SOME DESIGN ASPECTS OF CONSTANT RESISTANCE BILATERAL AMPLIFIERS

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ABSTRACT

A practical constant resistance bilateral amplifier circuit is designed using a floating negative impedance converter together with a grounded negative impedance converter in a bridged-T network configuration.

An experimental investigation is made to show that the amplifier circuit performs in accordance with its theoretical predictions.

Finally, potential application of this amplifier in two-wire telephone repeaters is considered.

CHAPTER I

INTRODUCTION

The rapidly growing field of integrated circuits has probably been the most significant stimulus behind the recent surge of interest in linear active networks. Discrete active components, such as the transistor, are rapidly losing ground to integrated circuits, such as the operational amplifier, in solving the problems associated with the realization of complex linear network functions. Yet, in today's telephone industry, the latest vintage of the two-wire negative impedance repeater design still uses the transistor as the basic active element.

In this paper, we will apply Antoniou's [1] proposed floating negative impedance converter, using operational amplifiers, to derive a constant resistance bilateral amplifier with potential application to two-wire telephone repeaters.

As background information, the theory of active networks and of negative impedance converters will be presented in Chapters II and III. Chapter IV will cover the E-type negative impedance repeaters used in two-wire telephone circuits. The theory of operational amplifiers will be given in Chapter V, and Chapter VI will introduce a practical constant resistance bilateral amplifier circuit which will be compared to the present-day E6 repeater.

CHAPTER II

ACTIVE NETWORKS

In general, an active network is composed of passive elements (like the inductor, capacitor and resistor) and active elements (like the vacuum tube, transistor and other solide-state devices). Most physical active elements are complex non-linear devices. However, if the AC signal level is small compared to the DC bias, then an active element can be replaced by its linear equivalent model over a limited band of frequencies.

Active networks basically contain two different kinds of sources, independent sources and controlled sources (or dependent sources).

2.1 Independent Sources

There are two basic models of independent sources which are classified by their voltage-current characteristics. These are the voltage source and the current source.[2]

The ideal voltage source is assumed to deliver energy with a specified terminal voltage, v(t), which is independent of the output current flow from the source. In practice, however, the internal resistance, R, of the voltage source will make the terminal voltage dependent on the magnitude of the output current. To approach the ideal voltage source, the load resistance, R_L, should be much greater than R as shown in Figure 2-1 (a).

The ideal current source is assumed to deliver energy with a specified current, i(t), through the terminals. This current is independent of the terminal voltage of the source. The internal resistance, R, of the nonideal current source will make the terminal current dependent on the magnitude of the output voltage. To approach the ideal current source, the load resistance, R_L, should be much smaller than R as shown in Figure 2-1 (b).

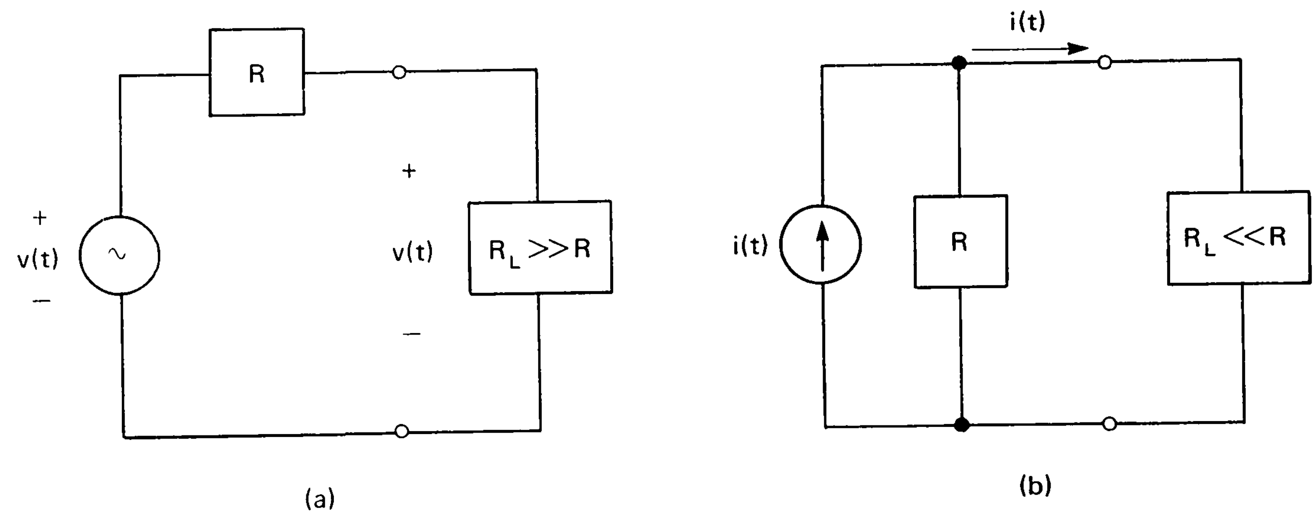


Figure 2-1. The conditions which approximate ideal independent sources. (a) the voltage source; (b) the current source.

Comparison of the two independent sources indicates that the voltage and current sources are duals of each other. The current and voltage roles are interchanged in the two sources resulting in other dual quantities such as open and short circuit, series and parallel, etc.

2.2 Controlled Sources

The controlled source is a unilateral active two-port device having two pairs of terminals, one controlled and one controlling. One of the terminal variables at the controlled port has a single valued dependence on a terminal variable at the controlling port.[3] The controlled source is unilateral in the sense that the input variable controls the output, but that conditions at the output have no influence on the input. Basically, there are four different types of controlled sources. For convenience, these idealized two-port devices will be designated as transducers.[3]

2.2.1 Current-to-Voltage Transducer

The current-to-voltage transducer (CVT) is an ideal controlled source in which the output voltage is proportional to the input current. The voltage-current relationship of the CVT, shown in Figure 2-2 (a), is given by

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ r & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \qquad (2.1)$$

Thus, the CVT has infinite input and output admittances, a forward transfer impedance equal to r, and no reverse transmission. Although the input power is zero, the CVT can supply unlimited output power and as a result exhibits infinite power gain.[3]

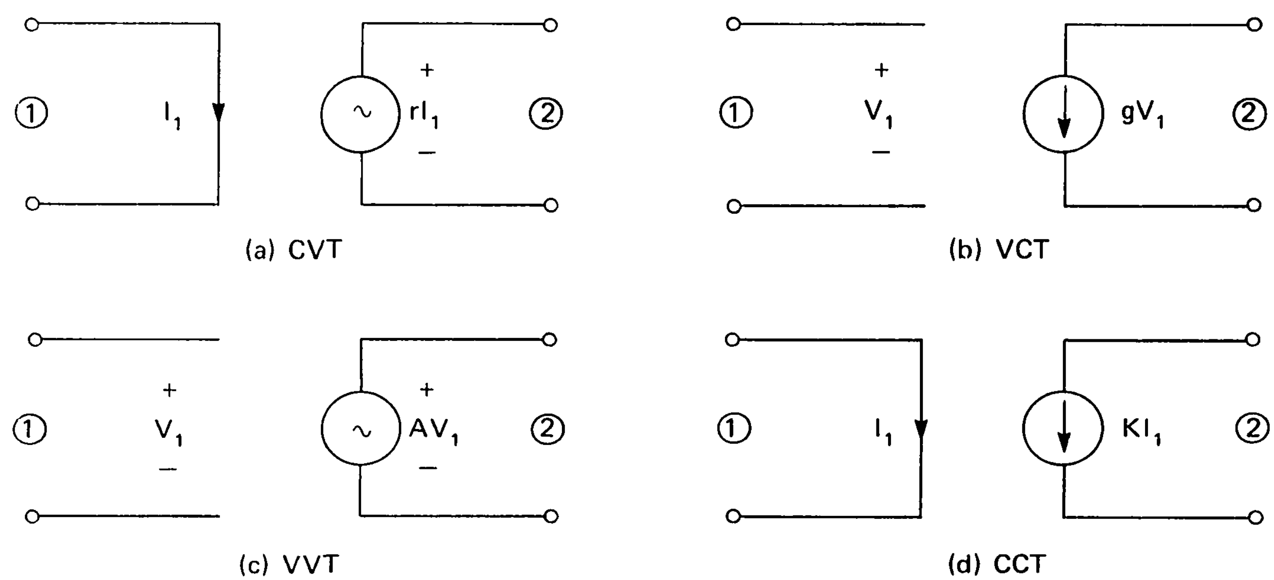


Figure 2-2. The four types of controlled sources. [3]

2.2.2 Voltage-to-Current Transducer

The voltage-to-current transducer (VCT) is an ideal controlled source in which the output current is dependent upon the input voltage. The voltage-current characteristics of the VCT, shown in Figure 2-2 (b), is given by the following relation: [3]

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ g & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \tag{2.2}$$

Thus the VCT has infinite input and output impedances, a forward transfer admittance equal to g, and no reverse transmission. The VCT also exhibits infinite power gain as it can supply unlimited power with zero-input power. [3]

2.2.3 Voltage-to-Voltage Transducer

The voltage-to-voltage transducer (VVT), also known as the voltage amplifier, is an ideal unilateral two-port in which the output voltage is dependent upon the input voltage. The voltage-current relationship, shown in Figure 2-2 (c), is represented by the following matrix equation:

$$\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ A & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ I_2 \end{bmatrix} \tag{2.3}$$

Thus, the VVT has a zero input admittance and output impedance, a forward transmission equal to A, and an infinite power gain. [3]

2.2.4 Current-to-Current Transducer

The current-to-current transducer (CCT), also known as the current amplifier, is an ideal unilateral controlled source in which the output current is dependent upon the input current. The voltage-current relationship, shown in Figure 2-2 (d), is represented by the following matrix equation:

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ K & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix} \qquad (2.4)$$

Thus, the CCT has a zero input impedance and output admittance, a forward current transfer ratio of K, and an infinite power gain.[3]

From the four elementary transducers just defined, the constraint constants (also called the gain) r, g, A and K are usually assumed to be real. A and K are commonly known as the voltage transfer ratio and the current transfer ratio respectively.[3]

It is common practice to represent the voltage-controlled voltage source (VVT) by a triangle, the vertex of which indicates the direction of power transmission. Figure 2-3 shows two typical representations of the VVT. The first symbol represents a floating VVT whereas the second figure represents a grounded VVT.[3]

2.3 The Nullator and Norator

Classical network theory takes for granted that an n-port is characterized by a set of n independent equations between the 2n voltages and currents at the ports. Carlin and Youla[4] suggested the possibility of extending the domain of n-ports by characterizing a set of independent equations that are larger or smaller than n. To prove their point, they introduced two new one-port singular elements which they called the Nullator and the Norator.

However, a few years later, Tellegen[5] showed that these singular elements are mathematical concepts only without any physical content. As a consequence,

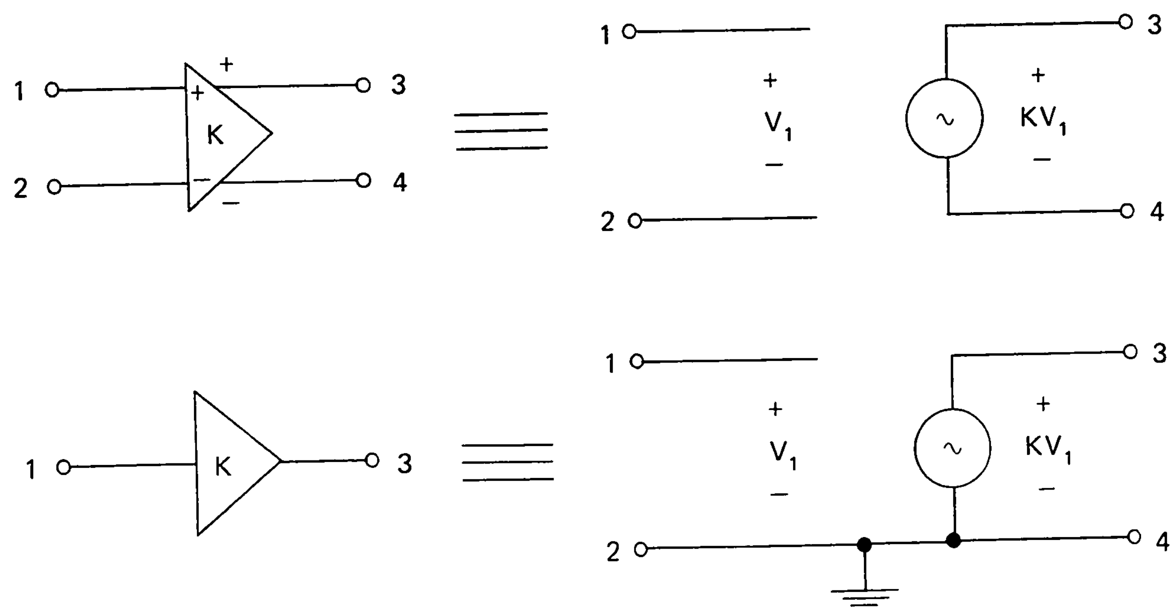


Figure 2-3. Commonly used symbolic representations of the voltage-controlled voltage source.[3]

from a physical point of view, it is impossible to extend the domain of n-ports.

In more recent years, the nullator and the norator have been found useful in representing transistors and operational amplifiers. [3] By using the equivalent nullator-norator representations they can be used for generating realizable circuits.

The nullator, shown in Figure 2-4 (a), is a 2-terminal element defined by the relation V = I = O. From this definition, the nullator is seen to be a bilateral lossless one-port element which can neither sustain a voltage nor pass a current.

The norator, shown in Figure 2-4 (b), is a 2-terminal element for which the terminal variables, V and I, are completely arbitrary, it can sustain a voltage and pass a current which are completely independent of each other. Thus, it follows that the norator is a non-reciprocal 2-terminal element. [3]

Carlin [6] showed that the singular elements cannot be derived from a limiting operation on physically realizable network components. However, a pathological two-port called a nullor exists which is obtainable by a limiting process. One pair of terminals of this device consists of a nullator whereas the other port behaves like a norator. The symbolic representation of the nullor is shown in Figure 2-4 (c).

Since the nullator and the norator form a basic set of active elements such as the nullor which is theoretically a physically realizable element, equivalent circuits using such devices can be formed. However, in physically realizable circuits all the voltages and currents are uniquely and definitely determined. Thus, in the equivalent representation of a physically realizable circuit, the nullator and the norator must always occur in pairs. [3] The number of nullators in a network should always equal to the number of norators in the same network.

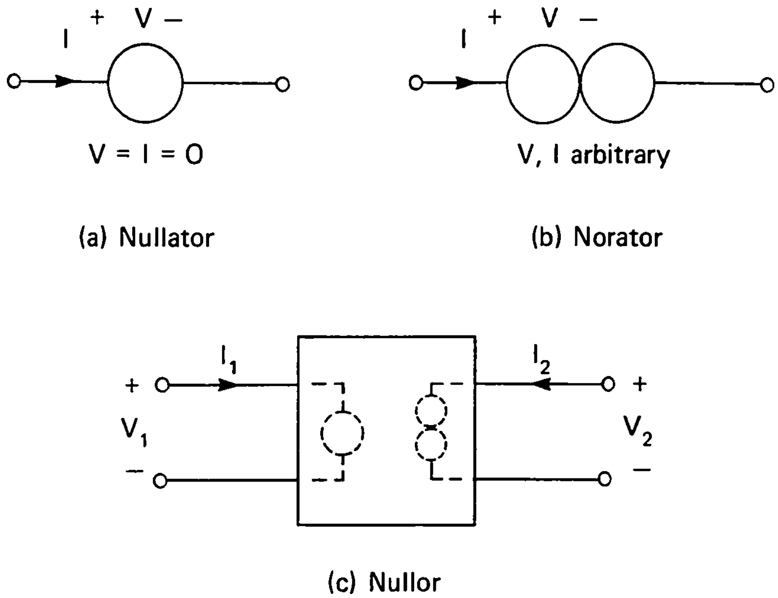


Figure 2-4. Symbolic representations of Singular Elements.

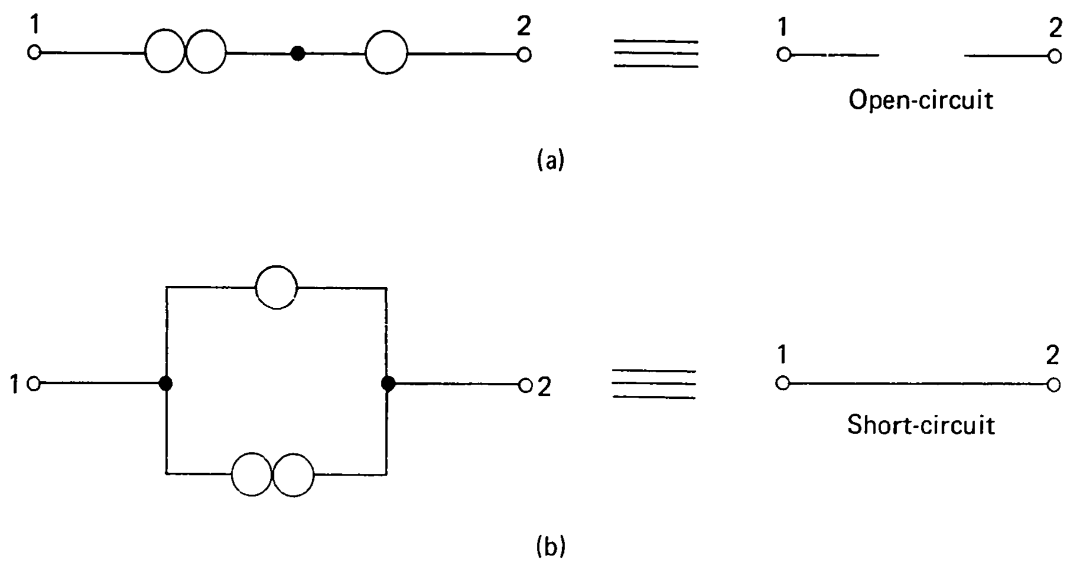


Figure 2-5. Two Nullator-Norator Identities.

It is interesting to note that the dual of a nullator is a nullator and the dual of a norator is a norator. Thus, dual representations of a planar network can be easily constructed.

Two important nullator-norator identities are shown in Figure 2-5. The series combination of Figure 2-5 (a) is equivalent to an open circuit whereas the parallel combination is equivalent to a short circuit as shown in Figure 2-5 (b). These identities can be used to generate additional nullator-norator equivalent circuits which will facilitate the realization of transistor circuits.

2.4 Controlled Source Representation

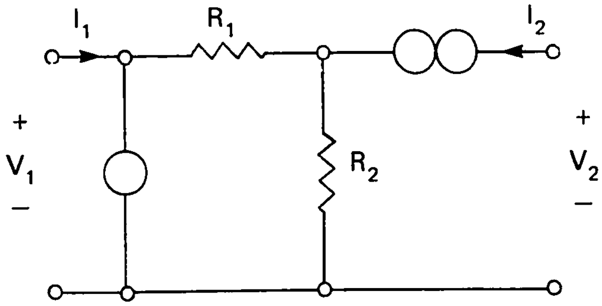
Equivalent circuits representing each of the four types of controlled sources as proposed by Davies [7] are shown in Figure 2-6. Each basic circuit employs a single nullator and a single norator.

To prove that these models actually do represent the controlled sources, consider the circuit of Figure 2-6 (b) which is the basic nullator-norator model of the voltage-to-voltage transducer. Since the voltage across the nullator is zero, then the voltage across resistor R_1 is equal to V_1 and the current through $R_1 = V_1/R_1$. The current flowing through the nullator is by definition equal to zero; thus the current through resistor R_2 is also equal to V_1/R_1. From this it follows that

$$V_2 = \frac{V_1}{R_1}(R_1 + R_2) = (1 + \frac{R_2}{R_1}) V_1 \qquad (2.5)$$

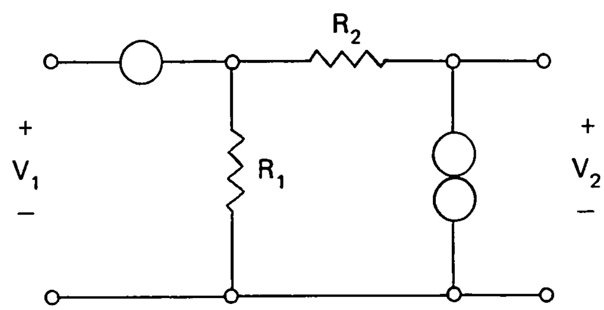
This equation is identical to the matrix equation 2.3 with $A = 1 + (R_1/R_2)$. Thus, the pertinent network behaves as a voltage amplifier. A similar analysis can be carried out to justify the operation of the remaining circuits.

The gain of the current and voltage amplifiers of Figure 2-6 is always greater than one. A unity gain current amplifier is obtained from Figure 2-6 (a) by



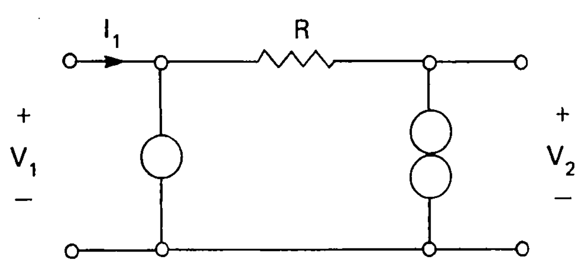
$$I_2 = -\left(1 + \frac{R_1}{R_2}\right) I_1$$

(a) CCT



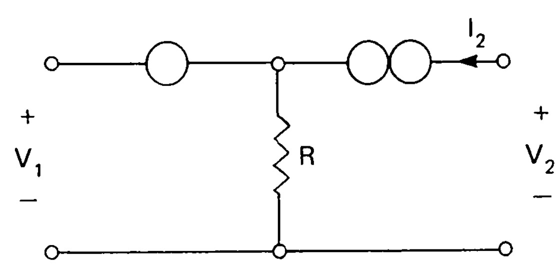
$$V_2 = \left(1 + \frac{R_2}{R_1}\right) V_1$$

(b) VVT



$$V_2 = -RI_1$$

(c) CVT



$$I_2 = -\frac{1}{R} V_1$$

(d) VCT

Figure 2-6. The four basic nullator-norator models of controlled sources.

letting R_1 = O and R_2 be infinite. Likewise, the model of a unity gain voltage amplifier is obtained from Figure 2-6 (b) if R_1 is infinite and R_2 is equal to zero. The models of these unity gain amplifiers are shown in Figure 2-7. The unity gain current amplifier is identical to the controlled source model of an ideal grounded base transistor whose output current is equal and opposite to the input current. An idealized transistor is assumed to have the following parameters: r_e and r_b tend to zero, r_c tends to infinity and the gain tends to unity. Thus, it follows that the unity gain current amplifier of Figure 2-7 (a) can be realized by a common base transistor as shown in Figure 2-8. Similarly, the unity gain voltage amplifier of Figure 2-7 (b) can be realized by a common-collector transistor as shown in Figure 2-9. Since the output polarity of the voltage is the same as the input polarity, this type of circuit is called a unity gain noninverting type voltage-to-voltage Transducer, or more commonly as an emitter follower.

Using the open-circuit nullator-norator identity shown in Figure 2-5 (a), the alternate realization of a cascaded emitter follower is arrived at as shown in Figure 2-10. The biasing arrangement for the transistors is excluded for simplicity. While such a cascade leads to high input impedance and low output impedance, the phase-shift problem worsens due to charge storage in the transistors used. Diode D_1 as shown in Figure 2-10 was suggested by Gile [8] primarily for compensating the base-emitter offset voltage of transistor T_1 and for temperature compensation.

2.5 Nonideal Controlled Sources

In reality, practical controlled sources have finite input, output and feedback impedances. Figure 2-11 gives the simplest approximations of the nonideal transducers. In analysing nonideal controlled sources, they can be treated as ideal sources to which parasitic impedances have been connected. However the finite

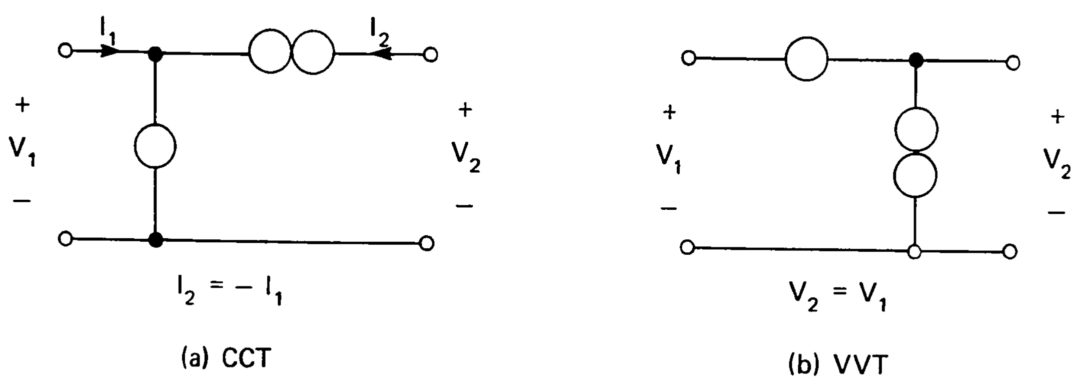


Figure 2-7. Unity gain nullator-norator models of controlled circuits.

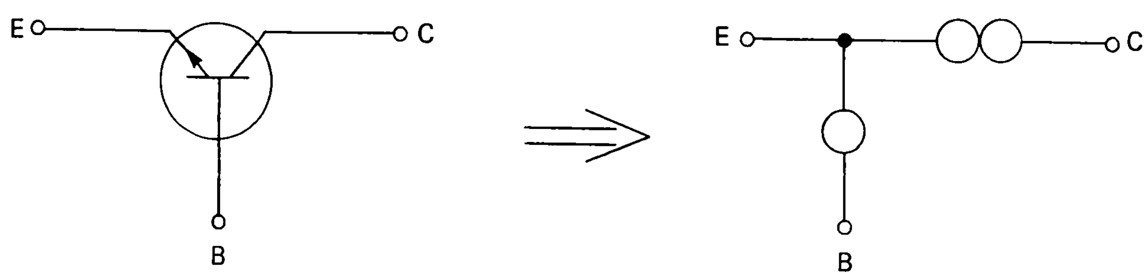


Figure 2-8. The nullator-norator equivalent representation of an ideal transistor.

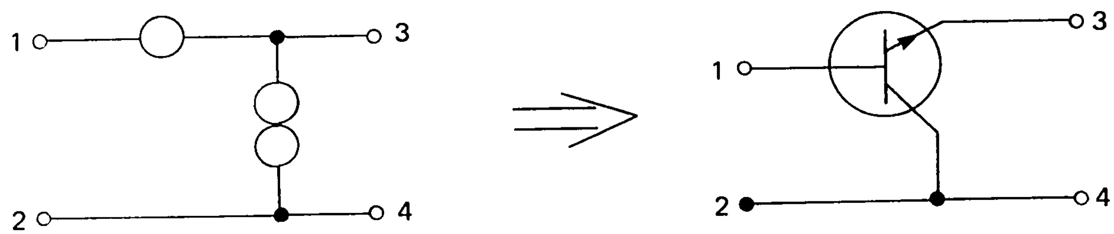


Figure 2-9. Transistorized realization of a unity gain noninverting type voltage-to-voltage transducer.

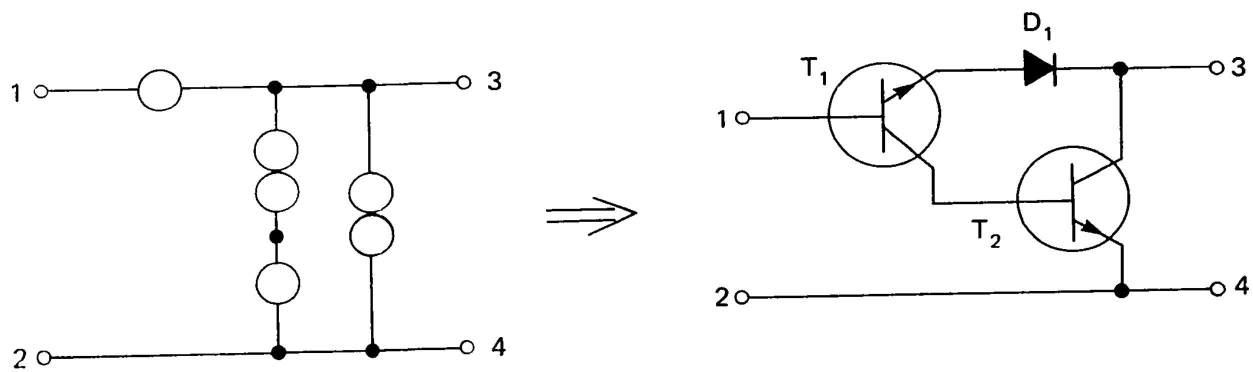


Figure 2-10. A modified unity gain emitter follower.

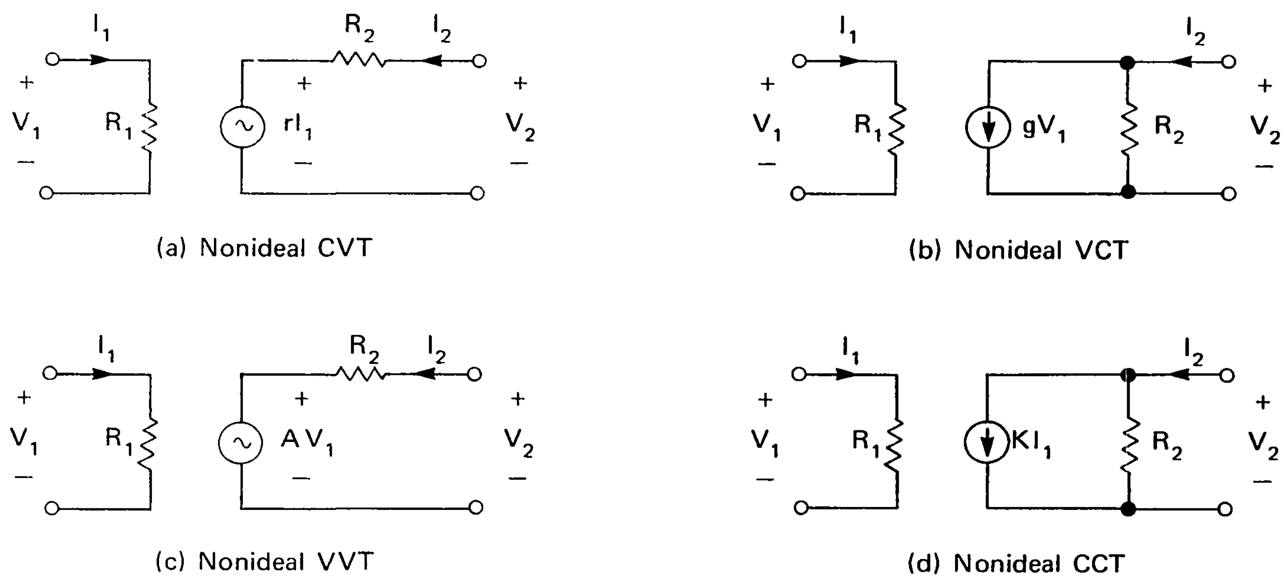


Figure 2-11. Nonideal controlled sources.

input and output impedances tend to make the power gain finite. In addition, the device gives up its unilateral property if feedback impedance is present. [3]

Another type of nonidealness, which creates an additional problem, is that the gain of a practical transducer is a frequency-dependent function instead of being a real constant. Thus the useful operation of these devices is restricted to the lower end of the frequency spectrum. [3]

CHAPTER III

THE NEGATIVE IMPEDANCE CONVERTER

The impedance converter is a device used to convert a terminating impedance. The most popular type of impedance converter is the negative impedance converter (NIC) which is a two-port device operating on the principle that any impedance Z(s) placed across one of its ports will appear as −Z(s) at the other port.

3.1 Definition of an Ideal Converter

An ideal impedance converter is a two-port network which, when terminated at one-port by a driving-point impedance $Z_L(s)$, presents at the other port an input impedance directly proportional to $Z_L(s)$ for all frequencies.

To derive the necessary requirements of such a two-port network, consider an arbitrary two-port which is terminated at its port 2 by a load impedance $Z_L(s)$ as shown in Figure 3-1. The two-port network transmission parameters are given by the following matrix relation

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} \tag{3.1}$$

The input impedance $Z_{11}(s)$ is obtained from

$$Z_{11}(s) = \frac{V_1}{I_1} \quad \text{and} \quad V_2 = -I_2 Z_L(s)$$

to be

$$Z_{11}(s) = \frac{AZ_L(s) + B}{CZ_L(s) + D} \tag{3.2}$$

The requirement for this two-port to be an ideal impedance converter is that for all possible terminations

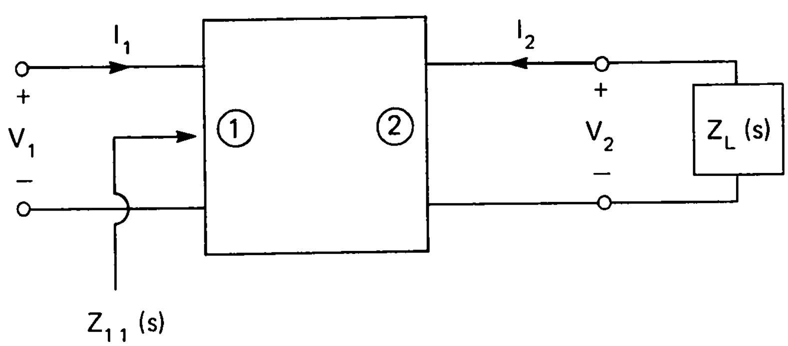


Figure 3-1. A general two-port network terminated at one port by an impedance Z_L (s).

$$Z_{11}(s) = K(s)\, Z_L(s) \tag{3.3}$$

where K(s) is a predetermined real rational function. The conditions which satisfy both equations 3.2 and 3.3 are

$$B = C = 0\ ; \quad A \neq 0\ ; \quad D \neq 0\ ; \text{ and } K(s) = \frac{A}{D}$$

With these conditions, the driving-point impedance $Z_{22}(s)$ at port 2, with port 1 terminated by $Z_L(s)$ is given by

$$Z_{22}(s) = \frac{D\, Z_L(s) + B}{C\, Z_L(s) + A} = \frac{D}{A} Z_L(s) = \frac{1}{K(s)} Z_L(s) \tag{3.4}$$

Thus, an ideal converter is a bilateral device since it can convert impedance in both directions. The proportionality factor K(s) of the ideal converter is known as the Conversion Factor.

3.2 Ideal Negative Impedance Converters

When the conversion factor of the ideal converter is made equal to a negative real constant, then this type of an active two-port network becomes an ideal negative impedance converter (NIC). This leads to two main possibilities, i.e. $K(s) = -1/K$ or $K(s) = -K$. The first condition can be realized by setting $A = -1/K$. Consequently, since by definition $A/D = K(s)$ this leads to $D = 1$. In the second condition, $A/D = -K$, setting $A = 1$ makes $D = -1/K$. With these two conditions, we get the following matrix equations:

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} -1/K & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} \text{ and } \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1/K \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} \tag{3.5}$$

The first set of equation 3.5 yields $V_2 = K(-V_1)$ and $(-I_2) = I_1$.

Thus, this device inverts the voltage polarity while leaving the normal direction of current unchanged. This type of converter is called a voltage inversion type negative impedance converter (VNIC).

The second matrix of equation 3.5 yields $V_2 = V_1$ and $(-I_2) = K(-I_1)$. Thus, this device inverts the current while keeping the voltage polarities unchanged. In other words, the current flow at one of the ports is reversed with respect to the flow that would normally be encountered in a purely passive device. This type of converter is called a current inversion type negative impedance converter (CNIC).

Converting the transmission parameters of equation 3.5 to their equivalent h-parameters using the following identities: $h_{11} = B/D$, $h_{12} = |F|/D$, $h_{21} = -1/D$ and $h_{22} = C/D$, we get

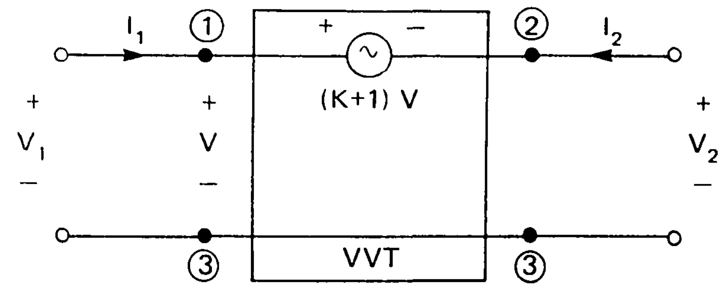
$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 0 & -1/K \\ -1 & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ K & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix} \qquad (3.6)$$

Thus, if h_{12} and h_{21} of the H matrix are both negative, the device is a VNIC. Consequently, when h_{12} and h_{21} are both positive, the device is a CNIC.

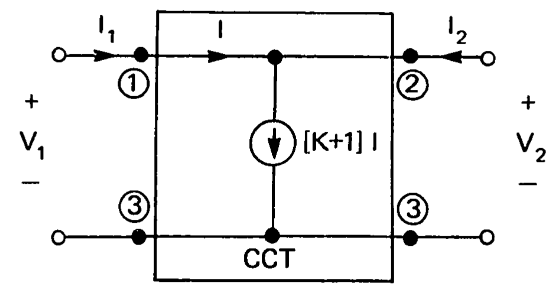
Using the positive gain VVT and the negative gain CCT described in Chapter II, the equivalent representations of the negative-impedance converter are shown in Figure 3-2. In order that these models behave as negative impedance converters, the VVT and the CCT must have a gain greater than unity.

3.3 Nullator-Norator Representation

Replacing the controlled sources in Figure 3-2 by their nullator-norator equivalent circuits of Figure 2-6, the VNIC and CNIC equivalent representations are easily derived. Consider first the equivalent controlled source representation of the CCT of Figure 2-6 (a). Letting $K = R_1/R_2$, the equivalent representations



$V_2 = -KV_1, \; I_1 = -I_2$



$V_1 = V_2, \; I_2 = KI_1$

Figure 3-2. Equivalent representations of the negative-impedance converter using controlled sources.

of this CCT and the NIC of Figure 3-2 are both shown in Figure 3-3 for comparison. Note from Figure 3-3 that the current in node 2 of (a) is equal to the current in node 3 of (b). In addition, the current in node 3 of (a) is the same as that of node 2 of (b). Thus, nodes 2 and 3 of the nullator-norator circuit of Figure 2-6 (a) have to be switched around before inserting it into the NIC model representation of Figure 3-2. The resulting Type I CNIC model representation is shown in Figure 3-4. Since there is no current flowing through the nullator, the voltage drop across resistor $R_1 = I_1 R_1$ and the drop across $R_2 = I_2 R_2$. The voltage drop across R_1 equals that across R_2 because the voltage across the nullator is zero. Thus $I_1 R_1 = I_2 R_2$ and it follows that $I_2 = (R_1/R_2)\ I_1 = K\ I_1$. Hence the conversion factor for this Type I CNIC is equal to R_1/R_2. The second equation defining the NIC behavior, $V_1 = V_2$, is obtained directly from Figure 3-4 since there is no voltage drop across the nullator.

Consider next the equivalent controlled source representation of the VVT circuit of Figure 2-6 (b). Again letting $K = R_1/R_2$, the equivalent controlled source representations of this VVT and the one used in the NIC of Figure 3-2 are both shown in Figure 3-5 for comparison. Note from Figure 3-5 that the voltage drop between nodes 1 and 2 of (a) is equal to the voltage drop between nodes 2 and 3 of (b). Also, the voltage drop between nodes 2 and 3 of (a) is the same as the drop between nodes 1 and 2 of (b). Thus, nodes 1 and 3 of the nullator-norator circuit of Figure 2-6 (b) have to be interchanged before inserting it into the NIC model representation of Figure 3-2. The resulting Type I VNIC model representation is shown in Figure 3-6. Comparing this model with Figure 3-4 reveals that one is the dual of the other.

From Figure 3-6, with no current flowing through the nullator, the current through R_1 is equal to the current through R_2. With no voltage across the nullator the current through R_1 is V_1/R_1 and the current through R_2 is $-(V_2/R_2)$

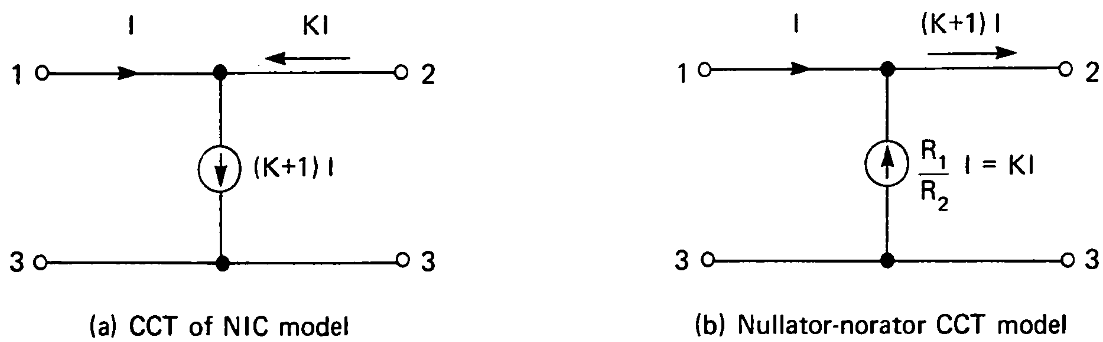


Figure 3-3. Comparison between controlled source models of Figures 3-2 and 2-6 (a).

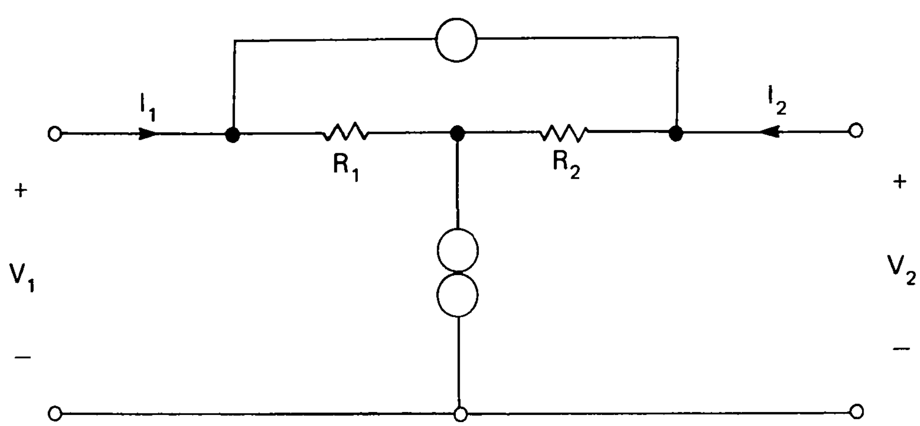
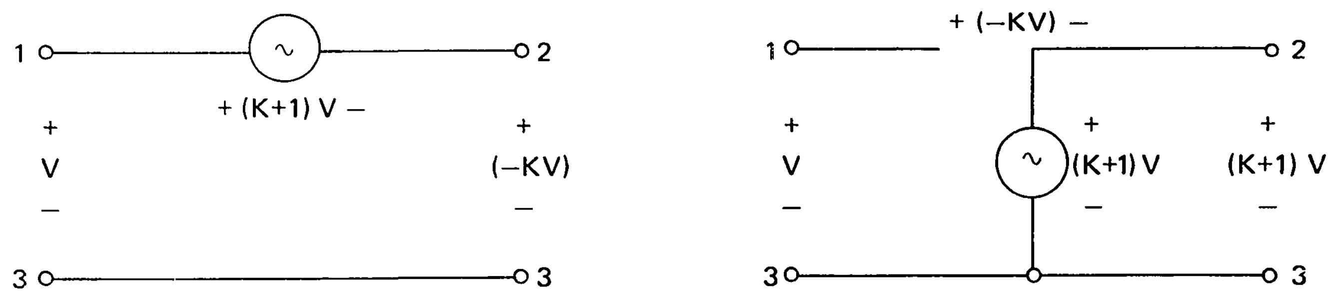


Figure 3-4. Type I CNIC nullator-norator representation.



(a) VVT of NIC model

(b) Nullator-norator VVT model

Figure 3-5. Comparison between controlled source models of Figures 3-2 and 2-6 (b).

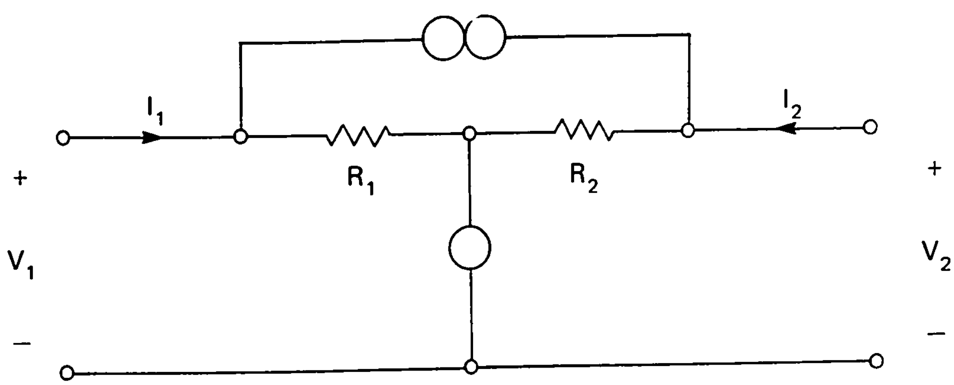


Figure 3-6. Type I VNIC nullator-norator representation.

as it is in opposite direction to the current through R_1. Since the current through R_1 is equal to the current through R_2, then $(V_1/R_1) = -(V_2/R_2)$ which leads to $V_2 = -(R_2/R_1)\ V_1 = -KV_1$.

To arrive at the second equation which defined the NIC of Figure 3-2, consider the current flow at the junction of R_1 and the norator. Letting the current towards the norator equal to I, the current through resistors R_1 and R_2 = $(I_1 - I)$. The sum of currents flowing towards the node at the junction of R_2 and the norator is $I + (I_1 - I) + I_2 = 0$, which leads to $I_1 = -I_2$.

3.4 Transistor Circuit Realization

One of the useful applications of the nullator-norator representation is the ability to construct a transistor circuit realization from the original equivalent circuit by simple manipulations of the model. The equivalence of the unity gain controlled source model to the ideal transistor was shown earlier in Figures 2-8 and 2-9. This similarity suggests a method of converting a nullator-norator circuit into an equivalent transistor circuit.

The method is really quite simple. All the nullators and norators should first be grouped into pairs of a nullator and a norator having a common junction. Next, each pair is replaced by a transistor as follows: the junction of the pair corresponds to the emitter terminal of the transistor, the other end of the nullator matches the base terminal of the transistor and finally the other end of the norator corresponds to the collector of the transistor.

Consider the type I CNIC model of Figure 3-4. As it stands, the nullator-norator pair do not have a common junction. However, using the identities of Figure 2-5, open-circuit nullator-norator models can be added to the circuit in such a way as to produce nullator-norator pairs with a common junction. Adding an open-circuit equivalent nullator-norator pair to the Type I CNIC equivalent circuit of Figure 3-4 yields the transistor realization circuit shown in Figure 3-7. In a similar fashion, the Type I VNIC transistor equivalent circuit of Figure 3-6 is obtained as shown in Figure 3-8.

In the transistor circuits of Figures 3-7 and 3-8 the biasing arrangements are not included. If more accurate analysis of these NIC circuits is desired, then the transistor can be further replaced by its equivalent small-signal model.

3.5 Nonideal Negative Impedance Converters

In practice, the zero elements of the NIC matrices are difficult to achieve. Nonzero values of these parameters, namely the h_{11} and h_{22} of the H matrix and the B and C of the F matrix, may counteract the NIC properties of the device. Usually, these parasitic elements have relatively small non-zero values. However, the NIC can be made to approach its ideal characteristics by adding input and output impedances in such a way as to cancel the parasitic elements.

Considering the H matrix definition of the NIC, the representation of a nonideal NIC is given in Figure 3-9 with the parasitic immittances h_{11} and h_{12} shown as series and shunt impedances added to the ideal NIC.[3]

Terminating port 2 of the nonideal NIC with an impedance Z_L the input impedance at port 1 of the ideal NIC is

$$Z_{11} = \frac{V_1}{I_1} = \frac{-K\ V_2}{-I_2} = \frac{K}{I_2}\ \frac{(-I_2)}{h_{22} + \frac{I}{Z_L}} \qquad = -\ \frac{K}{h_{22} + \frac{I}{Z_L}} \tag{3.7}$$

and the input impedance at port 1 of the nonideal NIC becomes

$$h_{11} - \frac{K}{h_{22} + \frac{I}{Z_L}} \tag{3.8}$$

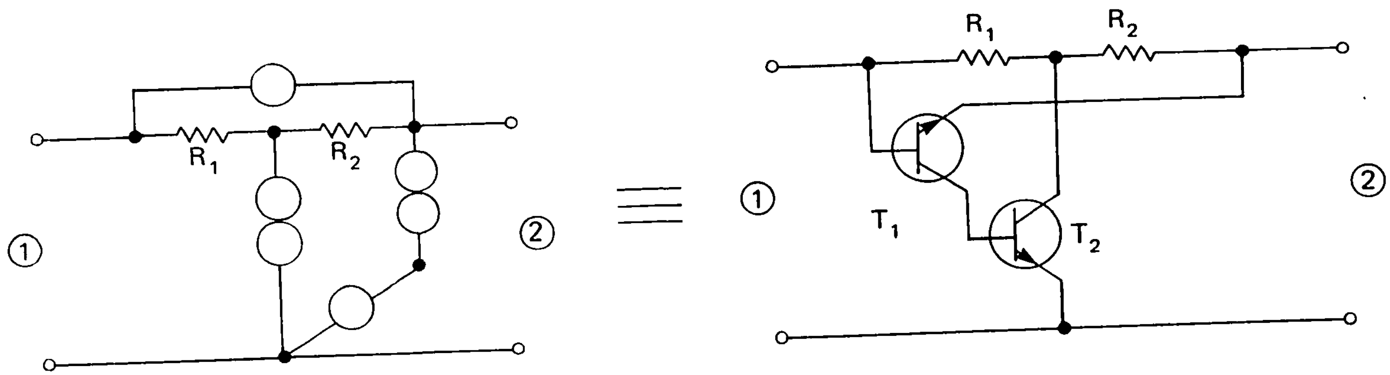


Figure 3-7. Transistorized realization of the Type I CNIC.

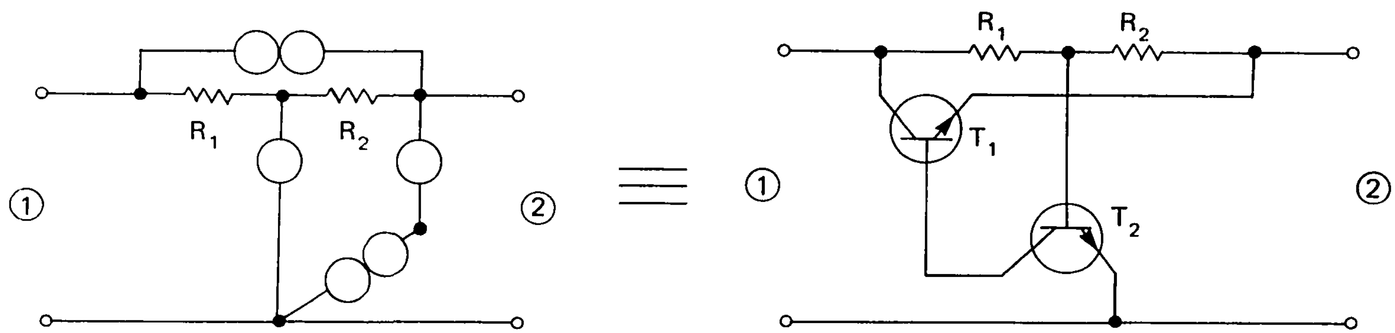


Figure 3-8. Transistorized realization of the Type I VNIC.

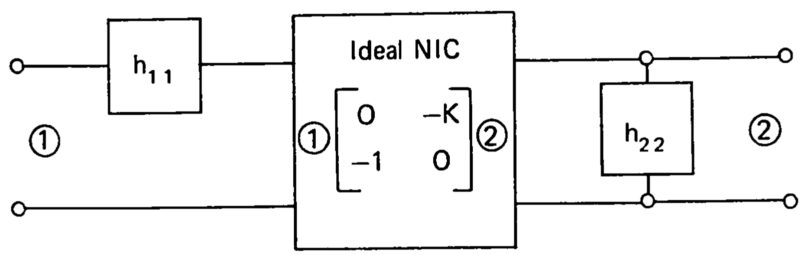


Figure 3-9. Representation of a nonideal NIC.

Assuming, for the present, that $h_{22} = 0$, then the input impedance of the nonideal NIC due to Z_L at port 2 is

$$h_{11} - K Z_L$$

Inserting an impedance Z_B in series with the load Z_L, the input impedance then becomes

$$h_{11} - K (Z_L + Z_B)$$

To cancel the effect of $h_{11} \neq 0$, Z_B can be chosen to be equal to h_{11}/K. Thus the converter behaves like an ideal NIC. Considering the effect of h_{22} assuming $h_{11} = 0$, the input impedance in equation 3.8 becomes equal to

$$-\frac{K}{h_{22} + \frac{1}{Z_L}} = -\frac{K Z_L}{1 + h_{22} Z_L} \qquad (3.9)$$

Connecting an impedance Z_A across the input port of the NIC, the input admittance becomes

$$\frac{1}{Z_A} - \frac{1 + h_{22} Z_L}{K Z_L} = \frac{1}{Z_A} - \frac{1}{K Z_L} - \frac{h_{22}}{K} \qquad (3.10)$$

Thus, to cancel the effect of $h_{22} \neq 0$, Z_A can be chosen to equal to K/h_{22}.

For the more general case where both h_{11} and h_{22} are not equal to zero, the possibility exists of cancelling the effects of these parasitic admittances by incorporating series and shunt impedances, Z_A and Z_B with the nonideal NIC as shown in Figure 3-10.

Defining the compensating impedances Z_A and Z_B in terms of the parasitic elements of the nonideal NIC, using the above derived expressions for $h_{11} = 0$,

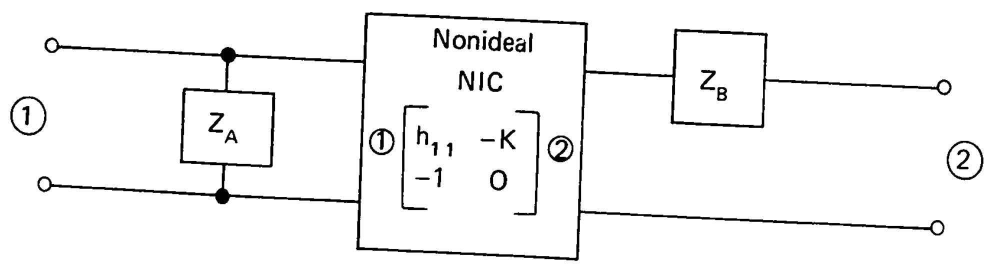


Figure 3-10. Compensation of a nonideal NIC.

$h_{22} \neq 0$ and $h_{11} \neq 0$, $h_{22} = 0$, we have

$$Z_A = \frac{A}{h_{22}} \quad \text{and} \quad Z_B = B\, h_{11} \tag{3.11}$$

where A and B are constants. The following expressions for the hybrid parameters of the composite two-port is derived by simple calculation to be equal to: [3]

$$h'_{11} = \frac{Z_A \left[h_{11} - \dfrac{K Z_B}{h_{22} Z_B + 1} \right]}{Z_A + h_{11} - \dfrac{K Z_B}{h_{22} Z_B + 1}} \tag{3.12}$$

$$h'_{22} = \frac{h_{22} - \dfrac{K}{h_{11} + Z_A}}{Z_B \left[h_{22} - \dfrac{K}{h_{11} + Z_A} \right] + 1} \tag{3.13}$$

where K is obtained from the h_{12}, h_{21} parameters i.e.

$$h_{12}\, h_{21} = (-K)\,(-1) = K$$

Setting h'_{11} and h'_{22} equal to zero and using the definitions for Z_A and Z_B given in equation 3.11, we have from (3.12):

$$h_{11} - \frac{K Z_B}{h_{22} Z_B + 1} = 0$$

which leads to

$$\frac{1}{B} = K - h_{11}\, h_{22} \tag{3.14}$$

from (3.13):

$$h_{22} - \frac{K}{h_{11} + Z_A} = 0$$

which leads to

$$A = K - h_{11}\, h_{22}$$

thus $A = \dfrac{1}{B}$ or $AB = 1$ (3.15)

The necessary conditions for compensating a nonideal NIC are given by equations 3.14 and 3.15. Further conditions can be derived by computing $h'_{12}\, h'_{21}$ and setting the product greater than zero to guarantee NIC action. Thus it is possible to design ideal NIC's in practice by using compensating impedances as derived above and by adjusting the values of the h parameters of the nonideal NIC. In general, since the h parameters of a practical NIC are complex functions of frequency, perfect compensation can only be achieved over a small band in the low-frequency range.[3]

CHAPTER IV

NEGATIVE IMPEDANCE REPEATERS

In today's telephone industry, voice-frequency transmission facilities consist of either two-wire or four-wire circuits. Although the use of four-wire trunk circuits is becoming more common, many trunks are still two-wire for economic reasons. In order to meet the loss requirements of telephone circuits, it is often necessary to add gain to the transmission path. On two-wire circuits, three main problems are encountered. The added gain must be independent of the direction of transmission; in other words, the two-wire circuit must retain its bilateral transmission properties. The second problem to overcome is that the inserted two-wire amplifier must not impair dial pulses. Finally, the added equipment must be able to withstand the relatively high voltage used for signalling the customer.

In order to overcome these special problems, the concept of the negative impedance converter described in Chapter III was used to develop what is commonly known as the E-type negative impedance repeater.

4.1 History

Before the E-type repeater was developed, the 22-type repeater was the standard two-wire bilateral means of amplifying voice signals in the Bell System.[9] The standard repeater consisted of two hybrid coils with balancing networks, two separate amplifiers, and two equalizers. The hybrids were used to split the two-directions of transmission and the separate amplifiers inserted gain for each direction. The equalizers were used as required to adjust independently the equalization for each direction. To prevent oscillation in the feedback loop consisting of the two hybrids and the two amplifiers in tandem, the impedances connected to the conjugate hybrid ports are closely matched by the balancing networks.

The first E-type repeater introduced in 1948 was known as the E1 repeater. It was developed for use in exchange areas where only moderate transmission gains (up to 10dB) were required. It is a bilateral repeater consisting of only one tube-type amplifier with its associated gain adjusting network and a line transformer as shown in Figure 4-1. The amplifier unit of the E1 repeater consisted of a dual triode gain circuit which behaved as a negative impedance converter. The gain adjusting network contained a variety of resistors, inductors and capacitors which could be strapped together in various combinations by the installer to set the required gain.

A few years after the first E1 repeaters were put in service, the installers encountered difficulties in strapping the gain options on those repeaters that were mounted on the top section of the bays. As a result, the E2 repeater was introduced which was electrically identical to the E1 repeater but had an additional plug-in capability for easier strapping.

The E2, like the E1, is essentially a two-terminal network operating on the principal of inserting a negative resistance in series with the telephone line. This type of repeater is known as a series type because the negative impedance is produced by connecting the output of an amplifier back in series with its input. This negative impedance is classified as open-circuit stable since it does not oscillate when its two terminals are open-circuited.[10]

The E2 repeater compared to the 22-type hybrid repeater is simple in design, easy to install and maintain, and preserves the d-c continuity of the line in which it is inserted. On the other hand, the E2 repeater introduced a substantial impedance irregularity in the line. Even with careful attention to spacing and gain adjustment, variations in cable characteristics with changes in temperature caused large variations in the loss of the circuit.

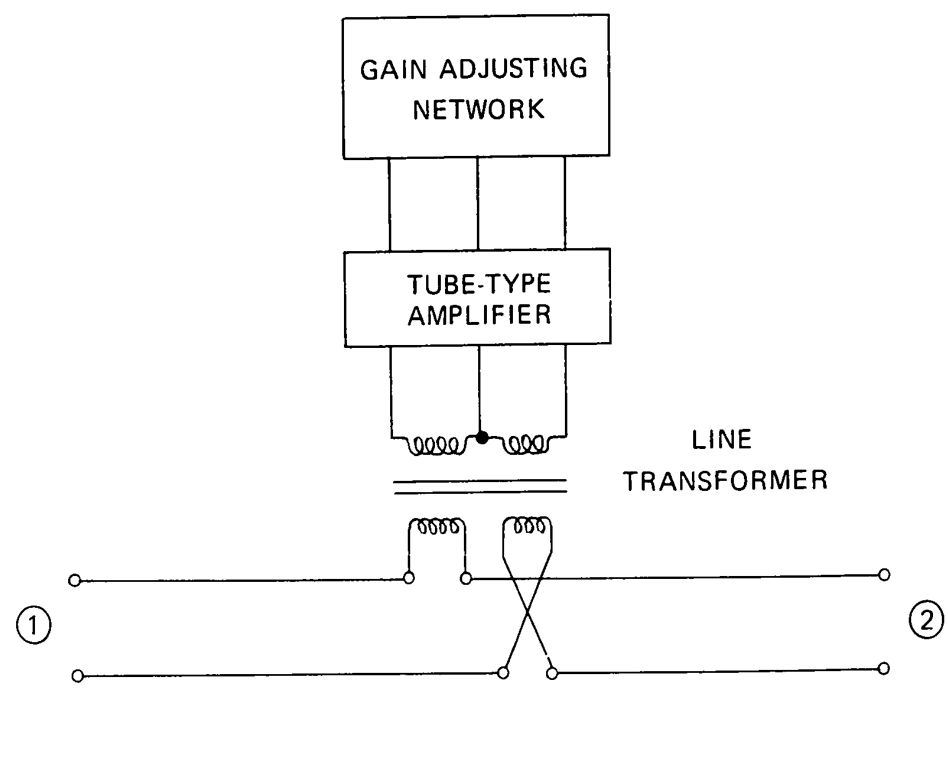


Figure 4-1. The E1 or E2 telephone repeater.

Consequently, the new E23 repeater was developed in 1954 consisting essentially of the E2 series repeater with the addition of a shunt negative impedance element called the E3 repeater. The E3 is a two-terminal network operating on the principal of inserting a negative impedance across the telephone line. This type of repeater is known as a shunt type because the negative impedance is obtained by connecting the output of an amplifier back in shunt with its input. This negative impedance element has been classified as the short circuit stable type because it will not oscillate when its two terminals are short circuited.[10] The E3 shunt repeater has not been used alone in the telephone industry because it tends to be unstable when its terminals are open-circuited. Usually, a telephone circuit is AC open-circuited at both ends when the line is in the idle condition. On the other hand, the E2 series repeater is usually more stable during the idle condition.

4.2 The E23 Electron Tube Repeater

The E23 repeater represented a considerable advance over the series-type negative impedance repeater. The addition of the E3 shunt-type converter to the existing E2 repeater eliminated the impedance discontinuities and the resulting undesirable reflection effects which were introduced earlier by the series-type repeaters. The effectiveness of this technique is attested by the fact that within six years, over one million E-type repeaters were installed in the Bell System.[11]

The negative impedance of the E23 repeater must be closely matched to the cable impedance in order to produce the desired gain and return loss characteristic. This matching is done by strapping components of both the series and shunt complex impedance networks shown in Figure 4-2. The required strapping varies with the amount of gain required, the cable facility with which the repeater is associated and the length of cable end-section to which the repeater is connected. Particular network connections are specified by extensive strapping charts thus

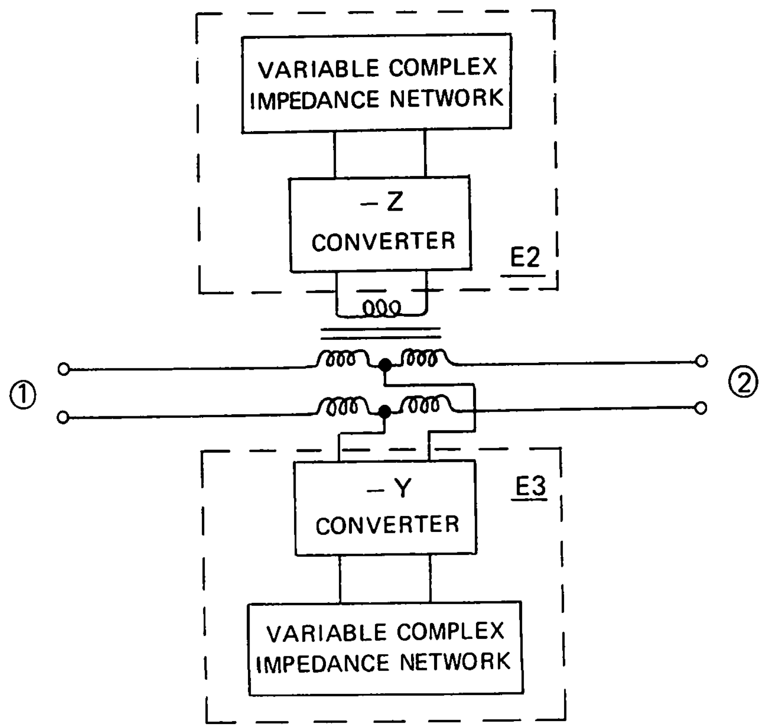


Figure 4-2. The E23 telephone repeater.

making it a time-consuming operation on the initial installation and on subsequent maintenance changes.[11]

The E23 repeater is arranged in a standard bridged-T network configuration with mutual coupling as shown in Figure 4-3 (a). The image impedance and insertion loss of the network are related to the series and shunt elements as follows:[11]

$$\text{Image Impedance} = Z_I = \sqrt{Z_A \; Z_B} \tag{4.1}$$

$$\text{Insertion Loss (dB)} = 20 \log_{10} \left| \frac{1 - \sqrt{\dfrac{Z_A}{4Z_B}}}{1 + \sqrt{\dfrac{Z_A}{4Z_B}}} \right| \tag{4.2}$$

In order to insert the E23 repeater into a distortionless transmission line without introducing impedance discontinuities, the image impedance of the repeater would have to match the characteristic impedance of the line. This matching is accomplished by relating the series and shunt elements of the equivalent bridged-T structure to the characteristic impedance of the line by a proportionality constant N. Thus, setting $Z_A = -N\, Z_0$ and $Z_B = -Z_0 / N$, the image impedance of the repeater becomes

$$Z_I = \sqrt{Z_A \; Z_B} = \sqrt{(-N\, Z_0) \left(\frac{-Z_0}{N}\right)} = Z_0 \tag{4.3}$$

where Z_0 is the characteristic impedance of the line. The equivalent bridged-T structure is shown in Figure 4-3 (b). Substituting the above expressions for Z_A and Z_B into the insertion loss equation 4.2, keeping in mind that the insertion gain is the negative of the insertion loss in dB, the gain of the repeater is expressed as:

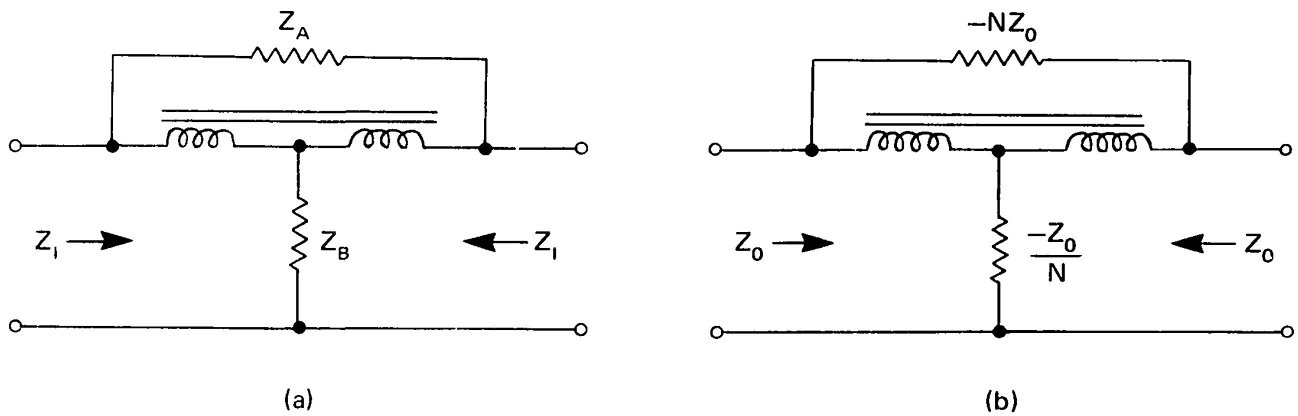


Figure 4-3. Equivalent bridged-T network configuration.

$$\text{Insertion Gain} = -\text{ Insertion Loss} = 20\log_{10}\left|\frac{1+\frac{N}{2}}{1-\frac{N}{2}}\right| \qquad (4.4)$$

Note that if N = 2, the insertion gain is infinite and the repeater will be unstable.

The E23 repeater has a basic limitation on its application to telephone offices; being an electron tube device, it requires a 130-volt DC plate supply in addition to the universally available 48-volt DC filament source. This limits the use of the repeater to offices where a 130-volt supply exists or where the cost of installing such a supply could be justified.[11]

4.3 The E6 Transistorized Repeater

The E6 repeater is a transistorized voice repeater designed to reduce the transmission losses of exchange-area trunk circuits with superior performance over the E23 repeater. It provides lower net loss circuits with higher return loss on good quality cables, requires only 48-volt unfiltered central-office battery, has low power consumption and takes only about half the bay space of an E23 repeater.

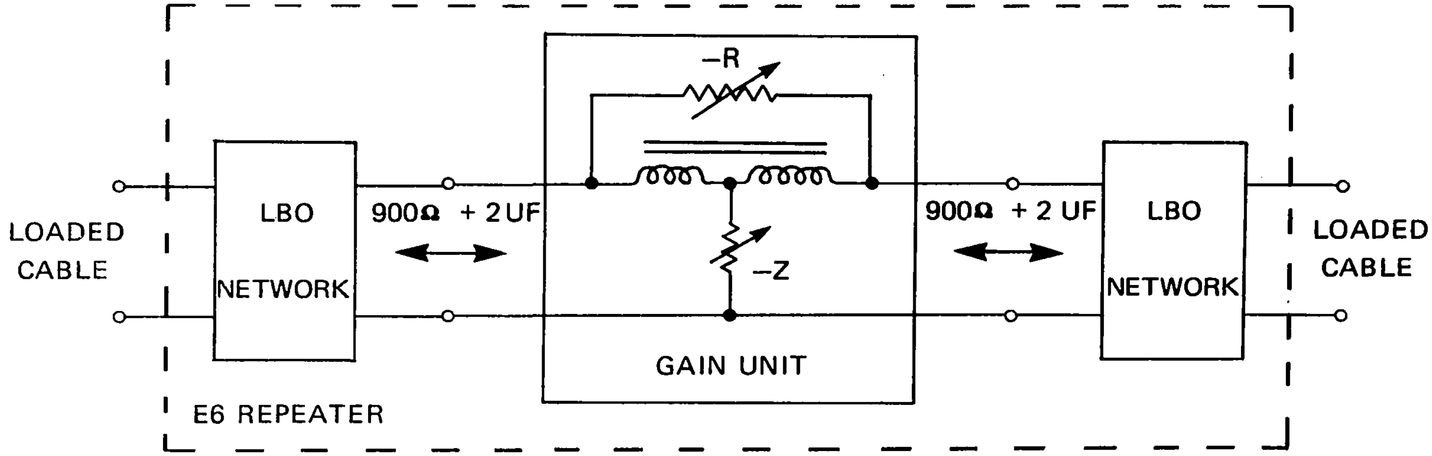
Up to 12dB transmission gains can be provided by the E6 when used as an intermediate repeater. DC supervisory signals and most low-frequency signaling currents are passed by the E6 without serious impairment and without the aid of auxiliary bypassing equipment.

Compared to the E23 repeater, the overall principle of operation is substantially different. The impedance of the E23 is made to match the "book" value of cable-pair impedance by the use of adjustable networks consisting of resistors, inductors and capacitors. The desired gain and impedance characteristics are adjusted by interconnecting these elements by strapping soldered wires to their terminals. In the E6 repeater, the impedance-matching function is performed by a passive line building-out (LBO) network which is separate from the gain unit. Once the LBO network has been adjusted, the gain of the E6 can be varied in small steps without readjustment of the LBO network. All adjustments on both the gain and LBO units are made by tightening or loosening screws to close or open electrical contacts. Thus, no strapping or soldering is required for adjusting the E6 repeater in the field.

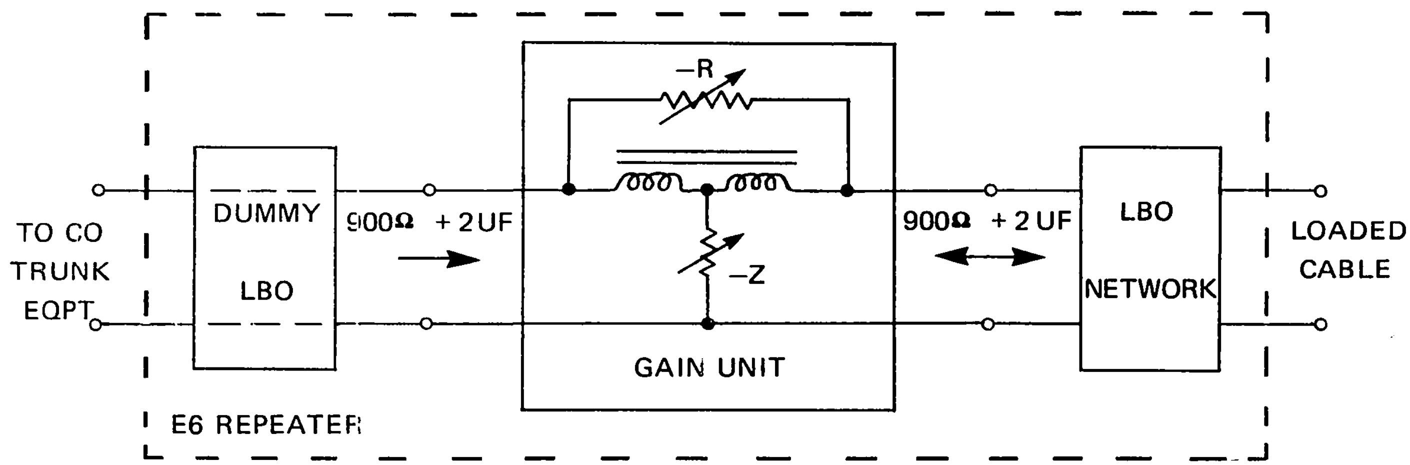
The E6 repeater can be used in both intermediate and terminal applications as shown in Figure 4-4. For intermediate applications, the repeater consists of a gain unit and two LBO networks as shown in Figure 4-4 (a). For terminal repeater applications, one of the LBO networks is replaced by a dummy network which permits a through connection from the shelf directly to one side of the gain unit. This arrangement is shown in Figure 4-4 (b). The LBO transforms the frequency dependent impedance of loaded cable pairs to a constant impedance of 900 ohms in series with 2 microfarads. Thus the gain unit always operates into this fixed impedance.

The gain unit circuit is the active amplifier portion of the repeater which is adjusted to have an image impedance equal to that of the LBO network. This 900 ohms in series with 2 microfarads is also the same impedance as the compromise hybrid networks used at exchange switching points in the Central Office (CO).

Thus, in contrast to the E23 repeater, the impedance-conversion, impedance-matching and gain adjusting functions of the E6 repeater are allocated to separate portions of the repeater. The impedance matching and adjusting functions are accomplished in the LBO networks while the gain adjustment is relegated to the series and shunt elements of the gain unit. [11] The latter is composed of two separate transistorized negative impedance converters interconnected in a bridged-T



(a) Intermediate Application



(b) Terminal Use

Figure 4-4. The E6 Repeater application arrangements.

arrangement. The remainder of this chapter will deal mainly with the theory of the E6 repeater gain unit circuit.

4.4 Theory of the E6 Repeater Gain Unit

One of the two negative impedance converters of the gain unit is essentially connected in series with the cable pair while the other is connected across the mid-points of the line transformer.

Ideally, the series-connected converter is a four-terminal network having an input impedance which is the negative of the terminating resistances connected across its output. As discussed earlier, it is possible to obtain circuits having this negative impedance conversion property by coupling the output of an amplifier back to its input. Consider the ideal CCT introduced in Chapter II having a forward current transfer ratio of K, a zero input impedance and a infinite output impedance. By connecting this ideal current amplifier as shown in Figure 4-5, an open-circuit stable negative impedance is seen at the source terminals AB.

The voltage across CD = I (K−1) Z_L and since the input impedance of the CCT is equal to zero, the voltage across AB is equal to minus the voltage across CD, i.e. $V_{AB} = -V_{CD} = I\,(1-K)\,Z_L$ and the impedance Z_{AB} of the circuit is given by

$$Z_{AB} = \frac{V_{AB}}{I} = \frac{I\,(1-K)\,Z_L}{I} = (1-K)\,Z_L \qquad (4.5)$$

Thus, for current amplification ratios of K greater than one, the input impedance at terminals AB becomes negative and directly proportional to the output impedance at terminal CD. Note that if terminals AB are open-circuited, the feedback between output and input is opened and the amplifier remains stable.

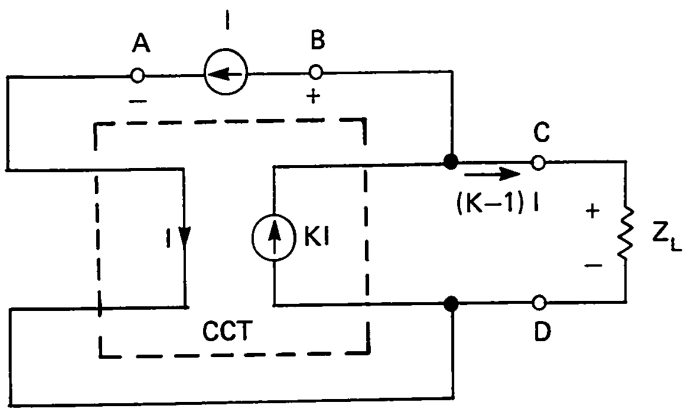


Figure 4-5. Equivalent controlled source representation of a series-connected Negative Impedance Converter.

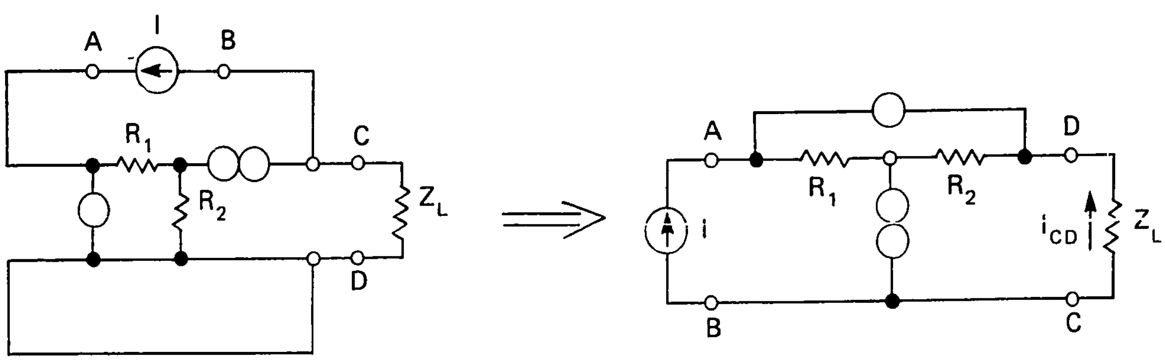


Figure 4-6. Nullator-norator model of the series-type NIC.

Substituting the nullator-norator model of the CCT given in Figure 2-6 (a), the equivalent model of the E6 repeater series-connected converter is obtained as shown in Figure 4-6. This is identical to the Type I CNIC nullator-norator representation arrived at earlier in Figure 3-4.

In Chapter III, the current relation for this circuit was found to be given by $I_2 = (R_1/R_2)\, I_1 = K I_1$. Thus, since $V_{AB} = -V_{DC} = -I_{CD}\, Z_L$, it follows that

$$Z_{AB} = \frac{V_{AB}}{I} = \frac{-\left(\frac{R_1}{R_2}\right) I\, Z_L}{I} = -\left[\frac{R_1}{R_2}\right] Z_L = -K\, Z_L \qquad (4.6)$$

For the unity gain CCT of Figure 2-7 (a), R_1 goes to zero, R_2 tends to infinity and Figure 4-6 reduces to that shown in Figure 4-7 (a). Using the nullator-norator representation of an ideal transistor given in Figure 2-8, the transistor realization of the E6 repeater series converter is obtained as shown in Figure 4-7 (b). Note that the output terminals CD are reversed with respect to the input terminals AB. Thus in order to correct this, an output transformer is used as shown in Figure 4-7 (c).

If the ratio of the transformer in Figure 4-7 (c) is equal to n, then the voltage across CD will be nIZ_L. The voltage across the winding $V_T = nV_{CD} = n^2 IZ_L$ and is equal to minus the voltage across AB. Thus the impedance at terminals AB is given by:

$$Z_{AB} = -\frac{n^2 IZ_L}{I} = -n^2 Z_L \qquad (4.7)$$

By interchanging the load impedance Z_L and the external current source I in Figures 4-5 and 4-7, the shunt-connected four-terminal negative impedance con-

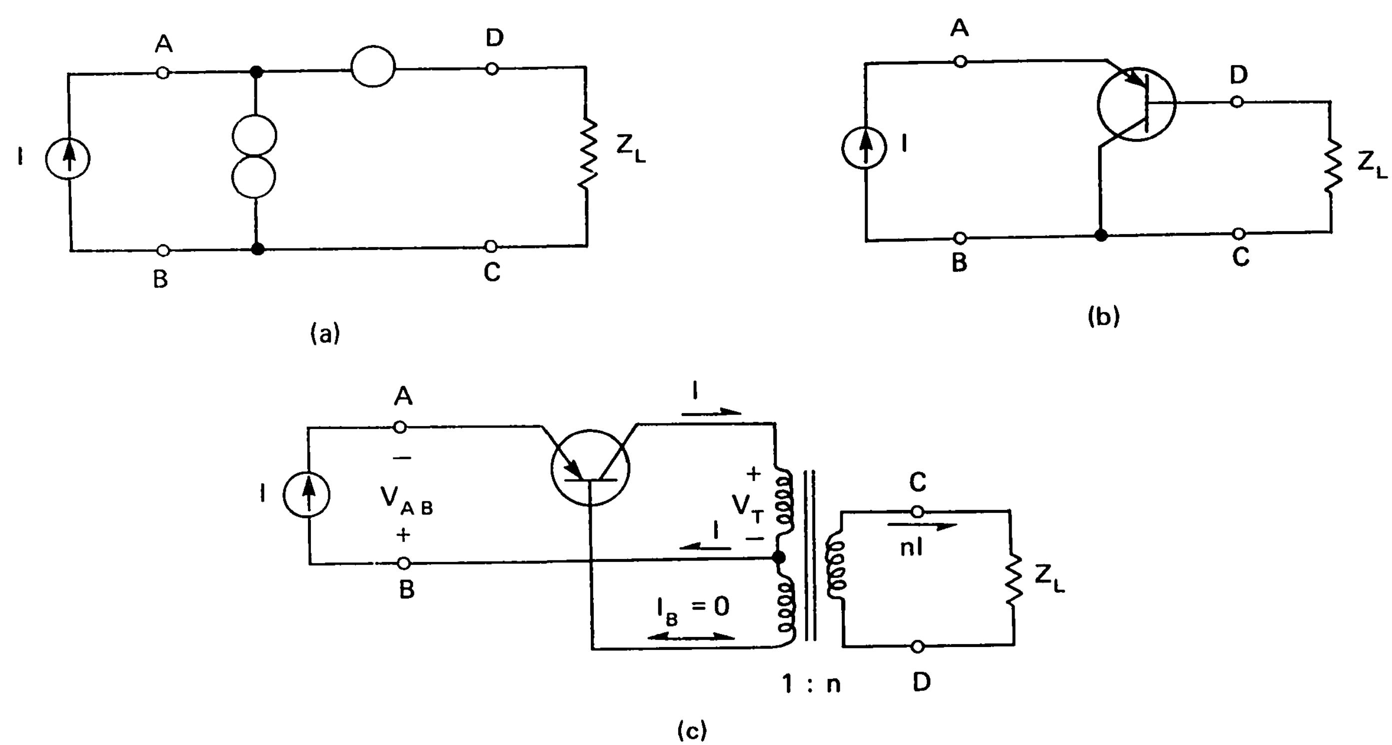


Figure 4-7. Transistor realization of the series-connected NIC.

verter is obtained as shown in Figure 4-8. The voltage across AB is equal to IZ_L/n. Since $V_T = -V_{AB}$, the voltage across the transformer winding is equal to $-IZ_L/n$. Thus, the voltage across CD is given by $V_{CD} = V_T/n = -IZ_L/n^2$ and the impedance Z_{CD} of the circuit is:

$$Z_{CD} = \frac{V_{CD}}{I} = \frac{-IZ_L/n^2}{I} = -\frac{1}{n^2} Z_L \tag{4.8}$$

Opening the external circuit at CD will still permit feedback between output and input thus making the shunt converter unstable in the open-circuit condition. On the other hand, if CD is short circuited no voltage, V_T, can develop across the transformer winding and the circuit will not oscillate. Thus the shunt converter is short-circuit stable and open-circuit unstable.

Consider the circuit of Figure 4-9 in which a series-type NIC is inserted in a transmission line having a characteristic impedance equal to Z_0. This increases the current in the line but creates a large impedance mismatch at the point where it is inserted. Without the insertion of the negative resistance $-R_S$, the current I_1 in the line is equal to $E/(Z_{L1} + Z_{L2}) = E/2Z_0$. With the insertion of $-R_S$, the current I_2 becomes $E/(Z_{L1} + Z_{L2} - R_S) = E/(2Z_0 - R_S)$. Thus, the resulting insertion gain is given by

$$\text{Gain} = 20 \log_{10} \frac{I_2}{I_1} = 20 \log_{10} \left| \frac{2Z_0}{2Z_0 - R_S} \right| \tag{4.9}$$

Note that the circuit remains stable as long as the magnitude of R_S is less than twice the resistance component of Z_0. As a practical example, if the magnitude of R_S is equal to Z_0, then equation 4.9 yields

$$\text{Gain} = 20 \log_{10} \left| \frac{2Z_0}{2Z_0 - Z_0} \right| = 20 \log_{10} 2 = 6\text{dB} \tag{4.10}$$

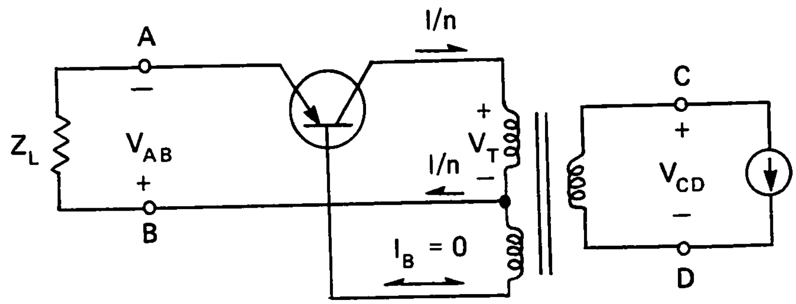


Figure 4-8. Transistor realization of the shunt-type NIC.

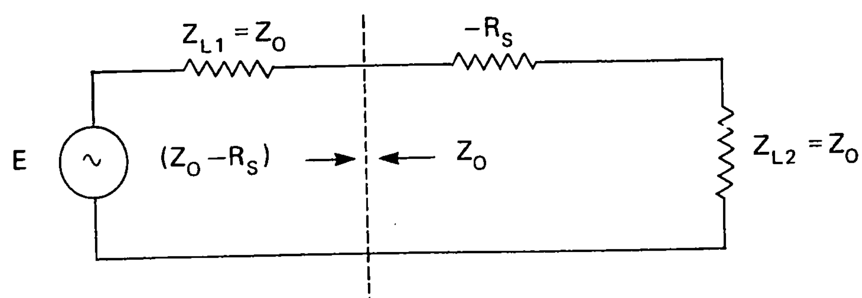


Figure 4-9. Insertion of a series-type NIC on the line.

Thus a 6dB current gain is delivered to Z_{L2}. At the point of insertion, the impedance equals $(Z_0 - Z_0)$ or zero ohms and the return loss at this point is equal to zero dB which is very poor.

In a similar fashion, a shunt negative impedance of the short-circuit stable type can be inserted across a transmission line to introduce gain as shown in Figure 4-10. The current I_1 flowing through Z_{L2} before the insertion of $(-Z_{SH})$ is again equal to $E/(Z_{L1} + Z_{L2})$. After insertion, the current $I_2 = V/Z_{L2}$, where V is the voltage drop across the line impedance Z_{L2} and is related to the source voltage E by the following voltage divider equation:

$$V = \frac{\dfrac{(-Z_{SH})(Z_{L2})}{Z_{L2} + (-Z_{SH})}}{Z_{L1} + \dfrac{(-Z_{SH})(Z_{L2})}{Z_{L2} + (-Z_{SH})}} E = \frac{(-Z_{SH})(Z_{L2})}{Z_{L1}Z_{L2} + (Z_{L1} + Z_{L2})(-Z_{SH})} E$$

from this, I_2 is found to be equal to

$$I_2 = \frac{V}{Z_{L2}} = \frac{(-Z_{SH})}{Z_{L1}Z_{L2} + (Z_{L1} + Z_{L2})(-Z_{SH})} E$$

and the current gain of the circuit becomes

$$\text{Gain} = 20 \log_{10} \left| \frac{(Z_{L1} + Z_{L2})(-Z_{SH})}{Z_{L1}Z_{L2} + (Z_{L1} + Z_{L2})(-Z_{SH})} \right| \qquad (4.11)$$

This circuit is stable as long as the magnitude of Z_{SH} is less than the sum of the conductance of Z_{L1} and Z_{L2}. Again, if Z_{L1}, Z_{L2} and the magnitude of Z_{SH} are made equal to the characteristic impedance of the line Z_0, the gain using equation 4.11 is equal to 6dB with a zero dB return loss giving 100% reflection.

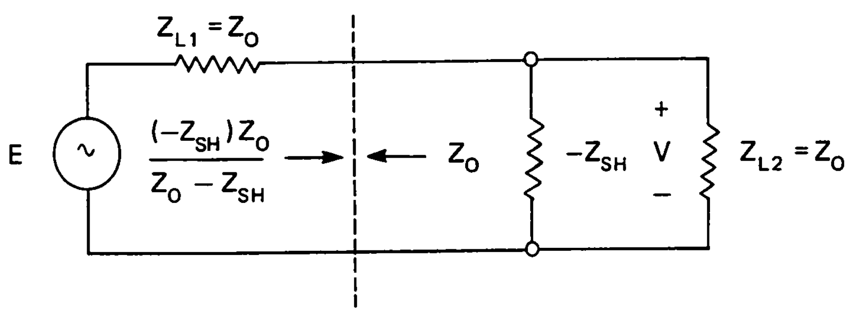


Figure 4-10. Insertion of a shunt-type NIC on the line.

Thus, an NIC inserted in series with a circuit lowers the impedance and increased the generator current. On the other hand, a shunt inserted NIC will increase the impedance of the circuit and decrease the current from the generator. Both types, however, introduce impedance irregularities at the point of insertion. As mentioned earlier, this problem was first overcome in the E23 repeater by combining the two types of converters in a bridged-T structure as shown in Figure 4-3. By making both series and shunt negative impedances have the same phase angle as the image impedance and by using the same ratio, N, for determining the magnitudes of the negative elements, this bridged-T structure was obtained with no impedance mismatch to the line and with a gain determined by the value of N in equation 4.4.

In the E6 repeater, however, it was found that it is not necessary to make both series and shunt negative impedances have the same phase angle as the desired image impedance. Equation 4.1 can be written as

$$|Z_I| \angle\theta = \sqrt{|Z_A||Z_B|} \angle\frac{A+B}{2} \tag{4.12}$$

Thus, only the sum of angles A and B of impedances Z_A and Z_B need be made equal to twice the angle of the desired image impedance. In other words, phase angles A and B do not have to be alike, provided their sum equals twice the desired angle of the image impedance.

The series and shunt converters shown in Figures 4-7 and 4-8 do not require the use of a center-tapped transformer to obtain the desired feedback. Push-pull type amplifier circuits can provide the feedback paths of the proper phase and have the advantage of cancelling even harmonic distortion products. In addition, they have higher output for a given transistor rating. Using single transistors in each side of the push-pull circuits to Figures 4-7 (c) and 4-8, the simplified E6 repeater gain circuit shown in Figure 4-11 is obtained. The T1 Line Transformer is added in order to remove DC supervisory voltages from the negative impedance circuits and to permit dial pulsing to go through without serious impairment.

In the series converter of Figure 4-11, the balancing inductor connected across the variable resistance network, R_S, is used to maintain stability over a wide frequency range by keeping the impedance at this point lower than that at the line terminals. Capacitors C1 and C2 provide the feedback path and are sufficiently large to introduce negligible impedance over the voice-frequency range.

The shunt converter is bridged across the cable pair at the midpoints of the line transformer primary windings as shown in Figure 4-11. The initial approach to block direct currents from the shunt converter of the E23 repeater was to insert two large capacitors in the line leads. This created a compromise problem since large, low-impedance coupling capacitors degrade DC loop pulsing while small capacitors have an effect on the NIC especially at the low-frequency end of the voice band. As a result, in the E6 repeater, a transformer with a single blocking capacitor is used which has the advantages of saving component space, having less effect on DC pulsing and isolating the shunt network from the line. In addition, this transformer coupling gives a high degree of isolation from the effects of power-induced longitudinal voltages which may occur on the transmission line.

From equation 4.12, the desired image impedance can be obtained provided the series and shunt phase angles are made equal to twice the image impedance angle. In the E6 repeater gain unit, angle A of the series converter is designed to be equal to zero, while the shunt converter angle B, is made equal to twice the image impedance angle. This is done by operating the series converter to provide pure negative resistance to the line and by introducing a fixed R–C circuit in

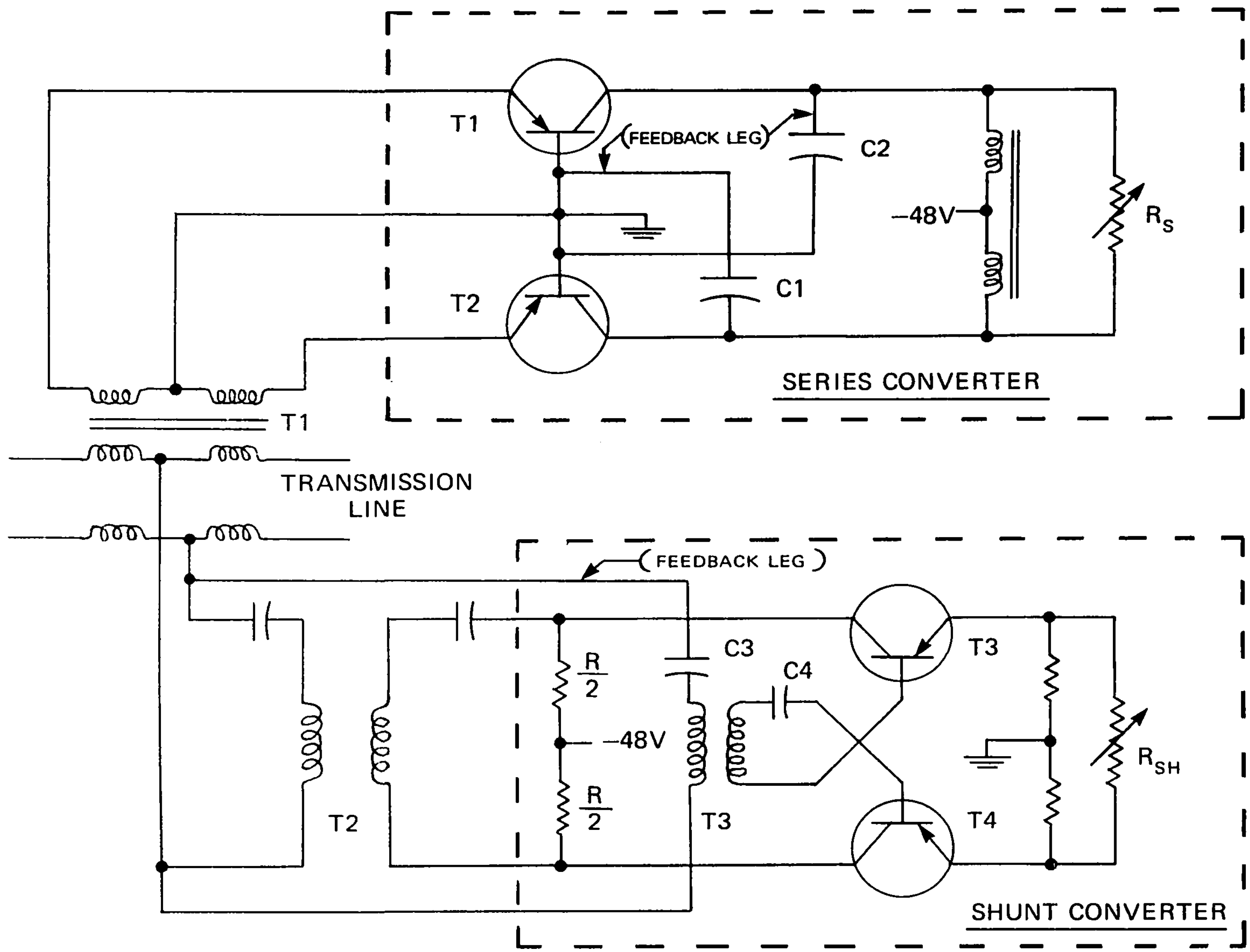


Figure 4-11. Simplified circuit of the E6 repeater gain unit.

the feedback path of the shunt converter to provide the desired phase angle. In Figure 4-11, this phase shift is accomplished by C3, C4 and the two R/2 resistors in series.

CHAPTER V

THE OPERATIONAL AMPLIFIER

To solve the problem of realizing a practical negative impedance converter, two active elements were used in the preceding chapter. The election tube was applied in the E23 repeater and when the transistor was developed, it was used in the E6 repeater. In today's electronics market, another active element called the Operational Amplifier has rapidly gained increased popularity over the transistor. One reason for its success is its high open loop gain; as a result, the sensitivity problems normally associated with the gain of the other active elements are essentially eliminated when using the operational amplifier. Another reason for its popularity is its ready availability in monolithic integrated circuit form as an off-the-shelf item at very reasonable cost.

5.1 Definition

The operational amplifier is a high-gain DC voltage amplifier composed of a dependent voltage source having an output which is proportional to the potential difference between two isolated input terminals. Theoretically, it is an infinite gain VVT whose output voltage polarity is opposite to one of the inputs and is the same as the other input terminal. [3]

The controlled-source representation and the symbol denoting the operational amplifier are both shown in Figure 5-1. Ideally, this type of active element has an infinite input impedance and a zero output impedance. Thus, the input excitation is independent of the power drawn by the amplifier and at the same time it can supply any amount of power as required by the load. In practice, the DC voltage gain of an integrated-circuit operational amplifier is not infinite but is extremely high; generally in the range from one thousand to one million. Connecting terminal 2 to ground transforms the operational amplifier into a VVT with a

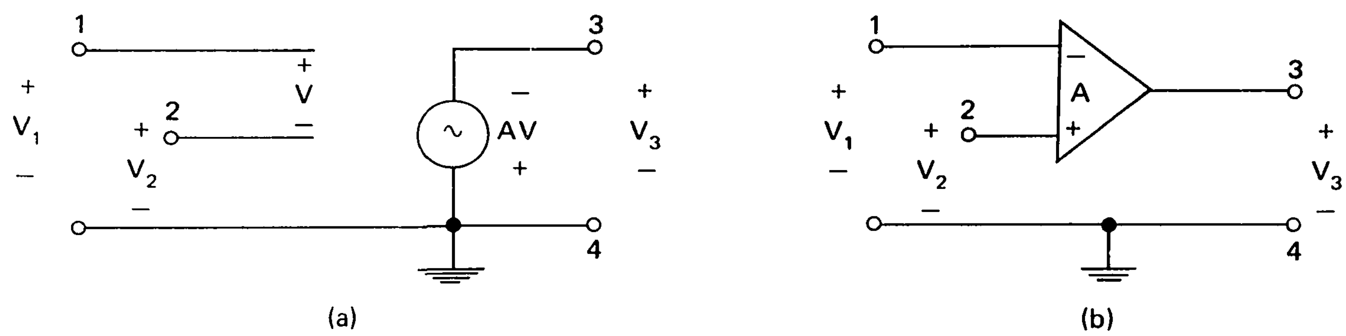


Figure 5-1. The operational amplifier (a) controlled-source, and (b) symbolic representations.

large negative gain implying a "single-ended" operation. When both input terminals are used, it is called a differential input operational amplifier. [3] For ideal performance, the output voltage of the amplifier should be equal to zero when there is no potential difference between the input terminals; this is commonly known as having a "zero offset". Most present-day manufacturers claim their device to have a very low DC offset and practically no drift with time over the required temperature range. Thus, for all practical purposes, zero-offset can be assumed.

The versatility and advantages of the operational amplifier stem from the use of negative feedback to control response characteristics. By providing sufficient gain, the closed-loop amplifier response becomes solely a function of the feedback components. In general, negative feedback improves the performance of the operational amplifier. Two basic feedback arrangements are shown in Figure 5-2. In Figure 5-2 (a), the current flowing through R_1 is equal to the current flowing through R_2 i.e., since the input impedance of the operational amplifier is very large, the input current can be assumed to be negligible. Using this assumption together with the relation $V_2 = -AV$ obtained from Figure 5-1 (a), the following equation is obtained:

$$\frac{V_1 - V}{R_1} = \frac{V - V_2}{R_2} = \frac{V + AV}{R_2} = \frac{(1 + A)}{R_2} V \tag{5.1}$$

$$V_1 - V = \frac{R_1}{R_2} (1 + A) V \Rightarrow V = \frac{V_1}{1 + \frac{R_1}{R_2} (1 + A)} \tag{5.2}$$

Thus, as the voltage gain A tends to infinity, the input voltage V tends to zero.

Consider next the circuit of Figure 5-2 (b), the circuit equations are given as:

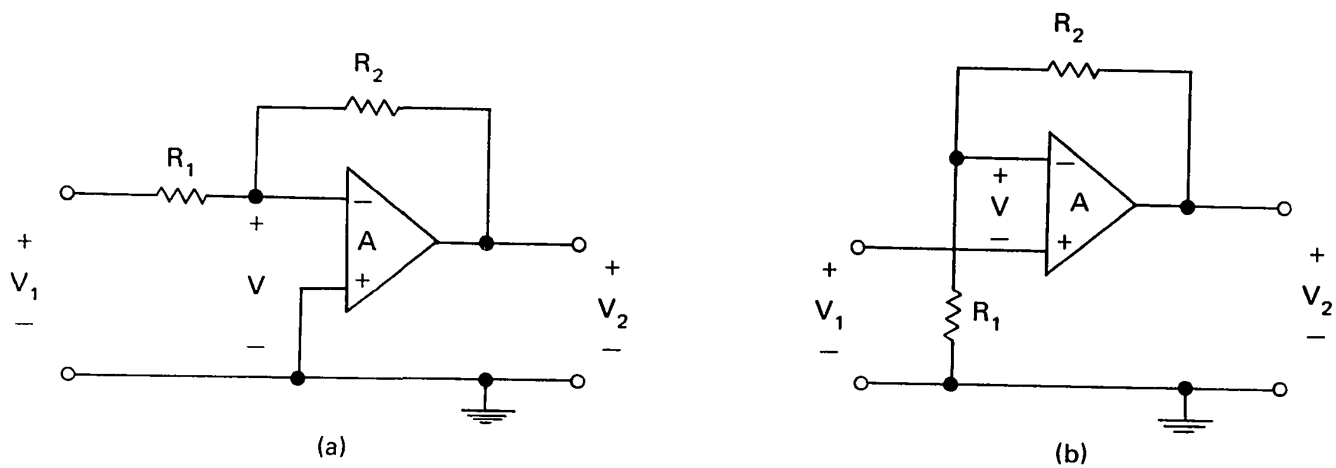


Figure 5-2. Operational amplifiers with negative feedback arrangements.

$$\frac{V_1 + V}{R_1} = -\frac{(V_1 + V) - V_2}{R_2} = -\frac{V_1 + V + AV}{R_2} = -\frac{V_1 + (1 + A)\,V}{R_2} \quad (5.3)$$

$$\frac{R_2}{R_1}(V_1 + V) = -V_1 - (1 + A)\,V \quad \Rightarrow \quad V = -\frac{\frac{R_2}{R_1} + 1}{(1+A) + \frac{R_2}{R_1}}\,V_1 \quad (5.4)$$

Thus, here again, as A tends to infinity, V goes to zero. Using the relations $V_2 = -AV$ and $(1/A) = 0$ in equations 5.2 and 5.4, the closed loop gain of these two circuits is found to be

$$V_2 = -AV = -\frac{V_1}{\frac{1}{A} + \frac{R_1}{R_2}\left(\frac{1}{A} + 1\right)} \quad \Rightarrow \quad \frac{V_2}{V_1} = -\frac{R_2}{R_1} \quad (5.5)$$

$$V_2 = -AV = +\frac{\frac{R_2}{R_1} + 1}{\left(\frac{1}{A} + 1\right) + \frac{1}{A}\frac{R_2}{R_1}}\,V_1 \quad \Rightarrow \quad \frac{V_2}{V_1} = \frac{R_2 + R_1}{R_1} \quad (5.6)$$

Thus, the closed loop gain of both the inverting type circuit of Figure 5-2 (a) and the non-inverting type circuit of Figure 5-2 (b) depend only on the external resistors R_1 and R_2, providing the DC voltage gain, A, is large.

5.2 Nullator-Norator Representation

In reality, the operational amplifier is a nonideal device. It has a frequency-dependent voltage gain whose magnitude starts from a very high value at DC (in the range of 80 to 120 db) and then monotonically decreases for higher frequencies. The bandwidth of most practical amplifiers starts at DC and rolls off to unity gain at typically 10 MHz or more. Like all physical devices, the amplifier has maximum limits of positive and negative output voltage over a large dynamic range, usually from ±10 to ±100 volts. The input and output impedances are finite and nonzero quantities; typical values are 100 kilohms and 100 ohms respectively. Figure 5-3 (a) gives the controlled-source representation of a nonideal operational amplifier. In most cases, the feedback resistor R_F is very large and can be neglected.[3] Neglecting R_F, the practical model of the operational amplifier is obtained as shown in Figure 5-3 (b).

The transmission matrix of the nonideal operational amplifier of Figure 5-3 (b) is obtained from:

$$f_{11} = \left.\frac{V_1}{V_2}\right|_{I_2 = 0} = \frac{V_1}{-AV_1} = -\frac{1}{A} \tag{5.7}$$

$$f_{12} = \left.\frac{V_1}{-I_2}\right|_{V_2 = 0} = \frac{V_1}{\frac{-AV_1}{R_0}} = -\frac{R_0}{A} \tag{5.8}$$

$$f_{21} = \left.\frac{I_1}{V_2}\right|_{I_2 = 0} = \frac{I_1}{-AV_1} = \frac{I_1}{-AI_1 R_I} = -\frac{1}{AR_I} \tag{5.9}$$

$$f_{22} = \left.\frac{I_1}{-I_2}\right|_{V_2 = 0} = \frac{\frac{V_1}{R_I}}{\frac{-AV_1}{R_0}} = -\frac{R_0}{AR_I} \tag{5.10}$$

Thus,

$$F = \begin{bmatrix} -1/A & -R_0/A \\ -1/AR_I & -R_0/AR_I \end{bmatrix} \tag{5.11}$$

For the case of an ideal operational amplifier, the input resistance, R_I, and

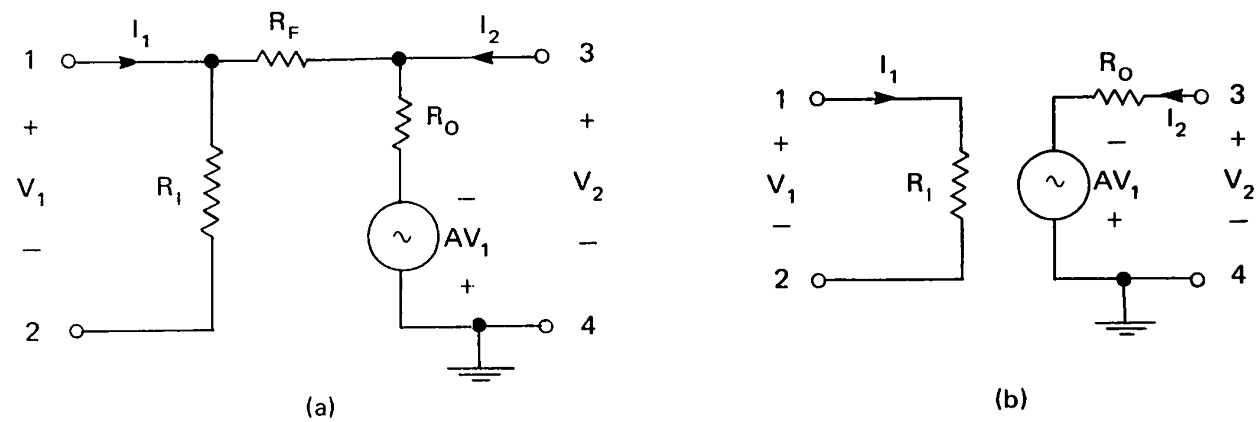


Figure 5-3. Controlled-source model of a nonideal operational amplifier.

the DC open loop gain, A, both tend to infinity whereas the output resistance, R_0 tends to zero. Thus, all the elements in the transmission matrix F of equation 5.11 becomes equal to zero and the matrix relation defining the ideal operational amplifier becomes:

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} \qquad (5.12)$$

Earlier, in Figure 2-4 (c), the two-port network defined as a nullor was introduced consisting of a nullator at the input port and a norator at the output terminals. From the defining equations of the nullator ($V_1 = I_1 = 0$) and the norator (V_2, I_2 arbitrary), the input-output relationship of this two-port can be written as [3]

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

which is identical to the null transmission matrix of the ideal operational amplifier given by equation 5.12. Thus, the nullor represents an ideal operational amplifier as shown in Figure 5-4.

In earlier chapters, the nullator-norator circuit model was used in forming duals and in obtaining transistorized realizations. Similarly, the nullator-norator model of Figure 5-4 can be used for constructing duals of operational amplifier circuits and for realizing nullator-norator circuits using operational amplifiers.

5.3 Realization of Controlled Sources

From the four basic nullator-norator models of controlled sources shown in Figure 2-6, the VVT and the CVT models are easily converted to their operational amplifier equivalent circuits by direct substitution of the grounded model

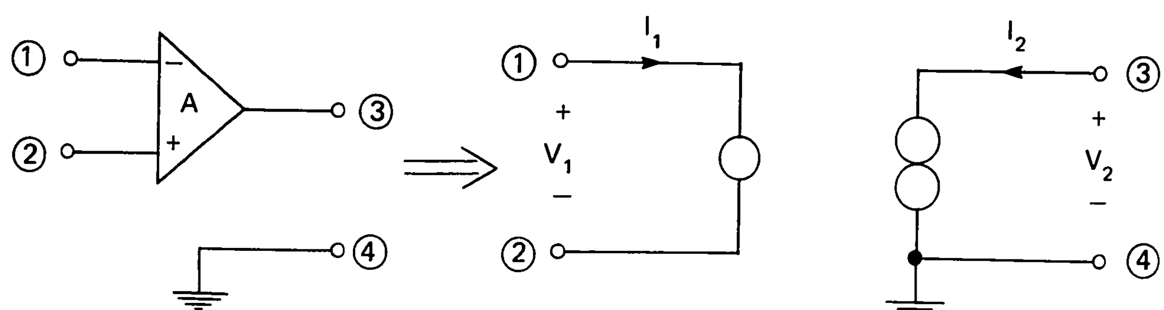


Figure 5-4. Nullator-norator model of an ideal operational amplifier.

of Figure 5-4. These realizations are shown in Figure 5-5.

Direct conversion of the CCT and VCT models using the equivalent representation of Figure 5-4 is not possible without a floating power supply. Thus, these circuits having a nongrounded output can be easily obtained by interchanging the load resistor, R_L, with the norator in the models of Figures 2-6 (a) and (d).[3] The final realizations are given in Figure 5-6.

The equations defining the controlled sources can be obtained by inspection of the operational amplifier equivalent circuits of Figures 5-5 and 5-6. Keeping in mind that the input current to the amplifier is negligible and the input voltage $V = -V_2/A$ approaches zero (providing the magnitude of A is large enough), the voltage current relationships are easily derived. For the VVT of Figure 5-5 (a), the current, I, flowing through R_2 and R_1 is equal to $V_2/(R_2 + R_1)$. Thus the voltage $V_1 = IR_1 = V_2R_1/(R_1 + R_2)$ which leads to the VVT equation of Figure 2-6 (b) i.e.

$$V_2 = \frac{R_1 + R_2}{R_1} V_1 = \left(1 + \frac{R_2}{R_1}\right) V_1 \tag{5.13}$$

From Figure 5-5 (b), the CVT equation is obtained directly by inspection, i.e.,

$$I_2 = \frac{V_1 - V_2}{R} = \frac{0 - V_2}{R} = -\frac{V_2}{R} \tag{5.14}$$

In Figure 5-6 (a) the current flowing through R_2 is equal to $I_1 + I_2$. Since $V_1 = 0$, the voltage drop across R_2 is equal to minus the voltage across R_1

Thus, $(I_1 + I_2)\ R_2 = -I_1\ R_1$ which leads to the CCT equation:

$$I_2 = -\left(1 + \frac{R_1}{R_2}\right) I_1 \tag{5.15}$$

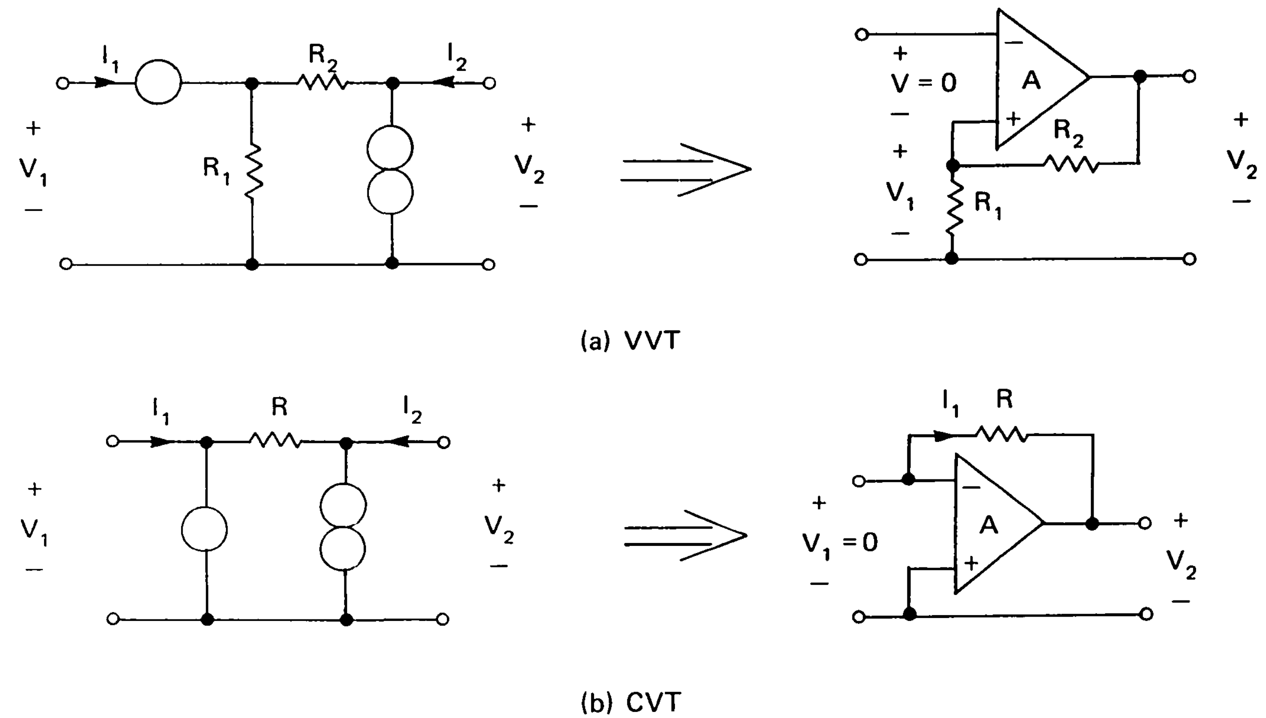


Figure 5-5. Operational amplifier realizations of the VVT and CVT controlled sources.

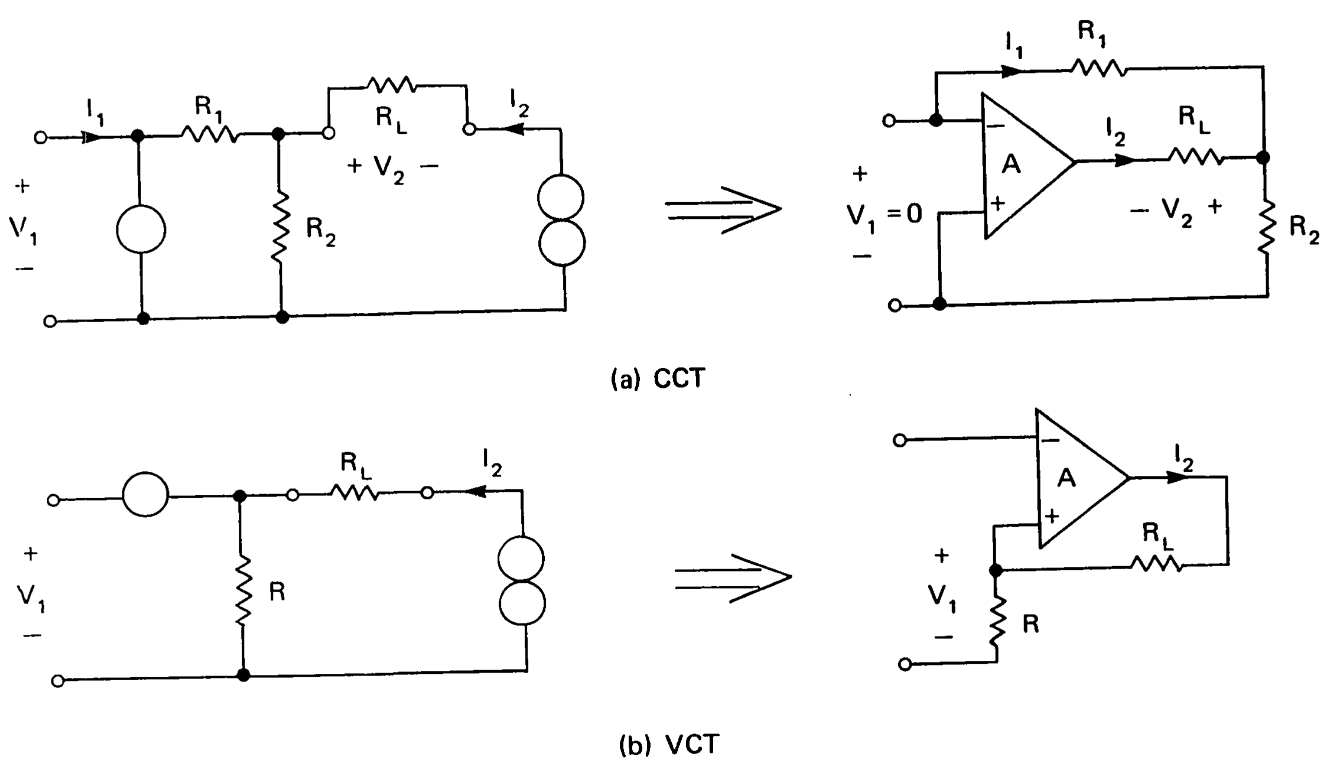


Figure 5-6. Operational amplifier realizations of the CCT and VCT controlled sources.

Finally, the VCT equation of Figure 5-6 (b) is seen by inspection to be

$$V_1 = I_2 R \quad \text{or} \quad I_2 = \frac{V_1}{R} \tag{5.16}$$

Equations 5.13 to 5.16 are identical to those given in Figure 2-6 of Chapter II.

5.4 Stability Properties of grounded Negative Impedance Converters

The Type I CNIC nullator-norator representation derived earlier in Chapter III Figure 3-4, can be directly converted into its equivalent operational amplifier circuit as shown in Figure 5-7 (a).

In Chapter IV, the series-type converter of the E6 repeater was found to be open-circuit stable (OCS) whereas the shunt-connected converter was seen to be short-circuit stable (SCS). To prove that port 1 of Figure 5-7 (a) is OCS while port 2 is SCS, consider the equivalent controlled source circuit of Figure 5-7 (b) in which, for simplicity, the input and output impedances of the operational amplifier have been omitted. Terminating port 2 by a load reistance R_L, the input impedance, Z_{I1}, seen at port 1 is derived as follows:

$$AV - I_2 R_2 - I_2 R_L = 0 \quad \Rightarrow \quad I_2 = \frac{AV}{R_2 + R_L} \tag{5.17}$$

$$-V_1 + V - I_2 R_L = 0 \quad \Rightarrow \quad -V_1 + V - \frac{AV}{R_2 + R_L} R_L = 0$$

from which,
$$V = \frac{(R_2 + R_L)\ V_1}{R_2 + (1-A) R_L} \tag{5.18}$$

$$-V_1 + I_1 R_1 - AV = 0 \Rightarrow -V_1 + I_1 R_1 - \frac{A\ (R_2 + R_L)}{R_2 + (1-A)\ R_L} V_1 = 0 \tag{5.19}$$

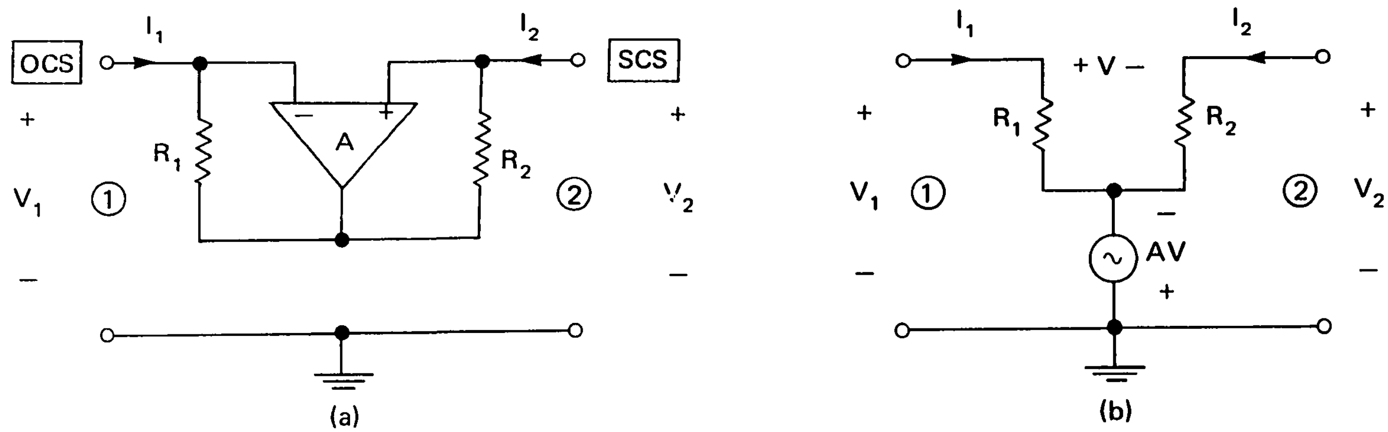


Figure 5-7. A grounded CNIC circuit and its equivalent controlled source representation.

Equation 5.19 can be rewritten as

$$I_1 R_1 [R_2 + (1-A) R_L] = [R_2 + (1-A) R_L + A(R_2 + R_L)] V_1$$

Thus,

$$Z_{I1} = \frac{V_1}{I_1} = \frac{R_1 R_2 + R_1 R_L (1-A)}{R_L + (1+A) R_2} \tag{5.20}$$

In practice, the magnitude of the voltage gain A is frequency dependent and can be approximated by the Bode Plot shown in Figure 5-8. The open loop gain, expressed as a function of the complex frequency variable $s = \sigma + jw$, is given by

$$A(s) = \frac{A}{(s+w_1)(s+w_2)(s+w_3)} \tag{5.21}$$

where w_1, w_2 and w_3 are the first, second and third cutoff frequencies respectively. Thus, when the operational amplifier is used in the feedback configuration, the stability of the closed-loop system becomes a major problem.

To insure stability in feedback applications, one solution is to internally compensate the open loop gain function A(s) by introducing an additional cutoff frequency to the circuit. This is done by inserting an RC circuit in tandem between the first and second stages of the amplifier such that the compensated open loop gain $A_C(s)$ becomes equal to

$$A_C(s) = \frac{w_C}{s + w_C} A(s) = \frac{w_C A}{(s+w_C)(s+w_1)(s+w_2)(s+w_3)} \tag{5.22}$$

where $w_C = 1/RC$ and capacitor C is usually chosen such that w_C is less than w_1 for feedback stability. This makes w_C the dominant pole of the compensated operational amplifier and the open loop gain is given by

$$A_C(s) = \frac{w_C A}{s + w_C} \tag{5.23}$$

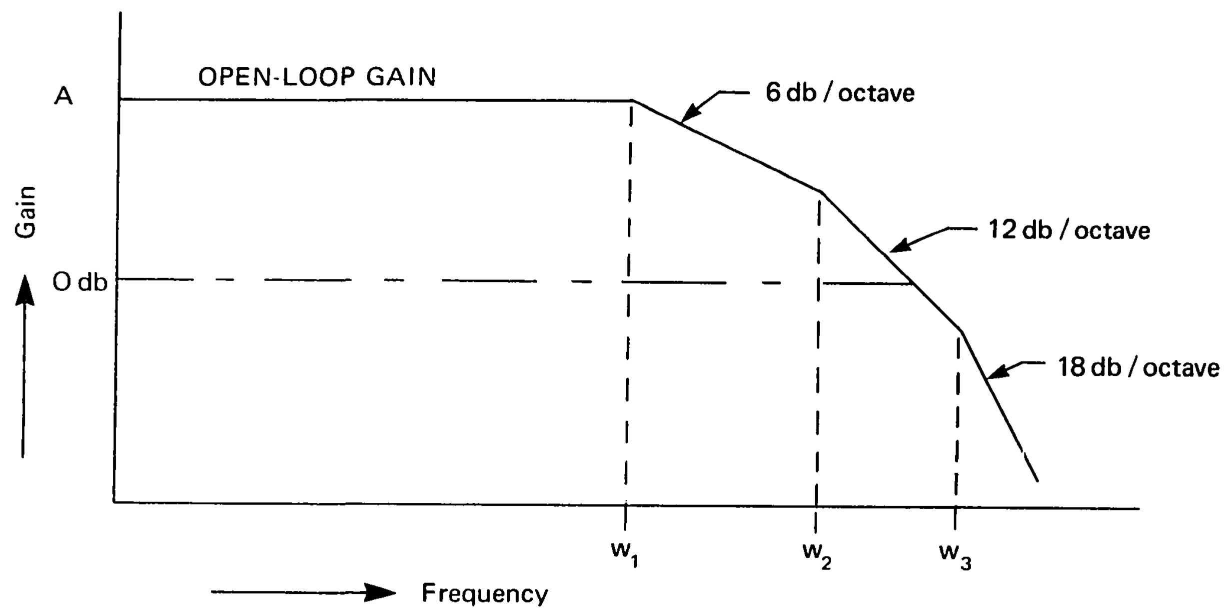


Figure 5-8. Frequency Response of a typical operational amplifier.

Substituting this relation for A in equation 5.20, we have

$$Z_{I1}(s) = \frac{R_1(R_2+R_L)(s+w_C) - w_C A R_1 R_L}{(R_2+R_L)(s+w_C) + w_C A R_2} = R_1 \frac{s+w_C a}{s+w_C b} \quad (5.24)$$

where

$$a = 1 - \frac{AR_L}{R_2+R_L} \quad \text{and} \quad b = 1 + \frac{AR_2}{R_2+R_L} \quad (5.25)$$

For stability, the poles and zeroes of equation 5.24 should lie in the left half (LH) of the complex frequency plane s. The zero occurs at $s_1 = -w_C a$ while the pole occurs at $s_2 = -w_C b$. Since "b" is always positive, there are no possible poles in the right half (RH) plane. However, "a" can attain negative values, as seen by equation 5.25, which will introduce a zero in the RH plane making the NIC unstable. To ensure stability, the following condition should be satisfied.

$$\frac{AR_L}{R_2+R_L} < 1 \quad \text{or} \quad \frac{R_2}{R_L} + 1 > A \quad (5.26)$$

It follows that the CNIC of Figure 5-7 is stable when (R_2/R_L) tends to infinity; i.e. when R_L tends to zero. Thus, port 2 of the CNIC is short circuit stable (SCS).

In a similar manner, terminating port 1 by a load resistance, R_S, the input impedance, Z_{I2}, seen at port 2 is given by

$$Z_{I2} = \frac{R_1 R_2 + R_2 R_S (1+A)}{R_S + R_1 (1-A)} \quad (5.27)$$

Substituting equation 5.23 for A, we get

$$Z_{I2}(s) = \frac{R_2(R_1+R_S)(s+w_C) + w_C A R_2 R_S}{(R_1+R_S)(s+w_C) - w_C A R_1} = R_2 \frac{s+w_C a}{s+w_C b} \quad (5.28)$$

where

$$a = 1 + \frac{AR_S}{R_1 + R_S} \quad \text{and} \quad b = 1 - \frac{AR_1}{R_1 + R_S} \tag{5.29}$$

To ensure stability, the pole at $s_2 = -\, w_C b$ should lie in the LH plane; thus the following condition should be satisfied

$$\frac{AR_1}{R_1 + R_S} < 1 \quad \text{or} \quad 1 + \frac{R_S}{R_1} > A \tag{5.30}$$

It follows that the CNIC is stable when R_S tends to infinity. Thus, port 1 is open circuit stable (OCS).

CHAPTER VI

CONSTANT RESISTANCE BILATERAL AMPLIFIERS

In the E6 telephone repeater described in Chapter IV, the bridged-T network configuration with mutual coupling, as shown in Figure 4-3, was used to avoid mismatch between the transmission line and the inserted amplifier. Using the same type of configuration, Antoniou [1] proposed a bilateral constant-resistance amplifier circuit which would eliminate the need for mutual coupling. This circuit is shown in Figure 6-1.

To prevent impedance discontinuities, it was seen in section 4.2 that the image impedance, Z_I, of the amplifier would have to match the characteristic impedance of the transmission line. This is done in Figure 6-1 by setting $Z_A = R$ and $Z_B = R_0{}^2/R$. Thus, the image impedance of the bilaterai amplifier circuit, using equation 4.1, becomes

$$Z_I = \sqrt{Z_A Z_B} = \sqrt{R \frac{R_0{}^2}{R}} = R_0 \tag{6.1}$$

where R_0 is the characteristic resistance of the line.

In Figure 6-1, letting $E_2 = 0$, the matrix equation is given by

$$\begin{bmatrix} E_1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 2R_0 + R_0{}^2/R & -R_0{}^2/R & -R_0 \\ -R_0{}^2/R & 2R_0 + R_0{}^2/R & -R_0 \\ -R_0 & -R_0 & 2R_0 + R \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} \tag{6.2}$$

Solving this set of equations yields

$$I_2 = \frac{E_1}{2(R_0 + R)} \tag{6.3}$$

From this, the voltage delivered to the load is

$$V_2 = I_2 R_0 = \frac{R_0}{2(R_0 + R)} E_1 \tag{6.4}$$

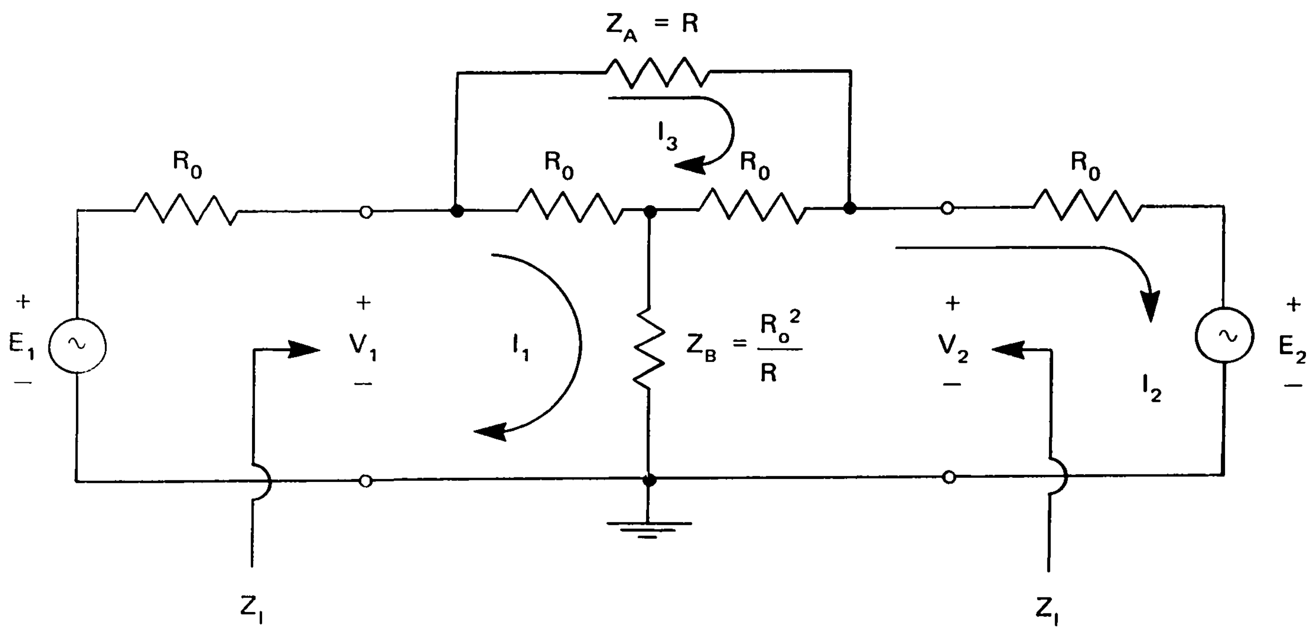


Figure 6-1. A bilateral constant-resistance amplifier.

Without the insertion of the proposed circuit, the voltage V_{2A} delivered to the load is $E_1 R_0 / (R_0 + R_0) = E_1 / 2$. Thus, the insertion loss is given by

$$\text{Insertion loss} = 20 \log \frac{E_{2A}}{E_2} = 20 \log \frac{R_0 + R}{R_0} \qquad (6.5)$$

If the inserted network furnishes gain, then E_2 is greater than E_{2A} and the insertion loss is negative. In order to avoid talking about negative loss, it is customary to write [12]

$$\text{Insertion gain} = 20 \log \frac{E_2}{E_{2A}} = 20 \log \frac{R_0}{R_0 + R} \qquad (6.6)$$

Equation 6.6 indicates that in order to have gain, R should have a negative value smaller than R_0. Note that if $-R = R_0$, the insertion gain becomes infinite and the amplifier will oscillate.

If R is negative in equation 6.4, then for the gain (V_2/E_1) to be greater than unity, the magnitude of R should be greater than $R_0/2$. Thus, for an amplification between unity and infinity, the following condition should be satisfied:

$$\frac{R_0}{2} < -R < R_0 \qquad (6.7)$$

In equation 6.2, the impedance matrix is symmetrical about the principle diagonal. Consequently, if E_1 is replaced by a short circuit in figure 6-1, the voltage across the load R_0 is given by

$$V_1 = \frac{R_0}{2(R_0 + R)} E_2 \qquad (6.8)$$

which is similar to equation 6.4. Thus, for negative values of R, the circuit of Figure 6-1 behaves like a bilateral constant resistance amplifier provided condition (6.7) is met.

6.1 A Floating Negative Impedance Converter

In order to simulate the two negative resistances $(-R)$ and $(-R_0{}^2/R)$ of Figure 6-1 without using transformers, a floating negative impedance converter (FNIC) is required for the former while the latter can be generated by the grounded NIC introduced earlier in Figure 5-7. An FNIC circuit was proposed by Antoniou [1] using differential-input operational amplifiers as shown in Figure 6-2 (a).

To obtain the chain matrix parameters, consider the equivalent controlled source circuit given in Figure 6-2 (b), where it is assumed that the amplifiers have finite open-loop gains, infinite input impedances, and zero output resistances. The chain matrix, $[A_T]$, is defined by equation 3.1 to be

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} = \begin{bmatrix} A_T \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} \tag{6.9}$$

From Figure 6-2 (b), the following loop equations are obtained using Kirchoff's voltage law:

$$-V_1 + I_1 R - A_2 V_4 = 0 \tag{6.10}$$

$$V_3 + I_1 R + I_L R = 0 \tag{6.11}$$

$$A_2 V_4 + I_L (2R + Z_L) - A_1 V_3 = 0 \tag{6.12}$$

$$V_4 + I_2 R - I_L R = 0 \tag{6.13}$$

$$-V_2 + I_2 R - A_1 V_3 = 0 \tag{6.14}$$

From equation 6.9, by definition,

$$A = \left.\frac{V_1}{V_2}\right|_{I_2 = 0} \tag{6.15}$$

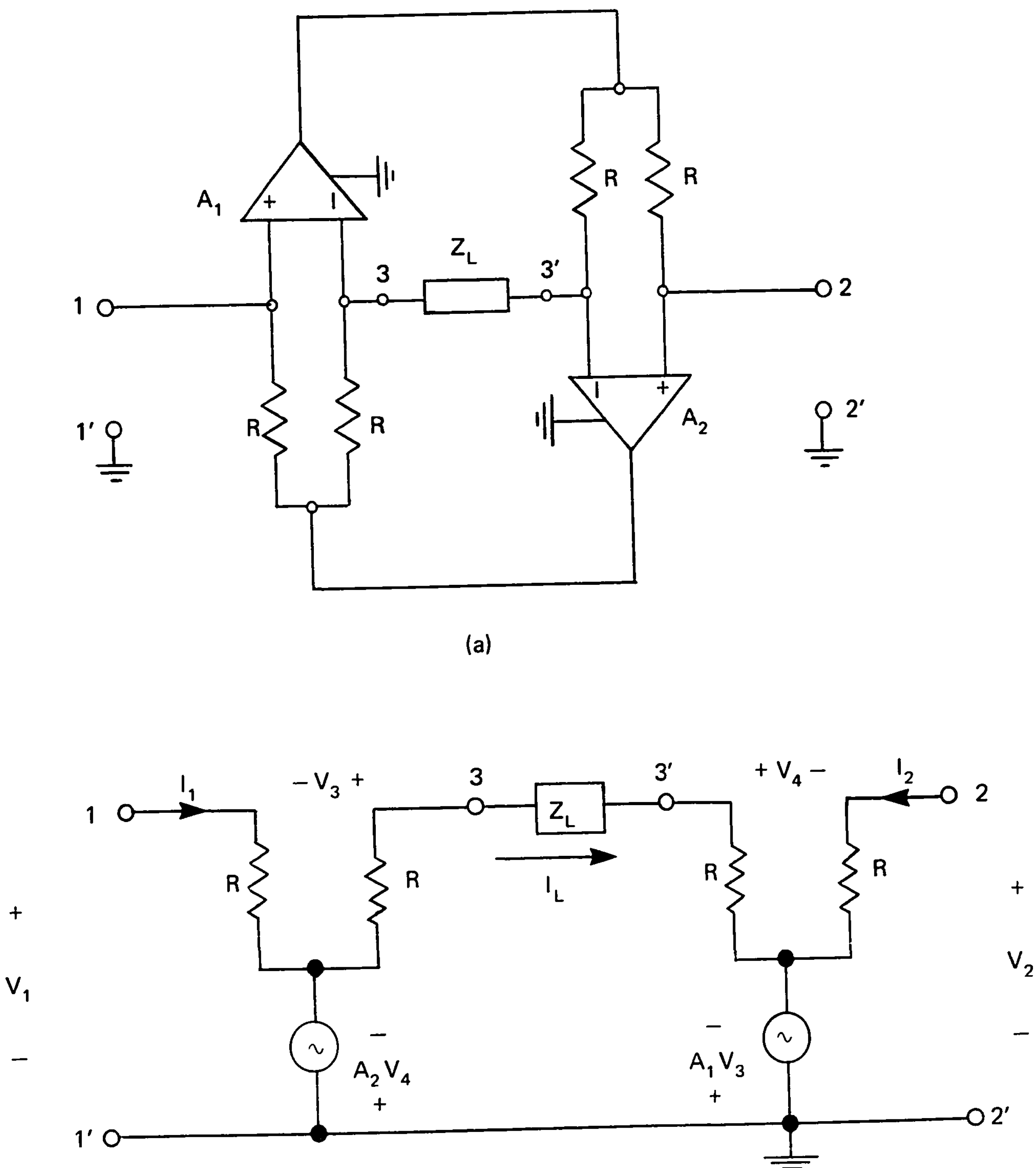


Figure 6-2. A floating negative impedance converter and its equivalent controlled source model.

With $I_2 = 0$, (6.13) and (6.14) reduce to

$$V_4 = I_L R \tag{6.16}$$

$$V_2 = -A_1 V_3 \tag{6.17}$$

from (6.10) and (6.11), $$-V_1 = V_3 + I_L R + A_2 V_4 \tag{6.18}$$

from (6.16) and (6.18), $$-V_1 = V_3 + (1+A_2) R I_L \tag{6.19}$$

from (6.12) and (6.16), $$R I_L = \frac{A_1 V_3}{A_2 + 2 + \frac{Z_L}{R}} \tag{6.20}$$

(6.20) into (6.19) yields

$$-V_1 = \frac{A_2 + 2 + \frac{Z_L}{R} + (1+A_2) A_1}{A_2 + 2 + \frac{Z_L}{R}} V_3 \tag{6.21}$$

(6.21) and (6.17) into (6.15) gives

$$A = \frac{A_2 + 2 + \frac{Z_L}{R} + A_1 + A_1 A_2}{A_1 A_2 + 2 A_1 + \frac{A_1 Z_L}{R}} \tag{6.22}$$

As A_1 and A_2 tend to infinity, dividing the numerator and denominator of equation 6.22 by $A_1 A_2$, gives

$$A = \frac{0 + 0 + 0 + 0 + 1}{1 + 0 + 0} = 1 \tag{6.23}$$

From equation 6.9, by definition,

$$B = \left. \frac{V_1}{-I_2} \right|_{V_2 = 0} \tag{6.24}$$

With $V_2 = 0$, (6.14) reduces to $I_2 \, R = A_1 \, V_3$ (6.25)

and the following additional voltage equation is obtained from Figure 6-2 (b):

$$A_2 \, V_4 + (R+Z_L) \, I_L + V_4 = 0 \tag{6.26}$$

from (6.13) and (6.26)

$$I_L = \frac{A_2 + 1}{A_2 + 2 + \frac{Z_L}{R}} \quad I_2 \tag{6.27}$$

from (6.12) and (6.18)

$$V_1 = (R+Z_L) \, I_L - (1+A_1) \quad V_3 \tag{6.28}$$

(6.25) and (6.27) into (6.28) yields

$$V_1 = \frac{(R+Z_L) \, (A_2 + 1)}{A_2 + 2 + \frac{Z_L}{R}} \, I_2 - \frac{(1+A_1) \, R}{A_1} \, I_2$$

which reduces to

$$B = \frac{V_1}{-I_2} = \frac{R \, (A_1 + A_2 + 2 + \frac{Z_L}{R}) - A_1 \, A_2 \, Z_L}{(A_2 + 2 + \frac{Z_L}{R}) \, A_1} \tag{6.29}$$

Here again, as A_1 and A_2 tend to infinity,

$$B = \frac{0 - Z_L}{1 + 0 + 0} = - Z_L \tag{6.30}$$

To find the element C of the chain matrix in equation 6.9,

$$C = \left. \frac{I_1}{V_2} \right|_{I_2 = 0} \tag{6.31}$$

from (6.11) and (6.20)

$$-I_1 R = \frac{A_2 + 2 + \frac{Z_L}{R} + A_1}{A_2 + 2 + \frac{Z_L}{R}} V_3 \tag{6.32}$$

from (6.17) and (6.32) we get

$$C = \frac{I_1}{V_1} = \frac{A_1 + A_2 + 2 + \frac{Z_L}{R}}{RA_1 (A_2 + 2 + \frac{Z_L}{R})} \tag{6.33}$$

As A_1 and A_2 tend to infinity,

$$C = \frac{0 + 0 + 0 + 0}{R + 0 + 0} = 0 \tag{6.34}$$

Finally, to find the element D

$$D = \frac{I_1}{-I_2} \quad V_2 = 0 \tag{6.35}$$

substituting equations (6.25) and (6.27) into (6.11) yields

$$\frac{R}{A_1} I_2 + I_1 R + \frac{R (A_2 + 1)}{A_2 + 2 + \frac{Z_L}{R}} I_2 = 0$$

from which

$$D = \frac{I_1}{-I_2} = \frac{A_1 A_2 + A_1 + A_2 + 2 + \frac{Z_L}{R}}{A_1 A_2 + 2A_1 + \frac{A_1 Z_L}{R}} \tag{6.36}$$

As A_1 and A_2 go to infinity,

$$D = \frac{1 + 0 + 0 + 0 + 0}{1 + 0 + 0} = 1 \tag{6.37}$$

Thus, as A_1 and A_2 tend to infinity, the chain matrix is given by

$$\begin{bmatrix} A_T \end{bmatrix} = \begin{bmatrix} 1 & -Z_L \\ 0 & 1 \end{bmatrix} \qquad (6.38)$$

which represents a floating impedance equal to $-Z_L$. It follows that the circuit of Figure 6-2 (a), with Z_L removed, corresponds to an FNIC having a unity conversion factor with terminals 1 and 2 as the input port and terminals 3 and 4 as the output port.[1]

6.2 Stability Properties

It was shown in section 5.4 that the grounded NIC is stable when one port is short circuited (SCS) or when the other port is open circuited (OCS). Antoniou[1] proved that the FNIC of Figure 6-2 (a) is subject to the same stability properties. For the sake of completeness, the highlights of his proof will be given in this section.

Terminating the FNIC with R_{L1}, R_{L2}, and R_{L3}, as shown in Figure 6-3, the driving-point impedance at the plier entry is given by

$$Z(s) = \frac{N(s)}{D(s)} = \frac{K_1 + K_2\,(A_1+A_2) + K_3\,A_1\,A_2}{K_4 + K_5\,(A_1+A_2) + K_6\,A_1\,A_2} \qquad (6.39)$$

where

$$K_1 = (2R + R_{L3})\,(R + R_{L1})\,(R + R_{L2}) \qquad (6.40)$$

$$K_2 = R\,(R + R_{L1})\,(R + R_{L2}) \qquad (6.41)$$

$$K_3 = (R_{L1} + R_{L2} - R_{L3})\;R^2 \qquad (6.42)$$

$$K_4 = (2R + R_{L3})\,(R + R_{L2}) \qquad (6.43)$$

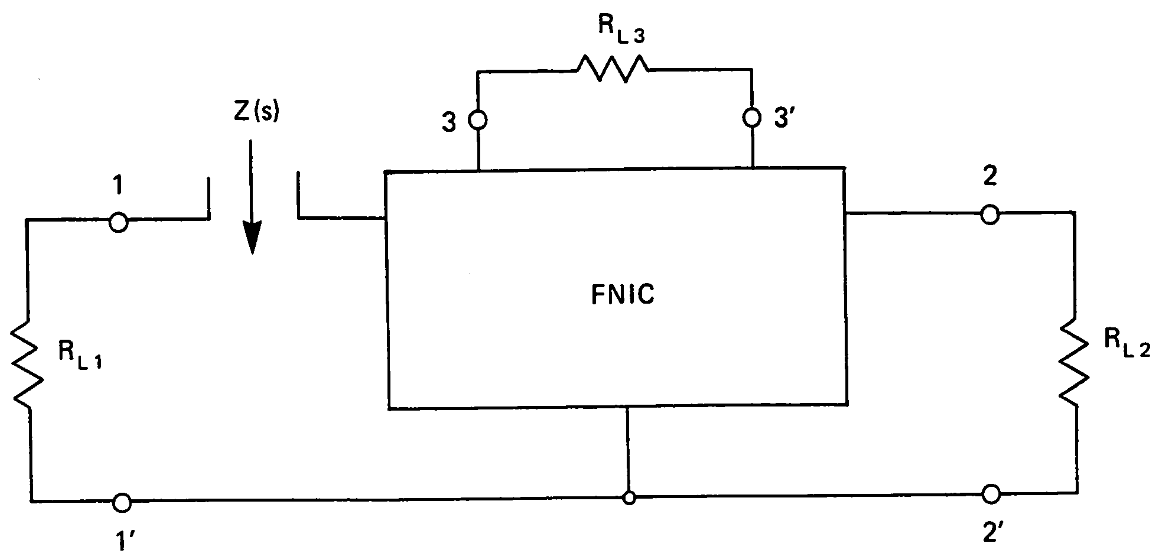


Figure 6-3. Plier entry to a terminated FNIC circuit.

$$K_5 = R\,(R + R_{L2}) \tag{6.44}$$

$$K_6 = R^2 \tag{6.45}$$

Using equation 5.23, the open loop gains of the two amplifiers, expressed as a function of the complex frequency variable "s", are given by

$$A_1 = \frac{w_1\, G_1}{s + w_1} \qquad \text{and} \qquad A_2 = \frac{w_2\, G_2}{s + w_2} \tag{6.46}$$

Substitution of these expressions in equation 6.39 yields

$$Z(s) = \frac{N(s)}{D(s)} = \frac{as^2 + bs + c}{ds^2 + es + f} \tag{6.47}$$

where

$$a = K_1 \tag{6.48}$$

$$b = K_1\,(w_1 + w_2) + K_2\,(G_1\, w_1 + G_2\, w_2) \tag{6.49}$$

$$c = K_1 + K_2\,(G_1 + G_2) + K_3\, G_1\, G_2 \quad w_1\, w_2 \tag{6.50}$$

The natural frequencies of the driving-point impedance Z(s) are the roots of the characteristic polynomial N(s) which, in this case, is of the second degree in "s". Thus, a sufficient condition for stability is that all the coefficients a, b and c be positive.

From equations (6.40) and (6.48), "a" is always positive. Also, from equations (6.40), (6.41), and (6.46), "b" is positive. However, from equations (6.42) and (6.50), "c" can attain negative values. Further analysis shows that the condition to make "c" positive reduces to

$$R_{L1} + R_{L2} > R_{L3} \tag{6.51}$$

It follows from condition (6.51) that the FNIC circuit of Figure 6-2 (a) is stable either when R_{L1} and/or R_{L2} tend to infinity, or when R_{L3} equals zero. The former indicates that the input port is open-circuit stable (OCS), while the latter proves that the output port is short-circuit stable (SCS).

6.3 Laboratory Tests on an Actual Circuit

With the introduction of the FNIC circuit in section 6.1, the constant resistance bilateral amplifier of Figure 6-1 can now be physically realized. The series-connected negative resistance (−R) of the bridged-T configuration can be realized using the proposed FNIC circuit of Figure 6-2 (a), while the shunt element ($-R_0{}^2/R$) can be generated using the grounded NIC circuit of Figure 5-7.

One such circuit, designed to match a 600 ohm line, is shown in Figure 6-4. The reason for choosing all the NIC resistors to be equal to 600 ohms, for both shunt and series converters, was only for convenience. The main restriction in the selection of these resistors is that R_1 be made equal to R_2 in the shunt converter while R_3 to R_6 should be equal for the series converter. This is essential so that both converters would have the required unity conversion factor.

The internally compensated μ A741C operational amplifier was used throughout the circuit of Figure 6-4. There was no special reason for selecting this particular type of amplifier except that it was readily available, has a high gain and does not require external components for frequency compensation.

In Figure 6-4, the FNIC consists of A_1, A_2, R_3, R_4, R_5, and R_6 with input terminals 1 and 2 and output terminals 3 and 3′. The input impedance of the FNIC is equal to the negative value of the output resistance, R_S. Thus, R_S can be selected to have any value provided condition (6.7) is met. In this particular circuit, R_0 = 600 ohms; hence, the selection of R_S is limited to the following:

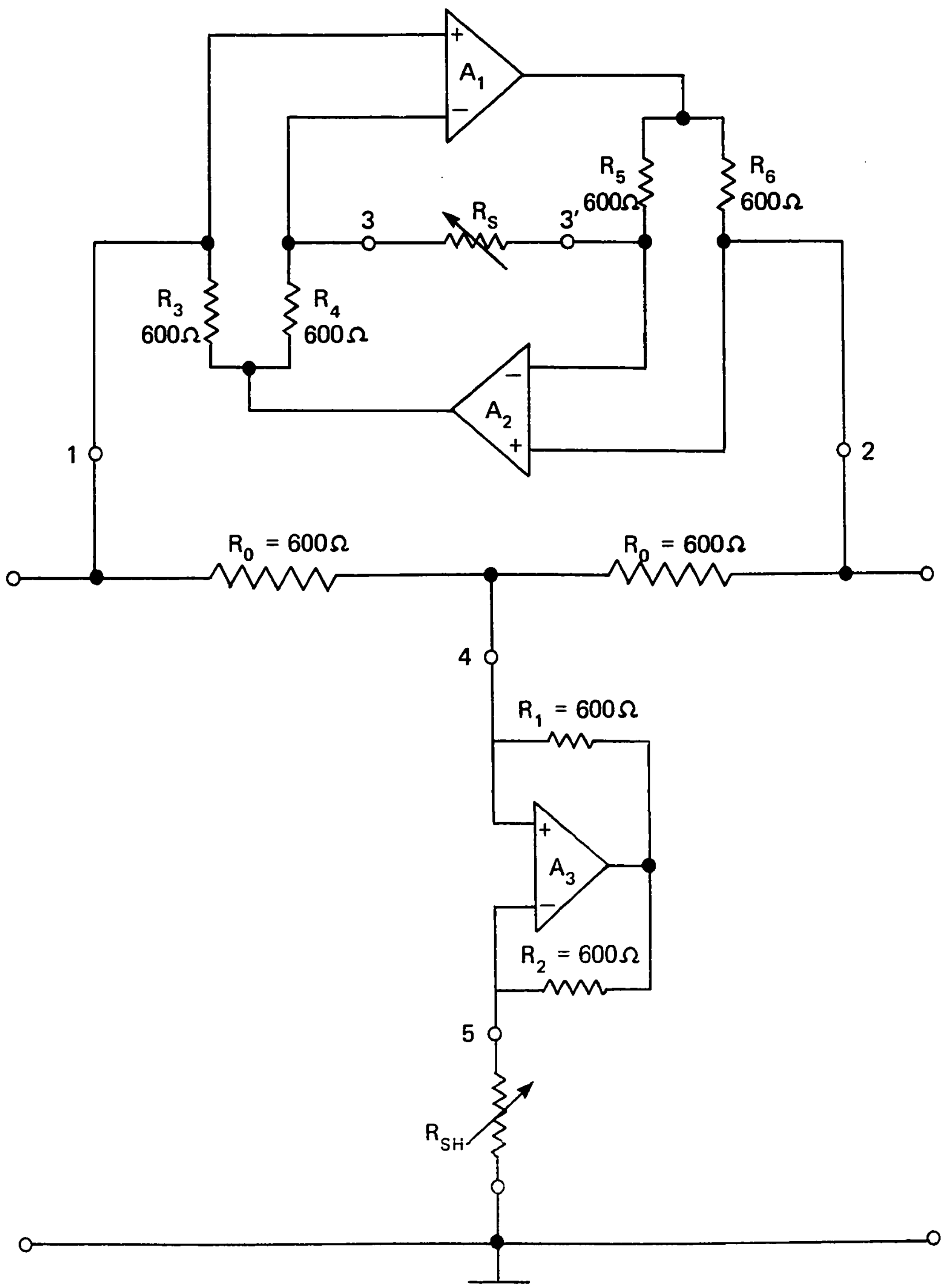


Figure 6-4. A 600-ohm constant-resistance bilateral amplifier.

$$300\ \Omega < R_S < 600\ \Omega \qquad (6.52)$$

In fact, if R_S is chosen to be greater than 600 ohms, in equation 6.6, the insertion gain becomes negative signifying that the amplifier behaves as an attenuator. This was tried in the laboratory by interchanging R_S and R_{SH}. Originally R_S = 470 ohms and R_{SH} = 750 ohms produced a gain of 13.75 dB. When, these resistors were interchanged, the unit introduced an 8 dB loss when inserted into the same line.

Another restriction on the resistance R_S of the series converter is dictated by the stability condition (6.51) of the FNIC. In this case, since $R_{L1} = R_{L2}$= 600 ohms and $R_{L3} = R_S$, the condition for stability requires that R_S be less than 1200 ohms. Thus, unless the line is short-circuited, or the line impedance drops below twice the magnitude of R_S, the FNIC will remain stable. Since the condition for amplification, (6.52), also satisfies the stability condition, $R_S < 1200$ ohms, the former will be the dominant restriction for the resistance R_S in the FNIC of Figure 6-4.

The grounded NIC in Figure 6-4 consists of R_1, R_2 and A_3 with the input between terminals 4 and ground and the output between terminals 5 and ground. The impedance at the input of the NIC is equal to the negative of the resistor R_{SH} connected at the output port of the NIC. From Figure 6-1, for the image impedance of the bilateral amplifier to match the characteristic impedance R_0 of the line, the restriction on the resistance R_{SH} is given by

$$R_{SH} = \frac{R_0^{\,2}}{R_S} \qquad (6.53)$$

Thus, the selection of the value of R_{SH} is dependent upon satisfying equation 6.53.

The stability condition for the grounded NIC will be obtained by applying the same plier entry method used for deriving the stability properties of the FNIC in section 6-2. Terminating the input and output ports of Figure 5-7 (b) by R_{L1} and R_{L2} respectively, the loop equations are given by

$$-V(s) + (R_{L1} + R_1)\, I_1 - A V = 0 \qquad (6.54)$$

$$AV - (R_2 + R_{L2})\, I_2 = 0 \qquad (6.55)$$

$$-V + R_1 I_1 - R_2 I_2 = 0 \qquad (6.56)$$

From these equations, straight forward analysis yields

$$Z(s) = \frac{V(s)}{I_1} = \frac{K_1 + K_2 A}{K_3 + K_4 A} \qquad (6.57)$$

where

$$K_1 = (R_1 + R_{L1})(R_2 + R_{L2}) \qquad (6.58)$$

$$K_2 = R_2 R_{L1} - R_1 R_{L2} \qquad (6.59)$$

$$K_3 = R_2 + R_{L2} \qquad (6.60)$$

$$K_4 = R_2 \qquad (6.61)$$

Using equation 5.23 to express the open loop gain A of the amplifier as a function of s in equation 6.57, we get

$$Z(s) = \frac{N(s)}{D(s)} = \frac{as + b}{cs + d} \qquad (6.62)$$

where

$$a = K_1 \qquad (6.63)$$

$$b = K_1 w_C + K_2 A w_C = w_C A (\frac{K_1}{A} + K_2) \qquad (6.64)$$

$$c = K_3 \qquad (6.65)$$

$$d = K_3 w_C + K_4 w_C \qquad (6.66)$$

A sufficient condition for stability is that the roots of the characteristic polynomial N(s) lie in the LH of the s-plane. Thus "a" and "b" should be positive. Equations 6.58 and 6.63 show that "a" is always positive. However, K_2 in equation 6.64 can attain negative values as indicated by (6.59). The condition for stability is seen from equation 6.64 to be $K_2 \geqslant 0$; since K_1/A is always positive. Thus from equation (6.59) the stability condition becomes

$$R_2 R_{L1} - R_1 R_{L2} \geqslant 0 \quad \text{or} \quad R_{L2} \leqslant \frac{R_2}{R_1} R_{L1} \qquad (6.67)$$

For the grounded NIC circuit of Figure 6-4, $R_2 = R_1 = 600$ ohms, $R_{L1} = R_0{}^2/R_S$ and $R_{L2} = R_{SH}$. This leads to the condition:

$$R_{SH} \leqslant \frac{R_0{}^2}{R_S} \qquad (6.68)$$

Comparing this with equation 6.53, we see that the only value that resistance R_{SH} can have is $R_0{}^2/R_S$ if the amplifier is to be stable and, at the same time, be able to match the characteristic impedance of the line. Thus, since R_S can only have values between 300 and 600 ohms, R_{SH} is limited by:

$$600\,\Omega < R_{SH} < 1200\,\Omega \qquad (6.69)$$

To evaluate the proposed bilateral amplifier experimentally, the circuits shown in Figures 6-4 and 6-5 were designed and put together by the author. Figure 6-5 is a convenient method for measuring insertion gain (or loss). With the switch in the lower position, the desired oscillator reference level is adjusted and

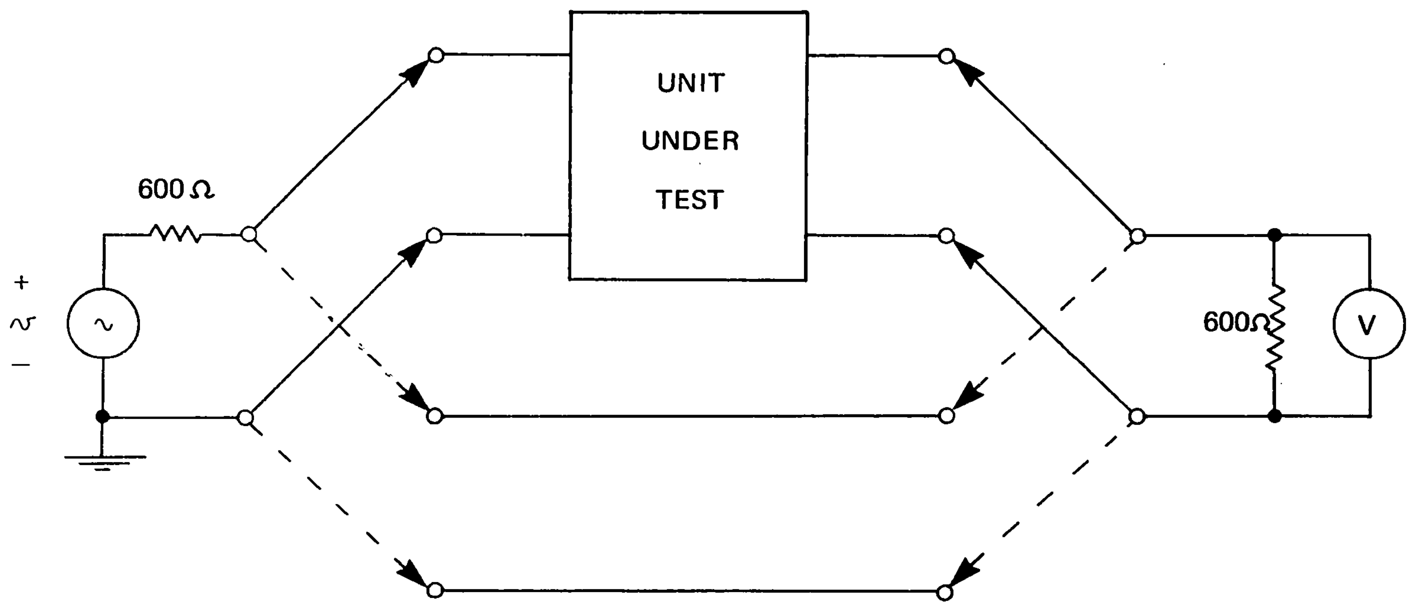


Figure 6-5. Test set-up for measuring insertion gain (or loss).

measured in dB on the vacum tube voltmeter (VTVM). When the switch is thrown to the upper position, the unit under test (UUT) is inserted between the oscillator and the VTVM as shown. Any gain or loss introduced by the UUT will be read directly on the VTVM in dB. The difference between this reading and the reference level gives the insertion loss (or gain) of the UUT.

In the constant resistance bilateral amplifier (CRBA) circuit of Figure 6-4 the biasing arrangements have been omitted for simplicity. In the lab, a 3-terminal power supply was used having a +15.5 V, a −15.5 V and a ground for providing the necessary DC bias.

6.3.1 Gain Setting

The gain of the CRBA is set in the following manner. If a gain of 9 dB is required, equation 6.6 can be written as

$$9 \text{ dB} = 20 \log \frac{R_0}{R_0 - R_S}$$

from which

$$\frac{1}{1 - \frac{R_S}{R_0}} = \log_{10}^{-1} \; 0.45 = 2.82$$

which leads to $R_S = 0.645\, R_0 = 387\Omega$

and $R_{SH} = \frac{R_0{}^2}{R_S} = \frac{(600)^2}{387} = 931\Omega$

Thus, theoretically, if we set R_S = 387 ohms and R_{SH} = 931 ohms, the gain of the CRBA will be 9 dB. In the lab, R_S was set at 390 ohms, R_{SH} was at 910 ohms and the gain was found to be 9.1 dB. The input reference signal was 1000 Hz at 0 dB.

6.3.2 Bilateral Property

The ability of the CRBA to amplify in both directions was verified by interchanging the oscillator and the VTVM in Figure 6-5. This test was made for a number of different gain settings as shown in Table 6.1. The gain in the east-west direction is denoted as "E-W Gain" while that in the west-east direction is denoted as "W-E Gain". The calculated gain in the last column is obtained from equation 6.6. For example, in the first test, R_S = 330 ohms and the gain is calculated to be

$$\text{Insertion gain} = 20 \log_{10} \frac{600}{600-330} = 6.94 \text{ dB}$$

From Table 6.1, we see that the E-W gain is identical to the W-E gain indicating that the CRBA has an excellent bilateral property.

6.3.3 Frequency Response

Setting the resistance R_S at 390 ohms, R_{SH} at 910 ohms and the reference signal level at 0 dB, the frequency response of the CRBA was obtained as shown in Table 6.2.

The gain is flat from 100 Hz up to 10 KHz after which it starts to drop until it reaches 0 dB at 50 KHz. Thus, for voice frequency telephone applications where the band of interest is 200 Hz to 3.5 KHz, the CRBA can readily be used. The reduction in gain of the CRBA at higher frequencies is mainly due to the bandwidth limitation of the compensated μA741C amplifiers. Should a wider bandwidth be required then a different type of operational amplifier could be selected. For application in the two-wire repeater, the μA741C bandwidth is more than adequate.

TABLE 6.1

BILATERAL PROPERTY OF AMPLIFIER

R_S Ω	R_{SH} Ω	E-W Gain dB	W-E Gain dB	Calculated Gain dB
330	1100	6.8	6.8	6.94
390	910	9.1	9.1	9.12
470	750	13.75	13.75	13.30
560	680	20.5	20.5	23.52

TABLE 6.2

FREQUENCY RESPONSE OF AMPLIFIER

Frequency Hz	Gain Hz	Deviation from 1KHz Gain Hz
100	9.1	0
1K	9.1	–
10K	9.05	–0.05
15K	8.5	–0.6
18K	7.8	–1.3
20K	7.1	–2.0
25K	5.5	–3.6
30K	4.1	–5.0
35K	2.8	–6.3
40K	1.8	–7.3
45K	0.9	–8.2
50K	0	–9.1

6.3.4 Gain Linearity

An oscilloscope was connected across the VTVM of Figure 6-5 to detect the minimum input level at which clipping of the output signal from the CRBA would occur. With R_S = 330 ohms, R_{SH} = 1100 ohms and the input signal set at 1000 Hz, the oscillator level was increased and the output deviation from the 0 dB input reading on the VTVM was recorded. This is shown in Table 6.3 from which we see that clipping starts when the input signal is between 7 and 8 dB. On the oscilloscope, the clipping started to occur when the input signal was 7.2 dB.

To improve the linearity range of the CRBA, one method is to increase the supply voltage of the operational amplifiers. The μ A741C is rated at ± 18 volts. Thus, by increasing the supply voltage to ± 18 volts from ± 15.5 volts, the CRBA can amplify input signals larger than 7.2 dB without distortion.

6.4 Two-wire Telephone Repeater Considerations

One way to see whether the CRBA can find potential use in the two-wire telephone repeater is to compare it with the gain unit of the E6 repeater discussed in section 4.4.

The first thing that strikes us as a major obstacle is that if Figure 6-4 were inserted in the transmission line, it will introduce a ground on the "ring" conductor. This is not allowed because the "tip and ring" of a cable pair have to pass DC supervisory currents from one end to the other in order to indicate the state of the far end subscriber set, be it "off-hook" or "on-hook". If there is a ground in the middle, it will drain the supervisory currents and the switching machine in the central office (CO) will lose vital information. Thus, some means is required to

TABLE 6.3

GAIN LINEARITY OF AMPLIFIER

Input Signal dB	Gain dB	Deviation from 0 dB signal dB
0	6.8	0
5	6.8	0
7	6.8	0
8	6.65	–0.15
9	6.4	–0.40
10	6.2	–0.60

$R_S = 330\ \Omega$

$R_{SH} = 1100\ \Omega$

isolate the ground from the cable pair. One way of doing this is to add transformers to the shunt circuit in a similar manner as the E6 shunt converter shown in Figure 4-11. The secondary of transformer T2 can be connected to the input of the grounded NIC of Figure 6-4 while the secondary of T3 with C4 can be connected across terminals 5 and ground of the NIC.

At first glance, one would assume that since the series converter is an FNIC, line transformer T1 is not needed when the CRBA is to replace the E6 gain unit. However, without T1, undesirable DC supervisory voltages will be generated at the FNIC while dial pulses will be distorted as they pass through the CRBA. Hence T1 is still necessary to remove the DC voltages and to permit dial pulsing to pass without serious impairment.

Thus far, it would seem that the CRBA could be used as a two-wire telephone repeater provided all the bulky and expensive passive components presently used in the E6 are added to the CRBA. In fact, the CRBA has a distinct disadvantage over the E6 gain unit. A DC to DC converter is required to convert the 48 volt CO supply to the required ±18 volt for biasing the μ A741C operational amplifiers.

Finally, another important factor to consider before trying to replace the E6 gain unit by the CRBA is that the operational amplifiers should be protected against the effects of foreign potentials due to lightning surges. Such surges may occur from either tip and ring wires to ground or between tip and ring wires. Thus, any new repeater should be able to tolerate voltages up to 600 volts.

CHAPTER VII

CONCLUSION

A practical constant resistance bilateral amplifier (CRBA) circuit has been described using an FNIC and a grounded Type I-CNIC to simulate the negative series and shunt elements of the bridged-T network configuration. An experimental investigation has shown that the amplifier performs in accordance with its theoretical predictions.

An attempt was made to find potential application of the CRBA in the two-wire voice-frequency telephone repeater. This was done by comparing the CRBA circuit with the gain unit of the E6 transistorized repeater. The result has shown that in order to meet the systems requirements of two-wire transmission lines, most of the expensive passive components in the E6 gain unit will have to be incorporated into the CRBA operational amplifier circuit.

To summarize, even though the modified CRBA can be used in two-wire telephone repeater applications, there is no economic justification in replacing the existing E6 repeater. In fact, the CRBA requires an additional 18 volt power converter for biasing purposes.

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