# Alternative Quantum Interpretations

# and

# The Two-Slit System

by

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A Thesis

in

The Department

of

Mathematics and Statistics

Presented in Partial Fulfillment of the Requirements

for the Degree of Master of Science at

Concordia University

Montreal, Quebec, Canada

September, 1994

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ISBN 0-612-01337-5

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**ABSTRACT**

**Alternative Quantum Theories and The Two-Slit System**

Jean-Guy Blouin

A basic experiment of quantum mechanics, the two-slit experiment, will be re-examined using alternative descriptions of quantum mechanics. The various classical and quantum theories needed will be introduced, and the two-slit system will then be solved using Feynman's path integral formulation.

The resulting wave function will then serve as the starting point for both Bohm's quantum potential interpretation and Nelson's stochastic quantum interpretation. The observed interference pattern in the two-slit experiment will be reinterpreted in terms of the quantum potential, eliminating the need for the duality principle.

## ACKNOWLEDGEMENTS

I would like to take the opportunity to thank those who have either helped or supported me in the past, including Prof. A. Boyarsky, without whom I would never have heard or have thought of investigating this topic; my good friend Natasha Cloutier as well as Cynthia Parkinson, who suffered tremendously while typing this thesis; my loving parents, whose financial help kept me from starving; and finally, to all my good friends who normally have to endure me.

CONTENTS

## Introduction

The classical interpretation of quantum mechanics (henceforth denoted CQM) assumes that a physical state of a certain system is completely determined by a wave function which allows a probabilistic description of the examined model. Thus no clear evolution of a system can be determined, as only the evolution of the probabilities can be obtained. As a consequence, no clear trajectories of particles can be theoretically observed.

Although CQM is quite adequate for the current description of the microscopic world, it is by no means certain that it will remain so in the future. The existence of other quantum interpretations might therefore prove useful in expanding the microscopic knowledge of the universe. Two such interpretations will be applied to the two-slit system, namely Bohm's quantum potential interpretation and Nelson's stochastic interpretation, both of which allow the existence of trajectories. These two theories will be applied to the two-slit system.

The present paper contains two parts: **part one** contains the physical background necessary to comprehend the methods used to solve the two-slit experiment, while **part two** will solve the experiment, giving the quantum potential and the forward drift, leaving out as few steps as possible.

## Part One

## General Physics

### Introduction

Part one contains the classical and quantum theories needed for the second part. Chapter 1 deals with the classical theories of Lagrange, Hamilton and Hamilton-Jacobi needed to understand Chapter 2, which deals with quantum mechanics. The different theories discussed here include a short review of Schroedinger's CQM, Bohm's quantum potential interpretation, another short review of Nelson's stochastic mechanics, and finally Feynman's path integral formulation, specifically needed to solve Schroedinger's equation for the two-slit system.

# Chapter 1

## Classical Physics

### 1. Introduction

Chapter 1 introduces the basic elements of classical mechanics that are necessary for the comprehension of the following material. All the subjects covered are developed in [7], [8].

A fundamental notion in mechanics is that of particle, or material point. The position of a particle in $\Re^3$ is determined by its position vector $\mathbf{r}$, with coordinates $x, y, z$. The derivative of $\mathbf{r}$ with respect to the time $t$

$$\mathbf{v} = \frac{d}{dt}\mathbf{r}$$

is called the vector velocity of the particle. Differentiating once again with respect to $t$ will give the vector acceleration. Henceforth, differentiation with respect to $t$ will be indicated by a dot: $\mathbf{v} = \dot{\mathbf{r}}$.

Determining the position of a system consisting of $N$ particles requires $N$ position vectors, or $3N$ coordinates, called the degrees of freedom if they uniquely define the position. It might prove more useful to use coordinates other than the cartesian ones, say $q_1, q_2, \ldots, q_s$, to fully determine the position of a system. The $\{q_i\}_{i=1,\ldots,s}$ are called the generalized coordinates, and their derivatives $\{\dot{q}_i\}_{i=1,\ldots,s}$ the generalized velocities. The complete knowledge of the $q_i$'s and $\dot{q}_i$'s is enough to fully determine the state of the system and to predict its future.

### 2. Lagrangian Mechanics

Lagrange's equations of motion are derived using Hamilton's principle of least action which states that the motion of a system of particles from time $t_1$ to the time $t_2$ is such that the integral

$$\mathbf{S} = \int_{t_1}^{t_2} L(q_1, \ldots, q_s,\ \dot{q}_1, \ldots, \dot{q}_s, t)dt \qquad (1.2.1)$$

must have a stationary value. The function $L(q_1,\ldots,q_s,\ \dot{q}_1,\ldots,\dot{q}_s,t) \equiv L(q,\dot{q},t)$ is called the Lagrangian, and $S$ the action.

Suppose the system possesses only one degree of freedom, and let $q = q(t)$ be the function for which $S$ has a minimum. Hamilton's principle can be restated as

$$\delta \mathrm{S} = \delta\left(\int_{t_1}^{t_2} L(q,\dot{q},t)dt\right) = 0, \tag{1.2.2}$$

or, performing the variation,

$$\delta \mathrm{S} = \int_{t_1}^{t_2} \left[\frac{\partial L}{\partial q}\delta q + \frac{\partial L}{\partial \dot{q}}\delta \dot{q}\right]dt = 0, \tag{1.2.3}$$

where $\delta\dot{q} = \frac{d}{dt}\delta q$, and $\delta q$ is the variation of the function $q(t)$.

Integrating by parts the second term of (1.2.3) yields

$$\delta \mathrm{S} = \frac{\partial L}{\partial \dot{q}}\delta q \,\Big|_{t_1}^{t_2} + \int_{t_1}^{t_2} \left[\frac{\partial L}{\partial q} - \frac{d}{dt}\frac{\partial L}{\partial \dot{q}}\right]\delta q dt = 0; \tag{1.2.4}$$

the first term of which vanishes, since $\delta q(t_1) = \delta q(t_2) = 0$. The value of the integral must be null for all values of $\delta q$, from which

$$\frac{\partial L}{\partial q} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) = 0. \tag{1.2.5}$$

If there is more than one degree of freedom, then there will be $s$ equations of the form

$$\frac{\partial L}{\partial q_i} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) = 0 \quad i = 1,\ldots,s. \tag{1.2.5'}$$

If the Lagrangian of the system is known, then (1.2.5') gives the equations of motion of the system of particles as a system of $s$ second order differential equations requiring $2s$ initial conditions; say $s$ initial positions and velocities. The above constitutes a more intuitive approach to Lagrange's equations. For a more formal approach, the reader is referred to [16], theorem 6.8, where one can go directly from (1.2.2) to (1.2.5).

The Lagrangian function of a system is defined as the difference between its kinetic and its potential energies. References [7] and [8] give two different ways of determining the Lagrangian. Thus it is simply stated that

$$L = T - V, \tag{1.2.6}$$

where $T$ is the kinetic energy and $V$ the potential energy.

Taking the total derivative of the Lagrangian with respect to time,

$$\frac{dL}{dt} = \sum_i \frac{\partial L}{\partial q_i}\dot{q}_i + \sum_i \frac{\partial L}{\partial \dot{q}}\ddot{q}_i + \frac{\partial L}{\partial t},$$

which, using Lagrange's equation to replace the $\frac{\partial L}{\partial q_i}$'s can be written

$$\frac{dL}{dt} = \sum_i \dot{q}_i \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) + \sum_i \frac{\partial L}{\partial \dot{q}_i}\ddot{q}_i + \frac{\partial L}{\partial t} = \sum_i \frac{d}{dt}\left(\dot{q}_i \frac{\partial L}{\partial \dot{q}_i}\right) + \frac{\partial L}{\partial t},$$

or

$$\frac{d}{dt}\left(\sum_i \dot{q}_i \frac{\partial L}{\partial \dot{q}_i} - L\right) + \frac{\partial L}{\partial t} = 0.$$

The quantity inside the brackets is called the energy $E$.

Thus

$$\frac{dE}{dt} = -\frac{\partial L}{\partial t}.$$

If the Lagrangian is time independent (as will be assumed from now on), then $\frac{dE}{dt} = 0$, and the energy is thus a conserved quantity.

For a system of particles, the Lagrangian is given by

$$L = \sum_a \frac{m_a v_a^2}{2} - V(\mathbf{r}_1, \mathbf{r}_2, \ldots) \tag{1.2.7}$$

where the potential $V$ depends only on the position vectors of the particles. Or, in generalized coordinates,

$$L(q, \dot{q}) = T(q, \dot{q}) - V(q).$$

Using Euler's theorem for homogeneous functions,

$$\sum_i \dot{q}_i \frac{\partial L}{\partial \dot{q}_i} = \sum_i \dot{q}_i \frac{\partial T}{\partial \dot{q}_i} = 2T,$$

so that

$$E = T(q, \dot{q}) + V(q), \tag{1.2.8}$$

or, in cartesian coordinates,

$$E = \sum_a \frac{m_a v_a^2}{2} + V(\mathbf{r}_1, \mathbf{r}_2, \ldots). \tag{1.2.8'}$$

Space being homogeneous, it is logical to assume that a parallel displacement of a system in space should not affect its physical properties. In generalized coordinates, $\frac{\partial L}{\partial \dot{q}_i} = p_i$ is called the generalized momentum, and $\frac{\partial L}{\partial q_i} = F_i$ the generalized force. Lagrange's equations are then written $\dot{p}_i = F_i$, which, in cartesian coordinates (in which case the generalized momenta are equivalent to the components of the vectors $\mathbf{p}_a$) are equivalent to Newton's law of motion $\dot{\mathbf{p}}_a = \mathbf{F}_a$.

## 3. Hamilton's Equations

Lagrangian mechanics requires the knowledge of the generalized coordinates and velocities. It is also possible to represent the equations of motion in a form involving the generalized coordinates and momenta, passage from one set of coordinates to the other being performed via the Legendre transformations.

Taking the total differential of the Lagrangian yields

$$dL = \sum_i \frac{\partial L}{\partial q_i} dq_i + \sum_i \frac{\partial L}{\partial \dot{q}_i} d\dot{q}_i.$$

But, by definition, $\frac{\partial L}{\partial \dot{q}_i} = p_i$, and applying Lagrange's equations, $\frac{\partial L}{\partial q_i} = \dot{p}_i$, so that

$$dL = \sum \dot{p}_i dq_i + \sum p_i d\dot{q}_i. \tag{1.3.1}$$

Rewriting the second term as

$$\sum_i p_i d\dot{q}_i = d(\sum_i p_i \dot{q}_i) - \sum_i \dot{q}_i dp_i,$$

(1.3.1) becomes

$$dL = \sum_i \dot{p}_i dq_i + d(\sum_i p_i \dot{q}_i) - \sum_i \dot{q}_i dp_i,$$

or

$$d(\sum_i p_i \dot{q}_i - L) = -\sum_i \dot{p}_i dq_i + \sum_i \dot{q}_i dp_i.$$

The quantity inside the brackets is called the Hamiltonian of the system,

$$H(p, q, t) = \sum_i p_i \dot{q}_i - L. \tag{1.3.2}$$

If the equations defining the generalized coordinates do not explicitly depend on time and if the forces can be derived from a potential, then $H \equiv E$. Thus,

$$dH = -\sum_i \dot{p}_i dq_i + \sum_i \dot{q}_i dp_i = \sum_i \frac{\partial H}{\partial p_i} dp_i + \sum_i \frac{\partial H}{\partial q_i} dq_i,$$

from which

$$\dot{q}_i = \frac{\partial H}{\partial p_i} \tag{1.3.3a}$$

$$\dot{p}_i = -\frac{\partial H}{\partial q_i} \tag{1.3.3b}$$

which constitute Hamilton's equations of motion.

## 4. The Hamilton-Jacobi Equations

The choice of the generalized coordinates $q_i$ is by no means limited by some constraint. It is therefore possible to choose other coordinates $Q$, which might depend on the old ones, say $Q_i = Q_i(q, t)$. These transformations (sometimes called "point transformations") do not alter either Lagrange's or Hamilton's equations of motion. However, the Hamiltonian formulation also allows the use of the generalized momenta $p_i$ as coordinates, which would suggest the use of new variables defined as

$$Q_i = Q_i(q, p, t) \tag{1.4.1a}$$

$$P_i = P_i(q, p, t) \tag{1.4.1b}$$

which should satisfy Hamilton's equations of motion for some new Hamiltonian $K = K(P, Q)$:

$$\dot{Q}_i = \frac{\partial K}{\partial P_i} \tag{1.4.2a}$$

$$\dot{P}_i = -\frac{\partial K}{\partial Q_i}. \tag{1.4.2b}$$

Transformations (1.4.1a) and (1.4.1b) for which a new Hamiltonian $K$ can be found such that (1.4.2a) and (1.4.2b) are valid are called "canonical transformations".

Consider the action $S$ given by (1.2.1), and its variation (1.2.4), where $\delta S$ depends on the upper bound $t_2$ (i.e. $\delta q(t_2) \neq 0$), and $\delta(q(t_1)) = 0$. Thus (1.2.4) becomes, after replacing $\frac{\partial L}{\partial \dot{q}_i}$ by $\dot{p}_i$,

$$\delta S = \sum_i p_i \delta q_i, \tag{1.4.3}$$

(since the second term of (1.2.4) vanishes), from which

$$\frac{\partial S}{\partial q_i} = p_i. \tag{1.4.4}$$

Consider next the action as a function depending explicitly on time. Then

$$\frac{dS}{dt} = \frac{\partial S}{\partial t} + \sum_i \frac{\partial S}{\partial q_i}\dot{q}_i = \frac{\partial S}{\partial t} + \sum_i p_i \dot{q}_i.$$

However, from (1.2.1),

$$\frac{dS}{dt} = L,$$

so that

$$L = \frac{\partial S}{\partial t} + \sum_i p_i \dot{q}_i,$$

or

$$\frac{\partial S}{\partial t} = L - \sum_i p_i \dot{q}_i,$$

or, finally, using (1.3.2),

$$\frac{\partial S}{\partial t} = -H. \tag{1.4.5}$$

From (1.4.4) and (1.4.5),

$$dS = \sum_i p_i dq_i - H dt,$$

and the action can then be written as

$$S = \int [\sum_i p_i dq_i - H dt],$$

whose variation is given by

$$\delta S = \delta \int [\sum_i p_i dq_i - H dt] = 0, \tag{1.4.6}$$

which the new Hamiltonian $K$ must also satisfy in the new coordinates:

$$\delta \int [\sum_i P_i dQ_i - K dt] = 0. \tag{1.4.7}$$

Equivalence of the two conditions (1.4.6) and (1.4.7) does not however mean that the integrands are equal. Equivalence will result if the two integrands differ only by a function $F$ depending on the coordinates, momenta and time. Thus

$$\sum_i p_i dq_i - H dt = \sum_i P_i dQ_i - K dt + dF,$$

or

$$dF = \sum p_i dq_i - \sum P_i dQ_i + (K - H)dt. \tag{1.4.8}$$

Thus

$$p_i = \frac{\partial F}{\partial q_i} \tag{1.4.9a}$$

$$P_i = -\frac{\partial F}{\partial Q_i} \tag{1.4.9b}$$

$$K = H + \frac{\partial F}{\partial t} \tag{1.4.9c}$$

and (1.4.9c) gives the required form for the new Hamiltonian.

Taking $F = S$ implies that $K = 0$, since $\frac{\partial S}{\partial t} = -H$. Setting $K = 0$ also ensures that the new coordinates $P$ and $Q$ are constant in time, since $\dot{Q} = 0$ and $\dot{P} = 0$ (by (1.4.2)). Thus using (1.4.4), (1.4.9c) finally becomes

$$\frac{\partial S}{\partial t} + H(q_i, \ldots, q_s, \frac{\partial S}{\partial q_1}, \ldots, \frac{\partial S}{\partial q_s}, t) = 0, \tag{1.4.10}$$

which is known as the Hamilton-Jacobi equation.

# Chapter 2

# Quantum Physics

## 1. Introduction

Chapter 2 deals with the different quantum interpretations applicable to the present problem. The basic interpretation of quantum mechanics (CQM) will be dealt with in the first section, to be followed in order by the formulations of Bohm, Nelson, and finally, Feynman. For a fuller understanding, the reader is referred to [3], [4], [5], [9], [15] for a richer description of CQM, to [1], [2], [11] for Bohm's interpretation, to [10] for Nelson's interpretation and finally, to [6] for Feynman's interpretation; although, in the last case, many books on elementary quantum field theory will contain a small section on the subject (see [13] for example).

## 2. CQM

Classical mechanics as elaborated in Chapter 1 allows the description of a macroscopic world (such as a planetary system) with sufficient precision for the current needs, but suffers great shortcomings when applied to microscopic objects.

Light was originally thought by Newton to be composed of small particles, until wave-like properties were later to complicate the picture (interference, diffraction). A study of blackbody radiation led Planck to state that for an electromagnetic (EM) wave of frequency $v$, the only possible energy states are whole multiples of $hv$, bringing about a quantization of the energy. Einstein then postulated that light is composed of a stream of photons, each of which possesses energy $hv$. Thus wave-like parameters (angular frequency $\omega = 2\pi v$ and wave number $\mathbf{k}$, with $|\mathbf{k}| = 2\pi/\lambda$) and particle-like parameters (energy $E$ and momentum $\mathbf{p}$) coexist and are connected by the Planck-Einstein equations:

$$E = hv = \hbar\omega \tag{2.2.1a}$$

$$\mathbf{p} = \hbar\mathbf{k} \tag{2.2.1b}$$

where $\bar{h} = h/2\pi$, $h$ being Planck's constant: $h \approx 6.62 \cdot 10^{-34}$ Joule · second.

CQM states that both wave-like and particle-like aspects must be investigated for a full description, as can be seen in the two-slit experiment (see figure II.2.1). A source $S_1$ emits a light beam that hits a screen $S_2$ pierced with two slits $A$ and $B$ which illuminates an observation screen $S_3$. If one of the screens is blocked, say $A$, then a diffraction pattern $I_1(x)$ appears. However, if both slits are open, the observed pattern $I(x)$ does not correspond to the sum of the separate patterns:

$$I(x) \neq I_1(x) + I_2(x),$$

see figure II.2.2.

If photons are emitted one by one from the source $S_1$, the same diffraction pattern $I$ will appear on the screen $S_3$, although each photon produces a localized impact. Thus both wave-like and time-like properties are seen to coexist, bringing about a wave-particle duality.

It can also be deduced that it is impossible to observe the interference pattern $I$ and to find out from which slit the photon is coming, thus destroying the classical idea of trajectory. Since the individually emitted photons hit the screen in an aleatory manner, a probabilistic model seems appropriate for the description of the behaviour of a photon.

A study of the absorption and emission spectrum of atoms led to a quantization of the energy, which in turn led de Broglie to postulate that material particles, just like photons, possess wave-like properties, so that all particles satisfy (2.2.1), henceforth called the de Broglie-Einstein relations. (2.2.1b) yields

$$\lambda = \frac{h}{|\mathbf{p}|}, \tag{2.2.2}$$

which is called the de Broglie relation. It gives the wavelength $\lambda$ of a matter wave associated with the motion of a material particle having momentum $\mathbf{p}$.

Thus, in quantum mechanics, a particle is fully described by a wave function $\psi(r,t)$, which is related to the behaviour of the particle by a probability density $\rho(\mathbf{r},t)$ through Born's postulate: the probability $P(\mathbf{r},t)$ that a particle associated with a wave function $\psi(\mathbf{r},t)$ be located at the instant $t$ in a volume element $d^3\mathbf{r} = \mathbf{dxdydz}$ situated at the point $\mathbf{r}$ is given by

$$dP(\mathbf{r},t) = C \mid \psi(\mathbf{r},t) \mid^2 d^3\mathbf{r} = C\rho(\mathbf{r},t)d^3\mathbf{r}, \tag{2.2.3}$$

$C$ being a normalization constant (henceforth assumed to be one).

For a particle of mass $m$ under the influence of a potential $V(\mathbf{r},t)$, the wave function satisfies Schroedinger's equation

$$i\bar{h}\frac{\partial}{\partial t}\psi(\mathbf{r},t) = -\frac{\bar{h}^2}{2m}\nabla^2\psi^*(\mathbf{r},t) + V(\mathbf{r},t)\psi^*(\mathbf{r},t). \tag{2.2.5}$$

Multiplying (2.2.4) by $\psi^*(\mathbf{r},t)$ and (2.2.5) by $-\psi(\mathbf{r},t)$ and then adding yields

$$i\bar{h}\frac{\partial}{\partial t}[\psi^*(\mathbf{r},t)\psi(\mathbf{r},t)] = -\frac{\bar{h}^2}{2m}[\psi^*(\mathbf{r},t)\nabla^2\psi(\mathbf{r},t) - \psi(\mathbf{r},t)\nabla^2\psi^*(\mathbf{r},t)]$$

which can be written

$$\frac{\partial}{\partial t}\rho(\mathbf{r},t) + \frac{\bar{h}}{2mi}[\psi^*(\mathbf{r},t)\nabla^2\psi(r,t) - \psi(\mathbf{r},t)\nabla^2\psi^*(\mathbf{r},t)] = 0 \tag{2.2.6}$$

where $\rho(\mathbf{r},t) = \mid \psi(\mathbf{r},t) \mid^2$ is the probability density. Letting $\mathbf{J}(\mathbf{r},t) = \frac{\bar{h}}{2mi}[\psi^*\nabla\psi - \psi\nabla\psi^*]$, (2.2.6) becomes

$$\frac{\partial}{\partial t}\rho(\mathbf{r},t) + \nabla\cdot\mathbf{J}(\mathbf{r},t) = 0, \tag{2.2.7}$$

which is known in fluid mechanics as the equation of continuity, suggesting a reinterpretation in terms of a "probability fluid" whose density and movement are described by $\rho(\mathbf{r},t)$, the probability density, and $\mathbf{J}(\mathbf{r},t)$, the current density.

One last concept to be dealt with is Heisenberg's uncertainty principle, which gives an estimate on the limitations of giving a deterministic interpretation of the microscopic world. The uncertainty principle states that if a measurement of position is made with accuracy $\Delta x$ and a measurement of momentum is made with accuracy $\Delta p$, then the product of the two errors is never smaller than $\bar{h}/2 : \Delta x\Delta p \geq \bar{h}/2$. It is thus impossible for an experiment to simultaneously determine the exact values of position and momentum. Using the de Broglie relation (2.2.2), the uncertainty principle becomes $\Delta x\Delta k \geq 1/4\pi$, where $k = \frac{2\pi}{\lambda}$ is the wave number. This states that it is impossible to simultaneously describe particle-like properties and wave-like properties. There is also a statement of the uncertainty principle relating time and frequency. $\Delta t \Delta \nu \geq 1/4\pi$, which, with the help of Planck's relation (2.2.10) can be written $\Delta E \Delta t \geq \bar{h}/2$. Mathematically speaking, it is possible to write down a generalized uncertainty relation in terms of standard deviation, $\Delta A \Delta B \geq \frac{1}{2} \mid \langle i[A,B] \rangle \mid$, where $\Delta A, \Delta B$ are the standard deviations of physical quantities $A$ and $B$, and $\langle i[A,B] \rangle$ is the expectation value of the commutator of their hermitian operators.

## 3. Bohm's Interpretation

A quantum interpretation was developed in 1951 by David Bohm which allowed a greater insight into the world of the infinitely small. An account follows for the one particle case, generalization to many particles being trivial (for references, see [1], [2], [11]).

As with CQM, the basic starting point is the Schroedinger equation (2.2.4)

$$i\bar{h}\frac{\partial \psi}{\partial t} = -\frac{\bar{h}^2}{2m}\nabla^2 \psi + V(\mathbf{r})\psi. \qquad (2.2.4)$$

The wave function $\psi$ is now written in the form

$$\psi = R \exp\,(iS/\bar{h}), \qquad (2.3.1)$$

where $R, S$ are real. Inserting (2.3.1) into (2.2.4) and then taking the real and imaginary parts yields the following equations for $R$ and $S$:

$$\frac{\partial R}{\partial t} = -\frac{1}{2m}(R\nabla^2 S + 2\nabla R \nabla S) \qquad (2.3.2a)$$

$$\frac{\partial S}{\partial t} = -[\frac{(\nabla S)^2}{2m} + V(\mathbf{r}) - \frac{\bar{h}^2}{2m}\frac{\nabla^2 R}{R}], \qquad (2.3.2b)$$

which, letting $P(r) = R^2(\mathbf{r})$, can be rewritten

$$\frac{\partial P}{\partial t} + \nabla \cdot (P\frac{\nabla S}{m}) = 0 \qquad (2.3.3a)$$

$$\frac{\partial S}{\partial t} + \frac{(\nabla S)^2}{2m} + V(\mathbf{r}) + U(\mathbf{r}) = 0 \qquad (2.3.3b)$$

where

$$U(\mathbf{r}) = \frac{-\bar{h}^2}{2m}\frac{\nabla^2 R}{R}. \qquad (2.3.4)$$

Remembering that $P = R^2 = 1 + P$, equation (2.3.3a) can be compared with the equation of continuity (2.2.7) expressing the conservation of probability, and $P\frac{\nabla S}{m}$ is then interpreted as the mean current of the particle.

Equation (2.3.3b) is equivalent to the Hamilton-Jacobi equation (1.4.10) for a 1 particle system if $V + V$ is interpreted as the total potential for the system.

Equation (2.3.4) is thus interpreted as a quantum potential which acts on the particle in addition to the classical potential $V$. Thus (2.3.3b) is still considered to be the Hamilton-Jacobi equation for a particle moving with velocity $\mathbf{v} = \nabla S(r)/m$ and with potential $V + U$. The total force acting on the particle is then given by $F = -\nabla(V(\mathbf{r}) + U(\mathbf{r})) = F + F_q$, where $F_q = -\nabla U$ can be thought of as a quantum force derived from a quantum potential. It is thus possible to find trajectories by integrating $\mathbf{v}$ or by integrating Newton's law of motion

$$m\frac{d^2\mathbf{r}}{dt^2} = -\nabla(V(\mathbf{r}) + U(\mathbf{r})), \tag{2.3.5}$$

indicating the usefulness of the quantum potential.

Bohm's quantum potential will be used in part two to illustrate how the interference patterns in the two-slit experiment might arise in terms of spatial anomalies.

4. Nelson's Stochastic Mechanics: The Forward Drift

Nelson's stochastic mechanics uses a (Markovian) diffusion process satisfying the Langevin stochastic differential equation (see [14]) given by

$$dq(t) = B(q(t), t)dt + \sqrt{2v}d\omega(t) \tag{2.4.1}$$

with diffusion coefficient $v = \bar{h}/2m$, forward drift

$$B(\mathbf{r}, t) = \frac{\bar{h}}{m}[\text{Re } (\frac{\nabla\psi}{\psi}) + \text{ Im } (\frac{\nabla\psi}{\psi})], \tag{2.4.2}$$

and probability density $\rho(\mathbf{r}, t) =\mid \psi(\mathbf{r}, t) \mid^2$ instead of a solution to Schroedinger's equation as its basic model.

Solving (2.4.1) gives a stochastic process whose probability density is identical to that obtained from a normalized wave function $\psi$ (i.e. (2.2.3) with $C = 1$).

Thus stochastic mechanics and CQM are experimentally indistinguishable. However, a solution of (3.4.1) gives a path of the diffusion, thus allowing the existence of trajectories (see [10]).

Although no stochastic trajectories will be determined in part two, the general configuration of the forward drift will be sufficient to explain the interference patterns in the two-slit experiment.

## 5. The Forward Drift and The Quantum Potential

As in section 3, let

$$\psi = R \exp\,(iS/\hbar). \tag{2.3.1}$$

Inserting in (2.4.2), the forward drift can then be written as

$$B(\mathbf{r},t) = \frac{\hbar}{m}[\frac{\nabla R}{R} + \frac{\nabla S}{\hbar}]. \tag{2.5.1}$$

Differentiating (2.5.1) yields

$$\frac{m}{\hbar}\nabla B = -\frac{2m}{\hbar^2}U - \frac{(\nabla R)^2}{R^2} + \frac{\nabla^2 S}{\hbar}, \tag{2.5.2}$$

where (2.3.4) was used on the LHS. Thus a relation linking the forward drift and the quantum potential is found.

## 6. Path Integrals

Feynman's path integral formulation is introduced, as it will prove useful in the determination of the wave function in the two-slit experiment. For a complete description of the theory, the reader is referred to [6].

In CQM, $\psi(b, t_b)$ is the probability amplitude that a particle is at the point $q_b$ at time $t_b$. The probability that the particle is situated there is then given by (2.2.5):

$$P(b, t_b) = \mid \psi(b, t_b) \mid^2 .$$

Feynman's path integral formulation of quantum mechanics is based on the notion of a propagator $K(b, a)$ defined as the probability amplitude for a transition from a point $q_a$ at time $t_a$ to a point $q_b$ at time $t_b$. As in (2.2.3), the probability that a particle goes from a point $q_a$ at time $t_a$ to a point $q_b$ at time $t_b$ is given by

$$P(b,a) = \mid K(b,a) \mid^2 . \tag{2.6.1}$$

The propagator is found by taking the sum over all possible paths from $q_a$ to $q_b$. It is found in [6] to be given by

$$K(b,a) = \int_a^b \exp\left[\frac{i}{\hbar} S(b,a)\right] Dq(t), \tag{2.6.2}$$

where $S(b,a)$ is defined by (1.2.1) and $D$ is used to distinguish between the usual integral and the path integral. For the full form, see [6].

Suppose the time interval between $t_a$ and $t_b$ is divided into two, say at time $t_c : t_a < t_c < t_b$. The action (1.2.1) can then be written as

$$S(b,a) = S(b,c) + S(c,a), \tag{2.6.3}$$

and the propagator as

$$K(b,a) = \int_{q_c} K(b,c) K(c,a) Dq_c. \tag{2.6.4}$$

Thus the transition from $(q_a, t_a)$ to $(q_b, t_b)$ can be seen as a transition from $(q_a, t_a)$ to all available points $q_c$ at a time $t_c$ followed by a transition from $(q_c, t_c)$ to $(q_b, t_b)$.

The propagator $K(b,a)$ is actually a wave function describing the system at the end of its evolution. Thus, $K(b,a) \equiv \psi(q_b, t_b)$ is actually the wave function corresponding to the solution of the Schroedinger equation. It is thus possible to solve Schroedinger's equation simply by finding the propagator for the system in question, as will next be done for the two-slit system.

# Part Two

## The Two-Slit System

### Introduction

Part two deals specifically with the two-slit system and uses the formulations introduced in part one to solve the problem. Expressions for both the forward drift and the quantum potential will be found. Chapter 3 deals with the one-slit system. Chapter 4 will use the results of Chapter 3 and expand them to the two-slit system. For simplicity, the experiment will be considered as taking place in the $x - t$ plane.

# Chapter 3

## The One-Slit System

### 1. Introduction

Suppose a particle (say $e^-$) is emitted at time $t = 0$ from the origin $x = 0$. At the time $t = T$ the particle goes through a slit screen with the slit centered at $x_0 > 0$ and with length $2b$. For $t > T$, the particle is diffracted by the slit. At $t = \Upsilon + T, \Upsilon > 0$, the particle is situated at the position $x$ (see figure III.1.1). The wave function for this particular situation will now be computed using path integrals.

### 2. The Wave Function

A particle travelling from $x = 0$ at $t = 0$ to $x = x$ at $t = \Upsilon + T$ encounters an obstacle (the slit screen) at $t = T$. The action for the system must be then written as in (2.6.3) and the propagator as in (2.6.4), with the point $c$ indicating the position of the particle as it passes through the slit (in this case, $c \equiv y$).

Suppose that at time $t = T$ the particle is situated at a distance less than $\pm b$ of $x_0$ (see figure III.1.1). The propagator (or, equivalently, the wave function $\psi(x,t)$) can be obtained by integrating over the length of the slit. Thus (2.6.4) can then be written in this case as

$$\psi(x, \Upsilon) = \int_{-b}^{b} K(x + x_0, T + \Upsilon; x_0 + y, T) \cdot K(x_0 + y, T; 0, 0)dy, \qquad (3.2.1)$$

where the first term describes the particle as it goes from the slit to its final position, and the second term describes the particle from the origin to the slit. As the particle is not influenced by an external source or potential, it then moves as a free particle, so that the Lagrangian is given by $(1/2)m\dot{x}^2$. The particles that get through the slit also move as free particles, since they are not acted upon by a classical potential.

The propagator (2.6.2) for a free particle with the action (1.2.1) given by

$$S = \int_{t_a}^{t_b} L(x,t)dt = \int_{t_a}^{t_b} \frac{m}{2}\dot{x}^2 dt$$

is expressed as

$$K(b,a) = \left(\frac{m}{2\pi i\hbar(t_b - t_a)}\right)^{\frac{1}{2}} \exp\left[\frac{\text{im}\,(x_b - x_a)^2}{2\hbar(t_b - t_a)}\right]. \tag{3.2.2}$$

Inserting (3.2.2) into (3.2.1), the wave function becomes

$$\psi(x,\Upsilon) = \int_{-b}^{b} \{(\frac{m}{2\pi i\hbar\Upsilon})^{\frac{1}{2}}\} \exp\left[\frac{\text{im}\,(x-y)^2}{2\hbar\Upsilon}\right]\cdot$$
$$\{(\frac{m}{2\pi i\hbar T})^{\frac{1}{2}} \exp\left[\frac{\text{im}\,(x_0+y)^2}{2\hbar T}\right]\}dy,$$

or

$$\psi(x,\Upsilon) = \int_{-b}^{b} \frac{m}{2\pi i\hbar\sqrt{\Upsilon T}} \cdot \exp\left[\frac{\text{im}}{2\hbar}\left(\frac{(x-y)^2}{\Upsilon} + \frac{(x_0+y)^2}{T}\right)\right]dy. \tag{3.2.3}$$

Consider next the function $G(y)$ defined by

$$G(y) = \begin{cases} 1, & \text{for } -b \le y \le b \\ 0, & \text{for } \mid y \mid > b. \end{cases}$$

Introducing this function in (3.2.3), the integral can be written as

$$\psi(x,\Upsilon) = \int_{-\infty}^{\infty} \frac{mG(y)}{2\pi i\hbar\sqrt{\Upsilon T}} \exp\left[\frac{\text{im}}{2\hbar}\left(\frac{(x-y)^2}{\Upsilon} + \frac{(x_0+y)^2}{T}\right)\right]dy.$$

Consider instead $G(y)$ to be defined by a Gaussian function,

$$G(y) = \exp\left[\frac{-y^2}{2b^2}\right], \tag{3.2.4}$$

so that approximately two-thirds of the area under the curve lies between $-b$ and $b$. Introducing this soft slit approximation in $\psi$, the wave function becomes

$$\psi(x,\Upsilon) = \int_{-\infty}^{\infty} \frac{m}{2\pi i\hbar\sqrt{\Upsilon T}} \exp\left[\frac{\text{im}}{2\hbar}\cdot\left(\frac{(x-y)^2}{\Upsilon} + \frac{(x_0+y)^2}{T}\right)\right] \exp\left[\frac{-y^2}{2b^2}\right]dy,$$

which can be rewritten as

$$\psi(x,\Upsilon) = \frac{m}{2\pi i\hbar\sqrt{\Upsilon T}} \exp\left[\frac{\text{im}}{2\hbar}\left(\frac{x^2}{\Upsilon} + \frac{x_0^2}{T}\right)\right]\cdot$$
$$\int_{-\infty}^{\infty} \exp\left[\left(\frac{\text{im}}{2\hbar\Upsilon}\right) + \frac{\text{im}}{2\hbar T} - \frac{1}{2b^2}\right]y^2 + \frac{\text{im}}{\hbar}\left(\frac{x_0}{T} - \frac{x}{\Upsilon}\right)dy. \tag{3.2.5}$$

This integral is of the form

$$\int_{-\infty}^{\infty} \exp\,[\alpha x^2 + \beta x]dx = \sqrt{\frac{\pi}{-\alpha}}\,\exp\,[\frac{-\beta^2}{4\alpha}] \quad \text{for Re}(\alpha) \leq 0, \tag{3.2.6}$$

so that, integrating (3.2.4), the wave function is given by

$$\psi(x,\Upsilon) = \frac{m}{2\pi i\sqrt{\Upsilon T}}\sqrt{\frac{\pi}{\frac{1}{2b^2} - \frac{\text{im}}{2\hbar}(\frac{1}{\Upsilon} - \frac{1}{T})}} \cdot \exp\,[\frac{\text{im}}{2\hbar}(\frac{x^2}{\Upsilon} + \frac{x_0^2}{T})]\cdot$$

$$\exp\,\{[-(\frac{\text{im}}{\hbar})^2(\frac{-x}{\Upsilon} + \frac{x_0}{T})^2][4(\frac{\text{im}}{2\hbar}(\frac{1}{\Upsilon} - \frac{1}{T}) - \frac{1}{2b^2})]^{-1}\},$$

which can finally be written as

$$\psi(x,\Upsilon) = \sqrt{\frac{m}{2\pi i\hbar}}[\Upsilon T(\frac{1}{\Upsilon} + \frac{1}{T} + \frac{i\hbar}{mb^2})]^{-\frac{1}{2}}\cdot$$

$$\exp\,\{\frac{\text{im}}{2\hbar}[\frac{x^2}{\Upsilon} + \frac{x_0^2}{T} - (\frac{x_0}{T} - \frac{x}{\Upsilon})^2(\frac{1}{\Upsilon} + \frac{1}{T} + \frac{i\hbar}{mb^2})^{-1}]\}. \tag{3.2.7}$$

Equation (3.2.6) thus gives the wave function for a particle of mass $m$ emitted from the origin and passing through a slit of diameter $2b$ centered at $x_0$ at time $T$ and stopping at the position $x$ at time $T + \Upsilon$. Using Born's postulate (2.2.3), it is thus possible to find the probability that the particle is located in a particular volume element. It is however impossible to gather more information about probable trajectories for the particle using CQM as the theory does not allow such notions.

It will however be possible to gather more information about the movement of the particle by using the formulations of Bohm and Nelson, both of which are outlined in part one.

## 3. Bohm's Interpretation

The main goal in this section is to find the quantum potential given by (2.3.4). In order to do so, the wave function must first be expressed in the form (2.3.1) in order to find $R$ and $S$, knowledge of which would also allow the determination of possible trajectories.

As a first step, rewrite equation (3.2.6) as

$$\psi(x,\Upsilon) = \frac{mb}{\sqrt{2\pi\hbar}}\sqrt{\frac{-\hbar\Upsilon T - imb^2(\Upsilon+T)}{(\hbar\Upsilon T)^2 + m^2b^4(\Upsilon+T)^2}}\cdot$$
$$\exp\left[\frac{-m^2b^2(x_0\Upsilon - xT)^2}{2(\hbar\Upsilon T)^2 + 2m^2b^4\hbar(\Upsilon+T)^2}\right]\cdot$$
$$\exp\left[\frac{\text{im}}{2\hbar}\left(\frac{x^2}{\Upsilon} + \frac{x_0^2}{T} - \frac{m^2b^4(\Upsilon+T)(x_0\Upsilon - xT)^2}{\Upsilon T[(\hbar\Upsilon T)^2] + m^2b^4(\Upsilon+T)^2}\right)\right], \tag{3.3.1}$$

so that only the square root need be considered. Let

$$\Gamma(\Upsilon) = \sqrt{-\hbar\Upsilon T - \text{ im } b^2(\Upsilon+T)/[(\hbar\Upsilon T)^2 + m^2b^4(\Upsilon+T)^2]}$$
$$= \frac{1}{\sqrt{(\hbar\Upsilon T)^2 + m^2b^4(\Upsilon+T)^2}}\cdot\sqrt{\propto + i\beta}, \tag{3.3.2}$$

where $\alpha = -\hbar\Upsilon T$ and $\beta = -mb^2(\Upsilon+T)$. Let $z = \alpha + i\beta = w(\cos\,\theta + i\,\sin\,\theta)$. Then $z^{\frac{1}{2}} = w^{\frac{1}{2}}[\cos\,(\frac{\theta}{2} + k\pi) + i\,\sin\,(\frac{\theta}{2} + k\pi)]; k = 0, 1.$

Only $k = 0$ needs to be considered since $k = 1$ will not alter the results. From the definition of $z$, it is clear that $\theta$ and $w$ are given by

$$\theta = \tan^{-1}\left(\frac{\beta}{\propto}\right) = \tan^{-1}\left(\frac{mb^2(\Upsilon+T)}{\hbar\Upsilon T}\right)$$

and

$$w = \sqrt{\propto^2 + \beta^2} = \sqrt{(\hbar\Upsilon T)^2 + m^2b^4(\Upsilon+T)^2}.$$

Thus

$$\sqrt{\propto + i\beta} = \sqrt{w}\left(\cos\,\left[\frac{1}{2}\tan^{-1}\left(\frac{\beta}{\propto}\right)\right] + i\,\sin\,\left[\frac{1}{2}\tan^{-1}\left(\frac{\beta}{\propto}\right)\right]\right),$$

and

$$\Gamma(\Upsilon) = w^{-\frac{1}{2}}\cdot\left(\cos\,\left[\frac{1}{2}\tan^{-1}\left(\frac{\beta}{\propto}\right)\right] + i\,\sin\,\left[\frac{1}{2}\tan^{-1}\left(\frac{\beta}{\propto}\right)\right]\right). \tag{3.3.3}$$

For simplification, let

$$F(\propto,\beta) = \frac{1}{2}\tan^{-1}\left(\frac{\beta}{\propto}\right),$$

$$G(x,\Upsilon) = \frac{m}{2\hbar}\left(\frac{x^2}{\Upsilon} + \frac{x_0^2}{T} - \frac{m^2b^4(T+\Upsilon)(x_0\Upsilon - xT)^2}{\Upsilon T[(\hbar\Upsilon T)^2 + m^2b^4(\Upsilon+\Upsilon)^2]}\right),$$

$$H(x,\Upsilon) = \frac{mbw^{-\frac{1}{2}}}{\sqrt{2\pi\hbar}}\exp\left[\frac{-m^2b^2(x_0\Upsilon - xT)^2}{2(\hbar\Upsilon T)^2 + 2m^2b^4(\Upsilon+T)^2}\right].$$

Then the wave function becomes, after expanding the exponential,

$$\psi(x,\Upsilon) = H(x,\Upsilon)(\cos\ F(\propto,\beta) + i\ \sin\ F(\propto,\beta)) \cdot (\cos\ G(x,\Upsilon) + i\ \sin\ G(x,\Upsilon)),$$

which can be written

$$\psi(x,\Upsilon) = H(x,\Upsilon)[\cos\ [F(\propto,\beta) + G(x,\Upsilon)] + i\ \sin\ [F(\propto,\beta) + G(x,\Upsilon)]].$$

Thus $\psi(x,\Upsilon) = T + iW$, and $\psi$ can now be written in the form (2.3.1), where

$$R^2 = T^2 + W^2$$
$$S = \bar{h}\ \tan^{-1}(\frac{W}{T}).$$

Thus

$$R = H(x,\Upsilon) \tag{3.3.5a}$$

$$S = \bar{h}\ \tan^{-1}[\tan[\frac{1}{2}\ \tan^{-1}(\frac{mb^2(\Upsilon+T)}{\bar{h}\tau T}) + G(x,\Upsilon)]]. \tag{3.3.5b}$$

Rewriting (3.3.5a) as

$$R = q(\Upsilon)e^{r(x,\tau)}, \tag{3.3.6}$$

the quantum potential then becomes

$$U(x,\Upsilon) = \frac{-\bar{h}^2}{2m}\frac{\nabla^2 R}{R} = \frac{-\bar{h}^2}{2m}(\nabla^2 r(x,\Upsilon)) + (\nabla r(x,\Upsilon))^2)$$
$$= \frac{m}{2}(b\bar{h}T)^2 \cdot (\frac{1}{[(\bar{h}\Upsilon T)^2 + m^2 b^4(\Upsilon+T)^2]} - \frac{m^2b^2(x_0\Upsilon - xT)^2}{[(\bar{h}\Upsilon T)^2 + m^2b^4(\Upsilon+T)^2]});$$

where, recalling, $m$ is the mass, $\bar{h}$ Planck's constant, $2b$ is the slit size, $x_0$ is the distance from the center of the slit to the line pasing through $x = 0$, and $T$ is the time that the particle takes to reach the slit screen (see figure III.1.1).

## 4. The Forward Drift

The forward drift is defined in equation (2.4.2) as

$$B(x,t) = \frac{\bar{h}}{m}[\text{Re}\ (\frac{\nabla\psi}{\psi}) + \ \text{Im}\ (\frac{\nabla\psi}{\psi})].$$

Writing $\psi$ as in (2.3.1), the forward drift can be rewritten as

$$B(x,t) = \frac{\hbar}{m}\left(\frac{\nabla R}{R} + \frac{1}{\hbar}\nabla S\right),$$

where $R$ is given by (3.3.6) and $S$ by (3.3.5b). Thus

$$B(x,t) = \frac{\hbar}{m}(\nabla r + \nabla G), \tag{3.4.1}$$

or, filling in the details,

$$\begin{aligned} B(x,t) =& \frac{\hbar}{m}\Big[\frac{m^2 b^2 T(x_0 \Upsilon - xT)}{(\hbar \Upsilon T)^2 + m^2 b^4 (\Upsilon, T)^2} \\ &+ \frac{m}{\hbar}\Big(\frac{x}{\Upsilon} + \frac{m^2 b^4 (\Upsilon + T)(x_0 \Upsilon - xT)}{\Upsilon[(\hbar \Upsilon T)^2 + m^2 b^4 (\Upsilon, T)^2]}\Big)\Big]. \end{aligned} \tag{3.4.2}$$

## 5. The One-Slit Experiment

Figures III.5.1 and III.5.2 show two views of the quantum potential with the slit situated at the origin, while figure III.5.2 shows a gradient plot of the quantum potential. The forward drift is shown in figure III.5.3, where the slit is once again situated at the origin.

As a particle emitted from a source (situated to the left of the slit, see figure III.1.1) passes through the slit, it encounters the quantum potential where it is strongest, keeping it from deviating to the sides. As it travels away from the slit, the effect of the quantum potential diminishes, allowing the particle to travel freely. The particle thus seems to be attracted to regions where the quantum potential is strongest, i.e. it can be thought of as travelling on the quantum potential surface. In the regions where the quantum potential is lowest, the particle is pushed towards regions where the quantum potential is stronger, as indicated by the forward drift (forward being in the $x$-direction, i.e. from left to right).

The forward drift thus gives a measure of how the particle is pushed around by the quantum force. A positive forward drift pushes the particle to the right, whereas a negative forward drift pushes it to the left.

Putting the two concepts together allows one to infer how a particle may travel. After the particle has passed through the slit (coming from the left), it encounters a region where the forward drift is stronger. It tends to keep the particle in the center, where the quantum potential is stronger. A particle deviating to the side, where the quantum potential is lower, is pushed back in the other direction towards the quantum potential plateau. As the particle continues along its path, the quantum potential stabilizes as the forward drift weakens, allowing the particle to scatter. If the experiment would be repeated with many (similar) particles, they would all encounter the same quantum potential and forward drift, and so would all scatter as they travelled, with more particles ending up at the center and less towards the sides. Figure III.5.4 shows a slice of the quantum potential which is in agreement with the scattering.

The quantum potential can then be thought of as the evolution of the particle density, more particles appearing where it is weaker (where the forward drift pushes them back towards stronger regions).

## Chapter 4

## The Two-Slit System

### 1. Introduction

Suppose, as in Chapter 3, that a particle is emitted at time $t = 0$ from the origin $x = 0$. At time $t = T$ the particle encounters a screen containing two slits, so that the particle passes through one of the slits. At $t = T + \Upsilon, \Upsilon > 0$, the particle is situated at the position $x$ (see figure IV.1.1). We shall proceed as in Chapter 3 in order to find the wave function.

### 2. The Wave Function

A particle reaching the slit screen at $t = T$ has two possible ways of getting through, as it can pass by either slit. Thus actually two events need to be considered: the particle passes through slit $A$, and the particle passes through slit $B$. Both contributions must be taken into account. A particle passing through slit $A$ and reaching a point $x$ at $t = T+\Upsilon$ is described by $\psi_A(x, \Upsilon)$ while a particle passing through slit $B$ and reaching the point $x$ at $t = T + \Upsilon$ is described by a wave function $\psi_B(x, \Upsilon)$. Thus the total wave function for a particle reaching point $x$ is expressed as

$$\psi(x, \Upsilon) = \psi_A(x, \Upsilon) + \psi_B(x, \Upsilon). \tag{4.2.1}$$

Using the path integral formalism, (4.2.1) becomes

$$\psi(x, \Upsilon) = \int_{-b}^{b} K(x + x_{0B}, T + \Upsilon; x_{0A} - y', T) \cdot K(x_{0A} - y', T; 0, 0)dy' +$$
$$\int_{-b}^{b} K(x+x_{0B}, T + \Upsilon; X_{0B} + y, T)K(x_{0B} + y, T; 0, 0)dy.$$

Now, since $x_{0A} = -x_{0B}$,

$$\psi(x, \Upsilon) = \int_{-b}^{b} K(x + x_{0B}, T + \Upsilon; -x_{0B} - y', T) \cdot K(-x_{0B} - y', T; 0, 0)dy' +$$
$$\int_{-b}^{b} K(x+x_{0B}, T + \Upsilon; x_{0B} + y, T)K(x_{0B} + y, T; 0, 0)dy.$$

The second integral (i.e. $\psi_B(x,\Upsilon)$) is the same as was dealt with in section 3.2, since slit $B$ is the same slit that was considered in Chapter 3. Thus the wave function $\psi_B(x,\Upsilon)$ is given by (3.2.7) or equivalently by (3.3.1), and the functions $R$ and $S$ (henceforth denoted by $R_B$ and $S_B$) are given by (3.3.5a) and (3.3.5b) respectively, letting $x_0 \equiv x_{0B}$.

The method for solving the first integral is exactly the same as in section 3.2. Thus, using (3.2.2) to expand the propagators,

$$\psi_A(x,\Upsilon) = \int_{-b}^{b} \{(\frac{m}{2\pi i\hbar\Upsilon})^{\frac{1}{2}} \exp[\frac{\text{im }(x+2x_{0B}+y')}{2\hbar\Upsilon}]\} \cdot \{(\frac{m}{2\pi i\hbar T}) \exp[\frac{\text{im}(x_{0B}+y')^2}{2\hbar T}]\} dy'. \tag{4.2.3}$$

For simplification, let $x + 2x_{0B} \equiv \gamma$. Then

$$\psi_A(x,\Upsilon) = \int_{-b}^{b} \frac{m}{2\pi i\hbar\sqrt{\Upsilon T}} \exp[\frac{\text{im}}{2\hbar}(\frac{(\gamma+y')^2}{\Upsilon} + \frac{(x_{0B}+y')^2}{T})] dy'.$$

The Gaussian function (3.2.4) is once more used, allowing (4.2.3) to be written as

$$\psi_A(x,\Upsilon) = \frac{m}{2\pi i\hbar\sqrt{\Upsilon T}} \int_{-\infty}^{\infty} \exp[\frac{\text{im}}{2\hbar}(\frac{\gamma^2}{\Upsilon} + \frac{x_{0B}^2}{T})] \cdot \exp[(\frac{\text{im}}{2\hbar T} + \frac{\text{im}}{2\hbar\Upsilon} - \frac{1}{2b^2})y'^2 + \frac{\text{im}}{\hbar}(\frac{\gamma}{\Upsilon} + \frac{x_{0B}}{T})y'] dy'.$$

This integral is again of the form (3.2.6), so that, after simplifying, the wave function is expressed as

$$\psi_A(x,\Upsilon) = \sqrt{\frac{m}{2\pi i\hbar}} [\Upsilon T(\frac{1}{\Upsilon} + \frac{1}{T} + \frac{i\hbar}{mb^2})]^{\frac{1}{2}} \cdot \exp[\frac{\text{im}}{2\hbar}[\frac{\gamma^2}{\Upsilon} + \frac{x_{0B}^2}{T} - (\frac{\gamma}{\Upsilon} + \frac{x_{0B}}{T})^2(\frac{1}{\Upsilon} + \frac{1}{T} + \frac{i\hbar}{mb^2})]]. \tag{4.2.4}$$

The wave function for the two-slit experiment can now be written as $\psi(x,\Upsilon) = \psi_A(x,\Upsilon) + \psi_B(x,\Upsilon)$, where $\psi_A(x,\Upsilon)$ is given by (4.2.4), and $\psi_B(x,\Upsilon)$ by (3.2.7) or (3.3.1). However, in order to find the quantum potential, we will want to write $\psi_A(x,\Upsilon)$ in the form (2.3.1).

3. Bohm's Interpretation

Proceeding as in section 3.3, rewrite $\psi_A(x,\Upsilon)$ in the form

$$\psi_A(x,\Upsilon) = \frac{mb}{\sqrt{2\pi\hbar}}\sqrt{\frac{-\hbar\Upsilon T - \text{ im } b^2(\Upsilon+T)}{(\hbar\Upsilon T)^2 + m^2b^4(\Upsilon+T)^2}} \cdot$$
$$\exp\left[\frac{-m^2b^2(\gamma T + x_{0B}\Upsilon)^2}{2(\hbar\Upsilon T)^2 + 2m^2b^4(\Upsilon+T)^2}\right] \cdot$$
$$\exp\left[\frac{\text{im}}{2\hbar}\left(\frac{\gamma^2}{\Upsilon} + \frac{x_{0B}^2}{T} - \frac{m^2b^4(\Upsilon+T)(\gamma T + x_{0B}\Upsilon)^2}{\Upsilon T[(\hbar\Upsilon T)^2 + m^2b^4(\Upsilon+T)^2]}\right)\right], \tag{4.3.1}$$

which is similar in shape to equation (3.3.1). The root being the same, it can then be expanded similarly as (3.3.2). Thus once again, $\psi$ can be written, after expanding the exponential, as

$$\psi_A(x,\Upsilon) = H_A(\gamma,\Upsilon)[\cos\,(F_A(\propto,\beta) + G_A(\gamma,\Upsilon)) + i\,\sin\,(F_A(\propto,\beta) + G_A(\gamma,\Upsilon))],$$

where

$$F_A(\propto,\beta) = \frac{1}{2}\tan^{-1}\left(\frac{\beta}{\propto}\right) = \frac{1}{2}\tan^{-1}\left(\frac{mb^2(\Upsilon+T)}{\hbar\Upsilon T}\right)$$

$$G_A(\gamma,\beta) = \frac{m}{2\hbar}\left(\frac{\gamma^2}{\Upsilon} + \frac{x_{0B}^2}{T} - \frac{m^2b^4(\Upsilon+T)(\gamma T + x_{0B}\Upsilon)^2}{\Upsilon T[(\hbar\Upsilon T)^2 + m^2b^4(\Upsilon+T)^2]}\right)$$

$$H_A(\gamma,\Upsilon) = \frac{mb\omega^{-\frac{1}{4}}}{\sqrt{2\pi\hbar}}\exp\left[\frac{-m^2b^2(\gamma T + x_{0B}\Upsilon)^2}{2(\hbar\Upsilon T)^2 + 2m^2b^4(\Upsilon+T)^2}\right].$$

Using (3.3.4),

$$R_A = H_A(\gamma,\Upsilon) \tag{4.3.2a}$$

$$S_A = \hbar\tan^{-1}\left[\tan\left[\frac{1}{2}\tan^{-1}\left(\frac{mb^2(\Upsilon+T)}{\hbar\Upsilon T}\right) + G_A(x,\Upsilon)\right]\right], \tag{4.3.2b}$$

where $\gamma = x + 2x_{0B}$.

Rewriting (4.3.2a) as

$$R_A = q(\Upsilon)e^{r_A(x,\Upsilon)}, \tag{4.3.3}$$

the quantum potential for slit $B$ becomes

$$U_A(x,\Upsilon) = \frac{-\hbar^2}{2m}\frac{\nabla^2 R_A}{R_A} = \frac{-\hbar^2}{2m}(\nabla^2 r_A(x,\Upsilon) + (\nabla r_A^{(x,\Upsilon)}))^2$$

and the forward drift

$$B_A(x,\Upsilon) = \frac{\bar{h}}{m}[\frac{\nabla R_A}{R_A} + \frac{1}{\bar{h}}\nabla S_A] = \frac{\bar{h}}{m}(\nabla r_A + \nabla G_A).$$

The quantum potential for slit $A$, plotted in figure IV.3.1, is similar to the quantum potential for slit $B$, with the exception that it is centered on slit $A$; similarly with the forward drift (figure IV.3.2).

## 4. The Two-Slit System

We now have the wave function $\psi(x,\Upsilon)$ given by $\psi(x,\Upsilon) = \psi_A(x,\Upsilon) + \psi_B(x,\Upsilon)$, or

$$\psi(x,\Upsilon) = R_A \exp[\frac{i}{\bar{h}}S_A] + R_B \exp[\frac{i}{\bar{h}}S_B], \tag{4.4.1}$$

where $R_A, S_A$ are given by (4.3.2) and $R_B, S_B$ by (3.3.5). To find the quantum potential and the forward drift, the wave function must first be written in the form

$$\psi(x,\Upsilon) = R^* \exp[\frac{i}{\bar{h}}S^*].$$

Expanding (4.4.1) yields

$$\psi(x,\Upsilon) = [R_A \cos(\frac{S_A}{\bar{h}}) + R_B \cos(\frac{S_B}{\bar{h}})] + i[R_A \sin(\frac{S_A}{\bar{h}}) + R_B \sin(\frac{S_B}{\bar{h}})],$$

from which

$$R^* = \sqrt{R_A^2 + R_B^2 + 2R_A R_B \cos(\frac{S_A - S_B}{\bar{h}})} \tag{4.4.2a}$$

$$S^* = \bar{h}\tan^{-1}[\frac{R_A \sin(S_A/\bar{h}) + R_B \sin(S_B/\bar{h})}{R_A \cos(S_A/\bar{h}) + R_B \cos(S_B/\bar{h})}]. \tag{4.4.2b}$$

Writing $S_A/\bar{h} \equiv s_A$, $S_B/\bar{h} \equiv s_B$ and with the help of (3.3.6) and (4.3.3), (4.4.2) can be rewritten

$$R^* = q(\Upsilon)\sqrt{\exp(2r_A) + \exp(2r_B) + 2\exp(r_A + r_B)\cos(s_A - s_B)} \tag{4.4.3a}$$

$$S^* = \bar{h}\tan^{-1}[\frac{\exp(r_A)\sin(s_A) + \exp(r_B)\sin(s_B)}{\exp(r_A)\cos(s_A) + \exp(r_B)\cos(s_B)}]. \tag{4.4.3b}$$

The quantum potential for the two slit system is then given by

$$U(x,\Upsilon) = \frac{-\bar{h}^2}{2m}\frac{\nabla^2 R^*}{R^*},$$

and the forward drift by

$$B(x, \Upsilon) = \frac{\bar{h}}{m}[\frac{\nabla R^{*}}{R^{*}} + \frac{1}{\bar{h}}\nabla S^{*}].$$

Figures IV.4.1 and IV.4.2 show two views of the quantum potential where the scale is similar to that of figures III.5.1 and II.5.2, while figures IV.3.3 and IV.3.4 show the same views of the quantum potential much closer to the slit screen. The forward drift is plotted (using the same scale as figure IV.3.1) in figure IV.4.5, while figure IV.4.6 shows the forward drift closer to the slit (using the same scale as figure IV.3.3).

The quantum potential for the two slit experiment is seen in figures IV.4.3 and IV.4.4 to be made up of the two one slit quantum potentials situated left and right of the region where they interact. Farther away from the slit screen, the stable regions vanish as the interactions between the two one slit potentials grow into a single wildly fluctuating region. Figure IV.4.7 shows three slices of the quantum potential (where the top one is taken near the slit screen and the bottom one at the back of figure IV.4.1) which show the increasing fluctuations. The forward drift (figures IV.4.5 and IV.4.6) exhibits a similar behaviour, as shown in figure IV.4.8, which shows the equivalent slices in the forward drift.

A particle emitted from a source situated on a line passing midway between the two slits goes through one of the slits. As in the one slit case, the particle is then acted upon by the forward drift, which either pushes the particle towards the high interaction zone of the quantum potential, or propels it towards the side, where it will eventually encounter the interaction zone.

Once in this area, the forward drift propels the particle forward in an erratic manner as it is continuously pushed around. Farther away, the magnitude of the quantum potential diminishes. However, fluctuations in the forward drift still push the particle sideways, although not as strongly. Farther away still, the forward drift regularizes in a way that allows the particle to continue on its way inside a region of stable potential.

All the similar particles released in the same manner would react in the same way to the quantum potential and the forward drift. Thus, when the effects of the quantum potential are weaker, the particles are pushed by the forward drift into different regions of stable quantum potential, scattering as they continue on their way. It is thus possible to explain the interference pattern (e.g. Figure IV.4.7) using a particle-only approach, as undulatory aspects are not necessary in the quantum potential interpretation of quantum mechanics.

## 5. Conclusion

It is possible, with the help of alternative quantum descriptions, to gather additional information on a quantum system (in this case, the one and two slit systems) such that the underlying quantum structure (e.g. interference) is not tampered with. In Bohm's quantum potential interpretation, the wave function is reinterpreted as a mathematical representation of a field which exerts a quantum force on the system (in this case, on the individual particles in the systems). The elements of the system (e.g. mass, slit width and separation) are combined by the quantum potential in a system-defined spatial structure which influences the system, in this case pushing or attracting a particle. Another system-dependant structure resulting from Nelson's stochastic mechanics, the forward drift, indicates how the quantum potential acts on the system. It then appears that the spatial properties of a system are intimately linked to the system itself (different particles generating different quantum potentials), bringing about a space-mass relationship reminiscent of relativistic theories.

Although current classical quantum formulations (such as Schroedinger's CQM) give a sufficient interpretation of the microscopic world, it can by no means be ascertained that this will remain so in the future. The current formulations must then be developed further, or alternative ones must be brought into play. The two interpretations used fall into this second category. The basic results of CQM remain valid in Bohm's quantum potential interpretation, although some reinterpretation might be required (see [3]). However, additional information is added, such as the theoretical description of particle trajectories. The same is true in the case of Nelson's stochastic formulation, although stochastic quantum formulations allow a generalisation to relativistic formulations (see [12] for a richer description of stochastic quantum theories), while Bohm's formulation is as yet a strictly non-relativistic theory. It is nevertheless worth considering the development of these (or other) quantum interpretations, as they might prove useful in the resolution of some unforeseen difficulties which might arise in the usual interpretation of quantum mechanics.

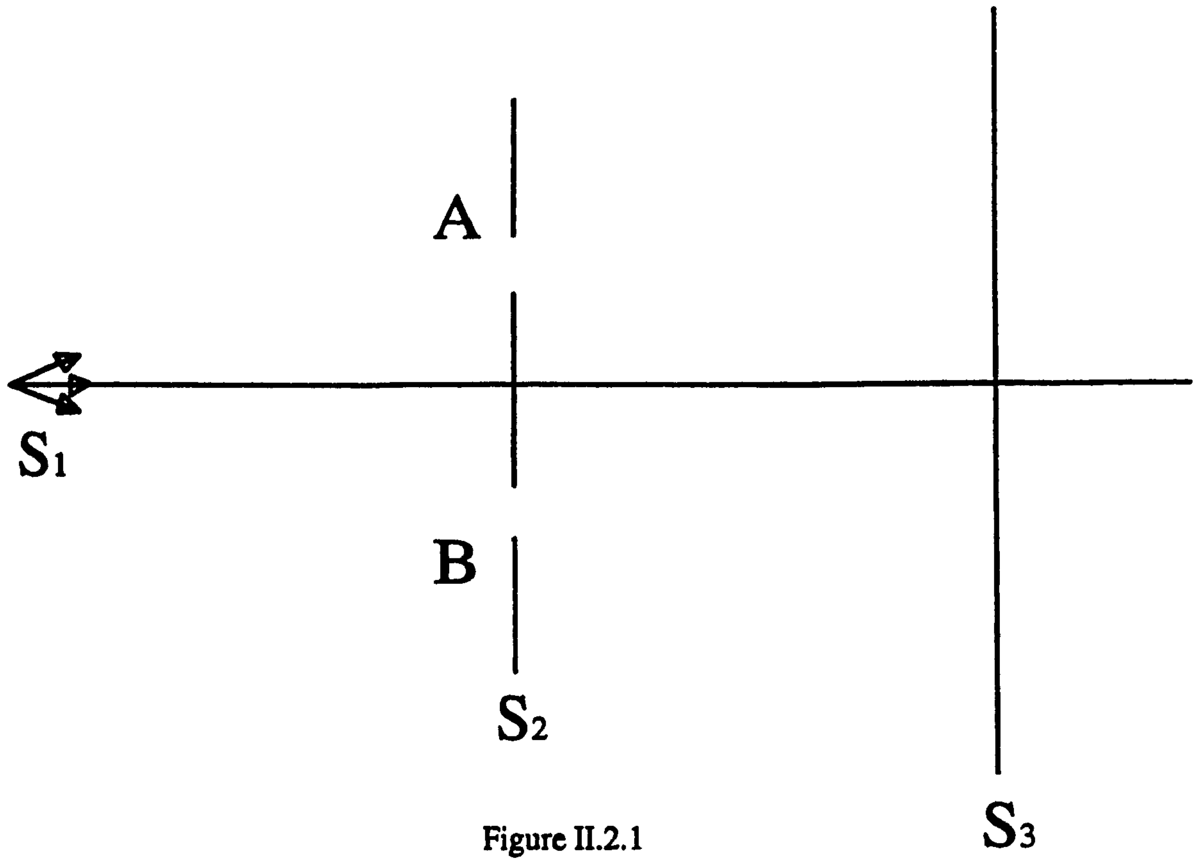



Figure II.2.1

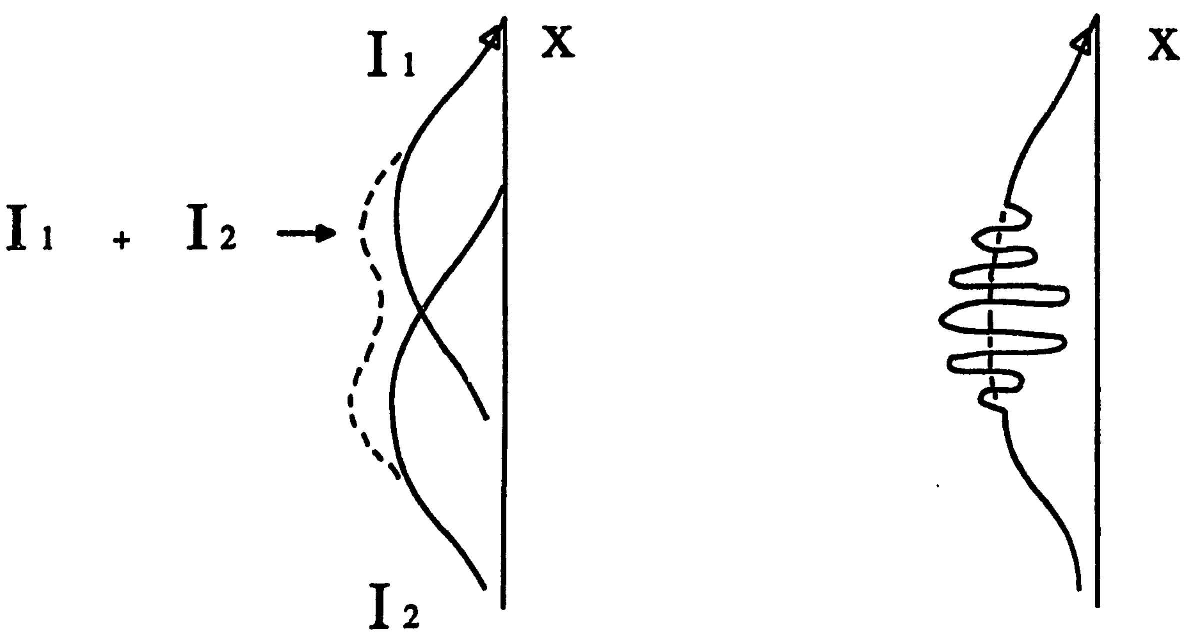



Figure II.2.2

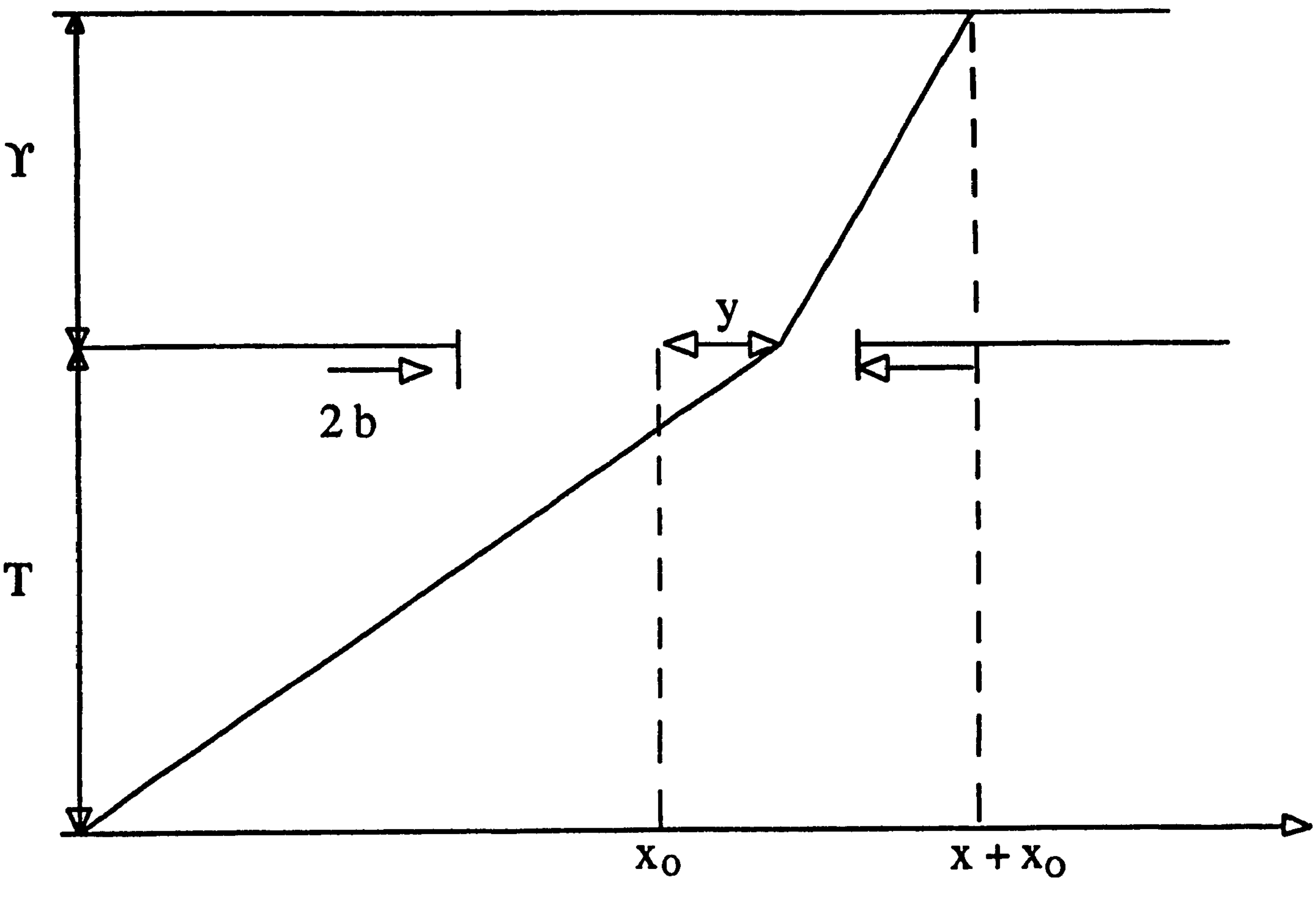



Figure III.1.1

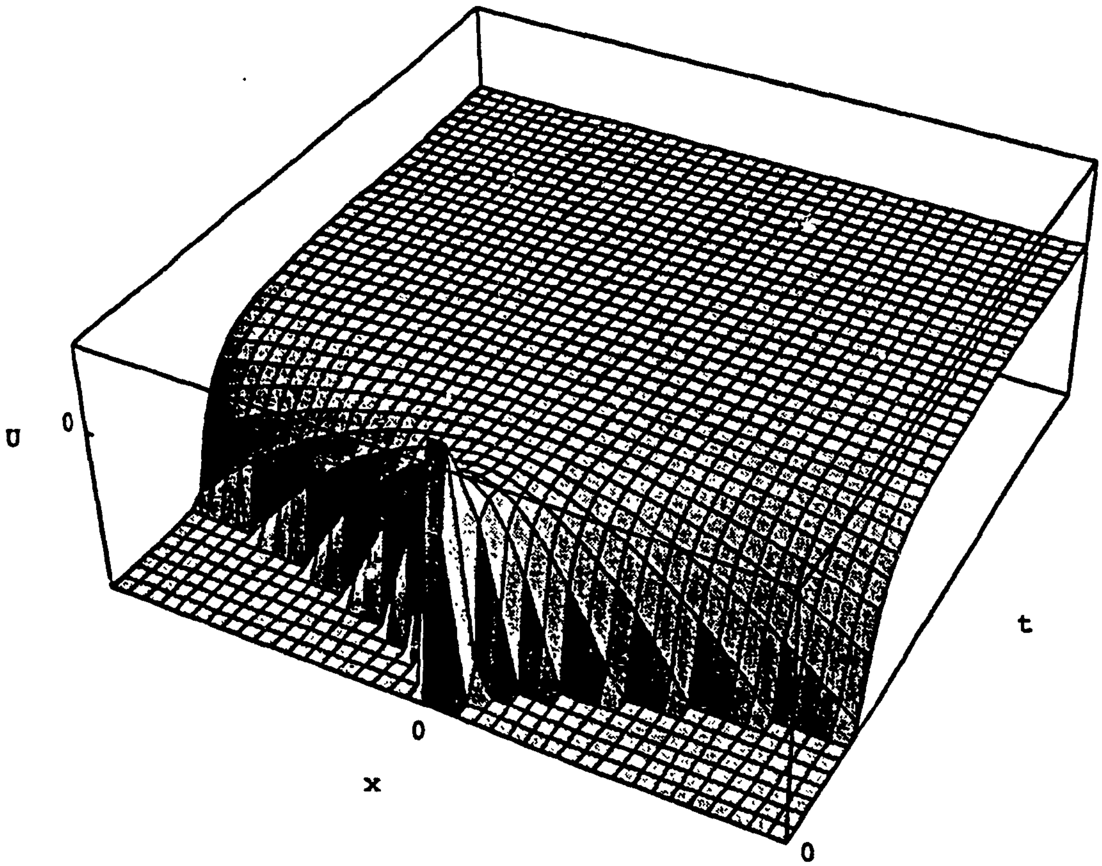



Figure III.5.1

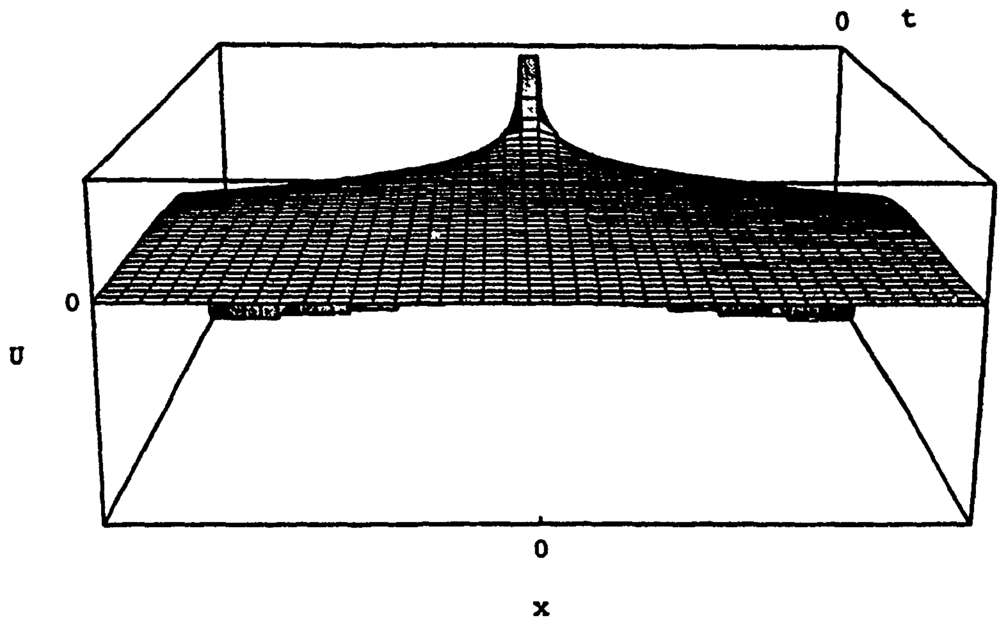



Figure III.5.2

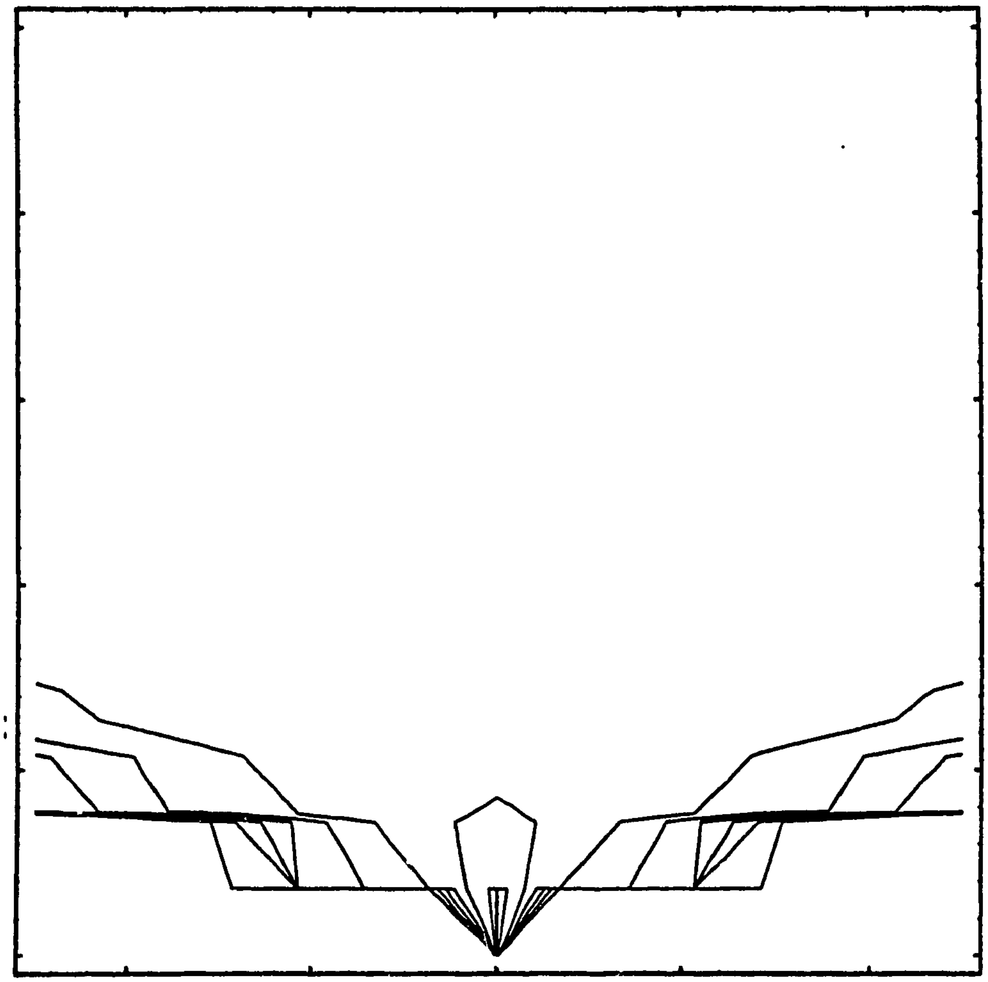


Figure III.5.3

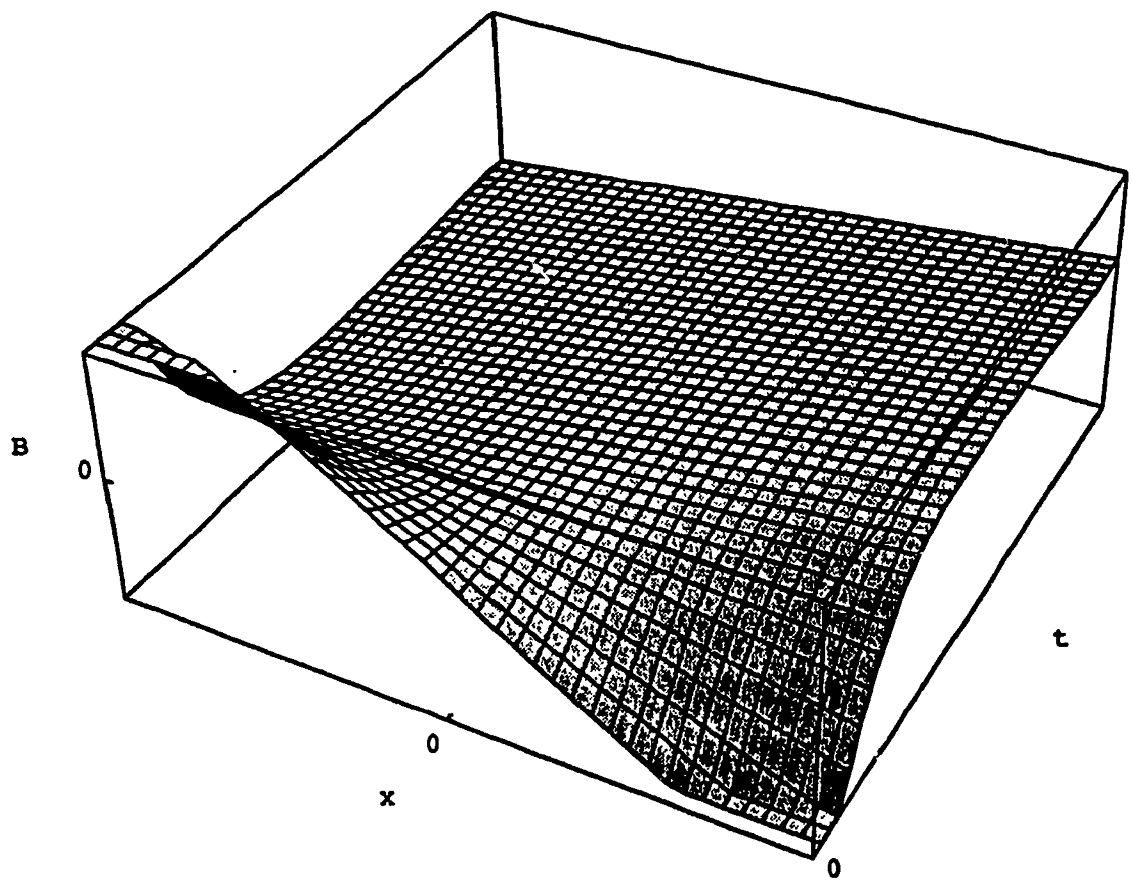



Figure III.5.4

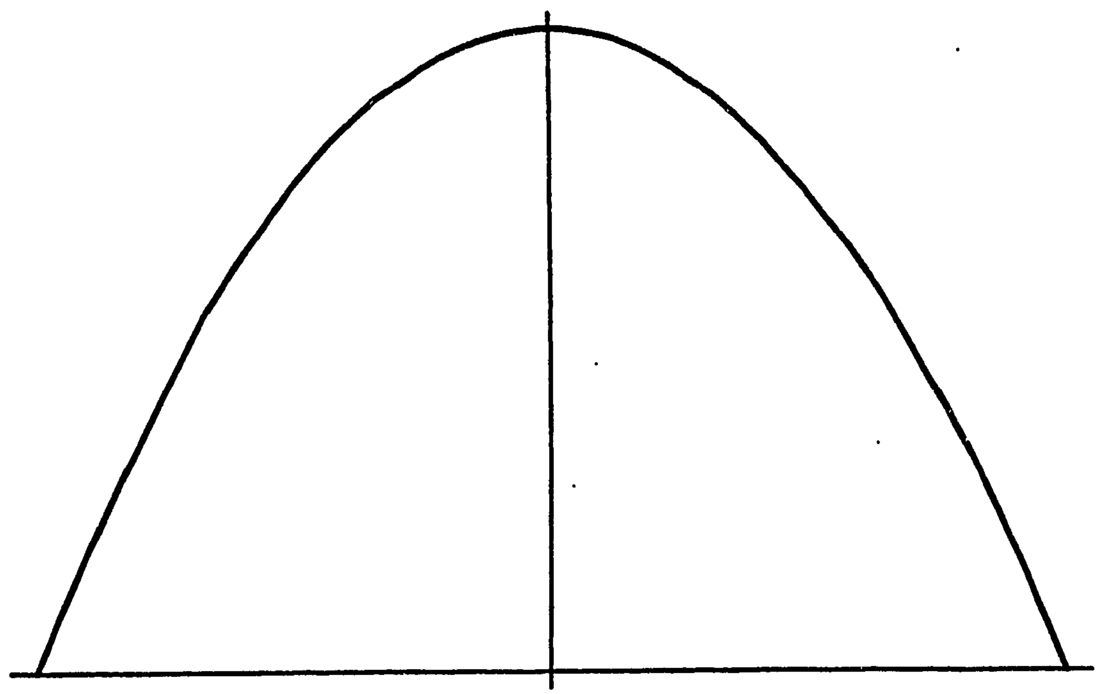


Figure III.5.5

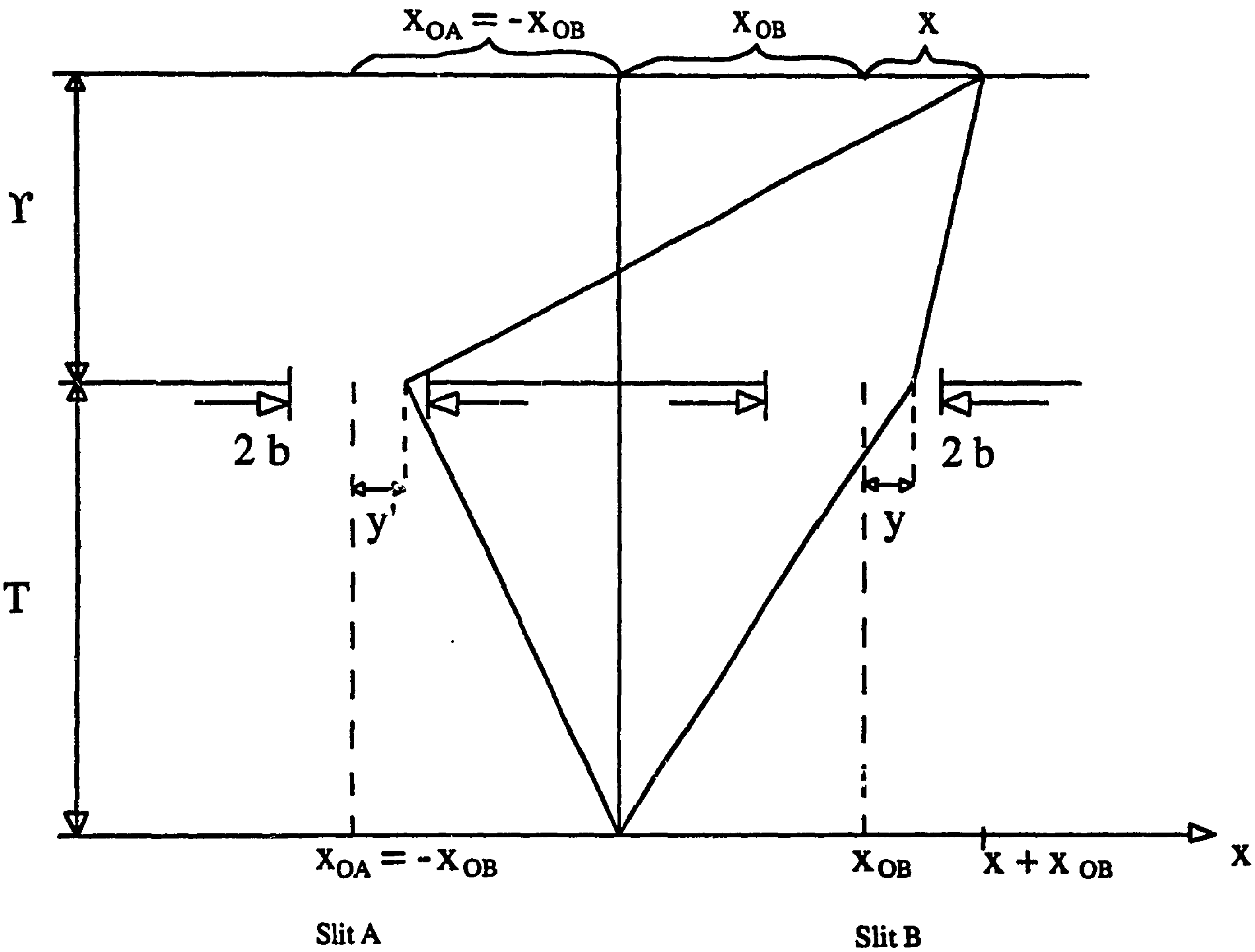



Figure IV.1.1

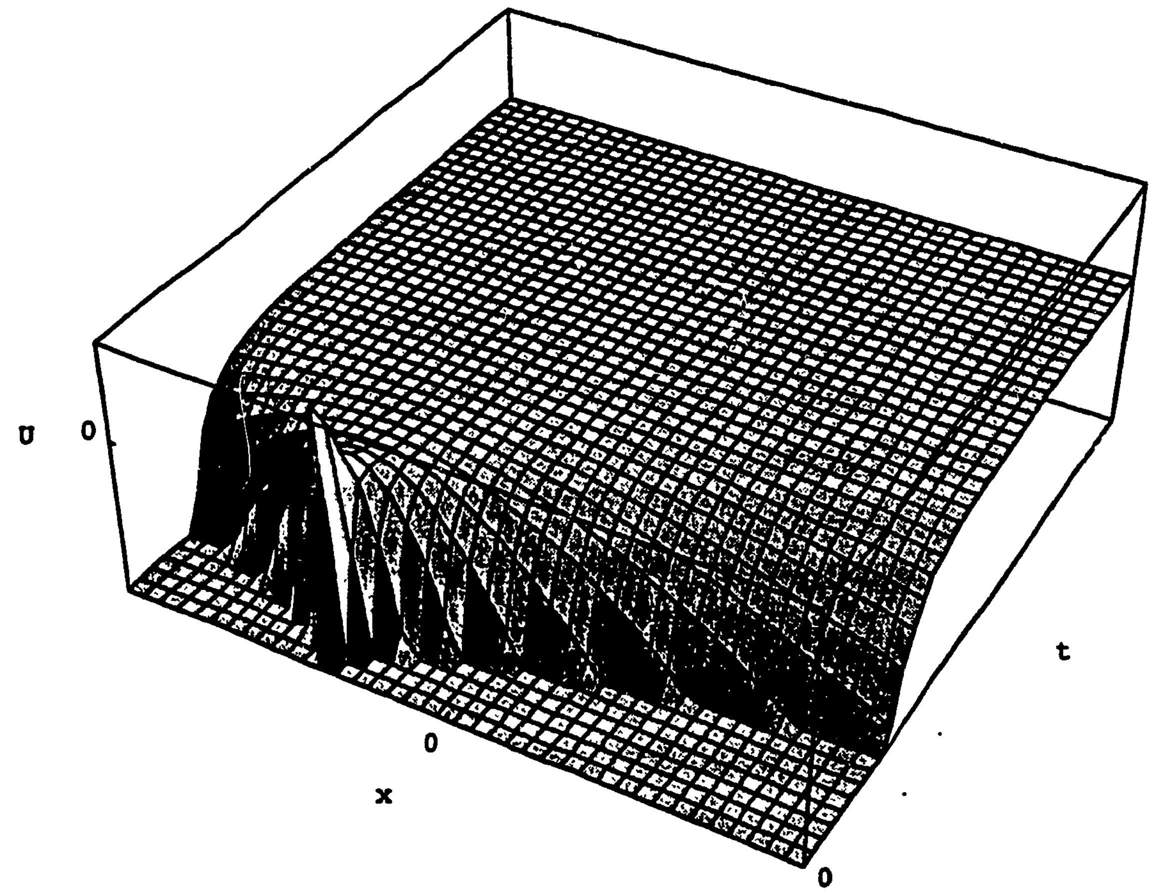



Figure IV.3.1

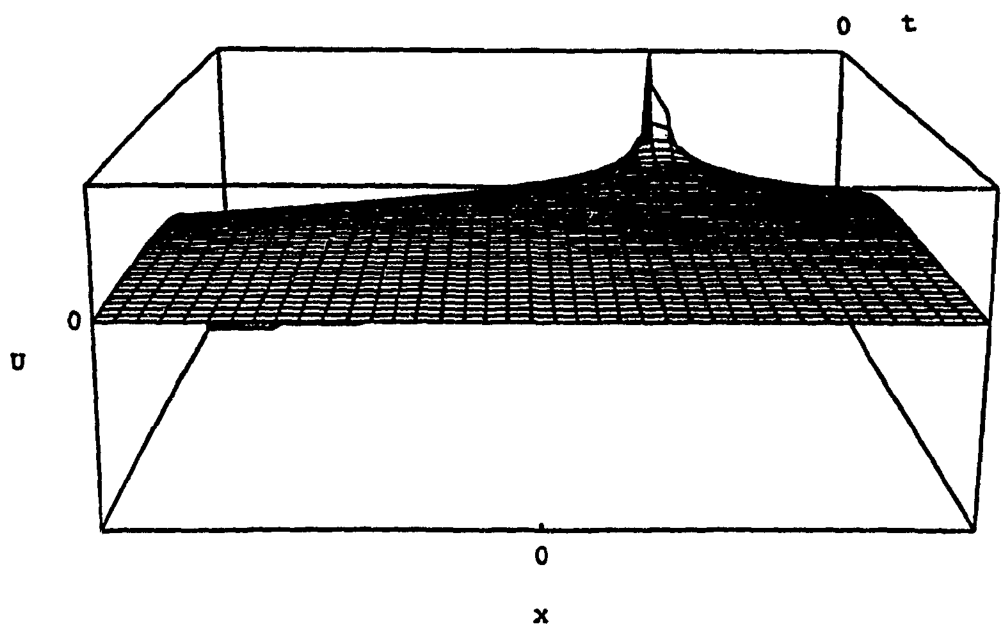
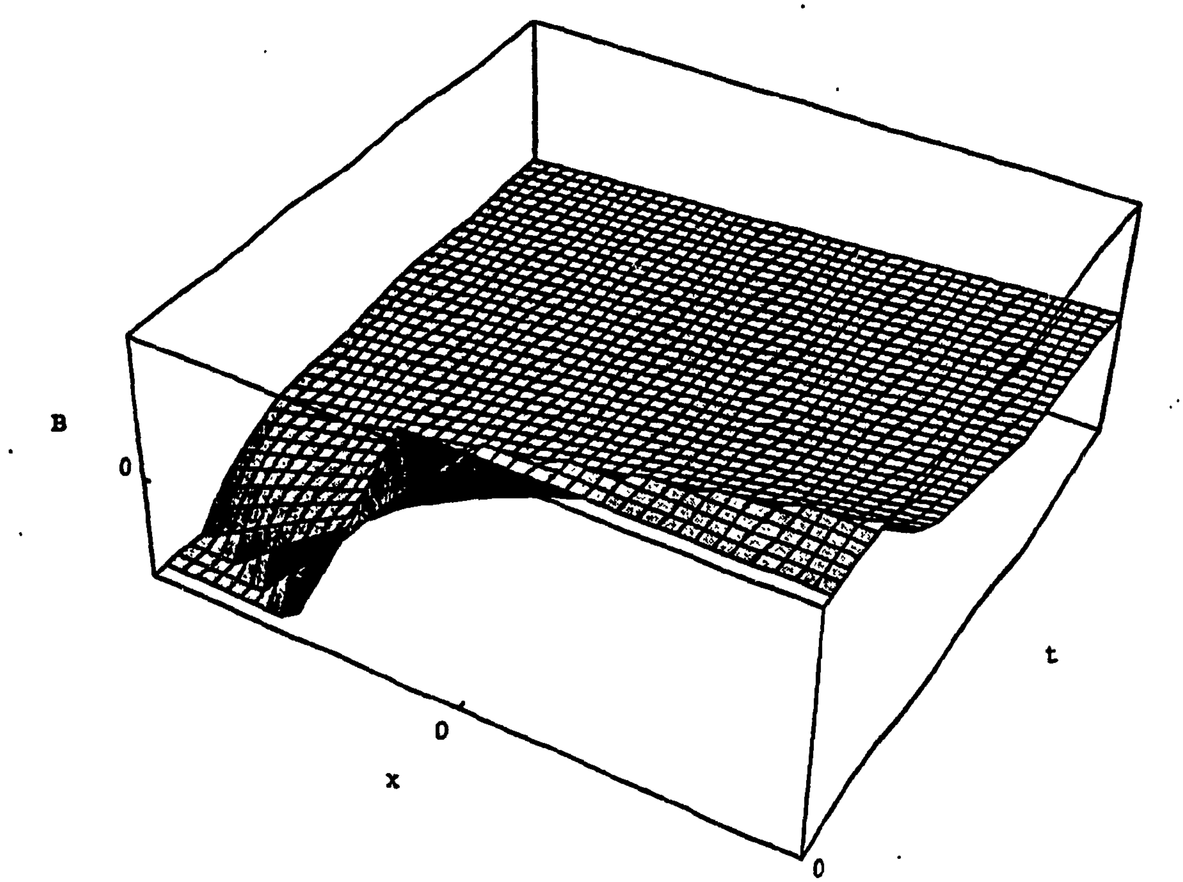



Figure IV.3.2

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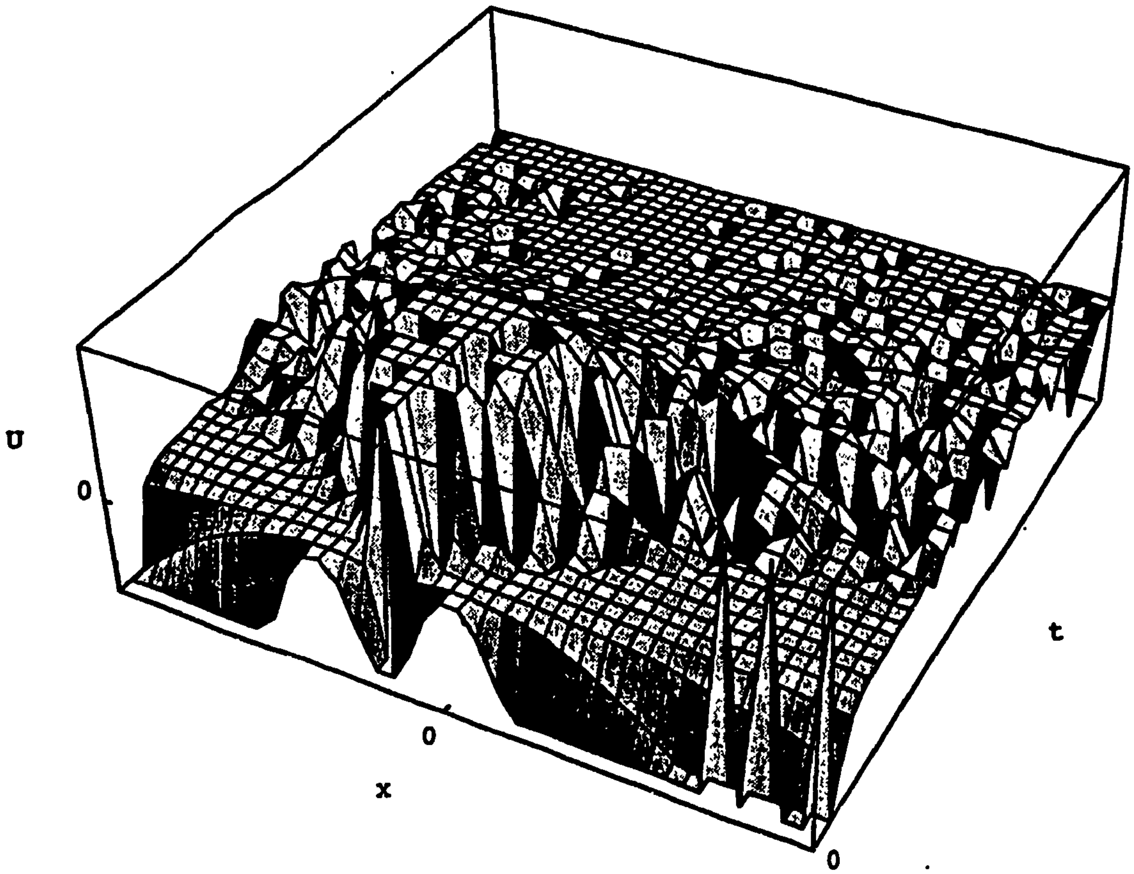



Figure IV.4.1

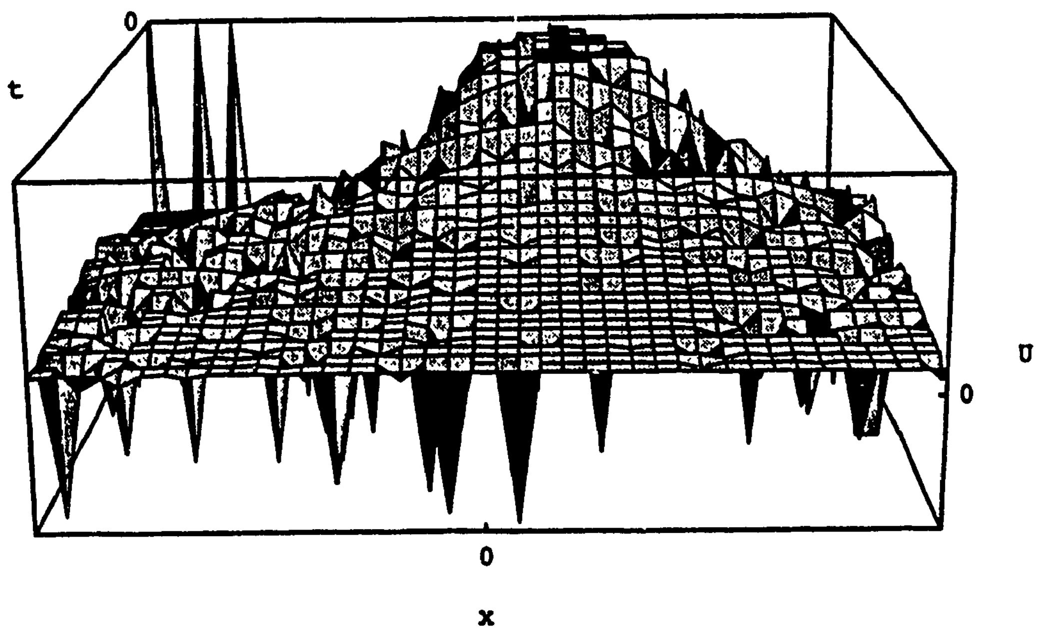



Figure IV.4.2

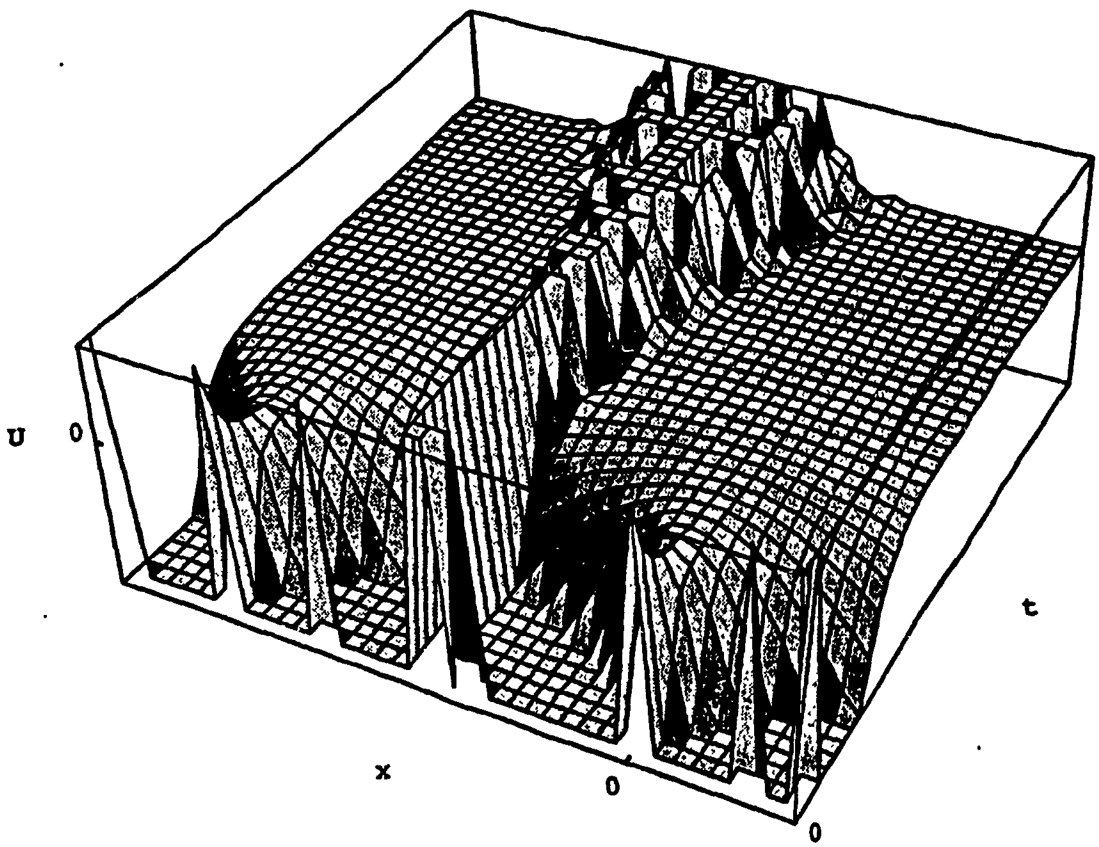



Figure IV.4.3

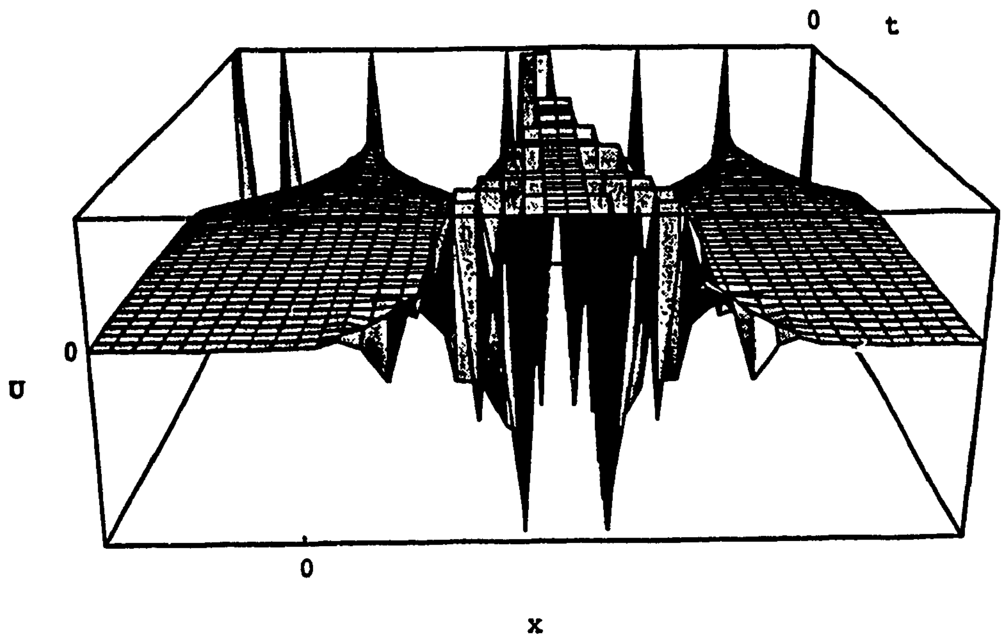



Figure IV.4.4

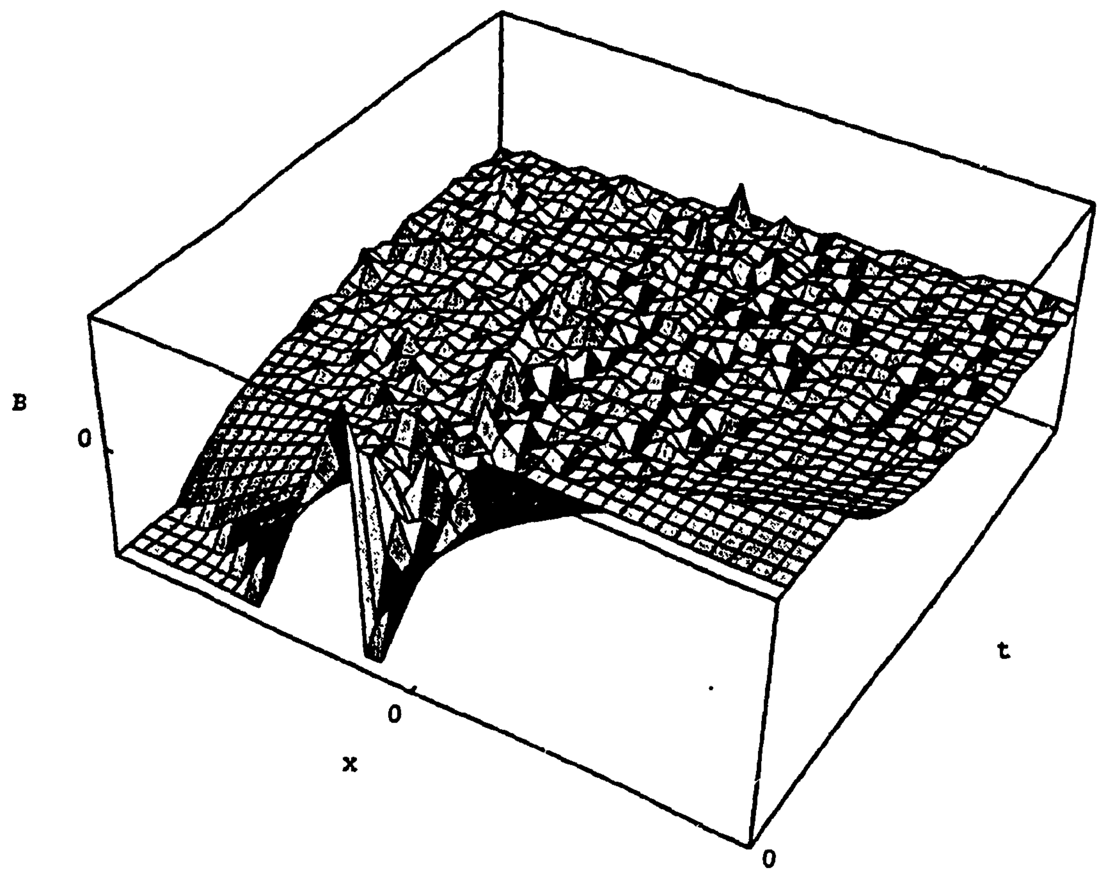
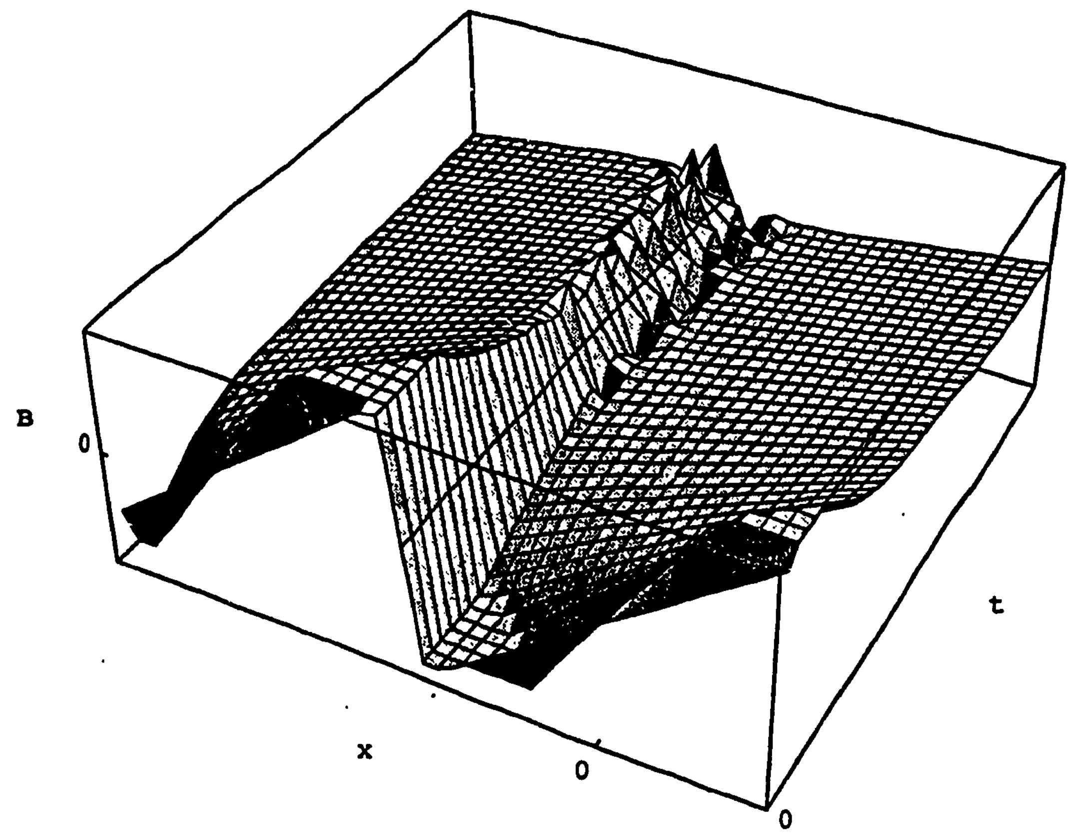



Figure IV.4.5

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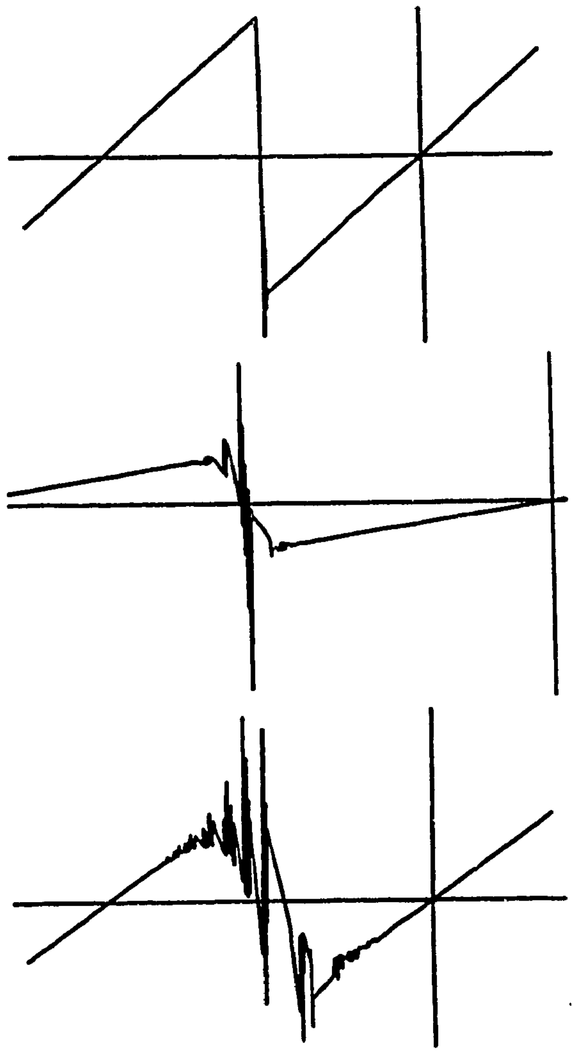

Figure IV.4.7

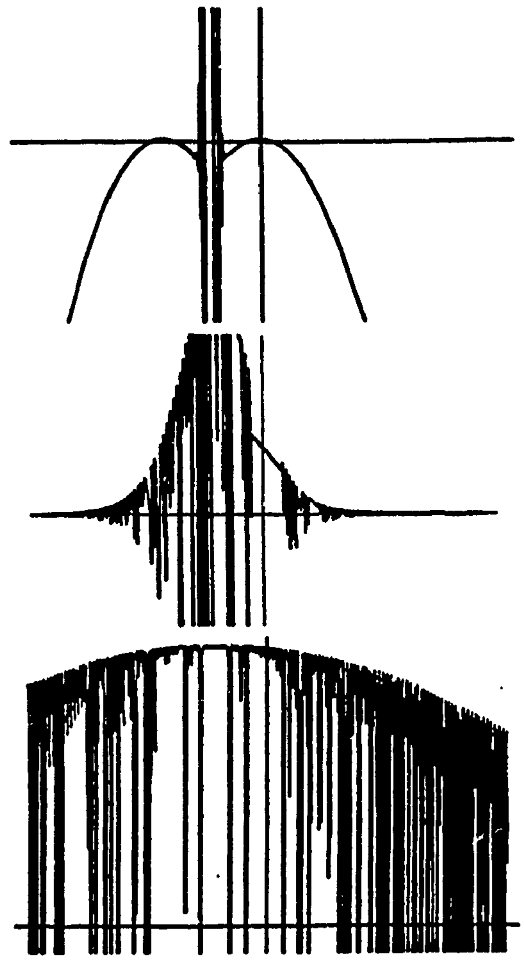

Figure IV.4.8

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