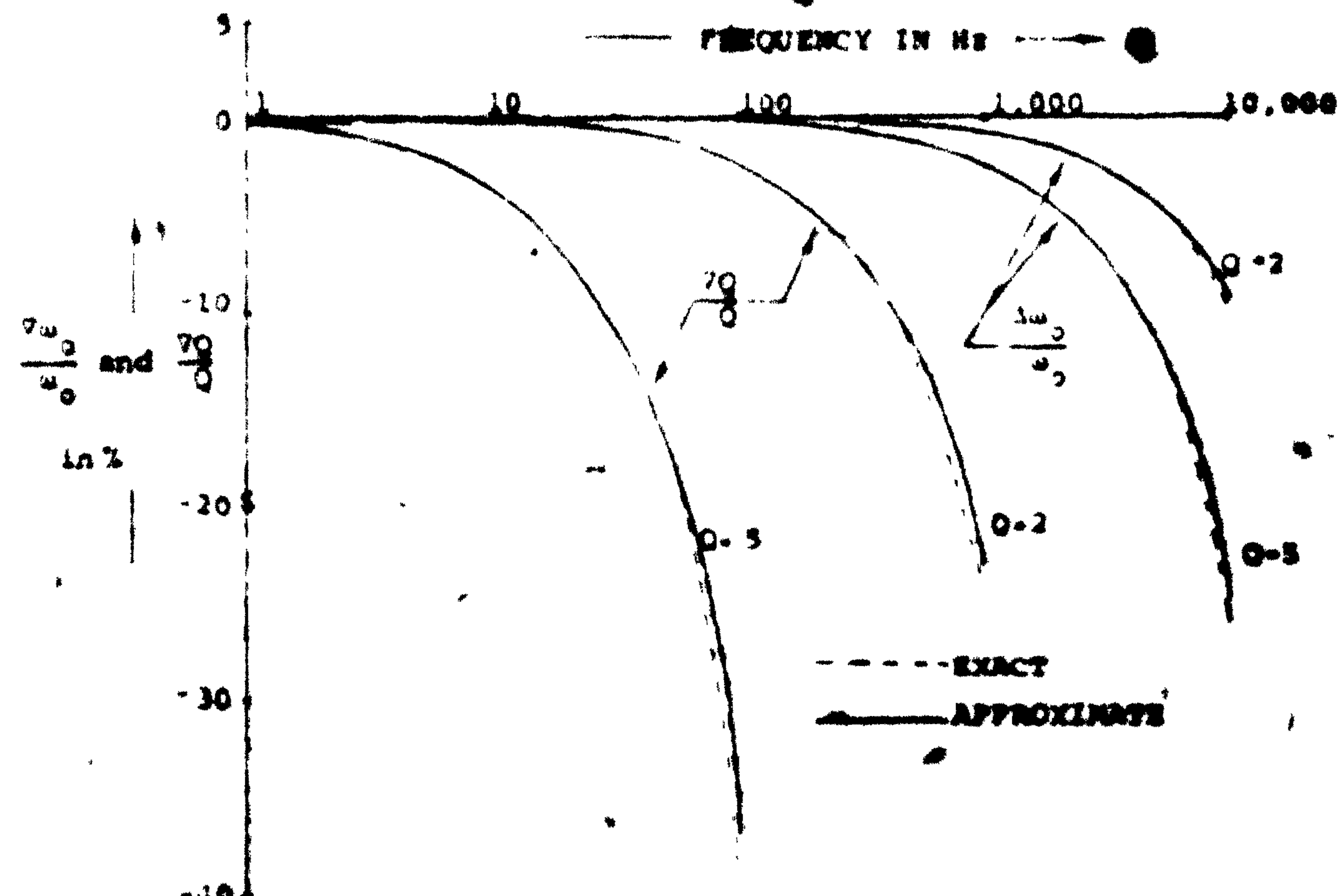

FIGURE 2.5

$\frac{\Delta\omega_o}{\omega_o}$ AND $\frac{\Delta Q}{Q}$ VERSUS FREQUENCY PLOTS FOR THE EQUAL CAPICITOR BP FILTER NETWORK OF DECOMPOSITION IIa.

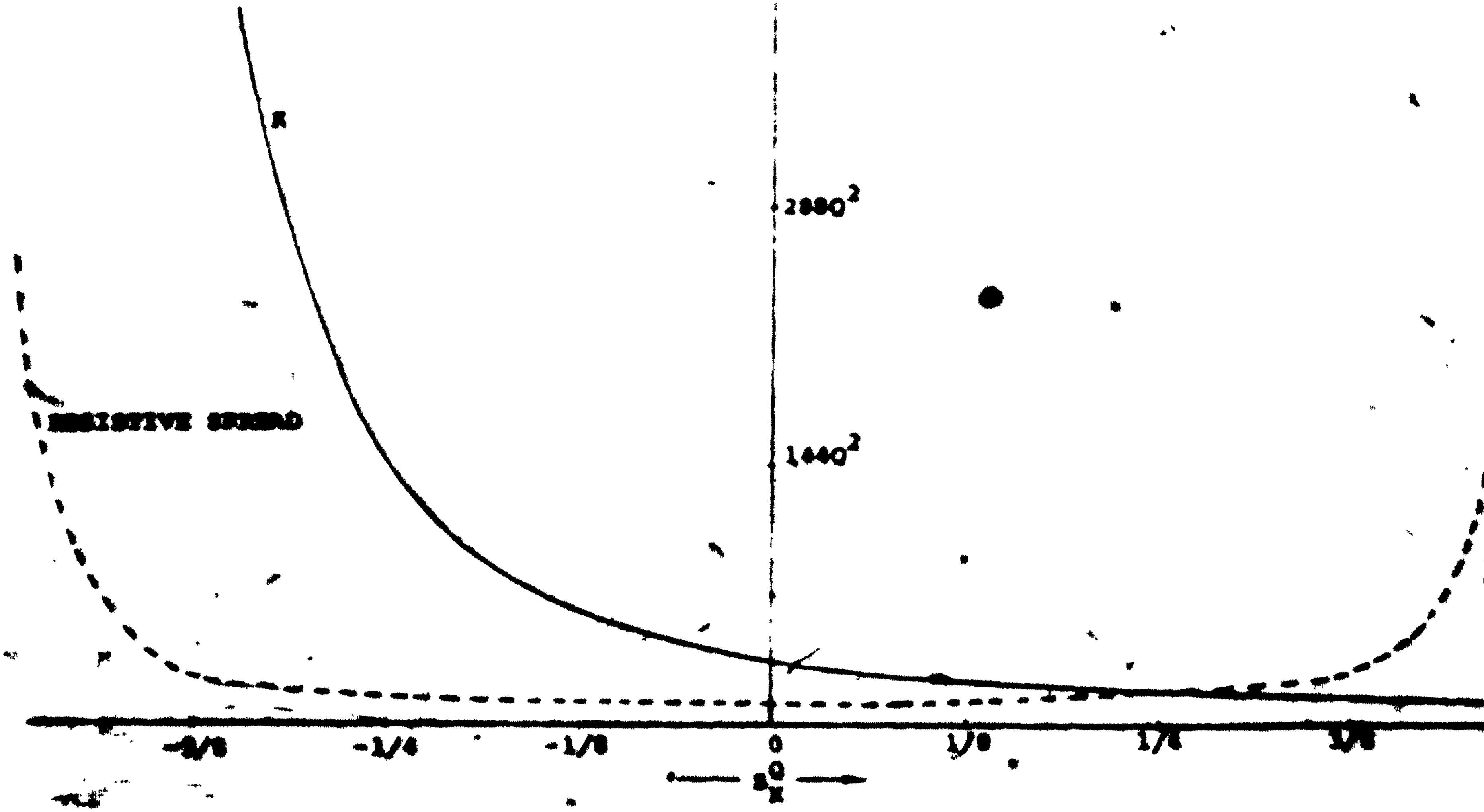

FIGURE 2.6

'K' AND RESISTIVE SPREAD VERSUS Q-SENSITIVITY PLOTS FOR BP FILTER NETWORK OF DECOMPOSITION IIa, CAPACITIVE SPREAD=UNITY

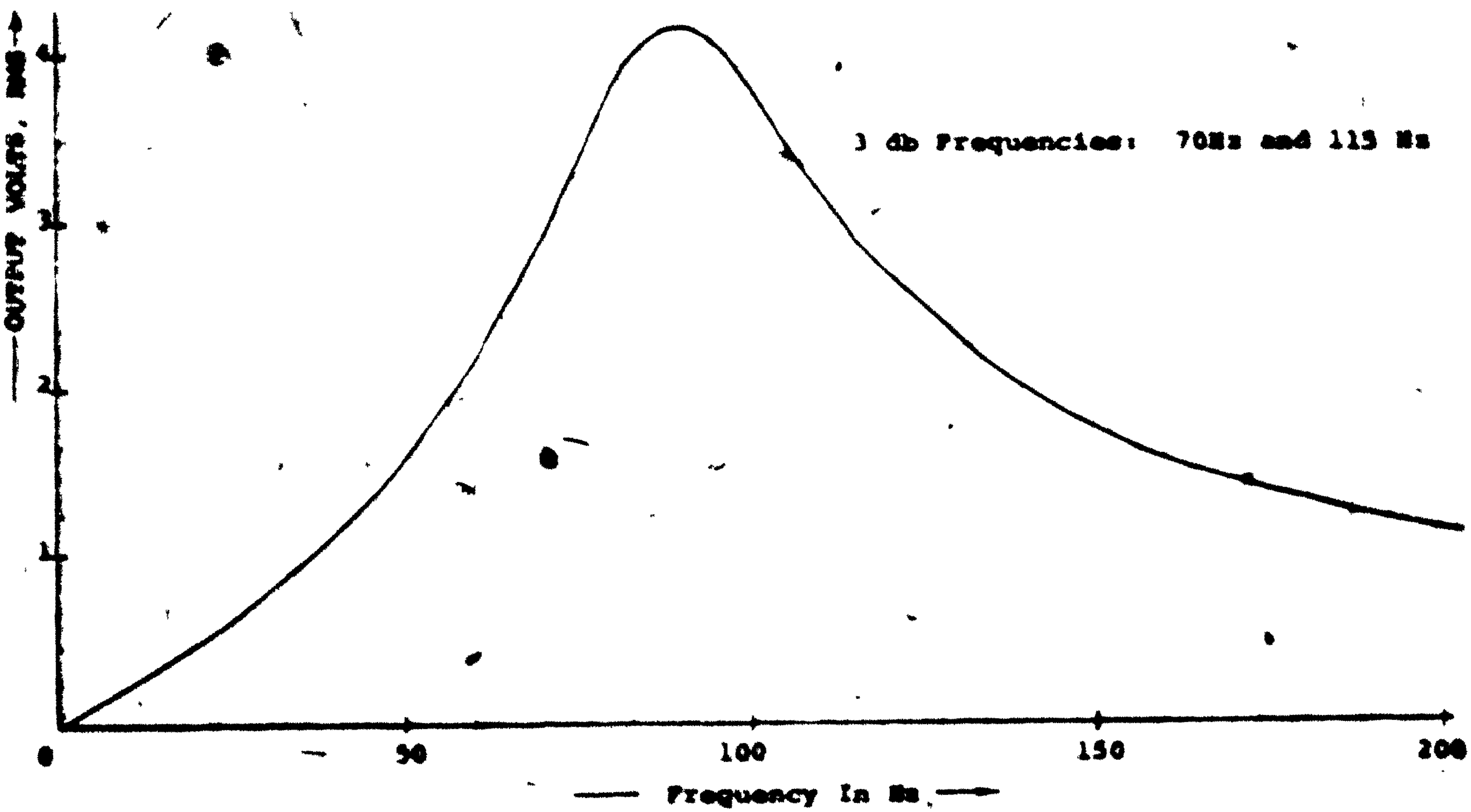

FIGURE 2.7

FREQUENCY RESPONSE OF BP FILTER NETWORK FOR DECOMPOSITION IIa, Q=2.0, ω_0= 90Hz, CAPACITIVE SPREAD: UNITY

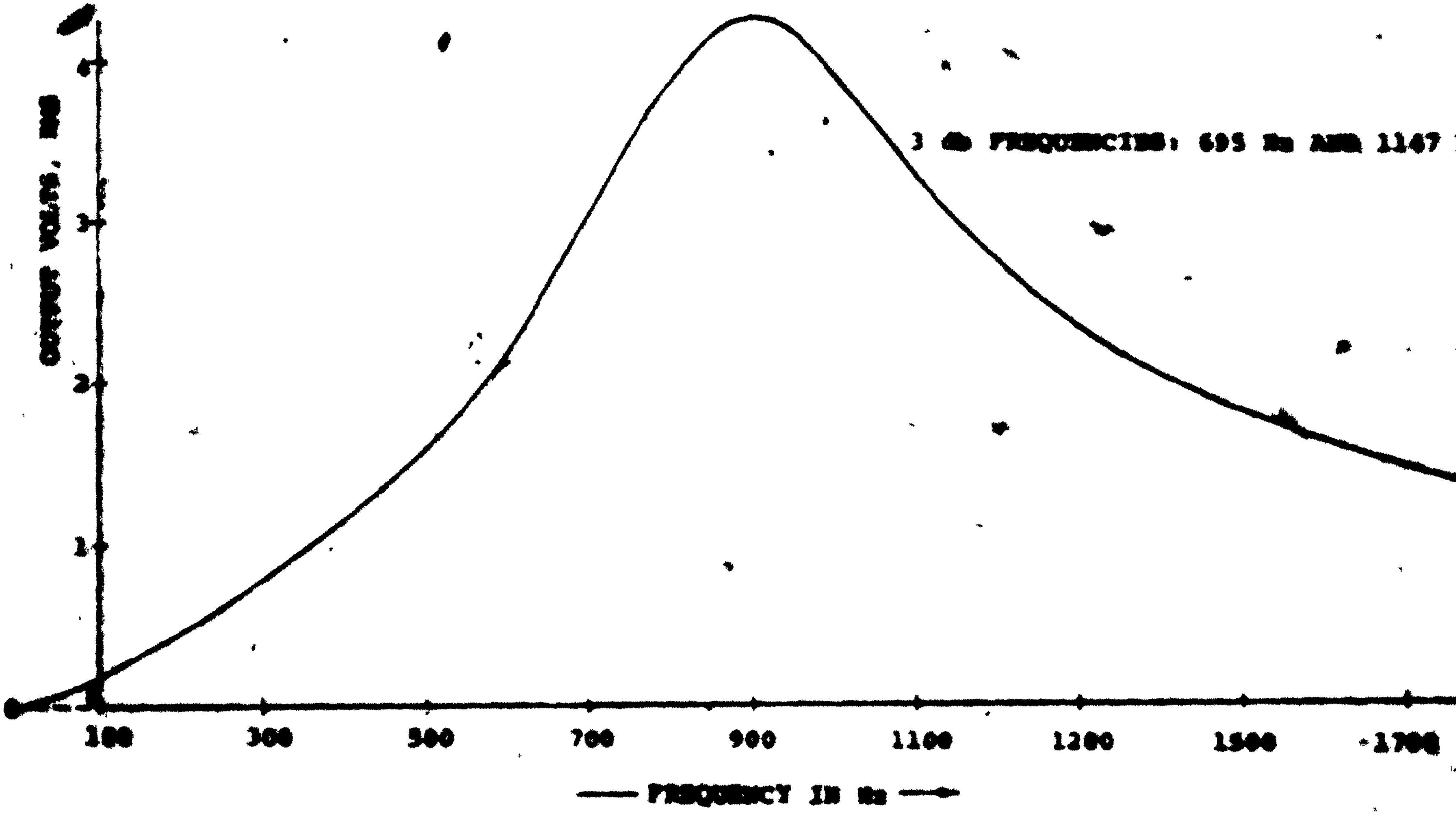

FIGURE 2.8

FREQUENCY RESPONSE OF BP FILTER NETWORK FOR DECOMPOSITION IIa,

Q=2.0, ω_0=903Hz, CAPACITIVE SPREAD: UNITY

The same circuit was also tested at $\omega_o = 900$ Hz. Only the capacitors were changed to 3.9 KPF. In this case R_1 had to be reduced to about 50% its original value to restore Q to the design value. This variation is mainly due to the effect of amplifier poles. The response of this circuit is as shown in Figure (2.8)

b) <u>Unequal-Capacitors Realization:</u>

$$(C_{f_1} \neq C_{f_2})$$

The design equations are as given in Eq^n(2.43).

The sensitivities are:

$$S^Q_K = 0 \tag{2.59a}$$

$$S^Q_{C_{f_1}} = -S^Q_{C_{f_2}} = -\frac{(p_2-1)\ (4Q^2-1)}{4p_2(4Q^2-1)+2} \tag{2.59b}$$

$$S^Q_{G_a} = -\frac{4Q^2-1}{4p_2(4Q^2-1)+4} \tag{2.59c}$$

$$S^Q_{G_1} = \frac{p_2(4Q^2-1)}{2p_2(4Q^2-1)+2} \tag{2.59d}$$

$$S^Q_{G_2} = -\frac{(2p_2-1)(4Q^2-1)}{4p_2(4Q^2-1)+4} \tag{2.59e}$$

$$|S^{\omega_0}_{\mu_1}| < 16 \times 10^{-5} \qquad \text{for } p_2 = 2 \qquad (2.59f)$$

$$|S^{\omega_0}_{\mu_2}| < 32 \times 10^{-5} . \qquad Q = 2 \qquad (2.59g)$$

$$S^{\omega_0}_{K} = -\frac{1}{2} + \frac{p_2 - 1}{4p_2(p_2(4Q^2-1)+1)} \qquad (2.59h)$$

$$S^{\omega_0}_{C_{f_1}} = S^{\omega_0}_{C_{f_2}} = -\frac{1}{2} . \qquad (2.59i)$$

$$S^{\omega_0}_{G_a} = \frac{1}{2} - \frac{1}{4p_2(4Q^2-1)+4} \qquad (2.59j)$$

$$S^{\omega_0}_{G_1} = \frac{1}{2p_2(4Q^2-1)+2} \qquad (2.59k)$$

$$S^{\omega_0}_{G_2} = \frac{1}{2} - \frac{1}{4p_2(4Q^2-1)+4} \qquad (2.59l)$$

$$|S^{\omega_0}_{\mu_1}| < 0.3 \times 10^{-5} \qquad \text{for } p_2 = 2 \qquad (2.59m)$$

$$|S^{\omega_0}_{\mu_2}| < 0.3 \times 10^{-3} \qquad Q = 2 \qquad (2.59n)$$

For this realization, from eq^{ns} (2.56), we get for $p_2 = 2$,

$$\frac{\tilde{\omega}_0}{\omega_0} = \frac{\sqrt{1 + \frac{32Q^2-4}{[illegible]}}}{\sqrt{1 + \frac{1}{[illegible]} + \frac{[illegible]}{[illegible]}\left(\frac{1}{2Q} + 8Q\right)}}$$

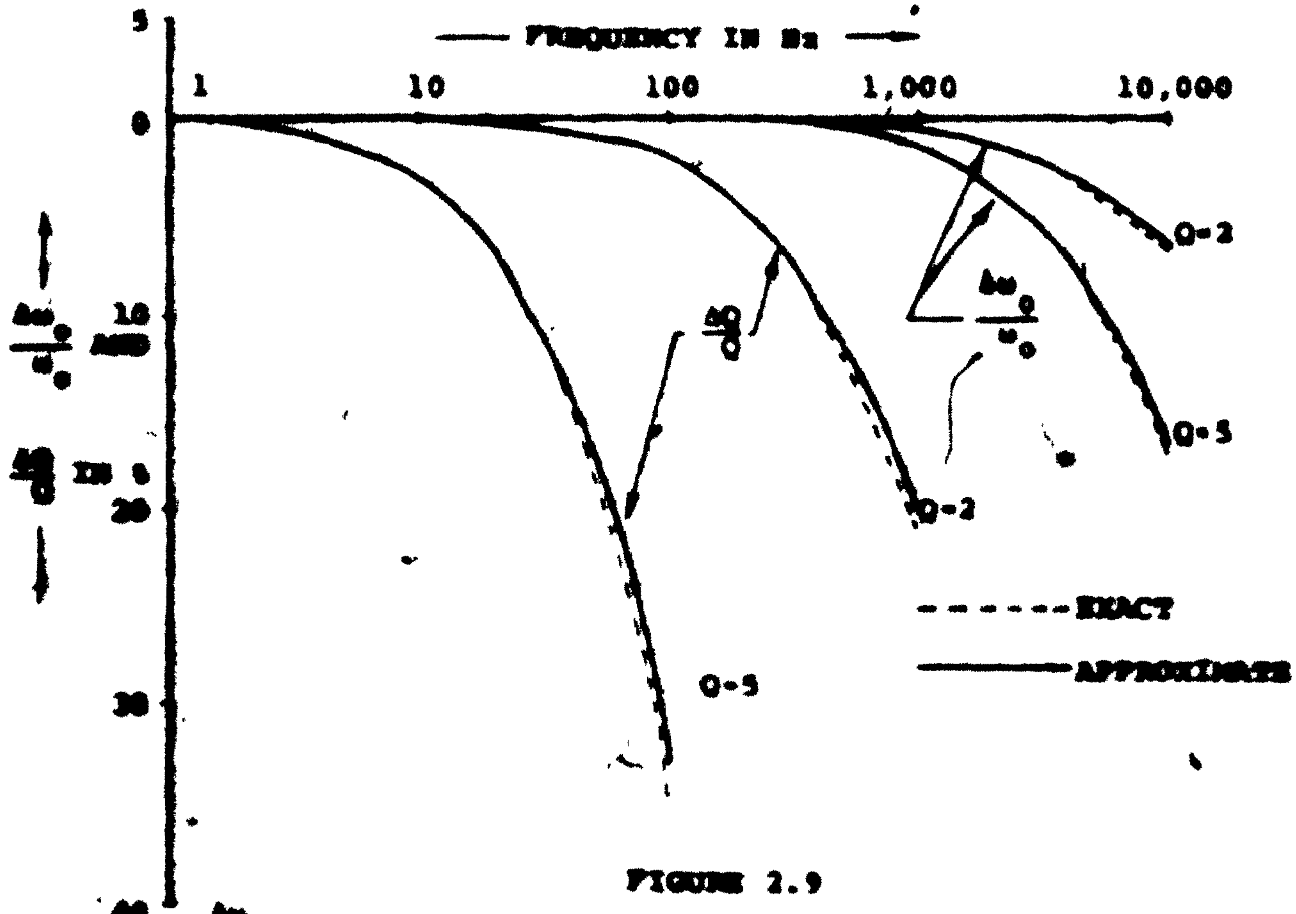

FIGURE 2.9

$\frac{\Delta\omega_0}{\omega_0}$ AND $\frac{\Delta Q}{Q}$ VERSUS FREQUENCY PLOTS FOR UNEQUAL CAPACITOR BP FILTER

NETWORK OF DECOMPOSITION IIa

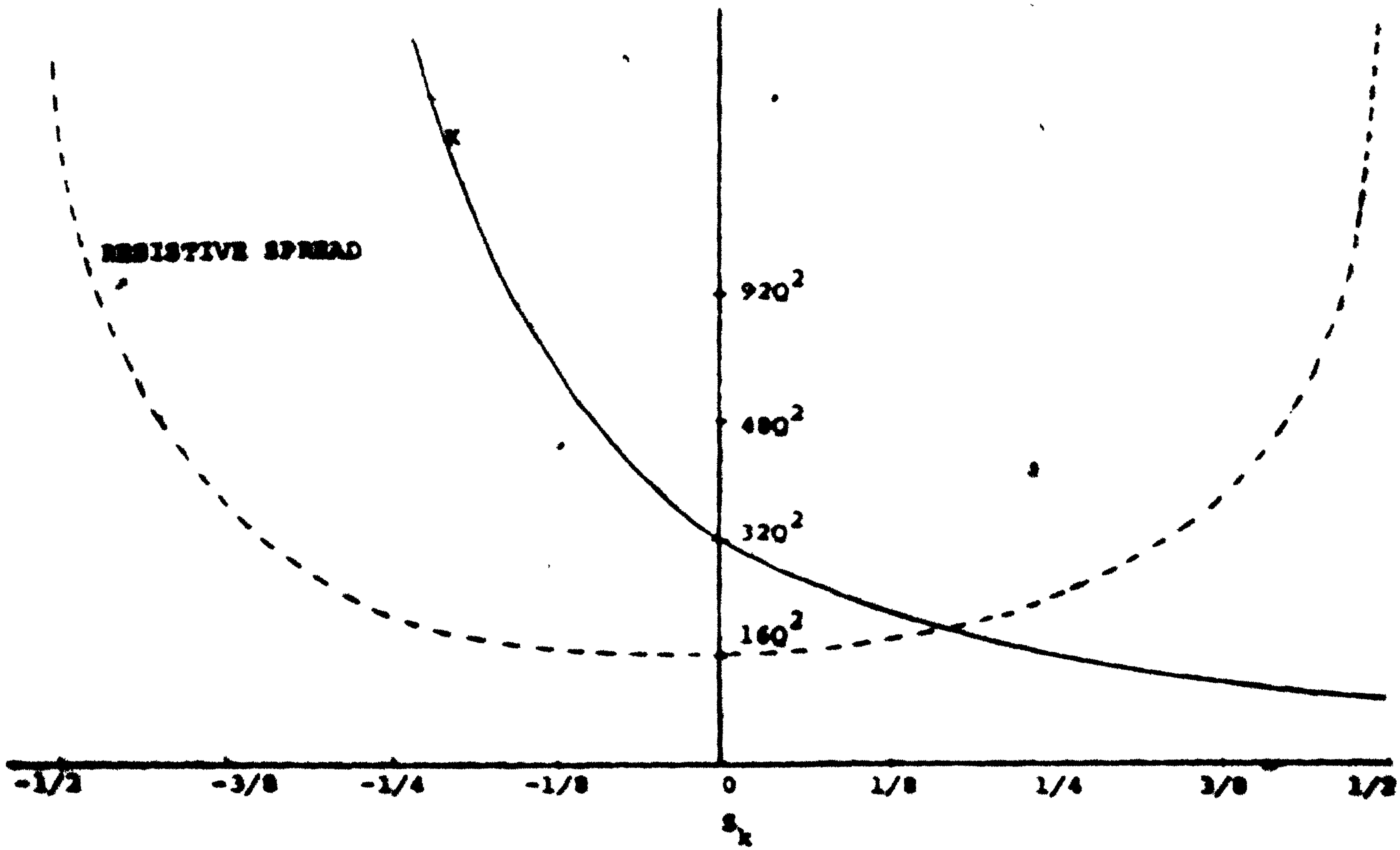

FIGURE 2.10

K AND RESISTIVE SPREAD VERSUS Q-SENSITIVITY PLOTS FOR RC FILTER NETWORK OF DECOMPOSITION IIa, CAPACITIVE SPREAD = 2

$$\text{and} \quad \frac{\hat{Q}}{Q} = \frac{\sqrt{1 + \frac{32Q^2-4}{\mu_o}}\ \sqrt{1 + \frac{1}{\mu_o} + \frac{\omega_o}{\mu_o\omega_c}(\frac{1}{2Q} + 8Q)}}{\frac{1}{2}(1 + \frac{1}{\mu_o}) + \frac{4Q^2-1}{2(8Q^2-1)} + \frac{2Q^2}{8Q^2-1}(1 + \frac{32Q^2-4}{\mu_o} + \frac{12Q^2-5}{\mu_o}\frac{\omega_o}{\omega_c})}$$

Expanding the right hand side, and considering first order terms only, we get:

$$\frac{\hat{\omega}_o}{\omega_o} = 1 + \frac{32Q^2-5}{2\mu_o} + \frac{\omega_o}{2\mu_o\omega_c}(\frac{1}{2Q} + 8Q) \qquad (2.60a)$$

$$\text{and} \quad \frac{\hat{Q}}{Q} = 1 + \frac{8Q^2-2}{\mu_o} - \frac{\omega_o Q}{\mu_o\omega_c}(32Q^2 - 9 - \frac{1}{4Q^2}) \qquad (2.60b)$$

Variations in ω_o and Q due to the effect of the pole of the OA are obtained from eq (2.60). The approximate and exact varations are plotted for various frequencies in Fig. 2.9 .

This network also can be used to obtain realizations for various specified values of S_K^Q. The variations in the values of K and the element spread are given in Fig. 2.10 .

2.6.5 DECOMPOSITION II_b:

K appears in s and s^o terms only:

It has been reported in the literature [18], [19] that this decomposition can be obtained from decomposition II_a using RC-CR transformation. It may be pointed out once again that RC-CR transformation in general may lead to unacceptable capacitive element spread. Further the number of capacitors in the transformed circuit would depend upon the number of resistors present in the original circuit. This in general shall be more than two in number and this may lead to third or higher degree D(s) $(D_3(s))$ where the Q-sensitivity with respect to passive elements tends to be high. Networks with this decomposition with constrained capacitive spread have not been reported in the literature so far. Therefore, this decomposition shall be discussed and a network for its realization shall be given.

D(s) is of the type

$$D(s) = s^2\alpha + s(\beta_1 + K\beta_2) + \gamma_1 + K\gamma_2 \qquad (2.61a)$$

and $$Q = \frac{\sqrt{\alpha(\gamma_1 + K\gamma_2)}}{\beta_1 + K\beta_2} \qquad (2.61b)$$

$$\omega_o = \sqrt{\frac{\gamma_1 + K\gamma_2}{\alpha}} \qquad (2.61c)$$

$$S_K^Q = \frac{1}{2}\,\frac{K\gamma_2}{\gamma_1 + K\gamma_2} - \frac{K\beta_2}{\beta_1 + K\beta_2} \qquad (2.61d)$$

$$S_K^{\omega_o} = \frac{1}{2}\,\frac{K\gamma_2}{\gamma_1 + K\gamma_2} \qquad (2.61e)$$

$$S_K^Q + S_K^{\omega_o} = \frac{K\gamma_2}{\gamma_1 + K\gamma_2} - \frac{K\beta_2}{\beta_1 + K\beta_2} \qquad (2.61f)$$

$$S_K^Q - S_K^{\omega_o} = -\frac{KD_2}{D_1 + KD_2} \qquad (2.61h)$$

with the sensitivity bounds being:

$$-1 < S_K^Q < 1/2 \qquad (2.61i)$$

$$0 < S_K^{\omega_o} < 1/2 \qquad (2.61j)$$

$$-1 < S_K^Q + S_K^{\omega_o} < 1 \qquad (2.61k)$$

$$-1 < S_K^Q - S_K^{\omega_o} < 0 \qquad (2.61l)$$

Q-sensitivity magnitude with respect to passive elements shall be less than unity in two capacitor realizations while it may be high in three or more capacitor realizations. ω_o-sensitivity with respect to passive elements is less than unity. $S_K^{\omega_o}$ in this case is exactly similar to that of decomposition I_b and as such its magnitude shall be almost equal to 1/2 if not prescribed. If it is prescribed, K would get fixed and this would fix the realizable value of Q. The networks obtained from this decomposition too are not suitable for prescribing $S_K^{\omega_o}$.

S_K^Q, if not prescribed, is quite low, usually its magnitude being lower than 1/2. It is possible to prescribe $S_K^Q = 0$ in this decomposition.

Once either S_K^Q or $S_K^{\omega_o}$ is prescribed, bounds on the remaining sensitivity get even more restricted according to the bounds on their sum and difference.

A three capacitor Realization is given as shown in Figure 2.11.

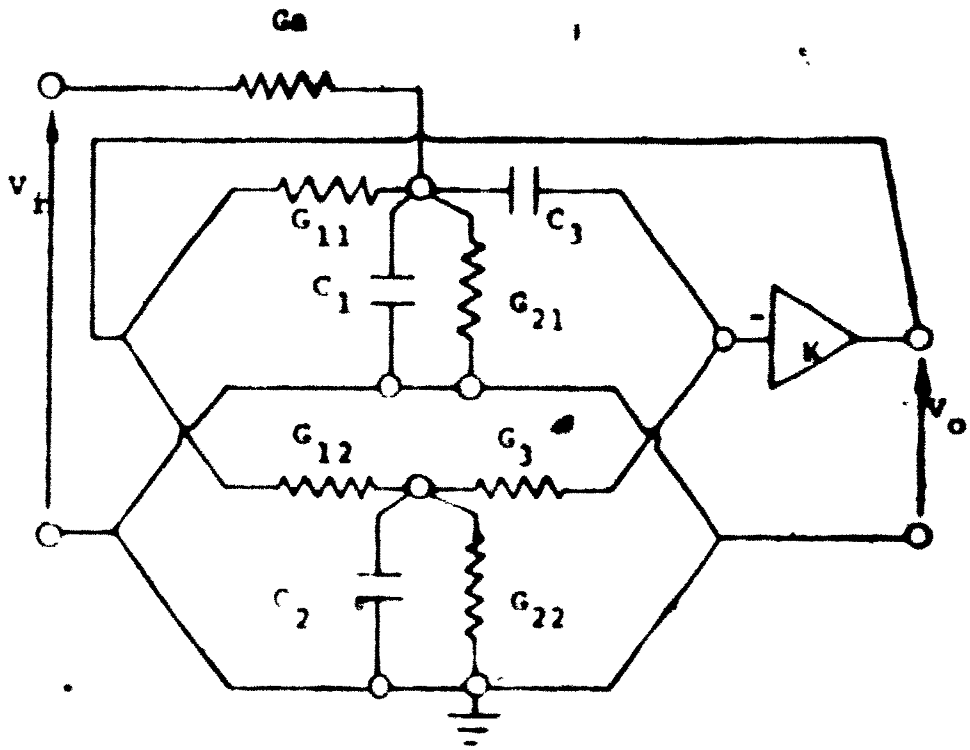

FIG. 2.11 BAND PASS FILTER REALIZATION FOR DECOMPOSITION IIb

Analysis yields:

$$T(s) = \frac{V_o}{V_i} = \frac{N(s)}{D(s)}$$

where $N(s) = -KsC_3G_a(sC_2+G_{12}+G_{22}+G_3)$ (2.62a)

$$D(s) = sC_3(sC_2+G_{12}+G_{22}+G_3)(sC_1+G_a+G_{11}+G_{21})+G_3(sC_2+G_{12}+G_{22}) \times (sC_1+sC_3+G_a+G_{11}+G_{21})+KsC_3G_{11}(sC_2+G_{12}+G_{22}+G_3)+KG_{12}G_3(sC_1+sC_3+G_a+G_{11}+G_{21})$$ (2.62b)

For $\frac{G_{12}+G_{22}+G_3}{C_2} = \frac{G_a+G_{11}+G_{21}}{C_1+C_3}$ (2.63)

one negative real pole would cancel one negative real zero of the T(s) thereby giving:

$$N(s) = -KsC_2C_3G_a$$ (2.64a)

$$D(s) = s^2C_1C_2C_3+sC_2C_3(G_a+G_{11}+G_{21})+sC_2G_3(C_1+C_3)+G_3(C_1+C_3)(G_{12}+G_{22})+KsC_2C_3G_{11}+KG_{12}G_3(C_1+C_3)$$ (2.64b)

$$Q = \frac{\sqrt{C_1C_2C_3(C_1+C_3)(G_{12}+G_{22}+KG_{12})G_3}}{C_2C_3(G_a+G_{21}+G_{11}+KG_{11})+C_2G_3(C_1+C_3)}$$ (2.64c)

$$\omega_o = \sqrt{\frac{G_3(C_1+C_3)(G_{12}+G_{22}+KG_{12})}{C_1C_2C_3}}$$ (2.64d)

$$S_K^Q = \frac{1}{2}\,\frac{KG_{12}}{G_{12}(K+1)+G_{22}} - \frac{KG_{11}C_3}{C_3(G_a+G_{21}+G_{11}+KG_{11})+G_3(C_1+C_3)}$$ (2.64e)

$$S_K^{\omega_o} = \frac{1}{2}\,\frac{KG_{12}}{G_{12}+G_{22}+KG_{12}}$$ (2.64f)

For $S_K^Q = 0$

$$K+1 = \frac{G_a+G_{21}}{G_{11}} + \frac{G_3}{G_{11}}\left(1+\frac{C_1}{C_3}\right) - 2\frac{G_{22}}{G_{12}} \qquad (2.65)$$

One set of design equations for this realization is:

$$G_a = G_{21} = \frac{a(a-\eta)}{2(a-\xi)} \qquad (2.66a)$$

$$G_{11} = \frac{a\eta}{a-\xi} \qquad (2.66b)$$

$$G_{22} = G_{12} = \frac{3-a}{\xi(3-a)-2} \qquad (2.66c)$$

$$G_3 = 3-a \qquad (2.66d)$$

$$C_1 = \frac{a}{a-\xi} \qquad (2.66e)$$

$$C_2 = \frac{(3-a)^2}{\xi(3-a)-2} \qquad (2.66f)$$

$$C_3 = \frac{a}{\xi} \qquad (2.66g)$$

$$K = \frac{3}{\eta} - 4 \qquad (2.66h)$$

the realizability conditions being:

$$3 > a > \xi > \frac{2}{3-a}$$

and $\eta < a$

Where 'a', η and ξ are arbitrarily chosen positive constants. These design equations with the network of Fig. (2.11) realize a T(s) given by:

$$T(s) = \frac{-Ks(a-\eta)/2}{s^2+s(K\eta+3)+K+2} \qquad (2.67)$$

where $Q = \frac{\sqrt{K+2}}{K\eta+3}$, $\omega_o = \sqrt{K+2}$

$$S_K^Q = 0 \qquad \text{for} \qquad K = \frac{3}{\eta} - 4$$

$$Q \text{ realized} = \frac{\sqrt{\frac{3}{\eta}-2}}{6-4\eta} \qquad (2.68)$$

η can be calculated from eq. (2.68)

for $\quad \eta << 1$

$$\eta \approx \frac{1}{12Q^2}$$

and $\quad K \approx 36Q^2 - 4$

Transfer Function gain is:

$$-K\,\frac{a-\eta}{2} \times \frac{Q}{\omega_o}$$

$$= -K\,\frac{a-\eta}{2} \times \frac{Q}{\sqrt{K+2}} \simeq -K\,\frac{aQ}{2\sqrt{K+2}}$$

$$\approx -\frac{36Q^2\,aQ}{12Q} \approx -3aQ^2$$

Element spread shall vary according to the selection of 'a' and ε.

For one such selection of 'a' = 1.5 and ε = 1.4

Element spread is: Capacitive: 1:21

Resistive: $1:1/(10\eta)$

Transfer function gain is $-4.50Q^2$ at the centre frequency.

A realization with $S_K^Q = 0$ can be obtained from the network of Fig. 2.3 belonging to decomposition IL_a with unequal capacitors by applying RC-CR transformation. In doing so K is of the order $32Q^2$ and the capacitive spread is of the order Q^2 which are not acceptable for high-Q circuits.

2.8.6 DECOMPOSITION IIc:

K_1 appears in s^2 and s^0 terms only

a). **Realization:**

D(s) is of the type

$$D(s) = s^2(\alpha_1 + K\alpha_2) + s\beta + \gamma_1 + K\gamma_2 \qquad (2.69a)$$

and

$$Q = \frac{\sqrt{(\alpha_1 + K\alpha_2)(\gamma_1 + K\gamma_2)}}{\beta} \qquad (2.69b)$$

$$\omega_o = \frac{\sqrt{\gamma_1 + K\gamma_2}}{\sqrt{\alpha_1 + K\alpha_2}} \qquad (2.69c)$$

$$S_K^Q = \frac{1}{2}\,\frac{K\alpha_2}{\alpha_1 + K\alpha_2} + \frac{1}{2}\,\frac{K\gamma_2}{\gamma_1 + K\gamma_2} \qquad (2.69d)$$

$$S_{K_1}^{\omega_o} = \frac{1}{2}\,\frac{K\gamma_2}{\gamma_1 + K\gamma_2} - \frac{1}{2}\,\frac{K\alpha_2}{\alpha_1 + K\alpha_2} \qquad (2.69e)$$

$$S_K^Q - S_K^{\omega_o} = \frac{K\alpha_2}{\alpha_1 + K\alpha_2} \qquad (2.69f)$$

$$S_K^Q + S_K^{\omega_o} = \frac{K\gamma_2}{\gamma_1 + K\gamma_2} \qquad (2.69g)$$

the sensitivity bounds being

$$0 < S_K^Q < 1 \qquad (2.70a)$$

$$-1/2 < S_K^{\omega_o} < 1/2 \qquad (2.70b)$$

$$0 < S_K^Q - S_K^{\omega_o} < 1 \qquad (2.70c)$$

$$0 < S_K^Q + S_K^{\omega_o} < 1 \qquad (2.70d)$$

Passive network having transfer function of the type $\frac{s^2+1}{s^2+s/Q_p+1}$, where Q_p = Passive Q, can be obtained by using at least three capacitors [33]. As such this type of decomposition can be realized with at least three capacitors with one pole being cancelled by one zero of the T(s). Q- and ω_o- sensitivities with respect to all the network parameters shall be computed through the coefficient sensitivities as described in [27].

For D(s), S_K^Q, if not prescribed, shall be very nearly equal to unity. If prescribed, S_K^Q fixes K which in its turn fixes the realizable value of Q. Hence networks obtained from this decomposition are not suitable for the prescription of S_K^Q.

Fialkow Gerst conditions dictate that

$$\alpha_2 \leq \alpha_1 \qquad (2.71a)$$

and

$$\gamma_2 \leq \gamma_1 \qquad (2.71b)$$

For

$$\frac{\alpha_2}{\alpha_1} = \frac{\gamma_2}{\gamma_1}$$

which can always be realized, we get

$$S_K^{\omega_o} \equiv 0$$

If

$$\frac{\alpha_2}{\alpha_1} \neq \frac{\gamma_2}{\gamma_1} ,$$

then $S_K^{\omega_o}$ shall not be zero, but shall be small.

It is always possible to prescribe $S_K^{\omega_o} = 0$ with respect to D(s), for which the condition $\frac{\alpha_2}{\alpha_1} = \frac{\gamma_2}{\gamma_1}$ has to be satisfied during realization. Using the general configuration of Figure 1.2, the active network for this decomposition can be obtained, which is a Band Pass Filter shown in Figure 2.12.

Analysis yields:

$$T(s) = \frac{V_o}{V_i} = \frac{-Ksc_3G_a(sc_2 + G_1 + G_3)}{s^2C_1G_3(sC_2+G_1+G_3)(K+1)+sC_3(G_a+G_2)(sC_2+G_1+G_3) +sC_2G_3(sC_1+sC_3+G_a+G_2)+G_1G_3(sC_1+sC_3+G_a+G_2)(K+1)} \tag{2.72}$$

One negative real axis pole zero pair cancels in T(s) if

$$\frac{C_2}{C_1 + C_3} = \frac{G_1 + G_3}{G_a + G_2}$$

After the pole zero pair cancellation we get:

$$T(s) = \frac{-KsC_2C_3G_a}{s^2C_1C_2C_3(K+1)+s[C_3C_2(G_a+G_2)+C_2G_3(C_1+C_3)]+G_1G_3(K+1)(C_1+C_3)} \tag{2.73}$$

the condition $\frac{\alpha_2}{\alpha_1} = \frac{\gamma_2}{\gamma_1}$ is also satisfied thus giving $S_K^{\omega_o} = 0$.

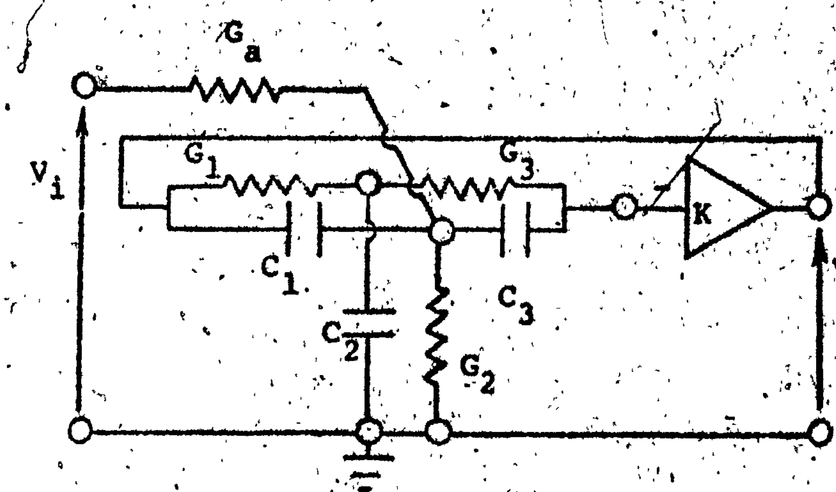

FIG. 2.12 BAND PASS FILTER REALIZATION FOR DECOMPOSITION IIc

Also letting

$$C_1C_2C_3 = G_1G_3(C_1 + C_3)$$

gives

$$\omega_o = 1$$

The design equations for this realization are

$$G_a = G_2 = \frac{G_1\ [b^2\eta - 4(\eta + 1)]}{8} \tag{2.74a}$$

$$G_3 = G_1\frac{[b^2\eta - 4(\eta + 1)]}{4(\eta + 1)} \tag{2.74b}$$

$$C_1 = G_1\frac{b^2\eta - 4(\eta + 1)}{2b} \tag{2.74c}$$

$$C_2 = \frac{b}{2}G_1 \tag{2.74d}$$

$$C_3 = \frac{[b^2\eta - 4(\eta + 1)]\ G_1}{2b\eta} \tag{2.74e}$$

$$K = bQ - 1 \tag{2.74f}$$

$$\omega_o = 1 \tag{2.74g}$$

where G_1 can be chosen arbitrarily and b and η are positive arbitrarily chosen constants. The necessary condition for realization is

$$b^2 > 4 + \frac{4}{\eta} \tag{2.75}$$

The negative real axis pole cancelling with negative real axis zero is

$$s = -\varepsilon_1 = \frac{-b\eta}{2(\eta + 1)} \qquad (2.76)$$

For D(s)

$$S_K^{\omega_o} = 0 \qquad (2.77a)$$

$$S_K^Q = \frac{K}{K + 1} < 1 \qquad (2.77b)$$

$$\text{Gain Q-Sensitivity Product} = \frac{K^2}{K + 1} \qquad (2.77c)$$

Transfer function gain at pole frequency =

$$-\frac{K}{K + 1} \times \frac{bQ}{4} = -\frac{K}{4} = -\frac{bQ}{4} + \frac{1}{4} \qquad (2.77d)$$

The capacitive spread has been plotted against η for various values of 'b' in Figure 2.13 . From these curves it is clear that for a maximum capacitive spread of 5* the least value 'b' can have, is $\sqrt{6}$. For any given capacitive spread and a given gain Q-sensitivity product it is possible to select 'b' and 'η' from these curves. If the element spread is not a consideration then the value of 'b' can approach but cannot equal 2, that is

$$K > 2Q - 1 \qquad (2.78)$$

Network of Figure 2.12 , used to obtain decomposition IIc shall be studied further in detail for stability during activation, Q- and ω_o- sensitivities with respect to the open loop gain of the OAs

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* In general it is preferable to keep the maximum capacitive spread of 10[34]. However, in this thesis, the maximum capacitive spread shall be specified at five.

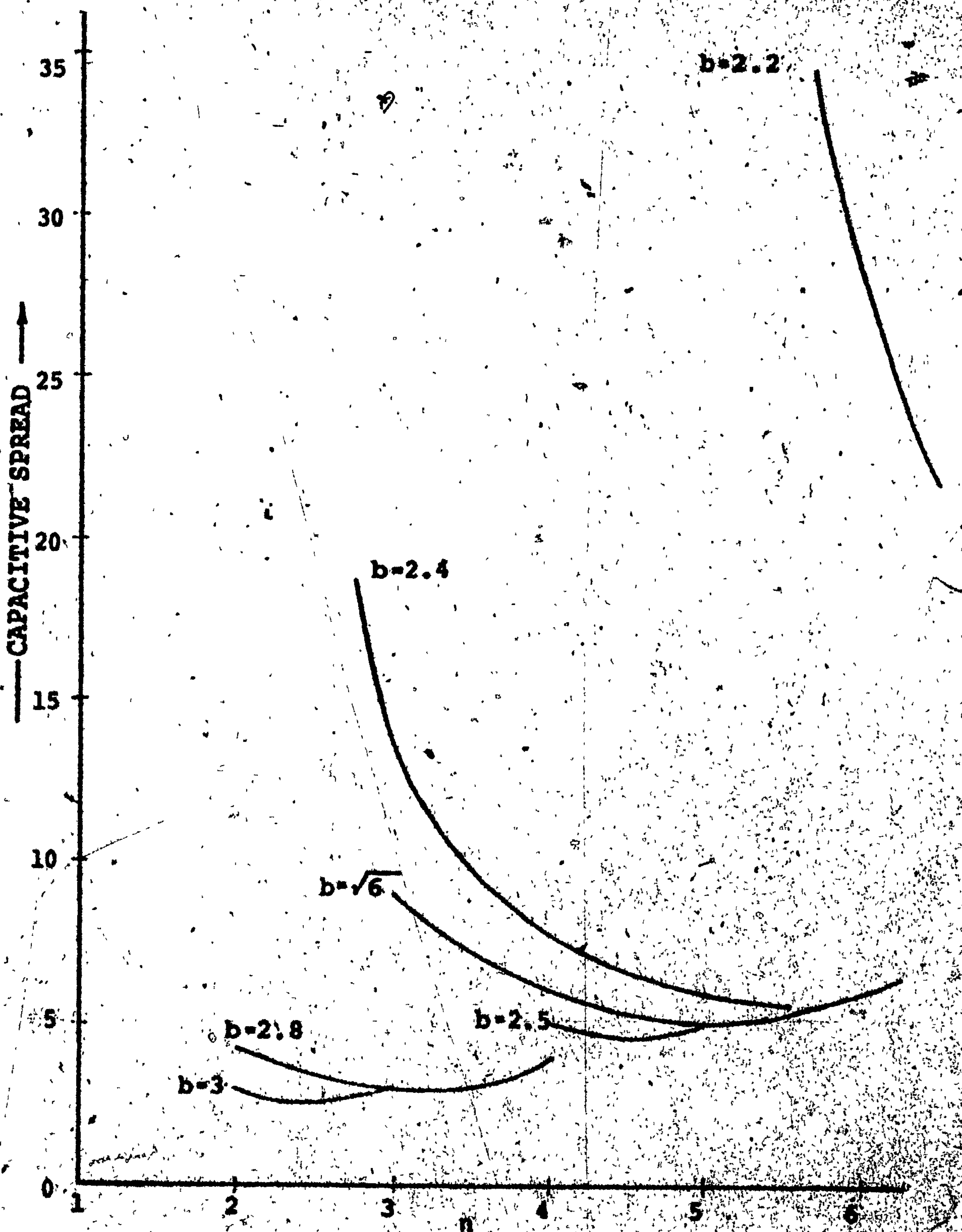


FIGURE 2.13

CAPACITIVE SPREAD VERSUS η PLOTS FOR VARIOUS VALUES OF 'b' FOR THE BP FILTER NETWORK OF DECOMPOSITION IIc and othor resistors realizing the closed loop gain K for D(s). Q- and ω_o-sensitivities with respect to all the network parameters shall be obtained for $D_3(s)$. Further the effect of OA pole on ω_o and Q shall be considered. Experimental results are also given.

Replacing the amplifier in Figure (2.12) by the network of Figure (2.5), the analysis yields

$$T(s) = \frac{N(s)}{D(s)} = \frac{V_o}{V_i}$$

where

$$N(s) = -sC_3G_aG_{31}(G_{11}+G_{21})(sC_2+G_1+G_3)\mu_1\mu_2 \quad (2.79a)$$

$$\begin{aligned} D(s) = {} & [\mu_1\mu_2G_{21}G_{41}+\mu_2G_{41}(G_{11}+G_{21})+\mu_1G_{21}(G_{31}+G_{41}) \\ & +(G_{11}+G_{21})(G_{31}+G_{41})] \times [sC_3(sC_2+G_1+G_3) \\ & (sC_1+G_a+G_2)+G_3(sC_2+G_1)(sC_1+sC_3+G_a+G_2)] \\ & +\mu_1\mu_2G_{31}(G_{11}+G_{21})[s^2C_1C_3(sC_2+G_1+G_3) \\ & +G_1G_3(sC_1+sC_3+G_a+G_2)] \end{aligned} \quad (2.79b)$$

where

$$K = 1+\frac{G_{21}}{G_{11}} = 1+\frac{G_{31}}{G_{41}} \quad (2.80a)$$

if

$$G_{11}G_{31} = G_{21}G_{41} \quad (2.80b)$$

and

$$\mu_1 >> 1+\frac{G_{11}}{G_{21}} \quad (2.80c)$$

$$\mu_2 >> 1+\frac{G_{31}}{G_{41}} = K \quad (2.80d)$$

also

$$\epsilon_1 = \frac{G_a+G_2}{C_1+C_3} = \frac{G_1+G_3}{C_2} \quad (2.80e)$$

b). Stability During Activation:

From eq. (2.79) it is clear that for the nominal values of the network parameters one negative real axis pole always cancels with one negative real axis zero of the T(s). Also from eq. (2.79) it can be seen that D(s) shall have no negative term after cancellation, and as such this network is stable during activation.

c). Q- and ω_o-Sensitivities for D(s):

For the sake of convenience, Q- and ω_o-sensitivities with respect to μ and with respect to the resistors realizing the amplifier will be computed for D(s). Subsequently Q- and ω_o-sensitivities with respect to all the network parameters for $D_3(s)$ shall be presented.

After the cancellation of one pole with one zero, D(s) from eq. (2.79) is

$$\begin{aligned} D(s) = {} & [s^2C_1C_2C_3+G_1G_3(C_1+C_3)][\mu_1\mu_2G_{31}(G_{11}+G_{21})+\mu_1\mu_2G_{21}G_{41} \\ & +\mu_2G_{41}(G_{11}+G_{21})+\mu_1G_{21}(G_{31}+G_{41})+(G_{11}+G_{21})(G_{31}+G_{41})] \\ & +s[C_2C_3(G_a+G_2)+C_2G_3(C_1+C_3)] \times [\mu_1\mu_2G_{21}G_{41}+\mu_2G_{41} \\ & (G_{11}+G_{21})+\mu_1G_{21}(G_{31}+G_{41})+(G_{11}+G_{21})(G_{31}+G_{41})] \end{aligned} \quad (2.81a)$$

and

$$Q = \frac{\sqrt{C_1C_2C_3G_1G_3(C_1+C_3)}}{C_2C_3(G_a+G_2)+C_2G_3(C_1+C_3)} \times \frac{\mu_1\mu_2G_{31}(G_{11}+G_{21})+\mu_1\mu_2G_{21}G_{41}+\mu_2G_{41}(G_{11}+G_{21})+\mu_1G_{21}(G_{31}+G_{41})+(G_{11}+G_{21})(G_{31}+G_{41})}{\mu_1\mu_2G_{21}G_{41}+\mu_2G_{41}(G_{11}+G_{21})+\mu_1G_{21}(G_{31}+G_{41})+(G_{11}+G_{21})(G_{31}+G_{41})} \quad (2.81b)$$

$$\omega_o = \sqrt{\frac{G_1G_3(C_1+C_3)}{C_1C_2C_3}} \tag{2.81c}$$

$$S_x^{\omega_o} = 0 \text{ for } x = \mu_1, \mu_2, G_{11}, G_{21}, G_{31}, \& G_{41} \tag{2.81d}$$

$$S_{\mu_1}^Q = \frac{\mu_1\mu_2G_{31}(G_{11}+G_{21})[\mu_2G_{41}(G_{11}+G_{21})+(G_{11}+G_{21})(G_{31}+G_{41})]}{[\mu_1\mu_2G_{31}(G_{11}+G_{21})+\mu_1\mu_2G_{21}G_{41}+\mu_2G_{41}(G_{11}+G_{21})+\mu_1G_{21}(G_{31}+G_{41})+(G_{11}+G_{21})(G_{31}+G_{41})] \times [\mu_1\mu_2G_{21}G_{41}+\mu_2G_{41}(G_{11}+G_{21})+\mu_1G_{21}(G_{31}+G_{41})+(G_{11}+G_{21})(G_{31}+G_{41})]} \tag{2.81e}$$

$$S_{\mu_2}^Q = \frac{\mu_1\mu_2G_{31}(G_{11}+G_{21})[\mu_1G_{21}(G_{11}+G_{21})+(G_{11}+G_{21})(G_{31}+G_{41})]}{[\mu_1\mu_2G_{31}(G_{11}+G_{21})+\mu_1\mu_2G_{21}G_{41}+\mu_2G_{41}(G_{11}+G_{21})+\mu_1G_{21}(G_{31}+G_{41})+(G_{11}+G_{21})(G_{31}+G_{41})] \times [\mu_1\mu_2G_{21}G_{41}+\mu_2G_{41}(G_{11}+G_{21})+\mu_1G_{21}(G_{31}+G_{41})+(G_{11}+G_{21})(G_{31}+G_{41})]} \tag{2.81f}$$

$$S_{G_{11}}^Q = \frac{\mu_1\mu_2G_{11}G_{31}+\mu_2G_{11}G_{41}+G_{11}(G_{31}+G_{41})}{\mu_1\mu_2G_{31}(G_{11}+G_{21})+\mu_1\mu_2G_{21}G_{41}+\mu_2G_{41}(G_{11}+G_{21})+\mu_1G_{21}(G_{31}+G_{41})+(G_{11}+G_{21})(G_{31}+G_{41})} - \frac{\mu_2G_{11}G_{41}+G_{11}(G_{31}+G_{41})}{\mu_1\mu_2G_{21}G_{41}+\mu_2G_{41}(G_{11}+G_{21})+\mu_1G_{21}(G_{31}+G_{41})+(G_{11}+G_{21})(G_{31}+G_{41})} \tag{2.81g}$$

$$S^{Q}_{G_{21}} = \frac{\mu_2 G_{41} G_{11} + G_{11}(G_{31}+G_{41})}{\mu_1\mu_2 G_{21}G_{41} + \mu_2 G_{41}(G_{11}+G_{21}) + \mu_1 G_{21}(G_{31}+G_{41}) + (G_{11}+G_{21})(G_{31}+G_{41})}$$

$$- \frac{\mu_1\mu_2 G_{11}G_{31} + \mu_2 G_{41}G_{11} + G_{11}(G_{31}+G_{41})}{\mu_1\mu_2 G_{31}(G_{11}+G_{21}) + \mu_1\mu_2 G_{21}G_{41} + \mu_2 G_{41}(G_{11}+G_{21}) + \mu_1 G_{21}(G_{31}+G_{41}) + (G_{11}+G_{21})(G_{31}+G_{41})} \quad (2.81h)$$

$$S^{Q}_{G_{31}} = \frac{\mu_1\mu_2 G_{31}(G_{11}+G_{21}) + \mu_1 G_{21}G_{31} + G_{31}(G_{11}+G_{21})}{\mu_1\mu_2 G_{31}(G_{11}+G_{21}) + \mu_1\mu_2 G_{21}G_{41} + \mu_2 G_{41}(G_{11}+G_{21}) + \mu_1 G_{21}(G_{31}+G_{41}) + (G_{11}+G_{21})(G_{31}+G_{41})}$$

$$- \frac{\mu_1 G_{21}G_{31} + G_{31}(G_{11}+G_{21})}{\mu_1\mu_2 G_{21}G_{41} + \mu_2 G_{41}(G_{11}+G_{21}) + \mu_1 G_{21}(G_{31}+G_{41}) + (G_{11}+G_{21})(G_{31}+G_{41})} \quad (2.81i)$$

$$S^{Q}_{G_{41}} = - \frac{\mu_1\mu_2 G_{31}(G_{11}+G_{21})\left[\mu_1\mu_2 G_{21}G_{41} + \mu_2 G_{41}(G_{11}+G_{21}) + \mu_1 G_{21}G_{41} + G_{41}(G_{11}+G_{21})\right]}{\left[\mu_1\mu_2 G_{31}(G_{11}+G_{21}) + \mu_1\mu_2 G_{21}G_{41} + \mu_2 G_{41}(G_{11}+G_{21}) + \mu_1 G_{21}(G_{31}+G_{41}) + (G_{11}+G_{21})(G_{31}+G_{41})\right] \times \left[\mu_1\mu_2 G_{21}G_{41} + \mu_2 G_{41}(G_{11}+G_{21}) + \mu_1 G_{21}(G_{31}+G_{41}) + (G_{11}+G_{21})(G_{31}+G_{41})\right]} \quad (2.81j)$$

For the nominal values of the elements and with the approximations mentioned these Q-sensitivities are (for $\mu_1 = \mu_2 = \mu$)

$$S^{Q}_{\mu_1} \approx 1/\mu \quad (2.82a)$$

$$S^Q_{\mu_2} \approx K/\mu \qquad (2.82b)$$

$$S^Q_{G_{11}} \approx 1/(K+1) \qquad (2.82c)$$

$$S^Q_{G_{21}} \approx -\frac{1}{K+1} \qquad (2.82d)$$

$$S^Q_{G_{31}} \approx K/(K+1) \qquad (2.82e)$$

$$S^Q_{G_{41}} \approx -K/(K+1) \qquad (2.82f)$$

d). <u>Q- and ω_o-Sensitivities for $D_3(s)$:</u>

From section 2.5 and eq. (2.79), we have

$$\delta_2 = \frac{C_1C_3(G_1+G_3)(K+1)+C_2C_3(G_a+G_2)+C_2G_3(C_1+C_3)}{C_1C_2C_3(K+1)} \qquad (2.83a)$$

$$\delta_1 = \frac{(K+1)G_1G_3(C_1+C_3)+C_2G_3(G_a+G_2)+C_3(G_a+G_2)(G_1+G_3)}{C_1C_2C_3(K+1)} \qquad (2.83b)$$

$$\delta_o = \frac{G_1G_3(G_a+G_2)}{C_1C_2C_3} = \epsilon_1 \qquad (2.83c)$$

It is to be noted that ϵ_1 is not a function of K.

From (2.83, we obtain

$$S_K^{\delta_2} = -\frac{K}{(K+1)^2}\left[\frac{G_a+G_2}{C_1} + \frac{G_3(C_1+C_3)}{C_1C_3}\right]$$

$$= -\frac{bQ-1}{b^2Q^2}\left[\frac{b}{2} + \frac{b}{2}\right] = -\frac{bQ-1}{bQ^2} \qquad (2.84a)$$

$$S_K^{\delta_1} = -\frac{K}{(K+1)^2}\left[\frac{G_3(G_a+G_2)}{C_1C_3} + \frac{(G_a+G_2)(G_1+G_3)}{C_1C_2}\right]$$

$$= -\frac{bQ-1}{b^2Q^2}\left[\frac{b^2\eta}{4(\eta+1)} + \frac{b^2\eta}{4(\eta+1)}\right] = -\frac{\eta(bQ-1)}{2Q^2(\eta+1)} = -\epsilon_1 x \frac{bQ-1}{bQ^2} \qquad (2.84b)$$

$$S_K^{\delta_o} \equiv 0 \qquad (2.84c)$$

where

$$\epsilon_1 = \frac{b\eta}{2(\eta+1)}$$

for nominal values of elements.

From eq. (2.30) of section 2.5 and eq. (2.84).

$$S_K^{\omega_o} \equiv 0 \qquad (2.85a)$$

$$S_K^Q = \frac{-Q(2Q-\epsilon_1)\left[-\frac{bQ-1}{bQ^2}\right]-Q[2\epsilon_1 Q-1]\left[-\epsilon\frac{bQ-1}{bQ^2}\right]}{2Q(1+\epsilon_1^2)-2\epsilon_1}$$

$$= \frac{2(Q - 1/b) - \varepsilon_1(1 - 1/bQ) + 2\varepsilon_1^2(Q - 1/b) - \varepsilon_1(1 - 1/bQ)}{2Q(1 + \varepsilon_1^2) - 2\varepsilon_1}$$

$$= \frac{2(Q - 1/b)(1 + \varepsilon_1^2) - 2\varepsilon_1(1 - 1/bQ)}{2Q(1 + \varepsilon_1^2) - 2\varepsilon_1} = 1 - \frac{1}{bQ} \tag{2.85b}$$

≈ 1 for a high Q, and $b > 1$

$$S_K^{\varepsilon_1} \equiv 0 \tag{2.85c}$$

μ_1, μ_2, G_{11}, G_{21}, G_{31} and G_{41} realize K and control K only. Since Q- and ω_o- sensitivities with respect to K do not change very much from D(s) to $D_3(s)$ therefore it is assumed that $S_{\mu_1}^Q$, $S_{\mu_1}^{\omega_o}$, $S_{\mu_2}^Q$, $S_{\mu_2}^{\omega_o}$, $S_{G_{11}}^Q$, $S_{G_{11}}^{\omega_o}$, $S_{G_{21}}^Q$, $S_{G_{21}}^{\omega_o}$, $S_{G_{31}}^Q$, $S_{G_{31}}^{\omega_o}$, $S_{G_{41}}^Q$ and $S_{G_{41}}^{\omega_o}$ do not change very much for $D_3(s)$.

e). <u>Effect of the pole of OA on Q and ω_o:</u>

In order to study the effect of the pole of OA on Q and ω_o, it is assumed that the desired elements are available and therefore one pole always cancels with one zero of T(s). For the nominal values of the elements, from eq. (2.79), after one pole zero pair cancellation, we get

$$D(s) = s^2\left[1+\frac{K/(K-1)}{\mu_1(K+1)} + \frac{K}{(K+1)\mu_2}\right] + \frac{s}{Q}\left[1+\frac{K/(K-1)}{\mu_1} + \frac{K}{\mu_2}\right]$$

$$+\left[1+\frac{K/(K-1)}{\mu_1(K+1)} + \frac{K}{(K+1)\mu_2}\right]$$

where terms with $\mu_1\mu_2$ in their denominators have been neglected compared to others.

For

$$K >> 1 ,$$

and

$$\mu_1 = \mu_2 = \mu$$

D(s) becomes

$$D(s) = s^2(1 + 1/\mu)+(s/Q)(1 + (K+1)/\mu) + 1 + 1/\mu \qquad (2.86)$$

Replacing μ by $\frac{\mu_o\omega_c}{\omega_o s + \omega_c}$ gives

$$D(s) = s^3 \frac{\omega_o}{\mu_o\ \omega_c} + s^2[1+\frac{1}{\mu_o} + \frac{(K+1)\omega_o}{Q\ \mu_o\ \omega_c}]+\frac{s}{Q}[1+\frac{K+1}{\mu_o} + \frac{\omega_o Q}{\mu_o\ \omega_c}]$$
$$+ 1 + \frac{1}{\mu_o}$$

$$= s^3 \frac{\omega_o}{\mu_o\ \omega_c} + s^2[1+\frac{1}{\mu_o} + \frac{b\omega_o}{\mu_o\omega_c}] + \frac{s}{Q}[1+\frac{bQ}{\mu_o} + \frac{\omega_o Q}{\mu_o\ \omega_c}]$$
$$+ 1 + \frac{1}{\mu_o} \qquad (2.87)$$

Comparing this equation term by term with eq. (2.55), we get

i) $$\frac{\omega_o^2}{\varepsilon_2\ \hat{\omega}_o^2} = \frac{\omega_o/GB}{1 + 1/\mu_o}$$

Since

$$\frac{\hat{\omega}_o}{\omega_o} \approx 1$$

$$\varepsilon_2 = \frac{GB}{\omega_o}(1 + 1/\mu_o) >> 1$$

ii) $$\frac{1}{\hat{\omega}_o^2} + \frac{1}{\hat{Q}\,\hat{\omega}_o\epsilon_2} = \frac{1 + 1/\mu_o + b\omega_o/(\mu_o\omega_c)}{1 + 1/\mu_o}$$

Neglecting the second term compared with the first, for $Q \gg 1$, $\epsilon_2 \gg 1$, gives

$$\frac{\hat{\omega}_o^2}{\omega_o^2} = \frac{1 + 1/\mu_o}{1 + 1/\mu_o + (b\omega_o)/(\mu_o\,\omega_c)}$$

Expanding the R.H. side and considering first order terms only we get

$$\frac{\hat{\omega}_o}{\omega_o} \approx 1 - \frac{b\omega_o}{2\mu_o\,\omega_c} \tag{2.88}$$

iii) $$\frac{1}{\epsilon_2} + \frac{1}{\hat{Q}\,\hat{\omega}_o} = \frac{1/Q[1 + bQ/\mu_o + (\omega_o Q)/(\mu_o\,\omega_c)]}{1 + 1/\mu_o}$$

$$\frac{\hat{Q}\,\hat{\omega}_o}{Q\,\omega_o} = \frac{1 + 1/\mu_o}{1 + bQ/\mu_o}$$

and

$$\frac{\hat{Q}}{Q} = \frac{\sqrt{1 + 1/\mu_o}\,\sqrt{1 + 1/\mu_o + (b\omega_o)/(\mu_o\,\omega_c)}}{1 + bQ/\mu_o}$$

Expanding the R.H. side and considering first order terms only, we get

$$\frac{\hat{Q}}{Q} \approx 1 + \frac{b\omega_o}{2\mu_o\omega_c} - \frac{bQ - 1}{\mu_o} \tag{2.89}$$

$\frac{\Delta\omega_o}{\omega_o}$ and $\frac{\Delta Q}{Q}$ due to the effect of the pole of the OA are computed from equations (2.88) and (2.89) for various values of ω_o and are given in Figures (2.14) and (2.15) respectively. Use of OA μA 741 has been assumed in these computations. Actual variations in

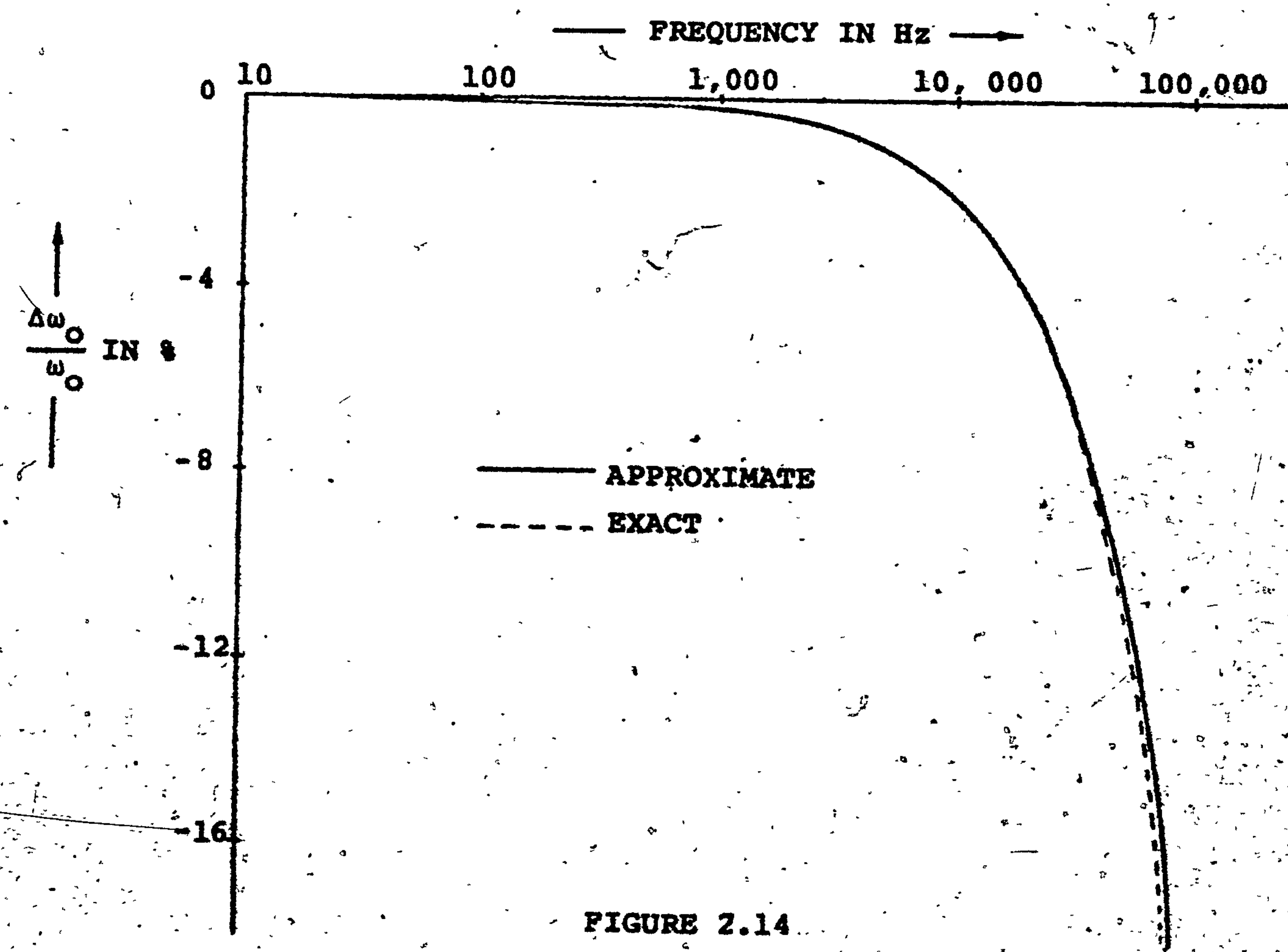


FIGURE 2.14

$\frac{\Delta\omega_o}{\omega_o}$ VERSUS FREQUENCY PLOTS FOR NETWORKS OF DECOMPOSITION IIc

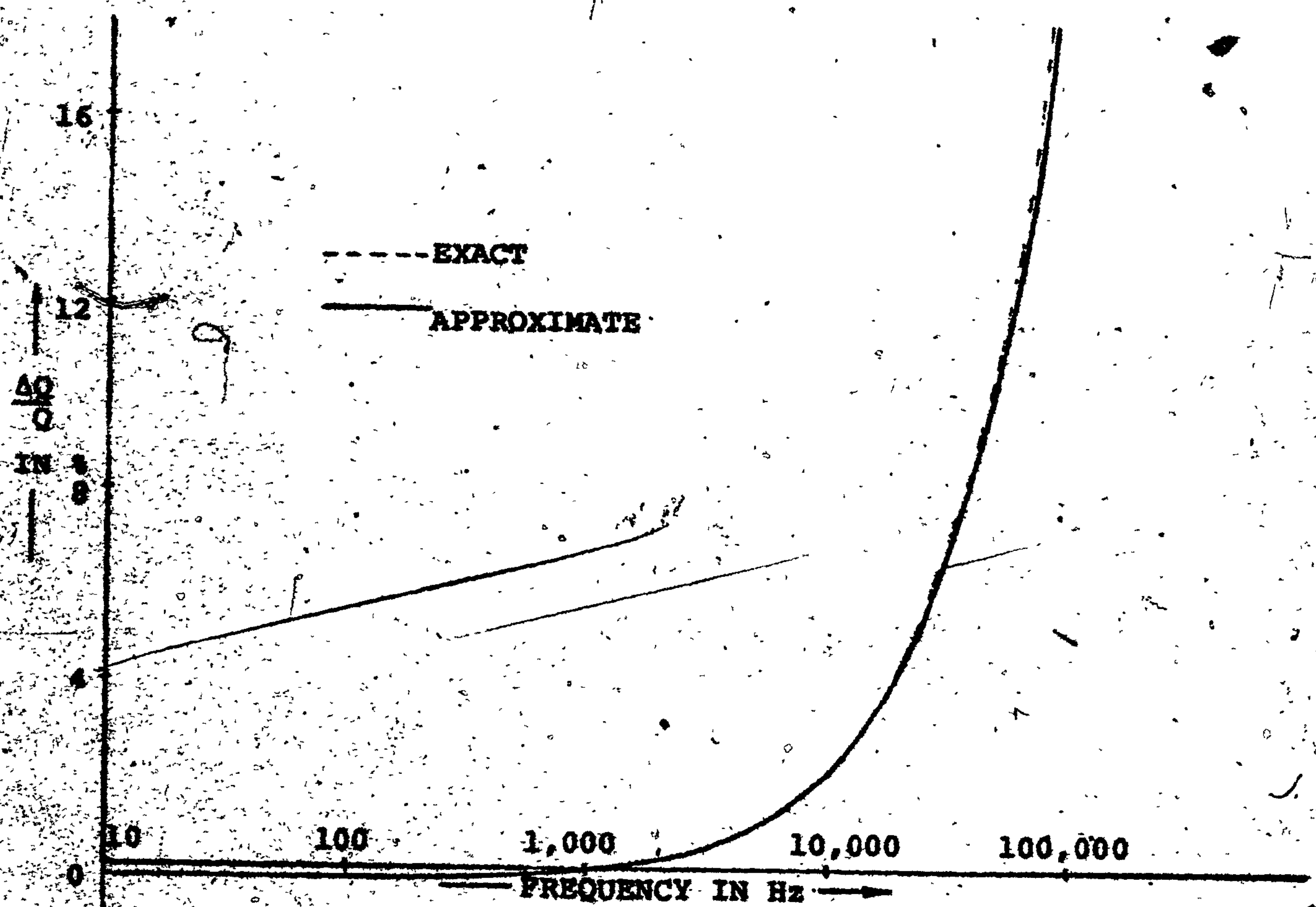


FIGURE 2.15

$\frac{\Delta Q}{Q}$ VERSUS FREQUENCY PLOTS FOR NETWORKS OF DECOMPOSITION IIc

Q and ω_o due to the effect of the pole of the OA are within 4% at the higher frequencies.

f). <u>Implementation of Band Pass Filter and Experimental Results:</u>

$Q = 100$, $b = 2.5$ and $\eta = 5$ and $G_1 = 1$ have been chosen to give

$$G_a = G_2 = \frac{7.25}{8}, \quad G_3 = \frac{7.25}{24}, \quad C_1 = \frac{7.25}{5}$$

$$C_2 = 1.25, \quad C_3 = \frac{7.25}{25}, \quad K = 2.5Q-1 = 249$$

For D(s), the sensitivities are:

$$S_K^{\omega_o} = 0 \qquad S_K^Q = 1 - \frac{1}{2.5Q} = 1 - 0.004 = 0.996$$

$$S_{\mu_1}^Q \approx 5 \times 10^{-6} \qquad S_{\mu_2}^Q \approx 1.25 \times 10^{-3}$$

$$S_{G_{11}}^Q \approx 0.004 \qquad S_{G_{21}}^Q \approx -0.004$$

$$S_{G_{31}}^Q \approx 1.0 \qquad S_{G_{41}}^Q \approx -1.0$$

For $D_3(s)$, the sensitivities are:

$$S_K^{\varepsilon_1} = 0, \quad S_K^{\omega_o} = 0, \quad S_K^Q \approx 1/4.24, \quad KS_K^Q \approx \frac{2.5Q}{4.24}, \quad KS_K^{\omega_o} \approx 0$$

$$S_{C_1}^{\varepsilon_1} \approx -0.4 \qquad S_{C_1}^{\omega_o} \approx -0.3 \qquad S_{C_1}^Q \approx -0.5Q$$

$S^{\varepsilon_1}_{C_2} \approx 0.6$ $\quad S^{\omega_o}_{C_2} \approx -0.24$ $\quad S^{Q}_{C_2} \approx 0.5Q$

$S^{\varepsilon_1}_{C_3} \approx -0.04$ $\quad S^{\omega_o}_{C_3} \approx 0.47$ $\quad S^{Q}_{C_3} \approx -0.08Q$

$S^{\varepsilon_1}_{G_1} \approx 0.38$ $\quad S^{\omega_o}_{G_1} \approx 0.28$ $\quad S^{Q}_{G_1} \approx -0.38Q$

$S^{\varepsilon_1}_{G_a} = S^{\varepsilon}_{G_2} \approx 0.26$ $\quad S^{\omega_o}_{G_a} = S^{\omega_o}_{G_2} \approx 0.125$ $\quad S^{Q}_{G_a} = S^{\omega_o}_{G_2} \approx 0.4Q$

$S^{\varepsilon_1}_{G_3} \approx +0.23$ $\quad S^{\omega_o}_{G_3} \approx 0.44$ $\quad S^{Q}_{G_3} \approx -0.11Q$

As can be seen the Q-sensitivities only are high and not the others as expected [19].

For tuning the circuit G_3 is varied to obtain the proper design frequency and G_1 is then varied to obtain the proper Q. This network after denormalization at $\omega_o = 10{,}000$ rad/sec. and a resistance ratio of 10,000 gives the designed parameters as

$\omega_o = 10{,}000$ rad/sec. $\quad Q = 100 \quad b = 2.5 \quad \eta = 5$

$R_1 = 10$ KΩ $\quad R_2 = R_a = \frac{80}{7.25}$ KΩ $= 11.0345$ KΩ

$R_3 = \frac{240}{7.25}$ KΩ $= 33.1034$ KΩ

$C_1 = 14500$ PF $\quad C_2 = 12500$ PF $\quad C_3 = 2900$ PF

$K = 249$

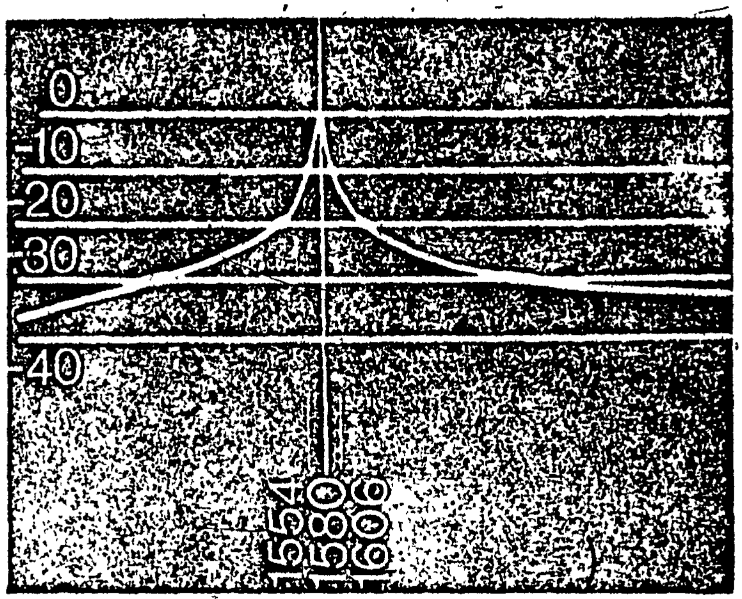
The used parameter values were

$R_1 = 10.03$ KΩ $\quad R_2 = R_a = 11.05$ KΩ $\quad R_3 = 33.1$ KΩ

$C_1 = 14{,}490$ PF $\quad C_2 = 12{,}500$ PF $\quad C_3 = 2907$ PF

$R_{11} = 2.66$ KΩ $\quad R_{21} = 9.9$Ω $\quad R_{41} = 2.59$ KΩ and

$R_{31} = 9.8$ KΩ



FREQUENCY RESPONSE OF BAND PASS FILTER (Q=100)

OF DECOMPOSITION IIc

FIGURE 2.16

a) ±10V,22° C.

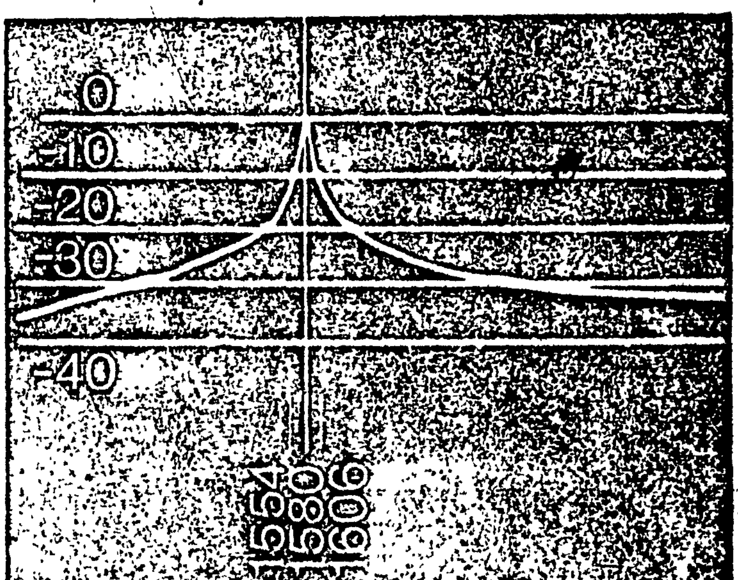


FIGURE 2.16

b) ±15V, 22° C.

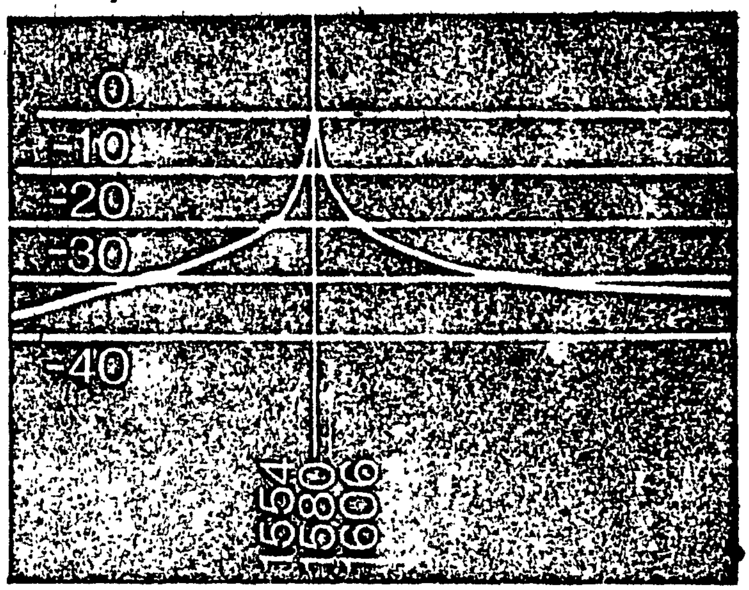


FIGURE 2.16

c) ±10V, 70° C.

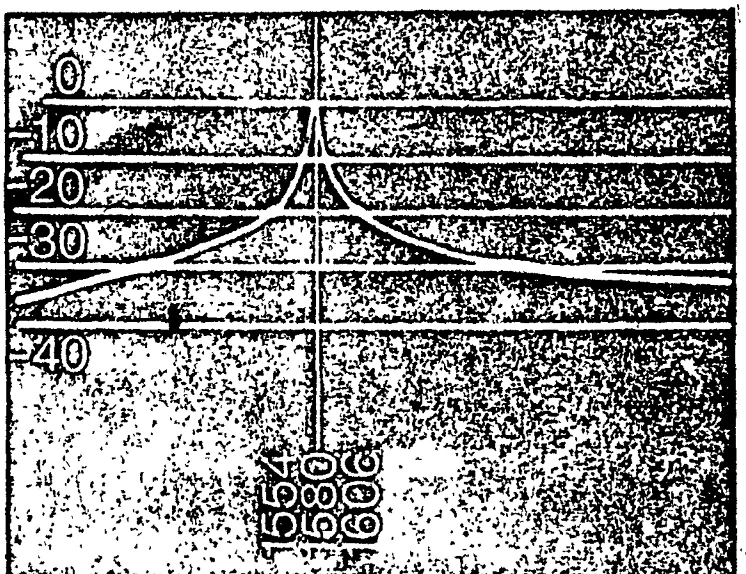


FIGURE 2.16

d) ±15V, 70° C.

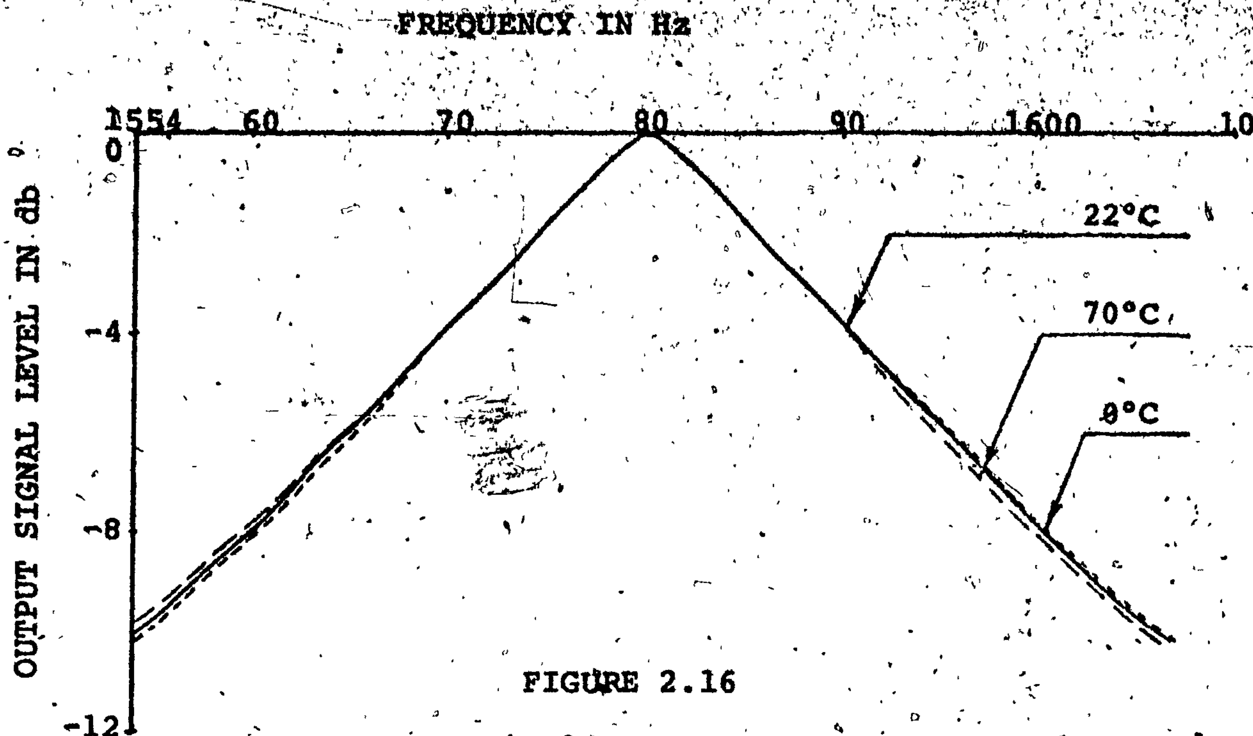


FIGURE 2.16

e) ±10V

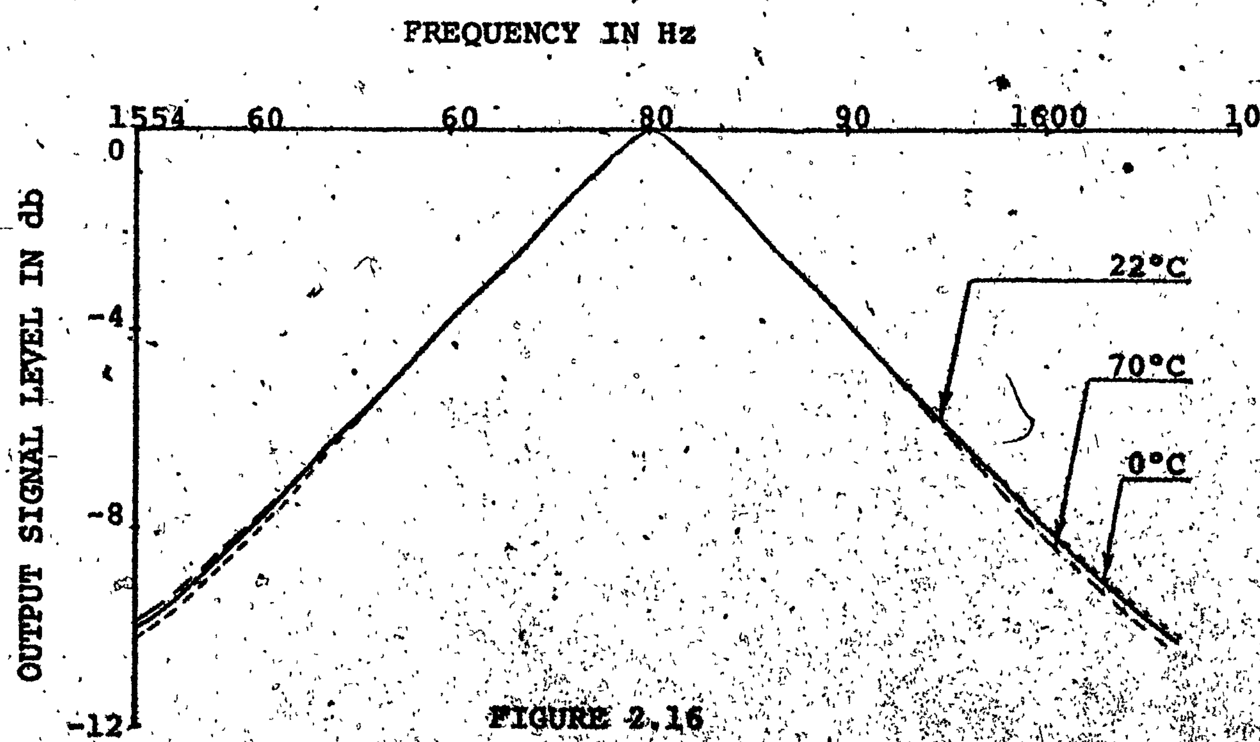


FIGURE 2.16

f) ±15V

This network was tested and it gave the following results.

$Q = 98.75$

$\omega_o = 1580$ Hz

3db. freq. $\omega_1 = 1572$ Hz $\omega_2 = 1588$ Hz

10db. freq. $\omega_3 = 1554$ Hz $\omega_4 = 1606$ Hz

The realized value of Q is within 1.25% and that of ω_o is within 1.0%. The variation can be attributed to the 1% passive elements used.

Due to the higher Q-sensitivities with respect to the passive elements, networks obtained by this decomposition are suitable for hybrid type of realization, where tracking of passive elements can be achieved.

The above network was tested for two supply d.c. voltages, ±10V and ±15V and three temperatures (only the OAs were placed in the oven), room temperature (22°C) and 70°C and the photographs of the response were taken. No appreciable difference in the response was seen during these variations. Photographs of various responses are given in Figure (2.16). In order to observe the effect of temperature and power supply variations on the response, values, as experimentally measured, are plotted in Figure 2.16 between the two 10-db points in the response.

g). <u>More Band Pass Filter Networks:</u>

The basic twin T network used in decomposition IIc can be used to realize various filter networks. Lifting of one or more of the grounded elements and connecting them to the input point assures no change in the D(s) ,[6],but this changes the N(s). This technique can

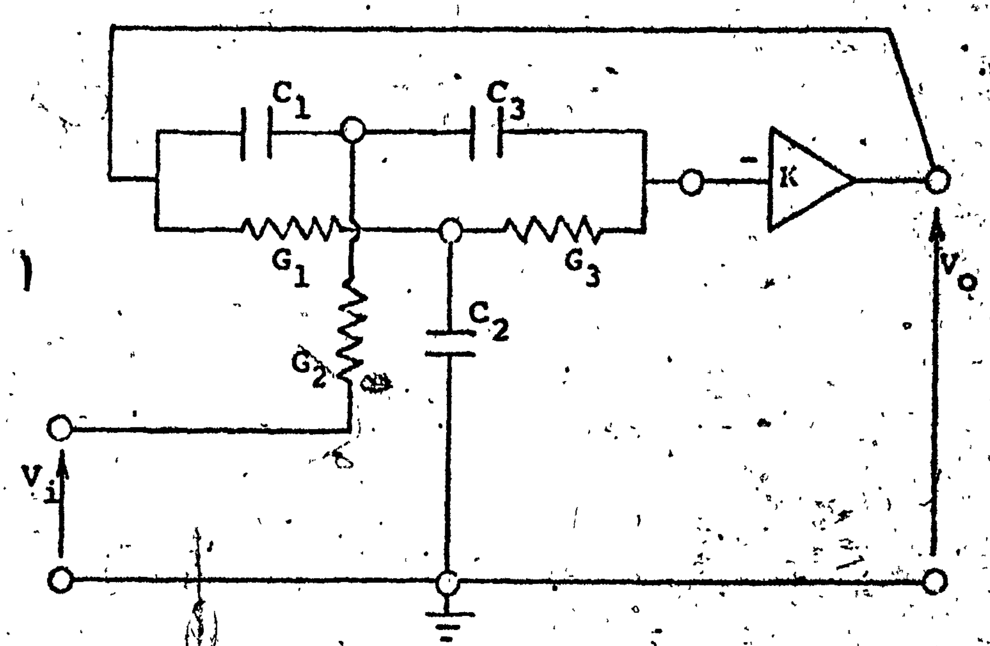


FIG. 2.17 SOME MORE BAND PASS FILTER REALIZATIONS FOR DECOMPOSITION IIc

(a)

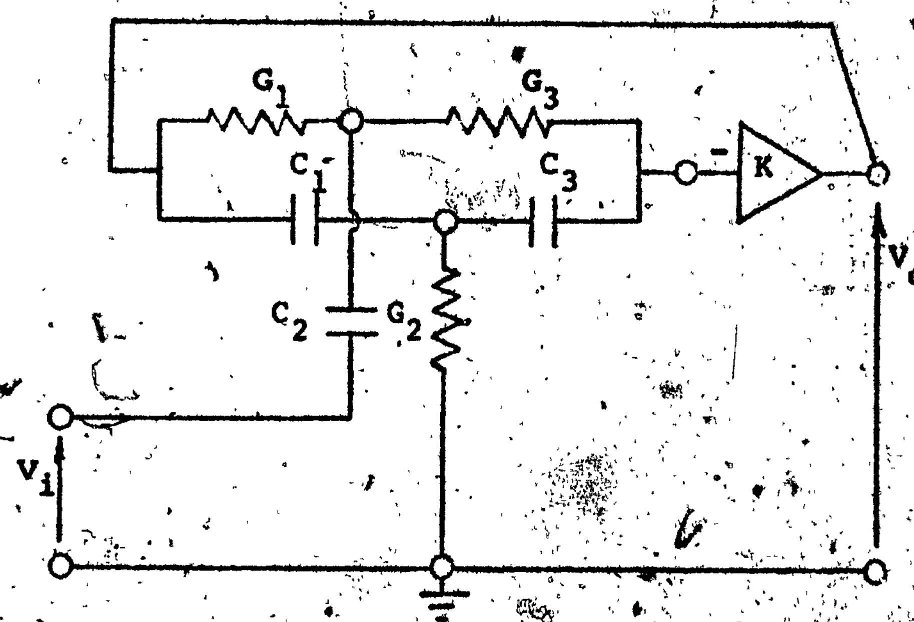


FIG. 2.17 (b)

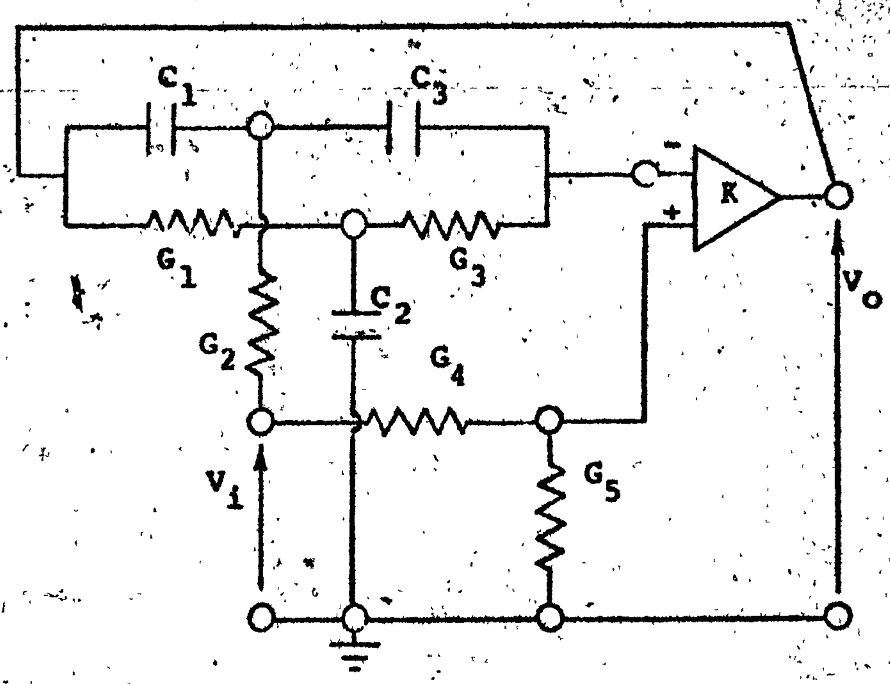


FIG. 2.18 SOME ALL PASS AND NOTCH FILTER REALIZATIONS FOR DECOMPOSITION IIc

(a)

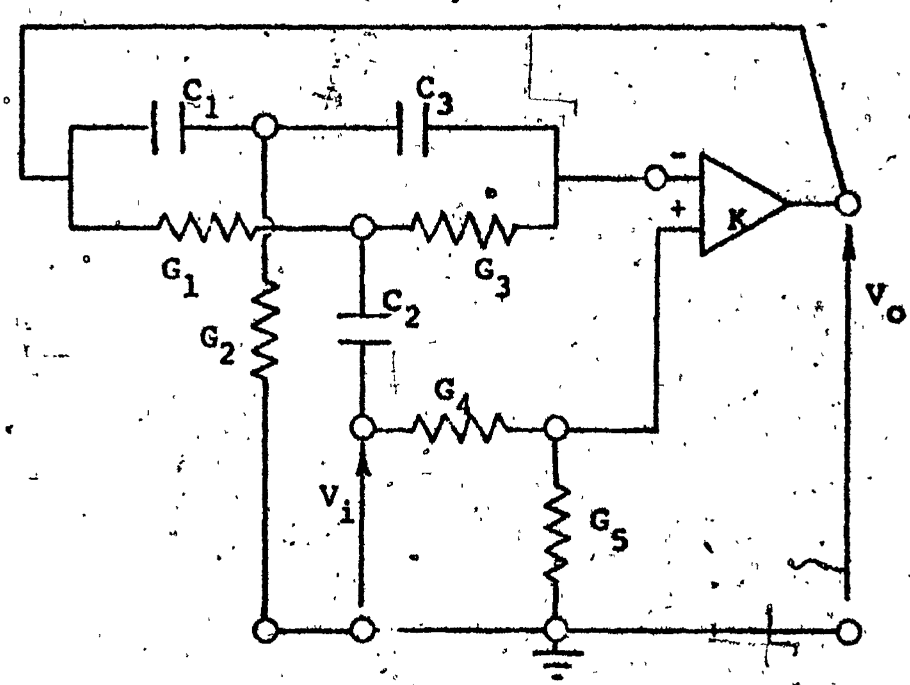


FIG. 2.18 (b)

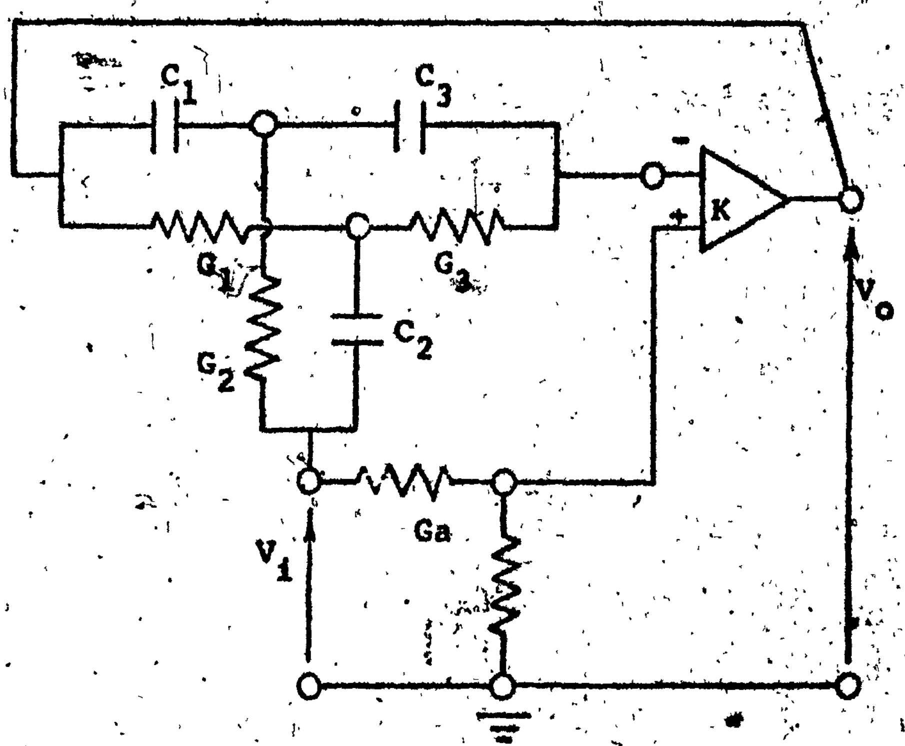


FIG. 2.18 (c)

TABLE 2.3

SOME MORE BAND PASS NETWORKS FOR DECOMPOSITION IIc

NETWORK OF	$T(s)=\frac{N(s)}{D(s)}$	ONE POLE ZERO PAIR CANCELS IF	ELEMENT VALUES
FIGURE 2.17a	$N(s)=-KsC_3G_2(sC_2+G_1+G_3)$ $D(s)=sC_3(sC_1+G_2)(sC_2+G_1+G_3)+$ $G_3(sC_2+G_1)(sC_1+sC_3+G_2)+$ $K\{s^2C_1C_3(sC_2+G_1+G_3)+$ $G_1G_3(sC_1+sC_3+G_2)\}$	$\frac{C_2}{G_1+G_3}=\frac{C_1+C_3}{G_2}$	$G_2=G_1\{b^2\eta-4(\eta+1)\}/4$ $G_3=G_1\{b^2\eta-4(\eta+1)\}/\{4(\eta+1)\}$ $C_1=G_1\{b^2\eta-4(\eta+1)\}/2b$ $C_2=\frac{b}{2}G_1$ $C_3=G_1\{b^2\eta-4(\eta+1)\}/2b\eta$ $K=bQ-1$ $\omega_o=1$
FIGURE 2.17b	$N(s)=-KsC_2G_3(sC_1+sC_3+G_2)$ $D(s)=sC_3(sC_1+G_2)(sC_2+G_1+G_3)+$ $G_3(sC_2+G_1)(sC_1+sC_3+G_2)+$ $K\{s^2C_1C_3(sC_2+G_1+G_3)+$ $G_1G_3(sC_1+sC_3+G_2)\}$	$\frac{C_2}{G_1+G_3}=\frac{C_1+C_3}{G_2}$	$G_2=G_1\{b^2\eta-4(\eta+1)\}/4$ $G_3=G_1\{b^2\eta-4(\eta+1)\}/\{4(\eta+1)\}$ $C_1=G_1\{b^2\eta-4(\eta+1)\}/2b$ $C_2=\frac{b}{2}G_1$ $C_3=G_1\{b^2\eta-4(\eta+1)\}/2b\eta$ $K=bQ-1$ $\omega_o=1$

TABLE 2.4

SOME ALL PASS AND NOTCH FILTER REALIZATIONS FOR DECOMPOSITION IIc

NETWORK OF	$T_v(s)\ \frac{N(s)}{D(s)}$	One Pole Zero Cancels if	Element Values
Fig. 2.18 (a)	$N(s)=\frac{KG_4}{(K+1)(G_4+G_5)}\left(s^2-s\frac{C_2C_3G_2\frac{G_5}{G_4}-C_2G_3(C_1+C_3)}{C_1C_2C_3}+\frac{G_1G_3(C_1+C_3)}{C_1C_2C_3}\right)$ $D(s)=s^2+s\frac{C_2C_3G_2+C_2G_3(C_1+C_3)}{(K+1)C_1C_2C_3}+\frac{G_1G_3(C_1+C_3)}{C_1C_2C_3}$ All Pass If $G_5=\frac{G_4}{K+1}\left(1+\frac{G_3(C_1+C_3)(K+2)}{C_3G_2}\right)$ Null If $G_5=G_4\frac{G_3(C_1+C_3)}{C_3G_2}$	$\frac{C_2}{G_1-G_3}=\frac{C_1+C_3}{G_2}$	$G_2=\frac{G_1[b^2n-4(n+1)]}{4}$ $G_3=\frac{G_1[b^2n-4(n+1)]}{(4(n+1))}$ $C_1=\frac{G_1[b^2n-4(n+1)]}{2b}$ $C_2=\frac{b}{2}G_1$ $C_3=\frac{G_1[b^2n-4(n+1)]}{2n}$ $K=bQ-1,\ \omega_o=1$ G_4 may be chosen abtrarily to give G_5 as per the dsired relation.
Fig. 2.18 (b)	$N(s)=\frac{KG_4}{G_4+G_5}\left(s^2-s\frac{\frac{G_5}{G_4}C_2G_3(C_1+C_3)-C_2C_3G_2}{C_1C_2C_3}\ \frac{G_1G_3(C_1+C_3)}{C_1C_2C_3}\right)$ $D(s)=s^2+s\frac{C_2C_3G_2+C_2G_3(C_1+C_3)}{(K+1)C_1C_2C_3}+\frac{G_1G_3(C_1+C_3)}{C_1C_2C_3}$ All Pass If $G_5=G_4\left(\frac{1}{K+1}+\frac{K+2}{K+1}\cdot\frac{C_3G_2}{G_3(C_1+C_3)}\right)$ Null If $G_5=G_4\frac{C_3G_2}{G_3(C_1+C_3)}$	$\frac{C_2}{G_1+G_3}=\frac{C_1+C_3}{G_2}$	$G_2=\frac{G_1(b^2n-4(n+1))}{4}$ $G_3=\frac{G_1(b^2n-4(n+1))}{(4(n+1))}$ $C_1=\frac{G_1(b^2n-4(n+1))}{2b}$ $C_2=\frac{b}{2}G_1$ $C_3=\frac{G_1(b^2n-4(n+1)\}}{2bn}$ $K=bQ-1,\ \omega_o=1$ G_4 may be chosen arbitrarily to give G_5 as per the desired relation.
Fig. 2.18 (c)	$N(s)=\frac{KG_4}{(K-1)(G_4-G_5)}\left(s^2-s\frac{G_5}{G_4}\frac{C_2G_3(C_1+C_3)+C_2C_3G_2}{C_1C_2C_3}-\frac{G_1G_3(C_1+C_3)}{C_1C_2C_3}\right)$ $D(s)=s^2+s\frac{C_2C_3G_2+C_2G_3(C_1+C_3)}{(K+1)C_1C_2C_3}+\frac{G_1G_3(C_1+C_3)}{C_1C_2C_3}$ All Pass If $G_4=(K+1)G_5$ Null If $G_5=0$, G_4 arbitrary	$\frac{C_2}{G_1+G_3}=\frac{C_1+C_3}{G_2}$	$G_2=\frac{G_1(b^2n-4(n+1))}{4}$ $G_3=\frac{G_1(b^2n-4(n+1))}{(4(n+1))}$ $C_1=\frac{G_1(b^2n-4(n+1))}{2b}$ $C_2=\frac{b}{2}G_1$, $C_3=\frac{G_1(b^2n-4(n+1))}{2bn}$ $K=bQ-1,\ \omega_o=1$ G_5 may be chosen arbitrarily to give G_4 as per the desired relation.

be employed to achieve other Band Pass Networks shown in Table (2.3). The Q- and ω_o-sensitivities and their GSPs with respect to the active parameter remain as before.

It is noted that for all these Band Pass Networks, prescription of $S_K^{\omega_o}=0$ and the minimization of S_K^Q, simultaneously reduces $\left.\frac{\Delta T(s)}{T(s)}\right|_{s=j\omega_o}$ [17][35] as well.

h). <u>All Pass and Notch Filter Networks:</u>

All pass networks have been reported in the literature [6], [36-38]. [36 38] realize only negative real axis poles while [6] utilizes decomposition III. No all pass networks using decomposition IIc have been reported in the literature to the author's knowledge so far.

Table 2.4 gives some all pass and null networks from the basic twin T network. These networks have the same D(s) as in those of table (2.3) and thus have same Q- and ω_o-sensitivities with respect to the active parameter with $S_K^{\omega_o}= 0$ and with an optimized S_K^Q or GQ-SP. Also the zero-Q(Q_z) and zero-ω_o sensitivities with respect to the active parameter are zero.

i). <u>Low Pass Filter Network:</u>

The basic twin T used as shown in Figure (2.19) gives a Low Pass Network realization.

Analysis yields

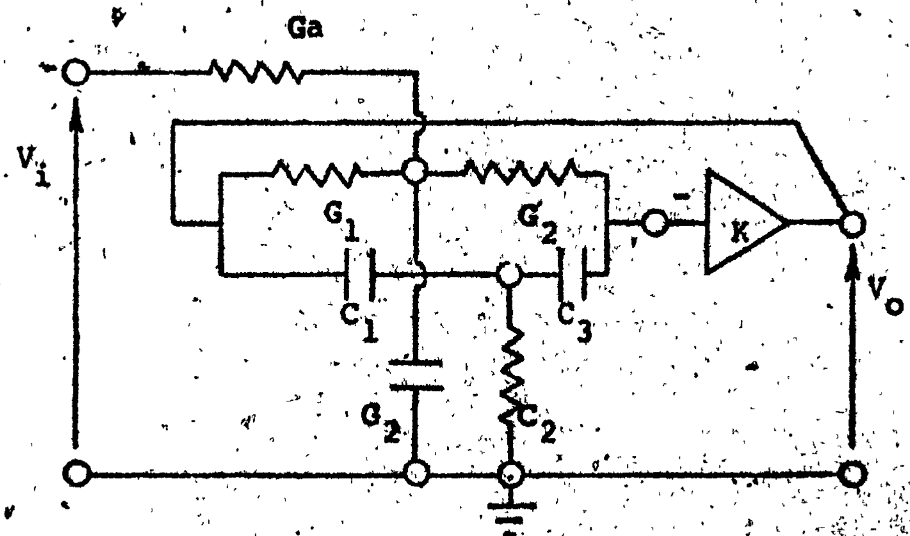


FIG. 2.19 LOW PASS FILTER REALIZATION FOR DECOMPOSITION IIc

$$T(s) = \frac{-KG_aG_3(sC_1 + sC_3 + G_2)}{s^2C_1C_3(sC_2+G_a+G_1+G_3)(K+1)+sC_3G_2(sC_2+G_a+G_1+G_2) +sC_2G_3(sC_1+sC_3+G_2)+G_3(sC_1+sC_3+G_2)[(K+1)G_1+G_a]} \tag{2.90}$$

One negative real axis pole zero pair cancels if

$$\frac{C_2}{G_a+G_1+G_3} = \frac{C_1+C_3}{G_2}$$

After one pole zero pair cancellation the T(s) is

$$T(s) = \frac{-KG_aG_3(C_1+C_3)}{s^2C_1C_2C_3(K+1)+sC_2C_3G_2+sC_2G_3(C_1+C_3)+G_3(C_1+C_3)[(K+1)G_1+G_a]}$$

$$= \frac{-(K/(K+1)(G_aG_3(C_1+C_3))/\ C_1C_2C_3}{s^2+s\ \dfrac{C_2C_3G_2+C_2G_3(C_1+C_3)}{C_1C_2C_3(K+1)} + \dfrac{G_3(C_1+C_3)[G_a+(K+1)G_1]}{C_1C_2C_3(K+1)}} \tag{2.91a}$$

It may be noted that

$$\frac{\alpha_2}{\alpha_1} \neq \frac{\gamma_2}{\gamma_1} \quad \text{and hence} \quad S_K^{\omega_o} \neq 0$$

and

$$\omega_o = \sqrt{\frac{G_3(C_1+C_3)[G_a+(K+1)G_1]}{C_1C_2C_3(K+1)}} \tag{2.91b}$$

$$Q = \frac{\sqrt{C_1 C_2 C_3 (K+1) G_3 (C_1+C_3) [G_a+(K+1)G_1]}}{C_2 C_3 G_2 + C_2 G_3 (C_1+C_3)} \qquad (2.91c)$$

$$S_K^{\omega_o} = -\frac{K}{2(K+1)} \times \frac{G_a}{G_a+G_1(K+1)} \qquad (2.91d)$$

$$S_K^Q = \frac{1}{2}\,\frac{K}{K+1} + \frac{1}{2}\,\frac{KG_1}{G_a+G_1(K+1)} \qquad (2.91e)$$

One set of design equations is

$$G_3 = G_1 \qquad (2.92a)$$

$$G_2 = (n+1)G_1 \qquad (2.92b)$$

$$C_1 = Q(n+1)\frac{G_1}{K+1} \qquad (2.92c)$$

$$C_2 = 2Q(1+n)G_1\frac{(2+G_a/G_1)}{K+1} \qquad (2.92d)$$

$$C_3 = Q(1+1/n)\,G_1/(K+1) \qquad (2.92e)$$

$$K+1 = \frac{G_a}{2G_1}\left[-1+\sqrt{1+16Q^2(1+1/n)(G_1/G_a)(1+2(G_1/G_a))}\,\right] \qquad (2.92f)$$

where G_1 and n may be chosen arbitrarily as positive numbers.

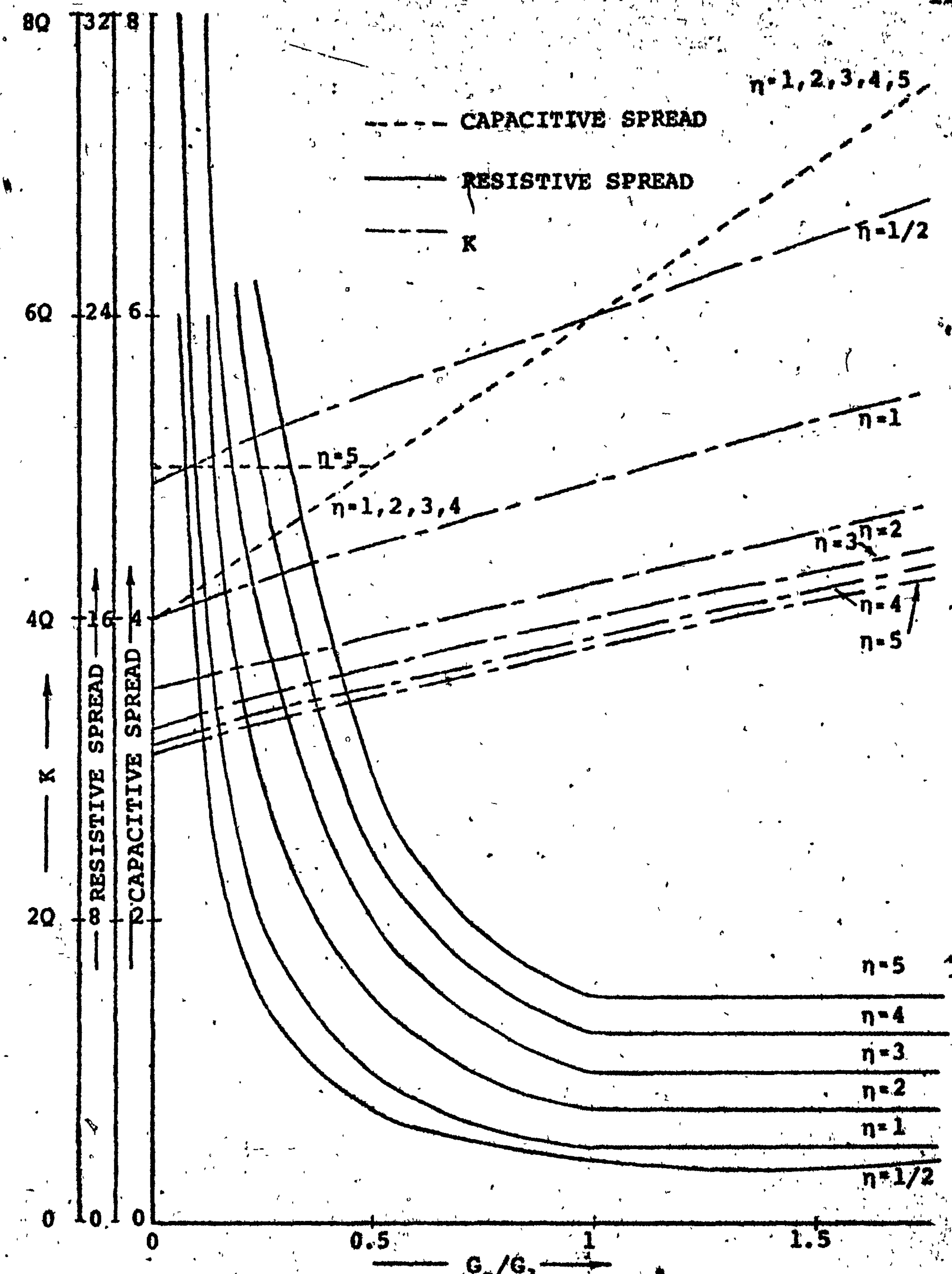


FIGURE 2.20

K, CAPACITIVE SPREAD AND RESISTIVE SPREAD VERSUS G_a/G_1 PLOTS FOR LP FILTER NETWORK OF DECOMPOSITION IIc

With the assumptions $K >> 1$ and $Q >> 1$, the design curves have been plotted with K, capacitive spread and resistive spread against $\frac{G_a}{G_1}$ in Figure (2.20). From these design curves it is possible to select a suitable $\frac{G_a}{G_1}$ for a prescribed value of K, for the capacitive spread being as low as possible. The least value of K available for a capacitive spread of five is 3.1Q for $\eta = 5$, $G_a/G_1 \rightarrow 0$. Since S_K^Q is very nearly equal to unity K also gives GQ-SP approximately.

For the network of Figure (2.19), $S_K^{\omega_o} \neq 0$ but it is very small if not prescribed. If $S_K^{\omega_o}$ is prescribed, this fixes K, as G_a/G_1 has to be within certain limits and this in turn would fix the realized Q. S_K^Q is very nearly equal to unity if not prescribed. This too, if prescribed, would fix K, and as a result the realized Q. On the other hand it is possible to optimize K for a given Q while the capacitive spread is being controlled, and this in turn optimizes GQ-SP. $S_K^{\omega_o}$ is of the order 1/2Q. This makes the variation in T(s) dependent on $\frac{\Delta Q}{Q}$ only.

j). High Pass Filter Network:

RC-CR transformation of the network shown in Figure (2.19) gives the network of Figure (2.21) and this realizes a high pass network.

Analysis yields

$$T(s) = \frac{-Ks^2C_aC_3(sC_2+G_1+G_3)}{s^2C_3(sC_2+G_1+G_3)[C_a+(K+1)C_1]+sC_3G_2(sC_2+G_1+G_3) + sC_2G_3(sC_1+sC_3+sC_a+G_2)+G_1G_3(sC_1+sC_3+sC_a+G_2)(K+1)} \quad (2.93)$$

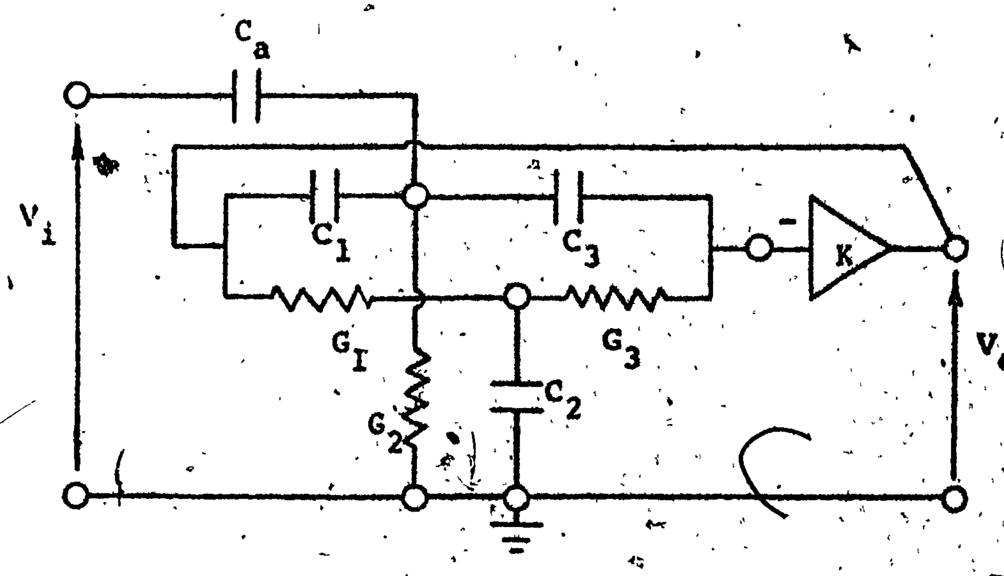


FIG. 2.21 HIGH PASS FILTER REALIZATION FOR DECOMPOSITION IIc

One negative real axis pole zero pair cancels if

$$\frac{C_a+C_1+C_3}{G_2} = \frac{C_2}{G_1 + G_3}$$

After one pole zero pair cancellation, T(s) is

$$T(s) = \frac{-Ks^2C_a/[C_a+(K+1)C_1]}{s^2 + s\,\dfrac{C_3G_2 + G_3(C_a+C_1+C_3)}{C_3[C_a+C_1(K+1)]} + \dfrac{G_1G_3(K+1)(C_a+C_1+C_3)}{C_2C_3[C_a+C_1(K+1)]}} \qquad (2.94a)$$

Once again it may be noted that $\frac{\alpha_2}{\alpha_1} \neq \frac{\gamma_2}{\gamma_1}$ and hence $S_K^{\omega_o} \neq 0$.

Also

$$\omega_o = \sqrt{\frac{G_1G_3(K+1)(C_a+C_1+C_2)}{C_2C_3[C_a+(K+1)C_1]}} \qquad (2.94b)$$

$$Q = \frac{\sqrt{C_2C_3[C_a+(K+1)C_1]G_1G_3(K+1)(C_a+C_1+C_2)}}{C_2C_3G_2 + C_2G_3(C_a+C_1+C_3)} \qquad (2.94c)$$

$$S_K^{\omega_o} = \frac{(1/2)K}{K+1} \cdot \frac{C_a/C_1}{K+1+C_a/C_1} \qquad (2.94d)$$

$$S_K^Q = \frac{1}{2}\,\frac{K}{K+1} + \frac{1}{2}\,\frac{K}{(K+1)+C_a/C_1} \qquad (2.94e)$$

$S_K^{\omega_o}$ is not zero, but is very small if not prescribed. If it is prescribed it fixes K, for a permissible value of C_a/C_1, and this fixes the realized Q. Similarly S_K^Q is almost equal to unity if not prescribed. If it is prescribed this would fix K for any given value of C_a/C_1 and this in turn would fix realized Q. On the other hand it is possible to optimize K for any given capacitive spread and thus minimize GQ-SP in this realization.

One set of design equations for this realization is

$$C_3 = C_1 \tag{2.95a}$$

$$C_2 = (n + 1)C_1 \tag{2.95b}$$

$$G_2 = \frac{C_1}{2Q}[K + 1 + C_a/C_1] \tag{2.95c}$$

$$G_1 = \frac{nC_1}{2Q} \times \frac{K+1+C_a/C_1}{2 + C_a/C_1} \tag{2.95d}$$

$$G_3 = \frac{C_1}{2Q} \times \frac{K+1+C_a/C_1}{2 + C_a/C_1} \tag{2.95e}$$

$$K+1 = \frac{C_a}{2C_1}\left[-1 + \sqrt{1 + 16Q^2(1 + 1/n)(C_1/C_a)(1 + 2(C_1/C_a))}\right] \tag{2.95f}$$

where C_1 and n may be chosen arbitrarily as positive numbers.

Design curves have been plotted in Figure (2.22), with K, the capacitive spread and the resistive spread against C_a/C_1 with the assumption $K \gg 1$, $Q \gg 1$. From these curves it is possible to obtain the desired value of C_a/C_1 for a prescribed K or GQ-SP with the capacitive spread being the lowest possible. From these curves it is possible to say that for the network of Figure (2.20), least possible

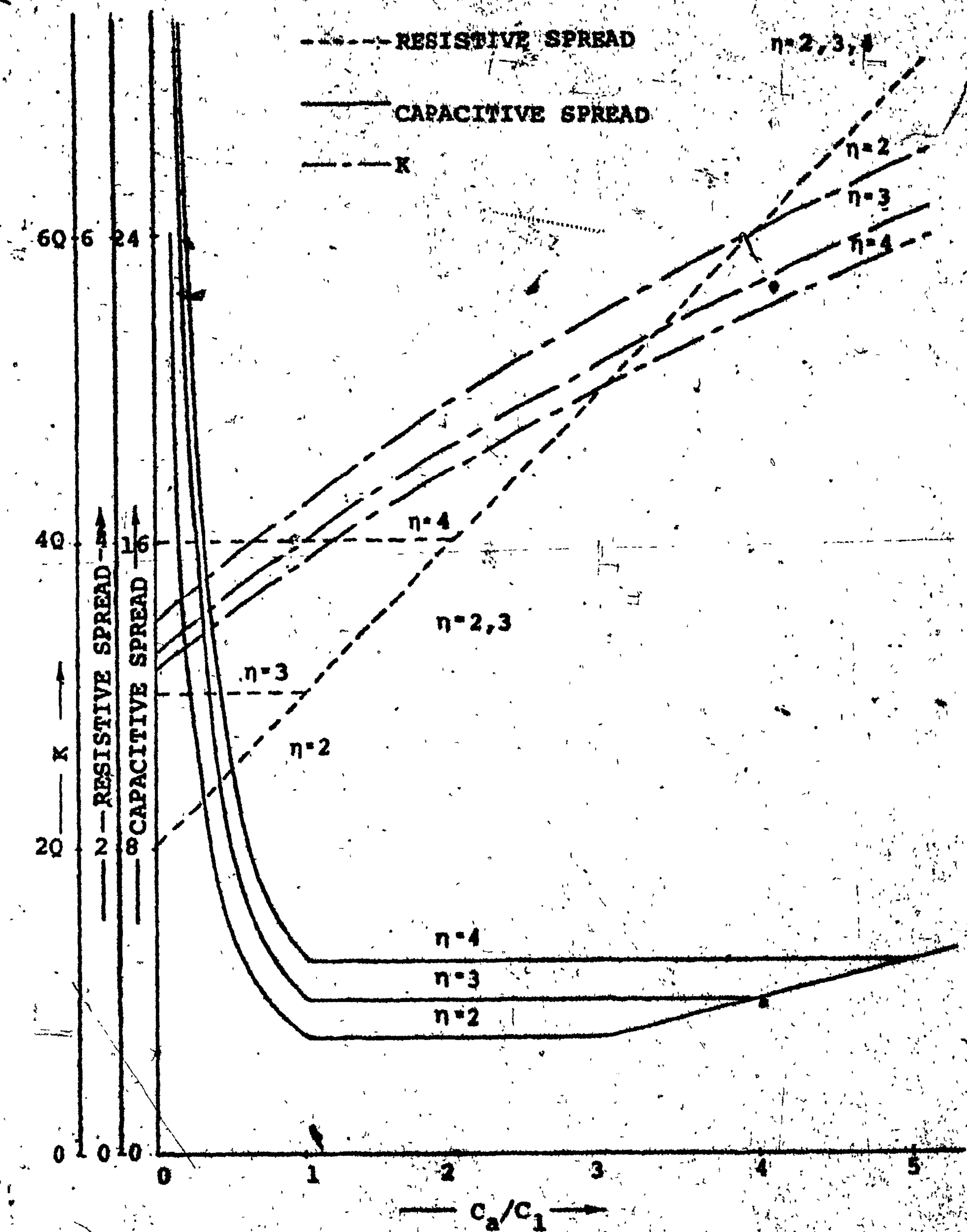


FIGURE 2.22

K, CAPACITIVE SPREAD AND RESISTIVE SPREAD VERSUS C_a/C_1 PLOTS FOR HP FILTER NETWORK OF DECOMPOSITION IIc value of K and GQ-SP shall be 3.86Q approximately for a capacitive spread of five and with a value $C_a/C_1 = 3$. $S_K^{\omega_o}$ is of the order 1/3 Q. This makes the variation in T(s) dependent on $\frac{\Delta Q}{Q}$ only.

2.8.7 DECOMPOSITION III:

In this decomposition K appears in s^2, s and s^o terms. D(s) can be of two types

$$D(s) = s^2(\alpha_1 + K\alpha_2) + s(\beta_1 + K\beta_2) + \gamma_1 + K\gamma_2$$

and

$$D(s) = s^2(\alpha_1 + K\alpha_2) + s(\beta_1 - K\beta_2) + \gamma_1 + K\gamma_2$$

These shall be discussed separately.

a). Ks Term is Positive:

$$D(s) = s^2(\alpha_1 + K\alpha_2) + s(\beta_1 + K\beta_2) + \gamma_1 + K\gamma_2 \quad (2.96a)$$

from which

$$Q = \frac{\sqrt{(\alpha_1 + K\alpha_2)(\gamma_1 + K\gamma_2)}}{\beta_1 + K\beta_2} \quad (2.96b)$$

$$\omega_o = \frac{\sqrt{\gamma_1 + K\gamma_2}}{\sqrt{\alpha_1 + K\alpha_2}} \quad (2.96c)$$

$$S_K^Q = \frac{1}{2}\frac{K\alpha_2}{\alpha_1 + K\alpha_2} + \frac{1}{2}\frac{K\gamma_2}{\gamma_1 + K\gamma_2} - \frac{K\beta_2}{\beta_1 + K\beta_2} \quad (2.96d)$$

$$S_K^{\omega_o} = \frac{1}{2}\frac{K\gamma_2}{\gamma_1 + K\gamma_2} - \frac{1}{2}\frac{K\alpha_2}{\alpha_1 + K\alpha_2} \qquad (2.96e)$$

$$S_K^Q + S_K^{\omega_o} = \frac{K\gamma_2}{\gamma_1 + K\gamma_2} - \frac{K\beta_2}{\beta_1 + K\beta_2} \qquad (2.96f)$$

$$S_K^Q - S_K^{\omega_o} = \frac{K\alpha_2}{\alpha_1 + K\alpha_2} - \frac{K\beta_2}{\beta_1 + K\beta_2} \qquad (2.96g)$$

with the sensitivity bounds being

$$-1 < S_K^Q < 1 \qquad (2.96h)$$

$$-\frac{1}{2} < S_K^{\omega_o} < \frac{1}{2} \qquad (2.96i)$$

$$-1 < S_K^Q + S_K^{\omega_o} < 1 \qquad (2.96j)$$

$$-1 < S_K^Q - S_K^{\omega_o} < 1 \qquad (2.96k)$$

Passive network having a T(s) of the type

$$T(s) = \frac{s^2 + as + 1}{s^2 + s/Q_p + 1}$$

where $a \ll 1/Q_p$, is available in general with 3-capacitors with one pole cancelling with one zero of the T(s). $S_K^{\omega_o} = 0$ can be prescribed in this case. Prescribing S_K^Q shall fix K and as a result the realizable value of Q shall get fixed. Hence, prescribing S_K^Q is not advisable in this decomposition.

The network of Figure (2.23) gives a realization of this decomposition.

Analysis yields

$$T(s) = \frac{-KsC_3G_a\ (sC_2 + G_1 + G_3)}{sC_3(sC_2+G_1+G_3)(sC_1+G_a+G_2+G_4)+G_3(sC_2+G_1)(sC_1+sC_3+G_a+G_2+G_4)+K[sC_3(sC_1+G_4)(sC_2+G_1+G_3)+G_1G_3(sC_1+sC_3+G_a+G_2+G_4)]} \quad (2.97)$$

for $\dfrac{G_1 + G_3}{C_2} = \dfrac{G_a + G_2 + G_4}{C_1 + C_3}$ one zero shall cancel with one pole. After one pole-zero cancellation in T(s), we get

$$D(s) = s^2C_1C_2C_3(K+1)+s[C_2C_3G_4(K+1)+C_2C_3(G_a+G_2)+C_2G_3(C_1+C_3)] + G_1G_3(K+1)(C_1+C_3) \quad (2.98a)$$

and

$$Q = \frac{\sqrt{C_1C_2C_3G_1G_3(C_1 + C_3)}(K + 1)}{C_2C_3G_4(K + 1)+C_2C_3(G_a + G_2)+C_2G_3(C_1 + C_3)} \quad (2.98b)$$

$$\omega_o = \sqrt{\frac{G_1G_3(C_1 + C_3)}{C_1C_2C_3}} \quad (2.98c)$$

$$S_K^Q = \frac{K}{K + 1} - \frac{KC_2C_3G_4}{C_2C_3G_4(K+1)+C_2C_3(G_a+G_2)+C_2G_3(C_1 + C_3)} \quad (2.98d)$$

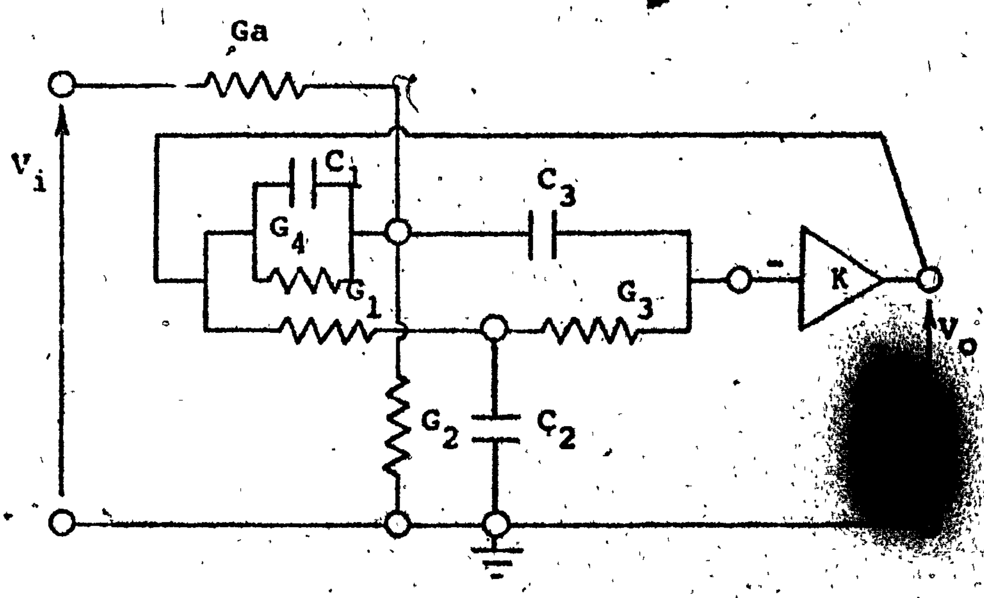


FIG. 2.23 BAND PASS FILTER REALIZATION FOR DECOMPOSITION IIIa

$$S_K^{\omega_o} = 0 \qquad (2.98e)$$

also $$(K+1)\left[\sqrt{C_1C_2C_3G_1G_3(C_1 + C_3)} - QC_2C_3G_4\right]$$

$$= Q[C_2C_3(G_a + G_2) + C_2G_3(C_1 + C_3)] \qquad (2.98f)$$

For a given Q, K shall be the smallest if $G_4 = 0$, in which case the decomposition corresponds to IIc. Also on the other hand for a given K, Q shall be largest if $G_4 = 0$.

A close look at the network as well as T(s) of eq. (2.97) confirms that decomposition IIc is a special case of this decomposition with $G_4 = 0$. Compared to IIc, this decomposition shall have a larger K for a given Q and a smaller Q for a given K and a larger resistive spread.

Assuming the elemental values for the realization of this decomposition being the same as that of the realization for decomposition IIc, we shall relate the value of G_4, the additional resistor required and the pole Q of this circuit 'Q_{3a}' with the Q of the corresponding network of decomposition IIc.

For $$C_1 = \frac{7.25}{5}, \quad C_2 = \frac{5}{4}, \quad C_3 = \frac{7.25}{25}, \quad G_1 = 1. \quad G_2 = G_a = \frac{7.25}{8}$$

$$G_3 = \frac{7.25}{24}, \quad K_1 = 2.5Q - 1$$

from eq. (2.98)

$$D(s) = s^2 + s[1/Q + G_4/1.45] + 1 \qquad (2.99a)$$

and $$Q_{3a} = \frac{1}{1/Q + G_4/1.45} = \frac{Q}{1 + QG_4/1.45} \qquad (2.99b)$$

Letting $G_4 = \dfrac{1.45}{Q \times 100}$ shall give

$$Q_{3a} = \frac{Q}{1.01} \qquad \text{i.e. } 1\% \text{ variation} \qquad (2.100a)$$

$$S_K^{Q_{3a}} = \frac{2.5Q - 1}{2.525Q} \qquad (2.100b)$$

Resistive spread: 1: 100Q/1.45 (2.100c)

Gain Q-sensitivity product $= (2.5Q-1)(100/101)[1 - 1/(2.5Q)]$

$$\approx (2.5Q - 1)^2/(2.5 \times 1.01Q) \qquad (2.100d)$$

$$< 2.5Q$$

This realization can be made comparable to the realization for decomposition IIc at the expense of resistive spread. Q- and ω_o-sensitivities for $D_3(s)$ can be computed in the same manner as in decomposition IIc and they shall be of the same order.

b). <u>Ks-Term is Negative:</u>

D(s) is of the type

$$D(s) = s^2(\alpha_1 + K\alpha_2) + s(\beta_1 - K\beta_2) + \gamma_1 + K\gamma_2 \qquad (2.101a)$$

from which

$$Q = \frac{\sqrt{\alpha_1 + K\alpha_2}\,\sqrt{\gamma_1 + K\gamma_2}}{\beta_1 - K\beta_2} \tag{2.101b}$$

$$S_K^Q = \frac{1}{2}\,\frac{K\alpha_2}{\alpha_1 + K\alpha_2} + \frac{1}{2}\,\frac{K\gamma_2}{\gamma_1 + K\gamma_2} + \frac{K\beta_2}{\beta_1 - K\beta_2} \tag{2.101c}$$

$$S_K^{\omega_o} = \frac{1}{2}\,\frac{K\gamma_2}{\gamma_1 + K\gamma_2} - \frac{K\alpha_2}{\alpha_1 + K\alpha_2} \tag{2.101d}$$

$$S_K^Q + S_K^{\omega_o} = \frac{K\gamma_2}{\gamma_1 + K\gamma_2} + \frac{K\beta_2}{\beta_1 - K\beta_2} \tag{2.101e}$$

$$S_K^Q - S_K^{\omega_o} = \frac{K\alpha_2}{\alpha_1 + K\alpha_2} + \frac{K\beta_2}{\beta_1 - K\beta_2} \tag{2.101f}$$

with the sensitivity bounds being

$$0 < S_K^Q < \infty \tag{2.101g}$$

$$-1/2 < S_K^{\omega_o} < 1/2 . \tag{2.101h}$$

$$0 < S_K^Q + S_K^{\omega_o} < \infty \tag{2.101i}$$

$$0 < S_K^Q - S_K^{\omega_o} < \infty \tag{2.101j}$$

Passive networks having a transfer function of the type $\dfrac{s^2 - a_1 s + a_o}{s^2 + b_1 s + b_o}$ where $a_1 > 0$, $b_o \geq a_o$ and $b_1 > 2\sqrt{b_o}$, in general are realizable with more than three capacitors and as such this is not discussed further in this thesis.

2.7 SUMMARY AND DISCUSSIONS:

At this stage it is possible to summarize the various properties of the decompositions considered in this chapter and also discuss some of the realizations.

i) Decompositions Ia and Ib:

These are suitable only when S_K^Q or $S_K^{\omega_o}$ is not prescribed. Q, S_K^Q and $S_K^{\omega_o}$ are interrelated and as such one cannot be prescribed independently of the other. The value of K required for a realization using these decompositions is high of the order $8Q^2$.

ii) Decompositions IIa and IIb:

For these decompositions $S_K^{\omega_o}$ cannot be prescribed independently of Q. However, these are suitable for the prescription of S_K^Q only. In fact S_K^Q can be specified to be zero. The amplifier gain K required is high of the order $32Q^2$. Q- and ω_o-sensitivities with respect to passive elements are low. Therefore, these networks are suitable for implementation by discrete elements. Pole of the OA has considerable effect on Q and ω_o for higher frequencies. Therefore these networks are suitable for low Q realizations at low frequencies (Q = 5 and ω_o = 100 Hz).

iii) Decomposition IIc:

For this decomposition, $S_K^{\omega_o}$ can be specified to be zero and simultaneously S_K^Q or GQ-SP can be minimized, while controlling the element spread as well. Thus the deviation in the transfer function also is minimized. The value of K for the networks of this decomposition is of the order Q and therefore these networks realize higher Q for a given K compared to other decompositions considered in this chapter.

Regarding the realizations, Band Pass, All Pass and Notch Filters have been realized with $S_K^{\omega_o}$ specified to be zero, and minimized S_K^Q or GQ-SP of $Q\sqrt{6}$ with a maximum capacitive spread of five. Low Pass and High Pass filters have been realized with a low $S_K^{\omega_o}$ (of the order 1/2 Q), and minimized S_K^Q or GQ-SP of 3.1Q and 3.86Q respectively. All these realizations are of the non-minimal capacitors type. The passive sensitivities are high (of the order of Q) and therefore the networks belonging to this decomposition are suitable for implementation by hybrid IC technology.

iv) Decomposition III:

This decomposition also can give a high Q realization with $S_K^{\omega_o}$ specified to be zero. However, the value of K required for a given Q is higher than that required in decomposition IIc.

CHAPTER III

POSITIVE FEEDBACK SINGLE AMPLIFIER NETWORKS

CHAPTER III

POSITIVE FEEDBACK SINGLE AMPLIFIER NETWORKS

3.1 INTRODUCTION

In this chapter the decomposition Ic of D(s) [19] shall be considered. The corresponding networks use only one amplifier with positive feedback. The bounds on Q-sensitivity with respect to amplifier gain are derived. In general, more than two capacitors are required for these realizations. The corresponding third degree denominator polynomial $D_3(s)$ is examined for Q- and ω_0-sensitivities.

Several networks belonging to this type of decomposition have been reported in the literature [8-13]. All of these networks use coefficient matching technique to obtain the realizations. A synthesis procedure that does not require matching coefficients is presented in this chapter. A method of optimizing Q-sensitivity or GQ-SP is given. Using this method it is possible to obtain the desired realization with either S_K^Q or GQ-SP being prescribed or minimized while the element spread is maintained within specified limits. ω_o-sensitivity in this case is zero.

3.2 SENSITIVITIES AND THEIR BOUNDS:

In decomposition Ic amplifier gain K appears in s terms only in D(s). A general biquadratic D(s) for decomposition Ic is

$$D(s) = s^2 + \frac{\omega_o}{Q} s + \omega_o^2$$

which may be re-written as

$$D(s) = s^2 + (b + \frac{1}{Q}) s + 1 - Kb_1 s \qquad (3.1)$$

where

$$b_1 = b/K$$

Further eq. (3.1) may be re-written as

$$D(s) = s^2\alpha + s(\beta_1 - K\beta_2) + \gamma \qquad (3.2)$$

where α, β_1, β_2 and γ are functions of the resistors and the capacitors in the network and 'b' is an arbitrary positive constant.

Also $\alpha = 1, \ \gamma = 1, \ \beta_1 - K\beta_2 = 1/Q$

$$Q = \frac{\sqrt{\alpha\gamma}}{\beta_1 - K\beta_2} \qquad (3.3a)$$

$$\omega_o = \frac{\sqrt{\gamma}}{\sqrt{\alpha}} \qquad (3.3b)$$

and

$$S_K^Q = \frac{K\beta_2}{\beta_1 - K\beta_2} \qquad (3.3c)$$

$$S_K^{\omega o} = 0 \qquad (3.3d)$$

Solving eq. (3.3) gives

$$S_K^Q = \frac{K\beta_2}{\beta_1 - K\beta_2} = \frac{K\beta_2 Q}{\sqrt{\alpha\gamma}} = \frac{Q}{\sqrt{\alpha\gamma}}\left[\beta_1 - \frac{\sqrt{\alpha\gamma}}{Q}\right]$$

$$= Q\,\frac{\beta_1}{\sqrt{\alpha\gamma}} - 1 \qquad (3.4)$$

Considering the passive part of D(s), we know that its roots shall be simple, that is

$$\frac{\beta_1}{2} > \sqrt{\alpha\gamma} \tag{3.5}$$

From (3.4) and (3.5), we get

$$S_K^Q = Q\frac{\beta_1}{\sqrt{\alpha\gamma}} - 1 > 2Q - 1 \tag{3.6}$$

Now the bounds on S_K^Q are:

$$2Q - 1 < S_K^Q < \infty$$

This is the same as found in [19].

3.3 REALIZABILITY CONDITIONS AND REALIZATIONS:

We shall use the general network shown in Figure 1.4 for realizations. Also we shall first focus our attention to Band Pass and Low Pass realizations only, while the others shall be dealt with later.

3.3.1 THE BAND PASS FILTER NETWORKS:

a). Realization

A general B P, T(s) shall be

$$T(s) = \frac{Kas}{s^2 + (b + \frac{1}{Q} - b_1K)\, s + 1} \tag{3.7}$$

where

$$b_1K = b$$

and

$$S_K^Q = bQ$$

Dividing the numerator N(s) and denominator D(s) by $(\frac{s}{\epsilon_1} + 1)$, gives, upon comparison with the network of Figure 1.4, one of the identifications as follows:

$$-Y_{21a} = \frac{as}{\frac{s}{\epsilon_1} + 1} \qquad (3.8a)$$

$$-Y_{12b} = \frac{b_1' s}{\frac{s}{\epsilon_1} + 1} = \frac{\frac{b}{K} s}{\frac{s}{\epsilon_1} + 1} \qquad (3.8b)$$

$$Y_{11b} + Y_{22a} = \frac{s^2 + (b+1/Q)\ s + 1}{\frac{s}{\epsilon_1} + 1} \qquad (3.8c)$$

Hereafter the problem changes to that of passive synthesis with an exact realization of given RC networks from [39]. Using Bialkow Gerst synthesis procedure [39], the following identification is possible

$$Y_{11b} = \frac{s^2 + b/K\ s}{\frac{s}{\epsilon_1} + 1} \qquad (3.9a)$$

$$Y_{22a} = \frac{s\left[b(1-1/K)+1/Q\right] + 1}{\frac{s}{\epsilon_1} + 1} \qquad (3.9b)$$

$$-Y_{12b} = \frac{b}{K} s \,/ (\frac{s}{\epsilon_1} + 1) \qquad (3.9c)$$

$$-Y_{12a} = as \,/\, (\frac{s}{\epsilon_1} + 1) \qquad (3.9d)$$

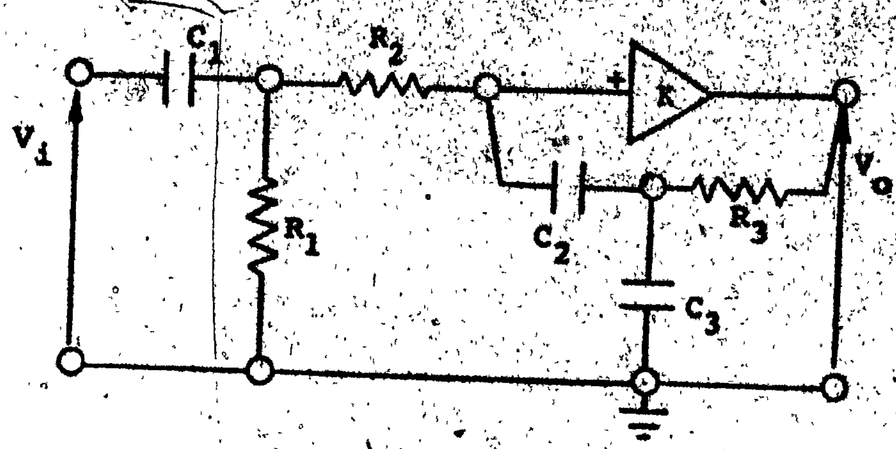


FIG. 3.1 BAND PASS FILTER NETWORK FOR DECOMPOSITION Ic

The realization is as shown in Figure 3.1.

where

$$a = b(1-1/K) + 1/Q \tag{3.10a}$$

$$C_1 = \frac{\left[b(1-1/K)+1/Q\right]^2}{b(1-1/K)+1/Q - 1/\varepsilon_1} \tag{3.10b}$$

$$C_2 = \frac{b}{K} \tag{3.10c}$$

$$C_3 = \frac{b}{b/\varepsilon_1 - K} \tag{3.10d}$$

$$R_1 = \frac{b(1-1/K)+1/Q - 1/\varepsilon_1}{b(1-1/K) + 1/Q} \tag{3.10e}$$

$$R_2 = \frac{1/\varepsilon_1}{b(1 - \frac{1}{K}) + 1/Q} \tag{3.10f}$$

$$R_3 = \frac{K(b/\varepsilon_1 - K)}{b^2} \quad \text{and} \tag{3.10g}$$

$$\omega_o = 1 \tag{3.10h}$$

b). <u>Realizability Conditions:</u>

Realizability conditions for the above realization are

$$\frac{b}{\varepsilon_1} > K > 0 \tag{3.11a}$$

$$b(1-1/K)+ 1/Q> 1/\varepsilon_1 > 0 \tag{3.11b}$$

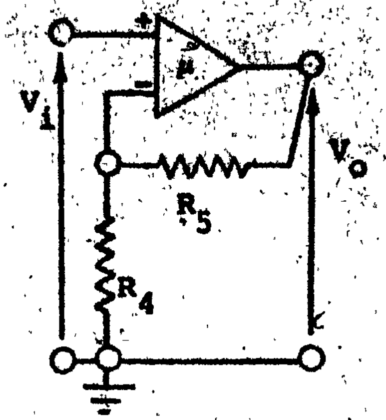


FIG. 3.2 POSITIVE GAIN AMPLIFIER REALIZATION

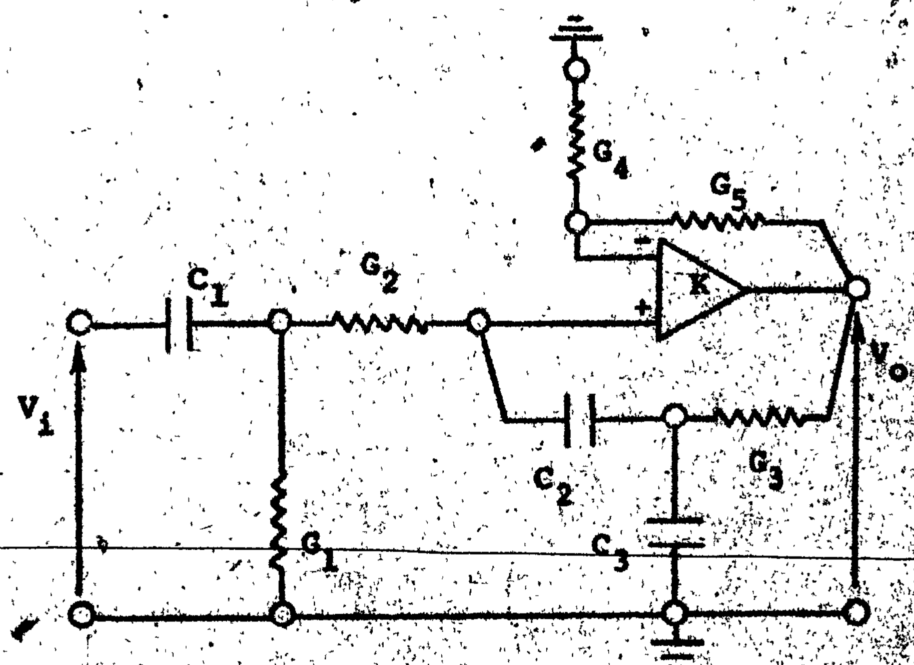


FIG. 3.3 BAND PASS FILTER REALIZATION WITH OA FOR DECOMPOSITION 4c

c). Q - and ωo Sensitivities for D(s):

The positive gain amplifier of gain K shall be realized by the network of Figure 3.2.

Analysis gives

$$K = \frac{V_o}{V_i} = \frac{1 + R_5/R_4}{1 + \frac{1 + R_5/R_4}{\mu}} \approx 1 + R_5/R_4$$

for $\mu >> 1+R_5/R_4 = K$ (3.12)

where μ is the open loop gain of the operational amplifier.

Replacing the amplifier in Figure (3.1) by the network of Figure (3.2) gives the network of Figure (3.3).

Analysis gives

$$T(s) = \frac{N(s)}{D(s)} = \frac{V_o}{V_i}$$

where

$$N(s) = sC_1G_2(sC_2 + sC_3 + G_3) \times \frac{1 + G_4/G_5}{1 + \frac{1 + G_4/G_5}{\mu}}$$

(3.13a)

$$D(s) = s^2C_2C_3(sC_1 + G_1 + G_2) + s[C_2G_3(sC_1 + G_1 + G_2) + C_1G_2(sC_2 + sC_3 + G_3) - C_2G_3(sC_1 + G_1 + G_2) \times \frac{1 + G_4/G_5}{1 + \frac{1 + G_4/G_5}{\mu}}] + G_1G_2(sC_2 + sC_3 + G_3)$$

(3.13b)

For

$$\varepsilon_1 = \frac{G_1 + G_2}{C_1} = \frac{G_3}{C_2 + C_3} \qquad (3.14)$$

One zero at $s=-\varepsilon_1$ cancels with one pole at $s=-\varepsilon_1$ giving a second degree D(s) as

$$D(s) = s^2C_1C_2C_3 + s[C_1C_2G_3(1 - \frac{1+G_4/G_5}{1+\frac{1+G_4/G_5}{\mu}} + C_1G_2(C_2+C_3)] + G_1G_2(C_2+C_3) \qquad (3.15a)$$

and

$$N(s) = sC_1G_2(C_2+C_3) \times \frac{1 + G_4/G_5}{1 + \frac{1+G_4/G_5}{\mu}} \qquad (3.15b)$$

$$Q = \frac{\sqrt{C_1C_2C_3G_1G_2(C_2+C_3)}}{C_1C_2G_3(1 - \frac{1+G_4/G_5}{\mu}) + C_1G_2(C_2+C_3)} \qquad (3.15c)$$

$$\omega_o = \sqrt{\frac{G_1G_2(C_2+C_3)}{C_1C_2C_3}} \qquad (3.15d)$$

From (3.15) and (1.3) we get

$$S_K^Q = bQ \qquad (3.16a)$$

$$S_K^{\omega_o} = 0 \qquad (3.16b)$$

$$S_\mu^Q = \frac{bKQ}{\mu} \qquad (3.16c)$$

$$S_\mu^{\omega_o} = 0 \qquad (3.16d)$$

$$S_{G_4}^Q = bQ(1 - 1/K) \qquad (3.16e)$$

$$S^{\omega_o}_{G_4} = 0 \qquad (3.16f)$$

$$S^{Q}_{G_5} = -bQ(1 - 1/K) \qquad (3.16g)$$

$$S^{\omega_o}_{G_5} = 0 \qquad (3.16h)$$

The approximation $\mu >> \frac{G_4+G_5}{G_5}$ has been used to derive the sensitivities in equation (3.16). Since G_4, G_5 and μ affect only K in the T(s), Q- and ω_o- sensitivities for $D_3(s)$ shall be derived with respect to K and other passive elements only and not with respect to μ, G_4 and G_5.

It may be seen that Q does not change appreciably with large variations in μ, but it is highly sensitive for any variations in G_4 or G_5 (which control K) and this is to be anticipated.

d). Sensitivities for $D_3(s)$:

From eq. (3.1), $D_3(s)$, in general can be represented by

$$\begin{aligned} D_3(s) &= (s+\varepsilon_1)\,(s^2 + \frac{\omega_o}{Q}s + \omega_o^2) \\ &= (s+\varepsilon_1)\,[s^2 + s(b+1/Q-b_1K) + 1] \\ &= s^3+s^2\,[\varepsilon_1 + b + 1/Q - Kb_1] + \\ &\quad s[1 + \varepsilon_1(b+\frac{1}{Q} - Kb_1)] + \varepsilon_1 \end{aligned} \qquad (3.17)$$

where ε_1 is the negative real axis pole of third order T(s).

From section 2.5, $D_3(s)$ can also be represented by

$$D_3(s) = s^3 + \delta_2 s^2 + \delta_1 s + \delta_o \qquad (2.55)$$

Comparing these two equations gives

$$\delta_2 = \varepsilon_1 + b + 1/Q - Kb_1 = \varepsilon_1 + 1/Q \qquad (3.18a)$$

$$\delta_1 = 1 + \varepsilon_1(b + 1/Q - b_1 K) = 1 + \frac{\varepsilon_1}{Q} \qquad (3.18b)$$

and

$$\delta_o = \varepsilon_1 \qquad (3.18c)$$

$$S_K^{\delta_2} = \frac{-bQ}{\varepsilon_1 Q + 1} \qquad (3.18d)$$

$$S_K^{\delta_1} = \frac{-\varepsilon_1 bQ}{Q + \varepsilon_1} \qquad (3.18e)$$

$$S_K^{\delta_o} = 0 \qquad (3.18f)$$

From equation (2.30) of sections (2.5) and (3.18) it can be shown that

$$S_K^Q = bQ \qquad (3.19a)$$

$$S_K^{\omega_o} = 0 \qquad (3.19b)$$

$$S_K^{\varepsilon_1} = 0 \qquad (3.19c)$$

and these are the same as for D(s).

Q-, ω_o- and ε_1- sensitivities with respect to all the passive elements are obtained from eq. (3.13) and section (2.5) and these are

$$S_{C_1}^Q = \frac{-2Q^2\varepsilon_1[b\varepsilon_1(1-1/K)-1] - Q[2\varepsilon_1^2 - b\varepsilon_1(1-1/K)+1] + \varepsilon_1}{2Q(1+\varepsilon_1^2) - 2\varepsilon_1} \qquad (3.20a)$$

$$S_{C_1}^{\omega_o} = \frac{Q[b\varepsilon_1(1-1/K)-1] + \varepsilon_1}{2Q(1+\varepsilon_1^2) - 2\varepsilon_1} \qquad (3.20b)$$

$$S_{C_1}^{\epsilon_1} = - \frac{Q[1 + \epsilon_1{}^2 + b\epsilon_1(1-1/K) - 1]}{Q(1 + \epsilon_1{}^2) - \epsilon_1} \tag{3.20c}$$

$$S_{C_2}^{Q} = \frac{2Q^2\epsilon_1[K-2+\frac{K\epsilon_1}{b}+b\epsilon_1(1-\frac{1}{K})]-Q\epsilon_1[\epsilon_1(K-2)-\frac{K}{b}+b(1-\frac{1}{K})]-\frac{K\epsilon_1^2}{b}}{2Q(1 + \epsilon_1{}^2) - 2\epsilon_1} \tag{3.20d}$$

$$S_{C_2}^{\omega_o} = \frac{Q[\epsilon_1{}^2(K - 2)-K\epsilon_1/b - b\epsilon_1(1-1/K)] + K\epsilon_1{}^2/b}{2Q(1 + \epsilon_1{}^2) - 2\epsilon_1} \tag{3.20e}$$

$$S_{C_2}^{\epsilon_1} = \frac{Q[-\epsilon_1{}^2(K-1)+K\epsilon_1/b+b\epsilon_1(1-1/K)-1]-\epsilon_1{}^2K/b+ \epsilon_1}{Q(1 + \epsilon_1{}^2) - \epsilon_1} \tag{3.20f}$$

$$S_{C_3}^{Q} = \frac{-2Q^2\epsilon_1(K-1+\frac{K\epsilon_1}{b})+Q(\epsilon_1{}^2K-\frac{K\epsilon_1}{b}+1) + \epsilon_1(\frac{K\epsilon_1}{b} - 1)}{2Q(1 + \epsilon_1{}^2) - 2\epsilon_1} \tag{3.20g}$$

$$S_{C_3}^{\omega_o} = \frac{-Q[\epsilon_1(K-1) + 1+K\epsilon_1/b + \epsilon_1{}^2]-K\epsilon_1{}^2/b +\epsilon_1}{2Q(1 + \epsilon_1{}^2) - 2\epsilon_1} \tag{3.20h}$$

$$S_{C_3}^{\epsilon_1} = \frac{Q\epsilon_1[\epsilon_1(K - 1)-K/b] + K\epsilon_1{}^2/b}{2Q(1 + \epsilon_1{}^2) - 2\epsilon_1} \tag{3.20i}$$

$$S_{G_1}^{Q} = \frac{(2Q^2+ \epsilon_1Q - 1)[\epsilon_1 - \frac{1}{b(1-1/K)+1/Q}]}{2Q(1 + \epsilon_1{}^2) - 2\epsilon_1} \tag{3.20j}$$

$$S_{G_1}^{\omega_o} = \frac{(\epsilon_1Q - 1)[\epsilon_1 - \frac{1}{b(1-1/K)+1/Q}]}{2Q(1 + \epsilon_1{}^2) - 2\epsilon_1} \tag{3.20k}$$

$$S_{G_1}^{\varepsilon_1} = \frac{Q[\varepsilon_1 + b(1-1/K)] / [b(1-1/K) + 1/Q]}{Q(1+\varepsilon_1^2) - \varepsilon_1} \tag{3.20l}$$

$$S_{G_2}^{Q} = \frac{-2Q^2[2\varepsilon_1+b(1-1/K)-1/(b(1-1/K)+1/Q)]+Q[b\varepsilon_1(1-1/K)+\varepsilon_1/(b(1-1/K)+1/Q)]+\varepsilon_1-1/(b(1-1/K)+1/Q)}{2Q(1+\varepsilon_1^2) - 2\varepsilon_1} \tag{3.20m}$$

$$S_{G_2}^{\omega_o} = \frac{Q[2-b\varepsilon_1(1-1/K)+\varepsilon_1/(b(1-1/K)+1/Q)]-\varepsilon_1-1/(b(1-1/K)+1/Q)}{2Q(1+\varepsilon_1^2) - 2\varepsilon_1} \tag{3.20n}$$

$$S_{G_2}^{\varepsilon_1} = \frac{Q[\varepsilon_1^2+b\varepsilon_1(1-1/K)-1-\varepsilon_1/(b(1-1/K)+1/Q)]+1/(b(1-1/K)+1/Q)}{Q(1+\varepsilon_1^2) - \varepsilon_1} \tag{3.20o}$$

$$S_{G_3}^{Q} = \frac{(2Q^2 - \varepsilon_1 Q)\ [\varepsilon_1 + b(1 - 1/K)]}{2Q(1+\varepsilon_1^2) - 2\varepsilon_1} \tag{3.20p}$$

$$S_{G_3}^{\omega_o} = \frac{Q\varepsilon_1[\varepsilon_1 + b(1 - 1/K)]}{2Q(1+\varepsilon_1^2) - 2\varepsilon_1} \tag{3.20q}$$

$$S_{G_3}^{\varepsilon_1} = \frac{Q[1 - b\varepsilon_1(1 - 1/K)] - \varepsilon_1}{Q(1+\varepsilon_1^2) - \varepsilon_1} \tag{3.20r}$$

It can be seen that, the Q-sensitivities with respect to the passive elements are high (of the order of Q). This makes these networks suitable only for implementation by hybrid IC technology.

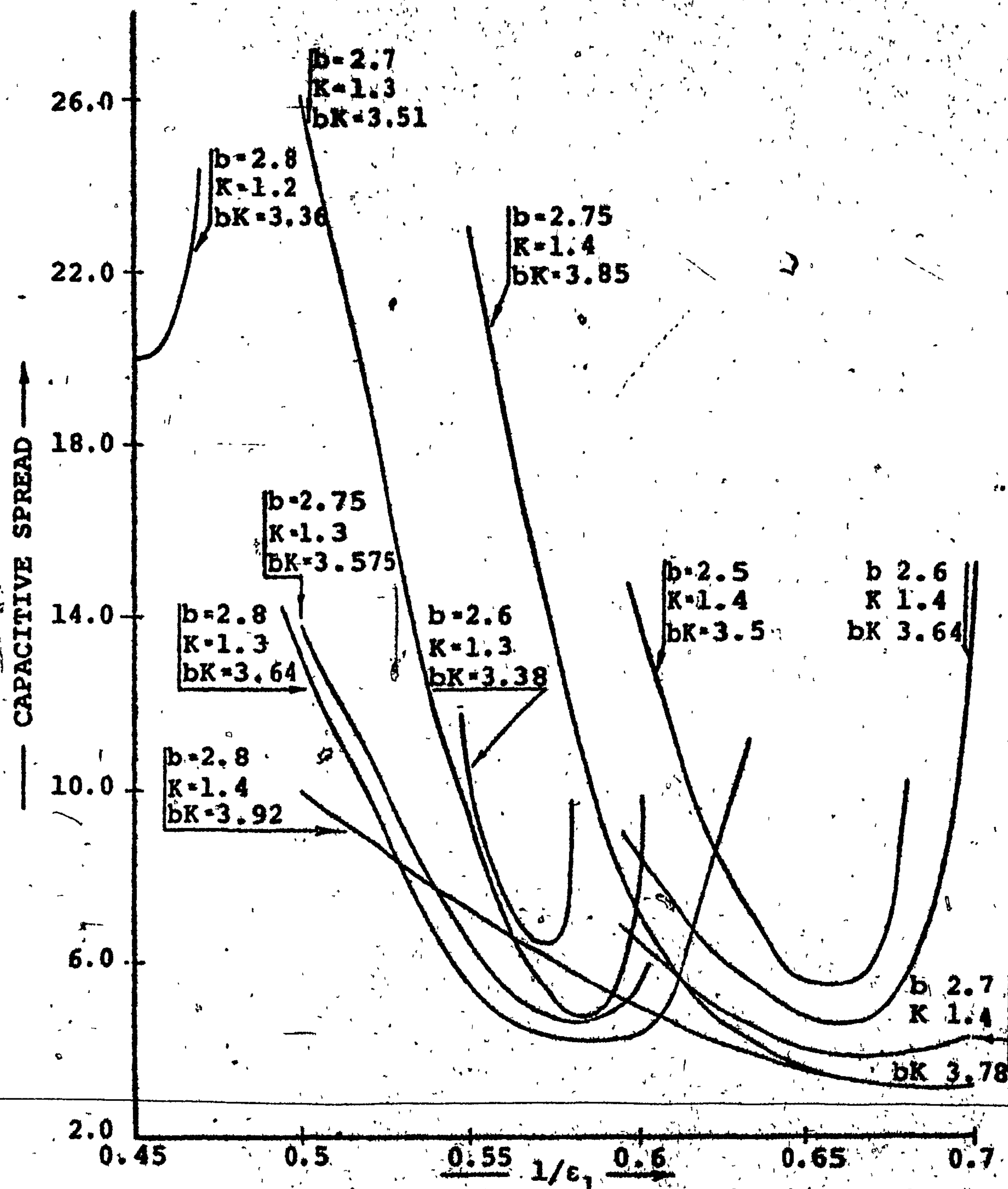


FIGURE 3.4

CAPACITIVE SPREAD VERSUS $1/\varepsilon_1$ PLOTS FOR VARIOUS VALUES OF b AND 'K' FOR BP FILTER OF DECOMPOSITION Ic

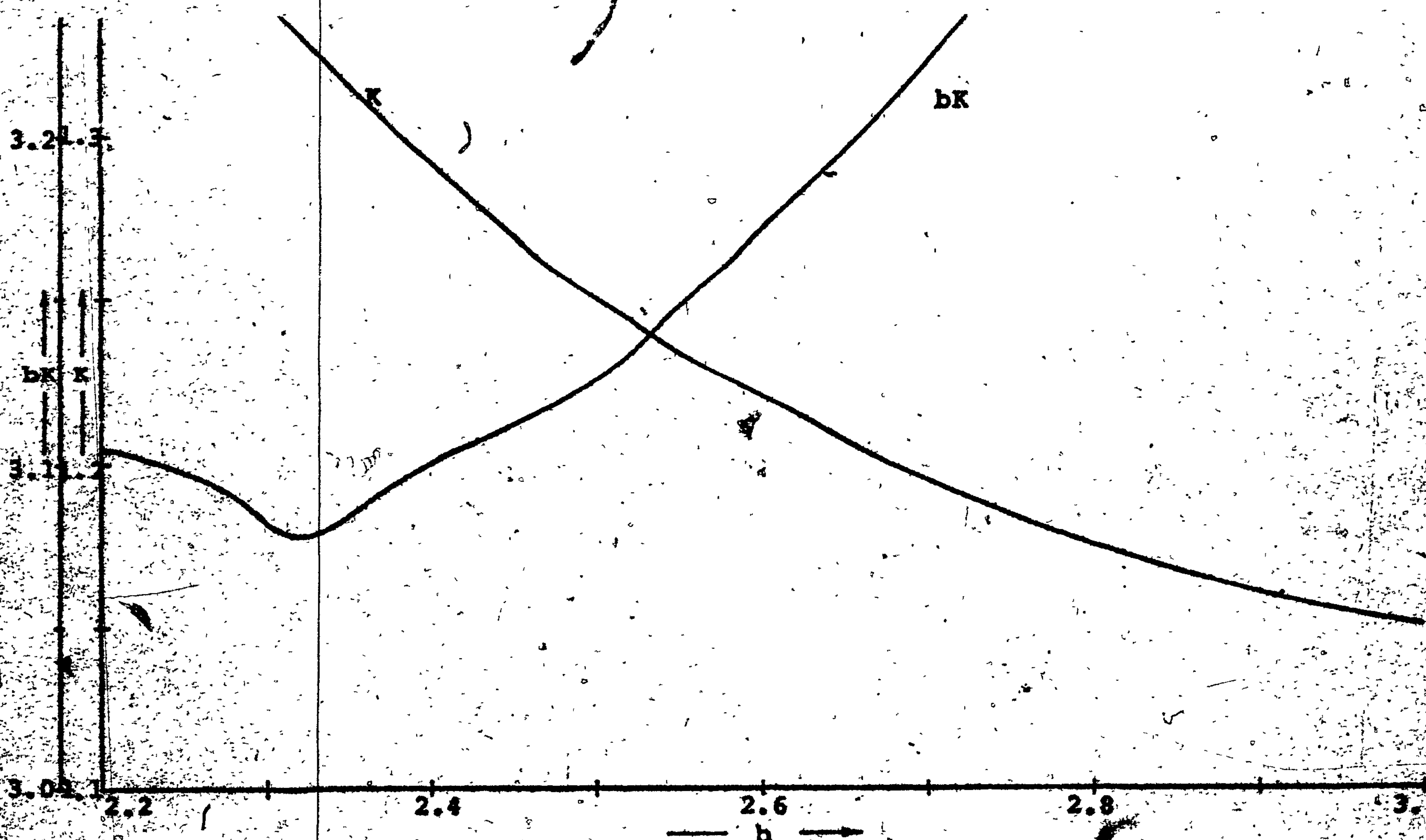


FIGURE 3.5

K AND LEAST bK VERSUS 'b' PLOTS FOR UNRESTRICTED ELEMENT SPREAD FOR NETWORKS OF DECOMPOSITION Ic e). Prescription and Minimization of S_K^Q and GQ-SP:

The choice of ϵ_1 is important as this affects the element spread, the values of b and bK. Figure (3.4) gives a family of curves giving capacitive spread versus $1/\epsilon_1$ for various values of b and K. From these curves it is possible to choose the values of $1/\epsilon_1$ and K for a given b and a given maximum capacitive spread. Similarly it is also possible to choose the values of b, K and $1/\epsilon_1$ for a given value of GQ-SP and a desired capacitive spread. It can be predicted that for the network of Figure (3.1) the least value of GQ-SP obtainable is about 3.51Q when the maximum capacitive spread is 4.8 for which, b = 2.7 and K = 1.3.

Further K and bK have been plotted in Figure (3.5) against 'b' to obtain the least possible value of GQ-SP for unrestricted element spread. From Figure (3.5) it can be seen that, for the network of Figure (3.1) the absolute minimum GQ-SP obtainable is about 3.075Q for unrestricted element spread.

f). Stability:

Positive feedback circuits have a tendency to become unstable which is clear from the presence of a negative term in the D(s) and also from high Q-sensitivities with respect to all the parameters of the network. Since the amplifier gain variation does not affect the pole frequency it is possible to use passive elements close to the

the designed values and a smaller value of K than the designed value to assure stability of the circuit. Later on K may be adjusted to get the desired value of Q. This simple scheme assures the stability of this circuit. This also permits a simple tuning procedure.

g). Effect of the Pole of OA on Q and ω_o:

From eq. (3.13), the D(s) is

$$D(s) = s^2 + s\left[b + \frac{1}{Q} - \frac{b}{1 + \frac{K}{\mu}}\right] + 1 \tag{3.13}$$

Replacing μ by $\frac{\mu_o\ \omega_C}{\omega_o s + \omega_C}$

where

ω_o is the designed pole frequency

ω_C is the pole of OA

μ_o is the d.c. gain of OA

$\mu_o\ \omega_C$ is the gain band width product of the OA

we get

$$D(s) = s^3(K\omega_o/(\mu_o\ \omega_C)) + s^2\left[1 + \frac{K}{\mu_o} + (b+1/Q)\frac{K\omega_o}{\mu_o\omega_C}\right] + s\left[1/Q + (b+1/Q)K/\mu + K\omega_o/(\mu_o\omega_C)\right] + 1 + K/\mu_o \tag{3.21}$$

comparing eq. (3.21) with the eq. (2.55) gives

i) $\frac{1}{\hat{\omega}_o^2\ \epsilon_2} = \frac{K\omega_o/(\mu_o\ \omega_C)}{1 + K/\mu_o}$

Since $\omega_o \approx \hat{\omega}_o = 1$

$$\varepsilon_2 \approx (1 + K/\mu_o)\,(\mu_o\,\omega_c)/(K\omega_o) \gg 1 \qquad (3.22)$$

ii) $$\frac{1}{\hat{\omega}_o^{\,2}} + \frac{1}{\varepsilon_2 \hat{Q}\hat{\omega}_o} = \frac{1 + K/\mu_o + (b+1/Q)(K\omega_o)/(\mu_o\,\omega_c)}{1 + K/\mu_o}$$

Second term on the L.H.S. can be neglected for $\varepsilon_2 \gg 1$, $Q \gg 1$ and $\hat{\omega}_o \approx 1$, compared to the first term and this gives

$$\frac{\hat{\omega}_o^{\,2}}{\omega_o^{\,2}} = \frac{1 + K/\mu_o}{1+K/\mu_o + (b+1/Q)\,(K\omega_o)/(\mu_o\omega_c)}$$

or $$\frac{\hat{\omega}_o}{\omega_o} = \frac{\sqrt{1 + K/\mu_o}}{\sqrt{1 + \frac{K}{\mu_o} + (b + \frac{1}{Q})\,\frac{K\omega_o}{\mu_o\,\omega_c}}}$$

$$\approx 1 - 1/2\,(b+1/Q)\,(K\omega_o)/(\mu_o\,\omega_c) \qquad (3.23)$$

iii) $$\frac{1}{\varepsilon_2} + \frac{1}{\hat{Q}\hat{\omega}_o} = \frac{1/Q + (b+1/Q)\,K/\mu_o + (K\omega_o)/(\mu_o\,\omega_c)}{1 + \frac{K}{\mu_o}}$$

or $$\hat{Q}\hat{\omega}_o = \frac{1 + \frac{K}{\mu_o}}{1/Q + (b+1/Q)\,K/\mu_o}$$

or $$\frac{\hat{Q}\,\hat{\omega}_o}{Q\,\omega_o} = \frac{1 + K/\mu_o}{1 + (bQ + 1)\,K/\mu_o}$$

or $$\frac{\hat{Q}}{Q} = \frac{\sqrt{1+K/\mu_o}\,\sqrt{1+K/\mu_o + (b+1/Q)\,(K\omega_o)/(\mu_o\,\omega_c)}}{1 + (bQ + 1)\,\frac{K}{\mu_o}}$$

$$\approx 1 + \frac{1}{2}(b + \frac{1}{Q})\frac{K\omega_o}{\mu_o\,\omega_c} - \frac{KbQ}{\mu_o} \qquad (3.24)$$

$\frac{\Delta\omega_o}{\omega_o}$ and $\frac{\Delta Q}{Q}$ versus frequency plots due to the effect of the pole of OA are obtained from eqs. (3.23) and (3.24) for an OA μA741 type and these have been given in Figures 3.6 and 3.7 respectively. Actual variations differ from the approximate ones within 3% at the higher frequencies.

h). Realization and Experimental Results:

This network was built up using discrete elements and tested. The results are summarized below

a) Designed Values:

$b = 2.7$

$K = 1.3$

$Q = 100$

$\omega_o = 10{,}000$ rad/sec = 1592 Hz

$C_1 = 80{,}034$ PF

$C_2 = 20{,}769$ PF

$C_3 = 98{,}504$ PF

$R_1 = 791\ \Omega$

$R_2 = 9.209\ k\Omega$

$R_3 = 488.8\ \Omega$

$1/\varepsilon_1 = 0.583$

Q- and ω_o- sensitivities for D(s) are

$S_K^Q = 2.7Q \qquad S_K^{\omega_o} = 0$

$S_\mu^Q \approx 1.755\times10^{-5}Q \qquad S_\mu^{\omega_o} = 0$

$S_{G_4}^Q \approx 0.623Q \qquad S_{G_4}^{\omega_o} = 0$

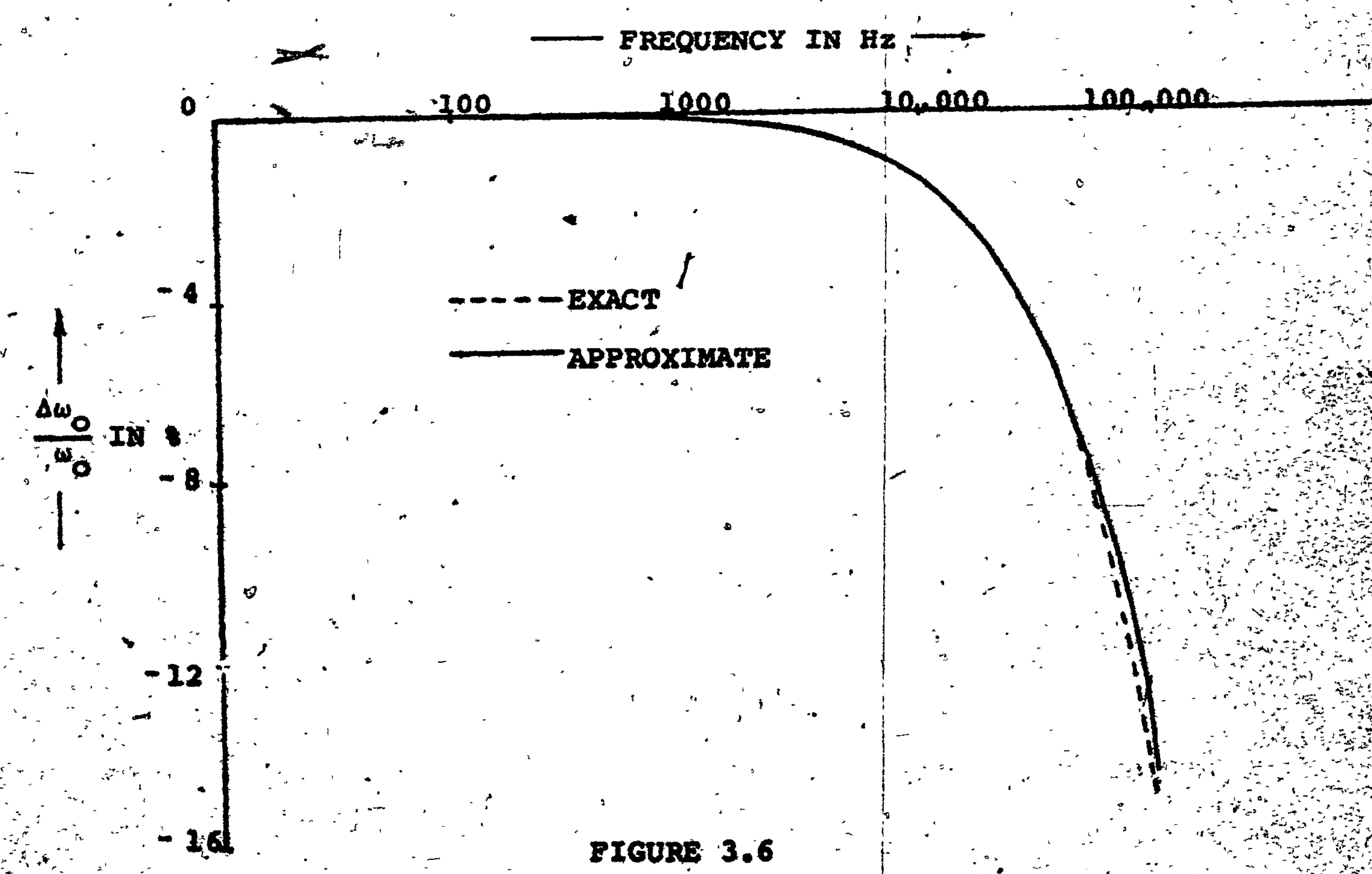


FIGURE 3.6

$\frac{\Delta\omega_o}{\omega_o}$ VERSUS FREQUENCY PLOTS FOR NETWORKS OF DECOMPOSITION Ic

(Q=100)

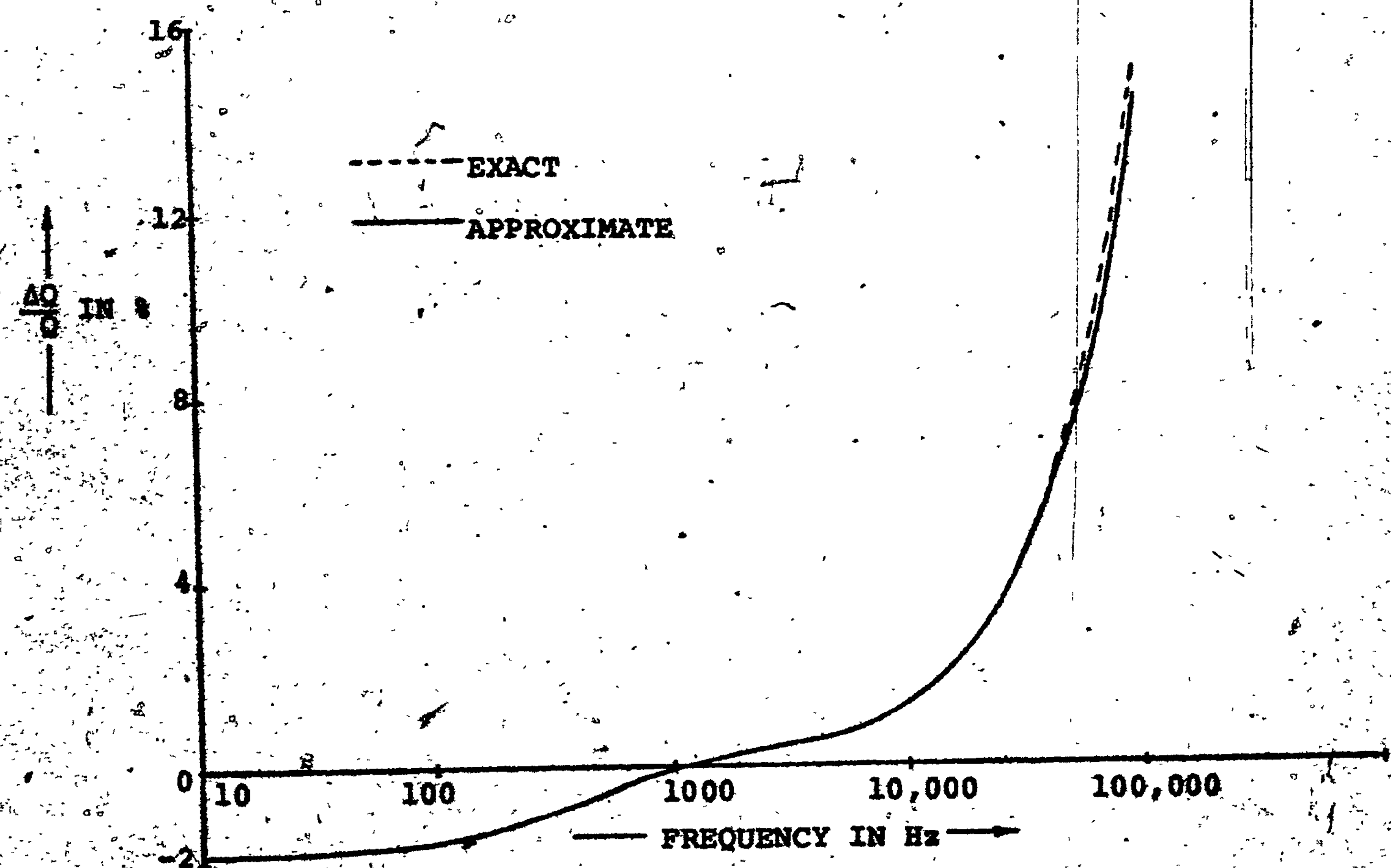


FIGURE 3.7

$\frac{\Delta Q}{Q}$ VERSUS FREQUENCY PLOTS FOR NETWORKS OF DECOMPOSITION Ic

(Q=100)

$S^{Q}_{G_5} \approx -0.623Q \quad S^{\omega_o}_{G_5} = 0$

$KS^{Q}_{K} = 3.51Q \quad KS^{\omega_o}_{K} = 0$

Q- and ω_o- sensitivities for $D_3(s)$ are

$S^{Q}_{K} = 2.7Q \quad S^{\omega_o}_{K} = 0 \quad S^{\varepsilon_1}_{K} = 0$

$S^{Q}_{C_1} \approx -0.03044Q-0.737 \quad S^{\omega_o}_{C_1} \approx 0.0089 \quad S^{\varepsilon_1}_{C_1} \approx 1.018$

$S^{Q}_{C_2} \approx 0.52Q-0.244 \quad S^{\omega_o}_{C_2} \approx -0.501 \quad S^{\varepsilon_1}_{C_2} \approx -0.547$

$S^{Q}_{C_3} \approx -0.482Q-0.286 \quad S^{\omega_o}_{C_3} \approx -0.509 \quad S^{\varepsilon_1}_{C_3} \approx 0.0183$

$S^{Q}_{G_1} \approx 0.0275Q+0.718 \quad S^{\omega_o}_{G_1} \approx 0.31 \quad S^{\varepsilon_1}_{G_1} \approx 0.936$

$S^{Q}_{G_2} \approx -0.62Q+0.795 \quad S^{\omega_o}_{G_2} \approx 0.46 \quad S^{\varepsilon_1}_{G_2} \approx 0.036$

$S^{Q}_{G_3} \approx 0.586Q+0.239 \quad S^{\omega_o}_{G_3} \approx 0.511 \quad S^{\varepsilon_1}_{G_3} \approx -0.0218$

$KS^{Q}_{K} = 3.51Q \quad KS^{\omega_o}_{K} = 0$

Resistor R_1 is varied to bring one negative real axis pole close enough to one negative real axis zero so that the response of the designed network is not affected. K initially is maintained lower than the designed value to ensure stability. Thereafter it is adjusted slowly to obtain the desired Q.

Element Spread:

Capacitive: 4.74:1

Resistive :18.84:1

Transfer function gain at centre frequency = 0.823Q

b). <u>Actual Values:</u>

R_1 = 792 Ω, R_2 = 9.16K Ω, R_3 = 496 Ω

C_1 = 80,100 PF, C_2 = 20,700 PF, C_3 = 98,500 PF

K = 1 + 312/1014 = 1.3077

ω_o = 1574 Hz

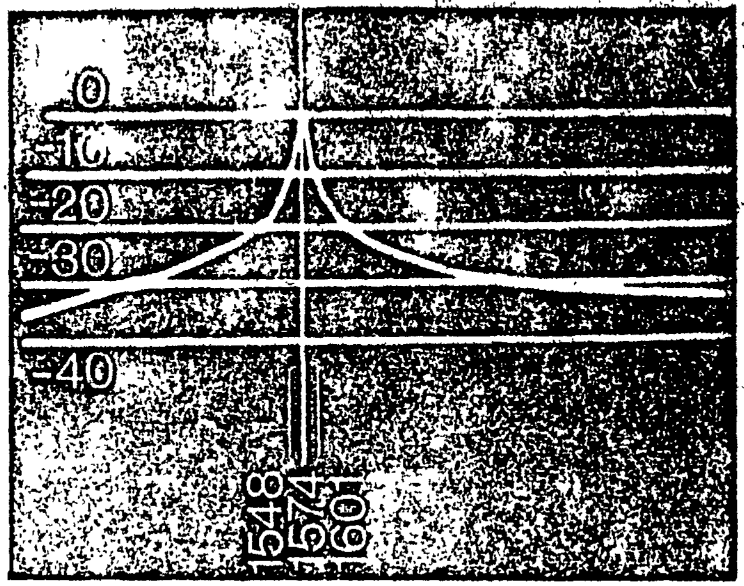
3db frequencies, 1566 Hz and 1582 Hz

10db frequencies, 1548 Hz and 1601 Hz

Realized Q = 98.4

Q and ω_o variations from their designed values are 1.6% and 1.2% respectively. This is due to the 1% discrete passive elements used for the realization.

Power supply used was ±10V and ±15V. Also the OA was placed in an oven and the temperature was controlled. The response was observed and photographs taken at 22°C (room temperature) and 70°C for both the power supplies. No appreciable change in the response was observed during the power supply or temperature variation. Thus the experimental results closely confirm the theoretical ones. Photographs of the response under various conditions are given in Figure (3.8). In order to observe the effect of temperature and power supply variations on the response, values as experimentally measured are plotted in Figure 3.8 between the two 10 db points in the response.



FREQUENCY RESPONSE OF BAND PASS FILTER NETWORK

OF DECOMPOSITION Ic (Q=100)

FIGURE 3.8

a) ±10V, 22° C.

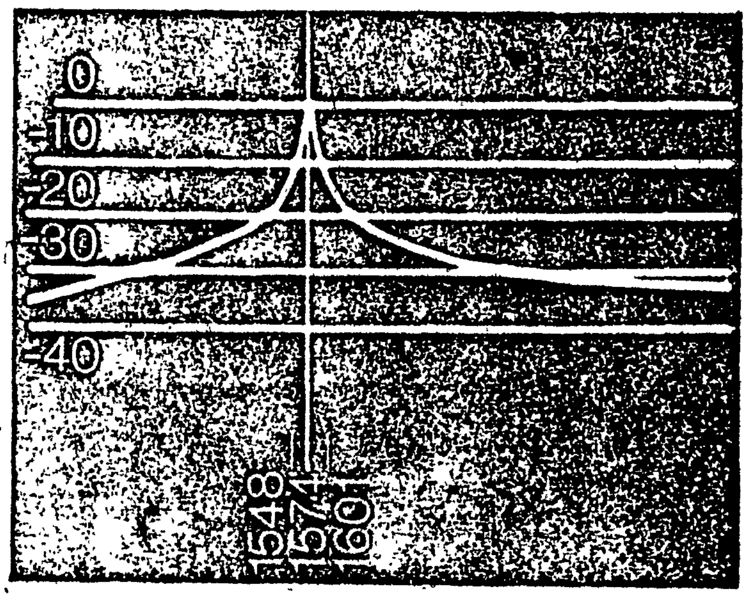


FIGURE 3.8

b) ±15V, 22° C.

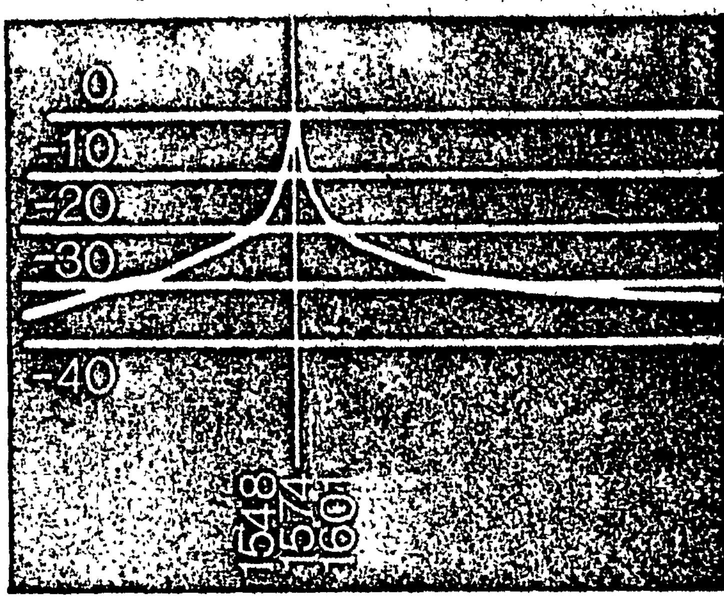


FIGURE 3.8

c) ±10V, -70° C.

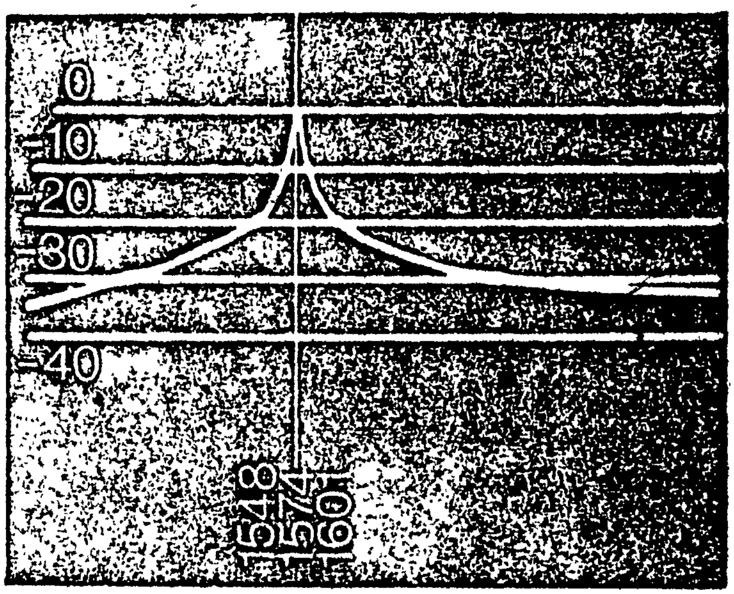


FIGURE 3.8

d) ±15V, 70°C.

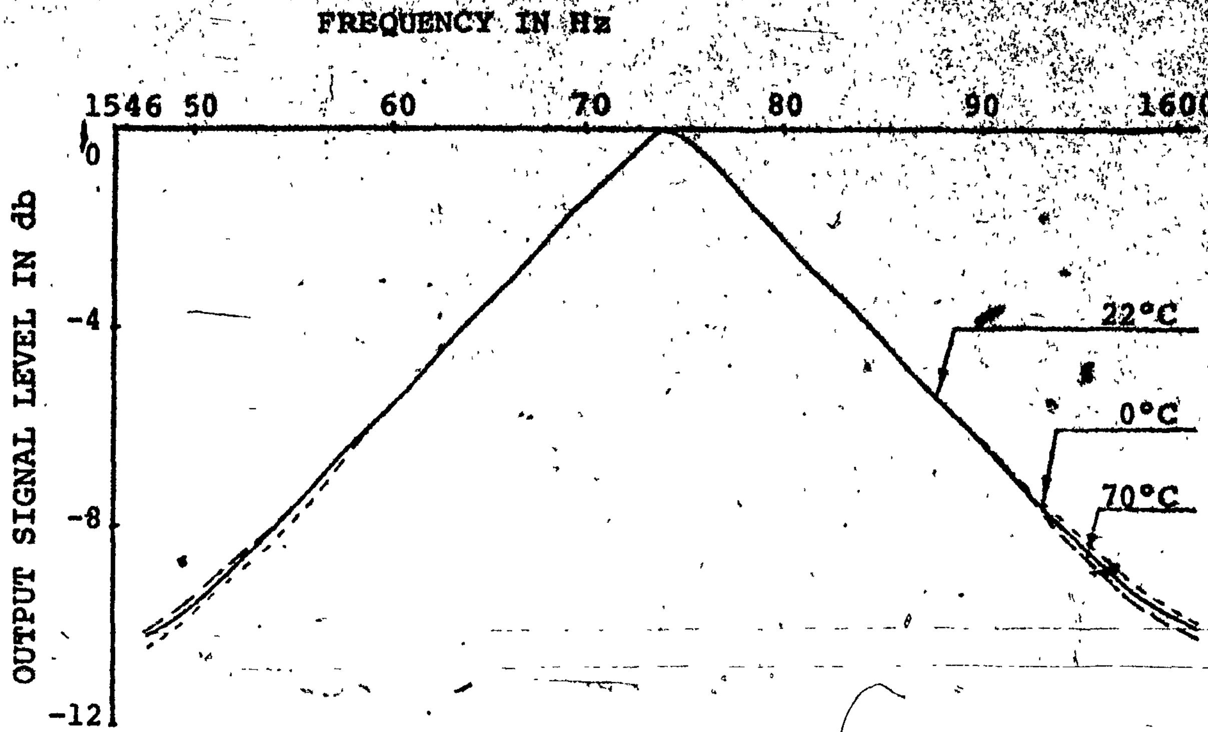


FIGURE 3.8

e) ±10V

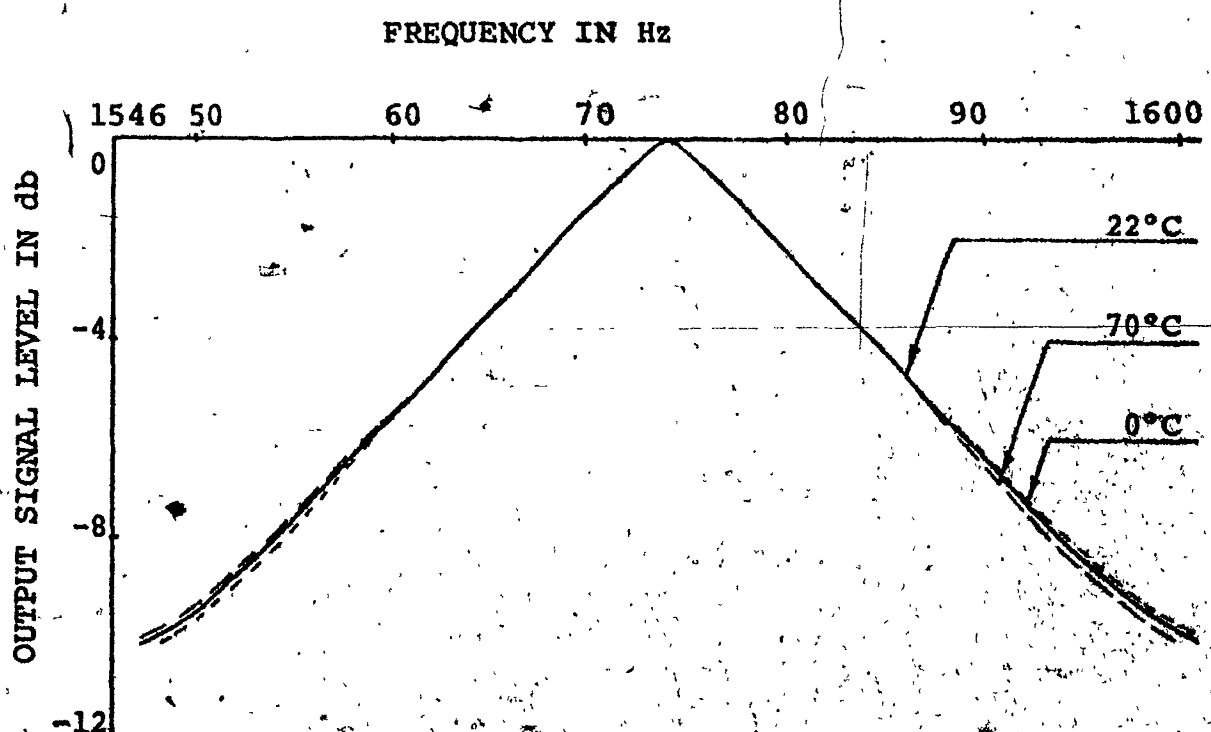


FIGURE 3.8

f) ±15V

3.3.2 THE LOW PASS FILTER NETWORK:

a). Realization:

A general L.P., T(s) is

$$T(s) = \frac{N(s)}{D(s)} = \frac{Ka}{s^2 + (b + 1/Q - b_1K)s+1} \quad (3.25)$$

where

$$b_1K = b$$

and

$$S_K^Q = bQ ,$$

'a' and 'b' being arbitrary positive constants.

Dividing both the N(s) and D(s) of (3.25) by $(s+\varepsilon_1)$ and comparing with the network of Figure (1.4), gives one of the identifications as follows:

$$Y_{11b} = \frac{s^2+(b-c+1/Q)s}{s+\varepsilon_1} \quad (3.26a)$$

$$Y_{22a} = \frac{cs + 1}{s + \varepsilon_1} \quad (3.26b)$$

$$-Y_{12b} = \frac{b/K\ s}{s+\varepsilon_1} \quad (3.26c)$$

$$-Y_{12a} = \frac{a}{s+\varepsilon_1} \quad (3.26d)$$

The realization is shown in Figure (3.9) for which the design equations are

$$R_1 = (c\varepsilon_1-1)/c \quad (3.27a)$$

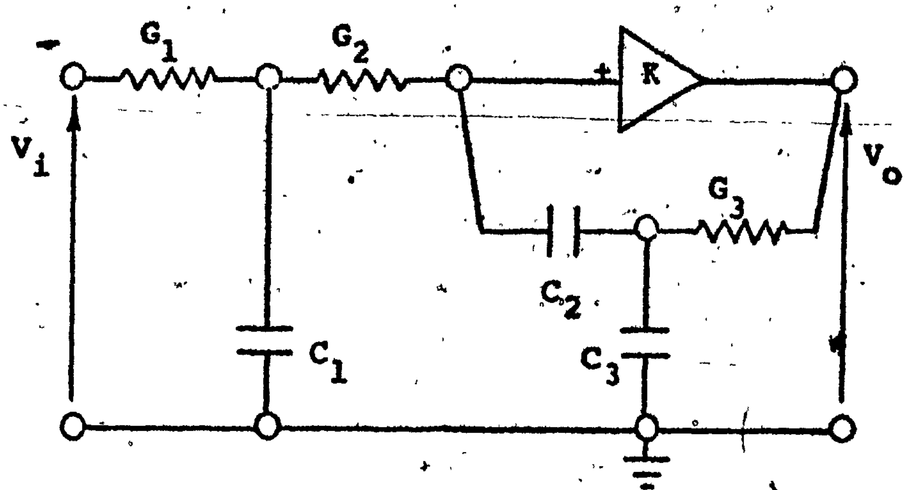


FIG. 3.9 LOW PASS FILTER REALIZATION FOR DECOMPOSITION Ic

$$R_2 = \frac{1}{c} \tag{3.27b}$$

$$R_3 = \frac{b-c-\varepsilon_1+1/Q}{(b-c+1/Q)^2} \tag{3.27c}$$

$$C_1 = \frac{c^2}{c\varepsilon_1-1} \tag{3.27d}$$

$$C_2 = \frac{b-c+1/Q}{\varepsilon_1} \tag{3.27e}$$

$$C_3 = \frac{b-c+1/Q}{b-c-\varepsilon_1+1/Q} \tag{3.27f}$$

$$K = \frac{b}{b-c+1/Q} \tag{3.27g}$$

$$a = 1 \tag{3.27h}$$

$$\omega_o = 1 \tag{3.27i}$$

and

$$T(s) = \frac{V_o}{V_i} = \frac{K(s+\varepsilon_1)}{(s+\varepsilon_1)[s^2+s(b+1/Q-b_1K)+1]} \tag{3.27j}$$

b). <u>Realizability Conditions:</u>

The realizability conditions are

$$1/c < \varepsilon_1 < \text{[illegible]}1/Q \tag{3.28}$$

c). Q- and ω_o-Sensitivities for D(s):

Replacing the amplifier of Figure 3.9 by the network of Figure 3.2 , the Analysis gives

$$T(s) = \frac{V_o}{V_i} = \frac{N(s)}{D(s)}$$

where

$$N(s) = G_1G_2(sC_2+sC_3+G_3) x \frac{1+G_4/G_5}{1+\frac{1+G_4/G_5}{\mu}} \qquad (3.29a)$$

$$D(s) = s^2C_2C_3(sC_1+G_1+G_2)+s[C_2G_3(sC_1+G_1+G_2)$$
$$(1 - (1+G_4/G_5)/(1 + (1+G_4/G_5)/\mu))$$
$$+C_1G_2(sC_2+sC_3+G_3)] + G_1G_2(sC_2+sC_3+G_3) \qquad (3.29b)$$

For

$$\varepsilon_1 = \frac{G_1 + G_2}{C_1} = \frac{G_3}{C_2 + C_3} \qquad (3.30)$$

One zero at $s = -\varepsilon_1$ cancels with one pole at $s = -\varepsilon_1$ giving a second degree D(s) as

$$D(s) = s^2C_1C_2C_3+s[C_1C_2G_3(1-\frac{1+G_4/G_5}{1+\frac{1+G_4/G_5}{\mu}})$$
$$+C_1G_2(C_2+C_3)] + G_1G_2(C_2+C_3) \qquad (3.31a)$$

$$N(s) = G_1G_2(C_2+C_3) x \frac{1+G_4/G_5}{1+\frac{G_4/G_5}{\mu}} \qquad (3.31b)$$

$$Q = \frac{\sqrt{C_1C_2C_3G_1G_2(C_2+C_3)}}{C_1C_2G_3\left(1-\frac{1+G_4/G_5}{1+\frac{1+G_4/G_5}{\mu}}\right) + C_1G_2(C_2+C_3)} \quad (3.31c)$$

$$\omega_o = \sqrt{\frac{G_1G_2(C_2+C_3)}{C_1C_2C_3}} \quad (3.31d)$$

For $\mu >> 1+\frac{G_4}{G_5}$

$$K = 1 + G_4/G_5 \quad (3.31e)$$

From eqs. (3.31) and (1.3) the sensitivities are

$$S_K^Q = bQ \quad (3.32a)$$

$$KS_K^Q = bKQ \quad (3.32b)$$

$$S_K^{\omega_o} = 0 \quad (3.32c)$$

$$S_\mu^Q \approx \frac{bKQ}{\mu} \quad (3.32d)$$

$$S_\mu^{\omega_o} = 0 \quad (3.32e)$$

$$S_{G_4}^Q = -S_{G_5}^Q \approx bQ(1-1/K) \quad (3.32f)$$

$$S_{G_4}^{\omega_o} = S_{G_5}^{\omega_o} = 0 \quad (3.32g)$$

The approximation, $\mu >> 1+G_4/G_5$ has been used to derive (3.32). Since G_4, G_5 and μ affect only K in the T(s), Q- and ω_o- sensitivities for $D_3(s)$ shall be derived with respect to K and other passive elements only, and not with respect to μ, G_4 and G_5.

Once again it may be seen that Q does not vary appreciably for large variations in μ but it is highly sensitive for variations in G_4 and G_5 (which control K) and this is anticipated.

d). Sensitivities for $D_3(s)$:

From section (2.5) and eq. (3.29) we get

$$\delta_2 = \frac{C_1C_2G_3(1-K)+C_2C_3(G_1+G_2)+C_1G_2(C_2+C_3)}{C_1C_2C_3} \tag{3.33a}$$

$$\delta_1 = \frac{C_2G_3(G_1+G_2)(1-K)+G_1G_2(C_2+C_3)+G_2G_3C_1}{C_1C_2C_3} \tag{3.33b}$$

$$\delta_0 = \frac{G_1G_2G_3}{C_1C_2C_3} \tag{3.33c}$$

and the sensitivities are

$$S^{Q}_{C_1} = -\frac{Q(2\varepsilon_1Q-1)(c\varepsilon_1-1)}{2Q(1+\varepsilon_1^{2})-2\varepsilon_1} \tag{3.34a}$$

$$S^{\omega_o}_{C_1} = \frac{Q(c\varepsilon_1-1)}{2Q(1+\varepsilon_1^{2})-2\varepsilon_1} \tag{3.34b}$$

$$S^{\varepsilon_1}_{C_1} = \frac{-\varepsilon_1Q(\varepsilon_1+c)+\varepsilon_1}{Q(1+\varepsilon_1^{2})-\varepsilon_1} \tag{3.34c}$$

$$\frac{2Q^2\varepsilon_1(c+\varepsilon_1)-\varepsilon_1 Q(1+c\varepsilon_1)}{b-c+1/Q}+2Q^2\varepsilon_1(c\varepsilon_1-1)$$

$$S_{C_2}^{Q}=\frac{-Q\varepsilon_1(c+\varepsilon_1)+\varepsilon_1}{2Q(1+\varepsilon_1^{2})-2\varepsilon_1} \qquad (3.34d)$$

$$S_{C_2}^{\omega_o}=\frac{-Q\left[c\varepsilon_1+\varepsilon_1^{2}-\frac{c\varepsilon_1^{2}-\varepsilon_1}{b-c+1/Q}\right]+\varepsilon_1}{2Q(1+\varepsilon_1^{2})-2\varepsilon_1} \qquad (3.34e)$$

$$S_{C_2}^{\varepsilon_1}=\frac{-Q\left[c\varepsilon_1+1+\frac{(c\varepsilon_1+1)\varepsilon_1}{b-c+1/Q}\right]}{Q(1+\varepsilon_1^{2})-\varepsilon_1} \qquad (3.34f)$$

$$S_{C_3}^{Q}=\frac{\frac{-2\varepsilon_1 Q^2(c+\varepsilon_1)+\varepsilon_1 Q(1+c\varepsilon_1)}{b-c+1/Q}+Q(1+\varepsilon_1^{2})-2\varepsilon_1+1}{2Q(1+\varepsilon_1^{2})-2\varepsilon_1} \qquad (3.34g)$$

$$S_{C_3}^{\omega_o}=\frac{-Q\left\{1+\varepsilon_1^{2}+\frac{\varepsilon_1(c\varepsilon_1-1)}{b-c+1/Q}\right\}+\varepsilon_1}{2Q(1+\varepsilon_1^{2})-2\varepsilon_1} \qquad (3.34h)$$

$$S_{C_3}^{\varepsilon_1}=\frac{Q\varepsilon_1(c\varepsilon_1+1)}{(b-c+1/Q)\left[Q(1+\varepsilon_1^{2})-\varepsilon_1\right]} \qquad (3.34i)$$

$$S_{G_1}^{Q}=\frac{(\varepsilon_1-1/c)(2Q^2+\varepsilon_1 Q-1)}{2Q(1+\varepsilon_1^{2})-2\varepsilon_1} \qquad (3.34j)$$

$$S_{G_1}^{\omega_o} = \frac{(\varepsilon_1 Q+1)(\varepsilon_1 - 1/c)}{2Q(1+\varepsilon_1^2) - 2\varepsilon_1} \tag{3.34k}$$

$$S_{G_1}^{\varepsilon_1} = \frac{Q(1-\varepsilon_1/c) - 1/c}{Q(1+\varepsilon_1^2) - \varepsilon_1} \tag{3.34L}$$

$$S_{G_2}^{Q} = \frac{-2Q^2(2\varepsilon_1 + c - 1/c) + Q(2 + \varepsilon_1/c + \varepsilon_1 c) - 1/c}{2Q(1+\varepsilon_1^2) - 2\varepsilon_1} \tag{3.34m}$$

$$S_{G_2}^{\omega_o} = \frac{Q[2+1/c-c] - 1/c}{2Q(1+\varepsilon_1^2) - 2\varepsilon_1} \tag{3.34n}$$

$$S_{G_2}^{\varepsilon_1} = \frac{Q[\varepsilon_1 c + \varepsilon_1^2 - 1 - \varepsilon_1/c] - \varepsilon_1 + \varepsilon_1/c}{Q(1+\varepsilon_1^2) - \varepsilon_1} \tag{3.34o}$$

$$S_{G_3}^{Q} = \frac{2Q^2(c+2\varepsilon_1) + Q(\varepsilon_1^2 - 2 - c\varepsilon_1 - 2\varepsilon_1) + \varepsilon_1}{2Q(1+\varepsilon_1^2) - 2\varepsilon_1} \tag{3.34p}$$

$$S_{G_3}^{\omega_o} = \frac{Q\varepsilon_1(\varepsilon_1 + c) - \varepsilon_1}{2Q(1+\varepsilon_1^2) - 2\varepsilon_1} \tag{3.34q}$$

$$S_{G_3}^{\varepsilon_1} = \frac{Q(1-\varepsilon_1 c)}{Q(1+\varepsilon_1^2) - \varepsilon_1} \tag{3.34r}$$

$$S_K^Q = bQ \tag{3.34s}$$

$$S_K^{\omega_o} = 0 \tag{3.34t}$$

$$S_K^{\epsilon_1} = 0 \tag{3.34u}$$

$$KS_K^Q = bKQ \tag{3.34v}$$

$$KS_K^{\omega_o} = 0 \tag{3.34w}$$

It can be seen once again, that Q-sensitivities with respect to passive elements are high (of the order of Q). This makes these networks suitable for implementation by hybrid IC technology.

e). Prescription and Minimization of S_K^Q and GQ-SP:

Once again the choice of ϵ_1 is important as it affects the element spread, the values of b and bK. Figure (3.10) gives a family of curves giving capacitive spread versus ϵ_1 for various values of b and c. From these curves it is possible to choose the values of ϵ_1 and c for a given b and a given maximum capacitive spread. Similarly it is also possible to select values of 'b', 'c' and ϵ_1 for a given value of GQ-SP and a maximum capacitive

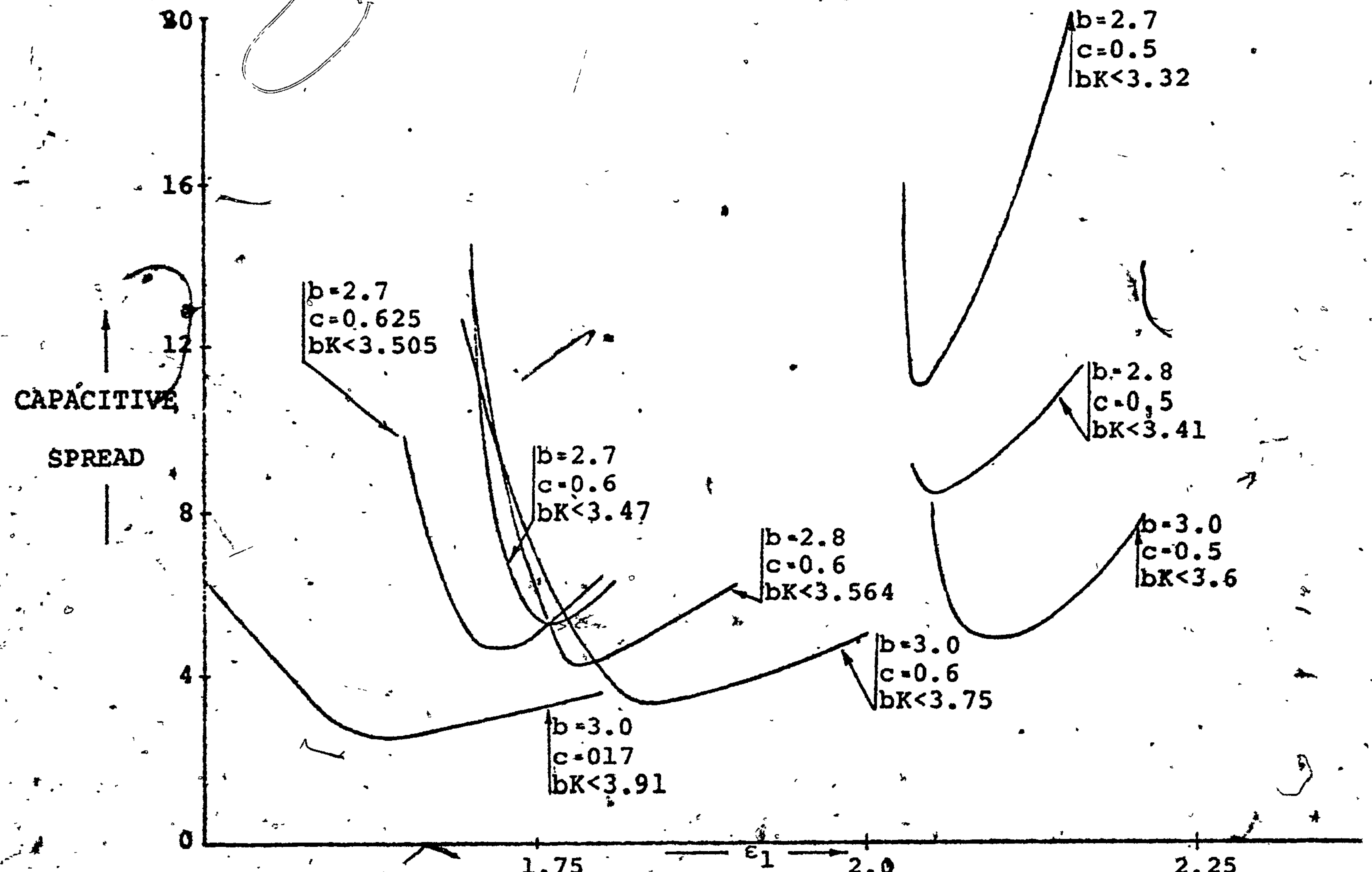


FIGURE 3.10

CAPACITIVE SPREAD VERSUS 'ε_1', PLOTS FOR VARIOUS VALUES OF b' AND 'c' FOR LP FILTER NETWORK OF DECOMPOSITION Ic.

spread. From these curves of Figure 3.10 it can be predicted that for a given maximum capacitive spread of 5, least GQ-SP obtainable for the network of Figure 3.9 would be 3.51Q approximately. For least GQ-SP, b = 2.7, $K = \frac{2.7}{2.075+1/Q}$, c = 0.625, $bK = \frac{7.29}{2.075+1/Q}$ are the approximately chosen values while capacitive spread is 4.73, $\varepsilon_1 = 1.715$.

For unrestricted element spread the plots of K and bK against b are the same as in Band Pass network case and are given in Figure 3.5. Thus for the low pass network of Figure 3.9 least GQ-SP (for unrestricted element spread) obtainable is about 3.075Q.

f). Stability:

This network too tends to become unstable because of the presence of a negative term in D(s) and this shows up in high Q-sensitivities with respect to passive elements. Once again the amplifier gain K, which does not affect the pole frequency, may be used to ensure the stability by using a lower value of K than the designed value while the passive elements chosen shall be close to the design values. Later on K may be adjusted to tune the circuit to the desired Q.

g). Effect of the pole of OA on Q and ω_o:

Since D(s) of the Low Pass filter is unchanged from that of the Band Pass filter, the variation in Q and ω_o due to the effect of the pole of OA remains unchanged for the same values of b and K. These variations can be read from Figures (3.6) and (3.7).

h) Realization for $Q=100, \omega_o=1592Hz$

For $b = 2.7$, $C = 0.625$, $K = \frac{2.7}{2.075 + 1/Q} \approx 1.295$ and $Q = 100$ the realization for $\omega_o = 10{,}000$ rad/sec and resistor ratio of 10,000 is

C_1 = 56,818 PF

C_2 = 12,193 PF

C_3 = 55,600 PF

R_1 = 1.1 K Ω

R_2 = 16.0 K Ω

R_3 = 862.6 Ω

Sensitivities for D(s) are:

$S_K^Q = 2.7Q$ $\qquad S_K^{\omega_o} = 0$

$S_\mu^Q \approx 1.748 \times 10^{-5}Q$ $\qquad S_\mu^{\omega_o} = 0$

$S_{G_4}^Q \approx 0.615Q$ $\qquad S_{G_4}^{\omega_o} = 0$

$S_{G_5}^Q \approx -0.615Q$ $\qquad S_{G_5}^{\omega_o} = 0$

$$KS_K^Q = 3.4965Q \qquad KS_K^{\omega_o} = 0$$

Sensitivities for $D_3(s)$ are:

$$S_K^Q = 2.7Q \qquad S_K^{\omega_o} = 0 \qquad S_K^{\epsilon_1} = 0$$

$$S_{C_1}^Q \approx -0.2264Q-0.679 \qquad S_{C_1}^{\omega_o} \approx 0.0664 \qquad S_{C_1}^{\epsilon_1} \approx -1.132$$

$$S_{C_2}^Q \approx 0.517Q-0.0224 \qquad S_{C_2}^{\omega_o} \approx -0.504 \qquad S_{C_2}^{\epsilon_1} \approx 0.00358$$

$$S_{C_3}^Q \approx -0.475Q-0.29 \qquad S_{C_3}^{\omega_o} \approx -0.509 \qquad S_{C_3}^{\epsilon_1} \approx 0.143$$

$$S_{G_1}^Q \approx 0.02108Q+0.725 \qquad S_{G_1}^{\omega_o} \approx 0.278 \qquad S_{G_1}^{\epsilon_1} \approx 0.936$$

$$S_{G_2}^Q \approx -0.624Q+0.79 \qquad S_{G_2}^{\omega_o} \approx 0.47 \qquad S_{G_2}^{\epsilon_1} \approx 0.653$$

$$S_{G_3}^Q \approx 0.585Q+0.243 \qquad S_{G_3}^{\omega_o} \approx 0.511 \qquad S_{G_3}^{\epsilon_1} \approx -0.017$$

$$GQ\text{-}SP = KS_K^Q = 3.4965Q \qquad G\omega_o\text{-}SP = KS_K^{\omega_o} = 0$$

Resistor R_1 shall be varied to bring one negative real axis pole close enough to negative real axis zero so that no effect is observed in the response of the network in the frequency range of interest. K shall be maintained below the designed value so as to assure stability while starting. Now

K shall be adjusted to obtain the desired Q.

Element Spread:

Capacitive: 4.73:1

Resistive : 18.57:1

Transfer Function Gain = 1.295 at s = 0.

3.3.3 HIGH PASS FILTER NETWORK:

a). Realization:

A general H.P., T(s) is

$$T(s) = \frac{N(s)}{D(s)} = \frac{aKs^2}{s^2 + s(b + 1/Q - b_1K) + 1} \qquad (3.35)$$

where

$$b_1 = b/K$$

and

$$S_K^Q = bQ$$

and

'a' and 'b' are arbitrary positive constants.

Dividing both N(s) and D(s) of eq. (3.41) by $(s+\epsilon_1)$ and comparing with the network of Figure 1.4 gives one of the identifications as follows:

$$Y_{11b} = \frac{b/K\ s + 1}{s + \epsilon_1} \qquad (3.36a)$$

$$-Y_{12b} = \frac{b/K\ s}{s + \epsilon_1} \qquad (3.36b)$$

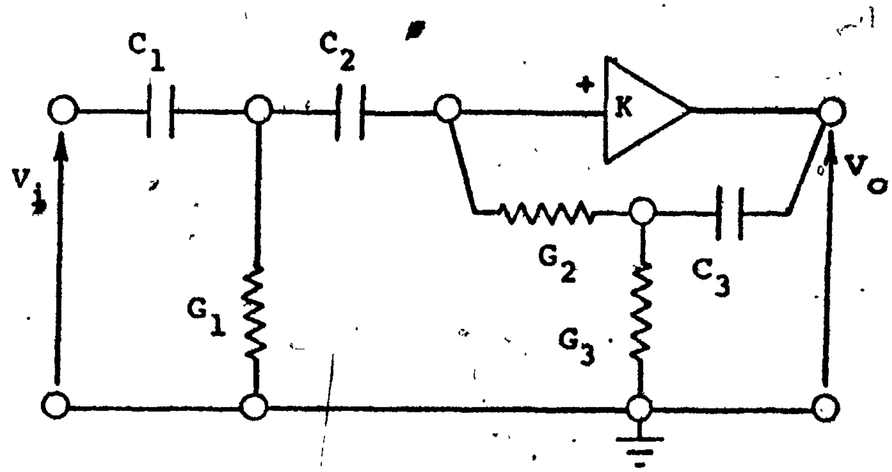


FIG. 3.11 HIGH PASS FILTER REALIZATION FOR DECOMPOSITION Ic

$$Y_{22a} = \frac{s^2 + (b - b/K + 1/Q)s}{s + \varepsilon_1} \qquad (3.36c)$$

$$-Y_{12a} = \frac{s^2}{s + \varepsilon_1} \qquad (3.36d)$$

$$a = 1 \qquad (3.36e)$$

and

$$T(s) = \frac{V_o}{Vi} = \frac{Ks^2(s + \varepsilon_1)}{(s+\varepsilon_1)[s^2 + (b + 1/Q - b_1K)s + 1]}$$

The network for this identification is shown in Figure (3.11). The design equations for it are:

$$C_1 = \frac{b - b/K + 1/Q}{b - b/K - \varepsilon_1 + 1/Q} \qquad (3.37a)$$

$$C_2 = \frac{b - b/K + 1/Q}{\varepsilon_1} \qquad (3.37b)$$

$$C_3 = \frac{(b/K)^2}{b/K\ \varepsilon_1 - 1} \qquad (3.37c)$$

$$R_1 = \frac{b - b/K - \varepsilon_1 + 1/Q}{(b - b/K + 1/Q)^2} \qquad (3.37d)$$

$$R_2 = \frac{K}{b} \qquad (3.37e)$$

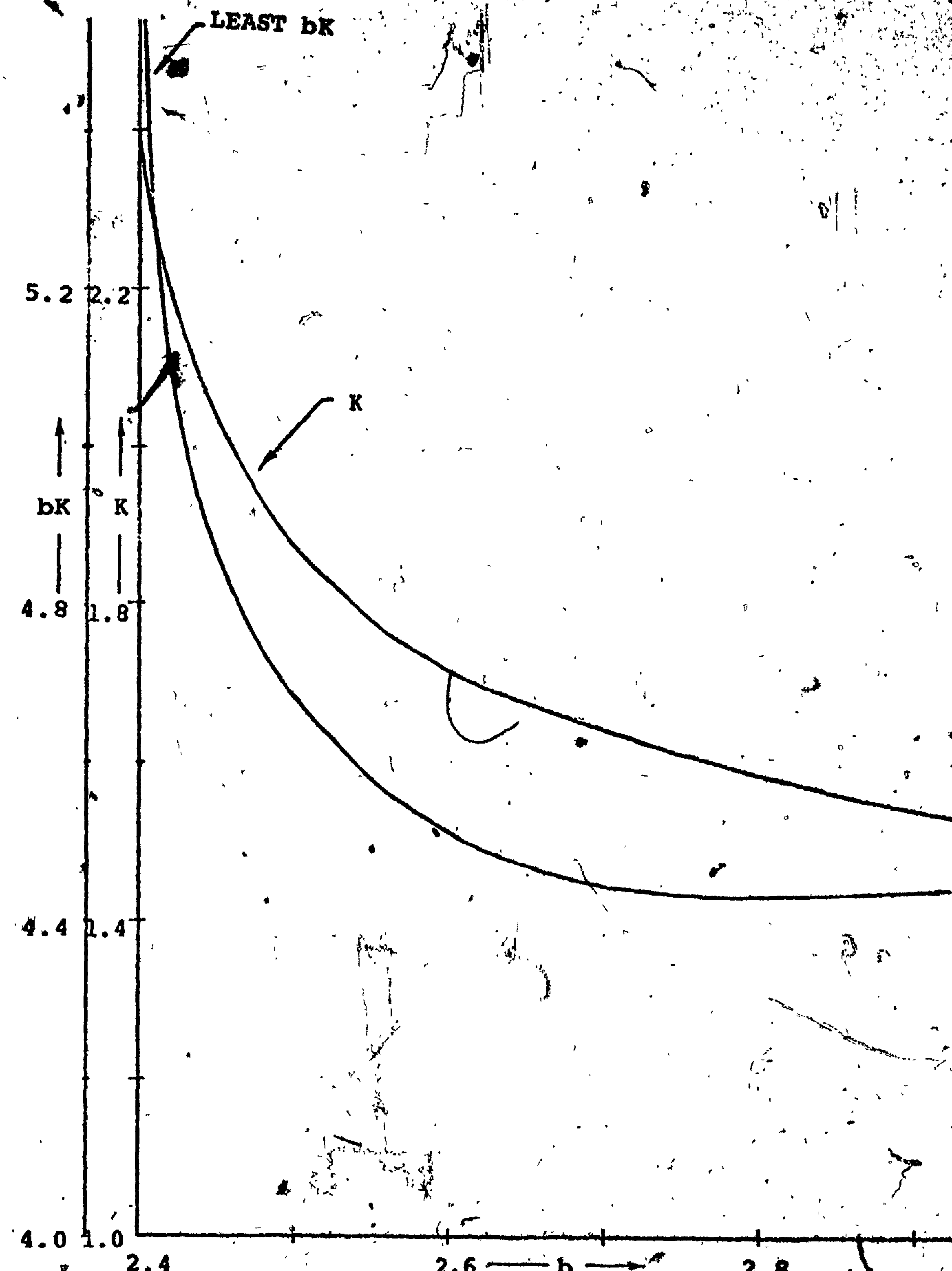


FIGURE 3.12

K AND bK VERSUS b PLOTS FOR HP FILTER NETWORK OF DECOMPOSITION Ic, WITH A MAXIMUM CAPACITIVE SPREAD OF FIVE.

$$R_3 = \varepsilon_1 - K/b \tag{3.37f}$$

$$\omega_o = 1 \tag{3.37g}$$

b). Realizability Conditions:

$$K/b < \varepsilon_1 < b - b/K + 1/Q \tag{3.38}$$

c). Q- and ω_o- Sensitivities for Second Degree D(s), prescription and minimization of S_K^Q and GQ-SP:

$$S_K^Q = bQ \tag{3.39a}$$

$$S_K^{\omega_o} = 0 \tag{3.39b}$$

$$KS_K^Q = KbQ \tag{3.39c}$$

$$KS_K^{\omega_o} = 0 \tag{3.39d}$$

For a maximun capacitive spread of five, K and bK were plotted against b in Figure(3.12). From the plots of Figure (3.12) it is possible to say that for a maximum capacitive spread of five, least GQ-SP obtainable is about 4.43Q and this occurs for b = 2.77-2.8 and K = 1.582-1.6. From these plots it is possible to obtain a realization once either S_K^Q or GQ-SP is prescribed for a given maximum capacitive spread of five.

Once again for unrestricted element spread the plots are the same as in Figure (3.6). Lower GQ-SP is obtainable for a higher capacitive spread.

Similar results are also obtainable by an RC-CR transformation from the L.P. network of section 3.6.2.

This high pass network shall not be discussed any further as other networks with lower GQ-SP, and lower capacitive spread have already been reported [12] in the literature even though they all use coefficient matching technique for the realization.

3.3.4 NOTCH FILTER NETWORK:

From the low pass network shown in Figure (3.8) lifting the capacitor C_3 off the ground and connecting it to the input terminal gives the network shown in Figure (3.13). Analysis gives

$$N(s) = KG_1G_2(sC_2+sC_3+G_3) + Ks^2C_2C_3(sC_1+G_1+G_2) \quad (3.40a)$$

$$D(s) = s^2C_2C_3(sC_1+G_1+G_2)+s[C_2G_3(sC_1+G_1+G_2)(1-K) +C_1G_2(sC_2+sC_3+G_3)]+G_1G_2(sC_2+sC_3+G_3) \quad (3.40b)$$

The D(s) of this network is exactly the same as in the low pass network. One pole zero cancellation shall occur if

$$\frac{-C_1}{G_1+G_2} = \frac{C_2+C_3}{G_3} \quad (3.41)$$

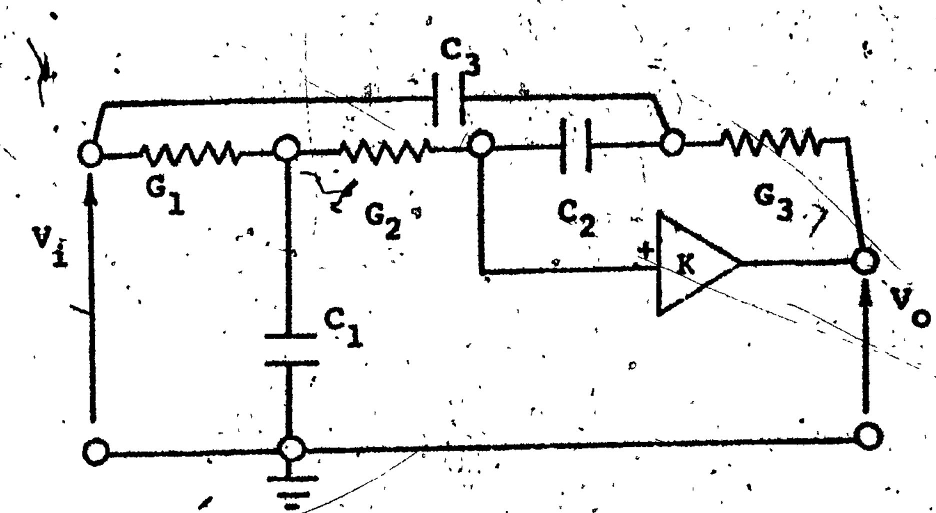


FIG. 3.13 NOTCH FILTER REALIZATION FOR DECOMPOSITION Ic

N(s) and D(s) after one pole zero cancellation are

$$N(s) = K[s^2 + \frac{G_1G_2(C_2+C_3)}{C_1C_2C_3}] \qquad (3.42a)$$

$$D(s) = s^2+s[G_3/C_3(1-K)+G_2(C_2+C_3)/(C_2C_3)]$$
$$+ G_1G_2(C_2+C_3)/(C_1C_2C_3) \qquad (3.42b)$$

$$\text{Notch frequency} = \sqrt{-\frac{G_1G_2(C_2+C_3)}{C_1C_2C_3}} \qquad (3.48)$$

This notch frequency is independent of the active parameter K and it has sensitivities with respect to passive elements lower than or equal to 1/2.

This network has exactly the same denominator as that of the L.P. network of Figure (3.8) and for the same element values, this shall have all the properties namely realizability conditions, sensitivities, gain sensitivity products etc. of the low pass network and as such no further detailed discussion of this network is considered necessary.

3.3.5 ALL PASS FILTER NETWORK:

A general all pass T(s) is given by

$$T(s) = \frac{N(s)}{D(s)} = \frac{Ka(s^2 - \omega_o/Q\, s + \omega_o^2)}{s^2 + \omega_o/Q\, s + \omega_o^2}$$

$$= \frac{Ka[s^2 - (b + 1/Q - b_1K)s + 1]}{s^2 + (b + 1/Q - b_1K)s + 1} \qquad (3.43)$$

where

$$b_1 = b/K$$

and

'a' and 'b' are arbitrary positive constants.

Dividing both N(s) and D(s) of eq. (3.48) by $(s + \epsilon_1)$ and upon comparison with the network of Figure (1.4) gives the identification as:

$$-Y_{21a} = \frac{a[s^2 - (b + 1/Q - b_1 K)\, s + 1]}{s + \epsilon_1} \qquad (3.44a)$$

$$-Y_{12b} = \frac{b/K\ s}{s + \epsilon_1} \qquad (3.44b)$$

$$Y_{22a}+Y_{11b} = \frac{s^2 + (b + 1/Q)s + 1}{s + \epsilon_1} \qquad (3.44c)$$

Three terminal network, Na, which realizes right half s-plane zeros, in general for high Q realizations, would require higher order open circuit voltage transfer function for network Na thereby making the degree of D(s) higher than three and therefore this realization is not considered any further.

3.4 COMPARISON WITH OTHER NETWORKS & GENERAL COMMENTS:

In this chapter a few network realizations have been given. These have the following advantages over the ones given in the literature [8 - 13].

a) A rigorous passive synthesis procedure has been adopted and no coefficient matching is to be resorted to obtain the realization.

b) S_K^Q or GQ-SP can be prescribed, or minimized.

c) Realizable Q is almost independent of the element spread.

d) Band Pass, Low Pass and Notch networks realize lower GQ-SP than the existing networks in the literature so far to the best of the knowledge of the author.

Band Pass and Notch filter networks of decompositions Ic and IIc upon comparison give the following:

a) GQ-SP realized for the networks belonging to decomposition IIc is $Q\sqrt{6}$ for D(s) and this becomes even lower for $D_3(s)$, while GQ-SP for the networks of decomposition Ic, is 3.51Q. Thus GQ-SP is lower for the networks of decomposition IIc.

b) Networks of decomposition IIc do not have any problems regarding the stability.

c) Networks of decomposition Ic, if proper care is exercised at the time of switching on for the first time have an easier tuning procedure compared to the networks of decomposition IIc.

CHAPTER IV

SUMMARY AND DISCUSSIONS

CHAPTER IV

SUMMARY AND DISCUSSIONS

This thesis has discussed realizations of second order open circuit voltage transfer functions by single amplifier active RC networks. These realizations have been obtained with prescribed sensitivities for Q or ω_o with respect to the amplifier gain and with minimized gain sensitivity product of Q or ω_o. Further the element spread in the resulting networks has been constrained to lie within acceptable limits.

Second order active RC-filter networks have been classified on the basis of possible decompositions of the denominator polynomials of their transfer functions. Bounds on Q- and ω_o- sensitivities with respect to any network parameter, namely, capacitor, resistor or the amplifier gain, are obtained for each decomposition. Since usually variations in the amplifier gain are larger compared to the variations in passive elements, only the problem of prescribing the Q- or ω_o- sensitivity with respect to the amplifier gain has been considered.

Chapter II discusses single amplifier RC-filter networks using negative feedback. Decompositions Ia and Ib, where the active parameter K appears in only one of the terms s^2 or s^o respectively, are found unsuitable for Q- or ω_o- sensitivity prescription because these sensitivities can not be prescribed independently of Q. Decomposition IIa and IIb, where the active parameter appears in only s^2 and s or s and s^o terms respectively, are suitable for prescribing the Q- sensitivity only, since ω_o- sensitivity can not be prescribed independently of Q. For these decompositions Q-sensitivity can be prescribed to be zero, while ω_o- sensitivity remains very nearly equal to 1/2. Minimal capacitor realization is obtained for decomposition IIa. Amplifier gain required for the decompositions Ia, Ib, IIa and IIb is high (of the order of Q^2) and this fact renders these decompositions useful for low Q and low frequency applications only. However, passive sensitivities are low for these networks and thus they may be suitable for implementation with discrete elements.

Decomposition IIc, where the active parameter K appears in s^2 and s^o terms only, yields high Q realizations when ω_o- sensitivity is either very low, of the order of 1/2Q as in Low Pass and High Pass filters, or it is prescribed to be zero as in Band Pass, All Pass and Notch filters. It is shown that a minimal capacitor realization is not possible for decomposition IIc sensitivities for all the filter realizations belonging to this decomposition are high and therefore these are suitable for implementation by hybrid IC technology where the variation in Q and ω_o due to the variations in the passive elements for changes in temperature can be made zero in view of excellent tracking property of the thin film RC components. The Q-sensitiivty or GQ-SP for these networks is minimized while the capacitive spread is constrained to lie within at most five. The BP, Notch and AP filter networks have been obtained for a minimized GQ-SP of $Q\sqrt{6}$ while ω_o-sensitivity is prescribed to be zero, and LP and HP filter networks have been obtained for a minimized GQ-SP of 3.1Q and 3.86Q respectively while ω_o-sensitivity is of the order of 1/2Q. To the author's knowledge, the GQ-SP's obtained are smaller than those of other negative feedback single amplifier networks reported in the literature so far.

Decomposition III, where the active parameter K appears in all the three s^2, s and s^0 terms, gives a high Q realization, when ω_o-sensitivity is prescribed to be zero. However, value of the amplifier gain K is higher in this case than that required in deocmposition IIc(which happens to be the optimum), and therefore this decomposition realizes a lower Q for the same K and is not discussed further.

Chapter III discusses single amplifier networks using positive feedback. These belong to the decomposition Ic, where the active parameter appears in s term only in the second degree denominator polynomial of the transfer function. A synthesis procedure has been used to obtain Low Pass, High Pass, Band Pass and Notch filter networks. For these networks ω_o-sensitivity is always zero, while Q-sensitivity or GQ-SP has been minimized. For Low Pass, Band Pass, and Notch filters GQ-SP's of 3.52Q or lower along with maximum capacitive spread of five have been obtained. In view of the fact that the passive sensitivities are high in these realizations, these networks should be implemented by hybrid IC technology.

Design curves have been given for filter networks corresponding to the decompositions IIc and Ic. It is possible, from these design curves, to obtain the desired realization readily when either Q-sensitivity or GQ-SP is prescribed or minimized while ω_o- sensitivity is either zero or very low. Minimization of both Q- and ω_o- sensitivities in turn minimizes the variation in the transfer function T(s).

Band Pass filters, one each for decompositions IIa (with $S_K^Q = 0$), IIc and Ic (with $S_K^{\omega_o} = 0$), were studied for stability during activation and for the effect of the pole of the OA. It is found that the network belonging to decomposition IIa is adversely affected by the pole of the OA at frequencies above 100 Hz, even for low values of Q, while the networks corresponding to the decompositions IIc and Ic are relatively unaffected by the pole of the OA at the frequencies considered (few KHz). These networks (for Q=2, ω_o= 90 Hz for decomposition IIa, and for Q=100, ω_o= 1592 Hz for decompositions IIc and Ic) were built using discrete elements and tested in the laboratory at supply d.c. voltages of ±10V and ±15V and with the OA at temperatures of $0^{o}C.$, $22^{o}C.$ (room temperature) and $70^{o}C.$ The test results confirm the theoretical predictions.

Thus this study clearly establishes that high Q single amplifier active RC networks can be realized in practice when first order ω_o-sensitivities with respect to amplifier gain is prescribed to be zero, while simultaneously requiring the Q-sensitivity or GQ-SP to be minimum along with a capacitive spread constrained to lie within acceptable limits.

Only the open circuit voltage transfer functions have been considered and realized in this thesis. But the same procudure could be readily extended to realize short circuit current transfer functions using the property of network transposition (40).

The main effort of this thesis has been in the area of single amplifier realizations. A systematic study of multi-amplifier realizations from the point of view of the considerations discussed in this thesis may be desirable.

Only first order sensitiivty effects have been taken into account in this thesis. However, higher order sensitivities may play a significant role in determining the deviations $\frac{\Delta Q}{Q}$ and $\frac{\Delta\omega_o}{\omega_o}$ and hence the overall performance of the filter.

This suggests that an attempt should be made to realize single amplifier RC-filter networks when $\frac{\Delta Q}{Q}$ or $\frac{\Delta \omega_o}{\omega_o}$ is prescribed.

In the literature, other figures of merit (41-43) have also been defined in connection with active networks. It may be worthwhile to explore the possibility of realizing networks with one or more of these figures of merit prescribed.

In chapter II, it is shown, that, for the same denominator polynomial, several Band Pass, All Pass and Notch filter networks are possible. This means that a number of networks exist with the same prescribed value for S_K^Q or $S_K^{\omega_o}$ and having the same optimized GSP. Several other such networks may also be generated by using the principle of switching of terminals (44), (45). It shall be interesting to examine the possibilities in this approach.

In conclusion, the author hopes that, in view of the importance of single amplifier active RC filters, the results of the investigations carried out in this thesis would prove useful to the filter designers.

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