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Structural Spaced Review: A Case Study

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Abstract

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This study evaluates an innovative review system which accents the development of "Structure" of mathematical concepts. The technique utilizes distributed or "Spaced" review sessions in which numerous aspects of past concepts are highlighted and interwoven with related concepts into a cohesive mathematical system of "generalized essential relationships". The evidence for the need for such a system is provided by a substantial Literature Review.

The case study centres designed around two College mathematics classes in which the SSR technique is used. Four mechanisms were used for the evaluation:

- A detailed report of several Class Presentations (in which students show their ability to cope with review problems at the black board).

- A detailed Questionnaire, which elicited quantitative and qualitative data regarding the students' perceptions of the SSR method.

- A specialized test designed to evaluate their knowledge of structure.

- Clinical Interviews based on the Structure Test, with a small cross-section of students from each class.

The study suggests certain advantages of the review technique. The students claimed that they learned more in the course than they might have done in the "traditionally taught" class; they experienced an increased confidence in mathematics and better preparedness for the final exam. The majority also suggested that they were better able to understand the material and did not have to resort to merely memorizing techniques.

Several students expressed a certain anxiety in coping with the Class Presentations, but the consensus was that this provided a valuable tension to the classes. Other pros and cons are discussed in the study.

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TABLE OF CONTENTS

CHAPTER 1

Statement of the Purpose

The usual approach to teaching mathematics concepts at the High School, College and University levels is to adopt one point of view, because of the linear nature of the subject and time constraints in the classroom. Consequently, students learn concepts from one narrow perspective when they are taught initially. Students usually do not encounter the same concepts until they are needed again at a much later date or in preparation for a class test. For example quadratic functions are taught at one point, quadratic equations at another and quadratic graphs at a third stage, in a seemingly disconnected manner.

The above approach follows a tradition in which references to memory are avoided or expressed in negative aspects of repetition and mindless drill. Indeed, as Byers and Erlwanger (1985) observe, the trend today is to ignore the role of memory in the learning and understanding of mathematics (see also Skemp (1976), Ushinskii (1978), MacDonald (1975)). The alternative has been to stress the spiral approach so that a concept is encountered several times in different contexts. However the spiral approach lacks a cohesive element because of the long time intervals between the teaching of connected topics. The author has observed that college texts and teachers have a tendency to use this same spiral approach with little or no attention to the inadequacies of such a technique, particularly in relation to the lower level courses.

At John Abbott College, for example, the average pass rate in the Functions course (one of the lower level mathematics courses) was 43% over the period 1984 to 1987. Of course there are many possible reasons for this state of affairs. However, the author has found from experience, in several informal studies over a five year period, that one important cause appears to be a poor grasp of concepts resulting from several factors associated with review. One factor is that teachers have little or no appreciation of the nature or role of review in mathematics learning. Consequently, review is often a haphazard affair that is undertaken occasionally when a previously taught concept is needed for a new topic or when reviewing for tests. Even in such cases review is rarely systematic or concerned with structural relationships.

The thesis of the proposed study is that most mathematics concepts at the High School level and beyond are too subtle to be grasped by the majority of students who are taught in the manner described above. That is, infrequent exposure to a concept from a given perspective may not be adequate for students. Indeed, it is conjectured that poor performance resulting from an incomplete or partial understanding of concepts is one of the products of the mode of teaching. A related conjecture is that this mode of teaching may also account for distortion of concepts described by Byers and Erlwanger (1985):

> **"The meaning the learner constructs may not be the meaning the teacher wants him to construct... in the absence of guidance a student may develop an entirely erroneous conception of mathematics."**

In a similar vein Ginsberg (1977) argues that:

> "Typically children's errors are based on systematic rules...**children's faulty rules have sensible origins. Usually they are distortions or misinterpretations of sound procedures.**

In a sense, the misconceptions referred to here have counterparts in the memory distortions that occur in story telling described by Bartlett (1932). The underlying point in the above situations is that faulty rules, errors or distortions arise in "the absence of guidance". We conjecture that the guidance required here is systematic review whereby previously learned concepts are encountered from varied viewpoints. The intention being to highlight structural relations and to diminish the importance of irrelevant detail. Thus, the aim of review would be to maximize opportunities for coherent retention.

The importance of coherent retention is also shared by several educators (in particular Krutetskii (1976), Smirnov (1973) and Butler and Wren (1941)). Krutetskii found that a 'good memory' is essential in mathematics. He observed, in particular, that capable students tend to remember the **structure** or **generalized essential relations** of concepts and problems whilst weak students in contrast tend to remember largely the specific and often irrelevant details of a problem. For example, in one study capable students recalled 86% of the structure and 2% of detail after 6 months. In contrast, weak students were unable to recall structure at all after 1 week. On the other hand, Krutetskii states that some of the weak students "were distinguished by a good memory in other school subjects". Could it be that his weak students had little or no guidance in how to learn and remember structure in mathematics (though they may have mastered such skills in other subject areas)?

As Byers and Erlwanger observed, the difference between good and poor students may lie in how they remember:

> "... a good student organizes his mathematical knowledge in a way that minimizes cognitive strain. He is able to strike a balance between memory and deduction. He knows, for instance, which formulas have to be remembered, which partially remembered and partially deduced, and which can be left to be derived as needed. He uses mnemonic devices successfully, and is able, in fact, to devise his own. **A poor student cannot do this; so he tries to remember by brute force a multitude of rules, facts, and procedures. Evidently remembering mathematics is an important skill distinct from the subject as such."**

One way in which the weaker student might be helped lies in the role of "repeated executions of actions" as highlighted by Smirnov (1973) in a powerful qoutation from his chapter dedicated to the "Significance and Functions of Repetition" as follows:

> "Repeated executions of actions are not exact copies of the first execution... they are considerable modifications of former actions, important changes of the initial activity. The action seems to come alive and develop and is in perpetual motion and change.
> **The changes can be clearly traced in the repeated performance of thinking activity. Each new comprehension of the same material results in the revelation of new aspects not noticed earlier, leads to a fuller, deeper and more exact understanding and reveals new connections and relations. The whole of the material is often comprehended from a new point of view, acquires a new aspect, a new meaning. Sometimes only repetition makes it possible to understand the material as a whole."**

Clearly, Smirnov's view of "repetition" as **"repeated performance of thinking activity"** is far removed from its negative connotations as "blind drill". Indeed, repetition becomes a purposeful activity necessary for a deeper and more exact understanding of essential relationships.

In a similar way, Butler and Wren (1941) argue the case for a systematic approach called "spaced learning" where:

> "Drill should be distributed in relatively small amounts at recurring intervals which should become more widely spaced as time goes on. **This principle of spaced learning, as contrasted with the idea of complete immediate mastery, is exceedingly important** and is coming to be widely recognized in the organization of textbooks and instructional material."

The important point here is the notion of **spaced learning** in contrast to complete, immediate mastery.

The above discussion so far leads to the general conjecture that the use of a review system which combines the key aspects of the propositions of both Smirnov and Butler and Wren might be particularly beneficial; especially when Krutetskii's findings regarding structure versus detail are taken into account. In other words the combination of the principles of teaching for **structure** rather than detail, via the mechanisms of **"repeated performance"** and **"spaced learning"** may have a positive effect on student learning. Such a technique should counteract the negative effects of forgetting and partial knowledge that probably lead to distortions and errors.

Variations of the above review technique have been used over the past ten semesters by the author. It is evident from these informal studies that review has to be organized in a systematic manner focusing mainly on structure (hence the term "Structural Spaced Review", or SSR, has been coined). This extended experience has shown that the technique does seem to have a potential to enhance understanding.

It is pertinent at this stage to detail the Objectives of the Study:

Statement of Objectives

The proposed study is exploratory in nature and is conducted in the form of a case study. Its general objective is to develop one variation of the Structural Spaced Review technique, to observe its use in a classroom setting and to observe some of its effects on the students' performance and their reactions to the technique.

Some general questions that might be answered by the study are as follows:

(i) Can some form of review as suggested above help students and in what ways?

(ii) How would SSR affect good students who apparently remember structural relations of concepts?

(iii) How would students in the "weak" and "capable" categories react to such a technique. Would they find it too demanding or boring?

(iv) Would they gradually become aware of its beneficial effects?

(v) What are some of the negative side-effects on the teacher and his/her teaching? Is the technique too demanding?

(vi) What are the negative effects on the students and their learning of mathematics?

(vii) How does it affect what they learn and remember?

(viii) Do students perform better on tests?

(ix) Does the technique seem to change their day to day responses in class?

(x) Does the technique take up too much class time?

The specific objectives of the study are:

1. To describe the review technique and its application in detail over one term in order to distinguish it as clearly as possible from a corresponding description of standard classroom teaching. [See Sections 3.1 and 4.1].

2. To observe in an exploratory way some of the possible effects of the technique on:

(a) The students. with regard to their study habits, their attitudes to review and the time spent on review. These observations will be ongoing throughout the course and an extensive questionnaire will be given towards the end of the course. [see Sections 3.2 and 4.2].

(b) The performance of students in a specialized test, by looking at their results and evidence of any significant knowledge of structure of concepts. Follow-up interviews will take place with a cross-section of students form each of the two groups. [see Sections 3.3 and 4.3].

CHAPTER 2

Review of the Literature

This review first demonstrates that the dilemma regarding "Meaningful" versus "Rote" Learning is truly an antique problem which still plagues educational theorists today. Comments are then made on the Spiral Approach and its inherent weaknesses (see p. 14). The traditional teaching mechanism of Massed Practice is balanced against a different technique developed and used in numerous High School texts by the American writer John Saxon (p. 15). There follows (p. 17) an account of several researchers' studies on the effects of spaced reviews, and subsequently, pertinent commentary on the Role of Memory (p. 25).

Apparent weaknesses in some of the studies are discussed (p. 26) and a section follows (p. 29) giving five researchers' definitions of the key concepts associated with review. Since the Soviet researcher Krutetskii's work on the Psychology of Mathematical Abilities is so important a foundation to this research, a brief critique of his thinking on Mathematical Memory is given (p. 31). The review terminates with a section (p. 34) consolidating some relevant insights hinging on the ideas of students' development of appropriate structures to facilitate learning by Bartlett, Byers, Erlwanger, Piaget and Skemp among others.

Meaningful versus Rote Learning: A Century of Conflict

The use of review strategies has been the subject of research and discussion for many years. The key to the discussion of this topic has hinged on two categories of learning: "meaningful" versus "rote" learning. Over a hundred years ago K.D. Ushinskii (1824-1870), a respected Soviet Pedagogical Anthropologist commented in an essay entitled "Man as the Object of Education" (translation 1978): "The new pedagogy, in contrast to the old, scholastic pedagogy, has put mechanical memory and mechanical learning too low; such memorizing, or learning from rote, however, still remains the material basis of any teaching..." (p. 164) He, however, concurs with the opposing view and quotes Montaigne, "Savoir par coeur n'est pas savoir" (to know by heart is not to know). He warns that though "to remember is not the same thing as to know", and continues: "it is not so easy to draw a line between these two psychic phenomena as it would seem." (p. 170)

Ushinskii makes the suggestion that "what can be done mechanically should be done mechanically so that one has the time to concern oneself with what requires other treatment." He concludes that both schools of thought (schools biased to Relational or Instrumental Understanding to use Skemp's definitions) have their good and bad sides and that both are wrong in their extremism and one-sidedness.

Jumping forward a century we find that Butler and Wren (1941) mirror Ushinskii's comments:

> The old pedagogy undoubtedly laid too much emphasis upon memorization and mechanical learning to the consequent neglect of meanings... The fallacy of this point of view is the tacit assumption that memorization and automatization imply understanding.... The 'new pedagogy', in its extreme form, takes the position that meanings alone have value... This point of view overlooks the important element of fixation without which it would be manifestly impossible to organize and relate concepts or to carry on any process at a reasonable level of efficiency." (p. 150)

They go on to suggest that an enlightened present-day view of mathematics instruction must reject both of these extreme positions as untenable. Drill must be recognized an an essential means of attaining some of these outcomes, just as a strong emphasis on Concepts must be regarded as essential.

The Butler and Wren text may have been misunderstood because of the excessive reference to the term "drill", though they were at pains to insist that "children should not be drilled on procedures which they do not understand." (p. 150)

Indeed they made a good case for the institution of a mathematics program in which, "integrated review would necessarily run systematically throughout the entire program, giving strength and coherence to the entire structure through continual interassociation of components." (p. 154) They point out that such review helps a student to develop a sense of "unity of the whole which might otherwise be lacking" and through making the necessary association of ideas he will be aided "not only in remembering them but in understanding them and appreciating their interrelations."

Fifty years later the dilemma between "drill" and "conceptualization", or what we prefer to term "structural" learning still exists. James T. Fey (1981), in a summary of Perspectives from Three National Surveys in the United States discussed the "back to basics" pedagogy prevalent in the last two decades: "The most frequently mentioned features of back to basics pedagogy are drill, repetition and hard work." In the selection of quotes from teachers appear the following revealing comments:

- I dislike our book, not enough drill, it's modern math. We adopted a new book ... it has more drill, more basics and I like it.

- There is abundant evidence to show that we are encouraging superficial learning in some of our [best students]. Sure they can do well on tests. Our materials on hand encourage this. The algebra book, for instance, is pure abstraction. The really good memorizer can go right through and not really have it all.

- No algebra should be taught in junior high. Fortunately, nature is on my side and very little algebra can be taught in junior high. (pp. 21-22)

They go on to say that the difficulty is as much one of traditional expectations as one of curriculum development:

> Students seem to expect emphasis on facts and memorization in mathematics, along with the neat closure that comes from a discipline with well-defined procedures and 'right' answers. In one science class a site visitor suggested to students that they might find D'Arcy Thompson's book on Growth and Form an interesting guide to modelling of scientific observations by mathematics. The observer commented, however, that it was clear from the student responses that this was not regarded as mathematics. (p. 22)

Here again we see the conflict between, on the one hand, a desire to impart a clear understanding of the subject, its structural relationships and interconnections, and on the other hand, the decidedly hollow goal of teaching a collection of rules and procedures to be learned to a level of near mechanical proficiency.

A recent newspaper article (The Gazette, April 2, 1989) on young Montrealers' views of high school education and curriculum, quotes a student who feels a need for a course in logical reasoning: "We can spit out formulae in math but we really don't know what they mean. We need to learn how to stop and THINK."

Indeed, research tends to suggest that this ability to "think" is lacking, or at best is shrouded by an indoctrination to "spit out formulae." The "student/professor" problem springs immediately to mind. In this research (Clement, Lochhead and Monk, 1981), 150 freshman engineering students at a major state university were asked the following question: Write an equation using the variables S and P to represent the following statement: "There are six times as many students as professors at this University". Use S for the number of students and P for the number of professors. Only 63% of them were successful, (68% of the errors were reversals: P = 6S instead of S = 6P). Lochhead (1980) had previously posed a similar problem to 202 university faculty. They were asked to write an English sentence that gives the same information as the equation A = 7S, where A is the number of assemblers in a factory and S the number of solderers. He found that only 65% gave correct interpretations. Clement, Lochhead and Monk used the same problem in 1981 with 83 engineering students and found a 29% success rate. The latter finding underscores the difficulties experienced by a wide cross-section in the area of translating from algebra to English.

The Spiral Approach and Textbook Reviews: Pros ands Cons

Students' inability to think, or to put it in other words, to see the structural relationships of the subject, may be an inevitable by-product of the spiral teaching method. Such is the assertion of Hart (1981), who states that:

> "In the secondary school we tend to believe that the child has a fund of knowledge on which we can build the abstract structure of mathematics. The child may have an amount of knowledge but it is seldom as great as we expect. The spiral curriculum in which it is intended that we build the fundamentals of mathematics may in practice have to be reduced to two dimensions and not three i.e. continual reiteration of the same points. The teacher may not in fact be building but **re-teaching** and so has to return time and again to what was thought to be already assimilated by the child. The re-teaching is very often seen as a revision exercise and does not take into account the necessary introduction of the topic. If one knows that the topic is new and is being presented for the first time one approaches it slowly. If on the other hand one thinks the child is in possession of certain facts then the approach is rather different.

Textbooks must also bear some of the brunt of some of the criticism of this "review" approach. They generally provide some revision exercises which may be arranged in some organized pattern. Indeed, textbook layout has, in general, changed very little since Butler and Wren wrote half a century ago:

> Numerous textbook publishers in recent years recognize the need for systematic maintenance work and make provision for it through sets of drill exercises, diagnostic inventory tests, cumulative reviews, and the like, placed at strategic points throughout the texts. In some cases these exercises have evidently been prepared hastily and with little attention to their validity or suitability."

Saxon (1982), a current American author of secondary school Math texts and proponent of the RAYG technique (Review As You Go), describes most modern high school mathematics texts as being full of dis-jointed spiral review sections. Accurately reflecting modern practice, Saxon points out that, "Most teachers ignore these spastic reviews, because they hinder understanding rather than help." In the spiral approach, topics are presented in units and are not encountered again until they are needed in the development of more sophisticated concepts; this circumstance usually arises well after the student has forgotten the initial concept.

Massed Practice versus the Saxon Approach

According to Butcher (1975) : "Traditional massed practice drills only the concepts and skills related to a single topic. Generally a teacher teaches a topic, assigns practice on just that topic, discusses the assignment and then begins teaching the new topic." This technique actually encourages rote learning.

Saxon is quite blunt in his analysis of such a system: "This approach to mathematics instruction does not work, has never worked and will never work. It is unsatisfactory because insufficient time is allotted for the assimilation of each new topic, and, as the next topic is attacked, understanding of the first topic quickly slips away."

Saxon claims that his "incremental approach" produces "outrageously higher test score results, and the students have a markedly more positive attitude toward mathematics." He summarizes the technique as follows: "Both teachers and textbook authors have overlooked the obvious. Why not review continuously? Why should we present 25 problems of the new kind? Why not present only three or four problems of the new kind, along with 25 review problems?"

In his "Introduction" to his text Algebra I An Incremental Development , Saxon (1981) adresses the student:

> ... If you find that a particular problem is troublesome, get help at once because the problem won't go away. It will appear again and again in future Problem Sets.... The Problem Sets contain all the review that is necessary. Your task is to work all the problems in every Problem Set.... The repetition is necessary to permit all students to master all of the concepts, and then the application must be practiced for a long time to insure retention. This practice has an element of drudgery to it, but it has been demonstrated that people who are not willing to practice fundamentals often find success elusive. Ask your favorite athletic coach for his opinion on the necessity of practising fundamental skills.

To demonstrate Saxon's "Incremental Development Procedure", six problem sets in his text Algebra I were chosen at random (numbers 32, 51, 89, 105, 109, 121). The corresponding ratio of questions from that lesson set versus questions from previous lessons were: 4:26, 6:24, 9:2_, 3:26, 3:28, 6:23. This sample indicates that a mean value of 83% of the exercises from the problem sets are related to material covered in previous lessons.

Hartzler (1984), in discussing the Oklahoma City Public Schools' plan using Saxon's procedures, points out that RAYG exercise assignments were routinely used many years ago but "Evidently recent literature and methods courses have allowed the issue to fall between the cracks." He cited Butcher's work on distributed homework practice as further evidence to support Saxon's procedures.

The Spacing of Reviews and its Effect on Retention:

Butcher proposes a distributed practice model in which proportionately more emphasis is on practice with previously learned concepts and skills than with new content. In the 1988 NCTM Yearbook (Coxford Ed.) Holdan summarized Butcher's technique thus: "Instead of assigning thirty trinomials to factor into the product of two binomials in one assignment, assign fifteen the first day, ten the next day and five the following day .... This results in homework assignments that structurally provide students with reinforcement over time, lessen the effects of forgetting due to interference of new learning, and make the information more accessible when it needs to be remembered. Generally speaking, spaced practice encourages active processing of information instead of rote learning often associated with massed practice."

There may be a caveat here which is not entirely dispelled by the literature on distributed practice, that it may be instilling a better ability to remember procedures but still not escaping the ogre of rote learning. The twelve times table is certainly well imbedded in our minds through endless drill, but this does not necessarily imply an understanding of the meaning of multiplication.

Hartzler (1984) in a section entitled "When using Saxon's books..." makes a point which may justify the above skepticism: "When explaining a new lesson, concentrate on how to do the problem. Cover the bulk of the <u>why</u> on the next day." Could it be that students become satisfied with an ability to deal with the "how" to the exclusion of the "why"? Hartzler may not be justified in his claim that, "A year of this approach leaves students thinking like algebraists in addition to remembering the skills." Hartzler's discussion of the effects of RAYG on students of lower versus higher ability raises some doubts concerning the true effects of this system:

> The benefits of a RAYG approach are more pronounced for the student of lower ability ... it should be clear why the high achiever has less need for RAYG for learning low-order skills than lo achievers: high achievers have already learned more and have more facts and concepts already in their minds with which to <u>associate</u>

> new learning .. the low achiever cannot learn as well by association, having less with which to associate, and must learn by review.

The use of the term "review" here is somewhat suspect, the low achiever may merely be improving his faculty of rote-memorizing to the exclusion of structural learning, since he apparently lacks the essential ability to associate new to past learning.

Burns (1958) is one of the few researchers found in a thorough search of ERIC documents who has moved away from the more mechanical type of review (which would seem to lead naturally to what Ushinskii described as "mechanical memory"). He conducted a study with sixth graders in which he provided thought-provoking review questions on common and decimal fractions, spaced throughout instruction. His intention was to introduce pupils to the idea that a variety of procedures might be used to complete the exercises and hence help them to form "valid generalizations and to formulate meaningful explanations", rather than rote-learned responses devoid of understanding.

Four examples of his "review study questions" are given below:

1. How can 7 1/4 be changed to 6 5/4 ?
2. Do you have to change 2 3/5 to an improper fraction in order to find the product of 2 3/5 x 5 ?
3. How do you know, before dividing, that the quotient to 7/8 divided by 2/3 is greater than one?
4. How would you explain why .2 and .20 and .200 have the same value?

The teachers using his "review study" approach indicated that the pupils' enjoyment of the questions grew during the year and expressed a preference for the review study questions as opposed to textbook reviews. They reported that the questions "held the interest of the class better and did a better job of reinforcing arithmetical concepts, stimulating thinking, and emphasizing meaning and understanding." The statistical part of his analysis did reveal that, while his review system required no more than 5 to 10 percent of classroom arithmetic time there was a statistically significant difference in performance between the test and control groups. It is apparent that Burns has demonstrated the value of the process of spaced structural review.

The positioning of the reviews in spaced practice and its effect on retention have important potential applications to learning theory and instructional effectiveness. Ausubel (1968) has suggested that, if adequate attention were paid to considerations such as optimal review, retention losses might not be inevitable.

In a study by Pressey, Robinson and Horrocks (1959) it was determined that approximately two-thirds of the concepts learned in high school and college courses are forgotten within two years. It is a small consolation that intellectual skills appear to be retained somewhat better than verbal information (Davis and Rood, 1947).

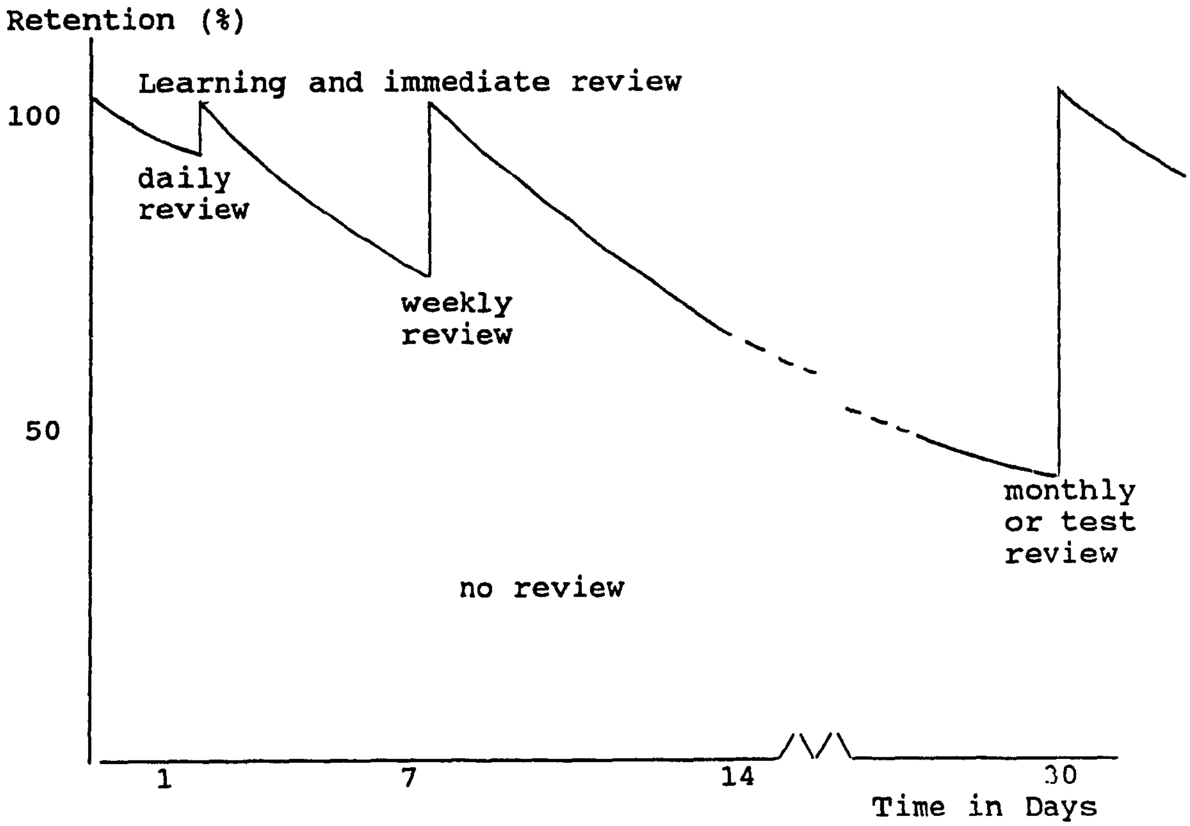
The Spitzer Study (H.F. Spitzer, 1939) shows that the forgetting of "meaningful" material follows the same curve as the traditional Ebbinghaus model. In Spitzer's experiment he formed ten groups of 360 students: eight experimental and two control. All ten groups read two articles, then a test was administered, but only to groups I and II. Groups III to VIII took a test at the same time, but their test had little relation to the content of the article. Groups III to VIII were tested at different time intervals: after 1, 2, 14, 21, 28 and 63 days respectively.

The exponential decline demonstrated that the amount retained (as a percent of amount learned) was 55% after one day, 35% after seven days and reduced to 15% after 63 days.

Skeptics of this research legitimately question the term "meaningful material" as potentially "meaningless". A percentage recall of 55% after one day does not convey an impression that the student taking part in the study could see relevant personal meaning in the text (see later discussion of Bartlett's Study). In effect, there is a close parallel here with the work of Ebbinghaus on nonsense syllables. There is, however, a significant difference worthy of highlighting related to degree of recall per unit of time: The forgetting of nonsense syllables drops off at a much faster rate than more "meaningful material."

Indeed it has been conjectured (MacDonald, 1989) that the greater the meaning to the individual, the slower the retention loss, (see also references to Skemp later).

MacDonald's retention model is based on spaced reviews and is demonstrated by the graph below (MacDonald, 1981).



Graph of Retention versus Time

MacDonald recommends a combination of immediate review after learning a concept, then further reviews after one day, one week and one month.

Ausubel (1968) stated that the issue concerning the best temporal positioning of reviews is still unresolved and needs more research. Horwitz (1975) focussed the spotlight further by stressing that: "Little experimental research has been conducted on the relationship between practice trials and the retention of meaningful material despite the immense practical significance for classroom learning."

Gay (1973), who herself conducted a brief study on temporal positioning of reviews, admits that: "While the effects of reviews have been inadequately investigated for verbal information, they have been totally ignored for intellectual skills such as mathematical rule learning; it has not even been experimentally demonstrated that reviews are beneficial for this type of learning."

Gay's study attempts to fill this wide gap in research by investigating whether reviews are effective for rule-application retention and if the temporal positioning of reviews is an important factor. In her second experiment, four groups were given different review treatments: Groups 1, 2 and 3 received two reviews; students in Group 4 received no review. Students in Group 1 received reviews one and 2 days following original learning; students in Group 2 one and seven days after original learning, and in Group 3, six and seven days after learning. Group 2, which was given reviews one and seven days after learning was found to have significantly better retention than the other groups.

Ausubel (1966) indicated that each type of review has its own relative advantage; an early review has a superior consolidating effect on the highly available material, whereas later (delayed) reviews involves superior relearning of forgotten material. Thus two such reviews can be combined profitably for the learner.

Reynolds and Glasser (1964) echo Ausubel's comments in their findings that "variations in repetition had only transitory effects upon retention, but that spaced review produced a significant facilitation in retention of the reviewed material."

A later study by Bausell and Moody (1972) followed Reynolds and Glasser's lead in the research of effects of programed review materials, but applied the technique to elementary mathematics instruction as opposed to Biology. They confirmed Reynolds and Glasser's findings with statistical findings showing that the group using reviews retained significantly more of the modular arithmetic instruction than the control group. They concluded: "It is believed by the present authors that more widespread use of programed material as review vehicles would result in an efficacious addition to the elementary mathematics curriculum."

Spaced Review was the subject of a report commissioned by the U.S. Office of Naval Research (Underwood, 1968). The conclusion reached indicated that the influences of time intervals of a few seconds to several minutes between practices had little influence on the rate of learning. However, long intervals (24 hours or more) between blocks of instruction had an enormous influence on the effects of instruction. Underwood suggests that such a practice schedule "innoculates" the learner against forgetting and prevents interference with other learning tasks.

It is most pertinent to raise procedures commonly used in the teaching of typing at this juncture. A recent keyboarding/typing text (Basic Keyboarding and Typewriting Applications by Robinson, 1983) includes the following summary under "Lesson Organization":

> Keyboard/typewriting teachers have stated their preferences for teaching no more than two or three letter keys per day ... and for providing periodic review lessons to consolidate students' accumulated learning. A brisk pace of instruction, with planned pauses in between learning, for tying together the separate learnings into one integrated whole, seems to produce the best results. These pauses in new learning do not mean that no learning occurs. Rather, they permit needed emphasis on speeding up keystroking and on weaving spacing, returning and shifting movements into the total pattern of keyboarding.

Later, in discussing the sequence of presenting letter keys, the author recommends "introducing the letter keys in ten lessons, with six intervening recall/reinforcement lessons in which no new learnings are presented or encountered." (This represents 38% of class time spent on "recall/reinforcement", a far cry indeed from current procedures in the teaching of other subjects).

The Role of Memory

Another study in the area of retention suggests that there is significant difference between retention and recall and that this difference should be carefully considered when conducting retention research (Bruce, 1974). In the Bruce experiments, retention did not appear to be as much of a problem as did recall. His experiments in non-intentional forgetting indicated that storage and retrieval were two different phenomena. Bruce suggested that information may be in the memory, but there can simply be a failure to "access" the appropriate information.

Perhaps more important than the distinction between storage and recall, the Bruce experiments indicated that selective rehearsal or review may be at the root of a number of situations in which at least short-term memory is facilitated. The tests were not conclusive, but it was suggested that, "the real value and essential nature of rehearsal or review is to identify retrieval information or cues in order to locate and extract information from memory."

An important aspect of storage was omitted in Bruce's research; that of the length, or quantity of material to be learned. Nelson and Lawson (1972) attempted to assess the relative importance of this variable in relation to forgetting. Their experiment suggests that the length of material to be retained in long-term memory affects storage rather than forgetting. The implication is that once something is really learned (stored), it is not likely to be easily forgotten and that humans can only store a limited amount of information per exposure to new material.

<u>Weaknesses of the Studies</u>

This lack of attention to the function and role of memory is one criticism that can be levelled at much of the research so far described. Is memory to be used merely as a storehouse or a bird-cage (to use Plato's model as described by Marshall and Fryer (1978)); a place where multitudinal species are to be rapidly pushed without attention to separating the various species into an orderly system? Should there not be an overt intention to arrange our "traces" in such a way that they are suitably interconnected and built into a clear structure? Or is it intended that we automatize relatively detailed processes and reactions to the exclusion of any development of structure? Each process has its place and its advantages. Returning full-circle to Ushinskii's comments, "to remember is not the same thing as to know ... it is not easy to draw a line between these two psychic phenomena as it would seem at first."

A key weakness of many of the aforementioned studies is that they tend to err on the "remember" side of that line rather than the "know" side. The almost universal use of the term "review" in these cases may mislead the reader into believing the student has a clear understanding of the structure of the subject; he does "know" and "understand." Whereas, in reality, it may indeed be the case that the student merely "remembers" or "repeats." Such criticism is not intended to denigrate their efforts but to point out a potentially serious oversight: Review is rarely explicitly defined. Saxon states, "Review is necessary if students are to assimilate and retain the abstractions of algebra." His description of his "incremental development" technique ("introduce topics in increments and practice every topic in every problem set") alludes to what Skemp (1976) describes as using "rules without reasons". Skemp (1971) describes such an approach as achieving "Instrumental Understanding", which he defines as "the ability to apply an appropriate remembered rule to the solution of a problem without knowing how the rule works" (p. 45). Saxon himself states that Algebra is a "fundamental skill" that "must be practiced for a long time to insure retention."

In the Gay study subjects were tested at the "review sessions". In these sessions, if they answered problems successfully or not they were "shown the problems and their answers". Successful students were given other problems requiring the use of another rule. The unsuccessful students were "presented with a new example." Again the implication being that the accent was on repetition with the goal of mastering skills.

Reynolds and Glasser use the terms repetition and review as one and the same. In fact, they call their two experimental groups: Massed <u>Repetition</u> Group and Spaced <u>Review</u> Group. Once again, no attempt is made to define "review" and the spaced review sessions they describe are, in actuality, spaced tests, along the lines of the Gay experiment using programed instruction units. One must also be skeptical about Reynolds' and Glasser's suggestions that the material taught to the students was "complex and meaningful" since none of the students had received previous exposure to biology (and none had had previous experience with programed instruction from teaching machines).

Hoover in an article entitled "Review and Drill: Valuable But Widely Misused Teaching Techniques" (1970) makes a comment that reflects the inadequacies of much research on so-called "review". He refers to the late William Burton's observation that he had seen several hundred reviews in progress, but practically none that had ammounted to anything more than drill. It would seem appropriate therefore at this stage to discuss the researchers' definitions of review, repetition and drill.

Definitions of Review, Repetition and Drill

It is apparent, thus, that "review" is considered by much of the research community as being synonymous with "repetition", "rehearsal" or merely "mindless drill". Butler and Wren, however, describe review in such a way that Ushinskii's dividing line is more clearly visible:

> Review is sometimes mistakenly identified with drill because they are both characterized by repetition and because they both aim at the fixation of reactions, concepts, or relationships. In spite of these common characteristics, however, it is a mistake to regard their functions as identical. Drill is concerned with automatization of relatively detailed processes and reactions. Review, on the other hand, has a dual function... Review is usually concerned with more or less comprehensive units of a subject, whereas drill

> is generally upon details.... Review aims at fixation and retention... through the deliberate processes of organizing, systematizing and relating elements and of generalizing and applying principles rather than through reducing reactions to the plane of automatic respon s. (pp. 153-154)

The Burns' study is much closer to the latter interpretation of the term review. As mentioned earlier, the teachers in his study found that Burns' review study questions "did a better job of reinforcing arithmetical concepts, stimulating thinking and emphasizing meaning and understanding."

It is interesting to note also that Smirnov (1973) uses the term "repetition", though his description of its use is much closer to the term "review" as defined by Butler and Wren:

> Repeated executions of actions are not exact copies of the first execution... Each new comprehension of the same material results in the revelation of new aspects not noticed earlier, leads to a fuller, deeper and more exact understanding and reveals new connections and relations... Sometimes only repetition makes it possible to understand the material as a whole."(p. 243)

His further comment shows a definite intent to demonstrate that "repetition" is far removed from "drill": "If repetition is correctly organized, not only does it not lead to mechanical memorization but, on the contrary, is of great help in avoiding mechanical memorization. The attempt to contrast repetition and understanding is absolutely unjustified."

Hoover attempts to clarify the distinction between review and drill:

> Review, literally, means a re-view or re-look at something. Its basic function is to extend or broaden original concepts or feelings to related situations, i.e. to develop a **new** view relative to previous learnings.

Drill, on the other hand is seen by Hoover as being "designed to extend or polish **skill** learnings. It is appropriate when a more or less fixed pattern of response is needed."

Sechenov (1947) alludes to the relationship between review and understanding when he states that "in the realm of learning, only assimilated and comprehensible material can be reproduced." He further states:

> Memorization of impressions and their repetition are as closely linked as effect and cause... the more frequently an object is seen, the greater the chances of observing it from different aspects, the more complete and differentiated becomes its image - the concept of it. Consequently, if we approach the mental content of man from the point of view of the content of his memory, it will become clear that its development derives from the repetition of impressions under the greatest possible diversity of the conditions of perception, both subjective and objective." (p. 308)

Experience might be seen as the architect of a well-structured memory.

Ushinskii affirms the importance of repetition and variation in repetitions: "Varied combinations of the same material are more useful than repeating in precise order what has been learned." (p. 427) He also refers to two types of repetition: passive repetition (the subject is passive and perceives again what he perceived earlier) and active repetition (the subject plays an active role and reproduces independently impressions perceived previously). In the same vein, he goes on to say, in favour of student's independent thinking: "Even if one assumes that the student understands the idea explained to him by the teacher, he will never commit it so firmly to memory as when he himself reasons it out." (p. 422)

Krutetskii's Research on Mathematical Memory

Krutetskii's research (1976) does not dwell on the ideas of repetition or review, but he sheds light on the learning process from a different perspective. In his study, a survey of 56 mathematics teachers was made to assess components of mathematical abilities in order of importance. The "Ability to Generalize" was voted the most important by 55 of the 56.

In his subsequent research he analyses, through extensive interviews with students across the ability range, how this generalizing ability varies. He detects a marked difference in the memories of mathematically "capable" as opposed to "inept" students:

> ... mathematically inept students are distinguished by a poor memory for generalized mathematical material, abstract mathematical relations and symbols, particularly problem types...
> Most capable students remember the type and general character of the operations they have solved but they do not remember a problem's specific data or numbers... the memory of a mathematically able student is markedly selective: the brain retains not all of the mathematical information that enters it, but primarily that which is "refined" of concrete data' and which represents generalized and curtailed structures. (pp. 299-300)

He therefore characterizes capable students as those who are able to recall what he terms **"generalized essential relations"** or **"curtailed structures"**, and the incapable students as those who, instead, can only recall details to the exclusion of any structure.

In a particular study (pp. 295-298), 8 problems were given to 26 capable students. The percentage of generalized essential relations recalled (for use in a similar problem) after 6 months was 86% , concrete data from the original problem was recalled at the 2% level and unnecessary data was completely forgotten.

In contrast, a group of mathematically incapable students showed complete inability to reproduce an experimental problem type a week later, to say nothing of 6 months later. Students in the latter category tended to attempt to remember complex definitions, proofs and the like by heart, reproducing them verbatim.

Krutetskii demonstrates these extremes by referring to an instance of each: A particular weak pupil made efforts to master difference-of-square problems like 113^2-112^2. Success came only after help was given by the experimenter. but in a week "she had forgotten the mathematical relationship, although she remembered that the numbers 113 and 112 were part of the problem" (concrete data recalled, structure forgotten). (p. 298)

His research team showed the other side of the coin in the following commentary:

> We conducted the following simple experiment time and time again: a capable pupil would do a problem of a definite type, and in two or three months he would be given a problem of the same type (but not the one he had done earlier), which he had not to our knowledge done either in class or at home. Often a "feeling of familiarity" would come to the pupil: he would believe he had done **this problem** already (not one of the same type but **the same one**). (p. 296)

Here the obverse is true: the structure is retained and the concrete data forgotten, even to the extent that the original concrete information is seen as indistinguishable from the new data.

This degree of ability to generalize has a corresponding significant effect on the problem solving capabilities of the student. Again to quote Krutetskii: "...an ability to transfer quickly to thinking in "curtailed" structures in problem-solving distinguishes mathematically able students".

To exemplify this his research team followed the detailed course of reasoning of students in the different categories of capability using the same problem for each student. In a particular problem in which 7 identifiable links in logic were needed to reach the solution, the capable student was able to reach the solution in 43 seconds by jumping steps in the logic and only verbalizing 4 links. At the other extreme, an incapable student took over 11 minutes to solve the same problem. In the process this student used all 7 links, made 15 errors in judgement or operation, 7 unnecessary repetitions and 4 irrelevant conclusions. Furthermore the interviewer had to intervene 3 times. (p. 274)

In conclusion to his chapter on Characteristics of Information Retention Krutetskii makes the important point that:

> We note that we are speaking of the strength or weakness of memory for **mathematical generalizations.** We stress that it is not a matter of incapable pupils' having a poor memory in general and not even of their having a poor memory for generalizations...many of them make satisfactory, even excellent progress in their other subjects...these pupils show a good memory in non-mathematical areas...with a good recall not only for factual, concrete material but also for ideas, patterns of reasoning, generalizations and conclusions. Thus a pupil's memory may be good in general, and its insufficiency may be detected only with operations with mathematical material.

<u>The Development of Learning Structures</u>

In her book "Piaget Sampler" Campbell (1976) quotes Jean Piaget in a chapter dedicated to his thoughts on development and learning: "My first conclusion is that learning of structures seems to obey the same laws as the natural development of these structures. In other words, learning is subordinated to development and not vice-versa..." This is a familiar Piagetian concept, but one which should not be overlooked. Piaget continues his argument attempting to determine whether effective learning has taken place. He brings up three points, the second of which is of interest in this context:

> The second question is, "How much generalization is possible?" What makes learning interesting is the possible transfer of a generalization. When you have brought about some learning, you can always ask yourself whether this is an isolated piece in the midst of the child's mental life, or if it is a really dynamic structure which can lead to generalizations... Learning is only possible when there is active

> assimilation...and I think that without this activity there is no possible didactic or pedagogy which significantly transforms the subject. (pp. 76-77)

To put these comments in simpler language in the context of mathematical thinking, Piaget is suggesting that:

1. The student must be at the right stage of mathematical maturity in order to absorb the new learning effectively.

2. The student must have available, at the time of new learning, an appropriate **"dynamic structure"** with which he can actively assimilate such learning.

The implication being that, if these two conditions are not met then the learning will be ineffective and the subject will be left with that "isolated piece" in the midst of his mental life.

Skemp is also a proponent of this "dynamic structure" approach also. He distinguishes between two types of understanding (Skemp 1976): Relational Understanding and Instrumental Understanding. The former he describes as "knowing both what to do and why" and the latter as using "rules without reason". In a 1979 article, he elaborates Relational Understanding stating that it "consists primarily in relating a task to an appropriate schema". He makes the interesting comment that relational mathematics is harder to learn but "it is easier to remember". This harks back to the student quoted earlier who complained that "we can spit out formulae in math but we really don't know what they mean. We really need to learn how to stop and THINK". (The Gazette, April 2, 1989). Skemp's schema, or what he also terms "conceptual structure" (1976), is also akin to Krutetskii's idea of "generalized essential relations". He indicates the considerable value of schema to the student when he states: "the more complete a pupil's schema, the greater his feeling of confidence in his own ability to find new ways of "getting there" without outside help" (1976). There is also an interesting point he alludes to concerning schemas:

> Paradoxically, in a schema there is more to learn but less to remember: more to learn, because higher order concepts are involved,and more connections; but less to remember because once learned it forms a cohesive whole, from which an infinitely large number of particular plans can be derived (Skemp 1979).

Bruner (1962) certainly concurs regarding the value of developing such schemata or "structured patterns":

> Perhaps the most basic thing that can be said about human memory, after a century of intensive research, is that unless detail is placed into a structured pattern, it is rapidly forgotten. ...Organising facts in terms of principles and ideas from which they may be inferred is the only way of reducing the quick rate of loss of human memory.

Bartlett's study on Remembering (1932) also places considerable emphasis on the idea of schematic memory and shows potential weaknesses of restricted schemata. He demonstrated that his subjects (British college students), who were asked to read folk tales from unfamiliar cultures and later reproduce them from memory, appeared to make a fundamental "effort after meaning". They assimilated the new material into existing "schemata", which are organized representations of past experience embodying the subjects view of reality. In other words the resulting representations were distortions of the original stories.

Bartlett specified three basic types of distortion:

1. Levelling (Unfamiliar details faded): "Without some general setting or label as we have repeatedly seen, no material can be assimilated or remembered."

2. Sharpening: Some particular details are retained in exaggerated form.

3. Rationalizing: Passages of the original story tended to become distorted so that they fit in with the individual's expectations - "an effort after meaning". Often concepts unfamiliar to the British culture were not well-remembered and many subjects tended to tack on a "moral" to the story since this was a widely accepted practice in stories with which they were familiar.

The first and third of these points once again demonstrate the importance of a cohesive structure into which the student can successfully implant new learning. As Byers and Erlwanger (1985) state:

> Memory is not detailed but schematic. The act of remembering is an active 'process of reconstruction'. Consequently, although remembering produces detailed information which appears to be correct to the individual, it may seriously distort the original material (p. 272).

Erlwanger's research (1973) on the effects of an "Individually Programmed Instruction" technique on a particular "bright" student, further underscore the importance of students having a well-developed structural understanding of mathematical concepts. Benny, a twelve year-old, who was considered by his teacher to be the best in the class, had the freedom to work a great deal on his own in a system designed to minimize teacher involvement. Here the "Bartlett effect" was at work and Benny developed his own extraordinary (though in its own way logical) way of dealing with manipulations of fractions. He regarded mathematics as a "wild goose chase" for answers embodied in the teacher's key. He also manufactured his own rules which he applied consistently and was quite ready to explain. For example, on the one hand he found that .3 x .4 = .12, but on the other hand .3 + .4 = .07. Similarly, though 2/4 + 2/4 = 4/4 = 1, Benny also asserted that 2/1 + 1/2 = 3/3 = 1. As Byers and Erlwanger (1984) state:

> Apparently Benny 'generalized' multiplication rules for decimals and fractions to cover certain cases of addition [as shown above]. To account for unacceptable answers he also 'generalized' his discovery that the same answer could be expressed in different forms into a 'theory of relativity' of mathematical **content**.

The latter research also demonstrates the inherent danger of allowing students to wander unchecked: They are prone to develop erroneous learning structures which are subsequently exceedingly difficult to eradicate. Erlwanger noted (1973) that after spending forty-five minutes on two days per week for eight consecutive weeks with Benny explaining the correct mathematical processes, he returned to the school five weeks later and presented him with the same questions. His answers were as they had been on the first day.

Ginsburg (1977) gives a poignant explanation for Benny's difficulties: "Typically children's errors are based on systematic rules. ...Children's faulty rules have sensible origins. Usually they are distortions or misrepresentations of sound procedures." It is evident that these distortions are also most resistant to change.

As Byers and Erlwanger (1985) conclude in this context:

> **The meaning the learner constructs may not be the meaning his teacher wants him to construct. ...in the absence of adequate guidance a student may develop an entirely erroneous conception of mathematics.**

It is the question of such "adequate guidance" via a suitable review mechanism that is the subject of this thesis.

CHAPTER 3

Procedures

This chapter is broken down into three sections, each of which relates to a specific objective. There follows a brief synopsis of each section:

3.1: SSR Procedure

This section gives a step-by-step breakdown of the Structural Spaced Review technique. To make the system more comprehensible, a precise account is given of the teaching of a specific topic and how the review method is used in the subsequent elaborations of that topic. The section terminates with a tentative description of standard classroom teaching.

3.2: Questionnaires

In this sub-division, the pilot questionnaire is given and there follows a discussion of the various categories of investigation that it inspired. From these general categories the more sophisticated, three-part, fifty-nine question evaluation was born. This final questionnaire is given in full and there follow some technical details concerning the questionnaire.

3.3: Structure Tests and Interviews

In this section, the purpose of the structure tests is discussed along with the interview procedure. The structure tests then follow by course heading, each test being composed of five questions. After each question, the preliminary interview questions are presented followed by a list of underlying intentions.

3.1 SSR PROCEDURE

[This section relates to Objective 1: "To describe the review technique and its application in detail over one term in order to distinguish it as clearly as possible from a corresponding description of standard classroom teaching."]

The mechanics of the Structural Spaced Review technique are described in the following paragraphs:

(a) The two courses pertinent to this study were College Algebra (201-211) and Applied Math for Commerce (201-101C). The course outlines included in Appendix 1.

(b) The classes were taught, by the author, over a fifteen week period, meeting three times per week for lessons of one hour and twenty minutes.

(c) The value of constant review was stressed to the students and the "P/9/7/5/2" system of structural spaced review was explained to them in the third week of the course.

The review technique functions as follows:

(i) Lessons are numbered from 1 to 45 (fifteen weeks, three classes per week).

(ii) Students were required to keep an index, by lesson number, of the material covered in each class. They also numbered their notes according to the lesson no. (e.g. 4A, 4B, 4C etc. for the three or more pages of notes of lesson number 4).

(iii) For any particular lesson, the student was encouraged to review the work covered in five previous lessons by application of the P/9/7/5/2 system. The review expected for lesson number 30 will be used as an example:

P:	Previous	(i.e. Review the work from the Previous lesson; Lesson 29)
9:	30 X .9 = 27	(i.e. Review the work from Lesson 27)
7:	30 X .7 = 21	(i.e. _ Lesson 21)
5:	30 X .5 = 15	(i.e. _ Lesson 15)
2:	30 X .2 = 6	(i.e. _ Lesson 6)

The numbers 9,7,5,2 have been chosen because they give a reasonable spacing between the lesson numbers of the review. Assuming we consider that the review procedure is used in every lesson, then the following table gives the number of reviews for a selection of lesson numbers. However, in practice, reviews only take place in two out of three lessons, we should consider the number of reviews to be about two thirds of the figure in the last column:

Lesson no.	When Reviewed	Number of reviews
5	6,7,9,10,23-27	9
15	16,17,21,22,29,30	6
25	26,28,35,36	4
35	36,39	2

The total breakdown of Lesson # versus which lessons are reviewed, is given in the following table (lesson nos. 5, 15, 25 and 35 are highlighted):

THE P/9/7/5/2 SYSTEM: WHICH LESSONS ARE REVIEWED

LES.#	P	9	7	5	2
6	**5**	**5**	4	3	1
7	6	6	**5**	4	1
8	7	7	6	4	2
9	8	8	6	**5**	2
10	9	9	7	**5**	2
11	10	10	8	6	2
12	11	11	8	6	2
13	12	12	9	7	3
14	13	13	10	7	3
15	14	14	11	8	3
16	**15**	14	11	8	3
17	16	**15**	12	9	3
18	17	16	13	9	4
19	18	17	13	10	4
20	19	18	14	10	4
21	20	19	**15**	11	4
22	21	20	**15**	11	4
23	22	21	16	12	**5**
24	23	22	17	12	**5**
25	24	23	18	13	**5**
26	**25**	23	18	13	**5**
27	26	24	19	14	**5**
28	27	**25**	20	14	6
29	28	26	20	**15**	6
30	29	27	21	**15**	6
31	30	28	22	16	6
32	31	29	22	16	6
33	32	30	23	17	7
34	33	31	24	17	7
35	34	32	**25**	18	7
36	**35**	32	**25**	18	7
37	36	33	26	19	7
38	37	34	27	19	8
39	38	**35**	27	20	8
40	39	36	28	20	8
41	40	37	29	21	8
42	41	38	29	21	8
43	42	39	30	22	9
44	43	40	31	22	9
45	44	41	32	23	9

It should be pointed out that when a test is given in a particular lesson (e.g. Lesson no. 15), then the review of that lesson implies a review of that test (which itself entails work from previous lessons). In the above table, it can therefore be seen that the test given on lesson 15 could be reviewed as many as six times, in lessons 16,17,21,22,29 and 30.

An inevitable consequence of this review procedure is that earlier work (e.g. from day 5 in the above example), will be reviewed more times than material covered in later lessons. This is intended to counter the detrimental effects that the passage of time has on memory.

(d) A detailed example follows showing the process as viewed in the context of the teaching and reviewing of specific material:

Sketching Sine and Cosine curves

(i) The teaching context.

This topic appears after students have been exposed to a number of manipulations using sine and cosine functions: They have used the definitions sinA=y/r, cosA=x/r to find the values of trig. functions of angles terminating in various quadrants. The A S T C concept of the sign of the functions in the quadrants has been introduced. They have been exposed to the "special angles" and the two triangles from which they emmanate. The calculator is used whenever possible in these situations and they are taught the manipulations required to find angles from function values (given restrictions on the angle), as well as the converse.

(ii) The lesson on sketching y=AsinBx and y=AcosBx.

The pattern of the lesson is as follows:

1. graph y=sinx using special angle values and further values from the calculator.

2. define Amplitude and Period in the context of the graph

3. show relationship between y=sinx and y=Asinx by tabulation

4. graph y=Asinx for differing "A" values (if A is positive then the curve is "regular"; if A is negative then the curve is "flipped" in the x axis)

5. show relationship between y=sinx and y=sinBx by tabulation

6. graph y=sinBx for differing "B" values

7. summarize graphing of both types indicating Amplitude and Period

8. combine the concepts and graph y=AsinBx by first establishing Amplitude and Period and setting out a skeleton axis showing the parameters for the sketching of one period (i.e. the graph of y=4sin3x has Amplitude 4 and Period 2pi/3; since A is positive the curve is "regular")

9. extend the graphing to differing domain limits (e.g. "graph y=5sinx/2 for -2pi<x<pi)

10. repeat the above steps for the development of graphing y=AcosBx).

(iii) The Reviews.

For the purposes of example, let us suppose that this topic is reviewed four times. The following is a breakdown of the type of questions asked and the discussion which is prompted following the review (be it a class presentation or a quiz):

Review 1:

Question:

Sketch one period of the curve y=3cos4x

Discussion: (after the student/s have attempted the question)

-what are the amplitude and period?

-why does the curve not pass through the origin?

-how can we find the x intercepts?

Review 2:

Question:

(a) Sketch two periods of y=2sin8x, showing all intercepts.

(b) Sketch one period of 5y=2sin8x.

Discussion:

-The same questions can be posed as for Review 1 for part (a); the essentials of the graph (i.e. the structure) being emphasized.

-The added complexity of dividing the function in part (b) by 5 can now be brought into play, since by this stage in the course, function notation has been taught.

Review 3:

Question:

(a) Sketch one period of 4y=3sin5x, showing intercepts.

(b) Sketch y=sinx and y=cosx on the same axes for the domain [0,2pi]. Indicate for which x values sinx=cosx.

Discussion:

-The first question can be approached in the same way as in Review 2.

-The second question now links the concepts of graphing to the solution of trigonemetric equations (where trig equations have been taught a short time before Review 3).

A crucial link can thus more forcefully be made between two hitherto disconnected subject areas, (sketching of trigonometric functions and solving simple trig. equations), thus further reinforcing the structure of both concepts.

Review 4:

Question:

(a) Sketch y=7sin2pix on [-2,1].

(b) Solve the equation sin2x = sinx, on [0,2pi] by graphing y=sinx and y=sin2x on the same axes for that domain.

Discussion:

-part (b) further demonstrates how a bond can now be forged between double angle formulas, trig. equations and graphing of trig. functions.

(e) The students were questioned on the review material on an average of two out of three lessons per week. These questions usually took the form of presentations in front of the class (Class Presentations), otherwise the whole class was given a written quiz . The Class Presentations (CPs) were organized thus: Four or five students were chosen at random from those present and they were asked to go to the board to tackle a problem each. This is treated as a test situation; each CP being evaluated out of 3% of the term mark. The students were given a little assistance if they made small errors or had difficulty starting a problem. If the student was subsequently unable to tackle the problem successfully, then a substitute was called from the remainder of the class (which is kept on "stand-by" until all the questions are being dealt with correctly). After the class presentation or quiz, a discussion or "review" period followed as indicated in the example above. In general the

Class Presentations had a duration of 10 to 15 minutes, and the discussion/review following the presentation took a further 5 to 15 minutes.

Naturally, questions were very much encouraged from the students, so the suggestions made in the examples concerning the fostering and development of structure are by no means exhaustive. In fact, one might go so far as to say that these brief discussion/review periods were particularly useful for the students to manipulate the concepts under discussion into a firm, cohesive structure. It presented them with an opportunity to ask that 'nagging question' and clear a little more 'mist' from an unstable structure.

(f) Three tests were given during the term which were of a cumulative nature (i.e. they included material from the early part of the course, not just that which has been covered since the previous test). The mark-value of the tests also increased; typically the weighting being close to 40:70:100.

Standard Classroom Teaching

This is not a fixed technique which is easy to describe, but what may be called "Standard Classroom Teaching" includes certain essential elements which distinguish it clearly from the SSR technique:

Concepts are usually learnt from a narrow perspective at their initial introduction. Typically, in a standard lesson, the previous homework is discussed at the beginning of the class and then the new lesson is given. The role of memory, review and understanding is generally neglected, especially in the teaching of lower level courses and where students have weak backgrounds. It may not be overly cynical to describe the teacher in such a class as one whose task is to fill a vessel by regular additions of approximately equal quantities of knowledge.

It is normal in such a procedure for the student not to encounter a particular concept until it is needed again at a much later date, by which time it is usually forgotten or confused in an unstable mental structure. Such 're-encounters' may take the form of erratic reviews given before a test or final exam. Reviews otherwise take the form of mere repetition (without an accent on understanding, or interconnecting of concepts) or, at worst, mindless drill exercises.

Standard classroom teaching is also typified by the covering of topics in disconnected 'units' after which 'unit tests' are administered to test the knowledge gained in that particular unit. The next, isolated unit is subsequently dealt with in the same fashion. In consequence, subjects which would benefit from interconnecting into a cohesive structure are, instead, taught in a seemingly disconnected manner. For example, in a Functions course, quadratic equations, the quadratic formula, quadratic functions and parabolas would generally be taught in this sort of independent isolation.

In defense of this kind of teaching process it is tempting to suggest that the onus is on the student to develop a suitable review mechanism to overcome its inadequacies. The reality is, however, that:

(a) Few students have such capacities;

(b) Even fewer recognize that they need some review technique to assist them in the learning process, and

(c) The teacher is more likely to be able to provide the required interconnections needed to build a cohesive structure than even the most determined and initiated students.

3.2 QUESTIONNAIRES

[This section relates to Objective 2(a): "To observe, in an exploratory way, some of the possible effects of the technique on the students with regard to their study habits, their attitudes to review and the time spent on review."]

As an initial step, a brief pilot questionnaire was developed and administered to a class in December 1988. It was intended to use the results of this pilot study to develop a more sophisticated questionnaire which would be administered in May of 1989 in the study proper.

The pilot concentrated on two areas: Student reaction to (a) the SSR technique and (b) the Class Presentations. The final question was left open-ended to elicit other areas of potential investigation. The questionnaire was given as a voluntary, take-home assignment to students in a Calculus II Science class at John Abbott College. The students were asked to omit their names so that the comments would be anonymous. The response rate was 50%. As mentioned earlier, these responses appear under Appendix 3.

<u>The Pilot Questionnaire</u>

1. Has the SSR review system changed your study habits?

   - if so, in what way?

2. Did you use the system on a regular basis?

   - if so, how much time per week?

3. What did you gain (or lose) from:

   (a) Preparing for the class presentations?

   (b) Doing the class presentations?

   (c) Watching the class presentations?

4. What adaptions do you think should be made to the review system?

5. Do you have any other suggestions or comments?

The results of this pilot were carefully considered by the author in collaboration with Lois Baron, an Education specialist from Concordia University and the questionnaire-proper was developed over the first four months of 1989. With the students' reactions to the pilot as a guide, a 59-question evaluation was created. This was divided into three sections. Section I included 41 questions with a Likert scale for responses. Section II involved 9 questions which were specific in nature and used conventional categorized responses. Section III included 4 ranking questions and 5 open-ended questions to permit the students maximum freedom in their responses.

The areas investigated in the questionnaire as a whole were divided into eight specific categories, though the questions were posed in a random fashion. The categories were: (i) The Review System; (ii) the Class Presentations; (iii) Understanding and Learning; (iv) Feelings; (v) Exams or Tests; (vi) Marks; (vii) Time, and (viii) Miscellaneous. These categories are used in the analysis of the questionnaire data in the next chapter. It was natural that some questions should overlap categories. The reasons for the choice of categories and some details of the questions asked are given in the following paragraphs:

(i) The Review System

We wished to establish the students' impressions of the value of the SSR system from several perspectives: Did they feel that they were understanding the material better because of the technique? Did they feel better prepared for tests and the final exam? Was the system stressful in terms of extra work and time?

(ii) The Class Presentations

It was evident from the pilot questionnaire that the CPs presented a form of barrier to the students. We wished, therefore, to establish the positive and negative effects the procedure had on the students: How nerve-wracking was the experience? Was the anxiety outweighed by the value in terms of learning? Was doing/watching a CP a useful way to learn? Was the time dedicated to CPs and their mark value appropriate?

(iii) Understanding and Learning

A major thrust of this research is to provide a more stimulating environment in which students might learn and understand more. Thus, the questions posed attempted to elicit such information: Was doing and watching CPs a good way to learn? Did the SSR system encourage understanding as opposed to just memorizing? Did the system help to show how the various concepts were inter-related?

(iv) Feelings

Here we wished to establish whether, on the one hand, there were unduly negative connations to the CP experience as opposed to increased confidence in self and the material, on the other hand.

(v) Exams or Tests

In this category we wished to know whether the CPs and the review system in general, better prepared students for tests and the final exam.

(vi) Marks

Considering the marks-conscious student body, we wanted to find out how the students felt about the weighting of marks in the evaluation process and also some information about past marks and predicted mark in the course.

(vii) Time

In the development of a new approach to teaching, the proper use of time is always a concern. What did students think of the split of the lesson into Class Presentations, discussion and lecture time? Also, did the extra preparation time needed from the student infringe on any other area?

(viii) Miscellaneous

This section collected together some useful, but isolated pieces of information like gender of student, impressions of the textbook etc..

Further discussion of the questionnaire is to follow, but it is appropriate to show it at this stage:

The Questionnaire

JOHN ABBOTT COLLEGE MAY 1989

STRUCTURAL SPACED REVIEW (SSR) AN EVALUATION

The purpose of this questionnaire is to assess your feelings about the teaching methods used in this course. The results of the questionnaire will be taken into account in making any changes to the course in future years. As the questionnaire is anonymous, please be as honest as you can in answering the questions.

MARK ALL ANSWERS IN SECTIONS I AND II ON THE GENERAL ANSWER SHEET.

SECTION 1

DIRECTIONS: Please mark on your answer sheet the letter which corresponds to the degree to which you agree or disagree with each of the following statements. If a statement does not apply to you or you have no answer for it, just leave it blank. Use the following scale in making your choices.

A. Strongly disagree
B. Disagree
C. Undecided
D. Agree
E. Strongly agree

1. The review system was a good way of using class time.

2. The amount of class time allotted to the CPs was just right.

3. The CPs made me nervous.

4. It would have been better to have written work (e.g. quizzes) than CPs.

5. Watching CPs was boring.

6. I am satisfied with the distribution of marks used in this course (9%=CPs, 41%=class tests, 50%=final exam).

7. Considering the amount of time I prepared, the CPs should be worth more marks.

8. Doing a CP was a good way for me to learn.

9. Watching the CPs of others was a good way for me to learn.

10. The problems given in the CPs were not too difficult.

11. The 9/7/5/2 system helped me understand the material better than by the ways I have been taught in other maths courses.

12. I liked the review system so much in this course, that I am using it in other courses too.

13. The time spent on reviewing for this course infringed on study time for other courses.

14. The time spent on reviewing for this course infringed on my leisure time.

15. I did not mind my peers watching me do my CP.

16. I would recommend this course over more traditionally-taught maths courses.

17. Any anxiety I had about doing the CPs was balanced by the positive effects on my learning in this course.

18. I learned from the mistakes others made in their CPs.

19. I hated the experience of other students watching me do a CP.

20. I worked harder in this course than in most courses I have taken.

21. The effort to keep up-to-date in this course was worth it.

22. I will be less nervous for the final exam because of the way this course prepared me.

23. The SSR system used in this course has "turned me on" to maths more.

24. Because of the CP experience in this course, I feel more comfortable speaking up in other courses.

25. As time went on in this course, I felt more comfortable doing a CP.

26. The SSR system allowed me to understand how to solve problems rather than just memorizing the techniques.

27. There was too much time spent on review rather than figuring out new problems.

28. The review method used in this course sure beats cramming for the final exam.

29. I feel more prepared for the final exam in this course than in most other courses I have taken.

30. If I were to take other maths courses, I would use the review system on my own.

31. I did more memorizing in this maths course than in most other maths courses I have taken.

32. The SSR system really helped me understand how the concepts were inter-related.

33. I did not mind when another student came up to the board and made corrections to my work.

34. The SSR system was one of the best learning experiences I have ever had.

35. Having both assignments and reviews to do for homework was heavy.

36. Not knowing when the CPs would be given was O.K.

37. I developed increased confidence in maths through the methods used in this course.

38. As time went on, I was better able to confront new maths problems because of the techniques used for learning in this course.

39. I would definitely choose a maths course taught by the SSR method over another maths course.

40. As nerve-wracking as the CP experience may have been, it was well worth it.

41. If I were to teach maths, I would definitely use the SSR system.

SECTION II

Continue on the general answer sheet in answering the following questions. Please be as honest as possible in your responses. Choose only one response for each question. If you cannot answer a question, leave it blank.

42. Gender:

    A. female
    B. male

43. I have taken this course the following number of times:

    A. once
    B. twice
    C. three times

44. My grades generally are:

    A. mostly 90 to 100
    B. mostly 80 to 89
    C. mostly 70 to 79
    D. mostly 60 to 69
    E. under 60

45. What grade do you expect to get in this course?

    A. 90 to 100
    B. 80 to 89
    C. 70 to 79
    D. 60 to 69
    E. under 60

46. The number of hours I spent in preparing for this course per week was approximately:

    A. 0-1 hour
    B. 1-2 hours
    C. 2-4 hours
    D. 4-6 hours
    E. more than 6 hours

47. Which was the hardest for you?

    A. preparing for the CPs
    B. doing a CP
    C. watching a CP
    D. having another student make corrections on my CP

48. Was the textbook useful in this course?

    A. yes
    B. no

49. When did you do most of your reviewing?

    A. right after class
    B. right before class
    C. the evening before class
    D. other

50. On the average, how many lessons did you usually review?

    A. one
    B. two
    C. three
    D. four
    E. five

(Please see over for Section III)

SECTION III

DIRECTIONS: PLEASE ANSWER THE FOLLOWING QUESTIONS ON THE QUESTIONNAIRE ITSELF

1. Please rank order the following in the order of which you learned the most from in this course. Use all the numbers from 1-6. Do not use the same number twice.

   ___ the lectures
   ___ the CP sessions
   ___ assignments
   ___ tests
   ___ class quizzes

2. Please rank order what was most beneficial to you in using the SSR method. Use the numbers 1-3. Do not use the same number twice.

   ___ learning the derivations of problems
   ___ it made it easier to memorize procedures
   ___ passing tests

3. If you had one and one-half hours of class time, how would you split up the time in minutes for each activity below?

   ___ CPs
   ___ review
   ___ lecture

4. If you could divide up the marks for the course, how many marks would you assign to each activity below out of a possible 100 marks?

   ___ CPs
   ___ class tests
   ___ final exam

5. What do you think is the 'method behind the madness' of the P/9/7/5/2 system?

6. Why do you think the course included the CPs?

7. The thing I liked best about the SSR system was (give your reasons):

8. The thing I liked the least about the SSR system was (give your reasons):

9. Any final positive comments, problems and/or frustrations with respect to the nature of the way this course was taught:

Having produced a protoype of the questionnaire, it was given to a volunteer student who had responded to the pilot questionnaire to see what adjustments might be made to make the evaluation tool as clear as possible. In consequence, a few ambiguities and subtleties were ironed out before giving the questionnaire to the two classes.

At the beginning of May 1989, approximately two weeks prior to the end of the course, the questionnaires were administered by a Faculty of Professional Development Services (FPDS) representative of the college. Twenty-one students in the Algebra 211 class and nineteen in 101C wrote the questionnaire. The procedure took about half-an-hour, and took place in the absence of the teacher.

Subsequently the General Answer Sheets were processed by the Management Information Systems Department. Using a "Computerized Assessment of Teaching Systems" (or "CATS") program, a printout was produced showing the number and percentage of students in each category of the Likert Scale questions in Section I. The information from Section II was typed by the FPDS in tabular form and the Section III responses were typed by question number. The anonymity of the students was thus preserved. Several months after the questionnaire was given, the file of responses was given to the author, though no names appear on the answer sheets.

Two pairs of questions were used in the questionnaire to test the consistency of student responses. In one pair, two similar questions were phrased differently:

Question 17: Any anxiety I had about doing the CPs was balanced by the positive effects on my learning in this course.

Question 40: As nerve-wracking as the CP experience may have been, it was well worth it.

In the other pair, the questions had opposite implications:

Question 15: I did not mind my peers watching me do my CP.

Question 19: I hated the experience of other students watching me do a CP.

Each student response was checked to see whether there was an illogical response to one or more of these pairs. This is further discussed in Section 5.1 (Questionnaire Results and Analysis).

3.3 STRUCTURE TESTS AND INTERVIEWS

[This section relates to Objective 2(b): "To observe, in an exploratory way, some of the possible effects of the technique on the performance of students in a specialized test, by looking at their results and evidence of any significant knowledge of structure of concepts."]

The purpose of the structure tests (and follow-up interviews) was to evaluate the students' knowledge of structure in the problems posed. Could they see the "generalized essential relations" needed in the deciphering and answering the questions posed? Or were they relying on a superficial knowledge of the mechanics of problem solution? Was there evidence that their approach to mathematics was one in which they aimed at what might be termed "sufficient performance" where understanding was a low priority which might be an occasional spin-off, or were there indications that there was a clear understanding of the concepts needed in each problem and a concomitant high performance?

The problems given in the structure tests were of two essential types: Those which vered on the side of a particular subtlety and were thus intended to tease out the students knowledge of structure and those which were more "mechanical" in nature. Questions of the latter type (of which there are perhaps an excessive number in North American testing) were included to underscore the difficulty of establishing an in-depth understanding from the students' answers. The teasing out of their knowledge of structure, in this case, was left for the individual interviews. The Structure tests were given two weeks prior to the end of the course in the form of a "bonus quiz" for the term. A time of forty-five minutes was allowed for the test, which was ample for most students.

The student interviews were given in the last week of the course and were of ten to fifteen minutes in duration. Five students were selected from each group according to their ability levels so that a there was a range from high to low abilities, (unfortunately, there was a tape malfunction with Robert, a 101C student, so that file had to be abandoned). The interviews were conducted on a one-to-one basis in a private office. The student was presented with the the question sheet of the structure test which he had written a week previously, along with his answers form the test (which he had not yet seen). A blank sheet of paper was provided for extra calculations.

A general review class was in progress during the interviews (reviewing a previous final exam with attached answer sheets). Students were told that there would be random interviews during the period and that these were purely for personal interest and had no mark value whatsoever. Each selected student was led from the classroom to the nearby interview room and escorted back after the interview. Being the last week of class, the students were quite relaxed and they seemed to be quite at ease during the sessions. The intention of the interviews was to start from a framework of specific questions and then, follow the student's flow of thinking as seemed appropriate. In this sense, there was a basic structure with an open-ended nature.

The questions given in the two structure tests are given below. Then, under the title "Interview" appear the interview questions and subsequently the title "Intentions" heads the group of reasons behind the preceeding questions. This format is used since there is a close link between the objective of the structure test questions and those of the interview.

The Structure Tests and Interviews

I. College Algebra: 201-211

1. In what way is the solution to $x^2 - 6x + 8 = 0$ connected to the graphing of the parabola $y = x^2 - 6x + 8$?

Interview

Questions:

(i) What is the difference between these two?
(i.e. $x^2 - 6x + 8 = 0$ and $y = x^2 - 6x + 8$)

(ii) How did you go about this question?

(iii) What would the graph of $y = x^2 - 6x + 8$ look like?

Intentions

-[Does the student realize the essential relationship between the distinct concepts of quadratic equations and quadratic functions?]

-[Can the student connect the two, realizing that the solution to $ax^2 + bx + c = 0$ gives the x intercepts of the parabola, which occurs at $y = 0$?]

2. Answer the questions below and explain the similarities and differences between the part (i) and part (ii) questions:

(a) (i) Simplify: $x/2 - 3/x - 4$; (ii) Solve: $x/2 - 3/x = 4$

(b) (i) Factor: $-x^2 - 7x - 12$;

(ii) Solve: $-x^2 - 7x - 12 = 0$

Interview

(i) What is the difference between parts (i) and (ii) of these questions?

(ii) Does x have a value which we can calculate in either or both parts?

(iii) How did you get your answers in part (ii) of each question?

Intentions

-[Does student realize the conceptual difference between expressions and equations?]

-[Is there realization that x has no fixed value in an expression, though it may have a value in an equation?]

-[Does student recognize the difference between a quadratic polynomial and a quadratic equation?]

-[Is there realization that there is no x value solution to the factor problem?]

3. If $f(x) = x^2 - x + 2$

(a) Explain, in words, what is meant by $f(x-1)$.

(b) Find $f(x-1)$.

(c) If $f(x-1) = 0$, find x.

Interview

(i) What does $f(x)$ mean?

(ii) How is $f(x-1)$ different?

(iii) How do you find $f(x-1)$?

(iv) How do you solve $f(x-1) = 0$?

Intentions

-[Does student understand the concept of Input/Output or Domain/Range?]

-[Is student aware that the new input, $x - 1$, is operated on in the same way as x in the original function definition?]

-[Does student follow through with the correct procedure for evaluating $f(x-1)$?]

-[Is there evidence of notational confusion?]

-[Does student incorrectly perceive $f(x-1) = 0$ as being equivalent to: $x - 1 = 0$?]

-[Since the correct solution to part (c) leads to a quadratic equation with no real solution, does this deter the student who reaches that stage in his answer?]

4. Identify the following as true or false; explaining the reason for your decision:

(a) $3x^{-1} = 1/(3x)$

(b) $(3x - 2)/3 = x - 2$

(c) If $f(x) = x^2$; then $f(2x) = 2x^2$

Interview

(i) What should I write in order to get: $1/(3x)$?

(ii) How would I have to change $(3x - 2)/3$ in order to get $x - 2$ as an answer?

(iii) How did you do the $f(2x)$ question?

Intentions

-[Does student see that the exponent only affects the x, as opposed to 3x?]

-[Is there a clear understanding of the "cancelling" process?]

-[Is $f(2x)$ viewed as $2f(x)$?]

-[Is $2x^2$ seen as equivalent to $(2x)^2$, and subsequently $4x^2$?]

5. How many possible solutions could there be to the quadratic formula? How does this relate to the different number of possible x intercepts in the graph of a parabola?

Interview

(i) What is the quadratic formula?

(ii) How many x intercepts could a parabola have? Show by graphing.

(iii) Is there any connection between the possible number of x intercepts of a parabola and the possible number of solutions to the quadratic formula?

(iv) What might the quadratic function of this parabola be? (Sketch shown of a parabola passing through -1 and 3, opening upwards).

(v) How many x intercepts might $y = (x)(x+2)(x-3)$ have and where would they be?

Intentions

-[Does the student see that the key to this question is the value of the determinant? $D < 0$, $D = 0$, or $D > 0$ give 0, 1 and 2 solutions respectively.]

-[Can the student reach this conclusion by considering various possible parabolas; with 0, 1 or 2, x intercepts?]

-[How does the student relate the quadratic formula and the parabola?]

-[Can he estimate a quadratic function, given a sketch?]

-[If successful at the latter question, can the student give the x intercepts of a factored cubic function?]

II. Applied Math for Commerce: 201-101C

1. A loan of $1200 is taken at 16% compounded monthly for 2 years. Given the Amortization Table below, complete the first Payment line ' d explain, in words, the relationship between the figures you enter:

Pay't #	Pay't	Interest	Reduc in Prin	Principal
0	-	-	-	1200
1				

Interview

(i) Why did you select this particular formula? (Referring to the method used to find the payment. There being some confusion as to whether a loan is a present or future value of an annuity.)

(ii) When you calculated Interest, what Period did you use and how did you calculate it?

(iii) Can you explain the relationship between Payment, Interest and Reduction in Principal and the logic behind the table?

Intentions

-[Can student use the appropriate formula for finding the Payment of an annuity, via Present rather than Future Value?]

-[Is the student able to explain the relationship adequately?]

2. How many 7-digit telephone numbers can be made in which the first digit is 4 and the last is even, if:

(a) Repeat digits are allowed; (b) Repeats are not allowed.

Interview

(i) Why are the numbers (of ways to make each choice) multiplied and not added?

(ii) How did you get the number in the first box?

(iii) Can you explian how to preceed and in which order?

(iv) How does "repeats not allowed" change the situation?

Intention

-[Does student demonstrate comprehension of the multiplication principal?]

-[Does the student correctly consider choice restrictions first?]

3. In the following Linear Programming problem, clearly define your variables, set up the constraints and the objective function, sketch the system and explian, in words the significance of the feasible region:

A Publishing Company produces 4 times as many paperbacks as hardbacks. In one year the demand for paperbacks is no more than 5000 and the demand for hardbacks is at least 1000.

Interviews

(i) How did you define the variables?

(ii) How did you determine the inequalities/equations?

(iii) What is the objective function in this problem?

(iv) What exactly is the meaning of the feasible region?

Intentions

-[Can student define the variables precisely?]

-[Is the inequality correctly interpreted?]

-[Is the student able to identify that there is no objective function defined in the problem?]

-[Can the student adequately explain the significance of the solution region?]

4. A Bridge hand has at least two hearts.

(a) What is meant by this?

(b) What is the complement of 'at least 2 hearts'?

(c) Using part (b), find the probability of getting a Bridge hand with at least two hearts

<u>Interview</u>

(i) Can you explain how you interpreted "at least two"?

(ii) What is the meaning of the term "complement", generally and specifically in this question?

(iii) How did you find the required probability?

<u>Intentions</u>

-[Can student interpret this as '2 to 13 hearts' as opposed to '1 or 2' or 'exactly 2'?]

-[Is the student able to correctly interpret the term 'complement' in this context?]

-[Does the student perceive the true structure of the problem and use the complement concept to correctly reach the answer?]

5. Three companies receive two newspapers each day; the Gazette and the Daily News. Company A receives 2 Gazettes and 3 copies of the Daily News, Company B gets 4 Gazettes and 2 Daily News and Company C 2 Gazettes and 5 Daily News. The Gazette costs 50c and the Daily News 40c.

From this information, construct two matrices P and Q, such that the product PQ is a matrix which contains the respective cost to each Company, in dollars, for one day's supply of newspapers.

Interview

(i) How did you go about finding P and Q?

(ii) How did you arrange to get an answer in dollars?

(iii) If you had to use a row matrix for the 50c, 40c, how would you do the problem?

Intentions

-[Is the student able to organize the information into appropriate matrices?]

-[Can he produce a suitable product matrix containing the necessary information?]

CHAPTER 4

Results

This chapter is divided into three sections, each of which relates to a sub-divisioin of Chapter 4. A short summary of each section follows:

4.1: SSR Procedure

In this section an attempt will be made to give the reader a flavour of the Structural Spaced Review technique in action: There is a brief report of several Class Presentations and Review sessions in each of the two groups under investigation. The section terminates with general comments and observations regarding the students' reactions to the use of these procedures throughout the course.

4.2: Questionnaires

This section presents the eight categories of the questionnaire data. Questions with extreme responses are highlighted in bold print. After each category of questions comes an analysis and discussion. The section terminates with a summary of the salient points.

4.3: Structure Tests and Interviews

Here the performance of the students on the specialized Structure Test is reported, the accent being on their knowledge of structure. More detailed commentary is made concerning the nine students with whom personal interviews were conducted concerning their responses to this test.

4.1 SSR PROCEDURE

This section will impart to the reader a clearer view of the "the review technique and its application over one term" (Objective 1). The mechanics of the technique have already been described in Section 3.1 under "Design of the Study", along with a tentative description of "standard classroom teaching"; it now requires that the flesh and blood be added to the skeletal framework, so that some life is breathed into the system.

Over the semester of the study, the author kept a detailed record of each question given for the Class Presentations along with the student evaluation for each question and a brief account of individual and class reactions to the use of the technique. Since it would be impractical and tedious to present all this information, a selection of material has been made to permit the reader to have, what is hopefully, an adequate insight into the process.

To this end, four days of class have been selected from each of the two courses under investigation: Algebra 211 and Applied Math 101C. The questions given as Class Presentations for each of these classes are given, along with the student's answers and comments about the review session given at the end of each class presentation. The lessons chosen are spread over the semester so that there is a sense of progression. At the end of this section there is a summary of general observations concerning the students' reactions to the use of the SSR procedure.

By its very nature, this section yields information pertaining to Objective 2(a) which seeks information concerning the effects of the technique on the students, "with regard to their study habits, their attitudes to review and the time spent on review". Though the Questionnaire is the major source of information for these areas of investigation, the forthcoming remarks may add significantly to the material amassed from the student responses to the Questionnaire.

The Review Technique in action: Algebra 211

In this class, Class Presentations were given on the following Lesson numbers: 2,4,7,9,11,12,15,16,17,19,20,22, 25,28,29,31,33,34,36,37,39,40,41,42,44 and 45 (a total of 26 lessons out of a maximum of 45). The four lessons selected as examples for this particular course are Lessons 9, 17, 28 and 41. The Lesson numbers and the related lessons from which the Class Presentations were taken, are summarized in the table below, which is a subset of the full table shown under section 4.1.

LES.#	P	9	7	5	2
9	8	8	6	5	2
17	16	15	12	9	3
28	27	25	20	14	6
41	40	37	29	21	8

(To refresh the reader's mind: The left column gives the lesson number; the "P" column refers to the previous lesson; the "9" column directs the student to the class represented by the rounded integer given by the calculation: 0.9 x Lesson # (e.g. for Lesson #28, 0.9 x 28 = 25.2, which is rounded to 25); the "7" column yields 20 for the example of Lesson #28 (0.7 x 28 = 19.6); the "5" and "2" column operate accordingly.)

In the process of explaining the "Results" of each question, references will be made to the general procedures of the Class Presentations so that the reader may achieve a more in-depth understanding of the whole procedure.

Lesson #9: Questions from Lessons 8, 6, 5 and 2.

(Note: The first question, instead of being called Question #1, is called Question #8 and pertains to material from Lesson #8. Similarly, Q6 is taken from Lesson #6 etc.. This procedure gives the student a direct reference to the appropriate pages of class notes (as mentioned earlier, they are encouraged to sequence their notes by Lesson number). Since the Review/Class Presentation procedure is an "open book" exercise, this assists the student to rapidly access pertinent notes, procedures, formulas etc.. It should be stressed that it is intended that some preparatory ground work has been done before the onset of the class. The question numbering was seen by past students to be a helpful reference, even after the CP sessions, in particular for follow-up or out of class questions).

Q8

Simplify: (a) $(3x^{-1}y^3)^3$

(b) $(-5xy)^2(x^2y^4)$

Result: The first part was answered correctly, but the "-5" in the second part was treated incorrectly; the students answer being: $-25x^4y^6$. The student was asked to explain the origin of the 27 in part (a) - correctly specifying that it came from 3^3, but when asked for the derivation of the -25 in part (b), he explained that this was "-5^2", seeming quite adamant that this was the correct process. The student was awarded a mark of 3/3 for only a minor error.

After returning to his seat, another student volunteered and corrected this error. At the end of the presentations (when all the students had completed their respective questions and returned to their seats), this point was raised by several students. References were made to the two distinct concepts: -5^2 and $(-5)^2$ in order to help in clarification. The point was taken by most of the class, but it was apparent that not all of the students were entirely convinced. Further encounters with similar questions (on the presentations of Lessons 11,12,15,16,40,41 and 42) acted as further catalysts for acceptance of the correct procedure.

Q6

Combine like terms: (a) $3x^2 - 2x + 4 - (5x^2 - 7x + 7)$

(b) $x^2yz - xy^2 + 3xyz - 6x^2yz$

Result: Correct answer, mark 3/3. The point of the distinctness of the terms was stressed in the review session and there were no questions. This topic was generally well received and understood by the vast majority of the students.

Q2

Identify into which category or categories the following real numbers fall (Rational, Irrational, Natural or Integral): 1/2, 0.555...,-65 and 8.

Result: The student, in this case admitted she had not studied this particular lesson and asked if someone else could answer instead. She was encouraged to attempt the question, but refused. Without malice, she was given 0/3 and another student was selected at random from the class list. It should be stressed that the students knew the marking procedure: 0 for a question not attempted; 1 for an attempt in which several fundamental errors are made; 2 for a significant minor error and 3 (the maximum) for a perfect answer or one with only a minor mistake. Naturally, the student's mark is a confidential matter between the teacher and the student, so the Class Presentation mark is only made known to the student upon request after the class.

The second student correctly categorized the four numbers, but gave only one category for each (Rational, Rational, Integral and Natural respectively). When asked if there were any that fell into more than one category, he said that he didn't think so, then reconsidered and added "Integral" to the last number (8). He was awarded 2 and a volunteer changed the answer of "Rational" for the number 0.555... to "Irrational". A further volunteer changed that back to "Rational" and completed the categorizations by adding "Rational" to -65 and "Integral" and "Rational" to 8. Since the procedure is to encourage volunteers, they are given a mark of 3 if their correction is entirely right, otherwise their attempt is accepted with appreciation but they are given no penalty in the form of a mark (which would otherwise be less than the maximum of 3). In this instance, therefore, the first volunteer was given no mark, and the second was awarded 3.

There followed a fairly lively discussion about the categorization of the different number types, a common difficulty being the confusion between repeating decimals, terminating decimals, and non-repeating decimals. This was to some extent clarified, further examples were to appear in the CPs of lessons 9 and 11.

<u>Lesson #17</u>: Questions from Lessons 16, 15, 12, 9 and 3.

<u>Q16</u>

(a) Express $8x^3 - 27$ as a difference of cubes and then factor it.

(b) Factor fully: $x^6 - y^6$

Result: The first student had some difficulty with both parts of this question, writing, for example: $8(x)^3 - 3^3$ and then: $8(x - 3)^3$, for the first. When asked to write the general formula for the difference of cubes he was in some confusion. Subsequently the correct formula was provided, but this did not serve to clarify. A second student was selected, who was able to correctly represent part (a) as: $(2x)^3 - (3)^3$, but then had difficulty in the substitution into the general formula, even with some assistance. A third student was called up and completed the first part correctly and was in the process of correctly answering the third by factoring as: $(x^3 - y^3)(x^3 + y^3)$ and then factoring each part, but was interrupted by the end of the Presentation. The latter student was given 3 and the preceeding ones appropriate lesser marks.

During the later discussion of these two problems, the subtleties of the process were further elaborated and the two seemingly different solutions to part (b) - one following the process used by the last student, and the other achieved by finding the difference of cubes first were both explained and the two answers reconciled. There was a request for another example which was duly given and solved once the students had had a few minutes to attempt it themselves. Further questions of this type appeared in the CP of Lesson #31.

Q15

Factor: (a) $18x^2 + 9x - 2$; (b) $-40x^2 + 52x + 12$

Result: The student answered both parts correctly. There were questions about the procedure for both and some confusion regarding the extraction of the common factor of -2 in part (b) and the breakdown of the middle term. Lesson #15 was to be encountered again in the Presentations of lessons 22 and 29.

Q12

Simplify: $$\frac{3x^{-1} - (2x)^{-1}}{(4x)^{-1} - 4x^{-1}}$$

Result: The first student was inconsistent in her interpretation of the exponents. Upon questioning she corrected the inconsistency, but then had problems proceding from that point. She instead shifted the $3x^{-1}$ to the denominator and expressed it as 3x, doing the same with the other terms. Asked how she got this, she gave an explanation that showed interference with the expression: $\frac{3x^{-1}\ (2x)^{-1}}{(4x)^{-1}\ 4x^{-1}}$. A second student was chosen who correctly found denominators for the two parts, but made a minor algebraic error in the subsequent steps. This error was pointed out by a group of students during the review process. The confusion between the question and its related "multiplication expression", as shown above, was the subject of some debate in this review stage. Further clarification and discussion, on this point, took place after the CPs pertaining to Lesson #23 (which were given at the beginning of Lessons 33 and 45), since Lesson #23 was the day of the second class test and questions of this nature appeared on that test.

<u>Q9</u>

Simplify: (a) $\dfrac{1}{\dfrac{2}{3}}$ (b) $\dfrac{1}{\dfrac{2}{3}}$ (c) $\dfrac{3^{-1} - 2^{-1}}{3^{-1}\ 2^{-1}}$

Result: This question was answered correctly on the first attempt, though there were several murmurings about it amongst the students. During the review came the question as to why the numerator could not be "shifted to the denominator and the exponents made positive". Parallel examples of a simpler nature were taken to exemplify the error of such a process (one such example being $(6^{-1}-3^{-1})/1$. It was shown not to yield the same result as $1/(6x3)$). References were also made to the structural differences between such expressions as: $(2 - x)/2$ and $(2x)/2$, where simplification precedures are significantly different (this being a major stumbling block for many students).

Q3

Demonstrate, by using numerical examples, that the associative property applies to addition and multiplication, but not to subtraction and division.

Result: The first student confused the associative property with the distributive property and was unable to recall the former when this was pointed out. The second student selected successfully gave numerical examples of each. The opportunity was taken to demonstrate that, in certain circumstances the latter two processes would "appear to work" if certain specific values were chosen (for example: 3-3 = 3-3), but this did not amount to a "proof" of the truth of the property, but merely a demonstration of an exception; on the contrary, evidence of any circumstance which could demonstrate that a proposition was false would be sufficient to discredit that proposition.

Lesson #28: Questions from Lessons 27, 25, 20, 14 and 6.

Q27

Solve using Algebra:

(a) The sum of two consecutive, even integers is 30.
Find the larger integer.

(b) A Natural number plus its reciprocal gives 37/6.
Find the reciprocal.

Result: Three students attempted this question; the third being successful. The first student tried $x + (x + 1) = 30$ and then realized that the answer was not integral. When prompted, he observed that he had taken consecutive integers, as opposed to consecutive even integers. He subsequently solved the question, but after setting up the equation for part (b), could not solve it. The second student achieved an erroneous solution to the quadratic equation resulting from the original equation: $x + 1/x = 37/6$ (i.e. $6x^2 - 37x + 6 = 0$). The third student was successful in finding the solutions to the equation and the appropriate answer to the question.

There was almost unanimous agreement in the class that this type of "word problem" was particularly difficult, though the solutions were followed by the class as a whole. The procedure of adhering to the restrictions imposed in the question was talked over, since there was a tendency to "panic and get any equation down that looked reasonable" (to quote one of the students). The point of relevance of mathematics to the real world was discussed as a means of stimulating interest in so called "word problems". The class seemed somewhat sympathetic to the notion that real questions were more akin to word problems than the rather synthetic questions that are generally posed. The author took the opportunity of discussing, in brief, the ideas of Krutetskii and Skemp regarding understanding of "Structure" versus "Detail", and that, in their view, the true mathematician is able to "see the wood and is not deflected by the individual trees". There were a few sparkles in students' eyes, so this deviation was probably not in vain.

Q25

Solve and check your solution:

(a) $1/(x + 2) - 1/(x + 1) = 1/12$

(b) $1/(x - 1) = 4 - 1/(1 - x)$

Result: The first student was unable to cope with the conversion of part (a) into a quadratic equation, but with a couple of hints about the procedure, she managed the solution. However she became bogged down in the algebra of part (b). Another student was called up who was also unable to further the solution. A volunteer then successfully arrived at the answer: $x = 1$, but was unable to reconcile the check which produced: $1/0 = 4 - 1/0$.

After the CP had ended several other students exclaimed that no answer was possible to the second question since "1" was not a possible answer. There was general agreement that the check was indeed a valuable mechanism. A more initiated student also pointed out that, upon careful consideration, one could observe that the original question was impossible, since it could be rewritten: $1/(x-1) = 4 + 1/(x-1)$, which suggested that "some quantity equalled 4 + that same quantity, which doesn't make sense". This was followed up by presenting an equivalent equation: $y = 4 + y$, to which the class agreed there was no possible solution.

<u>Q20</u>

Simplify: $1 + \dfrac{3}{x + 1/x}$

Result: This turned out to be a particularly difficult question for the majority of the students in the class as became evident in the results to the test which was given on the 22nd lesson. The difficulties experienced by the three students who attempted it were a prelude to this. The first student became tangled in an attempt to simplify the denominator (x + 1/x) and, despite an attempt on the teacher's behalf to disentangle him, he gave up. The second student was able to simplify the complex fraction, but could not properly include the "1" within the answer. At this point a third student volunteered, but made some errors in combining the two terms. By this time the CP had been going for over ten minutes and was brought to a close. The question was written out afresh, once some board space had been vacated and the problem was tackled stage by stage. Though the class accepted that they could follow the procedure, they expressed scepticism at their ability to do a similar problem themselves, at which point one was provided and , after a few minutes, explained with the help of the class. Again a few sparkles in eyes around the room indicated that there was more than a superficial understanding and that problems of this nature were well within their capabilities.

Q14

Factor: (a) $-x^2 + x + 6$; (b) $-12x^2y - 2xy + 30y$

Result: The student attempting problem (a) made two fundamental errors: He immediately changed the signs so that the expression read: $x^2 - x - 6$, and having factored into: $(x - 3)(x + 2)$, then proceeded to say that: $x = -2$ or 3. When asked if this problem was an expression or an equation, he replied that it was an expression, but was not deterred in his answer. His approach to the second part was inconsistent with this process: He correctly extracted a common factor, then factored the remaining expression. By this time the CP had come to an end (since he had moved rather slowly on each question), and the moment for review had arrived. Several of the more vocal students explained the errors in part (a) and the time was ripe to underline the difference between the processes of factoring expressions and solving equations via the mechanism of factoring. Another similar pair of questions to those given were posed and the class assisted in the solutions.

Q6

Simplify: $$\left[\frac{(-2x^{-1})^2}{4y^{-2}}\right]^{-2}$$

Result: The question was correctly answered by the first student, who "flipped" numerator and denominator and changed the exponent to positive 2 (one of the techniques used in teaching this concept). During the review there was a request to show the procedure "the other way" (in which the exponent -2 is used to operate independently on both the numerator and the denominator and the process continues accordingly). The request was accomodated and several questions on some subtleties of the procedure were answered.

Lesson 41: Questions from Lessons 40, 37, 21 and 8.

Q40

If $x = \dfrac{-b \text{ +/- } \text{SQRT}(b^2 - 4ac)}{2a}$ Show that $ax^2 + bx + c = 0$

Result: The first student attempted to start by squaring the right side of the first equation, but even after a hint became bogged down. His replacement had obviously applied himself to this proof (given in class 40) and was able not only to produce a clear solution, but could justify the steps he took.

Q37

Sketch, Showing intercepts and vertex to 1 decimal place:
$y = -x^2 + x + 7$

Result: Apart from an error in substitution into the quadratic formula (which became evident to the student since it implied an illogical inconsistency), this question was successfully answered. The student's error and subsequent correction were used to advantage in thc subsequent review session, as an example of the consequence of having a logical, rational approach to mathematics; an antedote to confused frustration.

Q29 (This lesson number was omitted, since the class was used as a period in which a previous final exam was reviewed).

Q21

(a) Simplify: x/2 - 5/x + 3; (b) Solve: x/2 - 5/x + 3 = 0

Result: The bait was laid here for a repeat of the previous CP in which there had been some interference between the distinct concepts of expressions and equations. Upon solving part (a), the student had actually "found a value for x", but upon starting the second question, he looked quizzically at the first answer, uttered an expletive, and quickly erased the "=0" that he had added and all the subsequent steps. He then realized that he had in fact answered the second question already, but continued to repeat his previous answer again.

There were again a few questions about the distinction between the two problems and the different approaches to the solutions.

Q8

Simplify, expressing your answer in positive exponents only:

$$\frac{(3x^{-1})^2 - 2x^{-1}}{(2x^{-3}y)^{-1}}$$

Result: The student fell in the trap of merely shifting the terms from numerator to denominator and vice versa. Asked if this was permitted he looked a little worried but said: "Yes.. I think so!" (It should be stated in passing that the mere fact of the teacher asking a question was not always cause for the student to assume he had made an error; on the contrary, there were many occasions in which the students were asked to justify their steps and it was hoped that the teachers gestures and facial expression did not "give away" an impression of "right or wrong").

In this instance, no further comment was made concerning the erroneous response and, as was often the case, volunteers were encouraged to make any appropriate corrections. One did in fact attempt to correct this answer, but made a further error in dealing with the two integers within the parentheses in the numerator and the denominator (i.e. the student wrote $3x^{-2}$, as opposed to $9x^{-2}$ for the first term). In reviewing this question there was a healthy interest in the procedures and reasoning behind them. A particular student stated a rule thus: "If there's a plus or minus between parts of the top (i.e. numerator), or parts of the bottom, then you can't flip". Such rules were encouraged and the importance of students reaching such rules was emphasized as a crucial step to a clear understanding. The author made some comments concerning the development of individual "Learning Structures" and their value in "personalizing" concepts.

<u>The Review Technique in action: Applied Math 101C</u>

In this class, Class Presentations were given on the following Lesson numbers: 3,5,7,8,9,10,13,14,16,18,21,23, 24,26,28,30,32,33,35,36,38,40,42,43 and 45 (a total of 27 lessons out of a maximum of 45). The four lessons selected as examples for this particular course are Lessons 7, 18, 32 and 40. The Lesson numbers and the related lessons from which the Class Presentations were taken, are summarized in the table below, which is a subset of the full table shown under section 4.1.

LES.#	P	9	7	5	2
7	6	6	5	4	1
18	17	16	13	9	4
32	31	29	22	16	6
40	39	36	28	20	8

It should be pointed out that this course is quite different, in a pedagogical sense, from the Algebra 211 course. In the latter course there exists more of a continuum in terms of the subject matter; on the other hand, the 101C course is divided into four independent sections: Business math (compound interest, annuities etc.); Probability basics; Matrices and Linear programming and Functions review (see the course outlines in Appendix 1). In this sense, the review procedure might be seen as taking on a somewhat different role - not only providing a reinforcement mechanism for structural development within each of the four individual sections, but also providing a crucial element of keeping past knowledge "alive" in the students' minds.

<u>Lesson #7</u>: Questions from Lessons 6, 5, 4, and 1.

<u>Q6</u>

Find the present value of an investment of $500 at the end of each quarter for 3 years at 6% compounded quarterly.

Result: On the initial attempt, the student used the simple interest formula; confusing, at the same time, that formula with a somewhat similar one for simple discount. When it was pointed out that the question dealt with regular payments and compound interest, a further "hybrid" formula appeared and another student was asked to continue the question. He was successful, though he had a little difficulty with the calculator procedure.

The discussion which centred around this question was primarily concerned with the present value idea and its practical meaning. Some students expressed some difficulty with the use of the calculator. Alternative calculator procedures to achieve the solution were discussed and some of this concern was diffused.

<u>Q5</u>

Find the amount of the annuity in Question 6.

Result: This question was answered correctly by the first student. There was no expression of difficulty by the class, they seemed to be much more in tune with the concept of future value than present value. The comparison between these two concepts (present versus future value) had been made in talking over the previous question, so it is likely that some conceptual difficulties had already been eradicated in the discussion of the latter question.

Q4

$5000 is invested at 7% compounded daily on November 7th 1988 until today (February 6th 1989). Find the Amount.

Result: This question hinged around the calculation of the number of days between the two dates given. A chart giving day numbers for each date was available to the students and the student doing this question used an appropriate procedure and completed the calculation. There were several questions during the review concerning the technique of, as it were, "bridging" between two different years, to which a time-line explanation received contented responses.

Q1

$5000 is invested at 7% simple interest on November 7th 1988 until today (February 6th 1989). Find the Amount.

Result: The student doing this question neglected the fact that these dates are in different years, but applied the right formula. (A case of recollection of detail, but neglect of structure, or Instrumental rather than Relational

Understanding ?). When asked how he got the number of days, he merely showed the subtraction procedure.

The explanation of the previous answer paved the way for the correct response to this one during the review.

Lesson #18: Questions from Lessons 16, 13 and 9.

Q17 (The class attended a workshop on math applications on Lesson #17, so there was no CP from that day).

Q16

(a) List the events for the sample space for two tosses of a coin.

(b) Find the number of ways for 115 persons to select their seats in a cinema with 200 seats. Give your answer in factorial form.

Result: The first attempt was inadequate, since the student had no clear idea of the meaning of "sample space" and was obviously guessing at the permutation question. The second student answered part (a) correctly, but interpreted (b) as a combination, though he gave the correct answer in terms of a permutation. The distinction between the two concepts was again explained during the subsequent review. It was clear from the pupils of the pupils that, though they acknowledged understanding, that in fact the distinction was not entirely clear to all of the class. Since no more questions were forthcoming, it was decided to await further examples in later classes.

Q13

Sketch a Venn Diagram showing the set:
A intersection (B union C')

Result: This question was answered well (the class seemed particularly confident with the visual nature of Venn diagrams in general). Some questions concerning the differences between intersections and unions and their visual interpretations arose from a handful of students as a result of this question.

Q9

Draw up the first two payment lines of the Amortization table for a loan of $1000 at 8% compounded monthly for 5 years.

Result: The first student mistook the loan for a future, as opposed to a present value, though she used the correct procedure in developing the table. Inevitably, instead of reaching a final Principal of $0, she had another figure.

She was left to continue on the wrong track since she felt confident that the $1000 was a future amount. During the review, the fact was stressed that any loan represents a present value and the question was reworked. There were questions concerning the relationship between the entries, but one could sense a certain satisfaction amongst the students with "knowing what to do" as opposed to understanding the underlying reasons for the necessary calculations.

Q4 (The 4th lesson was a workshop on asking questions in class given by the author and a John Abbott Counsellor, hence no presentation could be given pertaining to this class).

Lesson #32: Questions were from Lessons 31, 29, 22 and 6.

Q31

Sketch: 8x + 2y >= 16 where x,y >= 0

Result: The line 8x + 2y = 16 was correctly sketched, but the region was initially shown to be under, as opposed to over, the line. The student was clearly in some confusion about this, since she had not yet encountered an "open" region. At one point she actually said (concerning the region below the line): "I know this is wrong, but it (the region) has to be here!" She was encouraged to check with appropriate points, but could not convince herself that her answer should have been the open region above the line.

An example of this nature had not been given in the lecture on this this topic and it was interesting to see the class reaction to the idea of an open solution region. With further examples of the same type there was a realization of the extension to the general logic of closed regions. The lecture to follow concerned this topic so this served as a particularly useful introduction.

Q29

If 4 items cost a total of $124 to produce and 6 items cost $130 to produce, and the selling price is $5 per item, sketch the Cost and Revenue functions (assuming that they are linear) and find the Break Even Point.

Result: This question was answered with punctilious precision by the student called. There were questions about the derivation of the Cost function from the information given and one concerning the positioning of two different functions on the graph and the meaning of the vertical axis in this context.

Q22

If I select a group of 7 at random from a class of 15, find the probability that the group contains two particular students. Find the precise answer (i.e. no decimal approximation).

Result: The first attempt at this problem resulted in some considerable confusion to which some constructive suggestions only added. In this case, since the student had taken a good deal of time trying various techniques, a volunteer was taken form the class. She was able to produce an appropriate procedure for the solution, but had difficulties processing the answer and simplifying it. (The answer simplifies to: $7x8^2/(11x13x15)$, requiring a good deal of simplification). The CP had ended before she could complete the answer.

There was general agreement that the origin of the answer in combination format posed no problem, but clarification was needed on the mechanics of simplifying the factorials, which was duly provided.

Q16 (This question was omitted form the CP since the other four questions required time-consuming responses).

Q6

How long would it take to increase an investment by 25% if the investment were at:
(a) 10% compounded monthly;
(b) 10% simple interest?

Result: The response to this question was interesting. The students had seen questions of the first type before, but the easier second type had not been presented previously. The first student was successful with the first, but drew a blank in the second. Some suggestions did not help the student who might have been categorized as and "Instrumental Learner" - one who relied on absorbing formulas and attempting to fit the question into the formula category. The next student called up was definitively in the "Relational" category. She did not come prepared to write out the answer, but had been watching carefully as the first attempt was in progress. She first wrote out the Amount formula for Simple Interest and stared at it for a few moments ($A = P(1 + rt)$), then erased it and replaced it by: $I = Prt$ and proceeded to establish a value for P, extrapolate the value of I, enter the value of r and reach the solution. When asked if the first formula she had used would yield the same result, she proceeded to solve it accordingly.

In the review session following this CP there was an expression of some surprise as to "how easy it was" in reference to the second part, and the opportunity was taken to encourage the students to set down the facts and "have a go", since the question would not have been asked if it was outside their capacities.

<u>Lesson 40</u>: Questions from Lessons 39, 36, 20 and 8.

<u>Q39</u>

Solve the given system by Gauss-Jordan elimination and give 2 particular solutions:

$$\begin{aligned} x + y + z &= 6 \\ -x - y - z &= -6 \\ 3x + 2y + z &= 18 \end{aligned}$$

Result: The matrix solution to the problem was reached by the student and, with a modest prompt, the general solution appeared, but the two particular solutions were beyond the student. Since the CP was about to be brought to a close a volunteer was taken who quickly produced two elementary particular solutions.

There were a few students interested in the physical significance of both the general and particular solutions and several requests to re-explain the derivation of the particular solutions. As occasionally happened during these sessions, there were several pupils whose pupils lit up in the: "Oh, that's what this is all about" expressions.

<u>Q36</u>

Solve the given system by Gauss-Jordan Elimination:

$$x + 5y = 7$$
$$x + 7y = 15$$

Result: Apart from a minor error in dealing with the fractions, which was caught en-route, this was well answered, and the student successfully verified the answers in the two equations upon request. There were no questions.

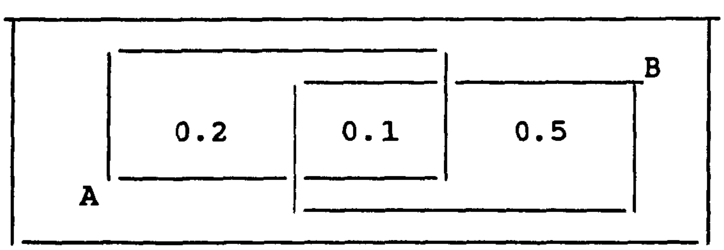
<u>Q28</u> (To optimize use of time, this question was omitted).

<u>Q20</u>

Given the Probability Venn Diagram shown find:

(a) P(A intersection B); (b) P(A/B); (c) P(B/A).

Also determine if A and B are independent.



Result: The first three parts were done with panache, but the student was unable to recall the meaning of "independent" and could only suggest that they were not independent since the two sets were linked. Another student was called and wrote a somewhat confused definition of independence; a third student showed that they were Mutually Exclusive claiming that this was the same as Independence. Since the time had elapsed for the CP, the question was thrown open to the class and a very useful discussion ensued on the differences between the various categories. It was no surprise to the author that this fairly subtle topic had not been at all well absorbed in the "first sitting".

Q8

Joe bought a house and gave a 20% downpayment. He took a mortgage for 25 years at 12% compounded monthly. If his monthly payments were $1023, find the original purchase price of the house.

Result: A question concerning not only the Present value, but including the complication of a downpayment had been given two classes previously. However, the first attempt yielded the correct figure for the Present value of the mortgage, but not the purchase price. With a little encouragement, the student was able to reach the answer. Apart from a couple of questions on the logic of dividing the Present value by 80%, there seemed to be a good understanding of the procedure and logic.

General Comments and Observations of the SSR Procedure

These samples of four lessons of Class Presentations are, in the opinion of the author, a realistic representation of a fair cross-section of the entire collection of class presentations. As seen in these examples, some questions were answered correctly on the first attempt and others took at least three attempts by different students before the true response emerged, and a few of the conceptually more difficult questions were not successfully answered, even after several attempts, within the time constraints of the CPs. Of the errors students made, the majority could be considered "typical", and both author and students agreed that the refining process of observing flaws and correcting them was a salutory process. Several of the more confident students made such comments in class and still others in the private of a one-to-one encounters. There was definitely a sense of nervousness regarding the procedure, and an evident sigh of relief from those not selected for the preliminary CP, but, in the main, there was a feeling that, despite the extra pressure, the experience was worthwhile. Some even went so far as to say that they appreciated the extra pressure, since it gave them a greater drive to learn. (Further comments on the CPs can be seen in Section 4.2, in the Questionnaire Results).

From an organizational point of view, the procedure presents quite a challenge to the teacher, since it is far removed from the procedure of standard classroom practice:

Keeping track of up to five questions simultaneously requires a certain alacrity, as does the keeping of the records of who has taken over a particular question, who has volunteered for another etc.. However, one quickly adapts to the procedure and the advantages of a fast-moving, more vibrant classroom environment become evident. During the classes in which there is neither a CP, nor another diversion such as a quiz or an assignment question from a student, it is evident that the time passes more slowly and the concentration of the students wanes. After the first few CPs, there develops a greater sense of unity in the class: students realize that everybody makes mistakes at some point and this further diffuses the pressure of being up at the board in the company of four other students. It is also very interesting to note the loosening-up of question-asking, since past errors or sources of confusion reappear and there is regular reclarification of subtle concepts which can often lead to interference. It was many a time that a student signalled that "Oh, now I get it" expression at some stage of the review process.

4.2 QUESTIONNAIRES

Amalgamation of Questionnaire results:

The responses of the two groups to the questionnaire were closely allied and they were consequently amalgamated for the purposes of presentation and analysis. The table below shows the mean responses by question for both groups. The last column gives the difference of the two means (101C mean, less 211 mean). The mean of the means is noted at the base of the table. For the purposes of this analysis, the standard Likert scale numbers were used as letter-equivalents (i.e. response A = 1, B = 2... E = 5).

Mean Responses

Ques #	211	101C	Diff	Ques #	211	101C	Diff
1	4.3	4.3	0.0	21	4.0	4.0	0.0
2	3.7	3.4	-0.3	22	4.1	4.2	0.1
3	3.0	3.8	0.8	23	3.1	3.3	0.2
4	2.7	2.8	0.1	24	2.8	2.7	-0.1
5	2.4	2.0	-0.4	25	3.6	2.9	-0.7
6	3.4	3.9	0.5	26	3.9	3.9	0.0
7	2.2	2.6	0.4	27	2.0	1.9	-0.1
8	3.8	4.1	0.3	28	4.5	4.4	-0.1
9	3.7	3.8	0.1	29	3.9	3.5	-0.4
10	3.2	3.1	-0.1	30	3.7	3.8	0.1
11	3.9	3.8	-0.1	31	2.8	2.3	-0.5
12	2.7	2.8	0.1	32	3.5	3.5	0.0
13	2.3	2.6	0.3	33	3.7	3.1	-0.6
14	2.4	2.8	0.4	34	3.1	3.3	0.2
15	3.2	2.7	-0.5	35	3.0	2.9	-0.1
16	4.1	4.1	0.0	36	3.3	2.8	-0.5
17	3.4	3.2	-0.2	37	4.0	3.5	-0.5
18	4.1	3.6	-0.5	38	3.8	3.6	-0.2
19	2.4	2.8	0.4	39	4.0	3.9	-0.1
20	3.8	3.9	0.1	40	3.8	3.7	-0.1
				41	3.7	3.8	0.1

[mean of differences = 0.046 or 1.2% on the 5-point scale]

It is worth reiterating at this point that, even though the two groups of students are one year apart in their studies and mathematical maturity they exhibit surprisingly similar responses to the questions posed. Moreover, evidence from the responses to part III of the questionnaire suggests that they are raising the same points regarding the SSR and CP process as did the Calculus II students (in the pilot Questionnaire) who are a further year above the 101C students in their studies. (The sequence of math courses is: 211-Algebra, 311-Trigonometry, 101-Applied Math, 103-Calculus I and 203-Calculus II).

Response consistency check:

As mentioned in section 3.2, there was a consistency-check built into the questionnaire in two pairs of questions (numbers 17 and 40 along with 15 and 19). Each student response was checked to see if there was a significant difference in responses thus: In the similar pair of questions (17 and 40), a variation of more than one point on the scale was considered significant. In the pair with opposite implications (15 and 19), a variation of more than one point from the opposite response was taken to be significant (i.e. a response of B for one was expected to be followed by a response of C,D or E for the other etc.). The data on the consistency check are as follows:

Similar Questions:

Question 17: Any anxiety I had about doing the CPs was Balanced by the positive effects on my learning in this course.

Question 40: As nerve-wracking 's the CP experience may have been, it was well worth it.

Number of identical responses: 17

Number of responses varying by one unit: 19

Number of responses varying by more than one unit: 3

Opposite Questions

Question 15: I did not mind my peers watching me do my CP.

Question 19: I hated the experience of other students watching me do a CP.

Number of identical (opposite) responses: 19

Number of responses varying by one unit: 16

Number of responses varying by more than one unit: 4

To summarize, there were 7 responses which could be considered inconsistent, among the 78 responses to the two pairs of questions. These responses were made by 7 students, that is to say each of them made an inconsistent response to only one of the given pair of questions. Also, 35 of the 78 responses (45%) varied from the expected by one unit.

It should be emphasized that there were subleties of phraseology in the questions (e.g. the term "anxiety" versus "nerve-wracking" in questions 17 and 40 and the semantic emphasis of "I did not mind" versus "I hated" in the other pair of questions). Also, there was a potential confusion in answering a question posed in the negative (e.g. disagreeing with "I do not mind my peers watching me" implies that the person "does mind").

After careful consideration of the above data and comments, and consultations with two questionnaire specialists, it was decided to retain all the original data intact, as opposed to either rejecting that data considered inconsistent, or eliminating all the responses of any student from the data bank.

Format and Layout of Response Data:

For the sake of clarity in the data printout (i.e. responses A to E), the modal response letter is printed in bold. The parenthesized letter corresponds to the response expected by the author in a pre-test run he took a week before the questionnaires were given to the students. In cases where there were more than 60% of the students responding in either extreme (i.e. 60% within the categories A and B on the "Disagree" side or D and E on the agree side), this question is highlighted in **bold** print. The figure of 60% has been chosen because: (a) it is an indication of a significantly skewed response and (b) there are a modest number of responses in this category and are therefore considered worthy of note.

As mentioned in the previous chapter, the questionnaire responses were sorted into eight essential categories: Review System, Class Presentations, Understanding or

Learning, Feelings, Exams or Tests, Marks, Time, and Miscellaneous. Some questions from Section I appear under more than one of the categories, for example Question 8: "Doing a CP was a good way for me to learn" appears under the two headings: "Class Presentations" and "Understanding or Learning". Questions from Section II tended to be more specific, so there was a one-to-one relationship (question to category). The responses to the nine questions in Section III, (five of which were open-ended) were reported under only one category, though some comments transcended that category. This was done to avoid excessive verbosity and repetition of lengthy responses.

Discussion and Interpretation of Data

After each category of data there follows a discussion and analysis. In referring to student responses, ordered quintuplets are used to give percentage responses (which are rounded to the nearest integer for convenience). For example: (0,8,40,47,5) represents 0% of the students responded "Strongly Disagree" (response A), 8% "Agree" (response B)... and 5% "Strongly Agree" (response E). The questionnaire section terminates with a summary of the major points emanating from the results.

Questions pertaining to: REVIEW SYSTEM

Question	Response Count/Percentage Disagree <------> Agree				
1. The review system was a good way of using class time.	A	B	C	**D**	(E)
	1	0	2	20	17
	2.5	0.0	5.0	50.0	42.5
11. The 9/7/5/2 system helped me understand the material better than by the ways I have been taught in other maths courses	A	B	C	D	**(E)**
	1	2	10	11	15
	2.6	5.1	25.6	28.2	38.5
12. I liked the review system so much in this course, that I am using it in other courses too.	A	B	**(C)**	D	E
	5	10	14	6	4
	12.8	25.6	35.9	15.4	10.3
16. I would recommend this course over more traditionally-taught maths courses.	A	B	C	D	**(E)**
	1	2	5	11	20
	2.6	5.1	12.8	28.2	51.3
22. I will be less nervous for the final exam because of the way this course prepared me.	A	B	C	**(D)**	E
	0	2	3	21	14
	0.0	5.0	7.5	52.5	35.0
23. The SSR system used in this course has "turned me on" to maths more.	A	B	C	**(D)**	E
	3	5	14	17	1
	7.5	12.5	35.0	42.5	2.5
26. The SSR system allowed me to understand how to solve problems rather than just memorizing the techniques.	A	B	C	**D**	(E)
	0	3	8	19	10
	0.0	7.5	20.0	47.5	25.0
27. There was too much time spent on review rather than figuring out new problems.	A	**(B)**	C	D	E
	10	24	3	3	0
	25.0	60.0	7.5	7.5	0.0
28. The review method used in this course sure beats cramming for the final exam.	A	B	C	D	**(E)**
	0	0	3	15	22
	0.0	0.0	7.5	37.5	55.0

Note: Bold letter indicates modal response
Parenthesized letter indicates expected response

	Disagree		<------>		Agree
29. **I feel more prepared for the final exam in this course than in most other courses I have taken.**	A	B	C	(D)	E
	1	4	7	18	9
	2.6	10.3	17.9	46.2	23.1
30. **If I were to take other maths courses, I would use the review system on my own.**	A	B	C	(D)	E
	1	3	10	17	9
	2.5	7.5	25.0	42.5	22.5
32. The SSR system really helped me understand how the concepts were inter-related.	A	B	C	(D)	E
	0	3	16	19	2
	0.0	7.5	40.0	47.5	5.0
34. The SSR system was one of the best learning experiences I have ever had.	A	B	C	(D)	E
	4	5	11	14	5
	10.3	12.8	28.2	35.9	12.8
35. Having both assignments and reviews to do for homework was heavy.	A	B	C	(D)	E
	2	15	7	15	1
	5.0	37.5	17.5	37.5	2.5
37. **I developed increased confidence in maths through the methods used in this course.**	A	B	C	(D)	E
	0	4	6	21	8
	0.0	10.3	15.4	53.8	20.5
38. **As time went on, I was better able to confront new maths problems because of the techniques used for learning in this course.**	A	B	C	(D)	E
	0	3	11	22	4
	0.0	7.5	27.5	55.0	10.0
39. **I would definitely choose a maths course taught by the SSR method over another maths course.**	A	B	C	D	(E)
	0	1	11	18	10
	0.0	2.5	27.5	45.0	25.0
41. If I were to teach maths, I would definitely use the SSR system.	A	B	C	(D)	E
	1	1	15	13	10
	2.5	2.5	37.5	32.5	25.0

Note: Bold letter indicates modal response
Parenthesized letter indicates expected response

(subsequent questions from section II)

49. When did you do most of your reviewing?

A. right after class
B. right before class
C. the evening before class
D. other

	#	%
A.	1	2.5
B.	14	35.0
(C)	21	52.5
D.	4	10.0

50. On the average, how many lessons did you usually review?

A. one
B. two
C. three
D. four
E. five

	#	%
A.	2	5.0
B.	12	30.0
(C)	16	40.0
D.	3	7.5
E.	7	17.5

Note: Bold letter indicates modal response
Parenthesized letter indicates expected response

(Subsequent questions, overleaf, are from Section III)

5. What do you think is the "method behind the madness" of the P/9/7/5/2 system?

- I think that it helps a student to review his work much better because even if they do not want to do homework, they literally have to. I like this method. It helped me because if I did not have to do it, I would not have done it.

- I think it was a way of teaching students to review a little at a time instead of cramming for a test. Also to show students their success.

- To keep the student up to date with his/her previous, assignments, lectures, etc...

- From Mr. Salusbury's point of view, I imagine the method was to bring back work and review it. But to me it added to the confusion of the work itself.

- To help us review everything that we had done over the year.

- It is decent, but I rarely ever study so it is not as beneficial to me.

- To review everything a little at a time - To enforce need to understand concepts and see how easy it is to mix concepts up with others.

- To review randomly through past lectures.

- To narrow down the day to day work, yet still get a good - even review.

- To keep us reviewing the work constantly, to help us remember previous lessons and know our work better to help study for exam.

- I believe it helps the students to recall and re-enter the knowledge learned from previous classes in order to help the students with the exam.

- I think that we use this system so that we review the present material, but at the same time go back and review that material that has been covered at the beginning of the course.

- It is basically used to encourage students to review past work.

- To review past work easily without too much work.

- I think it is to make it easier for the student to study, when reviewing for quizzes or CP. It helps in knowing what he is talking about.

- It helps you to review all the work you have done in the beginning of the term which you may have already forgotten.

- Strongly agree with it, find it beneficial.

- The method ensures everything is covered at least twice (once in lecture, once in review).

- Reviewing.

- It gives you a widespread of material to cover to keep it fresh in your mind.

- A student will not learn new concepts without forgetting the old.

- It is simply a system designed to encourage students to constantly review material to better its understanding.

- It ensures a way to review the entire course in a structural format.

- There could not be a better method. I like it a lot because we are always looking back at what we have learned since the beginning of this class.

- It is a great idea and hopefully you will continue using it in the future.

- You keep on reviewing the entire 15 weeks of class, which keeps your mind thinking about past events in the course.

- Considering that I loathe the CP's, I am not interested in the "method behind the madness". I assume it is to assure that every topic gets reviewed.

- To give a general review of past material to make sure all previous material is reviewed.

- I think it is to help the student review past material in an orderly way, including all the material covered in the course. It is to prevent a student from cramming for exams and experience the ability to retain the material learned.

- I think the method is that you always end up doing work that has been taught the previous lesson as well as further back in the beginning of the year. Does not give you the chance to forget too much.

- By using the system, the student is reviewing not only previous or mid-semester material, but the material from the beginning of the course as well.

- I think this is a very good way to review for the presentation the next day. It gets you ready by already knowing what the CP's are going to be about.

- The bottom line is that it is a great review system.

- It was very helpful because eventually, I realized that everything was being reviewed and by reviewing I learned my mystakes.

- This method is used because it ensures the students that they will not completely forget material learnt early in the year. It keeps information fresh on the mind, so that cramming for the exam can be avoided, to a certain extent.

7. The thing I liked the best about the SSR system was (give your reasons):

- I feel that the SSR system was good in all aspects.
- I feel much less worried about the final exam because I have spent all semester reviewing for it.
- It gave me a chance to see problems I had troubles with in the past.
- Being able to go over past lessons before class.
- It was inventive and kept data fresh in my memory.
- Learned from others. Reviewed work throughout semester.
- You did not forget the things you learned in the beginning of the year.
- I knew exactly what I was expected to review.
- I understand the different material better, I do not get previous sections confused with others anymore and I fully understand each, whereas before I only understood bits and pieces.
- The way it made me remember old formulae and the way it helped me get more marks.
- You know where to look when you are looking for something (organization).
- I liked this system not only because it made it easier to study, but it kept me very organized throughout the course.
- It made me dig back into past work and see where I went wrong, etc.. It made me remember old techniques which I could have forgotten.
- I did not like the SSR system.
- It takes maybe 10 minutes to be done.
- It was easier for me to learn, it was kind of a jolt to keep me going.
- It really helped you review your work in the best way because if you do one for every class, you end up reviewing for all past classes.

- It was an effective way to review.
- Got a chance to review everything at least once.
- I knew what kind of questions were going to be asked on the test.
- The way I could remember things.
- The 9/7/5/2 system. It gave me something to study, otherwise, I would probably just read over everything and learn nothing.
- Forced me to study notes of previous classes. I learned a lot from watching others do CP's. I especially liked when professor reviewed CP questions.
- It prepares us well for the final exam.
- I learned new ways to study - understand the main points of a reading.
- It made me study in a much better way. Also it played a very big role in passing my exams.
- The thing I like the best was that you were always reviewing different subjects. This made it possible for us not to forget any particular subjects.
- I do not know what the SSR system is but I learned a lot in this class, if that says anything.
- Was I was always forced to review past material, and it was harder to forget a past concept.
- I liked the way that all material was covered in review. The problems I did not understand at first became simple in the end.
- The fact that we were able to review as well as learn new things.
- We never forget what we learned in the beginning of the course; helps prepare for the final exam.
- Learned new ways to study, organize time, and try to stop procrastination.

- (i) helped to understand problems thoroughly (ii) helped to understand and memorize more material than ever before (iii) helped you to realize <u>all</u> of your mistakes and questions.
- (1) helped understand the previous sessions more thoroughly (2) helps me to correct my answers.
- (1) helped understand the previous sessions (2) helped to correct past mistakes in problems.
- It ensured me that I would not forget material. Also if I did not understand one part of the course, when it came to the test, other questions would appear on it which I would understand, therefore the possibility of failing is less.
- Not much to say it is just part of the course which we had to review with.

8. The thing I liked the least about the SSR system was (give your reasons):

- I did not find anything wrong with the SSR system.
- It was time consuming.
- Having to go up in front of the class and do it.
- It was sometimes boring and monotonous.
- Sometimes it went a little slow, especially when student did not know what he/she was doing (how to solve problem at board).
- It made me work (I am lazy).
- I like to look over previous days, and do problems I did not like to write down one question from each day even if I had no difficulties with it.
- My understanding of everything is much clearer.
- If I did not know how to perform the CPs he would kick me out instead of going over the problem with me. I know I was supposed to know it but sometimes I could not help but not remember.
- Rushing before class to prepare the CP review which I did and was not collected by teacher.
- I can not think of anything I did not like about SSR.
- Nothing.
- The least I liked about it, is that it would put the student on the spot and even if the student knew how to do the problem, nervous tension may set in and the student may do the problem wrong thus losing marks (not gaining any).
- You get tired to review the same thing because sometimes it was day 30 for the whole week.
- The fact that he never took any of the CP's in.
- Going up to the board to do the problems.
- In a fit of frustration, teacher decided to mark out (as written) of 12 when students were lax about doing reviews at home.

- Being called up for a CP - however, since I took the course, I should follow through with it, even if I do not like doing work in front of the class.

- Being corrected by peers - nervous - more time is needed to do longer problems - lessen pressure to finish within time specified (give more time).

- Sometimes the CP's would go on for too long.

- Nothing.

- Too much time spent after class on it - too much pressure.

- When the teacher would put up 4 different problems and some of them were really easy and some were extra hard. It was not fair for the person to do the hard problem and besides you, you got someone doing a simple little problem.

- <u>CLASS PRESENTATIONS</u>.

- The thing I liked the least was that when I had a problem with a new concept, it was hard to choose where the time should be spent. (on reviewing or doing more exercises involving the new concept.

- It was a bit time consuming in relation to the other courses.

- No complaints.

- There was a lot of material to prepare for each test, rather than being tested on what we have learned since the last one.

- Some lectures were a <u>bit</u> boring but that is all. It was a learning experience.

- NOTHING!

- Everything was beneficial to me.

- Well obviously there was three times more material on the last test to study than on the first. It is hard to keep a grip on such a lot of material. But the final is like that so I guess this way is better than just forgetting about material.

- The time consuming 9/7/5/2 thing.

9. Any final positive comments, problems and/or frustrations with respect to the nature of the way this course was taught:

- (only one) I feel that D. Salusbury is a fine teacher and his methods of teaching helped me greatly in understanding and also, he made me feel comfortable in front of the class.

- I find that Mr. Salusbury explains extremely well and if I had to take another math course it would be his; I feel he knows how to talk to people.

- Using it as a review procedure for yourself was great, but doing a CP in front of the class was not.

- Sir Dave makes some pretty awkward jokes but all in all it is pretty decent.

- Maybe more challenging questions or problems could have been included.

- Never dull, always have to think, should not teach this course before ten A.M.

- 1) This is the first time I ever took interest in/or understood math. 2) I felt pressured to keep up to date and prepare for the class ahead. 3) I felt I was neglecting other homework because I have so little time at the end of the day and there was a lot of math work to do.

- I really like the way the class was taught. I enjoyed coming to class and I feel I like math a lot more. I will not be afraid to ask questions anymore when I do not understand, and I wish more teachers, especially in math, would teach their classes as this one was taught.

- No, I believe the way the class was run made me understand a whole lot more than what a normal math class would.

- J.G.S. (Jolly Good Show) Too much on tests last and 2nd to last. Should have bonus. Question on every test, good thing we did not have to memorize the poem.

- I felt that this was the first math course where I stayed on top of my work, and kept to date with all my work. I think this is the first math course that I took that did not frustrate me.

- I feel I did enough work, though in the class tests I feel my ability was not shown. I did poorly on the tests, but was able to complete successfully all my CPs. I feel somewhat anxious to go in to the final, I believe because somehow I could not relax and enjoy math, but always worried about my performance.

- I just wish that quizzes instead of CP's had been given.

- It was a great course, the way he taught it and delivered the CP's and SSR, made me work really hard! Thanks Sir David.

- I think every math teacher should use the SSR system.

- I truly enjoyed this course and I am recommending it to my friends.

- (+) 1. Mandatory attendance, excellent policy; motivation. 2. Cheerful disposition of teacher, very welcome (and rare). 3. Good coverage of material in time provided. (-) 1. Some lecturing went by too fast (eg. graphing, parabolas, functions) 2. Why can we not sharpen pencils with pencil sharpener?

- I thought that the course was fun but I did not do enough CP, and did not have a lot of time to do the test.

- I found the tests in class to be <u>more</u> difficult than what we discussed in class for the most part.

- A better textbook (one that actually makes sense) would be nice but it was fun to blow good money on it was it not?

- I think it is a good way to teach this course because one is constantly being given the same material and eventually it sinks in. I have taken this course once before and although the material is the same, it was more easy to grasp (mainly because of this system).

- I found that the overall way this course was taught to be very good. I would take other math courses taught in the same method. I also thought the bonus quiz idea was very good.

- It was very well organized and it could not have been taught by a better teacher than Mr. Salusbury.

- No, but I would like to mention that this course helped me a great deal in preparing for the final and also for the future. Good work chap.

- Yes - on the tests, not enough time was given. The tests were more like 2-2.5 tests!! - Not enough time spent on reviewing - Not enough tests (suggest 4 - 5)

- I generally thought this course was well taught. One thing the problems on the test are always harder than the ones the teacher explained on the board.

- I hate class presentation, I thought everything else was terrific. Mr. Salusbury is funny and very entertaining. His lectures were always very clear and the material and organizational techniques (2A, 2B) were very easy to understand.

- The problem with the CP is that they add an extra pressure that is not needed. It is hard for an instructor to imagine the pressure of being in front of 20 people doing something (you are not secure with), it is different than being in front of the 20 and having a thorough knowledge of the subject. I think the class CP prevents some people from taking the class to begin with. (I was warned about them by other students).

- I think that this course has helped me to learn math and present math in a very simple way. If I knew that a teacher would use such a method of teaching I would definitely take the course.

- None. I felt the course was presented in a very well organized and thought out manner

- The CP's were nerve-wrecking and I dreaded hearing my name, on the other hand, I learned from my and others' mistakes. I think more time should have been put towards going over the questions. The bonus quiz after a test was a good idea, gave me hope.

- I think that this course gave us a good way to organize our work with the index. It was <u>very</u> organized! I thought Mr. Salusbury went over work until <u>everyone</u> understood.

- I think this is how all math courses should be taught. It is too bad all teachers can not teach like this.

- No, David Salusbury was a great teacher, one of the best I ever had, and if I can I will try and get him as a teacher again. His methods are quite refreshing and helpful and I will try and use them for other subjects.

- No, David Salusbury was a great teacher, understanding and also caring to the success of all his students.

- I think that this course was taught in an effective way. The encouragement of asking questions which were emphasized early on in the course, helped. I did not like the fact that very few part marks are allotted on tests. Even if the answer is not correct, marks should be allotted for effort.

- The jokes and enlightenment were much enjoyed.

Review System: Discussion and Interpretation

There appears to be a strong response to the first question: "the review system was a good way of using class time" (3,0,5,50,42). To a slightly lesser extent in Q11 the students claimed that the review system helped them "understand the material better" than the ways they had been taught in other maths courses (3,5,26,28,39). 73% of the students also agreed, in Q26, that the SSR system allowed them to "understand how to solve problems rather than just memorizing techniques" (0,7,20,48,25) and Q28 and 29 respectively, show a strong preference for the review technique as opposed to cramming for the fina exam (0,0,7,38,55) and general preparedness for the final (3,10,18,46,23).

When asked whether they "developed increased confidence in maths through the methods used in this course", there was once again a strong bias on the "agree" side in the responses: (0,10,15,54,21). Also, when asked if they felt "better able to confront new maths problems because of the techniques used in this course", there was another response weighted to the right: (0,8,27,55,10). A lesser, though significantly positive response came in Q32 to the statement: "the SSR system really helped me understand how the concepts are inter-related": (0,8,40,47,5). Q39 also shows a preference for a course "taught by the SSR method over another maths course": (0,3,27,45,25) and slightly fewer students agreed (in Q30) that they would use the review system on their own in other maths courses: (3,7,25,43,22).

Responses were more neutral with respect to using the review system in other courses - Q12: (13,26,36,15,10), "turning students on" to mathematics - Q23: (8,12,35,43,3) and the excess workload of "having both assignments and reviews to do for homework" - Q35: (5,38,17,38,2).

In Section II, Q49 indicates that the majority of the reviewing was done on the evening before class or right before class (53% and 35% respectively) and question 50 indicates that the number of lessons reviewed was, on average, below the five recommended implicitly by the P/9/7/5/2 system (the mean and modal response being 3.0).

The Section III responses show a wide breadth of attitudes. Question 5 ("What do you think is the "method behind the madness" of the P/9/7/5/2 system?") elicited a range of thirty-five responses from the negative: "From Mr. Salusbury's point of view, I imagine the method was to bring back work and review it. But to me it added to the confusion of the work itself." To the neutral: "It is decent, but I rarely ever study so it is not as beneficial to me." To the more positive: "It was very helpful because eventually, I realized that everything was being reviewed and by reviewing I learned my mistakes." And the practical: "To keep us reviewing the work constantly, to help us remember previous lessons and know our work better to help study for exam."

The majority of the responses is favourable, though students seem to have a "marks mentality" and in consequence, their perception of strengthening structural relationships between past and present material is limited. In a sense, they perceive the more functional practicalities such as preventing or avoiding cramming for the final exam and "to narrow down the day to day work, yet still get a good - even review."

Question 7 stated: "The thing I liked the best about the SSR system was (give your reasons)." Perhaps because the question was loaded in the positive, so were the answers. Again we saw a range of responses from the practical to the more considered:

- "The way it made me remember old formulae and the way it helped me get more marks."

- "I don't know what the SSR system is but I learned a lot in this class, if that says anything."

- "I knew exactly what I was supposed to review."

- "1. Helped to understand problems thoroughly.
  2. Helped to understand and memorize more material than ever before.
  3. Helped you realize <u>all</u> of your mistakes and questions."

- "Learned from others. Reviewed work throughout the semester."

- "The 9/7/5/2/ system. It gave me something to study, otherwise, I would probably just read over everything and learn nothing."

- "Forced me to study notes from previous classes. I learned a lot fom watching others do CP's. I especially liked when professor reviewed CP questions."

- "It was easier for me to learn, it was kind of a jolt to keep me going."

Question 8 attempted to reset the balance by stating the opposite of Question 7: "The thing I liked least about the SSR system was (give your reasons)."

Of thirty-four responses, eleven were more positive in nature (e.g. "It made me work [I am lazy]", "I cannot think of anything I did not like about SSR", "**NOTHING**", "Some lectures were a bit boring but that is all. It was a learning experience"). There were six comments referring to the pressure and nervousness instilled by the Class Presentations (e.g. "Having to go up in front of the class and do it"). A further six responses suggested that the CP sessions were boring or time consuming ("It was sometimes boring and monotonous"; "Sometimes it went a little slow, especially when a student did not know what he/she was doing"; "You get tired to review the same thing because sometimes it was day 30 for a whole week"; "sometimes the CPs would go on for too long").

Question 9 asked: "Any final positive comments, problems and/or frustrations with respect to the nature of the way this course was taught?" The responses naturally showed individual biases, but they were overwhelmingly positive and there were some significant points made regarding the success of the review system. A few selected quotations follow:

- "Using it as a review procedure for yourself was great."
- "This is the first time I ever took interest in/or understood math."

- "...the way the class was run made me understand a whole lot more than what a normal math class would."

- "(only one) I feel that D. Salusbury is a fine teacher and his methods of teaching helped me greatly in understanding and also, he made me feel comfortable in front of the class."

- "I think this is how all math courses should be taught."

Questions pertaining to: CLASS PRESENTATIONS

Question	Response Count/Percentage Disagree		<------>		Agree
2. **The amount of class time allotted to the CPs was just right.**	A 1 2.5	B 6 15.0	C 8 20.0	**(D)** 19 47.5	E 6 15.0
3. The CPs made me nervous.	A 3 7.5	B 11 27.5	C 3 7.5	**(D)** 13 32.5	E 10 25.0
4. It would have been better to have written work (e.g. quizzes) than CPs.	(A) 9 22.5	**B** 10 25.0	**C** 10 25.0	D 4 10.0	E 7 17.5
5. **Watching CPs was boring.**	**A** 13 32.5	**(B)** 13 32.5	C 7 17.5	D 7 17.5	E 0 0.0
7. **Considering the amount of time I prepared, the CPs should be worth more marks.**	A 2 5.0	**(B)** 27 67.5	C 5 12.5	D 5 12.5	E 1 2.5
8. **Doing a CP was a good way for me to learn.**	A 1 2.6	B 4 10.3	C 3 7.7	**D** 16 41.0	(E) 15 38.5
9. **Watching the CPs of others was a good way for me to learn.**	A 1 2.5	B 7 17.5	C 5 12.5	**(D)** 16 40.0	E 11 27.5
10. The problems given in the CPs were not too difficult.	A 1 2.5	B 13 32.5	C 7 17.5	**(D)** 17 42.5	E 2 5.0
15. I did not mind my peers watching me do my CP.	A 7 17.9	B 7 17.9	C 5 12.8	**(D)** 17 43.6	E 3 7.7
17. Any anxiety I had about doing the CPs was balanced by the positive effects on my learning in this course.	A 4 10.0	B 6 15.0	C 9 22.5	**(D)** 16 40.0	E 5 12.5
18. **I learned from the mistakes others made in their CPs.**	A 1 2.5	B 5 12.5	C 2 5.0	**(D)** 22 55.0	E 10 25.0

Note: Bold letter indicates modal response
Parenthesized letter indicates expected response

	Response Count/Percentage				
	Disagree	<------>			Agree
19. I hated the experience of other students watching me do a CP.	A 8 20.0	(B) 14 35.0	C 8 20.0	D 6 15.0	E 4 10.0
24. Because of the CP experience in this course, I feel more comfortable speaking up in other courses.	A 5 12.5	B 12 30.0	(C) 13 32.5	D 9 22.5	E 1 2.5
25. As time went on in this course, I felt more comfortable doing a CP.	A 6 15.0	B 7 17.5	C 6 15.0	(D) 13 32.5	E 8 20.0
33. I did not mind when another student came up to the board and made corrections to my work.	A 4 10.0	B 5 12.5	C 8 20.0	(D) 17 42.5	E 6 15.0
36. Not knowing when the CPs would be given was O.K.	A 5 12.5	B 7 17.5	C 12 30.0	(D) 13 32.5	E 3 7.5
40. **As nerve-wracking as the CP experience may have been, it was well worth it.**	A 3 7.5	B 1 2.5	C 9 22.5	(D) 17 42.5	E 10 25.0

(subsequent question is from section II)

47. **Which was the hardest for you?**

		#	%
A. preparing for the CPs	A.	4	10.5
B. doing a CP	(B)	26	68.4
C. watching a CP	C.	5	13.2
D. having another student make corrections on my CP	D.	3	7.9

Note: Bold letter indicates modal response
Parenthesized letter indicates expected response

(Subsequent questions, overleaf, are from Section III)

6. Why do you think the course included the CPs?

- To make everyone do some type of review work.
- Same reason as stated in #5 i.e. (I think it was a way of teaching students to review a little at a time instead of cramming for a test. Also to show students their success).
- I think the course included the CPs because it helped the students remember their work.
- I honestly do not know.
- To see if this type of system was any good.
- So that we will have a chance to see past work and it will remain fairly fresh in our memories by the time the final exam rolls around.
- To better prepare students for tests/exams. To enforce rules in math. To get a better understanding of lessons. To learn from others' mistakes.
- To get students to concentrate under pressure.
- To get students to prepare for the class ahead of time and see how well you retain the knowledge learned.
- To review material, have a short period as if we were doing a test and to make sure everyone gets review if they do not do it on their own.
- To help us get involved with the class and participate more.
- In order to keep reviewing so that it will sink into our thick skulls.
- To help us prepare for our exam so that when the time to start studying for the exam comes, we do not have to re-learn what we did at the start.
- To create a nervous tension to do one's homework. I also believe it was included because you can see your mistakes when you are up on the board, though, I do believe a friendly atmosphere should be present before students do CPs.
- So that students would always be reviewing which would increase chances of doing well on the final exam.

- To review materials of early class so that we would be ready for test.

- It was an aid in our fully understanding the subject of the course. To prepare us for exam.

- So you would be forced to review.

- To review, and for the teacher to know if we understood what he was teaching before he went on to something else.

- To ensure revision of material.

- To give students the "initiative?" to review their work.

- To ensure you keep on top of things.

- As a way to review with pressure (as in tests).

- There are many key difficult concepts in this course, many of which must be constantly reviewed to understand.

- To ensure that we review; to help us learn from other people's mistakes; to help us learn from our own mistakes.

- To review what we have learned since the beginning of the year and to help us understand better what was not well understood.

- Because it made you review old material therefore you are never behind in your work. Also you will not have to cram for your final exam.

- So that you always have pressure on you to do your homework and to study until you have learned it.

- I believe it was a chance for the teacher to test us on our skill at any time of the week. He was able to see if his students were learning or not.

- I think it included CP's to make sure that we review. I hate CP's. If there were no CP's in Mr. Salusbury's class, I would make sure to get him as a teacher for the rest of my math courses at Abbott. Because he does these CP's I am going to have to find other teachers, which I do not want to do. I think that class quizzes once or twice a week for a minimal amount of marks would have been much more beneficial because everyone would have gained something from it. It sounds really stupid, but it really was a traumatic experience, one which I am not going to repeat.

- To enforce people to review and I hope for the instructor to locate problem areas, where he might want to review again.

- I think the course includes CP so that the individual can experience as many problems as possible. He can learn from past mistakes and the common errors are found and stressed. The other students learn from the errors made at the board. Thus, the CP's prove beneficial for all students.

- An interesting not a boring way of learning, made the lectures seem shorter. You always had to be ready, kept you on your toes.

- It was a good way to review. My only critic is that it makes a person like myself, who is not all that understanding of math, feel even less intelligent.

- I think they included CP's so it will give us a chance to review previous work. So it will be always fresh in our minds.

- There is no point in learning work and taking notes if you are not going to review them. This is where the CP's come in.

- To help us realize the mistakes we were doing. To realize that all students were probably making the same errors and to correct our faults.

- To help us review on a regular basis, the material learned in class.

- The CP's ensured that everyone would get some form of review. If the system was voluntary then a vast majority of the students would not review. However, the CP's virtually force you to review old material. The instructor becomes more aware of personal difficulties. One also learns from the mistakes he/she makes, therefore the CP's are an effective way of learning.

- To make us keep up to date and study and to put the fear of God in us. Every mistake I made on the board, I will not soon forget, as it is scratched in my memory due to the embarrassment.

Class Presentations: Discussion and Interpretation

The reponses in this category exhibit the conflict of a beneficial process versus an anxiety-provoking experience. The consensus was that the amount of time allotted for CPs was "just right", Q2: (2,15,20,47,15) and that the mark for the CPs was appropriate (see Q7).

The majority considered that "Doing a CP was a good way for me to learn", Q8: (2,10,8,41,39) and that they "learned from the mistakes that others made on their CPs", Q18: (2,13,5,55,25). Over 3/4 of the students agreed that: "As nerve-wracking as the CP experience may have been, it was well worth it", Q40: (7,3,23,42,25) and slightly less considered that the anxiety was "balanced by the positive effects on my learning in this course", Q17: (10,15,23,40,12)

Opinions varied on whether "The CPs made me nervous", Q3: (7,28,7,33,25) and whether "it would have been better to have written work (e.g. quizzes) than CPs, Q4: (22,25,25,10,18). There was general disagreement that "watching the CPs was boring", Q5: (32,33,18,17,0).

The responses to Questions 15 and 19 (opposing questions used in the consistency check) showed an overall consistency in their results: The mode of the responses to Q15: "I did not mind my peers watching me do a CP" was D, whereas the modal response for Q19: "I hated the experience of other students watching me do a CP" was B.

Question 6 in Section III asked: "Why do you think the course included CPs." Of the forty responses, twenty-three made mention of the word "review" (e.g. "to make everyone do some type of review work" and "to enforce people to review and I hope for the instructor to locate problem areas where he might want to review again"). There were also references to memory: "...it helped students to remember their work"; "to get students to prepare for the class ahead of time and see how well you retain the knowledge learned." Some made a link between review, understanding and learning: "To review what we have learned since the beginning of the year and to help us understand better what was not well understood"; "There are many key difficult concepts in this course many of which must be constantly reviewed to understand"; "To ensure that we review; to help us learn from other people's mistakes; to help us learn from our own mistakes."

A handful of other related comments also appeared on the positive side: "To better prepare for tests/exams"; "...it was included because you can see your mistakes when you are up at the board"; "so that you always have pressure on you to do your homework and to study until you have learned it"; "To help you get more involved in the class and participate more".

There were four comments that included negative connotations: "I honestly don't know"; "I hate CPs... it sounds really stupid, but it really was a traumatic experience, one which I am not going to repeat"; "It is a good way to review. My only critic is that it makes a person like myself, who is not all that understanding of math, feel even less intelligent" and finally: "To make us keep up to date and study and put the fear of God into us. Every mistake I made at the board, I will not soon forget as it is scratched in my memory due to the embarrassment."

Questions pertaining to: UNDERSTANDING or LEARNING

Question	Response Count/Percentage				
	Disagree	<------>			Agree
8. **Doing a CP was a good way for me to learn.**	A 1 2.6	B 4 10.3	C 3 7.7	D 16 41.0	(E) 15 38.5
9. **Watching the CPs of others was a good way for me to learn.**	A 1 2.5	B 7 17.5	C 5 12.5	(D) 16 40.0	E 11 27.5
17. Any anxiety I had about doing the CPs was balanced by the positive effects on my learning in this course.	A 4 10.0	B 6 15.0	C 9 22.5	(D) 16 40.0	E 5 12.5
18. **I learned from the mistakes others made in their CPs.**	A 1 2.5	B 5 12.5	C 2 5.0	(D) 22 55.0	E 10 25.0
26. **The SSR system allowed me to understand how to solve problems rather than just memorizing the techniques.**	A 0 0.0	B 3 7.5	C 8 20.0	D 19 47.5	(E) 10 25.0
31. **I did more memorizing in this maths course than in most other maths courses I have taken.**	(A) 7 17.9	B 15 38.5	C 6 15.4	D 8 20.5	E 3 7.7
32. The SSR system really helped me understand how the concepts were inter-related.	A 0 0.0	B 3 7.5	C 16 40.0	(D) 19 47.5	E 2 5.0

Note: Bold letter indicates modal response
Parenthesized letter indicates expected response

(Subsequent questions, overleaf, are from Section III)

1. Please rank order the following in the order of which you learned the most from in this course. Use all the numbers from 1-6. Do not use the same number twice.

	RANKING					
	1	2	3	4	5	6
The lectures	30	5	3	-	1	1
The CP sessions	5	20	6	3	5	1
The textbook	-	4	5	5	3	23
Assignments	2	8	10	5	13	2
Tests	1	-	13	18	5	4
Class quizzes	2	1	5	9	13	10

2. Please rank order what was most beneficial for you in using the SSR method. Use the numbers 1-3. Do not use the same number twice.

	RANKING		
	1	2	3
Learning tne derivations of problems	11	9	19
It made it easier to memorize procedures	16	12	11
Passing tests	12	18	9

Understanding or Learning: Discussion and Interpretation

In this category there were seven questions and the majority exhibited extreme responses. Questions 8,9, and 18 all related to learning form CPs by "doing a CP" (Q8); "watching the CPs of others" (Q9) and "learning from the mistakes others made" (Q18). The responses all vered on the "agree" side (D being the modal response in each case) and the aggregate response was (2,14,8,46,30). In effect, 3/4 of the calss had strong feelings about the value of the CPs for learning. Q17 asked whether the anxiety of doing the CPs "was balanced by the positive effects on my learning in this course". The students' replies showed a further, though lesser, bias on the positive side, though 1/4 of the class fell in the opposing camp: (10,15,23,40,12).

Questions 26 and 32 show marked agreement with the value of the SSR system; in the former for understanding versus memorizing: (0,8,20,47,25) and in the latter in the context of helping the student "understand how the concepts are inter-related": (0,8,40,47,5). Concerning the question of "more memorizing in this course than in most other maths courses" (Q31), the weight of the responses is significantly left of centre: (18,38,15,21,8). This harks back to Skemp's comment that, though relational mathematics may be harder to learn, "it is easier to remember" (Skemp 1979).

We now move on to the two ranking questions of Section III: In Q1, students were asked to rank according to learning value. The lectures were ranked first by 30 and second by 5 and the CP sessions first by 5 and second by 20. In general the modal rankings for the tests, assignments, class quizzes and textbook were low (4th, 5th, 5th and 6th respectively). In percentage terms, 88% placed lectures in first or second place and 63% placed the CP sessions as first or second; quite a strong endorsement of the value of these two processes.

The spread of ranking order in Q2 is less marked, though noteworthy: There were three categories in this question and the modal group in the first ranking was the category: "It made it easier to memorize procedures" (which was placed first by 41%, second by 31% and third by 29%). Passing tests appeared strongest in the second rank, (the corresponding percentages being: 31,46 and 23), which might be considered surprising in a marks-orientated student body. "Learning the derivations of problems" was considered most important by 28%; ranked second by 23% and last by 49%. It might be argued that "Learning the derivations of problems" is a preliminary step to making it "easier to memorize procedures" and "passing tests", but, clearly the latter two consequences are considered to have more value to the students in tangible terms.

Questions pertaining to: FEELINGS

Question	Response Count/Percentage				
	Disagree	<------>			Agree
3. The CPs made me nervous.	A	B	C	(D)	E
	3	11	3	13	10
	7.5	27.5	7.5	32.5	25.0
15. I did not mind my peers watching me do my CP.	A	B	C	(D)	E
	7	7	5	17	3
	17.9	17.9	12.8	43.6	7.7
19. I hated the experience of other students watching me do a CP.	A	(B)	C	D	E
	8	14	8	6	4
	20.0	35.0	20.0	15.0	10.0
22. **I will be less nervous for the final exam because of the way this course prepared me.**	A	B	C	(D)	E
	0	2	3	21	14
	0.0	5.0	7.5	52.5	35.0
24. Because of the CP experience in this course, I feel more comfortable speaking up in other courses.	A	B	(C)	D	E
	5	12	13	9	1
	12.5	30.0	32.5	22.5	2.5
25. As time went on in this course, I felt more comfortable doing a CP.	A	B	C	(D)	E
	6	7	6	13	8
	15.0	17.5	15.0	32.5	20.0
29. **I feel more prepared for the final exam in this course than in most other courses I have taken.**	A	B	C	(D)	E
	1	4	7	18	9
	2.6	10.3	17.9	46.2	23.1
33. I did not mind when another student came up to the board and made corrections to my work.	A	B	C	(D)	E
	4	5	8	17	6
	10.0	12.5	20.0	42.5	15.0
37. **I developed increased confidence in maths through the methods used in this course.**	A	B	C	(D)	E
	0	4	6	21	8
	0.0	10.3	15.4	53.8	20.5

Note: Bold letter indicates modal response
Parenthesized letter indicates expected response

	Response Count/Percentage				
	Disagree	<------>			Agree
38. **As time went on, I was better able to confront new maths problems because of the techniques used for learning in this course.**	A	B	C	(D)	E
	0	3	11	22	4
	0.0	7.5	27.5	55.0	10.0
40. **As nerve-wracking as the CP experience may have been, it was well worth it.**	A	B	C	(D)	E
	3	1	9	17	10
	7.5	2.5	22.5	42.5	25.0

Note: Bold letter indicates modal response
Parenthesized letter indicates expected response

Feelings: Discussion and Interpretation

Six of these nine questions concerned the feelings related to the Class Presentations. In Question 3, it was agreed by 58% of the students that "CPs made me nervous", though 53% of respondents to Q25 agreed that: "As time went on in this course, I felt more comfortable doing a CP". Moreover, in Q40, 68% agreed that: "As nerve-wracking as the CP experience may have been, it was well worth it".

On the more extreme positive side, Q29 shows 79% agreeing that they "feel more prepared for the final exam in this course than in most other courses I have taken" and 88% in Q22 agreed that would feel "less nervous for the final exam" because of the way the course had prepared them. Q37 indicates "increased confidence in maths through the methods used in this course" (0,10,15,54,21) and Q38 shows 65% of the students agreeing that they were "better able to confront new maths problems because of the techniques used for learning in this course."

Questions relating to: EXAMS or TESTS

Question	Response Count/Percentage				
	Disagree	<------>			Agree
4. It would have been better to have written work (e.g. quizzes) than CPs.	(A) 9 22.5	**B** 10 25.0	**C** 10 25.0	D 4 10.0	E 7 17.5
22. **I will be less nervous for the final exam because of the way this course prepared me.**	A 0 0.0	B 2 5.0	C 3 7.5	(D) 21 52.5	E 14 35.0
28. **The review method used in this course sure beats cramming for the final exam.**	A 0 0.0	B 0 0.0	C 3 7.5	D 15 37.5	(E) 22 55.0
29. **I feel more prepared for the final exam in this course than in most other courses I have taken.**	A 1 2.6	B 4 10.3	C 7 17.9	(D) 18 46.2	E 9 23.1

Note: Bold letter indicates modal response
Parenthesized letter indicates expected response

Exams or Tests: Discussion and Interpretation

The first question in this category indicates a preference by 28% of the students for written work rather than CPs, as opposed to 48% for CPs rather than written work (24% remaining undecided). This is not a resounding note in favour of CPs, but and indication that more students (20% or 8 out of 40) consider the procedure to be more valuable.

The remaining three questions concern preparedness for the final exam. Here there is strong support for the way the course has prepared the students. An aggregate response to Questions 22, 28, and 29, (which concern being "less nervous for the final"; beating "cramming for the final" and preparedness for the final versus other courses) is: (1,5,11,45,38), signifying 83% who either "Agree" or "Strongly Agree".

Questions pertaining to: MARKS

Question	Response Count/Percentage				
	Disagree	<------>			Agree
6. I am satisfied with the distribution of marks used in this course (9%=CPs, 41%=class tests, 50%=final).	A	B	C	(D)	E
	2	5	7	16	10
	5.0	12.5	17.5	40.0	25.0
7. Considering the amount of time I prepared, the CPs should be worth more marks.	A	(B)	C	D	E
	2	27	5	5	1
	5.0	67.5	12.5	12.5	2.5

(subsequent questions are from section II)

44. My grades generally are:

		#	%
A. mostly 90 to 100	A.	0	0.0
B. mostly 80 to 89	B.	3	7.5
C. mostly 70 to 79	(C)	22	55.0
D. mostly 60 to 69	D.	13	32.5
E. under 60	E.	2	5.0

45. What grade do you expect to get in this course?

		#	%
A. 90 to 100	A.	2	5.
B. 80 to 89	B.	4	10.0
C. 70 to 79	(C)	17	42.5
D. 60 to 69	D.	16	40.0
E. under 60	E.	1	2.5

Note: Bold letter indicates modal response
Parenthesized letter indicates expected response

(The subsequent question, overleaf, is from Section III)

4. If you could divide up the marks for the course, how many marks would you assign to each activity below out of a possible 100 marks?

	CPs	CLASS TESTS	FINAL EXAM
Mean Response	15	45	40

Marks: Discussion and Interpretation

Questions 6 and 7 indicate good support for the distribution of marks in the course and the relative weighting of the Class Presentation mark. The remaining questions indicate that the students' general grades are normally distributed (Q44: mean 70.8%), as are the expected grades from their current math course (Q45: mean 71.9%). The proximity of the grades is some indication that expectancy in mathematics is in close accord with their general grades.

Question 4 from Section III shows a preference for reducing the final exam weighting from 50% to 40% and increasing the value of the class presentations from 9% to 15% and a corresponding adjustment in the value of class tests from 41% to 45%. This can be taken as a desire for the time and energy expended on CPs to be reflected by a slightly higher percentage.

Questions pertaining to: TIME

Question	Response Count/Percentage				
	Disagree	<------>			Agree
1. **The review system was a good way of using class time.**	A 1 2.5	B 0 0.0	C 2 5.0	**D** 20 50.0	(E) 17 42.5
2. **The amount of class time allotted to the CPs was just right.**	A 1 2.5	B 6 15.0	C 8 20.0	**(D)** 19 47.5	E 6 15.0
13. **The time spent on reviewing for this course infringed on study time for other courses.**	A 4 10.0	**(B)** 23 57.5	C 5 12.5	D 6 15.0	E 2 5.0
14. **The time spent on reviewing for this course infringed on my leisure time.**	A 4 10.0	**(B)** 22 55.0	C 3 7.5	D 8 20.0	E 3 7.5
27. **There was too much time spent on review rather than figuring out new problems.**	A 10 25.0	**(B)** 24 60.0	C 3 7.5	D 3 7.5	E 0 0.0

(subsequent question is from section II)

46. The number of hours I spent in preparing for this course per week was approximately:

		#	%
A. 0-1 hour	A.	12	30.0
B. 1-2 hours	B.	9	22.5
C. 2-4 hours	**(C)**	14	35.0
D. 4-6 hours	D.	5	12.5
E. more than 6 hours	E.	0	0.0

Note: Bold letter indicates modal response
Parenthesized letter indicates expected response

(The subsequent question, overleaf, is from Section III)

3. If you had one and one-half hours of class time, how would you split up the time in minutes for each activity below?

	CPs	REVIEW	LECTURE
Mean Response	18	24	48

<u>Time: Discussion and Interpretation</u>

Four of the five questions from Section I in this category concerned the review technique. In Q1 93% agreed that "the review system was a good way of using class time"; 85% of the respondents to Q27 disagreed with the statement that: "there was toomuch time spent on review rather than figuring out new problems" and 63% considered the amount of time allotted to CPs per se was "just right".

There was disagreement that the time spent on reviewing infringed on study time for other courses , Q13: (10,58,12,15,5) or leisure time , Q14: (10,55,7,20,8). This conclusion is complemented by the results to question 46 which asked the students to specify the approximate number of hours spent per week on preparing for the course: It yielded close to a bimodal response: 30% spending 0-1 hour, 22% 1-2 hrs, 35% 2-4 hrs and 13% 4-6 hrs. In passing, we might note that the recommended study/homework time by the Ministry of Education is 4 hrs per week; the evidence suggesting that the burden of preparation for class is not excessive (the mean of the response data being 2.2 hrs).

Question 3 of Section III shows general support for the status quo, but a little pressure for slight adjustments: CPs take 10 to 20 minutes (the mean of the recommended times being 18 mins), the subsequent review takes 5 to 15 minutes (mean recommendation: 24 minutes) and the lecture 45 to 65 minutes (48 minutes recommended).

If one considers that, whenever relevant, review is blended with lectures (see SSR Procedure - Section 4.1), then the students' suggestions are much in accord with the status quo.

Questions pertaining to: MISCELLANEOUS

Question	Response Count/Percentage				
	Disagree		<------>		Agree
20. **I worked harder in this course than in most courses I have taken.**	A 1 2.5	B 4 10.0	C 6 15.0	**(D)** 18 45.0	E 11 27.5
21. **The effort to keep up-to-date in this course was worth it.**	A 1 2.5	B 1 2.5	C 5 12.5	**D** 23 57.5	(E) 10 25.0

(subsequent questions are from section II)

42. Gender:

		#	%
A. female	A.	13	34.2
B. male	B.	25	65.8
	NR.	2	

43. I have taken this course the following number of times:

		#	%
A. once	A.	25	64.1
B. twice	B.	10	25.6
C. three times	C.	4	10.3

48. Was the textbook useful in this course?

		#	%
A. yes	**(A)**	23	59.0
B. no	B.	16	41.0

Note: Bold letter indicates modal response
Parenthesized letter indicates expected response

Miscellaneous: Discussion and Interpretation

The two Section I questions (20 and 21) both concern effort: Q20 asks if the student "worked harder in this course than in most courses", to which the response was 73% in the affirmative. Q21 asks if "the effort to keep up-to-date in this course was worth it", to which 83% agreed.

The Section II questions seek a breakdown by gender (Q42) and the number of repeats of the course under study (Q43). This information has potential usefulness in future cross-referencing of results. The last question (Q48), on the opinon the students have of the textbook showed a somewhat ambivalent response (59% pro and 41% con). This lends support to the lowest rating given the textbook in Q1 of section III (see the category "Understanding and Learning").

Summary of Questionnaire Results

The outstanding responses to the questionnaire were in the following areas: (i) The Structural Spaced Review technique, (ii) Understanding and Learning and (iii) Class Presentations. The following comments will atempt to link the areas of advantage and disadvantage to the students among these three areas. It is worth reiterating that the Structural Spaced Review technique was born out of a desire to engender in the students a greater understanding of the mathematics they were studying and the Class Presentations were a subsidiary tool used to provide a certain "activating" element within the review process.

The student responses indicate strong support for the SSR process when matched with the more traditional teaching methods. Particular advantages were seen in the context of preparedness for the final exam, increased confidence in mathematics and understanding the material better rather than merely memorizing techniques. In reference to these latter points, both Skemp and Krutetskii's viewpoints are supported: The building of a cohesive structure of generalized essential relationships (Relational Understanding), is a goal worthy of achievement; though it may not be an easy goal to attain. Whereas the converse seems to be true for Instrumental Understanding - the product of attempts at understanding merely by isolated fact and formula learning; such a process of learning "rules without reason" would appear to have severe limitations. The student responses, therefore, indicate some success at achieving <u>Relational</u> rather than <u>Instrumental</u> Understanding.

In the majority of cases, the Class Presentations appear to have augmented student awareness and comprehension for both those doing the CP and those watching. There is agreement that this procedure adds a tension to the class which does not exist in classrooms where the more traditional forms of teaching take place. This has, according to the students, several positive and a few negative spin-offs: The procedure is seen as a valuable part of the lesson; it is a catalyst to comprehension and assists students in linking related concepts; it reduces the necessity for excessive memorization and, to quote a pilot questionnaire response (to question 1): "it kept everything 'warm`." As a consequence, students felt a reduction in exam tension and an increased confidence for tackling tests and the final exam and a concommitant increase in grades.

But all that glistens is not gold: there was another side to the coin. Though in the minority, there were several negative comments concerning the SSR and CP procedures, mainly centering on the latter. The procedure was seen by these students to be too time-consuming in terms of preparation time as well as class time. The CPs engendered negative feelings in some respondents who found them to be too stressful and, in consequence, counter-productive. However it should be pointed out that the overall response was that this extra stress was more than adequately balanced by the advantages of the procedure.

4.3 STRUCTURE TESTS AND INTERVIEWS

This section sheds light on Objective 2 (b) in which we were: "To observe in an exploratory way some of the possible effects of the SSR technique on the performance of students in a specialized test, by looking at their results and evidence of any significant knowledge of structure of concepts." The structure test evidence will be bolstered by further evidence from the follow-up interviews which were conducted with a small cross-section of students from each group.

The evidence from the two structure tests and interviews is presented by course and is broken down by question number; each of the tests having five questions. After each question, the "Intentions" are repeated, then the results are laid out as follows: First some general statistics on the class results appear. These are followed by several of the more revealing student responses, spread through the ability scale. These are used to highlight points on structural understanding and/or lack of such understanding.

Subsequently, the interviews are discussed, once again using the focal point of structural knowledge. Excerpts of the interviews are used to underscore certain points. (The reader is reminded that the full interview protocols can be found in Appendix 2).

Results of The Structure Tests and Interviews

I. College Algebra: 201-211

Q1. In what way is the solution to $x^2 - 6x + 8 = 0$ connected to the graphing of the parabola $y = x^2 - 6x + 8$?

Intentions

-[Does the student realize the essential relationship between the distinct concepts of quadratic equations and quadratic functions?]

-[Can the student connect the two, realizing that the solution to $ax^2 + bx + c = 0$ gives the x intercepts of the parabola, which occurs at $y = 0$?]

Class Response

The students' marks were (in ascending order): 0,0,0,0,0,5,5,8,8,10,10,12,12,15,15,18,20,20; 2 students omitted the question. The mean of the responses was 8.8.

At the lower end of the mark scale were responses like:

-"Both Parabola"

-"because (-b +/- sqrt(b^2-4ac))/2a is related to $y = x^2 - 6x + 8$ or $x^2 - 6x + 8 = 0$."

-"The way the solution $x^2 - 6x + 8 = 0$ is connected to the graph of $y = x^2 - 6x + 8$ are by coordinates:

$x^2 - 6x + 8 = 0$	$y = x^2 - 6x + 8$
$(x - 2)(x - 4) = 0$	$y = (x - 4)(x - 2)$
$= 2, 4$	$y = 2, 4$"

-"You use the equation to find the x and y intercepts and the vertex"

Certainly there are inklings of some understanding here, but evidence of a lack of a clear structure. There is evidence that the quadratic function $y = x^2 - 6x + 8$, is not seen as distinct from the quadratic equation $x^2 - 6x + 8 = 0$ even though the point had been discussed in several of the classes and class presentations concerning this topic.

At the other end of the ability scale are responses such as the two below:

-"$x^2 - 6x + 8 = 0$ is connected in the way that the y coordinate is equal to 0 and to find the x coordinates at $y = 0$, all one has to do is factor $x^2 - 6x + 8 = 0$."

-"$0 = (x - 2)(x - 4)$, this solution is the answer to the x intercepts of the graph of $y = x^2 - 6x + 8$.
[The student proceeded to demonstrate this by sketching the parabola]."

Though the former of these two answers is somewhat lacking in clarity, this is evidence of a clearer knowledge of the structural relationship between the function and the quadratic equation.

<u>Interview</u>

Questions:

(i) What is the difference between these two?

(i.e. $x^2 - 6x + 8 = 0$ and $y = x^2 - 6x + 8$)

(ii) How did you go about this question?

(iii) What would the graph of $y = x^2 - 6x + 8$ look like?

The evidence from the interviews underscores the points already made in the previous section of "Class Responses"; it may even sound an alarm bell to alert us to the shallow understanding that many of our students have of some of the concepts we "teach".

Though the five students interviewed are spread across the ability range of the class (though the class may not be a subset representing the ability range of the student population), there is a strong indication of as blurr in the students' minds as to the distinction between a quadratic equation and a quadratic function. Witness the written and verbal responses of Michael (a student close to the top of the class):

-"The two equations are connected by the fact that if one was to plot the solution of the first equation on a graph, the points would be the same as if the second equation were graphed

$x^2 - 6x + 8 = 0$	$y = x^2 - 6x + 8$
$(x - 2)(x - 4) = 0$	$y = (x - 4)(x - 2)$
$x = 2, 4$	$y = 2, 4$"

The relevant section of the interview protocol is as follows:

T. In question No. 1, how do you regard the difference between these two?

S. The one equal to 0 is a parabola and by factoring you get $x = 2$ or 4.

T. In the other one you've got $y = 2$ or 4, how did you get that?

S. I really don't know.

T. How would it look if you graphed this one? $[(x - 4)(x - 2) = 0]$

S. [student does correct graph of the **parabola**: $y = (x - 4)(x - 2)$].

Another student (Steven), in his written answer states:

-"The way that it's connected is that the y has a value of zero and so $x^2 - 6x + 8 = 0$ is the same as saying: $y = x^2 - 6x + 8$. They have the exact same graph design. They look exactly alike as a graph."

When asked in the interview, he reinforces this confusion of seeing no difference between the function and the equation when he says: "Basically, I think they are the same." A further indication of interference of another concept concerning graphing appears at the end of the discussion of this question, when he asks himself how he would find the y values of the quadratic equation (of which there are none!): "I imagine the slopes would be the same."

Marlene points us to a potential source of confusion: The meaning of the terminology. In the interview discussion of the graph of the function she exhibits a misunderstanding of the term "intercept":

T. When you say x intercepts for this one, ($x^2 - 6x + 8 = 0$), can you explain what you mean?

S. You just simplify first, and get $x - 4 = 0$ or $x - 2 = 0$ and then you find the x values.

T. When you talk about x intercepts what do you mean?

S. I mean the x values.

T. So, the x value is like an x intercept?

S. Yes.

2. Answer the questions below and explain the similarities and differences between the part (i) and part (ii) questions:

(a) (i) Simplify: $x/2 - 3/x - 4$; (ii) Solve: $x/2 - 3/x = 4$

(b) (i) Factor: $-x^2 - 7x - 12$; (ii) Solve: $-x^2 - 7x - 12 = 0$

Intentions

-[Does student realize the conceptual difference between expressions and equations?]

-[Is there realization that x has no fixed value in an expression, though it may have a value in an equation?]

-[Does student recognize the difference between a quadratic polynomial and a quadratic equation?]

-[Is there realization that there is no x value solution to the factor problem?]

Class Response

The students' marks (in ascending order): 0,0,5,10,10,12,12,12,12,12,13,13,14,14,16,16,17,18,19,20. The mean was 11.9.

Again we present the evidence from particular student responses from "weaker to stronger":

-"2 (a) (i) $x/2 - 3/x - 4$ (ii) $x/2 - 3/x = 4$

$x/2x - 3/2x - 4/2x$ $x/2x - 3/2x = 4/2x$

$= (x - 1)/2x$

(b) (i) $-x^2 - 7x - 12$ (ii) $-x^2 - 7x - 12 = 0$

$-1(x + 3)(x + 4)$ $-(x + 4)(x + 3) = 0$

$-(x^2 + 3x + 4x + 12)$ $-(x^2 + 3x + 4x + 12) = 0$"

Here we see a considerable amount of confusion, though the beginnings of some understanding in part (b). Another response is as follows:

-"2(a) In the first part you simplify without changing the question, where the second part is an equation so the equation changes:

(i) $x/2 - 3x - 4$

$= x^2/2x - 6/2x - 8x/2x$

$= x^2 - 8x - 6$

(ii) $x/2 - 3/x = 4$

$x^2/2x - 6/2x = 8x/2x$

$(x^2 - 6)/2x = 8x/2x$

$(x^2 - 6)/2x \,.\, 2x/8x$

$= (x^2 - 6)/8x$

$= (x - 3)(x + 3)/8x$

2(b) The two equations end up with the same answers, there are two different ways to solve them:

(i) $-x^2 - 7x - 12$

$= -(x^2 + 7x + 12)$

$= -(x + 4)(x + 3)$

(ii) $-x^2 - 7x - 12 = 0$

$-x(x + 3) + 4(x + 3)$

$-(x + 4)(x + 3)$"

Again, there are indications here of a modest comprehension of the processes, but much confusion as to which technique to apply in which circumstance. This "interference" between two related, but distinct concept areas might be considered as a necessary prelude to the achievement of a clear structural understanding.

At the other end of the balance are the following two answers:

-"Part (i) is an expression and we cannot solve for x
Part (ii) is an equation and both sides can be manipulated to solve for x.
Both problems use the same numbers but are treated differently.
[The student then went on to give the solutions to all four which were correct bar an algebraic error in 2 (a) part (ii)]."

-"2 (a) (i) Expression (ii) Equation
[Followed by the correct solutions to each]
2 (b) (i) Expression (ii) Equation
[again followed by the right solutions]."

The evidence is clear that these two had a good knowledge of "The generalized essential relationships" (to quote Krutetskii), and were able to apply their knowledge to reach the correct responses. It may be worth indicating that, whenever the opportunity presented itself, this distinction between expressions and equations had been underlined with few questions coming from the students to indicate misunderstanding, but even so, the class mean for this particular question was only 11.9.

<u>Interview</u>

(i) What is the difference between parts (i) and (ii) of these questions?

(ii) Does x have a value which we can calculate in either or both parts?

(iii) How did you get your answers in part (ii) of each question?

The indications from the interviews are that the students have a clear knowledge of the distinction betwen the categories "expression" and "equation", but that it may only be "skin deep". The evidence to support this doubt comes from the two stronger students.

In the test Steven had written the following answer to 2 (b) part (i) (Factor: $-x^2 - 7x - 12$):

-"$-x^2 - 7x - 12 = -(x^2 + 7x + 12)$."

The subsequent interview went as follows:

T. Could you factor this further (in 2 (b) part (i))?

S. Yes. $(x + 3)(x + 4)$.

T. Would you stop there or would you find any values for x?

S. Yes, x could be -3 or -4.

T. So, you see part (i) and part (ii) as almost identical?

S. Yes, I guess. I don't know why.

Marisa, who was also successful in answering this question, does not appear to have the strength of her convictions with regard to x not having a particular value in the expressions. When asked if x has a value in part (i) of 2 (b), she responds with a long silence, then avoids the issue by stating that: "You have to find the factors." When pressed again to say whether x has a specific value, she replies: "No, I don't think so."

Marlene, on the other hand, exhibits more confidence in stating that: "I see x having a value here (in the equation), and not here (in the expression)." She even added a subtlety when asked if x could be anything (in reference to the equation): "Yes, except 0."

3. If $f(x) = x^2 - x + 2$

(a) Explain, in words, what is meant by f(x-1).

(b) Find f(x-1).

(c) If f(x-1) = 0, find x.

Intentions

-[Does student understand the concept of Input/Output or Domain/Range?]

-[Is student aware that the new input, x - 1, is operated on in the same way as x in the original function definition?]

-[Does student follow through with the correct procedure for evaluating f(x-1)?]

-[Is there evidence of notational confusion?]

-[Does student incorrectly perceive f(x-1) = 0 as being equivalent to: x - 1 = 0?]

-[Since the correct solution to part (c) leads to a quadratic equation, does this deter the student who reaches that stage in his answer?]

Class Response

The students' marks (in ascending order) were: 0,0,0,0,2,3,4,7,8,8,8,8,10,12,14,14,15,15,17,20, and the mean was 8.3.

There was a wide variety of responses to this question, and a good deal of evidence of notational confusion. In question 3 part (a) in particular, this was very evident:

-"f(x - 1): This means take the function of x and subtract 1 from it."

-"in the function, x has no value. After receiving a value for x, we insert that value where x was."

Part (b) was started correctly by the majority of the respondents. The most common error was the misinterpretation of f(x - 1) as f(x) - 1; a resounding note to demonstrate that the structural significance of function terminology was missed by a large proportion of the class.

The final part (c) question was omitted by all but a handful of students. Of those who attempted the question, only four correctly interpreted it, but then two got lost in the mechanics of simplification or solving the quadratic formula. It was a mild surprise to the author that this question received such a poor response, since similar questions had been attempted in class and followed by a good deal of discussion. It was evident that the class had had difficulty with the conceptualizations needed and had, in consequence, acquiesced in their lack of comprehension.

Interview

(i) What does $f(x)$ mean?

(ii) How is $f(x-1)$ different?

(iii) How do you find $f(x-1)$?

(iv) How do you solve $f(x-1) = 0$?

Three of the five interviewees gave correct answers on the test to parts (a) and (b). All five, however, gave correct responses to these questions during the interviews. (The reader is reminded that the question paper is presented "anew" to the student, and is referred to, if it is deemed useful, only **after** the student has been posed questions).

Micheal's answers demonstrate this surprising "about face": His written answers to parts (a) and (b) are:

-"f(x - 1) means that a function of a number at (x - 1) gives you a result. If the number is put into a machine in will get "(x - 1)ed":
$f(x - 1) = (x^2 - x - 2 - 1) = (x^2 - x - 3)$."

The interview reveals the change of heart:

T. In question No. 3, how do you see f(x)?

S. You plug x into the equation, or use whatever number x is, like here it changes to (x - 1).

T. So how did you go about finding f(x - 1)?

[At this juncture the student is shown his test answer sheet].

S. [Student considers his previous answer for a few moments.]
I...I don't know...what did I do? Actually, where x is I should plug in (x - 1), and multiply them out.

T. So what would that give you?

S. Pause...$(x - 1)^2 - (x - 1) - 2$. I think I messed up this one on the test!

This is an interesting example of what Piaget called "equilibration"; a state in which the student is wavering in his choice of which concept to apply to a certain situation; what one might call an intermediary stage between the old erroneous stage and the new, correct stage. In Krutetskii's terms, this might be seen as a sub-step in the process of development of his "generalized essential relationships".

The responses to part (c) were quite varied. Steven, who had shown a good grasp of the other two parts, sped quickly to the correct answer:

-"$f(x - 1) = 0$

$x^2 - 2x + 1 - x + 1 - 2 = 0$

$x^2 - 3x = 0$

$x = 0$ or 3."

However, emulating Michael's change of heart in a previous question, Steven exhibits some considerable confusion in the interview:

T. So, how would you solve part (c)?

S. All you do is plug in 0 into these and you get $0^2 - 0 - 2$ which gives -2.... So, $f(x - 1) = f(x)$, and if you're giving that a value of 0.

T. So, if $f(x - 1)$ would equal 0, then $f(x)$ would equal 0 as well?

S. Yes.

T. So, 0 would equal $x^2 - x - 2$?

S. But you can't have $0 = -2$, can you?
[student reviews his work]
I'm not sure what's going on.

Marleen, on the other hand, had a change of heart in the direction of right: To part (b) she correctly found $f(x - 1)$ to be $x(x - 3)$, but then continued to fall into that ever-tempting trap: $x = 0$, $x = 3$.

To part (c) she answered thus:

-"If $f(x - 1) = 0$ then $x = 3$."

When asked in the interview:

T. And in part c, what were they asking here?
[If $f(x -1) = 0$, find x.] You got 3 as an answer, how did you get that?

S. I just looked up at part b, if $x = 0$ in that equation, then I figured x could be 3 as well.

T. So, is it equal to both 0 and 3 or just 3?

S. [pause] I think it's both.

T. Why is it equal to both?

S. Because both would work out.

Stephane's written response was a demonstration of many students' confusion with the structure of the problem in question:

-"$f(x - 1) = 0$ $\underline{x = 1}$

Proof $f(x) = 1$

$f(x - 1) = 1 - 1 = 0$."

There was another surprising about-face during the interview, (where Stephane shows his rudimentary technique for finding solutions to quadratic equations). Part (b) had been discussed and he had come up with the correct procedure and answer, the interview proceeded as follows:

T. So, how would you answer part c?

S. You'd get $x^2 - 3x = 0$.

T. What could x be?

S. 0.

T. How do you get that?

S. Well, $0^2 - 3(0) = 0$.

T. Is there any other solution?

S. [pause]... x could be equal to 3.

T. How did you see that?

S. 3^2 is 9... - 3 x 3 gives 0.

T. Right, so those are the two possibilities. Is there any other way of doing that or were you just trying some numbers?

S. I just throw numbers in and see what happens.

This last comment is particularly fascinating. It highlights exactly what Krutetskii means when he describes his weaker students as lacking in the ability to generate "generalized essential relations". Here is an example of a student who, in mark terms at least, is the weakest of the five interviewees; a student who nevertheless has a reasonable grasp of some quite sophisticated concepts; but one who nevertheless relies on the extremely restrictive device of testing numbers in quadratic equations to "see what happens". He has failed to develop the necessary structure needed to generalize his solutions, and this significant handicap coupled with notational difficulties has had a braking effect on his progress.

4. Identify the following as true or false; explaining the reason for your decision:

(a) $3x^{-1} = 1/(3x)$

(b) $(3x - 2)/3 = x - 2$

(c) If $f(x) = x^2$; then $f(2x) = 2x^2$

Intentions

-[Does student see that the exponent only affects the x, as opposed to 3x?]

-[Is there a clear understanding of the "cancelling" process?]

-[Is f(2x) viewed as 2f(x)?]

-[Is $2x^2$ seen as equivalent to $(2x)^2$, and subsequently $4x^2$?]

Class Response

The students' marks (in ascending order) were: 0,5,7,7,10,12,13,14,14,14,14,14,14,15,15,18,18,20,20; one student omitted the question. The mean mark was 12.8.

In response to 4 (a) there were few students who considered the equation true, but a handful of individuals were attempting to apply a false "instrumentally learned" rule:

-"4 (a) True, because a negative goes on the bottom when you switch it."

-"4 (a) True, you flip it to lose the negative exponent."

A few correct responses to the question were somewhat confused in the following cases:

-"4 (a) False because the negative power only affects the thing to its immediate right."

-"4 (a) False because only the x is being multiplied by the negative exponent."

Clearer understanding is reflected by such answers as:

-"4 (a) False, because the negative exponent only effects the x element."

-"4 (a) The -1 only effects the x not the 3."

The success rate with 4 (b) was slightly higher, though some of the reasons given raise some doubts about a clear conceptual structure:

-"4 (b) False - The whole equation is not divided by 3."

-"False, you do not divide when things add or subtract in an equation."

A clearer picture of the reason for the response of "false" is given by a smaller number of students:

-"False, 3 is not a factor of -2 therefore cannot be factored out."

-"False because you cannot cancel without multiplication

Ex $\frac{3x - 2}{3} = x - 2$ false $\frac{3(x - 2)}{3} = (x - 2)$ true."

About half those who attempted 4 (c) on function notation gave the correct response, but this old chestnut was hard for some students to crack:

-"This is <u>true</u> because the function is x^2 and if it is doubled it is $2x^2$."

-"True because x^2 is plugged in to the x which gives $(2)(x^2)$ which simplifies to $2x^2$."

-"True because the x value is multiplied by 2."

On the other side of the coin were responses exhibiting the development of clear structures:

-"4 (c) false because 2x times itself is $4x^2$ not $2x^2$."

-"False, because the whole 2x [student drew arrow and circled 2x in the expression f(2x)] has to be squared NOT just x."

-"False, unless a bracket is put around the (2x) then squaring it all---> $(2x)^2$."

One correct response, followed by an erroneous explanation might shed some light on the students' confusion with regard to this question:

-"4 (c) False, because $2x^2 = 4x^2$."

In the classroom discussion of this concept, this ambiguity often occurred in the students' minds. Only after repeated, spaced exposures to the distinction between $2x^2$ without the parentheses and $(2x)^2$ with the parentheses, did some individuals appear to see the distinction. This also harps back to question 4 (a) where the $3x^{-1}$ was apparently seen by several students to be indistinguishable from $(3x)^{-1}$.

Interview

(i) What should I write in order to get: 1/(3x)?

(ii) How would I have to change (3x - 2)/3 in order to get x - 2 as an answer?

(iii) How did you do the f(2x) question?

All the respondents who were asked gave convincing explanations to the first two interview questions. The function notation question, however, gave rise to two interesting replies (the other three had a clear understanding of the meaning of f(2x)). In these two cases the students answers were consistent with their written responses.

Stephane wrote that the suggestion was true:

-"Because the function is multiplying x^2 by 2."

In the interview he reinforced this by commenting:

-"You double the x so you double the answer."

Michael had written that the suggestion was:

-"True it makes logical sense."

In the interview he stated: [bold numbers stressed]

-"if $f(x) = \mathbf{1}x^2$, then $f(\mathbf{2}x) = \mathbf{2}x^2$."

These "logical" responses by the two latter students are inconsistent with their verbal responses to question 3 (b), which they had given just a few minutes earlier! On the other hand, they are consistent with their written responses to those same questions; further evidence of that "intermediary stage" in their cognitive processes in which the interference of erroneous concepts has temporarily taken control again. This type of "negative interference" is somewhat symptomatic of Bartlett's research into students' cognitive adaptions of stories which are foreign to their culture (see the elaboration of this point in Chapter 2).

5. How many possible solutions could there be to the quadratic formula? How does this relate to the different number of possible x intercepts in the graph of a parabola?

Intentions

-[Does the student see that the key to this question is the value of the determinant? $D < 0$, $D = 0$, or $D > 0$ give 0, 1 and 2 solutions respectively.]

-[Can the student reach this conclusion by considering various possible parabolas; with 0, 1 or 2, x intercepts?]

-[How does the student relate the quadratic formula and the parabola?]

-[Can he estimate a quadratic function, given a sketch?]

-[If successful at the latter question, can the student give the x intercepts of a factored cubic function?]

Class Response

The students' marks (in ascending order) were: 0,0,3,4,5,5,7,7,7,8,8,8,10,10; the question was omitted by 6 students. The mean was 5.9.

It is evident from the class mean and the number of omissions (which were not apparently due to lack of time), that this question caused major difficulties. A good deal of flexibility was permitted in the answer, but no student was able to demonstrate that he had achieved a clear understanding of the needed concepts and their interrelations. The graphing of parabolas was one of the last topics discussed in the course, and it is the author's tentative opinion that the students did not have sufficient time to properly absorb the concepts and develop a clear, cohesive structure. This suggestion may be upheld by the type of responses given:

-"2 possible solutions. It will either be a parabola which opens upwards or downwards."

-"There are two possible solutions to the Quad formula and it does not change the number of x intercepts in the parabola, there are always two."

-"Part 1: 2 solutions
Part 2: because you use the x intercept with the formula $(4ac - b^2)/4a$."

-"There are usually 2 answers a positive and negative but there may be none if there is no solution.
Well if their are two answers a positive and a negative one then the parabola opens up or down, or if you have no answer its a no solution parabola."

-"There are 16 possible solutions to a quadratic formula, which relates to the different number of x intercepts in the graph of a parabola. This is possible because through the formula the x - int. must have a y - int. both of which are infinite."

<u>Interview</u>

(i) What is the quadratic formula?

(ii) How many x intercepts could a parabola have? Show by graphing.

(iii) Is there any connection between the possible number of x intercepts of a parabola and the possible number of solutions to the quadratic formula?

(iv) What might the quadratic function of this parabola be? (Sketch shown of a parabola passing through -1 and 3, opening upwards).

(v) How many x intercepts might y = (x)(x+2)(x-3) have and where would they be?

These questions was asked at the end of, what was for teacher and student, rather a long interview and in some cases the latter questions were omitted.

Three of the four respondents specified that there were two possible solutions to the quadratic formula and, in consequence, two x-intercepts in the graph of the parabola. Marlene extended this logic by suggesting that of these two, one would be positive and the other negative. In the interview, however, it became evident that there were several other subtleties that she was hiding up her sleeve:

T. On to the last question, how many solutions could there be to the quadratic formula?

S. Two real solutions, a positive and a negative, and no solution at all if you get a negative number in the square root.

T. You don't see any other possibilities for solutions, just the positive and negative versions?

S. Yes.

T. Maybe I can go on to the parabola idea, how many x intercepts could there be?

S. 2.

T. Does it always have to cross through the x axis in two places?

S. Yes.

T. Could it have 0 or 1 x intercept?

S. Yes, it could be further up here then it wouldn't have any x intercepts.

T. So, that wouldn't have any x intercepts?

S. Not crossing the x axis, no.

T. Do you see any connection between the number of x intercepts of the parabola and the number of possible solutions to the quadratic formula?

S. Yes, you'd have both the negative and the positive.

T. So, you'd always have a negative and a positive?

S. Yes.

T. For both, the parabola and the quadratic formula? or just one of them?

S. For both.... No, sorry. Just to the quadratic formula, because you can have a parabola that goes through both negatives.

T. But if the parabola doesn't touch the x axis, what would that imply about the quadratic formula?

S. There's no real solution??

T. How would that happen?

S. If you had a negative number in the square root.

Here is evidence of a lightly shrouded understanding which still has an element of concept-confusion. Marlene, along with several other students, has not had sufficient experience to see all the inter-relations of the relevant concepts so that she might be enabled to give accurate responses to the questions posed. As mentioned earlier, when discussing the Class Response, this topic was tackled in the latter part of the course. In consequence the review time allotted to it was particularly limited. Marlene's excellent effort in the interview, where she was nudged in various directions (as opposed to the sketchy, imprecise written answer), can be seen as an inevitable by-product of her limited experience.

The other inteerviewees demonstrated similar verbal-explanation capacities versus written weaknesses. Marisa, for example, who wrote in the test:

-"5 (a) 2 solutions possible

(b) there are also 2 possible ans. for x int."

She remained constant in her belief that there were only two possible solutions. she was, however, able to extend the logic to the successful sketching of the cubic:

$y = (x + 4)(x - 1)(x - 6)$.

II. Applied Math for Commerce: 201-101C

1. A loan of $1200 is taken at 16% compounded monthly for 2 years. Given the Amortization Table below, complete the first Payment line and explain, in words, the relationship between the figures you enter:

Pay't #	Pay't	Interest	Reduc in Prin	Principal
0	-	-	-	1200
1				

Intentions

-[Can student use the appropriate formula for finding the Payment of an annuity, via Present rather than Future Value?]

-[Is the student able to explain the relationship adequately?]

Class Response

The students' marks (in ascending order) were: 0,0,4,5,7,7,7,15,16,16,17,17,17,18,19,20,20,20,20; the mean was 12.9.

As the marks reflect, the majority of the students had a good grasp of the necessary process, though the explanations were often inadequate or misleading. One student, for example, performed the correct calculation for the payment ($58.76) then commented:

-"amount of payment without interest"

Though, of course the calculation of the regular payments is based on the interest. Another made the correct calculations and found the principal after the first period and then stated that this figure was: "whats left of payments". It is tempting to over trivialize these points, but evidence from class discussions and the student interviews gives strong suggestions that students are often satisfied with mimmicking explanations and techniques without an in-depth understanding of their meaning, (again we refer the reader to Skemp's classifications of Instrumental versus Relational learning).

As the marks indicate, there is a wide gap in the type of responses to this question between "the capable" and "the incapable" mathematicians (if we succumb to the temptation to use Krutetskii's terminology). The weaker brethren made significant conceptual errors in treating the question:

-One student found the Reduction in Principal by subtracting the Interest from the previous Principal (the Interest should be subtracted from the Payment).

-Another treated the loan as a future value as opposed to a present value, though she used the correct logic and explanation in the tabulation.

-A third demonstrated a state of considerable interference by treating the Loan as a Regular Payment and the Present Value of the Loan as the Regular Payment. He then went on to the erroneous explanation that the Reduction of Principal was found by adding (as opposed to subtracting) the Interest to the Payment.

The remaining students either omitted the question or experienced no success in evaluating the key ingredient, the regular payments.

<u>Interview</u>

(i) Why did you select this particular formula? (Referring to the method used to find the payment. There being some confusion as to whether a loan is a present or future value of an annuity.)

(ii) When you calculated Interest, what Period did you use and how did you calculate it?

(iii) Can you explain the relationship between Payment, Interest and Reduction in Principal and the logic behind the table?

The students exhibited some nervousness in answering these questions, which was evident in the laughs of Shaila and Paula. The author believes that this was intended to hide a lack of in-depth unde~standing, though these two students were able to give sufficient explanations to justify their written responses. Shaila, for example treated the Loan as a future value (A), even though she had used the corrrcct procedure and described the future value as "A".

Xanthe, who was also able to produce the right table, was unclear as to the function of interest in the problem. In her written answer she specified that the Reduction in Principal and the Principal column figures were arrived at "not considering interest". When asked in the interview if the new Principal took into account interest, she replied with an overly-nervous:

-"No. Interest does not appear in this column (indicating the Reduction in Principal column).

Mark demonstrated the results of Instrumental Learning (or "rules without reasons" to use Skemp's terminology). He had a significant misunderstanding of the origin and purpose of the table. The interest charge should have been calculated on the previous principal, but his technique is laid out here:

T. When you calculated the interest here, can you tell me how you did it?

S. Uh...I found the regular payments, 58.76 and then I multiplied that number by 16% [not the correct process].

T. Why did you take 16% of the payment?

S. Because that's the rule.

T. How would the interest be built up - from the payments?

S. Because it's the interest off the payments.

The tell-tale comment is all too familiar to mathematics teachers: "**Because that's the rule**", (hopefully because they have heard students use the expression, and not because they themselves have become users of the same term).

2. How many 7-digit telephone numbers can be made in which the first digit is 4 and the last is even, if:
(a) Repeat digits are allowed; (b) Repeats are not allowed.

Intention

-[Does student demonstrate comprehension of the multiplication principal?]
-[Does the student correctly consider choice restrictions first?]

Class Response

The students' marks (in ascending order) were: 5,5,7,7,12,13,13,14,15,15,15,16,17,17,18,18,20,20,20; the mean being 14.1.

Part (a) of this question was answered correctly by 8 students. The most common error, (made by 4 students), was considering that the last "box" could be filled in 4, as opposed to 5 ways. In discussions with individual students (and in the interviews), it became apparent that this error was most likely caused by neglecting 0 as being an even digit.

The other 2 errors were:
-Confusing the number of choices for the first digit (i.e. 1), with the digit 4 itself, there being 2 such errors.
-Entering "9" as the number of choices for the non-restricted digits - again caused by the neglect of the digit 0.

Question 2(b) was successfully answered by 5 of the 14 respondents. There was a host of different types of errors, only one of which was made by more than one student: 3 students neglected the fact that the second restriction reduced the number of availably digits to 8 and wrote: "9 8 7 6 5" for the second to sixth position as the number of choices as opposed to the correct response of: "8 7 6 5 4".

The other errors fell into numerous categories, with answers such as:

-"1x7x6x5x4x3x4",

-"10C1 9C5/10C7",

-"1x8x7x6x5x4x5",

-"1x8x7x6x5x4x1",

-"1C1 + 10P5 +5P1" and

-"1x9x8x7x6x5x2".

These underscore the difficulty that students have with this type of question, where interference with other related questions and concepts is frequent.

Interview

(i) Why are the numbers (of ways to make each choice) multiplied and not added?

(ii) How did you get the number in the first box?

(iii) Can you explain how to preceed and in which order?

(iv) How does "repeats not allowed" change the situation?

The errors made by the four interviewees in this question were of minor proportions. There was some confusion as to whether 0 was an even digit or not, and Mark, for example neglected to notice that he had already used an even digit (i.e. 4) in part (b) of the question. Perhaps the most revealing coment was made by Xanthe, when she was asked: "Why did you multiply these numbers as opposed to adding them?", she replied with a nervous laugh:

-"Because that's what you're supposed to do!"

When pressed further: "If you added them what would be wrong with the logic?", came a rapid, defensive:

-"Because adding them would not make sense."

It was evident from this response that the student would prefer not to be asked again, most likely for lack of knowledge than lack of will, so the point was not pursued.

The danger here should be evident: If students are not reminded, or asked to give the structural background to concepts, there is a strong tendency for them to become expert at processes, but ignorant of the reasoning behind the processes. At worst, the student may seem to be resistent to trying to understand the reasoning behind the processes, if they can perform to a suitable degree of precision. The third possibility is that the student forgets the reasoning once he has developed that "suitable degree of precision".

3. In the following Linear Programming problem, clearly define your variables, set up the constraints and the objective function, sketch the system and explian, in words the significance of the feasible region:

A Publishing Company produces 4 times as many paperbacks as hardbacks. In one year the demand for paperbacks is no more than 5000 and the demand for hardbacks is at least 1000.

Intentions

-[Can student define the variables precisely?]

-[Is the inequality correctly interpreted?]

-[Is the student able to identify that there is no objective function defined in the problem?]

-[Can the student adequately explain the significance of the solution region?]

Class Response

The students' marks (in ascending order) were: 0,0,0,4,5,6,7,11,12,12,12,12,13,13,14,14,15,15,16; the mean was 9.5.

15 students correctly defined the variables; the rest specified "x" as just "paperbacks" and "y" as "hardbacks", neglecting the term "number of...". Though this might be considered to be a minor error, the fact that the variables represented a quantity (in this case, a positive integer) was regularly stressed in class, since an ambiguous representation of the variable often misled the students into making incorrect assumptions.

The inequalities x <= 5000 and y >= 1000 were achieved by all but 3 respondents, but the other constraint relating x and y (i.e. x >= 4y), was a source of some considerable difficulty. 8 students claimed that "x = 4y"; neglecting completely the term "at least" in the question. 3 students wrote: "x <= 4y". 5 students gave the correct inequality and there were miscellaneous answers by the remaining 3:

-"x = 4 (5000) y = 1 (1000)."

-"4y <= 5000 y >= 1000."

-"C = 4x + y."

The question regarding the objective function was ignored by 14 of the respondents; one student actually stated that: "Objective Function not applicable" (this was the only correct response). The remaining 4 gave different interpretations of the Objective Function:

-"Objective Function = 5000x + 1000y = ."

-"objective function: to maximize number of books produced."

-"4y = x OBJECTIVE FUNCTION."

-"z = P = 4y = x."

The interpretation of the Feasible Region was partially dependent on the success of the student in producing a convincing graph representing the constraints of the system. Only 4 were able to do this, of which only one student was able to give a reasonable explanation of the significance of the Feasible Region:

-"Feasible region represents # of paperbacks & hardbacks which should be produced according to demand."

Six students correctly represented the constraints: $x <= 5000$ and $y >= 1000$, but either ommitted, or erroneously interpreted the further constraint: $x >= 4y$.

It is evident that this type of Linear Programming problem poses a series of daunting obstacles to students at this level. There is a history of research in the area of interpreting inequalities (the "student-professor" problem springs to mind, viz. Lochhead, J., 1980 which was discussed in Chapter 2). When teaching this topic, the teacher was struck with a strong tendency on the part of the students to favour the "Instrumental" rather than the "Relational" approach to learning the necessary concepts. There were occasions during CPs in which discussions took place regarding such problems, but it was evident that there was a "panic-like" attitude to the solutions in which the tendency was to generate equations and inequalities with very little forethought. It is the author's view that the development of clear structural relationships in this particular area would require much more time than was available in this course.

Interviews

(i) How did you define the variables?

(ii) How did you determine the inequalities/equations?

(iii) What is the objective function in this problem?

(iv) What exactly is the meaning of the feasible region?

One of the students had difficulty finding the suitable variables and admitted considerable confusion. Two of the three remaining were successful in establishing the appropriate inequalities and gave adequate reasons for their choice of the inequalities. Xanthe wrote 4y >= x as opposed to 4y <= x and when asked how she established that she gave a justification for 4y = x, but rushed via specious reasoning to establish the incorrect inequality.

Xanthe's logic continued to be somewhat suspect when she was asked to indicate the solution region; her constraints would have led her to an open region, but she now selected the correct region bounded by the sides of a triangle. When then asked what was the meaning of the region, came the reply:

-"The region? [pause]
It makes you happy. [laughs]
It's a diagram, it doesn't make a lot of sense. Well, it does if you pick a point and you want to solve the equation."

It was evident that the student was a little nervous, but Skemp's categorizations of learning once again spring to mind in the context of this type of answer.

In the context of the objective function (which, in this problem was not defined), there were some interesting comments. Xanthe felt that there had to be such a function and suggested that it would be "the one with x and y... because it's the only equation there that relates the two variables."

Paula reached the same conclusion and had actually written: "z = x >= 4y." When asked what that meant, this was the response:

-"[laughs] Well, I was a bit confused about that, I think...I think that's the objective function...Yep I'm pretty sure."

Shaila was perhaps more realistic in her indecision:

T. OK. Well, it asks for the objective function, what would that be in this question?

S. I didn't know if we were maximizing or minimizing. Well, I wasn't sure...it says supplier should minimize expenses and maximize profits, but we have two of them: the x and the y and I wasn't sure what to do with them.

Here we see the difficulty caused by the asking of a question which has no answer; the objective function could not be found since there was insufficient data. This was a foreign circumstance for the class, and as mentioned earlier, there was only on student of the entire 19 who wrote the test who pointed out that it was not possible to give an answer to such a question. It may be worth noting that Xanthe made a comment after the interview "off camera". In essence the gist was as follows: She hated this test (i.e. the Structure Test) because she had to explain the reasons for her answers; she much preferred to "just do the questions - give the answers". The implication here is that she (and no doubt many other students) preferred to be tested on the knowledge and memory of procedures and techniques rather than being asked "why".

4. A Bridge hand has at least two hearts.

(a) What is meant by this?

(b) What is the complement of 'at least 2 hearts'?

(c) Using part (b), find the probability of getting a Bridge hand with at least two hearts

Intentions

-[Can student interpret this as '2 to 13 hearts' as opposed to '1 or 2' or 'exactly 2'?]

-[Is the student able to correctly interpret the term 'complement' in this context?]

-[Does the student perceive the true structure of the problem and use the complement concept to correctly reach the answer?]

Class Response

The students' marks (in ascending order) were: 0,0,4,4,4,5,5,5,7,7,8,9,11,12,12,13,13,16,16; the mean being 7.9.

To part (a) of this question 13 students expressed "at least two" as being "two or more" with 3 students ellaborating that the maximum was 13 hearts. Three other students merely rephrased the question, but gave no explanation for the phrase. One student explained thus:

-"From a bridge hand of 13 cards there is a possibility of having two cards of Heart."

Another calculated the probability of getting 1 or 2 hearts, and the last student omitted the question completely.

In part (b), when asked for the "Complement of: 'at least two hearts'", the response success rate dropped dramatically. There were 7 correct interpretations (including the glib: "A bridge hand which doesn't have at least two hearts!"). Six students attempted, with varying degrees of success, tho find the Probability of getting a bridge hand with at least two hearts.

The remainder gave differing, erroneous interpretations, for example:

-"The complement of at least two heart could be getting two cards of spade, diamond, etc."

-"The complement is one heart."

The difficulties experienced in the first two parts of the question exacerbated the students' success in the last part. Even though similar types of questions had been discussed in class, not one student was able to set up the correct procedure for the correct answer. Even those who had correctly interpreted the complement in the previous question were not successful. One such student, for example wrote: 13C0 39C13/52C13 (which would emanate from the interpretation: '0 or less than 1 heart'). The individual responses were as diverse as the students, a few having apparent logical bases (for example one individual whose answer to part (b) v... "the complement is one heart", made a logical extension to that answer; his part (c) answer being:

-" 1 - 13C1 39C12/52C13."

There is an impression in the class responses to this question, that there is a peripheral understanding of the necessary concepts and in particular that of the term "at least". As demonstrated in the Linear Programming problem, the students are a good way off from the development of the necessary generalized essential relationships that would allow them to cope with problems of this complexity.

<u>Interview</u>

(i) Can you explain how you interpreted "at least two"?

(ii) What is the meaning of the term "complement", generally and specifically in this question?

(iii) How did you find the required probability?

All four of the students were sucessful in their explanation of the meaning of "at least two hearts", Paula for example wrote: "At least two is two hearts or more (up to 13). 2 <= Hearts <= 13." In the interview she restated this point: "It could be equal to two or more up to or equal to 13."

The question regarding the complement posed greater difficulties. Paula had written: "The complement is one", and stuck with this line in the interview until asked:

T. Is there any other situation that could be part of the complement?

S. Well, I guess it could be Spades or Clubs or Diamonds, any other number or suit.

T. So, if I had no Hearts, would that be part of the complement?

S. Uh... Ya.

Shaila misinterpreted the question as a probability question and wrote: "1 - P(0 or 1 heart)." This was the starting step for the part (c) answer. In the interview she gave the correct interpretation: "Well the complement of "at least 2" is having 1 Heart or 0 Hearts." When shown her answer to part (b), she admitted some confusion between the two parts of the question.

This "confusion" may have in fact been a matter of interference since Xanthe also made a similar suggestion in her written answer:

-"Complement means opposite so at least two hearts the opposite is at most 2 hearts = 1 - prob(of at least 2 hearts)."

This interference of closely related concepts was not uncommon in this context; in class, on several occasions, students had interpreted "Complement of a set" as being indistinguishable from "The Probability of the Complement", most likely since the concept of Complement was most often used in the context of Probability.

In calculating the probability, the students' responses were consistent with their answers to parts (a) and (b).

5. Three companies receive two newspapers each day; the Gazette and the Daily News. Company A receives 2 Gazettes and 3 copies of the Daily News, Company B gets 4 Gazettes and 2 Daily News and Company C 2 Gazettes and 5 Daily News. The Gazette costs 50c and the Daily News 40c.

From this information, construct two matrices P and Q, such that the product PQ is a matrix which contains the respective cost to each Company, in dollars, for one day's supply of newspapers.

Intentions

-[Is the student able to organize the information into appropriate matrices?]

-[Can he produce a suitable product matrix containing the necessary information?]

Class Response

The students' marks (in ascending order) were: 5,5,8,18,18,18,18,20,20,20,20,20,20,20,20,20,20,20,20; the mean being 17.4.

There is evidence from the student marks that this question was well-received. The three weaker responses were varied: One student attempted to find the profit for each company and produced a 4 X 4 matrix which had the aura of a solution to a linear programming problem. This was a case of mis-categorizing the question type - an occasional and significant cause of student difficulties. Students regularly experienced difficulties of this type, though often to a lesser degree, in particular in the situation of writing tests where the material is not easily identifiable in terms of the subject area, as it would be, for example in the standard classroom setting.

<u>Interview</u>

(i) How did you go about finding P and Q?

(ii) How did you arrange to get an answer in dollars?

(iii) If you had to use a row matrix for the 50c, 40c, how would you do the problem?

The first two interview questions were answered successfully by all the interviewees, bar Xanthe who neglected to make the necessary adjustment to produce an answer in dollars, as opposed to cents.

Part (iii) was also well received; the students realizing that expressing the cost matrix as a row matrix, would require a corresponding adjustment to the supply matrix. Even Mark, who was the weakest in the group, was able to produce the needed matrix:

T. If I asked you to put the cost into a row of matrix, would it be possible to do the question?

S. I don't think so.

T. Not even when you get a 1 by 2 like this, multiplying something? What could you multiply that by?

S. You'd have to multiply it by a 2 by 3.

T. What would the 2 by 3 look like?

S. It would be 2 rows and 3 columns.

T. What numbers would fit into it?

S. [gives correct response]

As far as conceptualization is concerned, the students appeared to quickly absorb the subtleties of matrix multiplication. It is evident, from their responses to this problem, that they seem to have a good grasp of the structural relationships.

<u>Summary of the Structure Tests and Interviews</u>

These twin mechanisms for delving deeper into the students' thinking patterns were most revealing. They uncovered numerous weaknesses and, on the other hand, showed several hidden strengths among the students. It was particularly evident from the Interviews that "one should not always believe what one reads": Students, on several occasions came up with much clearer explanations of their reasoning than was evident from their written responses. The converse was also true: What appeared to be a written demonstration of a reasonable understanding of structure was overturned by a verbal explanation exhibiting significant confusion (the third question in each of the tests were good examples of these inconsistent responses).

Krutetskii's research into differing abilities of students to make generalizations is supported by the findings of several interviews. The weaker students in the two groups (Mark in the 101 class and Stephane in the 211 class), exhibited a lack of ability to generalize and a preference for reliance on rules (witness Mark's erroneous Amortization Table in question 1), or the rudimentary "trial and error" method for finding solutions (recall Stephane's "throw in some numbers and see what works" technique for solving quadratic equations).

There is much evidence in the student errors and misconceptions of the distortion of concepts described by

Byers and Erlwanger (1985) and Ginsberg (1977). It would seem appropriate to repeat two qoutes from these researchers (which were given in Chapter 1):

> **"The meaning the learner constructs may not be the meaning the teacher wants him to construct... in the absence of guidance a student may develop an entirely erroneous conception of mathematics."**
>
> In a similar vein Ginsberg argues that:
>
> "Typically children's errors are based on systematic rules...**children's faulty rules have sensible origins. Usually they are distortions or misinterpretations of sound procedures.**

It is also tempting to think that Skemp's two categories of Learning: Instrumental and Relational, have been given more credence by this investigation. One might even claim evidence of students who use both of these learning devices in parallel. Xanthe, for example, demonstrates a good structural understanding in some contexts (e.g. questions 4 and 5), but is searching in the dark for reasons to support her results in other cases.

CHAPTER 5

Conclusions

This study has scratched the surface of a subject that has hitherto been neglected by all but a few researchers (viz. Burns, Butcher and Saxon); the area of the effects of a certain review technique on students structural understanding. In the Burns study (1960) "review study questions" were used to "stimulate thinking and emphasize meaning and understanding" and the teachers using the technique agreed that they considered they did a "better job of reinforcing arithmetical concepts". Butcher and Saxon attempted to evaluate the effects of giving "spaced" assignments (as opposed to the traditional "massed practice" assignments that teachers usually give at the end of a class or unit). They claim that students using their technique have improved performance; though Saxon, in particular has been shunned by many in the education community for neglecting "students' understanding" in the use of his technique.

The study might be seen as a first attempt to combine the advantages of Burns' review study questions with an element of the Butcher/Saxon approach. A significant departure from those mechanisms being the use of what might be called an "activating element" - Class Presentations.

This process involves the students in the review process in a more intrusive manner than the aforementioned methods.

Other researchers have extracted evidence from varying investigations to direct us to the reasons for students' weaknesses, or helped to categorize students according to their aptitudes to cope with the learning process. Krutetskii, for example, has made significant commentary regarding the categorizing of students by way of their mathematical abilities by comparing their abilities to generalize mathematical concepts, but he offered no mechanisms for the improvement of the status quo. It would seem appropriate to "move out of the laboratory" and try some of the medicines in the field.

This attempt to "use the medicine" may have produced some results worthy of mention and, of course, raised a good number of questions. The three basic branches of the study - the report on the classroom application of the SSR procedure, the results of the Questionnaires and the evidence form the Structure Tests and Interviews, have all had their part in revealing valuable information and raising queries worthy of further investigation.

Like a child embarking on a long journey, one is tempted to ask "Have we arrived yet?" - "No there is a long way to go still!", comes the reply. So it is with research: we move a stone, but a good part of the mountain remains. In this context, the Structure tests and follow-up Interviews revealed the magnitude of the task of effectively and efficiently transferring the many subtleties that make up the subjects we teach to our students.

In class the SSR procedure seemed to benefit the students, though they were not fond of coming to the board for their Class Presentations. The Questionnaires appear to support this suggestion. The students claimed that they learned more in the course than they might have done in the "traditionally taught" class; they experienced an increased confidence in mathematics and better preparedness for the final exam. The majority also suggested that they were better able to understand the material and did not have to resort to merely memorizing techniques. Indeed if the definition of "understanding" were accepted to be:

"The realization of a circumstance owing to previous experience in a similar circumstance",

(to quote a participant at a recent Learning seminar), then it is reasonable to suppose that a system such as the SSR technique would provide a series of "similar circumstances" and hence provide a backbone for the development of clear understanding.

On the negative side of student reaction to the SSR procedure, a minority found the Class Presentation technique very stressful, though others commented that it actually stimulated them into positive action. Some commented that watching CPs was boring, and that they took up too much class time, but they were again in the minority.

The Structure Tests and Interviews took us down to the grass roots with the worms and cobwebs; a close-up view of that attractive weed-free lawn. The evidence there is that there are still many sources of confusion, outcomes of significant interference as well as the occasional pleasant surprise: Though the students have had greater opportunities to see the same type of problems from different angles and hopefully percieve the subtleties and structural inter-connections with other concepts, many showed evidence of continued confusion, or the ill effects of trying to learn without understanding - Skemp's infamous Instrumental Learning. And what were the surprises? In more than a handful of occasions, students who gave impressions, on paper, of not knowing what they were doing were considerably more lucid in person during the interviews.

It would be appropriate at this juncture to make reference to the those questions, among the ten general questions, posed in Chapter 1 (under Objectives) that might be answered by the study. Two of the ten were outside the ambit of the study and are therefore grouped with further questions raised by the study (which appear on p. 228)

Questions (i), (iv), (vii) and (ix) are somewhat interrelated, so the questions are here repeated and followed by a cumulative response:

(i) Can some form of review as suggested above [in the description of the SSR technique] help students and in what ways?

(iv) Would they gradually become aware of its beneficial effects?

(vii) How does it affect what they learn and remember?

(ix) Does the technique seem to change their day to day responses in class?

The Questionnaire and the record of the Class Presentations provides a relevant response to these questions. To questions (i) and (vii) the answer appears to be as follows: The students benefited in terms of increased confidence; an ability to understand the material better rather than merely memorizing techniques and what might be termed an increased "durability" to their conceptual memory; an ability to "keep it warm" for longer, to quote one student.

As far as their awareness of its beneficial effects is concerned, the Class Presentations indicated a steady augmentation of student capacity to properly interpret the structural relationships. There were also comments in the Questionnaire that the technique helped the students "to keep on top of things".

The day to day reactions of the students did indeed seem to be changed by the use of the technique. A workshop on "asking questions in class" is always given at the beginning of the course, and this, coupled with the review discussions following the Class Presentations, sparked a gradually increasing interest in the students. Perhaps the most pertinent comment ralates to the point mentioned earlier concerning increased confidence. The Questionnaire showed that over 65% of the sample either "Agreed" or "Strongly Agreed" that:

"I developed increased confidence in maths through the methods used in this course". (Q37)

"As time went on, I was better able to confront new maths problems because of the techniques used for learning in this course". (Q38)

Questions (v), (vi) and (x) have been grouped together also since a common response appears pertinent:

(v) What are some of the negative side-effects on the teacher and his/her teaching? Is the technique too demanding?

(vi; What are the negative effects on the students and their learning of mathematics?

(x) Does the technique take up too much class time?

The major difficulty that might be encountered by teacher and student is relate` to question (x); the delicate issue of time. This is something of a tight-rope that must be walked with careful consideration of the question of balance: Too much time spent on Class Presentations and the subsequent review/discussion sessions inhibits the teacher's ability to adequately impart the following lecture. However one quickly develops the aptitude to curtail the CP and review session in time to move on to the new material. Keeping track of "who did what" (particularly when several students have attempted the same question) is a demanding occupation, but with care and attention this is soon second nature. In certain classes where the volume of the material is greater, it is often useful to print the CPs in advance and hand them out immediately, this also serving as a useful record for the students (this procedure was often used when teaching Cal II for example).

As a response to the suggestion that the procedure uses too much class time, it is appropriate to give this example: Suppose the "traditional mathematics lesson" on a particular topic were to include ten examples in a massed practice session following an explanation. Due to time constraints the SSR teacher would have to pare the ten examples down to, say four during that lesson, but, (and this might be considered to be the strength of the system), several more examples would appear during later Class Presentations.

Moreover, at that later date, there would be an opportunity to link and contrast them with related concepts.

Apart from the time factor there is the "anxiety" factor of Class Presentations that might be considered a potential negative effect. In Question 3 of the Questionnaire 58% of the students agreed that the CPs made them nervous (35% disagreed). However in Q8 80% agreed that: "Doing a CP was a good way for me to learn", and later (in Q40) 67% agreed that: "as nerve-wracking as the CP experience may have been, it was well worth it". Noteably 23% were indifferent to the latter question and only 10% disagreed indicating that, for them, the stress was not worthwhile. Though a small proportion, this 10% is of some concern and mechanisms to further reduce the tension are worthy of further investigation.

(viii) Do students perform better on tests?

There was a strong consensus among the students that better test and final exam performance was one of the significant benefits of the SSR technique. For example 70% agreed that: "I feel more prepared for the final exam in this course than in most other courses I have taken". (Q29) Responses to questions concerning being "less nervous" for the final and avoiding "cramming" for the final received even stronger affirmative responses. Verbal comments from the students from past courses might aptly be epitomized by the student's comment (see p. 129): "It made me study in a much better way. Also it played a very big role in passing my exams."

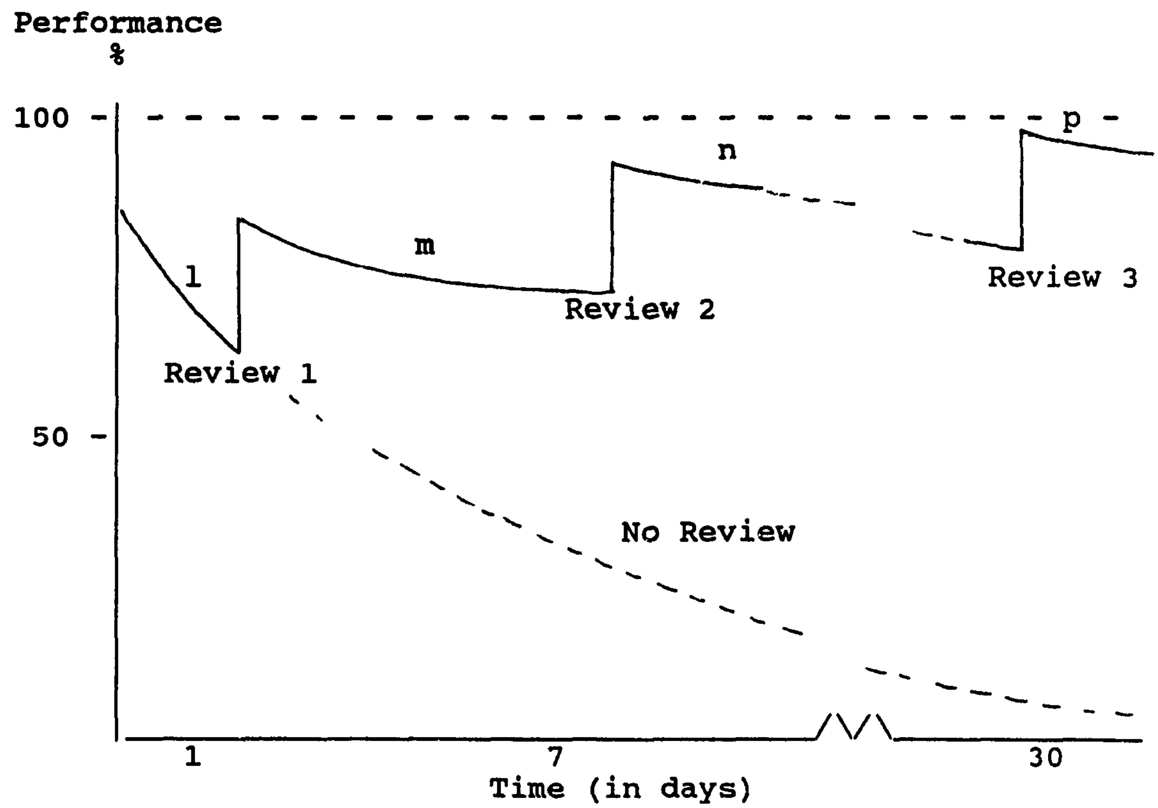
The author was partly inspired to do this research by the work of Rita MacDonald (whose Retention versus Time graph is shown on page 22). An initial hypothesis related to this graph still remains in the back reaches of the mind, and it may be appropriate to make mention of it here. The hypothesis could be expressed in two parts:

1. The better one learns a concept the longer it takes the ability to use that concept to fade from the memory.

2. Structural Spaced Review improves one's ability to deal with a concept, and in consequence, one's "performance" improves.

In graphical terms, these hypotheses might be translated as shown overleaf. The title of the graph relates MacDonald's research to that of the author. Since there have been discussions between the two regarding the graph itself, it is called the "Salusbury-MacDonald Hypothesis".

The Salusbury-MacDonald Hypothesis



The significance of the graph might be explained as follows:

1. The reviews are spaced in ever increasing time periods (as used in the SSR method).
2. The performance does not start at the 100% mark, but at some lower level (this will vary by individual).
3. The performance increases after each review, since the student becomes more aware of the necessary interconnections of the concept with which he/she is dealing.

4. The "drop-off" in performance is less after each successive review. Hence, in mathematical terms, the slopes of the curves l, m, n, and p decrease at a slower rate respectively.
5. The performance of the individual approaches an "asymptotic" value of 100%. The rapidity of the approach being dependent on the individual and the type of review experienced.

This model remains at the hypothesis stage and one of the recommendations is that some effort might be made to substantiate it.

As mentioned earlier, the study gives rise to a number of questions worthy of further consideration (the first three questions emanate from the general Objectives; previously questions (ii) and (iii)):

(a) How would SSR affect good students who apparently remember structural relations of concepts?

(b) How would students in the "weak" and "capable" categories react to such a technique. Would they find it too demanding or boring?

(c) Are we, and should we, be teaching for understanding?

(d) Are we, and should we, be testing for understanding?

(e) Should we change the way we test, including some form of oral testing?

(f) Does the SSR or hybrid system actually increase understanding, and if so how can it be measured?

(g) Could the SSR technique be used in a more pressured teaching environment, such as University?

(h) How would students react to the use of the system in a less pressured teaching environment such as High School or Elementary School?

(i) Would the Butcher/Saxon approach, by itself, be as effective or less effective than the SSR technique?

(j) Could Textbooks be adapted to bolster the SSR method?

(k) What are the long-term effects of the SSR technique? Do students remember concepts and structural relationships better than those who have not undergone such teaching?

(l) How can the anxiety of Class Presentations be reduced?

Recommendations

1. In order to give statistically valid conclusions regarding the advantages of the SSR system a study should be undertaken on a larger sample of students; ideally one which is split into two groups, an experimental group, in which some version of the SSR technique is used, and a control group which undergoes "standard classroom" teaching, or variations of current

practice of classroom teaching. The present study seems to indicate that the technique encourages a better development of conceptual structure in the minds of the students, but this conclusion remains at the intuitive level and would benefit from further investigation on the large scale.

2. A more substantial survey, as suggested above, might also shed some light on the differences in the reactions of the "capable" versus the "incapable" students (to use Krutetskii"s terminology). It is crucial to get a clear evaluation of the effects of the review system on the students across that range. The development of a properly balanced approach to teaching classes with a wide range of ability levels (which is almost the status quo in these post-60's anti-streaming educational environment) is dependent on such an evaluation.

3. The "Distributed Assignment" techniques used by Saxon and Butcher might be integrated into the P/9/7/5/2 system to reinforce its effect. The reader will recall that this procedure involves: "Distributing homework exercises related to a particular topic over several days... as compared to a massed practice approach to assigning homework exercises" (to quote Butcher's summary in the Dissertation Abstracts). Saxon's several mathematics texts utilize this technique, where

close to 80% of the end-of-section exercises relate to material covered in previous sections (see further details in Chapter 2).

4. The SSR system might be attempted at different levels of education (Primary and Secondary for example, where there would be less time constraints on the use of the system). Also, the author believes that the system could easily be adapted for use in other subject areas. There seems to be no reason why, for example, the technique could not be used in such widely divergent areas as English, History, Social Studies and Languages, as well as in the Sciences. In fact, there could be significant benefits in any subject area in which inter-related concepts are put forward.

5. Variations on the P/9/7/5/2 system could be tested to establish a more effective system. In particular, if one were to know what was to be taught class by class in a particular course, then the system could be tailor made to maximize the effects of review. One could also move in the other direction and reduce the rigor of the system by specifying, after each class, the material to be reviewed in the next class.

6. The "Salusbury-MacDonald Review Curve" hypothesis would benefit from some substantiating research. This might be achieved by pre- and post-testing students at each stage of the review process.

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APPENDIX 1

<u>Course Outlines</u>

Algebra 211

Applied Math 101C

Course Outline | John Abbott College | Fall 1989

MATH 201-211
(McKeague)

1. Objectives:

This course is designed for students who need to review or re-learn the basic algebra concepts and skills. It will prepare students to go on to more advanced topics and more sophisticated treatments.

2. Course Content:

Section	Topic	Exercises		
1.1	Basic Definitions	p. 6	# 1-89	J4
1.2	Real Nos.	p. 13	# 3-91	J4
1.3	Inequalities	p. 19	# 1-13	J2
			45-61	J4
1.4	Real Nos., Properties	p. 26	# 3-95	J4
1.5	Addition & Subtr.	p. 33	# 1-97	J4
1.6	Multiplication and Division of Real Nos.	p. 41	# 1-97	J4
2.1	Linear Equations	p. 53	# 3-55	J4
2.2	Linear Inequalities	p. 59	# 1-45	J4
3.1	Exponents I	p. 94	# 1-89	J4
3.2	Exponents II & Sci. Notation	p. 102	# 1-89	J4
3.3	Polynomials, Sums & Diffs.	p. 109	# 1-53	J4
3.4	Mult. of Polynomials	p. 118	# 1-73	J4
3.5	Factoring: G.C.F. & Grouping	p. 124	# 1-49	J4
3.6	Factoring Trinomials	p. 131	# 1-73	J2
3.7	Special Factoring	p. 138	# 1-61	J4
3.8	Factoring Review	p. 141	# 3-55	J4
4.1	Basic Props. of Rational Expressions	p. 159	# 3-51	J4
4.2	Division of Polynomials	p. 167	# 1-49	J4
4.3	Multiplication & Division of Rational Expressions	p. 174	# 3-51	J4
4.4	Addn. & Subtr.of Rational Expressions	p. 182	# 3-75	J4
4.5	Complex Fractions	p. 187	# 3-35	J4
4.6	Equations involving Rationals	p. 193	# 1-49	J4
4.7	Number Word Problems	p. 199	# 1-7	J2

Course Content (Cont'd)

Section	Topic	Exercises
5.3	Simplified Form for Radicals	p. 224 # 1-13 J2
5.4	Addn. & Subtraction of Radicals	p. 229 # 1-17 J2 29-37 J2
5.5	Mult. & Division of Radicals	p. 233 # 3-51 J4
6.2	Solving Quadratic Eqns. by Formula	p. 270 # 3-35 J2
7.1	Graphing in Two Dimensions	p. 300 # 1-49 J4
7.2	Slope of a Line	p. 308 # 1-29 J4
7.3	Equation of a Straight Line	p. 315 # 3-51 J4
8.1	Systems of Linear Equations in 2 Variables	p. 340 # 1-55 J2
9.1	Graphing Parabolas	p. 384 # 1-33 J2
10.1	Relations and Functions	p. 415 # 1-33 J2
10.2	Function Notation	p. 423 # 1-47 J2

Note: "J2" means "by jump of 2" e.g. 1-11 J2 is 1,3,5...11.
"J4" means "by jump of 4" e.g. 3-27 J4 is 3, 7,...,27.

3. Bibliography:

ALGEBRA WITH TRIGONOMETRY FOR COLLEGE STUDENTS 2nd Ed., McKeague. Harcourt Brace Jovanovich

Cost: Approx. $50.00 (Available from Bookstore)

Calculator: A calculator equipped with trig & log functions is required.

4. Methods:

i) Course Structure

This course consists of 45 lessons: 15 weeks of three 1.5 hour classes per week. Each class will consist of a combination of one or more of the following: lectures, class presentations, quizzes, tests and study sessions.

Methods (Cont'd)

ii) Attendance:

Consistant attendance is strongly recommended. There is a department regulation that "nine hours of missed class constitutes a failure. The enforcement of this rule is at the discretion of the teacher."

iii) Department Policy on Academic Dishonesty:

The Mathematics Department considers any form of cheating to be a serious offence. Cheating includes, but is not limited to using unauthorized material, viewing another student's test while the test is being given, copying another person's work, and allowing your own work to be copied. If you are caught cheating you should expect to be penalized.

5. Evaluation: The Final Grade is computed as follows:

i) If a student's class mark is < 45%, the student is NOT eligible to write the final exam, and receives the class mark as a Final Grade.

ii) A student must write the final exam if the student's class mark is ≥ 45%. Any student obtaining less than 45% on the final exam automatically fails the course, in which case the final exam mark is the Final Grade; otherwise the Final Grade is computed from 50% class mark and 50% final exam.

6. Teacher Availability: Office hours are posted outside of his/her office door.

Course Outline   John Abbott College   Fall 1989

MATH 201·101
C O M M E R C E

1. Objectives:

Specific Objectives: The student should be able to:

(a) review and strengthen his/her mathematical skill in functions and algebra;

(b) learn some new mathematical concepts (as listed in the Course Content) related to business, commerce and finance.

General Objectives:

A student who successfully completed this course should be able to:

(a) use terminologies from business, commerce and finance to converse fluently in the business world;

(b) read financial statements and current articles pertaining to the business world;

(c) make rational business decisions that he/she may encounter in his/her everyday life.

2. Course Content

No. of Weeks	Topics	Sections
3	Simple Interest and Discount	5.1
	Compound Interest (incl. finding "n" by use of logarithms)	5.2
	Annuities	5.4
	Present Value of an Annuity: Amortization	5.5
2	Linear Models	2.2
	Slope and Equation of a Line	2.3
	Linear Mathematical Models	2.4
	Quadratic Equations	1.9
	Quadratic Models	3.1

Course Content (Cont'd)

No. of Weeks	Topics	Sections
	Graphing Linear Inequalities	7.1
2	Mathematical Models, for Linear Programming	7.2
	Solving Linear Programming Problems Graphically	7.3
	Systems of Linear Equations	6.1
	Solution of Linear Systems by the Gauss-Jordan Method	6.2
2	Basic Matrix Operations	6.3
	Multiplication of Matrices	6.4
	Matrix Inverses	6.5
	Applications (Solving Systems only)	6.6
	Sets	8.1
2	Venn Diagrams	8.2
	Permutations and Combinations	8.3
	Basics of Probability	8.4
3	Probability of Alternative Events	8.5
	Conditional Probability	8.6
	Applications of Counting	9.2

N.B. The order of these topics and their time specifications are subject to change by your instructor. Relevant problems will be suggested by your instructor.

3. Bibliography: MATHEMATICS WITH APPLICATIONS, Fourth Edition Authors: Lial & Miller

   Course Cost: Text: Approximately $40.00.

   A pocket-calculator with the usual arithmetic operations and with logarithmic and exponential operations (including y^x) is necessary.

4. Methods:

This course meets 3 times a week as scheduled for 1 1/2 hours each session. Classes are primarily lectures, with some discussion and problem-solving. This is usually accompanied by assignments of homework which normally amounts to three or four hours per week.

5. Evaluation: The Final Grade is computed as follows:

i) If a student's class mark is < 45%, the student is NOT eligible to write the final exam, and receives the class mark as a Final Grade.

ii) A student must write the final exam if the student's class mark is $\geq$ 45%. Any student obtaining less than 45% on the final exam automatically fails the course, in which case the final exam mark is the Final Grade; otherwise the Final Grade is computed from 50% class mark and 50% final exam.

Grading:

Term work will consist of a combination of one or more of: assignments, quizzes, class presentations and class tests.

The time of each test will be announced by the instructor in advance.

Math Dept. Regulation: "Nine hours of missed classes constitutes a failure." The enforcement of this regulation is up to each instructor.

Assignments:

Homework assignments are an important part of the learning process. As a consequence, the success of a student depends on how many exercises he has attempted and completed. A student is advised to work on the suggested assignments as soon as possible following the lecture, as the material will be fresher in his or her mind. Repetition of past assignments is also a key element in success in this course.

6. Teacher Availability:

Each teacher will announce and post office hours at the beginning of the semester. See your teacher's office door for further details.

7. Department Policy on Academic Dishonesty

The Mathematics Department considers any form of cheating to be a serious offence. Cheating includes, but is not limited to using unauthorized material, viewing another student's test while the test is being given, copying another person's work, and allowing your own work to be copied. If you are caught cheating you should expect to be penalized.

APPENDIX 2

<u>Interview Protocols</u>

<u>MATH 211</u>

Michael

Steven

Marisa

Marlene

Stephane

<u>APPLIED MATH 101C</u>

Xanthe

Mark

Paula

Shaila

MATH 211

Interview Protocol - Michael

<u>Q1</u>

T. In question No. 1, how do you regard the difference between these two?

S. The one equal to 0 is a parabola and by factoring you get $x = 2$ or 4.

T. In the other one you've got $y = 2$ or 4, how did you get that?

S. I really don't know.

T. How would it look if you graphed this one? [$(x - 4)(x - 2) = 0$]

S. [student does correct graph of the **parabola**: $y = (x - 4)(x - 2)$].

Tape blurred.
Student confused about the difference between quadratic equation and quadratic function.

<u>Q2</u>

T. In question No. 2, is there a difference between the two parts of question a?

S. In part (i) you have to find a common denominator, and in part (ii) you have to multiply it by the lowest common denominator.

T. Can you find an x value in either part?

S. Only in the equation because you have " = 0".

T. Can you therefore, continue with this: $x^2 - 8x - 6 = 0$?

S. You could use the quadratic formula.

T. In part b, is it the same story?

S. You can find the x values only in part (ii).

Q3

T. In question No. 3, how do you see f(x)?

S. You plug x into the equation, or use whatever number x is, like here it changes to (x - 1).

T. So how did you go about finding f(x - 1)?

[At this juncture the student is shown his test answer sheet].

S. [Student considers his previous answer for a few moments.]
I...I don't know...what did I do? Actually, where x is I should plug in (x - 1), and multiply them out.

T. So what would that give you?

S. Pause...$(x - 1)^2 - (x - 1) - 2$. I think I messed up this one on the test! [student had interpreted f(x - 1) as f(x) - 1 when doing the test].

T. OK and when f(x - 1) = 0, how do you get an answer to that?

S. You'd have to use the quadratic formula for that.

Q4

T. And in question No. 4, you say it's false. What would you have to write to get 1/(3x)?

S. It would have to be $(3x)^{-1}$.

T. I see and in part b, you say it's false because 3 is not a common factor of the whole numerator. How could you change the left side in order to get x - 2?

S. You'd have to have 3x - 6 on top.

T. In part c, you say the statement is tru , how did you think that one out?

S. If $f(x) = 1x^2$, then $f(2x) = 2x^2$.

Q5

T. In question No. 5, how did you figure that one out?

S. For the quadratic formula there are two possible answers because of the "b^2 plus or minus".

T. I wonder, if I were to draw a quadratic function for you which goes through the points -1 and 3, could you write down the formula for that one?

S. [long pause]

T. If you go back to the question 1 it was:
$y = (x - 2)(x - 4)$,
so, what would be the equation for this new one?

S. $y = (x - 1)(x + 3)$

T. I see...is it possible for a parabola to have anything other than two x intercepts?

S. Nope.

T. OK, that was very interesting. Thank you very much!

MATH 211

Interview Protocol - Steven

Q1

T. Do you see a difference between these two?

S. Basically, I think they are the same.

T. And, so, how would you answer the question itself?

S. [Tape blurred.]

T. So, what would the graph look like for that one?

S. You have to find the points first, I guess.

T. Do you want to show me on here what it looks like?

S. You get (x - 4)(x - 2), so x = 4 or x = 2.

T. How is that connected to the graphing of the parabola then?

S. I'm not really sure.

T. There's a 0 in one and a y in the other, you said they're essentially the same, or not?

S. I think so, but I don't really know.

T. How would you graph the one that is equal to 0?

S. I really don't know what this is. [Student points to the quadratic function and then the quadratic equation.] Where would I find the y value for this one? [indicating quadratic equation]

T. I don't know.

S. I imagine the slopes would be the same.

Q2

T. Let's go on to the next one. Do you see any essential difference between question 2(a), part (i) and part (ii)?

S. One is an equation and one is just an expression.

T. What does that mean in other words?... Does x have a particular value in either case, in the expression and/or the equation?

S. I don't think so. It shouldn't anyway.

T. What would be the value of x in each of these cases?

S. I got the same thing, but the first had a lowest common denominator and the second one didn't, because this [indicates the 2x multiplying both sides of the equation] is cancelled out, and this [indicating LCD] stays in.

T. Do you see x having a value in either of these cases?

S. Here, I guess, in the quadratic equation, I'd use the quadratic formula, I guess.

T. And then, you could find the values?

S. Yes.

T. And in the expression can you find the x values or not?

S. That's the furthest I got it. I don't know how to get it any further.

T. And is it a similar thing in part b?

S. This is an equation. I broke it down and found the x values. This one here is an expression. I couldn't get it any further (2 (b) part (i)).

T. Could you factor this further (in 2 (b) part (i))?

S. Yes. $(x + 3)(x + 4)$.

T. Would you stop there or would you find any values for x?

S. Yes, x could be -3 or -4.

T. So, you see part (i) and part (ii) as almost identical?

S. Yes, I guess. I don't know why.

Q3

T. Let's go on to question No. 3. How do you see the idea of $f(x)$, what does it mean to you?

S. $f(x)$ is this [indicates $x^2 - x - 2$].

T. How is $f(x - 1)$ different?

S. This would be $(x - 1)^2 - (x - 1) - 2$, so wherever you had an x you now have $(x - 1)$.

T. So, how would you solve part (c)?

S. All you do is plug in 0 into these and you get: $0^2 - 0 - 2$ which gives -2.... So, $f(x - 1) = f(x)$, and if you're giving that a value of 0.

T. So, if $f(x - 1)$ would equal 0, then $f(x)$ would equal 0 as well?

S. Yes.

T. So, 0 would equal $x^2 - x - 2$?

S. But you can't have $0 = -2$, can you?
[student reviews his work]
I'm not sure what's going on.

Q4

T. Let's go down to question No. 4. I just wanted to look at part (c). Can you elaborate a bit on your answer?

S. If $f(x) = x^2$, so you can say $f(2x)$, all you're doing is multiplying the x by 2 and you mulitply this side by 2 also [indicates right-hand side as $2x^2$].

Q5

T. And in question No. 5, this thing about the quadratic formula, what is the quadratic formula?

S. [Student correctly writes out quadratic formula.]

T. At the same time, can you draw me a few parabolas and tell me how many x intercepts there could be for a parabola?

S. [Student draws a parabola with two x intercepts and then draws several vertical lines through parabola.]
Only one. (x intercept)

T. So, the x intercept is an x value?

S. Yes, that follows the x axis.

T. That one you've drawn, for example, how many x intercepts does it have?

S. An infinite number, [he seems to be referring to the number of different vertical lines that can be drawn through the parabola] because it only passes through one point......
[tape blurred]

T. Could the quadratic formula have an infinite number of solutions as well?

S. It depends what values you're plotting, doesn't it? Whatever value you plug in you get a different number.

T. What different types of solutions could you have? [pause] Do you see any connection between this business of the x intercepts and the quadratic formula?

S. [long pause]

T. These a, b, and c values, are they connected to the parabola in any way?

S. It all comes from the equation. Say you had:
$x^2 + 2x + 1$,
This is your "a" value, this is your "b" value and this is your "c" value. [indicates x^2, 2x and 1 respectively]

T. So, what would the x value be in this case?

S. -2 plus or minus 0,...you can't do it. How can you have square root of 0?

T. You can't do that?

S That's undefined.

T. The root of 0 is undefined?

S. Yes,...I think so!

T. OK, that's given me a very good insight. Thank you very much.

MATH 211

Interview Protocol - Marisa

Q1

T. What's the difference between these two?

S. This one is an equation, the one equal to 0. We have to find the x integers.

T. What do you mean by the x integers of the graph?

S. You plotted to find what the x's are.

T. Is that what you mean by the integers?

S. I think that's what I meant.

T. Did you mean intercepts?

S. Oh, yes. I meant intercepts. [student's mother tongue is French]

T. What would the graph look like?

S. Student draws correct graph of parabola: $y = x^2 - 6x + 8$

Q2

T. What's the difference between these two questions: 2a(i) and 2a(ii)?

S. In both you find a common denominator. In the expression it doesn't cancel but in the equation, it does. In part (ii) you find x using the quadratic formula.

T. How about question b, does x have a value in part (i)?

S. [long pause]

T. Could x be anything there or does it have to be something particular?

S. You have to find the factors.

T. In part (ii) you found x had two possible values, does it have values in part (i) as well?

S. No, I don't think so.

Q3

T. Let's look at question no. 3. How do you see this thing $f(x)$? What does it mean to you?

S. Well, $f(x)$ is $x^2 - x - 2$.

T. And $f(x - 1)$ how is that different from $f(x)$?

S. You put $(x - 1)$ in the place of x.

T. And in part c you've got $f(x - 1) = 0$, how do you get this: "$x = (x - 1)^2 - (x - 1) - 2 = 0$" ?

S. I put x would equal this and I worked it out.

T. Does that mean $x = 0$ or this other stuff equals 0?

S. Yes, this equals 0 (indicates latter).

Q4

T. In no. 4 you said $3x^{-1}$ is not equal to $1/(3x)$, so how could you get $1/(3x)$?

S. Pause....You'd have to have $(3x)^{-1}$.

T. In part b, you say it's false, that you cannot cross out the 3's, because of the negative sign 3 is not a common factor. What would you have to have in order to get $x - 2$ as an answer?

S. If you have $x - 2$ in brackets, then you can cross out the 3's.

Q5

T. OK, let's move on to question no. 5. How did you do this one?

S. I put 2 because you can come up with two possible solutions to the quadratic formula.

T. How are they connected, the quadratic formula and the parabolas?

S. There's two possible answers.

T. If I gave you something different like a parabola going through -1 and 5, what might be the function for that one? Can you write it out?

S. No... would it be like x^2 and so on?

T. In question No. 1, you graphed it going through 2 and 4 and the function factored as $y = (x - 2)(x - 4)$

S. So this one would be $(x + 1)(x - 5)$?

T. Yes, great! If I graph something like this that has three x intercepts, say -4, 1 and 6, how would you write that out...if you follow the same logic?

S. $y = (x + 4)(x - 1)(x - 6)$?

T. Yes, exactly. That's a cubic. OK, very good! Thanks.

MATH 211

Interview Protocol - Marlene

<u>Q1</u>

T. Do you see a difference between these two?

S. This is a quadratic equation and this is a quadratic function. [correctly identified]

T. How did you go about answering this?

S. I just took the x intercepts to find both x's for each one. That's why I took it to be the same as on the graph.

T. What would the graph of this function look like? Where would it carve through the x axis?

S. At 4 and 2.

T. When you say x intercepts for this one, ($x^2 - 6x + 8 = 0$), can you explain what you mean?

S. You just simplify first, and get $x - 4 = 0$ or $x - 2 = 0$ and then you find the x values.

T. When you talk about x intercepts what do you mean?

S. I mean the x values.

T. So, the x value is like an x intercept?

S. Yes.

<u>Q2</u>

T. In question No. 2, what do you see as the major difference between part (i) and part (ii)?

S. Part (i) is an expression and part (ii) is an equation.

T. In both cases?

S. Yes.

T. Does x have a value that we can calculate in either or those or in both?

S. I see x having a value here (in equation) and not here (in expression).

T. So, what could x be here? Anything?

S. Yes, except 0.

T. And the same thing in question b?

S. Yes.

T. How did you find the x values in part (ii)?

S. Well, I just simplified and factored....Oops, I made a mistake in this one: the numbers are supposed to add to -8 and multiply to -6.

Q3

T. In question No. 3, what does f(x) mean to you?

S. The value of x is substituted into this equation.

T. And f(x -1), what would that mean?

S. You substitute it [i.e. x - 1] into the equation and get: $f(x - 1) = (x - 1)^2 - (x - 1) - 2$.
[Student simplifies to x (x - 3).]

T. When you got down to this last step, you said: x = 0 or 3, why did you conclude that?
[student treated the expression like a quadratic equation].

S. You just figure it out like you do with a quadratic formula. So, x = 0 or x = 3.

T. And in part c, what were they asking here?
[If f(x -1) = 0, find x.] You got 3 as an answer, how did you get that?

S. I just looked up at part b, if x = 0 in that equation, then I figured x could be 3 as well.

T. So, is it equal to both 0 and 3 or just 3?

S. [pause] I think it's both.

T. Why is it equal to both?

S. Because both would work out.

Q4

T. In question No. 4 (c), how do you see f(2x)?

S. Here you're squaring x in that equation [$f(x) = x^2$]. So, you have to do the same for 2x. So, $(2x)^2 = 4x^2$.

Q5

T. On to the last question, how many solutions could there be to the quadratic formula.

S. Two real solutions, a positive and a negative, and no solution at all if you get a negative number in the square root.

T. You don't see any other possibilities for solutions, just the positive and negative versions?

S. Yes.

T. Maybe I can go on to the parabola idea, how many x intercepts could there be?

S. 2.

T. Does it always have to cross through the x axis in two places?

S. Yes.

T. Could it have 0 or 1 x intercept?

S. Yes, it could be further up here then it wouldn't have any x intercepts.

T. So, that wouldn't have any x intercepts?

S. Not crossing the x axis, no.

T. Do you see any connection between the number of x intercepts of the parabola and the number of possible solutions to the quadratic formula?

S. Yes, you'd have both the negative and the positive.

T. So, you'd always have a negative and a positive?

S. Yes.

T. For both, the parabola and the quadratic formula? or just one of them?

S. For both.... No, sorry. Just to the quadratic formula, because you can have a parabola that goes through both negatives.

T. But if the parabola doesn't touch the x axis, what would that imply about the quadratic formula?

S. There's no real solution??

T. How would that happen?

S. If you had a negative number in the square root.

T. OK, that's great. Thank you very much.

MATH 211

Interview Protocol - Stephane

Q1

T. What's the difference between these two fellows in question 1?

S. The one with the y looks like a parabola.

T. Could you find the x value for either of them?

S. [long pause]...Use the quadratic formula.

T. For which one?

S. The one equal to 0.

T. Could you factor either of them? [long pause]...You're looking for two numbers that multiply to...?

S. 8.

T. And add to?

S. -6... -2 and -4. So, $(x - 2)(x - 4) = 0$.

T. So, what would the x values be?

S. 2 or 4.

T. Could we use this information in that parabola? [long pause] How would it look if you graphed that one? [long pause] Would those two x values be intercepts or anything special on the parabola? [pause] Would the parabola go up or down? [long pause] And would it cross the x axis anywhere?

S. [Student unable to answer any of the above questions.]

S. I think the y intercept would be 8.

[Student draws parabola above x axis, passing through 8 on y axis.]

Q2

T. In No. 2, how do you see the difference between part (i) and part (ii) of question a?

S. One is an equation, the other is an expression.

T. For this one that's an equation, can you find an x value?

S. Yep.

T. And can you find an x value here [in the expression]?

S. Nope.

T. Can you go further in your answer: "$x^2/(2x) - 6/(2x) - (8x)/(2x)$"?

S. Nope.

T. 2x is a common denominator for each part, isn't it?

S. Yes, it is.

T. And that's fully simplified?

S. Yep.

T. And in question b you factored in part (i). Is this as far as you can go?

S. Yep.

T. And in part (ii), can you try this one?

S. [long pause]..You'd have to use the quadratic formula.

T. And do you come up with values for x?

S. Yep.

Q3

T. OK...In question No. 3, how do you see f(x)?

S. Whatever value you get for x you square it, subtract it, and subtract two.

T. So, when you have to find f(x - 1)?

S. You get $(x - 1)(x - 1) - (x - 1) - 2 \ldots = x^2 - 3x$ [correct answer as opposed to previous erroneous interpretation of f(x - 1) as f(x) - 1]

T. So, how would you answer part c?

S. You'd get $x^2 - 3x = 0$.

T. What could x be?

S. 0.

T. How do you get that?

S. Well, $0^2 - 3(0) = 0$.

T. Is there any other solution?

S. [pause]... x could be equal to 3.

T. How did you see that?

S. 3^2 is 9... - 3 x 3 gives 0.

T. Right, so those are the two possibilities. Is there any other way of doing that or were you just trying some numbers?

S. I just throw numbers in and see what happens.

Q4

T. I see..In question No. 4, you said the statement was false, then how could I get 1/(3x)?

S. $(3x)^{-1}$.

T. Great. And in part b, you said it's false because "you cannot remove a 3 from this problem. On top the 3 affects the x and not the 2". If I wanted something that gave me x - 2 as an answer, what would it be?

S. [pause]...(3x - 6)/3.

T. OK, great. In part c, you said the statement was true, what's your thinking in that one?

S. You double the x so you double the answer. [student wrote $f(2x) = 2x^2$ instead of $(2x)^2$].

T. You didn't do No. 5?

S. No, I didn't have time. [student seemed disinterested in continuing].

T. OK Stephane, that was very interesting, thank you very much.

APPLIED MATH

Interview Protocol - Xanthe

Q1

T. In question No. 1, why did you pick this particular formula?

S. I have to know what the regular payment is. This figures out the regular payment, so that's why I used that formula. [formula for regular payments of annuity].

Then I found the amount using compound interest and plugged it into the annuity formula.

T. How did you get this interest figure of $16.00?

S. (Gives correct reasoning for interest calculation.)

T. What's the relationship between the payment, interest, and reduction in principal.

S. (Student gives correct interpretation.)

T. Does the new pricipal take into account interest?

S. No. Interest does not appear in this column (indicating reduction column).

Q2

T. In question No. 2, why did you multiply these numbers as opposed to adding them?

S. Because that's what you're supposed to do!

T. If you added them, what would be wrong with the logic?

S. Because adding them would not make sense.

T. How did you go about choosing the numbers in these boxes?

S. The first number has to be 4, so you can only have one choice there, and the last is even, it's just 2, 4, 6 or 8. But, actually we already said the first number is 4, so you can only have 2, 6 or 8.

T. But, repeats are allowed here.

S. Oh, yes! So, there are 4 numbers for this box. And 0 to 9 means 10 digits for the other boxes.

T. And how does this "no repeats" in part (b) change the answer?

S. It decreases the amount of numbers you can have for each digit. So, it's going to be a smaller number of choices.

T. And what was the order in which you did this question?

S. I did 1 (first box), 3 in the last box (2, 6 or 8). That's two of the digits I've used, so that's 8 left. So, that's 8, 7, 6, 5, 4, 3.

T. And the even digits are just 2, 4, 6 and 8?

S. Yes.

Q3

T. On to the next one.

In question No. 3, how did you go about picking your values for x and y?

S. First of all, I've got paperbacks and hardbacks, that's my two items, so I have a choice x or y, one or the other. Since paperbacks was first, I put it as x, and y is the second one, hardbacks.

T. And these inequalities, how did you pick those out?

S. Well, it says there are 4 times as many paperbacks as hard-backs, so that's how I get the first inequality: 4y is > or = x. [inequality should be reversed]

T. How did you go about checking that?

S. Well, I took h and p instead x and y, and I figured out that p is 1, h is 4.

T. So, that's how you got 4h = 1p?

S. That's how I got the h and p to relate and then I switched them back into x and y and got 4y > or = x.

T. And with the other inequalities you graphed it accordingly, did you?

S. Yep.

T. And is this the solution region in the triangle or is it somewhere else?

S. [student looks at graph and figures out appropriate region-based on incorrect inequality-signalling correctly that region]

T. And what does the region mean to you?

S. The region? [pause]
It makes you happy. [laughs]
It's a diagram, it doesn't make a lot of sense. Well, it does if you pick a point and you want to solve the equation.

T. So, if I pick a point here, for example, does it mean it fits into all these?

S. Yes, it fits into all those equations.

T. And I also asked for the objective function, what would that be in this case?

S. I've no idea what the objective function is. Usually, the objective function, I thought referred to money. Isn't that what we usually use?

T. Maybe.

S. But there's no money talk here.

T. Would there be an objective function then?

S. Well, there has to be.

T. Well, would one of these be the objective function?

S. If I was to take a guess, I'd say it's the one with x and y, so, 4y > or = x. Now, you're going to ask me why, right? Because it's the only equation there that relates the two variables.

<u>Q4</u>

T. OK. Fair deal. Let's have a look at No. 4.

S. Oh, dear! I hate Math and I hate probability!

T. It's says, "what is meant by at least 2 Hearts"?

S. "At least 2" means you can have 2 Hearts, 3 Hearts, 4 Hearts, up to 13 Hearts, because you only have 13 cards in your hand.

T. And what is the complement of "at least 2"?

S. The complement is 0 or 1 Heart.

T. In the calculation you're doing one part for 1 Heart and the other for 0 Hearts and then adding them, are you?

S. Yep.

T. And over here, you're saying the probability is one minus those two pieces. Why do you say that?

S. Well, I know that's the procedure, but I didn't have time to do it.

<u>Q5</u>

T. OK. And in this last question, No. 5, how did you go about finding P and Q?

S. Well, in my original matrix, I think I had them the wrong way around, so I had to switch them, so it was a 2 by 3. I mean a 3 by 2. So, I get a 3 by 2 multiplied by a 2 by 1 matrix giving me a 3 by 1.

T. Is there any way you could have operated with the 2 by 3 matrix without changing it to a 3 by 2?

S. You could write this one as a 1 by 2 instead of a 2 by 1 but then there is no way of multiplying them. Oh, just a minute! You could switch them around and multiply them so you get a 1 by 2 multiplied by a 2 by 3. Ya, that would work.

T. And what would be the dimension of your result?

S. A 1 by 3.

T. And in this question, it asks you something about the units, how can you make sure the answers appear in dollars?

S. Well, uh...you can change the 50 to $.50 that is, .50 and the 40 to a .40 or else divide all your answers by 100.

T. OK. Well, thank you very much. That was very interesting!

APPLIED MATH

Interview Protocol - Mark

Q1

T. In question No. 1, can you tell me why you used this formula to find the regular payments?

S. Well, that's the present value formula and you have to use that to find the regular payments.

T. When you calculated the interest here, can you tell me how you did it?

S. Uh...I found the regular payments, 58.76 and then I multiplied that number by 16% [not the correct process].

T. Why did you take 16% of the payment?

S. Because that's the rule.

T. How would the interest be built up - from the payments?

S. Because it's the interest off the payments.

T. And what is the relationship between these three numbers here?

S. You subtract the interest from the payment to get the reduction in principal.

T. And how did you get this last figure in the principal column?

S. Well, you subtract the reduction in principal from the previous principal.

Q2

T. OK. Let's look down at the second one. What's the process you used in filling in these boxes?

S. First number is a 4, so there's only one in the first box, and repeats are allowed, so it's 10, 10, 10, 10, 10, and the last digit has to be even, so there are 5 even digits. So, that's how I get the five.

T. And in the second one, how did you do the calculation?

S. Well, I put 1 in the first box because the digit has to be 4, and then 5 in the last box like the previous question and then 8, 7, 6, 5, 4. But I wasn't really sure about this one.

T. And this last box, it would still be 5 possible digits, would it?

S. Yes.

T. And which would they be, those digits?

S. It would be: 2, 4, 6, 8 or 0.

T. And the fact that you've used one of those in the first box, does that change it?

S. Oh, yes! That means you can't use 4 again. So, that would be 4 in the last box instead.

Q3

T. Oh, I see.
And let's go down to question No. 3. How did you choose the variables? You said x is year and y is production.

S. I'm really not sure about this problem. I'm having troubles with these.

Q4

T. OK. Let's go on to question No. 4 about the "bridge hand". It says a "bridge hand" has "at least 2 Hearts", what does this mean? You said "at least 2"; there are more than one possibilities. What would they be these possibilities?

S. [long pause... no comment]

T. You're not sure about this one?

S. No, not really.

Q5

T. Let's go down to the last one here. How did you decide on these matrices?

S. First I wrote everything down, Company A, B and C, Gazette and Daily News, and then I slipped it into a matrix and the cost was $.50 and $.40 and so I multiplied them and came up with this 3 by 1 cost matrix.

T. If I asked you to put the cost into a row of matrix, would it be possible to do the question?

S. I don't think so.

T. Not even when you get a 1 by 2 like this, multiplying something? What could you multiply that by?

S. You'd have to multiply it by a 2 by 3.

T. What would the 2 by 3 look like?

S. It would be 2 rows and 3 columns.

T. What numbers would fit into it?

S. [gives correct response]

T. That's great, Mark. Thank you very much.

APPLIED MATH

Interview Protocol - Paula

Q1

T. In this first question, the amortization table, how did you select this formula to get your 58.76?

S. Well, that's the formula you use to find the payments.

T. And what does the 1200 here represent?

S. Well, that's the original principal.

T. OK. And can you explain the relationship between these figures?

S. You take the 58.76 and subtract the interest to get your reduction in principal.

T. Why do you subtract the interest?

S. Because it's being amortized [laughs], or whatever, and you're taking the interest which is [pause] accumulated and then you're subtracting it from the payment.

T. OK. And how does this 42.76 affect the 1200?

S. Well, it reduces the principal.

Q2

T. Let's move on to question No. 2, the probability question. Can you tell me how you got the 10,000,000 there?

S. There are 10 digits available and since you're allowed repeats you can use those digits more than once, so you get:
10 x 10 x 10 x 10 x 10 x 10 x 10 which is 10,000,000.

T. And let's look down to the second one. Why are all the numbers so different in that case? Repeat digits are not allowed.

S. Well, you should do the conditions first [student gives correct interpretation and explanation].

Q3

T. Let's look at the third one here. How did you choose your two variables here?

S. Well, you're talking about producing and exact number of paperbacks and hardbacks.

T. And these inequalities? Can you explain the x > or = to 4y?

S. Well, at first it's confusing because it says at least 4 times as many paperbacks as hardbacks, but if you substitute, like here, I took the number 3 and tried it, y = 3, so 4 x 3 is 12 which is the x.

T. OK.

S. And the x which is the paperbacks has to be 4 times as many as the hardbacks.

T. And how do you interpret this "at least" business?

S. It's says "at least", so you have to have more than 4 times, so it's greater than or equal to.

T. And when you did the graph, which of these things did you put in the graph?

S. Well, I put x and y and x and y > 0.

T. And this x > or = 4y? I see a z written next to that, what does that mean?

S. [laughs] Well, I was a bit confused about that, I think...I think that's the objective...function.

T. You're not sure?

S. Yep, I'm pretty sure.

Q4

T. Let's go on to question No. 4, the question about the "bridge hand". Here you said "at least 2 Hearts" is 2 Hearts or more up to 13, how did you decide on that?

S. Well, you have 13 cards and it has to be "at least 2", so that means it could be equal to 2 or more up to or equal to 13.

T. And the complement of at least 2 is 1, you say, how did you get that?

S. Everything but 2 that means 1 Heart.

T. Is there any other situation that could be part of the complement?

S. Well, I guess it could be Spades or Clubs or Diamonds, any other number or suit.

T. So, if I had no Hearts, would that be part of the complement?

S. Uh... Ya.

T. And when you went on to the calculation of 1 minus this [i.e. 1 minus the probability of 1 heart] , can you explain it to me?

S. Well, "at least 2 Hearts" means 2 or 3 or 4 up to 13, so, an easy way of doing it is to take the complement and say 1 minus the complement.

Q5

T. OK. Let me ask you a quick question on the last one, No. 5. Why did you choose to put the 2 matrices in this particular format?

S. Well, I just did it like that and I thought I'd check it afterwards, I wondered if it would work another way but it seemed to work out.

T. OK. Thank you very much, that's very interesting.

APPLIED MATH

Interview Protocol - Shaila

Q1

T. In question No. 1, why did you use this particular formula?

S. Well, it's the present value formula for annuities, and I need the regular payments.

T. So, "A" represents the present value?

S. "A" represents the present value, yes. [Student uses correct formula but uses the letter "A" for present value instead of "P".]

T. So, you came up with a figure of $21.99?

S. Yes, that's the regular payment. [Correct calculation but because of calculator error, student reaches wrong answer for regular payment.]

T. In the table, how did you calculate the $16.00 for interest?

S. [Student gives correct response.]
...and from then I subtracted the interest from the payment to give me the Net.

T. Why would you subtract those, the payment from the interest?

S. [Long pause]....Well, uh... you have a certain amount and that gives the interest... Well, why is it you make these amortization tables in the first place? [student laughs] Well, you pay the $16.00 of interest from your original payment so, that's how you get the Net or the Reduction in Principal. Then, from the $1200 you take away the $5.99 to get that [indicates the new Principal].

Q2

T. OK. Let's have a look at the next one, the telephone number problem. How did you choose these numbers?

S. The first number has to be 4, so out of the 10 digits there is one to choose from. Then, the last number must be even, so there are four possibilities: 2, 4, 6 or 8. The others, there are 10 different digits so, the're all tens.

T. Is 0 an even digit?

S. I wasn't sure about that one, but maybe it should be five possibilities instead of four.

T. In part (b), how come you get so many different numbers?

S. Well, the first digit there's only one way to choose it, and the last digit there are three ways because 4 is already used. And then we go to the second digit, and since two of the digits have already been used, there are only 8 possibilities, then 7, 6, 5 and 4.

Q3

T. In this paperbacks problem, why did you choose this for x and y?

S. I didn't understand this one too well. I chose x to be the number of paperbacks supplied and y the number of hardbacks.

T. OK. And how did you get this inequality x > or = 4y?

S. Since you can have 4 times as many paperbacks as hardbacks, I was thinking back to the question we had in the test where there were 3 times as many boys as girls and it was the same situation we had there. So, I just substituted and used the same logic for this question. And from that I just isolated y to get y > or = 1/4x. [Student omitted to reverse inequalities.]

T. So, when you switched to y >, how did you do it?

S. I just divided both sides by 4 and then switched the x and y.

T. And if you had to graph these, how would they look?

S. Well, first you'd have to find the intercepts for x = 4y. If x = 0, then y would = 0/4, I think that would be the origin. But I'm a bit confused when you get 0 over 4, I'm not sure if it's undefined or not. That's what really threw me off, I wasn't sure if it was a vertical line or a horizontal line.

T. And these ones here, would they be graphed? Here you wrote: D = 5000 < or = x, and D = 1000 > or = y.

S. Well, "D" is the demand so, x could be > or = 5000 and the demand for hardbacks y, so we get "D" = 1000, which is > or = y. But I didn't know where to put this on the graph. I didn't really understand how to do this.

T. OK. Well, it asks for the objective function, what would that be in this question?

S. I didn't know if we were maximizing or minimizing. Well, I wasn't sure...it says supplier should minimize expenses and maximize profits, but we have two of them: the x and the y and I wasn't sure what to do with them.

Q4

T. OK,...let's go on to question No. 4, where it asks for the meaning of "at least 2". How did you interpret that?

S. Well, the complement of "at least 2" is having 1 Heart or 0 Hearts.

T. OK. But you wrote "1 - P (0 or 1 Heart)", why did you use probabilities?

S. [Pause]...Well, I think I was doing question (c) and then I went back to question (b). Ya, I think I got the two parts mixed up. [Student gave essentially correct response to question (c), but did confuse this answer with the question (b) answer.]

Q5

T. Let's go on to question No. 5. How did you decide to put the numbers in the matrices like this?

S. Well, initially I did it the other way. But it didn't work out, so I switched them around so that I got a 3 by 2 matrix multiplying a 2 by 1 matrix.

T. And why did you change the 50 cents into .50?

S. Because you want the answer in dollars.

T. If I were to say I was going to put .5 and .4 in a row matrix, could you multiply that by something to come up with the answer?

S. [Long pause]...

T. Could you multiply the row matrix by something to get your answer?

S. Well, the other one would have to be 2 by something... Oh, ya, a 2 by 3.

T. And what would the result be?

S. A 1 by 3 matrix.

T. And what numbers would you put in the 2 by 3 matrix?

S. [Student does correct multiplication and comes up with correct answer.]

T. That's great. Thank you very much indeed. That's very interesting.

APPENDIX 3

Pilot Questionnaire Responses

The questions are restated, in order, and followed by all the student responses.

Question 1:

Has the P/9/7/5/2 review system changed your study habits?
- if so, in what way?

Responses:

- The system has improved rather than changed my study habits (in math, at any rate, I still don't study at all in physics or chem). The r.s. has enabled me to learn the entire content of the course rather than understand part by part and then forget everything until an exam. With the r.s. (which I intend to use on my own in later math courses) I can understand much better, and study longer amounts of time with facility.

- Yes, it has changed my habits in that, it forced me to review previous notes more often. That way, when the final comes, there wouldn't be as much to study. It has helped me realize where I stand...what do I know and what I don't !

- Yes, it has, because I always had to look back at all the material we covered. Even if I didn't "study it", I kept it in mind. This is good because now when I study I just have to practice not really teach myself things.

- It's a good system of review that covers the material well. Keeps one from forgetting early details.

- No, they have not.

- Yes, by doing the 9/7/5/2 system, I studied those parts plus I studied everything else as I took some examples from everything we had previously done. By doing the nine "9", I would also do 8, "7", I did [illegible] etc.

- It took some getting used to. But, once in place, it made the "big cram" for the final test a lot easier since I was familiar with the material. It kept everything "warm".

- Yes, it has. It forced me to study continuously everything I've learned from the beginning of the year , so I never really forgot what we did.

- Not really. I tend to review more, but I don't already have much time for anything more! It added onto the load. Maybe it helped for the exam, but we'll see.

- Yes, now I'm aware that keeping up to date for every class is very helpful. I know what I do and don't understand way before there is a test, and I can get help. I now will regularly review my class notes no matter what the course is.

Question 2:

Did you use the system on a regular basis?
- if so, how much time per week?

Responses:

- I would do all the regular assignments (about 1/2 hour a night on most school nights, and 2-3 hours on the weekend) and do the r.s. on a regular basis during my breaks at school (conveniently before math class) 3 times a week for about 45 minutes each time.

- Before every class (75% of the time), I would review my notes for about 35 minutes or so (usually the night before, but sometimes the morning of the class). I am more organized this way too (if I've labelled the days, and have an index for quick easy reference). 1.5 hours per week, on average.

- Not at all

- When I had a feeling you were going to give a C.P. (about twice a week).

- Two or three times per week. Usually about 1/2 hr. before class = 1 1/2 hrs. per week.

- No

- Yes, about one hour per week. Sometimes two hours when I was a bit shaky.

- Whenever I had time to review I used it. Once a week at most.

- Yes, I tried to use it before each class.
  It takes 30 mins. to 1 hr. depending on the material.

- I used it pretty well 2 or 3 times a week, depending on how much time I had.

- Yes. Out of 3 classes per week, I was, on the average, well prepared for a C.P. twice a week.

Question 3 (a):

What did you gain (or lose) from preparing for the CPs?

Responses:

- I lost several nails that were bitten off out of nervousness preparing for the CPs; other than that I thought the preparation (review system) was difficult because I was not used to it, but in the long run I will understand math much better than if we had no CPs.

- Preparing for CPs cost me some time, but it was worth it. Whenever I didn't review for a CP, the next time I tried to get back into the old stuff, I had troubles. A constant review is necessary, for falling behind in Cal II is NOT condusive to getting good marks. Without the CP system, which forced me to see my strong/weak points, I would have fallen into a false sense of security! (Thinking I Know it all and then blanking out on a test) - or the final!

  CPs tell you where you stand. Putting the pressure on, or "tightening the screws" is necessary. You'll only mess up a question at the board once! YOU'LL NEVER FORGET THE EXPERIENCE. Quote Norman Dobson: "Anything that will get you guys to work will help!"

I am better prepared going into this final, also, with your cumulative testing system, this enables me to constantly practice and review old work.

- Didn't

- Yes, it has, because I always had to look back at all the material we covered. Even if I didn't "study it", I kept it in mind. This is good because now when I study I just have to practice not really teach myself things.

- A great deal of anxiety and stress.

- A good review.

- Kept everything "warm".

- Helped me keep on top of things throughout the year. I'm very familiar with all the material because of the CPs.

- I knew what I knew and didn't know in the class material and I could then get help with what I didn't know.

Questions 3 (b) and 3 (c):

What did you gain (or lose) from (b) Doing the CPs and (c) Watching the CPs?

- Most were easy, so I had no problem, but there was one I fouled up and I was so mad at myself that I went and did math homework until it was coming out of my ears. Doing CPs in front of people isn't so bad because you tend not to be aware of them after a while. However, when you foul up, you think everyone is watching you.

  To tell you the truth, I never bothered watching other performances to a great extent, but whenever I did, I could always spot someone not using the review system. Most people performed quite well I thought.

- Doing the CPs made me nervous at times, but you get used to them after a while. If you go to the board and do the problems, you find out if you REALLY know what you're doing!

There's a world of difference between THINKING you know how to do a problem and actually doing it. It's harder than it sounds!

CPs were a real challenge; and sort of MINI TEST before every class session. More than once did I have butterfly in my stomach, but CPs were worth the effort you put in!

Constantly seeing the problems done on the board reinforces what you've learned. You won't forget so fast what was done in September, once you reach December! Also your review and going over those problems cleared up many misconceptions I had. Things I thought I knew were actually not very clear in my head. And if I still didn't understand AFTER you explained the problem, I made a note of it, and came to pay a visit to your office.

In short, keep up your system: patent it quickly, before one of your students begins to teach and use SIR DAVID'S MARVELLOUS system!...before you get the credit!

- Doing the CPs gave chance to "quiz" oneself on abilities and know what to look over for tests.

  See different ways of doing problems (some are an overkill, others are better).

- Getting down as much stuff as I know quickly and only the important things. I think I know what it feels like when someone is about to have a nervous breakdown.

  Seeing that other people are in the same boat as me.

- It helped your confidence when you could do them but confused you when you couldn't.

- After CPs are done they are useful for review.

- I gained a good sense of what it means to be nervous. I don't enjoy looking stupid in front of everybody. Of course, this feeling of stupidity probably was my incentive to know everything we had done until then and made me study harder.

  Unfortunately, since many of us make mistakes, my brain was subject to many false answers by students before they were corrected by Sir David. Like everything, when you see something done wrong enough times, you start to do the same things wrong yourself.

- Practice under pressure. Excitement.

  Not very useful. Couldn't understand writing or else too cramped space to read/learn from properly.

- I learned to work a bit quicker in a test situation.

  It was a good review.

- Since I am a very nervous/worryish type of person, I hated this!

  Watching CPs took pressure off doing them and showed to me that "if he/she can do it, I should be able to do it too".

- Trying to control my nerves in front of a whole class helped me stay calm during my tests (maybe not calm, but calmer).

  Just plain learning.

Question 4:

What adaptions do you think should be made to the review system?

Responses:

- A meaningful answer to such a question takes more than 53,8 cm^2 of paper to write on, but I can tell you that the CPs should be used on a more regular basis, perhaps three or four CPs at a time twice a week. In my other math classes, this teacher would answer questions from students for the first 1/2 hour; I found this method ineffective.

- The review system is fine, though perhaps you could make the fifth CP question come from the day PRIOR to the previous day. (So students have time to pick up the concepts and do the problems in the book - adequate preparation.) I became nervous when I still had problems with the last day's work; and a CP was on it (sometimes). Otherwise, the system is fine.

- None.

- Perhaps not mark them. The embarrassment of messing up in front of the class is enough of a punishment to make one work hard.

- None.

- I preferred just a review of everything we had done until then. I liked taking any examples from anywhere in the book and doing them. The variety of doing more types of examples (from anywhere in the book) was more interesting. I suggest on day "n", the review should be the sequence (1, 2, 3, 4...n).

- Very good method. Maybe more time for review and <u>less</u> assignments. I found little time to review properly.

- I really can't think of any.

- I was satisfied.

<u>Question 5:</u>

Do you have any other suggestions or comments?

<u>Responses:</u>

- Because the questions were not of interest to me, having CPs for the first 20 mins. of class forces everyone to use the review system and keeps everyone's interest.

- On the whole, I find your teaching methods very effective and I encourage you to continue asking for feedback, whether it's positive ("You're so good!") or negative ("You're a son of a [expletive]"). Keep up the good work!

- No other comments. These 2 semesters were quite an experience for me!

- CPs are a great way for a "lazy" or otherwise "busy" student (4 sciences), to do their much needed practice of problems. Thanks!

- The way the marks are divided is good.

- The CPs should not be all of such a difficult calibre. To do well, a student needs to feel that at least he/she has the ability to do well is he/she works at it. The way this semester went, I found that many people despaired, felt that they were idiots and gave up, even though they were working hard. People need some boosters.

- I found the notes and CPs given in class very useful and they covered the material thoroughly. (The textbook was superfluous.)

- Very ingenious (different) marking system. Thanks for the memories.

- I just would like to say that this is a very demanding course and it is time consuming. Trying to keep up in 6 other courses plus a job, plus having some time on the weekend is next to impossible. I had a lot of trouble with the course.

- No.

- I think the system is great! I plan on using the system in all my courses. I think it will help me a great deal. I've never been so up-to-date at the end of a semester on what we've done from the beginning of it. It's been a great help. I think the final should go quite well because of it. Keep it up!