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STATE PENSION SCHEMES, GROWTH AND DISTRIBUTION: A STUDY OF MODEL SPECIFICATION

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ABSTRACT

STATE PENSION SCHEMES, GROWTH AND DISTRIBUTION:
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This thesis is concerned with the influence of the theoretical framework on the study of actual economic issues. The primary objective is to see which, if any, of three alternative theoretical models best summarizes the actual working of a capitalist economy. The secondary objective is to draw some conclusions about the economic influence of the phenomenon which is studied, in this case a state old-age pension scheme.

Three alternative long-run, steady-state models of a state old-age pension scheme in a capitalist economy are built incorporating neo-classical, neo-Keynesian and neo-Marxian assumptions respectively. In each case, the implications for growth and for distribution in the long run are studied.

The models are tested in terms of their predictions on the relationship between a state old-age pension scheme and the saving ratio using an international, cross-section sample of western, industrialized countries. Saving functions incorporating neoclassical, neo-Keynesian and neo-Marxian assumptions are estimated and the results evaluated individually on both statistical and economic criteria. The econometric models are further tested against each other using the Davidson-MacKinnon "J test" (joint test) for model selection.

On the basis of the tests performed, the negative relationship between the size of a state pension scheme and the saving ratio predicted by neoclassical theory is rejected by the evidence, though it is not possible to say whether this relationship is negative or simply non-existent. It is concluded that the neoclassical model must be rejected on economic grounds but that the test performed is not powerful enough to discriminate between the neo-Keynesian and neo-Marxian models. Some patterns do emerge from the study, however, and their implications for further research are discussed.

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PREFACE

This thesis was motivated by a dissatisfaction with the way in which the theoretical framework is chosen for studies of applied economics. I have sensed (and I am no doubt not alone in this) that the sensitivity of the results of applied economic studies to the theoretical framework in which they are set means that a reader can never ~~be~~ as confident of the significance of the results as the writer appears to be in presenting them. While this contingency is no doubt understood intellectually by most economists, it is often lost in the interpretation. More serious is the fact that the non-professional reader--journalists, politicians, civil servants and even researchers in other disciplines--will not normally have the background in economics to evaluate the conclusions.

The problem is not so much that the reader is not aware that conclusions follow from assumptions but rather that he does not know what alternatives there are and what differences their use would imply. In this thesis I try to fill that gap, first by showing what the use of three important alternative models of a capitalist economy implies qualitatively for the study of an actual economic phenomenon (in this case, a state old-age pension scheme), and secondly by making an empirical test of the models so as to provide some evidence as to which model is most appropriate for use as a framework. While strong conclusions to the second objective are dependent (as always) on the data used and the inevitable assumptions made, it will be sufficient if this thesis can shed

CHAPTER I

INTRODUCTION

Background

It is said that with age comes disillusion; but we do not so much loose an illusion as change it. By "illusion" is meant that internal view of the world which we, as individuals and as economists, have about what we cannot experience directly. It is "illusion" only in the sense that we cannot confirm the vision in it's entirety--we do not have the time nor the ability. But that does not make such a view any the less necessary: we cannot act in the world without some summary view, or <u>model</u>, of it because we need a way of organizing and making sense of experience. Yet, while we need this world-view to interpret facts, it must ultimately adapt to them.

In what follows we use the terms "model", "theory", (and to a lesser extent) "hypothesis", and "paradigm" interchangeably to refer to that unifying intellectual construct which serves as a framework for the study of positive (as opposed to normative) phenomena in economics. In this thesis, we are concerned with <u>alternative</u> theories or models of the same real world phenomena. Thus, in economics, one theory incorporates certain definitions of basic agents, assumptions about their behaviour, as well as descriptions of the basic structure of the entire system. An alternative theory identifies different agents, looks for different forms of basic behaviour and puts the pieces together differently. (Although the distinction will not always be observed, in what follows we

some light on the broader issue by way of clarification.

I want to thank the Faculty of Commerce and Administration, Concordia University, for the support I have been given, both moral and financial throughout the writing of this thesis. I am particularly indebted to Ms. Mary Mullins, Ms. Susan Altimas and Ms. Irene Mazis as well as the other members of the Commerce Academic Support Staff for their patience and cooperation in typing the numerous drafts which preceded the final thesis. I wish also, to thank my supervisor, Professor Stephen A. Marglin and the other members of my committee, Professors A. Anastasopoulos and J. Breslaw for their guidance, their support at critical junctures, and their forebearance with my more than usual number of delays and missed deadlines. I am particularly grateful to Mr. George Tsoublekas for his help with computations and the time he devoted to discussions with me about the relationship of economic models and tests. Finally, and most important, I want to thank my wife, Margaretha Thorndahl. It was out of our many discussions of these and related issues to which she brought her knowledge of social anthropology and animal behaviour that I came to clarify my own ideas. More important was her support: it is to her and to our daughter Liv that I dedicate this thesis.

will use the term "theory" to refer to a unifying framework at a relatively abstract and general level and the term "model" when we wish to emphasize the particular and concrete form of theory. A "model", then, will often find its expression mathematically.)

In the realm of intellectual history the total eclipse of a unifying theory is a rare phenomenon. A framework theory becomes established only after it has demonstrated the ability to explain a wide variety of apparently unconnected activities. If the theory is fundamentally unsound, this will become evident only very slowly, as a body of contradictory evidence accumulates. Though the skirmishing is often over points of apparent logical contradiction, the real failure of a unifying theory is finally its inability to explain reality. A bad theory is bad because it does not deal with the world as it is.

Every theory idealizes the world to some extent. This is a necessary part of the abstraction and simplification that allows us to generalize. Since intellectuals are often much better at dealing with ideas than they are at dealing with facts, their fascination with the constructs often means that these ideal structures tend to take on lives of their own and develop independently of our knowledge of the real world. While this explains how theories have the permanence that they do, it also shows why they become overly rigid and lose touch with reality.

Systematic re-evaluation of its intellectual foundations against reality is, in principle, a central tenet of every scientific discipline, but in practice the process is somewhat different. Phenomena which are not easily accounted for by the received theory lead usually to patching and adaptation rather than to radical revision. It takes more than a few isolated inconsistencies to bring an entire unifying theory into question, such is the resilience of the received theory.

To most researchers an isolated example of inconsistency between theory and fact is either explained away as a failure of the method or leads to a minor revision of the theory; for a few, however, it will be the occasion for a fundamental re-evaluation of the entire intellectual structure of the discipline. This is why in every discipline there are always some dissidents who explicitly reject the orthodox hypothesis and advance alternatives of their own. For the most part such critical activity results in a strengthening of the received theory; occasionally it leads to a new paradigm.

In the study of economics today there is, in some quarters, dissatisfaction with the received theory, the neoclassical model of economic activity. Attacks on the neoclassical model are not new; Keynes's General Theory marked a very successful one almost fifty years ago. However, in the twenty-five years which followed its publication, the breach was, in the minds of most economists, successfully repaired. More recently, however, events have suggested that the synthesis was less successful than it first appeared. For some economists this has meant a return to the "classical" neoclassical (i.e., pre-Keynes) model,[1] a complete purge

[1]Terminology can be confusing. In this thesis the term "neoclassical" refers to a model of macroeconomic behaviour characterized by autonomous, utility maximizing economic agents and a full-employment equilibrium. This is what Keynes referred to as the "classical" model (1936, p. 3 ff.). In our terminology "classical" refers to models which find their roots in the work of Ricardo and concentrate on class and the distribution of power in their explanation of economic reality. In this typology, the "neo-Keynesion" and "neo-Marxian" models are both "classical".

of the Keynesian heresy. While personalities should be associated with intellectual movements with caution, it is hard to avoid the name of Milton Friedman in this connection. The true "neoclassical" model which his work epitomizes emphasizes an economic system composed of autonomous, rationally-calculating, utility-maximizing individuals, no one of whom (save perhaps the state) can significantly impose its will on the system. The long-run equilibrium towards which such an economy tends is characterized by full employment of all resources, in which possibilities meet preferences through the medium of the price system. This is the neoclassical description of a capitalist economy.

Such a return to the orthodox model has not satisfied all economists, however, and the challenge to the neoclassical model has been renewed, this time on more than one front. It is again the appropriateness of the theory in explaining real-life phenomena which motivates the challengers.

One set of critics speak with the mantle of Keynes. These neo-Keynesians[1] emphasize the revolutionary message of the General Theory and argue that neoclassical theory has drained economics of its power to deal with real-world problems. These critics have maintained that however logical neoclassical theory may be, it is not a valid representation of a capitalist economy. Kaldor (1966, p. 295), for example, speaks of "the intellectual sterility engendered by the methods of neoclassical economics". These writers argue that class and expectations, (especially expectations of capitalists) are more useful tools in economic analysis than notions of perfect markets and utility maximization.

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[1] Joan Robinson and Nicholas Kaldor are the best known representatives of this group. See the survey by Eichner and Kregel (1975).

These proponents of a neo-Keynesian theory are not alone in their criticism of the neoclassical orthodoxy. The neo-Marxian tradition, which finds its roots in the writings of Marx, has existed for more than one hundred years as an intellectual underground in the English-speaking world; yet its positive, as opposed to its ideological aspects, have only recently begun to be recognized. This model shares with the neo-Keynesian model an emphasis on class but unlike the latter sees the continuing struggle between capital and labour over the level of real wage, rather than the expectations of capitalists, as a fundamental construct of economic analysis.

These three theories and their differences are the raw material for this thesis. In the section which follows we will spell out our objectives.

Objectives of the Thesis

This thesis has two objectives. The first, and primary one, is to examine the question ~~of~~ which, if any, of the three models described above--the neoclassical, neo-Keynesian, and neo-Marxian--provides the best tool for analyzing actual economic issues. It is both a theoretical and empirical exercise in that the three models will be developed in a particular aspect and the predictions tested against real-world data. The testing will be done in a framework of statistical model selection in order that we can use the results to make inferences about the underlying models.

The second objective is to study the economic effect of state old-age pension schemes. This is not a separate project but rather a joint product of the first. The particular economic phenomenon that we will use for the study of model selection is the relationship between state pension schemes and saving. The results of these tests should allow us to draw conclusions about the economic effect of state pension schemes and saving, a matter which is of considerable importance for economic policy.

A caveat is in order. The method we use builds on the particular, in this case on the theoretical influence of state pension schemes on distribution, growth and saving. While our conclusions will be based on the results of empirical tests of these three theories, it would be inappropriate to suggest that our results, no matter how clear they may seem, can conclusively answer the larger question. This is not simply a question of modesty but rather an acknowledgement that there are, in any single work in economics, too many assumptions, too many gaps and too few hard facts to justify using the results to turn the intellectual world on its ear.

We make no claims to be able to settle a question as significant as this one. The problem of what is the appropriate framework theory in economics has a history as old as the subject itself, and while one view is ascendent now, it was not that way once and may not be again--all this is quite apart from anything that might be contributed here. This study will be a success if we can in some way clarify the issues and marshall some evidence in favour of one or another of these three (or even some yet-to-be-identified) alternative models of a capitalist economy. To expect more would show too little respect for the limitations of the discipline.

In the next section we provide a definition and institutional description of state old-age pension schemes. In the concluding section of this chapter we give an outline of the thesis.

State Pension Schemes

The state has always taken some responsibility for organizing the support of those unable to provide for themselves. In the last century and a half this has increasingly meant support for the aged. It is not hard to see why. During this period, medical advances, in particular stricter public health standards, led to a marked increase in life expectancy. At the same time as people began to look forward to a longer life span, changes in the labour market made it increasingly difficult to work for wages beyond the age of sixty-five. Moreover, with the fragmentation of the extended family, people could no longer depend on their children for support in old age. Making provision for one's own old age by saving has always been the mark of financial, if not moral, prudence, but in practice it is a standard that relatively few people have been able to attain. Widespread poverty among the aged has had a persistence that is a constant reminder of the unfulfilled promises of increasing productivity. As industrialization has proceeded it has become clear that maldistribution is a continuing problem, one that cannot be solved by simply increasing average output. Despite what the theorists have advocated, events have often led spontaneously to a political solution. In the case of poverty among the aged, the response has been the same in almost every industrialized country--a system of state pensions and social assistance designed to assure people a minimum level of financial support in old age.

The keystone to a system of state organized support for the aged has, in most countries, been some form of contributory pension which we will call a state old-age pension scheme or, in short, a state pension scheme. In one sense, state pension schemes are similar to private pension plans. A worker makes contributions to the scheme--usually matched by his employer--based on employment earnings.[1] Having contributed for some minimum qualifying period, a worker is entitled to a pension related in some specified manner to contributions made[2] for the period of retirement.[3] But there the similarity to private plans ends. Because these schemes are state sponsored[4] they have features which pri-

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[1]In most schemes contributions are proportional to earnings up to some maximum assessable level of wages. This maximum is often related to the median wage. In a few countries only the employer is assessed but in any case the basis for assessment is still the payroll. In only one country, the Netherlands, do employers make no contribution. (Pechman et al., 1968, Appendix C).

[2]The formula in most countries is often weighted to the years of highest earning (like many private formula pensions [see Asimakopulos and Weldon, 1970]) but insofar as people work most their adult lives there is a clear relationship between contribution and benefits.

[3]In some countries there is no retirement test; benefits are paid when a contributor reaches a certain age, retired or not. In many countries these shcemes have, like private pension plans, survivors' and disability coverage features; these will be disregarded in this study.

[4]It's important to distinguish clearly between state pension schemes and other forms of state support for the aged such as universal pensions and social assistance. Universal pensions are paid at a flat rate to all citizens or residents and are financed out of general revenues. Soical assistance payments are based on need and are financed out of general revenues. One program does not preclude the existence of another; Canada, for instance, has all three. See Pechman et al., (1968, Appendix C); for further details of state support for the old-age, see U.S. Social Security Administration, Social Security Programs Throughout the World (various dates).

vate insurers cannot offer. In most state pension schemes, benefits are indexed to the cost of living or to the level of average money wages. In some schemes benefits are linked to factors quite independent of contributions--to marital status for instance. Perhaps the most characteristic political feature of a state pension scheme is the pay-as-you-go method of finance. Unlike private pension plans which must hold net assets or other independent securities against the actuarial liability for future pensions (funding), state schemes can always rely on the taxing power of government to finance pension benefits.[1] Consequently, there is only a statutory link between contributions and benefits; in actuality a state pension scheme is a tax-transfer program in which current pension benefits are paid out of the proceeds of an earmarked tax. For this reason membership in the scheme must be compulsory, which means that contributions are in fact, if not in name, a tax. Looked at from a broader perspective the state is in fact administering a system of compulsory intergenerational transfers from the young active members of society to the old and retired.

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[1]In the long run, if population growth is steady and the average level of benefits is set so that it will just be covered by current contributions, this is no problem. This would also be true of private pension plans as well, except that a private plan cannot count on always gaining new members to finance pension benefits and so must hold a stock of capital directly or through debt instruments to assure that all pensions purchased at any date will be honoured. Of course population growth is not steady and so when the proportion of pensioners to contributors increases the state must make up the shortfall either by increasing the contribution ratio (this has been the practice in the U.S.) or by subsidizing the scheme out of general revenues (this would be in addition to any statutory contribution the state makes to the scheme). There could also be periods when contributions exceed payment. This is generally the case when a scheme is first instituted. The excess contribution may then be "invested" in state securities. For most governments borrowing from the state pension scheme is a substitute for state borrowing and it would be wrong to consider these securities in the state pension scheme's portfolio as funding since there is no reason to assume that state receipts generated in this manner are invested in social or other capital projects.

It is the duality of state pension schemes--the quasi-private saving characteristic on the one hand and the underlying feature of forced intergenerational transfers on the other--which makes them particularly interesting to economists. Emphasis on one or the other aspect has led to wide differences in opinion as to the ultimate economic effect of these schemes. It is the hybrid nature of these schemes which makes them ideally suited for the purpose of emphasizing the differences between alternative macroeconomic models.

Plan of the Thesis

The plan of the thesis is as follows. There are three major parts to the study. Part one is devoted to theory of state pension schemes. In chapter II we present an analytic framework consisting of the three alternative long-run, steady-state growth models. In chapters III through V these three models are the basis of alternative models of the economic effects of examining the effects on growth and a distribution state pension scheme. In part two, the theoretical models are tested empirically. First, in chapter VI, the empirical literature on state pension schemes is reviewed and the particular test statistic used, the aggregate saving ratio, is discussed. Next, in chapters VII, VIII, and IX, saving functions incorporating a state pension scheme in each of a neoclassical, neo-Keynesian, and neo-Marxian model are estimated and evaluated separately. Finally, in chapter X, a new statistical test of model selection, the Davidson-Mackinnon J-test, is introduced and used to make a comparative evaluation of the three models in a formal hypothesis-testing framework. Part three (chapter XI) provides a conclusion to the thesis.

PART ONE

THEORY

CHAPTER II

THE THEORETICAL FRAMEWORK

Introduction

In this chapter is presented a set of long-run growth models which provides the framework for the neoclassical, neo-Keynesian and neo-Marxian models of state pension schemes to be developed in chapters III, IV and V.

Marglin (forthcoming) has developed a set of three growth models which highlights the differences between the neoclassical, the neo-Keynesian, and the neo-Marxian conceptions of the operation of a capitalist economy. The models are simple and particularly well-suited to the objectives of this study. The simplicity of the models is directly apparent, as we shall see below, and reflects the fact that these are one-sector, steady-state models. Whether the simplicity of the models--which is gained by abstracting considerably from reality--is appropriate for this study is open to question. If we were interested solely in making theoretical comparisons between the models their realism would not be an issue; for such a purpose simple models are clearly preferable since they let the important differences between the models stand out sharply. Whether such simple models can be the basis for making judgements at the <u>empirical</u> level is another matter and deserves some discussion.

The first simplifying assumption of the Marglin models is that <u>one good</u> serves as both capital good and consumption good. The prototype of the Marglin model is a corn economy in which part of the harvest is held back from consumption to provide seed corn (investment) for the following year's harvest. Modelling a corn economy with one good is not a simplification but rather an accurate representation: one good--corn--is in fact consumption good and capital good. Here, however, we wish to use the model to draw conclusions about the operation of advanced industrialized economies which are characterized by a striking variety of commodities ranging from personal services provided by unaided labour to goods involving years of development both in conception and manufacture. In principle this variety can be modelled but the cost is considerable; such models are complex and the mathematical restrictions needed to ensure meaningful solutions are strong. For certain purposes such complexity might be desirable, but in this case it is not necessary because the questions which concern us can be answered with highly aggregated data--we will work with such broad economic aggregates as output and employment, saving and consumption, profits and wages. Since our primary interest is in the long-run characteristics of a growing economy, there is little loss in realism and considerable gain in simplicity by using a one-sector model.

The arguments just presented in defence of the use of a one-sector model are essentially the arguments advanced for the validity of macroeconomics. As Fisher (1969) has shown, the mathematical conditions for rigorous aggregation cannot normally be met in most economic applications but that does not mean that there is no place for aggregate economics; it simply means that we must keep the limitations of our assumptions in mind when we generalize or apply our results. The same applies to the assumption of steady-state growth. The question is whether "steady state" is a suitable assumption for the questions that interest us. On

The Basic Marglin Model

Let us now turn to the Marglin set of models. The three models have a common core of accounting and technical relations; we begin with the accounting relations.

In a discrete time period t, total production per worker is either consumed or invested. Letting X_t be gross output per worker in period t, C_t per worker consumption, and a_1 the capital-output coefficient--assumed for the moment to be fixed--we have

$$X_t = C_t + a_1 X_{t+1} \tag{2.1}$$

where X_{t+1} is total output of period t+1 divided by the number of workers in period t. In steady-state growth (which implies constant returns to scale) $X_{t+1}/X_t = (1+g)$ for any t where g is the constant growth rate of total output, total consumption and total capital. Substituting in (2.1) and dropping the time subscript in steady-state growth we can write

$$X = C + a_1 X (1+g) . \tag{2.2}$$

If we let a_o be the labour-output coefficient, we can normalize on labour by using the following identity,

$$X a_o \equiv 1 . \tag{2.3}$$

Multiplying (2.2) by a_o and using (2.3) gives

$$a_o X \equiv 1 = a_o C + a_1 a_o X(1+g)$$

or

$$1 = a_o C + a_1(1+g) . \tag{2.4}$$

This is the allocation equation.

The price of a unit of output in period t, p_t, is made up of the unit wage cost and unit gross profit

$$p_t = a_o W_t + a_1(1+r)p_{t-1} \tag{2.5}$$

where r is the profit rate and W the real wage. Note that depreciation the face of it is as equally unrealistic as the one-good assumption. For example, institutional rigidities in the short run do prevent real-world economics from maintaining a steady-state growth path and, in any case, major structural changes make quantitative comparisons over time meaningless in the very long run. However, in periods of the length we are interested in, measured in one or two decades, it is meaningful to speak of a long-run trend in the value of major economic aggregates and the stationariness of the growth rates.

The nature of state pension schemes confirms the appropriateness of the use of a steady-state model. These schemes operate on the assumption of a certain underlying stability in the long-run growth characteristics of an economy. Indeed, the self-financing feature of these schemes requires that the contribution rate and benefit calculation formula, which must be fixed in advance, be such as to generate a balance between receipts and disbursements within the planning horizon. This implies that relatively firm predictions are made about demographic and economic trends. Though revisions are made to the terms of the schemes in practice they are made only at long intervals and then only after it has become clear that there has been a permanent change in underlying trends;[1] this is not inconsistent with the assumption of steady-state economic growth <u>within</u> such periods.

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[1] Revision of terms is not always made just to bring the schemes back into balance with shifts in the underlying demographic and economic trends. See Diamond (1976) for a description of the way benefit liberalization chases contribution rate revision in a rising spiral in the United States Social Security System.

is assumed to be 100% per period; hence the number one in the expression $(1+r)$. This is a natural assumption for a corn economy or a circulating-capital economy. It holds true more generally if the period is defined to be long enough for capital to be entirely used up--an appropriate-enough assumption for fixed capital over the course of a decade. (This interpretation is consistent with our justification for a one-good model.)

If price is stable over time $p_t = p_{t-1} = p$ and we have.

$$p = a_o W + a_1 (1+r)\, p \;. \tag{2.6}$$

Set p, the unit of account, to unity (since there is only one good); hence

$$1 = a_o W + a_1 (1+r) \;. \tag{2.7}$$

This is the distribution equation or fundamental equation of value.

Equations (2.4) and (2.7) are the core accounting equations of the three models. As written, they are simply identities. If, however, a_o and a_1 are assumed to be fixed for technical reasons, they characterize growth and distribution when one in each of the pairs of variables (C,g), (W,r) is specified. This is most easily seen in figure 2.1, where graphs of (2.4) and (2.7), drawn respectively in g-C space and r-W space, are superimposed upon one another. Note that the economic restrictions that neither the share of consumption ($a_o C$) or wages ($a_o W$) be less than zero or greater than one means that both lines have the same length as well as the same position.

As the model stands the four variables (C, g, W, r) are only determined to the extent that there is one defined linear relation between the first pair and a formally identical relation between the second pair; if no additional variables are introduced, two more suitably determined relations are needed to close the model. As we shall see below this can

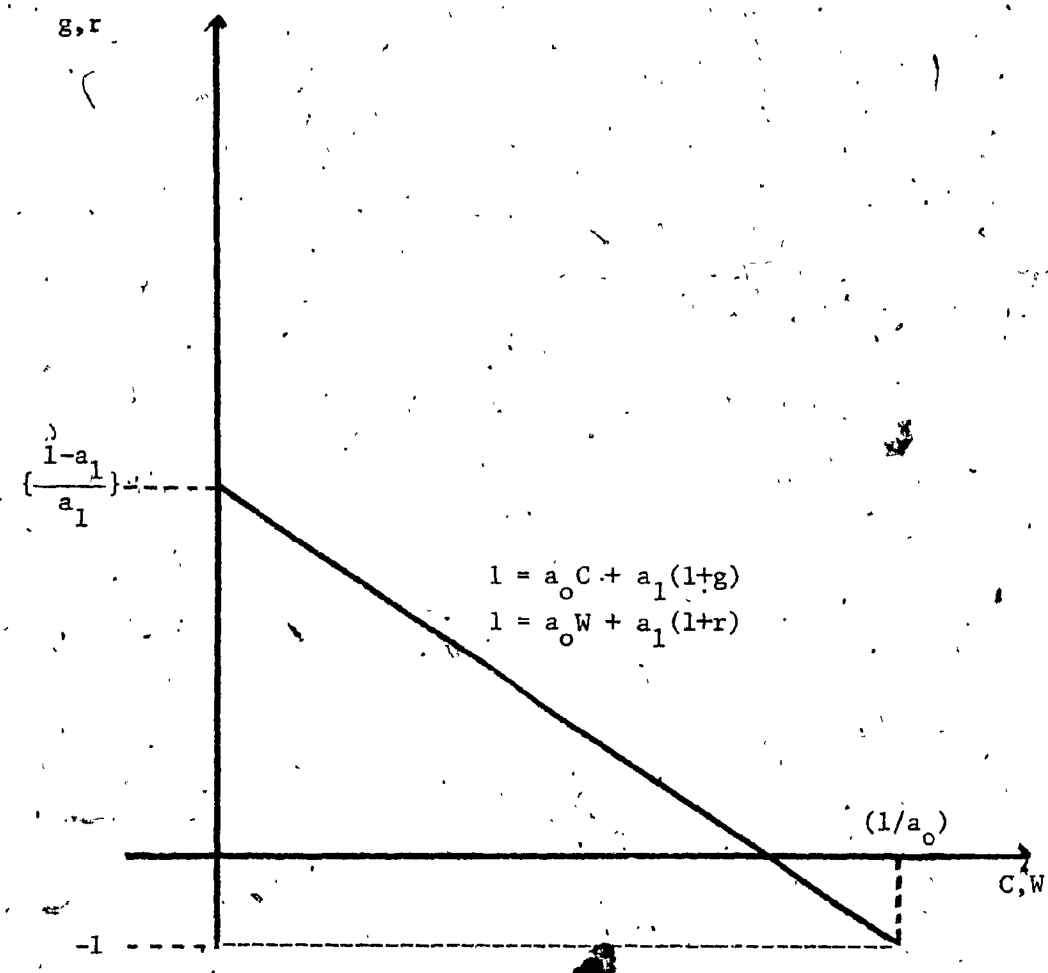

Figure 2.1. Allocation and distribution relations under fixed coefficients in steady-state growth.

be done in more than one way depending on the structural and behavioural relations we want to emphasize.

These two equations (2.4) and (2.7) are the complete common core of the three models if production coefficients are assumed to be fixed. There is, however, no reason to limit ourselves to fixed production coefficients. Since we emphasize the behavioural (i.e., social) differences between the three models, there is nothing particularly neoclassical about variable production coefficients in this framework and so a generalization to include variable coefficients is appropriate. To do this we need two additional equations. One is a production function: output per worker ($1/a_o$) is a function of capital per worker (a_1/a_o)

$$1/a_o = f(a_1/a_o) \quad . \qquad (2.8)$$

This intensive function is derived from a production function having the conventional properties of continuity, constant returns to scale and, assuming divisibility and additivity, positive but declining marginal products. The second assumption is perfect competition in factor markets, i.e.,

$$1+r = f'(a_1/a_o) \qquad (2.9)$$

or, equivalently,

$$w = f(\cdot) - (a_1/a_o)f'(\cdot) \quad . \qquad (2.10)$$

We can recapitulate at this point, bringing the equations together for convenience

$$1 = a_oC + a_1(1+g) \qquad (2.4)$$

$$1 = a_oW + a_1(1+r) \qquad (2.7)$$

$$1/a_o = f(a_1/a_o) \qquad (2.8)$$

$$1+r = f'(a_1/a_o) \quad . \qquad (2.9)$$

The first equation is a statement about how the whole of output is functionally allocated: $a_o C$ is the proportion of per-worker output consumed and $a_1(1+g)$ is the proportion invested (measured as gross investment). The second equation is a statement about the distribution of the value of per-worker output: $a_o W$ is the wage share and $a_1(1+r)$ the gross profit share. If a_o and a_1 are fixed, these two equations alone form the core of all the models. If production coefficients are variable, there are two additional variables but also two additional equations: (2.8) and (2.9). (In this case figure 2.1 is no longer generally true but can be interpreted as showing the solution values of a_o and a_1 [by the intercepts] given the equilibrium values of C, g, W, and r.)

In concluding this section it should be emphasized again that as the term is used here there is nothing peculiarly "neoclassical" about a smooth production function and marginal productivity factor pricing. These are properties shared, in principle, by all three models. This means that a number of theoretical criticisms, in particular those raised against the theory of marginal productivity factor pricing, the use of smoothly differentiable production functions, and the assumption of long-run equilibrium (steady-state) growth which are associated with the name of Professor Joan Robinson (1953-54, 1962, 1970) and other members of the Cambridge School do not form the basis for the neo-Keynesian model. The view taken in this study is that the characteristic assumptions of a long-run growth model in the Keynesian tradition have to do with the determinants of saving and investment and not with production. In particular it is the notions of fixed and identifiable saving propensities by class (Pasinetti, 1962) and the independent investment function (Robinson, 1962) which are peculiar to neo-Keynesian models. Although neo-Keynesian models are not normally presented with what are often called "neoclassical" production functions (apart from anything else variable production coefficients are not necessary to assure stability if variations in the distribution of income do the job) nothing of the essence of the models is lost by adding the assumption of smoothly differentiable production functions. Moreover, by adding that assumption we maintain the advantage of keeping as much common as possible among the models, and highlight the important differences. These same comments hold true for neo-Marxian models which are normally presented as having two sectors with fixed coefficients within each sector (see, e.g., Jones, 1975, chapter 5).

In the next three sections we will show how the core of the growth model, both fixed and variable-coefficients versions, can be closed in three different ways using alternative sets of assumptions about how a capitalist economy works. In the next section the model will be closed in a characteristically neoclassical fashion. In subsequent sections, neo-Keynesian and then neo-Marxian assumptions will be used to close the model.

The Neoclassical Growth Model

The two characteristic assumptions of a neoclassical growth model are (1) full employment and (2) intertemporal utility maximization on the part of all households. The second assumption is usually applied in such a way as to imply that all households have the same homogeneous utility functions. This allows differences of wealth to be ignored and a representative household to stand for all households. This is the method that will be used here but it should be remembered that such a strong assumption is not necessary; what is necessary is that there be no household which has both a) income solely from profits in its lifetime, and b) a saving propensity s_c such that $s_c > a_1 r s_w$ where $a_1 r$ is the share of profits in national income and s_w is the average saving propensity of all the other households. The implications of relaxing this assumption are explored in the section on the neo-Keynesian model.[1]

In order to develop the notion of intertemporal choice in the neo-classical model, we need first to expand the model to include a time dimension. Assume that there are always two generations living in any time period t, one working, the other retired (see figure 2.2). Looked at over time, the generations overlap. In period t, generation T-1 is spending the second half of its life retired while, concurrently, generation T is spending the first half of its life working. In period T+1 there is a similar overlap, only now it is generation T that is retired and generation T+1 that is working. This pattern continues ad infinitum. The only thing that does change from period to period is the size of each generation: each one is (1-n) as large as the one before it, where $\bar{n}$ is the exogenous growth rate of population.

(A realistic generalization would be to a three-overlapping-generations model corresponding to an individual lifetime three generations long. In the first two period of life (ages 20 to 40, and 41 to 60) people work, the difference between the two being that in the first period consumption includes the consumption of minor children who are not otherwise represented in the model. The third period (age 61 to 80) is one of retirement. In general, consumption requirements would differ

[1]There is a third, implicit assumption (emphatically not made in the neo-Keynesian model) that all saving is automatically translated into investment.

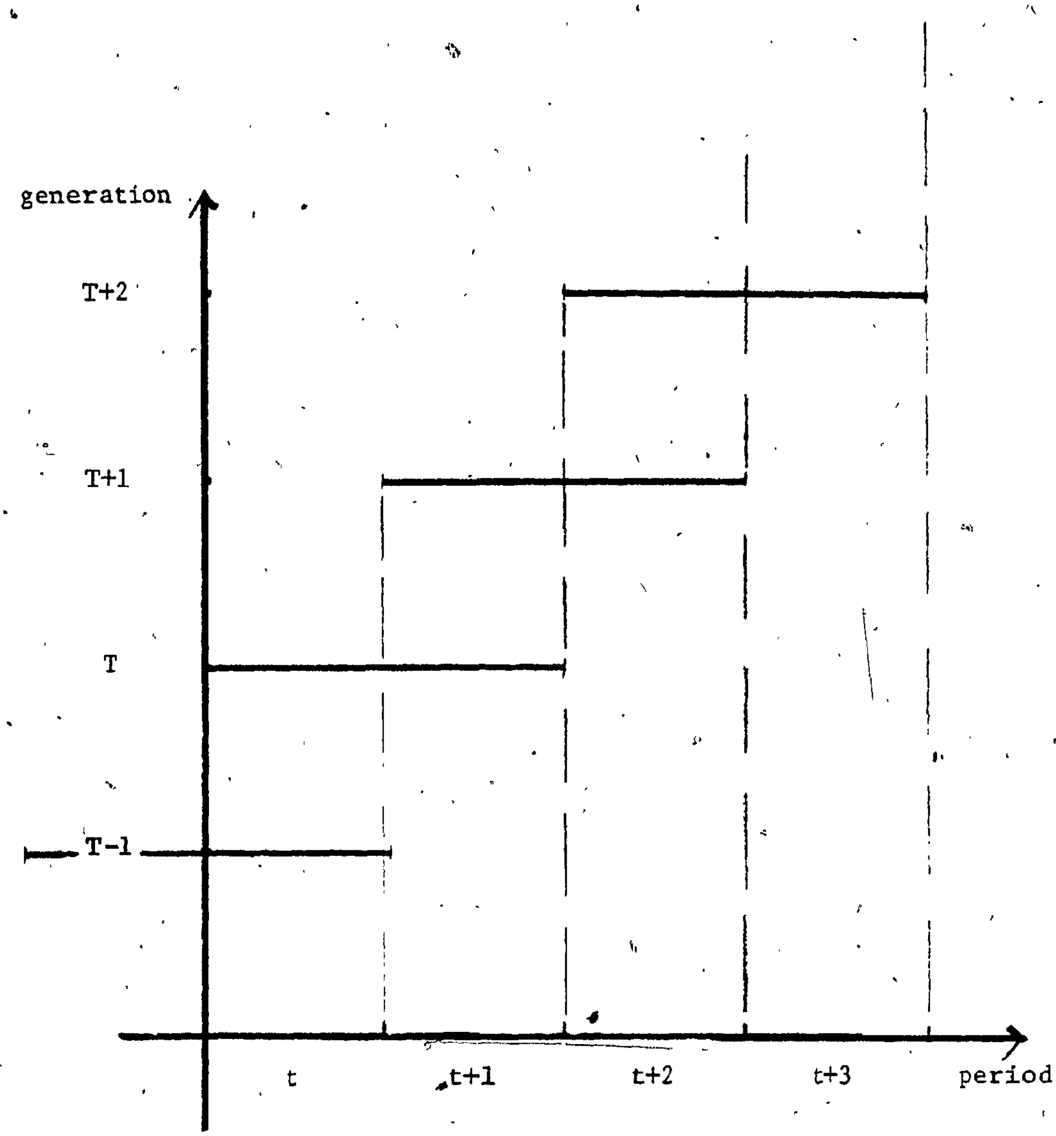

Figure 2.2. The structure of overlapping generations.

in the three periods. In the working years, the burden of child-rearing would make consumption requirement in period one greater than in period two, ceteris paribus, while in retirement the fact that life expectancy is less than the full twenty years of the period means that, on average the representative man would consume less than a full period's worth of consumption goods in comparison to period two. It would not be difficult to incorporate these systematic differences of consumption requirements into the parameters of the lifetime utility function. Such changes would complicate the model but apart from making quantitative estimates more easily interpretable there would be no essential changes in the results as compared to a two-generation model. Since only qualitative results are being sought in this part, the two-generation model will be used.)

In steady-state growth each generation behaves in an identical fashion to every preceding and to every subsequent generation--only the size of each generation differs. We can, consequently close the neo-classical model with reference to the economic behaviour of a representative, which we will call a household, just starting out its life in period t. Both the real wage, W, and the profit rate, r, (the rate of return on investment) are given. Assuming that work effort is fixed, the single economic decision that the household must make is to choose for itself a pattern of lifetime consumption. This is a standard two-period constrained utility-maximization problem.

Let C^1 be the representative household's first period consumption and C^2 be its second period consumption; $u(C^1, C^2)$ is a utility function defined over C^1 and C^2; W is the real wage--earned in the first period only--r is the profit rate on savings invested from period one to period two. The formal statement of the problem is

$$\text{maximize } u(C^1, C^2)$$
$$\text{subject to } C^1 + C^2/(1+r) = W \, . \tag{2.11}$$

From the first-order conditions, the solution takes the general form of the budget constraint and two demand equations

$$C^1 = C^1(W,r) \tag{2.12}$$
$$C^2 = C^2(W,r) \, . \tag{2.13}$$

It is conventional to place restrictions on the signs of the partial derivatives of the demand functions to assure normality of consumption in both periods. This requires that the own-price effects be non-positive i.e.,

$$\frac{\partial C^1}{\partial(1+r)} \leq 0$$
$$\frac{\partial C^2}{\partial(1/[1+r])} \leq 0 \, .$$

Intertemporal utility maximization adds two new variables, C^1 and C^2 and two equations, (2.12) and (2.13), to the basic model.

It will be convenient to define one additional variable before closing the model, but, before turning to that, an important distinction must be made. Henceforth in the neoclassical model the term "worker" refers to the household in the first period of its life; in the second period of its life the household (which we can easily think of as being a married couple) is a <u>retiree</u> or a <u>retired worker</u>. Because of this distinction there is a possibility that some of the variables may be misinterpreted. All of the intensive variables--C, C^1, C^2, W, $1/a_0$, a_1/a_0--are defined in <u>per-worker</u> terms; however, that is not necessarily the same as the <u>average of a worker</u>. Some are the same; for example, W, the per-worker wage remains, by definition, the average real wage. However, C, per-worker consumption, is not the same as C^1, the average con-

consumption of a worker; C is greater than C^1 by the amount of each worker's "share" of the consumption of retirees. Specifically, in steady-state growth[1]

$$C^1 + C^2(1+g) = C .$$

We have described the intertemporal utility maximization of the household in terms of two demand equations which, in turn, imply an equilibrium level of saving. However, from the point of view of a growth model, what is important for the continuity of the economy is investment. In the neoclassical model this creates no problem since saving is assumed to be automatically translated into investment. (In a neo-classical world there is no such thing as long-run insufficient demand.)

The real saving of a household in its working period is defined as follows

$$S^1 \equiv W - C^1 . \tag{2.14}$$

Multiplying S^1 by the labour-capital ratio, (a_o/a_1), gives gross saving of period t (i.e., capital of period t+1) in terms of the capital of period t which is of course, by definition, equal to one plus the growth rate, i.e.,

$$1+g = (a_o/a_1)S^1 . \tag{2.15}$$

Equation (2.15) will be called the saving-investment growth equilibrium equation, "saving" because it has been developed from the first-order conditions of intertemporal equilibrium of the representative worker

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[1]Note that this is not a definition; it is only true when households are in intertemporal equilibrium (it can be derived from the saving-investment growth equilibrium condition [2.15] which is developed below). The equation is presented here simply to help make clear the distinction between C, C^1 and C^2. See also Marglin (forthcoming, Chap. 2).

and "investment" because there being no independent investment demand function in the neoclassical model, all saving being automatically translated into investment. Growth, in other words, is the rate of saving per unit capital.

The neoclassical model is closed with the assumption of full-employment growth. Letting $\bar{n}$ be the exogenous growth rate of the labour force (equivalently, in the long run, population), we have

$$g = \bar{n} . \tag{2.16}$$

It will be useful to have functional forms for the demand functions; in this way the neoclassical system can be solved explicitly. If we let utility function $u(\cdot)$ take the logarithmic form[1]

$$u = \beta \log C^1 + (1-\beta) \log C^2 \qquad 0 < \beta < 1 \tag{2.17}$$

we have

$$C^1 = \beta W \tag{2.18}$$

$$C^2 = (1-\beta)(1+r)W \tag{2.19}$$

Note the special properties of this utility function:

1. the own-price effect of each period's consumption is non-positive,

$$\frac{\partial C^1}{\partial(1+r)} = 0, \qquad \frac{\partial C^2}{\partial(1/[1+r])} = -\frac{(1-\beta)}{(1+r)^2} W < 0$$

2. consumption is proportional to wealth measured in that period (See [2.18] and [2.19]

3. saving is proportional to the real wage, i.e.,

$$S^1 = W - \beta W = (1-\beta)W$$

Using the logarithmic utility function, the complete neoclassical

[1]The right-hand side is formally identical to the logarithmic form of the Cobb-Douglas production function but to avoid confusion the term "logarithmic" will be used in referring to the utility function (cf. Intriligator, 1971, p. 148). Because this is a utility function, no generality is lost by normalizing the coefficients.

model can be written as follows

$$1 = a_o C + a_1(1+g) \tag{2.4}$$

$$1 = a_o W + a_1(1+r) \tag{2.7}$$

$$1/a_o = f(a_1/a_o) \tag{2.8}$$

$$1+r = f'(a_1/a_o) \tag{2.9}$$

$$C^1 = \beta W \tag{2.18}$$

$$C^2 = (1-\beta)(1+r)W \tag{2.19}$$

$$S^1 = (1-\beta)W \tag{2.20}$$

$$1+g = (a_o/a_1)S^1 \tag{2.15}$$

$$g = \bar{n} \tag{2.16}$$

In the variable-coefficients model there are nine endogenous variables--C, g, W, n, a_o, a_1, C^1, C^2 and S^1--and one exogenous variable--$\bar{n}$. In the fixed-coefficients model equations (2.8) and (2.9) are suppressed and a_o and a_1 are specified exogenously.

We are now in a position to characterize the neoclassical equilibrium. We can begin with the fixed-coefficients model. With the appropriate substitutions it can be reduced to two equations, a full-employment growth equilibrium condition (F-E curve)

$$g = \bar{n} \tag{2.16}$$

and the saving-investment growth equilibrium condition in reduced form (S-I curve)[1]

$$1+g = (a_o/a_1)(1-\beta)W = (1/a_1)(1-\beta)[1-a_1(1+r)] \tag{2.21}$$

combining the two gives the equilibrium solution

$$\bar{n} = (1/a_1)(1-\beta)[1-a_1(1+r)] - 1$$

The nature of the equilibrium can be best presented graphically. In figure 2.3 the saving-investment growth equilibrium condition (the

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[1]Substitute (2.20) and then (2.4) into (2.15).

S-I curve) is a downward-sloping straight line in g-r space.[1] It intersects the vertical full-employment (F-E) curve just once (so long as $\bar{n} < [(1-\beta)/a_1]-1$) with positive values for both $(1+r)$ and $(1+g)$.

The economic meaning of an equilibrium defined at a <u>downward</u> sloping S-I curve is that the steady-state equilibrium is <u>unstable</u>. This is easily demonstrated. Consider the pressures in the economy if we are at point B on the S-I curve to the left of the long-run equilibrium solution at point A. Along the S-I curve households are, by definition, in intertemporal equilibrium; however, at point B the actual growth rate of capital and employment has fallen below the labour-force growth rate and, thus, there will be a growing pool of unemployed labour. This will tend to depress wages. However, in general equilibrium, as wages fall the equilibrium profit rate rises along the S-I curve.[2] As the profit rate rises along the S-I curve the actual growth rate falls further. In the limit, the growth rate will fall to -1 and the economy will collapse. A parallel argument holds for any point to the right of A on the S-I curve; here profits will tend to collapse (to -1), wages to rise to exhaust the value of output $(1/a_o)$ and the growth rate to rise to its maximum rate of $(1-\beta)/a_1-1$. (One uses the term "tend" because, unlike a

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[1]Note that along the S-I curve $dg/dr = -(1-\beta) < 0$. The downward-sloping and linear character of this S-I curve is a special consequence of assuming a logarithmic utility function <u>and</u> fixed production coefficients. If <u>either</u> of these assumptions is relaxed the S-I curve becomes curvilinear. See e.g., pp. 31-32 and pp. 34-38 below.

[2]From fundamental equation of value (2.7),

$$\frac{dr}{dW} = -(a_o/a_1) < 0 .$$

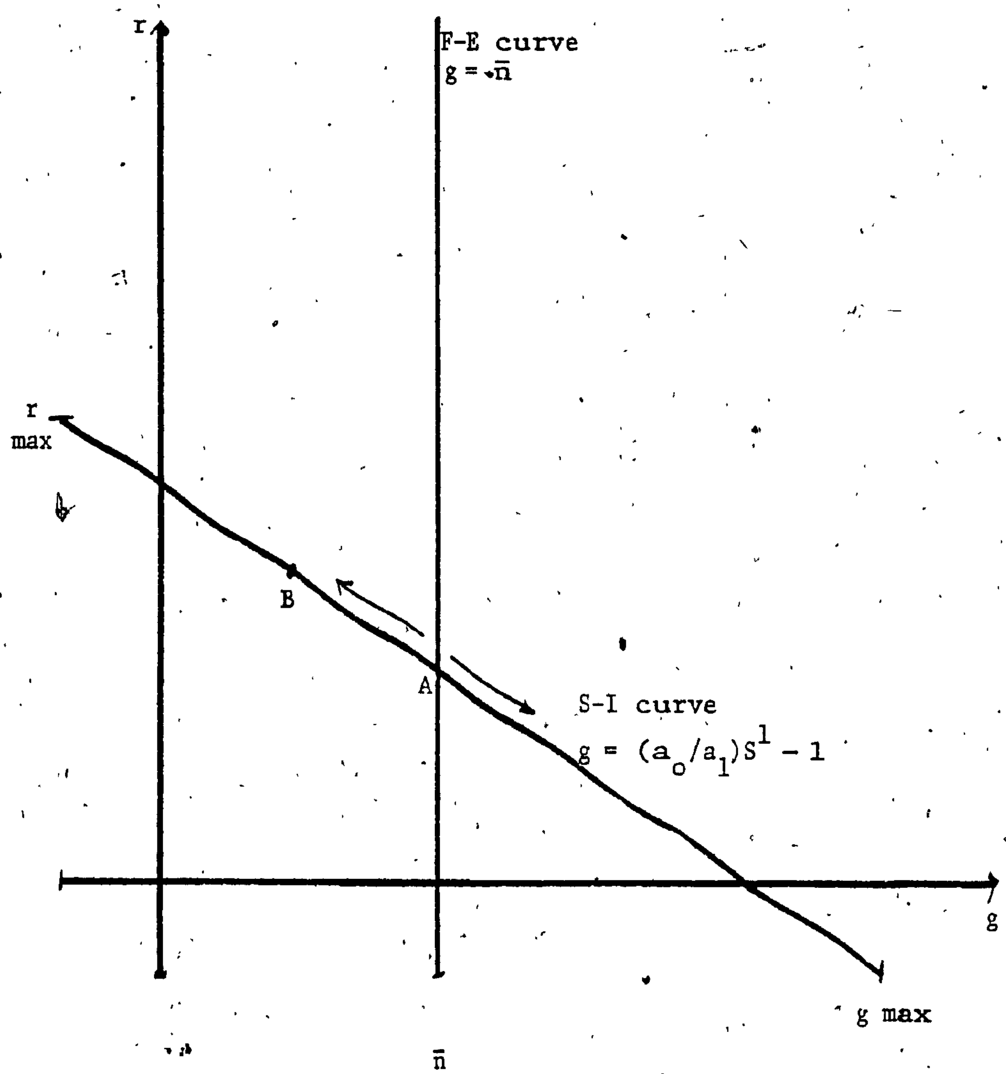

Figure 2.3. Equilibrium in the fixed-coefficients neoclassical model with a logarithmic utility function.

collapse of the growth rate to which growing unemployment is no real barrier, a growth rate in excess of the equilibrium growth rate will be quickly choked off by tightness in the labour market. Some additional mechanism might be introduced to show exactly how this would happen [in a monetary model it would, of course, be by inflation] but that is a secondary issue.) To summarize, then, we can state that the steady-state equilibrium in a neoclassical model with fixed production coefficients and a logarithmic utility function is unstable.

The instability of this neoclassical model is problematic as a description of the real world because it is not the sort of behaviour we normally see in real-world economies. This suggests that it would be worthwhile to see how the model might be altered to give a stable equilibrium. In this connection we can see a similarity between the instability of this neoclassical model and the instability of the Harrod-Domar growth model, the so-called "knife-edge" problem. The way in which that model, suitably generalized, served as a starting point for whole families of stable-growth models will serve as an example (see Hahn and Matthews, 1964). As Hahn and Matthews point out, instability is properly thought of as how the out-of-equilibrium adjustment mechanism is specified (ibid., p. 27). In the Harrod model, in particular, instability is a result of the way investment decisions are taken (the accelerator) and not just (as Solow [1956, pp. 161-162], for example, suggests) a consequence of fixed saving and production coefficients. Solow shows that a variable-coefficients production function will generate a model with a stable equilibrium solution but, as Hahn and Matthews (1964) make clear, that is only one way. The neoclassical model developed here is a case in point. It might be natural to attribute its instability ing our assumption of fixed production coefficients, (2.21) is now written

$$1+g = a_o/a_1[1-\beta(r)]W(r) \ . \qquad (2.22)$$

The slope of this S-I curve is

$$\partial g/\partial r = -(a_o W/a_1)\beta'(r) - a_1\text{[illegible]}a_o[1-\beta(r)]$$

which changes sign from positive to negative as r runs from its minimum to its maximum value.

This S-I curve is depicted in figure 2.4. What is striking is the backward-bend toward the origin (A-C). The important characteristic of this curve is that along its positively-sloped protion, an intersection with the F-E curve (e.g., point B) is a point of <u>stable</u> long-run equilibrium.[1] Thus we find that we do not need variable <u>production</u> coefficients in a neoclassical model to have a stable long-run equilibrium--flexibility in saving-consumption coefficients will do the job equally well.

This last point is made because there are several statements in Solow's 1956 paper (p. 162) which have led subsequent writers to see

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Constrained maximization gives as the demand for first-period consumption

$$C^1 = \left\{\frac{\rho}{\rho^{1/\delta}+(1-\rho)(1+r)^{1-\delta}}\right\}^{1/\delta} . W \ .$$

and we can, without loss of generality, call the expression in the square brackets β, noting that now β is a negative function of the rate of profit, $\beta'(r) < 0$. $\beta(r)$, moreover, can be shown to be continuous and bounded between zero and one. (It can be shown that the logarithmic utility function is a special case of the function. As $\delta \to 0$ this function becomes, in the limit, the Cobb-Douglas utility function [Arrow et al, 1961].)

[1]The stability analysis is given below in the discussion of the variable production coefficients models (p. 38).

simply to inflexibility in its parameters. But such a view would be mistaken. The production coefficients are, it is true, fixed by assumption and the saving coefficient $(1-\beta)$ seems also to be fixed (in this case on account of the particular form of the utility function). In fact, the saving propensity is only fixed in the sense that saving is proportional to wages but not in the Harrod sense that saving is proportional to total income. Instability here is, as in the Harrod model, better analyzed in terms of how the adjustment mechanism is specified and not simply attributed to rigidity in the parameters. In this model the problem is that if saving is not exactly what is required for full-employment growth, the resulting pressure on the real wage in conjunction with the particular form of the saving function leaves households to behave in such a way that the economy moves even further away from its full-employment growth path. The saving rate is flexible, but just not flexible in the right way.

We must not belabour this emphasis on the adjustment mechanism (as against flexibility in the parameters) because flexibility is not irrelevant. What is at issue is the <u>nature and degree</u> of flexibility in the structural parameters of the model under consideration because this will in fact determine the adjustment mechanism. For example, if we can make the saving rate a little more flexible, we may be able to achieve a stable equilibrium in a fixed production coefficients model. For this purpose we need only use a more general utility function, such as the constant-elasticity-of-marginal-utility function in which the "β" of (2.18) is now a bounded and decreasing function of r.[1] Maintain-

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[1]The constant-elasticity-of-marginal-utility function is written as follows (Intriligator, 1971, p. 148)

$$U = \frac{\rho(C^1)^{1-\delta}}{1-\delta} + \frac{(1-\rho)(C^2)^{1-\delta}}{1-\delta}, \qquad 0 < \rho,\ \delta < 1 .$$

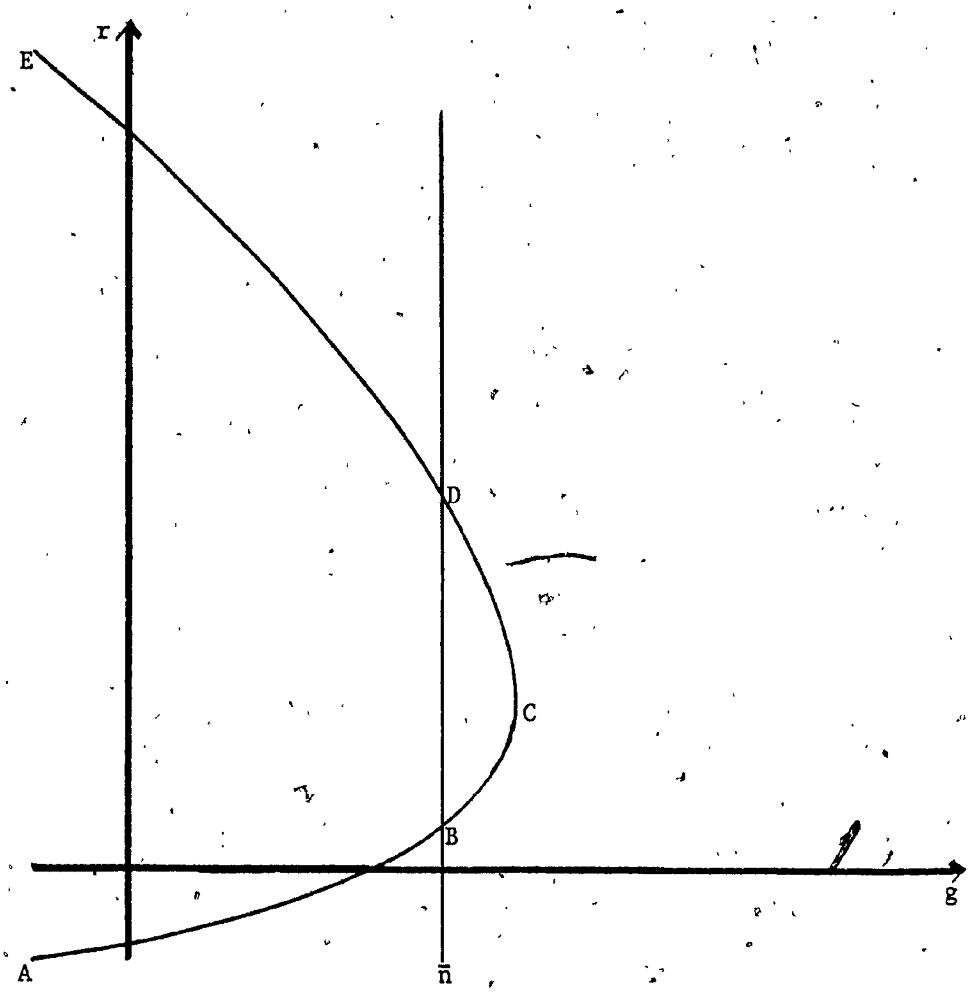

Figure 2.4. Steady-state growth equilibrium in a neoclassical model with a CEMU utility function.

variable production coefficients as the hallmark of the "neoclassical" growth model:

> One usually thinks of the long run as the domain of the neoclassical analysis, the land of the margin ..... The bulk of this paper is devoted to a model of long-run growth which accepts all the Harrod-Domar assumptions except that of fixed proportions. Instead I suppose that a single composite commodity is produced by labor and capital under the standard neoclassical condition. The price-wage-interest reactions play an important role in this neoclassical adjustment process ..... [emphasis added]

As has been made clear above this is not the view taken in this study. This is in a sense only a question of definition of terms but it bears repeating lest the results presented here be misinterpreted on account of terminological confusion.

Finally, we present without proof the neoclassical growth model with variable coefficients. Assume again a logarithmic utility function but now allow a_o and a_1 to vary and add a production function and the marginal-productivity factor pricing condition

$$1/a_o = f(a_1/a_o) \tag{2.8}$$

$$1+r = f'(a_1/a_o) \; . \tag{2.9}$$

The S-I curve (2.21) is now written

$$1+g = \frac{a_o(r)}{a_1(r)}(1-\beta)W(r) \; . \tag{2.23}$$

Totally differentiating (2.8) and (2.9) and using the following definition (Allen, 1967, p. 48)

$$\sigma = \frac{f'[f-(a_1/a_o)f']}{(a_1/a_o)ff''} > 0$$

where σ is the elasticity of substitution, we can then write

$$\partial a_o/\partial r = a_1\sigma/W \; , \; \partial a_1/\partial r = -a_1\sigma/(1+r) \; . \tag{2.24}$$

Hence along (2.23)

$$\partial g/\partial r = (1-\beta)\{\sigma[1+(a_oW/a_1[1+r])] - 1\} \qquad (2.25)$$

since from (2.7) $\quad \partial W/\partial r = -a_1/a_o$.

There are now three qualitatively different patterns to the S-I curve, corresponding to inelastic ($0<\sigma<1$), unit-elastic ($\sigma=1$) and elastic ($\sigma>1$) substitution. (Because of the way in which it has been defined, the elasticity parameter has been restricted to positive values but it is easy to see that, in the limit, as σ approaches zero in (2.25)

$$\partial g/\partial r = -(1-\beta)$$

which is simply the slope of the S-I relation with fixed coefficients.)

The three new S-I curves (along with the S-I relation with fixed coefficients case shown for comparative purposes) are presented in figure 2.5. We will discuss them in turn.

When $\sigma\to 0$ the S-I curve is linear and downward-sloping to the right [curve (a)]. As was discussed above, the long-run equilibrium can only be unstable. The shape of the S-I curve changes qualitatively as soon as we allow for variable production coefficients.

As σ increases from its lower limit at zero, the S-I curve becomes, in turn, a backward-bending curve [$0<\sigma<1$--curve (b) in figure 2.5]; a positively-sloped linear function [$\sigma=1$--curve (c)];and finally, a positively-sloped concave function ($\sigma>1$--curve (d)].

The reason for the positively-sloped portions of the new S-I curves involves changes in the equilibrium capital-labour ratio as the profit rate falls. Consider, for example, a movement along S-I curve (b) from a point on its negatively-sloped portion, say A, to a point on its positively-sloped portion, say B. Initially as r falls along (b) from A, the real wage rises (2.7) as does saving per worker (2.20). In-

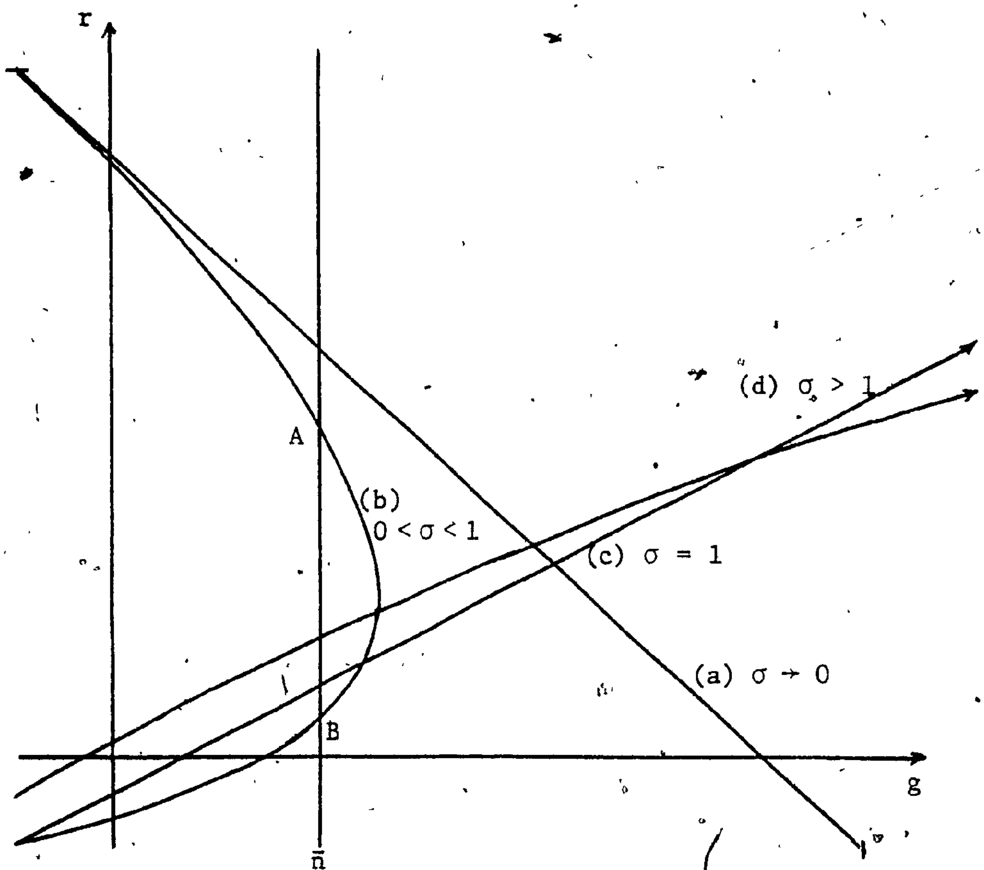

Figure 2.5. The saving–investment curve for various values of the elasticity of substitution (logarithmic utility function).

creasingly, however, as the real wage rises and the profit rate falls, the cost-minimizing capital-labour ratio rises. This means that the increased saving per worker associated with a lower profit rate is, at some point, no longer translated into a higher growth rate of capital. This point is the elbow of the S-I curve (b). As the profit rate continues to fall beyond this point, the resulting increases in the capital-labour ratio overwhelm the increases in saving per worker and the growth rate continues to fall, eventually to -1 where no amount of saving per man is sufficient to supply the capital needed to provide output. This positive relationship between r and g is also characteristic of curves (c) and (d), corresponding to values of $\sigma \geq 1$, and is so along the entire range of these curves.[1] The positive slope of these curves is significant for what it means for the stability of the long-run equilibrium.

We can characterize the equilibria on these new S-I curves. Consider first the case of inelastic substitution ($0 < \sigma < 1$) represented by curve (b). Because of the backward bend, there are, in general, two points of long-run equilibrium[2]; in figure 2.5, points A and B.

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[1]When $\sigma=1$ the production function is Cobb-Douglas and factor shares are constant. Thus, proportional changes in the profit-wage ratio are matched by proportional changes in the capital-labour ratio such that, the cost minimizing capital-labour ratio falls at just the rate required to allow the growth rate to increase linearly with the profit rate. If σ takes on any value greater than one, the anchor point of the S-I curve progressively shifts up the $r = -1$ axis and the curve itself becomes concave to the g axis. It keeps, however, its positive slope throughout because now the high elasticity of substitution means that every increase in the profit ratio causes labour to be substituted for capital at a faster rate than the declining wage reduces saving per man.

[2]If the F-E curve is too far to the right, there may be no long-run equilibrium. There is also the special case of a single, tangential, intersection at the elbow of the curve.

The stability of equilibria along positively-sloped portions of the S-I curve is easily demonstrated. Consider a point on the S-I curve (b) to the left of point B. Here households are in intertemporal equilibrium but the actual growth rate is lower than the full-employment rate, $\bar{n}$. Excess supply in the labour market will tend to drive the wage rate down and, as a result, the profit rate up. Since on this section of the S-I curve $\partial g/\partial r$ is positive, the growth rate increases until full-employment growth is reached at point B, at which time the labour-market clears. A parallel argument would shows how market pressure forces g down to its equilibrium value at B if we find ourselves to the right of B along a positively-sloped portion of the S-I curve. Since the S-I curves for the cases of elastic substitution ($\sigma \geq 1$) are positively-sloped along their entire lengths, and since they are not bounded to the right, a long-run equilibrium always exists, is unique, and is stable.

This concludes the presentation of the neoclassical growth model. Although the model seems to be sensitive to changes in specification in both production possibilities and preferences, when we rule out cases involving unstable equilibria we find that the model is not so amorphous as it may seem. Though there may or may not be flexibility in production coefficients, or there may or may not be flexibility in preference coefficients, flexibility in one or the other assures that, if it exists at all, there is a stable long-run growth equilibrium with full employment in the neoclassical model.

In the next two sections the two "classical" models, the neo-Keynesian and the neo-Marxian are presented.

fusion. Because there are many successors to the Keynesian legacy, it is important to state clearly which of these traditions we are concerned with. As we stated earlier, when we speak of the neo-Keynesian model we are referring to the Cambridge tradition which is associated with such names as Joan Robinson, Nicholas Kaldor, and Luigi Pasinetti.[1] There is another school of "post-Keynesian" thought which is associated with the names of Clower, Leijonhufvud, Barro and Grossman, among others, which takes up a very different strand of Keynes's thought. In this view, it is not a case of capitalists and workers, and saving and investment, but rather, a development of the analysis of markets which do not clear, where "quantity signals" as well as "price signals" are important conceptual tools (Muellbauer and Portes, 1979). Since full employment is clearly not an assumption of other "Keynesian" tradition, it would not seem fair to characterize it as "neoclassical." On the other hand, it is firmly in the neoclassical tradition in the sense of maintaining the assumption of utility maximization by households. Rather than trying to fit this tradition into our framework it would be more appropriate simply to take note of it and make it clear that it is not a model we are considering. We can now return to our development of the neo-Keynesian model.

Unlike the neoclassical model in which the growth rate is determined independently of income distribution, the neo-Keynesian model incorporates direct link between the growth rate and the distribution of income between capital and labour. This follows from the Keynesian hypothesis that the active and determining forces in an economy are the

[1] See Eichner and Kregel (1975) for a review of this Keynesian tradition.

The Neo-Keynesian Growth Model

An assumption common to both the neo-Keynesian and neo-Marxian models is that society is divided into two economic classes.[1] An economic class is a group of economic agents distinguished by the nature of its source of income. Members of the capitalist class have as their sole source of income profits; members of the working class have labour income (wages) though, if their saving propensity is greater than zero, they have some profit income as well. An important characteristic of these class-based models is that membership in a class is essentially permanent and hereditary. This distinguishes economic classes from the "generation-classes" of the neoclassical model; in that model a household earns wages in the first period of its life and profits (on its savings) in the second period. However, since every household goes through the same metamorphosis, a definition of class based solely on income source would make little sense.

Two further points in connection with the definition of class should be borne in mind. The first is that in terms of absolute numbers, the size of the capitalist class is irrelevant because the income of capitalists is independent of their labour. (This is easy to see if we think of the capitalist class as being composed of corporations, whose saving ratio is the average profit retention ratio.) Second, in order that the capitalist class not be "expropriated" by a high-saving working class, certain conditions on the relative size of saving propensities of the two classes must be fulfilled. These will be defined below.

Before continuing, we should clear up possible terminological con-

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[1]This assumption can be easily generalized to n-classes although in the long-run there will never be more than one capitalist class. If there are at any time more than one, the one with the highest saving propensity will eventually squeeze out the others.

independent decisions of savers on the one hand and investors on the other. Having these two behavioural relations in place of the one, combined, saving-investment relation in the neoclassical model[1] means that the growth rate must become a dependent variable in order that the model not be over-determined.[2] In a short-run framework the demonstration of this point was the revolutionary message of Keynes's General Theory: in a static monetary economy with sticky money wages, there is no assurance that general equilibrium will involve the full employment of capital and labour. In a long-run growth model with only one asset, the broad conclusion is the same. However, the mechanism is somewhat different (liquidity preference is no longer a consideration), and the units are different: in the long-run model the employment gap is defined in terms of the growth rates of labour and employment rather than their absolute levels.

We can begin the development of what we will call the neo-Keynesian model by making the simplifying assumption that workers consume all their current income. As Pasinetti(1962) has shown, this is not a very restrictive assumption and we can make use of his results to relax it subsequently.

Saving behaviour of capitalists can be summarized in a propensity to save parameter, s_c, which is defined as the ratio of saving of capitalists to their profit income. Assume s_c is fixed (or at least independent of the other variables of the model) and the definition be-

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[1]In the sense that the saving relation alone is independent in the neoclassical model. Investment is, by assumption, identical to saving.

[2]Note that this is in contrast to the full-employment growth assumptions of Kaldor and Pasinetti who, in other respects, must be considered neo-Keynesian writers.

The neo-Keynesian saving and investment functions are depicted in g-r space in figure 2.6.[1] We can discuss the existence and stability of equilibrium with reference to the diagram.

Given a smooth and monotonic investment function fulfilling the conditions above, a sufficient condition for equilibrium is that investment demand be positive when the profit rate is zero (Marglin, forthcoming, chapter 4). In this case the investment function intersects the saving function just once at point A. Stability in this model requires that investment intentions be realized, not, as might be expected, saving intentions. If that assumption is made, an equilibrium such as we have defined in which the investment function cuts the saving function from below is stable (ibid.)

It should be pointed out at this point that the natural growth rate, $g=\bar{n}$, has no place in the neo-Keynesian model. In a steady-state equilibrium, the economy will grow at the natural rate only by coincidence. This is quite consistent with Keynes's own view, that there is no natural economic mechanism, as there is in the neoclassical model, which assures full-employment growth. The model can be extended to deal with such cases (for clearly, in the very long run, g^* and $\bar{n}$ cannot continue to diverge) though this would take us too far afield. The interested reader is referred to Marglin (forthcoming, chapter 5).[2]

To complete the neo-Keynesian model we add the core accounting and

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[1] See Robinson (1962, p. 48)

[2] See also Robinson (1962, pp. 51-63) for a taxonomy of neo-Keynesian steady states.

comes a statement about saving behaviour

$$S_t = s_c \cdot P_t \qquad 0 < s_c \leq 1$$

where S_t is saving and P_t profits at time t. Dividing both sides by a measure of capital K_t, and making the steady-state assumption gives

$$S/K = s_c(P/K)$$

or

$$g = s_c r \tag{2.26}$$

where g in this case represents growth rate of the supply of capital.

There is nothing particularly Keynesian about the saving function (2.26) (and in fact it will also be used in the neo-Marxian model below); however, the investment function is peculiar to the Keynesian tradition. One of Keynes's important insights in the General Theory was the demonstration that the rate of investment was highly sensitive to businessmen's expectations. In a long-run growth model, the rate of growth of capital replaces the rate of investment and expectations are reduced to expectations of profit: "The central mechanism of our model is the desire of firms to accumulate, and we have assumed that it is influenced by the expected rate of profit. The rate of investment that they are planning for the future is, therefore, higher the greater the rate of profit on investment (estimated on the basis of current prices)." (Robinson, 1962, p. 47). In steady-state growth, the expected rate of profit can be represented by the current rate of profit and thus we can write, following Robinson

$$g = i(r) \qquad i' > 0 \tag{2.27}$$

We further assume that i" is negative (the desired rate of accumulation becomes less sensitive to increases in the expected rate of profit at high profit rates).

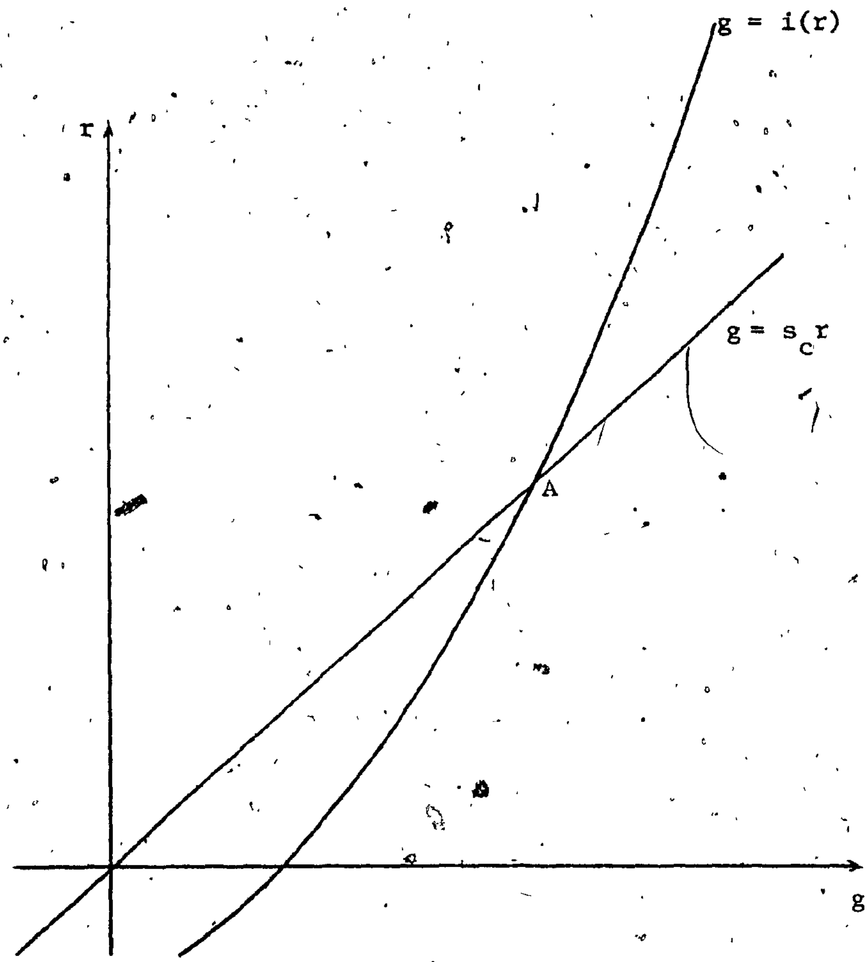

Figure 2.6. Steady-state growth equilibrium in the neo-Keynesian model.

class conflict in the sense that their behaviour and their behaviour alone determines the profit and wage rates and also the rate of growth of employment; in the neo-Marxian model, on the other hand, wages (and consequently profits and the growth rate) are determined in the continuing struggle between capitalist class and the working class.

Value, in the modern sense of relative prices, is determined in the neo-Marxian model by the wage bargain. The determinants of the wage bargain are themselves outside the model and can be conveniently divided into long-run and short-run factors. The long-run factors are those which provide the context for the wage bargain, essentially the general state of historical development of the economy, the most important features of which are the culturally-accepted standard of living of the working class on the one hand, and the normal conditions of work on the other (specifically the length of the working day and intensity of labour). Short-run factors are those which reflect the immediate economic and social circumstances, viz. the relative bargaining strength of the working class and the capitalist class.

Just as for the other two models, the neo-Marxian model uses the common core of accounting production and factor market relations equations:

$$1 = a_0C + a_1(1+g) \tag{2.4}$$

$$1 = a_0W + a_1(1+r) \tag{2.7}$$

$$1/a_0 = f(a_1/a_0) \tag{2.8}$$

$$1+r = f'(a_1/a_0) \tag{2.9}$$

(Again, if production coefficients are fixed, a_0 and a_1 become parameters of the system and equations (2.8) and (2.9) drop out.) To complete the system we add two equations. The first defines the saving production relations:

$$1 = a_o C + a_1(1+g) \tag{2.4}$$

$$1 = a_o W + a_1(1+r) \tag{2.7}$$

$$1/a_o = f(a_1/a_o) \tag{2.8}$$

$$1+r = f'(a_1/a_o)\ . \tag{2.9}$$

In the fixed-coefficients model a_o and a_1 are constants and equations (2.8) and (2.9) are dropped. It is easy to see that in the fixed-coefficients model, once equilibrium values of g and r are determined, C and W follow directly. It is a little more complicated with variable production coefficients. First the equilibrium value of the capital-labour ratio is determined from (2.9); this in turn determines output per worker through (2.8). From these we get a_o and a_1 and the rest follows.

These results can be generalized to allow for worker saving. Since the question of worker saving in the neo-Keynesian model is examined in chapter IV, we will defer consideration of this extension until then.

The Neo-Marxian Growth Model

The neo-Marxian model is similar to the neo-Keynesian model in that it is also based on the concept of a two-class society. It is similar as well in the way that it summarizes saving behaviour of a class in a single, fixed saving propensity. There the similarity ends: where the neo-Keynesian model is closed with an equation describing the behaviour of capitalists, the neo-Marxian model is closed with an equation defining the real wage. We can summarize the difference between the two models thus--in the neo-Keynesian model the capitalists have won the behaviour of capitalists which, since there is no independent investment function in the neo-Marxian model, also describes investment

$$g = s_c r \quad . \tag{2.26}$$

The second summarizes the outcome of the wage bargain

$$W = \bar{C} \tag{2.28}$$

where $\bar{C}$ is the culturally-determined subsistence level of consumption toward which, in the long run, the real wage is pressed.

Substituting (2.28) into (2.7) gives

$$r = 1/a_o(1-a_o\bar{C}) -1 \quad . \tag{2.29}$$

In this model the equilibrium value of r is determined directly once we know $\bar{C}$. If production coefficients are fixed, equilibrium r is a simple linear function of $\bar{C}$; if production coefficients are variable, equilibrium r is determined simultaneously with the equilibrium values of a_o and a_1 in the three-equation subsystem consisting of equations (2.29), (2.8) and (2.9). In either case the equilibrium value of r is a function of $\bar{C}$ alone; the rest, including the equilibrium values of g and C, follow.[1]

The neo-Marxian equilibrium is shown in g-r space as a horizontal line reflecting a profit rate fixed, effectively, in the labour market [curve (a)], crossing the saving function [curve (b)], at point A (figure 2.7). Parametric shifts in the real wage act directly on the growth rate. An increase in the real wage depresses the growth rate, a decrease raises it. In this model, as in the neo-Keynesian, the equilibrium growth rate is an outcome of economic decisions and need bear no relationship to natural growth rate, $\bar{n}$. In other words, continuing full employment is

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[1]Note that C will be larger than $\bar{C}$ unless $s_c = 1$ (i.e. if capitalists choose to consume some of their profits).

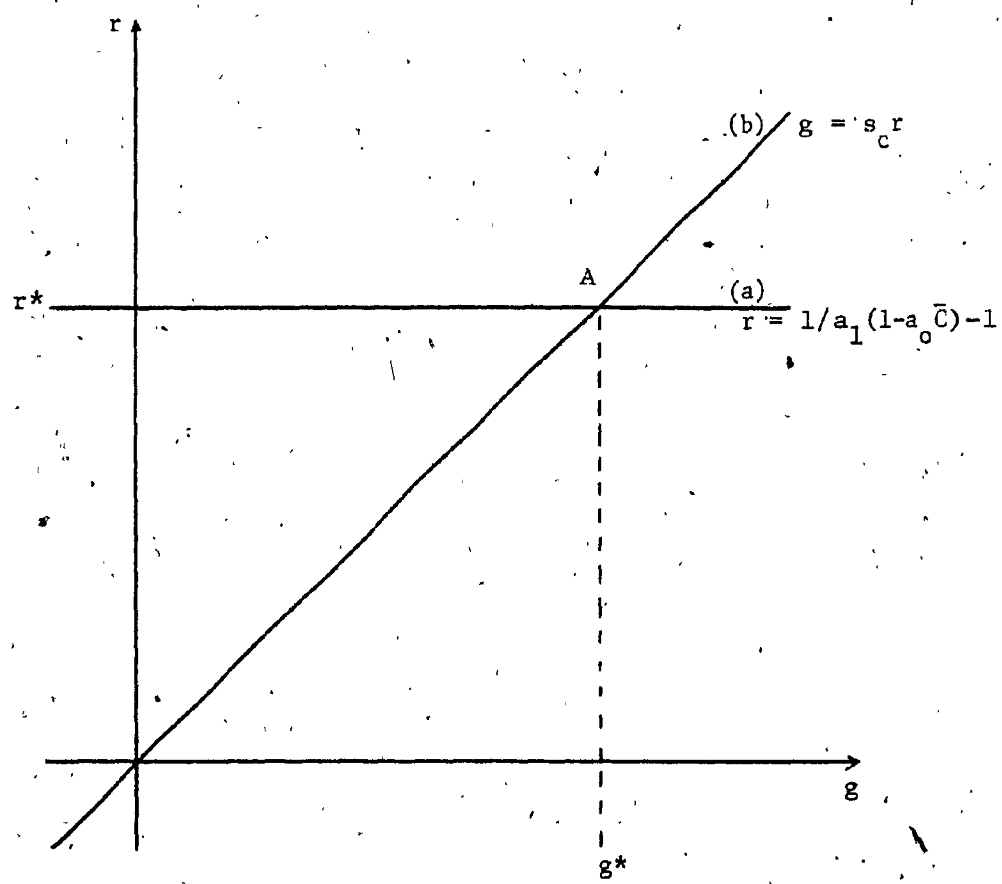

Figure 2.7. Steady-state growth equilibrium in the neo-Marxian model.

not assumed. Any discrepency between g and $\bar{n}$ is interpreted to mean that the capitalist sector either shrinks/expands within the larger economy (which is assumed to include non-capitalist modes of production), or involves emigration/immigration, or the export/import of capital. The reader is referred to Marglin (forthcoming, chapter 4) for further details.

The stability analysis of this model can be handled very briefly. Because of the central role of the subsistence wage, $W=\bar{C}$, at all times we are on (2.29) [curve (a) in figure 2.7]. If we are off the saving function, this simply means that capitalists are saving more (to the right of A) or less (to the left of A) than they wish to. As they adjust their saving to the desired level, the economy moves to the long-run equilibrium at point A.

As in the previous model, workers are assumed not to save. That may seem to be an unduly restrictive assumption. In fact, it is not. As Pasinetti (1962) has shown, so long as the saving propensity of workers as a class is less than $s_c a_1 r$, the saving function (2.26) continues to hold. In the world contemplated by the neo-Marxian model, any saving done by workers as a class is likely to be small and well within these limits.

Conclusion

In this chapter we have outlined the set of three long-run growth models developed by Marglin (forthcoming). In the next three chapters each of these models will be used as the starting point for developing, in turn, neoclassical, neo-Keynesian and neo-Marxian theories of a state pension scheme. With three theoretical models in hand we are then in a position to see which one best explains the facts.

CHAPTER III

THE NEOCLASSICAL MODEL OF A STATE PENSION SCHEME

Introduction

In the neoclassical growth model presented in chapter II, continuity through time was shown to be a consequence of the saving of households. Although saving is done by households solely for the purpose of financing their own retirement, the effective result is that they hold back, out of wages, the capital required for the following period's production. The mechanism by which the amount of capital supplied by households balances the amount needed for full-employment growth is found in the price system, in particular the prices of capital (profit) and labour (wages) which are the solutions to the set of equations which describe the neoclassical general equilibrium system.

The introduction of a pay-as-you-go state pension scheme disturbs a system in equilibrium because it provides a claim--the discounted value of future pension benefits--which satisfies the needs of households but which is not in fact represented by a stock of real capital. Because pension benefits provide the same service to households as real capital, participation in a pension scheme tends to displace real saving. This is shown to have the consequence, under reasonable assumptions, of raising the equilibrium profit rate in the economy and (if production coefficients are variable) reducing equilibrium capital intensity.

The analysis is continued on a per-worker basis. Each worker contributes a proportion, τ, of his wages to the state pension scheme where

τ is fixed by the state.[1] The self-financing feature of the state pension scheme means that, in the long run pension benefits are set at such a level that payments are equal to contributions collected in the same period. Thus in long-run, steady-state growth, the state pension scheme offers a prospective pension benefit of $\tau w(1+g)$ in return for a worker's contribution τw. Note that despite its formal similarity to a private pension plan, a state pension scheme is, by virtue of the compulsory nature of the contribution, a tax-transfer program (and for this reason the contribution is referred to as a tax in what follows).[2] In particular, the degree to which a worker participates in a given scheme is not a matter of choice.

In the neoclassical model, a household has choice variables C^1 and C^2, consumption in each of its two life-periods. The representative household chooses C^1 and C^2 so as to maximize a two-period utility function, U, subject to a life-time wealth constraint which now incorporates the state pension scheme.

$$\text{Maximize } U = \beta \ln C^1 + (1-\beta)\ln C^2$$
$$\text{subject to } C^1 + C^2/(1+r) = W(1-\tau) + \tau W(1+g)/(1+r). \quad (3.1)$$

A change in the size of a state pension scheme--that is, any change in the tax rate, τ—disturbs the steady-state growth equilibrium

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[1] τ must be between 0 and 1; in practice it will be less than 0.2. Note that in many schemes the worker's payroll tax is matched (or partially matched) by the employer. In such a case the tax rate, τ, is the ratio of the sum of the worker's and the employer's taxes to the sum of the nominal wage plus the employer's payroll tax.

[2] In the first years of the scheme's inception, pension benefits are paid to those who have made no previous contributions. This is normally how a state pension scheme becomes unfunded. (Kotlikoff, 1979a, p. 237).

because the pension tax τW, being a substitute for real saving,[1] displaces it. At any given profit rate an increase in τ reduces real saving per worker; the larger the change in τ, the greater the displacement of real saving. We can trace the implications of this change for equilibrium growth and profit rates by using the framework developed in chapter II. The key relationship in this framework is the saving-investment growth equilibrium condition.

Solving the constrained maximization problem set out in (3.1) gives

$$C^1 = \beta W[(1-\tau) + \tau(1+g)/(1+r)] .$$

Thus we now define real gross saving per worker as

$$S^1 = W(1-\tau) - C^1 = W\{(1-\tau) - \beta[(1-\tau) + \tau(1+g)/(1+r)] . \quad (3.2)$$

Multiplying the left-hand side of (3.2) by the labour-capital ratio a_o/a_1 gives real gross saving per unit of current capital which is, by definition, $1+g$:

$$1 + g = a_o/a_1 W\{(1-\tau) - \beta[(1-\tau) + \tau(1+g)/(1+r)\} . \quad (3.3)$$

This saving-investment growth equilibrium condition is similar to equation (2.21) which was derived in chapter II. There are two important differences. The first is the appearance of $(1-\tau)$ on the right-hand

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[1] Pension contribution and private saving are not perfect substitutes: apart from other considerations the rate of return to pension contributions is, in steady-state growth and in the absence of technical progress, equal to the growth rate of employment; this will equal r, the equilibrium profit rate, only by coincidence. The degree to which they are not substitutes is measured by the factor $(1+g)/(1+r)$ in the household life-time wealth constraint.

side of (3.3)--this measures the saving-displacement effect of a state pension scheme. The second is the appearance of g on the right-hand side of 3.3. This second appearance of g, or more precisely, the factor $(1+g)/(1+r)$, measures the degree to which real saving is displaced by the state pension scheme. This we will call, after Kotlikoff (1979a), the life-time wealth effect. Rewriting (3.3) to make g an explicit function of the other variables gives

$$g = \frac{a_0 W(1-\tau)(1-\beta)(1+r)}{a_1(1+r) + \beta\tau a_0 W} - 1 . \tag{3.4}$$

We can examine the effect of a change in τ on a steady-state growth equilibrium by tracing the induced shift of the saving-investment (S-I) curve (3.4) against the full-employment (F-E) curve, $g=\bar{n}$, in g-r space.

Whatever the shape of the S-I curve, an increase in τ shifts it to the left. From (3.4)

$$\frac{\partial g}{\partial \tau} = - \frac{[a_0 W(1-\beta)(1+r)][a_1(1+r)+\beta\tau(a_0 W)] + [a_0 W(1-\tau)(1-\beta)(1+r)\beta a_0 W]}{[a_1(1+r) + \beta\tau a_0 W]^2} < 0. \tag{3.5}$$

However, to know how the shift will translate into new equilibrium values of g and r we require information about the shape of the S-I curve. In chapter II we showed that the shape of the curve without a state pension scheme depends upon the nature of technical substitution in production. The same is true when we add a state pension scheme. Accordingly, the rest of this chapter is devoted to showing how changes in a state pension scheme affect growth and factor distribution under different assumptions about the elasticity of substitution.

Fixed Factor Coefficients

If there are no possibilities for factor substitution in production ($\sigma=0$), a_0 and a_1 are, by definition, fixed numbers and g in the S–I curve (3.4) condition is a function of r and W alone.

Consider equation (3.4). If $\tau=o$ this reduces to equation (2.21)

$$g = a_0/a_1 \ (1-\beta)W$$

which is a straight line sloping down to the right in g–r space. Maintaining the assumption of fixed production coefficients but allowing τ to take on any positive value causes a quantitative change in the shape of the S-I curve: it bends backwards to the origin. This qualitative change in the S-I curve is identical to the change in the S-I curve that the introduction of variable production coefficients caused in chapter II. The effect of the introduction of a state pension scheme on the S–I curve in a fixed-coefficients model is shown in figure 3.1, curve (a) representing the no-state-pension case and curve (b), the state-pension case.

The characteristic of a state pension scheme which causes the change in the S-I curve is the result of the life-time wealth factor $(1+g)/(1+r)$; were that to be held constant then the effect of changes in τ would simply be to cause (2.21) to shift in a parallel fashion to the left. The peculiar influence of the state pension scheme on the shape of the S-I curve is explained by observing the growing weight of the influence of the life-time wealth factor as r falls from its maximum value (point A in figure 3.1) to its minimum value ($r=-1$) along (b). It is convenient in doing this to refer directly to the S-I equation in implicit form (3.3). (Note, however, that the right-hand side of (3.3) does not <u>explain</u> g since g is also an argument on the right-hand side.) If we start at point A, where r has its maximum value and let the profit rate fall, the resulting

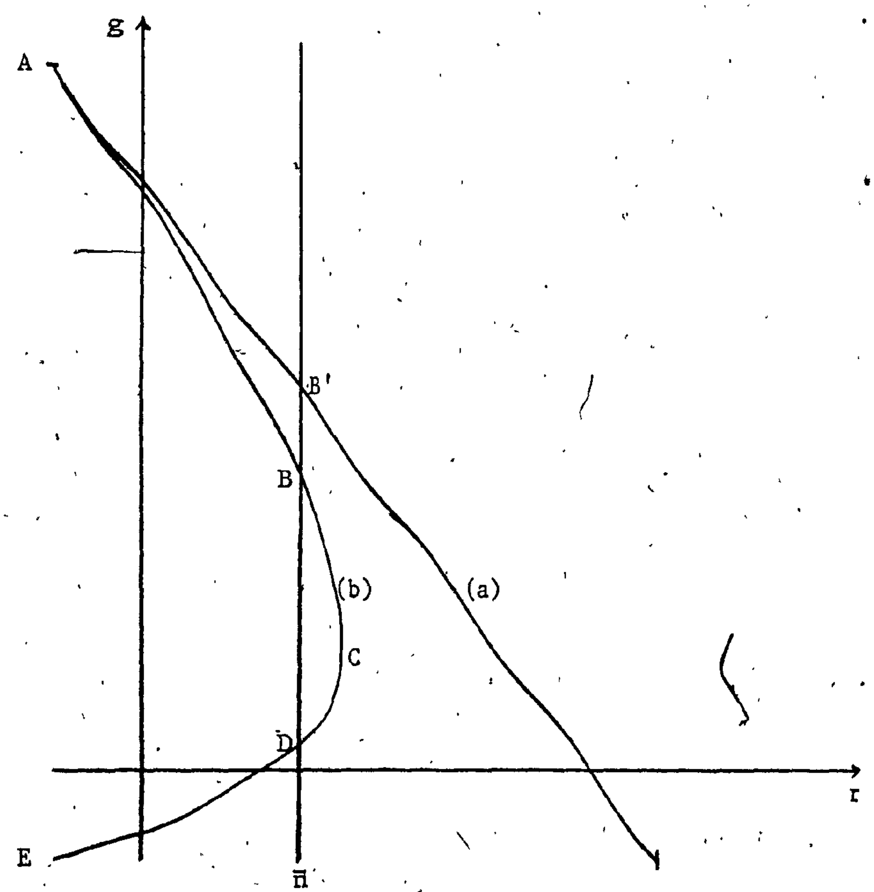

Figure 3.1. The effect of the introduction of a state pension scheme in the fixed-coefficients neoclassical model.

such an exceptional utility function as the logarithmic (for example, point B' in figure 3.1).

When the model is generalized to include a state pension scheme an unstable growth equilibrium (point B) remains but there is, in addition, a stable equilibrium at point D in the range where the S-I curve bends back on itself.[1] The existence of this stable equilibrium means that the fixed-coefficients growth model might have some application to the interpretation of long-run growth in real economies. (It was the instability of the original Harrod-Domar models that motivated subsequent work to develop growth models characterized by stable equilibrium; e.g., Solow [1956] and Kaldor [1955-56].)

As always, we must be careful about applying such highly abstract models to the real world. For instance, while there is little doubt that capitalist economies have been characterized by an increasing stability as they have matured, and that this change has been paralleled by a relative growth in state social welfare programs (one of the most important being state pension schemes), it would be unwise to conclude for this reason that the mechanics described in the model presented here are a valid representation of the mechanics actually at work. In particular, it would be inappropriate to attribute to these associated events, namely, growing social welfare programs and increasing stability, a strict cause and effect relationship of the sort implied by the model. We can cite two reasons. One is that a fixed-technical-coefficients model is not a good representation of a modern capitalist economy. It is stretching credibility to maintain that there are no possibilities

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[1] This assumes that the equilibria exist at all. If the F-E curve is to the right of point C there will be no long-run equilibrium.

increase in W (through [2.7]) causes g to rise; hence the S-I curve slopes down to the right. However, as the profit rate continues to fall, the numerator and denominator of the life-time wealth factor $(1+g/1+r)$ rise and fall respectively. As this continues as $(1+r)$ approaches zero, the present value of pension wealth and, consequently, total life-time wealth increases without limit and, as a consequence, first-period consumption, which is proportional to total life-time wealth,[1] also tends to increase without limit. This does not actually happen for, in attempting to realize such spending plans with a finite real wage, households are forced to draw resources from real saving, which, in the limit, drives the growth rate down to its minimum value, -1, as r falls to -1.[2] The actual point of inflection occurs where the increasing positive influence on current consumption of the life-time wealth effect overwhelms the increases in current real resources--which can only come from finite increases in the real wage--and, as a result, resources devoted to real savings are cut back (point C).

The backward bend in the S-I curve resulting from the incorporation of a state pension scheme in the model has an important theoretical implication. We have seen that neoclassical growth models with fixed factor coefficients have unstable steady-state growth equilibria with

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[1]This can be seen in equation (3.2).

[2]Wealth is $W\{(1-\tau)+\tau\frac{1+g}{1+r}\}$; household saving out of life-time wealth is $(1-\beta)W\{\cdot\}$ a positive function of the present value of pension benefits $\tau W\ (\frac{1+g}{1+r})$. However real saving is the difference between net wages and current consumption:

$$w\{(1-\tau)-B\left[(1-\tau)+\tau\left(\frac{1+g}{1+r}\right)\right]\}$$

which is an unbounded negative function of pension wealth. As the representative household reacts to the fall in the profit rate, it tries to reduce its real saving below zero. But what might be possible for one household cannot be done by every household without driving production and the growth rate to -1. Consequently along the S-I curve, r and g go to -1 together.

for factor substitution in economies with a wide range of industries and processes such as are characteristic of the modern world. Moreover, even if particular industries do have fixed factor coefficients, the possibility of a change in the pattern of resources devoted to different industries means that, in the aggregate, production coefficients are variable. The second, and more serious, difficulty with a literal interpretation of this model is that it is not at all clear how the introduction of a state pension scheme would take an economy from an unstable growth equilibrium (point B' in figure 3.1) to a stable equilibrium (point D). Indeed, the introduction of a state pension scheme would, normally lead from an unstable equilibrium at point B' to the equally unstable equilibrium on the new S-I curve at point B (and this ignores the fact that the initial shock would be destabilizing in itself). This second objection is a particularly serious one.

It is unrealistic to think that an unstable model can be useful in explaining economic behaviour. For this reason we will confine ourselves in what follows to stable equilibrium. Unfortunately, in the case of the fixed-production-coefficients model, even the new-found stable equilibrium in the presence of a state pension scheme is somewhat suspect since only unstable equilibria would have been possible in the period preceeding the introduction of the scheme. We will continue with analysis of the fixed-coefficients model for completeness, but this drawback to its application must be borne in mind.

We can now return to the model and to determine the incidence of a state pension scheme. We are interested in the sign of the expression $\frac{dr}{d\tau}$. This can be expressed, using the implicit function rule, as

$$\frac{dr}{d\tau} = -\frac{\partial g/\partial r}{\partial g/\partial \tau}$$

saving lost by the "crowding-out" effect of the state pension contribution. This result is illustrated in figure 3.2 as the movement from point A to point B.

Finally, it should be pointed out that while the mechanism described above assures that the loss of saving on account of the state pension is compensated for by an increased incentive to save, there is, none the less, a limit to the size of state pension scheme an economy can support in a fixed-coefficients growth model. As we have seen above, equation (3.5), as τ increases from 0 to 1, the S-I curve progressively flattens against the axis $g = -1$ and the maximum growth rate that saving can support declines. This is illustrated by the progression $C_1 \ldots C_n$ in figure 3.2. Setting the partial $\partial g/\partial r$ equal to zero gives

$$g^{max} = \frac{\sqrt{\beta\tau} - \beta\tau}{a_1(1-\beta\tau)} - 1 \ .$$

So long as $g^{max} > \bar{n}$ (curves C_1 and C_2 in figure 3.2) there are two steady-state growth equilibria, the one with the lower profit rate being stable. As τ continues to increase there comes a point ($\tau=\tau^*$) of tangency between the S-I curve and the F-E curve (point C); as τ increases beyond this critical point, no steady-state equilibrium exists. That is to say, a state pension scheme can be so large, and the life-time wealth induced current consumption so high, that there simply are not the resources available to replace the saving "crowded out" by the scheme whatever the profit rate.

In the next section we study the incidence of a state pension scheme in an economy with variable production coefficients.

where g is defined by (3.4). As we saw (3.5) the numerator of this expression is uniformly negative, but the sign of the denominator depends upon the value of r. Since the S-I curve is backward-bending, it has, looked at from the left-hand side, a maximum value measured along the g-axis. The value of r associated with this inflection point which we will call r* can be shown to be[1]

$$r^* = \frac{\beta\tau - \sqrt{\beta\tau}}{a_1(\beta\tau - 1)} - 1 \; .$$

For values of r greater than r^*, S-I has a negative slope (point A to point C in figure 3.1) and for values of r less than r^* S-I has a positive slope (point C to point E in figure 3.1). The negatively-sloped range corresponds to equilibria that are unstable and the positively-sloped range corresponds to stable equilibria. Since we are interested in stable equilibria only, we limit ourselves to considering the range of S-I defined by

$$-1 \le r \le r^* = \frac{\beta\tau - \sqrt{\beta\tau}}{a_1(\beta\tau - 1)} - 1 \quad .$$

Over this range $\partial g/\partial r$ is positive and consequently

$$\frac{dr}{d\tau} > 0 \; .$$

Note that although an increase in the size of the state pension scheme will tend to depress the growth rate ($\partial g/\partial \tau < 0$) in partial equilibrium, the profit rate must change in order to maintain the full-employment rate of growth which is an assumption in the neoclassical model and, consequently, in long-run equilibrium it is the profit rate only which adjusts. It is the increase in the profit rate which provides the incentive to households to increase their real saving to make up for the

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[1] Set the partial derivative $\partial g/\partial r$ equal to zero and solve for r. There are two roots of which only one makes economic sense.

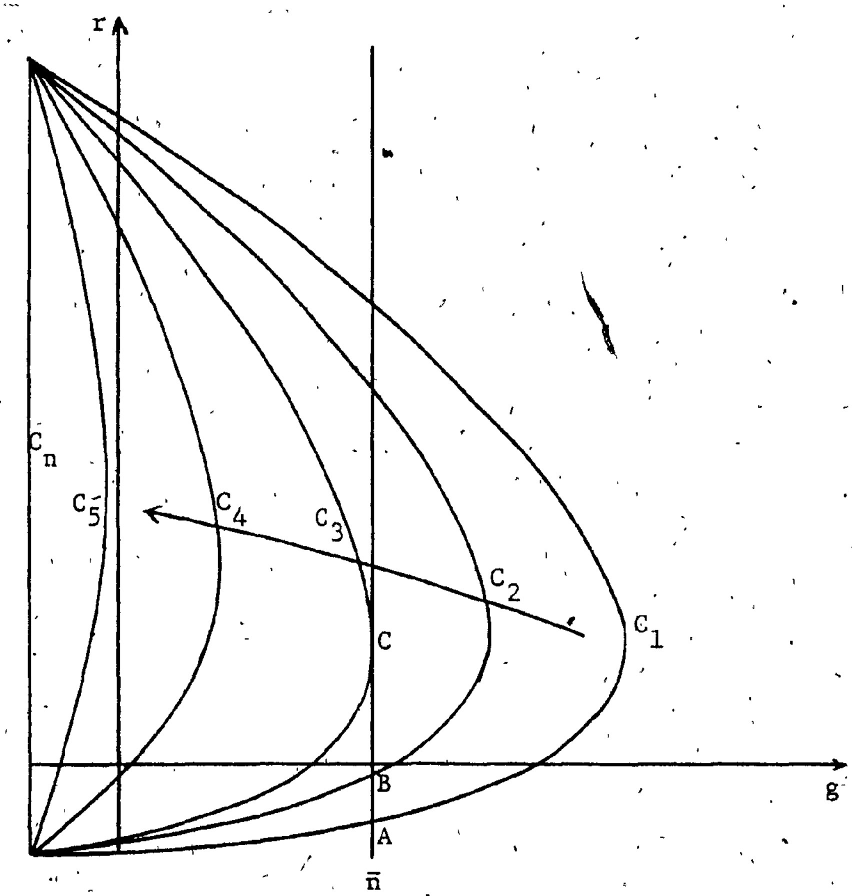

Figure 3.2. The effect of increases in the size of a state pension scheme in a fixed-coefficients neoclassical model.

Variable Factor Coefficients

If production coefficients are variable, a movement along the saving investment (S-I) curve reflects not only changes in saving behaviour as the profit rate varies but also changes in the capital-labour ratio as firms react to the changing relative cost of the two factors of production. As we saw in chapter II, as substitution possiblities (measured by the elasticity of substitution parameter σ) change from being non-existent ($\sigma \to o$) to being perfect ($\sigma \to \infty$), the S-I relation changes shape, each one being characteristic of a particular value or range of σ (refer to figure 2.5). As we saw in the last section, a state pension scheme changes the characteristic shape of the S-I curve when $\sigma=0$. Where σ is greater than zero a state pension scheme also influences the shape of the S-I curve although not so radically as in the case of fixed production coefficients. It is convenient to consider three cases separately: first inelastic substitution, $0<\sigma<1$; second, unitary elasticity (the Cobb-Douglas production function), $\sigma=1$; and third, elastic substitution, $\sigma>1$.

Inelastic Substitution

If the elasticity of substitution lies between zero and one the S-I curve has the same backward-bending shape without a state pension scheme that is characteristic of the S-I curve in the fixed-coefficients model with a state pension scheme. The introduction of a state pension scheme in the inelastic-substitution model simply flattens the S-I curve somewhat against the g = -1 axis in the same way that progressive increases in the size of the state pension scheme flatten the fixed-coefficients S-I curve against the axis. As in that model there are, in general, two equilibria, one stable and one unstable, the stable one being associated with a positive sign on $dr/d\tau$.

This can be shown algebraically. Again we are interested in the sign of $dr/d\tau$. By the implicit function rule

$$dr/d\tau = -\frac{\partial g/\partial\tau}{\partial g/\partial r}$$

The general expression for S–I as presented in equation (3.4) is still valid though we must bear in mind that the technical coefficients a_o and a_1 are now functions of the profit rate. The partial $\partial g/\partial\tau$ is still negative (3.5) but the sign of the partial with respect to the profit rate is ambiguous:

$$\frac{\partial g}{\partial r} = \frac{(1-\tau)(1-\beta)}{[a(r)+\beta\tau(1-\alpha(r))]^2}\ \{[\alpha(r)+\beta\tau(1-\alpha(r))][1+r)(-\frac{\partial\alpha(r)}{\partial r}+(1-\alpha(r)]$$
$$-(1-\alpha(r))(1+r)(1-\beta\tau)\frac{\partial\alpha(r)}{\partial r}\} \qquad (3.7)$$

Where $\alpha(r)$ is $a_1(1+r)$, the share of gross profit is gross national income. The influence on the sign can be isolated by making the following substitution.[1]

$$\frac{\partial\alpha(r)}{\partial r} = a_1(1-\sigma) \quad > 0$$

which gives

$$\frac{\partial g}{\partial r} = \frac{(1-\tau)(1-\beta)}{[\alpha(r)+\beta\tau(1-\alpha(r))]^2}\ \{[\alpha(r)+\beta\tau(1-\alpha(r))][(1-\alpha(r))-\alpha(r)(1-\sigma)]$$

$$-\alpha(r)(1-\alpha(r))(1-\beta\tau)(1-\sigma)\} \qquad (3.8)$$

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[1]This follows from $(\partial a_1/\partial r = -a_1\sigma/(1+r)$ which was developed in chapter II, equation (2.24).

This is $\lessgtr 0$ as

$$\alpha(r) \lessgtr \frac{-(\sigma-2\beta\tau)-\sqrt{\sigma^2 - 4\beta\tau\sigma + 4\beta\tau}}{2\beta\tau - 1}$$

Unitary Elasticity of Substitution

If the elasticity of substitution is one, the production function is Cobb-Douglas and factor shares are constant $(\alpha(r)=\bar{\alpha})$. Consequently, we can write the S-I relation

$$g = \frac{(1-\bar{\alpha})(1-\tau)(1-\beta)(1+r)}{\bar{\alpha} + \beta\tau(1-\bar{\alpha})} - 1 \tag{3.9}$$

This is linear in g-r space with a constant slope

$$\frac{\partial g}{\partial r} = \frac{(1-\bar{\alpha}r)(1-\tau)(1-\beta)}{\bar{\alpha}r + \beta\tau(1-\bar{\alpha}r)} \tag{3.10}$$

and lies to the left of S-I curve corresponding to the no-pension case. Unlike the fixed-coefficients case, the introduction of a pension scheme makes no qualitative change in the saving-growth relation: it simply rotates to the left (figure 3.3). Since neither the domain nor the range of this S-I curve has an upper bound, we know that a steady-state growth equilibrium always exists. Consequently

$$\left.\frac{dr}{d\tau}\right|_{\sigma=1} = -\frac{\partial g/\partial\tau}{\partial g/\partial r} > 0 \tag{3.11}$$

for any value of τ over the entire domain since S-I is linear. Since proportional changes in the capital-labour ratio are equal to proportional changes in the wage-profit ratio, increased per-capita saving as the profit rate falls (and wages rise) is more than compensated for by an increased demand for capital in production.[1]

---

[1]Mathematically, this is because per capita saving is a linear function of the profit rate but the per capita demand for capital has the form

$$\frac{a_1}{a_0} = \frac{[\alpha(r)-1]}{\sqrt{(1+r)/\alpha(r)}}$$

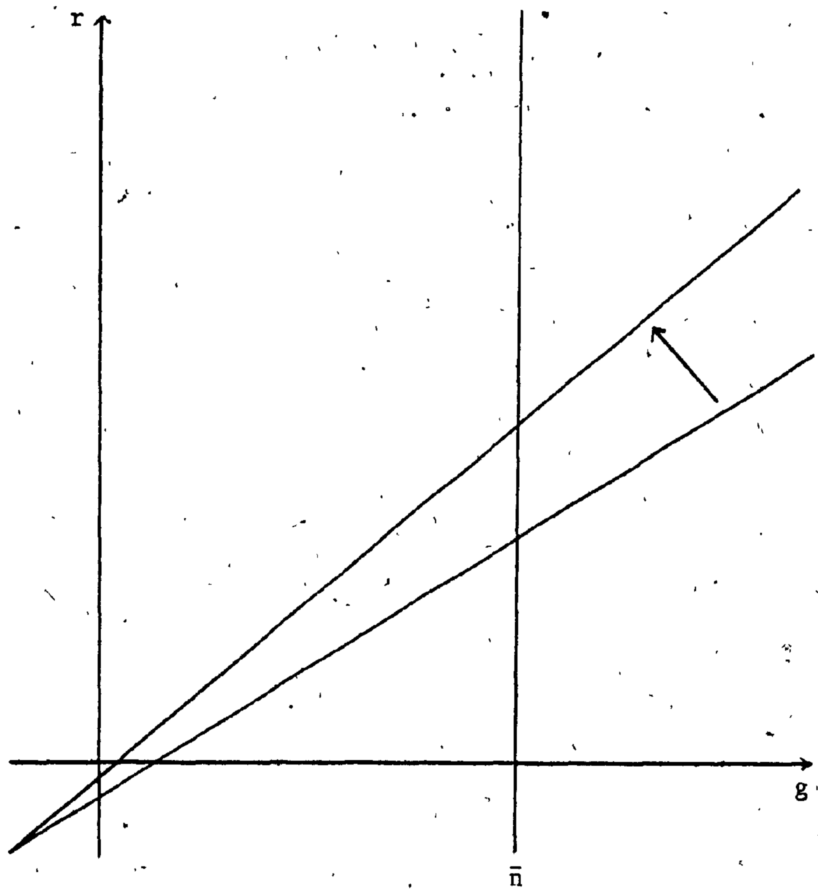

Figure 3.3. The effect of an increase in the size of a state pension scheme in a neoclassical model with unitary elasticity of substitution.

Elastic Substitution

Factor shares are a negative function of the rate of return to that factor if the elasticity of substitution exceeds one

$$\frac{\partial\alpha(r)}{\partial r} = a_1(1-\sigma) < 0 \qquad (3.12)$$

Equation (3.8) continues to hold but now its sign is uniformly positive and consequently

$$\frac{dr}{d\tau} > 0$$

Graphically, this is similar to the case of unitary elasticity: the positively-sloped but now curvi-linear S-I curve rotates counter-clockwise with an increase in the state pension scheme.

Conclusion

If the initial equilibrium in a neoclassical steady-state growth model is a stable one, then a small increase in the size of the state pension plan will, in the long run, cause the equilibrium profit rate to rise. This is a general result and is independent of the elasticity of substitution in production. A particularly interesting corollary is that the state pension scheme has the special property that its existence alone provides the possibility of a stable-state equilibrium in the normally unstable fixed-coefficients-of-production growth model: the introduction of a state pension scheme into a fixed production coefficient model has the same effect as allowing a small degree of substitutability between the factors of production; in both cases there is, at relatively low levels of the profit rate, a positive relationship between the profit rate and the growth rate. The usefulness of such a model for explaining real-world phenomena remains, however, as the model is silent about how one goes from the unstable to the stable equilibrium. Changes in

CHAPTER IV

THE NEO-KEYNESIAN MODEL OF A STATE PENSION SCHEME

Introduction

The neo-Keynesian model reflects a view of the world in which capitalists' behaviour alone determines the characteristics of the steady-state growth path. This is clearly the case in the model presented in chapter II: the model is closed by the intersection of the investment function and the aggregate saving function; capitalists are, by definition, solely responsible for investment decision as well as being the only savers. In that model the growth rate and the profit rate are fixed by choices that capitalists make; what the workers get as a wage is determined as a residual. This is in contrast to the neoclassical model in which the preferences of all economic agents count in the determination of equilibrium and in which the values of the dependent variables are determined simultaneously.

In this chapter, we will show that even if workers do account for some aggregate saving they cannot, within broad limits of behaviour, influence the important characteristics of the long-run growth equilibrium. Consequently, their reactions to a policy change such as the introduction or change in a state pension scheme have no influence on the characteristics of long-run growth in a capitalist economy.

factor shares on account of changes in the size of a state pension scheme depend on the elasticity of substitution. An increase in the profit rate means an increase in the capital's share of income if the elasticity of substitution is less than one, and a decrease in capital's share if the elasticity of substitution is greater than one; an elasticity of substitution of one means that factor shares will be independent of factor returns.

No Worker Saving

If workers do not save, the effect of the introduction (or change) in a state pension scheme is straightforward. Since the pension scheme works by taxing one group (workers) to transfer to another (retired workers) the only effect that an increase in state pension scheme can have is to reduce the consumption of those members of the working class currently working and increase the consumption of those retired. There will be no effect whatsoever on gross wages, the profit rate or the growth rate, or on the distribution of income or consumption between classes.

(In almost every society some provision is made for the elderly--intra-family transfers or some form of private or institutional charity are common examples--and one could expect that increases in a state pension system would be accompanied by reduction in transfers through these traditional channels. Actually, it has been argued that it has been the decline in traditional means of old-age support--especially intra-family transfers--following the growth of day-wage labour that has led to a political demand for old age support of the sort studied here. But, whether a state pension makes up for lost transfers or supplants them, the effect is the same: the long-run characteristics of the economy as a whole are unchanged.)

Worker Saving

The neo-Keynesian model can be generalized to account for worker saving. This is an appropriate extension to make since a change in a state pension scheme can be assumed to have some repercussion on workers' saving insofar as workers save for old age.

capital, respectively. Taking the first equality and inverting we have

$$\frac{Kc}{Sc} = \frac{Kw}{Sw} . \qquad (4.4)$$

Multiplying both sides by the common rate of profit, r gives

$$\frac{Pc}{Sc} = \frac{Pw}{Sw} \qquad (4.5)$$

and substituting the expression for Sc and Sw in (4.1) we have

$$\frac{Pc}{s_c Pc} = \frac{Pw}{s_w(W + Pw)} \qquad (4.6)$$

from which

$$s_c Pw = s_w(W + Pw) = Sw . \qquad (4.7)$$

Thus we can write total saving (4.1)

$$S \equiv s_w(W + Pw) + s_c Pc = s_c Pw + S_c Pc ,$$

or

$$S = s_c P \qquad (4.8)$$

from which follows (2.26) when we divide both sides of (4.7) by the capital stock, K

$$S/K = g = S_c r \qquad (2.26)$$

within the limits on s_w and s_c specified in (4.2) (Pasinetti, 1962, pp. 127-128).[1]

---

[1]In a model which was a precursor to Pasinetti's, Kaldor (1955-56) wrote per-worker total saving as

$$s_c Pc + s_w W .$$

It was Pasinetti's contribution that he recognized that worker saving implies worker capital and worker profits (1962). The restriction of workers' saving propensity has been made explicit and its implication investigated by Samuelson and Modigliani (1966). Samuelson and Modigliani have also developed a dual to the Pasinetti result for steady-state growth when $s_w \geq \alpha \cdot s_c$. In this case workers are accumulating more quickly than capitalists (because they save out of wages as well as profits) and in the long-run they own all but a vanishing share of the capital. Under such circumstances we revert, effectively, to a one-class neoclassical model in which growth is characterized by the relation $g = s_w/a_1$. Samuelson and Modigliani call this the "anti-Pasinetti" range.

Assume that workers save at a fixed rate on income s_w.[1] Aggregate per-worker saving, S., is now the sum of capitalists' saving and workers' saving

$$S = Sc + Sw = s_c Pc + s_w (W + Pw) \tag{4.1}$$

where Sc and Sw are per-worker saving of capitalists and workers, Pc and Pw per-worker profits of capitalists and workers. Pasinetti (1962) and Samuelson and Modigliani (1966) have shown that if the profit rate earned by workers on their capital is the same as that earned by capitalists <u>and</u>

$$s_w < \alpha s_c \tag{4.2}$$

where α is the net profit share, the aggregate saving relation can still be expressed by (2.26); i.e., it is independent of worker saving.

We can review Pasinetti's argument briefly. In long-run equilibrium, with the limits on s_w and s_c specified, the growth rates of capital stock of both capitalists and workers are equal to the growth rate of total capital, i.e.

$$\frac{Sc}{Kc} = \frac{Sw}{Kw} = \frac{S}{K} = g \tag{4.3}$$

where Kc, Kw, and K are capitalists' capital, workers' capital and total

---

[1]The fixed saving rate can be taken to represent rule-of-thumb behaviour (Jones 1975, pp. 154-155) as opposed to life cycle utility maximization of the neoclassical variety. Given, on the one hand, the uncertainties of future wage and profit rates, employment, working life and life span and, on the other, the fact that for most personal saving people tend to be locked into long-term fixed payment saving contracts (e.g. mortgage payments, company pensions, life insurance contracts and certain tax-exempt saving plans) such a representation is not unrealistic. If workers save in such a manner during their working years and then dissave in retirement, s_w for the whole class of workers represents a weighted average of a high saving rate in the early years and a negative rate in retirement. If we assume population growth, wage and profit rates are stable (as they would be in long-run equilibrium) the consumption profile over a lifetime is fixed, then there is a one-to-one relationship between saving out of wage income by working workers and saving out of all income by all workers. This point is developed below.

than some critical level); only the saving propensities of capitalists matter.[1] There is, however, one essential difference if workers save, and that reflects the fact that in long-run equilibrium some portion of the capital stock is owned by workers. Specifically, in steady-state growth the ratio of workers' capital to capitalists' capital is equal to the ratio of workers' saving to capitalists' saving

$$\frac{Kw}{Kc} = \frac{s_w(W + Pw)}{s_c Pc} = \frac{s_w(W + rKw)}{s_c rKc}$$

where Kw and Kc are per-worker workers' capital and capitalists' capital respectively. Cross-multiplying gives

$$\frac{s_c rKc}{Kc} = s_c r = \frac{s_w(W + rKw)}{Kw}$$

and combining with (2.26) gives

$$s_c r = g = \frac{s_w(W + rKw)}{Kw} \tag{4.9}$$

The ratio of workers' saving to workers' capital is always equal to the growth rate as determined by capitalists' saving. Moreover, since the equilibrium values of g, r and W are determined independently of workers' behaviour, the constancy of the ratio on the right-hand side of (4.9) is maintained by Kw varying with s_w. Specifically, given the equilibrium values of g, r and W in (4.9)

$$Kw = \frac{s_w W}{g - s_w r} \tag{4.10}$$

i.e., Kw is a positively-sloped function of s_w within the domain $0 \leq s_w < \alpha s_c$.

---

[1]This literally is true in steady-state growth. However, stability conditions do depend upon the value of s_w. See Marglin (forthcoming, chapter 4).

Pasinetti, and Samuelson and Modigliani studied the distributive properties of a Kaldorian two-class growth model in which the steady-state growth rate is given exogenously by the rate of growth of the labour force.[1] In other words, theirs were full employment models in the same sense as a neoclassical model: full-employment growth is assumed. In this sense the Kaldorian-Pasinetti model is a hybrid. Since the neo-Keynesian model we have specified has an independent investment function, the assumption of full employment growth would over-determine our model; consequently one relation must be dropped. None of the essentials of Pasinetti's results are lost if we substitute our Robinsonian investment function for his full-employment growth condition. Having done that we find that the generalized neo-Keynesian model <u>with worker saving</u> is closed with the same two equations it was closed with in chapter II:

$$g = s_c r \tag{2.26}$$

$$g = i(r) \tag{2.27}$$

with additional condition

$$s_w < \alpha s_c \tag{4.2}$$

This maintains the essential Pasinetti result: even though workers save, if there exists one class whose income is accounted for solely by profits, the saving propensity of workers drops out of the aggregate saving function. The characteristics of steady-state growth with which we have concerned ourselves--the equilibrium values of r, g, W and C--are independent of the saving propensity of workers (so long as s_w is less

[1] "The purpose of this essay is to present a more logical reconsideration of the whole theoretical framework regarded as a system of <u>necessary</u> relations to achieve full employment" (Pasinetti, 1962 p. 103) [Emphasis added].

Similarly, workers' profits are an increasing function of s_w

$$Pw = rKw = \frac{rs_wW}{g - s_wr} \tag{4.11}$$

We can review the nature of the long-run growth equilibrium in the neo-Keynesian model with worker saving. In figure (4.1) we have reproduced the neo-Keynesian saving and investment functions in g-r space. The only difference between this diagram and the one presented in chapter II (figure 2.6) is that here the saving function has a vertical component at $g = s_w/a_1$ which reflects the range over which the Pasinetti result does not hold, the "anti-Pasinetti" range (see p.71, n.1 above). The intersection of the investment function and the saving function in this "anti-Pasinetti" range is unstable and for this reason alone must be dismissed from further consideration.[1] In the region of point of intersection A the analysis of the neo-Keynesian with worker saving is identical to the analysis of the neo-Keynesian model without worker saving, which was presented in chapter II. The important conclusion is that, given the production function, the real wage, per-worker consumption, and the rates of profit and growth are determined by the behaviour of the capitalist class alone. Within wide limits the preferences and behaviour of workers are of no account in determining the general characteristics of the long-run growth equilibrium.

There is, however, one way in which workers can influence distribution. Variation in the saving rate of workers does directly affect the division of profit and consumption between capitalists and workers. From (4.10)

$$\frac{Kw}{K} = \frac{s_wW}{K(g-s_wr)} \tag{4.12}$$

[1] This was the criterion that was used in a similar case in the neoclassical model.

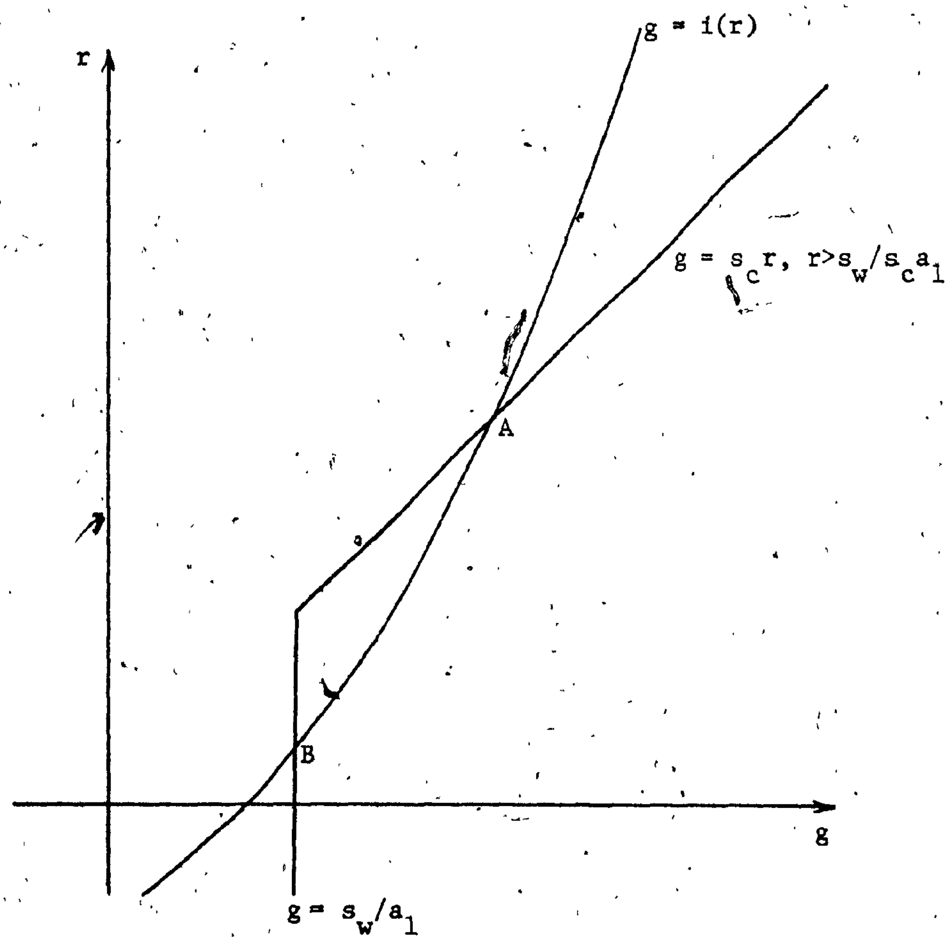

Figure 4.1. Neo-Keynesian equilibrium with worker saving.

Up to the limit $s_w = \alpha s_c$ (at which point Kw/K = 1 and workers own all but a vanishing share of capital) this equation shows the proportion of total profits accruing to workers as a function of the saving rate of workers. Since in long-run equilibrium

$$\frac{Pw}{P} = \frac{Kw}{K} \tag{4.13}$$

we can also derive expressions for Pw, Pw/W, and Sw from (4.12)

$$Pw = \frac{s_w WP}{K(g - s_w r)} = \frac{s_w Wr}{(g - s_w r)} \tag{4.14}$$

$$Pw = \frac{s_w r}{(g - s_w r)} \tag{4.15}$$

$$Sw = s_c Pw = \frac{s_c s_w Wr}{(g - s_w r)} \tag{4.16}$$

where the left-hand equality in (4.16) was derived in (4.7).

One final step is to reconcile our assumptions that (1) the saving propensities of workers as a class can be summarized in a single fixed saving coefficient s_w and that (2) individual worker households save positively during their working years and then dissave out of the accumulated capital to support themselves in retirement. The story goes something like this. Imagine that each worker saves a fixed percentage of wage income, x, each year investing his savings at the ruling rate of profit. Upon retirement he buys an annuity that pays him a fixed amount N per year until he dies. Since over a lifetime, each worker's saving nets to zero[1] it is clear that a positive level of aggregate worker saving can only result from growth of the working class. In fact, it can be shown that if the working class grows at a constant rate, there is a stable relationship between the net saving and the gross income (wages and profits) of

[1]It should be noted that there is an implicit assumption here that each worker saves only to spread consumption over his own lifetime and not to make bequests to heirs.

the entire class of workers, working and retired, in any given year. We can go a step further. Given r, we can posit a relationship between s_w and the level of consumption of retired workers. Writing Cwr for per-worker consumption of retired workers we have, given a fixed lifespan, work pattern, profit and growth rates

$$Cwr = b\ Sw \tag{4.17}$$

where b is a constant and is, in particular, independent of s_w.[1] Substituting (4.16) into (4.17) and using (2.26)

$$Cwr = \frac{b s_c s_w W}{(s_c - s_w)} \tag{4.18}$$

---

[1] The representative worker saving X = xW per year and investing it at r has accumulated, at the end of a working life of

$$\sum_{i=1}^{\phi}[X(1+r)^i] = X[\frac{(1+r)^{\phi}-1}{r}]$$

This will provide him with an annuity of $N = X[\frac{(1+r)^{\phi}-1}{r}] \quad [\frac{r}{1-(1+r)^{z}}]$ for his z years of retirement. Consequently, the annual retirement consumption of any worker is proportional to his annual saving. Since employment grows at a steady rate we can add over all age cohorts to find a fixed ralationship between Cwr and Sw. Letting the oldest living generation be the numéraire then

$$Cwr = \sum_{j=0}^{z-1} Y(1+g)^j = X[\frac{(1+r)^{\phi}-1}{r}] \; [\frac{r}{1-(1+r)^{z}}] \sum_{j=0}^{z-1}(1+g)^j$$

and

$$Sw = \sum_{j=z}^{z+\phi} X(1+g) = X \sum_{j=z}^{z+\phi}(1+g)$$

Hence

$$Cwr = [\frac{(1+r)^{\phi}-1}{r}][\frac{r}{1-(1+r)^{z}}] \frac{\sum_{j=0}^{z-1}(1+g)^j}{\sum_{j=z}^{z+\phi}(1+g)^j} Sw$$

and we can write Cwr = b Sw where

$$b = [\ .\ ]\ [\ .\ ]\ \frac{\Sigma\ (\ .\ )}{\Sigma\ (\ .\ )}$$

is a fixed number given only r, g, ϕ, z .

Consumption of retired workers is an increasing function of the average propensity to save of workers.

The relationship between s_c, s_w, Sw and Cwr can be shown graphically. In figure 4.2(a) we consider the case of Sc = 1 and in figure 4.2(b) the more general case of 0 < Sc < 1. In each figure the distance between the upper and lower horizontal axes represents per-worker output-income which is fixed given equilibrium value of r and the production function. Reading from the centre to extreme right we see how the components of income change as s_w goes from 0 to α. From the centre to the extreme left we can see how the components of expenditure change as s_w goes from zero to α.[1] The shaded portion of Cw represents Cwrc.

Looking first at figure 4.2(a) some general features stand out. Since $s_c = 1$, Sc = Pc and Sw = Pw for all values of s_w. Moreover, since r is given P is fixed and Pw + Pc is constant for all values of s_w. Similarly the residually determined qualities of W and C are equal and fixed for any value of s_w. The curvi-linear portion of P and S reflects the non-linear relation between s_w and Sw [equation (4.16)]. The diagram makes it clear that in steady-state growth workers as a class cannot change the real wage nor their total consumption by saving more. They can, of course, increase their total income by ΔPw, but that in itself has limited significance because to do so they must, as a class, increase their saving by the same amount. (ΔPw equal to ΔSw is a consequence of assuming $s_c = 1$. Capitalists set the terms of each class saving by their own behaviour; in general, from equation [4.6], ΔPw = 1/ScΔSw.)

The one reason that workers do have for saving is evident and is

[1]There is no reason, of course, why s_w could not exceed α but at any value of s_w beyond α the equilibrium profit rate is no longer independent of s_w. See Samuelson and Modigliani (1966).

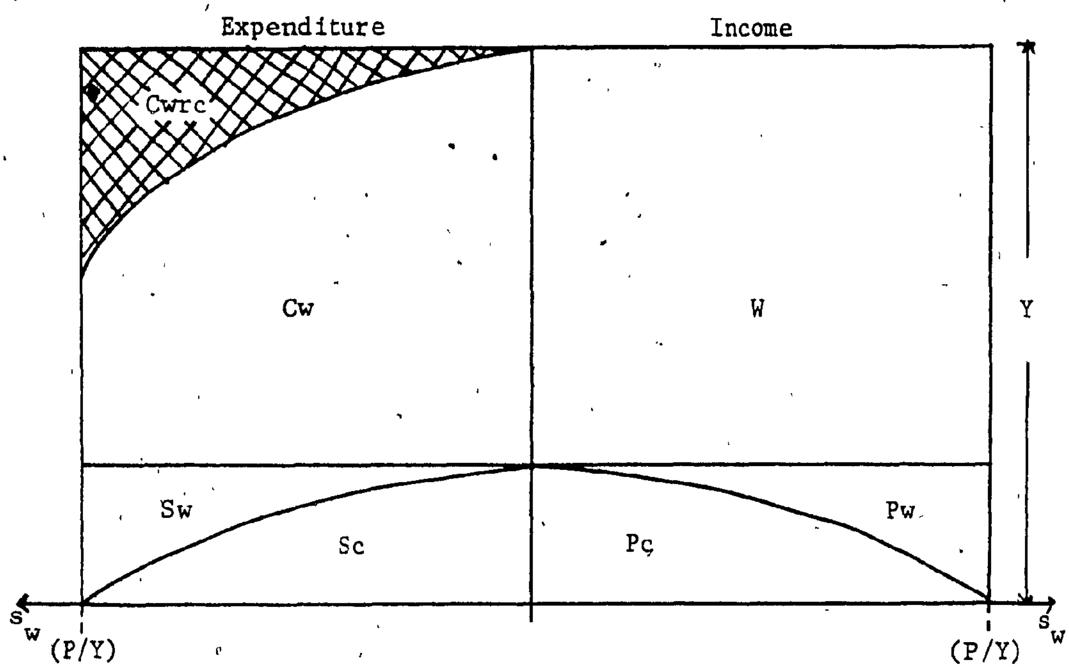

Figure 4.2(a). $s_c = 1$

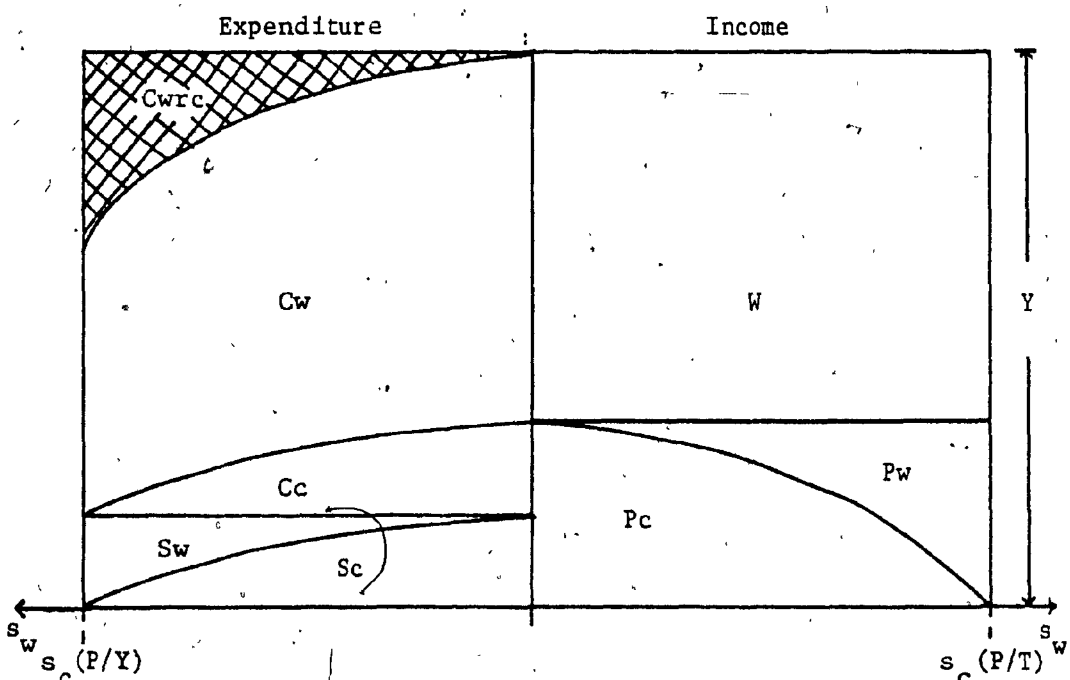

Figure 4.2(b). $0 < s_c < 1$

Figure 4.2. Variations in s_w and the distribution of national income and expenditure.

explained by equation (4.18). An increase in s_w reallocates consumption from working years to retirement years. Given the terms at which they can save (i.e., r, itself estimated by capitalists' behaviour) the higher the propensity to save of workers as a class, the larger is the share of workers' consumption that is being enjoyed by the retired members of that class. This is represented in the diagram by the cross-hatched area Cwr. This is reflected in increased profits of workers as a class, an increase that is attained at the expense of the capitalist class. However, since the increase is "purchased" at the cost of increased saving, the working class as a whole does not increase its absolute amount of consumption.

If capitalists consume part of their profits, then an increase in s_w erodes capitalists' consumption as well as capitalists' saving. Figure 4.2(b) shows the same relation between s_w and the division of per-worker income and expenditure. In this diagram s_c is 0.5 so that total saving is one-half total profits but total consumption is larger than W. There is the same curvi-linear relationship between s_w and Pw, Sw and Cwr as in figure 4.2(a), but now s_w has an influence on the amount of consumption, Cc, which capitalists as a class can enjoy. As s_w increases, capitalists' consumption falls. The reason for this is clear: Cc must always be a fraction $(1-s_c)$ of Pc. As workers increase their saving they "take over" a corresponding larger share of a fixed amount of profits; as this happens, the capitalists' share of profits shrinks and with it the capitalists' consumption.[1] In this case the workers increase their retirement consumption to some extent at the expense of

[1] Limited, as always in this analysis, by the condition $s_w < \alpha s_c$ (4.2).

capitalists' consumption. This is an example of the Keynes-Kalecki "widow's cruse" in the hands of workers rather than capitalists (see Kaldor, 1955-56, p. 368, n. 34).

The Impact of a State Pension Scheme

Since contributions and benefits of a state pension scheme are wage-related any impact that the introduction of a state pension scheme may have in the neo-Keynesian model will be through changes in the behaviour of workers, and in particular, through changes in the workers' propensity to save, s_w. As we have seen, a change in s_w does influence the distribution of income and expenditure between classes; it cannot, however, influence the broad characteristics of a long-run growth equilibrium--the real wage and profit rates, the level of aggregate output and consumption, or the growth rate. Only if a change in the size of a state pension scheme causes workers to increase their saving to such a degree that s_w exceeds αs_c would we see a change in the long-run growth equilibrium. While this is not an impossibility, it would be a fortuitous event, much more a question of the values of particular parameters being close to some critical values than the particular nature of the shock itself. Such an occurrence can not be considered to have any theoretical significance. In any case, as we pointed out above, in the "anti-Pasinetti" range of the neo-Keynesian model, the long-run equilibrium is unstable and may be ruled out of consideration for that reason alone.

Conclusion

In a neo-Keynesian world, capitalists control more than the means of production; by their behaviour they set the terms by which all other groups can save and, to a more limited extent, earn income. If workers do save and are free to respond in their saving behaviour to changes in a state pension scheme, they can only influence the distribution of income and consumption between classes. They have no influence whatsoever over the distribution of income between profits and wage. Aggregate saving, income and consumption as well as the growth rate remain unchanged whatever the response of workers to a state pension scheme so long as the economy is in the Pasinetti range $s_w < \alpha s_c$.

CHAPTER V

THE NEO-MARXIAN MODEL OF A STATE PENSION SCHEME

Introduction

The wage bargain plays a very special role in the Marxian tradition. The set of simultaneous equations which define the neo-Marxian growth model are triangular in the sense that, given capitalists' saving propensity as a class, the real wage determines the value of all the other endogenous variables in the system. A state pension scheme can, then, only affect the solution of the model through an influence on the saving propensity of capitalists[1] or on the wage bargain. We can rule out the former for a program designed for workers, so we are left, by elimination, with the conclusion that a state pension scheme is part of the wage bargain. In this chapter we will show, with reference to Marx and later writers in his tradition, that the impact of a state pension scheme on the wage bargain is the key to assessing its long-run effect. In particular we will show that an increase in the size of a state pension scheme represents an initial gain by labour at the expense of capital which is reflected in a reduction in the profit and growth rates in the long run but that in historical time, in the very long run, this gain is slowly eroded and there is a tendency for the economy to return to the original equilibrium.

[1]Workers' saving propensity may also be affected but as the Pasinetti theorem shows this has no effect on the solution of the model. (See Chapter IV above).

of the wage bargain as is the real wage; the class struggle can centre on either:

> In their attempts at reducing the working day to its former national dimensions, or, where they cannot enforce a legal fixation of a normal working day, at checking overwork by a rise of wages, ... working men fulfill only a duty to themselves and their race. They only set limits to the tyrannical usurpations of capital. Time is the room of human development. A man who has no free time to dispose of, whose whole lifetime, apart from the mere physical interruptions by sleep, meals, and so forth, is absorbed by his labour for the capitalist, is less than a beast of burden ... the whole history of modern industry shows that capital, if not checked, will reckelessly and ruthlessly work to cast down the whole working class to this utmost state of degradation (VPP, pp. 53-54).

Marx makes it clear that he thought that economic power lay primarily in the hands of capitalists but that for political reasons the workers could find support, reluctant though it may be, in the organs of the state.

> As to the limitation of the working day, ... it has never been settled except by legislative interference. Without the working men's continuous pressure from without that interference would never have taken place. ... [and certainly not in the private wage bargain] ... This very necessity of general political action affords the proof that in its merely economic action [emphasis added] capital is the stronger side. (VPP, pp. 58-59).

There is an apparent paradox here. The state represents political power in a capitalist economy and the capitalist class is the beneficiary of the way production is organized, yet it would be an error to say that the state and the capitalist class are identical. Although Marx often spoke loosely of the state as the ruling committee of the capitalist class, he did recognize that an analytic distinction must be made between the two. In particular, there are times when it is necessary for the state to act against the immediate interests of the capitalist class, and even against the long-run interests of a particular sector of the capitalist

State Pension Schemes and the Wage Bargain

There are no direct references to state old age pension schemes in the writings of Marx for the obvious reason that such programs did not exist in his lifetime. However, the elements of a Marxist theory of a state pension scheme can be found, first, by analogy to his treatment to other elements of the wage bargain such as the length of the working day, and second, in Marx's recognition of the distinction between the political roles of the state on the one hand and the dominant capitalist class on the other.

In the early, pure, stages of capitalism, such as it was in England in the mid-nineteenth century, the relative power of the capitalist class was so great as to reduce the working class close to biological subsistence. Capitalists had no interest in the health or even the life-span of the workers so long as labour power was forthcoming and cheap.

> A quick succession of unhealthy and short-lived generations will keep the labour market as well supplied as a series of vigorous and long-lived generations. (VPP p. 57)[1]

Because so few workers lived into healthy retirement, pensions as such were not an issue in the mid-nineteenth century (retirement, in fact, was considered by Marx as a sub-category of pauperism [Capital, Vol. 1, chap. 25, pp. 643-644]).[2] The physical exploitation of the worker was primarily the result of long working days and so the workers' first efforts to fight back were directed at shortening the working day (Capital, Vol. 1, chap. 10). The length of the working day is as much an element

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[1]References cited (VPP, page) are to Marx, Value, Price and Profit, Aveling edition, as published by International Publishers, 1935.

[2]References to Capital are cited (Capital, Volume, chapter and page). The edition is that of International Publishers, 1967.

class, in order to maintain the political hegemony of the capitalist class as a whole. This intervention characterizes the history of condition of the working class. Indeed, capitalists, individually, may recognize the necessity of such interventions. For example, Marx cites a petition to Parliament of a group of manufacturers who wanted to see a law to limit child labour, the abuses of which they acknowleged but found themselves unable to correct acting individually (presumably for fear of losing competitive advantage):

> Much as we deplore the evils before mentioned, it would not be possible to prevent them by any scheme of agreement between the manufacturers ... some legislative enactment is wanted. (Capital, Vol. 1, chap. 10, p. 270, n. 2).

When the state intervenes to change some specific condition of the wage bargain in favour of workers, there is a cost in terms of reduced production. Who bears it? Since labour is already pressed to its limit (by definition), most, if not all, the cost must be borne by capital. One can look at it a different way, but the result is the same. If we consider work-ameliorative costs as system-maintenance costs and, as such, as similar to costs of police, defence and legal administration, then these costs must be attributed to the class which benefits most by the system. In a capitalist system it is the capitalist class which benefits most. State old-age pension schemes are a form of work-amelioration and as such the cost must come out of profits.

Historical evidence for this interpretation of Marx can be found in the reactions of the Marxian socialists to the first modern state welfare schemes which were introduced by Bismarck in Germany in the late nineteenth century. The initial reaction of the Marxian socialists of the time was one of hostility: they argued that the true interests of the working class could only be served by the creation of a new economic order and that reforms in the present system were an illusion--"nothing but a tactical means designed to lure the workers away from the correct path".[1]

While these initial reactions against the schemes were in the revolutionary Marxian tradition, (Rimlinger, 1971, pp. 122-125), as it became clear that the workers were not ready to follow that path but, in fact, readily welcomed these programs,[2] the Marxian socialists were forced to choose between revolution and reform. The crisis caused a split in the socialist party with the revisionists accepting the new idea that capitalism was not in danger of imminent collapse. If this were the case, they concluded, then the working class had to be prepared for a much longer struggle and that meant taking advantage of whatever immediate improvements were offered (Rimlinger, 1971, pp. 125-126). They now took the view that given the circumstances--the circumstances being the continued existence of the capitalist state--state welfare schemes, appropriately financed, operated to the material advantage of the working class.

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[1] From a declaration of the [Marxist] Social Democratic Congress of 1883 which met in Copenhagen; cited by Rimlinger (1971, p. 123).

[2] Parallels to the German experience can be found in the support workers have given similar state initiatives in other countries, such as the following incident illustrates: In 1936, the Republicans campaigned against US old age social security with, among other things, pay-envelope inserts attacking the system. The Social Security Branch responded by distributing explanatory leaflets at factory gates with the aid of labour unions. Altmyer (1966, p. 69).

To answer the question of what were appropriate financial arrangements, we can consider the arguments the socialists used to ensure that employers paid some, if not all, of the direct costs of these programs. Rimlinger (1971, p. 126) summarizes the socialists' arguments against worker contributions:

> First, the worker is unable to pay because he is getting only a subsistence wage. Second, the worker ought not to pay because his insecurity is not his fault but that of the capitalist system, which is operated for the benefit of the capitalists. Third, if benefits must be financed out of wages, they can never be adequate. Fourth, the more heavily the employers are burdened, the sooner capitalism will collapse. Finally, since labor is the sole source of value, employer contributions are nothing but a small partial repayment of what belongs to the worker in the first place.

We need not be concerned with the normative arguments; they were ideological holdovers. What the socialists did recognize was that since the benefits and costs of these social insurance programs were potentially direct modifications to the wage bargain, legal incidence was equivalent to economic incidence. This is the basis of the neo-Marxian analysis of state welfare schemes and is neatly summarized in the first of the arguments cited above--"the worker is unable to pay because he is only getting a subsistence wage".

The most recent statement of the neo-Marxian[1] analysis of state social welfare schemes can be found in Gough (1975). His argument is, essentially, that the state protects the dominant political position of the capitalist class by enforcing certain economic concessions (broadly speaking, social welfare measures) to the benefit of the working class at the expense of the capitalist class. If we follow the details of the

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[1]The historical split in the socialist parties has had its parallels in Marxian theory, which has also, over the past century, been purged of many of its deterministic elements and leaving it, in its current neo-Marxian form, as a positive theory about how capitalist economy operates.

analysis we can see that it parallels the story Marx told in VPP which we have summarized above, the main difference being that with the passage of time the focus of the struggle has changed from hours of work to elements of the social wage.

The relevant part of Gough's analysis is as follows. The capitalist class is not unified but is broken into a number of factions or blocs which are unable to organize themselves effectively; as a result of this weakness in the dominant class the state is forced to step in (ibid, pp. 64-65). Although the state does serve the interests of the dominant capitalist class, the state is not identical to the capitalist class and so its interpretation of the interests of the capitalists class may differ from that class's own interpretation of its interests, especially its immediate interests (ibid, p. 65): Gough continues

> This 'unstable equilibrium of compromise' [between the state and the capitalist class] provides the basis for the whole series of social and economic reforms extracted by the working class in the post-war 'welfare states' of advanced capitalist societies, which yet leaves untouched the political power of capital and the repressive apparatus of the state on which it is ultimately based .... all governments of whatever political complexion depend for their survival under bourgeois democracy on their ability to offer certain reforms and concessions to the struggles of the dominated classes. (Ibid., pp. 65-66).

This need for concessions has been aggravated by the "continuous proletarianization of the population" (ibid., p. 67) which has, despite a higher absolute standard of living for workers, led to the situation where

> the security provided by family production and the possession of a minimum quantum of means of production is no longer present. As a result, collective provision often by the state is more and more necessary, especially since industrialization creates new 'diswelfares' (industrial diseases) and further sources of insecurity (redundancy and shake-out). (Ibid., p. 67).

as well as a need for higher quality labour power (education) and a strong tendency to urbanization (ibid., p. 67). All of this has "resulted in a growing socialization of the costs of production in the shape of growing state expenditure". (ibid., p. 67). In fact, much of the expenditure does not result in any fundamental change in the functional distribution of income (for example, "social security and health services are elements of variable capital when consumed by the productive workforce" (ibid., p. 71) but some of it does, especially pensions for the elderly unproductive groups, which are social expenses (luxuries or Marxian department III output) and represent an additional charge on the share of capital (ibid., p. 71).

We can take a closer look at state pension schemes in this framework of analysis. First, we must recognize that, despite being generally categorized as social expenses, a part of state pension expenditure, is simply "redistribution within the working class as a whole from workers to dependents" (ibid., p. 73) (Unemployment insurance is an obvious example of this sort of transfer.) The role of the state in these cases is simply to act as an organizer, providing a form of insurance that can not be provided privately for reasons of moral hazard and adverse selection. Gough and other writers in the Marxian tradition are vague about just how one distinguishes the <u>intra</u>-class transfer element from the <u>inter</u>-class transfer element (Gough, p. 71) but the implication seems to be that legal incidence of direct taxes corresponds to economic incidence.[1] Thus, the workers' share of contributions is an intra-class transfer and the employers' portion an inter-class transfer. The triangular form of the neo-Marxian model in which the exogenous

[1] In discussing the financing of the nationalized industries Gough writes: "If the resulting deficits are financed from general taxation the cost is passed on to the working population whilst the benefits are reaped by the capitalist sector". (Gough, 1975, p. 78).

determination of the real wage is the key element, lends support to this view as do the political disputes over the division of legal incidence of contribution to social insurance schemes.[1]

The financial share of the cost of social welfare schemes that the state imposes on capital represents what the state judges to be the price of the continued maintenance of the capitalist system in the current historical circumstances.[2] Put another way, the actual outcome is the result of the balance of power between the two classes which can be measured by the degree to which the working class manages to formally shift the financing of the scheme onto capital.

Before turning to the formal analysis, we should consider how the passage of historical time influences the long-run effect we have described. We have argued that the establishment of a state pension scheme represents a gain by the working class at the expense of profits. Whatever the outcome, the analysis begins with a one-time increase in the real wage. Since this increase occurs <u>outside</u> the system which makes up the neo-Marxian economic model, nothing that happens in the workings of the model can reverse it. On the other hand, it is a fundamental assumption of the neo-Marxian model that the real wage tends to the subsistence level of consumption. Although the institution of a state pension scheme increases the real wage, it does not, <u>ipso facto</u>, increase

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[1]See Bryden (1974, passim) for a description of the political struggle in Canada over the financing of old-age pension schemes. Riminger (1971, pp. 125-129) summarizes the debates in Germany over the terms of finance of the Bismarckian program.

[2]"... social insurance is not primarily insurance for workers, but a kind of insurance for capitalists and corporations." (O'Connor, 1973, p. 138).

the subsistence level of consumption. It may do so to some extent but there is no reason to think that the culturally-determined level of subsistence will be raised by the full amount of the gain that a scheme represents. Although the direct impact of an increase in a state pension scheme cannot easily be reversed by capitalists--in any case, it represented the price that had to be paid in the immediate historical context for civil and industrial peace--that does not mean that capitalists will not use every means at their disposal to take back the lost profits.

Capitalists as a class have many means at their disposal to resist any permanent diminution of the historical level of the profit rate. Some are direct, such as labour-saving technical change and the encouragement of immigration of cheap labour, both of which have the effect of increasing the reserve army; others are indirect and are expressed through the control the capitalist class has over money prices and the banking system. None are captured in our model, since they bear, as did the initial state-pension increase, on the wage bargain, and that is, in our model, an exogenous factor. Whatever the methods, the net effect of the capitalists' actions is to slowly erode the gains that the working class has managed to achieve;

> The law of capitalist accumulation ... states that the very nature of accumulation excludes every diminution in the degree of exploitation of labour, and every rise in the price of labour, which could seriously imperil the continual reproduction, on an over-enlarging scale, of the capitalist relation. (Capital, vol. 1, chap. 25, p. 621).

Such erosion may not be complete, and it will certainly be slow, but it will tend to press the real wage back down to its initial level.

The Economic Effects of a State Pension Scheme

We begin the formal analysis with the neo-Marxian model outlined in Chapter II. The basic equations of allocation, distribution and production are unchanged

$$1 = a_oC + a_1(1+g) \tag{2.4}$$

$$1 = a_oW + a_1(1+r) \tag{2.7}$$

$$1/a_o = f(a_1/a_o) \tag{2.8}$$

$$1+r = f'(a_2/a_o) \tag{2.9}$$

as is the saving-investment identity

$$g = s_cr. \tag{2.26}$$

The wage-bargain equation (2.28) is now modified to take account of a state pension scheme:

$$W = \overline{C} + m(t)T \tag{5.1}$$

where $\overline{C}$ is, as before, the subsistence level of per-worker consumption, now identified with the workers' standard of living <u>before</u> the introduction of a state pension, T is the pension payroll tax in real terms and (m)t is a factor which represents that portion of the cost of the scheme which workers have, at time t, managed to shift onto capitalists. The factor m is assumed to be positive and less than or equal to one. Following our discussion in the last section, the value of m at the inception of the scheme, $m(t_o)$, can be approximated by the employers' share of the wage tax. As time passes from the inception of the scheme, this burden will, by one means or another, be shifted back onto workers; hence m'(t) is negative.

Fixed Production Coefficients.

If a_o and a_1 are fixed, equations (2.8) and (2.9) drop out and equations (2.4), (2.7), (2.26) and (5.1) represent the complete system. Substituting (5.1) into (2.7) gives

$$r = 1/a_1 - (a_o/a_1)(\overline{C} + m(t)T) - 1 \tag{5.2}$$

Consequently, the long-run impact is

$$dr/dT = -(a_o/a_1)m(t) < 0, \tag{5.3}$$

and thus, from (2.26)

$$dg/dT = -s_c(a_o/a_1)m(t) < 0. \tag{5.4}$$

That is to say, the new long-run steady-state equilibrium will have lower growth and profit rates than the initial equilibrium. The response of m to t, historical time, means that in the very long run, as m "evaporates", g and r tend to their original values.

In figure 5.1 the horizontal profit-as-surplus line, (a)—equation (5.2)—shifts downward at the inception of a state-pension scheme. The initial impact on the long-run growth equilibrium is the movement of the intersection of this line with the saving-investment identity, (b)—equation (2.28)—from point A to point B. Both g and r fall. If the saving function is "classical", s_c=1, then $\Delta g = \Delta r$; if, however, capitalists consume out of their profits as well as save the change in the growth rate is proportionally lower: $\Delta g = s_c \Delta r$.

Nothing has been said about worker saving; this is consistent with the Marxian view that the real wage is always being pressed down to subsistence—at which level there would be little margin for saving. This is most apparent if we set T = 0 and s_c=1; then,

$$C = W = \overline{C}.$$

Total per-worker consumption, (which is in this case average consumption)

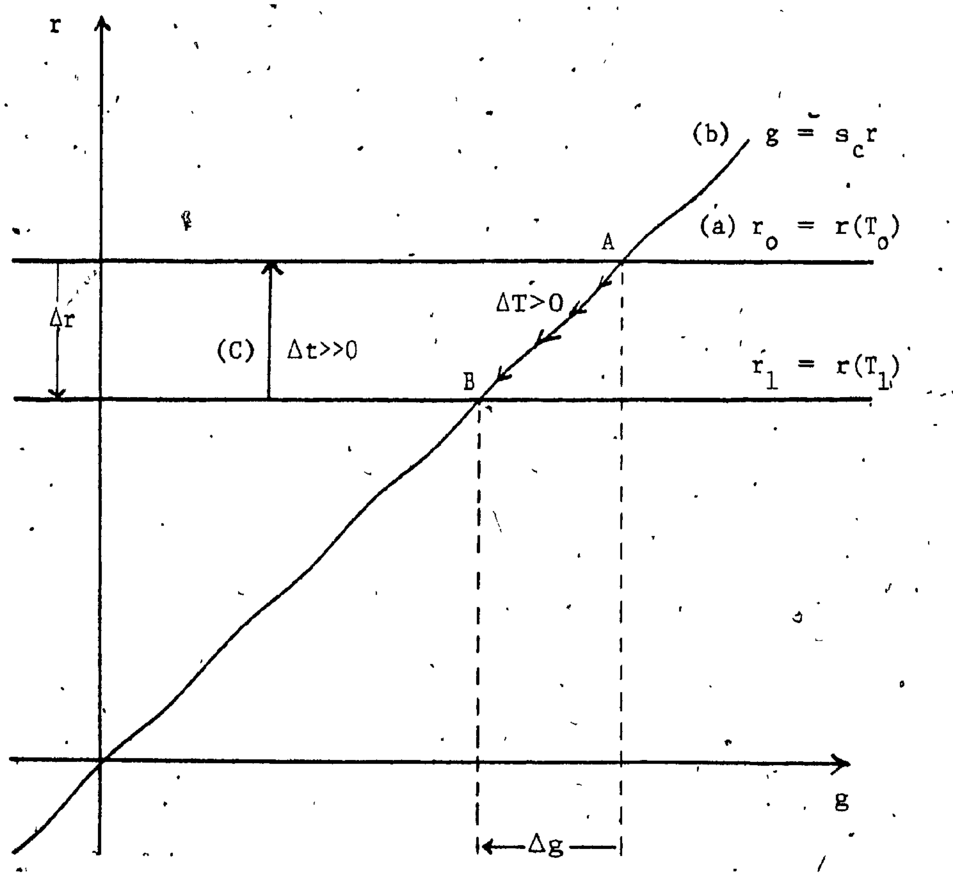

Figure 5.1. An increase in the state pension in a fixed-coefficients neo-Marxian model.

is identical to the real wage, i.e., it already is at the level of subsistence.

Finally, as indicated by the arrow (C), with the passage of time the profit-surplus line would drift back up towards its initial position as capitalists succeed in recovering what they lost in the bargaining over the financing of the state pension scheme. This happens in historical time or what we will call the very long run.

Variable Production Coefficients.

If production coefficients are variable, we have two more variables and two more equations than we had in the fixed-coefficients case. Remembering now that a_o and a_1 are functions of r, we can determine, the long-run impact, dr/dT most easily by totally differentiating (5.2)

$$dr = -(1/a_1)^2(\partial a_1/\partial r)dr - (\overline{C} + m(t)T \{a_1(\partial a_o/\partial r) - a_o(\partial a_1/\partial r)\} dr$$

$$- (a_o/a_1)m(t)dT. \qquad (5.5)$$

However, when we make the substitutions

$$\partial a_1/\partial r = -a_1\sigma/(1+r)$$

$$\partial a_o/\partial r = a_1\partial/W \qquad (2.24)$$

where σ is the elasticity of substitution parameter, and recall that $\overline{C} + m(t)T = W$, (5.5) reduces to

$$dr/dT = -[a_o(r)/a_1(r)]m(t) < 0 \qquad (5.6)$$

which is identical in form to dr/dT in the fixed-coefficients case (5.3). However, at the new equilibrium profit rate, the technical coefficients

(a_0, a_1) will differ from the original equilibrium and thus the net effect on the profit and growth rates will differ from the fixed coefficients case. In general, if ΔT is positive, Δr is negative, and therefore the labour-capital ratio $[a_0(r)/a_1(r)]$ will have fallen. Thus dr/dT will be smaller in absolute value in the variable-coefficients case than it will be in the fixed-coefficients case.

Conclusion

In the neo-Marxian view of the world, the characteristics of long-run growth depend on the outcome of the struggle between capital and labour over the terms of the real wage. There are occasions when the power of the working class, relative to that of capital, is great enough to wrest certain concessions through the medium of the state. A state pension scheme is such a concession and represents, approximately to the degree it is financed by capital, an increase in the real wage. With such an increase in the real wage, there will be a decrease in the equilibrium profit, and growth rates, in the new long-run.

In historical time an additional factor enters. Because it is in the nature of a capitalist that the real wage tends to a culturally-defined level of subsistence a large part of the state-pension-induced gain of the working class will, in the very long run, be eroded as capital, through the many means at its disposal, regains what it had lost at a moment of temporary weakness.

PART TWO

TESTS

CHAPTER VI

TESTING THE ECONOMIC EFFECTS OF STATE PENSION SCHEMES

Introduction

In part one we developed three alternative long-run models of a state pension scheme. In part two we present the results of an empirical test of the question of which of these three models provides the best description of the real world. In this chapter we will devote ourselves to the question of test design.

In the next section we review the empirical literature both for evidence of the validity of one or other of the three models and for guidance as to which method of testing would be most appropriate for our purposes. In the sections which follow we discuss the test statistic we have chosen, the aggregate saving ratio, and the nature of the data we will use in our tests. There is a brief concluding section.

Review of the Empirical Literature

There have been a number of empirical studies which bear on the question of the economic influence of state pension schemes. Some of these studies are of state pension schemes themselves and some are of related phenomena. In this section we review this literature for

1. Evidence it provides for the validity of the various models we have studied.

2. Directions it suggests in designing a new comparative test of the three models.

We will review the literature first and then consider the two questions.

In studying the long-run economic effects of state pension schemes, researchers have emphasized different aspects. We can identify from categories in which the long-run economic effects of state pension schemes and related economic phenomena can be classified.

1. Intertemporal redistribtion
2. Vertical redistribution
3. Factor distribution
4. Saving and growth

We will review the empirical literature under each of these headings in turn.

<u>Intertemporal Redistribution.</u>

State pension schemes provide intertemporal transfers by providing a way of exchanging contributions during the working years for annuities during retirement. Since from an individual participant's point of view this is similar to investing savings at interest, it is appropriate to compare the yield (insofar as it is meaningful) on state pension contributions to that available on more conventional forms of retirement saving.

Brittain (1972, pp. 158-168) has simulated the U.S. Social Security system to find the lifetime yield to contributions (employee and employer portions) for the typical contributor. Brittain makes the assumption that benefit levels will continue to be raised to maintain the same historical relationship between average benefits and average wages; moreover, he makes alternative assumptions about average real earning growth, family composition, population and mortality trends, and the age at which full-time work is begun. The yield to Social Security contributions is the rate which equalizes the accumulated taxes and discounted benefits in any reference year. Brittain's estimates are summarized in table 6.1.

There is considerable variation in real yield depending on the assumptions used; but all are within the range 2¾% – 6¼%, the lower bound of which is well above the yield for savings accounts over the past 50 years, the upper bound considerably below the real yield on industrial common shares over the entire period and most subperiods between 1920 and 1965 (ibid., 1972, pp. 163, 168). For reasons of risk and lack of familiarity with the stock market, holding discretionary savings in common shares is not a practicable alternative for most small savers. Brittain himself draws no strong conclusion from these computations but if one makes the not-unreasonable-assumption that the yield on savings deposits is the relevant benchmark yield,[1] it follows that U.S. Social Security provides an intertemporal transfer at terms superior to those which could be achieved in the market.

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[1] Other fixed income forms of saving such as life insurance or private pensions would offer real yields comparable to those of savings deposits. No attempt has been made to find estimates of the real yield to private housing for, while an important form of savings for many people, it would not be a substituted at the margin for other forms of retirement savings because of the strong social determinants of investment in private housing as well as indivisibilities in acquisition and capital market rigidities. After the house has been purchased the long-term contract implied by a mortgage leaves almost no room for marginal changes in the rate of house equity acquisition. Munnell (1974, chapter 3) excludes increases in home equity in constructing a savings series to be used as the dependent variable in her investigation of the influence of social security on private saving; she is never very clear about why she does this but the implication is that changes in real estate saving do not provide a great deal of information about the relative importance of retirement saving (Munnell, 1974, p. 45). (Strict neoclassicists have disputed this emphasis on the lack of substitutability between assets; see Upton [1975, p. 1091]). Casual empiricism would suggest that residential housing is a major form of retirement savings.)

TABLE 6.1

ESTIMATED REAL RATES OF RETURN ON U.S. SOCIAL SECURITY CONTRIBUTIONS FOR RECIPIENTS OF AVERAGE EARNING UNDER VARIOUS ASSUMPTIONS

(percentages)

Type of cost projection[a], starting age and family composition	Growth rate in real earnings	
	2 percent	3 percent
Low cost, age 18		
Man[b]	2.92	3.83
Woman[c]	3.43	4.35
Couple[d]	4.52	5.51
Low cost, age 22		
Man[b]	3.53	4.58
Woman[c]	4.06	5.11
Couple[d]	5.23	6.28
High cost, age 18		
Man[b]	2.78	3.68
Woman[c]	3.28	4.21
Couple[d]	4.38	5.32
High cost, age 22		
Man[b]	3.42	4.46
Woman[c]	3.92	4.98
Couple[d]	5.12	6.16

SOURCE: Brittain (1972) Table 6-5, page 167

[a] 'Low cost' is high birth rate, high mortality; 'high cost' is low birth rate, low mortality (alternative projections developed by the U.S. Social Security Administration).

[b] Single man or married man with wife who worked.

[c] Single woman or married woman with nondependent husband.

[d] Couple eligible for wife's benefit.

Vertical Redistribution

Vertical income redistribution is built into state pension schemes whenever the implicit yield is related to a participant's lifetime income or with some other objective characteristic which is itself correlated with income. Brittain (1972) has shown that there is a strong bias in U.S. Social Security to low income participants.[1] Some of his results are presented in table 6.2. It is clear that, *ceteris paribus*, the higher the taxable earning level, the lower the implicit yield.

These results suggest that, in the U.S. Social Security System, there is a progressive element in the structure of benefits. This analysis is not complete, however, because there are a number of other important considerations which might, if taken into account, change the conclusions. For example, the age at which a career is begun can be critical. Postponing entry into the labour force by as few as four years (e.g., university) has a striking effect on the yield to contribution in both Canada and the U.S. In the Canadian plan, for instance, the calculation of pension benefits is a simple linear function of contributions. By starting work later one's contributions and pension benefits fall proportionally. However, were the date of contribution considered (as it is implicitly in calculating a compound yield), benefits should fall much more for contributions dropped in the early years than for contributions dropped in later years. Since this does not happen in the Canada/Quebec scheme, starting work later, e.g., going to university, raises the implicit yield of the contributions. Some other characteristics to which the yield would be sensitive have been mentioned by Brittain but are

[1] See also Aaron (1967, pp. 63-67).

TABLE 6.2

IMPLICIT LIFETIME YIELDS TO U.S. SOCIAL SECURITY

CONTRIBUTIONS UNDER VARIOUS ASSUMPTIONS AND EARNINGS LEVELS

(percentages)

Starting age, growth rate of earnings, and family composition[a]	Taxable earnings levels				
	$2,000	$4,000	$6,000	$6,600	Mean
Starting age 18					
(a) Growth rate of 2%					
Man	3.78	2.82	2.39	2.30	2.92
Woman	4.27	3.34	2.93	2.85	3.43
Couple	5.34	4.43	4.02	3.94	4.52
(b) Growth rate of 3%					
Man	4.72	3.73	3.30	3.21	3.83
Woman	5.22	4.26	3.84	3.76	4.35
Couple	6.28	5.36	4.97	4.88	5.51
Starting age 22					
(a) Growth rate of 2%					
Man	4.46	3.43	2.98	2.89	3.53
Woman	4.97	3.96	3.53	3.44	4.06
Couple	6.10	5.13	4.71	4.62	5.23
(b) Growth rate of 3%					
Man	5.51	4.47	4.02	3.92	4.58
Woman	6.00	5.01	4.57	4.48	5.11
Couple	7.17	6.18	5.75	5.66	6.28

SOURCE: J. Brittain The Payroll Tax for Social Security (1972), Table 6-6, p. 170

[a] "Man" denotes a single man or a married man whose wife worked; "woman" denotes a single woman or a married woman with nondependent husband; "couple" denotes a couple eligible for the wife's benefits.

not taken into account. The lower mortality of low income people reduces the yield from that calculated (Brittain, 1972, p. 196n). The benefit schedule favours families in which only the husband (or wife) works; since a non-working wife is more typical of middle class families, this characteristic would further favour this group of participants (ibid., p. 174). Finally, assuming that employees bear the full burden of both employee and employer taxes, the reduced tax payable by the self-employed gives this group an advantage vis-à-vis employees. How this advantage distributes by income class depends upon the distribution of self-employed by income class (ibid., p. 196n).

The most commonly adduced evidence of the redistributive impact of government activity comes from fiscal incidence studies. The attractiveness of these studies lies in their comprehensive nature which allows the total redistributive impact of government to be numerically evaluated. Furthermore, because of the way in which the results are built up, it is possible to isolate the incidence of any particular program. For instance, in Gillespie's (1976) study, the social security sector (Unemployment Insurance, Old Age Insurance, and Guaranteed Income Supplement, Canada/Quebec Pension Plan, and Family Allowances) is found to be, on standard incidence assumptions, strongly redistributive whereas the net impact of the whole government sector is shown not to be very redistributive. The method is straightforward: each tax and each expenditure is distributed across income classes on the basis of some shifting hypothesis; in the case of Canada/Quebec Pension Plans the contributions fall proportionally on labour income (up to the maximum) and benefits are assumed to be enjoyed by the recipients as received.[1,2]

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[1] In the year for which the study was done, 1969, benefits under the Canada/Quebec Pension Plan were very small although contributions

Just how appropriate are these shifting assumptions? There seems to be little disagreement that the wage taxes which finance social insurance programs are regressive over the whole range of income and particularly burdensome in the conventional use of the term for low earners who have little or no relief as they have from income taxes (see Britain, 1972, passim). As for the distribution of benefits, as Gillespie himself acknowledges, it is by no means self-evident that those who receive the pension cheques are those who ultimately benefit (although he does take that as his standard hypothesis). To make the point he applies as an alternative expenditure incidence hypothesis what Musgrave (1968) has called the "prudent humanitarian hypothesis." Under this hypothesis, social security transfer payments are credited not to the recipients but instead to more well-to-do households. The rationale is that these programs are designed not for the purpose of redistribution, but rather to minimize the cost of ensuring that minimum level of social welfare that the rich feel is their humanitarian obligation to provide (Gillespie, 1976, p. 427; Musgrave, 1968, pp. 323-346). Gillespie does not provide comparative figures showing the fiscal incidence of the social security sector of government separately but the importance of incidence assumptions (and perforce, the economic model underlying them) is revealed in table 6.3 where the total fiscal incidence of government under the standard expenditure incidence assumptions is compared to the total fiscal incidence under the prudent humanitarian

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were collected at full rate. Under standard incidence assumptions, this sector will tend to show up as more redistributive to lower income groups as the plan matures and benefits rise to meet contributions.

[2]See also Musgrave and Musgrave (1973, pp. 670-672) for some U.S. results.

TABLE 6.3

FISCAL INCIDENCE, ALL LEVELS OF GOVERNMENT, CANADA, 1969, UNDER ALTERNATIVE SOCIAL SECURITY EXPENDITURE ASSUMPTIONS

(percentages of broad income)

Incidence Assumption	Family Broad Money Income Class								
	under $2,000	$2,000 2,999	$3,000 3,999	$4,000 4,999	$5,000 5,999	$6,000 6,999	$7,000 9,999	$10,000 14,999	$15,000 & over
Standard assumption	200.7	93.9	48.2	27.5	15.6	8.6	1.3	-7.8	-10.8
Prudent humanitarian assumption	133.5	61.4	37.0	24.8	13.9	6.2	1.7	-3.8	- 6.3

SOURCE: Gillespie (1976) Tables 3 & A-11 hypothesis. Although only the one set of expenditures is altered in the table, the overall fiscal incidence changes considerably. Clearly the results are very sensitive to the incidence assumptions. Rather than using these somewhat arbitrary assumptions it would be preferable in a study of the overall distributive impact of government to use a general equilibrium framework based on more fundamental behavioural assumptions.

There is a more serious error in the design of fiscal incidence studies which arises from their static nature (Polinsky, 1973). Looking at cross-section data, as these studies do, we see the relatively rich (the workers) being taxed for the benefit of the relatively poor (the retired). But, whether or not contributors would have freely saved for their old age in the absence of a pension scheme, given the plan they are conscious of the contributions going toward a future pension. (Certainly governments encourage this belief and in many schemes the final pension bears some relation to aggregate contributions.) Thus, only if the provisions of the plan are such as to make the implicit rate of return to contributions higher for the poor than the rich (measurement on a lifetime basis) could we say that state pension schemes are redistributive. Though this may be the case (see above for rates of return to U.S. Social Security), fiscal incidence studies cannot demonstrate it.

Factor Income Redistribution.

A state pension scheme may, by changing private saving, influence capital accumulation in the economy. If the level of capital intensity falls and there is a relationship between the value of the marginal product of a factor and its price, we would expect that the introduction of state pension scheme programs would result in a change in the rate of factor returns turning against labour and in favour of capital; wage rates would be lower, profit and interest rates higher.

Using the results of regressions run with aggregate U.S. time series data, Feldstein (1974, pp. 919-920, 922-924) estimates that, without Social Security, the wage rate would be 15 percent higher and the interest rate 28 percent lower.[1] These results depend critically on the assumption that state pension schemes reduce private saving. We turn next to the empirical evidence of the influence of state pensions on saving.

Saving and Growth.

In the years immediately following the publication of Keynes's General Theory underconsumption was considered a serious threat to economic growth in the long run and stability in the short run. At that time pay-as-you-go state pension schemes were seen as having a stimulating effect on consumption by transferring income from the economically active who have positive saving propensities to retirees who have very low or even negative saving propensities. Although the schemes were deliberately designed to be fully or substantially self-financing and to appear to the public as analogous to private pension schemes[2] the lifetime savings features of the plans were not, at first, taken

[1]Although his rough estimate shows large changes in factor returns, the assumption of constant-returns-to-scale Cobb-Douglas technology implies constant factor shares (Feldstein, 1974, p. 924). The changes in factor returns are compensated for by changes in factor intensity.

[2]This was even true of some universal pension systems. The original Canadian program was a universal, flat rate pension yet care was still taken to earmark slices of certain general taxes to maintain the illusion of a self-financing program. (In practice, however, deficits in the Old Age Security Fund were automatically made up out of general revenues with no change in the auto-financing formula). (Bryden, 1974, pp. 105, 190-191).

seriously by economists. It was not until the 1950s, with the development of the new life cycle and permanent income consumption theories, that contributions to state pension schemes began to be looked at as substitutes for free saving (with the actual degree of substitution left as an empirical question).[1] The first, albeit indirect, evidence was advanced in the mid-1960s. The results were rather unexpected from the point of view of neoclassical theory.

In 1965 Cagan (1965) published a study of the influence of pension coverage on savings behaviour. He found that, far from pension contributions being substitutes for private saving, they were in fact associated with increased private saving: one complements the other.[2]

Cagan hypothesized a recognition effect to explain the phenomenon:

> The accompanying increase in . . . other saving may . . . be explained in terms of the attitude of people during their working years toward preparing for retirement. Pension coverage draws attention to the problems of providing for retirement . . . (ibid., p. 53).

Cagan studied the connection between private pension plan coverage and saving, both voluntary contracts in contrast to a compulsory state pension scheme. It is so often cited in connection with the economic effect of state pension schemes on saving that it is worth examining in some detail here.

Disaggregating saving into Discretionary Saving--bank deposits, securities--Contractual Saving (CS)--life insurance and mortgage repay-

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[1] For example Friedman (1957, p. 123) has pointed out that the rate of substitution is theoretically indeterminate because, on the one hand, the more restrictive social security asset will be worth less, dollar for dollar than savings held in the form of tangible assets that can be freely disposed of, yet on the other, the implicit rate of return on contributions to an unfunded social security plan is generally higher than the rate available to the individual in the market.

[2] See also Andersen (1967) and Katona (1965).

ments--and Pension Saving (PS), Cagan found that Discretionary and Contractual Saving ratios (the saving component as a proportion of income) were greater, on average, for households which were covered by pensions. In two regressions

$$CS/Y = a_1(PS/Y) \tag{6.1}$$

$$DS/Y = b_1(PS/Y) - b_2(CS/Y) \tag{6.2}$$

where CS/Y is the other contractual saving ratio, PS/Y the pension saving ratio and DS/Y the discretionary saving ratio, the coefficients a_1 and b_1 were both positive and significant. This provided the most direct evidence for the complementarity thesis. There was other statistical evidence that supported this conclusion although there were some anomolous results as well: for example, Cagan found that households reporting partially vested pension rights had significantly higher other-saving ratios than households reporting full vested rights who, in turn, had other-saving ratios lower than these reporting no vested rights.

One serious fault in Cagan's study is the way in which the sample was drawn. Although large, it was unrepresentative and self-selected: members of Consumers Union were invited to fill out a long questionnaire about their household spending behaviour. There was, moreover, no attempt to identify differences between the characteristics of members of Consumers Union and the general population (which would no doubt be considerable, particularly in matters of social class and saving behaviour) to identify the bias from self-selection.

Other questions arise from the statistical techniques used. Munnell (1974) has devoted an entire chapter of her book on saving and social security to a critical reappraisal of Cagan's results emphasizing the manner in which he has analyzed the data rather than the data itself. Her central point is that running the sort of regressions he did, with the constant terms suppressed and without any attempt to account for other systematic influences such as an income, age, or wealth, means that the estimated coefficients have no particular significance (Munnell 1974, p. 81). She demonstrates this using a more carefully screened subsample of Cagan's data augmented by data from an additional survey of the same population, the results of which allow her to construct a saving variable of greater accuracy than the respondent-supplied estimates of the survey used by Cagan. First she reran regressions (6.1) and (6.2) and got the same positive signs on a_1 and b_1 as Cagan found. Next, using a model devised from a fully specified theory of saving (the Ando-Modigliani life cycle hypothesis) she shows that the results change markedly: reported R^2 is very low, never greater than 0.04, and the t-values on individual coefficients are insignificant in half the regressions. In all cases but one the sign of pension coverage is negative, not positive, as Cagan's hypothesis requires; in fact the value of the coefficient is very close to zero in all cases, and never of more than marginal statistical significance (t-value at its highest is 1.2). As a confirmation of the life cycle hypothesis, this evidence is weak; on the other hand it demonstrates that Cagan's conclusions of a complementary between pension coverage and saving are not warranted.

One anomaly that the new data uncovered in Cagan's regression model was not followed up by Munnell. This arose from Cagan's special treatment of extreme observations. For almost all his regressions, Cagan had excluded from his sample households reporting ratios of other saving to income in excess of 50 percent in absolute value. He justified this in part as a way of excluding what he judged to be gross reporting errors and in part to allow sample to fit a normal curve which would justify stronger interpretations of the tests of significance. He acknowledges that the exclusion changed the value of certain estimated coefficients; however, as it was the sign he was interested in he did not consider this a serious distortion. He argued

> . . . the reasons for each exclusion seem legitimate for our purposes, so that the results do not seem to be biased and indeed should be more reliable. (Cagan, 1965, p. 19).

Moreover, he continued,

> . . . we are interested in differences between normal saving patterns of groups that differ only in their pension coverage and unusual financial circumstances or habits, whatever the cause, only hide the basic differences we are looking for. (Ibid., p. 19).

Even though the excluded questionnaires accounted for only about 5 percent of the sample they had the effect of doubling the partial coefficient of the PS/Y ratio run against DS/Y (from .10 to .21, both with t-ratios greater than 3) and tripling the partial regression coefficient of other contractual saving (from .23 to .73 both with t-ratios greater than 23 [Cagan, 1965, p. 21, table 4]). In two comparable regressions Munnell finds coefficients showing similar peculiar behaviour: the direction of the changes are different, and in the case of b_2 the sign changes; the magnitude of the various changes are comparable (Munnell, 1974, p. 86, table 5-5). Despite this instability and despite the fact that, as she acknowledges, Cagan's justification for excluding extreme observations does not hold for the improved data, Munnell chose to use the reduced sample for the specious reason that the average saving ratios for the new data are "closest to Cagan's" (Munnell, 1974, p. 86). Cagan had some justification for excluding extreme ratios because his saving ratios were based on respondents' reported changes in components of household balance sheets. Having the results of an additional survey of the same sample group, Munnell was able to construct her saving ratio variables by year-end changes of net worth, a procedure much less subject to measurement error. Munnell, as she herself admits (ibid., p. 85), has no basis for choosing the restricted sample. The volatility of the results which are evident in the two regressions she does run using both the restricted and unrestricted data sets suggests that the extreme observations are a source of most of the variation in the sample. Their exclusion makes any conclusions based on the restricted data set highly suspect.

More recent studies of the relationship between state pension schemes and saving using a variety of data bases and statistical techniques have been grounded in more general theories of household behaviour. A number of studies based on the life cycle hypothesis of saving (Munnell, 1974; Feldstein, 1974; Feldstein, 1977; Feldstein and Pellechio, 1979; Kotlikoff, 1979b) provide evidence that state pension schemes do reduce private saving as predicted; on the other hand Leimer and Lesnoy (1980) in attempting to replicate Feldstein's (1974) U.S. time-series study found that correction of a programming error and the use of wider variety of reaction assumptions in the calculation of state pension wealth lead them to the conclusion that state pension schemes (in the U.S., at least) have no impact on saving.[1] Similarly Barro and MacDonald (1979) using a hybrid of the life cycle hypothesis and Friedman's permanent income hypothesis find that international evidence, contrary to Feldstein (1977), is generally inconclusive but points to the no-effect conclusion.[2] Other evidence points in the same direction: Diamond (1976) concludes on

[1]Boyle and Murray (1979), using Canadian time-series data, come to the same conclusion.

[2]These two studies are examined in some detail in chapter VII.

the basis of a U.S. Social Security system survey that:

> "a large fraction of units [households] have little more than Social Security or are relying on a continued ability to earn throughout their lifetimes. One would expect that rational lifetime accumulation patterns would give much wider financial asset holding in the economy since models of lifetime accumulation do not normally imply savings for retirement which is this small except with very large discount factors (Diamond 1976, p. 16).

Using two independent longitudinal data surveys Diamond calculates wealth to income ratios that are "suggestive" that a sizeable fraction of American workers would not follow sensible savings plans in the absence of Social Security (ibid., p. 27).

It is clear that the jury is still out on the question of state pension schemes and saving. The debate centres on questions of method (though the weakness of the data would suggest caution in making any conclusions on this topic) which in turn reflect different views about the appropriate underlying model to use in the analysis.

Discussion

There is little in the empirical literature on state pension and related schemes that can be used as evidence for one or other of the three economic models we are studying.

The studies concerned with the effects on intertemporal and vertical distribution of state pension schemes are all set in a partial-equilibrium framework and as such are of limited usefulness for our purposes. While his results are interesting, the work of Brittain (1972) is directly applicable only to the US Social Security system. Though his methods could be used to examine other national schemes, the conclusions in every case would depend always on the specific terms of the scheme being studied; no general conclusion about incidence can be drawn from his work. As for the fiscal incidence studies, in the question that concerns us, they assume what we wish to test. In particular, the incidence of taxes and benefits is assumed, rather than tested. Gillespie's (1976) comparison of the standard incidence assumption with the "prudent humanitarian" incidence assumption[1] demonstrates the importance of the question we are trying to answer; viz., how does the choice of an underlying economic theory influence the results of applied research? It does not, however, provide any evidence, one way or the other, to answer it.

A study of factor returns might be a useful method of empirically distinguishing among the three models. However, in Feldstein's (1974) paper, the section on factor returns and US Social Security is simply an application of the results of an empirical, time series study of the saving ratio. Again, it assumes what we wish to test.[2]

The saving-ratio studies cited above are the only ones which provide any useful evidence for one or the other models. Most are, either explicity or implicitly, set in a partial-equilibrium, neoclassical framework. The studies done by Munnell, Feldstein, Felstéin and Pellechio all came to the conclusion that the evidence confirmed the neoclassical model. Kotlikoff's results gave partial confirmation of the neoclassical

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[1] We note, in passing, that as applied to state pension schemes, the "prudent humanitarian" hypothesis of Musgrave (1968) is, in fact, what we have called the neo-Marxian theory.

[2] We should emphasize that Feldstein meant this section to be no more than an application. All of Feldstein's studies of state pension schemes have used the saving ratio as the test statistic.

model but Barro and MacDonald, Boyle and Murray, Diamond, and Leimer and Lesnoy all found the evidence gave no support to the neoclassical model as we have defined it. Since all the studies were carried out in a neoclassical framework,[1] in none of these cases are the results applicable to a test of either the neo-Keynesian or neo-Marxian model except insofar as one of these theoretical models can be shown to have a reduced form and set of predicted coefficients consistent with the statistical model used in one or other of these studies.[2] As we shall show in chapters VIII and IX, the reduced form of neither of these models bear any relationship to the econometric framework of the studies cited and thus they cannot be used as evidence for either the neo-Keynesian or neo-Marxian model.

Because we are interested in testing the fundamental assumptions of macroeconomic models, any useful study for this purpose must be carried out in a full-equilibrium framework and at the highest level of abstraction consistent with having meaningful data. Moreover, if we wish to make a formal, statistical comparison of the three models, they must each be tested using a common dependent variable. What this means in practice is that the dependent variable must be a well-defined, highly-

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[1] Barro and MacDonald used a different version of the neoclassical model, one based on the permanent income consumption hypothesis and which leads to a different theoretical prediction about the sign of the partial derivative of saving with respect to the size of the scheme. Boyle and Murray, and Leimer and Lesnoy attempted to replicate Feldstein's work using different data or more refined methods but in both cases they used the neoclassical model. Diamond's study was less formal, but he, too, interpreted his results against the neoclassical, life cycle model.

[2] Though not done in the context of the study of state pension schemes, Modigliani (1970) did attempt a comparative test of the neoclassical and neo-Keynesian models. His method and results are discussed toward the end of chapter VIII below.

aggregated national income statistic, preferably in ratio form.

The choice of dependent variable for the study is made by elimination. We consider three serious alternatives, the profit rate, the growth rate, and the saving rate. There are strong theoretical reasons for choosing the profit rate. In particular, the sign of the derivative of the profit rate with respect to the size of the state pension scheme is unambiguously different in the three theoretical models (see chapters III, IV and V). However, the well-known problem of measuring the economic profit rate[1] is sufficient reason for not using it as a test variable and explains why (in all likelihood) it has been so rarely used in empirical economics. Following the lines of our theoretical investigation, the growth rate is another candidate for dependent variable. There is no literature which uses the growth rate as a dependent variable, probably, (though this cannot be proven) because the adoption of the neoclassical model as a framework for study by most researchers studying this topic rules out state-pension-induced changes in the growth rate by assumption.[2] (Indeed, as we shall show in chapter VIII the growth rate is the principal explanatory variable in studies of state pension schemes in a neoclassical framework.) We are left, then, with the saving ratio.

Our review reveals that every study of state pension schemes which in any way bears on the question of the validity of an underlying economic theory uses the aggregate saving ratio--specifically, the ratio of net private saving to national income--as the dependent variable. It

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[1]Finding an accurate measure of aggregate economic depreciation and of the capital stock are the two most difficult aspects of the problem. There have been two recent studies aggregates profit rates (Nordhaus, 1974; King, 1975) but in neither case did the authors venture beyond the confines of a single country.

[2]Although it does not bear directly on our problem, an interesting attempt to show a connection between state activity and growth can be found in Bacon and Eltis (1978).

is not surprising that the saving ratio has been chosen. Because it is a ratio, measurement problems involving intertemporal and international (as the case may be) comparisons are minimized. Just as important from the point of view of researchers studying state pension schemes in the context of the neoclassical model, there is a direct theoretical relationship between the size of a state pension scheme and the saving ratio in the neoclassical model.[1]

For our purposes, the saving ratio is the best choice as test statistic. It does not have the disadvantages of either the profit or growth rates. There is a literature on the relationship between the state pension scheme and the saving ratio which provides a benchmark for our study (but which is, admittedly, only of direct relevance to testing the neoclassical model). Moreover, the saving ratio has the advantages with respect to measurement (referred to in the last paragraph) to recommend it. Most important, the saving ratio lends itself to being a dependent variable in reduced forms of the neo-Keynesian and neo-Marxian models as well as in the neoclassical model.

In the next section we give a brief overview of the theoretical relationship between the state pension scheme and the saving ratio in each of the three models.

The Relationship Between State Pension Schemes and the Saving Ratio in the Three Models

If we take the well-known steady-state growth equilibrium condition[1]

$$s = g.a_1 \qquad (6.1)$$

[1]This is developed in chapter VII.

[2]This is better known as the Harrod-Domar consistency condition (Solow, 1970, p. 9; Hahn and Matthews, 1964, p. 5) though neither of the original authors used it in the way we will.

where s is the saving-income ratio, g the growth rate and a_1 the capital-output ratio, we can classify the three models we have studied by the way in which the right-hand side of this equation is specified.

If we define a model in which growth rate is determined exogenously by the rate of labour-force growth, $\bar{n}$, and the equilibrium capital-output ratio is a function of the profit rate[1]--itself a function of preferences, β, technology, σ, and policy variables such as the size of a state pension scheme, τ--we have the <u>neoclassical</u> model in three equations:

$$s = g.a_1 \tag{6.1}$$

$$g = \bar{n} \tag{2.16}$$

$$a_1 = a_1(r) = a_1[r(\beta, \sigma;\tau)] \tag{6.2}$$

or, more simply in reduced form

$$s = \bar{n}.a_1(\beta, \sigma; \tau) \quad . \tag{6.3}$$

We know from chapter III that $dr/d\tau > 0$ in stable equilibrium. Combining this with the standard result when marginal product factor pricing holds, $da_1/dr < 0$, then

$$\frac{ds}{d\tau} = \bar{n}(da_1/d\tau) = \bar{n}(da_1/dr)\frac{dr}{d\tau} < 0 \ . \tag{6.4}$$

Equations (6.3) and (6.4) form the basis of the test of the neoclassical model of state pension schemes (chapter VII).

The two "classical models", the neo-Keynesian and neo-Marxian differ from the neoclassical in that the units of behaviour are not individuals but classes. In the simplest case there are two classes, distinguished, among other things, by their fixed propensities to save.

[1]The neoclassical model developed in chapters II and III did not express a_1 explicitly in functional form, but there is no reason that it could not have done so. Expressing a_1 in functional form is not necessary for testing purposes either; it is done here simply for purposes of exposition.

Equilibrium in these models can be characterized within a certain range[1] by the following relation

$$g = s_c r \tag{2.26}$$

where s_c is the saving propensity of the capitalist class. Substituting in equation (6.1) we have

$$\begin{aligned} s &= s_c r\, a_1 \\ &= s_c \alpha(r) \end{aligned} \tag{6.5}$$

where $\alpha(r)$ is the share of net profits in net national income, in general a function of the profit rate. In this formulation the growth rate no longer enters explicitly as a determinant of the saving ratio.

As written equation (6.5) is common to both "classical" models. They differ in how the equilibrium value of r is determined. In the <u>neo-Keynesian</u> model, the investment function

$$g = i(r) \tag{2.27}$$

closes the model. Since both (6.5) and (2.27) are functions of r, they can, in principle, be combined, expressing s as a function of other variables.

$$s = \Sigma(\cdot) \ . \tag{6.6}$$

Because the functional form of (2.27) has not yet been specified, we can say little about the functional form of Σ or its arguments. We can say, following the analysis of chapter IV that no measure of a state pension scheme will appear in the right-hand side of (6.6). The test of the neo-Keynesian model is reported in chapter VIII.

In the <u>neo-Marxian</u> model the profit rate is a function of the real wage W which in turn is function of the size, T, and age, t, of a state

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[1]The interpretation of equation (2.26) differs, for the neo-Keynesian and neo-Marxian models out of equilibrium; see chapter II. As an equilibrium condition (2.26) is common to both models so long as $s_w < s_c\alpha$ where s_w is the propensity to save of the working class and α is the net profit share (Pasinetti, 1962).

growth, we are bound to assume Harrod-neutral (labour-augmenting) technical progress as this is the only form compatible with steady state growth (Hahn and Matthews, 1964, pp. 51-52)[1].

Mathematically, Harrod-neutral (or labour-augmenting) technical progress has the useful property that it is additive with employment growth (which is, in the neoclassical model, identical to labour-force growth). Thus in the neoclassical model, where we wrote

$$g = \bar{n} \tag{2.16}$$

we can now write

$$g = \bar{n} + \bar{h} \tag{6.9}$$

where $\bar{h}$ is the rate of Harrod-neutral technical progress; g is now measured simply by the rate of growth of national income[2]. Harrod-neutral technical progress is also assumed in the test of the neo-Marxian model, though the context is somewhat different as the assumption does not bear directly on the measurement of variables. The question is taken up in chapter IX.

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[1] We should point out that the theoretical convenience of this assumption, just as the theoretical convenience of the steady-state assumption itself is not an argument for its empirical relevance (Hahn and Matthews, 1964, p. 53). However, in the context of the models we are working with, the assumption is not an inappropriate one. (See chapter II.)

[2] Modigliani (1966 p. 168-169) working from the macroeconomic foundations of the life cycle hypothesis and making reasonable assumptions about the values of parameters, shows that "at least for rates of growth in the empirically relevant range, the relation between the saving ratio and the rate of growth is quite similar whether the growth is due to population growth only, productivity only, or, consequently, some mixture of the two" (ibid., p. 168). This is an important result because we have not investigated here the implications that Harrod-neutral technical progress has for the demand functions in the neoclassical model.

pension scheme (chapter V)

$$r = \phi[W(T,t)] \ . \tag{6.7}$$

Substituting in (6.5) we have

$$s = s_c \alpha(T,t) \ . \tag{6.8}$$

As we shall show in chapter IX, the signs of ds/dT and ds/dt depend on the value of σ, the elasticity of substitution, but unless the elasticity is unitary[1], the two derivatives will have opposite signs. Equation (6.8) is the basis of the test of the neo-Marxian model of state pension schemes which is reported in chapter IX.

A Note on Technical Progress

Up to now the theoretical development has assumed no technical progress. The growth rate, g, represents the growth of total output and total capital. Apart from one-time changes in the technical coefficients, per-worker output, $1/a_0$, and per-worker capital stock, a_0/a_1 remain stationary. These assumptions are no longer reasonable when dealing with real-world data; we must consider how technical progress (i.e., productivity growth) is to be accounted for in our empirical models. As it happens, the need for an assumption about the form of technical progress arises directly in connection with neoclassical model and indirectly in connection with the neo-Marxian model. In the neo-Keynesian model, explanatory variables are used only in ratio form so accounting explicitly for technical progress does not arise in this case.

In a sense, the choice of a method for dealing with technical progress has already been made. By making the assumption of steady-state

[1]If elasticity is unitary, the underlying production function is Cobb-Douglas and α is unresponsive to changes in real wage including those caused by changes in the size of a state pension scheme.

The Sample and the Basic Data

For a test of the nature we wish to make that is concerned with the fundamental characteristics of a capitalist economy, we need a sample which offers the greatest degree of variation possible in the independent variables. Since state pension schemes are normally national in scope, and changes in the terms of the plans are neither that frequent nor that large, the needed variation can best be found in an international cross-section sample.

Unfortunately, over such a sample there will be many large and relevant differences between units of observation. To a limited extent we can take these differences into account in the econometric models, but no international sample could be large enough to provide the degrees of freedom necessary for a fully-specified model. To limit as much as possible the potential error implied by a less-than-fully specified model, our sample includes only countries at a comparable level of development. Thus, the countries chosen for the sample are all western and industrialized.

The use of a common sample for the tests of each of the three models permits a comparative test. In the remainder of this section the basic data, that on saving ratios and state pension schemes, is discussed. Additional data that is used in testing individual models is introduced in the chapter in which it is first used.

The sample is based on decade observations on thirteen countries for each of the 1960s and the 1970s. The saving ratios are taken as decade averages to eliminate cyclical effects, and are constructed with, where possible, up to 9 annual observations. The pension variable (and most of the other variables) are decade-representative observations, Cost). For the 1960s, the observations are for the fiscal year ended 1965 and for the 1970s, the fiscal year ended 1974. The data is not supplied directly; in this case we have multiplied "pensions as a percentage of total benefits" by "total benefit expenditure in national currency units" (ibid., table 8) and divided the product by GDP (ibid., appendix) to get PEN. Note that this measure of the pension variable is not exactly the correct one for either the neoclassical or the neo-Marxian models. The necessary adjustments to these basic data are described in the chapters VII and IX.

The statistical characteristics of s and PEN can be summarized as follows:

	s	PEN
Mean	0.1077	0.0506
Standard Deviation	0.0390	0.0194
Range	0.0406 - 0.1977	0.0213 - 0.0880

In the actual regressions, the data is weighted to reduce heteroscedasticity (this procedure and the source of the weights is described in chapter VII). Weighted, the statistical characteristics of the data change somewhat. Since it is the variation in the weighted data that is statistically significant, we show the mean and standard deviation of the weighted data as well:

	s	PEN
Mean	0.1010	0.0461
Standard Deviation	0.0360	0.0179

The coefficient of correlation between s and PEN is 0.1813 unweighted, and 0.5568 weighted.

usually taken at the mid-point in the decade--1965 for the 1960s, 1974 for the 1970s.

The thirteen OECD countries that make up the sample were chosen on the basis of availability of reliable data and indicated above, that the country be western and industrialized. On account of missing data two observations in the 1960s have been lost; there are, thus, 24 observations in all. The countries in the sample are: Australia, Austria, Belgium, Canada, Finland (1970s only), France, West German, Italy (1970s only), Netherlands, Norway, Sweden, UK, USA.

For the saving ratio, s, OECD, National Accounts of OECD Countries, 1950-1978, Volume II (cited OECD, National Accounts, 1950-1978) is the source. Because data on pure private saving is incomplete, private plus public corporate saving is used. This is defined as the sum of corporate (private and public), private non-profit institutions serving households, and household saving; general government saving is excluded. On the basis of the few countries for which there are data including and excluding public corporate saving, it would seem that its inclusion does not make much of a difference.[1] The denominator of the saving ratio is national income. The observations are averages of the actual ratios for the 9 year periods 1961-1969 and 1970-1978; exceptions are Norway 1962-1969, Austria 1964-1969 and 1970-1977, Australia 1970-1977, and Finland 1975-1978.

The basic state pension scheme variable is PEN, the ratio of state pension benefits paid to GDP. The source of PEN is ILO, Cost of Social Security, 9th International Inquiry (table 8 and appendix) (cited ILO,

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[1]The correlation coefficients for the saving ratios based on the two definitions are .9947 and .9939 for the 1960s and 1970s respectively (5 observations and 6 observations).

CHAPTER VII

TESTING THE NEOCLASSICAL MODEL

Introduction

In the neoclassical model the aggregate saving ratio can be decomposed into the product of an exogenous growth rate, $\bar{g}$,[1] and an endogenous capital-output ratio, a_1. In equilibrium, the capital-output ratio reflects both individual preferences[2] (as summarized in the parameter β) and production possibilities (as summarized in the elasticity of technical substitution parameter σ). The size of the state pension scheme, τ, affects this equilibrium and thus we write the reduced form saving function in steady-state growth as[3]

$$s = \bar{g}.a_1(\beta,\sigma;\tau) \quad (7.1)$$

In the orthodox neoclassical interpretation, a_1, and consequently s, is a negative function of τ,[4] i.e.,

$$\frac{ds}{d\tau} = \bar{g}\frac{da_1}{d\tau} < 0 \quad (7.2)$$

on the grounds that state pension wealth displaces real wealth (Feldstein, 1974). Although Feldstein (1974, 1976, 1977) also recognizes the possibility that an "induced retirement effect" might account for a net positive

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[1] In a model without technical progress, $\bar{g} = \bar{n}$, the rate of labour force growth. In a model with technical progress, $\bar{g} = \bar{n} + \bar{h}$, where $\bar{h}$ is the rate of Harrod-neutral technical progress (equations [6.9]).

[2] In which case we equivalently speak of a_1 as the wealth-income ratio.

[3] Cf. equation (6.3)

[4] CF. equation (6.4).

In the next section the neoclassical saving model incorporating a state pension scheme is developed in econometric form. Following that are sections on the data used as explanatory variables and on the results of the estimation. There is a short concluding section.

Econometric Specification

Regression analysis using international cross-section data is the basis for the test; the point of departure is the pioneering work on international differences in saving rates of Houthakker (1965) and Modigliani (1970). Modigliani's study is particularly important because in it he develops from a clear theoretical basis the testable implications of the neoclassical model in long-run equilibrium growth.

A clear theoretical development of a testable model is important given the fact that the empirical literature is not at all conclusive about the validity of the neoclassical model for the study of state pension schemes.[1] For example Feldstein begins with a multiplicative model. He

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in chapter III there is a range along the saving-investment equilibrium condition where g is a declining function of r and thus

$$\frac{dr}{d\tau} < 0$$

Since in a one-good model $\frac{da_1}{dr}$ is always negative $\left(\frac{da_1}{dr} = -\frac{\alpha a_1}{1+r} \text{ where } 0 \leq \hat{\alpha}\right)$ the sign $\frac{ds}{d\tau}$ can be positive. Specifically

$$\frac{ds}{d\tau} = \bar{g}\,\frac{da_1}{dr}\,\frac{dr}{d\tau} \gtrless 0$$

as the share of profits in national income, α, is greater or less than

$$\frac{-(\sigma-2p\tau) - \sqrt{\sigma^2 - 4\beta\tau\sigma + 4\beta\tau}}{2\beta\tau - 1}.$$

(See also Modigliani [1974, p. 15] for a discussion of the theoretical ambiguity of the sign $\frac{ds}{dr}$.) This line of argument is pursued in appendix B.

[1]For international cross-section studies see the papers by Feldstein, and Barro and MacDonald referred to above. A similar variety of conclusions can be found in time series studies (Feldstein, 1974; Munnell, 1974; Barro, 1978; Leimer and Lesnoy, 1980) and those using panel data (Feldstein and Pellechio, 1979; Kotlikoff, 1979b). In a major review article Danziger, Haveman and Plotnick (1981) find the same inconclusiveness.

association between the size of a state pension scheme and the saving ratio, his[1] is the first clear statement of the neoclassical displacement hypothesis[2] in the context of state pension schemes. (In chapter III we showed that the effect of a change in the size of a state pension scheme on the equilibrium profit rate need not be positive. It can be shown that this ambiguity of sign holds for the saving ratio as well.[3] It must be emphasized however that non-negative relationship between the size of a state pension scheme and the saving ratio can only occur if the macro-equilibrium with which it occurs is unstable. Since such a situation would not be characteristic of real world economies this possibility is of limited practical interest.)

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[1]Feldstein's (1974, p. 907 ff.) immediate purpose was to make a clear distinction between the neoclassical model on the one hand and certain ad hoc theories (Cagan's "recognition effect" and Katona's "goal gradient" hypothesis) which had come to prominence in the 1960s as attempts to explain an apparent positive relationship between pensions and saving. (See also Munnell, 1974.)

[2]Barro and MacDonald (1979) dispute this interpretation of the neoclassical model. In their version (which we might entitle "super-neoclassical") the behavioural unit is not the household but rather the genealogical family with an infinite life span. Because the utility maximizing unit cuts across generations, attempts by the state to influence the intergenerational balance of resources will be counteracted by intrafamily transfer. For example, because of the concern of parents for their children's welfare, the benefits of a state pension scheme will not be consumed but rather will be saved and passed on as bequests to children in order to compensate them for the taxes which they, as a generation, pay to finance their parents' benefits. In other words, though the taxes of the pension scheme may displace saving, they do so only to the extent that benefits augment it; thus the net effect on real saving is zero. To call this a variation of the neoclassical theory is to make the neoclassical theory a meta-theory. The position taken in this thesis is that to be of any use in explaining economic behaviour economic models cannot be infinitely flexible. Although there may be secondary influences at work, at the core of the neoclassical model is the view that maximizing takes place at the individual level. A model based on the family line with an infinite life span is not a neoclassical model in our lexicon.

[3]An obvious example is the case of fixed technical coefficients: if K/Y is fixed for technical reasons then $\frac{ds}{d\tau} = \frac{da_1}{d\tau} = 0$ by definition. More in- interesting is the case of inelastic technical substitution. As we saw

argues (1977, p. 181) that equation (7.1) could be estimated by a model in the form

$$s = \bar{g}(\alpha_0 + \alpha, \ t + \dots) \tag{7.3}$$

where "the composite term $\alpha_0 + \alpha_1 + \dots$ reflects the equilibrium ratio of wealth to national income." Despite this clear statement of the form of the regression equation, he goes on to argue (p. 182) that disequilibrium factors raise questions about how equation (7.3) should be generalized, and deals with the problem by using a simple linear regression specification "to avoid further arbitrary non-linearities." His basic regression model is

$$s = \beta_0 + \beta_1 g + \beta_2 P2 + \beta_3 LAFP65$$
$$+ \beta_4 MW + \beta_5 RW + \beta_6 LE65$$

where in our notation P2 is the ratio of state pension benefits per pensioner to per capita national income, LAFP65 is the labour force participation of the aged, MW the ratio of minors to working age adults, RW the ratio of aged persons to working age adults, and LE65 life expectancy at the beginning of the period defined as aged and the β's the coefficients to be estimated. While such a linearization of a non-linear theoretical specification is quite common and one we will ourselves use,[1] it may have created its own problems. For example, using Modigliani's data from the 1950s for 15 western countries, Feldstein (1977) finds that R2 has the negative sign predicted by neoclassical theory. On the other hand Barro and MacDonald (1979), using essentially the same linear specification and

[1] It can be rationalized by taking a Taylor expansion of the non-linear model.

basic sample but with a slightly different selection of variables and pooled cross-section/time-series observations found that the conventional neoclassical theoretical conclusion had no empirical support. They conclude somewhat pessimistically that " . . . any desired sign for the social security variable in a cross-country consumer expenditure equation can be picked by judicious choice of specification (p. 287)." No doubt, some of this sensitivity to specification changes is due to collinearity among the explanatory variables, a problem of the sample. It may, however, be caused by our disregarding certain multiplicative and other non-linear relationships demanded by the model. By paying closer attention to the development of the econometric model we will have more confidence in the correctness of the specification and thus come to a firmer conclusion.

As we did in chapter III, the theoretical development starts by positing an intertemporal utility maximization model of household behaviour. Because of its usefulness for empirical research the particular model used is the life cycle hypothesis of Modigliani and his collaborators. In this model the consumption of a household at age T is hypothesized to be proportional to v , the present value of resources accruing to the household over the remainder of its life

$$c^T = \Omega^T v^T \qquad (7.4)$$

where Ω^T, the proportionality factor, is dependent upon the form of the utility function, the profit rate and the present age, T, of the household.[1] This can be rewritten in terms of current and future wage income (w , w^*)

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[1]This development of the life cycle hypothesis follows Ando and Modigliani (1963). It can be shown to be equivalent to the simple life cycle hypothesis used in chapter III; see appendix A.

and net worth carried over from the previous period, a_{-1}, as

$$c^T = \Omega^T w^T + \Omega^T (N-T) w^{*T} + \Omega^T a^T_{-1} \qquad (7.5)$$

where N is the household's expected life span.

From the point of view of a household, contributions to a state pension scheme count as additions to household wealth which, like real net worth, will accumulate through the working years to be used to finance consumption in retirement. We can incorporate this in the life cycle consumption function by simply adding a state pension wealth term ($spsw^T_{-1}$) to equation (7.5)

$$c^T = \Omega^T w^T + \Omega^T (N-T) w^{*T} + \Omega^T a^T_{-1} + \Omega^T spsw^T_{-1} \qquad (7.6)$$

State pension scheme wealth is the present value of expected pension benefits of the household less the present value of contributions (taxes) yet to be paid. In an unfunded scheme there is no corresponding real asset but it can, in principle, be evaluated. (Feldstein, 1974; Leimer and Lesnoy, 1980). In general it is a function of the growth, profit and contribution rates.[1]

Assuming that Ω^T is the same for all households of age T we can aggregate within age groups

$$C^T = \Omega^T W^T + (N-T)\Omega^T W^{*T} + \Omega^T A^T_{-1} + \Omega^T SPSW^T_{-1} \qquad (7.7)$$

where upper case letters represent aggregates for the age cohort age T.

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[1]See below p. 139 ff.

The cohort relations can be further aggregated across age groups in any time period t to give

$$C_t = \tilde{\alpha}_1 W_t + \tilde{\alpha}_2 W^*_t + \tilde{\alpha}_3 A_{t-1} + \tilde{\alpha}_4 SPSW_{t-1} \quad (7.8)$$

where C_t , W_t , ... etc., are the national income aggregates in period t . Under the assumption of steady state growth the time subscripts have been dropped from the coefficients.

Although each of the coefficients, $\tilde{\alpha}_1$, $\tilde{\alpha}_2$, $\tilde{\alpha}_3$ and $\tilde{\alpha}_4$ is a weighted average of the vector $[\Omega^1 , \ldots \Omega^T , \ldots \Omega^N]$, since the patterns of weights -- $[W^1 , \ldots W^T , \ldots W^N]$, $[W^{*1} , \ldots W^{*T} , \ldots W^{*N}]$, etc. -- differ across age cohorts, the $\tilde{\alpha}_i$'s differ. In particular the wage coefficients, $\tilde{\alpha}_1$ and $\tilde{\alpha}_2$, can be expected to differ significantly from the asset coefficients $\tilde{\alpha}_3$ and $\tilde{\alpha}_4$. Insofar as the typical household's life cycle accumulation plan varies from the fixed contribution pattern normally required in a state pension scheme the coefficients $\tilde{\alpha}_3$ and $\tilde{\alpha}_4$ will differ[1] but since they are both household asset coefficients we would expect them to be roughly of the same order of magnitude.

We follow Ando and Modigliani in assuming future wage income to be proportional to current wage income

$$W^*_t = \mu W_t \ . \quad (7.9)$$

This gives

$$\alpha_1 = \tilde{\alpha}_1 + \mu\tilde{\alpha}_2 \, , \alpha_2 = \tilde{\alpha}_3 \text{ and } \tilde{\alpha}_3 = \tilde{\alpha}_4 \ .$$

The long-run implications of the life cycle consumption function are developed in terms of the steady state saving ratio. Following Modigliani

[1]Saving for the downpayment on a house or consumer durables would be an example of lumps in the life cycle saving plan. Such behaviour is typical of young households. (Kotlikoff, 1979b, p. 401.)

The form of equation (7.12) is intuitively consistent with the neoclassical theory. The expression in curly brackets is simply the capital-output ratio as a function of its determinants.[1] The partial derivative of the saving ratio with respect to the size of the state pension scheme is negative as we saw in chapter III.[2] In the absence of a state pension scheme the second term on the right-hand side vanishes leaving the neoclassical steady state relationship between the saving ratio and growth which was derived by Modigliani (1970).

Before going further, it would be instructive to make a rough check on the reasonableness of (7.12). Modigliani (1966, pp. 176-177) suggests using estimates from US time series regressions to compare the calculated value of S/Y with the actual value. For the US we use $\alpha_1 = .65$, $\alpha_2 = .08$, $r = .04$ (ibid.), and the actual growth rate in the 1950s of .032. If we let SPSW = 0 , we get the Modigliani formula which yields (cf. Modigliani, 1966, table 1, p. 169)

$$s = \frac{g(1-\alpha_1)}{g+\alpha_2 - \alpha_1(r)} = .1302 .$$

This is a little higher than the actual US saving ratio for the period (.099), a difference of approximately 30 percent, but Modigliani points out that differences of this order can be accounted for by the exclusion of additions to the stock of consumer durables from the reported ratio (approximately 15 to 20 percent of private saving according to him [1970, p. 203]).

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[1] See equation (7.1) and equation (6.3).

[2] We would expect the partial derivative $\partial s/\partial$ (SPSW/Y) to be negative since SPSW is a substitute for real assets in the consumption function [(7.6), etc.]. Note the difference between $\partial s/\partial$(SPSW/Y) and the total derivative $ds/d\tau$ which is also negative in stable equilibrium (equation [7.2]) but which in terms of (7.12) involves induced changes in r , α_1 , α_2 , and α_3 . See below, appendix B.

(1966, p. 172) and making the necessary changes to incorporate the state pension wealth term

$$S_t = Y_t - C_t = Y_t - \alpha_1(W_t + rA_{t-1}) - (\alpha_2 - \alpha_1 r)A_{t-1} - \alpha_3 SPSW_{t-1}$$

or

$$S_t = (1-\alpha_1)Y_t - (\alpha_2 - \alpha_1 r)A_{t-1} - \alpha_3 SPSW_{t-1} \ . \qquad (7.10)$$

Saving is defined in the usual sense as the difference between income and consumption expenditure. In this particular case it is real private net saving and corresponds to the change in aggregate real private net worth, $\Delta A_t = A_t - A_{t-1}$. (Behaviourally, of course, a private household counts as part of its saving the current contributions it makes to the state pension scheme. This is the basis of the neoclassical displacement hypothesis.)

Continuing, in ratio form,

$$S_t/Y_t = s_t = (1-\alpha_1) - (\alpha_2 - \alpha_1 r)\frac{A_{t-1}}{Y_t} - \alpha_3 \frac{SPSW_{t-1}}{Y_t} \ . \qquad (7.11)$$

Since $S_t/A_{t-1} = \frac{\Delta A_t}{A_{t-1}} = g$,[1] $A_{t-1} = S_t/g$ and, after some manipulation

$$s = g\left\{\frac{(1-\alpha_1) - \alpha_3(SPSW/Y)}{g + \alpha_2 - \alpha_1 \cdot r}\right\} . \qquad (7.12)$$

(In steady state growth the time subscripts can be dropped so long as we bear in mind that stocks are measured at the end of the period preceding the corresponding flows.)

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[1]Thus saving is measured net of replacement investment.

We want to test the neoclassical proposition that (7.12) adequately explains the variation of the saving ratio across countries when state pension wealth is taken into account. Adding a country index i to the variables g and p, and a normally distributed random error term ε_i, and simplifying the terminology by letting $p_i = SPSW_i/Y_i$, $\overline{A} = 1 - \overline{\alpha}_1$, $\overline{B} = \overline{\alpha}_2 - \overline{\alpha}_1(\overline{r})$ and $\overline{C} = -\alpha_3$ (where bars represent means for the sample),

$$s_i = \frac{(\overline{A}+\overline{C}.p_i)}{(g_i+B)} g_i + \varepsilon_i . \qquad (7.13)$$

Treating A, B and C in this manner implicitly assumes not only that their variation can be included with regression disturbance in the single error term, but also that the variation in A, B and C is much less important, relative to the variation in g and p, in explaining s.[1]

The non-linear functional form of equation (7.13) requires that the coefficients $\overline{A}$, $\overline{B}$ and $\overline{C}$ be estimated using a non-linear estimation procedure, in this case TSP 'FRML.' Using such a procedure we would expect, relying on the US time series parameter values referred to above, the estimated values of $\overline{A}$, $\overline{B}$ and $\overline{C}$ to be of the following order of magnitude:

$$\overline{A} = 1 - \tilde{\alpha}_1 \cong .35$$

$$\overline{B} = \overline{\alpha}_2 - \overline{\alpha}_1 r \cong .071 - (.65 \times .04) = .045$$

$$\overline{C} = -\overline{\alpha}_3 \cong -\overline{\alpha}_2 \cong -.071 .$$

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not 12 1/2 percent, but a replication of the calculation in the text with $\alpha_2 = \alpha_3 = .064$ and SPSW = 1 gives, to four decimal places, the same result: $\hat{s} = .1307$.

[1]The assumption is a standard one in cross-section studies; see Modigliani (1970), Feldstein (1977), Barro and MacDonald (1979). Although a random coefficients estimation model might seem appropriate, the cost in loss of degrees of freedom in estimating the variances of the coefficient would be very high given the small size of the sample (note that the A_i's, B_i's and C_i's are correlated). In any case OLSQ is still unbiased if not most efficient in this situation. (Theil, 1971, p. 623).

The next step is to recompute s including the correction for state pension scheme wealth. Before doing this however we must consider how the exclusion of state pension wealth in the original time series regression has biased Modigliani's estimate of α_2. Historically, the measured value of α_2 in the US has been roughly 4:1. Assume that the true values of α_2 and α_3 (both being propensities to save out of assets) are the same. If, as we calculate below, SPSW/Y for the US was about 0.5 in the 1950s then US social security must have displaced about 1/8 of private wealth. In other words the true "household wealth" that households were responding to in making saving decisions was roughly 12 1/2 percent higher than the series reported for household assets. This implies a 12 1/2 percent overestimate of α_2 in the time series study; it should be .071 not .08. Using the value of .071 for α_2 and α_3 and the value of .5 for SPSW/Y we get, substituting into (7.12),

$$s = .032\left\{\frac{(1 - .65) - .071(.5)}{.032 \quad + .071 - (.65 \times .04)}\right\} = .1307 \ .$$

This is surprisingly close to the first estimate which takes no account of state pension wealth and thus provides, on the same criterion, a rough validation of (7.12).[1]

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[1]The .5 value for SPSW/Y might be a little low. If, as Kotlikoff (1979a) has suggested, social security could have displaced about 20 percent of US private wealth, the SPSW/Y ratio should be about 1 (ignoring general equilibrium effects). Some independent calculations support this. Leimer and Lesnoy (1980, table B3) have made various estimates of state pension wealth ("net social security wealth") for the US; in 1955 these ranged from 469 to 901 billion 1972 dollars. A representative value of 412 billion dollars (642 billion 1972 dollars) compared to national income that year of 360 billion current dollars (OECD, Nat. Acct.) implies a SPSW/Y ratio of 1.14. ~~In an~~other estimate made in 1976 Kaplan (quoted in Kotlikoff, 1979a, p. 241) found a US social security deficit of 1.4 trillion dollars which compared to national income that year of 1.5 trillion dollars gives a SPSW/Y ratio of .93. Unity would seem to be a representative value. Using unity, the over estimate of α_2 is 25 percent

In order to compare our results to those in the literature, a linearization of (7.13) around the means of the observations--the familiar simple linear form presented by Modigliani and followed by Feldstein and Barro and MacDonald--is also developed.[1] Begin by defining the two partial derivatives

$$\frac{\partial s_i}{\partial g} \equiv s_g = \frac{-\bar{B}}{(g+\bar{B})^2}\,[\bar{A}+\bar{C}p_i] \quad . \tag{7.14}$$

$$\frac{\partial s_i}{\partial p} \equiv s_p = \frac{\bar{C}g_i}{g_i+\bar{B}} \quad . \tag{7.15}$$

Taking a Taylor expansion of the right-hand side of (7.13) around $(\bar{g}, \bar{p})$, the mean values of these variables in the sample, dropping the derivatives of the second order and higher we get

$$s_i \cong s(\bar{g},\bar{p}) + s_g(\bar{g},\bar{p})[g_i - \bar{g}] + s_p(\bar{g},\bar{p})[p_i - \bar{p}] + \varepsilon_i \quad .$$

$$= [s(\bar{g},\bar{p}) - \bar{g}.s_g(\bar{g},\bar{p}) - \bar{p}.s_p(\bar{g},\bar{p})]$$

$$+ s_g(\bar{g},\bar{p})\cdot g_i + s_p(\bar{g},\bar{p})\cdot p_i + \varepsilon_i \tag{7.16}$$

or more simply

$$s_i = \gamma_o + \gamma_1 g_1 + \gamma_2 p_i + \varepsilon_i \tag{7.17}$$

where ε_i is assumed to be a normally distributed random variable with mean zero.

[1]See Modigliani (1970, pp. 201-202).

assumption that the growth of national income is equal to the profit rate, SPSW can be shown to be proportional to current contributions (which are equal to current pension expenditure in a pay-as-you-go scheme.[1] As a first approximation we might use the ratio PEN/Y (where PEN is aggregate pension benefits) as a proxy for SPSW/Y , the two differing by some unknown factor. The difficulty with using PEN alone as a proxy for SPSW is that the factor of proportionality between the two is not independent of the growth rate. Roughly speaking, the faster an economy grows the larger the pension that can be paid at a given contribution level. Of course, maintaining our assumption that the profit rate equals the growth rate, this would also imply that the higher pension benefits would be discounted at a higher rate, but in fact the relationship is a bit more complex than that. Kotlikoff[2] has developed an expression for the factor, F , which when multiplied by the annual per capita contribution PEN/Y, gives SPSW/Y:

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[1] In a pay-as-you-go state pension scheme contributions accumulate at the growth rate, not the profit rate. If future benefits and contributions are discounted at the profit rate, the resulting expression becomes very complex unless the profit and growth rates are assumed to be equal. If they differ there is created what Kotlikoff (1979b) calls the "life-time wealth increment." Kotlikoff found no evidence for such an increment in the US using panel data and we will assume that the same is true for the other countries in our sample. If, in the long-term, the growth rate does differ from the profit rate in a particular country, measured SPSW will differ from true SPSW. To get some idea of the magnitude of the potential error we can refer to the simulation model of Kotlikoff (1979a, p. 246) in which he calculates that a one percent yield differential has about a five percentage point impact on steady state capital stock--in order of magnitude about one-fifth the impact of the displacement effect alone. It is not likely that the profit rate in any country, correctly measured, differs from the growth rate much more than that. In any case, we will assume that such yield differentials occur randomly with a zero mean over the sample.

[2] Kotlikoff (1979a, Appendix b). We have changed his notation slightly. Note also that Kotlikoff's expression is SPSW per age-zero worker. In his terminology SPSW is the "deficit of an unfunded social security system." His formula is based on the assumption that the profit rate is equal to the <u>aggregate</u> growth rate (natural growth plus productivity growth).

Using (7.13), (7.14) and (7.15) and substituting the point estimates of $\overline{A} = 1 - \alpha_1 = .35$, $\overline{B} = \alpha_2 - \alpha_1(r) = .071 - .65(.04) = .045$, and $\overline{C} = -\alpha_3 \cong -\alpha_2 = -.071$ referred to above, and the mean values of g and p from our 1950's sample, $\overline{g} = .045$ and $\overline{p} = .92$, we get rough estimates of what we should expect in fitting (7.17) to international cross-section data:

$$\hat{\gamma} = \frac{.045}{.045+.045}[.35-.071(.92)] - .045\frac{[.045]}{(.045+.045)^2}[.35-.071(.92)]$$

$$+ .92\frac{[. \quad (.045)]}{(.045+.045)}$$

$$= .1038$$

$$\hat{\gamma}_1 = \frac{(.045)[.35-.071(.92)]}{(.045+.045)^2} = 1.5816$$

$$\hat{\gamma}_2 = \frac{-.071(.045)}{(.045+.045)} = -.0355 .$$

(The calculated value of s using these parametric values and the means of the observed independent variables is .142 and compares to the mean of the observed saving ratios of .123, a difference of the expected order of magnitude.)[1]

A measure of the ratio of state pension wealth to national income, p, requires some development. State pension wealth (SPSW) is the present value of state pension benefits less contributions yet to be made for all living members of the population. In steady state growth and under the

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[1]Calculations along the same lines using the mean observed values of g and p from the 1960s-1970s combined sample gives $\hat{\gamma}_0 = .1051$, $\hat{\gamma}_1 = 1.490$ and $\hat{\gamma}_2 = -.0334$. The value of $\hat{s}$ arrived at using these values and the means of the independent variables is .113 and compares to the mean of the observed s of .108.

$$F = \frac{1}{1-e^{-nT}} [T - (D-T)(WR)e^{-nD}] \qquad (7.18)$$

where T is the retirement age, D the age of death (both counted from the beginning of the earning period), WR the ratio of the labour force to the the number of retirees; e is the base of the natural logarithms. Note also that

$$WR = \frac{\int_0^T e^{-na} da}{\int_T^D e^{-na} da} = \frac{1-e^{-nT}}{e^{-nT}-e^{-nD}} \qquad (7.19)$$

Values of the function F are presented for representative range of values of n , the population growth rate, in table 7.1. As it happens, given realistic values of the parameters, F is almost perfectly correlated with n , the coefficient of correlation being .99998: in fact there is almost a perfect linear relation between the two over the range. More interesting is that WR (which is the inverse of the old age dependency ratio) is also highly correlated to F (correlation coefficient .9931). This justifies the use of other researchers (Barro and MacDonald, 1979; Feldstein, 1977) have made of the ratio of benefits per pensioner to per capita income as the proxy for state pension wealth.[1] Still, using WR as a proxy for F is imperfect since the relation between the two is not linear (WR is more sensitive to changes in n as n gets larger), and, more serious, the use of WR makes the SPSW overly dependent on current conditions.[2]

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[1]Since (PEN/R) ÷ (Y/POP) = (PEN/Y)(POP/R) ≅ (PEN/Y)(WR), where R is the number of retirees, W the number of workers, and POP the total population.

[2]There is a view however that steady state assumptions are themselves inappropriate. This is developed in the discussion of the proper measure of growth rate below.

Two variations of the pension variable, $p = (\frac{SPSW}{Y})$, are used in the regression. P1 is $\frac{PEN}{Y} \cdot F$ where F is calculated from the formula (7.18) using n, the population growth rate and P2 is the one used in the Feldstein and Barro and MacDonald studies -- $\frac{PEN}{Y} \cdot L/M$--where L/M is the <u>measured</u> ratio of workers to retirees. Not surprisingly in our sample the two measures are highly correlated:

Sample: \ Correlation coefficient:	P1 : P2
1950s	.9457
1960s and 1970s	.8718

It is reassuring to find that for both sample periods P2 is about one-fifth the magnitude of P1, as indicated in table 7.1 for the relevant range of n.

Although this model is derived in terms of the growth rate g, Modigliani (1970, pp. 210-215) argues that, in general, both in <u>and</u> out of steady-state growth, the proper variable is the dependency ratio--the ratio of non-workers to workers (see also Leff, 1969). This is decomposed into two ratios, one the ratio of retirees to workers, RW, and the other the ratio of minor children to workers, MW. In balanced growth there is a unique relationship between each of RW and MW and g; hence the use of g in the theory; out of balanced growth we should use the true measures of the 'weight' of non-productive to productive population. The estimated coefficients on both should be negative: in the case of RW because a higher value represents relatively more dissavers; in the case of MW because a high proportion of minor children implies

TABLE 7.1

COMPARISON OF THE TRUE VALUE OF STATE PENSION WEALTH FACTOR F WITH THE APPROXIMATION WR
For Various Values of n(= r-g)

n	F	WR
$\frac{1}{4}$ %	30.3755	3.2352
$\frac{1}{2}$	30.7538	3.4920
$\frac{3}{4}$	31.1346	3.7728
1	31.4940	4.0800
$1\frac{1}{4}$	31.8598	4.4163
$1\frac{1}{2}$	32.2293	4.7847
$1\frac{3}{4}$	32.6021	5.1885
2	32.9566	5.6316
$2\frac{1}{4}$	33.3284	6.1162
$2\frac{1}{2}$	33.6988	6.6467
correlation coefficient -	n:F	n:WR
	.9998	.9934

Note: $$F = \frac{1}{1-e^{-nT}}\{T-(D-T)(WR)e^{-nD}\}$$

$$WR = (1-e^{-nT})/(e^{-nT}-e^{-nD})$$

$T = 45 - 20$, $D = 60 - 20$ (in order to measure from age of entry into labour force)

a) relatively more younger families who might be expected to be in the lower saving period of their productive lives, and b) more (unmeasured) saving in human capital in the form of education of dependent minors.

Finally, three additional variables are used in the estimation of the linear form of the saving function. The first two were introduced by Feldstein (1977). The existence of a retirement test (RT) for pension benefit eligibility effectively puts a tax on labour earnings than they otherwise might. The expected sign of this variable is ambiguous since ceteris paribus early retirement should reduce saving but, on the other hand, the prospect of early retirement may induce higher saving over the working years to maintain the desired level of consumption in retirement. A second variable, labour force participation over age 65 (LAFP65) is a more direct measure of the "induced retirement effect."

The third additional variable is per capita income (Y), generally used in inverted form (YINV).[1] It was first introduced by Modigliani (1970) and has been used as well by Feldstein and Barro and MacDonald. Modigliani originally used YINV in an attempt to replicate Houthakker's results and, as his theory predicted, found it insignificantly different from zero. He used it again in his fully specified linear model but found that it became insignificant after his sample was partitioned by level of development (1970, p. 218, table 4).

The life cycle hypothesis is posted on the homogenity of consumption with respect to resources of the household. Recall, earlier in this chapter,

$$c^T \cdot \Omega^T v^T \tag{7.4}$$

[1]This has the effect of reducing heteroscedasticity [Feldstein (1977, p. 192)].

where Ω^T the proportionality factor is dependent upon the form of the utility function, the profit rate and the age of the household. By definition, aggregate saving is not independent of income;

$$S_t = (1-\alpha_1)Y_t - (\alpha_2 - \alpha_1 r)A_{t-1} - \alpha_3 \text{ SPSW}_{t-1} \tag{7.10}$$

but the saving ratio is

$$s_t = (1-\alpha_1) - (\alpha_2 - \alpha_1 r)\left(\frac{A_{t-1}}{Y_t}\right) - \alpha_3\left(\frac{\text{SPSW}_{t-1}}{Y_t}\right) \ . \tag{7.11}$$

In neither (7.11) nor the form written in terms of the growth rate (7.12) does Y or YINV appear.

Feldstein (1977) rationalizes the use of income level in the neo-classical model by arguing that higher per capita income induces higher saving because high income induces early retirement. (His results only confirm this partially: the effect can only be seen at the low end of the income scale.) We can generalize Feldstein's argument by considering Y or YINV[1] as proxies for life cycle consumption preferences. There presumably exists a utility function which would substantiate this interpretation.

Finally, it should be pointed out that in a linear format YINV comes much more directly from a naive Keynesian consumption function:

$$\begin{aligned} S \equiv Y - C &= Y - (a + bY + c\text{PEN} + \dots) \\ &= (1-b)Y - a - c\text{PEN} - \dots \\ \therefore\ s &= (1-b) - a\text{YINV} - c \cdot p - \dots \ . \end{aligned}$$

This is not to argue that neither Y nor YINV should appear in a neoclassical saving function but rather that there is nothing peculiarly neoclassical

[1]Though not both together as Feldstein has done in some regressions.

about the variables and thus their appearance without full theoretical justification cannot strengthen the case for the neoclassical model. This observation carries particular weight if Y or YINV turn out to have significant explanatory power in a saving regression.

On the theory that the variance of the mean is proportional to the size of the sample from which it is taken, the observations in the OLSQ regressions are weighted by the square root of country size.[1] (Houthakker, 1965, p. 213; Modigliani, 1970.) Since the error term is assumed to be additive in the non-linear model, in addition to the dependent variable the entire expression on the right-hand side is weighted as a unit.

Data

In addition to the 1960s-1970s sample, a 1950s sample is used in the test of the neoclassical model. The definition and source of the data for the saving ratio and the basic pension variable, PEN, in the 1960s-1970s sample have been described in chapter VI. In the next three paragraphs we describe in detail the saving ratio variable and the basic PEN variable used in the 1950s sample. The rest of the section is devoted to a description of the sources and definitions for both samples of the remaining data used in the test of the neoclassical model.

All the data for the 1950s sample come from two sources, Modigliani (1970) and Barro and MacDonald (1979). Most come from Modigliani and correspond closely to his 'Western' sub-set. (Two subsequent tests [Feldstein, 1977; Barro and MacDonald, 1979] of the influence of state pension schemes on saving in a neoclassical framework have used essentially

[1]Barro and MacDonald report that a direct estimate of the relation between population and the variance of the error term indicates that a somewhat less skewed weighting scheme would have been more appropriate. They found, however, that the difference in the reported results between population weights and the estimated weights was negligible (p. 286, n. 11].

the same sample.) The data on state pension schemes is adapted from Barro and MacDonald (1979, data appendix).

The data on saving ratios, s, for the 1950s sample come originally from the United Nations, Yearbook of National Accounts Statistics, primarily the 1959 and 1960 editions (cited UN, National Accounts, year). Details of the definitions are found in Modigliani (1970) and Houthakker (1965). It should be noted that the saving ratio is based on the flow of private income: accordingly private saving is defined as the sum of personal and corporate saving and private income the sum of personal disposable income plus corporate saving. Where these data were not available private income is defined as national income less all personal and corporate income taxes, plus transfers; private saving in these cases is simply private income less consumption expenditure. (Modigliani, 1970, p. 205.)

For the 1950s the basic state pension variable PEN is adapted from data collected and modified by Barro and MacDonald (1979) and represents averages for the relevant sample period. Beginning with expenditure for old age and invalidity pensions taken from ILO, Yearbook of Labour Statistics, various years, 1953-1961 (cited ILO, Yearbook, year) Barro and MacDonald created a variable SS for each country: expenditure on old age and invalidity benefits per population over age 65, divided by per capital GDP. Data on the proportion of total population over 65 (OLD) was taken from the United Nations, Demographic Yearbook, various years, 1951-1970 (cited UN, Demographic Yearbook, year), and GDP was taken from OECD, National Accounts (1950-1958), except for Australia in which cases the source was UN National Accounts (various years). Dividing SS by OLD gives PEN, pension expenditure as a proportion of gross domestic product.

Following Modigliani the aggregate growth rate, g, is calculated from beginning and ending national income data using the following formula both samples LE65, were mid-decade interpolations from data found in the UN, Demographic Yearbook (various years).

The state pension variable P1 is constructed by multiplying the ratio of state pension benefits to national income, PEN, by the factor, F, which was defined in the previous section. P2 is simply PEN multiplied by LM, the ratio of workers to retirees.

Data on state pension schemes with a retirement test RT comes from Pechman et al. (1968, appendix C) and from US Social Security Administration Social Security Programs Throughout the World, 1971 (cited US, SSPTW, year). The first source gives information for 1967 and is used for both the 1950s and 1960s. The second source is for 1970 and is used for the 1970s sample. (Apart from the retirement test for the new Canada Pension Plan in the 1970s it was assumed that there were no changes.) Labour force participation of those over age 65, LAFP65, was taken from the ILO, Yearbook (various dates).

The state pension variable does not correspond exactly to that required by the theory. There are two problems. One is that we are using current relative benefits as a proxy for future benefits. Insofar as the steady-state assumption is valid this is not a serious difficulty.[1] A second problem is that measured benefits refer to all pensions paid by the state, not just those financed directly by a wage tax. Neoclassical theory implicitly assumes that it is the earmarked contribution that brings the state pension scheme into the ambit of the life cycle budget process. The veil of government is presumed to hang over other public sector tax-transfer programs (which, in the limit, should include all government activity). The data used mixes these two levels insofar as pensions are concerned. It may be that in their behaviour

[1]We can safely ignore this problem in an international cross-section study. In time series analysis it can be a source of real difficulty. See Leimer and Lesnoy (1980) for a sensitivity analysis of prospective benefits in the context of a US time series study.

$$g_i = \{Y_{ti} / Y_{oi}\}^{\frac{1}{(x_i-1)}} - 1 \qquad (7.20)$$

where x_i is the number of years in the sample. Modigliani used real private income for the 1950s; GDP in purchasers' values, volume indices (OECD, National Accounts, 1958-1978) is used for the 1960s and 1970s.

A measure of population growth, n, is needed for the computation of the state pension wealth factor, F. Modigliani's source (1950s sample) is the UN, Demographic Yearbook; for the 1960s and 1970s the source is OECD, National Accounts (1950-1978) using the same formula (7.20) as was for g above, substituting population for national income.

LM is the ratio of workers to retirees. For the 1950s it is the inverse of Modigliani's variable RW retirees to workers which is taken from the UN, Demographic Yearbook (as was the ratio of minors to workers, MW). For the 1960s and 1970s, OLD, which is the ratio of those over 65 to the total population (taken from the ILO, Yearbook [1979, table 1]) and ODEP which is the ratio of those under 15 and over 65 to the total population (taken from ILO, Cost [appendix]), were used to generate LM, RW, and MW, and LM according to the following formulae:

$$RW = OLD / (1 - ODEP) \qquad (7.21)$$

$$MW = (ODEP - OLD) / (1 - ODEP) \qquad (7.22)$$

$$LM = 1/RW \qquad (7.23)$$

Retirement age, R, is taken from Pechman et al. (1968, appendix C) and represents age requirement for full old age benefit in 1967. This was used for all samples. Age of death, D, is life expectancy at age 65, LE65, plus 65. Note that for the purposes of Kotlikoff's formula for F, an adjustment is made to R and to D to make them correspond to the number of years from entry into the labour force which is taken to be age 20. For

TABLE 7.2

SAVING REGRESSION RESULTS FOR
THE NON-LINEAR NEOCLASSICAL MODEL

Regression	Sample	$\overline{A}$	$\overline{B}$	$\overline{C}$	n	$\overline{r}^2$	ser
	1950s						
7.2.1	P1	0.0917 (0.0434)	0.00268 (0.01146)	0.0377 (0.0186)	13	.9389	.0172
7.2.2	P2	0.0527 (0.0189)	-0.00665 (0.00472)	0.2888 (0.0551)	13	.9609	.0138
	1950s; including Japan						
7.2.3	P1	0.3894 (0.0698)	0.0849 (0.0270)	-0.0579 (0.0222)	14	.9630	.0185
7.2.4	P2	0.3937 (0.0750)	0.0873 (0.0291)	-0.2909 (0.1256)	14	.9597	.0193
	1960s - 1970s						
7.2.5	P1	0.1052 (0.0669)	0.0623 (0.0638)	0.1213 (0.0909)	24	.7935	0.026
7.2.6	P2	0.1234 (0.0519)	0.0162 (0.0208)	20.82 (12.11)	24	.6802	0.033

Note: The saving ratio, s, is the dependent variable. The numbers in parentheses are the standard errors of the estimated coefficients. All regressions were run using weighted data.

households do not in fact make the distinction between earmark-tax-financed and general-tax-financed programs but if that is the case then all government expenditure should be included as a variable. This however would be a different experiment. In order to clean the data and test the proposition advanced here a dummy variable for the existence of a universal non-contributory pension, UP, is included in the linear regressions. The source is Pechman et al. (1968) for the 1950s and US, SSPTW (1971; 1977) for the 1960s-1970s sample.

The source of national income, Y, and its inverse, YINV, is data on GDP per capita in US dollars at 1975 prices and exchange rates found in OECD, National Accounts (1950-1978).

In some regressions, the growth rate, g, is decomposed into productivity growth, PG, and structural measures of population growth such as RW and MW. PG is measured simply by the difference between g and n, the population growth rate.

For the 1950s the weights (WGT) are the square roots of the products of population and the number of years used to generate the decade-average for the saving ratio (as calculated by Modigliani). For the 1960s and 1970s the weights are simply the square roots of population (taken from OECD National Accounts [1950-1978]).

The Estimated Neoclassical Saving Function

The results of the non-linear estimation of the neoclassical model (equation 7.13) are presented in table 7.2. The regressions were run using the 1950s western sample, the 1950s sample including Japan, and the 1960s-1970s sample; all regressions were run using the two measures of the state pension scheme P1 and P2. These results indicate that the neoclassical model does not satisfactorily explain the economic effect of state pension schemes on saving.

In all six regressions the estimates of $\overline{A}$ have the correct sign and order of magnitude though the samples which exclude Japan are a little low, ranging from 0.0527 to 0.1234, as compared to a predicted value of 0.35. This discrepancy is not excessive considering the manner in which the predicted value was arrived at. The estimates of $\overline{B}$ differ more across the samples (-0.00665 to 0.0873) and have relatively large standard errors but with one exception they are within two standard errors of the predicted value of 0.045. The overall fit of the regressions is good with $\overline{r}^2$ well over .9 for 4 of the regressions, and not dropping below .68 for the composite 1960s-1970s sample. (The relatively high $\overline{r}^2$ in these regressions reflects the influence of the weighting scheme.)

The variation in sign and magnitude of the estimates of $\overline{C}$ requires some discussion. The variation in magnitude of the estimates of $\overline{C}$ between the P1 specification and the P2 can be substantially accounted for by the different definition of these two state pension scheme variables.[1] But the change in sign--positive in regressions 7.2.1, 7.2.2, 7.2.5, and 7.2.6, negative in regressions 7.2.3 and 7.2.4--would seem to be a question of the sample.[2] Unfortunately, such sensitivity to the inclusion of a single observation, Japan, suggests that our model is at best not

[1] For the 1950s sample to estimate of $\overline{C}$ using P2 is 8 times the size of the P1 estimate which is not unexpected given the relative magnitude (1:5) and high correlation (.946) of the two variables in the sample. In the 1960s-1970s sample the estimate of $\overline{C}$ using P2 is 170 times as large as that using P1. Although the relative average magnitude of these two measures of the state pension is about the same as it is for the 1950s sample, their correlation is smaller (.8718). This and the much higher relative standard error of the estimates may explain the wide discrepancy in the order of magnitude of the two estimated coefficients.

[2] The possibility of a positive correlation between s and the size of a state pension scheme was discussed at the beginning of this chapter. However, even if we were to allow for the unstable macro-equilibrium which the positive correlation implies, it must be noted that that discussion was concerned with the sign of the <u>total derivative</u> of s with respect to p. $\overline{C}$ represents $-\alpha_3$ (where α_3 is the marginal propensity to consume out of

TABLE 7.3

SAVING REGRESSION RESULTS FOR THE LINEAR NEOCLASSICAL MODEL

Regression	C	g	PG	RW	MW	YINV	DECD	P1	LE65	UP	RT	n	$\bar{r}^2$	ser
1950s														
7.3.1	0.071 (0.012)	.951 (.256)										13	.5154	0.015
7.3.2	0.338 (0.105)		-0.611 (0.377)	-0.811 (0.277)	-0.180 (0.096)							13	.6270	0.013
7.3.3	0.286 (0.110)		-0.031 (0.631)	-0.759 (0.272)	-0.118 (0.105)			0.037 (0.030)				13	.6489	0.013
7.3.4	0.191 (0.104)		0.332 (0.555)	-0.558 (0.251)	-0.035 (0.098)	0.139 (0.069)		0.043 (0.025)				13	.7479	0.011
7.3.5	0.156 (0.027)		-0.355 (0.520)	-0.495 (0.169)		0.150 (0.059)		0.048 (0.021)				13	.7754	0.010
7.3.6	0.012 (0.307		-0.852 (0.821)	-0.468 (0.211)		0.220 (0.088)		0.086 (0.053)	0.0075 (0.0190)	0.0062 (0.0166)	0.021 (0.019)	13	.7289	0.012
1960s-1970s														
7.3.7	0.022 (0.107)		2.11 (1.08)	-0.346 (0.302)	0.017 (0.148)		0.003 (0.019)	0.051 (0.016)				24	.3927	0.028
7.3.8	0.033 0.042)		2.12 (1.04)	-0.366 (0.242)			0.003 0.018)	0.050 (0.015)				24	.4242	0.027
7.3.9	-0.017 (0.039)		2.79 (0.91)	-0.533 (0.211)		3.23 (1.08)	0.028 (0.017)	0.042 (0.013)				24	.5930	0.023
7.3.10	0.035 (0.040		2.01 (.77)	-0.355 (0.226)				0.051 (0.013)				24	.4522	0.027
7.3.11	-0.041 (0.032)	1.94 (0.61)						0.049 (0.011)				24	.4918	0.026

Note: Private saving ratio is dependent variable. All regressions were run using weighted data. The numbers in parentheses are the standard errors of the estimated coefficients.

very robust, at worst inappropriate.[1] Whatever the answer, this is a good reason for comparing the results of this study with those of the two others which examine the same issue using international cross-section samples (Feldstein, 1977; Barro and MacDonald, 1979). Apart from other differences of specification and sample, Feldstein's sample includes Japan, Barro and MacDonald's exclude it. Since these two studies both use linear regression models we begin by considering our own estimates of the linearized version of the neoclassical model. The starting point for this comparison is the linear approximation (7.17) to the true non-linear relationship between the size of a state pension scheme and the saving ratio. To the two-variable linear relationship have been added a number of other variables which have been suggested as relevant to the determination of the saving ratio (Modigliani; Feldstein).

The results of the estimates of the linear version are collected in table 7.3. Despite different selections of variables in these regressions, the results are, by and large, consistent with those of the nonlinear regression. In general, the basic neoclassical predictions of Modigliani are confirmed: s is positively correlated with the growth rate, however measured. On the other hand, the state pension variable belies the neoclassical model: it is in every case positive and, with three exceptions, significantly so.

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state pension wealth), the partial derivative, and thus should always be negative. A possible statistical reconciliation of the positive values of C in the context of an unstable neoclassical model is advanced below (Appendix B) for its theoretical interest, but on economic grounds we must find our explanation elsewhere or else abandon the neoclassical model.

[1] Japan was the second largest country in the expanded sample. Its saving rate was by far higher than all the other countries in the sample: 21.05 percent against a mean for the other 13 countries of 12.33 percent (standard deviation 2.3 percent) and its state pension wealth variable (P2) was by far the lowest: .009 against a mean for the rest of the sample of .17 (standard deviation .06). Japan was excluded from the primary sample in this study on the grounds that it is culturally different from the other,

Regression 7.3.1 reproduces Modigliani's (1970) result: in a neoclassical model the saving ratio is a positive function of the growth rate. Regression 7.3.2 breaks g into productivity growth and the age structure of population. As expected, productivity growth, PG, has the same general effect as overall growth (though with less statistical significance). Similarly the old age dependency ratio RW and, less significantly, the minor's dependency ratio MW have the expected negative influence on the saving ratio. The statistical influence of YINV (7.3.4-7.3.6 and 7.3.9) shows clearly in these regressions: in all cases the fit improves considerably with its inclusion. In regression 7.3.6 three additional variables are included, life expectancy at age 65 (LE65), the existence of a universal pension scheme (UP) and the existence of retirement test (RT). Not surprisingly LE65 is insignificant given the inclusion of MW and RW. The insignificance of the universal pension dummy suggests that any pension scheme will have the same influence regardless of its method of finance. Such a result is quite consistent with the rational life cycle planning basis of the neoclassical model. The retirement-test dummy RT turns out to be not significantly different from zero. In regressions 7.3.7 to 7.3.9 a decade dummy (DECD) is included to allow for structural change over the two decades of the 1960s-1970s sample. In no case is it significant. In fact dropping it and MW improves the fit somewhat; compare $\bar{r}^2$ in regressions 7.3.6 and 7.3.10. Indeed, replacing PG and RW by G alone improves the fit even more, regression 7.3.11. This is in contrast to behaviour of $\bar{r}^2$ for a similar change in specification in the 1950s sample (7.3.1 to 7.3.2). Yet against all this in these regressions if the fact

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western, industrialized countries sampled. It would be much more reasonable to attribute the significant difference in the saving ratio between Japan and the other Western countries to cultural differences than to differences in state pension arrangements, particularly when the rest of the sample taken separately pointedly does <u>not</u> confirm the neoclassical hypothesis.

TABLE 7.4

COMPARATIVE SAVING REGRESSION RESULTS
FOR THE LINEAR NEOCLASSICAL MODEL

Regression		C	g	RW	MW	LAFP65*	P2 (state pension scheme variable)	BPA/Y (state pension scheme variable)	YINV	Y/100	LE65	RT	n	$\bar{r}^2$	ser
7.4.1	West & Japan	0.520 (0.266)	0.747 (0.628)	-0.86 (0.56)	-0.012 (0.191)		-0.1440 (0.2329)		-0.234 (0.396)	-0.008 (0.007)	-0.011 (0.018)	0.032 (0.024)	14	0.9282	0.0117
F4.1	Feldstein	NR	1.93 (0.41)	-0.94 0.46	0.22 (0.13)			-0.086 (0.038)	-37.9 (18.4)	-0.58 (0.30)	2.22 (1.22)	2.48 (1.48)	15	0.95	NR
7.4.2	West & Japan	0.341 (0.230)	0.165 (0.423)	-0.318 (0.357)	0.105 (0.172)		0.0928 (0.1326)		0.231 (0.104)		-0.020 (0.017)	0.0068 (0.0129)	14	0.9225	0.0122
F4.2	Feldstein	NR	1.88 (0.48)	-0.54 (0.48)	-0.24 (0.15)			-0.071 (0.044)	-7.1 (10.0)		2.64 (1.41)	1.64 (1.55)	15	0.92	1.67
7.4.3	West & Japan	0.432 (0.230)	1.501 (0.348)	-1.113 (0.378)	-0.307 (0.169)	-0.0035 (0.0023)	-0.4393 (0.2296)				0.012 (0.020)		14	0.9123	0.0130
F1.1	Feldstein	NR	1.34 (0.17)	-1.09 (0.27)	-0.22 (0.07)	-0.25 (0.07)		-0.104 (0.022)					15		1.03

Notes: Private saving ratio is dependent variable. All regressions were run using weighted data. The numbers in parentheses are the standard errors of the estimated coefficients.

*: LAFP65 is treated as an endogenous variable using the two-stage least squares technique. In regression 7.4.3, g, RW, MW, YINV, RT, LE65 and P2 are used as independent variables in the first run.

NR: Not reported.

Source: F4.1, F4.2, F1.1 - Feldstein (1977).

that the sign of the pension variable (P1) is always positive (though not always significantly different from zero). The positive sign is strong evidence in favour of rejecting the neoclassical model.

We can now compare these results to other studies. Two of Feldstein's regressions are reproduced in table 7.4 as F4.1 and F4.2. His observations were weighted in a similar way to ours and the sample differed only by his inclusion of Costa Rica, Japan, Spain and Trinidad and exclusion of Denmark, Luxembourg and New Zealand. Almost all of Feldstein's coefficient estimates are significantly different from zero by the usual criteria and the overall fit is good. We do not have the original data for comparative purposes, but for most variables apart from the state pension variables, he drew on Modigliani as did we. The pension variable used in these regressions is benefits per person over 65 to average per capita income (BPA/Y) which corresponds most closely to our variable P2.[1]

Since with this weighting scheme Japan is relatively important in Feldstein's sample, it was added to our sample for the purposes of this comparison. Comparable regressions to Feldstein's F4-1 and F4-2 are 7.4.1 and 7.4.2 in table 7.4. Apart from the theoretically unimportant differences in magnitude[2] of the coefficient estimates, the results are broadly consistent with one another. The sign of the coefficients on MW and P2 in regression 7.4.2 and on LE65 in regressions 7.4.1 and 7.4.2 differ from the corresponding variables in the Feldstein equations but the size of the standard errors suggest that these differences are not significant in a statistical sense. Only the sign and significance of the coefficient of YINV in regression 7.4.2 cannot be reconciled with the Feldstein results.

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[1]Although not shown, the regressions using P1 are almost identical.

[2]The different order of magnitude of the coefficients would seem to be explained by systematic shifts of the decimal point in the observation.

Although it might be interesting to pursue this difference, it will not be done since, as we indicated above, Feldstein's data are not available.

Still, while the results of our replication of Feldstein's work are not statistically inconsistent with his, they are not strongly supportive either. In particular, the characteristically neoclassical negative sign on the state pension variable does not find support in our regressions 7.4.1 and 7.4.2. Though in one (7.4.1), the sign on the coefficient of P2 is negative, in the other (7.4.2) it is positive; more important, neither is significantly different from zero in a statistical sense. The instability of the sign on this coefficient indicates that the results are very sensitive to the sample and the specification. Only in regressions in which labour force participation of those over age 65 (LAFP65) is endogenous are we able to confirm Feldstein's results (our regression 7.4.3 as compared to F1-1 of Feldstein, both in table 7.4).

The one other study which bears examination in this context is that of Barro and MacDonald (1979). This was a pooled time-series/cross-section analysis of consumption and state pension schemes for the period of the 1950s using a 16 country western sample. The same weighting scheme was used in this study as in ours and Feldstein's.

Not surprisingly, since the decade averages of the state pension variable used by Barro and MacDonald form the basis for our state pension variables, their results are similar to ours. For the specification closest to ours they report the following (1979, table 1, equation 2):

$$c/y = \underset{(0.05)}{0.62} - \underset{(0.09)}{0.20}\, G/Y + \underset{(0.15)}{1.09}\, OLD - \underset{(0.20)}{1.19}\, \rho$$

$$+ \underset{(0.002)}{0.007}\, U - \underset{(0.04)}{0.01}\, Y_{t-1}/Y_t - \underset{(0.02)}{0.09}\, SS + \underset{(0.007)}{0.044}\, 1/Y$$

where C/Y is consumption expenditure to GDP, G/Y government purchases to

GDP, OLD, the ratio of over 65s to total population, ρ the real per capita growth rate for the period 1951-1975, U, the unemployment rate, Y_{t-1}/Y_t, the ratio of lagged to current real per capita GDP, SS, the ratio of state pension benefits paid per person over 65 to per capita GDP and 1/Y the reciprocal of real per capita GDP in constant U.S. dollars; r^2 is 0.61 and s.e.r. is 0.019. Standard errors are in parentheses. A corresponding equation for our sample (excluding Japan as have Barro and MacDonald) is

$$\begin{aligned} s = \underset{(0.027)}{0.126} - \underset{(0.515)}{0.354}\,PG - \underset{(0.170)}{0.307}\,RW \\ + \underset{(0.111)}{0.254}\,P2 + \underset{(0.060)}{0.130}\,YINV; \end{aligned} \tag{7.18}$$

$$r^2 = .7772$$

where PG is productivity growth. The definition selection of variables differs between the two specifications, the most noteworthy difference being Barro and MacDonald's inclusion of G/Y reflecting their 'super-neoclassical' view of individual economic behaviour which sees through the veil of government.

In comparing our results to those of Barro and MacDonald, we find differences in the signs[1] of the constant term and the coefficients of productivity growth and the inverse of per capita income (see below); on the other hand the signs of the state pension coefficients agree:[2] the size of the state pension scheme is positively associated with saving. In regressions which exploited the time series dimension of their data, Barro and MacDonald did get results closer to those of Feldstein in that they found a negative association between the size of the state pension

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[1]Note that Barro and MacDonald estimate a consumption function.

[2]Barro and MacDonald (p. 286) report that Sterling (1977) supports these findings: " . . . with labor force participation behaviour not held constant his estimates indicate a positive effect of social security on saving."

and saving,[1] though in this case, as they recognize, the conceptual differences between that model and Feldstein's are greater than in the case of the simple cross-section. Not having Feldstein's data, Barro and MacDonald are unable to reconcile their results with his. As they are quoted above (p. 131), they conclude that the question of the relationship between state pensions and saving cannot be answered in a cross-country sample since the results are so sensitive to changes in the variable set.[2]

One striking result of these comparisons is the importance played by per capita income (Y and YINV) as an explanatory variable in these linear regressions. Statistically it has the effect of improving the fit considerably: compare, for example, regressions 7.3.3 and 7.3.8 with the identical regressions including YINV, 7.3.4 and 7.3.9. In the first case $\bar{r}^2$ increases from .6489 to .7479, and in the second from .4242 to .5930. While as we pointed out above the presence of YINV may be rationalized in a neoclassical model, it is not a peculiarly neoclassical variable, and so the statistical effect of using this variable in these regressions must be taken in account in assessing the results as evidence for the neoclassical model.

Conclusion

In this chapter a long-run neoclassical saving function incorporating a state pension scheme has been developed. Special care has been

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[1]Note though, that such time series results are very sensitive to the way in which the state pension wealth variable is constructed (Leimer and Lesnoy, 1980). This sensitivity is not such a problem in a simple cross-section study.

[2]Their term is "specification." In fact, both Feldstein, and Barro and MacDonald use a simple linear regression form.

taken to find the correct functional form and to properly measure the state pension variable. The model, moreover, has been adapted to facilitate comparison with similar studies on the subject.

We have found that on economic criteria the neoclassical model does not adequately explain the data. Although the exploratory power of the regressions measured statistically is high, and, as predicted, the growth rate is consistently correlated with the saving ratio, the results are not acceptable on economic grounds. For the variable of interest, the statistical evidence shows conclusively using two functional forms for the regressions and a variety of specifications that, over a sample of western capitalist economies, the economic effect of a state pension scheme is opposite to that predicted by neoclassical theory.

We conclude that the neoclassical model of a state pension scheme is rejected by the data.

$$g = i(r) \qquad i' > 0 \tag{2.27}$$

though it was understood that the investment function must cut the saving function from below in g-r space if there is to be a stable equilibrium. Over the relevant range such an investment function can be approximated by the following linear formulation:

$$g = D + E\,r \qquad \left\{\begin{matrix} 0 < D \\ 0 < E < s_c \end{matrix}\right\} \tag{8.2}$$

where the restrictions on D and E assure a stable equilibrium.

Multiplying (8.2) by a_1, the capital-output ratio, gives, in equilibrium

$$ga_1 = s = Da_1 + E(P/Y) \tag{8.3}$$

Combining equation (8.3) with (8.1) we have

$$s = s_c\{\frac{a_1 D}{s_c - E}\} . \tag{8.4}$$

Note the similarity between (8.4) and the corresponding nonlinear neoclassical reduced form saving function (7.12). Each is a product of a parameter of the respective system--g in the case of the neoclassical model, s_c in the case of the neo-Keynesian model--and an expression in the form of a quotient with that same parameter in the denominator. There, of course, the similarity ends: the two functions have no variables in common and in particular, no state pension scheme variable appears in (8.4).

The next step is to determine which of the variables in (8.4) are to be taken as observations and which are to be estimated. In principle, D, E and s_c are all parameters of the system and a_1 is a variable; however,

CHAPTER VIII

TESTING THE NEO-KEYNESIAN MODEL

Introduction

As we saw in chapter VI the neo-Keynesian saving function can be written as follows in the Pasinetti range[1]

$$s = s_c . P/Y \qquad (8.1)$$

where s is the ratio of aggregate net saving to national income, s_c the propensity to save of the capitalist class, and P/Y the net profit share. In the neo-Keynesian model the size of state pension scheme does not enter into the determination of the aggregate saving ratio. To distinguish this model from the neo-Marxian model, which shares the classical saving function, we must incorporate the neo-Keynesian investment function into a reduced form saving function. The next section is concerned with the development of this model for estimation purposes. In the third section the results of the estimation are presented and discussed. There is a brief concluding section.

Econometric Specification

In addition to a saving function (8.1) we need to specify an investment function in order to close the neo-Keynesian model. In chapter II, the neo-Keynesian investment function was specified very generally as

[1] $s_c > s_w / (P/Y)$, where P/Y is the net profit share.

Predicted values of $\overline{D}$ and $\overline{E}$ are defined by the limits imposed in (8.2), (s_c in this case should be interpreted as the weighted mean of s_c for the sample). In addition we would expect $\overline{D}$ to be less than the weighted mean of g for the sample. Since the weighted mean of s_c is .233 and of $\overline{g}$ is 0.037, we would expect the estimated values of $\overline{D}$ and $\overline{E}$ to fall within the following ranges:

$$0 < \overline{D} < 0.037$$

$$0 < \overline{E} < 0.237 .$$

Although we could linearize (8.5) in the same manner as we did in chapter VII this will not be done for reasons that will become evident in the next few paragraphs. Briefly, since we have a problem in independently measuring s_c, the capitalist's propensity to save, the estimating equation must be modified.

In principle the capitalist's propensity to save should be easy to establish, perhaps by identifying it with the retention ratio of the corporate sector (Kaldor, 1966). Unfortunately such data are not available. A good proxy is the ratio S/P where S is aggregate saving and P aggregate profit. In fact S/P is too good because S/P and s_c are identical:

$$s \equiv (S/P)(P/Y) = s_c(P/Y) . \qquad (8.6)$$

The first equality is an identity, the second a feature of the classical (Pasinetti) model in which $s_c = g/r = S/P$ is assumed to be fixed. Unfortunately this identity introduces a direct dependency between the right and the left hand role of the regression. This can be made clearer by rearranging (8.6)

$$s_c = S/P = s/(P/Y) . \qquad (8.7)$$

in this model, when D, E and s_c are fixed for a particular country, then r and thus a_1 can have, normally, only one value. (Remember that the size of a state pension scheme cannot fill the role of shift parameter as it did in the neoclassical model because here it does not enter into the determination of a macro-equilibrium.) If we estimate (8.4) using D, E, s_c and a_1 as observations, almost any sample would fit since the regression would in fact have no a-priori structure. Consequently there would be no way of judging the results on purely economic grounds.

We can give the model some structure by following the method used in estimating the neoclassical model and letting D and E be coefficients to be estimated. That is to say, taking observations on the capitalists' saving ratio, we then estimate the means of D and E over the sample in the same way as we estimated the means of A, B and C in the neoclassical model.[1]

Along the lines followed for the neoclassical model we should thus estimate this version of (8.4)

$$s_i = s_{ci}\{\frac{a_{1i}\cdot\bar{D}}{s_{ci}-\bar{E}}\} + \varepsilon_i \tag{8.5}$$

where s_i, s_{ci} and a_{1i} are observations on the corresponding variables for country i, $\bar{D}$ and $\bar{E}$ are coefficients of the system representing the means of the D_i's and E_i's and ε_i is normally distributed random error term.[2]

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[1] In the neoclassical model A, B and C come out of a utility function whereas D and E represent the parameters of an investment function. On the other hand, the motivation (and justification) for allowing them to be estimated rather than to be observed is essentially the same: in neither model are these parameters really measurable.

[2] This is now, in principle, a random coefficients model. We will, however, for the reasons cited in chapter VII, p. 137, n.1, estimate this model using OLSQ.

Indeed, S/P is measured by using the second equality in (8.7), that is, by dividing s, the saving ratio, by P/Y, the profit share. We can thus rewrite (8.5)

$$s_i + \varepsilon_i = (s_i + \varepsilon_i)/(P/Y) \left\{ \frac{a_{1i}\,\overline{D}}{(s_i + \varepsilon_i)/(P/Y) - \overline{E}} \right\} \tag{8.8}$$

and rearrange to get

$$s_i = \overline{D}\,a_{1i} + \overline{E}(P/Y)_i + \varepsilon_i \tag{8.9}$$

where s_i, a_{1i} and $(P/Y)_i$ are observations on the corresponding variables for country i, $\overline{D}$ and $\overline{E}$ are coefficients of the system representing the means of the D_i's and E_i's, and ε_i is a normally distributed random error term.

Before turning to the estimation of this model we ought to consider if the form we have imputed to the investment function has imposed its own problems. Essentially, what we have done up to this point is to prepare a test of the neo-Keynesian model based on the investment function. The complexity of the algebraic manipulation may have obscured this but it can be seen more clearly if we multiply both sides of the linear approximation of the investment function (8.2) by the capital output ratio K/Y

$$g(K/Y) = (\dot{K}/Y)(D + Er)$$

$$S/Y = s = D(K/Y) + E(P/Y)\ . \tag{8.10}$$

This may be an unduly restrictive form of the investment function. One possible extension would involve adding per-capita income or its inverse to (8.9) as an explanatory variable in the same manner as Modigliani, Feldstein, et al., have done for the neoclassical model. The addition of YINV, could be rationalized in the following manner.

is actually fixed, but only that estimate value represents the mean of the s_{ci}'s of the population.[1]

Since different equilibria are characterized, ceteris paribus, by different values of the profit share, we need not even specify an investment function. We simply measure P/Y for each element of the sample.

Thus, the model to be estimated is

$$s = s_c (P/Y)_i + \varepsilon_i \quad (8.13)$$

We would expect that the estimated coefficient on (P/Y) to be between zero and one and of the order of .25 which is the measured value of s_c for the sample based on s and P/Y.

Unfortunately, since (8.13) is based solely on the 'classical' saving function it may be difficult if not impossible to use the results of estimating (8.13) to distinguish between the neo-Keynesian and neo-Marxian models.

As for the neoclassical model, all regressions are run using weighted data. The method is the same as that described in chapter VII, p. 146 and p. 150.

Data

The same data for the dependent variable is used for neo-Keynesian model as was used for the neoclassical model—see chapter VI and chapter VII. Only the 1960-1970s sample is used because of difficulty in obtaining good capital stock data for the 1950s. The measurement and sources of the new variables is discussed here.

The definition of the proxy for the capitalists' propensity to save and the adaptation of the regression equation to avoid regression of a

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[1] As in the previous model, this implies a random coefficients' model; again we will approximate it with OLSQ.

If the linear investment function had a term for the labour-capital ratio

$$g = D + E \cdot r + H(L/K) \tag{8.11}$$

then the corresponding equation with s as the dependent variable would be

$$s = D\,(K/Y) + E\,(P/Y) + H\,(L/Y) \tag{8.12}$$

where L/Y is YINV, the inverse of output per man. The inclusion of the labour-capital ratio is an attempt to allow the investment function to shift over the sample. If the investment function is the same across the sample, L/K will add no new information since r and L/K move together, given a common production function.

Adding variables to the investment function is one way of dealing with the restrictiveness of the linearity assumption. By assuming that (the means of) D, E, and H are fixed for the sample, we have chosen to let a shifting[1] saving function trace out the investment function. An alternative, more radical, is to disregard the form of the investment function altogether.

We can proceed as follows. If we begin with the saving function (8.1)[2]

$$s = s_c(P/Y) \tag{8.1}$$

we could assume that a fixed saving function for the population is traced out by a shifting investment function. Again, we need not assume that s_c

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[1] Actually, in this case the saving function is rotating about the origin.

[2] This is equivalent to the Pasinetti saving function $g = s_c \cdot r$ with both sides multiplied by K/Y.

The Estimated Neo-Keynesian Saving Function

The results of the estimated neo-Keynesian saving function (investment basis) are presented in table 8.1. The estimated coefficients of the basic specification are given in line 8.1.1. Both the coefficient on P/Y and the coefficient on KTOY, while significantly different from zero, are outside the range of predicted values. The coefficient on KTOY represents E, the slope of the investment function, and a value of .415 is more than 50 percent greater than the mean of s_c for the sample. This combined with a negative value of D, the intercept, implies an investment function which cuts the saving function from above in g-r space. That is to say, it represents an unstable equilibrium.

None of the other variants of this model would lead us to change this conclusion. We can review them briefly. Adding YINV confirms the shift hypothesis. If there were no investment function shift, the observations on YINV and P/Y would be perfectly collinear and their coefficients would have high standard errors. In fact, YINV improves the fit considerably. Regressions 8.1.3 through 8.1.9 test the homogeneity hypothesis. Although 8.1.3 suggests that the investment is homogeneous, the addition of a decade dummy 8.1.4 makes both the constant and the dummy significant. On the other hand, allowing the decade dummy to influence the slope of the dependent variables as in 8.1.5, 8.1.6 and 8.1.7 shows that the influence of any systematic shift in structure from decade to decade is best captured by allowing the slope of the profit share variable to take on two values, one for each decade.[1] When this is done the constant and dummy become insignificant (8.1.8 and 8.1.9).

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[1] DP/Y is the vector product of DECD and P/Y. DKTOY and DYINV are formed in a similar manner with KTOY and YINV.

variable upon itself were discussed in the last section. Thus, instead of s_c or its proxy S/P, the net profit share (P/Y) alone is used in estimating the first two variants of the neo-Keynesian saving function. The profit share data were obtained by subtracting from unity the wage share as given in the United Nations Yearbook of National Accounts 1978, volume II, table 4, "Compensation of Employees As a Percentage of GNP." As ~~is~~ well known, this measure of profit share is conceptually weak in that the complement of the wage share also includes the income of proprietorships, part of which must properly be classified as wages. Unfortunately, any worthwhile correction of published figures is beyond the scope of this study.

Data on the capital-output ratio is based on work by Balassa (1979). Balassa sought to measure capital per man in different countries. To avoid the well-known difficulties of measuring true economic capital, he used cumulated gross fixed investment per capita for the period 1955-1971 in constant U.S. dollars as a proxy. To create separate observations for the mid-points each of the 1960s and 1970s observation periods we have both divided the Balassa capita stock proxy by $(1 + G60)^6$ and multiplied it by $(1 + G70)^3$ where G60 and G70 are the 1960s and 1970s GDP growth rates as defined above, p. 148, eq. (7.20). The effect of this is to create two sets of observations from one, one for 1965, the other for 1974, the mid-points of the two decades. The implicit assumption in this is that capital stock grows at the average rate of GDP growth over each sample period. Finally we have converted the capital per man observation to capital-output ratios by dividing each by the corresponding output per man observation, Y.[1]

[1]See chapter VII, p. 150 for definition and source.

TABLE 8.1

SAVING FUNCTION REGRESSION RESULTS--NEO-KEYNESIAN MODEL (INVESTMENT BASIS)

Regression	C	DECD	P/Y	DP/Y	KTOY	DKTOY	YINV	DYINV	n	$\bar{r}^2$	ser
8.1.1			0.415 (0.080)		-0.068 (0.031)				24	.4426	0.0269
8.1.2			0.368 (0.085)		-0.764 (0.031)		1.568 (1.131)		24	.4650	0.0264
8.1.3	0.044 (0.084)		0.455 (0.111)		-0.044 (0.056)				24	.4237	0.0274
8.1.4	-0.033 (0.063)	0.044 (0.010)	0.605 (0.091)		-0.134 (0.047)				24	.6783	0.0204
8.1.5			0.539 (0.064)	0.102 (0.022)	-0.137 (0.027)				24	.7093	0.0194
8.1.6			0.451 (0.087)	0.177 (0.112)	-0.154 (0.054)	-0.020 (0.059)	2.873 (1,404)	0.011 (1.610)	24	.8530	0.0138
8.1.7			0.480 (0.045	0.127 (0.016)	-0.169 (0.019)		3.005 (0.593)		24	.8663	0.0132
8.1.8	0.002 (0.067	0.011 (0.098)	0.450 (0.103)	0.167 (0.134)	-0.155 (0.081)	-0.026 (0.095)	2.891 (1.659)	-0.039 (1.865)	24	.8349	0.0146
8.1.9	0.012 (0.050)	-0.015 (0.056)	0.456 (0.096)	0.160 (0.127)	-0.169 (0.034)		2.984 (0.634)		24	.8522	0.0139

Note: The saving ratio, s, is the dependent variable. The numbers in parentheses are the standard errors of the estimated coefficients. All regressions were run using weighted data.

Since we cannot accept results that imply an unstable macro-economic equilibrium, these results would seem to be sufficient to disqualify the neo-Keynesian model from further consideration. Before dismissing the model however we should consider that our test might not have done full justice to it. Specifically, the assumption that the model is well described by a family of parallel investment functions intercepting one country saving function might be inappropriate. Ideally we should like to allow both saving and investment functions to vary by country but our data will not fully permit it. One alternative, developed above, is to take the other extreme assumption and attempt to identify a fixed global saving function intercepting a set of individual country investment functions. The results of this estimation are presented in table 8.2. Unlike the first results these are not inconsistent with the neo-Keynesian model.[1]

In line 8.2.1 the basic specification is estimated. Not surprisingly the estimate of $\bar{s}_c$ is .24 which is roughly the mean of the calculated value of that variable. The overall fit is not good ($\bar{r}^2$ equal to .3472) but when one considers that we are dealing here with a one-variable model without an intercept, it is quite reasonable. Regression 8.2.2 tests the homogenity postulate and finds that the constant is just twice its standard error. This is inconclusive and in any case may simply indicate the presence of a measurement error in the dependent variable. (Recall that in the previous table, where the constant was significantly different from zero it too was negative.) The next regression gives support to

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[1] Note that these two approaches to estimating a neo-Keynesian model are themselves mutually inconsistent. One assumes that the salient difference between countries lies in the capitalists' saving propensity; the other looks for the difference in the degree of animal spirits. We would need considerably more data to take an intermediate approach.

TABLE 8.2

SAVING FUNCTION REGRESSION RESULTS--NEO-KEYNESIAN MODEL (SAVING BASIS)

Regression	C	DECD	P/Y	DP/Y	n	$\bar{r}^2$	ser
8.2.1			0.240 (0.014)		24	.3472	0.0291
8.2.2	-0.999 (0.047)		0.470 (0.109)		24	.4336	0.0271
8.2.3	-0.169 (0.048)	0.030 (0.011)	0.593 (0.105)		24	.5687	0.0237
8.2.4			0.221 (0.019)	0.039 (0.027)	24	.3760	0.0285
8.2.5	-0.084 (0.070)	-0.111 (0.089)	0.406 (0.155)	0.326 (0.205)	24	.5980	0.0229

Note: The saving ratio, s, is the dependent variable. The numbers in parentheses are the standard errors of the estimated coefficients. All regressions were run using weighted data.

this view, for when a decade dummy is added (8.2.3) the constant turns out to be identical to that in 8.1.4 and the coefficient on the decade dummy is 8.2.3 is reasonably close to the estimate in the first table.

The last two regressions confirm this. When the constant and dummy are dropped and the coefficient on PTOY is allowed to take a different value for each decade (8.2.4), there is little improvement over the original model. When the constant and decade dummy are restored (8.2.5), neither they nor the slope dummy are significantly different from zero, all of which suggests that in terms of the statistical significance of individual coefficients and overall explanatory power, 8.2.3 is most satisfactory. While it can be construed as a neo-Keynesian result, it does not provide conclusive evidence for the model. As it stands, it cannot be distinguished from a neo-Marxian saving function.

In this connection, these last results do bear comparison to a similar test carried out by Modigliani. Modigliani (1970, pp. 221-225) estimated the Kaldorian version of the neo-Keynesian saving function (Kaldor, 1955-1956; 1966)

$$s = s_L + (s_p - s_L) \cdot P/Y \qquad (8.14)$$

where s_L and s_p are the propensities to save out of wages and profit respectively. (The Kaldor model differs theoretically from the Pasinetti form of the neo-Keynesian model in that here the saving propensities attach to <u>sources</u> of income rather than economic class;[1] however, for empirical purposes equation (8.14) is identical to (8.13) with the addition of a constant term.) Using 14 western countries (essentially our 1950s sample)

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[1]The two forms cannot be reconciled functionally except when $s_L = 0$ in which case $s_p = s_c$.

Modigliani (1970, p. 223, table 6, row [E]) got the following result:

$$100s = \underset{(3.2)}{-0.4} + \underset{(8.4)}{35.2}\ P/Y;$$

$$\bar{r}^2 = .4489$$

where the numbers in parentheses are the standard errors.

In Modigliani's interpretation this equation implies an average propensity to save out of wages across the sample of approximately zero and a propensity to save out of profits of 1/3. This is a very 'classical' result and it is hard to understand Modigliani's remark that "the estimated coefficients do not make much sense" (p. 224). They make sense both in terms of Kaldor's model of saving and Pasinetti's. Interpreting these results in terms of the latter model, (8.13), this implies that the capitalists' propensity to save is .35, an estimate that does not differ qualitatively from our results on the 1960s-1970s sample in table 8.2. Note in this connection that the saving ratio in the latter sample does not approach the theoretical private-saving-to-private-income definition as closely as does Modigliani's, a difference which would also explain the significant constant term in our regressions. In any event Modigliani's results are consistent with the neo-Keynesian model (saving basis) as we have developed it.[1]

Modigliani's development and conclusions in this section are worth examining. He purports to show that the Kaldorian model does not stand

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[1]Note that Modigliani's dependent variable is 100s. We are not able to replicate Modigliani's results since he does not provide details of his P/Y series. It would seem however from the text that his definition of P/Y excludes any government income share both in the numerator and in the denominator.

up to his neoclassical results. He does this by adding the growth rate (G) the old age dependency ratio (RW), and the inverse of per capita income (YINV) as explanatory variables to (8.14). Since these (in various combinations) have the effect of making the coefficient on P/Y insignificantly different from zero, he concludes that subject to the limitations of the data "the outcome (of these) tests confirms our earlier (i.e., neoclassical) results, whereas it provides no support for the hypothesis that the saving ratio is controlled or even significantly affected by the functional distribution of income."[1]

In our view these results support no such conclusion. First, on its own terms the Kaldorian equation does fit Modigliani's data (and ours). Second, the fact that the distributional coefficient can be made insignificant by the addition of other variables is no more than a statistical artifact. Any particular regression coefficient can be made insignificant by the addition of spurious but appropriately correlated variables. The fact that the variables added are in some sense 'neoclassical' is irrelevant. A contrary example can make this clear. In this case we use the 1960s-1970s sample and our own neoclassical results presented in chapter VII. A number of neoclassical specifications were used (see table 7.3): in every case the addition of P/Y caused the coefficients on the "neoclassical" growth variables to become negative and in two cases, significantly so. Moreover, the coefficients on P/Y were in all cases significantly different from zero with estimated values between 0.61 and 1.04. One typical result is the following[2] (standard errors in parentheses):

[1]Modigliani (1970, p. 227). [2]Cf. regression 7.3.8.

$$s = -\ 0.169 + 0.853\ P/Y - 3.119\ PG - 0.242\ RW + 0.009\ DECD + 0.024\ P1$$
$$\quad (0.052)\ (0.180)\qquad (1.321)\qquad (0.168)\qquad (0.012)\qquad (0.011)$$
$$(\bar{r}^2 = .7288)\ . \tag{8.14}$$

Even more interesting is the results of a regression which add neoclassical variables g and P1 to a neo-Keynesian model--in this case, the investment basis version represented by regression 8.1.7. The augmented regression is

$$s = 0.357\ P/Y + 0.128\ DP/Y - 0.161\ KTOY + 3.333\ YINV + 0.007\ P1 + 0.750\ g$$
$$\quad (0.177)\qquad (0.028)\qquad (0.023)\qquad (0.884)\qquad (0.010)\qquad (1.131)$$
$$(\bar{r}^2 = .8874)\ . \tag{8.15}$$

In this case the "neoclassical" variables add nothing: there is very little change in the neo-Keynesian coefficients and the "neoclassical" variables are statistically insignificantly different from zero--indeed the standard errors are 50 percent greater than the estimated coefficients. It would be tempting to use these results to reject the general neoclassical model (quite apart from our rejection of the neoclassical model of a state pension scheme in chapter VII). But, even leaving aside the criticisms of the neo-Keynesian model investment basis, such a conclusion would be unwarranted on strictly statistical grounds.

Modigliani's purpose (p. 244) was to show that distributive considerations add nothing to a neoclassical explanation. With his data this was so. On the other hand, with our data we arrived at the opposite conclusion. The method has some intuitive appeal. It is not, however, grounded in statistical theory, and so, even if we had not been able to counter his conclusion with a contrary example it would not be possible to give any meaningful interpretation to the results of this method. For this reason it must be disregarded. In chapter X we introduce an econometric method of model selection that is grounded in statistical theory, and will thus allow us to make some judgement about the validity of the three models we are testing.

Conclusion

Two versions of the neo-Keynesian saving model have been developed in this chapter, one based on the neo-Keynesian investment function and the other based on the neo-Keynesian (classical) saving function. Consistent with the theory developed in chapter IV, in neither version of the neo-Keynesian model does a variable representing a state pension scheme appear.

The estimated reduced-form saving function (investment basis) is not supportive of the neo-Keynesian model. Although the theory is very vague about the functional form and the specific explanatory variables, if the saving function (investment basis) developed here is in any way representative of the function implied by the model, these results can be rationalized only by a neo-Keynesian model with an unstable equilibrium. Since a model with an unstable equilibrium is not a valid basis for a representation of reality, rejection of the model is indicated.

Still, the investment may have been misspecified. On this possibility a neo-Keynesian model based directed on the classical saving function was estimated. The results were not as strong in a statistical sense as the investment-function version, but they were economically plausible. All the same the use of the classical saving function as a structural basis of the reduced form, means that these results cannot be distinguished from the reduced form of a general neo-Marxian model of saving.

On the basis of these results, the neo-Keynesian model can be tentatively accepted, conditioned on the results of a test of model selection which includes a fully specified neo-Marxian saving function.

CHAPTER IX

TESTING THE NEO-MARXIAN MODEL

Introduction

The neo-Marxian model shares the classical (Pasinetti) saving function with the neo-Keynesian model

$$s = s_c \alpha \tag{2.26}$$

where s is the saving ratio, s_c, the saving propensity of the capitalist class, and α the net profit share. This can be written

$$s = s_c' \{1-[W(T,t)/Y]\}$$

where W is the real wage and Y per-worker output.

In chapter V we showed that in the neo-Marxian model there is a positive relationship between W, the real wage, and the size of a state pension scheme, and a negative relationship, over historical time, between W and the age of the scheme. In this chapter we test this theoretical prediction.

In the next section the analysis of chapter V is extended to provide the specification of an econometric relationship between the size and age of a state pension scheme and saving ratio, s. Following this is a section describing the additional data used for the neo-Marxian test and another section in which the results of the test are presented and analyzed. There is a brief concluding section.

productivity change itself. As we shall show the two are functionally linked in the neo-Marxian model.

Although increases in the intensity of work and the length of the working day accounted for a considerable part of productivity increases in the early days of capitalism, today it is structural changes in the organization of work, including increased mechanization, which are the prime determinants of productivity growth. More important from a neo-Marxian perspective is that, whatever its technical characteristics, productivity growth is a consequence of the initiatives of the owners of capital. The importance of this point in the context of the analysis of long-run productivity growth lies in the reactions of workers: since workers do not control the process of change in the organization of work, they tend, in the first instance, to resist them, and, later, to demand compensation for those changes they do finally accept. This has implications for the neo-Marxian analysis of the division of the fruits of technical progress.

From the point of view of capitalists we can identify three factors motivating them to make changes in the organization of work leading to increased productivity. The first is the purely economic desire of individual capitalists to economize inputs for a given output, the second is their desire to change the organization of work (generally through automation) to enhance their control over the process, and the third, a factor motivating capitalists as a class, is their recognition that a steady rise in the rate of output makes possible increases in the real wage, which in turn dull the natural desire of the working class for a radical change in the economic system. The relative importance of these three factors is an empirical question outside the scope of this study,

Econometric Specification

At the core of the neo-Marxian model is the statement that the average real wage tends to the subsistence level of consumption. This statement is not directly verifiable. Moreover, it must not be taken to imply that the real wage is constant either across countries or through time. Marx himself was very clear that subsistence is a cultural concept, not a biological one, and therefore does not imply a fixed real wage at all times and in all places (Capital, vol. 1, chap. 22). What the neo-Marxian approach does mean is that we should look for the extra-economic factors in the determination of the real wage. Specifically, the real wage must be analyzed in the context of a continuing class struggle in concrete historical circumstances.

The way in which the real wage changes has to do with the nature of the balance of class power. Apart from revolutionary changes, the factors underlying the secular improvement of the real wage in the last century in most western countries have been technical progress (the growth of per-capita output) and what we might call improvements in the social wage, those elements of consumption, often, but not always, collective, which are provided by the state. These state-provided transfers include payments made under programs such as state pension schemes insofar as they are not financed by a direct tax on wages. The relationship between the size of these programs and the saving ratio is our primary concern in this section but before turning to that we must deal first with the question of technical progress in the neo-Marxian model.

The neo-Marxian analysis of productivity growth is concerned not only with the distribution of the fruits of the increased per-capita output but also with its relationship with the factors influencing the but as a working hypothesis we will accept the following: insofar as factors one and two motivate technical progress, the increased production would be enjoyed solely by capitalists. However, the third factor can be considered as the price of the two, the net result being a rough sharing of the fruits of per-capita growth. In other words the real wage will tend to rise at the same rate as per-capita output. The record over the past one hundred years bears this hypothesis out, which is not surprising since it is only in this way that, in the long run, the game can continue to be played. Thus, as a first approximation, we assume that, ceteris paribus, in any country, the ratio of the real wage to per-worker output is stable over time; in other words, that the wage share does not vary over time on account of technical progress.[1]

The wage share, of course, does differ across countries. Insofar as the technical relations of production in every country can be described by a single production function,[2] the wage share differs across countries because the real wage differs across countries. This is another way of saying that the concept of subsistence is a relative one, relative, that is, to the average level of productivity; in a country where workers have been more successful in the struggle over the terms of the real wage,

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[1]Although theoretical simplicity is in no way binding on real-world behaviour, it is worth noting that only through such an equal sharing of the fruits of per-capita output increases, i.e., Harrod-neutral technical progress, can we appropriately use a steady-state model for the analysis of long-run growth. (See chapter VI, pp. 122-123.)

[2]While the notion of a single world-wide production function may seem a little far-fetched, such an assumption is only a short extension of the idea of a single national production function. Assumptions of this nature are a necessary (and usually implicit) part of most aggregate economic studies. Though it was perhaps less obvious there, in the development of the neo-classical econometric model a single representative preference function was assumed. We will make the assumption of a single, underlying production function for our sample of western industrialized countries in our test of the neo-Marxian model.

this will be reflected in a higher wage share. Thus, for a neo-Marxian explanation of international differences in the wage share, we can return to an analysis of the terms of the wage bargain.

Although numerous factors enter into the determination of the real wage, in countries at a similar level of development the factors referred to above under the heading of the social wage are particularly important in this context. This was the central point in the analysis presented in chapter V. The state, acting in the long-term interests of the owners of capital, will, from time to time, impose certain system costs on capitalists in the form of social welfare schemes in order to maintain social peace.

As we argued, the direct long-run incidence of the schemes can be measured, as a first approximation, by the employers' share of the scheme's financial cost.

We must repeat that this discussion is to be read against the backdrop of a regime of steady, Harrod-neutral technical progress. A change in the real wage resulting from, for instance, the introduction of a state pension scheme, is a one-time change. While this real wage change will lead to a change in average productivity (if production coefficients are variable) this too is a one-time change. Thus, the <u>rate of growth</u> of productivity will, after the change, be the same as it was before, in the long run.

In order to show how a change in the real wage occasioned by a change in the terms of a state pension scheme affects the saving ratio, we first summarize the way in which the real-wage change works its way through the economy in the long-run. For simplicity we consider only the direct long-run effect. The example we will use is that of an increase in the size of a state pension scheme, as measured by the change in the payroll tax percentage.

As we saw in chapter V, the increase in the real wage leads to a fall in the profit rate and then, through the saving-investment identity, to a fall in the growth rate. It is through its effect on the growth rate that the state pension change affects the saving ratio.

To see how the saving ratio is affected, we return to the Pasinetti saving function, i.e., the saving-investment identity in the neo-Marxian model

$$g = s_c . r \tag{2.26}$$

Since $g = \Delta K/K = S/K$, then, as we saw in chapter VI,

$$s = S/Y = (S/K)(K/Y)$$

$$= s_c rK/Y = s_c \alpha \tag{6.5}$$

or

$$s = s_c[1-(W/Y)] \tag{9.1}$$

In the neo-Marxian model the real wage, W, is a function of the size of a state pension scheme. Thus, in Chapter V we wrote

$$W = C^* + m(t)T \tag{5.1}$$

where C^* is the subsistence level of average consumption, T the pension payroll tax in real terms and m(t) a number which represents the degree to which the tax is a gain by the workers from capitalists (m is positive but is understood to be a declining function of the age, t, of the scheme). Although W is a positive function of T, the wage share is not necessarily so.

In order to see how a change in T affects s we write (9.1) as

$$s = s_c[1-a_o W(T)] \tag{9.2}$$

where a_o is the labour-output ratio (and inverse of Y). From (5.1)

$$\frac{dW}{dT} = m < 0$$

then, differentiating (9.2) with respect to T, we have[1]

$$\frac{ds}{dT} = \frac{ds}{dW}\frac{dW}{dT} = [(\partial s/\partial W) + (\partial s/\partial a_o)(da_o/dW)]m$$

$$= s_c a_o m[\sigma-1] \qquad (9.3)$$

which is $\gtrless 0$ as $\sigma \gtrless 1$.

In an international cross-section sample, the elasticity of subsitution parameter, σ, refers to the underlying international production function. As (9.3) makes clear, in the event that this production function has the Cobb-Douglas form (i.e., $\sigma = 1$) the saving ratio will be completely unresponsive to changes in the real wage. Although such a singular occurence is unlikely, what (9.3) does mean is that without knowledge of the underlying production function we cannot predict the effect a change in the size of a state pension scheme will have on the saving ratio.

We turn now to the actual specification of the econometric model. From (9.2) we can see that, as in the neo-Keynesian model, the saving ratio in the neo-Marxian model is the product of a fixed saving propensity and the profit share. The difference is that in this model the profit share is, in general, a function of the real wage which is in turn a function of, among other things, the size and age of a state pension scheme. Moreover, because changes in the real wage act in a parametric fashion on s, the reduced form of the neo-Marxian model is simply (9.2).

Again, as we did for the neo-Keynesian model, we might wish to take two approaches to estimation, one using estimates of s_c across the sample, the other assuming s_c to be fixed. Unfortunately the first method

[1]From (2.24) and (2.7): $da_o/dW = -a_o\sigma/W$.

cannot be used. As we saw in chapter VIII the measurement of s_c is not independent of s. In the neo-Keynesian model, this meant that the reduced form of the model was simplified considerably. Because the structure of the neo-Marxian model is different, use of the same method would lead to s dropping out of (9.2) and our being left with an identity. Thus, we must treat s_0 as a constant to be estimated.[1]

The characteristic difference between this model and the neo-Keynesian model is that profit share in this model is affected by parametric changes in the real wage. Thus, we must go behind the wage share and write (9.2) in terms of its identifiable determinants. Substituting (5.1) into (9.2) we get

$$s = s_c\{1-a_0[\bar{C} + m(t)T]\}$$

$$= s_c[1 - \bar{C}/Y - m(t)T/Y] \qquad (9.4)$$

We will assume that over our sample the wage shown has a common mean, i.e.

$$\overline{\bar{C}/Y} = H \qquad (9.5)$$

where H is a constant between 0 and one, and a systematic component of variation around this mean that can be attributed to the size and age of the state pension schemes, i.e.

$$m(t)T/Y = Q(PN_i, AGE_i) \qquad (9.6)$$

where PN_i measures the size of the pension scheme in country i and AGE_i its age. The sign of $\partial Q/\partial PN$ depends on the elasticity of substitution but in any case it will have the opposite sign to $\partial Q/\partial AGE$.[2]

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[1] cf. equation (8.13) in the neo-Keynesian test.

[2] Judging by the left-hand side of (9.6), the signs of the partials of Q would seem to be self-evident. This is not so however because Y is not independent of t or T.

If we add a random error term ε_i, we have the following estimating equation

$$s = s_c[\ 1 - H - Q(PN_i,\ AGE_i) - \varepsilon_i\ ]$$

$$= s_c(\ 1 - H) - s_cQ(PN_i,\ AGE_i) - s_c\varepsilon_i. \qquad (9.7)$$

We can simplify (9.7) by assuming that Q(.) has a linear, separable form. Noting that $s_c(1 - H)$ is a constant and that $s_c\varepsilon_i$ is a randomly distributed error term with a constant variance if ε_i has the same characteristics, we can write

$$s = \lambda_o + \lambda_1\ PN_i + \lambda_2\ AGE_i + \mu_i \qquad (9.8)$$

where λ_o is positive, and less than one and λ_1 and λ_2 are of indeterminate though opposite signs.[1]

As the final step before estimating (9.8), we must establish an appropriate way of measuring PN and AGE.

The measurement of PN would seem to be straightforward, judging by the left-hand side of (9.6), i.e., use T/Y. However Y is, in principle, quite responsive to a change in T and thus the ex-post measurement of T/Y is endogenous. We will use T/Y as an explanatory variable but a better choice is the ratio T/W which does not pose the same difficulty.

The variable AGE is more straightforward. The simplest expedient is to use the absolute number of years the state pension scheme has been in existence. However, since the underlying function in (t) is a decay function, the logarithm of age, LAGE might be more appropriate.

Properly speaking, each increment in the size of a state pension scheme should mark the starting point for measuring the age of that

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[1]If we have prior knowledge of the elasticity of substitution (σ) of the underlying production function, we can identify the signs of λ_1 and λ_2. See equation (9.3) above.

increment. In the present test, AGE will be measured from the inception of the scheme since we have only the current observation on the size variable. This approximation should not cause any particular difficulty insofar as the current measure of PN is typical of the scheme over its life time.[1]

Data

For the dependent variable, s, the same data and the same weighting scheme is used as was used for estimating the neoclassical and neo-Keynesian model, only the 1960-1970s sample is used.

The source of the basic data for the two pension variables is the same as for the neoclassical model--see chapter VI pp. 124-126. We use the basic variable PEN for T/Y and simply multiply it by the wage share (1 - P/Y) as defined in Chapter VIII) to create T/W.[2] Although the model calls for using the tax, data availability requires that we use pension expenditure. So long as the assumptions of steady-state growth are not seriously violated this does not create a difficulty.

The AGE of a state-pension scheme is measured from the date the first law for the scheme was passed[3] to 1965 (for the 1960s sample) and to 1975 (for the 1970s sample). The variable is used both directly (AGE) and

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[1]On the basis of a comparison of observations on PN from the 1960s to the 1970s in our sample for those countries which are represented in each decade, this is a reasonable assumption; the correlation coefficient for the two sources is 0.880.

[2]PEN is defined as the ratio of pension expenditure to GDP; see Chapter VI, pp. 125-126.

[3]The source is US SSPTW (1977).

in logarithmic form (LAGE). For schemes that did not exist in a target year--Canada for 1965, Australia for 1965 and 1975--the logarithm was set to zero.[1]

The Estimated Neo-Marxian Saving Function

The results of the estimation of the neo-Marxian model are presented in table 9.1. They are, in general, good and consistent with the predictions of the model. The constant term is positive and less than one, and the coefficients on the T/W and T/Y have the opposite sign to the coefficients on AGE and LAGE. The overall fit is good, with $\bar{r}^2$ ranging from .3374 to .5585.

All the same, these results are curious for what is implied by the signs of the pension coefficients. The positive sign on T/W implies that the elasticity of substitution of the underlying production function is greater than one.[2] This is by no means impossible but it is the less rather than the more likely possibility.[3] Still, unless we know for certain that substitution is inelastic for the underlying production function, we can take these results as being consistent with the neo-Marxian model.

We can examine the regressions one by one. Regression 9.1.1 uses T/W as the size variable and LAGE (the log of AGE as the age variable. All the coefficients are significantly different from zero at the .5% level and $\bar{r}^2$ is quite high, .5162. A comparison with regression 9.1.2

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[1]Despite not having a contributory pension scheme, Australia does have a measured pension scheme by our definition because of payments made under its social assistance program.

[2]See equation (9.3).

[3]E.g. Arrow et al (1961).

TABLE 9.1

SAVING FUNCTION REGRESSION RESULTS FOR THE BASIC NEO-MARXIAN MODEL

REGRESSION	C	DECD	T/W	T/Y	AGE*	LAGE*	n	$\bar{r}^2$	ser
9.1.1	0.082 (0.021)		0.898 (0.175)			-1.475 (0.654)	24	0.5162	0.0251
9.1.2	0.081 (0.025)			1.492 (0.404)		-1.349 (0.785)	24	0.3374	0.0293
9.1.3	0.052 (0.014)		1.026 (0.223)		-0.074 (0.037)		24	0.4960	0.0256
9.1.4	0.084 (0.020)	-0.019 (0.011)	1.016 (0.180)			-1.510 (0.625)	24	0.5585	0.0239

Note: The saving ratio, s, is the dependent variable. The number in parentheses are the standard errors of the estimated coefficients. All regressions were run using weighted data.

*Because of the low absolute value of the estimated coefficients on AGE and LAGE the estimates have been multiplied by 100.

Since the measurement ET/W is closer to the theoretically correct measure of the size of a state pension scheme than is T/W, we would expect an improvement in the statistical significance of the size variable and perhaps the other variables as well. In fact, the results in table 9.2 tell the opposite story. Compared regression by regression to the results in table 9.1, the regressions using ET/W were worse in every respect. The overall fit as measured by $\bar{r}^2$ is lower in each case, and the statistical significance of all the estimates are poorer; in fact, only the estimate of the coefficient on ET/W is significantly different from zero. The implication of this is clear: the statement made in the theoretical development of the neo-Marxian model that the measure of the size of a scheme can be identified with the employers' share would seem to be contrary to the evidence.[1]

The results reported in table 9.2 are not sufficient to reject the neo-Marxian model, however. The overall neo-Marxian predictions are still confirmed by the results reported in this table even if statistically they are no longer as convincing. For this reason, we must consider the first regressions, those reported in table 9.1, as being the more supportive of the neo-Marxian prediction that state pension schemes represent a burden to capital, the impact of which is reversed in the very long run. However, now we must declare our ignorance of particular mechanisms by which this effect is transmitted.

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[1]We have not considered errors in the measurement of the employers' share. Apart from ceilings and non-linearities in the contribution formula, many schemes provide for a direct subsidy from the state. We have ignored the state contribution in our division of shares between labour and capital which means that we implicitly assume that the burden of the state contribution is divided in the same proportion as the direct contribution, i.e., differently as the capital-labour share differs. Alternatives might have been to divide the government share 50:50 or to attribute it all to capital or all to labour. The arbitrary nature of all these possibilities indicates the weakness of these methods, and, indeed, the general problem of testing the neo-Marxian prediction that legal incidence is economic incidence without considering the entire state sector.

TABLE 9.2

SAVING FUNCTION REGRESSION RESULTS FOR THE NEO-MARXIAN MODEL ADJUSTED FOR EMPLOYERS' SHARE

REGRESSION	C	DECD	ET/W	T/Y	AGE*	LAGE*	n	$\bar{r}^2$	ser
9.2.1	0.100 (0.026)		1.008 (0.295)			-1.158 (0.790)	24	.2982	0.0302
9.2.2	0.071 (0.016)		0.872 (0.339)		-0.015 (0.040)		24	.2319	0.0316
9.2.3	0.103 (0.026)	-0.010 (0.013)	1.084 (0.313)			-1.158 (0.798)	24	.2846	0.0305

Notes: [as for table 9.1]

*[as for table 9.1]

In regression 9.1.4 a decade dummy has been added as an explanatory variable. It is not statistically different from zero at the 5% level which should not surprise us since we have, in effect, already accounted for decade in the LAGE variable. On the other hand, it is curious that the inclusion of DECD has affected neither the value nor the standard error of the coefficient on LAGE itself which suggests that the decade dummy represents some other systematic influence on the saving ratio which is not directly related to time. The improvement it makes on the overall statistical fit parallels the improvement it has caused in the other models. We can conclude that DECD represents some general environmental change, one which is not directly captured in any of our models. Given the nature of our sample in which two observations on essentially the same list of countries are taken at decade intervals, the use of a decade dummy as a standard procedure is indicated.

In the theoretical model developed in chapter V, the impact of the state pension scheme was related to the employer's share of its cost. The employers' share was identified with the initial value of m $(=m(t_0))$. In the way we have estimated the model, we have ignored this dimension and absorbed the mean of $m(t_0)$ into our estimate of the coefficient of T/W. We can, however, test the hypothesis that the impact of a state pension scheme in a neo-Marxian model is a function of the employers' share of the pension scheme contributions.

In table 9.2, regressions 9.1.1, 9.1.3, and 9.1.4 are rerun using ET/W instead of T/W, where ET/W is simply T/W adjusted for the proportion of state pension scheme financing borne by employers.[1]

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[1]The source is US SSPTW (1971); there is one observation for each country, i.e., one observation serves for both decades. The governments' share of financing was ignored in calculating the employers' share.

demonstrates the point made earlier that T/W is a better measure of the size of the pension scheme than T/Y. Individually, the coefficients do not change qualitatively with the use of a different explanatory variable,[1] but the overall explanatory power of the regression falls considerably, with $\bar{r}^2$ dropping from .5162 to .3374. All three coefficients are less significant when T/Y is used.

Regression 9.1.3 substitutes AGE for LAGE. Qualitatively the results do not change but, on balance, AGE is marginally less satisfactory as a measure of the age of a state pension scheme. It is not hard to see why, since the exponential decay implied by the use of LAGE is intuitively a more appropriate measure of capitalists' ability to recover the profits lost by the acceptance of a state pension scheme.

The implications of the coefficients on AGE and LAGE are both intuitively reasonable. The mean size of a state pension scheme as measured by T/W is .051 (i.e., 5.1%). Using the point estimates in regression 9.1.1 this implies a change of (.051 x 1.026 =) 0.052 in the saving ratio—roughly one half of the mean saving ratio. A measure of recovery of the entire 5% is not very meaningful for an exponential function (it works out to 3,700 years and would in all likelihood, take us well beyond the capitalist stage of development) but the estimates indicate that one percentage point of the saving ratio would be recovered in 5 years and 2½% (half the initial change) in 60 years. The linear measure of decay (regression 9.1.3) shows that the effect of a change of one percentage point of T/W (one fifth the mean) is recovered in 14 years; in other words that capitalists manage to win back the entire profits margin lost to the average scheme in 70 years.

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[1]The coefficient on T/Y bears the expected relation to the coefficient on T/W when we recall that W/Y is, on average, .55 for the sample.

Conclusion

In this chapter we have built and tested a neo-Marxian saving function which incorporates the influence of a state pension scheme. We have tested both the basic neo-Marxian prediction that a state pension will affect the saving ratio as well as the stronger statement that the effect will be related to the employers' share of the formal financing of the scheme.

The results are generally supportive of the neo-Marxian model. We have found that the size of a state pension scheme is positively associated with the saving ratio and that its age is negatively associated. These results imply that the production function underlying the sample has an elasticity of substitution greater than unity. As for the specific prediction that the effect is related to the size of the employers' share of pension contributions, the evidence is not supportive.

In the test of the neo-Keynesian model, we found that the particular form of the saving function which was consistent with evidence was of a general nature that could have been derived from a neo-Marxian model. Since we have found in this chapter that a more fully-specified neo-Marxian saving function is confirmed by the data, we might be led to reject the neo-Keynesian model altogether in favour of the neo-Marxian model. Such a conclusion would be premature however, since we have not yet compared these last two specifications in the framework of a formal test of model selection. That test is the subject of the next chapter.

CHAPTER X

A COMPARATIVE TEST OF THE THREE MODELS

Introduction

In the previous three chapters we have tested, in turn, a neoclassical, neo-Keynesian, and neo-Marxian model of a state pension scheme. All three models "explain" the variation in the saving ratio in a statistical sense to a greater or lesser degree. Using, for example, the usual statistical criteria for model selection, $\bar{r}^2$, the regression on the common sample which best represent the different variations of the three models are ranked as follows:

Rank	Model	Regression	$\bar{r}^2$
1	neo-Keynesian, investment basis	8.1.7	.8663
2	neoclassical, non-linear	7.2.5	.7935
3	neo-Keynesian, saving basis	8.2.5	.5980
4	neoclassical, linear	7.3.9	.5930
5	neo-Marxian, basic	9.1.4	.5585
6	neo-Marxian, employers' share	9.2.1	.2982

The neo-Keynesian model, investment basis, followed closely by the non-linear neoclassical model would seem to provide the best statistical explanations of the saving ratio; the two neo-Marxian models are at the bottom of the list. However, as a ranking criterion for economic models, as opposed to purely predictive statistical models, $\bar{r}^2$ is clearly insufficient, since on economic grounds we must eliminate, first, the neo-Keynesian model, investment basis as indicating an unstable model, second, both neoclassical models as having the wrong sign on the state pension scheme, and third, the neo-Marxian model adjusted for employers' share as being inconsistent with the basic neo-Marxian model.[1] This leaves us with only the neo-Keynesian model, saving basis and the basic neo-Marxian model. On the basis of $\bar{r}^2$ alone we should choose the neo-Keynesian model. Is this an appropriate procedure?

Pesaran (1974) points out that the use of $\bar{r}^2$ as a model selection criterion involves the assumption that one of the tested models is the "true" one. Since it is rare, he argues, that an economic model will have all the characteristics required for the use of classical regression techniques, let alone that one of the models under consideration is the "true model"--whatever that may be--comparing $\bar{r}^2$ or any other statistic based on residuals is not the correct procedure for model selection.[2] Rather, he concludes,

> ... it seems more appropriate to treat the problem of choosing among alternative models as an hypothesis testing problem rather than as an arbitrary definition of what the "true model" should be. (Pesaran, 1974, p. 154).

Briefly, the hypothesis-testing method involves taking each alternative model in turn as the maintained hypothesis and then using the data and an alternative hypothesis to see if the maintained hypothesis still explains the phenomenon we are studying. Pesaran and Deaton (1978, pp. 678-79) summarize the strength of this strategy in comparison to methods relying upon relative fit:

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[1] See chapters VII, VIII and IX.

[2] "The assumptions underlying the classical regression models are made, not because they are optimal from the point of view of economic theory, but because they are extremely convenient for estimation and hypothesis testing purposes" (Pesaran, 1974, p. 154). A trail of such simplifying assumptions can be traced through the econometric tests of chapters VII, VIII and IX.

> By making such tests, we are using the hypotheses in the same way that one usually uses the data; for example, the formulation of a previously unconsidered hypothesis can lead to new inferences about existing models just as would the discovery of new data. This highlights a basic feature of empirical methodology, that hypotheses are responsible for organizing data in order to yield meaningful information and that, without such organization, observations are meaningless, if not impossible. We thus consider that not only are [such] procedures ... necessary to make comparisons between hypotheses, but that the ability to make meaningful inferences about the truth of any single hypothesis demands the presence of at least one non-nested alternative. In econometrics, we never have a maintained hypothesis which we believe with certainty; we must always use the models we possess to organize the evidence in different ways and to ask whether the patterns which result are consistent with the views we currently hold.
>
> It is important that notions of the absolute fit or performance of individual models play no part in the analysis. Indeed, it should be clear from the previous discussion that, apart from the nested case, we regard such indicators as meaningless. In considering whether an alternative hypothesis, together with the data, contains sufficient information to reject the currently maintained hypothesis, the question of whether that alternative "fits" well or badly, even if meaningful, is certainly irrelevant. An hypothesis, which one would not wish to consider seriously in its own right, can be a perfectly effective tool for disproving an alternative, even if that alternative may in some respects seem much more promising. It is thus important that tests between non-nested hypotheses or models should encompass the possibility of rejecting both .... This is notably not the case for tests which compare relative fits, for example comparisons of R^2 statistics or likelihoods. Nor is it true of Bayesian procedures which, in the absence of discriminatory prior information, reduce to comparisons of likelihoods.

Since we have no single maintained hypothesis, the strategy of model selection by hypothesis testing is an appropriate one and is the one we will use.

The actual method that we will use is the Davidson-MacKinnon J-test for model selection (Davidson and MacKinnon, 1981). Its objectives and assumptions of the J-test are the same as those advocated by Pesaran (Pesaran, 1974; Pesaran and Deaton, 1978) but computationally it is considerably simpler.[1]

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[1]Davidson and MacKinnon (1981, pp. 790-792) provide a demonstration of the practical equivalence of their test to that of Pesaran and Deaton.

4. Neoclassical, linear, regression 7.3.9,

$$s = -\underset{(0.039)}{0.017} + \underset{(0.017)}{0.028}\,\text{DECD} + \underset{(0.91)}{2.79}\,\text{PG} - \underset{(0.211)}{0.533}\,\text{RW} + \underset{(1.08)}{3.23}\,\text{YINV}$$

$$+ \underset{(0.013)}{0.042}\,\text{P1}.$$

Despite the radically different approach to the model selection implied in the use of the J-test, the results reported in table 10.1 are not strikingly different from those suggested by the use of the $\bar{r}^2$ criterion: a model is always rejected if the model used as the alternative hypothesis has a higher $\bar{r}^2$. Thus the neo-Keynesian model, investment basis ($\bar{r}^2 = .8663$) rejects all the other models. On the other hand some models are rejected by alternative models having a lower $\bar{r}^2$. For instance, the neo-Keynesian model, saving basis is rejected by both the basic neo-Marxian model and the linear neoclassical model, both of which have lower $\bar{r}^2$'s than the tested hypothesis. Effectively, the J-test is a more conservative criterion than a single dimension criterion such as $\bar{r}^2$; as a consequence, the two candidate models are rejected.[1]

What is implied about the candidate models that they are rejected? Essentially what it means is that they add too little to the explanation of the alternative hypothesis. Again, this is not simply a matter of

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[1]We have used the 5 percent (two-tail) level of significance for our tests; for the candidate models this implies a critical value for t of 2.093. (Because of the presence of $\hat{B}$ as a explanatory variable in these regressions, the number of regressors is one greater than the normal number for the tested models; thus, for candidate models, in table 10.1, K = 5.) Since the tests are based on asymptotic distributions, a higher critical value might be appropriate. In a sample test using 82 time series observations, Davidson and MacKinnon (1981, p. 792) suggest a critical value of 2.5. By the same reasoning, we might use an even higher level, say 2.75, with our smaller sample. If we do, the neo-Keynesian, saving basis model is no longer rejected by the neo-Marxian nor the neo-Marxian by the neo-Keynesian, saving basis, model. What is striking though is that now the neoclassical linear model no longer rejects the neo-Marxian but continues to reject the neo-Keynesian, saving basis, though the latter has a higher $\bar{r}^2$ than the former. Note that our general conclusions are not changed since both candidate models are still rejected by the neo-Keynesian model, investment basis.

J-Test Results

Only two models survive the economic plausibility tests. For the purposes of the J-test we will use the particular variation of each of these models which report the highest $\bar{r}^2$ from among the family of models. This is, admittedly, an ad hoc procedure given our stated views on the validity of using $\bar{r}^2$ for model selection, but it is justified in this case since the candidate models are finally tested in a formal hypothesis-testing framework, viz., the J-test itself. The two candidate models are (standard errors in parentheses):

1. Neo-Keynesian, saving basis, regression 8.2.5,

$$S = -\underset{(0.070)}{0.084} - \underset{(0.089)}{0.111}\ DECD + \underset{(0.155)}{0.406}\ P/Y + \underset{(0.205)}{0.326}\ DP/Y$$

2. Neo-Marxian, basic version, regression 9.1.4,

$$S = \underset{(0.020)}{0.084} - \underset{(0.011)}{0.019}\ DECD + \underset{(0.180)}{1.016}\ T/W + \underset{(0.625)}{1.510}\ LAGE.$$

The results reported are those of the regressions run over the common 1960s-1970s sample of 24 observations.

Since it is legitimate in the context of the J-test to use models which have been rejected on economic grounds as alternative hypotheses, we also include the neo-Keynesian model, investment basic (on account of its very high $\bar{r}^2$) and the linear neoclassical model (because of its popularity in economic research).[1] The two versions used are

3. Neo-Keynesian, investment basis, regression 8.1.7

$$S = \underset{(0.045)}{0.480}\ P/Y + \underset{(0.016)}{0.127}\ DP/Y - \underset{(0.019)}{0.169}\ KTOY + \underset{(0.593)}{3.005}\ YINV$$

---

[1] The linear neoclassical model is compared to the non-linear version below.

The method of model selection proposed by Davidson and MacKinnon is very simple. It involves seeing whether calculated values of y_i (using the alternative hypothesis) are statistically closer to y_i than the explanatory model H_o. Thus if we run the following regression

$$y_i = (1-\alpha)A(X_i, \pi) + \alpha\hat{B}_i + \varepsilon_i \tag{10.3}$$

where $\hat{B}_i = B(Z_i, \hat{\gamma})$ and $\hat{\gamma}$ is the maximum likelihood estimate of γ, we can test the truth of A by a conventional t-test or maximum likelihood ratio test of whether or not $\alpha = 0$ (if H_o is true, the true value of α is zero). Note that $\hat{B}_i$ is asymptotically independent of the error term ε_i since it is a function of exogenous variables Z_i and parameter estimates $\hat{\gamma}$, where Z_i are independent of ε_i by assumption, the latter asymptotically so (ibid., 1981, p. 782). The test on α is referred to as the J-test.[1]

It is important to note that the t-statistic from the J-test is conditional on the truth of H_o. Thus the estimate of α will not necessarily converge asymptotically to one if H_1 is true. In order to test for the truth of H_1 we must reverse the positions of H_o and H_1 in (10.1)-(10.3) and repeat the test. The interpretation of the pair of tests is clear if one hypothesis is accepted and the other rejected. However, after the pair of tests we might find that both hypotheses are rejected or that both are accepted. The first possibility is an interesting one and reflects a strength of the test that two false hypotheses can disprove each other. The second possibility is, in view of the non-nested assumption, seemingly illogical but reflects the likelihood that there exists some underlying equivalence between the models.

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[1] Asymptotic properties of the test are provided in Davidson and MacKinnon (1981, section 2).

In the next section the model selection procedure of Davidson and MacKinnon is described. In the section following that, the test is applied to the problem of choosing between the neo-Keynesian model, saving basis, and the basic neo-Marxian model. As an illustration of the application of the test which allows for the use of clearly "wrong" models in disproving merely "poor" models, the neoclassical linear and neo-Keynesian, investment basis models are to be included in the test. There is a short concluding section.

The Davidson-MacKinnon J-Test

Consider a single model which purports to explain the variation in some variable y.[1] Let this be written as the null hypothesis, H_o. Thus,

$$H_o : y_i = A(X_i , \pi) + \varepsilon_{oi} \tag{10.1}$$

where y_i is the i observation on y , X_i some vector of exogenous variables, π is a vector of parameters and ε_{oi} is NID$(0 , \sigma_o^2)$. Consider also an alternative explanation of the variation in y :

$$H_1 : y_i = B(Z_i , \gamma) + \varepsilon_{1i} \tag{10.2}$$

where Z_i is another vector of exogenous variables and γ another vector of paramaters. The error term ε_{1i} is assumed to NID$(0 , \sigma_1^2)$ if H_1 is true. Neither H_o nor H_1 need to linear models.

Finally, assume that neither H_o nor H_1 is nested within the other which means that the truth of H_o implies the falsity of H_1 and vice versa.

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[1]The next few paragraphs follow closely Davidson and MacKinnon (1981), especially section 1. The interested reader is referred to the original article for further details.

TABLE 10.1

PAIRWISE J-TEST STATISTICS FOR REPRESENTATIVE MODELS

Tested Hypothesis	Alternative Hypothesis			
	Neo-Keynesian (saving)	Neo-Marxian (basic)	Neo-Keynesian (investment)	Neoclassical (linear)
1. Neo-Keynesian (saving)-8.2.5	.5980	0.535 (2.285) R	0.998 (6.196) R	0.682 (3.440) R
2. Neo-Marxian (basic)-9.1.4	0.618 (2.731) R	.5585	0.911 (6.914) R	0.732 (2.601) R
3. Neo-Keynesian (investment)-8.1.7	0.010 (0.039)	0.158 (0.958)	.8663	0.121 (0.641
4. Neoclassical (linear)-7.3.9	0.945 (4.052) R	0.748 (1.863)	0.962 (5.907) R	.5930

Note: Entries on the diagonal are $\bar{r}^2$. The first element in each off-diagonal cell is the estimated value of α from the J-test regression (10.3), and the second element (in parentheses) is the t-statistic on α; R indicates that at the 5 percent level of significance the tested hypothesis is rejected.

relative $\bar{r}^2$. In principle, a model with a respectable $\bar{r}^2$ might leave unexplained just that variation that the tested hypothesis deals well with; in such a case a model would survive the test. (As an example consider the linear neoclassical model as a tested hypothesis [row 4]. It is clearly rejected by the neo-Keynesian model, saving basis, but not by the neo-Marxian model, though these two models have $\bar{r}^2$ of approximately equal value.) If the alternative hypothesis has a significantly higher $\bar{r}^2$ than the tested hypothesis, it is, while not impossible, highly unlikely that there will be enough unexplained variation, whatever its pattern, for the tested hypothesis to explain. It will be, rightly, rejected.

The spoiler throughout this test (particularly of the adjustment in the critical value of t referred to in the last footnote made) is the neo-Keynesian model, investment basis. For interest's sake, it and the linear neoclassical models, are tested in rows 3 and 4. Not surprisingly, as the model which has so unambiguously rejected the two candidate models, it itself is rejected by none. In the context of the theory we have no explanation. The coefficient values of this model are consistent only with an unstable neo-Keynesian model.[1] Whether these results can lead us somewhere is dealt with in the next chapter. But on the basis of the experiment conducted we must conclude that none of the models we have developed is, for either economic or statistical reasons, a valid representation of a capitalist economy.

It remains only to compare the two neoclassical models. Again, as we have indicated for the J-test on the neo-Keynesian, investment basis, and linear neoclassical, this is for theoretical interest only, since both models are ruled out on economic grounds.

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[1]As indeed are probably the neoclassical results; see appendix B.

The specifications of the two neoclassical versions are

1. Non-linear neoclassical, regression 7.2.5

$$s = \left\{\frac{0.1052 + 0.1213P1}{g + 0.0623}\right\} g \tag{7.2.5}$$

2. Linear neoclassical, regression 7.3.9

$$\begin{aligned} s = &- 0.017 + 0.028DECD + 2.79PG \\ &- 0.533RW + 3.23YINV + 0.042P1 \end{aligned} \tag{7.3.9}$$

The pairwise test results are as follows:

Tested Hypothesis	Alternative Hypothesis	
	Non-linear	Linear
Non-linear	.7935	0.932 (2.729) R
Linear	0.006 (0.056)	.5930

Again, the elements on the diagonal are the values of $\bar{r}^2$ and in the off-diagonal, first the estimated value of α and then the value of the t-statistic in parentheses. The results are counter-intuitive. The low-$\bar{r}^2$ linear model rejects, at the 5 percent level of significance, the high-$\bar{r}^2$ non-linear model. The explanation is probably the following. The non-linear model has a higher $\bar{r}^2$ than the corresponding linear model simply because it is non-linear.[1] However, the extra information in the particular version of the linear model, especially the inclusion of YINV, means that the better

[1]This is borne out by a comparison between the non-linear and a simpler linear neoclassical model. In a pairwise J-test of the non-linear model against the following version of the linear neoclassical model,

$$s = 0.0345 + 2.009PG - 0.355RW + 0.051P1$$

fit counts for little against additional, valid dimensions of explanation in the linear model. Put differently, what this tells us is that the niceties of fit that come with a non-linear function are of less significance than getting, if only approximately, a more complete set of explanatory variables.

Conclusion

In this chapter, the two surviving models of a saving function, the neo-Keynesian, saving basis, and the neo-Marxian, basic version, have been tested using the Davidson-MacKinnon J-test for model selection. On the basis of this test neither model can be accepted as providing an adequate explanation of a macroeconomic saving function.

We conclude that the experiment which we have conducted offers insufficient evidence for choosing any one of the three models we have studied over the others. What this means for the broader question of model selection and for the specific question of the economic effect of a state pension scheme is dealt with in the next chapter.

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the values of the t-statistics were: 0.808 when the non-linear model is the tested hypothesis; and -0.959 when the linear model is the tested hypothesis. Neither rejects the other though the non-linear model has a far higher $\bar{r}^2$: .7935 as against .4522 for the linear model. Apart from using PG and RW for g, the linear model is, in fact, a limiting case of the non-linear; see (7.16).

PART THREE

CONCLUSION

CHAPTER XI

THEORIES AND FACTS

Summary

In part one of this thesis, we developed three alternative models of a state pension based on neoclassical, neo-Keynesian, and neo-Marxian foundations. All three models were set in a long-run, steady-state framework.

In part two, we tested the models, both individually and comparatively. The test statistic used was the aggregate saving ratio and the sample, an international cross-section of western, industrialized countries. On the basis of economic criteria we eliminated the neoclassical model entirely and one version each of the neo-Keynesian and neo-Marxian models. In a test of model selection, we rejected the two remaining candidate models.

We thus conclude that the experiment we have performed does not, on strict statistical grounds, allow us to choose any of the three models as an adequate representation of reality. Moreover, on the same economic and statistical grounds, we cannot draw any firm conclusion in respect of our second objective, the economic effect of a state pension scheme.

The fact that our experiment was formally inconclusive does not mean that we have learned nothing about the questions we have set out to investigate. In the next and final section we make a somewhat freer interpretation of our results to see what they suggest, first, about the question of model selection, and, second, about the economic effects of a

state pension scheme. Because of their speculative nature, these comments do not have the force of conclusions, but can indicate the direction of fruitful research.

A Freer Interpretation

Our primary objective in this thesis has been to find evidence that would help evaluate the empirical validity of the three basic models of a capitalist economy--the neoclassical, the neo-Keynesian, and the neo-Marxian. While we concluded that on a combination of strict economic and statistical criteria none of the models is supported by the evidence, some patterns do emerge from the tests. In the next few paragraphs we speculate about the direction research might take, first, on the question of model selection and second, on the question of the economic effects of state pension schemes.

The fact that both the neo-Keynesian, saving basis, and the basic neo-Marxian models reject each other is curious. The neo-Marxian model is economically, though not statistically, nested within the neo-Keynesian model. Both are derived from a Pasinetti (classical) saving function: the neo-Keynesian model explains the saving ratio directly in terms of the profit share; the basic neo-Marxian model goes behind the profit share to its state pension determinants. One would expect either that one rejects the other, or that neither rejects the other. The first outcome would indicate a successful experiment; the second that the data does not allow discrimination. Mutual rejection is harder to explain given the nature of the two models. The problem may lie in the way we have set up the test. Because of the size of the sample, the critical value we have used for the t-statistic may have been too low for a test of which

We turn finally to the question of the economic effects of a state pension scheme. Two alternative interpretations present themselves. In the first, we take note of the regularity with which a positive sign turns up on the state pension variable. This, in turn, can be, taken in one of two ways. One is that the neo-Marxian model is, broadly speaking, correct: the other is that there is a fourth model, incorporating in some way an incentive effect,[1] which has been overlooked. As far as policy is concerned, both point to the same conclusion: The popular concern that state pension schemes "crowd out" real saving would seem to be misplaced. Although the policy conclusions are the same, the research implications are not. If one leans to the neo-Marxian interpretation, the rejection by the neo-Keynesian model, investment basis, remains a serious obstacle. If one argues for the fourth-model approach, one risks the charge of ad-hocery unless a fully specified basic model is advanced. It is by no means an impossible task to create a plausible story but since there is so little to work with, in comparison to the traditions we have drawn on, there is a long way to go.

The second alternative explanation looks more fruitful. Again, it turns on the neo-Keynesian model, investment basis, and it is conditional on the instability implied by that model being appropriately dealt with. If we start with that model, the addition of state pension variables adds nothing--they are, statistically, insignificantly different from zero. Disregarding for the moment the instability implied by neo-Keynesian investment function, the implication is clear: State pension schemes have no effect on the saving ratio. As a policy matter, one then should be concerned only with social criteria; the economic effect of these schemes is nil. The implication for research is similar; state pension schemes are not important as economic phenomena.

[1]For example, Cagan's (1965) "recognition effect".

only the asymptotic properties are proven. Consequently, we should have used a higher critical value of t, say 2.75.[1] Had we done so neither test would have rejected the other. We could thus have concluded (as we might have intuitively expected) that there is not enough data to discriminate between the two models.[2] This comment is really about the J-test, the small sample properties of which have not been developed.

Whatever conclusion we draw about the two saving-basis models, we are faced with a more compelling result: the clear rejection of all models by the neo-Keynesian model, investment basis, on any reasonable statistical criterion. As has been made clear, the acceptance of this model in our framework implies a neo-Keynesian world with an unstable equilibrium. Still, it fits very well. Moreover, adding a state pension scheme adds nothing: the state pension variable P1 has a t-statistic of 1.4 when added, with g, to the neo-Keynesian model, investment basis.[3] As was made clear in chapter VIII, there is no theoretical basis in econometrics for drawing conclusions from these results. They do, however, suggest that there is need for further research on the nature of the neo-Keynesian investment function. In particular, they indicate that as a guide to the choice of variables, the neo-Keynesian model is a very useful one--more useful than the orthodox neoclassical model with its emphasis on the growth rate and quasi-assets (such as the state pension scheme), and one with no greater a theoretical disability. If the problem of its indicating an unstable equilibrium can be adequately dealt with, the neo-Keynesian model will reward further research.

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[1] See p. 203 n.1, chapter X.

[2] Changing the critical value ex post is not a legitimate statistical procedure. It is suggested for what it implies for subsequent applications of the J-test.

[3] See chapter VIII, p. 177. Adding P1 alone gives even worse results (t=0.385); using T/W, the neo-Marxian pension variable, has an almost identical effect.

APPENDIX A

A DEMONSTRATION OF THE EQUIVALENCE OF THE MARGLIN AND MODIGLIANI LIFE CYCLE MODELS

The Marglin version of the life cycle models which is the basis for the neoclassical model of a state pension scheme developed in chapter III, is defined in terms of the present value of life time resources measured in period one, W_A. Given W_A and a utility function, the utility-maximizing levels of consumption for every period are uniquely determined.

In the Modigliani version of the life cycle model, consumption each period is defined as a proportion of remaining life-time wealth measured in that period, W_{OT}, with the factor of proportionality a function of the form of the utility function, the profit rate and the present age, T, of the household.

To demonstrate the equivalence of these two versions of the life cycle model it is sufficient to show that the Marglin model can be restated in the Modigliani form. We begin by showing the equivalence for a two-period model.

In a two-period model with a state pension scheme, the present value of life-time resources in the Marglin model is defined as

$$W_A = W\{(1-\tau) + \tau(1+g)/(1+r)\} \tag{A-1}$$

where W is the real wage (earned in period one only), τ the state pension contribution rate, g the growth rate, and r the profit rate. Using the two-period logarithmic utility function it can easily be shown that[1]

$$C^1 = \beta W_A \tag{A-2}$$

$$C^2 = (1+r)(1-\beta)W_A \tag{A-3}$$

[1] See chapter III.

where C^1 and C^2 are consumption in periods one and two respectively, and β is the parameter of the utility function.

The Marglin demand functions can be restated in the form required by the Modigliani model. As stated, consumption in the first period (A-2), is already in a form consistent with the Modigliani definition since in period one clearly

$$W_A = W_{01} .$$

Thus, in period one,

$$C^1 = \beta W_{01} ;$$

first-period consumption is proportional to first-period wealth, the factor of proportionality, β being a function of the form of the utility function as required by the Modigliani definition.

Wealth in the second period as measured in the Modigliani version is the state pension benefit to be received in that period plus the accumulated value of first-period saving (second period labour income is nil in this two-period model). Thus

$$\begin{aligned} W_{02} &= \tau W(1+g) + [W(1-\tau) - C^1](1+r) \\ &= \{\tau W(1+g)/(1+r) + W(1-\tau) - \beta W_A\}(1+r) \\ &= \{W_A - \beta W_A\}(1+r) \\ &= (1+r)(1-\beta)W_A \end{aligned}$$

which is identical to second-period consumption as defined in the Marglin model (A-3), i.e.,

$$C^2 = W_{02}$$

and the same requirement of proportionality is maintained, in this case the factor being simply unity.

We have shown that the two-period Marglin life cycle model can be restated in terms of the Modigliani definition of the model. It can be shown that in an N-period model, the Marglin model can be restated in the following form:

$$C^1 = \beta_1 W_{01}$$

$$C^2 = [\beta_1/(\beta_2 + \ldots +\beta_N)] W_{02}$$

$$C^T = [\beta_{T-1}/(\beta_T + \ldots \beta_N)] W_{OT}$$

$$C^N = W_{ON}$$

where $\beta_1 \ldots \beta_T \ldots$ are the parameters of an N-period logarithmic utility function. As required by the Modigliani model, the factors of proportionality are functions of age, T and the form of the utility function.

This demonstration can be generalized to account for the influence of the profit rate on the β's. In the examples above, the factors of proportionality were independent of the profit rate. This, however, was simply a consequence of using a logarithmic utility function. With a more general utility function such as the CEMU utility function introduced in chapter II, the factors of proportional are, in general, a function of the profit rate as well as age and the form of the utility function. With such a generalization, the complete equivalence of the Marglin and Modigliani life-cycle models is demonstrated.

APPENDIX B

A RATIONALIZATION OF A POSITIVE STATE-PENSION-SCHEME COEFFICIENT IN THE CONTEXT OF AN UNSTABLE NEOCLASSICAL MODEL

Early in chapter VII (page 128, n.3) the possibility of a positive sign on the total derivative $ds/d\tau$ was advanced. Since a positive association between the saving ratio and the size of a state pension scheme could only occur in an unstable long-run equilibrium, this line of thought was not pursued in the development of the econometric neoclassical saving function. In the light of the results of the estimation in chapter VII, the possibility begins to look more interesting. Of course, the instability that the positive derivative implies means that any rationalization of these empirical results can have only theoretical interest. It is not a confirmation of a useful neoclassical model.

We begin with an important distinction. The theoretical discussion about the possibility of a positive correlation between s and the size of a state pension scheme involved the sign of the total derivative of s with respect to τ. $\bar{C}$ in equation (7.13) represents $-\alpha_3$ in equation (7.12) (where α_3 is the marginal propensity to consume out of state pension wealth) and thus, should always be negative. The issue can be made clearer if we write out the total derivative of s with respect to p (=SPSW/Y) using equation (7.13)[1].

$$\frac{ds}{dp} = \frac{\partial s}{\partial p} + \frac{\partial s}{\partial r}\frac{dr}{dp}$$

$$= \bar{c} + \frac{\partial s}{\partial r}\frac{dr}{dp} = -\alpha_3 + \frac{\partial s}{\partial r}\frac{dr}{dp}$$

[1]This is the total derivative of the reduced form of the saving function; at the beginning of chapter VII the total derivative is based on the structural equation (7.1). Note also τ is a proxy for p.

$\bar{c}$ is always negative but both $\frac{\partial s}{\partial r}$ and $\frac{dr}{dp}$ have ambiguous signs, and, in particular, the sign of $\frac{dr}{dp}$ will depend on the value of σ and r.

Since we have estimated a single equation model, what might be happening is the following. The values of r and p are such in our sample that $\frac{ds}{dp}$ is positive (i.e., we are in an unstable long-run equilibrium) but since we have estimated the means rather than observed the values of the coefficients A_i, B_i and C_i, the estimate of $\bar{C}$ which is in theory $\frac{\partial s}{\partial p}$, has been forced to take up the positive sign of the total derivative thus overwhelming the negative sign of the partial derivative. (See table 7.2) In principle this hypothesis could be checked by estimating [cf. equation (7.13)]

$$s_i = g_i\{\frac{(1-\alpha_{1i}) - c^*\alpha_{2i}\, p_i}{g_i+\alpha_{2i} - (\alpha_{1i}\cdot r_i)}\} \tag{B.1}$$

(where α_{1i}, α_{2i} and r_i are <u>observed</u> not estimated) and confirming that c* is roughly equal to one i.e., that the marginal propensity to consume out of state pension wealth is approximately equal to the marginal propensity to consume out of real wealth. Such a full test is precluded here by the absence of a consistent set of observations on the α_i's (these are in fact time series consumption function coefficients) <u>and</u> the profit rate r, but a limited test of this proposition is available.

Swamy (1968) has estimated the coefficients of the dynamic saving function of the form

$$S_t = \alpha_1 S_{t-1} + \beta_1 \Delta Y_t \tag{B.2}$$

where S and Y are per capita saving and per capita income. Modigliani (1974, p. 17, n.5) has shown that if the approximation $\Delta A = S_{t-1}$ is used, equation (B.2) is equivalent to the following variant of the life cycle hypothesis

$$C_t = \acute{\alpha}_1 Y_t + \acute{\alpha}_2 A_t \tag{B.3}$$

where $\acute{\alpha}_1 = 1-\beta_1$ and $\acute{\alpha}_2 = 1-\alpha_1$.

Assuming that a consumption function based on Y is a close substitute for one based on W as required by our theory, Swamy's point estimates of $\acute{\alpha}_1$ and $\acute{\alpha}_2$ can be used to estimate the following variant of (B.1)[1,2]

$$s_i = g_i\left\{\frac{(1-\acute{\alpha}_{1i}) - c^*\acute{\alpha}_{2i}p_i}{g_i + \acute{\alpha}_{2i} - b^*\acute{\alpha}_{1i}}\right\} + \varepsilon_i \tag{B.4}$$

Since observations on r_i are not available its mean, b*, is estimated and examined for reasonableness.

Swamy estimated time series coefficients for the period 1950-1964 for a collection of countries which included 8 of our 1950s sample.[3] The point estimates of b* and c* along with their standard errors for this subset of our 1950s sample are

$$\hat{b}^* = 0.022 \quad (0.060)$$

$$\hat{c}^* = 0.833 \quad (2.64)$$

---

[1] The series for $\acute{\alpha}_2$ has been corrected for the exclusion of state pension wealth in Swamy's estimates along the lines described in chapter VII. Thus, assuming that the over-estimate of the marginal propensity to consume out of household wealth is roughly proportional to the ratio of state pension wealth to national income and taking the US figures as the benchmark, the "adjusted" $\acute{\alpha}_2$,

$$\acute{\alpha}'_{2i} = [1 - \frac{(SPSW/Y)i}{(SPSW/Y)us} .2]\acute{\alpha}_{2i} .$$

[2] Note that the negative sign is written explicitly in the numerator of (B.1) and (B.4) in contrast to its treatment in (7.13).

[3] All the large-weight countries are kept. The subset is made up of Australia, Austria, Belgium, Canada, France, West Germany, The Netherlands, U.S.A.. Running non-linear regression (7.13) using this sample of 8 countries gives the following point estimates of the coefficients (standard error in parentheses) $\bar{A}$, 0.0651 (0.0341); $\bar{B}$, -0.0041 (0.00895); $\bar{C}$, -0.0445 (0.016). Compared to regression 7.2.1 in table 7.2, the change in sign of the estimate of $\bar{B}$ and $\bar{C}$ is evidence of the limitations of this sample.

APPENDIX C

DATA

The basic data is listed in the order in which it is introduced in the thesis:

s PEN WEIGHT g PG RW MW LAFP65[1] LE65 RT UP Y P/Y KTOY AGE EMPS[2].

For reference the following calculated variables are also listed:

F P1 P2 YINV PENW T/W[3] ET/W

The numbers alongside each list identify the decade and the country as follows:

1960s	1970s	
1	16	Australia
2	17	Austria
3	18	Belgium
4	19	Canada
-	21	Finland
7	22	France
8	23	Germany
-	24	Italy
10	25	Netherlands
11	26	Norway
12	27	Sweden
14	29	UK
15	30	US

Only the variables needed for the neoclassical test were drawn

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[1] 1950s sample only; see p. 221.

[2] Employers' share of pension contributions.

[3] Note that T/Y is PEN.

The inclusion of Japan for which Swamy also had estimates gives the following results

$$\hat{b}^* = -0.043 \quad (0.036)$$

$$\hat{c}^* = \ 0.217 \quad (2.01) \ .$$

The results are strongly suggestive if not conclusive. The estimate of b^* (i.e., $\bar{r}$) has the wrong sign but is within 2 standard errors of its "expected" value of .04 in the 8 country subset and within 2 standard errors of a reasonable mean value (.03) for the sample including Japan. The estimate of C^* is within two standard errors of the required value of unity although the standard error is relatively large.

Given the approximations involved in estimating the $\hat{\alpha}_{2i}$'s, the use of the unadjusted figures (see footnote 1 p. 136) may be more appropriate.[1] Estimating equation (B.4) above using Swamy's "raw" estimates of α_{2i} gives the following results:

$$\hat{b}^* = 0.0369 \quad (0.0096)$$

$$\hat{c}^* = 0.864 \quad (0.510)$$

These results are even closer to what would be expected if the explanation of an indeterminate sign of the estimate of $\bar{C}$ given above is true.

These results are interesting and in a sense, validate the neoclassical model. However, to accept them as such is to accept a model which provides only for an unstable equilibirum. If the instability problem could be dealt with in some other way, then these results might be of use. Until then they cannot be taken very seriously.

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[1]Most of Swamy's estimates are of a reasonable order of magnitude, i.e., implying marginal propensities to consume out of income between 0.6 and 0.9 and marginal propensities to consume out of assets in the neighbourhood of 0.1. Some, however, are of doubtful validity. Australia has a mpc out of income of 0.184 and Belgium has a marginal propensity to consume out of assets (unadjusted) of 0.533. Replicating the "reasonableness check" of chapter VII using Swamy's adjusted US estimates of $\hat{\alpha}_1$ and $\hat{\alpha}_2$ gives a calculated value of $\hat{s}$ of .1295, which is about right.

from the 1950s sample. The countries are the following:

1	Australia
2	Austria
3	Belgium
4	Canada
5	Denmark
6	France
7	Germany
8	Italy
9	Luxembourg
10	Netherlands
11	New Zealand
12	Norway
13	United States
14	Japan

The definitions and sources of the data can be found in the following chapters:

Chapter VI - s PEN

VII - WEIGHT g PG RW MW LAFP65 LE65 RT UP Y; F P1 P2

VIII - P/Y KTOY

IX - AGE EMPS; T/W ET/W.

	S	PEN	WGT
1	0.1194	0.0246	11,122
2	0.1105	0.0627	7,215
3	0.1300	0.0341	9,367
4	0.0921	0.0213	19,325
7	0.1212	0.0334	48,310
8	0.1320	0.0622	57,971
10	0.1516	0.0606	12,127
11	0.0721	0.0399	3,694
12	0.0668	0.0424	7,662
14	0.0815	0.0352	54,033
15	0.8330	0.0276	191,889
16	0.[illegible]505	0.0311	13,379
17	0.1225	0.0732	7,525
18	0.1535	0.0518	9,742
19	0.1222	0.0280	22,072
21	0.0556	0.0519	4,666
22	0.1247	0.0619	52,118
23	0.1203	0.0773	61,976
24	0.1977	0.0757	54,913
25	0.1427	0.0880	13,439
26	0.0531	0.0714	3,961
27	0.0406	0.0711	8,136
29	0.0694	0.0468	56,000
30	0.7080	0.0433	210,410

	G	PG	RW
1	0.05127	0.03142	0.137964
2	0.04518	0.03986	0.194331
3	0.04794	0.04248	0.192756
4	0.05537	0.03737	0.129813
7	0.05562	0.04483	0.199841
8	0.04567	0.03671	0.164580
10	0.04977	0.03702	0.139550
11	0.04512	0.03714	0.175987
12	0.04481	0.03785	0.193825
14	0.02848	0.02267	0.189628
15	0.04307	0.03022	0.154515
16	0.03603	0.01921	0.140442
17	0.04205	0.04030	0.238925
18	0.03692	0.03431	0.211356
19	0.04368	0.03126	0.136150
21	0.03338	0.03034	0.157270
22	0.04131	0.03488	0.227707
23	0.03034	0.02803	0.241379
24	0.03084	0.02399	0.188679
25	0.03656	0.02770	0.151181
26	0.04353	0.03764	0.204800
27	0.01810	0.01377	0.246512
29	0.02307	0.02202	0.210863
30	0.02949	0.02108	0.172956

	MW	LE65
1	0.477544	14.1
2	0.380472	13.7
3	0.382047	13.6
4	0.573765	15.1
7	0.389984	14.4
8	0.362137	13.7
10	0.468167	15.0
11	0.403791	15.1
12	0.312199	14.9
14	0.358359	13.9
15	0.517726	14.6
16	0.439336	14.0
17	0.389739	13.8
18	0.365931	13.7
19	0.428795	15.5
21	0.326409	13.5
22	0.364650	15.2
23	0.326019	13.8
24	0.383648	14.9
25	0.423622	15.3
26	0.395200	15.6
27	0.303876	15.6
29	0.386581	14.2
30	0.399371	15.5

	P/Y	KTOY	AGE
1	.48	1.018140	1
2	.53	1.187420	27
3	.52	0.932547	41
4	.48	1.113840	1
7	.53	1.185240	55
8	.49	1.043160	76
10	.47	0.910313	52
11	.50	1.182410	29
12	.42	1.080890	52
14	.40	1.217940	57
15	.42	0.983486	30
16	.41	1.184460	1
17	.49	1.143070	37
18	.46	0.926103	51
19	.45	1.251760	10
21	.48	1.316270	38
22	.48	1.262760	65
23	.43	1.125870	86
24	.47	1.025220	56
25	.41	0.969176	62
26	.44	1.311430	39
27	.38	1.183330	62
29	.35	1.254300	67
30	.37	1.142600	40

	RT	UP	Y
1	0	1	4857
2	1	0	3314
3	1	0	4406
4	0	1	5179
7	0	0	4397
8	0	0	5208
10	0	0	4412
11	0	1	5066
12	0	1	[illegible]723
14	1	0	3361
15	1	0	6013
16	0	1	6267
17	1	0	5078
18	1	0	6551
19	0	1	7239
21	1	1	5868
22	0	0	6448
23	0	0	6900
24	0	0	3597
25	0	0	6177
26	0	1	6764
27	0	1	8430
29	1	0	4136
30	1	0	7273

	EMPS	F	P1
1	0.	32.5275	0.800177
2	0.50	30.1637	1.891260
3	0.57	30.1363	1.027650
4	0.50	33.2443	0.708104
7	0.72	30.7961	1.028590
8	0.50	30.7178	1.910650
10	0.	31.9009	1.933200
11	0.70	31.6403	1.262450
12	0.67	31.1324	1.320020
14	0.50	30.3358	1.067820
15	0.50	31.7283	0.875702
16	0.	32.0372	0.996356
17	0.50	29.6675	2.171660
18	0.57	29.7495	1.541020
19	0.50	32.0865	0.898423
21	0.70	29.7167	1.542300
22	0.72	30.7378	1.902670
23	0.50	29.7531	2.299910
24	0.67	30.6362	2.319160
25	0.	31.4684	2.769220
26	0.70	31.4702	2.246970
27	0.67	31.0298	2.206220
29	0.50	29.7595	1.392750
30	0.50	31.4975	1.363840

	P2	YINV	PENW
1	0.178307	0.0205888	0.0473077
2	0.322646	0.0301750	0.1334040
3	0.176908	0.0226963	0.0710417
4	0.164083	0.0193087	0.0409615
7	0.167133	0.0227428	0.0710638
8	0.377931	0.0192012	0.1219610
10	0.434253	0.0226655	0.1143400
11	0.226721	0.0197394	0.0798000
12	0.218754	0.0148743	0.0731034
14	0.185626	0.0297530	0.0586667
15	0.178623	0.0166306	0.0475862
16	0.221443	0.0159566	0.0527119
17	0.306372	0.0196928	0.1435290
18	0.245084	0.0152648	0.0959259
19	0.205655	0.0138141	0.0509091
21	0.330006	0.0170416	0.0998077
22	0.271841	0.0155087	0.1190380
23	0.320243	0.0144928	0.1356140
24	0.401210	0.0278009	0.1428300
25	0.582083	0.0161891	0.1491530
26	0.348633	0.0147842	0.1275000
27	0.288425	0.0118624	0.1146770
29	0.221945	0.0241779	0.0720000
30	0.250353	0.0137495	0.0687302

	ETW
1	0.
2	0.031350
3	0.019437
4	.010650
7	.024048
8	.031100
10	0.
11	0.027930
12	.028408
14	.017600
15	.013800
16	0.
17	0.036600
18	.029526
19	.014000
21	.036330
22	.044568
23	.038650
24	.050719
25	0.
26	0.049980
27	.047637
29	.023400
30	.021650

	S	PEN	WGT
1	0.1320	0.0204	67.5
2	0.1211	0.0382	63.0
3	0.1360	0.0278	53.8
4	0.1067	0.0169	145.3
5	0.0828	0.0305	31.4
6	0.0910	0.0311	309.2
7	0.1493	0.0527	360.4
8	0.1401	0.0276	292.0
9	0.1394	0.0396	1.9
10	0.1538	0.0310	98.2
11	0.1117	0.0344	15.6
12	0.1402	0.0195	24.4
13	0.0990	0.0146	1520.9
14	0.2105	0.0009	634.9

	G	PG	RW
1	0.0268	0.0043	0.15
2	0.0585	0.0556	0.22
3	0.0222	0.0159	0.20
4	0.0446	0.0182	0.14
5	0.0592	0.0529	0.18
6	0.0453	0.0356	0.21
7	0.0813	0.0694	0.17
8	0.0534	0.0481	0.15
9	0.0462	0.0384	0.16
10	0.0656	0.0545	0.16
11	0.0292	0.0082	0.17
12	0.0252	0.0161	0.18
13	0.0321	0.0154	0.16
14	0.0993	0.0897	0.10

	MW	LAFP65	LE65
1	0.66	33.2	13.7
2	0.48	24.8	13.5
3	0.48	16.2	13.3
4	0.76	34.9	14.8
5	0.59	33.2	14.5
6	0.54	35.6	14.1
7	0.50	25.0	13.4
8	0.58	35.8	14.4
9	0.41	28.3	13.3
10	0.70	25.9	15.0
11	0.75	25.1	15.0
12	0.56	39.9	15.1
13	0.67	38.0	14.6
14	0.79	56.4	15.0

	RT	Y
1	0	977
2	1	432
3	1	909
4	0	1,273
5	0	719
6	0	893
7	0	642
8	0	385
9	0	882
10	0	513
11	0	958
12	0	707
13	1	1,718
14	0	224

	F	P1	P2
1	32.7322	0.6677370	0.136000
2	29.6952	1.1343600	0.173636
3	30.1188	0.8373040	0.139000
4	34.5323	0.5835960	0.120714
5	30.8260	0.9401930	0.169444
6	30.5454	0.9499610	0.148095
7	31.0206	1.6347900	0.310000
8	30.2399	0.8346220	0.184000
9	30.3486	1.2018000	0.247500
10	31.6574	0.9813780	0.193750
11	33.0990	1.1386100	0.202353
12	31.8620	0.6213090	0.108333
13	32.2956	0.4715160	0.091250
14	31.4350	0.0282915	0.009000

	YINV
1	0.1023540
2	0.2314810
3	0.1100110
4	0.0785546
5	0.1390820
6	0.1119820
7	0.1557630
8	0.2597400
9	0.1133790
10	0.1949320
11	0.1043840
12	0.1414430
13	0.0582072
14	0.4464290

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