SHORT-TERM OPTIMAL OPERATION OF A HYDROELECTRIC SYSTEM

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ABSTRACT

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The problem of optimizing the operation of the Alcan hydroelectric system in the Saguenay region of Quebec is examined. In particular, consideration is given to finding the optimal water release policy for a short-term planning horizon. The problem is solved using nonlinear programming techniques. The novel feature of this short-term model is the way that the objective function is defined using the concept of 'future generating potential'. A description of this concept and how it is used to solve the short-term problem is given. To put the problem in its proper prospective, the implementation of short-term optimization procedures at four reservoir systems are given as examples. Also included is a brief discussion on the hierarchical approach to solving reservoir management problems and a brief description of mathematical programming techniques currently in use.

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"If any of you does not know how to meet any particular problem he has only to ask God - who gives generously to all men without making them feel foolish or guilty - and he may be quite sure that the necessary wisdom will be given him"

James 1:5

I dedicate this dissertation to my Lord and Saviour Jesus Christ, who guided me and gave me wisdom when I most needed it.

I am grateful for a supportive family; for my father who encouraged me to seek the highest goals, for my mother who prayed and cared for me and for my brother who was always prepared to help me.

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CHAPTER 1

INTRODUCTION

1.1 MOTIVATION

The days of inexpensive energy are gone and the demand for energy is continually increasing. The last two decades have seen an increase in the number of power generating stations, particularly nuclear and thermal plants. Regions endowed with an abundance of water resources and suitable terrain have developed hydroelectric generating stations. Hydro power has become the most preferable option (if available) because water is a renewable resource and it does not have the problem of disposing nuclear waste or the ever increasing cost of fuel. It is a clean form of energy that has minimal operating costs when compared to its alternatives. The major drawback of hydro power is the large initial cost in developing and implementing the system. However, the long term benefits far outweigh any initial inconvenience. A perfect example is the James Bay power development project. The development and construction costs have been estimated at 15 billion dollars. However, if these costs are repaid over a 100 year period by 7.5 million consumers, the cost per consumer per year is only $20.

The cost of operating a hydroelectric generating system is almost insignificant when the revenue from the power generated is considered. In the past, hydro systems were operated according to rules obtained from the system characteristics and previous experience. There was little incentive to develop sophisticated methods of increasing power production. However, with the skyrocketing cost of energy and the efficiency of computers the situation changed.

The shortfall in energy production from hydro plants has to be made up from more expensive forms of energy. Almost three decades ago, the possibility of increasing the operating efficiencies of a hydro system at a reasonable cost were considered. Little [16] was the first to develop a feasible scheme using dynamic programming techniques. His results showed that by using optimization procedures, the system's generating capacity could be increased at minimal cost. It is estimated that productivity can be increased by as much as two percent. In actual dollars and cents, this represents 3.5 million dollars of extra power annually for the Alcan Power System. The combined total cost of development, implementation and operation of optimization procedures is significantly less than 3.5 million dollars.

1.2 OBJECTIVES

The primary objectives of this work are to introduce the reader to the area of hydroelectric system optimization and to review the present state-of-the-art. The purpose of the dissertation is to present the concepts and give an understanding of the nature and scope of the problem. It is for this reason that all unnecessary detail has been excluded. The interested reader is directed to the list of references in the Appendix for papers describing the various aspects of the problem.

1.3 OUTLINE

The problem of optimizing a hydroelectric power system is introduced in Chapter 2. This includes a detailed description of the hierarchical approach used to solve the problem.

Chapter 3 examines the various mathematical techniques presently being used in system optimization. A brief description of the major techniques is followed by a comparison and discussion of their suitability in solving the various aspects of the reservoir management problem.

In Chapter 4, we examine four different multireservoir systems currently employing optimization procedures. The objective of this chapter is to show how mathematical programming techniques are implemented in actual systems. These is also a discussion on the advantages and disadvantages of using a particular technique.

Finally in Chapter 5, we examine the Alcan hydroelectric system in detail. The emphasis is on showing how a short-term optimization policy is actually implemented. This includes a discussion on the concept of future generating potential. A procedure for aggregating some of the system variables to reduce the dimension of the problem is also given.

The Appendix at the end of the dissertation contains a partial listing and classification of papers that deal with the subject of system optimization.

CHAPTER 2

THE RESERVOIR MANAGEMENT PROBLEM

2.1 INTRODUCTION

The reservoir management problem is concerned with finding a water release policy that controls the water levels in such a manner that system operating objectives are satisfied and system constraints are not violated. The objectives and the constraints are different for every system.

A system usually consists of a number of reservoirs connected by rivers. The reservoir elevations are maintained by dams and are controlled by sluice gates. Normally there are powerhouses associated with each dam,and run-of-the-river powerhouses along the interconnecting rivers. The power generated at these powerhouses is then transmitted to the consumer via transmission lines.

The system's operating objectives are to find the water release policy that makes the most efficient use of the stored water. In most cases, this is interpreted to mean that a strategy is found that produces as much energy as possible with the available water. This is not a trivial problem as there are many factors to be considered, such as constraints imposed by the system and demands imposed by the consumer.

A very important consideration is the nature of the water inflows that continually replenish the reservoirs. The stochastic of these inflows presents a number of modelling and computational difficulties [24]. During the peak season, the inflow rate is very high while during winter, it is very low. Reservoirs are used to store the spring inflows so that there will be sufficient water to generate the required power during the winter season.

The constraints imposed by the physical nature of the system are also an important consideration. There are upper and lower bounds on the volumes stored in the reservoirs and on the amount of water that can be discharged. The generating capabilities of the powerhouses are also instrumental in determining how water is to be released. In most cases, the power generated is a function of the reservoir elevation and the rate at which water is discharged.

The above and other associated system requirements require a thorough knowledge of the system characteristics before any water release strategy can be considered. Once the system characteristics are known, a comprehensive optimization philosophy can be developed. The next two sections address themselves to the above problems.

2.2 THE ALCAN HYDROELECTRIC SYSTEM

In this section, the Alcan hydroelectric system will be described in general terms. A mathematical model of the system is presented in Chapter 5. The Alcan system is typical of many hydro systems and serves as a point of reference in later discussions on the implementation of optimization schemes. Although each system is unique, many of the techniques used in developing optimization procedures for the Alcan system are applicable to other systems.

The Alcan system is located in the Saguenay region of Quebec (see Figure 2.1). There are three major reservoirs and one minor reservoir in the system. The primary purpose of Lac Manouane, the

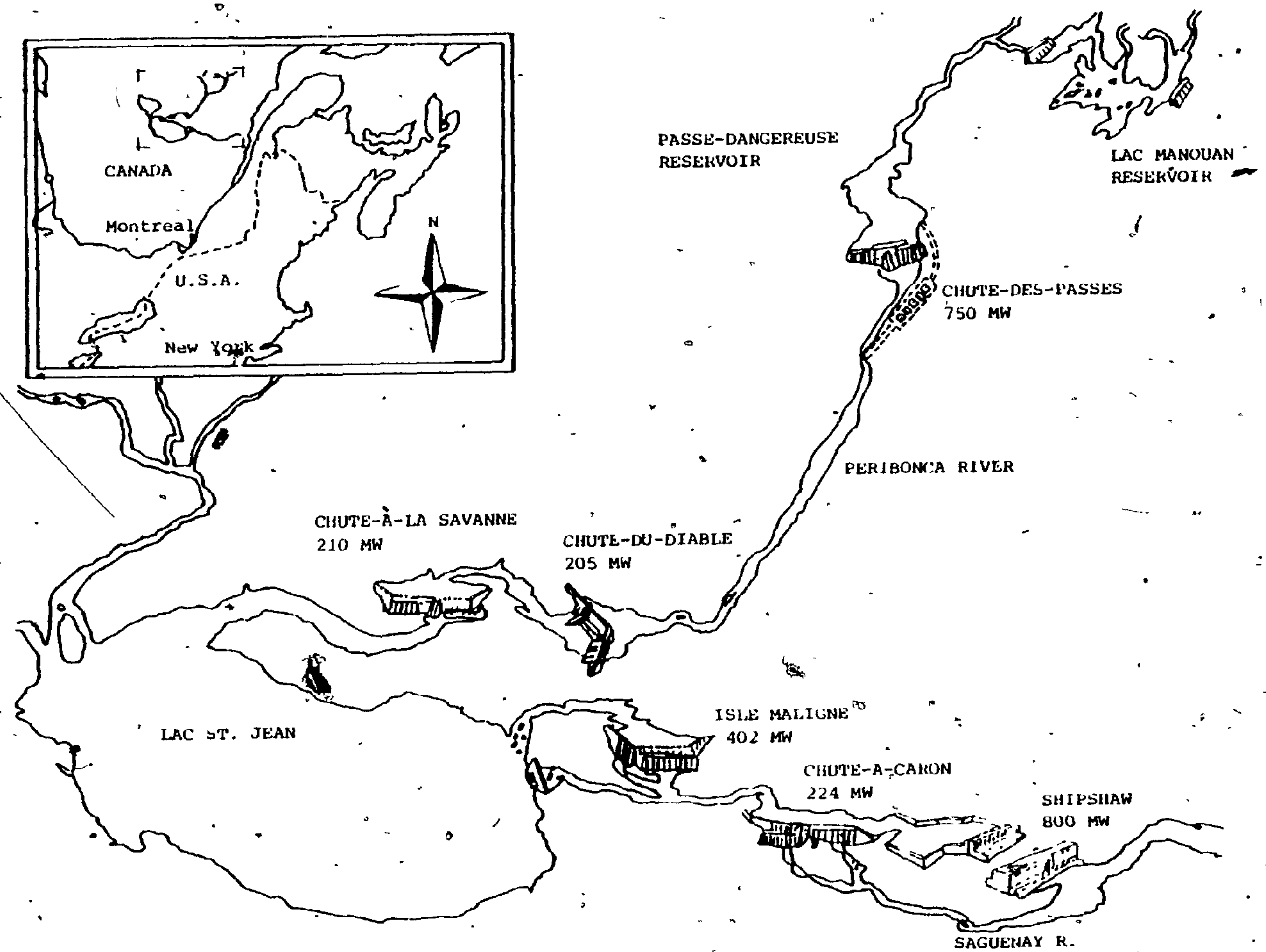

FIGURE 2.1 The Alcan Hydroelectric System [21]

furthest upstream reservoir, is water storage, as there is no powerhouse associated with it; the water is transferred directly through a canal to the Passes Dangereuse Reservoir. The water leaves this reservoir either through the Chute-des-Passes powerhouse or the spillways. It then continues down the Peribonca River to Diable, a minor reservoir. The water levels here are controlled by discharges at the Chute-du-Diable powerhouse. Further downstream, the water passes through Chute-à-la-Savanne, a run-of-the-river powerhouse, and then flows into Lac St. Jean. Water leaving Lac St. Jean enters the Saguenay River either through the Isle-Maligne powerhouse or a spillway. The final two powerhouses of the system are the parallel stations at Chute-à-Caron and at Shipshaw.

The inflows that fill these reservoirs come from a watershed that covers a total area of 73,100 square kilometers. One third of the water entering the system arrives during the spring run-off (April 1 to June 15). During this period great care must be taken in operating the system, because the inflows during this ten week period are more than the total live storage capacity of the system. Table 2.1 summarizes the watershed data for the individual reservoirs. It is the variability of these uncontrolled inflows (see Table 2.2) and their stochastic nature that make the process of obtaining a solution to the long-term optimization problem so difficult.

The power generated by the hydro system is primarily used to supply the energy required by the aluminum smelters of Alcan. Over the years, as the aluminum production facilities increased, so did the number of powerhouses. Table 2.3 gives the pertinent powerhouse data. It would seem that, if aluminum production is to be increased, one is

TABLE 2.1 Watershed Data [21]

Watershed	Area (km²)	Average inflow 1953-1977 (m^3/s)	Average annual inflow 1953-1977 ($10^9 m^3$)	Average inflow 1 April -15 June 1953-1977 ($10^9 m^3$)	Live storage capacity ($10^9 m^3$)
1. Lac Manouan (LM)	5,000	113.14	3.57	1.23	2.6
2. Passes Dangereuse (PD)	11,000	248.59	7.84	2.90	5.21
3. Diable	9,700	210.98	6.67	2.80	-
4. Chute-à-la-Savanne (CS)	1,300	35.48	1.12	0.48	-
5. Lac St-Jean	46,100	833.63	26.36	11.57	5.44
6. System (total) (SYS)	73,100	1441,82	45.56	18.98	13.25

TABLE 2.2 Variability of System Uncontrolled Inflow (1953-1977) [21]

Statistic	Annual volume ($10^9 m^3$)	Volume 1 April to 15 June ($10^9 m^3$)
Average	45.56	18.81
Minimum	36.02	14.24
Maximum	54.83	28.71
Range	18.81	14.47
Standard deviation	5.33	3.83
Coefficient of variation	0.117	0.204

TABLE 2.3 Power Station Data [21]

Powerhouse	Year completed	Average net head (meters)	Installed turbine capacity (MW)	Number of turbines
Chute-des-Passes	1960	143.3 -195.1	750.	5
Chute-du-Diable	1951	33.5	205.	5
Chute-à-la-Savanne	1953	33.5	210.	5
Isle-Maligne	1925	33.5	402.	12
Shipshaw	1943	64.0	896.	12
Chute-à-Caron	1930	48.8	224.	4
Total			2,687.	43

faced with two equally undesirable and costly options, namely building new powerhouses or purchasing power from other sources. However, the full impact of implementing these options can be lessened somewhat by fine tuning the system. This procedure can, with a relatively small initial cost, increase the system's energy generating capabilities by approximately two percent.

There are two aspects involved in fine tuning a hydroelectric reservoir system. One aspect is that of determining the physical characteristics of the various components of the system which have an influence on operating efficiencies. The other aspect involves finding a procedure for determining the optimal water release policies.

This present work is concerned with finding the best possible water release policy using a mathematical model that incorporates many of the system characteristics.

2.3 PROBLEM DISCUSSION

Hydroelectric systems have been operating efficiently for many decades without the benefit of computers or optimization programs. The operating policies in use were based on past experience and on human judgement. In order to increase the operating efficiency, more data about the system's characteristics must be included in the decision-making process. With more sophisticated data-gathering schemes, the logical approach would be to use computers to operate the system.

There are a number of preliminary and essential steps to be taken before computers can be used. The first step is to obtain a mathematical model of the system that describes the system as accurately as possible. This, however, can only be done by extensive data collection. The available data is then used to develop the required mathematical relationships, such as formulae relating reservoir volume to elevation and power generated to reservoir elevation and discharge. Expressions must also be obtained to describe the system's physical constraints. The more data that is available, the easier this process becomes.

The next step is to define the objectives of the optimization process. These objectives must then be translated into a mathematical expression. This is particularly difficult if the reservoir system has several conflicting objectives. One method of solving this problem is to obtain expressions for each objective and assign weights to the relative importance of each objective. These weights may be functions that vary according to the seasons.

In every system there are operating constraints that reflect the physical limitations of the system. There are hard constraints which cannot, or must not be violated, and soft constraints, usually reflecting desirable situations, which if violated slightly are not cause for concern. In some cases, conflicting objectives can be modelled as soft constraints.

A mathematical model containing all the system characteristics and objectives can become cumbersome and unmanageable. Breaking up the global model into a number of submodels, each reflecting the various objectives of the global model, not only simplifies the optimization procedure but also gives a deeper insight into the system's characteristics. In each of these submodels the objective function, and even some system constraints, take on different forms. The following section considers the problem of finding a comprehensive optimization strategy.

In general, the only quantities that are transferable from system to system are the optimization principles and the methods of solution, since the differences in the physical nature and desired objectives of each system result in different mathematical models.

2.4 PLANNING MODELS

2.4.1 Introduction

The overall objective of an optimization scheme is to produce as much energy as possible while satisfying minimum power requirements. As is the case in large engineering problems, the solution is best obtained by dividing the problem into subproblems. This is precisely what is done to solve the reservoir management problem.

The global optimization problem is solved using four models, each of which is concerned with a different length of planning horizon. By planning horizon we refer to the length of time for which the model is valid. There are four basic planning horizons, namely long-term, medium-term, short-term, and very-short-term. Each of these models has an objective function that reflects the nature of the subproblem. The relationship between the four models is indicated in Figure 2.2.

The long-term model has a planning horizon of one to ten years that can be used to determine the optimal amount of water to be released during each month. The medium-term model, which can

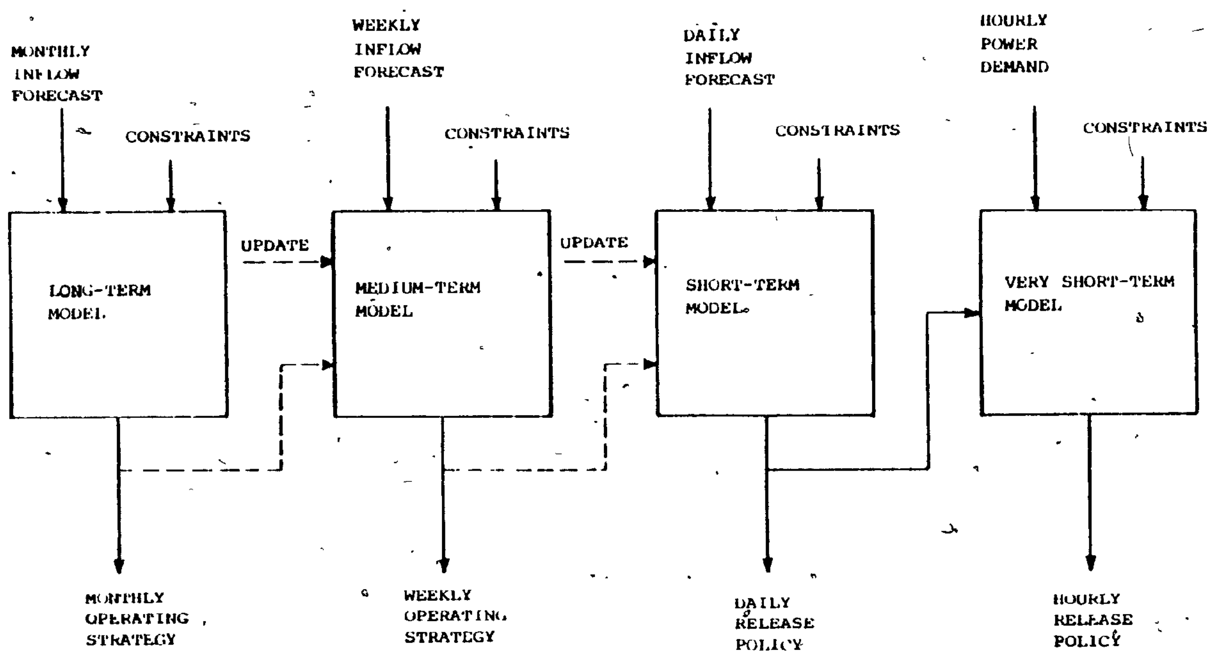

FIGURE 2.2 Reservoir System Operation Model [4]

incorporate the results of the long-term optimization, provides the framework that can be used to determine the total amount of water to be released each week over a period of a few months. Aside from the different lengths of the planning horizon and basic interval, the problem formulations are the same for the long-term and medium-term models.

A corresponding similarity exists between the short-term and very-short-term models. The short-term optimization procedure uses the results of the medium-term optimization to find the optimal amount of water to be released each day over a planning horizon of one week or so. These results are then used in conjunction with the very-short-term procedure to find the optimal hourly water release policy for each day.

The global optimization model can be considered as a sequence of planning models. The definitions of each model vary from system to system and are usually defined in accordance with the characteristics of a particular system. Although some utilities make no distinction between long-term and medium-term models or between short-term and very-short-term models, the overall hierarchy of optimization procedures remains the same.

We next consider the different planning models, each of which has its own peculiarities. Assumptions that hold for one model may not necessarily hold for another. Due to the similarities between the long-term and medium-term models, and between the short-term and very-short-term models, the focus will be on the long-term models and short-term models.

2.4.2 Long-term Models

The long-term (yearly) optimization program determines the optimal monthly water release strategy that maintains or exceeds the minimum power requirements throughout the planning horizon. These reservoir release policies are determined off-line using available historical inflow data and forecasts of future inflows. The historical inflow data is used primarily to determine the statistics of the highly stochastic inflows. The modelling of these inflows is given in [22] and [23].

The most critical time of year, in terms of controlling reservoir levels, is the spring season when the snow begins to melt. Since the stored water in a reservoir represents future generating potential, the objective during this time period is to store as much water as possible while minimizing spillage. Spilling water is occasionally necessary so as to avoid exceeding maximum elevation and discharge constraints.

The way that the water is used in the spring and summer seasons determines whether or not there is sufficient water to generate power during the 'dry' winter months. Water stored in an upstream reservoir has more value, in terms of generating potential, than a downstream reservoir. In the Alcan system, water that is stored in the Passes Dangereuse Reservoir is used to generate power at five stations whereas water from Lac St. Jean generates power at only two stations. It is therefore highly desirable to store as much water as possible in the upstream reservoirs and discharge water from downstream reservoirs so that future generating potential can be maximized.

There are a number of assumptions that can be made in the formulation of the long-term model that simplify and speed up the calculation of an optimal water release policy. The time needed for water to travel between powerhouses (transport delays) can be neglected. The actual physical characteristics of the system can also be exploited to simplify the problem formulation. For example, since there is no powerhouse between the upper two reservoirs (Lac Manouane and Passes Dangereuse), they can be considered as a single large reservoir for some seasons. Furthermore, from simulations, it was noticed that the minor reservoir, Diable, remains close to its upper elevation bound during the spring and winter seasons. Thus, some variables can be replaced by constant values, thereby simplifying calculations.

The medium-term (seasonal) model is basically a refinement of the long-term model. In general, all the assumptions made in the long-term model are applicable to the medium-term model. The length of the medium-term planning horizon is usually determined by the length of the climatological seasons. For example, there are three seasons for the Alcan system, spring (high inflows), summer (average inflows), and winter (low inflows). The reservoir levels at the start and at the end of each season can be calculated from the optimal long-term strategy. Using these initial and final reservoir level as constraints, the medium-term water release policy can be found by taking the long-term strategy as an initial solution and refining it until an optimal weekly or bi-weekly policy is found.

2.4.3 Short-term Models

The short-term models are significantly different from the long-term models. This is due primarily to that fact that some assumptions made in the long-term models are no longer valid and that a number of additional considerations must be taken into account.

The major difference between the long-term and short-term models can be found in the formulation of the objective function. The solution to the long-term optimization problem results in an operating strategy that produces sufficient energy to meet the monthly power requirements. Since in the short-term problem there is absolutely no benefit in generating excess power beyond the minimum requirements, the objective in the short-term problem is to produce the minimum required power in the most efficient way. A detailed discussion of this problem is found in Chapter 5.

Another significant change is found in the system model. In the short-term model the transport delays can no longer be neglected, thus complicating the mass-balance equation. For example, if extra water is needed at Shipshaw and if it could only be supplied by Lac Manouane, it would have to be released eight days before it is required at Shipshaw since it takes about eight days for water to travel from Lac Manouane to the Shipshaw powerhouse.

A simplification found in the short-term model arises from the fact that the inflows and power demands are more or less deterministic. Although the inflows still exhibit stochastic properties, they can be predicted fairly accurately for the entire short-term planning horizon from weather forecasts. Unlike most utilities, the power demand for the Alcan system is a deterministic quantity that is to a large extent determined by the aluminum production schedules and contractual arrangements.

The short-term operating strategy attempts to remain within the guidelines obtained from long-term or medium-term optimization policies. There are, however, situations when unforseen circumstances force a deviation in the normal operating routine. For example, consider the extreme case when the temperature drops below the freezing point for a number of days during the spring period. The subsequent reduction of the inflow rate and increase in power demand may reduce reservoir levels significantly. Another contingency occurs when a generator fails or is shut down for maintenance. In this case, an alternate generator, perhaps from another powerhouse, must compensate for this shortfall in power production. If the long-term constraints are severely violated during the implementation of a short-term policy, the long-term policy may have to be recalculated.

The very-short-term model is a refinement of the short-term model. Its objective is to regulate the power production to meet the varying hourly demand. Unlike the previous models, this policy is calculated and implemented on-line.

Along with the considerations of the short-term model, there are also constraints imposed by the generation characteristics of each machine and the operation of sluice gates to consider. The very-short-term model is completely deterministic with any unexpected short fall in power production being accommodated by maintaining sufficient spinning reserve.

2.5 STATUS OF CURRENT RESEARCH

The majority of hydroelectric systems have sophisticated methods of determining the system status and consequently have also developed mathematical models that describe their systems. Although most systems have developed an overall reservoir management philosophy, it is only recently that sophisticated mathematical techniques have been used to increase the system operating efficiency. At present, long-term and medium-term policies have been implemented by many power utilities. On the other hand, the solution of the short-term problem is still very much in the developmental stage with only very few systems using mathematical programming techniques for short-term operation.

After a brief examination of the mathematical techniques currently in use (Chapter 3), we look at a few systems that have implemented short-term optimization procedures (Chapter 4).

CHAPTER 3

MATHEMATICAL PROGRAMMING TECHNIQUES

3.1 INTRODUCTION

Each stage of power system optimization has its own unique characteristics. Consequently, certain programming techniques are better suited to solve a specific stage of the optimization problem than others. At present, linear programming (LP), dynamic programming (DP) and non-linear programming (NLP) are the main mathematical programming techniques that are in use. Numerous variations and combinations of these techniques have been developed to simplify the process of obtaining optimum solutions.

In this chapter we consider these techniques and their application to the reservoir management problem and then compare the strengths and weaknesses of each method. The chapter concludes with a brief discussion on how to deal with stochastic inflows. The Appendix gives a list of papers that describe the finer points of the various programming techniques.

3.2 DESCRIPTION OF PROGRAMMING TECHNIQUES

3.2.1 General Problem Formulation

The optimal reservoir release strategies are obtained using mathematical programming techniques. The typical mathematical programming problem in R^n is concerned with optimizing a given real valued function (known as the objective function) on a given subset S of R^n (referred to as the feasible set).

In symbols,

find $\underset{\sim}{x}^*$

such that

$$f(\underset{\sim}{x}^*) = \min_{\underset{\sim}{x} \in S} f(\underset{\sim}{x}).$$

By a slight abuse of notation we write this as

$$\min \{ f(\underset{\sim}{x}) | \underset{\sim}{x} \in S \in R^n \}.$$

Maximization problems need not be discussed separately because the problem of maximizing the objective function is equivalent to minimizing the negative of the objective function (i.e. min $\{-f(\underset{\sim}{x}) | \underset{\sim}{x} \in S\}$).

3.2.2 Linear Programming

A linear programming problem is a special case of the general programming problem. In linear programming, the objective function is a linear function and the constraints defining the feasible set are all linear equalities and/or linear inequalities. Since every real number can be expressed as the difference of two non-negative real numbers, we can transform every linear programming problem into the following standard form

$$\min \underset{\sim}{c}'\underset{\sim}{x}$$

subject to

$$\underset{\sim}{A}\,\underset{\sim}{x} \le \underset{\sim}{b}$$
$$\underset{\sim}{x} \ge \underset{\sim}{0}$$

where

$\underset{\sim}{c} \in R^n$ is a given vector

$\underset{\sim}{b} \in R^m$ is a given vector

$\underset{\sim}{A} \in R^m \times R^n$ is a given matrix

$\underset{\sim}{x} \in R^n$ is the vector of decision variables.

The solution to the LP problem is usually obtained by using the simplex method or its modifications. There are a number of good texts on linear programming that describe in detail how solutions can be obtained using this method (i.e. see [11], [18]).

3.2.3 Dynamic Programming

Dynamic programming techniques are particularly well-suited to solving problems that have multi-stage processes [6]. At each stage of such a process, a decision is taken according to some rule or policy which maximizes the objective function. This optimal policy is found by making use of the principle of optimality which says that "an optimal policy has the property that whatever the initial state and decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision" [3].

The problem of finding the best water release policy for a hydroelectric power system is of the following form:

$$\text{Max } F = \sum_{k=1}^{N} f\,(\underset{\sim}{x}_k, \underset{\sim}{u}_k) + \phi\,(\underset{\sim}{x}_{N+1})$$

subject to specific constraints

where

F = the value of the global objective function resulting from a certain policy

f_k = the value of the objective function at stage k

$\underset{\sim}{x}_k$ = the vector of state variables at stage k

$\underset{\sim}{u}_k$ = the vector of control variables at stage k

$\phi(\cdot)$ = the terminal cost

N = the number of stages

Notice that there are no restrictions placed on the nature of the function f_k: it may be either linear or nonliner. Dynamic programming can be used to solve the above problem provided that the decision at each of the N stages is dependent on only the events of that stage and all subsequent events (backward dynamic programming). The solution can then be obtained by successively solving the renewal equation

$$J_k(\underset{\sim}{x}_k) = \underset{\underset{\sim}{u}_k}{\text{Max}} \{f_k(\underset{\sim}{x}_k, \underset{\sim}{u}_k) + J_{k+1}(\underset{\sim}{x}_{k+1})\} \quad k = 1, \ldots, N$$

with the starting condition

$$J_{N+1}(\underset{\sim}{x}) = \phi(\underset{\sim}{x}).$$

To reduce the dimensionality problems, the possible decisions at each stage are discretized. The dimension of the problem is MxN, where M is the number of possible decisions at each stage and N is the number of stages. Reducing M may result in a saving of computation and storage requirements but at the same time may give an unrealistic solution.

There have been many successful attempts at modifying the classical dynamic programming formulation to reduce storage and computational requirements, the most popular of which are discrete differential dynamic programming [12] and state incremental dynamic programming [9], [15]. The basic idea in these modified forms is as follows. Instead of examining all possible solutions, an initial feasible trajectory is chosen and the dynamic programming algorithm is then applied by only considering possible decisions that lie within a corridor of the previous trajectory. This process is continued until no better trajectory can be found.

3.2.4 Nonlinear Programming

The general nonlinear programming problem can be stated as follows:

minimize $f(\underset{\sim}{x})$

subject to

$g_i(\underset{\sim}{x}) \leq 0$ for $i = 1,\ldots,m$

$h_i(\underset{\sim}{x}) = 0$ for $i = 1,\ldots,\ell$

where

$f(\cdot)$ is the objective function to be minimized,

$g_i(\cdot)$, $i = 1,\ldots,m$ are the inequality constraints

$h_i(\cdot)$, $i = 1,\ldots,\ell$ are the equality constraints.

A point $\underset{\sim}{x}^*$ is an optimal solution if $\underset{\sim}{x}^*$ satisfies the constraints and $f(\underset{\sim}{x}^*) \leq f(\underset{\sim}{x})$ for all $\underset{\sim}{x}$ in the feasible region.

There are many methods available to solve the above problem, the most popular and efficient of which are those using conjugate gradients.

There are many ways of handling constraints using conjugate gradient methods,the preferred ones being projected gradients, reduced gradients and augmented penalty functions [1].

The reduced gradient and projected gradient methods suffer from the limitation that the constraints should be linear if the methods are to be efficient. If the constraints are nonlinear, the calculation of an optimal solution becomes complicated and difficult since each point generated by the algorithm has to be checked to see if it is in the feasible region.

Unlike the projected gradient and reduced gradient methods, the augmented penalty function methods are well suited to handling nonlinear constraints. The basic idea of this method is to increase the value of the objective function if a constraint is violated. If a conjugate gradient method is used to minimize the objective function, the augmenting functions should be continuously differentiable. Mathematically, the augmented penalty function method can be expressed as

$$\text{Minimize } f(x) + \alpha\{\max\,\{0,\ g(x)\}\}^2 + \beta\ h^2(x)$$

where α and β are weights applied to the penalty which are increased with each iteration.

3.3 COMPARISON OF THE VARIOUS TECHNIQUES

The three techniques described in the previous section have all been used at one time or another to find optimal water discharge trajectories. The first methods to be used in solving reservoir management problems used linear and dynamic programming techniques [16], [17], with nonlinear programming techniques [10] only appearing in the last few years.

The main drawback to using linear programming techniques is that the objective function and constraints are required to be linear. This, however, has not proven to be too much of a deterrent as can be seen by the number of variations of linear programming techniques that have been used to optimize system operation. The majority of these variations are based on the linearization of nonlinear functions. To deal with the stochastic nature of the problem, stochastic linear programming techniques and chance-constrained linear programming techniques were developed [8], [20]. As with all linear programming based techniques, these two methods also suffer from the 'curse of dimensionality'.

Dynamic programming techniques are also limited by dimensionality considerations. The dimensionality problems, however, can be reduced to some degree by using discrete differential dynamic programming or state incremental dynamic programming. The advantage of dynamic programming over other methods is that it is much better suited to take into account the stochastic nature of the problem, provided that the inflows are not temporally correlated [20]. (The temporal correlation coefficients for a Hydro-Quebec system were found to lie between ± .28 and we can therefore assume that the inflows are temporally uncorrelated [19]). A strength of dynamic programming techniques is that they always give an optimal feedback solution. This is particularly useful when suboptimal decisions are made at any stage since a modified optimal strategy can be obtained for the subsequent stages without much recalculation.

A method frequently used to overcome the dimensionality problems associated with linear and dynamic programming techniques is the method of successive approximations [3]. In this method, the original problem is first divided into M subproblems, each corresponding to the operation of a single reservoir. Then, starting with an initial solution that does not violate any constraints, the problem is solved by optimizing one subproblem at a time while keeping the variables of the other M-1 subproblems unchanged. This process is continued, incorporating the results of the previous subproblem optimization, until all the subproblems have been optimized. Thus the original problem is reduced to solving a sequence of M lower order subproblems.

Nonlinear programming techniques can only handle deterministic problems and do not calculate an optimal feedback strategy. However, nonlinear programming techniques have a number of significant advantages over dynamic programming techniques. The main advantage is that computation and storage requirements of nonlinear programming techniques are significantly less than those required by dynamic programming techniques, thus giving nonlinear programming techniques the ability to solve larger problems. Dynamic programming techniques require that all variables be discretized, which can lead to errors. This, however, is not the case with nonlinear programming techniques since all variables are treated as continuous. With new and more powerful nonlinear programming algorithms being developed, the ability to handle larger and more complicated problems will increase the acceptability of nonlinear programming techniques as a means to solving reservoir management problems.

The three basic techniques outlined above have all been used to some extent to obtain optimal water release policies. The use of mathematical programming techniques can only be justified if their use results in a simplified and/or a more efficient operation of the hydro system. Consequently, a number of variations of the basic techniques, as well as a number of hybrid methods, have found their way into the hands of systems analysts. Over the last two decades, dynamic programming techniques have gained the widest acceptance. There are a number of reasons for this [15], the primary one being that dynamic programming techniques are able to handle the stochastic nature of the inflows more readily that other methods. Furthermore, dynamic programming techniques are also better known than nonlinear programming techniques. This is due in part to the fact that it is only recently that effective and efficient nonlinear programming algorithms capable of handling large systems have become available. Nonlinear programming techniques will continue to gain wider acceptance as their potential as a feasible alternative to dynamic programming techniques become known to systems analysts .

3.4 STOCHASTIC OPTIMIZATION

The problem of handling the stochastic nature of reservoir inflows has received considerable attention. Although many optimization schemes assume a priori knowledge of inflows, the effectiveness of a particular method is determined under realistic operation conditions. There are two methods that have been tried, with varying degrees of success, to solve this problem, namely implicit stochastic optimization (ISO) and explicit stochastic optimization (ESO) [5]. The major drawback of both procedures is that they require large amounts of computer time and storage. Furthermore, the procedures are not usually adaptive, that is, incorporating new data requires a complete recalculation. They can, however, be made to operate in an adaptive mode by using successive approximations [24], although this modification may destroy some of the desirable properties of dynamic programming if dynamic programming is used.

The implicit stochastic optimization procedure is based on the idea that it is possible to find a relationship between the system state and inflows (independent variables) and the decisions to be implemented (dependent variables). This relationship, called a decision function, can then be used to find the optimal water releases for a given set of conditions. This decision function can be obtained by first finding the optimal strategy for a number of representative inflow sequences (from data, either historical, or synthetically created), and then doing multiple variate analysis to find the function that gives the best estimate for a set of conditions. The problem with the ISO procedure is that poor estimates of the discharges may result under extreme inflow conditions and it is under precisely these conditions that good estimates are needed.

The ESO procedure incorporates the stochastic nature of the problem into the actual optimization procedure. The first step in the procedure is to use available data to estimate the inflow probabilities at each stage, and then to obtain the value of the objective function for all possible conditions. Then, using a dynamic programming algorithm, the objective function is maximized at all stages and a table of decisions is created for a specified set of conditions. Hence, given a set of conditions, an estimate of the best decision can be obtained.

Both the ISO and ESO procedures are complicated and time-consuming. On the other hand, the efficiency and speed of some nonlinear programming techniques can be used to obtain a more flexible and adaptive procedure. Using up-to-date forecasts, an estimate of a probable inflow sequence for the forthcoming year can be obtained and an optimal strategy can be determined. This strategy can be updated as often as new forecasts predict an inflow sequence that deviates from the projected inflow sequence. This method is still untried and it remains to be seen how useful it may actually be.

CHAPTER 4

IMPLEMENTATION OF OPTIMIZATION TECHNIQUES AT VARIOUS SYSTEMS

4.1 INTRODUCTION

In this chapter we examine four systems using short- and very-short-term optimization techniques and also consider, to some degree, how the mathematical techniques of the previous section are implemented. The systems under consideration were chosen because they represent a wide variety of optimization philosophies. A list of references giving a detailed description of these and other systems can be found in the Appendix.

The first system to be considered is the Shasta-Trinity portion of the Central Valley Project (CVP) in Northwestern California which has two multi-purpose reservoirs. The second system is an isolated portion of the Hydro-Quebec system on the Abitibi River. Next we look at the River Dee Project in England which uses optimization techniques primarily for flood control. Finally, we consider a subsystem of the Swedish State Power Board which is a hydro-thermal system. A brief description of each of the above systems is followed by a few comments.

4.2 THE CENTRAL VALLEY PROJECT

The overall optimization philosophy of the Central Vally Project (CVP) aligns itself closely to the one which was described in Chapter 2. Becker et. al. [2] define a short-term deterministic model that calculates optimal reservoir releases for the system. The eventual objective of the project is to achieve a practical real-time optimal operations policy.

The optimal reservoir release policy for the medium-term model is calculated using a combination of dynamic and linear programming, which using their particular method, does not require an initial feasible solution and convergence is ensured without any particular restrictions. The optimal monthly release policy provides the input data and the guidelines for the optimization of the short-term model. This deterministic model is continuously updated by data from streamflow forecasting obtained from a computer simulation that models components of the hydrological cycle. The model incorporates transport delays into the optimization procedure and the objective function to be optimized is approximated by a weighted linear function of the releases. By assuming that the daily reservoir head variations are small, the optimization routine is reduced to a number of runs using linear programming from day to day over a period of a month. The daily operations policy is subsequently selected from the set of releases that maximizes the stored potential energy.

Chu and Yeh [4] complete the description of the CVP model by considering a real-time very-short-term operations model for a single reservoir system. The objective of their model is to maximize the daily power generation subject to the hourly power schedules and the system constraints. The objective function is nonlinear and assumed to be concave while the constraints are both linear and nonlinear. To solve the problem, nonlinear programming techniques are used. The basis of the algorithm is to solve the primal and dual problems to find the saddle point from two directions using a Lagrangian procedure. A saddle point is found when the primal objective function is minimized and the dual objective function is maximized [1]. An optimal distribution policy to schedule water released through the penstocks is then implemented to complete the model.

Chu and Yeh [4] make a number of comments based on the numerical results and the computational experience obtained from using their algorithm. They experienced some computational difficulties in applying Lagrangian procedures to solve practical problems. For example, the proposed algorithm requires one set of initial solutions for the primal and another for the dual. Furthermore, their algorithm converges very slowly after a few initial rapid jumps. Chu and Yeh believe that the Langrangian procedures, in conjunction with decomposition schemes, can be used to optimize an hourly model in a multi-reservoir system provided that overall convergence can be maintained.

Some comments about the CVP model are in order at this point. The structure of the overall model greatly simplified the solution of the problems at the various stages. However, there are many questions that must be considered. In the short-term model, although conveyance lags are considered, linear programming is still used to maximize the objective function. The use of linear programming may give improved results over the previously used ad hoc procedures but whether they are optimal or near optimal is another question. The degree of optimality is dependent on accuracy of the linear approximations of the nonlinear constraints and the nonlinear objective function. The solution to the very-short-term model is a step in the right direction but much work is still required before it can be effectively used on an on-line basis. At present, the convergence rate is still too slow to be effective for on-line use, particularly if the algorithm is to be used for multi-reservoir optimization. The use of Lagrangian procedures, especially in conjunction with decomposition techniques is very cumbersome and maintaining convergence would be difficult. The use of a concave objective function may guarantee optimality but at the same time may not truly represent the system.

Although, Becker et al. [1] had originally proposed an LP-DP algorithm for the CVP very-short-term model, Chu and Yeh [4] have shown that an NLP approach is more suited to solving the problem. Although their NLP algorithm has some weaknesses, NLP seems to be the best approach to take in solving the short-term and the very-short-term problem.

4.3 THE ABITIBI RIVER SYSTEM

Drouin [7] shows how Hydro-Quebec has developed and implemented a very-short-term strategy that optimizes water releases at several powerhouses in series on the Abitibi River. This system is isolated and weakly connected to the main power grid since the power is generated at 25 hertz. Consequently, there is very little benefit in generating extra power and hence the optimal strategy is calculated off-line every week.

The deterministic mathematical model takes into consideration transport delays, reservoir levels, power generation characteristics and volume and flow constraints at each generating station. The objective function to be maximized is the daily gross energy, which is proportional to the product of the total flow times the reservoir elevation. In this model, the total flow is considered to be the sum of the actual flow through the turbines and any spillage that may occur. Incremental dynamic programming is used to determine the optimal strategy. To overcome dimensionality problems, the stations are optimized in pairs (the furthest upstream station serving as a balancing station in each case). Then, by successive approximation, the optimal solution is obtained. The model considers twenty-four one-hour periods per day, where the decision variables are the flows and the volumes are the states of the system.

The idea of using a balancing station seems to be an effective way to overcome the dimensionality problems that plague multi-reservoir systems. The drawback is that the balancing reservoir is restricted to being the furthest upstream and must also be large enough to handle all the variations arising from optimizing the operation of the reservoirs in pairs.

This model is designed to be used off-line, where the optimal reservoir release policy is normally calculated several days before implementation is effected. This is an acceptable procedure for this model only because the primary objective is to meet the forecast power requirements by releasing the minimum amount of water. There is much wastage of resources in using such a model since there is no distinction made in the objective function between spillage and water passing through the turbines. The choice of such an objective function is the major difference between this system model and the models of other systems.

4.4 RIVER DEE SYSTEM

The River Dee Research Program in England was set up to develop methods of finding an optimal water release policy for the River Dee System. The primary purpose of the program was to efficiently operate a multi-purpose reservoir sytem for water supply and flood control. A full description may be found in Jamieson and Wilkinson [13].

The approach used to solve the problem was basically the hierarchical approach described in Chapter 2 even though the system objectives are very different from those of the Alcan system. The ultimate purpose of the program is to define reservoir releases on an hour by hour basis, primarily for flood control. The result of the project was the development of a digital simulation package (DISPRIN) that calculates water releases using on-line processing of data obtained from a sophisticated telemetry scheme. DISPRIN can be operated in two modes, an exploratory mode and an operational mode. The exploratory mode is used for the sole purpose of parameter identification. In the operational mode, the program monitors the entire sytem at half-hour intervals to establish the existing state of the system, to forecast the future state of the system and finally, to decide the optimal releases from the reservoirs. The objective function varies according to whether a low, normal or high flow condition exists. The model is continually updated using half-hourly hydrograph forecasts and then dynamic programming is used to find the optimal water releases. Multi-dimensional look up tables help extend the computer's capabilities in analyzing the continuously updated information.

The main contribution of this research program is the analysis of hydrology and the development of hydrograph forecasting that accurately and frequently determines the state of the system. Although the extreme fluctuations of the River Dee System are not present in most power systems, much can be learned on how the state of the system can be accurately estimated.

4.5 THE SWEDISH STATE POWER BOARD SYSTEM

Tyren [25] describes the short-term and very-short-term optimization procedures in use at the Swedish State Power Board. The Board uses both hydraulic and thermal power to meet the electricity demands of its customers. The objective of the procedure is to minimize the cost of producing thermal power by maximizing transmission losses. The entire system consists of ten rivers, most of them having a series of stations separated by regulating reservoirs. To determine an optimal strategy, two models are used.

The first model is used to optimize the power system operation on a weekly basis. The week is divided into fourteen stages consisting of seven fourteen-hour daytime intervals and seven night-time intervals of ten hours each. In this model the transport delays are neglected. The objective is to maximize the total hydraulic power produced at all the stations during the entire week. The complexity of the system requires that the solution be obtained by solving a sequence of sub-problems. The basic algorithm used is based on linear programming and successive approximations. The algorithm proceeds as follows:

1. Find a feasible point
2. Calculate the gradient of the nonlinear benefit function at this point
3. Use linear programming to take a step in the feasible region
4. Check if the new point maximizes the benefit. If not, reduce the step size and go to step 2.
5. Use successive approximation to obtain a solution for the entire system.

The very-short-term model finds an optimal water release strategy for a twenty-four hour period with one-hour intervals. The transport delays cannot be disregarded in this model. To simplify computer programming and to reduce the number of computations, the time for water to travel between stations is approximated by an integer. Thus transport delays are accounted for by manipulating subscripts. Since the water released during the final hours of a daily period affects the economy of the following day, each twenty-four hour period is calculated as if it were only one in a series of identical periods. The results from the hourly optimization procedure are then used to select the units which are to be used to generate the required power.

The implementation of the above models has been successfully completed. In contrast to the hierarchical approach, the results obtained from the weekly model are not generally used to solve the daily problem. The primary purpose of the weekly model is to assist in making decisions regarding inter-utility power exchanges. The discharge trajectory of this model is used, however, to find an initial feasible solution for the daily problem.

The principle shortcoming of the method described above can be attributed to the use of linear programming, which severely limits the ability to calculate accurate solutions. However, this degree of inaccuracy can be tolerated since the deterministic treatment of the problem does not reflect the stochastic nature of the problem. Furthermore, the large amounts of computer resources needed makes this method unsuitable for on-line use.

4.6 SUMMARY

In this chapter we have considered some of the current procedures in use that optimize the operation of hydroelectric reservoirs. The description of the various sytems shows how mathematical programming techniques can be used in solving reservoir management problems for a wide variety of systems. The literature describing such operations is very limited. There are two possible explanations: either there is very little to report or perhaps the producers of hydroelectric power are reluctant to divulge any information. On the other hand, it has only been during the last decade that the development of long- and medium-term procedures have been producing results.

CHAPTER 5

SHORT-TERM OPTIMIZATION OF THE ALCAN HYDROELECTRIC SYSTEM

5.1 INTRODUCTION

In the previous chapters, we examined the problem formulation, the mathematical techniques available and finally, the implementation of optimization procedures at some systems. In this chapter we examine the development of a short-term mathematical optimization procedure that increases generation capacity at the Alcan hydroelectric system on the Saguenay River. The results obtained from the long-term and medium-term models are stated and the system description of chapter two is expanded.

The key to solving the short-term problem lies in the way the objective function is defined. This function uses the concept of future generating potential. The final sections of this chapter develop this concept and also show how the dimension of the model can be reduced through aggregation.

5.2 MATHEMATICAL MODEL

A brief description of the Alcan hydroelectric system was given in Chapter 2. In this section, we present a simplified mathematical model for the sytem that can be used for testing the nonlinear programming algorithms. A schematic diagram of the system is given in Figure 5.1.

The principal system model components are four reservoirs, the five power station equivalent representation of the six power stations, the transport delays between reservoirs and stations, the discharges and

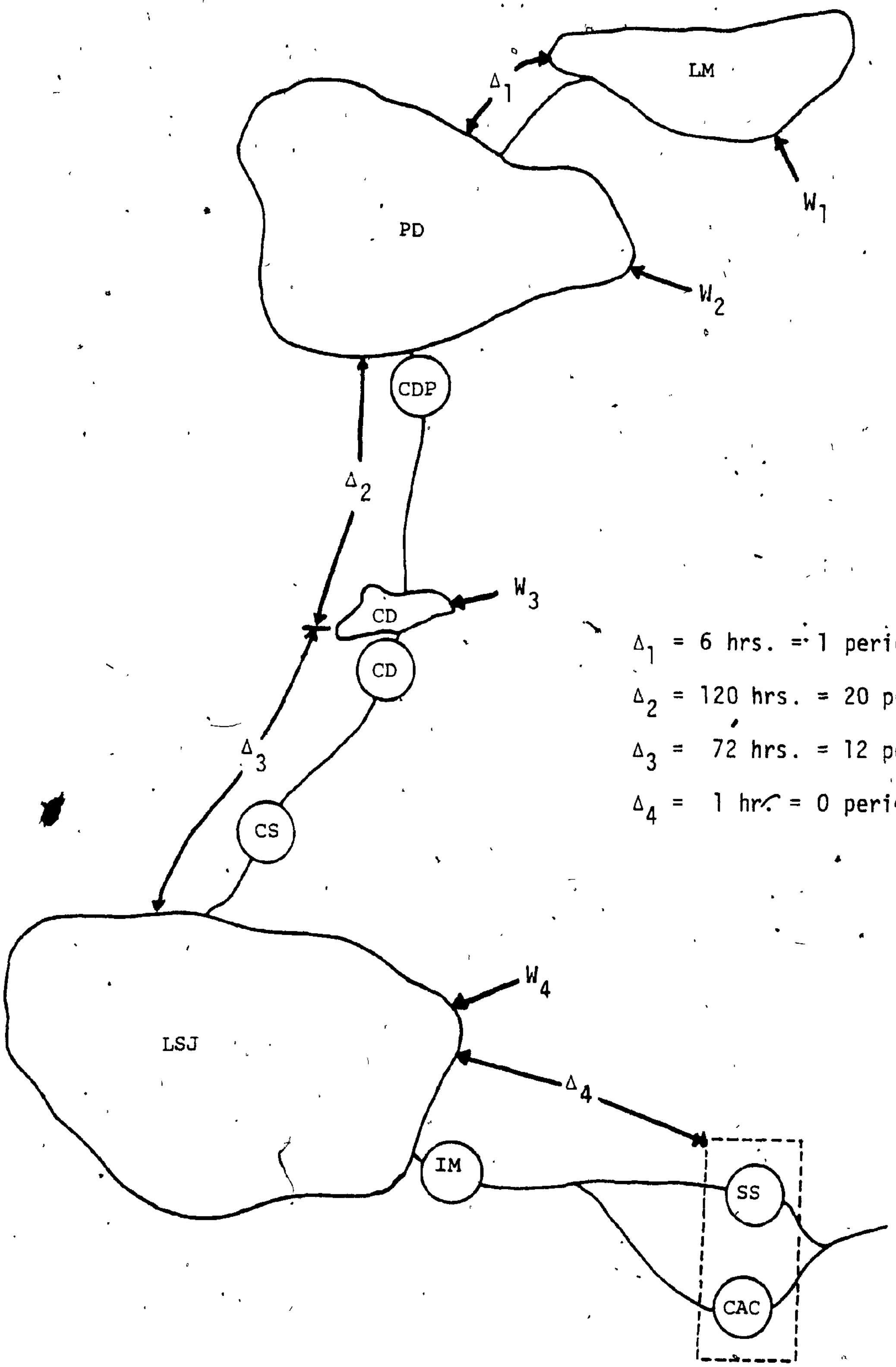

FIGURE 5.1 Schematic Diagram of the Alcan Hydroelectric System

the inflows throughout the system. The principal equations governing the system operation are the mass-balance equations which can be written as

$$V_{i,j+1} = V_{i,j} + K \cdot T \cdot (D_{i-1,j-\Delta_i} + W_{i,j} - D_{i,j})$$

$$i = 1,\ldots,4 \qquad j = 1,\ldots,N$$

where

$V_{i,j}$ = Volume of the i-th reservoir at the beginning of the j-th period.

$D_{i,j}$ = Discharge rate from the i-th reservoir during the j-th period (Note that $D_{o,j} = 0$ for all j).

$W_{i,j}$ = Rate of uncontrolled inflows into the i-th reservoir during the j-th period.

K = A physical constant.

T = Time increment in days.

Δ_i = Transport delay between the (i-1)st reservoir and the i-th reservoir, as a multiple of T.

N = Length of the planning period as a multiple of T.

The power generated at each station can be expressed as a function of the average elevation and the discharge from each reservoir. The exception is the aggregated stations at Shipshaw and Chute-à-Caron [SS/CAC] where the power generated is only a function of the discharge. The general mathematical formulation is

$$P_{i,j} = F_{i,j}\,(AE_{i,j},\ D_{i,j})$$

where

$P_{i,j}$ = Power generated at station i during period j.

$AE_{i,j} = (E_{i,j+1} + E_{i,j})/2$

= Average elevation of reservoir i during period j and E_{ij} is a nonlinear function of V_{ij}.

The physical constraints of the system are in general constant upper and lower bounds on the reservoir volumes and discharges. The exceptions are the upper discharge bound of Lac Manouane (LM) which is a nonlinear function of the reservoir's volume, and the upper discharge bound of Lac St. Jean (LSJ), which is a nonlinear function of the elevation.

The actual values of the constants used, the representative values of power generated, and reservoir volumes and elevations are not crucial to the understanding of the problem and hence are not considered. However, the transport delays between reservoirs are crucial to the development of the short-term model and are indicated in Figure 5.1. The basic period length for the short-term model is taken to be the greatest common divisor of all the delays, specifically six hours.

5.3 RESULTS OF THE LONG-TERM AND MEDIUM-TERM OPTIMIZATION

The following discussion is restricted to the results obtained from the nonlinear programming approach to the solution of the long-term and short-term problem at the Alcan system. For the most part these results are consistent with the results obtained using dynamic programming. The method of solution of the short-term problem is a modification of the long-term solution using nonlinear programming.

The objective of the long-term problem is to find the water discharge policy that maximizes the total power generated during the specified planning horizon while satisfying the system constraints. The variables are the discharges of each reservoir. Suppose a planning horizon of one year with half-monthly periods is being considered; then the dimension of the problem is 96 (24 periods x 4 reservoir discharges). Since the volume of water entering the system from inflows annually is more than three times the system storage capacity (see Table 2.1) [21]; the power generated is essentially determined by the inflows.

The inflows into the system are of a stochastic nature with a high inflow rate in the spring and minimal inflows during winter. Therefore, the medium-term problem formulation was used to analyze the system characteristics during these extreme seasons. The results obtained were useful in determining the behaviour of some of the reservoirs and these results were incorporated into the long-term model. The assumption that the transfer of water from one reservoir to another is instantaneous simplified calculations greatly.

The final result of the long-term and medium-term optimization is that target volumes for each reservoir for each week are obtained. These target volumes are then sent as input to the short-term problem.

5.4 SHORT-TERM PROBLEM FORMULATION

The short-term problem is in many respects very different from the long-term problem. About the only things that they have in common are the constraints on power generation, reservoir volumes and discharges. Even the mass balance equations are different because transport delays must be considered. There is the added complication of having to account for generator outages and individual power generation characteristics of each machine. This is offset somewhat by the fact that inflows are less random, and power generation expressions can be linearized due to small daily reservoir elevation variations.

The main focus, however, will be on finding a way to implement the objective function. The objective function for the short-term is to meet the load demand while minimizing the loss of potential energy. In other words, the objective is to maximize the potential energy at the end of the planning period.

Let us, however, first consider how the transport delays are to be implemented. As mentioned earlier, the basic time interval is a six-hour period (i.e. the weekly planning horizon is made up of six-hour periods). The length of the planning horizon suggested for the short-term problem is normally one week. However, in order for the problem formulation to be meaningful, all the water released at the furthest upstream reservoir should have worked its way through the system during the planning period. In the present case, the result is a planning horizon of 8.25 days (i.e. 33 six-hour periods). Since the last quarter day has a minimal effect on the optimization process, a planning horizon of only 8 days need be considered.

However, the short-term problem addresses itself to the problem of finding a daily water release policy, not a six-hour policy. Furthermore, the model becomes inexact, due to the transport delays, if the basic time interval is taken to be a day. This apparent conflict can be resolved by maintaining the basic six-hour interval but at the same time requiring that the discharges during each of the four six-hour intervals in any one day be equal. Not only does this proposal solve the apparent conflict, it also reduces the order of the problem. The problem as it originally was stated has 128 (32 intervals x 4 reservoirs) independent discharge variables. Implementing the proposed changes reduces the dimension to 32 independent variables, while not compromising the accuracy of the solution. The gradient expression, however, must be modified but this is not too difficult. Since the dimension of the problem has been reduced, there is a corresponding decrease in computation time.

5.5 FUTURE GENERATING POTENTIAL

5.5.1 Concept

There are many ways to define future generating potential. A factor common to all definitions is the desire to find the set of reservoir volumes that has the potential of generating the greatest amount of power.

One method of calculating this generating potential is to determine the relative value of water stored in each reservoir. As previously stated in Chapter 2, water that is stored in the furthest upstream reservoir is more valuable than water at a downstream reservoir. If expressions for these relative weights could be obtained, they could be incorporated into the model and become a factor in obtaining the optimum water release policy.

The problem with this method is that it is difficult to implement even though it is conceptually simple. The difficulty lies in the fact that the relative values of water in each reservoir is a function of the total water available which, in turn, is a function of past discharges. Furthermore, these relative weights vary significantly from season to season. For example, the water stored in Lac Manouane (LM) is roughly twice as valuable as the water stored in Passes Dangereuses (PD) during the spring season and roughly three times as valuable during the winter season. Trying to incorporate such a scheme would not only increase computation time but also introduce errors into the program since the relative values are only approximations.

An alternative method would be to calculate how much power could be generated from the final reservoir volumes and incorporate this calculation into the objective function. The beauty of this method is that, not only is it simple to implement, but it also implicitly achieves the objective of the previous proposal.

There are two important factors to note before this scheme can be implemented. The first is that the minimum power requirements must be met and secondly, there is no benefit for generating extra power during the planning period. This is in contrast to the long-term planning period in which the objective is to maximize power generation. The constraints for the short-term planning period remain the same, namely the power, volume and discharge constraints. The objective is now to find the reservoir volumes at the end of the planning period that have the greatest potential for generating power.

5.5.2 Problem Formulation

The best way to find out which set of reservoir volumes has the greatest potential is to actually calculate how much power can be produced. This is very easily accomplished by allowing the reservoirs to drain in an optimal fashion to their lower bounds without violating the discharge constraints. The power generated in this process is an accurate indication of the future generating potential for a set of reservoir volumes. The inflows during the drainage period (future planning period) are set to zero since it is impossible to drain the reservoirs with inflows present. The minimum power requirements during this period are also set to zero since it is desired that the reservoirs are drained as naturally as possible. Furthermore, the inclusion of inflows and minimum power requirements serve no useful purpose.

The question of finding a suitable length for the future planning period and the interval length was determined by repeated runs of the long-term optimization program modified for this purpose. The minimum period length was determined by the time it took to drain the reservoirs without violating discharge constraints. The minimum number of stages to accomplish this was two. No advantage was obtained by dividing the planning period any further. Hence, the future planning period was of approximately a four month duration and was divided into two intervals. Thus for every set of final reservoir volumes, the future generating potential could be calculated.

The computer program to solve the short-term problem and the one to determine the future generating potential are derived from the same code. Thus, the two algorithms are easily combined. The final product is a program that has eight one-day intervals and two two-month intervals. Individually, the objective of the eight-day period is to meet the minimum power requirements and that of the four-month period is to find the actual generating potential from the reservoir volumes obtained at the end of the eight-day period. However, the combined objective is to meet the daily minimum power requirements in such a way that future generating potential is maximized.

5.5.3 Mathematical Basis

Let us now examine the mathematical basis for the previous discussion. Consider a short-term problem with N stages and a future generation problem with M stages. The objective of the combined problem can be stated as

$$\max_{\underset{\sim}{D}_1,\ldots,\underset{\sim}{D}_N} J(\underset{\sim}{V}_{N+1})$$

subject to PC,VC, DC during period 1,...,N

where

J(·) = Future generating potential

PC = Power constraints

VC = Volume constraints

DC = Discharge constraints.

The future generating potential aspect of the problem can be expressed as

$$J(\underset{\sim}{V}_{N+1}) = \max_{\underset{\sim}{D}_{N+1},\ldots,\underset{\sim}{D}_{N+M}} \sum_{i=N+M}^{N+M} P_i \cdot t_i$$

subject to VC,DC

where

P_i = Power generated in i-th period

t_i = Length of i-th period.

Combining the two results we get:

$$\max_{\underset{\sim}{D}_1,\ldots,\underset{\sim}{D}_N} \left\{ \max_{\underset{\sim}{D}_{N+1},\ldots,\underset{\sim}{D}_{N+M}} \sum_{i=N+1}^{N+M} P_i \cdot t_i \right\}$$

subject to PC, VC, DC during 1,...,N

VC, DC during N+1,...,N+M

which can be simplified as

$$\max_{\underset{\sim}{D}_1,\ldots,\underset{\sim}{D}_{N+M}} \sum_{i=N+1}^{N+M} P_i \cdot t_i$$

subject to VC, DC during 1,..., N+M

PC during 1,...,N.

5.5.4 Reducing the Dimension of the Model

The additional calculation of the future generating potential increases the dimension of the problem by eight (2 stages x 4 reservoirs), from 32 to 40. The order of the problem can be reduced by making use of the fact that the final set of reservoir volumes are known (i.e. the lower bounds due to drainage). Since the first ($\underset{\sim}{V}_1$) and the final ($\underset{\sim}{V}_{N+M+1}$) reservoir volumes are known, we only need to calculate the discharges that determine $\underset{\sim}{V}_2,\ldots,\underset{\sim}{V}_{N+M}$ to obtain the complete volume trajectory. In other words, there are only N+M-1 independent discharge vectors. The final discharge vector can be determined from

$$\underset{\sim}{D}_{N+M} = \underset{\sim}{V}_{N+M+1} - \underset{\sim}{V}_{N+M}.$$

This final discharge vector must be calculated for each iteration if the future generating capability is to be known. With this latest modification, the order of the problem has been reduced to 4 (M+N-1) or 36 variables. As in the previous case, where time periods were aggregated, the gradient must also be modified to account for this reduction in the dimension of the system.

5.6 SUMMARY

In this chapter we have shown the development of a short-term optimization problem formulation. This formulation is based on the concept of trying to maximize future generating potential while maintaining minimum power requirements during the short-term planning horizon. Another important aspect of this chapter was showing how the dimension of the system model is reduced by aggregating variables. With the significant reduction in computer requirements, this program can be run to update discharge policies as often as new inflow data and/or new power requirements become available. Thus, in this chapter we have once again demonstrated the versatility of nonlinear programming techniques.

CHAPTER 6

CONCLUDING REMARKS

The purpose of this dissertation was to introduce some of the concepts and techniques used in optimizing the short-term operation of a hydroelectric system. This was accomplished by stating the overall optimization philosophy and describing some of the available mathematical techniques. Although, the primary focus of this dissertation was the implementation of short-term optimization procedures at the Alcan hydroelectric system, the survey of developments at four other systems illustrated the current state-of-the-art.

The hierarchical approach to solving the reservoir management problem successfully isolates the various distinctive aspects of system optimization, and thereby significantly simplifying the calculation of an optimal water release strategy. Whereas in the past dynamic programming techniques were the primary mathematical tools used in system optimization, nonlinear programming techniques are now gaining increasing popularity, which is primarily due to their efficiency and flexibility in finding optimal reservoir release strategies. The fact that nonlinear programming techniques are less sensitive to dimensionality problems than other techniques, makes nonlinear programming techniques especially suitable for solving the short- and very-short-term optimization problems.

Although the long-term and medium-term models are being used to optimize system operations, there is still much work remaining before the short-term models, and particularly the very-short-term models, are successfully used in system optimization. The use of the concept of 'future generating potential' to minimize the loss of potential energy during the planning horizon is a novel approach to solving the short-term problem. The advantage of using this approach is that, while being simple to implement, it gives an accurate estimate of the future generating potential.

Further research on improving data gathering methods and nonlinear programming algorithms will contribute to developing more efficient system operation procedures.

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APPENDIX

This appendix contains a bibliography that describes various aspects of reservoir management. They have been classified into the following categories:

LP	Linear Programming
DP	Dynamic Programming
NLP	Nonlinear Programming
LT	Long-Term Planning
MT	Medium-Term Planning
ST	Short- and Very-Short-Term Planning
SURVEY	Comparisons or a survey of some of the techniques being used.
MP	Multi-purpose Reservoirs

Table A-1 indicates under which each of the categories the listed papers fall. For some papers, the distinction between categories is hazy and therefore the most suitable grouping has been selected. The "SURVEY" category refers to survey papers as well as those papers which give comparisons of techniques and those which contain good literature reviews. Papers of a general nature have no specific categories indicated.

TABLE A-1 Classification of References Listed in Bibliography

PAPER	LP	DP	NLP	LT	MT	ST	SURVEY	MP
1						X		X
2					X			
3		X			X			
4		X		X				
5		X		X				
6				X				X
7	X	X			X	X		
8		X				X		
9		X						
10		X					X	
11			X					
12		X			X			X
13		X						
14		X			X			X
15		X		X				X
16			X			X		
17							X	X
18				X				
19		X		X				
20		X		X				
21						X		
22	X			X				
23								
24		X			X		X	
25		X				X		
26			X					
27								
28								
29		X			X			
30				X				X
31				X				X
32		X			X			
33		X			X			
34	X	X						
35			X					

TABLE A-1 Classification of References Listed in Bibliography (Cont'd)

PAPER	LP	DP	NLP	LT	MT	ST	SURVEY	MP
36.	X							
37								
38			X					
39								
40								
41		X						
42		X		X				X
43	X	X		X				X
44		X						X
45		X						X
46		X		X				X
47		X						
48		X						
49		X				X		
50	X							
51	X	X						
52								
53		X						
54		X		X	X	X		
55	X			X				X
56			X					
57		X						
58	X		X					
59	X			X				
60	X							
61	X						X	
62	X	X					X	
63	X							X
64		X						
65		X						X
66				X				
67	X			X				
68	X							
69	X			X				
70	X			X				

TABLE A-1 Classification of References Listed in Bibliography (Cont'd)

PAPER	LP	DP	NLP	LT	MT	ST	SURVEY	MP
71	x			x				
72			x					
73			x					
74			x					
75	x			x				
76	x			x				x
77								x
78	x	x		x			x	
79		x						x
80		x		x				
81		x						x
82		x						
83		x		x				x
84		x					x	x
85		x					x	
86		x					x	
87	x	x			x			
88		x		x				x
89				x				
90	x				x	x		
91								x
92	o	x		x				

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