SHORT-TERM LOAD FORECASTING IN AN ELECTRIC POWER SYSTEM

by

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ABSTRACT

The problem of load forecasting is studied, first reviewing several methods, then selecting one for application to an actual power system, using real data. The selected method models load by a deterministic and a residual component, the latter containing a weather-dependent variable as well as uncertainty. The model is represented by a Fourier series and difference equation which are put in state space form. Model parameters are identified from three weeks of past load and temperature data using the Fletcher Powell method. These identified parameters then are used in the Kalman prediction algorithms with temperature forecasts as input to forecast load into the fourth week. The basic forecast is for lead times of 1 to 72 hours.

To gain further information, several other forecasts are made, using parameters identified from data of single weeks, one of these forecasts with lead times up to 168 hours.

The method is shown to produce an acceptable forecast of good accuracy with reasonable computing time. Ideas for extensions and modifications are outlined on closing.

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Symbols used in Chapter 2

Section 2.1

$\underline{u}$	system input signal vector
$\underline{w}$	system noise vector
$\underline{v}$	measurement noise vector
$\underline{z}$	measured output vector of the system
$\underline{x}$	output vector of the process
$\underline{B}$	parameter vector of the process
$\underline{\hat{B}}$	parameter vector of the model
$\underline{\hat{z}}$	output vector of the model; estimate of $\underline{z}$
$\underline{e}$	error in estimation of $\underline{z}$

Section 2.2

z	load at a chosen time of day
a	the constant portion of base load
F(t)	a polynomial function representing annual variation of base load, at the t-th week
b_1, b_2, b_3, b_4	weather effect parameters
d	day-of-week correction on load
T	atmospheric temperature
W	cooling power of the wind
L	illumination index
P	precipitation rate

Section 2.3

$z(k,t)$	load on the k-th day at the t-th hour
$a(k,t)$	base load component of $z(k,t)$
$f_1(T_k)$	function of temperature T_k
$f_2(L_k)$	function of illumination L_k
$b(k,t)$	weighting factor for temperature
$c(k,t)$	weighting factor for illumination
$\underline{Z}(k,t)$	elements $z(k,t)$ represented as a matrix

Section 2.4

$z(t)$	load at hour t
$\hat{y}_p(t)$	Fourier series approximating the periodic component of $z(t)$
T	the current hour
$\underline{a}$	Fourier coefficient vector for $\hat{y}_p(t)$
$\underline{f}(t)$	sinusoidal vector of Fourier series $\hat{y}_p(t)$
W	smoothing constant
$\hat{z}(t)$	estimated load at hour t
ℓ	lead time
$\hat{z}(\ell)$	forecasted load at lead time ℓ
$\underline{h}$	smoothing vector for updating the Fourier coefficients
$\underline{L}$	transition matrix defined by $f(t) = \underline{L}\, f\,(t-1)$

Symbols used in Chapters 3 to 6

$\{*, +, \cdot\}$	the set of $*$, $+$, $\cdot$
Q	variance of noise

$\begin{bmatrix} \underline{x}_p \\ \underline{x}(t) \end{bmatrix}$	the state space variable (vector)
$\underline{x}_p$	the periodic parameter vector, $[x_p^0, x_p^1, .., x_p^{2n_p}]^T$
$\underline{x}(t)$	the stochastic state vector, $[x_1(t), x_2(t), .., x_n(t)]^T$
$\underline{\phi}(t)$	the periodic argument vector, $[1, \sin(2\pi/24)t, .., \sin(2\pi/24)n_p t$ $..\cos(2\pi/24)t, .., \cos(2\pi/24)\ 2n_p t]^T$
$\underline{A}$	state transition matrix in state space model
$\underline{B}$, $\underline{H}$	input transition matrix in state space model
$\underline{D}$	disturbance transition matrix in state space model
$\underline{u}(t)$	input vector in state space model
$\underline{w}(t-1)$	noise vector in state space model
$\underline{d}$	$[d_1, 0, .., 0]^T$, disturbance transition vector in state space model. $\underline{D}.\underline{w}(t-1) = \underline{d}\ w(t-1)$, eqns. (3.11), (3.12)
$\underline{c}$	measurement vector in state space model
T	Dorval temperature at time t
$\hat{T}$	normal Dorval temperature at time t (section 3.5)
t	time in integral hours
$y_p(t)$	the periodic component of load, MW, at time t
u(t)	the non-linear temperature-dependent input to the load process at time t
z(t)	load, MW, at time t
y(t)	the residual component of load, MW, at time t
a_i	retrogressive parameter, coefficient of y(t-i), where i is a positive integer

n	positive integer, largest value of i in a_i or $y(t-i)$
b_j	parameter, coefficient of $u(t-j)$, where j is a non-negative integer
$\underline{a}$	retrogressive parameter vector, $[a_1, a_2, \ldots, a_n]^T$
$\underline{b}$	weather-dependent parameter vector, $[b_0, b_1, \ldots, b_m]^T$
m	non-negative integer, largest value of j in b_j or $u(t-j)$
$w(t-1)$	white noise, MW, at time $t-1$
x_p^0	constant component of $y_p(t)$ in the Fourier series representing $y_p(t)$
x_p^i	periodic parameter, sine coefficient of i-th harmonic in Fourier series representing $y_p(t)$, where $i = 1$ indicates fundamental frequency of one per 24 hours
n_p	integer, largest value of i in x_p^i, $\sin(2\pi i/24)t$, $x_p^{n_p+i}$ or $\cos(2\pi i/24)t$
$x_p^{n_p+i}$	periodic parameter, cosine coefficient of i-th harmonic in Fourier series representing $y_p(t)$, where $i = 1$ indicates fundamental frequency of one per 24 hours
$u_T(t)$	component representing actual heating or cooling effect in calculation of $u(t)$
$u_{\hat{T}}(t)$	component representing normal heating or effect in calculation of $u(t)$
J	mean square error in estimation of $y(t)$; approximates Q on convergence of parameter identification program
$\underline{\theta}$	the parameter vector $[x_p^0, \ldots, x_p^{2n_p}, a_1, \ldots, a_n, b_0, \ldots, b_m]^T$

$\underline{\psi}(t)$	a matrix of retrogressive sinusoidal functions $[\phi(t-1), .., \phi(t-n)]^T$
$\underline{P}(t+1\|t)$ $\underline{S}(t+1)$	covariance matrix of error in estimation, at time t+1, given data up to time t
$\underline{K}(t+1)$	Kalman gain at time t+1
$\hat{\underline{x}}(t+1\|t)$	estimate of stochastic state space variable $\underline{x}(t+1)$ given data up to time t
$\hat{z}(t+1\|t)$	estimate of load z(t+1) given data up to time t
$\hat{\underline{x}}(t\|t)$	filtered value of $\underline{x}(t)$
V_f, V_p	variance of error in filtering, predicting, load z(t)
$\underline{P}(\ell+n+1\|n)$ $\underline{W}(\ell+n+1)$	covariance matrix of error in prediction with lead time ℓ given data up to time n+1
$z(t)$	load, MW, at hour t
$y_p(t)$	periodic component of z(t)
$y(t)$	residual component of z(t)
$u(t)$	non-linear temperature-dependent variable contained in y(t)
$\underline{x}(t)$	non-periodic part of state space vector
$\underline{A}$	state transition matrix
$\underline{B}$	control transition matrix
$\underline{d}$	disturbance transition vector
$\underline{c}$	measurement vector
$\underline{\phi}(t)$	periodic argument vector
$\underline{x}(t)$	periodic parameter vector
$\underline{u}(t)$	control input sequence, temperature-dependent variable vector

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CHAPTER 1

INTRODUCTION

1.1 Importance of Forecasting Load

The general object of an electric power system is to supply the various commercial, industrial, military, agricultural and domestic customers with power on demand or by contract, reliably, at steady voltage and constant frequency, and at the same time to accomplish these results at minimum overall cost. Generating equipment, as well as transformers, breakers, water storage, transmission lines, etc., must be ready to supply the required power. In the planning and operation of a power system then, load forecasting is essential.

1.2 Lead Time of the Forecast

The interval between the most recent load data, and the estimated future load, is called "lead time". The purpose of the forecast tends to determine the lead time. In very short term, which we shall arbitrarily consider here as milliseconds to minutes, generating machinery in operation is held under continuous automatic control. While this automatic control could be considered a system involving a very short-term forecast in real time, calling for corresponding action, checking the results of this action, possible correction of the forecast on the basis of these results, and performing these operations continuously, such is not the usual concept: the term actually used is "load frequency control". Analysis of the system could be done also, in non-real time, involving the time constants of transformers, lines, motors, generators, boilers, turbines, etc. This analysis, in non-real time, could be treated as forecasting also. Here, however, we shall consider the minimum lead time for a "forecast" as one hour.

There are, of course, no rigid rules defining lead times [2, 7, 9] of short term, medium term and long term load forecasts. Here, to clarify terminology used in this work but without intending a definition, we shall categorize lead times according to the purpose of the forecast as:

<u>long term</u>, 2 years to 10 years or more;
<u>medium term</u>, one week to 2 years; and
<u>short term</u>, one hour to one week.

1.3 Importance of the Short Term Forecast and its Relation to Medium and Long Term Forecasts

1.3.1 Long term

It is important to realize that the revenue of a power utility is basically proportional to sold energy, while capital cost, and to a lesser extent other costs, tend to be proportional to peak load (power) since load capacity of the capital equipment must adequately match this peak load, with a minimum of reserve. To accomplish this latter objective, the benefit of a long-term forecast is obvious. After the decision is made to proceed with construction of a power plant or even with a major extension of an existing plant, several years must elapse before equipment is financed, designed, built, installed and ready to take load. As described above then, the long-term forecast is used for <u>planning construction of facilities not yet existing, or for replacement of existing equipment</u>.

1.3.2 <u>Medium term</u>

Over a term of one week to two years, a utility can negotiate contracts with neighbouring utilities to buy or sell power, or can contract with customers to charge for power on the basis of peak load.

Also, gas turbine or diesel generators can be purchased in relatively short delivery time, or existing ones can be transported to a different location within the bounds of the power system, to take anticipated sharp peaks in load.

In systems using hydraulic power, water storage can be managed on the basis of spring thaw perhaps, and flood threat, coordinated with the anticipated energy demand.

Maintenance of equipment must be scheduled, altering any previous schedules according to any emergencies.

The medium-term forecast therefore bridges the gap between the long-term and short-term forecasts.

1.3.3 Short term

Generating machinery such as hydraulic turbines, boilers, gas turbines, nuclear reactors, etc., must be brought-on or dropped-off the line to take care of hour-to-hour and day-to-day changes due to weekday industrial load, weekend slack, evening lighting and cooking, weather changes affecting "heating" or "cooling" load, etc. Assuming capital expansion has been held to an economic limit on the basis of the long-term forecast, there will not be much excess capacity available at times of peak load. On the other hand, at times of low load, the machinery of lowest operating cost should be running. More specifically, for best economy, load and load swings must be shared among the various generating machinery according to its incremental cost (that is the increment in cost corresponding to the increment in load on the particular generating equipment). The location of the equipment affects the incremental cost, because of transmission line loss.

In general, generating machinery requires warm-up time before it can take load. A boiler, for example, should have several hours from cold start. A load forecast therefore is needed, having corresponding lead time so that sufficient "spinning reserve" can be provided. This spinning reserve must be ready to take forecasted increase in load plus provision for loss [1] of the largest generating unit or of that transmission capacity which renders unavailable the greatest amount of generating capacity.

Load forecasts sometimes concentrate on predicting peaks, which of course are very important, because peak load determines the maximum demand on total equipment in any particular period. To minimize overall cost on an incremental cost basis as mentioned above, however, a comprehensive short-term forecast must predict the entire load pattern throughout the range of the forecast lead times.

In summary then, the short-term forecast is used to schedule the operation of existing equipment. Or more fully, the short-term forecast takes the equipment existing as planned by the long-term forecast, and the flexibility provided by using the medium-term forecast, and permits scheduling operation of this equipment, in economic and reliable combinations, so that load frequency control can be applied to supply power demand, at a steady voltage, constant frequency and minimum overall cost. Thus, the object of a power system described in the opening statement of this introduction is accomplished.

Computer solution is most useful in the short-term [7] forecasts because they are required frequently and quickly.

1.4 The Specific Electric Power System to be Considered

1.4.1 General system characteristics

Real data short-term forecasting, in this thesis, will be done for the "Hydro-Québec" system. This system is of large geographical area, but with most of the load in a relatively concentrated portion of that area, much of the power being generated at hydro-electric stations remote from the load (2,500 miles of 735,000 volt lines). There are only two thermal stations, one nuclear, the other burning fossil fuel. It has been necessary to develop considerable expertise, to handle long distance transmission with the attendant problems of high voltage, tendency to instability, icing of high tension transmission lines, and significant transmission loss of primary power (of the order of 10%).

1.4.2 Load characteristics

The total load on the system is fairly large, ranging over a year from 3,000 to 10,000 megawatts approximately. The winter peak load is higher than the summer peak, probably due to the fact that Montreal's temperatures are generally lower than those of most cities of the United States for example, both summer and winter. Relatively then, summer "air conditioning and refrigeration" load tends to be low, and winter "heating" load high. Also, while the daily load has the usual primary and secondary peaks, the daily maximum in winter normally occurs in the afternoon, due to switching on lights, preparation for supper, etc., rather than near noon. As power systems gain a higher proportion of industrial load, the drop between these two peaks tends to decrease [4]. Hydro-Québec, however, has a distinct load drop between the peaks and a rapid rise to the evening peak. Annual peak load occurs near Christmas, before minimum winter temperatures are reached. The 24-hour trend of load is similar on non-holiday weekdays except that Mondays tend to have the largest weekday load, and also the largest range of load. As is to be expected, load on holidays and weekends is smaller and somewhat different from weekdays.

Some typical loads are shown in graph form in Figs. 1.1 to 1.5. These graphs cover weekends and weekdays, both summer and winter.

1.5 Scope and Content of the Thesis

Experiments using real data will be carried out for the month of January 1972 only. The basic experiment will consist of forecasting hourly load with lead times of 1 up to 72 hours, zero lead time corresponding to 11 p.m. on January 25, 1972.

Load will be represented by a mathematical model consisting of the sum of a periodic and a residual component, arbitrarily choosing the model order (no optimizing). Parameters of this model will be estimated by the Fletcher Powell method, and the forecast itself will be made, using the Kalman filter state estimation and prediction algorithms. Tuesdays, Wednesdays, Thursdays

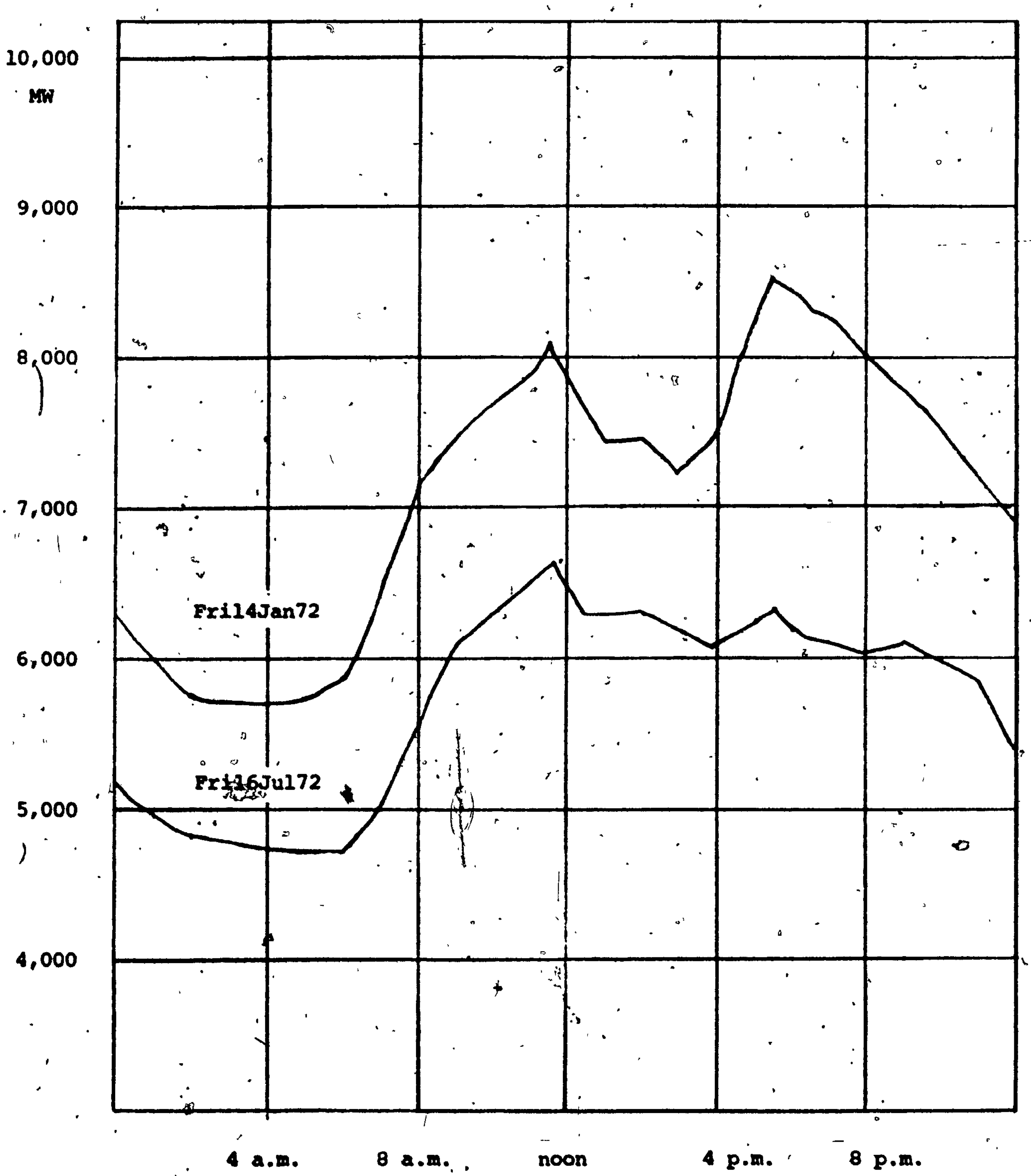

Fig. 1.1 - Hydro-Québec Primary Load

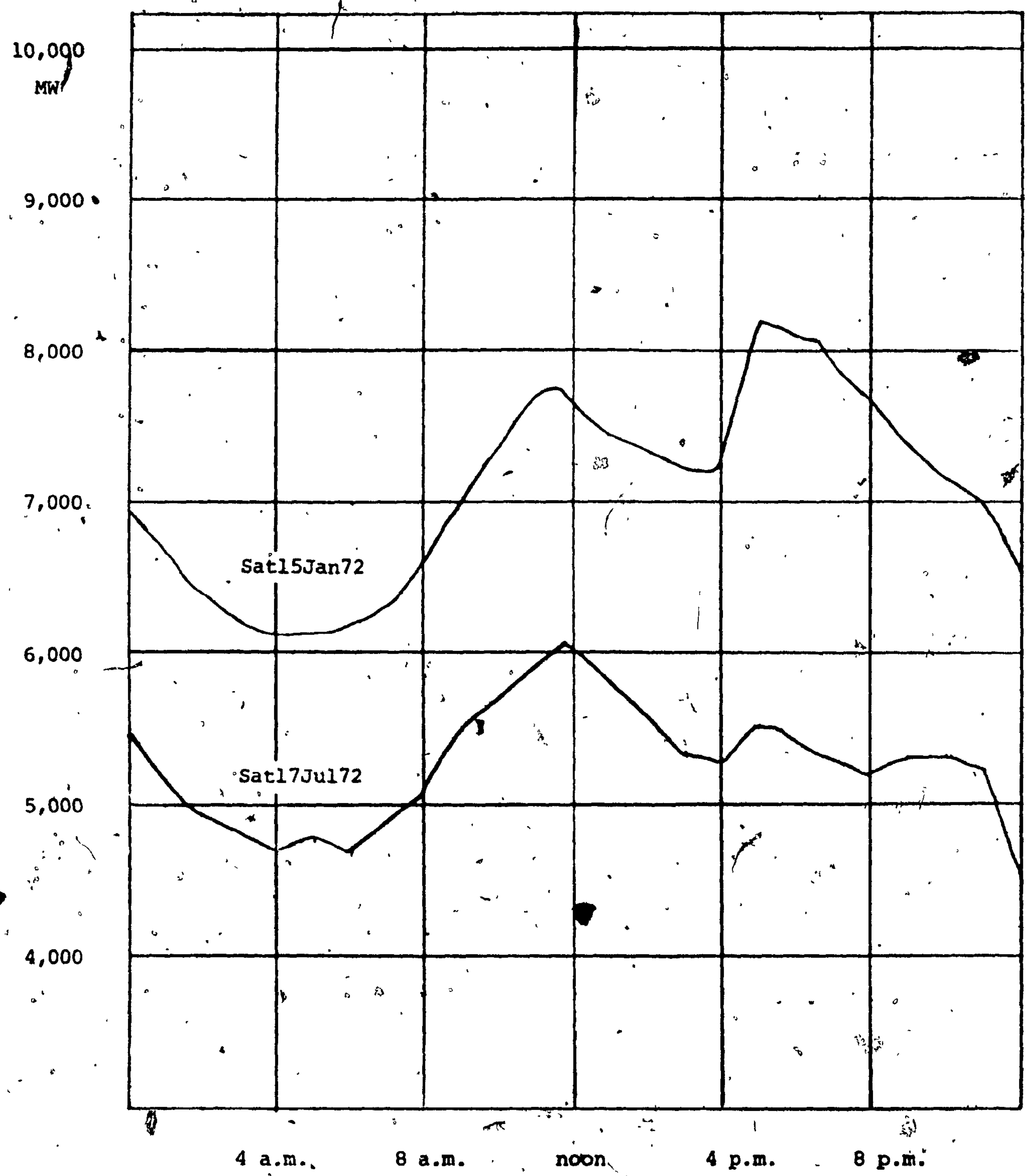

Fig. 1.2 - Hydro-Québec Primary Load

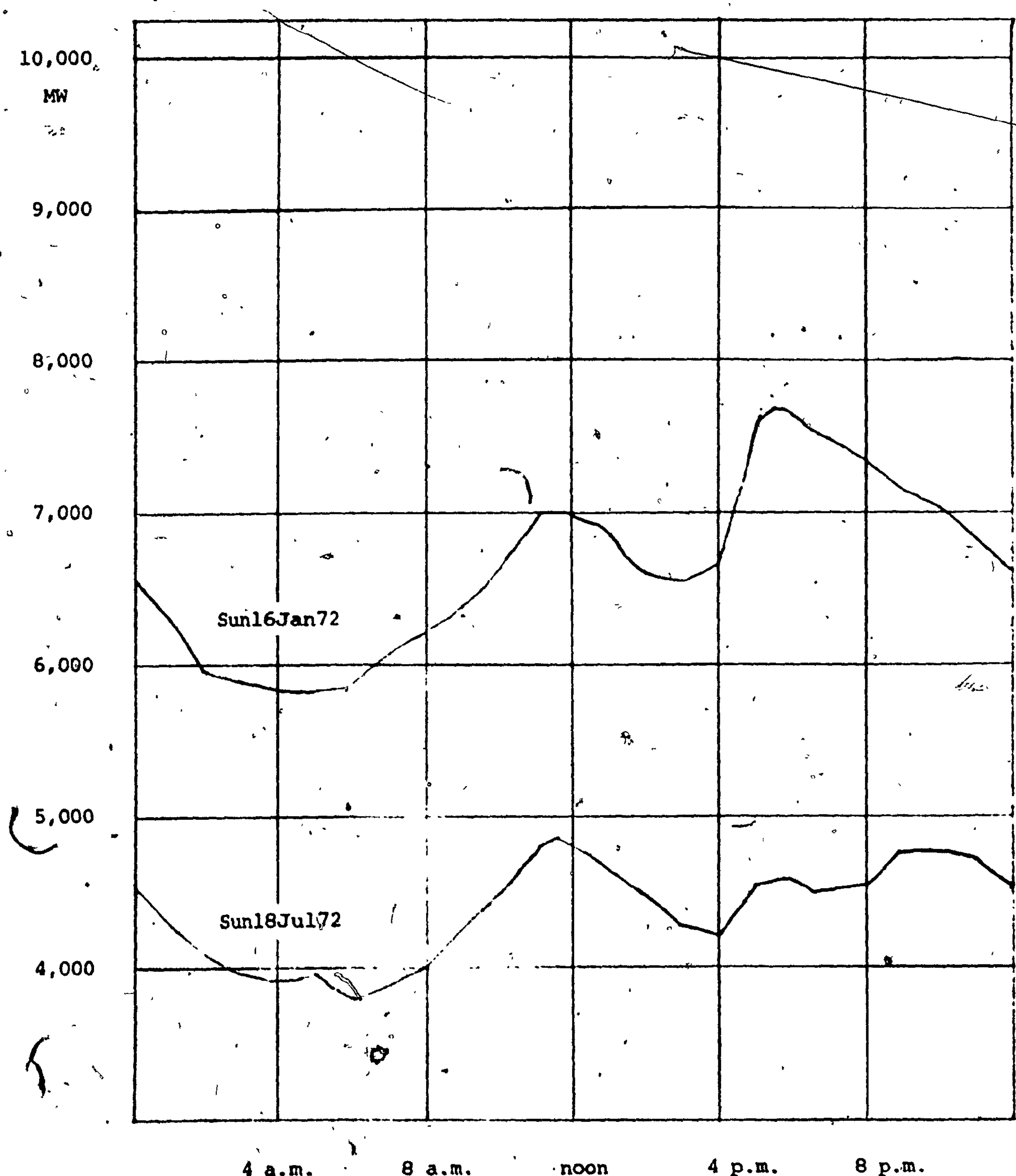

Fig. 1.3 - Hydro-Québec Primary Load

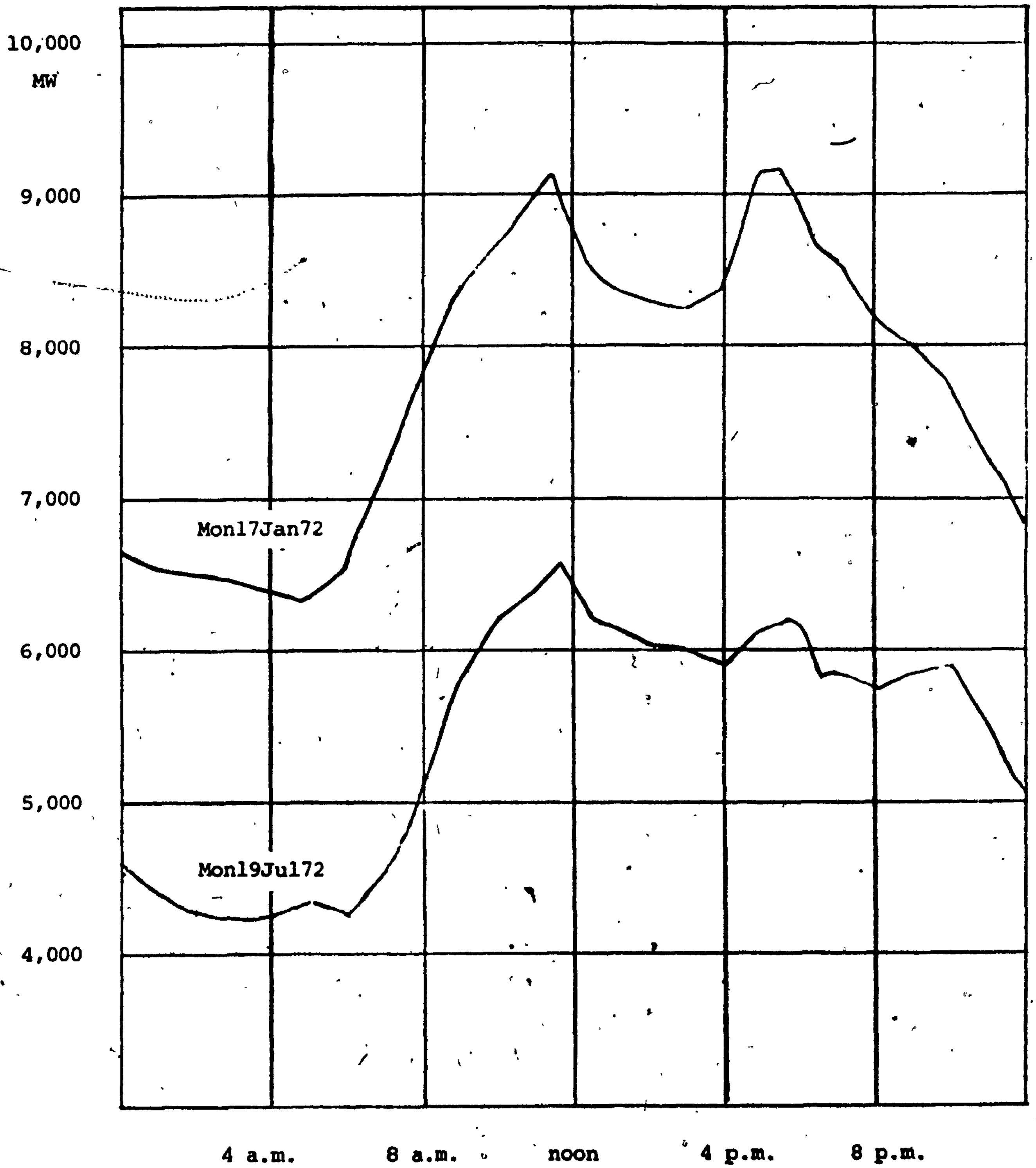

Fig. 1.4 - Hydro-Québec Primary Load

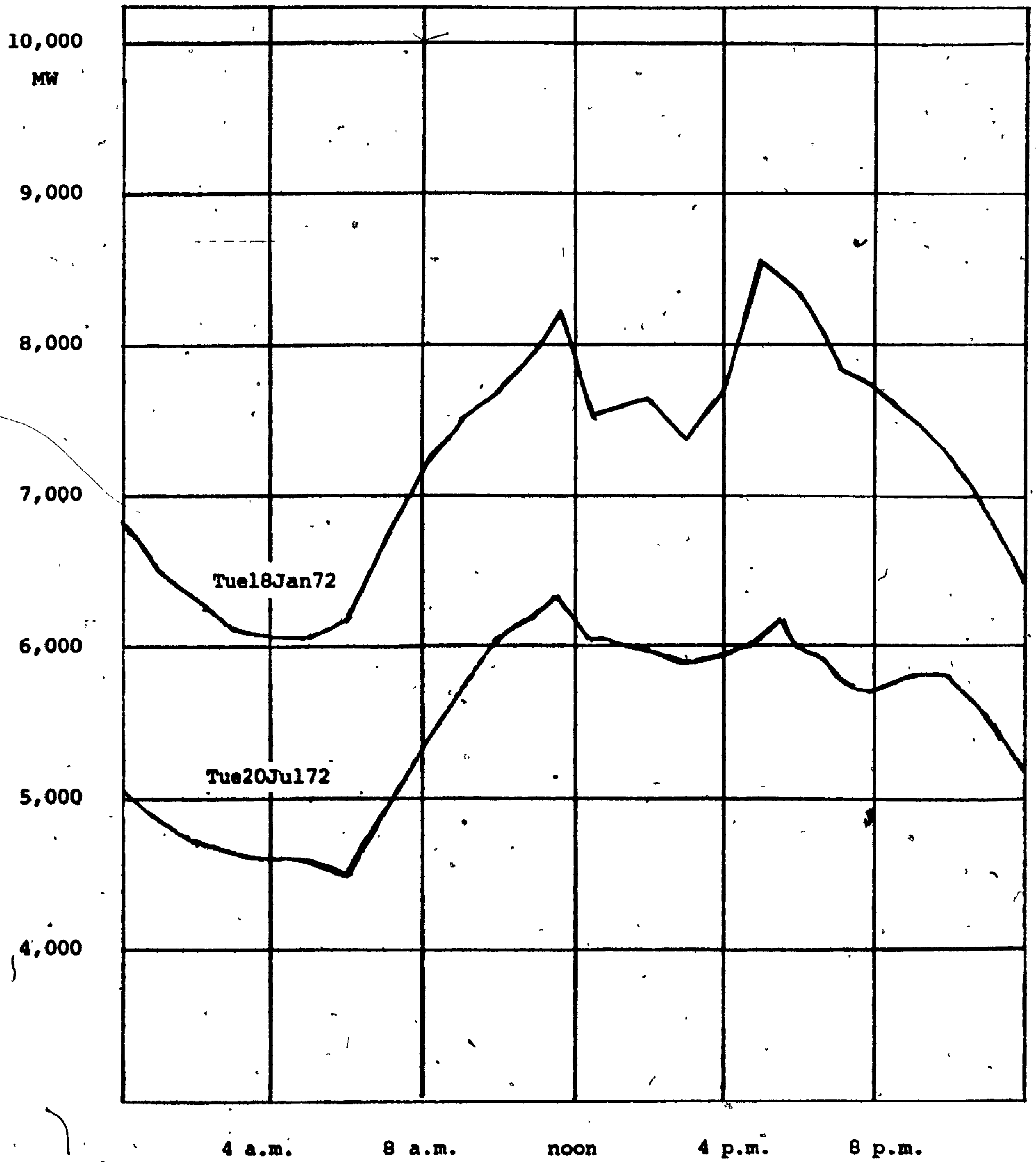

Fig. 1.5 - Hydro-Québec Primary Load

and Fridays only will be considered in the forecast. Mondays, weekends and holidays would require different models.

For analysis of the basic forecast, some other experiments will be undertaken, still limited to Tuesdays, Wednesdays, Thursdays and Fridays of January 1972.

In Chapter 2, some forecasting methods will be described briefly, giving references so that further information can be obtained. At the end of Chapter 2, some statistical checking methods will be listed. Specific statistical checking, however, will not be done in this thesis.

Chapter 3 will describe the selected forecasting method in detail. Results will be presented in Chapter 4, followed by discussion and conclusions in Chapter 5. Chapter 6 will close with suggestions for modifications and extensions based on results obtained, and for further work.

CHAPTER 2

SOME SHORT-TERM LOAD FORECASTING METHODS

2.1 General Remarks

No attempt will be made to cover all possibilities of short-term forecasting methods, nor combinations of them, because there are many, nor to describe any in full detail. References given, however, will permit obtaining further information. Reference [25] in particular outlines existing methods, giving other references. Methods confined to forecasting of peak load are given in references [6, 7], but will not be covered here.

In general, short-term load forecasting involves past load data, past weather data, and usually forecasted weather. Methods vary in emphasis placed on load correlation with past load versus weather factors. Because weather forecasts may not be available in the appropriate form and with desired accuracy, some authors [1, 2] advocate forecasting without using weather forecasts at all.

Early methods relied heavily on hand calculations and human judgement. More recently, emphasis has turned to computer computation with provision for human intervention under abnormal conditions.

It is important for an operator to recognize when conditions become abnormal unexpectedly. For this purpose among others he has use for a continual calculation of the variance of forecast error. Earlier methods concentrated on the load forecast without regard to its credibility. More recently, the demand for an indicator of forecasting accuracy has been recognized.

Some methods treat the load data as a time series to be extrapolated. Alternatively, the load is represented by mathematical functions with the concept of a load system containing a process having inputs and outputs as in Fig. 2.1. If the degree of reliability of the forecast is to be estimated, system noise and/or measurement noise are inputs to this process. For an electric power utility specifically, the input may be weather factors, and the output load. Once the form of mathematical representation has been decided, its parameters must be estimated, and this parameter estimation becomes the crux of the entire forecasting procedure. The parameters can be estimated using explicit calculation, correlation techniques [10] ~~for~~ example, in which case an analog rather than digital computer may be preferable.

An additional concept can be added: the process can be modelled. The mathematical model is fed input equivalent to that of the process (see Fig. 2.2). The output of the model is compared with the system output, the difference between these outputs being the error.

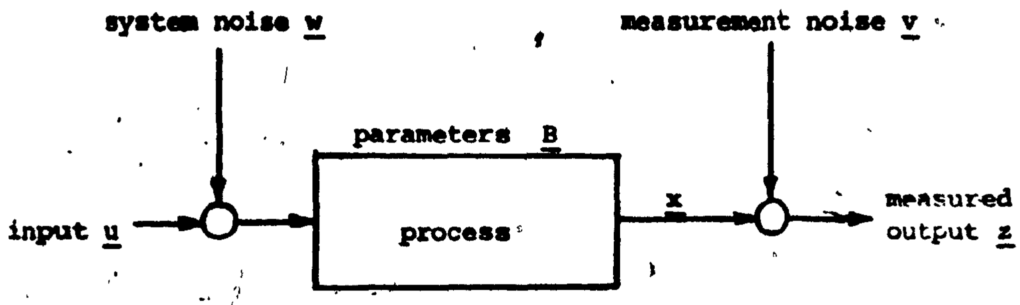

Fig. 2.1 - Process with System Parameters B, Input u, Measured Output z

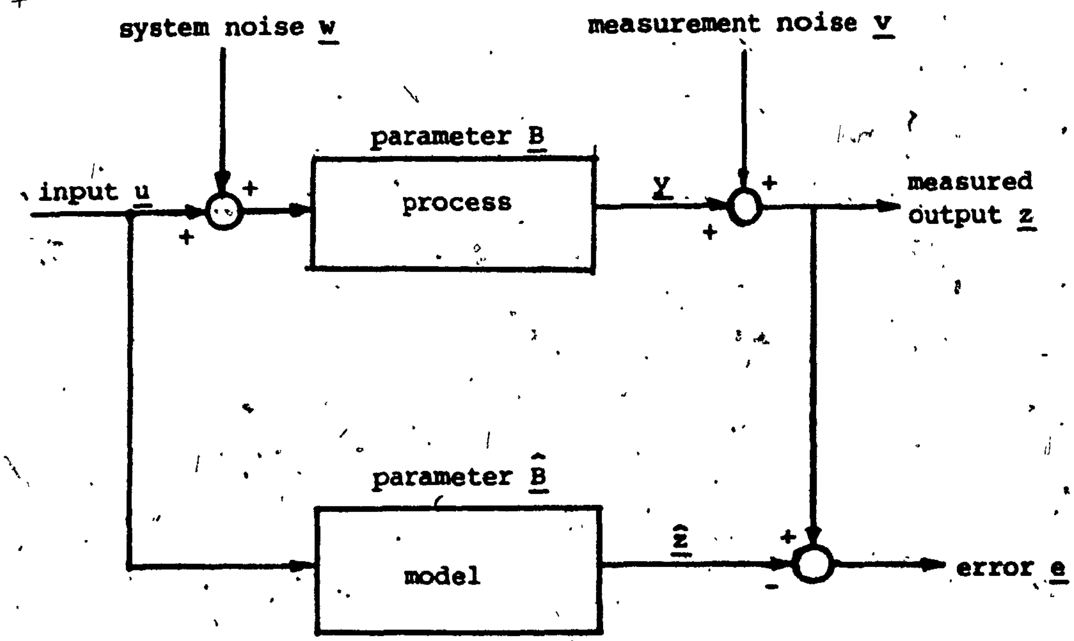

Fig. 2.2 - Modelling of a Process

The object is to obtain a mathematical model of the appropriate form in the first place, then to adjust the parameters of this model so as best to match the system.

In this thesis, the iterative adjustment of parameters, after a mathematical structure has been chosen, will be called "identification". This term will be used for convenience even though uncertainty is involved and the model has not necessarily been optimized. Identification can be done using an analog computer if parameter adjustment during solution is kept slow. Usually, however, a digital computer is employed.

As we said, the parameters are to be identified so that the model best matches the system. Criteria for this identification depend on the amount of a priori statistical information available and used concerning

(a) effect of parameter error,

(b) the parameters themselves, and

(c) the noise.

In the order of greatest information used, the identifications are called Bayes (or minimum risk), maximum likelihood, Markov or least squares [10]. In many cases no statistical information is available a priori, so that the least squares identification is used, perhaps with the statement being made that maximum likelihood applies if certain conditions are found to pertain. Least squares estimation simply means identification so that the mean square of the error is minimized. For a fuller treatment on this subject, see for example Reference [10].

2.2 Combination of Base Load and Weather Factors

In the simplest form, combining base load with weather forecasts [1], a linear combination of the base load for the particular day of a particular week and of the various forecasted weather factors is used. The base load is estimated by "vertical" averaging of past load.

The coefficients of the weather factors are determined [1] as follows:

Load on a particular system is represented as:

$$z = a + b_1T + b_2W + b_3L + b_4P + F(t) + d \qquad (2.1)$$

where z is load at a chosen time of day;

a, b_1, b_2, b_3 and b_4 are constants;

d is a day-of-the-week correction;

T is temperature;

W represents cooling power of wind;

L is illumination index;

P is precipitation rate; and

F(t) is a polynomial function of the time of year for a particular week. It accounts for variations in the base load with the time of year. Thus, a+F(t) is the base load for the t-th week.

The coefficients b_1, b_2, b_3 and b_4 and the day-of-the-week correction d then are found by linear regression, from known past values of the dependent variable z and a priori information on a+F(t). Having solved for these coefficients, eqn. (2.1) is extrapolated, using forecasted values of T, W, L and P to obtain the forecasts of z.

If it is considered necessary to use non-linear coefficients, it is possible to obtain a graphical solution [1], using an equation similar to (2.1) having coefficients representing non-linear effects of the various weather factors on load.

2.3 Spectral Expansion

2.3.1 Forecasting load without weather forecasts

Load forecasting by spectral expansion, in its original form, involved past weather data but no weather forecasts. Since the basic load pattern tends to repeat itself each day, it is possible to consider the time series for each day or for overlapping segments of a day as being a member of an ensemble of time series. In this way we have a non-stationary process made up of an ensemble of sample functions.

In the simplest form, the time series for a whole day is made up of loads at equal intervals during the day, that is, the day consists of a single segment. If the intervals are one hour, load on the k-th day at the t-th hour is:

$$z(k,t) = a(k,t) + f_1(T_k)\, b(k,t) + f_2(L_k)\, c(k,t) + \dots \tag{2.2}$$

where $f_1(T_k)$, $f_2(L_k)$, etc. are functions of temperature T_k, illumination L_k, etc.;

$a(k,t)$ is base load; and

the factors $b(k,t)$, $c(k,t)$, etc., allow for the varying importance of the weather parameters with the 24 hours-of-day.

Time series for adjacent weekdays are constructed in this manner. Thus, each load vector is linearly dependent upon the vectors $\underline{a}$, $\underline{b}$, $\underline{c}$, etc.

All these elements then are approximated in the form of a spectral expansion [1] with an error term. Now, putting the load elements $z(k,t)$ in the form of a matrix $\underline{Z}(k,t)$ and at the same time retaining them and the error, etc., in spectral expansion, a matrix equation is obtained. It can be shown [1] that least squares minimization of the error results in a matrix eigenvector equation. Furthermore, its eigenvalues and eigenvectors enable identification of the 24 sets of coefficients $a(k,t)$, $b(k,t)$, etc., and evaluation of $z(k,t)$ for each hour of the k-th day, where k corresponds to the current day. Thus, hourly load is evaluated for the current 24-hour day, meaning that hourly load forecasts can be made with lead times up to 24 hours.

2.3.2 Forecasting load using weather forecasts

As mentioned, the above method uses past weather data, but no weather forecasts. Lijesen and Rosing [20] have made an important modification in forecasting New Brunswick Electric Power Commission load. Load is modelled as the sum of three components, two comprising base load, with weather effects accounted for in the residual component. Spectral expansion is applied to this residual, but in such a way as to incorporate the average forecasted weather effects for the particular day as a whole, at the same time making the proper hourly distribution of these forecasted effects on the basis of recent weather data.

2.4 Adaptive Parameters Applied to a Fourier Series Fitting Function

A method using no weather forecasts has been employed by Christiaanse [2]. The load is modelled by a fitting function plus a noise component. The noise is substantially separated from the weekly trend of load by taking a one-week moving average, which average eliminates all cyclic variations with periods of an integral number of weeks. This moving average is then subtracted from the load at each hour. The function fitted to this difference (which does contain the weekly cyclic variation) is a Fourier series with a period of one week. Weekends are included for ease of implementation. The series represents the weekly cycle of the hourly load, so that load at time t is estimated as

$$\hat{z}(t) = \hat{y}_p(t) + \text{error} \qquad (2.3a)$$

where $\hat{y}_p(t) = c + \sum_{i=1}^{n_p} [a_i \sin(2\pi/168)i\,t$

$$+ b_i \cos(2\pi/168)i\,t]$$

or

$$\hat{y}_p(t) = \underline{a}^T(T) \cdot \underline{f}(t) \qquad (2.3b)$$

where $\hat{y}_p(t)$ is the Fourier series approximating the periodic component of load $z(t)$ at time t in integral hours ..., -1, 0, 1, 2, ...;

T is the current hour, i.e. the last hour for which load is known; and

$\underline{f}(t) = [\sin \omega t, \cos \omega t, \ldots, \sin n_p \omega t, \cos n_p \omega t]^T$

with $\omega = 2\pi/168$.

The parameters forming $\underline{a}(T)$ are identified by weighted least squares, i.e. by minimization of

$$\sum_{j=0}^{N} W^j \, [z(T-j) - \underline{a}^T(T) \, \underline{f}(t-j)]^2$$

where W is the weighting constant [2]; and

N is a large integer.

After this initial identification, the parameters are updated <u>hourly</u>, using a transition matrix and smoothing vector calculated on-line.

In the fitting function it is not necessary to include each harmonic from 1 to n_p. The most useful harmonics can be ascertained from the power spectrum, as in Fig. 4 of Reference [2]. Final selection of harmonics to be used, along with best value of weighting constant W, are determined by trial-and-error forecasts.

Hourly load then is modelled as:

$$z(t) = \underline{a}^T \, \underline{f}(t) + e(t) \qquad (2.4a)$$

where $e(t)$ is noise, or error of the modelled load;

$z(t)$ is estimated load at time t.

The corresponding forecast algorithm is:

$$z_\ell(T+\ell) = \underline{a}^T(T) \, \underline{f}(T+\ell) \qquad (2.4b)$$

where ℓ is lead time.

In the above algorithm, the locally-constant coefficients $\underline{a}(T)$ are updated hourly according to:

$$\underline{a}(T) = \underline{L}^T \underline{a}(T-1) + \underline{h} [z(T) - \hat{z}(T)] \tag{2.5}$$

where $\underline{h}$ is a smoothing vector obtainable from Z transforms using:

$$\underline{h} = [\sum_{j=0}^{\infty} W^j \underline{f}(-j) \underline{f}^T(-j)]^{-1} \underline{f}(0) \tag{2.6}$$

and

$\underline{L}$ is the transition matrix defined by

$$\underline{f}(t) = \underline{L} \underline{f}(t-1)$$

where, for Fourier fitting function,

$$\underline{L} = \begin{bmatrix} 1 & & & & \\ & \begin{matrix} \cos \omega_1, & \sin \omega_1 \\ -\sin \omega_1, & \cos \omega_1 \end{matrix} & & & \\ & & \begin{matrix} \cos \omega_2, & \sin \omega_2 \\ -\sin \omega_2, & \cos \omega_2 \end{matrix} & & \\ & & & \ddots & \\ & & & & \begin{matrix} \cos \omega_m, & \sin \omega_m \\ -\sin \omega_m, & \cos \omega_m \end{matrix} \end{bmatrix} \tag{2.7}$$

2.5 The BLS Seasonal Factor Method (1966), U.S. Department of Labor

The BLS seasonal factor method (1966) is utilized by the Bureau of Labor Statistics [14], U.S. Department of Labor, for seasonal correction of unemployment statistics. It comprises:

use of moving averages to separate trend-cycle from seasonal and irregular in a multiplicative model;

use of vertical averaging to separate seasonal from irregular;

extending centered moving averages closer to the two ends of available data by approximation from data for the most recent year;

searching of the irregular for trend-cycle component;

using weighted moving averages;

substitution for outliers; and

calculation of mean and variance of the residuals to calculate "credence factors".

The calculations are done in three phases, each with two iterations, so that the final seasonally adjusted data are a sixth approximation. The whole operation was designed for a small computer (IBM 1401 or 1460) with two tape units.

The computer program is available from the U.S. Department of Labor. It is used by Ontario Hydro for load forecasting: past load data, corrected for weather factors, are treated as a time series and separated into trend, periodics, and irregular. Load is then forecasted by compensating the trend and periodics for forecasted weather factors. The load forecasts are made for key times-of-day and are interpolated.

2.6 Box and Jenkins Approach

2.6.1 General remarks

Box and Jenkins methods [19] belong in any summary involving processing of statistics. In general, their approach [9] is to examine the data for irregularities that would affect results; suppress, remove, or compensate for these irregularities; use the modified data to get results; check the results statistically; and modify or repeat, if required.

2.6.2 Identification

In particular, for mathematical simulation of a process the auto- and cross-correlation functions of the input and measured output are examined. Differencing of data is carried out so that the correlation functions are stable, indicating stationarity. Stability is considered satisfactory when any oscillations of a correlation function versus sample time difference decrease rapidly with increase in absolute value of this time difference. The data thus modified are transformed by a whitening filter, designed using the auto-correlation function of the modified input. The cross-correlation function of the transformed input and output then is proportional to the impulse response, leading after similar treatment of estimated noise to an approximate mathematical model of the process, including rough parameter identification. Values of the parameters then may be improved if required, using an iterative procedure. Statistical checks are applied to the auto-correlation of the residuals and to their cross-correlation with the above transformed input.

2.6.3 Forecasting

The process thus is represented by a diff[illegible] equation with parameters identified. For forecas[illegible] Box and Jenkins apply this difference equation directly [19], always using the most up-to-date information step by step. From this fundamental application of the difference equation, two other methods have been developed for forecasting, one using integrated current and past random shocks and the other using weighted average of past data. These latter two methods are useful in providing insight [24] into the nature of the forecast.

Box and Jenkins methods ~~have~~ been applied to load forecasting of the New Brunswick Electric Power Commission load [24].

2.7 Kalman Filter State Estimation and Prediction

The Kalman filtering and prediction algorithms are applied, after a model of the system has been obtained, with parameters identified. After such identification, the model equations are put in state space form so that existing theory formulated by Kalman [12] can be directly applied. Use of the state form is natural also when we remember that the state of a system at any time is defined [22] as the minimum set of numbers which, along with all subsequent inputs to the system, is sufficient to determine the subsequent behaviour of the system.

Using the Kalman filtering algorithms [2], the correct state is estimated, taking into account all available past values of the state ("infinite memory"). Then, using the Kalman prediction algorithms, the future state at any chosen time (though one step at a time may be all that is desired) is forecasted. The future output is calculated from the state. Basically, the Kalman filter adjusts the initial estimate of the state in proportion to the error between the newly available output and the output calculated from this initial estimate of the state, the factor of proportionality (Kalman gain) being dependent on the covariance of the error. Since the Kalman filter method will be actually used in this work, all the pertinent equations and algorithms for filtering and prediction as well as for identification will be covered in detail in the following chapter.

2.8 Statistical Checks

In any of the above methods of analyzing data, after the results including residuals are obtained, the auto-correlation function of the residuals as well as the cross-correlations of the residual with the input and with the output of the process can be examined to see if they conform at least approximately with the conditions assumed for the method. Auto-correlations of the residuals can be used to evaluate "goodness of fit" quantitatively, and furthermore to modify the model where required. Such techniques are described by Box and Jenkins [19].

Other techniques described by Box and Jenkins are:

overfitting which will assess whether a model is of correct order, for example;

change in parameter values; and

the cumulative periodogram to check for periodicity remaining in the residuals.

CHAPTER 3

METHOD TO BE USED

3.1 General Description

A mathematical model [3] will be used for the load system, as previously suggested, consisting of a periodic component $y_p(t)$ represented by a Fourier series, and a stochastic residual component represented by an auto-regressive moving average series. This second part will contain system noise, assumed to be white (uncorrelated), and a non-linear temperature-dependent input $u(t)$. That is,

$$z(t) = y_p(t) + y(t) \tag{3.1a}$$

$$= y_p(t) + [\sum_{i=1}^{n} a_i\, y(t-i) + \sum_{j=0}^{m} b_j\, u(t-j) + w(t-1)] \tag{3.1b}$$

The difference between the measured load and the output of this model will be the error. In performing identification, this error will be minimized in the mean square sense, using starting values of parameters and adjusting them on the basis of the gradient vector of the error with respect to the parameters. For this parameter adjustment, the Fletcher Powell scientific subroutine will be used, solving for all the parameters simultaneously. While a mean square minimization will be used, a maximum likelihood estimate of parameters will result, as long as the system noise can be considered Gaussian and the data sample is large enough.

After the parameters of the model are thus identified, the model will be used in state space form, together with the Kalman filter prediction equations, to predict the state of the model. From this forecasted state, the forecasted load will be calculated.

3.2 The Load Model

In more detail, the mathematical model [3] of the system load will be as follows:

$$z(t) = y_p(t) + y(t) \qquad (3.2a)$$

where

$$y_p(t) = [x_p^0 + \sum_{i=1}^{n_p} \{ x_p^i \sin(2\pi i/24)\ t$$

$$+ x_p^{n_p+i} + \cos\ (2\pi i/24)\ t\}] \qquad (3.2b)$$

and

$$y(t) = [\sum_{i=1}^{n} a_i\ y(t-i) + \sum_{j=0}^{m} b_j\ u(t-j) + w(t-1)] \qquad (3.2c)$$

and $z(t)$ is load in megawatts at hour t;

$y_p(t)$ is the periodic deterministic component at hour t;

$y(t)$ is the stochastic component at hour t;

x_p^0 is the constant term of the Fourier series periodic component;

x_p^i are the n_p sine coefficients of this Fourier series having a period of 24 hours;

$x_p^{n_p+i}$ are the n_p cosine coefficients of the Fourier series;

a_i are the n auto-regressive coefficients of the "ARMA" series in y(t);

b_i are the m+1 coefficients of the "ARMA" series temperature-dependent input u(t-j);and

w(t-1) is white noise of variance Q.

The parameters to be identified are the sets

$$\{a_i, b_j, x_p^i, x_p^{n_p+i}\}$$

as well as the scalar Q. These parameters will vary with the seasons, but will be assumed constant throughout the period supplying data for identification of parameters plus the maximum lead time of the forecast, that is, over a period of about four weeks. The input u(t) is a non-linear temperature-dependent variable to be described in more detail under "Processing of Raw Data".

3.3 The Model in State Space Form

The model will be put in a state space form suggested by Meditch [16]. This form is selected because the state variables correspond closely to the physical variables, helping in visualization of the system. The stochastic portion, $\underline{x}(t)$, of the state vector will be formulated first, adding the periodic component afterwards.

Meditch's state equation considering only the stochastic portion of the state variable is:

$$\underline{x}(t) = \underline{A}\,\underline{x}(t-1) + \underline{H}\,\underline{u}(t-1) + \underline{D}\,\underline{w}(t-1) \tag{3.3a}$$

where the stochastic state variable $\underline{x}(t)$ is an n vector

$\underline{A}$ is an nxn matrix;

$\underline{H}$ is an nxr matrix;

$\underline{D}$ is an nxp matrix;

$\underline{u}(t)$ is an r vector; and

$\underline{w}(t)$ is a p vector.

Choosing $\underline{H}$ and $\underline{A}$ as follows,

$$\underline{H} = \begin{bmatrix} h_1 & h_2 & \cdot & \cdot & h_r \\ 0 & 0 & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & \cdot & \cdot & 0 \end{bmatrix} \tag{3.3b}$$

$$\underline{A} = \begin{bmatrix} a_1 & a_2 & \cdot & \cdot & a_n \\ 1 & 0 & \cdot & \cdot & 0 \\ 0 & 1 & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & \cdot & 1 & 0 \end{bmatrix} \tag{3.3c}$$

the stochastic state equation becomes:

$$\begin{bmatrix} x_1(t) \\ \cdot \\ \cdot \\ \cdot \\ x_n(t) \end{bmatrix} = \begin{bmatrix} a_1 & a_2 & \cdot & \cdot & a_n \\ 1 & 0 & \cdot & \cdot & 0 \\ 0 & 1 & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & \cdot & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t-1) \\ \cdot \\ \cdot \\ \cdot \\ x_n(t-1) \end{bmatrix} +$$

$$\begin{bmatrix} h_1 & h_2 & \cdot & \cdot & h_r \\ 0 & \cdot & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & \cdot & \cdot & 0 \end{bmatrix} \begin{bmatrix} u_1(t-1) \\ \cdot \\ \cdot \\ \cdot \\ u_r(t-1) \end{bmatrix} +$$

$$\begin{bmatrix} d_{11} & d_{12} & \cdot & \cdot & d_{1p} \\ d_{21} & \cdot & \cdot & \cdot & \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ d_{n1} & \cdot & \cdot & \cdot & d_{np} \end{bmatrix} \begin{bmatrix} w_1(t-1) \\ \cdot \\ \cdot \\ \cdot \\ w_p(t) \end{bmatrix} \tag{3.4}$$

or, expanding the above further,

$$\begin{aligned} x_1(t) = {} & a_1x_1(t-1) + a_2x_2(t-1) + \cdot\cdot\cdot + a_nx_n(t-1) \\ & + h_1u_1(t-1) + h_2u_2(t-1) + \cdot\cdot\cdot + h_ru_r(t-1) \\ & + d_{11}w_1(t-1) + d_{12}w_2(t-1) + \cdot\cdot\cdot + d_{1p}w_p(t-1) \end{aligned} \tag{3.5a}$$

$$x_2(t) = x_1(t-1) + d_{21}w_1(t-1) + \ldots + d_{2p}w_p(t-1) \tag{3.5b}$$

$$x_3(t) = x_2(t-1) + d_{31}w_1(t-1) + \ldots + d_{3p}w_p(t-1) \tag{3.5c}$$

$$\begin{array}{cccc} \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \end{array}$$

$$x_n(t) = x_{n-1}(t-1) + d_{n1}w_1(t-1) + \ldots + d_{np}w_p(t-1) \tag{3.5d}$$

Now we expand eqn. (3.2c) to show the residual component of $z(t)$ as a difference equation in expanded form:

$$y(t) = a_1y(t-1) + a_2y(t-2) + \ldots + a_ny(t-n) + b_0u(t) + b_1u(t-1) + \ldots + b_mu(t-m) + w(t-1) \tag{3.6}$$

and we see by comparing eqns. (3.5) and (3.6) that eqn. 3.6 is equivalent to the stochastic state space equation, provided that:

$$x_1(t) = y(t) \tag{3.7}$$

$$\begin{bmatrix} x_1(t-1) \\ x_2(t-1) \\ \cdot \\ x_n(t-1) \end{bmatrix} = \begin{bmatrix} y(t-1) \\ y(t-2) \\ \cdot \\ y(t-n) \end{bmatrix} \tag{3.8}$$

$$\begin{bmatrix} u_1(t-1) \\ u_2(t-1) \\ \cdot \\ u_r(t-1) \end{bmatrix} = \begin{bmatrix} u(t) \\ u(t-1) \\ \cdot \\ u(t-m) \end{bmatrix} \quad \text{where } r = m+1, \tag{3.9}$$

$$\underline{H} = \begin{bmatrix} h_1 & h_2 & \cdot & \cdot & h_r \\ 0 & 0 & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & \cdot & \cdot & \cdot & 0 \end{bmatrix} = \begin{bmatrix} b_0 & b_1 & \cdot & \cdot & b_m \\ 0 & 0 & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & \cdot & \cdot & \cdot & 0 \end{bmatrix} = \underline{B}$$

(3.10)

$$\underline{D} = \begin{bmatrix} d_{11} & d_{12} & \cdot & \cdot & d_{1p} \\ d_{21} & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ d_{n1} & \cdot & \cdot & \cdot & d_{np} \end{bmatrix} \begin{bmatrix} d_{11} & 0 & \cdot & \cdot & 0 \\ 0 & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & \cdot & \cdot & \cdot & 0 \end{bmatrix}, \tag{3.11a}$$

$$\text{and} \quad \begin{bmatrix} w_1(t-1) \\ w_2(t-1) \\ \cdot \\ \cdot \\ w_p(t-1) \end{bmatrix} = \begin{bmatrix} w(t-1) \\ 0 \\ \cdot \\ \cdot \\ 0 \end{bmatrix}, \tag{3.12a}$$

or, for a simpler equivalent to eqns. (3.11a), (3.12a) use:

$$\underline{d} = \begin{bmatrix} d_1 \\ 0 \\ \cdot \\ \cdot \\ 0 \end{bmatrix}, \tag{3.11b}$$

and w(t-1), scalar. (3.12b)

To assist in computer implementation, we shall make $m = n-1$. This equality can always hold by using some zero vector components if necessary. Also, $d_1 = 1$ is satisfactory. To summarize then, the state space equation representing the stochastic portion of the state space variable is:

$$\underline{x}(t) = \underline{A}\ \underline{x}(t-1) + \underline{B}\ \underline{u}(t-1) + \underline{d}\ w(t-1) \tag{3.13}$$

Hence, advancing one hour in time, recalling from eqn. (3.2) that $y_p(t)$ is deterministic, meaning that $\underline{x}_p$ is constant while calculations are performed, and adding an equation to define the measured output as a linear function of the state variables, we have:

$$\underline{x}_p(t+1) = \underline{x}_p(t) \tag{3.14a}$$

$$\underline{x}(t+1) = \underline{A}\ \underline{x}(t) + \underline{B}\ \underline{u}(t) + \underline{d}\ w(t) \tag{3.14b}$$

$$z(t) = \underline{\phi}^T(t)\ \underline{x}_p(t) + \underline{c}^T\ \underline{x}(t) \tag{3.14c}$$

Where the complete state space variable is:

$$\begin{bmatrix} x_p \\ \underline{x}(t) \end{bmatrix} = [x_p^0, x_p^1, \ldots, x^{2n_p}, x_1(t), x_2(t), \ldots, x_n(t)]^T \qquad (3.14d)$$

$$\underline{A} = \begin{bmatrix} a_1 & a_2 & . & . & a_n \\ 1 & 0 & . & . & 0 \\ 0 & 1 & . & . & 0 \\ . & . & . & . & . \\ . & . & . & 1 & 0 \end{bmatrix}, \qquad (3.14e)$$

$$\underline{B} = \begin{bmatrix} b_0 & b_1 & . & . & b_n \\ 0 & . & . & . & 0 \\ . & . & . & . & . \\ 0 & . & . & . & 0 \end{bmatrix}, \qquad (3.14f)$$

$$\underline{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ . \\ x_n(t) \end{bmatrix} = \begin{bmatrix} y(t) \\ y(t-1) \\ . \\ y(t-n+1) \end{bmatrix}, \qquad (3.14g)$$

$$\underline{u}(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \\ . \\ u_n(t) \end{bmatrix} = \begin{bmatrix} u(t+1) \\ u(t) \\ . \\ u(t-n+2) \end{bmatrix}, \qquad (3.14h)$$

$$\underline{c} = \underline{d} = \begin{bmatrix} 1 \\ 0 \\ \cdot \\ 0 \end{bmatrix}$$, this value of $\underline{c}$ being required to satisfy eqn. (3.2) (3.14i)

$$\underline{\phi}(t) = \begin{bmatrix} 1 \\ \sin\ (2\pi/24)\ t \\ \sin\ (2\pi/24)\ 2t \\ \cdot \\ \sin\ (2\pi/24)\ n_p t \\ \cos\ (2\pi/24)\ t \\ \cdot \\ \cos\ (2\pi/24)\ 2n_p t \end{bmatrix}, \quad \text{and} \tag{3.14j}$$

$$\text{and } \underline{x}_p = \begin{bmatrix} x_p^0 \\ \cdot \\ \cdot \\ \cdot \\ x_p^{2n_p} \end{bmatrix} \tag{3.14k}$$

For convenience we shall define also

$$\underline{a} = [a_1, a_2, \ldots, a_n]^T \tag{3.14l}$$

$$\underline{b} = [b_0, b_1, \ldots, b_{n-1}]^T \tag{3.14m}$$

3.4 Raw Data Required

The measured hourly load z(t) and hourly values of the non-linear temperature-dependent function u(t) will be required data, the latter being calculated from raw temperature data. For parameter identification, the above data will be required for three weeks (Tuesdays to Fridays only), preceeding the week into which the basic load forecast is to be made, that is, for the first three weeks of January 1972. In addition, however, the data will be required for the fourth (4-day) week of January 1972. These data for the fourth week will consist of u(t) as perfect temperature forecasts, and z(t) for evaluating the load forecast. Thus we are assuming the system is statistically stationary over a four-week period, or nearly so, and at the same time that the three-week period will give a large enough statistical sample (288 data points) to be representative. The three-week sample period, for identification is the same as that used by Galiana [3] and compares with four weeks found optimum by Matthewman and Nicholson [1], using the spectral expansion technique in their case.

Such hourly load data have been obtained from Hydro-Québec for January 1972. To calculate hourly values of u(t), however, we need not only hourly temperatures for the four weeks of January 1972, but for each of the ten Januarys from 1962 to 1971 inclusive. This more extensive requirement is due to the fact that ten-year hourly average temperatures will be used in calculating values of u(t). An eleven-year record of hourly temperatures for January 1962 to January 1972 at Dorval, therefore, has been obtained from the Department of Atmospheric Environment. Dorval temperatures are considered representative because so much of the Hydro-Québec system load is concentrated around Montreal, of which Dorval is a suburb.

3.5 Processing of Raw Data

Ten-year hourly average January temperatures will be calculated from the hourly temperatures for January 1962 to 1971 inclusive. To obtain a smoother progression of temperatures from hour to hour, seven-day moving averages of these ten-year averages will be taken. We shall call the resulting data "smoothed ten-year hourly average temperatures", or, for use in this thesis, simply "normal temperatures $\hat{T}$". For example, normal temperature for 10 a.m. on January 15, 1972, would be obtained as one seventh of the sum of ten-year averages for 10 a.m. on January 12 to 18 inclusive. Such moving averages are justified on the basis that seasonal temperature change during a one-week interval is quite small, in fact the change [5] is about 5°F for the entire month of January.

As mentioned in eqn. (3.2), the load model contains a non-linear function, u(t), described by Galiana [3] on a chart divided into nine classes, depending on whether actual temperature T and normal temperature $\hat{T}$ are individually below 60°F, above 70°F, or between these two values. Here, however, we shall define $u(t)$ algebraically rather than on a chart, as follows:

The deviation of heating or cooling effect from that encountered at normal temperature is

$$u(t) = u_T(t) - u_{\hat{T}}(t) \tag{3.15a}$$

where actual heating or cooling effect is

$$\begin{aligned} u_T(t) &= T - 70.0,\ T > 70.0 \quad \text{as cooling} \\ &= 60.0 - T,\ T < 60.0 \quad \text{as heating} \\ &= 0.0, \text{ otherwise} \end{aligned} \tag{3.15b}$$

and correspondingly normal heating or cooling effect is

$$\begin{aligned} u_{\hat{T}}(t) &= \hat{T} - 70.0,\ \hat{T} > 70.0 \quad \text{as cooling} \\ &= 60.0 - \hat{T},\ \hat{T} < 60.0 \quad \text{as heating} \\ &= 0.0, \text{ otherwise.} \end{aligned} \tag{3.15c}$$

3.6 Starting Values for Parameters

Accuracy of parameter starting values obviously will affect the amount of computer time necessary to obtain convergence. On implementing the computer programming, we would expect to re-calculate parameters weekly, using parameters calculated one week as starting values for the next calculation, and relying on this procedure for satisfactory computing time.

The Fletcher Powell scientific subroutine does not guarantee a global but only a local minimum (of J described in the following section) on successful convergence. Therefore, we want a set of reasonable starting values for the initial calculation.

We plan to use 19 parameters in our model: a_1, a_2, b_0, b_1, x_p^0, x_p^1, . . ., x_p^{14}, meaning second order in $\underline{a}$ and $\underline{b}$ and seven harmonics for $\underline{x}_p$. Optimization could require different model order, but as mentioned in Section 1.5, we are arbitrarily choosing model order.

To estimate the effect of the temperature-dependent variable u(t) on load, 24-hour averages of load z(t) will be compared with the corresponding 24-hour averages of u(t) for each of the 12 days in the three-week period described in section 3.4. Referring to eqn. (3.2), 24-hour averages are chosen, to average out the sine and cosine terms, concentrating the effect of u(t) on z(t). For our purpose we can arbitrarily consider $b_1 = 0.0$ as a starting value. A rough value of b_0 then can be obtained from the above 24-hour averages, by linear regression [17].

Having this rough value of b_0, load compensated for temperature effect can be obtained as $z(t) - b_0\, u(t)$ for each of the 288 data points in our sample. The next step is to obtain "vertical" averages for each of the 24 hours of day. For example, to obtain the vertical average for 10 a.m., $z(t) - b_0\, u(t)$ is averaged over the 12 days in our data sample. These 24 hourly values then will contain the periodic component plus a stochastic variable. Their average approximates x_p^0. The rest of the periodic content can be estimated using the Fischer-Hinnen [17] method of selected ordinates, calculating up to the seventh harmonic as planned, but with one exception. This exception is the arbitrary setting of coefficients for fifth and seventh harmonics at zero, to avoid awkward interpolation for ordinates at intervals of fifths and sevenths of a day.

Thus, we obtain rough values for 17 of the planned 19 parameters. Strictly, according to eqn. (3.2), our calculation of b_0 assumed zero combined effect from a_1 and a_2, and we could set $a_1 = a_2 = 0.0$ for starting values. Rather, however, we choose to use rough non-zero values based on results of previous work by Galiana [3].

The actual starting values of the 19 parameters, for the initial calculation, will be given in Chapter 4 (Table 7).

3.7 Identification of Parameters

The recursive relation [3] that will be used to solve for parameter values by minimization of J, the mean square error in estimation of y(t) is:

$$\underline{\theta}_{k+1} = \underline{\theta}_k + \underline{G}_k \; \delta J/\delta\underline{\theta}_k \tag{3.20a}$$

where

$$\underline{\theta} = [x_p^0, \ldots, x_p^{2n_p}, a_1, \ldots, a_n, b_0, \ldots, b_m]^T \tag{3.20b}$$

$$J = 1/N \sum_{t=1}^{N} (z(t) - \underline{\phi}^T(t)\ \underline{x}_p - \underline{a}^T\underline{z}(t) - \underline{b}^T\underline{u}(t+1)$$

$$+ \underline{a}^T\underline{\psi}(t)\underline{x}_p)^2 \tag{3.20c}$$

where

$$\underline{\psi}(t) = \begin{bmatrix} \underline{\phi}(t-1) \\ \cdot \\ \cdot \\ \underline{\phi}(t-n) \end{bmatrix}, \quad \underline{z}(t) = \begin{bmatrix} z(t-1) \\ \cdot \\ \cdot \\ z(t-n) \end{bmatrix},$$

and $\underline{G}_k$ is a matrix determined by the iterative procedure to be used. Other variables and parameters are defined in eqn. (3.14).

As previously stated, the Fletcher Powell double precision scientific subroutine, DFMFP, will be used. As can be seen from eqn. (3.20a), calculation of the gradient of J with respect to each of $\underline{x}_p$, $\underline{a}$ and $\underline{b}$ is necessary. In fact, this calculation must preceed each iteration of the Fletcher Powell subroutine. These required partial derivatives of J for recursive relation (3.20) are:

$$\delta J/\delta \underline{x}_p = 1/N \sum_{t=1}^{N} 2e(t)\ [\underline{\psi}^T(t)\ \underline{a} - \underline{\phi}(t)] \tag{3.21a}$$

$$\delta J/\delta \underline{a} = 1/N \sum_{t=1}^{N} 2e(t)\ [\underline{\psi}(t)\ \underline{x}_p - z(t)] \tag{3.21b}$$

$$\delta J/\delta \underline{b} = 1/N \sum_{t=1}^{N} 2e(t)\ [-\underline{u}(t+1)] \tag{3.21c}$$

where error

$$e(t) = y(t) - \underline{a}^T\ \underline{y}(t) - \underline{b}^T\ \underline{u}(t+1)$$

where

$$\underline{y}(t) = [y(t-1), \ldots, y(t-n)]^T$$

or

$$e(t) = z(t) - \underline{\phi}^T(t)\ \underline{x}_p - a^T\underline{z}(t) - \underline{b}^T\underline{u}(t+1) + \underline{a}^T\underline{\psi}(t)\ \underline{x}_p \tag{3.21d}$$

The Fletcher Powell subroutine DFMFP requires calculation of the above three gradient equations in a user's program. The Fletcher Powell method searches for deepest descent initially, converting later to the Newton-Raphson (second-order) algorithm for close convergence. While solving for the parameters, the value of J is calculated on each iteration. For large N, as can be seen from eqns. (3.20c) and (3.21d), J is the variance of the residuals. Using eqns. (3.141), (3.14m) and (3.14h) to compare eqns. (3.21d) and (3.2c), J, on minimization, will be a close approximation of Q as long as we have a good model with good identification of parameters. This value of Q will be required later for calculating the theoretical co-variance matrix of error in filtering and in predicting.

3.8 Actual Forecasting of Load

3.8.1 Filtering algorithms

The forecasting algorithms begin by filtering of data to suppress noise, providing a more meaningful base from which to project the forecast.

The Kalman filtering algorithms, shown in block diagram form in Fig. 3.1, are well known [12, 16] and are as follows:

$$\underline{S}(t+1) \text{ or } \underline{P}(t+1|t) = \underline{A}\,\underline{P}(t|t)\,\underline{A}^T + \underline{d}\,Q\,\underline{d}^T \tag{3.22a}$$

with starting value $P(0|0)$

$$\underline{K}(t+1) = \underline{S}(t+1)\,\underline{c}\,(\underline{c}^T\,\underline{S}(t+1)\,\underline{c})^{-1} \tag{3.22b}$$

$$\underline{P}(t+1|t+1) = \underline{S}(t+1) - \underline{K}(t+1)\,\underline{c}^T\,\underline{S}(t+1) \tag{3.22c}$$

$$\underline{\hat{x}}(t+1|t) = \underline{A}\,\underline{\hat{x}}(t|t) + \underline{B}\,\underline{u}(t) \tag{3.22d}$$

with starting value $\underline{\hat{x}}(0|0) = [0]$

$$\hat{z}(t+1|t) = \underline{c}^T\,\underline{\hat{x}}(t+1|t) + \underline{\phi}^T(t+1)\,\underline{x}_p \tag{3.22e}$$

$$\underline{\hat{x}}(t+1|t+1) = \underline{\hat{x}}(t+1|t) + \underline{K}(t+1)\,[z(t+1) - \hat{z}(t+1|t)] \tag{3.22f}$$

and the variance of the error in filtering $\hat{z}(t+1|t)$ is

$$V_f = \underline{c}^T \underline{P}(t+1|t)\,\underline{c} \quad \text{or} \quad \underline{c}^T \underline{S}(t+1)\,\underline{c} \qquad (3.22g)$$

In the above algorithms, $\underline{A}$, $\underline{d}$, Q, $\underline{c}$, $\underline{x}(t)$, $\underline{B}$, $\underline{u}(t)$, $z(t)$, $\underline{\phi}(t)$, $\underline{x}_p(t)$ have been defined previously under "The Model in State Space Form", eqn. (3.14). $\underline{P}(t+1|t)$ is the covariance matrix of the filtering error in estimating the state vector at time t+1, given data up to time t.

$\underline{K}(t+1)$ is the "Kalman gain" at time t+1 (the meaning being evident from the algorithms).

$\hat{\underline{x}}(t+1|t)$ is the estimate of the state vector at time t+1, given data on $z(t)$ up to time t.

$\hat{z}(t+1|t)$ is the filtered estimate of $z(t+1)$ at time t+1, given actual data up to time t.

V_f is the variance of the error in filtering $z(t)$.

3.8.2 Prediction algorithms

After filtering, the Kalman prediction algorithms, also well known [12, 16], are applied. The are shown in block form in Fig. 3.2, and listed following:

$$\underline{W}(\ell+n+1) = \underline{P}(\ell+n+1|n) = \underline{A}\,\underline{P}(\ell+n|n)\,\underline{A}^T + \underline{d}\,Q\,\underline{d}^T$$

or

$$\underline{A}\,\underline{W}(\ell+n)\,\underline{A}^T + \underline{d}\,Q\,\underline{d}^T \qquad (3.23a)$$

with starting value, $\underline{P}(n|n)$, the last covariance matrix calculated in filtering n sets of data;

$$\hat{\underline{x}}(\ell+n+1|n) = \underline{A}\,\hat{\underline{x}}(\ell+n|n) + \underline{B}\,\underline{u}(\ell+n) \qquad (3.23b)$$

with starting value, $\underline{x}(n\ n)$, the last stochastic state vector calculated in filtering n sets of data;

$$\hat{z}(\ell+n+1|n) = \underline{c}^T\ \hat{\underline{x}}(\ell+n+1|n) + \underline{\phi}^T(\ell+n+1)\ \underline{x}_p \qquad (3.23c)$$

where, in each of the above three equations, ℓ is lead time in hours.

The variance of error in predicting $\hat{z}(\ell+n+1|n)$ is:

$$V_p = \underline{c}^T\ \underline{W}(\ell+n+1)\,\underline{c} \qquad (3.23d)$$

3.8.3 Reduction of filtering algorithms

Now substituting our values of the measurement and disturbance vectors,

$$\underline{c} = \underline{d} = \begin{bmatrix} 1 \\ 0 \\ \cdot \\ \cdot \\ 0 \end{bmatrix} \text{ as defined in eqn. (3.14i)}$$

and, letting $\underline{P}(t+1|t+1) = \{p_{ij}\} =$

$$\begin{bmatrix} p_{11} & \cdot & \cdot & p_{1j} & \cdot & \cdot & p_{1n} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ p_{i1} & \cdot & \cdot & p_{ij} & \cdot & \cdot & p_{in} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ p_{n1} & \cdot & \cdot & p_{nj} & \cdot & \cdot & p_{nn} \end{bmatrix}$$

$$\underline{S}(t+1) = \{s_{ij}\}$$

$$\underline{W}(t+1) = \{w_{ij}\}$$

$$\underline{K}(t+1) = \begin{bmatrix} k_1 \\ \cdot \\ k_i \\ \cdot \\ k_n \end{bmatrix} (t+1)$$

The filtering algorithms, eqn. (3.22), reduce to:

$$\underline{S}(t+1) = \underline{A}\ \underline{P}(t|t)\ \underline{A}^T + \begin{bmatrix} Q & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ 0 & \cdot & \cdot & 0 \end{bmatrix} \tag{3.24a}$$

$$\underline{K}(t+1) = \begin{bmatrix} 1 \\ s_{21}/s_{11} \\ \cdot \\ \cdot \\ s_{n1}/s_{11} \end{bmatrix} \tag{3.24b}$$

$$\underline{P}(t+1|t+1) = \{p_{ij}\} = \{s_{ij} - k_i\, s_{1j}\}$$

or

$$p_{ij} = s_{ij} - k_i s_{1j}, \quad i = 1, \ldots, n; \; j = 1, \ldots, n \tag{3.24c}$$

$$\hat{\underline{x}}(t+1|t) = \underline{A}\,\hat{\underline{x}}(t|t) + \underline{B}\,\underline{u}(t) \tag{3.24d}$$

with starting value $\hat{\underline{x}}(0|0) = 0$.

$$\hat{z}(t+1|t) = \hat{x}_1(t+1|t) + \underline{\phi}^T(t+1)\,\underline{x}_p \tag{3.24e}$$

$$\hat{\underline{x}}(t+1|t+1) = \hat{\underline{x}}(t+1|t) + \underline{K}(t+1)\,[z(t+1) - \hat{z}(t+1|t)] \tag{3.24f}$$

$$V_f = s_{11} \tag{3.24g}$$

3.8.4 Reduction of prediction algorithms

Similarly, the prediction algorithms, eqn. (3.23), reduce to:

$$\underline{W}(\ell+n+1) = \underline{A}\,\underline{W}(\ell+n)\,\underline{A}^T + \begin{bmatrix} Q & \cdot & 0 \\ \cdot & \cdot & \cdot \\ 0 & \cdot & 0 \end{bmatrix} \tag{3.25a}$$

$$\hat{\underline{x}}(\ell+n+1|n) = \underline{A}\,\hat{\underline{x}}(\ell+n|n) + \underline{B}\,\underline{u}(n) \tag{3.25b}$$

with starting value $\hat{\underline{x}}(n|n)$

$$\hat{z}(\ell+n+1|n) = \hat{x}_1(\ell+n+1|n) + \underline{\phi}^T(\ell+n+1)\,\underline{x}_p \tag{3.25c}$$

$$V_p = w_{11} \tag{3.25d}$$

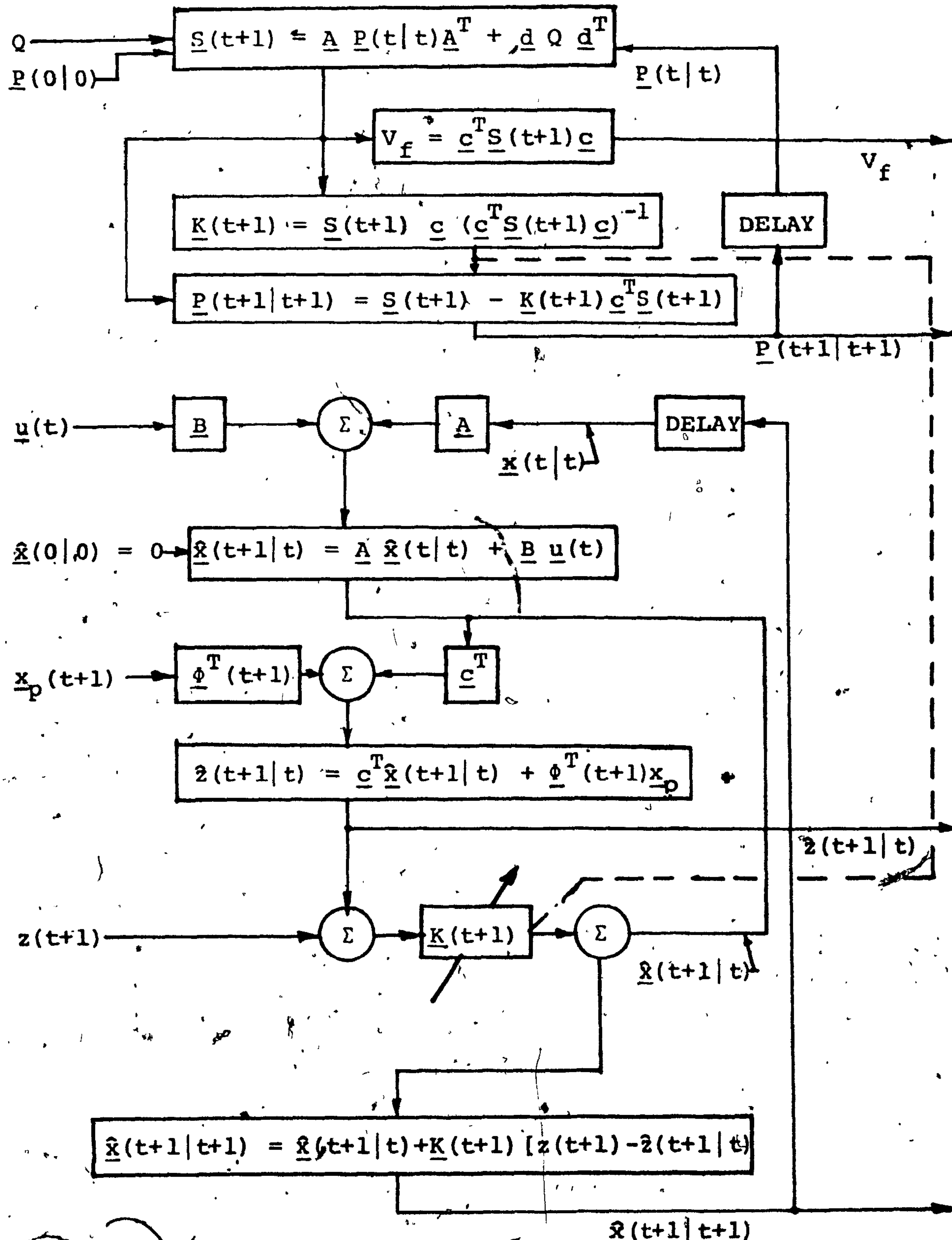

Fig. 3.1 - Kalman Filtering Algorithm

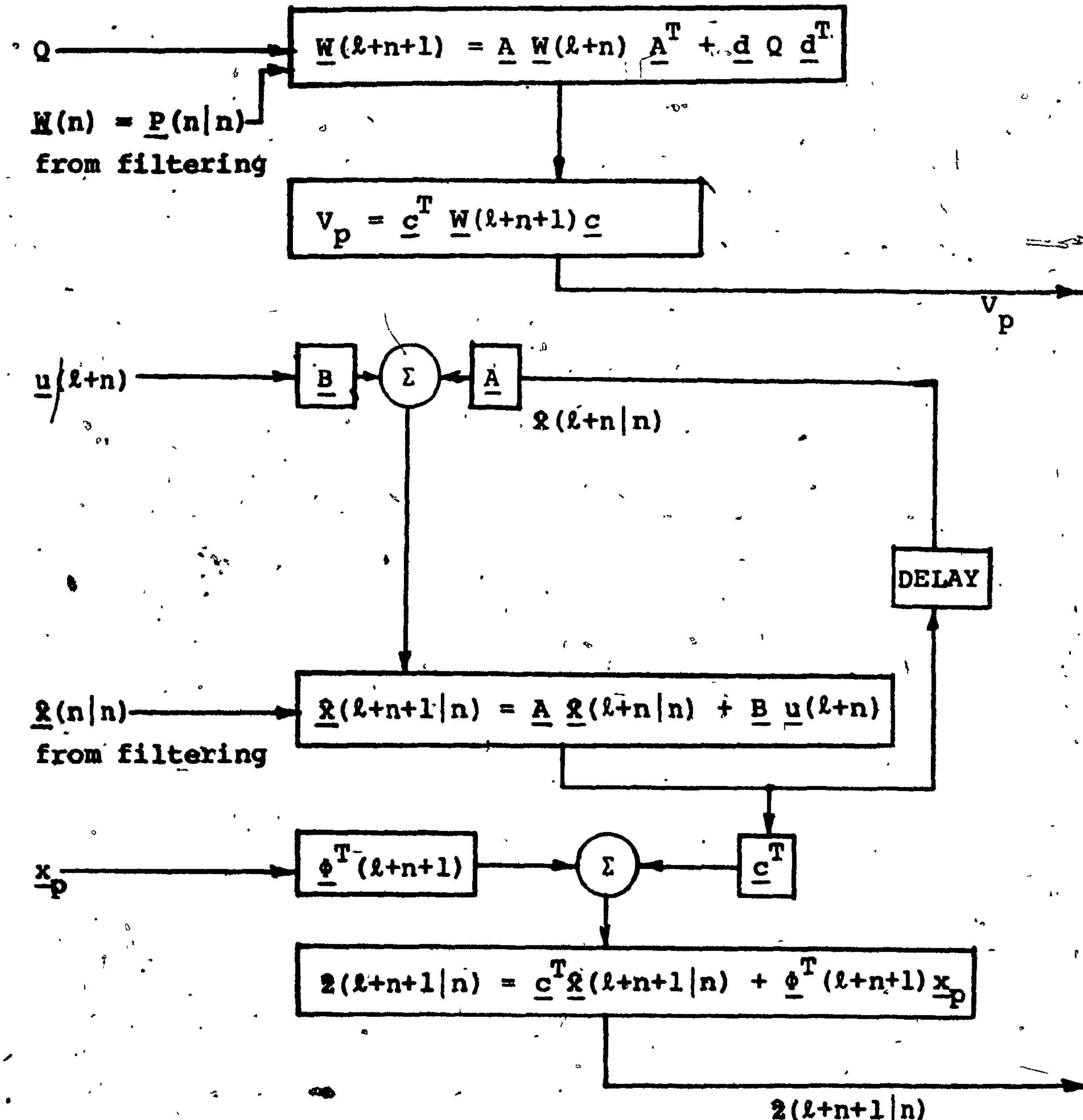

Fig. 3.2 - Kalman Forecasting Algorithms

CHAPTER 4

EXPERIMENTS AND RESULTS

4.1 The Required Data

As previously stated, January 1972 was chosen for investigation involving identification of system parameters. Accordingly, input data are tabulated as follows:

Table 1: Dorval ten-year January hourly average temperatures, °F, 1972 to 1971.
These averages were calculated from the ten-year record of January temperatures, 1962 to 1971 inclusive.

Table 2: Dorval normal temperatures, T °F, January 4 to 28, 1972.
These smoothed averages were obtained, as explained in section 3.5, by taking seven-day moving averages of Table 1.

Table 3: Dorval hourly temperatures, °F, January 1972.

Table 4: Dorval hourly values of temperature-dependent variable u(t), January 4 to 28, 1972.
These values of u(t) were calculated from Tables 2 and 3, using eqn. (3.15).

Table 5: Hydro-Québec hourly load, MW, January 1972.

TABLE 1

Dorval 10-year January Hourly Average Temperatures, 1962 to 1971

date \ hour	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
1	17	17	17	16	15	14	12	11	9	9	10	12	12	13	13	14	13	13	12	12	11	11	10	9
2	9	9	8	8	8	7	8	8	9	10	10	12	13	14	15	15	15	14	14	15	14	14	14	14
3	14	13	14	14	15	15	15	16	15	17	19	20	22	23	24	24	24	23	22	21	20	21	20	20
4	17	16	16	15	15	15	14	14	13	14	15	16	18	19	19	19	18	18	17	17	18	17	17	17
5	18	18	18	18	17	17	17	17	17	17	19	19	20	20	21	20	19	19	18	17	17	16	16	16
6	14	15	15	15	14	14	14	13	12	13	13	14	15	16	17	17	17	15	15	14	14	14	14	14
7	15	14	14	14	15	15	15	14	14	14	15	16	17	18	19	19	18	16	16	15	15	14	14	14
8	13	13	12	12	11	12	12	12	11	12	13	14	15	15	17	17	17	16	16	15	15	14	13	13
9	13	12	12	11	11	10	11	11	11	12	13	14	15	16	16	16	16	15	15	14	14	14	14	14
10	14	14	15	15	14	14	14	13	13	13	14	15	15	16	16	16	15	15	14	13	13	13	12	11
11	11	10	10	9	8	8	7	7	7	8	8	10	11	12	13	13	13	12	12	11	10	10	9	8
12	8	7	6	6	6	6	6	5	6	6	7	9	9	11	11	11	11	11	10	10	10	10	9	9
13	8	8	8	8	8	7	8	8	9	10	12	13	15	16	16	17	16	14	14	13	13	13	12	11
14	11	11	11	10	10	10	10	10	10	10	11	12	13	14	15	15	14	14	13	13	13	13	13	12
15	12	12	11	12	12	11	11	11	10	10	11	11	12	13	13	14	13	11	10	10	9	9	7	6
16	5	3	2	1	0	-1	0	-2	-2	0	2	3	5	7	8	8	8	7	6	7	6	6	7	7
17	7	8	8	8	7	8	8	8	7	8	10	12	14	14	14	14	14	13	11	11	11	10	9	9
18	8	8	7	6	6	6	4	5	5	7	8	10	11	12	13	14	14	13	13	13	12	13	13	12
19	13	13	12	11	11	12	12	12	12	13	14	16	16	17	18	18	18	17	16	16	16	15	15	15
20	15	16	15	15	15	16	17	18	17	17	19	20	21	21	22	22	22	21	20	20	19	19	18	17
21	16	15	14	13	13	12	12	12	12	14	16	17	18	19	20	21	20	20	19	19	20	20	18	19
22	18	20	20	20	19	19	19	18	19	19	21	22	23	23	24	24	24	23	21	20	18	18	17	18
23	17	17	17	17	16	17	17	17	17	18	18	19	20	19	20	21	21	20	19	18	18	17	17	17
24	16	17	16	16	15	15	15	15	16	17	18	19	21	22	22	22	22	22	22	22	21	22	22	21
25	21	20	19	18	17	16	16	17	17	19	20	21	22	23	22	23	23	21	21	20	18	17	16	15
26	15	14	14	13	13	13	13	13	13	14	16	16	16	17	18	19	19	18	18	17	18	17	17	16
27	16	16	16	16	15	14	13	13	12	11	12	12	13	14	14	15	15	14	13	13	12	12	11	10
28	9	9	8	7	7	6	5	4	5	5	6	7	8	9	10	10	9	9	9	9	9	9	8	8
29	7	7	6	6	6	6	7	7	6	7	8	10	11	12	13	14	14	14	14	14	14	14	14	14
30	14	14	14	14	14	14	14	14	15	15	15	16	18	19	20	21	20	20	19	18	18	17	14	12
31	12	10	10	8	8	7	7	6	7	9	10	12	14	15	16	17	17	16	15	15	14	14	13	13

TABLE 2

Dorval Normal Temperatures $\hat{T}$, 4 to 28Jan72

date \ hour	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
4	15	15	14	14	14	14	14	13	13	13	14	16	17	17	18	18	18	17	16	16	16	15	15	15
5	14	14	14	14	13	13	14	13	13	14	15	16	17	18	19	19	18	17	17	16	16	16	15	15
6	15	15	14	14	14	14	14	14	13	14	15	16	17	18	19	19	18	18	17	16	16	16	15	15
7	15	15	14	14	14	14	14	13	13	14	15	16	17	17	18	18	17	16	16	15	15	15	14	14
8	14	14	14	13	13	13	13	12	12	13	14	15	16	16	17	17	16	16	15	14	14	14	13	13
9	12	12	12	12	11	11	11	11	11	11	12	13	14	15	16	16	15	14	14	13	13	13	12	12
10	12	11	11	11	10	10	10	10	10	11	12	13	14	15	16	16	15	14	14	13	13	13	12	11
11	11	11	11	10	10	10	10	10	9	10	11	12	13	14	15	15	15	14	13	13	12	13	12	11
12	11	11	10	10	10	10	10	9	9	10	11	12	13	14	14	15	14	13	13	12	12	12	11	10
13	10	9	9	9	9	8	8	8	7	8	9	10	11	12	13	13	13	12	11	11	11	11	10	9
14	9	8	8	8	7	7	7	7	7	7	9	10	11	12	13	13	13	12	11	11	10	10	9	9
15	9	8	8	7	7	7	7	7	6	7	9	10	11	12	13	13	13	12	11	11	11	10	10	9
16	9	9	8	8	8	8	7	8	7	8	10	11	12	13	14	14	14	13	12	12	12	11	11	10
17	10	10	9	9	9	9	9	9	9	9	11	12	13	14	15	15	15	14	13	13	12	12	12	11
18	11	11	10	9	9	9	9	9	9	10	11	13	14	15	15	16	16	15	14	14	13	13	12	12
19	12	12	11	11	10	10	10	10	10	11	13	14	15	16	17	17	17	16	15	15	15	14	14	14
20	14	14	13	13	13	13	13	13	13	14	15	17	18	18	19	19	19	18	17	17	16	16	15	15
21	15	15	14	14	14	14	14	14	14	15	16	18	19	19	20	20	20	19	19	18	18	18	17	17
22	17	17	16	16	15	15	15	16	16	17	18	19	20	21	21	22	21	21	20	19	19	18	17	17
23	17	17	16	16	16	15	16	16	16	17	18	19	20	21	21	22	21	21	20	20	19	18	18	18
24	17	17	17	16	16	15	15	15	15	16	17	18	19	20	20	21	20	20	19	19	18	17	17	17
25	16	16	16	15	15	14	14	14	14	15	16	17	18	18	19	19	19	18	18	17	16	16	15	15
26	15	14	14	13	13	12	12	12	12	13	14	15	16	17	17	18	17	17	16	16	16	15	15	14
27	14	14	13	13	13	12	12	12	12	13	14	15	16	16	17	18	17	17	16	16	16	15	15	14
28	13	13	12	12	11	11	11	11	11	11	13	14	15	16	16	17	17	16	15	15	15	14	13	13

In the above table, Tuesdays, Wednesdays, Thursdays, Fridays are in groups of 4

TABLE 3

Dorval Hourly Temperatures T, °F, January 1972

date \ hour	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
1	-3	-2	-7	-7	-8	-7	-6	-2	-3	-4	-2	1	0	2	2	6	6	6	8	8	8	9	10	11
2	11	13	14	14	15	16	17	17	18	19	20	21	23	24	29	29	29	26	26	26	26	26	27	33
3	31	32	30	29	29	28	27	27	26	27	28	29	30	31	33	34	33	32	32	31	31	29	26	21
4	23	24	22	17	18	20	20	18	17	16	15	15	15	15	15	15	15	15	15	14	14	14	13	14
5	14	13	13	12	12	11	10	7	8	6	5	4	5	5	6	8	4	3	2	0	-1	-5	-6	-7
6	-9	-7	-7	-6	-5	-5	-5	-7	-7	-5	1	3	4	7	10	13	14	14	10	10	8	6	7	7
7	8	9	9	9	10	11	18	18	18	18	21	21	22	23	24	24	25	26	27	31	30	29	26	24
8	18	13	10	6	4	2	0	-2	-2	-3	-2	-2	-1	-1	0	1	-1	-3	-2	-2	-3	1	3	4
9	5	8	10	13	15	9	19	6	7	13	26	27	26	28	31	34	38	37	35	36	38	38	38	38
10	38	38	38	39	38	37	37	36	36	35	36	37	37	37	37	38	36	34	35	34	34	35	33	31
11	31	31	32	33	33	32	35	33	28	31	39	39	39	39	39	38	37	36	35	34	33	33	32	32
12	30	31	32	32	30	30	30	30	30	32	32	32	34	34	34	33	32	30	30	30	30	30	31	31
13	31	34	34	35	35	36	36	37	38	38	39	40	40	41	42	43	43	44	44	44	39	33	32	30
14	27	26	24	19	14	12	11	9	8	8	8	9	10	10	12	13	12	10	9	11	11	12	14	13
15	13	13	8	9	12	14	16	15	14	12	11	10	10	10	9	9	8	6	5	5	4	4	2	2
16	2	1	-1	-2	-3	-5	-6	-7	-8	-9	-8	-7	-8	-7	-7	-7	-7	-9	-10	-10	-10	-10	-8	-9
17	-6	-6	-4	-1	2	3	4	5	6	7	9	11	8	10	17	21	22	23	22	22	24	25	25	26
18	28	29	29	28	27	25	23	23	25	29	30	32	33	31	32	34	35	39	39	36	36	37	37	38
19	38	37	38	43	44	43	42	41	41	42	37	38	37	36	32	30	28	27	26	25	24	22	22	20
20	18	19	19	20	21	16	12	11	11	12	11	12	13	14	14	14	13	14	12	10	9	9	9	9
21	10	11	11	12	12	7	7	9	7	5	4	4	4	5	6	8	7	6	3	4	3	3	-1	-3
22	2	0	0	0	0	-2	-3	-3	-3	-1	1	5	8	11	16	19	26	34	35	35	36	36	35	35
23	34	35	35	36	36	36	36	42	41	39	39	38	38	37	37	37	36	35	35	34	33	32	32	31
24	31	29	27	25	25	24	24	24	23	22	22	22	26	25	25	26	25	23	24	24	25	26	27	28
25	31	33	38	38	37	35	37	37	42	37	36	34	31	28	27	26	22	17	14	11	8	7	4	3
26	1	0	-2	-4	-5	-5	-5	-5	-5	-4	-2	-1	0	1	3	4	3	3	3	4	5	4	4	2
27	3	2	3	3	2	3	1	0	-1	0	1	2	3	5	8	10	11	10	11	12	15	15	16	17
28	17	18	17	17	16	16	16	11	10	10	8	8	7	8	9	9	9	9	10	9	8	7	7	8
29	7	7	7	9	8	11	7	3	6	9	12	13	15	15	16	18	18	16	15	17	20	20	20	20
30	21	20	19	18	18	17	17	17	19	19	20	20	20	20	21	22	21	20	19	19	17	14	11	10
31	9	7	6	9	5	2	2	2	2	7	9	11	13	14	16	17	16	13	12	11	12	12	10	13

TABLE 4

Dorval Hourly Values of Temperature-Dependent Variable u(t), 4 to 28Jan72

date \ hour	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
4	-8	-10	-8	-3	-4	-6	-6	-5	-4	-3	-1	1	2	2	3	3	3	2	1	2	2	1	2	1
5	0	1	1	2	1	2	4	6	5	8	10	12	12	13	13	11	14	14	15	16	17	21	21	22
6	24	22	21	20	19	19	19	21	20	19	14	13	13	11	9	6	4	4	7	6	8	10	8	8
7	7	6	5	5	4	3	-4	-5	-5	-4	-7	-6	-6	-6	-6	-6	-8	-10	-11	-16	-15	-14	-12	-10
8	-4	1	4	7	9	11	13	14	14	16	16	17	17	17	17	16	17	19	17	16	17	13	10	9
9	7	4	2	-1	-4	2	-8	5	4	-2	-14	-14	-12	-13	-16	-19	-23	-23	-21	-23	-25	-25	-26	-26
10	-26	-27	-27	-28	-28	-27	-27	-26	-26	-24	-24	-24	-23	-22	-22	-23	-21	-20	-21	-21	-21	-22	-21	-20
11	-20	-20	-22	-23	-23	-23	-25	-24	-19	-21	-28	-27	-26	-25	-24	-23	-22	-22	-22	-21	-21	-21	-20	-21
12	-19	-20	-22	-22	-20	-21	-21	-21	-21	-22	-21	-20	-21	-20	-20	-19	-18	-17	-17	-18	-18	-18	-20	-21
13	-21	-25	-25	-27	-27	-28	-28	-30	-31	-30	-30	-30	-29	-29	-29	-30	-30	-32	-33	-33	-28	-23	-22	-21
14	-18	-18	-16	-12	-7	-5	-4	-2	-1	-1	1	1	1	2	1	0	1	2	2	0	-1	-2	-5	-4
15	-4	-5	[illegible]	-2	-5	-7	-10	-9	-8	-5	-2	0	1	2	4	4	5	6	6	6	7	6	8	7
16	7	[illegible]	9	10	11	13	13	15	15	17	18	18	20	20	21	21	21	22	22	22	22	21	19	19
17	16	16	13	10	7	6	5	4	3	2	2	1	5	4	-2	-6	-7	-9	-9	-9	-12	-13	-13	-15
18	-17	-19	-19	-19	-18	-16	-14	-14	-16	-19	-19	-19	-19	-16	-17	-18	-20	-25	-25	-22	-23	-24	-25	-26
19	-26	-25	-27	-33	-34	-33	-32	-31	-31	-31	-24	-24	-22	-20	-15	-13	-11	-11	-11	-10	-10	-8	-8	-6
20	-5	-5	-6	-7	-8	-3	1	2	2	2	4	5	5	4	5	5	6	4	5	7	7	7	6	6
21	5	4	3	2	2	7	7	5	7	10	12	14	15	14	14	12	13	13	16	14	15	15	18	20
22	15	17	16	16	15	17	18	19	19	18	17	14	12	10	5	3	-5	-14	-15	-16	-18	-18	-18	-18
23	-17	-18	-19	-20	-20	-21	-21	-26	-25	-22	-21	-19	-18	-16	-16	-15	-15	-14	-15	-15	-14	-14	-14	-13
24	-14	-12	-11	-9	-9	-9	-9	-9	-8	-6	-5	-4	-7	-6	-5	-5	-5	-3	-5	-6	-7	-9	-10	-11
25	-15	-17	-22	-23	-22	-21	-23	-23	-28	-23	-20	-17	-13	-10	-8	-7	-3	1	4	6	8	9	11	12
26	14	14	16	17	18	17	17	17	17	17	16	16	16	16	14	14	14	14	13	12	11	11	11	12
27	11	12	10	10	11	9	11	12	13	13	13	13	13	11	9	8	6	7	5	4	1	0	-2	-3
28	-4	-5	-5	-5	-5	-5	-5	0	1	1	5	6	8	8	7	8	8	7	5	6	7	7	6	5

In the above table, Tuesdays, Wednesdays, Thursdays, Fridays are in groups of 4

TABLE 5

Hydro-Quebec Hourly Load z(t), January 1972, in 10 MW Units

date \ hour	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
1	610	586	562	544	532	521	522	528	533	560	594	612	611	596	577	573	572	643	633	619	598	587	576	560
2	538	516	496	487	480	476	480	484	494	517	548	579	603	608	613	599	613	702	690	683	663	642	630	633
3	611	574	557	551	547	549	554	587	610	646	695	738	742	736	709	710	719	826	803	794	757	744	717	671
4	633	600	581	574	570	574	593	635	680	733	762	801	797	770	766	761	775	883	858	838	814	788	754	720
5	672	637	619	612	616	614	624	670	719	748	769	807	799	774	762	749	773	885	862	849	831	811	777	745
6	695	666	654	640	633	643	663	704	751	779	794	820	811	775	763	765	789	894	870	850	834	804	783	739
7	898	666	642	626	624	628	635	636	732	772	798	825	813	784	778	770	793	885	849	836	816	785	758	725
8	680	655	633	610	607	607	611	624	639	687	733	766	770	753	739	722	742	841	829	812	783	757	724	699
9	655	635	619	601	588	586	578	578	579	608	644	685	703	695	672	657	676	746	733	722	706	687	666	638
10	604	585	570	566	565	562	582	634	701	746	784	814	797	750	737	731	738	842	816	790	768	744	713	675
11	631	609	580	582	578	579	595	640	702	741	757	777	760	731	727	718	733	832	798	777	760	739	717	680
12	627	598	581	574	569	567	579	630	695	727	751	769	759	726	719	712	741	840	813	789	762	740	712	676
13	632	606	587	582	569	570	588	628	693	722	739	765	744	719	710	699	724	821	807	780	762	748	730	672
14	629	598	575	573	571	574	589	647	721	755	773	796	789	756	751	750	755	846	852	836	808	788	765	731
15	686	657	642	616	610	611	617	625	650	700	735	770	763	743	733	718	721	819	810	793	767	738	715	703
16	654	622	594	586	581	580	582	593	594	624	652	682	696	684	656	651	662	761	765	749	734	714	705	686
17	665	651	645	641	636	626	652	716	778	837	871	901	877	838	828	821	836	914	880	859	819	797	775	734
18	681	650	630	613	609	610	620	677	734	755	773	803	790	760	766	736	771	860	839	789	781	756	729	693
19	647	616	603	592	579	589	607	651	701	738	771	786	776	749	746	740	754	843	839	819	795	770	752	715
20	664	637	626	616	607	609	624	684	733	762	778	803	804	776	782	773	796	879	876	852	834	820	788	747
21	692	652	637	626	616	621	633	681	747	780	794	810	799	761	755	758	768	859	867	856	832	811	779	752
22	704	672	649	635	634	636	638	652	665	707	745	763	759	745	727	716	722	801	787	771	744	717	690	665
23	630	598	577	569	541	546	539	543	547	575	612	657	661	642	622	606	621	697	697	674	655	640	620	602
24	585	579	572	567	558	565	586	635	706	751	786	821	798	762	748	738	753	855	831	814	786	765	745	708
25	660	628	616	606	604	604	633	685	733	779	797	821	808	772	778	777	794	876	863	846	839	819	791	755
26	708	684	657	648	651	653	676	721	773	808	837	861	850	815	810	803	814	884	891	874	850	832	799	760
27	712	689	671	662	661	654	664	725	775	809	827	855	834	807	798	777	796	882	884	865	839	816	786	753
28	690	676	662	645	644	642	664	694	749	795	815	836	827	799	790	778	781	856	863	850	830	794	763	740
29	685	650	632	626	628	628	630	642	659	701	736	753	759	726	712	702	702	781	796	775	749	731	701	677
30	639	607	591	573	573	568	566	568	581	614	646	675	680	667	650	639	652	720	733	715	697	683	671	648
31	626	614	594	596	591	596	616	661	721	777	824	860	821	796	780	767	777	862	858	842	804	790	769	710

Also, in Fig. 4.1, Hydro-Québec loads are plotted separately for 1st week, 2nd week, 3rd week and 4th week (see following section).

4.2 Description of 4-Day Weeks, Used for Data and Analysis

For convenience we shall continue to write dates in January 1972, in unpunctuated and unspaced form, e.g.: 6Jan72 or Wed19Jan72. Also, we define 4 to 7Jan72 as 1st week, 11 to 18Jan72 as 2nd week, etc., all as in Table 6.

4.3 Starting Values for Parameters

Using the method described in section 3.6, starting values of the 19 parameters were obtained, as listed in Table 7.

4.4 Identification of Parameters

Program "POWEL5", incorporating the Fletcher Powell scientific subroutine DFMFP, is listed in Appendix A1 to A8.

This program was applied to data contained in Tables 4 and 5 for the 1st week (96 data points), using the starting values in Table 6. For computer output showing values of parameters at the start, during iteration and on solution, see Appendix A9 to A11. This computer output gives also the number of iterations performed and the computation time required in obtaining the results.

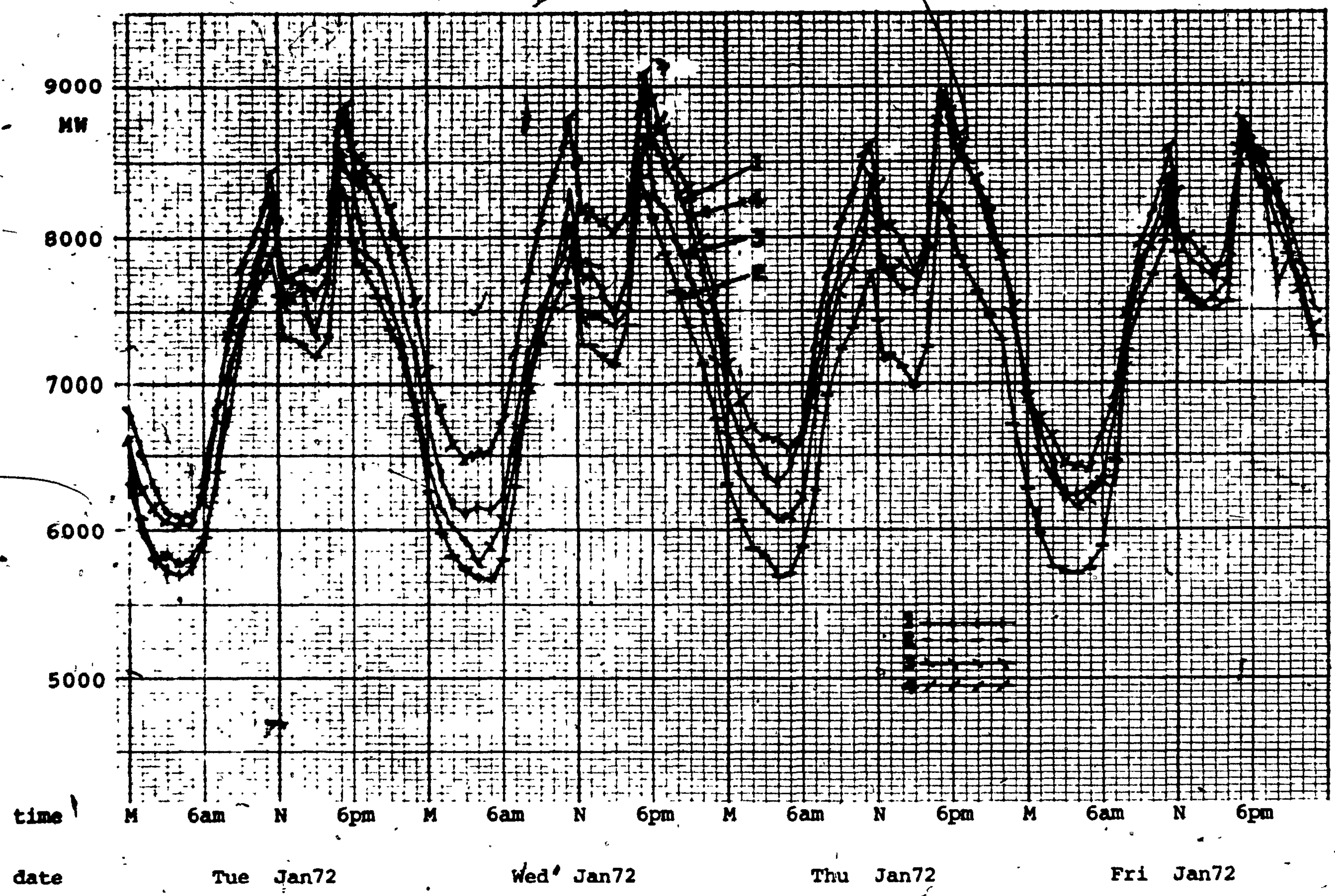

Fig. 4.1 - Loads for 1st Week, 2nd Week, 3rd Week and 4th Week

TABLE 6

Description of 4-day Weeks, January 1972, in Unspaced Format

1st week	2nd week	3rd week	4th week	3 weeks
Tue4Jan72	Tue11Jan72	Tue18Jan72	Tue25Jan72	1st week
Wed5Jan72	Wed12Jan72	Wed19Jan72	Wed26Jan72	2nd week
Thu6Jan72	Thu13Jan72	Thu20Jan72	Thu27Jan72	3rd week
Fri7Jan72	Fri14Jan72	Fri21Jan72	Fri28Jan72	combined

TABLE 7

Starting Values for Parameters of Model defined in Eqn.(3.2) with $m = n = 2$, and $n_p = 7$

Parameter	Starting Value
a_1	0.25
a_2	0.25
b_0	13.2
b_1	0.0
x_p^0	7351.0
x_p^1	-929.8
x_p^2	-376.2
x_p^3	115.4
x_p^4	52.0
x_p^5	0.0
x_p^6	-31.3
x_p^7	0.0

Parameter	Starting Value
x_p^8	-282.2
x_p^9	248.5
x_p^{10}	-217.2
x_p^{11}	54.7
x_p^{12}	0.0
x_p^{13}	-49.6
x_p^{14}	0.0

Further parameters then were identified as follows:

from data of 2nd week (96 data points) with results of 1st week as starting values,

from data of 3rd week (96 data points) with results of 2nd week as starting values,

from data of 3 weeks (288 data points) with results of 2nd week as starting values,

also from data of 4th week (96 data points) for use in analysis.

For computer outputs corresponding to these four computations, see Appendix A12 to A26.

Also, a partial computation was made with starting values alternately 10% lower and 10% higher than the already-obtained solution values (Appendix A23) using three weeks' data (288 data points). Computation was not allowed to continue to convergence and is shown during iteration in Appendix A27 to A29.

Table 8 lists all the parameters obtained as above, and the corresponding loads are plotted in Fig. 4.1.

4.5 Forecasting Load, Using Identified Parameters

4.5.1 Computer program listing

For a listing of program "PREDICT", incorporating the Kalman filtering and prediction algorithms, i.e. eqns. (3.24) to (3.27) inclusive, see Appendix A32 to A37.

TABLE 8

Identification of Parameters, January 1972

Parameters	using data of					
	1st week 04 to 07Jan72	2nd week 11 to 14Jan72	3rd week 18 to 21Jan72	averages 1st 3 weeks	three weeks	4th week 25 to 28Jan72
a_1	0.105	0.124	0.210	---	0.302	0.237
a_2	0.266	0.296	0.328	---	0.390	0.294
b_0	-3.076	-3.642	2.368	---	2.495	-13.18
b_1	6.875	11.815	1.895	---	1.850	17.91
x_p^0	7421.8	7262.2	7392.8	7358.9	7355.8	7599.4
x_p^1	-980.18	-956.39	-935.81	-957.46	-950.55	-856.81
x_p^2	-389.45	-426.56	-423.35	-413.12	-405.30	-390.49
x_p^3	115.88	147.24	123.52	128.88	130.56	107.51
x_p^4	15.73	-6.80	8.80	5.91	5.34	25.64
x_p^5	-95.21	-105.97	-89.80	-96.99	-100.40	-71.34
x_p^6	1.64	-12.17	-23.31	-11.28	-13.27	-20.54
x_p^7	21.74	18.31	12.23	17.43	20.26	13.37

TABLE 8
(continued)

Identification of Parameters, January 1972

Parameters	using data of					
	1st week 04 to 07Jan72	2nd week 11 to 14Jan72	3rd week 18 to 21Jan72	averages 1st 3 weeks	three weeks	4th week 25 to 28Jan72
x_p^8	-225.46	-269.72	-235.42	-243.53	-270.41	-325.22
x_p^9	178.26	206.09	196.42	193.59	189.56	218.06
x_p^{10}	-192.99	-180.71	-169.72	-181.14	-177.01	-200.40
x_p^{11}	112.54	85.32	69.02	88.96	91.33	61.53
x_p^{12}	51.01	80.90	68.96	66.96	65.73	57.71
x_p^{13}	-51.22	-30.27	-10.69	-30.72	-33.01	-8.04
x_p^{14}	-28.19	-46.26	-39.15	-37.87	-40.29	43.85
Q	15619	9195	8411	---	14635	7024

4.5.2 The basic forecast into 4th week

Fig. 4.2 shows the basic forecast into 4th week, using parameters identified from data of three weeks. Computer output is in Appendix A38 to A40.

4.5.3 Experimental forecasts into 4th week

Figs. 4.3 to 4.6 show the load forecast again into 4th week, using parameters identified respectively from data of single weeks, 1st, 2nd, 3rd, and 4th in succession (4th week for analysis -- these parameters are not available in an actual forecast). Corresponding computer outputs are in Appendix A41 to A52.

4.5.4 Experimental forecast with lead times up to 168 hours

Fig. 4.7 shows a sixth forecast with lead times up to seven full days, using parameters identified from data of 1st week. Computer output is in Appendix A53 to A58.

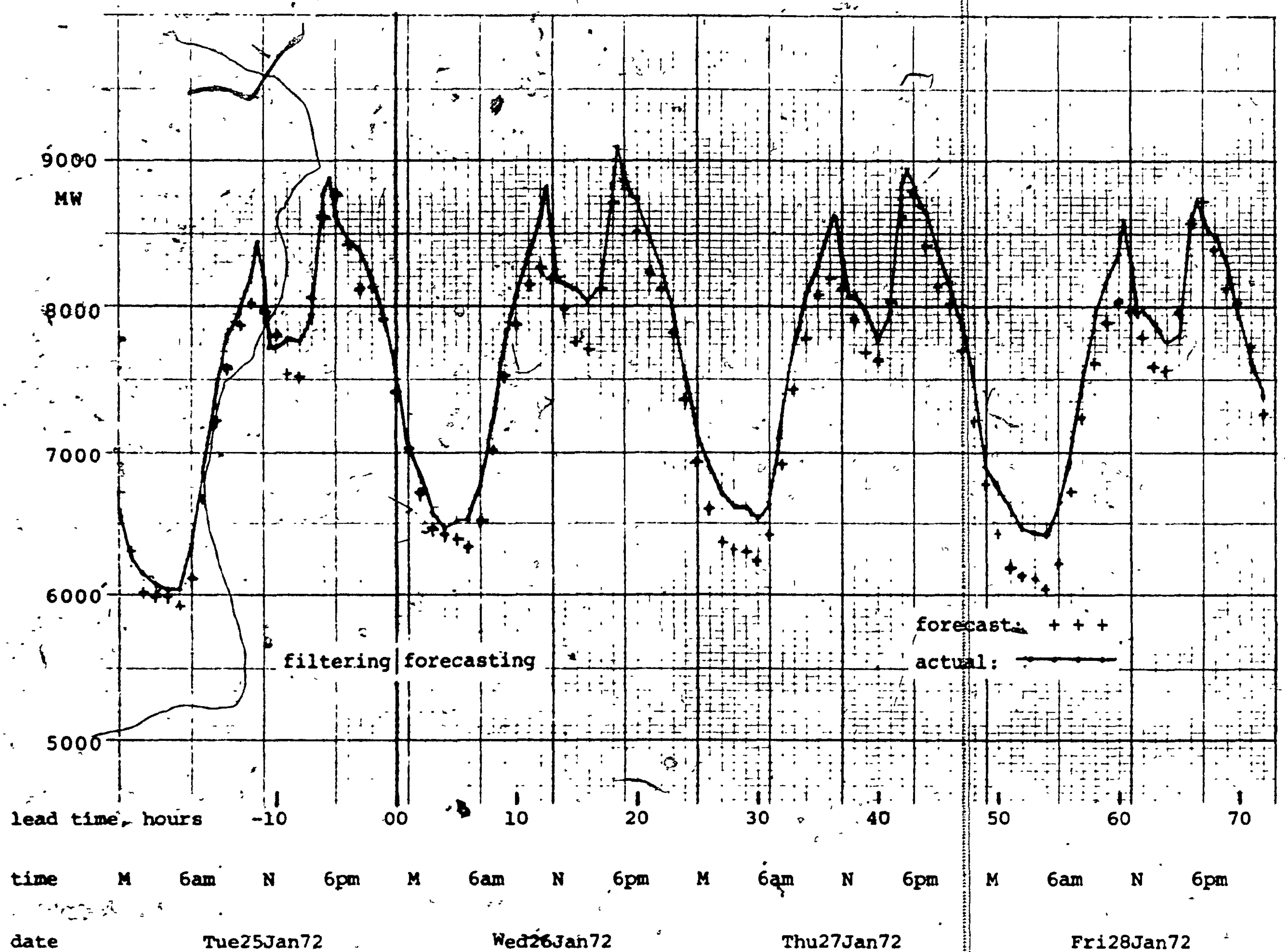

Fig. 4.2 - Forecasting in 4th Week, using Parameters identified from Data of three Weeks

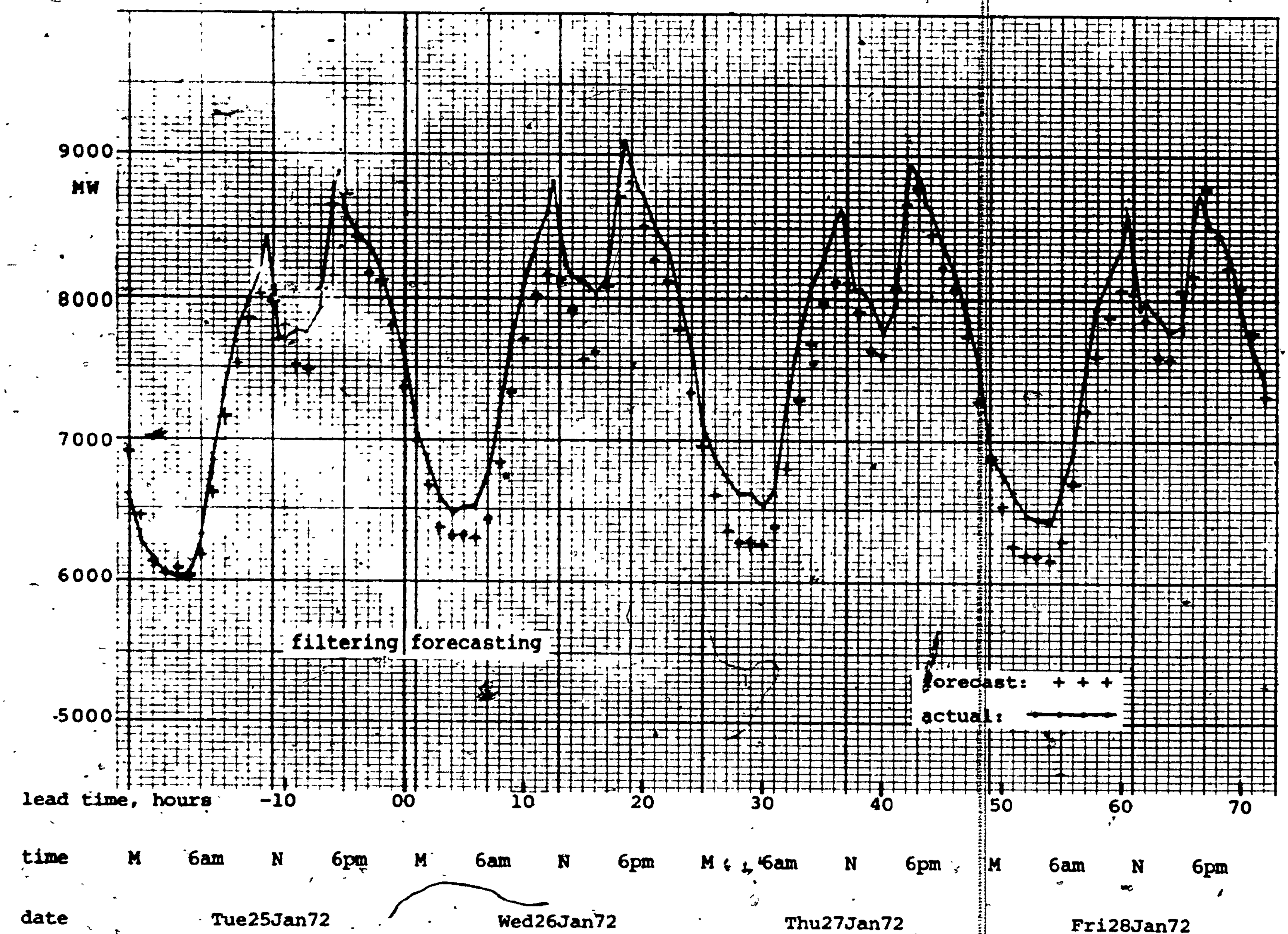

Fig. 4.3 - Forecasting in 4th Week, using Parameters identified from Data of 1st Week

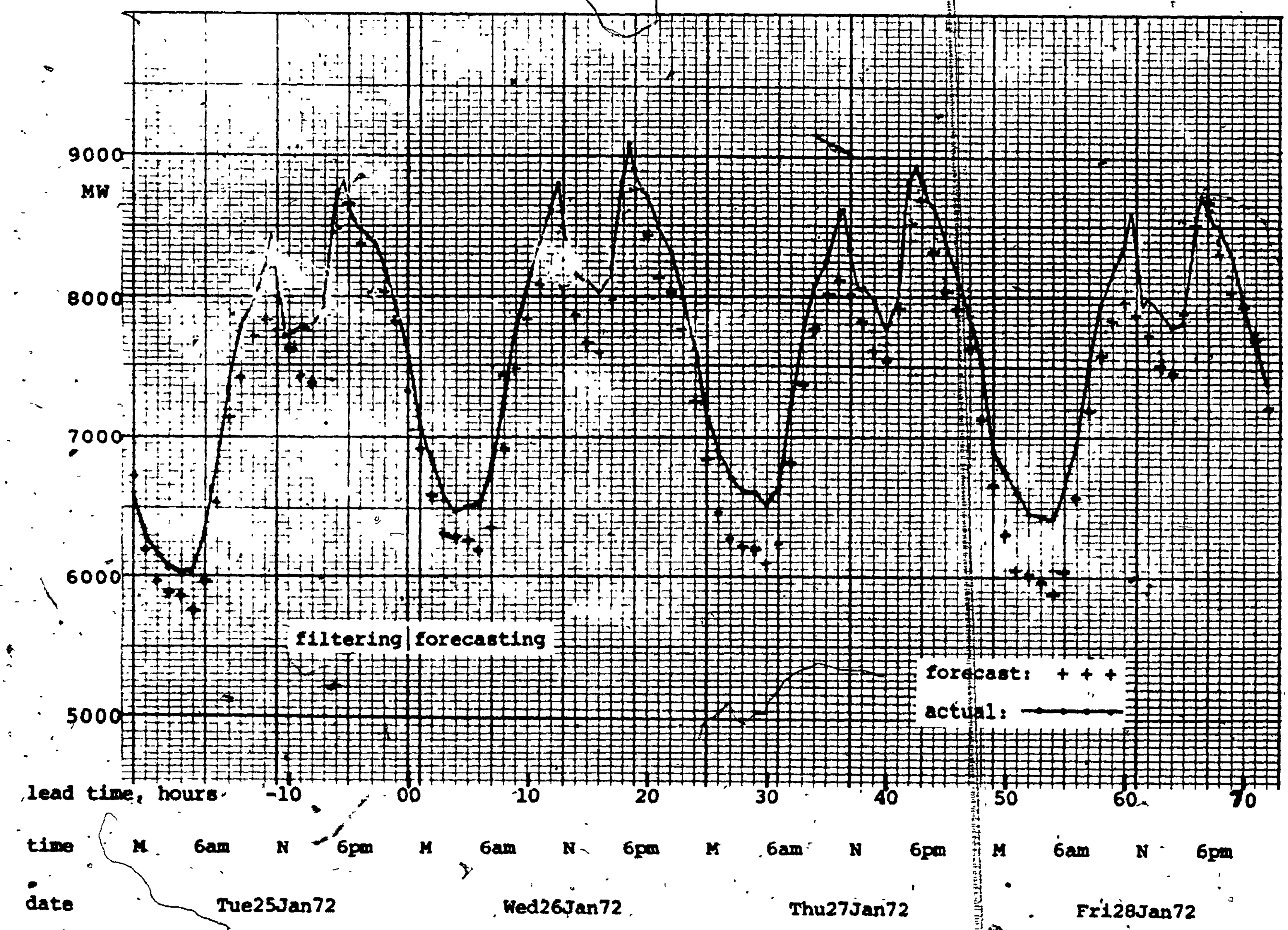

Fig. 4.4 - Forecasting in 4th Week, using Parameters identified from Data of 2nd Week

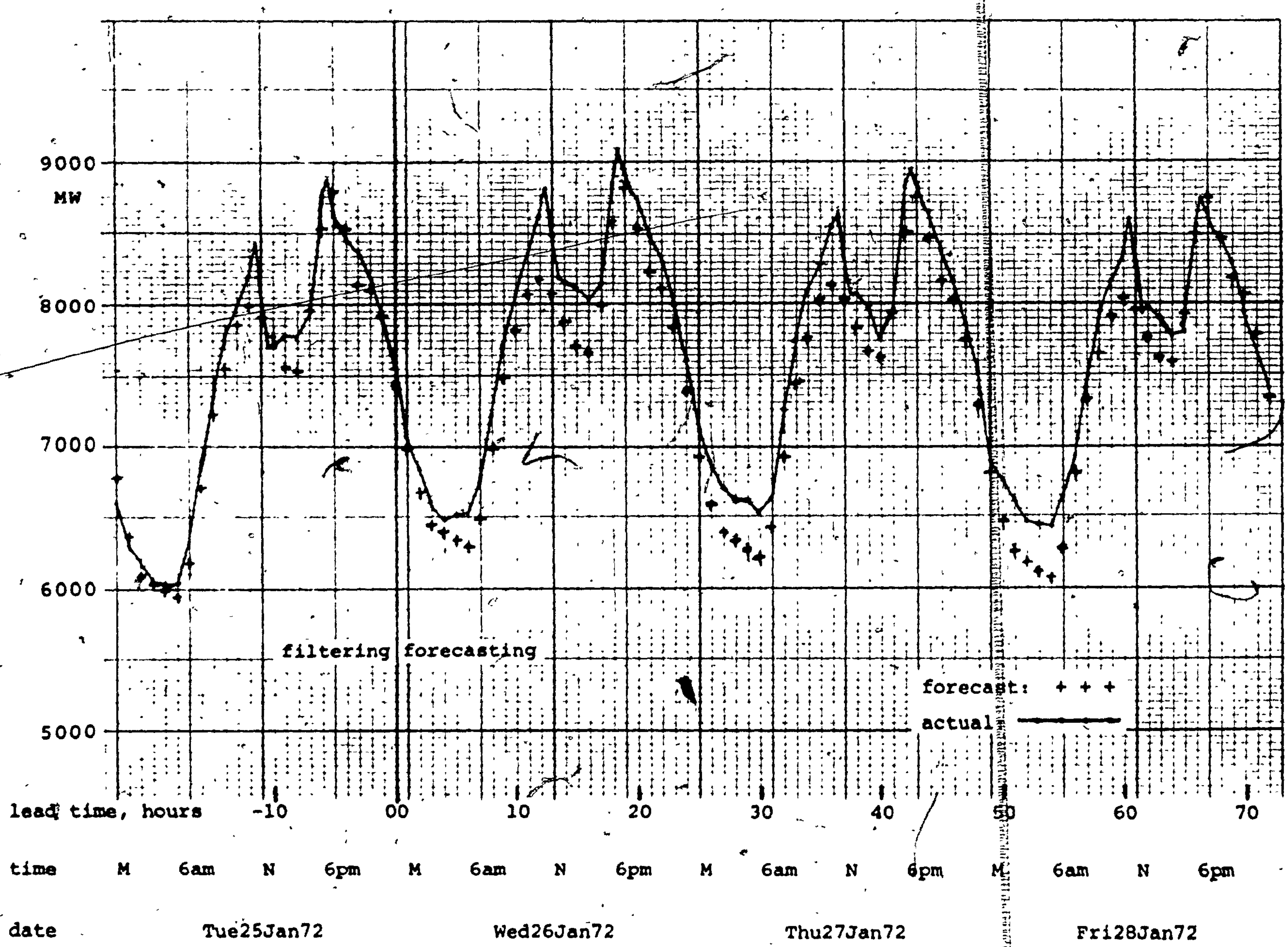

Fig. 4.5 - Forecasting in 4th Week, using Parameters identified from Data of 3rd Week

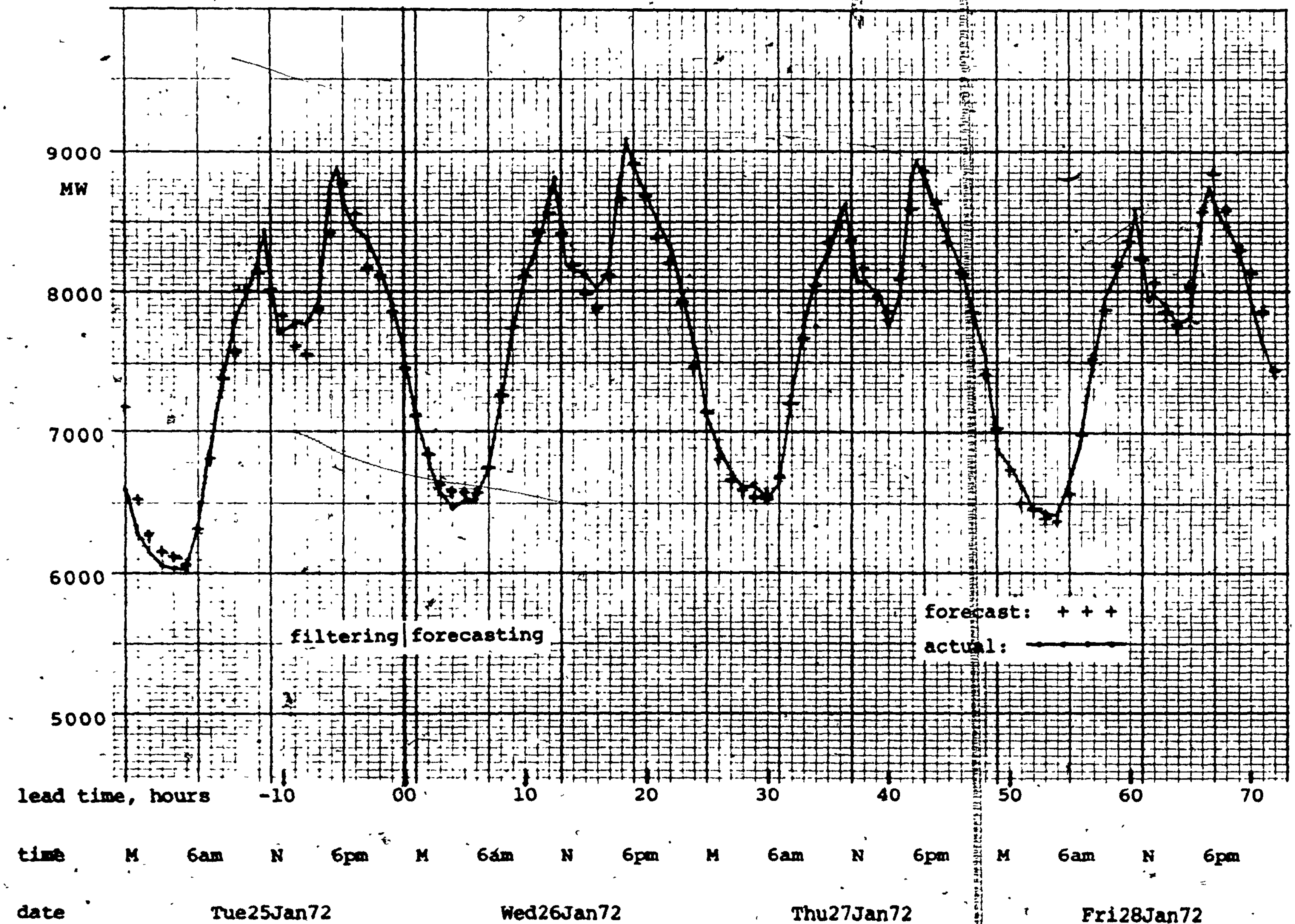

Fig. 4.6 - Forecasting in 4th Week, using Parameters identified from Data of 4th Week itself

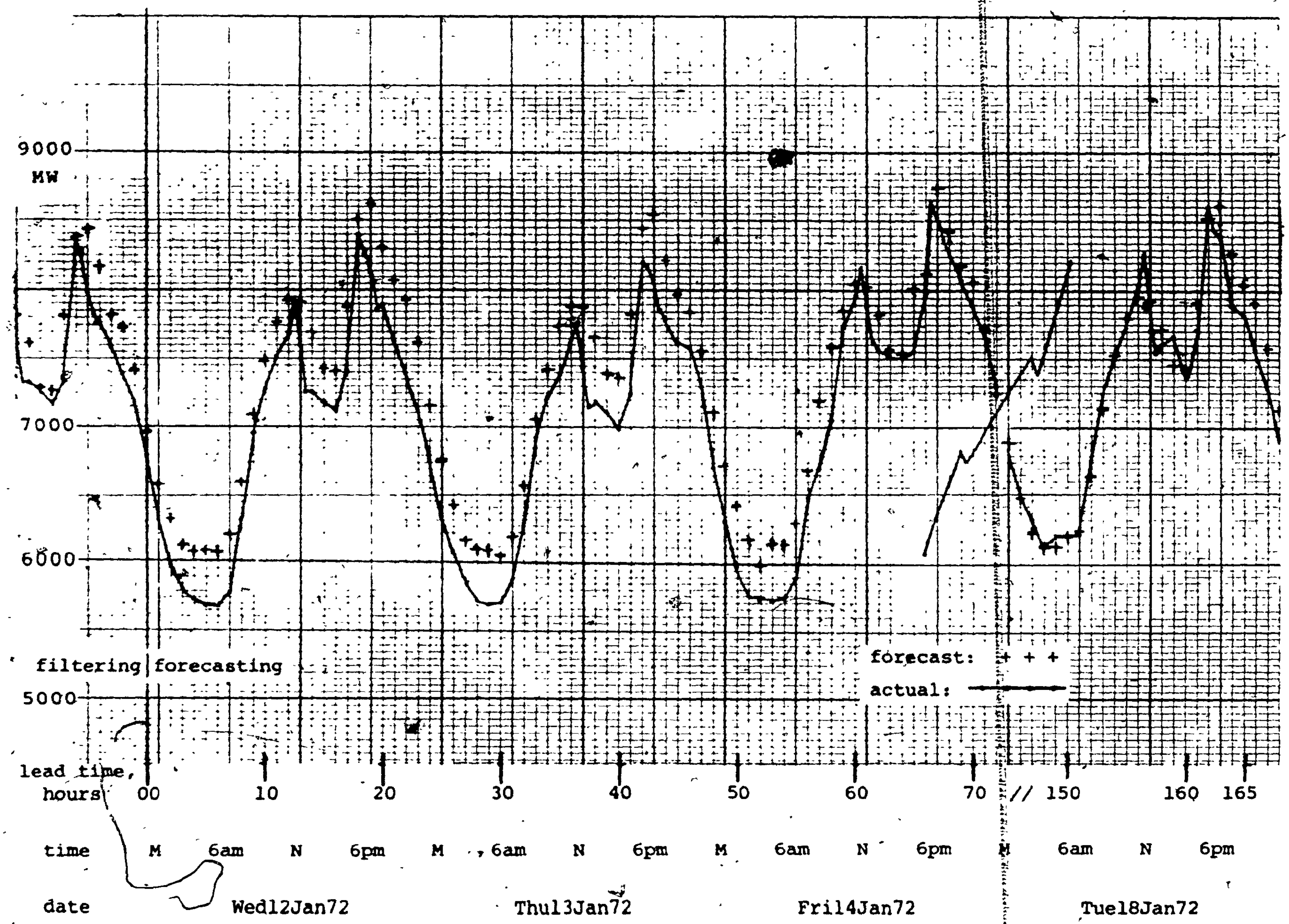

Fig. 4.7 - Forecasting Load beginning in 2nd Week and with Lead Times up to one full Week, using Parameters identified from Data of 1st Week

CHAPTER 5

DISCUSSION AND CONCLUSIONS

5.1 Discussion of Results

5.1.1 Calculation of temperature-dependent variable u(t)

As described in eqn. (3.15), any noise in normal temperature $\hat{T}$ of Table 2 affects calculation of u(t). In fact, for January at Dorval where the record maximum temperature [5] is 44.8°F on 26Jan43, eqn. (3.15) reduces to $u(t) = \hat{T} - T$. The effect of u(t) on modelling of load z(t) can be visualized by reference to eqns. (3.2a), (3.6) and Table 8. Potential for reducing such noise is illustrated by the fact that normal Dorval mean temperatures for January drop gradually throughout the month by about 5°F [5], corresponding to 0.16°F per day. Averaging of historical temperatures could begin with a twenty-year exponential average. After this initial calculation, it would not be necessary to store twenty-year temperatures since exponential weighting requires storing only the previous weighted average, with consequent smoothing.

5.1.2 Parameter starting values

The starting values of Table 7 produced convergence to the identified values for 1st week (96 data points) in 100 iterations requiring 96 seconds of CPU time. Computer output is in Appendix A9 to A11. Starting values could have been applied if desired, for identification using three weeks' data. If they had been applied, required CPU time can be estimated as 300 to 450 seconds (Appendix A30, A31 and Table 9). Thus, we have demonstrated that starting values, which are only roughly estimated, result in satisfactory computing time for off-line calculation in at least this one case.

5.1.3 Identification of parameters

Starting values for the initial calculation are required once only and, therefore, are not very important, provided that the following two questions can be answered in the affirmative:

(A) Can we use parameters, identified one week ago from data of the three preceeding weeks, as starting values for a similar off-line calculation this week, with reasonable requirements on the computer for computation time?

(B) Will various sets of parameter starting values give identical results?

Table 9 summarizes the CPU time required when using previously calculated sets of parameters as starting values for the new computation. In actual practice, we would identify parameters using data from a three-week (288 data points) sample, using starting values identified from data of a similar sample one week earlier. This procedure can be expected to give starting values closer to final values than those we used for Table 9. On the conservative side, however, we can allow around 7 minutes of

TABLE 9

Identification using previously calculated Parameters as Starting Values

Parameter Starting Values from Data of	New Parameters Computed for	Required Iterations	Required CPU Time, Seconds	Time per Iteration, Seconds	Time per Data Point, Seconds
see Section 3.6	1st week	100	96	1.0	1.0
1st week	2nd week	175	173	1.0	1.8
2nd week	3rd week	191	196	1.0	2.0
2nd week	three weeks	118	402	3.4	1.4

CPU time to calculate parameters for a three-week (288 data point) sample, using parameters identified from a similar computation made one week earlier as starting values. Strictly speaking, of course, this statement applies only to January 1972.

In any case, CPU time of approximately 7 minutes compares very favourably with 25 to 30 minutes reported by Galiana [2]. Factors contributing to the improvement probably include the following:

(1) We may have used starting values closer to the final values. Galiana apparently calculated starting values by weighted least squares estimation. We are assuming use of parameters from the previous identification are to be used as starting values for the current identification.

(2) We used a CDC 6400 computer which has greater word length than the IBM 360/65 used by Galiana (double precision in both cases).

(3) In the user program calculating the gradient of J, we limited the number of subroutines, programming directly some of the simple calculations for which scientific subroutines are available.

(4) We increased the tolerance "EPS" in the Fletcher Powell scientific subroutine DFMFP from 10^{-13} to 10^{-5}. As can be seen from the values of parameters during iteration in Appendix A10 to A31, a larger value could be used in individual cases, but "ESP=0.1E-04"

was adequately small for a variety of starting values and input data (the results are not all included in the Appendix). In actual implementation, the value of EPS as well as the iterations limit would be adjusted to suit.

Thus, requirement (A), at least within the limits of our experiments, is satisfied.

Concerning requirement (B), we still have no guarantee, but computer outputs in Appendix A20 to A23 and A27 to A31 show that various parameter starting values do yield identical results. Additional experiments, not included in the Appendix, were run, and in every case results were independent of starting values.

Appendix A27 to A29 shows the effect of offsetting starting values alternately 10% lower and higher than final values. Computation was stopped at 140 iterations, corresponding to nearly 500 seconds of CPU time. Progress in the proper direction, however, is evident. Favouring of certain parameters in succession in the search for steepest descent can be observed.

5.1.4 <u>Order of the model</u>

System roots, m, corresponding to parameter values a_1 and a_2 listed in Table 8, can be calculated from the characteristic equation $m^2 - a_1 m - a_2 = 0$, and they are:

0.571 and -0.466 for data of 1st week
0.610 and -0.486 for data of 2nd week
0.687 and -0.477 for data of 3rd week
0.793 and -0.491 for data of three weeks
and 0.674 and -0.437 for data of 4th week

For any pair of these roots to be complex, a_2 would have to be negative. Complex roots would indicate the residual model fitting some of the harmonic which belongs to the periodic. Also, x_p^7 and x_p^{14} are consistently significant in size. The above observations do not prove, however, that the 7th harmonic is justified. It is interesting to note in Table 8 that a_2 is consistently greater than a_1 and b_1 is sometimes of greater magnitude than b_0. The above observations suggest trial of higher order in <u>a</u> and perhaps in <u>b</u>, though 8th harmonic also could be tried. Aside from trial and error, further information could be obtained by statistical testing, such as outlined in section 2.6.

5.1.5 <u>Forecasting</u>

The basic forecast into 4th week using parameters identified from data of three weeks, is illustrated in

Fig. 4.2. The afternoon peak, so important in operation, is forecasted at the correct time. It is apparent that the peaks might be even more clearly defined by modifying the program to calculate the forecast half an hour each side of noon and 6 p.m. The greatest error, at 2 a.m. and 6 a.m. of 28Jan72, is 422 MW (as shown in Appendix A40, for lead times of 51 and 55 hours). As a maximum, this error of 4.8% of peak load for the day (8719 MW at 5:30 p.m.) can be tolerated. The computer output for this basic forecast is in Appendix A38 to A40. The first, second, fourth, and fifth columns under "forecasting" list the lead time in hours, and the following quantities calculated from the Kalman prediction algorithms of eqn. (3.23): periodic component, forecasted load, and standard deviation of prediction error. The other four columns list quantities available only in retrospect in any actual forecast and are calculated here for evaluation and analysis. They are actual values of load, prediction error, mean error, and standard deviation of this error. The mean error is simply the arithmetic average of error up to lead time t:

$$\text{mean error} = (1/t) \sum_{i=1}^{t} e_i \qquad (5.1a)$$

$$\text{where } e = \text{error} = (Z,\ \text{forecast}) - (Z,\ \text{real}) \qquad (5.1b)$$

The actual standard deviation is obtained using the standard formula [17] incorporating Bessel's correction:

$$\text{ACT. S.D.} = [(\sum_{i=1}^{t} e_i^2 - (\sum_{i=1}^{t} e_i)^2/t)/(t-1)]^{\frac{1}{2}} \qquad (5.2)$$

Under "filtering", the eight columns are similar to those under "forecasting".

In the basic forecast, the actual standard deviation does not exceed 151 calculated from the Kalman prediction algorithms. The mean error, however, is not zero, and we can calculate the root mean square error as a criterion for the quality of the forecast. From eqns. (5.1) and (5.2), applying Bessel's correction [17],

$$(\sum_{i=1}^{t} e_i^2)/(t-1) = (\text{ACT. S.D.})^2 + [(\sum_{i=1}^{t} e_i)/t]^2[t/(t-1)]$$

so that rms error = $[(\text{ACT. S.D.})^2 + (\text{MEAN ERR.})^2(t/(t-1)]^{\frac{1}{2}}$

(5.3)

For example, for the entire forecast, the rms prediction error is:

$$[126.40^2 + 203.10^2\ (72/71)]^{\frac{1}{2}} = 240 \text{ MW}$$

This rms error of 2.6% of peak load of 9,099 MW is acceptable. Parameters, identified from data of the individual four weeks of January, clearly are not constant, as can be seen in Table 8. Even the mean and the variance (Q) are not constant. Our intent, however, was to update the parameters weekly, recognizing that they would change, yet hoping that the changes would be small enough to give a satisfactory forecast. Our basic forecast is indeed acceptable. With these qualifications then, our assumption of effective statistical stationarity is valid for our purpose.

Forecasts made, using parameters identified from one week's data, were intended for analysis of the basic forecast. These forecasts are reasonably successful on their own however, as shown in Figs. 4.3 to 4.7 and the left portion of Table 10. In particular, the forecasts in Figs. 4.5 and 4.2, using parameters (Table 8) identified from data of the single previous week, and three weeks, are similar. Therefore, 96 data points may be sufficient if the parameters identified from them are used to modify those previously calculated.

The forecast in Fig. 4.7 has lead times up to 168 hours, or one full week. It, like all the other forecasts, depends for its success on validity of parameters throughout the forecast period.

The forecast in Fig. 4.6 can be accomplished only in retrospect since the parameters are identified from data of the forecasted week itself: these data are unavailable until the week has passed. The quality of the forecast shows that the identified parameters are indeed valid for the particular week, and that our mathematical model using them describes the system well. In particular, the mean

TABLE 10

Summary of Actual Standard Deviations, Means, and rms Errors for five forecasts into 4th week, with improvement that would result if we could substitute 7599.4 for the single parameter x_p^0, leaving other parameters unchanged.

Parameters identified from Data of	Refer to Fig.	Actual Standard Deviation	Actual Mean Error	Actual rms Error	Altered Mean Error	Change in Mean Error	Altered rms Error
1st week 04 to 07Jan	4.3	127	215	251	36	178	159
2nd week 11 to 14Jan	4.4	137	302	334	-35	337	142
3rd week 18 to 21Jan	4.5	155	214	266	08	207	128
three weeks combined	4.2	126	203	240	-41	244	127
4th week 25 to 28Jan	(4.6)	(88)	(0)	(88)	(0)	(0)	(88)

error is approximately zero and the actual standard deviation approximates that calculated by the Kalman prediction algorithms during the forecast. There is some day-to-day variation which indicates that models for Tuesday, Wednesday, Thursday and Friday could be made slightly different, and such a possibility will be considered shortly.

Now we shall consider weather factors other than temperature in 4th week in relation to the other three weeks of Jan72. Weather records [5] show that 4th week was windier and sunnier. As can be seen from Figs. 4.1 and Table 11, sunset and onslaught of evening lighting load coincided closely and increasingly so as the month progressed. In the Montreal area, wind has been found to increase winter heating load considerably [8]. Therefore, we should expect x_p^0, in Table 8, for data of 4th week, to be high. Changes in illumination tend to give changes in temperature in the same direction. In winter then, the effect on load of such temperature changes may be compounded with changes in lighting load. Thus, wind and illumination may explain the large b_0, b_1, 4th week. Many electric utilities treat illumination as an important weather factor in load forecasts, particularly if a significant portion of the load is domestic and commercial [4] as applies to Hydro-Québec. It appears that our forecast in 4th week would be more accurate if we included wind and illumination as weather factors in our model.

TABLE 11

Time of Sunset, Dorval, January 1972

Date in Jan72	Time of Sunset p.m., EST
1	4:21
2	4:22
3	4:23
4	4:24
5	4:24
6	4:26
7	4:27
8	4:28
9	4:29
10	4:30
11	4:31
12	4:32
13	4:34
14	4:35
15	4:36
16	4:38
17	4:39
18	4:40
19	4:42
20	4:43
21	4:45
22	4:46
23	4:48
24	4:49
25	4:50
26	4:52
27	4:53
28	4:55
29	4:56
30	4:57
31	4:59

Generally, in our forecasts much of the error can be seen as a shift in the mean for the forecasted week or for an individual day in that week. The mean of $z(t)$ through one, or any integral number of days, consists of the mean x_p^0 of the periodic component plus the mean of $\underline{c}^T\underline{x}(t)$ as can be seen from eqns. (3.2a) and (3.14c). For zero-mean noise, the mean of $\underline{c}^T\underline{x}(t)$ is determined by weather factors once $\underline{A}$ and $\underline{B}$ are fixed, as can be seen from eqn. (3.13). In short then, the mean of $z(t)$, through any integral number of days, is x_p^0 adjusted for off-normal weather. If a pattern of weekly mean load has been established from past load data on the basis of normal weather, we can use these mean values to adjust x_p^0 for the forecasted week. Similarly, if we know how the mean loads for the various days of the week normally compare, x_p^0 can be adjusted daily on their basis. We are thus recognizing that the system parameters are not thoroughly constant throughout the data plus forecast period, and are applying adjustment to a single parameter as an approximation to their change. The right side of Table 10 shows the improvement in forecasts that would result if somehow we were able to anticipate 7599.4 MW as a good value to use for x_p^0, when forecasting into 4th week. This hypothetical improvement can be visualized in the graphs listed in the second column of this table with all forecast points raised the amount listed in column 7.

5.2 Concluding Remarks

Forecast of load through 26 to 28Jan72 is satisfactory on the basis of accuracy and computer time for off-line computation of parameters, identifying these parameters using data of the first three 4-day weeks of Jan72. The daily peak load is forecasted with timing as close as discernible from hourly forecasts.

Calculation of normal temperatures, required in determining the temperature-dependent variable u(t), could be improved by taking an exponentially-weighted average of temperatures through the past twenty years. This procedure would provide some smoothing and eliminate the need for storing historical temperatures.

Concerning identification of parameters, solution values were found to be identical for all starting values tried, corresponding to any particular set of data (load z(t) and temperature-dependent variable u(t)). Also, parameters identified, using a three-week set of data, were found to be suitable starting values for a similar calculation one week later, off-line central processor time required for such a weekly calculation being approximately 7 minutes. This computing time was obtained using a CDC 6400 computer.

It must be emphasized that our load forecasts are based on perfect temperature forecasts. We are, therefore, relying on accuracy and availability of long-term temperature forecasts.

Statistical checking and trial-and-error procedures would be required to determine optimum order of the model. Values of identified parameters suggest, however, that third order could be tried concerning parameters $\{a_i\}$ and perhaps $\{b_j\}$ and eighth harmonic in the periodic component. These trials would be made, not necessarily expecting improvement of the model in each case but to search for an optimum.

Reference to weather records indicated that the load forecast through 26 to 28Jan72 would have been improved by including wind speed and probably illumination as weather factors, in addition to temperature already accounted for in the load model.

Also, predominant error in the forecasts appeared to be representable as a shift in average forecast of load over one or several days. Information available for pattern of weekly mean load, and comparative mean loads for typical days-of-week, therefore, could be used to adjust just one parameter x_p^0, leaving values of other parameters as identified.

The basic forecast was for load through 26 to 28Jan72, using parameters identified from data of the preceeding three 4-day weeks. Forecasts made for analysis of the basic forecast, using parameters identified from data of one 4-day week, however, turned out to be reasonably accurate in themselves, including one forecast with lead times up to one full week. Possibly therefore, parameters identified from the most recent single week's data could be used as an adjustment on parameters identified in the previous calculation, with a net reduction in CPU time required for the computations.

CHAPTER 6

SUGGESTIONS FOR FURTHER WORK

6.1 Statistical Analysis for Further Insight, Evaluation and Possible Improvement

6.1.1 Statistical checking

Statistical checking, as outlined in section 2.6, would give further insight into the system. In conjunction with such checking, trial-and-error experiments, varying the order of the model (e.g. using a_1, a_2, a_3, b_0, b_1, b_2, instead of a_1, a_2, b_0, b_1, and using up to eighth harmonic instead of seventh) might be conducted.

6.1.2 Separation of load into four components or more

Load at forecast lead time ℓ is:

$$z(\ell+n) = \text{forecasted load} + \text{error in the forecast}$$
$$= \hat{z}(\ell+n|n-1) + [z(\ell+n) - \hat{z}(\ell+n|n-1)]$$

or, from eqn. (3.23),

$$z(\ell+n) = \text{periodic component} + \underline{c}^T\ \underline{B}\ \underline{u}(\ell+n-1)$$
$$+ \underline{c}^T\ \underline{A}\ \underline{\hat{x}}(\ell+n-1|n-1) + \text{error}$$

The forecasting program can be altered to print out all four of these components. We already have print-outs of two. All four then could be plotted so as to observe their individual contributions to the make-up of the load. If desired, further separation could be made, into retrogressive parts.

6.1.3 Forecasting load without using weather forecast

In addition to separation into four components as above, identification of parameters and forecasting could be done setting $b_0 = b_1 = 0$, and $\underline{u}(t) = [0]$. This procedure would enable comparison of forecasting with and without weather forecasts. It would give different results from $\hat{z}(\ell+n) - \underline{c}^T \underline{B}\, \underline{u}(\ell+n-1)$, obtainable from section 6.1.2.

6.2 Implementation

6.2.1 Continuous forecasting through weekends, Mondays and holidays

In actual practice, forecasts of hourly load would be updated each hour, and parameters would be updated off-line each week. Computer implementation to the extent of providing a continuous forecast through Saturdays, Sundays, holidays, and weekdays including Mondays is beyond the scope of the thesis. Work done in the thesis, however, indicates that no serious problems should be expected.

6.2.2 Forecasting in summer, spring and fall

Forecasting other than in January 1972 also is beyond the scope of this thesis. In summer, as can be seen for example in Figs. 1.1 to 1.5, the afternoon peak is much subdued in comparison to winter.

Since annual peak loads tend to occur in summer and winter, most preliminary work on load forecasting tends to concentrate on those seasons. In spring and fall, however, both load and weather exhibit their maximum rates of change from week to week. Therefore, more care will be required regarding statistical stationarity. Also, heating and cooling load may be encountered in the same week. Therefore, in general a compensating coefficient will be required, depending on whether heating or cooling load applies on calculating u(t). It is to be hoped this coefficient can be constant throughout any particular spring or fall season.

6.2.3 <u>Allowance in load forecast for expected and unexpected events</u>

The operators may anticipate changes in generating requirements due to early factory closings on Election Days, etc. Work can be done on how to adjust the load forecast to use such information, since such changes are an anomaly to our forecasting method. One of the most common of such changes is a contract to buy or sell power through tie-lines to other electric utilities. In general, a forecast into such periods might be executed normally but treated as preliminary, for adjustment by simply adding the anticipated load disturbance to it.

Concerning unexpected events, anomaly detection is the first step. Galiana [3] includes some remarks on such detection and modification of the load forecast until return of operation to normal.

6.3 Considerations on Calculation of Temperature-Dependent Variable u(t)

6.3.1 Heating and cooling load in same week

An additional coefficient is required in general, as mentioned in section 6.2.2.

6.3.2 Interpolation of available forecasts of maximum and minimum temperature

Reliance on accuracy and availability of long-term forecasts of temperature is a serious limitation of our method. The electric utility usually has its own meteorologist. Also, the local weather office usually issues forecasts of minimum and maximum temperatures with lead time up to two days or more. From Dorval, estimated times of occurrence of these maxima and minima are available by telephone.

Work can be done on interpolation of hourly temperatures from available forecasts of maxima and minima, using them in the load forecast. Examination from an accuracy viewpoint of records of such forecasts of maxima and minima issued from Dorval indicates that such work is worthwhile.

6.3.3 Obtaining normal temperatures by filtering of historical data

Normal temperatures $\hat{T}$, Table 2, could be obtained as suggested in section 5.1.1, i.e. by exponentially weighted averages followed by moving averages.

To go further, however, historical temperature data might be represented by an autoregressive moving average (ARMA) series, filtered, applying the Kalman filter. This method would give statistically correct results and reduce the somewhat erratic variation seen in Table 2.

6.4 Use of Weather Factors other than Temperature for Load Forecasting

As suggested in Chapter 5, it looks worthwhile to take into account wind velocity and illumination as well as temperature as weather factors affecting load. In summer, another weather factor increasingly used [4] is humidity, because of its effect on refrigeration and air conditioning load. Possibly this factor should be used instead of wind velocity for summer forecasts of Hydro-Québec load.

6.5 Separation of Periodic and Residual Components

6.5.1 Galiana

Galiana [3] describes two methods of periodic component separation: (a) data pre-filtering and (b) weighted least squares identification of periodic component parameters $\{x_p^i\}$ followed by iteration.

6.5.2 Panuska

Panuska [21] has suggested 24-hour differencing from which parameters $\{a_i\}$ and $\{b_j\}$ can be identified for estimation, $\rho(t)$, of the residual component. Then the periodic parameters $\{x_p^i\}$ can be identified from $\hat{y}_p(t) = z(t) - \hat{y}(t)$. Iteration between the periodic and stochastic components follows to convergence.

6.5.3 Averaging of three sets of periodic parameters, and iteration

In Table 8, for the particular Hydro-Québec data of January 1972, the periodic parameters identified using three weeks' data were closely equivalent to the average of those identified individually using data of each of the three single weeks. Such a hypothesis could be tested as follows:

As a preliminary, parameters would be identified weekly, using data from the single preceeding week. The periodic parameters so obtained then would be modified by averaging-in with those (stored) from two weeks and three weeks ago. The estimate $\hat{y}_p(t)$ of the periodic component would be calculated from the result and subtracted from load data to give the estimated residual component, enabling identification of the residual component parameters, $\{a_i\}$ and $\{b_j\}$. For this latter identification, three weeks' data would be used. Iteration between residual and periodic components then would proceed to convergence.

6.5.4 Kalman filtering of periodic parameters, and iteration

With Kalman filtering of periodic parameters as an extension, the parameters would be identified using data from the single preceeding week, as in section 6.5.3. The periodic parameters, however, would be filtered by Kalman filter to yield a one-step prediction. The predicted parameters would be a forecast of those applicable to the forecast week. From them, the estimate of $\hat{y}_p(t)$ would be calculated and the remaining procedure would be as in section 6.5.3.

Such ideas are apparently supported by Lijesen and Rosing under "Further Developments" in reference [20].

6.5.5 Separation of periodic component as a table of values, followed by iteration

24-hour moving averages subtracted from load data, then averaged vertically, would produce a set of 24 values to be used as an approximation $\hat{y}_p(t)$ of the periodic component. Alternatively, the more comprehensive BLS seasonal factor method [14] could be used.

Subtracting the above from load data, $\{a_i\}$ and $\{b_j\}$ could be identified from the difference. These values then would be used to calculate an approximation $\hat{y}(t)$ of the residual.

Subtracting the sum of the above two approximations from load data, an error would be obtained. Treatment of this error as in the first paragraph of section 6.5.5 would yield a correction on $\hat{y}_p(t)$. Iteration would continue to convergence.

6.6 Use of New Fletcher Method

A program ZXPOWL for minimizing a function of n variables, using a modified Powell algorithm, is obtainable from the following address as part of the "IMSL" package:

International Mathematical Statistical Library
Suite 510
6200 Hillcroft
Houston, Texas 77036.

Financial resources do not permit use of ZXPOWL in this thesis. For actual implementation in a power system, however, saving in CPU time could make purchase well worth while.

6.7 Hargreaves and Panuska

Recursive least squares identification can be performed, using additional adaptive gain applied for parameters of the periodic component only [23, 26]. This gain is a function of estimated variance and approaches unity as iterations proceed.

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APPENDIX

APPENDIX

Listing of Parameter Identification Program "POWEL5"

```
      PROGRAM POWEL5(INPUT,OUTPUT)
      DIMENSION Z(500),U(500),X(26),GRAD(26),H(430)
      DIMENSION XX(26),ZZ(500),UU(500)
      DOUBLE PRECISION X,Z,U,F,GRAD,H
      COMMON/DATA11/Z,U/ /NA,NB,NXP,NT,ND
      EXTERNAL FUNCT
C
C SPECIFY NUMBER OF DATA POINTS,ORDER OF MODEL,
C AND READ STARTING VALUES OF PARAMETERS
      TIME1=SECOND(TIME1)
      NS=1
      NJ=4
      ND=NS*NJ*24
      NA=2
      NB=2
      NXP=15
      NT=NA+NB+NXP
      READ 10,(XX(I),I=1,NT)
   10 FORMAT(8F10.3)
      DO 8 I=1,NT
    8 X(I)=XX(I)
      DO 9 I=1,NT
    9 X(I)=XX(I)
C
C PRINT STARTING VALUES OF PARAMETERS
      PRINT 7
    7 FORMAT(1H1,*     *,///)
      PRINT 11
   11 FORMAT(10X,*BELOW ARE PARAMETER STARTING VALUES,IN THE FOLLOWING
     1FORMAT*,//)
      PRINT 12
   12 FORMAT(24X,*A1*,11X,*A2*,11X,*B0*,11X,*B1*,/,12X,*          XP0
     1 *,6X ,* XP1              XP2              XP3  *,/,14X,*  *,6X,* XP4   *7X
     1,* XP5  *,7X,* XP6  *,7X,* XP7  *,/,10X,*            *,16X,* XP8   *7X
     1,* XP9  *,7X,* XP10 *,/,22X,* XP11 *,7X,* XP12 *,7X,* XP13 *,7X,*
     1XP14 *)
      PRINT 30
      PRINT 13,(X(I),I=1,NT)
   13 FORMAT(17X        ,4(1X,D12.5),/,2(17X,4(1X,D12.5),/),30X,3(1X,D12.5
     1),/,17X,4(1X,D12.5),/)
C
C READ LOAD DATA,Z,AND TEMPERATURE-DEPENDENT DATA,U
      READ 15,(ZZ(I),I=1,ND)
   15 FORMAT(10X,12F5.0)
      READ 16,(UU(I),I=1,ND)
   16 FORMAT(12F5.1)
      DO 4 I=1,ND
      Z(I)=ZZ(I)
    4 U(I)=UU(I)
C
      PRINT 50
```

```
 50 FORMAT(1H1,10X,*BELOW ARE PARAMETER VALUES DURING ITERATION,IN THE
   1 FOLLOWING FORMAT*,///)
    PRINT 60
 60 FORMAT(24X,*A1*,11X,*A2*,11X,*B0*,11X,*B1*,/,12X,*NUMBER      XP0
   1 *,6X ,* XP1               XP2                XP3  *,/,14X,*OF*,6X,* XP4  *7X
   1,* XP5  *,7X,* XP6  *,7X,* XP7  *,/,10X,*ITERATION*,16X,* XP8  *7X
   1,* XP9  *,7X,* XP10 *,/,22X,* XP11 *,7X,* XP12 *,7X,* XP13 *,7X,*
   1XP14 *)
    PRINT 29
 29 FORMAT(44X,*U*,/)
  C
    LIMIT=60
    EPS=0.1E-04
    EST=10000
    IT=LIMIT
    CALL DFMFP(FUNCT,NT,X,F,GRAD,EST,EPS,LIMIT,IER,H)
    PRINT 32
 32 FORMAT(1H1,10X,*RESULTS*,//)
    PRINT 33,ND
 33 FORMAT(10X,*NUMBER OF DATA POINTS=*,I3)
    IF(IER .EQ. 0) GO TO 18
    IF(IER .EQ. 1) GO TO 20
    PRINT 17,IER
 17 FORMAT(10X,*IER=*,I2)
     GO TO 22
 18 PRINT 19
 19 FORMAT(10X,*CALCULATION TERMINATED BECAUSE CONVERGENCE OBTAINED*)
     GO TO 22
 20 PRINT 21
 21 FORMAT(10X*CALCULATION TERMINATED AT ITERATIONS LIMIT*)
 22 PRINT 23,LIMIT
 23 FORMAT(//,10X,*NUMBER OF ITERATIONS=*,I4)
    PRINT 24,EPS
 24 FORMAT(10X,*EPS=*,E7.1)
    PRINT 31,IT
 31 FORMAT(10X,*ITERATIONS LIMIT=*,I3)
    TIME2=SECOND(TIME2)
    TT=TIME2-TIME1
    PRINT 25,TT
 25 FORMAT(10X,*CALCULATION TIME=*,F10.3)
    PRINT 26
 26 FORMAT(10X,*BELOW ARE PARAMETER SOLUTION VALUES,IN THE FOLLOWING
   1FORMAT*,//)
    PRINT 12
    PRINT 28
 28 FORMAT(44X,*Q*,/)
    PRINT 13,(X(I),I=1,NT)
 30 FORMAT(10X,* *,/)
    PRINT 27,F
 27 FORMAT(37X,D16.10)
    STOP
    END
```

```
      SUBROUTINE PUNCT(N,X,VAL,GRAD,KOUNT)
      DIMENSION Z(500),U(500),A(5),B(5),XP(16),X(26),PSI(80),PHI(16),
     1F5(5),GRAD(26),F8(16)
C
      DOUBLE PRECISION Z,U,A,B,XP,X,GRAD,VAL,PSI,PHI,F1,F2,F3,F4,F5,P,
     2F6,F7,F8
C
      COMMON/DATA11/Z,U
      COMMON NA,NB,NXP,NT,ND
C NOW PRINT PARAMETERS AT INTERVALS OF 10 ITERATIONS,OR MORE
      IF(KOUNT .EQ. 70) PRINT 27
      IF(KOUNT .EQ. 150) PRINT 27
   27 FORMAT(1H1,*    *,///)
      IF(KOUNT-(KOUNT/10)*10 .EQ. 0) GO TO  1
      GO TO 2
    1 PRINT 70,KOUNT,(X(I),I=1,N)
   70 FORMAT(12X,I4,1X,4(1X,D12.5),/,2(17X,4(1X,D12.5),/),30X,3(1X,D12.5
     1),/,17X,4(1X,D12.5))
    2 VAL=0.0
      DO 30 I=1,NT
   30 GRAD(I)=0.0
C
      DO 3 I=1,NA
    3 A(I)=X(I)
C
      DO 4 I=1,NB
      K=I+NA
    4 B(I)=X(K)
C
      DO 5 I=1,NXP
      K=I+NA+NB
    5 XP(I)=X(K)
C
C
      DO 6 I=1,NA
      NI=24-I
      T=NI
      CALL TEPER(NXP,T,PHI)
      DO 7 J=1,NXP
      K= J+(I-1)*NXP
    7 PSI(K)=PHI(J)
    6 CONTINUE
C
      DO 10 I=24,ND
      T=I
      F1=P(I)
C
      CALL TEPER(NXP,T,PHI)
      F2=0.0
      DO 11 J=1,NXP
   11 F2=F2+PHI(J)*XP(J)
C
      F3=0.0
      DO 12 J=1,NA
```

```
      K=I-J
12    F3=F3+A(J)*Z(K)
C
      F4=0.0
      DO 13 J=1,NB
      K=I-J+1
13    F4=F4+B(J)*U(K)
      DO 40 J=1,NA
40    F5(J)=0.0
      DO 15 J=1,NA
      P=0.0
      DO 14 J1=1,NXP
      K=J1+(J-1)*NXP
14    P=P+PSI(K)*XP(J1)
15    F5(J)=P
C
      F6=0.0
      DO 16 J=1,NA
16    F6=F6+F5(J)*A(J)
C
      F7=F1+F6-F2-F3-F4
      VAL=VAL+F7**2
      DO 18 J=1,NA
      K=I-J
18    GRAD(J)=GRAD(J)+2.0*F7*(F5(J)-Z(K))
C
C
      DO 19 J=1,NB
      K=I-J+1
      K1=J+NA
19    GRAD(K1)=GRAD(K1)-2.0*F7*U(K)
C
      DO 22 J=1,NXP
22    F8(J)=0.0
      DO 20 J=1,NXP
      DO 21  JN=1,NA
       K=J+(JN-1)*NXP
      F8(J)=F8(J)+PSI(K)*A(JN)
21    CONTINUE
      K1=J+NA+NB
      GRAD(K1)=GRAD(K1)+2.0*F7*(F8(J)-PHI(J))
20    CONTINUE
      NA1=NA-1
      DO 23 J=1,NA1
      K=NA-J
      DO 24  J1=1,NXP
       JN1=J1+K*NXP
      JN2=JN1-NXP
24    PSI(JN1)=PSI(JN2)
23    CONTINUE
C
      DO 25 J=1,NXP
C
```

```
25   PSI(J)=PHI(J)
10   CONTINUE
     VAL=VAL/(ND-23)
     IF(KOUNT-(KOUNT/10)*10 .EQ. 0) GO TO  9
     GO TO 17
  9  PRINT  8,VAL
  8  FORMAT(37X,D16.10,/)
 17  DO 26J=1,NT
 26  GRAD(J)=GRAD(J)/(ND-23)
     RETURN
     END




     SUBROUTINE TEPER(N,T,PHI)
     DOUBLE PRECISION PHI  ,PI
     DIMENSION PHI(16)
C
     PI=3.14159265359
C    INITIALIZATION
     DO 10  I=1,N
10   PHI(I)=0.0
     PHI(1)=1.0
     M=(N-1)/2
     K=0
C
     DO 20  I=1,M
     K=I+1
     PHI(K)=DSIN(2.0*T*FLOAT(I)*PI/24.0)
     L=K+M
     PHI(L)=DCOS(2.0*T*FLOAT(I)*PI/24.0)
20   CONTINUE
C
     RETURN
     END




     SUBROUTINE DFMFP(FUNCT,N,X,F,G,EST,EPS,LIMIT,IER,H)
     DOUBLE PRECISION X,F,FX,FY,OLDF,HNRM,GNRM,H,G,DX,DY,ALFA,DALFA,
    1AMBDA,T,Z,W
     DIMENSION H(430),X(26),G(26)
     CALL FUNCT(N,X,F,G,KOUNT)
     IER=0
     KOUNT=0
     N2=N+N
     N3=N2+N
     N31=N3+1
```

```
    1 K=N31
      DO 4 J=1,N
      H(K)=1.00
      NJ=N-J
      IF(NJ)5,5,2
    2 DO 3 L=1,NJ
      KL=K+L
    3 H(KL)=0.00
    4 K=KL+1
    5 KOUNT=KOUNT+1
      OLDF=F
      DO 9 J=1,N
      K=N+J
      H(K)=G(J)
      K=K+N
      H(K)=X(J)
      K=J+N3
      T=0.00
      DO 8 L=1,N
      T=T-G(L)*H(K)
      IF(L-J)6,7,7
    6 K=K+N-L
      GO TO 8
    7 K=K+1
    8 CONTINUE
    9 H(J)=T
      DY=0.00
      HNRM=0.00
      GNRM=0.00
      DO 10 J=1,N
      HNRM=HNRM+DABS(H(J))
      GNRM=GNRM+DABS(G(J))
   10 DY=DY+H(J)*G(J)
      IF(DY)11,51,51
   11 IF(HNRM/GNRM-EPS)51,51,12
   12 FY=F
      ALFA=2.00*(EST-F)/DY
      AMBDA=1.00
      IF(ALFA)15,15,13
   13 IF(ALFA-AMBDA)14,15,15
   14 AMBDA=ALFA
   15 ALFA=0.00
   16 FX=FY
      DX=DY
      DO 17 I=1,N
   17 X(I)=X(I)+AMBDA*H(I)
      CALL FUNCT(N,X,F,G,KOUNT)
      KOUNT=KOUNT+1
      IF(KOUNT.GT.LIMIT) GO TO 50
      FY=F
      DY=0.00
      DO 18 I=1,N
```

```
18 DY=DY+G(I)*H(I)
   IF(DY)19,36,22
19 IF(FY-FX)20,22,22
20 AMBDA=AMBDA+ALFA
   ALFA=AMBDA
   IF(HNRM*AMBDA-1.E10)16,16,21
21 IER=2
   LIMIT=KOUNT
   RETURN
22 T=0.D0
23 IF(AMBDA)24,36,24
24 Z=3.D0*(FX-FY)/AMBDA+DX+DY
   ALFA=DMAX1(DABS(Z),DABS(DX),DABS(DY))
   DALFA=Z/ALFA
   DALFA=DALFA*DALFA-DX/ALFA*DY/ALFA
   IF(DALFA)51,25,25
25 W=ALFA*DSQRT(DALFA)
   ALFA=DY-DX+W+W
   IF(ALFA)250,251,250
250 ALFA=(DY-Z+W)/ALFA
   GO TO 252
251 ALFA=(Z+DY-W)/(Z+DX+Z+DY)
252 ALFA=ALFA*AMBDA
   DO 26 I=1,N
26 X(I)=X(I)+(T-ALFA)*H(I)
   CALL FUNCT(N,X,F,G,KOUNT)
   KOUNT=KOUNT+1
   IF(KOUNT.GT.LIMIT) GO TO 50
   IF(F-FX)27,27,28
27 IF(F-FY)36,36,28
28 DALFA=0.D0
   DO 29 I=1,N
29 DALFA=DALFA+G(I)*H(I)
   IF(DALFA)30,33,33
30 IF(F-FX)32,31,33
31 IF(DX-DALFA)32,36,32
32 FX=F
   DX=DALFA
   T=ALFA
   AMBDA=ALFA
   GO TO 23
33 IF(FY-F)35,34,35
34 IF(DY-DALFA)35,36,35
35 FY=F
   DY=DALFA
   AMBDA=AMBDA-ALFA
   GO TO 22
36 IF(CLOF-F+EPS)51,38,38
38 DO 37 J=1,N
   K=N+J
   H(K)=G(J)-H(K)
   K=N+K
37 H(K)=X(J)-H(K)
```

```
      IER=0
      IF(KOUNT-N)42,39,39
   39 T=0.D0
      Z=0.D0
      DO 40 J=1,N
      K=N+J
      W=H(K)
      K=K+N
      T=T+DABS(H(K))
   40 Z=Z+W*H(K)
      IF(HNRM-EPS)41,41,42
   41 IF(T-EPS)56,56,42
   42 IF(KOUNT-LIMIT)43,50,50
   43 ALFA=0.D0
      DO 47 J=1,N
      K=J+N3
      W=0.D0
      DO 46 L=1,N
      KL=N+L
      W=W+H(KL)*H(K)
      IF(L-J)44,45,45
   44 K=K+N-L
      GO TO 46
   45 K=K+1
   46 CONTINUE
      K=N+J
      ALFA=ALFA+W*H(K)
   47 H(J)=W
      IF(Z*ALFA)48,1,48
   48 K=N31
      DO 49 L=1,N
      KL=N2+L
      DO 49 J=L,N
      NJ=N2+J
      H(K)=H(K)+H(KL)*H(NJ)/Z-H(L)*H(J)/ALFA
   49 K=K+1
      GO TO 5
   50 IER=1
      LIMIT=KOUNT
      RETURN
   51 DO 52 J=1,N
      K=N2+J
   52 X(J)=H(K)
      CALL FUNCT(N,X,F,G,KOUNT)
      KOUNT=KOUNT+1
      IF(KOUNT.GT.LIMIT) GO TO 50
      IF(GNRM-EPS)55,55,53
   53 IF(IER)56,54,54
   54 IER=-1
      GO TO 1
   55 IER=0
   56 LIMIT=KOUNT
      RETURN
      END
```

Identification of Parameters from Data of 1st Week, using Initial Starting Values of Table 6, Section 4.2

BELOW ARE PARAMETER STARTING VALUES,IN THE FOLLOWING FORMAT

A1	A2	B0	B1
XP0	XP1	XP2	XP3
XP4	XP5	XP6	XP7
	XP8	XP9	XP10
XP11	XP12	XP13	XP14

.250000D+00	.250000D+00	.132000D+02	0.
.735100D+04	-.929800D+03	-.376200D+03	.115400D+03
.522000D+02	0.	-.313000D+02	0.
	-.282200D+03	.248500D+03	-.217200D+03
.547000D+02	0.	-.496000D+02	0.

BELOW ARE PARAMETER VALUES DURING ITERATION,
IN THE FOLLOWING FORMAT

NUMBER OF ITERATION	A1	A2	B0	B1
	XP0	XP1	XP2	XP3
	XP4	XP5	XP6	XP7
		XP8	XP9	XP10
	XP11	XP12	XP13	XP14
			J	
0	.25000D+00	.25000D+00	.13200D+02	0.
	.73510D+04	-.92980D+03	-.37620D+03	.11540D+03
	.52200D+02	0.	-.31300D+02	0.
		-.28220D+03	.24850D+03	-.21720D+03
	.54700D+02	0.	-.49600D+02	0.
			.4418282391D+05	
10	.81940D-01	.46264D+00	-.14594D+02	.17992D+02
	.73887D+04	-.94820D+03	-.38039D+03	.11771D+03
	.23637D+02	-.93010D+02	.82054D+01	.31758D+02
		-.26389D+03	.21446D+03	-.20049D+03
	.10482D+03	.55355D+02	-.47221D+02	-.34986D+02
			.1839063466D+05	
20	-.15193D-02	.38356D+00	-.43348D+01	.85174D+01
	.74134D+04	-.96970D+03	-.38523D+03	.11726D+03
	.22103D+02	-.78218D+02	-.22270D+01	.21268D+02
		-.23824D+03	.18340D+03	-.19524D+03
	.10374D+03	.44785D+02	-.49798D+02	-.25918D+02
			.1655211815D+05	
30	-.30118D-01	.41431D+00	-.77719D+00	.48321D+01
	.74282D+04	-.97590D+03	-.38611D+03	.11642D+03
	.20154D+02	-.85071D+02	-.56408D+00	.21480D+02
		-.23042D+03	.18426D+03	-.19606D+03
	.10605D+03	.45934D+02	-.50905D+02	-.27131D+02
			.1638263939D+05	
50	.10073D+00	.27542D+00	-.32706D+01	.70187D+01
	.74220D+04	-.97783D+03	-.38764D+03	.11641D+03
	.14931D+02	-.98838D+02	.25076D+01	.21809D+02
		-.22815D+03	.17970D+03	-.19286D+03
	.11450D+03	.53976D+02	-.50834D+02	-.28123D+02
			.1564627794D+05	
60	.10471D+00	.26509D+00	-.30288D+01	.68216D+01
	.74221D+04	-.97982D+03	-.38953D+03	.11583D+03
	.15729D+02	-.95155D+02	.16324D+01	.21732D+02
		-.22582D+03	.17803D+03	-.19289D+03
	.11255D+03	.51080D+02	-.51126D+02	-.28096D+02
			.1561927187D+05	

```
 70    .10448D+00   .26607D+00  -.30719D+01   .68694D+01
       .74218D+04  -.98016D+03  -.38945D+03   .11590D+03
       .15726D+02  -.95206D+02   .16434D+01   .21746D+02
                   -.22546D+03   .17824D+03  -.19301D+03
       .11254D+03   .50993D+02  -.51208D+02  -.28181D+02
                          .1561914463D+05

 80    .10452D+00   .26591D+00  -.30759D+01   .68749D+01
       .74218D+04  -.98018D+03  -.38945D+03   .11588D+03
       .15729D+02  -.95212D+02   .16395D+01   .21744D+02
                   -.22546D+03   .17826D+03  -.19299D+03
       .11254D+03   .51011D+02  -.51221D+02  -.28188D+02
                          .1561914309D+05

 90    .10452D+00   .26591D+00  -.30758D+01   .68749D+01
       .74218D+04  -.98018D+03  -.38945D+03   .11588D+03
       .15728D+02  -.95212D+02   .16393D+01   .21744D+02
                   -.22546D+03   .17826D+03  -.19299D+03
       .11254D+03   .51011D+02  -.51221D+02  -.28188D+02
                          .1561914309D+05

100    .10452D+00   .26591D+00  -.30758D+01   .68749D+01
       .74218D+04  -.98018D+03  -.38945D+03   .11588D+03
       .15728D+02  -.95212D+02   .16393D+01   .21744D+02
                   -.22546D+03   .17826D+03  -.19299D+03
       .11254D+03   .51011D+02  -.51221D+02  -.28188D+02
                          .1561914309D+05

RESULTS

NUMBER OF DATA POINTS= 96
CALCULATION TERMINATED BECAUSE CONVERGENCE OBTAINED
NUMBER OF ITERATIONS= 101
EPS= .1E-04
ITERATIONS LIMIT=130
CALCULATION TIME=     95.618
BELOW ARE PARAMETER SOLUTION VALUES,IN THE FOLLOWING  FORMAT

          A1           A2           B0           B1
          XP0          XP1          XP2          XP3
          XP4          XP5          XP6          XP7
                       XP8          XP9          XP10
          XP11         XP12         XP13         XP14
                              Q

       .10452D+00   .26591D+00  -.30758D+01   .68749D+01
       .74218D+04  -.98018D+03  -.38945D+03   .11588D+03
       .15728D+02  -.95212D+02   .16393D+01   .21744D+02
                   -.22546D+03   .17826D+03  -.19299D+03
       .11254D+03   .51011D+02  -.51221D+02  -.28188D+02

                          .1561914309D+05
```

Identification of Parameters from Data of 2nd Week, using those identified from Data of 1st Week as Starting Values

BELCW ARE PARAMETER STARTING VALUES,IN THE FOLLOWING FORMAT

A1	A2	B0	B1
XP0	XP1	XP2	XP3
XP4	XP5	XP6	XP7
	XP8	XP9	XP10
XP11	XP12	XP13	XP14

.10500D+00	.26600D+00	-.30760D+01	.68750D+01
.74218D+04	-.98018D+03	-.38945D+03	.11588D+03
.15728D+02	-.95212D+02	.16390D+01	.21744D+02
	-.22546D+03	.17826D+03	-.19299D+03
.11254D+03	.51011D+02	-.51221D+02	-.28188D+02

BELOW ARE PARAMETER VALUES DURING ITERATION,
IN THE FOLLOWING FORMAT

NUMBER OF ITERATION	A1	A2	B0	E1
	XP0	XP1	XP2	XP3
	XP4	XP5	XP6	XP7
		XP8	XP9	XP10
	XP11	XP12	XP13	XP14
		J		
0	.10500D+00	.26600D+00	-.30760D+01	.68750D+01
	.74218D+04	-.98018D+03	-.38945D+03	.11588D+03
	.15728D+02	-.95212D+02	.16390D+01	.21744D+02
		-.22546D+03	.17826D+03	-.19299D+03
	.11254D+03	.51011D+02	-.51221D+02	-.28188D+02
		.5151060986D+05		
10	.18311D+00	.56655D+00	-.71377D+01	.11994D+02
	.74160D+04	-.97222D+03	-.40196D+03	.14185D+03
	-.49744D+01	-.10809D+03	-.19203D+02	.15896D+02
		-.23334D+03	.18917D+03	-.18503D+03
	.82645D+02	.90124D+02	-.19245D+02	-.60720D+02
		.1159230639D+05		
20	.19134D+00	.51057D+00	-.27601D+01	.84485D+01
	.74081D+04	-.95725D+03	-.41484D+03	.14856D+03
	.25877D+01	-.10451D+03	-.11291D+02	.15604D+02
		-.24872D+03	.20132D+03	-.18285D+03
	.88975D+02	.73917D+02	-.32341D+02	-.46868D+02
		.1016806756D+05		
40	.15436D+00	.51470D+00	-.17621D+01	.77304D+01
	.73930D+04	-.93710D+03	-.41711D+03	.14714D+03
	-.13485D+01	-.10451D+03	-.11034D+02	.17168D+02
		-.26853D+03	.20261D+03	-.18318D+03
	.87894D+02	.76452D+02	-.32229D+02	-.46791D+02
		.1000398795D+05		
50	.12858D+00	.48283D+00	-.27028D+01	.90352D+01
	.73615D+04	-.93903D+03	-.41921D+03	.14716D+03
	-.19427D+01	-.10426D+03	-.11186D+02	.17673D+02
		-.26352D+03	.20473D+03	-.18310D+03
	.87888D+02	.75373D+02	-.33367D+02	-.46830D+02
		.9979091625D+04		
60	.49158D-01	.38309D+00	-.59700D+01	.13458D+02
	.72647D+04	-.94352D+03	-.42557D+03	.14740D+03
	-.37936D+01	-.10323D+03	-.11599D+02	.19601D+02
		-.24840D+03	.21136D+03	-.18290D+03
	.87433D+02	.71809D+02	-.37471D+02	-.47003D+02
		.9727014871D+04		

70	.47155D-01	.38497D+00	-.41931D+01	.11608D+02
	.72612D+04	-.94686D+03	-.42476D+03	.14653D+03
	-.20909D+01	-.10414D+03	-.11905D+02	.17059D+02
		-.24443D+03	.20935D+03	-.18293D+03
	.87333D+02	.73051D+02	-.34952D+02	-.45371D+02
		.9698185567D+04		
80	.21533D-01	.38461D+00	-.31945D+01	.10776D+02
	.72475D+04	-.94454D+03	-.42534D+03	.14249D+03
	-.37863D+01	-.10716D+03	-.11419D+02	.17031D+02
		-.26165D+03	.20274D+03	-.18285D+03
	.85914D+02	.76004D+02	-.32667D+02	-.47890D+02
		.9553687231D+04		
110	.11778D+00	.27693D+00	-.45243D+01	.12632D+02
	.72460D+04	-.95836D+03	-.42557D+03	.14225D+03
	-.51665D+01	-.10490D+03	-.10489D+02	.18763D+02
		-.26930D+03	.20371D+03	-.18210D+03
	.86874D+02	.77483D+02	-.32731D+02	-.43855D+02
		.9265129189D+04		
120	.11998D+00	.28636D+00	-.35662D+01	.11711D+02
	.72535D+04	-.95815D+03	-.42745D+03	.14823D+03
	-.70941D+01	-.10621D+03	-.12059D+02	.17984D+02
		-.27013D+03	.20680D+03	-.18011D+03
	.84744D+02	.80623D+02	-.30671D+02	-.45991D+02
		.9203018638D+04		
140	.12427D+00	.29547D+00	-.36354D+01	.11809D+02
	.72622D+04	-.95639D+03	-.42655D+03	.14724D+03
	-.67908D+01	-.10598D+03	-.12162D+02	.18274D+02
		-.26971D+03	.20608D+03	-.18071D+03
	.85335D+02	.80916D+02	-.30264D+02	-.46251D+02
		.9195198842D+04		
150	.12423D+00	.29552D+00	-.36419D+01	.11815D+02
	.72622D+04	-.95639D+03	-.42656D+03	.14724D+03
	-.67960D+01	-.10597D+03	-.12165D+02	.18385D+02
		-.26971D+03	.20609D+03	-.18071D+03
	.85318D+02	.80899D+02	-.30265D+02	-.46259D+02
		.9195197259D+04		
160	.12423D+00	.29552D+00	-.36420D+01	.11815D+02
	.72622D+04	-.95639D+03	-.42656D+03	.14724D+03
	-.67958D+01	-.10597D+03	-.12165D+02	.18305D+02
		-.26972D+03	.20609D+03	-.18071D+03
	.85318D+02	.80899D+02	-.30265D+02	-.46259D+02
		.9195197258D+04		
170	.12423D+00	.29552D+00	-.36420D+01	.11815D+02
	.72622D+04	-.95639D+03	-.42656D+03	.14724D+03
	-.67958D+01	-.10597D+03	-.12165D+02	.18305D+02
		-.26972D+03	.20609D+03	-.18071D+03
	.85318D+02	.80899D+02	-.30265D+02	-.46260D+02
		.9195197258D+04		

RESULTS

NUMBER OF DATA POINTS= 96
CALCULATION TERMINATED BECAUSE CONVERGENCE OBTAINED

NUMBER OF ITERATIONS= 175
EPS= .1E-04
ITERATIONS LIMIT=180
CALCULATION TIME= 173.148
BELOW ARE PARAMETER SOLUTION VALUES,IN THE FOLLOWING FORMAT

A1	A2	B0	B1
XP0	XP1	XP2	XP3
XP4	XP5	XP6	XP7
	XP8	XP9	XP10
XP11	XP12	XP13	XP14

Q

.124230D+00	.295520D+00	-.364280D+01	.118150D+02
.726220D+04	-.956390D+03	-.426560D+03	.147240D+03
-.679580D+01	-.105970D+03	-.121650D+02	.183050D+02
	-.269720D+03	.206090D+03	-.180710D+03
.853180D+02	.808990D+02	-.302650D+02	-.462600D+02

.9195197258D+04

Identification of Parameters from Data of 3rd Week, using those identified from Data of 2nd Week as Starting Values

BELOW ARE PARAMETER STARTING VALUES,IN THE FOLLOWING FORMAT

A1	A2	B0	B1
XP0	XP1	XP2	XP3
XP4	XP5	XP6	XP7
	XP8	XP9	XP10
XP11	XP12	XP13	XP14

.124000D+00	.296000D+00	-.364200D+01	.118150D+02
.726220D+04	-.956390D+03	-.426560D+03	.147240D+03
-.679600D+01	-.105970D+03	-.121650D+02	.183050D+02
	-.269720D+03	.206090D+03	-.180710D+03
.853180D+02	.888990D+02	-.302650D+02	-.462600D+02

BELOW ARE PARAMETER VALUES DURING ITERATION,
IN THE FOLLOWING FORMAT

	A1	A2	B0	B1
NUMBER	XP0	XP1	XP2	XP3
OF	XP4	XP5	XP6	XP7
ITERATION		XP8	XP9	XP10
	XP11	XP12	XP13	XP14
			J	
0	.12400D+00	.29600D+00	-.36420D+01	.11815D+02
	.72622D+04	-.95639D+03	-.42656D+03	.14724D+03
	-.67960D+01	-.10597D+03	-.12165D+02	.18305D+02
		-.26972D+03	.20609D+03	-.18071D+03
	.85318D+02	.80899D+02	-.30265D+02	-.46260D+02
			.1987574319D+05	
10	.36543D+00	.45270D+00	.14612D+02	-.13424D+02
	.72664D+04	-.95615D+03	-.42490D+03	.14079D+03
	.45409D+01	-.91867D+02	-.22780D+02	.12867D+02
		-.26750D+03	.20659D+03	-.17500D+03
	.78824D+02	.71000D+02	-.14438D+02	-.38640D+02
			.1069327870D+05	
20	.29653D+00	.48728D+00	.58668D+01	-.45844D+01
	.72749D+04	-.95078D+03	-.41772D+03	.12616D+03
	.11543D+02	-.92244D+02	-.21660D+02	.13425D+02
		-.25642D+03	.20822D+03	-.17031D+03
	.64712D+02	.69011D+02	-.10555D+02	-.38945D+02
			.9895942285D+04	
30	.23498D+00	.43230D+00	.86282D+01	-.65766D+01
	.72896D+04	-.94119D+03	-.41218D+03	.13053D+03
	.67138D+01	-.93821D+02	-.20986D+02	.14043D+02
		-.22671D+03	.19548D+03	-.17375D+03
	.65772D+02	.68072D+02	-.71325D+01	-.40539D+02
			.9801911026D+04	
50	.19194D+00	.40408D+00	.70933D+01	-.46252D+01
	.73037D+04	-.95239D+03	-.42002D+03	.12784D+03
	.82515D+01	-.92894D+02	-.21469D+02	.13505D+02
		-.22841D+03	.19976D+03	-.17094D+03
	.67464D+02	.70533D+02	-.82883D+01	-.38595D+02
			.9756869555D+04	

70	.11805D+00	.35130D+00	.62831D+01	-.29305D+01
	.73281D+04	-.96119D+03	-.41994D+03	.12699D+03
	.83882D+01	-.93834D+02	-.21403D+02	.13427D+02
		-.23639D+03	.19868D+03	-.17187D+03
	.67905D+02	.70189D+02	-.88810D+01	-.38986D+02
			.9720829681D+04	
80	.11729D+00	.35575D+00	.74878D+01	-.41041D+01
	.73263D+04	-.95327D+03	-.41879D+03	.12585D+03
	.79672D+01	-.93779D+02	-.21397D+02	.13458D+02
		-.23640D+03	.19648D+03	-.17279D+03
	.67083D+02	.69900D+02	-.83484D+01	-.38523D+02
			.9704441708D+04	
90	.10508D+00	.34905D+00	.60989D+01	-.25582D+01
	.73301D+04	-.95016D+03	-.42346[illegible]03	.12669D+03
	.78815D+01	-.92487D+02	-.22123[illegible]2	.12994D+02
		-.24011D+03	.19794D+03	-.17136D+03
	.67968D+02	.69485D+02	-.88073D+01	-.38339D+02
			.9697813366D+04	
100	.10156D+00	.34460D+00	.63970D+01	-.27968D+01
	.73311D+04	-.95037D+03	-.42202D+03	.12603D+03
	.75666D+01	-.92890D+02	-.21836D+02	.13219D+02
		-.23887D+03	.19750D+03	-.17209D+03
	.67669D+02	.70051D+02	-.89425D+01	-.38712D+02
			.9695390372D+04	
110	.24796D+00	.34252D+00	.64818D+01	-.28076D+01
	.73310D+04	-.95041D+03	-.42205D+03	.12594D+03
	.74890D+01	-.92891D+02	-.21819D+02	.13251D+02
		-.23901D+03	.19755D+03	-.17216D+03
	.67625D+02	.70159D+02	-.89930D+01	-.38774D+02
			.9077458431D+04	
120	.17432D+00	.29046D+00	.50194D+01	-.31169D+00
	.73791D+04	-.93990D+03	-.41750D+03	.12333D+03
	.11145D+02	-.90714D+02	-.20242D+02	.13628D+02
		-.22806D+03	.19588D+03	-.16783D+03
	.67244D+02	.71642D+02	-.83246D+01	-.38539D+02
			.8536368855D+04	
130	.20426D+00	.32121D+00	.33658D+01	.10654D+01
	.73909D+04	-.93508D+03	-.42230D+03	.12326D+03
	.75916D+01	-.90183D+02	-.23539D+02	.12116D+02
		-.23030D+03	.19511D+03	-.17063D+03
	.69177D+02	.68828D+02	-.10585D+02	-.39013D+02
			.8419257598D+04	

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140     .21122D+00   .32933D+00   .25639D+01   .16709D+01
        .73927D+04  -.93503D+03  -.42345D+03   .12359D+03
        .88354D+01  -.90021D+02  -.23245D+02   .12283D+02
                    -.23387D+03   .19615D+03  -.16988D+03
        .68744D+02   .69000D+02  -.10683D+02  -.39206D+02
                          .8411514624D+04

150     .21028D+00   .32843D+00   .23826D+01   .18867D+01
        .73927D+04  -.93578D+03  -.42330D+03   .12350D+03
        .88927D+01  -.89867D+02  -.23347D+02   .12222D+02
                    -.23550D+03   .19645D+03  -.16976D+03
        .68968D+02   .68985D+02  -.10664D+02  -.39120D+02
                          .8411005815D+04

170     .21036D+00   .32848D+00   .23682D+01   .18952D+01
        .73928D+04  -.93581D+03  -.42335D+03   .12352D+03
        .87955D+01  -.89801D+02  -.23307D+02   .12234D+02
                    -.23542D+03   .19642D+03  -.16972D+03
        .69015D+02   .68963D+02  -.10685D+02  -.39146D+02
                          .8410983583D+04

190     .21035D+00   .32848D+00   .23683D+01   .18951D+01
        .73928D+04  -.93581D+03  -.42335D+03   .12352D+03
        .87957D+01  -.89801D+02  -.23307D+02   .12234D+02
                    -.23542D+03   .19642D+03  -.16972D+03
        .69015D+02   .68962D+02  -.10685D+02  -.39146D+02
                          .8410983583D+04

RESULTS

NUMBER OF DATA POINTS= 96
CALCULATION TERMINATED BECAUSE CONVERGENCE OBTAINED
NUMBER OF ITERATIONS= 191
EPS= .1E-04
ITERATIONS LIMIT=200
CALCULATION TIME=   196.120
BELOW ARE PARAMETER SOLUTION VALUES,IN THE FOLLOWING  FORMAT

          A1           A2           B0           B1
          XP0          XP1          XP2          XP3
          XP4          XP5          XP6          XP7
                       XP8          XP9          XP10
          XP11         XP12         XP13         XP14
                                Q

        .21035D+00   .32848D+00   .23683D+01   .18951D+01
        .73928D+04  -.93581D+03  -.42335D+03   .12352D+03
        .87957D+01  -.89801D+02  -.23307D+02   .12234D+02
                    -.23542D+03   .19642D+03  -.16972D+03
        .69015D+02   .68962D+02  -.10685D+02  -.39146D+02

                          .8410983583D+04
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Identification of Parameters from Data of three Weeks, using those identified from Data of 2nd Week as Starting Values

BELOW ARE PARAMETER STARTING VALUES,IN THE FOLLOWING FORMAT

A1	A2	B0	B1
XP0	XP1	XP2	XP3
XP4	XP5	XP6	XP7
	XP8	XP9	XP10
XP11	XP12	XP13	XP14

.12400D+00	.29600D+00	-.36420D+01	.11815D+02
.72622D+04	-.95639D+03	-.42656D+03	.14724D+03
-.67960D+01	-.10597D+03	-.12165D+02	.18305D+02
	-.26972D+03	.20609D+03	-.18071D+03
.85318D+02	.80899D+02	-.30265D+02	-.46260D+02

PARAMETERS,I.E. RETRO AND HARMONIC COEFFICIENTS
APPLYING TO DIFFERENCE EQUATION REPRESENTING THE SYSTEM
ARE PRINTED BELOW IN PART,IN THE FOLLOWING FORMAT.

	A1	A2	B0	B1
NUMEER	XP0	XP1	XP2	XP3
OF	XP4	XP5	XP6	XP7
ITERATION		XP8	XP9	XP10
	XP11	XP12	XP13	XP14
		J		
0	.12400D+00	.29600D+00	-.36420D+01	.11815D+02
	.72622D+04	-.95639D+03	-.42656D+03	.14724D+03
	-.67960D+01	-.10597D+03	-.12165D+02	.18305D+02
		-.26972D+03	.20609D+03	-.18071D+03
	.85318D+02	.80899D+02	-.30265D+02	-.46260D+02
		.1984828816D+05		
10	.33084D+00	.48077D+00	.33904D+02	-.31572D+02
	.72737D+04	-.95628D+03	-.41738D+03	.12879D+03
	.93743D+01	-.95919D+02	-.15255D+02	.21208D+02
		-.27013D+03	.19995D+03	-.17453D+03
	.95415D+02	.53590D+02	-.36382D+02	-.33337D+02
		.2008889595D+05		
20	.27007D+00	.41139D+00	.34911D+01	.42242D+00
	.72927D+04	-.95425D+03	-.40217D+03	.12826D+03
	.58784D+01	-.10067D+03	-.13287D+02	.20237D+02
		-.26899D+03	.18966D+03	-.17562D+03
	.91727D+02	.66624D+02	-.32801D+02	-.40226D+02
		.1498839490D+05		
30	.24585D+00	.39310D+00	.44404D+01	.60373D-01
	.73105D+04	-.95293D+03	-.40452D+03	.13189D+03
	.59513D+01	-.10012D+03	-.13425D+02	.20459D+02
		-.26885D+03	.19058D+03	-.17677D+03
	.91807D+02	.65105D+02	-.33174D+02	-.39525D+02
		.1493587960D+05		
50	.28612D+00	.37815D+00	.27327D+01	.20101D+01
	.73621D+04	-.95028D+03	-.40723D+03	.12393D+03
	.20195D+01	-.10313D+03	-.12575D+02	.18970D+02
		-.26958D+03	.18979D+03	-.17612D+03
	.89414D+02	.72964D+02	-.31870D+02	-.44201D+02
		.1476290736D+05		
60	.30205D+00	.38947D+00	.24784D+01	.16803D+01
	.73538D+04	-.95066D+03	-.40586D+03	.13043D+03
	.56332D+01	-.10027D+03	-.13246D+02	.20244D+02
		-.26977D+03	.18967D+03	-.17697D+03
	.91522D+02	.65394D+02	-.32964D+02	-.40379D+02
		.1463613706D+05		

70	.30248D+00	.39050D+00	.24947D+01	.18454D+01
	.73558D+04	-.95054D+03	-.40525D+03	.13049D+03
	.53647D+01	-.10040D+03	-.13278D+02	.20268D+02
		-.27020D+03	.18955D+03	-.17700D+03
	.91341D+02	.65735D+02	-.33007D+02	-.40262D+02
		.1463540342D+05		
80	.30238D+00	.39034D+00	.24970D+01	.18480D+01
	.73557D+04	-.95055D+03	-.40527D+03	.13057D+03
	.53418D+01	-.10040D+03	-.13271D+02	.20262D+02
		-.27036D+03	.18855D+03	-.17701D+03
	.91326D+02	.65738D+02	-.32993D+02	-.40282D+02
		.1463539337D+05		
90	.30232D+00	.39036D+00	.24946D+01	.18497D+01
	.73558D+04	-.95055D+03	-.40530D+03	.13056D+03
	.53389D+01	-.10040D+03	-.13273D+02	.20264D+02
		-.27041D+03	.18956D+03	-.17701D+03
	.91325D+02	.65734D+02	-.33005D+02	-.40285D+02
		.1463539260D+05		
100	.30232D+00	.39036D+00	.24946D+01	.18497D+01
	.73558D+04	-.95055D+03	-.40530D+03	.13056D+03
	.53390D+01	-.10040D+03	-.13273D+02	.20264D+02
		-.27041D+03	.18956D+03	-.17701D+03
	.91325D+02	.65734D+02	-.33005D+02	-.40285D+02
		.1463539260D+05		
110	.30232D+00	.39036D+00	.24946D+01	.18497D+01
	.73558D+04	-.95055D+03	-.40530D+03	.13056D+03
	.53390D+01	-.10040D+03	-.13273D+02	.20264D+02
		-.27041D+03	.18956D+03	-.17701D+03
	.91325D+02	.65734D+02	-.33005D+02	-.40285D+02
		.1463539260D+05		

RESULTS

NUMBER OF DATA POINTS=288
CALCULATION TERMINATED BECAUSE CONVERGENCE OBTAINED

NUMBER OF ITERATIONS= 118
EPS= .1E-04
CALCULATION TIME= 401.523
BELOW ARE PARAMETER SOLUTION VALUES,IN THE FOLLOWING FORMAT

A1	A2	B0	B1
XP0	XP1	XP2	XP3
XP4	XP5	XP6	XP7
	XP8	XP9	XP10
XP11	XP12	XP13	XP14
		Q	

.30232D+00	.39036D+00	.24946D+01	.1[illegible]1
.73558D+04	-.95055D+03	-.40530D+03	.1[illegible]3
.53398D+01	-.10040D+03	-.13273D+02	.2[illegible]2
	-.27041D+03	.18956D+03	-.17[illegible]3
.91325D+02	.65734D+02	-.33005D+02	-.402[illegible]02

.1463539260D+05

Identification of Parameters from Data of 4th Week, using closely approximated Starting Values

BELCW ARE PARAMETER STARTING VALUES,IN THE FOLLCWING FORMAT

A1	A2	B0	P1
XP0	XP1	XP2	XP3
XP4	XP5	XP6	XF7
	XP8	XP9	XP10
XP11	XP12	XP13	XF14

.236000D+00	.294000D+00	-.131810D+02	.179050D+02
.759940D+04	-.356810D+03	-.390490D+03	.107510D+03
.256380D+02	-.713350D+02	-.205390D+02	.133680D+02
	-.325220D+03	.218060D+03	-.200400D+03
.615290D+02	.577050D+02	-.803800D+01	-.438450D+02

BELOW ARE PARAMETER VALUES DURING ITERATION,IN THE FOLLOWING FORMAT

	A1	A2	B0	B1
NUMBER	XP0	XP1	XP2	XP3
OF	XP4	XP5	XP6	XP7
ITERATION		XP8	XP9	XP10
	XP11	XP12	XP13	XP14
		J		
0	.23600D+00	.29400D+00	-.13181D+02	.17905D+02
	.75994D+04	-.85681D+03	-.39049D+03	.10751D+03
	.25638D+02	-.71335D+02	-.20539D+02	.13368D+02
		-.32522D+03	.21806D+03	-.20040D+03
	.61529D+02	.57705D+02	-.80380D+01	-.43845D+02
		.7023849211D+04		
10	.23690D+00	.29374D+00	-.13180D+02	.17904D+02
	.75994D+04	-.85681D+03	-.39049D+03	.10751D+03
	.25638D+02	-.71334D+02	-.20540D+02	.13367D+02
		-.32522D+03	.21806D+03	-.20040D+03
	.61529D+02	.57705D+02	-.80373D+01	-.43845D+02
		.7023835660D+04		
20	.23689D+00	.29372D+00	-.13182D+02	.17906D+02
	.75994D+04	-.85681D+03	-.39049D+03	.10752D+03
	.25638D+02	-.71335D+02	-.20539D+02	.13368D+02
		-.32522D+03	.21807D+03	-.20040D+03
	.61528D+02	.57705D+02	-.80395D+01	-.43846D+02
		.7023835549D+04		
30	.23689D+00	.29372D+00	-.13181D+02	.17904D+02
	.75994D+04	-.85681D+03	-.39049D+03	.10751D+03
	.25638D+02	-.71334D+02	-.20539D+02	.13368D+02
		-.32522D+03	.21806D+03	-.20040D+03
	.61529D+02	.57705D+02	-.80383D+01	-.43845D+02
		.7023835534D+04		
40	.23689D+00	.29372D+00	-.13181D+02	.17905D+02
	.75994D+04	-.85681D+03	-.39049D+03	.10751D+03
	.25638D+02	-.71335D+02	-.20539D+02	.13368D+02
		-.32522D+03	.21806D+03	-.20040D+03
	.61529D+02	.57705D+02	-.80380D+01	-.43845D+02
		.7023835533D+04		
60	.23689D+00	.29372D+00	-.13181D+02	.17905D+02
	.75994D+04	-.85681D+03	-.39049D+03	.10751D+03
	.25638D+02	-.71335D+02	-.20539D+02	.13368D+02
		-.32522D+03	.21806D+03	-.20040D+03
	.61529D+02	.57705D+02	-.80380D+01	-.43845D+02
		.7023835533D+04		

RESULTS

NUMBER OF DATA POINTS= 96
CALCULATION TERMINATED BECAUSE CONVERGENCE OBTAINED

NUMBER OF ITERATIONS= 66
EPS= .1E-04
ITERATIONS LIMIT=100
CALCULATION TIME= 65.421
BELOW ARE PARAMETER SOLUTION VALUES,IN THE FOLLOWING FORMAT

A1	A2	B0	B1
XP0	XP1	XP2	XP3
XP4	XP5	XP6	XP7
	XP8	XP9	XP10
XP11	XP12	XP13	XP14

Q

.23689D+00	.29372D+00	-.13181D+02	.17905D+02
.75994D+04	-.85681D+03	-.39049D+03	.10751D+03
.25638D+02	-.71335D+02	-.20539D+02	.13368D+02
	-.32522D+03	.21806D+03	-.20040D+03
.61529D+02	.57705D+02	-.80380D+01	-.43845D+02

.7023835533D+04

Partial Identification of Parameters from Data of three Weeks, with Starting Values alternately 10% Low and High

BELOW ARE PARAMETER STARTING VALUES,IN THE FOLLOWING FORMAT

A1	A2	B0	B1
XP0	XP1	XP2	XP3
XP4	XP5	XP6	XP7
	XP8	XP9	XP10
XP11	XP12	XP13	XP14

.272800D+00	.429000D+00	.224500D+01	.203500D+01
.662020D+04	-.104560D+04	-.364770D+03	.143620D+03
.480500D+01	-.110440D+03	-.119460D+02	.222900D+02
	-.243370D+03	.208520D+03	-.159310D+03
.180460D+03	.591610D+02	-.363060D+02	-.362560D+02

BELOW ARE PARAMETER VALUES DURING ITERATION,
IN THE FOLLOWING FORMAT

NUMBER OF ITERATION	A1	A2	B0	B1
	XP0	XP1	XP2	XF3
	XP4	XP5	XP6	XP7
		XP8	XP9	XF10
	XP11	XP12	XP13	XP14
		J		
0	.272000+00	.429000+00	.224500+01	.203580+01
	.66202D+04	-.10456D+04	-.364770+03	.143620+03
	.480500+01	-.110440+03	-.119460+02	.222900+02
		-.243370+03	.208520+03	-.159310+03
	.10046D+03	.591610+02	-.363060+02	-.362560+02
		.6528013764D+05		
10	.446450+00	.54528D+00	.16608D+02	-.163730+02
	.662040+04	-.10324D+04	-.385550+03	.130460+03
	.645420+01	-.882950+02	-.153180+02	.175160+02
		-.246690+03	.200120+03	-.177110+03
	.856350+02	.735930+02	-.279410+02	-.457520+02
		.1824230842D+05		
20	.428390+00	.57161D+00	.837550+01	-.804700+01
	.662060+04	-.10010D+04	-.411950+03	.126960+03
	.665450+01	-.997180+02	-.132100+02	.204260+02
		-.254100+03	.186450+03	-.185830+03
	.887960+02	.663070+02	-.328220+02	-.397710+02
		.1699350430D+05		
30	.419250+00	.572070+00	.441640+01	-.409100+01
	.661940+04	-.95719D+03	-.404010+03	.136120+03
	.670480+01	-.982710+02	-.122910+02	.216850+02
		-.264430+03	.189540+03	-.171150+03
	.865150+02	.668220+02	-.362240+02	-.381110+02
		.1668258569D+05		
60	.893720+01	-.33066D+01	.500720+01	-.47476D+01
	.661650+04	-.95726D+03	-.399120+03	.133600+03
	.621650+01	-.100640+03	-.130910+02	.206330+02
		-.26446D+03	.191740+03	-.175820+03
	.917270+02	.655850+02	-.327260+02	-.401190+02
		.1062358452D+08		
70	.463120+00	.529790+00	.499760+01	-.469190+01
	.661660+04	-.956720+03	-.399930+03	.133140+03
	.625830+01	-.999440+02	-.131200+02	.205950+02
		-.264630+03	.191360+03	-.176560+03
	.911620+02	.659950+02	-.325340+02	-.402870+02
		.1655601815D+05		

80	.46314D+00	.52978D+00	.50552D+01	-.47467D+01
	.66171D+04	-.95443D+03	-.40137D+03	.13317D+03
	.62485D+01	-.10023D+03	-.13113D+02	.20603D+02
		-.26536D+03	.19066D+03	-.17656D+03
	.91582D+02	.65832D+02	-.32570D+02	-.40233D+02
			.1655449319D+05	
90	.46312D+00	.52978D+00	.50805D+01	-.47717D+01
	.66177D+04	-.95275D+03	-.40090D+03	.13315D+03
	.62552D+01	-.10029D+03	-.13105D+02	.20613D+02
		-.26590D+03	.19087D+03	-.17656D+03
	.91256D+02	.65798D+02	-.32595D+02	-.40210D+02
			.1655407213D+05	
100	.46313D+00	.52975D+00	.50828D+01	-.47740D+01
	.66183D+04	-.95257D+03	-.40080D+03	.13312D+03
	.62455D+01	-.10024D+03	-.13100D+02	.20622D+02
		-.26596D+03	.19091D+03	-.17665D+03
	.91355D+02	.65812D+02	-.32638D+02	-.40188D+02
			.1655398911D+05	
110	.46382D+00	.52789D+00	.51760D+01	-.48674D+01
	.66569D+04	-.95255D+03	-.40051D+03	.13425D+03
	.56345D+01	-.99987D+02	-.13139D+02	.20718D+02
		-.26612D+03	.19045D+03	-.17737D+03
	.91342D+02	.65502D+02	-.32187D+02	-.39378D+02
			.1655170830D+05	
120	.46428D+00	.52374D+00	.53326D+01	-.50220D+01
	.67783D+04	-.95254D+03	-.39986D+03	.13599D+03
	.52695D+01	-.10000D+03	-.13121D+02	.20687D+02
		-.26659D+03	.18933D+03	-.17846D+03
	.91243D+02	.65496D+02	-.32325D+02	-.39736D+02
			.1653360167D+05	
130	.46632D+00	.50446D+00	.60593D+01	-.57432D+01
	.73390D+04	-.95244D+03	-.39686D+03	.14404D+03
	.35336D+01	-.10027D+03	-.13046D+02	.20561D+02
		-.26874D+03	.18415D+03	-.18349D+03
	.90794D+02	.65432D+02	-.32896D+02	-.41250D+02
			.1640270846D+05	
140	.46628D+00	.50386D+00	.60707D+01	-.57502D+01
	.73340D+04	-.95242D+03	-.39684D+03	.14429D+03
	.33863D+01	-.10027D+03	-.13051D+02	.20546D+02
		-.26872D+03	.18412D+03	-.18366D+03
	.90775D+02	.65394D+02	-.32939D+02	-.41352D+02
			.1640256759D+05	

Identification of Parameters from Data of three Weeks, with third Set of Starting Values

BELOW ARE PARAMETER STARTING VALUES,IN THE FOLLOWING FORMAT

A1	A2	B0	E1
XP0	XP1	XP2	XP3
XP4	XP5	XP6	XP7
	XP8	XP9	XP10
XP11	XP12	XP13	XP14
.250000D+00	.250000D+00	.143200D+02	0.
.734370D+04	-.929800D+03	-.376240D+03	.115380D+03
.522220D+02	-.860510D+02	-.313000D+02	.178680D+02
	-.282200D+03	.248520D+03	-.217190D+03
.547460D+02	.672190D+02	-.496000D+02	-.209800D+01

BELOW ARE PARAMETER VALUES DURING ITERATION,IN THE FOLLOWING FORMAT

	A1	A2	B0	E1
NUMBER	XP0	XP1	XP2	XP3
OF	XP4	XP5	XP6	XP7
ITERATION		XP8	XP9	XP10
	XP11	XP12	XP13	XP14
0	.250000D+00	.250000D+00	.143200D+02	0.
	.734370D+04	-.929800D+03	-.376240D+03	.115380D+03
	.522220D+02	-.860510D+02	-.313000D+02	.178680D+02
		-.282200D+03	.248520D+03	-.217190D+03
	.547460D+02	.672190D+02	-.496000D+02	-.209800D+01
		.3880974345D+05		
10	.191460D+00	.482120D+00	-.215350D+02	.260430D+02
	.734500D+04	-.933460D+03	-.387280D+03	.128760D+03
	.112330D+01	-.106370D+03	-.592480D+00	.230680D+02
		-.278960D+03	.221450D+03	-.188450D+03
	.944820D+02	.651100D+02	-.215810D+02	-.702220D+02
		.1955567126D+05		
30	.293630D+00	.382760D+00	-.313600D+01	.764800D+01
	.734780D+04	-.939480D+03	-.400060D+03	.131100D+03
	.536010D+01	-.101020D+03	-.111610D+02	.208110D+02
		-.276190D+03	.199060D+03	-.177430D+03
	.912650D+02	.655960D+02	-.311580D+02	-.432820D+02
		.1485981004D+05		
60	.302390D+00	.390250D+00	.249840D+01	.184210D+01
	.735520D+04	-.950740D+03	-.405280D+03	.130560D+03
	.537380D+01	-.100420D+03	-.132760D+02	.202600D+02
		-.270270D+03	.189610D+03	-.176990D+03
	.913030D+02	.657330D+02	-.330130D+02	-.402810D+02
		.1463542085D+05		

70	.30219D+00	.39044D+00	.24914D+01	.18537D+01
	.73558D+04	-.95055D+03	-.40532D+03	.13057D+03
	.53461D+01	-.10041D+03	-.13270D+02	.20260D+02
		-.27041D+03	.18953D+03	-.17701D+03
	.91319D+02	.65732D+02	-.33012D+02	-.40294D+02
			.1463539363D+05	
80	.30232D+00	.39036D+00	.24946D+01	.18497D+01
	.73558D+04	-.95055D+03	-.40530D+03	.13056D+03
	.53390D+01	-.10040D+03	-.13273D+02	.20264D+02
		-.27041D+03	.18956D+03	-.17701D+03
	.91325D+02	.65734D+02	-.33005D+02	-.40285D+02
			.1463539260D+05	
90	.30232D+00	.39036D+00	.24946D+01	.18497D+01
	.73558D+04	-.95055D+03	-.40530D+03	.13056D+03
	.53390D+01	-.10040D+03	-.13273D+02	.20264D+02
		-.27041D+03	.18956D+03	-.17701D+03
	.91325D+02	.65734D+02	-.33005D+02	-.40285D+02
			.1463539260D+05	
100	.30232D+00	.39036D+00	.24946D+01	.18497D+01
	.73558D+04	-.95055D+03	-.40530D+03	.13056D+03
	.53390D+01	-.10040D+03	-.13273D+02	.20264D+02
		-.27041D+03	.18956D+03	-.17701D+03
	.91325D+02	.65734D+02	-.33005D+02	-.40285D+02
			.1463539260D+05	

RESULTS

NUMBER OF DATA POINTS=288
CALCULATION TERMINATED BECAUSE CONVERGENCE OBTAINED

NUMBER OF ITERATIONS= 101
EPS= .1E-04
CALCULATION TIME= 341.276
BELOW ARE PARAMETER SOLUTION VALUES,IN THE FOLLOWING FORMAT

A1	A2	B0	B1
XP0	XP1	XP2	XP3
XP4	XP5	XP6	XP7
	XP8	XP9	XP10
XP11	XP12	XP13	XP14
		Q	

.30232D+00	.39036D+00	.24946D+01	.18497D+01
.73558D+04	-.95055D+03	-.40530D+03	.13056D+03
.53390D+01	-.10040D+03	-.13273D+02	.20264D+02
	-.27041D+03	.18956D+03	-.17701D+03
.91325D+02	.65734D+02	-.33005D+02	-.40285D+02
		.1463539260D+05	

Listing of Forecasting Program "PREDICT"

```
      PROGRAM PREDICT(INPUT,OUTPUT)
       DOUBLE PRECISION Z1,U1
      DIMENSION U1(500),Z1(500)
       COMMON NXP
      COMMON/DATA/Z,U,XP,B,A,ND,Q,NA,A1
C
      DIMENSION Z(500),U(500),A(2,2),XP(20),B(2),X(30),X1(5),P(2,2)
      DIMENSION A1(2,2)
C
C     SPECIFY TOTAL NUMBER OF DATA POINTS,AND NUMBER OF HOURLY FORECASTS
C     THEN READ LOAD DATA,AND TEMPERATURE-DEPENDENT VARIABLE,U
      NF=24
      ND=96
      NP=72
      NSF=ND-NP-NF+1
      NFF=ND-NP
      READ 1,(Z(I),I=1,ND)
    1 FORMAT(10X,12F5.0)
      READ 2,(U(I),I=1,ND)
    2 FORMAT(12F5.1)
      DO 9 I=1,ND
      Z1(I)=Z(I)
      U1(I)=U(I)
    9 CONTINUE
C
C     SPECIFY PARAMETERS
C     THIS PROGRAM IS FOR NA=NB=2
      NXP=15
      NA=2
      NB=NA
      Q=7023.84
      NT=NA+NB+NXP
      READ3,(X(I),I=1,NT)
    3 FORMAT(8F10.3)
      DO 4 I=1,NXP
    4 XP(I)=X(I)
      A(1,1)=X(NXP+1)
      A(1,2)=X(NXP+2)
      A(2,1)=1.
      A(2,2)=0.
      B(1)=X(NA+NXP+1)
      B(2)=X(NA+NXP+2)
      PRINT 13
   13 FORMAT(10X,*BELOW ARE PARAMETER VALUES,IN FOLLOWING FORMAT*,//)
      PRINT 14
```

```
14 FORMAT(24X,*A1*,11X,*A2*,11X,*B0*,11X,*B1*,/,12X,*            XP0
  1 *,6X ,* XP1          XP2              XP3   *,/,14X,*   *,6X,* XP4  *7X
  1,* XP5  *,7X,* XP6  *,7X,* XP7  *,/,10X,*              *,16X,* XP8  *7X
  1,* XP9  *,7X,* XP10 *,/,22X,* XP11 *,7X,* XP12 *,7X,* XP13 *,7X,*
  1XP14 *,/)
   PRINT 15,(X(I),I=16,19),(X(I),I=1,15)
15 FORMAT(17X       ,4(1X,F12.5),/,2(17X,4(1X,F12.5),/),30X,3(1X,F12.5
  1),/,17X,4(1X,F12.5),/)
   PRINT 16,Q
16 FORMAT(37X,F13.2)
C
C  INITIALIZE MATRICES A,P,A1,VECTOR X,THEN PROCEED WITH FILTERING
C
C  INITIALIZATION OF P
   DO10I=1,NA
   DO10J=1,NA
10 P(I,J)=0.0
   DO 11 I=1,NA
11 P(I,I)=10000.
C  CALCULATE A1=(A,TRANSPOSE)
   DO 12 I=1,NA
   DO 12 J=1,NA
12 A1(I,J)=A(J,I)
   DO 8 I=1,NA
 8 X1(I)=0.0
C
   CALL FILTER(X1,P,NA,NSF,NFF)
C
C  NOW PROCEED WITH FORECASTING
   CALL PREDI(X1,P,NP,A,B,A1,XP,NA,U,Q,Z,ND)
   END

   SUBROUTINE MULT(A,B,R,N)
   DIMENSION A(N,N),B(N,N),R(N,N)
C
C  THIS SUBROUTINE CALCULATES MATRIX PROCUCT,R=A*B
   DO 1 I=1,N
   DO 1 J=1,N
 1 R(I,J)=0.0
   DO 2 I=1,N
   DO 2 K=1,N
   S=0.0
   DO 3 J=1,N
 3 S=S+A(I,J)*B(J,K)
 2 R(I,K)=S
   RETURN
   END
```

```
      SUBROUTINE FILTER(X1,P,N,NSF,NFF)
      COMMON/DATA/Z,U,XP,B,A,ND,Q,NA,A1
       COMMON NXP
      DIMENSION Z(500),U(500),XP(20),B(2),A(2,2),A1(2,2)
      DIMENSION P(2,2),S(2,2),X1(2),R(2,2),K(2),XS(2)
      REAL K
C
C CALCULATE S(K+1)=P(K+1/K)=A*P(K/K)*(A,TRANSPOSE)+D*Q*(D,TRANSPOSE)
      ERR=0.0
      ERRS=0.0
      DO 50 I=NSF,NFF
      CALL MULT(A,P,R,N)
      CALL MULT(R,A1,S,N)
      S(1,1)=S(1,1)+Q
C
C CALCULATE K(K+1)=S(K+1)*C*(((C,TRANSPOSE)*S(K+1)*C)**-1))
      DO 13 J=1,N
      K(J)=S(J,1)/S(1,1)
   13 CONTINUE
C
C CALCULATE P(K+1/K+1)=S(K+1)-K(K+1)*(C,TRANSPOSE)*S(K+1)
      DO 5 J=1,N
      DO 5 L=1,N
      P(L,J)=S(L,J)-K(L)*S(1,J)
    5 CONTINUE
C
C CALCULATE X(K+1/K)=A*X(K/K)+H*U(K),
C WHERE U(K) IS THE SET OF U(K+1-J)
      DO 9 J=1,N
    9 XS(J)=0.0
      DO 6 J=1,N
      X=0.0
      DO 1 I1=1,N
    1 X=X+A(J,I1)*X1(I1)
    6 XS(J)=X
      DO 8 J=1,N
    8 X1(J)=XS(J)
      DO 2 J=1,N
    2 X1(1)=X1(1)+B(J)*U(I-J+1)
      IF(I .EQ. 1) GO TO 14
      GO TO 20
   14 PRINT 15
   15 FORMAT(///,29X,*FILTERING*,/)
      PRINT 18
   18 FORMAT(10X,*HOUR*,1X,*PERIODIC*,1X,*Z,REAL*,1X,*Z,FILTERED*,2X,*ST
     1D.DEV.*,2X,*ERROR*,2X,*MEAN ERR.*1X,*ACT.S.D.*,/)
C
C CALCULATE ZP(K+1/K)=(C,TRANSPOSE)*X1(K+1/K)+PSI
   20 L=1
      CALL PERIOD(I,NXP,PSI,XP,L)
      ZP=PSI+X1(1)
C
C CALCULATE X1(K+1/K+1)=X1(K+1/K)+K(K+1)*(Z(K+1)-ZP(K+1/K)
      DO 3 J=1,N
```

```
3 X1(J)=X1(J)+K(J)*(Z(I)-ZP)
C
C CALCULATE THEORETICAL ERROR OF FILTERED ESTIMATE,
C THEN ACTUAL ERROR,ITS MEAN,AND ITS STD. DEV.
  VA=S(1,1)
  SD=SQRT(VA)
  ER=Z(I)-ZP
  ERR=ERR+ER
  ERRS=ERRS+ER**2
  AI=FLOAT(I)
  BI=AI-1.0
  AV=ERR/AI
  IF(I .GT. 1) GO TO 40
  ASD=-999.99
  GO TO 42
40 ASD=SQRT(ERRS/BI-(ERR**2)/AI/BI)
C
C PRINT THE FILTERED DATA
42 PRINT 44,I,PSI,Z(I),ZP,SD,ER,AV,ASD,(X1(J),J=1,N)
44 FORMAT(10X,I3,7F9.2,8X,2F9.2)
50 CONTINUE
  RETURN
  END

  SUBROUTINE PERIOD(J,NXP,PSI,XP,IND)
C
C THIS SUBROUTINE CALCULATES THE PERIODIC COMPONENT PSI OF LOAD Z
  DIMENSION XP(20),PHI(20)
  T=J
  DO 1 I=1,NXP
1 PHI(I)=0.0
  PHI(1)=PHI(1)+1.0
  M=(NXP-1)/2
  PI=3.14159265
  DO 5 I=1,M
  K=I+1
  PHI(K)=SIN(PI*T*FLOAT(I)/12.0)
  L=K+M
5 PHI(L)=COS(PI*T*FLOAT(I)/12.0)
  PSI=0.0
  DO 10 I=1,NXP
10 PSI=PSI+PHI(I)*XP(I)
  RETURN
  END
```

```
      SUBROUTINE PREDI(X1,W,N,A,B,A1,XP,NA,U,Q,Z,ND)
C
C THIS SUBROUTINE FORECASTS LOAD ON COMPLETION OF FILTERING
      COMMON NXP
      DIMENSION R(2,2),XS(2),P(2,2)
      DIMENSION Z(500),U(500),XP(20)
      DIMENSION X1(2),W(2,2),A(2,2),B(2),A1(2,2)
C
      ERR=0.0
      ERRS=0.0
      DO 50 I=1,N
      DO 1 J=1,NA
C
C SET X(N/N)=0.
      X=0.0
C
C CALCULATE X(L+N+1/N)=A*X(L+N/N)+B*U(L+N)
      DO 2 K=1,NA
    2 X=X+A(J,K)*X1(K)
    1 XS(J)=X
      DO 6 J=1,NA
    6 X1(J)=XS(J)
      DO 3 J=1,NA
      K=ND-N+I-J+1
    3 X1(1)=X1(1)+B(J)*U(K)
      IF(I-(I/48)*48 .EQ. 1) GO TO 9
      IF(I-(I/24)*24 .EQ. 1) PRINT 8
    8 FORMAT(*  *,//)
      GO TO 20
    9 PRINT 11
   11 FORMAT(1H1,///,28X,*FORECASTING*,/)
      PRINT 18
   18 FORMAT(10X,*HOUR*,1X,*PERIODIC*,1X,*Z,REAL*,1X,*Z,FORECAST*,2X,*ST
     1D.DEV.*,2X,*ERROR*,2X,*MEAN ERR.*2X,*ACT.S.D.*,08X,*W(1,1)*,3X,*W(
     11,2)*,5X,*X1(1)*,4X,*X1(2)*,/)
   20 L=2
C
C CALCULATE PERIODIC,AND
C Z(L+N+1/N)=(C,TRANSPOSE)*X(L+N+1/N)+PERIODIC
      M=I+ND-N
      CALL PERIOD(M,NXP,PSI,XP,L)
      ZP=PSI+X1(1)
C
C CALCULATE COVARIANCE MATRIX,W(L+N+1)=A*W(L+N)*(A,TRANSPOSE)+
C D*Q*(C,TRANSPOSE),WITH STARTING VALUE W(N/N)=P(N/N)
      CALL MULT(A,W,R,NA)
      CALL MULT(R,A1,W,NA)
C
C CALCULATE THEORETICAL STD. DEV. OF PREDICTION ERROR,
C THEN ACTUAL ERROR,ITS MEAN,AND ITS STD. DEV.
      W(1,1)=W(1,1)+Q
      VAR=W(1,1)
      SD=SQRT(VAR)
      ER=Z(M)-ZP
```

```
      ERR=ERR+ER
      ERRS=ERRS+ER**2
      AI=FLOAT(I)
      BI=AI-1.0
      AV=ERR/AI
      IF(I .GT. 1) GO TO 40
      ASD=-999.99
      GO TO 42
   40 ASD=SQRT(ERRS/BI-(ERR**2)/AI/BI)
C
C     PRINT RESULTS
   42 PRINT 44,I,PSI,Z(M),ZP,SD,ER,AV,ASD,W(1,1),W(1,2),X1(1),X1(2)
   44 FORMAT(10X,I3,7F9.2,8X,4F9.2)
   50 CONTINUE
       RETURN
       END
```

Basic Forecast: Forecasting Load in 4th Week, using Parameters identified from Data of three Weeks

BELOW ARE PARAMETER VALUES,IN FOLLOWING FORMAT

A1	A2	B0	B1
XP0	XP1	XP2	XP3
XP4	XP5	XP6	XP7
	XP8	XP9	XP10
XP11	XP12	XP13	XP14

.30200	.39000	2.49500	1.85000
7355.80000	-950.55000	-405.30000	130.56000
5.33900	-100.40000	-13.27300	20.26400
	-270.41000	189.56000	-177.01000
91.32500	65.73400	-33.00500	-40.28500

14635.39

FILTERING

HOUR	PERIODIC	Z,REAL	Z,FILTERED	STD.DEV.	ERROR	MEAN ERR.	ACT.S.D.
1	6764.30	6600.00	6727.12	130.65	-127.12	-127.12	
2	6430.28	6282.00	6302.16	126.79	-20.16	-73.64	75.64
3	6208.28	6160.00	6012.27	120.98	147.73	.15	138.55
4	6162.56	6058.00	5991.82	120.98	66.18	16.66	117.84
5	6143.11	6035.00	5995.14	120.98	39.86	21.30	102.58
6	6088.80	6037.00	5922.40	120.98	114.60	36.85	99.34
7	6269.04	6329.00	6115.37	120.98	213.63	62.10	112.65
8	6764.74	6854.00	6662.47	120.98	191.53	78.28	113.89
9	7274.36	7333.00	7211.85	120.98	121.15	83.04	107.49
10	7635.47	7787.00	7580.37	120.98	206.63	95.40	108.61
11	7904.74	7973.00	7881.72	120.98	91.28	95.03	103.05
12	8025.65	8207.00	8024.83	120.98	182.17	102.29	101.42
13	7944.35	8079.00	7960.30	120.98	118.70	103.55	97.21
14	7752.81	7722.00	7814.71	120.98	-92.71	89.53	107.12
15	7536.61	7784.00	7541.05	120.98	242.95	99.76	110.56
16	7490.90	7772.00	7521.21	120.98	250.79	109.20	113.29
17	7890.39	7943.00	8051.01	120.98	-108.01	96.42	121.69
18	8492.34	8758.00	8614.93	120.98	143.07	99.01	118.56
19	8646.65	8627.00	8758.35	120.98	-131.35	86.89	126.77
20	8311.21	8463.00	8430.32	120.98	32.68	84.18	123.98
21	8038.65	8385.00	8108.63	120.98	276.37	93.33	127.91
22	7933.12	8187.00	8134.48	120.98	52.52	91.48	125.13
23	7649.11	7913.00	7905.52	120.98	7.48	87.82	123.50
24	7181.71	7553.00	7411.26	120.98	141.74	90.07	121.29

FORECASTING

HOUR	PERIODIC	Z,REAL	Z,FORECAST	STD.DEV.	ERROR	MEAN ERR.	ACT.S.D.
1	6764.30	7075.00	7035.23	120.98	39.77	39.77	
2	6430.28	6839.00	6717.06	126.37	121.94	80.86	58.10
3	6208.28	6569.00	6465.81	139.14	103.19	88.30	43.06
4	6162.56	6478.00	6424.13	142.73	53.87	79.69	39.15
5	6143.11	6509.00	6399.03	146.34	109.97	85.75	36.51
6	6088.80	6526.00	6344.37	148.01	181.63	101.73	50.97
7	6269.04	6755.00	6520.95	149.27	234.05	120.63	68.31
8	6764.74	7212.00	7015.48	149.97	196.52	130.12	68.70
9	7274.36	7725.00	7522.99	150.44	202.01	138.11	68.59
10	7635.47	8074.00	7882.40	150.72	191.60	143.46	66.84
11	7904.74	8369.00	8147.65	150.91	221.35	150.54	67.62
12	8025.65	8612.00	8264.58	151.02	347.42	166.94	85.95
13	7944.35	8496.00	8180.58	151.09	315.42	178.36	92.02
14	7752.81	8151.00	7985.61	151.14	165.39	177.44	88.48
15	7536.61	8103.00	7763.15	151.16	339.85	188.27	95.01
16	7490.90	8033.00	7710.31	151.18	322.69	196.67	97.75
17	7890.39	8142.00	8105.84	151.19	36.16	187.23	102.34
18	8492.34	8835.00	8704.11	151.20	130.89	184.10	100.17
19	8646.65	8909.00	8853.78	151.20	55.22	177.31	101.74
20	8311.21	8736.00	8511.33	151.21	224.67	179.68	99.59
21	8038.65	8498.00	8229.19	151.21	268.81	183.92	98.99
22	7933.12	8320.00	8117.14	151.21	202.86	184.79	96.69
23	7649.11	7989.00	7827.28	151.21	161.72	183.78	94.59
24	7181.71	7603.00	7358.39	151.21	244.61	186.32	93.34
25	6764.30	7117.00	6937.78	151.21	179.22	186.03	91.39
26	6430.28	6886.00	6601.30	151.21	284.70	189.83	91.61
27	6208.28	6705.00	6374.68	151.21	330.32	195.03	93.81
28	6162.56	6616.00	6322.83	151.21	293.17	198.54	93.91
29	6143.11	6613.00	6300.74	151.21	312.26	202.46	94.60
30	6088.80	6535.00	6240.54	151.21	294.46	205.53	94.46
31	6269.04	6637.00	6419.75	151.21	217.25	205.90	92.90
32	6764.74	7250.00	6918.86	151.21	331.14	209.82	94.03
33	7274.36	7751.00	7433.19	151.21	317.81	213.09	94.44
34	7635.47	8094.00	7798.48	151.21	295.52	215.51	94.07
35	7904.74	8269.00	8070.41	151.21	198.59	215.03	92.72
36	8025.65	8550.00	8193.57	151.21	356.43	218.96	94.37
37	7944.35	8339.00	8114.49	151.21	224.51	219.11	93.06
38	7752.81	8067.00	7921.62	151.21	145.38	217.17	92.57
39	7536.61	7977.00	7697.99	151.21	279.01	218.75	91.88
40	7490.90	7769.00	7641.20	151.21	126.80	216.45	91.85
41	7890.39	7963.00	8028.31	151.21	-65.31	209.58	100.80
42	8492.34	8822.00	8621.24	151.21	200.76	209.37	99.58
43	8646.65	8842.00	8765.42	151.21	76.58	206.28	100.45
44	8311.21	8649.00	8417.56	151.21	231.44	206.86	99.34
45	8038.65	8387.00	8126.67	151.21	260.33	208.04	98.53
46	7933.12	8159.00	8003.41	151.21	155.59	206.90	97.74
47	7649.11	7863.00	7701.48	151.21	161.52	205.94	96.90
48	7181.71	7525.00	7214.18	151.21	310.82	208.12	97.05

FORECASTING

HOUR	PERIODIC	Z,REAL	Z,FORECAST	STD.DEV.	ERROR	MEAN ERR.	ACT.S.D.
49	6764.30	6899.00	6779.63	151.21	119.37	206.31	96.86
50	6430.28	6756.00	6428.18	151.21	327.82	208.74	97.40
51	6208.28	6615.00	6192.72	151.21	422.28	212.93	100.95
52	6162.56	6452.00	6135.30	151.21	316.70	214.92	100.99
53	6143.11	6436.00	6107.53	151.21	328.47	217.07	101.22
54	6088.80	6422.00	6046.19	151.21	375.81	220.01	102.56
55	6269.04	6642.00	6219.64	151.21	422.36	223.69	105.21
56	6764.74	6935.00	6722.40	151.21	212.60	223.49	104.26
57	7274.36	7489.00	7242.81	151.21	246.19	223.89	103.36
58	7635.47	7948.00	7613.85	151.21	334.15	225.79	103.47
59	7904.74	8154.00	7899.73	151.21	254.27	226.27	102.64
60	8025.65	8358.00	8038.00	151.21	320.00	227.83	102.49
61	7944.35	8265.00	7975.70	151.21	289.30	228.84	101.93
62	7752.81	7993.00	7800.30	151.21	192.70	228.26	101.20
63	7536.61	7898.00	7595.45	151.21	302.55	229.44	100.81
64	7490.90	7776.00	7560.16	151.21	215.84	229.22	100.03
65	7890.39	7808.00	7967.39	151.21	-159.39	223.25	110.33
66	8492.34	8562.00	8573.70	151.21	-11.70	219.69	113.23
67	8646.65	8628.00	8727.49	151.21	-99.49	214.92	116.94
68	8311.21	8496.00	8392.56	151.21	103.44	213.28	118.82
69	8038.65	8297.00	8122.74	151.21	174.26	212.72	118.04
70	7933.12	7943.00	8020.61	151.21	-77.61	208.57	122.21
71	7649.11	7630.00	7737.62	151.21	-107.62	204.12	127.01
72	7181.71	7397.00	7265.87	151.21	131.13	203.10	126.40

Forecasting Load in 4th Week, using Parameters identified from Data of 1st Week

BELOW ARE PARAMETER VALUES,IN FOLLOWING FORMAT

A1	A2	B0	B1
XP0	XP1	XP2	XP3
XP4	XP5	XP6	XP7
	XP8	XP9	XP10
XP11	XP12	XP13	XP14

.10500	.26600	-3.07600	6.87500
7421.80000	-980.18000	-389.45000	115.88000
15.72800	-95.21200	1.63900	21.74400
	-225.46000	178.26000	-192.99000
112.54000	51.01100	-51.22100	-28.18800

15619.14

FILTERING

HOUR	PERIODIC	Z,REAL	Z,FILTERED	STD.DEV.	ERROR	MEAN ERR.	ACT.S.D.
1	6876.52	6600.00	6922.36	128.21	-322.36	-322.36	
2	6534.52	6282.00	6449.56	127.76	-167.56	-244.96	109.46
3	6280.01	6160.00	6132.65	124.98	27.35	-154.19	175.24
4	6218.38	6059.00	6054.74	124.98	3.26	-114.83	163.31
5	6223.86	6035.00	6086.64	124.98	-51.64	-102.19	144.22
6	6194.28	6037.00	6043.15	124.98	-6.15	-86.18	134.82
7	6328.05	6329.00	6189.05	124.98	139.95	-53.88	149.84
8	6736.83	6854.00	6608.02	124.98	245.98	-16.40	174.60
9	7220.11	7333.00	7160.29	124.98	172.71	4.62	175.06
10	7611.87	7787.00	7530.29	124.98	256.71	29.83	183.30
11	7902.94	7973.00	7860.18	124.98	112.82	37.37	175.68
12	8057.01	8207.00	8025.29	124.98	181.71	49.40	172.61
13	8029.44	8079.00	7986.11	124.98	92.89	52.74	165.70
14	7829.94	7722.00	7813.37	124.98	-91.37	42.45	163.79
15	7556.34	7784.00	7515.35	124.98	268.65	57.53	168.29
16	7530.19	7772.00	7490.23	124.98	281.77	71.55	171.98
17	8010.11	7943.00	8058.46	124.98	-115.46	60.55	172.59
18	8620.32	8758.00	8651.90	124.98	106.10	63.08	167.78
19	8732.75	8627.00	8726.84	124.98	-99.84	54.50	167.28
20	8415.32	8463.00	8446.45	124.98	16.55	52.60	163.04
21	8180.59	8385.00	8173.19	124.98	211.81	60.19	162.66
22	8047.20	8187.00	8111.03	124.98	75.97	60.90	158.78
23	7720.88	7913.00	7816.36	124.98	96.64	62.46	155.31
24	7265.75	7553.00	7363.89	124.98	189.11	67.73	154.08

FORECASTING

HOUR	PERIODIC	Z,REAL	Z,FORECAST	STD.DEV.	ERROR	MEAN ERR.	ACT.S.D.
1	6876.52	7075.00	6998.76	124.98	76.24	76.24	
2	6534.52	6839.00	6673.20	125.66	165.80	121.02	63.33
3	6280.01	6569.00	6375.73	130.35	193.27	145.10	61.20
4	6218.38	6478.00	6320.35	130.54	157.65	148.24	50.36
5	6223.86	6509.00	6323.22	130.92	185.78	155.75	46.73
6	6194.28	6526.00	6301.69	130.95	224.31	167.17	50.31
7	6328.05	6755.00	6431.79	130.99	323.21	189.46	74.75
8	6736.83	7212.00	6841.33	130.99	370.67	212.12	94.31
9	7220.11	7725.00	7325.01	130.99	399.99	232.99	108.18
10	7611.87	8074.00	7715.95	130.99	358.05	245.50	109.40
11	7902.94	8369.00	8009.43	130.99	359.57	255.87	109.33
12	8057.01	8612.00	8156.96	130.99	455.04	272.46	119.05
13	8029.44	8496.00	8128.36	130.99	367.64	279.79	117.00
14	7829.94	8151.00	7929.23	130.99	221.77	275.64	113.47
15	7556.34	8103.00	7655.96	130.99	447.04	287.07	117.96
16	7530.19	8033.00	7622.85	130.99	410.15	294.76	118.04
17	8010.11	8142.00	8095.85	130.99	46.15	280.14	129.22
18	8620.32	8835.00	8709.52	130.99	125.48	271.54	130.56
19	8732.75	8989.00	8819.26	130.99	89.74	261.98	133.56
20	8415.32	8736.00	8503.04	130.99	232.96	260.52	130.16
21	8180.59	8498.00	8262.78	130.99	235.22	259.32	126.98
22	8047.20	8320.00	8118.35	130.99	201.65	256.70	124.53
23	7720.88	7989.00	7795.06	130.99	193.94	253.97	122.37
24	7265.75	7603.00	7329.26	130.99	273.74	254.79	119.75
25	6876.52	7117.00	6954.03	130.99	162.97	251.12	118.66
26	6534.52	6886.00	6599.88	130.99	286.12	252.47	116.46
27	6280.01	6705.00	6356.55	130.99	348.45	256.02	115.68
28	6218.38	6616.00	6283.78	130.99	332.22	258.74	114.43
29	6223.86	6613.00	6286.17	130.99	326.83	261.09	113.08
30	6194.28	6535.00	6263.03	130.99	271.97	261.45	111.13
31	6328.05	6637.00	6379.81	130.99	257.19	261.32	109.26
32	6736.83	7250.00	6798.50	130.99	451.50	267.26	112.62
33	7220.11	7751.00	7282.41	130.99	468.59	273.36	116.26
34	7611.87	8094.00	7683.37	130.99	410.63	277.48	116.88
35	7902.94	8269.00	7975.20	130.99	293.80	277.87	115.18
36	8057.01	8550.00	8131.10	130.99	418.90	281.78	115.93
37	8029.44	8339.00	8103.31	130.99	235.69	280.54	114.56
38	7829.94	8067.00	7909.65	130.99	157.35	277.30	114.75
39	7556.34	7977.00	7634.43	130.99	342.57	278.97	113.72
40	7530.19	7768.00	7599.77	130.99	168.23	276.20	113.61
41	8010.11	7963.00	8070.37	130.99	-107.37	266.85	127.17
42	8620.32	8822.00	8667.55	130.99	154.45	264.17	126.88
43	8732.75	8842.00	8783.88	130.99	58.12	259.38	129.16
44	8415.32	8649.00	8457.76	130.99	191.24	257.83	128.06
45	8180.59	8387.00	8224.37	130.99	162.63	255.71	127.39
46	8047.20	8159.00	8067.67	130.99	91.33	252.14	128.28
47	7720.88	7863.00	7741.36	130.99	121.64	249.36	128.38
48	7265.75	7525.00	7272.88	130.99	252.12	249.42	126.93

FORECASTING

HCUR	PERIODIC	Z,REAL	Z,FORECAST	STD.DEV.	ERROR	MEAN ERR.	ACT.S.D.
49	6876.52	6899.00	6871.79	130.99	27.21	244.89	129.55
50	6534.52	6756.00	6526.86	130.99	229.14	244.57	128.24
51	6280.81	6615.00	6257.03	130.99	357.97	246.79	127.94
52	6218.38	6452.00	6198.61	130.99	253.39	246.92	126.68
53	6223.86	6436.00	6193.39	130.99	242.61	246.84	125.46
54	6194.28	6422.00	6169.89	130.99	252.11	246.94	124.27
55	6328.05	6642.00	6298.62	130.99	343.38	248.69	123.80
56	6736.83	6935.00	6692.04	130.99	242.96	248.59	122.67
57	7220.11	7489.00	7203.29	130.99	285.71	249.24	121.67
58	7611.87	7948.00	7597.32	130.99	350.68	250.99	121.33
59	7902.94	8154.00	7892.72	130.99	261.28	251.16	120.29
60	8057.01	8358.00	8065.78	130.99	292.22	251.85	119.38
61	8029.44	8265.00	8042.46	130.99	222.54	251.37	118.44
62	7829.94	7993.00	7863.20	130.99	129.80	249.41	118.48
63	7556.34	7898.00	7593.09	130.99	304.91	250.29	117.73
64	7530.19	7776.00	7569.09	130.99	206.91	249.61	116.92
65	8010.11	7808.00	8054.52	130.99	-246.52	241.98	131.31
66	8620.32	8562.00	8665.67	130.99	-103.67	236.74	137.07
67	8732.75	8628.00	8780.15	130.99	-152.15	230.94	144.08
68	8415.32	8496.00	8450.72	130.99	45.28	228.21	144.77
69	8180.59	8297.00	8218.24	130.99	78.76	226.04	144.82
70	8047.20	7943.00	8084.49	130.99	-141.49	220.79	150.33
71	7720.88	7630.00	7764.63	130.99	-134.63	215.78	155.10
72	7265.75	7397.00	7310.11	130.99	86.89	213.99	154.75

Forecasting Load in 4th Week, using Parameters identified from Data of 2nd Week

BELOW ARE PARAMETER VALUES,IN FOLLOWING FORMAT

A1	A2	B0	B1
XP0	XP1	XP2	XP3
XP4	XP5	XP6	XP7
	XP8	XP9	XP10
XP11	XP12	XP13	XP14

.12400	.29600	-3.64200	11.81500
7262.20000	-956.39000	-426.56000	147.24000
-6.79600	-105.97000	-12.16500	18.30500
	-269.72000	206.09000	-180.71000
65.31000	80.89900	-30.26500	-46.26000

9195.20

FILTERING

HOUR	PERIODIC	Z,REAL	Z,FILTERED	STD.DEV.	ERROR	MEAN ERR.	ACT.S.D.
1	6668.51	6600.00	6722.78	101.12	-122.78	-122.78	
2	6320.88	6282.00	6193.48	100.29	88.52	-17.13	149.41
3	6099.47	6160.00	5956.27	95.89	203.73	56.49	165.59
4	6065.24	6058.00	5879.62	95.89	178.38	86.96	148.31
5	6037.01	6035.00	5865.50	95.89	169.50	103.47	133.64
6	5943.79	6037.00	5754.86	95.89	282.14	133.25	140.03
7	6114.53	6329.00	5963.51	95.89	365.49	166.43	155.06
8	6665.96	6854.00	6532.52	95.89	321.48	185.81	153.67
9	7233.96	7333.00	7150.18	95.89	182.82	185.47	143.75
10	7598.64	7787.00	7415.80	95.89	371.20	204.05	147.71
11	7841.97	7973.00	7704.74	95.89	268.26	209.89	141.46
12	7939.13	8207.00	7835.48	95.89	371.52	223.35	142.72
13	7835.50	8079.00	7751.91	95.89	327.09	231.33	139.64
14	7634.23	7722.00	7621.45	95.89	100.55	221.99	138.64
15	7425.51	7784.00	7421.36	95.89	362.64	231.37	138.44
16	7380.17	7772.00	7378.85	95.89	393.15	241.48	139.73
17	7776.63	7943.00	7861.46	95.89	81.54	232.07	140.75
18	8399.60	8758.00	8494.03	95.89	263.97	233.84	136.75
19	8577.34	8627.00	8672.46	95.89	-45.46	219.14	147.54
20	8233.56	8463.00	8365.30	95.89	97.70	213.07	146.15
21	7948.59	8385.00	8032.40	95.89	352.60	219.72	145.67
22	7857.66	8187.00	8045.34	95.89	141.66	216.17	143.13
23	7587.36	7913.00	7821.38	95.89	91.62	210.75	142.23
24	7107.55	7553.00	7335.22	95.89	217.78	211.05	139.11

FORECASTING

HCUR	PERIODIC	Z,REAL	Z,FORECAST	STD.DEV.	ERROR	MEAN ERR.	ACT.S.D.
1	6668.51	7075.00	6912.75	95.89	162.25	162.25	
2	6320.88	6839.00	6591.17	96.63	247.83	205.04	60.52
3	6099.47	6569.00	6314.69	101.13	254.31	221.47	51.38
4	6065.24	6478.00	6294.79	101.39	183.21	211.90	46.11
5	6037.01	6509.00	6267.21	101.96	241.79	217.88	42.11
6	5943.79	6526.00	6188.76	101.91	337.24	237.77	61.59
7	6114.53	6755.00	6354.81	101.97	400.19	260.98	83.24
8	6665.96	7212.00	6908.47	101.98	303.53	266.30	78.52
9	7233.96	7725.00	7477.28	101.98	247.72	264.23	73.71
10	7598.64	8074.00	7840.72	101.98	233.28	261.14	70.18
11	7841.97	8369.00	8086.59	101.98	282.41	263.07	66.89
12	7939.13	8612.00	8172.25	101.98	439.75	277.79	81.66
13	7835.50	8496.00	8066.40	101.98	429.60	289.47	88.80
14	7634.23	8151.00	7864.45	101.98	286.55	289.26	85.32
15	7425.51	8103.00	7653.82	101.98	449.18	299.92	92.00
16	7380.17	8033.00	7594.87	101.98	438.13	308.56	95.36
17	7776.63	8142.00	7979.43	101.98	162.57	299.97	98.89
18	8399.60	8835.00	8606.63	101.98	228.37	296.00	97.41
19	8577.34	8909.00	8778.47	101.98	130.53	287.29	101.99
20	8233.56	8736.00	8434.03	101.98	301.97	288.02	99.33
21	7948.59	8498.00	8136.61	101.98	361.39	291.51	98.13
22	7857.66	8320.00	8026.40	101.98	293.60	291.61	95.76
23	7587.36	7989.00	7758.94	101.98	230.06	288.93	94.44
24	7107.55	7603.00	7262.40	101.98	340.60	291.09	92.96
25	6668.51	7117.00	6844.58	101.98	272.42	290.34	91.08
26	6320.88	6886.00	6477.08	101.98	408.92	294.90	92.22
27	6099.47	6705.00	6272.04	101.98	432.96	300.01	94.25
28	6065.24	6616.00	6217.70	101.98	398.30	303.52	94.34
29	6037.01	6613.00	6184.55	101.98	428.45	307.83	95.50
30	5943.79	6535.00	6098.85	101.98	436.15	312.11	96.72
31	6114.53	6637.00	6243.25	101.98	393.75	314.74	96.22
32	6665.96	7250.00	6812.44	101.98	437.56	318.58	97.11
33	7233.96	7751.00	7383.39	101.98	367.61	320.07	95.96
34	7598.64	8094.00	7764.69	101.98	329.31	320.34	94.51
35	7841.97	8269.00	8010.14	101.98	258.86	318.58	93.69
36	7939.13	8550.00	8111.30	101.98	438.70	321.92	94.48
37	7835.50	8339.00	8008.06	101.98	330.94	322.16	93.17
38	7634.23	8067.00	7815.12	101.98	251.88	320.31	92.61
39	7425.51	7977.00	7600.20	101.98	376.80	321.76	91.83
40	7380.17	7768.00	7536.76	101.98	231.24	319.50	91.77
41	7776.63	7963.00	7913.42	101.98	49.58	312.91	99.94
42	8399.60	8822.00	8512.58	101.98	309.42	312.83	98.72
43	8577.34	8842.00	8692.52	101.98	149.48	309.03	100.66
44	8233.56	8649.00	8330.15	101.98	318.85	309.26	99.50
45	7948.59	8387.00	8040.19	101.98	346.81	310.09	98.52
46	7857.66	8159.00	7905.97	101.98	253.03	308.85	97.78
47	7587.36	7863.00	7629.48	101.98	233.52	307.25	97.33
48	7107.55	7525.00	7121.01	101.98	403.99	309.26	97.30

FORECASTING

HCUR	PERIODIC	Z,REAL	Z,FORECAST	STD.DEV.	ERROR	MEAN ERR.	ACT.S.D.
49	6668.51	6899.00	6657.95	101.98	241.05	307.87	96.77
50	6320.88	6756.00	6299.59	101.98	456.41	310.84	98.06
51	6099.47	6615.00	6050.20	101.98	564.80	315.82	103.38
52	6065.24	6452.00	6017.78	101.98	434.22	318.10	103.67
53	6077.01	6436.00	5970.68	101.98	465.32	320.88	104.64
54	5943.79	6422.00	5885.74	101.98	536.26	324.86	107.71
55	6114.53	6642.00	6046.75	101.98	595.25	329.78	112.77
56	6665.96	6935.00	6579.20	101.98	355.80	330.25	111.79
57	7233.96	7489.00	7196.60	101.98	292.40	329.58	110.90
58	7598.64	7948.00	7569.14	101.98	378.86	330.43	110.12
59	7841.97	8154.00	7827.41	101.98	326.59	330.37	109.16
60	7939.13	8358.00	7961.37	101.98	396.63	331.47	108.57
61	7835.50	8265.00	7872.07	101.98	392.93	332.48	107.95
62	7634.23	7993.00	7708.64	101.98	284.36	331.70	107.24
63	7425.51	7898.00	7508.77	101.98	389.23	332.62	106.62
64	7380.17	7776.00	7470.36	101.98	305.64	332.19	105.82
65	7776.63	7808.00	7877.30	101.98	-69.30	326.02	116.20
66	8399.60	8562.00	8502.26	101.98	59.74	321.98	119.87
67	8577.34	8628.00	8681.73	101.98	-53.73	316.37	127.51
68	8233.56	8496.00	8318.48	101.98	177.52	314.33	127.67
69	7948.59	8297.00	8037.69	101.98	259.31	313.54	126.90
70	7857.66	7943.00	7946.78	101.98	-3.78	309.00	131.56
71	7587.36	7630.00	7686.55	101.98	-56.55	303.85	137.64
72	7107.55	7397.00	7205.09	101.98	191.91	302.30	137.30

Forecasting Load in 4th Week, using Parameters identified from Data of 3rd Week

BELOW ARE PARAMETER VALUES, IN FOLLOWING FORMAT

A1	A2	B0	B1
XP0	XP1	XP2	XP3
XP4	XP5	XP6	XP7
	XP8	XP9	XP10
XP11	XP12	XP13	XP14

.21000	.32800	2.36800	1.89500
7392.80000	-935.81000	-423.35000	123.52000
8.79600	-89.80100	-23.30700	12.23400
	-235.42000	196.42000	-169.72000
69.81500	68.96200	-10.68500	-39.14600

8410.98

FILTERING

HOUR	PERIODIC	Z,REAL	Z,FILTERED	STD.DEV.	ERROR	MEAN ERR.	ACT.S.D.
1	6820.83	6600.00	6785.55	99.64	-185.55	-185.55	
2	6483.07	6282.00	6355.57	97.15	-73.57	-129.56	79.18
3	6281.31	6160.00	6081.59	91.71	78.41	-60.23	132.48
4	6220.00	6058.00	6032.13	91.71	25.87	-38.71	116.43
5	6166.77	6035.00	5997.18	91.71	37.82	-23.40	106.48
6	6114.71	6037.00	5942.59	91.71	94.41	-3.77	106.69
7	6324.96	6329.00	6171.54	91.71	157.46	19.27	114.89
8	6834.66	6854.00	6711.73	91.71	142.27	34.64	114.91
9	7327.28	7333.00	7222.27	91.71	110.73	43.10	110.44
10	7660.50	7787.00	7562.01	91.71	224.99	61.29	118.96
11	7914.42	7973.00	7852.79	91.71	120.21	66.64	114.24
12	8022.87	8207.00	7997.42	91.71	209.58	78.55	116.48
13	7919.26	8079.00	7912.63	91.71	166.37	85.31	114.15
14	7733.28	7722.00	7778.38	91.71	-56.38	75.19	116.03
15	7566.17	7784.00	7578.02	91.71	205.98	83.91	116.79
16	7530.30	7772.00	7540.47	91.71	231.53	93.13	118.72
17	7862.44	7943.00	7963.99	91.71	-20.99	86.42	118.23
18	8437.44	8758.00	8530.42	91.71	227.58	94.26	119.43
19	8674.65	8627.00	8778.95	91.71	-151.95	81.30	129.08
20	8412.64	8463.00	8528.61	91.71	-65.61	73.96	129.86
21	8120.45	8385.00	8146.43	91.71	238.57	81.80	131.57
22	7991.90	8187.00	8100.78	91.71	86.22	82.00	128.40
23	7735.14	7913.00	7906.50	91.71	6.50	78.72	126.44
24	7272.23	7553.00	7423.40	91.71	129.60	80.84	124.09

FORECASTING

HCUR	FERIODIC	Z,REAL	Z,FORECAST	STD.DEV.	ERROR	MEAN ERR.	ACT.S.D.
1	6820.83	7075.00	6992.84	91.71	82.16	82.16	-
2	6483.07	6839.00	6670.26	98.71	168.74	125.45	61.22
3	6281.31	6569.00	6440.94	99.73	128.06	126.32	43.32
4	6220.00	6478.00	6385.40	100.64	92.60	117.89	39.18
5	6166.77	6509.00	6328.84	101.61	180.16	130.35	43.90
6	6114.71	6526.00	6277.89	101.88	248.11	149.97	82.07
7	6324.96	6755.00	6485.90	102.06	269.10	166.99	72.38
8	6834.66	7212.00	6995.54	102.13	216.46	173.18	69.25
9	7327.20	7725.00	7487.05	102.17	237.95	180.37	68.28
10	7660.50	8074.00	7819.50	102.19	254.50	187.79	68.51
11	7914.42	8369.00	8070.34	102.19	298.66	197.87	73.09
12	8022.87	8612.00	8175.74	102.20	436.26	217.73	97.94
13	7919.26	8496.00	8070.53	102.20	425.47	233.71	110.06
14	7733.28	8151.00	7882.21	102.20	268.79	236.22	106.16
15	7566.17	8103.00	7710.06	102.20	392.94	246.67	110.01
16	7530.30	8033.00	7668.48	102.20	364.52	254.03	110.29
17	7862.44	8142.00	7998.29	102.20	143.71	247.54	110.08
18	8437.44	8835.00	8571.31	102.20	263.69	248.44	106.87
19	8674.65	8909.00	8805.39	102.20	103.61	240.82	109.04
20	8412.64	8736.00	8538.04	102.20	197.96	238.67	106.56
21	8120.45	8498.00	8238.18	102.20	259.82	239.68	103.97
22	7991.90	8320.00	8105.22	102.20	214.78	238.55	101.60
23	7735.14	7989.00	7844.96	102.20	144.04	234.44	101.20
24	7272.23	7603.00	7382.48	102.20	220.52	233.86	99.02
25	6820.83	7117.00	6929.79	102.20	187.21	231.99	97.38
26	6483.07	6886.00	6590.85	102.20	295.15	234.42	96.21
27	6281.31	6705.00	6386.01	102.20	318.99	237.96	95.74
28	6220.00	6616.00	6319.87	102.20	296.13	239.65	94.60
29	6166.77	6613.00	6265.52	102.20	347.48	243.37	95.03
30	6114.71	6535.00	6209.18	102.20	325.82	246.11	94.58
31	6324.96	6637.00	6419.63	102.20	217.37	245.19	93.14
32	6834.66	7250.00	6933.93	102.20	316.07	247.40	92.47
33	7327.20	7751.00	7431.53	102.20	319.47	249.59	91.88
34	7660.50	8094.00	7768.88	102.20	325.12	251.81	91.40
35	7914.42	8269.00	8024.87	102.20	244.13	251.59	90.05
36	8022.87	8550.00	8134.90	102.20	415.10	256.13	92.85
37	7919.26	8339.00	8032.78	102.20	306.22	257.48	91.92
38	7733.28	8067.00	7844.92	102.20	222.08	256.55	90.85
39	7566.17	7977.00	7670.24	102.20	306.76	257.84	90.01
40	7530.30	7768.00	7623.97	102.20	144.03	254.99	90.65
41	7862.44	7963.00	7945.38	102.20	17.62	249.20	96.88
42	8437.44	8822.00	8513.63	102.20	308.37	250.61	96.13
43	8674.65	8842.00	8743.52	102.20	98.48	247.08	97.77
44	8412.64	8649.00	8472.03	102.20	176.97	245.48	97.20
45	8120.45	8387.00	8165.18	102.20	221.82	244.96	96.15
46	7991.90	8159.00	8023.00	102.20	136.00	242.59	96.43
47	7735.14	7863.00	7753.35	102.20	109.65	239.76	97.33
48	7272.23	7525.00	7275.63	102.20	249.17	239.96	96.29

FORECASTING

HCUR	PERIODIC	Z,REAL	Z,FORECAST	STD.DEV.	ERROR	MEAN ERR.	ACT.S.D.
49	6820.83	6899.00	6812.98	102.20	86.02	236.81	97.79
50	6483.07	6756.00	6463.70	102.20	292.30	237.92	97.10
51	6281.31	6815.00	6254.11	102.20	360.89	240.33	97.66
52	6220.08	6452.00	6186.66	102.20	265.34	240.82	96.76
53	6166.77	6436.00	6129.91	102.20	306.09	242.05	96.24
54	6114.71	6422.00	6075.25	102.20	346.75	243.99	96.39
55	6324.96	6642.00	6282.37	102.20	359.63	246.09	96.76
56	6834.66	6935.00	6801.78	102.20	133.22	244.07	97.05
57	7327.20	7489.00	7306.75	102.20	182.25	242.99	96.53
58	7660.50	7948.00	7649.68	102.20	298.32	243.94	95.95
59	7914.42	8154.00	7918.75	102.20	235.25	243.79	95.13
60	8022.87	8358.00	8042.02	102.20	315.98	245.00	94.78
61	7919.26	8265.00	7953.55	102.20	311.45	246.09	94.37
62	7733.28	7993.00	7779.35	102.20	213.65	245.56	93.69
63	7566.17	7898.00	7618.78	102.20	279.22	246.10	93.02
64	7530.30	7776.00	7588.76	102.20	187.24	245.18	92.58
65	7862.44	7808.00	7924.51	102.20	-116.51	239.61	102.22
66	8437.44	8562.00	8500.21	102.20	61.79	236.92	103.77
67	8674.65	8628.00	8734.05	102.20	-106.05	231.80	111.17
68	8412.64	8496.00	8470.37	102.20	25.63	228.77	113.14
69	8120.45	8297.00	8179.49	102.20	117.51	227.16	113.10
70	7991.90	7943.00	8052.98	102.20	-109.98	222.34	119.29
71	7735.14	7630.00	7796.13	102.20	-166.13	216.87	127.09
72	7272.23	7397.00	7328.09	102.20	68.91	214.81	127.39

Forecasting Load in 4th Week, using Parameters identified in Retrospect from Data of 4th Week itself

BELCW ARE PARAMETER VALUES, IN FOLLOWING FORMAT

A1	A2	B0	B1
XP0	XP1	XP2	XP3
XP4	XP5	XP6	XP7
	XP8	XP9	XP10
XP11	XP12	XP13	XP14
.23700	.29400	-13.18100	17.90500
7599.40000	-856.81000	-390.49000	107.51000
25.63800	-71.33500	-20.53900	13.36800
	-325.22000	218.06000	-200.40000
61.52900	57.70500	-8.03800	-43.84500

7023.84

FILTERING

HOUR	PERIODIC	Z,REAL	Z,FILTERED	STD.DEV.	ERROR	MEAN ERR.	ACT.S.D.
1	6994.14	6600.00	7190.54	91.92	-590.54	-590.54	
2	6712.83	6282.00	6526.70	88.49	-244.70	-417.62	244.54
3	6519.01	6160.00	6293.68	83.81	-133.68	-322.97	238.27
4	6467.25	6058.00	6154.95	83.81	-96.95	-266.47	224.99
5	6440.31	6035.00	6122.16	83.81	-87.16	-230.61	210.70
6	6401.88	6037.00	6062.18	83.81	-25.18	-196.37	206.28
7	6605.05	6329.00	6330.15	83.81	-1.15	-168.48	202.24
8	7098.56	6854.00	6818.53	83.81	35.47	-142.99	200.65
9	7579.04	7333.00	7396.71	83.81	-63.71	-134.18	189.54
10	7932.88	7787.00	7592.67	83.81	194.33	-101.33	206.70
11	8240.42	7973.00	8000.67	83.81	-27.67	-94.63	197.35
12	8382.71	8207.00	8143.03	83.81	63.97	-81.41	193.65
13	8256.72	8079.00	8003.32	83.81	75.68	-69.33	190.46
14	8026.63	7722.00	7823.42	83.81	-101.42	-71.62	183.19
15	7816.83	7784.00	7623.21	83.81	160.79	-56.13	186.44
16	7715.13	7772.00	7561.92	83.81	210.08	-39.49	192.02
17	7966.35	7943.00	7888.81	83.81	54.19	-33.98	187.31
18	8499.93	8758.00	8438.00	83.81	320.00	-14.31	199.96
19	8753.57	8627.00	8783.22	83.81	-156.22	-21.78	197.03
20	8525.33	8463.00	8554.79	83.81	-91.79	-25.28	192.41
21	8236.60	8385.00	8182.65	83.81	202.35	-14.44	194.01
22	8071.27	8187.00	8119.42	83.81	67.58	-10.71	190.14
23	7784.76	7913.00	7866.23	83.81	46.77	-8.21	186.15
24	7359.19	7553.00	7467.76	83.81	85.24	-4.32	183.06

FORECASTING

HOUR	PERIODIC	Z,REAL	Z,FORECAST	STD.DEV.	ERROR	MEAN EPR.	ACT.S.D.
1	6994.14	7075.00	7114.69	83.81	-39.69	-39.69	
2	6712.83	6839.00	6854.25	86.13	-15.25	-27.47	17.28
3	6519.01	6569.00	6633.48	90.99	-64.48	-39.81	24.62
4	6467.25	6478.00	6590.35	91.89	-112.35	-57.94	41.47
5	6440.31	6509.00	6575.17	92.62	-66.17	-59.59	36.10
6	6401.88	6526.00	6562.51	92.85	-36.51	-55.74	33.64
7	6605.05	6755.00	6765.81	92.99	-10.81	-49.32	35.09
8	7098.56	7212.00	7263.82	93.04	-51.82	-49.63	32.50
9	7579.04	7725.00	7749.83	93.06	-24.83	-46.88	31.50
10	7932.08	8074.00	8103.24	93.08	-29.24	-45.11	30.22
11	8240.42	8369.00	8424.69	93.08	-55.69	-46.08	28.85
12	8382.71	8612.00	8553.61	93.08	58.39	-37.37	40.82
13	8256.72	8496.00	8425.19	93.08	70.81	-29.05	49.27
14	8026.63	8151.00	8198.98	93.08	-47.98	-30.40	47.61
15	7816.83	8103.00	7997.57	93.08	105.43	-21.34	57.74
16	7715.13	8033.00	7883.63	93.09	149.37	-10.67	70.24
17	7966.35	8142.00	8114.44	93.09	27.56	-8.43	68.64
18	8499.93	8835.00	8657.39	93.09	177.61	1.91	79.73
19	8753.57	8909.00	8906.68	93.09	2.32	1.93	77.48
20	8525.33	8736.00	8688.35	93.09	47.65	4.22	76.11
21	8236.60	8498.00	8394.55	93.09	103.45	8.94	77.28
22	8071.27	8320.00	8199.74	93.09	120.26	14.00	79.06
23	7784.76	7989.00	7922.09	93.09	66.91	16.30	78.03
24	7359.19	7603.00	7461.23	93.09	141.77	21.53	80.49
25	6994.14	7117.00	7134.41	93.09	-17.41	[illegible]	79.18
26	6712.83	6886.00	6820.60	93.09	65.40	[illegible]	78.09
27	6519.01	6705.00	6660.83	93.09	44.17	22.55	76.70
28	6467.25	6616.00	6586.00	93.09	30.00	22.82	75.28
29	6440.31	6613.00	6547.22	93.09	65.78	24.30	74.35
30	6401.88	6535.00	6532.83	93.09	2.17	23.56	73.17
31	6605.05	6637.00	6684.51	93.09	-47.51	21.27	73.06
32	7098.56	7250.00	7193.73	93.09	56.27	22.36	72.14
33	7579.04	7751.00	7668.84	93.09	82.16	24.18	71.76
34	7932.08	8094.00	8042.65	93.09	51.35	24.97	70.82
35	8240.42	8269.00	8353.87	93.09	-84.87	21.84	72.20
36	8382.71	8550.00	8501.16	93.09	48.84	22.59	71.30
37	8256.72	8339.00	8374.56	93.09	-35.56	21.01	70.95
38	8026.63	8067.00	8166.51	93.09	-99.51	17.84	72.67
39	7816.83	7977.00	7967.48	93.09	9.52	17.63	71.72
40	7715.13	7768.00	7857.83	93.09	-89.83	14.94	72.80
41	7966.35	7963.00	8095.71	93.09	-132.71	11.34	75.49
42	8499.93	8822.00	8595.71	93.09	226.29	16.46	81.61
43	8753.57	8842.00	8864.87	93.09	-22.87	15.54	80.86
44	8525.33	8649.00	8622.51	93.09	26.49	15.79	79.93
45	8236.60	8387.00	8355.22	93.09	31.78	16.15	79.05
46	8071.27	8159.00	8138.32	93.09	20.68	16.25	78.17
47	7784.76	7863.00	7860.67	93.09	2.33	15.[illegible]	77.34
48	7359.19	7525.00	7412.22	93.09	112.78	17.97	77.78

FORECASTING

HOUR	PERIODIC	Z,REAL	Z,FORECAST	STD.DEV.	ERROR	MEAN ERR.	ACT.S.D.
49	6994.14	6899.00	7019.18	93.09	-120.18	15.15	79.46
50	6712.83	6756.00	6737.12	93.09	18.88	15.22	78.64
51	6519.01	6615.00	6501.44	93.09	113.56	17.15	79.06
52	6467.25	6452.00	6457.72	93.09	-5.72	16.71	78.35
53	6440.31	6436.00	6398.63	93.09	37.37	17.10	77.64
54	6401.88	6422.00	6374.06	93.09	47.94	17.67	77.02
55	6605.05	6642.00	6564.74	93.09	77.26	18.76	76.73
56	7098.56	6935.00	6991.21	93.09	-56.21	17.42	76.68
57	7579.04	7489.00	7528.00	93.09	-39.00	16.43	76.36
58	7932.08	7948.00	7878.92	93.09	69.08	17.34	76.00
59	8240.42	8154.00	8178.57	93.09	-24.57	16.63	75.54
60	8382.71	8358.00	8359.18	93.09	-1.18	16.33	74.93
61	8256.72	8265.00	8231.73	93.09	33.27	16.61	74.34
62	8026.63	7993.00	8051.48	93.09	-58.48	15.39	74.34
63	7816.83	7898.00	7855.23	93.09	42.77	15.83	73.82
64	7715.13	7776.00	7759.44	93.09	16.56	15.84	73.23
65	7966.35	7808.00	8028.94	93.09	-220.94	12.20	78.37
66	8499.93	8562.00	8571.13	93.09	-9.13	11.87	77.81
67	8753.57	8628.00	8841.21	93.09	-213.21	8.52	81.97
68	8525.33	8496.00	8583.32	93.09	-87.32	7.11	82.18
69	8236.60	8297.00	8297.02	93.09	-.02	7.00	81.58
70	8071.27	7943.00	8127.70	93.09	-184.70	4.26	84.16
71	7784.76	7630.00	7860.45	93.09	-230.45	.96	88.08
72	7359.19	7397.00	7447.68	93.09	-50.68	.24	87.67

Forecasting Load Beginning in 2nd Week, with Lead Time up to one Full Week, using Parameters identified from Data of 1st Week

BELOW ARE PARAMETER VALUES,IN FOLLOWING FORMAT

A1	A2	B0	B1
XP0	XP1	XP2	XP3
XP4	XP5	XP6	XP7
	XP8	XP9	XP10
XP11	XP12	XP13	XP14
.10500	.26600	-3.07600	6.87500
7421.80000	-980.18000	-389.45000	115.88000
15.72800	-95.21200	1.63900	21.74400
	-225.46000	178.26000	-192.99000
112.54000	51.01100	-51.22100	-28.18800

15619.14

FILTERING

HOUR	X1(1)	X1(2)	PERIODIC	Z.REAL	Z.FILTERED	STD.DEV.	ERROR
1	-565.52	-40.02	6876.52	6311.00	6937.43	128.21	-626.43
2	-446.52	-565.52	6534.52	6088.00	6390.51	127.76	-302.51
3	-479.01	-446.52	6280.01	5801.00	6009.95	124.98	-208.95
4	-400.38	-479.01	6218.38	5818.00	5972.25	124.98	-154.25
5	-443.86	-400.38	6223.86	5780.00	5967.95	124.98	-187.95
6	-400.28	-443.86	6194.28	5794.00	5950.20	124.98	-156.20
7	-378.05	-400.28	6328.05	5950.00	6091.09	124.98	-141.09
8	-336.83	-378.05	6736.83	6400.00	6489.01	124.98	-89.01
9	-197.11	-336.83	7220.11	7023.00	6979.83	124.98	43.17
10	-198.87	-197.11	7611.87	7413.00	7437.99	124.98	-24.99
11	-332.94	-198.87	7902.94	7570.00	7771.46	124.98	-201.46
12	-284.01	-332.94	8057.01	7773.00	7860.15	124.98	-87.15
13	-427.44	-284.01	8029.44	7602.00	7806.86	124.98	-204.86
14	-516.94	-427.44	7829.94	7313.00	7608.73	124.98	-295.73
15	-286.34	-516.94	7556.34	7270.00	7291.30	124.98	-21.30
16	-350.19	-286.34	7530.19	7180.00	7267.68	124.98	-87.68
17	-682.11	-350.19	8010.11	7328.00	7807.95	124.98	-479.95
18	-297.32	-682.11	8620.32	8323.00	8369.83	124.98	-46.83
19	-757.75	-297.32	8732.75	7975.00	8434.21	124.98	-459.21
20	-648.32	-757.75	8415.32	7767.00	8173.00	124.98	-406.00
21	-585.59	-648.32	8180.59	7595.00	7827.88	124.98	-232.88
22	-659.20	-585.59	8047.20	7388.00	7734.70	124.98	-346.70
23	-546.88	-659.20	7720.88	7174.00	7417.10	124.98	-243.10
24	-467.75	-546.88	7265.75	6798.00	6958.39	124.98	-160.39

FORECASTING

HOUR	X1(1)	X1(2)	PERIODIC	Z.REAL	Z.FORECAST	STD.DEV.	ERROR
1	-279.83	-467.75	6876.52	6265.00	6596.70	124.98	-331.70
2	-221.68	-279.83	6534.52	5975.00	6312.84	125.66	-337.84
3	-171.21	-221.68	6280.01	5813.00	6108.80	130.35	-295.80
4	-158.46	-171.21	6218.38	5735.00	6059.92	130.54	-324.92
5	-151.30	-158.46	6223.86	5686.00	6072.57	130.92	-386.57
6	-133.85	-151.30	6194.28	5674.00	6060.43	130.95	-386.43
7	-132.18	-133.85	6328.05	5794.00	6195.87	130.99	-401.87
8	-127.06	-132.18	6736.83	6304.00	6609.77	130.99	-305.77
9	-126.14	-127.06	7220.11	6954.00	7093.96	130.99	-139.96
10	-122.06	-126.14	7611.87	7274.00	7489.80	130.99	-215.80
11	-133.40	-122.06	7902.94	7514.00	7769.54	130.99	-255.54
12	-129.71	-133.40	8057.01	7688.00	7927.29	130.99	-239.29
13	-122.39	-129.71	8029.44	7594.00	7907.05	130.99	-313.05
14	-130.28	-122.39	7829.94	7256.00	7699.65	130.99	-443.65
15	-124.82	-130.28	7556.34	7185.00	7431.52	130.99	-246.52
16	-125.60	-124.82	7530.19	7117.00	7404.58	130.99	-287.58
17	-118.21	-125.60	8010.11	7411.00	7891.90	130.99	-480.90
18	-117.59	-118.21	8620.32	8401.00	8502.73	130.99	-101.73
19	-106.46	-117.59	8732.75	8133.00	8626.29	130.99	-493.29
20	-106.41	-106.46	8415.32	7894.00	8308.91	130.99	-414.91
21	-107.64	-106.41	8180.59	7619.00	8072.95	130.99	-453.95
22	-108.82	-107.64	8047.20	7396.00	7938.38	130.99	-542.38
23	-104.73	-108.82	7720.88	7119.00	7616.15	130.99	-497.15
24	-113.84	-104.73	7265.75	6762.00	7151.91	130.99	-389.91

FORECASTING

HOUR	X1(1)	X1(2)	PERIODIC	Z,REAL	Z,FORECAST	STD.DEV.	ERROR
25	-118.60	-113.84	6876.52	6318.00	6757.93	130.99	-439.93
26	-111.82	-118.60	6534.52	6059.00	6422.70	130.99	-363.70
27	-135.89	-111.82	6280.01	5870.00	6144.11	130.99	-274.11
28	-135.06	-135.89	6218.38	5823.00	6083.32	130.99	-260.32
29	-150.08	-135.06	6223.86	5690.00	6073.78	130.99	-383.78
30	-149.50	-150.08	6194.28	5703.00	6044.79	130.99	-341.79
31	-162.37	-149.50	6328.05	5882.00	6165.68	130.99	-283.68
32	-159.26	-162.37	6736.83	6282.00	6577.57	130.99	-295.57
33	-168.60	-159.26	7220.11	6926.00	7051.51	130.99	-125.51
34	-198.77	-168.60	7611.87	7218.00	7413.09	130.99	-195.09
35	-152.70	-198.77	7902.94	7390.00	7750.25	130.99	-360.25
36	-159.94	-152.70	8057.01	7645.00	7897.07	130.99	-252.07
37	-131.69	-159.94	8029.44	7442.00	7897.75	130.99	-455.75
38	-165.02	-131.69	7829.94	7187.00	7664.91	130.99	-477.91
39	-160.39	-165.02	7556.34	7102.00	7395.94	130.99	-293.94
40	-167.38	-160.39	7530.19	6986.00	7362.81	130.99	-376.81
41	-171.53	-167.38	8010.11	7244.00	7838.58	130.99	-594.58
42	-171.42	-171.53	8620.32	8211.00	8448.90	130.99	-237.90
43	-184.04	-171.42	8732.75	8073.00	8548.71	130.99	-475.71
44	-187.54	-184.04	8415.32	7795.00	8227.78	130.99	-432.78
45	-208.16	-187.54	8180.59	7616.00	7972.43	130.99	-356.43
46	-197.78	-208.16	8047.20	7478.00	7849.42	130.99	-371.42
47	-162.85	-197.78	7720.88	7296.00	7558.04	130.99	-262.04
48	-157.36	-162.85	7265.75	6719.00	7108.39	130.99	-389.39

FORECASTING

HOUR	X1(1)	X1(2)	PERIODIC	Z,REAL	Z,FORECAST	STD.DEV.	ERROR
49	-147.85	-157.36	6876.52	6287.00	6728.67	130.99	-441.67
50	-127.68	-147.85	6534.52	5975.00	6406.84	130.99	-431.84
51	-124.21	-127.68	6280.01	5750.00	6155.79	130.99	-405.79
52	-122.32	-124.21	6218.38	5734.00	6096.06	130.99	-362.06
53	-104.03	-122.32	6223.86	5713.00	6119.83	130.99	-406.83
54	-74.83	-104.03	6194.28	5741.00	6119.46	130.99	-378.46
55	-57.60	-74.83	6326.05	5888.00	6270.45	130.99	-382.45
56	-46.69	-57.60	6736.83	6470.00	6690.14	130.99	-220.14
57	-31.04	-46.69	7220.11	7207.00	7189.07	130.99	17.93
58	-23.46	-31.04	7611.87	7551.00	7588.41	130.99	-37.41
59	-17.00	-23.46	7902.94	7731.00	7885.94	130.99	-154.94
60	-6.29	-17.00	8057.01	7961.00	8050.72	130.99	-89.72
61	-2.00	-6.29	8029.44	7889.00	8027.44	130.99	-138.44
62	-.09	-2.00	7829.94	7560.00	7829.84	130.99	-269.84
63	11.13	-.09	7556.34	7506.00	7567.46	130.99	-61.46
64	7.02	11.13	7530.19	7501.00	7537.21	130.99	-36.21
65	2.54	7.02	8010.11	7553.00	8012.65	130.99	-459.65
66	1.34	2.54	8620.32	8460.00	8621.66	130.99	-161.66
67	5.66	1.34	8732.75	8524.00	8738.41	130.99	-214.41
68	15.93	5.66	8415.32	8358.00	8431.25	130.99	-73.25
69	2.58	15.93	8180.59	8081.00	8183.17	130.99	-102.17
70	5.54	2.58	8047.20	7883.00	8052.74	130.99	-169.74
71	2.36	5.54	7720.88	7652.00	7723.24	130.99	-71.24
72	-17.29	2.36	7265.75	7311.00	7248.46	130.99	62.54

FORECASTING

HOUR	X1(1)	X1(2)	PERIODIC	Z.REAL	Z.FORECAST	STD.DEV.	ERROR
73	23.53	-17.29	6876.52	6814.00	6900.05	130.99	-86.05
74	-63.47	23.53	6534.52	6499.00	6471.05	130.99	27.95
75	-68.84	-63.47	6200.01	6304.00	6211.17	130.99	92.83
76	-98.21	-68.84	6218.38	6125.00	6120.17	130.99	4.83
77	-101.75	-98.21	6223.86	6088.00	6122.12	130.99	-34.12
78	-109.97	-101.75	6134.28	6099.00	6084.32	130.99	14.68
79	-105.55	-109.97	6328.05	6200.00	6222.50	130.99	-22.50
80	-93.83	-105.55	6736.83	6768.00	6643.00	130.99	125.00
81	-83.97	-93.83	7220.11	7238.00	7136.14	130.99	101.86
82	-85.71	-83.97	7611.87	7548.00	7526.15	130.99	21.85
83	-105.13	-85.71	7902.94	7726.00	7797.82	130.99	-71.82
84	-103.65	-105.13	8057.01	8031.00	7953.36	130.99	77.64
85	-111.10	-103.65	8029.44	7903.00	7918.34	130.99	-15.34
86	-121.10	-111.10	7829.94	7601.00	7708.84	130.99	-107.84
87	-103.27	-121.10	7556.34	7658.00	7453.07	130.99	204.93
88	-100.58	-103.27	7530.19	7357.00	7429.61	130.99	-72.61
89	-104.55	-100.58	8010.11	7709.00	7905.56	130.99	-196.56
90	-96.43	-104.55	8620.32	8598.00	8523.88	130.99	74.12
91	-128.55	-96.43	8732.75	8387.00	8604.20	130.99	-217.20
92	-144.18	-128.55	8415.32	7894.00	8271.13	130.99	-377.13
93	-133.51	-144.18	8180.59	7808.00	8047.08	130.99	-239.08
94	-134.30	-133.51	8047.20	7558.00	7912.90	130.99	-354.90
95	-139.63	-134.30	7720.88	7289.00	7581.25	130.99	-292.25
96	-140.46	-139.63	7265.75	6934.00	7125.29	130.99	-191.29