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**Significance tests for contingency tables analysis.  
A new procedure proposed involving compound Poisson distribution.**

**Nathalie Audet**

**A Thesis  
in  
The Department  
of  
Mathematics and Statistics**

**Presented in Partial Fulfilment of the Requirements  
for the Degree of Master of Science at  
Concordia University  
Montreal, Quebec, Canada**

**June 1991**

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ISBN 0-315-68745-2

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## **Abstract**

**Significance tests for contingency tables analysis.**

**A new procedure is proposed involving compound Poisson distribution.**

Nathalie Audet, M.Sc.

The analysis of contingency tables involves study of the relationship among the cross-classified categories. Since 1900, many attempts to find the best statistical test of independence have been made. The most common tests are Karl Pearson's  $X^2$ -test and Ronald Fisher's exact test. Even though they have been widely used, they are subject of many controversies for some specific problems.

In this thesis, we bring a new test that can be used in the special case of comparative trial (1 fixed margin). For small samples, we suggest trinomial distribution whereas the compound Poisson distribution is used for larger samples. Probability tables are given in the appendix.

## **Acknowledgements**

First I would like to thank Dr. Y.H. Wang for sharing his statistical knowledge with me as a teacher and as my thesis supervisor. I thank him for suggesting the topic and providing the assistance essential to the completion of this research. I am also grateful to him for encouraging me to do graduate studies, for his precious help to get financial support and for his patience.

Thanks to the consultants of the computing center whom have been of great help and I acknowledge FCAR for the financial support.

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## INTRODUCTION

The first application over the years that could be considered as part of the statistical science is certainly the collection of data. From a start as simple as the population count, the collection of data gained popularity and eventually wakened researchers' curiosity to the relationship between attributes. In particular, categorical data turned out to be so useful for more than one branch of science that its corresponding analysis rapidly became of great importance. This thesis will emphasize on the different tests of inference for such data and their creators, with special regard to Mr Karl Pearson's original idea and its generalizations since then. We will concentrate on the analysis of  $2 \times 2$  contingency tables.

But first, we would like to introduce to you some historical aspects of statistics for our own interest and we hope the reader's as well. History of any science has never been easy to retrieve given the innumerable evolution's changes with years. If we look too far back in time, we may find similar basic notions but the general meaning differs almost completely. Results of any resemblance with statistical analysis before the 17<sup>th</sup> century were certainly biased by social restrictions like state and religious rules. It is a difficult task of retracing exact information, but one thing that authors agree on is that the word "statistics" itself has not always been used for the purpose we have today. In the mid-16<sup>th</sup> century, people associated this word with studies on statecraft and political economy. The absence of numerical

information in those earlier texts on the subject demonstrate the non-relation with contemporary statistics. This science misled the historians to proclaim the name of the Father of statistics. The real statistical applications as it is known today could be traced back to the 14<sup>th</sup> and 15<sup>th</sup> century but was not recognized as statistics yet.

Among the 14<sup>th</sup> and 15<sup>th</sup> century statistical applications, you find inventories, balance of sheets but mostly demographic studies. Collection of data for population count (according to sex, age, deaths, births etc ...) was common practice but they experienced poor elaboration of the analysis and of the interpretation of the results. At that period they were confined with collection and tabulation of data. Despite the simplicity of these surveys, important conclusions may have been brought from them such as forbidding polygamy after a sex ratio study. We must wait until the 17<sup>th</sup> century before we observe modern census, it actually progressed rapidly and these vital statistics became the foundation of a new science the Political Arithmetic. This term appeared as many attempts were made to apply some mathematical theory to real statistical problems. So the Father of statistics should be looked for in the real ancestor of statistics i.e. in Political Arithmetic. John Graunt has often been granted of this title since he enhanced this movement with his book on *Bills of Mortality* [Gregg (1973)]. The lack of scientific material restricted those Political Arithmetic researchers to calculations but on the other hand they were forced to reason more about their data which formed great interpreters.

The more convincing the interpreters, the better the chance of being respected by other humans who did not have statistical knowledge. Soon Statistics as it is known today was recognized as a part of applied mathematics with great possibilities for various types of studies. The diversity of its applications was one of the reason for the necessity to change the name Political Arithmetic to Statistics which occurred in 1768. This stealing of the name Statistics, up to then, applied to state affairs was not without some controversies. This change of name obviously created fights concurrently for the reserve of the term Political Arithmetic and of the original meaning of the word Statistics. Both of them lost their cause, the latter to the split into different branches and the former to the name "Statistics" which was eventually adopted throughout Europe. We now realize the importance of this change since the methods employed for Political Arithmetic can be usefully applied to different disciplines and not just to political issues.

The emergence of this new science rapidly captivated the interest of scientists and other human beings. By the 18<sup>th</sup> and the 19<sup>th</sup> century, statistics contributed to surveys, demography analysis, genetics as well as to actuarial and agriculture purposes. In the mid-19<sup>th</sup> century, well-known scientists such as Laplace, Gauss and DeMoivre, worked on the improvement of statistical methods to surely obtain confidence of the users among which they found as many sceptical minds as we do today. Also, in the 19<sup>th</sup> century, Florence Nightingale certainly deserves to be mentioned. She probably influenced many statisticians (such as Karl Pearson) by innovating in the collection, tabulation, interpretation, and graphical display of

descriptive social statistics.

Continuing in time leads to the most important contributor to this thesis subject; Karl Pearson. Two major statisticians at the beginning of this century were involved with contingency tables, Karl Pearson and G.U. Yule, but Pearson's methods always brought less controversies, were less criticized. Karl Pearson, the founder of the journal *Biometrika*, spent many years on biostatistics (biometry) research. Pearson concentrated, for a long period, on the analysis of categorical data.

Observations recorded in terms of prespecified categories i.e. categorical data possess ancient roots. This method figures among the oldest way of collecting data, especially with population count. Nowadays, surveys in social science and medicine mostly deal with categorical data. As the popularity of these data had a steady growth, there has been a tendency to find a way to summarize categorical data, create a good presentation. It is mostly in the last century that researchers looked upon better classification and elaboration of the categorical data analysis. Types of tables with cells containing joint occurrences of all the possible combinations of responses have been proposed. The advantages of a table over a list are that it takes less space and that it offers a better first-sight idea i.e. allows to examine how frequently the various combinations occur.

Pearson certainly noted the importance of a good classification representation before making any analysis of association. In particular, for non-quantitative attributes, he first assumed that specific ordering corresponding to a real quantitative

scale in the attributes was necessary in order to obtain a measure of degree of correlation or association. Later, he discovered that the results of the different orders came out in rather striking agreement. He came to the conclusion that the order by which we classify our attributes has no influence on the dependence of two characteristics. He named this new concept the contingency of which the attraction was that it does not demand for any scaling in the classification of attributes. This notion came out especially for categories that are not measurable quantitatively. Karl Pearson specified: "any measure of the total deviation of the classification from independent probability is a measure of its contingency". [Pearson, K. (1956)]

Related to this new terminology, Karl Pearson created the contingency table which originally meant that the order of the sub-groups is of no importance and since then, the expression has been used universally. A contingency table consists of a finite collection of mutually exclusive and exhaustive cross-classification of categories upon which the population (i.e. well-defined set of people or things) is classified. Those categories may be qualitative (nominal), quantitative or ordinal. Members of the population may respond to a characteristic like hair colour with the categories brown, red, blond, etc.. which is qualitative or may correspond to one of the five quantitative categories of age (0-18, 18-30, 30-50, ...). The third type, the ordinal, is actually a subset of the other two. It consists of attributes that involve rating, ordering (ex evaluating a subject from very bad to very good (1 to 5) or measure social class with "upper", "middle" and "lower"). Each characteristic forming the table may offer as many categories as it needs and these form the

dimension of the table.

If there is only one characteristic with three categories then we obtain a one-way table with dimension three. Whereas a two-way table consists of two ways of classification (2 characteristics), let's say row-classification has  $r$  categories and column classification has  $c$  categories then we get a  $r \times c$  table and the notation is similar to the matrices. The simplest case of two-way table is the  $2 \times 2$  contingency table which has two characteristics each of them with two categories. This dichotomous type of table owes its popularity to its connection to the failure-success process which is common in many surveys (presence or absence of a characteristic ex. male or female, smoker or non-smoker.. ). The  $2 \times 2$  type will be mostly our only concern in this study.

In summary, the contingency tables give a cross-classification of categorical data. Each cross-tabulated set of categories form cells for which entries  $n_{ij}$  are the frequencies or counts resulting of the collected data from the population or from a sample of it if the whole population is not available as it is often the case. Along with the frequency in each cell, the table also includes each row and column totals ( $n_{i\cdot}$  and  $n_{\cdot j}$ , respectively) known as the marginal totals along with the total of all frequencies  $N$ , ( $N = \sum_{i=1}^r \sum_{j=1}^c n_{ij}$ ).

Depending on the survey's method and purposes, the table may be settled in three different ways. Barnard (1947) exposed three abstract pictures (experiments) to describe his idea of the existence of three distinct sampling schemes that lead to

$2 \times 2$  contingency tables.

(1) Independent trial (2 fixed marginals)

(2) Comparative trial (1 fixed marginal)

(3) Double dichotomy (0 fixed marginal)

Each case must be considered separately when doing analysis, they impose different theoretical conditions on the test employed. Many textbooks and authors have a tendency to not separate the different cases and consequently use non-valid assumptions. As Barnard mentioned [Barnard, G.A. (1947a)], a good statistician is recognized in his (her) ability in associating a concrete case to an abstract picture while being aware of its limit of validity. Almost all tests exposed in this thesis will be given a specific application according to the different sampling scheme. Since they have been so often ignored, a description of each sampling scheme is worth some space. Barnard's types will be explained for the  $2 \times 2$  case according to the following notation:

	I	II	
A	a	c	m
B	b	d	n
	r	s	N

Independence Trial

In this case, no randomization takes place, both margins are fixed. It is the basis of Fisher's exact test that will be elaborated later and the classic concrete example is an experiment of Fisher himself. The purpose of this example often named the tea-tasting lady, is to determine whether or not a lady can tell whether the milk or the tea has been put first in the cup. The number of cups filled with milk first and the number of cups filled with tea first are fixed which satisfies the first condition i.e. the subjects coming from the two categories have the same properties but come from different processes. The second condition is the random order the cups will be randomly presented to the lady. Also, the lady's guess is regarded as equally determinate. The two margins are fixed, a test is performed to determine whether or not the two characteristics are independent. In this type of scheme, the table is completely identify by the knowledge of the entry of one of the four cells (for the  $2 \times 2$  case). The probability of a given result is therefore obtained with the hypergeometric distribution.

$$P(a|m, n, r, s) = \frac{\binom{m}{a} \binom{n}{b}}{\binom{N}{r}} = \frac{m!n!r!s!}{N!a!b!c!d!} .$$

### Comparative Trial

A margin is fixed but the second is random. The selection of the fixed number of subjects must be random and drawn from a statistically stable population. As an example, suppose a test is to be made to determine if a relation exists between the sex of a person and the smoking habit. Thus, in a population of  $N$  people, there

are exactly  $m$  women and  $n$  men and then, we collect how many of each group are smokers( $r$ ) and non-smokers( $s$ ) (which is the random characteristic). This scheme implies two independent binomial samples of size  $m$  and  $n$  with parameters  $p_1$  and  $p_2$ . Under the null hypothesis of independence ( $p_1 = p_2 = p$ ), we have:

$$P(a, b|m, n, p) = \binom{m}{a} \binom{n}{b} p^r (1-p)^{(N-r)}.$$

### Double Dichotomy

The values of both margins is random. In realistic cases, this may happen when the output of two different processes are mixed and then tested. This case is not as widely used as the previous ones, it appears more frequently in biostatistical studies. For example, it may test the independence of two characteristics of a given population of animals. The probability associated with this type is the multinomial:

$$P(a, b, c, d) = \frac{N!}{a!b!c!d!} (p_{11})^a (p_{12})^c (p_{21})^b (p_{22})^d$$

where  $p_{ij}$  = probability of the  $ij^{th}$  cell.  $\diamond$

As was brought up in the previous discussion of the sampling schemes, once we have a valid contingency table, the next step is the analysis. The purpose of the analysis is to determine if a relationship exists between characteristics and also give a measure of the association among variables, if any (only the first aspect will be considered here). The whole process starts with a significance test created

from observations with a statistical hypothesis. A test statistic formed from the observations decides if there is enough evidence as to reject the null hypothesis (assertion made about the data). This thesis concentrates on test statistic that deals with the independence assumption.

Many tests of independence and coefficients of association have been implemented since 1900, the year that Pearson's celebrity intensified with the  $X^2$  statistical test. This test was later found to be useful in comparing two proportions (or independence of two characteristics) as well as goodness-of-fit test. From this statistic, emerged many different arguments and discussions of its reliability, it has been modified to an apparently better test more than once for various reasons. Of course many tests arise from Pearson's  $X^2$ -test but we also encountered in the past 90 years, creation of many other statistical tests by different authors and some will be discussed in chapter 2.

Significance tests are unfortunately imperfect, they indicate whether attributes are independent or not but say nothing about how strong their dependence is. So, the lack of the significance tests lies in the absence of a measure of association. Consequently, along with the development of tests on independence, coefficients of associations made their appearance. Even though they existed before Pearson's  $X^2$ -statistic, most notable ones were found in the 19<sup>th</sup> and the 20<sup>th</sup> century.

At the beginning of the century, the analysis of contingency tables raised some sceptical minds. The conclusions brought from this underdeveloped method were

not fully trusted in comparison to the more accepted methods such as ANOVA analysis. But now it has much improved, for two reasons; first the advance in technology facilitate calculations and the studies need no more to be restricted to  $2 \times 2$  tables. Secondly, trustable elaborated methodology in categorical data slowly appeared which eventually classified the contingency table analysis in one of the most popular methods of data analysis of various branches of science.

With the contingency tables, the statistical science has proved, once again, that its presence in other sciences is indispensable for their evolution. Another interesting aspect concerning contingency tables lies in its recent past with increasing research and considering the tests of independence, pursuing search of the non-yet found perfect test is a challenge for the statisticians.

In chapter two, a description of the different tests that have been used since 1900 will be given. Starting with the  $X^2$ -test and its variations, particularities of each tests are exposed. Another famous test besides Pearson's statistic is the Fisher's exact test which did and still bring controversies. Other exact tests are also presented along with other tests which are not as popular as the first two mentioned.

Finally, in chapter three, we propose a new approach for the case  $m = n$  inspired by McNemar's test [McNemar (1947)] for paired dichotomous data. For the small sample, our test is an exact test in the sense of having the exact distribution for the test statistic. For the large sample, we use compound Poisson approximation

based on a result obtained in Wang (1989).

## CHAPTER TWO

### Statistical Tests

In this chapter will be displayed the main statistical procedures proposed for contingency tables, concentrating on tables of dimensions  $2 \times 2$ . Starting with the  $X^2$ -statistic and all its variations, followed by other tests of different origins. Each test will be discussed briefly in its own application. Throughout this chapter, and the others as well, we will use  $X^2$  to designate the test statistic and  $\chi^2$  to designate the chi-square distribution.

The notation will be according to the following:

	I	II	
A	a	c	m
B	b	d	n
	r	s	N

#### Chi-square test

Statistical inference exists now for many years but its emergence was certainly with the appearance of the  $X^2$ -statistic of Karl Pearson in 1900. As was mentioned before, Pearson devoted a considerable amount of his time on the contingency table

analysis. It follows that he searched for a significance test that would comment on the dependence among categories. He created a discrete test that has an approximate continuous distribution, the  $\chi^2$  distribution. The general form of the  $X^2$ -test is given by

$$(2.1) \quad X^2 = \sum_{i=1}^k \frac{(O_i - E_i)^2}{E_i}$$

where  $O_i$  = observed value of the  $i^{th}$  cell,  $E_i$  = expected value of the  $i^{th}$  cell under  $H_0$  and  $k$  = number of cells.

The statistic (2.1) has an asymptotic  $\chi^2$  distribution with specific degrees of freedom which will be elaborated shortly. First let's extend on this assertion of the  $\chi^2$  distribution according to our particular subject of matter, the contingency tables.

In a sample used for contingency tables purposes, we encounter units that belong to a single one of the  $k$  mutually exclusive and exhaustive cross-classifications of categories. Consider a  $1 \times k$  contingency table, if we let  $X_1, X_2, \dots, X_k$  be the number of observations in the  $i^{th}$  cell occurring with probability  $p$ , then the joint distribution of  $X_1, X_2, \dots, X_k$  is multinomial i.e.

$$P(X_1 = x_1, X_2 = x_2, \dots, X_k = x_k) = \frac{N!}{x_1! \cdot x_2! \cdots x_k!} p_1^{x_1} \cdot p_2^{x_2} \cdots p_k^{x_k}$$

$$\text{where } x_i \geq 0, \quad \sum_{i=1}^k x_i = N.$$

where  $N$  is the total number of observations in the sample. Since  $E(X_i) = Np_i$  and  $V(X_i) = Np_i(1 - p_i)$  then

$$Z_i = \frac{X_i - Np_i}{\sqrt{Np_i(1 - p_i)}} \sim N(0, 1)$$

$$\text{and } \sum_{i=1}^k Z_i^2 \sim \chi^2_k.$$

Then it can be shown that when  $N$  is large

$$\sum_{i=1}^k \frac{(X_i - Np_i)^2}{Np_i} \sim \chi^2_{k-1}.$$

Since one parameter is determined as the others are known (because of a linear restriction placed upon the observed cell counts),  $k - 1$  parameters are free to vary, consequently, the statistic approximate a  $\chi^2$  distribution with  $k - 1$  degrees of freedom.

For a  $r \times c$  contingency table, this expression leads to the statistic:

$$(2.2) \quad X^2 = \sum_{i=1}^r \sum_{j=1}^c \frac{(O_{ij} - E_{ij})^2}{E_{ij}} \sim \chi^2_{(r-1)(c-1)}.$$

Under the hypothesis of independence of two characteristics  $A$ ,  $B$ , the proportions of those in each category  $B_j$ , ( $j = 1, 2, \dots, c$ ) are expected to be equal with respect to one specific category  $A_i$ , ( $i = 1, 2, \dots, r$ ) i.e.

$$\frac{p_{ij}}{p_{.j}} = \frac{p_{ik}}{p_{.k}} = \frac{p_{i.}}{p_{..}} = p_i. \quad j, k = 1, 2, \dots, c \quad i = 1, 2, \dots, r.$$

This implies that in equation (2.2),  $E_{ij} = Np_{ij} = Np_{i\cdot}p_{\cdot j} = N(n_{i\cdot}/N)(n_{\cdot j}/N) = (n_{i\cdot}n_{\cdot j})/N$ ,  $n_{i\cdot}$  represents the marginal total for the  $i^{th}$  row,  $n_{\cdot j}$  represents the marginal total for the  $j^{th}$  column.

Consequently, for the  $2 \times 2$  contingency table with notation as specified at the beginning of this chapter, the  $X^2$  statistic becomes

$$X^2 = \frac{N(ad - bc)^2}{r \cdot s \cdot m \cdot n} .$$

The value of the  $X^2$ -statistic is compared to the corresponding one of the  $\chi^2$ -distribution for a decision upon the null hypothesis. Remember that this statistic approximates the  $\chi^2$  distribution. For this statistic to be *exactly* distributed as a  $\chi^2$  distribution it would require  $N$  to be infinite. This detail incited some authors to impose restrictions on the number of observations in each cell in order to use the  $X^2$ -test. Different guidelines were brought up, among them was Cochran (1954). He doubted the validity of the  $X^2$ -test if the expected number for 20% of the cells was less than 5 or if it was less than 1 for at least 1 cell. Other authors would simply require that the expected frequencies be at least 5 in each cell. If these conditions were not satisfied, they would either use another statistical test or combine categories together if similarity exists. Later in the seventies, it has been shown that small frequencies were not affecting the test enough as to ignore it. Though caution must be in order, if the  $X^2$ -test is used with small expected frequencies, you must expect a conservative test i.e.  $X^2$  will not reject the null hypothesis as often as it should, produce high probability of type II error.

Each  $X^2$ -statistic is associated with particular degrees of freedom and significant level. The investigator himself (herself) determines the significance level before the experiment. It consists of the maximum value of the probability of rejecting the null hypothesis ( $H_0$ ) when it is true that he (she) is willing to accept. A test that is not significant at the 5% level means that the difference between the observed and the expected values could not be attributed to chance i.e. a difference at least as large as that obtained could occur in the future with a probability of at least 0.05.

On the other hand, each contingency table under consideration establishes its own degrees of freedom. Under the hypothesis of independence, the degrees of freedom of the chi-square distribution which approximate  $X^2$  for contingency tables is simply the number of independent terms in equation (2.1) *given* that the row and column marginal totals are fixed. Rare is the literature on this topic, which does not diminish its importance for all that, a small modification in the degrees of freedom may alter notably the result of the test. Confusions existed when people started to use the  $X^2$  statistic, validity of the test was doubted because of unsuitable degrees of freedom. Pearson himself associated the degrees of freedom with the number of cells minus 1 i.e.  $n_{d.f.} = rc - 1$ , where  $r$  and  $c$  are the number of categories in the first and second characteristic. But this assignation soon became inappropriate in many situations. Fisher (1922) was actually the first one to indicate 1 as the degrees of freedom for the  $2 \times 2$  contingency table inspired by previous experimental results of G.U. Yule (1922) which shown some discrepancies of the  $\chi^2$ -test. If the degrees of freedom was taken as one instead of 3 then the  $\chi^2$ -test was much more

in accordance with other tests of significance. Fisher proved that the inaccuracy of the  $\chi^2$ -test was due to the incoherence of Pearson's degrees of freedom. Pearson added, a few years later, that the value of  $n_{d.f.}$  ought to be reduced as parameters are known i.e. if they can be estimated from the given data. We now attribute the degrees of freedom for each sampling scheme separately.

### (1) 2 fixed margins

For a  $r \times c$  table, the marginal totals of the row gives the values for  $r$  of the  $O_i$ , i.e. for an entire column. The  $(c - 1)$  marginal totals of the other columns fix  $(c - 1)$  other cell values such that the degrees of freedom are then the number of automatically determined cell entries subtracted from the total number of cells.

$$n_{d.f.} = rc - r - (c - 1) = (r - 1)(c - 1) .$$

### (2) 1 fixed margin

For a  $r \times c$  contingency table,  $r$  of the observed values are given by the marginal totals (given fixed marginal totals for the  $r$  rows). At this point, the degrees of freedom are  $(rc - r)$ . Under the hypothesis of independence, we test  $p_1 = p_2 = \dots = p_c = p$  which implies  $c - 1$  parameters to be estimated from the data. Consequently, we subtract  $c - 1$  from the previous degrees of freedom.

$$n_{df} = rc - r - (c - 1) = (r - 1)(c - 1) .$$

### (3) 0 fixed margin

Since no marginal totals are known,  $n_{df} = rc$ . Under the null hypothesis  $p_{ij} = p_i.p_j$   $i = 1, \dots, r$   $j = 1, \dots, c$ ,  $r$  probabilities must be estimated ( $p_i$ ) and  $(c - 1)$  probabilities for the column totals. The resulting degrees of freedom are:

$$n_{df} = rc - r - (c - 1) = (r - 1)(c - 1) .$$

It has been found that the degrees of freedom for  $r \times c$  contingency tables are  $(r - 1)(c - 1)$  under the null hypothesis and this, independently of the sampling scheme.

With the significance level and the degrees of freedom, we obtain a unique value of  $\chi^2$ -distribution eventually compared to the experimental value of  $X^2$  in order to make decision as to accept or reject the null hypothesis. The  $X^2$ -statistic sees its utility for contingency tables in two different ways. To test whether frequencies of the categories are equal for a given characteristic or, test the independence of the two characteristics. These two situations are mathematically equivalent. In the case of  $2 \times 2$  contingency tables,  $X^2$ -statistic tests either  $H_0 : p_{11} = p_{21}$  for independence or,  $H_0 : p_{.1} = p_{.2}$  where  $p_{.1} = r/N$  and  $p_{.2} = s/N$  for homogeneity. If  $X_k^2 \leq \chi_k^2$  then accept  $H_0$  otherwise reject  $H_0$

Pearson's test received plenty of comments upon its validity (or its infallibility) according to various factors. Since its appearance in the inference world, authors criticized and brought many alternatives. In the following, we shall display some extensions of Pearson's original  $X^2$ -test.

## EXTENSIONS OF THE $\chi^2$ -TEST

(1) Yates (1934)

Yates continuity correction

Since the discrete  $X^2$ -statistic is exactly distributed as a  $\chi^2$  distribution when  $N$  is infinite, the  $\chi^2$  approximation might break down for small samples. In that case, the test would be more conservative than desired, then, a correction is in order. Yates performed simple experiments to compare the probabilities given by the  $\chi^2$ -test and the exact probabilities obtained using a discrete distribution. He actually used Fisher's exact test (hypergeometric distribution) when dealing with contingency tables. The discrepancies were found to be quite large except at the tails. Yates suggested "If we group the  $\chi$  distribution, taking the half units of derivations from expectation as group boundaries, we may expect to obtain a much closer approximation to the distribution." He claimed that the approximation would be better if .5 was added to the terms  $(O_i - E_i)$  that are negative and subtract .5 if they are positive, and this correction applies before the values  $(O_i - E_i)$  are squared. According to the  $2 \times 2$  contingency table notation, the corrected statistic is as follows:

$$X^2_{c1} = \frac{N(|ad - bc| - N/2)^2}{m \cdot n \cdot r \cdot s}$$

This test has not been widely used since it has been shown to be useless in the comparative trial and double dichotomy cases. Plackett (1964) concluded that this continuity correction is inappropriate for  $2 \times 2$  comparative trial. Grizzle (1967)

cited that the  $X^2$ -statistic produces a conservative test and that the Yates correction produces a test that is so conservative as to be almost useless in both 2 x 2 comparative trial and double dichotomy. Yates himself admits that when the binomial distribution is used as the exact distribution, the correction does not improve the approximation especially with unsymmetrical cases.

**(2) Cook (1981)**

Cook's correction

Cook (1981) looked at the comparative trial case where  $m \neq n$  and suggested the following:

$$X^2_{c2} = \frac{N(|ad - bc| - h/2)^2}{m \cdot n \cdot r \cdot s}$$

where  $h$  = highest common factor of the two fixed marginals ( $m, n$ ).

**(3) Pirie and Hamdan (1972)**

Pirie and Hamdan correction

The following was based on an idea from Cox (1970) who showed that the best approximation of a discrete distribution with random variable  $U$  with a continuous distribution is by the following

$$P_D(-r < U < r) = P_D(-r + b' \leq U \leq r - b') \approx P_N(-r + \frac{1}{2}b' \leq U \leq r - \frac{1}{2}b')$$

where  $P_D$  : probability under the discrete distribution,  $P_N$  : probability under the normal distribution and  $U$  assumes equally spaced values  $a + ib'$  where  $i$  is integer-valued.

For the case of  $m = n = \frac{1}{2}N$  in comparative trial,  $(ad - bc)$  takes values which change in steps of  $b' = \frac{1}{2}N$  so, Pirie and Hamdan (1972) suggested the test

$$X_{c3a}^2 = \frac{N(|ad - bc| - N/4)^2}{m \cdot n \cdot r \cdot s} .$$

For the double dichotomy, they suggested

$$X_{c3b}^2 = \frac{N(|ad - bc| - .5)^2}{m \cdot n \cdot r \cdot s} .$$

Here the argument of Pirie and Hamdan starts with the multinomial distribution. Let  $p_{ij}$  ( $i, j = 1, 2$ ) be the probabilities that an observation falls in the  $ij^{th}$  cell. The hypothesis of independence is  $H_0 : p_{11}p_{22} = p_{12}p_{21}$  against the alternative hypothesis  $H_1 : p_{11}p_{22} \neq p_{12}p_{21}$ . Define  $U = n_{11}n_{22} - n_{12}n_{21}$  where  $n_{ij}$  ( $i, j = 1, 2$ ) be the frequencies of each  $ij^{th}$  cell. The random variables  $n_{11}, n_{12}, n_{21}, n_{22}$  have a multinomial distribution with parameters  $p_{11}, p_{12}, p_{21}, p_{22}$ . After some manipulations, we obtain, under the null hypothesis,  $E(U) = 0$  and  $V(U) = n^2(n-1)p_{11}p_{22}$ . Using the maximum likelihood estimators under  $H_0$ , we have  $\hat{\sigma}^2 = (n-1)n_{.1}n_{1.}n_{2.}n_{.2}/n^2$ . Following Cox's method,  $U$  has values spaced by unity ( $b' = 1$ ) so the approximate continuity correction gives the statistic  $[U - 1/2]/\hat{\sigma}$ . In the chi-square form for large samples along with our notation, we get  $X_{c3b}^2$  as defined above.

#### (4) Rao (1973)

#### Dandekar correction

This different idea for correcting  $X^2$  was first suggested by D.V. Dandekar but was exposed in one of Rao's publication. Rao describes this new  $X^2$  correction for

fixed marginals that has been proved to be equally good as the Yates' correction. This statistic involves three  $X^2$  values,  $X^2_{-1}$ ,  $X^2_0$  and  $X^2_1$  where  $X^2_0$  is the Pearson  $X^2$ -statistic for the observed configuration whereas  $X^2_{-1}$  and  $X^2_1$  are Pearson  $X^2$ -statistics obtained respectively by reducing and increasing the smallest frequency in the table by unity without altering the marginal totals. The corrected  $X^2$  becomes

$$X^2_{c4} = X^2_0 - \frac{X^2_0 - X^2_{-1}}{X^2_1 - X^2_{-1}} \cdot (X^2_1 - X^2_0) \sim \chi^2_{(r-1)(c-1)} .$$

This statistic, as the previous ones, follows the  $\chi^2$ -distribution with specified degrees of freedom. Rao affirms that the Dandekar's statistic is slightly better than the Yates'  $X^2_c$ , but its more complex calculations makes it less popular.

#### (5) Schouten and al. (1980)

#### Schouten and al. correction

This correction may be used with the  $2 \times 2$  comparative trial only and its goal is to be less conservative than the  $X^2$ -test and consequently more powerful. He defines the random variable  $V = ad - bc$  and let  $v + h$  be the smallest attainable value of  $V$  greater than  $v$ . If  $V'$  is normally distributed with same mean and variance as  $V$  then

$$P(V \leq v) \simeq P(V' \leq v + \frac{1}{2}h).$$

Since  $GCD(n, m) \leq h \leq \min(m, n) \leq \frac{1}{2}N$ , the corrected test statistic is as follows:

$$X^2_{c5} = \frac{N(|ad - bc| - \frac{1}{2} \min(m, n))^2}{m \cdot n \cdot r \cdot s} .$$

GCD stands for the greatest common divisor of  $n$  and  $m$  which is used in the Cook's correction.

**Upton (1982)**

### Adjusted chi-square

The  $X^2$ -test is equivalent to the following  $U$ -test using the normal distribution.

$$U = \frac{(a - rm/N)}{\sqrt{mnrs/N^3}} \sim N(0, 1) \quad U^2 = X^2 .$$

This statistic implies that the variance of  $(a - rm/N)$  is  $mnrs/N^3$ . Using the unbiased variance  $mnrs/N^2(N - 1)$ , we would get the following adjusted test statistic.

$$X^2_a = \frac{(N - 1)(ad - bc)^2}{r \cdot m \cdot n \cdot s} .$$

The difference between  $X^2_a$  and  $X^2$  will obviously decrease as  $N$  gets large.

**Hoel (1962)**

### Likelihood ratio test

This statistic compares the likelihood function under the null hypothesis to the one under the alternative by taking the ratio of the former to the latter.

$$\lambda = \frac{L(p)_{p \in \omega}}{L(p)_{p \in \Omega}}$$

Say a table of  $r \times c$  cells each having probability  $p_{ij}$  and frequency  $n_{ij}$  for  $i = 1, \dots, r$  and  $j = 1, \dots, c$  with  $\sum_{i=1}^r \sum_{j=1}^c p_{ij} = 1$  and  $\sum_{i=1}^r \sum_{j=1}^c n_{ij} = n$  then we have the following likelihood function:

$$L(p) = p_{11}^{n_{11}} \cdots p_{1c}^{n_{1c}} p_{21}^{n_{21}} \cdots p_{2c}^{n_{2c}} \cdots p_{r1}^{n_{r1}} \cdots p_{rc}^{n_{rc}}$$

Now let's form the likelihood ratio, under the null hypothesis (independence assumption):  $p_{ij} = p_{i\cdot}p_{\cdot j}$  and under  $H_1 : p_{ij} = n_{ij}/N$ , we obtain:

$$\lambda = \frac{\prod_{i=1}^r \prod_{j=1}^c (p_{i\cdot}p_{\cdot j})^{n_{ij}}}{\prod_{i=1}^r \prod_{j=1}^c (\frac{n_{ij}}{N})^{n_{ij}}}.$$

Taking the logarithm function of  $\lambda$ :

$$\ln \lambda = \sum_{i=1}^r \sum_{j=1}^c n_{ij} \ln \left( \frac{N p_{i\cdot} p_{\cdot j}}{n_{ij}} \right).$$

Since  $E_{ij} = N p_{i\cdot} p_{\cdot j}$  and  $O_{ij} = n_{ij}$ , we have:

$$-2 \ln \lambda = -2 \sum_{i=1}^r \sum_{j=1}^c O_{ij} \ln \left( \frac{E_{ij}}{O_{ij}} \right).$$

And this last statistic has an asymptotic chi-square distribution, the proof follows. Let  $x_{ij} = O_{ij} - E_{ij}$ , which is the difference between the observed and the expected frequency in the  $ij^{\text{th}}$  cell.

$$-2 \ln \lambda = 2 \sum_{i=1}^r \sum_{j=1}^c (x_{ij} + E_{ij}) \ln \left( \frac{x_{ij} + E_{ij}}{E_{ij}} \right)$$

And by the Taylor's expansion of the logarithmic function, we obtain:

$$-2 \ln \lambda = 2 \sum_{i=1}^r \sum_{j=1}^c (x_{ij} + E_{ij}) \left[ \frac{x_{ij}}{E_{ij}} - \frac{1}{2} \left( \frac{x_{ij}}{E_{ij}} \right)^2 + \frac{1}{3} \left( \frac{x_{ij}}{E_{ij}} \right)^3 \dots \right]$$

$$-2 \ln \lambda = \sum_{i=1}^r \sum_{j=1}^c \frac{x_{ij}^2}{E_{ij}} - \frac{1}{3} \sum_{i=1}^r \sum_{j=1}^c \frac{x_{ij}^3}{E_{ij}^2} + \dots$$

thus,

$$-2 \ln \lambda \approx \sum_{i=1}^r \sum_{j=1}^c \frac{x_{ij}^2}{E_{ij}} = \sum_{i=1}^r \sum_{j=1}^c \frac{(O_{ij} - E_{ij})^2}{E_{ij}}$$

which implies that:

$$Y^2 = -2 \ln \lambda \approx X^2 \sim \chi^2_{(r-1)(c-1)}$$

If the data are compatible with  $H_0$  then  $Y^2$  tends to be small. Again with the restriction of a large sample,  $Y^2$  has an approximate  $\chi^2$ -distribution. Its value does not in general vary much from the usual  $X^2$ -statistic at the tails of the distribution or when  $N$  is large.

### Improved likelihood ratio test

Williams(1976) shows that if a scale factor  $q$  is multiplied to the likelihood ratio test statistic  $L = -2 \log \lambda$  (where  $\lambda$  is the ratio of maximized likelihoods under the null and alternative hypotheses) it improves its approximation to the  $\chi^2$  distribution. The  $q$ -value is chosen such that the null distribution of the resulting statistic has the same moments as  $\chi^2_{v_{d.f.}}$ . If, under the null hypothesis,  $E(L) = \nu q$  where  $\nu$  is the number of degrees of freedom associated with  $L$ , then the improved likelihood ratio test is performed by comparing  $q^{-1}L$  with  $\chi^2$  distribution with  $\nu$  degrees of freedom. William gives formulas to determine a numerical value for  $q$  using the maximum likelihood estimators of expected cell frequencies and log linear models. For contingency tables with  $\nu = (r - 1)(c - 1)$ , we get:

$$q = 1 + \frac{(N \sum_{i=1}^r E_{i.}^{-1} - 1)(N \sum_{j=1}^c E_{.j}^{-1} - 1)}{6N\nu}$$

If the expected values are estimated by the following

$$\hat{E}_{i\cdot} = \frac{N}{r} \text{ and } \hat{E}_{\cdot j} = \frac{N}{c} .$$

Then we obtain

$$q = 1 + \frac{(r^2 - 1)(c^2 - 1)}{6N\nu} .$$

William then confirms that the modified statistic (with the presence of the factor  $q$ ) is better approximated by its asymptotic  $\chi^2$  distribution.

Agresti (1984)

### Partitioning $X^2$ -test

A variation of the  $X^2$ -test is found in its partition, obviously this test applies to tables of dimensions higher than  $2 \times 2$ . Since the sum of  $k$  independent  $\chi^2_1$  variable is distributed as a  $\chi^2_k$  we can always partition a  $X^2$ -statistic into other  $X^2$ -statistics which components may present a different type of association. Consider a  $2 \times 2$  contingency table where we want to test whether the two drugs A, B give different intensity of side effect. The second characteristic consists of two categories; no side effect, and some effect. The Pearson statistic in this case, with one degree of freedom, can be decomposed into sums of chi-square statistics if the second characteristic is later divided into three categories such as no side effect, moderate side effects and important side effects. It might be of interest to test for an evidence or not of the difference in side effects by the two drugs for these three categories using a  $X^2$ -statistic with two degrees of freedom or even test the side effect categories two by two with a  $X^2$ -statistic with one degree of freedom. "Such

a partitioning may show that the association in a table primarily reflects differences between certain rows or groupings of rows." (Agresti(1984)).

## **EXACT TESTS**

"The term exact refers to the use of an exact discrete distribution in computing the significance level associated with a test procedure; it does not refer to the size of the procedure being exactly as specified". [Storer and Kim (1990)]

### **(1) Fisher's exact test**

This test also called the conditional exact test is based on discrete probabilities, no approximations are involved as with the  $X^2$ -test. The exact probability distribution of the observed frequencies is used. This test consists of calculating the probabilities of the more extreme sets of frequencies while keeping marginals fixed and, depending on the comparison of their sum and the significance level chosen, the null hypothesis is accepted or rejected. Let's describe the test for a  $2 \times 2$  table. Assume 'a' is the smallest cell frequency of the table. The probability of obtaining  $a, b, c, d$  with fixed margins is simply the probability of obtaining 'a' given  $m, n, r, s$ , (since the values of  $b, c, d$  will be automatically known as the value of 'a' and of the margins are defined). And this probability follows the hypergeometric distribution,

$$P(a|m, n, r, s) = \frac{\binom{m}{a} \binom{n}{b}}{\binom{N}{r}} = \frac{m!n!r!s!}{a!b!c!d!N!}$$

We add to this value the probabilities of the more extreme cases, that is  $P(x|m, n, r, s)$  for  $o \leq x < a$ , to obtain the  $P$ -value. If the  $P$ -value is small then it is unlikely that this result would have occurred by chance and we reject the null hypothesis of independence. This decision is often made regarding to a predetermined significance level  $\alpha$ , so in other words, a  $P$ -value greater than  $\alpha$  indicates no evidence of association between variables.

Whereas the  $X^2$ -test involves two-ways analysis i.e.  $H_0 : p_{1.} = p_{2.}$  VS  $H_1 : p_{1.} \neq p_{2.}$ , the Fisher's exact test gives the following  $H_0 : p_{1.} = p_{2.}$  against  $H_1 : p_{1.} \leq p_{2.}$ . To get equivalence of a two-tail test, Fisher's  $P$ -value must be doubled, then comparison with the  $X^2$ -test may be performed. This test has been created especially as an alternative for the  $X^2$ -test when cell frequencies are so small that the  $\chi^2$  approximation is no longer reliable. And because marginals are kept fixed while calculating the probability value, people tend to apply this test with the  $2 \times 2$  independence trial (two fixed margins) only. It has even though been selected as the most adequate test in different cases. The lack of a better test incited people to wrongly use Fisher's exact test for any type of sampling scheme which implies a more conservative test than desired.

**(2a) Suissa and Shuster (1985)**      Exact unconditional test

In the following case, an alternative test to Fisher's is presented for the  $2 \times 2$  comparative trial i.e. a test that involves the comparison of two binomial samples. This test is intended for small and moderate samples sizes. The effect of the nuisance

parameter  $p$  is minimized by maximizing the size of the test over the domain of  $p$ .

$$T_1 = \sup_{0 \leq p \leq 1} \sum_{x=0}^m \sum_{y=0}^n \binom{m}{x} p^m (1-p)^{m-x} \binom{n}{y} p^n (1-p)^{n-y} \bullet$$

$$I[|Z(x, y, m, n)| \geq |Z(a, b, m, n)|]$$

Where  $I[\cdot]$  is the indicator function and

$$Z(x, y, m, n) = (x/m - y/n) / \left[ \frac{x/m(1-x/m)}{m} + \frac{y/n(1-y/n)}{n} \right]^{\frac{1}{2}} \bullet$$

Reject  $H_0 : p_1 = p_2$  if  $T_1 \leq \alpha$  at the  $100(1 - \alpha)\%$  significance level. .

**(2b) Storer and Kim (1990)**      Approximate unconditional test

Storer and Kim modified the last statistic by first taking the maximum likelihood estimate under  $H_0$  for the p-value and secondly, they used the pooled variance in the indicator function. They called it the approximate unconditional test.

$$T_2 = \sum_{x=0}^m \sum_{y=0}^n \binom{m}{x} \hat{p}^x (1-\hat{p})^{m-x} \binom{n}{y} \hat{p}^y (1-\hat{p})^{n-y} \bullet$$

$$I[|Z(x, y, m, n)| \geq |Z(a, b, m, n)|]$$

$$\hat{p} = \frac{a+b}{m+n} \quad Z(x, y, m, n) = \left( \frac{x}{m} - \frac{y}{n} \right) / \left[ \left( \frac{1}{m} + \frac{1}{n} \right) \hat{p} (1 - \hat{p}) \right]^{\frac{1}{2}}$$

Reject  $H_0 : p_1 = p_2$  if  $T_2 \leq \alpha$ .

**(3) Liddell (1976)**

For the comparison of two binomial samples, Liddell developed an exact test using the maximum likelihood estimate for p. His research especially followed the unacceptance of the conditional exact test, with or without randomization, in his own field, epidemiology which mostly deals with cases of one fixed margin. Since his type of research often deal with low frequencies because of medical ethics and great cost of such trials, a test involving continuous approximation would also be inadequate. He based his test on the following probability:

$$P(a, b|m, n, p) = H(a|r, m, n) \cdot B(r|N, p) = B(a, m, p) \cdot B(b, n, p)$$

Where H represents the hypergeometric distribution and B represents the binomial distribution. The procedure is to calculate, for every possible pair of values  $x, y$  the probability  $P(x, y|m, n, \hat{p})$  where  $\hat{p} = \frac{r}{N}$ . This together with the value of  $U = (\frac{x}{m} - \frac{y}{n})$  so to obtain the cumulative probability of U. In other words the test statistic becomes:

$$T_l = \sum_{x=0}^m \sum_{y=0}^n B(x, m, \hat{p}) \cdot B(y, n, \hat{p}) \cdot I[|(\frac{x}{m} - \frac{y}{n})| \geq |(\frac{a}{m} - \frac{b}{n})|]$$

Reject  $H_0 : p_1 = p_2$  if  $T_l \leq \alpha$  at the  $100(1 - \alpha)\%$  significance level.

D'Agostino (1972,1988)

Pooled t-test

This test comes from the similarity of the  $X^2$ -test and the F-test of ANOVA analysis. The only difference lies in the variance but asymptotically, they are equivalent. Again for the case of 2 x 2 contingency table, the  $X^2$ -test may be written

this form:

$$X^2 = \frac{(a/m - b/n)^2}{s/N(1 - s/N)(1/m + 1/n)}$$

Proof: The last equation may be written as

$$\left( \frac{an - bn}{mn} \right)^2 / \left( \frac{r(N-r)}{N^2} \cdot \frac{(m+n)}{mn} \right).$$

Since  $m + n = N$  and  $N - r = s$ , we obtain

$$(a(b+d) - b(a+c))^2 / \frac{(r \cdot s \cdot m \cdot n)}{N}$$

after simplification, we get equation (2.1)

$$\frac{N(ad - bc)^2}{r \cdot s \cdot m \cdot n} . \diamond$$

The F-test would have a slightly different denominator (pooled variance)

$$F = \frac{(a/m - b/n)^2}{\left( \frac{N}{N-k} \right) \cdot s/N(1 - s/N)(1/m + 1/n)}$$

where  $k$  relates to the number of independent dichotomous populations related to ANOVA analysis ( $k = 2$  for  $2 \times 2$  contingency tables).

Since  $t = \sqrt{F}$  then the t-statistic becomes,

$$t = \left[ \frac{N-2}{N} \right]^{\frac{1}{2}} \cdot \frac{N(ad - bc)}{[m \cdot n \cdot r \cdot s]^{\frac{1}{2}}} .$$

And this value is compared with the t-value of the Student's distribution table given a specific significance level.

Cox(1953) suggested a test based on Poisson distribution. If  $X \sim \text{Poisson}(\lambda)$ , then for some unit of time:

$$P(X \geq x) = P(\chi^2_{2x} < 2\lambda)$$

$$P(X \geq x + 1) = P(\chi^2_{2x+2} < 2\lambda)$$

From these statements,  $2\lambda$  may be treated as a chi-squared with  $2x + 1$  degrees of freedom. Taking the ratio of 2 parameters  $\lambda_1$  and  $\lambda_2$  from 2 independent observations  $x_1$  and  $x_2$  along with the degrees of freedom we obtain the following statistic

$$\left[ \frac{2\lambda_1}{2x_1 + 1} \right] / \left[ \frac{2\lambda_2}{2x_2 + 1} \right] \sim F(2x_1 + 1, 2x_2 + 2)$$

Cox found this statistic quite accurate even with small values of  $\lambda$ .

Gart was inspired by Cox's method to develop a statistic that would be more appropriate with contingency tables i.e. using binomial and multinomial distributions. In an analogous way, he applied the Beta approximations in the binomial and multinomial as the chi-square was applied to the Poisson in Cox method. For the multinomial case, the vector of probabilities  $(p_1, p_2, \dots, p_k)$  may be treated as a multivariate beta which itself can be derived as a function of independent chi-square random variables. In the  $2 \times 2$  cases, we can use the ratio of two chi-square variables to obtain a variable with an F distribution. Thus, after some manipulations, in the

double dichotomy case, using the Z-transformation for the logarithm of an F-variate leads to the test statistic:

$$X^2 = \frac{[\ln((a + \frac{1}{2})/(b + \frac{1}{2})) - \ln((d + \frac{1}{2})/(c + \frac{1}{2}))]^2}{(a + \frac{1}{2})^{-1} + (b + \frac{1}{2})^{-1} + (c + \frac{1}{2})^{-1} + (d + \frac{1}{2})^{-1}} \sim \chi^2_1$$

This test applies to tables with large cell values.

### Goodman's (1964)

### Goodman's test

If  $a$  and  $b$  denote the numbers of successes observed in two independent sets of  $m$  and  $n$  Bernoulli trials, respectively. Let  $p_1$  and  $p_2$  denote the true success probabilities associated with each set of trials we then have the following.

$$\frac{\left(\frac{a}{m} - \frac{b}{n}\right) - E\left(\frac{a}{m} - \frac{b}{n}\right)}{\sqrt{Var\left(\frac{a}{m} - \frac{b}{n}\right)}} \sim N(0, 1)$$

Under the null hypothesis  $H_0 : p_1 = p_2$  we have:

$$E\left(\frac{a}{m} - \frac{b}{n}\right) = 0 \quad V\left(\frac{a}{m} - \frac{b}{n}\right) = \frac{p_1(1-p_1)}{m} + \frac{p_2(1-p_2)}{n}$$

Goodman uses the following estimates for the probabilities included in the variance  $p_1 = a/m$  and  $p_2 = b/n$ . Standardization gives,

$$Z = \left(\frac{a}{m} - \frac{b}{n}\right) \Bigg/ \sqrt{\frac{ac}{m^3} + \frac{bd}{n^3}} \sim N(0, 1).$$

Goodman suggested that the square of this statistic can be compared to the values of the table of the chi-square distribution with one degree of freedom to test the independence  $p_1 = p_2$ .

### Normal approximation

This test is similar to Goodman's except that it uses a common estimate for the probabilities.

The null hypothesis is  $H_0 : p_1 = p_2$  and the two probabilities are  $\bar{p}_1 = a/m$   $\bar{p}_2 = b/n$ . By normal approximation

$$\bar{p}_1 - \bar{p}_2 \sim N\left(p_1 - p_2, \frac{p_1(1-p_1)}{m} + \frac{p_2(1-p_2)}{n}\right)$$

Under the null hypothesis  $H_0 : p_1 = p_2 = \bar{p}$ , the test statistic becomes

$$Z = \frac{\bar{p}_1 - \bar{p}_2}{\sqrt{\bar{p}(1-\bar{p})(1/m + 1/n)}} \sim N(0, 1) \quad \bar{p} = \frac{a+b}{m+n}$$

Notice that  $Z^2 = X^2$ .

**Kullback (1962)**

M.D.I.S.

This test is based on a minimum discrimination information statistic and its asymptotic distribution, the chi-square. The minimum discrimination information statistic (m.d.i.s.) used by Kullback is the following

$$I = \sum_{i=1}^k p_i \ln \frac{p_i}{\pi_i}$$

where  $k$  is the number of cells,  $p_i$  represents the probabilities of a distribution that is a member of a family  $\Omega$  of distributions of interest satisfying certain constraints

(given the target population) and  $\pi_i$  are the probabilities of a distribution that will be determined according to the problem of interest (it may be either a specified, an estimated, or a second observed distribution).

For categorical type of data we obtain:

$$\hat{I} = \sum_{i=1}^n f_i \ln \frac{f_i}{np_i}$$

which is, when multiplied by -2, exactly the likelihood ratio with

$p_i$  : probability of an observation from the  $i^{th}$  cell under the null hypothesis.

$f_i$ : observed frequency of the  $i^{th}$  cell.

This test adjust to the different types of tables.

### 0 fixed margin

$$2\hat{I} = 2 \sum_{i=1}^r \sum_{j=1}^c f_{ij} \ln \frac{f_{ij}}{(f_{i.} f_{.j})/n} \sim \chi^2_{(r-1)(c-1)}$$

### 1 fixed n.argin

$$2\hat{I} = 2 \sum_{i=1}^r f_{i.} \ln \frac{f_{i.}}{np_{i.}} \sim \chi^2_{(r-1)}$$

### 2 fixed margins

$$2\hat{I} = 2 \sum_{i=1}^r \sum_{j=1}^c f_{ij} \ln \frac{f_{ij}}{np_{ij}} \sim \chi^2_{(rc-1)}$$

The m.d.i.s. possesses 3 properties, additivity, convexity and asymptotically distributed as  $\chi^2$ .

Since Fisher's test has been found not always adequate, Rice (1988) created a new probability model for determining exact P-value without approximation when sample is large and without randomization. Rice designated this test for the comparison of binomial proportions that comes from two samples of prescribed size. Rice's procedure gives the probability of obtaining a difference in sample proportions at least as large as that observed under the hypothesis of independence.

He describes two models, one treats the case of an unknown and unconstrained parameter  $p$  and the other deals with prior knowledge about  $p$ .

We assumed that the unknown value of  $p$  in a particular experiment represents a random number from a  $U[0,1]$  distribution and let  $\hat{p} = \frac{r}{N}$  be the maximum likelihood estimators of  $p$  based on a sample of size  $N$ . Under the null hypothesis, we obtain the following two probabilities:

$$P(p|\hat{p}) = \frac{P(\hat{p}|p)}{\int_0^1 P(\hat{p}|p) dp}$$

$$P(a, b|m, n, p) = \binom{m}{a} \binom{n}{b} p^a (1-p)^{N-a-b}.$$

After some manipulations, the P-value for a particular table is then given by:

$$P(x, y|m, n, \hat{p}) = \frac{\binom{m}{x} \binom{n}{y} (r + x + y)! (2N - x - y - r)! / (2N + 1)!}{(r!(N - r)!) / (N + 1)!}$$

We then use the following as the P-value:

$$P = \sum_{x=0}^m \sum_{y=0}^n P(x, y|m, n, \hat{p}) \quad \text{if} \quad \left| \frac{x}{m} - \frac{y}{n} \right| \geq \left| \frac{a}{m} - \frac{b}{n} \right|$$

$P = 0$  otherwise .

$P$  is the probability, when  $H_0$  : independence is true , that future values of  $\left| \frac{a}{m} - \frac{b}{n} \right|$  will equal or exceed the observed.

If one has prior knowledge about the parameter, then the probability  $P(x, y|m, n, \hat{p})$  is conditioned by a different estimate of  $p$  which might be based on both previous and present experiments.

Haberman (1981)

#### Test based on canonical correlation

Consider a  $r \times c$  contingency table with frequencies  $n_{ij}$ ,  $1 \leq i \leq r$ ,  $1 \leq j \leq c$ . Assume that the  $n_{ij}$  have a multinomial distribution with sample size  $N$  and with cell probabilities  $p_{ij}$ . Finally, denote the observed relative frequency of the  $ij^{th}$  cell by  $f_{ij} = n_{ij}/N$ ,  $f_{i\cdot} = n_{i\cdot}/N$ ,  $f_{\cdot j} = n_{\cdot j}/N$  . The canonical correlation test is a test of the independence hypothesis:

$$p_{ij} = p_{i\cdot} p_{\cdot j}, \quad 1 \leq i \leq r, \quad 1 \leq j \leq c$$

To perform the test, we assign scores  $x_i$  and  $y_j$  to the row and column totals such that the mean of the row score and of the column score are both 0 and the observed variances are both 1.

$$\sum_{i=1}^r f_{i\cdot} x_i = \sum_{j=1}^c f_{\cdot j} y_j = 0$$

$$\sum_{i=1}^r f_{i\cdot} x_i^2 = \sum_{j=1}^c f_{\cdot j} y_j^2 = 1$$

Then subject to these constraints, the correlation  $R_1$  of the scores of row and column variables is maximized.

$$R_1 = \sum_{i=1}^r \sum_{j=1}^c f_{ij} x_i y_j$$

It has been shown that if the independence model holds then  $N R_1^2$  converges in distribution to an F distribution with  $(r-1)(c-1)$  degrees of freedom as  $N \rightarrow \infty$  which may be used for testing.

## **CHAPTER THREE**

### **Our proposed Statistic**

In this chapter, we will present a new procedure for testing the equality of two proportions of two independent populations. It is one of the problems discussed in chapter 1 (comparative trials) for which the contingency table technique is applicable.

Our procedure is inspired by McNemar's test which is a method of analyzing matched-pair dichotomous data. A short review and criticisms of McNemar's test can be found in Wang (1991). In that paper, a new procedure for handling matched-pair dichotomous data was proposed. Our new procedure is based on the method described in that paper.

In this present thesis, our procedure can be used in case of contingency tables of dimension two with dichotomous characteristics. Original categorical data should be collected in pairs or otherwise the data of the two independent populations will be randomly put in pairs. Regarding to our previous model of contingency tables, we consider, for our new proposal, the cases with fixed margins of equal sizes (say,  $m = n$ ) for one characteristic.

To visualize, we will cite here an experiment performed in Denver, USA. Two researchers, Karen Caplocitz Barrett of Colorado State University and her co-researcher Carolyn Zahn-Waxler of the national Institute of Mental Health, studied the responses of two-year-olds (22 boys and 22 girls) to a faked accident. The children were given a rag doll to play with while the researcher left the room. Almost immediately, the doll's leg came off. It was meant to happen, but the child didn't realize that. The children fell into one of two categories, which Barrett labelled "amenders" and "avoiders". The amenders immediately tried to fix the doll, and, failing that, rushed to tell the adult what had happened. The avoiders, on the other hand, didn't want to look the adult in the eye afterward. Instead, they turned away. They didn't try to fix the doll quickly, if at all. [The Gazette in Montreal, Sunday, March 3rd 1991]. If we classify a boy (a girl) to be "1" if he (she) fits into the avoiders category and to be "0" otherwise. According to the previous type of contingency tables, we would have :

**Table 3.1**

Sex	Reaction		
	0	1	
Girls	7	15	22
Boys	13	9	22
	20	24	44

We rearrange the contingency table in Table 3.1 for the new test presented in this chapter. For the given example, it would be the following with 22 pairs  $\{(X_i, Y_i) : i = 1, \dots, 22\}$  of observations.

**Table 3.2**

		Boys(Y)	
		1	0
Girls(X)	1	6	9
	0	3	4
		9	13
			22

Where 0 and 1 correspond to the two categories of the reaction.

Our procedure, will be based on analyzing the contingency table of the form exposed in Table 3.2. Let  $X_1, X_2, \dots, X_n$  and  $Y_1, Y_2, \dots, Y_n$  be two independent random samples from two Bernoulli populations with

$$P(X_i = j) = \begin{cases} p_1 & \text{if } j = 1 \\ 1 - p_1 & \text{if } j = 0 \end{cases} \quad \text{and} \quad P(Y_i = j) = \begin{cases} p_2 & \text{if } j = 1 \\ 1 - p_2 & \text{if } j = 0 \end{cases}$$

for all  $i = 1, 2, \dots, n$ .

We pair them as  $(X_1, Y_1), (X_2, Y_2), \dots, (X_n, Y_n)$  as if they were from a matched sample. Each of  $(X_i, Y_i)$  takes values in the set

$$\{(x, y) : (0, 0), (0, 1), (1, 0), (1, 1)\}$$

with probabilities

$$P\{(X_i, Y_i) = (x, y)\} = \begin{cases} (1 - p_1)(1 - p_2) & \text{if } (x, y) = (0, 0) \\ p_1(1 - p_2) & \text{if } (x, y) = (1, 0) \\ (1 - p_1)p_2 & \text{if } (x, y) = (0, 1) \\ p_1p_2 & \text{if } (x, y) = (1, 1) \end{cases}$$

**Remark 3.1** If only the categorized data  $a = \sum_{i=1}^n X_i$ ,  $b = n - \sum_{i=1}^n X_i$ ,  $c = \sum_{i=1}^n Y_i$ ,  $d = n - \sum_{i=1}^n Y_i$  are available, to obtain the paired values, we randomize  $X_1, X_2, \dots, X_n$  and  $Y_1, Y_2, \dots, Y_n$  as follows. First, we generate  $2n$   $U(0, 1)$

random variables  $W_1, W_2, \dots, W_n$  and  $Z_1, Z_2, \dots, Z_n$ . Next, we order them  $W_{(1)}, W_{(2)}, \dots, W_{(n)}$  and  $Z_{(1)}, Z_{(2)}, \dots, Z_{(n)}$ . Finally, denote  $X_i = 1$  if  $W_i$  is in the set  $W_{(1)}, W_{(2)}, \dots, W_{(a)}$  and similarly  $Y_i = 1$  if  $Z_i$  is in the set  $\{Z_{(1)}, Z_{(2)}, \dots, Z_{(c)}\}$ ,  $X_i = 0$  and  $Y_i = 0$ , otherwise.

Let  $n_{ij}$  represent the sample frequencies in the  $ij^{th}$  cell (i.e. the number of pairs with first coordinate  $i$  and second coordinate  $j$ ), then we have the following contingency table:

**Table 3.3**

		Y		
		0	1	
X	0	$n_{00}$	$n_{01}$	$n_{0.}$
	1	$n_{10}$	$n_{11}$	$n_{1.}$
		$n_{.0}$	$n_{.1}$	$n$

The corresponding probabilities can be expressed as follows:

**Table 3.4**

		Y		
		0	1	
X	0	$p_{00}$	$p_{01}$	$p_{0.}$
	1	$p_{10}$	$p_{11}$	$p_{1.}$
		$p_{.0}$	$p_{.1}$	1

In the notation of  $n_{ij}$ , we have  $n_{1.} = \sum_{i=1}^n X_i$  and  $n_{.1} = \sum_{i=1}^n Y_i$ .

Let  $D_i = X_i - Y_i$ , then  $D_i$  are independently and identically distributed trinomial random variables. Under the null hypothesis  $H_0 : p_1 = p_2$ , the random

variable  $D_i$  has the following distribution.

$$P(D_i = j) = \begin{cases} p & \text{if } j = -1 \\ 1 - 2p & \text{if } j = 0 \\ p & \text{if } j = 1 \end{cases}$$

Where  $p = p_2(1 - p_1) = p_1(1 - p_2)$  for all  $i = 1, 2, \dots, n$ .

We now define our statistic

$$S_n = \sum_{i=1}^n D_i = n_{10} - n_{01}.$$

Let's now define the likelihood function of  $D_1, D_2, \dots, D_n$ , in terms of  $n_{10}$  and  $n_{01}$ , to show that our test statistic  $S_n$  is a generalized likelihood ratio test.

(3.1)

$$L(p_{10}, p_{01}) = p_{10}^{n_{10}} p_{01}^{n_{01}} (1 - p_{10} - p_{01})^{n - n_{10} - n_{01}} \quad \text{for all } 0 \leq n_{10} + n_{01} \leq n .$$

It follows that

$$(3.2) \quad \begin{aligned} \frac{\partial \ln L}{\partial p_{10}} &= \frac{n_{10}}{p_{10}} - \frac{(n - n_{10} - n_{01})}{1 - p_{10} - p_{01}} \\ \frac{\partial \ln L}{\partial p_{01}} &= \frac{n_{01}}{p_{01}} - \frac{(n - n_{10} - n_{01})}{1 - p_{10} - p_{01}} . \end{aligned}$$

Setting (3.2) to be zero, we obtain the relation

$$\frac{n_{10}}{p_{10}} = \frac{n_{01}}{p_{01}} .$$

Hence, the maximum likelihood estimates of  $p_{10}$  and  $p_{01}$  under the whole parameter space  $\Omega = \{(x, y) : 0 \leq x, y \leq 1\}$  are

$$\hat{p}_{10} = \frac{n_{10}}{n} \quad \text{and} \quad \hat{p}_{01} = \frac{n_{01}}{n} . \quad \diamond$$

Now under the restricted parameter space  $\omega = \{(x, y) : 0 \leq x = y \leq 1\}$ , we have

$$(3.3) \quad L(p) = p^{n_{10}+n_{01}}(1-2p)^{n-n_{10}-n_{01}} \quad 0 \leq p = p_{10} = p_{01} \leq 1 .$$

From (3.3) it follows

$$(3.4) \quad \frac{\partial \ln L}{\partial p} = \frac{(n_{10} + n_{01})}{p} - \frac{2[n - (n_{10} + n_{01})]}{1-2p} .$$

Setting (3.4) equals to zero, we obtain

$$\hat{p} = \frac{(n_{10} + n_{01})}{2n} .$$

Therefore, the generalized likelihood ratio is

$$\lambda = \frac{\sup_{(p_{10}, p_{01}) \in \omega} L(p_{10}, p_{01})}{\sup_{(p_{10}, p_{01}) \in \Omega} L(p_{10}, p_{01})} = \left( \frac{n_{10} + n_{01}}{2n_{10}} \right)^{n_{10}} \left( \frac{n_{10} + n_{01}}{2n_{01}} \right)^{n_{01}} .$$

**Theorem 1.** *The set*

$$C = \{(n_{10} - n_{01}) : |n_{10} - n_{01}| > k\}$$

*is the critical region for the generalized likelihood-ratio test for testing the two-sided hypothesis:*

$$\mathbf{A} : H_0 : p_{1.} = p_{.1} \quad VS \quad p_{1.} \neq p_{.1}$$

**Proof:** We will show that  $\lambda$  is strictly increasing in  $|n_{10} - n_{01}|$ .

Define

$$f(x, y) = \left(\frac{x+y}{2x}\right)^x \left(\frac{x+y}{2y}\right)^y \quad \text{for } x, y \geq 0 .$$

Since  $f(x, y) = f(y, x)$ , only one half will be considered i.e.  $x - y = \delta \geq 0$

$$g(\delta, y) = \left(1 - \frac{\delta}{2(\delta + y)}\right)^{\delta+y} \left(1 + \frac{\delta}{2y}\right)^y .$$

Take the logarithms on both sides, then

$$\ln g(\delta, y) = (\delta + y) \ln \left(1 - \frac{\delta}{2(\delta + y)}\right) + y \ln \left(1 + \frac{\delta}{2y}\right) .$$

We now take the partial derivative w.r.t.  $\delta$

$$\frac{\partial}{\partial \delta} \ln g(\delta, y) = \ln \left(1 - \frac{\delta}{2(\delta + y)}\right) < 0 \quad \text{for all } \delta \geq 0 .$$

This last statement proves that  $g(x, y)$  is a strictly monotone decreasing function of  $|x - y|$ . And since the second partial derivative w.r.t.  $\delta$  is always negative and that  $\frac{\partial}{\partial \delta} \ln g(\delta, y) = 0$  at  $x = y$  i.e.  $\delta = 0$  then  $g(x, y)$  attains its maximum value 1 at  $x = y$ .

Consequently, the likelihood ratio  $\lambda$  is a strictly decreasing function in  $|n_{10} - n_{01}|$  and the critical region for the generalized likelihood-ratio test is given by C.

**Remark (3.2)** Because  $p_{1.} = p_{10} + p_{11}$  and  $p_{.1} = p_{01} + p_{11}$  the two-sided hypothesis A is equivalent to the following.

$$\mathbf{A}' : H_0 : p_{10} = p_{01} \quad \text{VS} \quad H_1 : p_{10} \neq p_{01}$$

**Remark (3.3)** For the one-sided hypotheses.

$$\mathbf{B}: H_0: p_{1.} = p_{.1} \quad \text{VS} \quad H_1: p_{1.} > p_{.1}$$

$$\mathbf{C}: H_0: p_{1.} = p_{.1} \quad \text{VS} \quad H_1: p_{1.} < p_{.1}$$

or their equivalents:

$$\mathbf{B}': H_0: p_{10} = p_{01} \quad \text{VS} \quad H_1: p_{10} > p_{01}$$

$$\mathbf{C}': H_0: p_{10} = p_{01} \quad \text{VS} \quad H_1: p_{10} < p_{01}$$

The generalized likelihood-ratio tests have the critical regions of the form :

$$C = \{(n_{10} - n_{01}) : n_{10} - n_{01} \geq k\} \quad (\text{for } \mathbf{B} \text{ or } \mathbf{B}')$$

$$C = \{(n_{10} - n_{01}) : n_{10} - n_{01} \leq k\} \quad (\text{for } \mathbf{C} \text{ or } \mathbf{C}')$$

To find the exact distribution of  $n_{10} - n_{01}$ , we use the fact that it is the sum of  $n$  independent and identically distributed trinomial random variables  $D_1, D_2, \dots, D_n$  taking values -1, 0, 1 with probabilities  $p, 1 - 2p, \text{ and } p$  respectively. For example, if  $n = 3$ , then  $S_n = n_{10} - n_{01}$  has the distribution

$$P\{S_n = -3\} = P\{S_n = 3\} = p^3$$

$$P\{S_n = -2\} = P\{S_n = 2\} = 3(1 - 2p)p^2$$

$$P\{S_n = -1\} = P\{S_n = 1\} = 3p(1 - 2p)^2 + (1 - 2p)^3$$

$$P\{S_n = 0\} = 6(1 - 2p)p^2 + (1 - 2p)^3 .$$

We have tabulated the cumulative probabilities in Table A.1 for  $S_n = n_{10} - n_{01}$  for  $p = .01(.01).25$  .

Exact tests are often restricted to small samples due to the tedious computations as the sample size increases. An approximation to a different distribution overcomes this problem but requires large sample sizes since it is established as  $n \rightarrow \infty$ . Now that we have made our suggestion for sample sizes less than or equal to 30, we will present the distribution that is an extension of Poisson distribution which is intended for larger sample sizes.

We will start with the Poisson distribution to get to the compound Poisson distribution which leads to our proposition. The former distribution was named after the French mathematician, Simeon Denis Poisson (1781-1840). Initially viewed as an approximation to the hard-to-compute binomial probabilities when sample is large, the Poisson distribution has now become one of the most important of all probability models.

In real world phenomena, we often encounter processes in which the interest is the number of events occurring in some unit of time. In any type of such counting processes, the random variable  $X$  denoting the number of occurrences is often assumed to have a Poisson distribution. For example, say  $X$  is the number of customers entering a bank during a unit of time or  $X$  denotes the number of car

accidents (or even the number of insurance claims) in some unit of time, then in these cases,  $X$  follows a Poisson distribution.

$$P(X = x) = \frac{e^{-\lambda} \lambda^x}{x!} \quad x = 0, 1, 2, \dots .$$

Actually, what we need for our particular problem, is the extension of the Poisson distribution often defined as a random sum of random variables called the compound Poisson distribution. Let  $\{X_i : i = 1, 2, \dots, n\}$  be a sequence of independent identically distributed random variables and  $N$  be a Poisson random variable independent of  $X_i$ , then

$$Y = \sum_{i=1}^N X_i$$

is said to have a compound Poisson distribution with parameter  $\lambda = E(X)$  and compounding distribution  $\{P(X_i = j); j = 0, 1, 2, \dots\}$ . If  $X_i \equiv 1$  for all  $i$ , then  $Y$  ( $Y = N$ ) has the Poisson distribution.

This result will be used as the distribution of our statistic for large samples for testing the previously defined hypotheses. The following theorem 2 is a special case of theorem 7 in Wang (1989). We shall present it in the form suitable for our purpose. For the proof we refer readers to Wang (1989).

**Theorem 2.** If  $np_1(1 - p_2) = np_2(1 - p_1) \rightarrow \delta$  as  $n \rightarrow \infty$ , then

$$\lim_{n \rightarrow \infty} P(S_n = k) = P(Y = k)$$

for all  $k = 0, 1, 2, \dots$  where  $Y$  follows the compound Poisson distribution with parameter  $\lambda = 2\delta$ .

The probabilities of  $Y$  have been computed by the following with a maximum of  $n$  as large as the computer can handle:

$$P\{Y = k\} = \sum_{n=1}^{80} P\{S_N = k | N = n\} P\{N = n\}$$

and they have been tabulated in Table A.2 for

$$\lambda = \begin{cases} .02 & \text{to} & .1 & \text{by} & .02 \\ .1 & \text{to} & 1 & \text{by} & .05 \\ 1 & \text{to} & 2 & \text{by} & .1 \\ 2 & \text{to} & 8 & \text{by} & .2 \\ 8 & \text{to} & 15 & \text{by} & .5 \end{cases}$$

We defined our statistic as

$$S_n = \sum_{i=1}^n D_i = n_{10} - n_{01}$$

where the  $D_i$ 's are independent and identically distributed trinomial random variables with distribution

$$P\{D_i = j\} = \begin{cases} p & \text{if } j = -1 \\ 1 - 2p & \text{if } j = 0 \\ p & \text{if } j = 1 \end{cases}$$

where  $p = p_1(1 - p_2) = p_2(1 - p_1)$  under the null hypothesis  $p_1 = p_2$ . Since  $p$  is the product of two probabilities where  $p_1 = p_2$  under the independence assumption, we have:

$$0 \leq p \leq \frac{1}{4}$$

To use Table A.2, we need two values;  $\lambda$  and C. The C value is given by our statistic  $S_n = n_{10} - n_{01}$ . Since  $n_{10} - n_{01} = \sum_{i=1}^n X_i - \sum_{i=1}^n Y_i$  we may use the entries as in Table (3.1), which avoids randomization that is necessary if the entries of a table of the type of Table (3.3) are unknown.

For the parameter  $\lambda$ , we must find an estimate for  $p$  since  $\lambda = 2np$ . In order to estimate  $p_1 = p_2 = p'$ , we use the minimum variance unbiased estimator

$$(3.5) \quad \hat{p}' = \frac{\sum_{i=1}^n X_i + \sum_{i=1}^n Y_i}{2n} .$$

We now apply our test statistic to our girls/boys example. We want to test the independence of the two characteristics, sex and reaction upon a broken toy. If "1" represents the avoiders category, then the null hypothesis is as follows

$$H_0 : p_1 = p_2$$

where  $p_1, p_2$  are the probabilities of falling in the avoiders category for the girls and boys respectively. According to Table (3.1) we have

$$S_n = \sum_{i=1}^n X_i - \sum_{i=1}^n Y_i = 15 - 9 = 6 .$$

And according to equation (3.5) we have

$$\hat{p}' = \frac{\sum_{i=1}^n X_i + \sum_{i=1}^n Y_i}{2n} = \frac{15 + 9}{2 \cdot 22} = 0.545$$

then,

$$\hat{p} = \hat{p}_1(1 - \hat{p}_2) = \hat{p}'(1 - \hat{p}') = 0.248 .$$

Since we have a sample less than 30, following the rules we would use the trinomial distribution. Regarding our tables, for a sample of 22 with  $\hat{p} = .248$  we get a P-value of 0.0481.

Even though the sample is less than 30, it would be interesting to find the P-value using the compound Poisson distribution.

Using  $C=6$  and  $\lambda = 2n\hat{p} = 10.91 \approx 11$ , we obtain from the Table A.2, the following P-value

$$P = 0.048 .$$

Thus, both tests give the same P-value and the conclusion is that the null hypothesis of independence is rejected at the 5% level. In Denver, the two researchers came to the same conclusion but neither the methods or the numerical results were ever mentioned in the article.

On the other hand, the chi-square statistic gives a probability of 0.074 for the two-sided test. To compare it with the previous one-sided results , we must divide this value by 2, i.e. the P-value with the chi-square test is 0.037 for one-sided which is not as accurate as the result obtained with the compound Poisson distribution when compared to the exact value.

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**Periodicals abbreviations**

**Annals of Mathematical Statistics:** Ann. Math. Statist.

**Annals of Statistics:** Ann. Statist.

**Annals of Probability (The)** Ann. of Prob.

**Applied Statistics:** Appl. Statist.

**Communications in Statistics:** Comm. Statist.

**International Statistical Review:** Int. Statist. Rev.

**Journal of the American Statistical Association:** JASA

**Journal of the Royal Statistical Society:** J. Roy. Statist. Soc.

**National Journal of Statistics and Operations Research:** Nat. J. Statist. Oper. Res.

**New Journal of Statistics and Operations Research:** N. J. Statist. Oper. Res.

**Philosophical Magazine:** Philos. Mag.

**Philosophical Transactions of the Royal Society of London:** Philos. Trans. Roy. Soc.

**Proceedings of the Royal Statistical Society:** Proc. Roy. Statist. Soc.

**Psychological Bulletin:** Psychol. Bull.

**Review of International Statistical Institute:** Rev. Int. Statist. Inst.

**Royal Society Philosophical Transactions:** Roy. Soc. Philos. Trans.

## APPENDIX A

**Table A.1 Trinomial Probability Tables**

		<b>p</b>				
N	k	.01	.02	.03	.04	.05
2	-2	0.0001	0.0004	0.0009	0.0016	0.0025
	-1	0.0197	0.0388	0.0573	0.0752	0.0925
	0	0.9803	0.9612	0.9427	0.9248	0.9075
	1	0.9999	0.9996	0.9991	0.9984	0.9975
	2	1.0000	1.0000	1.0000	1.0000	1.0000
3	-3	0.0000	0.0000	0.0000	0.0001	0.0001
	-2	0.0003	0.0012	0.0026	0.0045	0.0069
	-1	0.0291	0.0565	0.0822	0.1062	0.1287
	0	0.9709	0.9435	0.9178	0.8938	0.8712
	1	0.9997	0.9988	0.9974	0.9955	0.9931
	2	1.0000	1.0000	1.0000	0.9999	0.9999
	3	1.0000	1.0000	1.0000	1.0000	1.0000
4	-3	0.0000	0.0000	0.0001	0.0002	0.0005
	-2	0.0006	0.0022	0.0049	0.0084	0.0126
	-1	0.0382	0.0731	0.1049	0.1337	0.1598
	0	0.9618	0.9269	0.8951	0.8663	0.8402
	1	0.9994	0.9978	0.9951	0.9916	0.9874
	2	1.0000	1.0000	0.9999	0.9998	0.9995
	3	1.0000	1.0000	1.0000	1.0000	1.0000
5	-3	0.0000	0.0001	0.0002	0.0006	0.0010
	-2	0.0010	0.0036	0.0077	0.0131	0.0194
	-1	0.0471	0.0888	0.1256	0.1580	0.1864
	0	0.9529	0.9112	0.8744	0.8420	0.8136
	1	0.9990	0.9964	0.9923	0.9869	0.9806
	2	1.0000	0.9999	0.9998	0.9994	0.9990
	3	1.0000	1.0000	1.0000	1.0000	1.0000
6	-4	0.0000	0.0000	0.0000	0.0000	0.0001
	-3	0.0000	0.0001	0.0005	0.0010	0.0019
	-2	0.0014	0.0052	0.0110	0.0184	0.0268
	-1	0.0557	0.1035	0.1445	0.1795	0.2094
	0	0.9443	0.8965	0.8555	0.8205	0.7906
	1	0.9986	0.9948	0.9890	0.9816	0.9732
	2	1.0000	0.9999	0.9995	0.9990	0.9981
	3	1.0000	1.0000	1.0000	1.0000	0.9999
	4	1.0000	1.0000	1.0000	1.0000	1.0000

## P

		.01	.02	.03	.04	.05
N	k					
7	-4	0.0000	0.0000	0.0000	0.0001	0.0002
	-3	0.0000	0.0002	0.0008	0.0017	0.0031
	-2	0.0019	0.0071	0.0147	0.0241	0.0347
	-1	0.0640	0.1174	0.1618	0.1987	0.2294
	0	0.9360	0.8826	0.8382	0.8013	0.7706
	1	0.9981	0.9929	0.9853	0.9759	0.9653
	2	1.0000	0.9998	0.9992	0.9983	0.9969
	3	1.0000	1.0000	1.0000	0.9999	0.9998
	4	1.0000	1.0000	1.0000	1.0000	1.0000
8	-4	0.0000	0.0000	0.0000	0.0001	0.0003
	-3	0.0001	0.0004	0.0012	0.0025	0.0045
	-2	0.0025	0.0092	0.0187	0.0302	0.0429
	-1	0.0721	0.1305	0.1777	0.2158	0.2467
	0	0.9279	0.8695	0.8223	0.7842	0.7533
	1	0.9975	0.9908	0.9813	0.9698	0.9571
	2	0.9999	0.9996	0.9988	0.9975	0.9955
	3	1.0000	1.0000	1.0000	0.9999	0.9997
	4	1.0000	1.0000	1.0000	1.0000	1.0000
9	-4	0.0000	0.0000	0.0001	0.0002	0.0005
	-3	0.0001	0.0005	0.0017	0.0035	0.0062
	-2	0.0032	0.0114	0.0230	0.0365	0.0511
	-1	0.0800	0.1429	0.1923	0.2311	0.2618
	0	0.9200	0.8571	0.8077	0.7689	0.7382
	1	0.9968	0.9886	0.9770	0.9635	0.9489
	2	0.9999	0.9995	0.9983	0.9965	0.9938
	3	1.0000	1.0000	0.9999	0.9998	0.9995
	4	1.0000	1.0000	1.0000	1.0000	1.0000
10	-4	0.0000	0.0000	0.0001	0.0003	0.0008
	-3	0.0001	0.0008	0.0022	0.0047	0.0082
	-2	0.0039	0.0138	0.0274	0.0430	0.0594
	-1	0.0876	0.1545	0.2057	0.2449	0.2751
	0	0.9124	0.8455	0.7943	0.7551	0.7249
	1	0.9961	0.9862	0.9726	0.9570	0.9406
	2	0.9999	0.9992	0.9978	0.9953	0.9918
	3	1.0000	1.0000	0.9999	0.9997	0.9992
	4	1.0000	1.0000	1.0000	1.0000	1.0000
11	-5	0.0000	0.0000	0.0000	0.0000	0.0001
	-4	0.0000	0.0000	0.0002	0.0005	0.0011
	-3	0.0001	0.0010	0.0029	0.0061	0.0104
	-2	0.0047	0.0164	0.0320	0.0495	0.0676

## P

N	k	.01	.02	.03	.04	.05
12	-1	0.0950	0.1655	0.2180	0.2572	0.2868
	0	0.9050	0.8345	0.7820	0.7428	0.7132
	1	0.9953	0.9836	0.9680	0.9505	0.9324
	2	0.9999	0.9990	0.9971	0.9939	0.9896
	3	1.0000	1.0000	0.9998	0.9995	0.9989
	4	1.0000	1.0000	1.0000	1.0000	0.9999
	5	1.0000	1.0000	1.0000	1.0000	1.0000
	-5	0.0000	0.0000	0.0000	0.0000	0.0001
	-4	0.0000	0.0001	0.0003	0.0007	0.0015
	-3	0.0002	0.0013	0.0037	0.0076	0.0128
13	-2	0.0056	0.0191	0.0367	0.0561	0.0757
	-1	0.1022	0.1759	0.2293	0.2683	0.2972
	0	0.8978	0.8241	0.7707	0.7317	0.7028
	1	0.9944	0.9809	0.9633	0.9439	0.9243
	2	0.9998	0.9987	0.9963	0.9924	0.9872
	3	1.0000	0.9999	0.9997	0.9993	0.9985
	4	1.0000	1.0000	1.0000	1.0000	0.9999
	5	1.0000	1.0000	1.0000	1.0000	1.0000
	-5	0.0000	0.0000	0.0000	0.0001	0.0002
	-4	0.0000	0.0001	0.0004	0.0010	0.0020
14	-3	0.0002	0.0016	0.0046	0.0093	0.0153
	-2	0.0065	0.0219	0.0415	0.0626	0.0837
	-1	0.1092	0.1857	0.2398	0.2784	0.3064
	0	0.8908	0.8143	0.7602	0.7216	0.6936
	1	0.9935	0.9781	0.9585	0.9374	0.9163
	2	0.9998	0.9984	0.9954	0.9907	0.9847
	3	1.0000	0.9999	0.9996	0.9990	0.9980
	4	1.0000	1.0000	1.0000	0.9999	0.9998
	5	1.0000	1.0000	1.0000	1.0000	1.0000
	-5	0.0000	0.0000	0.0000	0.0001	0.0003
15	-4	0.0000	0.0001	0.0005	0.0013	0.0026
	-3	0.0003	0.0020	0.0056	0.0111	0.0181
	-2	0.0075	0.0247	0.0463	0.0691	0.0914
	-1	0.1160	0.1950	0.2494	0.2875	0.3146
	0	0.8840	0.8050	0.7506	0.7125	0.6854
	1	0.9925	0.9753	0.9537	0.9309	0.9086
	2	0.9997	0.9980	0.9944	0.9889	0.9819
	3	1.0000	0.9999	0.9995	0.9987	0.9974
	4	1.0000	1.0000	1.0000	0.9999	0.9997
	5	1.0000	1.0000	1.0000	1.0000	1.0000

N	k	.01	.02	.03	.04	.05
16	-4	0.0000	0.0001	0.0006	0.0016	0.0032
	-3	0.0004	0.0024	0.0067	0.0130	0.0210
	-2	0.0085	0.0277	0.0512	0.0755	0.0989
	-1	0.1226	0.2038	0.2584	0.2957	0.3220
	0	0.8774	0.7962	0.7416	0.7043	0.6780
	1	0.9915	0.9723	0.9488	0.9245	0.9011
	2	0.9996	0.9976	0.9933	0.9870	0.9790
	3	1.0000	0.9999	0.9994	0.9984	0.9968
	4	1.0000	1.0000	1.0000	0.9999	0.9996
	5	1.0000	1.0000	1.0000	1.0000	1.0000
17	-5	0.0000	0.0000	0.0001	0.0002	0.0005
	-4	0.0000	0.0002	0.0008	0.0020	0.0040
	-3	0.0004	0.0029	0.0078	0.0150	0.0240
	-2	0.0095	0.0307	0.0561	0.0819	0.1061
	-1	0.1290	0.2122	0.2667	0.3033	0.3286
	0	0.8710	0.7878	0.7333	0.6967	0.6714
	1	0.9905	0.9693	0.9439	0.9181	0.8939
	2	0.9996	0.9971	0.9922	0.9850	0.9760
	3	1.0000	0.9998	0.9992	0.9980	0.9960
	4	1.0000	1.0000	0.9999	0.9998	0.9995
18	5	1.0000	1.0000	1.0000	1.0000	0.9999
	-6	0.0000	0.0000	0.0000	0.0000	0.0001
	-5	0.0000	0.0000	0.0001	0.0003	0.0007
	-4	0.0000	0.0002	0.0010	0.0024	0.0048
	-3	0.0005	0.0034	0.0091	0.0172	0.0271
	-2	0.0107	0.0338	0.0610	0.0880	0.1132
	-1	0.1352	0.2200	0.2743	0.3102	0.3346
	0	0.8648	0.7800	0.7257	0.6898	0.6654
	1	0.9893	0.9662	0.9390	0.9120	0.8868
	2	0.9995	0.9966	0.9909	0.9828	0.9729
19	3	1.0000	0.9998	0.9990	0.9976	0.9952
	4	1.0000	1.0000	0.9999	0.9997	0.9993
	5	1.0000	1.0000	1.0000	1.0000	0.9999
	6	1.0000	1.0000	1.0000	1.0000	1.0000
	-6	0.0000	0.0000	0.0000	0.0000	0.0001
	-5	0.0000	0.0000	0.0001	0.0003	0.0008
	-4	0.0000	0.0003	0.0012	0.0030	0.0057

## P

N	k	.01	.02	.03	.04	.05
19	2	0.9994	0.9961	0.9896	0.9806	0.9697
	3	1.0000	0.9997	0.9988	0.9970	0.9943
	4	1.0000	1.0000	0.9999	0.9997	0.9992
	5	1.0000	1.0000	1.0000	1.0000	0.9999
	6	1.0000	1.0000	1.0000	1.0000	1.0000
	-6	0.0000	0.0000	0.0000	0.0000	0.0001
	-5	0.0000	0.0000	0.0001	0.0004	0.0010
	-4	0.0000	0.0004	0.0014	0.0035	0.0067
	-3	0.0007	0.0045	0.0118	0.0218	0.0335
	-2	0.0130	0.0400	0.0706	0.1000	0.1265
	-1	0.1472	0.2346	0.2881	0.3222	0.3451
	0	0.8528	0.7654	0.7119	0.6778	0.6549
	1	0.9870	0.9600	0.9294	0.9000	0.8735
	2	0.9993	0.9955	0.9882	0.9782	0.9665
	3	1.0000	0.9996	0.9986	0.9965	0.9933
20	4	1.0000	1.0000	0.9999	0.9996	0.9990
	5	1.0000	1.0000	1.0000	1.0000	0.9999
	6	1.0000	1.0000	1.0000	1.0000	1.0000
	-6	0.0000	0.0000	0.0000	0.0001	0.0002
	-5	0.0000	0.0000	0.0002	0.0005	0.0013
	-4	0.0000	0.0004	0.0017	0.0041	0.0078
	-3	0.0009	0.0051	0.0132	0.0242	0.0368
	-2	0.0142	0.0432	0.0754	0.1058	0.1327
	-1	0.1529	0.2413	0.2943	0.3276	0.3496
	0	0.8471	0.7587	0.7057	0.6724	0.6504
	1	0.9858	0.9568	0.9246	0.8942	0.8673
	2	0.9991	0.9949	0.9868	0.9758	0.9632
	3	1.0000	0.9996	0.9983	0.9959	0.9922
	4	1.0000	1.0000	0.9998	0.9995	0.9987
	5	1.0000	1.0000	1.0000	0.9999	0.9998
	6	1.0000	1.0000	1.0000	1.0000	1.0000
21	-6	0.0000	0.0000	0.0000	0.0001	0.0002
	-5	0.0000	0.0000	0.0002	0.0007	0.0015
	-4	0.0000	0.0005	0.0020	0.0048	0.0089
	-3	0.0010	0.0058	0.0147	0.0266	0.0402
	-2	0.0155	0.0464	0.0801	0.1114	0.1388
	-1	0.1584	0.2477	0.3001	0.3325	0.3538
	0	0.8416	0.7523	0.6999	0.6675	0.6462
	1	0.9845	0.9536	0.9199	0.8886	0.8612
	2	0.9990	0.9942	0.9853	0.9734	0.9598
	3	1.0000	0.9995	0.9980	0.9952	0.9911
	4	1.0000	1.0000	0.9998	0.9993	0.9985

## P

		.01	.02	.03	.04	.05
N	k					
22	5	1.0000	1.0000	1.0000	0.9999	0.9998
	6	1.0000	1.0000	1.0000	1.0000	1.0000
	-6	0.0000	0.0000	0.0000	0.0001	0.0003
	-5	0.0000	0.0000	0.0003	0.0008	0.0018
	-4	0.0001	0.0006	0.0023	0.0055	0.0101
	-3	0.0011	0.0065	0.0163	0.0291	0.0435
	-2	0.0167	0.0496	0.0847	0.1168	0.1446
	-1	0.1638	0.2538	0.3055	0.3371	0.3577
	0	0.8362	0.7462	0.6945	0.6629	0.6423
	1	0.9833	0.9504	0.9153	0.8832	0.8554
	2	0.9989	0.9935	0.9837	0.9709	0.9564
	3	0.9999	0.9994	0.9977	0.9945	0.9899
	4	1.0000	1.0000	0.9997	0.9992	0.9982
23	5	1.0000	1.0000	1.0000	0.9999	0.9997
	6	1.0000	1.0000	1.0000	1.0000	1.0000
	-6	0.0000	0.0000	0.0000	0.0001	0.0003
	-5	0.0000	0.0001	0.0003	0.0010	0.0022
	-4	0.0001	0.0007	0.0027	0.0062	0.0113
	-3	0.0013	0.0073	0.0179	0.0317	0.0469
	-2	0.0181	0.0529	0.0893	0.1221	0.1502
	-1	0.1691	0.2595	0.3105	0.3413	0.3613
	0	0.8309	0.7405	0.6895	0.6587	0.6387
	1	0.9819	0.9471	0.9107	0.8779	0.8498
	2	0.9987	0.9927	0.9821	0.9683	0.9531
	3	0.9999	0.9993	0.9973	0.9938	0.9887
	4	1.0000	0.9999	0.9997	0.9990	0.9978
24	5	1.0000	1.0000	1.0000	0.9999	0.9997
	6	1.0000	1.0000	1.0000	1.0000	1.0000
	-6	0.0000	0.0000	0.0000	0.0001	0.0004
	-5	0.0000	0.0001	0.0004	0.0011	0.0025
	-4	0.0001	0.0008	0.0031	0.0070	0.0127
	-3	0.0014	0.0080	0.0196	0.0343	0.0503
	-2	0.0194	0.0561	0.0938	0.1273	0.1556
	-1	0.1742	0.2650	0.3153	0.3452	0.3646
	0	0.8258	0.7350	0.6847	0.6548	0.6354
	1	0.9806	0.9439	0.9062	0.8727	0.8444
	2	0.9986	0.9920	0.9804	0.9657	0.9497
	3	0.9999	0.9992	0.9969	0.9930	0.9873
	4	1.0000	0.9999	0.9996	0.9989	0.9975
	5	1.0000	1.0000	1.0000	0.9999	0.9996
	6	1.0000	1.0000	1.0000	1.0000	0.9999

## P

N	k	.01	.02	.03	.04	.05
25	-7	0.0000	0.0000	0.0000	0.0000	0.0001
	-6	0.0000	0.0000	0.0000	0.0002	0.0005
	-5	0.0000	0.0001	0.0004	0.0013	0.0029
	-4	0.0001	0.0010	0.0035	0.0079	0.0140
	-3	0.0016	0.0089	0.0214	0.0369	0.0537
	-2	0.0208	0.0593	0.0982	0.1323	0.1608
	-1	0.1792	0.2702	0.3197	0.3489	0.3677
	0	0.8208	0.7298	0.6803	0.6511	0.6323
	1	0.9792	0.9407	0.9018	0.8677	0.8392
	2	0.9984	0.9911	0.9786	0.9631	0.9463
	3	0.9999	0.9990	0.9965	0.9921	0.9860
	4	1.0000	0.9999	0.9996	0.9987	0.9971
	5	1.0000	1.0000	1.0000	0.9998	0.9995
	6	1.0000	1.0000	1.0000	1.0000	0.9999
	7	1.0000	1.0000	1.0000	1.0000	1.0000
26	-7	0.0000	0.0000	0.0000	0.0000	0.0001
	-6	0.0000	0.0000	0.0001	0.0002	0.0006
	-5	0.0000	0.0001	0.0005	0.0016	0.0034
	-4	0.0001	0.0011	0.0039	0.0088	0.0155
	-3	0.0018	0.0097	0.0231	0.0396	0.0571
	-2	0.0222	0.0625	0.1025	0.1371	0.1658
	-1	0.1840	0.2752	0.3239	0.3523	0.3706
	0	0.8160	0.7248	0.6761	0.6477	0.6294
	1	0.9778	0.9375	0.8975	0.8629	0.8342
	2	0.9982	0.9903	0.9769	0.9604	0.9429
	3	0.9999	0.9989	0.9961	0.9912	0.9845
	4	1.0000	0.9999	0.9995	0.9984	0.9966
	5	1.0000	1.0000	0.9999	0.9998	0.9994
	6	1.0000	1.0000	1.0000	1.0000	0.9999
	7	1.0000	1.0000	1.0000	1.0000	1.0000
27	-7	0.0000	0.0000	0.0000	0.0000	0.0001
	-6	0.0000	0.0000	0.0001	0.0003	0.0007
	-5	0.0000	0.0001	0.0006	0.0018	0.0038
	-4	0.0001	0.0013	0.0044	0.0097	0.0169
	-3	0.0019	0.0106	0.0249	0.0422	0.0604
	-2	0.0236	0.0657	0.1068	0.1418	0.1706
	-1	0.1887	0.2800	0.3278	0.3555	0.3733
	0	0.8113	0.7200	0.6722	0.6445	0.6267
	1	0.9764	0.9343	0.8932	0.8582	0.8294
	2	0.9981	0.9894	0.9751	0.9578	0.9396
	3	0.9999	0.9987	0.9956	0.9903	0.9831
	4	1.0000	0.9999	0.9994	0.9982	0.9962

## P

N	k	.01	.02	.03	.04	.05
28	5	1.0000	1.0000	0.9999	0.9997	0.9993
	6	1.0000	1.0000	1.0000	1.0000	0.9999
	7	1.0000	1.0000	1.0000	1.0000	1.0000
	-7	0.0000	0.0000	0.0000	0.0000	0.0001
	-6	0.0000	0.0000	0.0001	0.0003	0.0008
	-5	0.0000	0.0001	0.0007	0.0020	0.0043
	-4	0.0001	0.0014	0.0049	0.0107	0.0185
	-3	0.0021	0.0115	0.0268	0.0449	0.0638
	-2	0.0250	0.0689	0.1110	0.1464	0.1752
	-1	0.1933	0.2845	0.3315	0.3585	0.3758
	0	0.8067	0.7155	0.6685	0.6415	0.6242
	1	0.9750	0.9311	0.8890	0.8536	0.8248
	2	0.9979	0.9885	0.9732	0.9551	0.9362
	3	0.9999	0.9986	0.9951	0.9893	0.9815
29	4	1.0000	0.9999	0.9993	0.9980	0.9957
	5	1.0000	1.0000	0.9999	0.9997	0.9992
	6	1.0000	1.0000	1.0000	1.0000	0.9999
	7	1.0000	1.0000	1.0000	1.0000	1.0000
	-7	0.0000	0.0000	0.0000	0.0001	0.0002
	-6	0.0000	0.0000	0.0001	0.0004	0.0010
	-5	0.0000	0.0002	0.0008	0.0023	0.0049
	-4	0.0002	0.0016	0.0054	0.0117	0.0200
	-3	0.0024	0.0124	0.0286	0.0476	0.0671
	-2	0.0265	0.0720	0.1151	0.1508	0.1797
	-1	0.1977	0.2888	0.3350	0.3614	0.3782
	0	0.8023	0.7112	0.6650	0.6386	0.6218
	1	0.9735	0.9280	0.8849	0.8492	0.8203
30	2	0.9976	0.9876	0.9714	0.9524	0.9329
	3	0.9998	0.9984	0.9946	0.9883	0.9800
	4	1.0000	0.9998	0.9992	0.9977	0.9951
	5	1.0000	1.0000	0.9999	0.9996	0.9990
	6	1.0000	1.0000	1.0000	0.9999	0.9998
	7	1.0000	1.0000	1.0000	1.0000	1.0000
	-7	0.0000	0.0000	0.0000	0.0001	0.0002
	-6	0.0000	0.0000	0.0001	0.0004	0.0011
	-5	0.0000	0.0002	0.0009	0.0026	0.0054
	-4	0.0002	0.0018	0.0060	0.0128	0.0216
	-3	0.0026	0.0134	0.0305	0.0503	0.0703
	-2	0.0279	0.0752	0.1191	0.1551	0.1840
	-1	0.2021	0.2929	0.3383	0.3640	0.3805
	0	0.7979	0.7071	0.6617	0.6360	0.6195
	1	0.9721	0.9248	0.8809	0.8449	0.8160

p

N	k	.01	.02	.03	.04	.05
	2	0.9974	0.9866	0.9695	0.9497	0.9297
	3	0.9998	0.9982	0.9940	0.9872	0.9784
	4	1.0000	0.9998	0.9991	0.9974	0.9946
	5	1.0000	1.0000	0.9999	0.9996	0.9989
	6	1.0000	1.0000	1.0000	0.9999	0.9998
	7	1.0000	1.0000	1.0000	1.0000	1.0000

p

		.06	.07	.08	.09	.10
N	k					
2	-2	0.0036	0.0049	0.0064	0.0081	0.0100
	-1	0.1092	0.1253	0.1408	0.1557	0.1700
	0	0.8908	0.8747	0.8592	0.8443	0.8300
	1	0.9964	0.9951	0.9936	0.9919	0.9900
	2	1.0000	1.0000	1.0000	1.0000	1.0000
3	-3	0.0002	0.0003	0.0005	0.0007	0.0010
	-2	0.0097	0.0130	0.0166	0.0207	0.0250
	1	0.1498	0.1693	0.1875	0.2044	0.2200
	0	0.8502	0.8307	0.8125	0.7956	0.7800
	1	0.9903	0.9870	0.9834	0.9793	0.9750
	2	0.9998	0.9997	0.9995	0.9993	0.9990
	3	1.0000	1.0000	1.0000	1.0000	1.0000
4	-4	0.0000	0.0000	0.0000	0.0001	0.0001
	-3	0.0008	0.0012	0.0018	0.0025	0.0033
	-2	0.0176	0.0230	0.0290	0.0354	0.0421
	-1	0.1834	0.2047	0.2238	0.2411	0.2565
	0	0.8166	0.7953	0.7762	0.7589	0.7435
	1	0.9824	0.9770	0.9710	0.9646	0.9579
	2	0.9992	0.9988	0.9982	0.9975	0.9967
	3	1.0000	1.0000	1.0000	0.9999	0.9999
	4	1.0000	1.0000	1.0000	1.0000	1.0000
5	-4	0.0001	0.0001	0.0002	0.0003	0.0004
	-3	0.0017	0.0027	0.0038	0.0052	0.0069
	-2	0.0265	0.0342	0.0424	0.0509	0.0597
	-1	0.2114	0.2333	0.2524	0.2692	0.2838
	0	0.7886	0.7667	0.7476	0.7308	0.7162
	1	0.9735	0.9658	0.9576	0.9491	0.9403
	2	0.9983	0.9973	0.9962	0.9948	0.9931
	3	0.9999	0.9999	0.9998	0.9997	0.9996
	4	1.0000	1.0000	1.0000	1.0000	1.0000
6	-4	0.0002	0.0003	0.0005	0.0007	0.0010
	-3	0.0031	0.0047	0.0066	0.0089	0.0115
	-2	0.0361	0.0460	0.0561	0.0665	0.0768
	-1	0.2350	0.2567	0.2753	0.2911	0.3046
	0	0.7650	0.7433	0.7247	0.7089	0.6954
	1	0.9639	0.9540	0.9439	0.9335	0.9232
	2	0.9969	0.9953	0.9934	0.9911	0.9885
	3	0.9998	0.9997	0.9995	0.9993	0.9990
	4	1.0000	1.0000	1.0000	1.0000	1.0000
7						

p

N	k	.06	.07	.08	.09	.10
7	-5	0.0000	0.0000	0.0001	0.0001	0.0001
	-4	0.0003	0.0006	0.0009	0.0014	0.0020
	-3	0.0049	0.0073	0.0101	0.0133	0.0170
	-2	0.0461	0.0578	0.0697	0.0815	0.0930
	-1	0.2548	0.2760	0.2937	0.3085	0.3209
	0	0.7452	0.7240	0.7063	0.6915	0.6791
	1	0.9539	0.9422	0.9303	0.9185	0.9070
	2	0.9951	0.9927	0.9899	0.9867	0.9830
	3	0.9997	0.9994	0.9991	0.9986	0.9980
	4	1.0000	1.0000	0.9999	0.9999	0.9999
8	5	1.0000	1.0000	1.0000	1.0000	1.0000
	-5	0.0000	0.0001	0.0001	0.0002	0.0003
	-4	0.0006	0.0010	0.0016	0.0023	0.0033
	-3	0.0071	0.0103	0.0141	0.0184	0.0231
	-2	0.0561	0.0696	0.0829	0.0958	0.1082
	-1	0.2717	0.2921	0.3088	0.3225	0.3339
	0	0.7283	0.7079	0.6912	0.6775	0.6661
	1	0.9439	0.9304	0.9171	0.9042	0.8918
	2	0.9929	0.9897	0.9859	0.9816	0.9769
	3	0.9994	0.9990	0.9984	0.9977	0.9967
9	4	1.0000	0.9999	0.9999	0.9998	0.9997
	5	1.0000	1.0000	1.0000	1.0000	1.0000
10	-5	0.0001	0.0001	0.0002	0.0004	0.0006
	-4	0.0009	0.0016	0.0025	0.0036	0.0050
	-3	0.0097	0.0138	0.0186	0.0239	0.0296
	-2	0.0661	0.0810	0.0954	0.1092	0.1223
	-1	0.2862	0.3056	0.3213	0.3341	0.3446
	0	0.7138	0.6944	0.6787	0.6659	0.6554
	1	0.9339	0.9190	0.9046	0.8908	0.8777
	2	0.9903	0.9862	0.9814	0.9761	0.9704
	3	0.9991	0.9984	0.9975	0.9964	0.9950
	4	0.9999	0.9999	0.9998	0.9996	0.9994
11	5	1.0000	1.0000	1.0000	1.0000	1.0000
	-6	0.0000	0.0000	0.0000	0.0001	0.0001
	-5	0.0001	0.0002	0.0004	0.0006	0.0010
	-4	0.0014	0.0023	0.0036	0.0051	0.0070
	-3	0.0125	0.0177	0.0235	0.0298	0.0364
	-2	0.0759	0.0920	0.1074	0.1218	0.1352
	-1	0.2986	0.3171	0.3318	0.3437	0.3534
12	0	0.7014	0.6829	0.6682	0.6563	0.6466
	1	0.9241	0.9080	0.8926	0.8782	0.8648
	2	0.9875	0.9823	0.9765	0.9702	0.9636

## P

		.06	.07	.08	.09	.10
N	k					
11	3	0.9986	0.9977	0.9964	0.9949	0.9930
	4	0.9999	0.9998	0.9996	0.9994	0.9990
	5	1.0000	1.0000	1.0000	0.9999	0.9999
	6	1.0000	1.0000	1.0000	1.0000	1.0000
	-6	0.0000	0.0000	0.0001	0.0001	0.0002
	-5	0.0002	0.0004	0.0006	0.0010	0.0015
	-4	0.0020	0.0033	0.0049	0.0069	0.0093
	-3	0.0157	0.0218	0.0286	0.0358	0.0434
	-2	0.0855	0.1026	0.1186	0.1335	0.1472
	-1	0.3094	0.3270	0.3408	0.3519	0.3609
	0	0.6906	0.6730	0.6592	0.6481	0.6391
	1	0.9145	0.8974	0.8814	0.8665	0.8528
	2	0.9843	0.9782	0.9714	0.9642	0.9566
	3	0.9980	0.9967	0.9951	0.9931	0.9907
	4	0.9998	0.9996	0.9994	0.9990	0.9985
12	5	1.0000	1.0000	0.9999	0.9999	0.9998
	6	1.0000	1.0000	1.0000	1.0000	1.0000
	-6	0.0000	0.0000	0.0001	0.0002	0.0003
	-5	0.0003	0.0005	0.0009	0.0014	0.0021
	-4	0.0027	0.0044	0.0065	0.0090	0.0119
	-3	0.0190	0.0262	0.0339	0.0420	0.0503
	-2	0.0947	0.1126	0.1292	0.1443	0.1582
	-1	0.3189	0.3355	0.3485	0.3589	0.3674
	0	0.6811	0.6645	0.6515	0.6411	0.6326
	1	0.9053	0.8874	0.8708	0.8557	0.8418
	2	0.9810	0.9738	0.9661	0.9580	0.9497
	3	0.9973	0.9956	0.9935	0.9910	0.9881
	4	0.9997	0.9995	0.9991	0.9986	0.9979
	5	1.0000	1.0000	0.9999	0.9998	0.9997
	6	1.0000	1.0000	1.0000	1.0000	1.0000
13	-6	0.0000	0.0001	0.0002	0.0003	0.0004
	-5	0.0004	0.0008	0.0013	0.0020	0.0029
	-4	0.0035	0.0056	0.0082	0.0113	0.0148
	-3	0.0226	0.0307	0.0393	0.0482	0.0573
	-2	0.1036	0.1222	0.1391	0.1544	0.1683
	-1	0.3272	0.3429	0.3552	0.3650	0.3730
	0	0.6728	0.6571	0.6448	0.6350	0.6270
	1	0.8964	0.8778	0.8609	0.8456	0.8317
	2	0.9774	0.9693	0.9607	0.9518	0.9427
	3	0.9965	0.9944	0.9918	0.9887	0.9852
	4	0.9996	0.9992	0.9987	0.9980	0.9971
	5	1.0000	0.9999	0.9998	0.9997	0.9996

## P

N	k	.06	.07	.08	.09	.10
14	6	1.0000	1.0000	1.0000	1.0000	0.9999
	-7	0.0000	0.0000	0.0000	0.0000	0.0001
	-6	0.0001	0.0001	0.0002	0.0004	0.0007
	-5	0.0006	0.0011	0.0018	0.0027	0.0039
	-4	0.0045	0.0070	0.0101	0.0138	0.0179
	-3	0.0263	0.0353	0.0448	0.0545	0.0641
	-2	0.1122	0.1312	0.1484	0.1638	0.1777
	-1	0.3345	0.3495	0.3611	0.3703	0.3779
	0	0.6655	0.6505	0.6389	0.6297	0.6221
	1	0.8878	0.8688	0.8516	0.8362	0.8223
	2	0.9737	0.9647	0.9552	0.9455	0.9359
	3	0.9955	0.9930	0.9899	0.9862	0.9821
	4	0.9994	0.9989	0.9982	0.9973	0.9961
	5	0.9999	0.9999	0.9998	0.9996	0.9993
15	6	1.0000	1.0000	1.0000	1.0000	0.9999
	7	1.0000	1.0000	1.0000	1.0000	1.0000
	-7	0.0000	0.0000	0.0000	0.0001	0.0001
	-6	0.0001	0.0002	0.0003	0.0006	0.0009
	-5	0.0008	0.0014	0.0023	0.0035	0.0049
	-4	0.0056	0.0086	0.0122	0.0164	0.0211
	-3	0.0302	0.0401	0.0503	0.0607	0.0709
	-2	0.1204	0.1398	0.1571	0.1726	0.1863
	-1	0.3410	0.3553	0.3663	0.3751	0.3823
	0	0.6590	0.6447	0.6337	0.6249	0.6177
	1	0.8796	0.8602	0.8429	0.8271	0.8137
	2	0.9698	0.9599	0.9497	0.9393	0.9291
	3	0.9944	0.9914	0.9878	0.9836	0.9789
	4	0.9992	0.9986	0.9977	0.9965	0.9951
16	5	0.9999	0.9998	0.9997	0.9994	0.9991
	6	1.0000	1.0000	1.0000	0.9999	0.9999
	7	1.0000	1.0000	1.0000	1.0000	1.0000
	-7	0.0000	0.0000	0.0001	0.0001	0.0002
	-6	0.0001	0.0003	0.0005	0.0008	0.0012
	-5	0.0010	0.0018	0.0029	0.0044	0.0062
	-4	0.0068	0.0103	0.0145	0.0193	0.0245
	-3	0.0341	0.0448	0.0558	0.0668	0.0774
	-2	0.1282	0.1479	0.1653	0.1807	0.1944
	-1	0.3469	0.3604	0.3709	0.3793	0.3862
	0	0.6531	0.6396	0.6291	0.6207	0.6138
	1	0.8718	0.8521	0.8347	0.8193	0.8056
	2	0.9659	0.9552	0.9442	0.9332	0.9226
	3	0.9932	0.9897	0.9855	0.9807	0.9755

## P

N	k	.06	.07	.08	.09	.10
17	4	0.9990	0.9982	0.9971	0.9956	0.9938
	5	0.9999	0.9997	0.9995	0.9992	0.9988
	6	1.0000	1.0000	0.9999	0.9999	0.9998
	7	1.0000	1.0000	1.0000	1.0000	1.0000
	-7	0.0000	0.0000	0.0001	0.0002	0.0003
	-6	0.0002	0.0003	0.0006	0.0011	0.0016
	-5	0.0013	0.0023	0.0037	0.0054	0.0075
	-4	0.0081	0.0121	0.0169	0.0222	0.0279
	-3	0.0381	0.0496	0.0613	0.0727	0.0838
	-2	0.1357	0.1555	0.1730	0.1883	0.2019
	-1	0.3521	0.3651	0.3751	0.3832	0.3898
	0	0.6479	0.6349	0.6249	0.6168	0.6102
	1	0.8643	0.8445	0.8270	0.8117	0.7981
	2	0.9619	0.9504	0.9387	0.9273	0.9162
18	3	0.9919	0.9879	0.9831	0.9778	0.9721
	4	0.9987	0.9977	0.9963	0.9946	0.9925
	5	0.9998	0.9997	0.9994	0.9989	0.9984
	6	1.0000	1.0000	0.9999	0.9998	0.9997
	7	1.0000	1.0000	1.0000	1.0000	1.0000
	-8	0.0000	0.0000	0.0000	0.0000	0.0001
	-7	0.0000	0.0001	0.0001	0.0002	0.0004
	-6	0.0002	0.0005	0.0008	0.0014	0.0021
	-5	0.0017	0.0029	0.0045	0.0065	0.0090
	-4	0.0095	0.0141	0.0194	0.0252	0.0315
	-3	0.0422	0.0544	0.0667	0.0786	0.0900
	-2	0.1428	0.1628	0.1802	0.1955	0.2089
	-1	0.3569	0.3693	0.3789	0.3867	0.3931
19	0	0.6431	0.6307	0.6211	0.6133	0.6069
	1	0.8572	0.8372	0.8198	0.8045	0.7911
	2	0.9578	0.9456	0.9333	0.9214	0.9100
	3	0.9905	0.9859	0.9806	0.9748	0.9685
	4	0.9983	0.9971	0.9955	0.9935	0.9910
	5	0.9998	0.9995	0.9992	0.9986	0.9979
	6	1.0000	0.9999	0.9999	0.9998	0.9996
	7	1.0000	1.0000	1.0000	1.0000	0.9999
	8	1.0000	1.0009	1.0000	1.0000	1.0000
	-8	0.0000	0.0000	0.0000	0.0000	0.0001
	-7	0.0000	0.0001	0.0002	0.0003	0.0005
	-6	0.0003	0.0006	0.0011	0.0017	0.0026
	-5	0.0020	0.0035	0.0054	0.0077	0.0105
	-4	0.0109	0.0161	0.0220	0.0283	0.0351
	-3	0.0462	0.0592	0.0720	0.0843	0.0961

p

N	k	.06	.07	.08	.09	.10
20	-2	0.1496	0.1697	0.1870	0.2022	0.2154
	-1	0.3612	0.3732	0.3824	0.3899	0.3960
	0	0.6388	0.6268	0.6176	0.6101	0.6040
	1	0.8504	0.8303	0.8130	0.7978	0.7846
	2	0.9538	0.9408	0.9280	0.9157	0.9039
	3	0.9891	0.9839	0.9780	0.9717	0.9649
	4	0.9980	0.9965	0.9946	0.9923	0.9895
	5	0.9997	0.9994	0.9989	0.9983	0.9974
	6	1.0000	0.9999	0.9998	0.9997	0.9995
	7	1.0000	1.0000	1.0000	1.0000	0.9999
	8	1.0000	1.0000	1.0000	1.0000	1.0000
21	-8	0.0000	0.0000	0.0000	0.0001	0.0001
	-7	0.0001	0.0001	0.0002	0.0004	0.0007
	-6	0.0004	0.0008	0.0013	0.0021	0.0032
	-5	0.0025	0.0042	0.0064	0.0091	0.0122
	-4	0.0125	0.0182	0.0246	0.0315	0.0387
	-3	0.0503	0.0639	0.0772	0.0899	0.1019
	-2	0.1561	0.1762	0.1935	0.2084	0.2215
	-1	0.3652	0.3767	0.3856	0.3928	0.3988
	0	0.6348	0.6233	0.6144	0.6072	0.6012
	1	0.8439	0.8238	0.8065	0.7916	0.7785
22	2	0.9497	0.9361	0.9228	0.9101	0.8981
	3	0.9875	0.9818	0.9754	0.9685	0.9613
	4	0.9975	0.9958	0.9936	0.9909	0.9878
	5	0.9996	0.9992	0.9987	0.9979	0.9968
	6	0.9999	0.9999	0.9998	0.9996	0.9993
	7	1.0000	1.0000	1.0000	0.9999	0.9999
	8	1.0000	1.0000	1.0000	1.0000	1.0000
	-8	0.0000	0.0000	0.0000	0.0001	0.0002
	-7	0.0001	0.0002	0.0003	0.0005	0.0009
	-6	0.0005	0.0010	0.0017	0.0026	0.0038
23	-5	0.0029	0.0049	0.0074	0.0105	0.0139
	-4	0.0142	0.0204	0.0274	0.0348	0.0424
	-3	0.0544	0.0686	0.0823	0.0953	0.1075
	-2	0.1623	0.1824	0.1995	0.2144	0.2273
	-1	0.3688	0.3799	0.3885	0.3955	0.4013
	0	0.6312	0.6201	0.6115	0.6045	0.5987
	1	0.8377	0.8176	0.8005	0.7856	0.7727
	2	0.9456	0.9314	0.9177	0.9047	0.8925
	3	0.9858	0.9796	0.9726	0.9652	0.9576
	4	0.9971	0.9951	0.9926	0.9895	0.9861
24	5	0.9995	0.9990	0.9983	0.9974	0.9962
	6	0.9999	0.9998	0.9997	0.9995	0.9991

p

		.06	.07	.08	.09	.10
N	k					
22	7	1.0000	1.0000	1.0000	0.9999	0.9998
	8	1.0000	1.0000	1.0000	1.0000	1.0000
	-8	0.0000	0.0000	0.0001	0.0001	0.0002
	-7	0.0001	0.0002	0.0004	0.0007	0.0011
	-6	0.0006	0.0012	0.0020	0.0031	0.0046
	-5	0.0035	0.0057	0.0086	0.0119	0.0158
	-4	0.0159	0.0227	0.0302	0.0380	0.0461
	-3	0.0585	0.0732	0.0873	0.1006	0.1130
	-2	0.1682	0.1882	0.2053	0.2200	0.2327
	-1	0.3722	0.3829	0.3912	0.3980	0.4036
	0	0.6278	0.6171	0.6088	0.6020	0.5964
	1	0.8318	0.8118	0.7947	0.7800	0.7673
	2	0.9415	0.9268	0.9127	0.8994	0.8870
	3	0.9841	0.9773	0.9698	0.9620	0.9539
	4	0.9965	0.9943	0.9914	0.9881	0.9842
23	5	0.9994	0.9988	0.9980	0.9969	0.9954
	6	0.9999	0.9998	0.9996	0.9993	0.9989
	7	1.0000	1.0000	0.9999	0.9999	0.9998
	8	1.0000	1.0000	1.0000	1.0000	1.0000
	-8	0.0000	0.0000	0.0001	0.0002	0.0003
	-7	0.0001	0.0003	0.0005	0.0009	0.0014
	-6	0.0008	0.0014	0.0024	0.0037	0.0053
	-5	0.0040	0.0066	0.0098	0.0135	0.0177
	-4	0.0177	0.0251	0.0330	0.0413	0.0497
	-3	0.0625	0.0777	0.0921	0.1057	0.1183
	-2	0.1739	0.1938	0.2107	0.2252	0.2378
	-1	0.3753	0.3857	0.3938	0.4003	0.4058
	0	0.6247	0.6143	0.6062	0.5997	0.5942
	1	0.8261	0.8062	0.7893	0.7748	0.7622
	2	0.9375	0.9223	0.9079	0.8943	0.8817
	3	0.9823	0.9749	0.9670	0.9587	0.9503
	4	0.9960	0.9934	0.9902	0.9865	0.9823
	5	0.9992	0.9986	0.9976	0.9963	0.9947
24	6	0.9999	0.9997	0.9995	0.9991	0.9986
	7	1.0000	1.0000	0.9999	0.9998	0.9997
	8	1.0000	1.0000	1.0000	1.0000	0.9999
	-9	0.0000	0.0000	0.0000	0.0000	0.0001
	-8	0.0000	0.0001	0.0001	0.0002	0.0004
	-7	0.0001	0.0003	0.0006	0.0011	0.0017
	-6	0.0009	0.0017	0.0028	0.0043	0.0062

N	k	.06	.07	.08	.09	.10
25	-3	0.0665	0.0821	0.0969	0.1106	0.1234
	-2	0.1793	0.1991	0.2159	0.2302	0.2427
	-1	0.3781	0.3882	0.3961	0.4025	0.4079
	0	0.6219	0.6118	0.6039	0.5975	0.5921
	1	0.8207	0.8009	0.7841	0.7698	0.7573
	2	0.9335	0.9179	0.9031	0.8894	0.8766
	3	0.9804	0.9725	0.9641	0.9554	0.9466
	4	0.9953	0.9925	0.9890	0.9849	0.9804
	5	0.9991	0.9983	0.9972	0.9957	0.9938
	6	0.9999	0.9997	0.9994	0.9989	0.9983
	7	1.0000	0.9999	0.9999	0.9998	0.9996
	8	1.0000	1.0000	1.0000	1.0000	0.9999
	9	1.0000	1.0000	1.0000	1.0000	1.0000
	-9	0.0000	0.0000	0.0000	0.0001	0.0001
	-8	0.0000	0.0001	0.0001	0.0003	0.0005
	-7	0.0002	0.0004	0.0008	0.0013	0.0020
	-6	0.0011	0.0020	0.0033	0.0050	0.0071
26	-5	0.0053	0.0085	0.0124	0.0168	0.0217
	-4	0.0215	0.0299	0.0388	0.0479	0.0570
	-3	0.0704	0.0865	0.1015	0.1155	0.1283
	-2	0.1844	0.2011	0.2208	0.2350	0.2473
	-1	0.3808	0.3906	0.3983	0.4046	0.4098
	0	0.6192	0.6094	0.6017	0.5954	0.5902
	1	0.8156	0.7959	0.7792	0.7650	0.7527
	2	0.9296	0.9135	0.8985	0.8845	0.8717
	3	0.9785	0.9701	0.9612	0.9521	0.9430
	4	0.9947	0.9915	0.9876	0.9832	0.9783
	5	0.9989	0.9980	0.9967	0.9950	0.9929
	6	0.9998	0.9996	0.9992	0.9987	0.9980
	7	1.0000	0.9999	0.9999	0.9997	0.9995
	8	1.0000	1.0000	1.0000	0.9999	0.9999
	9	1.0000	1.0000	1.0000	1.0000	1.0000
	-9	0.0000	0.0000	0.0000	0.0001	0.0001
	-8	0.0000	0.0001	0.0002	0.0003	0.0006
	-7	0.0002	0.0005	0.0009	0.0015	0.0023
	-6	0.0013	0.0024	0.0038	0.0057	0.0080
	-5	0.0061	0.0096	0.0138	0.0185	0.0237
	-4	0.0235	0.0324	0.0417	0.0512	0.0606
	-3	0.0743	0.0908	0.1060	0.1201	0.1331
	-2	0.1894	0.2090	0.2254	0.2395	0.2516
	-1	0.3833	0.3929	0.4004	0.4065	0.4116
	0	0.6166	0.6071	0.5996	0.5935	0.5884
	1	0.8106	0.7910	0.7746	0.7605	0.7484

N	k	.06	.07	.08	.09	.10
27	2	0.9257	0.9092	0.8940	0.8799	0.8669
	3	0.9765	0.9676	0.9583	0.9488	0.9394
	4	0.9939	0.9904	0.9862	0.9815	0.9763
	5	0.9987	0.9976	0.9962	0.9943	0.9920
	6	0.9998	0.9995	0.9991	0.9985	0.9977
	7	1.0000	0.9999	0.9998	0.9997	0.9994
	8	1.0000	1.0000	1.0000	0.9999	0.9999
	9	1.0000	1.0000	1.0000	1.0000	1.0000
	-9	0.0000	0.0000	0.0000	0.0001	0.0002
	-8	0.0000	0.0001	0.0002	0.0004	0.0007
	-7	0.0003	0.0006	0.0011	0.0018	0.0027
	-6	0.0015	0.0027	0.0044	0.0065	0.0090
	-5	0.0068	0.0106	0.0152	0.0203	0.0259
	-4	0.0255	0.0348	0.0446	0.0544	0.0642
	-3	0.0782	0.0949	0.1105	0.1247	0.1377
	-2	0.1941	0.2136	0.2299	0.2438	0.2558
	-1	0.3857	0.3950	0.4023	0.4083	0.4133
28	0	0.6143	0.6050	0.5977	0.5917	0.5867
	1	0.8059	0.7864	0.7701	0.7562	0.7442
	2	0.9218	0.9051	0.8896	0.8753	0.8623
	3	0.9745	0.9652	0.9554	0.9456	0.9358
	4	0.9932	0.9894	0.9848	0.9797	0.9741
	5	0.9985	0.9973	0.9956	0.9935	0.9910
	6	0.9997	0.9994	0.9989	0.9982	0.9973
	7	1.0000	0.9999	0.9998	0.9996	0.9993
	8	1.0000	1.0000	1.0000	0.9999	0.9998
	9	1.0000	1.0000	1.0000	1.0000	1.0000
	-9	0.0000	0.0000	0.0001	0.0001	0.0002
	-8	0.0001	0.0001	0.0003	0.0005	0.0009
	-7	0.0003	0.0007	0.0013	0.0021	0.0032
	-6	0.0018	0.0031	0.0050	0.0073	0.0101
	-5	0.0076	0.0118	0.0167	0.0221	0.0280
	-4	0.0275	0.0374	0.0475	0.0577	0.0677
	-3	0.0820	0.0990	0.1147	0.1291	0.1421
	-2	0.1987	0.2180	0.2341	0.2479	0.2597
	-1	0.3879	0.3970	0.4042	0.4100	0.4149

## P

N	k	.06	.07	.08	.09	.10
29	7	0.9999	0.9999	0.9997	0.9995	0.9991
	8	1.0000	1.0000	0.9999	0.9999	0.9998
	9	1.0000	1.0000	1.0000	1.0000	1.0000
	-9	0.0000	0.0000	0.0001	0.0001	0.0003
	-8	0.0001	0.0002	0.0003	0.0006	0.0010
	-7	0.0004	0.0008	0.0015	0.0024	0.0036
	-6	0.0020	0.0036	0.0056	0.0082	0.0112
	-5	0.0085	0.0130	0.0182	0.0240	0.0302
	-4	0.0296	0.0399	0.0504	0.0609	0.0712
	-3	0.0857	0.1031	0.1189	0.1333	0.1465
	-2	0.2030	0.2222	0.2382	0.2518	0.2635
	-1	0.3900	0.3989	0.4059	0.4116	0.4164
	0	0.6100	0.6011	0.5941	0.5884	0.5836
	1	0.7970	0.7778	0.7618	0.7482	0.7365
	2	0.9143	0.8969	0.8811	0.8667	0.8535
	3	0.9704	0.9601	0.9496	0.9391	0.9288
	4	0.9915	0.9870	0.9818	0.9760	0.9698
	5	0.9980	0.9964	0.9944	0.9918	0.9888
	6	0.9996	0.9992	0.9985	0.9976	0.9964
30	7	0.9999	0.9998	0.9997	0.9994	0.9990
	8	1.0000	1.0000	0.9999	0.9999	0.9997
	9	1.0000	1.0000	1.0000	1.0000	0.9999
	-10	0.0000	0.0000	0.0000	0.0000	0.0001
	-9	0.0000	0.0000	0.0001	0.0002	0.0003
	-8	0.0001	0.0002	0.0004	0.0007	0.0012
	-7	0.0005	0.0010	0.0017	0.0028	0.0041
	-6	0.0023	0.0040	0.0063	0.0091	0.0123
	-5	0.0093	0.0142	0.0198	0.0259	0.0324
	-4	0.0317	0.0424	0.0533	0.0641	0.0746
	-3	0.0894	0.1070	0.1230	0.1375	0.1506
	-2	0.2072	0.2262	0.2421	0.2555	0.2671
	-1	0.3920	0.4007	0.4075	0.4131	0.4178
	0	0.6080	0.5993	0.5925	0.5869	0.5822
	1	0.7928	0.7738	0.7579	0.7445	0.7329
	2	0.9106	0.8930	0.8770	0.8625	0.8494
	3	0.9683	0.9576	0.9467	0.9359	0.9254
	4	0.9907	0.9858	0.9802	0.9741	0.9676
	5	0.9977	0.9960	0.9937	0.9909	0.9877
	6	0.9995	0.9990	0.9983	0.9972	0.9959
	7	0.9999	0.9998	0.9996	0.9993	0.9988
	8	1.0000	1.0000	0.9999	0.9998	0.9997
	9	1.0000	1.0000	1.0000	1.0000	0.9999
	10	1.0000	1.0000	1.0000	1.0000	1.0060

p

		.11	.12	.13	.14	.15
N	k					
2	-2	0.0121	0.0144	0.0169	0.0196	0.0225
	-1	0.1837	0.1968	0.2093	0.2212	0.2325
	0	0.8163	0.8032	0.7907	0.7788	0.7675
	1	0.9879	0.9856	0.9831	0.9804	0.9775
	2	1.0000	1.0000	1.0000	1.0000	1.0000
3	-3	0.0013	0.0017	0.0022	0.0027	0.0034
	-2	0.0296	0.0346	0.0397	0.0451	0.0506
	-1	0.2344	0.2477	0.2599	0.2710	0.2813
	0	0.7656	0.7523	0.7401	0.7290	0.7188
	1	0.9704	0.9654	0.9603	0.9549	0.9494
	2	0.9987	0.9983	0.9978	0.9973	0.9966
	3	1.0000	1.0000	1.0000	1.0000	1.0000
4	-4	0.0001	0.0002	0.0003	0.0004	0.0005
	-3	0.0043	0.0055	0.0068	0.0083	0.0100
	-2	0.0491	0.0562	0.0635	0.0708	0.0781
	-1	0.2703	0.2827	0.2937	0.3035	0.3123
	0	0.7297	0.7173	0.7063	0.6965	0.6877
	1	0.9509	0.9438	0.9365	0.9292	0.9219
5	2	0.9957	0.9945	0.9932	0.9917	0.9900
	3	0.9999	0.9998	0.9997	0.9996	0.9995
	4	1.0000	1.0000	1.0000	1.0000	1.0000
	-5	0.0000	0.0000	0.0000	0.0001	0.0001
	-4	0.0006	0.0008	0.0011	0.0014	0.0018
6	-3	0.0088	0.0109	0.0133	0.0159	0.0188
	-2	0.0685	0.0773	0.0860	0.0946	0.1030
	-1	0.2965	0.3076	0.3174	0.3259	0.3335
	0	0.7035	0.6924	0.6826	0.6741	0.6665
	1	0.9315	0.9227	0.9140	0.9054	0.8970
	2	0.9912	0.9891	0.9867	0.9841	0.9812
	3	0.9994	0.9992	0.9989	0.9986	0.9982
	4	1.0000	1.0000	1.0000	0.9999	0.9999
	5	1.0000	1.0000	1.0000	1.0000	1.0000
	-5	0.0001	0.0001	0.0002	0.0002	0.0003
	-4	0.0014	0.0019	0.0025	0.0033	0.0041
	-3	0.0144	0.0177	0.0212	0.0249	0.0289
	-2	0.0870	0.0970	0.1066	0.1160	0.1250
	-1	0.3162	0.3262	0.3348	0.3423	0.3489
	0	0.6838	0.6738	0.6652	0.6577	0.6511
	1	0.9130	0.9030	0.8934	0.8840	0.8750
	2	0.9856	0.9823	0.9788	0.9751	0.9711

## P

		.11	.12	.13	.14	.15
N	k					
7	3	0.9986	0.9981	0.9975	0.9967	0.9959
	4	0.9999	0.9999	0.9998	0.9998	0.9997
	5	1.0000	1.0000	1.0000	1.0000	1.0000
	-6	0.0000	0.0000	0.0000	0.0000	0.0001
	-5	0.0002	0.0003	0.0005	0.0006	0.0009
	-4	0.0027	0.0036	0.0047	0.0059	0.0073
	-3	0.0210	0.0253	0.0299	0.0346	0.0396
	-2	0.1042	0.1150	0.1252	0.1349	0.1441
	-1	0.3314	0.3404	0.3481	0.3548	0.3606
	0	0.6686	0.6596	0.6519	0.6452	0.6394
	1	0.8958	0.8850	0.8748	0.8651	0.8559
	2	0.9790	0.9747	0.9701	0.9654	0.9604
	3	0.9973	0.9964	0.9953	0.9941	0.9927
	4	0.9998	0.9997	0.9995	0.9994	0.9991
8	5	1.0000	1.0000	1.0000	1.0000	0.9999
	6	1.0000	1.0000	1.0000	1.0000	1.0000
	-6	0.0000	0.0001	0.0001	0.0001	0.0002
	-5	0.0005	0.0007	0.0009	0.0013	0.0017
	-4	0.0044	0.0058	0.0074	0.0092	0.0111
	-3	0.0281	0.0335	0.0390	0.0447	0.0504
	-2	0.1201	0.1312	0.1418	0.1517	0.1609
	-1	0.3435	0.3516	0.3586	0.3647	0.3700
	0	0.6565	0.6484	0.6414	0.6353	0.6300
	1	0.8799	0.8688	0.8582	0.8483	0.8391
	2	0.9719	0.9665	0.9610	0.9553	0.9496
	3	0.9956	0.9942	0.9926	0.9908	0.9889
	4	0.9995	0.9993	0.9991	0.9987	0.9983
	5	1.0000	0.9999	0.9999	0.9999	0.9998
	6	1.0000	1.0000	1.0000	1.0000	1.0000
9	-6	0.0001	0.0001	0.0002	0.0003	0.0004
	-5	0.0009	0.0012	0.0017	0.0022	0.0029
	-4	0.0066	0.0085	0.0107	0.0130	0.0156
	-3	0.0356	0.0419	0.0482	0.0547	0.0611
	-2	0.1345	0.1460	0.1566	0.1665	0.1757
	-1	0.3534	0.3608	0.3672	0.3727	0.3776
	0	0.6466	0.6392	0.6328	0.6273	0.6224
	1	0.8655	0.8540	0.8434	0.8335	0.8243
	2	0.9644	0.9581	0.9518	0.9453	0.9389
	3	0.9934	0.9915	0.9893	0.9870	0.9844
	4	0.9991	0.9988	0.9983	0.9978	0.9971
	5	0.9999	0.9999	0.9998	0.9997	0.9996
	6	1.0000	1.0000	1.0000	1.0000	1.0000

## P

N	k	.11	.12	.13	.14	.15
10	-7	0.0000	0.0000	0.0000	0.0001	0.0001
	-6	0.0002	0.0002	0.0004	0.0005	0.0007
	-5	0.0014	0.0020	0.0026	0.0035	0.0044
	-4	0.0092	0.0116	0.0144	0.0173	0.0205
	-3	0.0433	0.0504	0.0574	0.0645	0.0715
	-2	0.1477	0.1592	0.1699	0.1797	0.1888
	-1	0.3615	0.3684	0.3743	0.3795	0.3840
	0	0.6385	0.6316	0.6257	0.6205	0.6160
	1	0.8523	0.8408	0.8301	0.8203	0.8112
	2	0.9567	0.9496	0.9426	0.9355	0.9285
	3	0.9908	0.9884	0.9856	0.9827	0.9795
	4	0.9986	0.9980	0.9974	0.9965	0.9956
	5	0.9998	0.9998	0.9996	0.9995	0.9993
	6	1.0000	1.0000	1.0000	0.9999	0.9999
	7	1.0000	1.0000	1.0000	1.0000	1.0000
11	-7	0.0000	0.0000	0.0001	0.0001	0.0002
	-6	0.0003	0.0004	0.0006	0.0009	0.0012
	-5	0.0021	0.0029	0.0039	0.0050	0.0063
	-4	0.0121	0.0151	0.0184	0.0220	0.0258
	-3	0.0510	0.0588	0.0665	0.0740	0.0814
	-2	0.1598	0.1713	0.1819	0.1916	0.2005
	-1	0.3685	0.3749	0.3804	0.3853	0.3895
	0	0.6315	0.6251	0.6196	0.6147	0.6105
	1	0.8402	0.8287	0.8181	0.8084	0.7995
	2	0.9490	0.9412	0.9335	0.9260	0.9186
	3	0.9879	0.9849	0.9816	0.9780	0.9742
	4	0.9979	0.9971	0.9961	0.9950	0.9937
	5	0.9997	0.9996	0.9994	0.9991	0.9988
	6	1.0000	1.0000	0.9999	0.9999	0.9998
	7	1.0000	1.0000	1.0000	1.0000	1.0000
12	-7	0.0001	0.0001	0.0001	0.0002	0.0003
	-6	0.0005	0.0007	0.0010	0.0013	0.0018
	-5	0.0030	0.0041	0.0053	0.0068	0.0084
	-4	0.0153	0.0189	0.0228	0.0269	0.0312
	-3	0.0587	0.0670	0.0752	0.0832	0.0909
	-2	0.1708	0.1822	0.1927	0.2022	0.2110
	-1	0.3745	0.3805	0.3857	0.3903	0.3943
	0	0.6255	0.6195	0.6143	0.6097	0.6057
	1	0.8292	0.8178	0.8073	0.7978	0.7890
	2	0.9413	0.9330	0.9248	0.9168	0.9091
	3	0.9847	0.9811	0.9772	0.9731	0.9688
	4	0.9970	0.9959	0.9947	0.9932	0.9916

## P

N	k	.11	.12	.13	.14	.15
13	5	0.9995	0.9993	0.9990	0.9987	0.9982
	6	0.9999	0.9999	0.9999	0.9998	0.9997
	7	1.0000	1.0000	1.0000	1.0000	1.0000
	-8	0.0000	0.0000	0.0000	0.0000	0.0001
	-7	0.0001	0.0001	0.0002	0.0003	0.0005
	-6	0.0007	0.0010	0.0014	0.0019	0.0026
	-5	0.0041	0.0055	0.0070	0.0088	0.0108
	-4	0.0187	0.0229	0.0273	0.0320	0.0367
	-3	0.0663	0.0751	0.0837	0.0920	0.1000
	-2	0.1808	0.1922	0.2025	0.2119	0.2205
	-1	0.3797	0.3854	0.3903	0.3947	0.3985
	0	0.6203	0.6146	0.6097	0.6053	0.6015
	1	0.8192	0.8078	0.7975	0.7881	0.7795
	2	0.9337	0.9249	0.9163	0.9080	0.9000
	3	0.9813	0.9771	0.9727	0.9680	0.9633
14	4	0.9959	0.9945	0.9930	0.9912	0.9892
	5	0.9993	0.9990	0.9986	0.9981	0.9974
	6	0.9999	0.9999	0.9998	0.9997	0.9995
	7	1.0000	1.0000	1.0000	1.0000	0.9999
	8	1.0000	1.0000	1.0000	1.0000	1.0000
	-8	0.0000	0.0000	0.0000	0.0001	0.0001
	-7	0.0001	0.0002	0.0004	0.0005	0.0007
	-6	0.0010	0.0014	0.0020	0.0027	0.0035
	-5	0.0053	0.0070	0.0090	0.0111	0.0135
	-4	0.0223	0.0271	0.0320	0.0371	0.0423
	-3	0.0736	0.0829	0.0918	0.1004	0.1086
	-2	0.1901	0.2013	0.2115	0.2207	0.2291
	-1	0.3843	0.3897	0.3944	0.3986	0.4023
15	0	0.6157	0.6103	0.6056	0.6014	0.5977
	1	0.8099	0.7987	0.7885	0.7793	0.7709
	2	0.9264	0.9171	0.9082	0.8996	0.8914
	3	0.9777	0.9729	0.9680	0.9629	0.9577
	4	0.9947	0.9930	0.9910	0.9839	0.9865
	5	0.9990	0.9986	0.9980	0.9973	0.9965
	6	0.9999	0.9998	0.9996	0.9995	0.9993
	7	1.0000	1.0000	1.0000	0.9999	0.9999
	8	1.0000	1.0000	1.0000	1.0000	1.0000
	-8	0.0000	0.0001	0.0001	0.0001	0.0002
	-7	0.0002	0.0004	0.0005	0.0008	0.0010
	-6	0.0014	0.0020	0.0027	0.0036	0.0046
	-5	0.0067	0.0087	0.0110	0.0136	0.0163

		P				
N	k	.11	.12	.13	.14	.15
16	-4	0.0261	0.0314	0.0368	0.0423	0.0479
	-3	0.0808	0.0904	0.0996	0.1084	0.1167
	-2	0.1987	0.2097	0.2197	0.2287	0.2370
	-1	0.3884	0.3936	0.3981	0.4021	0.4056
	0	0.6116	0.6064	0.6019	0.5979	0.5944
	1	0.8013	0.7903	0.7803	0.7713	0.7630
	2	0.9192	0.9096	0.9004	0.8916	0.8833
	3	0.9739	0.9686	0.9632	0.9577	0.9521
	4	0.9933	0.9913	0.9890	0.9864	0.9837
	5	0.9986	0.9980	0.9973	0.9964	0.9954
	6	0.9998	0.9996	0.9995	0.9992	0.9990
	7	1.0000	0.9999	0.9999	0.9999	0.9998
	8	1.0000	1.0000	1.0000	1.0000	1.0000
	-8	0.0000	0.0001	0.0001	0.0002	0.0003
	-7	0.0003	0.0005	0.0008	0.0011	0.0014
	-6	0.0018	0.0026	0.0035	0.0046	0.0058
17	-5	0.0083	0.0106	0.0133	0.0162	0.0193
	-4	0.0300	0.0357	0.0416	0.0476	0.0535
	-3	0.0877	0.0976	0.1070	0.1160	0.1244
	-2	0.2066	0.2175	0.2273	0.2362	0.2442
	-1	0.3921	0.3971	0.4014	0.4052	0.4086
	0	0.6079	0.6029	0.5986	0.5948	0.5914
	1	0.7934	0.7825	0.7727	0.7638	0.7558
	2	0.9123	0.9024	0.8930	0.8840	0.8756
	3	0.9700	0.9643	0.9584	0.9524	0.9465
	4	0.9917	0.9894	0.9867	0.9838	0.9807
	5	0.9982	0.9974	0.9965	0.9954	0.9942
	6	0.9997	0.9995	0.9992	0.9989	0.9986
	7	1.0000	0.9999	0.9999	0.9998	0.9997
	8	1.0000	1.0000	1.0000	1.0000	0.9999
	-9	0.0000	0.0000	0.0000	0.0001	0.0001
	-8	0.0001	0.0001	0.0002	0.0003	0.0004
	-7	0.0005	0.0007	0.0010	0.0014	0.0019
	-6	0.0024	0.0033	0.0044	0.0057	0.0072
	-5	0.0099	0.0127	0.0157	0.0190	0.0224
	-4	0.0339	0.0401	0.0464	0.0527	0.0590
	-3	0.0945	0.1046	0.1142	0.1232	0.1318
	-2	0.2139	0.2246	0.2343	0.2430	0.2509
	-1	0.3954	0.4002	0.4044	0.4081	0.4114
	0	0.6046	0.5998	0.5956	0.5919	0.5886
	1	0.7861	0.7754	0.7657	0.7570	0.7491
	2	0.9055	0.8954	0.8858	0.8768	0.8682
	3	0.9661	0.9599	0.9536	0.9473	0.9410
	4	0.9901	0.9873	0.9843	0.9810	0.9776

p

N	k	.11	.12	.13	.14	.15
18	5	0.9976	0.9967	0.9956	0.9943	0.9928
	6	0.9995	0.9993	0.9990	0.9986	0.9981
	7	0.9999	0.9999	0.9998	0.9997	0.9996
	8	1.0000	1.0000	1.0000	0.9999	0.9999
	9	1.0000	1.0000	1.0000	1.0000	1.0000
	-9	0.0000	0.0000	0.0000	0.0001	0.0001
	-8	0.0001	0.0002	0.0003	0.0004	0.0006
	-7	0.0006	0.0010	0.0014	0.0019	0.0025
	-6	0.0030	0.0041	0.0054	0.0070	0.0087
	-5	0.0117	0.0149	0.0182	0.0218	0.0256
	-4	0.0380	0.0446	0.0512	0.0579	0.0644
	-3	0.1009	0.1113	0.1210	0.1301	0.1387
	-2	0.2207	0.2313	0.2408	0.2493	0.2571
	-1	0.3984	0.4031	0.4071	0.4107	0.4139
	0	0.6016	0.5969	0.5929	0.5893	0.5861
	1	0.7793	0.7687	0.7592	0.7507	0.7429
	2	0.8991	0.8887	0.8790	0.8699	0.8613
	3	0.9620	0.9554	0.9488	0.9421	0.9356
	4	0.9883	0.9851	0.9818	0.9782	0.9744
	5	0.9970	0.9959	0.9946	0.9930	0.9913
	6	0.9994	0.9990	0.9986	0.9981	0.9975
	7	0.9999	0.9998	0.9997	0.9996	0.9994
	8	1.0000	1.0000	1.0000	0.9999	0.9999
	9	1.0000	1.0000	1.0000	1.0000	1.0000
19	-9	0.0000	0.0000	0.0001	0.0001	0.0002
	-8	0.0002	0.0003	0.0004	0.0006	0.0008
	-7	0.0008	0.0012	0.0018	0.0024	0.0031
	-6	0.0037	0.0050	0.0066	0.0083	0.0103
	-5	0.0137	0.0171	0.0209	0.0248	0.0289
	-4	0.0420	0.0490	0.0560	0.0629	0.0698
	-3	0.1072	0.1177	0.1275	0.1367	0.1453
	-2	0.2271	0.2375	0.2468	0.2552	0.2629
	-1	0.4012	0.4057	0.4097	0.4131	0.4162
	0	0.5988	0.5943	0.5903	0.5869	0.5838
	1	0.7729	0.7625	0.7532	0.7448	0.7371
	2	0.8928	0.8823	0.8725	0.8633	0.8547
	3	0.9580	0.9510	0.9440	0.9371	0.9302
	4	0.9863	0.9829	0.9791	0.9752	0.9711
	5	0.9963	0.9950	0.9934	0.9917	0.9897
	6	0.9992	0.9988	0.9982	0.9976	0.9969
	7	0.9998	0.9997	0.9996	0.9994	0.9992
	8	1.0000	1.0000	0.9999	0.9999	0.9998
	9	1.0000	1.0000	1.0000	1.0000	1.0000

p

N	k	.11	.12	.13	.14	.15
20	-9	0.0000	0.0001	0.0001	0.0002	0.0003
	-8	0.0002	0.0003	0.0005	0.0008	0.0011
	-7	0.0011	0.0016	0.0022	0.0030	0.0039
	-6	0.0045	0.0060	0.0078	0.0098	0.0120
	-5	0.0157	0.0195	0.0236	0.0278	0.0322
	-4	0.0461	0.0534	0.0607	0.0679	0.0750
	-3	0.1132	0.1238	0.1337	0.1430	0.1516
	-2	0.2331	0.2433	0.2525	0.2608	0.2683
	-1	0.4038	0.4082	0.4120	0.4153	0.4184
	0	0.5962	0.5918	0.5880	0.5847	0.5816
	1	0.7669	0.7567	0.7475	0.7393	0.7317
	2	0.8868	0.8762	0.8663	0.8570	0.8484
	3	0.9539	0.9466	0.9393	0.9321	0.9250
	4	0.9843	0.9805	0.9764	0.9722	0.9678
	5	0.9955	0.9940	0.9922	0.9902	0.9880
	6	0.9989	0.9984	0.9978	0.9970	0.9961
	7	0.9998	0.9997	0.9995	0.9992	0.9989
	8	1.0000	0.9999	0.9999	0.9998	0.9997
	9	1.0000	1.0000	1.0000	1.0000	0.9999
21	-10	0.0000	0.0000	0.0000	0.0000	0.0001
	-9	0.0001	0.0001	0.0002	0.0002	0.0003
	-8	0.0003	0.0005	0.0007	0.0010	0.0014
	-7	0.0014	0.0020	0.0027	0.0036	0.0047
	-6	0.0053	0.0071	0.0091	0.0114	0.0138
	-5	0.0178	0.0220	0.0264	0.0309	0.0356
	-4	0.0501	0.0578	0.0654	0.0728	0.0801
	-3	0.1190	0.1297	0.1397	0.1490	0.1576
	-2	0.2387	0.2488	0.2578	0.2659	0.2733
	-1	0.4062	0.4104	0.4141	0.4174	0.4203
	0	0.5938	0.5896	0.5859	0.5826	0.5797
	1	0.7613	0.7512	0.7422	0.7341	0.7267
	2	0.8810	0.8703	0.8603	0.8510	0.8424
	3	0.9499	0.9422	0.9346	0.9272	0.9199
	4	0.9822	0.9780	0.9736	0.9691	0.9644
	5	0.9947	0.9929	0.9909	0.9886	0.9862
	6	0.9986	0.9980	0.9973	0.9964	0.9953
	7	0.9997	0.9995	0.9993	0.9990	0.9986
	8	0.9999	0.9999	0.9998	0.9998	0.9997
	9	1.0000	1.0000	1.0000	1.0000	0.9999
	10	1.0000	1.0000	1.0000	1.0000	1.0000
22	-10	0.0000	0.0000	0.0000	0.0001	0.0001
	-9	0.0001	0.0001	0.0002	0.0003	0.0005

p

N	k	.11	.12	.13	.14	.15
11	-8	0.0004	0.0006	0.0009	0.0013	0.0017
	-7	0.0017	0.0024	0.0033	0.0043	0.0055
	-6	0.0063	0.0083	0.0105	0.0130	0.0157
	-5	0.0200	0.0245	0.0292	0.0341	0.0390
	-4	0.0541	0.0621	0.0700	0.0776	0.0850
	-3	0.1246	0.1354	0.1454	0.1547	0.1633
	-2	0.2439	0.2539	0.2627	0.2707	0.2780
	-1	0.4084	0.4125	0.4161	0.4193	0.4222
	0	0.5916	0.5875	0.5839	0.5807	0.5778
	1	0.7561	0.7461	0.7373	0.7293	0.7220
	2	0.8754	0.8646	0.8546	0.8453	0.8367
	3	0.9459	0.9379	0.9300	0.9224	0.9150
	4	0.9800	0.9755	0.9708	0.9659	0.9610
	5	0.9937	0.9917	0.9895	0.9870	0.9843
	6	0.9983	0.9976	0.9967	0.9957	0.9945
	7	0.9996	0.9994	0.9991	0.9987	0.9983
	8	0.9999	0.9999	0.9998	0.9997	0.9995
	9	1.0000	1.0000	1.0000	0.9999	0.9999
	10	1.0000	1.0000	1.0000	1.0000	1.0000
24	-10	0.0000	0.0000	0.0001	0.0001	0.0001
	-9	0.0001	0.0002	0.0003	0.0004	0.0006
	-8	0.0005	0.0008	0.0011	0.0016	0.0021
	-7	0.0020	0.0029	0.0039	0.0051	0.0065
	-6	0.0073	0.0095	0.0120	0.0148	0.0177
	-5	0.0222	0.0271	0.0321	0.0372	0.0424
	-4	0.0581	0.0664	0.0745	0.0823	0.0899
	-3	0.1300	0.1408	0.1508	0.1601	0.1688
	-2	0.2489	0.2587	0.2674	0.2753	0.2824
	-1	0.4105	0.4145	0.4180	0.4211	0.4239
	0	0.5895	0.5855	0.5820	0.5789	0.5761
	1	0.7511	0.7413	0.7326	0.7247	0.7176
	2	0.8700	0.8592	0.8492	0.8399	0.8312
	3	0.9419	0.9336	0.9255	0.9177	0.9101
	4	0.9778	0.9729	0.9679	0.9628	0.9576
	5	0.9927	0.9905	0.9880	0.9852	0.9823
	6	0.9980	0.9971	0.9961	0.9949	0.9935
	7	0.9995	0.9992	0.9989	0.9984	0.9979
	8	0.9999	0.9998	0.9997	0.9996	0.9994
	9	1.0000	1.0000	0.9999	0.9999	0.9999
	10	1.0000	1.0000	1.0000	1.0000	1.0000

## P

N	k	.11	.12	.13	.14	.15
25	-7	0.0024	0.0034	0.0046	0.0060	0.0075
	-6	0.0083	0.0108	0.0136	0.0165	0.0197
	-5	0.0245	0.0297	0.0350	0.0404	0.0458
	-4	0.0621	0.0706	0.0789	0.0869	0.0946
	-3	0.1351	0.1460	0.1561	0.1654	0.1740
	-2	0.2536	0.2632	0.2718	0.2796	0.2866
	-1	0.4124	0.4163	0.4197	0.4228	0.4255
	0	0.5876	0.5837	0.5803	0.5772	0.5745
	1	0.7464	0.7368	0.7282	0.7204	0.7134
	2	0.8649	0.8540	0.8439	0.8346	0.8260
	3	0.9379	0.9294	0.9211	0.9131	0.9054
	4	0.9755	0.9703	0.9650	0.9596	0.9542
	5	0.9917	0.9892	0.9864	0.9835	0.9803
	6	0.9976	0.9966	0.9954	0.9940	0.9925
	7	0.9994	0.9991	0.9986	0.9981	0.9975
	8	0.9999	0.9998	0.9996	0.9995	0.9992
	9	1.0000	1.0000	0.9999	0.9999	0.9998
	10	1.0000	1.0000	1.0000	1.0000	1.0000
26	-10	0.0000	0.0001	0.0001	0.0002	0.0003
	-9	0.0002	0.0003	0.0004	0.0007	0.0009
	-8	0.0008	0.0012	0.0017	0.0023	0.0030
	-7	0.0029	0.0040	0.0053	0.0069	0.0086
	-6	0.0095	0.0122	0.0152	0.0184	0.0218
	-5	0.0269	0.0323	0.0379	0.0436	0.0492
	-4	0.0660	0.0747	0.0832	0.0914	0.0992
	-3	0.1401	0.1510	0.1611	0.1704	0.1790
	-2	0.2580	0.2675	0.2760	0.2836	0.2905
	-1	0.4142	0.4180	0.4214	0.4244	0.4270
	0	0.5858	0.5820	0.5786	0.5756	0.5730
	1	0.7420	0.7325	0.7240	0.7164	0.7095
	2	0.8599	0.8490	0.8389	0.8296	0.8210
	3	0.9340	0.9253	0.9168	0.9086	0.9008
	4	0.9731	0.9677	0.9621	0.9564	0.9508
	5	0.9905	0.9878	0.9848	0.9816	0.9782
	6	0.9971	0.9960	0.9947	0.9931	0.9914
	7	0.9992	0.9988	0.9983	0.9977	0.9970
	8	0.9998	0.9997	0.9996	0.9993	0.9991
	9	1.0000	0.9999	0.9999	0.9998	0.9997
	10	1.0000	1.0000	1.0000	1.0000	0.9999

P

N	k	.11	.12	.13	.14	.15
27	-7	0.0034	0.0047	0.0061	0.0078	0.0097
	-6	0.0107	0.0136	0.0169	0.0203	0.0239
	-5	0.0293	0.0350	0.0408	0.0467	0.0526
	-4	0.0698	0.0788	0.0874	0.0957	0.1037
	-3	0.1449	0.1559	0.1659	0.1752	0.1837
	-2	0.2622	0.2716	0.2800	0.2875	0.2943
	-1	0.4159	0.4196	0.4229	0.4258	0.4284
	0	0.5841	0.5804	0.5771	0.5742	0.5716
	1	0.7378	0.7284	0.7200	0.7125	0.7057
	2	0.8551	0.8441	0.8341	0.8248	0.8163
	3	0.9302	0.9212	0.9126	0.9043	0.8963
	4	0.9707	0.9650	0.9592	0.9533	0.9474
	5	0.9893	0.9864	0.9831	0.9797	0.9761
	6	0.9966	0.9953	0.9939	0.9922	0.9903
	7	0.9991	0.9986	0.9980	0.9973	0.9965
	8	0.9998	0.9996	0.9994	0.9992	0.9989
	9	1.0000	0.9999	0.9999	0.9998	0.9997
	10	1.0000	1.0000	1.0000	0.9999	0.9999
	11	1.0000	1.0000	1.0000	1.0000	1.0000
28	-11	0.0000	0.0000	0.0000	0.0001	0.0001
	-10	0.0001	0.0001	0.0002	0.0003	0.0004
	-9	0.0003	0.0005	0.0007	0.0010	0.0014
	-8	0.0011	0.0017	0.0023	0.0032	0.0041
	-7	0.0039	0.0053	0.0070	0.0089	0.0109
	-6	0.0119	0.0151	0.0186	0.0223	0.0261
	-5	0.0317	0.0377	0.0438	0.0499	0.0560
	-4	0.0736	0.0828	0.0916	0.1000	0.1080
	-3	0.1496	0.1605	0.1705	0.1798	0.1883
	-2	0.2662	0.2755	0.2837	0.2911	0.2978
	-1	0.4175	0.4212	0.4241	0.4272	0.4298
	0	0.5825	0.5788	0.5756	0.5728	0.5702
	1	0.7338	0.7245	0.7163	0.7089	0.7022
	2	0.8504	0.8395	0.8295	0.8202	0.8117
	3	0.9264	0.9172	0.9084	0.9000	0.8920
	4	0.9683	0.9623	0.9562	0.9501	0.9440
	5	0.9881	0.9849	0.9814	0.9777	0.9739
	6	0.9961	0.9947	0.9930	0.9911	0.9891
	7	0.9989	0.9982	0.9977	0.9969	0.9959
	8	0.9997	0.9995	0.9993	0.9990	0.9986
	9	0.9999	0.9999	0.9998	0.9997	0.9996
	10	1.0000	1.0000	1.0000	0.9999	0.9999
	11	1.0000	1.0000	1.0000	1.0000	1.0000
	-11	0.0000	0.0000	0.0001	0.0001	0.0001

## P

N	k	.11	.12	.13	.14	.15
29	-10	0.0001	0.0001	0.0002	0.0004	0.0005
	9	0.0004	0.0006	0.0008	0.0012	0.0016
	-8	0.0013	0.0020	0.0027	0.0036	0.0047
	-7	0.0045	0.0061	0.0079	0.0099	0.0122
	-6	0.0132	0.0167	0.0204	0.0243	0.0283
	-5	0.0341	0.0404	0.0467	0.0530	0.0593
	-4	0.0774	0.0867	0.0956	0.1041	0.1123
	-3	0.1541	0.1650	0.1750	0.1842	0.1927
	-2	0.2700	0.2792	0.2873	0.2946	0.3012
	-1	0.4190	0.4226	0.4258	0.4286	0.4311
	0	0.5810	0.5774	0.5742	0.5714	0.5689
	1	0.7300	0.7208	0.7127	0.7054	0.6988
	2	0.8459	0.8350	0.8250	0.8158	0.8073
	3	0.9226	0.9133	0.9044	0.8959	0.8877
	4	0.9659	0.9596	0.9533	0.9470	0.9407
	5	0.9868	0.9833	0.9796	0.9757	0.9717
	6	0.9955	0.9939	0.9921	0.9901	0.9878
	7	0.9987	0.9980	0.9973	0.9964	0.9953
	8	0.9996	0.9994	0.9992	0.9988	0.9984
	9	0.9999	0.9999	0.9998	0.9996	0.9995
	10	1.0000	1.0000	0.9999	0.9999	0.9999
	11	1.0000	1.0000	1.0000	1.0000	1.0000
	-11	0.0000	0.0000	0.0001	0.0001	0.0002
	-10	0.0001	0.0002	0.0003	0.0004	0.0006
	-9	0.0004	0.0007	0.0010	0.0014	0.0019
	-8	0.0016	0.0023	0.0032	0.0042	0.0054
	-7	0.0051	0.0068	0.0088	0.0111	0.0135
	-6	0.0145	0.0182	0.0222	0.0263	0.0305
	-5	0.0366	0.0431	0.0496	0.0562	0.0626
	-4	0.0811	0.0905	0.0996	0.1082	0.1164
	-3	0.1584	0.1693	0.1793	0.1884	0.1969
	-2	0.2737	0.2827	0.2907	0.2979	0.3044
	-1	0.4204	0.4240	0.4271	0.4298	0.4323
	0	0.5796	0.5760	0.5729	0.5702	0.5677
	1	0.7263	0.7173	0.7093	0.7021	0.6956
	2	0.8416	0.8307	0.8207	0.8116	0.8031
	3	0.9189	0.9095	0.9004	0.8918	0.8836
	4	0.9634	0.9569	0.9504	0.9448	0.9374
	5	0.9854	0.9818	0.9778	0.9737	0.9695
	6	0.9949	0.9932	0.9912	0.9889	0.9865
	7	0.9984	0.9977	0.9968	0.9958	0.9946
	8	0.9996	0.9993	0.9990	0.9986	0.9981
	9	0.9999	0.9998	0.9997	0.9996	0.9994
	10	1.0000	1.0000	0.9999	0.9999	0.9998
	11	1.0000	1.0000	1.0000	1.0000	1.0000

P

N	k	.11	.12	.13	.14	.15
30	-11	0.0000	0.0001	0.0001	0.0001	0.0002
	-10	0.0001	0.0002	0.0004	0.0005	0.0007
	-9	0.0005	0.0008	0.0012	0.0017	0.0023
	-8	0.0018	0.0026	0.0036	0.0048	0.0061
	-7	0.0058	0.0077	0.0098	0.0122	0.0148
	-6	0.0159	0.0199	0.0240	0.0283	0.0328
	-5	0.0391	0.0458	0.0526	0.0593	0.0658
	-4	0.0847	0.0943	0.1035	0.1121	0.1204
	-3	0.1626	0.1734	0.1834	0.1925	0.2009
	-2	0.2771	0.2860	0.2939	0.3010	0.3075
	-1	0.4218	0.4253	0.4283	0.4310	0.4334
	0	0.5782	0.5747	0.5717	0.5690	0.5666
	1	0.7229	0.7140	0.7061	0.6990	0.6925
	2	0.8374	0.8266	0.8166	0.8075	0.7991
	3	0.9153	0.9057	0.8965	0.8879	0.8796
	4	0.9609	0.9542	0.9474	0.9407	0.9342
	5	0.9841	0.9801	0.9760	0.9717	0.9672
	6	0.9942	0.9923	0.9902	0.9878	0.9852
	7	0.9982	0.9974	0.9964	0.9952	0.9939
	8	0.9995	0.9992	0.9988	0.9983	0.9977
	9	0.9999	0.9998	0.9996	0.9995	0.9992
	10	1.0000	0.9999	0.9999	0.9999	0.9998
	11	1.0000	1.0000	1.0000	1.0000	0.9999

## P

		.16	.17	.18	.19	.20
N	k					
2	-2	0.0256	0.0289	0.0324	0.0361	0.0400
	-1	0.2432	0.2533	0.2628	0.2717	0.2800
	0	0.7568	0.7467	0.7372	0.7283	0.7200
	1	0.9744	0.9711	0.9676	0.9639	0.9600
	2	1.0000	1.0000	1.0000	1.0000	1.0000
3	-3	0.0041	0.0049	0.0058	0.0069	0.0080
	-2	0.0563	0.0621	0.0680	0.0740	0.0800
	-1	0.2906	0.2990	0.3067	0.3137	0.3200
	0	0.7094	0.7010	0.6933	0.6863	0.6800
	1	0.9437	0.9379	0.9320	0.9260	0.9200
	2	0.9959	0.9951	0.9942	0.9931	0.9920
	3	1.0000	1.0000	1.0000	1.0000	1.0000
4	-4	0.0007	0.0008	0.0010	0.0013	0.0016
	-3	0.0118	0.0138	0.0160	0.0183	0.0208
	-2	0.0854	0.0927	0.0998	0.1068	0.1136
	-1	0.3201	0.3271	0.3333	0.3389	0.3440
	0	0.6799	0.6729	0.6667	0.6611	0.6560
	1	0.9146	0.9073	0.9002	0.8932	0.8864
	2	0.9882	0.9862	0.9840	0.9817	0.9792
	3	0.9993	0.9992	0.9990	0.9987	0.9984
	4	1.0000	1.0000	1.0000	1.0000	1.0000
5	-5	0.0001	0.0001	0.0002	0.0002	0.0003
	-4	0.0023	0.0029	0.0035	0.0043	0.0051
	-3	0.0218	0.0250	0.0284	0.0319	0.0355
	-2	0.1112	0.1191	0.1268	0.1341	0.1411
	-1	0.3401	0.3460	0.3513	0.3560	0.3603
	0	0.6599	0.6540	0.6487	0.6440	0.6397
	1	0.8888	0.8809	0.8732	0.8659	0.8589
	2	0.9782	0.9750	0.9716	0.9681	0.9645
	3	0.9977	0.9971	0.9965	0.9957	0.9949
	4	0.9999	0.9999	0.9998	0.9998	0.9997
6	5	1.0000	1.0000	1.0000	1.0000	1.0000
	-6	0.0000	0.0000	0.0000	0.0000	0.0001
	-5	0.0004	0.0006	0.0008	0.0010	0.0012
	-4	0.0051	0.0062	0.0074	0.0088	0.0102
	-3	0.0330	0.0372	0.0416	0.0461	0.0506
	-2	0.1335	0.1417	0.1495	0.1568	0.1638
	-1	0.3547	0.3598	0.3644	0.3686	0.3724
	0	0.6453	0.6402	0.6356	0.6314	0.6276
	1	0.8665	0.8583	0.8505	0.8432	0.8362

P

		.16	.17	.18	.19	.20
N	k					
7	2	0.9670	0.9628	0.9584	0.9539	0.9494
	3	0.9949	0.9938	0.9926	0.9912	0.9898
	4	0.9996	0.9994	0.9992	0.9990	0.9988
	5	1.0000	1.0000	1.0000	1.0000	0.9999
	6	1.0000	1.0000	1.0000	1.0000	1.0000
	-6	0.0001	0.0001	0.0002	0.0002	0.0003
	-5	0.0011	0.0014	0.0018	0.0023	0.0028
	-4	0.0088	0.0105	0.0124	0.0144	0.0165
	-3	0.0446	0.0497	0.0549	0.0600	0.0652
	-2	0.1528	0.1610	0.1687	0.1760	0.1829
8	-1	0.3658	0.3704	0.3745	0.3783	0.3817
	0	0.6342	0.6296	0.6255	0.6217	0.6183
	1	0.8472	0.8390	0.8313	0.8240	0.8171
	2	0.9554	0.9503	0.9451	0.9400	0.9348
	3	0.9912	0.9895	0.9876	0.9856	0.9835
	4	0.9989	0.9986	0.9982	0.9977	0.9972
	5	0.9999	0.9999	0.9998	0.9998	0.9997
	6	1.0000	1.0000	1.0000	1.0000	1.0000
	-7	0.0000	0.0000	0.0000	0.0000	0.0001
	-6	0.0002	0.0003	0.0004	0.0006	0.0007
9	-5	0.0022	0.0028	0.0034	0.0042	0.0050
	-4	0.0133	0.0156	0.0181	0.0207	0.0235
	-3	0.0562	0.0620	0.0677	0.0734	0.0790
	-2	0.1696	0.1777	0.1853	0.1924	0.1991
	-1	0.3747	0.3789	0.3827	0.3861	0.3893
	0	0.6253	0.6211	0.6173	0.6139	0.6107
	1	0.8304	0.8223	0.8147	0.8076	0.8009
	2	0.9438	0.9380	0.9323	0.9266	0.9210
	3	0.9867	0.9844	0.9819	0.9793	0.9765
	4	0.9978	0.9972	0.9966	0.9958	0.9950
10	5	0.9998	0.9997	0.9996	0.9994	0.9993
	6	1.0000	1.0000	1.0000	1.0000	0.9999
	7	1.0000	1.0000	1.0000	1.0000	1.0000
	-7	0.0000	0.0001	0.0001	0.0001	0.0002
	-6	0.0005	0.0007	0.0009	0.0012	0.0015
	-5	0.0037	0.0045	0.0055	0.0066	0.0079
	-4	0.0184	0.0213	0.0244	0.0276	0.0309

## P

N	k	.16	.17	.18	.19	.20
10	2	0.9325	0.9262	0.9200	0.9140	0.9081
	3	0.9816	0.9787	0.9756	0.9724	0.9691
	4	0.9963	0.9955	0.9945	0.9934	0.9921
	5	0.9995	0.9993	0.9991	0.9988	0.9985
	6	1.0000	0.9999	0.9999	0.9999	0.9998
	7	1.0000	1.0000	1.0000	1.0000	1.0000
	-7	0.0001	0.0002	0.0002	0.0003	0.0004
	-6	0.0009	0.0012	0.0016	0.0020	0.0025
	-5	0.0055	0.0067	0.0081	0.0096	0.0112
	-4	0.0239	0.0274	0.0310	0.0347	0.0385
	-3	0.0783	0.0850	0.0915	0.0978	0.1039
	-2	0.1972	0.2050	0.2123	0.2190	0.2253
	-1	0.3881	0.3917	0.3951	0.3981	0.4008
	0	0.6119	0.6083	0.6049	0.6019	0.5992
	1	0.8028	0.7950	0.7877	0.7810	0.7747
	2	0.9217	0.9150	0.9085	0.9022	0.8961
	3	0.9761	0.9726	0.9690	0.9653	0.9615
	4	0.9945	0.9933	0.9919	0.9904	0.9888
	5	0.9991	0.9988	0.9984	0.9980	0.9975
	6	0.9999	0.9998	0.9998	0.9997	0.9996
	7	1.0000	1.0000	1.0000	1.0000	1.0000
11	-8	0.0000	0.0000	0.0001	0.0001	0.0001
	-7	0.0002	0.0003	0.0004	0.0006	0.0007
	-6	0.0015	0.0020	0.0025	0.0031	0.0038
	-5	0.0077	0.0093	0.0110	0.0129	0.0149
	-4	0.0297	0.0337	0.0378	0.0419	0.0461
	-3	0.0886	0.0956	0.1023	0.1089	0.1151
	-2	0.2087	0.2163	0.2234	0.2300	0.2362
	-1	0.3934	0.3968	0.3999	0.402	0.4054
	0	0.6066	0.6032	0.6001	0.5972	0.5946
	1	0.7913	0.7837	0.7766	0.7700	0.7638
	2	0.9114	0.9044	0.8977	0.8911	0.8849
	3	0.9703	0.9663	0.9622	0.9581	0.9539
	4	0.9923	0.9907	0.9890	0.9871	0.9851
	5	0.9985	0.9980	0.9975	0.9969	0.9962
	6	0.9998	0.9997	0.9996	0.9994	0.9993
	7	1.0000	1.0000	0.9999	0.9999	0.9999
	8	1.0000	1.0000	1.0000	1.0000	1.0000
12	-8	0.0001	0.0001	0.0001	0.0002	0.0002
	-7	0.0004	0.0006	0.0007	0.0010	0.0012
	-6	0.0023	0.0029	0.0037	0.0045	0.0054
	-5	0.0102	0.0122	0.0143	0.0166	0.0189

p

N	k	.16	.17	.18	.19	.20
13	-4	0.0356	0.0401	0.0446	0.0491	0.0537
	-3	0.0984	0.1056	0.1125	0.1192	0.1255
	-2	0.2190	0.2265	0.2334	0.2398	0.2458
	-1	0.3979	0.4012	0.4042	0.4069	0.4094
	0	0.6021	0.5988	0.5958	0.5931	0.5906
	1	0.7810	0.7735	0.7666	0.7602	0.7542
	2	0.9016	0.8944	0.8875	0.8808	0.8745
	3	0.9644	0.9599	0.9554	0.9509	0.9463
	4	0.9898	0.9878	0.9857	0.9834	0.9811
	5	0.9977	0.9971	0.9963	0.9955	0.9946
	6	0.9996	0.9994	0.9993	0.9990	0.9988
	7	0.9999	0.9999	0.9999	0.9998	0.9998
	8	1.0000	1.0000	1.0000	1.0000	1.0000
14	-9	0.0000	0.0000	0.0000	0.0000	0.0001
	-8	0.0001	0.0001	0.0002	0.0003	0.0004
	-7	0.0007	0.0009	0.0012	0.0015	0.0019
	-6	0.0033	0.0041	0.0051	0.0061	0.0073
	-5	0.0130	0.0154	0.0179	0.0205	0.0232
	-4	0.0416	0.0465	0.0514	0.0563	0.0611
	-3	0.1077	0.1150	0.1220	0.1288	0.1352
	-2	0.2284	0.2356	0.2424	0.2486	0.2545
	-1	0.4020	0.4051	0.4079	0.4105	0.4129
	0	0.5980	0.5949	0.5921	0.5895	0.5871
	1	0.7716	0.7644	0.7576	0.7514	0.7455
	2	0.8923	0.8850	0.8780	0.8712	0.8648
	3	0.9584	0.9535	0.9486	0.9437	0.9389
	4	0.9870	0.9846	0.9821	0.9795	0.9768
	5	0.9967	0.9959	0.9949	0.9939	0.9927
	6	0.9993	0.9991	0.9988	0.9985	0.9981
	7	0.9999	0.9999	0.9998	0.9997	0.9996
	8	1.0000	1.0000	1.0000	1.0000	0.9999
	9	1.0000	1.0000	1.0000	1.0000	1.0000

## P

N	k	.16	.17	.18	.19	.20
15	2	0.8836	0.8761	0.8690	0.8622	0.8558
	3	0.9524	0.9472	0.9419	0.9368	0.9317
	4	0.9840	0.9813	0.9784	0.9755	0.9724
	5	0.9956	0.9945	0.9933	0.9920	0.9906
	6	0.9990	0.9987	0.9983	0.9979	0.9973
	7	0.9998	0.9997	0.9997	0.9995	0.9994
	8	1.0000	1.0000	0.9999	0.9999	0.9999
	9	1.0000	1.0000	1.0000	1.0000	1.0000
	-9	0.0000	0.0001	0.0001	0.0001	0.0002
	-8	0.0003	0.0004	0.0005	0.0007	0.0009
	-7	0.0014	0.0018	0.0023	0.0029	0.0036
	-6	0.0057	0.0070	0.0085	0.0100	0.0117
	-5	0.0192	0.0223	0.0255	0.0277	0.0321
	-4	0.0535	0.0591	0.0646	0.0700	0.0754
	-3	0.1247	0.1322	0.1394	0.1462	0.1527
	-2	0.2446	0.2515	0.2579	0.2639	0.2694
	-1	0.4088	0.4117	0.4143	0.4167	0.4189
16	0	0.5912	0.5883	0.5857	0.5833	0.5811
	1	0.7554	0.7485	0.7421	0.7361	0.7306
	2	0.8753	0.8678	0.8606	0.8538	0.8473
	3	0.9465	0.9409	0.9354	0.9300	0.9246
	4	0.9808	0.9777	0.9745	0.9713	0.9679
	5	0.9943	0.9930	0.9915	0.9900	0.9883
	6	0.9986	0.9982	0.9977	0.9971	0.9964
	7	0.9997	0.9996	0.9995	0.9993	0.9991
	8	1.0000	0.9999	0.9999	0.9999	0.9998
	9	1.0000	1.0000	1.0000	1.0000	1.0000
	-10	0.0000	0.0000	0.0000	0.0000	0.0001
	-9	0.0001	0.0001	0.0002	0.0002	0.0003
	-8	0.0004	0.0006	0.0008	0.0010	0.0013
	-7	0.0019	0.0025	0.0031	0.0039	0.0047
	-6	0.0072	0.0087	0.0104	0.0122	0.0141
	-5	0.0226	0.0259	0.0294	0.0330	0.0367
	-4	0.0594	0.0653	0.0710	0.0767	0.0822
	-3	0.1325	0.1401	0.1473	0.1541	0.1606
	-2	0.2517	0.2585	0.2647	0.2706	0.2760
	-1	0.4117	0.4145	0.4170	0.4193	0.4214
	0	0.5883	0.5855	0.5830	0.5807	0.5786
	1	0.7483	0.7415	0.7353	0.7294	0.7240
	2	0.8675	0.8599	0.8527	0.8459	0.8394
	3	0.9406	0.9347	0.9290	0.9233	0.9178
	4	0.9774	0.9740	0.9706	0.9670	0.9633
	5	0.9928	0.9913	0.9896	0.9878	0.9859

p

N	k	.16	.17	.18	.19	.20
17	6	0.9981	0.9975	0.9969	0.9961	0.9953
	7	0.9996	0.9994	0.9992	0.9990	0.9987
	8	0.9999	0.9999	0.9998	0.9998	0.9997
	9	1.0000	1.0000	1.0000	1.0000	0.9999
	10	1.0000	1.0000	1.0000	1.0000	1.0000
	-10	0.0000	0.0000	0.0000	0.0001	0.0001
	-9	0.0001	0.0002	0.0003	0.0003	0.0005
	-8	0.0006	0.0008	0.0011	0.0014	0.0018
	-7	0.0025	0.0032	0.0040	0.0049	0.0059
	-6	0.0088	0.0106	0.0125	0.0146	0.0168
	-5	0.0260	0.0297	0.0335	0.0374	0.0413
	-4	0.0652	0.0713	0.0773	0.0831	0.0888
	-3	0.1398	0.1475	0.1547	0.1615	0.1680
	-2	0.2582	0.2649	0.2710	0.2767	0.2820
	-1	0.4143	0.4170	0.4195	0.4217	0.4238
	0	0.5857	0.5830	0.5805	0.5783	0.5762
	1	0.7418	0.7351	0.7290	0.7233	0.7180
	2	0.8602	0.8525	0.8453	0.8385	0.8320
	3	0.9348	0.9287	0.9227	0.9169	0.9112
	4	0.9740	0.9703	0.9665	0.9626	0.9587
	5	0.9912	0.9894	0.9875	0.9854	0.9832
	6	0.9975	0.9968	0.9960	0.9951	0.9941
	7	0.9994	0.9992	0.9989	0.9986	0.9982
	8	0.9999	0.9998	0.9997	0.9997	0.9995
	9	1.0000	1.0000	1.0000	0.9999	0.9999
	10	1.0000	1.0000	1.0000	1.0000	1.0000
18	-10	0.0000	0.0001	0.0001	0.0001	0.0002
	-9	0.0002	0.0003	0.0004	0.0005	0.0007
	-8	0.0008	0.0011	0.0015	0.0019	0.0023
	-7	0.0032	0.0041	0.0050	0.0061	0.0072
	-6	0.0106	0.0126	0.0148	0.0171	0.0195
	-5	0.0295	0.0335	0.0376	0.0417	0.0459
	-4	0.0709	0.0772	0.0833	0.0893	0.0951
	-3	0.1468	0.1545	0.1617	0.1685	0.1749
	-2	0.2642	0.2708	0.2768	0.2824	0.2875
	-1	0.4168	0.4194	0.4217	0.4239	0.4259
	0	0.5832	0.5806	0.5783	0.5761	0.5741
	1	0.7358	0.7292	0.7232	0.7176	0.7125
	2	0.8532	0.8455	0.8383	0.8315	0.8251
	3	0.9291	0.9228	0.9167	0.9107	0.9049
	4	0.9705	0.9665	0.9624	0.9583	0.9541
	5	0.9894	0.9874	0.9852	0.9829	0.9805
	6	0.9968	0.9959	0.9950	0.9939	0.9928

## P

		.16	.17	.18	.19	.20
N	k					
19	7	0.9992	0.9989	0.9985	0.9981	0.9977
	8	0.9998	0.9997	0.9996	0.9995	0.9993
	9	1.0000	0.9999	0.9999	0.9999	0.9998
	10	1.0000	1.0000	1.0000	1.0000	1.0000
	-10	0.0001	0.0001	0.0001	0.0002	0.0002
	-9	0.0003	0.0004	0.0005	0.0007	0.0009
	-8	0.0011	0.0015	0.0019	0.0024	0.0030
	-7	0.0040	0.0050	0.0061	0.0074	0.0087
	-6	0.0124	0.0147	0.0171	0.0197	0.0223
	-5	0.0331	0.0374	0.0417	0.0461	0.0504
	-4	0.0764	0.0829	0.0892	0.0953	0.1012
	-3	0.1535	0.1611	0.1683	0.1751	0.1815
	-2	0.2699	0.2763	0.2822	0.2876	0.2927
	-1	0.4190	0.4215	0.4238	0.4259	0.4279
	0	0.5810	0.5785	0.5762	0.5741	0.5721
	1	0.7301	0.7237	0.7178	0.7124	0.7073
	2	0.8465	0.8389	0.8317	0.8249	0.8185
	3	0.9236	0.9171	0.9108	0.9047	0.8988
	4	0.9669	0.9626	0.9583	0.9539	0.9496
	5	0.9876	0.9853	0.9829	0.9803	0.9777
	6	0.9960	0.9950	0.9939	0.9926	0.9913
	7	0.9989	0.9985	0.9981	0.9976	0.9970
	8	0.9997	0.9996	0.9995	0.9993	0.9991
	9	0.9999	0.9999	0.9999	0.9998	0.9998
	10	1.0000	1.0000	1.0000	1.0000	0.9999
20	-11	0.0000	0.0000	0.0000	0.0001	0.0001
	-10	0.0001	0.0001	0.0002	0.0002	0.0003
	-9	0.0004	0.0005	0.0007	0.0009	0.0012
	-8	0.0014	0.0019	0.0024	0.0030	0.0037
	-7	0.0049	0.0061	0.0074	0.0088	0.0103
	-6	0.0144	0.0169	0.0196	0.0223	0.0252
	-5	0.0367	0.0413	0.0458	0.0504	0.0550
	-4	0.0818	0.0885	0.0949	0.1011	0.1071
	-3	0.1598	0.1674	0.1746	0.1813	0.1877
	-2	0.2751	0.2814	0.2872	0.2925	0.2975
	-1	0.4211	0.4235	0.4257	0.4278	0.4297
	0	0.5789	0.5765	0.5743	0.5722	0.5703
	1	0.7249	0.7186	0.7128	0.7075	0.7025
	2	0.8402	0.8326	0.8254	0.8187	0.8123
	3	0.9182	0.9115	0.9051	0.8989	0.8929
	4	0.9633	0.9587	0.9542	0.9496	0.9450
	5	0.9856	0.9831	0.9804	0.9777	0.9748
	6	0.9951	0.9939	0.9926	0.9912	0.9897

## P

N	k	.16	.17	.18	.19	.20
21	7	0.9986	0.9981	0.9976	0.9970	0.9963
	8	0.9996	0.9995	0.9993	0.9991	0.9988
	9	0.9999	0.9999	0.9998	0.9998	0.9997
	10	1.0000	1.0000	1.0000	0.9999	0.9999
	11	1.0000	1.0000	1.0000	1.0000	1.0000
	-11	0.0000	0.0000	0.0001	0.0001	0.0001
	-10	0.0001	0.0002	0.0002	0.0003	0.0004
	-9	0.0005	0.0007	0.0009	0.0012	0.0015
	-8	0.0018	0.0024	0.0030	0.0037	0.0045
	-7	0.0059	0.0072	0.0087	0.0103	0.0120
	-6	0.0164	0.0192	0.0221	0.0251	0.0282
	-5	0.0404	0.0452	0.0499	0.0547	0.0594
	-4	0.0871	0.0939	0.1004	0.1067	0.1128
	-3	0.1657	0.1734	0.1805	0.1872	0.1935
	-2	0.2800	0.2862	0.2918	0.2971	0.3020
	-1	0.4230	0.4254	0.4275	0.4295	0.4314
	0	0.5770	0.5746	0.5725	0.5705	0.5686
	1	0.7200	0.7138	0.7082	0.7029	0.6980
	2	0.8343	0.8266	0.8195	0.8128	0.8065
	3	0.9129	0.9061	0.8996	0.8933	0.8872
	4	0.9596	0.9548	0.9501	0.9453	0.9406
	5	0.9836	0.9808	0.9779	0.9749	0.9718
	6	0.9941	0.9928	0.9913	0.9897	0.9880
	7	0.9982	0.9976	0.9970	0.9963	0.9955
	8	0.9995	0.9993	0.9991	0.9988	0.9985
	9	0.9999	0.9998	0.9998	0.9997	0.9996
	10	1.0000	1.0000	0.9999	0.9999	0.9999
	11	1.0000	1.0000	1.0000	1.0000	1.0000
22	-11	0.0000	0.0001	0.0001	0.0001	0.0002
	-10	0.0002	0.0002	0.0003	0.0004	0.0006
	-9	0.0006	0.0009	0.0012	0.0015	0.0019
	-8	0.0023	0.0029	0.0036	0.0045	0.0054
	-7	0.0069	0.0084	0.0101	0.0118	0.0137
	-6	0.0186	0.0216	0.0247	0.0279	0.0312
	-5	0.0440	0.0490	0.0540	0.0590	0.0639
	-4	0.0922	0.0991	0.1057	0.1121	0.1183
	-3	0.1714	0.1790	0.1861	0.1928	0.1991
	-2	0.2846	0.2906	0.2962	0.3014	0.3062
	-1	0.4247	0.4271	0.4292	0.4311	0.4329
	0	0.5753	0.5729	0.5708	0.5689	0.5671
	1	0.7154	0.7094	0.7038	0.6986	0.6938
	2	0.8286	0.8210	0.8139	0.8072	0.8009
	3	0.9078	0.9009	0.8943	0.8879	0.8817

## P

N	k	.16	.17	.18	.19	.20
23	4	0.9560	0.9510	0.9460	0.9410	0.9361
	5	0.9814	0.9784	0.9753	0.9721	0.9688
	6	0.9931	0.9916	0.9899	0.9882	0.9863
	7	0.9977	0.9971	0.9964	0.9955	0.9946
	8	0.9994	0.9991	0.9988	0.9985	0.9981
	9	0.9998	0.9998	0.9997	0.9996	0.9994
	10	1.0000	0.9999	0.9999	0.9999	0.9998
	11	1.0000	1.0000	1.0000	1.0000	1.0000
	-11	0.0001	0.0001	0.0001	0.0002	0.0002
	-10	0.0002	0.0003	0.0004	0.0006	0.0008
	-9	0.0008	0.0011	0.0015	0.0019	0.0023
	-8	0.0027	0.0035	0.0044	0.0053	0.0064
	-7	0.0080	0.0097	0.0115	0.0135	0.0155
	-6	0.0208	0.0240	0.0273	0.0308	0.0342
	-5	0.0476	0.0529	0.0580	0.0632	0.0682
	-4	0.0972	0.1042	0.1109	0.1174	0.1235
	-3	0.1769	0.1844	0.1915	0.1981	0.2043
	-2	0.2889	0.2949	0.3003	0.3054	0.3101
	-1	0.4264	0.4287	0.4308	0.4327	0.4344
	0	0.5736	0.5713	0.5692	0.5673	0.5656
	1	0.7111	0.7051	0.6997	0.6946	0.6899
	2	0.8231	0.8156	0.8085	0.8019	0.7957
	3	0.9028	0.8958	0.8891	0.8826	0.8765
	4	0.9524	0.9471	0.9420	0.9368	0.9318
	5	0.9792	0.9760	0.9727	0.9692	0.9658
	6	0.9920	0.9903	0.9885	0.9865	0.9845
	7	0.9973	0.9965	0.9956	0.9947	0.9936
	8	0.9992	0.9989	0.9985	0.9981	0.9977
	9	0.9998	0.9997	0.9996	0.9994	0.9992
	10	0.9999	0.9999	0.9999	0.9998	0.9998
	11	1.0000	1.0000	1.0000	1.0000	0.9999
24	-12	0.0000	0.0000	0.0000	0.0001	0.0001
	-11	0.0001	0.0001	0.0002	0.0002	0.0003
	-10	0.0003	0.0004	0.0006	0.0007	0.0010
	-9	0.0010	0.0014	0.0018	0.0023	0.0028
	-8	0.0033	0.0041	0.0051	0.0062	0.0074
	-7	0.0092	0.0111	0.0131	0.0152	0.0174
	-6	0.0230	0.0265	0.0300	0.0336	0.0373
	-5	0.0513	0.0567	0.0620	0.0673	0.0725
	-4	0.1020	0.1091	0.1159	0.1224	0.1286
	-3	0.1820	0.1895	0.1966	0.2031	0.2093
	-2	0.2930	0.2988	0.3042	0.3092	0.3138
	-1	0.4280	0.4302	0.4322	0.4341	0.4358

## p

		.16	.17	.18	.19	.20
N	k					
25	0	0.5720	0.5698	0.5678	0.5659	0.5642
	1	0.7070	0.7012	0.6958	0.6908	0.6862
	2	0.8180	0.8105	0.8034	0.7969	0.7907
	3	0.8980	0.8909	0.8841	0.8776	0.8714
	4	0.9487	0.9433	0.9380	0.9327	0.9275
	5	0.9770	0.9735	0.9700	0.9664	0.9627
	6	0.9908	0.9889	0.9869	0.9848	0.9826
	7	0.9967	0.9959	0.9949	0.9938	0.9926
	8	0.9990	0.9986	0.9982	0.9977	0.9972
	9	0.9997	0.9996	0.9994	0.9993	0.9990
	10	0.9999	0.9999	0.9998	0.9998	0.9997
	11	1.0000	1.0000	1.0000	0.9999	0.9999
	12	1.0000	1.0000	1.0000	1.0000	1.0000
	-12	0.0000	0.0000	0.0001	0.0001	0.0001
	-11	0.0001	0.0001	0.0002	0.0003	0.0004
	-10	0.0004	0.0005	0.0007	0.0009	0.0012
	-9	0.0013	0.0017	0.0022	0.0027	0.0034
	-8	0.0039	0.0049	0.0060	0.0072	0.0085
	-7	0.0105	0.0125	0.0147	0.0170	0.0194
	-6	0.0253	0.0290	0.0327	0.0365	0.0404
	-5	0.0549	0.0605	0.0660	0.0714	0.0767
	-4	0.1067	0.1139	0.1207	0.1273	0.1335
	-3	0.1870	0.1944	0.2014	0.2079	0.2141
	-2	0.2968	0.3026	0.3079	0.3128	0.3173
	-1	0.4294	0.4316	0.4336	0.4354	0.4371
	0	0.5706	0.5684	0.5664	0.5646	0.5629
	1	0.7032	0.6974	0.6921	0.6872	0.6827
	2	0.8130	0.8056	0.7986	0.7921	0.7859
	3	0.8933	0.8861	0.8793	0.8727	0.8665
	4	0.9451	0.9395	0.9340	0.9286	0.9233
	5	0.9747	0.9710	0.9673	0.9635	0.9596
	6	0.9895	0.9875	0.9853	0.9830	0.9806
	7	0.9961	0.9951	0.9940	0.9928	0.9915
	8	0.9987	0.9983	0.9978	0.9973	0.9966
	9	0.9996	0.9995	0.9993	0.9991	0.9988
	10	0.9999	0.9999	0.9998	0.9997	0.9996
	11	1.0000	1.0000	0.9999	0.9999	0.9999
	12	1.0000	1.0000	1.0000	1.0000	1.0000
26	-12	0.0000	0.0000	0.0001	0.0001	0.0002
	-11	0.0001	0.0002	0.0003	0.0004	0.0005
	-10	0.0005	0.0007	0.0009	0.0012	0.0015
	-9	0.0015	0.0020	0.0026	0.0032	0.0040
	-8	0.0045	0.0056	0.0069	0.0082	0.0097

N	k	.16	.17	.18	.19	.20
27	-7	0.0118	0.0140	0.0164	0.0188	0.0214
	-6	0.0277	0.0315	0.0355	0.0394	0.0434
	-5	0.0584	0.0642	0.0698	0.0754	0.0808
	-4	0.1112	0.1185	0.1254	0.1320	0.1383
	-3	0.1917	0.1991	0.2061	0.2125	0.2186
	-2	0.3005	0.3061	0.3113	0.3162	0.3206
	-1	0.4308	0.4329	0.4349	0.4366	0.4383
	0	0.5692	0.5671	0.5651	0.5634	0.5617
	1	0.6995	0.6939	0.6887	0.6838	0.6794
	2	0.8083	0.8009	0.7939	0.7875	0.7814
	3	0.8888	0.8815	0.8746	0.8680	0.8617
	4	0.9416	0.9358	0.9302	0.9246	0.9192
	5	0.9723	0.9685	0.9645	0.9606	0.9566
	6	0.9882	0.9860	0.9836	0.9812	0.9786
	7	0.9955	0.9944	0.9931	0.9918	0.9903
	8	0.9985	0.9980	0.9974	0.9968	0.9960
	9	0.9995	0.9993	0.9991	0.9988	0.9985
	10	0.9999	0.9998	0.9997	0.9996	0.9995
	11	1.0000	0.9999	0.9999	0.9999	0.9998
	12	1.0000	1.0000	1.0000	1.0000	1.0000
27	-12	0.0000	0.0001	0.0001	0.0001	0.0002
	-11	0.0002	0.0002	0.0003	0.0005	0.0006
	-10	0.0006	0.0008	0.0011	0.0014	0.0018
	-9	0.0018	0.0024	0.0030	0.0038	0.0046
	-8	0.0052	0.0064	0.0078	0.0093	0.0109
	-7	0.0132	0.0156	0.0181	0.0207	0.0235
	-6	0.0301	0.0341	0.0382	0.0424	0.0465
	-5	0.0620	0.0679	0.0737	0.0793	0.0848
	-4	0.1157	0.1230	0.1299	0.1365	0.1428
	-3	0.1962	0.2036	0.2105	0.2169	0.2229
	-2	0.3039	0.3095	0.3146	0.3194	0.3238
	-1	0.4321	0.4342	0.4361	0.4378	0.4394
	0	0.5679	0.5658	0.5639	0.5622	0.5606
	1	0.6961	0.6905	0.6854	0.6806	0.6762
	2	0.8038	0.7964	0.7895	0.7831	0.7771
	3	0.8843	0.8770	0.8701	0.8635	0.8572
	4	0.9380	0.9321	0.9263	0.9207	0.9152
	5	0.9699	0.9659	0.9618	0.9576	0.9535
	6	0.9868	0.9844	0.9819	0.9793	0.9765
	7	0.9948	0.9936	0.9922	0.9907	0.9891
	8	0.9982	0.9976	0.9969	0.9962	0.9954
	9	0.9994	0.9992	0.9989	0.9986	0.9982
	10	0.9998	0.9998	0.9997	0.9995	0.9994
	11	1.0000	0.9999	0.9999	0.9999	0.9998

## P

N	k	.16	.17	.18	.19	.20
28	12	1.0000	1.0000	1.0000	1.0000	0.9999
	-13	0.0000	0.0000	0.0000	0.0001	0.0001
	-12	0.0001	0.0001	0.0001	0.0002	0.0003
	-11	0.0002	0.0003	0.0004	0.0006	0.0008
	-10	0.0007	0.0010	0.0013	0.0017	0.0021
	-9	0.0022	0.0028	0.0036	0.0044	0.0053
	-8	0.0059	0.0073	0.0088	0.0104	0.0121
	-7	0.0146	0.0172	0.0199	0.0227	0.0256
	-6	0.0325	0.0367	0.0410	0.0453	0.0496
	-5	0.0655	0.0715	0.0774	0.0832	0.0888
	-4	0.1200	0.1273	0.1343	0.1409	0.1473
	-3	0.2006	0.2079	0.2147	0.2211	0.2271
	-2	0.3072	0.3127	0.3177	0.3224	0.3267
	-1	0.4333	0.4354	0.4372	0.4389	0.4405
29	0	0.5667	0.5646	0.5628	0.5611	0.5595
	1	0.6928	0.6873	0.6823	0.6776	0.6733
	2	0.7994	0.7921	0.7853	0.7789	0.7729
	3	0.8800	0.8727	0.8657	0.8591	0.8527
	4	0.9345	0.9285	0.9226	0.9168	0.9112
	5	0.9675	0.9633	0.9590	0.9547	0.9504
	6	0.9854	0.9828	0.9801	0.9773	0.9744
	7	0.9941	0.9927	0.9912	0.9896	0.9879
	8	0.9978	0.9972	0.9964	0.9956	0.9947
	9	0.9993	0.9990	0.9987	0.9983	0.9979
	10	0.9998	0.9997	0.9996	0.9994	0.9992
	11	0.9999	0.9999	0.9999	0.9998	0.9997
	12	1.0000	1.0000	1.0000	0.9999	0.9999
	13	1.0000	1.0000	1.0000	1.0000	1.0000
29	-13	0.0000	0.0000	0.0000	0.0001	0.0001
	-12	0.0001	0.0001	0.0002	0.0002	0.0003
	-11	0.0003	0.0004	0.0005	0.0007	0.0009
	-10	0.0009	0.0012	0.0016	0.0020	0.0025
	-9	0.0026	0.0033	0.0041	0.0050	0.0060
	-8	0.0067	0.0082	0.0098	0.0116	0.0134
	-7	0.0161	0.0188	0.0217	0.0246	0.0277
	-6	0.0349	0.0393	0.0437	0.0482	0.0526
	-5	0.0689	0.0751	0.0811	0.0869	0.0926
	-4	0.1241	0.1315	0.1385	0.1452	0.1515
	-3	0.2047	0.2120	0.2188	0.2251	0.2311
	-2	0.3103	0.3157	0.3207	0.3253	0.3296
	-1	0.4345	0.4365	0.4383	0.4400	0.4416
	0	0.5655	0.5635	0.5617	0.5600	0.5584
	1	0.6897	0.6843	0.6793	0.6747	0.6704

## P

N	k	.16	.17	.18	.19	.20
30	2	0.7953	0.7880	0.7812	0.7749	0.7689
	3	0.8759	0.8685	0.8615	0.8548	0.8485
	4	0.9311	0.9249	0.9189	0.9131	0.9074
	5	0.9651	0.9607	0.9563	0.9518	0.9474
	6	0.9839	0.9812	0.9783	0.9754	0.9723
	7	0.9933	0.9918	0.9902	0.9884	0.9866
	8	0.9974	0.9967	0.9959	0.9950	0.9940
	9	0.9991	0.9988	0.9984	0.9980	0.9975
	10	0.9997	0.9996	0.9995	0.9993	0.9991
	11	0.9999	0.9999	0.9998	0.9998	0.9997
	12	1.0000	1.0000	1.0000	0.9999	0.9999
	13	1.0000	1.0000	1.0000	1.0000	1.0000
	-13	0.0000	0.0000	0.0001	0.0001	0.0001
	-12	0.0001	0.0001	0.0002	0.0003	0.0004
	-11	0.0003	0.0005	0.0007	0.0009	0.0011
	-10	0.0010	0.0014	0.0018	0.0023	0.0029
	-9	0.0030	0.0038	0.0047	0.0057	0.0068
	-8	0.0076	0.0092	0.0109	0.0128	0.0148
	-7	0.0176	0.0205	0.0235	0.0266	0.0298
	-6	0.0373	0.0419	0.0465	0.0511	0.0556
	-5	0.0723	0.0786	0.0847	0.0906	0.0964
	-4	0.1282	0.1356	0.1426	0.1493	0.1556
	-3	0.2087	0.2160	0.2227	0.2290	0.2348
	-2	0.3133	0.3186	0.3235	0.3281	0.3323
	-1	0.4356	0.4375	0.4394	0.4410	0.4425
	0	0.5644	0.5624	0.5606	0.5590	0.5575
	1	0.6867	0.6814	0.6765	0.6719	0.6677
	2	0.7913	0.7840	0.7773	0.7710	0.7652
	3	0.8718	0.8644	0.8574	0.8507	0.8444
	4	0.9277	0.9214	0.9153	0.9094	0.9036
	5	0.9627	0.9581	0.9535	0.9489	0.9444
	6	0.9824	0.9795	0.9765	0.9734	0.9702
	7	0.9924	0.9908	0.9891	0.9872	0.9852
	8	0.9970	0.9962	0.9953	0.9943	0.9932
	9	0.9990	0.9986	0.9982	0.9977	0.9971
	10	0.9997	0.9995	0.9993	0.9991	0.9989
	11	0.9999	0.9999	0.9998	0.9997	0.9996
	12	1.0000	1.0000	0.9999	0.9999	0.9999
	13	1.0000	1.0000	1.0000	1.0000	1.0000

## P

N	k	.21	.22	.23	.24	.25
2	-2	0.0441	0.0484	0.0529	0.0576	0.0625
	-1	0.2877	0.2948	0.3013	0.3072	0.3125
	0	0.7123	0.7052	0.6987	0.6928	0.6875
	1	0.9559	0.9516	0.9471	0.9424	0.9375
	2	1.0000	1.0000	1.0000	1.0000	1.0000
3	-3	0.0093	0.0106	0.0122	0.0138	0.0156
	-2	0.0860	0.0920	0.0979	0.1037	0.1094
	-1	0.3257	0.3309	0.3356	0.3398	0.3438
	0	0.6743	0.6691	0.6644	0.6602	0.6563
	1	0.9140	0.9080	0.9021	0.8963	0.8906
	2	0.9907	0.9894	0.9878	0.9862	0.9844
	3	1.0000	1.0000	1.0000	1.0000	1.0000
4	-4	0.0019	0.0023	0.0028	0.0033	0.0039
	-3	0.0234	0.0262	0.0291	0.0321	0.0352
	-2	0.1202	0.1266	0.1328	0.1388	0.1445
	-1	0.3486	0.3527	0.3565	0.3600	0.3633
	0	0.6514	0.6473	0.6435	0.6400	0.6367
	1	0.8798	0.8734	0.8672	0.8612	0.8555
	2	0.9766	0.9738	0.9709	0.9679	0.9648
	3	0.9981	0.9977	0.9972	0.9967	0.9961
	4	1.0000	1.0000	1.0000	1.0000	1.0000
	5					
5	-5	0.0004	0.0005	0.0006	0.0008	0.0010
	-4	0.0060	0.0071	0.0082	0.0094	0.0107
	-3	0.0392	0.0430	0.0469	0.0508	0.0547
	-2	0.1478	0.1543	0.1604	0.1663	0.1719
	-1	0.3642	0.3678	0.3711	0.3741	0.3770
	0	0.6358	0.6322	0.6289	0.6259	0.6230
	1	0.8522	0.8457	0.8396	0.8337	0.8281
	2	0.9608	0.9570	0.9531	0.9492	0.9453
	3	0.9940	0.9929	0.9918	0.9906	0.9893
	4	0.9996	0.9995	0.9994	0.9992	0.9990
6	5	1.0000	1.0000	1.0000	1.0000	1.0000
	-6	0.0001	0.0001	0.0001	0.0002	0.0002
	-5	0.0015	0.0018	0.0022	0.0027	0.0032
	-4	0.0118	0.0135	0.0154	0.0173	0.0193
	-3	0.0551	0.0596	0.0641	0.0686	0.0730
	-2	0.1705	0.1768	0.1828	0.1884	0.1938
	-1	0.3758	0.3790	0.3819	0.3847	0.3872
	0	0.6242	0.6210	0.6181	0.6153	0.6128
	1	0.8295	0.8232	0.8172	0.8116	0.8062

## P

N	k	.21	.22	.23	.24	.25
7	2	0.9449	0.9404	0.9359	0.9314	0.9270
	3	0.9882	0.9865	0.9846	0.9827	0.9807
	4	0.9985	0.9982	0.9978	0.9973	0.9968
	5	0.9999	0.9999	0.9999	0.9998	0.9998
	6	1.0000	1.0000	1.0000	1.0000	1.0000
	-7	0.0000	0.0000	0.0000	0.0000	0.0001
	-6	0.0004	0.0005	0.0006	0.0007	0.0009
	-5	0.0034	0.0040	0.0048	0.0056	0.0065
	-4	0.0187	0.0211	0.0236	0.0261	0.0287
	-3	0.0702	0.0752	0.0802	0.0850	0.0898
	-2	0.1894	0.1955	0.2013	0.2068	0.2120
	-1	0.3848	0.3877	0.3904	0.3929	0.3953
	0	0.6152	0.6123	0.6096	0.6071	0.6047
	1	0.8106	0.8045	0.7987	0.7932	0.7880
	2	0.9298	0.9248	0.9198	0.9150	0.9102
	3	0.9813	0.9789	0.9764	0.9739	0.9713
8	4	0.9966	0.9960	0.9952	0.9944	0.9935
	5	0.9996	0.9995	0.9994	0.9993	0.9991
	6	1.0000	1.0000	1.0000	1.0000	0.9999
	7	1.0000	1.0000	1.0000	1.0000	1.0000
	-7	0.0001	0.0001	0.0002	0.0002	0.0003
	-6	0.0009	0.0012	0.0014	0.0017	0.0021
	-5	0.0060	0.0070	0.0081	0.0093	0.0106
	-4	0.0263	0.0293	0.0323	0.0353	0.0384
	-3	0.0844	0.0898	0.0950	0.1001	0.1051
	-2	0.2054	0.2113	0.2169	0.2222	0.2272
	-1	0.3922	0.3948	0.3973	0.3996	0.4018
	0	0.6078	0.6052	0.6027	0.6004	0.5982
	1	0.7946	0.7887	0.7831	0.7778	0.7728
	2	0.9156	0.9102	0.9050	0.8999	0.8949
	3	0.9737	0.9707	0.9677	0.9647	0.9616
	4	0.9940	0.9930	0.9919	0.9907	0.9894
	5	0.9991	0.9988	0.9986	0.9983	0.9979
	6	0.9999	0.9999	0.9998	0.9998	0.9997
	7	1.0000	1.0000	1.0000	1.0000	1.0000
9	-8	0.0000	0.0000	0.0000	0.0001	0.0001
	-7	0.0002	0.0003	0.0004	0.0005	0.0007
	-6	0.0018	0.0022	0.0027	0.0032	0.0038
	-5	0.0092	0.0106	0.0121	0.0137	0.0154
	-4	0.0343	0.0377	0.0411	0.0446	0.0481
	-3	0.0976	0.1032	0.1086	0.1139	0.1189
	-2	0.2192	0.2250	0.2304	0.2355	0.2403

## P

		.21	.22	.23	.24	.25
N	k					
10	-1	0.3982	0.4007	0.4031	0.4052	0.4073
	0	0.6018	0.5993	0.5969	0.5948	0.5927
	1	0.7808	0.7750	0.7696	0.7645	0.7597
	2	0.9024	0.8968	0.8914	0.8861	0.8811
	3	0.9657	0.9623	0.9589	0.9554	0.9519
	4	0.9908	0.9894	0.9879	0.9863	0.9846
	5	0.9982	0.9978	0.9973	0.9968	0.9962
	6	0.9998	0.9997	0.9996	0.9995	0.9993
	7	1.0000	1.0000	1.0000	0.9999	0.9999
	8	1.0000	1.0000	1.0000	1.0000	1.0000
	-8	0.0001	0.0001	0.0001	0.0002	0.0002
	-7	0.0005	0.0007	0.0008	0.0011	0.0013
	-6	0.0030	0.0036	0.0043	0.0051	0.0059
	-5	0.0129	0.0147	0.0166	0.0186	0.0207
11	-4	0.0423	0.0461	0.0500	0.0538	0.0577
	-3	0.1099	0.1156	0.1211	0.1264	0.1316
	-2	0.2313	0.2368	0.2421	0.2470	0.2517
	-1	0.4034	0.4057	0.4079	0.4100	0.4119
	0	0.5966	0.5943	0.5921	0.5900	0.5881
	1	0.7687	0.7632	0.7579	0.7530	0.7483
	2	0.8901	0.8844	0.8789	0.8736	0.8684
	3	0.9577	0.9539	0.9500	0.9462	0.9423
	4	0.9871	0.9852	0.9834	0.9814	0.9793
	5	0.9970	0.9964	0.9957	0.9949	0.9941
	6	0.9995	0.9993	0.9992	0.9989	0.9987
	7	0.9999	0.9999	0.9999	0.9998	0.9998
	8	1.0000	1.0000	1.0000	1.0000	1.0000
	-9	0.0000	0.0000	0.0000	0.0000	0.0001
	-8	0.0001	0.0002	0.0003	0.0003	0.0004
	-7	0.0010	0.0012	0.0015	0.0018	0.0022
	-6	0.0046	0.0054	0.0064	0.0074	0.0085

N	k	.21	.22	.23	.24	.25
12	7	0.9999	0.9998	0.9997	0.9997	0.9996
	8	1.0000	1.0000	1.0000	1.0000	0.9999
	9	1.0000	1.0000	1.0000	1.0000	1.0000
	-9	0.0000	0.0001	0.0001	0.0001	0.0001
	-8	0.0003	0.0004	0.0005	0.0006	0.0008
	-7	0.0015	0.0019	0.0023	0.0028	0.0033
	-6	0.0064	0.0075	0.0087	0.0100	0.0113
	-5	0.0214	0.0239	0.0266	0.0292	0.0320
	-4	0.0582	0.0627	0.0671	0.0715	0.0758
	-3	0.1316	0.1375	0.1431	0.1485	0.1537
	-2	0.2514	0.2567	0.2616	0.2662	0.2706
	-1	0.4117	0.4138	0.4158	0.4177	0.4194
	0	0.5883	0.5862	0.5842	0.5823	0.5806
	1	0.7486	0.7433	0.7384	0.7338	0.7294
	2	0.8684	0.8625	0.8569	0.8515	0.8463
	3	0.9418	0.9373	0.9329	0.9285	0.9242
	4	0.9786	0.9761	0.9734	0.9708	0.9680
13	5	0.9936	0.9925	0.9913	0.9900	0.9887
	6	0.9985	0.9981	0.9977	0.9972	0.9967
	7	0.9997	0.9996	0.9995	0.9994	0.9992
	8	1.0000	0.9999	0.9999	0.9999	0.9999
	9	1.0000	1.0000	1.0000	1.0000	1.0000
	-9	0.0001	0.0001	0.0002	0.0002	0.0003
	-8	0.0005	0.0006	0.0008	0.0010	0.0012
	-7	0.0023	0.0028	0.0034	0.0040	0.0047
	-6	0.0085	0.0099	0.0113	0.0129	0.0145
	-5	0.0260	0.0289	0.0318	0.0348	0.0378
	-4	0.0659	0.0706	0.0753	0.0798	0.0843
	-3	0.1414	0.1473	0.1529	0.1583	0.1635
	-2	0.2599	0.2650	0.2698	0.2743	0.2786
	-1	0.4151	0.4172	0.4191	0.4208	0.4225
	0	0.5849	0.5828	0.5809	0.5792	0.5775
	1	0.7401	0.7350	0.7302	0.7257	0.7214
	2	0.8586	0.8527	0.8471	0.8417	0.8365
	3	0.9341	0.9294	0.9247	0.9202	0.9157
	4	0.9740	0.9711	0.9682	0.9652	0.9622
	5	0.9915	0.9901	0.9887	0.9871	0.9855
	6	0.9977	0.9972	0.9966	0.9960	0.9953
	7	0.9995	0.9994	0.9992	0.9990	0.9988
	8	0.9999	0.9999	0.9998	0.9998	0.9997
	9	1.0000	1.0000	1.0000	1.0000	1.0000
14	-10	0.0000	0.0000	0.0000	0.0001	0.0001

## P

		.21	.22	.23	.24	.25
N	k					
15	-9	0.0002	0.0002	0.0003	0.0004	0.0005
	-8	0.0008	0.0010	0.0013	0.0015	0.0019
	-7	0.0032	0.0039	0.0046	0.0054	0.0063
	-6	0.0109	0.0125	0.0142	0.0160	0.0178
	-5	0.0307	0.0339	0.0371	0.0403	0.0436
	-4	0.0734	0.0783	0.0831	0.0878	0.0925
	-3	0.1504	0.1563	0.1619	0.1673	0.1725
	-2	0.2676	0.2726	0.2773	0.2816	0.2858
	-1	0.4182	0.4201	0.4220	0.4237	0.4253
	0	0.5818	0.5799	0.5780	0.5763	0.5747
	1	0.7324	0.7274	0.7227	0.7184	0.7142
	2	0.8496	0.8437	0.8381	0.8327	0.8275
	3	0.9266	0.9217	0.9169	0.9122	0.9075
	4	0.9693	0.9661	0.9629	0.9597	0.9564
	5	0.9891	0.9875	0.9858	0.9840	0.9822
	6	0.9968	0.9961	0.9954	0.9946	0.9937
	7	0.9992	0.9990	0.9987	0.9985	0.9981
	8	0.9998	0.9998	0.9997	0.9996	0.9995
	9	1.0000	1.0000	1.0000	0.9999	0.9999
	10	1.0000	1.0000	1.0000	1.0000	1.0000
16	-10	0.0000	0.0001	0.0001	0.0001	0.0002
	-9	0.0003	0.0003	0.0004	0.0006	0.0007
	-8	0.0012	0.0015	0.0018	0.0022	0.0026
	-7	0.0043	0.0052	0.0061	0.0070	0.0081
	-6	0.0134	0.0153	0.0173	0.0193	0.0214
	-5	0.0355	0.0389	0.0424	0.0459	0.0494
	-4	0.0806	0.0857	0.0907	0.0955	0.1002
	-3	0.1588	0.1647	0.1703	0.1757	0.1808
	-2	0.2746	0.2795	0.2840	0.2883	0.2923
	-1	0.4209	0.4228	0.4246	0.4262	0.4278
	0	0.5791	0.5772	0.5754	0.5738	0.5722
	1	0.7254	0.7205	0.7160	0.7117	0.7077
	2	0.8412	0.8353	0.8297	0.8243	0.8192
	3	0.9194	0.9143	0.9093	0.9045	0.8998
	4	0.9645	0.9611	0.9576	0.9541	0.9506
	5	0.9866	0.9847	0.9827	0.9807	0.9786
	6	0.9957	0.9948	0.9939	0.9930	0.9919
	7	0.9988	0.9985	0.9982	0.9978	0.9974
	8	0.9997	0.9997	0.9996	0.9994	0.9993
	9	1.0000	0.9999	0.9999	0.9999	0.9998
	10	1.0000	1.0000	1.0000	1.0000	1.0000

N	k	.21	.22	.23	.24	.25
17	-8	0.0016	0.0020	0.0025	0.0030	0.0035
	-7	0.0056	0.0066	0.0077	0.0088	0.0100
	-6	0.0162	0.0183	0.0205	0.0227	0.0251
	-5	0.0403	0.0440	0.0477	0.0514	0.0551
	-4	0.0876	0.0928	0.0979	0.1028	0.1077
	-3	0.1667	0.1726	0.1782	0.1835	0.1885
	-2	0.2810	0.2858	0.2902	0.2944	0.2983
	-1	0.4234	0.4252	0.4269	0.4285	0.4300
	0	0.5766	0.5748	0.5731	0.5715	0.5700
	1	0.7190	0.7142	0.7098	0.7056	0.7017
	2	0.8333	0.8274	0.8218	0.8165	0.8115
	3	0.9124	0.9072	0.9021	0.8972	0.8923
	4	0.9597	0.9560	0.9523	0.9486	0.9449
	5	0.9838	0.9817	0.9795	0.9773	0.9749
	6	0.9944	0.9934	0.9923	0.9912	0.9900
	7	0.9984	0.9980	0.9975	0.9970	0.9965
	8	0.9996	0.9995	0.9993	0.9991	0.9989
	9	0.9999	0.9999	0.9998	0.9998	0.9997
	10	1.0000	1.0000	1.0000	1.0000	0.9999
18	-11	0.0000	0.0000	0.0001	0.0001	0.0001
	-10	0.0001	0.0002	0.0002	0.0003	0.0004
	-9	0.0006	0.0008	0.0010	0.0012	0.0015
	-8	0.0022	0.0027	0.0032	0.0039	0.0045
	-7	0.0070	0.0082	0.0094	0.0107	0.0122
	-6	0.0190	0.0214	0.0238	0.0263	0.0288
	-5	0.0452	0.0491	0.0530	0.0569	0.0607
	-4	0.0943	0.0996	0.1048	0.1099	0.1147
	-3	0.1741	0.1799	0.1855	0.1907	0.1958
	-2	0.2869	0.2915	0.2959	0.3000	0.3038
	-1	0.4257	0.4274	0.4291	0.4306	0.4321
	0	0.5743	0.5726	0.5709	0.5694	0.5679
	1	0.7131	0.7085	0.7041	0.7000	0.6962
	2	0.8259	0.8201	0.8145	0.8093	0.8042
	3	0.9057	0.9004	0.8952	0.8901	0.8853
	4	0.9548	0.9509	0.9470	0.9431	0.9393
	5	0.9810	0.9786	0.9762	0.9737	0.9712
	6	0.9930	0.9918	0.9906	0.9893	0.9878
	7	0.9978	0.9973	0.9968	0.9961	0.9955
	8	0.9994	0.9992	0.9990	0.9988	0.9985
	9	0.9999	0.9998	0.9998	0.9997	0.9996
	10	1.0000	1.0000	0.9999	0.9999	0.9999
	11	1.0000	1.0000	1.0000	1.0000	1.0000

## P

N	k	.21	.22	.23	.24	.25
19	-10	0.0002	0.0003	0.0004	0.0005	0.0006
	-9	0.0008	0.0011	0.0013	0.0016	0.0020
	-8	0.0029	0.0035	0.0041	0.0049	0.0057
	-7	0.0085	0.0099	0.0113	0.0128	0.0144
	-6	0.0220	0.0246	0.0272	0.0299	0.0326
	-5	0.0500	0.0541	0.0582	0.0622	0.0662
	-4	0.1007	0.1062	0.1114	0.1166	0.1215
	-3	0.1810	0.1868	0.1923	0.1975	0.2025
	-2	0.2924	0.2969	0.3011	0.3051	0.3089
	-1	0.4278	0.4295	0.4311	0.4326	0.4340
	0	0.5722	0.5705	0.5689	0.5674	0.5660
	1	0.7076	0.7031	0.6989	0.6949	0.6911
	2	0.8190	0.8132	0.8077	0.8025	0.7975
	3	0.8993	0.8938	0.8886	0.8834	0.8785
	4	0.9500	0.9459	0.9418	0.9378	0.9338
	5	0.9780	0.9754	0.9728	0.9701	0.9674
	6	0.9915	0.9901	0.9887	0.9872	0.9856
	7	0.9971	0.9965	0.9959	0.9951	0.9943
	8	0.9992	0.9989	0.9987	0.9984	0.9980
	9	0.9998	0.9997	0.9996	0.9995	0.9994
	10	1.0000	0.9999	0.9999	0.9999	0.9998
	11	1.0000	1.0000	1.0000	1.0000	1.0000
	-11	0.0001	0.0001	0.0001	0.0002	0.0002
	-10	0.0003	0.0004	0.0005	0.0007	0.0008
	-9	0.0011	0.0014	0.0018	0.0021	0.0025
	-8	0.0036	0.0044	0.0051	0.0060	0.0069
	-7	0.0102	0.0117	0.0133	0.0150	0.0168
	-6	0.0250	0.0278	0.0307	0.0336	0.0365
	-5	0.0548	0.0591	0.0633	0.0675	0.0717
	-4	0.1069	0.1125	0.1178	0.1230	0.1279
	-3	0.1875	0.1933	0.1987	0.2039	0.2088
	-2	0.2974	0.3018	0.3060	0.3099	0.3136
	-1	0.4297	0.4313	0.4329	0.4343	0.4357
	0	0.5703	0.5687	0.5671	0.5657	0.5643
	1	0.7026	0.6982	0.6940	0.6901	0.6864
	2	0.8125	0.8067	0.8013	0.7961	0.7912
	3	0.8931	0.8875	0.8822	0.8770	0.8721
	4	0.9452	0.9409	0.9367	0.9325	0.9283
	5	0.9750	0.9722	0.9693	0.9664	0.9635
	6	0.9898	0.9883	0.9867	0.9850	0.9832
	7	0.9964	0.9956	0.9949	0.9940	0.9931
	8	0.9989	0.9986	0.9982	0.9979	0.9975
	9	0.9997	0.9996	0.9995	0.9993	0.9992

## p

		.21	.22	.23	.24	.25
N	k					
20	10	0.9999	0.9999	0.9999	0.9998	0.9998
	11	1.0000	1.0000	1.0000	1.0000	0.9999
	-12	0.0000	0.0000	0.0000	0.0001	0.0001
	-11	0.0001	0.0002	0.0002	0.0003	0.0003
	-10	0.0004	0.0006	0.0007	0.0009	0.0011
	-9	0.0015	0.0018	0.0023	0.0027	0.0032
	-8	0.0045	0.0053	0.0062	0.0072	0.0083
	-7	0.0119	0.0136	0.0154	0.0173	0.0192
	-6	0.0282	0.0311	0.0342	0.0373	0.0403
	-5	0.0595	0.0639	0.0683	0.0727	0.0769
	-4	0.1129	0.1185	0.1239	0.1291	0.1341
	-3	0.1937	0.1994	0.2048	0.2099	0.2148
	-2	0.3021	0.3064	0.3105	0.3143	0.3179
	-1	0.4314	0.4331	0.4346	0.4360	0.4373
	0	0.5686	0.5669	0.5654	0.5640	0.5627
	1	0.6979	0.6936	0.6895	0.6857	0.6821
	2	0.8063	0.8006	0.7952	0.7901	0.7852
	3	0.8871	0.8815	0.8761	0.8709	0.8659
	4	0.9405	0.9361	0.9317	0.9273	0.9231
	5	0.9718	0.9689	0.9658	0.9627	0.9597
	6	0.9881	0.9864	0.9846	0.9827	0.9808
	7	0.9955	0.9947	0.9938	0.9928	0.9917
	8	0.9985	0.9982	0.9977	0.9973	0.9968
	9	0.9996	0.9994	0.9993	0.9991	0.9989
	10	0.9999	0.9998	0.9998	0.9997	0.9997
	11	1.0000	1.0000	1.0000	0.9999	0.9999
	12	1.0000	1.0000	1.0000	1.0000	1.0000
21	-12	0.0000	0.0001	0.0001	0.0001	0.0001
	-11	0.0002	0.0002	0.0003	0.0004	0.0005
	-10	0.0006	0.0008	0.0010	0.0012	0.0014
	-9	0.0019	0.0023	0.0028	0.0034	0.0040
	-8	0.0054	0.0064	0.0074	0.0086	0.0098
	-7	0.0138	0.0157	0.0176	0.0197	0.0218
	-6	0.0313	0.0345	0.0377	0.0410	0.0442
	-5	0.0641	0.0687	0.0733	0.0777	0.0821
	-4	0.1187	0.1243	0.1297	0.1349	0.1400
	-3	0.1995	0.2051	0.2105	0.2156	0.2204
	-2	0.3065	0.3107	0.3147	0.3185	0.3220
	-1	0.4331	0.4347	0.4361	0.4375	0.4388
	0	0.5669	0.5653	0.5639	0.5625	0.5612
	1	0.6935	0.6893	0.6853	0.6815	0.6780
	2	0.8005	0.7949	0.7895	0.7844	0.7796
	3	0.8814	0.8757	0.8703	0.8651	0.8600

N	k	.21	.22	.23	.24	.25
22	4	0.9359	0.9313	0.9267	0.9223	0.9179
	5	0.9687	0.9655	0.9623	0.9590	0.9558
	6	0.9862	0.9843	0.9824	0.9803	0.9782
	7	0.9946	0.9936	0.9926	0.9914	0.9902
	8	0.9981	0.9977	0.9972	0.9966	0.9960
	9	0.9994	0.9992	0.9990	0.9988	0.9986
	10	0.9998	0.9998	0.9997	0.9996	0.9995
	11	1.0000	0.9999	0.9999	0.9999	0.9999
	12	1.0000	1.0000	1.0000	1.0000	1.0000
	-12	0.0001	0.0001	0.0001	0.0001	0.0002
	-11	0.0002	0.0003	0.0004	0.0005	0.0006
	-10	0.0008	0.0010	0.0012	0.0015	0.0018
	-9	0.0024	0.0029	0.0034	0.0041	0.0048
	-8	0.0064	0.0075	0.0087	0.0100	0.0113
	-7	0.0157	0.0178	0.0199	0.0221	0.0244
	-6	0.0345	0.0379	0.0413	0.0447	0.0481
	-5	0.0687	0.0734	0.0781	0.0826	0.0871
	-4	0.1242	0.1299	0.1353	0.1406	0.1456
	-3	0.2050	0.2106	0.2159	0.2209	0.2257
	-2	0.3106	0.3148	0.3187	0.3223	0.3258
	-1	0.4346	0.4361	0.4376	0.4389	0.4402
	0	0.5654	0.5639	0.5624	0.5611	0.5598
	1	0.6894	0.6852	0.6813	0.6777	0.6742
	2	0.7950	0.7894	0.7841	0.7791	0.7743
	3	0.8758	0.8701	0.8647	0.8594	0.8544
	4	0.9313	0.9266	0.9219	0.9174	0.9129
	5	0.9655	0.9621	0.9587	0.9553	0.9519
	6	0.9843	0.9822	0.9801	0.9779	0.9756
	7	0.9936	0.9925	0.9913	0.9900	0.9888
	8	0.9976	0.9971	0.9966	0.9959	0.9952
	9	0.9992	0.9990	0.9988	0.9985	0.9982
	10	0.9998	0.9997	0.9996	0.9995	0.9994
	11	0.9999	0.9999	0.9999	0.9999	0.9998
	12	1.0000	1.0000	1.0000	1.0000	0.9999
23	-13	0.0000	0.0000	0.0000	0.0001	0.0001
	-12	0.0001	0.0001	0.0002	0.0002	0.0003
	-11	0.0003	0.0004	0.0005	0.0007	0.0008
	-10	0.0010	0.0012	0.0015	0.0019	0.0023
	-9	0.0029	0.0035	0.0041	0.0049	0.0057
	-8	0.0075	0.0088	0.0101	0.0115	0.0129
	-7	0.0177	0.0199	0.0222	0.0246	0.0270
	-6	0.0377	0.0413	0.0448	0.0484	0.0519
	-5	0.0732	0.0780	0.0828	0.0874	0.0920

P

N	k	.21	.22	.23	.24	.25
24	-4	0.1295	0.1352	0.1407	0.1459	0.1510
	-3	0.2102	0.2157	0.2210	0.2260	0.2307
	-2	0.3145	0.3186	0.3224	0.3260	0.3294
	-1	0.4360	0.4375	0.4389	0.4403	0.4415
	0	0.5640	0.5625	0.5611	0.5597	0.5585
	1	0.6855	0.6814	0.6776	0.6740	0.6706
	2	0.7808	0.7843	0.7790	0.7740	0.7693
	3	0.8705	0.8648	0.8593	0.8541	0.8490
	4	0.9268	0.9220	0.9172	0.9126	0.9080
	5	0.9623	0.9587	0.9552	0.9516	0.9481
	6	0.9823	0.9801	0.9778	0.9754	0.9730
	7	0.9925	0.9912	0.9899	0.9885	0.9871
	8	0.9971	0.9965	0.9958	0.9951	0.9943
	9	0.9990	0.9988	0.9985	0.9981	0.9977
	10	0.9997	0.9996	0.9995	0.9993	0.9992
	11	0.9999	0.9999	0.9998	0.9998	0.9997
	12	1.0000	1.0000	1.0000	0.9999	0.9999
	13	1.0000	1.0000	1.0000	1.0000	1.0000
	-13	0.0000	0.0000	0.0001	0.0001	0.0001
	-12	0.0001	0.0002	0.0002	0.0003	0.0004
	-11	0.0004	0.0005	0.0007	0.0008	0.0010
	-10	0.0012	0.0016	0.0019	0.0023	0.0028
	-9	0.0035	0.0042	0.0049	0.0057	0.0066
	-8	0.0087	0.0101	0.0115	0.0130	0.0147
	-7	0.0198	0.0222	0.0246	0.0272	0.0297
	-6	0.0410	0.0447	0.0484	0.0520	0.0557
	-5	0.0776	0.0825	0.0874	0.0921	0.0967
	-4	0.1346	0.1403	0.1458	0.1511	0.1562
	-3	0.2151	0.2206	0.2258	0.2308	0.2354
	-2	0.3181	0.3221	0.3259	0.3294	0.3327
	-1	0.4374	0.4388	0.4402	0.4415	0.4427
	0	0.5626	0.5612	0.5598	0.5585	0.5573
	1	0.6819	0.6779	0.6741	0.6706	0.6673
	2	0.7849	0.7794	0.7742	0.7692	0.7646
	3	0.8654	0.8597	0.8542	0.8489	0.8438
	4	0.9224	0.9175	0.9126	0.9079	0.9033
	5	0.9590	0.9553	0.9516	0.9480	0.9443
	6	0.9802	0.9778	0.9754	0.9728	0.9703
	7	0.9913	0.9899	0.9885	0.9870	0.9853
	8	0.9965	0.9958	0.9951	0.9943	0.9934
	9	0.9988	0.9984	0.9981	0.9977	0.9972
	10	0.9996	0.9995	0.9993	0.9992	0.9990
	11	0.9999	0.9998	0.9998	0.9997	0.9996

## P

N	k	.21	.22	.23	.24	.25
25	12	1.0000	1.0000	0.9999	0.9999	0.9999
	13	1.0000	1.0000	1.0000	1.0000	1.0000
	-13	0.0000	0.0001	0.0001	0.0001	0.0002
	-12	0.0002	0.0002	0.0003	0.0004	0.0005
	-11	0.0005	0.0007	0.0009	0.0011	0.0013
	-10	0.0015	0.0019	0.0023	0.0028	0.0033
	-9	0.0041	0.0049	0.0057	0.0067	0.0077
	-8	0.0099	0.0114	0.0130	0.0147	0.0164
	-7	0.0219	0.0245	0.0271	0.0297	0.0325
	-6	0.0442	0.0480	0.0519	0.0557	0.0595
	-5	0.0819	0.0869	0.0918	0.0966	0.1013
	-4	0.1395	0.153	0.1508	0.1561	0.1611
	-3	0.2199	0.2253	0.2304	0.2353	0.2399
	-2	0.3215	0.3255	0.3292	0.3326	0.3359
	-1	0.4386	0.4401	0.4414	0.4427	0.4439
	0	0.5614	0.5599	0.5586	0.5573	0.5561
	1	0.6785	0.6745	0.6708	0.6674	0.6641
	2	0.7801	0.7747	0.7696	0.7647	0.7601
	3	0.8605	0.8547	0.8492	0.8439	0.8389
	4	0.9181	0.9131	0.9082	0.9034	0.8987
	5	0.9558	0.9520	0.9481	0.9443	0.9405
	6	0.9781	0.9755	0.9729	0.9703	0.9675
	7	0.9901	0.9886	0.9870	0.9853	0.9836
	8	0.9959	0.9951	0.9943	0.9933	0.9923
	9	0.9985	0.9981	0.9977	0.9972	0.9967
	10	0.9995	0.9993	0.9991	0.9989	0.9987
	11	0.9998	0.9998	0.9997	0.9996	0.9995
	12	1.0000	0.9999	0.9999	0.9999	0.9998
	13	1.0000	1.0000	1.0000	1.0000	1.0000
26	-13	0.0001	0.0001	0.0001	0.0002	0.0002
	-12	0.0002	0.0003	0.0004	0.0005	0.0006
	-11	0.0007	0.0008	0.0011	0.0013	0.0016
	-10	0.0019	0.0023	0.0028	0.0033	0.0039
	-9	0.0048	0.0057	0.0066	0.0077	0.0088
	-8	0.0112	0.0128	0.0146	0.0164	0.0182
	-7	0.0241	0.0268	0.0295	0.0324	0.0352
	-6	0.0474	0.0514	0.0554	0.0593	0.0632
	-5	0.0861	0.0912	0.0962	0.1011	0.1058
	-4	0.1443	0.1500	0.1556	0.1608	0.1659
	-3	0.2243	0.2297	0.2348	0.2397	0.2442
	-2	0.3248	0.3286	0.3323	0.3357	0.3389
	-1	0.4398	0.4412	0.4425	0.4438	0.4449
	0	0.5602	0.5588	0.5575	0.5562	0.5551

N	k	.21	.22	.23	.24	.25
27	1	0.6752	0.6714	0.6677	0.6643	0.6611
	2	0.7757	0.7703	0.7652	0.7603	0.7558
	3	0.8557	0.8500	0.8444	0.8392	0.8341
	4	0.9139	0.9088	0.9038	0.8989	0.8942
	5	0.9526	0.9486	0.9446	0.9407	0.9368
	6	0.9759	0.9732	0.9705	0.9676	0.9648
	7	0.9888	0.9871	0.9854	0.9836	0.9818
	8	0.9952	0.9943	0.9934	0.9923	0.9912
	9	0.9981	0.9977	0.9972	0.9967	0.9961
	10	0.9993	0.9992	0.9989	0.9987	0.9984
	11	0.9998	0.9997	0.9996	0.9995	0.9994
	12	0.9999	0.9999	0.9999	0.9998	0.9998
	13	1.0000	1.0000	1.0000	1.0000	0.9999
28	-14	0.0000	0.0000	0.0000	0.0001	0.0001
	-13	0.0001	0.0001	0.0002	0.0002	0.0003
	-12	0.0003	0.0004	0.0005	0.0006	0.0007
	-11	0.0008	0.0010	0.0013	0.0016	0.0019
	-10	0.0022	0.0027	0.0033	0.0039	0.0045
	-9	0.0055	0.0065	0.0076	0.0087	0.0099
	-8	0.0126	0.0143	0.0162	0.0181	0.0201
	-7	0.0263	0.0291	0.0320	0.0350	0.0380
	-6	0.0506	0.0547	0.0588	0.0629	0.0668
	-5	0.0902	0.0954	0.1005	0.1054	0.1102
	-4	0.1489	0.1546	0.1601	0.1654	0.1704
	-3	0.2286	0.2340	0.2390	0.2438	0.2483
	-2	0.3278	0.3317	0.3352	0.3386	0.3417
	-1	0.4409	0.4423	0.4436	0.4448	0.4460
	0	0.5591	0.5577	0.5564	0.5552	0.5540
	1	0.6722	0.6683	0.6648	0.6614	0.6583
	2	0.7714	0.7660	0.7610	0.7562	0.7517
	3	0.8511	0.8454	0.8399	0.8346	0.8296
	4	0.9098	0.9046	0.8995	0.8946	0.8898
	5	0.9494	0.9453	0.9412	0.9371	0.9332
	6	0.9737	0.9709	0.9680	0.9650	0.9620
	7	0.9874	0.9857	0.9838	0.9819	0.9799
	8	0.9945	0.9935	0.9924	0.9913	0.9901
	9	0.9978	0.9973	0.9967	0.9961	0.9955
	10	0.9992	0.9990	0.9987	0.9984	0.9981
	11	0.9997	0.9996	0.9995	0.9994	0.9993
	12	0.9999	0.9999	0.9998	0.9998	0.9997
	13	1.0000	1.0000	1.0000	0.9999	0.9999
	14	1.0000	1.0000	1.0000	1.0000	1.0000
	-14	0.0000	0.0000	0.0001	0.0001	0.0001

## P

N	k	.21	.22	.23	.24	.25
29	-13	0.0001	0.0002	0.0002	0.0003	0.0003
	-12	0.0003	0.0005	0.0006	0.0007	0.0009
	-11	0.0010	0.0013	0.0016	0.0019	0.0023
	-10	0.0026	0.0032	0.0038	0.0045	0.0052
	-9	0.0063	0.0074	0.0086	0.0098	0.0111
	-8	0.0140	0.0159	0.0179	0.0199	0.0220
	-7	0.0285	0.0315	0.0346	0.0376	0.0407
	-6	0.0538	0.0581	0.0622	0.0664	0.0704
	-5	0.0942	0.0995	0.1046	0.1096	0.1144
	-4	0.1533	0.1591	0.1646	0.1698	0.1748
	-3	0.2327	0.2380	0.2430	0.2477	0.2522
	-2	0.3308	0.3345	0.3380	0.3413	0.3444
	-1	0.4420	0.4434	0.4446	0.4458	0.4469
	0	0.5580	0.5566	0.5554	0.5542	0.5531
	1	0.6692	0.6655	0.6620	0.6587	0.6556
	2	0.7673	0.7620	0.7570	0.7523	0.7478
	3	0.8467	0.8409	0.8354	0.8302	0.8252
	4	0.9058	0.9005	0.8954	0.8904	0.8856
	5	0.9462	0.9419	0.9378	0.9336	0.9296
	6	0.9715	0.9685	0.9654	0.9624	0.9593
	7	0.9860	0.9841	0.9821	0.9801	0.9780
	8	0.9937	0.9926	0.9914	0.9902	0.9889
	9	0.9974	0.9968	0.9962	0.9955	0.9948
	10	0.9990	0.9987	0.9984	0.9981	0.9977
	11	0.9997	0.9995	0.9994	0.9993	0.9991
	12	0.9999	0.9998	0.9998	0.9997	0.9997
	13	1.0000	1.0000	0.9999	0.9999	0.9999
	14	1.0000	1.0000	1.0000	1.0000	1.0000

## P

N	k	.21	.22	.23	.24	.25
30	2	0.7634	0.7581	0.7532	0.7485	0.7441
	3	0.8424	0.8367	0.8312	0.8259	0.8209
	4	0.9019	0.8965	0.8913	0.8863	0.8815
	5	0.9430	0.9387	0.9344	0.9302	0.9260
	6	0.9692	0.9661	0.9629	0.9597	0.9565
	7	0.9846	0.9826	0.9804	0.9783	0.9760
	8	0.9929	0.9917	0.9904	0.9890	0.9876
	9	0.9970	0.9963	0.9956	0.9949	0.9940
	10	0.9988	0.9985	0.9982	0.9978	0.9973
	11	0.9996	0.9994	0.9993	0.9991	0.9989
	12	0.9999	0.9998	0.9997	0.9997	0.9996
	13	1.0000	0.9999	0.9999	0.9999	0.9998
	14	1.0000	1.0000	1.0000	1.0000	0.9999
	-14	0.0001	0.0001	0.0001	0.0001	0.0002
	-13	0.0002	0.0002	0.0003	0.0004	0.0005
	-12	0.0005	0.0007	0.0009	0.0011	0.0013
	-11	0.0014	0.0018	0.0022	0.0026	0.0031
	-10	0.0035	0.0042	0.0050	0.0058	0.0067
	-9	0.0080	0.0093	0.0107	0.0121	0.0137
	-8	0.0169	0.0191	0.0213	0.0236	0.0259
	-7	0.0330	0.0363	0.0396	0.0429	0.0462
	-6	0.0601	0.0646	0.0690	0.0733	0.0775
	-5	0.1020	0.1074	0.1126	0.1176	0.1225
	-4	0.1617	0.1674	0.1729	0.1781	0.1831
	-3	0.2404	0.2456	0.2505	0.2551	0.2595
	-2	0.3362	0.3398	0.3432	0.3464	0.3494
	-1	0.4439	0.4453	0.4465	0.4476	0.4487
	0	0.5561	0.5547	0.5535	0.5524	0.5513
	1	0.6638	0.6602	0.6568	0.6536	0.6506
	2	0.7596	0.7544	0.7495	0.7449	0.7405
	3	0.8383	0.8326	0.8271	0.8219	0.8169
	4	0.8980	0.8926	0.8874	0.8824	0.8775
	5	0.9399	0.9354	0.9310	0.9267	0.9225
	6	0.9670	0.9637	0.9604	0.9571	0.9538
	7	0.9831	0.9809	0.9787	0.9764	0.9741
	8	0.9920	0.9907	0.9893	0.9879	0.9863
	9	0.9965	0.9958	0.9950	0.9942	0.9933
	10	0.9986	0.9982	0.9978	0.9974	0.9969
	11	0.9995	0.9993	0.9991	0.9989	0.9987
	12	0.9998	0.9998	0.9997	0.9996	0.9995
	13	0.9999	0.9999	0.9999	0.9998	0.9998
	14	1.0000	1.0000	1.0000	0.9999	0.9999

**Table A.2 Compound Poisson Probability Tables**

		Lambda									
		0.02	0.04	0.06	0.08	0.10	0.15	0.20	0.25	0.30	0.35
C											
-3	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001
-2	0.000	0.000	0.000	0.001	0.001	0.002	0.004	0.006	0.009	0.012	
-1	0.010	0.019	0.029	0.038	0.046	0.067	0.087	0.104	0.121	0.137	
0	0.990	0.981	0.971	0.962	0.954	0.933	0.913	0.896	0.879	0.863	
1	1.000	1.000	1.000	0.999	0.999	0.998	0.996	0.994	0.991	0.988	
2	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999
3	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

		Lambda									
		0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
C											
-4	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.001
-3	0.001	0.001	0.002	0.002	0.003	0.003	0.004	0.005	0.006	0.006	0.006
-2	0.015	0.018	0.021	0.025	0.028	0.032	0.036	0.040	0.043	0.047	
-1	0.151	0.165	0.177	0.189	0.200	0.211	0.220	0.229	0.238	0.246	
0	0.849	0.835	0.823	0.811	0.800	0.789	0.780	0.771	0.762	0.754	
1	0.985	0.982	0.979	0.975	0.972	0.968	0.964	0.960	0.957	0.953	
2	0.999	0.999	0.998	0.998	0.997	0.997	0.996	0.995	0.994	0.994	
3	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999	0.999	
4	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

		Lambda									
		0.90	0.95	1.00	1.10	1.20	1.30	1.40	1.50	1.60	1.70
C											
-5		0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.001
-4		0.001	0.001	0.001	0.002	0.002	0.003	0.003	0.004	0.005	0.005
-3		0.007	0.008	0.009	0.011	0.014	0.016	0.019	0.022	0.025	0.028
-2		0.051	0.055	0.059	0.067	0.075	0.082	0.090	0.097	0.104	0.111
-1		0.253	0.260	0.267	0.279	0.290	0.300	0.308	0.316	0.323	0.330
0		0.747	0.740	0.733	0.721	0.710	0.700	0.692	0.684	0.677	0.670
1		0.949	0.945	0.941	0.933	0.925	0.918	0.910	0.903	0.896	0.889
2		0.993	0.992	0.991	0.989	0.986	0.984	0.981	0.978	0.975	0.972
3		0.999	0.999	0.999	0.998	0.998	0.997	0.997	0.996	0.995	0.995
4		1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999	0.999	0.999
5		1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

		Lambda									
		1.80	1.90	2.00	2.20	2.40	2.60	2.80	3.00	3.20	3.40
C											
-6		0.000	0.000	0.000	0.000	0.001	0.001	0.001	0.001	0.002	0.002
-5		0.001	0.001	0.002	0.002	0.003	0.004	0.005	0.006	0.007	0.008
-4		0.006	0.007	0.008	0.011	0.013	0.016	0.019	0.022	0.025	0.029
-3		0.031	0.034	0.037	0.044	0.050	0.057	0.063	0.070	0.076	0.082
-2		0.118	0.124	0.130	0.142	0.153	0.163	0.173	0.182	0.190	0.198
-1		0.336	0.341	0.346	0.354	0.362	0.368	0.374	0.378	0.383	0.387
0		0.664	0.659	0.654	0.646	0.638	0.632	0.626	0.622	0.617	0.613
1		0.882	0.876	0.870	0.858	0.847	0.837	0.827	0.818	0.810	0.802
2		0.969	0.966	0.963	0.956	0.950	0.943	0.937	0.930	0.924	0.918
3		0.994	0.993	0.992	0.989	0.987	0.984	0.981	0.978	0.975	0.971
4		0.999	0.999	0.998	0.998	0.997	0.996	0.995	0.994	0.993	0.992
5		1.000	1.000	1.000	1.000	0.999	0.999	0.999	0.999	0.998	0.998
6		1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

C	Lambda										
	3.60	3.80	4.00	4.20	4.40	4.60	4.80	5.00	5.20	5.40	
-8	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.001	0.001	0.001
-7	0.001	0.001	0.001	0.001	0.001	0.002	0.002	0.002	0.003	0.003	0.003
-6	0.003	0.003	0.004	0.004	0.005	0.006	0.007	0.008	0.009	0.010	
-5	0.010	0.011	0.013	0.015	0.017	0.018	0.020	0.022	0.024	0.026	
-4	0.032	0.035	0.039	0.043	0.046	0.050	0.053	0.057	0.060	0.064	
-3	0.088	0.094	0.100	0.106	0.111	0.116	0.121	0.126	0.131	0.136	
-2	0.205	0.211	0.218	0.224	0.229	0.235	0.240	0.244	0.249	0.253	
-1	0.390	0.394	0.396	0.399	0.402	0.404	0.406	0.408	0.410	0.412	
0	0.610	0.606	0.604	0.601	0.598	0.596	0.594	0.592	0.590	0.588	
1	0.795	0.789	0.782	0.776	0.771	0.765	0.760	0.756	0.751	0.747	
2	0.912	0.906	0.900	0.894	0.889	0.884	0.879	0.874	0.869	0.864	
3	0.968	0.965	0.961	0.957	0.954	0.950	0.947	0.943	0.940	0.936	
4	0.990	0.989	0.987	0.985	0.983	0.982	0.980	0.978	0.976	0.974	
5	0.997	0.997	0.996	0.996	0.995	0.994	0.993	0.992	0.991	0.990	
6	0.999	0.999	0.999	0.999	0.999	0.998	0.998	0.998	0.997	0.997	
7	1.000	1.000	1.000	1.000	1.000	1.000	0.999	0.999	0.999	0.999	
8	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	

C	Lambda									
	5.60	5.80	6.00	6.20	6.40	6.60	6.80	7.00	7.20	7.40
-9	0.000	0.000	0.000	0.001	0.001	0.001	0.001	0.001	0.001	0.001
-8	0.001	0.001	0.001	0.002	0.002	0.002	0.002	0.003	0.003	0.003
-7	0.004	0.004	0.005	0.005	0.006	0.006	0.007	0.008	0.008	0.009
-6	0.011	0.012	0.013	0.014	0.015	0.017	0.018	0.019	0.020	0.022
-5	0.028	0.031	0.033	0.035	0.037	0.039	0.041	0.044	0.046	0.048
-4	0.067	0.071	0.074	0.077	0.081	0.084	0.087	0.090	0.093	0.096
-3	0.140	0.144	0.149	0.153	0.157	0.160	0.164	0.168	0.171	0.175
-2	0.257	0.261	0.265	0.268	0.272	0.275	0.278	0.281	0.284	0.286
-1	0.414	0.415	0.417	0.418	0.419	0.421	0.422	0.423	0.424	0.425
0	0.586	0.585	0.583	0.582	0.581	0.579	0.578	0.577	0.576	0.575
1	0.743	0.739	0.735	0.732	0.728	0.725	0.722	0.719	0.716	0.714
2	0.860	0.856	0.851	0.847	0.843	0.840	0.836	0.832	0.829	0.825
3	0.933	0.929	0.926	0.923	0.919	0.916	0.913	0.910	0.907	0.904
4	0.972	0.969	0.967	0.965	0.963	0.961	0.959	0.956	0.954	0.952
5	0.989	0.988	0.987	0.986	0.985	0.983	0.982	0.981	0.980	0.978
6	0.996	0.996	0.995	0.995	0.994	0.994	0.993	0.992	0.992	0.991
7	0.999	0.999	0.999	0.998	0.998	0.998	0.998	0.997	0.997	0.997
8	1.000	1.000	1.000	0.999	0.999	0.999	0.999	0.999	0.999	0.999
9	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

C	Lambda										
	7.60	7.80	8.00	8.50	9.00	9.50	10.00	10.50	11.00	11.50	
-11	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.001	0.001	0.001
-10	0.000	0.000	0.001	0.001	0.001	0.001	0.002	0.002	0.002	0.002	0.003
-9	0.001	0.001	0.002	0.002	0.003	0.003	0.004	0.005	0.006	0.006	0.006
-8	0.004	0.004	0.004	0.006	0.007	0.008	0.009	0.011	0.012	0.014	
-7	0.010	0.010	0.011	0.013	0.015	0.018	0.020	0.022	0.025	0.028	
-6	0.023	0.024	0.026	0.029	0.033	0.037	0.040	0.044	0.048	0.052	
-5	0.050	0.052	0.055	0.060	0.065	0.071	0.076	0.081	0.086	0.091	
-4	0.099	0.102	0.105	0.112	0.119	0.125	0.131	0.137	0.143	0.148	
-3	0.178	0.181	0.184	0.192	0.199	0.205	0.211	0.217	0.222	0.228	
-2	0.289	0.292	0.294	0.300	0.305	0.310	0.315	0.319	0.323	0.327	
-1	0.426	0.427	0.428	0.430	0.433	0.434	0.436	0.438	0.439	0.441	
0	0.574	0.573	0.572	0.570	0.567	0.566	0.564	0.562	0.561	0.559	
1	0.711	0.708	0.706	0.700	0.695	0.690	0.685	0.681	0.677	0.673	
2	0.822	0.819	0.816	0.808	0.801	0.795	0.789	0.783	0.778	0.772	
3	0.901	0.898	0.895	0.888	0.881	0.875	0.869	0.863	0.857	0.852	
4	0.950	0.948	0.945	0.940	0.935	0.929	0.924	0.919	0.914	0.909	
5	0.977	0.976	0.974	0.971	0.967	0.963	0.960	0.956	0.952	0.948	
6	0.990	0.990	0.989	0.987	0.985	0.982	0.980	0.978	0.975	0.972	
7	0.996	0.996	0.996	0.994	0.993	0.992	0.991	0.989	0.988	0.986	
8	0.999	0.999	0.998	0.998	0.997	0.997	0.996	0.995	0.994	0.994	
9	1.000	1.000	0.999	0.999	0.999	0.999	0.998	0.998	0.998	0.997	
10	1.000	1.000	1.000	1.000	1.000	1.000	0.999	0.999	0.999	0.999	0.999
11	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

C	Lambda							
	12.00	12.50	13.00	13.50	14.00	14.50	15.00	
-14	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
-13	0.000	0.000	0.000	0.000	0.001	0.001	0.001	0.001
-12	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.002
-11	0.001	0.002	0.002	0.002	0.003	0.003	0.004	
-10	0.003	0.004	0.005	0.005	0.006	0.007	0.007	
-9	0.007	0.008	0.009	0.011	0.012	0.013	0.014	
-8	0.015	0.017	0.019	0.021	0.023	0.024	0.026	
-7	0.030	0.033	0.035	0.038	0.041	0.043	0.046	
-6	0.055	0.059	0.063	0.066	0.070	0.073	0.077	
-5	0.095	0.100	0.104	0.109	0.113	0.117	0.121	
-4	0.154	0.159	0.163	0.168	0.173	0.177	0.181	
-3	0.232	0.237	0.242	0.246	0.250	0.254	0.257	
-2	0.330	0.334	0.337	0.340	0.342	0.345	0.348	
-1	0.442	0.443	0.444	0.445	0.446	0.447	0.448	
0	0.558	0.557	0.556	0.555	0.554	0.553	0.552	
1	0.670	0.666	0.663	0.660	0.658	0.655	0.652	
2	0.768	0.763	0.758	0.754	0.750	0.746	0.743	
3	0.846	0.841	0.837	0.832	0.827	0.823	0.819	
4	0.905	0.900	0.896	0.891	0.887	0.883	0.879	
5	0.945	0.941	0.937	0.934	0.930	0.927	0.923	
6	0.970	0.967	0.965	0.962	0.959	0.957	0.954	
7	0.985	0.983	0.981	0.979	0.977	0.976	0.974	
8	0.993	0.992	0.991	0.989	0.988	0.987	0.986	
9	0.997	0.996	0.995	0.995	0.994	0.993	0.993	
10	0.999	0.998	0.998	0.998	0.997	0.997	0.996	
11	0.999	0.999	0.999	0.999	0.999	0.999	0.998	
12	1.000	1.000	1.000	1.000	1.000	1.000	1.000	
13	1.000	1.000	1.000	1.000	1.000	1.000	1.000	
14	1.000	1.000	1.000	1.000	1.000	1.000	1.000	