SENSITIVITY CONSIDERATIONS OF

SOME GROUNDED CAPACITOR ACTIVE CIRCUITS

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ABSTRACT

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An attempt has been made to investigate some active circuits with grounded capacitors with the view of obtaining low sensitivities with respect to variations in elements. This objective was achieved by analysis of a basic circuit model consisting of an operational amplifier and an RC network with grounded capacitor. Several variations of this model in cascaded form with zero, single and multiple feedback were studied in order to derive low-pass and high-pass active circuits with high Q and low Q sensitivity. Detailed analyses of the circuits, together with calculations using the thin-film circuit parameters, led to very useful circuit configurations with Q's up to 100 appearing together with low Q-sensitivities.

29 Jun 71 D.R. Smyth

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CHAPTER 1

INTRODUCTION

In this dissertation an attempt has been made to investigate some active circuits with grounded capacitors with the view of obtaining low transfer function sensitivity with respect to variations in elements. Moschytz [3] states that this problem of sensitivity minimization is one of the major problems of active RC network synthesis because of changes in components due to ambient conditions and becauseof the conditional stability of active RC networks. Although we do not use Moschytz's approach, we do use operational amplifiers in a new circuit approach.

Use of operational amplifiers in active RC networks is not new, as is seen from their use in References [3], [4], [8], [12], [15], [20], and many other recent articles and books on network synthesis. Two of the best and most practical source books on operational amplifiers are Burr-Brown's two handbooks, References [8] and [9], which contain numerous circuit configurations.

Use of state variables in circuit analysis appears to be a recent technique borrowed from linear systems theory. Newcomb, in a recent book [15], states that use of the state variable approach provides three basic advantages: (1) minimal capacitors, (2) use of operational amplifiers, and (3) reasonably low sensitivity.

In another paper, Kerwin, Huelsman, and Newcomb [12] discuss a theory for low sensitivity transfer function realization using state variable flow graphs. Kerwin, Huelsman, and Newcomb also demonstrate a well-known general second-order active network (Mitra [1]; Tow [7]; Tow [14]) which has a minimal number of capacitors. In particular, mention is made of a positive - gain grounded-capacitor integrator as shown in Fig. 1.1 (although no use is made of this specific integrator in realizing second-order networks).

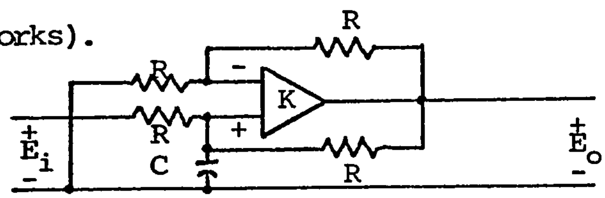


Fig. 1.1 - Positive-gain grounded-capacitor integrator.

Analysis of the circuit of Fig. 1.1 shows that the voltage transfer function is given by

$$T(s) = \frac{E_o(s)}{E_i(s)} = \frac{2K}{(2 + K)s\,CR + 4} \qquad (1.1)$$

where K is the closed-loop gain of the operational amplifier. Hence, at low frequencies, the circuit ceases to behave as an integrator, which is a serious limitation.

Washington, in a recent paper [5], evaluates various types of active filters including a multiple feedback bandpass type. Washington discusses his results of Q stability and frequency stability which indicate that the multiple feedback circuit is best when these two parameters must be controlled. Newcomb, Rao and Woodard [13] develop a minimal capacitor synthesis technique for integrated circuits; however, their approach is through use of gyrators rather than operational amplifiers.

Discussion of sensitivity occurs in almost all papers and books on active RC networks. Mitra, in a recent book [1], has a very extensive discussion of sensitivity definitions for quick reference. Mitra also discusses methods of sensitivity function minimization without maximizing loop gain which usually leads to stability problems. Moschytz, in recent paper [2], discusses a new figure of merit for hybrid integrated networks using single operational amplifiers. He suggests that the gain-sensitivity product is a more meaningful measure of Q stability than sensitivity alone. Geffe [19], discusses the dramatic fall of sensitivities in recent years due to active network research. He refers to macroscopic or realistic sensitivity versus differential sensitivity, the latter for which we must settle in order that the sensitivity problem be tractable mathematically, even though we are not dealing with purely differential quantities.

Kerwin, Huelsman, and Newcomb [12] state that grounded capacitors are best for integrated circuit devices. Since we are investigating a new active RC circuit with grounded capacitors and also study use of hybrid integrated circuits in achieving physically realizable models of our network, we have thus heeded the remarks about grounded capacitors mentioned above in a recent book edited by Huelsman [16]. According to Huelsman [16], grounding of capacitors simplifies the integrated circuit configuration by providing all capacitors with a common isolation junction, thereby saving considerable area.

CHAPTER 2

CIRCUIT ANALYSIS OF ACTIVE CIRCUITS

Several variations of a basic active RC network are analyzed. The basic network consists of an operational amplifier coupled directly to an RC network with grounded capacitor. A description of operational amplifiers is contained in Appendix A. All symbols used in the following analyses are defined for quick reference in Appendix B.

2.1 Basic Circuit

The basic circuit is shown in Fig. 2.1. For ease in presentation, the RC network is represented by the element shown in Fig. 2.2b with terminal 3-3' grounded in all cases.

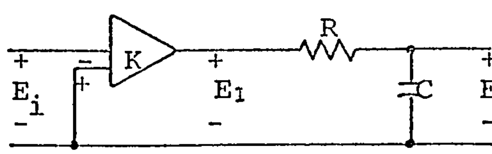


Fig. 2.1 - Basic Circuit.

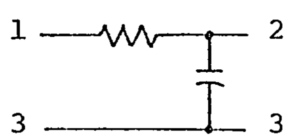


Fig. 2.2a - RC network.

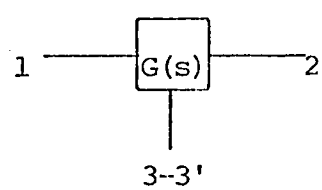


Fig. 2.2b - Equivalent network.

Using Mason's rule as stated in Kuo [18], the voltage transfer function of the circuit in Fig. 2.1 is given by

$$T(s) = \frac{E_o(s)}{E_i(s)} = -KG(s) \qquad (2.1)$$

where

$$G(s) = \frac{1}{1 + \alpha s} \qquad (2.2a)$$

$$\alpha = RC \qquad (2.2b)$$

$$K = \text{closed loop gain of amplifier} \qquad (2.2c)$$

and

$$s = \alpha + j\omega.$$

Use of the basic circuit of Fig. 2.1 in cascaded arrangements with and without feedback loops leads to quadratic functions in both the numerator and the denominator of the voltage transfer function as shown below. In all circuits analyzed, it is assumed that the input impedances to amplifiers are infinite, and that output impedances are zero. A summary of signal flow graphs for Figs. 2.3 to 2.13 is found in Appendix C.

2.2 Variations of Basic Circuit

The first version of the basic circuit is that of Fig. 2.3 which consists of two cascaded basic circuits with multiple feedback.

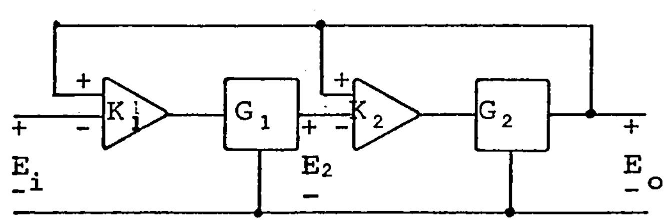


Fig. 2.3 - Cascaded basic circuit with multiple feedback.

For the circuit of Fig. 2.3, the voltage transfer function is given by

$$T_1(s) = \frac{E_o(s)}{E_i(s)} = \frac{1}{a_1 s^2 + b_1 s + c_1} \qquad (2.3)$$

where

$$a_1 = \frac{\alpha_1\alpha_2}{K_1K_2} \qquad (2.4a)$$

$$b_1 = \frac{\alpha_2 + \alpha_1 (1 - K_2)}{K_1K_2} \qquad (2.4b)$$

$$c_1 = \frac{1 + K_2 (K_1 - 1)}{K_1K_2} \qquad (2.4c)$$

$$\alpha_1 = R_1C_1 \qquad (2.4d)$$

and

$$\alpha_2 = R_2C_2 \; . \qquad (2.4e)$$

Reversal of the input leads to the two operational amplifiers of Fig. 2.3, that is, with the output voltage E_o feeding into the negative input terminals of the amplifiers and the input voltages E_i and E_2 feeding into the positive input terminals, leads to the circuit of Fig. 2.4 with the voltage transfer function given by

$$T_2(s) = \frac{E_o(s)}{E_i(s)} = \frac{1}{a_2 s^2 + b_2 s + c_2} \qquad (2.5)$$

where

$$a_2 = \frac{\alpha_1\alpha_2}{K_1K_2} \qquad (2.6a)$$

$$b_2 = \frac{\alpha_2 + \alpha_1 (1 + K_2)}{K_1K_2} \qquad (2.6b)$$

and

$$c_2 = \frac{1 + K_2 (K_1 + 1)}{K_1K_2} \; . \qquad (2.6c)$$

Note that equation (2.6a) is similar to equation (2.4a), but that equations (2.6b) and (2.6c) differ from (2.4b) and (2.4c) by a sign change.

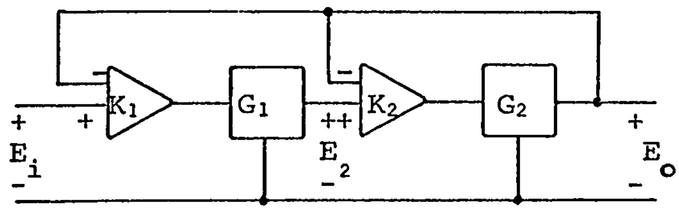


Fig. 2.4 - Circuit of Fig. 2.3 with reversal of input leads.

The circuit of Fig. 2.5 is two cascaded basic circuits without feedback.

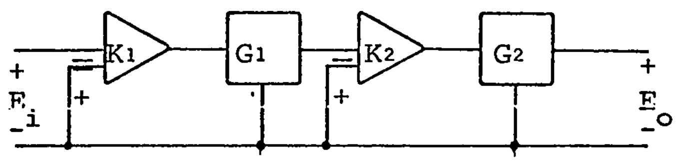


Fig. 2.5 - Cascaded basic circuit without feedback.

The voltage transfer function for the circuit of Fig. 2.5 is given by

$$T_3(s) = \frac{E_o(s)}{E_i(s)} = \frac{1}{a_3 s^2 + b_3 s + c_3} \qquad (2.7)$$

where

$$a_3 = \frac{\alpha_1 \alpha_2}{K_1 K_2} \qquad (2.8a)$$

$$b_3 = \frac{\alpha_1 + \alpha_2}{K_1 K_2} \qquad (2.8b)$$

and

$$c_3 = \frac{1}{K_1K_2} \quad . \tag{2.8c}$$

Reversal of the input leads to the two operational amplifiers in the circuit of Fig. 2.5 does not change equations (2.7), (2.8a), (2.8b), and (2.8c).

Another circuit variation is that of Fig. 2.3 with no feedback to the second operational amplifier. This is the circuit of Fig. 2.6.

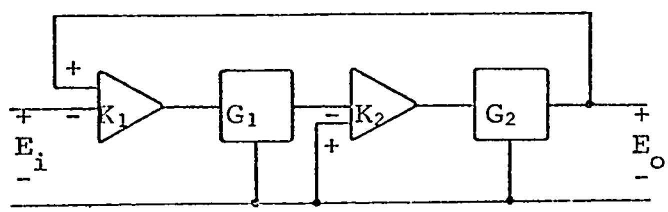


Fig. 2.6 - Cascaded basic circuit with single feedback to Op Amp 1.

The voltage transfer function of the circuit of Fig. 2.6 is given by

$$T_4(s) = \frac{E_o(s)}{E_i(s)} = \frac{1}{a_4s^2 + b_4s + c_4} \tag{2.9}$$

where

$$a_4 = \frac{\alpha_1\alpha_2}{K_1K_2} \tag{2.10a}$$

$$b_4 = \frac{\alpha_1 + \alpha_2}{K_1K_2} \tag{2.10b}$$

and

$$c_4 = 1 + \frac{1}{K_1K_2} \quad . \tag{2.10c}$$

The circuit of Fig. 2.7 also has a single feedback path, this time to the second operational amplifier only.

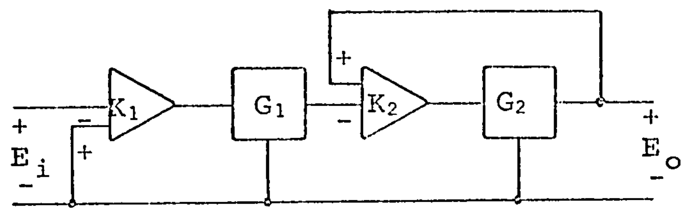


Fig. 2.7 - Cascaded basic circuit with single feedback to Op Amp 2.

The voltage transfer function of the circuit of Fig. 2.7 is given by

$$T_5(s) = \frac{E_o(s)}{E_i(s)} = \frac{1}{a_5 s^2 + b_5 s + c_5} \qquad (2.11)$$

where

$$a_5 = \frac{\alpha_1 \alpha_2}{K_1 K_2} \qquad (2.12a)$$

$$b_5 = \frac{\alpha_2 + \alpha_1 (1-K_2)}{K_1 K_2} \qquad (2.12b)$$

and

$$c_5 = \frac{1 - K_2}{K_1 K_2} . \qquad (2.12c)$$

The addition of an operational amplifier in the feedback paths of the circuits of Figs. 2.3 and 2.4 provides additional control of the coefficients a, b, and c of the voltage transfer function. The circuit of Fig. 2.8 is an example of this, as is the circuit of Fig. 2.9.

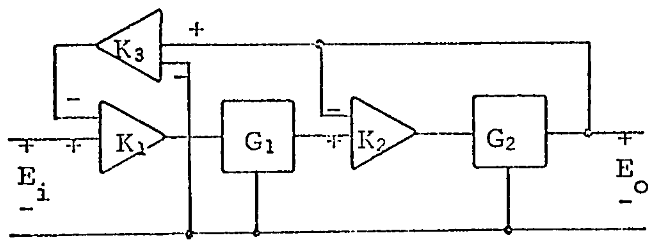


Fig. 2.8 - Circuit with additional amplifier in feedback path.

The voltage transfer function of the circuit of Fig. 2.8 is given by

$$T_6(s) = \frac{E_o(s)}{E_i(s)} = \frac{1}{a_6 s^2 + b_6 s + c_6} \quad (2.13)$$

where

$$a_6 = \frac{\alpha_1 \alpha_2}{K_1 K_2} \quad (2.14a)$$

$$b_6 = \frac{\alpha_2 + \alpha_1 (1 + K_2)}{K_1 K_2} \quad (2.14b)$$

and

$$c_6 = \frac{1 + K_2 (K_1 K_3 + 1)}{K_1 K_2} \quad . \quad (2.14c)$$

The second version consists of two additional operational amplifiers in the feedback path, as shown in Fig. 2.9.

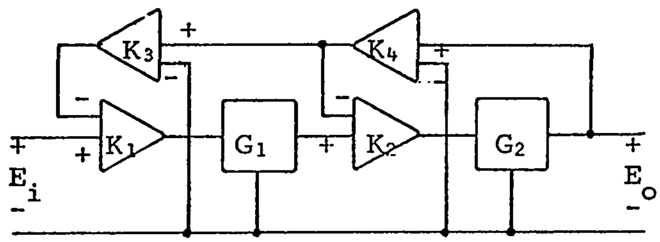


Fig. 2.9 - Two additional feedback amplifiers.

The voltage transfer function of the circuit of Fig. 2.9 is given by

$$T_7(s) = \frac{E_o(s)}{E_i(s)} = \frac{1}{a_7 s^2 + b_7 s + c_7} \tag{2.15}$$

where

$$a_7 = \frac{\alpha_1 \alpha_2}{K_1 K_2} \tag{2.16a}$$

$$b_7 = \frac{\alpha_2 + \alpha_1 (K_2 K_4 + 1)}{K_1 K_2} \tag{2.16b}$$

and

$$c_7 = \frac{K_2 K_4 (K_1 K_3 + 1) + 1}{K_1 K_2} . \tag{2.16c}$$

All the circuits of Figs. 2.3 to 2.9 inclusive are low-pass active RC networks. Several other circuits were investigated, using the same basic circuit of Fig. 2.1, in order to attempt the realization of more general biquad functions. The first of these circuits is that shown in Fig. 2.10.

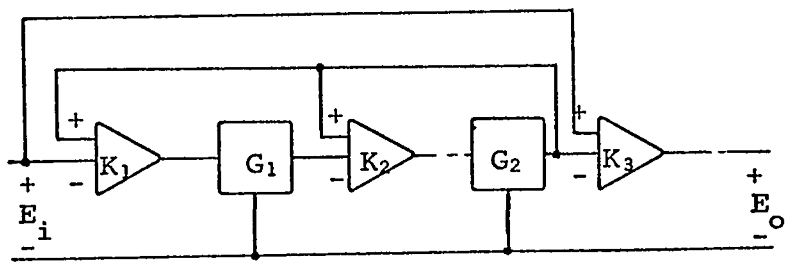


Fig. 2.10 - Restricted biquad realization.

The voltage transfer function of the circuit of Fig. 2.10 is given by

$$T_8(s) = \frac{E_o(s)}{E_i(s)} = K_3 \cdot \frac{a_5 s^2 + b_5 s + c_5}{a_1 s^2 + b_1 s + c_1} . \tag{2.17}$$

Note that the coefficients of the denominator of equation (2.17) are identical to those of the denominator of equation (2.3), and the coefficients of the numerator of equation (2.17) are identical to those of the denominator of equation (2.11).

The second circuit is that of Fig. 2.11 whose voltage transfer function is given by

$$T_9(s) = \frac{E_o(s)}{E_i(s)} = K_3 \cdot \frac{a_9 s^2 + b_9 s + c_9}{a_1 s^2 + b_1 s + c_1} \tag{2.18}$$

where

$$a_9 = \frac{\alpha_1 \alpha_2}{K_1 K_2} \tag{2.19a}$$

$$b_9 = \frac{\alpha_1 (1 - K_2) + \alpha_2 (1 - K_1)}{K_1 K_2} \tag{2.19b}$$

and

$$c_9 = \frac{1 + K_1 - K_2}{K_1 K_2} . \tag{2.19c}$$

Neither circuit can realize numerator and denominator independently, and are thus termed restricted biquad realizations.

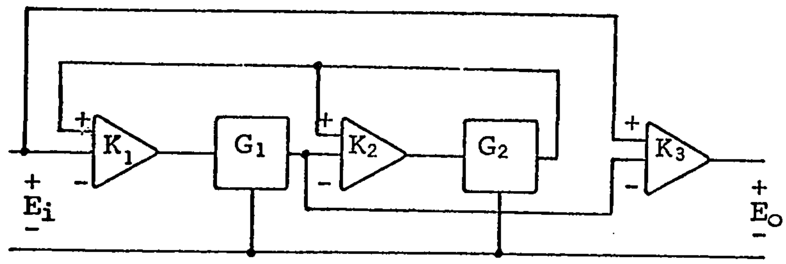


Fig. 2.11 - Second version of restricted biquad.

The circuit of Fig. 2.12 is a variation of the circuit of Fig. 2.8 with input voltages to the three amplifiers reversed.

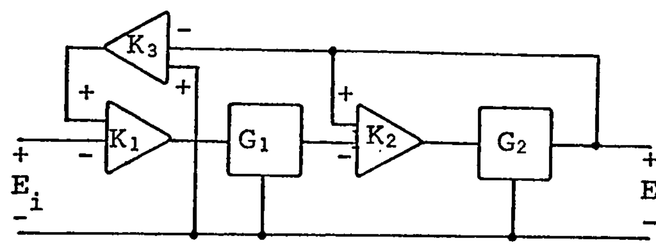


Fig. 2.12 - Circuit of Fig. 2.8 with reversed inputs.

The voltage transfer function of the circuit of Fig. 2.12 is given by

$$T_{10}(s) = \frac{E_o(s)}{E_i(s)} = \frac{1}{a_{10}s^2 + b_{10}s + c_{10}} \tag{2.20}$$

where

$$a_{10} = \frac{\alpha_1\alpha_2}{K_1K_2} \tag{2.21a}$$

$$b_{10} = \frac{\alpha_2 + \alpha_1(1 - K_2)}{K_1K_2} \tag{2.21b}$$

and

$$c_{10} = \frac{1 + K_2(K_1K_3 - 1)}{K_1K_2}\ . \tag{2.21c}$$

The circuit of Fig. 2.13 is a variation of the circuit of Fig. 2.9 with input voltages to three amplifiers reversed as shown.

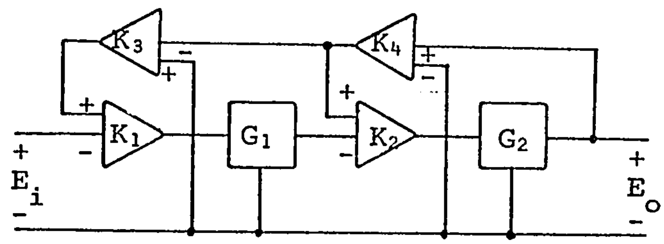


Fig. 2.13 - Circuit of Fig. 2.9 with reversed inputs.

The voltage transfer function of the circuit of Fig. 2.13 is given by

$$T_{11}(s) = \frac{E_o(s)}{E_i(s)} = \frac{1}{a_{11}s + b_{11}s + c_{11}} \qquad (2.22)$$

where

$$a_{11} = \frac{\alpha_1\alpha_2}{K_1K_2} \qquad (2.23a)$$

$$b_{11} = \frac{\alpha_2 + \alpha_1(1 - K_2K_4)}{K_1K_2} \qquad (2.23b)$$

and

$$c_{11} = \frac{K_2K_4(K_1K_3 - 1) + 1}{K_1K_2} \ . \qquad (2.23c)$$

2.3 Summary

The eleven circuits shown in Figs. 2.3 to 2.13 provide nine low-pass networks and two restricted biquads. Several of these are not adequate in providing suitable means of varying or controlling polynomial coefficients by means of the closed-loop gains of the operational amplifiers. The circuits of Figs. 2.5 and 2.6 fall in this category because, on inspection of Table 2.1, coefficients $\frac{b_3}{a_3}$ and $\frac{b_4}{a_4}$ respectively cannot be controlled by the closed-loop gains K_1 and K_2.

Those circuits which do provide control of the coefficients are those of Figs. 2.3, 2.4, and 2.7 to 2.13.

Table 2.1 provides a quick view of the circuit analyses carried out, whereas Table 2.2 provides a summary of the quality factor Q for the numerator and denominator polynomials of the various voltage transfer functions.

Note that a second-degree polynomial given in the form

$$P(s) = as^2 + bs + c \qquad (2.24)$$

can be made equivalent to the following form

$$P(s) = a(s^2 + Bs + \omega_n^2) \qquad (2.25)$$

where

$$B = \frac{b}{a} = \text{bandwidth} \qquad (2.26a)$$

and

$$\omega_n = \sqrt{\frac{c}{a}} = \text{centre frequency.} \qquad (2.26b)$$

The quality factor is thus defined as

$$Q = \frac{\omega_n}{B} = \frac{\sqrt{ac}}{b} . \qquad (2.27)$$

Table 2.3 provides a quick summary of the voltage transfer function evaluated at $\omega = \omega_n$. For ease in writing equations, the factor k is defined as

$$k = \frac{\alpha_2}{\alpha_1} . \qquad (2.28)$$

For the circuit of Fig. 2.1, the voltage transfer function is

$$T(s) = \frac{1}{1 + \alpha s} \tag{2.29}$$

which, when s =jω, is

$$T(j\omega) = \frac{1}{1 + j\omega\alpha} \; . \tag{2.30}$$

The magnitude of T(jω) when $\omega = \omega_n$, the centre frequency, is given by

$$|T(j\omega)|_{\omega=\omega_n} = \frac{1}{\sqrt{1 + \omega^2\alpha^2}} \; . \tag{2.31}$$

For the circuits of Figs. 2.3 to 2.9, 2.12, and 2.13, the voltage transfer function is of the form

$$T(s) = \frac{1}{as^2 + bs + c} \; , \tag{2.32}$$

which, when s =jω, is

$$T(j\omega) = \frac{1/a}{-\omega^2 + jB\omega + \omega_n^2} \; . \tag{2.33}$$

The magnitude of T(jω) when $\omega = \omega_n$ is thus given by

$$|T(j\omega)|_{\omega=\omega_n} = \frac{1}{aB\omega_n} = \frac{1}{b\sqrt{\frac{c}{a}}} \; . \tag{2.34}$$

For the circuits of Figs. 2.10 and 2.11, the voltage transfer functions are of the form

$$T(s) = H. \; \frac{a_2s^2 + a_1s + a_0}{b_2s^2 + b_1s + b_0} \tag{2.35}$$

When s = jω, equation (2.35) becomes

$$|T(j\omega)|_{\omega=\omega_n} = \frac{a_2H}{b_2} \cdot \frac{-\omega^2 + jB_a\omega + \omega_a^2}{-\omega^2 + jB\omega + \omega_n^2} \tag{2.36}$$

where

B_a, B are bandwidths,

ω_a, ω_n are centre frequencies

and H is a constant.

The magnitude of the transfer function T(s) in equation (2.35) evaluated at $\omega = \omega_n$ is given by

$$|T(j\omega)|_{\omega=\omega_n} = \frac{a_2 H}{b_1 \omega_n} \cdot \left(\sqrt{(\omega_a^2 - \omega_n^2)^2 + B_a^2 \omega_n^2} \right) \cdot \qquad (2.37)$$

TABLE 2.1 – Circuit analyses.

Fig.	T(s)	a	b	c
2.1	$\frac{1}{bs+c}$	0	$\frac{\alpha}{K}$	$\frac{1}{K}$
2.3	$\frac{1}{a_1s^2+b_1s+c_1}$	$\frac{\alpha_1\alpha_2}{K_1K_2}$	$\frac{\alpha_2+\alpha_1(1-K_2)}{K_1K_2}$	$\frac{1+K_2(K_1-1)}{K_1K_2}$
2.4	$\frac{1}{a_2s^2+b_2s+c_2}$	$\frac{\alpha_1\alpha_2}{K_1K_2}$	$\frac{\alpha_2+\alpha_1(1+K_2)}{K_1K_2}$	$\frac{1+K_2(K_1+1)}{K_1K_2}$
2.5	$\frac{1}{a_3s^2+b_3s+c_3}$	$\frac{\alpha_1\alpha_2}{K_1K_2}$	$\frac{\alpha_1+\alpha_2}{K_1K_2}$	$\frac{1}{K_1K_2}$
2.6	$\frac{1}{a_4s^2+b_4s+c_4}$	$\frac{\alpha_1\alpha_2}{K_1K_2}$	$\frac{\alpha_1+\alpha_2}{K_1K_2}$	$1+\frac{1}{K_1K_2}$
2.7	$\frac{1}{a_5s^2+b_5s+c_5}$	$\frac{\alpha_1\alpha_2}{K_1K_2}$	$\frac{\alpha_2+\alpha_1(1-K_2)}{K_1K_2}$	$\frac{1-K_2}{K_1K_2}$
2.8	$\frac{1}{a_6s^2+b_6s+c_6}$	$\frac{\alpha_1\alpha_2}{K_1K_2}$	$\frac{\alpha_2+\alpha_1(1+K_2)}{K_1K_2}$	$\frac{1+K_2(K_1K_3+1)}{K_1K_2}$
2.9	$\frac{1}{a_7s^2+b_7s+c_7}$	$\frac{\alpha_1\alpha_2}{K_1K_2}$	$\frac{\alpha_2+\alpha_1(K_2K_4+1)}{K_1K_2}$	$\frac{K_2K_4(K_1K_3+1)+1}{K_1K_2}$
2.10	$K_3 . \frac{a_5s^2+b_5s+c_5}{a_1s^2+b_1s+c_1}$	As in Figs. 2.3 and 2.7		
2.11	$K_3 . \frac{a_9s^2+b_9s+c_9}{a_1s^2+b_1s+c_1}$	As in Fig. 2.3 for denominator. For numerator, below:		
		$\frac{\alpha_1\alpha_2}{K_1K_2}$	$\frac{\alpha_1(1-K_2)+\alpha_2(1+K_1)}{K_1K_2}$	$\frac{1+K_1-K_2}{K_1K_2}$
2.12	$\frac{1}{a_{10}s^2+b_{10}s+c_{10}}$	$\frac{\alpha_1\alpha_2}{K_1K_2}$	$\frac{\alpha_2+\alpha_1(1-K_2)}{K_1K_2}$	$\frac{1+K_2(K_1K_3-1)}{K_1K_2}$
2.13	$\frac{1}{a_{11}s^2+b_{11}s+c_{11}}$	$\frac{\alpha_1\alpha_2}{K_1K_2}$	$\frac{\alpha_2+\alpha_1(1-K_2K_4)}{K_1K_2}$	$\frac{1+K_2K_4(K_1K_3-1)}{K_1K_2}$

TABLE 2.2 - Quality factor Q.

Fig.	Q(numerator)	Q(denominator)
2.3	-	$\frac{\sqrt{k}(1+K_1K_2-K_2)}{k+1-K_2}$
2.4	-	$\frac{\sqrt{k}(1+K_1K_2+K_2)}{k+1+K_2}$
2.5	-	$\frac{\sqrt{k}}{k+1}$
2.6	-	$\frac{\sqrt{k}(K_1K_2+1)}{k+1}$
2.7	-	$\frac{\sqrt{k}(1-K_2)}{k+1-K_2}$
2.8	-	$\frac{\sqrt{k}(1+K_1K_2K_3+K_2)}{k+1+K_2}$
2.9	-	$\frac{\sqrt{k}(K_1K_2K_3K_4+K_2K_4+1)}{k+1+K_2K_4}$
2.10	$\frac{\sqrt{k}(1-K_2)}{k+1-K_2}$	$\frac{\sqrt{k}(1+K_1K_2-K_2)}{k+1-K_2}$
2.11	$\frac{\sqrt{k}(1+K_1-K_2)}{1-K_2+k(1+K_1)}$	$\frac{\sqrt{k}(1+K_1K_2-K_2)}{k+1-K_2}$
2.12	-	$\frac{\sqrt{k}(1+K_1K_2K_3-K_2)}{k+1-K_2}$
2.13	-	$\frac{\sqrt{k}(1+K_1K_2K_3K_4-K_2K_4)}{k+1-K_2K_4}$

TABLE 2.3 - T(s) value at $\omega=\omega_n$, or $|T(j\omega_n)|$

Fig.	$\lvert T(j\omega_n)\rvert$		
2.3	$\lvert T_1(j\omega_n)\rvert$	=	$\dfrac{1}{\dfrac{k+1-K_2}{K_1K_2}\sqrt{\dfrac{1+K_2(K_1-1)}{k}}}$
2.4	$\lvert T_2(j\omega_n)\rvert$	=	$\dfrac{1}{\dfrac{k+1+K_2}{K_1K_2}\sqrt{\dfrac{k+K_2(K_1+1)}{k}}}$
2.5	$\lvert T_3(j\omega_n)\rvert$	=	$\dfrac{1}{\dfrac{k+1}{K_1K_2}\cdot\sqrt{\dfrac{1}{k}}}$
2.6	$\lvert T_4(j\omega_n)\rvert$	=	$\dfrac{1}{\dfrac{k+1}{K_1K_2}\sqrt{\dfrac{1+K_1K_2}{k}}}$
2.7	$\lvert T_5(j\omega_n)\rvert$	=	$\dfrac{1}{\dfrac{k+1-K_2}{K_1K_2}\sqrt{\dfrac{1-K_2}{k}}}$
2.8	$\lvert T_6(j\omega_n)\rvert$	=	$\dfrac{1}{\dfrac{k+1-K_2}{K_1K_2}\sqrt{\dfrac{1+K_2(K_1K_3+1)}{k}}}$
2.9	$\lvert T_7(j\omega_n)\rvert$	=	$\dfrac{1}{\dfrac{k+K_2K_4+1}{K_1K_2}\sqrt{\dfrac{K_2K_4(K_1K_3+1)+1}{k}}}$
2.10	$\lvert T_8(j\omega_n)\rvert$	=	$\left(\dfrac{K_3a_5\cdot}{b_1\cdot\sqrt{\dfrac{c_1}{a_1}}}\right)\sqrt{\left(\dfrac{c_5}{a_5}-\dfrac{c_1}{a_1}\right)^2+\dfrac{b_5^2}{a_5^2}\cdot\dfrac{c_1}{a_1}}$
2.11	$\lvert T_9(j\omega_n)\rvert$	=	$\left(\dfrac{K_3a_9\cdot}{b_1\cdot\sqrt{\dfrac{c_1}{a_1}}}\right)\sqrt{\left(\dfrac{c_9}{a_9}-\dfrac{c_1}{b_1}\right)^2+\dfrac{b_9^2}{a_9^2}\,\dfrac{c_1}{a_1}}$
2.12	$\lvert T_{10}(j\omega_n)\rvert$	=	$\dfrac{1}{\dfrac{k+1-K_2}{K_1K_2}\sqrt{\dfrac{1+K_2(K_1K_3-1)}{k}}}$
2.13	$\lvert T_{11}(j\omega_n)\rvert$	=	$\dfrac{1}{\dfrac{k+1-K_2K_4}{K_1K_2}\sqrt{\dfrac{1+K_2K_4(K_1K_3-1)}{k}}}$

CHAPTER 3

SENSITIVITY ANALYSIS OF ACTIVE CIRCUITS

The sensitivity functions are first defined and then determined for each of the circuits of Figs. 2.3 to 2.13.

3.1 Definitions

This dissertation is basically concerned with the sensitivity functions pertaining to the quality factor Q of the circuits, the centre frequency ω_n, and the bandwidth B. These three parameters have previously been defined in equations (2.27), (2.26b), and (2.26a) respectively.

By definition, the Q-sensitivity is given by

$$S_x^Q = \frac{\partial \ln Q}{\partial \ln x} = \frac{\partial Q/Q}{\partial x/x} \tag{3.1}$$

where x is any active or passive element in a circuit. As a result of manipulating equations (2.26a), (2.26b), and (2.27), the Q-sensitivity can be expressed as

$$S_x^Q = \frac{\partial \ln (\sqrt{ac}/b)}{\partial \ln x}$$

$$= \frac{x}{2}\left(\frac{a'}{a} + \frac{c'}{c}\right) - x\frac{b'}{b} \tag{3.2}$$

where a', b', and c' are the first partial derivatives of a, b, and c respectively with respect to x.

The centre-frequency or ω_n - sensitivity is given by

$$S_x^{\omega_n} = \frac{\partial \ln \omega_n}{\partial \ln x} \tag{3.3}$$

$$= \frac{x}{2}\left(\frac{c'}{c} - \frac{a'}{a}\right) .$$

The bandwidth or B-sensitivity is given by

$$S_x^B = x\left(\frac{b'}{b} - \frac{a'}{a}\right) . \tag{3.4}$$

3.2 Sensitivity Functions

Using the above definitions given by equations (3.2), (3.3), and (3.4), the sensitivity functions of the circuits of Figs. 2.3 to 2.13 are now found. These are summarized in Table 3.1 for S_x^Q, in Table 3.2 for $S_x^{\omega_n}$, and in Table 3.3 for S_x^B for the parameters a, b, c, α_1, α_2, K_1, K_2, K_3, and K_4. Since the voltage transfer function is of the form of equation (2.32), the following sensitivity functions are the same for all circuits under study, as shown by

$$S_a^Q = \tfrac{1}{2} \qquad S_b^Q = -1 \qquad S_c^Q = \tfrac{1}{2} \tag{3.5}$$

$$S_a^{\omega_n} = -\tfrac{1}{2} \qquad S_b^{\omega_n} = 0 \qquad S_c^{\omega_n} = \tfrac{1}{2} \tag{3.6}$$

$$S_a^B = -1 \qquad S_b^B = 1 \qquad S_c^B = 0 \tag{3.7}$$

Note also that

$$S_{\alpha_1}^{\omega_n} = S_{\alpha_2}^{\omega_n} = -\tfrac{1}{2} \tag{3.8}$$

and

$$S_{\alpha_2}^Q = -S_{\alpha_1}^Q \tag{3.9}$$

for all circuits.

TABLE 3.1 - Summary of S_x^Q.

Fig. 2.3	$S_{\alpha_1}^Q = \frac{1}{2} \cdot \frac{k - 1 + K_2}{k + 1 - K_2}$
	$S_{K_1}^Q = \frac{1}{2} \cdot \frac{K_1 K_2}{1 + K_1 K_2 - k}$
	$S_{K_2}^Q = -\frac{1}{2} \cdot \frac{K_1 K_2 - K_2 + 2}{1 + K_1 K_2 - K_2} + \frac{k + 1}{k + 1 - K_2}$
Fig. 2.4	$S_{\alpha_1}^Q = \frac{1}{2} \cdot \frac{k - 1 - K_2}{k + 1 + K_2}$
	$S_{K_1}^Q = \frac{1}{2} \cdot \frac{K_1 K_2}{K_1 K_2 + 1 + K_2}$
	$S_{K_2}^Q = -\frac{1}{2} \cdot \frac{K_1 K_2 + K_2}{1 + K_1 K_2 + K_2} + \frac{k + 1}{k + 1 + K_2}$
Fig. 2.5	$S_{\alpha_1}^Q = \frac{1}{2} \cdot \frac{k - 1}{k + 1}$
	$S_{K_1}^Q = S_{K_2}^Q = 0$
Fig. 2.6	$S_{\alpha_1}^Q = \frac{1}{2} \cdot \frac{k - 1}{k + 1}$
	$S_{K_1}^Q = S_{K_2}^Q = \frac{1}{2} - \frac{1}{2} \cdot \frac{1}{1 + K_1 K_2}$
Fig. 2.7	$S_{\alpha_1}^Q = \frac{1}{2} \cdot \frac{k + 3K_2 + 1}{k - K_2 + 1}$
	$S_{K_1}^Q = 0$
	$S_{K_2}^Q = \frac{k + 1}{k + 1 - K_2} - \frac{1}{2} - \frac{1}{2} \cdot \frac{1}{1 - K_2}$
Fig. 2.8	$S_{\alpha_1}^Q = \frac{1}{2} - \frac{1 + K_2}{k + 1 + K_2} = \frac{1}{2} \cdot \frac{k - 1 - K_2}{k + 1 + K_2}$
	$S_{K_1}^Q = \frac{1}{2} - \frac{1}{2} \cdot \frac{1 + K_2}{1 + K_2 (K_1 K_3 + 1)}$
	$S_{K_2}^Q = \frac{1}{2} - \frac{1}{2} \cdot \frac{1}{1 + K_2 (K_1 K_3 + 1)} + \frac{k + 1}{k + 1 + K_2}$

TABLE 3.1 – continued

Fig. 2.8 $$S^Q_{K_3} = \frac{1}{2} \cdot \frac{K_1K_2K_3}{1 + K_1K_2K_3 + K_2}$$

Fig. 2.9 $$S^Q_{\alpha_1} = \frac{1}{2} \cdot \frac{k - 1 - K_2K_4}{k + 1 + K_2K_4}$$

$$S^Q_{K_1} = \frac{1}{2} \cdot \frac{K_1K_2K_3K_4}{1 + K_1K_2K_3K_4 + K_2K_4} = S^Q_{K_3}$$

$$S^Q_{K_2} = -\frac{1}{2} - \frac{1}{2} \cdot \frac{1}{1 + K_2K_4(K_1K_3 + 1)} + \frac{k + 1}{k + 1 + K_2K_4}$$

$$S^Q_{K_4} = \frac{1}{2} \cdot \frac{K_1K_2K_3K_4}{K_1K_2K_3K_4 + K_2K_4 + 1} - \frac{K_2K_4}{k + K_2K_4 + 1}$$

Fig. 2.10 $S^Q_{\alpha_1}$, $S^Q_{K_1}$, $S^Q_{K_2}$ for denominator are same as those for Fig. 2.3. For numerator, the S^Q_x are same as those for Fig. 2.7.

Fig. 2.11 For denominator, the S^Q_x are same as those for Fig. 2.3. For numerator, the S^Q_x are as follows:

$$S^Q_{\alpha_1} = \frac{1}{2} - \frac{1 - K_2}{1 - K_2 + k(1 + K_1)}$$

$$S^Q_{K_1} = -\frac{1}{2} - \frac{1}{2} \cdot \frac{1 - K_2}{1 + K_1 - K_2} + \frac{1 - K_2 + k}{1 - K_2 + k(1 + K_1)}$$

$$S^Q_{K_2} = -\frac{1}{2} - \frac{1}{2} \cdot \frac{1 + K_1}{1 + K_1 - K_2} + \frac{1 + k(1 + K_1)}{1 - K_2 + k(1 + K_1)}$$

Fig. 2.12 $$S^Q_{\alpha_1} = \frac{1}{2} - \frac{1 - K_2}{k + 1 - K_2} = \frac{1}{2} \cdot \frac{k - 1 + K_2}{k + 1 - K_2}$$

$$S^Q_{K_1} = \frac{1}{2} - \frac{1}{2} \cdot \frac{1 - K_2}{1 + K_2(K_1K_3 - 1)}$$

$$S^Q_{K_2} = -\frac{1}{2} - \frac{1}{2} \cdot \frac{1}{1 + K_2(K_1K_3 - 1)} + \frac{k + 1}{k + 1 - K_2}$$

$$S^Q_{K_3} = \frac{1}{2} \cdot \frac{K_1K_2K_3}{1 + K_2(K_1K_3 - 1)}$$

TABLE 3.1 - Continued

Fig. 2.13

$$S^{Q}_{\alpha_1} = \frac{1}{2} \cdot \frac{k - 1 + K_2K_4}{k + 1 - K_2K_4}$$

$$S^{Q}_{K_1} = \frac{1}{2} \cdot \frac{K_1K_2K_3K_4}{1 + K_2K_4(K_1K_3 - 1)} = S^{Q}_{K_3}$$

$$S^{Q}_{K_2} = -\frac{1}{2} + \frac{1}{2} \frac{1}{1 + K_2K_4(K_1K_3 - 1)} + \frac{k + 1}{k + 1 - K_2K_4}$$

$$S^{Q}_{K_4} = \frac{1}{2} \cdot \frac{K_1K_2K_3K_4}{1 + K_2K_4(K_1K_3 - 1)} + \frac{K_2K_4}{k + 1 - K_2K_4}$$

TABLE 3.2 – Summary of $S_x^{\omega n}$.

Fig. 2.3	$S_{K_1}^{\omega n} = \frac{1}{2} \cdot \frac{K_1K_2}{1 + K_1K_2 - K_2}$
	$S_{K_2}^{\omega n} = \frac{1}{2} \cdot \frac{K_1K_2 - K_2}{1 + K_1K_2 - K_2}$
Fig. 2.4	$S_{K_1}^{\omega n} = \frac{1}{2} \cdot \frac{K_1K_2}{1 + K_1K_2 + K_2}$
	$S_{K_2}^{\omega n} = \frac{1}{2} \cdot \frac{K_1K_2 + K_2}{1 + K_1K_2 + K_2}$
Fig. 2.5	$S_{K_1}^{\omega n} = S_{K_2}^{\omega n} = 0$
Fig. 2.6	$S_{K_1}^{\omega n} = S_{K_2}^{\omega n} = -\frac{1}{2} \cdot \frac{1}{1 + K_1K_2} + \frac{1}{2}$
Fig. 2.7	$S_{K_1}^{\omega n} = 0$
	$S_{K_2}^{\omega n} = -\frac{1}{2} \cdot \frac{1}{1 - K_2} + \frac{1}{2}$
Fig. 2.8	$S_{K_1}^{\omega n} = \frac{1}{2} - \frac{1}{2} \cdot \frac{K_2 + 1}{1 + K_2(K_1K_3 + 1)}$
	$S_{K_2}^{\omega n} = \frac{1}{2} - \frac{1}{2} \cdot \frac{1}{1 + K_2(K_1K_3 + 1)}$
	$S_{K_3}^{\omega n} = S_{K_3}^{Q}$
Fig. 2.9	$S_{K_1}^{\omega n} = \frac{1}{2} - \frac{1}{2} \cdot \frac{K_2K_4 + 1}{1 + K_2K_4(K_1K_3 + 1)}$
	$S_{K_2}^{\omega n} = \frac{1}{2} - \frac{1}{2} \cdot \frac{1}{1 + K_2K_4(K_1K_3 + 1)}$
	$S_{K_3}^{\omega n} = S_{K_3}^{Q}$
	$S_{K_4}^{\omega n} = \frac{1}{2} \cdot \frac{K_1K_2K_3K_4 + K_2K_4}{1 + K_2K_4(K_1K_3 + 1)}$

TABLE 3.2 - continued

Fig. 2.10 For numerator, the $S_x^{\omega n}$ are same as those for Fig. 2.7.

For denominator, the $S_x^{\omega n}$ are same as those for Fig. 2.3.

Fig. 2.11 For denominator, the $S_x^{\omega n}$ are same as those for Fig. 2.3.

For numerator, the $S_x^{\omega n}$ are as follows:

$$S_{K_1}^{\omega n} = \frac{1}{2} - \frac{1}{2} \cdot \frac{1 - K_2}{1 + K_1 - K_2}$$

$$S_{K_2}^{\omega n} = \frac{1}{2} - \frac{1}{2} \cdot \frac{1 + K_1}{1 + K_1 - K_2}$$

Fig. 2.12

$$S_{K_1}^{\omega n} \doteq \frac{1}{2} + \frac{1}{2} \cdot \frac{K_2 - 1}{1 + K_2 (K_1K_3 - 1)}$$

$$S_{K_2}^{\omega n} = \frac{1}{2} - \frac{1}{2} \cdot \frac{1}{1 + K_2 (K_1K_3 - 1)}$$

$$S_{K_3}^{\omega n} = S_{K_3}^{\Omega}$$

Fig. 2.13

$$S_{K_1}^{\omega n} = \frac{1}{2} + \frac{1}{2} \cdot \frac{K_2K_4 - 1}{1 + K_2K_4 (K_1K_3 - 1)}$$

$$S_{K_2}^{\omega n} = \frac{1}{2} - \frac{1}{2} \cdot \frac{1}{1 + K_2K_4 (K_1K_3 - 1)}$$

$$S_{K_3}^{\omega n} = S_{K_3}^{\Omega}$$

$$S_{K_4}^{\omega n} = \frac{1}{2} \cdot \frac{K_1K_2K_3K_4 - K_2K_4}{1 + K_2K_4 (K_1K_3 - 1)}$$

TABLE 3.3 - Summary of S_x^B.

Fig. 2.3

$$S^B_{\alpha_1} = \frac{-k}{k + 1 - K_2}$$

$$S^B_{\alpha_2} = -\frac{1 - K_2}{k + 1 - K_2}$$

$$S^B_{K_1} = 0$$

$$S^B_{K_2} = -\frac{K_2}{k + 1 - K_2}$$

Fig. 2.4

$$S^B_{\alpha_1} = -\frac{k}{k + 1 + K_2}$$

$$S^B_{\alpha_2} = -\frac{1 + K_2}{k + 1 + K_2}$$

$$S^B_{K_1} = 0$$

$$S^B_{K_2} - \frac{K_2}{k + 1 + K_2}$$

Fig. 2.5

$$S^B_{\alpha_1} = \frac{-k}{k + 1}$$

$$S^B_{\alpha_2} = -\frac{1}{k + 1}$$

$$S^B_{K_1} = S^B_{K_2} = 0$$

Fig. 2.6

$$S^B_{\alpha_1} = -\frac{k}{k + 1}$$

$$S^B_{\alpha_2} = -\frac{1}{k + 1}$$

$$S^B_{K_1} = S^B_{K_2} = 0$$

Fig. 2.7

$$S^B_{\alpha_1} = -\frac{k}{k + 1 - K_2}$$

$$S^B_{\alpha_2} = -\frac{1 - K_2}{k + 1 - K_2}$$

$$S^B_{K_1} = 0$$

TABLE 3.3 - continued

Fig. 2.7 $S^B_{K_2} = -\dfrac{k+1}{k+1-K_2} + 1$

Fig. 2.8 $S^B_{\alpha_1} = -\dfrac{k}{k+1+K_2}$

$S^B_{\alpha_2} = -\dfrac{1+K_2}{k+1+K_2}$

$S^B_{K_1} = S^B_{K_3} = 0$

$S^B_{K_2} = \dfrac{K_2}{k+1+K_2}$

Fig. 2.9 $S^B_{\alpha_1} = -\dfrac{k}{k+1+K_2K_4}$

$S^B_{\alpha_2} = -\dfrac{1+K_2K_4}{k+1+K_2K_4}$

$S^B_{K_1} = S^B_{K_3} = 0$

$S^B_{K_2} = S^B_{K_4} = \dfrac{K_2K_4}{k+1+K_2K_4}$

Fig. 2.10 For numerator, the S^B_x are same as those for Fig. 2.7.

For denominator, the S^B_x are same as those for Fig. 2.3.

Fig. 2.11 For denominator, the S^B_x are same as those for Fig. 2.3.

For numerator, the S^B_x are as follows:

$S^B_{\alpha_1} = -\dfrac{k(1+K_1)}{1-K_2+k(1+K_1)}$

$S^B_{\alpha_2} = -\dfrac{1-K_2}{1-K_2+k(1+K_1)}$

$S^B_{K_1} = -\dfrac{1-K_2+k}{1-K_2+k(1+K_1)} + 1$

$S^B_{K_2} = -\dfrac{1+k(1+K_1)}{1-K_2+k(1+K_1)} + 1$

Fig. 2.12 $S^B_{\alpha_1} = \dfrac{-k}{k+1-K_2}$

TABLE 3.3 - continued

Fig. 2.12

$$S^{B}_{\alpha_2} = -\frac{1 - K_2}{k + 1 - K_2}$$

$$S^{B}_{K_1} = 0 = S^{B}_{K_3}$$

$$S^{B}_{K_2} = \frac{-K_2}{k + 1 - K_2}$$

Fig. 2.13

$$S^{B}_{\alpha_1} = \frac{-k}{k + 1 - K_2K_4}$$

$$S^{B}_{\alpha_2} = -\frac{1 - K_2K_4}{k + 1 - K_2K_4}$$

$$S^{B}_{K_1} = 0 = S^{B}_{K_3}$$

$$S^{B}_{K_2} = \frac{-K_2K_4}{k + 1 - K_2K_4} = S^{B}_{K_4}$$

3.3 Summary

Inspection of the sensitivity relationships determined above indicates that many of these are small, that is, equal to 0, $\pm \frac{1}{2}$, or $\pm$ 1. As for the remainder, most can be made small by using the following assumptions:

$$K_1 > K_2 > 1 \qquad (3.10)$$

$$k \simeq 1 \qquad (3.11)$$

$$K_1K_3 > 1 \qquad (3.12)$$

and

$$K_2K_4 > 1 \quad . \qquad (3.13)$$

Use of the above assumptions reduces all sensitivity functions in Tables 3.1, 3.2 and 3.3 to within manageable quantities, as shown in Chapter 4. The above assumptions also affect the quality factor Q as is also shown in Chapter 4.

Appendix A provides a further discussion of sensitivity functions in considering operational amplifiers. All sensitivity functions discussed so far in this chapter relate to the closed-loop gains K, whereas Appendix A takes into account the relationship with respect to open-loop gains A.

CHAPTER 4

CALCULATIONS USING THIN-FILM COMPONENTS

The use of thin-film components is especially suitable for constructing the circuits of Figs. 2.3 to 2.13 which have grounded capacitors in all cases. Therefore, calculations of circuit parameters such as sensitivity, quality factor, and magnitude of transfer functions are carried out below using typical thin-film component values.

4.1 Thin-Film Parameters

Study of various product bulletins such as [6], [10], [11], and [21] and sources such as Huelsman [16] and Whitney, Silis, and Barber [18] indicates the following typical values for thin-film components currently available: (1) resistors range from 10 ohms to 10 megohms; (2) capacitors range from 100 picofarads to 50 nanofarads. The closed-loop gain of the operational amplifiers has been selected to range up to 100. Use of the typical resistance and capacitance values stated above leads to the factor α ranging from 10^{-9} sec up to 0.5 sec.

4.2 Assumptions

Four basic assumptions have previously been made in the last chapter. These are:

$$K_1 > K_2 > 1 \tag{3.10}$$

$$K \simeq 1 \tag{3.11}$$

$$K_1 K_3 > 1 \tag{3.12}$$

and

$$K_2 K_4 > 1 \ . \tag{3.13}$$

Several other basic assumptions are now presented:

a) Since R_1, R_2, C_1, and C_2 are all made of homogeneous material, then the sensitivities with respect to variations in α_1 and α_2 track closely;

b) the closed-loop gains K_1, K_2, K_3, and K_4 can be set initially to within less than 1% [22] by the feedback and input resistors of the operational amplifiers;

c) resistors can be set to within 0.1%;

d) the closed-loop gain can be controlled quite accurately by one component only as seen by the circuit of Fig. A.6;

e) for realizable networks with transfer functions of the form of equation (2.32), the following two conditions apply:

$$b>0, \quad c>0 \ . \tag{4.1}$$

4.3 Calculations

Typical values of Q and Q-sensitivity are calculated and summarized in Table 4.1 for the circuits of Figs. 2.3 to 2.13. Magnitudes of the transfer function for the various circuits are calculated and summarized in Table 4.2.

4.4 Summary

Table 4.3 summarizes the findings of Tables 4.1 and 4.2 for high Q and corresponding $|T(j\omega_n)|$ values.

A look at the results shown in Table 4.1 indicates that the main sensitivity functions of significance are $S^Q_{\alpha_1}$, $S^Q_{\alpha_2}$, and $S^Q_{K_2}$. Since we have assumed that all resistive and capacitive components are made of homogeneous material, then the two functions $S^Q_{\alpha_1}$ and $S^Q_{\alpha_2}$, when added together, result in a net reaction due to changes in R_1, R_2, C_1, and C_1 of zero. Therefore, we are left to consider $S^Q_{K_2}$, or the effect on Q of variations in the closed-loop gain of operational amplifier 2, that is, K_2. The results in Table 4.1 indicate that both $S^Q_{K_2}$ and Q are increased with increases in K_2. This is especially evident for the circuits of Figs. 2.3, 2.7, 2.10, 2.11, and 2.12. This leads us to assume that the value of K_2 should perhaps be limited to ten or less, at least an order of 10 less than K_1. Limiting K_2, however, tends to limit the circuit gain $|T(j\omega_n)|$ at the centre frequency ω_n as shown in Table 4.2.

For high Q, low $S^Q_{K_2}$ and reasonable $|T(j\omega_n)|$, the circuits of Figs. 2.3, 2.6, 2.8, 2.9 (marginally because of low $|T(j\omega_n)|$), 2.10, 2.11, 2.12, and 2.13 (marginally because of low $|T(j\omega_n)|$), are adequate in meeting the prime objective of this dissertation. To be truly selective about the various circuits, we would have to state that the circuits of Figs. 2.6 and 2.12 meet our criteria of high Q with low $S^Q_{K_2}$, together with adequate $|T(j\omega_n)|$. The circuits of Figs. 2.7 and 2.9 are not adequate in that the $|T(j\omega_n)|$ is too low in both cases, that is, approximately zero.

Attempts to define $S_{K_2}^{Q}$ and $|T(j\omega_n)|$ in terms of Q through inspection of Tables 4.1, 4.2, and 4.3 indicate the following:

a) $S_{K_2}^{Q}$ is approximately zero for all circuits except that of Fig. 2.3 ($S_{K_2}^{Q} \propto Q/10$), Fig. 2.6 ($S_{K_2}^{Q} < 0.5$), Figs. 2.12 and 2.13 ($S_{K_2}^{Q} \propto Q/100$);

b) $|T(j\omega_n)|$ varies directly with Q for the circuits of Figs. 2.3, 2.4, 2.5, and 2.6, is zero for those of Figs. 2.7 and 2.9, and varies directly with Q/100 for those of Figs. 2.8, 2.12, and 2.13.

It is noted also in Table 4.1 that for the circuits of Figs. 2.10 and 2.11, the numerators of the transfer function cannot be set independently of the denominators. Because of this, the biquadratic cannot be fully realized.

A discussion of transfer function sensitivity is provided in Appendix D. Also included therein are calculations of typical $S_K^{T(j\omega)}$ and $S_A^{T(j\omega)}$. The sensitivity of the transfer functions with respect to open-loop gain for a circuit such as Fig. 2.12 is very low.

TABLE 4.1 - Summary of calculations of Q and S_x^Q .

Fig.	k	K_1	K_2	K_3	K_4	Q	$S_{\alpha_1}^Q$	$S_{\alpha_2}^Q$	$S_{K_1}^Q$	$S_{K_2}^Q$
2.3	1	1	1	0	0	1	.5	-.5	.5	1
	1	.01	.01	0	0	.5	0	0	0	0
	1	100	1	0	0	10	.5	-.5	.5	1.5
	10	100	10	0	0	100	9.5	-9.5	.5	9.5
	100	100	100	0	0	1000	100	-100	.5	100
2.4	1	1	1	0	0	.6	-.17	.17	.17	.33
	1	100	1	0	0	3.3	-.17	.17	.5	.17
	10	100	10	0	0	5	0	0	.5	0
	100	100	100	0	0	5	0	0	.5	0
2.5	1	1	1	0	0	.5	0	0	0	0
	10	1	1	0	0	.3	.5	-.5	0	0
	.1	1	1	0	0	.3	-.5	.5	0	0
2.6	1	1	1	0	0	.7	0	0	.25	.25
	1	100	1	0	0	5	0	0	.5	.5
	100	100	100	0	0	10	.5	-.5	.5	.5
	1	100	100	0	0	50	0	0	.5	.5

TABLE 4.1 - continued

Fig.	k	K_1	K_2	K_3	K_4	Q	$S^Q_{\alpha_1}$	$S^Q_{\alpha_2}$	$S^Q_{K_1}$	$S^Q_{K_2}$
2.7	1	1	1	0	0	0	2.5	-2.5	0	∞
	1	1	.01	0	0	.5	.5	-.5	0	0
	.01	1	.01	0	0	.1	.5	-.5	0	0
2.8	1	1	1	1	0	.6	-.17	.17	.17	0
	10	100	10	100	0	50	0	0	.5	0
	1	100	1	100	0	33	-.17	.17	.5	.17
2.9	1	1	1	100	100	1	-.5	.5	.5	-.5
	1	100	1	100	100	10	-.5	.5	.5	-.5
	1	100	10	100	100	3.3	-.5	.5	.5	-.5
	1	100	1	100	1	33	-.17	.17	.5	.17
2.10	1	1	1	0	0	0	2.5	-2.5	0	∞
(num.)	1	1	.01	0	0	.5	.5	-.5	0	0
	.01	1	.01	0	0	.1	.5	-.5	0	0
2.10	1	1	1	0	0	1	.5	-.5	.5	1
(denom.)	1	.01	.01	0	0	.5	0	0	0	0
	1	100	1	0	0	10	.5	-.5	.5	1.5
	10	100	10	0	0	100	9.5	-9.5	.5	9.5
	100	100	100	0	0	1000	100	-100	.5	100

TABLE 4.1 - continued

Fig.	k	K_1	K_2	K_3	K_4	Q	$S^Q_{\alpha_1}$	$S^Q_{\alpha_2}$	$S^Q_{K_1}$	$S^Q_{K_2}$
2.11	1	1	1	100	0	.5	.5	-.5	0	0
(num.)										
2.11	1	1	1	0	0	1	.5	-.5	.5	1
(denom.)	1	.01	.01	0	0	.5	0	0	0	0
	1	100	1	0	0	10	.5	-.5	.5	1.5
	10	100	10	0	0	100	9.5	-9.5	.5	9.5
	100	100	100	0	0	1000	100	-100	.5	100
2.12	1	1	1	1	0	1	.5	-.5	.5	1
	10	100	10	100	0	1000	9.5	-9.5	.5	10.5
	1	100	1	100	0	100	.5	-.5	.5	1.5
2.13	1	1	1	100	1	10	.5	-.5	.5	1.5
	1	100	1	100	1	100	.5	-.5	.5	1.5

TABLE 4.2 – Calculation of $|T(j\omega_n)|$.

Fig.	k	K_1	K_2	K_3	K_4	$\lvert T(j\omega_n)\rvert$
2.3	1	1	1	1	0	1
	1	.01	.01	0	0	0
	1	100	1	0	0	10
	10	100	10	0	0	100
	100	100	100	0	0	1000
2.4	1	1	1	0	0	.2
	1	100	1	0	0	.1
	10	100	10	0	0	.5
	100	100	100	0	0	5
2.5	1	1	1	0	0	.5
	10	1	1	0	0	.3
	.1	1	1	0	0	.3
2.6	1	1	1	0	0	.35
	1	100	1	0	0	5
	100	100	100	0	0	10
	1	100	100	0	0	50
2.7	1	1	1	0	0	
	1	1	.01	0	0	0
	.01	1	.01	0	0	0

TABLE 4.2 - continued

Fig.	k	K_1	K_2	K_3	K_4	$\lvert T(j\omega_n) \rvert$
2.8	1	1	1	1	0	.2
	10	100	10	100	0	.5
	1	100	1	100	0	.3
2.9	1	1	1	100	100	0
	1	100	1	100	100	0
	1	100	10	100	100	0
	1	100	1	100	1	.3
2.10				1		1
				10		10
				100		100
2.11				1		1
				10		10
				100		100
2.12	1	1	1	1	0	1
	10	100	10	100	0	10
	1	100	1	100	0	1
2.13	1	1	1	100	1	.1
	1	100	1	100	1	1

TABLE 4.3 - Inferences drawn from Tables 4.1 and 4.2 .

Fig.	Conditions found	For k and K's as below	Then Q is	And $T(j\omega_n)$ is
2.3	$K_2 \leqslant k+1$	$K_2 \to 0$ $k \to 0$	$Q \to 0.5$ $Q \to 100+$	$\|T\| \to 0$ $\|T\| \to \infty$
2.4	$K_2 \geqslant -(k+1)$	$k>1$ $K_1 \gtrsim 1$ $K_2 > 1$	$Q \to 5.0$	$\|T\| \to 5.0$
2.5		$k=1$ $k<1,\ k>1$ $k=0$	$Q \to 0.5$ $Q<0.5$ $Q=0$	$\|T\| = K_1K_2/2$ $\|T\| \to \infty$
2.6		$k>1$ $K_1>1$ $K_2>1$	$Q \to 50$	$\|T\| \to 50$
2.7	$K_2 \leqslant k+1$ $K_2<1$	$K_1 \geqslant 1$	$Q \to 0.5$	$\|T\| \to \infty$
2.8		$\geqslant 1$	$Q \to 50$	$\|T\| \to .5$
2.9		$\geqslant 1$	$Q \to 10$	$\|T\| \simeq 0$
2.10 (num.)			$Q \to 0.5$	$\|T\| \simeq K_3$
2.10 (denom.)		$K_2 \to 0$ $k \to 0$	$Q \to 0.5$ $Q \to 100$	
2.11 (num.)	$K_2 \leqslant K_1+1$		$Q \to 0.5$	$\|T\| \simeq K_3$
2.11 (denom.)	$K_2 \leqslant k+1$	$K_2 \to o$ $k \to 0$	$Q \to 0.5$ $Q \to 100$	
2.12	$K_2 \leqslant k+1$	$\geqslant 1$	$Q \to 1000$	$\|T\| \to 10$
2.13	$K_2K_4 \leqslant k+1$	$\geqslant 1$	$Q \to 100$	$\|T\| \to 1$

CHAPTER 5

CONCLUSIONS

The primary objective of using operational amplifiers and RC networks with grounded capacitors in order to derive active RC circuits with high quality factors and low sensitivity functions has been shown. Use of a basic circuit such as Fig. 2.1 in various configurations, Figs. 2.3 to 2.13, leads to high Q and low S_x^Q when typical thin-film components are used.

Two circuits in particular provide the two criteria stated quite adequately. These are the circuits of Figs. 2.6 and 2.12 wherein Q's of 50 and 100 respectively are obtained with very low $S_{K_2}^Q$'s and adequate $|T(j\omega_n)|$. With respect to $|T(j\omega_n)|$, the circuit of Fig. 2.6 is even better than that of Fig. 2.12 because of a Q of 50 at a $|T(j\omega_n)|$ of 50.

The use of operational amplifiers in differential input configurations as shown in Appendix A (Fig. A.5 for example) leads to an interesting result. By ensuring that the forward path has negative gains through each operational amplifier stage, which occurs in the circuits of Figs. 2.3, 2.5 to 2.7, 2.10 to 2.13, a higher quality factor is also ensured. Comparison of the Q for the circuits of Figs. 2.3 and 2.4, Figs. 2.8 and 2.12, and Figs. 2.9 and 2.13 indicates directly that the Q's of the circuits of Figs. 2.3, 2.12, and 2.13 are much higher than those of the circuits of Figs. 2.4, 2.8 and 2.9.

Use of the circuits of Figs. 2.3, 2.6, 2.12, and 2.13 for low-pass networks and the circuits of Figs. 2.10 and 2.11 for restricted biquad networks with high Q and low S_x^Q is recommended, using the analytical formulas derived in this dissertation. The circuits of Figs. 2.10 and 2.11, though yielding biquads, are still not adequate in that the numerator cannot be set arbitrarily, that is, idependently of the denominator. The nature of the coefficients is such that changes to the numerator affect the denominator and hence parameters such as Q and S_x^Q. Further work is justified in developing more generalized biquad networks.

Calculation of transfer function sensitivities such as $S_A^{T(j\omega)}$, that is, with respect to open-loop gains for the operational amplifiers was done for only one of the many circuits developed. This is discussed in Appendix D (see Fig. D.1) for the circuit of Fig. 2.12. Appendix A develops the background theory of operational amplifiers and indicates that S_A^K is approximately equal to K/A. Since the open-loop gain can be as large as 10^5, S_A^K is very small. Therefore, calculation of typical $S_A^{T(j\omega)}$ as shown in Appendix D indicates that these sensitivity functions are also very small.

As stated above, further work is required on generalized biquads. Another excellent project is the development of a general second-order active RC circuit (with grounded capacitors) using the circuits of this dissertation in the same way as Kerwin, Huelsman, and Newcomb [12].

APPENDIX A

FUNDAMENTALS OF OPERATIONAL AMPLIFIERS

The following is a summary of fundamentals of operational amplifiers taken mainly from a recent Burr-Brown bulletin [22]. Discussion of the sensitivity of the closed-loop gain with respect to changes in the open-loop gain is also presented.

A.1 Basic Model

The operational amplifier is a high-gain, dc - coupled amplifier with either a differential or single input and usually a single output. The basic amplifier model is shown in Fig. A.1 with amplifier symbols shown in Figs. A.2a and A.2b.

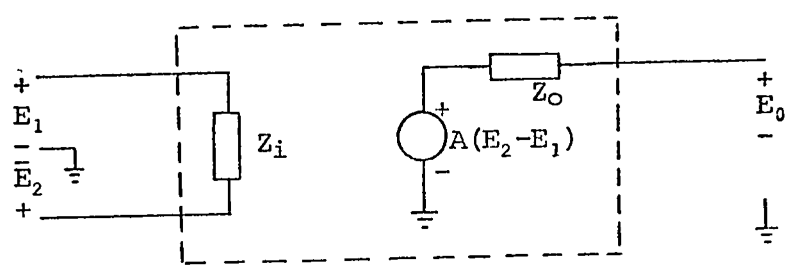


Fig. A.1 - Basic amplifier model.

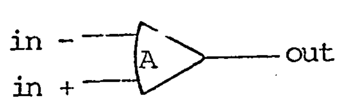


Fig. A.2a - Symbol of differential amplifier.

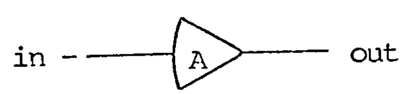


Fig. A.2b - Symbol of single input amplifier (inverting type).

In the above Figs. A.1, A.2a, A.2b, the symbols are defined as follows:

E_1, E_2	input voltages,
E_0	output voltages,
Z_i	input impedance,
Z_o	output impedance,
A	open-loop gain.

A.2 Fundamental Inverting Circuit

The fundamental inverting circuit is given in Fig. A.3.

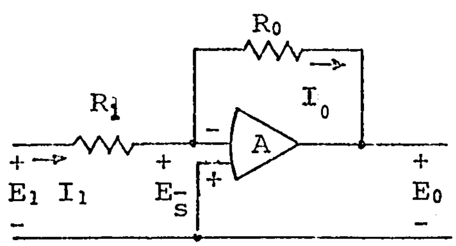


Fig. A.3 - Inverting circuit.

In the circuit of Fig. A.3, since Z_i is assumed infinite,

$$I_1 = I_0, \tag{A.1}$$

or

$$I_1 = \frac{E_1 - E}{R_1} = \frac{E_s - E_0}{R_0} = I_0 . \tag{A.2}$$

Since

$$E_0 = AE_s \tag{A.3}$$

equation (A.2) becomes

$$\frac{E_1 + \frac{E_0}{A}}{R_1} = \frac{\frac{-E_0}{A} - E_0}{R_0} \tag{A.4}$$

or

$$\frac{E_0}{E_1} = K = -\frac{AK_0}{K_0 + A + 1} \tag{A.5}$$

where

$$K_0 = \frac{R_0}{R_1} . \tag{A.6}$$

If we now determine the sensitivity of the gain K with respect to changes in the open-loop gain A, we find

$$S_A^K = \frac{\partial K/K}{\partial A/A} = \frac{K_0 + 1}{K_0 + A + 1} . \tag{A.7}$$

Since

$$K_0 << A, \tag{A.8}$$

then

$$S_A^K \simeq \frac{K_0 + 1}{A} . \tag{A.9}$$

A.3 Fundamental Non-Inverting Circuit

The fundamental non-inverting circuit is given in Fig. A.4.

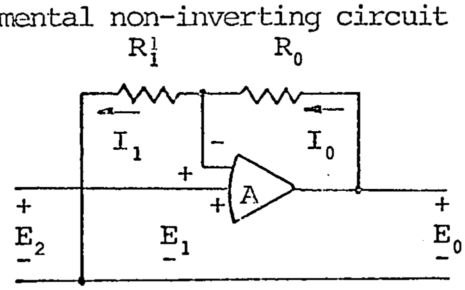


Fig. A.4 - Non-inverting circuit.

In the circuit of Fig. A.4,

$$I_1 = I_0 \tag{A.1}$$

and

$$E_1 = I_1R_1 = \frac{E_0 R_1}{R_1+R_0} \tag{A.10}$$

Since

$$E_0 = A(E_2-E_1), \tag{A.11}$$

if we combine equations (A.10) and (A.11), we obtain

$$E_2 = \frac{E_0}{A} + E_0 \cdot \frac{R_1}{R_1+R_0} \tag{A.12}$$

or

$$\frac{E_0}{E_2} = K_n = \frac{AK_{on}}{K_{on}+A} \tag{A.13}$$

where

$$K_{on} = \frac{R_1+R_0}{R_1} \quad . \tag{A.14}$$

If we now determine the sensitivity of the gain K with respect to changes in the open-loop gain A, we find

$$S_A^{K_n} = \frac{\partial K_n/K_n}{\partial A/A} = \frac{K_{on}}{K_{on}+A} \quad . \tag{A.15}$$

Since

$$K_{on} \ll A, \tag{A.16}$$

then

$$S_A^{K_n} \simeq \frac{K_{on}}{A} \quad . \tag{A.17}$$

A.4 Feedback Differential Amplifier

The circuit of Fig. A.5 is a typical feedback differential amplifier.

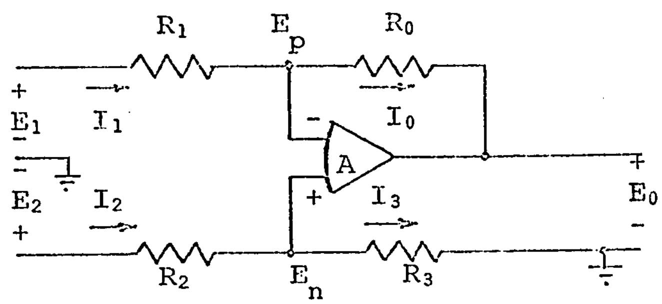


Fig. A.5 - Feedback differential amplifier.

Since the amplifier draws no current in Fig. A.5, we find that

$$I_0 = I_1 \tag{A.18}$$

$$I_2 = I_3 \tag{A.19}$$

$$I_1 = \frac{E_1-E_p}{R_1} = \frac{E_p-E_0}{R_0} \tag{A.20}$$

$$I_2 = \frac{E_n}{R_3} = \frac{E_2}{R_3+R_2} . \tag{A.21}$$

From equation (A.20), we get

$$E_0 = E_p\left(\frac{1}{R_0}+\frac{1}{R_1}\right) - \frac{R_0}{R_1} E_1 . \tag{A.22}$$

From Equation (A.21), we get

$$E_n = \frac{R_3}{R_3+R_2} E_2 . \tag{A.23}$$

Noting that for $A \to \infty$,

$$E_n = E_p \tag{A.24}$$

and letting

$$R_2 = R_1 \tag{A.25a}$$

and

$$R_3 = R_0 \qquad \text{(A.25b)}$$

for convenience, we can combine equations (A.22) and (A.23) to get

$$E_0 = \frac{R_0}{R_1}(E_2 - E_1) \ . \qquad \text{(A.26)}$$

The feedback differential amplifier of Fig. A.5 is advantageous because there is only one operational amplifier; however, it has disadvantages in that (1) it is difficult to vary the gain because both R_0's must be changed and (2) it has a low input impedance because it is unbalanced to ground. A better differential amplifier is shown by Fig. A.6 which has high input impedance and facility in changing gain (simply by varying R_1). The only disadvantage of the circuit of Fig. A.5 is the need for three operational amplifiers.

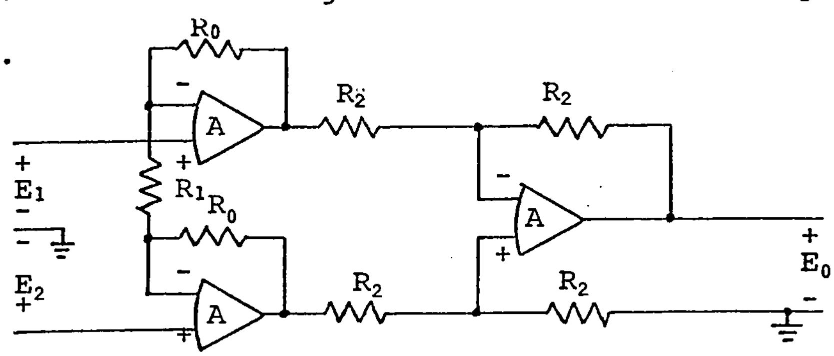


Fig. A.6 - Better differential amplifier.

A.5 Specifications of Operational Amplifiers

The open-loop gain A of a compensated operational amplifier is shown in Fig. A.7 which is a Bode plot. The closed-loop gain K is also shown by a dashed line. The frequency response of an open-loop operational amplifier is given by

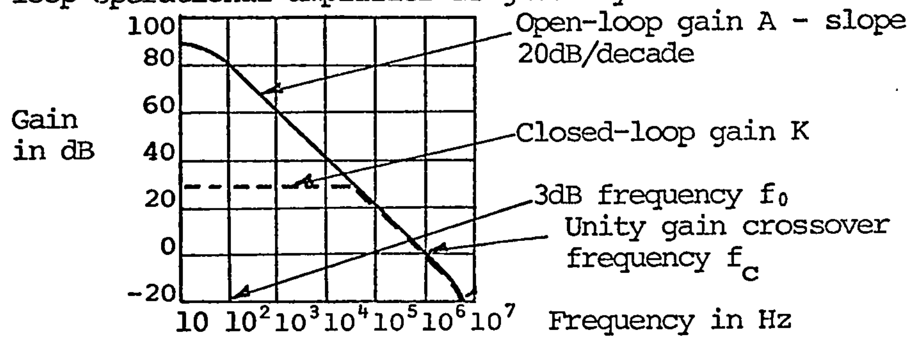


Fig. A.7 - Open-loop gain (compensated amplifier).

$$A(j\omega) = \frac{A_0}{1 + \frac{j\omega}{\omega_0}} \tag{A.27}$$

where A_0 is the gain at dc and ω_0 is the 3dB attenuation frequency in radians per second. Thus, for the inverting amplifier of Fig. A.3, the closed-loop gain is given by

$$K = \left(-\frac{A_0K_0}{K_0+1}\right) \Big/ \left(\frac{A_0}{K_0+1} + 1 + \frac{j\omega}{\omega_0}\right) . \tag{A.28}$$

The gain-bandwidth relationship defined by f_0A_0 which is the unity-gain bandwidth is a constant for any specific device.

Fig. A.8 indicates the phase response of a compensated operational amplifier having the loop gain shown in Fig. A.7.

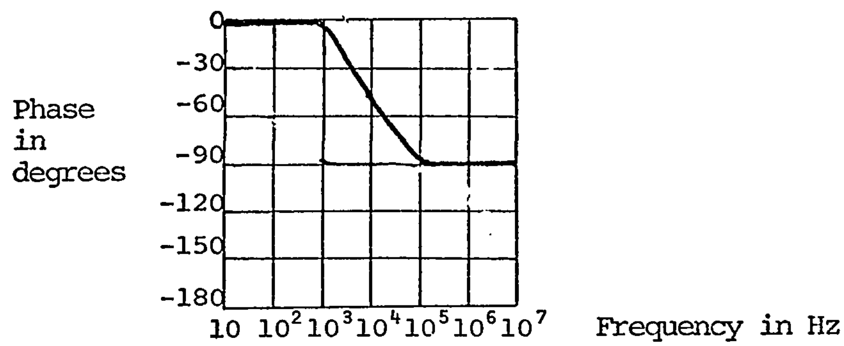


Fig. A.8. - Phase (compensated amplifier).

Uncompensated operational amplifiers have open-loop gains which range from 10^4 to 10^5 at dc, with useful frequency ranges up to 300 kHz and unity-gain bandwidths up to 15 MHz. Thus, for typical A of 30000, the useful bandwidth is 500 Hz. For a typical closed-loop gain of 100, the useful bandwidth is 150 kHz.

APPENDIX B

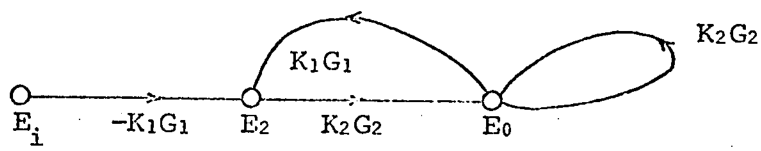
KEY TO SYMBOLS USED

a	Coefficient of polynomial
b	" " "
c	" " "
f	Frequency
j	$\sqrt{-1}$
k	Factor, or ratio of α_2 to α_1
s	Complex frequency variable
x	Circuit element or parameter
A	Open-loop gain of amplifier
B	Bandwidth
C	Capacitance
E	Voltage
G	RC - network transfer function
H	Constant
K	Closed-loop gain of amplifier
P	Polynomial
Q	Quality factor
R	Resistance
S_x^B	Sensitivity of bandwidth with respect to x
S_x^Q	Q-sensitivity with respect to x
$S_x^{\omega_n}$	Sensitivity of centre frequency with respect to x
T(s)	Voltage transfer function
$T(j\omega)$	" " " when $s=j\omega$
α	Time constant, RC
σ	Real part of complex frequency variable s
ω	Imaginary part of complex frequency variable s
ω_n	Centre frequency of network

APPENDIX C

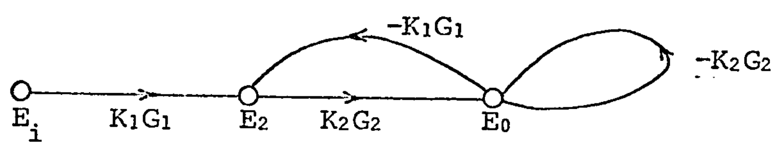
SUMMARY OF SIGNAL FLOW GRAPHS

Fig. 2.3



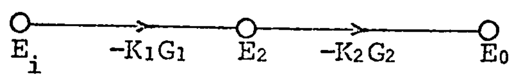
$$T_1(s) = \frac{E_0(s)}{E_i(s)} = \frac{K_1K_2G_1G_2}{1 + K_1K_2G_1G_2 - K_2G_2}$$

Fig. 2.4



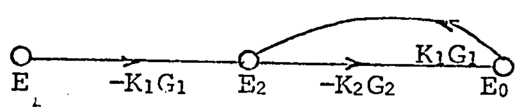
$$T_2(s) = \frac{E_0(s)}{E_i(s)} = \frac{K_1K_2G_1G_2}{1 + K_1K_2G_1G_2 + K_2G_2}$$

Fig. 2.5



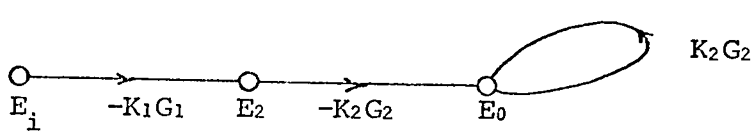
$$T_3(s) = \frac{E_0(s)}{E_i(s)} = K_1K_2G_1G_2$$

Fig. 2.6



$$T_4(s) = \frac{E_0(s)}{E_i(s)} = \frac{K_1K_2G_1G_2}{1 + K_1K_2G_1G_2}$$

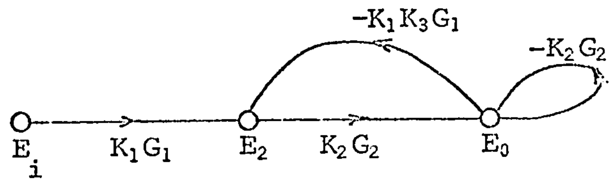
Fig. 2.7



$$T_5(s) = \frac{E_0(s)}{E_i(s)} = \frac{K_1K_2G_1G_2}{1 - K_2G_2}$$

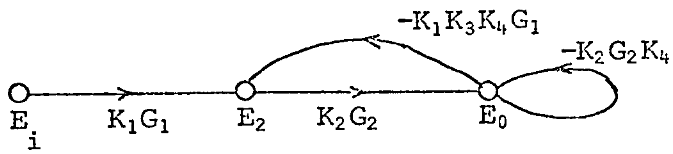
APPENDIX C - Continued

Fig. 2.8



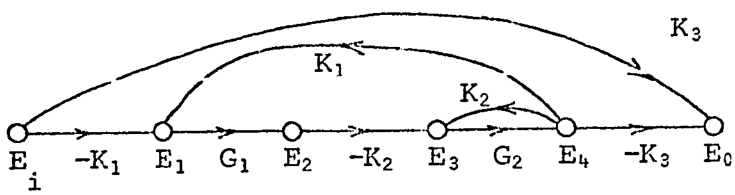
$$T_6(s) = \frac{E_0(s)}{E_i(s)} = \frac{K_1K_2G_1G_2}{1 + K_1K_2K_3G_1G_2 + K_2G_2}$$

Fig. 2.9



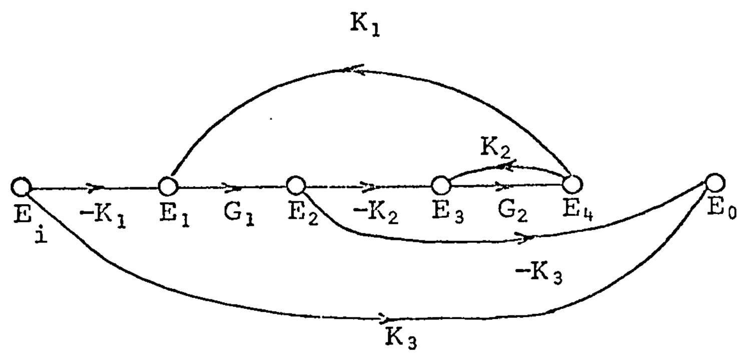
$$T_7(s) = \frac{E_0(s)}{E_i(s)} = \frac{K_1K_2G_1G_2}{1 + K_1K_2K_3K_4G_1G_2 + K_2K_4G_2}$$

Fig. 2.10



$$T_8(s) = \frac{E_0(s)}{E_i(s)} = \frac{K_3(1 - K_2G_2)}{1 + K_1K_2G_1G_2 - K_2G_2}$$

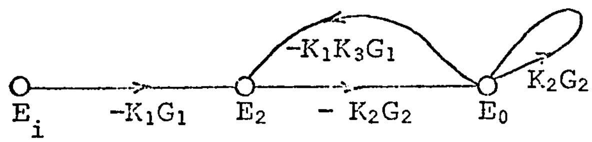
Fig. 2.11



$$T_9(s) = \frac{E_0(s)}{E_i(s)} = \frac{K_3(1 + K_1G_1 - K_2G_2)}{1 + K_1K_2G_1G_2 - K_2G_2}$$

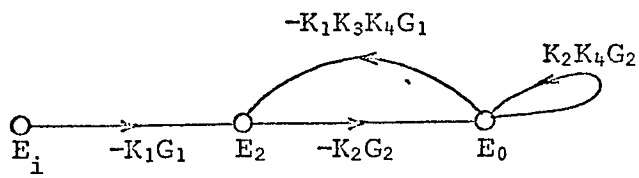
APPENDIX C – Continued

Fig. 2.12



$$T_{10}(s) = \frac{E_0(s)}{E_i(s)} = \frac{K_1K_2G_1G_2}{1 + K_1K_2K_3G_1G_2 - K_2G_2}$$

Fig. 2.13



$$T_{11}(s) = \frac{E_0(s)}{E_i(s)} = \frac{K_1K_2G_1G_2}{1 + K_1K_2K_3K_4G_1G_2 - K_2K_4G_2}$$

APPENDIX D

DISCUSSION OF TRANSFER FUNCTION SENSITIVITY

Transfer function sensitivity is defined as

$$S_x^{T(s)} = \frac{\partial \ln T(s)}{\partial \ln x} = \frac{\partial T(s)/T(s)}{\partial \; x/x} \; . \tag{D.1}$$

For voltage transfer functions of the form

$$T(s) = \frac{1}{as^2 + bs + c}, \tag{D.2}$$

the various sensitivity functions can be determined to be

$$S_a^{T(s)} = -as^2\,T(s) \tag{D.3}$$

$$S_b^{T(s)} = -bs\;T(s) \tag{D.4}$$

and

$$S_c^{T(s)} = -c\;T(s) \; . \tag{D.5}$$

Consider the circuit of Fig. D.1 which is essentially the circuit of Fig. 2.12 discussed in this dissertation.

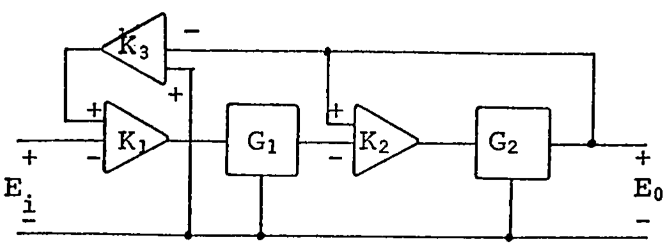


Fig. D.1 - Typical low-pass network.

The voltage transfer function is given by

$$T(s) = \frac{E_o(s)}{E_i(s)} = \frac{1}{as^2 + bs + c} \tag{D.6}$$

where

$$a = \frac{\alpha_1\alpha_2}{K_1K_2} \tag{D.7a}$$

$$b = \frac{\alpha_2 + \alpha_1(1 - K_2)}{K_1K_2} \tag{D.7b}$$

and

$$c = \frac{1 + K_2(K_1K_3 - 1)}{K_1K_3} \quad . \tag{D.7c}$$

The gain is given by

$$|T(j\omega_n)| = \frac{1}{\frac{k + 1 - K_2}{K_1K_2}\sqrt{\frac{1 + K_2(K_1K_3 - 1)}{k}}} \quad . \tag{D.8}$$

The sensitivity functions with respect to the closed-loop gains K_1, K_2, and K_3 are given by

$$S_{K_1}^{T(s)} = \frac{as^2 + bs + c_1}{as^2 + bs + c} \tag{D.9}$$

where

$$c_1 = \frac{1 - K_2}{K_1K_2} \;, \tag{D.10}$$

$$S_{K_2}^{T(s)} = \frac{as^2 + b_2s + c_2}{as^2 + bs + c} \tag{D.11}$$

where

$$b_2 = \frac{\alpha_2 + \alpha_1}{K_1K_2} \tag{D.12a}$$

and

$$c_2 = \frac{1}{K_1K_2} \;, \tag{D.12b}$$

and

$$S_{K_3}^{T(s)} = -\frac{K_3}{as^2 + bs + c} \quad . \tag{D.13}$$

Note that the sensitivity function with respect to the open-loop gain A is given by

$$S_A^{T(s)} = S_K^{T(s)} \cdot S_A^K \tag{D.14}$$

where S_A^K for typical differential input amplifiers such as shown in Appendix A (Fig. A.5) is approximately equal to K/A. Since the open-loop gain has a range of values up to 10^5, the sensitivity of K with respect to A can have values as small as 10^{-5}.

Therefore, the sensitivity function with respect to A is

$$S_A^{T(s)} \simeq 10^{-5} \; S_K^{T(s)} \quad . \tag{D.15}$$

Using the circuit of Fig. D.1, with

$$k = K_2 = 1 \tag{D.16a}$$

and

$$K_1 = K_3 = 100, \tag{D.16b}$$

we find that the Q-sensitivity functions are very low, the $|T(j\omega_n)|$ is approximately unity, and the sensitivity functions are

$$S_{K_1}^{T(j\omega)} \simeq 1 \tag{D.17a}$$

$$S_{K_2}^{T(j\omega)} \simeq 1 \tag{D.17b}$$

$$S_{K_3}^{T(j\omega)} \simeq -100 \tag{D.17c}$$

so that

$$S_{A_1}^{T(j\omega)} = S_A^{T(j\omega)} \simeq 10^{-5} \tag{D.18a}$$

and

$$S_{A_3}^{T(j\omega)} \simeq -10^{-3} \quad . \tag{D.18b}$$

Thus, the sensitivity functions with respect to the open-loop gains are very low, indicating that variations in open-loop gain will not affect the circuit gains too much.

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