REVIEW OF SOME SHEAR STRENGTH THEORIES FOR RECTANGULAR REINFORCED CONCRETE BEAMS

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ABSTRACT

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All types of shear failure are preceded by cracking in the concrete inclined to main axis of the member with the exception of horizontal shear in a compression flange. If shear cracks form, a beam may fail immediately by shear i.e. diagonal tension failure or after an increase in load in which case it may fail by shear-compression, bending or some other mode.

On the basis of researches carried out concerning the shear strength of concrete beams, it appears that inclined cracking due to shear does not terminate the work of a beam if there is sufficient web reinforcement, and the main tensile reinforcement is well anchored beyond the support. Beams, even with inclined cracks will be able to withstand further loading, until they reach the diagonal splitting strength limit due to what can be called, arch behaviour.

In this paper, mostly, we have attempted to recover the best solution to prevent the failure of beams due to shear.

ACKNOWLEDGEMENT

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NOTATION

f_{tc}	=	tensile strength of concrete under combined bi-directional tension and compression
f_{cc}	=	strength of concrete under combined bi-directional compression
f_s	=	tensile strength of steel
f_t	=	flexural tension stress
f_t'	=	diagonal tension strength of concrete
f_{tt}	=	strength of concrete under combined bi-directional tension
f_v	=	stress in web reinforcement
f_y	=	yield strength of reinforcement
h	=	total depth of beam
H	=	force in tension reinforcement
I	=	moment of inertia
jd	=	internal moment arm
K_1	=	constant
$L = \ell$	=	length of beam
ℓ'	=	span, measured center-to-center of supports
ℓ_n	=	clear span measured face-to-face of supports
M	=	bending moment
M_u	=	applied load moment at a section
M_s	=	ultimate shear-compression

$n = E_s/E_c$

R = Reaction

r = ratio

s = shear reinforcement spacing

u = bond stress

v = shear stress

V = shear force

V_a = shear in arch theory

V_c = shear force carried by concrete

V_h = part of shear force taken by the horizontal web reinforcement

V_s = shear force in shear theory

V_u = ultimate shear strength

V_v = part of shear force taken by the vetrical web reinforcement

Ω = perimeter of reinforcing bars

CHAPTER I

INTRODUCTION

CHAPTER 1

1.1 Objective

This report is prepared with the intention of presenting the readers certain facts about the behaviour of rectangular concrete beams as they approach or reach their shear capacities. Furthermore, a study was done on certain theories and/or studies relating to the matter and where possible certain comments are made on the subject. However, a detailed criticism and explanation as to errors and/or possible omissions in the theories (classical or otherwise) are beyond the scope of this report. The main objective is to establish the existence of several theories on shear. To confirm the fact that any or all of the theories may be employed in the solution to recognized to be safe and in according with the standards of design procedures. However, besides presenting one with the existing formulas and summaries of studies and results of tests done on concrete beams in shear, this report also pretents the accuracy of several methods and theories compared to one another.

It will become abvious that in all cases, the design procedures and formulas used to design for shear are very conservative. It will also become apparent that many of the classical solutions neglect certain properties of concrete such as "aggregate interlock" and in omitting these properties render the classical formulas even more safe than they need be. Yet, it will also become clear that such safety is a necessary thing considering the amount of unknowns when dealing with concrete as a building material. The reader will see that how close a design may get, formulas never go beyond a certain limit in ensuring a standard of safety.

1.2 Shear Strength of Concrete Beams

Extensive research work has been carried out over the past 20 years on tests and theoretical studies concerning the shear strength of concrete beams. Much of the research on shear has been devoted to the determination of shear cracking loads. Fig. 1.1 shows the development of shear cracking and diagonal tension failures which these failures in beams commonly referred to as "shear failures".

The factors influencing shear strength and formation of inclined cracks are numerous and complex. Many research papers have been written on the subject. By the nature of the problem a definitive conclusion is practically unattainable regarding ~~the correct~~ mechanism of inclined cracking that results from high shear. Bresler and Mac Gregor [5] have provided an excellent review and systematic correlation of the basic concept.

They noted that the influencing factors are:

1. The proportions and shape of the beam.
2. The structural restraints and the interaction of the beam with other components in the system.
3. The amount and arrangement of tension and compression reinforcement.
4. The size and spacing of transverse or inclined reinforcement.
5. The degree of prestressing, if any.
6. The load distribution and loading history.
7. The properties of concrete and steel.
8. The concrete placement and curing.
9. The enviroment history.

1.3 Theoretical Concept of Shear Strength

Let's consider a reinforced concrete beam as shown in Fig. 1.2 which has two symmetrical point loads. For this particular type of loading, the maximum moment and maximum shear both occur at the location of the load Fig. 1.3, and their raitio is $\frac{M}{V} = a$ (i.e. the shear span).

There is, in general, both bending moment and a shear force at each cross section along the shear span. For an element A Fig. 1.2a situated at the neutral axis no bending stress will occur and the element is in a state of pure shear. Hence this magnitude and direction of the principal stresses may be obtained from the conditions of equilibrium. Fig. 1.2b shows that such an element will develop unit diagonal tensile and compressive stresses of magnitude V on a plane 45^{o} with the planes where only the shearing stresses exist. For an element at B, it will have a compressive stress f_c in addition to shear. Such a stress produces diagonal compression as shown in Fig. 1.2c When Fig. 1.2c is combined with the shear effect of Fig. 1.2b to obtain the total stress, the diagonal tension on section m-m is reduced and the diagonal compression on section n-n is increased. Likewise, an element at C will carry a tension stress as well as a shear. Such a stress produces diagonal tensions as shown in Fig. 1.2d. When Fig. 1.2d is combined with the shear effect of Fig. 1.2b to obtain the total stress, the diagonal tension on section m-m is increased and the diagonal compression on section n-n is reduced. The combined diagonal stresses are not maximum on section m-m and n-n, tension being maximum on a stepper plane than m-m when the horizontal direct stress is tension or on a flatter plane when the horizontal direct stress is compression. Its diagrams are shown at Fig. 1.3.

It is now necessary to derive expressions for the variation of vertical shear stress on the cross-section of a reinforced concrete beam.

Consider a rectangular reinforced concrete beam which has a width b, height h, and depth d from the extreme fiber in compression to the center of gravity of the tension steel. This section is shown in Fig. 1.4a with the compression area above the neutral axis cross - hatched. We will take as a rigid body the portion of the beam (i.e. abcd), lying between the cross sections dx apart Fig. 1.4c. The horizontal force equilibrium on this body is:

$$\nu_y \, b \, dx = C_2 - C_1 \qquad 1.1$$

Where: ν_y = unit horizontal shear stress on a plane at a distance y from the neutral axis

C_1 and C_2 = result of the uniformly varying compressive stress

$$C_1 = \tfrac{1}{2}(f_{c1}+f_{c1y})\ b\ (kd - y) \qquad 1.2$$

$$f_{c1y} = (y/kd) f_{c1} \qquad 1.3$$

$$C_1 = \tfrac{1}{2} f_{c1}\ (1+{}^{y}/kd)\ (kd - y)\ b$$

$$= \tfrac{1}{2} f_{c1}\ bkd\ (1 - {}^{y^2}/{}_{kd^2}) \qquad 1.4$$

By letting: M_1 = bending moment on section 1-1

jd = moment arm of internal couple,

equilibrium gives:

$$M_1 = \tfrac{1}{2} f_{c1}.kdb.\ jd$$

so $$f_{c1} = 2M_1 / jkbd^2 \qquad 1.5$$

By substituing into Eq. 1.4 gives

$$C_1 = M_1/_{jd} \left[1 - y^2/(kd)^2 \right] \qquad 1.6$$

similarly

$$C_2 = M_2/_{jd} \left[1 - y^2/(kd)^2 \right] \qquad 1.7$$

substituing Eqs. 1.6 and 1.7 into eq. 1.1 gives

$$v_y = \left(\frac{M_2 - M_1}{dx}\right) \left(\frac{1}{bjd}\right) \left[1 - \frac{y^2}{(kd)^2}\right]$$

$$= \frac{V}{bjd} \left[1 - \frac{y^2}{(kd)^2}\right] \qquad 1.8$$

The variation of the vertical shear stress v_y in this equation is valid above the neutral axis for any value of y from o to kd; y is the distance to a point in the concrete where it is effective in carrying bending stress. As it is seen, proceeding from the extreme compression fiber to the neutral axis, the differential horizontal force C_2-C_1 increases to a maximum. Thus the maximum shear stress will be at neutral axis y=o.

$$v = \frac{V}{bjd} \qquad 1.9$$

This variation is shown at Fig. 1.4 d.

A major contribution to the understanding of reinforced concrete members in shear was made by Morsch between years 1902 and 1910. He pointed out that shear failures are the result of principal tensile stresses and that, even in a state of pure shear, with equal horizontal and vertical shear stresses, equal tensile stresses exist on planes at 45° to the neutral axis. He also developed the same equation for the nominal shearing stress:

$$v = \frac{V}{bjd}$$

The fact that this formula is still universally used today (with the minor change of omitting "j") shows the enormous influence some of the early poineers have had on modern design practice. The importance of eq. 1.1. is such that a closer look is justified. At any point in an isotropic, homogeneous concrete beam the principle tensile stress

f_t(max), can be related to the shear stress v, and flexural stress f_t, at that point by the equation:

$$f_t(\max) = \tfrac{1}{2}f_t + \sqrt{(\tfrac{1}{2}f_t)^2 + v^2} \qquad 1.10$$

Although equation 1.9 is not an accurate failure criterion for concrete, most texts on reinforced concrete adopt it, with an explanation of its approximate applicability. It is then argued that in the region of high v, f_t is relatively small, and thus f_t(max) or the diagonal tension stress is approximately equal to v. Therefore the nominal shearing stress as expressed by equation 1.9, can only be used as an indication of the diagonal tension stress and not as a quantity equal to it.

In the development of the expression for shearing stress and in its generalization as a measure of diagonal tension, the ability of concrete to resist some degree of tension is first neglected and then acknowledged. Such an inconsistency suggests a basic weakness in Eq. 1.9 [Ref. 4].

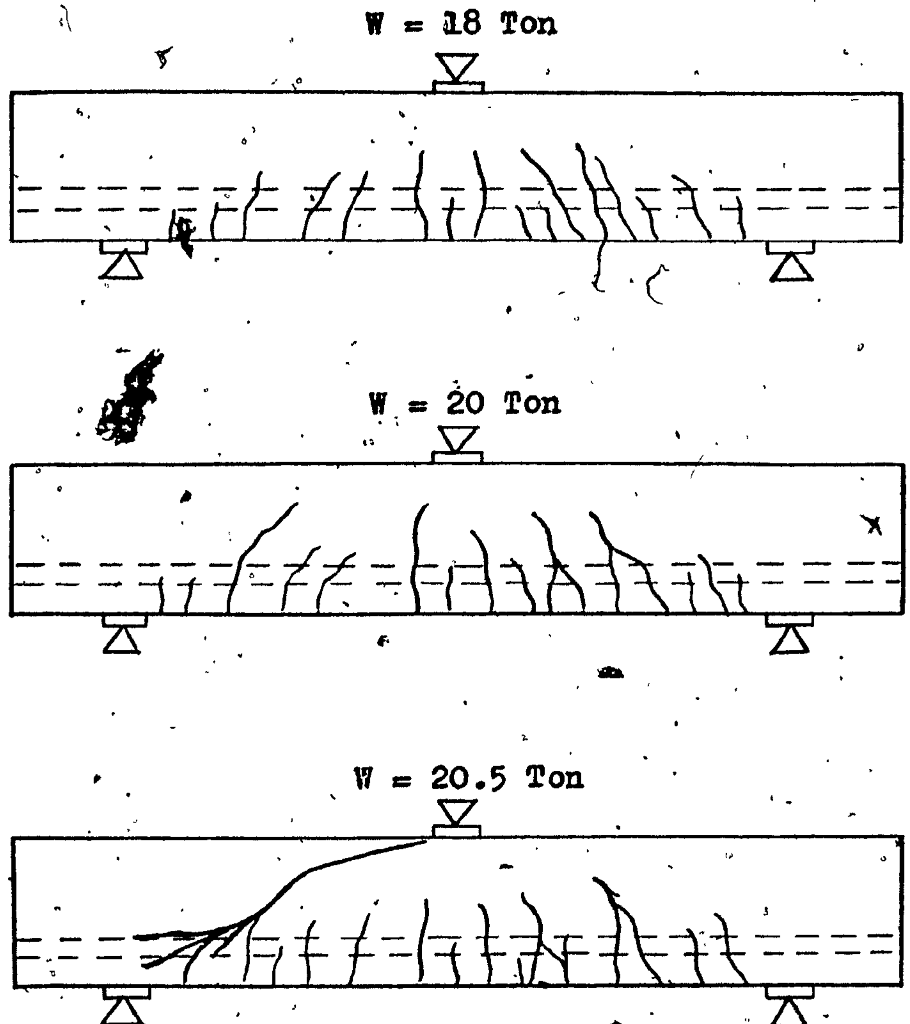


Fig. 1.1 - DEVELOPMENT OF SHEAR CRACKING

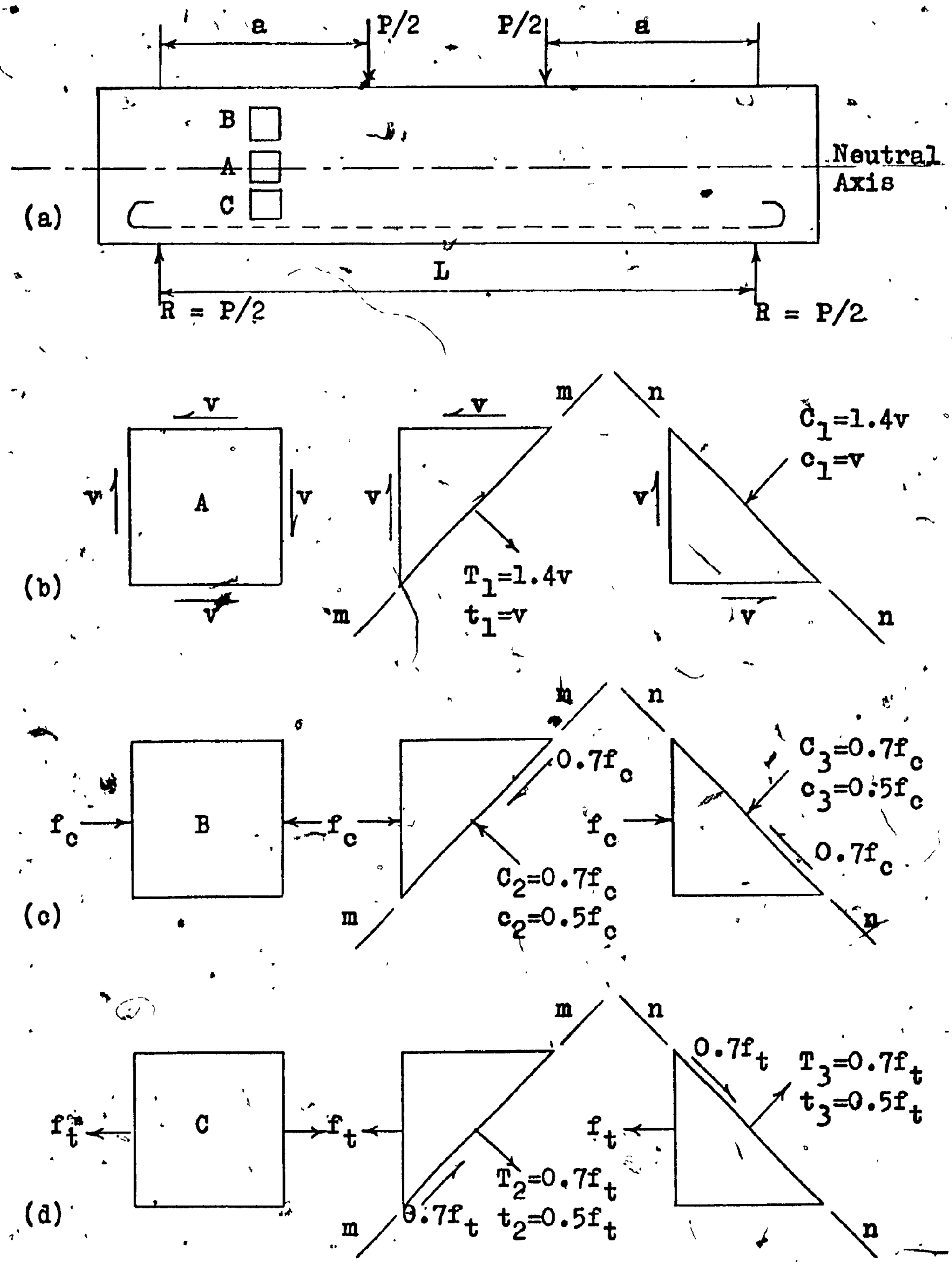


Fig. 1.2 - SIMPLY SUPPORTED REINFORCED CONCRETE BEAM

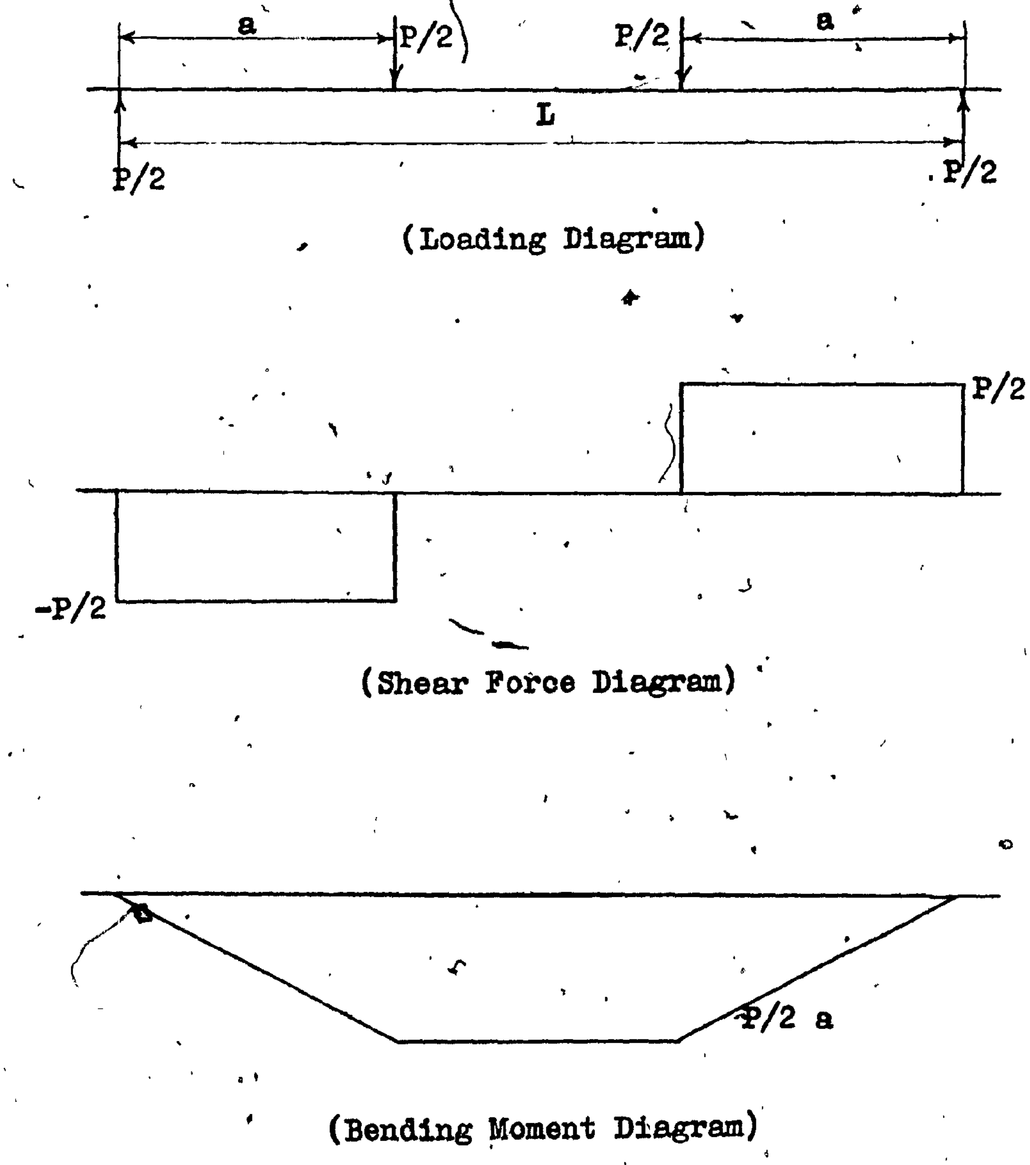


Fig. 1.3 - SHEAR AND BENDING MOMENT DIAGRAMS

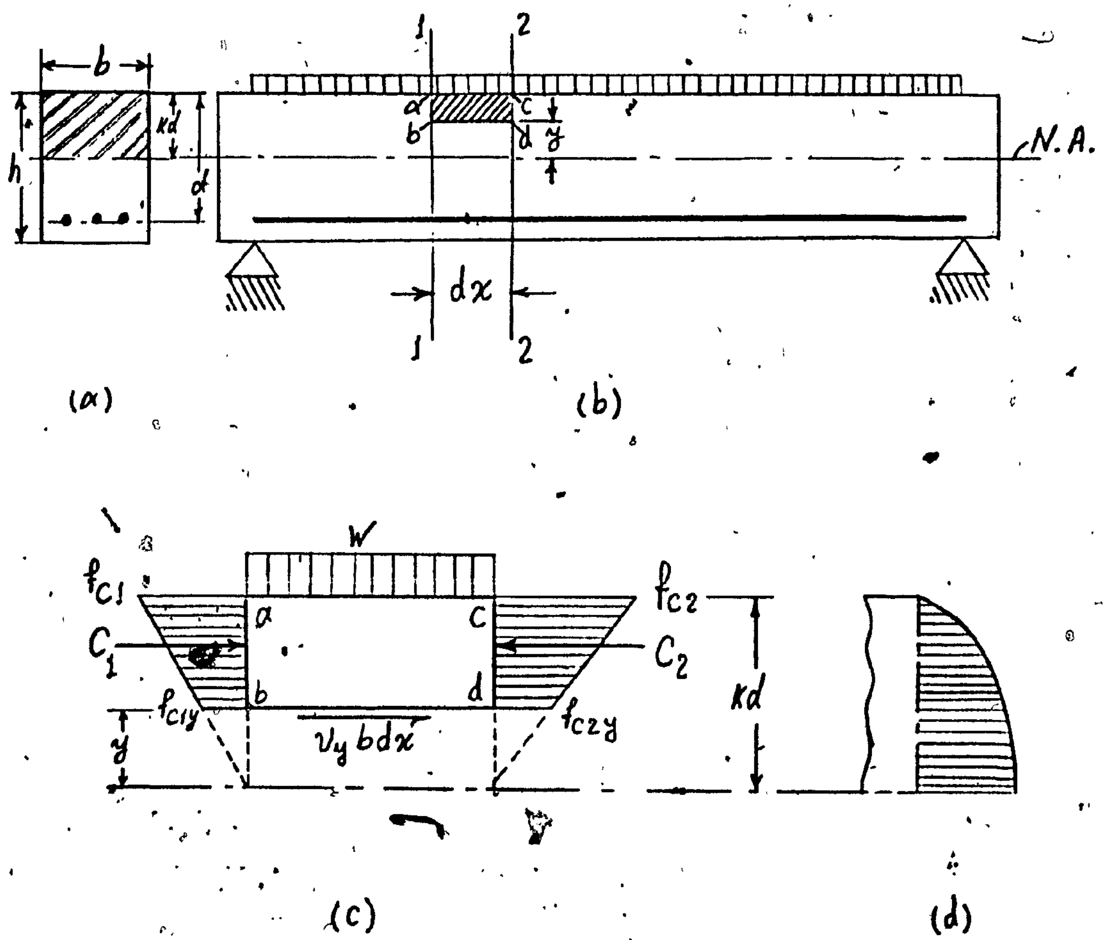


Fig. 1.4

CHAPTER 2

BEAMS WITHOUT WEB-REINFORCEMENT

2.1 Shear Failure of Beams Without Shear Reinforcement

Because designing a beam without web reinforcement is not a Common design, studies have been carried out [Ref: 14] concerning the ultimate load behaviour of beams. It was found that the study of beams without shear reinforcement is significant and relevant to beams with shear reinforcement.

As already we know the shear failure in some cases occurs nearly immediately after first shear cracks, while in other cases it will resist upto higher load prior to failure. It is because if the load capacity of beam is equal or less than the load initiating the shear cracking, failure occurs simultaneously with cracking. But if the load carrying capacity of a beam is greater than load initiating the shear cracking, an increase of load can be supported beyond initial shear cracking.

Some of the failure are considered hereunder in briefly.

2.1.1 Diagonal Tension

This kind of failure happens when shear cracking takes place. A shear crack starts as inclined continuation of an earlier flexural crack and extends to comperessive zone and backwards beyond and below of the origional flexural crack, and it will initiate the collaps when this crack extends through compresive zone while at the other end at level of main reinforcement it extends by tearing action. [Figs. 1.1

and 2.1] This kind of failure because of its tensile nature is called "Diagonal Tension" and is sudden, Fig. 2.4a. References 4,5,7,8 is used to study the failure, the ultimate load should be considered equal to the load causing the initial shear cracking.

2.1.2 Shear Compression

In Fig.2.4b a shear-compression failure has been plotted. If the load resistance of a beam is greater than the shear cracking load but less than the flexural capacity, failure occurs by shear-compression. Failure in all other cases is generally by flexural not by shear.

2.1.3 Flexural Failure

A flexural failure takes place by crushing of the concrete in the compression zone of a beam either before or after the yielding of the tensile reinforcement, and occasionally by breaking of the tensile reinforcement. Therefore it is easy to incorporate the ultimate cause of failure in the definition of flexural failure [Ref. 5], which is defined as: "A beam is said to have failed in flexure if it fails by crushing of the concrete or fracture of the reinforcement as a result of bending stresses".

2.1.4 Inter-relation between failures

Researches show [8] that there is an inter-relation between diagonal-tension, shear-compression and flexure, which can be expressed graphically by plotting M_u againt a/d.

As we discussed earlier if the load resistance of a beam is less than the required to initiate shear cracking, failure by diagonal tension occurs immediately after the formation of a shear crack. Failure by shear-compression occurs if the load resistance is greater than shear cracking but less than flexural capacity. In all other cases failure would be because of flexure.

The above have been expressed mathematically [7]

1) If $M_f \frac{d}{a} \leqslant V$, there is no shear cracking and $M_f = M_u$

2) If $M_s \frac{d}{a} \leqslant V \leqslant M_f \cdot \frac{d}{a}$, Shear cracking occurs and failure is in diagonal tension with $V_u = V$

3) If $V \leqslant M_s \cdot \frac{d}{a} \leqslant M_f \cdot \frac{d}{a}$ Shear cracking occurs and failure is in shear - compression with $M_u = M_s$

where M_s = ultimate shear-compression moment.

Because shear-compression equation cannot be applied at very small value of a/d, (if $a/d \to o$ then $V_u \to \infty$) so there is a limit which by Regan [Ref. 11] defines as $V_u \leqslant 1/6\ f_c bd$

For an ultimate design of a beam at all values of (a/d), the AC1-ASCE Committee [4] use the "shear cracking diagonal tension" equation which by decreasing the ratio of a/d, V increases. It is recognized that the ultimate loads of short beams can be considerably in excess of the shear cracking loads, just as it is predicted by the shear-compression theories.

2.2 General Categories of Failure

Experiences have shown [7] that the ratio of shear span to effective depth ($^a/d$) has a high significant influence in establishing shear strength. And if the other factors to be kept constant, the variation in shear capacity for a 2-point loaded symmetrical, simply supprted rectangular beam could be ploted like in Fig.2.2.

From this figure four general categories of failure may be identified.

1. Deep Beams: $a/d < 1$
2. Short Beams: $1 < a/d \leq 3$
3. Intermediate Length Beams $3 < a/d \leq 6$
4. Long Beams $6 < a/d$

In deep beams, Flexural stresses are less important than shear stresses. As shown in Fig. 2.3. There are compressions and tensions along and across the line joinning the load point and the reaction point. After appearance of inclined cracking, the beam may transforms into a tied-arch, which can fail in a number of ways [7], which are indicated in Fig. 2.3 b.

In short beams, an inclined crack is generally resulted by flexural crack which extends into the compression zone from the tension surface of the beam along the reinforcement, and then beams inclined and extends toward the nearest concentrated load, Fig. 2.4. When inclined cracking developed, failure will occur in one of the indicated modes : (1) Shear-tension failure, Fig.2.4a or (2) Shear-compression failure, Fig. 2.4b.

In intermediate length beams, inclined cracks do not develop before the beam fails in flexure. Several felxural cracks develop and create the "teeth" which are beam segments between these inclined cracks as shown in Fig. 2.5. Formation of inclined cracks and existance of "tooth" which acts as cantilever, represents the ultimate shear capacity of this kind of beams, this type of failure is called "diagonal tension failure" (7).

If the beam is relatively long, or if the precentage of tensile reinforcement is low, further increase in load will cause failure by crushing of the concrete at or near the location of maximum moment. Slightly inclined cracks may be present in addition to the flexural cracks at the section of maximum bending moment, before the failure occur. So, the ultimate strength of such a beam is independent from the size of the shear force and just depends on maximum bending moment capacity.

It can be concluded that qualitatively shear affects the behavior of beams without web reinforcement through the formation of a digonal tension crack. If a digonal tension crack does not form, the effect of shear is negligible. Collapse of the beam may occur simultaneously with the formation of the diagonal crack. On the other hand, a beam may be capable of resisting loads in excess of those causing the formation of the critical diagonal tension crack, and in such cases the final collapse is caused by shear-compression or by some secondary cause brought about by the presence of the diagonal tension crack. Quantitatively, shear limits the ultimate strength of a beam if a diagonal tension crack forms before the ultimate load is reached.

2.3 SHEAR STRENGTH OF BEAMS WITHOUT WEB REINFORCEMENT

In order to develop a formula from which the ultimate shear strength of a beam without web reinforcement may be predicated, 440 tests were studied by the ACI-ASCE Joint Committee on Shear and Diagonal Tension [4].

They indicated that shear capacity depends primarily on three variables, Viz., the precentage of longitudinal reinforcement ρ, the dimensionless quantity M/Vd, and the quality of concrete as expressed by the compressive strength

f'_c. Other variables do not have significant effects on shear strength.

By a series of Mathematical relation and to know

$$V = K_1 \frac{\overline{V}_u}{bd}$$

and $f_t(\max) = K_2 \sqrt{f'_c}$

They got following equation, 7 :

$$\frac{\overline{V}_u}{bd \quad f'_c} = \frac{K_2}{\frac{1}{2} \frac{K_2 \quad M_u \sqrt{f'_c}}{E_s \overline{V}_u \ d \rho} + \sqrt{\left(\frac{1}{2} \frac{K_2 \ M_u \sqrt{f'_c}}{E_s \ \overline{V}_u \ d}\right)^2 + K_1^2}} \qquad 2.1$$

Which variables in this equation are non-dimentional quantities $\overline{V}_u / (bd\sqrt{f'_c})$ and $(M'_u \sqrt{f'_c}) / (E_s \rho \overline{V}_u d)$. They got following equation upon using the correct value of E_s:

$$\frac{\overline{V}_u}{bd\sqrt{f'_c}} = 1.9 + 2500 \rho \frac{\overline{V}_u \ d}{M_u \sqrt{f'_c}} \leqslant 3.5$$

Which has been accepted by ACI Code as the shear (inclined cracking) strength of a beam without Web reinforcement.

So strength of such beam $\overline{V}_c$ becomes

$$\overline{V}_c = (1.9 \sqrt{f'_c} + 2500 \rho \frac{\overline{V}_u \ d}{\overline{M}_u}) \ bd \leqslant 3.5 \ bd\sqrt{f'_c}$$

and its nominal stress:

which is in fact identical to ACI Code formula (11-4). which states

$$v_c = 1.9\sqrt{f'_c} + 2500\, \rho_w \frac{V_u d}{M_u} \leqslant 3.5\sqrt{f'_c} \qquad 2.2$$

where V_u and M_u are design shear force and design moment and $\rho_w = \frac{A_s}{b_w d}$. For a rectangular section $b_w = b$. The value of $\frac{V_u d}{M_u}$ shall not be taken greater than 1.0 except when axial compression is present, which has the effect of limiting v_c at end near the point of inflection [4].

This analysis relates the nominal shearing stress, $v = V/bd$, to the three major variables known to influence it:

1. the nominal shearing stress v increases with increasing concrete strength
2. v decreases with increasing M/Vd
3. v increases with increasing ρ

Lightweight Concrete: This formula with a slight modification is valid for lightweight-concrete also. For "all-lightweight" concrete,

$$v_c = 0.75 \left[1.9\sqrt{f'_c} + 2500 \frac{\rho_w V_u d}{M_u}\right] \qquad 2.3$$

and for "sand-lightweight" concrete,

$$v_c = 0.85 \left[1.9\sqrt{f'_c} + 2500 \frac{\rho_w V_u d}{M_u}\right] \qquad 2.4$$

Linear interpolation is permitted when partial sand replacement is used.

2.5 Comparison of Shear Cracking Theories

The Morsch equation (i.e. $v = V/bjd$) doesn't give satisfactory agreement with test data, but the other theories like ACI. ASCE. eq.2.16, give acceptable correlations with test result, (6, 8, 12). Variations between the other theories in regard to mean value of V_{cal} / V_{test} correspond to the fact that some give lower bounds to the scatter of results while others give mean value. In the ACI-ASCE equation the two parameters are linked in the same secondary term with the result of if M/Vd is large the influence of ρ is negligible. Similarly if ρ is small this reduces the influence of M/Vd which is generally negligible in any case of $M/Vd > 1$. Briefly for all type of the beam except shortest one the term $2500 \rho Vd/M$ is negligible [4]. The treatment of reinforcement ratio ρ in ACI-ASCE gives difference meaning than the other theories especially when it is very small. As ρ tends to zero the Peterson, Regan and revised Smith values of V/bd tends to zero [4], while ACI-ASCE equation give $v = 1.9 \sqrt{f'_c} \simeq 2 \sqrt{f'_c}$ which is called (Simplified conservative Method)

2.6 Classification of Strength Limits

Extensive research work have been carried out by Dr. Z.A. Zielinski, [1,2,3], to find the best solution of preventing failure of beams, due to the load. He has classified the strength limits and the sequence of the cracks. He has identified five strength limits for reiforced concrete beams respectively:

1. First flexural Cracking;
2. Inclined Cracking;
3. Diagonal Splitting due to arch work;
4. Ultimate flexural failure;
5. Excessive deformation of beam.

And three types of cracks are representing the cracking strength limits of a beam, which are indicated in Fig.2.9.

The flexural cracks are the first cracks to appear, next inclined cracks appear which are of group B in shear and moment loaded zone of beam, and last cracks will be splitting cracks which may appear in beams under high shear, Fig.2.9.

2.7 Inclined Cracking

According to Dr.Z.A. Zielinski, [1], this type of cracks which conventionally are called shear cracks takes place as a result of the work and the bond which ties the reinforcement to the surrounding concrete.

He considers the beam after two cracks appear, assuming with a distance Δx, Fig.2.10. On the unit length d_x, there is bond stress U acting on the circumference of the bars Ω which in the region of the beam above the bars, causes the appearance of bi-axial stresses inclined under 45^0, and defined by

$$v_c = U \Omega / b$$

If we substitute bond stresses $U = V/\Omega jd$ in this equation, we get:

$$v_c = V/bjd = v$$

Clearly it shows the dependence of shear stress v on the bond stress U. If the reinforcement is unbonded so there is no bond resistance (i.e. U=0).

so, $v = v_c = v_t = 0$

So in this case there will not be inclined cracking. Briefly, beam with unbonded reinforcement can not have inclined cracks, on the contrary, existence of the bond, causes bi-directional inclined stress condition of $v_c = -v_t$ which are added to flexural normal stress due to the moment, and because flexural stress in the neutral axis is zero, $v_c = - v_t$ govern the strength of the beam. If these reach their limit strength, as

$$f_{sc} = v_c = - v_t \simeq 0.9\ f_{to}$$

the 45^0 inclined cracking will appear. But if the main reinforcement is well anchored beyond the support, it can remain in good condition till reaching the diagonal splitting at arch scheme.

The tests which have been carried out [1,2,3] show that the strain in concrete at the moment of cracking is between $\epsilon = 0.0001$ to 0.00015 and the accompanying stress in steel is only

$$f_s = \epsilon E = (0.0001 \text{ to } 0.00015) \times 30 \times 10^6$$
$$= 3000 \text{ to } 4500 \text{ psi}$$

which is just 5 to 8 percent of f_y.

So he found that it is not economical to try to eliminate inclined cracking by means of reinforcement.

Because we must assure the additional load carring capacity, beyond flexural and inclined cracking, he believes that even a quite small amount of the Web reinforcement (smaller than on shear theory is required) can satisfy us.

Shearing force V_s which can describe limit strength in inclined cracking and beam work is as follow. For beams without web reinforcement

$$V_s = V_c\ bjd = f_{sc} bjd \simeq 0.9\ f_{to} bjd$$

With web reinforcement

$$V_s = 0.9\ f_{to} bjd \left(1 + P_s \frac{3000}{f_{to}} \cos \alpha_s \right)$$

As we see there are differences between these formulas and those recommended by ACI-ASCE, we will consider these differences later.

2.8 Diagonal Splitting Due to Arch Work

For a beam with unbonded reinforcement three basic schemes of arch work can be identified as in Fig. 2.11, which relate to :

i. Uniformly loaded beam

ii. One or two point loaded beam. This beam is subject of diagonal splitting.

iii. Three or more point loaded beam.

According to Prof. Z.A. Zielinski [1,2,3] and referring to Fig.2.12, bi-directional stresses depend on the geometry of the support-segment and be defined for a beam without web reinforcement as:

$$f_{ct} = \frac{S}{bh} \cos \alpha = \frac{H}{bh} \qquad 2.5$$

$$f_{tc} = \frac{S}{ba_2} \sin \alpha = \frac{V}{ba_2} \qquad 2.6$$

$$\frac{f_{ct}}{f_{tc}} = \frac{Ha_2}{Vh} = \frac{a_2^2}{h^2} \qquad 2.7$$

When the concrete in the middle of the support-load reach f_{tc}, the diagonal splitting occurs. According to (Ref 1) and refer to Fig. 2.7 and equation 2.7 :

$$f_{tc} = \frac{f'_c}{\frac{f'_c}{f_{to}} + \frac{a_2^2}{h^2}} \qquad 2.8$$

then $V_a = f_{tc} \cdot a_2 b$

for beam without web reinforcement

For a beam with web reinforcement the formulas change to:

$$f_{ct} = \frac{H}{bh\left(1 + \frac{E_c}{E_s}\rho_h\right)} \qquad 2.9$$

$$f_{tc} = \frac{V}{ba_2\left(1 + \frac{6000}{f_{tc}}\rho_v\right)} \qquad 2.10$$

Where $\rho_h = \frac{A_{sh}}{bh}$ and $\rho_v = \frac{A_{sv}}{ba_2}$

Then splitting force V_a is

$$V_a = f_{tc}\, ba_2 \left(1 + \frac{6000}{f_{tc}}\rho_v\right) \qquad 2.11$$

For the beam has sufficient web, the work capability of the beam does not end here. It will work further until it reaches V_u the ultimate shear force. Then it accompany the ultimate flexural failure and causes diagonal cracked support-load segment under diagonal uni-axial compression only, while the web reinforcement carry all the tangent splitting force.

By following formula, after diagonal splitting occured, we can calculate the ultimate capacity.

$$V_v = \cos\alpha \sum_{i=1}^{n} A_{svi} \quad f_{yi} \qquad \text{(I)}$$

I. Portion of the reaction force which is carried by vertical web reinforcement (A_{sv})

$$V_h = \sin\alpha \sum_{i=1}^{n} A_{shi} \; f_{yi} \qquad \text{(II)}$$

II. Portion of reaction force which is carried by horizontal web reinforcement (A_{sh})

$$V_u = V_v + V_h \qquad \text{(III)}$$

III. Ultimate reaction force which could be carried at the support.

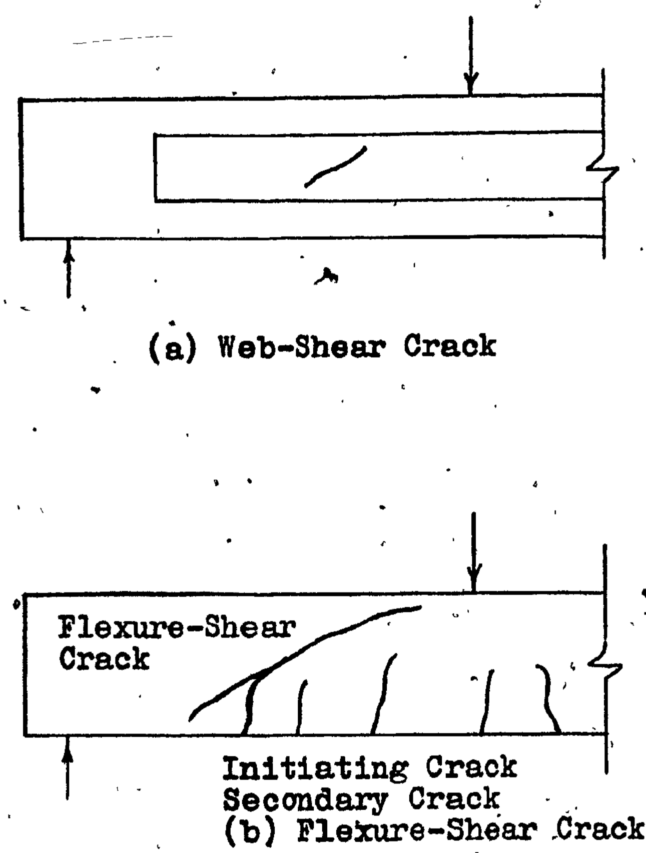


Fig. 2.1 - Type of inclined Cracks
(from Ref. 5)

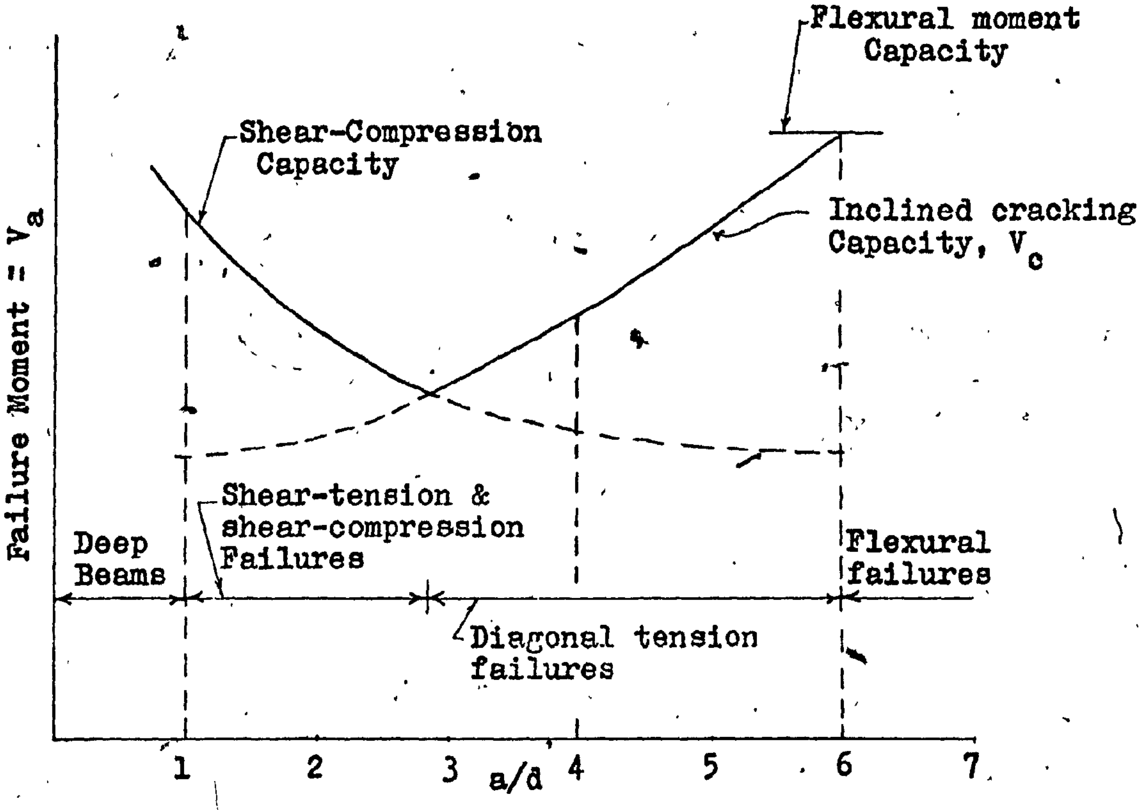
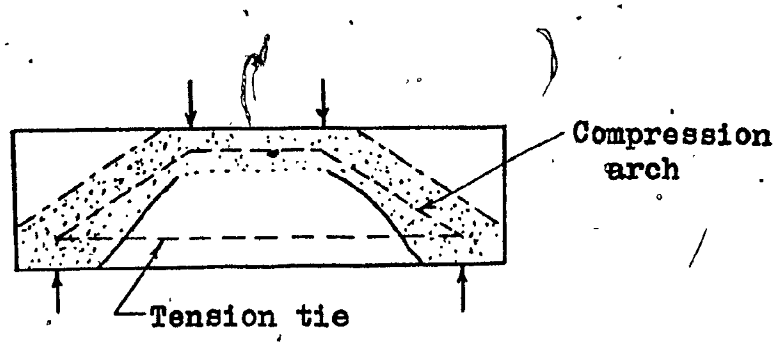
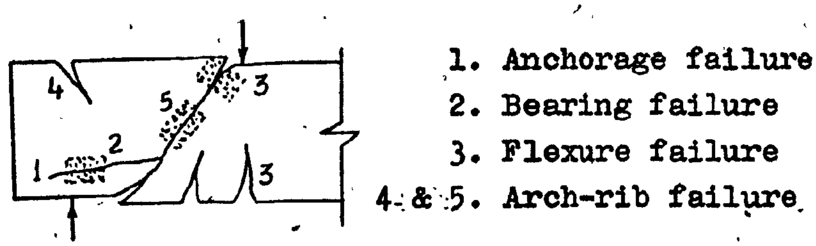


Fig. 2.2 - Variation in shear capacity with a/d for rectangular beams.

(Ref.5)

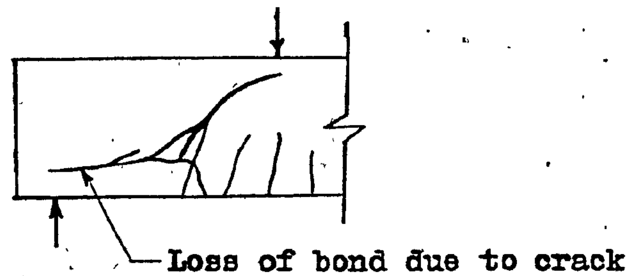


(a) Arch action

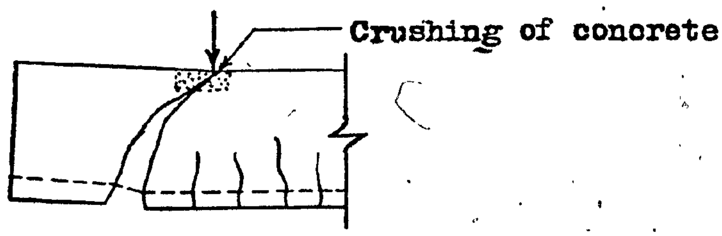


(b) Types of failure

Fig. 2.3 - Modes of failure in deep beams, $a/d \leq 1.0$ (adapted from Ref. 5)



(a) Shear - tension failure



(b) Shear - Compression failure

Fig. 2.4 - Typical shear failures in short beams a/d = 1 to 3 (adapted from Ref. 5)

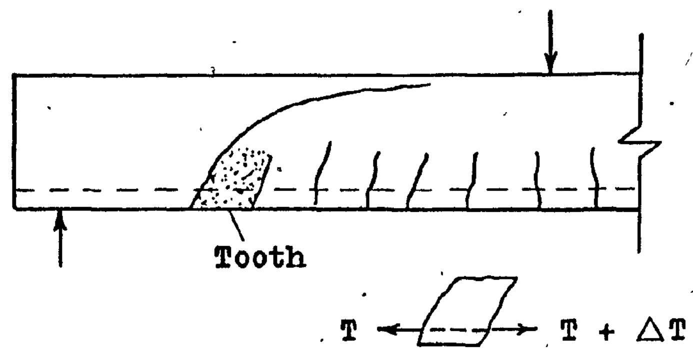


Fig. 2.5 - "Diagonal tension failure" or "tooth cracking failure" on intermediate length beams, a/d = 3 to 6.

(Ref.5)

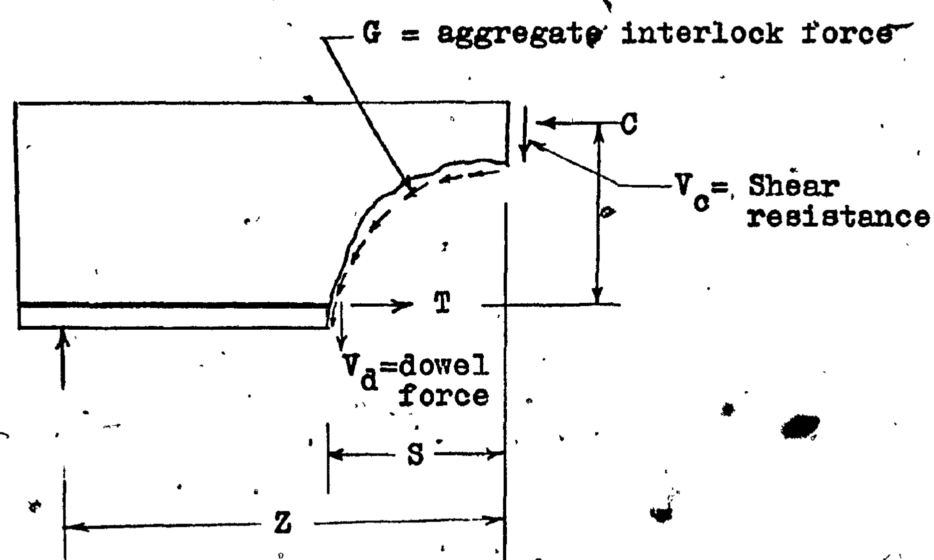


Fig. 2.6 - Redistribution of shear resistance after formation of inclined crack. (Ref.5)

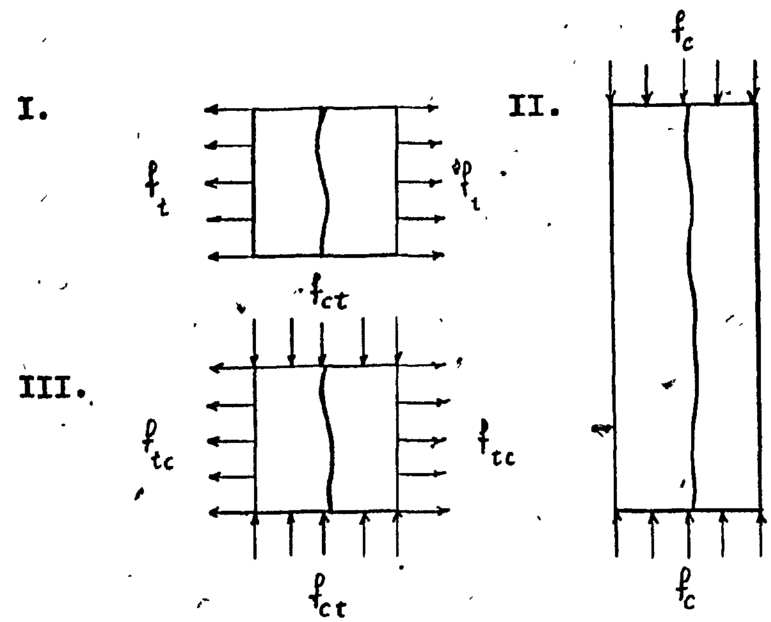


Fig. 2.7 - Basic causes of failure of concrete: (i) uniaxial tension stress, (ii) uni-axial compression stresses, (iii) bi-directional tension and compression stresses. (Ref.1)

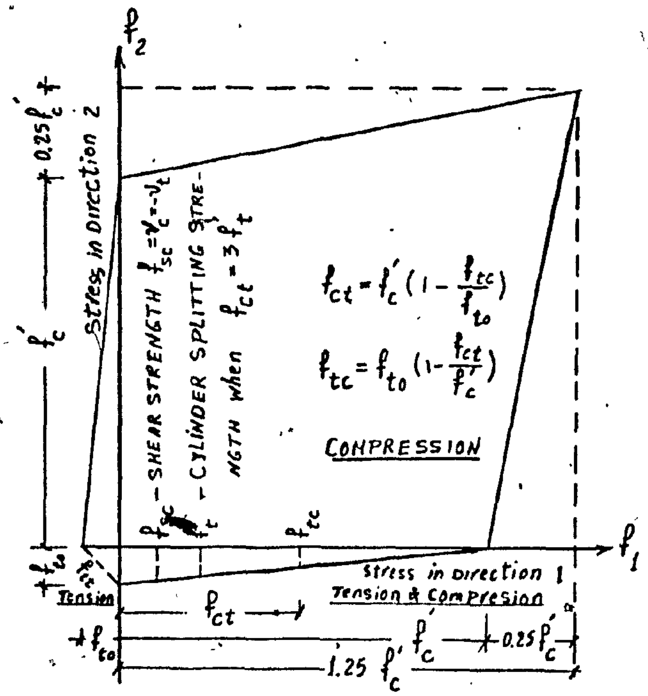


Fig. 2.8 - Combined bi-directional strength of concrete. (Ref.1)

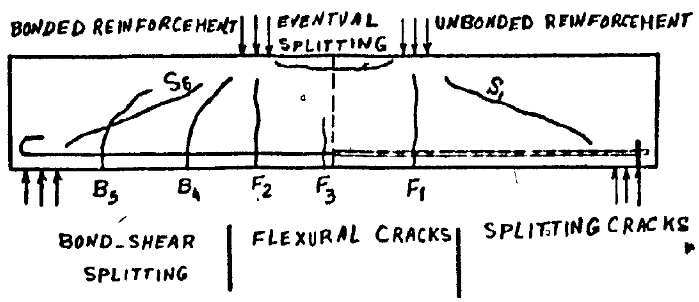


Fig. 2.9 - Generalized crack propagation in the beam with bonded or unbonded reinforcement. F_1-F_3 - flexural vertical cracks, B_4-B_5 - inclined cracks due to the bond and beam work, S_6-S_7 - diagonal splitting cracks due to arch work. Nos. 1,2,3, etc., show the order of crack appearance. (Ref.1)

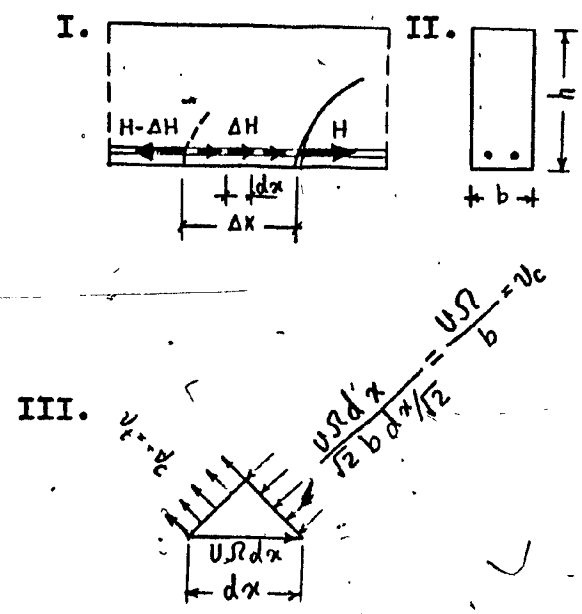


Fig.2.10 - Definition of bi-directional stress condition at inclined cracking of the beam caused by bond (i) segment of beam, (ii) cross-section, (iii) stress in concrete due to bond.
(Ref.1)

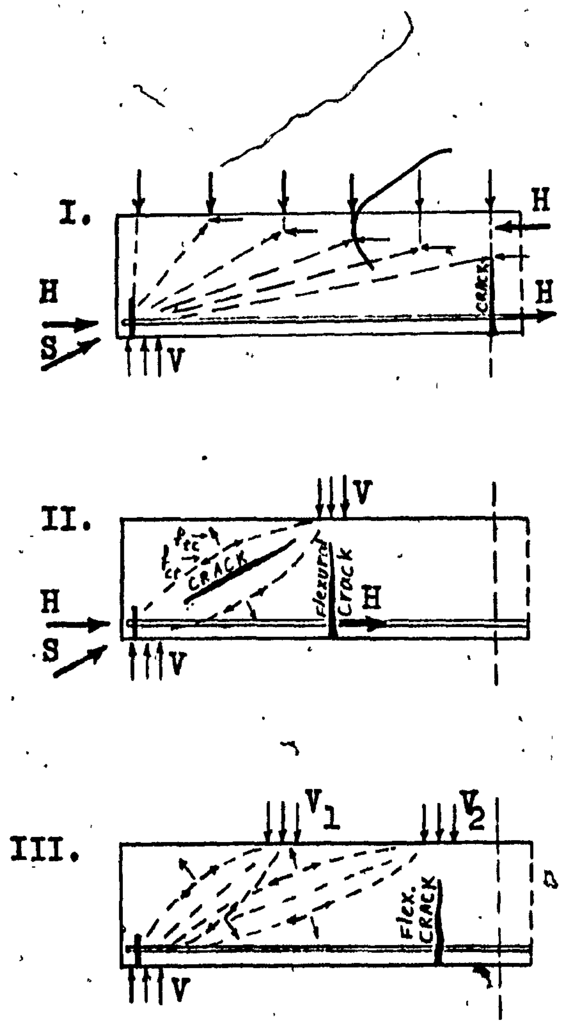


Fig. 2.11 - Basic loading schemes describing arch strength of reinforced concrete beam with unbonded reinforcement: (i) uniformly loaded beam, (ii) one or two-point loaded beams, (iii) three or more-point loaded beams.

(Ref.1)

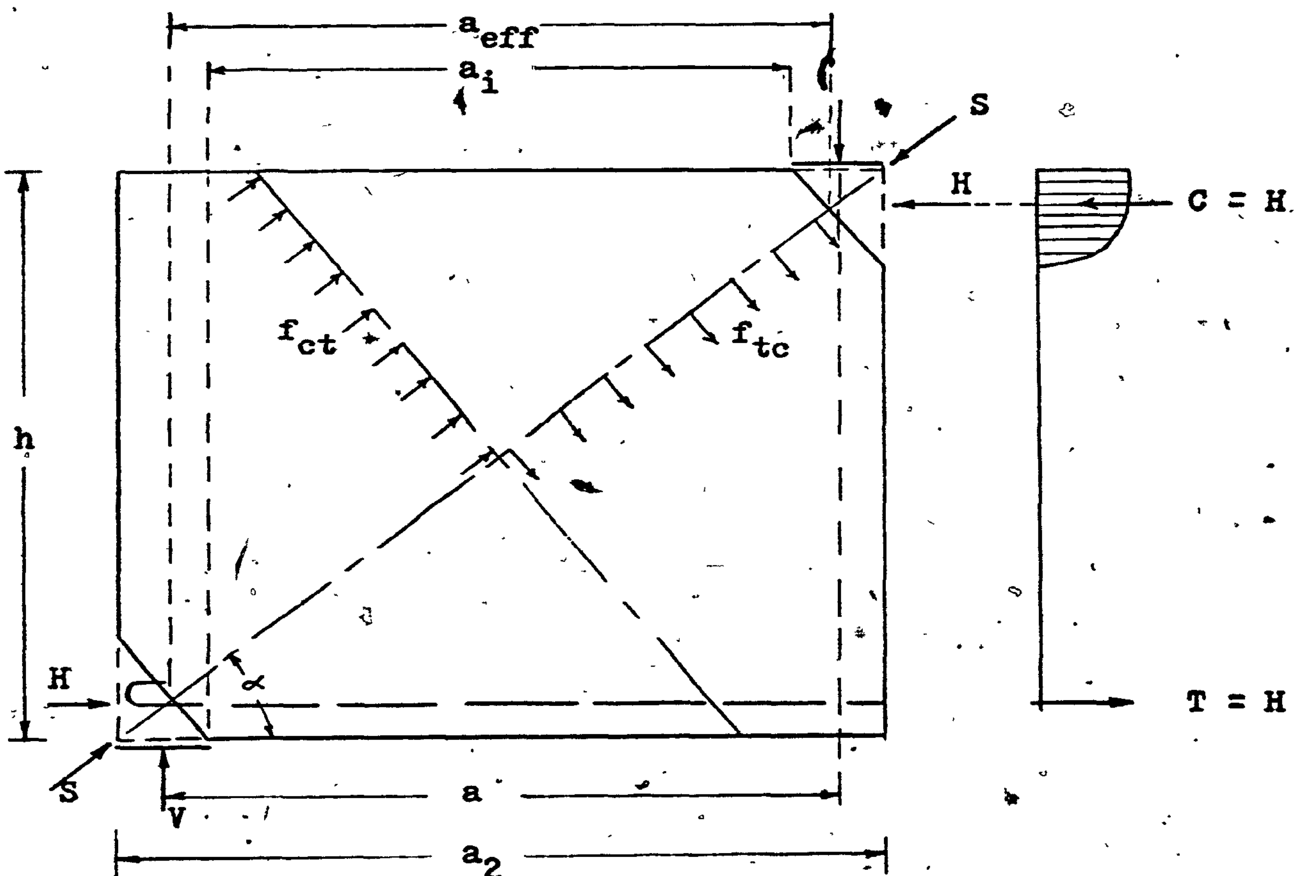


Fig.2.12 - SIMPLIFIED STRESS DISTRIBUTION UNDER DIAGONAL COMPRESSION

CHAPTER 3

REINFORCE CONCRETE BEAMS WITH WEB REINFORCEMENT

CHAPTER III

3.1 REINFORCED CONCRETE BEAMS WITH WEB REINFORCEMENT:

Economy of design demands, in most cases, that a flexural member be capable of developing its full moment capacity rather than having its strength limited by premature shear failure. This is also desirable in that stractures, if overloaded, should not fail in the sudden and explosive manner characteristic of many shear failures, but should show adequate ductility and warning of impending distress. The latter, as was pointed out, obtains for flexural failure caused by yielding of the longitudinal steel, which is preceded by gradual, excessive deflections and noticeable enlargement of cracks. If, then, the available shear strength by eq. (2.12) is not adequate, special shear reinforcement, known as "web reinforcement" is used to increase this strength.

3.2 FOUNCTION OF WEB REINFORCEMENT:

Web reinforcement may be in either of the forms as shown in Fig. (3.1).

1) Stirrups prependicular to the longitudinal reinforcement,
2) Stirrups making an angle of 45° or more with longitudinal reinforcement,

3) Longitudinal bars bent so that the axis of the bent bars makes an angle of 30^0 or more with the axis of longitudinal portion of the bar; and

4) Combination of (1) or (2) with (3).

spirals, including rectangular helices, are also permited by the 1971 ACI Code.

Web reinforcement has no noticeable effect prior to the formation of diagonal cracks. In fact, measurements show [5] that the web steel is practically free of stress prior to crack formation. After diagonal cracks have developed, web reinforcement increases the shear resistance of a beam in three separate way:

(1) Part of the shear force is resisted by the bars which traverse a particular crack.

(2) The presence of these same bars restrict the growth of diagonal cracks and reduces their penetration into the compression zone. This leaves more uncracked concrete available at the head of the crack for resisting the combined action of shear and compression.

(3) As seen in the cross section of Fig. (3.1), the stirrups are so arranged that they tie the longitudinal reinforcement into the main bulk of the concrete. This provides some measure of restraint against the splitting of concrete along the longitudinal reinforcement and increases the share of the shear force resisted in dowel action.

Once cracks have appeared, the transverse reinforcement begins to deform gradually as the load increases, until it yields. If the member has only a small amount of transverse reinforcement, failure may take place through one or several of transverse reinforcement bars. If the amount of transverse reinforcement is sufficient, the inclined cracking will be of little significance and failure will be due to flexure.

Providing suitable transverse reinforcement in beams that could fail in shear can greatly increase the shear capacity all usually can assure flexural failure under given loading conditions. Indeed, one of the principal objectives in providing web reinforcement is to eliminate shear as a mode of failure. This approach to the design of web reinforcement does not eliminate the shear problem but only changes its form, as the question is then focused on how much web reinforcement is required to prevent shear failure. A traditional answer to this question awaits the general theoretical solution of the problem. In the meantime, transverse reinforcement is usually designed for the "excess shear", generally defined as the difference between the design shear load and the acceptable inclined cracking load limit in a beam without transverse reinforcement. Thus, definition of this inclined cracking load is important, not only because of its fundamental significance as an essential stage in the mechanism of potential shear failure, but also as a crucial design parameter for a beam in which shear failure may be prevented.

The web reinforcement in a reinforced concrete beam can be compared with the diagonal members of a steel truss. If we consider the steel truss of Fig. 3.2a, we see that upper and lower cords are in compression and tension situation respectively and the diagonal members, called web members, are alternatively in compression and tension. Truss action in a reinforced concrete beam and the web reinforcement are shown in Fig. 3.2b. These web as we know will increase the shear strength of the beam. As it is seen the web reinforcement must be anchored in the compression zone of the concrete and is usually hooped around the longitudinal tension reinforcement.

Analogous truss action in Fig. 3.2b and d easily describe the action of inclined and vertical web reinforcement of the beam in Figs. 3.2c and e.

3.3 SHEAR FAILURE OF BEAMS:

3.3.1 Traditional Mörsch Truss Analogy

In the Mörsch truss analogy the internal structure of a beam containing shear cracks is assumed to act as a truss, in which the concrete compressive zone and the tension steel are the main chords, while the shear reinforcement and strips of web concrete inclined at 45° to the tension steel form the lattice Fig. (3.3).

The only equation used is that of vertical equilibrium, and in cases where the shear reinforcement consists of vertical stirrups only, this gives

$$V' = r.f_v.b.l_a \qquad 3.1$$

where V' = external shear force

A_v = area of one or group of stirrups, at one cross section

f_v = stress in stirrups

l_a = internal lever arm.

f_v is generally assumed, or at least implied to be equal to the yield stress f_y when $V = V_u$.

As experiences show equation gives a poor correlation with test results, and is often grossly over-conservative [8]. The theoritical objections to it are also considerable. The main ones being:

a) It ignores the ability of the concrete compressive zone to support shear.

b) It appears to predict that failure is caused by the shear reinforcement's reaching its yield stress, while in fact shear failure of beams with shear reinforcement is generally due to the compressive failure of the concrete above a shear crack.

c) The assumption that all web compressive forces, or in effect all shear cracks, are at 45° to the main steel is an over-simplification Fig. (3.4).

3.3.2 Empirical Adaption of the Truss Analogy-ACI-ASCE [4]

ASCE-ACI Committee 326, published the results of their findings in the form of a paper, in which they recommended the use of certain empirical formulas for design purposes. These recommendations are based on test data obtained from programmes of investigation conducted under the supervision of the committee, and on all other available data of other tests results. For beams with no web reinforcement, as mentioned before, the following formula was recommended.

$$v_u = \frac{V_u}{bd} = 1.9\sqrt{f'_c} + 2500\,\rho\frac{Vd}{M} \leq 3.5\sqrt{f'_c} \qquad 3.2$$

As for beams with web reinforcement, the committee recommended the continuation of the use of the "Truss-Analogy" since it has been giving reliable results. The relationship suggested for design is:

$$V_{uw} = V_u + Krf_y \qquad 3.3$$

where,

V_{uw} is the ultimate shear stress for beams with web reinforcement and should not be greater than $8\sqrt{f'_c}$

V_u is given by equation 3.2

f_y shall not exceed 60,000 p.s.i.

Krf_y shall not be less than 60 p.s.i

Although the committee recognized the advantages of "shear-moment" approach and its prospect, they did not recommend its use as a design criterion because the hypothesis was not fully developed.

It could be summarized from these theories, that shear reinforcement does not only help to carry part of the total shear, but it increases the ability of the compression zone to resist shear, helps prevent splitting of the beam at the level of the tension reinforcement, and keeps the beam as a unit giving ample warning of any impending collapse.

3.4 SHEAR STRENGTH OF BEAMS WITH WEB REINFORCEMENT

The shear strength of a reinforced concrete beam with transverse reinforcement can be defined in term of the shear carried by concrete, longitudinal steel, and transverse steel reinforcement. The presence of transverse reinforcement modifies the nature and extent of inclined cracking and therefore influences the amount of shear carried by the concrete V_c, and the longitudinal steel V_{sl}. However, as there is no general theory which would permit precise evaluation of these quantities the shear capacity of a beam with transverse reinforcement is usually obtained by adding the contribution of the web reinforcement to the capacity (including load) of a similar beam without web reinforcement [8].

When shear strength contributed by the web reinforcement is V_u'.

$$V_u = V_c + V_u' \qquad 3.4$$

By assuming that an inclined crack in 45^0 direction extends all the way from the longitudinal reinforcement to the compression surface and that intersects about n stirrups, Fig 3.5, we develop an expression for V'_u. Part of this amount which is carried by the stirrup is equal:

$$V'_u = n A_v f_y \sin\alpha \qquad 3.4$$

From trigonometry,

$$nS = d (\cot 45^o + \cot \alpha) = d (1 + \cot \alpha) \qquad 3.5$$

Then

$$V'_u = \frac{d(1+\cot\alpha)}{S} A_v f_y \sin\alpha = \frac{d(\sin\alpha + \cos\alpha)}{S} A_v f_y \qquad 3.6$$

Which its nominal unit would be

$$v'_u = \frac{(\sin\alpha + \cos\alpha)}{bS} A_v f_y \qquad 3.7$$

In vertical transverse reinforcement (i.e. $\alpha = 90^o$)

$$v'_u = \frac{A_v f_y}{bS} \qquad 3.8$$

According to ACI-ASCE joint Committee 326, the comparison of test result and calculation shows that these formulas are conservative. It is said that additional resistances which participate to help prevent failure but are not considered in computing shear strength are those due to "dowel action" and "aggregate interlock" Fig.3.6.

3.5 Requirements for shear Reinforcement

For designing a beam , lower and upper amount of web should be considered, because if the amount of web reinforcement is not sufficient it will yield immediately at the formation of the inclined cracks. If the amount of web to be high it will fail because of shear-compression before yielding of the web steel. So the amount of web should be optimized till both web reinforcement and compression zone continue to carry increasing shear after formation of inclined crack. In this case a gradual failure is ensured and stirrups restrain the growth of inclined cracks.

The ACI Code (ACI Formula 11.1) recommends following limitation.

Minimum web reinforcement area

$$\text{Min } A_v = 50\,\frac{b_w S}{f_y}$$

This amount provides

$$v'_u = \frac{A_v f_y}{b_w S} = \left(50\,\frac{b_w S}{f_y}\right)\left(\frac{f_y}{b_w S}\right) = 50 \text{ psi}$$

And for higher amount of web reinforcement it gives

$$V_u = 8\sqrt{f'_c} \qquad \text{(AC1-11.6.4)}$$

For design prpose, the minimum amount of web reinforcement is required wherever

$$V_u > \frac{V_c}{2} \qquad \text{(AC1-11.1.2)}$$

except for beams of total depth less than 10 in. Also the amount of minimum web reinforcement may be waived.

There are two basic values that can be used for the concrete capacity of V_c:

1) By the "simplified conservative method"

$$V_c = 2\sqrt{f'_c} \qquad \text{(AC1-11.4.1)}$$

2) By the "more exact method"

$$V_c = 1.9\sqrt{f'_c} + 2500\,\rho_w \frac{V_u d}{M_u} \qquad \text{(AC1-11.4.2)}$$

Where $\quad \propto \frac{V_u d}{M_u} < 1$

3.6 Discussion of Cracking Strength Limits
By Prof. Z.A. Zielinski

In order to study inclined cracking strength limits, comparison of arch theory and shear theory is carried out as follow:

$$M_u \simeq 0.35\ bh^2 f'_c \qquad 3.14$$

This moment expresses ultimate flexural strength by Concrete.

and $$M_s = V_a\ a = 0.9\ f_{to} b\ j\ d\ a \qquad 3.15$$

expresses the moment of a 45° inclined cracking due to beam work and bond with 2 point loaded.

By assuming $f_{to} \simeq 0.1\ f'_c$ and $jd = 0.57\ h$ we get

$$M_s = 0.06\ abh\ f'_c \qquad 3.16$$

which if we compare this one and (3.14) the ratio will give us a straight line valid for $a/h \leqslant 5.8$ (see Fig.3.7)

$$F_s = \frac{Ms}{M_u} = \frac{0.06\ ab\ hf'c}{0.35\ b\ h^2 f'_c} = 0.172\ {}^a/_h$$

M_u will define the difference regions of influence.

Region 1. for ${}^a/_h \geqslant 5.8$ defines beams which will fail by reaching ultimate flexural where web reinforcement is not required at all, and no stirrups is required for beam $l \geqslant 11.6\ h \simeq 12h$ of one-joint loaded.

Region 2. for $a/h < 5.8$ defines bonded beams which will be subjected to inclined cracking (by shear) and where web reinforcement is required.

For diagonal splitting (at arch work), M_a is its moment for a 2 point loaded, unbonded reinforced beam without web reinforcement.

$$M_a = V_a . a = f_{tc} \; ba^2 = \frac{fc \; a^2 b}{fc/f'_{to} + a^2/h^2} \approx \frac{fc \; a^2 b}{10 + a^2/h^2}$$

Region 3. for $a/h \leqslant 2.32$ to be seperated by the line F_a which is:

$$F_a = \frac{Ma}{M_u} = \frac{ba^2 \; fc}{(a^2/h^2 + 10) \; 0.35bh^2 f_c} = \frac{2.85}{10 + a^2/h^2} \cdot \frac{a^2}{h^2}$$

identifies unbonded beams which will fail by diagonal splitting at the arch scheme. For taking over the splitting force, stirrups will be required in this region. But for $a/h > 2.32$ there is no danger of inclined splitting due to arch work. No stirrups is necessary for one point loaded, unbonded beam of $l \geqslant 4.64h$.

According to researches carried out by Dr. Z.A. Zielinski [1] we can design a bonded, web reinforcement beam of $a/h = 2$ which expose 45^o inclined cracks due to beam work and later on splitting cracks.

Also investigation has been done by comparing shear strength in arch theory (V_a) and traditional shear theory (V_s) by author of Ref. 1.

Comparison gives:

$$F_3 = \frac{V_a}{V_s} = \frac{f_{tc}\ ab}{0.9\ f_{to} bjd} = \frac{f_c}{0.9\ f_{to}} \times \frac{a}{jd} \times$$

$$\frac{1}{f_c/f_{to} + a^2/h^2}$$

which again for $jd = 0.67\ h$ and $f_{to} = 0.1\ f_c$ gives:

$$F_3 = \frac{V_a}{V_s} = \frac{1.67}{10 + a^2/h^2} \cdot \frac{a}{h}$$

As it is seen in Fig. 3.8 for $1.5 < a/h < 5.8$ the shear force in arch theory is between 2 to 2.6 times higher than shear force defined on beam work and inclined cracking for beams.

Fig. 3.9 shows comparison of amount of reinforcement required for taking over of shear forces in arch theory (A_a) and shear theory (A_s)

$$F_4 = \frac{A_s}{A_a} = \frac{0.9\ f_{to}\ ab}{f_{tc}\ ab} = 0.09\ (10 + a^2/h^2)$$

It is seen again that the amount of reinforcement required in arch theory for beam of $1.5 < a/h < 5.8$ is much less than on traditional shear theory.

Another comparison has been done between required web reinforcement allowed by ACI (A_{co}) with that required by arch theory.

Shear stress in stirrups by ACI 318-63 is:

$$v_u = V_u - V_c$$

$$V_u = 10\phi\sqrt{f_c'} \qquad \text{where } \phi < 1$$

$$V_c = 3.5\phi\sqrt{f_c'}$$

then $$v_u = 6.5\phi\sqrt{f_c'} \simeq 5.5\sqrt{f_c'}$$

so $$\frac{A_{co}}{A_a} = \frac{5.5\sqrt{f_c'}\,ba}{f_{tc}\,ab} = \frac{5.5\sqrt{f_c''}}{f_c'}\left(10 + a^2/h^2\right)$$

which by increasing $f_c = 3000$ psi to 4000 psi; A_{co}/A_a decreases just 13% which means the function of A_{co}/A_a is almost same as Fig. 3.9. We find that the amount of required web which ACI especially considers for a beam of $2.32 < a/h < 5.8$ is much more than that which is required by arch theory, which of coure is not necessary at all.

3.7 Web Reinforcement Requirement (In Arch Theory)

Consideration has been done by Dr. Z.A. Zielinski in Ref.1 about a bridge beam loaded with a traveling single force P, with clear span l, as follows, for finding the amount of web required.

The ultimate maximum flexural moment,

Hence $$M = 0.35\ bh^2\ f_c' = \frac{Pl}{4}$$

$$P = \frac{1.4\ bh^2\ f_c'}{l}$$

so shear force at P, Fig. 3.10 will be,

$$V = \frac{P\ (l-a)}{l} = \frac{1.4\ bh^2\ f_c'\ (l-a)}{l^2}$$

The amount of web reinforcement which we need to provide load carring ability beyond the arch strength limit and able to take over the whole splitting force begining from 45° inclined cracking moment has been defined as:

$$A_a = \frac{V}{f_s} = \frac{1.4\ bh^2\ (l-a)}{l^2} \cdot \frac{f_c'}{f_s}$$

so the required reinforcement density will be.

$$\Omega_a = \frac{A_a}{a} = \frac{1.4\ bh^2\ (l-a)}{a\ l^2} \cdot \frac{f_c'}{f_s}$$

The web in any case will be required for beams of

$$^{a}/_{h} < 5.8 \quad \text{and} \quad ^{l}/_{h} < 11.6 \simeq 12,$$

Example : $l = 12h$, $b = {}^{h}/_{3}$, $f_c = 4000$ psi

and $f_s = 30.000$ psi.

For such a beam

$$A_a = \frac{1.4\ h^3}{3x12\ h} \cdot \frac{4000}{30000} \left(1 - \frac{a}{l} \right)$$

$$= 0.000\ 4\ 35h^2 \left(12 - \frac{a}{h}\right) = k \left(12 - \frac{a}{h}\right) \qquad 3.20$$

so $$\Omega_a = \frac{A_a}{a} = \frac{k}{a} \left(12 - \frac{a}{h}\right) \qquad 3.21$$

where $k = 0.000\ 435\ h^2$

Formula 3.20 and 3.21 have been ploted in solid lines in Fig. 3.10.

On the basis of ACI - 318 - 63, the amount of web and its density required are:

$$A_s = \left(\frac{V}{jbd} - 3.5\phi\sqrt{f_c}\right)\frac{ab}{f_s}$$

$$= \frac{V_a}{jd\ f_s} - \frac{3.5\ f_c\ ab}{f_s}$$

For assumption as before and $jd = 0.67h$ and $\phi = 0.85$ we get

$$A_s = k \left(13.2\ ^{a}/_{h} - 1.5\ ^{a^2}/_{h^2}\right)$$

Accordingly

$$n_s = \frac{A_s}{a} = \frac{k}{a} \quad (13.2\ ^a/_h - 1.5\ ^{a^2}/_{h^2})$$

These 2 formulas have been plotted in dotted lines in Fig. 3.10. By comparison of these diagrams (Fig.3.10) we see that the amount of web reinforcement which arch theory suggests is much less than web requirement in ACI - Code.

Furthermore, we see on the diagram the average of web reinforcement for this example could be reduced by more than half.

It must be mentioned again that, the arch scheme is possible only if the whole main reinforcement is carried and fully anchored beyond the support.

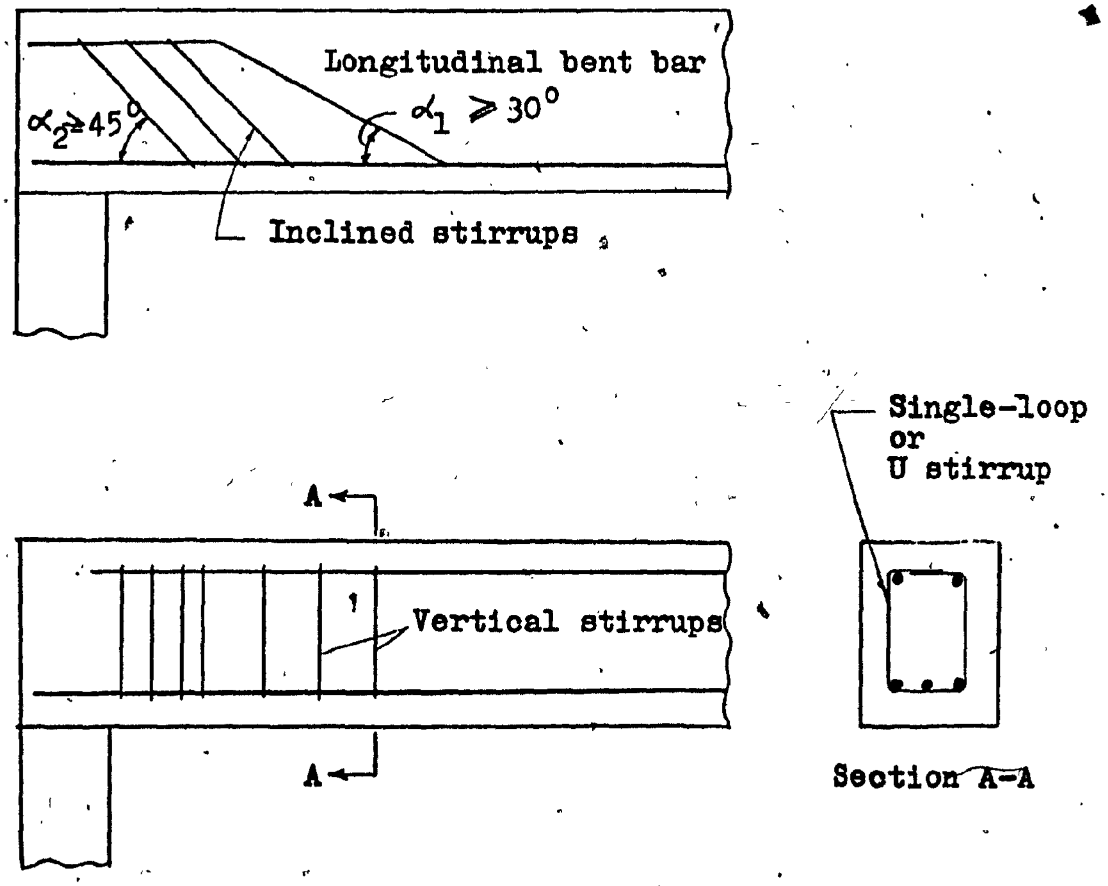
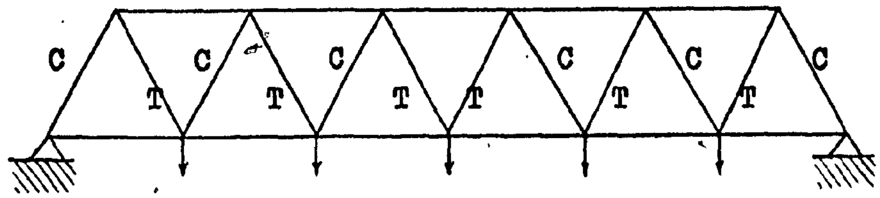
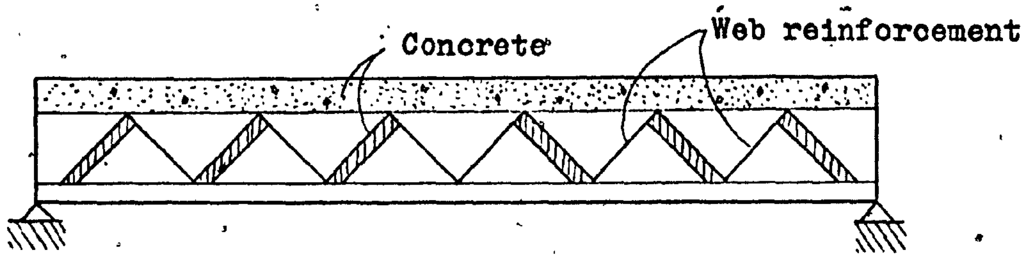


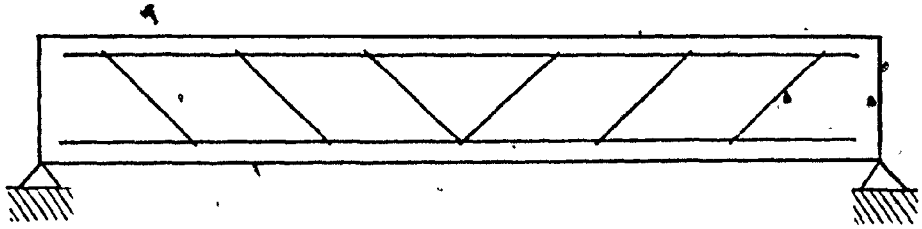
Fig. 3.1 - Typical stirrup and bent-bar arrangements. (Ref.7)



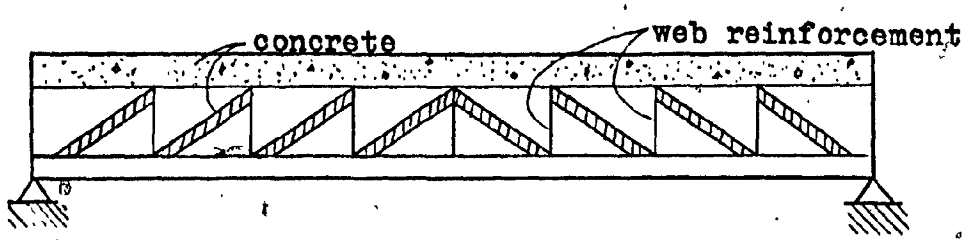
(a) A steel truss



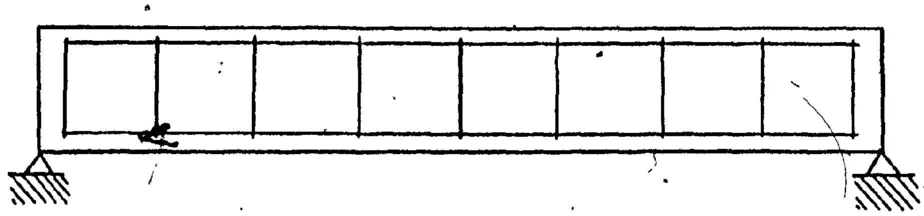
(b) Truss action in a reinforced concrete beam



(c) Reinforced concrete beam with inclined web reinforcement



(d) Truss action in a reinforced concrete beam



(e) Reinforced concrete beam with vertical web reinforcement.

Fig. 3.2 - Truss analogies
(Ref.8)

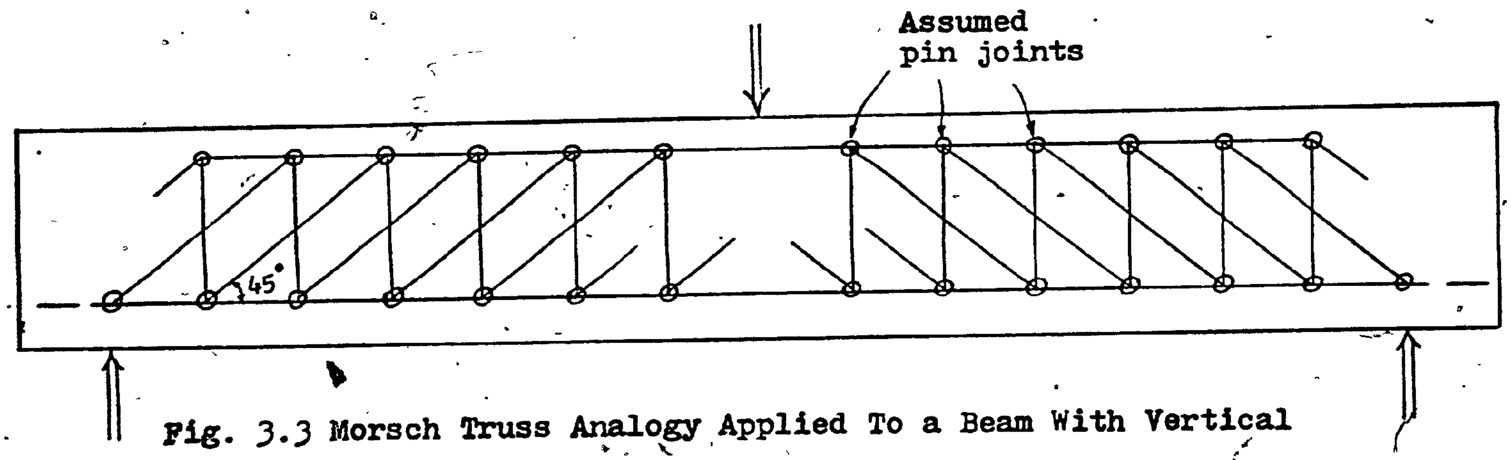


Fig. 3.3 Morsch Truss Analogy Applied To a Beam With Vertical Stirrups.

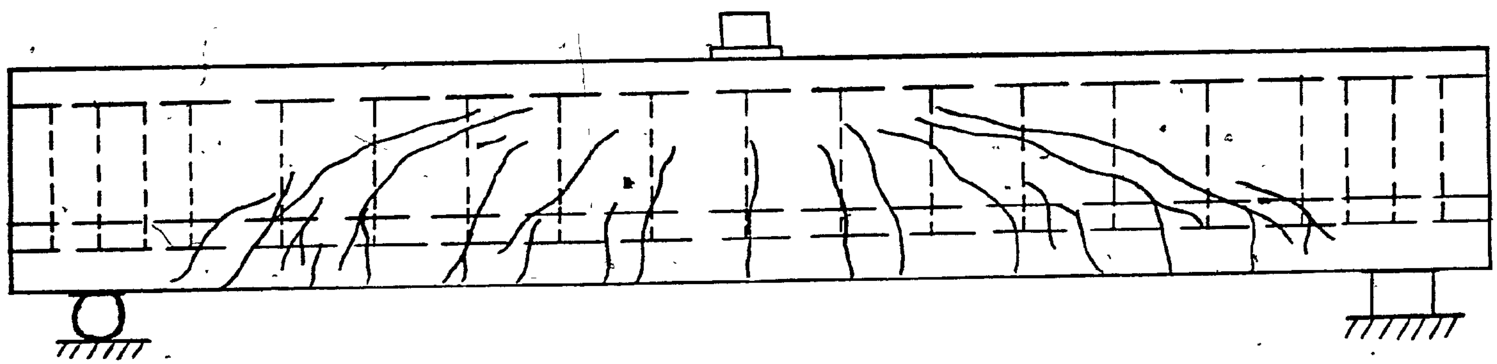


Fig. 3.4 Crack Pattern in a Beam With Vertical Stirrups-Penultimate Load Stage (Ref. 21)

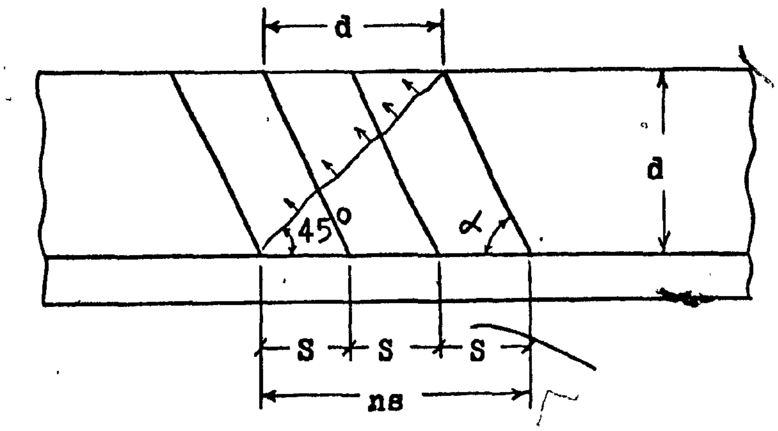


Fig. 3.5
Spacing of web reinforcement.
(Ref.7)

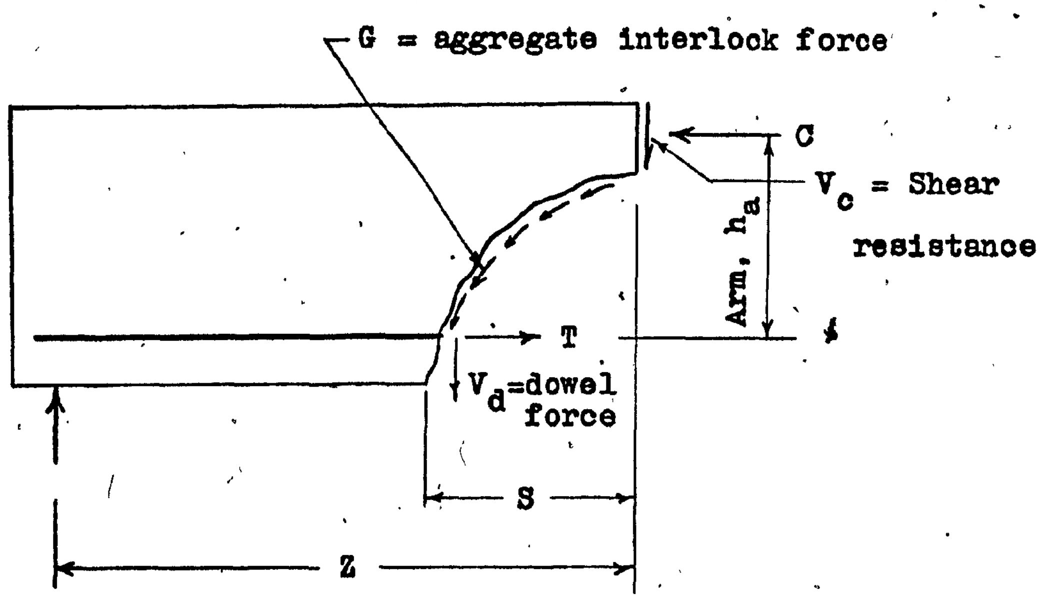


Fig. 3.6
Redistribution of shear resistance after formation of inclined crack.
(Ref.7)

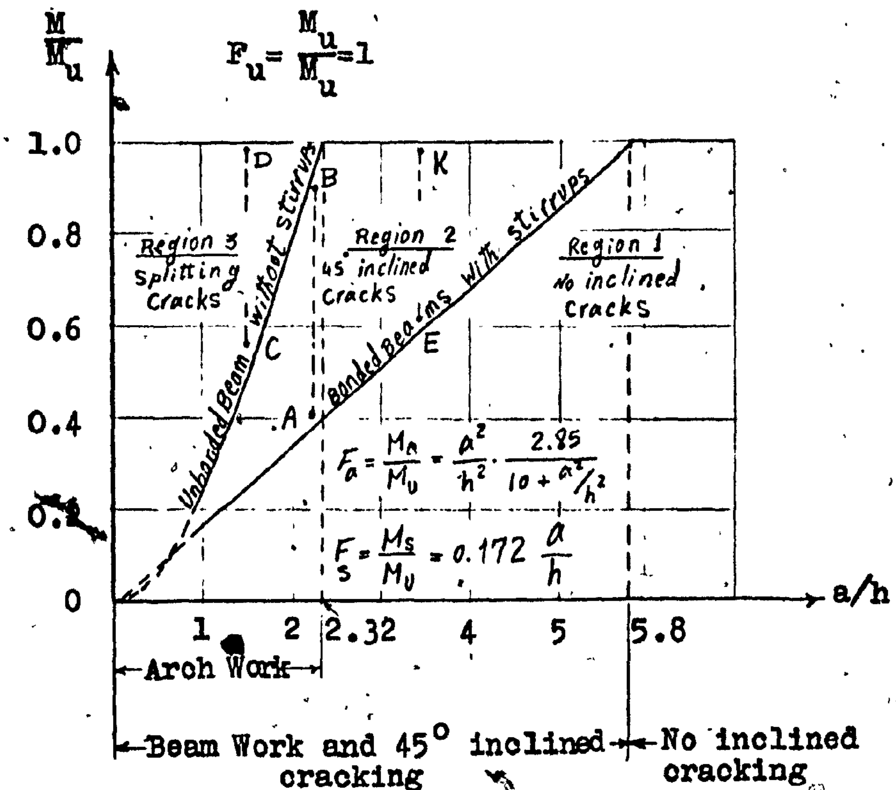


Fig. 3.7 - Graphical illustration of the relative cracking strength limits for two-point loaded beam in relation to a/h. (Ref.1)

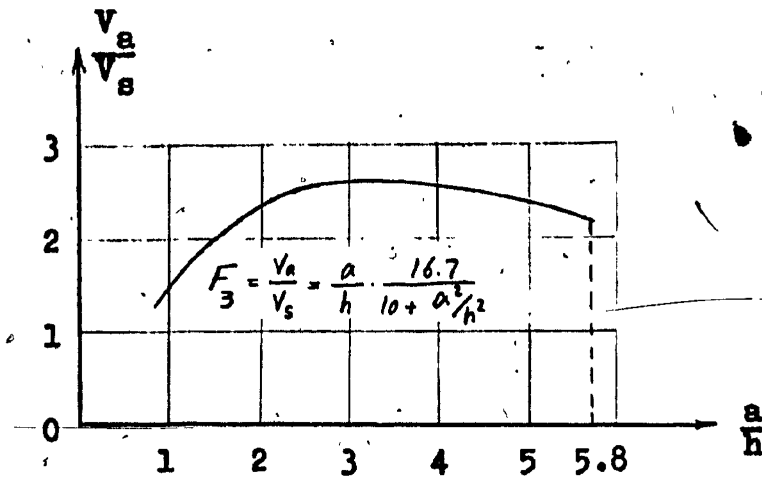


Fig. 3.8 - Comparison of the shearing force V_a at arch work with the shearing V_s at beam work and inclined cracking for beams without stirrups as related to a/h. (Ref.1)

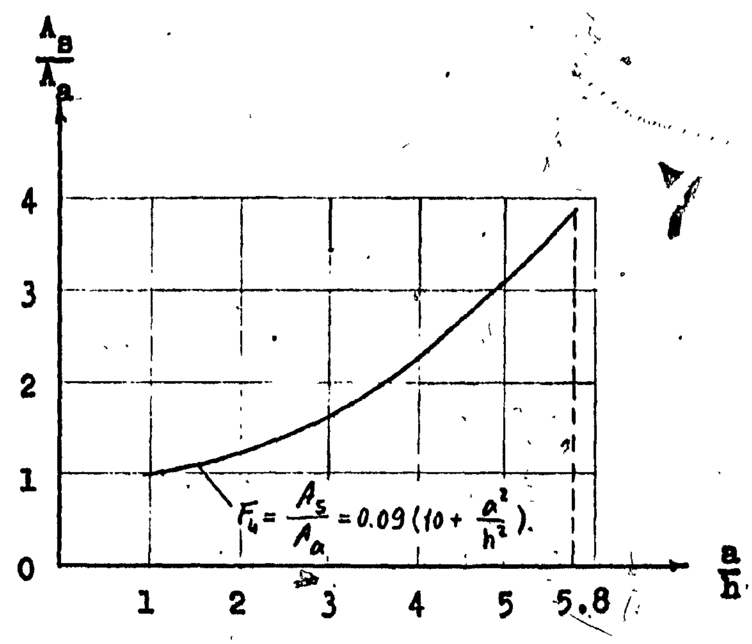


Fig. 3.9 - Comparison of the reinforcement amount A_s required on shear theory with the amount required on arch theory A_a as related to a/h.
(Ref.1)

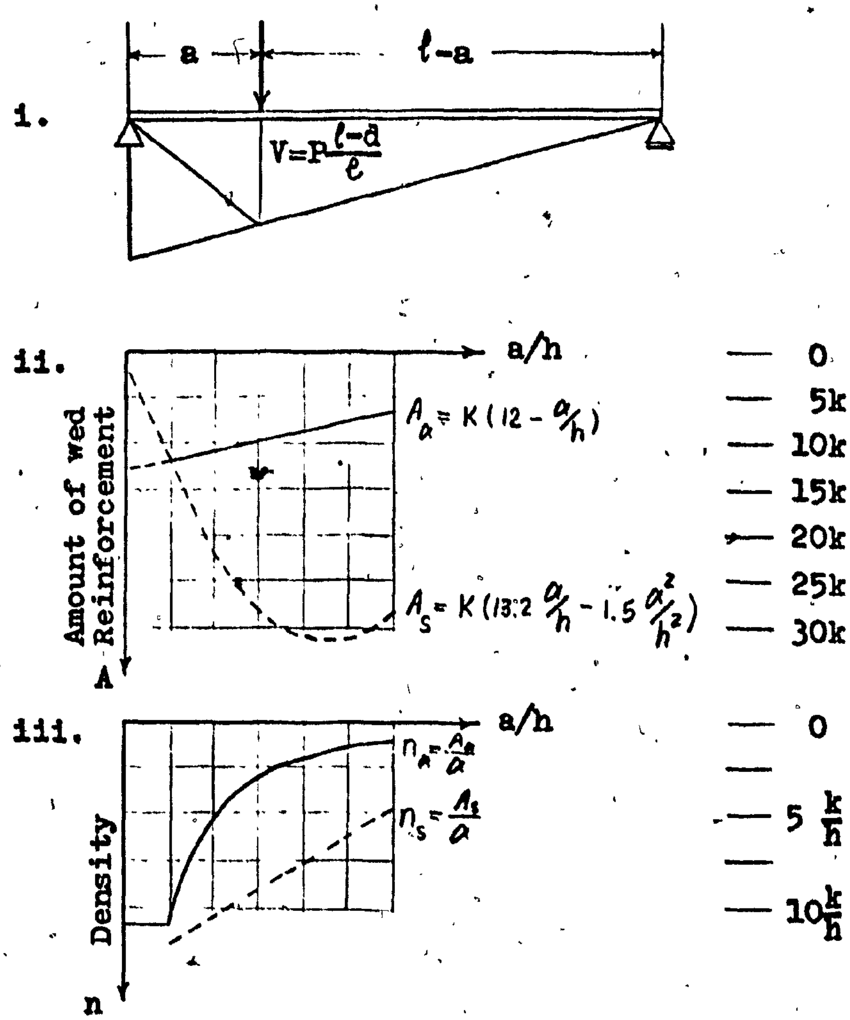


Fig. 3.10 - The relative maximum amount and the density of web reinforcement required on arch and shear theory for a rectangular bridge beam of $\ell = 12h$; $b=h/3$; $f'_c = 4000$ psi; $f_s = 30{,}000$ psi loaded with a single traveling force P: i) loading scheme; ii) required amount of web reinforcement A_a on arch and A_s on shear theory, iii) required density of web reinforcement.
(Ref.1)

SUMMARY AND CONCLUSION

CHAPTER IV

SUMMARY

Basically, there are three main causes of cracking failure of concrete in a beam:

Cause I. Uni-axial tension and tearing crack, tangent to stress.

Cause II. Uni-axial compression, leading to loss of cohesion and splitting along the compression.

Cause III. Bi-directional tension and compression leading to cracking tangent to tension

And in general there are four strength limits for the behaviour of a reinforced concrete beam.

(i) The limit strength A - first flexural cracking;

(ii) The limit strength B - inclined cracking;

(iii) The limit strength C - diagonal splitting; and

(iv) The limit strength D - ultimate, flexural failure.

The cracking strength limits of a beam can be represented basically through three types of cracks for beam with bonded or unbonded reinforcement.

The first cracks to appear are usually the flexural cracks, which are developed vertically and defind limit strength A. Usually there are fewer flexural cracks in the beam with unbonded reinforcement than in bonded beams.

Next to appear will be shear cracks which are in shear + moment loaded zone of beam and originally begin as vertical (up to main reinforcement level) but develop further as inclined (up to 45° in middle portion of the beam). However they are appearing due to the bond and the bi-axial compression and tension stress conditions and there is no this kind of cracks in beams with unbonded reinforcement. This group of cracks appear due to beam action, when beam with unbonded reinforcement, from the first moment of the flexural cracking, work as tied arches.

Last cracks, which may appear only in beams under high shear, will be diagonal splitting cracks directed from the support to the loading point. They appear due to arch work and reach of ultimate combined strength of concrete under bi-directional compression and tension, or uni axial direction compression strength. These cracks may appear in unbonded beams as well as in beams with bonded reinforcement. In a beam with bonded reinforcement, a diagonal splitting crack is usually connecting already existing shear cracks, but it also may intersect these cracks. A similar horizontal splitting cracks may appear in the compression zone of beam, in its portion under pure moment loading.

In discussion of cracking strength limit, we recovered that there is no danger of inclined cracking in beams without the web reinforcement for $\frac{a}{h} > 5.8$.

No stirrups will be required for one-point loaded beam of clear span $\ell \geq 11.6h \simeq 12h$.

No danger of inclined splitting due to arch work for beams with unbonded reinforcement when $\frac{a}{h} \geq 2.32$.

There will be no stirrups required for a single-point loaded beam with unbonded tensile reinforcement of clear span $\ell \geq 4.64h$.

By comparing the magnitudes of shear forces and required web reinforcement as defined in perposed arch theory and in the traditional shear theory it was seen that, for cases of $1.5 < \frac{a}{h} < 6$ the shearing force V_a defined on beam work and 45^{0} inclined cracking. Again, the comparison showed that the required amount of web reinforcement estimated on arch theory is, for beam of $1.5 < \frac{a}{h} < 5.8$ much less than the amount estimated on traditional shear theory.

The arch theory of Dr. Z.A. Zielinski is of major importance in view of the contents of this report. Although relatively new, it has however proven that certain omissions with regards to the properties of concrete, cause the classical design formulas to be more conservative than they need be. It was demonstrated that it does not pay to apply as much web reinforcement as the present day formulas call for. Thus Dr. Zielinski believes is due to the fact that certain characteristics of concrete beam (the arch behaviour) that contribute to the shear strength are not considered when designing for shear. Consequently much of the shear reinforcement may be termed as excessive, and could be trimmed down after a more accurate analysis and design. Suffice it to say that all tests done of less conservative formulas even the one presented by Dr. Zielinski show that inspite of a liberal attitude in design and a decrease in material use, the formulas remain to be safe and creditable without sacrificing safety.

CONCLUSION

As a result of pervious discussion it appears that beam with unbonded reinforcement from the very beginning, and beams with bonded main reinforcement, from the moment of the first cracking, work as in the arch scheme. And the inclined cracking due to shear does not terminate the work of the beam. If some web reinforcement is provided and main tensile reinforcement is well anchored beyond the support, beams will be able to withstand further load increase until they reach the point of diagonal splitting strength limit due to arch behaviour. Apparently, the number of stirrups required for later arch work is much less than that required in traditional shear theory.

The calculation of web reinforcement in the arch theory considered here is very simple. For any shear span "a" we just have to provide web reinforcement in strength of shear force V, when in case of traditional shear theory we have to provide web reinforcement in strength of $V.\frac{a}{jd}$, means $\frac{a}{jd}$ times more.

In the case of beam loaded uniformly, if main reinforcement is fully anchored beyond the support, there is no need for web reinforcement. However if such a beam is subject of early 45° inclined cracking, it will be advisable to provide some web reinforcement, in order to restrict the penetration of inclined cracks and to make the arch scheme possible. A nominal amount of web reinforcement in the strength of single shear force V distributed on whole region of inclined cracking "a" will be sufficient.

As mentioned before bond between main reinforcement and concrete is undesirable for inclined cracking of any beam. Bond existence is the main reason for the 45° inclined which we used to call shear cracking. The best way of eliminating inclined cracking would be the complete destruction of bond. Although the ultimate flexural strength is higher in beam with unbonded reinforcement it would be improper however to advise the complete elimination of bond. Presence of bond increases significantly the flexural cracking strength and increases the number of cracks but these cracks are reduced in width which are good for corrosion protection. However, for beams subjected to very high shear, and mainly for short beams with steady loading point, it would be advantageous to eliminate bond on bar portions situated in shear zones. This would eliminate the inclined cracking and not injure the segments subjected to inclined splitting due to arch scheme. For improvement of both flexural and inclined cracking strength, it may be advisable to use tension reinforcement composed of a few smaller diameter bars or wires having good bond strength and of big diameter round bars of high strength steel, with or without reduced bond strength, but with good end anchorages. This would produce good flexural cracking strength of beam and would earlier impose the more beneficial arch work.

With comparing the traditional shear theory and arch theory, first one is applicable only for bonded beam at early stage of elastic behaviour up to the moment of 45° inclined cracking, but ceases to be applicable afterward, when natural beam behaviour and new arch work replaces previous beam work. If so, then checking of the shear stress beyond the moment of reach of the limit stress $V = V_c = -V_t = 0.9f_{to}$ has also no validity, and accordingly, there is no explanation for admissible or ultimate allowed shear stress concept still used in the code requirements. Recognition of an arch work should allow beams of much higher shear stress if checked traditionally.

This report did not recommend one method of design over the other because it is felt that engineering besides being a science is by itself an art and that the designer must be left with a choice of design procedures. Where one engineer might be liberal and less willing to be very conservative, another may be conservative and less willing to be libral. In effect the options must always remain as a choice for the designer.

Shear by itself is much more complicated than any one report can summarize. This is due to the complicated nature and behaviour of the essential material, concrete. But by studying and examining certain aspects of concrete, more and more of the unknowns and uncertainties are being eliminated and thus with the course of the events formulas become less and less conservatives. The reader must bear in mind that none of the formulas are accused of being wrong, only that one is different than another.

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