THE CHILD'S CONCEPTION OF SPACE AND PAINTING: SOME OF PIAGET'S IDEAS RELATED TO A CEZANNE WATER-COLOUR

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ABSTRACT

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This thesis looks closely at Piaget's conclusions in The Child's Conception of Space and relates them to the spatial field in paintings. As well, the relations Piaget described which constitute our conception of topological and projective/euclidean space are related to a water-colour by Cézanne, Trees Forming a Vault, which is put into a brief historical perspective.

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CONTENTS

Introduction

The purpose of this thesis is to see how Piaget's explanation of the mental constructs we make based on our experience of everyday space can be related to the allusions to depth in a reproduction of a two-dimensional art work.

I began with an interest in pictorial space, in the general sense of the space special to paintings, whether representational or non-objective. This paper investigates one part of pictorial space; how we read a spatial field into a painting, and how relations between elements such as brush strokes add to its spatial complexity.

In the reading I did, Fernande Ste Martin's essay "L'elaboration des espaces"[1] suggested an aesthetic based on kinds of space such as the early practical spaces Piaget had investigated. This lead me to <u>The Child's Conception of Space</u>[2] and eventually to its conclusions. If we thought on a sub-logical level in terms of position, order, displacement, proximity and intensive measurement in a spatial field; if the image was not important; then this was the mental underpinning of the spatial field in painting. Whether the work was representational or not did not matter. There was a way of thinking spatially on a two dimensional surface, though not in terms of form like Arnheim. This way of thinking was not axiomatic, but sub-logical; e.g. not necessarily a system of perspective but less formulated. It explained some of how I felt when I painted and when I looked at painting.

In this paper I have used some of the conclusions in The Child's Conception of Space and related them to painting in two ways; how we conceive of a spatial field, and how we read into a painting, spatial relations which we have already constructed from our everyday experience. Each step in the evolution of these relations is explained for the general reader, and each is related to the reproduction of a Cezanne watercolour.

In Cezanne's late work the spatial indications are rich and subtle, and not based primarily on traditional linear perspective. Choosing one of his late works to discuss Piaget's ideas was both relevant, since this spatial complexity is a constant factor in Cezanne's work, and provocative, as this complexity seemed to be based on sub-logical spatial awareness rather than axiomatic rules.

The rest of this introduction will touch on the following areas: Suzi Gablik's Progress in Art,[3] which also relates Piaget's work to art; Arnheim's idea of visual thinking; the importance of Piaget as a thinker, and a clarification of the aims and limits of the paper.

Chapter II looks briefly at The Child's Conception of Space as a whole. Chapter III follows the relevant parts of his General Conclusions in detail; sublogical spatial awareness is a step between incoming sensation and formal logic. Mental images record our physical interaction with objects rather than a figural image. Our active minds construct spatial relations into which these mental images are assimilated, and these relations form spatial schemes. These operations of our mental activity are first sub-logical and concrete and eventually logical and axiomatic.

How can these mental images and constructs relate to a painting? The solution I chose is to relate the operations, which are described in terms of elements, to the brush strokes which can be considered as separate elements.

Chapter V puts the reproduction briefly in the context of the work of Cezanne in general, and his work in relation to changes in art at that time, remaining within the concerns of this paper. The conclusion suggests how ideas in the thesis might be extended, and reviews the theoretical aspects.

In Progress in Art, Suzi Gablik puts forward the thesis that changes in art history may be viewed as a series of transformations in modes of thinking; that in fact it is the transformational element in thinking that produces change in art. She uses Piaget's model of cognition. He has found a close relationship between the genesis of individual intelligence and of systems of knowledge built progressively through history.

She will use the clear and unbroken development of the transformations of space and the elements of geometric form, as an example to illustrate her thesis.

> Geometrization is a process which starts with tentative schemata and concepts drawn from Graeco-Roman antiquity and medieval scholastic tradtions, proceeds to the highly mathematicized art of the Renaissance, and ends with the propositional and deductive logic that characterized the more conceptual forms of recent art.[4]

She subdivides the history of art into three megaperiods to correspond to Piaget's stages of cognitive development. The art of the ancient and medieval world is related to a pre-operational stage, in which space is subjectively organized and imagery is static. Topological relations are characteristic; distance is based on proximity on a two dimensional plane, with no representation of depth. The Renaissance is equated with the stage of concrete operations and figures are related within an overall coordinated system. This is based on projective/euclidean relations, conserving viewpoint and the distance between objects. The modern period, from late Impressionism on, is at the stage of propositional thinking. In paintings, space is overall. Geometric forms are free of reference to empirical reality and become elements in a logical system.

She locates her ideas, including the question of progress in art, with reference to the understanding of the history of science as well as of art, of theories of the evolution of knowledge in both.

In comparison to the wide scope of Gablik's thesis, which applies Piaget's theories using geometrization as an example, I am looking more or less ahistorically at sublogical spatial awareness itself as a base through which we experience painting. My approach is more analytic; relations between brush strokes can indicate a feeling of depth, irrespective of the representational image they form.

On the whole, Gablik is thinking of relations between representational images as representing spatial awareness. In the twentieth century she is interested primarily in the relations between geometric forms, and these not necessarily on a two dimensional surface.

She seems to consider the Renaissance use of perspective as belonging both to concrete operational thinking and the logical or axiomatic stage. Though it is axiomatic, it is still related to representing concrete phenomena in the empirical world. On the other hand, in this paper I consider the rules of perspective to be axiomatic. And following the usage of The Child's Conception of Space, topological relations are considered operational thinking.

Arnheim considers art or any graphic or sculptural image to depict concepts through its form. Intelligence is involved in the act of perception itself, and he does not consider an intermediate category between percept and concept. This paper is based on this area suggested by Piaget.

Piaget is considered a seminal figure in the world of ideas today.[5] The model of understanding he has presented has to be taken into account one way or another. Though he has been criticized for his small samples, his work remains fruitful even in terms of how it generates antitheses. In art education his ideas relate art to understanding and suggest, for instance, that the possibilities of linear perspective be taught when the child has achieved the necessary level of cognitive growth. Though Piaget investigated perception rather than artistic creativity, the capacity to symbolize something in its physical absence, the creation of images, symbolic play, are all important to our understanding of art.

This paper has some limits and aims I would like to clarify. The discussion of the reproduction of Trees Forming a Vault is concerned only with a very limited part of its spatial complexity. The analysis of the relations between the brush strokes is on a very elementary level in terms of the possible complexity of their interrelations. Left aside is the whole complex interplay of light, tone, colour, and paper integrated by Cezanne in an overall system. In no way is this a comprehensive look at the reproduction, not to speak of Cézanne.

I am working from a reproduction of the Cezanne watercolour, which means that all kinds of information in the original are not available; size, surface, colour, tonality, all are changed. The reader is disadvantaged further because the colour xerox of the reproduction is another step removed from the original. For instance, the layers of transparent watercolour in the reproduction can still be distinguished; in the colour xerox they are muddled. The topological relation of order in enclosures, one next to the enclosed surface of the other, is lost. This thesis would be different if done from the original. Only through whatever the original has in common with the reproduction will the ideas discussed apply to both. However, I did work from the reproduction and not the xerox.

I am interested in the sub-logical spatial awareness of adults, yet, except for the conclusion, The Child's Conception of Space deals with the child. However, Piaget seems to assume that the evolution of projective/euclidean relations, complete by eleven or twelve years of age, is also the cognitive base for the adult. He describes the operations as they evolve, and I follow this order, relating each to the reproduction. Included are topological relations, which are devloped in early childhood; the point is that the adult viewer is enriched by them. In fact, it is interesting to pick out these very different relations as we are so used to perceiving projective/euclidean ones.

Chapter II: An Overview of The Child's Conception of Space

Let's briefly consider The Child's Conception of Space as a whole. Conceptual space, or the concepts we form about space, develops after sensory-motor space (the space we create through movement and our senses) and after perceptual space (the space we percive at a given moment). Around sixteen months of age, our first mental images begin. When the first conceptual relations form between them as they begin to be incorporated in a mental scheme, conceptual space starts.

The first conceptual spatial relations are what Piaget calls topological. (These differ from the topological relations of mathematics.) A very young child will copy a triangle, circle or rectangle as a closed figure without making any distinction between them. But this child does distinguish between open and closed figures, and whether one figure surrounds, overlies or borders another figure. These relations of proximity, separation, enclosure, and later, continuity and order, are topological. Size and shape are not relevant; the child's mental image is concerned only with the object's property of being bounded or connected. Each figure relates to the next in these ways, and is otherwise viewed as isolated and not partaking in an over-riding spatial scheme.

At the age of seven or eight, the child begins to develop a sub-logical projective/euclidean scheme of spatial concepts.

Projective relations operate with the idea of viewpoint and involve the changing shape of objects in response to changes in viewpoint. Euclidean relations set up a coordinate reference frame work, in which distances and lengths become meaningful, and the size or shape of an object becomes meaningful in relation to its position. Projective and euclidean concepts evolve interdependantly.

Piaget seems to assume that the projective/euclidean scheme is the most elaborate one constructed on a sub-logical level, and that most adults also operate within it. Any further developments continue on the level of formal or axiomatic logic, such as mathematics.

Most of the book is a detailed look at each step in the evolution of spatial concepts in children. Along with drawings, match sticks, maquettes, projected shadows, etc., are used to gauge the stages of development. The procedures used and the children's responses are described in detail.

About drawing, Piaget says that around the age of sixteen months

> there comes into being for the first time, the mental image which makes possible delayed imitation and, as a result of this, the first attempts at drawing.[6]

Drawing does not spring directly from motor activity, or from perception, but from the mental image which incorporates both.

He was aware that there are pitfalls in using drawings to gauge children's ideas of space.

> many others have shown that the structure of a drawing, as regards the third dimension, for example, is not entirely a translation of image representation.[7]

Image here means mental image. So he will restrict his use of drawings to "the general features of drawings based on simple everyday shapes" because in these "there can be no doubt that drawing does constitute a certain kind of representation".

He uses 'pictorial space' to mean the depiction of an awareness of depth on a two dimensional surface, and has no intention of dealing with drawing as art.

He then discusses the three stages of children's drawings investigated by Luquet. In the earliest, synthetic incapacity, or the lack of co-ordination of isolated elements, the child has hardly begun to construct topological relationships. For example, she or he will draw a head and torso with the legs and arms growing from the head. In the next stage, intellectual realism, the child draws everything that is there, putting two eyes in a profile head, rather than what is seen from one point of view. Topological relations are applied to all shapes and projective and euclidean ones are just emerging. In the third stage of visual realism the child endeavors to draw what is actually seen, including perspective, proportion and distance.

This is how pictorial space develops. The omissions in younger children's drawings are due to the stage of development of their spatial awareness, which is more important than any 'technical ineptitude of a motor character'.

Now I would like to try to clarify what 'image' and 'operation' mean for Piaget.

Image is always used to mean the mental image, and it "is a symbol in that it constitutes the semiotic instrument necessary in order to evoke and think what has been perceived". It designates the figurative aspect of a specific object rather than the object as a general concept. "The degree of resemblance between the image and the object signified varies widely...there is a schematization to suit the subject", (the subject or person fits the image into a conceptual scheme), "some characteristics are retained, others eliminated and others distorted."[8] In other words, the object is evoked by a mental image or schema which can resemble the figurative aspect of the object more or less.

The image becomes part of cognition, rather than simply representing perceived data, when it is based on a comorehension of the transformations accounting for the data; that is, when it is incorporated into a larger mental structure o or scheme. This occurs through the operations of our intelligence.

These operations evolve from our physical actions which are internalized -- incorporated into our mental processes -- in stages. In the earliest stage, up to four or five years of age, action is first recalled in imagination prior to being performed. From four or five to seven or eight years the internal schemata of the physical actions are coordinated further, though in a trial and error fashion. After seven or eight years, the schemata are coordinated sufficiently to be combined and explored mentally in alternate directions. These are the first operations, at a concrete and sub-logical level. By eleven or twelve years, the further coordination of operations reaches an abstract level, and they can be expressed as propositions. This is the final stage, the beginning of formal logical concepts.

The role of the image changes with each of these stages, and becomes increasingly subordinate as the operations become more highly organized.

Chapter III: Piaget's Conclusions

Here we will look at some of Piaget's ideas; such as how intuition is related to formal logic, how the external world can be replaced by mental concepts, and how this leads to concrete operations, which are spatio-temporal rather than logico-arithmetical. Conceptual space differs from perceptual space. The role of the image in concrete operations is not of prime importance.

Piaget begins by discussing the separation between intuition and logic or axiomatics:

> Every possible shade of transition has been suggested to connect elementary intuition with logical operations... in geometric reasoning there always remains some link with intuitive structure....Gonseth...suggests that the 'schema' formed by formal logic always retains traces of intuition, while the primary intuition requires some degree of schematization in order to possess a structure...even for mathematicians, intuition is far more than a system of perceptions or images. Rather it is the basic awareness of space, at a level not yet formalized.[9]

Piaget continues with a problem that becomes very interesting if we keep painting in mind;

> How can consciousness confront the external world so directly as to appear its perceptual or symbolic image, and then proceed to loosen all ties with externality so completely that it is able to replace it by concepts belonging entirely to the subject himself?[9]

In other words, how in our minds do we make the switch from perceptions which occur while we are seeing, feeling, moving objects in the everyday world, to symbols which we can manipulate on an abstract level of thinking? The answer lies in our own physical interaction with the object.

> It is precisely because it enriches and develops physical reality instead of merely extracting from it a set of ready-made structures, that action is eventually able to transcend physical limitations and create operational schemata which can be formalized and made to function in purely abstract, deductive fashion.[10]

That is, the everyday world as given is not meaningful in terms of space. It is our physical interactions with it that enable us as children to act mentally and construct specific spatial schemata. We are not that involved with the recording of a figural image of the object, but rather with recording the action we have taken, or can conceive of taking, with the object.

> From the rudimentary sensori-motor activity right up to abstract operations, the development of geometrical intuition is that of an activity, in the fullest sense; beginning with the adaptive actions which link it with the object, and at the same time assimilate the objects to its own functional structure...The image is at first no more than an internal imitation of previously performed actions, then later, of actions capable of being performed...Finally, at the level, first of concrete, then of abstract operations, action is once more apparent, this time in the richer, yet purer form of the operations themselves....purer, because from now on they go beyond the physical objects with which they are concerned.[10]

These operations introduce

> ...a new element into the classical debate opposing intuition to logic. This is that to the extent that actions are internalized as operations, the initial perceptual and empirical intuitions become rational and coherent, even before having been formalized as propositions. Thus the rigour of the system of concrete operations exceeds that of elementary intuition without reaching that of abstract operations, the basis of hypothetico-deductive propositions. This makes it necessary to introduce new gradations between intuition and logic, the chief of which is the logic of concrete

> operations, superior to pre-logical intuition and inferior to formal logic.[10]

That is, there is a level between elementary intuition and formal logic: the level of concrete operations.

He then compares spatio-temporal operations to logico-arithmetical ones.

> Exactly parallel with these operations there exists operations of a spatio-temporal or sub-logical character, and it is precisely these which constitute the idea of space....their function is to produce the concept of the object as such, in contrast to the collection of objects....They substitute the concept of proximity for that of resenblance, difference of order or position (especially the concept of displacement) for difference in general, and the concept of measurement for that of number....They form just as complete a system as do logico-arithmetical operations...sub-logical operations are accompanied by symbolic images (mental images or pictorial representations) which reflect them far more accurately (though not wholly adequately) than the images accompanying class or number concepts.[11]

In other words, along with operations that deal sub-logically with arithmetical concerns, there are others that deal with spatial awareness. These operations are meaningful in terms of order, position and displacement, proximity, and measurement. (Measurement here means intensive measurement only, and is explained further later on.)

> Sublogical operations...lead to the formation of continuous, unitary schemata. In other words, they lead to complete and continuous spaces...[12]

How the image of an object is integrated into a spatial field is confusing. Furth helps to clarify this by using the terms scheme and schema. The various signifiers of the object form a schema which in turn is integrated through the operations into an overall scheme, or "complete, continuous spaces".[13]

Piaget writes that it is essential to distinguish between perceptual and conceptual space. Perceptual space is the space we construct as we are perceiving things; conceptual space is how we imagine objects in their absence. He continues:

> Spatial concepts are internalized actions and not merely mental images of external things or events -- or even images of the results of the actions... from the initial appearance of thought right up to its ultimate, purely abstract form, the functional connections between the image (as signifier') and the relationship which it 'signifies'(the internalized actions) undergo continuous transformations.[14]

The image

> only plays the part of a 'signifier' or a symbol in relation to the actual process of getting to know the object. Nevertheless, this is an important function, since it is to the extent that the physical action can be recalled by means of an image that it can be conceptualized.[15]

At the level of concrete operations, not the image, but the operation is of prime importance.

To sum up, on a level of thought between incoming sense data and formal logic, there exists a sub-logical area of concrete operations where our awareness of space has become conceptual. Here the mental image may resemble more or less the specific object it represents, but its main reference is the physical interaction of the person with the object. These images are incorporated in relations of order, position and displacement, proximity and intensive measurement into larger spatial schemes of complete and continuous spaces by our mental operations.

This is the cognitive underpinning of the spatial field in paintings. Cézanne's Trees Forming a Vault (estate of Henry Pearlman, New York)[16] can be described in the terms of the above paragraph: It consists of elements which combine in schemata to form graphic images which more or less resemble the objects they represent and which are incorporated into the overall scheme of the painting in relations that are meaningful in terms of order, position... and which form a spatial field. But to tie in the ideas discussed in this section more concretely and specifically to a painting is difficult.

Piaget points out in The Mental Image of the Child, that while the relationship between the graphic and the mental image is more or less close, it is indirect, even in simple drawings. The graphic image means externalizing a previously internalized mental image. This involves the factor of concretization called for by a particular technique. He considers the role of the mental image in spontaneous drawing or the creative imagination a separate area of investigation which he has not investigated in either book.

Although it is difficult to relate the mental image specifically to the reproduction it is possible to connect the reproduction and the operations. In the third part of the conclusion, Piaget discusses the operations in terms of elements as well as objects in the everyday world, or in mathematical terminology. If we consider the brush strokes in the reproduction as separate elements, we can relate them to the operations. They will not be considered, except on a very elementary level, as specific entities each with its special functions, but as elements in a general system.

I chose a reproduction of <u>Trees Forming a Vault</u> because it was not based on a traditional use of linear perspective, and yet our spatial awareness was very important in our looking at it. Because the original water colour was composed of clearly legible brush strokes, it was possible to consider them as separate elements. Our spatial experience of the everyday world can be read into the relations between the brush strokes; this is apart completely from their aspect as figural images representing the everyday world.

Chapter IV; The Operations

The sub-logical operations we are concerned with in this paper are intensive rather than extensive or metrical. Intensive means a direct comparison of one element to another, for instance A is smaller than B, without any reference to any exterior or abstracted unit of measurement. (Comparison becomes extensive if two parts are compared in terms of the number of elements which each contains. And extensive comparison becomes metrical when a unit of measurement is used, and you can tell by how many units A is bigger than B.)

Children use purely intensive sublogical operations to form their rudimentary topological conception of space.

Topological Relationships

Piaget writes that the child begins with the notion of proximity as a given, and works out notions of order, surrounding or enclosure, separation, and continuity. Once these are linked together sufficiently so that they can be reversed as well, the level of concrete operations is reached. Reversal is similar to understanding that if A comes before B and B before C; then in reverse C will come before B, and B before A.

Let's think of the xerox in terms of the above notions and how they could be made visual by the brush mark. Proximity could be seen as one brush mark next to another, and separation as two brush marks not touching at any point.

The notion of enclosure involves complete and partial enclosure. Complete enclosure could be one brush mark completely inside another. A brush mark that partially lies within another brush mark would be partially enclosed. Order, which Piaget describes as a linear series of elements, would be a series of brushmarks in a linear order. Order can also apply to a series of enclosures, one coming after the next. If we consider the brush stroke as a small area enclosed by its edge, as if a line were drawn around its surface area, then the brush stroke too becomes an enclosure. A series of enclosures in order would then be a series of overlaid strokes, like the fine layers of paint on the right edge of the xerox at A.

Now that we have a visual vocabulary established, let us look at the operations which are based on them, and the relationships they constitute. We should distinguish here between notions, e.g. the idea of proximity; operations which are our mental activity and which establish a relationship, e.g. that of being 'between'; and the physical manifestation of this relationship, e.g. a green next to a blue brush mark.

If some of the following seems overly simple, keep in mind that they are concepts that are learned in childhood. These same groupings lead to much more complex structures as projective and euclidean operations.

1. The partition of sets and the addition of sub-sets

This first topological operation is the abstracting of the relationship of a whole to its parts; a continuous whole can be broken up into neighbouring elements, and these same elements can be made again into a continuous whole. A seven year old can conceive of breaking up a line into smaller pieces of line (though not into infinite points). At B on the xerox, the band across the road, which can be seen to be made up of separate yellow and green elements, is an example of a set and sub-sets.

II. Order of placement

This is the notion of linear order, formed through the progressive combining of elements in proximity, resulting in a particular sequence in one direction and its reverse. In other words, wxyz always goes from w to x to y to z and in reverse from z to y to x to w. Looking at the lower right corner of the xerox, the sequence of strokes can be read as CDE and the reverse as EDC. The awareness of their fixed sequence of position is an awareness of linear order.

III. The reciprocity of proximities

If x is next to y, then y is next to x. In the xerox at B, if yellow is next to green, then green is also next to yellow, going in the reverse order.

IV. Symmetrical interval relations

This is the relationship described by the word 'between'. It's the relationship and not the distance that's symmetrical. Just above and left of center, ~~the brush~~ brush mark at F is between G and H.

If y is between x and z in the order xyz going in either direction, the interval from x to z is always y, and is the same as the interval from z to x. In the xerox, the interval from H to G is F, and from G to H is F.

The idea of 'between' depends on understanding linear order, the second operation.

V. One-one multiplication of elements

If a line crosses a circle at a point x, the point x at the same time belongs to the circle and the line. This is a multiplicative operation, because the point has two functions at the same time. At I, the bleeding of the blue into the green belongs both to the green triangular mark and the blue line, and is in that sense multiplied.

Transparent brush marks which blend into others over or under them also exist as part of two or more elements at the same time.

VI. One-one multiplication of relations

This is the same as the preceding operation, but in terms of relationships rather than elements. The elements in one series are seen as corresponding to the elements of another. These relationships are "in general no more than the path followed by eye movements" connecting them.[17] That is, it is the eye that makes the connections between elements of the two series, rather than straight lines or other graphic or concrete phenomena.

On the xerox at C and J, we see two bands across the road and we can correlate the elements they are composed of in terms of colour or position. That is, we relate the two greens together because of their hue, or the green on the left with the yellow on the left, because of their position.

VII and VIII. One-many multiplication,
either of elements or relationships

This is a continuous extension of proximities, either through elements or relationships. One element is next to neighbouring elements which in turn are related to other neighbouring elements. On the xerox, F is next to G and H, which are in turn next to L, M and N, etc. As for relationships, we relate O to Q because of their similar hue, shape and inclination. Q then relates to P and R, and so a spreading network of relationships is established.

This ends the section of operations which constitute topological relationships which form topological space.

Topological relations and notions exist at the tactile level, as well as the visual, and can be conceived of by someone blind. A blind person cannot see the proximity of a green to a blue brush mark, but can still conceive through touch of the notion of proximity and the relations that operations make based on that notion, as well as the other topological notions of order, separation, enclosure and continuity, and the operations based on them.

But the following operations are based on experiences in the everyday world which are not available to someone without sight.[18] The blind do not have a viewing point, and are unaware of distances as spatial. (A walk along a street is thought of as a series of events in time.) A viewpoint and an awareness of distance are the base of projective and euclidean relationships. Newly sighted adults have to learn to see in what seems to me to be the projective/euclidean framework, and do so, if at all, with great difficulty. Our conception of projective/euclidean space grows out of our physical interaction with the everyday world -- but only if we can see. In this sense it is truly visual.

Sub-logical Operations Constituting Projective Relationships

These conserve the relationship between the position of the observer and the object seen. Projective concepts are constituted by the same operational groupings as topological ones, but with the addition of the concept of viewpoint.

In these sections on operations, I am describing what each step involves for a hypothetical person whose spatial awareness is evolving. At this point, for a child whose concepts of projective and euclidean space are just beginning to develop, in his or her drawing the same visual cues will begin to take on a new significance. An item drawn on the boundary of a closed shape, which as a topological relationship implied partial enclosure, now implies overlap.

Perceptual and conceptual space awareness can have an interval of several years in their development, as conceptual space develops more slowly. A young viewer might be aware of projective relations when looking at a painting, inasmuch as this might depend on perception,and yet draw in terms of topological relationships, as this depends on her or his own level of spatial concepts. In contrast, for an adult viewer information in a drawing or a painting could carry significance on a topological level and a projective/euclidean level, at the same time. She or he could conceivably see an item on the boundary of a closed shape as implying both partial enclosure and overlap.

How conceptual and perceptual spatial awareness interact in a viewer looking at a painting I can't answer. But it is clear the viewer does read the conceptual framework which exists in her or his head into the information given by the painting. Apart from the viewer,there is no space there.

I. Addition and subtraction
of projective elements

This is the series of changes a projective figure goes through as parts of the figure move and obscure other parts, resulting from changes in point of view. Shape becomes important.

In the reproduction, the presupposition of a point of view enables us to think of some brush strokes, like the one on the xerox at S, as interrupted because it would seem to disappear behind the tree trunk. Partial enclosure can be seen as overlap. Because of their changing yet similar shape and hue, we can read the many small, green triangular shapes (i.e. at O, P, Q, R, T and V) throughout the reproduction as if one green mark was moving across the surface and the brush recorded its changing shape due to its changing position.

II. Rectilinear order

Linear order of the elements here acquires significance as a straight line, because this is the only shape which remains unchanged throughout projective alterations; that is, it is always a straight line though its length and angle vary. It becomes a point when sighted from one end, which makes line of sight possible.

The series of brown strokes at the right of the road, V on the xerox, are in a straight line. How much we read into the angle of its projection (and how much it affects the reading of the overall space of the painting), become evident if you cover it.

III. Complementary perspective relations

This involves conservation of the relative position of two neighbouring elements, even with a change in viewpoint. The band at B across the road is green next to yellow from left to right. The relative position of green to yellow would remain the same from any viewpoint.

To take this further, the two bands at B and J are similar in shape; one is green-yellow, one yellow-green. The reversal of the position of the hues could imply a change in viewpoint. The relative position of yellow to green remains the same, but is reversed. This is what would occur in the everyday world if the two elements at B were seen from in front and then behind. Here both views are presented simultaneously, the change in position can be seen as implying a displacement of viewpoint.

IV. Symmetrical interval relations

This is similar to the preceding operation, but in terms of relations, not elements. In xyz and zyx, y remains 'between' and the interval from x to z is the same as the interval from z to x.

If we consider the yellow (E), pink (D), and green (C) strokes on the lower right, the pink would remain between the yellow and the green even if our viewpoint shifted theoretically 90° to the right; its relationship to the others remains the same.

V. One-one multiplication of elements

Here relations between different elements become meaningful in terms of left/right, above/below and/or before/behind. This leads to the concept of the plane. According to Piaget, a plane is a network whose intersections are established by two out of the three above relationships; that is, from one viewpoint the plane becomes a straight line because the intersections of the network all run either before/behind, or left/right and not above/below.

All brushmarks on the flat surface of the xerox are planes inasmuch as they are networks in the only two dimensions by the paper -- height and width. But the definition above deals with 'relations' and not dimensions, and this is where the difference lies between brushmarks as physical entities and brushmarks as elements meaningful in terms of spatial relationships in a conceptual framework we bring to the reproduction. The brushstrokes can be read as planes established by two out of three relations: This is why the green mark at W can be read as tilting. The whole xerox is now energized in terms of these three sets of relationships. Carried further, the multiplication of planes establishes a projective three dimensional space. In the reproduction this is the cumulative effect of the various tilts we read into the brush strokes.

This operation is a one-one multiplication because each element is established by two relationships but one relationship does not affect the other. In a one-many relationship, the one point is affected by its relations to more than one factor, which also influence each other. We will see this in operations VII and VIII.

VI. One-one multiplication
of relations

The height and width of the background here becomes related to the dimensions of the foreground; foreshortening enters. All the elements relate simultaneously in three separate ways; left/right, above/below and front/rear. This boils down to 'further x shorter' or that evely spaced intervals between points on a straight line are seen to decrease as it recedes as in the intervals between a ruler seen in perspective.

This is hard to exemplify, but the repetition of similar shapes in similar size through the reproduction such as at U and W, plays against our projective expectations.

VII and VIII. One-many multiplications
of elements and relations

One element is related to many elements, or one relation to many relations. This differs from the preceding projective operations in that one-many relations influence each other. Rather than the intervals on a single line of the preceding operation, we are now concerned with two lines, and the relations between points and intervals on each line to each other.

> one-many correspondences describe triangular structures, the simplest example of which is provided by a pair of lines meeting at the horizon, such as a railway line[19]

Consider the intervals between the sleepers in terms of the relations. Not only are the intervals governed by 'further x shorter' as in the last operation, but their width is governed by 'further x narrower'. We now have 'further x shorter x narrower' at the same time.

In the xerox, the converging sides of the road and the changing relationship between them exemplifies this, as do the converging sides of the brush stroke at H.

The Sub-logical Operations
Constituting Euclidean Space

If the subject can co-ordinate different viewpoints to construct projective relationships, she or he can also co-ordinate distances and construct euclidean relationships. The two develop interdependantly.

The conservation of distances between positions now occurs. In contrast to changes in the relationships between objects and to changes in shape related to point of view, euclidean space co-ordinates the positions to which the movements of the object are related. The conservation of distance leads to the development of an overall reference frame. In this frame, the connection between the size or the shape of the object, and its position becomes meaningful.

I. Addition and subtraction
of elements

An object in one position has a particular shape which is that object in that position. Whereas in projective relationships, the addition or subtraction of parts implied a viewpoint in which parts obscured other parts, here they imply the conservation of a whole object, in relation to its particular shape in this one particular position. As an example, we can take the green triangular shape at O, although any brush mark in the painting would do.

II. Placement and displacement
of objects

This distinguishes between elements and the positions they occupy. It introduces two concepts: the first is the mobility of the elements and their potential repositioning; the second is the fixity of the positions compared to the mobile elements. Think of chequers as compared to a chequerboard.

In the reproduction we see numerous green triangular shapes similar to O, such as those at P, Q, R, T and U. This repetition of brush marks of the same colour and similar size and shape could imply mobility, as if one single green triangular brush mark could move through the fixed positions of the reproduction, an analogue to the first projective operation.

III. Reciprocity of references

A series of shapes (or positions) of objects, in proximity and added together, starting at one point, obtain a certain term. And this term can also be arrived at through their neighbouring shapes (or positions). Think of two bead necklaces lying next to one another, or the points on two adjacent lines. This operation is the idea of length, without its being metrical. When arranged later in two or three dimensions, it leads to grids or the framework for euclidean space.

Let's look at the two series of painted shapes that begin with the brush marks at F and H and descend from them to the pale green area divided at X by a small painted line. The series of shapes on the left come to the same term (or length) as the series on the right. (This operation does not yield to being easily described in pictorial terms, but it is important as a step to the establishment of the awareness of height, width and depth in operation VI.)

The references are reciprocal because, for example, a bead three beads from the end of one necklace is in a reciprocal term with a bead in the same position on another necklace.

IV. Inclusion of intervals or distances

In the everyday world, irrespective of where the objects are, stationary positions remain an unchanging interval or distance apart, even if they are crossed by a moving object. We now have conservation of distance between positions, e.g. the distance between the positions of O and P is meaninful and fixed.

V. One-one multiplication of elements

Two linear series of elements multiplied together form a surface, and multiplied by a third, a volume. In the xerox, the points or brush marks along one line related to the points or brush marks along another line imply a surface; and related to the points along a third line can be seen as describing a volume. The horizontal bar at B describes a line; together with the vertical edge of R it implies a surface, if we think of all the points along the edge of one as related to the points along the edge of the other. The blue line at Y can be seen as receding (see projective operation II) and in relating to the surface between the points of the former lines suggests a volume. This volume is not a cube and not rectilinear, nor does it have to be so in any way.

VI. One-one multiplications of placement and displacement relations

This is the establishment of a non-metrical grid of relationships, a co-ordinate system occupying two or three dimensions simultaneously. (These grids do not necessarily have their axes at right angles.) Operation III is here extended to several dimensions.

Each point on the grid relates separately in terms of position and distance to each of the three axes, and these relationships do not influence each other.

The axes for height, width and depth form a conceptual framework which we bring to the xerox. Every brush mark is seen as positioned in relation to these three axes.

However, if for example we look at the brush mark at W, we see its position is still ambiguous, perhaps because unlike the everyday world, the xerox does not give us enough information to pinpoint its position exactly.

In this operation we can see the difference between operations and the relations they constitute. The operations have set up a framework which enables certain relationships to exist between brush marks; for example, that we can think of W as further back than B.

VII. One-many multiplication of the elements

One element relates to two or more other elements simultaneously; that is, each relationship affects the other or others. Piaget gives the concept of the triangle for two dimensions, and the pyramid for three. It's not so much the lines connecting the elements but the area between them that is shaped by its relations to all the elements.

The triangular area of the brush mark at R is formed by the relations between the elements on two of its sides. A pyramid is implied when we consider the possibility of B as a third side. That the shapes are so sketchy compared to geometrical figures does not stop us from reading our spatial concepts into them.

VIII. One-many multiplication of relations

This operational system is responsible for qualitative estimates of angle prior to measurement. It can roughly be described as realizing that if two angles of the same size are superimposed, their sides coincide.

When we look at the xerox, we are aware that the angles at the apex of R and Q are different in size, and that the saturated part of O and the brush mark at P are similar in angle.

This concludes the section of euclidean operations. Piaget says that the concepts of projective and euclidean relations develop together, are mutually interdependent and "fundamentally cognate".[20] This suggests that topological relations form one kind of space, and projective/euclidean relations form, together, another. Both topological and projective/euclidean space can be read into the reproduction. Tracing the elaboration of porjective/euclidean relations did not erase the topological relations I described earlier.

The same eight operational groupings constitute topological, projective and euclidean relationships. With the child's mental growth, "they acquire new significance, integrating topological relationships by giving them definite specifications".[21] The same structuring acquires different meanings as our spatial understanding grows.

These eight operations of sub-logical thought are, according to Piaget, the same for logical, mathematical thinking. Again the significance differs, while the structures of the operations remain the same. He suggests this is because the operations are no more than the groupings which can join, relate and multiply elements or relationships only within the limited number of possible combinations. The correspondence between the operations "is very interesting as regards the functional unity of the various operations of thought." [21]

If these sub-logical spatial relations are a form of thought, and are made visual in the reproduction, then the reproduction can be considered a form of visualized spatial thinking. This differs from Arnheim's 'visual thinking' which is concerned with the perception of the shape or form of the image, and doesn't allow for an intermediate area of sub-logical operations between incoming sense data and spatial concepts.

Chapter V; a Brief Look at Cézanne

Cézanne's paintings depict a shallow rather than deeply receding pictorial space. Braque in an interview said

> The acute angles in the paintings I did at L'Estaque in 1908 were the result of a new conception of space. I said goodbye to the 'vanishing point'. And to avoid any projections towards infinity I interposed a series of planes, set one on top of another, at a short distance from the spectator. It was to make him realize that objects did not retreat backwards into space but stood up close in front of one another. Cézanne had thought a lot about that...he had done away with distance and... after him infinity no longer exists.[22]

In his later paintings, Cézanne built this shallow pictorial space with an interwoven network of small planes, and touches of colour. It was "moduler" not "modeler"; a spatial field was articulated rather than the sculptural roundness of the objects being depicted, or a strictly logical euclidean space.

Rubin in "Cezannisme and the Beginnings of Cubism" points out that Picasso and Cubism owe much more to Braque, and Braque to Cézanne, than is generally acknowledged. Braque's low relief and 'passages' come directly from Cézanne. Rubin quotes Braque as saying most painters

> totally ignore that what is between the apple and the plate can be painted too...This in-between space (entre-deux) seems to me just as important as the objects themselves[23]

Rubin points out

> this is precisely the space bridged by Cézanne's passage. Thus what Braque described as a "materialization of a new space"...was in effect, the explicit articulation and radicalization of a Cezannian idea.[24]

This concept of a continuous field in which the space between the apple and the plate has its being, is carried over to the articulation of the spatial field of the painting. This was the foundation for the later development of Cubist space.

In this shallow spatial field, the role of the brush strokes increased. In Cézanne's earlier paintings the brush strokes were grouped within the boundaries of more or less delineated images, but in the later works the loosely arranged marks play a large role as separate elements, as we can see in Trees Forming a Vault.

Cézanne took from the Impressionists "the microcosm of the picture plane, its molecular tissue"[25] which was composed of distinct brush strokes. To the attitude of the Impressionists towards the natural world he added a formal structural approach. The structure of the image and of the spatial field of the painting are united in the small patches of colour made by the brush strokes. Renoir

> had said that Cézanne "could not put two spots of colour on a canvas without its already being very good". Picasso amended this by saying "without its already being a picture".[26]

Obviously the spatial field elaborated in Cezanne's late paintings depends on our spatial awareness, the overall spatial field we bring to the paintings.

Without being narrative, the reproduction of the Cézanne does not represent a static moment. No sharp sunlight or moving human is caught at a point in time. The trunks of the trees dissolve at the edges; the foliage is interwoven greens rather than specific plants. The paint brush's movement can be traced in the repetition of many of the greens, which can be thought of as repeated but changing images of the same element as it moves like a falling leaf across the paper. In sum, we are offered a record of transitions, a network of variable relations, rather than a static moment and images of discrete objects.

> Ici, au bord de la rivière, les motifs se multiplient, le même sujet vu sous un angle différent offre un sujet d'étude du plus puissant intérêt, et si varié que je crois que je pourrais m'occuper pendant des mois sans changer de place en m'inclinant tantôt plus à droite, tantôt plus à gauche.[27]

This sensitivity to the shifting nature of his subject matter found its way into Cézanne's painting.

In contrast to this, traditional perspective rests on a kind of snapshot convention. There is a single moment in time in which nothing moves, neither the artist or the scene depicted. The spectator, to see the painting at its most convincing as a depiction of euclidean space, must look from a fixed point, without moving, and with one eye closed. The Impressionists with their interest in shimmering light and atmosphere had already moved away from this. Gablik writes about twentieth century art:

> ...the forms converge and deflect themselves around the surrounding space. Single perceptual acts, rather than being scattered and disconnected, coalesce with each other into one sustained process; multiple appearances not only succeed but confirm, continue and complement each other. The fact is that modern artists achieve a synthesis of experience, movement and perception over time which is unknown in earlier art, and which transcends the single, fixed point of view.[28]

Ivins in Art and Geometry[29] points out that the Ancient

Greeks thought of the world as static and discontinuous objects unrelated to each other; this was reflected in their art. The West later grasped the relatedness of events and their continuity; Alberti's two point perspective tied together for the first time the spatial relationships between objects. Ivins explains that recently relations of change rather than a fixed external world have become an accepted paradigm for reality.

> Here being has countless forms, countless leaves, but being is subject to no dispersion; if I could ever succeed in grouping together all the images of being, all the multiple, changing images that in spite of everything, illustrate permanence of being...[30]

Cézanne's discontinuous tables and staggered wall moldings, etc., can be seen as an interest in rendering human perception rather than logical euclidean space. His work, although it takes place within a euclidean framework, is a soft approach, sub-logical rather than axiomatic; that is, not following the rules of linear perspective.

Loran in Cézanne's Composition[31] has made an investigation of some of Cézanne's paintings by means of photographs of the original sites. Photographs, like linear perspective, in which the size of the image varies in a direct ration to its closeness to the picture plane, seem distorted in comparison with how we perceive size in relation to distance, in the everyday world. (When photographs first came out showing vanishing lines spreading widely apart in the foreground, artists thought the photographs were fakes.)[32] We compensate for s changes in size, colour and shape due to changes in distance lighting, or point of view, so that euclidean spatial relations are not a representation of human perception. In The Eye and the Brain, Gregory explains size constancy. (The 'image' is the image on the retina.)

> The image of an object doubles in size whenever its distance is halved. This is a simple fact from geometric optics, and applies to a camera as it does to the eyes.... although the image grows as the distance of the object decreases it still looks almost the same size;consider an audience at a theatre -- the faces all look much the same size, and yet the images of the distant faces are far smaller than the nearer.[33]

He quotes Descartes on shape constancy:

> ...our judgements of shape clearly come from our knowledge, or opinion, as to the position of the various parts of the objects, and not in accordance with the pictures in the eyes; for these pictures normally contain ovals and diamonds when they cause us to see circles and squares.[33]

Loran, in comparing photographs of the sites with the paintings of Cézanne, rather inadvertantly found that in comparison, the paintings had roads tilted up towards the picture plane, enlarged backgrounds, buildings drawn from multiple viewpoints, etc. In comparison with either photographs or linear perspective, this handling of spatial relations is closer to human perception.

Conclusion

Piaget's work on how we conceive of the form of objects and the space around them explains at least partly why The Natural Way to Draw is a useful approach to teaching drawing. It urges the student to translate her or his muscular tensions and repose, the touching and handling of objects, the knowledge of form from multiple viewpoints, into a graphic image.

> Like contour, gesture is closely related to the tactile experience. In contour drawing you feel that you are touching the edge of the form with your finger (or pencil). In gesture drawing you feel the movement of the whole form in your body.[34]

This paper has touched on areas that could be extended.

In the rare event someone else grows into adulthood without sight and then has it restored, can Piaget's elaboration of the steps involved in spatial awareness be used to help him or her learn the necessary projective/euclidean sub-logical relations?

A long look at how Cézanne's work evolved in relation to sub-logical spatial awareness, as well as much fuller formal analyses of individual paintings, with an eye to a comparison of the use of oil paint, would be fruitful.

Using Piaget's description of the steps in the growth of sub-logical spatial awareness had enabled us to begin to relate each one to some of the relations between the brush strokes on an elementary level.

The simple yet so rich contribution of the topological relations could be otherwise overlooked as components of spatial awareness. As an example, let us look at the series of enclosures in order at A; the enclosure, partial or complete of one brush stroke by another, and another, and another..., all visible in the transparent water-colour, if indistinguishable, colour added separately to colour. This construction, so rich visually, can be found over and over again in this reproduction and others of Cezanne's water-colours. Using Piaget has halped to articulate the spatial complexity of these constructions, which is in turn a part of their visual complexity.

From the basic notions like proximity or separation, the growth of a complex scheme which positions each stroke in relation to height, width and depth in a euclidean framework, each and every step in the formation of spatial awareness could be read into the reproduction. The earlier topological relations were not erased by the later projective/euclidean ones; both kinds could be traced by an adult viewer. Also, one brush stroke can be read as having several different spatial relations. For example, the partial enclosure of topological relations becomes overlap in projective relations; the brush strokes at O and Q can be read one way and then the other. In addition, of course, all the relations described by this paper are not limited to the brush strokes mentioned, but can be found repeatedly throughout the reproduction. Because of all this, the reproduction of a water-colour with a limited number of brush strokes can offer, even on an elementary level, a wonderfully rich and complex variety of spatial relations.

Footnotes

1. Fernande Saint-Martin, Structures de l'espace pictural: essai, Collection Constantes, vol.17(Montréal, éditions HMH, 1968)pp.55-70.

2. Jean Piaget and Bärbel Inhelder, The Child's Conception of Space, trans. F.J. Langdon and J. L. Lunzer, The Norton Library(New York, W. W. Norton & Co.,1967.) Although I refer to Piaget only, in the text, this is not meant to overlook the contribution of Bärbel Inhelder.

3. Suzi Gablik, Progress in Art (New York, Rizzoli, 1977.)

4. Ibid., p. 13.

5. From an interview with David Pariser, Assistant Professor in Art Education, Faculty of Fine Arts, Concordia University, in Montreal on September 18, 1979.

6. Piaget, p. 12.

7. Ibid., p. 46.

8. Jean Piaget and Bärbel Inhelder, Mental Imagery in the Child; a Study of the Development of Imaginal Representation, trans. P.A. Chilton (New York, Basic Books, c1971),pp.381-82.

9. Piaget, The Child's Conception of Space, p. 448.

10. Ibid., p. 449.

11. Ibid., p. 450.

12. Ibid., p.459.

13. Hans G. Furth, Piaget and Knowledge; Theoretical Foundations (Englewood Cliffs, New Jersey, Prentice-Hall, c1969), p.264.

14. The Child's Conception of Space, p. 455.

15. Ibid.,p.455.

16. The xerox copies are of the reproduction in William Rubin, ed. Cezanne; the Late Work (New York, Museum of Modern Art, 1977),p.305.

17. The Child's Conception of Space, p. 466.

18. M. von Senden, Space and Sight; the Perception of Space and Shape in the Congenitally Blind Before and After Operation; trans. Peter Heath. (Glencoe, Illinois, Free Press, 1960).

19. The Child's Conception of Space, p. 473.

20. Ibid., p. 476.

21. Ibid., p. 480.

22. Gablik, Progress in Art, p.81-82.

23. Rubin, Cezanne; the Late Work, p. 198.

24. Ibid., p. 169.

25. Fritz Novotny, Painting and Sculpture in Europe; 1780-1880 (Harmondsworth, England, Penguin Books, 1971),p.370.

26. Rubin, Cézanne; the Late Work, p. 189.

27. Paul Cézanne, Correspondance, ed. John Rewald (Paris, Grasset, 1937)p.288.

28. Gablik, Progress in Art, p. 24.

29. William Ivins, Art and Geometry (New York, Dover, c1946).

30. Gaston Bachelard, The Poetics of Space, trans. Maria Jolas (Boston, Beacon Press, 1969)p. 201.

31. Erle Loran, Cézanne's Composition; Analysis of his Form with Diagrams and Photographs of his Motifs, 2d ed. (Berkeley, University of California Press, 1959).

32. Kurt Badt, The Art of Cezanne, trans. Sheila Ogilvie (Berkeley, University of California Press, 1965)p.4.

33. R. L. Gregory, The Eye and the Brain; the Psychology of Seeing , World University Library (New York, McGraw Hill, 1976) p.152.

34. Kimon Nicolaides, The Natural Way to Draw; a Working Plan for Art Study (Boston, Houghton Mifflin, c1969) p.16.

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Pl. 110. *Trees Forming a Vault.* Watercolor, $17\frac{1}{2} \times 24\frac{1}{2}$ in (44.5×62.2 cm). Estate of Henry Pearlman, New York

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