Taxation Policy and Portfolio Decisions Under Uncertainty

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Abstract

Taxation of risky capital income transfers risk, under certain circumstances, to the government constraint. This thesis examines how portfolio/savings decisions and the welfare of an investor are affected under three different assumptions about the disposal of risk by the state. In the first case, the tax revenue risk is disposed in a costless fashion; in the second situation, the risk is returned to the investor; in the third case, it is spread across generations.

I investigate the deadweight loss of capital income taxation under uncertainty in a portfolio/savings model with non-expected utility preferences. I find that the deadweight loss of a capital income tax is lower for an investor that allocates more of his savings to the risky asset relative to the safe asset. The deadweight loss of a capital tax increases with higher risk aversion parameter values; it also rises with higher elasticity of intertemporal substitution values as in certainty models.

I examine the issue of implementing an expenditure tax via the cash flow or the pre-payment method. I find that a pre-payment version of a consumption tax without the inclusion of excess returns will lead to a welfare loss. The utilization of a weighted average discount rate to evaluate the risky consumption tax revenue implies that the

household holds a risky asset that is perfectly correlated with the market portfolio. Investors that hold diversified portfolios and bear the risks of their own tax revenue fluctuations are indifferent as to the method of implementation of a consumption tax, while the investors who hold a less than perfectly diversified portfolio prefer the cash flow approach. A differential incidence analysis indicates sizeable welfare gains from a cash flow consumption tax. In terms of tax policy, I argue that the implementation of a consumption tax through the cash flow method alleviates the controversy over the pre-payment wage tax version.

Finally, I analyze the effects of taxation on risky human capital. The results of the previous analysis do not apply to this problem because the return depends on the amount invested. I find that a consumption tax or a wage tax creates a positive substitution effect on human capital.

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Chapter 1

Introduction

The earlier literature on the effects of taxation on risk taking activity (e.g. Domar and Musgrave (1944), Mossin (1968), and Stiglitz (1969)) assumed that the investors do not bear the risk that is passed on to the government. In such a framework, Domar and Musgrave (1944) established perhaps the most celebrated result in taxation policy, namely that the imposition of an income tax with full loss offset provisions may stimulate risk taking activity. In the Domar-Musgrave analysis a risk averse investor is confronted with a choice between expected income and risk. With the introduction of a loss offset income tax the investor receives a lower expected return and bears less risk. The government becomes a silent partner sharing part of the expected return and risk. They showed that in certain circumstances the investor may be able to increase the amount invested in the risky asset such that the resulting post-tax probability distribution is the same as prior to the imposition of the tax, without loss of satisfaction. The stimulating effects of taxation on risk taking activity have been confirmed in a more general one period expected utility setting (See Mossin (1968), Stiglitz (1969). Ahsan (1974)).

Even many years since its publication, the possibility that a certain capital income tax may induce an increase in the risky asset demand appears counter intuitive or even controversial to many public finance scholars. The most interesting challenge to the Domar-Musgrave phenomenon comes from Summers and Bulow (1984), Hamilton (1987) and Zodrow (1994). They argue against the taxation of risky capital income. They doubt the practicality of the risk shifting via the taxation system. Their belief is that investors, well educated about the diversification theorems in finance, eliminate all unsystematic risks with a portfolio of assets and while the aggregate or systematic risk does remain within the economic system; the investors are compensated for the systematic risk they bear with a higher expected return. Therefore sharing of risks through taxation of risky assets is not possible as the work of Eaton and Rosen (1980) dealing with risky wage income had earlier implied. In the asset choice framework, the state by taxing the risky capital income absorbs some of the expected return and some of the systematic risk. Yes, the state becomes a silent partner in one's portfolio. However, Summers et al. argue that this additional expected tax revenue cannot be used for the provision of public goods because it has no market value to the investor. They argued that the alleged stimulating effects on risk taking relied on the ability of the state to bear risks without cost. Gordon (1985) implied otherwise, that even though the risk remains within the private sector, in a perfect capital market, the government is able to collect sizeable tax revenues from the taxation of risky assets via taxation of pure profits and/or risk premium revenue without causing any additional distortions.

One contribution of my thesis is to re-examine the effects of capital income taxation on portfolio/savings behaviour when choices are made under uncertainty and under alternative mechanisms of disposal of the revenues by the state. The effects of tax policies are examined within a two period life cycle choice model of the Dreze-Modigliani (1972) and Sandmo (1968,1970) type. My conclusion is that the behavioral results depend on the revenue disposal policy of the government. First, I analyze the common case whereby the proceeds of the tax are spent on a public good, which enters the utility function in an additive way. This implies that the stochastic tax revenue has no additional effect on the decision variables and in particular on risky asset holdings. This has been the traditional approach. This modelling of investor's behaviour is used as a benchmark for comparison. An alternative approach is to examine the case whereby the government re-distributes the random tax revenue back to the investor in the same period the tax is collected as Gordon (1985) and Gordon and Wilson (1989) suggested. In this sense the investors ultimately bear the risks of their own tax revenue. A further alternative is to examine the case whereby the state returns a stochastic weighted average tax revenue to the investor. The stochastic per capita weighted average tax revenue returned to the investor is partly his own risky tax revenue and partly the past risky per capita tax revenue. This implies that the current generational investors do not bear their own tax revenue risks but they bear part of their tax revenue risks and part of the risks of the previous generations. I find that the behavioral response and welfare of the investor depends critically on the assumption about the disposition of the tax revenue.

Another contribution of my thesis is to propose a new methodology, which is different from the existing methodology of Stiglitz (1972) in the evaluation of risky tax revenue. I assume that the government either re-distributes proceeds of the tax back to the investors and we let them evaluate the risky tax revenue using an asset pricing model or the state acts as if it sells the proceeds of the risky tax revenue as a promissory note and uses the market value of the tax revenue for the provision of a public good that enters the utility function in an additive manner. This way the traditional approach and the alternative approaches can be compared more directly.

My thesis also contributes to the debate on the appropriate choice of tax base. I argue that a move toward a wage tax system from an expenditure tax system will result in welfare losses. Most proposals argue for switching the personal tax base to an expenditure tax system and away from income taxation. The expenditure tax is favoured over the income tax on the ground that the latter increases the price of future consumption and encourages current as against future consumption. This distortion in intertemporal prices is alleged to create a welfare loss for consumers. Several economists have gone a step further and argued for a tax on wage income on grounds of administrative feasibility and simplicity. My thesis indicates that a movement towards a wage tax system of the pre-payment type will lead to a reduction in welfare. I argue that at least for small investors, the private market is unlikely to provide the socially optimal degree of risk sharing. There is strong evidence that investors do not hold diversified portfolios due to market imperfections (i.e., transaction costs, borrowing constraints etc.). I find that the government, acting as a financial intermediary, can provide some insurance and increase the welfare of the undiversified person without affecting the perfectly diversified investor by choosing a cash flow consumption tax over the pre-payment wage tax system.[1]

Finally this dissertation examines the effects of taxation on human capital under uncertainty. I assume that the proceeds of the tax are spent on a public good. I examine the allocative effects of different tax policies on the decision making process of investors allocating time to accumulate human capital. Here the return on investment depends on the amount invested. Hence the results of the previous analysis do not apply. The recent proposals to alter our educational system in a way whereby students borrowing increases to finance their higher tuition fees is being viewed by some as unfair to the lower income families. One way to make the system more accessible to lower income families is to allow the interest to be tax deductible. This will reduce the cost of borrowing funds and stimulate human capital. Under uncertainty an income tax will stimulate human capital by more than the consumption tax much as is the case under certainty.

More specifically, chapter two examines whether more general capital income tax policies necessarily discourage risk taking. The positive analysis of capital income taxation under uncertainty has only been conducted in a one period model (Domar and

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[1]Examining the U.S income returns, Friend and Blume (1975) and later Gordon (1985) established that a large percentage of the population did not own equity. The holding of a non-diversified portfolio can be explained by dropping the perfect capital market assumption. Market imperfections such as transaction costs, borrowing constraints, asymmetric information, heterogenous expectations can cause investors to undertake private risks which are different than social risks.

Musgrave (1944), Mossin (1968), Feldstein (1969), Stiglitz (1969), Atkinson and Stiglitz (1980)). This past research considered the problem of allocating a fixed amount of wealth to a risky and a safe investment without worrying about the disposition of risky tax revenue. The introduction of portfolio/savings decisions causes capital income taxation policy to affect both the size and composition of the portfolio. Hence, extending the planning horizon into a two period model makes savings an endogenous choice variable and may alter the results of the previous analysis (See Ahsan (1989,1990)). Ahsan (1990c) has examined capital income taxation within this same traditional setting. This chapter reviews these results which are used as a benchmark for comparison.[2] The traditional analysis assumes that the proceeds of the tax are spent on a public good, which enters the utility function in an additive way.[3]

Chapter three examines the normative analysis of capital income taxation under uncertainty. Although the stimulating effects of certain capital income taxation policies on risk taking activity have been well documented in one period models, little if any work on the normative implications has been discussed in the literature, with the exception of Hamilton (1987), Gordon and Wilson (1989) and Koskela and Kanniainen

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[2] These results are based on the assumption that the investors do not care about the transfer of the risk to the governmental constraint. They act in a risk neutral manner towards revenue evaluation.

[3] Stiglitz (1972) assumes that the state has a risk averse outlook towards using risky tax revenue for the provision of public goods. The state's attitudes towards risk need not be the same as individuals belief towards risk in the consumption of private goods. Stiglitz model was a one period model.

(1984). This chapter adds an additional dimension to the normative implications of capital income taxation under uncertainty. I examine the decision making behaviour of an investor under the assumption that the government will re-distribute the random tax revenue back to the investor in the same period the tax is collected and indicate that the results of the previous chapter depends heavily on the assumption about the disposition of the tax revenue. The assumption that the government re-distributes back to the investor the tax revenue, stochastic where relevant, in the form of a uniform lump sum payment ultimately implies that the risk remains within the private sector, and hence investors ultimately bear the risk of tax revenue fluctuations. Gordon (1985) makes a similar assumption and states: "Given that the government absorbs a sizable fraction of the risk as a result of the taxes on corporate income, one might have expected the market risk premium to fall. However, the government cannot freely dispose of the risk it bears. Individuals must ultimately bear this risk, whether through random tax rates on other income, random government expenditures, or random government deficits."

One purpose of chapter three is to compare the behavioral response of the investor under the assumption that the state re-distributes the risky per capita tax revenue back to the household. Does the decision making process of the investor change if the risky tax revenue is returned in a stochastic lump sum fashion? What happens to the size and composition of the portfolio under various capital income tax policies? How do these results compare with the assumption that the risky tax revenue is used for the provision of public goods that enter the utility function in an additive manner? Are there any deadweight losses of taxing capital income under uncertainty? How do they compare with the deadweight losses in the certainty environment?

Even though the tax revenue from capital income is transferred back to the taxpayer the deadweight losses under certain capital income tax policies still exist. Chapter four attempts to provide a measure of the deadweight loss of capital income taxation under uncertainty in a two period portfolio/savings model with non-expected utility preferences. The existing literature dealing with the deadweight losses of capital income taxation is done in a certainty environment (Feldstein (1978), Boskin (1978), Summers (1981), Drifill and Rosen (1983)).[4] How are the deadweight losses of capital income taxation affected in the case where the investor can hold both a risky and a safe asset? The non-expected utility function allows us to separate the relative risk aversion parameter from the intertemporal elasticity of substitution. Through which path, risk aversion or intertemporal elasticity of substitution, does capital income taxation policy affect the optimal levels of consumption, savings, risk taking and deadweight losses? How do the deadweight losses of capital income taxation change when there is a change in the distribution of the asset structure?

Chapter five uses the non-expected utility preferences to examine the effects of capital income tax polices on portfolio/savings decisions. This chapter extends the previous

[4] An exception is Gordon and Wilson (1989) who examine the deadweight losses of capital income taxation in an uncertain environment.

analysis to an infinite horizon and examines the effects of various capital income tax policies on portfolio-savings choice using the non-expected utility function (Svennson (1989), Epstein and Zin (1989), Weil (1990))

Chapter three, four and five examined the behaviour of the investor when the government re-distributed the stochastic per capita lump sum tax revenue payment back to the household. In this sense the investors ultimately bear the risks of their own tax revenue fluctuations. These chapters can be thought as a special case in which the risky outcome is perfectly correlated for all investors (Atkinson and Stiglitz (1980). In this framework the state re-distributes the stochastic per capita tax revenue to the investor and this amount matches exactly the tax payment of the investor.

Chapter six examines a different policy from the previous chapters. I assume that the government re-distributes back to the investors only a part of the stochastic per capita lump sum risky tax revenue the current generation pays. The other part is transferred to the future generations. This implies that there is some risk sharing of tax revenue risks. The current generation receives, for sharing tax revenue risks, with past generations a share of the stochastic tax revenue of the previous lotteries in which they participated even though they were not yet born. I assume that the future generations share with the current investors the current "market" or "social" risks via an uncertain government budget constraint. This way the current generation absorbs a small fraction of the riskiness of the current tax revenue. The other fraction of the current tax revenue is assimilated by all future unborn generations. My argument relies on the idea that what is "systematic" risk at a certain point in time will become "idiosyncratic" risk when this is shared with all current and future unborn generations. Pooling risks across generations through tax revenue uncertainty resembles the diversification theorems of the finance literature. This assumption alters my previous results and new results appear in the literature. The analysis of this chapter is also a prelude to the choice of the tax base presented in the next two chapters.

One principal aim of chapter seven is to evaluate alternative methods of implementation of a consumption tax in a portfolio-savings model. Proposals for implementing a consumption tax can be classified into two alternatives. The first, the cash flow method, allows a tax deduction on savings while the entire proceeds upon withdrawal become taxable. The second method, known as the pre-payment approach, is a modification of the income tax system, where the "return" on an asset is exempted from taxation. It is this second method (pre-payment) that is most controversial. There are economists who argue that all capital income should be excluded from income taxation (i.e. Hamilton (1987), Zodrow (1994)) to make it equivalent to a consumption tax, and hence argue that a pre-payment method or a wage tax is equivalent to a consumption tax under uncertainty. There are others, who argue that one should retain the extraordinary capital gains component in the income tax base, and exempt the imputed riskless return on the entire savings (Ahsan (1990)) to obtain the equivalence between a consumption tax and an income tax, the new tax being baptized the "modified wage tax" (MWT). The argument put forward by Ahsan (1990) requires that capital gains or losses receive the same treatment under both cash flow and pre-payment versions; they both are subject to tax. This chapter derives the conditions which are necessary for a cash flow consumption tax (CFT) to become equivalent to a wage tax under uncertainty. I also obtain the requirements under which a modified wage tax is equivalent to a wage tax. Finally, I obtain the conditions under which a cash flow consumption tax is equivalent to the modified wage tax. Answering the question on the equivalence between a consumption tax and a "wage tax" or a "modified wage tax" in the presence of uncertainty will provide a new perspective on the current tax reform debate on consumption versus the income tax system.

The next chapter, eight, conducts a differential incidence analysis to examine the welfare gains of a cash flow consumption tax vis-a-vis a pre-payment wage tax under the assumption that the social discount rate is equal to the risk free rate. Using the risk free rate to discount the risky tax revenue implies that the state acts in a risk neutral manner or that the tax revenue risks are shared among the current and future generations. This discounting rate causes the cash flow consumption tax to be the preferred method of taxation and yields welfare gains. The differential incidence analysis will determine the magnitude of the welfare gains that can arise from a switch to a cash flow consumption tax from a pre-payment wage tax. If the gains are negligible then the consumption tax might not outweigh the added administrative complexity of the tax system and a pre-payment wage tax will be the front runner. Numerical calculations are presented using the non-expected utility hypothesis. We present results for two cases a) the investor evaluates the risky tax revenue in a risk neutral manner due to the risk sharing arrangement of the state and b) the investor is concerned with the uncertainty and takes explicit account of the risky tax revenue by using the optimal certainty equivalent future consumption in the evaluation of risky tax revenue. The certainty equivalent future consumption is endogenously determined from the preference structure of the household and used for tax revenue evaluation.

Chapter nine deals with the effects of taxation on risky human capital investments. Examination of the impact of taxation on human capital investment when the returns are uncertain has been very limited compared to the amount of research devoted to the investigation of taxation and physical capital. Research in the area of taxation and human capital investment under certainty is quite enormous relative to the literature under uncertainty.[5] Investigation in the area of uncertainty, taxation and human capital investments is only limited to the work of Eaton and Rosen (1980) who basically have shown that there are welfare gains from some risk sharing through taxation of risky wage income. These welfare gains have also been confirmed by Hamilton (1987).

A decision model which includes both physical and human capital investment decisions will provide more insight to tax policy analysis. I examine the allocative effects of

[5] References include Boskin (1975), Heckman (1976), Summers and Kotlikoff (1979), Sgontz (1982), Sgontz and Pogue (1985), Davies and Whalley (1991), Kaplow (1994)).

different tax policies on the decision making process of investors allocating time to accumulate human capital. The effects of tax policy on human capital are important for the following reasons. First, taxation policy may have an important and large impact on the quality of the work force as well as an effect on the amount of work effort as measured by "hours of work" (Eaton and Rosen (1980)). Secondly, income contingent loan-repayment programs that help finance higher education might be financed from payment of taxes (Freidman (1962)). Finally, human capital investment has unambiguously contributed to welfare and economic growth. Individuals investing in human capital acquire skills and knowledge. The acquisition of skills and knowledge is a significant factor in explaining economic growth (Becker (1964), Schultz (1971)).

Chapter ten reports the major findings of this thesis and provides suggestions for future work.

Chapter 2

Capital Income Taxation under Uncertainty

2.1 Introduction

Domar and Musgrave (1944) established perhaps the most celebrated result in taxation policy, namely that the imposition of an income tax with full loss offset provisions may stimulate risk taking activity. In the Domar - Musgrave analysis a risk averse investor is confronted with a choice between expected income and risk. With the introduction of a loss offset income tax the investor receives a lower expected return and bears less risk. The government becomes a silent partner and shares the risk. They showed that in certain circumstances the investor may be able to increase the amount invested in the risky asset such that the resulting post-tax probability distribution is the same as prior to the imposition of the tax. The stimulating effect of taxation on risk taking activity has been confirmed in a more general one period expected utility setting (See Mossin (1968), Stiglitz (1969). Ahsan (1974)).

This chapter examines as to whether more general capital income tax policies necessarily discourage risk taking. This chapter extends the analysis of the effects of capital income taxation in a two period life cycle expected utility model with portfolio/savings decisions.

I examine various capital income tax policies on current consumption and the size and composition of portfolio holdings. Both the uncompensated as well as the Hicks compensated effects are reported. The critical assumption of this chapter is, the traditional assumption, that the proceeds of the tax are spent on a public good that enters the utility function in an additive way. This implies that the state can dissipate risk in a cost-less fashion.

Section 2.2 presents the two period portfolio/savings life cycle budget constraint of a typical investor and the wealth effects are reported in section 2.2.1. Section 2.3 examines a tax on the excess return of the risky asset. Section 2.4 examines other capital income tax policies. A proportional capital income tax is examined in section 2.4.1. A tax on the imputed safe income is examined in section 2.4.2. In section 2.5 I compare the previous capital income taxes and I arrive at some conclusions. The next section, 2.6, investigates the effects of a separate tax on the risky income and one on the safe asset income. This may be viewed as a tax on uncertain corporate dividend income and a tax on the income of the safe asset, respectively.

2.2. The Portfolio/Savings Choice Model

Consider a young individual that works in the first period and earns a non-stochastic wage income (Y_{1t}). The young household in the first period allocates the wage income (Y_{1t}) to current consumption (C_{1t}) and savings (S_{1t}). Total savings may be held in the form of a risky asset (a_{1t}) and riskless investment (m_{1t}). The risky asset yields in the second period an uncertain before tax return of x_{t+1} with mean $E(x) > 0$ and variance σ_x^2. Limited liability restricts $x_{t+1} > -1$. The safe asset yields a risk free return of r. In the second period, when old the household consumes (C_{2t}) the gross proceeds from the riskless and risky investment and leaves no bequests. The period by period budget constraints are given by:

$$C_{1t} = Y_{1t} - (a_{1t} + m_{1t})$$

$$C_{2t} = a_{1t} (1 + x_{t+1}) + m_{1t} (1 + r)$$

or alternatively,

$$C_{2t} = (1 + r) S_{1t} + a_{1t} z_{t+1}$$

where $z_{t+1} = x_{t+1} - r$ is the excess return from the risky asset. The expected excess return is assumed to be positive (i.e, $E(z_{t+1}) > 0$) with variance σ_x^2.

2.2.1 The Wealth Effects

In order to evaluate the impact of the various capital income tax policies on the decision variables of the household we need to determine the wealth effects.[1] The result of this section will provide us with the necessary tools to sign the comparative static results of the problem.

The wealth effects can be determined and signed under the plausible hypothesis of decreasing absolute (DARA) and non-decreasing relative risk aversion (NDRRA).[2] In particular, we can show the following comparative static results:

$$\frac{\partial m_{1t}}{\partial Y_{1t}} > 0 \quad , \quad \frac{\partial a_{1t}}{\partial Y_{1t}} > 0 \quad , \quad 0 < \frac{\partial C_{1t}}{\partial Y_{1t}} < 1 \quad , \quad 0 < \frac{\partial S_{1t}}{\partial Y_{1t}} < 1$$

Under the DARA and NDRRA assumptions the safe asset, the risky asset and current consumption are normal goods. An increase (decrease) in first period exogenous income leads to an increase (decrease) in the safe and risky asset as well as current consumption. In addition, these behavioral assumptions guarantee that marginal propensity to consume lies between zero and one.

The behavioral hypothesis of decreasing absolute and non-decreasing relative risk aversion also leads to an important income elasticity result; namely that the wealth

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[1] The wealth effects have been derived by Ahsan (1989, 1990).

[2] For the derivation of the results in this section see Appendix I section A.I.1.

elasticity of the safe asset is greater than or equal to the wealth elasticity of savings which in turn, is greater than or equal to the wealth elasticity of the risky asset. Thus a one percentage increase in wealth will lead to a greater percentage increase in the safe asset than to total savings or to risky asset holdings Specifically,

$$\eta^m \geq \eta^s \geq \eta^a$$

For the special case of constant relative risk aversion (CRRA) the wealth elasticity of savings, risky asset, safe asset and consumption are all equal to unity. Consequently, the CRRA hypothesis will give us the sharpest results, in the comparative static analysis of the tax policy.

$$\eta^s = \eta^a = \eta^m = \eta^c = 1$$

2.3 The Domar-Musgrave Phenomenon (1944)

The Domar-Musgrave phenomenon (DM) can be replicated in a two period setting by imposing a loss offset proportional tax (t_z) on the excess return of the risky asset. Consider a young generation (t) investor who maximizes a separable expected utility defined over current and future random consumption by choosing current consumption C_{1t} and the amount invested in the risky asset a_{1t}:[3]

$$Max \quad V = g(C_{1t}) + E(h(C_{2t}))$$

where $g(C_{1t})$ and $h(C_{2t})$ are well behaved continuous functions and have positive (i.e. $g', h' > 0$) and diminishing marginal utility for current and future consumption (i.e. $g'', h'' < 0$), subject to the constraint:

$$C_{2t} = (1+r)S_{1t} + a_{1t}\,(1-t_z)\,z_{t+1}$$

The first order conditions with respect to current consumption and risky asset holdings are given by:

$$g' - (1 + r)\,E(h') = 0$$

$$E(h'(1-t_z)z_{t+1}) = 0$$

The first order conditions remain unaffected by this tax policy. The first condition states that at the optimum the expected marginal utility of current consumption is equal to the

[3] The assumption of a separable utility function is not necessary for the DM phenomenon to appear. The DM phenomenon is independent of the households preferences and can be derived under any well behaved utility function.

future value of expected marginal gain from future consumption. The second condition states that at the optimum the expected marginal gain from the risky asset is equal to that of the safe asset in terms of their contribution to future consumption. The effect of this tax on the current, future expected consumption and expected utility is given by:[4]

$$\frac{\partial C_{1t}}{\partial t_z} = 0 \quad , \quad \frac{\partial E(C_{2t})}{\partial t_z} = 0 \quad , \quad \frac{\partial V}{\partial t_z} = 0$$

This tax does not affect the optimal choices of current consumption and expected future consumption. The expected growth rate of consumption and that of wealth remain unaffected by the imposition of this tax. The tax does not cause any expected utility loss to the investor. This happens because the investor alters her (his) portfolio holdings the following way:

$$\frac{\partial a_{1t}}{\partial t_z} > 0 \quad , \quad \frac{\partial S_{1t}}{\partial t_z} = 0 \quad , \quad \frac{\partial \beta_{1t}}{\partial t_z} > 0$$

The individual investor increases the risky asset holdings and reduces the safe holdings in a way whereby total savings remain unchanged. This implies that the fraction of savings allocated to the risky asset increases and is measured by proportional risk taking $\beta_{1t}=\alpha_{1t}/S_{1t}$. It is optimal for the investor to respond by altering his portfolio in a way whereby the pre-tax optimal values continue to be realizable post tax. The exact effect of the tax on risky asset is:

[4] See Appendix I section A.I.4.

$$a_{1t}(t_z) = \frac{a_{1t}(0)}{(1-t_z)}$$

where $\alpha_{1t}(0)$ is the pre - tax risky asset investment.

The tax on excess returns reduces both the excess return and the variance of the risky asset return. This effect makes risky asset demand more attractive on the margin relative to the safe asset for any risk averse individual. If C_{1t}^*, C_{2t}^* were the pre-tax optimum consumption bundle the investor would increase the amount invested in the risky asset and reduce that of the safe asset to maintain the same level of satisfaction as prior to the imposition of the tax.

Hicks Compensated Effects of a Tax on Excess Returns:

Hicks compensation treats investors in a way whereby the change in the endowment of the investor is sufficient so that the investor can attain the same level of satisfaction as prior to the imposition of the tax. The required adjustment in first period income is given by:[5]

$$\frac{dY_{1t}}{dt_z}\bigg|_{\bar{V}} = 0$$

Although the investor is taxed on the excess return of the risky asset, the individual investor voluntarily alters the portfolio composition he (she) holds in order to face the

[5] See section A.I.4.1. of Appendix I.

same probability distribution without loss in expected utility and hence no need to be Hicks compensated. The utility compensated effects of the tax are the total effects reported above.

2.4 Capital Income Taxation Under Uncertainty

In this section we will examine the specific and Hicks compensated tax incidence effects of various capital income tax policies.[6] First a proportional full capital income tax of t_k will be imposed. Second we examine a tax on the imputed safe income t_r. A tax on the imputed safe income exempts the excess return component and therefore this capital income tax policy does not affect the variance of the security or portfolio of the investor. Third we compare the effects of a tax on the return of the safe income ($t_r = t_m$) with a tax on the risky income source (t_x). The budget constraint is given by:

$$C_{2t} = (1+r(1-t_r))S_{1t} + a_{1t}((1-t_x)x_{t+1} - (1-t_m)r)$$

[6] All analytical results are derived in appendix I. For explicit expression see the relevant section of Appendix I.

2.4.1. A Full Capital Income Tax (t_k)

The post tax intertemporal budget constraint is given by:

$$C_{1t} + \frac{C_{2t}}{(1+r(1-t_k))} = Y_{1t} + \frac{a_{1t}(1-t_k)z_{t+1}}{(1+r(1-t_k))}$$

From the budget constraint we observe that the capital income tax distorts the relative price of future consumption. The capital income tax increases the price of future consumption. Secondly, a full capital income tax reduces the excess return and variance of the risky asset. The overall effect of this tax on the decision variables cannot be determined apriori due to the conflicting substitution and income effects.[7]

Hicks Compensated Effects of a Capital Income Tax:

The full capital income tax reduces the investors income. For Hicks compensation the required adjustment in first period income is given by:[8]

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[7] See Section A.I.2 of appendix I.

[8] See section A.I.2.1 of appendix I.

$$\frac{dY_{1t}}{dt_k}\Big|_{\bar{V}} = \frac{r\,S_{1t}}{(1+\bar{r})}$$

$$where \quad \bar{r} = r(1-t_k)$$

Even though the tax base is full capital income, the income effects turn out to be proportional to the present value of the imputed secure income. This happens because part of the capital income tax falls on the excess return of the risky asset and this involves a substitution effect with no deadweight losses as the initial analysis indicated. The individual investor voluntary alters the portfolio composition he (she) holds in order to face the same probability distribution without loss in expected utility and hence no need to be Hicks compensated (i.e. no income effects for the taxation of excess returns). The utility compensated effects on current consumption:

$$\frac{\partial C_{1t}}{\partial t_k}\Big|_{\bar{V}} > 0$$

If investor is compensated to stay on the initial indifference surface the tax will induce preference towards current consumption and away from future consumption. This occurs because the tax increases, just as under certainty, the price of future consumption. Future consumption becomes more expensive hence the investor unambiguously increases current consumption.

The utility compensated effect on portfolio choice is given by:

$$\frac{\partial a_{1t}}{\partial t_k}\Big|_{\bar{V}} \gtrless 0 \ , \ \frac{\partial S_{1t}}{\partial t_k}\Big|_{\bar{V}} \gtrless 0 \ , \ \frac{\partial \beta_{1t}}{\partial t_k}\Big|_{\bar{V}} > 0$$

The effect on risk taking is ambiguous under the existing behavioral assumptions. The more expensive future consumption causes risk taking to decline since the risky asset is a mean of acquiring future consumption. In addition, the full capital income tax reduces the excess return and variance of the risky asset. This latter effect, exactly the DM phenomenon, makes the risky asset more attractive on the margin relative to the safe asset for a risk averse individual. The overall effect of a full capital income tax on risk taking activity cannot be determined without imposing additional restrictions in the investors' behaviour. Also the response of the investors' savings behaviour cannot be determined without additional restrictions. There are two effects operating on the individual's savings behaviour. First, since future consumption is more expensive now, total savings, a means of transferring consumption into the future, declines. Second, the Hicksian compensation increases savings. Thus, the two effects on savings are operating in the opposite direction.[9] What is even more interesting is that proportional risk taking unambiguously increases.

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[9] The impact on savings of a capital income tax depends on whether the compensation is paid in the first or second period. If the compensation is done in the first period, there is a stronger tendency for savings to increase. However, as was pointed out by Professor Davies, savings will decline if compensation is paid in the second period and savings are defined as first period income less consumption (i.e., $S = Y_1 - C_1$). I find that the timing of the compensation does not alter the effects of the capital tax on risky asset holdings, current consumption and expected future consumption. In this paper, I assume that the compensation is paid in the first period. Under this case, the impact of the capital tax on the pattern of savings is determined by the two opposing effects.

The portfolio becomes riskier. This implies that the elasticity of the risky asset with respect to the capital income tax exceeds the elasticity of savings with respect to the tax.

Special Case: CRRA

Under the special case of constant relative risk aversion the utility compensated results are even sharper. In fact, a sufficient condition for unambiguous conclusions is the reasonable assumption of a relative risk aversion parameters of greater than unity. This constrains the elasticity of substitution to be less than one. The elasticity of substitution determines the strength of the distortion that is created by the capital income tax on current consumption, risk taking and savings behaviour. Since the elasticity of substitution is in general found to be small (Hall (1988)) the effect of a capital income tax on current consumption should be of a small magnitude. The negative impact of the tax on risk taking activity will also be of a small magnitude if the elasticity of substitution is small. This allows the Domar- Musgrave phenomenon to dominate the effect and causes risk taking to increase. In addition, savings are not discouraged with a low elasticity of substitution. The Hicks compensation term dominates and increases savings. The result under CRRA indicate that savings and risk taking are encouraged for all relative risk aversion parameters greater than unity.[10] The utility compensated effects

[10] If the preference structure was of the non-expected utility type (see next chapter), then the distortionary effects of capital income tax operate through the elasticity of substitution and not through the relative risk aversion. Hence, since the elasticity of substitution is observed to be small (Hall 1988) the utility compensated effect of this tax could be close to the taxation of excess returns. A capital income tax will not distort

of the tax under CRRA and an elasticity of substitution of less than unity are as follows:[11]

For all $\theta \geq 1$

$$\frac{\partial a_{1t}}{\partial t_k}\Big|_{\bar{V}}^{CRRA} > 0 \ , \ \frac{\partial S_{1t}}{\partial t_k}\Big|_{\bar{V}}^{CRRA} > 0 \ , \ \frac{\partial \beta_{1t}}{\partial t_k}\Big|_{\bar{V}}^{CRRA} > 0$$

The Total Effects of Capital Income Taxation:

In order to obtain the full specific effects of capital income taxation, it is necessary to include the income effects. The total effect of the tax on the decision variables is given by:[12]

$$\frac{\partial X_{1t}}{\partial t_k} = \frac{\partial X_{1t}}{\partial t_k}\Big|_{\bar{V}} - \frac{rS_{1t}}{(1+\bar{r})}\frac{\partial X_{1t}}{\partial Y_{1t}}$$

$$\textit{where} \qquad X_{1t} = C_{1t} \ ; \ a_{1t} \ , \ S_{1t} \ , \ \beta_{1t}$$

Since all assets and consumption are normal goods in this model adding a negative term to the Hicks compensated effects only generates ambiguities. However, for the class of CRRA preferences we obtain unambiguous conclusion for values of the elasticity of

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consumption choices if the elasticity of substitution is zero.

[11] See section A.I.2.3 of appendix I.

[12] See section A.I.2.2 of appendix I

substitution less than unity:[13]

For all $\theta \geq 1$

$$\frac{\partial C_{1t}}{\partial t_k}|^{CRRA} < 0 \ , \ \frac{\partial E(C_{2t})}{\partial t_k}|^{CRRA} < 0 \ , \ \frac{\partial a_{1t}}{\partial t_k}|^{CRRA} > 0 \ , \ \frac{\partial S_{1t}}{\partial t_k}|^{CRRA} > 0 \ , \ \frac{\partial \beta_{1t}}{\partial t_k}|^{CRRA} > 0$$

For low values in the elasticity of substitution the distortionary effect, which is proportional to the elasticity of substitution, is weak. This allows the income effect to dominate, and reduces current consumption and increases savings. For the risk taking decision, the DM effect dominates all the negative terms and leads to an unambiguous stimulus. Proportional risk taking also increases.

An Interim Summary:

This chapter extends the portfolio choice model to include endogenous savings. I assumed that the tax revenue, stochastic where relevant, is used to provide a public good that enters the utility function in an additive manner. In other words private investors no longer face the risk taken on by the state via tax policy. This assumption is critical for these results. Even if this assumption is maintained different results from the standard pure asset choice model are detected. In the standard pure asset choice models, a full capital income tax increases the demand for the risky asset under NDRRA (e.g., Atkinson - Stiglitz (1980), p. 107). In the two period model we need stronger assumptions, namely that the relative risk aversion is of the constant type and have a

[13] See section A.I.2.4. of appendix I.

value of greater than one or that the elasticity of intertemporal substitution be less than one.

Similarly savings are encouraged from a capital income tax under the assumption of CRRA and a relative risk aversion parameter of greater than one, even though expected future consumption falls. This outcome confirm's Feldstein's (1978) belief. Referring to a tax cut he argued: "since the net price of future consumption has fallen, this extra quantity of future consumption can be achieved without an equiproportionate increase in savings. In fact, it is perfectly possible for future consumption to increase and for current savings to decrease" (p. S31).

2.4.2. Taxation of the Imputed Safe Income

The post tax intertemporal budget constraint is given by:[14]

$$C_{1t} + \frac{C_{2t}}{(1+r(1-t_r))} = Y_{1t} + \frac{a_{1t} z_{t+1}}{(1+r(1-t_r))}$$

This tax applies to the imputed secure income on the entire portfolio regardless of whether it is allocated to the safe or risky asset. Since the excess return is exempted from taxation capital losses will have to be borne by the investor alone. Hence, the government revenue is non-risky. Here the investor would be unable to merely adjust his portfolio holdings so as to face the same probability distribution as prior to the imposition of the tax. Therefore, the DM phenomenon is absent. From the budget constraint we observe that the capital income tax that exempts the excess return component distorts the relative price of future consumption just like the full capital income tax. The tax increases the price of future consumption.

Hicks Compensated Effects:

This tax clearly reduces the investor's wealth and causes income effects. The Hicksian compensation so that the investor can attain the same level of satisfaction as prior to the

[14] See Section A.I.3

imposition of the tax is, not surprisingly, the same present value of the imputed secure income as was the case with the full capital income tax. The utility compensated effect of the tax on current consumption:[15]

$$\frac{\partial C_{1t}}{\partial t_r}\Big|_{\bar{V}} > 0$$

The distortionary effect of the tax increases current consumption.

The effect on portfolio composition as well as size is:

$$\frac{\partial a_{1t}}{\partial t_r}\Big|_{\bar{V}} < 0 \ , \ \frac{\partial S_{1t}}{\partial t_r}\Big|_{\bar{V}} \gtrless 0 \ , \ \frac{\partial \beta_{1t}}{\partial t_r}\Big|_{\bar{V}} \lessgtr 0$$

The effect on savings is, similar to the case of full capital income, indeterminate. The effect on risk taking is unambiguous in this case due to the absence of the DM phenomenon. The dearer future consumption causes risk taking to decline. The behaviour of proportional risk taking in this case is ambiguous.

Special Case: CRRA

The utility compensated effects of the tax on the portfolio composition under CRRA:[16]

$$\frac{\partial a_{1t}}{\partial t_r}\Big|_{\bar{V}}^{CRRA} < 0 \ , \ \frac{\partial S_{1t}}{\partial t_r}\Big|_{\bar{V}}^{CRRA} \lessgtr 0 \ , \ \frac{\partial \beta_{1t}}{\partial t_r}\Big|_{\bar{V}}^{CRRA} < 0$$

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[15] See section A.I.3.1 of appendix I.

[16] See section A.I.3.2. of appendix I.

The results under CRRA indicate that risky asset demand is discouraged, savings are ambiguous but proportional risk taking declines unambiguously. The portfolio becomes less risky. However, for all relative risk aversion parameters greater than unity the following results emerge:

For all $\theta \geq 1$

$$\frac{\partial a_{1t}}{\partial t_r}\Big|_{\bar{V}}^{CRRA} < 0 \ , \ \frac{\partial S_{1t}}{\partial t_r}\Big|_{\bar{V}}^{CRRA} > 0 \ , \ \frac{\partial \beta_{1t}}{\partial t_r}\Big|_{\bar{V}}^{CRRA} < 0$$

Savings are encouraged. Savings increase because the elasticity of substitution is small. With a small elasticity of substitution the effect of the tax on current consumption is weak and the Hicksian income compensation dominates.

The Total Effects of a Tax on the Imputed Secure Income:

In order to obtain the full specific effects of the tax on the imputed secure income, it is necessary to include the income effects. The total effect of the tax on the decision variables is given by:[17]

$$\frac{\partial X_{1t}}{\partial t_r} = \frac{\partial X_{1t}}{\partial t_r}\Big|_{\bar{V}} - \frac{rS_{1t}}{(1+\bar{r})}\frac{\partial X_{1t}}{\partial Y_{1t}}$$

$$where \qquad X_{1t} = C_{1t} \ , \ a_{1t} \ , \ S_{1t} \ , \ \beta_{1t}$$

$$where \qquad \bar{r} = r(1-t_r)$$

[17] See section A.I.3.3. of appendix I.

Risky asset holding unambiguously declines since the income effect reinforces the substitution effect. For the other decision variables the income effect conflicts with the substitution effect. However, for the case of CRRA the effects are once again the sharpest. Especially for all relative risk aversion parameters greater than one we obtain:

For all $\theta \geq 1$

$$\frac{\partial C_{1t}}{\partial t_r}\Big|^{CRRA} < 0$$

For low values in the elasticity of substitution the distortionary effect on current consumption, which is proportional to the elasticity of substitution, is weak. This allows the income effect to dominate and, accordingly, reduces current consumption and increases savings. The effect of the tax on portfolio composition is:

$$\frac{\partial a_{1t}}{\partial t_r}\Big|^{CRRA} < 0 \ , \ \frac{\partial S_{1t}}{\partial t_r}\Big|^{CRRA} > 0 \ , \ \frac{\partial \beta_{1t}}{\partial t_r}\Big|^{CRRA} < 0$$

Proportional risk taking unambiguously declines. Hence, the proportion allocated to the safe asset increases.

2.5 A Comparison

The results of a full capital income tax and that of a tax on the safe imputed income are different due to the presence of the DM phenomenon in the case of a full capital income taxation as opposed to that of a tax on the imputed safe income.

The Hicks compensation effects of a full capital income tax policy and of an imputed safe income tax on current consumption indicate the same pattern, namely, a stimulus on current consumption due to the more expensive future consumption. However, the two tax policies affect portfolio choices differently. The Hicks compensated effects indicate that an imputed safe income tax discourages the risky asset while that of a full capital income tax the effect on the amount invested in the risky asset is indetermine except for CRRA and a relative risk aversion parameter of greater than unity in which case risk taking is stimulated. In addition, the Hicks compensated effects of both tax policies on savings is in general ambiguous except for the CRRA case with a relative risk aversion parameter of greater than unity whereby savings are encouraged. Finally, the Hicks compensated effects of an imputed safe income reduces proportional risk taking while that of a full capital income increases proportional risk taking.

For the total effect the only unambiguous conclusions can be derived under the assumption of a CRRA with a relative risk aversion parameter of greater than unity. Under this case a full capital income tax policy discourages current consumption while it encourages risk asset, savings and proportional risk taking. On the other hand, a tax on the imputed safe income discourages current consumption, risky asset demand and proportional risk taking and encourages savings.

2.6 Differential Taxes on the Assets

2.6.1. A Tax on the Return of the Risky Asset

The post tax intertemporal budget constraint is given by:[18]

$$C_{1t} + \frac{C_{2t}}{(1+r)} = Y_{1t} + \frac{a_{1t}((1-t_x)x_{t+1}-r)}{(1+r)}$$

From the budget constraint we observe that the tax on the return on the risky asset distorts the relative prices of the two assets. The tax does not cause an intertemporal distortion, as was the case of a capital income tax. The investor upon observing this tax substitutes (reallocates the portfolio) away from the less attractive asset (i.e risky asset) towards the more attractive asset (i.e. safe asset). On the other hand, this tax also reduces the return and variance of the risky asset. This makes the risky asset more attractive on the margin. Thus we should observe the DM phenomenon. Evidently the two forces are in conflict. The next section presents the qualitative results of this exercise.

Hicks Compensated Effects of a Tax on the Risky Return:

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[18] See section A.I.5. of appendix I.

The required adjustment in first period income is given by:[19]

$$\frac{dY_{1t}}{dt_x}\Big|_{\bar{V}} = \frac{a_{1t}}{(1-t_x)}\frac{r}{(1+r)}$$

With the above positive compensation amount the investor will remain on the same indifference surface as prior to the imposition of the tax. The investor faced with a different set of prices will alter her (his) decision rules. The utility compensated effects on current consumption:

$$\frac{\partial C_{1t}}{\partial t_x}\Big|_{\bar{V}} > 0$$

If investor is compensated to stay on the initial indifference surface the tax will induce preference towards current consumption and away from future consumption.

The utility compensated effect on portfolio choice is given by:

$$\frac{\partial a_{1t}}{\partial t_x}\Big|_{\bar{V}} \gtrless 0 \ , \ \frac{\partial S_{1t}}{\partial t_x}\Big|_{\bar{V}} \gtrless 0 \ , \ \frac{\partial \beta_{1t}}{\partial t_x}\Big|_{\bar{V}} \gtrless 0$$

The effect of this tax on all three portfolio decision rules are ambiguous. The less attractive risky asset causes the investor to reallocate his funds away from the risky asset and towards the safe asset. In addition, as mentioned previously, the tax also reduces the return and variance of the risky asset. This latter effect, exactly the DM phenomenon, makes the risky asset more attractive on the margin relative to the safe asset for a risk averse individual. The overall effects of a the tax on risk taking activity cannot therefore be determined. Also the response of the investors' savings behaviour cannot be

[19] See section A.I.5.1. of appendix I.

determined a priori and without additional restrictions, just as was the case with the full capital income tax. Savings, however, increase for the special case of CRRA and a relative risk aversion of greater than unity. The effect on proportional risk taking is also ambiguous due to the ambiguity of total savings and risky asset demand.

The Total Effects of a Tax on the Return of the Risky Asset:

In order to obtain the full specific effects of a tax on the return of the risky asset, it is necessary to include the income effects. The total effect of the tax on the decision variables is given by:[20]

$$\frac{\partial X_{1t}}{\partial t_x} = \frac{\partial X_{1t}}{\partial t_x}\Big|_{V=C} - \frac{r\, a_{1t}}{(1+r)(1-t_x)} \frac{\partial X_{1t}}{\partial Y_{1t}}$$

$$\textit{where} \qquad X_{1t} = C_{1t}\,,\ a_{1t}\,,\ S_{1t}\,,\ \beta_{1t}$$

The total effect of this policy on the decision variables is ambiguous due to the conflicting substitution and income effects except for the class of CRRA preferences. In the latter event we obtain unambiguous conclusions for less than unity values of the elasticity of substitution namely:

For all $\theta \geq 1$

[20] See section A.I.5.2. of appendix I.

$$\frac{\partial C_{1t}}{\partial t_x}\Big|^{CRRA} < 0\ ,\ \frac{\partial a_{1t}}{\partial t_x}\Big|^{CRRA} \gtrless 0\ ,\ \frac{\partial S_{1t}}{\partial t_x}\Big|^{CRRA} > 0\ ,\ \frac{\partial \beta_{1t}}{\partial t_x}\Big|^{CRRA} \gtrless 0$$

For low values in the elasticity of substitution the distortionary effect, which is proportional to the elasticity of substitution, is weak. This allows the income effect to dominate and reduce current consumption and increase savings. For the risk taking decision the DM effect does not dominate all the negative terms and leads to an ambiguous result. Proportional risk taking also is ambiguous.

2.6.2. A Tax on the Return of the Safe Asset

The post tax intertemporal budget constraint is given by:[21]

$$C_{1t} + \frac{C_{2t}}{(1+r(1-t_m))} = Y_{1t} + \frac{a_{1t}(x_{t+1} - r(1-t_m))}{(1+r(1-t_m))}$$

From the budget constraint we observe that the tax on the income of the safe asset alone distorts the relative prices of the two assets and creates intertemporal distortions. The tax on the safe asset's income also causes a distortion in the relative returns of the two assets. The investor upon observing this tax substitutes (reallocates the portfolio) away from the less attractive safe asset towards the more attractive risky asset. Since the mean and variance of the risky asset return remains unchanged, we do not anticipate the

[21] See section A.I.6. of appendix I.

appearance of the DM phenomenon. The next section presents the qualitative results of this exercise.

Hicks Compensated Effects of a the Tax on the Safe Asset Income:

For expected utility to remain unchanged the required adjustment in first period income is given by:[22]

$$\frac{dY_{1t}}{dt_m}\Big|_{\bar{V}} = \frac{r}{(1+\bar{r})} m_{1t}$$

$$where \qquad \bar{r} = r(1-t_m)$$

It is interesting to note that the above result depends critically as to whether the investor is a borrower or a lender. For a lender, a positive compensation amount is required for the investor to remain on the same indifference surface as prior to the imposition of the tax. However, for a borrower, the tax acts as a subsidy on the interest payment and we require to take away from the investor a part of her (him) endowment in order to keep her (him) on the same level of happiness.

The utility compensated effects on current consumption:

$$\frac{\partial C_{1t}}{\partial t_m}\Big|_{\bar{V}} > 0$$

If investor is compensated to stay on the initial indifference surface the tax will induce

[22] See section A.I.6.2. of appendix I.

a preference towards current consumption.

The utility compensated effect on portfolio choice is given by:

$$\frac{\partial a_{1t}}{\partial t_m}\Big|_{\bar{V}} \gtrless 0 \ , \ \frac{\partial S_{1t}}{\partial t_m}\Big|_{\bar{V}} \gtrless 0 \ , \ \frac{\partial \beta_{1t}}{\partial t_m}\Big|_{\bar{V}} \gtrless 0$$

The effect of this tax on all three portfolio decision rules is also ambiguous. The less attractive safe asset causes the investor to reallocate his funds towards the risky asset. However, due to the increase in the price of future consumption the amount invested in the risky asset must also fall. These two effects operate in opposite directions, and hence no definite conclusion can be arrived. The only tentative conclusion that can be made is that with a relative low elasticity of substitution the discouragement of the risky asset due to the more expensive future consumption will be of a small magnitude and hence risk taking might be stimulated with the imposition of this tax.

Also the response of the investor's savings behaviour cannot be determined a priori and without additional restrictions. Future consumption being more expensive causes total savings, a means of transferring consumption into the future, to decline. However, the Hicksian compensation increases savings for a lender and the overall effect cannot be determined. Savings, however, increase for the special case of constant relative risk aversion and a relative risk aversion of greater than unity. For a borrower, since the compensation is negative, the tax leads to unambiguous decrease in total savings. The effect on proportional risk taking is also ambiguous due to the ambiguity of total savings and risky asset demand.

The Total Effects of a Tax on the Income of the Safe Asset:

In order to obtain the full specific effects of a tax on the return of the risky asset, it is necessary to include the income effects. The total effect of the tax on the decision variables is given by:[23]

$$\frac{\partial X_{1t}}{\partial t_m} = \frac{\partial X_{1t}}{\partial t_m}\Big|_{V=C} - \frac{r\, m_{1t}}{(1+\bar{r})}\frac{\partial X_{1t}}{\partial Y_{1t}}$$

$$where \qquad X_{1t} = C_{1t}\ ,\ a_{1t}\ ,\ S_{1t}\ ,\ \beta_{1t}$$

The total effect of this policy on the decision rules is ambiguous due to the conflicting substitution and income effects. These effects also depend on the household being a lender or a borrower.

For a borrower a tax on the income of the safe asset acts like a subsidy on interest payments. This causes current consumption to unambiguously increase since the substitution effect reinforces the income effect. Also if we assume a relatively low elasticity of intertemporal substitution the tax will tend to stimulate investment in the risky asset by more than the substitution effect since in this case the income effect increases risky asset demand.

For a lender, a low value in the elasticity of substitution causes the intertemporal

[23] See section A.I.6.3 of appendix I.

distortionary effect, which is proportional to the elasticity of substitution, to be weak. This allows the income effect to dominate and reduces current consumption and increases savings. The effect on risk taking is in general ambiguous.

2.7. Concluding Remarks.

Our qualitative results indicate that a tax on the income of the risky asset causes the DM phenomenon to appear. The taxation of the risky asset reduces the risk and return of the asset. This will stimulate risky asset demand for a risk averse individual. However, risk taking may not increase since this tax also creates a static distortionary reallocation of portfolio holdings away from the less attractive risky asset. On the other hand, the taxation of the safe asset may also not necessarily lead to a stimulus in risk taking activity. Again there are two opposite effects operating, but no DM phenomenon. First the tax on the income of the safe asset causes the risky asset to be more attractive on the margin. However, due to the increase in the price of future consumption, the intertemporal distortion, risky asset demand will drop. If the distortion in relative rates of return is stronger than the intertemporal distortion, the risky asset demand will increase. The results of this chapter depend on the assumption that the proceeds of the tax are used to provide a public good that enters in the utility function in an additive way. Thus the riskiness of the tax revenue has no additional impact on the choice variables. The next chapter removes this assumption and I examine how these results are altered.

Chapter 3

Capital Income Taxation and Re-Distribution under Uncertainty

3.1 Introduction

In the previous chapter I assumed that the proceeds of the tax are spent on a public good, which enters the utility function in an additively separable way. This implies that the stochastic tax revenue has no additional effect on the decision variables and in particular on risky asset holdings. This chapter examines the behaviour of the investor when the government re-distributes risk back to the investor and indicates that the results of the previous chapter depend on this assumption.

I assume that the government re-distributes back to the investor the proceeds of the tax revenue, stochastic where relevant, in the form of a uniform lump sum payment. The re-distribution is done in the period in which the tax is collected. The per capita lump sum payment becomes:

$$G_t = \frac{\sum_{i=1}^{m} T_{2t}^{i}}{m}$$

where T^i_{2t} is the tax revenue collected from investor i in period 2, and m is the number of households. Note that with capital income tax, the tax is always due in period 2.

In order to focus on the no risk shifting role of the state I will assume initially that there is a common risky asset (i.e the market portfolio) which is bought by all current generational investors. As was indicated by Atkinson and Stiglitz (1980) this implies that all risks are perfectly correlated.[1] Furthermore, I assume that all investors are identical, with identical initial wealth. The per capita re-distribution is given by:

$$G_t = T_{2t}^*$$

where T_{2t}^* is the tax payment by a typical individual.

The state, in effect, returns the revenue raised by the tax system as a lump sum, "stochastic" if necessary, transfer to the investor in the same period in which the tax is collected.[2] This ultimately implies that the risk remains within the private sector, and hence investors ultimately bear the entire risk. Gordon (1985) makes a similar assumption and states: "Given that the government absorbs a sizable fraction of the risk as a result of the taxes on corporate income, one might have expected the market risk premium to fall. However, the government cannot freely dispose of the risk it bears. Individuals must ultimately bear this risk, whether through random tax rates on other income, random government expenditures, or random government deficits."

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[1] Summers and Bulow (1984), Hamilton (1987) and Gordon (1985) all assume that the households are identical and purchase the same securities.

[2] It would be of interest to assume that investors have heterogenous expectations and examine the implications of such a re-distributional policy.

How does this policy affect current consumption, the size and composition of the portfolio of the investor? Is the DM phenomenon observed? Are there any deadweight losses from the imposition of capital income taxation? If any how do the deadweight losses compare with the traditional Harberger Triangle? Are the deadweight losses of capital income taxation different from the certainty literature? How do differential taxes on the assets affect choice and welfare? How do these results compare with the previous analysis? These are the main questions that will be addressed in the chapter.

Section 3.2 starts with an equal proportional tax on all capital income. Initially the effects on the decision variables are reported. Section 3.3 presents the deadweight losses of capital income taxation under uncertainty with portfolio/savings choice. The deadweight losses are compared with the Harberger Triangle. We also compare them to Summers and Bulow (1984). Section 3.4 examines the remaining capital income tax policies. In section 3.4.1 I examine a tax on the imputed safe asset. Similar analysis as the full capital income tax is conducted. Section 3.4.2 examines the taxation of excess returns and some conclusion are made. Section 3.5 examines a differential tax on the two assets. The tax on the risky asset is examined in section 3.5.1 and the tax on the safe asset is examined in section 3.5.2. Section 3.7 offers some concluding remarks.

3.2. Capital Income Taxation and Decision Making

Under a full capital income tax the tax revenue the state returns to the household is:

$$T_{2t}^{*} = t_k \left(r\, S_{1t}^{*} + z_{t+1}\, a_{1t}^{*} \right)$$

Where a_{1t}^{*} and S_{1t}^{*} are aggregate portfolio choices. Notice that this re-distributional amount is a random variable depending on the outcome of x_{t+1}.

I continue to assume that the investor maximizes the two period expected utility by choosing the decision rules and is subject to the budget constraint given by:

$$C_{2t} = (1+\bar{r})S_{1t} + a_{1t}\,\overline{z_{t+1}} + T_{2t}^{*}$$

$$\begin{aligned} \textit{where} \quad \bar{r} &= r(1-t_k) \\ \overline{z_{t+1}} &= (1-t_k)\, z_{t+1} \end{aligned}$$

and T_{2t}^{*} is the per-capita lump sum stochastic tax revenue that is returned to the household. For a given T_{2t}^{*} the first order conditions of the problem are given by:

$$g' - (1+r(1-t_k))E(h') = 0$$

$$E(h' z_{t+1}) = 0$$

Two observations can be made from the optimality conditions and the risk disposal hypothesis. First, capital income taxation continues to affect the price of future consumption and causes the well known intertemporal distortion. Second, since the lump sum tax revenue re-distribution exactly cancels with the tax payment of the investor the investors ultimately bear the entire risk and as a result the DM phenomenon is not expected to appear.[3]

Even though it is clear that there is no risk sharing through the government policy I still need to examine the effect of this re-distribution scheme on the all the choice variables and compare them to the previous analysis.

The effect of this policy on current consumption is given by:

$$\frac{\partial C_{1t}}{\partial t_k}\Big|_{NRS} > 0$$

NRS stands for no risk sharing.

Current consumption increases due to the substitution effect. Future consumption being more expensive changes the households consumption plans and leads to a higher level of current consumption. The effect on the amount invested in the risky asset is:

$$\frac{\partial a_{1t}}{\partial t_k}\Big|_{NRS} < 0$$

Due to the absence of risk sharing, via the stochastic lump sum transfer, the policy leads to an unambiguous decline in the amount invested in the risky asset. The decline in the amount invested in the risky asset arises from the more expensive future consumption. The risky asset just as the safe asset are means of acquiring future consumption in this model. The effect of this policy on total savings is given by:

[3] See appendix II section A.II.1 for analytical results.

$$\frac{\partial S_{1t}}{\partial t_k}\Big|_{NRS} = -\frac{\partial C_{1t}}{\partial t_k}\Big|_{NRS} < 0$$

Total savings unambiguously drop. This happens because the increased current consumption is financed at the expense of savings (i.e from holding a lower risky and/or riskless asset). The effect of this policy on the safe asset is given by:

$$\frac{\partial m_{1t}}{\partial t_k}\Big|_{NRS} < 0$$

Therefore, this policy not only discourages the risky asset but discourages investment in the safe asset also. In addition, due to NDRRA preferences savings is discouraged more strongly than risky investment. Portfolio riskiness therefore increases.

$$\frac{\partial \beta_{1t}}{\partial t_k}\Big|_{NRS} \geq 0$$

Proportional risk taking remains constant only for the constant relative risk aversion case. Hence, it is seen that even when there is no risk sharing via tax policy, increased capital income taxation may indeed lead the investor to hold a riskier portfolio.

3.3. The Deadweight Loss of Capital Income Taxation

The policy of re-distributing the stochastic tax revenue back to the household leads to a decline in investor's welfare. The lump sum stochastic re-distribution is not sufficient to hold the investor on the same level of happiness as prior to the imposition of the tax. Capital income taxation creates a distortion in the intertemporal price of future consumption. The marginal deadweight loss of this policy is given by:

$$\frac{\partial V}{\partial t_k}\Big|_{NRS} = -t_k \; r \; E(h')\frac{\partial C_{1t}}{\partial t_k}\Big|_{NRS}$$

This is the marginal efficiency cost of capital income taxation under uncertainty. This deadweight loss of capital income taxation is analogous to the efficiency cost of a tax change using Arnold Herberger's (1971) measure which under certainty is given by:

$$DWL = -\sum_{i}^{n} T_i \; (\Delta X_i)$$

where T_i measures the value of the tax outstanding per unit of the ith activity and (ΔX_i) measures the change in the ith activity due to the tax change. Harberger developed this measure of deadweight loss under certainty. Under uncertainty, this measure is adjusted to the following expression:

$$DWL = -\sum_{i}^{n} T_i \; E(\Delta X_i)$$

The analogy of our result and that of Harberger is transparent. The term

$$T_i = T = t_k \, r \, E(h')$$

is the taxes paid on the riskless asset by the investor weighted by the marginal utility of future consumption.[4] The reason that this value is used for T_t can be seen by analyzing the optimality conditions of the investor. Investors are indifferent between more investment in the riskless and risky asset. Therefore, they value the after tax return on the two assets equally, in terms of the assets contribution to future consumption:

$$(1 - t_k)E(h'x_{t+1}) = (1 - t_k)rE(h')$$

Also, from the optimality conditions, they also value equally the tax payments:

$$t_k E(h'x_{t+1}) = t_k rE(h')$$

Hence the T_t can be measured from the taxes paid on the riskless investment.

Gordon and Wilson (1989) state:

"In equilibrium investors are indifferent between further investment in riskless and risky capital, implying that they value the two alternative after-tax return streams equally. But since the two streams of tax payments from the alternative assets are proportional to the two stream after-tax returns, they value equally the tax payments they make to the government on the two assets. Therefore the size of the tax distortion is the same on each asset, and can be measured most simply by the taxes paid on the riskless investment."

Hence, even though the investor pays on average more taxes this extra loss to investors is compensated for the benefit of having the state absorb some of the risk.

The second term representing the change in behaviour $E(\Delta X_t)$ is analogous to the term: This term shows the expected behaviour of current consumption due to a change in the

[4] This also has been used by Fullerton and Gordon (1983) and Joel Slemrod (1983).

$$E(\Delta X_i) = \frac{\partial C_{1t}}{\partial t_k}|_{NRS}$$

tax rate. Clearly, the magnitude of this term is determined by the elasticity of substitution and/or the relative risk aversion parameter. The lower the elasticity of substitution the smaller the deadweight loss of capital income taxation. Equivalently, the deadweight loss of capital income taxation decreases as the risk aversion parameter increases. In the next chapter, I will show that the deadweight loss of capital income taxation operates through the elasticity of substitution and not the risk aversion parameter. The path through which taxation policy, elasticity of substitution and/or risk aversion parameter, operates is important for public policy and measuring the deadweight loss under uncertainty. For example a reasonable value for the elasticity of intertemporal substitution maybe 0.1 but this value would generate an unreasonable relative risk aversion parameter of 10. But such a high value of the RRA parameter is unreasonable and yields no risk taking activity.

How does our analysis compare with Summers and Bulow (1984)? They use for the tax distortion term T_i the taxes due on a security that yields the weighted average rate of return. They use the following measure:[5]

$$T_i = t_k\ [\ (E(x) - r)\ \beta_{1t} + r\ ]\ E(h^i)$$

This implies that the deadweight loss from capital income taxation is extremely large

[5] This measure has also been used in many applied studies (See for example Feldstein (1978) and Fullerton, Shoven and Whalley (1978)).

since the weighted average return exceeds the risk free rate by at least 4 times for reasonable parameter values as shown in chapter 8. Summers and Bulow argue that most of the risk is due to random changes in the price of the asset. As a result, they argue, investors require a higher return to compensate for the higher risk of the asset. Under the existing corporate tax system a segment of this higher yield is taxed away and since changes in asset prices do not enter the corporate tax base, investors still bear the risk in asset prices. Accordingly, there is no benefit to investors from the reduced risk bearing to compensate for the significant tax payments made by them.

In our model, however, the Summers and Bulow argument does not hold because the personal capital income tax system includes capital gains/losses and hence the government does absorb some of the risk of either random asset price changes or random dividend income.[6]

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[6] An important limitation of our study is the two period framework as was the case with Summers and Bulow (1984) and Gordon (1985). Gordon and Wilson (1989) have extended the work to a multi-period framework and have shown that the deadweight loss of capital income taxation is even lower than the one of a two period model. The Harberger measure in a multi-period framework under uncertainty is given by:

$$DWL = - \sum_{i}^{n} T_i \; CE(\Delta X_i)$$

Since the certainty equivalent is lower then the expected change in behaviour the deadweight loss is smaller.

3.4. Other Capital Income Tax Policies

3.4.1 Taxation of the Imputed Safe Income

Under a tax that exempts the excess returns, the state returns to the household the following tax revenue in the second period:

$$T_{2t}^{*} = t_r \, r \, S_{1t}^{*}$$

The investor is subject to the following constraint:

$$C_{2t} = (1+\bar{r})S_{1t} + a_{1t}\,\overline{z_{t+1}} + T_{2t}^{*}$$

where T_{2t}^{*} is the per-capita lump sum stochastic tax revenue that is returned to the household. Since the lump sum re-distribution is non-stochastic one might expect the effects of this tax policy to be identical to the Hicks compensated effects of the previous chapter. However, this is true only for current consumption and the amount invested in the risky asset.[7] The effect on current consumption is:

$$\frac{\partial C_{1t}}{\partial t_r}\Big|_{NRS} = \frac{\partial C_{1t}}{\partial t_r}\Big|_{\bar{V}} > 0$$

Current consumption increases. On amount invested in the risky asset:

$$\frac{\partial a_{1t}}{\partial t_r}\Big|_{NRS} = \frac{\partial a_{1t}}{\partial t_r}\Big|_{\bar{V}} < 0$$

The amount invested in the risky asset falls.

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[7] See section A.II.2. of appendix II for derivation of these results.

On total savings:

$$\frac{\partial S_{1t}}{\partial t_r}\Big|_{NRS} = -\frac{\partial C_{1t}}{\partial t_r}\Big|_{NRS} < 0$$

Total savings fall since current consumption increases. In the previous chapter no conclusion could be reached on the effects of this tax policy on savings. The Hicks compensated effects of a imputed safe income tax on savings was indetermine; only for the CRRA case with a relative risk aversion parameter of greater than unity did savings unambiguously increase. In this case total savings move in the opposite direction due to the increased level of current period consumption. In this section there is no Hicks compensation but a lump sum transfer that is paid only in the second period, and that is not sufficient to maintain the former level of expected utility.

This policy just as the full capital income tax policy of the previous section not only discourages the risky asset but discourages investment in the safe asset also. Again due to NDRRA preferences savings is discouraged more strongly than risky investment. Portfolio riskiness therefore increases except for the CRRA case in which it remains unaffected.

What is more interesting is the deadweight loss of this tax policy. The lump sum re-distribution is not sufficient to hold the investor on the same level of happiness. A tax on the safe imputed income creates a distortion in the intertemporal price of future consumption as the full capital income tax policy. The deadweight loss of this policy is given by:

$$\frac{\partial V}{\partial t_r}\Big|_{NRS} = - t_r\, r\, E(h') \frac{\partial C_{1t}}{\partial t_r}\Big|_{NRS}$$

Interestingly, the deadweight losses of the tax on the imputed safe income are identical to the deadweight losses of a full capital income tax when $t_k = t_r$, even though the latter includes the capital gains or risk premium component. This clearly implies that adding the capital gains or risk premium to the tax base of capital income taxation does not cause any additional deadweight loss.

3.4.2. Taxation of Excess Returns

The per capita lump sum re-distribution is:

$$T^*_{2t} = t_z\, (\, z_{t+1}\, a^*_{1t}\,)$$

In the analysis of the specific incidence, it was noted that this tax induced no income effects, while the substitution effect was restricted to the DM phenomenon in the risky asset demand. The re-distribution of risk back to the household is even simpler; it also wipes out the DM effect. The tax is entirely neutral. This case has also been analyzed by Gordon (1985) in a more general framework. He concluded that " so long as (a) a risk-free investment would pay no taxes on net, and (b) taxes paid by an individual are returned to him in a lump sum fashion, eliminating income effects, then these tax have absolutely no effect on the equilibrium allocation" (p.12).

An Interim Summary

In many countries, capital gains are not included in the tax base, and if included the taxable component often attracts a lower rate than applicable to wages and salaries. Canadians, for example, enjoyed up to 1993 a lifetime exemption of $100,000 of taxable capital gains. This does not include the gains from the disposal of the principle dwelling. Since most capital gains/losses are taxed only on realization and given the endogeneity of the holding period length, the effective rate of a tax on capital gains falls well below the statutory level. On what grounds, would this leniency be rationalized given my results?

I find that in all cases, except the taxation of capital gains (or excess returns/extraordinary gains), I observe deadweight losses of capital income taxation. The deadweight losses of the imputed safe asset turned out to be identical to the deadweight losses of a full capital income tax policy for equal tax rates $t_r = t_k$. Ergo, the inclusion of excess returns in the tax base of the imputed safe income does not cause any additional deadweight loss to the investor and can generate more tax revenue to the government. Attention must be given to the fact that the taxation of excess returns even though it does not cause any additional deadweight loss is also independent of what the state does with the stochastic tax revenue. If the state returns the "stochastic" lump sum tax revenue from excess returns to the investor, in the same period that it was collected, the investor's welfare will remain unchanged. In this case, the investor will not re-allocate his (her) portfolio holdings. The amount invested in the risky asset will remain unchanged. If on the other hand, as chapter two indicated, the state uses the funds for the provision of a public good that enter the utility function additively, the investor is still indifferent in terms of welfare as with the re-distribution policy. However, in this case the investor increases the risky asset and reduces the safe asset so that he will face the same probability distribution as prior to the imposition of the tax. Accordingly, the investor is practically indifferent as to whether the state returns the funds in the form of risky tax revenue, in the same period it has been collected, or in the form of random government expenditures that are used to provide a public good that enters the utility function in an additive manner. The allocative effects, however, of the two policies are different as I observed. Hence the taxation of capital gains can bring additional tax revenue which can be used for the provision of a public good.

3.5 Differential Taxes on the Two Assets

3.5.1. A Tax on the Return of the Risky Asset

The per capita re-distribution amount is:

$$T_{2t}^{*} = t_x \, x_{t+1} \, a_{1t}^{*}$$

Where again a_{1t}^{*} is the aggregate asset choice. Notice again that this re-distributional amount is a random variable depending on the outcome of x_{t+1}.[8]

The investor maximizes the two period expected utility by choosing the decision rules and is subject to the budget constraint given by:

$$C_{2t} = (1+r)\,S_{1t} + a_{1t}\,[\,x_{t+1}(1-t_x) - r\,] + T_{2t}^{*}$$

where T_{2t}^{*} is the per-capita lump sum stochastic tax revenue that is returned to the household. For a given T_{2t}^{*} the first order conditions of the problem are given by:

$$g' - (1+r)\,E(h') = 0$$

$$E(h'((1-t_x)\,x_{t+1} - r)) = 0$$

A dividend tax does not affect the price of future consumption and hence does not create intertemporal distortion. This tax creates a static distortion in portfolio choice.

The effect of this policy on current consumption is given by:

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[8] This dividend tax may be viewed as a corporate income tax.

$$\frac{\partial C_{1t}}{\partial t_x}|_{NRS} > 0$$

Current consumption increases. Since this tax makes the risky asset relatively less attractive to the safe asset the investor would lower savings and hance increases current consumption. The effect on the amount invested in the risky asset is:

$$\frac{\partial a_{1t}}{\partial t_x}|_{NRS} < 0$$

The decline in the amount invested in the risky asset arises from the static distortion which changes the relative rate of returns of the two assets in favour of the safe asset. The safe asset becomes a more attractive way of acquiring future consumption in this model. The effect of this policy on total savings is given by:

$$\frac{\partial S_{1t}}{\partial t_x}|_{NRS} = - \frac{\partial C_{1t}}{\partial t_x}|_{NRS} < 0$$

Total savings unambiguously drop. As noted above this happens because the increased current consumption is financed at the expense of savings (i.e from holding a lower risky and/or riskless asset). The effect of this policy on the safe asset is given by:

$$\frac{\partial m_{1t}}{\partial t_x}|_{NRS} = \frac{\partial S_{1t}}{\partial t_x}|_{NRS} - \frac{\partial a_{1t}}{\partial t_x}|_{NRS} \gtrless 0$$

Therefore, the effect of this policy on the safe asset is not determined with the given preference structure. Since total savings fall in order to finance the increased current period consumption this first effect operates to reduce the demand for the safe asset. The second effect stimulates the demand for the safe asset and this happens because the demand for the risky asset falls. The net outcome depends on the relative magnitude of the two effects. The savings effect depends directly on the wealth elasticity of the risky asset and the elasticity of intertemporal substitution. A low value of both would cause the increased current consumption effect to be of a small magnitude. This would cause only the second term to be operational and thus cause an increase in the demand for the safe asset. The effect on portfolio riskiness is also ambiguous.

$$\frac{\partial \beta_{1t}}{\partial t_x}\Big|_{NRS} = \frac{1}{S}\,\frac{\partial a_{1t}}{\partial t_x}\Big|_{NRS} - \frac{a_{1t}}{S_{1t}^2}\,\frac{\partial S_{1t}}{\partial t_x}\Big|_{NRS} \lessgtr 0$$

The effect on proportional risk taking is not determined with this general preference structure except for the case whereby the total savings effect is weak (i.e a low elasticity of substitution). In such a case proportional risk taking will fall.

The Deadweight Loss of Taxing the Risky Asset

The deadweight loss of this policy is given by:

$$\frac{\partial V}{\partial t_x}\Big|_{NRS} = E(h' z_{t+1})\,\frac{\partial a_{1t}}{\partial t_x}\Big|_{NRS}$$

This is the efficiency cost of taxing the risky asset under uncertainty. The first term can be written as follows, from the optimality conditions:

$$T_t = T = E(h' z_{t+1}) = t_x E(h' x_{t+1}) > 0$$

But from the first order conditions I observe that the investor will be indifferent between further investment in riskless and risky asset. The investor values the two after-tax return streams equally. Hence in this case I have:

$$(1 - t_x)\, E(h' x_{t+1}) = r\, E(h')$$

As a result the size of this tax distortion term measured in terms of the riskless asset can be written as follows:

$$T_i = T = E(h' z_{t+1}) = \frac{r\, t_x}{(1-t_x)} E(h') > 0$$

As a result the size of the tax distortion increases by $rt_x/(1-t_x)$ times the marginal utility of future consumption. This is the certainty equivalent of the tax payment. The second term representing the change in behaviour $E(\Delta X_i)$ is analogous to the term:

$$E(\Delta X_i) = \frac{\partial a_{1t}}{\partial t_x}\Big|_{NRS}$$

This term shows the expected behaviour of asset choice due to a change in the tax rate. Clearly, the magnitude of this term is determined by the degree of quasi-concavity of the preference structure. Therefore, in the case of no risk sharing, there is a deadweight loss from taxing the risky asset.

3.5.2. A Tax on the Return of the Safe Asset

The final tax to be examined is a tax on the safe asset. The per capita re-distribution amount is:

$$T_{2t}^{*} = t_r \, r \, m_{1t}^{*}$$

Where again m_{1t}^{*} is the aggregate safe asset choice. This re-distributional amount is not a random variable.

The investor maximizes the two period expected utility by choosing the decision rules and is subject to the budget constraint given by:

$$C_{2t} = (1 + r(1 - t_m)) S_{1t} + a_{1t} \left[x_{t+1} - (1 - t_m) \, r \right] + T_{2t}^{*}$$

where T_{2t}^{*} is the per-capita lump sum stochastic tax revenue that is returned to the household. For a given T_{2t}^{*} the first order conditions of the problem are given by:

$$g' - (1 + r(1 - t_m)) E(h') = 0$$

$$E(h'(x_{t+1} - (1 - t_m) r)) = 0$$

A tax on the safe asset affects the price of future consumption and hence does create intertemporal distortions. In addition this tax creates a static distortion in portfolio choice. It makes the risky asset relatively more attractive vis-vis the safe asset.

The effect of this policy on current consumption is given by:

$$\frac{\partial C_{1t}}{\partial t_m}\Big|_{NRS} \gtrless 0$$

The effect of this policy is ambiguous on current consumption. The tax increases the price of the future consumption and hence stimulates the cheaper current consumption. However, the tax also reduces the return on the safe asset and the investor reallocates his portfolio away from the safe asset towards the risky asset. However, for marginal tax rates (i.e. t_m = o initially) I obtain:

$$\frac{\partial C_{1t}}{\partial t_m}\Big|_{NRS} > 0$$

The effect on the amount invested in the risky asset is:

$$\frac{\partial a_{1t}}{\partial t_m}\Big|_{NRS} \gtrless 0$$

Although one would expect the risky asset to increase from the imposition of this tax there is an additional opposite effect taking place. This second effect, is negative because the risky asset is a mean of acquiring future consumption and since future consumption is more expensive the investor reduces the demand. However, for a relatively low elasticity of substitution or a low wealth elasticity of the risky asset I expect that the static distortion will dominate and stimulate risk taking asset demand.

The effect of this policy on total savings is given by:

$$\frac{\partial S_{1t}}{\partial t_m}\bigg|_{NRS} = -\frac{\partial C_{1t}}{\partial t_m}\bigg|_{NRS} \gtrless 0$$

The effect on total savings is ambiguous, except for a marginal increase in the tax rate from $t_m = 0$ in which case total savings fall. The effect of this policy on the safe asset is given by:

$$\frac{\partial m_{1t}}{\partial t_m}\bigg|_{NRS} \gtrless 0$$

Therefore, the effect of this policy on the safe asset is not determined with the given preference structure. However, for a relatively low elasticity of substitution I expect the demand for the safe asset to fall since in this case the static distortion effect will dominate the intertemporal distortionary effect. The effect on portfolio riskiness is also ambiguous. Again in the case of a relatively low elasticity of substitution I expect the risky asset to increase and total savings to fall which will result in a riskier portfolio.

$$\frac{\partial \beta_{1t}}{\partial t_m}\bigg|_{NRS} = \frac{1}{S}\frac{\partial a_{1t}}{\partial t_m}\bigg|_{NRS} - \frac{a_{1t}}{S_{1t}^2}\frac{\partial S_{1t}}{\partial t_m}\bigg|_{NRS} \lessgtr 0$$

The Deadweight Loss of Taxing the Safe Asset

The deadweight losses of this policy is given by:

$$\frac{\partial V}{\partial t_m}|_{NRS} = r\,t_r E(h') \; \frac{\partial m_{1t}}{\partial t_m}|_{NRS}$$

This is the efficiency cost of taxing the safe asset under uncertainty. Again the analogy of our result and that of Harberger is transparent.

3.7. Concluding Remarks

Various tax policies have been investigated in a world of uncertainty. We find, as do Gordon and Wilson (1989), the deadweight loss of capital income taxation to be less than that found by economists who have ignored uncertainty. Including capital gains in the tax base of capital income taxation, does not alter the deadweight loss of capital income taxation in a world whereby no risk sharing by the government takes place. The results of this study depend on the government bearing risk at the same cost as private investors. If the state can bear risks more cheaply than private investors, the deadweight loss of taxing risky capital income will be even lower as was originally suggested by Tobin (1958) and later by Stiglitz (1969)). One avenue for the state to bear risk at a lower rate is through intergenerational risk sharing (Gordon and Varian (1988)). Another possibility which was suggested by Atkinson and Stiglitz (1980) occurs if risks are not perfectly correlated across individuals. In this case the above welfare cost of capital income taxation will be even lower.

The results of this study are conducted in a two period partial equilibrium setting. In this sense our results are limited. However, the results of this study are conducted under uncertainty. Most studies examining the welfare losses of capital income taxation are conducted in a world of certainty. In this sense, this is a first attempt to include normative issues in a two period life cycle model where results of great importance are uncovered. One early exception is Stiglitz (1972) but his model is for one period and assumes that the state can bear risks cheaper than individuals. Stiglitz (1972) uses risky tax revenue for the provision of a public good. However, he does not add the funds to the utility function in an additive way but assumes that the state just as the individuals has attitudes towards risk. The attitudes towards risks by the state are different from the attitudes of individuals. Richter and Wiegard (1990) also makes a similar assumption. Finally, Hamilton (1987) has conducted a normative analysis of capital income taxation but his results hold only for the special case of constant relative risk aversion and the assumption of an infinite horizon.

The effects of taxation on savings and welfare have been analyzed in the past only under certainty. Boskin (1978) and Feldstein (1978) compute the welfare cost of capital income taxation using the Harberger Triangle in a two period savings model under certainty. Boskin estimates a deadweight loss of 25% of first period savings. Feldtsein (1978) estimates a welfare loss from capital income taxation at about 20% of savings. This translates to a welfare loss of approximately 1.5% of National Income. His model like mine is a two period model but with labour in period one. However, my analysis is conducted under uncertainty with asset choice. Summer's (1980) model incorporates general equilibrium effects, production and a very high interest elasticity of savings. The most important general equilibrium effects is the increase in gross wages which is the outcome of an increase in capital intensity due to the elimination of capital income taxation. His steady state welfare gains from moving to a consumption tax system is estimated to be 10% of national income.

In the next chapter I provide additional insights into the deadweight loss of capital income taxation under uncertainty using the non-expected utility preference structure. I will present numerical simulation results in order to capture the magnitude of the deadweight loss of capital income taxation under uncertainty with portfolio choice.

Chapter 4

The Deadweight Loss of Capital Income Taxation

Non-Expected Utility Preferences and Some Numerical Results

4.1. Introduction

The previous chapter, dealing with no risk sharing role of the state indicated that there are deadweight losses under various capital income tax policies. In this chapter I attempt to provide a measure of the deadweight loss of capital income taxation under uncertainty in a two period portfolio/savings model with non-expected utility preferences. The existing literature dealing with the deadweight loss of capital income taxation are all done in a certainty environment (Feldstein (1978), Boskin (1978), Summers (1981), Drifill and Rosen (1983)).[1]

In section 4.2 I discuss the structure of preferences. In section 4.3 I provide an explicit measure of the deadweight loss of capital income taxation. In section 4.4 I provide some numerical results. In the concluding section 4.6 I suggest direction of future research.

[1] One exception is Gordon and Wilson (1989) who examine theoretically the deadweight loss of capital income taxation in an uncertainty environment.

4.2. The Preferences

I propose to compute the deadweight loss of capital income taxation using the recently advanced generalized isoelastic multi-period utility function which allows the separation of risk aversion and intertemporal substitution. This generalized utility function includes the corresponding expected cardinal utility function as a special case.[2] The deadweight loss of capital income taxation can also be computed using the traditional Von-Neumann Morgenstern intertemporal expected utility preferences. However, these preferences have a drawback. They constrain the relative risk aversion parameter to be the inverse of the elasticity of substitution in consumption. The empirical evidence indicates that the individuals displays a low elasticity of intertemporal substitution and a modest aversion to risk taking. The von Neumann-Morgenstern utility restrictions on preferences have been rejected on empirical grounds (Grossman and Shiller (1981), Hansen and Singleton (1983), Mehra and Prescott (1985) and Hall (1988)). The expected utility model cannot describe a household as having concurrently a low elasticity and a small degree of risk aversion. Hall (1988, p. 340) states that " ...the elasticity is unlikely to be much more above 0.1 and may well be zero." Based on this figure the von Neumann-Morgenstern utility would generate a relative risk aversion parameter of the order ten or greater. This is not consistent with the observed willingness of consumers to take risk and the existing

[2] Since we are working with the state of nature approach in the two period model these preferences collapse to the Ordinal certainty equivalent (OCE) preferences analyzed by Selden (1978, 1979). Extension to multi - period will not violate the principle of intertemporal consistency of plans (Johnsen and Donaldson (1985) since the Epstein and Zin (1989,1990), Weil (1990) preferences are used.

evidence on the value of the elasticity of substitution. Friend and Blume (1975) analyzing survey data on the financial characteristics of consumers found that the relative risk aversion is constant and in excess of unity. Recently, Epstein and Zin (1991) estimate the parameters of a multi-period savings-portfolio model which separates risk aversion from the elasticity of substitution. They found that the elasticity of intertemporal substitution to lie between 0.1 to 0.5 and a relative risk aversion parameter between 1.0 and 2

The household is assumed to make choices between current and certainty equivalent future consumption based on the elasticity of intertemporal substitution. The preference are described by:

$$U(C_1,CE(C_2)) = [C_1^{\theta} + \delta CE(C_2)^{\theta}]^{\frac{1}{\theta}}$$

where C_1 is current consumption, $CE(C_2)$ is the certainty equivalent of future consumption, $\delta=1/(1+\rho)$, where ρ is the rate of time preference, and $\eta = 1/(1-\theta)$ is the elasticity of intertemporal substitution, with $0 \neq \theta < 1$. θ may well be negative. The household computes the certainty equivalent future consumption depending on his risk preferences and then relying on his intertemporal substitutability combines current consumption with the certainty equivalent future consumption. The certainty equivalent of future consumption is given by:[3]

$$CE(C_2) = [\ E(C_2^{\alpha})\ ]^{\frac{1}{\alpha}}$$

where the constant relative risk aversion parameter is equal to $\gamma = 1-\alpha$. Since $\gamma \geq 0$, we require $\alpha \leq 1$ (again α may well be negative). Substituting the two expressions yields:

$$U(C_1, CE(C_2)) = [C_1^{\theta} + \delta E(C_2^{\alpha})^{\frac{\theta}{\alpha}}]^{\frac{1}{\theta}}$$

This generalized isoelastic utility function allows the separation of risk aversion parameter from the intertemporal elasticity of substitution. This generalized utility function includes the corresponding expected cardinal utility function as a special case. When $\alpha = \theta$ we obtain the familiar Von-Neumann Morgenstern utility function showing indifference to the timing of the resolution of uncertainty.

$$U(C_1, CE(C_2)) = [C_1^{\theta} + \delta E(C_2^{\theta})]^{\frac{1}{\theta}}$$

In addition, the risk neutral constant elasticity of intertemporal substitution RINCE

[3] The OCE preference (U) is invariant under any increasing monotonic transformation. Any positive monotonic transformation of the utility function will not change the optimality conditions and the decision variables.

preferences developed by Farmer (1990) occur when α is equal to one.[4] They yield the following non-expected utility function:

$$U(C_1,E(C_2)) = [C_1^{\theta} + \delta E(C_2)^{\theta}]^{\frac{1}{\theta}}$$

$$\textit{Since} \qquad CE(C_2) = E(C_2)$$

Furthermore, if the elasticity of substitution is zero then the non-expected utility function becomes of the fixed coefficient type whereby current and future consumption become perfect complementary commodities:

$$U(C_1,CE(C_2)) = Min\ [\ C_1\ ,\ \delta E(C_2^{\alpha})^{\frac{1}{\alpha}}\ ]$$

Finally, Kreps and Porteus (1978) show that investors exhibit preference for early (late) resolution of uncertainty if the utility function is convex (concave) in its second argument. Investors prefer early (late) resolution of uncertainty if $\alpha < (>)\ \theta$.

In the next section I examine the effects of a full capital income tax on the optimality conditions and then proceed to derive the deadweight loss expressions.

[4] These type of preferences are not defined under the von-Neumann Morgenstern expected utility since a zero relative risk aversion implies an infinite elasticity of substitution.

4.3. Capital Income Taxation and Optimality Conditions

The problem of the typical investor is to maximize utility subject to the budget constraint:

$$C_1 + \frac{C_2}{(1+r(1-t_k))} = Y_1 + \frac{(1-t_k)az}{(1+r(1-t_k))}$$

The first order condition with respect to current consumption:

$$C_1^{\theta-1} = \delta(1 + r(1-t_k))CE(C_2)^{(\theta-\alpha)}E(C_2^{\alpha-1})$$

The household will set the marginal utility of current consumption equal to the future value of the marginal utility of future consumption adjusted for its risk preference. Notice that if $\alpha = 1$ the investor acts in a risk neutral manner and the expected marginal utility of future consumption equals one and the investor still equates the marginal utility of current consumption with the marginal utility of the future certainty equivalent consumption. An interior solution still exists in intertemporal decisions. On the other hand, if the elasticity of substitution approaches zero we have intertemporal complementarity. The intertemporal consumption decision becomes of the fixed coefficient type. Notice that in this case there is no substitution effect of an interest tax and hence no deadweight loss for the consumer. Therefore I expect the welfare losses to decline as the elasticity of intertemporal substitution falls.

The first order condition with respect to risk taking:

$$E(C_2^{\alpha-1}(1-t_k)z) = 0$$

Obviously, this condition states that at the optimum the expected marginal gain from risk taking is equal to that of the riskless investment in terms of their contribution to future consumption. An interior solution is obtained if the expected return on the risky asset exceeds the return on the riskless asset. Notice that if the investor is risk neutral we obtain a corner solution whereby s(he) allocates all savings to the higher yielding risky asset and nothing to the lower yielding safe asset. At the other extreme of a very high risk aversion the investor allocates all the savings to the safe asset. This case will allow us to compare our results with certainty.

4.4. The Deadweight Loss and the Elasticity of Substitution

The deadweight loss of capital income taxation is given by:[5]

$$DWL = -\frac{r\, t_k\, \delta\, CE(C_2)^{\theta-\alpha}\, E(C_2^{\alpha-1})}{(C_1^{\theta} + \delta\, CE(C_2)^{\theta})} \frac{\partial C_1}{\partial t_k}\Big|_{NRS}$$

where the left hand side is the partial change in the indirect utility function with respect to the change in the capital income tax rate relative to after tax indirect utility function ($V(t_k)$). Notice that this DWL is different and more general then the one reported by Gordon and Wilson (1989), since the expected utility model is captured only in the special case whereby $\theta = \alpha$.

The effect of the tax on current consumption is given by:

$$\frac{\partial C_{1t}}{\partial t_k}\Big|_{NRS} = \frac{r\, S_{1t}}{(1+\bar{r})}\, \eta\, \frac{C_{1t}}{Y_{1t}} > 0$$

In terms of public policy, I can state that it is the elasticity of substitution, that determines the existence of deadweight loss from capital income taxation and not the relative risk aversion parameter. The latter affects the magnitude of the loss through the behaviour of savings and current consumption. Capital income taxation does not cause any distortion and there is no deadweight loss if the intertemporal elasticity of substitution is zero. The deadweight losses of capital income taxation remain if the

[5] See appendix III section A.III.1 for analytical results.

relative risk aversion parameter is extremely large and the elasticity of substitution is not equal to zero. Actually the deadweight loss increases with higher relative risk aversion parameters. Combining these two pieces of information, utilizing the first order conditions, and expressing the deadweight loss in terms of compensating variation I can write the deadweight loss in a more explicit form:[6]

$$DWL = \left(\frac{r}{(1+\bar{r})}\right)^2 t_k \, \eta \, \frac{S(t_k)}{Y_1} \left(\frac{C_1(t_k)}{Y_1}\right)^\theta \frac{1}{\left[\left(\frac{C_1(t_k)}{Y_1}\right)^\theta + \delta \left(\frac{CE(C_2(t_k))}{Y_1}\right)^\theta \right]}$$

The definition of compensating variation is the additional income the consumer would have to receive in the first period to compensate him (her) for the effects of the tax increase.[7] Notice that welfare remains unchanged even if the tax increases.

The DWL of capital income taxation does not depend on the endowment of the investor. There is an important omission in our model. The endowment of the investor represents the present value of future labour earnings. When a capital income tax is imposed the after tax interest rate falls. When the after tax interest rate falls, the endowment of the investor rises. The increase in current consumption expenditure due to this event is termed by Summers (1981) the "human wealth effect". As a result savings fall with income being held constant. The lower savings rate reduces future consumption and the

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[6] A positive linear transformation of the utility function will not affect this welfare measure.

[7] This measure of deadweight loss has been extensively used in the literature. See Diamond and McFadden (1974).

welfare of the household. The two period model does not capture this human wealth effect as was pointed out correctly by Summers. Future research should examine the human wealth factor.

4.5. Numerical Results on the Harberger Triangle

In order to obtain numerical figures on the deadweight loss I need to obtain closed form solutions. First I need to specify the distribution of the random variable. Since assuming a particular density function for the random variable makes generalization difficult, I have decided to conduct the experiment using the state of nature approach (Ahsan (1989)). Assuming two states of nature $x_1 > 0$ and $x_2 < 0$ with probability of p and (1-p) respectively, I may write the certainty equivalent future consumption as follows:

$$CE(C_2)=[pC_{21}^{\alpha} + (1-p)C_{22}^{\alpha}]^{\frac{1}{\alpha}}$$

where C_{2j} denotes future consumption in state j, and $\gamma = 1 - \alpha$ is as before the constant relative risk aversion parameter.

The constraints for the portfolio-savings problem are given by:

$$C_1 = Y - (a + m)$$

$$C_{21} = (Y - C_1)(1+r(1-t_k)) + az_1(1-t_k) \qquad z_1 = x_1 - r > 0$$

$$C_{22} = (Y - C_1)(1+r(1-t_k)) + az_2(1-t_k) \qquad z_2 = x_2 - r < 0$$

This problem leads to an optimal consumption level, risk taking, savings and certainty equivalent which are all linear in the endowment.[7]

[7] See chapter 8 and the corresponding appendix for a detailed analysis.

4.5.1. Parameter Selection

Secondly, I choose the parameters of the model based on empirical regularities. In addition I examine some extreme cases. The elasticity of substitution and the relative risk aversion parameter are allowed to vary according to the following table:

$1/(1-\theta)$	2.0	0.99	0.5	0.25	0.1
$\gamma=1-\alpha$	0.5	0.99	2.0	4.0	100

In this respect, our choice of the relative risk aversion parameter is made independently of the elasticity of substitution. The relative risk aversion is assumed to vary between 0.5 to a maximum of 4. A relative risk aversion parameter of .5 will give us approximate results of the risk neutral agent (RINCE preferences developed by Farmer (1990)). At this low relative risk aversion parameter the investor allocates most of his (her) savings to the higher yielding risky asset. I also examine the case whereby the relative risk aversion of the household is so large that he invests all his assets in the safe asset (i.e. 100). This represents the benchmark of complete certainty. Notice that he still chooses the consumption stream based on the intertemporal rate of substitution. The welfare results of this case will correspond to the two period model life cycle model under certainty (See Atkinson and Stiglitz (1980)). The elasticity of substitution is allowed to vary between 2 and .1 as was found by Hall (1988). The elasticity of substitution equal to 0.99 will yield results that correspond to the Cobb-Douglas case.

The Distribution of the asset, given the probability of the good state of 80% is shown below:

r	x1	x2	E(x)	E(z)	v(x)
0.50	3.7906	-0.1623	3	2.5	2.5
0.50	3.8660	-0.4641	3	2.5	3
1.25	3.7906	-0.1623	3	1.75	2.5
1.25	3.866	-0.4641	3	1.75	3

On the choice of the distribution of the assets I make the following observations. The value of the safe return is assumed to be 50% over a life cycle of 30 years which translates in an annual compounded rate of return of 1.35%. This annual rate corresponds to the real yield on government bonds. Also I choose the rate of 125% which corresponds to an annual rate of return of 2.25%. This increase in the risk free rate can be viewed as an increase in the risk premium on the government bonds. The rate on the risky asset is chosen initially to yield an average real annual rate of return of 4.75%. These rates have also been chosen by Ahsan (1989). Accordingly, this will allow one to make direct comparisons between the two studies. Then I examine a mean preserving increase in variance under both cases.

We estimate the deadweight loss of capital income taxation at the 33% rate and I also assume the discount factor to be $\delta = .95$.

4.5.2. The Deadweight Loss of Capital Income Taxation

The deadweight losses of a capital income tax system under the initial distribution are given by:

Table 4.1: Deadweight Losses (r=.50,E(x)=3,V(x)=2.5)

		Intertemporal Elasticity of Substitution				
		2.0	0.99	0.5	0.25	0.1
Relative Risk Aversion Parameter	0.50	0.043	0.036	0.015	0.006	0.002
	0.99	0.054	0.036	0.016	0.007	0.003
	2.00	0.066	0.036	0.017	0.008	0.003
	4.00	0.072	0.036	0.018	0.008	0.003
	100	0.075	0.036	0.018	0.009	0.004

The deadweight losses of capital income taxation are measured as the percentage change in initial endowment that is required to maintain the investor on her (his) initial level of satisfaction.[8] The losses range from zero to a maximum of 7.5%. An investor with an elasticity of substitution of 0.1 and a relative risk aversion of around 2 would require compensation equal to 0.3% of his initial endowment or wage income to be as well off as in the pre-tax position. This deadweight loss, under these parameter values, is small when compared to the welfare loss reported under certainty. Feldstein (1978) estimated

[8] This measure is given in page 76.

the welfare loss of capital income taxation at 1.9% of wage income (p. 46).[9] This translates to a welfare loss of approximately equal to 1.33% of National Income.[10] His model includes labour/leisure choice in the first period. My analysis does not capture labour/leisure distortions but includes uncertainty and asset choice. New observations from the simulation results include the following; First, the deadweight loss falls as the intertemporal elasticity of substitution decreases for any given level of risk aversion. As was explained previously, the role of the relative risk aversion parameter is to determine the certainty equivalent future consumption. The role of the elasticity of substitution is to determine the consumption path. With an extreme value of 100 for the relative risk aversion the investor invests almost all his funds in the safe asset. He (she) invests only a very small amount in the risky asset. In this case, the investor does not undertake any risk. For this investor the deadweight loss is the highest for a given intertemporal rate of substitution.[11] I observe that the deadweight loss of capital income taxation drops as the relative risk aversion falls. In terms of public policy, this implies that for the investor who holds the risky asset or a diversified portfolio (i.e., a combination of the risky and safe asset) the deadweight loss of capital income taxation will be smaller than the investor who holds only the safe asset. An investor with a high degree of risk aversion will invest most of his funds in the safe asset and therefore does not benefit from the state absorbing

[9] Boskin (1978), in a related study obtains a welfare loss from capital income taxation equal to 25% of savings. If we assume that investors save 10% of income then this translates to a 2.5% of endowment income.

[10] Assuming that wage income accounts for 70% of national income.

[11] It should be noted though that the deadweight loss does not rise much once RRA reaches the figure of 4.0.

some of the risk. For a given elasticity of substitution the deadweight loss of capital income taxation is the smallest for the least risk averse investor. Furthermore, for the special case of an elasticity of substitution equal to one, the welfare losses are equal to 3.6% and are independent of the degree of risk aversion.

A mean preserving increase in the variance causes the following effects on the deadweight losses of capital income taxation:

Table 4.2: Deadweight Losses ($r=.50, E(x)=3, V(x)=3$)

		Intertemporal Elasticity of Substitution				
		2.0	0.99	0.5	0.25	0.1
Relative Risk Aversion Parameter	0.50	0.051	0.036	0.016	0.007	0.002
	0.99	0.060	0.036	0.017	0.008	0.003
	2.00	0.070	0.036	0.017	0.008	0.003
	4.00	0.073	0.036	0.017	0.009	0.003
	100.0	0.075	0.036	0.018	0.010	0.004

A mean preserving increase in variance tends to increase the deadweight loss of capital income taxation for all values of relative risk aversion and the elasticity of intertemporal substitution. A mean preserving increase in variance will increase the safe asset holdings and reduce the risky asset holdings of the investor's portfolio. The increase in safe asset holdings will increase the losses. However, there is a small change in the deadweight loss of capital income taxation relative to the previous table. Notice that a mean preserving increase in variance does not affect the deadweight loss of an investor with an elasticity of substitution of one.

The next table represents an increase in the risk free rate. The deadweight losses of capital income taxation are given in Table 4.3. The distribution of the risky asset is the initial one with E(x)=3 and V(x)=2.5.

Table 4.3: Deadweight losses (r=1.25,E(x)=3,V(x)=2.5)

		Intertemporal Elasticity of Substitution				
		2.0	0.99	0.5	0.25	0.1
Relative Risk Aversion Parameter	0.50	0.207	0.129	0.055	0.024	0.009
	0.99	0.232	0.129	0.057	0.026	0.010
	2.00	0.255	0.129	0.058	0.027	0.010
	4.00	0.267	0.129	0.059	0.028	0.011
	100.0	0.277	0.129	0.060	0.028	0.011

The deadweight loss of capital income taxation dramatically increases for all elasticity of substitution and risk aversion parameters. For a representative investor with an elasticity of substitution of 0.5 and a risk aversion parameter equal to two the deadweight loss of capital income taxation is equal to 5.8% of his initial wealth. This deadweight loss is large when compared to the certainty literature. This translates to a welfare loss of 3.5% of National income. This implies that the welfare loss of capital income taxation will be bigger the smaller the gap between the expected return of the risky asset and the risk free asset. The deadweight loss of capital income taxation will be larger the smaller the risk premium. For a person with an elasticity of substitution equal to 1 the welfare losses are 13% percent. In the next table I conduct again a mean preserving increase in variance and find that the deadweight loss of capital income taxation is even bigger than the previous case.

Table 4.4: Deadweight Losses (r=1.25,E(x)=3,V(x)=3)

		Intertemporal Elasticity of Substitution				
		2	1	.5	.25	.1
Relative Risk Aversion Parameter	0.50	0.22	0.129	0.056	0.025	0.009
	0.99	0.241	0.129	0.058	0.026	0.010
	2.00	0.259	0.129	0.059	0.027	0.010
	4.00	0.268	0.129	0.059	0.028	0.011
	100.0	0.277	0.129	0.060	0.028	0.011

4.6 Concluding Remarks

The deadweight loss of capital income taxation has been analyzed in a partial equilibrium framework under uncertainty with portfolio choice. In addition, the deadweight loss of capital income taxation has been examined under the assumption that the stochastic tax revenue is returned to the investor, in the same period that is collected. This implies that the investors are the ultimate bearers of tax revenue risks. An explicit formula for calculating the deadweight losses has been developed under the assumption of CRRA and CIES for non-expected utility preferences. The important result of this study is that under uncertainty the relevant parameter for the existence of a deadweight loss is a non-zero elasticity of intertemporal substitution as in the certainty models. This arises because the non-expected utility preference structure separates the elasticity of substitution from the relative risk aversion parameter. Among our major findings is that investors who put more funds into the risky asset have a lower deadweight loss from capital income taxation than investors that hold most of their funds in the safe asset. An investor who allocates most of his savings to the risky asset will benefit from the state absorbing some of the risk through tax revenue uncertainty. This is an important result and the size of the deadweight loss depends on the relative risk aversion parameter. As a final remark, investors that have a high intertemporal rate of substitution are seen to be hurt the most by capital income taxation. Finally the deadweight losses of capital income taxation increase as the risk premium falls.

Future research should examine the deadweight losses of other capital income taxation policies. Among the most important one is the taxation of the risky asset versus the safe asset. Our theoretical results of the previous chapter indicated that the deadweight losses from static distortions could be extremely large. Future research awaits to see.

Future research should also examine a multi-period framework. Gordon and Wilson (1989) in a theoretical paper claim that the deadweight losses of capital income taxation are even smaller than previous researchers have found. Finally, research should also be extended to examine a general equilibrium model of the Summers type with uncertainty embodied into the model.

Chapter 5

Capital Income Taxation and the Infinite Horizon

5.1 Introduction

This chapter extends the previous analysis to an infinite horizon and examines the effects of various capital income tax policies on portfolio-savings choice using the non-expected utility preferences (Svennson (1989), Epstein and Zin (1989), Weil (1990)). In order to analyze the effects of capital income taxation in this setting I will restrict the preferences to be of the constant relative risk aversion (CRRA) and constant but separate elasticity of substitution (CIES) form. In addition, extending the model to an infinite horizon with CRRA and CIES yields results that are identical in form to the one period models of Atkinson and Stiglitz (1980) and hence we can contrast them to the two period model of Sandmo (1968) whereby there is interaction between portfolio size and composition. These preferences, will also permit us to distinguish the path, risk aversion or intertemporal elasticity of substitution, through which various capital income taxation policies affect the optimal levels of consumption, savings, risk taking and welfare.

In section 5.1 we present the model and conduct a few comparative static analysis. In section 5.2 we analyze alternative capital income tax structures. In section 5.3 I present my concluding remarks and direction for future research.

5.2 The Model

Epstein and Zin (1989,1990) and Weil (1990) independently advanced a generalized isoelastic multi-period utility function which allows the separation of risk aversion and intertemporal substitution parameters. This generalized isoelastic utility function includes the corresponding expected time additive cardinal utility function as a special case.

The utility function is defined over current consumption and the certainty equivalent random future utility.[1] First the investor computes the certainty equivalent stochastic future utility depending on the risk aversion behaviour. Secondly, relying on the intertemporal substitutability the agent combines current consumption with the certainty equivalent future utility through an aggregator function W to compute utility at time t with information I_t:[2]

$$U_t = W(C_t, E(U_{t+1}/I_t))$$

In order to determine the path, risk aversion or elasticity of substitution, through which taxation policy operates we need closed form solutions. In order to examine the various capital income tax polices we will use the CRRA and the constant but separate elasticity of intertemporal substitution (CIES), defined by the following functional:

[1] As seen in chapter 4, in a two period model the certainty equivalent future utility collapses to the certainty equivalent future consumption.

[2] This recursive form does not violate the principle of intertemporal consistency of plans (Johnsen and Donaldson (1985))

$$U_t = [\ C_t^{\theta} + \delta\ [E_t(U_{t+1}^{1-\gamma})]^{\theta/(1-\gamma)}\ ]^{1/\theta}$$

with $\gamma > 0$ and $0 \neq \theta < 1$.

The parameter δ is the subjective discount rate under certainty. The constant relative risk aversion parameter is equal to γ. The parameter θ is a measure of the intertemporal elasticity of substitution $\eta = 1/(1-\theta)$. Furthermore, Kreps and Porteus (1978) show that investors exhibit preference for early (late) resolution of uncertainty if the utility function is convex (concave) in its second argument. Investors prefer early (late) resolution of uncertainty if $1-\gamma < (>)\ \theta$. When $1 - \gamma = \theta$ we obtain the familiar von-Neumann Morgenstern utility function showing indifference to the timing of the resolution of uncertainty. In addition the risk neutral constant elasticity of intertemporal substitution RINCE preferences developed by Farmer (1990) occur when γ is equal to zero.[3]

Svensson (1989) has extended the portfolio choice model with non-expected utility in continuous time. Since the analysis to follow is formulated in continuous time we utilize his specification.[4]

Following Svensson (1989), the preferences in period t can be written in continuous time

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[3] These type of preferences are not defined under the von-Neumann Morgenstern expected utility since a zero relative risk aversion implies an infinite elasticity of substitution.

[4] Merton (1975) discusses the advantages of using continuous time formulation in portfolio theory.

by considering a small interval Δt and taking the limit when Δt approaches zero.

$$U(t) = \lim_{\Delta t \to 0} U(C(t), CE(U(t;t+\Delta t)); t, t+\Delta t)$$

where C(t) is consumption and $CE(U(t,t+\Delta t))$ is the certainty equivalent future utility. The certainty equivalent future utility is defined with the random utility function given by:

$$CE\ (U(t,t+\Delta t)) = V^{-1}[E_t V_t(U(t+\Delta t))]$$

where $E_t\ (V_t(U_{t+\Delta t}))$ is a risk utility function.

Assuming CRRA and CIES forms, the non-expected utility functional is given by:

$$U(C,CE(U);\ t,t+\Delta t) = e^{-\delta t}\ [\ C^{\theta}\Delta t\ +\ e^{-\delta\Delta t}CE(U)^{\theta}\ ]^{1/\theta}$$

where the intertemporal elasticity of substitution is given by $\eta = 1/(1-\theta)$ and the evaluation of the certainty equivalent utility function is given by $V(U) = U^{1-\gamma}$. Where γ is the relative risk aversion parameter.

Next, consider an investor who chooses consumption levels C(t) and the portfolio composition between a risky β(t) and a riskless asset (1-β(t)) in order to maximize the non-expected utility function defined previously.[5] The budget constraint facing the investor is:

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[5] The founder of the continuous time consumption-portfolio model is Merton (1969).

$$dW(t)=[\beta(t)(E(x)-r)+r]W(t)dt-C(t)dt+\beta(t)W(t)\sigma dz$$

where W(t) is wealth at time t, r is the return on the riskless asset, E(x) is the expected rate of return of the risky asset, σ is the standard deviation of the risky asset's return and dz is a stochastic Weiner process.[6] [7]

Explicit solutions for the optimal consumption and portfolio choice are derived in appendix IV section A.IV.3 using dynamic programming. The optimal level of C(t) and β(t) are given by the following expressions:

$$\frac{C(t)}{W(t)} = \mu(0) = [\ \frac{\delta}{1-\theta} - \frac{\theta}{(1-\theta)}\ (\ \frac{(E(x)-r)^2}{2\sigma^2\gamma} + r\ )\ ]$$

$$\beta(0)= \frac{(\ E(x)\ -r\ )}{\sigma^2\ \gamma}$$

As was shown by Svennson (1989) it is not the intertemporal elasticity of substitution that affects the optimal portfolio composition but the risk aversion and the parameters of the distribution.[8] [9] On the other hand, both the intertemporal elasticity of substitution and

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[6] See appendix IV section A.IV.1 and A.IV.2 for a rigorous analysis of the stochastic process and budget constraint.

[7] It is assumed that along the optimal path C(t), W(t) and W_0 are greater than zero.

[8] As can be seen if $\alpha < r$ the investor sells short some of the risky asset. If $\alpha - r > \sigma^2\gamma$ the investor will borrow funds and invest them in the risky asset.

[9] This is different from the multi-period model. In the multi-period model the elasticity of substitution affects the amount invested in the risky asset but does not affect the proportion just as in the infinite horizon case.

the relative risk aversion parameters affect the consumption stream.

Another interesting economic parameter that will be used in our analysis is the rate of growth of wealth per unit of time. From the budget constraint the expected rate of growth of wealth is given by:

$$E(\frac{dW/W}{dt}) = \rho(0) - \mu(0)$$

$$\rho(0) = \beta(0)\ (a - r) + r$$

The expected rate of growth of wealth is equal to the return of the composite asset (i.e. the weighted average return) less the consumption rate. The expected rate of growth of wealth is affected by risk aversion and the intertemporal rate of substitution.

The comparative static results of a change in the parameters of the model on risk taking activity suggest that a) as the degree of relative risk aversion increases the proportion invested in the risky asset falls, b) as the variance of the risky asset increases the investor reduces the risky asset demand, and c) an increase in the excess return of the risky asset increases the proportion invested in the risky asset and finally, d) the elasticity of substitution does not affect risk taking activity.

Changes in the parameters of the model on consumption activity indicate that a) an increase in the relative risk aversion parameter increases, leaves unchanged, or decreases current consumption as θ is greater than zero, equal to zero, or negative, respectively.

Given that the empirical evidence indicate a low elasticity of substitution, a large negative θ, we expect current consumption to drop with increasing risk aversion. Since risky asset holdings also drop with increases in aversion to risk, an increase in risk aversion leads to increases in the safe asset demand. Furthermore, since the optimal portfolio is constant through time let us define a composite portfolio that has a constant mean $\rho = r + \beta(E(x)-r)$ and a constant variance $\sigma^2(\rho) = \beta^2\sigma^2$. Merton (1969) suggested that one can visualize the original problem as one being reduced to a Phelps-Ramsey problem of optimal consumption under uncertain yield of a composite asset. The optimal consumption-wealth ratio is shown below:

$$\mu(0) = [\frac{\delta}{1-\theta} - \frac{\theta}{1-\theta} (\rho - \gamma \frac{\sigma^2(\rho)}{2})]$$

The following are some of the comparative static results of this problem. An increase in the mean of the composite asset will increase, leave unchanged, or decrease the level of consumption as the elasticity of intertemporal substitution is less than, equal, or greater than unity, respectively.

Following Merton (1969) we can derive the substitution and income effects arising from an increase in the mean of the composite portfolio. This is shown in appendix IV section A.IV.4.[10] We just present the results:

[10] In Appendix IV I follow the procedure of Merton (1969).

Substitution Effect :

$$\frac{\partial C(0)}{\partial \rho}\Big|_{\Delta J(W_0))} = -\eta\ W_0$$

Income Effect :

$$W_0$$

where W_0 is the initial endowment of the investor.

The substitution effect indicates that the investor will lower the level of current consumption and increase the savings level since a higher yield leads to a cheaper price of future consumption. The income effect on the other hand leads to an increase in current consumption. The overall effect cannot be determined a priori but depends on the elasticity of intertemporal substitution. In particular, if the elasticity of substitution is greater than one, equal, or less than one current consumption will fall, remain unchanged, or increase, respectively. A high elasticity of intertemporal substitution causes the negative substitution to outweigh the positive income effect leading to a lower level of current consumption. The opposite is true with a low level of intertemporal substitution. The degree of risk aversion plays no role in the determination of the outcome.

An increase in the variance of the composite asset will increase, not affect, or decrease current consumption as the elasticity of substitution is greater than, equal to, or less than unity, respectively.[11] The higher the degree of risk aversion the greater the impact of an increase in variance of the composite asset on current consumption. The substitution and income effects are given by:

$$\textit{Substitution Effect}:$$

$$\frac{\partial C(0)}{\partial \sigma^2(\rho)}\Big|_{\Delta J(W(0))} = \frac{\eta \ \gamma \ W(0)}{2}$$

$$\textit{Income Effect}:$$

$$-\gamma \frac{W(0)}{2}$$

An increase in the variance of the composite asset causes a positive substitution effect on current consumption. This effect is proportional to the elasticity of substitution and the degree of relative risk aversion. A low elasticity of intertemporal substitution (given the level of risk aversion) causes a smaller substitution effect. A lower degree of risk aversion also causes a smaller substitution effect. The income effect leads to a reduction in current consumption. The income effect is smaller in absolute value the lower the degree of risk aversion. The total effect depends on the positive substitution effect and the negative income effect. The degree of elasticity of intertemporal substitution determines the outcome of such an exercise. The role of the risk aversion parameter determines the magnitude of the change.

The final exercise, prior to examining the various taxation policies, deals with the

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[11] Svennson does not examine changes in the mean and variance of the composite portfolio.

relative effect of a concurrent increase in the mean of the asset and a reduction in variance. By examining this result we can state whether a percentage change in the mean of the composite asset leads to a greater percentage change in current consumption or the same percentage reduction in variance leads to a greater percentage change in current consumption. This analysis can also provide insights into capital ne taxation since such a tax reduces the mean and variance of the composite asset. Let us define the following elasticities:

Elasticity of Consumption with respect Mean:

$$E_1 = \frac{\rho}{C(t)}\frac{\partial C(t)}{\partial \rho}$$

Elasticity of Consumption with respect variance:

$$E_2 = \frac{\sigma^2(\rho)}{C(t)}\frac{\partial C(t)}{\partial \sigma^2(\rho)}$$

Finally, by examining the relative ratio of the two elasticities we get:

$$-\frac{E_1}{E_2} = \frac{2}{\gamma}\,CV$$

A value of greater than one would indicate that consumption responds more to a percentage increase in the mean of the composite asset than a percentage reduction in the variance of the composite asset and vice-versa. This ratio depends only upon the relative risk aversion parameter and the coefficient of variation. It is independent of the elasticity of intertemporal substitution. In particular, the ratio varies proportionately with the coefficient of variation and inversely with the relative risk aversion parameter. As an illustration, let CV = 1, then a high risk averse investor ($\gamma > 2$) would to respond to a reduction in the variance by more than a mean increase. A low risk aversion parameter ($\gamma < 2$) would indicate that consumption responds more to mean changes than to variance changes. Holding the relative risk aversion constant, an increase in the mean per unit of variance would lead to a greater response of consumption to a mean change than a variance change.

5.3 The Effect of Various Capital Income Tax Policies

In this section we consider the effects of a capital income tax t_k. This tax can be thought as an equal tax rate on both the risky and safe asset. We also consider a tax on the return of the risky asset t_x and a tax on the safe asset t_m on current consumption and portfolio choice under CRRA and CIES.[12]

5.3.1 A Capital Income Tax

A capital income tax affects the constraint in the following way:

$$dW(t)=(1-t_k)[\ \beta(t)(E(x)-r)+r\ ]W(t)dt-C(t)dt+(1-t_k)\beta(t)W(t)\sigma dz$$

The optimal consumption ratio and risk taking activity are given by:

The expected growth rate of wealth is:

[12] The results of this section are presented in appendix IV section A.IV.5.

$$\mu(t_k) = \mu(0) + (\eta - 1)\, t_k\, r$$

$$\beta(t_k) = \frac{\beta(0)}{(1-t_k)}$$

$$E\left(\frac{dW/W}{dt}\right) = (1-t_k)\, \rho(t_k) - \mu(t_k)$$

$$E\left(\frac{dW/W}{dt}\right) = \rho(0) - \mu(0) - \eta\, r\, t_k$$

The substitution and income effects of a capital income tax:

Substitution Effect :

$$\frac{\partial C(t_k)}{\partial t_k}\Big|_{\Delta J(W_0))} = \eta\, r\, W_O$$

Income Effect :

$$- W_0$$

where W_0 is the endowment of the investor.

The substitution effect indicates that the investor will increase the level of current consumption and decrease the savings level since the capital income tax makes future consumption more expensive relative to current consumption. The income effect on the other hand leads to a decrease in current consumption. The overall effect cannot be determined a priori but depends on the elasticity of intertemporal substitution. In par. ·ular, if the elasticity of substitution is greater than one, equal, or less than one current consumption will decrease, remain unchanged, or increase, respectively. A high elasticity of intertemporal substitution causes the positive substitution to outweign the negative income effect leading to a higher level of current consumption. The opposite is true with an intertemporal elasticity of substitution of less than unity which is the norm. Notice that the degree of risk aversion plays no role in the determination of the outcome.

A capital income tax also increases risk taking activity. The result is identical to the DM phenomenon. The investor reacts by increasing the proportion invested in the risky asset in order to encounter the same probability distribution as prior to the tax imposition. Private risk taking remains unchanged.[13] Finally the expected growth rate of wealth falls with a capital income tax. The reduction in the expected growth rate of wealth depends on the elasticity of substitution, the risk free rate and the capital income tax rate. Notice, that these results are different from the two period model.

5.3.2. A Tax on the Return of the Safe and the Risky Asset

A tax on the safe asset at a rate of t_m and a tax on the risky asset at a rate of t_x affects the budget constraint as follows:

$$dW(t) = [\ \beta(t)((1-t_x)E(x)-r(1-t_m)) + r(1-t_m)\ ]W(t)dt - C(t)dt + (1-t_x)\beta(t)W(t)\sigma dz$$

The optimal consumption ratio and risk taking activity:

[13] With CRRA and CIES utility function the portfolio decision becomes separable from the savings, and the analysis of the one period model applies.
Therefore, the portfolio decision in an infinite horizon model and that of one period model are identical.

$$\mu(t_x,t_m) = [\frac{\delta}{(1-\theta)} - \frac{\theta}{1-\theta}(\frac{(1-t_x)E(x)-r(1-t_m))^2}{2\sigma^2(1-t_x)^2\gamma} + (1-t_m)r)]$$

$$\beta(t_x,t_m) = \frac{(1-t_x)E(x) - (1-t_m)r}{\sigma^2(1-t_x)^2\gamma}$$

The Effect of a Tax on the Risky Asset

The effect of an increase in the tax on the risky asset on the consumption stream per unit of wealth is:

$$\frac{\partial\mu(t_x,t_m)}{\partial t_x} = (\eta - 1)\frac{((1-t_x)E(x)-r(1-t_m))}{\sigma^2(1-t_x)^3\gamma}\, r\,(1-t_m)$$

A tax on the risky asset will reduce the consumption stream per unit of wealth for all intertemporal elasticity of substitution values less than unity.

The substitution and income effects of the tax on the risky asset:

Substitution Effect :

$$\frac{\partial C(t_x,t_m)}{\partial t_x}\Big|_{\Delta J(W_0))} = \eta\; r\,(1-t_m)\,\frac{\beta(t_x,t_m)}{(1-t_x)}\,W_0$$

Income Effect :

$$-\,r\,(1-t_m)\,\frac{\beta(t_x,t_m)}{(1-t_x)}\,W_0$$

where W_0 is the endowment of the investor.

The substitution effect of this tax depends on both the elasticity of substitution and the relative risk aversion parameter. The higher the elasticity of substitution the greater the substitution effect of the tax on the consumption stream. Also the substitution effect of this tax on consumption varies inversely with the relative risk aversion parameter. The more risk averse the investor the less the proportion invested in the risky asset and the smaller is the substitution effect.

The effect of this tax on the risky investment is given by:

$$\frac{\partial\beta(t_x,t_r)}{\partial t_x} = \frac{\beta(t_x,t_m)}{(1-t_x)} - \frac{(1-t_m)r\,\beta(t_x,t_r)}{(1-t_x)}\frac{1}{(1-t_x)E(x)-(1-t_m)r}$$

The effect of a tax on the risky asset depends on the relative strengths of two opposite effects. The first is the relative inattractiveness of the risky asset vis-a-vis the safe asset (the second term in the above comparative static result). This discourages investment in the risky asset. The second effect is the DM phenomenon (the first term in the above equation). This encourages investment in the risky asset. The overall effect depends on the relative strength of the two effects. As a general result if the after tax risk premium defined as $(1-t_x)E(x) - (1-t_m)r$ on the risky asset is greater than the after tax return on the safe asset the risky asset will increase. If on the other hand, the risk premium is smaller than the after tax return on the safe asset then the risky asset will be discouraged.

The effect of the tax on the rate of growth of wealth is given by:

$$\frac{\partial E(\frac{dW/W}{dt})}{\partial t_x} = -(\eta+1)\, r\,(1-t_m)\, \frac{\beta(t_m,t_x)}{(1-t_x)} < 0$$

The expected growth rate of wealth will unambiguously fall from the imposition of this tax.

The Effect of a Tax on the Safe Asset

The effect of an increase in the tax on the safe asset on the steady state consumption stream per unit of wealth is given by:

$$\frac{\partial \mu(t_x,t_m)}{\partial t_m} = (\eta - 1)\, r\, [\, 1 - \frac{((1-t_x)E(x) - r(1-t_m))}{\sigma^2(1-t_x)^2\gamma}\,]$$

A tax on the safe asset will also reduce the consumption stream per unit of wealth for all intertemporal elasticity of substitution values less than unity and the investor is a lender. The substitution and income effects of the tax on the safe asset:

Substitution Effect :

$$\frac{\partial C(t_x,t_m)}{\partial t_m}\Big|_{\Delta J(W_0))} = \eta\, r\, (1 - \beta(t_x,t_m))\, W_0$$

Income Effect :

$$-\, r\, (1-\beta(t_x,t_m))\, W_0$$

where W_0 is the endowment of the investor.

The substitution effect of this tax depends on the elasticity of substitution and the proportion of wealth invested in the safe asset. The higher the elasticity of substitution the greater the substitution effect of the tax on the consumption stream. The substitution effect of this tax on consumption varies positively with the proportion invested in the safe asset which is determined by the relative risk aversion parameter. The more risk averse the investor the higher the proportion of wealth invested in the safe asset the bigger the substitution effect of the tax.

The effect of this tax on the risky investment is given by:

$$\frac{\partial \beta(t_x,t_r)}{\partial t_m} = \frac{r}{\sigma^2(1-t_x)^2\gamma} > 0$$

The effect of a tax on the risky asset yields an unambiguous conclusion. The tax encourages the proportion invested in the risky asset. This happens because the tax makes the safe asset less attractive.

The effect of the tax on the rate of growth of wealth is given by:

$$\frac{\partial E(\frac{dW/W}{dt})}{\partial t_x} = -r\left[\eta\ (1-\beta(t_x,t_m)) - \beta(t_m,t_x)\right]$$

The expected growth rate of wealth is ambiguous. This ambiguity arises because of the stimulus of the tax on the risky asset and on the weighted average rate. Notice that for an elasticity of substitution of zero the expected growth rate of wealth will increase from the imposition of this tax.

5.4 Conclusion and Future Research.

The comparative static analysis of alternative capital income tax policies indicate that the effect of the tax on the consumption stream of the investor depends on the value of the intertemporal rate of substitution. First a capital income tax reduces the consumption per unit of wealth for all less than one values of the elasticity of substitution. The capital income tax increases the proportion invested in the risky activity. Most importantly, the tax effect on consumption operates through the elasticity of intertemporal substitution and not the relative risk aversion parameter. The risk aversion of the investor affects the magnitude and not the direction of the comparative static results. The intertemporal elasticity of substitution does not affect portfolio choice as previously noted in the literature. Second a tax on the risky asset also reduces the consumption stream per unit of wealth for less than unity elasticity of substitution. The tax on the risky asset has an ambiguous effect on the proportion invested in the safe asset. The overall effect depends on the relative size of the after tax risk premium and the after tax return on the safe asset. Third the effect of the tax on the safe asset causes a reduction in the consumption stream for all elasticity of substitution values less than unity. The tax on the safe asset unambiguously stimulates the risky asset since it is more attractive now relative to the safe asset.

The results of the infinite horizon are comparable to the one period model. This happens due to the assumption of constant relative risk aversion and constant elasticity of substitution. However, the one period model constrains the elasticity of substitution to equal the inverse of the relative risk aversion parameter and hence even these results are altered.

Future work should concentrate on the optimal differentiation of risky versus non-risky income taxation. This will shed more light on the capital income taxation policy. In addition, using the methodology of Gordon and Wilson (1989) we can compute the deadweight losses of capital income taxation. Gordon and Wilson (1989) argued that the infinite horizon deadweight loss of capital income taxation is smaller than the one reported in the two period model.

Chapter 6

Capital Income Taxation and Tax Revenue Risks

6.1 Introduction

Chapters three and four examined the behaviour of the investor when the government re-distributed the "stochastic" per capita lump sum tax revenue back to the household. In this sense the investors ultimately bear the risks of their own tax revenue fluctuations. In all cases, except the taxation of capital gains (i.e. the excess returns or extraordinary gains), I find deadweight losses of capital income taxation. The deadweight loss of the imputed safe asset turned out to be identical to the deadweight loss of a full capital income tax policy for equal tax rates $t_r = t_k$. Ergo, the inclusion of excess returns in the tax base of the imputed safe income does not cause any additional deadweight losses to the investors and can generate additional tax revenue for the government.

Attention must be given to the observation that the taxation of excess returns, not only does not cause any additional deadweight loss, but is independent of what the state does with the stochastic tax revenue. If the state returns the stochastic lump sum tax revenue from excess returns to the investor, in the same period that it was collected, the investor's welfare will remain unchanged. In this case, the investor will not re-allocate his (her) portfolio holdings. The amount invested in the risky asset will remain unchanged. If on the other hand, as chapter two indicated, the state uses the funds for the provision of a public good that enters the utility function additively, the investor is still indifferent in terms of welfare. However, in this case the investor increases the risky asset and reduces the safe asset so that he will face the same probability distribution as prior to the imposition of the tax. Accordingly, the investor is practically indifferent as to whether the state returns the funds in the form of risky tax revenue, in the same period it has been collected, or in the form of random government expenditures that are used to provide a public good that enters the utility function in an additive manner. The allocative effects, however, of the two policies are different as we observed. There is one important difference between the two analyses; the government by taxing full capital income can generate additional tax revenue. Can the state use these additional dollars for the provision of a public good? This chapter addresses this question.

Chapters three and four can be thought as a special case in which the risky outcome is perfectly correlated for all investors. The state re-distributes the "stochastic" per capita tax revenue to the investor and this amount matches exactly the tax payment of the investor. This chapter examines a different case. I assume that the government re-distributes back to the investors a part of the stochastic per capita lump sum risky tax revenue the current generation pays. The other part is transferred to future generations. This implies that there is some sharing of tax revenue uncertainty.[1] The current

[1] Intergenerational risk sharing has been introduced by Gordon and Varian (1988) while Hamilton (1987) contributed to the normative implications of capital income taxation under uncertainty.

generation receives, for sharing tax revenue risks, with past generations a share of the "stochastic" tax revenue of the previous lotteries in which they participated even though they may not have been born yet.

We assume that the future generations share with the current investors the current "market" or "social" risks via tax policy and ultimately an uncertain government budget constraint. This way, the current generation absorbs a small fraction of the riskiness of the current tax revenue, the other fraction being assimilated by all future unborn generations. My argument relies on the idea that what is "systematic" risk at a certain point in time will become "idiosyncratic" risk when this is shared with all current and future unborn generations. Pooling tax revenue risks across generations resembles the diversification theorems of the finance literature.

Section 6.2 reviews the risk pooling concept. Section 6.3 applies the risk pooling argument to risky tax revenue. We also discuss different weights that can be used to discount tax revenue. Section 6.4 evaluates the risky capital income tax revenue using the security market approach. Section 6.5 examines the effects of this policy on current consumption, the size of the portfolio and its composition. Section 6.6 re-examines the deadweight losses of capital income taxation. Finally section 6.7 concludes.

6.2. Risk Pooling

The concept of risk pooling, not being a new concept in economic theory, was pioneered by the Nobel price winner Markowitz (1952). Risk pooling is defined as the reduction in the variance of a portfolio as the number of securities in a portfolio is increased. This reduction in variance occurs as long as the assets of the portfolio are imperfectly correlated. To illustrate, consider a portfolio consisting of n assets such that the share of the portfolio taken up by asset i is γ_i and the return per dollar invested is x_{it} in period t. The investor's total return on the portfolio is given by:

$$R_{pt} = \sum_{i=1}^{n} \gamma_i \, x_{it}$$

which is a random variable. The expected return from the portfolio of assets is:

$$E(R_{pt}) = \sum_{i=1}^{n} \gamma_i \, E(x_{it})$$

The expected return on the portfolio is a weighted average of the expected returns of the assets in the portfolio. More explicitly, the variance of the portfolio is:

$$Var(R_{pt}) = \sum_{i=1}^{n} \gamma_i^2 \, var(x_{it}) + \sum_{i=1}^{n} \sum_{\substack{j=1 \\ j \neq i}}^{n} \gamma_i \, \gamma_j \, cov(x_{jt}, x_{it})$$

What happens to the portfolio variance as we increase the number of assets? The variance of the portfolio decreases and approaches the average covariance of the securities. This famous result is due to Markowitz (1952). This result is intuitively obvious since the first term in the expression has n elements while the second term has n(n-1) elements. More specifically, consider taking the limit of the variance of the portfolio as the number of assets in the portfolio increases assuming without loss of generality that $\gamma_{it} = 1/n$ for all i.[2]

$$\lim_{n\to\infty} Var(R_{pt}) = \lim_{n\to\infty} \sum_{i=1}^{n} \gamma_i^2 \, var(x_{it}) + \lim_{n\to\infty} \sum_{i=1}^{n} \sum_{\substack{j=1 \\ j\neq i}}^{n} \gamma_i \, \gamma_j \, cov(x_{jt}, x_{it})$$

substituting $\gamma_{it} = 1/n$ yields:

$$\lim_{n\to\infty} Var(R_{pt}) = 0 + \lim_{n\to\infty} \sum_{i=1}^{n} \sum_{\substack{j=1 \\ j\neq i}}^{n} \frac{1}{n^2} cov(x_{jt}, x_{it})$$

The first term drops to zero, while the second term remains as non-zero. The second term represents the average covariance among all assets in the portfolio as seen from the following expression:

$$\lim_{n\to\infty} Var(R_{pt}) = \lim_{n\to\infty} \frac{1}{n^2} \sum_{i=1}^{n} \sum_{\substack{j=1 \\ j\neq i}}^{n} cov(x_{jt}, x_{it})$$

$$\operatorname*{Lim}_{n\to\infty} Var(R_{pt}) = \overline{\sigma_{ijt}}$$

When all asset returns are mutually uncorrelated the variance of the portfolio can be decreased to zero through diversification. Hence it is possible, as Bailey and Jensen (1972) argued, that private exceed social risks if the covariance among returns on a very

[2] This result holds as long as all γ_{it} approach zero as the number of securities increases to infinity.

large number of assets is zero and the investor cannot hold a very large number of assets due to transaction costs and indivisibility of assets. A typical example of such an asset is human capital.[3]

This analysis has been applied to the evaluation of risky public investment projects in an article by Bailey and Jensen (1972) arguing in favour of using higher discount rates for government projects than for private ones of comparable risk. A debate on the appropriate discount rate for public projects took place in the 1960s with the work of Hirshleifer (1964), Samuelson (1964) and Arrow (1965). The latter two argued for a lower discount rate. Arrow (1965) based his argument on "the law of large numbers" without discussion on the size of the risky project undertaken. Hirshleifer (1964) argued for a higher discount rate similar to that used in private investment. In the seventies the debate continued with Arrow and Lind (1970) supporting a lower discount rate for a different reason and, on the other hand Mayshar (1977) and Sandmo (1972)) arguing for a higher discount rate. Mayshar (1977) and Sandmo (1972) argued for a higher discount rate as long as capital markets are perfect. On the other hand, Arrow and Lind (1970) argued that the appropriate social discount rate for risky public investments is the risk free rate. Arrow and Lind relied on the state taking projects with uncorrelated returns, where each project is small in size for the generation of current taxpayers. The only discussion on the proper discount rate for the evaluation of risky tax revenue comes from

[3] As long as there are not economy wide risks this result has been shown by Eaton and Rosen (1981).

Hamilton (1987) who uses a weighted average as the appropriate discount rate for risky tax revenue. He supports his decision based on the existing work of Hirshleifer (1964), Mayshar (1977) and Sandmo (1972). In the next section we attempt to evaluate this risky tax revenue instead of treating it the same as risky public projects.

6.3. Implications for Tax Revenue Evaluation

Let us consider the entire group of generational tax payers. If the number of such generational risk sharers is sufficiently large for each to share in a small part of the current tax revenue risks, and if the tax revenue across generations is independently distributed, then the weighted average tax revenue across generations has a lower variance then the per capita stochastic tax payment of a single generation. In the limiting case the risk of tax revenue fluctuations for a single generation can be ignored. Atkinson and Stiglitz (1980) state:

"...there appear to be circumstances where the return to different assets which different individuals purchase is imperfectly correlated, where there may be possibilities for risk sharing effectively through the intermediation of the government, which are not possible through the market. Suppose at the other extreme from the previous example, that the returns are independently distributed then in the limiting case the lump sum payment is the expected revenue, evaluated at the equilibrium." (pg. 118)

Suppose that the state returns the following per capita "stochastic" tax revenue:[4]

$$R_{pt} = \sum_{i=1}^{n} \gamma_i \, T^{*}_{2t-i+1}$$

where n is the number of generations sharing the risks of tax revenue fluctuations and T_{2t-i+1} is the per capita stochastic tax revenue of generation t-i+1 paid in period 2. Notice

[4] I am assuming that the tax revenue that is passed onto future generations can earn interest. However, due to population growth it will cancel out if the latter equals the growth rate of population. Future research should examine this. However, as long as these rates are not stochastic our conclusions still remain.

that the current generational investor receives a fraction of the current tax revenue risks $\gamma_1 T_{2t}$. What he gets from the state in this period is less than what he pays to the state. But he also receives stochastic tax revenues from past tax revenue lotteries. The past tax revenue he receives is:

$$G_{pt} = \sum_{i=2}^{n} \gamma_i \ T^{*}_{2t-i+1}$$

He participated in these past lotteries when he was not yet born and hence could not have bought shares into these lotteries. In this sense, the state through this re-distributional policy allows the investor to participate in lotteries which he could not have otherwise bought.

For example with two generations (i.e., in a given time period t the young and old overlap) sharing the tax revenue risks we have the following payoff matrix:[5]

t=1	t=2	t=3	t=4	t=5
$\gamma_1 T_{20}$	$\gamma_2 T_{20}$			
	$\gamma_1 T_{21}$	$\gamma_2 T_{21}$		
		$\gamma_1 T_{22}$	$\gamma_2 T_{22}$	
			$\gamma_1 T_{23}$	$\gamma_2 T_{23}$

The first row refers to the time period. All other rows show how the per capita tax payment of a generational investor is distributed across time. Notice that the sum of the

[5] I am assuming that all generations are identical except in the stochastic events taking place across time. The assumption of identical individuals is standard in the public finance literature in order to examine efficient outcomes.

row entries represents the total per capita stochastic capital income tax paid by the typical young generation deciding in period t. Also notice that the sum of the columns are the lump sum re-distributional amounts each generation receives. For example, when the young generation is deciding their consumption stream in period t=1 it pays T_{21} of stochastic capital income tax to the state in the period 2. I assume that the state returns only a fraction of the per capita lump sum tax revenue (i.e. the fraction is γ_1) in the same period that the tax is collected (i.e. t=2). Thus the young generation when old (i.e. t=2) receive less stochastic return and less risk from his own risky tax revenue. This payment is shown in the second column third row as γ_1 T_{21}. The other fraction of his (her) tax revenue $\gamma_2 T_{21}$ is passed onto the next young generation who are deciding their consumption stream in period t=2. The young generation of period 1 will also receive in period 2 a fraction of the tax revenue of the previous generation, thus this young generation will receive a fraction of the stochastic tax revenue and risk of the previous generation's tax payment ($\gamma_2 T_{20}$). Accordingly, the next young generation in period t=2 will receive the fraction of their stochastic tax payment in period t=3 of γ_1 T_{22} and a fraction of the tax revenue of the previous generation, γ_2 T_{21}. Assuming this process continues each generation receives a well diversified stochastic tax revenue.

The following is the pay-off to each generational investor:

Time	Stochastic Lump Sum Return
t=1	$R_{p1} = \gamma_1 T_{21} + \gamma_2 T_{20}$
t=2	$R_{p2} = \gamma_1 T_{22} + \gamma_2 T_{21}$
t=3	$R_{p3} = \gamma_1 T_{23} + \gamma_2 T_{22}$

Generalizing then at any given period t the young generation receives the following tax revenue:

$$R_{pt} = \sum_{i=1}^{n} \gamma_i T^{*}_{2t-i+1}$$

Notice that if the tax revenue is non-stochastic, then the lump sum payment the investor receives matches the per capita tax payment.[6]

The γ_i are the weights placed on the current and past lotteries. The weights have the common property of adding up to 1, and being between zero and one.

$$0 < \gamma_i < 1$$

$$\sum_{i=1}^{n} \gamma_i = 1$$

[6] Notice also that this re-distributional policy will adversely affect the old generation at the time the policy is implemented. Thus, this policy like any other policy will have a transitional phase.

These weights are determined exogenously in this thesis. Future research should determine the optimal weighting scheme. Why would investors evaluate this tax revenue and not their own tax revenue fluctuations? How is this re-distributional policy possible? What are the implications in terms of decision making and in terms of welfare? What are the implications for the choice of tax base? These questions are important in terms of public policy and need to be answered.

Investors evaluating their own tax revenue is also a possibility but imagine, as an example, another Great Depression happening tomorrow. In this case, a lot of people would lose a great deal of financial as well as human wealth. In a system whereby there is capital income taxation, the tax revenue and tax payments from the capital income source will fall and the government will run a budget deficit and issue debt. In order for future generations to offset this budget deficit the current generation of taxpayers will receive a lump sum stochastic tax payment that is lower than their tax payment to the state. The current generation, however, participated in past tax revenue lotteries and hence will receive some stochastic tax revenue from previous generations. In other words, the current generation will not face their own tax revenue risks. The degree of risk sharing among generations is determined by a particular weighting scheme society selects. Gordon (1985), Summers and Bulow (1984) and Gordon and Wilson (1989) place a weight of one to the current generation. However, any other weighting scheme will allow current and future generations to share in the current economic risks through tax revenue uncertainty.[7]

Among the suggested weights we may include the equal weighting scheme. In this case:

$$\gamma_i = \frac{1}{n} \quad \forall\ i$$

The stochastic lump sum tax revenue returned to the investor is an equally weighted average tax revenue across generations. An alternative weighting scheme is a declining weighted average whereby:

$$\gamma_1 \; > \; \gamma_2 \; > \; \gamma_3 \; > \; \gamma_4 \; > \; etc.....$$

This declining weighting scheme suggests that the current generation weights its own "stochastic" lump sum tax revenue fluctuations more than the past stochastic tax revenue lotteries. We also have the Gordon (1985), Summers and Bulow (1985) weighting scheme whereby:

$$\gamma_i = 0 \quad \forall \quad i \neq 1$$

$$\gamma_1 = 1$$

The weighting scheme society chooses to place on stochastic tax revenue will have very strong implications for choosing the tax base of individuals. See the next two chapters of the thesis for further elaboration.

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[7] In addition, I am assuming that the current tax revenue risks are imperfectly correlated with the tax revenue risks of the next generation. In this sense tax revenue risks can be seen as a supplement to an undiversified portfolio across time.

We assume that the stochastic tax revenue which is evaluated by a typical investor is the weighted average tax revenue across generations and not the tax revenue it pays to the state. The investor is still concerned with the evaluation of this tax revenue and calculates the expected value of this portfolio, which obviously is given by:[8]

$$E(R_{pt}) = \sum_{i=1}^{n} \gamma_i \, E(T_{2t-i+1}) = E(T_{2t})$$

The variance of this portfolio is given by:

$$Var(R_{pt}) = \sum_{i=1}^{n} \gamma_i^2 \, var(T_{2t-i+1}) + \sum_{i=1}^{n} \sum_{\substack{j=1 \\ j \neq i}}^{n} \gamma_i \, \gamma_j \, cov(T_{2t-j+1}, T_{2t-i+1})$$

Assuming that the social risks across generations are identically and independently distributed:

$$cov(T_{2t-j+1}, T_{2t-i+1}) = 0 \qquad \forall \;\; i \neq j, \quad i,j \in [1,n]$$

the covariance of tax revenue across generations is zero[9]. Substituting this yields the following variance of the portfolio:

$$Var(R_{pt}) = \sum_{i=1}^{n} \gamma_i^2 \, var(T_{2t-i+1})$$

Assuming a non-time varying variance yields the following result:

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[8] We are assuming that all generations are identical except in the stochastic events taking place across time. The assumption of identical individuals is standard in the public finance literature in order to examine the efficient outcomes.

[9] Even if this is not true,the average covariance across time is relatively small enough to be ignored.

$$Var(R_{pt}) = Var(T_{2t}) \sum_{i=1}^{n} \gamma_i^2$$

The variance of the risky tax revenue is greatly reduced. If we take the case of Gordon (1985), Summers and Bulow (1984), Hamilton (1987), Gordon and Wilson (1989) of assuming $\gamma_1 = 1$ and $\gamma_j = 0$ for all $j \neq i$. Then we have:

$$Var(R_{pt}) = Var(T_{2t})$$

This has a very large magnitude compared to the previous one. How many generations will it take to reduce the variance of the portfolio to zero?

With two generations:

$$Var(R_{pt}) = Var(T_{2t}) [\gamma_1^2 + \gamma_2^2]$$

Three generations:

$$Var(R_{pt}) = Var(T_{2t}) [\gamma_1^2 + \gamma_2^2 + \gamma_3^2]$$

Four generations:

$$Var(R_{pt}) = Var(T_{2t}) [\gamma_1^2 + \gamma_2^2 + \gamma_3^2 + \gamma_4^2]$$

It will not take many generations to reduce the variance of the tax revenue approximately to zero. The equally weighting scheme will reduce this variance faster then a declining scheme. Notice the correspondence of this result to portfolio theory. In portfolio theory reviewed earlier in the chapter, an equally weighted scheme was assumed. An infinitely small amount is invested in each security. In our portfolio theory we can have a declining scheme. The choice of a particular weighting scheme can be obtained via some social welfare function or some political mechanism. We leave this for future research. Also public finance scholars can devote time and effort providing empirical research to the above claim.

6.4. Security Market and the Evaluation of the Risky Tax Revenue

Let us examine the issue in a slightly different context and assume that a security market exists. Next, suppose that the government will re-distribute this tax revenue, in the second period, to the investor. The investor wants this tax revenue evaluated and rushes to the stock market to get a price. The investor evaluates this security just like any other security. However, securities actually trading in the market cannot be considered a perfect substitute for the proposed one. In fact, this security has something special; it is defined to exist across time periods. The stochastic capital income tax revenue of interest to the current period generational investor is:

$$R_{pt} = \sum_{i=1}^{n} \gamma_i \, T_{2t-i+1}$$

where again the γ_i is the weight that is placed on current and past lotteries. The mean and variance of this security was discussed previously. Recall that the variance of the security falls as more and more generations participate in current tax revenue risks. What price are the investors willing to pay for this security ? I assume that the investors can sell this security at a price that is established in the market, and add it to their future consumption stream.

The market value of the tax revenue is usually used for the provision of a public good or service and not returned in a lump sum fashion to the investor to augment his future consumption stream. However, since we are concerned with efficiency issues in this chapter we will continue to assume that the per capita "stochastic" lump sum tax is redistributed back to the investor, in the second period. However, if the tax revenue is used for the provision of a public good then public finance scholars ought to conduct a differential incidence analysis to determine preference towards the ranking of a particular tax base. The answer to the preceding inquiry will critically depend on the value of the tax revenue. Such a differential incidence study is conducted in the following two chapters.

The most common approach in the finance literature has been to compute the price of a security by the "capital asset pricing model" (CAPM) of Lintner (1965), Mossin (1966) and Sharpe (1964). The CAPM allows us to evaluate the risky tax revenue flow. What price are the young investors willing to pay for this risky security? Investors that hold diversified portfolios do not price risky assets based on the variability of the risky asset but what matters to these investors is the covariance of the risky asset with the market portfolio.[10] Using the capital asset pricing model we can solve for the certainty equivalent of the risky tax revenue. This will reflect the price of the "risky" security. Using the CAPM the price of the bond will equal the expected return less the risk premium. The risk premium is a function of the covariance of the uncertain tax revenue with the market return times the price of risk. The following expression illustrates:

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[10] We are assuming that the investors are holding perfectly diversified portfolios. If the investor held less than perfectly diversified portfolio then capital income taxation would be welfare improving.

$$CE(R_{pt}) = E(R_{pt}) - \lambda \, cov(R_{pt}, x_{t+1})$$

$$\lambda = \frac{E(x) - r}{\sigma_x^2}$$

where x_{t+1} is the return on the market portfolio in the second period, E(x) is the expected return from the market portfolio, σ_x^2 is the variance of the market portfolio and λ is the market price of risk or the expected excess return of the market portfolio per unit of risk, and R_{pt} is as indicated previously. As seen above what is important in terms of evaluating this security is the covariance of the security with the market portfolio. What price are young investors willing to pay for the security whose risks are shared among all generations? Again using the CAPM to evaluate this new stream of revenue, the certainty equivalent of this tax revenue stream is:

$$CE(R_{pt}) = E\left[\sum_{i=1}^{n} \gamma_i T_{2t-i+1}\right] - \lambda \, cov\left[\sum_{i=1}^{n} \gamma_i T_{2t-i+1}, \lambda_{t+1}\right]$$

Expanding the terms and using the properties of covariance we get:

$$CE(R_{pt}) = E(R_{pt}) - \lambda \, cov\left[\gamma_1 T_{2t} + \gamma_2 T_{2t-1} + \gamma_3 T_{2t-2} + \ldots, x_{t+1}\right]$$

Since the market or social risks across generations are idiosyncratic the covariance of T_{2t} is the only relevant term in the evaluation of the risky tax revenue. Hence:

$$cov\ [\ \gamma_i T_{2t-i+1}\ ,\ x_{t+1}\ ] = 0 \qquad\qquad \forall\ \ i \neq 1$$

Substituting this into the certainty equivalent tax revenue we get:

$$CE(R_{pt}) = E(R_{pt}) - \lambda\, \gamma_1\, cov(T_{2t}, x_{t+1})$$

This is the price of "risky" tax revenue stream assuming that the household holds the market portfolio. Notice, that the second period "stochastic" per capita tax revenue of the t generation will have a risk premium that is lower than the market risk premium by γ_1 the weight placed on the risky tax revenue. Hence as more and more generations share the current tax revenue risks the lower the weight placed on the current tax revenue risks.

6.5. Capital Income Taxation and the Value of the Uncertain Tax Revenue

The full capital income tax revenue the state receives from the investor is:

$$T_{2t}^{*} = t_k \left(r S_{1t}^{*} + a_{1t}^{*} z_{t+1} \right)$$

Where a_{1t}^{*} and S_{1}^{*} are aggregate portfolio choices. We assume that this per capital risky tax revenue is shared between current and future generations. Evaluating this tax revenue using the CAPM yields the following market value:[11]

$$CE(R_{pt}) = t_k \left(r S_{1t}^{*} + (1 - \gamma_1) a_{1t}^{*} E(z) \right)$$

As long as there is intergenerational risk sharing, (i.e. $\gamma_1 < 1$) then the price of the security or the certainty equivalent of the tax revenue will increase relative to the price of a "risky" anticipation note equal to unity. The individual will be willing to pay a higher price for some tax revenue uncertainty.[12] If we take the weights of Gordon (1985), Summers and Bulow (1984) and Gordon and Wilson (1989) the value of this tax revenue is given by:

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[11] Substituting the stochastic tax revenue into the CAPM and after simple manipulations we get the value of the tax revenue. See appendix 5 also.

[12] For a person that holds an undiversified portfolio the risky tax revenue will also have a higher value. My results are independent of the investor's portfolio holdings.

$$CE(R_{pt}) = t_k \, r \, S_{1t}^*$$

The excess return of the risky asset has a zero market value. This however, is a special case in which each investor faces his own tax revenue risks and holds a perfectly diversified portfolio.[13]

We continue to assume that the investor maximizes the two period expected utility by choosing the decision rules and is subject to the budget constraint given by:

$$C_{2t} = (1 + \bar{r}) \, S_{1t} + a_{1t} \, \overline{z_{t+1}} + CE(R_{pt})$$

where $CE(R_{pt})$ is the market price of the risky tax revenue. This is how much he would have had he sold the security.

For a given $CE(R_{pt})$ the first order conditions of the problem are given by:

$$g' - (1+r(1-t_k))E(h') = 0$$

$$E(h' z_{t+1}) = 0$$

Capital income taxation distorts the relative prices of current and future consumption and creates an intertemporal distortion. This distortion is independent of the lump sum re-distributional policy of the state. However, the marginal utility of future consumption

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[13] Appendix V section A.V.2. derives the market value of other capital income tax policies.

now depends on the capital income tax rate, directly and through the lump sum re-distribution policy. Under this setting do we observe the DM phenomenon ? The lump sum tax revenue re-distribution does not cancel the tax payment of the investor and hence investors do not bear the entire current economic risk via tax revenue. Future generations also bear part of this burden. Given this risk sharing scheme what are the effect of this re-distribution scheme on the choice variables and how do they compare to our previous analysis ? And most importantly what is the effect of this policy on welfare and on the deadweight losses of capital income taxation ? The answers to these questions are given in the next section of this chapter.

As a preliminary analysis let us examine the investor's future consumption stream:

$$C_2 t = (1 + r) S_{1t}^{*} + a_{1t}^{*} \overline{z_{t+1}} + (1 - \gamma_1) a_{1t}^{*} E(z) t_k$$

A few things one can notice from this policy. The market price of the risky tax revenue is equal to the imputed safe income and a certain part of the expected value of the excess returns due to intergenerational risk sharing. The tax on the imputed safe income cancels out with the tax payment of the investor. The tax on excess returns remains. This tax remains whether there is risk sharing or not. This happens, because the investor acts as if (s)he sells the stochastic re-distributional stream and receives in exchange the non-stochastic market value of the tax revenue. However, we know from our previous analysis that the tax on excess returns is welfare neutral even though the allocative effects of this tax remain. Ignoring the last term we can conclude that the allocative effects of this policy are similar to the Hicks compensated effects analyzed in chapter two of the thesis. The last term is in a way the reward for sharing risks with the previous generations lotteries.

The effect of this policy on current consumption is given by:[14]

$$\frac{\partial C_{1t}}{\partial t_k}|_{RS} > 0$$

The effect of increasing the capital income tax rate can be decomposed into two components. The first effect is to make future consumption more expensive and as a result current consumption increases. This is the well known intertemporal substitution effect. There is a second effect in operation: the increase in the capital income tax rate increases the market price of the tax revenue and causes current consumption to increase further. If γ_1 is equal to one then this second effect is not observed and the only effect is the intertemporal substitution effect.

The effect on the amount invested in the risky asset is:

$$\frac{\partial a_{1t}}{\partial t_k}|_{RS} \gtrless 0$$

This policy leads to an ambiguous result in the amount invested in the risky asset. There are three effects operating. The first effect is the re-appearance of the DM phenomenon. The DM phenomenon re-appears because the tax falls on excess returns and remains with the investor, hence this tax alters the probability distribution of the risky asset. This happens because the stock market converts the "stochastic" transfer to a non-stochastic

[14] Appendix V section A.V.1 derives these results analytically.

market value. As a result the investor reacts by increasing the risky asset so that she faces the same pre-tax probability distribution as prior to the imposition of the tax. Notice that the DM phenomenon appears also if the state taxes only the excess returns. The second effect is the decline in risk taking activity due to the increased price of future consumption. These two effects are similar to the substitution effects observed in chapter 2 of the thesis. In chapter two we assumed that the tax revenue, stochastic or not, was allocated to a public good that enters the utility function additively. The third effect, absent in the previous analysis, is the increase in the market value of the tax revenue. This effect will also be absent if γ_1 is equal to one as Summers and Bulow (1984) argued. Notice that for CRRA preferences and an elasticity of intertemporal substitution of less than unity, the capital income tax will lead to an unambiguous increase in the risky asset holding. This substitution effect will be greater than the one presented in chapter 2 of the thesis. However, the effect on private risk taking is ambiguous and this confirms the Atkinson and Stiglitz (1980) result. However, for an individual with a very low elasticity of intertemporal substitution private risk taking will unambiguously increase.

The effect of this policy on total savings is given by:

$$\frac{\partial S_{1t}}{\partial t_k}|_{RS} = - \frac{\partial C_{1t}}{\partial t_k}|_{RS} < 0$$

Total savings unambiguously drop. This happens as a result of the increased current consumption. The effect of this policy on the safe asset is given by:

$$\frac{\partial m_{1t}}{\partial t_k}\Big|_{NRS} = \frac{\partial S_{1t}}{\partial t_k}\Big|_{RS} - \frac{\partial a_{1t}}{\partial t_k}\Big|_{RS} \lessgtr 0$$

Therefore, the effect of this policy on the safe asset is also ambiguous except in the case whereby the risky asset is encouraged. In this case savings fall and risk taking rises, which will lead to an unambiguous discouragement of the safe asset demand. Under this policy portfolio riskiness unambiguously increases given that savings fall and that risk taking increases. Or alternatively portfolio riskiness increases under the assumption of NDRRA:

$$\frac{\partial \beta_{1t}}{\partial t_k}\Big|_{RS} > 0$$

Also, this policy, under the assumptions of CRRA and CIES, and with an elasticity of intertemporal substitution of less than one, causes an unambiguous increase in current consumption, risk taking and proportional risk taking. The safe asset demand is unambiguously discouraged in this case.

6.6 The Welfare of the Investor with a Capital Income Tax

This re-distributional tax policy leads to an increase in welfare for marginal increases in the capital income tax rate. Capital income taxation creates a distortion in the intertemporal price of future consumption. The tax also reduces the current economic risks society is facing and transfers them to future generations. This way a marginal increase in the capital income tax rate from zero will not cause any distortions and provide some insurance to individuals. This resembles, somehow, the work of Eaton and Rosen (1981). However, they argued for a wage tax based on idiosyncratic risks across individuals within the same generation. I am arguing for a capital income tax rate based on idiosyncratic risks across generations. The welfare change due to an increase in the capital income tax rate is given by:

$$\frac{\partial V}{\partial t_k}\Big|_{RS} = -t_k\ r\ E(h')\frac{\partial C_{1t}}{\partial t_k}\Big|_{RS} + (1-\gamma_1)E(z)\,t_k\frac{\partial a_{1t}^{*}}{\partial t_k}\Big|_{RS} + (1-\gamma_1)a_{1t}^{*}E(z)E(h')$$

For initial t_k equal to zero the first two terms drop out. Notice that the last term remains and this is strictly positive for all $\gamma_1 < 1$. Thus for a marginal increase in the capital income tax rate from zero the investor's welfare unambiguously increases.

6.7. Conclusion

In the previous chapters capital income taxation created distortions and provided no additional insurance. I assumed that risks are perfectly correlated across individuals within the same generation. In this case private and current social or economic risks were equal. Capital income taxation cannot improve the efficiency at which the economy handles risks. Atkinson and Stiglitz (1980) make note that re-distributing "stochastic" per capita tax revenue to the investor and this amount matching the tax payment of the investor is a special case in that the risky outcome is perfectly correlated for all investors. Atkinson and Stiglitz (1980) state:

"The previous example is rather special in that the outcomes of the risky investment are perfectly correlated for all investors. Hence, there is no sense in which the government, through taxation, improves the efficiency with which the economy handles risk. On the other hand, there are strong reasons to believe that the capability of the market to share and spread risks is limited. Some of the reasons have to do with limited liability laws, i.e. the ability of individuals to default on their loans. Markets for human capital are, as a consequence, notoriously imperfect. The state, with its ability to tax, is not subject to some of the same limitations as are private lenders." (pg 119)

The present chapter takes a different prospective of the riskiness of tax revenue fluctuations. We view the riskiness of the tax revenue fluctuations of the current generation as minor when compared with the average across all generations. We show that by allowing some risks to be transferred the state budget can be welfare improving.

I assume that the government re-distributes back to the investors part of the "stochastic" per capita lump sum risky tax revenue the current generation pays. The other part is transferred to the future generations. The current generation receives, for sharing tax revenue risks, with past generations a "bonus". They will receive a share of the "stochastic" tax revenue of the previous lotteries in which they participated. I assume that the future generations share with the current investors the current "market" or "social" risks via an uncertain government budget constraint. This way, the current generation absorbs only a small fraction of the riskiness of the current tax revenue. The other fraction being assimilated by all future unborn generations. The argument relies on the idea that what is "systematic" risk at a certain point in time will become "idiosyncratic" risk when this is shared with all current and future unborn generations. Pooling risks across generations through tax revenue uncertainty resembles the diversification theorems of the finance literature.

We find that this re-distributional tax policy leads to an increase in welfare for marginal increases in the capital income tax rate. A marginal increase in the capital income tax rate from zero will not cause any distortions and provide some insurance to individuals. Capital income taxation creates a distortion in the intertemporal price of future consumption. But it also reduces the current economic risks society is facing and transfers them to future generations. Future generations will also benefit out of this policy; just like the current generation they too will receive part of the "stochastic" tax revenue of their own tax revenue and part of the past generations tax revenue lottery.

The analysis of this chapter is a prelude to the choice of the tax base presented in the next two chapters. This chapter argues that all the tax revenue which investors pay to the state has a market value that ranges between the value obtained with certainty to the expected value of the tax revenue. This in turn brings alive and strengthens the old argument of Domar-Musgrave (1944), Tobin (1958) and Stiglitz (1969) for risk sharing by the state with private investors.

Future research should derive results of other capital income tax policies. Particular attention can be given to the treatment of risky versus safe income taxation. An alternative interesting avenue for future research is to move towards an optimal capital income tax policy. One where the deadweight losses are offset with the insurance elements of tax policy. Also note that investors that hold undiversified portfolios would receive some form of insurance from the taxation of risky capital income.

Chapter 7

Cash Flow Consumption Tax or Pre-Payment Wage Tax

The Discount Rate Argument

7.1 Introduction

One principal aim of the present chapter is to evaluate alternative methods of implementation of a consumption tax in a portfolio-savings model. Proposals for implementing a consumption tax can be classified into two alternatives. The first, the cash flow method, allows a tax deduction on savings while the entire proceeds upon withdrawal become taxable; The second method, known as the pre-payment approach, is a modification of the income tax system, where the "return" on an asset is exempted from taxation. It is this second method (pre-payment) that has become most controversial. There are economists who argue that all capital income should be exempt from income tax (e.g., Hamilton (1987), Zodrow (1994)) to arrive to a consumption tax. There are others who argue that one should retain the extraordinary capital gains component in the income tax base, and exempt the imputed riskless return on the entire savings (Ahsan (1990)) to arrive to a consumption tax base, the new tax being baptized the "modified wage tax" (MWT). The argument put forward by Ahsan (1990) requires that capital gains or losses receive the same treatment under both cash flow and pre-payment versions; they both are subject to a tax.

Under what conditions does a cash flow consumption tax (CFT) become equivalent to a wage tax under uncertainty? When does a modified wage tax become equivalent to a wage tax? Is the cash flow consumption tax equivalent to the modified wage tax? Answering the question on the equivalence between a consumption tax and a "wage tax" or a "modified wage tax" in the presence of uncertainty will provide a new perspective on the current tax reform debate on consumption versus income tax system.

The answer to the above questions centres around the tax revenue evaluation. I depart from the traditional approach in evaluating risky tax revenue using the ad hoc discount rate methodology and propose a new methodology. I assume, just as the previous chapter, that the government acts as if it sells the proceeds of the risky consumption tax revenue as a promissory note and I let households evaluate this stream of revenue. The state uses the market value of the tax revenue to provide a public good that enters additively the investor's utility function. The investors in turn evaluate the uncertain tax revenue using the traditional capital asset pricing model from the theory of finance.

This chapter argues that, at least for small investors, the private market is unlikely to provide the socially optimal degree of risk sharing. There is strong evidence that investors do not hold diversified portfolios (e.g. Friend and Blume (1975), Gordon (1985)). Thus the government, acting as a financial intermediary, can increase welfare through taxation policy.

This chapter concludes that one ought to implement a consumption tax via the "modified wage tax" or the cash flow approach. The argument is different from previous work in this area. A pre-payment wage tax is not equivalent to a consumption tax because (a) the market value or certainty equivalent of the tax revenue obtained from the excess return of the risky asset is non-zero if people hold less than fully diversified portfolios and/or (b) households do not evaluate the two period random tax revenue stream they pay to the state but instead they evaluate and ultimately bear the average random tax revenue across all generations, alive or yet to be born. As with Gordon and Varian (1988) we assume that the future generations share with the current households the "market" or "social" risks via tax policy (i.e. an uncertain government budget constraint). This way, the current generation absorbs a small fraction of the riskiness of their own tax revenue. The other fraction is assimilated by all future unborn generations. This causes the certainty equivalent of the risky consumption tax revenue to exceed the tax revenue generated from a wage tax set at a rate of $t_c(1+t_c)^{-1}$, and as a result reduces the social discount rate. As we allow more and more unborn generations to share in risk, the social discount rate converges to the risk free rate. Again the argument relies on the idea that what is "systematic" risk at a certain point in time will become "idiosyncratic" risk when it is shared with all current and future unborn generations.

Section 7.2. examines the budget constraints under the alternative methods of implementing a consumption tax. Certain equivalency results are presented. Section 7.3 presents the ad hoc discount rate methodology which is used to evaluate risky tax revenue. Section 7.4 presents our alternative methodology of computing the certainty equivalent tax revenue. Section 7.5 uses a particular asset pricing model to evaluate the risky tax revenue. Section 7.6 re-evaluates the risky tax revenue using the average tax revenue across generations and concludes that the value of the tax revenue under a modified wage tax exceeds that of a pre-payment wage tax. Finally, in section 7.7 I arrive at some conclusions.

7.2 Risky Tax Revenue

This section considers the evaluation of risky tax revenue. The model is the same two period life cycle model, with a choice between a safe and a risky asset. No discussion on the preference structure of a household is needed. The period by period budget constraint of the young household in the absence of taxation is given by:

$$C_{1t} = Y_{1t} - (a_{1t} + m_{1t})$$

$$C_{2t} = a_{1t}(1+x_{t+1}) + m_{1t}(1+r)$$

The intertemporal budget constraint of the investor in the absence of taxation is given by:[1]

$$C_{1t} + \frac{C_{2t}}{(1+r)} = \left(Y_t + \frac{a_{1t} z_{t+1}}{(1+r)} \right)$$

where z_{t+1} is the excess return on the risky asset over the riskless, $z_{t+1}=x_{t+1}-r$. The present discounted value of the consumption stream is equal to the endowment income

[1] An alternative way of writing the budget constraint is as follows:

$$C_{2t} = (1 + r + \beta_{1t} z_{t+1}) (Y_{1t} - C_{1t})$$

where $\beta_{1t} = a_{1t}/S_{1t}$ is a measure of proportional risk taking. This simple re-formulation of the budget constraint states future consumption is equal to the proceeds from the investment in a composite portfolio that has a mean value equal to the weighted average return. Households choose current consumption and the proportion invested in the risky asset. As in Merton (1969) one can visualize the original problem as one being reduced to a Ramsey-Phelps problem of optimal consumption under an uncertain yield for a composite asset. In other words, the problem can be seen as having an uncertain price for future consumption.

and the present value of the excess return on investment.

A proportional cash flow consumption tax, with loss offset provision, affects the intertemporal budget constraint of the young investor as follows:

$$C_{1t} + \frac{C_{2t}}{(1+r)} = \frac{1}{(1+t_c)} \left(Y_{1t} + \frac{a_{1t} z_{t+1}}{(1+r)} \right)$$

A cash flow consumption tax falls both on period the current certain consumption C_{1t} and the second period uncertain consumption C_{2t} of the generation t investor. Under the wage tax the intertemporal budget constraint is:

$$C_t + \frac{C_{2t}}{(1+r)} = (1 - t_w)\, Y_t + \frac{a_t z_{t+1}}{(1+r)}$$

The wage tax falls on the endowment (i.e. wage income).

From first observation of the budget constraints, it is obvious that the equivalence between the two taxes is obtained if the pre-payment approach includes in the tax base the extraordinary capital gains and one obtains the "modified wage tax" (MWT) or "wealth tax".[2] Under a modified wealth tax the budget constraint is:

[2] This is equivalent in terms of ex post budget constraints. This does not imply full equivalence since the value of the tax revenue is not yet known under the alternative tax regimes.

$$C_{1t} + \frac{C_{2t}}{(1+r)} = (1 - t_d) [Y_{1t} + \frac{a_{1t} z_{t+1}}{(1+r)}]$$

This is the pre-payment method put forward by Ahsan (1989,1990).[3] The modified wage tax falls on the endowment Y_{1t} and the capital gains term. Under the MWT method, the government can collect a sizeable amount of tax revenue vis-avis the pre-payment wage tax set at a rate of $t_w = t_d$.

Alternatively, equivalence between the pre-payment wage tax and the CFT can also be obtained if one excludes capital gains from the cash flow or modified wage tax approach. If we exclude the extraordinary capital gains from the cash flow consumption tax (Bulow and Summer (1984), Hamilton (1987), Zodrow (1994)), the equivalence of a cash flow consumption or MWT and endowment tax is re-established. And therefore the results under certainty carry over to the world of uncertainty. The evaluation of the cash flow consumption tax revenue will equal the certain wage tax revenue for all $t_w = t_c/(1+t_c)$. But why exclude capital gains?

According to Summers and Bulow, the risky tax revenue has a zero market value. They argue that the certainty equivalent or market value of this risky tax revenue is negligible. This is in contrast to Gordon (1985) who argues that even though all risks remain with

[3] This method has been rejected by Zodrow (1994) mainly because of the administrative problems it can cause. Although one has to consider the administration costs of implementing a tax system, the other major concerns in public finance are the efficiency and equity issues of a tax system.

the private sector the government is able to collect a sizeable amount of tax revenue.[4] Summers and Bulow (1984) state about the sizeable tax revenue of the state:

"However, though Gordon (1985) implies otherwise, there is no free lunch here. Because the government is taking a claim of zero market value, it can only finance programs that have a certainty equivalent of zero. If the government wishes to spend the expected tax revenue of this tax, it must also impose another tax, with zero expected tax revenue but a high market value (i.e. a countercyclical tax). The imposition of this second tax would be burdensome on the corporate sector."

However, in this chapter we show that dismissing this revenue implies that the household holds the "market portfolio" or a portfolio that is perfectly correlated with the "market portfolio". Given the strong evidence to the contrary a wage tax is not equivalent to a consumption tax under uncertainty. Inclusion of the extraordinary capital gains in the wage tax base obtains the equivalence. This paper argues that tax base policies cannot be designed on the basis of perfect capital markets.

The state, under the modified wage tax receives tax revenue from certain wage income Y_{1t} and uncertain second period capital gains $a_{1t}z_{t+1}$. Under a cash flow consumption tax the state receives tax revenue from current consumption and uncertain future consumption. The second period cash flow consumption tax revenue or MWT will be uncertain whereas the wage tax falls on endowment income and is certain. Under a wage tax the present value of the tax revenue is:

[4] This paper will show that the market value of this tax revenue will be positive if all generations share in the current market risks.

$$T_{wt} = t_w \, Y_{1t}$$

This tax revenue stream is certain.[5] The tax revenue the state receives from the young t generation under the cash flow consumption tax is:

$$\text{1st Period.} \quad t_c \, C_{1t}$$

$$\text{2nd Period:} \quad t_c \, C_{2t}$$

Similarly under a modified wage tax the young household pays to the state the following tax revenue:

$$\text{1st Period:} \quad t_d \, Y_{1t}$$

$$\text{2nd Period:} \quad t_d \, a_{1t} \, z_{t+1}$$

There are two broad ways in which one may account for the riskiness of the tax revenue. First we can use a discount rate that is higher than the risk free rate. Alternatively, we can use the certainty equivalent of the tax revenue. Both of these methods should ideally be equivalent.

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[5] Even if this was uncertain the results in terms of revenue evaluation would not change under the two tax systems. Both tax systems pre-payment and cash flow offer insurance. Furthermore, endowment uncertainty is eliminated through aggregation if this risk is assumed to be of the unsystematic type (See Eaton and Rosen (1980a,b)).

7.3. The Discount Rate Approach: ρ

7.3.1. Evaluation of the Cash Flow Consumption Tax

The discounting method has been used by Hamilton (1987) and recently by Zodrow (1994). They both use the weighted average as the appropriate discount rate. They choose the discount rate on an ad hoc basis.[6] Letting the public discount rate be any arbitrary ρ and evaluating the risky tax revenue from the cash flow consumption tax we have:

$$E(T_{ct}) = t_c \left[C_{1t} + \frac{E(C_{2t})}{(1 + \rho)} \right]$$

or alternatively, after substituting the intertemporal budget constraint of the young household we get:

$$E(T_{ct}) = \frac{t_c}{(1+t_c)} Y_{1t} \left[1 - \frac{S_{1t}(t_c)}{Y_{1t}} \left(1 - \frac{1+r+\beta_{1t}(t_c)E(z)}{(1 + \rho)} \right) \right]$$

where $S_{1t}(t_c)$, $\beta_{1t}(t_c)$ are respectively the savings and proportional risk taking. Both

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[6] The only discussion on the proper discount rate for the evaluation of risky tax revenue comes from Hamilton (1987) who refers to the work from the literature of risky public investments (See Arrow and Lind (1970), Hirshleifer(1965), Mayshar (1977) and Sandmo (1972)). Arrow and Lind (1970) argued that the appropriate social discount rate for risky public investments is the risk free rate. They rely on the state taking projects with uncorrelated returns, and where each project is small in size for the generation of current taxpayers. Mayshar (1977), Hirshleifer (1965) and Sandmo (1972) argued for a higher discount rate; government should use discount rates similar to those used in private investment.

depend on the cash flow consumption tax rate.[7] Under an equal yield wage tax we have:

$$t_w^* = \frac{t_c}{(1+t_c)} [1 - \frac{S_{1t}(t_c)}{Y_{1t}} (1 - \frac{1+r+\beta_{1t}(t_c)E(z)}{(1 + \rho)})]$$

It is evident that the valuation of the risky tax revenue depends on the value of the public discount rate. It is clear, that setting $t_w = t_c(1+t_c)^{-1}$ will not in general equalize revenue. In particular, if the public discount rate is lower than the weighted average rate, $r + \beta_t(t_c)E(z)$, then the cash flow consumption tax rate raises more tax revenue than the wage tax in expected present value terms if the rates are set such that $t_w = t_c(1+t_c)^{-1}$. Thus a higher wage tax is needed in order to obtain the same amount of revenue as the cash flow consumption tax.[8] On the other hand, if the weighted average discount rate is used as a public discount rate then both methods yield identical tax revenue in the expected present value sense.[9] In contrast, if the public discount rate is equal to the expected rate of return on the risky asset, E(x), then setting tax rates in the conventional way results

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[7] If the utility function is of the iso-elastic form then Ahsan (1989) has shown that savings and proportional risk taking remain unaffected by a cash flow consumption tax.

[8] Zodrow (1994) argued that even if the government uses the safe rate of return to discount the risky tax revenue, Ahsan's "modified wage tax" is not the only method to design a wage tax system that is equivalent in present value terms to the cash flow tax. Zodrow insists that the pre-payment approach could still be used by increasing the wage tax rate until the revenue requirement is met. Zodrow argues incorrectly. Later in the paper we show that if the social discount rate is lower than the weighted average rate then individuals will prefer the modified wage tax approach and not the higher equivalent wage tax approach.

[9] This also implies that the state has the same opportunity cost as the private sector. Zodrow (1994) argues that the state can obtain tax revenue under the wage tax and invest them to yield a return equal to the average return in the economy. This view is not a plausible one. The role of the state is not to invest in an equity market. If at all, the role of the state is to arrange its policies so as to allow the efficient operation of the market.

in the wage tax generating more revenue in expected present value terms.[10] These observations provide us with the following result;

Proposition 7.1: If the social discount rate is equal to the weighted average rate, then a cash flow consumption tax is equivalent to a wage tax. If social discount rate is lower, then a larger amount of expected present discounted tax revenue can be raised with a cash flow consumption tax when compared with a wage tax at the rate of $t_w = t_c(1+t_c)^{-1}$.

7.3.2. Evaluation of the Modified Wage Tax

We have established that a wage tax is equivalent to a cash flow consumption tax if the weighted average tax discount rate is used. But can one obtain an equivalence between a wage tax and a "modified wage tax"? Letting the social discount rate, to evaluate the tax revenue from a "modified wage tax" be an arbitrary parameter k we have:

$$E(T_{dt}) = t_d \, [\; Y_{1t} + \frac{a_{1t}(t_d) \; E(z)}{(1 + k)} \;]$$

Implementing the consumption tax via the modified wage tax method generates more tax revenue in present expected value tax revenue than the endowment tax set at a rate of $t_w = t_d$ for any finite value of the social discount rate k.

7.3.3. Modified Wage Tax versus Cash flow Consumption Tax

[10] We shall show shortly that this is not possible with a cash flow consumption tax.

The previous result implies that the equivalence between the cash flow consumption tax and the modified wage tax may also become questionable. The cash flow consumption tax might be non-equivalent to a modified wage tax.[11] The only social discount rate that is capable of generating equivalence of the modified wage tax with the cash flow consumption tax is the risk free rate. This can be seen from the following condition. The present value cash flow consumption tax revenue is given by:

$$T_{ct} = t_c \left[C_{1t}(t_c) + \frac{C_{2t}(t_c)}{(1+r)} \right]$$

Substituting the intertemporal budget constraint in the above constraint yields:

$$T_{ct} = \frac{t_c}{(1+t_c)} \left[Y_{1t} + \frac{a_{1t}(t_c)\ z_{t+1}}{(1+r)} \right]$$

Setting the modified wage tax equal to $t_c(1+t_c)^{-1}$ does not establish equivalence since the behaviour of the household in terms of risky asset choice is not known a-priori. Under the modified wage tax the present value tax revenue is given by:

$$T_{dt} = t_d \left[Y_{1t} + \frac{a_{1t}(t_d)\ z_{t+1}}{(1\ +\ r)} \right]$$

If the amount invested in the risky asset is the same under a cash flow consumption tax as under the modified wage tax then the equivalence holds.[12]

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[11] This was originally discussed, in a certainty environment, in the Atkinson and Stiglitz (1980) textbook.

[12] We shall show that under CRRA the equivalence does in fact hold. The amount invested in the risky asset is unchanged from the pre-tax position, for both tax policies.

What determines which public discount rate to use? Why is it that the weighted average discount rate is the cut off point towards the revenue evaluation if the cash flow approach is used?[13] Why not use the expected return of the risky asset? Why not the risk free rate? In the next section, we follow a different methodology. The method followed avoids the pitfalls of choosing the discount rate in an ad hoc manner.

[13] Hamilton (1987) uses the weighted average return to conduct a differential incidence analysis.

7.4. The Certainty Equivalent Approach

In order to provide an answer to these policy questions we follow an alternative but equivalent approach. We use the certainty equivalent of tax revenue. Under suitable conditions this alternative method should be equivalent to the discounting method. This method can provide us with additional insights than the ad hoc discounting rules prevalent in the literature. In what follows the discount rate used to evaluate the tax revenue becomes the risk free rate since the certainty equivalent takes into account the riskiness of the tax revenue. Certainty equivalent of the tax revenue is the certain amount of tax revenue such that the public would be indifferent between paying this amount and allowing some uncertainty in the tax revenue. By risk aversion, the amount the public will pay in order not to transfer the risk onto the budget constraint is a positive risk premium. The certainty equivalent value of the tax revenue is then the expected value of the tax revenue less the risk premium.

How does one compute the certainty equivalent of a stream of uncertain tax revenue? The answer is given in the next section.

7.5. The Capital Asset Pricing Model

7.5.1. Evaluation of the Modified Wage Tax Revenue

In order to compute the certainty equivalent of a stream of revenue I use the traditional "capital asset pricing model" (CAPM) of Lintner (1965), Mossin (1966) and Sharpe (1964).[14] In order to use the CAPM to price the "risky" tax revenue we assume that the government behaves as if it sells the proceeds of the risky modified wage tax revenue as a "risky' promissory note that has a mean value of $E(T_{dt})$ and variance σ_{Tdt}^2. What price are the young investors willing to pay for this risky bond?[15] Investors that hold diversified portfolios do not price risky assets based on the variability of the risky asset but what matters to these investors is the covariance of the risky asset with the market portfolio. Using the capital asset pricing model we can solve for the certainty equivalent of the risky tax revenue. This will reflect the price of the "risky" bond. Using the CAPM the price of the bond will equal the expected return less the risk premium. The risk premium is a function of the covariance of the uncertain tax revenue with the market return times the price of risk. The following expression illustrates:

[14] The CAPM is derived from a mean-variance utility framework or by assuming a normal distribution for the returns of the risky asset, or even from the consumption based CAPM if the marginal utility of future consumption is perfectly correlated with the return of the market portfolio. Future research should use the consumption based CAPM or the arbitrage pricing model which are examples of more general asset pricing models.

[15] If the households view these bonds as a perfect substitute for the risky asset, then the bonds will command the same price as the risky asset.

$$CE(T_{dt}) = R_{dt} = E(T_{dt}) - \lambda \, cov(T_{dt}, r_{mt+1})$$

$$\lambda = \frac{E(r_m) - r}{\sigma_m^2}$$

where r_{mt+1} is the market return in the second period, $E(r_m)$ is the expected return from the market portfolio, σ_m^2 is the variance of the market portfolio and λ is the market price of risk or the expected excess return of the market portfolio per unit of risk, and T_{dt} is given by:[16]

$$T_{dt} = t_d \left[Y_{1t} + \frac{a_{1t}(t_d)\, z_{t+1}}{(1+r)} \right]$$

Substituting this modified wage tax revenue T_{dt} that the young generation household pays to the state into the covariance term and expressing it in terms of the correlation coefficient yields:

$$CE(T_{dt}) = E(T_{dt}) - t_d \frac{a_{1t}(t_d)E(z)}{(1+r)} \frac{\frac{(E(r_m)-r)}{\sigma_m}}{\frac{E(z)}{\sigma_x}} r_{xm}$$

where r_{xm} denotes the correlation coefficient between the return of the risky asset the investor holds and the return on the market portfolio.

Furthermore, in equilibrium the following condition holds:

[16] T_{dt} is the present value of the tax revenue the young generation pays to the state. The discount rate we use is the risk free rate. This is the proper discount rate to use when we follow the certainty equivalent approach.

$$\frac{E(r_m - r)}{\sigma_m} = \frac{E(x - r)}{\sigma_x}$$

This condition is derived from the capital market equilibrium condition. It states that in equilibrium the expected excess return per unit of risk of any security or portfolio is equal to the slope of the capital market line (i.e. the expected excess return per unit of risk of the market). If a portfolio has an expected excess return per unit of risk greater than the market portfolio per unit of risk and investors have homogeneous expectations the security will be in excess demand. This will cause the price of the security or portfolio to increase and the expected excess return to fall until the equality is re-established and vice versa. The CAPM model is assumed to derive an equilibrium price for all securities under consideration. The value of the promissory note becomes:

$$CE(T_{dt}) = E(\ T_{dt}\ ) - t_d\ \frac{a_{1t}(t_d)E(z)}{(1+r)}\ r_{xm}$$

In the case where the household invests in the market portfolio or alternatively in a "mutual fund" that is perfectly correlated with the market portfolio then

$$r_{mx} = 1$$

The certainty equivalent of the tax revenue from a modified wage tax then becomes equal to the expected value of the tax revenue less the risk premium, where the latter is exactly equal to the expected tax revenue from the extraordinary gain component.[17] Substituting the expected tax revenue into the certainty equivalent tax revenue yields:

$$R_{dt} = CE(T_{dt}) = t_d \, Y_{1t}$$

This is the only tax revenue that can be used to finance public projects which enters into the utility function in an additive way. Because the government is taking a claim of zero market value from the risk premium, it can correspondingly only finance programs that have a certainty equivalent equal to the pre-payment wage tax. The price of the "risky" note is equal to the consumption tax revenue the government would raise under certainty. This certainty equivalent tax revenue of a modified wage tax revenue is independent of the parameters of the model, such as the risk aversion of individuals.

Proposition 7.2: If capital markets are perfect, and the individual holds a perfectly diversified portfolio then a wealth tax or modified wage tax becomes equivalent to a pre-payment wage tax as in the certainty literature.

The evidence provided by Friend and Blume (1975), Mayshar (1979), and more recently by Gordon (1985) indicates that most households or investors hold less than fully diversified portfolios.

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[17] By implication, we immediately observe that as long as the correlation coefficient is less than unity then there are gains from a consumption tax relative to the wage tax. Inclusion of capital gains into the tax base of a wage tax is welfare improving at least for the small investor who does not diversify fully.

It is easy to see that if the household does not hold a perfectly diversified portfolios then the correlation coefficient between the market return and the portfolio return will be less than one.

$$r_{mx} < 1$$

If the household does not hold a perfectly diversified portfolio then the certainty equivalent of the risky tax revenue will be greater than the pre-payment wage tax that is set such that $t_w = t_d$

The certainty equivalent tax revenue in the case of a less than fully diversified portfolio holder is:

$$CE(T_{dt}) > t_w \, Y_{1t}$$

The certainty equivalent of the tax revenue, which is equal to the expected value of the tax revenue less the risk premium, is less than the expected tax revenue income from the extraordinary gain component. The value of the "risky" tax revenue exceeds the tax revenue generated by a wage tax set at the same rate as under certainty. [18]

Concluding this section we can state that individuals that hold undiversified portfolios

[18] All these results hold true with a cash flow consumption tax. Appendix 2 shows the results.

would be willing to pay a higher price for the "risky" tax revenue. However, for the investor that holds the market portfolio the "risky" tax revenue is perceived as a risky bond that is a perfect substitute with the risky asset. Hence the price of the "risky" note will be lower for an investor that holds the market portfolio. In addition, for a perfectly diversified investor, the implementation of a consumption tax as a modified wage tax or an endowment wage tax is a matter of irrelevance since his welfare is unaffected by the method of imposition of the tax.[19] However, for the individual that does not hold a completely diversified portfolio these "risky" taxes may serve to reduce the diversifiable portfolio risk that he is exposed to. Hence, such an investor is willing to pay a higher price for the "risky" bonds. If an economic system, contains both types of investors, then the undiversified investors are strictly better off with a modified wage tax[20] whereas one who holds the market portfolio is just as well off with a modified wage tax version or an endowment tax. A pareto optimal improvement in resources can be achieved via a modified wage tax .

[19] We show this result later in the paper.

[20] Preference of a particular tax method is shown later in the paper under a differential incidence analysis.

7.6. Intergenerational Risk Sharing

This section puts forward an argument for a modified wage tax that is based on intergenerational risk sharing. The intergenerational risk sharing argument is based on the seminal work of Gordon and Varian (1988). Gordon and Varian (1988) state:

> The objective of this paper is to explore the characteristics of an optimal government risk-sharing scheme, assuming that the government has the power to recommit future generations. Since there is an arbitrary large number of future generations to share in any particular lottery (e.g. today's recession), one might expect, by analogy with the diversification theorems in finance, that on efficiency grounds each generation ought to bear an arbitrary small share in the outcome of any lottery. (p. 186)

This argument is independent of the young investor's portfolio holdings. The investor can hold the market portfolio or any other portfolio of assets. We point out that even if the household held the market portfolio, and is exposed to "market" or "social" risks, a tax system that falls on uncertain returns reduces the current "market" risks and provides some insurance. The government by taxing the risky capital income absorbs a certain fraction of the expected return and the risk, while the investor receives a lower return it also bears a lower risk (i.e, similar to the Domar-Musgrave (1944) phenomenon). The risk is transferred to the budget constraint. We continue to assume that the risk remains in the system and that evaluation of the tax revenue is needed. However, with intergenerational risk sharing, the risk of current tax revenue fluctuations can be pooled across all generations.[21] We argue that what is considered as "social" or "market" tax

[21] The risk can be of the human capital type or the financial type.

revenue risk at any one date is idiosyncratic risk when pooled with the independent lotteries of many later generations.[22]

The state can reallocate wealth across generations cheaper through its debt management policy. Accordingly, when there is an unlucky outcome (e.g., a recession), this will generate a fall in the tax revenue and a fall in the financial wealth of a household.[23] The state by absorbing some of the "market" risk can run a deficit and create government debt.[24] The government has the ability to issue bonds at the risk free rate when the economy is hit by a recession. This debt is passed on to all future generations even though it was created by the current unlucky generation. Each future generation bears a small share in the risks of today's recession. The evidence indicate that governments do handle stochastic revenue from risky outcomes by allowing some uncertainty in the tax revenue. This view is supported by an earlier paper of Gordon (1985) who states that "This argument provides a rationale for high corporate tax rates, perhaps generous investment incentives, and a variable government deficit. It is intriguing that government policy has in fact evolved in this direction". On the other hand, the wage tax (pre-payment version) has no risk sharing elements since it falls on the endowment of a

[22] It would be of interest to compare the work of Gordon and Varian (1988) with the much earlier contribution of Arrow and Lind (1970).

[23] Comparing this with a wage tax, the latter does not cause tax revenue fluctuations but it reduces the financial wealth of a household by more than the consumption tax. The cash flow consumption tax or "modified wage tax" has a loss offset provision.

[24] When there is a lucky outcome (i.e., an economic boom), the state will generate more tax revenue, which can be used to retire debt.

household. Thus if a "recession" occurs then not only the "unlucky investors will have prepaid a tax on expected returns and will then obtain no deduction for the losses they incur, and if the "favourable" outcome occurs then "lucky" investors might become very rich and owe no additional tax liability on future consumption of their wealth" (See Blueprint for Basic Tax Reform pp. 128-9), thus creating "horizontal inequities". Large fluctuations in the wealth of the current and future generations would be observed. The pre-payment wage tax does not provide financial wealth insurance and does not pass the "market" or "social" risk onto the future generations since under this tax there is no fluctuation in the government budget from tax revenues.[25]

If we assume that all generations (n in number) share in the current "market" or "social" risk via a stochastic tax policy then the relevant tax revenue valuation is not what the investor pays to the state but the weighted average modified wage tax revenue across all generations, alive or yet to be born. In order to evaluate this stream of stochastic tax revenue we assume that the government sells a security that pays a weighted average over all generations stochastic modified wage tax revenue. The stochastic modified wage tax revenue of interest to the current period generational investor is given by:

[25] This assumes that risk is only of the financial type and not human capital risk. However, as stated previously both tax systems offer insurance when risk is of the human capital type. Only one tax system, the cash flow consumption tax, provides an added financial wealth insurance via the tax system which the pre-payment wage tax system does not.

$$A_{dt} = \sum_{i=0}^{n-1} \gamma_i T_{d(t-i)}$$

where γ_i is the weight of each lottery that is allocated to the generation born i periods later and the sum of the weights of each generations add to unity:

$$\sum_{i=0}^{n-1} \gamma_i = 1$$

For example if $\gamma_i = 1/2$, and there are two generations sharing tax revenue risk, then the stochastic tax revenue under consideration by the young steady state generation in period t is given by:

$$A_{dt} = \frac{T_{dt} + T_{dt-1}}{2}$$

where T_{dt-1} is the modified wage tax revenue of the currently alive old generation. The time period t-1 refers to the previous generation. The old generation alive in the current period with the t young generation. The government upon observing the stochastic outcome T_{dt-1} transfers $T_{dt-1}/2$ from the old generation in the current period to the t young generation. In the next period, the state transfers $T_{dt}/2$ tax revenue to the young t+1 generation from the current old t generation. The old generation in period 2 is the young t generation. In period 2 a t+1 generation is born. This way the young generation at time t shares in the past lottery and keeps some share of its own lottery. The expected value of the tax revenue is given:

$$E(A_t) = E\left[\sum_{i=0}^{n-1} \gamma_i T_{dt-i}\right] = E(T_{dt})$$

The variance of the tax revenue is given by:

$$var\ (A_t) = var\left(\sum_{i=0}^{n-1} \gamma_i T_{dt-i}\right) = var(T_{dt})\sum_{i=0}^{n-1} \gamma_i^2$$

Notice that the covariance term drops due to the assumption that the economic shocks are identically and independently distributed.

What price are young investors willing to pay for this security? Since each weight is less than one the variance of the tax revenue will fall with intergenerational risk sharing. However, as stated previously, in the theory of finance the variance of the security is not important when an evaluation is done. What is important is the covariance of the security with the market portfolio. What price are young investors willing to pay for the security who's risks are shared among all generations? Again using the CAPM to evaluate this new stream of revenue, the certainty equivalent of this tax revenue stream becomes:[26] Expanding the terms and using the properties of covariance we get:

[26] Future research should concentrate on the endogenous determination of the value of γ. This can be accomplished with some type of social welfare function or even via political mechanisms.

$$CE(T_{dt}) = E\left[\sum_{i=0}^{n-1} \gamma_i T_{dt-i}\right] - \lambda \, cov\left[\sum_{i=0}^{n-1} \gamma_i T_{dt-i} , r_{mt+1}\right]$$

$$CE(T_{dt}) = E(T_{dt}) - \lambda \, cov\left[\gamma_0 T_{dt} + \gamma_1 T_{dt-1} + \gamma_2 T_{dt-2} + \ldots , r_{mt+1}\right]$$

Since the market or social risks across generations is idiosyncratic the covariance of T_{dt} is the only relevant term in the evaluation of the risky tax revenue. Hence:

$$cov\left[\gamma_i T_{dt-i} , r_{mt+1}\right] = 0 \qquad \forall \quad i \neq 0$$

Substituting the above assumption into the certainty equivalent tax revenue we get:

$$CE(T_{dt}) = E(T_{dt}) - \lambda \, cov(\gamma_0 \, T_{dt} , r_{mt+1})$$

In this framework, individuals would increase the price of "risky" tax revenue stream. The evaluation of this "risky" tax revenue assuming that the household holds the market portfolio is:

$$CE(T_{mt}) = t_m \left[Y_{1t} + (1 - \gamma_0) \frac{a_{1t}(t_m)\, E(z)}{(1+r)} \right]$$

As long as there is intergenerational risk sharing, (i.e. $\gamma_0 < 1$) then the price of the bond or the certainty equivalent of the tax revenue will increase[27] relative to the price of the risky anticipation note without intergenerational risk sharing.[28] The individual will be willing to pay a higher price for some tax revenue uncertainty. The public discount rate for the evaluation of risky tax revenue is given by:

$$k = \frac{(1+r)}{(1-\gamma_0)} - 1$$

This is obtained by setting the present value of the risky tax revenue using the certainty equivalent method equal to the present value of the tax revenue using the discounting method and solving for the required rate of return. Notice that as γ_0 approaches zero the discount rate approaches the risk free rate. On the other hand as the weight approaches unity the social discount rate approaches infinity.

Proposition 7.5: Even if the household holds the market portfolio, intergenerational risk sharing will lead to a positive market evaluation of the risky tax revenue.

Proposition 7.6: The public discount rate becomes finite and monotonically decreases as we increase the number of intergenerational risk sharers.

These propositions are in accordance with an earlier article of Gordon (1985) stating that:

[27] The "risky" note will not be viewed as a perfect substitute of the risky asset with intergenerational risk sharing.

[28] The evaluation of the risky tax revenue assuming no intergenerational risk sharing.

"For this to be worthwhile (setting a high corporate tax rate, so that a large part of the risk goes to the government), however, the government must face lower costs than the private sector in reallocating risk. One situation where the government should find it cheaper is in the intergenerational reallocation of risk. In principle efficiency would require that even unborn generations share in the risk on existing capital. Yet these individuals do not trade currently in equity for the obvious reason that they are not yet alive. Also since they are not yet alive, there is no alternative way to set up a mutually beneficial contract."

If the number of unborn generations sharing in the current "social" or "market" risks is large then the social discount rate is equal to the risk free rate. Efficiency then requires that each generation ought to bear an arbitrary small share in the risks of today's recession. A similar theorem exists in the literature of finance, namely that of diversification.

Proposition 7.7: The social discount rate is equal to the risk free rate when all future unborn generations bear a small share of the "market" risks of today's lottery.[29]

The certainty equivalent value of the modified wage tax revenue is:

$$CE(T_{dt}) = R_{dt} = t_d \left[Y_{1t} + \frac{a_{1t}(t_d)E(z)}{(1+r)} \right]$$

The young generation now views the government as if it sells the proceeds of the modified wage tax revenue which is in the form of a weighted average across generations. This stream of tax revenue is also risky. The risk of the tax revenue is

[29] Intergenerational risk sharing would not hold in a model where the household has an infinite horizon as in Hamilton (1987). In fact, Hamilton's work (1987) is an extreme version of a household with altruism towards future generations.

however viewed as idiosyncratic when tax revenue uncertainty is shared with all generations, and hence we can use the risk free rate to discount the expected value of the tax revenue.

7.7. Concluding Remarks

This chapter has examined the evaluation of the cash flow consumption tax revenue of a two period life cycle investor under uncertainty and compared it with a pre-payment wage tax. Implementation of a consumption tax through a pre-payment method has occupied a central position in the agenda of tax analysts. I show that the utilization of a weighted average rate to discount risky cash flow consumption tax revenue implies that the household holds a perfectly diversified portfolio. I also show that an investor that holds less than perfectly diversified portfolio will value the tax revenue from a cash flow consumption tax higher than the endowment tax. I further show that if the cash flow consumption tax revenue is viewed as a security whose risks are shared equally with all generations the cash flow consumption tax or modified wage tax is the preferred method of taxation. With intergenerational risk sharing through tax policy, the social discount rate reduces to the risk free rate. This happens because all future generations are pre-committed to participating in the current "market" or "social" risks via tax revenue uncertainty. With intergenerational risk sharing a cash flow consumption tax or a "modified wage tax" becomes a front runner in the choice of tax base.[30]

In the next chapter we conduct a differential incidence analysis to examine the welfare gains of a cash flow consumption tax under the assumption that the social discount rate is equal to the risk free rate.[31] This will determine the extent of the welfare gains that can arise from a switch to a cash flow consumption tax from a wage tax. Is this intergenerational risk sharing argument worthwhile? If the gains are negligible then the insurance elements of a cash flow consumption tax will not outweigh the added administrative complexity of the tax system.

[30] Appendix VI evaluates the risky cash flow consumption tax revenue and I arrive at similar conclusions.

[31] In the appendix we show that the cash flow consumption tax is also of the idiosyncratic type if all generations share in the current market risks via tax revenue uncertainty.

Chapter 8

A Differential Incidence Analysis:

Cash Flow Consumption or Pre-Payment Wage Tax?

8.1. Introduction

Fundamental proposals for switching the tax base from income to consumption are being advocated by many economists and policy makers in recent years. The efficiency issue of switching the tax base has been examined mainly through the use and development of simulation models. Most simulation studies have concentrated in the income versus consumption tax choice under certainty. The results of most studies indicate large welfare gains from switching to tax base from income taxation to wage or consumption tax base.[1]

Turning to an uncertain environment, very few analytical and simulation results have been conducted to ascertain and evaluate the welfare implications of a consumption tax with the exception of Ahsan (1989) and Hamilton (1987). The most difficult aspect of the problem under uncertainty is the treatment of the risky tax revenue. Ahsan (1989)

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[1] For a literature review see the article entitled "On the Specification of Simulation Models for Evaluating Income and Consumption Taxes" by C. Ballard (1990) in Rose M.

utilizes a two period life cycle model of portfolio choice and Hamilton (1987) employs the continuous infinite horizon portfolio-savings model developed by Samuelson (1968) and Merton (1969). They conduct a differential incidence study to determine the welfare implications of a tax base switch. They find that the welfare gains of a consumption tax are of a small magnitude when compared to the literature under certainty. Ahsan uses a risk neutral approach to the risky tax evaluation and examines consumption versus income taxation, Hamilton (1987) examines a wage tax vis-a-vis a capital income tax.[2] In order to show the welfare gains of a consumption tax he assumes that the market value of the risky tax revenue from the capital income tax revenue is zero.[3] The assumption of Hamilton reduces the uncertainty problem to yield identical welfare results as the certainty tax results.[4]

This chapter presents a differential incidence analysis of a cash flow consumption tax versus a wage tax. Numerical calculations are presented using the non-expected utility hypothesis. We present results for two cases a) the household acts in a risk neutral manner due to intergenerational risk sharing and b) the household is concerned with the uncertainty and takes explicit account of the risky tax revenue by using the optimal certainty equivalent future consumption in the evaluation of risky tax revenue. The certainty equivalent future consumption is endogenously determined from the preference

[2] In his model a pre-payment wage tax is equivalent to a cash flow consumption tax due to the usage of an ad hoc discount rate.

[3] This argument was also presented earlier by Summers and Bulow (1984).

[4] Implicit in the work of Hamilton is the assumption of perfect capital markets.

structure of the household and used for tax revenue evaluation. We show that the welfare gains from a cash flow consumption tax are large even if we positively discount the uncertain tax revenue.

Section 8.2 discusses the preferences that will be used to conduct the differential incidence study. Section 8.3 presents the closed form solutions. Section 8.4 discusses the parameters used to simulate the welfare change. Section 8.5 presents numerical results on the optimal decision variables in the no tax case. Section 8.6 examines the effect of the tax systems on the choice variables. Section 8.7 sets out to measure the welfare change. Section 8.8 presents numerical results on the welfare losses due to a move to a wage tax system. Section 8.9 uses the certainty equivalent future consumption to evaluate the risky cash flow consumption tax revenue and welfare losses are observed even if we account for the risky tax revenue. Section 8.10 concludes and suggests future research.

8.2. The Preferences

A differential incidence analysis requires us to work with explicit functional forms. All studies that conduct a differential incidence study use the class of isoelastic von Neumann-Morgenstern intertemporal expected utility function. We propose to conduct the differential incidence analysis using the recently advanced generalized isoelastic multi-period utility function which allows the separation of risk aversion and intertemporal substitution.

The household is assumed to make choices between current and certainty equivalent future consumption based on the elasticity of intertemporal substitution. The preference are described by:

$$U(C_1,CE(C_2)) = [C_1^{\theta} + \delta CE(C_2)^{\theta}]^{\frac{1}{\theta}}$$

The certainty equivalent of future consumption is given by:

$$CE(C_2) = [\ E(C_2^{\alpha})\ ]^{\frac{1}{\alpha}}$$

The first order condition with respect to current consumption:

$$C_1^{\theta-1} = \delta(1+r)CE(C_2)^{(\theta-\alpha)}E(C_2^{\alpha-1})$$

The first order condition with respect to risk taking:

$$E(C_2^{\alpha-1}z) = 0$$

8.3. Closed Form Solutions: The No Tax Case

In order to obtain closed forms solutions we assume two states of nature $x_1 > 0$ and $x_2 < 0$ with probability of p and (1-p) respectively just as in chapter 4. Solving this problem leads to an optimal consumption level, risk taking, savings and certainty equivalent which are linear in the endowment.[5] In the no-tax world the optimal level of current consumption is:[6]

$$C_1(0) = b(r,x_1,x_2,p,\delta,\theta,\alpha)\ Y$$

Where the function b, the factor of proportionality, represents average or marginal propensity to consume. Marginal propensity to consume therefore depends on the parameters of the model such as the risk aversion parameter, the intertemporal elasticity of substitution parameter, on the risk free rate and on the distribution of the risky returns. Similarly we obtain:

[5] See appendix VII for the analysis.

[6] The Notation X(0), where $X=C_1,S,\alpha,\beta$ etc, indicates that the decision rules are for the no tax case. Generally the notation is $X(t_h)$, where h identifies the tax regime (i.e. h=w,r,c, etc.)

$$S(0) = (1 - b(r,x_1,x_2,p,\theta,\alpha))\ Y$$

$$a(0) = c(r,x_1,x_2,p,\theta,\alpha)\ Y$$

$$\beta(0) = \frac{a(0)}{S(0)} = \beta(r,x_1,x_2,p,\alpha)$$

respectively, for total savings, optimal investment in the risky asset and proportional risk taking. Proportional risk taking is evidently independent of the endowment income. More interestingly, proportional risk taking is independent of the elasticity of intertemporal substitution. This result is consistent with Svensson's (1989) extension of the non-expected utility in a continuous time framework. Proportional risk taking is determined only by the degree of risk aversion and not the intertemporal elasticity of substitution. The intertemporal elasticity of substitution affects the amount invested in the risky asset (a type of savings), but not the fraction invested out of total savings. Finally, the certainty equivalent future consumption can be expressed as a linear function of current consumption expenditures:

$$CE(C_2(0)) = k(r,x_1,x_2,p,\theta,\alpha)\ C_1(0)$$

We find that certainty equivalent future consumption is a linear function of current consumption.

8.4. The Parameters of the Model

We choose the parameters based on certain observed empirical regularities. The elasticity of substitution and the relative risk aversion parameter are allowed to vary according to the following table:

$1/(1-\theta)$	.5	.33	.09	.05	.01
$\gamma = 1-\alpha$	.5	.99	1.5	2	4

In this respect, our choice of the relative risk aversion parameter is made independently of the elasticity of substitution. The relative risk aversion is assumed to vary between 0.5 to a maximum of 4. The elasticity of substitution is allowed to vary between .5 to .01 as was found by Hall (1988).

The Distribution of the asset, given the probability of the good state of 80% is shown below:

r	x_1	x_2	E(x)	E(z)	v(x)
0.50	3.7906	-0.1623	3	2.5	2.5
0.50	3.8660	-0.4641	3	2.5	3
1.25	3.7906	-0.1623	3	1.75	2.5
1.25	3.866	-0.4641	3	1.75	3

8.5. Numerical Simulation Results

We first present the results of the no-tax world under the ($r=.5, E(x)=3, V(x)=3$) distribution of asset returns. This will allow us to compare the sensitivity of our results with the expected utility hypothesis. The table below shows the marginal propensity to consume over various risk aversion and intertemporal substitution parameters.

Table 8.1: Marginal Propensity to Consume: ($C_1(0)/Y$,MPC(0))

		Intertemporal Elasticity of Substitution				
		.50	.33	.09	.05	.01
Relative Risk Aversion Parameter	0.50	.683	.732	.794	.804	.812
	0.99	.645	.688	.743	.752	.760
	1.50	.621	.656	.703	.711	.717
	2.00	.606	.636	.678	.685	.691
	4.00	.582	.610	.636	.641	.646

From the table we can observe that average propensity to consume is stable over various parameters and ranges from a low of .58 to a high of .81. Marginal propensity to consume falls with increases in relative risk aversion holding the intertemporal elasticity constant. Also, marginal propensity to consume increases with a decrease in the intertemporal rate of substitution. In contrast, the expected utility model would make the following prediction. Decreasing the intertemporal elasticity of substitution would imply that the relative risk aversion increases and marginal propensity to consume decreases.

The next table shows total investment in both riskless and risky asset.

Table 8.2: Marginal Propensity to Save in the Assets (S(0)/Y, MPS(0))

	Intertemporal Elasticity of Substitution					
		.50	.33	.09	.05	.01
Relative Risk Aversion Parameter	0.50	.318	.268	.206	.196	.188
	0.99	.353	.312	.257	.248	.240
	1.50	.379	.344	.298	.289	.283
	2.00	.394	.364	.323	.315	.309
	4.00	.419	.396	.365	.359	.354

We observe that as risk aversion increases more is invested in the combined risky and safe asset. This occurs because the increase in risk aversion makes the investor concerned about the future and this leads to a reduction in current consumption and encourages the savings motive. The safe investment increases, and this outweighs the reduction in the risky asset due to the increase in risk aversion. Also as the elasticity of intertemporal substitution decreases the average propensity to invest in the assets falls. Total investment in the assets is not independent of the parameters of risk aversion and elasticity of substitution. Next table presents numerical figures on risky investment.

Table 8.3: Marginal Investment in the Risky Asset (a(0)/Y,MPI(0))

	Intertemporal Elasticity of Substitution					
		.50	.33	.09	.05	.01
Relative Risk Aversion Parameter	0.50	.483	.410	.313	.298	.285
	0.99	.411	.363	.299	.289	.280
	1.50	.304	.277	.239	.232	.227
	2.00	.232	.215	.190	.186	.182
	4.00	.112	.106	.098	.096	.095

As expected, risky investment falls with a higher relative risk aversion parameter. What is interesting to note is that the risky investment varies inversely with the elasticity of substitution as well. The smaller the elasticity of intertemporal substitution the lower the amount invested in the risky investment. The amount invested in the risky asset depends on both the elastic· · of substitution and the risk aversion parameter. Table 4 examines the riskless asset decision.

Table 8.4: Marginal Investment in the Riskless Asset

	Intertemporal Elasticity of Substitution					
		.50	.33	.09	.05	.01
Relative Risk Aversion Parameter	0.50	-.165	-.140	-.107	-.102	-.098
	0.99	-.058	-.051	-.042	-.041	-.039
	1.50	.074	.068	.058	.057	.055
	2.00	.162	.150	.133	.130	.127
	4.00	.307	.290	.267	.263	.259

First we observe that at a relative risk aversion below unity the individual borrows, and at relative risk aversion of greater than one the individual saves.[7] As expected, investment in the riskless asset rises continuously as risk aversion increases. Furthermore, as the elasticity of substitution decreases the amount invested in the riskless asset falls. Table 5 shows the behaviour of proportional risk taking.

Table 8.5: Proportional Risk Taking

Relative Risk Aversion				
.5	.99	1.5	2.00	4.00
1.52	1.16	.804	.589	.268

Proportional risk taking is independent of the elasticity of substitution. Proportional risk taking is independent of the elasticity because both total savings and risky investment change proportionally with the elasticity of substitution such that the ratio is constant. As expected, proportional risk taking falls as relative risk aversion increases. This happens because risky investment falls and the total investment rises with increased risk aversion. Proportional risk taking plays an important role in computing the weighted average rate of return that previous researchers have used. Next in table 6 we show the weighted average discount rate that Hamilton (1987) and Zodrow (1993) have used.

[7] The CRRA and CIES assumptions guarantee no bankruptcy (See Zeldes (1989)).

Table 8.6: Weighted Average Return on Investment

Relative Risk Aversion				
.5	.99	1.5	2.00	4.00
4.302	3.410	2.509	1.973	1.169

The weighted average return on investment is independent of the elasticity of substitution also. In addition the weighted average return falls as relative risk aversion increases. This makes the gap between the risk free rate and the weighted average return smaller. In the limit as relative risk aversion increases household does not invest in equity and the weighted average return will equal to the risk free rate.[8] However, it is clear that for a plausible values of relative risk aversion, say 2, the weighted average rate will lead to discounting the tax revenue at a very high rate vis-a-vis the risk free rate. The ratio between the weighted average and the risk free rate is close to 4 times. Due to the preference structure we next calculate the certainty equivalent future consumption.

[8] In the limit, what can be said about the social discount rate? What can be said about equivalence? It is obvious that if households are very risk averse taxation has no insurance elements.

Table 8.7: Certainty Equivalent Future Consumption ($CE(C_2)/Y$)

		Intertemporal Elasticity of Substitution				
		.50	.33	.09	.05	.01
Relative Risk Aversion Parameter	0.50	1.394	1.178	.904	.861	.824
	0.99	1.128	.997	.822	.793	.768
	1.50	.969	.882	.762	.741	.724
	2.00	.886	.819	.726	.710	.696
	4.00	.767	.727	.668	.658	.650

The certainty equivalent future consumption behaves as expected. As the degree of intertemporal substitution decreases certainty equivalent future consumption also decreases. Certainty equivalent future consumption is used for substitutability with current consumption. Also certainty equivalent varies with risk aversion. As expected, an increase in risk aversion increases the risk premium, thus reducing certainty equivalent future consumption. It is seen that only for $\eta = 0$ the certainty equivalent future and current consumption appear to converge.

Concluding this section we can summarize our findings as follows. First, all variables are found to be stable over all parameter values. A decrease in the intertemporal elasticity of substitution will increase current consumption and reduce savings, risky and riskless investment, and certainty equivalent future consumption. Proportional risk taking, it is to be noted, is independent of the elasticity of intertemporal substitution. As the relative risk aversion parameter increases, current consumption falls, risky investment and proportional risk taking also fall. Finally, as expected, certainty equivalent future consumption falls with risk aversion. However, as relative risk aversion increases total savings and riskless investment increases.

8.6. The Tax System[9]

8.6.1. A Cash Flow Consumption Tax

The cash flow consumption tax affects the intertemporal budget constraint the following way:

$$C_1 + \frac{C_2}{(1+r)} = \frac{1}{(1+t_c)} (Y + \frac{az}{(1+r)})$$

The consumption tax does not create intertemporal distortions; the effect of a cash flow consumption tax is to reduce current consumption by an income effect. This occurs such that gross consumption remains unchanged. The consumption tax by affecting the return to risk taking causes a positive substitution effect of the Domar-Musgrave type and a negative income effect. The total effect on risk taking activity is zero, under the joint assumption of CRRA and CIES. The positive substitution effect cancels out with the negative income effect. Private risk taking decreases, however, leading to a divergence between social and private risk taking. Savings also remain unaltered.

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[9] The comparative static results of this section are also derived in Ahsan (1989) using the expected utility model. Since these taxes do not create any intertemporal distortions but only wealth and portfolio re-allocation effects, the non-expected utility model does not provide an answer to the question of which parameter, elasticity of substitution or risk aversion, does the effect of taxation policy operate through. Earlier we found that the taxation of capital income operates through the elasticity of substitution, and not the risk aversion parameter. The role of the risk aversion parameter was to determine the magnitude but not the direction.

8.6.2. A Modified Wage Tax[10]

A Wealth tax or "Modified Wage Tax" affects the intertemporal budget constraint the following way:

$$C_1 + \frac{C_2}{(1+r)} = (1-t)\ (Y + \frac{az}{(1+r)})$$

The wealth tax, much like the cash-flow consumption tax, does not create intertemporal distortions. Current consumption is of course reduced by an income effect. The wealth tax, as with the consumption tax, affects the return and variance of the risky asset. Risk taking does not get affected with a modified wage tax under the twin assumptions of CRRA and CIES. Private risk taking decreases with the imposition of a wealth tax. The difference between this tax and the cash flow consumption tax lies on savings. Savings

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[10] In this section MWT is not necessarily treated as equivalent to the cash flow consumption tax. As discussed earlier, equivalence depends on the evaluation of the risky tax revenue. Here we are concerned only with the positive impact of a wealth tax.

decline with a wealth tax.[11] Proportional risk taking increases with a wealth tax since risky investment remains unchanged and savings decline. Individuals hold a riskier portfolio.

[11] This is the well known difference in the time path of taxes. Under the wealth tax (as with the wage tax) the agent pays in the first period a fraction of his endowment income $t_w Y$, while under a cash flow consumption tax he pays $t_c C_1$ in the first period and $t_c C_2$ in the second period. Given the consumption choice, the individual saves more under the cash flow in order to pay the higher tax in the second period. There can be controversy over the modified wealth tax and cash flow consumption tax also. Which method would a rational household prefer? In the case of perfect capital markets both methods are identical but as was pointed out correctly by Atkinson and Stiglitz (1980), preference to a particular structure depends on capital market imperfections. Atkinson and Stiglitz argue that a cash flow consumption tax will be preferred if the household is faced with liquidity constraints. The household that faces a borrowing constraint prefers the cash flow consumption tax method, because under this method the household can alter his tax liability by consuming less today and investing the proceeds in a lumpy project. The lower current tax liability will be invested in a risky asset. This option is not available with a pre-payment wage tax.

8.6.3. A Wage Tax (Endowment) Tax:

Under the wage tax the intertemporal budget constraint is given by:

$$C_1 + \frac{C_2}{(1+r)} = (1 - t_w)\, Y + \frac{az}{(1+r)}$$

Since the pre-payment wage tax falls on endowment we do not expect this tax to cause any distortions. The comparative static analysis of the problem leads to the following results. The wage tax does not create any intertemporal distortions much as the cash flow consumption tax; the effect of a wage tax is to reduce current consumption by an income effect. The wage tax, in contrast to the cash flow consumption tax, does not affect the return to risk taking and has no substitution effect of the Domar-Musgrave type . The endowment tax only has a negative income effect. A wage tax discourages risky asset holdings. The tax causes the same negative impact on private risk taking. One does not need to differentiate between social and private risk taking with this tax system.

The following table summarizes the findings of the three tax systems:

VARIABLES	NO TAX	CONSUMPTION TAX	WEALTH TAX	WAGE TAX
Risk Taking	$a(0)$	$a(t_c)=a(0)$	$a(0)$	$(1-t_w)a(0)$
Consumption	$C_1(0)$	$C_1(0)/(1+t_c)$	$(1-t)C(0)$	$(1-t_w)C_1(0)$
Savings	$S(0)$	$S(t_c)=S(0)$	$(1-t)S(0)$	$(1-t_w)S(0)$
Proportional Risk Taking	$\beta(0)$	$\beta(t_c)=\beta(0)$	$\beta(0)/(1-t)$	$\beta(0)$

In summary, the pre-payment approach (without the inclusion of capital gains) to implementing a consumption tax causes a reduction in current consumption, savings and the amount invested in the risky asset. The pre-payment wage tax leaves proportional risk taking unchanged.[12] The cash flow consumption tax does not affect the amount of invested in the risky asset and savings, but decreases current consumption. The wealth tax, on the other hand, reduces consumption and savings, but leaves risky investment unchanged. Consequently, proportional risk taking increases with a wealth tax.

[12] In a discussion paper version of Ahsan (1989), he recognized that a wage tax relative to a consumption tax, and under CRRA preferences, leads to lower values of these functions. This being independent of the chosen values of the pre-tax savings and risk taking activity.

8.7. The Welfare Measurement

In order to measure welfare losses that arise from the tax change we will use the compensating variation principle. The compensating variation is defined as the change in wealth required to maintain the individual at the same level of utility holding constant the value of tax revenue.

We start from a cash flow consumption tax and compute the optimal levels of current consumption, savings, risk taking activity, and certainty equivalent future consumption[13] over various values of the risk aversion parameter and the elasticity of substitution.[14] Furthermore we compute the maximized value of the non-expected utility function. The maximized utility level arising from a cash flow consumption tax is given by:

$$V(C_1(t_c),CE(C_2(t_c))) = \frac{1}{(1+t_c)} W(t_c)\ V(C_1(0),CE(C_2(0)))$$

where $V(C_1(0),CE(C_2(0)))$ is the indirect utility with no tax and $W(t_c)$ is the endowment of the household under a consumption tax. The cash flow consumption tax reduces the maximized non-expected utility level by the factor of $(1+t_c)^{-1}$ relative to the pre-tax

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[13] Notice that a cash flow consumption tax also reduces the certainty equivalent future consumption level because of the income effect.

[14] The effect of this tax on the decision variables was analyzed in the previous section. Basically a cash flow consumption tax leaves unchanged savings and the amount invested in the risky asset. The cash flow tax like the prepayment wage tax leaves unchanged proportional risk taking. Proportional risk taking increases only with a modified wage tax. The pre-payment wage tax discourages both savings and the amount invested in the risky asset.

maximized non-expected utility level. For a given t_c we then evaluate the risky tax revenue and compute equal value wage tax rates. The equal value tax revenue wage tax rates:

$$t_w^*(t_c) = \frac{t_c}{(1+t_c)} \left[1 - \frac{S(t_c)}{Y} \left(1 - \frac{1+r+\beta(t_c)E(z)}{(1+\rho)} \right) \right]$$

The equal value wage tax rates depend on risk aversion and the elasticity of substitution in contrast to the certainty model whereby equal value wage tax rates are set such that they are always equal to $t_c(1+t_c)^{-1}$.[15]

Next we use these equal value wage tax rates and compute the compute the maximized levels of current consumption, savings, risk taking activity, and certainty equivalent future consumption with a pre-payment wage tax without the inclusion of the capital gains. Finally, we compute the maximized value of the non-expected utility function with a wage tax:

$$V(C_1(t_w^*),CE(C_2(t_w^*))) = (1-t_w^*)\ W(t_w^*)\ V(C_1(0),CE(C_2(0)))$$

Equating the two indirect utilities and solving for the relative wealth terms yields the compensating variation measure:

[15] This equation is obtained by setting the value of the risky tax revenue equal to the value of the revenue from the wage tax and solving for the wage tax. The wage tax will be a function of the initial consumption tax, savings, proportional risk taking, the social discount rate and the weighted average rate of return.

$$\frac{W(t_w^*(t_c))}{W(t_c)} = \frac{1}{(1-t_w^*(t_c))(1+t_c)}$$

A movement to a wage tax base will lead to welfare losses if $W(t_w^*(tc)) > W(t_c)$, that is if the individual requires a higher initial endowment level in the new system (i.e the pre-payment wage tax version) than in the cash flow consumption tax system.[16] Substituting the equal value wage tax rates into the welfare measurement we obtain:

$$\frac{W(t_w)}{W(t_c)} = \frac{1}{1 + t_c\ MPS(t_c)\ [1 - \frac{1+r+\beta(t_c)E(z)}{(1+\rho)}]}$$

where $MPS(t_c)$ is marginal propensity to save with a cash flow consumption tax.

Proposition 8.1: If the social discount rate is equal to the weighted average rate then the equal value wage tax is set equal to $t_c(1+t_c)^{-1}$ and welfare remains unchanged from this tax substitution.

Proposition 8.2: If the social discount rate is lower than the expected weighted average discount rate, the individual will require a higher initial endowment in the wage tax (i.e., $W(t_w) > W(t_c)$) in order to be compensated. The pre-payment wage tax rate will lead to welfare losses.

As long as the social discount rate is lower than the weighted average rate there are welfare losses from a switch to a pre-payment wage tax. If the social discount rate is equal to the risk free rate then a higher initial consumption tax rate will increase the

[16] We could have compared the relative indirect utility levels under the two tax systems as in Ahsan (1989). The qualitative results would be the same.

welfare losses of a wage tax. A higher consumption tax rate will shift more of the risk onto the government constraint, and to future generations, and less on the households alive today.[17] Furthermore, a reduction in the elasticity of substitution will decrease the welfare losses of a wage tax. This happens because as elasticity of substitution decreases, so does the proportion saved. An increase in the relative risk aversion parameter reduces the welfare losses of a wage tax. Recall that as the relative risk aversion increases proportional risk taking falls; in the limit the weighted average discount rate collapses to the risk free rate and then the model becomes certain since risky investment is zero. Furthermore, as relative risk aversion increases savings also increase, but the reduction in proportional risk taking is much stronger. This implies that the welfare gains of a cash flow consumption tax will accrue to the less risk averse households. Using the risk free rate as the social discount rate the welfare losses become:

$$\frac{W(t_w^*(t_c))}{W(t_c)} = \frac{1}{1 - t_c \dfrac{MPI(t_c)E(z)}{(1+r)}}$$

where MPI(t_c) is marginal propensity to invest in the risky asset.

Proposition 8.3: The welfare losses of a wage tax are at a maximum if the social discount rate is the risk free rate.

[17] Of course this implies that the welfare gains are maximized at 100% consumption tax rate. However, if we allow labour - leisure choice then the consumption tax rate will be less than 100% and greater than zero. This occurs because the distortionary effect of a consumption tax on labour-leisure choice are equated to the insurance effects of a consumption tax.

The welfare losses arise because with the risk free rate as a social discount rate the state can raise more revenue in present value terms than a wage tax set at a rate of $t_c(1+t_c)^{-1}$. A wage tax set at a rate of $t_c(1+t_c)^{-1}$ yields equal utility with the cash flow consumption tax if the social discount rate is equal to the weighted average rate. Since more expected present value revenue can be raised with a cash flow consumption tax, the wage rate has to increase to satisfy the equal expected present value tax revenue criterion. This increase in the wage tax rate reduces utility and individuals demand a higher compensation because of the tax change. Households would be indifferent between a cash flow consumption tax and the wage tax if the state, in a wage tax system, gave them back the expected present value tax revenue from the risk premium which was generated with a cash flow consumption tax.

8.8. Numerical Results on Welfare Change

A consumption tax rate of 33% and a social discount rate equal to the risk free rate yields the following equal value wage tax rates:

Table 8.8: Equal Value Wage tax Rates (r=.50,E(x)=3,V(x)=3)

	Intertemporal Elasticity of Substitution					
		.50	.33	.10	.05	.01
Relative Risk Aversion Parameter	0.50	0.448	0.417	0.379	0.372	0.366
	0.99	0.418	0.398	0.373	0.368	0.364
	1.50	0.374	0.363	0.348	0.345	0.342
	2.00	0.344	0.337	0.327	0.325	0.324
	4.00	0.294	0.292	0.289	0.288	0.287

Examination of the equal value wage tax rates shows very stable values across all the relative risk aversion and elasticity of intertemporal substitution parameters. Zodrow (1994) argues that even if the individuals use the safe rate of return to discount the risky tax revenue, Ahsan's "modified wage tax" is not the only method to "design a wage-tax system that is equivalent in present value terms to the cash flow consumption tax". Zodrow argues that one can increase the pre-payment wage tax until the government obtains the same revenue as under the cash flow tax. This is precisely what is done above. However this wage tax is more burdensome to the household than a cash flow consumption tax. The pre-payment wage tax under certainty is equal to 25% for a 33% cash flow tax. If the social discount rate is equal to the weighted average rate a 2[illegible]% wage tax would yield equal expected utilities from the two taxes even under uncertainty. For example a household that has an elasticity of substitution of .01 and a relative risk aversion of 2, the equal value wage tax rate is 32.4%. The very high equal value wage tax rates shown in table 1 reduce the households pre-payment wage tax utility relative to the utility level obtained with a cash flow consumption tax. The welfare losses from switching to a wage tax are shown below:

Table 8.9: Welfare Losses $(r=.50, E(x)=3, V(x)=3)$

Intertemporal Elasticity of Substitution						
		.50	.33	.10	.05	.01
Relative Risk Aversion Parameter	0.50	1.362	1.289	1.211	1.197	1.186
	0.99	1.292	1.249	1.199	1.189	1.182
	1.50	1.201	1.180	1.152	1.150	1.143
	2.00	1.146	1.134	1.117	1.114	1.111
	4.00	1.066	1.062	1.057	1.056	1.055

The welfare losses range from a low of 5.5% to a maximum of 36%. For a household who's elasticity of substitution is .01 and relative risk aversion of 2, the welfare losses are estimated to be 11%. These welfare losses are large compared to the welfare losses in the existing literature on uncertainty and taxation policy. The welfare losses fall as the intertemporal elasticity of substitution decreases. As the intertemporal elasticity of substitution decreases, the investment in the risky asset falls, reducing the welfare losses. On the other hand as relative risk aversion increases less is invested in the risky asset causing a reduction in the welfare losses of a wage tax. A mean preserving decrease in the variance causes the following effects on welfare losses:

Table 8.10: Welfare Losses (r=.50,E(x)=3,V(x)=2.5)

Intertemporal Elasticity of Substitution						
		.50	.33	.1	.05	.01
Relative Risk Aversion Parameter	0.50	1.547	1.401	1.263	1.241	1.224
	0.99	1.456	1.367	271	1.254	1.241
	1.50	1.302	1.261	1.212	1.202	1.195
	2.00	1.211	1.189	1.161	1.155	1.151
	4.00	1.088	1.082	1.075	1.073	1.072

A mean preserving decrease in variance tends to increase the welfare losses of a wage tax. This occurs because of the increase in risk taking activity and savings. The welfare losses of moving to a pre-payment wage tax are very sizeable. For example a household with a relative risk aversion parameter of 2 and an elasticity of substitution of .01 the welfare losses amount to 15%. In the next table we show how an increase in the risk free rate affects the welfare losses of a wage tax. This case can be associated with an increased risk premium on government bonds.

Table 8.11: Welfare losses (r=1.25,E(x)=3,V(x)=2.5)

Intertemporal Elasticity of Substitution						
		.50	.33	.09	.05	.01
Relative Risk Aversion Parameter	0.50	1.143	1.119	1.091	1.086	1.082
	0.99	1.110	1.094	1.076	1.072	1.070
	1.50	1.078	1.069	1.057	1.055	1.053
	2.00	1.060	1.053	1.045	1.044	1.042
	4.00	1.030	1.027	1.023	1.023	1.022

As expected, the welfare losses of a pre-payment wage tax fall with an increase in the social discount rate. The welfare losses are still in the range of 2% to 14%. Comparing two households that differ in risk aversion we can state that the less risk averse households, as measured by the risk aversion parameter, have a stronger preference towards a cash flow consumption tax. Only for the highly risk averse households that do not undertake risky activities is the imposition of the tax through the cash flow approach or pre-payment approach is a matter of irrelevance.[18] The less risk averse investors undertake more risk and therefore are exposed more to "social" or "market" risks. The next table shows the welfare losses under the initial distribution. The welfare losses decrease rather slightly relative to the previous table because risk taking is less now, due to the increase in variance.

Table 8.12: Welfare Losses (r=1.25,E(x)=3,V(x)=3)

		Intertemporal Elasticity of Substitution				
		.50	.33	.09	.05	.01
Relative Risk Aversion Parameter	0.50	1.118	1.100	1.078	1.074	1.071
	0.99	1.087	1.076	1.063	1.060	1.058
	1.50	1.063	1.056	1.047	1.045	1.044
	2.00	1.048	1.043	1.037	1.036	1.035
	4.00	1.024	1.022	1.019	1.018	1.018

[18] They might have a preference towards a pre-payment wage tax due to administrative simplicity.

8.9. Risk Aversion Evaluation and Further Numerical Results

We also conduct the differential incidence analysis using the certainty equivalent of future consumption to discount the risky tax revenue. This method takes into account the riskiness of the tax revenue. But instead of relying on some asset pricing model and the market efficiency hypothesis, we use the certainty equivalent future consumption which is derived from the optimization problem of the household.

We start again from a cash flow consumption tax and compute the optimal levels of current consumption, savings, risk taking activity, and certainty equivalent future consumption over various values of the risk aversion parameter and the elasticity of substitution. Furthermore we compute the maximized value of the non-expected utility function. We then evaluate the risky tax revenue from a cash flow consumption tax using the certainty equivalent future consumption choice. The certainty equivalent tax revenue of a cash flow consumption tax is:

$$R_{ct} = t_c \left(C_t(t_c) + \frac{CE(C_{t+1}(t_c))}{(1+r)}\right)$$

where $CE(C_{t+1}(t_c))$ is the certainty equivalent future consumption as a function of the consumption tax rate. Setting this tax revenue equal the tax revenue under a wage tax yields the following equal value tax revenue wage tax rates:

$$t_w^* = t_c \left[\frac{C_1(t_c) + \frac{CE(C_2(t_c))}{1+r}}{Y}\right]$$

The equal value wage tax rates also depend on risk aversion and the elasticity of substitution, and are not independent as in the certainty model whereby equal value wage tax rates are set such that they are always equal to $t_c(1+t_c)^{-1}$. Assuming a consumption tax rate of 33% yields the following equal value wage tax rates for the distribution of returns of (r=.5,E(x)=3, V(x)=3):

Table 8.13: Equal Value Wage tax Rates (r=.50,E(x)=3,V(x)=3)

		Intertemporal Elasticity of Substitution				
		.50	.33	.10	.05	.01
Relative Risk Aversion Parameter	0.50	0.400	0.376	0.348	0.342	0.338
	0.99	0.347	0.336	0.321	0.318	0.316
	1.50	0.315	0.310	0.301	0.299	0.298
	2.00	0.297	0.293	0.288	0.287	0.286
	4.00	0.271	0.270	0.268	0.268	0.267

First, the equal value wage tax rates are very high relative to the wage tax rate of 25% under certainty. The equal value wage tax rates are very stable across all the relative risk aversion parameters and elasticity of intertemporal substitution parameters. Uncertainty is taken into account by using the certainty equivalent future consumption. Again high wage tax rates are very burdensome to the household. They discourage the optimal decision variables of the households. The wage tax rate of 25% obtained under certainty is achieved only for households that have an extremely high risk aversion behaviour.

From the table it is even possible, in this model of uncertainty and portfolio choice, that the equal value wage tax rate increases above the consumption tax rate. This occurs because of the sizeable tax revenue the government receives from a cash flow consumption tax revenue. As stated previously, the pre-payment wage tax under certainty is equal to 25% for a 33% cash flow tax. The very high equal value wage tax rates shown above reduce the households welfare relative to the welfare level obtained with a cash flow consumption tax. What is different with this analysis is that we have taken explicit account of the risky tax revenue and still obtain very high equal value wage tax rates. As shown previously, if we had used the expected present value of the tax revenue using the risk free rate the equal value wage tax rates were even higher than these. For example for a relative risk aversion parameter of 2 and an elasticity of substitution of .01 the equal value wage tax rate is 28.6% using the risk aversion approach while the risk neutral evaluation yields 32.4% wage tax rates. These rates correspond to a cash flow consumption tax of 33%.

Once the equal value wage tax rates are computed we determine the optimal consumption, savings, risk taking and certainty equivalent future consumption using these equal value wage tax rates. The effect of these high wage tax rates is to greatly dampen the optimal decision values. Next we compute the indirect utility function using these new values and compare the two indirect utility function: The relative welfare levels depends on the compensating variation principle and is given by:

Again a movement to a wage tax base will lead to welfare losses if $W(t_w(tc)) > W(t_c)$.

$$\frac{\bar{W}(t_w^*(t_c))}{W(t_c)} = \frac{1}{(1-t_w^*(t_c))(1+t_c)}$$

In other words, the individual would require a higher initial endowment level in the new system (i.e the pre-payment version) than in the cash flow consumption tax system to be equally well off.

The welfare losses from switching to a pre-payment wage tax are shown below:

Table 8.14: Welfare Losses (r=.50,E(x)=3,V(x)=2.5)

		Intertemporal Elasticity of Substitution				
		.50	.33	.10	.05	.01
Relative Risk Aversion Parameter	0.50	1.375	1.284	1.191	1.176	1.164
	0.99	1.217	1.181	1.138	1.130	1.124
	1.50	1.132	1.116	1.096	1.093	1.089
	2.00	1.092	1.083	1.072	1.070	1.068
	4.00	1.040	1.037	1.034	1.034	1.033

The welfare losses range from a low of 3% to a maximum of 38%. In addition, given the elasticity of substitution is .01 and relative risk aversion around 2 the welfare losses are estimated to be 7%. For the same values the risk neutral evaluation yields a welfare loss equal to 15%. The welfare losses again fall as the intertemporal elasticity of substitution decreases and as the relative risk aversion increases. Notice that for a fixed risk aversion parameter the decrease in the elasticity of substitution to zero still yields welfare losses under a pre-payment wage tax.

Examining a mean preserving increase in the variance yields the following welfare losses:

Table 8.15: Welfare Losses (r=.50,E(x)=3,V(x)=3)

		Intertemporal Elasticity of Substitution				
		.50	.33	.1	.05	.01
Relative Risk Aversion Parameter	0.50	1.253	1.206	1.152	1.143	1.135
	0.99	1.152	1.132	1.107	1.102	1.099
	1.50	1.097	1.087	1.075	1.073	1.071
	2.00	1.070	1.064	1.057	1.055	1.054
	4.00	1.032	1.030	1.028	1.027	1.027

A mean preserving increase in variance tends to reduce the welfare losses of a wage tax. This occurs because of the reduction in risk taking activity and savings. However, the welfare losses still persist and are sizeable. Finally in the next table we show how an increase in the risk free rate affects the welfare losses of a wage tax.

Table 8.16: Welfare Losses (r=1.25,E(x)=3,V(x)=3)

		Intertemporal Elasticity of Substitution				
		.50	.33	.09	.05	.01
Relative Risk Aversion Parameter	0.50	1.081	1.070	1.055	1.052	1.050
	0.99	1.050	1.044	1.037	1.035	1.034
	1.50	1.035	1.031	1.026	1.025	1.024
	2.00	1.026	1.023	1.020	1.019	1.019
	4.00	1.013	1.012	1.010	1.010	1.009

As expected, the welfare losses of a wage tax fall with an increase in the discount rate. The welfare losses are in the range of 1% to 8%.

8.10. Conclusion

It is well known that in the world of certainty and under the assumption of perfect capital markets the switch from a wage tax to a consumption tax will not affect the welfare of the household. The two taxes are equivalent in their effect on the budget constraint. However, the effect on savings is not the same since the time path of the tax revenue is different.[19]

Under uncertainty, however, there is an additional difference in the two tax systems if the risk arises from uncertain capital income. The difference is that the tax revenues under a consumption tax are uncertain while that of a wage tax are certain. The role of the state may become that of a risk sharing partner. If the household does not hold a fully diversified portfolio of assets s/he will prefer the cash flow approach to the implementation of a consumption tax. Alternatively, if the household invests in an asset that is perfectly correlated with the market portfolio then the cash flow consumption tax and the pre-payment wage tax may turn out to be equivalent as under certainty.

[19] Atkinson and Stiglitz (1980) argued that a household will prefer the cash flow consumption tax over the pre-payment wage tax method if s/he faces borrowing constraints and an investment opportunity appears in the future. According to them if a "lumpy" investment project (e.g, to start a business) becomes available the household, under a consumption tax, has the option of reducing her (his) tax bill by consuming less and investing the proceeds in the project. This option is not available under a pre-payment wage tax.

For illustration consider the following Pareto improvement example under uncertainty. Individuals that hold undiversified portfolios would be willing to pay a higher price for the "risky" cash flow consumption tax revenue. For the investor that holds the market portfolio the "risky" cash flow consumption tax revenue is perceived as a "risky" bond that is a perfect substitute with the "risky" asset. Hence the price of the "risky" note will be lower for an investor that holds the market portfolio. For the perfectly diversified investor the implementation of a consumption tax as a cash flow or wage tax prepayment version is a matter of irrelevance since his welfare is unaffected by the method of imposition of the tax. However, for the individual that does not hold a completely diversified portfolio these "risky" notes will reduce the portfolio risk. Hence, this investor is willing to pay a higher price for the "risky" bonds. Hence if an economic system, contains both types of investors, then the undiversified investors are strictly better off with a cash flow consumption tax whereas the investor that holds the market portfolio is just as well off with a cash flow version or a pre-payment wage tax version of a consumption tax. Hence a pareto optimal improvement in resources can be achieved via a cash flow consumption tax.

In addition, we have shown that even if the household holds the "market portfolio" there is a role for the state. This comes from the intergenerational risk sharing argument of Gordon and Varian (1988). All generations will participate in the current lottery or current "market" (or "social") risks with each generation, either living or yet to be born, bearing an arbitrary small share. The connection of the risk sharing arrangement operates from the loss offset provisions of tax policy and transfers the risks via a stochastic government budget balance onto future generations.

Chapter 9

Human Capital and Taxation Policy

9.1. Introduction

Human capital investment has unambiguously contributed to welfare and economic growth. Individuals investing in human capital acquire skills and knowledge. The acquisition of skills and knowledge is a significant factor in explaining the increased rate of economic growth and living standards. (See Becker (1964), Schultz (1971)).

The early economics literature dealing with human capital theory focused attention on decisions made under complete certainty (Becker (1964), Schultz (1971). It is well known that human capital decisions involve considerable risk and uncertainty.[1] In addition to uncertainty the decision maker is faced with imperfections in the capital market making it difficult if not impossible to borrow capital funds to finance the expenditure.

When households make decisions about investing time to acquiring skills and knowledge in order to increase their earning capacity, and thereby attain better future consumption opportunities, they incur not only considerable costs in terms of foregone current

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[1] The theory of human capital under uncertainty was elegantly presented in a pioneering study by Levhari and Weiss (1974). They examined the effects of increased uncertainty on human capital investment decisions.

earnings but are also faced with an uncertain stream of future earnings.

The uncertainty of human capital earnings fall into the following main categories. Quality of the services received from schooling are not precisely known at the time of the decision; also the decision maker has imperfect knowledge of the value of his abilities. Second, changing economic environments make the earnings capacity from human capital uncertain. Will human capital accumulation provide higher income and better employment opportunities given that unpredictable events alter future demand and supply conditions ? [2] The former is known as input uncertainty and the latter is output uncertainty.

In the face of an uncertain stream of future earnings it is also of importance to stress the imperfections in the capital market to provide funds for human capital investment. This has been discussed extensively (Becker (1964), Nerlove (1972), Stigler (1971), Schultz (1971), Friedman (1962), Friedman and Kuznets (1945)). Freidman (1962) makes clear the failure of the market to provide funds:

"A further complication is introduced by the inappropriateness of fixed money loans to finance investment in training. Such an investment necessarily involves much risk....Consequently if fixed money loans were made, and were secured only by expected future earnings, a considerable fraction would never be repaid. In order to make such loans attractive to lenders, the nominal interest rate charged on all loans would have to be sufficiently high to compensate for the capital losses on the defaulted loans. The high nominal interest rate would both conflict with usury laws and make the loans unattractive

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[2] Another uncertainty an individual faces is the length of working life.

to borrowers. The device adopted to meet the corresponding problem for other risky investments is equity investment plus limited liability on the part of the shareholders. The counterpart for education would be to "buy" a share in the individual's earning prospect: to advance him the funds needed to finance his training on condition that he agree to pay the lender a specified fraction of his future earnings...One way to achieve this result is for the government to engage in equity investment in human beings....The individual in return would agree to pay to the government in each future year a specified percentage of his earnings .... This payment could easily be combined with payment of income tax and so involve a minimum of additional administrative expense."

The failure of the market system due to the existence of severe moral hazard to provide insurance against the unforeseen future events has led to government intervention to reduce uncertainty and correct capital market imperfection through unemployment insurance benefits, income-contingent loan-repayment programs and taxation policy. Freidman (1962) suggests that private contracts do not exist because of the large transaction costs and argues for government intervention to improve the market imperfections in such a way as to make capital available.

This chapter deals with the effects of taxation on risky human capital investments. Examination of the impact of taxation on human capital investment when the returns are uncertain has been very limited compared to the amount of research devoted to the investigation of taxation and physical capital. A decision model which includes both physical and human capital investment decisions will provide more insight to tax policy analysis. The effects of taxation policy on human capital are important for the following reasons. First, taxation policy may have an important and large impact on the quality of the work force as well as an effect on the amount of work effort as measured by "hours of work" (Eaton and Rosen (1980b)). Secondly, income contingent loan-repayment programs that help finance higher education might be financed from payment of taxes (Freidman (1962)). Finally, taxation policies that discourage human capital investment will inevitably lead to a reduction in economic growth and welfare (see Becker (1964), Schultz (1971)).

Section 9.2 presents the model that is used to investigate the effects of alternative tax policies. Section 9.3 examines the wealth effects on the decision variables. Section 9.4 examines the impact of four well defined taxes on physical and human capital investments. These are, respectively, a cash flow consumption tax, a pre-payment wage tax, an interest tax, and an income tax. Furthermore, the utility compensated effects of taxation policy will be presented. The comparative static results will be based on the common assumptions of risk averse behaviour, namely the hypothesis of non-increasing absolute risk aversion and non-decreasing relative risk aversion. In Section 9.5 we conclude the analysis of taxation policy on human capital investments.

9.2. The Model and Preferences

In a pioneering study, Levhari and Weiss (1974) examined the effects of uncertainty on the investment in human capital. They developed a two-period Fisherian model where future wages are stochastic and conditional on the investment in human capital. This model is somewhat analogous to that of Leland (1968), Sandmo (1968) and Dreze and Modigliani (1972) analyzing the problems of portfolio-savings choice under uncertainty. The originality of their analysis is that the return to human capital is conditional on the amount of human capital investment and on the state of the world. The dependence of the return on the magnitude of human capital investment makes the model different from the portfolio/savings model, where typically the return is independent of the amount of investment. Therefore, taxation policies analyzed for non-human capital investment decisions may not bear any close relation to the effects on human capital investments.

In the first period the representative individual is endowed with exogenous income Y_{1t} which (s)he allocates to current consumption C_{1t}, investment in non-human capital S_{1t}, and human capital investments wH_{1t}. The investment in human capital is expressed in terms of foregone earnings. Letting w be the wage rate per unit of time and H_{1t} the amount of time spent on human capital from a total time endowment of H_{0t}, the foregone earnings in the first period is Wh_{1t}. More formally the first period constraint is given by: $C_{1t} = Y_{1t} - Wh_{1t} - S_{1t}$ where Y_{1t} is an exogenous level of income , i.e., $Y_{1t} = W_{1t} + Wh_{0t}$, In the second period the individual earns income from the investment in human capital. These earnings depend on the amount of time spent on accumulating human capital (H_{1t}) and on the state of uncertainty μ_{t+1}. Future human capital earnings are given by: $\mu_{t+1}F(H_{1t})$, where $F(H_{1t})$ is the human capital production function. The technology is assumed to be known. The human capital production function is characterized by positive and diminishing marginal returns to human capital represented by $F' > 0$ and $F'' < 0$, respectively. The variable μ_{t+1} represents the random element entering in a multiplicative way.[3] The random variable μ_{t+1} lies in the closed interval $[\alpha_{t+1},\beta_{t+1}]$ with mean $E(\mu)$. The random variable affects future earnings and marginal returns positively. These assumptions have been used by various researchers (e.g. Levhari and Weiss (1974), Eaton and Rosen (1980), Hamilton (1987), Brown and Kaufold (1988)).[4]

Proceeds for future consumption are also obtained from future exogenous income, Y_{2t}, and non - human capital investments, $S_{1t}(1+r)$, where r is the certain rate of return. More specifically, the second period constraint is given by: $C_{2t} = Y_{2t} + \mu_{t+1}F(H_{1t}) + (1+r)S_{1t}$ where C_{2t} is future consumption.

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[3] The multiplicative form has been used by Eaton and Rosen (1980a,b) for the input uncertainty case. This form has also been used by Brown and Kaufold (1988) to study the effects of unemployment insurance benefits on human capital investment. In the later context the random element represents the proportion of time spent employed and is determined by shocks to the economy. Therefore the shock represents output uncertainty.

[4] The assumption of the random variable affecting the marginal returns positively, implies a positive correlation between marginal and average rate of return to human capital. It also implies that as time allocated to human capital increases the variance of future earnings increases, namely the so called principle of increasing risk.

The individual is assumed to maximize his (her) satisfaction from current and future consumption by choosing the amount of human capital investment H_{1t} and current consumption C_{1t}. More formally:

$$Max \qquad V = g(C_{1t}) + E(h(C_{2t}))$$

where $g(C_{1t})$ and $h(C_{2t})$ are well behaved continuous functions and have a positive ($g', h' > 0$) and diminishing marginal utility of current and future consumption ($g'', h'' < 0$).

An optimum allocation must satisfy the overall constraint obtained by combining the first and second period constraints:

$$C_{1t} + \frac{C_{2t}}{(1+r)} = Y_{1t} + \frac{Y_{2t}}{(1+r)} + \left(\frac{\mu_{t+1} F(H_{1t})}{(1+r)} - wH_{1t} \right)$$

The first order conditions for an interior solution are given by:

$$g' - (1+r)E(h') = 0$$

$$E(h'(\mu_{t+1}F' - w(1+r))) = 0$$

or alternatively, we can write the second condition as follows:

$$E(h' z_{t+1}) = 0$$

where $z_{t+1} = \{\mu_{t+1}F' - w(1+r)\}$ represents the excess return on human capital investment.

The first condition states that at the optimum the marginal utility of current consumption is equal to the future value of expected marginal gain from future consumption. The second condition states that at the optimum the expected marginal gain from human capital is equal to that of physical capital in terms of their contribution to future consumption.[5]

9.3 The Wealth Effects

The wealth effects can be determined and signed under the plausible hypothesis of decreasing absolute and non-decreasing relative risk aversion.[6] In particular, we can show the following comparative static results.

For a non-borrower:

$$\frac{\partial S_{1t}}{\partial Y_{1t}} > 0 \quad , \quad \frac{\partial H_{1t}}{\partial Y_{1t}} > 0 \quad , \quad 0 < \frac{\partial C_{1t}}{\partial Y_{1t}} < 1 \quad , \quad 0 < \frac{\partial (S_{1t}+wH_{1t})}{\partial Y_{1t}} < 1$$

Under the DARA and NDRRA assumptions savings, human capital and current consumption are normal goods. An increase (decrease) in first period exogenous income leads to an increase (decrease) in human capital, savings and current consumption. In

[5] An interior solution is obtained if the expected return on human capital exceeds the return from non human capital. The assumption of risk aversion is sufficient to yield an interior solution.

[6] Appendix 8A1. examines in great detail the wealth effects on the decision variables.

addition, these behavioral assumptions guarantee that marginal propensity to consume lies between zero and one. Extending the analysis to a borrower, we cannot arrive at any meaningful analytical results of income effects on borrowing, and current consumption activity. However, if the individual is constrained not to borrow more than the present discounted value of his future non-stochastic, non-human income then borrowing activity decreases with increases in first period exogenous income under the assumptions of DARA and NDRRA.

For a borrower who is constrained such that: $B \leq Y_{2t}/(1+r)$, the comparative static results are given by:

$$\frac{\partial B_{1t}}{\partial Y_{1t}} < 0 \quad , \quad \frac{\partial H_{1t}}{\partial Y_{1t}} > 0 \quad , \quad 0 < \frac{\partial C_{1t}}{\partial Y_{1t}} < 1 \quad , \quad 0 < \frac{\partial (S_{1t}+wH_{1t})}{\partial Y_{1t}} < 1$$

The imposition of constraints on borrowing by financial institutions causes the household to view borrowing as an inferior commodity.

9.4. Taxation Policy and Decision Making

9.4.1. A Cash flow Consumption Tax

The post-consumption tax intertemporal budget constraint is given by:

$$C_{1t} + \frac{C_{2t}}{(1+r)} = \frac{1}{(1+t_c)}\left(Y_{1t} + \frac{Y_{2t}}{(1+r)} + \left(\frac{\mu_{t+1}F(H_{1t})}{(1+r)} - wH_{1t}\right)\right)$$

As is well known, a consumption tax does not distort the relative price of current and future consumption. Additional effects can also be detected from the imposition of the consumption tax. First, in the absence of any compensation, the consumption tax reduces the present discounted value of total human wealth thus creating income effects. Second, the tax reduces the return and variance of the random future earnings. The latter effect makes human capital more attractive on the margin relative to physical savings for a risk averse individual. The overall effect of a consumption tax cannot be determined outright because of the conflicting income and substitution effects. As it turns out, we need additional restrictions concerning the attitude towards risk in order to determine the allocative effects of a consumption tax.

The first order conditions of the optimization problem are given by:

$$g' - (1+r)E(h') = 0$$

The optimality conditions remain unaffected. This happens for two reasons. First, a consumption tax does not alter the relative price of future consumption and therefore,

$$E\left(\frac{h' z_{t+1}}{(1+t_c)}\right) = 0$$

does not create intertemporal distortions. Second, the tax reduces the expected return and the opportunity cost of human capital investment at the same rate, the tax does not alter disproportionately the relative return of human and physical capital.

Hicks Compensated effects of a Consumption Tax

The consumption tax reduces the households income. To obtain the compensated expected-utility results we require that the households first period endowment be adjusted by the following amount:

$$\frac{dY_{1t}}{dt_c}\Big|_{\bar{V}} = (1+t_c)^{-1}\left[\, Y_{1t} + \frac{Y_{2t}}{(1+r)} + wH_{1t}\left(\frac{F(H_{1t})}{H_{1t}\,F'} - 1\right)\right]$$

The expected utility compensated effects of a consumption tax on human capital investment is given by (see appendix 8A4.2):

$$\frac{\partial H_{1t}}{\partial t_c}\Big|_{\bar{V}} > 0$$

Proposition 9.1: Under the plausible behaviour of non-decreasing relative risk aversion and non-increasing absolute risk aversion, the substitution effect on human capital is positive.

Given the risk aversion hypothesis cited above, the substitution effect indicates an increased willingness to undertake human capital investment. This increased incentive to undertaking the risky activity emanates from the shifting of risk to the state (i.e., the reduction in variance). The tax lowers the random return and variance of human capital. Since the risk averse individual is utility compensated s/he undertakes more investment in the risky activity and hence less of the riskless current earnings.[7] We observe a result similar to the Domar-Musgrave phenomenon. The Domar-Musgrave phenomenon, states that the investor will increase the amount invested in the risky asset in order to encounter the same post-tax probability distribution as prior to the tax imposition. The difference between the present result and that of Domar-Musgrave lies on the dependence of the random earnings on investment. The dependence of the average rate of return on the investment (i.e., rate of return on investment is equal to ($\mu F(H)/Wh)-1$)) allows the individual to alter the moments of the probability distribution of the random earnings. An increase in human capital investment results in a decrease in the average rate of return to human capital. The adjustment, from the imposition of the tax, will be less than that in the pure asset choice. One can notice that without diminishing returns to the risky activity (i.e., if $F''=0$) we would exactly obtain the Domar-Musgrave phenomenon.

[7] As was pointed out by Professor Belzil this phenomenon is in contrast to a large class of dynamic models (search models) under uncertainty. In search models increasing risk raises the level of investment in the risky activity. In the two period human capital model examined in this paper the result is the opposite. In this model a mean preserving increase in variance of the risky income will reduce investment in the risky activity. In this paper investment in human capital (time spent in schooling) is a way of increasing future consumption. A mean preserving increase in risk decreases the risky investment. Risk averse households will increase the riskless current earnings. Hence a utility compensated wage tax reduces the variability of returns and as a consequence the risk averse household increases the time spent on accumulating human capital. This paper is also concerned as to whether the DM phenomenon holds when the investment depends on the amount invested. Future research should examine as to whether a wage tax affects dynamic risky investment decisions (i.e., search models) under uncertainty differently.

Differentiating $(1+t_c)^{-1}$ F(H) with respect to t_c we obtain the effects of a consumption tax on private risk taking:

$$\frac{\partial((1+t_c)^{-1}\ F(H))}{\partial t_c}\Big|_{V=C} < 0$$

Private risk taking unambiguously falls. Comparing this outcome with the portfolio choice model we can state that the utility compensated effects of a consumption tax on private human capital-risk taking decreases while that in portfolio choice models private risk taking remains unchanged with a consumption tax.

A consumption tax has the following effect on current consumption.

$$\frac{\partial C_{1t}}{\partial t_c}\Big|_{\bar{V}} > 0$$

Proposition 9.2: An expected utility compensated increase in the consumption tax leads to increased current consumption under decreasing absolute risk aversion.

The substitution effect is positive under the assumption of decreasing absolute risk aversion causing an increase in current consumption.[8] This result depends on the diminishing returns (i.e., F" < 0) of human capital investment. Because of diminishing returns and decreasing absolute risk aversion the tax creates a positive effect on current

[8] Notice that under constant absolute risk aversion the substitution effect is nil. Also, under the uncommon assumption of increasing absolute risk aversion the substitution effect on current consumption is negative.

consumption. This result is new and different from the portfolio choice model. In the portfolio choice model no substitution towards current consumption occurs (See Ahsan (1990)) since there is no intertemporal distortion and no dependency of returns on the investment.

From the definition of savings, (i.e. $S_{1t} = Y_{1t} - wH_{1t} - (1+t_c)C_{1t}$), the utility compensated effect of a consumption tax on savings is given by:

$$\frac{\partial S_{1t}}{\partial t_c} \Big|_{\bar{V}} > 0$$

An expected utility compensated increase in the consumption tax leads to an unambiguous increase in non-human savings. The discussion in the appendix indicates that there are two opposite effects. The increase in human capital and current consumption leads to a reduction in savings, however the first term, the income compensation, leads to an increase in savings. Consumption tax leads to an increase in savings if the first term, i.e., income compensating effect, dominates. This is precisely what happens and we arrive at proposition 9.3.

Proposition 9.3: An expected utility compensated increase in the consumption tax leads to increased physical savings.

Finally, we examine the effect of the consumption tax on the expected future consumption, and hence on the expected growth rate of consumption. The expected utility compensated effect of the consumption tax on expected future consumption is negative since all other decision variables increase (i.e. physical capital, human capital and current consumption).

Summarizing the discussion so far, we can state the following. An expected utility compensated increase in the consumption tax leads to (a) an unambiguous increase in the levels of human capital, current consumption and savings, (b) a reduction in expected future consumption and the expected growth rate of consumption.

The Total Effects of a Consumption Tax

The specific effect of an increase in the consumption tax on the decision rules is:[9]

$$\frac{\partial X_{1t}}{\partial t_c} = \frac{\partial X_{1t}}{\partial t_c}\Big|_{\bar{V}} - \frac{dY_{1t}}{dt_c}\Big|_{V=C} \frac{\partial X_{1t}}{\partial Y_{1t}}$$

$$where \qquad X_{1t} = C_{1t}\ ,\ H_{1t}\ ,\ S_{1t}$$

The first term on the right hand side represents the utility compensated effect of the consumption tax and the second term captures the income effect. Since all assets and current consumption are normal goods in this model adding a negative term to the positive Hicks compensated effects only generates ambiguities.

Continuing the analysis one step further we are able to offer the following unambiguous

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[9] We derive all the analytical results in appendix VIII. We present only the final form in the text. This is done for the purpose of clarity.

conclusions about the specific effects of a consumption tax on human capital.

Proposition 9.3: Under constant absolute risk aversion and increasing relative risk aversion an increase in the consumption tax will increase the level of human capital investment.

$$\frac{\partial H}{\partial t_c} > 0$$

This is obvious since under constant absolute risk aversion the income effect is absent, and the total effect coincides with the positive substitution effect.

Proposition 9.4: Under constant relative risk aversion an increase in the consumption tax does not affect human capital investment decisions.

$$\frac{\partial H}{\partial t_c} \Big|_{CRRA} = 0$$

This happens because the negative income effect is equal in magnitude to the positive substitution term.

The following table provides a summary of the specific effects of a consumption tax on human capital:

TYPE OF RISK AVERSION	**DRRA**	**CRRA**	**IRRA**
IARA	NA	NA	?
CARA	NA	NA	+
DARA	?	NIL.	?

The specific effects of a consumption tax on current consumption yields the following result:

Proposition 9.5: A consumption tax will reduce current consumption under the hypothesis of non-decreasing relative risk aversion.

$$\frac{\partial C_0}{\partial t_c} < 0$$

This occurs because the income effect outweighs the substitution effect under the hypothesis of non-decreasing relative and non-increasing absolute risk aversion. One can observe that under increasing relative and constant absolute risk aversion the substitution effect is absent and the total effect is equal to the negative income effect leading to a reduction in current consumption. In the special case of constant relative risk aversion the effect of an increase in the tax on current consumption is given by the following unambiguous expression:

$$\frac{\partial C_0}{\partial t_c}\Big|_{CRRA} = -\frac{C_0}{(1 + t_c)}$$

Proposition 9.6: While a consumption tax leads to a reduction in current consumption, gross consumption $(1+t_c)C_0$, however, remains unchanged.

The following table provides a summary of the effects of a consumption tax on current consumption:

TYPE OF RISK AVERSION	DRRA	CRRA	IRRA
IARA	NA	NA	?
CARA	NA	NA	-
DARA	?	-	-

A consumption tax has an ambiguous effect on savings since the income effect and the substitution effect operate in opposite directions. However, under CRRA the effect of an increase in the consumption tax on savings is given by:

$$\frac{\partial S}{\partial t_c}|_{CRRA} = 0$$

Proposition 9.8: Under CRRA savings remain unaffected by a consumption tax.

This happens because under CRRA, on the on hand the effect on human capital is nil, and secondly, gross current expenditure $C_0(1+t_c)$ remains unaffected by the tax.

The following table provides a summary of the effects of a consumption tax on the decision variables under CRRA:

Decision Rules	**CRRA**
Human Capital Investment	**No Effect**
Savings	**No Effect**
Current Period Consumption	**Declines**
Expected Future Consumption	**Declines**
Expected growth of Consumption	**No Effect**

We observe that under the hypothesis of constant relative risk aversion a consumption tax has no specific effects on human capital or savings but it deters current and expected future consumption to the same degree leaving the expected growth rate of consumption unaffected.

Finally, we compare the results with the two-period asset-choice model. We observe that risky activity, under an expected utility compensation, is stimulated in both models.[10] However, it is important to recognize that the effect on human capital is different from the amount needed to leave net risky income unaffected by taxation. In the human capital model the individual is able to alter the returns of the investment (i.e. uncertainty is endogenous) as opposed to the portfolio model where the returns are parametric. In addition, the utility compensated effect on current consumption is positive in the human capital model, while in the portfolio model it remains unaffected. This too is the result of the dependence of the returns to human capital on the investment. Furthermore in the human capital model, a utility compensated increase in the consumption tax leads to increased savings more if the returns did not depend on investment. In the human capital model a utility compensated increase in the tax rate leads to a reduction in expected future consumption activity, while in the traditional portfolio model expected future consumption is unaffected. Finally, note that the common CRRA assumption leads to specific results which are identical in both models.

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[10] Mossin (1968) was the first to indicate that the substitution effect is positive in the one period asset - choice model. We generalize the result to the human capital investment model.

9.4.2 A Wealth Tax

The intertemporal budget constraint under a wealth tax is given by:

$$C_{1t} + \frac{C_{2t}}{(1+r)} = (1-t_w)\left(Y_{1t} + \frac{Y_{2t}}{(1+r)} + \left(\frac{\mu_{t+1} F(H_{1t})}{(1+r)} - wH_{1t} \right)\right)$$

The equivalence between the consumption tax and a wealth tax can be established easily if the market is perfect and the consumption tax is set equal to $t_c = t_w/(1-t_w)$. The distortions introduced by this tax are identical to the consumption tax. Therefore, the effect of the wealth tax on human capital and current consumption is similar and no further discussion is warranted (See appendix 8A3.1). However, as pointed out by Ahsan (1990), the wealth tax has a different effect on savings. The difference stems from the difference in the time path of tax payments. Under a consumption tax one would expect individuals saving more in order to pay higher future taxes. Even though the time path of tax payments differ under the two regimes, the consumption path net of taxes is identical. From the definition of savings: $S_{1t} = (1 - t_w)(Y_{1t} - wH_{1t}) - C_{1t}$ we can see that an expected utility compensated wealth tax affects savings with two opposing forces. One effect indicates an increased willingness to undertake savings due to the Hicksian income compensation term and the second effect indicates a reduction in savings activity due to the increased level of human capital investment and current consumption. However, the first effect dominates.

Proposition 9.7: An expected utility compensated increase in the wealth tax leads to increased physical savings.

The **specific** effects of an increase in the wealth tax under CRRA on human capital, current consumption and savings are:

$$\frac{\partial H}{\partial t_w}\Big|_{CRRA} = 0$$

$$\frac{\partial C_0}{\partial t_w}\Big|_{CRRA} = -\frac{C_0}{(1-t_w)}$$

$$\frac{\partial S}{\partial t_w}\Big|_{CRRA} = -\frac{S}{(1-t_w)}$$

Human capital is unchanged; the household does not undertake more of the risky activity. Current consumption and savings fall. Evidently, savings unambiguously decreases for a lender under a wealth tax; for a borrower, borrowing decreases.

Proposition 9.8: A wealth tax under the hypothesis of constant relative risk aversion has no specific effects on human capital, but deters both current consumption and savings.

Thus while, under CRRA, a "cash flow" consumption tax does not affect savings, a wealth tax (a "pre-payment" consumption tax) does discourage savings, since savings are made out of net-of-tax earnings.

9.4.3. An Interest Tax

Investigating an interest income tax will allow us to extend the analysis to a broader tax base, the income tax base. The interest income tax affects the constraint as shown below:

$$C_{2t} = (1-t_w)\ [\ Y_{2t}+\mu_{t+1}F(H_{1t})\ ] + (1+r(1-t_r))\ (\ (1-t_w)[\ Y_{1t}-wH_{1t}\ ] - C_{1t}\ )$$

The first order conditions from the maximization problem is given by:

$$g' - (1+r(1-t_r))E(h') = 0$$

$$E(h'(1-t_w)(\mu_{t+1}F' - w(1+r(1-t_r)))) = 0$$

In this case the interest income tax creates two distortions. First, it alters the relative price between current and future consumption. An increase in the interest tax increases the price of future consumption (reduces the net return to physical capital) making future consumption dearer; this makes current consumption and human capital more attractive, on the margin. Second, the interest income tax affects the relative return between human and physical capital investment. The interest income tax reduces the opportunity cost of human capital making once more current consumption and human capital attractive.

The Hicks Compensated Effects of a Capital Income Tax

The interest tax also affects the household's income. To obtain the compensated expected-utility results we require that the households first period endowment be adjusted by the following amount:

$$\frac{dY_{1t}}{dt_r}\Big|_{\overline{V}} = \frac{r\,S_{1t}}{(1+r(1-t_r))(1-t_w)}$$

The Hicksian amount required for compensation depends on whether the investor is a lender or a borrower. If the household is a lender then the Hicksian compensation will be positive, however, if the investor is a borrower then the interest tax acts as a subsidy and hence a negative Hicksian compensation is required.

The substitution effect of an interest tax is given by:

$$\frac{\partial H}{\partial t_r}\Big|_{\overline{V}} > 0$$

The substitution effect is positive reflecting the tendency to increase human capital investment. As explained earlier this occurs because the opportunity cost of human capital falls and because future consumption becomes dearer. Note also that an interest income tax does not reduce the randomness of future consumption and therefore we do not find a Domar-Musgrave type of phenomenon. The substitution effect does not depend on the relative and absolute risk aversion parameters as it happened with the wealth or consumption tax. This leads us to proposition 9.11.

Proposition 9.11: The expected utility compensated increase in interest tax leads to an increase in human capital and the effect is independent of the risk aversion behaviour.

The substitution effect of the interest tax on current consumption is given by:

$$\frac{\partial C_{1t}}{\partial t_r}\Big|_{\bar{V}} > 0$$

The substitution effect depends on the normality or inferiority of savings. The assumption of normality of savings, and of physical capital is guaranteed under the assumption of decreasing absolute and non-decreasing relative risk aversion for a lender and for a borrower constrained not to borrow more than the present value of his (her) future non-stochastic, non-human, income, (See appendix 8A1.), lead to a positive substitution effect on current consumption from an increase in the interest tax. Therefore we arrive at the following conclusions;

For a lender:

$$\frac{\partial C_{1t}}{\partial t_r}\Big|_{V=C} > 0 \qquad as \qquad \frac{\partial S_{1t}}{\partial Y_{1t}} > 0$$

For a borrower:

$$\frac{\partial C_{1t}}{\partial t_r}\Big|_{V=C} > 0 \qquad as \qquad \frac{\partial B_{1t}}{\partial Y_{1t}} < 0$$

Proposition 9.12: Under the hypothesis of decreasing absolute and non-decreasing relative risk aversion, both a lender and a borrower (Constraint not to exceed the future non-stochastic income stream) each increase current consumption under an interest income tax holding expected utility constant.

Although the expected utility compensated effects of an interest income tax are stimulating for current consumption and human capital the outcome of such an exercise on savings is ambiguous.

In summary, a utility compensated increase in an interest tax will unambiguously stimulate investment in the risky asset and current consumption. The effect on savings is ambiguous for a lender, while savings decline for a borrower (i.e. borrowing increases).

The Total Effects of a Capital Interest Income Tax

The total effect on the decision variables of an increase in the interest income tax is given by (see appendix 8A4.4 for derivations):

$$\frac{\partial X_{1t}}{\partial t_r} = \frac{\partial X_{1t}}{\partial t_r}\Big|_{\bar{V}} - \frac{r\ S_{1t}}{(\ 1 + r\ (\ 1\ -\ t_r\ ))(1-t_w)}\ \frac{\partial X_{1t}}{\partial Y_{1t}}$$

$$where \qquad X_{1t} = H_{1t}\ ,\ C_{1t}\ ,\ S_{1t}$$

The income effect depends on whether the individual is a borrower or a lender. If the individual is a lender then the income effect discourages the choice variables. If the investor is a borrower then the income effect encourages the choice variables.

The effect on human capital depends on whether or not the substitution effect reinforces the income effect. If the individual is a lender then the income effect discourages human capital. This occurs because the after tax return from non human capital investment decreases, and under the hypothesis of decreasing absolute risk aversion, the agent will undertake less of the risky activity. If the investor is a borrower then the income effect operates in the same direction as the substitution effect. If the individual is a borrower then an increase in the interest income tax reduces the cost of borrowing funds and the income effect acts to encourage investment in human capital.

Proposition 9.13 : For a borrower, an interest income tax unambiguously increases human capital investment. For a lender, the effect is indeterminate due to the conflicting income and substitution effects.

The current consumption will unambiguously increase if the individual is a borrower and/or if borrowing is an inferior activity. On the other hand, for a lender the effect is indeterminate, The outcome depends upon the strength of the positive substitution and negative income effect. For a borrower with behavioral characteristics of decreasing absolute and non-decreasing relative risk aversion, and who faces a borrowing constraint, current consumption will unambiguously increase with an interest income tax. For a lender, the effect of the tax on consumption is ambiguous.

The effect on savings of an increase in the interest income tax is:

$$\frac{\partial S}{\partial t_r} = -(1-t_w)\frac{\partial wH}{\partial t_r} - \frac{\partial C_0}{\partial t_r}$$

The specific effects of an interest income tax on savings are unambiguous for a borrower. The interest income tax will discourage savings if the household is a borrower and if human capital, physical capital and current consumption are normal commodities. The interest income tax discourages savings for borrowers displaying characteristics of decreasing absolute and non decreasing relative risk aversion and are liquidity constrained.

Finally when comparing, the interest income tax with the wealth or consumption tax, we observe that the substitution effect of an interest income tax increase is positive, and that of a consumption or wealth tax are also positive. The reason for the income compensated interest tax to stimulate human capital arises from the reduction in the after tax rate of return to non human capital relative to human capital and from the intertemporal price distortion of current and future consumption. This makes human capital more attractive on the margin. An income compensated wealth or consumption tax does not affect the relative returns of human and physical capital as does an income compensated interest tax. An income compensated consumption or wealth tax stimulates human capital under nondecreasing relative risk aversion because it falls on random future earnings, and as a result, it reduces the riskiness of this form of investment relative to non human capital. If the aim of the authorities is to stimulate human capital, without distorting the relative prices and by providing some insurance then the desired result is achieved via a wage, wealth or consumption tax. The interest income tax can also stimulate human capital but at the expense of price distortions.

9.4.4. The Income Tax

An income tax falls on both wages and interest income. The income tax base explored in this analysis taxes interest income and wages at the same rate. This tax alters the agent's constraint as follows:

$$C_{1t} + \frac{C_{2t}}{(1+r(1-t_y))} = (1 - t_y)\ [\ Y_{1t} - wH_{1t} + \frac{Y_{2t} + \mu_{t+1} F(H_{1t})}{(1+r(1-t_y))}\ ]$$

First we observe that the income tax increases the relative price of future consumption. Secondly, the income tax reduces the present discounted value of human and non-human wealth. Finally, we observe that the tax also reduces the variance of future earnings.

The Hicks Compensated Effects of an Income Tax

The expected utility compensated income tax increase will affect human capital as follows:

$$\frac{\partial H_{1t}}{\partial t_y}\Big|_{V=C} = \frac{\partial H_{1t}}{\partial t_w}\Big|_{V=C} + \frac{\partial H_{1t}}{\partial t_r}\Big|_{V=C}$$

A utility compensated increase in the income tax rate will stimulate human capital under the assumption of non-decreasing relative risk aversion and non-increasing absolute risk aversion.

Comparing the two tax systems under equal expected utility, consumption tax and income tax, we observe that an income tax provides additional stimulus to human capital than a consumption tax. This happens because the income tax effect can be decomposed into a positive substitution wealth tax effect and a positive utility compensated interest tax effect. In fact, Hamilton (1987) argued that the socially optimum level of human capital is greater than the private undertaking given an optimal wage tax. He therefore, claimed that an additional stimulus on human capital can be achieved via an interest income tax in addition to an optimal consumption tax.

The utility compensated effects of an income tax on current consumption is positive. This effect is also composed of two individual wealth and interest tax effects. The wealth tax effect stimulates human capital and the interest tax effect reinforces the wealth tax effect. Therefore, as with human capital, an income tax leads to a greater impact on current consumption than a consumption tax.

As for the utility compensated effects on savings the results is ambiguous, in general. The outcome depends on the positive impact of a wealth tax on savings and the ambiguous impact of the interest tax on savings.

The Total Effect of an Income Tax

The total effect of an increase in the income tax on human capital can be decomposed into the wealth and interest tax effects.

$$\frac{\partial H_{1t}}{\partial t_y} = \frac{\partial H_{1t}}{\partial t_w}\Big|_{t_w=t_y} + \frac{\partial H_{1t}}{\partial t_r}\Big|_{t_r=t_y}$$

Where the partial derivatives and choice variables are evaluated at $t_y=t_w=t_r$. We therefore, do not require any additional calculus, we will simply combine the earlier analysis in order to evaluate the comparative static properties. In general, the income tax can stimulate human capital under the following conditions.

Proposition 9.15: For a borrower displaying constant relative risk aversion and non decreasing absolute risk aversion, an increase in the income tax rate will unambiguously stimulate human capital.

Proposition 9.16: Under constant absolute and increasing relative risk aversion an income tax will stimulate human capital. This result is not dependent on the individual being a borrower or a lender.

Comparing the consumption with the income tax under the assumption of CRRA, we can state that a consumption tax is neutral to its effects on human capital decisions. Furthermore, the consequences of a consumption tax are independent of the agent being a borrower or a lender. On the other hand, an income tax will unambiguously stimulate and discriminate in favour of human capital investment for a borrower. Under CRRA, an income tax has identical effects on human capital as an interest income tax.

The total effect of an increase in the income tax on current consumption can be decomposed also into the wealth and interest tax effects.

$$\frac{\partial C_{1t}}{\partial t_y} = \frac{\partial C_{1t}}{\partial t_w}\Big|_{t_w=t_y} + \frac{\partial C_{1t}}{\partial t_r}\Big|_{t_r=t_y}$$

where the partial and optimal values are evaluated at $t_y = t_w = t_r$.

9.5. Concluding Remarks

Analyzing alternative tax structures we find that a consumption tax creates a positive substitution effect on human capital, current consumption and savings and a reduction in expected future consumption under the assumptions of decreasing absolute and non-decreasing relative risk aversion. We show that under constant expected utility social risk taking rises. On the other hand, private risk taking falls. Given the evidence that private risks exceed social risks with human capital investment, the utility compensated wage tax represents an improvement in resource allocation. Combining the theoretical evidence, the increase in social risk taking, with the pioneering public finance work of Eaton and Rosen (1980a,b) on optimal taxation, a proportional tax on risky human capital income is optimal even if lump-sum taxation is feasible.[11] On the other hand, an interest income tax also causes a positive substitution effect on human capital. The positive substitution effect of an interest income tax does not depend upon the behavioral characteristics of the investor. Hamilton (1987) has shown that a positive interest tax is optimal, even with an optimal wage tax. In part this is due to the positive interest tax encouraging more investment in human capital. This happens because the state chooses a higher level of human capital investment than individuals choose with a consumption or pre-payment wage tax already imposed.

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[11] This optimality arises with the introduction of labour-leisure choice. This creates a trade-off between the deadweight losses of a utility compensated wage tax and the benefits from the provision of insurance via a lower variance of earnings.

Considering the specific effects of a consumption or wealth tax we further show that the effects are ambiguous in general for human capital. In the case of constant relative risk aversion, a wealth tax or "a pre-payment" version of the "cash flow" consumption tax discourages both current consumption and savings but leaves unchanged the level of risk taking. On the other side, a "cash flow" consumption tax leaves unchanged both savings and human capital under CRRA.

The interest tax encourages and discriminates in favour of risky human capital investments for borrowers. The effect of an interest tax on human capital is in general ambiguous for lenders.

Comparing the consumption tax with the income tax under the assumption of CRRA we can state that a consumption tax is neutral as to its effects on human capital decisions. Likewise, the consequences of a consumption tax are independent of the agent being a borrower or a lender. On the other hand, an income tax will unambiguously stimulate and discriminate in favour of human capital investment for a borrower. Under CRRA, an income tax has identical effects on human capital as an interest income tax.

There are a number of interesting avenues for future research in the area of human capital and taxation policy under uncertainty that this essay has not touched upon. The following is a small list.

Future research should be devoted to tax reforms and welfare. In particular, the conduct of a re-distributional policy of the type conducted previously will provide more insights into public policy. How does the state dispose the risky tax revenue from human capital?

The tax effects on the decision variables were conducted under the assumption of a perfect capital market whereby borrowing and lending rates are equal. However, the above analysis should be extended under imperfect capital markets as well. Research in this front has been conducted by Kodde and Ritzen (1985) but they assume certainty, and did not analyze the tax policy implications.

In addition the above analysis can be extended to an overlapping generational model (Nerlove et al (1984)). Examining an overlapping generational model of human capital whereby, the parent lives for one period and cares for his (her) children's future consumption will provide additional insights into tax base choice.

Chapter 10

Conclusion and Future Research

The traditional literature on the effects of taxation policy on portfolio decisions assumed that the investor does not bear the risk that is passed on to the government. This traditional approach has led to the prediction that capital income taxation with loss offset provisions may stimulate risky asset demand and investment. This result, was derived independently of the diversification possibilities open to the investors. In other words, the stimulating effects were independent of the type of risk an investor is exposed to (i.e., systematic or unsystematic type). However, if we assume that the investors hold diversified portfolios then they eliminate all the unsystematic risk, but they are still subject to the social or systematic risk. Indeed, the investors are compensated for the systematic risk they bear with a higher expected return. Is risk sharing through taxation possible under the case whereby the investor perfectly diversifies and as a result faces only market risks? Recently, some economists argued that social risks cannot be reduced further through the taxation of risky assets. In contrast, this is possible as the work of Eaton and Rosen (1980) dealing with risky human capital had earlier implied, where the risks may be viewed as idiosyncratic. In the latter framework, the taxation of the returns from these assets yields a tax revenue that is less risky than the private risks individuals are exposed to.

What about the case of small investors, who do not hold diversified portfolios due to the existence of capital market imperfections? Can the government, acting as a financial intermediary, increase welfare through taxation policy? This dissertation examines and synthesizes the effects of taxation policy on portfolio decisions under different types of risks. The main conclusion reached is that the outcome of a particular policy depends on how the state dissipates, if at all the risk of stochastic tax revenue.

I have initially examined the traditional approach whereby the proceeds of the tax on capital income are spent on a public good, which enters the utility function in an additive way. This implies that the stochastic tax revenue has no additional effect on the decision variables and in particular on risky asset holdings. This modelling of investor's behaviour is used as a benchmark for comparison. However, the decision model was extended from a fixed investable wealth to an intertemporal two period framework with endogenous savings (See Ahsan (1989),(1990)). I obtain different results from the one period asset choice model. However, in most cases, and under plausible hypothesis about the attitudes towards risk, I find that capital income taxation encourages risky asset demand, savings and proportional risk taking.

I next examine the various capital income tax policies in the portfolio/savings model under the assumption that the per capita tax revenue, "stochastic" where relevant, is returned to the investor in the same period in which the tax is collected. This implies that the investor ultimately bears all the risks in the fluctuations of the tax revenue. This alters the conclusions of the traditional analysis. I find that a full capital income tax discourages investment in the risky asset, the safe asset and savings, although proportional risk taking increases.

Furthermore, I show that if the stochastic tax revenue is returned to the household in the same period it was collected, capital income taxation causes a deadweight loss. Concentrating on the capital income tax, the re-distribution of tax revenue back to the investor does not eliminate the distortion in relative prices. Therefore, as under certainty, capital income taxation causes only a deadweight loss and does not provide insurance. However, only in one case I find, namely the taxation of capital gains or excess returns, does the deadweight loss of capital income taxation disappear. The deadweight loss of the imputed safe asset turns out to be identical to the deadweight loss of a full capital income tax policy for equal tax rates. Ergo, the inclusion of excess returns in the tax base of the imputed safe income does not cause any additional deadweight loss to the investors and can generate more tax revenue for the government. However, does this additional tax revenue have any market value? How do we measure the market value of risky tax revenue?

Another contribution of my thesis is to propose a new methodology, which is different from the existing methodology of Stiglitz (1972) in the evaluation of "stochastic" tax revenue. I assumed that the governmental tax revenue can be evaluated using the traditional capital asset pricing model from the theory of finance.

This way the stochastic tax revenue is priced like any other asset. One can use the market value of the tax revenue to provide a public good. This way the traditional approach and the alternative approaches can be compared more directly.

I next calculate the deadweight loss of capital income taxation using the non-expected utility preferences. The important result of this study is that under uncertainty the relevant parameter for the existence of deadweight losses is a non zero elasticity of intertemporal substitution as in the certainty models. This arises because the non-expected utility preference structure separates the elasticity of substitution from the relative risk aversion parameter. Numerical results indicate that the deadweight loss of capital income taxation is of a relatively small magnitude. I find that investors who put more funds into the risky asset face a smaller deadweight loss from capital income taxation than investors that hold most of their funds in the safe asset. This is an important result and depends on the relative risk aversion parameter. As a final remark, just like certainty models, investors that have a high intertemporal rate of substitution will be hurt the most by capital income taxation. Finally, I find that the deadweight losses of capital income taxation increase as the risk premium falls. Future research should examine the deadweight losses of other capital income taxation policies. Among the most important one is the taxation of the risky asset versus the safe asset. Eventually one can discuss optimal taxation of risky and safe asset when all risks are returned to the investor.

This dissertation also undertook the difficult task of evaluating alternative policies of disposing "stochastic" tax revenue. In particular, instead of returning, to the investor, his own personal "stochastic" tax revenue, the state may return a stochastic weighted average tax revenue. The stochastic per capita weighted average tax revenue returned to the investor is partly his own risky tax revenue and partly the risky per capita tax revenue of previous generations. This implies that the current generational investors do not bear the entire risk of tax revenue but in turn, part of the risks of the previous generations. This argument relies on the idea that what is "systematic" risk at a certain point in time will become "idiosyncratic" risk when this is shared with all current and future unborn generations. I claim that the riskiness of the tax revenue fluctuations can be diversified across generations by an appropriate re-distributional policy or debt management policy. I use the capital asset pricing model to evaluate the "new" stochastic tax revenue and find that the market value of the tax revenue increases. Hence, by allowing some risks to be transferred to the state budget, the investor's welfare improves for constant marginal tax rates. However, the capital income tax creates distortions and one ought to balance the distortions of the tax with the social risk reduction. This in turn, brings alive and strengthens the old argument of Domar-Musgrave (1944), Tobin (1968) and Stigiltz (1969) for a risk sharing by state with private investors.

Recently, the income taxation system has been under attack by several economists. Most proposals argue for switching the personal tax base to an expenditure tax system and away from income taxation. The expenditure tax is favoured over the income tax on the ground that the latter increases the price of future consumption and encourages current as against future consumption. This distortion in intertemporal prices is alleged to create a welfare loss for consumers. Several economists (Hall and Rabuska (1985), Bradford (1986), Mclure et al. (1988), and Zodrow (1994)) have gone a step further and argued for a tax on wage income on grounds of administrative feasibility and simplicity. However, Pechman (1990) being a strong supporter of the personal income tax system states about such a substitution "Most people would be appalled by a proposal to substitute a wage tax for income tax, yet that is essentially what expenditure tax proponents are advocating." (pg. 9).

My dissertation also contributes to the debate on the appropriate choice of tax base. I argued that a move toward a wage tax system will lead to welfare losses. A pre-payment wage tax is not equivalent to a consumption tax either because of (a) the market value or certainty equivalent of the tax revenue obtained from the excess return of the risky asset is non-zero if people hold less than fully diversified portfolios and/or (b) if the households do not evaluate the two period random tax revenue stream they pay to the state but instead they evaluate and ultimately bear the average random tax revenue across all generations, alive or yet to be born. I also find that a perfectly diversified investor is indifferent as to whether the consumption tax is imposed on a cash flow or pre-payment basis.

With intergenerational risk shifting, the market value of the "stochastic" consumption tax revenue rises. The increased consumption tax revenue causes the tax rate to fall relative to the tax rate relevant for a matching pre-payment wage tax. This lower tax rate and broader tax base yields more utility to the investor. Conducting a differential incidence analysis yields large welfare gains from a cash flow consumption tax vis-a-vis the pre-payment wage tax. In order to evaluate the gains I used the non-expected utility preference structure. I also expect that such individuals will prefer the income tax system to the pre-payment wage tax system. Future research should examine the desirability of an income tax to a pre-payment wage tax. Within my framework, this extension will throw more light onto the tax base debate.

I finally analyze the effects of alternative tax structures on risky human capital investment. Here the return on investment depends on the amount invested. Hence the results of the previous analysis do not apply. The major finding is that a consumption tax creates a positive substitution effect on human capital, current consumption and savings and a reduction in expected future consumption. I show that under constant expected utility social risk taking rises. On the other hand, private risks fall. Given the evidence that private risks exceeds social risks with human capital this utility compensated wage tax represents an improvement in resources. Comparing the consumption tax with the income tax we can state that a consumption tax may discourage human capital investment due to the conflicting income and substitution effects. On the other hand, an income tax will unambiguously stimulate human capital investment for a borrower. Future research should be devoted to examining the disposition of tax revenue risks from human capital. The usual approach was to view the tax revenue risks across individuals within the same generation as idiosyncratic. This is not true over the business cycle and even human capital risks can be of the systematic type. What is then the market value of the wage tax revenue? I find the use of the market value concept on tax revenue can yield results that are very different from the traditional approach of evaluating risky flows. Future research awaits to see answers to these important policy questions.

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APPENDIX I

A.I.1 The Wealth Effects

The effects of a change in exogenous income on the decision variables of a steady state generation in period t can be obtained by differentiating the optimum conditions with respect to Y_{1t} yielding:

$$[A]\begin{bmatrix} \dfrac{\partial C_{1t}}{\partial Y_{1t}} \\ \dfrac{\partial a_{1t}}{\partial Y_{1t}} \end{bmatrix} = \begin{bmatrix} (1+r)^2 E(h'') \\ -(1+r)E(h'' z_{t+1}) \end{bmatrix}$$

where the determinant of A > 0 and the elements of A are:

$$\begin{aligned} A_{11} &= g'' + (1+r)^2 E(h'') \\ A_{12} &= -(1+r)E(h'' z_{t+1}) \\ A_{22} &= E(h'' z_{t+1}{}^2) \end{aligned}$$

The effect of increasing wealth on risky asset demand is:

$$\frac{\partial a_{1t}}{\partial Y_{1t}} = - \frac{(1+r)\, g'' \, E(h'' z_{t+1})}{A}$$

Decreasing absolute risk aversion (DARA) guarantees that $E(h''z_{t+1})$ is positive.[1] This is a well known result. The risky asset is normal under DARA.

The effect of a change in wealth on savings is:

$$\frac{\partial S_{1t}}{\partial Y_{1t}} = \frac{g'' \, E(h'' z_{t+1}{}^2)}{A}$$

The effect on current consumption is given by:

$$\frac{\partial C_{1t}}{\partial Y_{1t}} = \frac{(1+r)^2 \, [\, (E(h'')E(h'' z_{t+1}{}^2) - E(h'' z_{t+1})^2 \,]}{A}$$

Current consumption will be a normal good if the numerator is positive. Under decreasing absolute and non-decreasing relative risk aversion the numerator will be positive. In order to see this re-write the expression as follows:

$$a[E(h'')E(h'' z_{t+1}^2) - E(h'' z_{t+1})^2] = E(h'')E(h'' C_{2t} z_{t+1}) - E(h'' z)E(h'' C_{2t})$$

Clearly, a sufficient condition for this term to be positive is that the term $E(h''z_{t+1}C_{2t})$ be non-negative. Therefore, the effect of an exogenous increase in first period wealth will increase current consumption by less under the assumption of DARA and NDRRA.

[1] It has been proven that $E(h''z)$ is greater than, equal, or less than zero as absolute risk aversion decreases, remains constant, or increases with future consumption respectively. Also $E(h''zC_1)$ is greater than, equal, or less than zero as relative risk aversion decreases, remains constant, or increases with increases in second period consumption.

The effect of a change in wealth on the safe asset is:

$$\frac{\partial m_{1t}}{\partial Y_{1t}} = \frac{g''[E(h'' z_{t+1}{}^2) + (1+r)E(h'' z_{t+1})}{A}$$

The effect of an increase in wealth on the safe asset is ambiguous. However, under the assumption of DARA and NDRRA the safe asset is a normal good for a lender. This can be seen as follows:

$$sign\ \frac{\partial m_{1t}}{\partial Y_{1t}} = -\ sign\ [E(h'' z_{t+1}{}^2) + (1+r)E(h'' z_{t+1})]$$

The right hand side expression can be written as follows:

$$S\ [E(h'' z_{t+1}{}^2) + (1+r)E(h'' z_{t+1})] = m\ E(h'' z_{t+1}{}^2) + E(h'' C_{2t} z_{t+1})$$

Assuming NDRRA the second term is less than or equal to zero. For a lender the first term is also negative. Thus the term:

$$[E(h'' z_{t+1}{}^2) + (1+r)E(h'' z_{t+1})] < 0$$

In conclusion, the safe asset is normal for a lender displaying DARA and NDRRA.

Elasticity Results.

The behavioral assumptions of non-decreasing relative risk aversion restricts the wealth elasticities. To see the exact restrictions consider the NDRRA hypothesis:

$$E(h'' z_{t+1} C_{2t}) \leq 0$$

Substituting the budget constraint of the investor yields the following result:

$$\eta^{m} > \eta^{s} \geq \eta^{a}$$

Under nondecreasing relative risk aversion and decreasing absolute risk aversion the wealth elasticity of the safe asset is greater than the wealth elasticity of savings, η^{s}, which in turn is greater than the wealth elasticity of the risky asset, η^{a}. For the CRRA case we obtain the following result:

$$\eta^{c} = \eta^{a} = \eta^{s} = 1$$

where η^{c} is the income elasticity of consumption in the first period.

A.I.2 The Full Capital Income Tax, t_k

A.I.2.1 The Hicks compensated effects:

To obtain the compensated expected-utility comparative static results we require that the investors first period income be adjusted such that:

$$dY_{1t} = \frac{rS_{1t}}{(1+\bar{r})}\, dt_k$$

Utilizing the above constant expected utility requirement and differentiating the first order conditions yields the following system of equations:

$$[\,A_k\,]\begin{bmatrix} \dfrac{\partial C_{1t}}{\partial t_k}\Big|_{\bar{V}} \\ \dfrac{\partial a_{1t}}{\partial t_k}\Big|_{\bar{V}} \end{bmatrix} = \begin{bmatrix} D_{1k} \\ D_{2k} \end{bmatrix}$$

where D_{1k} and D_{2k} are gi ·n by:

$$D_{1k} = rE(h') - \frac{a_{1t}}{(1-t_k)}(1+\bar{r})E(h''\overline{z_{t+1}})$$

$$D_{2k} = \frac{a_{1t}}{(1-t_k)} E(h''\overline{z_{t+1}}^{2})$$

The utility compensated effect of a full capital income tax on risky asset holdings is:

$$\frac{\partial a_{1t}}{\partial t_k}\Big|_{\bar{V}} = \frac{a_{1t}}{(1-t_k)} \left[1 - \frac{1}{\theta} \frac{\bar{r}}{(1+\bar{r})} \frac{C_{1t}}{Y_{1t}} \eta^a \right]$$

The effect of a capital income tax can be decomposed into two effects. One component is the DM phenomenon and the other component is due to the change in the relative price of current and future consumption.

The utility compensated effect of a full capital income tax on current consumption:

$$\frac{\partial C_{1t}}{\partial t_k}\Big|_{\bar{V}} = \frac{rS_{1t}}{(1+\bar{r})} \frac{1}{\theta} \frac{C_{1t}}{Y_{1t}} \eta^s$$

Current consumption is stimulated.

The expected utility compensated effect of the capital income tax on savings:

$$\frac{\partial S_{1t}}{\partial t_k}\Big|_{\bar{V}} = \frac{rS_{1t}}{(1+\bar{r})} \left[1 - \frac{1}{\theta} \frac{C_{1t}}{Y_{1t}} \eta^s \right]$$

The effect on total savings is ambiguous as explained in the main text.

The effect on proportional risk taking is given by:

$$\frac{\partial \beta_{1t}}{\partial t_k}\Big|_{\bar{V}} = \frac{\beta_{1t}}{(1-t_k)} \left[\frac{1}{(1+\bar{r})} - \frac{\bar{r}}{(1+\bar{r})} \frac{1}{\theta} [\eta^a - \eta^s] \right]$$

Proportional risk taking increases.

A.I.2.2 The Specific Effects.

The total effect of a capital income tax on the decision rules is given by:

$$\frac{\partial X_{1t}}{\partial t_k} = \frac{\partial X_{1t}}{\partial t_k}\Big|_{\bar{V}} - \frac{rS_{1t}}{(1+\bar{r})} \frac{\partial X_{1t}}{\partial Y_{1t}}$$

$$where \qquad X_{1t} = C_{1t}\,,\ a_{1t}\,,\ S_{1t}\,,\ \beta_{1t}$$

Where the first term is the Hicks compensated term and the second term is the income effect. The specific effects can be expressed as follows:

Risky asset holdings:

$$\frac{\partial a_{1t}}{\partial t_k} = \frac{a_{1t}}{(1-t_k)} \left[\, 1 - \frac{\bar{r}}{(1+\bar{r})} \eta^a \left[\, 1 - (1 - \frac{1}{\theta}) \,\right] \frac{C_{t1}}{Y_{1t}} \,\right]$$

Current consumption:

$$\frac{\partial C_{1t}}{\partial t_k} = \frac{rS_{1t}}{(1+\bar{r})} \frac{C_{1t}}{Y_{1t}} \left[\, \frac{1}{\theta} \eta^s - \eta^a \,\right]$$

For Savings:

$$\frac{\partial S_{1t}}{\partial t_k} = \frac{rS_{1t}}{(1+\bar{r})} \left[\, 1 - \left[\, 1 - (1 - \frac{1}{\theta}) \frac{C_{1t}}{Y_{1t}} \,\right] \eta^s \,\right]$$

For Proportional risk taking:

$$\frac{\partial \beta_{1t}}{\partial t_k} = \frac{\beta_{1t}}{(1-t_k)} \left[\, \frac{1}{(1+\bar{r})} - \frac{\bar{r}}{(1+\bar{r})} \frac{1}{\theta} \left[\, \eta^a - \eta^s \,\right] \,\right]$$

Proportional risk taking increases.

In order to obtain stronger results we will restrict our analysis to the constant relative risk aversion case. See next section of the appendix.

A.I.2.3 The CRRA Hypothesis.

The CRRA results are obtained by setting all the income elasticities equal to unity. Examining the Hicks compensated effects under CRRA yields:

For Risky Asset Holdings

$$\frac{\partial a_{1t}}{\partial t_k}\Big|_{\bar{V}}^{CRRA} = \frac{a_{1t}}{(1-t_k)}\left[\,1 - \frac{1}{\theta}\,\frac{\bar{r}}{(1+\bar{r})}\,\frac{C_{tl}}{Y_{1t}}\,\right]$$

Ambiguous.

For Current Consumption:

$$\frac{\partial C_{1t}}{\partial t_k}\Big|_{\bar{V}}^{CRRA} = \frac{rS_{1t}}{(1+\bar{r})}\,\frac{1}{\theta}\,\frac{C_{1t}}{Y_{1t}}$$

Increases.

For Savings:

$$\frac{\partial S_{1t}}{\partial t_k}\Big|_{\bar{V}}^{CRRA} = \frac{rS_{1t}}{(1+\bar{r})}\left[\,1 - \frac{1}{\theta}\,\frac{C_{1t}}{Y_{1t}}\,\right]$$

Ambiguous.

For proportional risk taking:

$$\frac{\partial \beta_{1t}}{\partial t_k}\Big|_{\bar{V}}^{CRRA} = \frac{\beta_{1t}}{(1-t_k)}\,\frac{1}{(1+\bar{r})}$$

Increases.

A.I.2.4 The Specific Effects under CRRA are:

Risky asset holdings:

$$\frac{\partial a_{1t}}{\partial t_k}\Big|^{CRRA} = \frac{a_{1t}}{(1-t_k)} \left[1 - \frac{\bar{r}}{(1+\bar{r})} \frac{1}{\theta} \frac{C_{tl}}{Y_{1t}} \right]$$

Increases for all $\theta > 1$.

Current consumption:

$$\frac{\partial C_{1t}}{\partial t_k}\Big|^{CRRA} = \frac{rS_{1t}}{(1+\bar{r})} \frac{C_{1t}}{Y_{1t}} \left[\frac{1}{\theta} - 1 \right]$$

Decreases for all $\theta > 1$. Income effect outweighs the substitution effect.

For Savings:

$$\frac{\partial S_{1t}}{\partial t_k}\Big|^{CRRA} = \frac{rS_{1t}}{(1+\bar{r})} \left[\left(1 - \frac{1}{\theta} \right) \frac{C_{1t}}{Y_{1t}} \right]$$

Increases for all $\theta > 1$.

For Proportional risk taking:

$$\frac{\partial \beta_{1t}}{\partial t_k}\Big|^{CRRA} = \frac{\beta_{1t}}{(1-t_k)} \frac{1}{(1+\bar{r})}$$

Increases and is independent of the elasticity of substitution.

A.I.3 The Imputed Safe Income Tax: t_r

A.I.3.1. The Hicks compensated effects:

To obtain the compensated expected-utility comparative static results we require that the investors first period income be adjusted such that:

$$dY_{1t} = \frac{rS_{1t}}{(1+\bar{r})} dt_r$$

The compensation is similar to the compensation required under the full capital income taxation. Utilizing the above constant expected utility requirement and differentiating the first order conditions yields the following system of equations:

$$[\,A_r\,] \begin{bmatrix} \dfrac{\partial C_{1t}}{\partial t_r}\Big|_{\bar{V}} \\ \dfrac{\partial a_{1t}}{\partial t_r}\Big|_{\bar{V}} \end{bmatrix} = \begin{bmatrix} D_{1r} \\ D_{2r} \end{bmatrix}$$

where D_{1r} and D_{2r} are given by:

$$D_{1r} = -\, r\, E(h')$$
$$D_{2r} = 0$$

The utility compensated effect of an imputed safe income tax on the risky asset is:

$$\frac{\partial a_{1t}}{\partial t_r}\Big|_{\bar{V}} = - a_{1t} \frac{1}{\theta} \frac{r}{(1+\bar{r})} \frac{C_{1t}}{Y_{1t}} \eta^a$$

The amount invested in the Risky asset is unambiguously discouraged.

The utility compensated effect of the tax on current consumption is:

$$\frac{\partial C_{1t}}{\partial t_r}\Big|_{\bar{V}} = \frac{rS_{1t}}{(1+\bar{r})} \frac{1}{\theta} \frac{C_{1t}}{Y_{1t}} \eta^s$$

Current Consumption is encouraged.

The utility compensated effect of the tax on total savings is:

$$\frac{\partial S_{1t}}{\partial t_r}\Big|_{\bar{V}} = \frac{rS_{1t}}{(1+\bar{r})} [1 - \frac{1}{\theta} \frac{C_{1t}}{Y_{1t}} \eta^s]$$

Result is ambiguous.

The utility compensated effects of the tax on proportional risk taking is given by:

$$\frac{\partial \beta_{1t}}{\partial t_r}\Big|_{\bar{V}} = \frac{r\beta_{1t}}{(1+\bar{r})} [\frac{C_{1t}}{Y_{1t}} \frac{1}{\theta} [\eta^s - \eta^a] - 1]$$

This result is also ambiguous.

A.I.3.2 The CRRA Hypothesis:

Setting the wealth elasticities equal to unity results in the following effects:

Risk Taking:

$$\frac{\partial a_{1t}}{\partial t_r}\Big|_{\bar{V}}^{CRRA} = - a_{1t} \frac{1}{\theta} \frac{r}{(1+\bar{r})} \frac{C_{1t}}{Y_{1t}}$$

Risk Taking is unambiguously discouraged.

Current Consumption:

$$\frac{\partial C_{1t}}{\partial t_r}\Big|_{\bar{V}}^{CRRA} = \frac{rS_{1t}}{(1+\bar{r})} \frac{1}{\theta} \frac{C_{1t}}{Y_{1t}}$$

Current Consumption is encouraged.

The utility compensated effect on savings is given by:

$$\frac{\partial S_{1t}}{\partial t_r}\Big|_{\bar{V}}^{CRRA} = \frac{rS_{1t}}{(1+\bar{r})} [1 - \frac{1}{\theta} \frac{C_{1t}}{Y_{1t}}]$$

Savings is encouraged for all relative risk aversion parameters greater than unity.

Proportional risk taking is:

$$\frac{\partial \beta_{1t}}{\partial t_r}\Big|_{\bar{V}}^{CRRA} = - \frac{\bar{r}}{(1+\bar{r})} \frac{\beta_{1t}}{(1-t_r)}$$

Proportional risk taking is discouraged.

A.I.3.3 The Specific Effects.

The total effect of this policy on the optimal choice variables is:

$$\frac{\partial X_{1t}}{\partial t_r} = \frac{\partial X_{1t}}{\partial t_r}\Big|_{V=C} - \frac{rS_{1t}}{(1+\bar{r})}\frac{\partial X_{1t}}{\partial Y_{1t}}$$

$$where \qquad X_{1t} = C_{1t}\ ,\ a_{1t}\ ,\ S_{1t}\ ,\ \beta_{1t}$$

The first term is the substitution effect and the second term the income effect.

On risk Taking:

$$\frac{\partial a_{1t}}{\partial t_r} = \frac{\partial a_{1t}}{\partial t_r}\Big|_{V=C} - \frac{rS_{1t}}{(1+\bar{r})}\frac{\partial a_{1t}}{\partial Y_{1t}} < 0$$

Risky asset holdings fall. The income effect operates in the same direction as the substitution effect.

Current Consumption:

$$\frac{\partial C_{1t}}{\partial t_r} = \frac{rS_{1t}}{(1+\bar{r})}\frac{C_{1t}}{Y_{1t}}\left[\frac{1}{\theta}\eta^s - \eta^c\right]$$

Result is ambiguous. The substitution effect operates in the opposite direction to the income effect.

For Savings:

$$\frac{\partial S_{1t}}{\partial t_r} = \frac{rS_{1t}}{(1+\bar{r})} \left[1 - \left[1 - \left(1 - \frac{1}{\theta} \right) \frac{C_{1t}}{Y_{1t}} \right] \eta^s \right]$$

Result is ambiguous.

For Proportional risk taking:

$$\frac{\partial \beta_{1t}}{\partial t_r} = \frac{\beta_{1t}}{(1-t_r)} \frac{\bar{r}}{(1+\bar{r})} \left[\frac{C_{1t}}{Y_{1t}} \frac{1}{\theta} [\eta^s - \eta^a] - 1 \right]$$

The result of the tax policy on proportional risk taking is also ambiguous. The CRRA results are easily obtained by setting all the income elasticities equal to unity.

A.I.4 The Risk Premium Tax, t_z:

A.I.4.1 The Hicks compensated effects.

The adjustment required to maintain the investor on the same level of utility is:

$$dY_{1t} = 0 \; dt_z$$

The change in the first period income, of the investor, required to attain the same level of satisfaction as prior to the imposition of the tax is zero. The household does not require any compensation. He adjusts his portfolio composition in such a way (i.e. reduces the safe and increases the risky asset holdings) as to face the same probability distribution as prior to the imposition of the tax. Utilizing the above constant expected utility requirement and differentiating the first order conditions yields the following system of equations:

$$\left[A_z \right] \begin{bmatrix} \dfrac{\partial C_{1t}}{\partial t_z}\Big|_{\bar{V}} \\ \dfrac{\partial a_{1t}}{\partial t_z}\Big|_{\bar{V}} \end{bmatrix} = \begin{bmatrix} D_{1z} \\ D_{2z} \end{bmatrix}$$

where D_{1z} and D_{2z} are given by:

$$D_{1z} = - \frac{a_{1t}}{(1-t_z)} (1+r) E(h'' \overline{z_{t+1}})$$

$$D_{2z} = \frac{a_{1t}}{(1-t_z)} E(h'' \overline{z_{t+1}}^2)$$

The utility compensated effect of a risk premium tax on the risky asset is:

$$\frac{\partial a_{1t}}{\partial t_z}\Big|_{\bar{V}} = \frac{a_{1t}}{(1-t_z)}$$

Risk taking unambiguously increases. The DM phenomenon.

The utility compensated effect of the tax on current consumption is:

$$\frac{\partial C_{1t}}{\partial t_z}\Big|_{\bar{V}} = 0$$

Current consumption is unaffected by this tax.

The effect on the other decision variables is given in the main text.

A.I.5 A Tax on The Income of the Risky Asset, t_x

A.I.5.1 The Hicks compensated effects:

To obtain the compensated expected-utility comparative static results we require that the investors first period income be adjusted such that:

$$dY_{1t} = \frac{r\, a_{1t}}{(1+r)(1-t_x)}\, dt_x$$

Utilizing the above constant expected utility requirement and differentiating the first order conditions yields the following system of equations:

$$[\, A_x \,] \begin{bmatrix} \frac{\partial C_{1t}}{\partial t_x}\big|_{\bar{V}} \\ \frac{\partial a_{1t}}{\partial t_x}\big|_{\bar{V}} \end{bmatrix} = \begin{bmatrix} D_{1x} \\ D_{2x} \end{bmatrix}$$

where D_{1x} and D_{2x} are given by:

$$D_{1x} = -\frac{a_{1t}}{(1-t_x)}(1+r)E(h''\overline{z_{t+1}})$$

$$D_{2x} = \frac{a_{1t}}{(1-t_x)}E(h''\overline{z_{t+1}}^{2}) + \frac{r\,E(h')}{(1-t_x)}$$

The utility compensated effect of a capital income tax on risky asset:

$$\frac{\partial a_{1t}}{\partial t_x}\Big|_{\bar{V}} = \frac{a_{1t}}{(1-t_x)} + \frac{r\,E(h')}{(1-t_x)}\frac{A_{x11}}{A_x}$$

The first term is the DM phenomenon and the other term is due to the change in the relative rate of returns of the two assets. The first term is positive while the second term is negative.

The utility compensated effects of the tax on current consumption:

$$\frac{\partial C_{1t}}{\partial t_x}\Big|_{\bar{V}} = \frac{r a_{1t}}{(1+r)(1-t_x)}\,\frac{1}{\theta}\,\frac{C_{1t}}{Y_{1t}}\,\eta^a$$

Current consumption is stimulated. Notice that in this case the effect of the tax on current consumption depends on the wealth elasticity of the risky asset as opposed to the wealth elasticity of total savings which was the case of a full capital income tax.

The expected utility compensated effect of the tax on savings:

$$\frac{\partial S_{1t}}{\partial t_x}\Big|_{\bar{V}} = \frac{r a_{1t}}{(1+r)(1-t_x)}\left[\,1 - \frac{1}{\theta}\,\frac{C_{1t}}{Y_{1t}}\,\eta^a\,\right]$$

The effect on total savings is ambiguous as explained in the main text. The first term represents the Hicksian compensation term while the second term is the negative of the tax effect on current consumption. Notice that under CRRA and an relative risk aversion parameter of greater than unity savings are encouraged.

The effect on proportional risk taking is given by:

$$\frac{\partial \beta_{1t}}{\partial t_x}\Big|_{\bar{V}} = \frac{\beta_{1t}}{(1-t_x)}\left[\,1 - \beta_{1t}\frac{r}{(1+r)}\left[\,1 - \frac{1}{\theta}\,\frac{C_{1t}}{Y_{1t}}\,\eta^a\,\right]\right] + \frac{1}{S_{1t}}\,\frac{rE(h')}{(1-t_x)}\,\frac{A_{x11}}{A_x}$$

Result is ambiguous.

A.I.5.2 The Specific Effects.

The total effect of a capital income tax on the decision rules is given by:

$$\frac{\partial X_{1t}}{\partial t_x} = \frac{\partial X_{1t}}{\partial t_x}\Big|_{\bar{V}} - \frac{r\, a_{1t}}{(1+r)(1-t_x)} \frac{\partial X_{1t}}{\partial Y_{1t}}$$

$$where \qquad X_{1t} = C_{1t}\, ,\; a_{1t}\, ,\; S_{1t}$$

Where the first term is the Hicks compensated term and the second term is the income effect. The specific effects can be expressed as follows:

Risky asset holdings:

$$\frac{\partial a_{1t}}{\partial t_x} = \frac{a_{1t}}{(1-t_x)} \left[\, 1 - \frac{r}{(1+r)} \frac{a_{1t}}{Y_{1t}} \eta^a \,\right] + \frac{r\, E(h')\, A_{x11}}{(1-t_x)\, A_x}$$

Not determined with the existing information even under CRRA. The effect is independent of the elasticity of substitution.

Current consumption:

$$\frac{\partial C_{1t}}{\partial t_x} = \frac{r\, a_{1t}}{(1+r)(1-t_x)} \frac{C_{1t}}{Y_{1t}} \left[\, \frac{1}{\theta} \eta^a - \eta^c \,\right]$$

The effect on current consumption can be determined only under CRRA. Under CRRA the effect of the tax on current consumption is determined by the elasticity of substitution. If the elasticity of substitution is greater than one, equal to unity, or less than unity, current consumption will unambiguously increase, remain unaffected, or fall, respectively.

For Savings:

$$\frac{\partial S_{1t}}{\partial t_x} = \frac{ra_{1t}}{(1+r)(1-t_x)} \left[1 - \eta^s + \frac{1}{\theta} \frac{C_{1t}}{Y_{1t}} [\eta^s - \eta^a] \right]$$

Under CRRA the effect on savings is determined by the relative risk aversion parameter.

A.I.6 A Tax on the Safe Assets Income, t_m

A.I.6.2 The Hicks compensated effects:

To obtain the compensated expected-utility comparative static results we require that the investors first period income be adjusted such that:

$$dY_{1t} = \frac{r\, m_{1t}}{(1+\bar{r})}\, dt_m$$

Utilizing the above constant expected utility requirement and differentiating the first order conditions yields the following system of equations:

$$[\, A_m \,] \begin{bmatrix} \dfrac{\partial C_{1t}}{\partial t_m}\Big|_{\bar{V}} \\ \dfrac{\partial a_{1t}}{\partial t_m}\Big|_{\bar{V}} \end{bmatrix} = \begin{bmatrix} D_{1m} \\ D_{2m} \end{bmatrix}$$

where D_{1m} and D_{2m} are given by:

$$D_{1m} = -\ r\ E(h')$$
$$D_{2m} = -\ r\ E(h')$$

The utility compensated effect of a tax on the income of the safe asset on risky asset holdings is:

$$\frac{\partial a_{1t}}{\partial t_m}\Big|_{\bar{V}} = - \frac{r\, E(h^{\prime})\, A_{m11}}{A_m} - \frac{1}{\theta}\, \frac{a_{1t} r}{(1+\bar{r})}\, \frac{C_{t1}}{Y_{1t}}\, \eta^a$$

The first term reflects increased relative attractiveness of the risky asset vis-a-vis the safe asset. The second term indicates a reduction in risk taking due to the more expensive future consumption.

The utility compensated effect of the tax on current consumption:

$$\frac{\partial C_{1t}}{\partial t_m}\Big|_{\bar{V}} = r\, \frac{m_{1t}}{(1+\bar{r})}\, \frac{1}{\theta}\, \frac{C_{1t}}{Y_{1t}}\, \eta^m$$

Current consumption is stimulated. Notice that the wealth elasticity of the safe asset determines the magnitude of the stimulus of current consumption expenditures.

The expected utility compensated effect of the tax on savings:

$$\frac{\partial S_{1t}}{\partial t_r}\Big|_{\bar{V}} = \frac{r m_{1t}}{(1+\bar{r})}\, [\, 1 - \frac{1}{\theta}\, \frac{C_{1t}}{Y_{1t}}\, \eta^m\,]$$

The effect on total savings is ambiguous except for the CRRA in which case savings increase for a lender with a relative risk aversion parameter that is greater than unity. As for a borrower, total savings decline under CRRA.

A.I.6.3 The Specific Effects.

The total effect of the tax on the decision rules is given by:

$$\frac{\partial X_{1t}}{\partial t_m} = \frac{\partial X_{1t}}{\partial t_m}\Big|_{\bar{V}} - \frac{r\, m_{1t}}{(1+\bar{r})} \frac{\partial X_{1t}}{\partial Y_{1t}}$$

$$\textit{where} \qquad X_{1t} = C_{1t}\ ,\ a_{1t}\ ,\ S_{1t}$$

Where the first term is the Hicks compensated term and the second term is the income effect. Notice that for a lender the income effects reduce current consumption, risk taking and savings. While for a borrower the income effects provide a stimulus on the decision variables due to the subsidy on the interest payments.

APPENDIX II

A.II.1 A Capital Income Tax, t_k

In order to determine the impact of this tax policy, we assume that the argument in $h'(C_{2t})$ is evaluated at $a_{1t}=a_{1t}^*$ and $S_{1t}=S_{1t}^*$. This lump sum transfer as stated in the main text cancels with the tax payment of the household. Hence, the investors future consumption level is given by:

$$C_{2t} = (1+r)S_{1t}^* + a_{1t}^* z_{t+1}$$

Differentiating the first order conditions totally yields the following system of equations:

$$[\,A_k\,]\begin{bmatrix} \frac{\partial C_{1t}}{\partial t_k}\Big|_{NRS} \\ \frac{\partial a_{1t}}{\partial t_k}\Big|_{NRS} \end{bmatrix} = \begin{bmatrix} D_{1k} \\ D_{2k} \end{bmatrix}$$

where the determinant of $A_k > 0$ and the elements of A are:

$$\begin{aligned} A_{11} &= g'' + (1+\bar{r})(1+r)E(h'') \\ A_{12} &= -(1+\bar{r})E(h''z_{t+1}) \\ A_{21} &= -(1+r)E(h''\overline{z_{t+1}}) \\ A_{22} &= (1-t_k)E(h''z_{t+1}^2) \end{aligned}$$

Notice that the determinant is not symmetric except for $t_k=0$.

And where D_{1k} and D_{2k} are given by:

$$D_{1k} = -\, r\, E(h')$$
$$D_{2k} = 0$$

Solving for the effect of the tax on risky asset holdings gives:

$$\frac{\partial a_{1t}}{\partial t_k}\Big|_{NRS} = -\,\frac{a_{1t}}{(1-t_k)}\frac{1}{\theta}\,\frac{r}{(1+r)}\,\frac{C_{1t}}{Y_{1t}}\,\eta^{a} < 0$$

Risky asset holdings fall since there is no risk sharing and future consumption is more expensive. Notice that risky asset holdings is discouraged less with a lower elasticity of substitution or a higher relative risk aversion parameter.

The effect on current consumption is:

$$\frac{\partial C_{1t}}{\partial t_k}\Big|_{NRS} = \frac{rS_{1t}}{(1+\bar{r})}\,\frac{1}{\theta}\,\frac{C_{1t}}{Y_{1t}}\,\eta^{s} > 0$$

Current consumption increases. The degree of increase depends positively on the elasticity of substitution and inversely with the relative risk aversion parameter.

The effect on savings is:

$$\frac{\partial S_{1t}}{\partial t_k}\Big|_{NRS} = -\,\frac{\partial C_{1t}}{\partial t_k}\Big|_{NRS} < 0$$

Savings fall since households increase current consumption.

The effect on the safe asset is:

$$\frac{\partial m_{1t}}{\partial t_k}\Big|_{NRS} = -\,\frac{m_{1t}}{(1-t_k)}\frac{1}{\theta}\,\frac{r}{(1+r)}\,\frac{C_{1t}}{Y_{1t}}\,\eta^{m} < 0$$

Safe asset holdings fall for a lender. For a borrower, borrowing increases to finance the higher current consumption expenditures.

The effect on proportional risk taking is:

$$\frac{\partial \beta_{1t}}{\partial t_k}\Big|_{NRS} = \frac{\beta_{1t}}{(1-t_k)} \frac{r}{(1+r)} \frac{1}{\theta} \frac{C_{1t}}{Y_{1t}} [\ \eta^s - \eta^a\ ] \geq 0$$

The ratio of risky assets to savings increases under NDRRA. This implies that the discouragement of the risky asset is smaller than the total savings.

To measure the deadweight losses of capital income taxation we differentiate the indirect utility function with respect to the tax rate and substitute the first order conditions. This gives us the desired result in the text.

A.II.2 A Tax On the Imputed Safe Income: t_r

In order to determine the impact of this tax policy, we proceed as the previous analysis. The argument of $h'(C_{2t})$ (i.e. future consumption) is evaluated at the equilibrium value $a_{1t}=a_{1t}^*$ and $S_{1t}=S_{1t}^*$. The investors future consumption level is:

$$C_{2t} = (1+r)S_{1t}^* + a_{1t}^* z_{t+1}$$

Differentiating the first order conditions yields the following system of equations:

$$[\, A_r \,] \begin{bmatrix} \dfrac{\partial C_{1t}}{\partial t_r}\Big|_{NRS} \\ \dfrac{\partial a_{1t}}{\partial t_r}\Big|_{NRS} \end{bmatrix} = \begin{bmatrix} D_{1r} \\ D_{2r} \end{bmatrix}$$

where D_{1r} and D_{2r} are given by:

$$D_{1r} = -\, r\, E(h')$$
$$D_{2r} = 0$$

The effect of the safe imputed income tax on the risky asset is:

$$\frac{\partial a_{1t}}{\partial t_r}\Big|_{NRS} = -\, a_{1t} \frac{1}{\theta} \frac{r}{(1+\bar{r})} \frac{C_{1t}}{Y_{1t}} \eta^a < 0$$

Risk Taking is unambiguously discouraged.

The effect of the tax on current consumption is:

$$\frac{\partial C_{1t}}{\partial t_r}\Big|_{NRS} = \frac{rS_{1t}}{(1+\bar{r})} \frac{1}{\theta} \frac{C_{1t}}{Y_{1t}} \eta^s > 0$$

Current Consumption is encouraged.

The effect of the tax on savings is:

$$\frac{\partial S_{1t}}{\partial t_r}\Big|_{NRS} = -\frac{\partial C_{1t}}{\partial t_r}\Big|_{NRS}$$

Total savings unambiguously fall just like the full capital income taxation.

The effect on the safe asset is:

$$\frac{\partial m_{1t}}{\partial t_r}\Big|_{NRS} = - m_{1t} \frac{1}{\theta} \frac{r}{(1+\bar{r})} \frac{C_{1t}}{Y_{1t}} \eta^m < 0$$

The safe asset holdings drop for a lender.

The effect on proportional risk taking is:

$$\frac{\partial \beta_{1t}}{\partial t_r}\Big|_{NRS} = \beta_{1t} \frac{r}{(1+\bar{r})} \frac{1}{\theta} \frac{C_{1t}}{Y_{1t}} [\ \eta^s - \eta^a\ ] \geq 0$$

The ratio of risky assets to savings increases under NDRRA again.

The effect of this policy on savings, safe asset and proportional risk taking are the same as the full capital income tax policy.

A.II.3 A Tax on the Risky Asset, t_x

In order to determine the impact of this tax policy, we assume again that the argument in $h'(C_{2t})$ is evaluated at $a_{1t}=a_{1t}^*$. This lump sum transfer as stated in the main text cancels with the tax payment of the household. Hence, the investors future consumption level is given by:

$$C_{2t} = (1+r)S_{1t}^* + a_{1t}^* z_{t+1}$$

Differentiating the first order conditions totally yields the following system of equations:

$$[\,A_x\,]\begin{bmatrix} \frac{\partial C_{1t}}{\partial t_x}\Big|_{NRS} \\ \frac{\partial a_{1t}}{\partial t_x}\Big|_{NRS} \end{bmatrix} = \begin{bmatrix} D_{1x} \\ D_{2x} \end{bmatrix}$$

where the determinant of $A_x > 0$ and the elements of A are:

$$\begin{aligned} A_{11} &= g'' + (1+r)^2 E(h'') \\ A_{12} &= -(1+r)E(h'' z_{t+1}) \\ A_{21} &= -(1+r)E(h'' \overline{z_{t+1}}) \\ A_{22} &= (1-t_x)E(h'' z_{t+1}^2) \end{aligned}$$

Notice again that the determinant is not symmetric except for $t_x=0$.

And where D_{1x} and D_{2x} are given by:

$$\begin{aligned} D_{1x} &= 0 \\ D_{2x} &= E(h' x_{t+1}) \end{aligned}$$

After simple manipulations we get the following effects:

The effect on risky asset holdings is:

$$\frac{\partial a_{1t}}{\partial t_x}\Big|_{NRS} = - E(h' x_{t+1}) \frac{A_{11}}{A} < 0$$

Risky asset holdings decrease. Since there is no risk sharing, the risky asset less attractive relative to safe asset.

The effect on current consumption is:

$$\frac{\partial C_{1t}}{\partial t_x}\Big|_{NRS} = \frac{r E(h')(1+r) E(h'' z_{t+1})}{A} > 0$$

Current consumption increases for initial t_x equal to zero. The degree of increase depends on the elasticity of substitution. For initial $t_x = 0$ we get the effect on current consumption as:

$$\frac{\partial C_{1t}}{\partial t_x}\Big|_{NRS} = \frac{r S_{1t}}{(1+r)} \frac{1}{\theta} \frac{C_{1t}}{Y_{1t}} \eta^a > 0$$

Current consumption increases. The degree of increase depends on the elasticity of substitution. The effect on savings is:

$$\frac{\partial S_{1t}}{\partial t_x}\Big|_{NRS} = - \frac{\partial C_{1t}}{\partial t_x}\Big|_{NRS} < 0$$

Savings fall since the investor increases current consumption.

The effect on the safe asset is:

The investment in the safe asset will increase if the elasticity of substitution is zero.

$$\frac{\partial m_{1t}}{\partial t_x}\bigg|_{NRS} = \frac{\partial S_{1t}}{\partial t_x}\bigg|_{NRS} - \frac{\partial a_{1t}}{\partial t_x}\bigg|_{NRS} \gtrless 0$$

The effect on proportional risk taking is:

$$\frac{\partial \beta_{1t}}{\partial t_x}\bigg|_{NRS} = \frac{1}{S_{1t}}\frac{\partial a_{1t}}{\partial t_x}\bigg|_{NRS} - \frac{a_{1t}}{S_{1t}^2}\frac{\partial S_{1t}}{\partial t_x}\bigg|_{NRS} \lessgtr 0$$

The ratio of risky assets to savings will decrease if the elasticity of substitution is zero or the relative risk aversion is extremely large.

A.II.4 A Tax on the Safe Asset, t_m

In order to determine the impact of this tax policy, we proceed as previously and evaluate the argument in $h'(C_{2t})$ at $m_{1t}=m_{1t}^*$. This lump sum transfer also cancels with the tax payment of the household. Hence, the investors future consumption level is given by:

$$C_{2t} = (1+r)S_{1t}^* + a_{1t}^* z_{t+1}$$

Differentiating the first order conditions totally yields the following system of equations:

$$[\, A_m \,] \begin{bmatrix} \dfrac{\partial C_{1t}}{\partial t_m}|_{NRS} \\ \dfrac{\partial a_{1t}}{\partial t_m}|_{NRS} \end{bmatrix} = \begin{bmatrix} D_{1m} \\ D_{2m} \end{bmatrix}$$

where the determinant of $A_m > 0$ and the elements of A are:

$$\begin{aligned} A_{11} &= g'' + (1+r)(1+\bar{r})E(h'') \\ A_{12} &= -(1+\bar{r})E(h'' z_{t+1}) \\ A_{21} &= -(1+r)E(h''\overline{z_{t+1}}) \\ A_{22} &= (1-t_m)E(h'' z_{t+1}{}^2) \end{aligned}$$

Notice again that the determinant is not symmetric except for $t_m=0$ initially.

And where D_{1m} and D_{2m} are given by:

$$\begin{aligned} D_{1m} &= -r\,E(h') \\ D_{2m} &= -r\,E(h') \end{aligned}$$

After simple manipulations we get the following effects:

The effect on risky asset holdings is:

$$\frac{\partial a_{1t}}{\partial t_m}\Big|_{NRS} = -\frac{rE(h')\,A_{m11}}{A_m} + \frac{rE(h')A_{m21}}{A_m} \gtrless 0$$

The effect on risky asset holdings cannot be determined. There is no risk sharing, hence the risky asset is more attractive relative to safe asset but in addition now future consumption is more expensive. The two effects operate opposite directions.

The effect on current consumption for initial tax t_m equal to zero is:

$$\frac{\partial C_{1t}}{\partial t_m}\Big|_{NRS} = \frac{r}{(1+r)}\,\frac{1}{\theta}\,\frac{C_{1t}}{Y_{1t}}\,m_{1t}\,\eta^m > 0$$

Current consumption increases for initial t_m equal to zero. The degree of increase depends on the elasticity of substitution.

The effect on savings for initial t_m equal to zero is:

$$\frac{\partial S_{1t}}{\partial t_m}\Big|_{NRS} = -\frac{\partial C_{1t}}{\partial t_m}\Big|_{NRS} < 0$$

Savings fall since the investor increase current consumption. The effect on the safe asset is:

$$\frac{\partial m_{1t}}{\partial t_m}\Big|_{NRS} = \frac{\partial S_{1t}}{\partial t_m}\Big|_{NRS} - \frac{\partial a_{1t}}{\partial t_m}\Big|_{NRS} \gtrless 0$$

The safe asset will decrease if the elasticity of substitution is zero.

The effect on proportional risk taking is:

$$\frac{\partial \beta_{1t}}{\partial t_m}\Big|_{NRS} = \frac{1}{S_{1t}} \frac{\partial a_{1t}}{\partial t_m}\Big|_{NRS} - \frac{a_{1t}}{S_{1t}^2} \frac{\partial S_{1t}}{\partial t_m}\Big|_{NRS} \lessgtr 0$$

The ratio of risky assets to savings will increase if the elasticity of substitution is zero or the relative risk aversion is extremely large, otherwise the effect is ambiguous.

APPENDIX III

A.III.1 A Capital Income Tax, t_k

We proceed just as the previous chapter and we assume that the argument in $h'(C_{2t})$ is evaluated at $a_{1t}=a_{1t}^*$ and $S_{1t}=S_{1t}^*$. This lump sum transfer cancels with the tax payment of the household and hence there are no income effects. Hence, the investors future consumption level is given by:

$$C_{2t} = (1+r)S_{1t}^* + a_{1t}^* z_{t+1}$$

Differentiating the first order conditions totally yields the following system of equations:

$$[\, A_k \,] \begin{bmatrix} \frac{\partial C_{1t}}{\partial t_k}\Big|_{NRS} \\ \frac{\partial a_{1t}}{\partial t_k}\Big|_{NRS} \end{bmatrix} = \begin{bmatrix} D_{1k} \\ D_{2k} \end{bmatrix}$$

where the determinant of $A_k > 0$ and where:

And where D_{1k} and D_{2k} are given by:

$$\begin{aligned} D_{1k} &= -\, r\ \delta\ CE(C_2)^{(\theta-\alpha)}\ E(C_2^{(\alpha-1)}) \\ D_{2k} &= 0 \end{aligned}$$

The effect on current consumption is:

$$\frac{\partial C_{1t}}{\partial t_k}\Big|_{NRS} = \frac{rS_{1t}}{(1+\bar{r})}\ \sigma\ \frac{C_{1t}}{Y_{1t}}\ \eta^{s} > 0$$

Current consumption increases. The degree of increase depends positively only on the elasticity of substitution. The relative risk aversion does not appear.

To measure the Deadweight Losses of capital income taxation we differentiate welfare with respect to the tax rate and substitute the first order condition. After simple manipulations this gives us the desired result in the text.

APPENDIX IV

A.IV.1 The Weiner Process and Ito's Lemma

A Weiner Process

Let the stock price behaviour over time follow a geometric Brownian motion. In discrete-time this implies that:

$$\frac{\Delta P(t)}{P(t)} = E(x)\ \Delta t + \sigma\ \epsilon_t \sqrt{\Delta t}$$

where E(x) is the expected return of the risky asset per unit time, ϵ_t is a standardized normal random variable (i.e., a normal random variable with mean zero and standard deviation equal to one), and σ is the standard deviation of the stock price. Both the mean and variance of the stochastic returns of the risky asset are assumed constant. The first term on the right-hand side of the equation represents the expected return provided by the stock on a short interval of time Δt. This term represents the trend of the stock price. The second term is the stochastic component of the return over time. The variance of the stochastic return is σ^2 Δt. The return of the risky asset is distributed normally with mean α Δt and standard deviation $\sigma\sqrt{\Delta t}$. The normality assumption of the asset return or the assumption of asset price following a lognormal distribution is consistent with limited liability. The term $\Delta z = \epsilon_t\sqrt{\Delta t}$ is known as a Wiener process. The Weiner process or Brownian motion describes in physics a particle that is affected by a large number of small molecular shocks. The Wiener process assumes that the values of Δz for any two different short intervals of time are independent. This term causes the stochastic movements over time of the asset return from the trend value that it follows.

Let us examine what the above conditions imply about the behaviour of $z=\Delta P/P$ the percentage change in the stock price over a finite period of time T. Suppose we break the interval T into n units of equal increments to $n=T/\Delta t$. Then the change of the capital gain (z) over this interval is given by (let $E(x)=0$):

$$z(s+T)-z(s)=\sum_{i=1}^{n}\epsilon_i(\Delta t)^{1/2}$$

Clearly the change in z over the interval is normal with mean zero and variance T. The variance of the change in the capital gains grows linearly with the time interval.

Ito's Lemma

A Taylor series expansion is the best way to illustrate Ito's Lemma. Consider a variable x that obeys a geometric Brownian motion with drift as indicated above: $dx = a\,x\,dt + \sigma\,x\,dz$ and consider a function f(x,t) that is differentiable. The total differential of this function is given by:

$$df = f_x\, dx + f_t\, dt + \frac{1}{2} f_{xx}\, dx^2 + \frac{1}{6} f_{xxx}\, dx^3 + \ldots$$

where f_x, f_t are the partial derivatives. In calculus the higher order terms disappear in the limit. However, if x follows the Ito process this is not true. In order to see this evaluate dx^2 and notice that all the terms do not vanish in the limit.

$$dx^2 = (a\, x\, dt + \sigma\, x\, dz)^2 =$$

$$dx^2 = a^2 x^2 dt^2 + 2\, a\, x^2 \sigma\, \epsilon_t\, x^2\, dt^{3/2} + \sigma^2 x^2 e_t^2 dt$$

Dividing both sides by dt and taking the limit as dt -> 0 leaves only the last term. All higher order terms vanish. Now substituting for dx^2 yields the total differential of a function which contains a stochastic weiner process:

$$df = f_x\, dx + f_t\, dt + \frac{1}{2} f_{xx}\, dx^2$$

The last term is absent in calculus.

A.IV.2 The Budget Constraint.

Let $N_i(t)$ be the number of shares of asset i bought in period t, $P_i(t)$ the price of the i-th asset. Let W(t) be the investors wealth in period t, C(t) consumption in period t and Y(t) labour income in period t.

The change in the investors wealth dW(t) is equal to the capital gains arising from changes in the price of the stock plus any non capital income (i.e labour income) less any consumption in period t per unit of time. The budget equation is as follows:

$$dW(t) = \sum_{i=1}^{n} N_i(t) dP_i(t) - C(t)dt + Y(t)dt$$

(1A.1)

Letting β(t) be the proportion of wealth invested in the risky asset we obtain:

$$dW(t) = \sum_{i=1}^{n} \beta_i(t) W(t) \frac{dP_i(t)}{P_i(t)} - C(t)dt$$

(1A.2)

Assuming that the asset prices $P_i(t)$ are generated by a geometric Brownian motion:

$$\frac{dP_i(t)}{P_i(t)} = E(x_i)dt + \sigma_i dz_i$$

(1A.3)

where $E(x_i)$ is the expected return of asset i, σ_i is the standard deviation of the asset i and dz_i is a Wiener stochastic process. Substituting (IV.3) into (IV.2) we obtain the continuous-time budget constraint of portfolio-consumption decision model:

$$dW(t) = \sum_{i=1}^{n} \beta_i(t) E(x_i) W(t)dt - C(t)dt + \sum_{i=1}^{n} \beta_i(t) W(t) \sigma_i dz_i$$

(1A.4)

For the two asset case it can be easily shown that (IV.4) becomes:

$$dW(t) = (\beta(t)(E(x)-r)+r)W(t)dt - C(t)dt + \beta(t)W(t)\sigma dz$$

A.IV.3 The Optimal Consumption and Portfolio Choice.

The investors' problem is to choose C(t) and β(t) to maximize his utility subject to the budget constraint. The CRRA and CIES utility function in discrete time is given by:[1]

$$U(C,CE(U);t,t+\Delta t) = e^{-\delta t}\left[\, C^{\theta} + e^{-\delta\Delta t}E_t(U^{(1-\gamma)})^{\frac{\theta}{(1-\gamma)}}\,\right]^{\frac{1}{\theta}}$$

(3A.1)

subject to:

$$dW(t) = [\beta(t)(E(x)-r)+r]W(t)dt - C(t)dt + \beta(t)W(t)\sigma dz$$

(3A.2)

and assuming C(t), W(t) > 0 and initial endowment of $W(0) = W_0 > 0$.

Given that the utility function is of the constant elasticity and constant relative risk aversion form we can guess that the indirect utility function is of the type:

$$J(W,t) = e^{-\delta t}I(W)$$

(3A.3)

[1] This section follows closely Svennson (1989) who is the founder of the non-expected utility in continuous time.

The indirect utility function must satisfy:

$$I(W(t)) = \lim_{\Delta t \to 0} \max_{c,\omega} \left[C^{\theta}\Delta t + e^{-\delta \Delta t} \left[E_t\, I(W(t+\Delta t))^{(1-\gamma)} \right]^{\frac{\theta}{(1-\gamma)}} \right]^{\frac{1}{\theta}}$$

(3A.4)

subject to the budget constraint. Given this type of preference structure and the homogenous budget constraint we may also guess that the form of the indirect utility function and the consumption function is proportional to wealth.

$$I(W) = A\ W, \quad A > 0 \qquad C(W) = B\ W, \quad B > 0$$

(3A.5)

where A , B are functions of the parameters of the model.

Substituting (3A.5) into (3A.4), applying Ito's Lemma, and taking the limit leads as to the Bellman equation of optimality.

$$0 = \underset{B,\beta}{Max} \left[(B/A)^{\theta} - \delta + \theta[(\beta(E(x)-r)+r) - B - \frac{\gamma\sigma^2\omega^2}{2}] \right]$$

(3A.6)

The first order conditions with respect to β and B are:

With respect β :

$$\theta \ (E(x)-r) \ - \ \frac{\theta \ \sigma^2 \ \beta \ \gamma}{2} \ = 0$$

With respect B :

$$\frac{\theta}{A} \ (B/A)^{(\theta-1)} \ - \ \theta \ = 0$$

(3A.7)

Solving for the portfolio choice we get:

$$\beta \ = \ \frac{(E(x) - r)}{\sigma^2 \ \gamma}$$

Portfolio choice is determined only by the risk aversion parameter and the moments of the distribution. No role is given the intertemporal elasticity of substitution.

Solving for B we get:

$$B \ = A^{\frac{\theta}{(1-\theta)}}$$

Substituting the values of β and B into the Bellman equation and solving for A we obtain:

$$b \ = \ [\ \frac{\delta}{(1-\theta)} \ - \ \frac{\theta}{(1-\theta)} \ [\ \frac{(E(x)-r)^2}{2\sigma^2\gamma} \ + r \ ]\ ]^{1-1/\theta}$$

(3A.7)

Since $C(t) = A^{\theta/(\theta-1)} W(t)$ we have:

$$\mu(0) \ = \ [\ \frac{\delta}{(1-\theta)} \ - \ \frac{\theta}{(1-\theta)} \ [\ \frac{(E(x)-r)^2}{2\sigma^2\gamma} \ + r \ ]\ ]$$

(3A.8)

the optimal proportion invested in the risky asset is:

$$\beta(t) = \beta(0) = \frac{(E(x)-r)}{\sigma^2 \gamma} \tag{3A.9}$$

A.IV.4 Income and Substitution Effects

We next derive the substitution and income effects of the Phelps-Ramsey problem of optimal consumption under uncertainty. Consumption per wealth can be re-written as follows:

$$\mu(0) = [\frac{\delta}{(1-\theta)} - \frac{\theta}{(1-\theta)} [\rho - \gamma \frac{\sigma^2(\rho)}{2}]] \tag{4A.1}$$

where $\rho = \beta(0)(E(x)-r)+r$ is the mean of the composite asset and $\sigma^2(\rho) = (\beta(0)\sigma)^2$ is the variance of the composite asset. Let z be a financial parameter (i.e. mean, variance, tax rate) then the substitution effect is given by:

$$\frac{\partial C(t)}{\partial z} \Big|_{J(W(0))=C} \tag{4A.2}$$

The income effect is given by:

Total Effect Less the Substitution Effect:

$$\frac{\partial C(t)}{\partial z} - \frac{\partial C(t)}{\partial z} \Big|_{J(W(0))=C} \tag{4A.3}$$

Differentiating totally the indirect utility function J(W(0)) = AW(0) and setting the result equal to zero yields:

$$0 = W(0) \frac{\partial A}{\partial z} \Big|_{J(W(0))=C} + A \frac{\partial W(0)}{\partial z} \Big|_{J(W(0))=C}$$

(4A.4)

Solving for the income compensation yields:

$$\frac{\partial W(0)}{\partial z} \Big|_{J(W(0))=C} = - \frac{W(0)}{A} \frac{\partial A}{\partial z} \Big|_{J(W(0))=C}$$

(4A.5)

From the consumption function $C(t) = A^{1/(\theta-1)} W(t)$ we can easily observe that the effect of the change in the financial parameter on A is given by:

$$\frac{\partial A}{\partial z} = \frac{(\theta-1)}{\theta} \mu(o)^{-(\frac{1}{\theta})} \frac{\partial \mu(0)}{\partial z}$$

(4A.6)

Substituting into the income compensation yields:

$$\frac{\partial W(0)}{\partial z} \Big|_{J(W(0))=C} = - \frac{W(0)}{A} \frac{(\theta-1)}{\theta} \mu(o)^{-(\frac{1}{\theta})} \frac{\partial \mu(0)}{\partial z}$$

(4A.7)

Consider $z=\rho$ then the substitution effect is given by:

$$\frac{\partial C(0)}{\partial \rho} \Big|_{J(W(0))=C} = W(0) \frac{\partial \mu(0)}{\partial \rho} + \mu(0) \frac{\partial W(0)}{\partial \rho} \Big|_{J(W(0))=C}$$

(4A.8)

Substituting the relevant information into the above expression yields the desired result:

$$\frac{\partial C(t)}{\partial z}\Big|_{J(W(0))=C} = -\eta\ W(0) < 0$$

(4A.9)

The substitution effect indicates that an increase in the mean of the composite asset will make future consumption more attractive and given a positive elasticity of substitution will lead to a reduction in current consumption. Following the procedure discussed above yields the income effect equal to W(O).

Consider $z=\sigma^2(\rho)$ then the substitution effect is given by:

$$\frac{\partial C(0)}{\partial \sigma^2(\rho)}\Big|_{J(W(0))=C} = W(0)\ \frac{\partial \mu(0)}{\partial \sigma^2(\rho)} + \mu(0)\ \frac{\partial W(0)}{\partial \sigma^2(\rho)}\Big|_{J(W(0))-C}$$

(4A.9)

Substituting we obtain the following substitution effect:

$$\frac{\partial C(0)}{\partial \sigma^2(\rho)}\Big|_{J(W(0))=C} = \frac{\gamma\ \sigma}{2}\ W(0)$$

(4A.10)

Since an increase in the variance makes future consumption more risky the investor substitutes towards current consumption. The income effect can be obtained as a residual from the total effect.

A.IV.5 Taxation Policies and Optimal Decisions

In this appendix we present the various Bellman equations of optimality under different tax policies. In order to derive the optimal quantities we follow the same procedure as appendix 3. The optimal choices are shown in the main text.

Choice under a Capital Income Tax

The bellman equation of optimality under a capital income tax is:

$$0 = \underset{B,\beta}{Max}\ [\ (B/A)^{\theta} - \delta + \theta[\ (1-t_y)\ (\beta(E(x)-r)+r) - B - \frac{\gamma\sigma^2(1-t_y)^2\beta^2}{2}]\ ]$$

(5A.1)

Choice under a Differentiated Capital Income Tax

Taxation of the Safe asset t_m and the Risky asset t_x. The bellman equation under a tax on the safe asset and the risky asset is:

$$0 = \underset{B,\beta}{Max}\,[\,(B/A)^{\theta} - \delta + \theta\,[\,\beta(E(x)(1-t_x)-r(1-t_m))+r(1-t_m) - B - \gamma\frac{\sigma^2\,(1-t_x)^2\,\omega^2}{2}\,]\,]$$

(5A.2)

APPENDIX V

A.V.1 A Capital Income Tax, t_k

In order to determine the impact of this tax policy, we assume that the argument in $h'(C_{2t})$ is evaluated at $a_{1t}=a_{1t}^*$ and $S_{1t}=S_{1t}^*$. This lump sum transfer as stated in the text does not cancel with the tax payment of the household. Hence, the investors future consumption level is given by:

$$C_{2t}^* = (1+r) S_{1t}^* + a_{1t}^* \overline{z_{t+1}} + (1-\gamma_1) a_{1t}^* E(z) t_k$$

Differentiating the first order conditions totally yields the following system of equations:

$$[A_k] \begin{bmatrix} \frac{\partial C_{1t}}{\partial t_k}|_{RS} \\ \frac{\partial a_{1t}}{\partial t_k}|_{RS} \end{bmatrix} = \begin{bmatrix} D_{1k} \\ D_{2k} \end{bmatrix}$$

where the determinant of $A_k > 0$ and the elements of A are:

$$\begin{aligned} A_{11} &= g'' + (1+\bar{r})(1+r) E(h'') \\ A_{12} &= -[(1+\bar{r}) E(h''\overline{z_{t+1}}) + (1+\bar{r})(1-\gamma_1) E(h'') E(z) t_k] \\ A_{21} &= -(1+r) E(h''\overline{z_{t+1}}) \\ A_{22} &= [E(h''\overline{z_{t+1}}^2) + t_k (1-\gamma_1) E(z) E(h''\overline{z_{t+1}})] \end{aligned}$$

Notice that the determinant is not symmetric except for $t_k=0$. Also notice that for very high capital income tax rates the second order conditions will not hold. In such a case we do not expect to find a solution to our problem.

And where D_{1k} and D_{2k} are given by:

$$D_{1k} = - r\, E(h') + \frac{a_{1t}^{*}}{(1-t_k)} A_{12} + \frac{a_{1t}^{*}}{(1-t_k)} (1+\bar{r})\, E(z)\, (1-\gamma_1)\, E(h'')$$

$$D_{2k} = \frac{a_{1t}^{*}}{(1-t_k)} [\, A_{22} - (1-\gamma_1)\, E(z)\, E(h'' \overline{z_{t+1}})\,]$$

Solving for the effect of the tax on risky asset holdings gives:

$$\frac{\partial a_{1t}}{\partial t_k}|_{RS} = \frac{a_{1t}^{*}}{(1-t_k)} - \frac{a_{1t}}{(1-t_k)} \frac{1}{\theta} \frac{r}{(1+\bar{r})} \frac{C_{1t}}{Y_{1t}} \eta^{a} + (1-\gamma_1) \frac{a_{1t}^{*}\, E(z)}{(1-t_k)(1+r)} \frac{a_{1t}^{*}}{Y_{1t}} \eta^{a} \lessgtr 0$$

The effect on current consumption is:

$$\frac{\partial C_{1t}}{\partial t_k}|_{RS} = - \frac{r\, E(h') A_{22}}{A} + (1-\gamma_1) \frac{a_{1t}^{*}\, E(z)}{(1-t_k)\,(1+r)} \frac{C_{1t}^{*}}{Y_{1t}^{*}} \eta^{c} > 0$$

The effect on savings is:

$$\frac{\partial S_{1t}}{\partial t_k}|_{RS} = - \frac{\partial C_{1t}}{\partial t_k}|_{RS} < 0$$

The effect on proportional risk taking is:

$$\frac{\partial \beta_{1t}}{\partial t_k}|_{RS} > 0$$

The ratio of risky assets to savings increases under NDRRA.

To measure the Deadweight Losses of capital income taxation we differentiate welfare with respect to the tax rate and substitute the first order condition. This gives us the desired result in the text.

A.V.2 Evaluation of other Capital Income Tax Policies

The Taxation of Excess Returns

The capital asset pricing model with intergenerational risk sharing is:

$$CE(R_{pt}) = E(R_{pt}) - \lambda\, \gamma_1 \, cov(T_{2t}, x_{t+1})$$

The per capital stochastic lump sum tax revenue from the excess return of the risky asset is:

$$T_{2t} = t_z \, a_{1t}^{*} \, z_{t+1}$$

The market value of this tax revenue assuming that the investor holds the market portfolio is given by:

$$CE(R_{pt}) = (1 - \gamma_1)\, t_z \, a_{1t}^{*} \, E(z)$$

The market value of the tax revenue from the taxation of excess returns is positive if γ_1 is less than one. The taxation of excess returns does not create any deadweight losses. The taxation of excess returns can generate revenue to the state. This revenue has a positive market value. If each generation faces his own tax revenue risks then the market value of this tax revenue is zero. In addition the allocative effects of this tax policy is transparent. Current Consumption and risk taking increases, while savings fall, proportional risk taking increases under the assumption of NDRRA and DARA. Expected future consumption also increases. Welfare in this case increases since the capital income tax is positively related to the value of the tax revenue.

The Taxation of the Imputed Safe Income

The capital asset pricing model with intergenerational risk sharing is:

$$CE(R_{pt}) = E(R_{pt}) - \lambda \gamma_1 cov(T_{2t}, x_{t+1})$$

The per capital stochastic lump sum tax revenue from the imputed safe income is:

$$T_{2t}^{*} = t_r \; r \; S_{1t}^{*}$$

The market value of this tax revenue is:

$$CE(R_{pt}) = r \; t_r \; S_{1t}^{*}$$

The market value of this tax revenue is the same as the tax payment of the investor. Hence the value of this tax revenue matches the tax revenue from capital income taxation only in the case in which the risk premium has a zero market value. In any other case the government is capable of collecting additional tax revenue.

The Taxation of the Risky Asset

The capital asset pricing model with intergenerational risk sharing is:

$$CE(R_{pt}) = E(R_{pt}) - \lambda \gamma_1 cov(T_{2t}, x_{t+1})$$

The per capital stochastic lump sum tax revenue from the risky asset is:

$$T_{2t}^{*} = t_x \; x_{t+1} \; a_{1t}^{*}$$

The market value of this tax revenue is:

The market value of this tax revenue is a weighted average between the two tax revenues. Notice that if γ_1 is equal to one the tax revenue that can be generated from the

$$CE(R_{pt}) = \frac{\gamma_1 \; r \; t_x \; a_{1t}^{\bullet}}{(1-t_x)} + (1-\gamma_1) \; t_x \; E(x) \; a_{1t}^{\bullet}$$

taxation of the risky asset is lower than the expected value of the tax revenue. The market value of this tax revenue without risk sharing is:

$$CE(R_{pt}) = \frac{r \; t_x \; a_{1t}^{\bullet}}{(1-t_x)}$$

In all other cases the market value of the tax revenue varies. If γ_1 equals zero the market value of the tax revenue is equal to the expected value of the tax revenue from the risky asset.

The Taxation of the Safe Asset

The capital asset pricing model with intergenerational risk sharing is:

$$CE(R_{pt}) = E(R_{pt}) - \lambda \, \gamma_1 \, cov(T_{2t}, x_{t+1})$$

The per capital stochastic lump sum tax revenue from the safe asset is:

$$T_{2t}^{\bullet} = t_r \; r \; m_{1t}^{\bullet}$$

The market value of this tax revenue is:

$$CE(R_{pt}) = r \; t_r \; m_{1t}^{\bullet}$$

The market value of this tax revenue is equal to the tax payment of the investor.

APPENDIX VI

The Capital Asset Pricing Model

Evaluation of the Cash Flow Consumption Tax Revenue

This evaluates the risky cash flow consumption tax revenue using the asset pricing model. We show that the outcome is the same as the modified wage tax. Starting from the capital asset pricing model we obtain:

$$CE\ (\ T_{ct}\ ) = R_{ct} = E\ (\ T_{ct}\ ) - \lambda\ cov\ (\ T_{ct}\ ,\ r_{mt+1}\ )$$

$$\lambda = \frac{E(r_m) - r}{\sigma_m^2}$$

where T_{ct} is given by:

$$T_{ct} = t_c\ [\ C_t(t_c) + \frac{C_{t+1}(t_c)}{(1+r)}\ ]$$

$$T_{ct} = \frac{t_c}{(1+t_c)}\ [Y_t + \frac{a_t(t_c)\ z_{t+1}}{(1+r)}]$$

Substituting the cash flow consumption tax revenue T_{ct} that the young generation household pays to the state into the covariance term and expressing it in terms of the correlation coefficient of the "risky" note yields:

$$CE(T_{ct}) = E(T_{ct}) - \frac{t_c}{(1+t_c)} \frac{a_t(t_c)E(z)}{(1+r)} \frac{\frac{(E(r_m)-r)}{\sigma_m}}{\frac{E(z)}{\sigma_x}} r_{xm}$$

Alternatively we can write the above equation in the following form:

$$CE(T_{ct}) = \frac{t_c}{(1+t_c)} Y_t + \frac{t_c}{(1+t_c)} \frac{a_t(t_c)E(z)}{(1+r)} [1 - \frac{\frac{(E(r_m)-r)}{\sigma_m}}{\frac{E(z)}{\sigma_x}} r_{xm}]$$

In the case, where the household invests in the market portfolio or alternatively, to a "mutual fund" that is perfectly correlated or "mimics" the market portfolio then:

$$r_{mx} = 1$$

in addition given:

$$\frac{E(r_m - r)}{\sigma_m} = \frac{E(x - r)}{\sigma_x}$$

Substituting the above two conditions into the certainty equivalent tax revenue we obtain:

$$R_{ct} = CE(T_{ct}) = \frac{t_c}{(1+t_c)} Y_t$$

Again this is the only tax revenue that can be used to finance public projects which enters into the utility function in an additive way. Even though the government taxes the consumption of the household and receives more tax revenue in expected present value. The government is taking a claim of zero market value from the risk premium and hence can only finance programs that have a certainty equivalent equal to the pre-payment wage tax. The price of the "risky" note is equal to the consumption tax revenue the government would raise under certainty. This certainty equivalent tax revenue of a cash flow consumption tax revenue is independent of the parameters of the model. The certainty equivalent tax revenue does not depend on the risk aversion of the individuals.

Hence, we observe that by using the weighted average discount rate for a social rate to discount the risky cash flow consumption tax revenue we are implicitly assuming that the household holds a perfectly diversified portfolio. The social discount rate used to evaluate the cash flow consumption tax cannot exceed the weighted average discount rate in a risk averse world.

Risk Sharing

If we assume .hat all generations (n in number) share in the current "market" or "social" risk via a stochastic tax policy then the relevant tax revenue valuation is the average cash flow consumption tax revenue across all generations. What price are the young investors willing to pay for a security who's risks are shared with all generations ? Using again the CAPM to evaluate this new stream of revenue, the certainty equivalent of this tax revenue stream becomes:

$$CE(T_{ct}) = E\ [\ \sum_{i=0}^{n-1} \frac{T_{ct-1}}{n}\ ] - \lambda\ cov\ [\ \frac{\sum_{i=0}^{n-1} T_{ct-i}}{n}\ ,\ r_{mt+1}\ ]$$

I am assuming that the an equally weighted average cash flow consumption tax revenue. Expanding the terms and using the properties of covariance we get:

$$CE(T_{ct}) = E(T_{ct}) - \frac{\lambda}{n}\ cov\ [\ T_{ct} + T_{ct-1} + T_{ct-2} + \ldots,\ r_{mt+1}\ ]$$

setting

$$cov\ [\ T_{ct-i}\ ,\ r_{mt+1}\ ] = 0 \qquad \forall\ i \neq 0$$

Substituting the cash flow tax revenue into the above expression and assuming that the household holds the market portfolio gives:

$$R_{ct} = \frac{t_c}{(1+t_c)}\left(Y_t + (1-\frac{1}{n})\frac{a_t E(z)}{(1+r)}\right)$$

The public discount rate for the evaluation of risky tax revenue is given by:

$$\rho = (1+r)\left[\frac{1+r+\beta E(z)}{1+r+(1-\frac{1}{n})\beta E(z)}\right]-1$$

This is obtained by setting the present value of the risky tax revenue using the certainty equivalent method equal to the present value of the tax revenue using the discounting method and solving for the required rate of return. Notice that as n approaches infinity the social discount rate approaches the risk free rate. On the other hand as n approaches unity the social discount rate converges to the weighted average rate.

APPENDIX VII

The No Tax World:

Given the preference structure:

$$V(C_1,CE(C_2)) = [C_1^{\theta} + \delta[\, pC_{21}^{\alpha} + (1-p)\, C_{22}^{\alpha})]^{\frac{\theta}{\alpha}}]^{\frac{1}{\theta}}$$

(A.1)

and the constraints:

$$C_1 = Y - (a + m)$$

$$C_{21} = (Y - C_1)(1+r) + az_1 \qquad z_1 = x_1 - r > 0$$

$$C_{22} = (Y - C_1)(1+r) + az_2 \qquad z_2 = x_1 - r < 0$$

(A.2)

Maximization of (A.1) with respect to current consumption (C_1), and amount invested in the risky asset (a) yields the following first order condition; With respect to current consumption:

$$C_1^{\theta-1} = \delta(1+r)CE(C_2)^{(\theta-\alpha)}E(C_2^{\alpha-1})$$

(A.3)

With respect to risk taking:

$$E(C_2^{\alpha-1}z) = 0$$

(A.4)

Defining the following quantities:

$$A_0 = [\ \delta(1+r)p(\frac{z_2 - z_1}{z_2})[p+(1-p)B_0^{-\alpha}]^{(\frac{\theta}{\alpha})-1}\ ]^{\frac{1}{\theta-1}}$$

$$B_0 = [\frac{-pz_1}{(1-p)z_2}]^{\frac{1}{1-\alpha}}$$

$$A_1 = A_0^{-1}(z_1 - B_0 z_2)$$

$$B_1 = B_0(z_1 - z_2)(1+r)$$

and solving for the optimal quantities we obtain for the amount invested in the risky asset:

$$a(0) = \frac{(1+r)(B_0 - 1)\ A_0^{-1}\ Y}{A_1 + B_1}$$

The amount invested in the risky asset depends on both risk aversion and the intertemporal substitution parameter.

For Current Consumption:

$$C_1(0) = \frac{B_1}{A_1 + B_1}\ Y$$

For Savings:

$$S(0) = \frac{A_1}{A_1 + B_1}\ Y$$

For proportional risk taking:

$$\beta(0) = \frac{a(0)}{S(0)} = \frac{(1+r)(B_0 - 1)}{(z_1 - B_0 z_2)}$$

Interestingly, proportional risk taking is independent of the elasticity of substitution. This result is consistent with Svensson's (1989) extension of the non-expected utility in a continuous time framework.

Finally the Certainty equivalent future consumption is given by:

$$CE(C_2) = [E(C_2^{\alpha})]^{\frac{1}{\alpha}}$$

$$\text{where} \quad C_{21} = A_0^{-1} C_1$$
$$C_{22} = (B_0 * A_0)^{-1} C_1$$

Following a similar procedure under the different tax regimes yields the following effects on the optimal quantities:

VARIABLES	NO TAX	CONSUMPTION TAX	WEALTH TAX	WAGE TAX
Risk Taking	a(0)	$a(t_c)=a(0)$	a(0)	$(1-t_w)a(0)$
Consumption	$C_1(0)$	$C_1(0)/(1+t_c)$	(1-t)C(0)	$(1-t_w)C_1(0)$
Savings	S(0)	$S(t_c)=S(0)$	(1-t)S(0)	$(1-t_w)S(0)$
Proportional Risk Taking	$\beta(0)$	$\beta(t_c)=\beta(0)$	$\beta(0)/(1-t)$	$\beta(0)$

APPENDIX VIII

A.VIII.1 The Wealth Effects

The effects of a change in exogenous income on the decision variables can be obtained by differentiating the optimum conditions with respect to Y_{1t} yielding:

$$[A]\begin{bmatrix} \dfrac{\partial C_{1t}}{\partial Y_{1t}} \\ \dfrac{\partial H_{1t}}{\partial Y_{1t}} \end{bmatrix} = \begin{bmatrix} (1+r)^2 E(h'') \\ -(1+r)E(h'' z_{t+1}) \end{bmatrix}$$

<8A1.1>

where the determinant of A > 0 and the elements of A are:

$$A_{11} = g'' + (1+r)^2 E(h'') < 0$$

$$A_{12} = A_{21} = -(1+r)\ E(h'' z_{t+1}) < 0$$

$$A_{22} = E(h' \mu_{t+1} F'') + E(h'' z_{t+1}^2) < 0$$

Solving for the optimum response of the endogenous variables we obtain:

$$\frac{\partial H_{1t}}{\partial Y_{1t}} = - \frac{(1+r)\ g''\ E(h'' z_{t+1})}{A}$$

<8A1.2>

Decreasing absolute risk aversion (DARA) guarantees that $E(h'' z_{t+1})$ is positive. Therefore, human capital is normal under DARA.

The effect of a change in the exogenous level of income on physical capital investment is given by:

$$\frac{\partial S_{1t}}{\partial Y_{1t}} = \frac{g'' \,[\, E(h'\mu_{t+1}F'') + E(h'' z_{t+1}^{2}) + w(1+r)\, E(h'' z_{t+1}) \,]}{A}$$

<8A1.3>

Alternatively, we can re-write <8A1.3> as follows:

$$\frac{\partial S_{1t}}{\partial Y_{1t}} = \frac{g''\, F''\, E(h'\mu_{t+1})}{A} + \frac{g''\, F'\, E(h'' z_{t+1}\mu_{t+1})}{A}$$

<8A1.4>

In order to sign the wealth effect on savings we need to sign the second term. In particular if $E(h'' z_{t+1}\mu_{t+1})$ is negative then savings will be normal.[1] This condition is sufficient but not necessary. However, this term is related to the behaviour of relative risk aversion since:

[1] An alternative interpretation of $E(h''z\mu)$ being negative is the following. An increase in μ causes an increase in future consumption and a reduction in the degree of risk aversion as measured by the absolute risk aversion. If the percentage increase in the random variable is greater than the percentage drop in the absolute risk aversion then the safe investment increases with increases in exogenous first period wealth. An elasticity of absolute risk aversion of less than or equal to one. An increase in μ can be associated with an improvement in future earnings and future consumption opportunities and under-decreasing absolute risk aversion the degree of risk aversion will fall as measured by the odds demanded. If the degree of risk aversion does not fall sufficiently more, the investor will increase both physical and human capital investments with exogenous wealth increases.

$$F(H)\, E(h' z_{t+1} \mu_{t+1}) = E(h'' z_{t+1} C_{2t}) - (1-r)\, S_{1t}\, E(h' z_{t+1}) - Y_{2t}\, E(h' z_{t+1})$$

Under non-decreasing relative risk aversion $E(h'' z_{t+1} C_{2t})$ is non-positive. Consequently, the following proposition holds.

Proposition 8.1a: For a non-borrower displaying decreasing absolute risk aversion and non-decreasing relative risk aversion savings are normal.

$$\frac{\partial S_{1t}}{\partial Y_{1t}} > 0$$

<8A1.5>

This result indicates that even though a risk avert lender will increase the amount of the risky activity if there is an increase in period-one wealth, he (she) will also increase the non risky activity under the additional assumption of non-decreasing relative risk aversion.[2]

For a borrower the analysis is more complicated and the results are not as straight forward. In order to determine whether borrowing increases with increases in wealth we need to sign the following expression:

[2] It can be shown that the behaviour of relative risk aversion imposes restrictions on the elasticity of absolute risk aversion. In particular, if the relative risk aversion is an increasing (decreasing) function of future consumption then the elasticity of absolute risk aversion is less than one (greater than one)

$$\frac{\partial B_{1t}}{\partial Y_{1t}} = - \frac{g''F' E(h'\mu_{t+1})}{A} - \frac{g' F' E(h'' z_{t+1}\mu_{t+1})}{A}$$

<8A1 6>

The first term is negative: therefore in order to determine whether the individual will increase or decrease his borrowing activity we need to sign the second term $E(h''z_{t+1}\mu_{t+1})$. Again by symmetry this term is related to the behaviour of relative risk aversion.

$$F(H)\ E(h''z_{t+1}\mu_{t+1}) = E(h''z_{t+1}C_{2t}) - (1-r)B_{1t}E(h''z_{t+1}) - Y_{2t}E(h''z_{t+1})$$

We cannot unambiguously sign the term $E(h''z_{t+1}\mu_{t+1})$ for a borrower displaying decreasing absolute risk aversion and increasing relative risk aversion. However, if borrowing is constrained not to exceed the present value of future non-stochastic earnings

$$B \leq \frac{Y_2}{1-r}$$

<8A1 7>

we obtain:

$$F(H)\ E(h''z_{t+1}\mu_{t+1}) = E(h''z_{t+1}C_{2t}) - (1-r)(B_{1t} - \frac{Y_{2t}}{(1-r)})E(h''z_{t+1})$$

Now the assumption of increasing relative risk aversion, and the condition that borrowing does not exceed the present value of future expected earnings guarantees that $E(h''z_{t+1}\mu_{t+1})$ will be negative. This implies that borrowing will decrease with increases of exogenous first period wealth. Hence borrowing is inferior if constraints are imposed on the individual.

$$\frac{\partial B_{1t}}{\partial Y_{1t}} < 0$$

<8A1.8>

Proposition 8.1b: For borrowers displaying non-decreasing relative and decreasing absolute risk aversion and constraint such that their borrowing does not exceed the present value of their non-stochastic, non-human, earnings makes savings normal.

Therefore increased exogenous first period wealth will lead to reductions in borrowing activity.[3]

The effect on current consumption is given by:

$$\frac{\partial C_{1t}}{\partial Y_{1t}} = \frac{(1+r)^2 \left[\left(E(h'')(E(h'\mu_{t+1}F'') + E(h''z_{t+1}^2)\right) - E(h''z_{t+1})^2 \right]}{A}$$

<8A1.9>

A sufficient condition for the $\partial C_{1t}/\partial Y_{1t}$ to lie between zero and one is that the term $E(h'')E(h''z_{t+1}^2) - [E(h''z_{t+1})]^2$ be positive. This term can be written as follows:

$$E(h'')\ E(h''z_{t+1}^2) - E(h''z_{t+1})^2 = F' \left[E(h'')\ E(h''z_{t+1}\mu_{t+1}) - E(h''z_{t+1})\ E(h''\mu_{t-1}) \right]$$

Clearly, a sufficient condition for this term to be positive is that the term $E(h''z_{t+1}\mu_{t+1})$

[3] This result although new in the literature of human capital has been recently uncovered by Ahsan (1989b) in a portfolio asset choice model.

be negative. Therefore, the effect of an exogenous increase in first period wealth will increase current consumption by less under the assumption of DARA and NDRRA.

A.VIII.2 The Consumption Tax.

A.VIII.2.1. The Income Effects.

Differentiating the first-order conditions with respect to first period exogenous income Y_{1t} yields the following income effects:

$$[\ \Delta\ ] \begin{bmatrix} \dfrac{\partial C_{1t}}{\partial Y_{1t}} \\ \dfrac{\partial H_{1t}}{\partial Y_{1t}} \end{bmatrix} = \begin{bmatrix} \dfrac{(1+r)^2 E(h'')}{(1+t_c)} \\ -\dfrac{(1+r)E(h''z_{t+1})}{(1+t_c)^2} \end{bmatrix}$$

<8A2.1>

Solving from <8A2.1> for the income effects yields the following outcomes:

$$\frac{\partial C_{1t}}{\partial Y_{1t}} = \frac{(1+r)^2\ [\ (E(h'')[E(h'\mu_{t+1}F'') + \dfrac{E(h''z^2_{t+1})}{(1+t_c)}] - \dfrac{E(h''z_{t+1})^2}{(1+t_c)})\ ]}{(1+t_c)^2\ \ \Delta}$$

<8A2.2>

and

$$\frac{\partial H_{1t}}{\partial Y_{1t}} = -\frac{(1+r)\ g''\ E(h''z_{t+1})}{(1+t_c)^2\ \ \Delta}$$

<8A2.3>

Both of these effects are positive given the assumptions of decreasing absolute and non-decreasing relative risk aversion. See previous appendix for details.

A.VIII.2.2. The Substitution Effects.

To obtain the compensated expected - utility comparative static results we require that the households wealth be adjusted such that:

$$dY_{1t} = (1+t_c)^{-1} \left[Y_{1t} + \frac{Y_{2t}}{(1+r)} + wH_{1t} \left(\frac{F(H)}{H_{1t}\, F'} - 1 \right) \right] dt_c$$

<8A2.4>

Utilizing the above constant expected utility requirement and differentiating the first order conditions yields the following system of equations:

$$[\,\Delta\,] \begin{bmatrix} \dfrac{\partial C_{1t}}{\partial t_c}\Big|_{\bar{V}} \\ \dfrac{\partial H_{1t}}{\partial t_c}\Big|_{\bar{V}} \end{bmatrix} = \begin{bmatrix} -\dfrac{(1+r)\; F(H)\; E(h'' z_{t+1})}{(1+t_c)^2\; F'} \\ \dfrac{F(H)\; E(h'' z_{t+1}^2)}{(1+t_c)^3\; F'} \end{bmatrix}$$

<8A2.5>

The utility compensated effect of a consumption tax on human capital is given by the following expression:

$$\frac{\partial H_{1t}}{\partial t_c}\Big|_{\bar{V}} = \frac{g'' F(H) E(h'' z_{t+1}^2)}{(1+t_c)^3 F' \Delta} + \frac{(1+r)^2\; F(H)\; [E(h'')E(h'' z_{t+1}^2) - E(h'' z_{t+1})^2]}{(1+t_c)^3\; F'\; \Delta}$$

<8A2.6>

Alternatively, utilizing the determinant of Δ, we can re-write <8A2.6> as follows:

$$\frac{\partial H}{\partial t_c}\Big|_{\bar{V}} = \frac{H_{1t}}{(1+t_c)} \frac{(F(H)/H_{1t})}{F'} \left(1 - \frac{\Delta_{11}E(h'\mu_{t+1})F''}{(1+t_c)\ \Delta} \right)$$

Note that we do not observe the Domar-Musgrave phenomenon in the human capital model as in the portfolio choice model. The difference stems from the dependence of the random earnings on human capital investment and on the deviation of average from marginal productivity. If F" was equal to zero then we would have obtained the Domar-Musgrave result. A constant marginal productivity of human capital then changes human capital (i.e., by $F(H)/(F'(1+t_c))$) and current consumption by zero. As Mossin (1968) states on pg 77 "the investor adjusts so that the expected net gain remains unchanged". The change in human capital under F" = 0 would be such that his net gain from the portfolio remains unchanged and the tax rate would not make any difference to the investor given the income comensation.

The utility compensated effect of a consumption tax on current consumption is given by the following expression:

$$\frac{\partial C_{1t}}{\partial t_c}\Big|_{\bar{V}} = -\frac{(1+r)\ F(H)\ F''\ E(h''z_{t+1})\ E(h'\mu_{t+1})}{(1+t_c)^3\ F'\ \Delta}$$

<8A2.7>

Under the assumption of non-decreasing relative and non increasing absolute risk aversion both <8A2.6> and <8A2.7> are positive. These are reported in the main text as propositions 1.1 and 1.4.

Examining the expected utility compensated effect of a consumption tax on savings through its definition, i.e. $S_{1t}=Y_{1t} - wH_{1t} - (1+t_c)C_{1t}$ we obtain:

$$\frac{\partial S_{1t}}{\partial t_c}\Big|_{\overline{V}} = \frac{\partial Y_{1t}}{\partial t_c}\Big|_{\overline{V}} - w\frac{\partial H_{1t}}{\partial t_c}\Big|_{\overline{V}} - \Big[(1+t_c)\frac{\partial C_{1t}}{\partial t_c}\Big|_{\overline{V}} + C_{1t} \Big]$$

Substituting <8A2.4>, <8A2.6> and <8A2.7>, and after some manipulation we obtain:

$$\frac{\partial S_{1t}}{\partial t_c}\Big|_{\overline{V}} = \frac{S_{1t} + Y_{2t}(1+r)^{-1}}{(1+t_c)} + \frac{F(H)\ F''}{\Delta\ (1+t_c)^2\ F'} E(h'\mu_{t+1})\ [\ g''\ w + (1+r)\ F'\ E(h''\mu_{t+1})$$

<8A2.8>

A.VIII.2.3. The Specific Effects.

Differentiating the first order conditions with respect to t_c and utilizing the utility compensated and income effects we arrive at the following taxation effects:

$$\frac{\partial H_{1t}}{\partial t_c} = \frac{\partial H_{1t}}{\partial t_c}\Big|_{\overline{V}} - (1+t_c)^{-1}\ [Y_{1t}+Y_{2t}(1+r)^{-1}+\frac{wF(H)}{F'} - wH_{1t}]\ \frac{\partial H_{1t}}{\partial Y_{1t}}$$

<8A2.12>

and

$$\frac{\partial C_{1t}}{\partial t_c} = \frac{\partial C_{1t}}{\partial t_c}\Big|_{\overline{V}} - (1+t_c)^{-1}\ [\ Y_{1t}+(1+r)^{-1}Y_{2t}+\frac{wF(H)}{F'} - wH_{1t}\ ]\ \frac{\partial C_{1t}}{\partial Y_{1t}}$$

<8A2.13>

A.VIII.2.4. The CRRA Hypothesis.

Utilizing <8A2.8> we obtain for human capital investment:

$$\frac{\partial H_{1t}}{\partial t_c} = \frac{[g''+(1+r)E(h'')]E(h''z_{t+1}C_{2t})}{(1+t_c)^2\Delta} - C_{1t}\frac{\partial H_{1t}}{\partial Y_{1t}} + \frac{(1+r)E(h''C_{2t})}{g''}\frac{\partial H_{1t}}{\partial Y_{1t}}$$

<8A2.14>

For Current Consumption:

$$\frac{\partial C_{1t}}{\partial t_c} = -C_{1t}\frac{\partial C_{1t}}{\partial Y_{1t}} - \frac{(1+r)E(h''C_{2t})}{(1+t_c)g''}\left[1-(1+t_c)\frac{\partial C_{1t}}{\partial Y_{1t}}\right] + \frac{(1+r)E(h''z_{t+1})E(h''z_{t+1}C_{2t})}{(1+t_c)^3\Delta}$$

<8A2.15>

From the definition of constant relative risk aversion:

$$E(h''\, z_{t+1}\, C_{2t}) = 0$$

$$where \quad RRA = -\frac{h''C_{2t}}{h'} = -\frac{g''C_{1t}}{g''}$$

Substituting the above conditions into <8A2.14> and <8A2.15> yields:

For human capital:

$$\frac{\partial H_{1t}}{\partial t_c} = 0$$

<8A2.16>

For current consumption:

$$\frac{\partial C_{1t}}{\partial t_c} = - \frac{C_{1t}}{(1 + t_c)}$$

<8A2.17>

For savings:

$$\frac{\partial S_{1t}}{\partial t_c} = - w \frac{\partial H_{1t}}{\partial t_c} - (1 + t_c) \frac{\partial C_{1t}}{\partial t_c} - C_{1t}$$

<8A2.18>

Under CRRA savings:

$$\frac{\partial S_{1t}}{\partial t_c} = 0$$

<8A2.19>

A.VIII.3 The Pre-Payment Wage Tax.

Since a pre-payment wage tax is equivalent to a consumption tax we present only the CRRA results:

A.8.3.3. The CRRA Hypothesis.

For human capital:

$$\frac{\partial H_{1t}}{\partial t_w} = 0$$

<8A3 1>

For current consumption:

$$\frac{\partial C_{1t}}{\partial t_w} = - \frac{C_{1t}}{(1 - t_w)}$$

<8A3.2>

The total effect of an increase in the consumption tax on savings is given by:

$$\frac{\partial S_{1t}}{\partial t_w} = - (Y_{1t} - wH_{1t}) - (1-t_w) w \frac{\partial H_{1t}}{\partial t_w} - \frac{\partial C_{1t}}{\partial t_w}$$

<8A3.3>

Under CRRA savings:

$$\frac{\partial S_{1t}}{\partial t_w} = - \frac{S_{1t}}{(1-t_w)}$$

<8A3.4>

A wealth tax leads to discouragement of savings under CRRA assumption. Note that $S_{1t}(1-t_w)$ remains unaffected by taxation.

A.VIII.4. The Interest Tax

A.VIII.4.1. The Income Effects.

Differentiating the first order conditions with respect to first period exogenous income results in the following system of equations:

$$[\ \Delta\ ] \begin{bmatrix} \dfrac{\partial C_{1t}}{\partial Y_{1t}} \\ \dfrac{\partial H_{1t}}{\partial Y_{1t}} \end{bmatrix} = \begin{bmatrix} (1+r(1-t_r))^2\ (1-t_w)\ E(h'') \\ (1+r(1-t_r))\ (1-t_w)\ E(h'' z_{t+1}) \end{bmatrix}$$

<8A4.1>

Solving from <8A4.1> for the income effects yields:

$$\frac{\partial C_{1t}}{\partial Y_{1t}} = \frac{(1-t_w)(1+r(1-t_r))^2\ [\ (E(h'')\,[\,(1-t_w)\,E(h'\mu_{t+1}F'') + E(h'' z^2_{t+1})\,] - E(h'' z_{t+1})^2}{\Delta}$$

<8A4.2>

and

$$\frac{\partial H_{1t}}{\partial Y_{1t}} = -\frac{(1-t_w)\ (1+r(1-t_r))\ g''\ E(h'' z_{t+1})}{\Delta}$$

<8A4.3>

A.VIII.4.2. The Substitution Effects.

To obtain the constant expected - utility comparative static results we require that the household be compensated according to:

$$dY_{1t} = (1 - t_w)^{-1} (1 + r(1 - t_r))^{-1} \, r \, S_{1t} \, dt_r$$

<8A4.4>

Utilizing the above constant expected utility requirement and differentiating the first order conditions with respect to t_r yields the following system of equations:

$$[\Delta] \begin{bmatrix} \frac{\partial C_{1t}}{\partial t_r}|_{\bar{V}} \\ \frac{\partial H_{1t}}{\partial t_r}|_{\bar{V}} \end{bmatrix} = \begin{bmatrix} -r \, E(h') \\ -(1-t_w) \, w \, r \, E(h') \end{bmatrix}$$

<8A4.5>

The utility compensated effect of an interest income tax on human capital is given by the following expression:

$$\frac{\partial H_{1t}}{\partial t_r}|_{\bar{V}} = - \frac{r \, E(h')}{\Delta} \, [\, g'' \, w \, (1-t_w) + (1+r(1-t_r)) \, (1-t_w) \, F' \, E(h'' \mu_{t+1}) \,]$$

<8A4.6>

The utility compensated effect of an interest income tax on current consumption is:

$$\frac{\partial C_{1t}}{\partial t_r}\Big|_{\bar{V}} = -\frac{r\,E(h')}{g''(1-t_w)}\,\frac{\partial S_{1t}}{\partial Y_{1t}}$$

<8A4.7>

Examining the expected utility compensated effect of an interest tax on savings through its definition, i.e. $S_{1t}=(1-t_w)(Y_{1t} - wH_{1t}) - C_{1t}$

$$\frac{\partial S_{1t}}{\partial t_r}\Big|_{\bar{V}} = (1-t_w)\frac{\partial Y_{1t}}{\partial t_r}\Big|_{\bar{V}} - (1-t_w)\,w\,\frac{\partial H_{1t}}{\partial t_r}\Big|_{\bar{V}} - \frac{\partial C_{1t}}{\partial t_r}\Big|_{\bar{V}}$$

Substituting <8A4.4> we obtain:

$$\frac{\partial S_{1t}}{\partial t_r}\Big|_{\bar{V}} = r\,S_{1t}\,(1+r)^{-1} - (1-t_w)\,w\,\frac{\partial H_{1t}}{\partial t_r}\Big|_{\bar{V}} - \frac{\partial C_{1t}}{\partial t_r}\Big|_{\bar{V}}$$

<8A4.8>

A.VIII.4.3. The Specific Effects.

The effects of an interest income tax on the decision variables can be obtained by differentiating the first order conditions with respect to t_r.

A.VIII.5. The Income Tax

A.VIII.5.1. The Specific Effects.

Differentiating the first order conditions with respect to t_y we obtain:

$$[\Delta]\begin{bmatrix}\dfrac{\partial C_{1t}}{\partial t_y}\\[2ex] \dfrac{\partial H_{1t}}{\partial t_y}\end{bmatrix} = \begin{bmatrix} -(1+r(1-t_y))E(h''[Y_{2t}+\mu_{t+1}F(H)+(1+r(1-t_y))(Y_{1t}-wH_{1t})+rS_{1t}\;]\;)-rE(h') \\ E(h''z_{t+1}[Y_{2t}+\mu_{t+1}F(H)+(1+r(1-t_y))(Y_{1t}-wH_{1t})+rS_{1t}])-(1-t_y)wrE(h') \end{bmatrix}$$

<8A5.1>

where $z_{t+1}=(1-t_y)(\mu_{t+1}F'-(1+r(1-t_y))w)$

We can easily observe that the total effect of an income tax can be decomposed into the wage tax effect and an interest tax effect at equal tax rates $t_w=t_r=t_y$.

$$\frac{\partial X_{1t}}{\partial t_y} = \frac{\partial X_{1t}}{\partial t_w}\Big|^{t_w=t_r=t_y} + \frac{\partial X_{1t}}{\partial t_r}\Big|^{t_w=t_r=t_y}$$

where is $X_{1t} = H_{1t}, C_{1t}, S_{1t}$, respectively.

A.VIII.5.2 The Substitution Effects.

The substitution effects of an income tax can also be decomposed into the wealth tax and interest income tax effect evaluated at equal tax rates $t_w = t_r = t_y$

$$\frac{\partial X_{1t}}{\partial t_y}\Big|_{\bar{V}} = \frac{\partial X_{1t}}{\partial t_w}\Big|_{\bar{V}}^{t_w=t_r=t_y} + \frac{\partial X_{1t}}{\partial t_r}\Big|_{\bar{V}}^{t_w=t_r=t_y}$$