CPPI for Fixed Income Securities:
Empirical Evidence in Canada

Eric Martin

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- Ancient 0289
- Linguistics 0290
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- General 0401
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- Comparative 0295
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- American 0591
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- Canadian (English) 0352
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Religion
- General 0318
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- Clergy 0319
- History of 0320
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Theology 0469

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American Studies 0323
Anthropology
- Archaeology 0324
- Cultural 0326
- Physical 0327

Business Administration
- General 0310
- Accounting 0272
- Banking 0770
- Management 0454
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Canadian Studies 0385
Economics
- General 0501
- Agricultural 0503
- Commerce-Business 0505
- Finance 0508
- History 0509
- Labor 0510
- Theory 0511

Folklore 0358
Geography 0366
Gerontology 0351
History
- General 0578
- Ancient 0579
- Medieval 0581
- Modern 0582
- Black 0328
- African 0331
- Asia, Australia and Oceania 0332
- Canadian 0334
- European 0335
- Latin American 0336
- Middle Eastern 0333
- United States 0337

History of Science 0585
Law 0398
Political Science
- General 0615
- International Law and Relations 0616
- Public Administration 0617

Recreation 0814
Social Work 0452
Sociology
- General 0626
- Criminology and Penology 0627
- Demography 0938
- Ethnic and Racial Studies 0631
- Individual and Family Studies 0628
- Industrial and Labor Relations 0629
- Public and Social Welfare 0630
- Social Structure and Development 0700
- Theory and Methods 0344

Transportation 0709
Urban and Regional Planning 0999
Women's Studies 0453

THE SCIENCES AND ENGINEERING

BIOLOGICAL SCIENCES
Agriculture
- General 0473
- Agronomy 0285
- Animal Culture and Nutrition 0475
- Animal Pathology 0476
- Food Science and Technology 0359
- Forestry and Wildlife 0478
- Plant Culture 0479
- Plant Pathology 0480
- Plant Physiology 0817
- Range Management 0777
- Wood Technology 0746

Biology
- General 0306
- Anatomy 0287
- Biostatistics 0308
- Botany 0309
- Cell 0379
- Ecology 0329
- Entomology 0353
- Genetics 0369
- Limnology 0793
- Microbiology 0410
- Molecular 0307
- Neuroscience 0317
- Oceanography 0416
- Physiology 0433
- Radiation 0821
- Veterinary Science 0778
- Zoology 0472

Biophysics
- General 0786
- Medical 0760

EARTH SCIENCES
Biogeochemistry 0425
Geochemistry 0996
Geodesy 0370
Geology 0372
Geophysics 0373
Hydrology 0388
Mineralogy 0411
Paleobotany 0345
Paleoecology 0426
Paleontology 0418
Paleozoology 0985
Palynology 0427
Physical Geography 0368
Physical Oceanography 0415

HEALTH AND ENVIRONMENTAL SCIENCES
Environmental Sciences 0768
Health Sciences
- General 0566
- Audiology 0300
- Chemotherapy 0992
- Dentistry 0567
- Education 0350
- Hospital Management 0769
- Human Development 0758
- Immunology 0982
- Medicine and Surgery 0564
- Mental Health 0347
- Nursing 0569
- Nutrition 0570
- Obstetrics and Gynecology 0380
- Occupational Health and Therapy 0354
- Ophthalmology 0381
- Pathology 0571
- Pharmacology 0419
- Pharmacy 0572
- Physical Therapy 0382
- Public Health 0573
- Radiology 0574
- Recreation 0575
- Speech Pathology 0460
- Toxicology 0383

Home Economics 0386

PHYSICAL SCIENCES
Pure Sciences
Chemistry
- General 0485
- Agricultural 0749
- Analytical 0486
- Biochemistry 0487
- Inorganic 0488
- Nuclear 0738
- Organic 0490
- Pharmaceutical 0491
- Physical 0494
- Polymer 0495
- Radiation 0754

Mathematics 0405
Physics
- General 0605
- Acoustics 0986
- Astronomy and Astrophysics 0606
- Atmospheric Science 0608
- Atomic 0748
- Electronics and Electricity 0607
- Elementary Particles and High Energy 0798
- Fluid and Plasma 0759
- Molecular 0609
- Nuclear 0610
- Optics 0752
- Radiation 0756
- Solid State 0611

Statistics 0463

Applied Sciences
Applied Mechanics 0346
Computer Science 0984

Engineering
- General 0537
- Aerospace 0538
- Agricultural 0539
- Automotive 0540
- Biomedical 0541
- Chemical 0542
- Civil 0543
- Electronics and Electrical 0544
- Heat and Thermodynamics 0348
- Hydraulic 0545
- Industrial 0546
- Marine 0547
- Materials Science 0794
- Mechanical 0548
- Metallurgy 0743
- Mining 0551
- Nuclear 0552
- Packaging 0549
- Petroleum 0765
- Sanitary and Municipal 0554
- System Science 0790

Geotechnology 0428
Operations Research 0796
Plastics Technology 0795
Textile Technology 0994

PSYCHOLOGY
General 0621
Behavioral 0384
Clinical 0622
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ABSTRACT

CPPI for Fixed Income Securities:

Empirical Evidence in Canada

Eric Martin

Portfolio Insurance (PI) is a trading intensive strategy that attempts to protect a portfolio from falling below a prespecified level in an adverse market and to add return in a favourable environment. The primary objective of the this research is to see whether or not portfolio insurance (PI) can be used successfully as an asset allocation and risk management tool for fixed income securities. Many strategies exist to realize this objective. We propose to use Constant Proportion Portfolio Insurance (CPPI). Specifically, we want to see if CPPI can be used to manage a Canadian bond portfolio so as to reduce losses in a falling market and to add gains in a favourable market. CPPI is based on the work of Hakanoglu (1989), Black & Jones (1987) and Perold (1986).

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To my parents

for their encouragement, support and love

TABLE OF CONTENTS

TABLES, EXHIBITS AND GRAPHS

INTRODUCTION

Portfolio insurance (PI), as defined by Hill, Jain & Wood (1988), generally refers to:

> *an investment strategy that attempts to alter the payoff pattern of a portfolio of risky assets in a manner that significantly reduces the risk of return below a minimum protected level over a horizon chosen for implementation of the insurance program. PI strategies have the capacity to modify returns by adding positive skewness to the return payoff pattern.* (Hill, Jain & Wood, in Fabozzi (Ed), 1989, p.727)

Pioneered in 1980 by Leland, O'Brien and Rubinstein Associates, PI is based on the option pricing theory. It tries to replicate the payoff function of a put option on a portfolio (Rubinstein & Leland, 1981 & Rubinstein, 1985) through dynamic trading with a position in cash (T-Bills) and a risky asset. To replicate the delta of a put option, risky assets are purchased with cash (sold for cash) as they are gaining (losing) in value. Thus, PI becomes a powerful asset allocation and a risk management instrument as it permits one to seek higher yielding investments in favourable markets and to reduce the risky component in the portfolio in declining markets. In the best case scenario, the portfolio participates in most of the upward movement and, in a worst case, it becomes invested only in risk-free assets. The portfolio is then immunized at a risk-free rate guaranteeing a minimum return. This is similar to contingent immunization (CI) (Leibowitz & Weinberger, 1981) except for one difference:

Once the portfolio is forced to immunize under CI, the portfolio can never be reactivated even if the risky asset recovers. CI is like a stop loss order (Fong & Tang, 1988).

PI gained a lot of popularity in the 80's. It was the centre of considerable attention in both academic literature and the investment profession. The positive skewness and the technical innovation (Rubinstein & Leland, 1981 & Rubinstein, 1985) are the primary attractions of PI. They are the chief reasons why so much attention has been accorded to it among the investment management community (Hill, Jain & Wood, 1988).

This interest can easily be understood if we consider that most investors prefer higher expected return, lower risk in terms of standard deviation, and finally, positively skewed payoffs patterns (Leland, 1980). Thus, it is not surprising to see that a lot of money was poured into different PI instruments in the past decade. Estimates (in US Dollars) are:

Year		Assets Under PI[1]
1984		: $4 Billion
1985		: $4-$8 Billion
1986		: $27 Billion
1987	(Oct)	: $60-$90 Billion
1987	(Dec)	: $45 Billion

Table I.1

However, the enthusiasm for PI came to a halt with the 1987 stock market crash. While many investors and regulators blamed PI for the crash and its severity, some PI users were disappointed by the returns they got on their investments (Hakanoglu et al., 1989). Many of the assumptions on which PI is based did not hold during the crash, making PI programs impossible to execute. Nevertheless, the promise of a protected portfolio remains attractive, albeit with a new awareness of the risks.

Over the years a lot has been written on PI. Many articles explained how PI works: [Rubinstein (1985), O'Brien (1988)]. Some identified who would benefit from PI: [Brennan & Schwartz (1988), Leland (1980)]. Others designed new

[1]Pensions and Investment Age, different issues. Estimates of the amount of assets under PI are probably understated because many firms run their own PI program in house without reporting their activity. Moreover, the figures stated above do not include the amount of assets under casual hedging strategies or under stop loss orders (a primitive PI instrument). Figures after 1987 could not be obtained.

strategies or proposed modifications to older products: [Hakanoglu et al., (1989), Black & Perold (1989), Estep & Kritzman (1988), Black & Jones (1987), Etzioni (1986)]. While many tried to establish the true cost of portfolio Insurance: [Zhu & Kavee (1988), Garcia & Gould (1987), Rendleman & McEnally (1987), Black & Hakanoglu (1987), Black & Rouhani (1987)]; others explained its effect on the companies pension funds and their corporate balance sheet and income statement: [Black & Jones (1988), Somes & Zurack (1987)]. In fact, in light of the new accounting rule FASB 87 for corporate pension funds, surplus insurance (SI), an outgrowth of PI, was suggested as an alternative to PI. SI is similar to PI except that the riskless asset is replaced by an asset that mimics the portfolio's liabilities. Thus, the asset becomes riskless relative to the liabilities of the fund. It is believed that SI will absorb PI in the future [Knisley (1987)]. Finally, while many tried to explain the effects of PI on the markets: [Grossman (1988)], or to blame PI for the crash or for its amplitude, others found no conclusive evidence supporting such claim: [Leland (1988), Malkiel (1988), Kelley & Ramaswami (1988), Rubinstein (1988), Leland & Rubinstein (1988)].

The different strategies and products that have been designed to date fall in the following categories: Stop loss order, purchase of exchange traded put options, creation of synthetic put options, dynamic hedging (using futures contract) and CPPI (Constant Proportion Portfolio Insurance). A comparison of these different strategies can be found in Appendix 1. The great majority of articles covering these products, however, concern the equity markets. Very little was published on fixed income securities (FIS). We intend to redress the problem of the paucity of available material concerning PI for FIS.

The model of interest is based on CPPI. It was proposed by Black & Jones (1987) and Perold (1986) as an alternative to the more complex approaches of PI based on option replications (Black & Perold, 1989). CPPI replaces the option model by a linear trading rule. Hakanoglu (1989) later applied CPPI in 1989 to FIS. We intend to build on Hakanoglu's model for our work.

This paper is divided into four sections. In chapter 1, we will cover the objectives of this thesis. Chapter 2 is a review of the literature where the concept of PI will be addressed. Potential users of PI will be discussed and a review of prior studies conducted on the performance of PI strategies will be presented. A detailed description of the CPPI strategy will follow and other important issues concerning PI will be covered. In chapter 3, we will describe the model we will use in this simulation. The methodology involved will be reviewed and the data we will use will be examined. Finally, the analysis of the results will be presented in chapter 4.

1. OBJECTIVES

The overall objective of this study is to see whether or not PI can be used in the bond market as an asset allocation and a risk management tool. Building on the work of Hakanoglu et al. (1989), Black & Jones (1987) and Perold (1986), we propose a CPPI model that uses both duration and/or convexity to allocate assets and to control the risk-returns parameters.

The models will be tested over a twelve year horizon that covered all kinds of markets -a bull, bear and stagnant markets; with high, medium and low volatility- and where the yield curve was sometimes upward sloping and, at other times, downward sloping, flat or kinked.

We aim to examine CPPI for fixed income securities in several ways. First, we will test CPPI for FIS in a duration context. This merely repeats the work of Hakanoglu (1989). Then, we also want to add convexity to the duration measure to see if we can improve the end results. Laddered portfolio and barbell portfolios will also be tested using CPPI. The results of all these strategies will be compared to buy and hold portfolios of similar initial duration. The objective behind CPPI is to enhance the total return over a fixed allocation strategy like a Buy & Hold portfolio because CPPI allows the portfolio duration to be extended in favourable interest rate environments and reduced in declining bond markets.

The major difference between Hakanoglu's model and ours is that we do not allocate funds between a risky and a riskless index. Instead, we change the composition of our portfolios to meet the desired overall duration. Our portfolios are thus invested in several bonds while subject to a set of constraints.

All these strategies will share the same database of bonds. Transactions will take place in the cash market because the interest rate futures market only began in the late 80's in Canada and lacks sufficient liquidity even today.

The results will be analyzed on a yearly basis. The average results will be compared over the whole twelve year horizon for all strategies.

Since very few studies have been conducted or published on the topic of PI for FIS, we have many unanswered questions. Specifically, we would like to know what are the costs implied by CPPI? Can these costs be reduced? What kind of performance can be expected over the long run? Which version of CPPI will be the best suited? How trading intensive will this strategy be? Can improvements be made? We hope to provide answers to all these questions at the end of this study.

Our study, besides being broader than Hakanoglu's, is also more complete for several reasons. First, we will test the model over a period of twelve years during which different kinds of market with varying volatility levels were experienced in interest rates. Hakanoglu's results were obtained during a bullish bond market for the most part. Second, CPPI will be used on a real portfolio, not just a bond index which we think can distort the performance. Third, we are extending Hakanoglu's work by adding elements that we hope will fine tune the results. Finally, we will test CPPI using real Canadian data, not simulated, aggregated (like an index) or American data.

This research will help determine if CPPI for FIS can be used successfully in the management of fixed income securities. If the results are positive, CPPI could become a valuable tool to bond managers in improving the returns on their portfolio while controlling for the higher risks. The study could thus change the way some pension funds manage their fixed income portfolios.

2. REVIEW OF THE LITERATURE

2.1 Portfolio Insurance: The Concept

As we mentioned previously, PI is a strategy that attempts to protect a portfolio from falling below a prespecified level while, at the same time, enabling it to participate in most of the upside.

Unfortunately, because of the nature of portfolio risk, portfolio protection was never offered to investors by insurance companies. Portfolios are just too highly correlated with one another to be insured by risk-pooling. So, PI was never possible the old fashioned way.

Therefore, if insurance companies could not underwrite portfolio insurance, the market eventually would. The development of the option pricing theory and option market has enabled investors to buy insurance on their portfolios. A manager holding an index portfolio of risky securities can now purchase a listed put option on the same index to guarantee a minimum value on his portfolio at the expiration date of the option. However, while listed options are useful to create new payoff patterns, they sometimes have characteristics that are not necessarily desirable for portfolio insurers:

1) Listed puts only have standard strike prices. This may be undesirable for a PI users if the strike prices available do not correspond to their desired level of protection.

2) Listed options may lack sufficient liquidity to implement PI without disturbing the market.

3) Position limits exist for listed options which renders impossible the implementation of any PI program on a large scale.

4) Listed options are american. American options are more expensive because they can be exercised early. PI users, however, do not need this early exercise privilege as they wish to insure their portfolio at the end of a specific horizon interval. Therefore, they do not wish to pay more for a privilege they do not need. The use of american put options unnecessarily increases the cost of the insurance program.

5) Finally, index options have a maximum maturity of 3 months. Equity options have a maximum maturity of 9 months. Some large companies now have options with up to two years in maturity but their liquidity is very low. The problem with all these options is that their maturity is either too short or that it does not exactly correspond to the desired maturity of the PI user. Moreover, rolling over the position after the

expiration of an option increases the cost of the insurance program.

The creative applications developed by financial engineers have enabled portfolio managers to replicate options with a position in a risky asset and cash (T-bills). Rubinstein & Leland (1981) and Rubinstein (1985) suggested that a portfolio of risky securities can effectively be insured by shifting assets to a risky component as the portfolio is rising in value and to a riskless asset as the portfolio is falling in value. The precise amounts to shift are determined using option valuation formulae. Typically, insuring a portfolio of risky assets with a synthetic put involves combining the long position in a risky asset with a replicated put option. First, a put option is replicated by selling short risky assets and investing the proceeds at the risk-free rate. As the stock falls, more risky assets are sold short and when the stock rallies, the investor covers the short position by liquidating the risk-free asset and buying the risky asset. The exact amounts invested in the risky and risk-free assets are determined by the delta[2] of the option

[2]The delta is the amount by which an option's price will change for a corresponding change in price by the underlying entity. For out of the money options, the delta is very small. For at the money options, the delta is about 0.5 and for deep in the money options, the delta approaches unity. For example, a delta of 0.7 for a call option means that the option will move by $0.70 if the price of the underlying stock changes by $1. In order to replicate this option, we will need to own 0.7 share of the stock financed by borrowing at

being replicated. When one combines a long position with a synthetic put option, the net position is long less than one share of the risky asset and some investment in the risk-free security. At expiration, the delta of the put option is either unity if the risky asset is below the desired level of protection or 0 if it is above. In the first case, the portfolio is completely in the risk-free asset and insurance is provided. In the latter, the portfolio is fully invested in the risky asset and participating in the rise in the market. Under perfect market conditions, PI will produce the same outcome as a protective put option strategy.

The purchase of a protective put option, option replication and stop loss order are three strategies that can be used to protect a portfolio from falling below a prespecified floor. Appendix 1 covers each of these strategies in greater length.

A fourth strategy for portfolio protection is CPPI. CPPI was developed by Black and Jones (1987) and Perold (1986) as an alternative to the more complex, fixed time horizon PI strategies such as put option replication. Like option

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the risk free rate. A put option with a delta of 0.7 will move by $0.70 if the price of the underlying stock changes by $1. In order to replicate this option, we will need to be short 0.7 share of the stock with the proceed invested at the risk-free rate. Notice that the delta of a put option moves in the opposite direction of the underlying security.

replication, CPPI is also a trading intensive strategy. CPPI invests in risky and riskless assets by following a linear trading rule to insure the portfolio from falling below a prespecified level. In order to insure a portfolio using CPPI, an investor needs to determine two parameters: The level of protection (floor) desired at the end of an investment horizon and a constant multiple (M). The choice in the size of these two parameters will be discussed in sections 2.4 and 2.5. The difference between the value of the portfolio and the floor is the cushion (C). The amount invested in the risky asset (E) is the product of the cushion by a constant multiple (E=MC) where the multiple (M) is greater than one. Because the multiple is always held constant, the size of the cushion is what determines the exposure to the risky asset. If the cushion grows, the exposure to the risky assets will be increased and the portfolio will participate in the market rise. If the cushion shrinks, less risky assets will be held. Portfolio protection is provided because holdings in the safe asset are increased at low wealth levels. The formula is slightly modified for bond portfolios to include the notion of duration.

A characteristic of CPPI is that it is time invariant. Contrary to other PI strategies based on an option pricing model, CPPI does not have a stated maturity. In fact, time is never an input. CPPI is used the same way whether on an ongoing basis or with an expiration date. Time invariance is important because many institutional investment portfolios do not have a predetermined maturity (Brennan and Schwartz, 1988).

An important property of any PI strategy is that it must be path independent. A path independent strategy is a strategy whose payoff depends only on the value of the portfolio at the end of the PI program and on the parameters of the hedge. The payoff does not depend on the particular path taken by the portfolio over the course of the hedging period. Path independence is a desirable property of any PI program. A strategy that is not path independent gives an uncertain payoff (Bookstaber and Langsam, 1988). Here is an example that will clarify the concept of path independence for the reader.

A manager who invests in a portfolio of risky securities with a stop loss order[3] is buying PI on his portfolio. This manager, however, risks suffering from severe path dependence: Let us assume that this manager invests $100 in a portfolio of risky securities and places a stop loss order at $95. In the event that the portfolio would fall in value, the manager would cut his loss to 5%. Let us assume two potential price

[3]A stop loss order is a conditional market order to sell the underlying security if its value drops to a given level.

paths for this portfolio. In the first case, the price will go from $100 to $110. In the second, the price of the portfolio of risky securities will first go from $100 to $93 and then straight to $110. In both cases, the portfolio of risky securities has achieved a return of 10%. But in the second case, the stop loss order was executed and the manager was left with $95 instead of $110. As the reader can see, if the prices rebound after the execution of the market order, the managers cannot benefit from the increase in the price of the risky securities. On the other hand, if the prices of the portfolio of risky securities keep on falling, the position of the manager is enhanced. Hence, the success of this PI strategy is dependant on the market movement subsequent to the execution of the market order. The main contributing factor to this path dependence is that all investment funds are transferred from risky to riskless asset at one time in an irreversible manner (Bird, Dennis & Tipett, 1988). Ideally, PI should work regardless of the subsequent move of the risky asset.

Fortunately, path independent strategies exist. The purchase of a listed put on a portfolio of risky securities is one such strategy. CPPI is another. Black and Perold (1989) showed that CPPI follows a weak form of path independence[4].

[4]CPPI follows a weak form of path dependence due to the transaction costs and the slippage involved.

Yet, CPPI can become path dependant if, after the portfolio becomes fully invested in the risky asset, it is not allowed, as prescribed by the model, to further increase its investments in the risky asset due to borrowing constraints. Such a situation, however, rarely occurs. Furthermore, such an insurance program would be ill designed to begin with because it would not offer much protection[5].

Finally, it is important to stress the key assumptions that lie behind PI, CPPI, CPPI for FIS and all other dynamic asset allocation strategies. For all these trading intensive strategies, prices must be continuous with no upward or downward price jumps. Liquidity in the financial assets is paramount: individual portfolio managers must be able to trade without affecting the market price. Lastly, an orderly market must be present where volatility does not fluctuate drastically (Leland, 1988). All these suppose that the market will move slowly (not jump) which will allow the manager to change his positions. If any of these assumptions are seriously violated, the effective implementation of any of these strategies will be undermined.

As for the potential impact of PI on the market, Grossman (1988) argues that a growing demand for PI increases the level of volatility in the price of risky asset. Because an

[5]Because the floor is too low.

investor using an insurance strategy will reduce equity exposure following a price fall (or bad news) and increase equity following a price rise (or good news), his holdings in the risky assets, compared to those of the value investors, will be a more volatile function of news announcements.

If the demanders of insurance use listed put options to implement their strategy, their net equity exposure will change automatically as equity prices change. But, if we assume for a minute that there are no offsetting suppliers of insurance, the sellers of puts will want to dynamically hedge their put options. They will accomplish this by selling equities as prices fall and buying equities when prices rise. This strategy is the same as creating synthetic insurance. If there is a sizeable demand for portfolio insurance, a sizeable number of puts will need to be hedged in the same manner as above. This will lead to a sizeable supply of equity offered upon the arrival of bad news or sizeable amount of equity demanded upon the arrival of good news.

But, because all investors (or a large number of investors) cannot sell (buy) equities all at the same time following bad (good) news or a price fall (rise), sellers of puts understand that the larger the number of portfolio insurers, the higher the future price volatility and the more expensive the hedging strategy will be. Therefore, recognizing that these strategies will be more expensive to carry out, the sellers will increase the price of put options.

Up to this point, we see that a net demand of insurance will raise the stock price volatility, whether insurance is implemented by a dynamic trading strategy of the investors themselves or by dynamic hedging of the put sellers. But, the difference between the two strategies is that the demand for PI may be much smaller with listed puts than with insurance implemented dynamically. When the demand for insurance increases, the price of listed puts will go up. Facing the true cost of portfolio insurance, the potential users of PI will lower their demands of insurance in the face of higher put prices. The increase in the cost of puts will dampen the demand for insurance. Grossman's point is that there are no such constraining force when insurance is created synthetically. Instead, the user of synthetic insurance attempts to replicate a put option based on some estimate of volatility (often based on the implied volatility of listed options) which ignores how many other investors intend to carry out a similar strategy. It is the ignorance of the size of the demand for insurance that may significantly raise the level of future volatility in the market and, as a result, significantly raise the cost of these strategies.

Hill and Jones (1988) also believe that PI raises the level of volatility in the market. By reinforcing market moves, PI will destabilize the market, at least on a short term basis, until enough capital can be committed by value based investors to take the opposite side. The increase in the level of volatility is due to the fact that PI is implemented in a very short time lag while value based strategies take more time to be carried out. It takes more time (and possibly some price concession) for value based investors to be induced to take the cpposite side. The authors, however, recognize that except in the case of a large stock index move, PI trades are designed to reduce the equity exposure gradually, relative to what might otherwise have resulted from shifting large amount of funds in a single transaction.

Hill and Jones suggest that the level of volatility due to PI would be reduced if the time lag to implement PI strategies were lengthened, the time lag to implement value based strategies were shortened or finally, the amount of money under value based strategies were increased. They also salute the adoption of "circuit breakers" in the futures and stock markets to allow the markets to cool off after large price swings.

Grossman (1988), on the other hand, favours a mechanism where enough information can be transmitted to value based investors to take the opposite side. If value investors are made aware that the selling is done for hedging purposes and not based on negative information, value based investors should be willing to commit more funds more rapidly to the market. This will reduce the price volatility caused by PI. Leland (1988) shares Grossman's view. He and his firm have even proposed a mechanism (sunshine trading) for portfolio insurers to preannounce their trading requirements.

After the 1987 stock market crash, many people blamed PI for the severity of the decline. In a survey by the Brady Commission (1988), PI, among other factors, was one of the most cited reason for the cause of the crash. But was PI really responsible for the crash? According to Leland (1988), the answer is no. PI is a reactive strategy for conservative investors. The market had already started to fall following bad economic news: Twin deficits, changes in the new tax legislation, increases of interest rates, etc. A more relevant question is: Did PI contribute significantly to the crash? Any investor reducing holdings of stocks, shorting stocks, covering long futures positions or initiating short futures positions contributed to the crash. Based on this, PI contributed to the crash. In fact, PI contributed to 15% of the volume on October 19. But, is 15% of the volume responsible for more than 15% of the decline. The question is difficult to answer. Leland says that it is hard to blame PI when there were so many other sellers. Rubinstein (1988) argues that it was well known well before the crash that there was between $60 and $80 billion of assets managed under PI. Therefore, the selling due to PI should have been expected. Much more dangerous to the market are investors who unexpectedly revise their positions in the same direction of the market, catching the market unprepared. Rubinstein also says that all markets around the world suffered severe declines. Yet, in most of these markets, PI plays a very small role (if any role at all). This can certainly not support the notion that PI was responsible for the crash. The Brady Commission acknowledges that PI played a role in the crash but does not identify it as the main culprit.

Instead, Leland (1988) blames the inability of market makers to provide short term liquidity:

> *"Relative to trading volume, short-term market making capital has fallen dramatically. The Brady Commission reports that the NYSE specialists had only one third the capital (relative to trading volume) in 1987 as they had in 1977... As the trading costs fell and more investors chose to institute hedging strategies, the potential for greater liquidity demands grew... Yet there was not a concomitant growth in the market-making capital which could meet the potential demand for liquidity... On October 19th, the amount of selling by PI alone far overwhelmed the total capital (estimated at $4 billion) of all specialists combined. And of course, there were many other sellers as well. In sum, the short-run supply of liquidity available from market-makers was incapable of handling the demands from liquidity traders."* (Leland, California Management Review, Summer 1988, pp.88-89).

Rubinstein also cites the undercapitalization of market makers and specialists as one reason for the market crash.

Thus, the effect of PI on the market is a controversial issue. The fact that the use of PI was reduced by 25% to 50% (see the figures in the introduction) following the market crash indicates at least that portfolio managers realize that PI can raise the level of market volatility.

2.2 Who Should Use Portfolio Insurance?

As we mentioned in our introduction, PI does not suit all investors. But, investors who prefer higher return to less, lower risk in terms of standard deviation to higher, and finally, positively skewed payoffs patterns to symmetric or negatively skewed payoffs may be attracted by the features of PI.

Leland also concluded that PI is well suited for all investors whose utility function is described as follows:

> *1- Investors who have average expectations, but whose risk tolerance increases with wealth more rapidly than average.*
> or
> *2- investors who have average risk tolerance, but whose expectations of returns are more optimistic than average.* (Leland, Journal of Finance, 1980, p.582)

According to Leland, both types of investors will want to hold PI.

> *Institutional investors falling in class 1 might include pension or endowment funds which at all cost must exceed a minimum value, but thereafter can accept reasonable risks... Investors falling in class 2 might include well diversified funds which believe themselves to have positive "α" (i.e. funds which expects on average to achieve excess returns by superior stock selections). In order to exploit these excess returns, but at the same time keep risk within tolerable levels, insured type strategies are optimal.* (Leland, Journal of Finance, 1980, p.580)

Investors in class 2 are likely to pursue a more aggressive strategy knowing that the downside risks are better controlled (Leland & Rubinstein, 1988 and Jacques, 1987). Both types of investors are candidates for PI. They share a utility function whose payoffs is convex at higher wealth level and concave at lower wealth level (Perold, 1988). This is also consistent with the rule of thumb many investors follow: Run with your winners and cut your losses or with a safety first approach (Leland, 1980).

Additional examples of investors who could benefit from portfolio insurance include portfolio managers whose investment performance is monitored on a periodic basis and where a poor performance would be heavily penalized (Brennan & Schwartz, 1988). Others include **corporation** who must add any surplus or deficit in their **corporate pension funds** to their own balance sheet and income statement (Dreher, 1988;

Wagner, 1988; Somes & Zurack, 1987 and Kritzman, 1986). In brief, any institution that is involved in asset/liability management could benefit from some form of portfolio or surplus insurance.

Because PI is not a true form of insurance where everyone can benefit from the pooling of independent risk but rather is insurance against a common market risk, PI can only be obtained from the market. For every buyer of PI there must be a seller. Those sellers are considered as value investors. They prefer "buying low" when the demand for risky asset is weak and "selling high" when the demand for those same assets is strong. To them selling low and buying high makes little sense. These investors do not need a floor on their investments. Rather, they favour capital gains realized by buying low and selling high. Their payoff function is said to be concave since the payoff increases at a decreasing rate with the value of the underlying risky asset (Perold, 1988). Black and Hakanoglu (1989) have simulated an experiment in a world with only PI buyers and sellers. They found that buyers make more money in trending markets, up or down, while sellers profit more in volatile non-trending market. Sellers also profit more in stationary markets, the higher the multiple (the trades are larger) and the lower the tolerance level (which increases the number of trades), because they buy low and sell high. Perold (1988) found similar results.

PI can also be useful in the FIS market. While bonds are less risky than stocks, the increasing volatility in the bond market in the past 15 years has been such that hedging strategies may be desirable to control risk. For institutions who strictly invest in fixed income securities, a program of PI based on duration can be a great tool to manage a fixed income portfolio against interest rate risk. For institutions who deal with asset/liability management in a FIS context, SI can provide the protection needed. PI (SI) will allocate funds to the best performing asset. Thereby, in a favourable interest rate environment, a manager can seek higher return by extending the portfolio (surplus) duration. Conversely, in a declining market, PI (SI) will systematically protect the value of the portfolio (surplus) from falling below a predetermined level by reducing the portfolio (surplus) duration. Thus, the minimum return will never be in a position of being violated and liabilities will be funded at their expiration even in an adverse bond market. PI in the FIS market becomes an asset-liability management tool. Example of institutions who can benefit from PI in a FIS environment include **life insurance companies** that must fund annuities and life insurance claims, **banks** who must fund guaranteed investment certificates (GIC) and deposits and finally, **mutual funds companies** who invest their funds in FIS with a certain objective in mind (i.e. beat a five year duration index for example). All these investors must meet a certain liability by the end of a time period (Brennan & Schwartz, 1988). PI for FIS can also be used to manage interest rate risk and credit risk simultaneously. This was demonstrated by Hakanoglu et al. (1989).

2.3 Review of Studies Conducted on Portfolio Insurance

Garcia and Gould (1987):

A study was conducted on the performance and the cost of PI implemented via synthetic options. Garcia and Gould tried to determine the cost of PI in terms of foregone returns. They also addressed the question of the distribution of returns and of the dispersion of the costs. The study also considered the effect of different floors, of different levels of interest rates and transaction costs.

In order to perform their study, Garcia and Gould generated returns for the S&P 500 for 240 overlapping years by taking, first, 20 January to January years (from January 1963 to December 1983), then 20 February to February years, and so on. Reinvested dividends and short term interest rates were used to evaluate the performance of two hedged portfolios. The PI models had a zero and a -5% floor. PI was implemented with a dynamic hedging strategy on the S&P 500 Index and T-Bills. Portfolios were rebalanced by first assuming no transaction costs and then by assuming one-way transaction cost of 0.5%. The portfolios with insurance are directly compared with a fully invested portfolio.

The results indicate that, on average, PI with a zero and a -5% floor underperformed a fully invested portfolio by 170 bps and 83 bps respectively. A static mix strategy (50%/50%) would have dominated a hedged portfolio with a -5% floor. To the extent that the transaction costs employed are too high, the long run cost may be biased upward. Simulations with no transaction cost would have dominated a fully invested portfolio by 51 bps and 68 bps for a zero and a -5% floor respectively.

Then, the authors divided the results into two groups. The years where the S&P 500 portfolio dominated the hedged portfolio (shortfall) and the years where the hedged portfolio outperformed the S&P 500 portfolio (excess). The average excess was 7.21% and 3.98% and the average shortfall 7.19% and 3.85% for the hedged portfolio with a floor of zero and -5% respectively. Ignoring transaction costs, the average excess was 4.29% and 2.09% and the average shortfall 4.49% and 2.13% respectively.

Looking at the distribution of these shortfalls and excesses, a hedged portfolio with a zero floor never had a shortfall greater than 20%. Its shortfall exceeded 10% in 21%

of the 177 years with a shortfall. The shortfall exceeded 15% in 6.2% of the years. The median of the distribution of the shortfall lied between -10 and -5%. The excesses, on the other hand, exceeded 15% in 17.2% and 10% in 48% of the 64 years respectively.

A hedged portfolio with a -5% floor, missed the upside by 10% in only 5 years (2.8%) and exceeded the S&P by 10% in 17.2% of the years with an excess (64). The median for the shortfall is between -5% and 0. The distribution of the returns were much more dispersed with a zero floor. This may be due to the high transaction costs. To the extent that transaction costs are too high, the distribution of the returns for a portfolio with a zero floor would be much less dispersed. Based on the above results and the fact that up years occur 2.75 times more often than a down year, PI with a -5% floor seems more appropriate than with a zero floor.

A high interest rate decreases the cost of PI as more of the portfolio can be allocated to the risky asset but it does not decrease the average shortfall. The authors believe that the advantages of high interest rates are offset by volatility changes. A high level of volatility for the risky asset has a mitigated effect: It raises the cost of PI in up years but decreases the cost in down years.

Finally, the authors found that PI is an expensive strategy. While PI is for a conservative set of investors, Garcia and Gould conclude that, based on their results, PI is more appropriate for less risk-averse investors: Over the long run, investors willing to tolerate losses of 5% per year will wind up with better results from PI than clients willing to lose nothing.

Zhu & Kavee (1988):

Zhu & Kavee studied the impact of under/overestimating the level of volatility in the use of synthetic put options. Like Grossman (1988), they found that the cost of insurance increases when the market volatility was underestimated. But worse, underestimating the volatility level can threaten the strategy from delivering the promised level of protection. When the volatility was slightly underestimated, the effect was trivial but when the volatility is seriously underestimated, the strategy can fail to provide the promised level of protection by a relatively important margin. For example, when the volatility was underestimated by 10%, the level of protection can be missed by as much as 8%.

The results arising from overestimating the market volatility also increases the cost of insurance but its effects on the level of protection provided is much less serious than with underestimating it. An overestimation means that we pay a higher premium for the protection which translates into a lower overall expected return. This is because a greater proportion of the portfolio is invested in the riskless asset. An overestimation of volatility seems preferable than an underestimation. Black and Rouhani (1989) came to similar conclusions.

Zhu and Kavee also compared the performance of CPPI and synthetic put options (SP) in the equity market. Using a Montecarlo simulation, the authors generated 1000 simulations with 250 lognormally distributed daily observations. They used the following assumptions: Annual mean return of 15%, paired with different values for market volatility; a tolerance level of 5%; rebalancing was based on market movement; the strike price was 100 for the synthetic option; the time interval was 1 year; the comparison was done versus a portfolio fully invested in the risky assets and CPPI; the CPPI portfolio had a multiple of 2.5 and a floor of 80.

The authors have found that both strategies manage to reduce downward risk and retain part of the upward gains. However, the ability to reshape the return distribution is slightly different for the two strategies. CPPI is able to provide a better protection than SP. With a market volatility of 15%, 31.2% of the total returns with a SP strategy were below a zero floor and 1.7% below a -5% floor. For CPPI, the figures were 14.4% and 4.1% respectively. The better performance of CPPI at reducing downward risk was achieved by giving up more upward gains. As for the average return, SP dominated CPPI by 88 bps. Relative to a fully invested portfolio in the risky assets, the total cost of insurance came to 300 and 388 bps for SP and CPPI respectively. In a separate study, Black and Rouhani (1989) found that neither strategy outperforms the other all the time. SP's payoff is greater when the market increases moderately. CPPI performs better if the market drops or increases by a small or a large amount.

In more volatile environment, both strategies become more expensive. Black and Rouhani explain that this is due to the fact that both strategies must be rebalanced more frequently which increases transaction costs. Furthermore, in more volatile environment, protecting the portfolio becomes more arduous. Both strategies can fail to deliver the protection they promised by a larger margin which increases the total cost.

Rendleman & McEnally (1987):

Rendleman & McEnally have attempted to determine the long term cost of PI (with futures) compared to an optimal portfolio designed to maximize the rate of growth over time.

In their paper, the authors also studied the impact of raising the floor, of changing the level of interest rates obtainable on the risk-free asset, of reducing the level of volatility on the risky asset and of lengthening the horizon period on the performance of PI. They compared the performance of a PI strategy to an optimal portfolio. They also considered the distribution of returns for PI vis-à-vis an optimal portfolio.

Based on several assumptions (like expected return, standard deviation for the risky asset, risk-free rate), the authors have generated 1000 years of returns. These returns were used to evaluate the average performance of a portfolio with (different levels of) insurance and of another that maximizes the compound rate of growth (optimal portfolio). The performance of these portfolios was recorded under varying market conditions. Transaction costs and taxes were ignored. The probability distribution of returns on risky assets was assumed to be lognormal. PI was implemented through the use of a dynamic hedging strategy.

A portfolio insured over a period of one year in a 6% interest rate environment, with a floor of -5%, an expected return of 12% and a standard deviation of 18% for the risky asset, would in the long term return 9.06%. In the same circumstances, an optimal portfolio would return 10.57%. The cost, the difference in foregone return, is 151 bps.

Under different assumptions, the performance and the long term costs of PI changes noticeably. For example, a lower floor will decrease the long term cost relative to the optimal portfolio because a higher proportion of the portfolio can be allocated to the risky asset. The cost will also fall as the investment horizon is lengthened. Finally, the cost of PI will be higher in a low interest rate environment because a larger percentage of the portfolio must be allocated to the risk-free asset to guarantee the floor at the end of the insurance horizon.

The higher cost of PI must be balanced with the benefits of protection in years of poor return. That is why the authors have looked at probability distributions. Based on 1000 simulated periods, they found that over an investment horizon of one year, in an interest rate environment of 6%, a PI strategy with a floor of -5%, an expected return of 12% and a standard deviation of 18% for the risky asset, would just meet the floor 16% of the time. Under these assumptions, the return of the optimal portfolio will fall below the minimum return in 27% of the cases. The insured portfolio will also outperform the optimal portfolio 41% of the time.

As for the cost of PI, the probability distribution changes as we modify the inputs. The probability of the insured portfolio just meeting the floor increases as the floor is raised. The frequency of the optimal portfolio falling below the floor and the probability of the insured portfolio outperforming the optimal portfolio both increase as level of protection is raised. Increasing the length of the investment horizon reduces the probability of the insured portfolio just meeting the floor. A longer investment horizon will also reduce the frequency of the optimal portfolio falling below the floor and the likelihood of the insured portfolio outperforming the optimal portfolio. Finally, reducing the level of interest rate increases the frequency of the insured portfolio of just meeting the floor.

The authors concluded that a portfolio managed with PI will generate a lower compound rate of growth than a comparable optimal portfolio. Given the high long term cost of foregone returns, PI is for highly risk averse investors.

Hakanoglu et al (1989):

Hakanoglu et al (or Hakanoglu) proposed to use a modified version of CPPI for fixed income securities. In this paper, the authors describe how they adjusted the CPPI model to include the notion of duration (we will cover the concept in greater details in section 2.4 of this paper). The objective is to systematically reduce the duration in bear markets so as to protect the portfolio from falling below a prespecified floor and to increase it in bull times to take advantage of rising prices. Furthermore, it is possible, as in Hakanoglu's simulation, to use instruments of different credit quality to take advantage of higher risk premium.

Hakanoglu tested his model over a period of 6 years from 1982 to 1987. He started with a portfolio of $100 (million), a floor of $90 and a multiple of 5. Initially, half of his portfolio was invested in the active asset and the other half in the reserve asset. The active asset consisted of a basket of investment grade corporate bonds of 6.2 years in duration and the reserve asset of treasury bonds of 5 years in duration. At the end of every week, the amount allocated in the active and the reserve asset was revised based on the CPPI relationship.

The results show that CPPI portfolio was successful in adding gains of 279 bps relative to a buy and hold strategy[6]. The average cost (or underperformance) relative to a fully invested portfolio in the basket of corporate bonds was 126 bps. Hakanoglu also varied the tolerance level to see which one would yield the best performance. Although the difference

[6]It is important to note that the comparison between the CPPI portfolio and the buy and hold (B&H) strategy is difficult to make given that the B&H portfolio consisted of 90% treasury bonds and 10% of corporate bonds. Thus, the comparison is hard to justify since, compared to the CPPI portfolio, the B&H strategy was invested in securities of smaller duration and of smaller risk premium (greater credit quality).

was small, the best performance was achieved with a tolerance level of 5%.

Even though the comparison with a B&H strategy provided by Hakanoglu is difficult to justify, his development of the CPPI model for FIS is interesting and warrants further testing.

2.4 Description of CPPI

In this section, we will define the concept of CPPI and CPPI for FIS and explain how the strategy works.

Traditional CPPI

CPPI, like most PI products allocates assets dynamically over time. It seeks to give investors the ability to limit downside risk while allowing some participation in upside markets. CPPI, like other dynamic asset allocation strategy can beat a fixed allocation strategy because they react to a rising market by increasing a portfolio's exposure to risky assets, and do just the opposite in a falling market.

The approach is easy to understand and straightforward to implement. Let us introduce some key concepts:

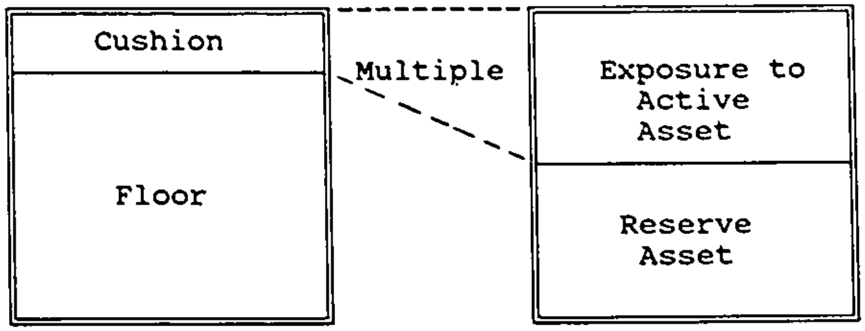
E=MC
E = Exposure C = Cushion M = Multiple

Traditional CPPI
Exhibit 2.4.1

Reserve Asset	:	(R)	Asset that has an acceptable minimum rate of return. It is often referred to as the safe or riskless part of the portfolio.
Active Asset	:	(A)	Asset whose expected return exceeds that of the reserve asset.
Floor	:	(F)	Lowest value of the portfolio or the present value of the liabilities.
Cushion	:	(C)	Portfolio value minus floor.
Exposure	:	(E)	Amount in the active asset.
Multiple	:	(M)	Exposure divided by cushion.
Tolerance	:		Percentage difference between actual and target exposure that triggers a trade.
Limit	:		Maximum percentage of the portfolio in the active asset.

First, in order to use CPPI, it is important to view our CPPI portfolio as two separate portfolios: one containing the reserve asset and the other the active asset. The allocation between the two asset classes depends on the floor, the cushion and the multiple. The floor is set at the lowest acceptable value the CPPI portfolio is allowed to take and the cushion is the difference between the portfolio value and the floor. The amount in the active asset, the exposure, is the product of the cushion times a predetermined, constant multiple. Exhibit 2.4.2 shows this relationship.

TOTAL PORTFOLIO VALUE ASSET ALLOCATION



CPPI PORTFOLIO
Exhibit 2.4.2.

The choice in the reserve asset will depend on the active asset. If the active assets are stocks, reserve assets could be treasury bonds or treasury bills; if on the other hand, the active assets are bonds (corporate or government), the reserve assets should be an immunized portfolio with a duration equal to the investment horizon (Fong & Tang, 1988). When the portfolio is part of a pension plan, SI should be used. The reserve asset becomes a portfolio of securities that mimics the pension liabilities (Black & Jones, 1988; Kritzman, 1986). The reserve portfolio is risk-free relative to the pension liabilities.

The next question is how do you determine how much to put in the reserve and in the active assets? This depends on the multiple and the desired floor of the fund which in turn depends on the risk aversion and the investment objectives of the investors and/or the liability stream of the pension fund.

The idea behind CPPI is to invest the cushion in the risky asset (with an expected rate of return greater than that of the reserve asset) and the floor at the risk-free rate. However, this means investing a relatively small portion of the CPPI portfolio in the active asset. The investors may wish to have a greater exposure to the latter especially if the floor can still be guaranteed with high probabilities. To increase the exposure, one can trade the assets in the two portfolios in a way that replicates a put option with a strike price equal to the floor. This will make sure that the portfolio value never falls below the floor while allowing the exposure to the risky asset to increase in favourable markets. However, imitating a put option requires using complex option formulas and dealing with a definite expiration date. CPPI provides a way to increase exposure to the risky asset, makes sure the portfolio will never fall below the floor and does not require any expiration date (Black, 1987).

As the words <u>Constant</u> and <u>Proportion</u> imply, CPPI requires the multiple to be held constant at all time. That is, the exposure to the risky assets is always a constant multiple times the cushion (E=MC). But, as the market changes, so do the exposure, the cushion and the multiple. This triggers trading in the active and in the reserve asset to restore the balance (multiple) between the cushion and the exposure. When the market goes up (falls), the manager must increase (decrease) his exposure to the risky asset to maintain the multiple constant. The manager is forced to sell (buy) some risk-free asset and purchase (sell) some risky asset until the multiple is restored. Hence, the investor becomes fully invested in the active asset in bull markets and in the reserve asset in bear markets. This insures the CPPI portfolio as the cushion is managed so as to never fall below zero in value. If the cushion falls to zero, the portfolio will become immunized, being only invested in the reserve asset. The actual allocation of the funds between the risky and the risk-free assets is determined by changing market conditions or the changing requirements of the portfolio managers. Let us use an example to demonstrate this relationship:

We have an initial portfolio of $100. Our initial floor (PV of the liabilities) is $90. With a multiple of 5 and a initial cushion of $10, the exposure to the risky asset is $50 (E=MC) while the remaining $50 are allocated to the risk-free asset. A week later, the prices have changed. The value of the portfolio is now $103 ($98). The risky asset is now worth $52 ($47) and the reserve asset $51. The floor has grown to $91 and the cushion to $12 ($7). In order to restore the multiple to 5, we must readjust the exposure to 5 times the cushion. In this case, we will readjust the portfolio to hold $60 ($35) in risky asset and $43 ($63) in the reserve asset.

The higher (lower) the multiple, the more (less) the investors will participate in market upside and the faster (slower) the portfolio will approach the floor when the market falls. The higher (lower) the multiple, the more (less) the portfolio will lose in a choppy market. This is because CPPI buys high and sells low. The more (less) risk averse the investor, the lower (higher) the multiple and/or the higher (lower) the floor. The higher the investor's expected return on the risky asset, the higher the multiple should be.

Since by definition, the exposure is always a constant multiple of the cushion, when the cushion approaches zero the exposure also approaches zero. This is true regardless of the size of the multiple being used. This will ensure that the CPPI portfolio will not fall below zero (Assuming no downward price jumps). As the cushion rises, the portfolio will eventually approach 100% in the risky asset. If the total portfolio is capped, it will have reached its limit and CPPI will be in a buy-and-hold stage (Black & Perold, 1989). If it is not, the whole portfolio will become leveraged: the portfolio will borrow at the risk-free rate and be completely (more than 100%) invested in the active asset. The limit states the maximum percentage of the portfolio in the active asset. Most funds have a limit of 100% (or a lower limit) and will not permit the CPPI portfolio to become leveraged.

Finally, CPPI is a trading intensive strategy. This means that a tolerance level is needed to avoid continuous rebalancing (which is not practical). This tolerance level is used as a filter to avoid whipsaws in choppy markets. This will reduce trading costs (commissions, buy high sell low) as rebalancing is triggered by small market fluctuations. The lower (higher) the tolerance level, the more (less) closely the exposure is related to the cushion, the more (less) frequent the trades but the smaller (larger) their size.

CPPI for FIS or Duration Adjusted CPPI

Unlike individual stocks whose prices are a function of many different factors, bond prices depend mainly on interest rates[7]: The amount by which bond prices vary is usually a function of duration, a proxy for interest rate sensitivity. The higher the duration, the larger the bond price fluctuation given an interest change[8]:

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[7]Additional factors affecting bond prices are:

- risk premium
- default premium
- maturity risk
- liquidity risk
- volatility risk
- inflation risk.

[8]Simon Benninga, "Numerical Techniques in Finance", MIT Press, 1989.

$$\Delta P \cong -D_{mod} \cdot P \cdot \Delta Y$$

Where $D_{mod} = \dfrac{D}{(1+YTM/k)}$

and

$$D = \frac{\sum_{t=1}^{n} PVCF_t \cdot t}{PVTCF \cdot k}$$

$PVCF_t$ = Present Value of the Cash flow in period t discounted at the yield to maturity
PVTCF = Price of the bond
t = the period when the cash flow is expected to be received (t = 1,...,n)
n = number of period until maturity
k = number of periods per year
YTM = Yield to Maturity
ΔY = Yield Change
In this study, the modified duration measure will be used for all simulations.

Modified Duration
Exhibit 2.4.3

Since the interest rate structure is normally upward sloping, higher returns can be expected in the longer range of the yield curve but at the cost of higher volatility. Thus, CPPI for FIS (or duration adjusted CPPI) assumes that higher expected return can be obtained in a flat or falling (rising) interest rate environment by extending (reducing) the duration of the portfolio. This assumes that the yield curve moves in a more or less parallel fashion.

Let us define the key concepts and then explain how our model works[9].

[9]The reader can refer to Appendix 10 to see how this equation was developed.

$$Ed_p = m_d c$$

Ed_p = Duration Exposure
m_d = Duration Adjusted Multiple
c = Proportional Cushion

CPPI for FIS
Exhibit 2.4.4

Duration of the Reserve	:	(D_r)	Reserve asset duration
Duration of the Active	:	(D_a)	Active asset duration
Multiple	:	(M)	First term in the Duration Adjusted Multiple equation
Duration Adjusted Multiple	:	(m_d)	Duration Exposure divided by the Proportional Cushion $m_d = \tilde{M} \cdot (D_a - D_r)$
Proportional Cushion	:	(c)	Cushion divided by the portfolio value
Duration Exposure times	:	(Ed_p)	Duration adjusted multiple proportional cushion
Portfolio Duration	:	(D_p)	Weighted Average of Activeand Reserve asset duration $D_p = D_r + Ed_p$
Limit	:		Maximum percentage of portfolio in the active asset
Tolerance	:		Percentage difference between actual and target duration that triggers a trade

The general concept of CPPI does not change: duration-adjusted CPPI shares many similarities with traditional CPPI. The manager will invest part of the portfolio in a <u>reserve asset</u> that has a minimum acceptable rate of return and the rest in the <u>active asset</u> which has an expected rate of return greater than that of the reserve asset. The <u>floor</u> is still set as the minimum value the portfolio can have or the present value of the liabilities in the case of the pension fund. The cushion, the difference between the CPPI portfolio and the floor, is invested in the active asset. The exposure is always kept at a constant multiple to the cushion.

The similarities, however, end here as subtle differences between traditional and duration adjusted CPPI quickly become apparent. The exposure, the multiple, the CPPI portfolio, the reserve and the active asset are all expressed in terms of duration. This is because CPPI for FIS manages the portfolio in a duration context where the exposure is measured in terms of the extension of the duration from the reserve asset to (a maximum of) that of the active asset (D_a-D_r) (Hakanoglu et al., 1989).

How do we determine the duration to use? This will depend on the liabilities of the fund and/or on the investment objectives of the fund managers. An investor with liabilities must at all cost meet the obligations of the fund. Thus, the structure of these liabilities will have a major influence on the choice of the reserve asset. In order to fund these liabilities, two conditions need to be met. First, the present value (PV) of the portfolio of securities must at least be equal to the PV of the liabilities. Second, the sensitivities (durations) of the PV of the assets and liabilities to interest rate changes should be the same. Therefore, the present value and the duration of the liabilities will determine the value of the floor and the duration of the reserve asset respectively. In the absence of liabilities, a fund manager will choose the duration of the reserve asset based on a minimum investment objectives. In both cases, the active asset will contain higher expected return-higher risk securities, usually located on the longer range of the yield curve, to allow the investor to generate a return above that of the reserve asset.

Let us explain the dynamics of duration adjusted CPPI. As the value of the active assets rise (fall) relative to that of the reserve asset, the proportional cushion will grow (shrink), causing the target duration of the portfolio (D_p) to increase (decrease) towards the duration of the active asset (D_a) [reserve asset (D_r)]. In the best case scenario, the CPPI portfolio becomes fully invested in the active asset. At the limit, in the worst case scenario, D_p approaches D_r; the value of the overall asset approaches that of the floor and the portfolio will become immunized. Thus, the duration adjusted CPPI strategy protects the value of the assets from falling below the value of the floor (assuming no downward price jump). All of this takes place through rebalancing. Since CPPI for FIS is based on keeping the duration exposure equal to the duration adjusted multiple times the proportional cushion, trading will be precipitated by: a change in the interest rates or a change in the yield spread between the active and the reserve assets (Hakanoglu et al., 1989)[10].

The rest remains the same as with traditional CPPI with similar consequences. The multiple and the floor depend on the risk aversion and the objectives of the investor and/or the liability stream of the pension fund. A tolerance level is needed to avoid continuous rebalancing. A limit may exist on the use of leverage in the CPPI portfolio.

2.5 Other Important Issues Concerning CPPI

Despite the attractive features of PI, several issues must be considered before implementation. The first issue is the cost of the strategy. As we have seen in the review of prior studies, the protection of a portfolio does not come free. The cost of PI implemented with a dynamic hedging strategy or with CPPI is comparable to the premium one would have to pay to buy a protective put on a portfolio (Donnelly, 1986). It also corresponds to the opportunity cost of not being fully invested in the active asset when the market rises or fully divested when the market declines.

The cost is paid as the strategy is implemented. For

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[10]Trading can also become precipitated if D_a-D_r is allowed to fluctuate. The reader can refer to Appendix 2 for an example of how the formula works. In this paper, D_a-D_r will be kept constant.

CPPI, the costs will depend on the desired level of downside protection (floor), the multiple, the actual return on both the risky and reserve asset, the tolerance level, the volatility and the pattern of volatility of both the risky and riskless asset (Black & Rouhani, 1989; Negrych & Senft, 1989).

The costs of PI take two forms: implicit costs and explicit costs (O'Brien, 1988). The implicit costs represent the foregone return in exchange for protection against downward risks. The explicit costs include transaction costs and the slippage (Buy high sell low, and bid-ask spread one has to pay in addition to the transaction costs when one buys and sells a security).

The costs are expected to grow when volatility increases because the frequency of trading will rise. A larger multiple will also increase transaction costs as the size of the trades will be bigger. The cost of PI is also likely to increase the more people use it. Grossman (1988) showed that market volatility will increase the more market participants practice PI through dynamic hedging. When all these PI users attempt to sell (buy) securities at the same time after a price fall (rise), the price is likely to fall (rise) further away from the current price level. This will increase the explicit costs of PI.

The second issue for CPPI is the size of the multiple. As we know, the higher (lower) the multiple, the more (less) the return on the portfolio will be tied to the performance of the risky asset. The higher (lower) the multiple, the more (less) investors will participate in market upside, the faster (slower) the portfolio will approach the floor in a falling market and, the more (less) the portfolio will lose in a choppy market. The higher (lower) the investor's expected return for the risky asset and the lower (higher) his expectation of volatility, the higher (lower) his multiple should be. However, if we continue to raise the multiple further, a smaller move in the market can put the portfolio below the floor before appropriate adjustments can be made. In fact, a move greater than **-1/m** will be sufficient to put the portfolio completely in the reserve asset. It is thus easy to see that as the multiple increases to ∞, CPPI becomes a stop loss strategy and stop loss orders imply path dependence (Black & Perold, 1989). Hence, after a certain size, increasing the multiple further does not increase the expected return any further[11]. Therefore, it is easy to understand that higher expected return can be obtained with a finite multiple than with an infinite multiple (Black &

[11]When the multiple is very large, a very small decline in the value of the risky asset (-1/m) will force the portfolio into immunization at the risk free rate. Increasing the multiple to a large number fails to increase the expected rate of return because the expected rate of return on the reserve asset is lower than that of the active asset.

Perold, 1989). This is because increasing the multiple also increases the volatility for both the cushion and the total return (Zhu & Kavee, 1988). Thus, the size of the multiple depends on the delicate balance between the investor's expected return and his risk aversion.

In the design of any CPPI program, an issue that is very interrelated with the size of the multiple is the size of the floor. This issue is important because the floor determines the level of protection as well as the size of the cushion which, in turn, determines the exposure to the risky asset when multiplied by the multiple (E=MC). A low floor provides the investor with less protection and a larger exposure to the risky asset. Conversely, a high floor offers a higher level of protection at the expense of a lower exposure to the risky asset. However, by combining a low floor with a low multiple or a high floor with a high multiple, the manager can create the same exposure while providing a reasonable level of protection. For example, a multiple of 5 and a floor of $90 for an portfolio worth $100 translates into an exposure of $50 to the risky asset. The same exposure can be reproduced with a multiple of 10 and a floor of $95. In the first case, both the sensitivity to the risky asset and the level of protection are lower. In the second example, the sensitivity to the risky asset is larger and so is the level of protection provided by a higher safety net. The higher risk of a higher multiple is offset, at least in part, by the higher protection of a higher floor. Thus, in the design of a CPPI program, it is important to consider the impact of the interrelationship between the multiple and the floor on the exposure. When the floor is not determined by the PV of the liabilities (for example, when the floor is used to track a benchmark index), the choice of the parameters for the floor and multiple should also depend on how correlated and how volatile the returns on the risky and reserve assets are. In a FIS environment, the returns are less volatile than for equities. Furthermore, the returns on securities of different duration but of equal credit risk are very much correlated. Thus, it will be interesting to test if a high floor and a high multiple can be used for FIS without running the chance of putting the floor in jeopardy. Appendix 9 summarizes the possible interactions between the size of the multiple and of the floor.

The third issue is the tolerance level. The presence of a tolerance level is necessary to avoid continuous rebalancing and eliminate unnecessary trading costs (commissions, buy high sell low) as rebalancing is triggered by small market fluctuations. But what should be the size of the tolerance level? A small tolerance level improves tracking accuracy at the expense of more frequent transaction costs and slippage. A large tolerance level, on the other hand, will reduce getting whipsawed in a choppy market. The drawback to a large tolerance level is that the filter will introduce some performance lag and diminish tracking accuracy when the market heads in one direction or another: any rebalancing will be executed only after the market rises or falls enough to push the portfolio outside the tolerance bounds. Here is an example:

In this demonstration, we will use tolerance levels of 1% and 5%. For the current week, the target and current duration for the CPPI portfolio is 5.10 years. A week later, the value and the duration of the portfolios have changed due to the new bond prices. Because the CPPI portfolio performed worse than the reserve portfolio, the duration exposure of the CPPI portfolio must be reduced towards that of the reserve portfolio. Based on CPPI, the new target duration of the CPPI portfolio is thus slightly reduced. In this example, let us say that the new target duration is now 5. Based on the new bond prices, the current duration of the CPPI portfolio is 5.07. If we use a tolerance level of 1% on either side of the target duration, adjustments only occur if the duration of the CPPI portfolio falls outside 4.95 and 5.05. In this case rebalancing is necessary. If we use a tolerance level of 5%, the target zone enlarges to 4.75 to 5.25 and no adjustments are necessary.

In brief, the lower (higher) the tolerance level, the more (less) closely the exposure is related to the cushion, the more (less) frequent the trades but the smaller (larger) their size. Hakanoglu (1989) found the optimal tolerance level to be 5% in his study on FIS. We think that a lower tolerance level might be in order given the safety of the investments involved and the fact that duration is affected in the same direction as the cushion by a change in interest rates.

Just like in the equity market, CPPI for FIS permits trading in the futures, the options and the cash market. This allows some flexibility to the manager as to the instruments that can be used. Liquidity and fair pricing, however, are paramount. In all cases however, the investor must monitor the sensitivity to interest rates carefully.

The major risk in implementing PI via a dynamic hedging strategy rather than with listed puts is that a catastrophic decline might occur before appropriate adjustments can be made. This was witnessed in the stock market crash of 1987. PI did not perform as expected for all PI users. This is because PI assumes an orderly market where liquidity is present, volatility does not fluctuate drastically and prices are continuous (Leland, 1988). To the extent that the bond market offers greater intrinsic value and security than stocks, a crash is much less likely. Nonetheless, the major drawback of PI implemented through a trading intensive strategy is that the manager cannot know in advance how much this strategy is going to cost.

Finally, PI, in a FIS world, really becomes a dynamic asset-liability management strategy where the amount invested in the risky securities is continuously revised so as to make sure the liabilities are funded or that the minimum value objective is attained at the end of the time horizon. CPPI for FIS can be used with a defined time horizon or as a perpetual strategy. In this experiment, we chose to use the latter because of its simplicity.

3. OUR MODEL

3.1 Description of Our Model

We propose to test CPPI on a portfolio of FIS with the objective to beat a fixed allocation strategy. The managers will try to outperform a fixed allocation strategy by increasing the duration of the CPPI portfolio in bull markets and reducing it in bear markets to protect its value from falling below a predetermined minimum (floor).

As we mentioned previously, CPPI allocates funds between a risky and a riskless asset. Usually, these assets are indices or portfolios that can be easily bought and sold. Diversified bond indices, however, cannot be easily traded (Douglas, 1990). This is a problem because CPPI is trading intensive. However, because duration is additive[12], it is possible to build, replicate or modify a bond portfolio so that it will have the same interest rate sensitivity as a liability stream or a minimum investment objective. This means that we no longer need bond indices for the CPPI portfolio since it is possible to combine individual bonds to

[12]The duration of a portfolio of bonds is the weighted duration of the component securities in the portfolio (Douglas, 1988).

achieve the duration of the CPPI portfolio[13].

As with traditional CPPI, CPPI for FIS also requires that we monitor the floor[14] in order to determine the proportional cushion (defined in section 2.4) and thus, the duration of the CPPI portfolio ($D_p = m_d c + D_f$). The reader can refer to Appendix 2 for an example of the calculations. If the CPPI portfolio outperforms (underperforms) the floor portfolio, the proportional cushion will grow (shrink) which will increase (reduce) the target duration of the CPPI portfolio towards that of the active (floor) portfolio. This new target duration will be achieved by changing the composition of the CPPI portfolio so as to meet the desired duration. In the best case scenario, the portfolio will have a duration equal to that of the active asset (if the portfolio is capped). In the worst case scenario, the portfolio becomes immunized with a duration and a value equal to that of the floor portfolio. All this implies that the manager needs to

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[13]Instead of trading between risky and riskless asset (index), individual bonds are traded to meet the desired target duration. The amount and the particular bonds to trade are determined using a linear program. The next section describes the objective function and constraints we will use in the simulation.

[14]In this simulation, since no liabilities are present, the value of the floor corresponds to the minimum level of protection desired. The duration of the floor depends on the minimum duration considered by the portfolio manager or the duration of the reserve asset. The floor portfolio will also be built by combining individual bonds together.

monitor the value and the duration of both the floor and the CPPI portfolio separately. Hence, as long as a manager controls the overall volatility exposure ($D_p = m_d c + D_r$) of the CPPI portfolio, he will have considerable freedom in building his portfolio in terms of maturity sector and structure with or without regard to distinctions between active and reserve assets (Hakanoglu et al., 1989). This is an advantage because the portfolio manager can build his portfolio to his liking (laddered, barbell, etc.).

Types of Portfolios

Because portfolio managers favour different strategies at one time or another, we propose to test different structures for our CPPI portfolio. The objective is to see if these structures can perform well in a CPPI framework. We propose to use a laddered structure, a barbell structure, and a portfolio where convexity is enhanced, and finally, a portfolio with none of these constraints. These portfolios will be built and managed by adding additional constraints to our linear program (LP) routine. The next section discusses the constraints and the objectives that will be used in the LP.

1) A non-structured Portfolio. The only constraints we will use are that the target duration of the portfolio, as determined by the CPPI, be respected. No bonds shall be held in an amount greater than a prespecified limit. This should give us a good indication of the kind of performance and transaction costs involved in a CPPI strategy.

2) A laddered portfolio. Because of the way the model functions, we cannot take advantage of any mispricing that an active portfolio manager would try to identify and take advantage of. Moreover, because we do not try to anticipate market swings, laddering the portfolio will limit the extent by which one can benefit or suffer from the gyrations of the yield curve. By structuring a portfolio with securities spread all along the yield curve, the portfolio will participate in all yield shifts, parallel and non parallel. A portfolio structured differently might not. This is important because a portfolio invested only in a certain area of the yield curve could show an excellent or a terrible performance depending on how the yield curve behaves at that segment. This could affect the performance of CPPI in a FIS framework, obscure the results we obtain and distort our conclusion. Moreover, laddered strategies, on a stand alone basis, may have attractive features for a portfolio manager: On average, laddered portfolios provide higher interest coupons in the long maturities (Bradley & Crane, 1975). They also provide the portfolio with less reinvestment risk by spreading out reinvestment over the full interest rate cycle as continuous cash flow maturing over time can be reinvested at the then current rates. To the extent that CPPI does not dictate selling these securities before maturity, we should witness this.

3) A portfolio where convexity is enhanced at a targeted duration level. The rationale for testing this portfolio is that enhancing convexity is a form of total return maximization (Douglas, 1990). The purpose of buying additional convexity is that convexity is beneficial in periods of high volatility. For a given duration, positive convexity cushions the price fall of a bond when the rates go up and accelerates the price increase of the bond when the rates fall. This assumes parallel yield shifts. However, the advantage of convexity can be mitigated by two factors. First, incremental convexity may fail to deliver its advantages when non parallel shift occur in the yield curve (Douglas, 1990). Second, transaction costs and the price paid for convexity may also reduce or even eliminate the advantages of convexity. There are also times when convexity is too dear: yield giveups to more convex issues is too expensive. In such case, convexity should be sold and less expensive less convex instruments should be bought. Unfortunately, our model cannot differentiate between those times. In any case, we still believe convexity can enhance the performance of a bond portfolio and, consequently, we will test to see if purchasing incremental convexity can increases the total return.

4) Barbell portfolio. A barbell approach anticipates that the best risk/reward ratio is achieved by balancing the defensive qualities of short term securities with the aggressive qualities of long term securities while avoiding intermediates. Because convexity rises at a faster rate than duration, building a barbell portfolio improves the convexity over a same duration-bullet. As additional convexity becomes more valuable in the face of changing interests rates, barbell portfolios can become a useful strategy. The barbell strategy will be profitable as long as the yield curve makes a parallel shift or the yields at the long end fall by more (or rises less) than those at the short end. Thus, in a way, a barbell strategy is a play on convexity. Unfortunately, convexity enhancement usually comes at the cost of yield giveups compared to bullet portfolios[15]. A legitimate question is: Can barbell portfolios be successful in a CPPI framework? Our tests will provide us with the answer.

We will also construct three Buy & Hold portfolios (B&H). First, the reserve portfolio will be built with a duration of 4 years. The reserve portfolio is important

[15]This is so as long as the yield curve is positively sloped. When the curve is flat, barbell portfolios increase convexity with no yield giveups. When the slope is inverted, convexity gains are associated with yield pickups (Diller, 1991).

because it will be used to monitor the value of the floor and, indirectly, determine the duration of the CPPI portfolio. Second, a B&H portfolio with 6 years duration will also be constructed to monitor the value of active asset. Since the active asset is managed as part of the CPPI portfolio, the active portfolio will only be needed later for comparison of the results with the CPPI portfolio. Finally, a B&H portfolio with 5 years duration will also be built. This portfolio represents the fixed allocation strategy. It is important because it is directly comparable to the CPPI portfolio since it will have the same initial duration as the CPPI portfolio. All these portfolios will be managed the same way as the non-structured portfolio, except for one difference: their duration will be kept **constant** (deviations of 1% will be tolerated). This implies that the weights of the components in the reserve portfolio will be revised from time to time as its duration changes due to the passage of time and to variations in the level of interest rates[16]. This is important because the duration of the CPPI portfolio is based, among other factors, on the difference between the duration of the reserve and the active portfolio (D_a-D_r). If D_a and D_r were allowed to fluctuate, the duration of the CPPI portfolio would vary. Its duration would fall outside our control and the

[16]Technically, these portfolios are not Buy & Hold portfolios if revisions are needed to keep their duration from drifting away from their fixed target duration.

results would be more difficult to analyze.

The performance of each of the CPPI portfolios will be compared to the Buy & Hold portfolios. These comparisons will be useful because they will show if the CPPI portfolio can outperform a fixed allocation strategy. Remember that a portfolio managed under CPPI is supposed to outperform a fixed Buy & Hold portfolio of equal initial duration because it reacts to bullish (bearish) market by increasing (decreasing) the overall duration. The comparisons will also be important when we study the distribution of the shortfalls and excesses relative to those Buy & Hold portfolios. Finally, we will be able to see the costs involved with CPPI in FIS.

3.2 Methodology

The investor will specify at the outset the horizon of the insured portfolio, the portfolio value and the floor, the multiple, the duration for the reserve and the active asset as well the tolerance level. The complete list of inputs can be found in appendix 3. The portfolios will be created by pursuing the following steps.

Objective Function:

At the beginning of the first week, we want to start with the best possible reserve and CPPI portfolio given the constraints we have. This is why we select bonds that will maximize the average yield to maturity on the respective portfolios[17]. The reader can refer to Appendix 4 to see the formulation of the objective function and constraints of the LP program. In every other week, our objective function will be to minimize transaction costs when rebalancing the portfolios becomes necessary. From week to week, the set of constraints will remain the same as during the first week. To see if rebalancing is necessary, the following steps are undertaken.

Steps

1-Read the new bond prices.

Value the reserve portfolio with the new bond prices.

Verify if any coupons were paid or if any bonds matured.

2-Calculate the duration of the reserve portfolio based on the new prices.

3-If the duration of the reserve portfolio falls outside its tolerance band, we will rebalance the reserve portfolio to its

[17]The calculations used to maximize the average yield to maturity of the portfolio are an approximation. However, we are satisfied with it given that many transactions will take place in the portfolio, not all bonds will be held to maturity and that the portfolio does not have a stated maturity.

target duration and reevaluate it after transaction costs.

4-Calculate the value of the CPPI portfolio. Verify if any coupons were paid or if any bonds matured.

5-Calculate the duration of the CPPI portfolio.

6-Based on the value of the reserve and the CPPI portfolio, calculate the cushion and determine the target duration of the CPPI portfolio.

7-Rebalance the portfolio if necessary. That is, only if the duration calculated in 5 falls outside the tolerance band around the new target duration determined in 6. The rebalancing will be subject to the following constraints:

Constraints:

1-The most important constraint for both the reserve and the CPPI portfolio is that each week, their duration is compelled to be within a tolerance band of their respective target duration. If the duration of either portfolio drifts away from the tolerance band, the portfolio will be rebalanced.

2-The initial value of the CPPI portfolio will be $100 Million. Varying floors with an initial value varying from

$90 to $97,5 million will be used. Floors with a lower initial value do not provide any meaningful protection in a FIS world.

3-The maximum amount in any bond held in the portfolio will be $10 million face value. This constraint will force the program to select a minimum of at least 10 bonds (given that the CPPI portfolio is worth $100 million initially). The reason for this maximum limit is that we want a minimum of diversification in the portfolio. For certain years, however, where the selection of bonds available is smaller, this maximum will be raised.

4-The portfolio must hold at all time a minimum number of bonds. For example, a minimum of 15 or 20 bonds. The rationale for this is that we would like to make sure our universe of bonds is well represented in our portfolios. This constraint will be relaxed for barbell, laddered and convex portfolios given the smaller number of bonds available and the additional constraints on the structure of these portfolios.

5-The minimum amount held in the portfolio in any bond will be $250,000 face value. The reason for a minimum is because we do not want the portfolio to have an insignificant position in any bond.

6-The portfolios will not be involved in short sales.

7-The program allows each week for up to $25,000 of idle cash. This is because in this simulation, as in real life, it may not be possible to invest the portfolio's funds to the last penny. The idle cash is invested in securities with one week to maturity. This constraint also helps the LP routine in its selection.

8-The Bid-Ask spread will include the commission rate. The spread that we will use is 10 Bps on either side of the quoted price. Thus, for a quoted price of 101.25, the simulated bid and ask will be 101.15 and 101.35 respectively. This spread may appear to be low, but it is the spread large institutions are accustomed to when dealing with large amount of money. In fact, for very liquid issue, the spread is actually smaller (5 Bps) on each side. We decided to use 10 Bps for all bonds because some of the issues we are dealing with are less liquid. The spread we are using seems appropriate considering that Fong & Fabozzi (1985) used 15 Bps in a simulation on US treasury securities in 1985 and that the spreads have narrowed since.

9-For certain portfolios, an additional constraint will be added. The convexity of the CPPI portfolio will be required to be greater than a certain amount. This amount will be raised in the years when more convex instruments become available. Since this is an enhancement strategy, the constraint will only be used for the CPPI portfolio.

10-No commission will be charged when a bond matures.

11-For the laddered portfolio, bonds will be classified by maturity. 8 to 9 maturity groups will be possible each year. A minimum of 5% and a maximum of 20% of the entire portfolio will be allocated in each of the maturity groups. To reduce transaction costs and to add flexibility, an additional margin of 1% can be added to the minimum and maximum bounds before rebalancing becomes necessary. See Appendix 7 for the list of maturity groups.

12-Buy & Hold portfolios will also be created to see if CPPI can achieve a higher return while it controls for the higher risk. These portfolios are similar to the reserve portfolio and will be managed in the same way. We will build Buy and Hold portfolios of different durations and rebalance them only when their respective durations drift away from the tolerance bands.

When the time comes to rebalance, the model will consider the securities already in the portfolio before making any changes so as to limit transaction costs. There will be weeks where no rebalancing will take place and others where only the reserve or the CPPI portfolio or both will need to be rebalanced. The proceeds of coupon and principal payments will be reinvested in the week that follows payment. The same tolerance level will be used for the entire simulation for both the reserve and the CPPI portfolio. The weekly performance of the reserve index, the CPPI portfolio and the Buy & Hold portfolios will be recorded for comparison later.

In practice, the model is flexible enough to allow the investment manager to change the parameters along the way depending on his expectations of the market. For our study, since our objective is not to predict interest rates, we will keep these parameters fixed during the entire horizon chosen.

3.3 Data

Our database was obtained from FRI Corporation. It contains 64 different government of Canada bonds. For each bond, we collected prices and yield to maturity on a weekly basis from January 1980 to December 1991. The list of governments bonds appears in Appendix 5. This 12 year horizon was chosen for two reasons: first, because it covers a period of complete bull and bear markets with all sorts of yield curves; second, because this period also had times where the markets experienced low, medium and very high volatility. This will be interesting because it will allow us to test CPPI in different environments. From the prices and the yield to maturity, the modified duration and convexity were computed for each bond each week.

When we selected the bonds for our database, we carefully looked for bonds sharing three different characteristics. By order of importance, we wanted to have bonds that matured all along the maturity spectrum, bonds that were trading frequently (liquid) and finally bonds with low, medium and large coupons. We also wanted to avoid bonds with embedded options. The task was not simple, especially in the early 80's, as not all maturities were available. We sometimes had to compromise by selecting less liquid bonds, especially in the early 80's. Finally, only bonds with a minimum of $100 Million outstanding were considered for purchase by the model.

We purposely stayed away from corporate and municipal bonds even if they would have generated better returns. The reasons for this is that these bonds often have embedded options, they are less liquid and they also contain other risk factors such as default risk or sinking fund risks, etc. that we did not want to deal with or monitor.

In the present study, riskiness is expressed strictly in terms of duration since higher expected return and higher interest rate risk usually go hand in hand with higher duration. Thus, our reserve portfolio has a low duration and the active portfolio a higher duration.

In order to make this study as realistic as possible, we provided for the program to invest the portfolio's idle cash in short term securities of one week in maturity. The amount invested in the short term securities will never exceed $25,000. This will help the linear program in its selection of the bonds.

Unfortunately, we were not able to collect short rates of one week in maturity for investments of less than $25,000. Therefore, we used an approximation that, we feel, reflects the rates available for investments in short term securities. The rates were determined in the following way: We took the average of the highest and lowest weekly bank rates during each year. We then subtracted 2 from the result: (H+L)/2-2. We used this average rate for all weeks during the same year. The effect of this approximation on the simulation will be minimal considering the relative weight of idle cash to the size of the portfolio ($100 Millions). The weekly bank rates were obtained from the Bank of Canada. The average rates are given in appendix 6.

We selected weekly data for many reasons. First, given the length of time we are covering, weekly prices seem justifiable. Second, using daily data would have raised the cost of this study significantly without adding much information about the performance/cost tradeoff for the strategy. Finally, Etzioni (1986) showed that semi-weekly or weekly rebalancing of the portfolio resulted in the best performance/cost tradeoff when testing his PI model.

4. ANALYSIS OF THE RESULTS

In the analysis of these results, we hope to provide answers to a number of questions:

1- We want to find out which multiple and which tolerance level are most appropriate. In order to answer this question, we will first compare the average performance of the portfolios over a twelve year horizon and then on a yearly basis. We will break down the results in bull and bear markets. We will compare the performance of the CPPI portfolios with the reserve, the active and a fixed allocation portfolio (of similar initial duration). The Sharpe measure will also be used to assess the performance of these portfolios on a risk adjusted basis.

2- We will look at the distribution of the performance to see if CPPI is able to reshape the return distribution of a portfolio. We will also verify how trading intensive the strategy is and how transaction costs can be reduced by increasing the tolerance level.

3- We will also use CPPI when building different portfolio structures. Since portfolio managers favour different strategies at different times, our objective is to see if CPPI can be used with a barbell, a laddered structure and with a portfolio where convexity is maximized.

The first thing to determine is the optimal size of the multiple. The choice in the size of the multiple is not obvious when the active and the reserve assets are highly correlated. The high correlation may require a higher multiple to increase the sensitivity to the risky asset and thus, to reallocate funds more rapidly to the best performing asset. However, if we modify the size of the multiple, we also need to adjust the floor to keep the initial duration exposure the same for all simulations (Refer to Appendix 2 for a review of the formula of the duration exposure). Thus, we ran simulations with 7 different combinations of multiple and floor. Table 4.1 includes the combinations of the parameters we used.

Each simulation was tested over one year for twelve different years. Every year, the parameters were restarted. The results in Table 4.2 indicate that the size of the multiple has very little effect on the general performance. Over a twelve year horizon, the difference in performance is about one basis point depending on the size of the multiple we used. For the full results, refer to Appendix 11. At

Combination of Parameters				
M=5	F=90.00	c=0.1000	m_d=10	D_p=5
M=7	F=92.86	c=0.0714	m_d=14	D_p=5
M=8	F=93.75	c=0.0625	m_d=16	D_p=5
M=10	F=95.00	c=0.0500	m_d=20	D_p=5
M=12	F=95.83	c=0.0417	m_d=24	D_p=5
M=15	F=96.67	c=0.0333	m_d=30	D_p=5
M=20	F=97.50	c=0.0250	m_d=40	D_p=5

Where:
- M = Multiple
- F = Initial Floor
- c = Initial Proportional Cushion
- m_d = Duration adjusted multiple
- D_p = Initial Portfolio Duration

The tolerance level used was 1%.

Table 4.1.

first glance, these results are surprising since we expected that a higher multiple would improve the general performance of the model.

If we compare these general results (using a multiple of 10) to the average performance of buy & hold portfolios, the model outperforms a Buy & Hold portfolio of 4 years in duration (B&H4) by 46 Bps. See Table 4.3. Compared to a B&H5, the model beats the B&H portfolio by 12 Bps. Finally, compared to a B&H6, the cost (in underperformance) is 13 Bps. Thus, the results clearly indicate that CPPI was able to improve the returns over a fixed B&H5[18]. Yet, the

[18]The results may appear to be small. However, over a period of 12 years, the compounding of 13 bps on a portfolio of $100,000,000 amounts to $1,473,864.

Comparison Between CPPI Portfolios of Different Multiple			
Size of the Multiple	Average Return	Average Std-Dev	Average[19] Sharpe Measure
5	13.453	0.929	0.934
7	13.468	0.926	0.936
8	13.460	0.923	0.925
10	13.461	0.920	0.919
12	13.463	0.917	0.909
15	13.458	0.912	0.885
20	13.434	0.905	0.824

Based on the Geometric Average

Table 4.2.

reader should notice that over a 12 year horizon, the average standard deviation and the Sharpe measure for the B&H5 and CPPI portfolios were virtually the same. Still, in order to gain a better insight of the subtleties involved, we will look at the average and yearly performance of these simulations in bull and bear markets. This may better help us to determine which size should be optimal for the multiple in fixed income world.

In order to do so, we divided the twelve years into two different groups: years of bull market and years of bear market. We subsequently further divided the two groups into

[19]The average Sharpe measure was computed by adding the yearly Sharpe measure and dividing the sum by the number of years involved. Thus, little resemblance may exist with the results one would obtain by dividing the difference in average return (between the risky and the riskless asset) by the average standard deviation.

Comparison Between the Model and Buy & Hold Portfolios

Portfolio	Average Return	Average Std-Dev	Average Sharpe Measure
B&H4	13.009	0.761	0.000
B&H5	13.339	0.940	0.920
M10	13.461	0.920	0.919
B&H6	13.590	1.140	1.327

B&H4: Reserve Portfolio
B&H5: Portfolio corresponding to a Fixed Allocation Strategy
M10: CPPI portfolio with a multiple of 10
B&H6: Active Portfolio
The Sharpe measure = $(R_i - r_f)/\sigma_i$
The risk-free rate used for the Sharpe measure corresponds to the rate of the reserve asset

Table 4.3.

subcategories: Strong, moderate and flat markets. Table 4.4. shows the categories.

Categories of Bull and Bear Markets

Strong Bull Market: 1982 1985	**Strong Bear Market:** 1981 1980 1987
Moderate Bull Market: 1991 1986 1989 1984	**Moderate Bear Market:** 1990 1983
Flat to Moderate Bull Market: Nil	**Flat to Moderate Bear Market:** 1988

Table 4.4.

Looking at the average performance during bull years, the CPPI portfolios showed a modest improvement over a B&H5. The gains increased with the size of the multiple. The average gain over a B&H5 was 1 and 6 bps for a multiple of 10 and 15 respectively. In bear markets, on the other hand, the CPPI portfolios were more successful in adding gains and in protecting the portfolio from falling in value. On average, the gains were 17 to 22 basis points higher than with a B&H5. Table 4.5 summarizes the results.

Average Performance in Years of Bull and Bear Market

Portfolios	Bull Market Average Return	Bull Market Average Std-Dev	Bull Market Average Sharpe Measure	Bear Market Average Return	Bear Market Average Std-Dev	Bear Market Average Sharpe Measure
B&H4	18.39	0.616	0.000	7.87	0.945	0.000
B&H5	20.27	0.756	2.482	6.81	1.123	-0.642
B&H6	21.93	0.920	3.798	5.82	1.360	-1.155
M5	20.26	0.762	2.444	7.03	1.097	-0.577
M10	20.28	0.767	2.446	7.03	1.077	-0.607
M15	20.33	0.762	2.462	6.98	1.058	-0.692

Based on the Geometric Average
B&H4: Reserve Portfolio
B&H5: Portfolio corresponding to a Fixed Allocation Strategy
B&H6: Active Portfolio
M5 : CPPI Portfolio with a Multiple of 5
M10 : CPPI Portfolio with a Multiple of 10
M15 : CPPI Portfolio with a Multiple of 15
The Sharpe measure = $(R_i - r_f)/\sigma_i$

Table 4.5.

If we examine the subcategories, we get some much more interesting results. Refer to Table 4.6. In strong bull markets, the model was helpful in improving the performance over B&H portfolios of 4 and 5 years in duration. The gain was more considerable the larger the size of the multiple. The average gain for those two years over a B&H5 was 9, 9 and 42 bps for portfolios with a multiple of 5, 10 and 15 respectively. In strong bear markets, the model managed to improve the performance over a B&H5 by a significant margin in 1981 (the year of the most severe bear market)[20]. In 1980, only the portfolios with a moderate multiple managed to improve the performance. In 1987, the performance was more or less the same as a B&H5 regardless of the size of the multiple being used. On average, over the three strong bear market years covered, the model outperformed a B&H5 by 48, 52 and 48 Bps for a multiple of 5, 10 and 15 respectively. Most of the gain came in 1981.

In moderate bull markets, the results are mixed. The performance of the CPPI portfolios was improved for most multiples in 1986. During the other three years of moderate bull markets, the model failed to improve the performance over a fixed B&H5. For all these three years the performance was worse (sometimes only slightly) when the multiple was increased. During the years of moderate advance, the B&H5 dominated the CPPI portfolio by 6, 7 and 10 Bps for a multiple of 5, 10 and 15 respectively. In moderate bear market, the performance of the CPPI portfolio was worse than a B&H5 by 6, 12 and 18 Bps for a multiple of 5, 10 and 15. Increasing the multiple worsened the performance.

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[20]The results of individual years appear in Appendix 11.

Comparison of the Performance Between a B&H5 Portfolio and CPPI Portfolios with Varying Multiples in Different Market Environments

Market\Multiple:	M5	M10	M15
Strong Bull Market:	+ 9	+ 9	+42
Strong Bear Market:	+48	+52	+48
Moderate Bull Market:	- 6	- 7	-10
Moderate Bear Market:	- 6	-12	-18
Flat to Moderate Bear Market:	- 4	- 5	-14

The comparison is based on the geometric average. The numbers (expressed in Bps) under the corresponding multiple represent by how much the CPPI portfolio outperformed or underperformed a fixed allocation strategy (B&H5).

B&H4: Reserve Portfolio
B&H5: Portfolio corresponding to a Fixed Allocation Strategy
B&H6: Active Portfolio
M5: CPPI Portfolio with a Multiple of 5
M10: CPPI Portfolio with a Multiple of 10
M15: CPPI Portfolio with a Multiple of 15

Table 4.6.

Finally, in a flat to moderately bear market, the model also failed to outperform a B&H5 by 4, 5 and 14 Bps for multiple of 5, 10 and 15 respectively.

These results were corroborated by the Sharpe measure in Table 4.7. The higher Sharpe measure for the CPPI portfolios (relative to the B&H5) also indicates that the CPPI portfolios were able to add gains in periods of strong bull markets and to protect the portfolios in strong bear markets. Furthermore, in these same markets, the Sharpe measure was also larger the higher the multiple. In moderate to flat markets, however, the lower Sharpe measure for the CPPI markets, however, the lower Sharpe measure for the CPPI portfolios (vis-à-vis the B&H5) also reaffirms the lower risk adjusted performance of the CPPI portfolios in those interest rate environments. A higher Sharpe measure was also established with lower multiples.

Comparison of the Performance of Several Portfolios in Different Market Environments

Average Return

Market:	B&H4	B&H5	B&H6	M5	M10	M15
Strong Bull Market:	26.30	29.71	32.99	29.80	29.90	30.13
Strong Bear Market:	6.17	4.29	2.70	4.78	4.81	4.78
Moderate Bull Market:	14.62	15.81	16.76	15.75	15.74	15.71
Moderate Bear Market:	9.50	9.01	8.48	8.95	8.89	8.83
Flat to Moderate Bear Market:	9.81	10.15	10.13	10.11	10.10	10.01

Sharpe Measure

Market:	B&H4	B&H5	B&H6	M5	M10	M15
Strong Bull Market:	0.000	3.333	5.317	3.381	3.463	3.649
Strong Bear Market:	0.000	-1.104	-1.815	-0.890	-0.887	-0.936
Moderate Bull Market:	0.000	2.057	3.039	1.976	1.937	1.868
Moderate Bear Market:	0.000	-0.548	-0.930	-0.641	-0.727	-0.833
Flat to Moderate Bear Market:	0.000	0.558	0.435	0.491	0.474	0.326

The comparison is based on the geometric average.
B&H4: Reserve Portfolio
B&H5: Portfolio corresponding to a Fixed Allocation Strategy
B&H6: Active Portfolio
M5 : CPPI Portfolio with a Multiple of 5
M10 : CPPI Portfolio with a Multiple of 10
M15 : CPPI Portfolio with a Multiple of 15
The Sharpe measure = $(R_i - r_f)/\sigma_i$

Table 4.7

Based on the above results, the choice of the size of the multiple is not clear. In very strong bull or bear markets, a high multiple significantly improves the performance while in moderate to flat markets, a smaller multiple seems preferable.

Table 4.7 is also interesting because it allows us to see the performance of all portfolios, including those of the reserve and the active portfolios, in different market environments. But more specifically, the comparison of the performance between the CPPI portfolios and the active portfolio is especially interesting in bull markets as it allows us to verify how well the CPPI portfolios can capture additional gains. Similarly, the comparison with the reserve portfolio is interesting in bear markets as it lets us see the kind of protection the CPPI portfolios can provide.

For example, in strong bull markets, the top portion of table 4.7 reveals that the CPPI portfolio (with a multiple of 15) added 42 bps relative to the B&H5. While the gains are substantial, they are still 286 bps lower than that of the active portfolio. In strong bear markets, the CPPI portfolios managed to gain about 50 bps relative to the B&H5 portfolios. Yet, relative to the performance of the reserve portfolio, these results are 137 bps inferior. Thus, this shows that while the CPPI portfolios were able to add gains in strong bull or bear markets, the model was only able to capture a fraction of the total gain. Still, these additional gains would not have been possible without CPPI.

If we consider the distribution of the excesses and the shortfalls of the CPPI portfolio compared to a B&H5 portfolios[21], it will allow us to see how CPPI is able to reshape the distribution of returns vis-à-vis the reference portfolio (in this case the comparison is especially relevant with a fixed allocation strategy of 5 years in duration)[22].

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[21]An excess is the difference between the CPPI and the reference portfolio when the CPPI portfolio dominates the reference portfolio. A shortfall is the difference between the reference portfolio and the CPPI portfolio when the reference portfolio outperforms the CPPI portfolio. The reference portfolios are the B&H with a duration of 4, 5 and 6 years. Garcia & Gould (1987) performed a similar study.

[22]Table 4.8 is divided into 6 sections. The top three sections correspond to the excesses measured against the reserve (B&H4), the fixed allocated (B&H5), and the active portfolio (B&H6) respectively. The bottom three sections correspond to the shortfalls registered against the reserve, the fixed allocated and the active portfolio respectively. Each excess (or shortfall) vis-à-vis the portfolio of reference was recorded in the interval in which it occurred to measure the distribution of excesses and shortfalls. Below the last interval of each section is the sum of all occurrences of excesses (or shortfalls) and the average excess (shortfall) realized against the portfolio of reference. For example, in the top left-end section, we see that the CPPI portfolio with a multiple of 5 (M5) dominated the reserve portfolio (B&H4) in 7 occasions by an average of 171 bps. The excesses are distributed as follows: One excess occurred in the 26 to 50 bps interval, two in the 51-100 range, one between 101-150, one between 151-200 and two in the 201+ interval.

Distribution of Excesses and Shortfalls compared to B&H4, B&H5 and B&H6 Portfolios

Excesses Interval	B&H4 Portfolios: B&H5	B&H6	M5	M10	M15
[0, 25]	0	0	0	0	1
[26, 50]	1	1	1	1	1
[51,100]	2	0	2	2	1
[101,150]	1	0	1	1	1
[151,200]	1	2	1	1	1
[201, ++]	2	4	2	2	2
Total	7	7	7	7	7
AVG (+)	172	322	171	173	177

Excesses Interval	B&H5 Portfolios: B&H6	M5	M10	M15
[0, 5]	0	1	1	0
[6,10]	0	1	1	0
[11,15]	0	1	0	0
[16,20]	0	0	1	0
[21,25]	0	0	1	1
[26,++]	6	1	1	3
Total	6	4	5	4
AVG (+)	175	57	43	67

Excesses Interval	B&H6 Portfolios: B&H5	M5	M10	M15
[0, 25]	1	1	1	0
[26, 50]	0	0	0	0
[51, 100]	3	2	2	2
[101,150]	0	0	0	0
[151,200]	0	0	0	0
[201, ++]	1	2	2	2
Total	5	5	5	4
AVG (+)	98	143	179	175

Shortfalls Interval	B&H4 Portfolios: B&H5	B&H6	M5	M10	M15
[0, -25]	1	1	1	1	1
[-26, -50]	0	0	0	0	0
[-51,-100]	3	0	3	1	2
[-101,-150]	0	0	0	2	1
[-151,-200]	0	2	0	0	0
[-201, --]	1	2	1	1	1
Total	5	5	5	5	5
AVG (-)	131	248	105	105	109

Shortfalls Interval	B&H5 Portfolios: B&H6	M5	M10	M15
[0, -5]	1	4	2	1
[-6,-10]	1	3	3	1
[-11,-15]	0	1	1	3
[-16,-20]	0	0	0	2
[-21,-25]	0	0	0	0
[-26, --]	4	0	1	1
Total	6	8	7	8
AVG (-)	98	6	11	16

Shortfalls Interval	B&H6 Portfolios: B&H5	M5	M10	M15
[0, -25]	0	1	1	2
[-26, -50]	1	0	0	0
[-51,-100]	2	2	1	1
[-100,-150]	1	2	3	3
[-151,-200]	1	0	0	1
[-201, --]	2	2	2	1
Total	7	7	7	8
AVG (-)	175	151	149	128

*All numbers are expressed in bps. The average excess or shortfall was computed from the geometric average.
B&H4: Reserve Portfolio
B&H5: Portfolio corresponding to a Fixed Allocation Strategy
B&H6: Active Portfolio
M5 : CPPI Portfolio with a Multiple of 5
M10 : CPPI Portfolio with a Multiple of 10
M15 : CPPI Portfolio with a Multiple of 15

Table 4.8.

Table 4.8 shows that the CPPI portfolios managed to achieve an excess over a B&H5 portfolio in 4 or 5 occasions with an average gain of 43 to 67 bps, depending on the size of the multiple we used. As the table shows, the distribution of those gains were mostly registered in the last interval, especially the larger the multiple. This supports the idea that in the years where the CPPI portfolios was useful, it was able to provide protection or to add value.

Looking at the shortfalls versus a B&H5 portfolio, CPPI portfolios experienced 7 or 8 shortfalls. They lost on average between 6 to 16 bps depending on the size of the multiple. The distribution changed as the multiple was raised: CPPI portfolios with higher multiples showed a more frequent occurrence of larger shortfalls. Yet, it is interesting to note that most of the shortfalls tended to be very small. In at least 63% of the time, the shortfall was less than 15 bps. This figure attains 75% with a multiple of 5. Notice that there were fewer excesses than shortfalls but they were larger on average.

Compared to the B&H6, the CPPI portfolios managed to achieve a greater average excess and a smaller average shortfall relative to the B&H5. These results improved as the multiple was raised. The distribution of the excesses and shortfalls was very similar for all portfolios. Compared to the B&H4 portfolio, the CPPI portfolios were only able to reduce the average shortfall leaving the average excess relatively unchanged. Thus, CPPI is able reshape the return distribution relative to a fixed allocation strategy.

This raises the question: Why increasing the multiple failed to increase total return during years of moderate and flat price change?

Transaction costs alone cannot explain these results. Transaction costs first decreased as the multiple was raised from 5 to 10. When we subsequently increased the multiple from 10 to 20, transaction costs rose but not enough to explain a difference of more than 1 or 2 bps. The frequency of trading grew as the multiple was raised but the increase was moderate and irregular across the multiples and across the years. So there must be another explanation.

We believe that the main reason for the lacklustre performance of higher multiples in years of flat to moderate price change is due to whipsaws. Whipsaws (Buy high and sell low) reduce the performance of the model by increasing trading costs. Assuming a parallel shift in the yield curve, a higher multiple will shift more assets after a market rises (falls) from low (high) to high (low) duration bonds. If the market reverses, high (low) duration bonds will lose more (gain less) in value than low (high) duration bonds; trading is again precipitated after the market reverses. A higher multiple will shift more assets more often and will cause more whipsaws than a lower multiple[23]. However, when the general trend of interest rates is clearly in one direction, the effect of whipsaws are less important as shifting to higher (lower) duration bonds more than offsets the costs of whipsaws.

Another reason that may explain to a smaller extent why higher multiples failed to improve performance is the fact that the components differed from portfolios to portfolios. Some bonds perform better than others given a parallel or a non parallel shift in the level of interest rates.

Thus, in the presence of highly correlated risky and risk-free securities[24], the choice in the size of the multiple should depend on the expectations of the investors regarding future interest rates. If the investor expects a large variation in the level of interest rates (or a strong trend), he should increase the multiple. Otherwise, a more moderate

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[23]As we mentioned previously in chapter 7 (and in appendix 9), raising the floor at the same time as we raise the multiple has two effects: First, it allows us to control for the initial duration exposure of the CPPI portfolio. Second, it increases the protection for the CPPI portfolio should the higher multiple increases the cost associated with higher whipsaws.

[24]Note that the reserve asset is risk-free relative to some investment objective.

multiple is probably preferable. We will continue the rest of this simulation with multiples of 10 and 15.

Transaction Costs:

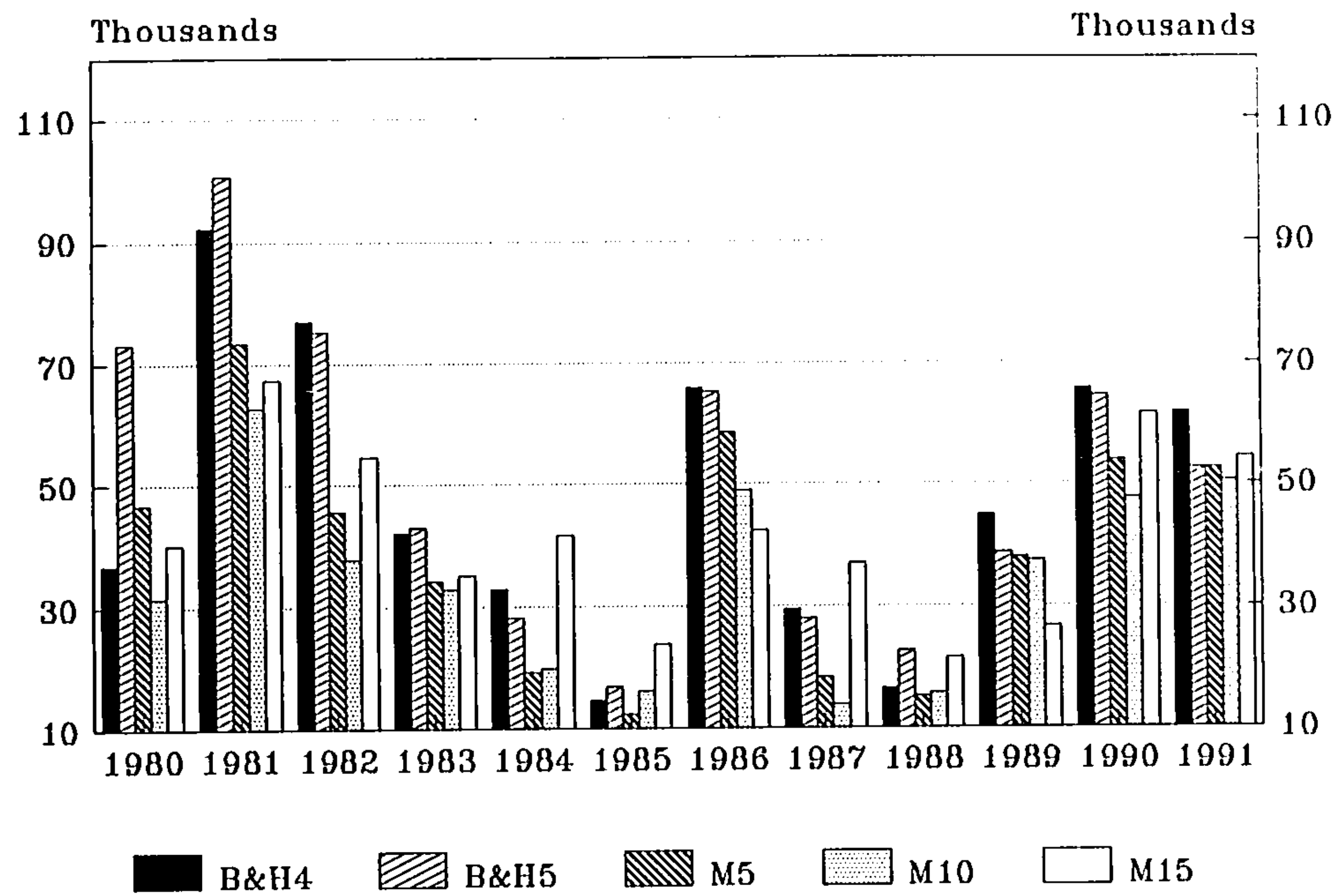
A final note on transaction costs. Transaction costs are unavoidable when rebalancing the portfolios becomes necessary. It was expected that the higher the multiple, the more frequent the rebalancing and thus, the higher the transaction costs and whipsaws. The results show that the above statement is only partially true.

Graph 11.1.

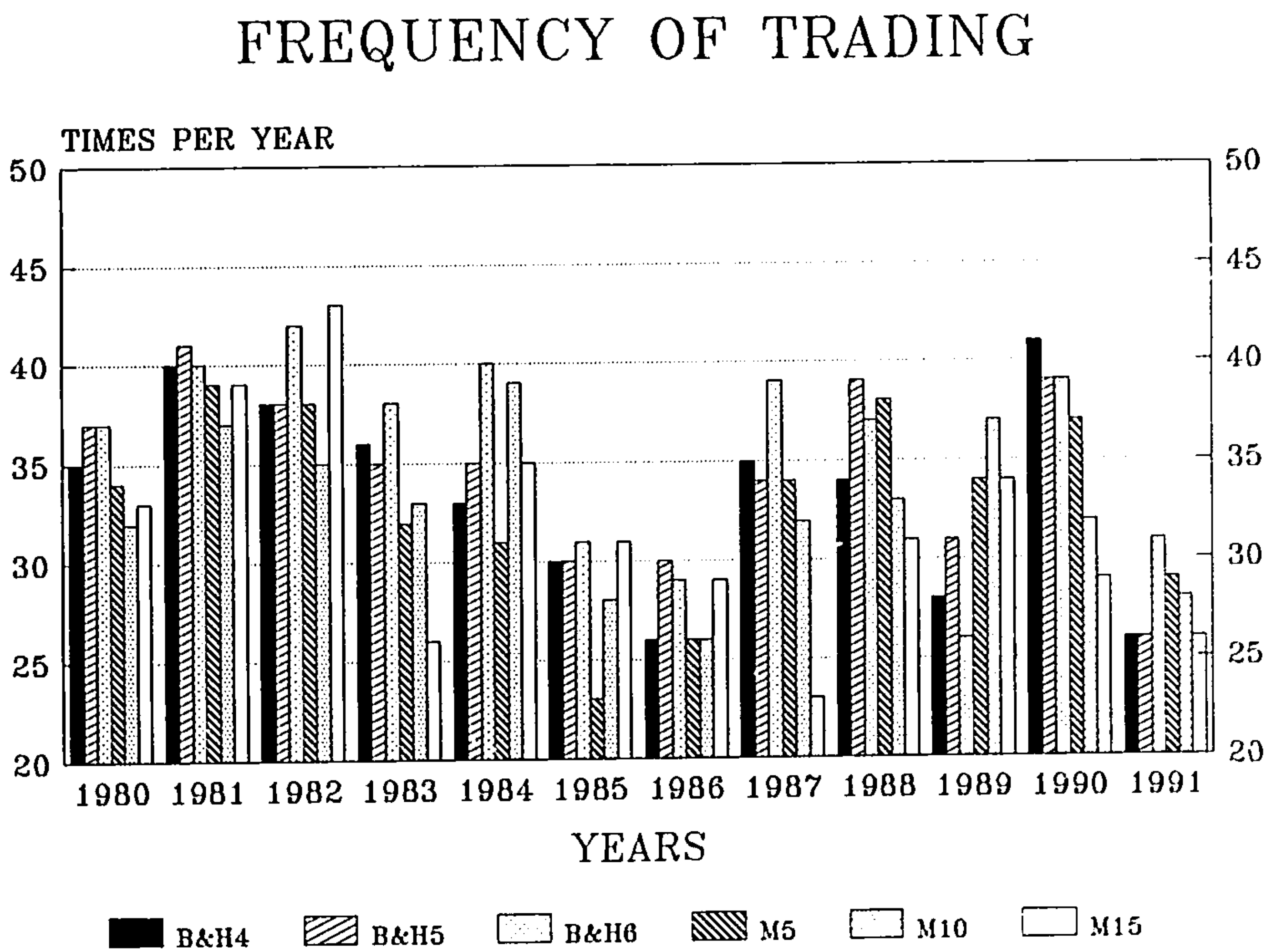
For low to moderate multiples, transactions cost are actually decreasing. For moderate to high multiple, transactions costs rise. The frequency of trading does not seem to explain these results completely. As we mentioned before, the frequency of trading increases as the multiple is raised, but the rise is very moderate and irregular across the higher multiples.

Graph 11.2

TRANSACTION COSTS



Graph 4.1



FREQUENCY OF TRADING
TIMES PER YEAR
50
45
40
35
30
25
20
1980 1981 1982 1983 1984 1985 1986 1987 1988 1989 1990 1991
YEARS
B&H4
B&H5
B&H6
M5
M10
M15

Graph 4.2

So what can explain what seems to be an anomaly? We have developed the following hypothesis: Duration is affected by two main factors: the passage of time and a change in the level of interest rates. A change in interest rates also affects the value of both the model and the reserve portfolio. The difference between the two values determines the cushion and thus, the target duration for the CPPI portfolio. Under the assumption of parallel shifts, when the interest rates go up (down), the value of the portfolios falls (goes up) and the cushion shrinks (expands) which, indirectly, reduces (increases) the target duration. Rebalancing becomes necessary when the actual duration falls outside the target interval. However, a change in the level of interest rates affects both the actual and the target duration in the same direction. Therefore, less trading is required to readjust the actual duration into the target range. We believe that for moderate multiples, the actual and the target duration are affected by about the same magnitude the closer we are to a multiple of 10. As we move away from a multiple of 10, more frequent trading and larger transactions are necessary to rebalance the portfolio into the target range. Notice that the transaction costs are always below those of B&H portfolios.

Tolerance Level

A tolerance level is defined as the percentage difference between the actual and the target duration that triggers a trade (Section 2.4). Using a multiple of 10 and 15, we tested CPPI with a tolerance level varying from 1% to 5% (1% to 4% for a multiple of 10). The objective was to find out which tolerance level was more appropriate in a FIS environment. A higher tolerance level decreased both the frequency of trading (measured by the number of weeks in a year where trading took place to rebalance a portfolio) and transaction costs. Using a multiple of 15, the frequency of trading was reduced by 7.3 and 9.5 times per year as we progressively raised the tolerance level from 1% to 2% and 5% respectively. For a multiple of 10, the reduction in the frequency was 9 and 10.2 respectively. Transaction costs fell on average from 6 to 8 thousand dollars a year when the tolerance level was increased from 1% to 2% and 5% respectively. The results were similar when a multiple of 10 was used. The reduction in transaction costs translated into a gain of less than 1 bps. Table 4.9 shows the overall performance with different tolerance level over the twelve years horizon. Using a multiple of 15, the best performance was achieved with a tolerance level of 3%. When a multiple of 10 was used, the best performance took place with a tolerance level 2%. Note that the difference in returns is very small.

The same results were confirmed by the Sharpe measure. Increasing the tolerance level further failed to improve the performance.

Comparison Between CPPI Portfolios with Different Multiples under Varying Tolerance Levels.

Tolerance Level	Performance 10	Performance 15	Std-Dev 10	Std-Dev 15	Sharpe Measure 10	Sharpe Measure 15	Trans. Costs 10	Trans. Costs 15	Frequency 10	Frequency 15
1%	13.46	13.46	0.920	0.912	0.919	0.885	34,499	38,233	32.67	31.58
2%	13.48	13.48	0.921	0.914	0.949	0.931	31,575	33,917	23.67	24.25
3%	13.44	13.49	0.921	0.918	0.931	0.948	30,456	31,952	22.58	23.17
4%	13.42	13.47	0.920	0.916	0.927	0.938	29,936	30,297	22.50	22.33
5%	-	13.44	-	0.916	-	0.926	-	29,902	-	22.08

The performance is based on the geometric average.

Table 4.9.

By raising the tolerance level, we expected the frequency and the transaction costs to fall. We also expected that a lag would be introduced in the performance of the model when the market would strongly head in one direction or another. Up to this point we were not surprised. What we did not expect is for whipsaws to play such an important role in periods of moderate bull and bear markets. In the twelve years covered, four years showed a performance that was significantly improved with a larger multiple. All four years took place in periods of moderate bull or bear markets. Table 4.10 shows these results.

Comparison of the Performance of CPPI Portfolios with Varying Multiples & Tolerance Levels in Years of Moderate Markets

	Multiple/Tolerance Level								
Market	10/1	10/2	10/3	10/4	15/1	15/2	15/3	15/4	15/5
Moderate Bull									
1991	17.90	18.14	18.23	18.31	17.83	18.07	18.17	18.28	18.38
1986	16.00	16.07	16.12	16.17	16.04	16.11	16.20	16.25	16.26
Moderate Bear:									
1990	8.15	8.18	8.19	8.15	8.11	8.10	8.15	8.14	8.19
1983	9.64	9.66	9.70	9.71	9.56	9.67	9.70	9.71	9.73
AVG Gain	-	+ 9	+14	+16	-	+ 9	+17	+21	+26

The comparison is based on the geometric average. The average gain realized by increasing the tolerance level is expressed in bps.

Table 4.10.

Based on the above, the choice in a tolerance level is not that clear cut. In very strong bullish and bearish markets, a low tolerance level is definitively more advantageous. In more moderate or flat markets, a higher tolerance level increases the performance because fewer transactions are required which reduces the chances for whipsaws. If the portfolio manager is confident about his expectations of the future market behaviour, changing the size of the tolerance level would improve his performance. For example, if the investor expects a large variation in interest rates, a low tolerance level should be used. For expectations of low variation, a higher tolerance level would improve the return. For others who do not have any expectations, using a multiple of 10 with tolerance of 2% or a multiple of 15 and a tolerance level of 3% would slightly improve the overall results. Finally, since CPPI is trading intensive, investors facing large transaction costs should possibly use a larger tolerance level.

CPPI With Special Portfolio Structure

In order to see if CPPI can be successfully used with portfolios of specific structure, we decided to build laddered, barbell and enhanced convexity portfolios (convex portfolio). For all these portfolios, we used a multiple of 10 with a tolerance level of 2% and a multiple of 15 with a tolerance level of 3%. The overall results appear in Table 4.11.

These results indicate that both the barbell and the convex portfolios showed performances that are worth investigating further. Both types of portfolios showed a performance that rivals that of the unstructured portfolios. The laddered portfolio, on the other hand, performed very poorly. The main reason is attributed to the high transaction costs and whipsaws associated with the frequent rebalancing (almost once a week). On average, transaction costs alone reduced the performance by as much as 42 Bps a year. This is very expensive when compared to the costs of less than 4 bps for portfolios with no required structure. The advantages of a laddered portfolio were more than offset by the high transaction required to maintain the laddered structure in a CPPI framework. Reducing the multiple from 15 to 10 improved the overall results slightly but not enough compared to the other portfolios. Given the high transaction costs and the poor overall performance of laddered portfolios, we will not spend any more time on the laddered structure. Instead, we will examine the barbell and convex portfolios in more detail.

Comparison Between Structured CPPI Portfolios and Buy & Hold Portfolios

Portfolio Multiple/Tolerance Level		AVG	STD-DEV	SM	TC	FREQ
Laddered	10/2	13.17	0.922	0.496	420924	48.3
Laddered	15/3	13.13	0.915	0.460	425109	48.2
Convex	10/2	13.44	0.916	1.066	75493	28.1
Convex	15/3	13.38	0.912	0.947	78094	26.3
Barbell	10/2	13.29	0.957	0.709	64557	23.8
Barbell	15/3	13.49	0.948	0.825	63135	22.7
B&H4		13.01	0.761	0.000	48268	34.5
B&H5		13.34	0.940	0.920	50294	34.6
B&H6		13.59	1.140	1.327	71119	35.8

AVG : Average Return (Geometric Average)
SM : Sharpe Measure
STD-DEV: Standard Deviation
TC : Average Transaction Costs
FREQ : Average Number of Trades in a year

Table 4.11.

Table 4.11 shows that the overall performance of the barbell portfolio improved by 20 bps when the multiple was raised from 10 to 15. For the convex portfolio, it was just the opposite. The performance was superior with a lower multiple. These results are also reflected in the Sharpe measure. This suggests that we could possibly obtain a superior performance by increasing the multiple further with a barbell portfolio and by reducing the multiple with a convex portfolio. For the convex portfolio, a multiple of 5 could possibly do the trick. The results in Table 4.11 also indicate that the transaction costs just about doubled compared to the non-structured CPPI portfolios while the frequency of transaction remained relatively stable. This can possibly be explained by the fact that fewer bonds were used to build these portfolios. With fewer bonds to choose from, larger transactions involving more than one bond may become necessary to adjust the CPPI portfolio at the target duration.

In strong bull markets, neither the barbell nor the convex portfolio managed to improve the performance over a B&H5. Refer to Table 4.12. Using a multiple of 10 and 15, the barbell portfolios underperformed a B&H5 portfolio by 37 and 29 bps respectively. The convex portfolios, on the other hand, showed returns that were 99 and 112 bps lower. Moreover, both strategies displayed a performance that was inferior to other non-structured CPPI portfolios. To the extent that we should expect more convex instruments (like barbell and convex portfolios) to dominate other portfolios in strong bull and bear markets, these results are very surprising. The price paid for convexity bonds must have been too dear, which adversely affected the return on these securities. Moreover, a yield shift in the (yield) curve could also have a played an negative role for both the barbell and the portfolio.

In strong bear markets, both the barbell and the convex portfolios outperformed the B&H5 portfolio. The improvement was 5 and 66 bps for the barbell portfolios and 13 and 9 bps for the convex portfolios. Yet, both convex portfolios failed to outperform the non-structured CPPI portfolios. Only the barbell portfolio with a multiple of 15 managed to add gains (of about 17 bps) relative to a non-structured CPPI portfolio (of same multiple and tolerance level). These results seem to support the idea that barbell portfolios perform better with a larger multiple as a faster shift from low (high) to high (low) duration securities is possible.

In more moderate bullish years, the performance was more encouraging for the convex strategy. Both convex portfolios outperformed a B&H5 portfolio by 22 and 8 bps respectively and the non-structured CPPI portfolios by an even wider margin. The improvement was consistent in three out of four years. The barbell portfolios, on the other hand, failed to beat a B&H5 portfolio. In moderate bear market, convex portfolios managed again to improve the portfolio by 49 and 48 bps over a B&H5 portfolio and the barbell portfolios by 23 and 27 bps. The performance of the convex portfolio, however, was not consistent across the two bearish years. In 1983, the

Comparison of the Performance of Several Portfolios in Different Market Environments

Average Return

Market	B&H5	M10	M15	C10	C15	B10	B15
	Portfolios						
Strong Bull Market	29.71	29.84	30.07	28.72	28.59	29.34	29.52
Strong Bear Market	4.29	4.83	4.82	4.38	4.42	4.34	4.95
Moderate Bull Market	15.81	15.79	15.76	16.03	15.89	15.63	15.66
Moderate Bear Market	9.01	8.92	8.92	9.50	9.49	9.24	9.28
Flat to Moderate Market	10.15	10.05	10.01	10.90	10.88	10.22	10.17

Sharpe Measure

Market:	B&H5	M10	M15	C10	C15	B10	B15
	Portfolios						
Strong Bull Market	3.333	3.418	3.602	2.511	2.450	2.727	2.856
Strong Bear Market	-1.104	-0.876	-0.909	-1.238	-1.258	-1.128	-0.814
Moderate Bull Market	2.057	2.042	1.985	2.223	1.944	1.541	1.586
Moderate Bear Market	-0.548	-0.690	-0.681	0.404	0.367	-0.202	-0.154
Flat to Moderate Market	0.558	0.393	0.326	1.763	1.723	0.674	0.591

The comparison is based on the geometric average.
B&H5: Portfolio corresponding to a Fixed Allocation Strategy
M10 : CPPI Portfolio with a Multiple of 10 and a Tolerance level of 2%
M15 : CPPI Portfolio with a Multiple of 15 and a Tolerance level of 3%
B10 : Barbell Portfolio with a Multiple of 10 and a Tolerance level of 2%
B15 : Barbell Portfolio with a Multiple of 15 and a Tolerance level of 3%
C10 : Convex Portfolio with a Multiple of 10 and a Tolerance level of 2%
C15 : Convex Portfolio with a Multiple of 15 and a Tolerance level of 3%
The Sharpe measure = $(R_i - r_f)/\sigma_i$

Table 4.12

convex portfolio added strong gains while in 1990, it underperformed the B&H5 by a wide margin. The barbell portfolios, on the other hand, showed a consistent performance in the two years. Finally, in flat markets, the convex and barbell portfolios both added gains. The Sharpe measure is relatively consistent with the observations noted in the three preceeding paragraphs.

The results of the convex and barbell portfolios are surprising. Theory tells us that a very convex bond will dominate a less convex one of equal duration in strong bull or bear market. It also says that convex bonds will underperform less convex securities in flat to moderate bull or bear market. In our results, just the opposite happened. In very strong bull and bear markets, the convex and barbell portfolios generally underperformed the B&H5 or the other CPPI portfolios by a wide margin. In flat and moderate bull and bear markets, these portfolios generally improved the performance by a relatively sound margin. This may be explained by the fact that the price paid for more convex instruments was too expensive, thus reducing the rate of return. The behaviour of the yield curve could also have a played a certain role: Depending on how the yield curve may twist at one section or another may positively or negatively impact the end results. These reasons could explain the very erratic performance of the convex portfolios from year to year (and for years within a same market: i.e. years of strong bull market) and the unexpected performance of barbell portfolios in certain markets. To a lesser extent, another explanation may be that the convex or barbell portfolios never contained many bonds. Our database contained only a few highly convex bonds (or a smaller selection for a barbell strategy). Moreover, we relaxed the constraints related to the minimum number of bonds and the maximum dollar value per bond to be held in the portfolio. Thus, the selection of a few bonds with a high proportion of the portfolio invested in each could have a very large impact on the performance of the portfolio.

Studying the distribution of excesses and shortfalls in Table 4.13, we find that performance of the convex portfolio was very erratic. Either convex portfolios were able to easily add strong gains relative to the B&H5 or they underperformed the B&H5 portfolio by a wide margin. Compared to non-structured CPPI portfolios, the excesses of the convex portfolios were more frequent (7 vs 5) and the average excess was larger. The shortfalls of convex portfolios, on the other hand, were very large. Most of them were located in the last two intervals and their average shortfalls was on average about 90 bps larger than those of non-structured CPPI portfolios. Raising the multiple from 10 to 15 did not modify the shape of the distribution of the excesses or of the shortfalls.

Relative to a B&H4, the underperformance was also usually very large. The average shortfall relative to the reserve portfolio was about 65 bps lower than those of other non-structured CPPI portfolios. Most of these shortfalls were distributed in the last three intervals. This supports the idea that the convex portfolios had very dispersed returns. These two paragraphs indicate that in the years where

Distribution of Excesses and Shortfalls compared to B&H4, B&H5 and B&H6 Portfolios

Excesses Interval	B&H4 Portfolios: B&H5	M10	C10	C15	B10	B15
[0, 25]	0	0	0	0	0	1
[26, 50]	1	1	0	0	3	2
[51,100]	2	2	0	1	1	1
[101,150]	1	1	3	3	1	1
[151,200]	1	1	3	2	1	1
[201, ++]	2	2	2	2	2	2
Total	7	7	8	8	8	8
AVG (+)	172	173	164	154	137	144

Excesses Interval	B&H5 Portfolios: M10	C10	C15	B10	B15
[0, 5]	0	0	1	2	4
[6,10]	1	0	0	1	1
[11,15]	0	0	0	1	1
[16,20]	1	1	0	1	0
[21,25]	1	0	1	1	0
[26,++]	1	6	5	1	1
Total	5	7	7	7	7
AVG (+)	43	84	95	18	46

Excesses Interval	B&H6 Portfolios: B&H5	M10	C10	C15	B10	B15
[0, 25]	1	1	0	0	1	1
[26, 50]	0	0	1	1	0	0
[51, 100]	3	2	1	1	3	3
[101,150]	0	0	0	0	1	0
[151,200]	0	0	1	1	0	0
[201, ++]	1	2	2	2	1	2
Total	5	5	5	5	6	6
AVG (+)	98	179	169	171	109	140

Shortfalls Interval	B&H4 Portfolios: B&H5	M10	C10	C15	B10	B15
[0, -25]	1	1	0	0	0	0
[-26, -50]	0	0	0	0	0	0
[-51,-100]	3	1	1	1	3	3
[-101,-150]	0	2	1	0	0	0
[-151,-200]	0	0	0	2	0	1
[-201, --]	1	1	2	1	1	0
Total	5	5	4	4	4	4
AVG (-)	131	105	170	170	158	113

Shortfalls Interval	B&H5 Portfolios: M10	C10	C15	B10	B15
[0, -5]	2	0	0	1	2
[-6,-10]	3	0	0	1	0
[-11,-15]	1	0	0	0	0
[-16,-20]	0	0	0	0	0
[-21,-25]	0	0	1	0	0
[-26, --]	1	5	4	3	3
Total	7	5	5	5	5
AVG (-)	11	100	113	38	34

Shortfalls Interval	B&H6 Portfolios: B&H5	M10	C10	C15	B10	B15
[0, -25]	0	1	0	0	0	0
[-26, -50]	1	0	2	1	0	0
[-51, -100]	2	1	2	3	1	1
[-101,-150]	1	3	0	0	2	2
[-151,-200]	1	0	1	1	1	1
[-201, --]	2	2	2	2	2	2
Total	7	7	7	7	6	6
AVG (-)	175	149	174	187	200	191

All numbers are expressed in bps. The average excess or shortfall was computed from the geometric average.
B&H4: Reserve Portfolio
B&H5: Portfolio corresponding to a Fixed Allocation Strategy
B&H6: Active Portfolio
M10 : CPPI portfolio with a multiple of 10 with no specific structure
C10 : Convex portfolio with a multiple of 10 and a tolerance level of 2%
C15 : Convex portfolio with a multiple of 15 and a tolerance level of 3%
B10 : Barbell portfolio with a multiple of 10 and a tolerance level of 2%
B15 : Barbell portfolio with a multiple of 15 and a tolerance level of 3%

Table 4.13

convexity was useful, it did some wonderful work but in the years where it did not, it seriously underperformed other portfolios. While convex portfolio improved the overall returns over a twelve year horizon, their year to year performance seems unpredictable.

The first thing we notice for barbell portfolios is that the excesses relative to B&H5 were also more frequent (7 vs 5) compared to non-structured portfolios. These excesses were relatively evenly distributed with a few more occurrences in the smaller gains. This is reflected in the average excess which stood at 18 and 46 bps for a multiple of 10 and 15 respectively. The shortfalls, on the other hand, at -34 and -38 bps were about twice the size as those of other CPPI portfolios. The distribution indicates that the shortfalls were either very small or relatively large.

Relative to the reserve portfolio, the shortfalls were fewer (4 vs 5) for non-structured CPPI portfolios. The distribution of these shortfalls was concentrated in the -51 to -100 interval. The average shortfall was -158 and -113 bps for a multiple of 10 and 15. Again, we can see that the performance of the barbell portfolio is sensitive to the size of the multiple we use. Vis-à-vis a B&H6 portfolio the average excess and shortfall were also softer than for the other portfolios.

Based on these results, both the convex and the barbell portfolios managed to reshape the return distribution vis-à-vis the B&H5 portfolio. Both portfolios were able to increase the frequency of excesses (relative to the B&H4 and B&H5 portfolios) compared to non-structured CPPI portfolios. The convex portfolios seemed to have more dispersed returns with either large excesses or shortfalls. The barbell portfolios, on the other hand, had more evenly distributed returns. While the return distribution of the barbell portfolio did not seem much affected by the size of the multiple, the average excess (shortfall) generally increased (declined) with a larger multiple.

Summary

We tested Constant Proportion Portfolio Insurance (CPPI) on portfolios of Government of Canada bonds over a period of one year for twelve different years. We compared the performance achieved against a fixed allocation strategy of equivalent initial duration. The overall results show that over a period of 12 years, a portfolio managed based on CPPI outperformed a fixed allocation strategy (B&H5) by 12 bps. By changing the parameters, we found that the performance of CPPI could be improved by combining either a multiple of 10 with a tolerance level of 2% or a multiple of 15 with a tolerance level of 3%. The overall gains relative to the reserve portfolio was 46 bps. The long term cost or underperformance viv-à-vis the active portfolio was only 13 bps.

We also found that CPPI does not perform equally well all the time. For example, a CPPI portfolio was much more successful in adding gains in strong bull or bear market than in a more moderate or flat environment. Furthermore, we found that by changing some parameters in these different market environments, the overall performance could be enhanced. In strong bull or bear markets, increasing the sensitivity to the active asset with a high multiple and a low tolerance level improved the results substantially. In more moderate or relatively flat markets, using a lower multiple and a higher tolerance level to reduce transaction costs and avoid costly whipsaws would increase the end results. We also found that CPPI was able to reshape the return distribution which shows that CPPI can add gains in bull markets and protect a portfolio in bear markets.

CPPI was also possible with different portfolio structures. In fact, a barbell and a convex structure even slightly improved the overall performance over those of non-structured CPPI portfolios. From our results, it appears that the overall performance of these portfolios could even be improved by increasing the multiple for the barbell portfolio and by reducing the multiple for the convex portfolio. The yearly performance of these portfolios, however, was a little more unpredictable as other important elements play a key role in the success of these strategies. Namely: The price paid to implement the strategy and the behaviour of the yield curve. Both elements are crucial. While these two strategies should not be used all the time, their implementation at the right time and their careful monitoring can add substantial returns.

A laddered strategy was also used in a CPPI setting. Unfortunately, the results we obtained were very disappointing due to the high transaction costs involved. Based on these results we do not recommend the use of laddered portfolio with a CPPI strategy.

Although CPPI is a trading intensive strategy, it only required between 22 and 33 trades per year on average depending on the parameters that were chosen. Transaction cost amcunted on average between 29 and 39 thousand dollars. This sum is not much if we consider that our portfolio was worth 100 million dollars at the beginning of every year. The trading costs represent less than 0,04% (4 bps) per year. With a barbell or a convex portfolio structure, the transaction costs almost doubled to as much as 78 thousand dollars on average (with a tolerance level of 2% and 3%), despite a relatively stable trading frequency. Yet, this still represents less than 0.08% (or 8 bps) of the total portfolio.

CONCLUSION

Using CPPI to manage a portfolio of highly correlated fixed income securities yielded mixed results. At first glance, it appears that CPPI is useful. Over the twelve year horizon, the model managed to improve the overall performance by 12 bps over a fixed allocation strategy (B&H5). The overall gain relative to the reserve portfolio was 46 bps and the overall cost relative to the active portfolio was 13 bps only. But this performance was not constant across all types of markets. In very strong bull or bear markets, there is no doubt that CPPI provided some very positive results, improving significantly the performance over a B&H5. The improvement was higher the larger the multiple and the smaller the tolerance level. The model was able to protect the portfolio in bear markets and enhance the performance in bull markets. In more moderate and in flat markets, the story is different. In most of these occasions, the CPPI portfolio performed worse than the B&H5 portfolio. It appears that a more moderate multiple would have been preferable. It also appears that using a higher tolerance level would have yielded a better return. The gain associated with a higher tolerance level is due not only to lower transaction costs and to a lower frequency of trading but also (and sometimes mainly) to a reduction of costly whipsaws. It is also important to note that in the years the CPPI portfolio underperformed a B&H5

portfolio, the difference in performance was very small whereas in the years where the CPPI portfolio outperformed the B&H5 portfolio, the improvements were significantly higher. This shows that CPPI was useful. Yet, in the years the CPPI portfolio was able to enhance the total return, it is important to note that CPPI was only able to capture a fraction of the gain.

Because portfolio managers prefer to use a different strategy or portfolio structure at certain times, we tested CPPI with different structures. Two portfolio structures are worth mentioning: The barbell and the convex portfolios. Both were used successfully in a CPPI framework. The two portfolios were able to show an overall performance that was respectable compare to the other CPPI portfolios. Their performance could possibly be improved with a higher multiple for the barbell portfolio and with a lower multiple for the convex portfolio. While both portfolios were able to slightly increase the overall performance and to reshape the return distribution with a higher frequency of excesses compared to non-structured CPPI portfolios, the convex and the barbell portfolio (at least managed blindly) were unpredictable from year to year. It appears that neither structure can dominate the other all the time even in a CPPI setting. The performance of these two portfolios is very dependant to the price paid for additional convexity and to shifts or twists in the yield curve. If these two elements are carefully monitored and appropriate adjustments taken, there is no doubt that these two portfolio structures can be used successfully in a CPPI environment. As for a laddered portfolio, it seemed difficult to use in a CPPI framework due to the high and frequent transaction costs that were necessary to maintain the laddered structure.

In light of the above results, CPPI is probably not as useful as the we thought it would be in the presence of highly correlated assets. It is very possible that using CPPI with investment grade corporate or municipal bonds as the active asset would have proved more successful since the correlation with the reserve asset would have been reduced. This would require the investor to monitor the default risk of these bonds. Furthermore, CPPI would probably show a better performance if used actively in conjunction with the investor's expectations of future interest rates. For example, the investor could change some of the parameters of the model based on his expectations of future interest rates. For instance, the investor would use a larger multiple and a lower tolerance level if a large variation in the level of interests rates is expected. The investor could also replace the active and the reserve asset by an instrument that is judged more appropriate (or use options and futures). If the investor's expectations are right, his portfolio would show a strong performance. If he is wrong, his performance would suffer, but much less given that he has some kind of safety net.

Given that we used a portfolio instead of a bond index, and that the components of the portfolio varied from time to time, the difference in the performance of the portfolios may be explained to a certain degree by the different bonds in the different portfolios. In order to eliminate the effect of the different portfolios, an active and a reserve bonds indices would have to be constructed. But this brings us back to Hakanoglu et al (1989) and the problem of trading bond indices.

These results should however be taken with a certain degree of latitude. If we consider that the results are analyzed at the end of each year, the period under which this simulation was tested is relatively short. It would have been interesting to test this simulation over a longer period of time. This would have helped us in our analysis when we studied the distribution of returns. It also would have helped to have more years of performance when we separated the periods of bull and bear markets in years of strong, moderate and relatively flat markets. It also would have been interesting to have more bonds, especially when we tested special portfolio structures. It is clear that having just a few bonds qualifying (for some portfolio structures) forced us to loosen some of the constraints in the linear program. This might have affected the results. Although this is out of the scope of this analysis, it would have been interesting to look at the yield curve more often. This would have helped us better determine the exact causes of why a portfolio performed the way it did in a given year or market. In spite of these limitations, we believe that the results and the conclusion suggested by these results are valid and could lead to suggestions for further research.

For example, in this simulation, we purposely decided to keep the duration of the reference portfolios fixed at a certain number of years. This means that the reserve, the active and the B&H5 portfolios, which we have all labelled as Buy & Hold, had to be readjusted over time to maintain their respective duration within a target range. The net effect of these readjustments was to keep the difference of the duration of the risky and riskless asset constant (D_a-D_r). If D_a-D_r had been allowed to fluctuate, their duration would have gradually drifted with time. Two consequences would have resulted for the CPPI portfolio: First, the duration of the CPPI portfolio would also have fallen with time. Second, the difference between D_a-D_r would eventually have grown (When a bond approaches its maturity, its duration falls more quickly than the duration of a longer term bond. Thus, D_a-D_r increases with time as D_r falls more quickly than D_a). This would have slightly increased the duration of the CPPI portfolio. These two consequences would have affected the performance of the CPPI portfolio and would have made more difficult the analysis of the effect of the multiple, the tolerance level, etc. While testing CPPI without adjusting for D_a-D_r can be simulated, the results would be harder to analyze. The researcher would have to isolate for fluctuations in D_a-D_r. It would be interesting to conduct such study for the practical implications in the field of bond portfolio management.

It would also be interesting to repeat this simulation with different durations, over a longer period of time and possibly with more bonds. Furthermore, testing CPPI for FIS in conjunction with other financial instruments could lead to interesting results. For example, it might be useful to include interest rate futures when a fully developped futures market is available. Furthermore, simulating CPPI for FIS with corporate and municipal bonds could yield interesting results. The researcher would, however, have to monitor default rates and adjust the effective duration of these bonds for any embedded options and default risk. A third subject would be to compare the performance of two PI strategies in FIS: Put option replication and CPPI. We believe that all these points represent areas for further research. Finally, the answers to these questions could have important implications for bond portfolio managers.

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APPENDIX 1

WHY CHOOSE CPPI

As we mentioned in the introduction, many different products were designed in the past to protect portfolios. Most of these instruments are based on one of these strategies: The purchase of exchange traded puts options, stop loss order, creation of synthetic options (calls and puts), dynamic hedging using futures contract and constant proportion portfolio insurance. In this section, we will quickly describe each strategies for the reader. Our study has led us to choose CPPI because it is the simplest, the most flexible and the easiest to implement. It does not require any estimate of volatility. It is not based on a complex formula and it is path independent.

PORTFOLIO INSURANCE WITH LISTED PUT OPTIONS

Description of the Strategy

In its simplest form, PI consists of the purchase of a protective put option on a portfolio of risky securities. At maturity, the put will be worth the maximum of:

$$\text{Max } [X-P,0]$$

and the value of a protected portfolio:

Max [P-X-P,P] = Max [X,P]

where: P: value of the Portfolio

X: the exercise price of the option (protection level)

If the market falls, the drop in the value of the portfolio will be offset by the gain in the value of the option. If the market rises, the portfolio value will also increase but the premium paid for the put option will be lost. The cost of this protection corresponds to the premium paid for the purchase of the put option. The protection is provided during the entire life of the option.

Advantages:

- The manager knows the cost of the insurance premium and pays for it at the beginning of the program.
- Does not require constant monitoring.
- Does not require assumptions about volatility or interest rates.
- Path independent.

Disadvantages:

- Tracking problem: because perfect correlation between the portfolio and the market index is difficult to obtain.

Most portfolios are not 100% correlated with the index.
- Only American options are available on the market.
- Listed options are not protected against normal cash dividends payments (in the case of stock index options).
- Maximum maturity of 9 months with most liquidity taking place in the first three months.
- The cumulative purchase of puts would result in a much greater cost than a longer term option.
- Confined to fixed interval and fixed exercise price.

General Comment:

Index put options are useful because they immediately indicate the price the market charges for the insurance protection. This form of insurance is readily available and is simple.

STOP LOSS STRATEGY

Description of Strategy

Conditional market order to sell a portfolio (of risky assets) if its value falls to a given level (floor). It allows the investor to make money if the market goes up and to cut losses approximately to a predetermined level should the market fall.

Advantages:

- Little trading cost. The strategy is not trading intensive.

Disadvantages:

- Path dependant: if the market falls, stop loss order is executed and the portfolio is sold. Then the fund manager needs to make a decision about when to get back into the market. If the market goes back up and the manager stays in cash he has lost the opportunity to gain back what he has lost. On the other hand, if the market keeps going down, the funds position is enhanced by the stop loss order. Thus, the success of this PI strategy is path dependant. Ideally, PI should work regardless of the subsequent movement in the market.

General Comment:

Represents a crude and not very useful form of PI. Cost is difficult to determine due to the high degree of path dependence.

PORTFOLIO INSURANCE WITH SYNTHETIC OPTIONS

Description of the Strategy:

The strategy was developed by Rubinstein and Leland (1981) and Rubinstein (1985). The objective is to replicate the price behaviour and the payoff of an option by continuously trading a risky and a riskless asset (T-Bills) based on an option pricing formula.

First, let us see how a synthetic option can be created. A call option can be replicated by investing in a stock (risky asset) and borrowing at the risk-free rate. When the market is rising, more stocks are purchased, financed by more borrowing. When the reverse happens, the strategy calls for selling the stocks and paying off borrowing or investing at the risk-free rate. The amount in the stocks and riskless asset is determined by the delta of the option being replicated. Trading in the stocks and risk-free assets will be precipitated with changes in the delta of the option. The idea is to be more invested in the stocks as the delta rises and less invested when it falls in value. At maturity, if the call is in the money (the delta is one), the investor is long one entire share financed by borrowing. To realize the same profit as the call holder, the investor sells the share and uses the proceed to repay the borrowing. The difference is the same as the profit realized by the call holder. If at maturity, the option is out of the money, no share of stocks is held in the portfolio. The amount borrowed still outstanding is due to the loss taken on trading the two assets and corresponds to the premium one would have had to pay for the privilege to own a real call option.

A put option can be replicated by shorting stocks and lending the proceed at the risk-free rate. When the market falls, more stocks are sold short with the proceeds being lent at the risk-free rate. When the market goes up, stocks are repurchased using the proceeds from the initial short sale. Again the exact position in the stocks and in the risk-free asset depends on the delta of the option being replicated. At maturity, if the put is in the money, one entire share of the stock will be held short with the proceed lent out. To obtain the same profit as the put holder, the investor liquidates the risk-free asset and covers the short position. The gain realized on the short sale of the short (plus the interest income of the risk-free asset) corresponds to the profit the put holder would have made. If, at maturity, the option is out of the money, the investor is fully invested in the shares of the risky asset: No share of the stocks is sold short as the shares were repurchased as the stock was heading higher. At maturity, there will be a small deficit in the lending due to the loss taken on trading the two assets. The deficit corresponds to the premium the investor would have had to pay for the purchase of a real put option. Thus, the payoff is the same as for a real put option.

Synthetic PI can be achieved by combining the position of a long stock and a synthetic put option in the underlying stock. The net position is long stock (but less than one share because part of a share in the stock was sold to replicate the put) and lending at the risk-free rate. When the market falls, shares of the stocks are sold and the proceeds is used to purchase risk-free assets. When the reverse happens, shares of the stocks are repurchased, financed by liquidating the risk-free assets. Insurance is provided because the portfolio is fully invested in stocks when the market rises (the delta of the put option is almost 0) and very little invested when the market falls (the delta of the put option grows close to 1). PI can also be achieved by combining a call option with lending.

It is important to note that the delta of an option is also affected by the time to maturity, the volatility in the underlying security as well as the interest rate and the possible dividend payments on the risky security. Furthermore, it is important that the transaction costs be relatively negligible, that borrowing and selling short be possible. The investor must also realize that it may not always be possible to replicate an option. For example, if there is a gap opening or if the risky asset collapses before the investor can readjust his positions.

Although it is probably impossible to find a strategy to exactly duplicate options in all circumstances, Rubinstein and Leland believe that replicating options by carefully readjusting the positions in the underlying risky security and a riskless asset is close enough in most conditions so as to make this strategy a valuable tool. In fact, many investors (option traders) use this strategy to arbitrage mispricing in options, to hedge some of the risk on their option positions or to replicate options where none exist.

Advantages:

- Allows PI where options market does not exist or to replicate an options if premium on existing options is judged to be too expensive.
- Allows portfolio manager to tailor make his own PI strategy.
- Creates a european option where none exists.
- Asymptotically path independent except near the maturity of the life of the option if the option is at the money. At that point, the delta of the option becomes very sensitive. A rapid move (up or down) by the risky asset

can keep the portfolio from participating in the rally or put it below its floor. The result is an increase in the cost of the strategy.

Disadvantages:

- Trading Intensive.
- Requires liquidity of financial Instruments.
- Requires Continuous prices. Gap opening or catastrophic price decline can hinder the user from replicating the option position.
- There may be some uncertainty regarding future volatility, interest rates and dividends (for stocks).
- Based on a complex formula.
- Options have an expiration date.

General Comment:

If the volatility during the implementation of the program is lower (higher) than what was expected at the beginning, the costs to carry out the strategy will be lower (higher). Grossman (1988) argues that the volatility, and thus the cost of this strategy, is dependant upon the demand for synthetic insurance.

PORTFOLIO INSURANCE WITH FUTURES (DYNAMIC HEDGING)

Description of the Strategy:

A fund manager who owns a portfolio of risky assets will trade index futures to adjust the exposure of his portfolio. If the market (of risky assets) declines, futures contracts will be shorted to effectively reduce the position in the portfolio. If the market goes up, the portfolio manager repurchases futures contracts to cover the short position. The amount of index futures shorted at any times also depends on the delta of the option being replicated. In fact, this strategy is the same as PI with a synthetic put except that the manager shorts futures contracts instead of selling some of the risky assets in his portfolio and purchasing risk-free assets. Again, the idea is to decrease (increase) the net exposure to risky assets when the market goes down (up).

Advantages:

- Low trading Costs in the futures market.
- Futures market are very liquid even for the largest institutional investors.
- Futures allow for the independence between the management of the risky assets in the portfolio and PI.

Disadvantages:

- Tracking Problem: perfect correlation between the portfolio and the market index is difficult to obtain. Most portfolios are not 100% correlated with the index.
- Continuous monitoring.
- Trading intensive.
- Futures mispricing and basis risk can raise costs. Depends on a high degree of correlation between the value of the index futures contracts and the value of the underlying index.
- Insurance cost is unknown at the beginning and is realized as the dynamic hedging strategy is implemented
- Requires volatility estimates.

General Comment:

Futures mispricing and basis risk can be an advantage or a disadvantage. This is the most popular strategy in the equity market.

APPENDIX 2

CALCULATIONS

	Week #1	Week #2	Week #3
1- Value of the Portfolio:	\$100	\$102	\$102
2- Value of the Floor:	\$ 90	\$ 91	\$ 93
3- Value of the Cushion:	\$ 10	\$ 11	\$ 9
4- Proportional Cushion (c): c=cushion/value of portfolio	.10	.1078	.0882
5- Multiple (M):	5	5	5
6- Duration of Active Asset (D_a):	7	7	7
7- Duration of Reserve Asset: (D_r):	5	5	5
8- Duration Adjusted Multiple (m_d): $m_d = M * (D_a - D_r)$	10	10	10
9- Duration of the Portfolio:	?	?	?

The multiple, the duration adjusted multiple and the duration of both the reserve and active portfolio are fixed. These are inputs chosen by the manager at the beginning of the program based on his investment objectives and his expectations of future interest rates.

Calculations:

Duration of the portfolio:

$$D_p = D_r + m_d*c$$

The portfolio is managed in terms of duration. The duration of the CPPI portfolio will oscillate between a minimum bound, D_r, and a maximum bound, D_a (if the portfolio is capped). This is the relationship the above formula expresses. If the manager is successful, the duration of the portfolio will be increased towards that of the active asset. If he is not, the duration of the portfolio will be reduced towards that of the reserve index. In the worst case scenario, the CPPI portfolio becomes immunized with a duration equal to that of the reserve asset. In the best case, the CPPI portfolio will have a duration equal to that of the active asset (if capped). The manager can revise his inputs at any time. Once the inputs have been chosen by the manager, the only indicator to watch is the proportional cushion because it is the only one that varies and that will cause the duration to change. These calculations are performed at the beginning of every week and will determine if rebalancing the portfolio is necessary.

Duration of the portfolio

Week #1:

$D_p = 5 + (10*0.1) = \underline{6}$.

Week #2:

$D_p = 5 + (10*0.1078) = \underline{6.08}$ We need to increase the duration of the portfolio.

Week #3:

$D_p = 5 + (10*0.0882) = \underline{5.88}$ We need to reduce the duration of the portfolio.

As one can see, when the portfolio outperforms (underperforms) the floor (reserve), we systematically increase (reduce) the duration of the portfolio. The calculations are straight forward. All inputs are given.

APPENDIX 3

LIST OF INPUTS

Starting Year?	1980
Ending Year?	1980
Initial Value of the Floor or Reserve Portfolio ($)?	95000000
Duration of the Reserve Portfolio?	4
Initial Value of the CPPI portfolio ($)?	100000000
Multiple?	10
Name of Result File?	result80
Transaction Cost for each bond ($100 Face Value)?	0.1
Duration of Active Portfolio?	6
Tolerance level?	0.01

For the CPPI portfolio:

Basis for Optimization (1 or 2)?	1
1) Max Initial YTM on portfolio for the first week & Min Transaction Costs during subsequent weeks	
2) Convexity & Min Transaction Costs during subsequent weeks	
Maximum quantity (par value) per bond?	400
Minimum quantity (par value) per bond?	0
Minimum number of bonds in the portfolio?	0
Preselection of the bonds based on its coupon size (Y or N)?	N

If Yes,

Coupons greater or equal to?

Coupons smaller or equal to?

Type of Portfolio (1,2,3,4)? 1

1) Barbell

Duration greater or equal to? 6

Duration smaller or equal to? 3.5

2) Bullet

Duration greater or equal to?

Duration smaller or equal to?

3) Laddered

Minimum Percentage?

Maximum Percentage?

Additional margin for flexibility (%)?

4) None

For the Floor or the Reserve Portfolio

Basis for Optimization (1 or 2)? 1

1) Max Initial YTM on portfolio for the first week

& Min Transaction Costs during subsequent weeks

2) Convexity

& Min Transaction Costs during subsequent weeks

Maximum quantity (par value) per bond? 400

Minimum quantity (par value) per bond? 0

Minimum number of bonds in the portfolio? 0

Preselection of the bonds based on the size of the coupon (Y or N)? N

If Yes,

Coupons greater or equal to?

Coupons smaller or equal to?

Type of Portfolio (1,2,3,4)? 4

1) Barbell

Duration greater or equal to?

Duration smaller or equal to?

2) Bullet

Duration greater or equal to?

Duration smaller or equal to?

3) Laddered

Minimum Percentage?

Maximum Percentage?

4) None

Are the results OK (Y or N)? Y

APPENDIX 4

OBJECTIVE FUNCTIONS AND CONSTRAINTS OF THE MODEL

Definition of the Variables:

V	= Value of the Portfolio before rebalancing
VV	= Value of the portfolio after transaction costs due to rebalancing
$X_{i(t-1)}$	= Quantity of bond i already held in the portfolio at time t minus 1
X_{it}^{p}	= Quantity of bond i to be purchased at time t
X_{it}^{s}	= Quantity of bond i to be sold at time t
Z_{it}	= Boulean Variable. This is to count the number of bonds held in the portfolio at time t
P_{it}	= Price of Bond i at time t
Y_{it}	= Yield to maturity of bond i at time t
D_{it}	= Duration of Bond i at time t
D_{r}	= Duration of Reserve Asset
D_{pt}	= Duration exposure for the CPPI portfolio at time t
C_{it}	= Convexity of Bond i at time t
TL	= Tolerance level specified by the manager at the beginning of the program.
IC_{t}	= Idle cash at time t. IC is invested in securities of one week in maturity

t = Current week. t is replaced by o for the initial week

Each bond unit has a $25000 face value. A unit of idle cash is worth a dollar.

D_p will vary on a weekly basis based on the performance of the portfolio and the reserve portfolio.

Both the reserve and the CPPI portfolios are created based on the first week's objective function and constraints. The reserve and CPPI portfolios are then managed using the objective and constraints of the every other week.

First Week[25]

Objective Function: Max $\Sigma_i^n (X_{io}{}^p Y_{io} P_{io})/V_o$

Max: Yield to Maturity of the Portfolio

Constraints:

$$\Sigma_i^n (X_{io}{}^p D_{io} P_{io})/V_o >= (1-TL)*D_{po}$$

$$\Sigma_i^n (X_{io}{}^p D_{io} P_{io})/V_o <= (1+TL)*D_{po}$$

Deviations of up to a certain percentage (1 to 5%) are

[25]No transaction costs were recorded in the first.

tolerated before rebalancing takes place. D_{po} is replaced by D_{ro} for the reserve portfolio.

[Remember: $D_p = D_r + m_d c$ and $m_d c = m \cdot (D_a - D_r) \cdot c$]

$$\Sigma \; (X_{io}^{p} P_{io} + IC_o) = V_o$$

$X_{io}^{p} \quad <= 400Z_i$

$X_{io}^{p} \quad >= 10Z_i$

$X_{io}^{p} \quad >= 0$

$\Sigma Z_{io} \quad >= 20$

$Z_{io} \quad = 0$ or 1

$IC_o \quad <= 25000$

** $\Sigma_i^{n} \; (X_{io}^{p} P_{io} C_{io}) \; / V_o >= 55$

We want the CPPI portfolio to have a convexity greater than a minimum number.

At the beginning of a new week, the duration and the value of the model and the reserve portfolios will be computed based on the new bond prices. Accrued interest will be considered in the evaluation. Based on the new information, the duration exposure (target) of the CPPI portfolio will be determined. If the duration of the model (reserve) portfolio falls outside the tolerance bound of the duration exposure (reserve duration), the model (reserve) portfolio will be rebalanced.

Every other Week:

Objective Function: Min $\Sigma_i^n (X_{it}^s + X_{it}^p)*0.1$

Min: Transaction Costs

Constraints:

$$\Sigma_i^n \; (X_{i(t-1)} - X_{it}^s + X_{it}^p)* P_{it}D_{it} \; /VV_t >= (1-TL) * D_{pt}$$

$$\Sigma_i^n \; (X_{i(t-1)} - X_{it}^s + X_{it}^p)* P_{it}D_{it} \; /VV_t <= (1+TL) * D_{pt}$$

Deviations of up to a certain percentage (1 to 5%) are tolerated before rebalancing takes place. D_{pt} is replaced by D_{rt} for the reserve portfolio.

$$\Sigma_i^n \; (X_{i(t-1)} - X_{it}^s + X_{it}^p) * P_{it}D_{it} + IC_t - (\Sigma_i^n (X_{it}^s + X_{it}^p) * 0.1) = VV_t$$

$X_{it} <= 400Z_i$

$X_{it} >= 10Z_i$

$X_{it} >= 0$

$\Sigma Z_{it} >= 20$

$Z_{it} = 0$ or 1

$IC_t <= 25000$

** $\Sigma_i^n \; (X_{i(t-1)} - X_{it}^s + X_{it}^p)* P_{it}C_{it} \; /VV_t >= 55$

We want the CPPI portfolio to have a convexity greater than this minimum number.

** This constraint will be used when we will test the model with convexity.

In order to build barbell portfolios, the bonds in the database will be screened based on the desired durations. No new constraints will be added into the model. Only the bonds that are preselected will be used by the linear program to build the barbell portfolios.

For laddered portfolios, an additional constraint will be added. A the beginning of every year, each bond will be classified based on its maturity into a group of bonds of similar maturities ($j = 1,...,m$). A minimum of 5% and a maximum of 20% of the portfolio will be allocated in each group. The amount allocated in each group is the sum invested in each bond of that group. 8 to 9 groups of maturity will be possible each year. See appendix 7 for the groups of maturity. The following equation formalizes the constraint:

$$\Sigma_{i=1}^{n}{}_{j=1}^{m} \; (X_i P_{it} \; / \; VV_t) >= 0.05$$

$$\Sigma_{i=1}^{n}{}_{j=1}^{m} \; (X_i P_{it} \; / \; VV_t) <= 0.20$$

APPENDIX 5

LIST OF GOVERNMENT OF CANADA BONDS

APPENDIX 5 List of Government of Canada Bonds

	Coupon Size	Maturity Date	1991	1990	1989	1988	1987	1986	1985	1984	1983	1982	1981	1980	Issue Date
							YEARS Amount Outstanding (in millions)								
1	12.25	15OCT82										150	150	150	01OCT80
2	8	15OCT82										475	475	475	15OCT77
3	11.75	15DEC82										875	875	875	11DEC79
4	13.75	15MAR83									300	300	300	300	31MAR80
5	4.5	01SEP83									1992	1992	1992	1992	01SEP58
6	9	15DEC83									350	350	350	350	01DEC80
7	10	01JUN84								1075	1075	1075	1075	1075	01FEB79
8	11.25	01JUL85							450	450	450	450	450	450	01JUN80
9	8.00	15DEC85							117	117	117	117	117	117	01OCT78
10	10.00	15MAR86						625	625	625	625				22FEB83
11	13	06JUN86						375	375	375					06MAR84
12	8	01OCT86						410	410	410	410	410	410	410	01OCT69
13	15.00	15MAR87					800	800	800	800	800	800			31MAR82
14	8.25	01JUL87					525	525	525	525	525	525	525	525	01JUL77
15	13.5	01SEP87					150	150	150	150					01AUG84
16	11.00	15DEC87					900	900	900	775	325	325			15DEC82
17	11.75	01FEB88				350	350	350	125	125					14NOV84
18	10.5	15MAR88				875	875	875	625	625	625				22FEB83
19	5	01JUN88				150	150	150	150	150	150	150	150	150	01JUN63
20	6.75	15FEB89			150	150	150	150	150	150	150	150	150	150	15FEB71
21	13.75	01AUG89			442	442	442	442	442						01MAR81
22	11.25	15DEC89			1075	1075	1075	1075	675	675	450	450	450	450	15DEC79
23	13.75	15MAR90		840	840	840	840	840	840	840					31MAY80
24	5.25	01MAY90		225	225	225	225	225	225	225	225	225	225	225	01MAY65
25	10.75	01SEP90		550	550	550	550	550	100	100	100				12JUL83
26	12.5	01FEB91	719	719	719	719	719								01FEB81

APPENDIX 5 List of Government of Canada Bonds

	Coupon	Maturity	YEARS Amount Outstanding (in millions)												Issue
	Size	Date	1991	1990	1989	1988	1987	1986	1985	1984	1983	1982	1981	1980	Date
27	9.25	01MAR91	725	725	725	550	550	375	375						15DEC85
28	8.5	01JUL91	1700	1700	1700	1700	1700	1700							23APR86
29	18	01OCT91	393	393	393	393	393	393	118						15OCT81
30	9.25	01JUL92	1400	1400	1300	1125	1125								21MAY87
31	15	01JUL92	399	399	399	399	399								01JUL82
32	5.75	01SEP92	225	225	225	225	225	225	225	225	225	225	225	225	01SEP66
33	11.75	15DEC92	2050	2050	2050	2050	2050	2050	2050	2050	2050	1100			22NOV82
34	10.75	01MAY93	1725	1625	1550	1550	1050	1050	1050	1050	1050				15MAY83
35	15.25	01JUN93	895	895	895	895	895	173							01JUN81
36	8.75	01FEB94	250	250	250	250	250	250							15DEC86
37	8.25	01MAR94	800	800	800	800	800								29JAN87
38	13.75	15MAY94	1200	1200	1200	1200	1200	1200	1200	1200					01JUN84
39	9.5	15JUN94	640	676	728	728	735	764	815	815	845	874	896	896	15JUN74
40	12.25	01FEB95	725	725	725	725	725	725	725	425					14NOV84
41	6.5	01OCT95	100	100	100	100	100	100	100	100	100	100	100	100	01OCT68
42	10	01OCT95	659	678	691	691	691	691	710	754	754	780	805	825	01OCT75
43	9.25	01MAY96	3300	3300	2000	1550	1550	475							10APR86
44	8.75	01JUN96	2175	2175	2175	2175	1975	1975							28APR86
45	3	15SEP96	55	55	55	55	55	55	55	55	55	55	55	55	15SEP36
46	8.25	01MAR97	1125	1125	1125	1125	1125								29JAN87
47	9.25	15MAY97	906	936	984	984	1002	1032	1032	1074	1074	1098	1122	1140	15MAT77
48	3.75	15MAR98	197	197	197	197	197	197	197	197	197	197	197	197	15SEP56
49	9.5	01OCT98	2200	2200	1500	1500									30JUN88
50	9	15OCT99	546	564	593	593	593	603	621	647	647	661	676	687	15OCT77
51	13.5	01DEC99	400	400	400	400	400	400	400	400	400	400	400	400	21SEP80
52	13.75	15MAR00	1050	1050	1050	1050	1050	1050	1050	1050	1050	1050	750	250	31MAR80
53	15	01JUL00	175	175	175	175	175	175	175	175	175	175	175		01JUL81

APPENDIX 5 List of Government of Canada Bonds

	Coupon Size	Maturity Date	YEARS Amount Outstanding (in millions) 1991	1990	1989	1988	1987	1986	1985	1984	1983	1982	1981	1980	Issue Date
54	9.75	15DEC00	509	516	538	538	538	550	550	572	572	584	597	606	15DEC78
55	13	01MAY01	1325	1325	1325	1325	1325	1325	1325	1325	1325	1325	1325	825	01MAY80
56	8.75	01FEB02	220	228	240	240	245	252	263	263	269	275	280	280	01FEB77
57	10	01MAY02	1850	1850	1850	1850	1850	1850	1850	1850	1850	1850	1850	1850	01MAT79
58	12	01MAR05	1775	1775	1775	1775	1775	1775	1775	1775	750				15OCT83
59	10	01JUN08	3450	3450	2475	550	325	325	325						15DEC85
60	9.5	01JUN10	2975	2525	650	650	325	325							10APR86
61	8.5	01JUN11	750	750	750	750	750								19FEB87
62	10.25	15MAR14	2550	850	850										15MAR89
63	11.25	01JUN15	2350	2350											01MAY90
64	10.5	15MAR21	700	700											15DEC90

APPENDIX 6

AVERAGE WEEKLY RATES

Year	Average Short Term Rate (1 week)	Year	Average Short Term Rate (1 week)
1980	11.77%	1986	8.22%
1981	15.95%	1987	6.44%
1982	11.32%	1988	7.85%
1983	7.67%	1989	9.91%
1984	9.62%	1990	10.89%
1985	8.29%	1991	7.51%

Table A6.1

APPENDIX 7

MATURITY GROUPS FOR LADDERED PORTFOLIOS

There are 9 maturity groups for each year except for 1980 and 1991 where there are only 8 groups. The groups were formed so that a minimum number of bonds are present in each group at all time. For example, in the year 1980, the groups 0-3 and 3-5 contain bonds that will mature in the next three years in the first group and between the third and the fifth year in the second respectively. The amount allocated in each group is based on a minimum and a maximum weight (specified in the inputs by the portfolio manager).

Maturity Groups for Laddered Portfolios

YEAR	MATURITY GROUPS								
1980:	0-3	3-5	5-8	8-11	11-16	16-19	19-21	21-23	
1981:	0-2	2-4	4-6	6-9	9-14	14-17	17-19	19-21	21-25
1982:	0-2	2-4	4-6	6-8	8-11	11-14	14-17	17-19	19-24
1983:	0-2	2-4	4-6	6-8	8-11	11-13	13-16	16-18	18-23
1984:	0-2	2-4	4-6	6-8	8-10	10-13	13-16	16-18	18-23
1985:	0-2	2-4	4-6	6-8	8-10	10-12	12-15	15-17	17-24
1986:	0-2	2-4	4-6	6-8	8-10	10-13	13-15	15-17	17-27
1987:	0-2	2-4	4-6	6-8	8-10	10-13	13-15	15-19	19-26
1988:	0-2	2-4	4-6	6-8	8-10	10-12	12-14	14-18	18-25
1989:	0-2	2-4	4-6	6-8	8-10	10-12	12-14	14-22	22-33
1990:	0-2	2-4	4-6	6-8	8-10	10-12	12-16	10-21	21-32
1991:	0-2	2-4	4-6	6-8	8-10	10-12	12-20	20-31	

Table A7.1

APPENDIX 8

MINIMUM CONVEXITY REQUIRED FOR THE CONVEX PORTFOLIOS

In this table, the minimum required portfolio convexity for the convex portfolios can be found for each year. As a mean of comparison, the minimum and average portfolio convexity realized for an un-structured portfolio are provided for each year.

Minimum and Average Convexity for Two Portfolios

Year	Convex Portfolio Minimum Convexity	Convex Portfolio Average Convexity	Unstructured CPPI Portfolio Minimum Convexity	Unstructured CPPI Portfolio Average Convexity
1980	43.0	50.4	41.2	46.2
1981	46.0	47.9	39.2	44.1
1982	49.0	54.2	41.5	46.5
1983	51.0	52.7	45.9	48.2
1984	50.0	52.0	39.4	42.5
1985	50.0	52.1	39.3	42.8
1986	53.0	55.9	48.0	50.7
1987	49.0	50.6	35.6	38.8
1988	51.0	51.2	38.1	39.9
1989	55.0	56.8	47.2	48.8
1990	58.0	60.9	52.4	56.7
1991	58.0	63.3	54.3	56.7

The information in this Table are based on a Convex and an Unstructured Portfolio with a multiple of 10 and a tolerance level of 2%.

Table A8.1

APPENDIX 9

EFFECT OF COMBINING MULTIPLE AND FLOOR OF VARYING SIZES

The following matrix summarizes the implications for the possible combinations between a multiple and a floor.

	Size of the Multiple: Small	Large
Low Floor/ High Cushion	- With a low multiple, the return on the portfolio is less tied to the performance of the risky asset. - A low floor provides a low level of protection - The combination of the low multiple with a high cushion translates into a moderate exposure to the risky asset. - For a risk averse investor - Adequate for active and reserve assets whose returns are not highly correlated. - Ideal for PI.	- With a high multiple, the return on the portfolio is more tied to the performance of the risky asset. - A low floor provides a low level of protection - The combination of the high multiple with a high cushion translates into a very high exposure to the risky asset. - This strategy provides an almost meaningless level of protection. PI becomes almost irrelevant. - For a very risk seeking investor.
High Floor/ Low Cushion	- With a low multiple, the return on the portfolio is less tied to the performance of the risky asset. - A high floor provides a high level of protection - The combination of the low multiple with a low cushion translates into a very low exposure to the risky asset. - For a highly risk averse investor.	- With a high multiple, the return on the portfolio is more tied to the performance of the risky asset. - A high floor provides a high level of protection - The combination of the high multiple with a low cushion translates into a moderate exposure to the risky asset. - For a averse investor. - Adequate for active and reserve assets highly correlated like FIS. - Ideal for PI.

Multiple-Floor Matrix
Exhibit A9.1

APPENDIX 10

EQUATIONS OF CPPI FOR FIS

$$Ed_p = m_d c$$

Ed_p = Duration Exposure
m_d = Duration Adjusted Multiple
c = Proportional Cushion

CPPI for FIS
Exhibit A10.1

The equations below were used for the development that eventually lead to the above relationship: (From Hakanoglu et al, 1989):

$$A = MC \qquad (1)$$

and

$$D_p = D_a f_a + D_r f_r \qquad (2)$$

Where:

$$f_a = \frac{A}{A+R} = \frac{A}{P} \qquad (3)$$

$$f_r = \frac{R}{A+R} = \frac{R}{P} \qquad (4)$$

Which leads to:

$$D_p = D_a \frac{MC}{P} + D_r \left(1 - \frac{MC}{P}\right) \qquad (5)$$

$$D_p - D_r = (D_a - D_r)\frac{MC}{P} \qquad (6)$$

$$Ed_p = D_p - D_r \qquad (7)$$

$$m_d = M\,(d_a - d_r) \qquad (8)$$

$$c = C/P \qquad (9)$$

Which gives:

$$Ed_p = m_d c \qquad (10)$$

Where:

Active Asset	: (A)
Reserve Asset	: (R)
Portfolio Value	: (P)
Cushion	: (C)
Duration of the Reserve	: (D_r)
Duration of the Active	: (D_a)
Multiple	: (M)
Duration Adjusted Multiple	: (M_d)
Proportional Cushion	: (c)
Duration Exposure	: (Ed_p)
Portfolio Duration	: (D_p)

APPENDIX 11

RESULTS

Initial Duration 5
Multiple 5
Floor (millions) 90
Tolerance Level (%) 1
Structure of PortfolioNo structure

	Annual				Duration		
Year	Return	Std-Dev	T-C	Freq	Min	Max	Avg
80	6.840	1.4289	46614	34	4.76	5.05	4.90
81	3.230	1.8079	73337	39	4.66	4.99	4.84
82	38.690	1.3376	43525	38	4.85	5.4	5.07
83	9.680	0.5741	34130	32	4.89	5.12	5.00
84	14.950	0.8126	19127	31	4.81	5.07	4.95
85	21.480	0.7486	12342	23	4.92	5.19	5.07
86	15.810	0.7094	58534	26	4.94	5.16	5.08
87	4.300	1.2518	18221	34	4.79	5.07	4.92
88	10.110	0.6112	15004	38	4.93	5.12	4.99
89	14.330	0.5161	37830	34	4.95	5.15	5.03
90	8.220	0.9087	53754	37	4.79	4.98	4.90
91	17.960	0.4458	52457	29	4.95	5.16	5.04
GAVG	13.453	0.9294	38739.6	32.9167			

Initial Duration 5
Multiple 7
Floor (millions) 92.857
Tolerance Level (%) 1
Structure of Portfolio No structure

	Annual				Duration		
Year	Return	Std-Dev	T-C	Freq	Min	Max	Avg
80	7.090	1.4283	38656	34	4.7	5.08	4.87
81	3.220	1.7814	66694	38	4.56	5.01	4.79
82	38.650	1.3364	33076	35	4.81	5.54	5.07
83	9.670	0.5758	34374	36	4.87	5.13	4.99
84	14.860	0.8086	18398	34	4.75	5.07	4.91
85	21.490	0.7491	13157	24	4.92	5.22	5.08
86	15.910	0.7115	53097	25	4.94	5.19	5.10
87	4.300	1.2400	15377	34	4.73	5.08	4.89
88	10.120	0.6123	14677	36	4.92	5.13	4.99
89	14.320	0.5177	37787	37	4.94	5.19	5.05
90	8.190	0.8998	50638	34	4.73	4.98	4.85
91	17.940	0.4476	51337	28	4.95	5.21	5.05
GAVG	13.468	0.9257	35605.7	32.9167			

Std-Dev :Annual Standard Deviation
T-C :Annual Transaction Cost
Freq :Annual Frequency of trading
GAVG :Geometric Average Return
All other averages are arithmetic

Initial Duration 5
Multiple 8
Floor (millions) 93.5
Tolerance Level (%) 1
Structure of PortfolioNo structure

	Annual				Duration		
Year	Return	Std-Dev	T-C	Freq	Min	Max	Avg
80	7.020	1.4220	36065	33	4.66	5.08	4.86
81	3.230	1.7693	65020	36	4.52	5.01	4.76
82	38.660	1.3357	34277	36	4.79	5.61	5.08
83	9.660	0.5762	33986	38	4.86	5.14	4.99
84	14.830	0.8052	18636	35	4.73	5.07	4.90
85	21.490	0.7508	13954	27	4.92	5.25	5.09
86	15.970	0.7113	50462	25	4.94	5.22	5.12
87	4.300	1.2347	14565	34	4.7	5.08	4.88
88	10.110	0.6120	15360	37	4.92	5.13	5.00
89	14.310	0.5180	37678	36	4.94	5.21	5.06
90	8.170	0.8957	49384	33	4.7	4.98	4.83
91	17.920	0.4480	50728	28	4.95	5.23	5.06
GAVG	13.460	0.9232	35009.6	33.1667			

Initial Duration 5
Multiple 10
Floor (millions) 95
Tolerance Level (%) 1
Structure of Portfolio No structure

	Annual				Duration		
Year	Return	Std-Dev	T-C	Freq	Min	Max	Avg
80	6.970	1.4121	31391	32	4.58	5.07	4.82
81	3.230	1.7462	62599	37	4.44	5.03	4.72
82	38.820	1.3383	37737	35	4.76	5.77	5.11
83	9.640	0.5754	32851	33	4.84	5.15	4.99
84	14.780	0.7995	19776	39	4.67	5.07	4.87
85	21.550	0.7543	16026	28	4.92	5.34	5.13
86	16.000	0.7143	48977	26	4.94	5.25	5.15
87	4.280	1.2261	13767	32	4.64	5.08	4.85
88	10.100	0.6125	15626	33	4.91	5.13	5.00
89	14.310	0.5200	37332	37	4.94	5.25	5.08
90	8.150	0.8879	47531	32	4.64	4.98	4.79
91	17.900	0.4491	50371	28	4.95	5.25	5.06
GAVG	13.461	0.9196	34498.7	32.6667			

Std-Dev :Annual Standard Deviation
T-C :Annual Transaction Cost
Freq :Annual Frequency of trading
GAVG :Geometric Average Return
All other averages are arithmetic

Initial Duration 5
Multiple 12
Floor (millions) 95.833
Tolerance Level (%) 1
Structure of Portfolio No structure

Year	Annual Return	Std-Dev	T-C	Freq	Duration Min	Max	Avg
80	6.910	1.4033	27698	33	4.54	5.05	4.78
81	3.230	1.7235	62922	37	4.37	5.05	4.67
82	38.980	1.3418	41805	40	4.73	5.91	5.14
83	9.620	0.5751	33114	29	4.84	5.15	4.99
84	14.750	0.7977	21334	37	4.62	5.07	4.86
85	21.600	0.7579	18921	31	4.93	5.43	5.17
86	16.020	0.7186	48911	26	4.94	5.29	5.18
87	4.290	1.2195	12870	29	4.61	5.08	4.84
88	10.060	0.6130	16743	32	4.91	5.13	5.00
89	14.310	0.5218	37072	38	4.94	5.3	5.11
90	8.130	0.8804	46260	30	4.58	4.98	4.75
91	17.880	0.4508	50498	28	4.95	5.26	5.07
GAVG	13.463	0.9169	34845.7	32.5			

Initial Duration 5
Multiple 15
Floor (millions) 96.667
Tolerance Level (%) 1
Structure of Portfolio No structure

Year	Annual Return	Std-Dev	T-C	Freq	Duration Min	Max	Avg
80	6.790	1.3833	40018	33	4.45	5.05	4.71
81	3.280	1.6947	67402	39	4.29	5.07	4.60
82	39.170	1.3454	54649	43	4.68	6.13	5.18
83	9.560	0.5747	32527	26	4.83	5.2	5.00
84	14.690	0.7930	24018	35	4.55	5.07	4.83
85	21.680	0.7634	25766	31	4.93	5.57	5.22
86	16.040	0.7226	51076	29	4.94	5.38	5.22
87	4.300	1.2095	12474	23	4.55	5.08	4.82
88	10.010	0.6145	18865	31	4.9	5.17	5.01
89	14.310	0.5246	36757	34	4.94	5.36	5.14
90	8.110	0.8688	44903	29	4.5	4.98	4.69
91	17.830	0.4531	50342	26	4.95	5.32	5.08
GAVG	13.458	0.9123	38233.1	31.5833			

Std-Dev :Annual Standard Deviation
T-C :Annual Transaction Cost
Freq :Annual Frequency of trading
GAVG :Geometric Average Return
All other averages are arithmetic

Initial Duration 5
Multiple 15
Floor (millions) 96.667
Tolerance Level (%) 2
Structure of Portfolio No structure

	Annual				Duration		
Year	Return	Std-Dev	T-C	Freq	Min	Max	Avg
80	6.760	1.3865	27247	26	4.47	5.05	4.72
81	3.360	1.6996	57809	30	4.29	5.07	4.61
82	39.040	1.3441	50195	31	4.66	6.1	5.17
83	9.670	0.5785	25674	21	4.91	5.25	5.05
84	14.630	0.7897	24307	22	4.54	5.07	4.82
85	21.690	0.7630	24088	23	4.93	5.56	5.23
86	16.110	0.7268	47179	23	4.94	5.4	5.25
87	4.320	1.2149	11842	21	4.59	5.08	4.83
88	10.020	0.6128	17137	20	4.89	5.13	5.00
89	14.220	0.5332	30751	25	4.95	5.43	5.17
90	8.100	0.8727	43014	25	4.54	4.98	4.72
91	18.070	0.4492	47756	24	4.95	5.36	5.13
GAVG	13.478	0.9143	33916.6	24.25			

Initial Duration 5
Multiple 15
Floor (millions) 96.667
Tolerance Level (%) 3
Structure of Portfolio No structure

	Annual				Duration		
Year	Return	Std-Dev	T-C	Freq	Min	Max	Avg
80	6.640	1.3912	20263	22	4.43	5.05	4.71
81	3.540	1.7112	54027	29	4.32	5.02	4.64
82	39.040	1.3441	50195	31	4.66	6.1	5.17
83	9.700	0.5849	20835	21	4.92	5.29	5.08
84	14.580	0.7892	24200	21	4.47	5.07	4.80
85	21.680	0.7635	22968	20	4.93	5.57	5.23
86	16.200	0.7307	45185	23	4.94	5.46	5.29
87	4.320	1.2149	11842	21	4.59	5.08	4.83
88	10.010	0.6135	14274	18	4.88	5.13	4.99
89	14.140	0.5380	31290	25	4.95	5.49	5.20
90	8.150	0.8781	41993	24	4.58	4.98	4.77
91	18.170	0.4514	46361	23	4.95	5.42	5.18
GAVG	13.494	0.9176	31951.9	23.1667			

Std-Dev :Annual Standard Deviation
T-C :Annual Transaction Cost
Freq :Annual Frequency of trading
GAVG :Geometric Average Return
All other averages are arithmetic

Initial Duration 5
Multiple 15
Floor (millions) 96.667
Tolerance Level (%) 4
Structure of Portfolio No structure

	Annual				Duration		
Year	Return	Std-Dev	T-C	Freq	Min	Max	Avg
80	6.500	1.3856	16668	21	4.46	5.05	4.72
81	3.580	1.7094	52231	28	4.23	5.02	4.62
82	38.690	1.3308	45045	25	4.61	5.96	5.12
83	9.710	0.5856	20837	21	4.92	5.35	5.12
84	14.570	0.7833	24578	21	4.47	5.07	4.78
85	21.600	0.7575	20123	20	4.93	5.48	5.17
86	16.250	0.7360	43594	23	4.94	5.51	5.33
87	4.320	1.2149	11842	21	4.59	5.08	4.83
88	10.040	0.6104	12456	18	4.88	5.13	4.97
89	14.110	0.5425	29363	23	4.95	5.46	5.20
90	8.140	0.8827	41969	24	4.62	4.98	4.80
91	18.280	0.4539	44857	23	4.95	5.48	5.23
GAVG	13.467	0.9161	30296.9	22.3333			

Initial Duration 5
Multiple 15
Floor (millions) 96.667
Tolerance Level (%) 5
Structure of Portfolio No structure

	Annual				Duration		
Year	Return	Std-Dev	T-C	Freq	Min	Max	Avg
80	6.740	1.3894	15737	19	4.37	5.05	4.71
81	2.940	1.6844	44760	27	4.15	5.02	4.60
82	38.740	1.3318	46834	25	4.66	6	5.11
83	9.730	0.5864	21361	22	4.92	5.4	5.16
84	14.470	0.7822	26291	21	4.46	5.07	4.75
85	21.600	0.7575	20123	20	4.93	5.48	5.17
86	16.260	0.7403	42014	23	4.94	5.56	5.37
87	4.320	1.2149	11842	21	4.59	5.08	4.83
88	9.980	0.6087	14566	18	4.81	5.13	4.95
89	14.110	0.5449	29299	23	4.95	5.48	5.22
90	8.190	0.8918	42561	23	4.63	5.02	4.84
91	18.380	0.4559	43432	23	4.95	5.54	5.28
GAVG	13.435	0.9157	29901.7	22.0833			

Std-Dev :Annual Standard Deviation
T-C :Annual Transaction Cost
Freq :Annual Frequency of trading
GAVG :Geometric Average Return
All other averages are arithmetic

Initial Duration	5
Multiple	20
Floor (millions)	97.5
Tolerance Level (%)	1
Structure of Portfolio	No structure

	------------**Annual**------------				--------**Duration**-------		
Year	**Return**	**Std-Dev**	**T-C**	**Freq**	**Min**	**Max**	**Avg**
80	6.560	1.3411	70747	35	4.25	5.05	4.58
81	3.410	1.6558	89435	39	4.15	5.1	4.51
82	39.670	1.3621	119289	45	4.61	6.66	5.26
83	9.410	0.5749	49984	27	4.76	5.3	5.00
84	14.600	0.7885	29690	32	4.47	5.08	4.78
85	21.760	0.7725	44457	38	4.91	5.81	5.31
86	15.800	0.7269	57734	37	4.9	5.52	5.28
87	4.210	1.1854	20556	32	4.41	5.07	4.76
88	9.960	0.6185	22692	30	4.9	5.25	5.04
89	14.300	0.5295	37581	31	4.95	5.48	5.21
90	8.060	0.8490	43691	28	4.37	4.98	4.59
91	17.860	0.4559	51328	31	4.95	5.44	5.10
GAVG	13.434	0.9050	53098.7	33.75			

Initial Duration	4
Multiple	Nil
Floor (millions)	Nil
Tolerance Level (%)	1
Structure of Portfolio	Buy & Hold (Reserve)

	------------**Annual**------------				--------**Duration**-------		
Year	**Return**	**Std-Dev**	**T-C**	**Freq**	**Min**	**Max**	**Avg**
80	7.690	1.1515	36774	35	3.96	4.04	3.99
81	5.550	1.5964	92221	40	3.96	4.04	3.98
82	33.970	1.0447	76991	38	3.96	4.04	4.01
83	9.790	0.4113	42042	36	3.96	4.04	4.00
84	14.210	0.6422	32800	33	3.96	4.04	4.00
85	19.060	0.6240	14436	30	3.96	4.04	4.02
86	14.300	0.5760	65919	26	3.96	4.04	4.01
87	5.290	1.0426	29316	35	3.96	4.04	4.00
88	9.810	0.4875	16483	34	3.96	4.04	3.99
89	13.510	0.4276	44784	28	3.96	4.04	4.00
90	9.210	0.7431	65695	41	3.96	4.04	4.00
91	16.500	0.3813	61760	26	3.96	4.04	4.01
GAVG	13.009	0.7607	48268	33.5			

Std-Dev :**Annual Standard Deviation**
T-C :**Annual Transaction Cost**
Freq :**Annual Frequency of trading**
GAVG :**Geometric Average Return**
All other averages are arithmetic

Initial Duration 5
Multiple Nil
Floor (millions) Nil
Tolerance Level (%) 1
Structure of Portfolio Buy & Hold (Fixed Allocation)

	------------Annual------------				--------Duration-------		
Year	Return	Std-Dev	T-C	Freq	Min	Max	Avg
80	6.940	1.4597	73154	37	4.95	5.05	5.00
81	1.680	1.8936	100718	41	4.95	5.05	4.98
82	38.570	1.3196	75315	38	4.95	5.05	5.01
83	9.730	0.5636	42884	35	4.95	5.05	5.00
84	15.090	0.8128	27988	35	4.95	5.05	5.00
85	21.420	0.7420	16766	30	4.95	5.05	5.02
86	15.610	0.7010	65337	30	4.95	5.05	5.01
87	4.320	1.2843	27882	34	4.95	5.05	4.98
88	10.150	0.6088	22632	39	4.95	5.05	4.97
89	14.390	0.5147	38557	31	4.95	5.05	4.99
90	8.290	0.9292	64598	39	4.95	5.05	4.99
91	17.970	0.4482	55257	26	4.95	5.05	5.02
GAVG	13.339	0.9398	50924	34.5833			

Initial Duration 6
Multiple Nil
Floor (millions) Nil
Tolerance Level (%) 1
Structure of Portfolio Buy & Hold (Active)

	------------Annual------------				--------Duration-------		
Year	Return	Std-Dev	T-C	Freq	Min	Max	Avg
80	4.410	1.8008	100307	37	5.94	6.08	5.99
81	0.410	2.2163	277384	40	5.94	6.07	5.97
82	43.130	1.6894	131947	42	5.94	6.06	6.01
83	9.640	0.7713	35061	38	5.94	6.06	6.00
84	15.920	0.9919	41685	40	5.94	6.06	6.00
85	23.570	0.8655	23677	31	5.94	6.06	6.00
86	17.120	0.8353	42402	29	5.94	6.06	6.02
87	3.320	1.5108	36942	39	5.94	6.06	5.97
88	10.130	0.7361	21509	37	5.94	6.06	5.96
89	15.390	0.6004	26583	26	5.94	6.06	5.99
90	7.340	1.1223	61501	39	5.94	6.06	5.99
91	18.620	0.5400	54432	31	5.94	6.06	6.00
GAVG	13.590	1.1400	71119	35.75			

Std-Dev :Annual Standard Deviation
T-C :Annual Transaction Cost
Freq :Annual Frequency of trading
GAVG :Geometric Average Return
All other averages are arithmetic

Initial Duration	**5**
Multiple	**10**
Floor (millions)	**95**
Tolerance Level (%)	**2**
Structure of Portfolio	**No structure**

	Annual				Duration		
Year	**Return**	**Std-Dev**	**T-C**	**Freq**	**Min**	**Max**	**Avg**
80	6.940	1.4179	28302	25	4.61	5.13	4.83
81	3.280	1.7493	56124	30	4.44	5.03	4.72
82	38.740	1.3354	36782	27	4.76	5.78	5.10
83	9.660	0.5777	26918	21	4.9	5.24	5.04
84	14.760	0.7961	19236	21	4.67	5.07	4.86
85	21.520	0.7535	15349	20	4.93	5.33	5.13
86	16.070	0.7194	46807	23	4.94	5.31	5.19
87	4.310	1.2217	12673	22	4.62	5.08	4.85
88	10.050	0.6114	13943	20	4.88	5.13	4.99
89	14.230	0.5278	30041	23	4.95	5.31	5.10
90	8.180	0.8919	44821	27	4.63	4.98	4.83
91	18.140	0.4490	47902	25	4.95	5.33	5.11
GAVG	13.475	0.9209	31574.8	23.6667			

Initial Duration	**5**
Multiple	**10**
Floor (millions)	**95**
Tolerance Level (%)	**3**
Structure of Portfolio	**No structure**

	Annual				Duration		
Year	**Return**	**Std-Dev**	**T-C**	**Freq**	**Min**	**Max**	**Avg**
80	6.970	1.4200	24418	20	4.61	5.19	4.84
81	2.800	1.7347	56345	28	4.38	5	4.69
82	38.790	1.3302	37754	24	4.73	5.72	5.09
83	9.700	0.5846	20837	21	4.92	5.29	5.08
84	14.690	0.7940	21449	21	4.61	5.07	4.84
85	21.510	0.7541	14362	20	4.93	5.29	5.13
86	16.120	0.7244	45258	23	4.94	5.36	5.23
87	4.320	1.2178	12711	21	4.6	5.08	4.84
88	10.060	0.6105	12564	18	4.86	5.13	4.98
89	14.110	0.5351	29481	25	4.95	5.37	5.13
90	8.190	0.8969	43526	25	4.68	4.99	4.86
91	18.230	0.4517	46763	25	4.95	5.38	5.16
GAVG	13.438	0.9212	30455.7	22.5833			

Std-Dev :Annual Standard Deviation
T-C :Annual Transaction Cost
Freq :Annual Frequency of trading
GAVG :Geometric Average Return
All other averages are arithmetic

Initial Duration	5
Multiple	10
Floor (millions)	95
Tolerance Level (%)	4
Structure of Portfolio	No structure

	--------------Annual--------------				--------Duration--------		
Year	**Return**	**Std-Dev**	**T-C**	**Freq**	**Min**	**Max**	**Avg**
80	6.830	1.4140	21883	19	4.5	5.19	4.82
81	2.910	1.7312	53182	27	4.38	5	4.69
82	38.220	1.3179	38669	25	4.65	5.69	5.03
83	9.710	0.5856	20836	21	4.92	5.34	5.11
84	14.770	0.7933	24568	22	4.62	5.07	4.84
85	21.540	0.7498	13346	20	4.93	5.25	5.09
86	16.170	0.7289	43725	23	4.94	5.41	5.27
87	4.330	1.2152	12771	20	4.59	5.08	4.83
88	10.080	0.6075	12948	18	4.81	5.13	4.96
89	14.110	0.5396	29472	25	4.95	5.44	5.18
90	9.150	0.9005	42208	25	4.72	5.04	4.89
91	18.310	0.4527	45627	25	4.95	5.44	5.21
GAVG	13.416	0.9197	29936.3	22.5			

Initial Duration	5
Multiple	10
Floor (millions)	95
Tolerance Level (%)	2
Structure of Portfolio	Barbell

	------------Annual------------				--------Duration-------		
Year	Return	Std-Dev	T-C	Freq	Min	Max	Avg
80	7.110	1.4424	43114	27	4.6	5.14	4.85
81	1.670	1.7690	161074	33	4.18	5.04	4.67
82	37.850	1.5544	162793	27	4.63	5.8	5.10
83	10.160	0.6291	40471	18	4.91	5.3	5.07
84	15.230	0.7773	18107	20	4.72	5.16	4.94
85	21.360	0.7776	12491	18	4.93	5.33	5.16
86	16.060	0.7336	75825	23	4.92	5.36	5.23
87	4.320	1.2299	26749	23	4.66	5.09	4.87
88	10.220	0.6080	12101	17	4.89	5.15	5.01
89	13.830	0.5158	88601	25	4.93	5.25	5.06
90	8.320	0.8967	65785	29	4.65	4.97	4.83
91	17.410	0.4969	67573	26	4.84	5.28	5.06
GAVG	13.285	0.9526	64557	23.8333			

Initial Duration	5
Multiple	15
Floor (millions)	96.667
Tolerance Level (%)	3
Structure of Portfolio	Barbell

	------------Annual------------				--------Duration-------		
Year	Return	Std-Dev	T-C	Freq	Min	Max	Avg
80	6.910	1.4195	40162	22	4.47	5.13	4.77
81	3.690	1.7455	133824	29	4.39	5.05	4.67
82	38.120	1.5593	192824	31	4.45	6.2	5.14
83	10.230	0.6433	26971	17	4.91	5.35	5.12
84	15.220	0.7745	26202	20	4.59	5.22	4.92
85	21.460	0.7870	17477	18	4.93	5.5	5.26
86	16.290	0.7463	69983	21	4.92	5.56	5.37
87	4.290	1.2109	25639	20	4.54	5.12	4.84
88	10.170	0.6088	12789	16	4.89	5.15	5.01
89	13.680	0.5224	88968	25	4.91	5.38	5.08
90	8.330	0.8869	62512	29	4.56	4.97	4.78
91	17.470	0.4739	60278	24	4.87	5.39	5.10
GAVG	13.490	0.9482	63135.8	22.6667			

Std-Dev :Annual Standard Deviation
T-C :Annual Transaction Cost
Freq :Annual Frequency of trading
GAVG :Geometric Average Return
All other averages are arithmetic

Initial Duration	5
Multiple	15
Floor (millions)	96.667
Tolerance Level (%)	3
Structure of Portfolio	Convex

	Annual				Duration		
Year	Return	Std-Dev	T-C	Freq	Min	Max	Avg
80	6.040	1.4203	40873	22	4.46	5.12	4.77
81	3.650	1.7229	143692	36	4.33	5.04	4.70
82	36.430	1.2607	123757	33	4.77	5.87	5.13
83	11.360	0.6215	39990	19	5	5.36	5.18
84	15.450	0.7917	99080	24	4.78	5.3	5.04
85	21.200	0.7258	36254	25	5.05	5.57	5.27
86	16.570	0.7412	94656	23	5.02	5.56	5.36
87	2.720	1.1997	127116	34	4.2	5.1	4.76
88	10.880	0.6209	25024	25	5	5.29	5.10
89	14.640	0.5034	40581	20	4.95	5.47	5.26
90	7.650	0.8706	106771	31	4.51	4.95	4.69
91	16.920	0.4662	59336	24	4.94	5.3	5.09
GAVG	13.383	0.9121	78094.2	26.3333			

Initial Duration	5
Multiple	10
Floor (millions)	95
Tolerance Level (%)	2
Structure of Portfolio	Convex

	Annual				Duration		
Year	Return	Std-Dev	T-C	Freq	Min	Max	Avg
80	7.130	1.4431	42512	26	4.59	5.14	4.85
81	3.310	1.7388	123435	35	4.44	5.01	4.77
82	37.010	1.2611	108148	34	4.85	5.67	5.12
83	11.290	0.6073	46369	21	5	5.36	5.15
84	15.450	0.7993	83533	26	4.86	5.31	5.06
85	20.930	0.7160	49652	25	5.05	5.41	5.19
86	16.770	0.7325	105001	24	5.01	5.46	5.26
87	2.770	1.2375	104217	38	4.48	5.1	4.85
88	10.900	0.6183	28108	27	4.99	5.24	5.08
89	14.840	0.4912	60346	19	4.95	5.33	5.18
90	7.740	0.8848	94769	34	4.6	4.95	4.78
91	17.090	0.4612	59833	28	4.94	5.2	5.07
GAVG	13.442	0.9159	75493.6	28.0833			

Std-Dev :Annual Standard Deviation
T-C :Annual Transaction Cost
Freq :Annual Frequency of trading
GAVG :Geometric Average Return
All other averages are arithmetic

Initial Duration	5
Multiple	10
Floor (millions)	95
Tolerance Level (%)	2
Structure of Portfolio	**Laddered**

	Annual				Duration		
Year	**Return**	**Std-Dev**	**T-C**	**Freq**	**Min**	**Max**	**Avg**
80	6.670	1.3855	295250	50	4.54	5.05	4.84
81	3.430	1.7588	71095	44	4.44	5.05	4.74
82	38.960	1.3621	320826	45	4.75	5.7	5.10
83	9.090	0.5734	392973	51	4.81	5.26	4.99
84	14.150	0.7923	597027	48	4.66	5.06	4.85
85	21.200	0.7392	121560	49	4.89	5.28	5.08
86	14.900	0.7219	230426	49	4.94	5.27	5.10
87	3.870	1.2356	470718	44	4.66	5.15	4.91
88	9.220	0.6172	1093136	52	4.75	5.25	4.92
89	13.580	0.5164	431617	49	4.91	5.26	5.07
90	8.560	0.8729	312677	50	4.6	4.96	4.81
91	18.700	0.4871	713781	49	4.83	5.28	5.07
GAVG	13.172	0.9219	420924	48.3333			

Initial Duration	5
Multiple	15
Floor (millions)	96.667
Tolerance Level (%)	3
Structure of Portfolio	**Laddered**

	Annual				Duration		
Year	**Return**	**Std-Dev**	**T-C**	**Freq**	**Min**	**Max**	**Avg**
80	6.040	1.3310	367873	49	4.37	5.05	4.68
81	3.400	1.7084	59208	43	4.28	5.05	4.62
82	39.170	1.3998	332279	44	4.62	6.12	5.17
83	8.980	0.5620	384304	51	4.73	5.29	4.97
84	13.780	0.7839	694723	47	4.56	5.06	4.81
85	21.340	0.7479	130452	49	4.89	5.49	5.15
86	15.550	0.7334	131773	49	4.94	5.37	5.18
87	3.870	1.2332	500297	46	4.66	5.22	4.89
88	9.260	0.6138	1058241	52	4.66	5.29	4.87
89	13.570	0.5235	421302	49	4.87	5.42	5.14
90	8.340	0.8569	301785	51	4.51	4.95	4.72
91	18.670	0.4835	719077	49	4.76	5.44	5.12
GAVG	13.133	0.9148	425110	48.25			

Std-Dev :Annual Standard Deviation
T-C :Annual Transaction Cost
Freq :Annual Frequency of trading
GAVG :Geometric Average Return
All other averages are arithmetic

Annual Return

DURATION	4	5	6	5	5	5	5	5	5	5	5	5	5
TYPE	B&H	B&H	B&H	PI	PI	PI	PI	PI	PI	PI	PI	PI	PI
MULTIPLE	nil	nil	nil	5	7	8	10	12	15	20	15	15	15
FLOOR	nil	nil	nil	90	92.8	93.3	95	95.8	96.6	97.5	96.6	96.6	96.6
TOL LEVEL	1%	1%	1%	1%	1%	1%	1%	1%	1%	1%	2%	3%	4%
STRUCTURE	No	No	No	No	No	No	No	No	No	No	No	No	No
YEAR													
1980	7.69	6.94	4.41	6.84	7.09	7.02	6.97	6.91	6.79	6.56	6.76	6.64	6.5
1981	5.55	1.68	0.41	3.23	3.22	3.23	3.23	3.23	3.28	3.41	3.36	3.54	3.58
1982	33.97	38.57	43.13	38.69	38.65	38.66	38.82	38.98	39.17	39.67	39.04	39.04	38.69
1983	9.79	9.73	9.64	9.68	9.67	9.66	9.64	9.62	9.56	9.41	9.67	9.7	9.71
1984	14.21	15.09	15.92	14.95	14.86	14.83	14.78	14.75	14.69	14.6	14.63	14.58	14.57
1985	19.06	21.42	23.57	21.48	21.49	21.49	21.55	21.6	21.68	21.76	21.69	21.68	21.6
1986	14.3	15.81	17.12	15.81	15.91	15.97	16	16.02	16.04	15.8	16.11	16.2	16.25
1987	5.29	4.32	3.32	4.3	4.3	4.3	4.28	4.29	4.3	4.21	4.32	4.32	4.32
1988	9.81	10.15	10.13	10.11	10.12	10.11	10.1	10.06	10.01	9.96	10.02	10.01	10.04
1989	13.51	14.39	15.39	14.33	14.32	14.31	14.31	14.31	14.31	14.3	14.22	14.14	14.11
1990	9.21	8.29	7.34	8.22	8.19	8.17	8.15	8.13	8.11	8.06	8.1	8.15	8.14
1991	16.5	17.97	18.62	17.96	17.94	17.92	17.9	17.88	17.83	17.86	18.07	18.17	18.28
GAVG	13.009	13.339	13.590	13.453	13.468	13.460	13.461	13.463	13.458	13.434	13.478	13.494	13.467
AVG T-C	48268	50924	71119	38740	35606	35010	34499	34846	38233	53099	33917	31952	30297
AVG Freq	33.50	34.58	35.75	32.92	32.92	33.17	32.67	32.50	31.58	33.75	24.25	23.17	22.33

DURATION: Initial Duration
TYPE: B&H (Buy & Hold Portfolio) or PI (Insured Portfolio based on CPPI)
MULTIPLE: Size of the Multiple
FLOOR: Size of the Floor in Million
TOL LEVEL: Tolerance Level
STRUCTURE: Structure of the Portfolio (None, Laddered, Barbell or Convex)
GAVG: Geometric Average Return
AVG T-C: Average Transaction Cost
AVG Freq: Average Trading Frequency

Annual Return

DURATION	5	5	5	5	5	5	5	5	5	5
TYPE	PI	PI	PI	PI	PI	PI	PI	PI	PI	PI
MULTIPLE	15	10	10	10	10	15	10	15	10	15
FLOOR	96.6	95	95	95	95	96.6	95	96.6	95	96.6
TOL LEVEL	5%	2%	3%	4%	2%	3%	2%	3%	2%	3%
STRUCTURE	No	No	No	No	Laddered	Laddered	Barbell	Barbell	Convex	Convex
YEAR										
1980	6.74	6.94	6.97	6.83	6.67	6.04	7.11	6.91	7.13	6.94
1981	2.94	3.28	2.8	2.91	3.43	3.4	1.67	3.69	3.31	3.65
1982	38.74	38.74	38.79	38.22	38.96	39.17	37.85	38.12	37.01	36.43
1983	9.73	9.66	9.7	9.71	9.09	8.98	10.16	10.23	11.29	11.36
1984	14.47	14.76	14.69	14.77	14.15	13.78	15.23	15.22	15.45	15.45
1985	21.6	21.52	21.51	21.54	21.2	21.34	21.36	21.46	20.93	21.2
1986	16.26	16.07	16.12	16.17	14.9	15.55	16.06	16.29	16.77	16.57
1987	4.32	4.31	4.32	4.33	3.87	3.87	4.32	4.29	2.77	2.72
1988	9.98	10.05	10.06	10.08	9.22	9.26	10.22	10.17	10.9	10.88
1989	14.11	14.23	14.11	14.11	13.58	13.57	13.83	13.68	14.84	14.64
1990	8.19	8.18	8.19	8.15	8.56	8.34	8.32	8.33	7.74	7.65
1991	18.38	18.14	18.23	18.31	18.7	18.67	17.41	17.47	17.09	16.92
GAVG	13.435	13.475	13.438	13.416	13.17219	13.13291	13.28515	13.48978	13.442	13.383
AVG T-C	29902	31575	30456	29936	420924	425109.5	64557	63135.75	75494	78094
AVG Freq	22.08	23.67	22.58	22.50	48.33	48.25	23.83	22.67	28.08	26.33

Annual Transaction Costs

DURATION	4	5	6	5	5	5	5	5	5	5	5	5	5
TYPE	B&H	B&H	B&H	PI	PI	PI	PI	PI	PI	PI	PI	PI	PI
MULTIPLE	nil	nil	nil	5	7	8	10	12	15	20	15	15	15
FLOOR	nil	nil	nil	90	92.8	93.3	95	95.8	96.6	97.5	96.6	96.6	96.6
TOL LEVEL	1%	1%	1%	1%	1%	1%	1%	1%	1%	1%	2%	3%	4%
STRUCTURE	No	No	No	No	No	No	No	No	No	No	No	No	No
YEAR													
1980	36774	73154	100307	46614	38656	36065	31391	27698	40018	70747	27247	20263	16668
1981	92221	100718	277384	73337	66694	65020	62599	62922	67402	89435	57809	54027	52231
1982	76991	75315	131947	43525	33076	34277	37737	41805	54649	119289	50195	50195	45045
1983	42042	42884	35061	34130	34374	33986	32851	33114	32527	49984	25674	20835	20837
1984	32800	27988	41685	19127	18398	18636	19776	21334	24018	29690	24307	24200	24578
1985	14436	16766	23677	12342	13157	13954	16026	18921	25766	44457	24088	22968	20123
1986	65919	65337	42402	58534	53097	50462	48977	48911	51076	57734	47179	45185	43594
1987	29316	27882	36942	18221	15377	14565	13767	12870	12474	20556	11842	11842	11842
1988	16483	22632	21509	15004	14677	15360	15626	16743	18865	22692	17137	14274	12456
1989	44784	38557	26583	37830	37787	37678	37332	37072	36757	37581	30751	31280	29363
1990	65695	64598	61501	53754	50638	49384	47531	46260	44903	43691	43014	41993	41969
1991	61760	:5257	54432	52457	51337	50728	50371	50498	50342	51328	47756	46361	44857
AVG T-C	48268	50924	71119	38740	35606	35010	34499	34846	38233	53099	33917	31952	30297

Annual Transaction Costs

DURATION	**5**	**5**	**5**	**5**	**5**	**5**	**5**	**5**	**5**	**5**
TYPE	**PI**	**PI**	**PI**	**PI**	**PI**	**PI**	**PI**	**PI**	**PI**	**PI**
MULTIPLE	**15**	**10**	**10**	**10**	**10**	**15**	**10**	**15**	**10**	**15**
FLOOR	**96.6**	**95**	**95**	**95**	**95**	**96.6**	**95**	**96.6**	**95**	**96.6**
TOL LEVEL	**5%**	**2%**	**3%**	**4%**	**2%**	**3%**	**2%**	**3%**	**2%**	**3%**
STRUCTURE	**No**	**No**	**No**	**No**	**Laddered**	**Laddered**	**Barbell**	**Barbell**	**Convex**	**Convex**
YEAR										
1980	15737	28302	24418	21883	295250	367873	43114	40162	42512	40873
1981	44760	56124	56345	53182	71095	59208	161074	133824	123435	143692
1982	46834	36782	37754	38669	320826	332279	162793	192824	108148	123757
1983	21361	26918	20837	20836	392973	384304	40471	26971	46369	39990
1984	26291	19236	21449	24568	597027	694723	18107	26202	83533	99080
1985	20123	15349	14362	13346	121560	130452	12491	17477	49652	36254
1986	42014	46807	45258	43725	230426	131773	75825	69983	105001	94656
1987	11842	12673	12711	12771	470718	500297	26749	25639	104217	127116
1988	14566	13943	12564	12948	1093136	1058241	12101	12789	28108	25024
1989	29299	30041	29481	29472	431617	421302	88601	88968	60346	40581
1990	42561	44821	43526	42208	312677	301785	65785	62512	94769	106771
1991	43432	47902	46763	45627	71[illegible]781	719077	67573	60278	59833	59336
AVG T-C	29902	31575	30456	29936	420924	425110	64557	63136	75494	78094

Sharpe Measure

DURATION	**4**	**5**	**6**	**5**	**5**	**5**	**5**	**5**	**5**	**5**	**5**	**5**	**5**
TYPE	**B&H**	**B&H**	**B&H**	**PI**	**PI**	**PI**	**PI**	**PI**	**PI**	**PI**	**PI**	**PI**	**PI**
MULTIPLE	**nil**	**nil**	**nil**	**5**	**7**	**8**	**10**	**12**	**15**	**20**	**15**	**15**	**15**
FLOOR	**nil**	**nil**	**nil**	**90**	**92.8**	**93.3**	**95**	**95.8**	**96.6**	**97.5**	**96.6**	**96.6**	**96.6**
TOL LEVEL	**1%**	**1%**	**1%**	**1%**	**1%**	**1%**	**1%**	**1%**	**1%**	**1%**	**2%**	**3%**	**4%**
STRUCTURE	**No**	**No**	**No**	**No**	**No**	**No**	**No**	**No**	**No**	**No**	**No**	**No**	**No**
YEAR													
1980	0	-0.514	-1.821	-0.595	-0.420	-0.471	-0.510	-0.556	-0.651	-0.843	-0.671	-0.755	-0.859
1981	0	-2.044	-2.319	-1.283	-1.308	-1.311	-1.329	-1.346	-1.340	-1.292	-1.289	-1.175	-1.152
1982	0	3.486	5.422	3.529	3.502	3.511	3.624	3.734	3.865	4.185	3.772	3.772	3.547
1983	0	-0.106	-0.194	-0.192	-0.208	-0.226	-0.261	-0.296	-0.400	-0.661	-0.207	-0.154	-0.137
1984	0	1.083	1.724	0.911	0.804	0.770	0.713	0.677	0.605	0.495	0.532	0.469	0.460
1985	0	3.181	5.211	3.233	3.244	3.237	3.301	3.351	3.432	3.495	3.447	3.432	3.353
1986	0	2.154	3.376	2.128	2.263	2.348	2.380	2.393	2.408	2.064	2.490	2.600	2.649
1987	0	-0.755	-1.304	-0.791	-0.798	-0.802	-0.824	-0.820	-0.819	-0.911	-0.798	-0.798	-0.798
1988	0	0.558	0.435	0.491	0.506	0.490	0.473	0.408	0.325	0.243	0.343	0.326	0.377
1989	0	1.710	3.131	1.589	1.565	1.544	1.539	1.533	1.525	1.492	1.332	1.171	1.106
1990	0	-0.990	-1.666	-1.089	-1.134	-1.161	-1.194	-1.227	-1.266	-1.354	-1.272	-1.207	-1.212
1991	0	3.280	3.926	3.275	3.217	3.169	3.118	3.061	2.935	2.983	3.495	3.699	3.921
AAVG	0	0.920	1.327	0.934	0.936	0.925	0.919	0.909	0.885	0.824	0.931	0.948	0.938

Sharpe Measure

DURATION	**5**	**5**	**5**	**5**	**5**	**5**	**5**	**5**	**5**	**5**
TYPE	**PI**	**PI**	**PI**	**PI**	**PI**	**PI**	**PI**	**PI**	**PI**	**PI**
MULTIPLE	**15**	**10**	**10**	**10**	**10**	**15**	**10**	**15**	**10**	**15**
FLOOR	**96.6**	**95**	**95**	**95**	**95**	**96.6**	**95**	**96.6**	**95**	**96.6**
TOL LEVEL	**5%**	**2%**	**3%**	**4%**	**2%**	**3%**	**2%**	**3%**	**2%**	**3%**
STRUCTURE	**No**	**No**	**No**	**No**	**Laddered**	**Laddered**	**Barbell**	**Barbell**	**Convex**	**Convex**
YEAR										
1980	-0.684	-0.529	-0.507	-0.608	-0.736	-1.240	-0.402	-0.549	-0.388	-0.528
1981	-1.549	-1.298	-1.585	-1.525	-1.205	-1.258	-2.193	-1.066	-1.288	-1.103
1982	3.582	3.572	3.624	3.225	3.663	3.715	2.496	2.661	2.411	1.951
1983	-0.102	-0.225	-0.154	-0.137	-1.221	-1.441	0.588	0.684	2.470	2.526
1984	0.332	0.691	0.605	0.706	-0.076	-0.549	1.312	1.304	1.551	1.566
1985	3.353	3.265	3.249	3.307	2.895	3.049	2.958	3.050	2.612	2.949
1986	2.648	2.460	2.512	2.565	0.831	1.704	2.399	2.666	3.372	3.063
1987	-0.798	-0.802	-0.797	-0.790	-1.149	-1.151	-0.789	-0.826	-2.036	-2.142
1988	0.279	0.393	0.409	0.444	-0.956	-0.896	0.674	0.591	1.763	1.723
1989	1.101	1.364	1.121	1.112	0.136	0.115	0.620	0.325	2.708	2.245
1990	-1.144	-1.155	-1.137	-1.177	-0.745	-1.015	-0.992	-0.992	-1.661	-1.792
1991	4.124	3.653	3.830	3.998	4.516	4.488	1.831	2.047	1.279	0.991
AAVG	0.928	0.949	0.931	0.927	0.496	0.460	0.709	0.825	1.066	0.947

Annual Trading Frequency

DURATION	4	5	6	5	5	5	5	5	5	5	5	5	5
TYPE	B&H	B&H	B&H	PI	PI	PI	PI	PI	PI	PI	PI	PI	PI
MULTIPLE	nil	nil	nil	5	7	8	10	12	15	20	15	15	15
FLOOR	nil	nil	nil	90	92.8	93.3	95	95.8	96.6	97.5	96.6	96.6	96.6
TOL LEVEL	1%	1%	1%	1%	1%	1%	1%	1%	1%	1%	2%	3%	4%
STRUCTURE	No	No	No	No	No	No	No	No	No	No	No	No	No
YEAR													
1980	35	37	37	34	34	33	32	33	33	35	26	22	21
1981	40	41	40	39	38	36	37	37	39	39	30	29	28
1982	38	38	42	38	35	36	35	40	43	45	31	31	25
1983	36	35	38	32	36	38	33	29	26	27	21	21	21
1984	33	35	40	31	34	35	39	37	35	32	22	21	21
1985	30	30	31	23	24	27	28	31	31	38	23	20	20
1986	26	30	29	26	25	25	26	26	29	37	23	23	23
1987	35	34	39	34	34	34	32	29	23	32	21	21	21
1988	34	39	37	38	36	37	33	32	31	30	20	18	18
1989	28	31	26	34	37	36	37	38	34	31	25	25	23
1990	41	39	39	37	34	33	32	30	29	28	25	24	24
1991	26	26	31	29	28	28	28	28	26	31	24	23	23
AVG	33.50	34.58	35.75	32.92	32.92	33.17	32.67	32.50	31.58	33.75	24.25	23.17	22.33

Annual Trading frequency

DURATION	**5**	**5**	**5**	**5**	**5**	**5**	**5**	**5**	**5**	**5**
TYPE	**PI**	**PI**	**PI**	**PI**	**PI**	**PI**	**PI**	**PI**	**PI**	**PI**
MULTIPLE	**15**	**10**	**10**	**10**	**10**	**15**	**10**	**15**	**10**	**15**
FLOOR	**96.6**	**95**	**95**	**95**	**95**	**96.6**	**95**	**96.6**	**95**	**96.6**
TOL LEVEL	**5%**	**2%**	**3%**	**4%**	**2%**	**3%**	**2%**	**3%**	**2%**	**3%**
STRUCTURE	**No**	**No**	**No**	**No**	**Laddered**	**Laddered**	**Barbell**	**Barbell**	**Convex**	**Convex**
YEAR										
1980	19	25	20	19	50	49	27	22	26	22
1981	27	30	28	27	44	43	33	29	35	36
1982	25	27	24	25	45	44	27	31	34	33
1983	22	21	21	21	51	51	18	17	21	19
1984	21	21	21	22	48	47	20	20	26	24
1985	20	20	20	20	49	49	18	18	25	25
1986	23	23	23	23	49	49	23	21	24	23
1987	21	22	21	20	44	46	23	20	38	36
1988	18	20	18	18	52	52	17	16	27	25
1989	23	23	25	25	49	49	25	25	19	26
1990	23	27	25	25	50	51	29	29	34	[illegible]
1991	23	25	25	25	49	49	26	24	28	24
AVG	22.08	23.67	22.58	22.50	48.33	48.25	23.83	22.67	28.08	26.[illegible]

Performance in Different Market Environments

DURATION	4	5	6	5	5	5	5	5	5	5	5	5	5
TYPE	B&H	B&H	B&H	PI	PI	PI	PI	PI	PI	PI	PI	PI	PI
MULTIPLE	nil	nil	nil	5	7	8	10	12	15	20	15	15	15
FLOOR	nil	nil	nil	90	92.8	93.3	95	95.8	96.6	97.5	96.6	96.6	96.6
TOL LEVEL	1%	1%	1%	1%	1%	1%	1%	1%	1%	1%	2%	3%	4%
STRUCTURE	No	No	No	No	No	No	No	No	No	No	No	No	No
YEAR													
STRONG BULL													
1982	33.97	38.57	43.13	38.69	38.65	38.66	38.82	38.98	39.17	39.67	39.04	39.04	38.69
1985	19.06	21.42	23.57	21.48	21.49	21.49	21.55	21.6	21.68	21.76	21.69	21.68	21.6
GAVG	26.30	29.71	32.99	29.80	29.79	29.79	29.90	30.00	30.13	30.41	30.08	30.07	29.86
MODERATE BULL													
1991	16.5	17.97	18.62	17.96	17.94	17.92	17.9	17.88	17.83	17.86	18.07	18.17	18.28
1986	14.3	15.81	17.12	15.81	15.91	15.97	16	16.02	16.04	15.8	16.11	16.2	16.25
1989	13.51	14.39	15.39	14.33	14.32	14.31	14.31	14.31	14.31	14.3	14.22	14.14	14.11
1984	14.21	15.09	15.92	14.95	14.86	14.83	14.78	14.75	14.69	14.6	14.63	14.58	14.57
GAVG	14.62	15.81	16.76	15.75	15.75	15.75	15.74	15.73	15.71	15.63	15.75	15.76	15.79
STRONG BEAR													
1981	5.55	1.68	0.41	3.23	3.22	3.23	3.23	3.23	3.28	3.41	3.36	3.54	3.58
1980	7.69	6.94	4.41	6.84	7.09	7.02	6.97	6.91	6.79	6.56	6.76	6.64	6.50
1987	5.29	4.32	3.32	4.30	4.30	4.30	4.28	4.29	4.30	4.21	4.32	4.32	4.32
GAVG	6.17	4.29	2.70	4.78	4.86	4.84	4.81	4.80	4.78	4.72	4.80	4.82	4.79
MODERATE BEAR													
1990	9.21	8.29	7.34	8.22	8.19	8.17	8.15	8.13	8.11	8.06	8.10	8.15	8.14
1983	9.79	9.73	9.64	9.68	9.67	9.66	9.64	9.62	9.56	9.41	9.67	9.70	9.71
GAVG	9.50	9.01	8.48	8.95	8.93	8.91	8.89	8.87	8.83	8.73	8.88	8.92	8.92
FLAT TO MODERATE													
1988	9.81	10.15	10.13	10.11	10.12	10.11	10.1	10.06	10.01	9.96	10.02	10.01	10.04

Performance in Different Market Environment

DURATION	**5**	**5**	**5**	**5**	**5**	**5**	**5**	**5**	**5**	**5**
TYPE	**PI**	**PI**	**PI**	**PI**	**PI**	**PI**	**PI**	**PI**	**PI**	**PI**
MULTIPLE	**15**	**10**	**10**	**10**	**10**	**15**	**10**	**15**	**10**	**15**
FLOOR	**96.6**	**95**	**95**	**95**	**95**	**96.6**	**95**	**96.6**	**95**	**96.6**
TOL LEVEL	**5%**	**2%**	**3%**	**4%**	**2%**	**3%**	**2%**	**3%**	**2%**	**3%**
STRUCTURE	**No**	**No**	**No**	**No**	**Laddered**	**Laddered**	**Barbell**	**Barbell**	**Convex**	**Convex**
YEAR										
STRONG BULL										
1982	38.74	38.74	38.79	38.22	38.96	39.17	37.85	38.12	37.01	36.43
1985	21.6	21.52	21.51	21.54	21.2	21.34	21.36	21.46	20.93	21.2
GAVG	29.89	29.84	29.86	29.61	29.78	29.95	29.34	29.52	28.72	28.59
MODERATE BULL										
1991	18.38	18.14	18.23	18.31	18.7	18.67	17.41	17.47	17.09	16.92
1986	16.26	16.07	16.12	16.17	14.9	15.55	16.06	16.29	16.77	16.57
1989	14.11	14.23	14.11	14.11	13.58	13.57	13.83	13.68	14.84	14.64
1984	14.47	14.76	14.69	14.77	14.15	13.78	15.23	15.22	15.45	15.45
GAVG	15.79	15.79	15.78	15.83	15.32	15.37	15.63	15.66	16.03	15.89
STRONG BEAR										
1981	2.94	3.28	2.80	2.91	3.43	3.40	1.67	2.69	3.31	3.65
1980	6.74	6.94	6.97	6.83	6.67	6.04	7.11	6.91	7.13	6.94
1987	4.32	4.31	4.32	4.33	3.87	3.87	4.32	4.29	2.77	2.72
GAVG	4.65	4.83	4.68	4.68	4.65	4.43	4.34	4.95	4.38	4.42
MODERATE BEAR										
1990	8.19	8.18	8.19	8.15	8.56	8.34	8.32	8.33	7.74	7.65
1983	9.73	9.66	9.70	9.71	9.09	8.98	10.16	10.23	11.29	11.36
GAVG	8.96	8.92	8.94	8.93	8.82	8.66	9.24	9.28	9.50	9.43
FLAT TO MODERATE										
1988	9.98	10.05	10.06	10.08	9.22	9.26	10.22	10.17	10.9	10.88

Performance in Bull & Bear Markets

DURATION	4	5	6	5	5	5	5	5	5	5	5	5	5
TYPE	B&H	B&H	B&H	PI	PI	PI	PI	PI	PI	PI	PI	PI	PI
MULTIPLE	nil	nil	nil	5	7	8	10	12	15	20	15	15	15
FLOOR	nil	nil	nil	90	92.8	93.3	95	95.8	96.6	97.5	96.6	96.6	96.6
TOL LEVEL	1%	1%	1%	1%	1%	1%	1%	1%	1%	1%	2%	3%	4%
STRUCTURE	No	No	No	No	No	No	No	No	No	No	No	No	No
YEAR													
BULL YEARS													
1982	33.97	38.57	43.13	38.69	38.65	38.66	38.82	38.98	39.17	39.67	39.04	39.04	38.69
1984	14.21	15.09	15.92	14.95	14.86	14.83	14.78	14.75	14.69	14.6	14.63	14.58	14.57
1985	19.06	21.42	23.57	21.48	21.49	21.49	21.55	21.6	21.68	21.76	21.69	21.68	21.6
1986	14.3	15.81	17.12	15.81	15.91	15.97	16	16.02	16.04	15.8	16.11	16.2	16.25
1989	13.51	14.39	15.39	14.33	14.32	14.31	14.31	14.31	14.31	14.3	14.22	14.14	14.11
1991	16.5	17.97	18.62	17.96	17.94	17.92	17.9	17.88	17.83	17.86	18.07	18.17	18.28
GAVG	18.39	20.27	21.93	20.26	20.25	20.25	20.28	20.30	20.33	20.36	20.34	20.35	20.30
BEAR YEARS													
1980	7.69	6.94	4.41	6.84	7.09	7.02	6.97	6.91	6.79	6.56	6.76	6.64	6.50
1981	5.55	1.68	0.41	3.23	3.22	3.23	3.23	3.23	3.28	3.41	3.36	3.54	3.58
1983	9.79	9.73	9.64	9.68	9.67	9.66	9.64	9.62	9.56	9.41	9.67	9.70	9.71
1987	5.29	4.32	3.32	4.30	4.30	4.30	4.28	4.29	4.30	4.21	4.32	4.32	4.32
1988	9.81	10.15	10.13	10.11	10.12	10.11	10.10	10.06	10.01	9.96	10.02	10.01	10.04
1990	9.21	8.29	7.34	8.22	8.19	8.17	8.15	8.13	8.11	8.06	8.10	8.15	8.14
GAVG	7.87	6.81	5.82	7.03	7.07	7.05	7.03	7.01	6.98	6.91	7.01	7.03	7.02

Performance in Bull and Bear Markets

DURATION	**5**	**5**	**5**	**5**	**5**	**5**	**5**	**5**	**5**	**5**
TYPE	**PI**	**PI**	**PI**	**PI**	**PI**	**PI**	**PI**	**PI**	**PI**	**PI**
MULTIPLE	**15**	**10**	**10**	**10**	**10**	**15**	**10**	**15**	**10**	**15**
FLOOR	**96.6**	**95**	**95**	**95**	**95**	**96.6**	**95**	**96.6**	**95**	**96.6**
TOL LEVEL	**5%**	**2%**	**3%**	**4%**	**2%**	**3%**	**2%**	**3%**	**2%**	**3%**
STRUCTURE	**No**	**No**	**No**	**No**	**Laddered**	**Laddered**	**Barbell**	**Barbell**	**Convex**	**Convex**
YEAR										
BULL YEARS										
1982	38.74	38.74	38.79	38.22	38.96	39.17	37.85	38.12	37.01	36.43
1984	14.47	14.76	14.69	14.77	14.15	13.78	15.23	15.22	15.45	15.45
1985	21.6	21.52	21.51	21.54	21.2	21.34	21.36	21.46	20.93	21.2
1986	16.26	16.07	16.12	16.17	14.9	15.55	16.06	16.29	16.77	16.57
1989	14.11	14.23	14.11	14.11	13.58	13.57	13.83	13.68	14.84	14.64
1991	18.38	18.14	18.23	18.31	18.7	18.67	17.41	17.47	17.09	16.92
GAVG	20.31	20.30	20.29	20.25	19.95	20.04	20.03	20.11	20.12	19.98
BEAR YEARS										
1980	6.74	6.94	6.97	6.83	6.67	6.04	7.11	6.91	7.13	6.94
1981	2.94	3.28	2.80	2.91	3.43	3.40	1.67	3.69	3.31	3.65
1983	9.73	9.66	9.70	9.71	9.09	8.98	10.16	10.23	11.29	11.36
1987	4.32	4.31	4.32	4.33	3.87	3.87	4.32	4.29	2.77	2.72
1988	9.98	10.05	10.06	10.08	9.22	9.26	10.22	10.17	10.90	10.88
1990	8.19	8.18	8.19	8.15	8.56	8.34	8.32	8.33	7.74	7.65
GAVG	6.95	7.04	6.97	6.97	6.78	6.62	6.92	7.24	7.14	7.15