STRUCTURAL DESIGN OF STEEL BINS

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ABSTRACT

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The purpose of this report is to prepare from various design methods available today, a review for the design of steel bins, to be used by design engineers.

The theories for calculating pressure exerted by stored material on bin structure that are developed so far, are still approximate and therefore the use of one over the other does not necessarily mean that one is better.

It is the intent of this report to sort out the advantages and disadvantages of the different methods, so that the designer can combine them to optimize his design.

The emphasis is on setting a compatible bin structural modeling and design procedure. This latter will include both strength and stability analysis for governing loadings, i.e., due to stored material (static and dynamic) when material is at rest and during discharge, respectively.

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TABLE OF CONTENTS

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF FIGURES

CHAPTER 1

INTRODUCTION

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INTRODUCTION

Bins are used to store different materials from food (such as sugar, flour, etc.), to industrial products (such as coal, clinker, cement, etc.), as a result of the increasing industrialization and a requirement of the processing technology.

Bins are of two main types: silos, also called deep bins, and bunkers or shallow bins.

They are usually constructed of steel or reinforced concrete, but can also be of other materials, and nowadays even of prefabricated plastic. They have at their base emptying hoppers and are enclosed at their upper part by a roof supporting device for filling them.

The main subject of this report will be : steel bins.

In the design process of the bins, two aspects are considered: their functional and structural design.

The functional design shall provide for adequate volume and shape of bins and also the selection of the appropriate type of bins to be used for specific materials to be stored.

The structural design considerations are stability and strength for different types of loads: static and dynamic; and also alternate methods of analysis.

In this report, a study of the cylindrical shell (single cell) unstiffened and stiffened is performed; then a study of the rectangular type of bins (single and multicell) is made.

Obviously, in the design of steel bins, the analysis would not be complete without a study of the buckling problem for cylindrical and rectangular shapes.

Finally, the requirements by the existing codes and their different implications in the design of the steel bins are considered and discussed.

The main objectives of this report in summary are:

1) An investigation of the state of the art;

2) An evaluation of the different methods of design;

3) A preparation of comprehensive guidelines for designers.

The subject is hardly exhausted and further research is necessary to provide optimum structural design in bins.

CHAPTER 2

FUNCTIONAL DESIGN OF BINS

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FUNCTIONAL DESIGN OF BINS

2.1 SHAPES AND TYPES OF BINS

Bins provide a storage function in the overall process system and have to be designed accordingly. The process requirements may range from a multiple silo proportioning system to a single load-out silo. In any case, the storage capacity of silos, and the required discharge rate must be determined.

2.2 SHAPES OF OUTLETS AND HOPPERS

The proper solution for selecting adequate outlets and hoppers is based on the analysis of the material flow properties and conditions during material discharge, and also on the geometry of the discharge outlet.

The relevant properties are:

- density
- angle of internal friction
- angle of wall friction (with corresponding values of static and kinetic coefficient of friction)
- moisture content, etc.

All the preceding characteristics of the stored material influence the forces exerted by material on the silo structure.

When the flow properties of the bulk material have been measured and are known, then the critical dimensions of the hopper outlet and the slope and shape of the hopper can be determined in order to allow the unobstructed flow of material.

2.3 MATERIALS OF CONSTRUCTION

The geometry and wall finishing of a silo and the type of discharging devices determine the type of flow pattern which then develops within the silo.

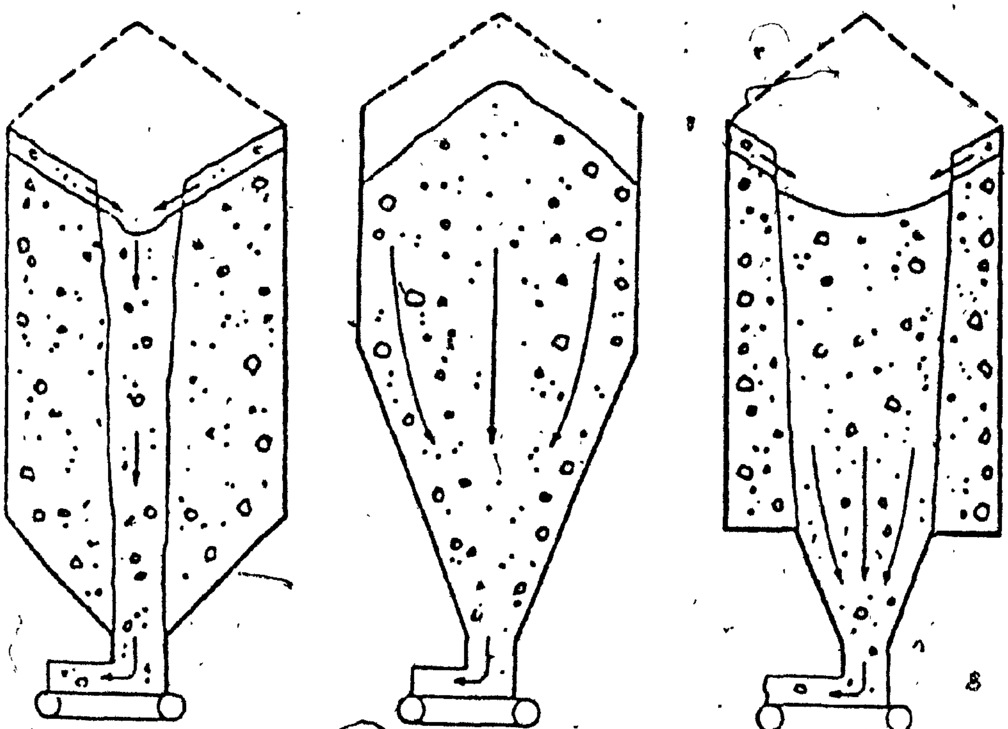
In the majority of existing silos, materials flow toward the feeder through a channel which is formed within the stationary solid. This is referred to as a funnel-flow (Fig. 2.1(a)).

Funnel flow bins are acceptable for coarse, free-flowing, chemically stable solids which do not segregate.

If the hopper of the bin is sufficiently steep and smooth and the feeder capable of drawing material across the whole area of the outlet, all the material flows whenever the feeder is in operation; this being referred to as the mass flow (Fig. 2.1(b)).

The mass flow bins are particularly well-suited to the storage of powders which tend to fluidize and of materials which cave or degrade in storage.

In many cases, adequate operation is obtained by using a short mass-flow hopper-feeder unit under a funnel-flow bin



a) Funnel Flow b) Mass Flow c) Expanded Flow

FIG. 2.1 TYPES OF FLOW

(Fig. 2.1(c)), so-called expanded-flow bins, useful in the storage of large quantities of non-degrading solids, such as ores.

The flow patterns affect the uniformity and consistency of the stream, segregation, degradation, etc., and also the pressures exerted on bin structures.

The qualitative and quantitative analyses of those pressures represent the main content in the structural design of bins. The problems of flows that occur in the flowing mass of granular solids, the various conditions which can arise in the flow and storage, also have special characteristics in terms of forces exerted on bin structures and may govern in the structural design of bins.

The efficient functional design of bins is aimed at preventing those problems, or at least, minimizing them, but due to the range of variable characteristics of bulk materials they cannot be totally eliminated and consequently, have to be considered in the structural design of bins.

The two typical problems of flow relevant to the structural design, are:

a) No flow: a stable arch or rathold develops within the solid,and flow ceases;

b) Erratic flow: momentary arches form within the solid, ratholes empty out partly or completely, then collapse.

A knowledge of the flow properties of stored material and the problems of flow is essential to determine the dimensions and geometry required for gravity flow (the functional design of bins), and the forces exerted on bin structures (the structural design of bins).

CHAPTER 3

DESIGN OF BINS - LOADINGS

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DESIGN OF BINS - LOADINGS

3.1 GENERAL THEORIES

The design of bins and the consideration of the stresses due to the confined material are the responsibility of the civil engineer, and forms the subject of this report.

Regarding the relatively high stresses acting upon the walls under the effect of the thrust and of the friction exerted on these walls by the stored materials, several proposals have been made for the purpose of defining them.

The first bin designers calculated the walls of the cells as though they were acted upon by a liquid of the same density as the stored material.

Then other designers sought to extend to bins the theory of the calculation of the thrust of silos on a retaining wall. The problem of calculating the pressure of material on bin walls is somewhat similar to the problem of the retaining wall, but is not so simple. The Rankine theory used for retaining walls is applicable to shallow bins with smooth walls where the plane of rupture cuts the material surface, but will not apply to deeper bins or bins with rough walls. It should be remembered that Rankine assumes a granular mass of unlimited extent. [1]

Later, Prante, in Germany and Airy in France, proved on the one hand, that these methods of calculation were mistaken, and on the other hand, that the thrust of the stored material and the pressure on the bottom of the bin do not increase indefinitely and tend towards defined limits.

Then, some authors such as Janssen, Koenen and Morsch put forward theories based on the hypothesis of the invariability of the ratio P/q of the horizontal thrust due to the material to the vertical pressure exerted by the latter.

Finally, M. and A. Reimbert, thanks to new methods of measuring pressures, performed some very accurate tests on reduced scale models of bins of various shapes, and also on actual silos. The result of those tests has shown that at the low depths of the bins, the above ratio p/q is not constant.

Nowadays, the most commonly used methods are: Janssen's classical method, and Reimbert's method. Therefore, those two methods are the ones that will be considered in this report.

Material stored in a bin applies lateral forces to the side walls, vertical force (through friction) to the side walls, vertical forces to the horizontal bottoms and both normal and frictional forces to the inclined surfaces. The static values of these forces, resulting from materials at rest, are all modified during the withdrawal of the material. In general, all forces will increase, so that the loads during withdrawal tend to control the design.

Forces applied by stored materials may also be affected by moisture changes, by compaction, and by settling, which may accompany alternate expansion and contraction of the walls during daily or seasonal temperature change.

An approach to the calculation of a bin's loads would involve the conditions of material flow during emptying. Such an approach has not yet been perfected. Therefore, the equations derived for static forces may be combined with experimental data to approximate the pressure increases occurring during the material withdrawal.

The procedure involves determining the static pressures and then multiplying these forces by an "overpressure" factor to obtain the design pressures.

3.2 STATIC LOADS

As mentioned in Section 3.1, two methods for determining static pressures are used: Janssen's method and Reimbert's method. The first one is the more popular in the United States. However, experiments show Janssen's method to be unconservative in some cases, whereas, Reimbert's method is reported to give pressures closer to the tests results.

Before analyzing the two methods mentioned above, the pressures exerted by a stored pulverulent mass shall be defined first.

When a pulverulent material is poured onto a plane, it heaps up into a volume conical in shape, the generatrices of which form a specific angle ϕ with the horizontal.

If this material is poured into a closed space such as a bin, it exerts pressures on the walls and on the bottom of it, the resultant of which is the thrust, which, due to the friction of the material on the walls, is oblique in relation to the surface of the wall. This thrust has therefore two components, one N normal to the wall considered, and the other tangential T parallel to the wall. The normal pressure is also called lateral thrust. (Fig. 3.1 and Fig. 3.2). If ϕ' is the angle of friction of the material on the walls, the corresponding coefficient of friction is $\tan \phi'$.

As a function of the oblique thrust Q defined above, the two components normal N and tangential T are:

$$N = Q \cos \phi$$

$$T = Q \sin \phi$$

Therefore

$$T = N \tan \phi$$

T is therefore the load balanced by the friction corresponding to the thrust N.

At a given depth inside the bin, the load on the bottom or total vertical pressure, is the difference between the

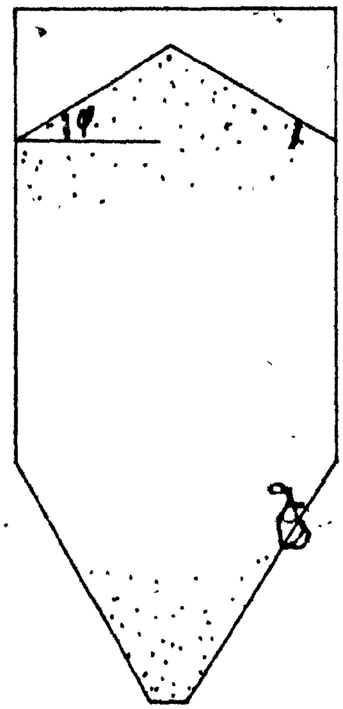


FIG. 3.1 ANGLE OF REPOSE OF MATERIAL

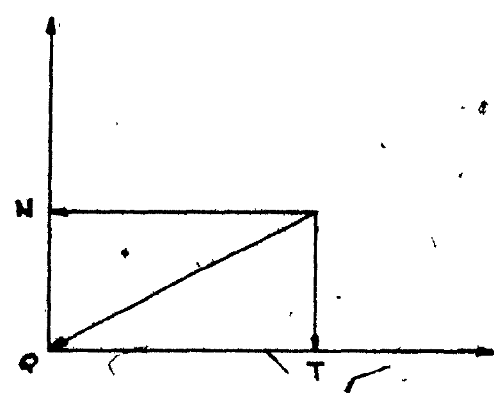


FIG. 3.2 COMPONENTS N AND T OF THRUST Q ON WALL

total weight of the stored material and the total load balanced by the friction of the material on the walls.

Now the tests performed in order to determine the vertical pressure on the bottom of a silo at various depths inside this silo, show that the pressure increases with the depth, but that as a result of the friction of the material on the walls, it is only a fraction of the weight of the stored material, and this friction is such that at large depths, the pressure reaches a maximum and remains constant.

The curve representing pressures obtained by plotting depths against pressures, is shown in Figure 3.3.

The same observation is made about the curve representing the lateral pressure on the walls, Figure 3.4.

3.2.1 Janssen's Method, Figure 3.5

The vertical static unit pressure at depth z below the surface is:

$$q = \frac{\gamma r}{\mu' k}\left(1 - e^{-\mu' k z / r}\right) \qquad (3.1)$$

where

γ = weight per unit volume for stored material

r = hydraulic radius $\left(\frac{\text{area}}{\text{perimeter}}\right)$ of horizontal cross-section of the inside of the bin

μ' = coefficient of friction between stored material and wall = $\tan\phi'$

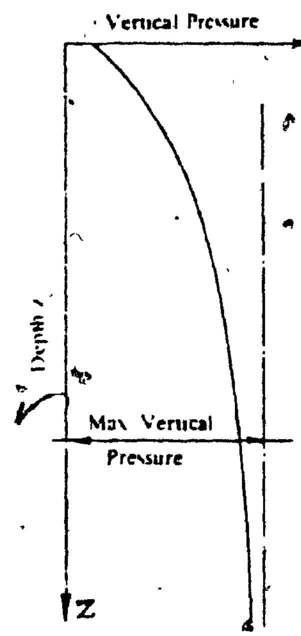


FIG. 3.3 VERTICAL PRESSURE CURVE

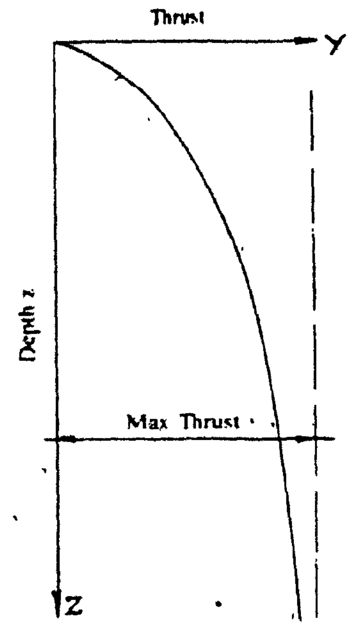


FIG. 3.4 HORIZONTAL PRESSURE CURVE

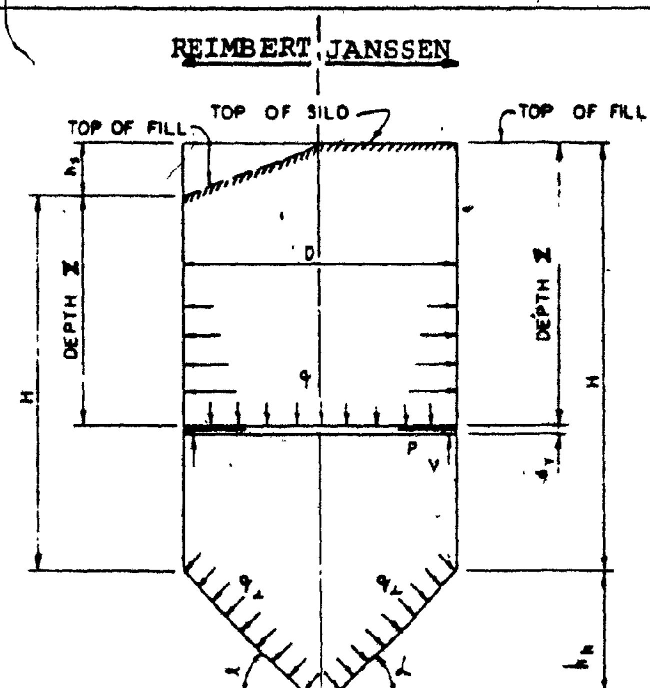


FIG. 3.5 SILO DIMENSIONS FOR USE IN REIMBERT'S AND JANSSEN'S EQUATIONS

k = ratio of p to q

The lateral static unit pressure at depth z is

$$p = \frac{\gamma r}{\mu'} (1 - e^{-\mu' k z / r}) = qk \qquad (3.2)$$

where

$$k = \frac{p}{q}$$

The ratio $\frac{p}{q}$ is assumed by Janssen to be constant at all depths and has the value

$$k = \frac{p}{q} = \frac{1 - \sin\phi}{1 + \sin\phi} \tan^2 \left(\frac{\pi}{4} - \frac{\phi}{2}\right) \qquad (3.3)$$

At the limit

$$p_{max} = q_{max} \times k = \frac{\gamma r}{\mu'} = \frac{\gamma r}{\tan \phi'} \qquad (3.4)$$

For circular bins,

$$r = \frac{S}{L} = \frac{\pi D^2}{4} \times \frac{1}{\pi D} = \frac{D}{4}$$

where

S = area of the bin

L = perimeter of the bin

D = diameter of the bin

For polygonal bins, $r = \frac{D}{4}$, assuming a circular shape of equivalent area to the polygonal shape.

For square bins or for shorter wall "a" of rectangular bins:

$$r = \frac{a}{4}$$

For the long wall "b" of rectangular bins

$$r = \frac{a'}{4}$$

where

$$a' = 2\,ab - \frac{a^2}{b} \quad \text{(See Figure 3.6)}$$

It should be noted that the Janssen theory based on the hypothesis that the ratio p/q is constant, also agrees that in a horizontal plane, the lateral pressure "p" on the walls is also constant. But such a proposition is only true in the case of a cylindrical bin or of an undefined bin between two parallel walls of infinite length. Moreover, the law concerning the distribution of pressures along plane walls of a bin of a polygonal section is not known, therefore, it would not be possible to make the summation

$$\int_0^z p_z \, dz$$

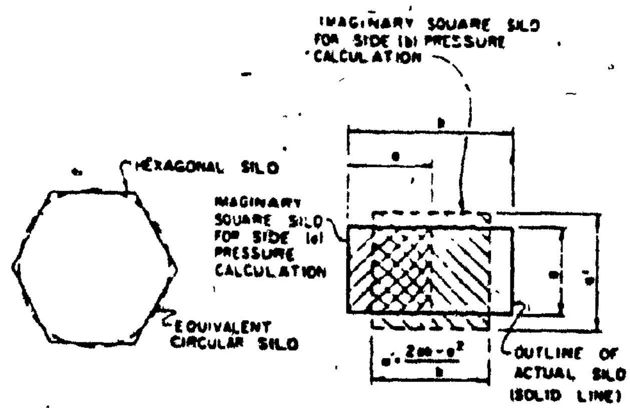
3.2.2 Reimbert's Method

The vertical static unit pressure at depth z below the surface is

$$q = \gamma[z(\frac{z}{c} + 1)^{-1} + \frac{h_s}{3}] \qquad (3.5)$$

The lateral static unit pressure at depth z is

$$p = p_{max}[1 - (\frac{z}{c} + 1)^{-2}] = \varepsilon \; p_{max} \qquad (3.6)$$



a) Polygonal bin b) Rectangular bin

FIG. 3.6 EQUIVALENT SILO SHAPES

p_{max} is the maximum Lateral unit pressure

c is the "characteristic abscissa"

$$p_{max} = \frac{\gamma D}{4 \tan \phi'} \tag{3.7}$$

and

$$c = \frac{D}{4 \tan \phi' \tan^2 (\frac{\pi}{4} - \frac{\phi}{2})} - \frac{h_s}{3} \tag{3.8}$$

For polygonal bins or bins having more than four sides

$$p_{max} = \frac{\gamma r}{\tan \phi'} \tag{3.9}$$

and

$$c = \frac{L}{\pi} [\frac{1}{4 \tan \phi' \tan^2 (\frac{\pi}{4} - \frac{\phi}{2})}] - \frac{h_s}{3} \tag{3.10}$$

"r" has the same values used in Janssen's theory.

For rectangular bins - on shorter wall "a"

$$(p_{max})_a = \frac{\gamma a}{4 \tan \phi'} \tag{3.11}$$

and

$$c_a = \frac{a}{\pi \tan \phi' \tan^2 (\frac{\pi}{4} - \frac{\phi}{2})} - \frac{h_s}{3} \tag{3.12}$$

For rectangular bins - on longer wall "b"

$$(p_{max})_b = \frac{\gamma a'}{4 \tan \phi'} \qquad (3.13)$$

and

$$c_b = \frac{a'}{\pi \tan \phi' \tan^2 (\frac{\pi}{4} - \frac{\phi}{2})} - \frac{h_s}{3} \qquad (3.14)$$

where

$$a' = \frac{2\,ab - a^2}{b}$$

For design purposes, the granular material is usually assumed level at the top of the bin, therefore:

$$h_s = 0$$

It should be observed that the theory developed by Reimbert takes into consideration the variation of the ratio p/q with depth and also with the shape of the bins, (see Figures 3.7 and 3.8). Before Reimbert, Pleissner found also that the ratio p/q was not constant. [1]

3.2.3 Static Pressure on Flat Bottoms

The Equations (3.1) and (3.5) give the pressure on a flat bottom. However, for rectangular silos these equations give different bottom pressure for areas next to the short and long sides. An approximation currently used is to assume the pressure, q_a, computed using r (from Janssen) or c (from Reimbert) for the short side "a", to act on the area A_a as c shown in Figure 3.9.

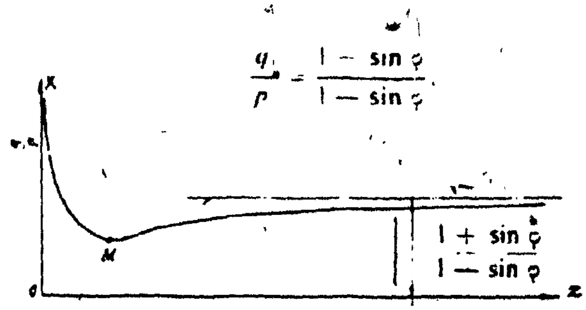


Fig. 3.7 VARIATION OF RATIO p/q WITH DEPTH AND SHAPE OF BINS

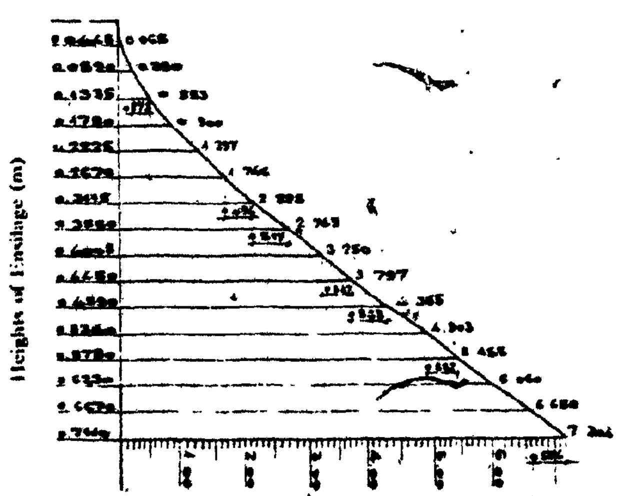


Fig. 3.8 LOADS (Kg) BALANCED BY FRICTION ON THE WALLS OF SQUARE SILO 0.150 x 0.150 m.

Similarly, pressure q_b, computed using r or c for side "b", is assumed to act on area A_b.

3.2.4 Static Forces - Vertical Friction

For circular, square and regular polygonal bins, the total static frictional force per foot-wide vertical strip of wall above depth z is approximately:

By Reimbert's method:

$$V = \frac{(\gamma z - q)A}{L} \tag{3.15}$$

By Janssen's method:

$$V = \frac{(\gamma z - 0.8q)A}{L} \tag{3.16}$$

where

A = area of the horizontal cross-section of the bin

L = Perimeter of the horizontal cross-section of the bin

For rectangular bins, lateral pressures differ for the long and short walls; hence, side friction and vertical pressures also differ. The friction loads may be approximated by Equations (3.15) and (3.16), when terms q, A and L, respectively are substituted by q_a, A_a and <u>a</u> for side "a", and by q_b, A_b and <u>b</u> for side "b", Figure 3.9.

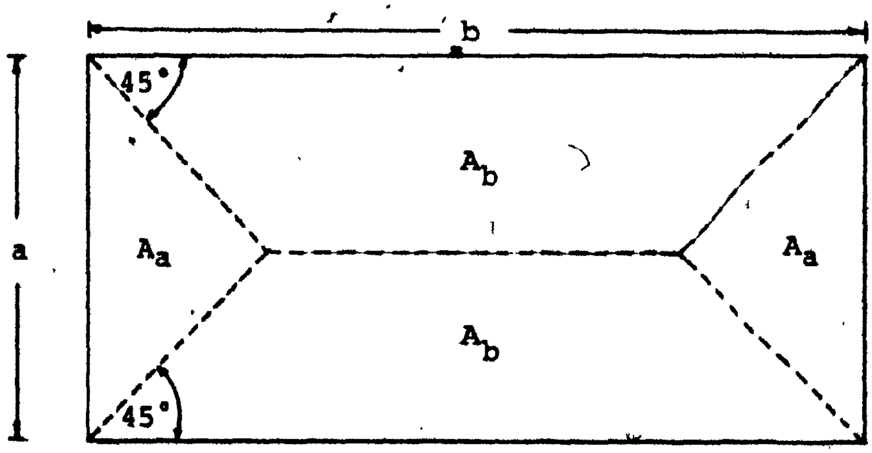


Fig. 3.9 FLAT BOTTOM OF RECTANGULAR BIN

3.2.5 Static Pressures on Silo Hoppers

The static horizontal pressures, p, and vertical pressures, q, on inclined hopper walls are calculated by the Janssen or Reimbert formulas. The hydraulic radius r, may be reduced within the hopper depth, but usually is assumed constant and equal to that of the bin. The static unit pressure normal to the inclined surface, at depth z from the top of the fill is

$$q_{\alpha} = p \sin^2\alpha + q \cos^2\alpha \qquad (3.17)$$

Before, the stresses in hopper bins were most easily calculated by graphic methods or by a combination of algebraic and graphic methods. [1]

3.3 DYNAMIC LOADS

The calculation of vertical and lateral forces exerted on bin walls from stored material was presented for the static condition, when the stored material is at rest.

In general, there will be an increase in these forces over the static values when the stored materials are in the state of flow, during filling, discharging or when arches formed within material are collapsing. These phenomena drew attention only after many bin structures, that were designed for static condition only, exhibited visible signs of damage and failure. Many tests and observations reveal that these loads are dynamic in character, i.e. time-varying.

In order to confirm this fact, reference is given to Figures 3.10 and 3.11, depicting results from experiments performed at the beginning of the century [1] and in recent years [2]. Other experiments were performed using special techniques (such as piezo-electricity, extensometry, etc.) in order to measure the actual stresses in bin walls vs. the calculated ones, on actual bins or on models of bins. All these measurements have helped to evaluate the necessary factors, called the "over-pressure factors", by which the static loads are multiplied in order to obtain the realistic governing design loads.

It has also been established from these experiments and actual observations that the dynamic effects, when they appear, are associated with the discharge of material rather than with filling.

Figure 3.12 [3] shows over-pressure coefficients from two tests which indicate the difficulty in assessing the exact value.

Various authors and codes state different values of the factors and are, in general, very precautious about the actual values to be used in the design, almost always restricting their use to funnel type flow.

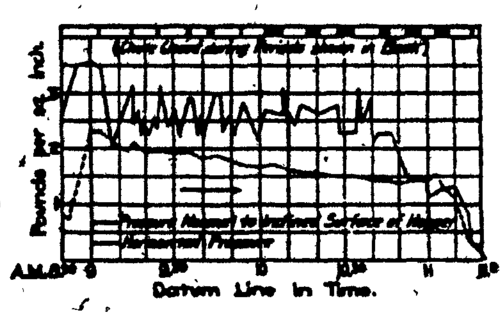
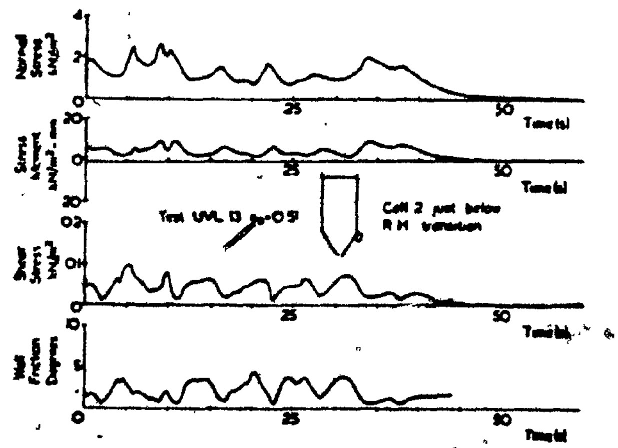
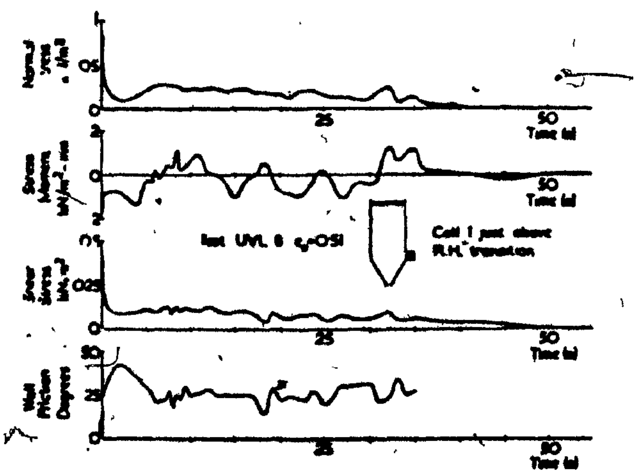


FIG. 3.10 LATERAL AND VERTICAL PRESSURES, BIN-EMPTYING, DIAPHRAGM ON INCLINED HOPPER SURFACE AND ON BOTTOM OF BIN WALL



a) Wall Stress Variations on a Cell at the Top of the Hopper Wall in Test UVL 13



b) Wall Stress Variations on a Cell at the Bottom of the Bin Wall in Test UVL 8

FIG. 3.11 WALL STRESSES PLOTTED AGAINST TIME IN TWO CONTINUOUS-DISCHARGE TESTS ON DENSE SAND WITH THE 60 DEG. HOPPER

The state-of-the art of this part of bin structural design, at the present time, is described even with such statements as "the uncertainty" and "the generally admitted ignorance" [3]. Surveying the references it can be concluded that an adequate technique in analyzing bins for dynamic loads is not available.

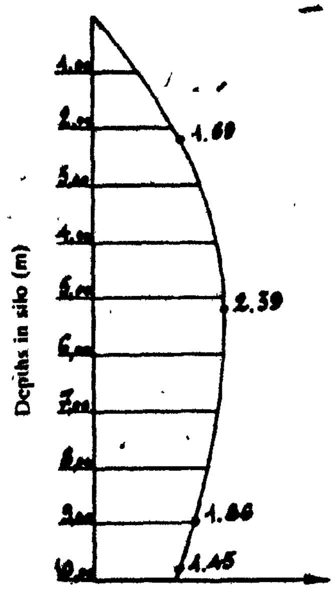
In order to evaluate the dynamic response of the bin structure (i.e. the resulting stresses and displacements which are also time-varying due to the dynamic loads), it is necessary to solve the equation of motion of the system, which is represented as a single circular ring (exactly how the long cylindrical shells are designed for axisymmetric horizontal loads), Figure 3.13.

A bin ring segment is a single-degree-of-freedom system, with the radial displacements Δ being the single displacement component and with all essential properties, namely, its mass M, its elastic property-stiffness K, its energy-loss mechanism - damping C, as well as the loading p(t), assumed to be concentrated in a single physical element (Figure 3.4), whose equation of motion is:

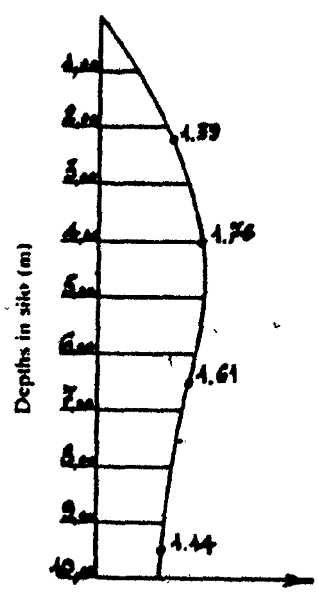
$$m\ddot{\Delta}(t) + C\dot{\Delta}(t) + K\Delta(t) = p(t) \qquad (3.18)$$

The mass is given by:

$$m = \frac{A\gamma}{g} \qquad (3.19)$$



a) First Test



b) Second Test

FIG. 3.12 COEFFICIENTS OF OVER-PRESSURE

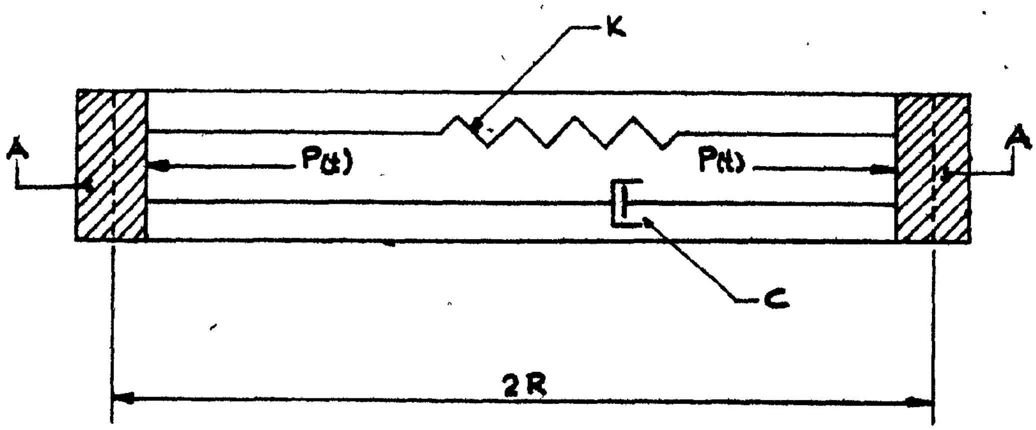


FIG. 3.13 MODEL FOR BIN RING SEGMENT

with A being the ring cross-section with unit weight γ.

To evaluate the damping is very difficult. It is always the case with structural damping, usually assumed to be of the order of 5% of the critical damping. The critical damping is defined as the smallest amount of damping for which no oscillation occurs in the free vibration (i.e., motion taking place with $p(t) = 0$).

The stiffness K is given by:

$$K = \frac{EA}{r^2} \tag{3.20}$$

The natural circular frequency of the ring in pure radial vibrations is:

$$\omega = \frac{4}{r} \frac{Eg}{\gamma} \tag{3.21}$$

A circular ring can also have the higher modes of extensional vibration, analogous to the longitudinal vibrations of pneumatic bins, with circular frequencies [4].

$$\omega_i = \frac{4}{r}\sqrt{\frac{Eg}{\gamma} \quad 1 + i^2} \tag{3.22}$$

where <u>i</u> denotes the number of wavelengths to the circumference. In flexural vibrations, for the case when loading is not perfectly axisymmetric, the circular frequency of the bin's ring segment is:

$$\omega_i = \frac{16}{r^2}\sqrt{\frac{Eg}{\gamma} \frac{I}{A} \frac{i^2(1-i^2)^2}{1 \; i^2}} \tag{3.23}$$

Obviously, for each particular steel bin structure, the preceding frequencies could be determined and then compared with the frequency of the applied load $\overline{\omega}$.

For rectangular cross-sections of bins, the evaluation of natural frequencies is similar, although not as straight-forward as for circular cross-sections, and the use of computers is suggested as the most efficient way in assessing their natural frequencies.

Designating then, the ratio of the applied load frequency to the natural free-vibration frequency of the system with

$$\beta = \frac{\overline{\omega}}{\omega} \tag{3.24}$$

and using the extensively worked out analysis and charts of a single degree of freedom (SDOF) system's response to various types of dynamic loading, the dynamic magnification factor (which represents the ratio of the resulting maximum dynamic response vs. the static response) can be determined. This is of real interest in the structural design of bins.

The type of dynamic loading (horizontal pressure in this case) and its frequency are not positively determined.

Jenike and Johanson [5] state that the wall stresses are time-varying in cycles during continuous flow, i.e., that the loading can be approximated as periodic.

Winter [6] states that experiments reveal "the static pressures to be exceeded by a factor of 2.0 to 4.0" according to the type of flow-funnel or mass type.

It can be concluded (at this stage of the state-of-the-art of bins' design for dynamic loads) that in the case of funnel-flow types, the maximum dynamic magnification factor of 2.0 should be used, (which is in agreement with the recommendation of the few available codes for the design of bins/silos) and in the case of mass-flow types, the maximum dynamic magnification factor of 4.0 should be used.

When the loading becomes more accurately defined, the approximate dynamic magnification factor could be determined for each particular case.

CHAPTER 4

STRUCTURAL DESIGN OF STEEL BINS

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STRUCTURAL DESIGN OF STEEL BINS

Once the different pressures acting on the walls of the bin have been defined,the designer is ready to determine the thickness of the bin's walls.

4.1 CYLINDRICAL SHELLS

The calculation of the size of the vertical walls of cylindrical bins does not present special difficulties. Beyond the self-weight of the walls, they are acted upon by two main forces: the lateral pressure and the vertical friction force due to the friction of the material on the walls, which generates vertical compression stresses on the walls.

If p_z is the lateral pressure and r the internal radius of the bin, the thickness of the wall must be designed to withstand a circumferential tensile or hoop stress of:

$$f_t = \frac{p_z . r}{t} \tag{4.1}$$

For small bins, stiffeners may not be required. For large and deep bins, because the cylindrical wall may buckle under vertical compressive loads, and may bend under bending moments induced from uneven distribution of the wall pressure due to dynamic effects and due to eccentrically located outlets, etc., ring stiffeners or rings and vertical stiffeners are recommended.

After calculation of the hoop stress as mentioned before, the vertical pressure will be calculated.

The longitudinal compressive stress is, following Reimbert's theory:

$$f_c = \frac{V}{t} = \frac{(\gamma_z - q)\pi r^2}{2\pi r t} = \frac{(\gamma_z - q) r}{2t} \tag{4.2}$$

It should be noted that the Equation (4.2) applies only when $\gamma_z > q$.

To find the maximum stress in the plate at the level considered, f_t and f_c may be combined in the form:

$$f_t' = f_t + \nu f_c \tag{4.3}$$

$$f_c' = f_c + \nu f_t \tag{4.4}$$

where

ν = 0.3, Poisson's ratio for steel.

f_t' = maximum tensile stress in the plate.

f_c' = maximum compressive stress in the plate

f_t = circumferential tensile stress or hoop stress

f_c = longitudinal compressive stress

Equations (4.3) and (4.4) are derived from the Saint Venant's maximum normal strain theory.

For maximum compressive stresses in the bin walls, consideration must be given to the portions of bin walls at the stiffener and to those portions of bin walls which lie between stiffeners. The load distributions and their corresponding stresses may be calculated as follows:

$$q_s = \frac{(A_s + 2b_e t)F_{cs}}{(A_s + 2b_e t)F_{cs} + (b - 2b_e)t\,F_{cu}} \tag{4.5}$$

$$q_u = q - q_s \tag{4.6}$$

$$f_{cs} = \frac{q_s}{n(A_s + 2b_e t)} \tag{4.7}$$

$$f_{cu} = \frac{q_u}{n(b - 2b_e)t} \tag{4.8}$$

where

F_{cs}, F_{cu} = allowable compressive stress of stiffened and unstiffened shell respectively

f_{cs}, f_{cu} = vertical compressive stress of stiffened and unstiffened shell respectively

b = horizontal spacing of vertical stiffeners

b_e = effective width of the plate

n = number of vertical stiffeners

Consideration should be given to the thermal stresses in the case of bins receiving hot fill. Any temperature drop may cause ring tension in the silo wall. The only possible relief of the ring tension due to temperature drop is by compression of the stored material. In large diameter steel bins for granular material, the temperature stresses are carefully examined because the granular material is not incompressible material. The hoop stress caused by 100° F drop in temperature for granular material is as high as 19.5 ksi. However, since fine material is generally easy to compact, the temperature stresses can be ignored if the temperature differential is less than, say 100°F.

4.1.1 Stiffened Shells

The portions of bin shells near the stiffeners have greater load-carrying capacities. If "n" is the total number of equally spaced vertical stiffeners, and "b_e" is the effective width of the shell on each side of the stiffeners, as shown in Figure 4.1 [6], the effective width "b_e" will be:

$$b_e = 0.95t\left(1 - 0.475 \frac{t}{b} \frac{Es}{F_{cs}}\right) \frac{Es}{F_{cs}} \qquad (4.9)$$

F_{cs} is the allowable stress of the stiffener and plate column determined by the appropriate column formula with the vertical spacing "ℓ" of the horizontal stiffeners as the column length. Since the allowable stress F_{cs} and the effective width "b_e" are interdependent (because of the effect of "b_e" on the column's radius of gyration) it is necessary to use a trial-and-error method by assuming "b_e", to calculate F_{cs}, or vice versa. After F_{cs} and b_e have been determined, the capacity of the stiffened shells of the bin can be calculated as:

$$q_s = n F_{cs} (A_s + 2b_e t) \qquad (4.10)$$

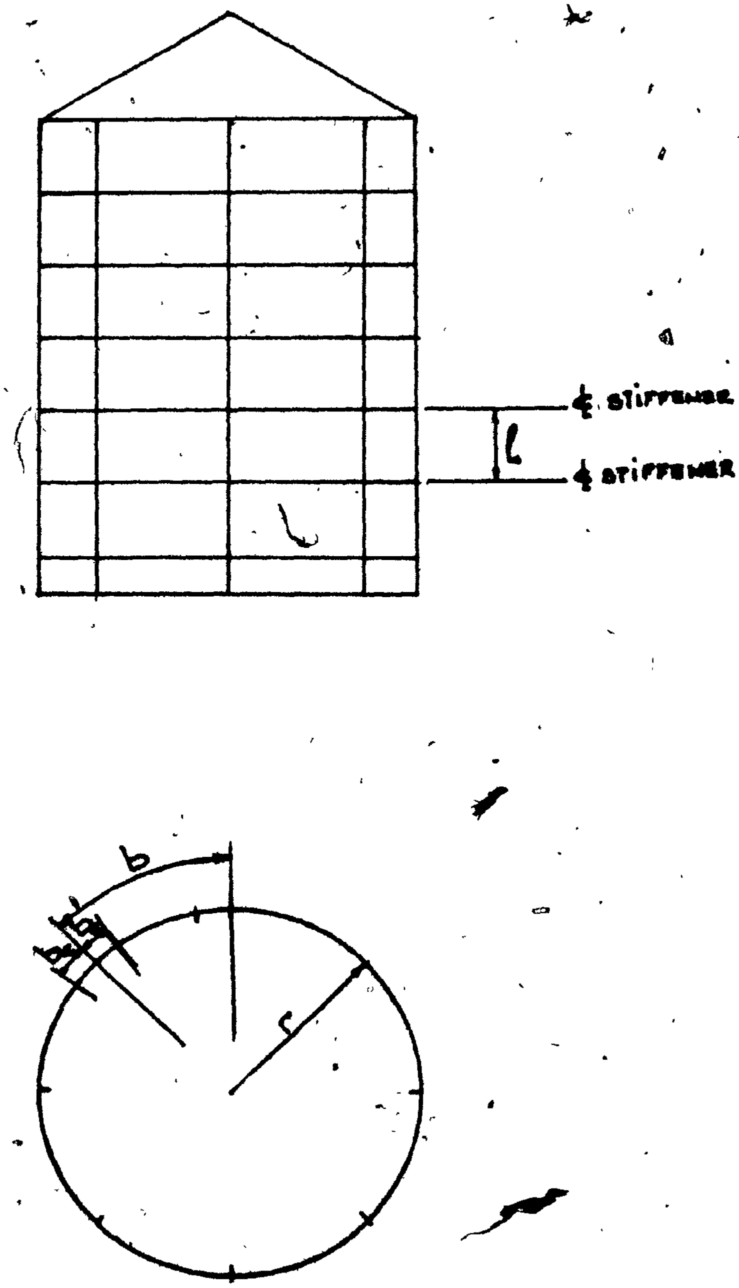

FIG. 4.1 STIFFENED SHELL

For quick and approximate evaluation, b_e may be assumed to be 30 to 40 t, where t is the thickness of the plate.

It should be noted that the above effective width is taken as approximately equal to that of a flat plate. In the case of a cylindrical panel, the load-carrying capacity will be increased due to its curvature. Let λ_1 be the slenderness ratio of the stiffener and shell column, and $\lambda_2 = 20\sqrt{r/t}$, where "r" is the shell radius in feet, and "t" the wall thickness in inches, then the combined slenderness ratio can be calculated by:

$$\lambda = \frac{\lambda_1 \lambda_2}{\sqrt{\lambda_1^2 + \lambda_2^2}} \tag{4.11}$$

4.1.2 Unstiffened Shells

a) The portions of the bin shells not influenced by the stiffeners but which are in direct contact with the stored material, can be considered as restraint against buckling. This assumption is based on the pressure of material in contact with the shells preventing any inward buckling. Therefore, the maximum tensile or compressive stresses f_{max} = e Fts can be used where Fts is the allowable tensile stress and e is the efficiency of the double-welded butt joint.

b) The portions of the bin shells which are not influenced by the stiffeners and are not in contact with the stored material as in the case of arching formed above, may buckle if the loads increase beyond their carrying capacities. For that case consider the allowable buckling stress F_{cu} for the unstiffened portions with a slenderness ratio of λ_2. The strength of the unstiffened shells of the bin can be calculated:

$$q_u = n\ F_{cu}\ t(b - 2b_e) \qquad (4.12)$$

4.1.3 Stiffeners

Based upon previous experience of the designers of cement bins, and upon the facts that arching and uneven distribution of lateral pressure in the horizontal section during filling and emptying are possible, stiffeners are always recommended. Vertical spacing of the horizontal stiffeners of 10 to 15 feet, and horizontal spacing of the vertical stiffeners of 4 to 10 feet may be recommended as the normal practice.

The vertical stiffener can be designed as a portion of a stiffener-and-shell column, as discussed in stiffened shells. The horizontal stiffeners should be designed to support the vertical stiffeners against buckling. The minimum width of such a horizontal stiffener to be recommended here is one-hundredth of the diameter of the bin, with a central filling or central discharging device. For bins with non-central fillings or non-central discharging devices, the horizontal stiffeners should be designed for one-sided pressure effect. The recommendation by many designers for the design of each horizontal stiffener is:

$$M = 0.04\ \gamma r^2\ d_e\ \ell\ \tan\phi, \text{for } \ell < 2r \tan\phi \qquad (4.13)$$

$$M = 0.08\ \gamma r^3\ d_e\ \tan^2\phi, \text{ for } \ell > 2r \tan\phi \qquad (4.14)$$

where

d_e = the eccentric distance of the outlet from the centre of the bin, and

M = the moment in the stiffener

In no case, shall the width of the horizontal stiffener be less than one-hundredth of the diameter of the bin, as recommended for bins with central filling or central discharging devices.

4.1.4 Hoppers

The cells of the silos are terminated at their lower part by hoppers (funnels), their shape being usually that of a truncated cone in the case of cylindrical cells, of pyramidal shape, in the case of rectangular cells, in order to permit complete discharge of the stored material through the discharge trap placed at the lowest point.

In calculating the thickness of the walls of the hoppers, it is assumed that the stored pulverulent materials transmit to the walls of the hoppers the vertical pressure which they exert at the level of the connection to the vertical walls, i.e., at the level of the junction of the walls of the cells to the walls at these hoppers.

The following loads are considered:

a) The vertical pressure exerted by the stored material at the lower level of the vertical walls.

b) The weight of the stored material filling the hopper.

c) The weight of the devices fixed onto the hopper (if any).

The plates forming the cone will be subjected to both the hoop tension and the meridional tension. Janssen's or Reimbert's expression for both lateral and horizontal pressure may be used, but some modifications of a minor nature will be necessary. For application, we will use Janssen's expressions. The mean hydraulic radius "r" should be replaced by half the appropriate cone radius $r_1/2$ and the coefficient of friction on the wall μ' by the corresponding internal friction coefficient, μ. Those modifications take into account the fact that in this area the material is sliding against the material located outside the radius r_1, instead of sliding against the bin wall, Figure 4.2.

The lateral pressure would therefore be:

$$P = \frac{\gamma r_1}{2\mu}\left[1 - e^{-2\mu k z/r_1}\right] \qquad (4.15)$$

giving a hoop tension of:

$$T_H = P r_1 \qquad (4.16)$$

resulting in a hoop stress of:

$$f_H = \frac{Pr_1}{t} \qquad (4.17)$$

Similarly, the longitudinal tension would take the form:

$$T_L = \frac{W_1 \operatorname{cosec}\phi}{2\pi r_1} \qquad (4.18)$$

In Equation (4.18)

$$W_1 = q\pi r_1^2 + \gamma r_1^2 \pi \frac{z'}{3} + \text{ self weight}$$

$q\pi r_1^2$ = vertical pressure of stored material at lower level

where

$$q = \frac{\gamma r_1}{2\mu K}\left[1 - e^{-2\mu kz/r}\right] \qquad (4.19)$$

γr_1^2 = weight of the stored material

self - weight = weight of the hopper

resulting in a meridional stress of:

$$f_L = \frac{W_1 \operatorname{cosec}\phi}{2\pi r_1 t} \qquad (4.20)$$

Both f_h and f_L will be a maximum at the waist, and a minimum at the outlet.

4.1.5 Roofs

The roof should be a self-supporting structure. It may be flat, or in the form of a cone or dome. However, in view of

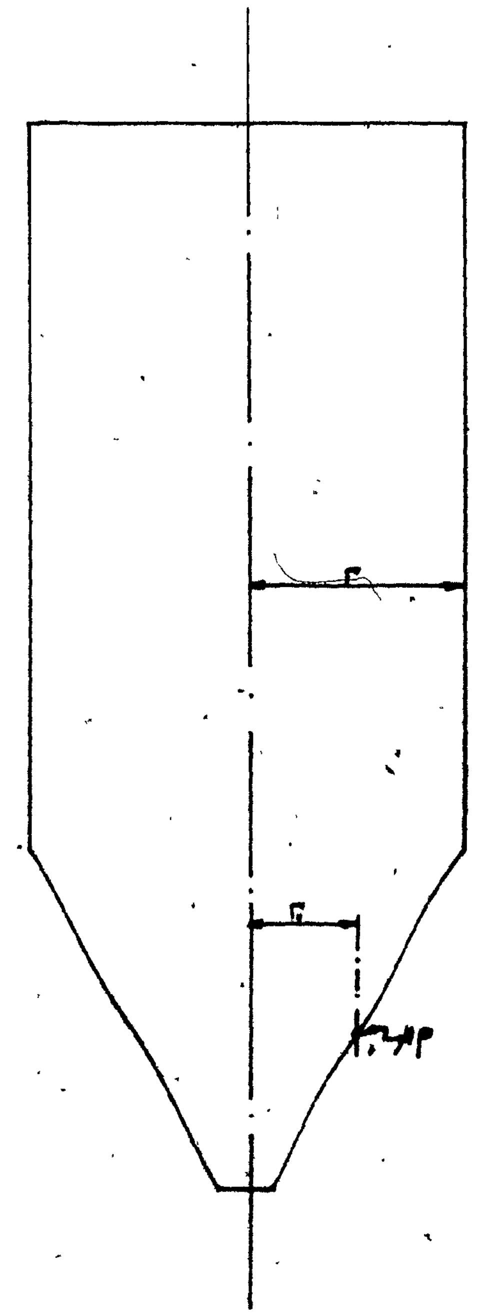

FIG. 4.2 PRESSURE ON HOPPER

the fact that a partial vacuum may develop, above the stored material due to the arching action, larger vents should be provided to prevent any inward buckling of the silo walls or of the roof, itself, from this cause.

4.2 RECTANGULAR SHELLS

4.2.1 Single Cell

In the horizontal plane, the walls of the rectangular bins are acted upon by bending moment stresses due to the direct thrust of the material and by tensile stresses due to the reactions of adjacent walls. In the vertical plane they are acted upon by the same stresses as those defined for the cylindrical bins.

The walls should therefore be designed for compound bending with tension.

The walls of the cells will be considered in horizontal layers of the unit height, assumed as independent from each other and assuming that each cell is independent of the adjoining cells. Each layer of unit height will therefore be studied as a horizontal closed framework acted upon by internal horizontal pressures p_a and p_b, Figure 4.3.

The tensile stress in a wall is the resultant of the horizontal reactions of support of the two adjacent walls. The tensile stresses are therefore:

$$T_a = \frac{P_b \times a}{2} \quad \text{for side "a"} \qquad (4.21)$$

$$T_b = \frac{P_b \times b}{2} \quad \text{for side "b"} \qquad (4.22)$$

For calculation of the bending moments in the walls, the formulae applicable to the frames subjected to uniform pressures but differing as between the small and large sides, are used.

Let I_a, and I_b be the moments of inertia respectively of the walls of length "a" and "b" and for a horizontal slice of unit height and p_a and p_b the thrusts due to the material on the walls "a" and "b".

The stiffness coefficient k is calculated as:

$$k = \frac{I_b}{I_a} \times \frac{a}{b}.$$

The bending moment at the corners is:

$$M_A = M_B = M_C = M_D = \frac{p_a \ a^2 k + p_b \ b^2}{12(k+1)} \qquad (4.23)$$

The moments in the bays are:

$$M_a = \frac{p_a \ a^2}{8} - |M_A| \qquad (4.24)$$

$$M_b = \frac{p_b \ b^2}{8} - |M_A| \qquad (4.25)$$

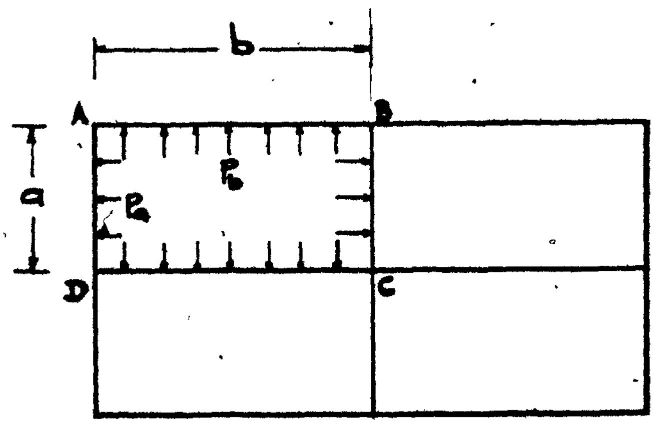

FIG.4.3 RECTANGULAR SHELLS

If the bin is square, the four walls are identical and:

$$a = b \qquad I_a = I_b \qquad \text{therefore} \qquad k = 1$$

Equations (4.23), (4.24) and (4.25) become (with $p_a = p_b = p$)

$$M_A = \frac{pa^2}{12} \tag{4.26}$$

$$M_a = M_b = \frac{pa^2}{8} - \frac{pa^2}{12} = \frac{pa^2}{24} \tag{4.27}$$

It should be noted that in order to calculate the bending moments acting on the walls of a rectangular bin one must necessarily know the moments of inertia of the walls and consequently, the thickness of these walls. But the thickness is a function of the moments to be calculated. Therefore, a sufficiently exact determination of the bending moment is only achieved by successive approximation.

A method suggested by Reimbert [3] to get around that difficulty is to use the formulae for continuous girders with gussets in the shape of frames, which he says gives direct and satisfactory results.

4.3 BUCKLING

In horizontal planes, both circular and rectangular steel bins are under axial tension and there is no danger of buckling.

In vertical planes, the uniformly distributed compressive loading due to the own weight of bin walls, the frictional force of the stored material and roof and equipment loads, buckling of the bin walls may occur at a certain value of the compressive load.

4.3.1 Cylindrical Bins [7]

The buckling stress for cylindrical bins is given by the classical solution for axially compressed cylinders.

$$\sigma_{cr} = \frac{Eh/a}{[3(1-\nu^2)]^{\frac{1}{2}}} \tag{4.28}$$

Where h is the wall thickness and a the average surface radius, (with $h << a$).

For $\nu = 0.3$ (steel bins), Eq. (4.28) becomes

$$\sigma_{cr} = 0.605\ Eh/a \tag{4.29}$$

The critical values of axial stresses for cylinders subjected to axial compression are conveniently expressed as a function of the so-called Batdorf parameter Z

$$Z = \frac{L^2}{ah}(1-\nu)^{\frac{1}{2}} \tag{4.30}$$

with the corresponding values of factor K_a, as shown in Figure 4.4, and given by

$$K_a \equiv \frac{L^2 h}{\pi^2 D}\sigma_{cr} \tag{4.31}$$

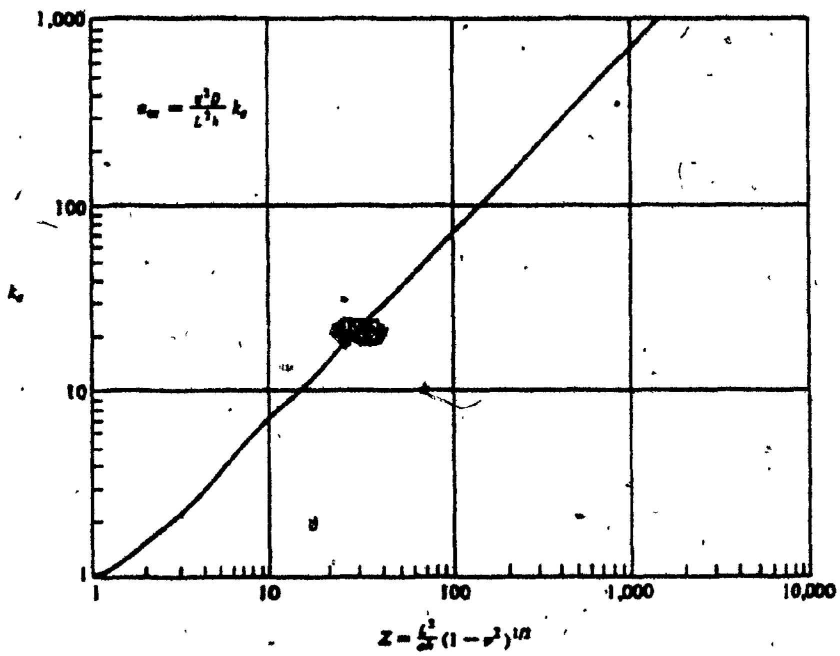

FIG. 4.4 CRITICAL VALUES OF AXIAL STRESSES FOR CYLINDERS SUBJECTED TO AXIAL COMPRESSION

$$D \equiv \frac{Eh^3}{12(1-\nu^2)} \tag{4.32}$$

For the values of $Z > 2.85$, long cylinders, the values of ν_{cr} given by Figure 4.4 and Eq. (4.28) are the same.

($Z < 2.85$ represents the short cylinders).

4.3.2 Rectangular Bins [3]

The walls of the rectangular bins are composed of rectangular plates that are subjected to vertical and horizontal axial forces.

The vertical axial forces are compressive and horizontal are tensile, Figure 4.5(a), although for multicell bins they can sometimes be compressive, due to the difference in cell conditions (some full, some empty), so in the worst general case, the bin wall plates are subjected to in-plane compression in two directions, Figure 4.5(b).

The forces in the two directions are inter-related by non-dimensional constant R by letting [2]

$$\frac{P_y}{a} = R\,\frac{P_x}{b} \tag{4.33}$$

where

R = load ratio

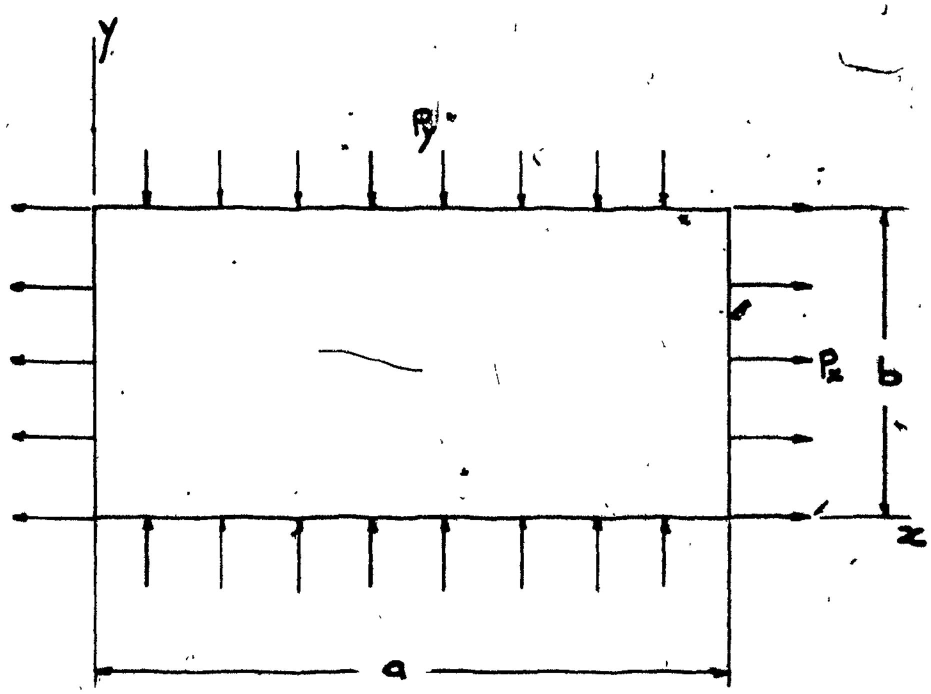

FIG. 4.5(a) COMPRESSIVE AND TENSILE FORCES ON BIN WALL

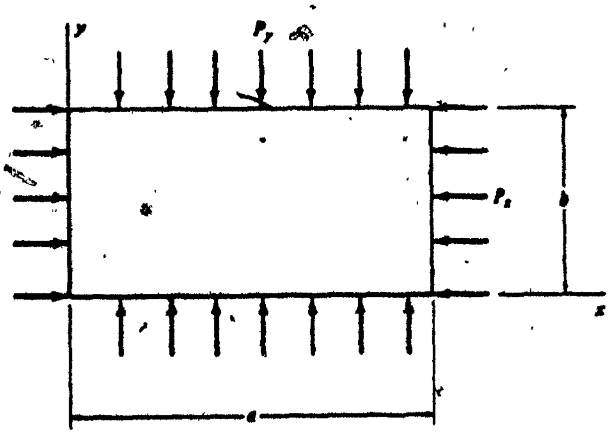

FIG. 4.5(b) COMPRESSIVE FORCES IN TWO DIRECTIONS

For the case of simply supported edges, the buckling force is given by

$$P_x = K_{cc} \frac{\pi^2 D}{b} \tag{4.34}$$

where

$$K_{cc} = \frac{[(m\, b/a)^2 + n^2)]^2}{(m\, b/a)^2 + Rn^2} \tag{4.35}$$

For given values of the load ratio R and plate aspect ratio a/b, the values of m and n (positive integers) may be chosen by trial to give the smallest buckling force P_{cr}.

For a square plate, for example, $a/b = 1$. Then, for $R = 1$, 0, and -1, respectively, Equation (4.35) gives, for m, n, and K_{cc} for minimum K_{cc}, the values

R	m	n	K_{cc}
1	1	1	2
0	1	1	4
-1	2	1	8.33

Negative values of R, of course, signify tensile loading in the y-direction. As would be expected on intuitive grounds, the addition of a tensile load in the transverse direction is seen to have a stabilizing influence.

The results of a stability analysis for combined loading are frequently presented in terms of so-called interaction curves. Such curves are illustrated, e.g., in Figure 4.6, for the case a/b = 1. [2]

The coordinates in the graph are $\sigma x/\sigma x_{cr}$ and $\sigma y/\sigma y_{cr}$ where:

$$\sigma_x \equiv \frac{P_x}{bh}, \quad \sigma_y \equiv \frac{P_y}{ah} \tag{4.36}$$

and σx_{cr} and σy_{cr} are the critical values of σx and σy, respectively, when each is acting alone.

From Equations (4.33) and (4.36),

$$R = \frac{\sigma y}{\sigma x}$$

The solid lines in the interaction plot represent minimum eigenvalues, and the dotted ones represent higher eigenvalues. The negative values of the parameter $\sigma y/\sigma y_{cr}$ represent tensile loading in the y-direction.

Interaction curves for an extensive variety of combinations of compression, bending, and shear loading are given by Gerard and Becker, [8] in graphical form.

4.3.3 Compression Ring Between Cylinder and Hopper

When the transition from a cylinder to a cone is made abruptly, a compression ring must be provided to resist the

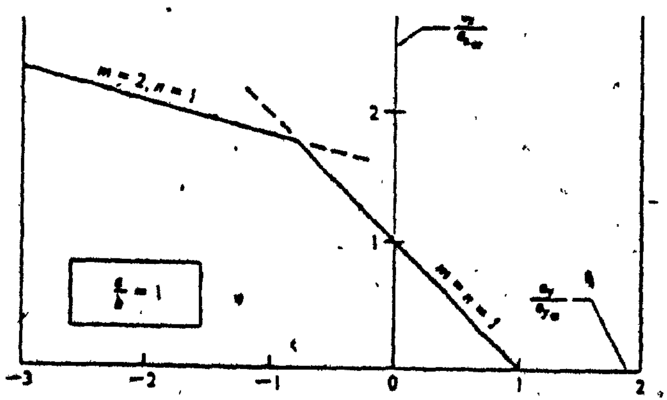

FIG. 4.6 INTERACTION CURVES FOR PLATE SUBJECTED TO IN-PLANE COMPRESSION IN TWO DIRECTIONS

horizontal inward pull from the cone, as shown in Figure 4.7. This steel ring should be designed for an allowable stress of 10,000 psi [9]. This relatively low value is used to minimize deflection and hence the secondary bending stress. But, the compression ring must be checked particularly for buckling. Using a factor of safety of 3, in Lévy's formula for buckling of a ring under uniform pressure:

$$T_{H(critical)} = \frac{3EI}{R_r^3} \tag{4.37}$$

where

T_H = horizontal component of T2, lb. per in., where T_2 = meridional force, lb. per in, Figure 4.8.

R_r = centroidal radius of a ring, in.

E = Young's modulus, psi

I = minimum moment of inertia, in^4

Georges Manning [10], Fellow of the Institution of Civil Engineers, believes that the ring becomes unstable when the external loading exceeds the critical value of Equation (4.37).

In their Handbooks, the British Steel Corporation and the British Steel Piling International Foundation are even more conservative than Manning. [10] They recommend a safe working load:

$$T_{H(critical)} = \frac{1.5\ EI}{R_R^3} \tag{4.38}$$

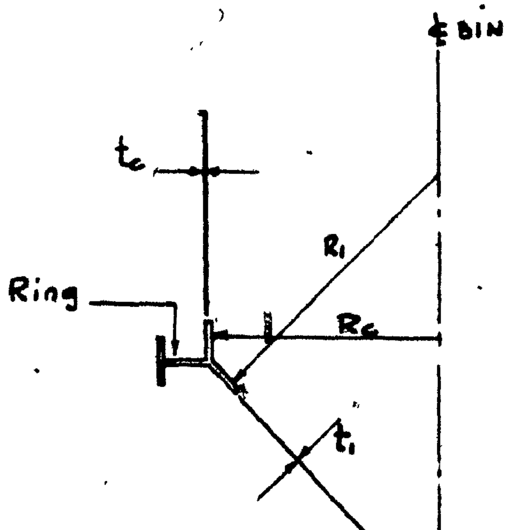

FIG. 4.7 CYLINDER-TO-CONE TRANSITION

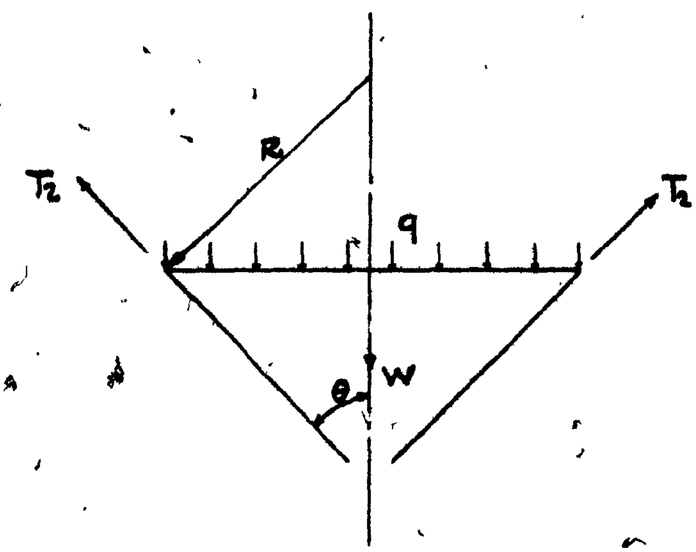

FIG. 4.8 FORCES ON SUSPENDED BOTTOM

CHAPTER 5

CONCLUSION

CHAPTER 5

CONCLUSION

For many years bins have been designed according to the most known general theories: Janssen's method and Reimbert's method. The fact is that these two methods are approximate - e.g., the ratio p/q is assumed constant by Janssen, the vertical friction is underestimated by Reimbert and neither theory accounts for the dynamic effects. This explains the reliance on empirical results, in engineering practice.

Further confirmation of the above-mentioned facts is relected by the absence of adequate Code provisions on the subject. In general, Codes include guidance that can be used by designers in their calculation with prudence and judgement. There are no Canadian or American Codes on Structural design of steel bins.

The ACI Code and also the German and Russian Codes refer in their section on bins to concrete bins, or to general theories that can be used for both concrete and steel.

In addition, there are only a very few references for the design of structural steel bins. The most currently mentioned are: Ketchum [1] the oldest, and more recently, Reimbert [3]. Even those, are based on assumptions and on empirical test results.

The purpose of this report was to organize the available materials on the subject, (i.e., to review the state of art) and to provide guidelines which will be up-dated as soon as new methods or theories are developed.

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