A QUASI-FINITE ELEMENT APPROACH FOR THE DETERMINATION OF THE TEMPERATURE DISTRIBUTION IN A SINGLE POINT CUTTING TOOL

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ABSTRACT

A quasi-finite element approach for the determination of the steady state thermal field in a tool-chip-workpiece system in orthogonal cutting is presented. The three mechanisms of heat transmission are considered in the analysis of the mathematical model. Only a few temperature measurements at certain discrete points in the continuum are needed to serve as boundary conditions.

A FORTRAN program based on the mathematical model is prepared and tested. The model and the analytical technique are verified for rectangular plates with given boundary conditions. Results agree with the known closed form solution and the maximum error is found to be less than one percent.

The method has been applied to the real problem of determining the isotherms in a tool-chip-workpiece of single point cutting operation. The results compare very well with the available experimental measurements under similar cutting conditions.

The technique presented here can be extended to include temperature fields of multi-point cutting tools under steady and transient heat transmission.

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NOMENCLATURE

A	Area of heat flow for an element, in.
B1	=d for M=1, h_o for M=2 and E for M=3, where M indicates the mode of heat transfer as explained below.
B2	=h_1 for M=2
B3	=h_2 for M=2
d	Heat flow path between the centroids of two adjacent elements, in.
E	Emissivity coefficient.
F	View factor.
H	An NU×NU square matrix.
h_0, h_1, h_2	Coefficients of best curve fit for convection.
K	Thermal conductivity BTU/(in.sec.F°)
k_0, k_1, k_2	Coefficients of the best curve fit for conduction.
N	Total number of nodes connected to the i'th node.
NA	Basic node number.
NB	Node number to which basic node is connected.
NHC	Number of heat connections.
NU	Total number of unknown nodes in the idealized thermal field.

NOMENCLATURE (cont'd)

M	Mode of heat transfer. M=1, convection M=2, conduction M=3, radiation
MC	Machine code vector.
Q	Heat flow, BTU/sec.
β	An NU×1 vector.
σ	Stefan-Boltzman constant.
θ	Temperature, F°.

Subscripts

i,j	Node numbers; $A_{i,j}$ is the area of heat flow between the i'th and the j'th element; $h_{i,j}$ is the convective heat transfer coefficient between the i'th and the j'th elements.

Superscripts

c	conduction
r	radiation
v	convection

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CHAPTER 1

INTRODUCTION

In the course of numerous studies, it has been recognized that the temperature field of a tool-chip-workpiece system is an important factor in determining the causes of tool wear [1, 2, 3]*. Sadek [4], showed that the crater topography and the temperature distribution on the rake face are closely interrelated.

Different research groups have been formed with the main objective to provide reliable and relevant techniques for determining the temperature distribution of the tool-chip-workpiece system, as well as to relate the cutting temperature fields, machinability, tool wear, surface finish and other machining parameters.

The existing methods for temperature measurement in metal cutting fall into five categories:

a) Thermoelectrical methods.

b) Colorimetric methods.

c) Electrical analogy methods.

d) Radiation methods.

e) Analytical methods.

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*Numbers in brackets [] designate references at end of text.

1.1 Thermoelectrical Methods

In this category, the cutting temperature is determined either by measuring the thermal e.m.f. of the tool-chip-workpiece couple [5,6,7,8,9,10] or by the aid of a thermocouple attached to the rake face near the cutting edge [11,12,13] or inserted in fine holes in the cutting tool [14]. The reliability of these methods are doubtful since the measured thermal e.m.f. depends not only on the temperature but also on the state of stress of the thermocouple.

1.2 Colorimetric Method

The most significant characteristic of this category is the application of the thermosensitive paints utilizing their property for colour change at certain temperature [15,16]. This method is only practicable to measure the average temperature of accessible surfaces up to 500°C.

1.3 Electrical-Analogy Methods

Maximum temperature zone is determined through the analogy between the electric and thermal fields [17]. The major drawback of this method is the fact that the effect of the temperature change on the heat conductivity of the tool as well as the heat losses to the surroundings are not taken into consideration.

1.4 Radiation Methods

This category is only applicable to obtain the temperature distribution of accessible surfaces. The main measuring device used here is the total pyrometer [18, 19] which differs from one method to another depending on its construction. Infra-red photography are also classified under this category [20].

1.5 Analytical Methods

Due to the obvious limitations of experimental methods, extensive attempts to solve the problem analytically have been made. These include the finite difference method [21], heat balance of the cutting process [16], and closed form solutions [22] with the assumption of constant thermal material properties and negligible heat losses. The drawbacks of these methods lie in the fact that the heat losses and the influence of temperature changes on the heat conductivity of the tool are not taken into consideration.

A review of measuring and calculating methods leads to a conclusion that none of the existing methods is universally applicable. The need for a reliable and practical technique is therefore evident. The well known finite element method [23] should provide a reliable computerized technique for the determination of the thermal field in the cutting tool with the aid of only a few measured points as boundary condition. However, in this method the geometry of the element is a determined factor. Therefore, the nodes have to be exactly defined with reference to a system of axes. Moreover, convection and radiation which appear as boundary conditions are not easily implemented. In this work, a quasi-finite element approach is introduced. In this approach the elements are represented by their centroids, where as in the classical finite element the elements are visualized as connected to each other through nodes. Therefore, the geometry of the element is not a determined factor and the compatibility is not necessarily satisfied. For practical applications, the quasi-finite element approach offers the advantage of a considerable reduction in the memory and computing time requirements as well as the consideration of the three modes of heat transmission.

CHAPTER 2

THE QUASI-FINITE ELEMENT APPROACH

The classical finite element method permits almost all problems of structural stress analysis, or the analysis of such field problems as heat transfer and fluid flow, to be represented in a mathematical form suitable for solution on a digital computer. Complicated structures or systems as shown in Fig.(1), can be visualized as an assemblage of structural elements interconnected at a discrete number of nodal points. If the various relationships for the individual elements are known, it is possible to derive the properties and study the behaviour of the assembled structure.

2.1 Mathematical Model

Fig.(2) shows an element 'i' connected to an element 'j' such that it receives heat in the three modes of heat transfer: a) conduction, b) convection and c) radiation.

In the case of radiation, the two elements are physically apart from each other. In the case of convection, one of the elements is usually a fluid.

The heat flow into the i'th element from the surrounding elements is given by:

$$Q^c + Q^v + Q^r \qquad (1)$$

where, Q^c Conductive heat flow from N^c elements.

Q^v Convective heat flow from N^v elements.

Q^r Radiative heat flow from N^r elements.

The different modes of heat flow are given by:

$$Q^c = \sum_{j=1}^{N^c} \frac{A^c_{ij}\ K_{ij}}{d_{ij}} (\theta_j - \theta_i)$$

$$Q^v = \sum_{j=1}^{N^v} A^v_{ij}\ h_{ij}\ (\theta_j - \theta_i)$$

$$Q^r = \sum_{j=1}^{N^r} A^r_{ij}\ E_{ij}\ \{(\theta_j+460)^4-(\theta_i+460)^4\} \qquad (2)$$

The above relations can be reduced to:

$$Q^c = \sum_{j=1}^{N^c} C^c_{ij}\ (\theta_j - \theta_i) \qquad (3.a)$$

$$Q^v = \sum_{j=1}^{N^v} C^c_{ij}\ (\theta_j - \theta_i) \qquad (3.b)$$

$$Q^r = \sum_{j=1}^{N^r} C^r_{ij}\ (\theta_j - \theta_i) \qquad (3.c)$$

where,

$$C^{c}_{ij} = (A^{c}_{ij} \;\; K_{ij})/(d_{ij})$$

$$C^{v}_{ij} = A^{v}_{ij} \;\; h_{ij}$$

$$C^{r}_{ij} = A^{r}_{ij} \;\; E_{ij} \;\{(\theta_j+460)^2+(\theta_i+460)^2\}+\{(\theta_j+460)+(\theta_i+460)\}$$

$$K_{ij} = (2 \;\; K_i \;\; K_j)/(K_i + K_j)$$

For a steady state condition:

$$Q^{c} + Q^{v} + Q^{r} = 0 \qquad (4)$$

hence, the heat flow into the i'th unknown nodal element can be formulated as follows:

$$\sum_{j=1}^{N^c} C^{c}_{ij}(\theta_j-\theta_i) + \sum_{j=1}^{N^v} C^{v}_{ij}\;(\theta_j-\theta_i) + \sum_{j=1}^{N^r} C^{r}_{ij}(\theta_j-\theta_i) = 0 \qquad (5)$$

which can be rearranged to read:

$$\theta_i\{\sum_{j=1}^{N^v} C^{c}_{ij}+\sum_{j=1}^{N^v} C^{v}_{ij}+\sum_{j=1}^{N^r} C^{r}_{ij}\}= \sum_{j=1}^{N^c} C^{c}_{ij}\theta_j+\sum_{j=1}^{N^v} C^{v}_{ij}\theta_j+\sum_{j=1}^{N^r} C^{r}_{ij}\theta_j \qquad (6)$$

As the subscript 'i' indicates an unkown node, while the 'j' (where $j \neq i$) can indicate an unknown as well as a known or a boundary node, whose temperature is known at all times. Therefore, the total number of nodes 'N' could be written as:

$$N = N^c + N^v + N^r \tag{7}$$

where, $N^c = N^c_n + N^c_u$

$$N^v = N^v_n + N^v_u$$

$$N^r = N^r_n + N^r_u$$

N_n represents the total number of known nodes.

N_u represents the total number of unknown nodes.

Substituting in Eqn.(6),

$$\theta_i \{ \sum_{j=1}^{N^c} C^c_{ij} + \sum_{j=1}^{N^v} C^v_{ij} + \sum_{j=1}^{N^r} C^r_{ij} \}$$

$$-\{ \sum_{j=1}^{N^c_u} C^c_{ij}\, \theta_j + \sum_{j=1}^{N^v_u} C^v_{ij}\, \theta_j + \sum_{j=1}^{N^r_u} C^r_{ij}\, \theta_j \}$$

$$= \{ \sum_{j=1}^{N^c_n} C^c_{ij}\, \theta_j + \sum_{j=1}^{N^v_n} C^v_{ij}\, \theta_j + \sum_{j=1}^{N^r_n} C^r_{ij}\, \theta_j \} \tag{8}$$

The right hand side of Eqn.(8) is presumably a known value if the C_{ij}'s are not functions of the temperature.

An equation similar to Eqn.(8) can be written for every unknown node. This leads to an 'NU' equation for the 'NU' unknown nodes. These equations could be written in the following matrix form:

$$[H]\cdot\{\theta\} = \{\beta\} \quad (9)$$

The H-matrix is symmetric with strong positive diagonal entries. It is a sparsely populated matrix.

The H-matrix is assembled together with the β-vector as shown in the block diagram of Fig.(3), and by using a direct or an iterative technique, the temperature of the unknown nodes can be obtained.

If the entries of the H-matrix and the β-vector are dependent on the values of the unknown temperatures, the computational procedure starts by assuming approximate values for the θ-vector from which the H-matrix as well as the β-vector can be determined. Solving Eqn.(9), an improved θ-vector can be obtained. If this improvement is not within an acceptable tolerance, a second iteration is performed.

The initial entries of the θ-vector for the unknown nodes may be taken as the average temperature based on all the given known temperatures.

2.2 The Variation of the Coefficients of Heat Transfer Due to a Change in Temperature

In evaluating C^c_{ij}, C^v_{ij} and C^r_{ij}, the effect of temperature changes must be taken into consideration, since their values as given in Eqn.(3) are functions of temperature. For the conduction mode, the ith and the jth are represented by a second order polynomial as follows:

$$K_i = k_0 + k_1 \theta_i + k_2 \theta_i^2$$

$$K_j = k_0 + k_1 \theta_j + k_2 \theta_j^2$$

The combined effect is therefore:

$$K_{ij} + (2 K_i K_j)/(K_i + K_j) \qquad (10)$$

In the same manner, the combined coefficient of convective heat transfer is given by:

$$h_{ij} = h_0 + h_1 \theta_i + h_2 \theta_i^2 \qquad (11)$$

In practive, the coefficient h_{ij} varies from one point to the other on the surface depending on the pressure, velocity, density and the temperature of the adjacent fluid. It is a function of Reynold's, Nusselt's and Praudl's Numbers. The variation from point to point is usually rapid and it becomes necessary to read in a coefficient for every node.

Usually, the coefficient k_0, k_1, k_2 and h_0, h_1, h_2, are obtained from experimental data using a curve fitting technique.

Appendix I shows the curve fitting technique used for Eqn.(10) and Eqn.(11), the flow chart, the computer program and the material characteristics used to obtain these coefficients.

For the radiation mode, the emissivity factor is given by:

$$E_{ij} = \sigma\varepsilon_{ij} F_{ij} \tag{12}$$

where, is the Stefan-Boltzman constant
equal to 0.173×10^{-8} BTU/(ft^2)(hr)(degree Rankin)^4
or 0.333718×10^{-14} BTU/(in^2-sec.degree Rankin)^4

Referring to Fig.(4), $F_{ij} A_i = F_{ji} A_j$

where,

$$F_{ij} = \frac{1}{A_i} \int_{A_j} \int_{A_i} \frac{\cos\alpha_i \cos\alpha_j}{r^2} \, dA_i \, dA_j$$

$$F_{ij} \neq F_{ji}$$

$$F_{ji} = \frac{1}{A_j} \int_{A_i} \int_{A_j} \frac{\cos\alpha_i \cos\alpha_j}{r^2} \, dA_j \, dA_i$$

The coefficient ε_{ij} should be treated the same way as the coefficient of conductivity. Due to the low range of temperature dealt with in the tool-chip-workpiece system, the combined effect of convection and radiation are considered to obtain the coefficients of the best curve fit for convection.

2.3 Restriction Due to the Configuration of the Element

The basic concept of the finite element method is to replace the actual complex structure by an equivalent model made up from discrete structural elements having known properties that can be expressed mathematically. For a three-dimensional system, elements with a rectangular cross-section would facilitate the calculation for: a) exposed areas between elements, b) the length of the path between the centroids, c) location of the centroids of different elements within the body with respect to the defined axes.

Therefore, there is a maximum of three heat connections for convection in the case of a rectangular element located at the corner of a body. There is a maximum of six heat connections for conduction in the case of a rectangular element located within the body and surrounded by elements on each side. There is a maximum of eight heat connections for radiation assuming that those elements are the ones that are directly exposed to the radiation circle, and any element beyond that region will receive a negligible amount of radiational heat.

Furthermore, elements within a body, need not be of the same size all through the system. Areas of concentration such as forces, heat, stresses, etc., require smaller element sizes than other elements in the body.

2.4 Restriction Due to the Method of Numbering Nodes

In the finite element approach, a node represents the centroid of an element. Numbering of nodes will identify the elements as related to the system under study. Thus, for any element, all other elements that are connected to it by a heat connection can be identified by their label or by a certain number assigned to them.

No sequential ordering is required for adjacent elements, but the following rule for numbering the nodes should be followed: Start first by numbering the unknown nodes. Then continue by numbering the known nodes. For example, if the system is composed of 48 unknown nodes and 29 known nodes or boundary nodes, for a total of 77 nodes in the system, then, the nodes that carry the numbers from 1 to 48 are all unknown nodes or elements, and the nodes that carry the numbers from 49 to 77 are all boundary or known elements.

This rule or constraint is considered to facilitate the programming logics in the formation of the H-matrix.

2.5 Test Problem

The steady state thermal field in a thin rectangular plate is considered. Two sets of boundary conditions are investigated:

a) The plate is subjected to a heat source of 100°F on one edge, while the other edges are kept at 0°F.

b) The plate is subjected to a heat source of 500°F at its center, while the edges are kept at 0°F.

In both cases, the coefficient of thermal conductivity is assumed constant.

To investigate the validity of the mathematical model for the quasi-finite element approach, the closed form solution and the finite difference method for the same boundary conditions are also considered.

Case 1

For the closed form solution, Fig.(5), the equation governing the thermal field in a steady state condition is given by:

$$\nabla^2 \theta = 0 \tag{13}$$

The boundary conditions are expressed by:

$$\theta(0,y) = 0$$

$$\theta(a,y) = 0$$

$$\theta(x,0) = 0$$

$$\theta(x,b) = T = 100°F$$

In this case, the solution is given by:

$$\theta(x,y) = \frac{2T}{\pi} \sum_{n=1}^{\infty} \frac{1}{n}\{1-(-n)^n\} \cosh\frac{n\pi b}{a} \sin\frac{n\pi x}{a} \sinh\frac{n\pi y}{a} \tag{14}$$

For the finite different method, the approximation at the i^{th} mesh point is described by:

$$(\theta_{i+1,J} + \theta_{i,J+1} + \theta_{i-1,J} + \theta_{i,J-1}) - 4\theta_{ij} = 0 \tag{15}$$

For the quasi-finite element approach, the plate is divided into elements as shown in Fig.(6). The temperature distribution field is shown in Fig.(7).

The computer results obtained using the three approaches: the closed form, the finite difference and the quasi-finite element, are shown in Appendix II. Close agreement is noticed, Table (1). The maximum error is found to be of the order of 0.5%.

Case 2

The quasi-finite element approach is applied in the second case. The plate was divided into elements, Fig.(6). The results are plotted in Fig.(7). It is noticed that the finite difference relation given in Eqn.(15) is satisfied at every unknown element except for the elements surrounding the central heat source. This discrepancy is due to the discontinuity of the function $\theta(x,y)$ at that point.

CHAPTER 3

THE ALGORITHM METHOD

This chapter discusses the procedures and steps required for the solution of the quasi-finite element approach. It also considers the maximum permissible number of elements as related to indexing methods and the size of the computer's memory.

3.1 Computational Procedure

The block diagram in Fig.(3) illustrates the computational procedure. The input data layout records, the flow chart, and the FORTRAN program for the test problem are given in Appendix III. The procedure involves the preparation of input data information, validation rules, the preparation of the H-matrix and β-vector, storing the data in the computer's memory, and the solution of the H-matrix.

3.1.1 Preparation of Input Data

The system under consideration is divided into elements. The elements are identified as known and unknown elements according to the boundary conditions of the system. The nodes are labeled following the numbering rule mentioned in Chapter 2. This has been manually processed for this problem.

For each heat connection to an unknown node, a record (set of data information) is assigned. Additional records are necessary to indicate the areas and lengths of path between the centroids of the adjacent elements as well as the modes and the coefficients of heat transfer. Appendix III shows the suggested record layout of heat connection per unknown node that is to be used as input information. Initial values for the unknown nodes temperature, i.e. the assumed θ-vector, are computed as the average temperature of all known nodes.

3.1.2 Validation Rules

Validation rules are considered to fulfil the following conditions:

a) Any number in the system should not exceed the total number of nodes prescribing the system.

b) Any number assigned to an unknown node should not exceed the total number of unknown nodes.

c) The total number of heat connections per unknown node should not exceed 3 in the case of convection, 6 in the case of conduction and 8 in the case of radiation.

d) The entries in the field representing the mode of heat transfer should be either 1, 2 or 3.

Any violation of these rules will force a print-out of an appropriate message and an immediate job termination.

3.1.3 Preparation of the H-Matrix and the β-Vector

Since the formation of the H-matrix and the β-vector are dependent on the θ-vector, the solution of Eqn.(9) is obtained iteratively. An initial vector $\tilde{\theta}$ is assumed to construct the [H] and the{β}. The solution for [H]{θ}={β} is obtained using the Gaussian elimination method. If the deviation of{θ}-{$\tilde{\theta}$} is within an acceptable tolerance, the solution is considered satisfactory. Otherwise, the {$\tilde{\theta}$} is replaced by {θ}and the [H] and {β}are recalculated. The process is repeated until the tolerance is acceptable. More attention should be given to the method of data storage since the calculation of the[H] matrix might be repeated several times.

3.1.4 Data Storage

No special sequential rule is required for numbering the elements. In order to facilitate the scanning operation in the 'DO LOOP' statement, special locations in the memory should be considered as follows:

a) Two-dimensional array N stores the identification of all elements attached to element i, which are classified according to their mode of heat transfer. The accumulated number of heat connections per node are also stored for data validation and various steps of computation.

b) Two-dimensional array C stores all other data information in their different shape for each unknown element i. The data is grouped for each heat transfer mode.

The data storage in this form ensures the best retrieval of input data.

3.1.5 Solution of the H-Matrix

The Gaussian elimination method is applied to solve the matrix Eqn.(9). The flow chart in Appendix III shows the method of solution.

3.2 Memory Allocation and Capacity Constraint

It is preferable to determine the capacity of the computer memory in terms of number of unknown elements in the complex under study. For N unknown elements, the requirements of the quasi-finite element mathematical model for storing are as follows:

Two{θ} vectors	→ 2N
[N] array	→ 20N
[C] array	→ 40N
[H] array	→ N^2
{M} vector	→ N

where, {M} is the material code vector.

For a computer that requires two words for a floating variable and one word for a fixed variable, the total requirement of the computer memory is:

$$(2N + 105N) \text{ words} \qquad (16)$$

Denoting the total available storage space in the memory by 'n', thus,

$$2N^2 + 105N = n$$

which gives,

$$N = \frac{-105 + \{(105)^2 + 8n\}^{1/2}}{4}$$

or

$$N = (n/2)^{1/2}, \text{ for } n>10,000 \qquad (17)$$

For an average machine with n = 15,000, the maximum permissible mumber of unknown elements is equal to 65.

These points are sufficient for the determination of temperature distribution in two-dimensional problems. However, the number of elements required for a three-dimensional problem is largely increased and therefore, the indexing and partitioning techniques should be so considered to overcome the capacity constraint of the computer memory.

3.3 Indexing and Partitioning Techniques

In the tool-chip-workpiece system, the greater part of calculations are carried out to obtain the exact solution of the [H] matrix. The number of elements in the system are therefore limited by the size of the computer's memory. Under normal conditions the maximum number of elements (unknown) is given by Eqn.(17). Indexing, data manipulation and partitioning techniques are integrated in the main FORTRAN program given in Appendix IV to increase the number of unknown elements in the tool-chip-workpiece system.

3.3.1 Indexing

An N×N matrix requires N^2 locations to be stored sequentially in the computer's memory. Considering different sizes of square matrices with their associated vectors, the elements of the upper triangle are stored sequentially as shown in Fig.(8). Two relations can be derived:

a) The maximum number of elements that exists in the upper triangle and its associated vector are equal to:

$$N(N + 3)/2 \tag{18}$$

b) An element A_{ij} in a matrix can be located by the intersection of its corresponding row 'i' and column 'j' for i, j = 1, 2, 3, ...N. Therefore, the location of any element in the upper triangle of a square matrix is a function of its row and column as follows:

$$H_{ij} = (N + 1)(I - 1) - I(I - 1)/2 + J \tag{19}$$

where I and J are the location of the row and the column in the matrix.

Due to the symmetry of the H-matrix, only the upper triangle is required to obtain a solution. Introducing the two relations given in Eqns.(18) and (19), the requirements of the model for storage are reduced to:

One $\{\theta\}$ vector	$\rightarrow$	N
[N] array	$\rightarrow$	20N
[C] array	$\rightarrow$	40N
[H] array+$\{\theta\}$	$\rightarrow$	N(N+3)/3
$\{M\}$ vector	$\rightarrow$	N

Hence, Eqn.(17) becomes:

$$N = \frac{-106 + ((106)^2 + 4n)^{1/2}}{2} \qquad (20)$$

Thus, for an average machine with n = 15,000 words, the maximum permissible number of unknown elements is equal to 80.

3.3.2 Manipulation of Data

Data manipulation permits further increase in the maximum permissible number of unknown elements. Input information can be stored on tapes or discs. The required portion for calculation is only read and located in the memory of the computer. The required modifications for the [C] and the [N] arrays are shown in the flow chart given in Appendix IV. In this case, the main requirements of memory locations are as follows:

One $\{\theta\}$ vector	→	N
[N] array	→	3N
[C] array	→	0
[H] array $+\{\theta\}$	→	N(N+3)/2
{M} vector	→	N

Hence, Eqn.(17) becomes:

$$N = \frac{-9 + \sqrt{(9)^2 + 4n}}{2}$$

and therefore, the maximum permissible number of elements is increased to 118 unknown elements for the same memory size. Appendix IV shows a flow chart illustrating the required modifications.

3.3.3 Partitioning

Partitioning is a well known technique used in solving large matrices. The matrix is divided into segments. Each segment is called for separately from its storage location to perform the different steps of calculations. This concept is applied to solve the H-matrix. The upper triangle of the matrix is divided into segments.

The size of each segment does not exceed the maximum available memory allocations. Considering the data given in Section 3.3.2, the maximum permissible size of any segment should not exceed:

$$(n - 5N)/2 \qquad (22)$$

This is considered in the main program in Appendix IV.

CHAPTER 4

THE QUASI-FINITE ELEMENT APPROACH FOR THE DETERMINATION OF THE TEMPERATURE DISTRIBUTION IN A SINGLE POINT CUTTING TOOL IN AN ORTHOGONAL CUTTING OPERATION

A tool-chip-workpiece system as shown in Fig.(9), is considered. The system is sliced into nine layers of different thicknesses. Each layer is mainly broken down into rectangular elements while the shape of the surface elements follow the boundary of each corresponding layer. The various elements are numbered as shown in Fig.(10)* following the same concept described in Chapter 2. The thermal properties of the different material in the system are calculated. A curve fitting technique is applied to obtain the coefficients of the polynomial passing through these data giving the thermal properties of the different material at specified temperatures. Appendix I shows an example of the calculated coefficients for the workpiece material.

In the quasi-finite element approach, the determination of the temperature at a few locations in the system is

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* The dimension of the insert and the numbering of the whole system representing the tool-chip-workpiece are shown in Appendix V.

required to serve as the boundary conditions. The best location of the points where temperatures are measured is in the contact zone area and depends on the nature of the cutting operation. Kuester's [14] experimental results are considered as a guide line to estimate the temperature at different boundary locations. Consequently, the boundary conditions of the tool-chip-workpiece system can be considered as follows:

a) The temperatures of the tool shank elements nearest to the insert are taken as 200°F.

b) The temperatures of the workpiece elements may then be considered to vary between 200°F and 550°F depending on their respective distances from the cutting edge.

c) The temperature of the surrounding atmosphere as represented by the layers such as 1 and 9 are assumed to be ambient.

d) Temperature at central element of contact area is 900°F.

The coefficients of the polynomials giving the thermal properties of the different material, the temperatures of the boundary elements, areas and path lengths between the adjacent elements, and the mode of heat transfer represent the input data shown in Appendix V.

4.1 Solving the Algorithm

The main program given in Appendix IV is used to solve the problem. Minor changes are introduced to conform to the computer CDC 3300. Such changes affect only the speed of data manipulation in the computer. In spite of these modifications, the computation time is relatively long. The use of the existing software packages for solving huge matrices is suggested. The IBM package "MATLAN" is used only to solve the H-matrix. The MATLAN program is shown in Appendix V.

For practical purposes, the solution is limited to two iterations. The difference in nodal temperatures at the end of the second iteration varies between 0° and 5.75°F. Additional iterations are expected to improve the results only approximatley 1%. The computation time for the IBM computer model 75 is approximately 58 minutes per iteration. Therefore, the tolerance reached at the end of the second iteration is considered acceptable.

4.2 Discussion of Results

Figs.(11) to (15) show the computed isotherms in the tool-chip-workpiece system. Although in the quasi-finite element approach the maximum cutting temperature and its location are estimated from previous experimental results, more accurate and reliable temperature distribution can be achieved if the temperature at several points are known experimentally within the tool-chip-contact area as well as on the surface. Also, accurate information is required regarding the effect of temperature changes on the heat properties of the cutting tool and workpiece. A detailed knowledge of the dynamics of the surrounding media such as air or coolant and their boundary layer behaviours would make the results more exact.

The following observations are made from the analysis of the results shown in Figs.(11) to (15):

a) The highest temperature gradient occurs in the contact zone, approximately at a distance of 0.046 inches from the cutting edge. This agrees well with the established results that the maximum crater depth occurs at the point of maximum cutting temperature. This may be referred to in Fig.(17) (after [3]) where the location of the maximum crater wear is measured at a distance of 0.04 inches from the cutting edge.

b) The isotherm contours in the tool-chip-workpiece system agree in general with Kuester's experimental results shown in Fig.(16).

c) The equal temperature lines follow the same features of the contour lines of equal crater depth measured on carbide inserts [3]. The rate of temperature decrease along the cutting edge is much lower than that in the normal direction as seen in Fig.(15). The same feature may be observed in Fig.(17), also where the crater wear decreases at a much greater rate in the direction normal to the cutting edge than along the edge.

The above observations lead to the conclusion that the temperature distribution in the tool-chip-workpiece system does not depend on the intensity or the progression of the crater wear. This statement is qualified by the fact that there is a close interrelationship between the crater topography and the temperature distribution on the rake face [4], and further the location of the deepest point of the crater is independent of time [3] as shown in Fig.(18). Hence, it may be stated that the temperature distribution as calculated by the quasi-finite element approach is also time independent. This is of significant importance in the study of temperature in metal cutting operations and its relationship to the topography of crater wear and tool life.

CONCLUSIONS

The problem of obtaining the temperature distribution in single point orthogonal cutting operations was considered and a quasi-finite element technique was employed to obtain reliable results. The results presented agree well with the available experimental data and also relate closely to the topography of the crater wear and tool life.

In the classical finite element approach [23], the elements are visualized as connected to each other through nodes. The temperature at these nodes are determined so that the heat balance and the compatibility requirements for each element are satisfied. The nodes have to be exactly defined in reference to a convenient system of axes. Convection and radiation appear as boundary conditions and usually they are not easily implimented in the classical approach. In the quasi-finite element approach, however, the geometry of the element is not a determined factor and the compatibility is therefore not necessarily satisfied. Furthermore, the quasi-finite element approach offers the following advantages:

a) For the same number of elements, a considerable reduction is achieved regarding the memory and the computing time requirement.

b) The temperature dependence of the heat properties of the material and its isotropic characteristics are easily accounted for.

In order to provide the industry research laboratories with a reliable routine for the determination of the temperature fields in cutting tools, the following areas of future investigations are recommended:

a) The study of the dynamics of the surrounding media of the cutting action such as air and commercial coolants and their boundary layer behaviour.

b) The study of the time dependancy of the cutting tool isotherms and their relationship to crater wear progression.

c) The implementation of the method to multi-point cutting tools.

e) Development of computerized procedure for automatic preparation of input data cards.

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F I G U R E S A N D T A B L E S

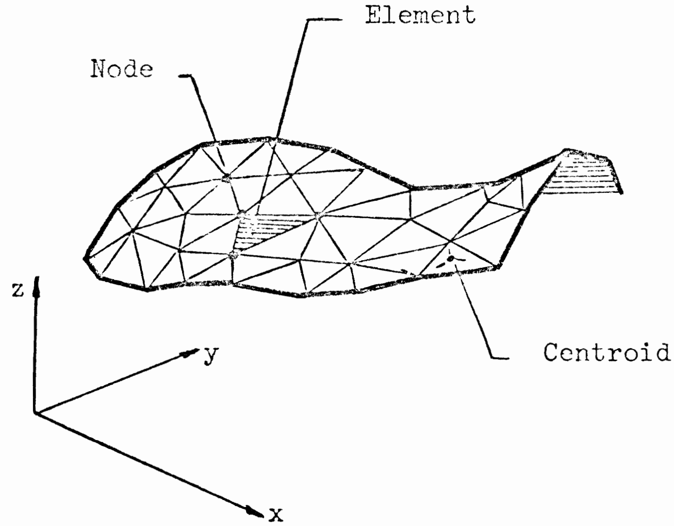


Fig. 1 -- The definition of centroid, element and node in an arbitrary complex surface.

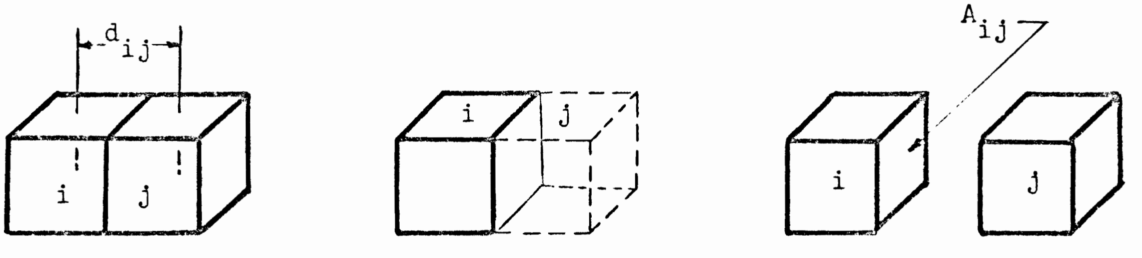


Fig. 2 -- The relation between adjacent elements in the three modes of heat transfer.

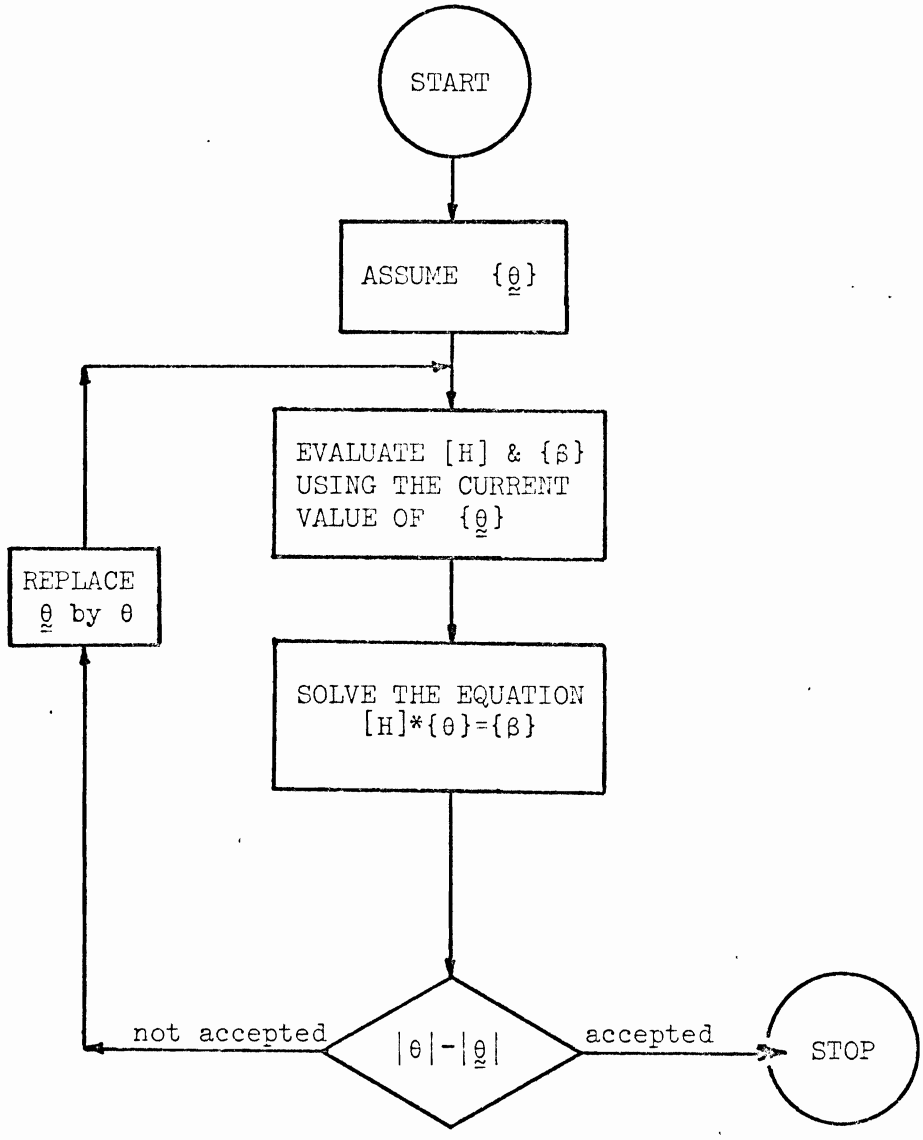


Fig. 3 -- The Block Diagram

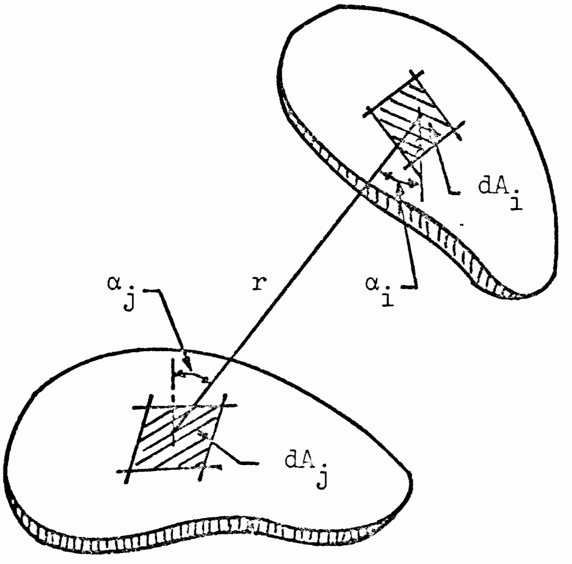
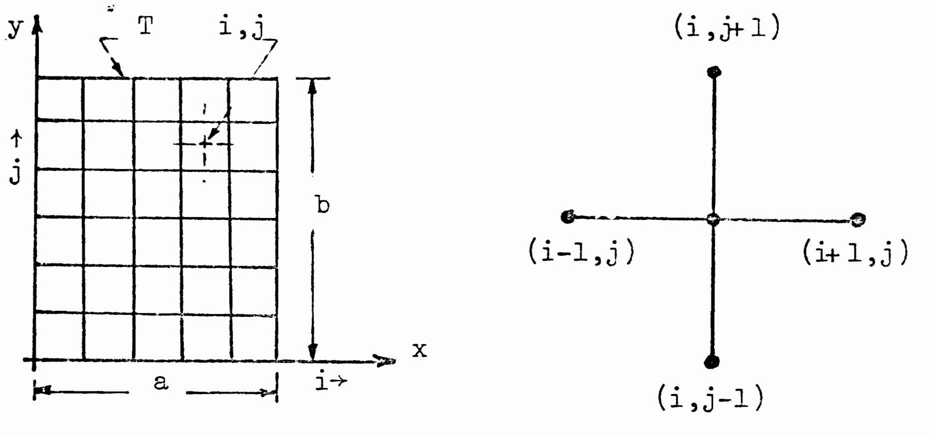


Fig. 4 -- The view factor in the radiation mode.



One edge of a rectangular plate exposed to a temperature of 100°F, while, the other edges are maintained at zero degree.

Fig. 5 -- The test problem

CASE 1

	71	72	73	74	75	76	
70	43	44	45	46	47	48	62
69	37	38	39	40	41	42	61
68	31	32	33	34	35	36	60
67	25	26	27	28	29	30	59
66	19	20	21	22	23	24	58
65	13	14	15	16	17	18	57
64	7	8	9	10	11	12	56
63	1	2	3	4	5	6	55
	49	50	51	52	53	54	

CASE 2

	71	72	73	74	75	76	77	
70	42	43	44	45	46	47	48	63
69	35	36	37	38	39	40	41	62
68	28	29	30	31	32	33	34	61
67	22	23	24	49	25	26	27	60
66	15	16	17	18	19	20	21	59
65	8	9	10	11	12	13	14	58
64	1	2	3	4	5	6	7	57
	50	51	52	53	54	55	56	

Boundary conditions for case 1:

a) Temperature of nodes from no. 49 to 54 are equal to 100°F

b) Temperature of nodes from no. 55 to 76 are equal to zero

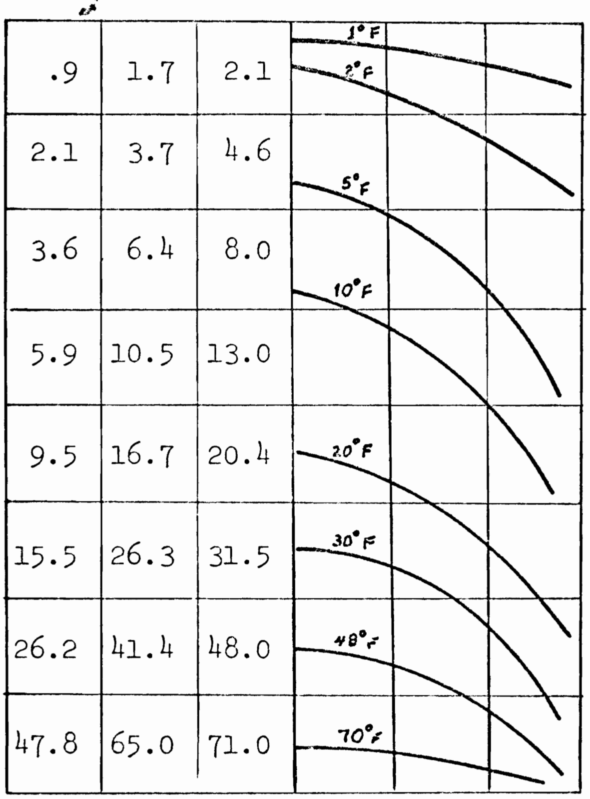
Boundary conditions for case 2:

a) Temperature of nodes from no. 50 to 77 are equal to zero

b) Temperature of the central node is equal to 500°F

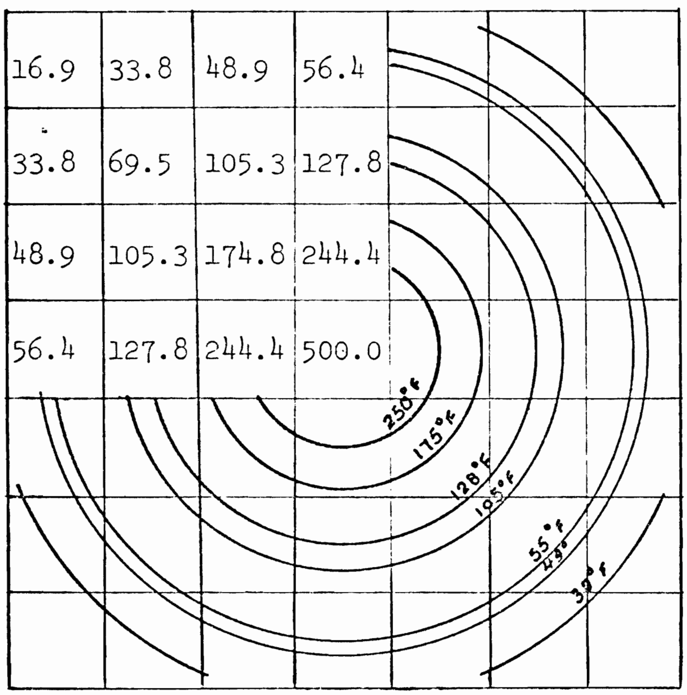
Fig. 6 -- Numbering of nodes for the test problem.

CASE 1



.9	1.7	2.1
2.1	3.7	4.6
3.6	6.4	8.0
5.9	10.5	13.0
9.5	16.7	20.4
15.5	26.3	31.5
26.2	41.4	48.0
47.8	65.0	71.0

CASE 2



16.9	33.8	48.9	56.4
33.8	69.5	105.3	127.8
48.9	105.3	174.8	244.4
56.4	127.8	244.4	500.0

Fig. 7 -- Computer results and temperature distribution in test problem.

2×2

1	2	3
	4	5

3×3

1	2	3	4
	5	6	7
		8	9

4×4

1	2	3	4	5
	6	7	8	9
		10	11	12
			13	14

5×5

1	2	3	4	5	6
	7	8	9	10	11
		12	13	14	15
			16	17	18
				19	20

6×6

1	2	3	4	5	6	7
	8	9	10	11	12	13
		14	15	16	17	18
			19	20	21	22
				23	24	25
					26	27

7×7

1	2	3	4	5	6	7	8
	9	10	11	12	13	14	15
		16	17	18	19	20	21
			22	23	24	25	26
				27	28	29	30
					31	32	33
						34	35

From the analysis, the following relations could be derived:-

a) An element H_{ij} is represented by

$$(N+1)(I-1) - I(I-1)/2 + J$$

b) Total number of elements of the upper triangle plus the associated vector is given by :-

$$N(N+3)/2$$

Fig. 8 -- Sequential allocation of elements in symmetrical matrices

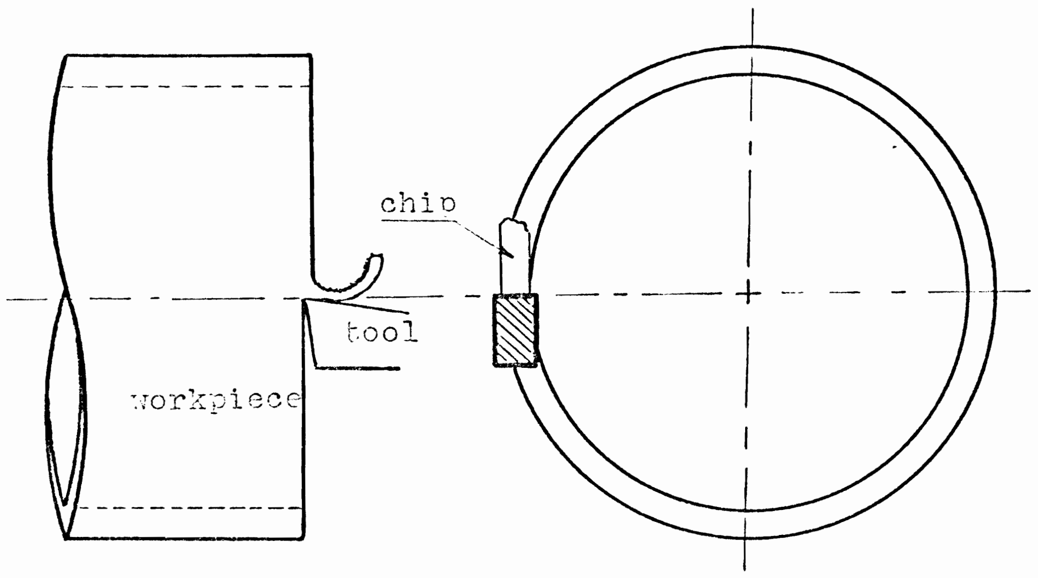
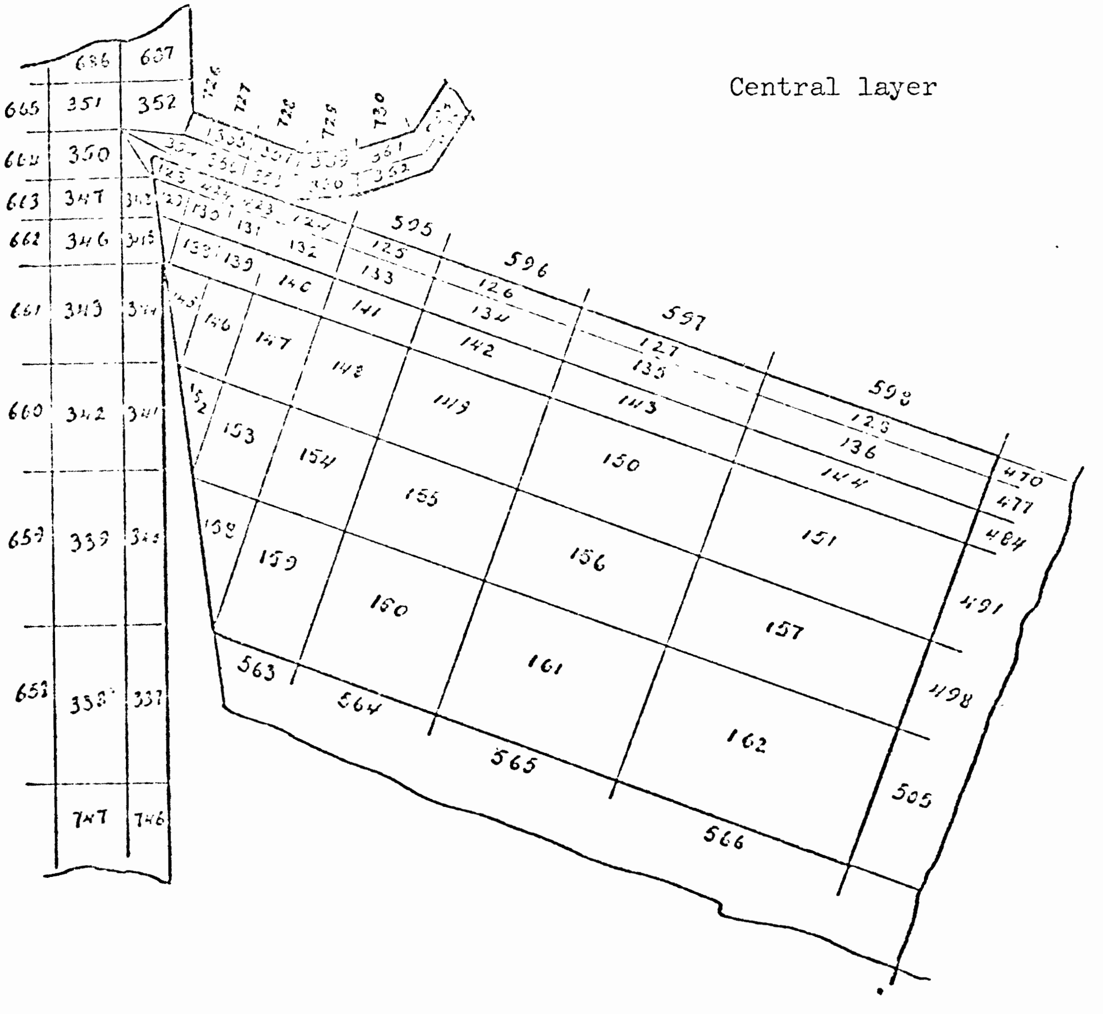


Fig. 9 -- Tool-chip-workpiece system in orthogonal cutting.



Central layer

Fig. 10 -- Slice sample for numbering of nodes for the tool-chip-workpiece system.

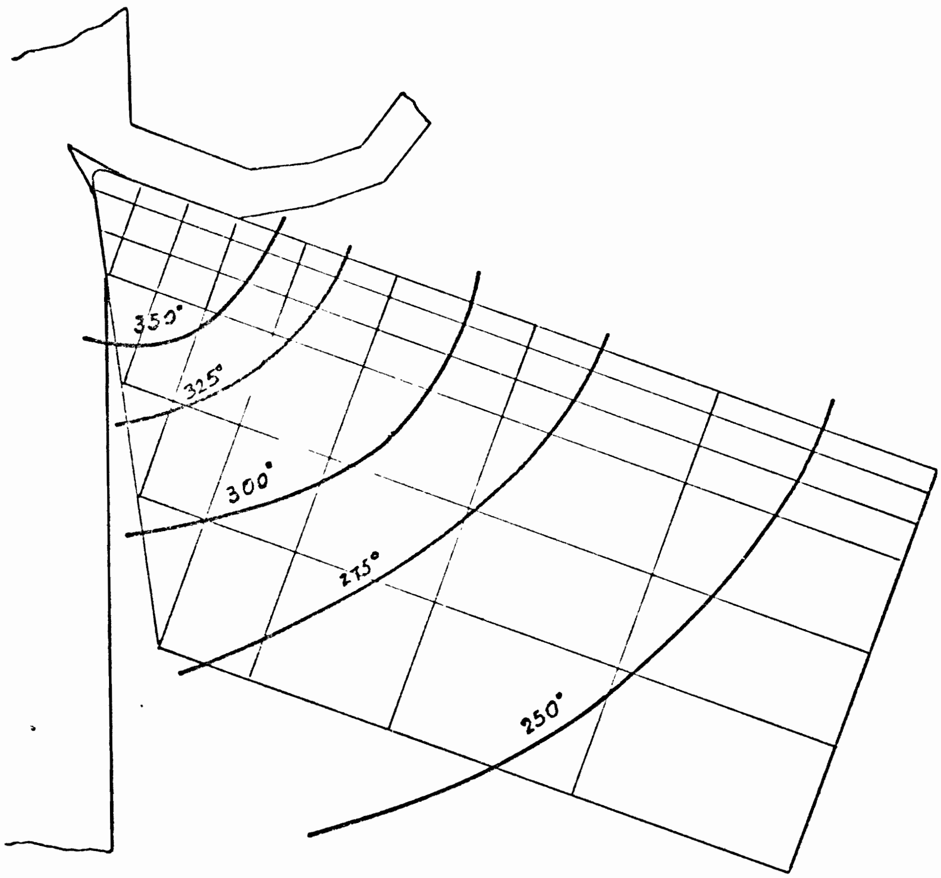


Fig. 11 -- Temperature distribution -- layer no.2 & 8

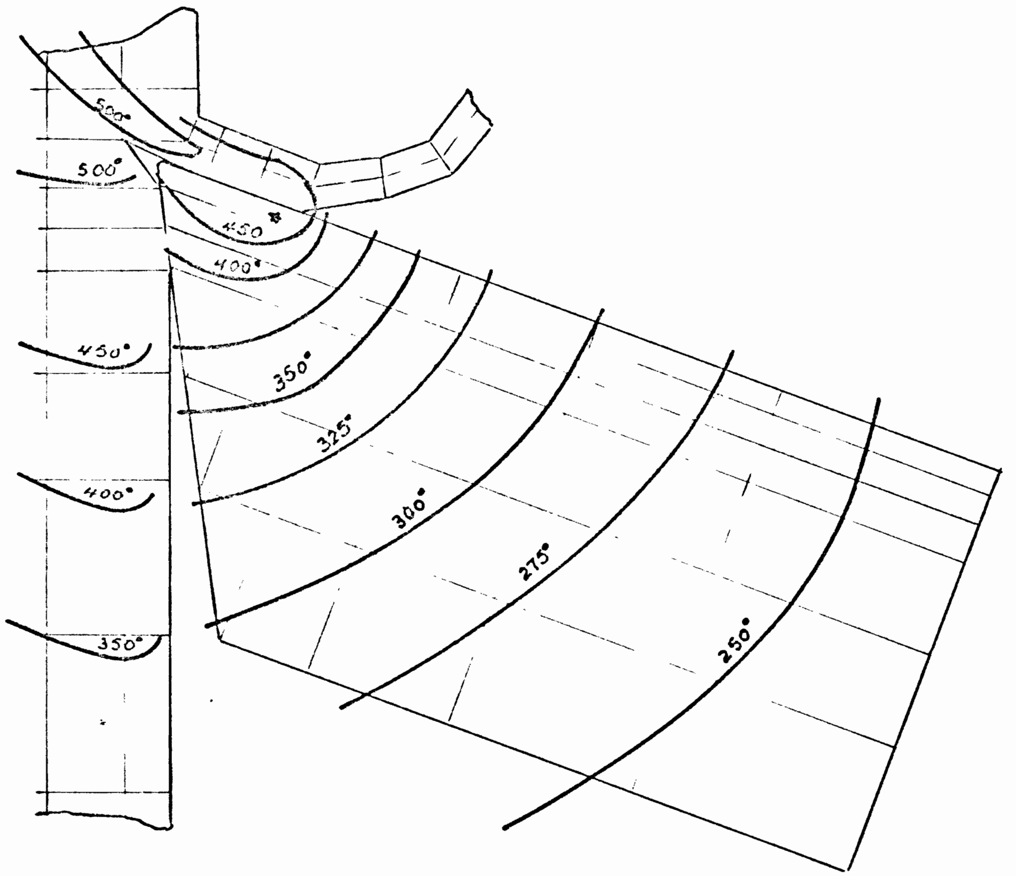


Fig. 12 -- Temperature distribution -- layer no. 3 & 7

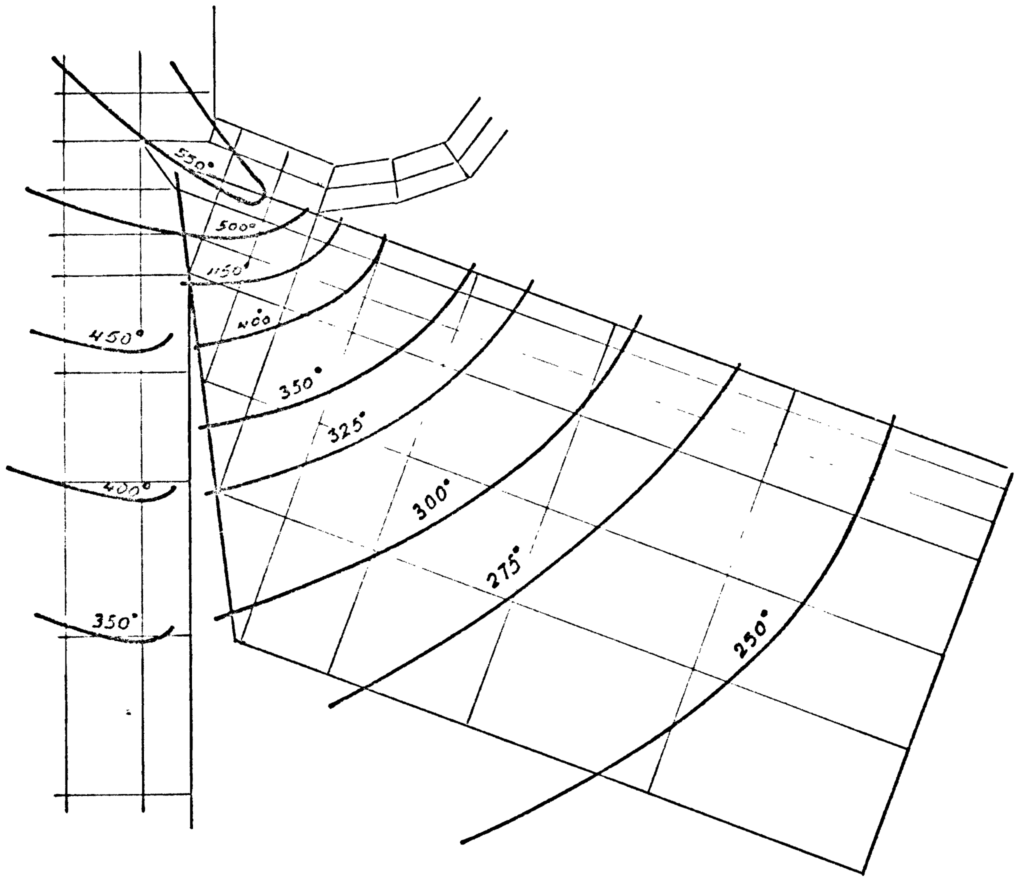


Fig. 13 -- Temperature distribution -- layer no.4 & 6

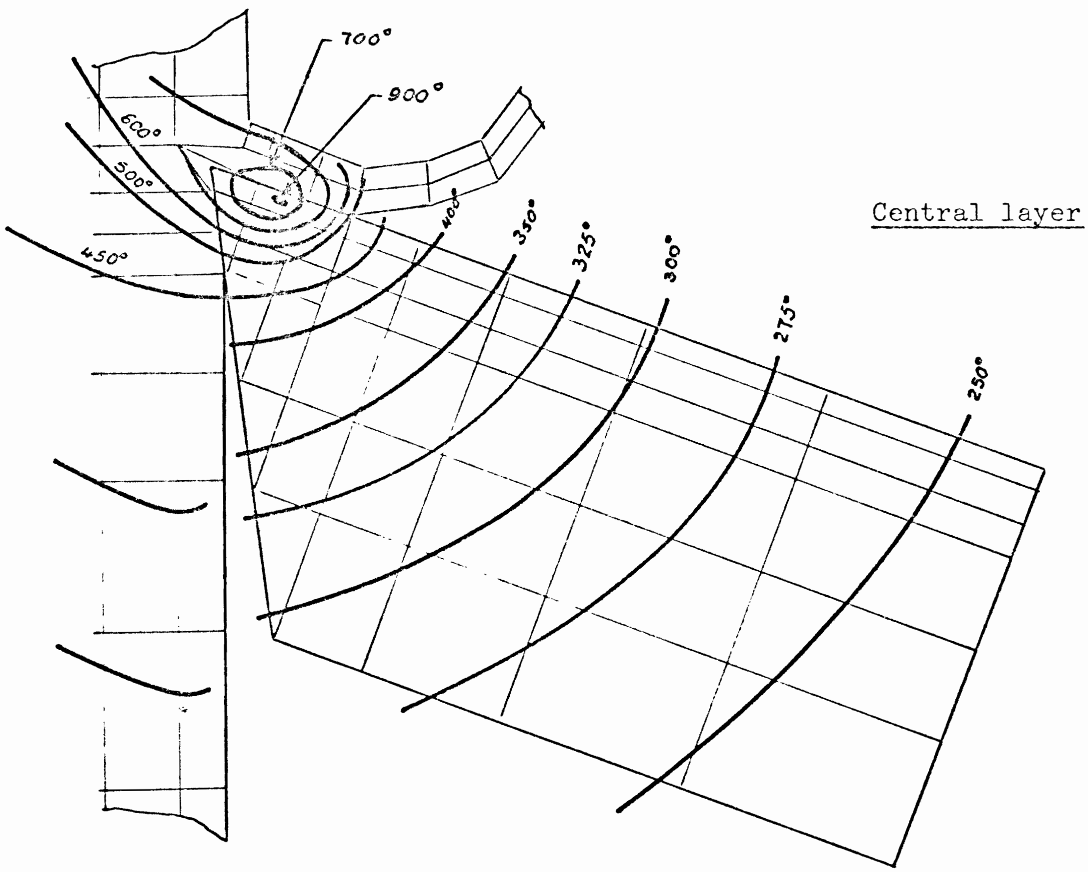


Fig. 14 -- Temperature distribution -- layer no.5

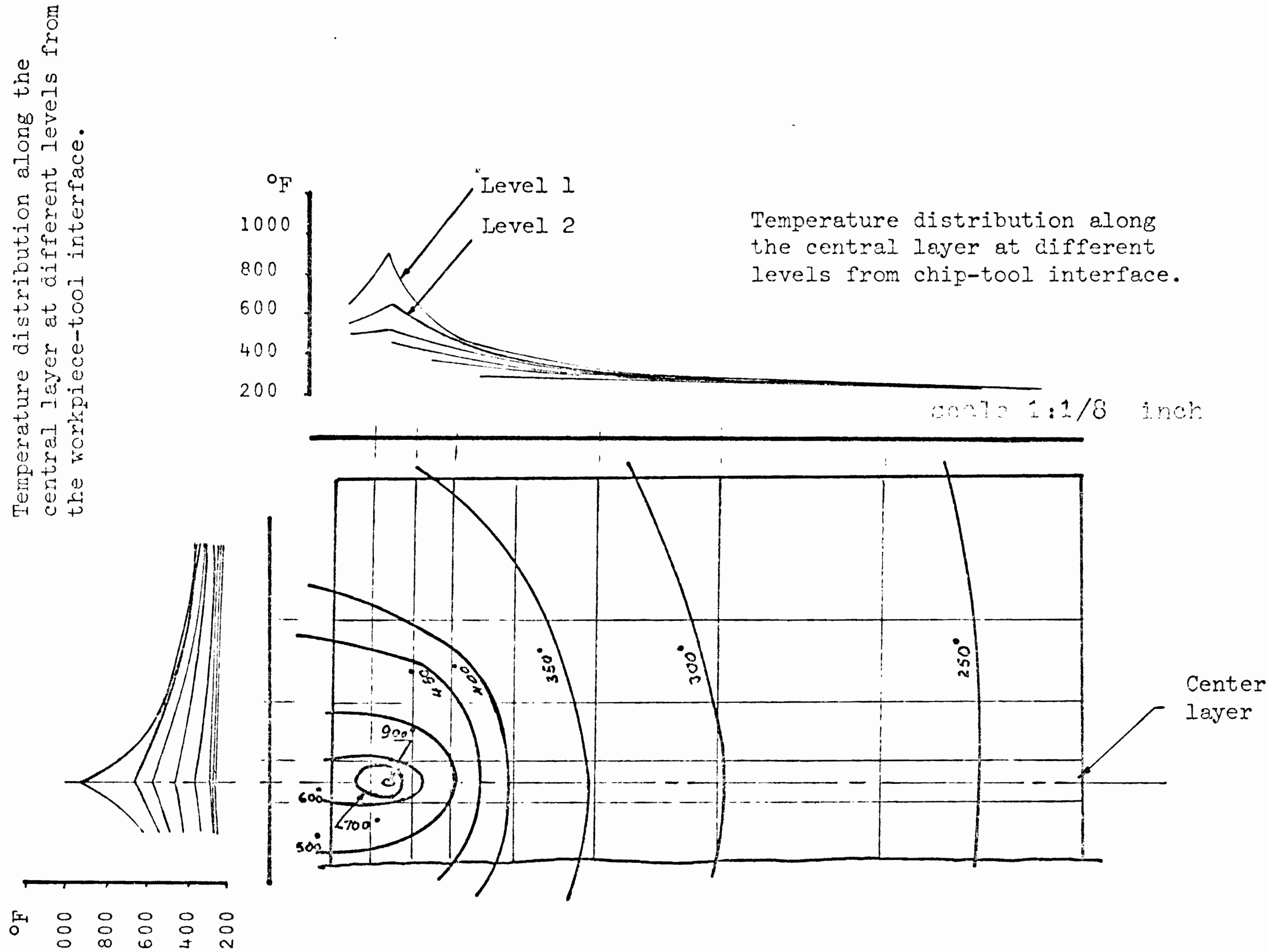


Fig. 15 -- Temperature distribution -- top view

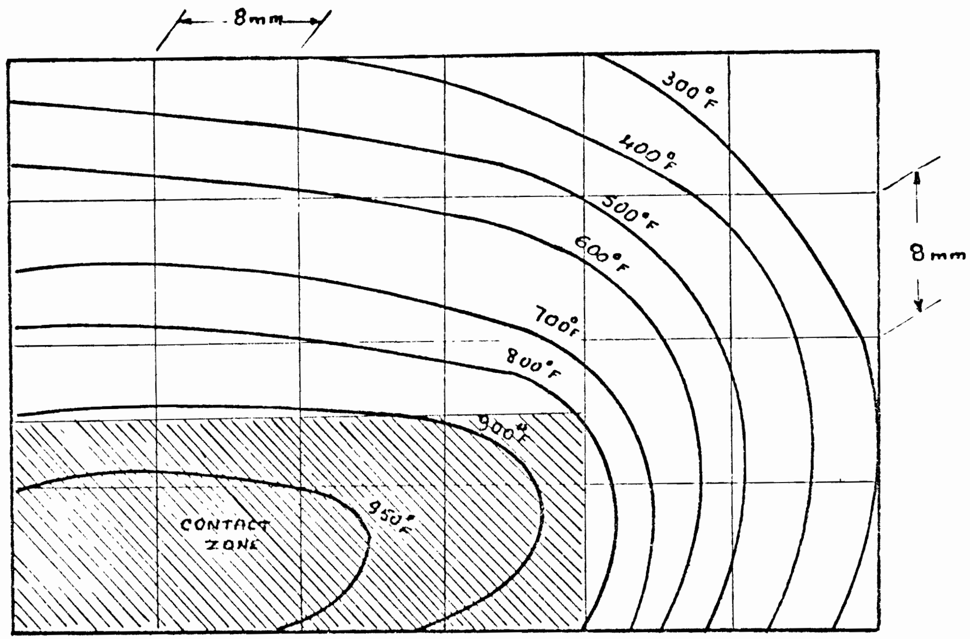


Fig. 16 -- Experimental results obtained by Kuester [14]

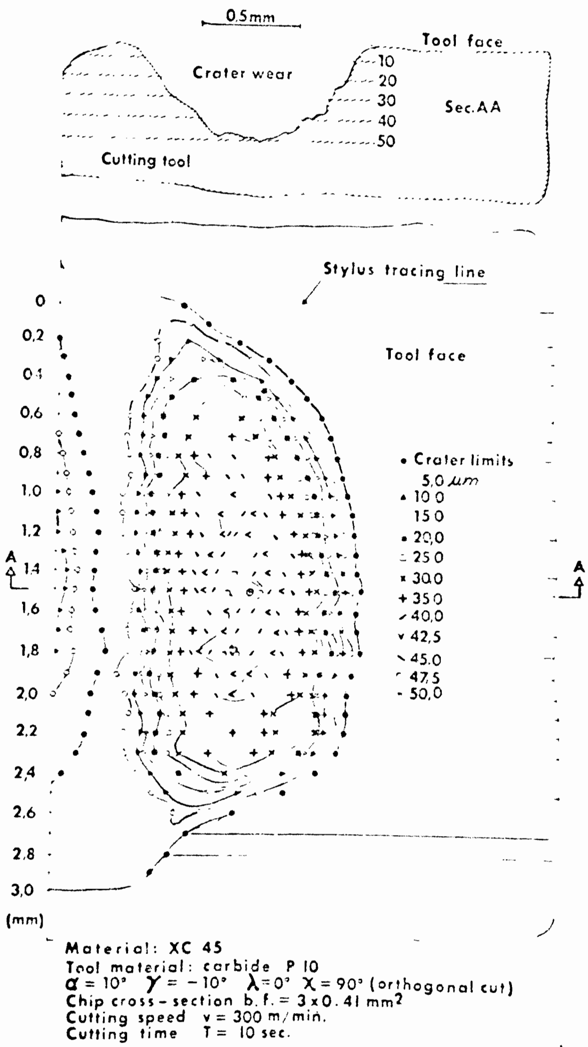
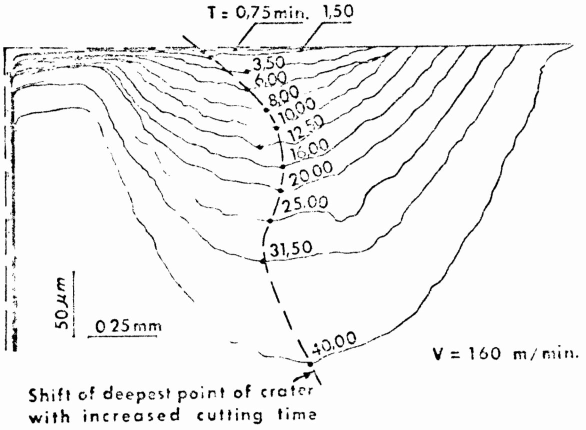
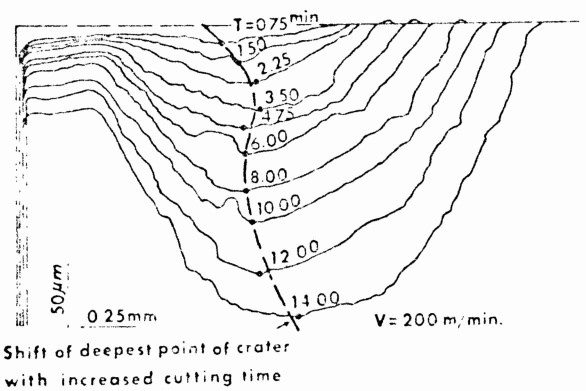


Fig. 17 -- Experimental results of contour lines of equal crater depth measured on carbide inserts. (after OSMAN, [3])





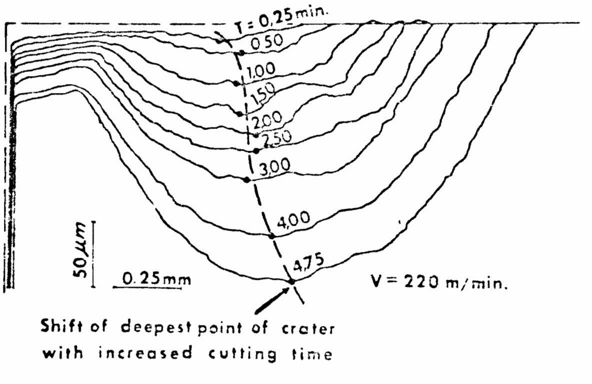


Fig. 18 -- Location of deepest point of the crater.

(after OSMAN [3])

TABLE 1

Comparison between results obtained by;quasi approach, close form and finite difference methods.

node	*	**	***	node	*	**	***
1	47.8	47.7	47.7	25	5.9	5.7	5.9
2	65.5	65.8	65.5	26	10.5	10.3	10.5
3	71.0	71.6	71.0	27	13.0	12.7	13.0
4	71.0	71.6	71.6	28	13.0	12.8	13.0
5	65.0	65.8	65.0	29	10.5	10.3	10.5
6	47.8	47.8	47.8	30	5.9	5.7	5.9
7	26.2	25.4	26.2	31	3.6	3.5	3.6
8	41.4	41.4	41.4	32	6.4	6.3	6.4
9	48.0	48.3	48.0	33	8.0	7.8	8.0
10	48.0	48.3	48.0	34	8.0	7.8	8.0
11	41.4	41.4	41.4	35	6.4	6.3	6.4
12	26.2	25.5	26.2	36	3.6	3.5	3.6
13	15.5	15.0	15.5	37	2.1	2.0	2.1
14	26.3	26.0	26.3	38	3.7	3.6	3.7
15	31.5	31.5	31.5	39	4.6	4.5	4.6
16	31.5	31.5	31.5	40	4.6	4.5	4.6
17	26.3	26.1	26.3	41	3.7	3.6	3.7
18	15.5	15.1	15.5	42	2.1	2.0	2.1
19	9.5	9.2	9.5	43	0.9	0.9	0.9
20	16.7	16.4	16.7	44	1.7	1.6	1.7
21	20.4	20.2	20.4	45	2.1	2.0	2.1
22	20.4	20.2	20.4	46	2.1	2.0	2.1
23	16.7	16.4	16.7	47	1.7	1.6	1.7
24	9.5	9.3	9.5	48	0.9	0.9	0.9

* quasi approach, ** close form solution, *** finite difference

APPENDIX I

CURVE FITTING

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Material specification.

Material	Steel, AISI 1040	
Chemical composition	C	0.37 - 0.44 %
	Mn	0.60 - 0.90 %
	P	0.040 % max.
	S	0.050 % max.

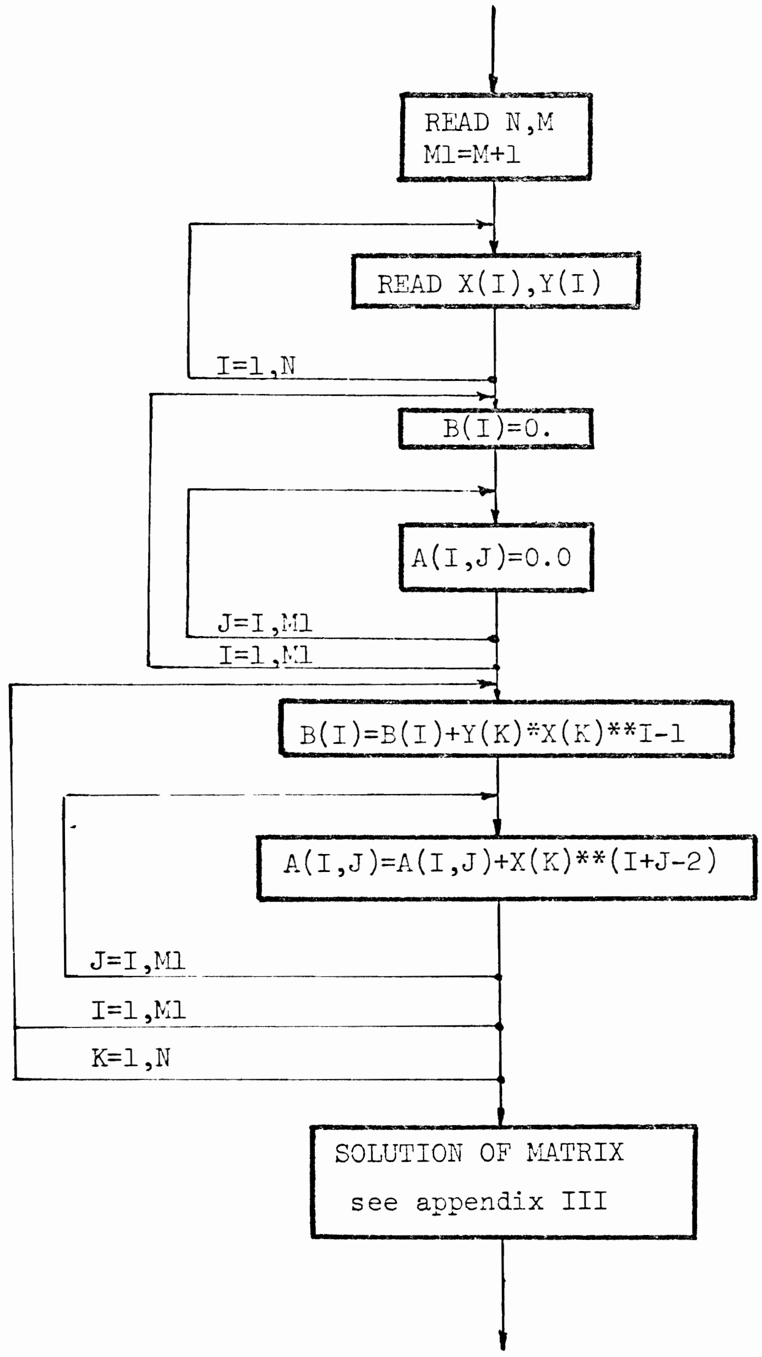
Thermal conductivity *

Material tested 0.415 C, 0.643 Mn, 0.063 Ni, 0.12 Cu, tr Cr

Temp. range	BTU/sec/ft²°F/in
32	.100
212	.098
392	.093
572	.088
752	.081
932	.073
1112	.065
1292	.058
1472	.048
1832	.052
2192	.057

* ASM, Metal Handbook, data from the national Phisical Lab. Jour. Iron and Steel Inst., 1946 N° II.

Flow chart for curve fitting



INPUT DATA (Work-piece)

NUMBER OF POINTS = 9
ORDER OF POLYNOMIAL = 2

TEMP.	THERMAL CONDUCTIVITY
32	30.000
212	29.400
392	27.900
572	26.400
752	24.300
932	21.900
1112	19.500
1292	17.400
1472	14.400

COEFFICIENTS OF THE POLYNOMIAL

3.04529E 01-5.28086E-03-3.85802E-06

VERIFICATION

TEMP.	THERMAL CONDUCTIVITY
32	30.280
212	29.160
392	27.790
572	26.170
752	24.300
932	22.180
1112	19.810
1292	17.190
1472	14.320

APPENDIX II

RESULTS OF THE TEST PROBLEM

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INPUT DATA (Case 1)

TOTAL NUMBER OF UNKNOWN NODES = 48

TOTAL NUMBER OF KNOWN NODES = 28

TOTAL NUMBER OF HEAT CONNECTIONS = 110

TOTAL NUMBER OF MATERIAL CODES = 1

TOLERANCE LIMIT = ,001000

THERMAL CONDUCTIVITY (BTU/HR*FT*DEGREE F)

MATERIAL CODE	(K1)	(K2)	(K3)
1	8,00000E 00	0	0

DATA OF NODES

A) INITIAL TEMPERATURES

UNKNOWN NODES	TEMP	MACHINE CODES	KNOWN NODES	TEMP	MACHINE CODES
1	21.4	1	49	100.0	1
2	21.4	1	50	100.0	1
3	21.4	1	51	100.0	1
4	21.4	1	52	100.0	1
5	21.4	1	53	100.0	1
6	21.4	1	54	100.0	1
7	21.4	1	55	0	1
8	21.4	1	56	0	1
9	21.4	1	57	0	1
10	21.4	1	58	0	1
11	21.4	1	59	0	1
12	21.4	1	60	0	1
13	21.4	1	61	0	1
14	21.4	1	62	0	1
15	21.4	1	63	0	1
16	21.4	1	64	0	1
17	21.4	1	65	0	1
18	21.4	1	66	0	1
19	21.4	1	67	0	1
20	21.4	1	68	0	1
21	21.4	1	69	0	1
22	21.4	1	70	0	1
23	21.4	1	71	0	1
24	21.4	1	72	0	1
25	21.4	1	73	0	1
26	21.4	1	74	0	1
27	21.4	1	75	0	1
28	21.4	1	76	0	1
29	21.4	1			
30	21.4	1			
31	21.4	1			
32	21.4	1			
33	21.4	1			
34	21.4	1			
35	21.4	1			
36	21.4	1			
37	21.4	1			
38	21.4	1			
39	21.4	1			
40	21.4	1			
41	21.4	1			
42	21.4	1			
43	21.4	1			
44	21.4	1			

45	21.4	1
46	21.4	1
47	21.4	1
48	21.4	1

B) HEAT CONNECTIONS

A = AREA IN SQUARE INCH FOR HEAT CONNECTION

B1 = PATH LENGHT (INCH) IN CASE OF CONDUCTION
= EMMISIVITY FACTOR IN CASE OF RADIATION

B1,B2,B3 = HEAT.TRANSFER CONVECTIVE COEFFICIENTS

NA = BASIC NODE NUMBER (UNKNOWN)

NB = NODE NUMBER TO WHICH NODE IS CONNECTED

M = 1,CONVECTION
= 2,CONDUCTION
= 3, RADIATION

	A	B1	B2	B3	NA	NB	M
1	.6	.3	0	0	1	2	2
2	.6	.3	0	0	1	7	2
3	.6	.3	0	0	1	63	2
4	.6	.3	0	0	1	49	2
5	.6	.3	0	0	2	50	2
6	.6	.3	0	0	2	3	2
7	.6	.3	0	0	2	8	2
8	.6	.3	0	0	3	51	2
9	.6	.3	0	0	3	4	2
10	.6	.3	0	0	3	9	2
11	.6	.3	0	0	4	52	2
12	.6	.3	0	0	4	5	2
13	.6	.3	0	0	4	10	2
14	.6	.3	0	0	5	53	2
15	.6	.3	0	0	5	6	2
16	.6	.3	0	0	5	11	2
17	.6	.3	0	0	6	54	2
18	.6	.3	0	0	6	55	2
19	.6	.3	0	0	6	12	2
20	.6	.3	0	0	7	8	2
21	.6	.3	0	0	7	13	2
22	.6	.3	0	0	7	64	2
23	.6	.3	0	0	8	9	2
24	.6	.3	0	0	8	14	2
25	.6	.3	0	0	9	10	2
26	.6	.3	0	0	9	15	2
27	.6	.3	0	0	10	11	2
28	.6	.3	0	0	10	16	2
29	.6	.3	0	0	11	12	2
30	.6	.3	0	0	11	17	2
31	.6	.3	0	0	12	56	2
32	.6	.3	0	0	12	18	2

33	.6	.3	0	0	13	14	2
34	.6	.3	0	0	13	19	2
35	.6	.3	0	0	13	65	2
36	.6	.3	0	0	14	15	2
37	.6	.3	0	0	14	20	2
38	.6	.3	0	0	15	16	2
39	.6	.3	0	0	15	21	2
40	.6	.3	0	0	16	17	2
41	.6	.3	0	0	16	22	2
42	.6	.3	0	0	17	18	2
43	.6	.3	0	0	17	23	2
44	.6	.3	0	0	18	57	2
45	.6	.3	0	0	18	24	2
46	.6	.3	0	0	19	20	2
47	.6	.3	0	0	19	25	2
48	.6	.3	0	0	19	66	2
49	.6	.3	0	0	20	21	2
50	.6	.3	0	0	20	26	2
51	.6	.3	0	0	21	22	2
52	.6	.3	0	0	21	27	2
53	.6	.3	0	0	22	23	2
54	.6	.3	0	0	22	28	2
55	.6	.3	0	0	23	24	2
56	.6	.3	0	0	23	29	2
57	.6	.3	0	0	24	58	2
58	.6	.3	0	0	24	30	2
59	.6	.3	0	0	25	26	2
60	.6	.3	0	0	25	31	2
61	.6	.3	0	0	25	67	2
62	.6	.3	0	0	26	27	2
63	.6	.3	0	0	26	32	2
64	.6	.3	0	0	27	28	2
65	.6	.3	0	0	27	33	2
66	.6	.3	0	0	28	29	2
67	.6	.3	0	0	28	34	2
68	.6	.3	0	0	29	30	2
69	.6	.3	0	0	29	35	2
70	.6	.3	0	0	30	59	2
71	.6	.3	0	0	30	36	2
72	.6	.3	0	0	31	32	2
73	.6	.3	0	0	31	37	2
74	.6	.3	0	0	31	68	2
75	.6	.3	0	0	32	33	2
76	.6	.3	0	0	32	38	2
77	.6	.3	0	0	33	34	2
78	.6	.3	0	0	33	39	2
79	.6	.3	0	0	34	35	2
80	.6	.3	0	0	34	40	2
81	.6	.3	0	0	35	36	2
82	.6	.3	0	0	35	41	2
83	.6	.3	0	0	36	60	2
84	.6	.3	0	0	36	42	2
85	.6	.3	0	0	37	38	2
86	.6	.3	0	0	37	43	2
87	.6	.3	0	0	37	69	2
88	.6	.3	0	0	38	39	2
89	.6	.3	0	0	38	44	2
90	.6	.3	0	0	39	40	2
91	.6	.3	0	0	39	45	2
92	.6	.3	0	0	40	41	2

93	.6	.3	0	0	40	46	2
94	.6	.3	0	0	41	42	2
95	.6	.3	0	0	41	47	2
96	.6	.3	0	0	42	61	2
97	.6	.3	0	0	42	48	2
98	.6	.3	0	0	43	44	2
99	.6	.3	0	0	43	71	2
100	.6	.3	0	0	43	70	2
101	.6	.3	0	0	44	45	2
102	.6	.3	0	0	44	72	2
103	.6	.3	0	0	45	46	2
104	.6	.3	0	0	45	73	2
105	.6	.3	0	0	46	47	2
106	.6	.3	0	0	46	74	2
107	.6	.3	0	0	47	48	2
108	.6	.3	0	0	47	75	2
109	.6	.3	0	0	48	62	2
110	.6	.3	0	0	48	76	2

SOLUTION S STEPS

ITERATION	MAX. DEVIATION	PERMISSIBLE
1	49.572771	.001000
1	0	.001000

SOLUTION
≠≠≠≠≠≠≠≠

1	47.8	49	100.0
2	65.0	50	100.0
3	71.0	51	100.0
4	71.0	52	100.0
5	65.0	53	100.0
6	47.8	54	100.0
7	26.2	55	0
8	41.4	56	0
9	48.0	57	0
10	48.0	58	0
11	41.4	59	0
12	26.2	60	0
13	15.5	61	0
14	26.3	62	0
15	31.5	63	0
16	31.5	64	0
17	26.3	65	0
18	15.5	66	0
19	9.5	67	0
20	16.7	68	0
21	20.4	69	0
22	20.4	70	0
23	16.7	71	0
24	9.5	72	0
25	5.9	73	0
26	10.5	74	0

27	13.0	75	0
28	13.0	76	0
29	10.5		
30	5.9		
31	3.6		
32	6.4		
33	8.0		
34	8.0		
35	6.4		
36	3.6		
37	2.1		
38	3.7		
39	4.6		
40	4.6		
41	3.7		
42	2.1		
43	.9		
44	1.7		
45	2.1		
46	2.1		
47	1.7		
48	.9		

1	1.0	1.0
2	2.0	1.0
3	3.0	1.0
4	4.0	1.0
5	5.0	1.0
6	6.0	1.0
7	1.0	2.0
8	2.0	2.0
9	3.0	2.0
10	4.0	2.0
11	5.0	2.0
12	6.0	2.0
13	1.0	3.0
14	2.0	3.0
15	3.0	3.0
16	4.0	3.0
17	5.0	3.0
18	6.0	3.0
19	1.0	4.0
20	2.0	4.0
21	3.0	4.0
22	4.0	4.0
23	5.0	4.0
24	6.0	4.0
25	1.0	5.0
26	2.0	5.0
27	3.0	5.0
28	4.0	5.0
29	5.0	5.0
30	6.0	5.0
31	1.0	6.0
32	2.0	6.0
33	3.0	6.0
34	4.0	6.0
35	5.0	6.0
36	6.0	6.0
37	1.0	7.0
38	2.0	7.0
39	3.0	7.0
40	4.0	7.0
41	5.0	7.0
42	6.0	7.0
43	1.0	8.0
44	2.0	8.0
45	3.0	8.0
46	4.0	8.0
47	5.0	8.0
48	6.0	8.0

1	.903	5
2	1.626	5
3	2.026	5
4	2.025	5
5	1.624	5
6	.901	5
7	1.992	5
8	3.585	5
9	4.466	5
10	4.464	5
11	3.592	5
12	1.987	5
13	3.497	7
14	6.283	7
15	7.816	7
16	7.814	7
17	6.277	7
18	3.488	7
19	5.742	7
20	10.279	7
21	12.749	7
22	12.746	7
23	10.269	7
24	5.727	7
25	9.260	7
26	16.421	7
27	20.222	7
28	20.216	7
29	16.405	7
30	9.236	7
31	15.055	7
32	26.065	7
33	31.561	7
34	31.554	7
35	26.042	7
36	15.017	11
37	25.487	15
38	41.455	15
39	48.356	17
40	48.347	17
41	41.424	15
42	25.427	15
43	47.828	21
44	65.842	21
45	71.648	21
46	71.641	21
47	65.814	21
48	47.744	29

Results of the finite difference method

100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
0	47.8	65.0	71.0	71.0	65.0	47.8	0
0	26.2	41.4	48.0	48.0	41.4	26.2	0
0	15.5	26.3	31.5	31.5	26.3	15.5	0
0	9.5	16.7	20.4	20.4	16.7	9.5	0
0	5.9	10.5	13.0	13.0	10.5	5.9	0
0	3.6	6.5	8.0	8.0	6.5	3.6	0
0	2.1	3.7	4.6	4.6	3.7	2.1	0
0	.9	1.7	2.1	2.1	1.7	.9	0
0	0	0	0	0	0	0	0

APPENDIX III

INPUT DATA INFORMATION

Contents | Page

INPUT DATA INFORMATION

Record layout.

Program ALLEGRIA for the steady state temperature analysis.

1- General:-

Total number of cards   one card.

NU	NN	NHC	NMC	TOL
I5	I5	I5	I5	I5

where, NU → No. of unknown nodes.
NN → No. of known nodes.
NMC → No. of material codes.
NHC → No. of heat connections.
TOL → Acceptable tolerance.

2- Material:-

Total number of cards   NMC cards.

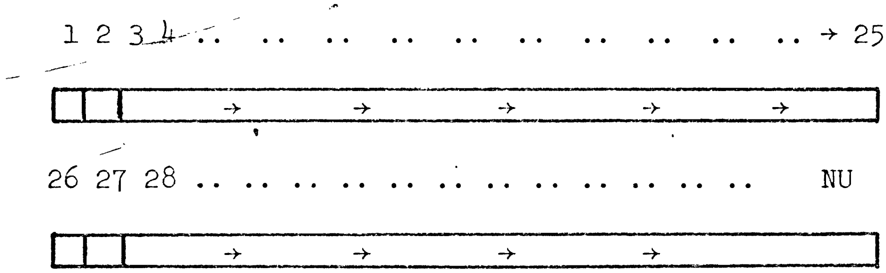
	AK1	AK2	AK3
1			
2			
↓	↓	↓	↓
NMC			
	F10	F10	F10

AK1, AK2, and AK3 are the coefficient of the best curve fit.

Record layouts (cont.)

3- Unknown nodes:-

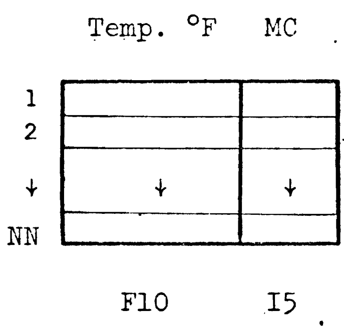
Total number of cards (NU/25) cards.



These entries are filled with 1, 2, 3, → NMC, depending on the material of the particular node.

4- Boundary Conditions:- (known nodes)

Total number of cards NN cards.



where, MC represents the material code.

These entries are filled with the temperatures and the material codes of the particular known node.

Record layouts (cont.)

5- Heat connections:-

	A	B1	B2	B3	NA	NB	M
1							
2							
↓	↓	↓	↓	↓	↓	↓	↓
NHC							
	F10	F10	F10	F10	I5	I5	I5

where;

A ≡ Area in square inch for heat connection.

B1≡ Path lenght (inch) in case of conduction.

≡ View factor in case of radiation.

NA≡ Basic node number (unknown) < NU

NB≡ Node number to which basic node is connected > NA

M ≡ Mode of heat transmission.

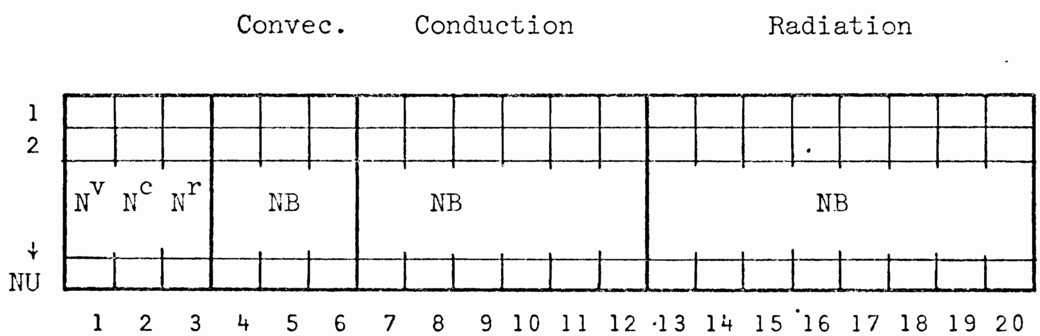
M=1, for convection.
M=2, for conduction.
M=3, for radiation.

In case of convection, B1, B2, and B3 are the convective heat transfer coefficients.

Storage of data in memory:-

Due to the limitations of number of connections,(3 connections for convection, 6 connections for conduction, and 8 connections for radiation), the data cards are processed in such a manner that the following N-matrix and C-matrix are formed.

N-matrix:-



where,

N^r Total number of heat connections in the convection mode for i=1→NU

N^c Total number of heat connections in the conductive mode for i=1→NU

N^r Total number of heat connections in the radiation mode for i=1→NU

By implication, they indicate the "unknown node" number.

Memory allocation 20 NU words.

Dimensioning N(NU,20)

C-matrix:

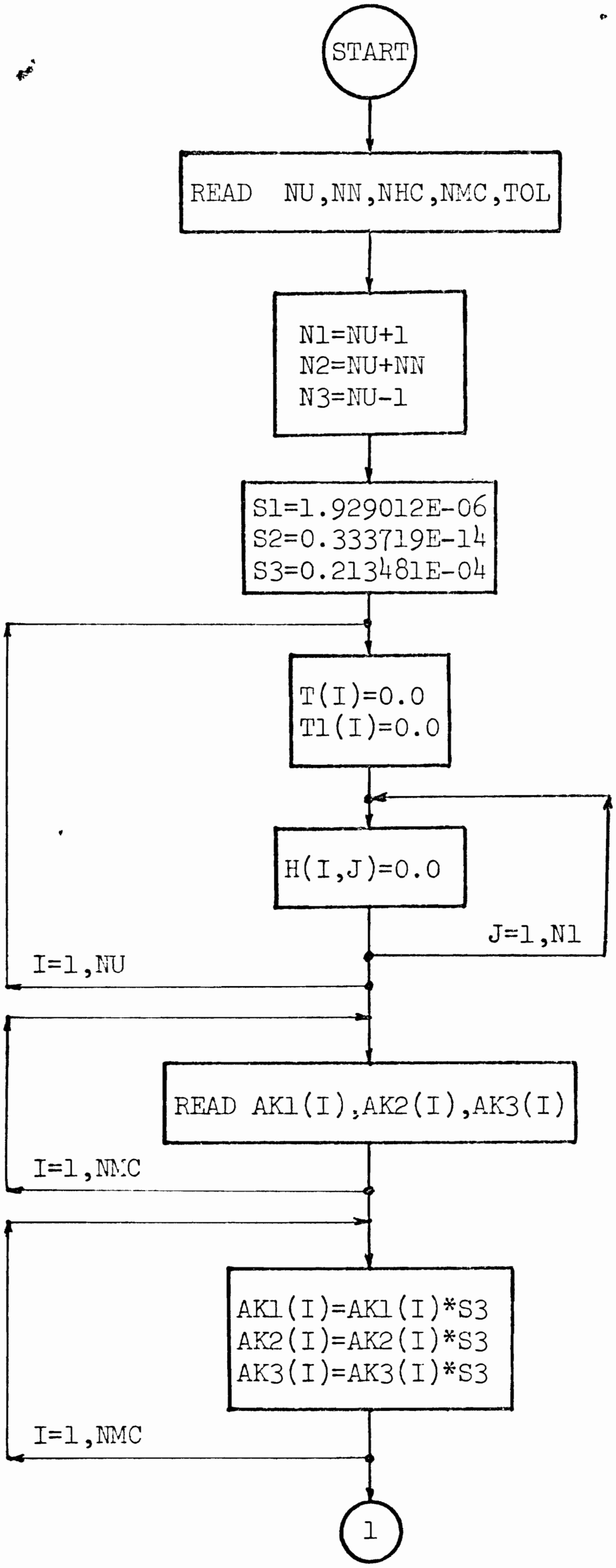
	A	h_o	h_1	h_2	A → →	A → →	A	d	A	d	→ → →	A	d	A	ε	A	ε	→ → →	A	ε
	1	2	3	4	5 → →	9 → →	13	14	15	16	→ → →	23	24	25	26	27	28	→ → →	39	40
1																				
2																				
3																				
↓ ↓ ↓			3 connections						6 connections							8 connections				
NU																				

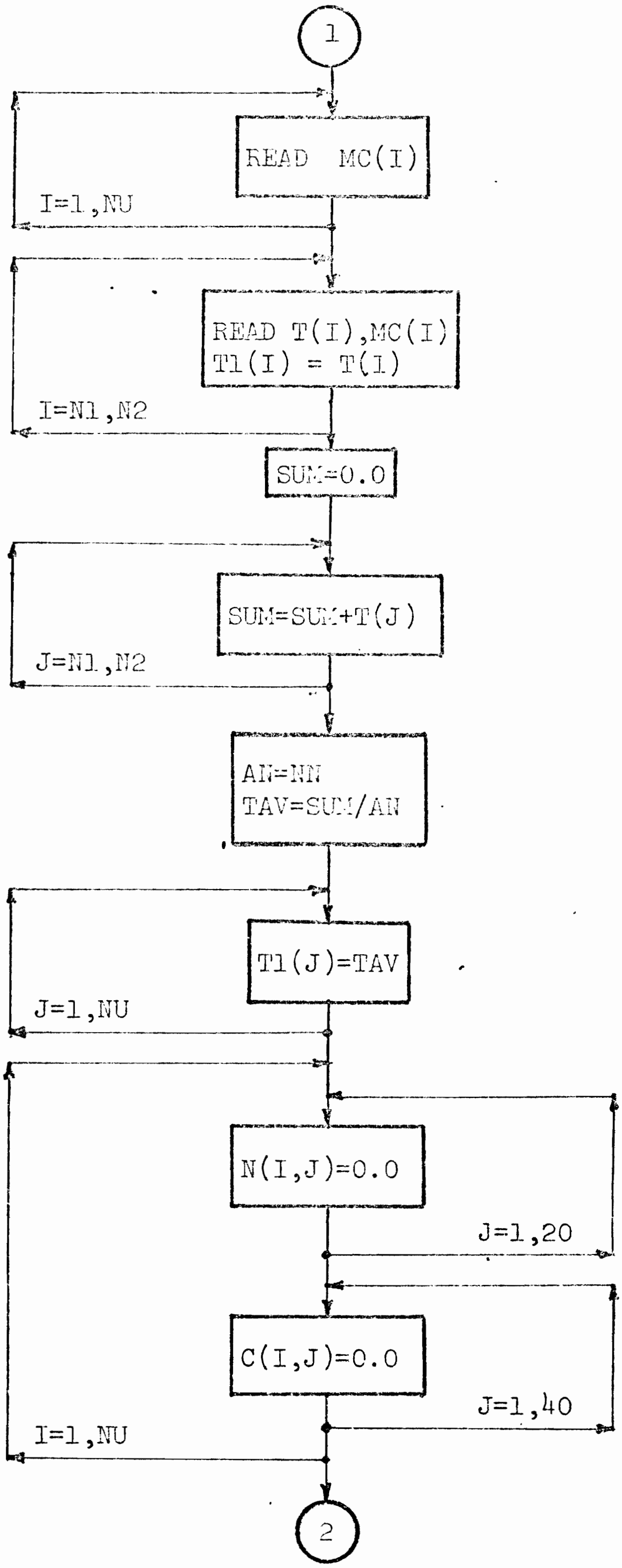
Convection Conduction Radiation

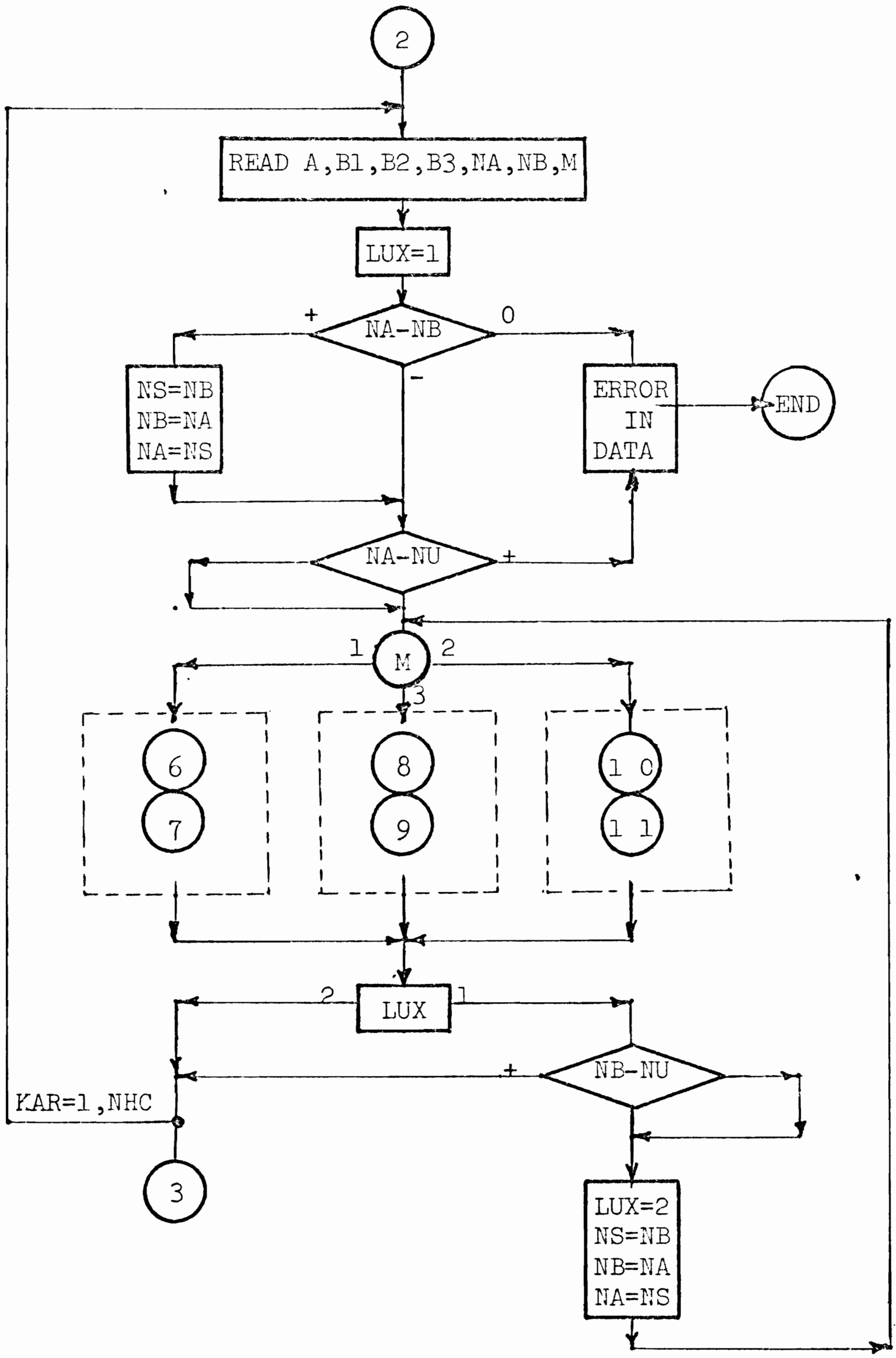
DIMENSION: C(NU,40)

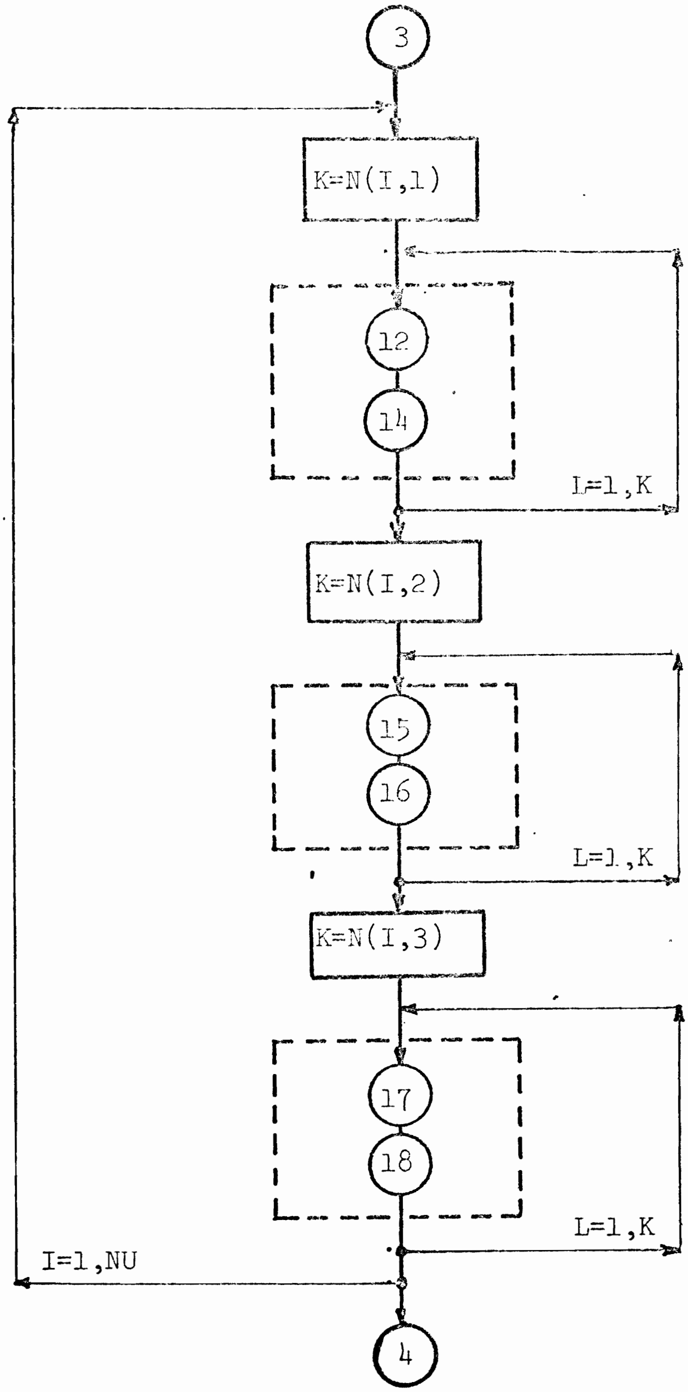
Memory Allocation 40 NU words.

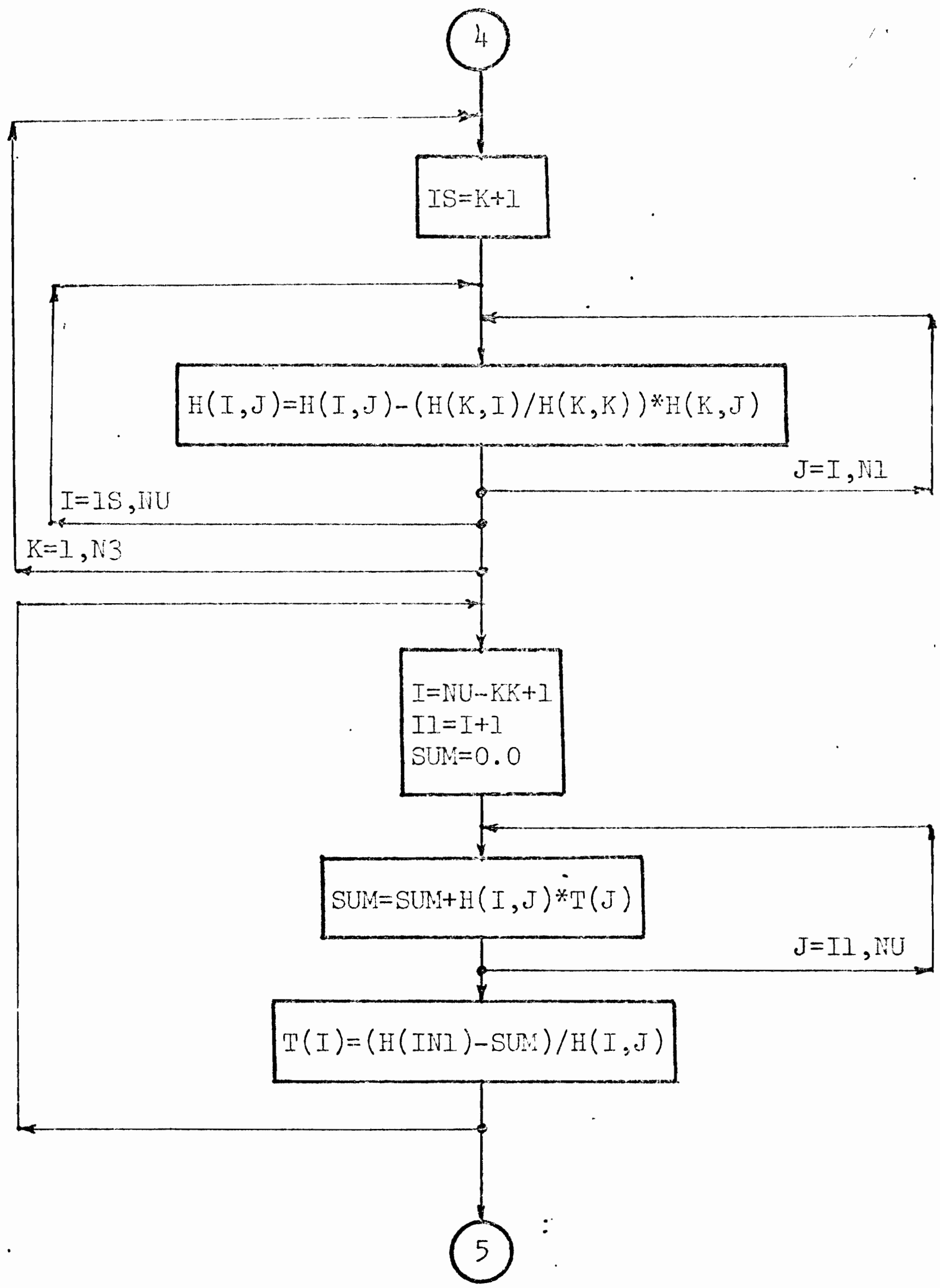
Flow chart


START
READ NU,NN,NHC,NMC,TOL
N1=NU+1
N2=NU+NN
N3=NU-1
S1=1.929012E-06
S2=0.333719E-14
S3=0.213481E-04
T(I)=0.0
T1(I)=0.0
H(I,J)=0.0
J=1,N1
I=1,NU
READ AK1(I),AK2(I),AK3(I)
I=1,NMC
AK1(I)=AK1(I)*S3
AK2(I)=AK2(I)*S3
AK3(I)=AK3(I)*S3
I=1,NMC
1

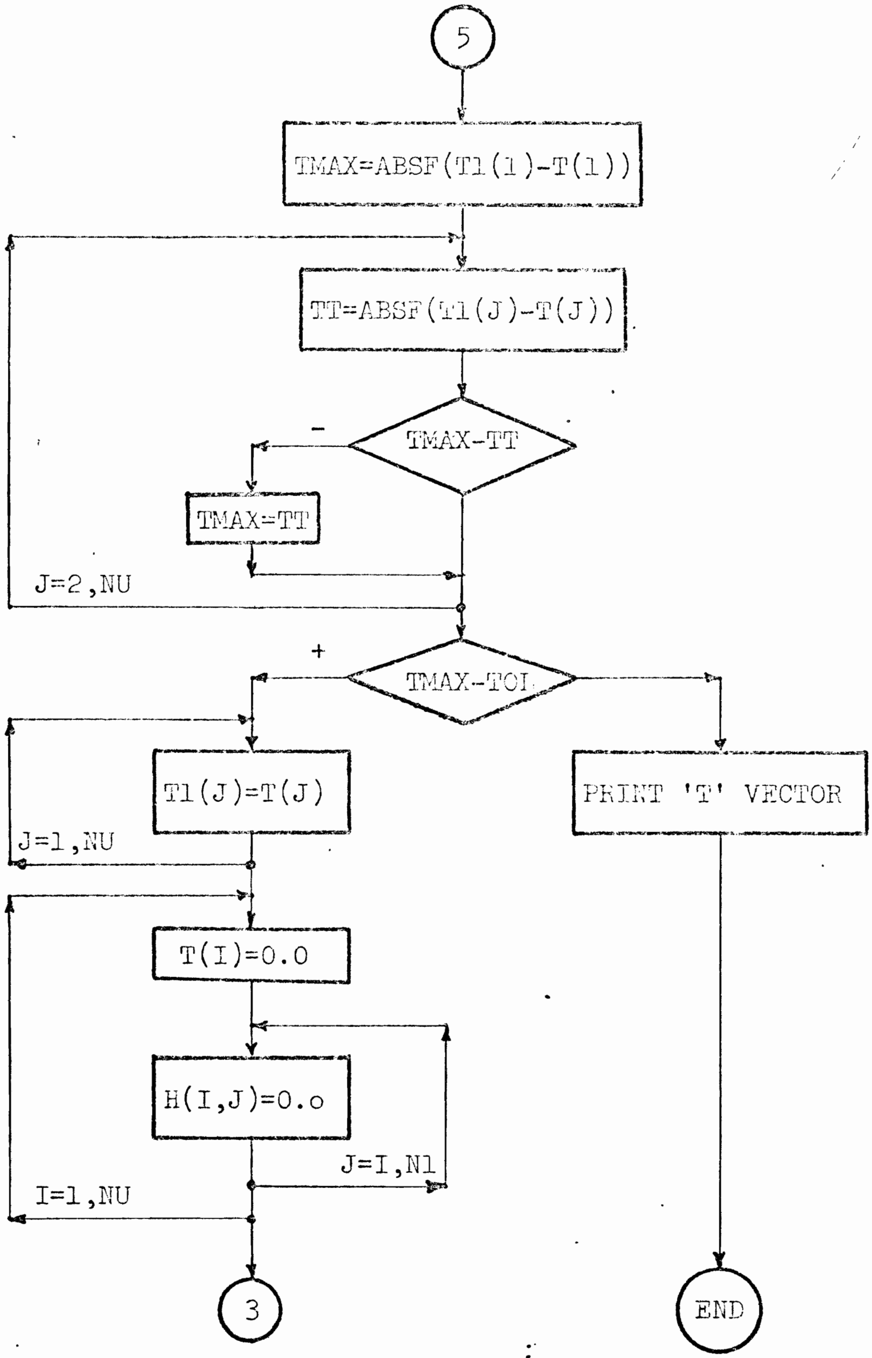

1
READ MC(I)
I=1,NU
READ T(I),MC(I)
T1(I) = T(I)
I=N1,N2
SUM=0.0
SUM=SUM+T(J)
J=N1,N2
AN=NN
TAV=SUM/AN
T1(J)=TAV
J=1,NU
N(I,J)=0.0
J=1,20
C(I,J)=0.0
J=1,40
I=1,NU
2

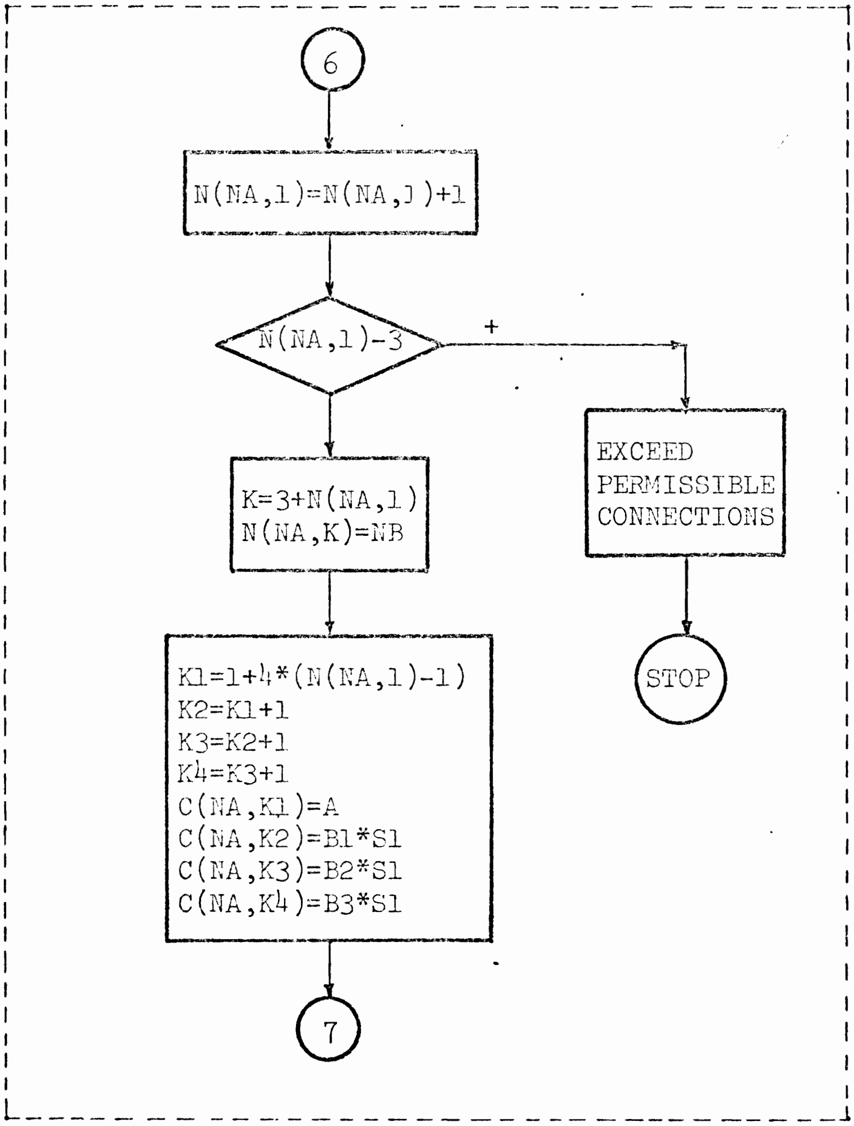

2
READ A,B1,B2,B3,NA,NB,M
LUX=1
NA-NB
+
0
-
NS=NB
NB=NA
NA=NS
ERROR
IN
DATA
END
NA-NU
+
M
1
2
3
6
7
8
9
1 0
1 1
LUX
2
1
NB-NU
+
KAR=1,NHC
3
LUX=2
NS=NB
NB=NA
NA=NS


3
K=N(I,1)
12
14
L=1,K
K=N(I,2)
15
16
L=1,K
K=N(I,3)
17
18
L=1,K
I=1,NU
4



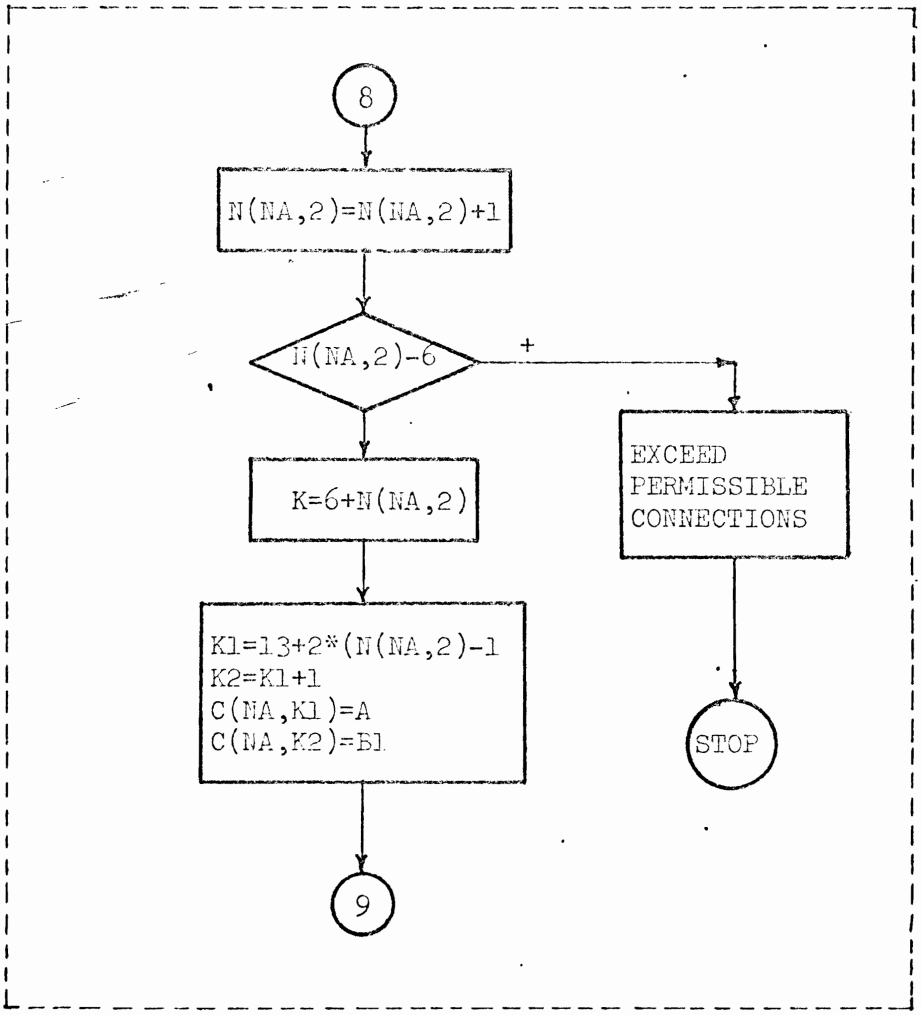
GAUSSIAN BACKWARDS SUBSTITUTION


5
TMAX=ABSF(T1(1)-T(1))
TT=ABSF(T1(J)-T(J))
TMAX-TT
-
TMAX=TT
J=2,NU
TMAX-TOL
+
T1(J)=T(J)
J=1,NU
PRINT 'T' VECTOR
T(I)=0.0
H(I,J)=0.o
J=I,N1
I=1,NU
3
END



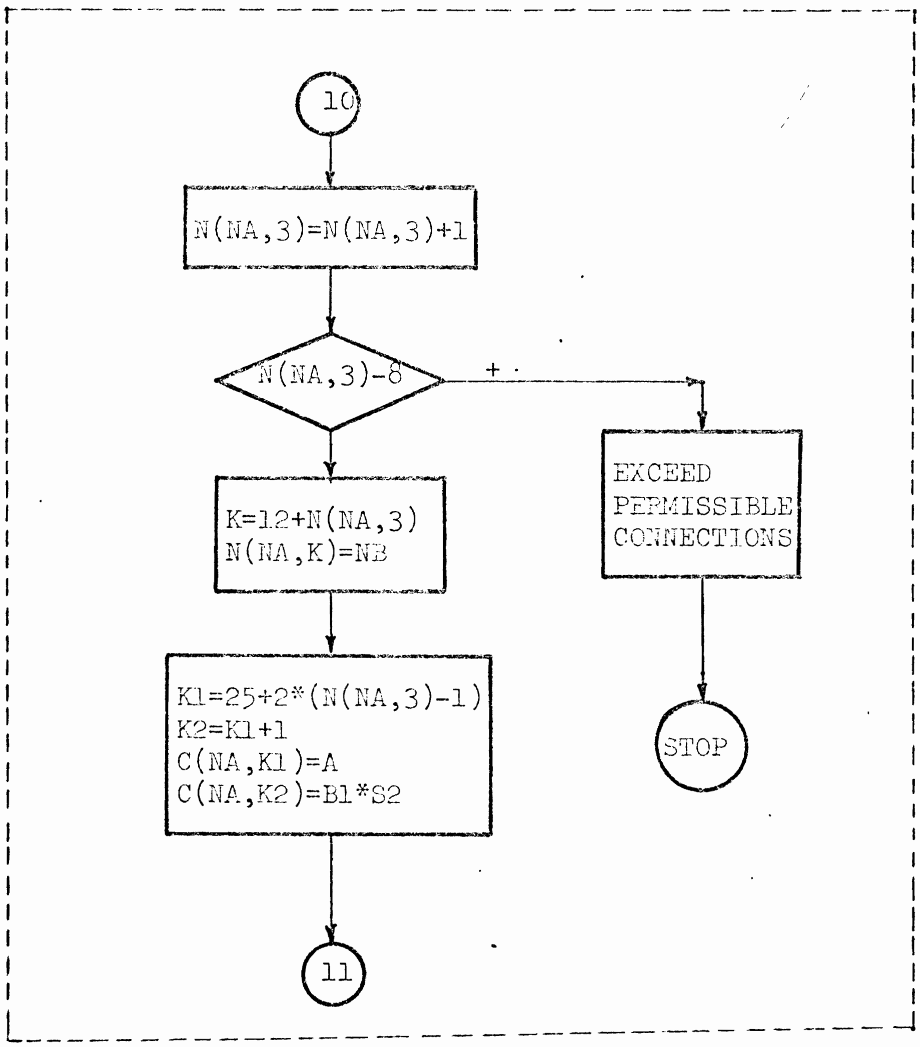
M = 1

CONVECTION.



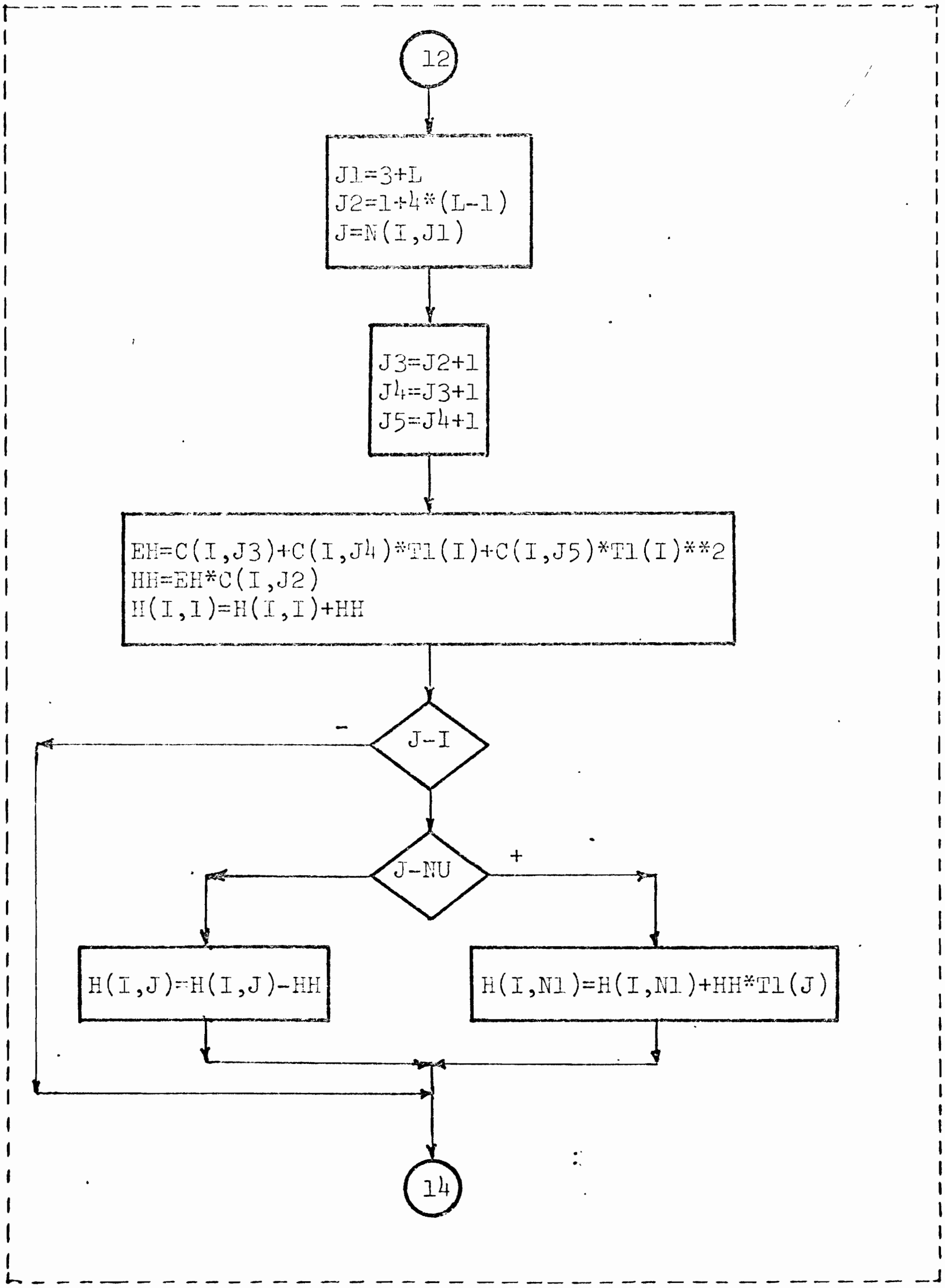
M = 2

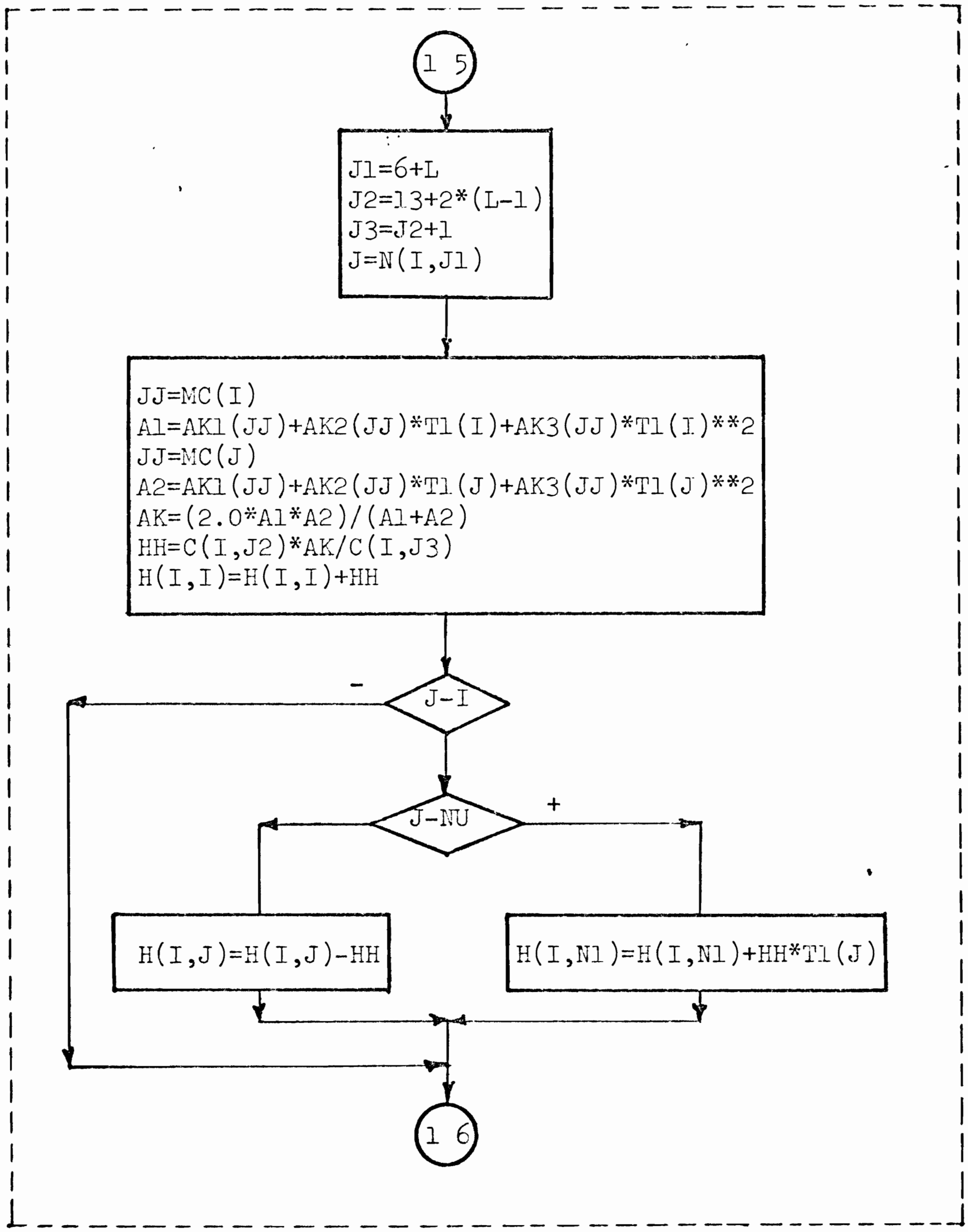
<u>CONDUCTION.</u>



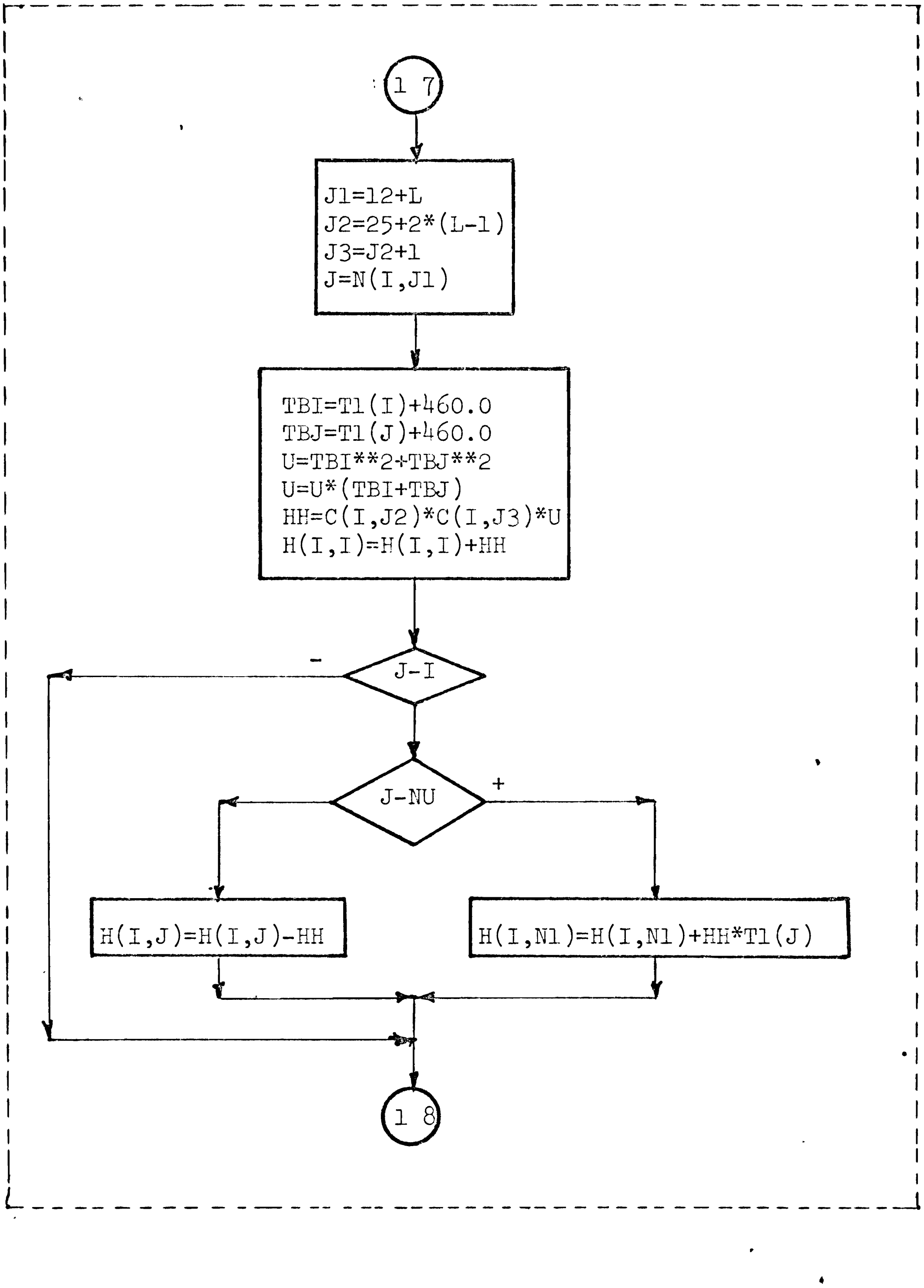
M = 3

RADIATION


12
J1=3+L
J2=1+4*(L-1)
J=N(I,J1)
J3=J2+1
J4=J3+1
J5=J4+1
EH=C(I,J3)+C(I,J4)*T1(I)+C(I,J5)*T1(I)**2
HH=EH*C(I,J2)
H(I,I)=H(I,I)+HH
J-I
-
J-NU
+
H(I,J)=H(I,J)-HH
H(I,N1)=H(I,N1)+HH*T1(J)
14



15
J1=6+L
J2=13+2*(L-1)
J3=J2+1
J=N(I,J1)
JJ=MC(I)
A1=AK1(JJ)+AK2(JJ)*T1(I)+AK3(JJ)*T1(I)**2
JJ=MC(J)
A2=AK1(JJ)+AK2(JJ)*T1(J)+AK3(JJ)*T1(J)**2
AK=(2.0*A1*A2)/(A1+A2)
HH=C(I,J2)*AK/C(I,J3)
H(I,I)=H(I,I)+HH
-
J-I
+
J-NU
H(I,J)=H(I,J)-HH
H(I,N1)=H(I,N1)+HH*T1(J)
16



17
J1=12+L
J2=25+2*(L-1)
J3=J2+1
J=N(I,J1)
TBI=T1(I)+460.0
TBJ=T1(J)+460.0
U=TBI**2+TBJ**2
U=U*(TBI+TBJ)
HH=C(I,J2)*C(I,J3)*U
H(I,I)=H(I,I)+HH
-
J-I
+
J-NU
H(I,J)=H(I,J)-HH
H(I,N1)=H(I,N1)+HH*T1(J)
18

```
FORTRAN (3.2)/MASTER

      PROGRAM ALLEGRIA
      DIMENSION T(101),T1(101),H(50,51),N(50,20),C(50,40)
      DIMENSION AC(101),AK1(5),AK2(5),AK3(5)
C
C     FORMAT STATEMENTS
C
500   FORMAT(4I5,F7.6)
501   FORMAT(3F10.2)
502   FORMAT(25I2)
503   FORMAT(F10.1,I5)
504   FORMAT(4F10.3,3I5)
550   FORMAT(1H1////////////////23X,
     150HA FINITE ELEMENT APPROACH FOR THE DETERMINATION OF/23X,
     210(5H-----)///23X,32HTHE  TEMPERATURE  DISTRIBUTION  ,
     318HIN A SINGLE  POINT/23X,10(5H-----)///27X,
     440HCUTTING TOOL (STEADY STATE TEMPERATURES)/27X,
     510(4H----)////////////////60X,10HA. MAZZAWI/60X,5(2H--)///60X,
     510HJAN.  1970/60X,5(2H--))
551   FORMAT(1H1///16X,10HINPUT DATA/16X,5(2H--)///)
552   FORMAT(///10X,34HTOTAL  NUMBER OF UNKNOWN  NODES  =,I5//10X,
     134HTOTAL  NUMBER  OF  KNOWN NODES    =,I5//10X,
     234HTOTAL NUMBER OF HEAT CONNECTIONS =,I5//10X,
     334HTOTAL  NUMBER OF MATERIAL  CODES =,I5//10X,
     434HTOLERANCE LIMIT                  =,F8.6)
553   FORMAT(///16X,22HTHERMAL CONDUCTIVITY  ,20H(BTU/HR*FT*DEGREE F),
     1/16X,14(3H---),///10X,8HMATERIAL/12X,4HCODE,12X,4H(K1),16X,
     24H(K2),16X,4H(K3)/)
554   FORMAT(/12X,I3,3X,3(5X,E15.5))
555   FORMAT(1H1///40X,15HDATA  OF  NODES/40X,5(3H===))
556   FORMAT(///10X,24HA) INITIAL  TEMPERATURES/10X,6(4H----))
557   FORMAT(/10X,7HUNKNOWN,10X,7HMACHINE,13X,5HKNOWN,11X,7HMACHINE)
558   FORMAT(11X,5HNODES,4X,4HTEMP,4X,5HCODES,14X,5HNODES,4X,
     14HTEMP,4X,5HCODES///)
559   FORMAT(12X,I3,4X,F5.1,6X,I1,17X,I3,4X,F5.1,6X,I1)
560   FORMAT(12X,I3,4X,F5.1,6X,I1)
561   FORMAT(1H1///10X,21HB)  HEAT  CONNECTIONS/10X,5(4H----))
562   FORMAT(///10X,44HA  = AREA IN SQUARE INCH FOR HEAT CONNECTION)
563   FORMAT(/10X,45HB1 = PATH LENGHT (INCH) IN CASE OF CONDUCTION/
     112X,41H = EMMISIVITY FACTOR IN CASE OF RADIATION)
564   FORMAT(/10X,48HB1,B2,B3 = HEAT TRANSFER CONVECTIVE COEFFICIENTS
565   FORMAT(/10X,32HNA = BASIC NODE NUMBER (UNKNOWN))
566   FORMAT(/10X,43HNB = NODE NUMBER TO WHICH NODE IS CONNECTED)
567   FORMAT(/10X,17HM   = 1,CONVECTION/12X,15H = 2,CONDUCTION/12X,
     112X,15H = 3, RADIATION)
568   FORMAT(///26X,1HA,11X,2HB1,11X,2HB2,11X,2HB3,4X,
     12HNA,3X,2HNB,3X,1HM//)
569   FORMAT(10X,I3,4F12.1,3I5)
570   FORMAT(///10X,19H***ERROR IN DATA***/10X,
     122HAT HEAT CONNECTION NO.,I3)
571   FORMAT(///10X,24H***ERROR INDATA***  M =,I3)
572   FORMAT(///10X,
     159H***YOU EXCEEDED NO. OF PERMISSIBLE CONNECTIONS FOR M =,I3)
573   FORMAT(1H1/////16X,36HREPRESENTATION  OF HEAT  CONNECTIONS/16X,
     112(3H---)/////10X,4HNO.,3X,10HCONVECTION,7X,10HCONDUCTION,16X,
```

```
         29HRADIATION/)
  574    FORMAT(/10X,I4,4X,3I3,3X,3I3,3X,6I3,3X,8I3)
  575    FORMAT(1H1///16X,16HSOLUTION S STEPS/16X,8(2H--),///10X,
        19HITERATION,5X,14HMAX. DEVIATION,5X,11HPERMISSIBLE/)
  576    FORMAT(/10X,I6,12X,F10.6,10X,F8.6)
  577    FORMAT(1H1///16X,8HSOLUTION/16X,8H≠≠≠≠≠≠≠≠///)
  578    FORMAT(/10X,I4,3X,F8.1,20X,I4,F8.1)
  579    FORMAT(/10X,I4,3X,F8.1)
  C
  C      INPUT DATA
  C
         READ(60,500) NU,NN,NHC,NMC,TOL
         WRITE(61,550)
         WRITE(61,551)
         WRITE(61,552) NU,NN,NHC,NMC,TOL
         NNN=400
         N1=NU+1
         N2=NU+NN
         N3=NU-1
         N4=(NU*(NU+3))/2
         S1=1.929012E-06
         S2=0.333719E-14
         S3=0.231481SE-04
         DO 1 I=1,NU
         T(I)=0.0
         T1(I)=0.0
         DO 1 J=1,N1
         H(I,J)=0.0
  1      CONTINUE
         WRITE(61,553)
         DO 2 I=1,NHC
         READ(60,501) AK1(I),AK2(I),AK3(I)
         WRITE(61,554) I,AK1(I),AK2(I),AK3(I)
  2      CONTINUE
         DO 3 I=1,NMC
         AK1(I)=AK1(I)*S3
         AK2(I)=AK2(I)*S3
         AK3(I)=AK3(I)*S3
  3      CONTINUE
         READ(60,502) (MC(I),I=1,25)
         READ(60,502) (MC(I),I=26,50)
         DO 4 I=N1,N2
         READ(60,503) T(I),MC(I)
         T1(I)=T(I)
  4      CONTINUE
         SUM=0.0
         DO 5 J=N1,N2
         SUM=SUM+T(J)
  5      CONTINUE
         AN= NN
         TAV=SUM/AN
         DO 6 J=1,NU
         T1(J)=TAV
  6      CONTINUE
```

```
      WRITE(61,555)
      WRITE(61,556)
      WRITE(61,557)
      WRITE(61,558)
      DO 9 I=1,NU
      II=NU+I
      IF(II-N2)7,7,8
7     WRITE(61,559) I,T1(I),MC(I),II,T1(II),MC(II)
      GO TO 9
8     WRITE(61,560) I,T1(I),MC(I)
9     CONTINUE
      DO 11 I=1,NU
      DO 10 J=1,20
      N(I,J)=0.0
10    CONTINUE
      DO 11 J=1,40
      C(I,J)=0.0
11    CONTINUE
      WRITE(61,561)
      WRITE(61,562)
      WRITE(61,563)
      WRITE(61,564)
      WRITE(61,565)
      WRITE(61,566)
      WRITE(61,567)
      WRITE(61,568)
      DO 36 KAR=1,NHC
      READ(60,504) A,B1,B2,B3,NA,NB,M
      WRITE(61,569) KAR,A,B1,B2,B3,NA,NB,M
      LUX=1
      IF(NA-NB)15,12,14
12    WRITE(61,570) KAR
      GO TO 999
14    NS=NB
      NB=NA
      NA=NS
15    CONTINUE
      IF(NA-NU)17,17,16
16    WRITE(61,570) KAR
      GO TO 999
17    CONTINUE
      IF(M-1)18,21,18
18    IF(M-2)19,24,19
19    IF(M-3)20.27,20
20    WRITE(61,571) M
      GO TO 999
21    N(NA,1)=N(NA,1)+1
      IF(N(NA,1)-3)23,23,22
22    WRITE(61,572) M
      GO TO 999
23    K=3+N(NA,1)
      N(NA,K)=NB
      K1=1+4*(N(NA,1)-1)
      K2=K1+1
```

FORTRAN (3.2)/MASTER

```
      K3=K2+1
      K4=K3+1
      C(NA,K1)=A
      C(NA,K2)=B1*S1
      C(NA,K3)=B2*S1
      C(NA,K4)=B3*S1
      GO TO 30
24    N(NA,2)=N(NA,2)+1
      IF(N(NA,2)-6)26,26,25
25    WRITE(61,572) M
      GO TO 999
26    K=6+N(NA,2)
      N(NA,K)=NB
      K1=12+2*(N(NA,2)-1)
      K2=K1+1
      C(NA,K1)=A
      C(NA,K2)=B1
      GO TO 30
27    N(NA,3)=N(NA,3)+1
      IF(N(NA,3)-8)29,29,28
28    WRITE(61,572) M
      GO TO 999
29    K=12+N(NA,3)
      N(NA,K)=NB
      K1=25+2*(N(NA,3)-1)
      K2=K1+1
      C(NA,K1)=A
      C(NA,K2)=B1*S2
30    CONTINUE
      IF(LUX-1)31,31,35
31    IF(NB-NU)32,32,35
32    LUX=2
      NS=NB
      NB=NA
      NA=NS
      GO TO 17
35    CONTINUE
36    CONTINUE
C
C     THE FORMATION OF THE H-MATRIX
C
      ITER=0
      WRITE(61,575)
51    CONTINUE
      DO 67 I=1,NU
      K=N(I,1)
      DO 56 L=1,K
      J1=3+L
      J2=1+4*(L-1)
      J=N(I,J1)
      J3=J2+1
      J4=J2+2
      J5=J2+3
      EM=C(I,J3)+C(I,J4)*T1(I)+C(I,J5)*T1(I)**2
```

FORTRAN (3.2)/MASTER

```
      HH=EH*C(I,J2)
      H(I,I)=H(I,I)+HH
      IF(J-I)55,52,52
52    IF(J-NU)54,54,53
53    H(I,N1)=H(I,N1)+HH*T1(J)
      GO TO 55
54    H(I,J)=H(I,J)-HH
55    CONTINUE
56    CONTINUE
      K=N(I,2)
      DO 61 L=1,K
      J1=6+L
      J2=13+2*(L-1)
      J=N(I,J1)
      J3=J2+1
      JJ=MC(I)
      A1=AK1(JJ)+AK2(JJ)*T1(I)+AK3(JJ)*T1(I)**2
      JJ=MC(J)
      A2=AK1(JJ)+AK2(JJ)*T1(J)+AK3(JJ)*T1(J)**2
      AK=(2.0*A1*A2)/(A1+A2)
      HH=C(I,J2)*AK/C(I,J3)
      H(I,I)=H(I,I)+HH
      IF(J-I)60,57,57
57    IF(J-NU)59,59,58
58    H(I,N1)=H(I,N1)+HH*T1(J)
      GO TO 60
59    H(I,J)=H(I,J)-HH
60    CONTINUE
61    CONTINUE
      K=N(I,3)
      DO 66 L=1,K
      J1=12+L
      J2=25+2*(L-1)
      J3=J2+1
      J=N(I,J1)
      TBI=T1(I)+460.0
      TBJ=T1(J)+460.0
      U=TBI**2+TBJ**2
      U=U*(TBI+TBJ)
      HH=C(I,J2)*C(I,J3)*U
      H(I,I)=H(I,I)+HH
      IF(J-I)65,62,62
62    IF(J-NU)64,64,63
63    H(I,N1)=H(I,N1)+HH*T1(J)
      GO TO 65
64    H(I,J)=H(I,J)-HH
65    CONTINUE
66    CONTINUE
67    CONTINUE
C
C     APPLYING THE GAUSSIAN ELEMINATION METHOD TO SOLVE THE H-MATRIX
C
      DO 81 K=1,N3
      IS=K+1
```

```
      DO 81 I=IS,NU
      DO 81 J=I,N1
      H(I,J)=H(I,J)-(H(K,I)/H(K,K))*H(K,J)
81    CONTINUE
      DO 83 KK=1,NU
      I=NU-KK+1
      I1=I+1
      SUM=0.0
      DO 82 J=I1,NU
      SUM=SUM+H(I,J)*T(J)
82    CONTINUE
      T(I)=(H(I,N1)-SUM)/H(I,I)
83    CONTINUE
C
C     ITTERATION PROCEEDURE FOR TOLERANCE JUSTIFICATION
C
      TM=T1(1)-T(1)
      TMAX=ABSF(TM)
      DO 92 J=2,NU
      TM=T1(J)-T(J)
      TT=ABSF(TM)
      IF(TMAX-TT)90,91,91
90    TMAX=TT
91    CONTINUE
92    CONTINUE
      IF(TMAX-TOL)96,96,93
93    DO 94 J=1,NU
      T1(J)=T(J)
94    CONTINUE
      DO 95 I=1,NU
      T(I)=0.0
      DO 95 J=I,N1
      H(I,J)=0.0
95    CONTINUE
      ITER=ITER+1
      WRITE(61,576) ITER,TMAX,TOL
      GO TO 51
C
C     OUTPUT
C
96    WRITE(61,576) ITER,TMAX,TOL
      WRITE(61,577)
      DO 99 I=1,NU
      II=I+NU
      IF(II-N2)97,97,98
97    WRITE(61,578) I,T(I),II,T(II)
      GO TO 99
98    WRITE(61,579) I,T(I)
99    CONTINUE
999   END
```

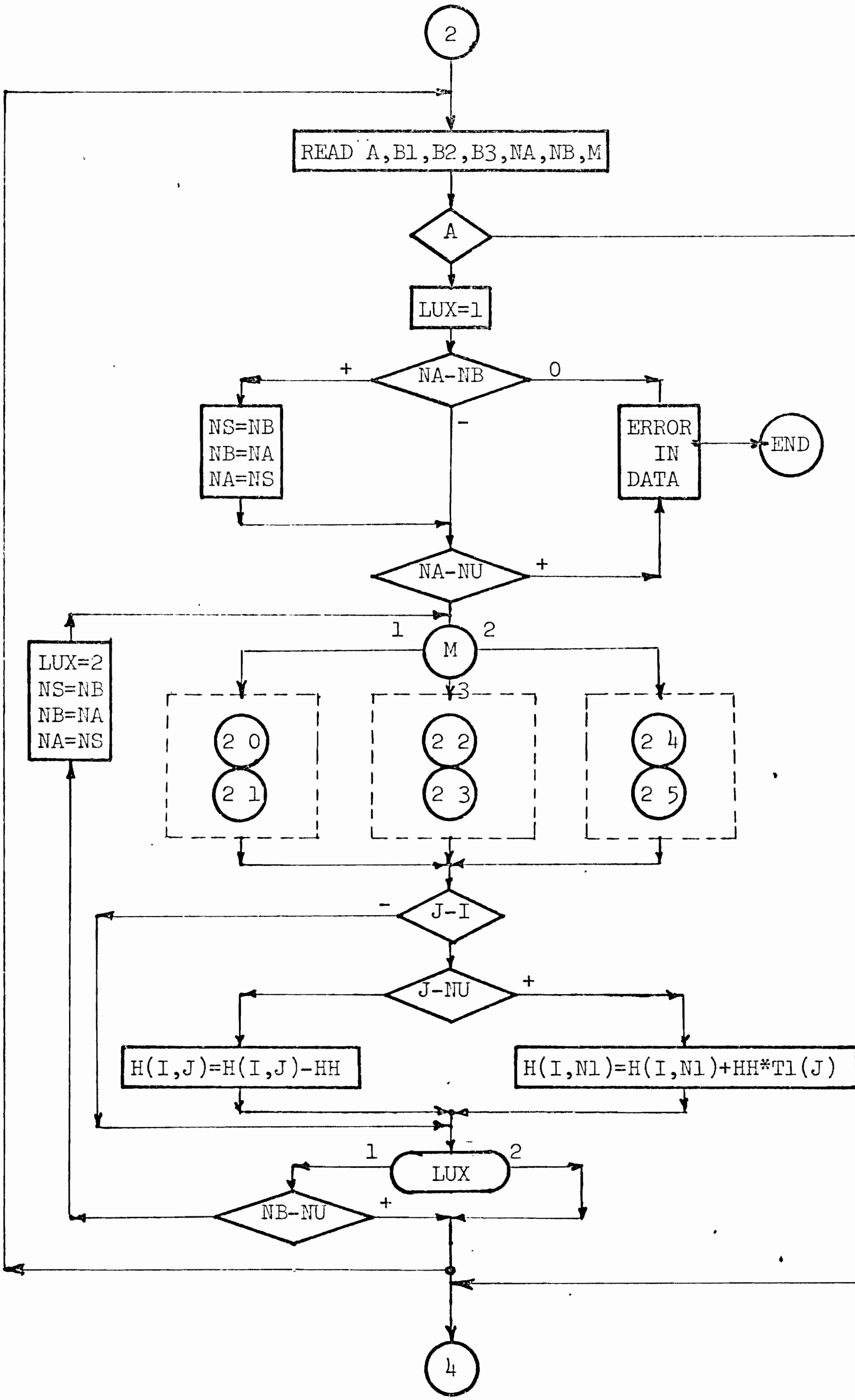
PROGRAM VARIABLES

25745	A	01674	H	26006	JJ
26007	A1	26003	HH	25763	K
26011	A2	25720	I	25764	K1
26015	AK	26031	I1	25765	K2
25625	AK1	25740	II	25766	K3

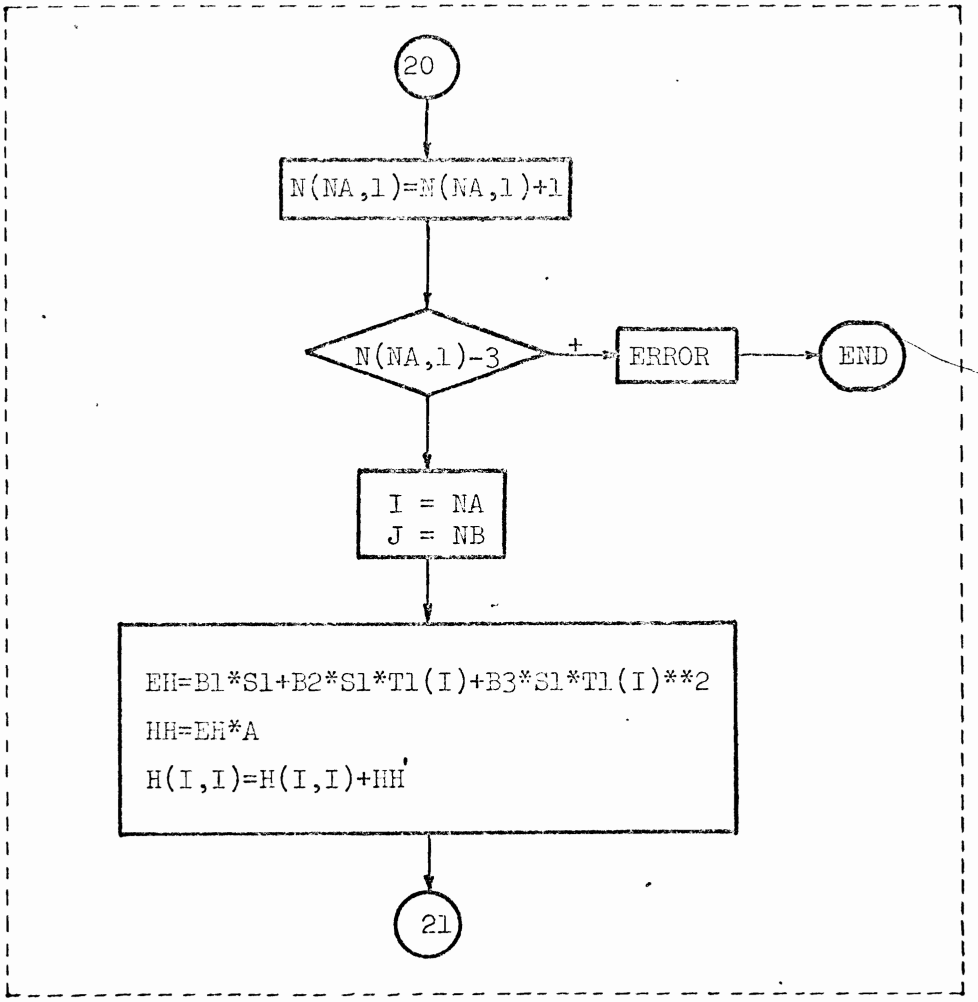
APPENDIX IV

MODIFICATION OF THE PROGRAM

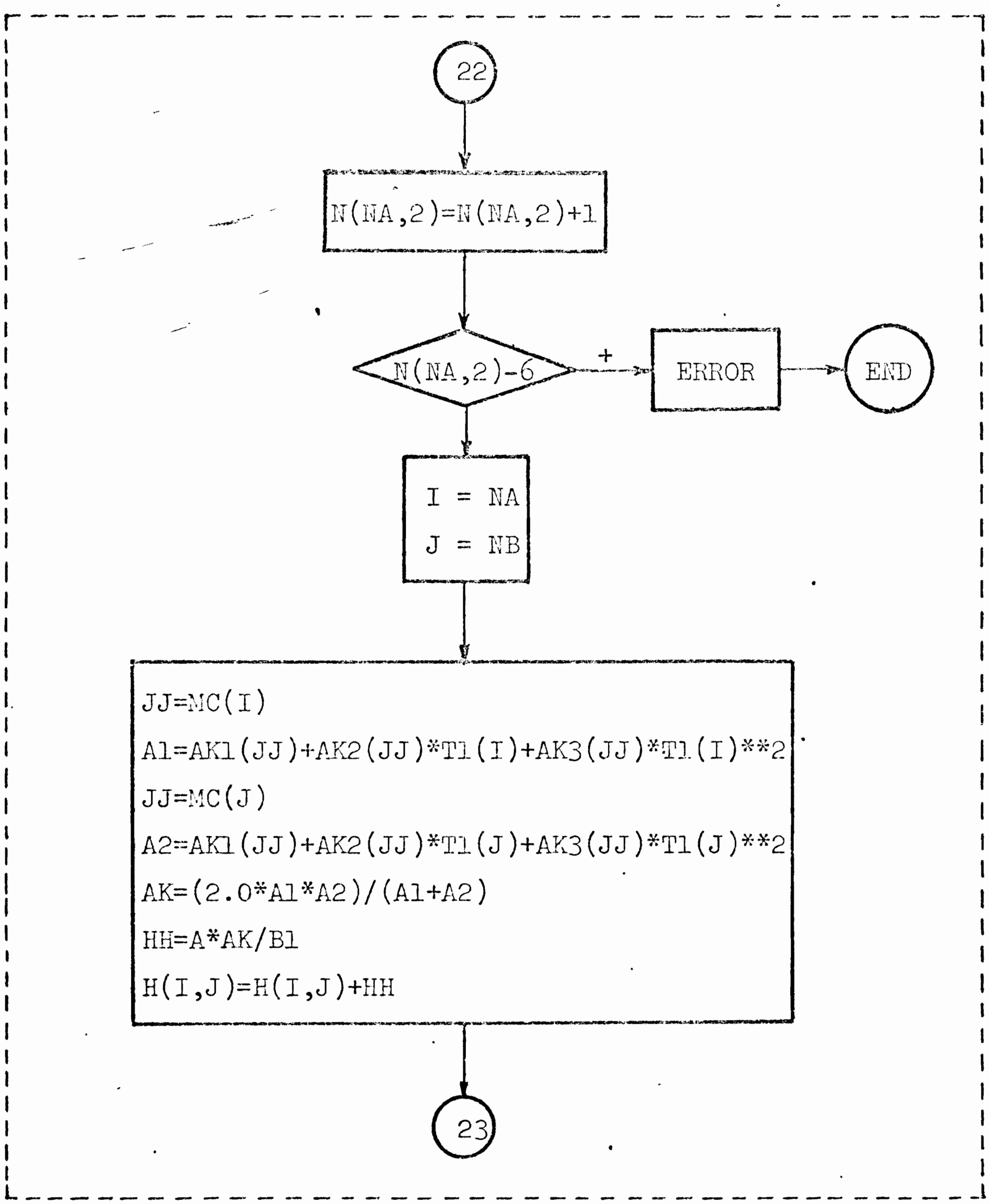
Contents | Page



2
READ A,B1,B2,B3,NA,NB,M
A
LUX=1
NA-NB
+
0
-
NS=NB
NB=NA
NA=NS
ERROR IN DATA
END
NA-NU
+
LUX=2
NS=NB
NB=NA
NA=NS
M
1
2
3
2 0
2 1
2 2
2 3
2 4
2 5
J-I
-
J-NU
+
H(I,J)=H(I,J)-HH
H(I,N1)=H(I,N1)+HH*T1(J)
LUX
1
2
NB-NU
+
4

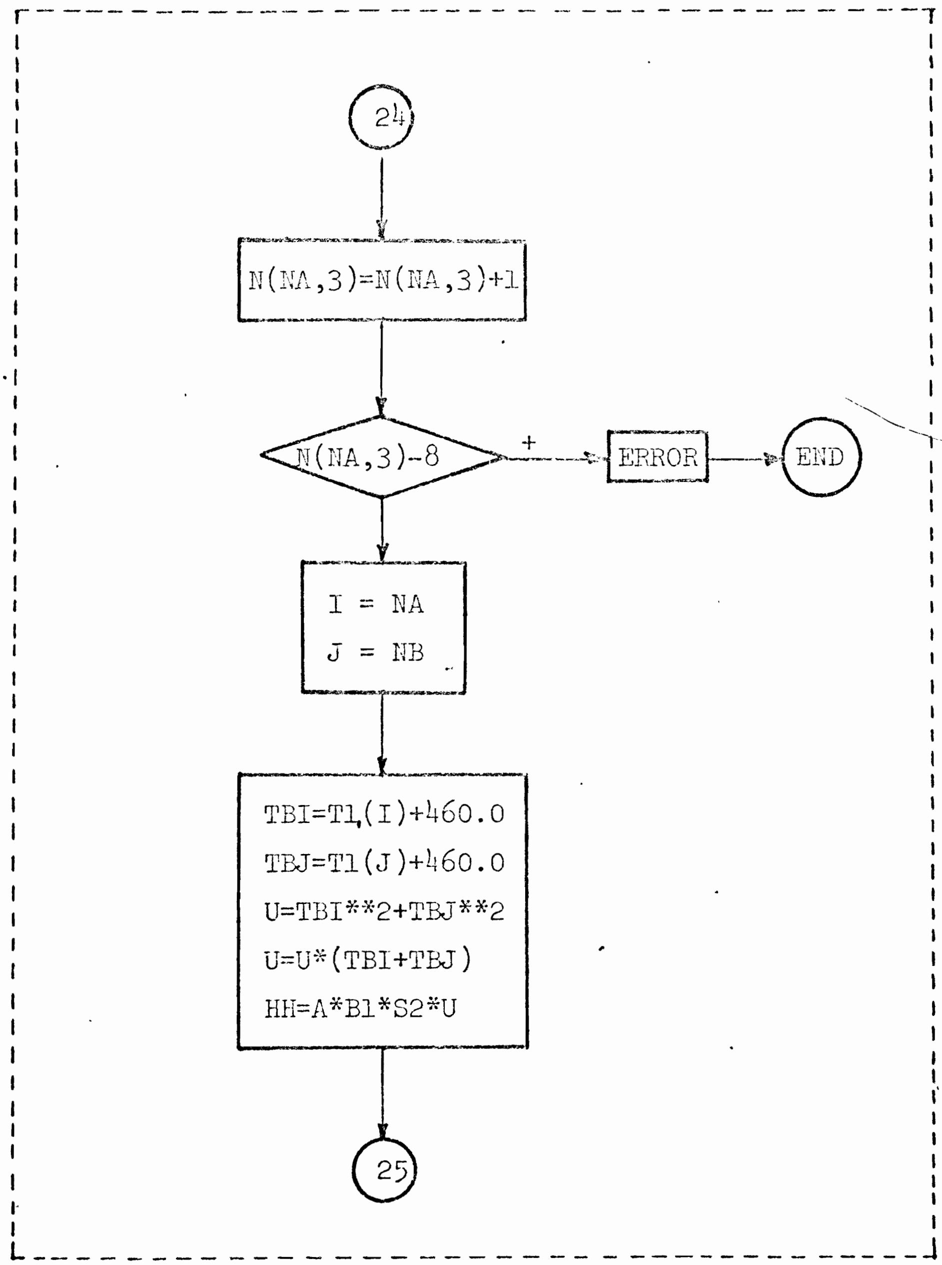


M = 1
CONVECTION



M = 2

CONDUCTION



M = 3

RADIATION

```
FORTRAN (3.2)/MASTER

      PROGRAM ALLEGRIA    (Main program)
      DIMENSION T(101),T1(101),H(1230),N(50,3)
      DIMENSION MC(101),AK1(5),AK2(5),AK3(5)
      DIMENSION NSEG(20),NLAST(20)
      DIMENSION NACCSEG(20),NACCLST(20)
      DIMENSION XROW(101)
C
C     FORMAT STATEMENTS
C
500   FORMAT(4I5,F7.6)
501   FORMAT(3F10.2)
502   FORMAT(25I2)
503   FORMAT(F10.1,I5)
504   FORMAT(F10.4,3F10.3,3I5)
550   FORMAT(1H1////////////////23X,
     150HA FINITE ELEMENT APPROACH FOR THE DETERMINATION OF/23X,
     210(5H-----)///23X,32HTHE   TEMPERATURE   DISTRIBUTION   ,
     316HIN A SINGLE  POINT/23X,10(5H-----)///27X,
     440HCUTTING TOOL (STEADY STATE TEMPERATURES)/27X,
     510(4H----)////////////////60X,10HA. MAZZAWI/60X,5(2H--)///60X,
     610HJAN. 1970/60X,5(2H--))
551   FORMAT(1H1///16X,10HINPUT DATA/16X,5(2H--)///)
552   FORMAT(///10X,34HTOTAL  NUMBER OF UNKNOWN  NODES  =,I5//10X,
     134HTOTAL  NUMBER  OF  KNOWN NODES    =,I5//10X,
     234HTOTAL NUMBER OF HEAT CONNECTIONS =,I5//10X,
     334HTOTAL  NUMBER OF MATERIAL  CODES =,I5//10X,
     434HTOLERANCE LIMIT              .      =,F8.6)
553   FORMAT(///16X,22HTHERMAL CONDUCTIVITY  ,20H(BTU/HR*FT*DEGREE F),
     1/16X,14(3H---),///10X,8HMATERIAL/12X,4HCODE,12X,4H(K1),16X,
     24H(K2),16X,4H(K3)/)
554   FORMAT(/12X,I3,3X,3(5X,F15.5))
555   FORMAT(1H1////[illegible]X,15HDATA  OF   NODES/[illegible]X,5(3H---))
556   FORMAT(///10X,24HA) INITIAL  TEMPERATURES/10X,6(4H----))
557   FORMAT(/1[illegible]X,[illegible] [illegible]CHINE)
558   FORMAT(11X,5HNODES,4X,4HTEMP,5X,5HCODES,14X,5HNODES,4X,
     14HTEMP,4X,5HCODES///)
559   FORMAT(12X,I3,[illegible]X,F5.1,6X,I1,1[illegible]X,I3,4X,F5.1,6X,I1)
560   FORMAT(12X,I3,4X,F5.1,6X,I1)
561   FORMAT(1H1///1[illegible]X,21HC)  HEAT  CONNECTIONS/10X,5(4H----))
562   FORMAT(///10X,[illegible]H A  = AREA IN SQUARE INCH FOR HEAT CONNECTION)
563   FORMAT(/10X,[illegible]H L = PATH LENGHT (INCH) IN CASE OF CONDUCTION/
     112X,41H = EMISSIVITY FACTOR IN CASE OF RADIATION)
564   FORMAT(/10X,40HHB1,B2,[illegible] = HEAT TRANSFER CONVECTIVE COEFFICIENTS )
565   FORMAT(/10X,32HMA = BASIC CODE NUMBER (UNKNOWN))
566   FORMAT(/10X,[illegible]HNB = NODE NUMBER TO WHICH NODE IS CONNECTED)
567   FORMAT(/10X,17HM   = 1,CONVECTION/12X,1[illegible]H = 2,CONDUCTION/12X,
     11H  = 3, RADIATION)
568   FORMAT(///20X,1HA,11X,2HB1,11X,2HB2,11X,2HB3,4X,
     12HMA,3X,2HNB,3X,1HM//)
569   FORMAT(10X,I3,4F12.4,3I5)
570   FORMAT(///10X,19H***ERROR IN DATA***/10X,
     122HAT HEAT CONNECTION NO.,I3)
571   FORMAT(///10X,24H***ERROR INDATA***   M =,I3)
572   FORMAT(///10X,
```

```
        154H***YOU EXCEEDED NO. OF PERMISSIBLE CONNECTIONS FOR M =,I3)
 573     FORMAT(1H1/////16X,36HREPRESENTATION  OF HEAT  CONNECTIONS/16X,
        112(3H---)/////10X,4HU.N.,3X,10HCONVECTION,7X,10HCONDUCTION,16X,
        29HRADIATION/)
 574     FORMAT(/10X,I4,4X,3I3,3X,3I3,3X,6I3,3X,8I3)
 575     FORMAT(1H1///16X,16HSOLUTION S STEPS/16X,8(2H--),///10X,
        19HITERATION,5X,14HMAX. DEVIATION,5X,11HPERMISSIBLE/)
 576     FORMAT(/10X,I6,12X,F10.6,10X,F8.6)
 577     FORMAT(1H1///16X,8HSOLUTION/16X,8H≠≠≠≠≠≠≠≠///)
 578     FORMAT(/10X,I4,3X,F8.1,20X,I4,F8.1)
 579     FORMAT(/10X,I4,3X,F8.1)
 580     FORMAT(1H1///10X,16HB) PARTITIONING/10X,4(4H----)///10X,
        17HSEG NO.,5X,7HROW/SEG,5X,7HACC.ROW,4X,9HLAST WORD,4X,
        28HACC.LAST///)
 581     FORMAT(10X,I5,4I12)
 582     FORMAT(///6X,10H***FAULT =,3I6)
 C
 C       INPUT DATA
 C
         READ(60,500) NU,NN,NHC,NMC,TOL
         WRITE(61,550)
         WRITE(61,551)
         WRITE(61,552) NU,NN,NHC,NMC,TOL
         NNN=400
         N1=NU+1
         N2=NU+NN
         N3=NU-1
         N4=(NU*(NU+3))/2
         S1=1.929512E-06
         S2=0.333719E-14
         S3=0.2314615E-04
         DO 1 I=1,NU
         T(I)=0.0
         T1(I)=0.0
 1       CONTINUE
         DO 101 J=1,N4
         H(J)=0.0
 101     CONTINUE
         WRITE(61,553)
         DO 2 I=1,NHC
         READ(60,501) AK1(I),AK2(I),AK3(I)
         WRITE(61,554) I,AK1(I),AK2(I),AK3(I)
 2       CONTINUE
         DO 3 I=1,NHC
         AK1(I)=AK1(I)*S3
         AK2(I)=AK2(I)*S3
         AK3(I)=AK3(I)*S3
 3       CONTINUE
         READ(60,502) (MC(I),I=1,25)
         READ(60,502) (MC(I),I=26,50)
         DO 4 I=N1,N2
         READ(60,503) T(I),MC(I)
         T1(I)=T(I)
 4       CONTINUE
```

FORTRAN (3.2)/MASTER

```
      SUM=0.0
      DO 5 J=N1,N2
      SUM=SUM+T(J)
 5    CONTINUE
      AN=NN
      TAV=SUM/AN
      DO 6 J=1,NU
      T1(J)=TAV
 6    CONTINUE
      WRITE(61,555)
      WRITE(61,556)
      WRITE(61,557)
      WRITE(61,558)
      DO 9 I=1,NU
      II=NU+I
      IF(II-N2)7,7,8
 7    WRITE(61,559) I,T1(I),MC(I),II,T1(II),MC(II)
      GO TO 9
 8    WRITE(61,560) I,T1(I),MC(I)
 9    CONTINUE
C
C     PARTITIONING
C
      K=1
      KK=0
      NN=0
      DO 203 I=1,NU
      J=I+1
      NN=NN+J
      IF(NG-NN)201,201,200
200   NN=NN-J
      NSEG(K)=KK
      NLAST(K)=NN
      NN=J
      K=K+1
      KK=0
201   CONTINUE
      KK=KK+1
      IF(I-NU)203,202,203
202   NSEG(K)=KK
      NLAST(K)=NN
      NPART=K
203   CONTINUE
      NP=NPART/2
      DO 204 I=1,NP
      J=NPART-I+1
      NN=NLAST(I)
      NLAST(I)=NLAST(J)
      NLAST(J)=NN
      NN=NSEG(I)
      NSEG(I)=NSEG(J)
      NSEG(J)=NN
204   CONTINUE
      IOO=0
```

```
      IG=0
      DO 205 I=1,NPART
      IQQ=IQQ+NSEG(I)
      IQ=IQ+NLAST(I)
      NACCSEG(I)=IQQ
      NACCLST(I)=IQ
205   CONTINUE
      WRITE(61,560)
      DO 206 I=1,NPART
      WRITE(61,561) I,NSEG(I),NACCSEG(I),NLAST(I),NACCLST(I)
206   CONTINUE
C
C     PREPARATION OF THE H-MATRIX
C
      WRITE(61,561)
      WRITE(61,562)
      WRITE(61,563)
      WRITE(61,564)
      WRITE(61,565)
      WRITE(61,566)
      WRITE(61,567)
      WRITE(61,568)
      ITER=0
      REWIND 10
      REWIND 11
10    CONTINUE
      DO 39 KAR=1,NHC
      IF(ITER)111,110,111
110   READ(60,564) A,B1,B2,B3,NA,NB,M
      WRITE(61,569) KAR,A,B1,B2,B3,NA,NB,M
      WRITE(10,564) A,B1,B2,B3,NA,NB,M
      GO TO 11
111   READ(10,564) A,B1,B2,B3,NA,NB,M
11    CONTINUE
      LUX=1
      IF(NA-NB)15,12,14
12    WRITE(61,570) KAR
      GO TO 999
14    NS=NB
      NB=NA
      NA=NS
15    CONTINUE
      IF(NA-NU)17,17,16
16    WRITE(61,570) KAR
      GO TO 999
17    CONTINUE
      IF(M-1)18,21,18
18    IF(M-2)19,24,19
19    IF(M-3)20,27,20
20    WRITE(61,571) M
      GO TO 999
21    N(NA,1)=N(NA,1)+1
      IF(N(NA,1)-3)23,23,22
22    WRITE(61,572) M
```

```
      GO TO 999
23    I=NA
      J=NB
      EH=B1*S1+B2*S1*T1(I)+B3*S1*T1(I)**2
      HH=EH*A
      LOC=N1*(I-1)-(I*(I-1))/2
      III=LOC+I
      VALUE=HH
      WRITE(11) III,VALUE
      GO TO 30
24    N(NA,2)=N(NA,2)+1
      IF(N(NA,2)-6)26,26,25
25    WRITE(61,572) M
      GO TO 999
26    I=NA
      J=NB
      JJ=MC(I)
      A1=AK1(JJ)+AK2(JJ)*T1(I)+AK3(JJ)*T1(I)**2
      JJ=MC(J)
      A2=AK1(JJ)+AK2(JJ)*T1(J)+AK3(JJ)*T1(J)**2
      AK=(2.0*A1*A2)/(A1+A2)
      HH=A*AK/B1
      LOC=N1*(I-1)-(I*(I-1))/2
      III=LOC+I
      VALUE=HH
      WRITE(11) III,VALUE
      GO TO 30
27    N(NA,3)=N(NA,3)+1
      IF(N(NA,3)-8)29,29,28
28    WRITE(61,572) M
      GO TO 999
29    I=NA
      J=NB
      TBI=T1(I)+460.0
      TBJ=T1(J)+460.0
      U=TBI**2+TBJ**2
      U=U*(TBI+TBJ)
      HH=A*B1*S2*U
      LOC=N1*(I-1)-(I*(I-1))/2
      III=LOC+I
      VALUE=HH
      WRITE(11) III,VALUE
30    CONTINUE
      IF(J-I)35,32,32
32    IF(J-NU)34,34,33
33    III=LOC+N1
      VALUE=HH*T1(J)
      WRITE(11) III,VALUE
      GO TO 35
34    III=LOC+J
      VALUE=-HH
      WRITE(11) III,VALUE
35    CONTINUE
      IF(LUX-1)36,36,38
```

```
36      IF(NB-NU)37,37,38
37      LUX=2
        NS=NB
        NB=NA
        NA=NS
        GO TO 17
38      CONTINUE
39      CONTINUE
        REWIND 10
        REWIND 11
        IZERO=0
        IONE=1
        LU=30
        DO 61 K=1,NPART
        DO 49 I=1,NNN
        H(I)=0.0
49      CONTINUE
        IQQ=NLAST(K)
        IP=NACCLST(K)-IQQ+1
        IQ=NACCLST(K)
        LOC=IQ-IQQ
54      READ(11) III,VALUE
        IF(EOFCKF(11)-2)59,55,50
55      IF(III-IP)58,57,56
56      IF(III-IQ)57,57,58
57      III=III-LOC
        H(III)=H(III)+VALUE
58      CONTINUE
        GO TO 54
59      CONTINUE
        REWIND 11
        J=LOCATEF(LU,K,IZERO)
        GO TO (51,449) J
449     IFAULT=101
300     WRITE(61,582) IFAULT,J,K
50      CALL ABNORMAL
51      BUFFER OUT (LU,IONE)(H(1),H(NNN))
301     J=UNITSTF(LU)
        IF(J-2)301,53,52
52      IFAULT=102
        GO TO 300
53      CONTINUE
61      CONTINUE
        DO 666 K=1,N3
        L=1
62      IF(L-NPART)63,162,162
63      IQQ=NACCSEG(L)
        IF(K-IQQ)65,65,64
64      L=L+1
        GO TO 62
65      GO TO (67,66) LOCATEF(LU,L,IZERO)
66      IFAULT=66
        GO TO 300
67      BUFFER IN (LU,IONE)(H(1),H(NNN))
```

```
      DO 68 I=1,N1
      XROW(I)=0.0
68    CONTINUE
      LOC=NACCLST(L)-NLAST(L)
      IQ=N1*(K-1)-(K*(K-1))/2
      ISS=NACCSEG(L)
      IS=K+1
69    IF(UNITSTF(LU)-2)69,71,70
70    IFAULT=70
      GO TO 300
71    CONTINUE
      DO 72 I=K,N1
      IN=IQ+I-LOC
      XROW(I)=H(IN)
72    CONTINUE
      DO 73 I=IS,ISS
      LOCI=N1*(I-1)-(I*(I-1))/2
      DO 73 JJ=I,N1
      IJ=LOCI+JJ-LOC
      H(IJ)=H(IJ)-(XROW(I)/XROW(K))*XROW(JJ)
73    CONTINUE
      GO TO (75,74) LOCATEF(LU,L,IZERO)
74    IFAULT=74
      GO TO 300
75    BUFFER OUT (LU,IONE)(H(1),H(NNN))
76    IF(UNITSTF(LU)-2)76,77,74
77    CONTINUE
      L=L+1
160   IF(L-NPART)78,78,161
78    GO TO (80,79) LOCATEF(LU,L,IZERO)
79    IFAULT=79
      GO TO 300
80    BUFFER IN (LU,IONE)(H(1),H(NNN))
      ISS=NSEG(L).
      IS=NACCSEG(L)
      LOC=NACCLST(L)-NLAST(L)
150   IF(UNITSTF(LU)-2)150,152,151
151   IFAULT=151
      GO TO 300
152   CONTINUE
      DO 153 II=1,ISS
      I=IS-ISS+II
      LOCI=N1*(I-1)-(I*(I-1))/2
      DO 153 JJ=I,N1
      IJ=LOCI+JJ-LOC
      H(IJ)=H(IJ)-(XROW(I)/XROW(K))*XROW(JJ)
153   CONTINUE
      GO TO (155,154) LOCATEF(LU,L,IZERO)
154   IFAULT=154
      GO TO 300
155   BUFFER OUT (LU,IONE)(H(1),H(NNN))
156   IF(UNITSTF(LU)-2)156,157,154
157   CONTINUE
      L=L+1
```

```
      GO TO 160
161   CONTINUE
666   CONTINUE
162   CONTINUE
      GO TO (164,163) LOCATEF(LU,NPART,IZERO)
163   IFAULT=163
      GO TO 300
164   BUFFER IN (LU,IONE)(H(1),H(NNN))
      LOC=NACCLST(NPART)-NLAST(NPART)
      IQ=K
165   IF(UNITSTF(LU)-2)165,167,166
166   IFAULT=166
      GO TO 300
167   CONTINUE
      DO 81 KK=IQ,N3
      LOCK=N1*(KK-1)-(KK*(KK-1))/2
      IS=KK+1
      DO 81 I=IS,NU
      LOCI=N1*(I-1)-(I*(I-1))/2
      DO 81 J=I,N1
      KI=LOCK+I-LOC
      KKK=LOCK+KK-LOC
      KJ=LOCK+J-LOC
      IJ=LOCI+J-LOC
      H(IJ)=H(IJ)-(H(KI)/H(KKK))*H(KJ)
81    CONTINUE
      GO TO (169,168) LOCATEF(LU,NPART,IZERO)
168   IFAULT=168
      GO TO 300
169   BUFFER OUT (LU,IONE)(H(1),H(NNN))
170   IF(UNITSTF(LU)-2)170,172,171
171   IFAULT=171
      GO TO 300
172   CONTINUE
      DO 89 K=1,NPART
      KKK=NPART-K+1
      IQQ=NLAST(KKK)
      IQ=NACCLST(KKK)
      LOC=IQ-IQQ
      GO TO (85,50) LOCATEF(LU,KKK,IZERO)
85    BUFFER IN (LU,IONE)(H(1),H(NNN))
86    IF(UNITSTF(LU)-2)86,87,50
87    CONTINUE
      IS=NSEG(KKK)
      DO 89 I=1,IS
      II=NACCSEG(KKK)-I+1
      LOCI=N1*(II-1)-(II*(II-1))/2
      I1=II+1
      SUM=0.0
      DO 88 J=I1,NU
      IJ=LOCI+J-LOC
      SUM=SUM+H(IJ)*T(J)
88    CONTINUE
      IJ=LOCI+N1-LOC
```

```
      KI=LOCI+II-LOC
      T(II)=(H(IJ)-SUM)/H(KI)
39    CONTINUE
C
C     ITERATION PROCEEDURE FOR TOLERANCE JUSTIFICATION
C
      TM=T1(1)-T(1)
      TMAX=ABSF(TM)
      DO 92 J=2,NU
      TM=T1(J)-T(J)
      TT=ABSF(TM)
      IF(TMAX-TT)90,91,91
90    TMAX=TT
91    CONTINUE
92    CONTINUE
      IF(TMAX-TOL)96,96,93
93    DO 94 J=1,NU
      T1(J)=T(J)
94    CONTINUE
      DO 95 I=1,NU
      T(I)=0.0
95    CONTIUUE
      DO 995 J=1,N4
      H(J)=0.0
995   CONTINUE
      ITER=ITER+1
      WRITE(61,576) ITER,TMAX,TOL
      DO 956 I=1,NU
      DO 956 J=1,3
      N(I,J)=0
956   CONTINUE
      GO TO 10
C
C     OUTPUT
C
96    WRITE(61,576) ITER,TMAX,TOL
      WRITE(61,577)
      DO 99 I=1,NU
      II=I+NU
      IF(II-N2)97,97,98
97    WRITE(61,578) I,T(I),II,T(II)
      GO TO 99
98    WRITE(61,579) I,T(I)
99    CONTINUE
999   END
```

PROGRAM VARIABLES

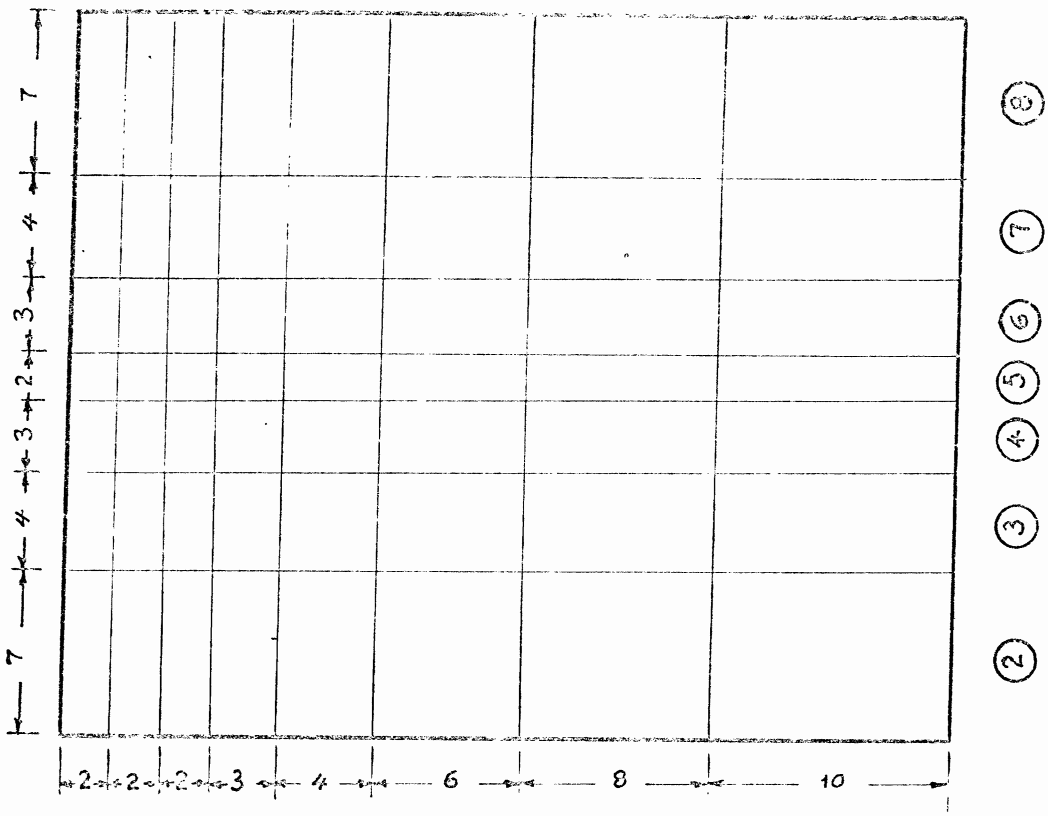
07755	A	10032	IFAULT	07755	KAR	077
10005	A1	07741	II	10053	KI	07
10007	A2	10000	III	10055	KJ	077
10013	AK	10043	IJ	07744	KK	073
07175	AK1	10041	IN	10054	KKK	077
07207	AK2	10026	IONE	10034	L	077
07221	AK3	10030	IP	07777	LOC	076
07734	AN	07751	IQ	10042	LOCI	072
07760	B1	07750	IQQ	10052	LOCK	076
07762	B2	10037	IS	10027	LU	074

APPENDIX V

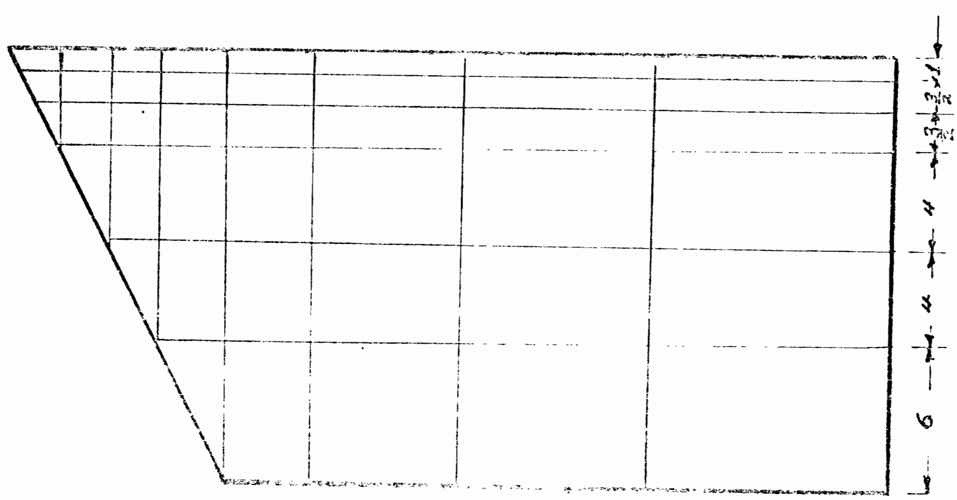
IMPLIMENTATION TO THE TOOL-CHIP-WORKPIECE SYSTEM

Contents Page

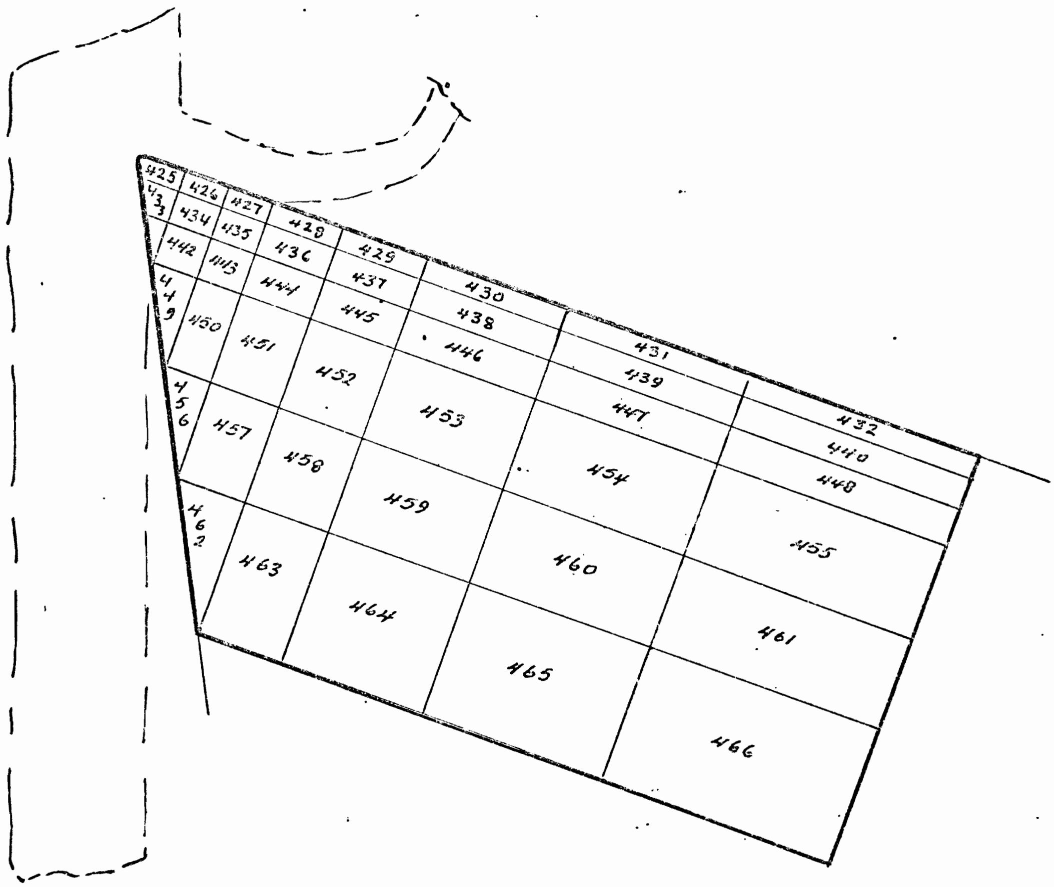
DIMENSION (× 1/64 inches)



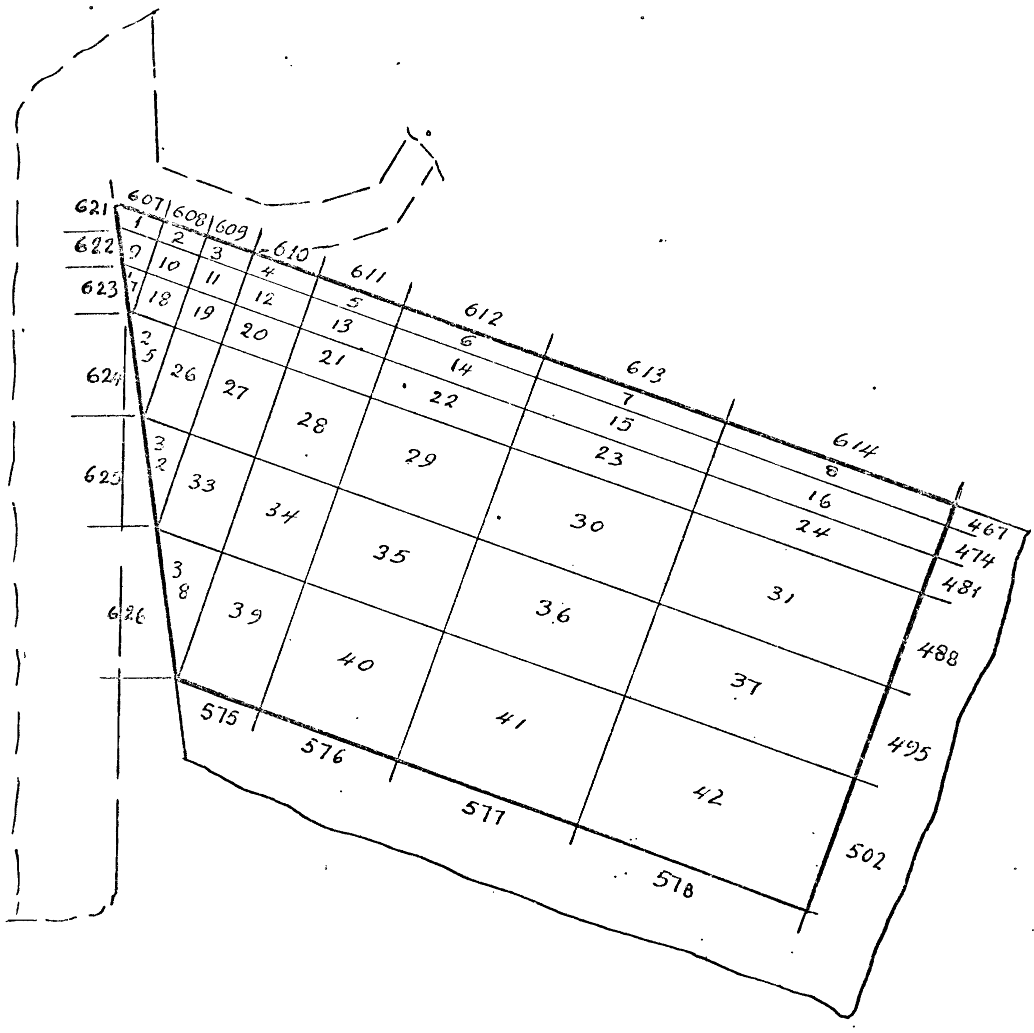
PLAN



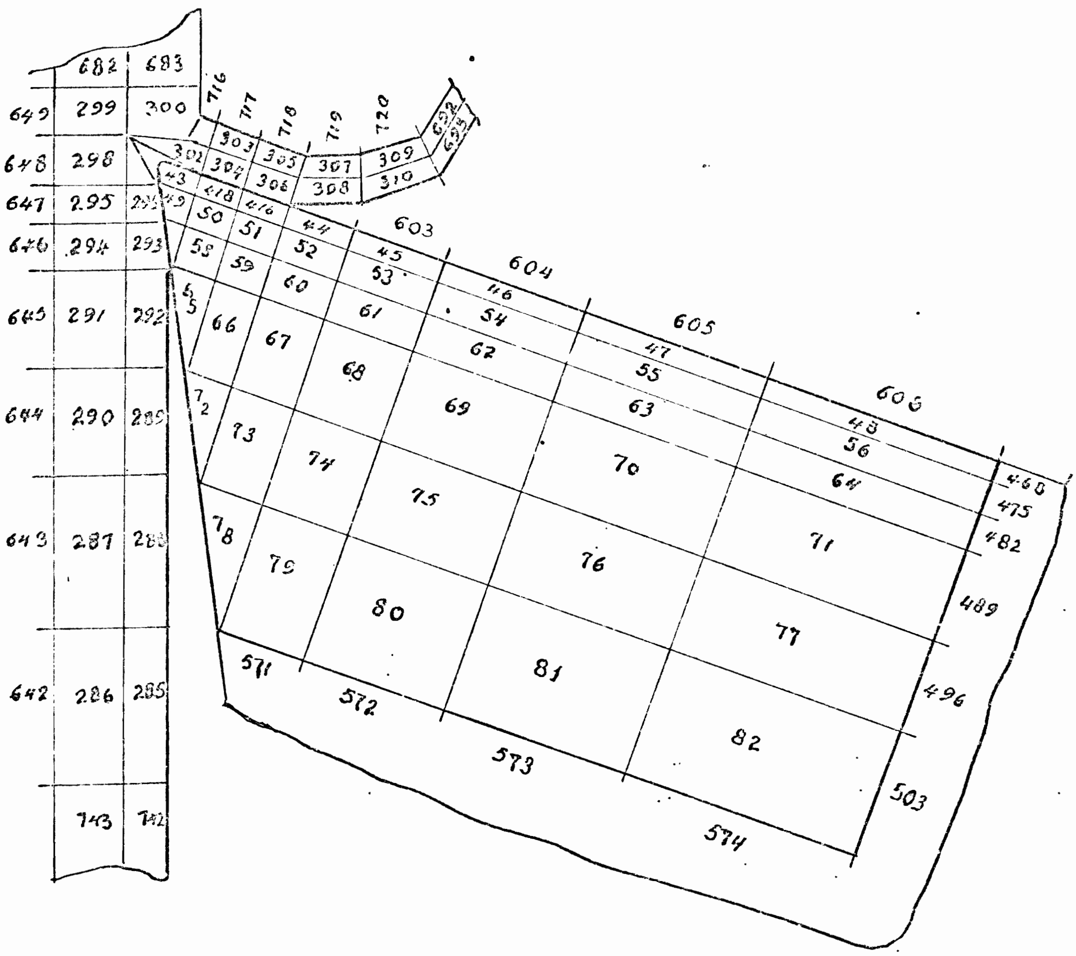
SIDE VIEW



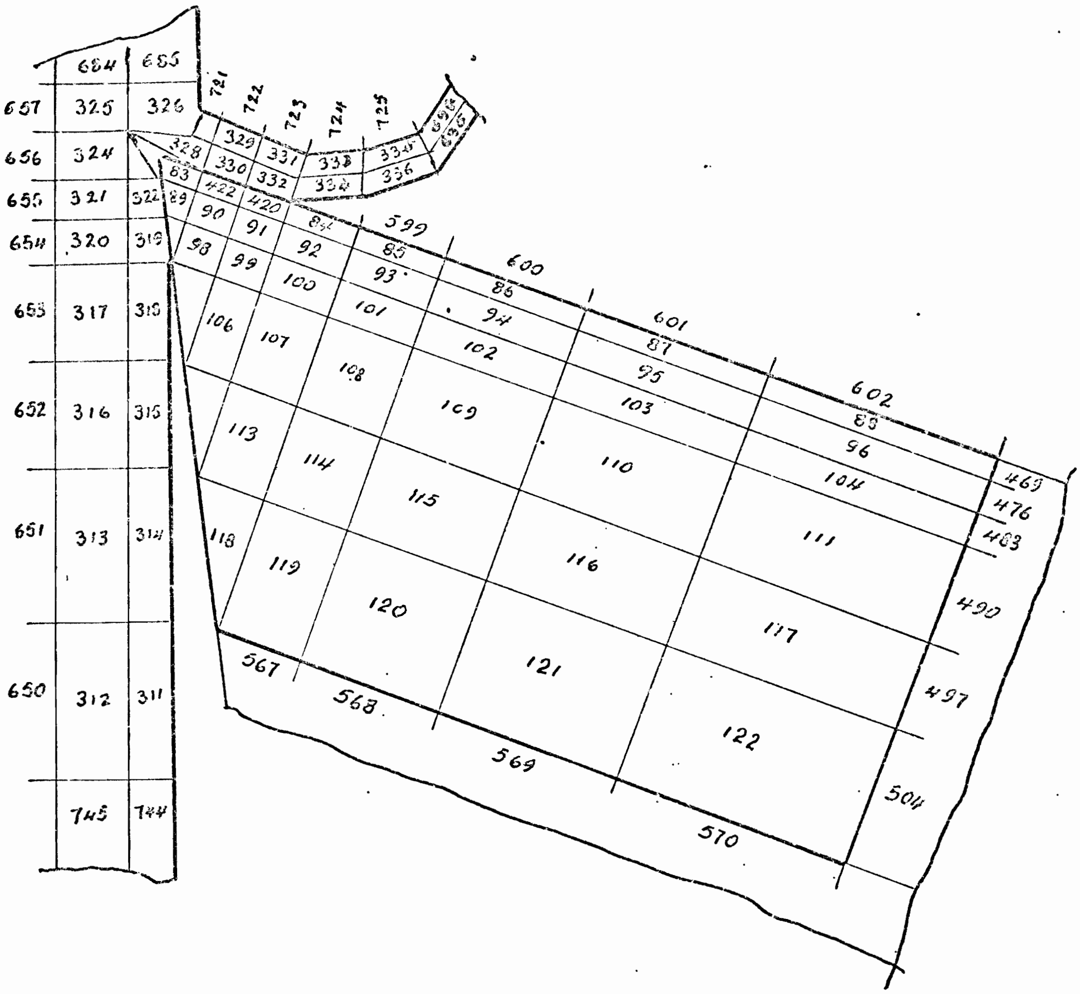
Numbering of nodes -- Layer no.1 -- Air



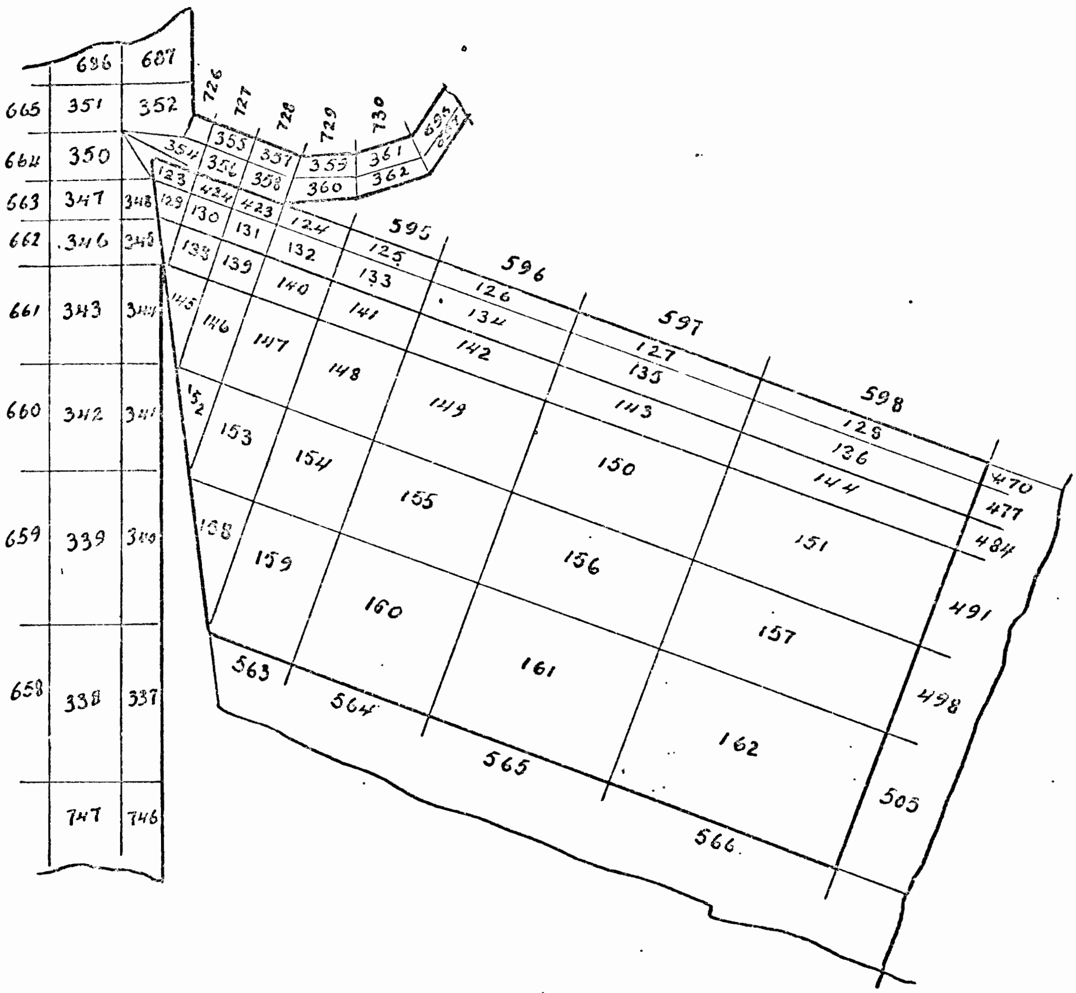
Numbering of nodes -- Layer no.2 -- Thickness 7/64 "



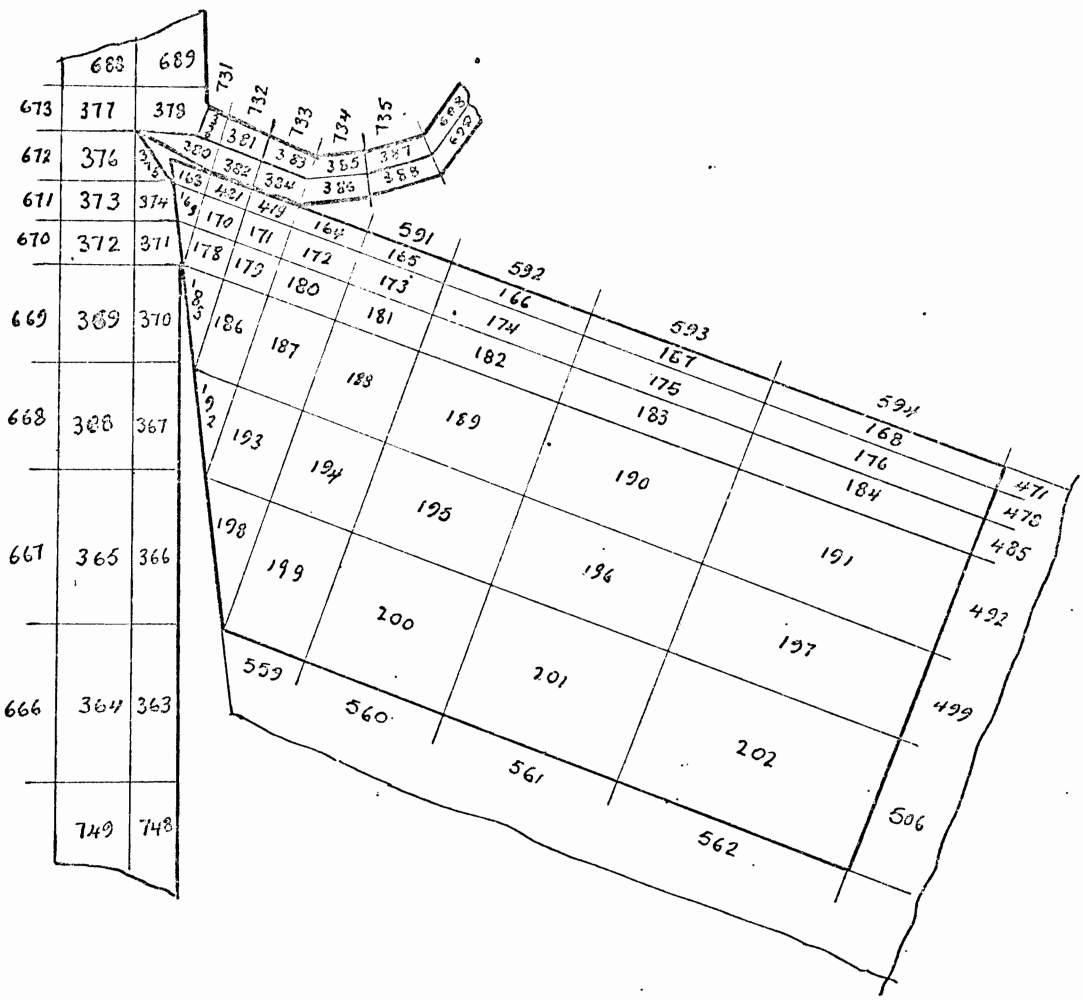
Numbering of nodes -- Layer no.3 -- Thickness 4/64



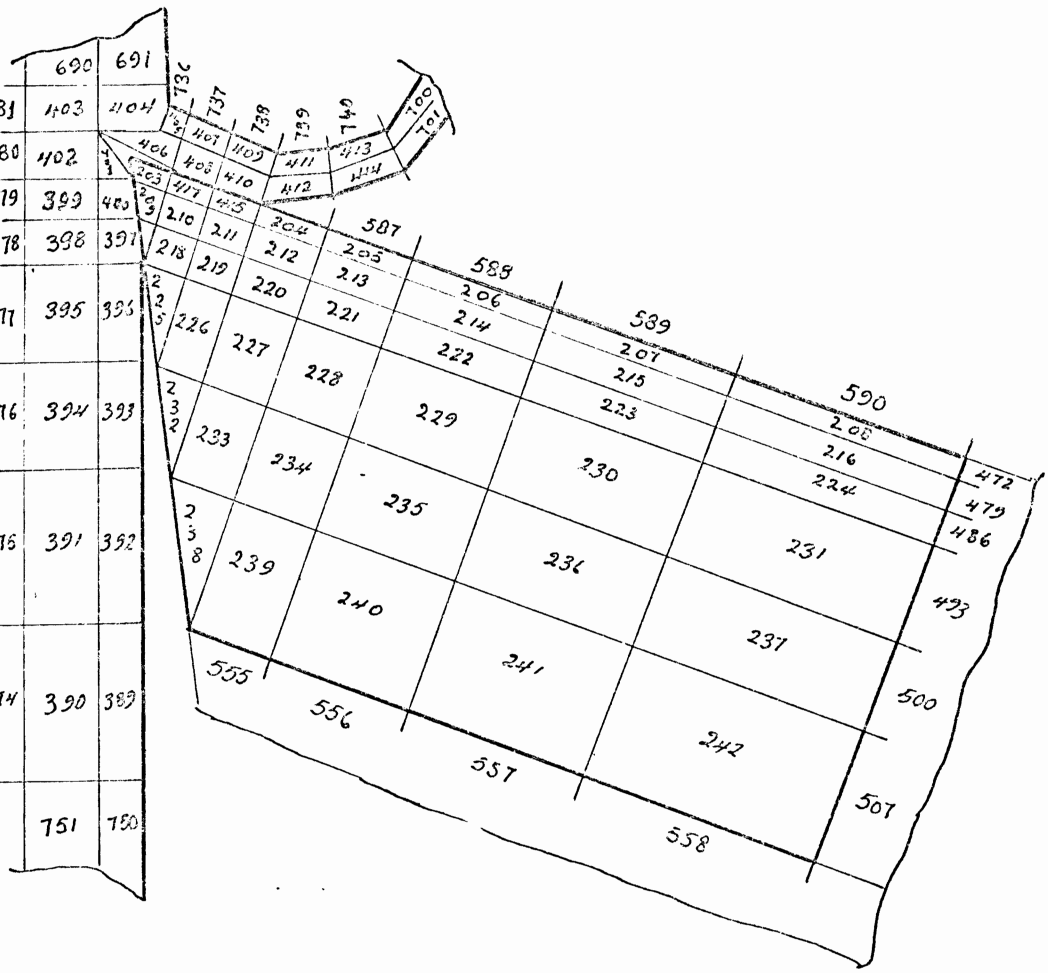
Numbering of nodes -- Layer no. 4 -- Thickness 3/64"



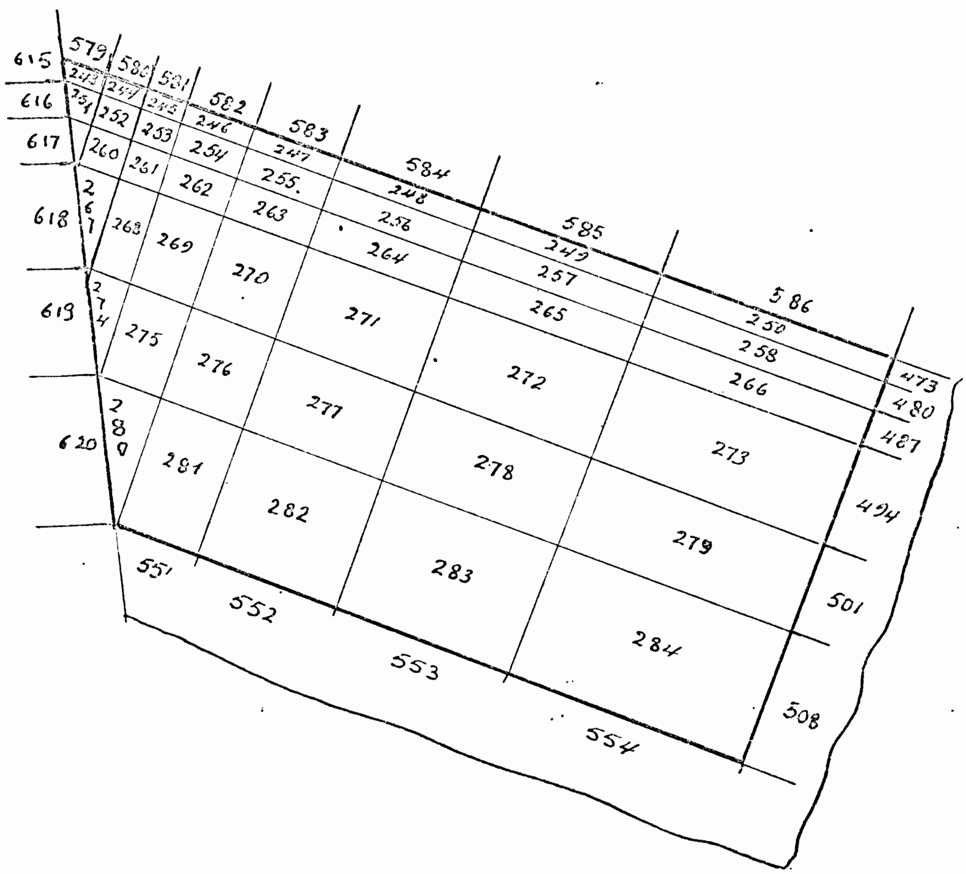
Numbering of nodes --- Layer no. 5 -- Thickness 2/64"



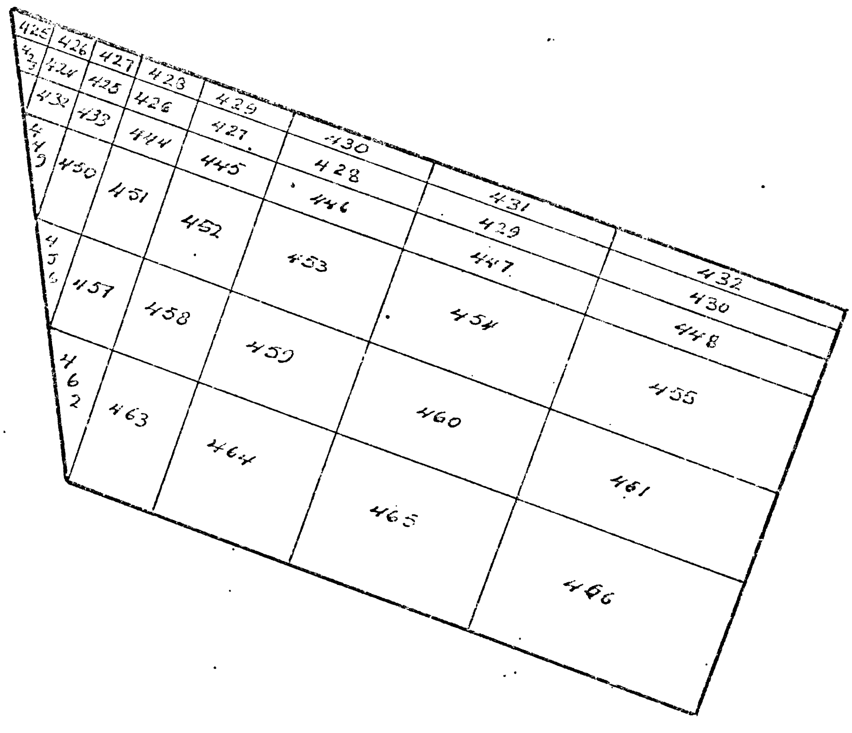
Numbering of nodes -- Layer no. 6 -- Thickness 3/64"



Numbering of nodes --- Layer no.7 --- Thickness 4/64"

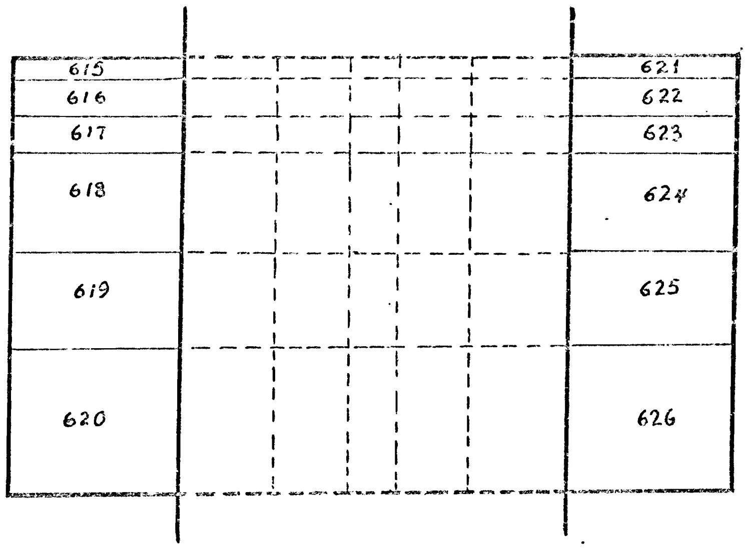


Numbering of nodes -- Layer no.8 -- Thickness 7/64"

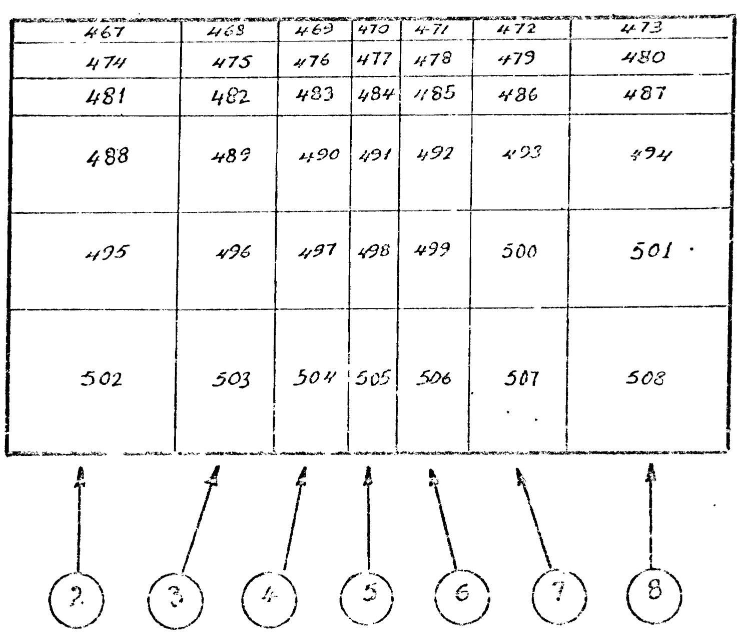


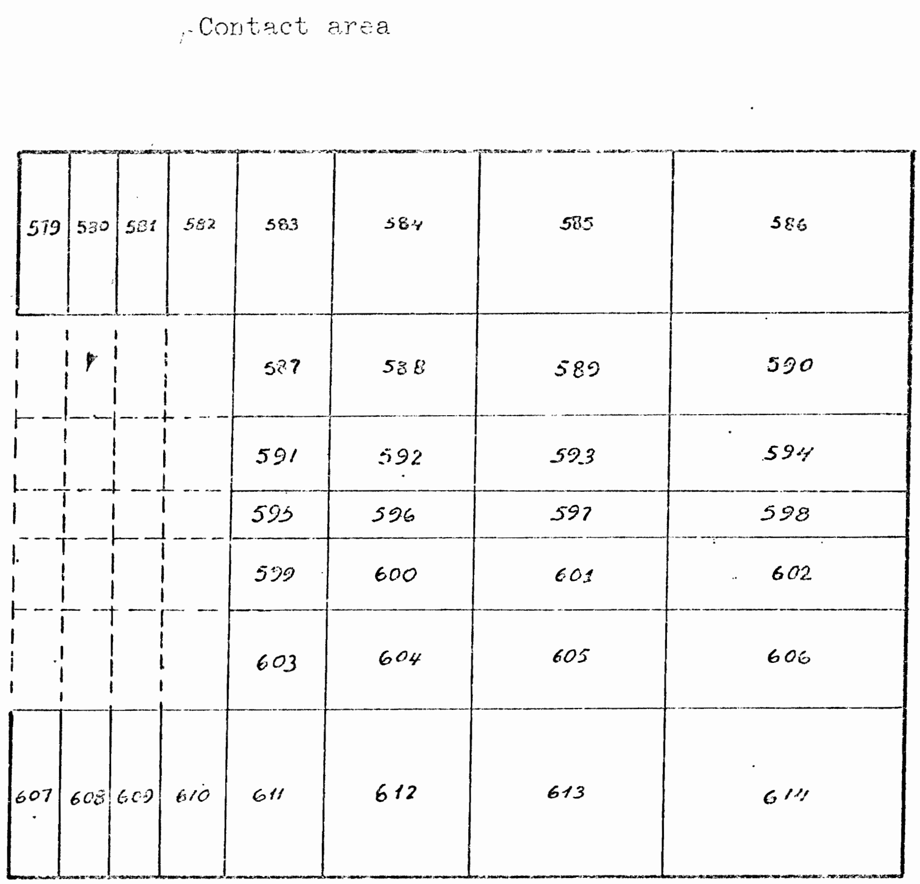
Numbering of nodes -- Layer no.9 - Air

Numbering of nodes -- Front view



Numbering of nodes -- Back view





Numbering of nodes -- Top view

551	552	553	554	(8)
555	556	557	558	(7)
559	560	561	562	(6)
563	564	565	566	(5)
567	568	569	570	(4)
571	572	573	574	(3)
575	576	577	578	(2)

numbering of nodes -- (bottom view)

INPUT DATA (only a sample is given)

TOTAL NUMBER OF UNKNOWN NODES = 423

TOTAL NUMBER OF KNOWN NODES = 328

TOTAL NUMBER OF HEAT CONNECTIONS = 1462

TOTAL NUMBER OF MATERIAL CODES = 5

TOLERANCE LIMIT = .001000

1	.0700	.2000	0	0	1	2	2
2	.0175	.5500	0	0	1	43	2
3	.1050	.1300	0	0	1	9	2
4	.0175	0	0	0	1	425	1
5	.0784	0	0	0	1	621	1
6	.1400	0	0	0	1	607	1
7	.0700	.2000	0	0	2	3	2
8	.0200	.5500	0	0	2	418	2
9	.1400	.1250	0	0	2	10	2
10	.0200	0	0	0	2	426	1
11	.1400	0	0	0	2	608	1
12	.0700	.2500	0	0	3	4	2
13	.0200	.5500	0	0	3	416	2
14	.1400	.1250	0	0	3	11	2
15	.0200	0	0	0	3	427	1
16	.1400	0	0	0	3	609	1
17	.0700	.3500	0	0	4	5	2
18	.0300	.5500	0	0	4	44	2
19	.2100	.1250	0	0	4	12	2
20	.0300	0	0	0	4	428	1
21	.2100	0	0	0	4	610	1
22	.0700	.5000	0	0	5	6	2
23	.0400	.5500	0	0	5	45	2
24	.2800	.1250	0	0	5	13	2
25	.0400	0	0	0	5	429	1
26	.2800	0	0	0	5	611	1
27	.0700	.7000	0	0	6	7	2
28	.0600	[illegible]	0	0		[illegible]	2
:			0				2
1456	[illegible]						
1457	.0600	.1200	0	0	[illegible]	[illegible]	2
1458	.0200	.2500	0	0	423	124	2
1459	.0200	.2500	0	0	423	419	2
1460	.0200	.2000	0	0	423	424	2
1461	.0400	.1000	0	0	423	358	2
1462	.0400	.1250	0	0	423	131	2

DATA OF NODES

A) INITIAL TEMPERATURES (only a sample is given)

UNKNOWN NODES	TEMP	MACHINE CODES	KNOWN NODES	TEMP	MACHINE CODES
1	190.0	1	424	900.0	1
2	190.0	1	425	80.0	5
3	190.0	1	426	80.0	5
4	190.0	1	427	80.0	5
5	190.0	1	428	80.0	5
6	190.0	1	429	80.0	5
7	190.0	1	430	80.0	5
8	190.0	1	431	80.0	5
9	190.0	1	432	80.0	5
10	190.0	1	433	80.0	5
11	190.0	1	434	80.0	5
12	190.0	1	435	80.0	5
13	190.0	1	436	80.0	5
14	190.0	1	437	80.0	5
15	190.0	1	438	80.0	5
16	190.0	1	439	80.0	5
17	190.0	1	440	80.0	5
18	190.0	1	441	80.0	5
19	190.0	1	442	80.0	5
20	190.0	1	443	80.0	5
21	190.0	1	444	80.0	5
22	190.0	1	445	80.0	5
23	190.0	1	446	80.0	5
24	190.0	1	447	80.0	5
25	190.0	1	448	80.0	5
26	190.0	1	449	80.0	5
27	190.0	1	450	80.0	5
28	190.0	1	451	80.0	5
29	190.0	1	452	80.0	5
30	190.0	1	453	80.0	5
31	190.0	1	454	80.0	5
32	190.0	1	455	80.0	5
33	190.0	1	456	80.0	5
34	190.0	1	457	80.0	5
35	190.0	1	458	80.0	5
36	190.0	1	459	80.0	5
37	190.0	1	460	80.0	5
38	190.0	1	461	80.0	5
39	190.0	1	462	80.0	5
40	190.0	1	463	80.0	5
41	190.0	1	464	80.0	5
42	190.0	1	465	80.0	5
43	190.0	1	466	80.0	5
44	190.0	1	467	200.0	3

?) PARTITIONING

SEG NO.	ROW/SEG	ACC.ROW	LAST WORD	ACC.LAST
1	3	8	3364	3364
2	14	22	5733	9097
3	15	37	5925	15022
4	15	52	5700	20722
5	16	68	5832	26554
6	16	84	5576	32130
7	17	101	5644	37774
8	18	119	5661	43435
9	20	139	5910	49345
10	21	160	5775	55120
11	23	183	5819	60939
12	25	208	5725	66664
13	29	237	5858	72522
14	34	271	5797	78319
15	45	316	5895	84214
16	107	423	5885	90099

THERMAL COEFFICIENTS (BTU/HR*FT*DEGREE F)

MATERIAL CODE	(K1)	(K2)	(K3)
1	1.97952E 01	2.38095E-03	0
2	3.04529E 01	-5.23086E-03	-3.85802E-06
3	3.42468E 01	-8.67651E-03	-2.38830E-06
4	1.92593E 00	2.48592E-03	6.28282E-06
5	2.13441E 00	2.77224E-03	6.16338E-06

(first iteration)

```
STMT    MATLAN STATEMENT

  1     MAIN
  2     READ       (H,V),DSF=9
  3     PRIVIL     V
  4     EXUPPTR    H,UP
  5     TRANS      UP,LOW
  6     ATTRIB     SYMMET,SYM=1
  7     CHECK
  8     ADD        H,LOW,SYMMET
  9     CANCEL     UP,LOW,H
 10     DIV        SYMMET,V,RESULT
 11     WRITE      RESULT,FORMAT=A5
 12     END
```

Results for Iteration 1 & 2

Node	Iter. 1	Iter. 2	Tolerance
1	383.72	382.06	1.66
2	379.80	378.26	1.54
3	371.39	370.01	1.38
4	356.45	355.43	1.02
5	334.42	333.84	.58
6	306.71	306.42	.29
7	276.70	276.61	.09
8	249.21	249.23	.02
9	381.84	380.24	1.60
10	377.96	376.49	1.47
11	369.61	368.30	1.31
12	355.29	354.33	.96
13	333.71	333.16	.55
14	306.33	306.05	.28
15	276.51	276.44	.07
16	249.11	249.14	.03
17	376.97	375.49	1.48
18	372.96	371.61	1.35
19	365.13	363.95	1.18
20	352.01	351.14	.87
21	331.57	331.05	.52
22	305.08	304.82	.26
23	275.87	275.81	.06
24	248.76	248.79	.03
25	360.26	359.20	1.06
26	353.92	353.01	.91
27	343.41	342.71	.70
28	325.71	325.27	.44
29	301.45	301.22	.23
30	273.95	273.90	.05
31	247.66	247.71	.05
32	334.74	334.18	.56
33	326.95	326.50	.45
34	314.59	314.28	.31
35	293.75	293.61	.14
36	268.68	268.64	.04
37	244.47	244.54	.07
38	305.09	304.85	.24
39	296.06	295.91	.15
40	280.08	280.02	.06
41	259.39	259.39	0
42	238.70	238.78	.08
43	478.44	475.28	3.16
44	398.78	397.01	1.77
45	351.33	350.74	.59
46	312.53	312.26	.27
47	278.17	278.11	.06
48	249.56	249.58	.02

49	476.45	473.55	2.90
50	462.67	459.50	3.17
51	439.40	436.32	3.08
52	393.99	392.43	1.56
53	349.61	349.05	.56
54	312.05	311.80	.25
55	277.97	277.92	.05
56	249.46	249.49	.03
57	464.72	462.16	2.56
58	441.71	439.46	2.25
59	418.69	416.65	2.04
60	384.44	383.22	1.22
61	344.63	344.17	.46
62	310.59	310.37	.22
63	277.29	277.25	.04
64	249.10	249.13	.03
65	400.33	399.10	1.23
66	385.70	384.67	1.03
67	364.17	363.44	.73
68	330.27	330.02	.25
69	306.70	306.54	.16
70	275.28	275.26	.02
71	247.99	248.04	.05
72	352.24	351.71	.53
73	340.75	340.34	.41
74	329.75	329.51	.24
75	298.01	297.92	.09
76	269.75	269.74	.01
77	244.75	244.82	.07
78	311.48	311.30	.18
79	300.96	300.86	.10
80	282.26	282.23	.03
81	260.09	260.10	.01
82	238.91	239.00	.09
83	540.64	538.79	1.85
84	422.72	421.22	1.50
85	361.66	361.05	.61
86	315.54	315.34	.20
87	278.89	278.85	.04
88	249.69	249.75	.06
89	522.76	521.52	1.24
90	523.32	521.13	2.19
91	481.55	479.09	2.46
92	416.54	415.23	1.31
93	359.84	359.28	.56
94	315.02	314.81	.21
95	278.68	278.65	.03
96	249.60	249.66	.06
97	496.15	495.05	1.10
98	482.82	481.40	1.42
99	451.17	449.63	1.54

100	403.70	402.68	1.02
101	354.93	354.44	.49
102	313.38	313.17	.21
103	277.96	277.95	.01
104	249.24	249.30	.06
105	423.75	422.87	.88
106	404.01	403.22	.79
107	377.01	376.38	.63
108	342.68	342.33	.35
109	308.82	308.67	.15
110	275.86	275.86	0
111	248.13	248.19	.06
112	359.98	359.56	.42
113	345.33	344.98	.35
114	326.36	326.14	.22
115	298.76	298.69	.07
116	270.14	270.15	.01
117	244.87	244.95	.08
118	313.30	313.14	.16
119	301.74	301.64	.10
120	282.83	282.83	0
121	260.35	260.37	.02
122	238.99	239.09	.10
123	637.09	635.35	1.74
124	434.64	433.34	1.30
125	364.64	364.09	.55
126	316.33	316.14	.19
127	279.09	279.04	.05
128	249.75	249.78	.03
129	577.00	577.04	.04
130	638.09	635.78	2.31
131	519.86	517.85	2.01
132	426.36	425.18	1.18
133	362.71	362.19	.52
134	315.79	315.59	.20
135	278.87	278.82	.05
136	249.65	249.70	.05
137	520.82	520.88	.06
138	521.55	520.66	.89
139	470.73	469.51	1.22
140	410.59	409.65	.94
141	357.63	357.13	.50
142	314.09	313.90	.19
143	278.14	278.11	.03
144	249.29	249.34	.05
145	434.14	433.45	.69
146	410.13	409.44	.69
147	380.33	379.73	.60
148	345.12	344.74	.38
149	309.37	309.20	.17

150	270.01	270.01	0
151	248.18	248.23	.05
152	361.63	361.25	.38
153	346.02	345.70	.32
154	325.68	325.45	.23
155	298.78	298.70	.08
156	270.23	270.23	0
157	244.91	244.99	.08
158	313.31	313.17	.14
159	301.57	301.48	.09
160	282.81	282.81	0
161	260.39	260.41	.02
162	239.02	239.12	.10
163	540.96	539.18	1.78
164	423.22	421.70	1.52
165	362.57	361.92	.65
166	315.95	315.72	.23
167	279.03	278.98	.05
168	249.75	249.79	.04
169	523.07	521.91	1.16
170	522.99	520.94	2.05
171	481.40	478.98	2.42
172	417.01	415.67	1.34
173	360.83	360.22	.61
174	315.42	315.19	.23
175	278.82	278.76	.06
176	249.65	249.70	.05
177	495.35	494.32	1.03
178	482.57	481.26	1.31
179	451.20	449.70	1.50
180	404.20	403.15	1.05
181	356.13	355.59	.54
182	313.76	313.54	.22
183	278.10	278.05	.05
184	249.29	249.34	.05
185	423.79	422.95	.84
186	404.12	403.36	.76
187	377.52	376.88	.64
188	344.33	343.92	.41
189	309.10	308.91	.19
190	275.97	275.95	.02
191	248.18	248.23	.05
192	359.28	358.87	.41
193	344.36	344.02	.34
194	324.43	324.18	.25
195	298.39	298.29	.10
196	270.17	270.16	.01
197	244.91	244.98	.07
198	312.42	312.26	.16
199	300.86	300.76	.10
200	282.53	282.52	.01

201	260.34	260.35	.01
202	239.01	239.11	.10
203	478.53	475.50	3.03
204	399.81	398.04	1.77
205	353.50	352.76	.74
206	313.46	313.14	.32
207	278.51	278.43	.08
208	249.69	249.68	.01
209	476.50	473.72	2.78
210	462.80	459.75	3.05
211	439.75	436.77	2.98
212	395.10	393.54	1.56
213	352.11	351.41	.70
214	312.97	312.66	.31
215	278.31	278.23	.08
216	249.58	249.59	.01
217	464.76	462.27	2.49
218	441.87	439.71	2.16
219	419.13	417.17	1.96
220	385.75	384.53	1.22
221	348.31	347.71	.60
222	311.47	311.17	.30
223	277.60	277.55	.05
224	249.22	249.24	.02
225	400.60	399.36	1.24
226	386.08	385.04	1.04
227	365.81	365.04	.77
228	338.61	338.14	.47
229	307.21	306.99	.22
230	275.54	275.50	.04
231	248.09	248.14	.05
232	350.40	349.88	.52
233	337.95	337.54	.41
234	320.68	320.40	.28
235	297.13	296.99	.14
236	269.84	269.82	.02
237	244.83	244.90	.07
238	309.47	309.30	.17
239	298.83	298.71	.12
240	281.69	281.64	.05
241	260.09	260.09	0
242	238.96	239.04	.08
243	383.89	382.41	1.48
244	379.93	378.51	1.42
245	371.61	370.33	1.28
246	356.95	355.97	.98
247	335.49	334.83	.66
248	307.93	307.54	.39
249	277.28	277.15	.13
250	24[illegible].32	249.32	0

251	382.03	380.63	1.40
252	378.05	376.71	1.34
253	369.78	368.57	1.21
254	355.75	354.81	.94
255	334.79	334.15	.64
256	307.58	307.20	.38
257	277.09	276.97	.12
258	249.32	249.32	0
259	376.72	375.42	1.30
260	372.88	371.66	1.22
261	365.17	364.07	1.10
262	352.34	351.48	.86
263	332.65	332.05	.60
264	306.45	306.10	.35
265	276.45	276.33	.12
266	248.96	248.97	.01
267	359.97	358.98	.99
268	353.53	352.66	.87
269	343.27	342.56	.71
270	326.68	326.19	.49
271	303.19	302.90	.29
272	274.52	274.44	.08
273	247.86	247.88	.02
274	332.63	332.07	.56
275	324.19	323.74	.45
276	312.97	312.63	.34
277	295.25	295.06	.19
278	269.12	269.07	.05
279	244.63	244.68	.05
280	303.39	303.13	.26
281	294.81	294.62	.19
282	280.06	279.96	.10
283	259.55	259.54	.01
284	238.80	238.86	.06
285	324.51	323.72	.79
286	326.86	326.18	.68
287	373.51	372.58	.93
288	375.31	374.34	.97
289	429.21	428.22	.99
290	430.72	429.87	.85
291	460.84	459.67	1.17
292	460.34	458.97	1.37
293	475.67	473.44	2.23
294	485.04	483.71	1.33
295	499.55	498.29	1.26
296	489.64	487.36	2.28
297	507.23	505.77	1.46
298	516.72	515.79	.93
299	531.09	530.38	.71
300	533.52	531.52	2.00

301	522.28	518.84	3.44
302	524.34	521.18	3.16
303	506.58	501.75	4.83
304	500.40	495.37	5.03
305	495.10	489.56	5.54
306	486.58	480.83	5.75
307	509.57	504.44	5.13
308	509.20	504.08	5.12
309	520.23	516.33	3.90
310	520.90	517.03	3.87
311	327.81	327.08	.73
312	331.57	330.97	.60
313	378.29	377.50	.79
314	378.70	377.81	.89
315	430.94	430.03	.91
316	431.90	431.10	.80
317	464.27	463.27	1.00
318	465.47	464.41	1.06
319	495.04	494.11	.93
320	496.22	495.47	.75
321	513.40	512.86	.54
322	514.54	513.89	.65
323	526.61	526.24	.37
324	530.84	530.59	.25
325	547.79	547.65	.14
326	560.74	559.39	1.35
327	562.24	559.81	2.43
328	562.74	560.53	2.21
329	560.67	557.19	3.48
330	561.63	558.18	3.45
331	534.66	529.98	4.68
332	528.53	523.63	4.90
333	528.72	524.08	4.64
334	528.40	523.77	4.63
335	530.17	526.51	3.66
336	530.78	527.18	3.60
337	329.14	328.42	.72
338	334.29	333.74	.55
339	381.21	380.49	.72
340	380.15	379.31	.84
341	431.94	431.10	.84
342	434.13	433.44	.69
343	467.61	466.75	.86
344	468.17	467.26	.91
345	506.81	506.79	.02
346	502.99	502.62	.37
347	521.96	521.88	.08
348	532.78	533.52	.74
349	535.11	535.39	.28
350	541.06	541.33	.27

351	561.99	562.61	.62
352	582.57	581.86	.71
353	601.15	600.01	1.14
354	601.98	601.08	.90
355	637.20	636.66	.54
356	696.75	700.05	3.30
357	569.66	565.89	3.77
358	573.10	569.44	3.66
359	538.22	533.85	4.37
360	538.23	533.85	4.38
361	535.62	532.36	3.26
362	536.18	532.95	3.23
363	327.79	327.05	.74
364	331.56	330.96	.60
365	378.75	377.96	.79
366	378.82	377.94	.88
367	430.75	429.84	.91
368	431.73	430.97	.76
369	464.39	463.44	.95
370	465.54	464.52	1.02
371	494.74	493.87	.87
372	496.16	495.49	.67
373	513.49	512.98	.51
374	514.25	513.63	.62
375	524.51	524.12	.39
376	531.27	531.05	.22
377	547.96	547.85	.11
378	560.82	559.50	1.32
379	562.37	560.01	2.36
380	562.86	560.71	2.15
381	560.86	557.47	3.39
382	561.90	558.56	3.34
383	534.75	530.11	4.64
384	528.72	523.85	4.87
385	528.30	523.72	4.58
386	528.02	523.42	4.60
387	529.12	525.72	3.40
388	529.71	526.35	3.36
389	324.36	323.57	.79
390	326.71	326.03	.68
391	372.57	371.63	.94
392	374.65	373.65	1.00
393	428.96	427.96	1.00
394	430.47	429.62	.85
395	460.81	459.66	1.15
396	460.30	458.96	1.34
397	475.65	473.49	2.16
398	485.05	483.76	1.29
399	499.61	498.40	1.21
400	489.58	487.39	2.19

401	507.14	505.76	1.38
402	516.78	515.91	.87
403	531.15	530.48	.67
404	533.57	531.62	1.95
405	522.33	518.98	3.35
406	524.40	521.30	3.10
407	506.61	501.93	4.68
408	500.45	495.55	4.90
409	494.94	489.52	5.42
410	486.51	480.86	5.65
411	508.60	503.53	5.07
412	508.24	503.16	5.08
413	517.37	513.48	3.89
414	517.66	513.79	3.87
415	463.24	458.52	4.72
416	463.05	458.25	4.80
417	482.66	478.26	4.40
418	482.55	478.03	4.52
419	509.02	504.98	4.04
420	508.53	504.47	4.06
421	558.15	554.97	3.18
422	557.60	554.33	3.27
423	569.23	565.87	3.36