A Semantics for Literary Texts

Marie Reyes

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Abstract

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Marie Reyes, Ph.D.
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The aim of this work is to develop a semantics for literary texts based on the notion of kind. Kinds are interpretations of count nouns such as "person" and "dog". They allow us to interpret quantifiers and talk counterfactually about its members. I give a mathematical formalization of the notion of kind which leads to the category of kinds. This category provides an adequate semantics for a formal system of many sorted higher order intensional logic. My basic thesis is that we need kinds to interpret count nouns of literary texts and members of kinds to interpret proper names. In this context, I discuss problems of existence, possibility and identity which permeate all literature.

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To Gonzalo and Luis-Emilio.

Contents

Introduction

In this introduction, I will present an outline of the main aspects of the semantics for literary texts developed in this thesis. I shall divide these aspects, with a certain degree of arbitrariness, into metaphysical aspects and linguistic aspects.

Metaphysical aspects

1. We have a real world, our world, and worlds of fiction corresponding to stories, novels, plays, etc., under consideration.
2. Each world is determined by
   (a) A set of factual and counterfactual situations preordered by the relation of "including whatever goes on in".
   (b) A set of basic kinds relative to these situations. Basic kinds may be viewed as sets of "urelements" whose members appear in situations of the world in question. In our world, for instance, dog and **person** are examples of basic kinds.
   (c) A category of kinds built from basic kinds.
3. A situation of a world is not a complete state of that world, but only a partial one.
4. Modalities such as necessity and possibility apply to predicates of kinds of a given world only.
5. In general, kinds do not belong to more than one world. However, abstract kinds such as **natural number** whose members do not appear in any situation of any world may be kinds of several worlds. On the other hand, kinds of a world whose members appear in some situations of that world, cannot be kinds of any other world.

6. A basic kind of any world of fiction may be viewed as a set of "urelements" in the real world, whose members do not appear in any situation of the real world. When a kind is thus "brought" to our world, we will say that it is "frozen".

7. Kinds and members of kinds belonging to different worlds may be compared by stipulating "counterparts" for those kinds and members that we want to compare.

Linguistic aspects

1. We introduce a language for a modal sort theory to refer to all kinds of the real world as well as kinds of the worlds of fiction under consideration. In this language, variables and proper names are sorted.

2. Sorts are interpreted as kinds; quantifiers are assumed to range over members of kinds, and proper names are interpreted as members of kinds. Since a member of a kind may appear in different situations of a given world, we do not need "counterparts" to deal with counterfactual situations in that world. It is possible to talk counterfactually about any member of a kind of a world in that world.

3. Terms and formulas are interpreted as maps between kinds and predicates of kinds, respectively.

4. It is possible to transfer our theories about kinds and their members of our world, partially or totally, to the corresponding "counterparts" in the world of the story. I postulate that "the logic of English" is totally transferred. I view this transfer mechanism as a formulation of Aristotle's mimesis.

5. We may talk, in our world, about kinds and their members of other worlds by first "freezing" them. Whatever is true "situationless" in the story is stipulated to hold at every situation of our world. We cannot therefore genuinely talk counterfactually about members of these "frozen" kinds.

6. The logic of contingent or modally free propositions is intuitionistic, whereas the logic of modally closed propositions is classical.

This thesis is divided into three chapters.

Chapter 1 is based on two term papers that I wrote in 1986: one on the semantics of Montague and the other, on the semantics of Gupta. These papers were elaborations of talks given by Professor Gonzalo E. Reyes at McGill and Montréal universities in 1985 and 1986. The aims of these talks were, to present on the one hand, a simplified version of Montague's semantics and, on the other hand, a Boolean-valued version of the semantics of Gupta. After a rather detailed presentation of these semantics there follows a section of criticisms and evaluation.

Chapter 2 develops a semantics of kinds for a many sorted, higher order intensional language. After a brief introduction, I state and give arguments for a series of theses on reference and generality which motivate the introduction of kinds. Some of these theses are due to Gupta and Macnamara who follow previous work of Geach, Bressan and medieval logicians. A mathematical formalization of the notion of kind leads to the category of kinds exposed in the following section. I then show in detail how our language can be interpreted in this category. As an application, I show how to formulate notions such as "de re", transparency, etc. This chapter is based on work done in collaboration with Professor Reyes and it is an application of his topos-theoretic approach to reference and modality [29] to the particular case of set-valued functors on a preordered set.

In Chapter 3, I apply the semantics of kinds to literary texts. I first discuss and criticise Parsons's theory of fiction. I then postulate kinds as the interpretations of the count nouns of the story and members of these kinds as the interpretations of proper names. These kinds are relative to the set of situations of the world of the story and allow us to use correctly our quantifiers and speak counterfactually about their members. Some of these kinds are "counterparts" of kinds of our world and I show how to transfer the relevant "background beliefs". I discuss problems of existence, possibility and identity in literary texts and show how several notions like character, surrogate, etc., can be defined in my theory. Finally I look at myths, stories and metamorphoses from the view point of my theory.

Chapter 1

Two semantics for intensional logic

In this chapter, to motivate the desirability of two main features of the system studied in Chapter 2, I shall describe the logical system of Montague [24] and the logical system of Gupta [14] in the simplified versions of G. E. Reyes, as exposed in my papers [30,31]. These features are that the logic considered is of *higher order* and *many-sorted.*

1.1 The semantics of Montague

1.1.1 Introduction

The aim of Montague [24] is "to present in a rigourous way the syntax and the semantics of a fragment of a certain dialect of English". To achieve this goal, Montague translates first the English expressions of the sample into an intensional logic that uses the theory of higher order and the λ-calculus; then he interprets this logic in the theory of sets. Montague wants a "rich" language which will permit him to mirror a good deal of the semantics. This approach gives him the means to discuss some problems whose solutions are achieved by looking sometimes at the syntactical side of the language and sometimes at the semantical side of the language.

Montague imposes on his logic a very fruitful constraint: if two expressions of the natural language belong to the same grammatical category, then their translations in intensional logic should belong to the same sort and their interpretations in sets should belong to the same kind (see for instance Dowty et al. [11, pages 260–262]).

Furthermore, Montague wants his intensional logic to deal with problems of opacity versus transparency. An opaque context is one for which substitutions of equals for equals result in different truth values. On the other hand, a context is transparent if it allows such substitutions without changing truth values. He also wants his intensional logic to allow for the possibility of the *de re* and *de dicto* readings of a sentence in relevant contexts. I discuss and illustrate these points with some examples.

The constraint on translation can be understood as follows:

1. John overtakes Mary, therefore Mary walks slower than John (it is clear that some further assumptions have to be made for a "logical" deduction, for instance, "both Mary and John are walking", etc.).

2. John overtakes nobody, therefore nobody walks slower than John.

1. and 2. have the same grammatical form. In fact "Mary" and "nobody" belong to the same grammatical category: noun phrase (NP). Therefore their translation into intensional logic should belong to the same sort. If we interpret "Mary" as a person, then "nobody" and "Mary" belong to different kinds since "nobody" cannot be interpreted as a person. On the other hand if we think of "Mary" as a set of properties, then "nobody" can also be thought of as a set of properties, namely the properties that nobody has. In this way we can represent "Mary" and "nobody" as belonging to the same sort. Nevertheless they have different logical structure: "Mary" is translated as $Mary = \lambda PP(m)$ and interpreted as the set of properties that Mary has, whereas "nobody" is translated as $nobody = \lambda P \forall x \neg P(x)$ and interpreted as the set of properties that nobody has. Obviously 1. is valid and 2. is invalid. I will formalize this argument later on, see section 2.4.1.

Since Frege, logicians have translated "nobody" as a quantifier and "Mary" as a constant in first order logic. With the introduction of higher order logic, Montague can translate, for example "John", "John and Mary", "nobody", "the teacher of Plato" into expressions belonging to the same sort so that the phrases "John runs", "nobody runs", "the teacher of Plato runs" can all be analyzed in the same way. The possibility of forming NPs of the sort "John or Mary, but not Jane" and therefore of connecting both uses of "and" in NPs and "and" in sentences "John and Mary, but not Jane went to the market", "John went to the market, or Mary went to the market..." was exploited by Keenan and Faltz [16]. I shall come back to this question later.

A different aspect of this discussion of logical form is found in the following example. From "John finds a thrush" we can certainly deduce "there are thrushes". On the other hand from "John seeks a unicorn" we cannot possibly deduce "there are unicorns". Once again, "find" and "seek" belong to the same grammatical category: verb phrase (VP), so their translations will be of the same sort and their interpretations will belong to the same kind, but their logic should be different. Nothing in the form of the verbs help to differentiate between the two. A solution will be reached only if the semantical part of the language is taken into account. Therefore Montague introduces the notion of "meaning postulates". This last example is quite intricate and brings about notions of transparency versus opacity, of *de re* readings versus *de dicto* readings.

These notions are dealt with by introducing into intensional logic "intensions" whose interpretations are functions. The idea is simply that some properties of functions depend on the whole graph of the functions in question. An example would be "f is increasing at 0". It is not enough to know the value $f(0)$ to decide whether f is increasing at 0 or not; we could have a function g which coincides with f at 0 but which is decreasing at 0. On the other hand, there are properties such as "f is positive at 0" for which knowledge of the value $f(0)$ suffices. Let us be more specific.

To solve the problems of transparency versus opacity and of differentiation between the two readings mentioned above, Montague interprets the VPs "very high" in the theory of higher order. Let us look at the sentence, "the temperature rises" (this is considered a paradigm by Montague). Let the interpretation of "the temperature" be a function $T \longrightarrow R$, where T is the set of moments of time and R is the set of reals. One can ask at time t_0: "Is the temperature rising"? Since the answer is "yes" or "no", it is then natural to interpret "the temperature rises" as a function $T \longrightarrow \{0,1\}$ and consequently to interpret "rises" as a function $R^T \longrightarrow \{0,1\}^T$.

We now consider the classical example: "George IV wished to know whether Scott was the author of Waverley". Let $\phi(x)$ be the context "George

IV wished to know whether x was the author of Waverley". Although $\phi(Scott)$ is true, and "Scott equals the author of Waverley" is also true, $\phi(\textit{the author of Waverley})$ is certainly false. The above remarks suggest that "Scott" and "the author of Waverley" should be interpreted as different functions which happen to coincide at our world. In other words, the notion of equality involved is not one of identity, but rather of coincidence and no logical problems arise from the fact that two different functions have different properties.

A context $\phi(x)$ is transparent if from $x equal y$, and $\phi(x)$ one can conclude $\phi(y)$. A context is opaque if that conclusion cannot be drawn. I think that the existence of opaque contexts is a normal feature of everyday language. Not only do opaque contexts occur in expressions like "wish to know", "believe", "necessary", but as Keenan and Faltz [16] have shown, expressions like "with Fred", "for Mary" introduce opaque contexts. For instance, it may well happen that the people who are working in a room are exactly those who are talking. Nevertheless, we cannot conclude that the people who are working with Fred are those who are talking with Fred. Contrary to what Keenan and Faltz believe, even the expression "in the park" can create opaque contexts, as the following example of M. Barr's indicates: "those who are doing research are those who are publishing" does not imply "those who are doing research in the park are those who are publishing in the park". This seems to indicate the ubiquity of opaque contexts in natural languages, independently of the occurrence of modal and epistemic operators.

Montague's higher order logic provides a nice way to tackle the problem of descriptions. Descriptions are difficult to handle in first order logic and Russell's analysis can not be applied to all descriptions. Descriptions which occur in contexts where a *de re* reading (primary occurrence) and a *de dicto* reading (secondary occurrence) are possible can be analyzed à la Russell. But, as G. E. Reyes has remarked: "Ponce de Leon was looking for the fountain of youth" seems to have only one possible reading, namely a *de re* reading: $\exists x(x\ \textit{is a fountain of youth} \wedge \textit{Ponce de Leon was looking for}\ x \wedge \forall y(y\ \textit{is a fountain of youth} \rightarrow y = x))$. This reading makes the sentence false, even though Ponce de Leon was really looking for the fountain of youth! First order logic seems incapable of dealing with non-existent objects which are required to handle the logic of fables, fairy tales and literature in general. In higher order logic as we will see shortly we can obtain the two readings mentioned above by correlating "the fountain of youth" with a set of properties that the fountain of youth has. This solution could, for instance, offer the possibility of correlating Hamlet, Sherlock Holmes, etc. with a set of properties as Parsons [25] has emphasized. I shall discuss the theory of fictional objects of Parsons in Chapter 3.

Montague works with a modal higher order theory, namely a higher order theory with two modal operators: an operator of necessity $\Box$, read as "it is necessary that" and an operator of possibility $\Diamond$, read as "it is possible that". In this context, I now mention another aspect of descriptions that played an important role in the discussions of Quine [28] and others on quantification and modality. Let us suppose that we are speaking about horse races. Let

$$a = \textit{the winner of the second race}$$

and the context

$$\phi(x) = \Box x \textit{ wins the second race.}$$

Obviously $\phi(a)$ is valid but strangely enough this does not imply that $\exists x \phi(x)$. On the other hand, if we take

$$a = \textit{Lucky Strike},$$

then $\exists x \phi(x)$ is valid. This indicates that the equality considered has to be handled with care, as I mentioned at the beginning of this section, and that the interaction between modal operators and quantifiers is intricate. Furthermore constants and interpretations of descriptions must not have the same logical form, so ϕ(*the winner of the second race*) is quite different from ϕ(*Lucky Strike*). I will return to descriptions in the course of my work, since they exemplify many problems that will be discussed.

1.1.2 The language of modal higher order theory and its interpretation

In this section I introduce the language of modal higher order theory of Montague's intensional logic and interpret it in sets. I define ***sorts*** and ***terms*** by recursion as follows:

Sorts

1. U is a ***basic sort***
2. Ω is a ***sort***

3. If X and Y are *sorts*, so is Y^X
4. Nothing else is a sort.

Terms of a given sort are defined by recursion as follows (where $t : X$ is an abbreviation for "t is a term of sort X"):

1. *Basic constant terms* $c \in Con_X$ are *terms* of sort X, for instance, $John \in Con_{\Omega(\Omega^U)}$, $j \in Con_U$
2. If $\alpha \in Var_X$, then α is a *term* of sort X, where Var_X is a countable set, for each sort X
3. If $\alpha \in Var_X$ and $t : Y$, then $\lambda\alpha t : Y^X$
4. If $t : Y^X$ and $s : X$ then $t(s) : Y$
5. $\top$ and $\perp$ are *terms* of sort Ω
6. If $t : X$ and $s : X$, then $t = s : \Omega$
7. If $\phi : \Omega$ and $\psi : \Omega$, then $\phi \, \Delta \, \psi : \Omega$, where $\Delta \in \{\wedge, \vee, \rightarrow\}$
8. If ϕ is a *term* of sort Ω, then so are $\forall\alpha\ \phi$ and $\exists\alpha\ \phi$
9. If ϕ is a *term* of sort Ω, then so are $\Box\phi$ and $\Diamond\phi$
10. Nothing else is a *term*.

Montague introduces in his language the tense operators F that can be thought of as "it will be the case that ϕ" and P that can be thought of as "it has been the case that ϕ". I will not introduce them although it could be done straightforwardly. If ϕ is a term of sort Ω, we let $\neg\phi \equiv \phi \longrightarrow \perp$. The *formulas* are by definition the terms of sort Ω. The connectives, quantifiers and modal operators are all understood in the usual way. The expression $t(s)$ is understood as denoting the value of the function denoted by t for the argument denoted by s. If α is a variable of sort X, $\lambda\alpha t$ is understood as denoting that function from the objects of sort X which takes as value for any such object a the object denoted by t when α is understood as denoting a. I now interpret this language in sets by choosing:

1. An arbitrary non empty set W that can be thought of as the set of possible worlds.

2. An arbitrary non empty set E that can be thought of as the set of individuals or entities.

3. A function m which interprets the basic constants: $Con_X \xrightarrow{m} \|X\|$, where $\|X\|$ is defined by recursion as follows: $\|\Omega\| = 2^W$ where
$$2 = \{0,1\}\ ,\ \|U\| = E^W \text{ and } \|X^Y\| = \|X\|^{\|Y\|}.$$

An interpretation is a triple $[W,\ E,\ m]$. We define, for every term $t : X$ and every $g : Var_X \longrightarrow \|X\|$, $\|t\|_g \in \|X\|$ which is the interpretation of t under the assignment g as follows:

1. If $c \in Con_X$, then $\|c\|_g = m(c) \in \|X\|$.
2. If $\alpha \in Var_X$, then $\|\alpha\|_g = g(\alpha) \in \|X\|$
3. If $t : Y$ and $\alpha \in Var_X$, then $\|\lambda\alpha\ t\|_g : \|X\| \longrightarrow \|Y\|$ is defined by $$\|\lambda\alpha\ t\|_g(a) = \|t\|_{g(a/\alpha)} \text{ where } g(a/\alpha)(\beta) = \begin{cases} g(\beta) & \text{if } \alpha \neq \beta \\ a & \text{if } \alpha = \beta \end{cases}$$
4. If $\|t\|_g \in \|Y\|^{\|X\|}$ and $\|s\|_g \in \|X\|$, then $\|t(s)\|_g = \|t\|_g(\|s\|_g) \in \|Y\|$

I will interpret $\top$ and $\perp$ later when I introduce the forcing. In order to interpret an equality between two terms, we first define by recursion on sorts and for every sort X a set $\mid X \mid$ and a "canonical" map
$$can_X : \|X\| \longrightarrow \mid X \mid^{W}$$
as follows: $\mid \Omega \mid = 2$, $\mid U \mid = E$, $\mid Y^X \mid = \mid Y \mid^{(\mid X \mid^W)}$, $can_\Omega = Id$, $can_U = Id$, $can_{YX}(\phi) = \lambda w \lambda \alpha can_Y \phi(can_X^{-1}(\alpha))(w)$ where $\phi \in \|Y^X\|$.

I will make many abuses of language. For instance, I will use α as a variable of the language ($\alpha \in Var_X$) and then as a variable in the interpretation of the language ($\alpha : W \longrightarrow \mid X \mid$).

Proposition 1.1.2.1 *For every sort X, $can_X : \|X\| \simeq \mid X \mid^W$*

Proof. It is clear that can_Ω and can_U are bijections and if can_X and can_Y are bijections so then is can_{YX}. □

With the help of this proposition we interpret the equality between two terms. If $\|t\|_g \in \|X\|$ and $\|s\|_g \in \|X\|$, then $\|t = s\|_g(w) = 1$ if and only if $can_X(\|t\|_g)(w) = can_X(\|s\|_g)(w)$

To interpret terms of sort Ω I introduce the forcing notation that will be used throughout my work. Let $\phi : \Omega$, then $w_0 \Vdash \phi[g]$ iff $\|\phi\|_g(w_0) = 1$.

5. $w_0 \Vdash \top[g]$ *always* and $w_0 \Vdash \bot[g]$ *never*

6. $w_0 \Vdash t = s[g]$ iff $\|t = s\|_g(w_0) = 1$

7. We define $\Vdash$ for $\wedge$, $\vee$, *and* $\rightarrow$

   1. $w_0 \Vdash \phi \wedge \psi[g]$ iff $w_0 \Vdash \phi[g]$ and $w_0 \Vdash \psi[g]$,
   2. $w_0 \Vdash \phi \vee \psi[g]$ iff $w_0 \Vdash \phi[g]$ or $w_0 \Vdash \psi[g]$,
   3. $w_0 \Vdash \phi \longrightarrow \psi[g]$ iff $w_0 \Vdash \phi[g]$ implies that $w_0 \Vdash \psi[g]$

8. We define $\Vdash$ for $\exists$ *and* $\forall$

   1. $w_0 \Vdash \exists \alpha \phi[g]$ iff $\exists a \in \|X\| w_0 \Vdash \phi[g(a/\alpha)]$,
   2. $w_0 \Vdash \forall \alpha \phi[g]$ iff $\forall a \in \|X\| w_0 \Vdash \phi[g(a/\alpha)]$

9. We define $\Vdash$ for $\Box$ *and* $\Diamond$

   1. $w_0 \Vdash \Box \phi[g]$ iff $\forall w \in W w \Vdash \phi[g]$,
   2. $w_0 \Vdash \Diamond \phi[g]$ iff $\exists w \in W w \Vdash \phi[g]$

We remark that $w_0 \Vdash \neg \phi[g]$ iff $w_0 \Vdash \phi \longrightarrow \bot[g]$ is the same as $w_0 \nVdash \phi[g]$.

We define the notion of *validity* for this interpretation: if $M = [W, E, m]$, then $\theta(\alpha_1, \alpha_2, \ldots)$ is valid in M symbolized as $M \models \theta(\alpha_1, \alpha_2, \ldots)$

$$\text{iff } \forall w \in W \forall g \; w \Vdash \theta(\alpha_1, \alpha_2, \ldots)[g]$$

$$\text{iff } \forall w \in W \forall g \|\theta(\alpha_1, \alpha_2, \ldots)\|_g(w) = 1.$$

We remark that

$$M \models \theta(\alpha_1, \alpha_2, \ldots) \text{ iff } M \models \Box \forall \alpha_1, \alpha_2, \ldots \theta(\alpha_1, \alpha_2, \ldots)$$

$$\text{iff } M \models \forall \alpha_1, \alpha_2, \ldots \Box \theta(\alpha_1, \alpha_2, \ldots).$$

1.1.3 Description of a fragment of English and its translation into modal higher order theory

For the purpose of this exposition I will restrict myself to a fragment of the fragment studied by Montague. Furthermore my translation takes into account the simplifications that I have introduced in the modal higher order theory just described; for instance I have not introduced symbols for intension ($^\wedge$) and extension ($^\vee$) as Montague does in his intensional logic.

The fragment studied will contain basic expressions (B) belonging to the following categories: intransitive verbs (IV), common nouns (CN), names and pronouns (T), transitive verbs (TV). In section 1.1.1, I have presented a motivation for the requirement that basic expressions like "rise" should be of sort Ω^U; in the same way I could motivate the sort of the other categories introduced. Let us describe the fragment.

$B_{IV} = \{run,\ rise\}$	of sort Ω^U
$B_{CN} = \{man,\ unicorn\}$	of sort Ω^U
$B_T = \{John,\ Mary,\ he_0,\ he_1,\ he_2,\ \ldots\}$	of sort $\Omega^{(\Omega^U)}$
$B_{TV} = \{seek,\ find,\ be\}$	of sort $(\Omega^U)^{(\Omega^{(\Omega^U)})}$
$B_{IV/S} = \{believe\ that\}$	of sort $(\Omega^U)^\Omega$

A basic expression of the fragment is understood as a member of

$$\bigcup_{A \in Cat} B_A.$$

P_A will be the set of composite expressions of the category A. P_S is understood as containing all the statements of the fragment, which are of course of sort Ω. We remark that there are no basic expressions of sort Ω corresponding to the sort t in Montague, and of sort U corresponding to the sort e in Montague. The sets P_A are the smallest sets satisfying the syntactical rules and the corresponding translation rules given by Montague. We will write $T(\zeta)$ for the translation of ζ. If ζ is in B_A, then $T(\zeta) = \zeta$ except for the members of B_T and *be*. The translations for the members of B_T and *be* will be given if needed in the examples.

I present some examples where I tacitly use Montague's rules of syntactical derivation and translation to give an idea of what is involved; a more detailed version can be found in Reyes [30]. The last example presented will contrast the *de re* and the *de dicto* readings of the same statement.

1. "John runs". This example allows ambiguous syntactical derivations but unambiguous translations. "John runs" will be interpreted the same way no matter what syntactical analysis is used.

(a) John runs
↙ ↘
John run

(b) John runs
↙ ↘
John he_0 runs
↙ ↘
he_0 run

$$
\begin{array}{lll}
\text{(a) T(run)} & : & run \\
\quad \text{T(John)} & : & \lambda PP(j) \\
\quad \text{T(John runs)} & : & (\lambda PP(j))(run) \\
 & & = (run)(j) \text{ by } \lambda\text{- conversion} \\
\text{(b) T}(he_0) & : & \lambda PP(x_0) \\
\quad \text{T(run)} & : & run \\
\quad \text{T}(he_0 \text{ runs}) & : & run(x_0) \\
\quad \text{T(John runs)} & : & \lambda PP(j)(\lambda x_0(run(x_0))) \\
 & & = (\lambda x_0(run(x_0)))(j) = run(j)
\end{array}
$$

2. I contrast the two statements: "John is Mary" and "John is a man".

(a) John is Mary
↙ ↘
John be Mary
↙ ↘
be Mary

(b) John is a man
↙ ↘
John is a man
↙ ↘
be a man

$$
\begin{array}{llll}
\text{(a)} & \text{T(be)} & : & \lambda\mathcal{P}\lambda x\mathcal{P}(\lambda y(x = y)) \\
 & \text{T(Mary)} & : & \lambda PP(m) \\
 & \text{T(be Mary)} & : & \lambda\mathcal{P}\lambda x\mathcal{P}(\lambda y(x = y))(\lambda PP(m)) \\
 & & & = \lambda x(\lambda PP(m))(\lambda y(x = y) \\
 & & & = \lambda x(\lambda y(x = y))(m) \\
 & & & = \lambda x(x = m) \\
 & \text{T(John)} & : & \lambda PP(j) \\
 & \text{T(John is Mary)} & : & \lambda PP(j)(\lambda x\,(x = m)) \\
 & & & = (\lambda x(x = m))(j) \\
 & & & j = m \\
\text{(b)} & \text{T(a man)} & : & \lambda Q(\exists x(man(x) \wedge Q(x)))
\end{array}
$$

$$
\begin{array}{lcl}
\text{T(be)} & : & \lambda\mathcal{P}\lambda z\mathcal{P}(\lambda y(z = y))) \\
\text{T(is a man)} & : & \lambda z(\lambda Q(\exists x(man(x) \wedge Q(x))))(\lambda y(z = y)) \\
 & & = \lambda z(\exists x(man(x) \wedge (\lambda y(z = y))(x)) \\
 & & = \lambda z(\exists x(man(x) \wedge (z = x)) \\
\text{T(John is a man)} & : & \lambda PP(j)(\lambda z(\exists x(man(x) \wedge (z = x))) \\
 & & = \lambda z(\exists x(man(x) \wedge (z = x))(j) \\
 & & = \exists x(man(x) \wedge j = x)
\end{array}
$$

3. I now analyze the two possible readings of the statement: "John seeks a unicorn". These two readings have different interpretations as we will see.

(a) John seeks a unicorn (*de dicto*)

$$
\begin{array}{l}
\text{John seeks a unicorn (\textit{de dicto})} \\
\swarrow \qquad \searrow \\
\text{John} \qquad \text{seeks a unicorn} \\
\qquad\qquad \swarrow \qquad \searrow \\
\qquad\qquad \text{seek} \qquad \text{a unicorn} \\
\qquad\qquad\qquad\qquad \swarrow \qquad \searrow \\
\qquad\qquad\qquad\qquad \text{a} \qquad \text{unicorn}
\end{array}
$$

$$
\begin{array}{lcl}
\text{T(unicorn)} & : & unicorn \\
\text{T(a unicorn)} & : & \lambda Q\exists x(unicorn(x) \wedge Q(x)) \\
\text{T(seek)} & : & seek \\
\text{T(seek a unicorn)} & : & seek(\lambda Q\exists x(unicorn(x) \wedge Q(x))) \\
\text{T(John)} & : & \lambda PP(j) \\
\text{T(John seeks a unicorn)} & : & \lambda PP(j)(seek(\lambda Q\exists x(unicorn(x) \wedge Q(x)))) \\
 & & = seek(\lambda Q(\exists x(unicorn(x) \wedge Q(x))))(j)
\end{array}
$$

And finally, we interpret $seek(\lambda Q(\exists x(unicorn(x) \wedge Q(x))))(j)$ as

$$(m(seek)(\|\lambda Q\exists x(unicorn(x) \wedge Q(x))\|_g))(m(j)) \in \|\Omega\|$$

where $m(seek) : \|\Omega\|^{(\|\Omega\|^{\|U\|})} \to \|\Omega\|^{\|U\|}$,

$$\|\lambda Q\exists x(unicorn(x) \wedge Q(x))\|_g : \|\Omega\|^{\|U\|} \to \|\Omega\|$$

and $m(j) \in \|U\|$.

(b) John seeks a unicorn (*de re*)

↙ ↘

a unicorn — John seeks him_0

↙ ↘ ↙ ↘

a — unicorn — John — seek him_0

↙ ↘

seek — he_0

T(he_0)	: $\lambda Q Q(x_0)$
T(seek)	: $seek$
T(seek him_0)	: $seek(\lambda Q Q(x_0))$
T(John seeks him_0)	: $\lambda P P(j)(seek(\lambda Q Q(x_0))$
	$= seek(\lambda Q Q(x_0))(j)$
T(a unicorn)	: $\lambda P(\exists x(unicorn(x) \wedge P(x)))$
T(J. seeks a unicorn)	: $\lambda P(\exists x(unicorn(x) \wedge P(x)))(\lambda x_0 seek(\lambda Q Q(x_0))(j))$
	$= \exists x(unicorn(x) \wedge (\lambda x_0 seek(\lambda Q Q(x_0))(j))(x))$
	$= \exists x(unicorn(x) \wedge seek(\lambda Q Q(x))(j))$

We interpret $\exists x(unicorn(x) \wedge seek(\lambda Q Q(x))(j)$ as

$$\|\exists x(unicorn(x) \wedge seek(\lambda Q Q(x))(j)\|_g .$$

1.1.4 Transparency and *de re*

In the last section, we derived the statement "John seeks a unicorn" in two different ways. We obtained two different translations of it, one corresponding to the *de re* reading and the other corresponding to the *de dicto* reading. These different readings can be paraphrased as follows: when John is seeking a unicorn he might (*de re*) or might not (*de dicto*) be seeking a particular unicorn. So natural language has been disambiguated by allowing for two different derivations and their corresponding different translations.

We remark that different derivations do not always lead to different translations as we have seen in the first example of the last section. If we consider the statement "John finds a unicorn" and we apply the same rules, we obtain two different translations:

1. $find\ (\lambda Q\ (\exists x\ (unicorn(x) \wedge Q(x))))(j)$
2. $\exists x\ (unicorn(x) \wedge find(\lambda Q\ Q(x))(j))$.

Unlike the case of "seek" there seems to be no ambiguity here. If John finds a unicorn, there must exist a particular unicorn that he finds. So there should be only one reading. Since the system was built to permit two readings it seems that there are no straightforward ways to stop the analysis that leads to 1. To solve this problem, Montague restricts the interpretation: he accepts only models where 1. and 2. are equivalent, namely he postulates in the language of the intensional logic that 1. and 2. are equivalent. Montague recognises many contexts where there exist such phenomena and he introduces as many postulates as there are expressions presenting this phenomenon. I call these postulates *transparency postulates*. The reasons for that name will become clear when we write some of these postulates.

I prefer not to use the name *meaning postulates* despite the fact that Dowty et al., for instance, have used it. Carnap [6] in 1947 introduced meaning postulates to deal with analytically true sentences, sentences which were true in virtue of the meaning of the words, but which could not be analyzed as logically true: true as a consequence of their syntactical form. To analyze "all bachelors are unmarried", Carnap's meaning postulate is

$$\forall x\,(B(x) \longrightarrow \neg M(x)),$$

where B stands for bachelors and M for married. A model would then be admissible only if that sentence is true in the model. In other words we restrict the models to the ones that make that sentence true. Montague does not consider postulates that relate the meaning of two words, apart from one exception when he analyzes "seek" as "try to find", but rather he considers postulates that articulate the logic of the expressions considered as, for example, in the case of "find".

I shall reformulate some of Montague's transparency postulates. In Montague's terminology the elements of E^W are called "individual concepts", the elements of 2^W are called "propositions" and the elements of $(2^W)^{(E^W)}$ are called "intensional properties". Since I choose a fragment of the fragment studied by Montague, I will mention only the postulates relevant to my fragment.

TP1 $m(\alpha) : W \longrightarrow E$ is a constant function. In other words $\exists e \in E$ such that $\forall w \in W m(\alpha)(w) = e$, where α is j or m.

TP2 $\forall w \in W(m(\delta)(a))(w) = 1 \Longrightarrow \exists e \in E\, \forall w' \in Wa(w') = e$, where δ is *man*, or *unicorn*

TP3 All members of B_{IV} except "rise" are transparent. Namely,

$$x = y \rightarrow (\delta(x) \leftrightarrow \delta(y)),$$

where δ is *run*.

TP4
1. $\delta(\lambda PP(x))(y)$ is transparent both in x and in y namely that $x = x' \wedge y = y' \rightarrow (\delta(\lambda PP(x))(y) \leftrightarrow \delta(\lambda PP(x'))(y'))$, where δ is *find*.
2. $\delta(\mathcal{P})(x) \leftrightarrow \mathcal{P}(\lambda y \delta(\lambda PP(y)(x)))$, where δ is *find*

TP5 *seek*$(\lambda PP(x))(y)$ is transparent in y.

TP6 *believe that* $(\sigma)(y)$ is transparent in y.

The equivalence of these postulates with Montague's postulates is shown in Reyes [30]. We remark that TP1 guarantees that names are rigid designators in contrast to descriptions: the denotation of "j" is the constant individual concept which picks the same individual (namely John) in each possible world. We cannot, in this language, give a syntactical version of TP1 and TP2 since we have no means of reaching the elements of E. We cannot, for example, write $\exists e \in E$ *such that*...

From TP4 we can deduce the following corollaries that we apply to the statement "John finds a unicorn"

1. $find(\mathcal{P})(j) \wedge \mathcal{P} = \mathcal{P}' \rightarrow find(\mathcal{P}')(j)$. We remark that $\mathcal{P} = \mathcal{P}'$ implies that $\forall P \mathcal{P}(P) = \mathcal{P}'(P)$, hence

$$find(\mathcal{P})(j) \leftrightarrow \mathcal{P}(\lambda y find \lambda PPy)(j) =$$

$$\mathcal{P}'(\lambda y find \lambda PPy)(j) \leftrightarrow find(\mathcal{P}')(j)$$

2. $find(\lambda Q \exists x(unicorn(x) \wedge Q(x)))(j)$

$$\leftrightarrow \exists x(unicorn(x) \wedge find \lambda PP(x)(j)).$$

This corollary says that there is only one reading, (*de re* reading) for "John finds a unicorn".

3. $find(\lambda Q \forall x(unicorn(x) \rightarrow Q(x)))(j)$

$$\leftrightarrow \forall x\,(unicorn(x) \rightarrow find \lambda PP(x)(j))$$

4. $find(\mathcal{P} \wedge \mathcal{P}')(j) \leftrightarrow find(\mathcal{P})(j) \wedge find(\mathcal{P}')(j)$

5. $find(\mathcal{P} \vee \mathcal{P}')(j) \leftrightarrow find\mathcal{P}(j) \vee find\mathcal{P}'(j)$

6. $find(\mathcal{P} \vee \neg\mathcal{P})(j) \leftrightarrow find\mathcal{P}(j) \wedge \neg find\mathcal{P}(j)$

These clauses assure us that the interpretation of "find" is a Boolean homomorphism which preserves existential and universal quantifiers. This is closely connected to the meaning postulates of Keenan and Faltz. These authors, however, impose their conditions at the level of the models only; their models have to be of a very special kind given in terms of complete and atomic Boolean algebras. From these clauses, it should be clear that such restriction is not necessary to formulate their insights.

1.1.5 Criticisms and conclusion

We can summarize this section on Montague's semantics by saying that higher order logic is needed to provide a satisfactory treatment of descriptions, of "formal grammar" (for instance the fact that "John but not Mary", "nobody" and "John" should belong to the same sort) and of opacity. However, all the sorts in the intensional logic of Montague are constructed from only one basic sort which is interpreted as the set of all the possible and actual entities needed to interpret the constants of that basic sort in the fragment under consideration. I point out some of the difficulties with this approach and shall briefly describe them.

The first thing that we notice is that counting does not apply to heaps or conglomerates of objects. As was argued by Frege, the same conglomerate which makes up an army could be counted as 1 army, 6 divisions, 18 brigades or 500,000 men. The same remark applies to all quantifiers. Although Montague never says so explicitly, he seems to be committed to "bare individuals" or objects, or things. Indeed, his "possible entities" may well be unicorns in one world, persons in another and minerals in still another possible world: the only link being the bare individual underlying the unicorn, the person and the mineral in the possible worlds under consideration. How can we make clear in such an approach what we are baptizing when we say "I baptize thee "John" in nomine..."? Are we baptizing the nose, the baby, the godfather with the baby, the baptismal robe, the set of molecules that constitute the body of the baby or what?

Another problem pointed out by Gupta is the following: How is it that airline companies count differently human passengers and persons? How can we explain this fact since in any given situation the human passenger coincides with the person? This problem is a modern version of the problem of heralds treated in the Middle Ages; see for instance Geach [13]. I mention a last problem which was considered by Keenan and Faltz: How to account for the fact that the same individual may be tall ...as a pygmy, but not tall ...as a man?

In the next section, I shall try to answer some of these questions while describing the semantics of Gupta.

1.2 The semantics of Gupta

1.2.1 Introduction

In the usual presentation of the semantics of quantified modal logic, common nouns and verb phrases (VP) which are translated as formulas with one argument (for instance, "run", "find an apple", "loves John" and "is red") are given the same interpretation. They belong to different grammatical and syntactical categories but the interpretation does not respect their grammatical and syntactical differences. Common nouns and verb phrases are interpreted as intensional properties $W \longrightarrow 2^{E^W}$, where W is the set of all possible worlds and E is the set of individuals. I shall consider only count nouns in my presentation, for instance "dog" and "person". Gupta's study proceeds in just this way and only mentions casually other types of common nouns. The idea that it would be better to keep apart the semantical categories of common nouns and verb phrases comes mainly from the following remarks. But before going into these remarks, I should say some words on the terminology that will be used.

Count nouns, for instance "dog", are translated as sorts, symbolized as *dog* and interpreted as kinds which are symbolized as dog. Verb phrases, like "run", are translated as formulas with one argument by $run(x)$ and interpreted as predicates by $\|run\|(g(x))$. I use sometimes the words property or unary relation instead of the word predicate. I shall use the expression *predicate of a kind* since, as will be explained shortly, all variables are sorted. If we use "run" which is translated as $run(x)$, we must know the sort of x or, in other words, we must know to which kind the predicate is "applied".

The predicates come equipped with a principle of application. The principle of application for predicates of a kind specifies when a member of the kind has the predicate. This principle is not an epistemic criterion and is independent of our capacity to decide if the thing has or not the property. According to Gupta, kinds also come equipped with a principle of application. For example, if we consider the kind **dog** then the principle of application says what should count as a dog: it specifies, for instance, that a bitch with its puppies does not count as a dog, and that the legs and the tail of a dog do not count as a dog.

Furthermore, Gupta postulates that kinds come equipped with a principle of identity. For instance if we take the kind **person**, the principle of identity says when a person in a possible world is the same as another person in another possible world. This principle of identity is not an epistemic criterion but a metaphysical counterpart of it. I quote here Gupta:

> *It is not the rule by which one* determines, *say, when an object is the same river as another object. It is rather the metaphysical counterpart of such an epistemic rule. The principle of identity for "river" is the rule* in virtue of which *an object at a time (and a world) is the same river as an object at another time (and a world).* [14, page 2]

The usual interpretation of count nouns such as Montague's takes into account only the principle of application and ignores the principle of identity. To illustrate this Gupta gives the following example. Let us consider a person who takes the plane twice in a week. This person is counted by the airlines companies as two passengers, even if these two passengers are the same person. In the usual semantics such as Montague's we cannot account for this fact. Passengers and persons are finally the same since all objects belong to a unique kind, namely the entities, and the only identity considered is the identity that comes with these entities.

In order to take into account the principle of identity, which intuitively will be a different principle for horses, for dogs and for persons, Gupta, following Bressan [5], introduced different sorts in the language. The sorts and their interpretation, the kinds, permit us to understand more clearly some problems related to quantification as well as other problems mentioned already in the preceding section. I shall in my presentation consider sorted elements only. Gupta, similarly to Montague, postulates a set of unsorted entities, elements, individuals. I believe that this postulate goes against the spirit of Gupta's own work.

1.2.2 Interpretation of count nouns

I describe here the interpretation of the count nouns with the help of the count noun "person". Gupta analyzes only kinds of the sort **person, dog, passenger** which are *separated* kinds. I will define this notion later, but intuitively we say that a kind is separated if when two members of the kind happen to coincide in one possible world then they are the same member. I shall therefore construct those kinds which are separated as sets of individual concepts starting from individuals in their possible worlds or possible circumstances. Members of kinds have total existence in Gupta's approach. I generalize ~~this approach~~ by constructing kinds such that their members have only partial existence.

I consider the family $(|\, person\, |_w)_{w \in W}$ where intuitively $|\, person\, |_w$ is the set of persons that exist in the possible world w and is given by the principle of application. I assume that the principle of identity of person gives an equivalence relation $=_{person}$ on

$$\bigsqcup_{w \in W} |\, person\, |_w = \{(p, w) \ : \ p \in |\, person\, |_w\}$$

such that

$$(p_1, w_1) =_{person} (p_2, w_2) \wedge w_1 = w_2 \rightarrow p_1 = p_2$$

We now define the notion of individual concept associated with the count noun "person". Let $p \in |\, person\, |_w$ and let us define a partial function on W, $\alpha^{person}_{<p,w>}$ whose domain is given by

$$w' \in dom\alpha^{person}_{<p,w>} \leftrightarrow \exists p' \in |\, person\, |_w \ (\alpha^{person}_{<p,w>}(w), w) =_{person} (p', w')$$

and whose value at w' is $\alpha^{person}_{<p,w>}(w') = p'$. We remark that p' is unique for a given w', by our assumption on $=_{person}$. I will use the following notation and definition:

$$\|person\|(w) = \{\alpha^{person}_{<p,w>} \ : \ p \in |\, person\, |_w\}$$

Finally we define the interpretation of "person" as

$$\|person\| = \bigcup_{w \in W} \|person\|(w)$$

In general if K is a count noun we define

$$\|K\|(w) = \{\alpha^{K}_{<k,w>} \ : \ k \in | \ K \ |_w\}$$

and the interpretation of K as $\|K\| = \bigcup_{w \in W} \|K\|(w)$.

So far, we have succeeded in interpreting "person" without the help of unsorted individuals. We have also been able to deduce the notion of an individual concept attached to a particular member of a kind from the principle of identity and application. We can imagine that the individual concept attached to a particular person at w describes the trajectory or the history of that particular person through times and possible worlds or possible circumstances.

Proposition 1.2.2.1 $\|person\|(w) \simeq | \ person \ |_w$

Proof. We define the isomorphism by

$$ev_w(\alpha^{person}_{<p,w>}) = \alpha^{person}_{<p,w>}(w) = p$$

where $p \in | \ person \ |_w$.

(a) ev_w is surjective by construction, since we have constructed an individual concept attached to each $p \in | \ person \ |_w$ such that $\alpha^{person}_{<p,w>}(w) = p$.

(b) ev_w is injective, that is

$$ev_w(\alpha^{person}_{<p,w>}) = ev_w(\beta^{person}_{<p,w>}) \Rightarrow \alpha^{person}_{<p,w>} = \beta^{person}_{<p,w>}.$$

Let us assume the hypothesis:

$$\alpha^{person}_{<p,w>}(w) = \beta^{person}_{<p,w>}(w).$$

Let $\alpha^{person}_{<p,w>}(w') = p'$ and $\beta^{person}_{<p,w>}(w') = q'$ where w' is in their domain. By the construction of individual concepts we obtain

$$(\alpha^{person}_{<p,w>}(w), w) =_{person} (p', w'), \ (\beta^{person}_{<p.w>}(w), w) =_{person} (q', w')$$

which implies that $q' = p'$. Hence $\alpha^{person}_{<p,w>} = \beta^{person}_{<p,w>}$. More generally we have $\|K\| \overset{ev_w}{\simeq} | \ K \ |_w$ where K is a count noun. $\square$

Proposition 1.2.2.2 $(p_1, w_1) =_{person} (p_2, w_2)$ *if and only if*

$$\alpha^{person}_{<p_1,w_1>} = \alpha^{person}_{<p_2,w_2>}$$

if and only if

$$\exists \alpha \in \|person\|(w_1) \cap \|person\|(w_2)$$

such that $\alpha(w_1) = p_1$ *and* $\alpha(w_2) = p_2$. □

Corollary 1.2.2.3 $\coprod_{w \in W} \mid person \mid_w / =_{person} \stackrel{\Phi}{\simeq} \|person\| =$

$$\bigcup_{w \in W} \|person\|(w)$$

Proof. Let Φ be $[(p, w)] \longmapsto \alpha^{person}_{<p,w>}$. Φ is well defined and is injective by the previous proposition. Let $\alpha \in \|person\|(w)$, by definition

$$\alpha = \alpha^{person}_{<p,w>} = \Phi[(p, w)].$$

So Φ is surjective. □

Let us analyze the problem mentioned previously: the problem of passenger versus person. How will their different principles of identity affect the relations between $\mid passenger \mid_w$, $\mid person \mid_w$, $\|passenger\|(w)$ and $\|person\|(w)$?

First of all we remark that at a world w, $\mid passenger \mid_w \subseteq \mid person \mid_w$. This expresses the fact that a passenger at w is a person at w, that each (human) passenger is a person. We also remark that

$$(pa_1, w_1) =_{passenger} (pa_2, w_2) \Rightarrow (pa_1, w_1) =_{person} (pa_2, w_2),$$

where pa is an abbreviation for "passenger". Now let us compare $\alpha^{person}_{<pa,w>}$ and $\alpha^{passenger}_{<pa,w>}$. By the principle of application of what a passenger is it is clear that

$$dom(\alpha^{passenger}_{<pa,w>}) \subseteq dom(\alpha^{person}_{<pa,w>}),$$

and if $w' \in dom(\alpha^{passenger}_{<pa,w>})$ then

$$w' \in dom(\alpha^{person}_{<pa,w>}) \text{ and } \alpha^{passenger}_{<pa,w>}(w') = \alpha^{person}_{<pa,w>}(w').$$

Thus

$$\alpha^{passenger}_{<pa,w>} = \alpha^{person}_{<pa,w>} \upharpoonright dom(\alpha^{passenger}_{<pa,w>}),$$

where $\upharpoonright$ stands for the restriction of a function.

Let us consider the extension u of $\|passenger\|(w)$ into $\|person\|(w)$. This extension is injective; to each passenger we associate the "underlying person who is that passenger".

$$\|passenger\|(w) \overset{u}{\hookrightarrow} \|person\|(w)$$

$$\alpha^{passenger}_{<pa,w>} \longmapsto \alpha^{person}_{<pa,w>}$$

where $pa \in \mid passenger\mid_w \subset \mid person\mid_w$. The function u gives rise to a new function

$$U : \|passenger\| \rightarrow \|person\|$$

which need not be a monomorphism

$$\|passenger\| = \bigcup_{w\in W} \|passenger\|(w),\ \|person\| = \bigcup_{w\in W} \|person\|(w)$$

and the principles of identity are different. The same person who travels twice can then be counted as two different passengers because, so to speak, each one of these passengers has his own domain of existence...on the same person!

1.2.3 The language of a first order many sorted logic and its interpretation

Following our resolution to consider only sorted individuals, we now describe a first order language which will be many sorted, will have a sort operator, a designator operator and two modal operators. The non-logical symbols will fall under the following three heads.

1. The constant sorts: with each "atomic" count noun (for instance, "person", "passenger" and "dog") we associate a constant sort.

2. The n-ary sorted relation symbols: a relational symbol R comes equipped with a natural number n: the number of places of R; R is called "n-ary relation symbol"; R comes also equipped with an assignment of a constant sort K_i $i = 1, \ldots, n$; K_i is the constant sort of the i^{th} place of R. The operative effect of this assignment will be that only variables of the right sort can occupy a given place when forming formulas using R. We write $R \subset K_1 \times \ldots \times K_n$ to indicate the sorting of R.

3. The n-ary sorted operation symbols: an operation symbol F comes equipped with a natural number n which indicates the number of places of F; F is called an n-ary operation symbol; F comes equipped with constant sort K_i $i = 1, \ldots, n$, K_i being called the constant sort of the i^{th} place of F, and also with an additional constant sort K, called the sort of the values of F. We write $F : K_1 \times \ldots \times K_n \longrightarrow K$.

We remark that 2. and 3. have been taken from Makkai and Reyes [21].

Let us give an example of an R symbol, the binary relation symbol *is taller than*, and let us suppose that the constant sorts are *dog* and *cat*. We have then that *is taller than* $\subseteq$ *dog* $\times$ *cat*. The function u described before, which could be thought of as the function "underlying" is an example of an operation symbol.

For each sort K we have an infinite set of variables of sort K and a set of constants of sort K; we denote the variables and constants respectively by $x^K, y^K, \ldots$, and $c^K, d^K, \ldots$. We will not write the indices when the context of use is clear. The logical symbols are: $\neg$, $\rightarrow$, $\leftrightarrow$, $\vee$, $\wedge$, $\forall$, $\exists$, $\Diamond$, $\Box$. We define by simultaneous recursion the notion of term relative to a sort, the notions of sort, subsort and formula.

1. The constants and variables of sort K are terms of sort K.

2. If F is an operation symbol and if $t_1, \ldots, t_n$ are terms of the right constant sorts $K_1, \ldots, K_n$ than $F(t_1, \ldots, t_n)$ is a term of constant sort K.

3. If R is a relation symbol and $t_1, \ldots, t_n$ are terms of the right constant sorts $K_1, \ldots, K_n$ then $R(t_1, \ldots, t_n)$ is a formula.

4. If t and s are terms of the same sort then $t = s$ is a formula. (More generally we could require that t and s belong to a sort and a subsort of that sort respectively.)

5. If ϕ and ψ are formulas so are $\neg\phi, \Box\phi, \Diamond\phi, \phi \rightarrow \psi, \phi \leftrightarrow \psi, \phi \vee \psi, \phi \wedge \psi$.

6. If ϕ is a formula and x^K a variable of sort K then $\exists x^K \phi$ and $\forall x^K \phi$ are formulas

7. If x^K is a variable of sort K and ϕ a formula then $\{x^K : \phi\}$ is a sort called a subsort of K.

8. If K is a sort, then $\dagger K$ is a term of sort K.

I give an example to illustrate 7.

Let $K = person$ and $\phi = sick(x^{person})$, $\{x^{person} : sick(x^{person})\}$ is the count noun "person who is sick" formed from the (atomic) count noun "person" and the formula $sick(x^{person})$. We could also form the following count noun: "person who is sick who is red" where $K = person\ who\ is\ sick$ and $\phi = red(x^{person\ who\ is\ sick})$, then we have

$$\{x^{person\ who\ is\ sick} : red(x^{person\ who\ is\ sick})\}.$$

We interpret the language by choosing an arbitrary non empty set W thought of as the set of "possible worlds"; we associate to each sort K a family $(|K|_w)_{w \in W}$ thought of as the Ks which exist at w, and an equivalence relation $=_K$ on $\coprod_{w \in W} |K|_w$ in terms of which we define

$$\|K\| = \bigcup_{w \in W} \|K\|(w).$$

Notice that $\|K\|(w)$ can be recovered from $\|K\|$ and w as the set $\alpha \in \|K\|$ satisfying $w \in dom(\alpha)$. We remark that Montague has only one sort U interpreted as $\|U\| = E^W$ where E is thought of as the set of "possible individuals" rather than existing individuals. We introduce as many sorts as there exist count nouns.

When Gupta considers K given with a principle of identity, he really studies objects of the form $(\|K\|, \delta)$ where $\delta : \|K\| \times \|K\| \longrightarrow 2^W$ given by $\delta(\alpha, \beta) = \{w \in dom\alpha \cap dom\beta : \alpha(w) = \beta(w)\}$ for all $\alpha, \beta \in \|K\|$. Intuitively δ expresses the extent to which α and β coincide. We say that $(\|K\|, \delta)$ is separated if

$$\delta(\alpha,\beta) = \begin{cases} dom(\alpha) = \delta(\alpha,\alpha) & \text{if } \alpha = \beta \\ \emptyset & \text{if } \alpha \neq \beta \end{cases}$$

Since for separated $(\|K\|, \delta)$ δ is completely determined by $\epsilon(\alpha) = \delta(\alpha, \alpha)$, I write sometimes $(\|K\|, \epsilon)$ rather than $(\|K\|, \delta)$. We interpret count nouns as sets obtained this way. Such interpretation follows the intuition that if two persons coincide in one possible world then they coincide in all possible worlds belonging to their domain of existence. Notice however that this does not cover all common nouns: as already mentioned words like "water" and "temperature" which are not count nouns are not analyzed in this study. Gupta's semantics differs from Montague's semantics in that for each w, only the set of existing individuals is considered. For each w, Montague considers the set of all possible individuals E. In Gupta's semantics the $| K |_w$s may differ.

We now interpret the sorts, the sorted constants, the operation sorted symbols and the relation sorted symbols as follows:

1. Each constant sort K is interpreted as a separated set $(\|K\|, \epsilon_K)$

2. Each constant c of sort K is interpreted as a function

$$\|c\| : W \longrightarrow \|K\|$$

   such that $\|c\|(w) \in \|K\|(w)$

3. Each sorted operation symbol $F : K_1 \times \ldots \times K_n \longrightarrow K$ is interpreted as a map

$$\|F\| : \|K_1\| \times \ldots \times \|K_n\| \longrightarrow \|K\|$$

   satisfying the condition

$$\epsilon_{K_1}(\alpha_1) \cap \ldots \cap \epsilon_{K_n}(\alpha_n) \subseteq \epsilon_K(\|F\|(\alpha_1, \ldots, \alpha_n))$$

4. Each sorted relation symbol $R \subseteq K_1 \times \ldots \times K_n$ is interpreted as a map

$$\|R\| : \|K_1\| \times \ldots \times \|K_n\| \longrightarrow 2^W$$

   satisfying the condition

$$\|R\|(\alpha_1, \ldots, \alpha_n) \subseteq \epsilon_{K_n}(\alpha_1) \cap \ldots \cap \epsilon_{K_n}(\alpha_n)$$

I shall give an example of a constant, an operation symbol and a relation symbol.

To interpret a sorted constant such as "Socrates", as well as to interpret descriptions, we need a function

$$i^{*}_{person} : W \longrightarrow \|person\|.$$

such that $i^*_{person}(w) \in \|person\|(w)$ should be thought of as "the non-existent *person*". In terms of i^* we define the interpretation of "Socrates" as the function

$$\|Socrates\| : W \longrightarrow \|person\|$$

in the following way:

$$\|Socrates\|(w) = \begin{cases} Socrates \in \|person\| & w \in dom(Socrates) \\ i^*_{person}(w) & \text{if } w \notin dom(Socrates) \end{cases}$$

We have seen how to construct $(\|person\|, \epsilon)$ and $(\|passenger\|, \epsilon)$. We interpret the operation sorted symbol

$$u : passenger \longrightarrow person \text{ as } \|u\| : \|passenger\| \longrightarrow \|person\|$$

where $\|u\|(\alpha^{passenger}_{<pa,w>}) = \alpha^{person}_{<pa,w>}$. Clearly, the condition

$$\epsilon_{passenger}(\alpha^{passenger}_{<pa,w>}) \subseteq \epsilon_{person}(\|u\|(\alpha^{passenger}_{<pa,w>}))$$

is satisfied.

The relation sorted symbol *sick* $\subseteq$ *person* is interpreted as follows

$$\|sick\| : \|person\| \longrightarrow 2^W$$

which satisfies the condition

$$\|sick\|(\alpha^{person}_{<p,w>}) \subseteq \epsilon_{person}(\alpha^{person}_{<p,w>}).$$

Intuitively, the person is sick at most as long as she (or he) exists. The binary relation sorted symbol *is taller than* $\subseteq$ *tree* $\times$ *flower* is interpreted as

$$\|is\ taller\ than\| : \|tree\| \times \|flower\| \longrightarrow 2^W$$

which satisfies the condition

$$\|is\ taller\ than\|(\alpha, \beta) \subseteq \epsilon_{tree}(\alpha) \cap \epsilon_{flower}(\beta)$$

An assignment is a function $g : Var_K \longrightarrow \|K\|$ for each K. For a given assignment, we interpret sorts, terms and formulas by recursion in the usual fashion.

Given an assignment, we define forcing as a relation between a possible world and a formula as follows:

$$w \Vdash \phi(x_1^{K_1}, \ldots, x_n^{K_n})[g]$$

where $g(x_i^{K_i}) \in \|K_i\|(w)$ (equivalently $w \in dom(g(x_i^{K_i})) = \epsilon(g(x_i^{K_i}))$

I give some examples to illustrate the interpretation of formulas.

1. $w \Vdash x^K = y^K[g]$ if and only if $w \in \delta(g(x^K), g(y^K))$ which is the case just when
$$g(x^K)(w) = g(y^K)(w)$$

2. $w \Vdash u(x^{passenger}) = y^{person}[g]$ iff $\|u\|(g(x^{passenger})(w) = g(y^{person})(w)$ since $(\|person\|, \delta)$ is separated we can conclude that
$$\|u\|(g(x^{passenger})) = g(y^{person})$$

3. If $R \subseteq K_1 \times \ldots \times K_n$ and $w \in \epsilon(g(x_i^{K_i}))$ forall i, then
$$w \Vdash R(x_1^{K_1}, \ldots, x_n^{K_n})[g] \text{ iff } w \in \|R\|(g(x_1^{K_1}), \ldots, g(x_n^{K_n}))$$

4. $w \Vdash \neg\phi(x_1^{K_1}, \ldots, x_n^{K_n})[g]$ iff $w \not\Vdash \phi(x_1^{K_1}, \ldots, x_n^{K_n})[g]$

5. $w \Vdash \exists x^K \phi(x^K)[g]$ iff $\exists \alpha \in \|K\|(w)\ w \Vdash \phi(x^K)[g(\alpha/x^K)]$

6. $w \Vdash \Box\phi(x_1^{K_1}, \ldots, x_n^{K_n})[g]$ iff
$$\forall w' \in \bigcap_{x_i^{K_i} \in FV(\phi)} \epsilon(g(x_i^{K_i})) \quad w' \Vdash \phi(x_1^{K_1}, \ldots, x_n^{K_n})[g].$$

The restriction of the quantifier to the intersection means that we do not look at all the possible worlds but rather at those that belong to the common domain of existence of the $g(x_i^{K_i})$.

Proposition 1.2.3.1 $x^K = y^K \longrightarrow \Box(x^K = y^K)$

Proof. $w \Vdash ((x^K = y^K) \longrightarrow \Box(x^K = y^K))[g]$ if and only if

$$w \Vdash (x^K = y^K)[g] \Rightarrow w \Vdash \Box(x^K = y^K)[g]$$

if and only if

$$g(x^K)(w) = g(y^K)(w) \Rightarrow \forall w' \in \epsilon(g(x^K)) \cap \epsilon(g(y^K))\ w' \Vdash x^K = y^K[g].$$

Since $(\|K\|, \delta)$ is separated,

$$g(x^K)(w) = g(y^K)(w) \text{ iff } g(x^K) = g(y^K)$$

implies that $g(x^K)(w') = g(y^K)(w')$. $\square$

I now show that one half of the Barcan rule is satisfied, namely

$$1. \quad \Box\forall x\phi(x) \longrightarrow \forall x\Box\phi(x).$$

Then, I construct a model showing that the other half of the Barcan rule, namely

$$2. \quad \forall x\Box\phi(x) \longrightarrow \Box\forall x\phi(x)$$

is not satisfied.

1. Let M be a model and w a possible world. We must show that

$$w \Vdash \Box\forall x^K\ \phi(x^K)[g] \Longrightarrow w \Vdash \forall x^K\ \Box\phi(x^K)[g]$$

Let $\alpha \in \|K\|(w)$ and $w' \in \epsilon(\alpha)$ then we must show that $w' \Vdash \phi(x^K)[g(\alpha/x^K)]$. But by hypothesis, $w' \Vdash \forall x^K\ \phi(x^K)[g]$ since $w' \in \epsilon(g(x^K))$. We notice that

$$w' \in \epsilon(g(x^K)) = \epsilon(\alpha) \Longrightarrow \alpha \in \|K\|(w').$$

In particular, then $w' \Vdash \phi(x^K)[g(\alpha/x^K)]$.

2. Let $W = \{0,1\}$ and $|\ A\ |_0 = \{p_0\}$ and $|\ A\ |_1 = \{c_1, p_1\}$ we define $(\|A\|, \delta)$ where $\|A\| = \{c,p\}$, $p(0) = p_0$, $p(1) = p_1$, $c(1) = c_1$, it is clear that $(\|A\|, \delta)$ is separated. We see that $\|A\|(0) = \{p\}$ and that $\|A\|(1) = \{c,p\}$. We consider the following predicate:

$$\|P\| : \|A\| \longrightarrow 2^{\{0,1\}}$$

where

$$\|P\|(c) = \emptyset \subseteq \epsilon(c) = \{1\} \text{ and } \|P\|(p) = \{0,1\} \subseteq \epsilon(p) = \{0,1\}.$$

Now let us prove that

$$0 \not\Vdash (\forall x^A \Box P(x^A) \longrightarrow \Box \forall x^A P(x^A))[g]$$

or equivalently that

$$0 \Vdash \forall x^A \Box P(x^A)[g] \not\Longrightarrow 0 \Vdash \Box \forall x^A P(x^A)[g]$$

or that

$$0 \Vdash \forall x^A \Box P(x^A)[g] \text{ and } 0 \not\Vdash \Box \forall x^A P(x^A)[g].$$

Let us analyze the two members of this conjunction.

(a) $0 \Vdash \forall x^A \Box P(x^A)[g]$ if and only if

$$\forall \alpha \in \|A\|(0)\ 0 \Vdash \Box P(x^A)[g(\alpha/x^A)]$$

if and only if

$$\forall \alpha \in \|A\|(0) \forall w \in \epsilon(g(x^A))\ w \Vdash P(x^A)[g(\alpha/x^A)].$$

But since there is only p at $\|A\|(0)$, we have

$$\forall w \in \epsilon(p) = \{0,1\},\ w \Vdash P(x^A)[g(p/x^A)].$$

Hence, we obtain $0 \Vdash P(x^A)[g(p/x^A)]$ and $1 \Vdash P(x^A)[g(p/x^A)]$. We conclude that $\|P\|(p) = \{0,1\}$ which is true by the definition of P.

(b) $0 \not\Vdash \Box \forall x^A P(x^A)[g]$, let us suppose that $0 \Vdash \Box \forall x^A P(x^A)[g]$. We then have that

$$\forall w\ w \Vdash \forall x^A P(x^A)[g]$$

if and only if

$$\forall w \forall \alpha \in \|A\|(w)\ w \Vdash P(x^A)[g(\alpha/x^A)].$$

There are two cases to consider.

1. $w = 0, p \in \|A\|(0)\ 0 \Vdash P(x^A)[g(p/x^A)]$ hence $0 \in \|P\|(p)$.

2. $w = 1, \{p, c\} \doteq \|A\|(1)\ 1 \Vdash P(x^A)[g(p/x^A)]$ hence $1 \in \|P\|(p)$ but $1 \not\Vdash P(x^A)[g(c/x^A)]$ hence $1 \notin \|P\|(c) = \emptyset$ and we conclude that

$$0 \not\Vdash \Box \forall x^A P(x^A)[g].$$

We can imagine that A translates "animals on Joe's farm" and P translates "is a pig". We can read this affirmation as follows: "Every animal on Joe's farm is necessarily a pig" does not imply that "necessarily every animal on Joe's farm is a pig". Gupta [14, page 43] explains this as follows:

> *For although it may be true that "Every animal on Joe's farm is essentially a pig", for on Joe's farm there are only pigs, it does not follow that "It is necessary that every animal on Joe's farm is a pig". Joe might also have grown chickens on his farm.*

Gupta says that the two halves of the Barcan rule are false in his system. To show that 1. is false he gives the following example: "It is necessary that every bachelor is unmarried" is true, but "Every bachelor is necessarily unmarried" is false. I believe that the trouble with this example and with similar examples found in the literature is the following: since variables are sorted, the formalization of these sentences in our language is not uniquely determined. For instance, if "It is necessary that every bachelor is unmarried" is formalized as $\Box\forall x\ unmarried(x)$, where "x" is a variable of sort *bachelor* and "Every bachelor is necessarily unmarried" is formalized as $\forall x\Box\ unmarried(x)$, then our argument that 1. is true implies that the purported counterexample of Gupta is not a genuine one. On the other hand, we might formalize "It is necessary that every bachelor is unmarried" as $\Box\forall x\ bachelor(x) \longrightarrow unmarried(x)$, where "x" is a variable of sort *man*. If furthermore, "Every bachelor is necessarily unmarried" is formalized as $\forall x\, bachelor(x) \longrightarrow \Box unmarried(x)$, then Gupta's counterexample is correct and the argument that the first sentence implies the second is an example of what Mates [23, page 117] calls the "fallacy of the slipped modal operator". This example shows that formalization of sentences of natural languages has to be handled carefully.

We remark that any formula $\phi(x_1^{K_1},\ldots,x_n^{K_n})$ gives rise to a map

$$\|\phi(x_1^{K_1},\ldots,x_n^{K_n})\| \ : \|K_1\| \times \ldots \times \|K_n\| \longrightarrow 2^W$$

defined by

$$\|\phi(x_1^{K_1},\ldots,x_n^{K_n})\|(\alpha_1,\ldots,\alpha_n) =$$
$$\{w \in \bigcap_{i=1,\ldots,n} domg(x_i^{K_i}) \ : \ w\Vdash\phi(x_1^{K_1},\ldots,x_n^{K_n})[g(\alpha_i/x_i^{K_i})]\}.$$

This allows us to define the interpretation of a subsort $\{x^K : \phi\}$ as $(\|K\|,\delta_\phi)$ where $\delta_\phi \ : \|K\| \times \|K\| \longrightarrow 2^W$ and $\delta_\phi(\alpha,\beta) = \|\phi(x)\|(\alpha) \cap \delta(\alpha,\beta)$. Let us, for example, consider the sort *person* and the unary formula x *is sick*. We are restricting the domain of existence of a person to the extent that the person is sick (her domain of sickness, so to speak) to obtain the sort *person who is sick*.

The interpretation of the description $\dagger K$ is the function

$$\| \dagger K \| : W \longrightarrow \|K\|$$

defined by

$$\| \dagger K\|(w) = \begin{cases} \alpha & \text{if } \|K\|(w) = \{\alpha\} \\ i^*(w) & \text{if } \|K\|(w) \text{ is not a singleton.} \end{cases}$$

Notice that $\| \dagger K\|(w) \in \|K\|(w)$ for all w. In terms of descriptions we can think of the interpretation of "Socrates" already given as being the interpretation of the description "the person who is Socrates".

1.2.4 Criticisms and conclusion

Gupta works with first order logic only. I have already pointed out, in section 1.1.1 the necessity of having a higher order logic, and I shall not elaborate on that point here. Furthermore, Gupta deals only with necessarily existent objects. Usual kinds such as **person** and **dog** have members that come in and go out of existence. In my presentation, I have eliminated this limitation by introducing the coincidence relation and the predicate of existence.

Another difficulty with Gupta's approach is his notion of "possible K", where K is a kind. This notion is needed to define modal constancy of the interpretations of some count nouns. For instance, **person** is modally constant if and only if possible persons are persons. In my view, this notion of "possible K" is not cogent, as I shall argue presently. In my theory, every kind will be modally constant by definition, so to speak, and "possibility" and "necessity" will be applied only to predicates of kinds and not to kinds themselves. This way of considering "possibility" (and "necessity") agrees with the grammar of these notions. Suppose that we find an archeological site with skeletons of some anthropoids. If we are asked whether some are hominoids, we could naturally reply that "three of these skeletons are possibly hominoid skeletons" or "it is possible that three of these skeletons are hominoid skeletons", but we would not say "there are three possible hominoid skeletons". Similarly, we do not say that Mr. and Ms. X have twelve possible children, but rather that it is possible for Mr. and Ms. X to have twelve children. Independently of grammar, I question the cogency of this notion of "possible K" on the basis that neither the principle of application nor the principle of identity is well defined for them. In other words, I deny that "possible K" is a kind. As regards the principle of application, does an apple jelly count as a possible apple? Does a piece of junk of metal count as a possible car? As regards the principle of identity, Quine has already asked the relevant question: How many fat men are there in the door?

I can recover some of the intuitions of Bressan and Gupta on modal constancy by applying this notion to predicates rather than to kinds, which are for me, as already mentioned, automatically modally constant. Thus we could say, for instance, that "to be an apple" is modally constant as a predicate of `fruit`, although presumably will not be modally constant as a predicate of `ingredient in a recipe`.

This analysis shows the importance of distinguishing sharply between kinds and predicates of kinds.

Another problem that I detected in Gupta's analysis of common nouns is his distinction between common nouns such as "man" on the one hand and "man born in Jerusalem" on the other. According to Gupta [14, page 35], common nouns such as "man", "number" and "river" express sorts that give essential properties of objects, whereas "man born in Jerusalem" does not express such a sort, because "being born in Jerusalem" is not an essential property of any man. This puts the notorious problem of untangling essential and not essential properties at the very foundation of his theory. In the theory that I develop in Chapter 2 this problem is avoided.

A further problem with Gupta's semantics is his notion of individual concepts. I think that individual concepts introduce irrelevant problems at the foundational level and should be avoided. Let us recall that in the usual semantics (Montague [24], Gupta [14], Scott [32]), kinds are envisaged as sets whose members are individual concepts which are total or partial functions defined over situations. The trouble with this view is simply this: what is the principle of identity for kinds? Clearly, identity of two functions means identity of their values, but where do the values live, say for the kind **person**? We seem to need a kind other than **person** with new principles of application and identity to receive the values of the individual concepts. But **person** was precisely the kind needed to provide a notion of identity for their members! Furthermore, this new kind should again be a set of individual concepts whose principle of identity should be defined in terms of a new kind, etc.

On further reflection, we see that individual concepts are employed to do two related things for us: they specify the "domain of definition" of the member in question which is the domain of the function, and they specify when two members coincide or are coincident, which is given by the identity of the two functions. We shall see that there is no need to introduce individual concepts, and that whatever can be achieved with them can be achieved without them provided the notion of coincidence is taken as primitive.

Finally, although I have used the terms "principle of application" and "principle of identity" to follow Gupta, I believe that these notions are problematic, precisely because they seem to require unsorted entities or objects for their formulation. Thus, Gupta tells us:

> *Common nouns, like predicates, are true or false of objects. They divide all the objects in the world into two classes: those objects that fall under them, and those that do not. That is, common nouns, like predicates, supply a* principle of application.

I shall return to this question in Chapter 2.

Chapter 2

The semantics of kinds

I shall first explain the motivation that brought me to the logical system described in this chapter. Then I develop this logical system which is both *higher order* and *many sorted.*

2.1 Introduction

My logical system is based on an attempt to formalize the notions of reference and generality in natural languages (to borrow from the title of Geach [13]). More precisely, it is concerned with relations of the kind "A refers to B", where A is a proper name, and "A is true", "A is possible", where A is a statement. It is a remarkable property of natural languages that a proper name, say "John", picks up its reference uniquely every time that it is uttered, regardless of whether its reference is present at the moment of utterance, regardless of whether we know John's whereabouts at that moment, whether or not we are able to recognize John and whether or not we are referring to events that took place in the past or may take place in the future. My main concern will be the following *representation problem*: What structure should the reference have to accomplish the formidable task just described? I shall propose an answer based on an analysis of count nouns and kinds which are the interpretations of such count nouns along the lines of Gupta [14] and Macnamara [20], which in turn build on previous work of Geach [13], Bressan [5] and others.

The emerging picture is not literally the same as that presented by these authors: there are several points of disagreement (some minor, some major), but several of the basic intuitions are, I believe, the same.

2.2 Count nouns and kinds

I develop my analysis of count nouns and kinds by stating and discussing a series of *theses on reference.* Although this medieval practice has long gone out of fashion I believe it useful to understand the issues involved. My first thesis concerns proper names.

1. The reference of a proper name is rigid.

This thesis asserts that a proper name, say "Nixon" has the property of picking up its reference throughout all actual as well as possible situations, past, present and future in which Nixon appears or may appear. A biographer of Nixon would recognize the boy who grew up in California, the young politician who won his first election and the president who was forced to resign from office as being one and the same person in spite of the different times and situations. To explain Nixon's actions, the biographer should entertain a series of possibilities as to Nixon's motives and evaluate them critically. So he is forced to consider counterfactual situations about this very person, Nixon. For instance, he may be unsure of whether Nixon won his first election by fraudulent means, and he may discuss both the possibility that he used such means and the possibility that he did not to arrive at the truth of the matter. The thesis of rigidity has been forcefully argued by Kripke [17].

The fact that counterfactual situations have to be considered to describe real situations is exemplified in Classical Mechanics and in the Calculus of Variations: to describe the real trajectory of a body, we compute the Lagrangian of all its possible trajectories (most of which are not physically possible!) and we take as the real one that trajectory for which the Lagrangian has a stationary value. The possible is needed to describe the real. I shall restrict my attention to counterfactual situations which are consistent. I do not view proofs by reductio ad absurdum as constructing "counterfactual situations" in mathematics. For instance, in the proof that the square root of 2 is not a rational number, we start by assuming that the square root of 2 is a rational number and then we derive a contradiction. However there cannot be a situation where this assumption holds, since such an assumption is logically inconsistent.

2. Rigidity presupposes count nouns.

This thesis, which characterizes the approach I am developing, asserts that the only way of tracing the identity of Nixon throughout all real and possible situations (past or present), is by means of a count noun such as "person" or "man". Indeed, the boy in California, the young politician and the president remained one and the same person, although he successively stopped being a boy, a young politician and a president. We cannot, for instance, trace Nixon's identity through the molecules that compose his body, since those that constituted his body at the time of his youth were different from those that constituted his body at the time that he was a president. As Aristotle emphasized, change requires something to change. There must be a constancy that underlies the change, for instance as I just said, the boy changes and becomes an adult, but to understand the link between the two there must be something that stays the same. In fact, it is the same person who undergoes all these metamorphoses. To make this thesis more precise, I shall state claims on the interpretations of count nouns, the kinds.

Kinds are the interpretation of count nouns such as "person" , "dog" and "atom of oxygen". There are other nouns besides count nouns, for instance mass nouns such as money, clay, copper and oxygen, but we associate kinds only with count nouns. We can interpret expressions such as "three dogs", "two drops of water" and "three atoms of oxygen", as well as expression of generality such as "every dog", but we cannot interpret "three oxygens" or "two moneys". To interpret expressions of the first type, kinds should consist of members which are individuated and could be counted. This feature of kinds allows us, more generally, to interpret quantifiers and the equality symbol and thus to interpret first order languages.

3. Kinds are modally constant.

What this thesis says is that kinds are what remains "constant" from situation to situation and thus kinds are precisely the standard against which we can understand changes and modalities. In other words, kinds should be modally constant "by definition". Thus, man is a constant set, a set which does not change according to times and situations. As a consequence, the statement "all men are greedy" makes reference to all men independently of any situation. It will be true at a given situation independently of the men who appear in that situation, since it does not refer only to those men who appear in the situation, but to all men. On the other hand, man in

the Salle Wilfred Pelletier of the Place des Arts is an example of a variable set which changes according to times and circumstances. I believe that the context in which these notions can be adequately formulated is topos theory, but I will not go into these matters here. A discussion of modal constancy in this context can be found in Reyes [29].

4. Modalities apply only to predicates of kinds and not to kinds themselves.

This thesis concerns "the grammar" of modalities. Modalities such as possibility and necessity have been applied to predicates of kinds such as "x used fraudulent means to win the election" which is a predicate of the kind **person**, to form new predicates such as "x possibly used fraudulent means to win the election" which is again a predicate of **person**. Nevertheless, modalities have not been applied only to predicates but also to kinds themselves to form new "kinds" like **possible apple** or **possible man** (see Gupta [14]). The thesis claims that these attempts are wrongheaded and that modalities may be applied to predicates only. The thesis that modalities are not applicable to kinds is made plausible by the thesis 3., since kinds are precisely what remains constant in real and possible transformations. Independently of this, there are serious difficulties with any attempt to view "possible apple" as a count noun as we saw in Chapter 1.

5. The existence of opaque contexts is a normal and common feature of natural languages.

In spite of Frege, who thought the contrary, we know today that opaque contexts, also called non-referential contexts, can be generated with quite innocent looking expressions of the type "with Fred", "for Mary", "in the park", etc., (see Keenan and Faltz [16]) and not only from "epistemic" or "modal" operators. To mention just one example: it may happen that the people who are working in a room are precisely those who are talking. However it does not follow that the people who are working with Fred are those who are talking with Fred. These examples can be repeated ad infinitum, even with expressions like "in the park", contrary to the expectations of Keenan and Faltz.

It is not cogent to consider examples of "lack of substitutivity of equals for equals" as examples of two identical members of a kind having different properties! It follows that we need another notion, besides identity, to represent "equality" in these common phenomena.

Therefore, we postulate that kinds should come equipped with a relation of coincidence which stipulates when two members of a kind happen to coincide at a given situation. For an example of coincidence, take the kind proposition. Two different propositions may coincide at a situation in the sense that they have the same truth values for all situations which contain more information than the original one. We shall say that these propositions are coincident at that situation. Another example is given by the kind predicate of person in the above example, it may happen that the predicate expressed by "to be working" coincides at a given situation with the predicate expressed by "to be talking" in the sense that the people who are working are precisely those who are talking in every situation with more information than the original one, but clearly these predicates are not identical. The paradigm of coincidence in mathematics is given by two distinct functions f and g which have the same value at a given point, for instance, two functions defined on the reals which have the same value at 0. In this case we say that f and g coincide at 0 and it may well happen that in spite of this coincidence, f has the property of being increasing at 0, whereas g may be decreasing at 0. This example seems to have motivated the introduction of individual concepts in intensional logic to handle the existence of opaque contexts. Nevertheless, I do not consider individual concepts, since they present foundational problems as was shown in Chapter 1.

6. Kinds do not exhaust the semantical universe.

Central as they are in reference, kinds are too discrete to either exhaust or even generate the semantical universe. Indeed, the prototype of a kind in mathematics is natural number and kinds share the discreteness of natural numbers, precisely to accomplish the tasks that we require from them. On the other hand, space-time, which should be a member of the semantical universe is a connected structure and there is no good reason to suppose that we could define such a structure if we start from discrete kinds. The possibility of this "construction" of connected structures from discrete ones is analogous to the program of the "Arithmetization of Analysis" and may be subjected to the same criticisms. At any rate, my semantics should allow room for both connected and discrete structures and should allow means to compare them and study their relations. I believe that these desiderata offer a more fruitful approach, than reductionism.

I finish this section by pointing out that I will consider possible situations rather than possible worlds as the building blocks of my semantics. I view situations as interpretations of actual or possible partial descriptions of reality. The possibility of carrying out an action, for instance, depends only on the situations which are relevant to the action in question and not on the whole state of the universe. There is another argument for situations rather than possible worlds as a foundation for semantics: situations are ordered by inclusion and thus they are more structured. The situation, for instance, of John sitting on a sofa while his dog sits at his feet includes the situation of John sitting on a sofa. This intuition of a natural partial order between situations does not apply to possible worlds, since they are supposed to be the interpretations of complete possible descriptions of reality. For this reason, I shall avoid the expression "possible world" in the sequel. Let us notice that both Kripke's theory of the "idealized mathematician" and Husserl's notion of the horizon of an act use similar orderings: ordering between states of knowledge in one case, and ordering between determinations of the object of the act in the other.

2.3 The category of kinds

I propose a mathematical formalization of the notion of kind in the context of category theory. It is my conviction, to rephrase Montague [24], that philosophy, at this stage in history, has as its proper theoretical framework category theory. The very possibility of using category theory imposes, I believe, fruitful constraints on the logic of reference, to mention just one of them: the logic of quantification should be the standard one. Thus my approach will differ from others which change the logic of quantifiers to accommodate the modal operators, such as Montague's approach for instance.

The category of kinds is based on a partially ordered set that I think of as "possible situations". I view situations as interpretations of actual or possible partial descriptions of reality. My starting point is then a set $\mathcal{P} = (P, \leq)$ of "possible situations" partially ordered by inclusion: $V \leq U$ whenever V contains whatever "goes on" in U. In other words, considering situations as interpretations of partial descriptions of reality, V interprets a more complete description than U.

We define $\Omega(1) = \{D \subseteq P : D \text{ is downward closed}\}$. In other words, the members of $\Omega(1)$ are sets D of situations such that whenever $U \in D$ and

$V \leq U$, then $V \in D$. Whenever a situation belongs to D, so do all the situations which include it.

Proposition 2.3.0.1 *$\Omega(1)$ is a complete Heyting algebra.*

Proof. It is immediate that $\Omega(1)$ is closed under arbitrary unions and intersections. We define the implication as follows:

$$D_1 \Rightarrow D_2 = \bigcup\{D \in \Omega(1) \mid D \cap D_1 \subseteq D_2\}$$

It is easy to check the usual adjointness

$$D \subseteq D_1 \Rightarrow D_2 \quad \text{iff} \quad D \cap D_1 \subseteq D_2$$

Defining $0 = \emptyset$ and $1 = P$, we have that $(\Omega(1), \bigcup, \cap, \Rightarrow, 0, 1)$ is a distributive lattice with 0, 1, implication, and closed under arbitrary sups (and infs), namely, it is a complete Heyting algebra. □

We remark that in the symbol $\Omega(1)$, (1) is part of the symbol and does not have any independent meaning.

A *kind* is a couple (A, δ_A), where A is a set and δ_A is a *coincidence relation*, namely, a map $\delta_A : A \times A \longrightarrow \Omega(1)$ satisfying the properties:

$$\delta_A(a, a') = \delta_A(a', a)$$

$$\delta_A(a, a') \cap \delta_A(a', a'') \subseteq \delta_A(a, a'')$$

In terms of the coincidence relation, we can define $\epsilon_A : A \longrightarrow \Omega(1)$ by letting $\epsilon_A(a) = \delta_A(a, a)$.

A kind is then defined as a couple consisting of a set A together with a rule which associates, with every pair of elements of the set A, the set of situations in which the elements in question occur and happen to coincide. Thus, a member is coincident with itself precisely in those situations in which it occurs.

As an example, **proposition** is a kind, namely the set of propositions, together with the following coincidence relation: two propositions coincide at a situation if they have the same truth value at all situations having more information that the given one. The ϵ corresponding to the relation of coincidence associates $1 \in \Omega(1)$ with each proposition. We notice that if U is a situation in which two propositions have the same truth values and $V \leq U$, then V is also a situation in which the two propositions have the same truth value, since V is the interpretation of a more complete description of reality than U. This justifies our assumption that $\delta_A(a, a')$ is a downward closed subset of P, namely, a member of $\Omega(1)$.

As I mentioned in Chapter 1, I do not need, and do not want, individual concepts in order to develop my semantics, since coincidence, and therefore existence is built into the notion of a kind.

Besides kinds, I consider relations between the kinds, namely functions which preserve the coincidence relation. For instance, the connection between the kinds **baby**, **adult** and **person** is seen in my context as follows: we have a function, which is not representable in the natural language, from the set of babies into the set of persons, and similarly, from the set of adults into the set of persons, intuitively the function that associates with each baby or adult the person that underlies him. The identity between a baby and an adult is really an assertion about their underlying persons, namely that they are identical as members of the kind **person**.

Kinds constitute a *category* under the following definition of morphism: a *morphism* $f : (A, \delta_A) \longrightarrow (B, \delta_B)$ is a function $f : A \longrightarrow B$ such that $\delta_A(a, a') \subseteq \delta_B(f(a), f(a'))$ for all $a, a' \in A$.

As an example, consider the kinds **passenger** and **person**. We view their connections as follows: there is a function u from the set of passengers to the set of persons which associates with each passenger its "underlying" person. Then, u is a morphism of the category of kinds if at a given situation in which two passengers are coincident, which is the same as being equal in this case, then so are the "underlying" persons.

The identity function $1_A : A \longrightarrow A$ is a morphism

$$1_A : (A, \delta_A) \longrightarrow (A, \delta_A)$$

since $\delta_A(a, a') \subseteq \delta_A(1_A(a), 1_A(a')) = \delta_A(a, a')$.

The composition of two morphisms is again a morphism. Let

$$f : (A, \delta_A) \longrightarrow (B, \delta_B) \text{ and}$$

$$g : (B, \delta_B) \longrightarrow (C, \delta_C)$$

be two morphisms then $g \circ f$ is a morphism: we have

$$\delta_A(a, a') \subseteq \delta_B(f(a), f(a')),$$

since f is a morphism, and we have

$$\delta_B(f(a), f(a')) \subseteq \delta_C(g(f(a)), g(f(a'))),$$

since g is a morphism, hence we obtain

$$\delta_A(a, a') \subseteq \delta_C((g \circ f)(a), (g \circ f)(a')).$$

We remark that $(\Omega(1), \Leftrightarrow)$ is the kind **proposition**. The reasons for this will be clear later on.

Let $\mathcal{S}(\Omega(1))$ be the *category of kinds.*

Proposition 2.3.0.2 *$\mathcal{S}(\Omega(1))$ is a cartesian closed category.*

Proof. What this says is that $\mathcal{S}(\Omega(1))$ is a category having a terminal object 1, products and exponentials.

In fact, $1 = (\{*\}, \delta_1)$, where $\delta_1(*, *) = 1 \in \Omega(1)$ and the product of (A, δ_A) with (B, δ_B) is the diagram

$$(A, \delta_A) \xleftarrow{\pi_A} (A \times B, \delta_{A \times B}) \xrightarrow{\pi_B} (B, \delta_B)$$

where $\delta_{A \times B}((a, b), (a', b')) = \delta_A(a, a') \cap \delta_B(b, b')$, as can be easily checked.

The exponential of the kind (B, δ_B) to the kind (A, δ_A) is the diagram

$$(B^A, \delta_{B^A}) \times (A, \delta_A) \xrightarrow{ev} (B, \delta_B)$$

where

$$\delta_{B^A}(f, f') = \bigcap_{a, a' \in A} \{\delta_A(a, a') \Rightarrow \delta_B(f(a), f(a'))\}$$

and $ev(f, a) = f(a)$. In fact, to see this, we have to check the universal property of the morphism ev only.

Let $(C,\delta_C) \times (A,\delta_A) \xrightarrow{\phi} (B,\delta_B)$ be a morphism. We have to show the existence of a unique morphism $(C,\delta_C) \xrightarrow{h} (B^A,\delta_{B^A})$ such that the diagram

$$\begin{tikzcd}
(B^A,\delta_{B^A}) \times (A,\delta_A) \arrow[rr, "ev"] & & (B,\delta_B) \\
(C,\delta_C) \times (A,\delta_A) \arrow[u, "h \times 1_A"] \arrow[urr, "\phi"'] & &
\end{tikzcd}$$

commutes.

Notice that since morphisms are functions, such an h exists uniquely as a function, namely the exponential transpose of ϕ and we need only check that h is a morphism. But ϕ is a morphism, namely,

$$\delta_{C\times A}((c,a),(c',a')) \subseteq \delta_B(\phi(c,a),\phi(c',a'))\ \forall (c,a),(c',a') \in C \times A.$$

By definition of $\delta_{C\times A}$, we conclude that

$$\delta_C(c,c') \cap \delta_A(a,a') \subseteq \delta_B(\phi(c,a),\phi(c',a'))\ \forall c,c' \in C,\ \forall a,a' \in A.$$

By the adjointness property of $\Rightarrow$, this last relation is equivalent to

$$\delta_C(c,c') \subseteq \delta_A(a,a') \Rightarrow \delta_B(\phi(c,a),\phi(c',a'))\ \forall c,c' \in C,\ \forall a,a' \in A.$$

But $\phi(c,a) = h(c)(a)$ by definition of h and therefore

$$\delta_C(c,c') \subseteq \bigcap_{a,a' \in A} \{\delta_A(a,a') \Rightarrow \delta_B(h(c)(a),h(c')(a'))\} \text{ for all } c,c' \in C$$

But then, $\delta_C(c,c') \subseteq \delta_{B^A}(h(c),h(c'))$ for all $c,c' \in C$, by the very definition of δ_{B^A}, namely, h is a morphism. □

We remark that there is a *forgetful functor*

$$\mathcal{U} : \mathcal{S}(\Omega(1)) \longrightarrow \mathcal{S}$$

defined by $\mathcal{U}(A,\delta_A) = A$ and if $f : (A,\delta_A) \longrightarrow (B,\delta_B)$ then $\mathcal{U}(f) = f$ where $f : A \longrightarrow B$ and $\mathcal{S}$ is the category of sets.

Proposition 2.3.0.3 *The forgetful functor $\mathcal{U} : \mathcal{S}(\Omega(1)) \longrightarrow \mathcal{S}$ has both a left and a right adjoint and it preserves exponentials. In particular, it preserves the cartesian closed structure of $\mathcal{S}(\Omega(1))$.*

Proof. The left adjoint $(L \dashv \mathcal{U})$ is given by $L(A) = (A, \delta_\emptyset)$, where $\delta_\emptyset$ is defined by $\delta_\emptyset(a, a') = 0$ for all $a, a' \in A$ and the right adjoint $(\mathcal{U} \dashv R)$ is given by $R(B) = (B, \delta_1)$ where δ_1 is defined by $(\delta_1)(b, b') = 1$ for all $b, b' \in B$ with the obvious actions on functions $(L(f) = R(f) = f)$.

The fact that the forgetful functor preserves exponentials is obvious from the characterization of exponentials. □

2.4 The language of modal higher order theory and its interpretation

In this section I introduce the language of modal higher order theory and interpret it in the category of kinds $\mathcal{S}(\Omega(1))$. I define *sorts*, *terms* and *formulas* by recursion as follows:

Sorts

1. *Basic sorts* are *sorts*: passenger, person, dog, reading, etc.
2. Ω (proposition) is a *sort*
3. If X, Y are *sorts*, so are $X \times Y$ and Y^X
4. Nothing else is a *sort*.

Terms of a given sort are defined by recursion as follows, where we use $t : X$ as an abbreviation for "t is a term of sort X":

1. *Basic constant terms* $c \in Con_X$ are *terms* of sort X, for instance, $Mary \in Con_{\text{person}}$, run, $(to\ be\ a)\ bachelor \in Con_{\Omega^{\text{person}}}$, $owner \in Con_{\Omega^{\text{person} \times \text{dog}}}$, $underlying \in Con_{\text{person}^{\text{bachelor}}}$, etc.
2. If $x \in Var_X$, then x is a *term* of sort X, where Var_X is an infinite set, for each sort X
3. If $t : X$ and $s : Y$, then $< t, s >: X \times Y$
4. If $x \in Var_X$ and $t : Y$, then $\lambda x t : Y^X$
5. If $t : Y^X$ and $s : X$ then $t(s) : Y$
6. $\top$ and $\bot$ are *terms* of sort Ω

7. If $t : X$ and $s : X$, then $t = s : \Omega$ and $t \asymp s : \Omega$

8. If $\phi : \Omega$ and $\psi : \Omega$, then $\phi \Delta \psi : \Omega$, where $\Delta \in \{\wedge, \vee, \rightarrow\}$

9. If ϕ is a *term* of sort Ω, then so are $\forall\alpha\ \phi$ and $\exists\alpha\ \phi$

10. If ϕ is a *term* of sort Ω, then so are $\Box\phi$ and $\Diamond\phi$

11. Nothing else is a *term*

Formulas are defined to be terms of sort Ω.
If $t : X$, then we let $E(t) \equiv t \asymp t$.
If ϕ is a formula, we let $\neg\phi \equiv \phi \rightarrow \bot$.

We shall assume that we have defined the usual notions like "substitution of a variable by a term", "free variable of a term or a formula", "a term being free for a variable in a term or a formula", etc.

I now interpret the language by first interpreting sorts as kinds, i.e., as objects of the category $\mathcal{S}(\Omega(1))$. To interpret sorts in $\mathcal{S}(\Omega(1))$, it is enough to interpret basic sorts. In fact, once these sorts have been interpreted, we extend the interpretation $\| \ \|$ to all sorts as follows (recalling that $\mathcal{S}(\Omega(1))$ has finite products and exponentials): $\| \Omega \| = (\Omega(1), \Leftrightarrow)$.

Furthermore, if X and Y have been interpreted, then

$$\| X \times Y \| = \| X \| \times \| Y \|$$

$$\text{and } \| Y^X \| = \| Y \|^{\| X \|}$$

We notice that products and exponentials on the right hand sides refer to the cartesian closed structure of the category of kinds.

To simplify the notation I shall identify the sort X with the set underlying the kind $\| X \|$.

For each term $t : X$ and each sequence $\vec{x} = < x_1, \ldots, x_n >$ of distinct variables such that the free variables of t are among the elements of the sequence $\vec{x}$, we define by recursion a function

$$\| \vec{x} : t \| : X_1 \times \ldots \times X_n \longrightarrow X$$

I will use sometimes the notation $\vec{X}$ for $X_1 \times \ldots \times X_n$.

1. Basic constant terms $c \in Con_X$ are interpreted as members of X, $\| c \| \in X$. If $\vec{x}$ is any sequence of (distinct) variables of sorts $X_1, \ldots, X_n$, we let

$$\| \vec{x} : c \| : X_1 \times \ldots \times X_n \longrightarrow X$$

be the constant map which always takes value $\| c \|$.

2. If $x_i \in Var_X$, then $\| \vec{x} : x_i \| = \pi_i$, the i^{th} projection.

3. If $t : X$ and $s : Y$, then $\| \vec{x} : < t, s > \| = < \| \vec{x} : t \|, \| \vec{x} : s \| >$

4. If $x \in Var_X$ and $t : Y$, then

$$\| \vec{x} : \lambda x t \| : X_1 \times \ldots \times X_n \longrightarrow Y^X$$

is defined to be the exponential transpose of the function

$$\| \vec{x}x : t \| : X_1 \times \ldots \times X_n \times X \longrightarrow Y.$$

5. If $t : Y^X$ and $s : X$, then $\| \vec{x} : t(s) \| = ev(< \| \vec{x} : t \|, \| \vec{x} : s \| >)$, where $ev : Y^X \times X \longrightarrow Y$.

Remark 2.4.0.4 Since the sequence $\vec{x}x$ should consist of distinct variables, we should replace (in general) the variable x by, say, the first variable of the same sort as x which is different from all the x_is. However, to keep the notation as simple as possible, we assume that x is different from all the x_i.

We are now in a position to interpret formulas. We do this via a notion of *forcing*, which is essentially the same as that of Kripke's models, but with a new clause for modal operators. We define, by recursion on formulas, a relation $U \Vdash \phi[a_1, \ldots, a_n]$ between an element $U \in P$, a formula ϕ and a sequence $< a_1, \ldots, a_n >$ of elements of $X_1, \ldots, X_n$, such that the sorts of the free variables of ϕ are among $X_1, \ldots, X_n$. I will sometimes use the notation $\vec{a}$ for $a_1, \ldots, a_n$.

1. $U \Vdash \top[a_1, \ldots, a_n]$ *always* and $U \Vdash \bot[a_1, \ldots, a_n]$ *never*

2. Let $c \in Con_{\Omega X}$, $x : X$, $c(x) : \Omega$, then $U \Vdash c(x)[a_1, \ldots, a_n]$ iff $U \in \|c\|(a_1)$ where $a_1 \in X$

3. (a) $U \Vdash t = s[a_1, \ldots, a_n]$ iff

$$\| \vec{x} : t \| (a_1, \ldots, a_n) = \| \vec{x} : s \| (a_1, \ldots, a_n)$$

(b) $U \Vdash t \asymp s[a_1, \ldots, a_n]$ iff

$$U \in \delta_X(\| \vec{x} : t \| (a_1, \ldots, a_n), \| \vec{x} : s \| (a_1, \ldots, a_n))$$

4. (a) $U \Vdash \phi \wedge \psi[a_1, \ldots, a_n]$ iff $U \Vdash \phi[a_1, \ldots, a_n]$ and $U \Vdash \psi[a_1, \ldots, a_n]$
   (b) $U \Vdash \phi \vee \psi[a_1, \ldots, a_n]$ iff $U \Vdash \phi[a_1, \ldots, a_n]$ or $U \Vdash \psi[a_1, \ldots, a_n]$
   (c) $U \Vdash \phi \rightarrow \psi[a_1, \ldots, a_n]$ iff

$$\forall V \leq U \text{ if } V \Vdash \phi[a_1, \ldots, a_n] \text{ then } V \Vdash \psi[a_1, \ldots, a_n]$$

5. (a) $U \Vdash \exists x \phi[a_1, \ldots, a_n]$ iff there is an $a \in X$ such that

$$U \Vdash \phi[a_1, \ldots, a_n, a]$$

   (b) $U \Vdash \forall x \phi[a_1, \ldots, a_n]$ iff $\forall V \leq U \; V \Vdash \phi[a_1, \ldots, a_n, a]$ for all $a \in X$

6. (a) $U \Vdash \Diamond \phi[a_1, \ldots, a_n]$ iff $\exists V \; V \Vdash \phi[a_1, \ldots, a_n]$
   (b) $U \Vdash \Box \phi[a_1, \ldots, a_n]$ iff $\forall V \; V \Vdash \phi[a_1, \ldots, a_n]$

Proposition 2.4.0.5 *If $U \Vdash \phi[a_1, \ldots, a_n]$ and $V \leq U$, then $V \Vdash \phi[a_1, \ldots, a_n]$. This expresses the functoriality of the forcing relation.*

Proof. The proof is done by induction on formulas and is straightforward. To illustrate, I shall do the proof for $\neg$ and $\exists$.

($\neg$) : We have $U \Vdash \neg \phi[a_1, \ldots, a_n]$ and $V \leq U$, we show that $V \Vdash \neg \phi[\vec{a}]$. So given a $W \leq V$ we show that $W \not\Vdash \phi[\vec{a}]$. But by the definition of forcing for $\neg$ we have $\forall W' \leq U \;\; W' \not\Vdash \phi[\vec{a}]$. So in particular for $W \leq V$.

($\exists$) : We have $U \Vdash \exists x \phi[a_1, \ldots, a_n]$ and $V \leq U$, we show that $V \Vdash \exists x \phi[\vec{a}]$. Hence we show that $\exists b \in X$ such that $V \Vdash \phi[\vec{a}, b]$. But by hypothesis $\exists a \in X$ such that $U \Vdash \phi[\vec{a}, a]$ and by induction hypothesis $V \Vdash \phi[\vec{a}, a]$ and we take $b = a$.

We can finally define the basic notion of *validity* of a sentence, namely a formula without free variables. A sentence σ is *valid* (in $\mathcal{S}(\Omega(1))$) if and only if $1 \Vdash \sigma[\,]$, where $[\,]$ is the empty sequence. More generally, we say that a formula is *valid* in the category of kinds if and only if its universal closure, which is a sentence is valid in the category of kinds.

To finish this section, I shall describe a formal system MHT (for "modal higher order theory") based on Gentzen's sequents. These expressions, following Boileau and Joyal [4], will be of the form $\Gamma \vdash_{\vec{X}} \phi$, where Γ is a finite set of formulas of the language of modal higher order theory already described, ϕ a single formula and $\vec{X}$ a finite set of variables containing all the free variables of Γ and ϕ. We shall assume that these expressions satisfy the following rules. This system follows, in part, Lambek and Scott [18, page 134] which in turn is based on Gentzen's work (see Szabo [33,34]).

1. *Structural rules*

1.1 $\quad p \vdash_{\vec{X}} p$

1.2 $\quad \dfrac{\Gamma \vdash_{\vec{X}} p \qquad \Gamma \cup \{p\} \vdash_{\vec{X}} q}{\Gamma \vdash_{\vec{X}} q}$

1.3 $\quad \dfrac{\Gamma \vdash_{\vec{X}} q}{\Gamma \cup \{p\} \vdash_{\vec{X}} q}$

1.4 $\quad \dfrac{\Gamma \vdash_{\vec{X}} q}{\Gamma \vdash_{\vec{X},y} q}$

1.5 $\quad \dfrac{\Gamma \vdash_{\vec{X},y} \phi}{\Gamma[t/y] \vdash_{\vec{X}} \phi[t/y]}$

where t is free for y in ϕ and Γ

2. *Logical rules*

2.1 $\quad p \vdash_{\vec{X}} \top$ and $\bot \vdash_{\vec{X}} p$

2.2 $\quad r \vdash_{\vec{X}} p \wedge q$ iff $r \vdash_{\vec{X}} p$ and $r \vdash_{\vec{X}} q$

$\quad\quad p \vee q \vdash_{\vec{X}} r$ iff $p \vdash_{\vec{X}} r$ and $q \vdash_{\vec{X}} r$

2.3 $\quad p \vdash_{\vec{X}} q \rightarrow r$ iff $p \wedge q \vdash_{\vec{X}} r$

2.4 $\quad p \vdash_{\vec{X}} \forall x \phi$ iff $p \vdash_{\vec{X},x} \phi$

$\quad\quad \exists x \phi \vdash_{\vec{X}} p$ iff $\phi \vdash_{\vec{X},x} p$

provided that x is not a variable in $\vec{X}$.

2.5 $\vdash_{\vec{X}} \forall x(\sigma \vee \phi) \rightarrow \sigma \vee \forall x \phi$

provided that x is not free in σ

3. *Identity rules*

3.1 $\vdash_{\vec{X}} t = t$

3.2 $t = s \vdash_{\vec{X}} \tau[t/x] = \tau[s/x]$

provided that t and s are free for x in τ

3.3 $t = s \vdash_{\vec{X}} \phi[t/x] \rightarrow \phi[s/x]$

provided that t and s are free for x in ϕ

4. *Rules on special symbols*

4.1 $< a,b > = < c,d > \vdash_{\vec{X}} a = c$
$< a,b > = < c,d > \vdash_{\vec{X}} b = d$

4.2 $$\frac{\Gamma, \quad z = < x,y > \vdash_{\vec{X},x,y,z} \phi}{\Gamma \vdash_{\vec{X},z} \phi}$$

provided that x and y are not free in Γ or ϕ

5. *Coincidence rules*

5.1 $x \asymp y \vdash_{\vec{X}} y \asymp x$

5.2 $x \asymp y, \quad y \asymp z \vdash_{\vec{X}} x \asymp z$

5.3 $< a,b > \asymp < c,d > \vdash_{\vec{X}} a \asymp c$

5.4 $< a,b > \asymp < c,d > \vdash_{\vec{X}} b \asymp d$

5.5 $a \asymp c, b \asymp d \vdash_{\vec{X}} < a,b > \asymp < c,d >$

5.6 $f \asymp g, \quad x \asymp y \vdash_{\vec{X}} f(x) \asymp g(y)$

5.7 $$\frac{\Gamma, \quad x \asymp y \vdash_{\vec{X},x,y} f(x) \asymp g(y)}{\Gamma \vdash_{\vec{X}} f \asymp g}$$

provided that x and y are not free in Γ

5.8 $$\frac{p \vdash_{\vec{X}} q, \quad q \vdash_{\vec{X}} p}{\vdash_{\vec{X}} p = q}$$

6. *Rules for the λ -calculus*

6.1 $\vdash_{\vec{X}} \lambda x t(x) = t$

provided that x is not a variable in $\vec{X}$

6.2 $\quad \vdash_{\vec{X}} \lambda x\phi(t) = \phi[t/x]$

provided that t is free for x in ϕ

6.3 $$\frac{\Gamma \vdash_{\vec{X},x} t = s}{\Gamma \vdash_{\vec{X}} \lambda x t = \lambda x s}$$

7. *Rules for modal operators*

7.1 $\quad \Box\phi \vdash_{\vec{X}} \phi \qquad \phi \vdash_{\vec{X}} \Diamond\phi$

7.2 $\quad \Box\phi \vdash_{\vec{X}} \Box\Box\phi \qquad \Diamond\Diamond\phi \vdash_{\vec{X}} \Diamond\phi$

7.3 $\quad \phi \vdash_{\vec{X}} \Box\Diamond\phi \qquad \Diamond\Box\phi \vdash_{\vec{X}} \phi$

7.4 $\quad x = y \vdash_{\vec{X}} \Box(x = y)$

7.5 $$\frac{\phi \vdash_{\vec{X}} \psi}{\Box\phi \vdash_{\vec{X}} \Box\psi}$$

7.6 $$\frac{\phi \vdash_{\vec{X}} \psi}{\Diamond\phi \vdash_{\vec{X}} \Diamond\psi}$$

7.7 $\quad \vdash_{\vec{X}} \Diamond\phi \leftrightarrow \neg\Box\neg\phi$

7.8 $\quad \vdash_{\vec{X}} \Box\phi \vee \neg\Box\phi$

This completes my system. I have not tried to describe it in the simplest or most economical manner. In fact, a more economical system seems possible (see Lambek and Scott [18]).

In order to formulate a soundness theorem, we need the following definition: $\Gamma \models_X \phi$ if and only if

$$\forall U \in P\,\forall(a_1,\ldots,a_n) \in X_1 \times \ldots \times X_n\ U \Vdash \Gamma[a_1,\ldots,a_n] \Rightarrow U \Vdash \phi[a_1,\ldots,a_n]$$

where X is a finite set of free variables of sorts $X_1,\ldots,X_n$ containing the free variables of ϕ and Γ, and $U \Vdash \Gamma[a_1,\ldots,a_n]$ iff $U \Vdash \gamma[a_1,\ldots,a_n]$ for all $\gamma \in \Gamma$. Furthermore, we need the following lemmas.

Lemma 2.4.0.6 *(Projection) The following diagram is commutative:*

$$\begin{tikzcd}
X_1 \times \ldots \times X_n \times Z \arrow[r, "\|\vec{x}, z : t\|"] \arrow[d, "<\pi_1,\ldots,\pi_n>"'] & X \\
X_1 \times \ldots \times X_n \arrow[ur, "\|\vec{x} : t\|"'] &
\end{tikzcd}$$

where the free variables (FV) of t are among the elements of the sequence $\vec{x}$.

Proof. The proof proceeds by induction on terms. Let us illustrate this proof for the case of $\lambda xt : Y^X$ where $t : Y$ and $x \in Var_X$. We must prove that

$$\|\vec{x}, z : \lambda xt\| = \|\vec{x} : \lambda xt\| \circ < \pi_1, \ldots, \pi_n > .$$

By induction hypothesis, we have that

$$\|\vec{x}, z : t\| = \|\vec{x} : t\| \circ < \pi_1, \ldots, \pi_n > .$$

We remark that we also have

$$\|\vec{x}, x, z : t\| = \|\vec{x}, x : t\| \circ < \pi_1, \ldots, \pi_n >,$$

by hypothesis.

Since $X_1 \times \ldots X_n \times X \times Z$ is isomorphic to $X_1 \times \ldots \times X_n \times Z \times X$ we also obtain $\|\vec{x}, z, x : t\|$ and by definition the functions $\|\vec{x}, z : \lambda xt\|$ and $\|\vec{x} : \lambda xt\|$ are the exponential transpose of the functions $\|\vec{x}, z, x : t\|$ and $\|\vec{x}, x : t\|$. □

The lemma could also be written as $\|\vec{x}, z : t\|(\vec{a}, c) = \|\vec{x} : t\|(\vec{a})$ where $\vec{a} \in X_1 \times \ldots \times X_n$ and $c \in Z$. For terms of sort Ω (formulas), the preceding lemma can be formulated as follows: $U \Vdash \phi[a_1, \ldots, a_n]$, if and only if $U \Vdash \phi[a_1, \ldots, a_n, b]$ where $(a_1, \ldots, a_n, b) \in X_1 \times \ldots \times X_n \times Z$. The proof is done by induction on formulas.

Lemma 2.4.0.7 *(Substitution) If*

1. $\vec{x}y$ *are distinct variables,*
2. s *is free for* y *in* t, *namely that the substitution is proper,*
3. $FV(t[s/y]) \subseteq \vec{x}$,

then $FV(t(y)) \subseteq \vec{x}y$, $FV(s) \subseteq \vec{x}$ *and*

$$\|\vec{x} : t[s/y]\|(\vec{a}) = \|\vec{x}y : t(y)\|(\vec{a}, \|\vec{x} : s\|(a))$$

where $\vec{a} \in X_1 \times \ldots \times X_n$ *and* $a \in Y$.

Proof. The proof proceeds by induction on terms. Let us illustrate this proof for the case of $t(q) : Z$ where $t : Z^X$ and $q : X$. We must show that

$$\|\vec{x} : (t(q))[s/y]\|(\vec{a}) = \|\vec{x}y : (t(q))\|(\vec{a}, \|\vec{x} : s\|(\vec{a})).$$

By definition of substitution, we have

$$\|\vec{x} : (t(q))[s/y]\| = \|\vec{x} : t[s/y](q[s/y])\|.$$

By definition of $t(q))$ we have

$$\|\vec{x} : t[s/y](q[s/y])\| = \|\vec{x} : ev_X \circ < \|\vec{x} : t[s/y]\|, \|\vec{x} : q[s/y]\| > \|.$$

To apply the induction hypothesis we must verify 1., 2. and 3.

1. is verified by hypothesis.

2. is verified since the fact that s is free for y in $t(q)$ implies that s is free for y in t and s is free for y in q.

3. $FV(t[s/y]) \subset \vec{x}$ and $FV(q[s/y]) \subset \vec{x}$ since $FV((t(q))[s/y]) \subseteq \vec{x}$ by definition. Hence

$$\|\vec{x} : t[s/y]\|(\vec{a}) = \|\vec{x}y : t\|(\vec{a}, \|\vec{x} : s\|(\vec{a}))$$

and

$$\|\vec{x} : q[s/y]\|(\vec{a}) = \|\vec{x}y : q\|(\vec{a}, \|\vec{x} : s\|(\vec{a}))$$

and we thus obtain

$$\|\vec{x} : (t(q))[s/y]\|(\vec{a}) =$$
$$\|\vec{x} : ev_X \circ < \|\vec{x}y : t\|(\vec{a}, \|\vec{x} : s\|(\vec{a})), \|\vec{x}y : q\|(\vec{a}, \|\vec{x} : s\|(\vec{a})) > \|$$

and finally

$$\|\vec{x} : (t(q))[s/y]\|(\vec{a}) = \|\vec{x} : (t(q))\|(\vec{a}, \|\vec{x} : s\|(\vec{a})). \square$$

For terms of sort Ω (formulas), the preceding lemma can be formulated as follows: $U \Vdash \phi[s/y][\vec{a}]$ if and only if $U \Vdash \phi[\vec{a}, \|\vec{x} : s\|(\vec{a})]$. The proof proceeds by induction on the formulas.

We remark that if $y \in \vec{x}$ we can always change y to another variable not having appeared before, $t[s/y] = (t[z/y][s/z])$ where $z \notin FV(t)$ (see van Dalen [10]).

Theorem 2.4.0.8 (Soundness)
If $\Gamma \vdash_{\mathcal{X}} \phi$*, then* $\Gamma \models_{\mathcal{X}} \phi$.

Proof.(In sketch) Induction on proofs. For most axioms and rules of inference this is quite straightforward, the only tricky verifications being those connected with substitutions of terms.

To illustrate this proof, we verify it for the following rules 1.5, 3.2, 7.7 and 7.8.

1.5 We must show that $\Gamma[t/y] \models_{\mathcal{X}} \phi[t/y]$. Let U and $\vec{a} \in \vec{X}$ be given and assume $U \Vdash \Gamma[t/y][\vec{a}]$ we show that $U \Vdash \phi[t/y][\vec{a}]$. Since the hypotheses of the lemma of substitution are satisfied

$$U \Vdash \Gamma[t/y][\vec{a}] \text{ iff } U \Vdash \Gamma[\vec{a}, \|\vec{x} : t\|(\vec{a})].$$

But by hypothesis

$$\forall U,\ \forall \vec{a}, b\ U \Vdash \Gamma[\vec{a}, b] \Longrightarrow U \Vdash \phi[\vec{a}, b].$$

We choose then $b = \|\vec{x} : t\|(\vec{a})$ and we can conclude $U \Vdash \phi[\vec{a}, \|\vec{x} : t\|(\vec{a})]$.

3.2 We must show that $t = s \models_{\mathcal{X}} \tau[t/x] = \tau[s/x]$. Given U, $\vec{a}$ and assuming that $t = s$, we show that $U \Vdash (\tau[t/x] = \tau[s/x])[\vec{a}]$ or by definition of forcing that

$$\|\vec{x} : \tau[t/x]\|(\vec{a}) = \|\vec{x} : \tau[s/x]\|(\vec{a}).$$

The hypotheses being satisfied, we can apply the substitution lemma 2.4.0.7. Hence

$$\|\vec{x} : \tau[t/x]\|(\vec{a}) = \|\vec{x}, x : \tau\|(\vec{a}, \|\vec{x} : t\|(\vec{a}))$$

and

$$\|\vec{x} : \tau[s/x]\|(\vec{a}) = \|\vec{x}, x : \tau\|(\vec{a}, \|\vec{x} : s\|(\vec{a})).$$

But by hypothesis $U \Vdash t = s$ implies that $\|\vec{x} : t\|(\vec{a}) = \|\vec{x} : s\|(\vec{a})$, hence the conclusion.

7.7 We show that given U, $U \Vdash \neg\Box\neg\phi$ if and only if $U \Vdash \Diamond\phi$. But by definition

$$\begin{aligned} U \Vdash \neg\Box\neg\phi &\text{ iff } \forall V \leq U\ V \not\Vdash \Box\neg\phi \\ &\text{ iff } \forall V \leq U \neg(\forall W W \Vdash \neg\phi) \end{aligned}$$

$$\text{iff } \forall V \leq U\, \exists W\, W \not\Vdash \neg\phi$$

$$\text{iff } \forall V \leq U\, \exists W \neg(\forall W' \leq W\, W' \not\Vdash \phi)$$

$$\text{iff } \forall V \leq U\, \exists W \exists W' \leq W\, W' \Vdash \phi$$

$$\text{iff } \exists W\, W \Vdash \phi \text{ iff } U \Vdash \Diamond\phi.$$

7.8 We show that given U, $U \Vdash \Box\phi \vee \neg\Box\phi$. By definition of forcing, we must then show

$$U \Vdash \Box\phi \text{ or } U \Vdash \neg\Box\phi$$

equivalently

$$\forall V\, V \Vdash \phi \text{ or } \forall W \leq U\, W \not\Vdash \Box\phi$$

equivalently

$$\forall V\, V \Vdash \phi \text{ or } \forall W \leq U \neg(\forall V\, V \Vdash \phi)$$

equivalently

$$\forall V\, V \Vdash \phi \text{ or } \forall W \leq U \exists V\, V \not\Vdash \phi$$

and finally

$$\forall V\, V \Vdash \phi \text{ or } \exists V\, V \not\Vdash \phi$$

which is true.

2.4.1 Transparency and *de re* revisited

In this section, we will study some of the connections between $X \times X$ and $\Omega^{\Omega^X} \times \Omega^{\Omega^X}$ where X is not a sort derived from Ω (we can think that X is *person*, for instance). For convenience of notation, we shall write

$$t : X \longrightarrow Y$$

instead of $t : Y^X$.

A term

$$\Phi : \Omega^{\Omega^X} \times \Omega^{\Omega^X} \longrightarrow \Omega$$

gives rise to a new term

$$X \times X \longrightarrow \Omega$$

defined as follows

$$(w(\Phi))(p,q) = \Phi(\lambda P P(p), \lambda Q Q(q)),$$

where $p, q : X$ and $P : \Omega^X$.[5] Parsons uses the terminology of Meinong and calls $w(\Phi)$ the "watered down version of Φ".

I give the following definitions:

1. Φ is *transparent in the first argument* if and only if

$$\forall \mathcal{P}, \mathcal{P}', \mathcal{Q}\ \Phi(\mathcal{P}, \mathcal{Q}) \wedge \mathcal{P} \asymp \mathcal{P}' \longrightarrow \Phi(\mathcal{P}', \mathcal{Q})$$

   (Similarly for the second argument.) We illustrate this definition with the help of the next examples:

   (a) $\asymp (\mathcal{P}, \mathcal{Q})$ and $= (\mathcal{P}, \mathcal{Q})$ are transparent in the first and second arguments by the definition of $\asymp$ and $=$ respectively.

   (b) "$\mathcal{P}$ thinks about $\mathcal{Q}$" is transparent in the first argument but not in the second.

2. Φ is *de re in the first argument* if and only if

$$\forall \mathcal{P}, \mathcal{Q}\ \Phi(\mathcal{P}, \mathcal{Q}) = \mathcal{P}(\lambda p \Phi(\lambda P P(p), \mathcal{Q}))$$

   we assume that this definition holds for all $\mathcal{P}$ of the form $\lambda P P(p)$ where $p : X$, $P : \Omega^X$, $\lambda P P(p) : \Omega^{\Omega^X}$.

3. Φ is *de re in the second argument* if and only if

$$\forall \mathcal{P}, \mathcal{Q}\ \Phi(\mathcal{P}, \mathcal{Q}) = \mathcal{Q}(\lambda q \Phi(\mathcal{P}, \lambda Q Q(q)))$$

   we assume that this definition holds for all $\mathcal{Q}$ of the form $\lambda Q Q(q)$ where $q : X$, $P : \Omega^X$, $\lambda Q Q(q) : \Omega^{\Omega^X}$.

I give some examples that illustrate 3. and 2.

(a) "$\mathcal{P}$ finds $\mathcal{Q}$" is *de re* in the first and second argument.

(b) "$\mathcal{P}$ thinks about $\mathcal{Q}$" is *de re* in the first argument but not in the second.

(c) $\asymp$ is not *de re* in the first or second argument since $\mathcal{P} \asymp \mathcal{Q}$ and $\mathcal{P} = \mathcal{P}_1 \wedge \mathcal{P}_2$ does not imply that $\mathcal{P}_1 \asymp \mathcal{Q}$ and $\mathcal{P}_2 \asymp \mathcal{Q}$. We remark that if $\asymp$ was *de re* in the first argument we would have:

$$\mathcal{P} \asymp \mathcal{Q} = \mathcal{P}(\lambda p (\lambda P P p \asymp \mathcal{Q}))$$

$$\mathcal{P}_1 \wedge \mathcal{P}_2 \asymp \mathcal{Q} = \mathcal{P}_1 \wedge \mathcal{P}_2(\lambda p(\lambda PPp \asymp \mathcal{Q}))$$
$$\mathcal{P}_1 \wedge \mathcal{P}_2 \asymp \mathcal{Q} = \mathcal{P}_1(\lambda p(\lambda PPp \asymp \mathcal{Q})) \wedge \mathcal{P}_2(\lambda p(\lambda PPp \asymp \mathcal{Q}))$$
$$\mathcal{P}_1 \wedge \mathcal{P}_2 \asymp \mathcal{Q} = \mathcal{P}_1 \asymp \mathcal{Q} \wedge \mathcal{P}_2 \asymp \mathcal{Q}$$

which is clearly false. Similarly $=$ is not *de re* in the first or second argument.

I introduce the following notation and terminology

$$(\mathcal{P} \otimes \mathcal{Q})(\Phi) = \mathcal{P}(\lambda p \mathcal{Q}(\lambda q \Phi(\lambda PP(p), \lambda QQ(q)))).$$

We say that Φ has the *Fubini property* if and only if

$$\forall \mathcal{P}, \mathcal{Q}\ \Phi(\mathcal{P}, \mathcal{Q}) = (\mathcal{P} \otimes \mathcal{Q})(\Phi) = (\mathcal{Q} \otimes \mathcal{P})(\Phi).$$

I use the name "Fubini property" by analogy with Fubini's theorem

$$\int\int f = \int^p \int^q f(p,q)dq\,dp = \int^q \int^p f(p,q)dp\,dq$$

Proposition 2.4.1.1 *Φ is* de re *in both the first and the second argument if and only if Φ has the Fubini property. Namely*

$$\Phi(\mathcal{P}, \mathcal{Q}) = (\mathcal{P} \otimes \mathcal{Q})(\Phi) = (\mathcal{Q} \otimes \mathcal{P})(\Phi).$$

Proof. ($\Longrightarrow$:) Let $\mathcal{P}_\prime, \mathcal{Q}_\prime$ be given. Since Φ is *de re* in the first argument, we have

$$\Phi(\mathcal{P}_\prime, \mathcal{Q}) = \mathcal{P}_\prime(\lambda p \Phi(\lambda PP(p), \mathcal{Q})),\ \forall \mathcal{Q}.$$

In particular, let us take $\mathcal{Q} = \lambda QQ(q)$. We thus obtain

$$\Phi(\mathcal{P}_\prime, \lambda QQ(q)) = \mathcal{P}_\prime(\lambda p \Phi(\lambda PP(p), \lambda QQ(q)))\ \forall q \quad (*).$$

Since Φ is *de re* in the second argument we obtain

$$\Phi(\mathcal{P}_\prime, \mathcal{Q}_\prime) = \mathcal{Q}_\prime(\lambda q \Phi(\mathcal{P}_\prime, \lambda QQ(q))).$$

We replace $\Phi(\mathcal{P}_\prime, \lambda QQ(q))$ by $(*)$ and thus obtain

$$\Phi(\mathcal{P}_\prime, \mathcal{Q}_\prime) = \mathcal{Q}_\prime(\lambda q \mathcal{P}_\prime(\lambda p \Phi(\lambda PP(p), \lambda QQ(q)))) = (\mathcal{Q}_\prime \otimes \mathcal{P}_\prime)(\Phi).$$

Similarly we prove that

$$\Phi(\mathcal{P}_\prime, \mathcal{Q}_\prime)) = \mathcal{P}_\prime(\lambda p \mathcal{Q}_\prime(\lambda q \Phi(\lambda PP(p), \lambda QQ(q)))) = (\mathcal{P}_\prime \otimes \mathcal{Q}_\prime)(\Phi).$$

($\Longleftarrow$:) Let us prove that Φ is *de re* in the second argument. Let $\mathcal{P}_{\prime}$ and $\mathcal{Q}_{\prime}$ be given, we show that

$$\Phi(\mathcal{P}_{\prime}, \mathcal{Q}_{\prime}) = \mathcal{Q}_{\prime}(\lambda q \Phi(\mathcal{P}_{\prime}, \lambda QQ(q))).$$

By hypothesis

$$\forall \mathcal{P}, \mathcal{Q}\ \Phi(\mathcal{P}, \mathcal{Q}) = \mathcal{Q}(\lambda q \mathcal{P}(\lambda p \Phi(\lambda PP(p), \lambda QQ(q)))) \quad (**)$$

and taking $\mathcal{Q} = \lambda QQ(q)$ we have

$$\Phi(\mathcal{P}_{\prime}, \lambda QQ(q)) = \lambda QQ(q)(\lambda q \mathcal{P}(\lambda p \Phi(\lambda PP(p)(\lambda QQ(q)))).$$

By λ -conversion and evaluation we obtain

$$\Phi(\mathcal{P}_{\prime}, \lambda QQ(q)) = \mathcal{P}(\lambda p \Phi(\lambda PP(p), \lambda QQ(q)))$$

we replace in $(**)$ and obtain

$$\Phi(\mathcal{P}_{\prime}, \mathcal{Q}_{\prime}) = \mathcal{Q}_{\prime}(\lambda q(\Phi(\mathcal{P}_{\prime}, \lambda QQ(q)))). \square$$

For the following proposition, we define: X is *modally separated* if and only if

$$\forall x, x'(x \asymp x' \longrightarrow x = x')$$

where X is a sort and $x, x' : X$

Proposition 2.4.1.2 *If Φ is de re in the first argument which is of sort Ω^{Ω^X} where X is modally separated, then Φ is transparent in the first argument. (Similarly for the second argument.)*

Proof. Let $\mathcal{P}, \mathcal{P}', \mathcal{Q}$ be given, we must show that

$$\Phi(\mathcal{P}, \mathcal{Q}) \wedge \mathcal{P} \asymp \mathcal{P}' \longrightarrow \Phi(\mathcal{P}', \mathcal{Q})$$

or equivalently assuming $\mathcal{P} \asymp \mathcal{P}'$ we must show $\Phi(\mathcal{P}, \mathcal{Q}) \longleftrightarrow \Phi(\mathcal{P}', \mathcal{Q})$. We first show that

$$\lambda p \Phi(\lambda PP(p), \mathcal{Q}) \asymp \lambda p \Phi(\lambda PP(p), \mathcal{Q}).$$

By 5.7, it suffices to show

$$p \asymp q \vdash \Phi(\lambda PPp, \mathcal{Q}) \leftrightarrow \Phi(\lambda PPq, \mathcal{Q}).$$

But by hypothesis X is modally separated, therefore,

$$p \asymp q \vdash p = q.$$

So it suffices to show that

$$p = q \vdash \Phi(\lambda PPp, Q) \leftrightarrow \Phi(\lambda PPq, Q)$$

and this is a consequence of the axioms of identity 3.1 and 3.3. Using 5.6 and assuming that $\mathcal{P} \asymp \mathcal{P}'$ we conclude that

$$\mathcal{P}(\lambda p(\Phi(\lambda PP(p), Q))) \asymp \mathcal{P}'(\lambda p(\Phi(\lambda PP(p), Q))).$$

By hypothesis Φ is *de re* in the first argument, so we have

$$\Phi(\mathcal{P}, Q) = \mathcal{P}(\lambda p(\Phi(\lambda PP(p), Q))).$$

Similarly for $\mathcal{P}'$, and hence we have

$$\Phi(\mathcal{P}, Q) \longleftrightarrow \Phi(\mathcal{P}', Q). \square$$

I illustrate $\Phi(\mathcal{P}, Q) = (Q \otimes \mathcal{P})$ with the following example: $\mathcal{P} = \lambda PP(John)$ where $John : person$, $\Phi = finds$, $Q = the\ queen$, where *the queen* is translated by

$$\lambda Q(\exists x(x\ is\ a\ queen \wedge Q(x) \wedge \forall y(y\ is\ a\ queen \rightarrow x = y))).$$

$$\begin{aligned}\Phi(\mathcal{P}, Q) &= \lambda PP(John)(\lambda p(Q(\lambda q\Phi(\lambda PP(p), \lambda QQ(q))))) \\ &= \lambda p(Q(\lambda q\Phi(\lambda PP(p), \lambda QQ(q))))(John) \\ &= Q(\lambda q\Phi(\lambda PP(John), \lambda QQ(q)))\end{aligned}$$

Let us abbreviate the expression "is a queen" as "iaq"

$$\begin{aligned}&= \lambda Q(\exists x(x\ iaq \wedge Q(x) \wedge \forall y(y\ iaq \rightarrow x = y)))(\lambda q\Phi(\lambda PP(John), \lambda QQ(q))) \\ &= \exists x(x\ iaq \wedge (\lambda q\Phi(\lambda PP(John), \lambda QQ(q)))(x) \wedge \forall y(y\ iaq \rightarrow x = y)) \\ &= \exists x(x\ iaq \wedge \Phi(\lambda PP(John), \lambda QQ(x)) \wedge \forall y(y\ iaq \rightarrow x = y)) \\ &= \exists x(x\ iaq \wedge finds(\lambda PP(John), \lambda QQ(x)) \wedge \forall y(y\ iaq \rightarrow x = y))\end{aligned}$$

Proposition 2.4.1.3 *If Φ is de re in the first argument then*

1. $\Phi(\mathcal{P}_1 \vee \mathcal{P}_2, \mathcal{Q}) = \Phi(\mathcal{P}_1, \mathcal{Q}) \vee \Phi(\mathcal{P}_2, \mathcal{Q})$,

2. $\Phi(\mathcal{P}_1 \wedge \mathcal{P}_2, \mathcal{Q}) = \Phi(\mathcal{P}_1, \mathcal{Q}) \wedge \Phi(\mathcal{P}_2, \mathcal{Q})$,

3. $\Phi(\neg\mathcal{P}, \mathcal{Q}) = \neg\Phi(\mathcal{P}, \mathcal{Q})$,

4. $\Phi(\lambda P \exists x \phi, \mathcal{Q}) = \exists x \phi \Phi(\lambda P P(x), \mathcal{Q})$,

5. $\Phi(\lambda P \forall x \phi, \mathcal{Q}) = \forall x \phi \Phi(\lambda P P(x), \mathcal{Q})$.

Similarly for the second argument, we obtain the same equalities.

Proof. I just prove *1.*, the other proofs are done similarly. Since Φ is *de re* in the first argument we have

$$\Phi(\mathcal{P}_1 \vee \mathcal{P}_2, \mathcal{Q}) = (\mathcal{P}_1 \vee \mathcal{P}_2)(\lambda p \Phi(\lambda P P(p), \mathcal{Q})) =$$

$$\mathcal{P}_1(\lambda p \Phi(\lambda P P(p), \mathcal{Q})) \vee \mathcal{P}_2(\lambda p \Phi(\lambda P P(p), \mathcal{Q})) = \Phi(\mathcal{P}_1, \mathcal{Q}) \vee \Phi(\mathcal{P}_2, \mathcal{Q}),$$

by definition of the operations of the corresponding Heyting algebra, where the operations are defined pointwise. □

Corollary 2.4.1.4 $\Phi(\mathcal{P}_1 \vee \mathcal{P}_2, \mathcal{Q} \wedge \neg\mathcal{R}) =$

$$(\Phi(\mathcal{P}_1, \mathcal{Q}) \wedge \neg\Phi(\mathcal{P}_1, \mathcal{R})) \vee (\Phi(\mathcal{P}_2, \mathcal{Q}) \wedge \neg\Phi(\mathcal{P}_2, \mathcal{R})). \square$$

We remark, for instance, that the sentence "John or Mary finds Jane but not Bill" is equivalent to the sentence "John finds Jane, but he does not find Bill or Mary finds Jane, but she does not find Bill". Notice that in English we would not say "John finds not Bill", although "John finds Mary but not Bill" is perfectly grammatical. I do not claim that everything that is expressed in this logic may be directly expressed in ordinary English. We have already seen such an example in section 1.2.2 with "underlying person".

I now come back to the example mentionned in section 1.1.1. We will formalized the expressions "John overtakes Mary therefore Mary walks slower than John" and "John overtakes nobody therefore nobody walks slower than John".

We assume that the relation "overtakes(John, Mary)" is *de re* in the second argument (and the first).

$$\begin{aligned}\mathrm{T(John\ overtakes\ Mary)} : &\ \mathit{Mary}(\lambda x(\mathit{John\ overtakes}\ \lambda PPx)) \\ &= \lambda QQm(\lambda x(\mathit{John\ overtakes}\ \lambda PPx)) \\ &= \lambda x(\mathit{John\ overtakes}\ \lambda PPx)(m) \\ &= \mathit{John\ overtakes}\ \lambda PPm\end{aligned}$$

which can be written as $overtakes(j,m)$ under "the watered down version of" overtakes. Similarly, we write $walks\,slower(m,j)$ and we conclude:

$$overtakes(j,x) \not\longrightarrow walks\,slower(x,j).$$

Let us analyze "John overtakes nobody".

$$\begin{aligned}\mathrm{T(John\ overtakes\ nobody)} : &\ \mathit{nobody}(\lambda x(\mathit{John\ overtakes}\ \lambda QQx)) \\ &= \lambda P\forall x\neg Px(\lambda x(\mathit{John\ overtakes}\ \lambda QQx)) \\ &= \forall x\neg(\lambda x(\mathit{John\ overtakes}\ \lambda QQx))(x) \\ &= \forall x\neg \mathit{John\ overtakes}\ \lambda QQx \\ &= \forall x\neg overtakes(j,x)\end{aligned}$$

Similarly, we obtain $\forall x\neg walks\,slower(x,j)$ and clearly

$$\forall x\neg overtakes(j,x) \not\longrightarrow \forall x\neg walks\,slower(x,j).$$

We remark that from "John finds nobody" we can conclude "It is not the case that John finds somebody". The relation $finds(x,y)$ is *de re* in the second argument as well as in the first argument. But let us consider "John seeks nobody", we cannot conclude "It is not the case that John seeks somebody" as we would like since $seeks(x,y)$ is not *de re* in the second argument.

The logic of expressions such as "nobody", "everybody" and "somebody" translated respectively as $\lambda P\forall x\neg Px$, $\lambda P\forall xPx$, $\lambda P\exists xPx$ is reduced to the ordinary first order logic through the *de re* property of the relation. If the relation considered is not *de re* these expressions cannot be analyzed further.

2.4.2 Sub-kinds versus predicates

I shall now introduce the notions of a sub-kind of a given kind and of a predicate of a given kind.

A morphism $(B, \delta_B) \xrightarrow{u} (A, \delta_A)$ is a *sub-kind of* (A, δ_A) if and only if

1. u is injective, namely that if $u(b) = u(b')$ then $b = b'$
2. $\delta_B(b, b') = \delta_A(u(b), u(b')) \cap \epsilon_B(b) \cap \epsilon_B(b')$.

We remark that 2. implies that u is a morphism.

Any map $A \xrightarrow{\phi} \Omega(1)$ is a *predicate of* (A, δ_A).

Let us study the connections between the sub-kinds of a given kind ($Sub(A, \delta_A)$) and the predicates of a given kind ($Pred(A, \delta_A)$).

$$Sub(A, \delta_A) \xrightarrow{P} Pred(A, \delta_A) \xrightarrow{S} Sub(A, \delta_A)$$

Since a sub-kind of a given kind is given by the diagram $(B, \delta_B) \xrightarrow{u} (A, \delta_A)$, we will use indifferently the notations $P(u)$ and $P(A, \delta_A)$ for the predicate. We define

$$P(u)(a) = \{U : \exists b(U \in \epsilon_B(b) \wedge u(b) = a)\}.$$

Clearly $P(u)(a)$ is downward closed. We define

$$S(\phi) = (\{a : \phi(a) \neq \emptyset\}, \delta_{S(\phi)}) \xrightarrow{i} (A, \delta_A),$$

where i is the inclusion, $\delta_{S(\phi)}(a, a') = \phi(a) \cap \phi(a') \cap \delta_A(a, a')$.

We can easily prove that $S(\phi)$ is a sub-kind.

Theorem 2.4.2.1 *1.* $S(P(u)) \cong (B^*, \delta_{B^*})$, *where*

$$B^* = \{b \in B : \epsilon_B(b) \neq \emptyset\}$$

and $\delta_{B^*} = \delta_B \mid B^* \times B^*$

2. $P(S(\phi)) = \phi \cap \epsilon_A$

Proof. We first prove *1*. Given $(B, \delta_B) \xrightarrow{u} (A, \delta_A)$, and with the definitions given above we obtain that

$$S(P(u)) = (\{a : \exists b(\epsilon_B(b) \neq \emptyset \wedge u(b) = a)\}, \delta_{S(P(u))}).$$

Let $A^* = \{a : \exists b(\epsilon_B(b) \neq \emptyset \wedge u(b) = a)\}$. It is easily verified that

$$S(P(u)) \xrightarrow{i} (A, \delta_A)$$

is a sub-kind.

To prove that $S(P(u)) \cong (B^*, \delta_{B^*})$, we show

(a) $A^* \cong B^*$

(b) $\delta_{B^*}(b, b') = \delta_{S(P(u))}(u(b), u(b'))$:

(a) Since u is injective, its restriction, still to be denoted by "u",

$$u : B^* \rightarrow A^*,$$

is injective. Furthermore, it is surjective by definition of A^*.

(b) By definition

$$\delta_{S(P(u))}(u(b), u(b')) = P(u)(u(b)) \cap P(u)(u(b')) \cap \delta_A(u(b), u(b')).$$

So we must show that

$$\forall b, b' \in B^* \; \delta_B(b, b') = P(u)(u(b)) \cap P(u)(u(b')) \cap \delta_A(u(b), u(b')).$$

By definition of a sub-kind

$$\delta_B(b, b') = \epsilon_B(b) \cap \epsilon_B(b') \cap \delta_A(u(b), u(b')).$$

But $\epsilon_B(b) = P(u)(u(b))$ since $U \in P(u)(u(b))$ if and only if

$$\exists b'(U \in \epsilon_B(b') \wedge u(b) = u(b'))$$

if and only if $U \in \epsilon_B(b)$ (since u is injective).

To prove *2.*, we remark the following:

$$P(S(\phi),\delta_{S(\phi)})(a)=\{U:\exists a'\in S(\phi)(U\in\epsilon_{S(\phi)}(a')\wedge i(a')=a)\}=$$

$$\{U:a\in S(\phi)\wedge U\in\epsilon_{S(\phi)}(a)\}=\{U:\phi(a)\neq\emptyset\wedge U\in\phi(a)\cap\epsilon_A(a)\}$$

and this is equal to $\phi(a)\cap\epsilon_A(a)$. □

We say that ϕ is an *E-predicate of* (A,δ_A) if and only if $\phi\leq\epsilon_A$

Corollary 2.4.2.2 *(P,S) establishes a bijection between sub-kinds "without impossible members" and E-predicates of (A,δ_A).*

Proof. $PS=Id$ if and only if $PS(\phi)=\phi$ if and only if $\phi\leq\cancel{\epsilon_A,\text{ and }SP}=Id$ if and only if

$$SP(B,\delta_B)=(B,\delta_B)$$

if and only if

$$\epsilon_B(b)\neq\emptyset\ \forall b\in B.\ \square$$

I shall now present a syntactical proof of the fact that

$$(X,\delta_X)\overset{\check{ev}}{\to}(\Omega(1)^{\Omega(1)^X},\delta),$$

where δ is given by the exponential (see proposition 2.3.0.2), is a sub-kind.

1. The fact that $\check{ev}$ is injective can be translated as follows:

$$x=y\dashv\vdash_{x,y}\lambda PP(x)=\lambda PP(y)$$

(a) Let us prove that

$$x=y\vdash_{x,y}\lambda PP(x)=\lambda PP(y).$$

Using 3.3, we choose $\phi=\lambda PP(z)$, provided that x and y are free for z in ϕ. By substitution, we obtain

$$x=y\vdash_{x,y}\lambda PP[x/z]=\lambda PP[y/z].$$

Hence, we obtain

$$x=y\vdash_{x,y}\lambda PP(x)=\lambda PP(y).$$

(b) Let us prove that

$$\lambda PP(x) = \lambda PP(y) \vdash_{x,y} x = y.$$

We remark first that using 3.3 we can prove that

$$f = g \vdash_{f,g,x} f(x) = g(x).$$

Hence, if $t = \lambda PP(x)$ and $s = \lambda PP(y)$, by choosing $\phi = \lambda z(z = x)$ and with the usual proviso, we obtain

$$\lambda PP(x) = \lambda PP(y) \vdash_{x,y} \lambda PP(x)(\lambda z(z = x)) = \lambda PP(y)(\lambda z(z = x)).$$

By evaluation, we obtain

$$\lambda PP(x) = \lambda PP(y) \vdash_{x,y} \lambda z(z = x)(x) = \lambda z(z = x)(y).$$

And finally,

$$\lambda PP(x) = \lambda PP(y) \vdash_{x,y} x = y$$

2. We prove that $x \asymp y \dashv\vdash_{x,y} \lambda PP(x) \asymp \lambda QQ(y)$. This is a stronger condition which is not true in general for all sub-kinds, for instance, consider the sub-kind boy of the kind person. Nevertheless, this condition holds for the sub-kind under study.

(a) Let us prove that

$$x \asymp y \vdash_{x,y} \lambda PP(x) \asymp \lambda QQ(y).$$

By 6.1, we remark that

$$P(x) = \lambda PP(x)(P).$$

By 3.2 and choosing $\phi(z) = z \asymp Q(y)$ with the usual proviso, we obtain

$$P(x) = \lambda PP(x)(P), \phi[P(x)/z] \vdash \phi[\lambda PP(x)(P)/z].$$

Hence

$$P(x) = \lambda PP(x)(P), P(x) \asymp Q(y) \vdash_{x,y} \lambda PP(x)(P) \asymp Q(y).$$

Similarly, we apply 3.3 for

$$Q(y) = \lambda QQ(y)(Q) \textit{ and } \phi(z) = P(x) \asymp z.$$

Hence, letting $P(x) = \lambda PP(x)(P)$ be represented by $(*)$ in the following equation, we obtain

$$(*), P(x) \asymp Q(y), Q(y) = \lambda QQ(y)(Q) \vdash_{x,y} \lambda PP(x)(P) \asymp \lambda QQ(y)(Q),$$

which can be simplified as

$$P(x) \asymp Q(y) \vdash_{x,y} \lambda PP(x)(P) \asymp \lambda QQ(y)(Q).$$

By applying 5.6, we obtain

$$P \asymp Q, x \asymp y \vdash_{x,y} \lambda PP(x)(P) \asymp \lambda QQ(y)(Q).$$

Hence, using 5.7, we conclude that

$$x \asymp y \vdash_{x,y} \lambda PP(x) \asymp \lambda QQ(y).$$

To finish the proof and show the other side of the implication of 2., we need the following remark:

Remark 2.4.2.3 By applying 5.6, we obtain that

$$\lambda PP(x) \asymp \lambda QQ(y), P' \asymp Q' \vdash_{P',Q',x,y} P'(x) \asymp Q'(y)$$

and similarly that

$$\lambda PP(x) \asymp \lambda QQ(y), P'' \asymp Q'' \vdash_{P'',Q'',x,y} P''(x) \asymp Q''(y).$$

Let us then find two couples (P', Q') and (P'', Q'') such that

$$\vdash_{P',Q'} P' \asymp Q' \text{ and } \vdash_{P'',Q''} P'' \asymp Q''.$$

From this we can conclude that

$$\lambda PP(x) \asymp \lambda QQ(y) \vdash_{P',Q',P'',Q'',x,y} P'(x) \asymp Q'(y) \wedge P''(x) \asymp Q''(y).$$

To obtain the desired implication we then show that

$$P'(x) \asymp Q'(y) \wedge P''(x) \asymp Q''(y) \vdash_{P',Q',P'',Q'',x,y} x \asymp y.$$

(b) Let us prove that $\lambda PP(x) \asymp \lambda QQ(y) \vdash_{x,y} x \asymp y$. From the above remark, we must choose two couples.

We choose $(E, \lambda y\top)$ as the first couple. We must show that

$$\vdash E \asymp \lambda y\top.$$

Using 5.6, we obtain

$$\lambda PP(x) \asymp \lambda QQ(y), E \asymp \lambda y\top \vdash \lambda PP(x)(E) \asymp \lambda QQ(y)(\lambda y\top).$$

By evaluation and from the fact that $\vdash E(x) \asymp \top$ if and only if $\vdash E(x)$, we conclude that

$$\lambda PP(x) \asymp \lambda QQ(y), E \asymp \lambda y\top \vdash E(x).$$

By applying 5.7 and remembering that

$$\top = (\lambda y\top)(y),\ E(x) = (\lambda x E(x))(x)$$

and that

$$E = \lambda x E(x),$$

we conclude that

$$\lambda PP(x) \asymp \lambda QQ(y) \vdash_{x,y} E(x).$$

For the other couple, we take $(\lambda z(x \asymp z), \lambda z(x \asymp z))$. By applying 5.7 (with the axioms of $\asymp$ we have that $z_1 \asymp z_2 \vdash x \asymp z_1 \leftrightarrow x \asymp z_2$) and 5.6, we obtain

$$\lambda PP(x) \asymp \lambda QQ(y), \lambda z(x \asymp z) \asymp \lambda z(x \asymp z) \vdash_{x,y}$$
$$\lambda z(x \asymp z)(x) \asymp \lambda z(x \asymp z)(y).$$

Hence, in particular

$$\lambda PP(x) \asymp \lambda QQ(y) \vdash_{x,y} x \asymp x \rightarrow x \asymp y$$

and finally since $x \asymp x \leftrightarrow E(x)$, we conclude that

$$\lambda PP(x) \asymp \lambda QQ(y) \vdash_{x,y} x \asymp y.$$

2.4.3 Principles of application and identity

At the end of Chapter 1, I questioned Gupta's principles of application and identity on the basis that unsorted objects were needed to formulate them. Because of this, I have avoided any mention of these principles in this chapter. I would like to state my criticisms of these principles in more detail and show that there is a counterpart of these principles, suitably relativized, in my theory.

We first recall that kinds have been identified with certain sets having a coincidence relation defined in terms of the set-theoretical structures involved. Kinds such as **dog**, **person**, etc., are identified with sets of "urelements", namely, sets whose members do not have any further set-theoretical structure. On the other hand, the kind **proposition** was identified with a set of downward closed sets. Similarly, the kind **predicate** was identified with a set of functions. Members of these last kinds have set-theoretical structure, namely they are sets and functions, respectively.

My viewpoint is that the notion of set of "urelements" is basic and cannot be analyzed further. In particular I do not understand the sense in which these sets "come equipped with principles or rules of application or identity". In so far as I can see, rules and principles presuppose an already constituted domain of individuated members on which they operate. How then could they be constitutive elements of these domains? Assume, for instance, that the principle or rule of application required to constitute kinds is given by a set of conditions. It seems that this assumption leads to difficulties, since the statements belonging to the set of conditions involve quantifiers ranging over the members of further kinds which require, in turn, new rules or principles of application for their constitution. As an example, the principle of application for **dog** could be "is an animal with such and such DNA". But notice that we are using the kind **animal** whose principle of application should again be given in terms of further kinds. At one point we need to stop. On the other hand, if we assume that the principle of application is really a relation of membership, we seem to be at a loss to answer the question: a relation between what and what?, unless we use unsorted objects as the domain of membership, a domain that we must assume to have been constituted previous to any "principle of application" or "membership relation".

I view kinds as whatever is needed to interpret quantifiers and the equality symbol. They are the basic constitutive domains in terms of which we can define ordered pairs, relations, power sets, functions and other set-theoretical operations. Once the constitutive domains are available, we can formulate relativized principles of application and identity à la Gupta, by going to underlying kinds and considering the former kinds as predicates of the underlying ones, in accordance with the section 2.4.2. And predicates do come equipped with a principle of application! Consider for instance the kind dog. We can state a "principle of application" for doghood among animals. Such a principle could be, as suggested above, "is an animal with such and such DNA". This principle will imply, for instance, that a given cat is not a dog, etc. Similar remarks can be made about the principle of identity. In my theory these principles lose the basic role that Gupta assigned them. Indeed, they can be considered as particular theories about kinds. These theories and more general ones will play an important role in my next chapter since they formalize our background belief needed to understand literary texts. Principles of application and identity do not exhaust our theories about the world and its kinds.

Chapter 3

The logic of literary texts

In this last chapter, after a presentation of some of my "philosophical working hypotheses" and a sketch of Parsons's theory of fiction, I describe how I apply the theory developed in Chapter 2 to literary texts.

3.1 Preliminaries

3.1.1 Introduction

The main observation that motivated this work is that we can understand literary texts, although to do so we are forced, in the case of fairy tales and fables for instance, to allow for a wide variety of possibilities which seem to contradict our very system of background beliefs required for understanding the world. But if this is so, how can we understand literary texts? This question, inconspicuous as it might look, embodies in its formulation problems that have been recognized and discussed since the time of Aristotle. I do not confront this question directly, but remark that such an understanding presupposes an underlying logic of literary texts. My aim is then to develop a logic capable of dealing with problems of existence, possibility and identity which permeate all of literature and ordinary language. In fact most of these problems have appeared in attempts to set up a logic of ordinary language by logicians and philosophers. However, in literature, they seem "exacerbated" and lie at the heart of the literary phenomenon. I shall give some examples to illustrate the problems that will challenge me continually throughout my work.

Problems of existence, or rather problems of nonexistence, could be stated as follows:

> *Are there objects that don't exist? The orthodox, mainstream answer (in Anglo-American philosophy, anyway) is a resounding "No!—there's no such a thing as a thing that doesn't exist...."* Parsons [25, page 1]

How then can we talk meaningfully and truly about nonexistent objects or beings? Most people will agree that the following sentence is true: "Sherlock Holmes is not a real detective". But how could this be, if Sherlock Holmes does not exist? Furthermore, to follow the plot of a Conan Doyle story, we must assume the correctness of some arguments involving Dr. Watson, Moriarty and other equally nonexistent beings. Notice also, that Sherlock Holmes himself may want to refer to somebody who lacks existence "to a second degree" so to speak, but still he may refer to him in a coherent and truthful way, in a way that we can understand. For example, in the story "The adventure of the cardboard box", Sherlock Holmes refers to "an unsuccessful lover" who could have killed Mr. and Mrs. Browner. We discover later that there was no unsuccessful lover, and that Holmes had rejected this solution after receiving a certain answer to his telegram. This is an example of a non-existent being: an unsuccessful lover referred to by another non-existent being: Sherlock Holmes.

Problems of possibility arise at all levels in the study I want to pursue. Let us mention a paradigmatic one that I illustrate with the following example: it is common in a detective story for the detective, Sherlock Holmes, for instance, to consider various situations which are counterfactual to what really happens (in the story) in order to discover the culprit. In the story just mentioned above, Holmes says:

> *She might have buried the ears, and no one would have been the wiser. That is what she would have done if she had wished to shield the criminal. But if she does not wish to shield him she would give his name.*

A bit later in the text we read:

> *An unsuccessful lover might have killed Mr. and Mrs. Browner, and the male ear might have belonged to the husband.* [12, pages 192, 196]

We discover, later in the story, that Mr. Browner is the criminal who killed his wife and her lover who was successful, and that the ears were those of his wife and her lover, and not those of Mr. and Mrs. Browner. We need therefore to consider not only situations provided by the story, but also situations that are suggested in the story and are counterfactual to the story. To find out what is true, Sherlock Holmes finds out first what might be true. In detective stories in general, many plots about different characters are spelled out in order to discover who among them is the murderer. The story ends by rejecting all the plots but one. I give a last example found in "A Holiday for Murder" by Agatha Christie:

> *Poirot said, with a sudden ring of authority in his voice: "I have had to show you the possibilities! These are the things that might have happened! Which of them actually* did *happen we can only tell by passing from the outside appearance to the inside reality."* [7, pages 154–155]

The last problem to be considered here is the problem of identity. Metamorphoses in fairy tales force upon us several problems of change and identity. It is commonplace for a prince to become a frog and then a prince again, or for the companions of Ulysses to be turned into swine and back again into men. Or, in Tournier's novel "Gaspard, Melchior, Balthazar", for a king, Nabunassar III, to become in his old age the young king Nabunassar IV, who becomes in his old age the young king Nabunassar V, who becomes...the King is dead, long live the King, the perfect continuity for a royal family! In all these stories there is change. But what is it that changes and allows us to think that it had experienced these changes? It does not seem to be the prince (in my first example), since while a frog he cannot be a prince, his frog-state precludes prince-state. The same argument seems to show that what changes cannot be the frog. But if it is neither the prince nor the frog, what is it that changes? Similarly, the continuity of the royal family cannot be found in any of the Nabunassars. Further problems confront us with R. L. Stevenson's novel "Dr. Jekyll and Mr. Hyde". Is Dr. Jekyll the same as Mr. Hyde? If he is, how can we distinguish them? There is also the case of twins who are mentioned in a story in which no further specifications are added. How can we count them as two, as implied by the logic of the word "twins", when no means are provided in the story to tell them apart.

3.1.2 Philosophical preliminaries

I would like to emphasize that I am concerned only with the logic of literary texts; I am not concerned with question of esthetics or literary criticism. In particulars questions related to the author's intentions or to the reader's perception are left out of this enquiry. This is not a denial of the importance of these questions, but a delimitation of my field of analysis. Clarity in this field may, in fact, help to understand more clearly the nature of these other questions.

Before applying my theory to literary texts and explaining in more detail what the logic should take into account or should account for, I would like to describe as succinctly as possible my philosophical "mise en scène" so to speak. I remark that I have used the words "literary texts" and "background beliefs" at the beginning of the introduction 3.1.1. I shall try to specify what I mean by these expressions.

I use "literary texts" in the sense that Wellek and Warren [36] use "literary work of art".

> *The [literary] work of art, then, appears as an object of knowledge* sui generis *which has a special ontolological status. It is neither real (physical, like a statue) nor mental (psychological, like the experience of light or pain) nor ideal (like a triangle).* [36, page 156]

So a literary work of art is neither a state of mind nor an author's experience nor a reader's experience. A literary work of art is not to be confused with its calligraphy (even if in some cases the calligraphy could be part of the work) or with the sound patterns associated with the calligraphy (even if in some cases the sounds are in themselves part of the work).

Following Ingarden [15], Wellek and Warren consider that a literary work of art is a system which is made up of several strata each implying its own subordinate group. There is first a sound stratum from which a second stratum arises: the sentence patterns. From this structure arises a third stratum, that of the object represented, the "world of the novelist", the characters, the setting, the plot. Ingarden adds two more strata that according to Wellek and Warren could be incorporated into the third stratum. One of these strata consists in the different viewpoints of the events of the "world of the novelist", for example an event could be seen, heard or described by a character. The other stratum added is a stratum of "metaphysical qualities": the sublime, the tragic, the terrible, the holy.

Unlike a number or a triangle, the literary work of art was created at a certain point in time, is subject to change and even to destruction. The work of art can be recognized as being the same throughout its history, as having the same "structure" even if its structure is "dynamic", changes throughout the process of history.

> *The literary work of art...must be assumed to exist in collective ideology, changing with it, accessible only through individual mental experiences, based on the sound-structure of its sentences.*
> Wellek and Warren [36, page 156]

Without necessarily endorsing the analysis of Wellek and Warren, partly because I do not wholly understand it, I fully agree with them that a literary work of art has an existence of its own outside any experiences or mental states, outside the mere traces of ink on the pages, outside the mere echoes of sound. I remark also that my analysis will be restricted to the third stratum. This is the stratum where the problems of existence, possibility and identity that I study are located.

The expression "background beliefs" is a bit more difficult to tackle and I will be consciously more vague. Instead of only tackling that expression I will go around it and loosely describe my epistemic versus my metaphysical set up. First, I remark that I use the expression "background beliefs" instead of the more common expression "background knowledge" for the reasons given by Bellert [2]. I also remark that I do not want to question an expression like "background beliefs" that I keep at an intuitive level but I want to indicate that my study confronts many problems and one of them is grounded in the not so clear-cut distinction that has been drawn between epistemological and metaphysical problems or about what could be relevant to epistemology or to metaphysics.

In my study, and most of all in the questions of metaphysics versus epistemology, I will find myself largely in agreement with the outlook of Putnam [26]. I shall make use of his notions of "linguistic community", "stereotypes", "indexicality" and of his distinction between "artificial" and "natural kinds". With the help of these notions, I will interpret expressions like "background beliefs". I will try to draw clear distinctions between what I consider to be relevant to epistemology or relevant to metaphysics (I prefer to make clear mistakes!... but even that is not an easy task!). I will explain briefly what a stereotype is, emphasizing those aspects which are relevant to my study.

A stereotype is a conventional idea, which may be inaccurate, of what a certain thing looks like or acts like or is. For example, someone who has acquired the word "tiger" is required to know the conventional idea associated with "tiger" in his linguistic community. A speaker of that linguistic community, for instance, has to know that tigers are striped if his acquisition of the word "tiger" is to count as being successful. But this does not imply that "tigers are striped" is an analytic proposition. Tigers without stripes would not cease to be tigers, because what really counts is the internal structure of a tiger. We say that someone has acquired the word "tiger" if the two following conditions are fulfilled: (1) The person is able to use the word in a way that makes sense to the community. His use "passes muster" as better expressed by Putnam. People won't say of him: "he does not know what a tiger is". (2) The socially determined extension, in the sense of Putnam [26, page 247], of the word "tiger", in his idiolect, is the set of tigers.

What goes into the stereotype is then characteristically the kind of phenomenal properties which we have access to. Some of these properties could be inaccurate like when for example we endow wolves with wickedness. (It might take a movie like *Never cry "wolf"* to begin to change the stereotype of a given linguistic community!) Nevertheless, the fact that we can communicate successfully implies that most of our stereotypes must be fairly accurate. The stereotypes of a linguistic community are dynamic. They change, they evolve with the linguistic community. For example, the stereotype attached to the word "deer" has evolved from the Middle Ages to our present days, and also changes in our times from one linguistic community to another. Black [3] utilises a notion similar to that of stereotype: the tiger-system, or wolf-system or whatever-system of related common places built from the background beliefs of the linguistic community.

I should remark at this point how I will make use of the notion "stereotype". I will mainly use the words "our theory of gold", "our theory of tiger". These theories are expressions of the relevant background beliefs, they may be accurate or not and may contain, as in the case of "our theory of gold" the atomic number of gold or not. These theories may be as "scientific" as needed for the understanding of a given literary text. So in particular these theories would not only contain stereotypes to use Putnam's terminology. We reach an understanding of a literary text whenever we can give a coherent account in terms of our theories, regardless of the accuracy of these theories. I shall present in more detail, in the next sections, what I mean by "coherent account".

3.1.3 Parsons's theory of fiction

Parsons [25] is interested in the same problems that I deal with and has carefully developed both a formal system and a semantics which share some features with my system, but is sufficiently different from mine to make comparison fruitful. I shall briefly sketch his theory of fiction as described in his book "Nonexistent Objects".

Parsons develops his views in the context of a general theory of nonexistent objects. Nonexistent objects provide the stock from which some "fictional objects" will be selected. To understand his theory of fiction, I need therefore to say a few words about his general theory of nonexistent objects.

Meinong's theory of objects is at the origin of Parsons's theory of nonexistent objects. Both Husserl and Meinong tried to give an account of the elusive notion of intentionality formulated by their teacher Brentano. Whereas Husserl's account finds its expression in the theory of noemata in his phenomenology, Meinong tried to account for Brentano's intentionality by creating a theory of objects which includes existent and nonexistent objects. According to Meinong, if we think about something, then some object, maybe nonexistent, is such that we think about it.

The main idea is, roughly, that any set of properties determines a unique, usually nonexistent, object having those properties. As an example, "to be a mountain" and "to be golden" determines the famous golden mountain of Meinong. This account seems to fit nicely the paradigmatic example "John is thinking about the golden mountain": an object correlated with the set of properties "to be a mountain" and "to be golden" is such that John thinks about it. However, this principle, as formulated, ~~is too~~ brutal and in fact leads to contradictions; pretty much as Frege's original axiom that every property determines a set led to the contradiction discovered by Russell: consider the set of all sets which do not contain themselves, then this set has the curious property of containing itself precisely when it does not contain itself, by its very definition. Take, for instance, the properties "to be a mountain", "to be golden", "to be not golden" and "to exist". Clearly there cannot be any existing mountain that is golden and at the same time is not golden. This example of Parsons shows clearly the difficulties with Meinong's views: there cannot be an object correlated with the set of properties "to be a mountain", "to be golden", "to be not golden" and "to be existing" such that John thinks about it; in other words, there cannot be an existing mountain that is both golden and not golden such that John thinks about it.

Parsons's way out of these difficulties is to restrict the application of this principle to "nuclear properties" only, examples of which would include "to be a mountain", "to be golden", "to be not a mountain", "to be kissed by Socrates" but exclude "to exist", "to be fictional", "to be thought about", "to be possible". These last properties are called extra-nuclear properties and in fact considered as higher-order predicates in Parsons's system. Parsons's theory allows us to make a distinction between these types of properties, but I will not go into details, since my criticisms of Parsons do not depend on this distinction.

Parsons's theory proceeds by identifying certain objects of fiction, or fictional objects as he calls them, with certain nonexistent objects. The idea is to view Sherlock Holmes, for instance, as the object which has exactly those nuclear properties which are attributed to Sherlock Holmes in Conan Doyle's novels. This is the usual and traditional way of defining a character, in fact, the only way that has been considered. Parsons distinguishes two sorts of objects in a literary work that *occur in the story*: the "native" ones such as Sherlock Holmes and Dr. Watson in Conan Doyle's novels and the "immigrant" ones such as London and Gladstone. I shall assume in this chapter, for the sake of argument, that Gladstone meets Sherlock Holmes in one of Conan Doyle's novels; but I could have taken the case of the famous "real" Spanish violinist, Sarasate, born in 1844 and referred to by Sherlock Holmes in the story "The Red-Headed League". We notice that a fictional object such as Sherlock Holmes may be immigrant in a novel by another author. For instance, Réouven in the "Le détective volé" places Sherlock Holmes and Watson in the Paris of 1834.

The above identification of objects of fiction with objects determined by nuclear properties applies to native objects only. With respect to immigrant objects, it is the thesis of Parsons that "Gladstone" refers to the real man who was Prime Minister of England and that "London" refers to the real city of that name in England. In the same way Parsons would maintain that "Sherlock Holmes" refers to the fictional object of Conan Doyle's novels in any work in which he is an immigrant object. We remark then, that "fictional object" does not mean for Parsons "nonexistent object" but rather means "object occurring in fiction", since real objects occur in fiction as well as nonexistent ones in Parsons's theory. This thesis has the surprising consequence that we are forced to change the "logic of English" as the following considerations show. Assume that in the novel of Conan Doyle, Sherlock Holmes shakes hands with Gladstone. We would naturally conclude "from the logic of English" that Gladstone also shakes hands with Holmes, but this is not a property that we can attribute to the real Gladstone and hence we are forced to conclude that "A shakes hands with B" may be true, but that "B shakes hands with A" may be false.

Parsons develops a formal system and a corresponding semantics to handle the peculiar relations that may hold between real and nonreal objects. For real objects however, his system is true to the original intuitions about "the logic of English", and also for nonreal objects as for instance when Sherlock Holmes meets Dr. Watson. The problem arises only when there are extensional relations such as "meeting", "shaking hands" or "kissing" between real and nonreal objects.

When Parsons speaks of an object having a property "in a story", he means something like an object's having a property *according to* a story, where story is not understood as being a "possible world" as discussed in possible worlds semantics. An object described in a story is highly incomplete, in the sense that whether a property holds or not of that object is not always decidable in the story. For instance, in Conan Doyle's stories, there is no way to decide whether Holmes has a birthmark or not on his left leg and there would be no way to decide whether Holmes played or not on a Stradivarius if it had not been mentioned in one of the stories; this fact is related in "The adventure of the cardboard box". In possible worlds semantics, on the other hand, every object is complete, since the worlds are complete by definition.

Furthermore, Parsons introduces the notion of a surrogate object. A surrogate object is an object that is picked out by an expression of the form "the

London of Conan Doyle's novels" or "Sherlock Holmes's London". Clearly these objects are incomplete and consequently nonexistent. According to Parsons these objects do not *occur in* the story. So, as a consequence, we can say of these objects that *having a certain property in the story* does not imply for them that they have that property and that they are in the story, since according to Parsons, for instance, it is the real London that occurs in the story. The London of Conan Doyle's novels has certain properties that the real London does not have. Every immigrant object (N) has a surrogate object (the N of s, where s stands for story).

To finish this section on Parsons's theory I say a few words on what is meant by the expression "true in the story". According to Parsons what is true in a story is whatever the *maximal account* explicitly says, and nothing else, the maximal account being a "model" of our understanding of a story. The problem is how to arrive at this maximal account according to him. I quote Parsons:

> *Basically, the idea is that as the reader reads the story from start to finish a partial account is gradually developed as follows:*
>
> 1. *Typically, as a new sentence is read, that sentence is added to the account.*
> 2. *Typically, lots of other sentences are simultaneously added as well.*
> 3. *Often, sentences are removed from the account*
>
> *...The account is modified and expanded during the reading, and the final result may be called the* maximal account. [25, pages 176 and 175]

The way the sentences are added is not by logical inference since that way too many irrelevant sentences would be added and the crucial ones would fail to be added. A great deal of the sentences come from the reader's understanding of the world and are added automatically by the reader, but, of course, not in a way that contradicts the story. For instance, if we read in a story that 18,000 mermaids have invaded the Thames, we will not bring in our background beliefs and add to the account that mermaids do not exist. Parsons defines a maximal account as something that resembles a description of a possible world, but with at least two differences: it is typically highly incomplete, and it is occasionally impossible or inconsistent.

3.1.4 Criticisms of Parsons's theory

From my point of view, it is clear that there are several problems with Parsons's theory. I will mention some of those for which I think that my theory offers an answer.

1) Parsons talks about "objects" or bare individuals which do not belong to any kind, thus making himself the target of the various criticisms that I have formulated against any approach that uses bare individuals (see for instance, section 1.1.5).

2) Since characters such as Sherlock Holmes are defined by sets of properties of the type "is a detective", "lives on Baker street", "owns a Stradivarius", etc., it is impossible to talk counterfactually about them. In the story "The man with the twisted lip", the main character, Neville St. Clair, is present when the policemen forcefully made their way into the room in which he had been seen by his wife shortly before. Accepting the traditional way of defining characters by sets of properties, it is logically impossible for Neville St. Clair to have the property "not to have been in the room in question at that moment". Nevertheless, the whole story would be incomprehensible if we do not consider this possibility. In fact, it was this very possibility that eventually led Sherlock Holmes to solve the mystery of the disappearance of Neville St. Clair. This is a serious problem for all theories which use such a notion of character.

3) The properties of Sherlock Holmes change according to the situations of the story: he may be in London at his Club, or on a train, or in a coach, etc. Hence we need to make precise the sense in which he has those properties that are attributed to him in the novel. Parsons seems to consider only non-changing properties of Sherlock Holmes, for instance, "is a detective", "lives on Baker street", etc. The reason is, I believe, that since he does not take situations into account he would then end up with a set of properties of the type "is in London", "is away from London", "is on a train", "is in a coach", "is sleeping", "is awake", etc.

4) The change of "the logic" of the relations involved makes a normal or natural understanding of a literary text impossible. Normal readers, including children, will automatically conclude that Gladstone shakes hands with Holmes when reading that Holmes shakes hands with Gladstone; such

inferences are part of our very understanding of a language and cannot be laid aside when language is used in literary texts.

5) Another problem with the theory, also of a logical nature, is the way it handles quantifiers and hence the way it deals with questions of generality. Suppose that we read in a novel that a large group of people form a chain, that each person carries a flag in such a way that whenever somebody carries a blue flag, his neighbor carries a red flag and vice-versa. Parsons does not have enough fictional objects, corresponding to sets of properties, to interpret these statements correctly, since his quantifiers range over characters only. Characters are individualized fictional objects of the story and the members of a crowd are not individualized and hence they are not characters. Although Parsons discusses crowds in his book, he never does so in the context of quantifiers, a context which creates real difficulties for his approach, as the above example illustrates.

6) I question the possibility of understanding an inconsistent account of a literary text.

3.2 A semantics for literary texts

3.2.1 Introduction

I will apply the theory that was developed in Chapter 2 to literary texts. Specifically, I will deal with problems of reference and generality in literary texts and as a consequence I will shed some light on the problems of existence, possibility and identity which permeate all literature.

My main hypothesis is that the semantics requires kinds to interpret the count nouns of the text. For instance, the text may contain expressions like "men", as in Sherlock Holmes by Sir Arthur Conan Doyle, "troll", as in the Moumine stories of Tove Janssen, "Kabouter", as in "Leven en werken van de Kabouter" by Rien Poorvliet and Wil Huygen or "Lilliputians", as in "Gulliver's Travels" by Jonathan Swift, expressions which function as count nouns. Furthermore, the text usually contains expressions like "Sherlock Holmes", "Moumine", "Tomte Haroldson" or "Flimnap" which play the role of proper names in their respective stories, as well as expressions of the kind "the man with the twisted lip", which are ordinary descriptions. The text also contains expressions such as "meet" or "shake hands" which function as relational symbols (VP).

I then postulate the existence of kinds whose members are "urelements" to interpret the basic count nouns, as already mentioned, particular members of the corresponding kind to interpret proper names and relations to interpret relational symbols. I further postulate that relation symbols are interpreted with their ordinary "logic of English". Even if our knowledge of trolls is rather meager, we know that if a troll A meets another troll B, then B meets A. In section 3.2.3, I shall present a more precise version of my thesis. In the next section, I will describe where the kinds that I have postulated "live".

3.2.2 Situations "in literary texts"

A fundamental notion of the theory developed in Chapter 2 was the notion of *situation*. We need, then, to introduce the parallel notion of *situation in a story*. But first, let us introduce some concepts that were developed by Martínez Bonati [22].

Martínez Bonati distinguishes different levels, in a story, consisting, on the one hand, of what is said by the narrator and, on the other hand, of what is said by the characters; this does not exclude the possibility of the narrator being one of the characters. What is said by the narrator constitutes the "basic" level of narration-description. To make a very brief and naive summary of Martínez Bonati's ideas, one could say that this level consists of *assertive phrases*, as opposed to exclamative or interrogative phrases, *of a concrete and particular nature expressed by the narrator.*

I illustrate these ideas by means of an example given by Martínez Bonati [22, pages 68–69].

> *Pedro and Juan were slowly walking through the old part of the city. As they turned towards the river, the first pointed out a place with his extended arm, exclaiming "Another admirable temple!". "I see neither a temple nor anything admirable at all" said Juan dryly, without stopping.*
>
> *There was in fact a temple, a naked building of severe appearance. But the too frequent and too enthusiastic artistic-historical observations of Pedro had finished by exasperating his reserved companion. Even the noblest enthusiasm irritates when it is not kept within bounds.*

"Let us reach that boat", suggested Juan abruptly in an opaque voice, while looking in the direction of the still distant river. Pedro joined him in one leap and the two of them started to walk side by side at a good step. (Translated from the Spanish by G. E. Reyes.)

From what the characters say in the first paragraph, we cannot decide whether or not there is a temple in that place of the city. In the second paragraph, however, the narrator asserts indeed the existence of a temple: "a naked building of severe appearance". The reader, at this point, must then accept the existence of the temple such as described if he is to understand the story. Nevertheless, he is not committed in the same way to the general statement uttered by the narrator in this same paragraph: "Even the noblest enthusiasm irritates when not kept within bounds". These general statements describe the feelings or thoughts of the narrator, but they play no role in the narrative-descriptive level that we try to circumscribe. To find the narrator in a story is not always as straightforward as these two paragraphs exemplify. The narrator may be present or not in the story, he may even be one of the characters. There may be a three mouthed narrator or a hierarchy of included levels of narration-description, as for instance in Italo Calvino's novel "If on a winter's night a traveller". In the third paragraph of the example under consideration, what is said by Juan: "Let us reach that boat" implies the existence of a boat and counts as one of the sayings of the narrator that determines the level of narration-description. The main narrator is silent, he does not contradict or add any statements to what is said by Juan at this point. So the reader accepts the existence of a boat as part of the story. For Martínez Bonati, to understand literary phrases is to "unveil" the situation immanent to the phrase. Literature, for him, is the development of the situation immanent to the phrase, without the help of auxiliary determinations.

Keeping these distinctions of level in mind, we proceed to develop the notion "situation in a story". We remark that the story may present contradictions, the author may make mistakes. In this case, we automatically try to reinstate coherence: our further understanding of the story depends on that coherence which we must achieve. It is not my goal to explain how to reach this level which is needed for subsequent understanding and I shall take for granted that we can provide such a level.

I view *situation in a story* as the interpretation of actual and counterfactual partial descriptions of the "world" of the story under consideration. Some of the actual and some of the counterfactual partial descriptions are those which are contained in the narrative-descriptive level considered above. I used then the words "assertive phrases of a concrete and particular nature expressed by the narrator". The other actual and counterfactual partial descriptions whose interpretations make up the situations in a story are those which are needed to provide the coherence of the story and determine the range of possibilities allowed by the text. Thus a science fiction story allows for a wider range of possibilities than a realistic novel. These partial descriptions could be obtained from some descriptions which are deduced from the written text of the story with the help of the background beliefs. These last descriptions are not directly obtained from statements of the story and they were not considered by Martínez Bonati.

As I said above, my goal is not to describe the process which enables us to arrive at these descriptions. I take for granted that we can perform such a task. These descriptions will form part of what I call the "idealized story". I will describe some more features of the "idealized story" in the sections to come. The semantics that I develop is the semantics of this coherent "idealized story".

Let us remark that I do not use the notion of "possible world". Possible worlds are complete, whereas we deal only with partial descriptions. Indeed, incompleteness is a characteristic feature of literary works and it would be irrelevant and probably hopeless to try to complete those partial descriptions given by the narrative-descriptive level of the story. It seems important to point this out, since some authors (for instance Mates [23, page 252]) question the cogency of the notion of a counterfactual situation, precisely on the basis of the impossibility of stipulating a complete world in which this counterfactual situation is a factual situation. My notion is highly incomplete but consistent!

3.2.3 Language and interpretation

The fact that "troll", "Lilliputian" and "unicorn" play the grammatical role of count nouns whose interpretations should be kinds can be understood by the fact that we can count trolls, Lilliputians or unicorns, individuate them, make them appear in situations of the text and, in general, correctly apply our quantifiers to them. These kinds are not ordinary kinds of our world, but kinds relative to the set of situations of the "world of the story", more precisely, to the set of situations in a story as described in section 3.2.2.

Several questions arise: How should we interpret the count noun "man" in a story? And again, how should we interpret the proper name "Gladstone" in the Conan Doyle's story? Assuming that "man" is interpreted as a kind of the story, what is the connection between that kind and the kind **man** of the real world? Assuming that "Gladstone" is interpreted as a member of the kind **man** of the story, what is the connection between that member and the real Gladstone? Can we even compare them? How can we talk about Sherlock Holmes in our world? What does Sherlock Holmes refer to when he says "every man so and so"?

There are two apparently inconsistent intuitions with respect to general statements about men, for instance, in a story. On the one hand these statements should refer to real men since it is precisely the fact that literature reveals something about the real world that makes it important. This intuition seems to be correctly captured by Parsons in his theory. On the other hand, these general statements seem to refer to characters of the novel in question, since Holmes would certainly include Dr. Watson and Moriarty in the range of his quantifier "every man". There is a corresponding problem with the reference of proper names. Assume that Sherlock Holmes refers to Gladstone and his Irish policy. It seems then that Holmes refers to the real man and hence that the name "Gladstone" refers to the man who was Prime Minister of England. But if Holmes, on the other hand, shakes hands with Gladstone in one of Conan Doyle's novels, the name "Gladstone" seems to refer to a character of the novel. When Sherlock Holmes refers to Gladstone, he can say "the man whith whom I shook hands", which is a property that the "real" Gladstone does not have; the "real" Gladstone did not shake hands with Holmes. Therefore, he seems to refer to a native character of the novel who in some ways has something to do with the "real" Gladstone and who is a member of the kind **man** of the story.

Parsons's view that "Gladstone" refers to the "real" Gladstone forces him to give a non-standard "logic of English". Thus, although Holmes shakes hands with Gladstone, Gladstone does not shake hands with Holmes, and this makes literature rather incomprehensible.

My point of view is that "man" in the story refers to a native kind of the story and, similarly, "Gladstone" refers to a member of this kind. Furthermore, I postulate, as I mentioned above, "ordinary" relations between the members of these kinds, namely, relations which follow the ordinary "logic of English". Thus, if Holmes shakes hands with Gladstone, then Gladstone shakes hands with Holmes, since now both have the same "ontological status", namely, they both belong to the kind **man** of the story.

So far, so good, literature is now comprehensible, the logic is safe! But how does the real Gladstone relate to the Gladstone of the story? How does the kind **man** of the story relate to the kind **man** of the real world? To be able to answer these questions, we must first enrich the language introduced in Chapter 2, section 2.4.

In Chapter 2, I introduced symbols, basic sorts, for the count nouns of English, in the language. For instance, for the count noun "man" I introduced the basic sort *man* that I interpreted as the kind **man**. Since we will be dealing with both the real world and the world of the story, some means must be provided to know which kind we have in mind. First we remark that when we talk about kinds of the real world we mean those kinds which are the interpretation of sorts as described in section 2.4, namely the smallest class which contains the basic kinds such as **man**, **dog**, **bed**, **natural number**, etc., under consideration, and is closed under operations such as products, exponentiations, etc. For the count noun "man" I shall introduce the basic sort man_r to be interpreted as the kind $\mathbf{man}_r$ of the real world and another basic sort man_s to be interpreted as the native kind $\mathbf{man}_s$ of the world of the story. Similarly, I use $Gladstone_r$ as the constant of sort man_r whose reference is the real Gladstone and $Gladstone_s$ as the constant of sort man_s to refer to the Gladstone of the story who is a member of the kind $\mathbf{man}_s$. More generally, whenever a count noun k is mentioned in the story I introduce a basic sort k_s to be interpreted as the kind K_s of the world of the story. Similar remarks apply to proper names of persons, cities, etc. which are mentioned in the story, such as "London" in the Conan Doyle's novels. In this manner, we obtain L_s, which is the language of the story and we have, as before, the language of our world that we could symbolize as L_r.

Not all count nouns mentioned in the story have also interpretations as kinds of the real world. Some examples were already given: "troll", "Lilliputian", "unicorn", etc. I must also introduce new symbols for the constants which refer to relations of the story. For instance, for the relation "meet" between Sherlock Holmes and Dr. Watson I introduce the basic constant term $meet_s \in Con_{\Omega_s^{man_s \times man_s}}$ which I interpret as the corresponding relation of the story. We remark that the problem of the multiplication of the constants, as for instance $meet_s$ mentioned above, is already present in our world, since every relational symbol is sorted. As a consequence, the interpretation of "find" in "John finds Mary" is a relation between two persons and hence it differs from the interpretation of "find" in "the dog finds the bone" which is a relation between a dog and a bone. I believe that far from being artificial these different interpretations agree with our intuitions.

Let us try to understand some of the relations between these new symbols and the symbols already introduced in section 2.4. Since I have postulated that the logic of the relations should be the same whether we are in the real world or in the world of the story, we can conclude that the logic of *meet* is the same as the logic of $meet_s$, and since $Russell_r$ $meets_r$ $Gladstone_r$ implies that $Gladstone_r$ $meets_r$ $Russell_r$, so in the same way $Sherlock\ Holmes_s$ $meets_s$ $Gladstone_s$ implies that $Gladstone_s$ $meets_s$ $Sherlock\ Holmes_s$. We notice that we have two basic sorts for "proposition": Ω_r and Ω_s in the languages L_r and L_s respectively.

With the help of the new language and its interpretation we can now try to answer some of the questions related to the problem of individuating members of a given group and to the problem of the correct use of quantifiers. Returning to my example of Conan Doyle's novels, we ask the following question: What does Holmes refer to when he says "every man so and so"? My point of view is that "man" refers to the postulated kind $\mathbf{man}_s$, which is different from the kind $\mathbf{man}_r$ since it includes Holmes, Dr. Watson, Moriarty and other men which we do not want to include in the range of our quantifier "every man", although Sherlock Holmes certainly wants to include them in the range of his quantifier "every man".

But why not assume that we have a single set which contains both the real and counterfactual situations of the real world and the real and counterfactual situations of Conan Doyle's world? The answer is that we cannot assume that a situation belongs both to our world and to Conan Doyle's world, since the count noun "man" would have two different interpretations. We do not want to include Dr. Watson as a man when we refer to "every man", although Sherlock Holmes certainly wants to include him when he (Holmes) speaks about all men.

Crowds, in particular crowds of people, are considered by Parsons as characters by themselves and no means of individuating the members of a crowd are provided. I have already given an example where we show that we need to quantify over individual members of a crowd. I repeat that example here in order to discuss it. Suppose that we read in a novel that a large group of people or a crowd of people form a chain, that each person carries one flag in such a way that whenever somebody carries a blue flag, his neighbor carries a red flag and vice-versa. As already mentioned, Parsons does not have enough fictional objects, correlated to sets of properties, to interpret these statements correctly, since his quantifiers range over characters only. I do not have this problem, since I have postulated the kind $\mathbf{man}_s$ in the story. So in the example of the crowd, the quantifiers range over $\mathbf{man}_s$ and we have enough men to interpret that sentence.

A pair of twins presents us with similar problems. Let us suppose that nothing in the story permits us to find a difference between the two twins. The pair of twins is a member of the kind $\mathbf{person}_s \times \mathbf{person}_s$ of the form (t, t'), we can count them, they are two and they are automatically individuated by the kind $\mathbf{person}_s$. Even if, in the story, there is nothing that permits us to distinguish between t and t', we can talk counterfactually about them and assume that in a given situation of the world of the story, t, let us say, could have had blue eyes and t' could have had brown eyes.

In order to answer questions about the connection between the "real" Gladstone and the "real" men on the one hand, and the Gladstone and the men of the story on the other, we must first describe a way to associate sentences or theories of the world of the story with sentences or theories of the real world. Thanks to this association, we can compare sentences or theories of the real world with sentences or theories of the world of the story. This comparison will give us a mechanism to "transfer" parts of the "relevant background beliefs" to the world of the story to make that story coherent. If T_r is the set of "relevant background beliefs", we shall define a theory T_s which we impose as true in the world of the story. This theory then becomes part of the "idealized story". In general, we transfer the minimal amount of background beliefs that enables us to understand the story and make it coherent. We are not looking for a "maximal account" as Parsons would, but for a minimal idealized story!

We are now able to compare the real Gladstone with the Gladstone of the story by relating statements of the background beliefs about the real Gladstone with statements that are true about the Gladstone of the story.

But first, we need a definition. If ϕ_r is a formula in the original language generated from basic sorts of the form k_r (and excluding others), we let ϕ_s be the result of substituting in ϕ_r all variables of sort k_r by corresponding variables of sort k_s and all constants of sort k_r by corresponding constants of sort k_s (as $Gladstone_r$ and $Gladstone_s$). Let us illustrate this definition with the following example: if ϕ_r is $meet_r(x, Gladstone_r)$ where $x \in Var_{man_r}$, then ϕ_s is $meet_s(x, Gladstone_s)$, where $x \in Var_{man_s}$. We can now state the transfer mechanism as follows: if ϕ_r is a sentence (without free variables) which is true in some real situation of our world, we put $\Diamond\phi_s$ in T_s. Notice that we are really transferring modally closed sentences only. In fact, we cannot transfer ϕ_r itself if the computation of its truth values required a specific situation, since we cannot compare situations across worlds. On the other hand, modally closed sentences such as $\Diamond\phi_s$ satisfy the excluded middle (see section 2.4) and hence we do not require specific situations to compute their truth values. As an example, it just does not make sense to transfer the sentence "Gladstone succeeds with his Irish policy", although we could transfer the sentence "it is possible that Gladstone succeeds with his Irish policy". Transfering this sentence to the story assures us that Gladstone, contrary to other men of the story has the possibility of succeeding in carrying out his Irish policy.

To compare the real Gladstone with the Gladstone of the story we can then relate modally closed statements of the background beliefs (about the real Gladstone) with modally closed statements that are true about the Gladstone of the story. We could say that the Gladstone of the story is a "counterpart" of the real Gladstone. Notice, however, that in spite of obvious similarities with Lewis's notion of "counterpart" (see Lewis [19]) there are deep differences between these two notions. In the first place, our "counterpart" of Gladstone does not have all properties of the real Gladstone, but only some of them, a choice which depends on the story. In the second place, and this is the most important difference, our "counterpart" of Gladstone cannot occur in either real or counterfactual situations of our world. It is Gladstone himself who appears in counterfactual situations of the real world. Similarly, it is the Gladstone of the story himself who appears in the counterfactual situations of the story. This of course is a consequence of my thesis that proper names are rigid designators.

We notice that my formalization of kinds follows Kripke's paradigm of a dice as studied in "baby probability": to study the throw of a dice we must consider not only the actual situation of the dice showing 4, say, on its upper face, but also the possible situations of the dice showing 1, 2, 3, etc. on its upper face. In other words, we must associate a set of possible situations in which one and the same dice may occur. Similarly, as explained in section 2.3, we have associated with one and the same man, Gladstone, a set of possible situations in which *he* can occur.

Our theories about the kind **man**$_s$, where s is a given story of Conan Doyle, are given by the transfer mechanism applied to our theories of man obtained from our background beliefs about man. In the case of a "realistic" story by authors like Balzac, Zola and Mauriac, we can understand the story with our usual theories of man, since nothing said in the story clashes with our theories. In the case of fairy tales or science fiction, on the other hand, we transfer only conditions which do not contradict the story. For instance, to understand the story of the three little pigs, we postulate the kind **pig**, whose members have properties obtained from our theories of pigs...and of men! We transfer only the conditions which do not clash with the story. We do not transfer the knowledge that pigs do not talk! In Swift's novel, the conditions for being a "Yahoo" and a "Lilliputian" come mainly from the story itself and are supplemented by our anthropomorphic vision. In Conan Doyle's novels, we bring to the understanding of the story our theories of men almost entirely. Conan Doyle wants his men to "look like" our men...and the best way to obtain this result is to say nothing about "his" men since then he can be assured that our background beliefs will not clash with his stories.

In this sense, our understanding of the story does not contradict our background beliefs, as it was suggested in the introduction, provided that the relevant theories are only *partially* transferred.

Macnamara has pointed out the following difficulty with the notion of background beliefs and the transfer mechanism. Suppose that one of the clients of Dr. Watson complains about gout and he is told to drink less alcohol. According to my viewpoint, to understand Dr. Watson's advice, we must transfer those of our background beliefs about alcohol and gout which does not clash with the story. Since today we do not believe that alcohol causes gout, we cannot understand this advice. It seems that we need to bring the background beliefs of the relevant period of the story, for instance, the England of the last century in the case under consideration, to see how gout was thought to be related to alcohol and thus to understand Dr. Watson's advice. Similar remarks apply to novels which are supposed to take place, let us say, in the Middle Ages, even if these novels have been written by modern authors. It is amusing to remark that this example of gout and alcohol is also discussed by Zenon, the hero of "L'Oeuvre au noir" by Marguerite Yourcenar.

According to Putnam [27], it is impossible to describe kinds uniquely, essentially because we have non-standard models for first-order theories. As Makkai has pointed out, we do not require that theorem to make Putnam's point. It suffices to point out that theories, even formulated in higher order logic do not distinguish between isomorphic structures. Nevertheless, as Makkai has also remarked, Putnam's position does not take into account the fact that we seem to be able to describe kinds uniquely by pointing out some of their members. This possibility is not available in the world of fiction and therefore Putnam's argument seems correct in this realm. We cannot point to a man of the story and say "Hamlet" or "man". We cannot fix interpretations of count nouns (kinds) uniquely, even in the case of the authors providing a gestalt type for members of a given kind as, for instance, the drawings of a troll by Tove Janssen or of a kabouter by Rien Poorvliet and Wil Huygen. In these cases, if the drawings are slightly altered, the question of whether the drawings still represent a troll or not, a kabouter or not, does not make sense. Trolls and kabouters "lack internal structure".

We could draw a parallel between the kinds of the story and the artificial kinds of our world. I think that the suitably relativized principle of application of an artificial kind is given by a set of conditions of the type "a bed is an artefact to sleep in". What would be the "internal structure" of a bed in the same way that a particular type of DNA gives the internal structure of a dog? Of course, we could point to a bed and say "That is a bed" but this pointing does not fix membership in the kind as in the case of the basic kind dog for instance.

As an aside, let us remark that according to Putnam [26] one-criterion words, like, for instance, pediatrician = doctor specializing in the care of children, have a strong tendency to develop a "natural kind" sense with all the rigidity and indexicality attached to it. I think that this remark could also apply to many artificial kinds, and that, for example, children would learn the kind dog in the same way that they would learn the kind `chair`. Furthermore, I think that this is one of the reasons why we can deal with complicated artificial kinds postulated in Literature, in Philosophy and to a certain extent in Physics. Our mind has been organised to understand natural kinds!

At the beginning of this section I ask the question: How can we speak about Sherlock Holmes in our world? We remark that unfortunately, the sorts introduced already still do not provide the means to talk about Sherlock Holmes, Moumine, Flimnap, etc. in *our world*. In the next section we will see how we can accomplish this.

3.2.4 The forgetful functor: character

To be able to talk about Sherlock Holmes or Moumine in our world, we must "bring" in the kinds in which they appear to our world. The idea is simply this: we may "freeze" the kinds of the story by forgetting the coincidence relation. Furthermore, we must "freeze" the relations between the kinds to arrive at abstract relations between abstract sets. We notice, however, that abstract sets of "urelements" may be considered as a kind of the real world or of any world, by defining as principle of coincidence the trivial one which assigns the empty set of situations of our world or the world in question to any two members of the kind in question.

I call *character* the forgetful functor which associates with a kind of the story the set of its members and with a morphism of kinds the corresponding set-theoretical function.

I now introduce further basic sorts and constants which I interpret as these abstract sets of "urelements" and, which then may be viewed as abstract kinds of our world. Thus, for the count noun "troll", I introduce a further basic sort $troll_{s0}$ which is interpreted as the set of all trolls, namely, the underlying set of the kind $\mathtt{troll}_s$. In a similar vein, I introduce the new constant $Moumine_{s0}$ to be interpreted as Moumine, a member of the set of trolls, and the relational symbols to be interpreted as ordinary relations between abstract sets. In this way, we can talk in our world about the world of the story.

I think of these "frozen" kinds or sets as being the characters of a story, whence the name "character" for the forgetful functor. This notion of character of a story differs from that which has been used in theories of fiction, since characters as used in these theories correspond in our theory to members of these "frozen" kinds which are *individualized* in the story. As I emphasized at the beginning of this chapter, I do not believe that the consideration of characters in this restricted sense suffices to describe the logic of fiction: we may lack individualized characters to interpret our quantifiers correctly, and most importantly, we are not able to talk counterfactually about Moumine, Hamlet, Sherlock Holmes, etc.

The reason we need a new kind to talk about Sherlock Holmes in our world is that Sherlock Holmes cannot appear in any real or counterfactual situations of *our world*, as I already explained: we certainly do not include Sherlock Holmes when we refer to "every man", regardless of the situation. For similar reasons, if we were to find animals in our world having the stereotypes of unicorns, they *would not be* unicorns. This seems to be consistent with Kripke [17]. I quote here a passage of Agatha Christie's book "Elephants can remember":

> *"I don't know what's likely to happen," said Mrs. Oliver, "because, you see, in all the crimes that I write, I've invented the crimes. I mean, what I want to happen, happens in my stories. It's not something* that actually has happened *or* that could happen." The emphasis is mine. [8, page 94]

These "frozen" kinds give us the means to recapture part of Parsons's theory of characters. As in Parsons's theory, we do not have situations since we only consider abstract sets. However properties of members may be frozen to become "situationless" and hence either true or false. As an example, a situational property of Sherlock Holmes such as "Sherlock Holmes visits Miss Cushing" in "The adventure of the cardboard box" can be "frozen" into "At the beginning of his investigation Sherlock Holmes made a visit to Miss Cushing". I believe that every "situational" property can be rephrased in this way, since situations in a story are describable in the language at the narrative level, as mentioned in section 3.2.2. We thus have the means to speak about Sherlock Holmes in our world.

I emphasize, once again, that we cannot talk counterfactually about Sherlock Holmes *as a character*, since as a character, he belongs to an abstract set of our world. He cannot occur in any possible or actual situations of our world. This consequence of my theory explains the use of situationless statements as the only way to compare reality and fiction.

Besides the "frozen" kinds there are kinds which are abstract in any world in the sense that their members cannot appear in any situation of any world. Thus, these kinds can be interpreted in all worlds. As an example, since Holmes mentions natural numbers in "The adventure of the Musgrave ritual" in connection with measurements of a tree, we must postulate the kind **natural number**, to interpret his computations. However, this kind is the same abstract kind **natural number** that we need to interpret our computations. His notion of natural number is exactly the same as our notion of natural number. In fact, the kind **natural number** is a kind whose members never appear or occur in any situations of any world. In this sense their members are abstract in any world. That cannot be said about Holmes. Holmes is a man that lives in his world, the world of Conan Doyle's stories and the men of that world constitute a concrete kind of that world, although an abstract kind of ours. In his world, Holmes may appear in situations of his world, a natural number never.

A notion like "abstract kind" is not category-theoretical as the following simple consideration shows: the kind **cow** is isomorphic in the category of kinds to the set of singletons of cows with the transported coincidence relation. The first kind is "concrete" in the sense that it consists of concrete members, the cows, whereas the other is "abstract", consisting of sets. Similarly, the intuitive notion of existence or appearance in a situation is not category-theoretic since the transported existence, defined by the transported coincidence, is not the intuitive existence. In fact, cows may exist or appear in a situation, whereas sets such as singletons cannot! If we want to conclude, in spite of this, that our categorical notion of "existence", as coincidence, corresponds to the intuitive one for some kinds such as **cow**, we seem to be forced to postulate that the category of these kinds is skeletal; in other words, the only isomorphisms are identities. These considerations are consistent with some philosopher's insistence that "persons cannot be identified with sets, since sets are inert, do not have a sense of moral responsibility, etc." As an aside, let us remark that this is another instance of the contradiction between set-theory and category-theory that is far from being adequately dealt with!

To clarify the connections between a kind such as $\mathbf{troll}_{w_0}$ and an abstract kind such as **natural number**, I introduce the following terminology: a kind of a world is *fictitious* if its members cannot appear in situations of that world. Thus **unicorn** and $\mathbf{troll}_{s0}$ are fictitious kinds of our world, but $\mathbf{troll}_s$ is not a fictitious kind of the world of Tove Janssen's stories. On the other hand the kind **natural number** is fictitious in every world. Members of fictitious kinds are said to be fictitious.

Is the sentence "Hamlet is fictitious" true? In my theory, this sentence turns out to be ambiguous: if "Hamlet" refers to the member of the kind $\mathbf{man}_s$ in the world of the story, this sentence is trivially false. On the other hand, if "Hamlet" refers to the member of the kind $\mathbf{man}_{s0}$ of our world, namely, if it is a character, then the sentence is true. Notice that we have the means to eliminate the ambiguity of the sentence in question: "$Hamlet_s$ is fictitious" vs "$Hamlet_{s0}$ is fictitious". The world of the story in which, for instance, Hamlet dwells, is the "real world" for Hamlet, in which Polonius, Horatio, Ophelia also live. But contrasted to that world there is the world of the story, the play within the play, in which Gonzago, Baptista and Lucianus live. Just as we can talk about Hamlet in our world, Hamlet can talk about Gonzago in his world. In that world the sentence "Gonzago is fictitious" is true. We also remark that the property "fictitious" cannot belong to the set of conditions that determine the principle of application of for instance, $\mathbf{man}_{s0}$ since the principles of application of $\mathbf{man}_{s0}$ and $\mathbf{man}_s$ are the same and the members of $\mathbf{man}_s$ appear in the situations of the story. We see that "fictitious" is related to existence.

With the help of the constants already introduced, we can capture Parsons's notion of surrogate as follows. I define the surrogate kind of K_r to be K_{s0}, assuming of course that K_s is a kind of the world of a given story. We then say that the interpretation of a constant c_{s0} of sort k_{s0}, which is a member of the surrogate kind is the surrogate of the interpretation of the constant c_r which is a member of the real kind K_r. We assume that such a constant c_r is available. As an example, the interpretation of "the Denmark of Shakespeare", which is a member of $\mathbf{country}_{s0}$ is the surrogate of the interpretation of the constant $Denmark_r$. Nevertheless, the interpretation of Moumine's country is a member of $\mathbf{country}_{s0}$, but is not the surrogate of the interpretation of any c_r, since there is no c_r available in this case.

In the next section, I shall try to answer Parsons's argument for considering the "real" London as occurring in the story and I shall present some aspects of the old discussion of the problem of reality versus fiction.

3.2.5 Reality versus fiction

Parsons's argument in favour of his view that it is true that London (the real) is such that, according to the novels, Holmes lived in it, is just that he does not see any difference in the following referential situations:

> *I think I am inclined to accept it because I see no difference in the referential situations:*
>
> 1. *Telling a lie about Jimmy Carter*
> 2. *Telling a lie about Carter which is very long (e.g., book length).*
> 3. *Making up a story about Carter which is not intended to deceive anyone, and which contains falsehoods.*
> 4. *Writing a work of fiction in which Carter is a character.*
>
> [25, page 58]

Parsons maintains that there are cases of *1.* which can be reported as "Carter is such that according to what Parsons said he is P"(*), and similarly that at least some cases of *2.-4.* can be reported in such a fashion. We first notice that at least in *3.* and *4.*, and contrary to *1.*, specific mention of a story or a work of fiction is made. It seems to me that the correct way to report cases of *3.* should be "Carter is such that according to what *Parsons's story* said he is P", rather than "Carter is such that according to what Parsons said he is P", which means something quite different. In what follows, we shall ignore problems connected with the intentionality of the expression "says that P" and I refer the reader to the section 3.2.7 for a discussion of some of these problems.

I believe that Parsons is confusing the statement "Carter is such that according to Parsons he is P" with the statement "Carter is such that according to Parsons's story he is P". These statements could occur more easily in a logic textbook than in real life and I would like to rephrase them to make them more understandable. I shall use the constants $Carter_r$ and $Carter_s$ as explained in section 3.2.3 to refer to the real Carter and the Carter of the story, respectively. The first would give: "$Carter_r$ has the property Q_r", where x has the property Q_r iff Parsons said that x has the property P_r. This sentence is true iff Parsons actually said that Carter has the property P. On the other hand, to find out whether what Parsons says is true we should look at the real world to see whether the real Carter has the property P. The second statement would give: "$Carter_r$ has the property R_r", where x has the property R_r iff Parsons's story says that $Carter_s$ has the property P_s. In this case, R_r is a spurious property which any real person has whenever $Carter_s$ has the property P_s in Parsons's story. Now this sentence is true iff $Carter_s$ has the property P_s in Parsons's story iff "$Carter_s$ has P_s" is true in the world of the Parsons's story. In other words, to find out whether $Carter_r$ has the property R_r we must read the story in question.

The reader may have noticed the following peculiarity in the way I have handled the pronoun "he" in these examples: "he" in the first statement refers to the real Carter, whereas the "he" of the second refers to the Carter of Parsons's story. In fact, my theory leads to some complications when quantifiers, pronouns and possessive adjectives are used in contexts which blend fiction and reality and we shall study other examples. Let us suppose that the real Carter has written an autobiography. Does Carter refer to himself? Or does he refer to a member of the kind **man**$_s$ of a story? It seems to me that the answer to this question really depends on the status of the book in question: Is this a work of history or is this a work of fiction? My point of view is that in the first case, Carter refers to himself, whereas in the last, "Carter" refers to a native of the story. It is interesting to point out that an autobiographical novel may succeed in giving a true psychological portrait of the real person in spite of the fact that several statements which purportedly describe facts about him may be false. This is not so for autobiographies which must "stick" to facts. Such a distinction would be hard to understand if only the real person was referred to in both cases. In the following discussion, I shall assume, for the sake of argument, that Carter has written an autobiographical novel.

As another example, consider "In his autobiographical novel, Carter asserts that he kissed Marylin Monroe" or "Carter is such that, according to his autobiographical novel, he kissed Marylin Monroe". Let us concentrate on the second one. The question is: What does "Carter" refer to? The problem is that "his" and "he" cannot both refer either to the real Carter or to the Carter of the story. My viewpoint is that we are making an assertion about the real Carter, the man who wrote the autobiographical novel ("his" novel) and hence "he" cannot refer to the real Carter, but refers to the Carter of the novel. Once again, we seem to be forced to give a non standard interpretation to pronouns and possessive adjectives. In my language, we may express the example as follows: "$Carter_r$ is such that, according to his autobiographical novel, $Carter_s$ kissed $Marylin\ Monroe_s$. Once again, we have an example of a spurious property which holds of the real Carter (or for that matter of anybody) precisely when $Carter_s$ kissed $Marylin\ Monroe_s$ in the world of the novel.

For another example of the same type, consider "Denmark is such that, according to the story, Hamlet lived in it". On the one hand, the "it" refers to the Denmark of the story and not to the real country, since Hamlet lives in *it* and we want to keep the logic of the relations standard. On the other hand, "Denmark" refers to the real Denmark. The sentence may be paraphrased as "$Denmark_r$ is such that, according to the story, 'Hamlet lived in $Denmark_s$". This sentence, once again, assigns a spurious property to the real Denmark.

The next example is even more puzzling. Let us suppose that an historical-biographical novel about a given group of people has been written, but that everybody, mentioned in the novel, claims that the house attributed to him in the novel is quite different from his own. We have here a case of quantifiers, pronouns and possessive adjectives intermingled between fiction and reality. The way out of this riddle seems to be to enumerate the individuals which are in the range of the quantifiers and rephrase quantifiers by conjunctions. For instance, if the novel is about John, Peter and Andrew, we add in the language the corresponding constants $John_r$, $John_s$, $Peter_r$, $Peter_s$, $Andrew_r$ and $Andrew_s$. To simplify, we shall assume that the same proper names are used in the novel. Our sentence can then be paraphrased as the conjunction of three sentences of which the first could be: "$John_r$ claims that the house of $John_s$ in the novel is quite different from his own". Notice that we can express this difference as follows: for some properties ϕ_r which are true of $John_r$'s real house, the corresponding properties ϕ_s are not true of the $house_s$ of $John_s$ in the novel.

I conclude that we can do away with Parsons's immigrant objects, which are real objects occurring in the novel, to explain these and other phenomena already discussed. Parsons has only considered characters with proper names such as London and Gladstone for his immigrant objects. But I do not see why the egg eaten by Sherlock Holmes during one of his breakfasts should not also be considered as an immigrant object which should then be real. I believe that everything taken for granted from our world and individuated in a story should be considered by Parsons as an immigrant object.

If this is so, the problem of the change of the logic of English becomes even more acute: Sherlock Holmes eats a real egg which does not have the property of being eaten at all! In my context, on the other hand, the distinction between immigrant and native objects does not arise. Independently of the difficulties that Parsons's view implies about the change of the logic of English, I think that his distinction blurs the whole picture of fiction versus reality. I prefer to postulate that every character is native to the story, and that to understand the story we transfer most or part of our stereotypes about the real objects in question.

Let me add a few reflections on some aspects of fiction versus reality which may occur in colloquial statements.

If two people disagree about the existence of unicorns, the one that denies the existence seems to be in the platonic predicament explained by Quine [28, pages 1–3]. This is not so in my view: I believe that both arguers must assume the kind **unicorn** to start with so as to be able to understand expressions like "three unicorns", "twin unicorns" or "every unicorn has four legs" to which our stereotypes of unicorn apply. The real disagreement is between the existence of the members of this kind. One of the discutants would claim that it is an abstract kind, whereas the other would maintain that some members appear in real situations of our world. In other words, they agree on what should count as a unicorn, but disagree on the relation of coincidence or existence for the kind **unicorn**.

This distinction between abstract and concrete kinds is reminiscent of one due to Leibniz. According to Mates [23, page 9], Leibniz made the following distinction between "essential" and "existential" propositions. Any sentence "A is B" is equivalent to the corresponding sentence "AB is an entity". But the word "entity" is ambiguous; it can mean the same as "possible thing" or it can mean the same as "actually existing thing". If it is taken in the former sense, the sentences concerned are called "essential"; if it is taken in the latter sense, they are "existential". So, for instance, the sentence "Pegasus is a winged horse" is equivalent to the sentence "Pegasus, the winged horse, is an entity" which is true as an "essential" sentence but false as an "existential" sentence.

In the next section, I shall contrast myths and stories with the help of the reflections made on reality versus fiction and discuss the problem of metamorphosis in relation to questions of identity.

3.2.6 Myth, story and metamorphosis

There is a popular belief in the countryside of the north of Chile according to which there are men whose heads become owl-like birds at night and wander in the countryside, while these men at home still enjoy their sleep ...without their heads. During the day, such a being, called "chonchón", is a normal man.

This myth is not intended as a fairy tale. The chonchón is (according to this myth) a real living being which appears in situations of our world. So there are situations of our world in which the head of a man goes wandering under an owl form! We could explain this myth by saying that the theories of men of this culture and of this linguistic community, allow men with these extraordinary possibilities. Confronted with another linguistic community, these possibilities may be rejected and other theories of man adopted.

Now if we were to write a story about this myth, the case would be quite different. The intention would be to write a story, not a description of a (purported) fact about the real world. The men, or some of the men in that story, would have the strange capability of having their heads wander in owl form at night. But for that strange fact, they would be exactly like a man and act exactly as a man does. We would then postulate the kind **chonchón$_s$** which, depending on the story, could even be a sub-kind of the kind **man$_s$**. Only parts of our theories about men can be transferred. For instance, the background beliefs that a man needs his head to live cannot be transferred because we would then have a story which is not coherent. The notion of man$_s$ depends also on what is said in the story. If the story evolves during the years, then the notion of man might change to accommodate the new story. We could also transfer the criteria or decision procedure to find out whether a man is or is not a chonchón: when in the night a person hears the flight of an owl-like bird, that she does not see, she says to the bird "Come tomorrow to borrow some salt". Then, if the following morning a person comes to borrow salt, it is clear that this person is a chonchón.

Part of these reflections can also be applied in the case of Santa Claus. We may view this case either as a myth or as a story. The myth is told to children by parents who describe Santa Claus as a real man who brings real gifts to real children. The story (told by some progressive parents) is not intended to deceive them and concerns gifts of the story given by a man of the story to children of the story. It is instructive to analyze the logic of both the myth and the story according to my theory. The very first question that we must answer is: what is the grammatical role of "Santa Claus"? Is it a description or a proper name? Assume that it is a proper name. Then it is a rigid designator, and rigidity presupposes a count noun to be interpreted as a kind. But this kind cannot be a kind of our world, since Santa Claus cannot occur in situations of our world. Therefore it can only be a kind of a world of a story, say the kind **man**, of the world of the story of Santa Claus. To understand the story, we need to postulate further kinds like **child**, and **gift**, and we need to transfer part of our "background belief" to this world. Since the myth, just as in the case of the chonchón, is not meant as a story, but it purportedly deals with real children, real gifts, etc. it follows that "Santa Claus" in this case is not a proper name, but a description. The myth could then be stated as "There is a person such that so and so" and, as we all know, this statement is told by the parents knowing that it is false with the intention of making their children believe the myth. Indeed they hope that the description that they give of Santa Claus will be "realistic" enough to permit them to encompass Santa Claus in their stereotype of persons living in our world. The Santa Claus of our myth must be described in such a way that the child assumes that Santa Claus is a real man. A consequence of this analysis is that we cannot freely talk counterfactually about Santa Claus in the myth, although we could do so in the story.

We now turn our attention to the notion of metamorphosis. My theory has been designed to be able to accommodate for the metamorphosis in a very straightforward way. In section 2.[illegible] we have studied relations between kinds, namely functions which preserve the coincidence relation. I explained the connection between the kinds **baby**, **adult** and **person** with the help of a function, which is not representable in the natural language, from the set of babies into the set of persons, and similarly, from the set of adults into the set of persons: intuitively the function that associates with each baby or adult the person that underlies him. In this way, we may explain the metamorphosis of a baby into an adult, the problem being that the baby and the adult belong to different kinds. The identity between a baby and an adult is really an assertion about their underlying persons, namely that they are identical as members of the kind **person**. The kind **soul** of many religions is required to state metempsychosis, the suffering in hell, the resurrection. The man who lived and the resurrected (man) have the same underlying souls, they are identical as members of the kind **soul**.

As an aside, let us remark that van Baaren [1] says that in Bouddhism there is neither an immortal soul nor anything of this nature which survives after death and that could explain reincarnations. He then asks this question: But what does revive again? And he concludes that, according to our occidental categories of thought, it is impossible to answer that question. We need a certain continuity to explain change. Otherwise, as I mentioned at the beginning of this chapter, we cannot even understand the question "What is it that changes?"

Metamorphoses in fairy tales may be explained by considering further underlying kinds. The problem with this approach is that we do not always have such a kind at our disposal and to understand literary texts, we are sometimes forced to create some artificial kinds. But that does not seem such a difficult task for us.

We shall now study the case of some more metamorphoses. The movie "Lady Hawk" offers an interesting case of metamorphoses. Two lovers were condemned by a jealous bishop never to see each other under their human form. At dawn, the lady becomes a hawk and spends the day on the hand of the knight, her lover; at dusk, the hawk becomes a lady while the knight is metamorphosed into a wolf. The hawk does not know that it has gone through metamorphosis and that it has some relations to a being which is in love with the knight, similarly, the wolf does not know that it has gone through metamorphosis and that it has some relations to a being which is in love with the lady. Nevertheless, as humans, the knight knows that the hawk is his metamorphosed lady and the lady knows that the wolf is her metamorphosed knight...let me tell you that the story has a happy ending!

To understand this story, we must then postulate a kind which will explain what changes. The background beliefs of our linguistic community will probably not allow us to postulate the kind `soul`, since, in this linguistic community, animals are not supposed to have a soul (the same fate was shared by women not so long ago!). In the fairy tale the "Beauty and the Beast", the kind `soul` could explain the metamorphosis because the Beast had a very anthropomorphic behaviour and aspect. So we could say that the underlying soul of the Beast was the same as the underlying soul of the Prince. In "Lady Hawk" however, we must consider another kind. I suggest the kind `living being`. The living being which underlies the hawk is the same as the living being which underlies the lady.

In this particular example and contrary to usual fairy tales, let us notice that the theories of hawk and wolf are taken unchanged from our background beliefs and do not exhibit any anthropomorphic features.

We look now at another myth that comes from the region of Burgundy in France. I quote some lines of "La billebaude" by Henri Vincenot [35, pages 132–133], which describe a metamorphosis.

> *Nous sommes tombés sur l'insaisissable Dahut, cette fille folle d'amour, qui, depuis "le temps des grosses pierres" court les bois à la recherche d'un garçon, mais que personne n'a jamais vue, car aussitôt qu'un être humain la regarde, elle se transforme en une espèce de bête refouse, galeuse, affreuse. C'est à sa capture qu'on convie les commis un peu simplets, mais attention! celui qui la verra sous sa forme de fille folle d'amour tombera raide mort, ce qui donne à l'aventure un certain air de chevalerie".*

In this case the underlying kind could also be `living being`. The interesting aspect of this metamorphosis is that the metamorphosis takes place only under observation by a human eye. This is also what happens with the physical entities of Quantum Mechanics which under observation act differently. Maybe there is also a metamorphosis in this case! With the transmutation of elements there is definitively a metamorphosis involved. The caterpillar metamorphoses into a butterfly. It is the same insect that undergoes the metamorphosis. The clouds, rain and snow are metamorphosed into each other having all the time the same underlying substance H_2O. These last examples were given to show that metamorphosis is not a phenomenon whose unique domain is literature!

The story of Dr. Jekyll and Mr. Hyde presents another aspect of metamorphoses. Do "Dr. Jekyll" and "Mr. Hyde" rigidly refer to the same person or not? Nowadays, we would say that this is the case of a person having two different personalities. The names would then be used to name the two different personalities, members of the kind `personality`. They would refer rigidly to the given personality. The underlying person of Dr. Jekyll is the same as the underlying person of Mr. Hyde. The underlying kind `person` allows us to understand the metamorphosis of Dr. Jekyll into Mr. Hyde.

3.2.7 Intentional relations, descriptions, proper names and opacity

The reader may have noticed that I have avoided in this chapter expressions like "to look for", "to seek", "to believe", "to think about", etc. In fact I have tried consistently to avoid "intentional relations". This may seem surprising, since, following Montague, I interpreted such expressions in Chapter 1 and I could have done something similar in this chapter. We notice, however, that such interpretations presuppose very strong idealizations of which the following will be typical examples. The sentence "John believes that p" is interpreted as a relation between a man and a proposition. As a consequence, if "p" is logically equivalent to "q", then the interpretation of "p" is the same proposition as the interpretation of "q" and hence we conclude at once that "John believes that p" iff "John believes that q". The absurdity of this view becomes clear when we take "$1+1 = 2$" for "p" and a hopelessly complicated theorem of Algebraic Geometry for "q". Similarly, "John looks for Neruda" is interpreted as a relation between two persons. But it may well be that John is looking for Neruda but not looking for Neftalí Reyes, a state of affairs that is not allowed by my logic since Neruda and Neftalí Reyes rigidly refer to the same person. Furthermore we have remarked, see section 2.4.1, that from "John seeks nobody" we cannot conclude "It is not the case that John seeks somebody". This is another reason why I am not satisfied with the treatment offered to intentional relations and I have avoided studying them in my work.

In fact, I do not know of any successful attempt to deal with these types of relations in the literature. A recent and interesting proposal of Cresswell [9] in which propositions are structured (so that "p" and "q" of my example are interpreted as different propositions), fails precisely in his account of sentences of the type "John is looking for Neruda" as Macnamara has pointed out.

To illustrate this problem in literature, we will consider the story "The man with the twisted lip" by Conan Doyle. This is a story where two proper names "Neville St. Clair" and "Hugh Boone" are used in epistemic contexts. Holmes is looking for Neville St. Clair, but Neville St. Clair is Hugh Boone, nevertheless Holmes is not looking for Hugh Boone. On the other hand if Sherlock Holmes shakes hands with Neville St. Clair, he also shakes hands with Hugh Boone.

Some contexts involving intentional relations seem to be interpretable without undue idealization and I shall give two such examples. The first has already been discussed in Chapter 1, namely "George IV wished to know whether Scott was the author of *Waverly*". We may view this statement as one of the type "George IV wished to know whether p", where "p" is "Scott was the author of *Waverly*". Although "p" and "Scott was Scott" have the same truth value, their interpretations are different propositions. In fact the interpretation of "p" consists of the situations in which Scott is the author of Waverly, whereas the interpretation of "Scott was Scott" is the proposition which contains every situation as a member. Notice that this distinction is reminiscent of that between descriptions and proper names. We could even say that "Scott was Scott" "refers rigidly to the truth value true".

The second example is found in Oedipus Rex by Sophocles. In Oedipus Rex, a work that the Colombian Nobel Prize winner, Gabriel García Márquez, considers the greatest detective story ever written, Oedipus learns that the plague in Thebes is due to the anger of the gods, who are offended because the murderer of the old king Laius was never found and punished. Oedipus then starts looking for the murderer, a search that ends with the discovery that the murderer is himself. Although it is true that Oedipus was looking for the murderer, it is clear that he was not looking for himself. If we interpret "the murderer of the old king Laius" as an ordinary member of the kind man_s, then the reference of this description must be identical with Oedipus, since they happen to coincide in that world, and the property that the murderer has, namely that he is looked for by Oedipus, should also be a property of Oedipus, which is not the case: Oedipus does not have the property of being looked for by Oedipus. In order to understand this phenomenon, referred to as opacity, which is a quite normal feature of natural language, we must interpret "the murderer of the old king Laius" as a member of the kind whose members are sets of properties of the members of the kind man_s. In this case, "the murderer of the old king Laius" does not refer rigidly as the proper name "Oedipus" does. In fact, if the story had been different, Oedipus might not have been the murderer of the king; he could, for instance, have engaged somebody to kill him, although he could not have failed to be Oedipus.

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