SHEAR FORM FACTORS FOR VARIOUS CROSS-SECTIONS

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ABSTRACT

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SHEAR FORM FACTORS FOR VARIOUS CROSS-SECTIONS

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In this study, form factors for shear deformation of some geometric sections are evaluated.

When the deflection due to shear is sought for a beam with a given cross-section, a form factor is needed. The cross-sections for which the form factors are evaluated are rectangular, triangular, trapezoidal, circular sector, arc, circular segment, T and hexagonal.

Computer programs are developed for solving the numerical integration of the governing equations. The results of the analysis are tabulated and plotted in Chapter IV.

ACKNOWLEDGEMENTS

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NOTATION

NOTATION

A	Area of the cross-section
a	Side length of hexagonal section
$A(y)$	Variable area for calculation of Q
B	A specified width of cross-section and width of flange in T-section
b	Variable width of the layer on which shear stress distribution is sought
D	Depth of section
F_s	Form factor for shear
H	Height of stem of T-section and depth of trapezoidal section
I	Moment of inertia of the cross-section about its neutral axis
Q	First moment of area about neutral axis of the cross-section
q_1	Tangential stress on the section
R_1	Inner radius of arc section
R_2	Outer radius of arc section
R	Radius of the sector
t	Lower base of trapezoidal section
T_F	Thickness of the flange of the T-section
T_W	Thickness of the web of the T-section
V	Shear force acting on the section
x	Distance of the layer from a chosen axis
y	Distance of the layer from a chosen axis
y_0	Distance of the centroid of the arc or sector from the origin of the circle subtending it

$\bar{y}$	Distance of the centroid of the variable area from the neutral axis of the cross-section
$\bar{z}$	Distance of the centroid of the variable area from a chosen axis
α	Variable angle measured from a chosen axis.
β	Angle of arc or sector
γ	Shear strain
θ	Half of the angle of subtending arc or sector
σ	Normal or axial stress
τ	Shear stress
ε	Normal or axial strain

CHAPTER I

INTRODUCTION

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INTRODUCTION

Form factor is a constant by means of which the shear force on the section can be applied directly in the integrating formula to evaluate the strain energy, without the necessity of considering variation of the shear stress on the section. This factor, a constant for a given section, is tabulated for several symmetrical cross-sections in text books on applied elasticity and strength of materials. The sections usually cited include rectangular, I, box, circular and ring-shaped. In this study, an attempt is made to calculate the form factors for some sections symmetrical about only one axis, the only exception being that of hexagonal sections. The process of numerical integration is performed by the use of computer programs.

CHAPTER II

THEORY

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THEORY

2.1 STRAIN ENERGY

The work done by external forces in causing deformation is stored within the body in the form of strain energy.

We consider a rectangular prism of dimensions dx, dy, dz subjected to uniaxial tension. The front view of the prism is represented in Fig. 2.1. In evaluating the work done by stresses σ_x on either side of the element, it is noted that each stress acts through a different displacement.

The net work done on the element by force ($\sigma_x$dydz) is therefore:

$$dW = dU = \int_0^{\varepsilon_x} \sigma_x d\left(\frac{\partial u}{\partial x}\right) dydz = \int_0^{\varepsilon_x} \sigma_x d\varepsilon_x (dxdydz)$$

where

$$\frac{\partial u}{\partial x} = \varepsilon_x$$

dW is the work done on dxdydz, and dU is the corresponding increase in strain energy.

Designating the energy per unit volume (strain energy density) as U_0, for a linearly elastic material, we have:

$$U_0 = \int_0^{\varepsilon_x} \sigma_x \, d\varepsilon_x = \int_0^{\varepsilon_x} E\varepsilon_x \, d\varepsilon_x$$

or

$$U_0 = \tfrac{1}{2}E\varepsilon^2_x = \tfrac{1}{2}\sigma_x \varepsilon_x \qquad (2.1)$$

This quantity represents the shaded area in Fig. (2.2). When σ_x, σ_y and σ_z act simultaneously, the total work done is:

$$U_0 = \tfrac{1}{2}(\sigma_x \varepsilon_x + \sigma_y \varepsilon_y + \sigma_z \varepsilon_z) \qquad (2.2)$$

2.2 STRAIN ENERGY DUE TO SHEAR

The elastic strain energy associated with shear deformation is now analyzed by considering an element of thickness, dz, subject only to shearing stresses, τ_{xy} (Fig.2.3). From the figure, we note that shearing force, τ_{xy} dxdz, causes a displacement of γ_{xy} dy. The strain energy due to shear is $\tfrac{1}{2}(\tau_{xy}\, dxdz)(\gamma_{xy}\, dy)$, where the factor $\tfrac{1}{2}$ arises because the stress varies linearly with the strain from zero to its final value.

The strain energy density is therefore:

$$U_0 = \tfrac{1}{2}\tau_{xy}\gamma_{xy} = \frac{1}{2G}\tau^2_{xy} = \tfrac{1}{2}G\gamma^2_{xy} \qquad (2.3)$$

The total strain energy due to shear alone is:

$$U_0 = \tfrac{1}{2}(\tau_{xy}\gamma_{xy} + \tau_{yz}\gamma_{yz} + \tau_{xz}\gamma_{xz}) \qquad (2.4)$$

Given a general state of stress the total strain energy is found to be:

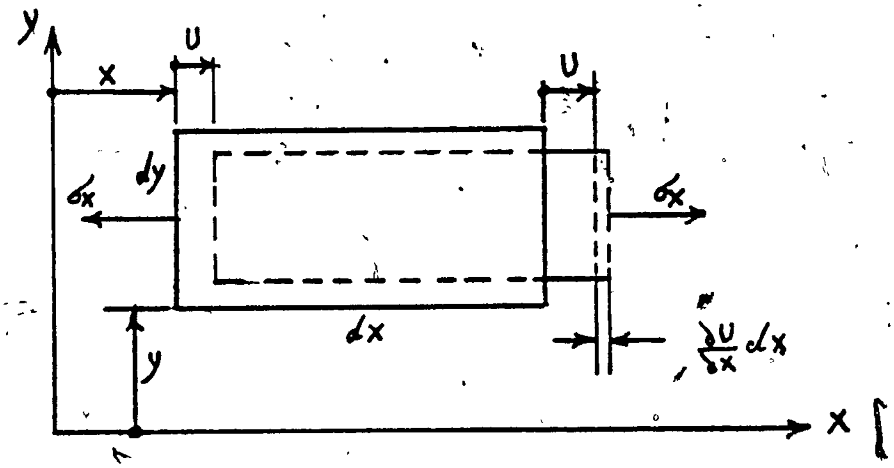

FIG.2.1 RECTANGULAR PRISM IN UNIAXIAL TENSION

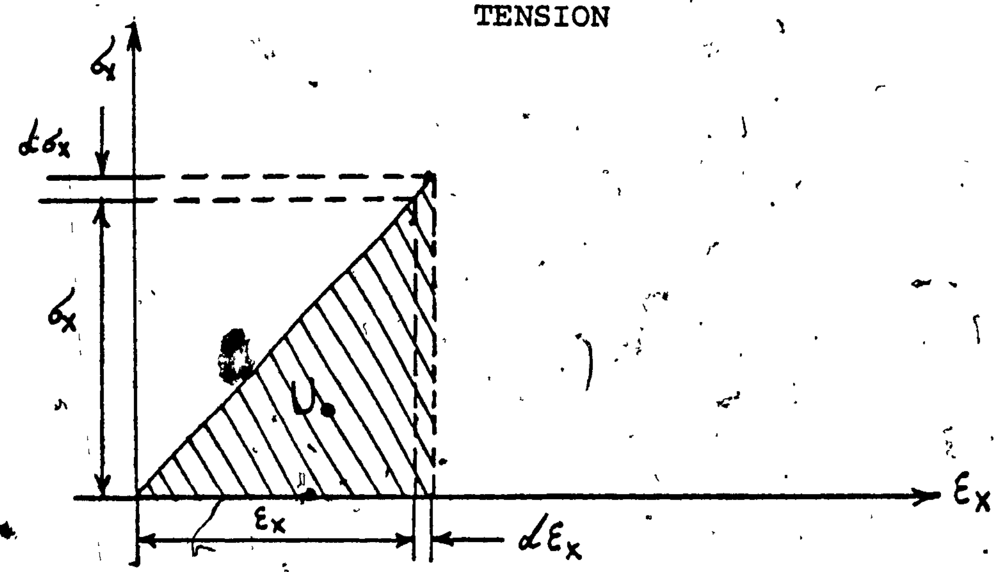

FIG.2.2 STRAIN ENERGY DENSITY

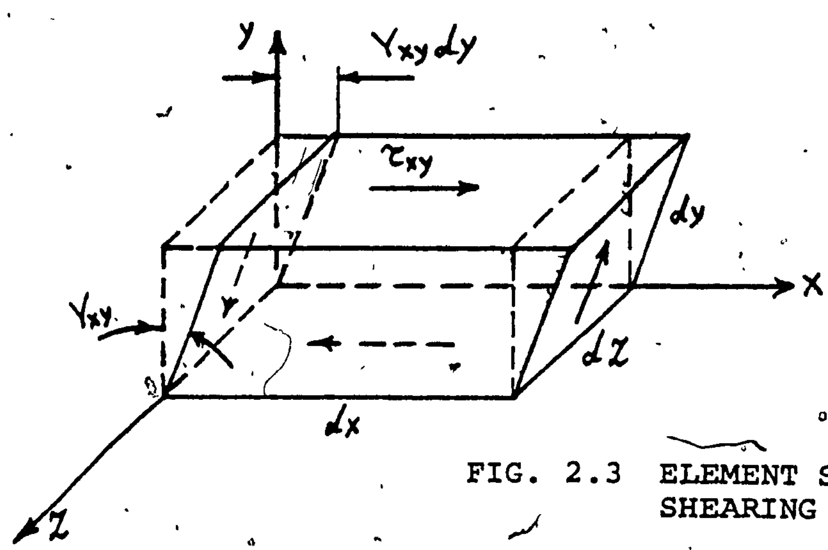

FIG. 2.3 ELEMENT SUBJECT TO SHEARING STRESSES

$$U_0 = \frac{1}{2}(\sigma_x\varepsilon_x + \sigma_y\varepsilon_y + \sigma_z\varepsilon_z + \tau_{xy}\gamma_{xy} +$$

$$+ \tau_{yz}\gamma_{yz} + \tau_{xz}\gamma_{xy}) \qquad (2.5)$$

2.3 RELATION OF FORM FACTOR TO STRAIN ENERGY

Strain energy methods are frequently employed to analyze the deflections of beams and other structural elements. Here, the strain energy due to shear will be evaluated in order to find the deflection of the beam caused by shear. The stress in a given section of a beam due to shear is given by

$$\tau = \frac{VQ}{Ib} \qquad (2.6)$$

From Equation (2.3), $U_0 = \tau^2/2G$.

Substituting τ from Equation (1.6) we have:

$$U_0 = \frac{V^2Q^2}{2GI^2b^2}$$

Integrating this expression over the volume of the beam of the cross-sectional area A, we obtain

$$U = \int \frac{V^2}{2GI^2} \left[\int \frac{Q^2}{b^2}\, dA\right] dx \qquad (2.7)$$

we denote

$$F_s = \frac{A}{I^2} \int \frac{Q^2}{b^2}\, dA \qquad (2.8)$$

This expression, F_s, is called the form factor for shear. When this factor is substituted in Equation (2.7), the strain energy can be expressed as:

$$U = \int \frac{F_s V^2 dx}{2GA} \tag{2.9}$$

So, as is seen, the form factor has to be determined for a given section in order to evaluate the strain energy due to shear, from which the deflection due to shear can be obtained.

The form factor is a dimensionless quantity which is unique for a given cross-section.

2.4 SHEAR STRESS DISTRIBUTION IN T - AND NON-RECTANGULAR SECTIONS

2.4.1 T-Section

Using the formula

$$\tau = \frac{VQ}{Ib} \tag{2.10}$$

the vertical shear stress distribution in the T-section, Fig. (2.4), is as shown. Its magnitude is maximum on the neutral axis. There is a discontinuity at the lower edge of the flange because of instant change of width from T_W to B, where T_W is the web width and B is the width of the flange. Of course, this just gives an approximation of the magnitude of the vertical shear in the flange and is closer to reality in the upper half of the flange.

In addition to the vertical shear, there is a horizontal shear (Fig. 2.5) given by the formula

$$\tau = \frac{V\ X(H_1 - T_F/2)T_F}{I\ T_F} = \frac{V(H_1 - T_F/2)}{I} X$$

which has its maximum value in the middle of the flange and has to be taken into account in the calculation of strain energy due to shear.

2.4.2 Non-Rectangular Sections

In applying Equation (2.10), the assumption has been made that the horizontal component of shear is equal to zero and the vertical component evenly distributed over the width of the section. In beams of triangular or circular sections, a uniform stress distribution cannot be assumed, since at any point on the boundary of the cross-section, the shear stress must be parallel to the boundary and on the vertical line of symmetry parallel to the vertical shear force V, Fig. (2.6).

The tangential shearing stress q_1 at points 1 and 2 of the cross-section, will be directed towards point 0 on the vertical line of symmetry of the cross-section, Fig. (2.7). It is generally assumed that the shearing stresses at all points on the layer A-A are also directed towards point 0 and that the vertical components of these stresses are all equal and uniformly distributed over the section.

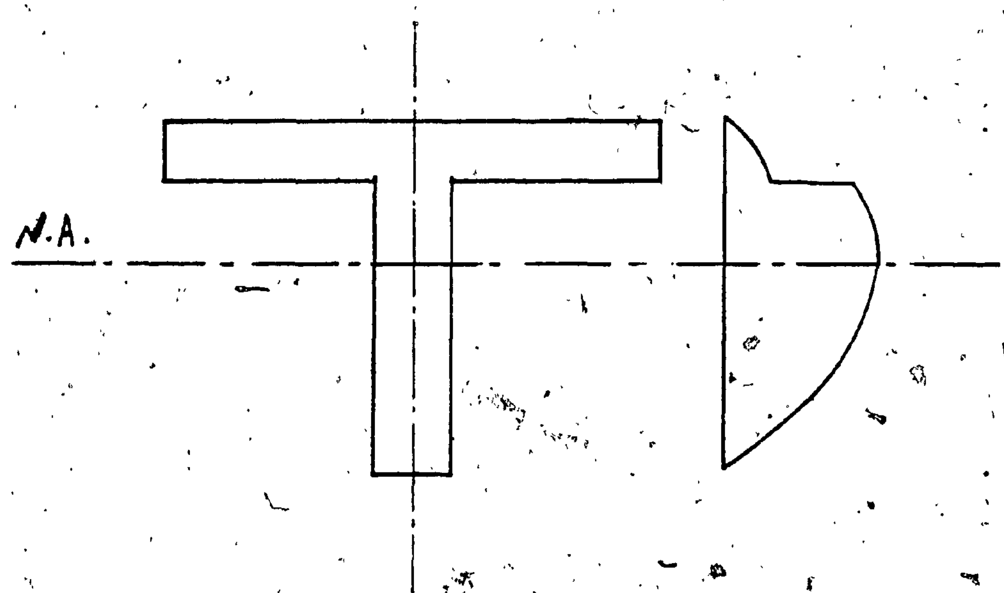

FIG. 2.4 SHEAR STRESS DISTRIBUTION IN T-SECTION

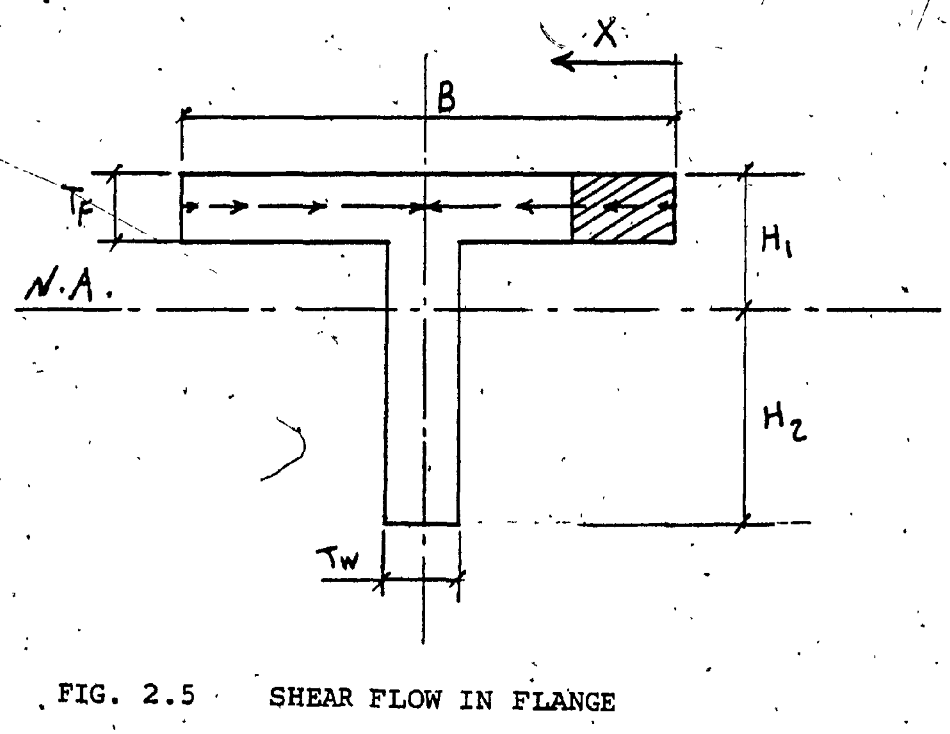

FIG. 2.5 SHEAR FLOW IN FLANGE

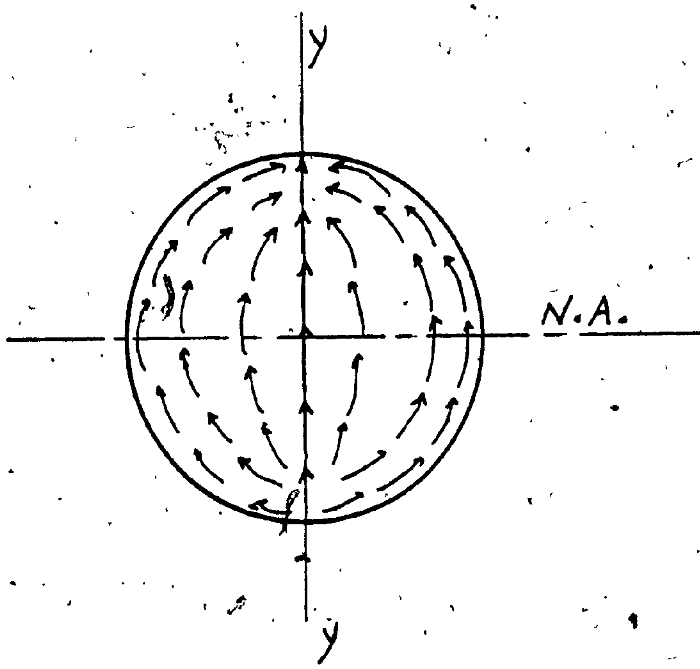

FIG. 2.6 STRESS DISTRIBUTION IN A CIRCULAR SECTION

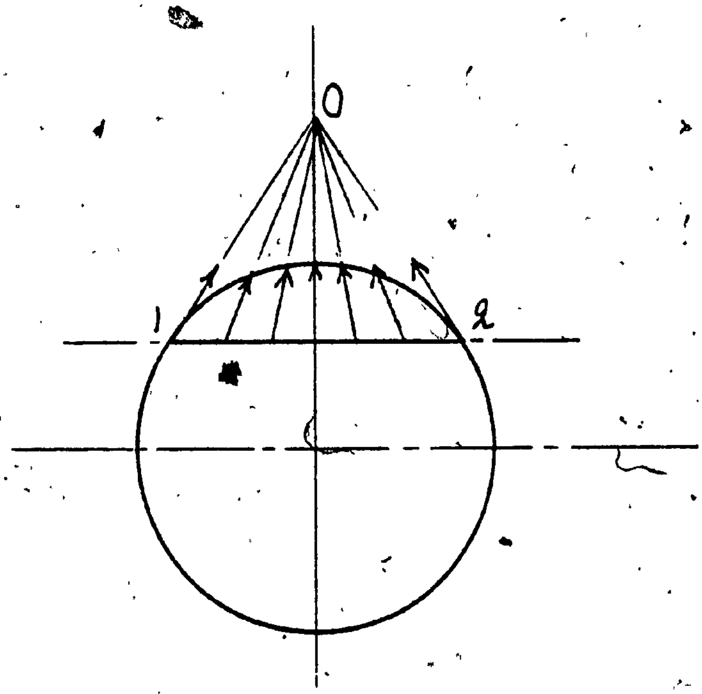

FIG. 2.7 STRESS DISTRIBUTION ON A TRANSVERSE LAYER IN CIRCULAR SECTION

Similarly, for a triangle considering layer A-A, shear stresses are directed towards the head of the triangle (Fig.2.8). We derive an expression for the distribution of the vertical shear stress across a circular section of diameter 2R, as shown in Fig. 2.9, in which

$$x = R\cos(\theta)$$

$$y = R\sin(\theta)$$

$$\frac{dy}{d\theta} = R\cos(\theta)$$

$$dy = R\cos(\theta)d\theta$$

The area of the elemental strip at a distance y from the neutral axis x-x becomes

$$dA = 2xdy = 2R\cos(\theta)[R\cos(\theta)]d\theta$$

$$dA = 2R^2\cos^2(\theta)d\theta$$

$$\tau = \frac{VQ}{Ib} = \frac{V}{Ib}\int ydA = \frac{V}{2R\cos(\theta_1)}\,\frac{4}{\pi R^4}\int_{\theta_1}^{\pi/2} R\sin(\theta)\,.$$

$$.\ [2R^2\cos^2(\theta)]d\theta \qquad (2.11)$$

We note that

$$\frac{d\cos(\theta)}{d\theta} = -\sin(\theta)$$

$$d\cos(\theta) = -\sin(\theta)d\theta$$

Substituting this in Equation (2.11) gives

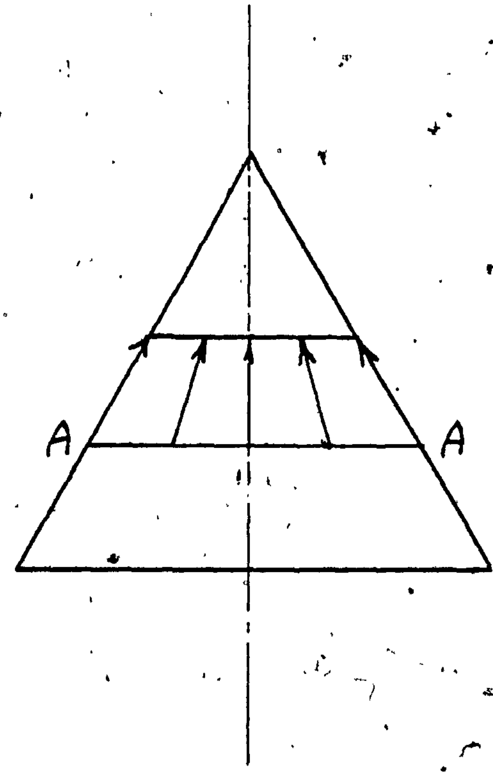

FIG. 2.8 STRESS DISTRIBUTION IN A TRIANGULAR SECTION

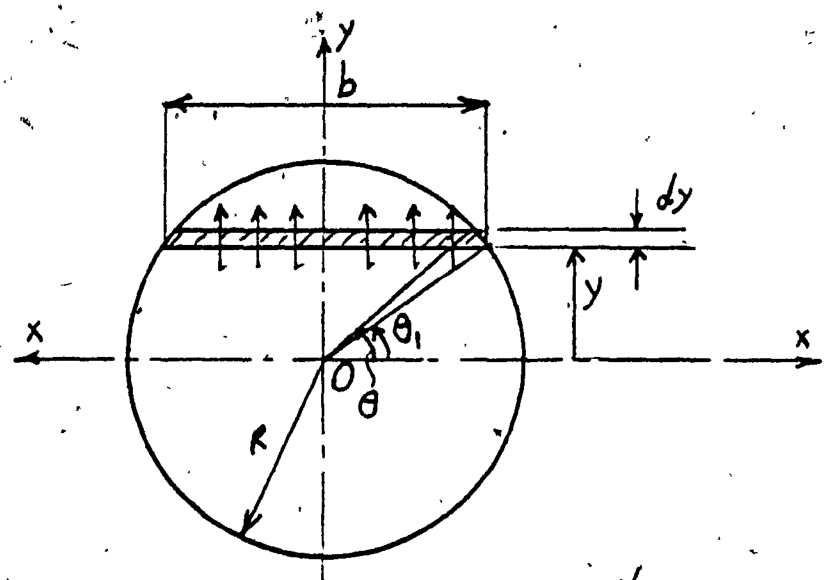

FIG. 2.9 COORDINATES FOR CIRCULAR SECTION

$$\tau = \frac{4V}{\pi R^2 \cos(\theta_1)} \int_{\theta_1}^{\pi/2} (-\cos^2(\theta) d \cos(\theta) =$$

$$= \frac{4V}{\pi R^2 \cos(\theta_1)} \left[- \frac{\cos^3(\theta)}{3}\right]_{\theta_1}^{\pi/2} =$$

$$= \frac{4V}{\pi R^2 \cos(\theta_1)} \left[0 + \frac{\cos^3(\theta_1)}{3}\right] = \frac{4V}{\pi R^2} \cdot \frac{\cos^2(\theta_1)}{3} \quad (2.12)$$

Equation (2.12) gives the shear stress for any section, y distant from the neutral axis. The maximum value obviously occurs on the neutral axes when $\theta_1 = 0$ which gives:

$$\tau_{max} = \left(\frac{4}{3}\right) \frac{V}{\pi R^2} = \left(\frac{4}{3}\right) \frac{V}{A} \quad (2.13)$$

For a thin ring, the shear stress distribution can be assumed at any point perpendicular to the radius or in a circumferential direction. Considering Figure 2.10, and the relation (3.7) derived in Chapter III:

$$Q = [A]\bar{y}$$

$$Q = \left(\frac{2}{3}\right) \sin(\theta) [R_2^3 - R_1^3]$$

$$\tau = \frac{VQ}{Ib} = \frac{\left(\frac{2}{3}\right) \sin(\theta) [R_2^3 - R_1^3] V}{\frac{\pi (R_2^4 - R_1^4)}{4} \, 2(R_2 - R_1)}$$

After simplification, this gives:

$$\tau = \frac{\left(\frac{4}{3}\right) \sin(\theta)}{t \times 2R_{av} \times \pi} \left[\frac{R_1^2 + R_1 R_2 + R_2^2}{R_2^2 + R_1^2}\right] V \quad (2.14)$$

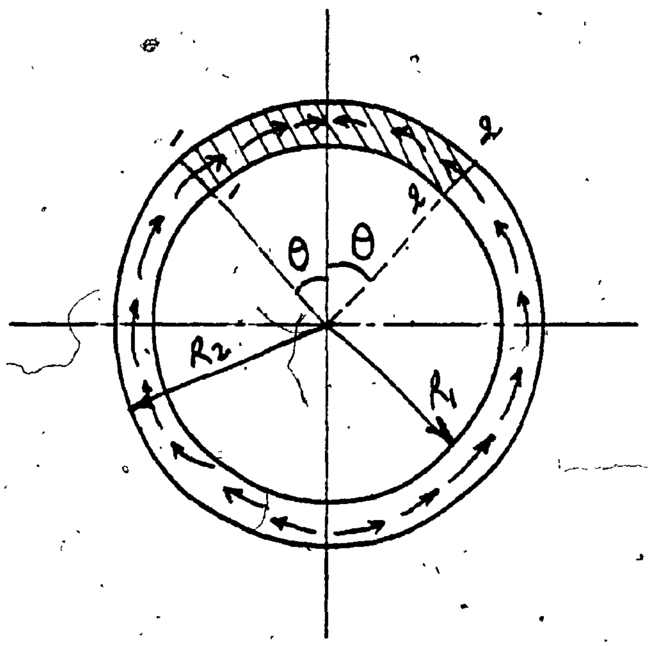

FIG. 2.10 SHEAR STRESS DISTRIBUTION IN A RING

where

$$R_{av} = \frac{R_1+R_2}{2}$$

and $t = R_2 - R_1$

The expression within the brackets tends to $\pi/2$, the thinner we make the ring, so we have:

$$\tau = \frac{V}{R_{av} \times t \times \pi} \sin(\theta) \qquad (2.15)$$

For $\theta = \pi/2$, i.e., on the neutral axis

$$\tau_{max} = \frac{V}{R_{av} \times t \times \pi} = (2)\ \frac{V}{A} \qquad (2.16)$$

2.5 DISTRIBUTION IN HEXAGON

We study the shear stress distribution in a hexagonal section by solving an example.

According to the expression (3.28), the first moment of the area of a trapezoidal section about the neutral axis is given by

$$Q(y) = \frac{2}{\sqrt{3}} \left((\tfrac{1}{4}) a^3 \sqrt{3} - (\tfrac{1}{2}) a y^2 \sqrt{3} + (\tfrac{1}{3}) y^3 \right)$$

$$\tau = \frac{VQ}{Ib} = \frac{16V}{5\sqrt{3}a^4} \left[\frac{(\tfrac{1}{4}) a^3 \sqrt{3} - (\tfrac{1}{2}) a y^2 \sqrt{3} + (\tfrac{1}{3}) y^3}{(a\sqrt{3} - y)} \right] \qquad (2.17)$$

When $y = 0$

$$\tau_0 = \frac{16V}{5\sqrt{3}a^4} (\tfrac{1}{4}a^2) = 0.461 \frac{V}{a^2}$$

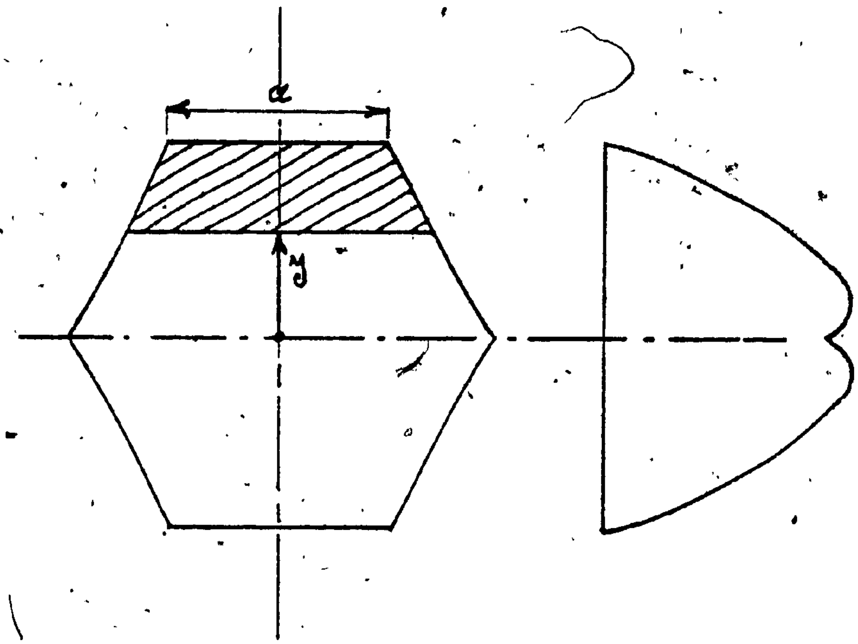

FIG. 2.11 SHEAR STRESS DISTRIBUTION IN A HEXAGONAL SECTION

$$\tau_{average} = \frac{V}{(\frac{1}{2})\,3\sqrt{3}a^2} = 0.386\ \frac{V}{a^2}$$

$$\tau_0 = \frac{0.461}{0.386}\ \tau_{average} = 1.2\ \tau_{average}$$

When $y = 0.2a$

$$\tau = 0.483\ \frac{V}{a^2}$$

As we see in this particular cross-section, the maximum shear does not occur at the neutral axis; the shear stress distribution is as shown in Figure 2.11.

TABLE 2.1 VALUE OF SHEAR STRESS ACROSS THE DEPTH OF THE HEXAGONAL SECTION

y	0	0.2a	0.4a	0.6a	0.8a	0.866a
τ	$0.461\frac{V}{a^2}$	$0.483\frac{V}{a^2}$	$0.436\frac{V}{a^2}$	$0.315\frac{V}{a^2}$	$0.097\frac{V}{a^2}$	0

CHAPTER III

DETERMINATION OF FORM FACTOR BASED ON STRAIN ENERGY

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DETERMINATION OF FORM FACTOR BASED ON STRAIN ENERGY

In this Chapter, the formula developed in the preceding chapter for the form factor will be applied to several cross-sections. The expressions developed for these sections will be solved numerically.

3.1 FORM FACTOR FOR RECTANGLE

Consider Fig.3.1:

$$Q = A\bar{y} = b(h-y)(h - \frac{(h-y)}{2})$$

or

$$Q = b/2(h^2-y^2)$$

$$F_s = \frac{A}{I^2} \int \frac{Q^2}{b^2} dA$$

$$F_s = \frac{9}{2bh^5} \int_{-h}^{h} \frac{1}{4}(h^2-y^2) dby$$

$$F_s = \frac{9b}{8bh^5} [h^4y+\frac{1}{5}y^5-\frac{2}{3}h^2y^3]_{-h}^{h}$$

$$F_s = \frac{6}{5}$$

3.2 FORM FACTOR FOR TRIANGLE

The x-axis is chosen along the neutral axis of the triangle with the y-axis positive downward as in Fig. 3.2.

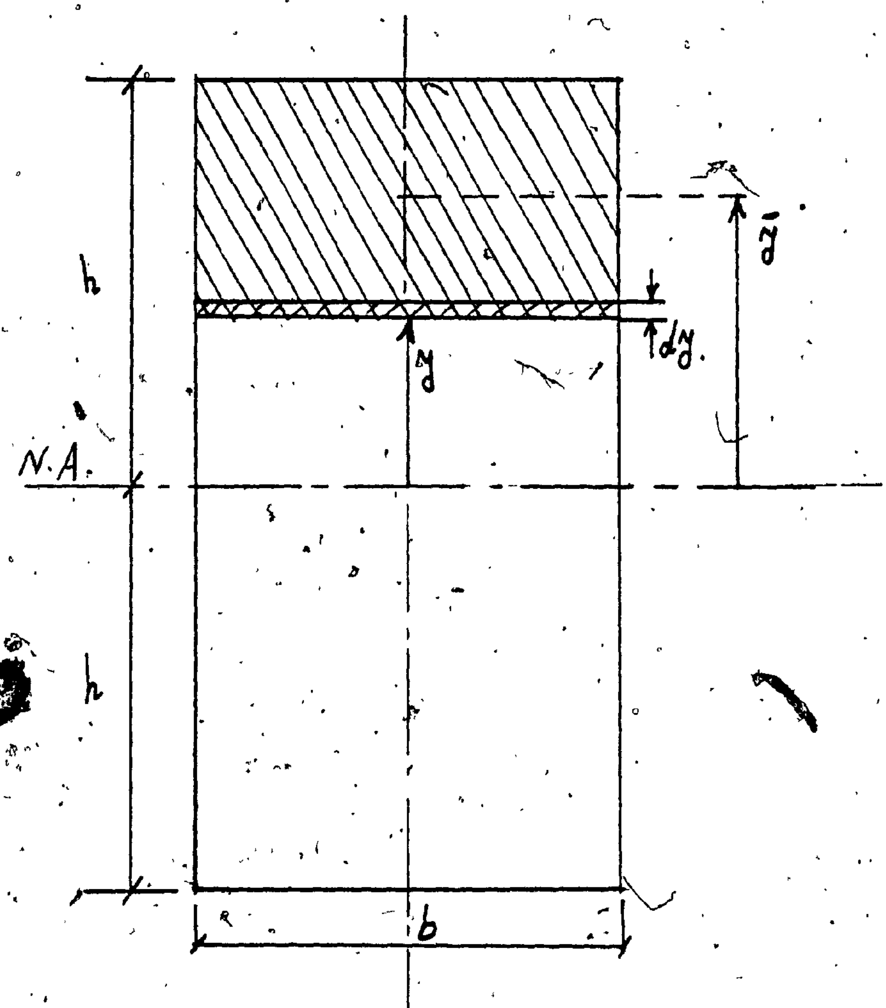


FIG. 3.1 VARIABLES FOR RECTANGULAR SECTION

From Figure 3.2:

$$Q(y) = A\overline{y} = \frac{1}{2}[L(y) \times (2h/3-y)]\overline{y}$$

where

$\overline{y}$ is the distance from the neutral axis to the center of the shaded area;

and

L(y) is the base of the shaded triangle expressed as a function of y.

$$\overline{y} = y + \frac{2h/3-y}{3}$$

Now, an expression should be found for L(y).

Assume

$$L(y) = A'y + B'$$

From an inspection of Fig. 3.2:

$$\text{Boundary conditions:} \begin{cases} L = B & y = -h/3 \\ L = 0 & y = 2h/3 \end{cases}$$

After applying the boundary conditions, we have:

$$L(y) = (-B/h)y + 2B/3$$

Thus, the final expression for Q becomes:

$$Q(y) = \frac{1}{2}\left(\frac{-B}{h}y+2B/3\right)(2h/3-y)\left(y+\frac{2h/3-y}{3}\right)$$

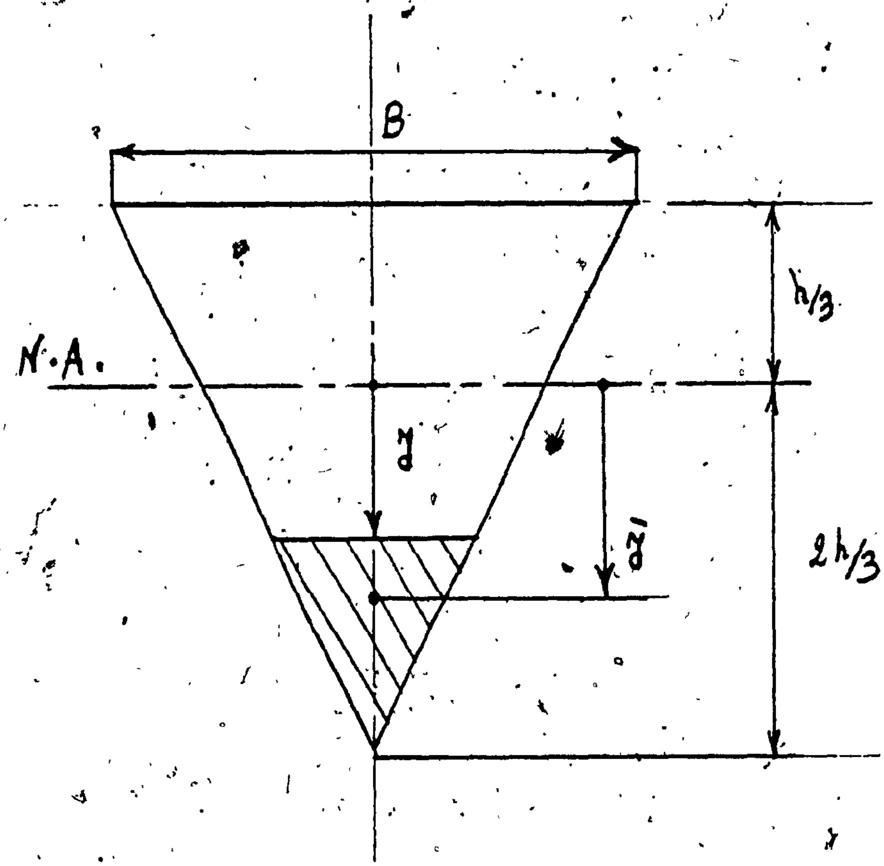


FIG. 3.2 VARIABLES FOR TRIANGULAR SECTION

$$F_s = \frac{A}{I^2} \int \frac{Q^2}{b^2}\, dA$$

$$A = \frac{1}{2} B \times h$$

$$I = \frac{Bh^3}{36}$$

$$F_s = \frac{A}{I^2} \int_{-h/3}^{2h/3} \frac{\left(\frac{1}{4}\right)\left(\frac{-B}{h}y+2\frac{B}{3}\right)^2 \left(2\frac{h}{3}-y\right)^2 \left(\frac{2y}{3}+2\frac{h}{9}\right)^2 \left(\frac{-B}{h}y+2\frac{B}{3}\right)}{\left(\frac{-B}{h}y+2\frac{B}{3}\right)^2}\, dy$$

After simplification we have:

$$F_s = \frac{A}{I^2} \int_{-h/3}^{2h/3} \left(2\frac{h}{3}-y\right)^2 \left(\frac{2y}{3}+2\frac{h}{9}\right)^2 \left(\frac{-B}{h}y+2\frac{B}{3}\right) dy \qquad (3.1)$$

If the values of an arbitrary triangle are substituted in Equation (3.1) and numerical integration is done, the value of the form factor becomes 1.2, which is independent of the dimensions of the triangle.

3.3 FORM FACTOR FOR TRAPEZOID

Consider Figure 3.3. The positive direction of the y-axis is chosen downward and the x-axis along the upper base.

According to Figure 3.4, to find the neutral axis of the trapezoid section, we have

$$\frac{x}{x + h} = \frac{t}{B}$$

$$x = \frac{ht}{B - t}$$

$$\frac{\frac{ht}{B - t}}{\frac{ht}{B - t} + h - y} = \frac{t}{L(y)}$$

$$L(y) = B - \frac{B - t}{h} y \qquad (3.2)$$

The above equation (3.2), gives the length of an elemental strip in terms of its distance from the upper base.

$$Ay_0 = \int_A y dA$$

Let

$$K = \frac{B - t}{h}$$

$$\int_0^h (By - \frac{B-t}{h}y^2) dy = y_0 (\frac{B+t}{2}h)$$

$$[\frac{By^2}{2} - \frac{K}{3}y^3]_0^h = y_0 (\frac{B+t}{2}h)$$

$$h_1 = y_0 = \frac{3Bh - 2Kh^2}{3(B+t)} \qquad (3.3)$$

After finding the position of the neutral axis, the moment of inertia about the neutral axis will be found.

From Figure 3.5, an expression may be developed for an elemental strip, with respect to its distance from the neutral axis. Assume:

$$L(y) = A'y + B'$$

with the boundary conditions $\begin{cases} L = B & y = -h_1 \\ L = t & y = h_2 \end{cases}$

After evaluating the constants A' and B' in the above equation:

$$L(y) = \frac{t-B}{h}y + \frac{th_1 + Bh_2}{h} \qquad (3.4)$$

Let:

$$M = \frac{th_1 + Bh_2}{h}$$

So:

$$L(y) = -Ky + M$$

$$I = \int_{-h_1}^{h_2} y^2 dA = \int_{-h_1}^{h_2} (-Ky+M)y^2 dy$$

$$I = \left[-\frac{K}{4}y^4 + \frac{M}{3}y^3\right]_{-h_1}^{h_2}$$

Finally:

$$I = -\frac{K}{4}h_2^4 + \frac{M}{3}h_2^3 + \frac{K}{4}h_1^4 + \frac{M}{3}h_1^3 \qquad (3.5)$$

To find the first moment of the area, considering Fig. 3.6 ,

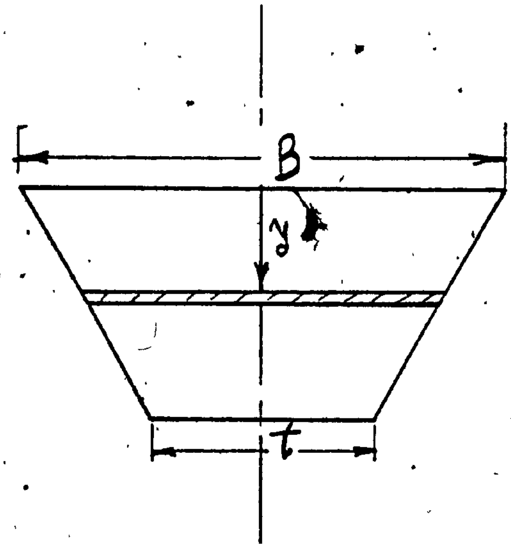


FIG. 3.3 COORDINATES FOR LOCATING NEUTRAL AXIS

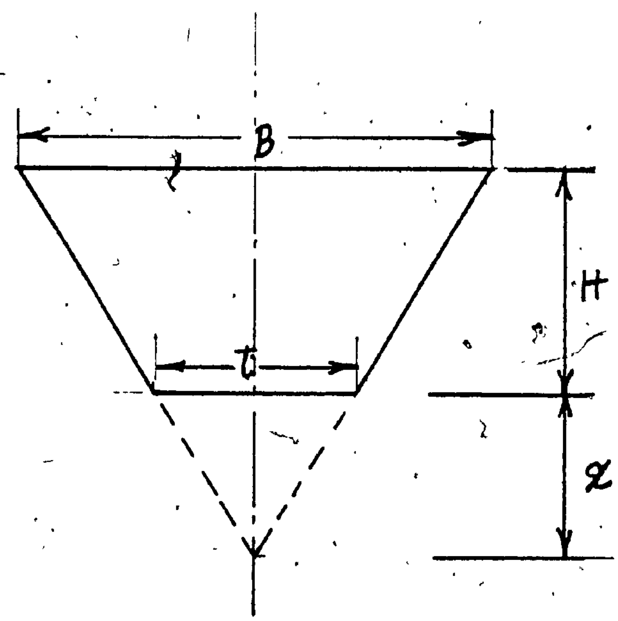


FIG. 3.4 TRAPEZOIDAL SECTION

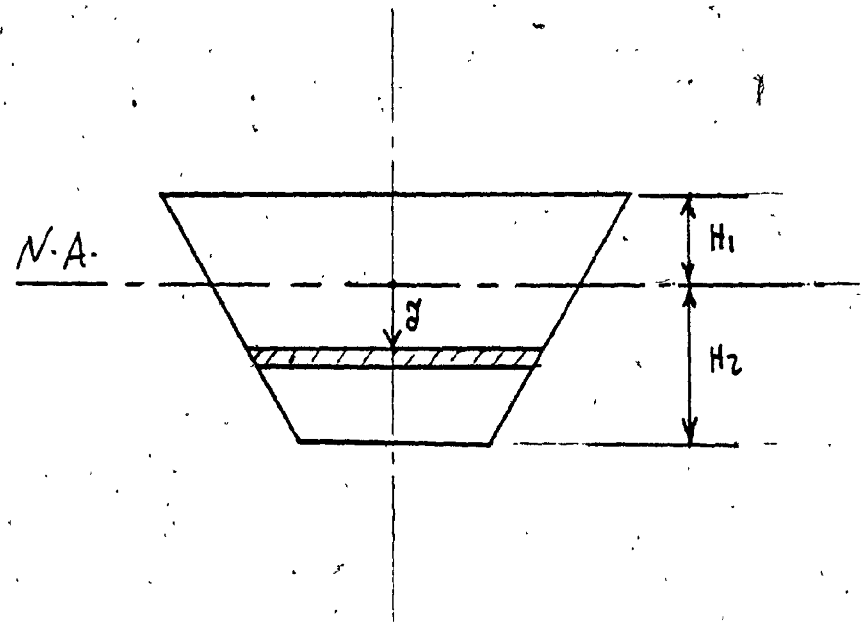


FIG. 3.5 TRAPEZOIDAL SECTION

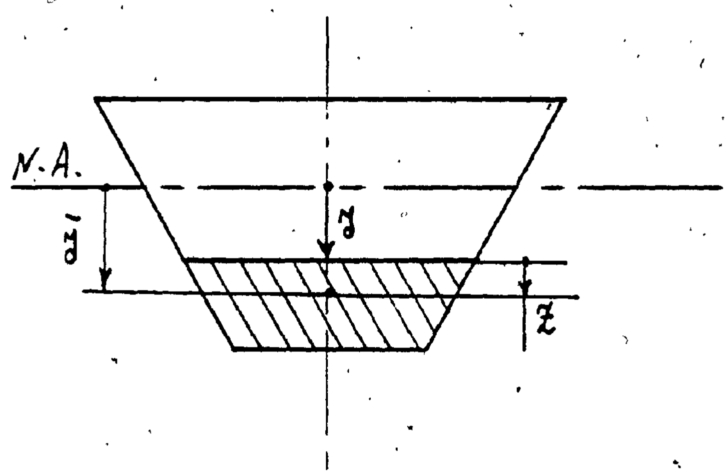


FIG. 3.6 VARIABLES FOR TRAPEZOIDAL SECTION

$$Q(y) = [A(y)]\bar{y} = [A(y)](y+Z(y))$$

in which

$$A(y) = \frac{L(y)+t}{2}(h_2-y)$$

Considering Equation (3.2):

$$Z(y) = \frac{3(-Ky+M)(h_2-y) - 2W(h_2-y)^2}{3(-Ky+M+t)} \quad (3.3)$$

in Equation (3.3)

$$W = \frac{(-Ky+M) - t}{(h_2-y)} \quad (3.4)$$

After inserting the values of K and M in Eq. (3.4) it is seen that y is eliminated and W = K.

Finally, we have:

$$Q(y) = [A(y)]\bar{y}$$

$$Q(y) = [\frac{-Ky\ M+t}{2}(h_2-y)]\ (y + Z(y))$$

$$F_s = \frac{A}{I^2} \int_{-h_1}^{h_2} \frac{[\frac{-Ky+M+t}{2}(h_2-y)]^2(y + Z(y))^2}{(-Ky+M)^2} (-Ky+M)\,dy$$

After simplification:

$$F_s = \frac{A}{I^2} \int_{-h_1}^{h_2} \frac{[(-Ky+M+t)(h_2-y)]^2(y + Z(y))^2}{4(-Ky+M)} dy \quad (3.5)$$

The above expression is integrated numerically and the results are shown in a table for various upper base to lower base ratios.

3.4 PROPERTIES OF ARC SECTION

3.4.1 Centroid of Circular Sector

Considering the circular sector of angle 2θ and choosing the coordinates as shown in Fig. 3.7, we consider elemental circular sectors which can be treated as triangles.

We have:

$$Ay_0 = \int_A y\,dA$$

$$\frac{2\theta}{2\pi}(\pi R^2)y_0 = \int_{\pi/2-\theta}^{\pi/2+\theta} \frac{1}{2} R^2 d\alpha \times \frac{2}{3} R \sin(\alpha)$$

$$R^2\theta y_0 = \frac{1}{3} R^3[-\cos(\theta)]_{\pi/2-\theta}^{\pi/2+\theta} = \frac{2}{3} R^3 \sin(\theta)$$

or

$$y_0 = \frac{2}{3} R \frac{\sin(\theta)}{\theta} \qquad (3.6)$$

3.4.2 Centroid of Arc

For an arc having outer radius of R_2 and inner radius of R_1 (Fig. 3.8), we have:

$$\Sigma Ay = Ay_0$$

$$A_2y_2 + A_1y_1 = Ay_0$$

$$A_2 = R_2^2\theta$$

$$A_1 = -R_1^2\theta$$

$$R_2^2\theta\left(\frac{2}{3}\frac{R_2\sin(\theta)}{\theta}\right) - R_1^2\theta\left(\frac{2}{3}\frac{R_1\sin(\theta)}{\theta}\right) = (R_2^2 - R_1^2)\, y_0$$

from which

$$y_0 = \frac{\frac{2}{3}\frac{\sin(\theta)}{\theta}(R_2^3 - R_1^3)}{(R_2^2 - R_1^2)} \qquad (3.7)$$

3.4.3 Moment of Inertia of Arc About Its Neutral Axes

First, we find the moment of inertia about the origin 0 for a circular sector of angle 2θ (Fig.3.9).

$$I = \int y^2 dA$$

$$I_x = \int_{\pi/2-\theta}^{\pi/2+\theta}\int_0^R (R_0 \sin(\alpha))^2 R_0\, dR_0 d\alpha$$

$$I_x = \int_{\pi/2-\theta}^{\pi/2+\theta} \frac{R^4 \sin^2(\alpha)}{4} d\alpha = \frac{R^4}{8}\left[\alpha - \frac{\sin(2\alpha)}{2}\right]_{\pi/2-\theta}^{\pi/2+\theta}$$

$$I_x = \frac{R^4}{8}[2\theta+\sin(2\theta)] \qquad (3.8)$$

So, for an arc of outer radius of R_2 and inner radius of R_1 one has:

$$I_x = I_{x_2} - I_{x_1} = \frac{1}{8}(R_2^4 - R_1^4)(2\theta+\sin(2\theta))$$

$$I_0 = I_x - Ay_0^2 = \frac{1}{8}(R_2^4 - R_1^4)(2\theta+\sin(2\theta)) -$$

$$- (R_2^2 - R_1^2)\theta\left[\frac{\frac{4}{9}\frac{\sin^2(\theta)}{\theta^2}(R_2^3 - R_1^3)^2}{(R_2^2 - R_1^2)^2}\right]$$

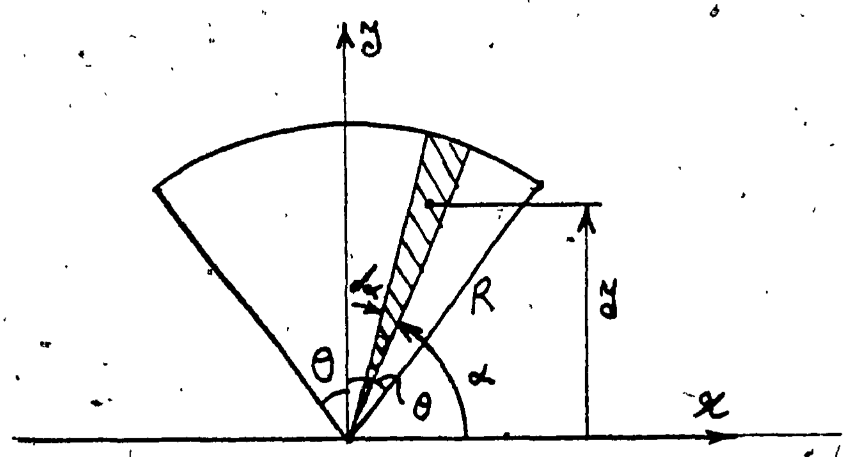


FIG. 3.7 VARIABLES FOR CIRCULAR SECTOR

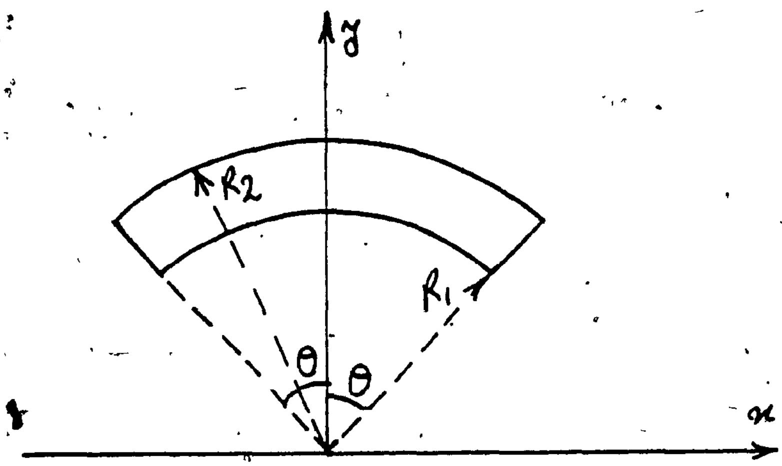


FIG. 3.8 ARC SECTION

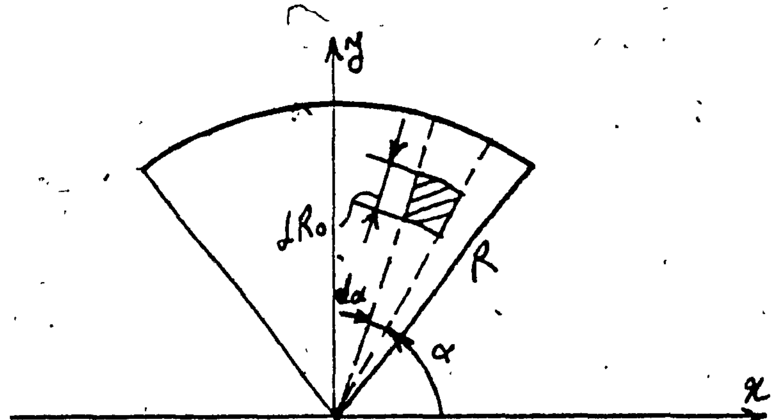


FIG. 3.9 VARIABLES FOR CIRCULAR SECTOR

or

$$I_0 = \frac{1}{8}(R_2^4 - R_1^4)(2\theta + \sin(2\theta)) - \left[\frac{\frac{4}{9}\frac{\sin^2(\theta)}{\theta}(R_2^3 - R_1^3)^2}{(R_2^2 - R_1^2)}\right] \quad (3.9)$$

where I_0 is the moment of inertia of the arc about its neutral axis.

3.5 FORM FACTOR FOR CIRCULAR SECTOR

$$F_s = \frac{A}{I^2} \int \frac{Q^2}{b^2}$$

$$A = \theta R^2$$

I can be determined using the formula (3.9) putting $R_1 = 0$.

The integration should be done in two parts:

Part (a) from 0 to $R \times \cos(\theta)$, and

Part (b) from $R \times \cos(\theta)$ to R.

Part (a)

The integration from $0 \rightarrow R \cdot \cos(\theta)$ for the section shown in Figure (3.10)

$$A = \theta R^2 - y^2 \times \tan(\theta)$$

$$Z = \frac{\theta R^2[\frac{2}{3}R\frac{\sin(\theta)}{\theta}] - y^2 \times \tan(\theta)[\frac{2}{3}y]}{\theta R^2 - y^2 \times \tan(\theta)} , \quad (3.10)$$

$$Q(y) = A(y)[Z-y_0]$$

where

y_0 is given by the formula (3.7) putting $R_1 = 0$,

and

Z is given by Equation (3.10).

$$X_1 = \int_0^{R\times\cos(\theta)} \frac{(Z-y_0)A^2(y^2)}{(2y\tan(\theta))^2}(2y\tan(\theta))dy \quad (3.11)$$

Part (b)

For this part, referring to Fig. (3.11)

$$A(y) = \alpha R^2 - y^2 \times \tan(\alpha)$$

$$Z = \frac{\alpha R^2[\frac{2}{3}R\frac{\sin(\alpha)}{\alpha}] - y^2 \times \tan(\alpha)[\frac{2}{3}y]}{\alpha R^2 - y^2 \tan(\alpha)} \quad (3.12)$$

where

$$\alpha = \cos^{-1}(\frac{y}{R})$$

$$X_2 = \int_{R\times\cos(\theta)}^{R} \frac{A^2(y)(Z-y_0)^2}{2y\tan(\alpha)} dy$$

At $\alpha = \frac{\pi}{2}$ or $y = 0$, there is a singular point in the integrand.

To remove singularity one has:

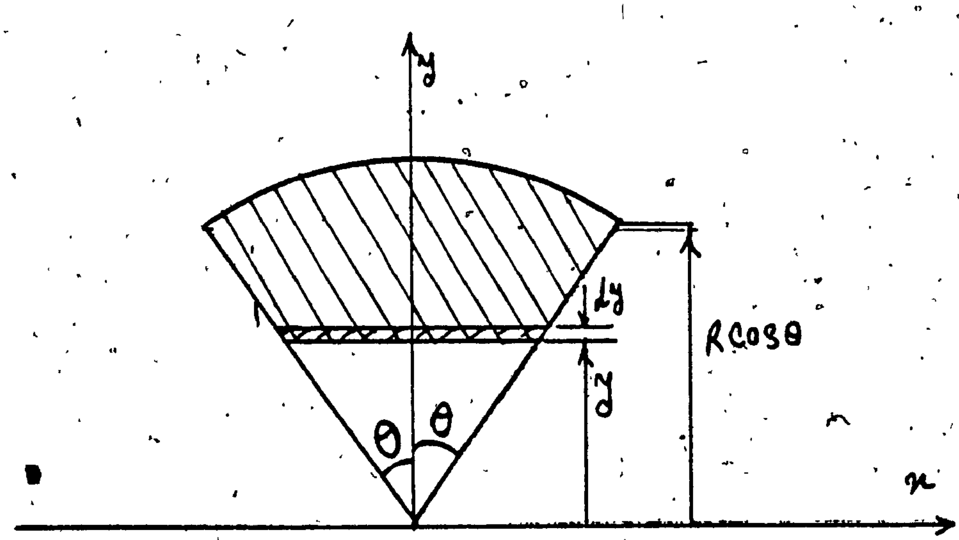


FIG. 3.10 VARIABLES FOR CALCULATION OF FORM FACTOR

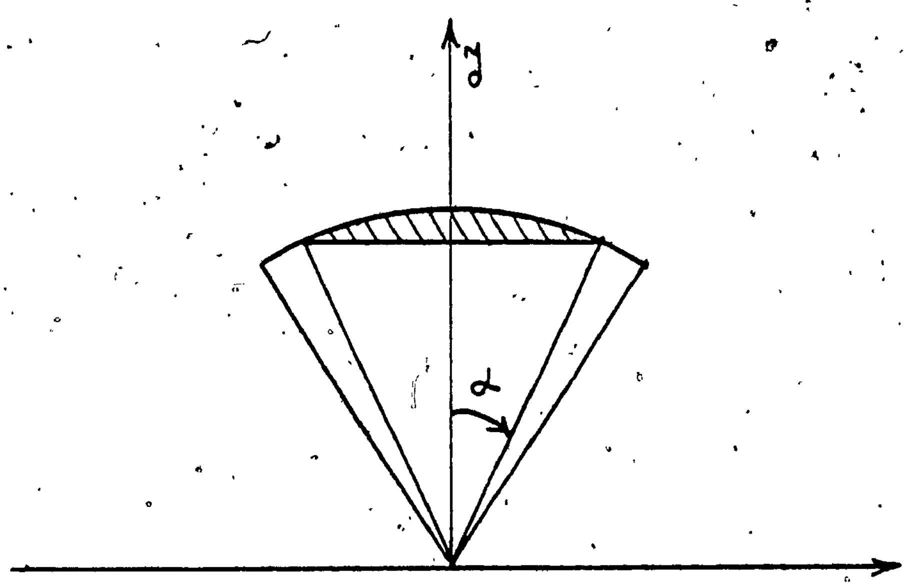


FIG. 3.11 VARIABLES FOR CALCULATION OF FORM FACTOR

$$\cos(\alpha) = \frac{y}{R}$$

$$\cos^2(\alpha) = \frac{1}{1 + \tan^2(\alpha)}$$

$$\tan(\alpha) = \sqrt{R^2/y^2 - 1}$$

$$y \times \tan(\alpha) = \sqrt{R^2 - y^2} = R \times \sqrt{1 - y^2/R^2}$$

So we have

$$X_2 = \int_{R\cos(\theta)}^{R} \frac{A^2(y)\,(Z-y_0)^2}{2R\sqrt{1-y^2/R^2}}\,dy \qquad (3.13)$$

where

Z is given by Eq. (3.12)

$$F_s = \frac{A}{I^2}\,(X_1+X_2)$$

For the solid circle $R_1 = 0$ and $\theta = \pi$ and $x_1 = 0$ the expression (3.11) is integrated numerically and the value of 10/9, which is the correct value for the circle, is obtained. Next, values of F_s for the circular sectors of different angles ($\theta \le \pi/2$) are obtained and are shown in the graph in the next Chapter.

3.6 FORM FACTOR FOR ARC

Considering the shear stress distribution in the ring in Chapter II, we first derive the form factor for a circular ring. For the shaded area, (Fig. 3.12), using Eq. (3.7)

$$Q = A\bar{y} = \frac{2}{3}\sin(\alpha)(R_2^3-R_1^3)$$

$$F_s = \frac{2A}{I^2}\int_0^{\pi/2}\frac{\frac{4}{9}\sin^2(\alpha)[R_2^3-R_1^3]^2}{4(R_2-R_1)^2}(R_2^2-R_1^2)d\alpha$$

$$F_s = \frac{2\pi(R_2^2-R_1^2)^2(R_2^3-R_1^3)^2}{\frac{\pi^2}{16}(R_2^4-R_1^4)^2(R_2-R_1)^2\times 9\times 2}\left[\frac{\pi}{2}\right]$$

After simplification we have:

$$F_s = \frac{8}{9}\left[\frac{R_2^2 + R_1R_2 + R_1^2}{R_2^2 + R_1^2}\right]^2$$

The expression in the bracket tends to 3/2 as R_1 approaches R_2 (the case of a ring), so for a thin ring:

$$F_s = \frac{8}{9}\left[\frac{3}{2}\right]^2 = 2$$

Next, we proceed to derive a formula for an arc. Considering Figure 3.13:

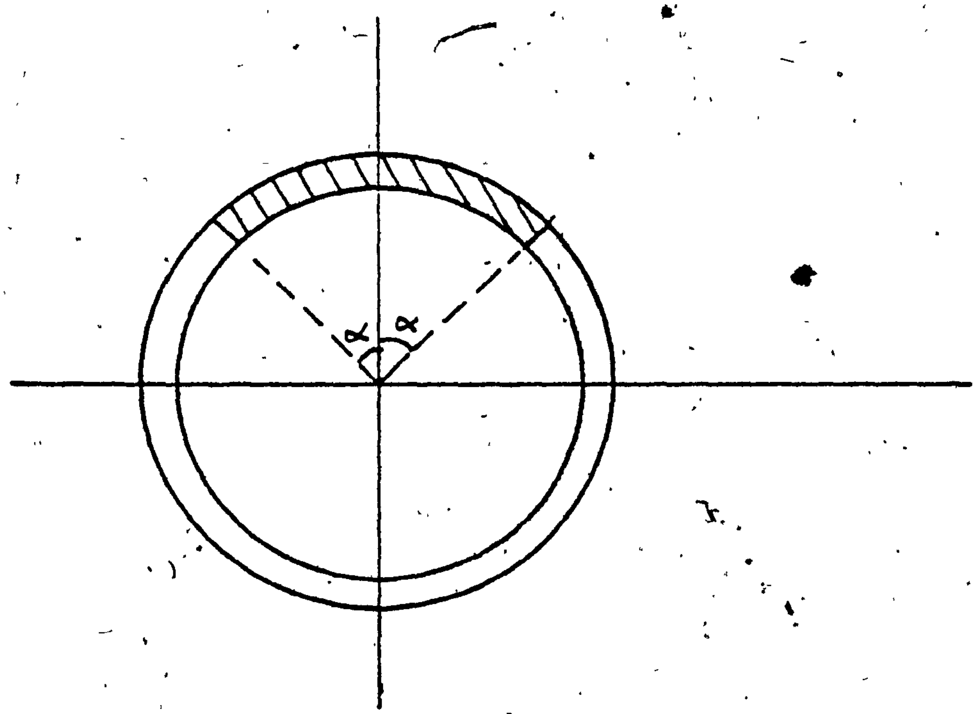


FIG. 3.12 RING SECTION

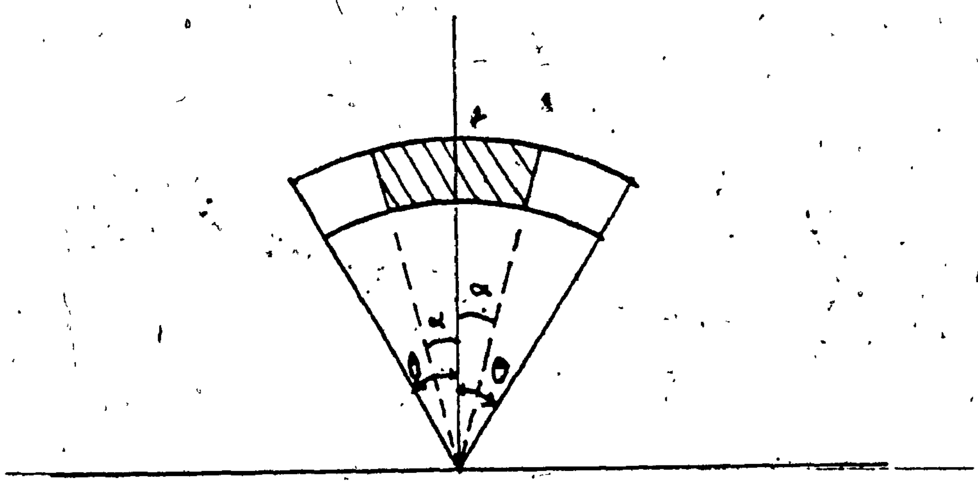


FIG. 3.13 ARC SECTION

$$A(\alpha) = \alpha(R_2^2 - R_1^2) \qquad dA = (R_2^2 - R_1^2)\,d\alpha$$

$$Z = \frac{(\frac{2}{3})\sin(\alpha)[R_2^3 - R_1^3]}{\alpha(R_2^2 - R_1^2)}$$

$$F_s = \frac{A}{I^2}\int_0^\theta \frac{A(\alpha)^2(Z-y_0)^2}{2(R_2-R_1)^2}(R_2^2 - R_1^2)\,d\alpha \qquad (3.14)$$

The above integration is evaluated numerically for $\pi/2 \leq \theta \leq \pi$, and the results are shown by means of a graph in the next Chapter.

3.7 FORM FACTOR FOR CIRCULAR SEGMENT

In calculating a form factor for a circular segment, the moment of inertia about the neutral axis will be found first.

3.7.1 Centroid of Circular Segment

Consider Fig. (3.14)

$$\Sigma A\bar{y} = Ay_0$$

$$Y_0 = \frac{\frac{2}{3}[\theta R^3\sin(\theta) - R^3\sin(\theta)\cos^2(\theta)]}{\theta R^2 - R^2\sin(\theta)\cos(\theta)} \qquad (3.15)$$

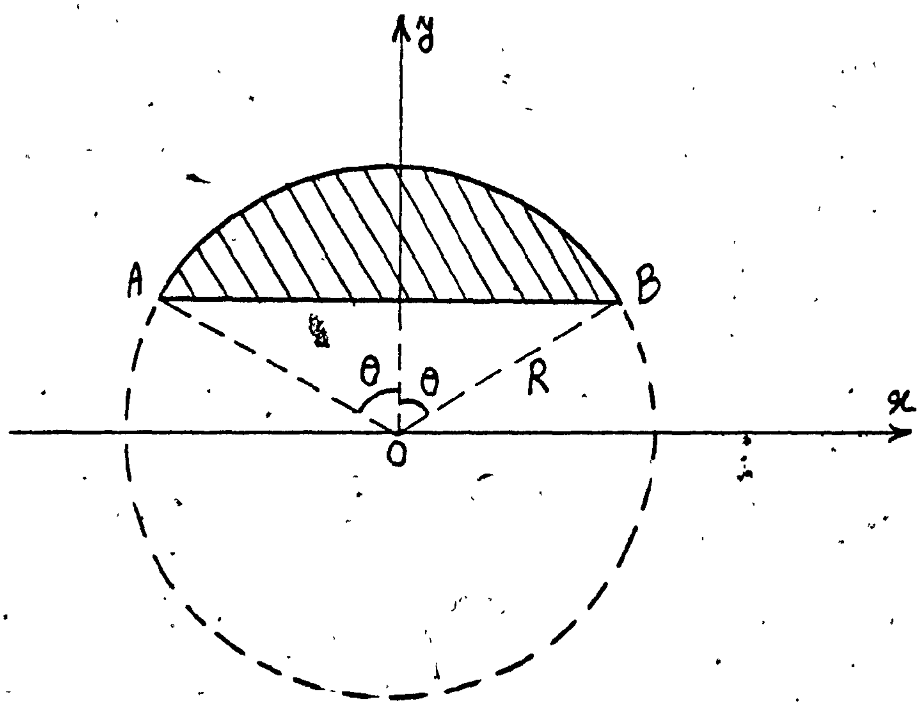


FIG. 3.14 CIRCULAR SEGMENT

3.7.2 Moment of Inertia of Circular Segment

Consider Fig. (3.14). The moment of inertia of the shaded area (circular segment) about point 0 is the moment of inertia of the circular sector AOB about point 0 minus the moment of inertia of a triangle AOB about a point 0, so:

$$I_x = \frac{R^4}{8}[2\theta+\sin(2\theta)] - R^4\sin(\theta)\cos^3(\theta)/2 \qquad (3.16)$$

$$I = I_x - y_0 A^2$$

or

$$I = I_x - [\theta R^2 - R^2\sin(\theta)\cos(\theta)]y_0^2 \qquad (3.17)$$

in which

y_0 is given by Eq.(3.15) and I is the moment of inertia of a circular segment about its neutral axis.

3.7.3 Calculation of Form Factor

Eq. (3.13) can be used to calculate the form factor.

$$F_s = \frac{A}{I^2} \int_{R\cos(\theta)}^{R} \frac{A^2(y)(Z-y_0)^2}{2R\sqrt{1 - y^2/R^2}} dy \qquad (3.18)$$

where

Z is given by Eq. (3.12), and

y_0 is given by Eq. (3.15).

Expression (3.18) is integrated numerically, and the results are shown in Chapter IV.

3.8 FORM FACTOR FOR T-SECTION

First, the location of the neutral axis with respect to the top of the flange will be found. Considering Fig.3.15, we have:

$$\Sigma Ay = Ay_0 = AH_1$$

$$H_1 = \frac{\Sigma Ay}{A} = \frac{B \times T_F \times T_F/2 + H \times T_W(T_F+H/2)}{BT_F + HT_W}$$

$$A = B \times T_F + H \times T_W$$

$$H_1 = \frac{B\,T_F^2/2 + HT_W T_F + H^2/2 \times T_W}{B \times T_F + H \times T_W} \qquad (3.19)$$

The moment of inertia with respect to the neutral axis (Fig.3.16)

$$H_2 = T_F + H - H_1$$

$$I = \frac{B \times T_F^3}{12} + B \times T_F(H_1 - T_F/2)^2 + T_W \times \frac{H^3}{12} +$$

$$+ (H_2 - H/2)^2 T_W \times H \qquad (3.20)$$

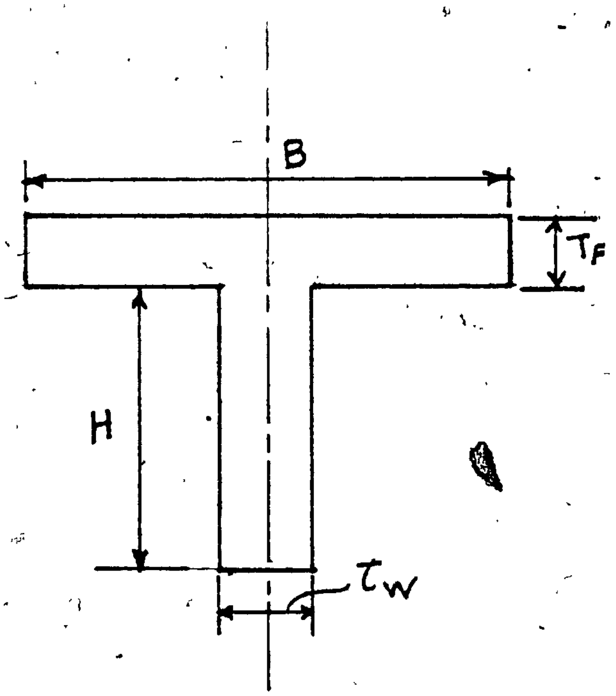


FIG. 3.15 T-SECTION

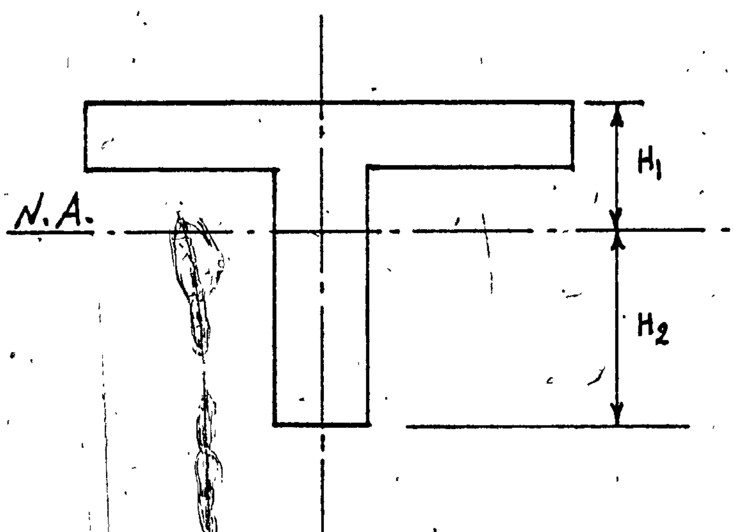


FIG. 3.16 T-SECTION

$$H_1 - T_F = K$$

Considering Eq. (3.20) and Fig. 3.17, we have

$$Z(y) = \frac{B \times T_F^2/2 + (K-y)T_W \times T_F + \frac{(k-y)^2}{2} \times T_W}{B \times T_F + (k-y)T_W} \quad (3.21)$$

$$A(y) = B \times T_F + (k-y)T_W$$

$$\overline{y} = H_1 - \overline{Z}(y)$$

$$Q_1 = [B \times T_F + (k-y)T_W](H_1 - Z(y)) \quad (3.22)$$

The above relation is valid for

$$H_1 - T_F \geq y \geq - H_2$$

For $H_1 \geq y \geq K$ we have

$$Q_1 = B/2(H_1^2 - y^2) \quad (3.23)$$

Now, the effect of the shear flow should be considered. Consider Fig. 3.18:

$$Q_2 = T_F \times y(H_1 - T_F/2) \quad (3.24)$$

Finally, we have

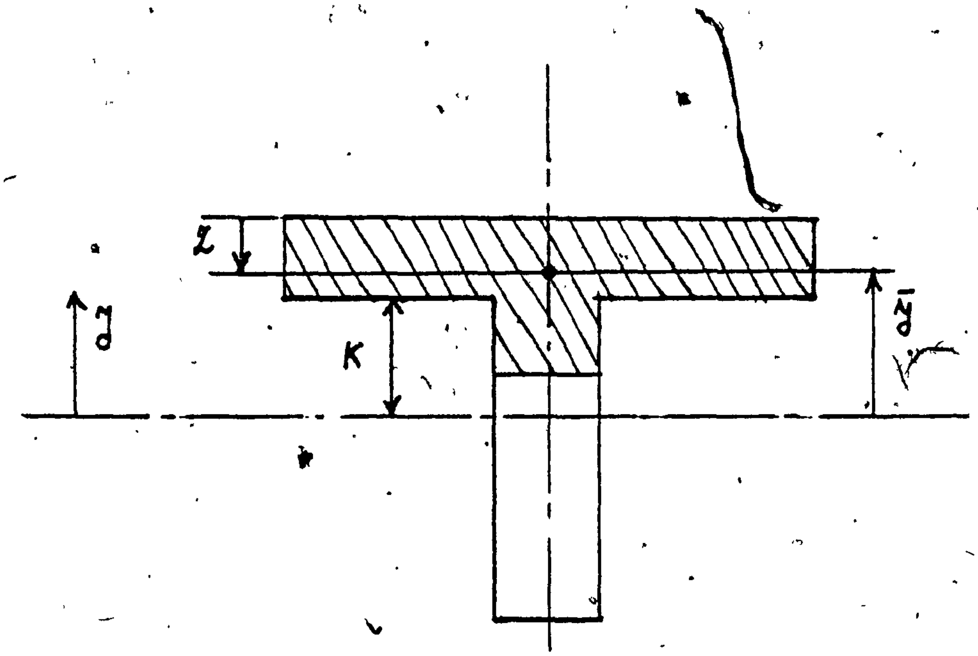


FIG. 3.17 VARIABLES FOR CALCULATION OF FORM FACTOR

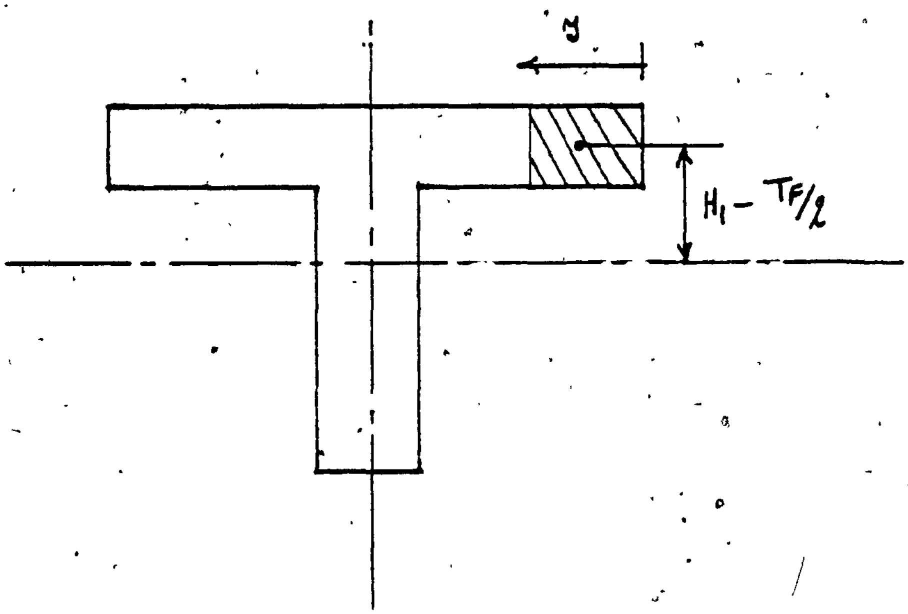


FIG. 3.18 VARIABLES FOR CALCULATION OF FORM FACTOR

$$X_1 = \int_{-H_2}^{K} \frac{[B \times T_F + (k-y)T_W]^2 (H - Z(y))^2}{T_W} dy \quad (3.25)$$

$$X_2 = \int_{K}^{H_1} \frac{(H_1^2 - y^2)}{4} Bdy \quad (3.26)$$

$$X_3 = 2\int_0^{B/2} \frac{T_F^2 \times y^2 (H_1 - T_F/2)^2}{T_F^2} T_F \times dy$$

$$X_3 = \frac{1}{12} \times T_F (H_1 - T_F/2)^2 B^3 \quad (3.27)$$

$$F_s = \frac{A}{I^2}(X_1 + X_2 + X_3)$$

3.9 FORM FACTOR FOR HEXAGON

Consider the hexagonal section of Figure 3.19, with side a.

I_0 = I of rectangle − 4 (I of shaded triangle about apex)

$$I_0 = \frac{1}{12} \times 2a(a\sqrt{3})^3 - 4[\tfrac{1}{2}a(\tfrac{1}{2}a\sqrt{3})^3 \times \tfrac{1}{4}] = \frac{5\sqrt{3}}{16} a^4$$

Consider an elemental strip of width b and thickness dy

$$dA = bdy$$

From similar triangles

$$b = \frac{2}{\sqrt{3}}(a\sqrt{3} - y)$$

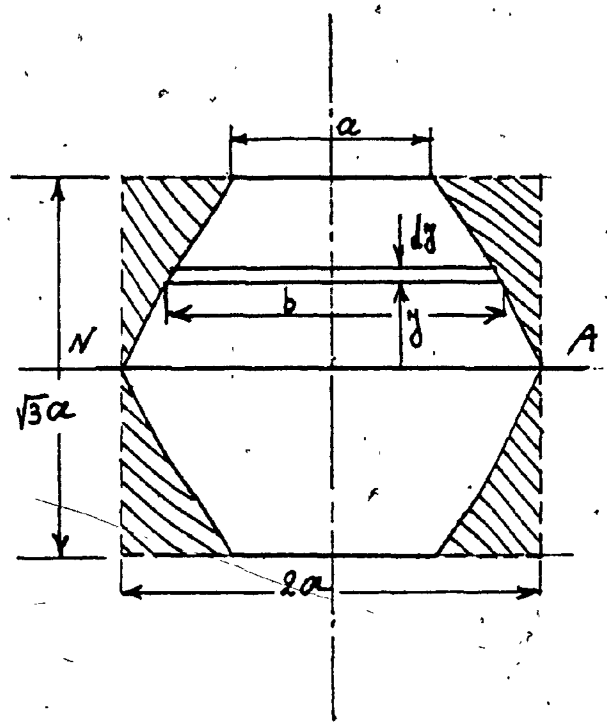


FIG. 3.19 HEXAGONAL SECTION

$$dQ = by\,dy = \frac{2}{\sqrt{3}}\, y\,(a\sqrt{3}-y)\,dy$$

$$Q(y) = \frac{2}{\sqrt{3}} \int_{y}^{\sqrt{3}a/2} (ay\sqrt{3}-y^2)\,dy$$

$$Q(y) = \frac{2}{\sqrt{3}} \left[\frac{1}{4} a^3\sqrt{3} - \frac{1}{2} ay^2\sqrt{3} + \frac{1}{3}y^3\right] \qquad (3.28)$$

Because of the symmetry of the section, twice the value obtained by integrating from zero to $\sqrt{3}\,a/2$ is used to calculate F_s.

$$\int_{-\sqrt{3}\,a/2}^{\sqrt{3}\,a/2} \frac{Q^2}{b^2}\,dA = 2\int_{0}^{\sqrt{3}\,a/2} \frac{Q^2}{b^2}\,dA$$

$$F_s = \frac{2A}{I^2} \int_{0}^{\sqrt{3}\,a/2} \frac{\left[\frac{2}{\sqrt{3}}\left(\frac{1}{4} a^3\sqrt{3} - \frac{1}{2}ay^2\sqrt{3} + \frac{1}{3}y^3\right)\right]^2}{\frac{2}{\sqrt{3}}\,(a\sqrt{3}-y)}\,dy \qquad (3.29)$$

The above integral is evaluated numerically and the results are shown in the next Chapter.

CHAPTER IV

RESULTS

CHAPTER IV

RESULTS

Equations obtained in Chapter III for a triangle, trapezoid, sector, arc, segment, T and hexagon are solved numerically and the results are shown in this Chapter.

4.1 TRIANGLE

For a triangle, the value of 1.2 was obtained independent of the geometry and dimensions of the triangle. The vertical shear stress is assumed uniformly distributed on transverse layers and the effect of the horizontal component of shear is neglected. The bigger the side angles of the triangles, the better is the approximation.

4.2 TRAPEZOID

For a trapezoid, the same assumptions as for the triangle were made and it was found that the form factor is independent of the height of the trapezoid and depends only on the ratio of the bases. It varies slightly from 1.2. For B = T, a rectangle results and the value of 1.2 is obtained. For T = 0, a triangle results and the value of 1.2 is obtained, which verifies the results obtained in Section 4.1. The highest values obtained was 1.204 for T/B = 0.2. So, 1.2 is a very good approximation for the form factor of the trapezoid. Table 4.1 shows the results obtained.

4.3 CIRCULAR SECTOR

First, to verify the formulas, the sector was changed to a full circle and the value of 10/9 which is given in the books was obtained. Then, for different angles of circular sectors, i.e., $10 \leq 2\theta \leq 180$ the form factor is found and is plotted in the Graph 4.1.

4.4 ARC

For arcs using the shear distribution as described in Section 2.3(b), first the arc was changed to a full thin ring and the value of 2 (the thinner the arc the closer the form factor approached 2) was obtained. Then, values of the form factor when $180 \leq 2\theta \leq 360$ were obtained. For $2\theta < 180$, the shear stress distribution deviates from being circumferentially distributed, as described in Section 2.3(b). Studying this distribution is beyond the scope of this report. For small values of 2θ and R_1 close to R_2, the arc section can be treated as a rectangle, assuming shear stress uniformly distributed on transverse layers and a value of 1.2 can be assumed.

In the Graph 4.2, the form factor is plotted versus the angle of the arc β and the ratio R_1/R_2 varies from 0.99 to 0.6. It is clear that the smaller the ratio R_1/R_2, the greater is the error resulting from the assumed shear distribution. However this gives a good approximation with R_1/R_2 having a value up to 0.6.

4.5 CIRCULAR SEGMENT

The results obtained for circular segments indicate that the range of variation of form factor is small for different values of D/B, as shown in Figure 4.1. The values of the form factor from $D/B = 0.5$ (that is a half-circle) to $D/B = 0.022$ is shown in Table 4.2.

4.6 T-SECTION

In a T-section, the value of the form factor varies widely, according to the dimension ratios of the section. We have four dimensions in a T-section, so we have three independent dimension ratios, namely B/T_W, T_F/D, D/T_W. In order to obtain form factors of the most possible practical T-sections, the ranges of variation of ratios are as follows:

$$30 \geq B/T_W \geq 5$$

$$0.4 \geq T_F/D \geq 0.02$$

$$30 \geq D/T_W \geq 5$$

The values of the form factor for different ratios of B/T_W, T_F/D and D/T_W are plotted in the Graphs 4.3 to 4.10. The form factor for the ratios between those given in the graphs can be found by interpolation.

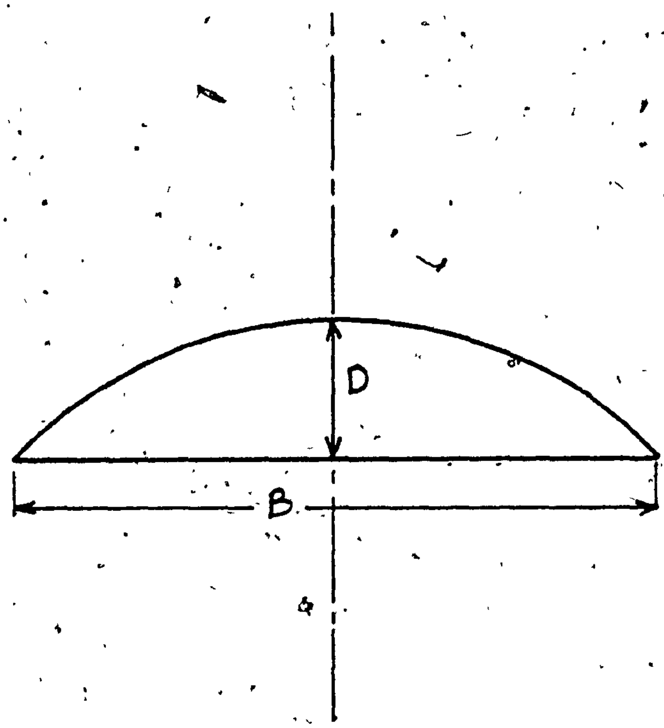


FIG.4.1 CIRCULAR SEGMENT

4.7 RESULTS FOR A HEXAGON

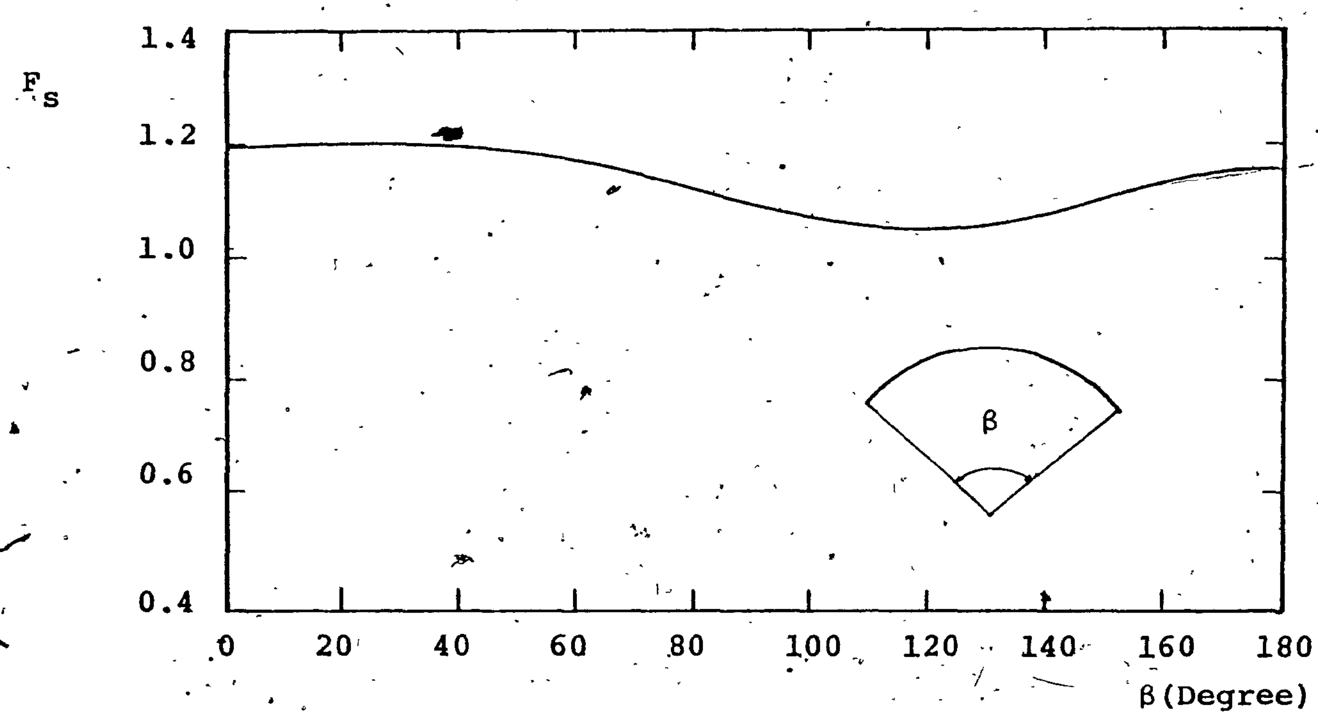
For a hexagon, the value of the form factor was estimated at 1.1097, which is very close to the value for a circle.(1.1111 ....). This is reasonable because in the limit a hexagon can be changed to a circle by increasing the number of sides.

TABLE 4.1 FORM FACTOR FOR TRAPEZOIDAL SECTION
t/B IS THE RATIO OF LOWER BASE TO
UPPER BASE

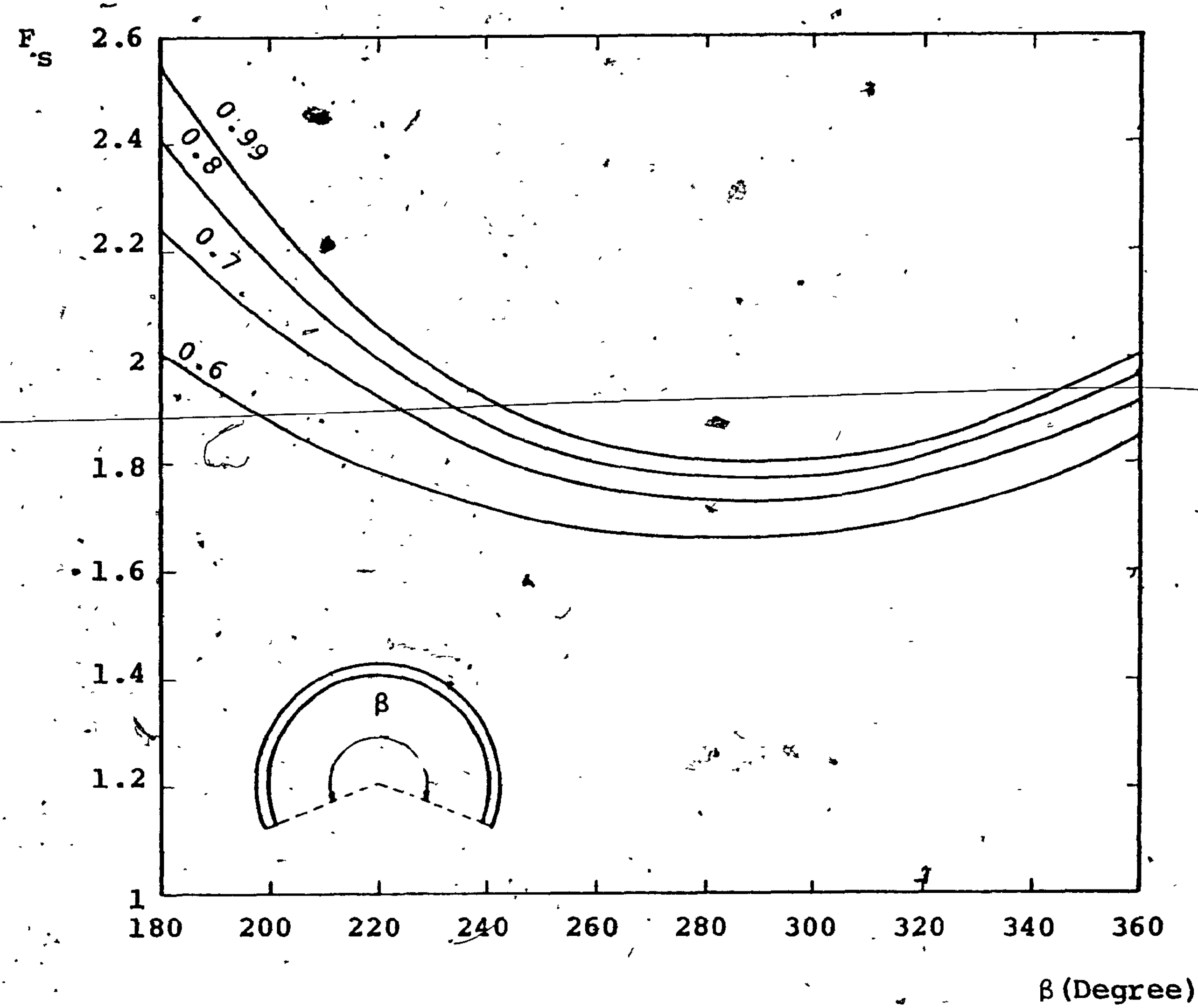
t/B	F_s
1.00	1.200
0.95	1.200
0.90	1.200
0.8	1.200
0.7	1.201
0.6	1.201
0.5	1.202
0.4	1.203
0.3	1.204
0.2	1.204
0.1	1.203
0.0	1.200

TABLE 4.2 FORM FACTOR FOR CIRCULAR SEGMENT D/B IS THE RATIO OF DEPTHS TO WIDTHS

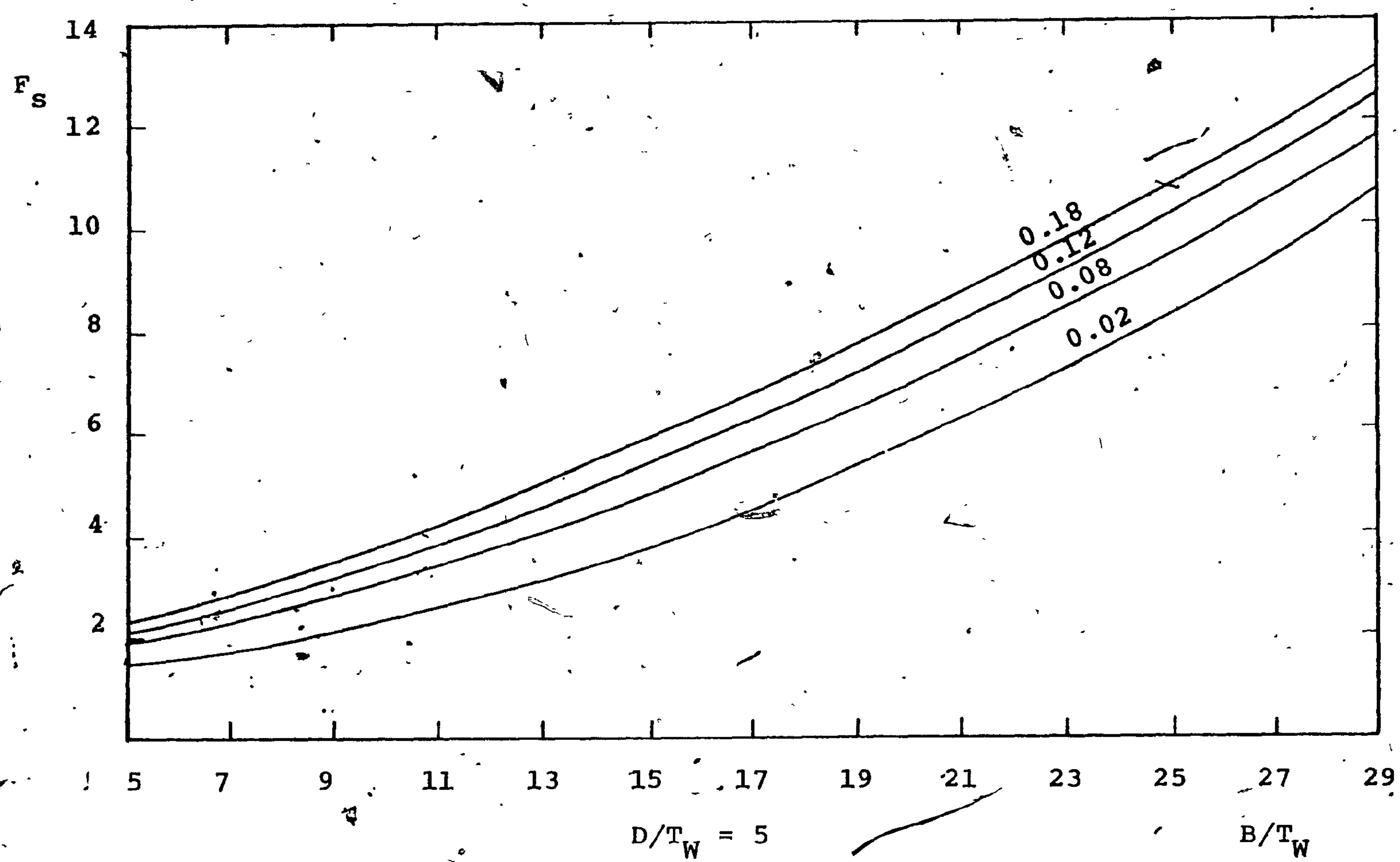
D/B	F_s
0.500	1.162
0.458	1.164
0.420	1.166
0.384	1.168
0.350	1.169
0.319	1.171
0.289	1.172
0.260	1.173
0.233	1.174
0.207	1.175
0.182	1.176
0.134	1.177
0.111	1.177
0.088	1.178
0.066	1.178
0.044	1.178
0.022	1.178



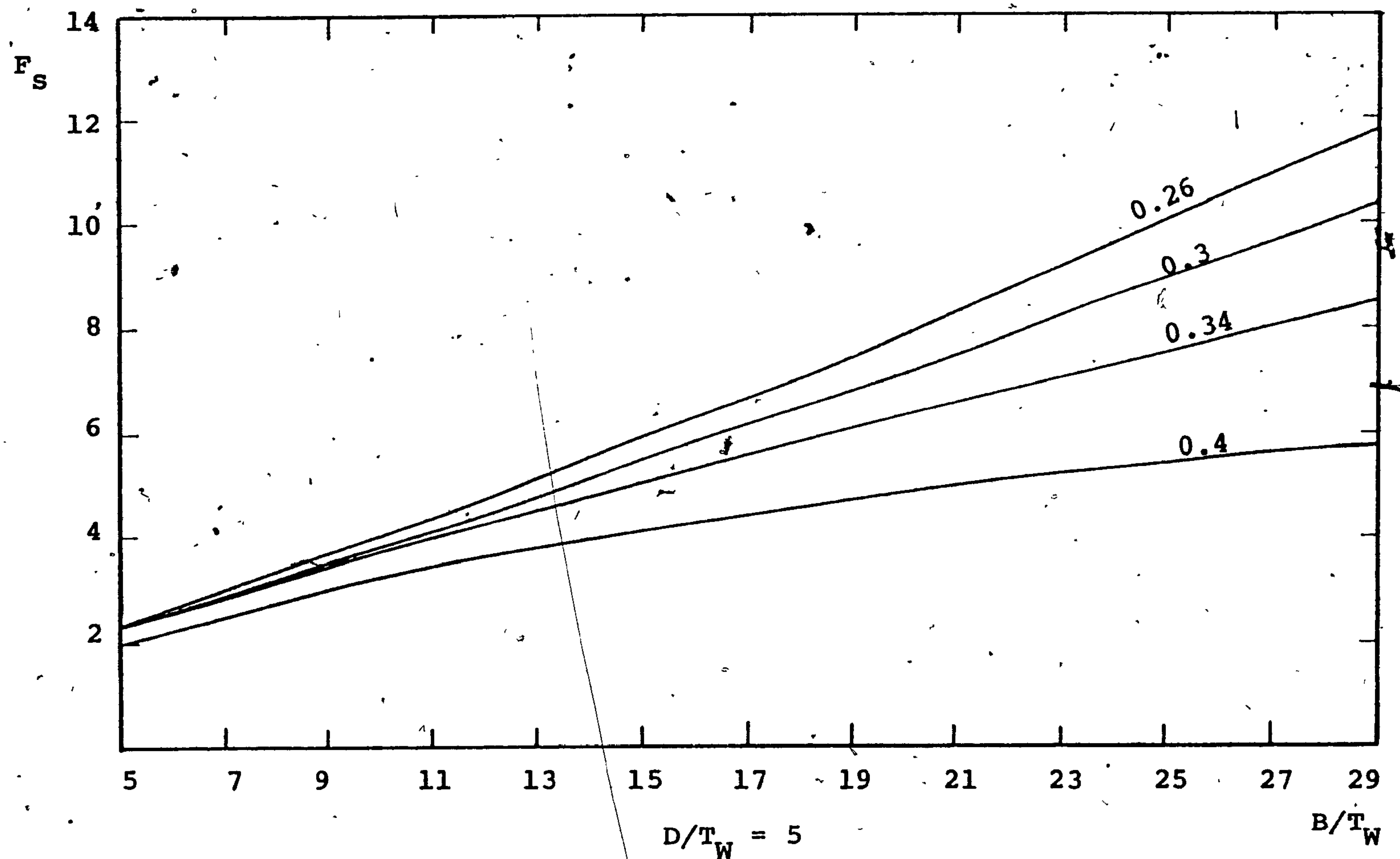
GRAPH 4.1 FORM FACTOR OF SECTOR



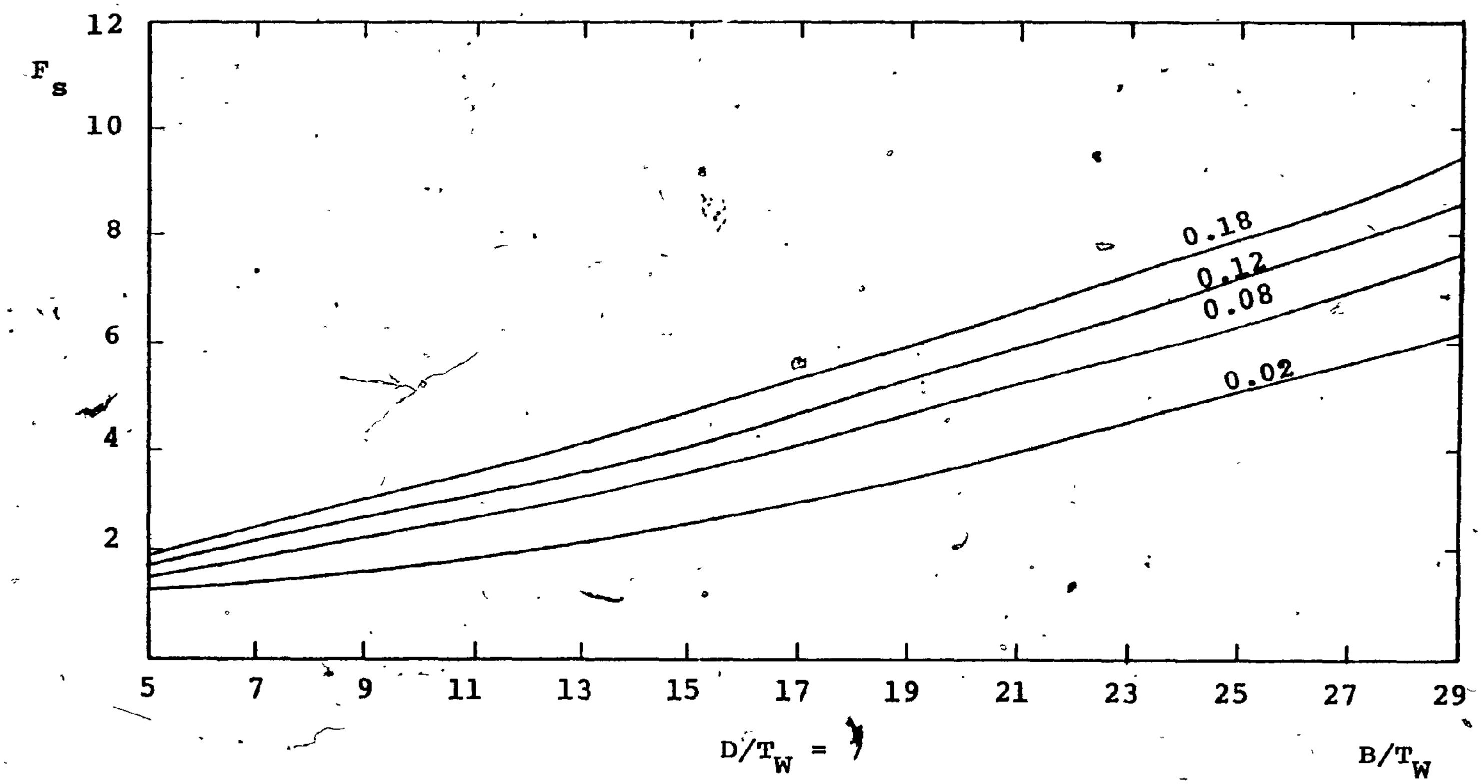
GRAPH 4.2. FORM FACTOR FOR ARC. NUMBERS ON THE CURVES ARE VALUES OF R_1/R_2



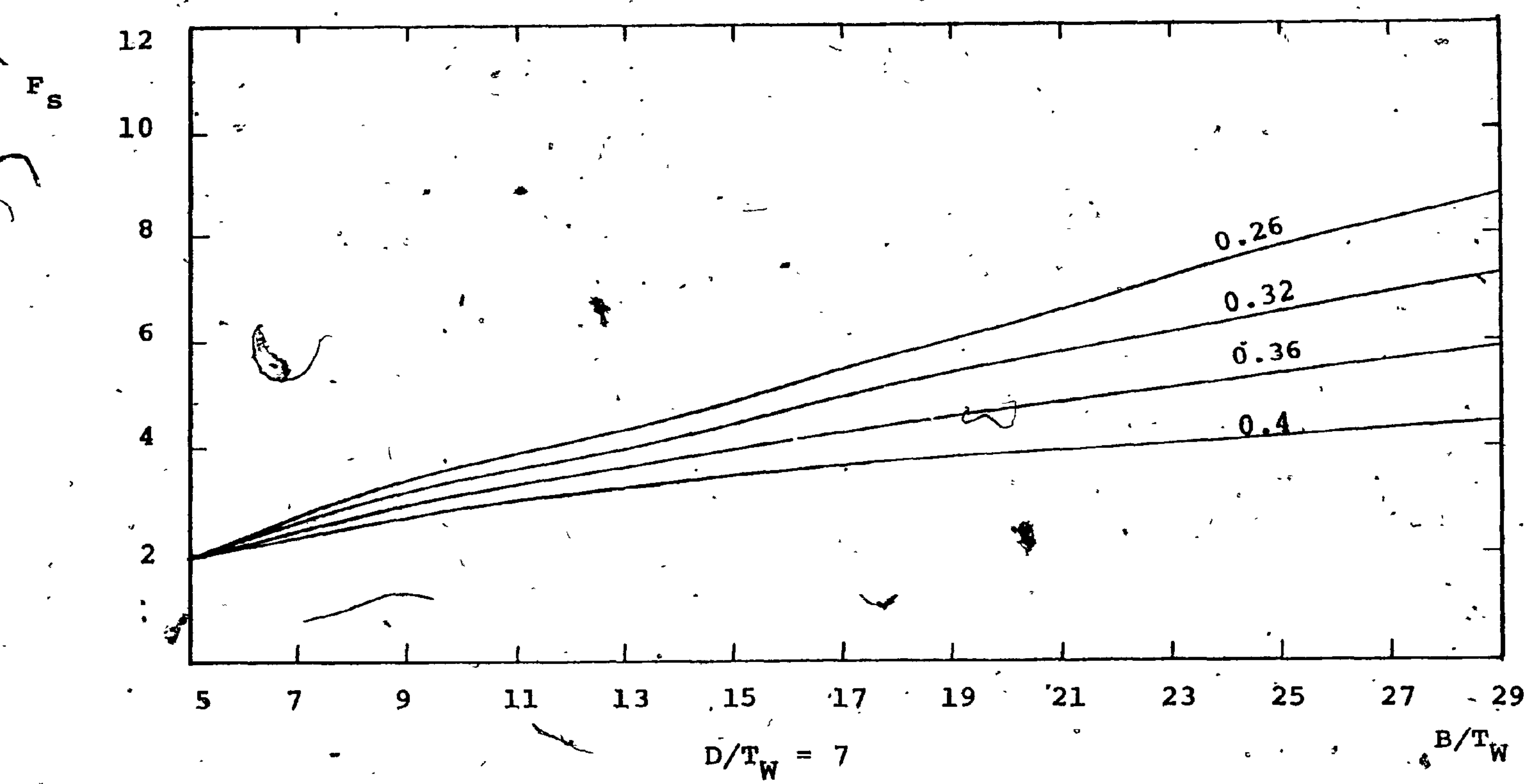
GRAPH 4.3(a) FORM FACTOR FOR T-SECTION. NUMBERS ON THE CURVES ARE THE VALUES OF T_F/D



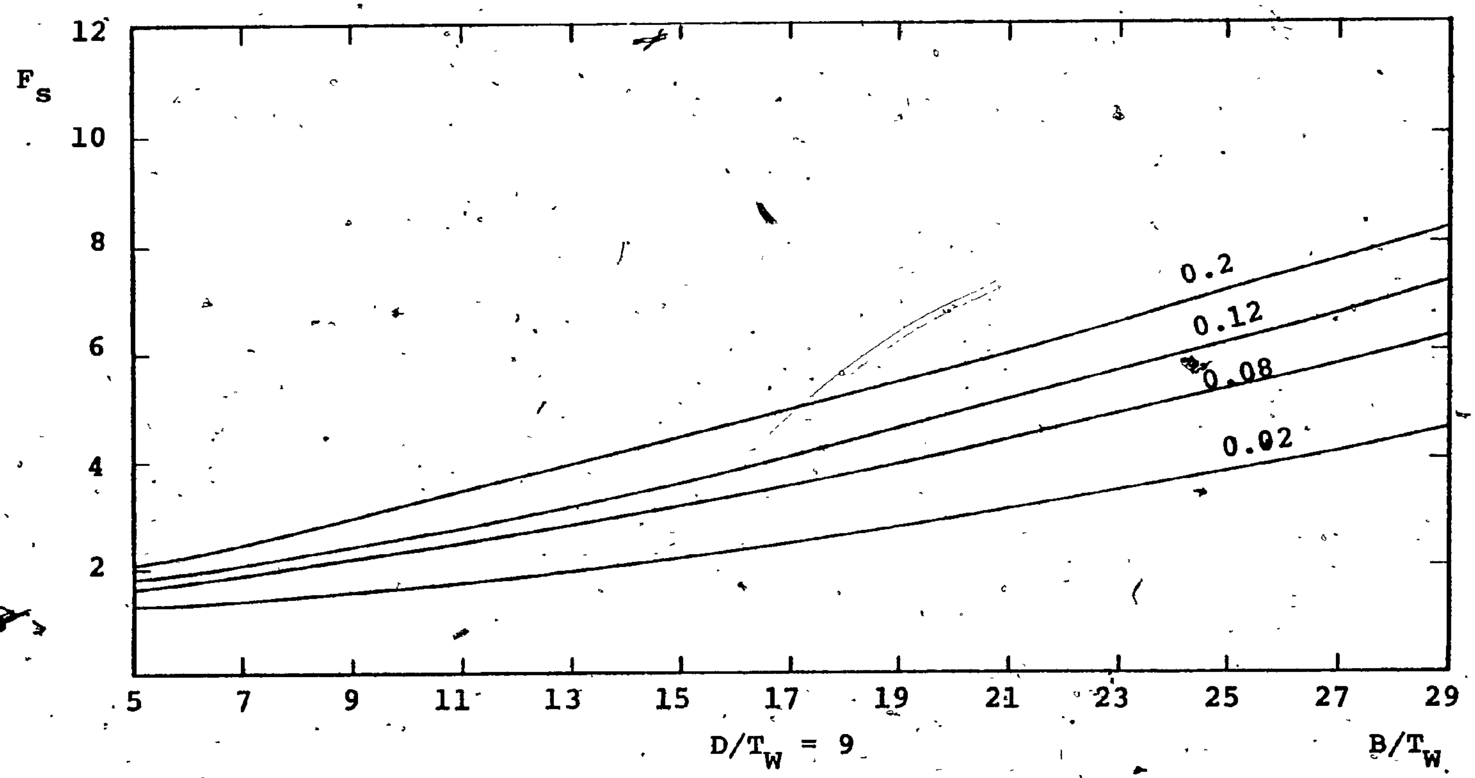
GRAPH 4.3(b) FORM FACTOR FOR T-SECTION. NUMBERS ON THE CURVES ARE THE VALUES OF T_F/D



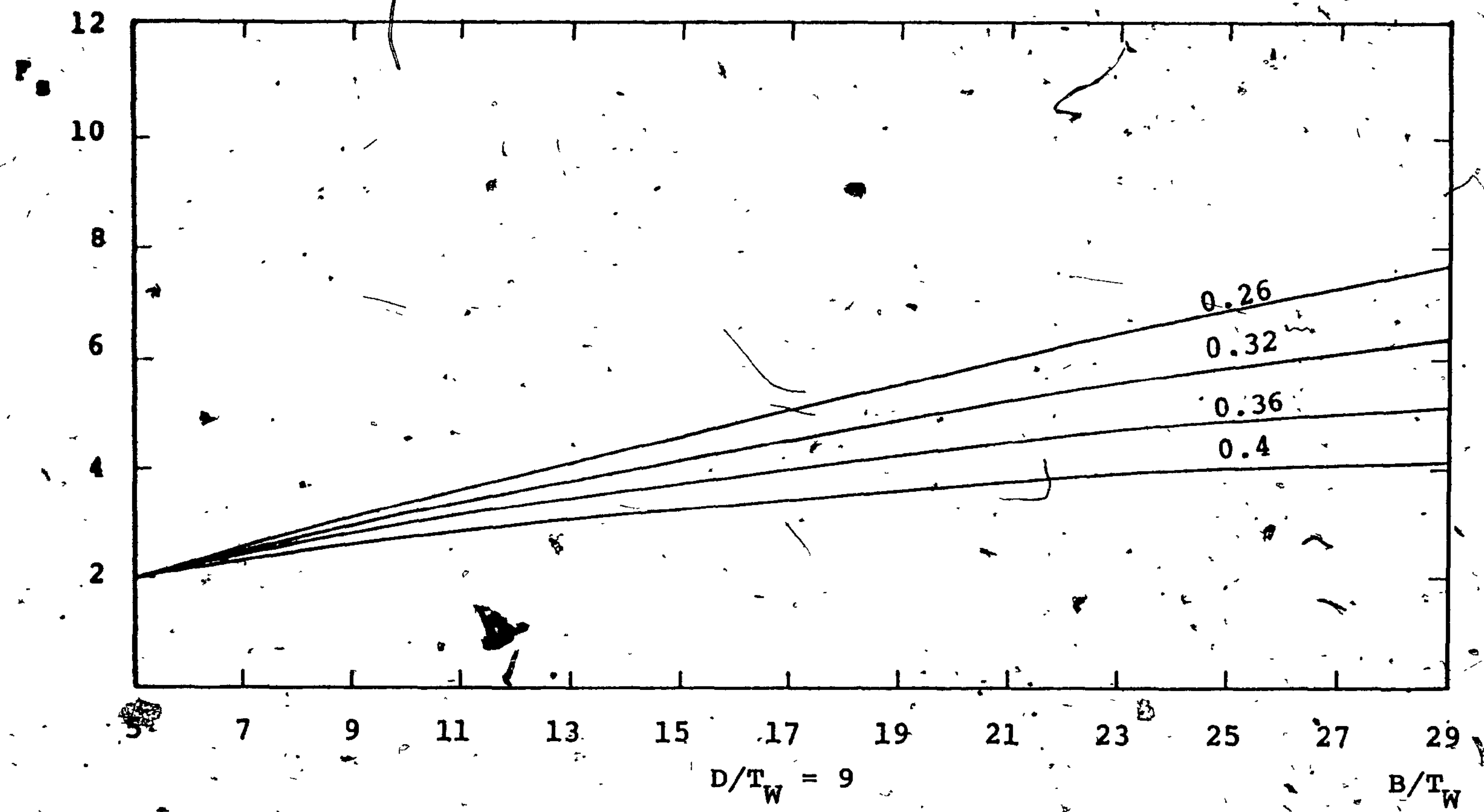
GRAPH 4.4(a) FORM FACTOR FOR T-SECTION. NUMBERS ON THE CURVES ARE THE VALUES OF T_F/D



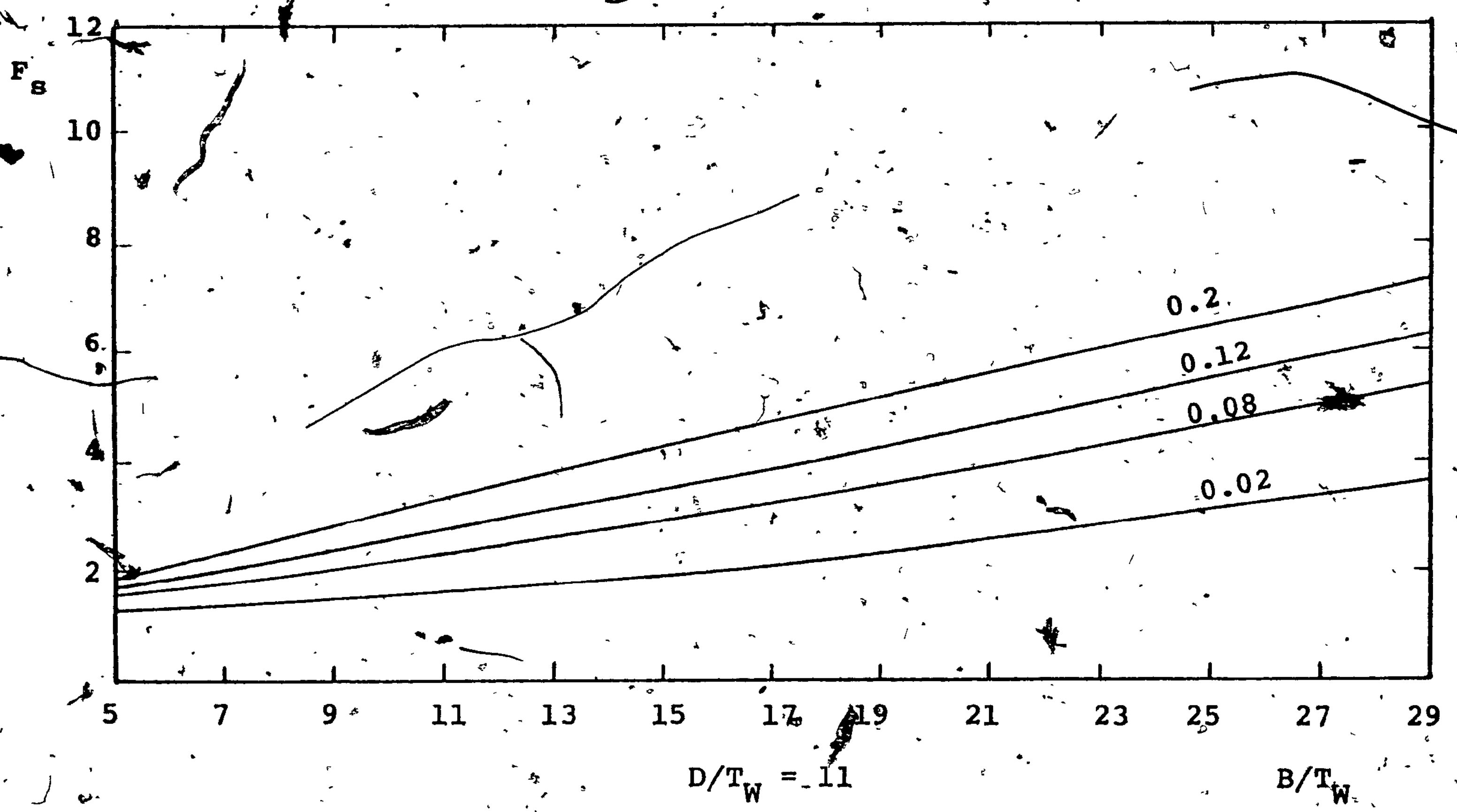
GRAPH 4.4(b) FORM FACTOR FOR T-SECTION. NUMBERS ON THE CURVES ARE THE VALUES OF T_F/D



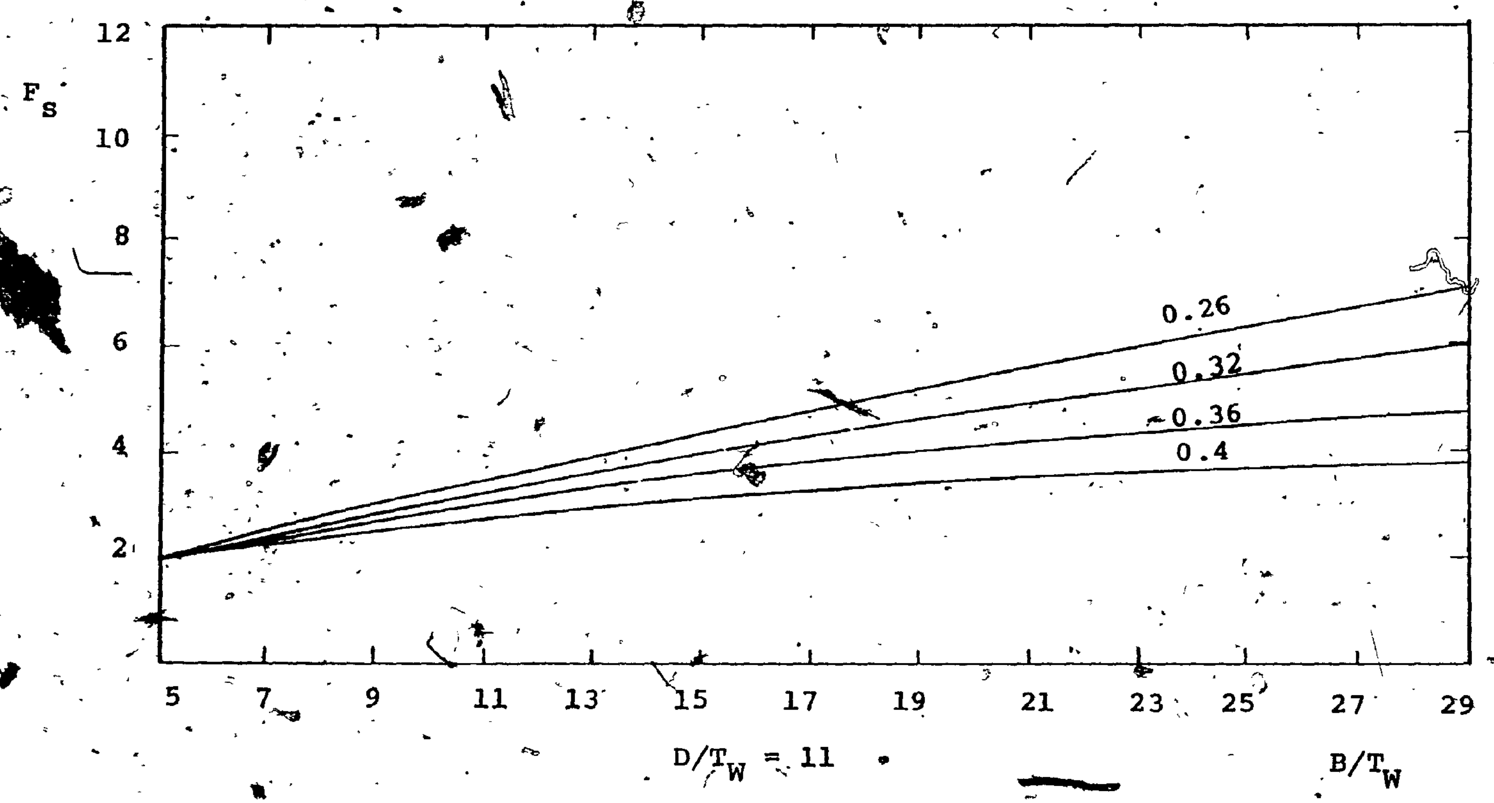
GRAPH 4.5(a) FORM FACTOR FOR T-SECTION. NUMBERS ON THE CURVES ARE THE VALUES OF T_{oF}/D



GRAPH 4.5(b) FORM FACTOR FOR T-SECTION. NUMBERS ON THE CURVES ARE THE VALUES OF T_F/D

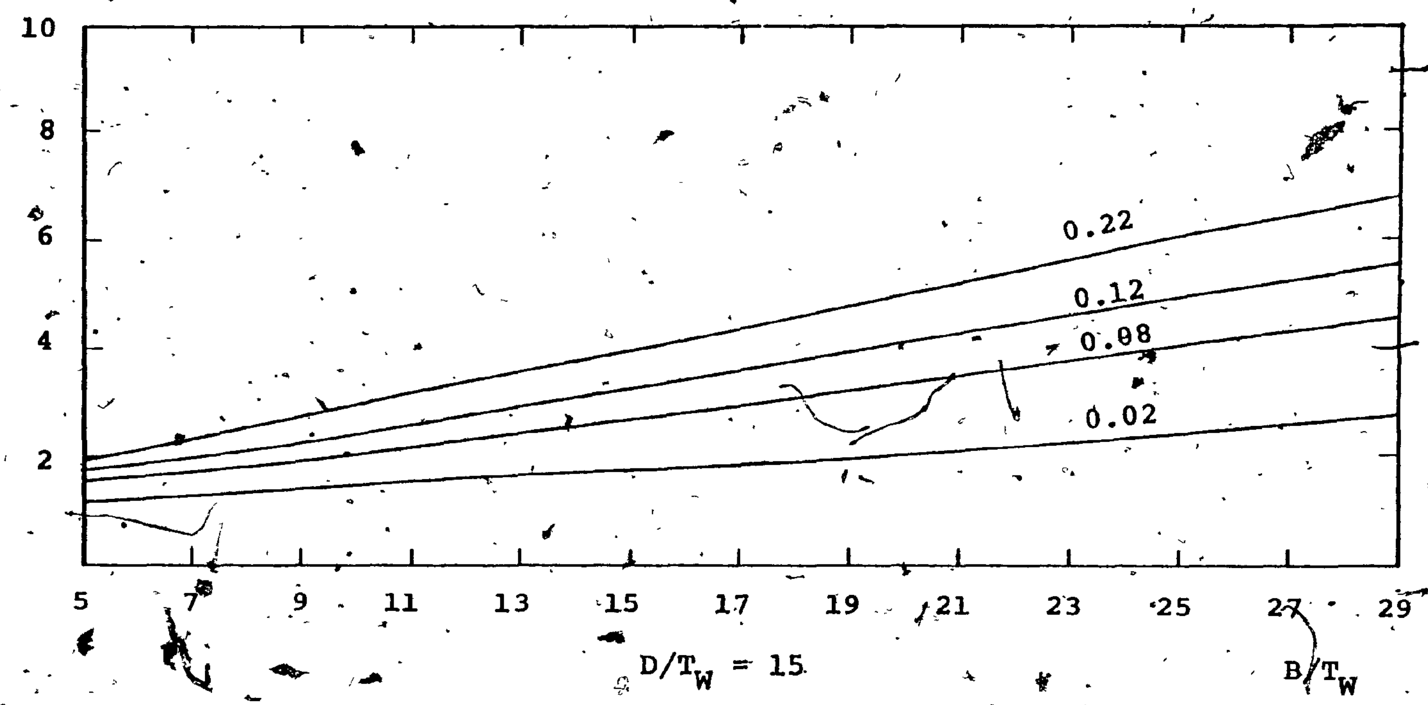


GRAPH 4.6(a) FORM FACTOR FOR T-SECTION. NUMBERS ON THE CURVES ARE THE VALUES OF T_F/D

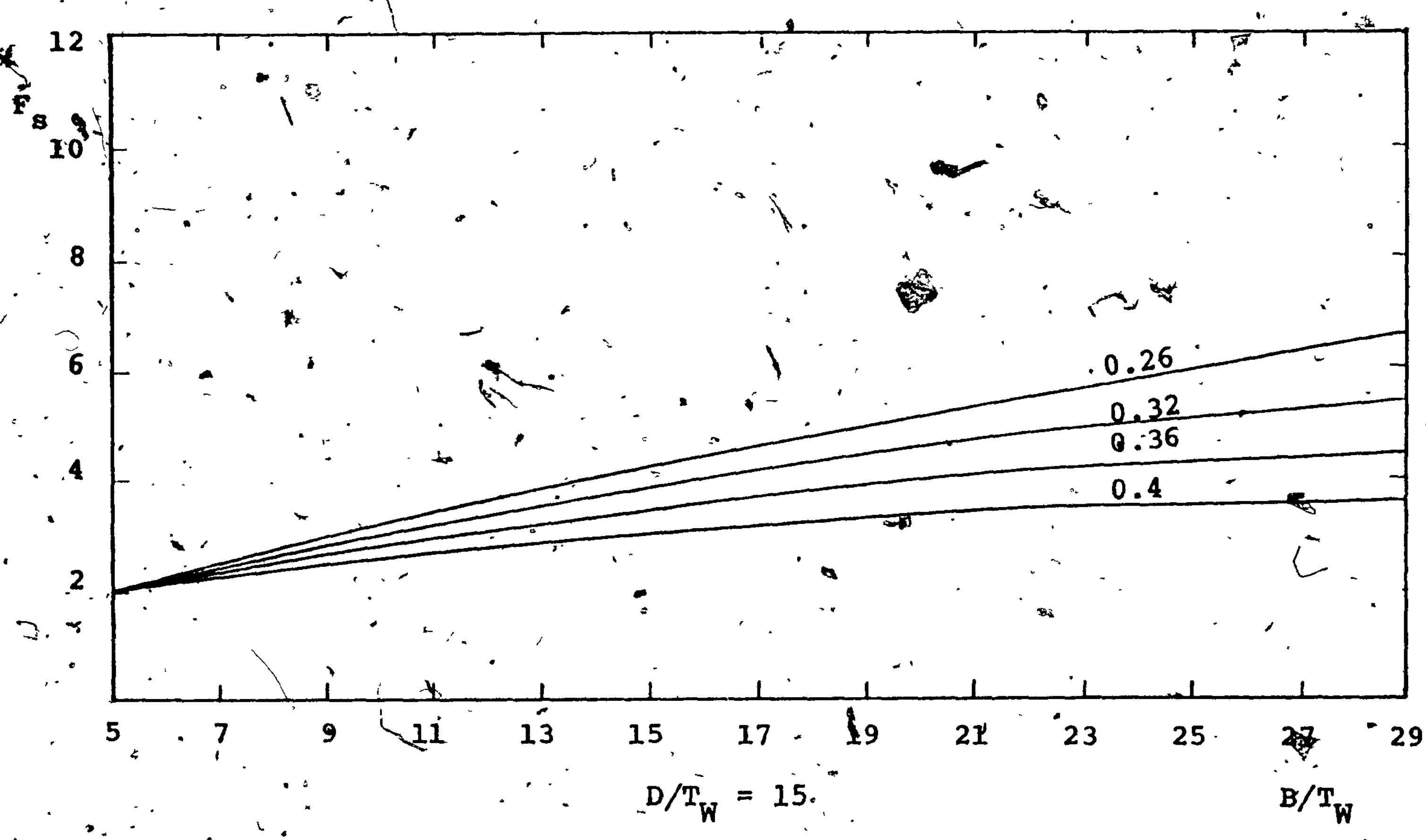


$D/T_W = 11$

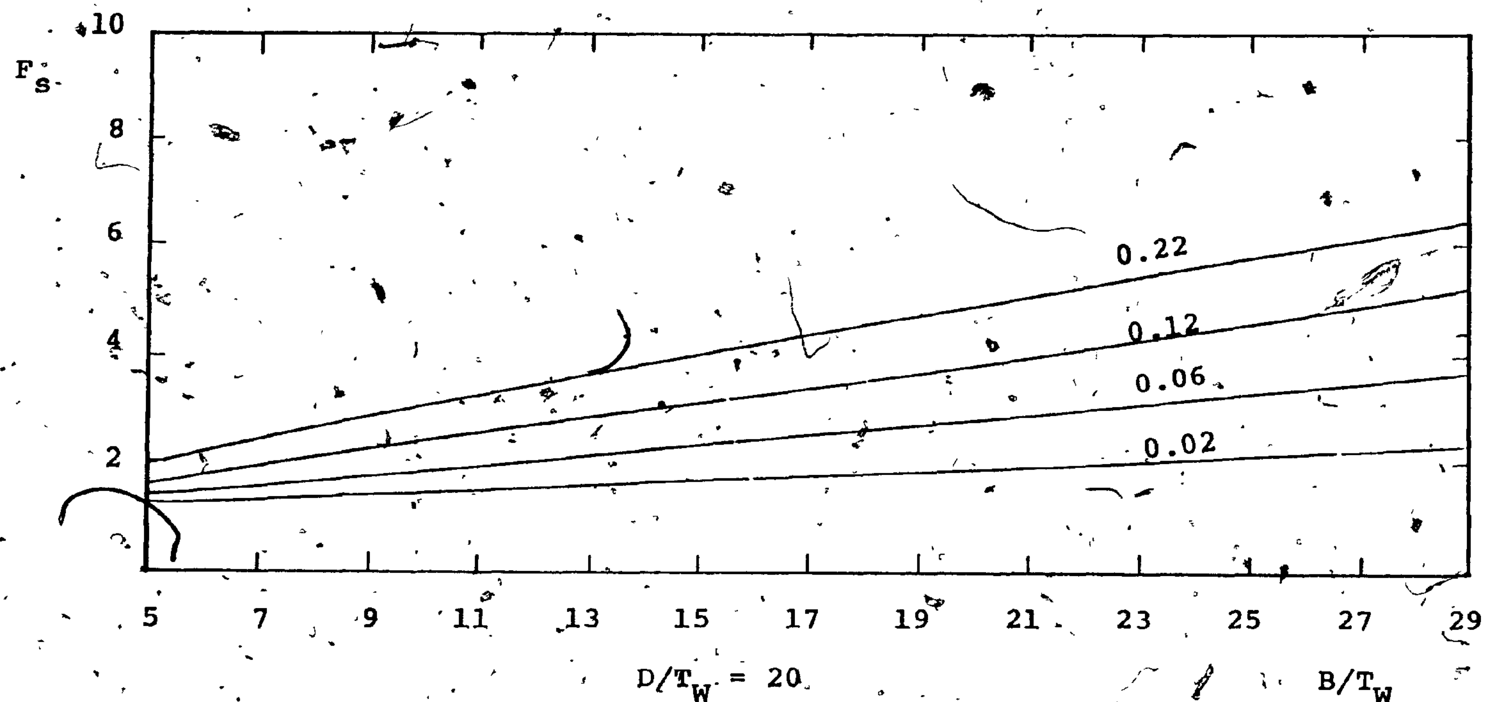
GRAPH 4.6(b) FORM FACTOR FOR T-SECTION. NUMBERS ON THE CURVES ARE THE VALUES OF T_F/D



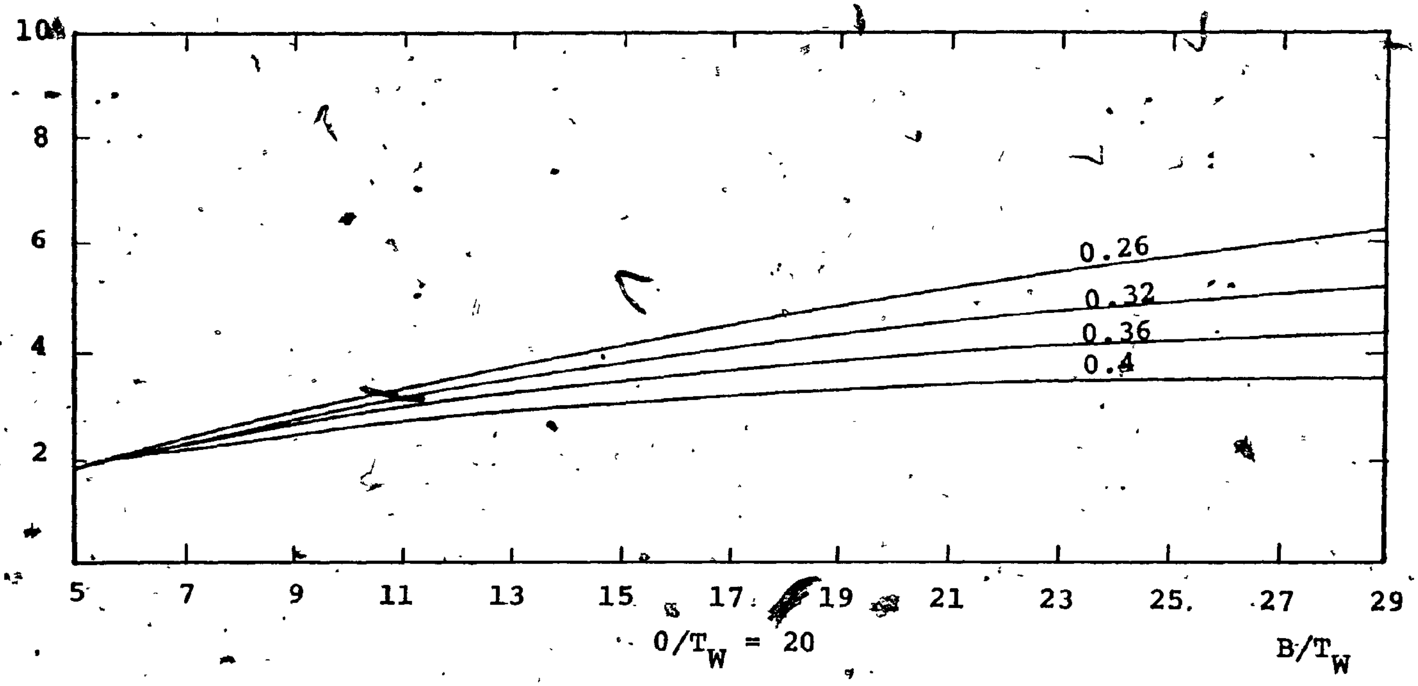
GRAPH 4.7(a) FORM FACTOR FOR T-SECTION. NUMBERS ON THE CURVES ARE THE VALUES OF T_F/D



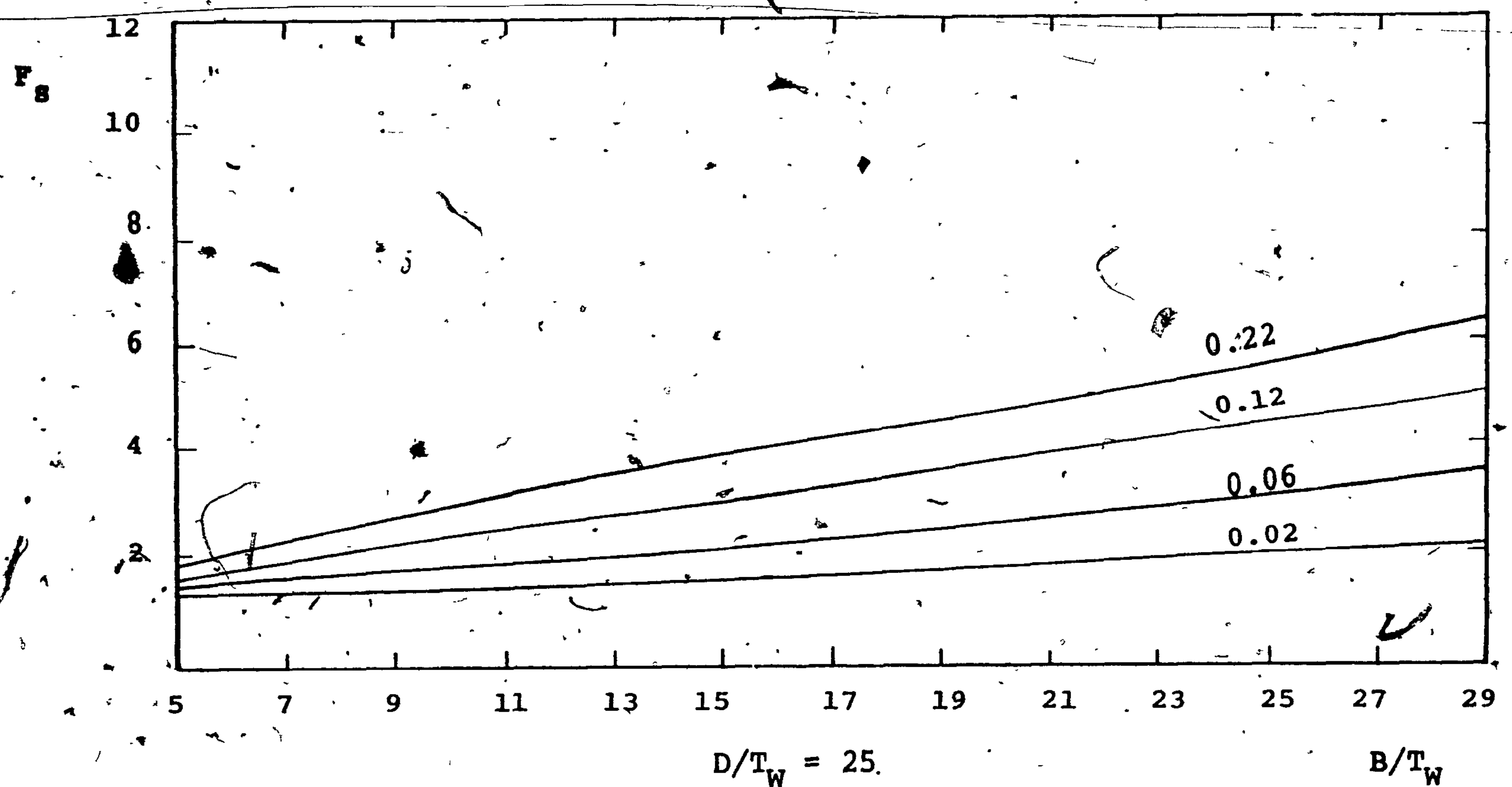
GRAPH 4.7(b) - FORM FACTOR FOR T-SECTION. NUMBERS ON THE CURVES ARE THE VALUES OF T_F/D



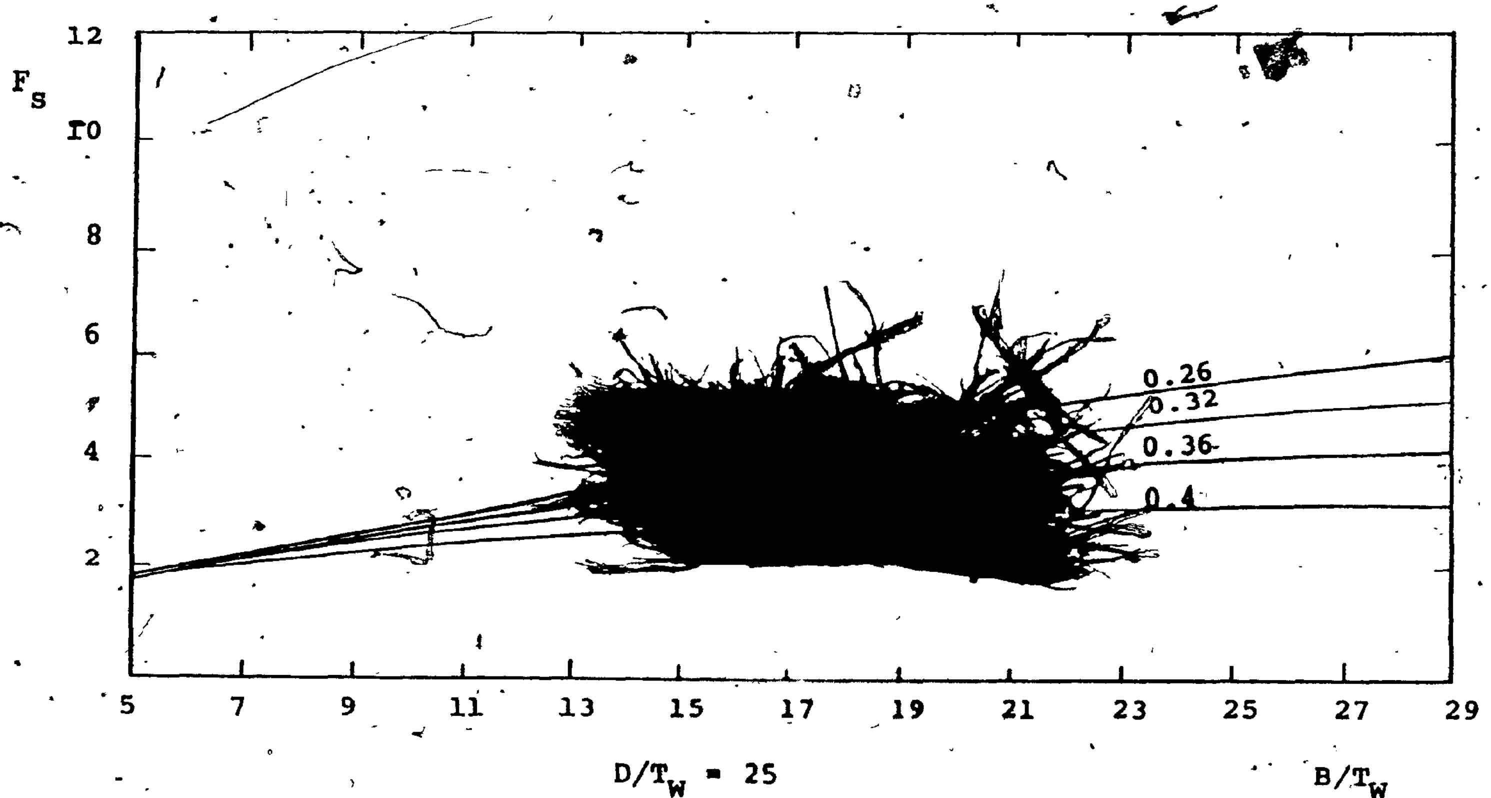
GRAPH 4.8(a) FORM FACTOR FOR T-SECTION. NUMBERS ON THE CURVES ARE THE VALUES OF T_F/D



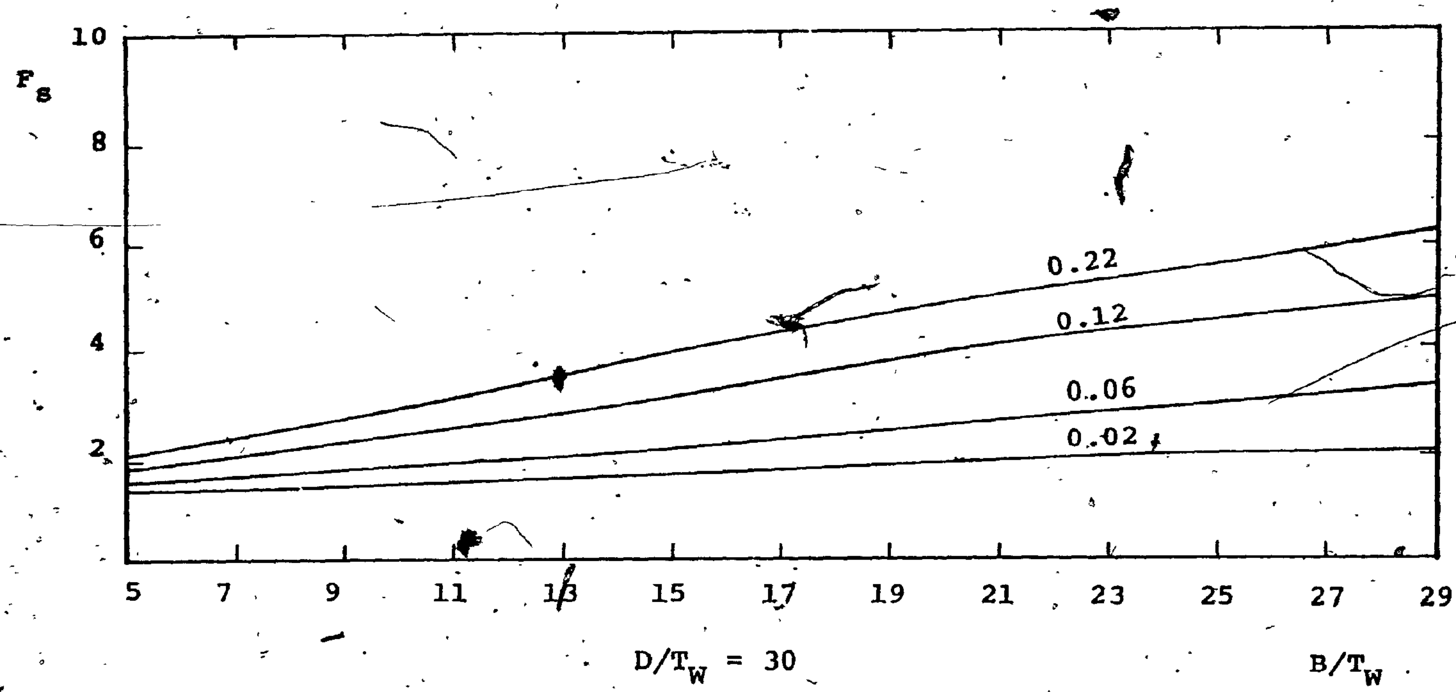
GRAPH 4.8(b) FORM FACTOR FOR T-SECTION. NUMBERS ON THE CURVES ARE THE VALUES OF T_F/D



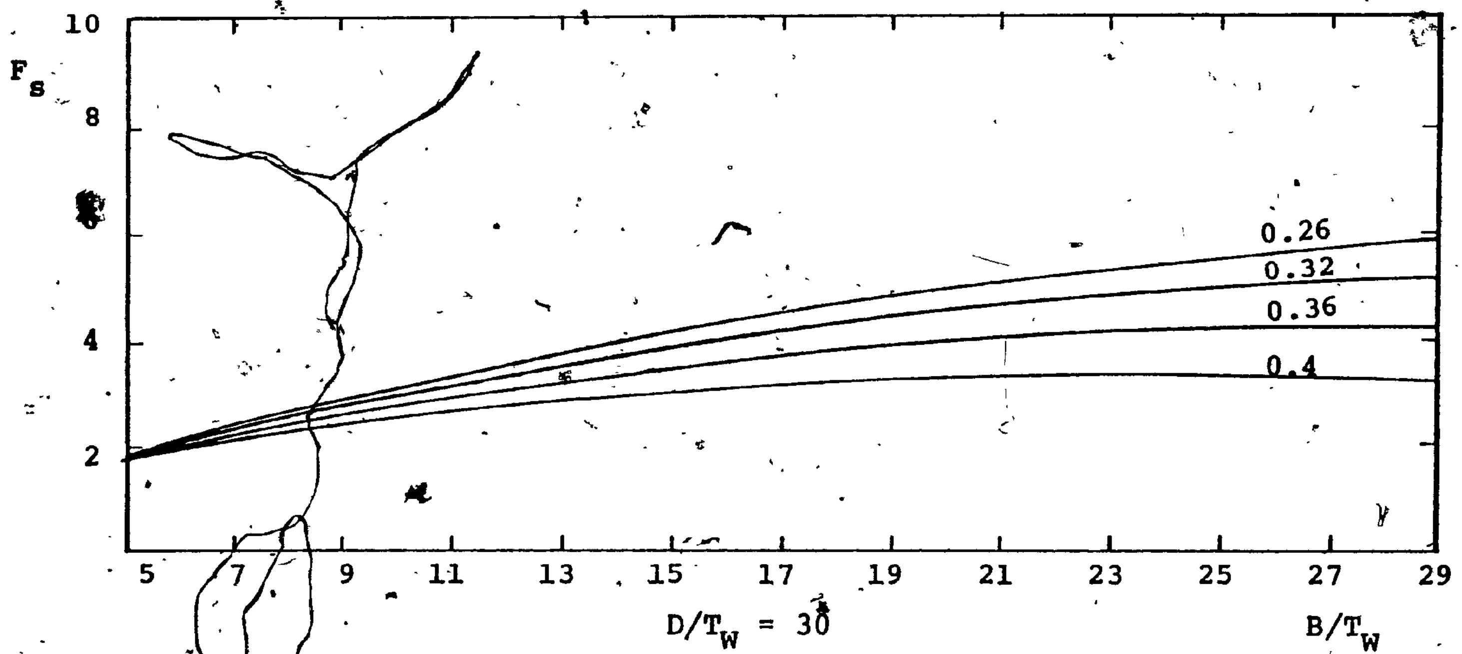
GRAPH 4.9(a) FORM FACTOR FOR T-SECTION. NUMBERS ON THE CURVES ARE THE VALUES OF T_F/D



GRAPH 4.9(b) FORM FACTOR FOR T-SECTION. NUMBERS ON THE CURVES ARE THE VALUES OF T_F/D



GRAPH 4.10(a) FORM FACTOR FOR T-SECTION. NUMBERS ON THE CURVES ARE THE VALUES OF T_F/D



GRAPH 4.10(b) FORM FACTOR FOR T-SECTION. NUMBERS ON THE CURVES ARE THE VALUES OF T_F/D

CHAPTER V

CONCLUSION

CHAPTER V

CONCLUSION

The following results are obtained from this study of the various cross-sections:

(1) For a triangle:

The form factor is independent of the geometry and has a value of 1.2.

(2) For a trapezoid:

The value of the form factor, F_s is independent of the depth of the section and depends upon the ratio T/B where T and B are the width of the lower base and upper base, respectively. For different values of T/B, F_s varies from 1.2 to 1.204. So, for a trapezoid, 1.2 is a good approximation regardless of the geometry.

(3) For a sector:

It ranges from 1.162 which is for half-circles, to 1.2 which is for angles less than 10°. This is reasonable because for small angles a sector is almost a triangle.

(4) For an arc:

The values obtained are for the arcs which subtend angles of $360 \geq 2\theta \geq 180$. Obtaining F_s for $2\theta < 180$ is beyond the scope of this report because of complicated shearing stress distribution across the arc.

(5) For segments of a circle:

The range of variation is small and F_s ranges from 1.162 to 1.178 for practical dimension ratios of D/B = 0.5 to D/B = 0.022.

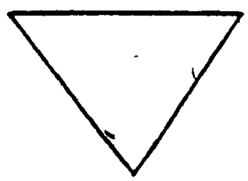
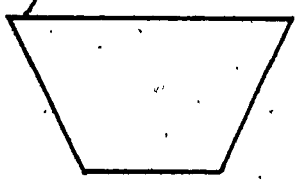
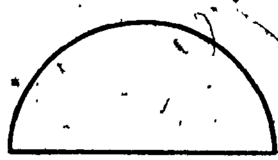
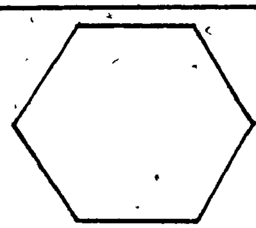
(6) For the T-Section:

The range of variation is wide and is strictly dependent on geometry.

(7) For a hexagon:

The value of 1.1097 was obtained, which is very close to the value of a full circle.

TABLE 5.1 FORM FACTOR FOR: A - TRIANGULAR SECTION
B - TRAPEZOIDAL SECTION; C - HALF CIRCLE;
D - HALF OF A RING; E - HEXAGONAL SECTION

Cross-Section		F_s
A		1.2
B		1.2*
C		1.162
D		2.5
		1.1097

* For trapezoidal section the maximum value of form factor obtained was 1.204.

REFERENCES

REFERENCES

[1] Ugural, A.C., Fenster, S.K., Advanced Strength and Applied Elasticity. American Elsevier Publ. Co. (1975).

[2] D.R. Axelrad, Strength of Materials For Engineers. Sir Isaac Pitman & Sons Ltd. (1959).

[3] Paradise, R.S., Problems in Strength of Materials. Blackie & Sons Limited. (1959).

APPENDIX A

COMPUTER PROGRAMS USED

A.1 PROGRAM FOR TRIANGLE

```
      PROGRAM TRI(INPUT,OUTPUT)
      EXTERNAL F
      COMMON B,H
      DO 100 I=1,4
      READ 1,B,H
    1 FORMAT(2F8.2)
      A=.5*B*H
      AI=B*H**3/36.
      X=DCADRE(F,-H/3.,2*H/3.,1.0E-9,1.0E-9,ERR,IER)
      PRINT 10,X,ERR,IER
   10 FORMAT(1H ,2F15.5,I5)
      FS=(A/AI**2)*X
      PRINT 11,FS
   11 FORMAT(1H ,"FS=",F10.5)
  100 CONTINUE
      STOP
      END
      FUNCTION F(Y)
      COMMON B,H
      F=((-B/H)*Y+2*B/3.)*((2*H/3.-Y)*(2*Y/3.+2*H/9.))**2/4.
      RETURN
      END
```

A.2 PROGRAM FOR TRAPEZOID

```
      PROGRAM TRAP(INPUT,OUTPUT)
      EXTERNAL F,Z
      COMMON B,H,T,AK,H1,H2,A
      B=20.
      H=25.
      R=1.
      N=1
      DO 100 I=1,N
      N=N+1
      IF(R-0.01) 21,20,20
20    T=B*R
      AA=(B+T)*H/2.
      AK=(B-T)/H
      H1=(3*B*H-2*AK*H**2)/(3*(B+T))
      H2=H-H1
      A=(T*H1+B*H2)/H
      AI=(-AK/4.)*H2**4+(A/3.)*H2**3+(AK/4.)*H1**4+(A/3.)*H1**3
      X=DCADRE(F,-H1,H2,1.0E-9,1.0E-9,ERR,IER)
      FS=(AA/AI**2)*X
      PRINT 11,R,FS
11    FORMAT(3H   ,"T/B=",F4.2,5H     ,"FS=",F8.6)
      R=R-0.05
100   CONTINUE
21    STOP
      END
      FUNCTION Z(Y)
      COMMON B,H,T,AK,H1,H2,A
      Z=(3*(-AK*Y+A)*(H2-Y)-2*((-AK*Y+A)-T)*(H2-Y))/(3*(-AK*Y+A+T))
      RETURN
      END
      FUNCTION F(Y)
      COMMON B,H,T,AK,H1,H2,A
      F=(((T-AK*Y+A)*(H2-Y)/2.)*(Y+Z(Y)))**2/(A-AK*Y)
      RETURN
      END
```

A.3 PROGRAM FOR SECTOR

```
      PROCRAM SEC(INPUT,OUTPUT)
      EXTERNAL F1,F2,F3,F4,F5
      COMMON R1,R2,TET,YQ,TETA,YB
      R2=40.
      TETA=190.
      R=0.
25    R1=R*R2
      N=1
      DO 100 I=1,N
      N=N+1
      TETA=TETA-10.
      IF(TETA.LT.10.) GO TO 25
11    TET=3.1415926554/360.*TETA
      A=(R2**2-R1**2)*TET
      YO=SIN(TET)*(R2**3-R1**3)/(A*1.5)
      AI=1./8.*(R2**4-R1**4)*(2*TET+SIN(2*TET))-4./9.*SIN(TET)**2*(R2**
     $3-R1**3)**2/A
      X=DCADRE(F1,R2*COS(TET),R2,1.0E-9,1.0E-9,ERR,IER)
      X1=DCADRE(F5,0.,R2*COS(TET),1.0E-9,1.0E-9,ERR,IER)
      X=X+X1
      PRINT 37,R,TETA,FS
37    FORMAT(3H    ,"R1/R2=",F5.3,5H     ,"TETA=",F5.1,5H     ,"FS=",
     $F8.6)
100   CONTINUE
50    TETA=370.
101   CONTINUE
25    STOP
      END
      FUNCTION F1(Y)
      COMMON R1,R2,TET,YO,TETA,YB
      AL=ACOS(Y/R2)
      IF(AL.EQ.0.) GO TO 30
      IF(Y.EQ.-R2) GO TO 30
      B=SQRT(1-Y**2/R2**2)
      A=AL*R2**2-R2*Y*B
      Z=(R2**3*B-R2*Y**2*B)/(F.5*A)
      F1=A**2*(Z-YB)**2/(2*R2*B)
      GO TO 40
30    F1=0.
40    RETURN
      END
      FUNCTION F5(Y)
      COMMON R1,R2,TET,YO,TETA
      IF(TETA.EQ.180.) GO TO 30
      IF(Y.EQ.0.) GO TO 30
      A=TET*R2**2-Y**2*TAN(TET)
      Z=(R2**3*SIN(TET)-Y**3*TAN(TET))/(1.5*A)
      F5=A**2*(Z-YO)**2/(2*Y*TAN(TET))
      GO TO 40
30    F5=0.
40    RETURN
      END
```

A.4 PROGRAM FOR ARC

```
      PROGRAM ARC(INPUT,CUTPUT)
      EXTERNAL F1,F2,F3,F4,F5
      COMMON R1,R2,TET,YO,TETA,YB
      R2=40.
      TETA=370.
      R=1.
      M=1
      DO 101 J=1,M
      M=M+1
      R=R-0.01
      IF(R.LT.0.5) GO TO 25
26    R1=R*R2
      N=1
      DO 100 I=1,N
      N=N+1
      TETA=TETA-10.
      IF(TETA.LT.10.) GO TO 25
11    TET=3.141592654/360.*TETA
      A=(R2**2-R1**2)*TET
      YO=SIN(TET)*(R2**3-R1**3)/(A*1.5)
      AI=1./9.*(R2**4-R1**4)*(2*TET+SIN(2*TET))-4./9.*SIN(TET)**2*(R2**
     $3-R1**3)**2/A
      X=DCADRE(F2,0.,TET,1.0E-9,1.0E-9,ERR,IER)
      FS=(A/AI**2)*X
      PRINT 37,R,TETA,FS
37    FORMAT(3H    ,"R1/R2=",F5.2,5H      ,"TETA=",F5.1,5H      ,"FS=",
     $F8.3)
100   CONTINUE
50    TETA=370.
      PRINT 38
38    FORMAT(//////)
101   CONTINUE
25    STOP
      END
      FUNCTION F2(ALF)
      COMMON R1,R2,TET,YO
      IF(ALF.EQ.0.) GO TO 30
      A=ALF*(R2**2-R1**2)
      Z=(R2**3-R1**3)*SIN(ALF)/(1.5*A)
      F2=(R2**2-R1**2)*A**2*(Z-YO)**2/(2*(R2-R1))**2
      GO TO 40
30    F2=0.
40    RETURN
      END
```

A.5 PROGRAM FOR SEGMENT

```
      PROGRAM SEC(INPUT,OUTPUT)
      EXTERNAL F1,F2,F3,F4,F5
      COMMON R1,R2,TET,YO,TETA,YB
      R2=40.
      TETA=190.
      R=0.
 26   R1=R*R2
      N=1
      DO 100 I=1,N
      N=N+1
      TETA=TETA-10.
      IF(TETA.LT:10.) GO TO 25
 110  TET=3.141592654/360.*TETA
      A=(R2**2-R1**2)*TET
      YO=SIN(TET)*(R2**3-R1**3)/(A*1.5)
      AA=A-R2**2*SIN(TET)*COS(TET)
      YB=(A*YO-R2**3*SIN(TET)*COS(TET)**2*(2./3.))/AA
      D=R2-R2*COS(TET)
      B=2*R2*SIN(TET)
      RA=D/B
      AI1=(R2**4/8.)*(2*TET+SIN(2*TET))-R2**4*SIN(TET)*COS(TET)**3/2.
      AI=AI1-AA*YB**2
      X=DCADRE(F1,R2*COS(TET),R2,1.0E-9,1.0E-9,ERR,IER)
      FS=(AA/AI**2)*(X)
      PRINT 38,RA,TETA,FS
 38   FORMAT(1X,5X,"D/B=",F4.3,5X,"TETA=",F5.1,5X,"FS=",F8.2)
 100  CONTINUE
 50   TETA=370.
 101  CONTINUE
 25   STOP
      END
      FUNCTION F1(Y)
      COMMON R1,R2,TET,YO,TETA,YB
      AL=ACCS(Y/P2)
      IF(AL.EQ.0.) GO TO 30
      IF(Y.EQ.-P2) GO TO 30
      B=SQRT(1-Y**2/R2**2)
      A=AL*R2**2-R2*Y*B
      Z=(R2**3*B-R2*Y**2*B)/(1.5*A)
      F1=A**2*(7-YB)**2/(2*R2*B)
      GO TO 40
 30   F1=0.
 40   RETURN
      END
```

A.6 PROGRAM FOR T-SECTION

```
      PROGRAM TEE(INPUT,OUTPUT)
      EXTERNAL F1,F2,Z
      COMMON B,TF,H,TW,AK,H1,H2
      TW=1.
      ALF=4.
      BET=0.
      GAM=4.
      DO 102 I=1,5
      GAM=GAM+1.
      D=GAM*TW
      DO 101 J=1,20
      BET=BET+0.02
      TF=BET*D
      H=D-TF
      DO 100 K=1,25
      ALF=ALF+1.
      B=ALF*TW
      H1=(B*TF**2/2.+H*TW*(TF+H/2.))/(B*TF+H*TW)
      H2=TF+H-H1
      A=B*TF+H*TW
      AI=B*TF**3/12.+B*TF*(H1-TF/2.)**2+TW*(H**3/12.)+(H2-H/2.)**2)*
     $TW*H
      AK=H1-TF
      X1=DCADRE(F1,-H2,AK,1.0E-9,1.0E-9,ERR,IER)
      X2=DCADRE(F2,AK,H1,1.0E-9,1.0E-9,ERR,IER)
      X3=TF*(H1-TF/2.)**2*B**3/12.
      X=X1+X2+X3
      FS=(A/AI**2)*X
      AW=TW*H
      AT=AW+B*TF
      R=AT/AW
      PRINT 11,GAM,BET,ALF,R,FS
11    FORMAT(1X,5X,"D/TW=",F5.2,5X,"TF/D=",F4.2,5X,"B/TW=",F5.2,5X,
     $"AT/AW=",F7.2,5X,"FS=",F6.3)
100   CONTINUE
      PRINT 28
28    FORMAT(/////)
      ALF=4.
101   CONTINUE
      BET=0.
102   CONTINUE
      STOP
      END
      FUNCTION Z(Y)
      COMMON B,TF,H,TW,AK,H1,H2
      Z=(B*TF**2/2.+(AK-Y)*TW*TF+((AK-Y)**2)*TW/2.)/(B*TF+(AK-Y)*TW)
      RETURN
      END
      FUNCTION F1(Y)
      COMMON B,TF,H,TW,AK,H1,H2
      F1=((B*TF+(AK-Y)*TW)*(H1-Z(Y)))**2/TW
      RETURN
      END
      FUNCTION F2(Y)
      COMMON B,TF,H,TW,AK,H1,H2
      F2=(B/4.)*(H1**2-Y**2)**2
      RETURN
      END
```

A.7 PROGRAM FOR HEXAGON

```
      PROGRAM HEX(INPUT,OUTPUT)
      EXTERNAL F
      COMMON B,S
      S=SQRT(3.)
      B=20.
      A=3*S*B**2/2.
      AI=5*S*B**4/16.
      X=DCADRE(F,0.,S*B/2.,1.0E-9,1.0E-9,ERR,IER)
      FS=(A/AI**2)*2*X
      PRINT 12,B,FS
12    FORMAT(1X,5X,"B=",F5.2,5X,"FS=",F7.4)
      STOP
      END
      FUNCTION F(Y)
      COMMON B,S
      AL=(2./S)*(B*S-Y)
      Q=(2./S)*(1./4.*S*B**3-S/2.*B*Y**2+1./3.*Y**3)
      F=Q**2/AL
      RETURN
      END
```