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A Cluster-Based Classification Method for the Recognition of Unconstrained Handwritten Numerals

Tien Dung La

A Major Technical Report

in

The Department

of (6)

Computer Science

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ABSTRACT

A Cluster-Based Classification Method for the Recognition of Unconstrained Handwritten Numerals

Tien Dung La

The advancement of character recognition has led to its many applications in various fields and it has become an effective means of processing large volumes of data. are several schemes in character recognition. immense variety' in handwritten characters, because of the none of existing methods has proved to be reliable in unconstrained numerals. totally recognizing recognition, by identifying its several characteristic attributes one can deduce the unknown character from one's knowledge. The simulation of such a process through the integration of a data base and the introduction of it into the computer will certainly narrow the gap between human and machine recognition. The objective of this study is to explore the applicability of a cluster-based algorithm to recognize the totally unconstrained handwritten numerals. In our proposed scheme, the information, about characters, that is different distinctive features, is organized into a data base and the combination of these features, in which one feature can complement the discriminant power of others is explored to enhance the recognition accuracy of current The results indicate that the enhancement of which . about recognition is accuracy,

Multi-directional Loci method used in our work, is significant.

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CHAPTER I

INTRODUCTION

1.1. Why is character recognition used in high volume data processing

In the past decades, we have witnessed an explosive growth of information in almost every aspect of our daily In order to cope with this accelerating speed of growth, digital computers and computer networks have been developed to generate, transmit, store, access and process immense volumes of information. The traditional ways of communication have rapidly become more and more electronic processes. Now we can have letters mailed electronically to almost anywhere in the world instantly. An entire library can be stored on magnetic disks and can be retrieved or printed out at remote sites. business, the complex networks of interbanking, credit cards, automatic teller and various types of saving accounts with different daily interest rates 'etc., have become a . reality due to the speed and precision of digital computers. With its financial gules and taxation increasingly complex every' year, the tax administration has become more and more dependent on digital computers for its operation. In industry, automation is on most planning agendas of corporations for efficiency and reliability. In research and development areas. Computer Aided Design (CAD) and

Computer Aided Manufacturing (CAM) have become indispensable tools for their versatilities and applications.

It seems that we have just entered an Information age in which man and his intelligent machines can carry out tasks much more complex and faster than ever before. The work forces have become increasingly more productive and the quality of life has been greatly improved.

However the communication between man and machine is still far from perfect. We need a more friendly man-machine interface. The current efforts in developping fifth generation computers and artificial intelligence are decisively aimed at solving this problem. Especially in the area of character recognition, during the last two decades, a lots of intensive researches have been done. It can provide a solution for processing large volumes of data automatically, e.g. source data entry, postal code reading [1], [2], [3] etc.

1.2. Advances in character recognition.

Owing to intensive research in the past decades, the recognition of machine printing has become a commercial reality for more than a decade. Faced with the fact that there are hundreds of type fonts and print fonts used in the world, special fonts which facilitate processing by optical readers and enhance recognition rates have been developed. Among the most mommonly known are the OCRA character set by

ANSI [4] and OCRB by European Computer Manufacturers Assocciation [5]. By Canadian standard, OCRA and OCRB character sets have been established [6], [7] together with an alphanumeric set [8]. Using these character sets, commercial OCR machines have become reliable tools for processing large volumes of bills, checks, credit card slips, medicare slips, cashier tapes and various paper forms.

Another challenging area of research is the development of reading machines for the blind, but the progress in this field has been hindered by the great number of fonts used in printings and the problem has not been completely solved. There exists commercial reading machines by various manufacturers (e.g., IBM, Control Data Corp.) for several common fonts only.

Besides the well-defined fonts and printed materials, there exists a great demand for handprinted character recognition systems. There are various possible applications, e.g., handwritten computer programs, drawings, checks, tax and medicare forms, vehicle and student registration forms, handwritten envelopes etc. Especially in the case of Zip codes, pressed by the need to process large quantities of mail, the research and development of reliable OCR techniques for mail-sorting has been carried on intensively and could be considered as one of the primary factors providing an impetus to the advancement.

€%.

However, dealing with handprinted characters is a much more difficult task for there is an infinite variation in handwritings because of a writer's own way and style of writing. Even human beings make about 4% mistakes in identifying isolate handwritten characters in the absence of context [9], [10]. It is clearly indicated that more reliable and sophisticated techniques are required for recognizing of handprints.

Significant progress has been made in this direction, a recognition accuracy of over 99% for constrained handprinted. FORTRAN texts has been reported [11], [12]. In Fujimoto et al. [12] as well as R.O. Duda and P.E. Hart [13] experiments, contextual information has been employed to improve recognition. Usually the context analysis follows one of the following approaches:

__ Table look-up : a table of keywords is used for references

Use of probabilities: the probability of occurrence of character pairs (bigrams) and character triplets (trigrams) in English text is used to verify the validity of a recognized character.

Another promising area is on-line character recognition.

On-line character recognition means that characters are recognized as they are written on a writing surface such as a graphic tablet or a CRT with a light pen etc. On-line

character recognition has several applications as computer-aided design [14], and handwritten programs [15].

1.3. What is a character recognition system

The functional block diagram of a typical handprinted character recogition system is shown in Fig. 1

Input characters are read and digitized by an optical scanner. The heart of a scanner is the image sensor array, the most popular one is the charge-coupled device (CCD). Usually the sensor analog output is binarized by applying a threshold that is optimal to the system. If the input is handwritten cursively then each character is located and separated from, the other characters under software control of the computer. Then some of the following preprocessing techniques can be applied on the resulting character matrix:

- _ smoothing : filling of holes and breaks in line segments elimination of noise : clean up isolated black spots
- thinning: a skeleton of character is obtained
- size and slant normalization.

Distinctive features are extracted from the preprocessed character for classification. Many different sets of features can be derived under different predefined extraction schemes. In the classification stage, usually based on extracted features and statistics of features obtained from a training set of samples or by its structural

characteristics, a classifier assigns an unknown character to its proper class. And of course, for text recognition we can apply context analysis to enhance recognition accuracy.



Fig. 1 Block diagram of a typical OCR system .

CHAPTER II

CHARACTER RECOGNITION METHODS

There are two main recognition approaches, namely:

- Statistical approach.
- Syntactic approach:

2.1. Statistical approach.

In this approach, a set of characteristic attributes, called features are extracted from characters and the recognition is usually done, by partitioning the feature space.

Suppose that N features are extracted from each input character and each set of N features can be considered as a feature vector X. The problem of classification is to assign each vector in the N-dimensional feature space to a proper character class. This task can be carried out by partitioning the feature space into mutually exclusive regions, where each region will correspond to a character class.

Let C_1 , C_2 ,..., C_t be character classes, and

$$X = \begin{cases} x_1 \\ x_2 \\ \vdots \\ x_f \end{cases}$$

be the feature vector, where x_i is the i-th feature component, and $D_j(X)$, $j=1,2,\ldots$, t denotes a function used to decide to which character class an unknown character belongs. This function is usually called discriminant function.

Then the feature vector X is in class C_i (X \in C_i) if $D_i(X) > D_j(X)$, i, j = 1, 2, ..., t $i \neq j$ That is for all X \in C_i , $D_i(X)$ must be the largest.

In a character recognition system, sometimes the number of extracted features is quite large (for example, Chinese and Japanese characters, etc.), then the reduction of complexity can be achieved by describing the complex character as a composition of simpler subpatterns. The syntactic and structural approaches have been proposed to meet this challenge.

2.2. Syntactic approach.

For a character recognition system, the structural information is important. This information can be represented in a hierarchical manner, that is, a character is described by simpler subpatterns (primitive elements) and each subpattern can be described in terms of even simpler subpatterns. This hierarchical representation is analogous to the syntactic structure of languages and the theory of formal languages [16] can be applied to recognize characters.

For a formal language, we can define a grammar G as :

 $G = (V_n, V_t, P, S)$ where

V_n: a set of nonterminals (variables)

 V_t : a set of terminals (constants).

P : set of productions or rewriting rules

S: the start or root symbol

The language generated by G, denoted by L(G), is the set of strings which satisfy two conditions:

_ each string is composed only of terminals.

_____each string can be derived from S by suitable applications of rules from the set P.

Example:

For the grammar $G = (V_n, V_t, P, S)$ where

 V_n : < S >

 $V_t : \langle a,b \rangle$

P: < S -> aSb, S -> ab >

Applying each production rule once, we obtain

S => aSb => aabb

In character recognition the features can be considered as terminals. Assume that there is a grammar such that the generated language consists of sentences (characters) which belong exclusively to one of the classes C_i ($1 \le i \le t$). Thus a given unknown character can be classified to belong to class C_i if and only if it is a sentence (character) of $L(G_i)$.

Some main features that can be considered as primitives :

- * Straight line segments : | / \
- * Curves : U A > C
- * Loops : 0
- * Branches : Y X K + F

In all the above mentioned approaches, the characteristic attributes, called 'features', are supposed to be invariant or less sensitive with respect to the variations and distortions of characters. Then the problem is what features should be extracted from the input character. This task is usually carried out based on either the importance of the features in characterizing characters or the contribution of features to the accuracy of recognition.

CHAPTER III

FEATURE EXTRACTION

3.1. Feature classification

The purpose of feature extraction is to find a transformation that maps the character images into a set of characteristics, called features, that contain some relevant or discriminatory information required for a recognition system.

Generally speaking, the features of a character class are the characterizing attributes of all characters which belong to that class. Due to the great variation in the handwritings, there are many possible ways to define a set of features. However, based on past research, features can be classified into the following main families (C Y. Suen [17]):

3.1.1. Numerical features

_ Distribution of points : the measurements of the distibution of points in the character matrix which provide the positional information, density, distance of certain points from reference points as well as crossings.

Transformation: a character matrix can be transformed into a vector, a series such as Fourier series, or a corresponding waveform.

Physical measurements: the respective width and height of the character can be obtained from the number of rows and columns occupied by the pattern.

3.1.2. Structural Features

Line segments and edges: Edges and line segments are detected from the character. From the above information, such features as line lengths, and line ends can be obtained.

Outline of the character: The contour of a character is traced.

Contour tracing can generate the following features :

- * Line tips, commonly known as terminals or end points.
- * Length of line segments, including perimeter.
- * Sharp angles, protrusions and spikes : > \wedge
- * Arcs, bends : O C
- * Splits : -- '
- * Loops, circles : · O
- * Points of inflection : 5
- * Concavities and convexities : 👢 🕽

Center-line of the character: By applying the thinning

process on a character outline, the center-line of a character, commonly called skeleton is obtained.

From the skeleton, the following features can be obtained:

- * Line tips."
- * Straight line segments, horizontal and vertical.
- * Diagonal lines, slants.
- * Concavities...
- * Loops.
- * Intersections, forks, branches, nodes.

Most handwritten characters are subject to the deformations caused by defects in equipment such as writing surfaces or writing instruments and human errors. The most common factors are:

- _ noise : bumps and gaps in lines, broken lines etc.
- _ distortion : rounding of corners, protrusion, dilation and shrinkage, etc.
- __ style variation : different shapes used to represent the same character
- _ translation: movement of the whole character or its component

rotation: change in orientation of character

In most character recognition problems which arise in practice, the determination of a complete set of discriminatory features is extremely difficult, if not impossible, because of the fact that most feature extraction methods are ad hoc in nature, due in part to our lack of understanding of the true character features and in part to the limitations of current technology.

So far none of the existing feature extraction schemes can claim to be the best or optimum. Therefore a workable character recognition machine should use a combination of methods, each of which may compensate for the shortcomings of the other.

3.2. Combination of features

An attempt has been made by Spanjersberg [18] to use a combination of different features for the recognition of handwritten digits.

In his work, Spanjersberg has employed three different schemes of feature extraction. Each scheme consists of a pre-processor for detecting features and a classifier.

The three schemes are :

Moments

Glucksman [18] characteristic loci

Views

The moment-feature is described as

$$\mathbf{M}_{pq} = \sum_{i=1}^{n} \sum_{j=1}^{m} \mathbf{z}_{ij} \cdot i^{p} \sqrt{j}^{q}$$

where

n, m : respective numbers of rows and columns of the character matrix

i, j : indicates rows and columns

 z_{ij} : binary indication of condition (1 or 0 in the binary image)

The moment of up to the fifth order was used with the condition (p + q \leq 5). The permutations of pq, by varying p and q from 0 to 5 with the condition (p + q \leq 5), give a total of 21 moments such as M₀₅, M₀₄,..., etc. Five of the moments are given fixed values and the 16 remaining moments are regarded as features.

Patterns from one class tend to cluster in a part of the 16-dimensional feature space. By introducing separating hyperplanes that divide the space between the centers of two clusters into two parts, a pattern is classified if each of the 9 decisions (out of 10 classes of numerals from 0 to 9) is in favor of the same class.

The view-features are obtained as the 4 outside views named as lower, upper, left and right views taken from a pattern. In this way four groups of features can be distinguished: discontinuities, slopes, end points and boundaries of 'island'. (see Fig. 2)

For each view of a pattern only one feature of each group is selected with a maximum of 6 features. Then the feature vector consists of 24 components. The classification is performed by using probability of occurrence of features. A pattern is classified into a class with the highest probability of occurrence of features.

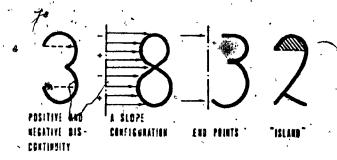


Fig. 2 Examples of features

In Glucksman's characteristic loci scheme [19], the components of feature vector are derived from the pattern's background called loci.

From a point in the pattern's background we project imaginary rays left, right, up and down. If the ray intersects a line of the pattern, the digit value assigned to that direction is 1, if more than one intersection 2 is assigned, and 0 is assigned for no intersection. Putting together these digit values, we have a four-digit code for each point of the pattern's background (see Fig. 3). And the feature space of 81 (34) dimensions can be defined.

In this scheme, the horizontal and vertical boundaries to the pattern form a rectangle called the characteristic rectangle and features are derived from loci inside this rectangle.

h The classification is done by using a discriminant function:

$$S = \sum_{i=0}^{81} x_i \cdot w_i$$

where

 x_i : component of feature vector.

 x_0 : fixed value, e.g. +1

w_i : weight factor.

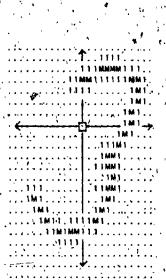


Fig. 3 Characteristic loci

Spanjersberg has employed three strategies in combinations of different systems.

For combination of two different schemes, two strategies are described as:

!series' combination : a pattern is classified only if both systems give the same recognition:

_ 'parallel' combination: a pattern is classified if both systems give the same recognition or if it is rejected by only one system.

In the third strategy, a combination of all three schemes, the correct recognition is chosen by the majority.

The results are classified into three categories: correct recognition, error and rejection. For all employed strategies, the error rates are reduced significantly from about 8% to about 2%. However only in the combination of all three schemes that the correct recognition rate is increased from about 90% to 92.8%.

3.2.1. Features used

In order to select a feasible combination, of features, we start with Glucksman's characteristic loci method because this scheme has the following properties:

The feature vector is insensitive to the translation of the pattern outline within the matrix of character

elements and to the size variation.

A break in the pattern outline can be tolerated if it doesn't eliminate the significant components of the feature vector.

_ It is possible to combine the characteristic loci in order to reduce the total number of components of feature vector.

____ The operations on distinct rows and columns (of the matrix of character elements) are independent, so there is a possibility for parallel processing.

A. L. Knoll [20] has done extensive experiments utilizing the Glucksman scheme for the recognition of handprinted characters on datasets from various sources such as Stanford Research Institute (SRI), Honeywell Corp., Highleyman [21]. In his experiments recognition rates for, numerals varied from 74% - 84%. He observed that the recognition rate decreased considerably for unconstrained samples because of such factors as missing background areas, large breaks and skews in the character strokes etc.

Knoll noted that it would be useful that additional features be of the same type as the characteristic loci features so they can be extracted using the same or slightly modified logic in the software. He has also suggested that features defined from points on the character outline as

well as from diagonal directions could be used.

The Characteristic Loci resolution can be increased by using other directions such as diagonal directions. This subject has been investigated intensively by Suen [22]. The new features are called Multi-directional Loci, and one may increase the vector directions in multiples of 2, e.g., 4, 8, 16, etc.

In our work, we have tried to experiment with features defined from 'black' points (points on the character outline). However, recognizing the fact that the thickness of character strokes could affect the feature-vector, we have limited to the boundary 'black' points only for two main reasons:

More efficient processing

Insensitive to the thickness of character outline.

We also tried with features defined from boundary points and using four diagonal directions. However the combination of 8 directional features defined from both white points and boundary points has proved to be most effective in our experiment (see Fig. 4).

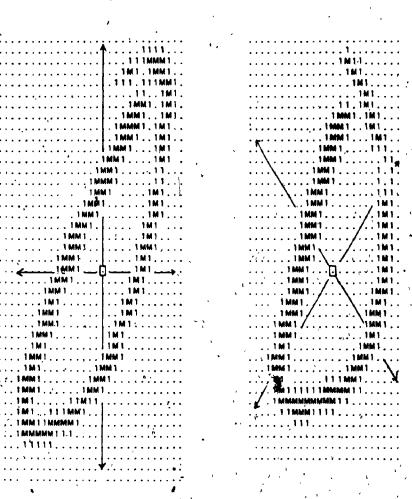


Fig. 4 Multi-directional loci

Zoning:

extraction scheme. Hussain et al. [23] divided a normalized 20 X 20 character matrix into 25 distintive regions 4X4. By density of black points in these regions, a printed character can be recognized. Shridhar and Badreldin [22] have used the horizontal and vertical projections of the contours in the pour quadrants of the square frame enclosing the character as features. Though these features are not sufficient for the identification of a numeral uniquely, they provide useful subclasses for further processing.

Recognizing the fact that zoning can give some useful positional information about a character, we tried to include it into the characteristic loci scheme.

In the case of handwritten numerals, there is a lot of variation in handwriting styles and irregularities as twisted lines, broken loops, etc., we find that the division of characteristic rectangle into four equal quadrants has not provided very consistent information. Instead the center of gravity of the character has a close relation with the distribution of black points. By drawing a vertical and a horizontal line through the center of gravity, we can divide the characteristic rectangle into four quadrants. The regions crossing the center of gravity, 3 rows for

horizontal direction and 1 column for vertical direction, belong to both adjacent quadrants, e.g. upper and lower quadrants or left and right quadrants.

The distributions of black points in these quadrants reflect the pattern in a more consistent way. It can give the information about the densities in the left, right, top and bottom quadrants.

With four different quadrants, we have a feature space of 81 x 4 dimensions for 4 directional loci or 81 x 8 dimensions for 8 directional loci. By the combination of features derived from both background loci and boundary points, the feature space has 81 x 4 x 2 and 81 x 8 x 2 dimensions for 4 and 8 Directional Loci respectively.

The positional information provides the distinction between left and right, top and bottom loci. It is useful in case of some numerals, for example 6 and 9.

Other features:

The center of gravity can be used not only as a reference point in zoning as described above, but from that point the enclosing character outline can be estimated, grossly as described below. The coordinates of the center of gravity indicate roughly where the character outline is centered, e.g. in the upper or lower part of the chacteristic-rectangle.

the center of gravity also provides other features which can be obtained as follows :

- 1 The number of crossings in the four directions left, right, up and down originating from the center of gravity. This information can give an approximate horizontal and vertical lines a character has. It is useful, for the discrimination between similar characters, in that they have close scores in the minimum-distance classifier used in the characteristic loci method. minimum-distance classifier computes the distance from an unknown pattern X to the prototype of each class and assigns the pattern to the class closest to it.
- The center of gravity of the character can generate another feature. The coordinates of center of gravity are computed in the following way :

$$X_{C} = 1/T \sum_{i=1}^{n} \sum_{j=1}^{m} X_{ij}.K_{ij}$$

$$Y_{c} = 1/T \sum_{i=1}^{n} \sum_{j=1}^{m} Y_{ij}.K_{ij}$$

where

n, m : number of rows and columns of the character matrix X_{ij}, Y_{ij} : X and Y coordinate of pixel P_{ij} $K_{ij} = 1$ for black point

= 0 for white point

$$T = \sum_{i=1}^{n} \sum_{j=1}^{m} K_{ij} = \text{total number of black points}$$

Then new coordinates X'_C and Y'_C with respect to the characteristic rectangle are:

$$x'_C = x_C - minj$$

$$Y'_{C} = Y_{C} - mini$$

where *

minj: indicates column number in which a black point is encountered first starting from the left of character matrix,

mini: indicates row number in which a black point is encountered first starting from the top.

Since the character matrices vary in size, then the new coordinates (X'_C & Y'_C) of the center of gravity are normalized by dividing by the width and height of the characteristic rectangle respectively and rounded to the first decimal. Putting together these first decimals, we have a code for the center of gravity.

For those examples shown in Fig. 5, with numeral 1, the center of gravity has normalized values of 0.5 and 0.5 for coordinates x and y respectively. The combination of 5 and 5 forms code 55 for the center of gravity of numeral 1. These codes reflect the symmetry and asymmetry of a character. Where codes 'llll' and '0000' are resulted from the number of crossings in four directions left, right, up

and down originating from the centers of gravity of numerals 0 and 1 respectively.

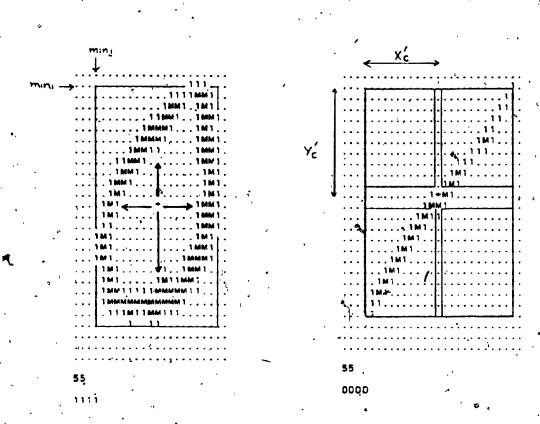


Fig. 5 Examples of feature extraction

CHAPTER IV

IMPLEMENTATION OF A CLUSTER-BASED CLASSIFICATION METHOD.

4.1. Data organization

In human recognition, a person tries to recognize a character by identifying its characteristic attributes, and recognition comes instantly. The recognition process is complicated and not well known yet. However he has to use his memory. Then he can deduce the unknown character from his knowledge about characters, different fonts, etc. acquired in his life-time. In case of confusion, mostly due to the poor candwriting, he will try to find particular features as well as contextual information that enable him to make the right distinction.

In the existing character recognition systems, usually only one set of features is employed. The basic problem is that none of the rules or schemes can cover the immense valiety of handwritten characters. Then evidently, by employing existing schemes, machine recognition is usually inferior to human recognition at the current stage.

However, we have to acknowledge that the fine distinction and intelligent interpretation of the immense variety of handwritten characters by human beings can be acquired only through a lengthy learning process. The simulation of such process by the integration of a data base

which contains information such as features extracted from many different characters, etc., and the storing of this information into the computer will certainly narrow the gap between human and machine intelligence in handwriting recognition.

In our work, we made an attempt to organize the information about characters, that is features, into a data base in order to cover the great variation in handwritings. We also tried to explore the possibility of using this information to enhance the power current recognition techniques.

Extracting features with different schemes we will get different sets of features. A classifier will make use of these features to assign an unknown character to its proper class. However in the recognition, we always have misclassification when a character is mistaken for another character.

We could consider feature extraction with a predefined scheme as viewing a character under a certain aspect of it. And certain subset of characters show similar characteristics under that aspect. However, when we change our view each member of this subset may have a quite distinct characteristic. Therefore, by combining different features obtained from different schemes, we hope to arrive at a finer distinction between characters in the process of

recognition.

In order to tackle variations in handwritings, we try to store the information about a character including its diffrent features and its proper classification in a data base for reference.

This data base is implemented by the use of an index-sequential file organization. Each record contains the information about a numeral in four fields. They are the predicted numeral which comes from Multi-directional loci scheme, the number of crossings projected from the center of gravity, the code derived from the normalized coordinates of the center of gravity and the actual identity of the numeral, respectively. The first three fields form the key for the record.

4.2. Classification method

Classifier for characteristic loci :

The pattern classification is performed by using a distance function. In our work a minimum-distance classifier is employed.

The Euclidean distance $D_{\hat{i}}$ between an unknown pattern X, and the i-th prototype $Z_{\hat{i}}$ is given by

$$\mathbf{p}_{i} = \sum_{r=1}^{r} (\mathbf{x}_{r} - \mathbf{z}_{ir})^{2}$$

where

 x_r is the r-th component of x_i is the r-th component of z_i and $x \in W_i$ if $D_{ii} < D_j$, for all $j \neq i$, for $i, j = 1, \dots, t$

The minimum-distance classifier is most effective when patterns of each class tend to cluster about a representative pattern for that class.

Clustering :

The common problem in pattern recognition is the lack of homogeneity among attributes belonging to a pattern class. For example, in handwritten characters we can distinguish several styles used by different writers.

Different characters possess different shapes; however they can be classified into a set of prototypes. The Japanese OCR researchers have often used templates which are actually the prototypes of patterns that they wanted to recognize. Suen [20] has investigated the important role of prototypes in character recognition. Clustering techniques are employed to obtain prototypes*(see Appendix B and C for examples of numeric prototypes).

Clustering techniques are used to partition a given set of entities into weld-separated subsets. These subsets contain all entities that are similar according to a predefined measure of similarity.

In order to define a cluster, it is necessary first to define a measure of similarity by which patterns can be assigned to the domain of a particular cluster center. There are various typical ways to define a measure of similarity, some of them are listed in Tou et al [25], e.g.

 \setminus _ The Maḥalanobis distance

$$D = (X - M) \cdot C^{-1} (X - M)$$

where

M/: mean feature vector.

 \mathbf{x}' : feature vector of unknown pattern'

C : the covariance matrix of a pattern population

It is a useful measure of similarity when statistical features are considered:

The nonmetric measure of similarity S(X,Z) = X'Z / ||X|| ||Z||

where

X : feature vector of unknown pattern

Y: feature vector of prototype

S(X,Z): the cosine of the angle between the vectors X and Z.

Euclidean distance

It is a commonly used measure of similarity because of its simple interpretation in terms of the concept of proximity.

Once the measure of similarity has been selected the choice of a clustering criterion is very important in defining a cluster.

The clustering criterion can be categorized into two main approaches, see e.g. Tou et al [25]:

Heuristic scheme

Performance-index scheme

In the heuristic scheme, a set of rules is specified. Then based on these rules and a chosen measure of similarity, the process of clustering is performed. In this scheme the Euclidean distance is commonly used as a measure of similarity and it is necessary to set a threshold in

order to define degrees of acceptable similarity in the clustering process.

In the performance-index scheme, a procedure is established to minimize or maximize the chosen performance-index. The most commonly used index is the sum of the square errors index (Tou et al [25]):

$$J = \sum_{j=1}^{N_C} \sum_{x \in S_j} ||x - M_j||^2$$

where

 N_{C} : number of clusters

Si : denotes j-th cluster

N; : number of samples in S;

 M_j : mean feature vector of the patterns in S_j = $1/N_j$ $\sum_{i=1}^{N_j} X_i$

In our work the Euclidean distance is chosen as measure of similarity for its simplicity and we employ the K-means algorithm for the clustering (Tou et al [25]).

The K-means algorithm consists of the following steps:

Choose K initial cluster centers

 $z_1(1)$, $z_2(1)$,..., $z_k(1)$ arbitrarily where index 1 indicates the first iteration.

_ Step 2

At the k-th iterative step the population of samples $\, X \,$ are distributed among $\, K \,$ cluster domains according to the relation

 $X \in S_j(k)$ if $||X - Z_j(k)|| < ||X - Z_j(k)||$, for all i=1,2,...,K, $i \neq j$

where

 $S_j(k)$ denotes the set of samples with cluster center as $Z_j(k)$, and index k indicates the k-th iteration.

_ Step 3

Compute the new cluster center $Z_{\hat{j}}(k+1)$ at iteration k+1 such that the performance-index

$$J_{j} = \sum_{x \in S_{j}(k)} || || || x - z_{j}(k+1)|| || ||^{2}||, j=1,...,K|$$

is minimized. The $Z_j(k+1)$ which minimizes the performance index is simply the sample mean of $S_j(k)$ and is given by

$$Z_{j}(k+1) = 1/N_{j} \sum_{K \in S_{j}(k)} X, \text{ for } j=1,...,K$$

where

 N_j : number of samples in $S_j(k)$

Step 4

If $Z_j(k+1) = Z_j(k)$ for j=1,...,K then the final K cluster centers have converged and the procedure is terminated. Otherwise go back to step 2.

In our work, the initial value of K'is set as 30 and X is a feature vector which consists of 81x8x2 components.

Then if the first two characters selected by the minimum-distance classifier belong to the same class then it is accepted as a recognized character, otherwise it is rejected and subjected to the verification process as described below.

The classification using Multi-directional loci scheme is intended to be used as a prediction and used as the first component in the key for the index-sequential file. The other two components are the number of crossings projected from the center of gravity and the code obtained from the normalized coordinates of the center of gravity with respect to the characteristic rectangle.

The index sequential file is used as a memory bank to store the information about a character including its different features and its proper classification.

Our strategy of verification consists of the following steps:

_ Take the first predicted character and form the key to access the index sequential file. If access is successful, then the character stored in the data field of the record is considered as the true identity of unknown character and the recognition is done. Otherwise proceed to the next step.

Take the second predicted character and form the key. If the record does exist then the unknown character is classified as mentioned in the previous step. Otherwise, the unknown character is not recognized; that is, it is rejected.

Examples of this process are shown on Fig. 6

In the character matrices, the boundary black points are indicated by digit 'l'. In the first case, the numeral 1 is predicted as 7 and 1. However with additional features ': '0000' and 43, where code '0000' were derived almost exclusively from numeral 1, the unknown numeral is correctly identified. In the second case, the numeral 7 is predicted as 1 and 7, but code '0111' cannot be obtained from numeral '1', then the second predicted numeral is identified as the correct one.



....1,111,

```
THE LIST OF MOST PROBABLE CHAR, IN DROER AS :
- CHARACTER = 7
- CHARACTER = 1
7 0000 43
RECOGNIZED CMARACTER AS : 1
 . 1 1 MMMMMM 1 1 1 1 . . . . . .
  . . 1 1 1 MANAGEMENT 1 1 . . . .
  11 V1 1000000 1 7.
   . .4..... 1 MMM1.
   , . . . . . . . . . 1MM1.
       .... 1 MMMM 1 .......
  . . . 1MM11
  ., ,111.......
THE LIST OF MOST PROBABLE CHAR, IN ORDER AS:
- CHARACTER = 1
- CHARACTER = 7
1 6111 45
7 0111 45
RECOGNIZED CHARACTER AS : 7
```

Fig. 6 Examples of classification

CHAPTER V

EXPERIMENTATION AND RESULTS

5.1. Datasets

In our work, samples of totally unconstrained handwritten numeric Zip codes were obtained from the American Post Office. These codes were written on the envelopes by different persons from various locations in US.

The hand addressed envelopes have shown that the humans have a tendency to write well segmented numerals. However, in handwritten numerals 2: 5 and 8, the last stroke used in forming the character tended to be drawn toward the following numeral. About 85% of the Zip codes are well segmented. For the rest, the segmentation is done using analysis of connected regions [26].

Originally Zip codes on the envelopes were digitized by an optical scanner with 16-grey levels, and were binarized by choosing an optimal threshold to the system, which was established by experimentation. The digitized images were segmented into individual character matrices. Some preprocessing techniques such as the elimination of noise and smoothing were used to clean up isolated black spots, bumps and gaps in line segments. (see Appendix A for examples of samples). In order to normalize the size of the character each component of feature vector X is normalized

using the following equation :

 $x'_i = x_i * 100 / Total$

where Total: total number of boundary points from which loci features are taken, or total number of white points in the characteristic rectangle when loci features are extracted from the white points. For details, see the description on page 23 and Fig. 4.

work, 4 and 8 Directional Loci features defined from boundary points of the character outline considered. However the combination of 8 Directional loci features defined from both boundary point and 'white point (denoted by 0 in the binary image) has proved to be most effective in our experiment. The statistics of features were obtained from a training set of 1090 samples and the clustering technique was applied to form prototypes. have tried up to 30 prototypes per numeral. Statistical values of these features were stored, as reference vectors. There were 333 new samples in the testing set, and 390 samples from the training set were used for testing. each sample three different sets of features were extracted. If a sample is rejected after the Chafacteristic Loci classification according to the established criteria, the cluster-based classification is applied on that sample ambiguity. . If the verification is not the successful, then that sample is not recognized.

5.2. Results

The results of recognition on training and testing datasets are listed below:

Table I Recognition Results using Multi-directional Loci

	Correct	Error	Reject
Training set	84.62 %	00.00 %	15.38 % 24.20 %

A. Confusion Table for training set

**	0	1,	2	3	4	5	' 6	7	8	9	Rej.	Tot.
Τ. Λ.Ι	59	<u>, </u>	<u> </u>	_	. 0	<u>, o </u>			I. A			·
1, 0	0	40	' ()	0	0	0	0	.0	0	0	! <u>1</u>	60
	ő	0	49	.0	Ö	Ö	. 0	ŏ	Ì	l . ŏ	18	67
3	ō	0	0	49	0	0	0	Ö	0	ĺ ō,	16	65
4	0	Ô	0 -	0	- 6	0	0	-0	٨٥	0	6`	13
5	0	0	0	0	0	26	O	0	0	, 0	3 .	29
6	0	0.	0	0	_0	0	23	0	0	0	1	24
1.7	0	0	0	0	ا 0 بمبا	, 0	0	40	0	l Q	5	45
8	0 -	0 ,	0	0	0	0	0	0	23	0	5	28
9 1	0	0 1	0	.0	0	0	0	, 0	0	. 15	3	18
· Im.	50	40.1	40	40		7.	2.3	40			<u> </u>	1300 1
Tot, l	59	40 1	49	49	6	26	23	40	1 23	15	60	390

B. Confusion Table, for testing set

	1	∢ .											
**	0	7	l	2	3	4	5	6	7	8	9	Rej.	Tot.
•		-											* (
0	44	1	0	0	0	0	1 0	0	0-+	Q	0	4	48
1	0	4	2	0	0	0	0	0	0	ÌÌ	0	0	1 42,1
2	0	1 (0	30	0	1 0	0	· 0	0	0	0	12	44
3	1 0	1	0	0	41	0	0	0	3	1	1. 0	34	1 79
4	. 0		0	1 0	0	. 4	Q.	1	3	0	0	2	10 1
5	0.	1	Q	0	0	1 0	1.7	1. 0	0	0	0	10	27
6	0	1 1	0	0	0	0	0	17	0 '	0	0	4	24
ל	0	1 (0	0	. 0	0	0	0	25	0	0	2	27
8	1	1 (3	0	0	0	0,	0,	0.	10	2	8 °	21
9	1 0	1.	0	0	0	0	0	0.	1	0	9	4	14
Tot.	45	4	2	30	41	4	17	18	33	12	11	80	333
												-	

Table II Recognition Results using Cluster-based classification

	Correct	Error	Reject
Training set * Testing set	99.23 %	00.77 %	00.00 %

. A. . Confusion Table for training set

											` .	
** ,	0	1	. 2	3	4	5	6	7	8	9	Rej.	Tot.
		•					,			`		
0 (60	0	0	\0	0	0	(O	0	0	0	0	60
1	0	41	0	D D	0	0	0	0	0	0.	0	41
2	0	0	67	0	0	0	0	0	0	0	0	67
3	0	0	1	62	0	0	0	1		1	0	65
4	0	0	0	0	13	0	0	0	0	- 0	0	13
5	0	0	0	0	0	29	0	0'	0	0	0	29
6	0	~ 0	0	0	0	7 0	24	0	0	0	0	24
7	0	0	0	0	0	0	0	45	0	0	0	45
8	0	0	0	0	0	0	0	0	28	~0	1 6	28
9	0	0	0	0	0	, 0	0	0	0	18	0	18
						•	•				·	
Tot.	60	41	68	62	13	29	24	46	28	19	0	390
•	_	'	1		,		•	•			'	

. B. Confusion Table for testing set

**	0	1	28	3	4	5	6	7	8	9	Rej.	Tot.
0	47	.0	0	0	0	0	0	0	0	1	0	- 48
ľľ	0	42	0	0	, 0	מ	0	0	0	0	0	42
2	0	0	39	3	0	0	0	1	1	0	0	44
3	0	2	0	65	1	0	0	3	2	1	5	79
4	0	0	0-0	, 0	•5	0	1	3	0	0	1	10
5	· 0	0	2	1	0	19	0	0	2	0	3	27
. 6	0	0	0	0 .	0	3	48	0	0	0	0	21
7	Ŏ	0	1	0	, 0	0	. 0	26	0	0	0	27
. 8	1	0	0 .	0 `	0 /	1	. 0	0 1	14	2	3	21
9	· 0.	0	Q I	0 .	0	0	0	3	0	10	1	14
			· · · · · · · · · · · · · · · · · · ·	U								
Tot.	48	44	42	69	6	.23	19	36	19	14	13	333

Although the direct comparison of recognition results of different researchers is virtually impossible owing to the wide range of experimental procedures, constraints imposed and datasets used, we would like to list some results of various authors as below. These results are reproduced from [27]:

Table III Recognition Results of handprinted numerals

Author	.Source of data	No. of samples Training Testing	Correct (%)	Error (%)	Reject (%)
Highleyman	Highleyman	same 500	×83.00	-	1
Knell	Highleyman	not fixed	73.70	16.50	9.80
Knoll	Munson	total = 1470	82.80	8.00	9 20
Ali et al.	Munson	480 840	92.98	3.57	3.45
Spanjersberg	Giro-document	30000 30000	92.80	2.70	4.50

CHAPTER VI

CONCLUSIONS

The cluster-based algorithm can be applied to recognize the totally unconstrained handwritten numerals. The results indicate that the combination of different features can enhance the recognition accuracy of existing methods such as the Multi-directional Loci scheme used in our experiment. However in order to arrive at a finer distinction, a systematic selection of distinctive features, in which one feature can complement the discriminant power of others, is required. The numerical features can be complemented by structural features or vice versa. By organizing the information about characters including its different characteristic attributes into a data base, the immense variety in handwritten characters could be tackled to a certain extent.

By using the Multi-directional loci scheme, we have obtained a recognition accuracy of 84.62% on the training set and 71.77% on the testing set. In another experiment, the Multi-directional loc. scheme was used to predict the possible classes of an unknown numeral. In this scheme, additional features (the number of pressings and the code deriving from the center of gravity of the character) were used to recognize an unknown as the of the predicted classes. Using this scheme, 99.23% of the training set

samples were correctly recognized; whereas, on the testing, set samples, 85.59% correct recognition was achieved. The enhancement of about 14% in recognition accuracy was obtained due to the usage of additional features.

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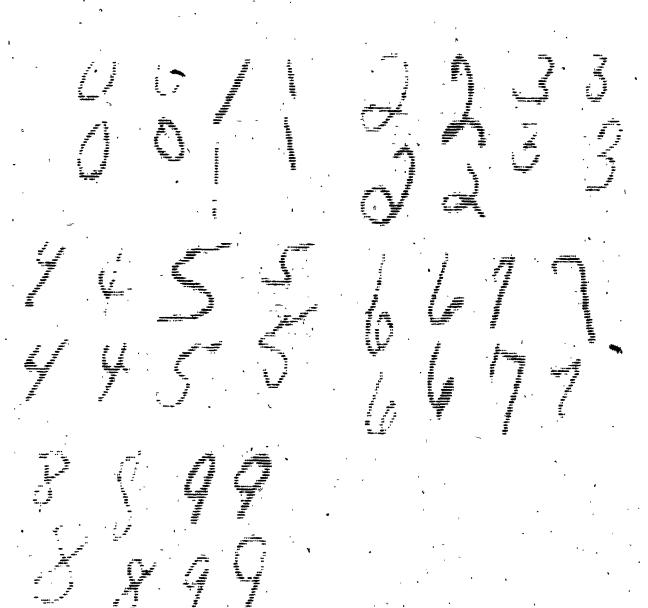
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APPENDIX A. Typical samples



APPENDIX B. Numeric prototypes

Frototype 01

\$co.co.co.uo# 02



Prototune 83



Prototype 21



| 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 | 1861 |

Prototype 22

APPENDIX C. Prototypes of numeral 3

