# The Dynamics of the Trade Balance and the Terms of Trade: A Frequency Domain Analysis

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# Abstract

**The Dynamics of the Trade Balance and the Terms of Trade: A Frequency Domain Analysis**

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Two well known international macroeconomic phenomena are the countercyclical movements of net exports, and the sluggishness with which the trade balance responds to changes in the terms of trade. Standard textbooks describe the dynamic relationship between output, the trade balance, and the terms of trade under the general rubric of the "J-curve", due to the fact that the graphical representation of the path of the nominal trade balance responding to a depreciation of the home currency resembles the letter J. Recent empirical and theoretical research in this area has stressed the importance of a general equilibrium perspective. Extant dynamic general equilibrium models of the J-curve are nonlinear rational expectations models, and the preferred empirical framework for evaluating models of this type has been calibration and simulation

Within this framework, researchers calibrate a model, approximate a solution, and then simulate the approximation to compare various properties of the simulated data (*i.e. moments*) to the same properties of the historical data. Typically, the comparison between actual data and simulated data has been done using time domain

statistics, and the full dynamic properties of the models are rarely investigated.

This thesis proposes a set of frequency domain test statistics that are convenient to apply within the calibration and simulation framework, and which summarize all of the information in the auto and cross-covariance functions of the variables under investigation. The diagnostic statistics are applied to evaluate the frequency domain properties of a general equilibrium international business cycle model of the J-curve. The theoretical model fails to replicate the multivariate frequency domain properties of output, the trade balance, and the terms of trade. As a result, the inclusion of a stochastic measurement error is considered as tool to reconcile the theory with the data.

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The work is dedicated to my family and especially to my daughter.

# Contents

# List of Tables

# List of Figures

# Chapter 1

# Introduction

Business cycle research has always represented an important part of empirical macroeconomics. Theoretically the often quoted argument that "all business cycles are alike" has suggested that a grand unifying theoretical framework may exist to understand and explain cyclical fluctuations in economic variables.[1] While most empirical research has been undertaken in a domestic framework, the growth in interdependencies between developed economies has sparked an interest in international aspects of business cycles.

In recent years, theoretical work on international business cycles has primarily taken place within the context of real business cycle theory. As initially conceived, real business cycle theory explained aggregate fluctuations as the consequence of fluctuations in the pace of technological change (Kydland and Prescott (1982) and Long and Plosser (1983)), although it was immediately clear that the methodology was more generally applicable to a wide variety of models. An interesting feature of the

[1] Lucas (1977) has persuasively argued along these lines

empirical evaluation of these models has been an almost exclusive reliance on *calibration and simulation* as the method of choice for comparing the model properties with the properties of historical time series. Within this framework, researchers calibrate (choose parameters), then simulate a theoretical model of interest and compare various properties of the simulated data (*i.e. moments)* to the same properties of the historical data. Within this framework Backus, Kehoe and Kydland (1993), have modeled international business cycle responces as arising from general equilibrium responces to technology shocks. They found evidence that an international version of a real business cycle model can account simultaneously for the familiar domestic co-movements and several international co-movements.

Most of this research has used time domain methods to characterize the data and evaluate the theories. While time domain methods are perhaps more intuitive to economic theorists, frequency domain methods may provide more useful information when examining the business *cycle* properties of macroeconomic time series. Many authors *define* the business cycle within the frequency domain, either as the existence of common peaks in univariate spectral densities across major macroeconomic aggregates, or as the presence of high pairwise coherencies at low frequencies where most macroeconomic time series with "typical" spectral shapes have power (Sargent, (1987)).This thesis argues that inference in the frequency domain is a useful way to explore the cyclical properties of general equilibrium international business cycle models, and proposes a set of frequency domain diagnostic tests for evaluating the dynamic properties of these models.

The diagnostic tests measure the distance between spectral and cross-spectral distribution functions estimated from historical time series, and spectral and cross spectral distribution functions estimated from simulated data. With these diagnostic statistics, it is possible to examine the distance between simulated and historical spectral distribution functions over a particular region of frequencies, in order to provide direct evidence as to the ability of macroeconomic models to "explain" the business cycle when evaluated over business cycle frequencies. As spectral densities and distribution functions incorporate all of the information in auto- and cross-covariance functions, these tests provide a convenient summary of the ability of the model to replicate the complete temporal behaviour of the endogenous variables in the model. The empirical methodology borrows extensively from the literature on univariate spectral shape tests for serial correlation developed by Durlauf (1993), and extended to multivariate situations in Willson (1993). In turn, the literature on shape tests was originally designed for examining hypotheses concerning probability density and distribution functions; the techniques proposed here are the frequency domain analogs of the two-sample tests for probability distribution functions discussed in Durbin (1973). Within the calibration framework, our statistics are the frequency domain analogs of the time domain procedures developed in Gregory and Smith (1991).

Several economists have used frequency domain procedures to address different issues related to a certain degree to the international business cycle. Dellas (1986), in an attempt to analyze the generation and transmission of economic fluctuations across countries, estimates the spectrum for four countries and also estimates the coherencies (done in a pairwise form) between all the possible pairs. Gerlach (1988), uses spectral methods to measure the correlation between output fluctuations for different exchange regimes. Altuğ (1989) estimates the theoretical spectra of a real business cycle model and uses them as a tool for estimating the model parameters. The two studies that are most closely related to our research are Sòderlind (1992) and Watson (1993). Watson (1993) suggests a number of frequency-domain procedures for evaluating general equilibrium models: however, he treats the theoretical model as necessarily false and subject to random measurement error, and attempts to ascertain the properties of the measurement error required to reconcile theory with the data. Sòderlind (1992) compares theoretical coherencies from the real business cycle model of Kydland (1988) with coherencies from the data, using the framework suggested in Gregory and Smith (1991).

Two applications of these diagnostic tests are considered. First, the test statistics are employed to evaluate the dynamic properties of the relationship between the trade balance, output, and the terms of trade. Recent empirical research within the calibration and simulation framework has suggested a striking empirical regularity, dubbed the "S-curve" by Backus, Kehoe, and Kydland, for the cross-correlation function between the trade balance and the terms of trade for the majority of industrialized countries. Chapter three of this dissertation investigates the frequency domain properties of their model. The empirical results suggest that the model properties are very sensitive to the approximation method employed in the solution of the model, and the frequency domain properties of the model differ significantly from the historical data in a wide variety of cases. As a second application, measurement error in trade statistics is considered as a possible reason for the empirical rejection of the model in Chapter 3. The frequency domain properties of the model under a variety of plausible measurement error specifications are considered.

The thesis will proceed as follows: Chapter 2 develops the diagnostic tests and discusses their application within the calibration and simulation framework, and within the context of classical frequency domain econometric procedures. Chapter 3 employs these tests to evaluate the Backus, Kehoe and Kydland (1991) model of the relationship between the terms of trade, output, and the trade balance. Chapter 4 provides some motivation for the existence of measurement error in trade statistics as a plausible reason for the empirical results in Chapter 3. In addition, Chapter 4 develops a theoretical relationship between the spectral matrix for the endogenous variables in the model without measurement error, and the spectral matrix for the same variables when it is assumed that they are observed with error. Chapter 5 concludes and suggests avenues for future research.

# Chapter 2

# Spectral Shape Diagnostics For Evaluating Business Cycle Models

This chapter develops a set of frequency domain diagnostic tests that can be used to evaluate the dynamic behavior of economic models. While these tests are potentially useful in a wide variety of situations, the methodology is ideally suited to the evaluation of nonlinear dynamic general equilibrium models that are often associated with the *real business cycle* literature. Within this class of models, the predominant empirical methodology is *calibration and simulation*, where researchers choose parameter values for the model under consideration, solve the model numerically using one of several possible approximation techniques, and then simulate the model to compare the properties of the model with the properties of historical time series that the model attempts to explain.

An important feature of this research framework is that formal multivariate estimation and inference procedures for comparing the model with the data are rarely applied, although it is well known that classical econometric procedures such as GMM and maximum-likelihood can be applied in many cases.[1] Instead, a fair degree of latitude is often employed in the selection of parameter values in the calibration stage, with information from a wide variety of aggregate and disaggregate sources often being combined in an *ad hoc* manner. As these models are primarily nonlinear, researchers must also choose a method for obtaining an *approximate* solution to the model, and in many cases the statistical properties of the approximate solutions vary substantially across solution techniques.[2] Once a model has been simulated, properties of the simulated time series (usually moments) are compared with the same properties of the historical time series to evaluate the validity of the model. In practise, this comparison is made using "the eyeball", or by using the variability in the simulated moment to gauge the distance between the historical moment and the mean of the model moment evaluated over a large number of simulations.

It is important to realize that the absence of a formal, completely specified framework for estimation and inference in the calibration and simulation framework is actually viewed as an advantage by some researchers. For example, Prescott (1986) argues

> "The models constructed within this theoretical framework are necessarily highly abstract. Consequently they are necessarily false, and statistical hypothesis testing will reject them. This does not imply, however that nothing can be learned from such quantitative theoretical exercises. I think much has already been learned and confidently predict that much more will be learned as other features of the environment are introduced."

[1] Gregory and Smith (1993) provide an excellent discussion of the relative merits of traditional estimation techniques such as GMM and ML, and calibration and simulation

[2] Taylor and Uhlig (1989) provide a striking illustration of the sensitivity of the properties of simulated time series to different approximation techniques

Along these lines, calibration and simulation exercises can provide important diagnostic information concerning the dimensions in which a model is consistent and/or inconsistent with the data, while avoiding the modelling of extra parameters. From this perspective, calibration and simulation exercises are not necessarily incompatible with formal econometric methods, but represent procedures for suggesting profitable modifications to existing models or new avenues for research.

One prominent characteristic of the majority of studies within the calibration and simulation framework is their reliance on time domain statistics to represent the dynamic characteristics of the economy under consideration. Typically, researchers only consider the first order autocorrelation coefficients, and higher order autocorrelations as well as cross-correlations at leads and lags, are rarely investigated.[3] As a result, the complete dynamic properties of these models are rarely compared with the data. This chapter develops a set of convenient diagnostic statistics for the evaluation of the dynamic properties of nonlinear dynamic general equilibrium models within the calibration and simulation framework.

The diagnostic statistics developed here are based on comparisons between spectral and cross-spectral distribution functions for simulated and historical data. For a univariate time series, the spectral density function is the fourier transform of the autocovariance function; for a bivariate time series, the cross-spectral density function is the fourier transform of the cross-covariance function. These frequency domain diagnostics summarize *all* of the information in the auto and cross-covariance functions for the time series under consideration, and therefore represent convenient

[3]Recent exceptions are Backus, Kehoe, and Kydland (1994) and Brandner and Neusser (1992).

summary measures of the distance between the model and the dynamic properties of the data.

Perhaps more importantly, focussing on frequency rather than time domain diagnostics can potentially provide important new insight into the dynamic behavior of economic models. Some authors *define* the business cycle in the frequency domain, either as the existence of a peak in the univariate spectral density at business cycle frequencies, or as the existence of high pairwise coherencies at business cycle frequencies (Sargent,1987 pp. 282). From this perspective, it seems natural to employ frequency domain statistics to evaluate the cyclical behavior of these models. The diagnostic tests developed here allow for the detailed analysis of the frequency domain properties of economic time series over any range of frequencies.

Finally, as the statistics are based on spectral distribution function estimates, rather than density function estimates, there is no need to choose a spectral window to ensure that the frequency domain estimates are consistent. Within the context of a diagnostic test, this is particularly useful because smoothing spectral estimates is often viewed as an "art" rather than a science, different spectral windows are often suggested for different time series (see Brockwell and Davis, (1991, pp. 117) or Priestley, (1981, Ch. 9)), and concentrating on the distribution function removes a degree of arbitrariness from the analysis. In addition, there is some evidence suggesting that the distribution function estimates have more attractive finite sample properties.[4]

This chapter will proceed as follows. Section 1 describes the theoretical motivation for the statistical analysis of economic time series within the frequency

[4]See Durlauf (1991) and the discussion in Section 2.3 of this chapter

domain. Section 2 develops the test statistics and discusses their relationship to the standard procedures for model evaluation within the calibration and simulation literature. Finally, section 3 discusses some of the additional statistical issues involved in the analysis of economic time series, how they are usually treated in the calibration and simulation literature, and potential impacts on the diagnostic tests suggested here.

## 2.1 The Frequency Domain Analysis of Economic Time Series

For a univariate time series $X_t$, the spectral density function is defined as the fourier transform of the autocovariance function, *i.e.*

$$f(\omega) = \frac{1}{\pi} \sum_{k=-\infty}^{k=\infty} \gamma(k) \cos(\omega k) \tag{2.1.1}$$

$$= \frac{1}{\pi} \left[ \gamma(0) + 2 \sum_{k=1}^{\infty} \gamma(k) \cos(\omega k) \right]. \tag{2.1.2}$$

where $\gamma(k) = Cov(X_t, X_{t+k})$.

The spectral density function is useful because it summarizes all of the information in the autocovariance function for $X_t$. The spectral density also has a useful physical interpretation, in that $f(\omega)d(\omega)$ represents the contribution to the variance of $X_t$ from components with frequencies in the range $(\omega, \omega + d\omega)$. As a result, the total area underneath the spectral density function represents the variance of the series, and the occurence of a peak in the spectral density of a series at a particular frequency will indicate an important contribution to the variance of the series at that frequency. Nerlove (1964) and Granger (1967) have argued that power spectra from most economic time series have a "typical spectral shape", with considerable power (peaks in the spectral density function ) at low frequencies.

To illustrate, consider the normalized spectral density function $f^*(\omega)$ obtained by dividing $f(\omega)$ by $\sigma_x^2$ where $\sigma_x^2$ is the variance of the series $X_t$. One can easily observe that $f^*(\omega)$ is the fourier transform of the **autocorrelation** function. If $X_t$ is a weak stationary process, the *Cramer Representation Theorem* ensures that the time series can be expressed as

$$X_t = \int_0^{\pi} \cos(\omega t)du(\omega) + \int_0^{\pi} \sin(\omega t)dv(\omega). \tag{2.1.3}$$

where $u(\omega)$ and $v(\omega)$ are uncorrelated continuous processes with orthogonal increments that are defined for all $\omega$ in the range $(0,\pi)$. In other words, the time series $X_t$ can be represented as the sum of a continuum of orthogonal sinusoidal terms. Orthogonality means that one can speak of the contribution of a particular frequency $\bar{\omega} \in [0,\pi]$ to the total variance of the time series

$$\cos^2(\omega)Edu(\omega)^2 + \sin^2(\omega)Edv(\omega)^2 = Edu(\omega)^2. \tag{2.1.4}$$

Orthogonality also implies that the normalized spectral density function gives a precise measure of how different intervals of cycles contribute to the volatility of the series. Consequently, spectral analysis is particularly suited to the study of cyclical characteristics of economic time series, as it determines how the variance of any economic time series is affected by cyclical movements and what the period of this cycle is.

For a weakly stationary multivariate time series $\{Z_t\}=\{X_t, Y_t\}$, the cross spectrum between $Y_t$ and $X_t$ is the fourier transform of the cross covariance function. *i.e.*,

$$f_{yx}(\omega) = \frac{1}{\pi} \sum_{k=-\infty}^{\infty} \gamma_{yx}(k) \cos(\omega k). \tag{2.1.5}$$

where $\gamma_{yx}(k)$ =Cov$(Y_t, X_{t+k})$.

The cross-spectral density is complex valued because $\gamma_{yx}(k)$ is not an even function. The real part of $f_{yx}$ is called the co-spectrum and is given by

$$c_{yx}(\omega) = \frac{1}{\pi} \sum_{k=-\infty}^{k=\infty} [\gamma_{yx}(k) + \gamma_{xy}(k)] cos(\omega k) \tag{2.1.6}$$

$$= \frac{1}{\pi}[\gamma_{yx}(0) + \sum_{k=1}^{\infty}[\gamma_{yx}(k) + \gamma_{xy}(k)] cos(\omega k)].$$

The complex part of $f_{yx}$ is called the quadrature spectrum and is given by

$$q_{yx}(\omega) = \frac{1}{\pi} \sum_{k=-\infty}^{k=\infty} [\gamma_{xy}(k) - \gamma_{yx}(k)] sin(\omega k) \tag{2.1.7}$$

$$= \frac{1}{\pi}[\sum_{k=1}^{\infty}[\gamma_{xy}(k) - \gamma_{yx}(k)] sin(\omega k)].$$

Note that $f_{yx}(\omega)$=$c_{yx}(\omega)$-$iq_{yx}(\omega)$

A multivariate version of the *Cramer Representation Theorem* for $Z_t$ implies

$$X_t = \int_0^{\pi} \cos(\omega t) du_x(\omega) + \int_0^{\pi} \sin(\omega t) dv_x(\omega) \tag{2.1.8}$$

$$Y_t = \int_0^{\pi} \cos(\omega t) du_y(\omega) + \int_0^{\pi} \sin(\omega t) dv_y(\omega). \tag{2.1.9}$$

This provides a useful decomposition of the covariance of two stochastic processes by specific frequencies.

To provide some intuition for the usefulness of the co- and quadrature spectra, it is useful to consider the relationship between the definitions (2.1.6) and (2.1.7), the *Cramer Representation* (2.1.8)-(2.1.9), and more traditional frequency domain quantities such as coherency that have been employed in the definition and analysis of business cycles. From the *Cramer Representation*, it is straightforward to show that

$$E du_x(\omega) du_y(\omega) = E dv_x(\omega) dv_y(\omega) = c_{yx} d\omega \tag{2.1.10}$$

and

$$E du_x(\omega) dv_y(\omega) = -E dv_x(\omega) du_y(\omega) = q_{yx} d\omega \tag{2.1.11}$$

As a result, $c_{yx}(\omega)$ represents the covariance between components that are "*in-phase*" in the two processes, while $q_{xy}(\omega)$ represents the covariance between components that are "*out of phase*" by an angle of $\frac{\pi}{2}$. A statistic that is often employed in business cycle analysis is the coherency function $\gamma_{yx}(\omega)$ which is defined as

$$\gamma_{yx}(\omega) = \frac{|c_{yx}(\omega)|^2 + |q_{yx}(\omega)|^2}{f_y(\omega) f_x(\omega))}, \tag{2.1.12}$$

and is often interpreted as the squared correlation between the time series at the specified frequency. Sargent (1987) argues that although the definition of "business cycle" varies substantially across authors, a definition that is implicit in much applied research requires high pairwise coherencies at business cycle frequencies. Rather than focussing on coherencies, the diagnostic tests presented below examine each element of the spectral matrix separately. As will be seen shortly, these tests are simpler and provide more detailed diagnostic information.

## 2.2 Diagnostic Test Statistics

### 2.2.1 Univariate Test Statistics

A natural estimate of the spectral density would appear to be the periodogram

$$I_I(\omega) = \frac{1}{\pi}\left[\hat{\gamma}(0) + 2\sum_{k=1}^{T-1}\hat{\gamma}(k)\cos(\omega k)\right]. \tag{2.2.13}$$

Unfortunately, it is well known that the periodogram is not a consistent estimate of the spectral density function. Consistent estimates can be obtained using appropriately weighted averages of neighbouring point spectral density estimates.[5] Alternatively, we may consider the periodogram based estimate of the spectral distribution function which is a consistent estimate of the true spectral distribution function due to the effects of averaging under the integral operator.

[5] Priestley Ch 8 (1981) provides a comprehensive summary of this literature.

More specifically, consider the normalized integrated periodogram estimate of the normalized spectral distribution function, *i.e.*

$$\frac{F_T(\lambda)}{F_1(\lambda)} = \int_0^\lambda \frac{I_I(\omega)}{\hat{\gamma}(0)} = \frac{1}{\pi}\left[1 + 2\sum_{k=1}^{T-1} \hat{\rho}(k)\frac{sin(k\pi t)}{k}\right]. \tag{2.2.14}$$

Normalization by the sample variance implies that diagnostic tests will be insenstive to differences in the variances of simulated and historical time series. This point will be discussed further in Section 2.3. Univariate spectral shape tests measure the deviations between the periodogram estimate of the normalized spectral distribution function, and the normalized spectral distribution function under the null hypothesis F. In particular, Dahlhaus (1985), building on earlier work by Durbin (1973) and Grenander and Rosenblatt (1957), has shown that for $\omega=\pi t$, $t\in[0,1]$,

$$U_T(t) = (\frac{T}{2\pi})^{\frac{1}{2}}\left[\frac{F_T(t)}{F_I(1)} - \frac{F(t)}{F(1)}\right] \tag{2.2.15}$$

converges weakly (in D[0,1]) to a zero-mean Gaussian process U(t) that is a transformation of tied-down Brownian motion. Morever, the limiting random function is a standard Brownian bridge when the null hypothesis is a flat spectrum, *i.e.* when $X_t$ is white noise.

This result provides two possible avenues for developing diagnostic tests for univariate spectra. Global spectral shape tests map this random function into a scalar random variable with a known asymptotic distribution, and provide useful summary statistics for spectral deviations across all frequencies. For example, when the null hypothesis is white noise, U(t) is a Brownian bridge, and the continuous mapping theorem implies that:

$$CVM_T = \int_0^T U_T^2(t)dt \Rightarrow_w CVM. \tag{2.2.16}$$

$$KS_T = sup_{[0\leq s,t\leq 1]}|U_T(t)| \Rightarrow_w KS.$$

$$K_T = sup_{[0\leq s,t\leq 1]}|(U_T(t) - U_T(s))| \Rightarrow_w K.$$

where CVM, KS, and K, are the Cramer von-Mises, Kolmogorov-Smirnov, and Kuiper statistics respectively. Unfortunately, for the business cycle models under consideration here, it is obviously the case that the white noise null hypothesis is inappropriate, and asymptotic distributions for test statistics in these situations are unknown.

Perhaps more problematic is the observation that for the majority of non-linear rational expectations models and their approximate solutions, it is generally not possible conveniently to represent spectral and cross-spectral distribution functions for the model directly as functions of the calibrated parameters, *i.e.* there is not representation for the null hypothesis. As it is possible to simulate the model, a test could theoretically be based on the difference between the periodogram estimate of the spectral distribution function from the model, and the spectral distribution function estimated from the historical time series. In this instance, the test would resemble a two-sample shape test for comparing probability distribution functions, Durbin (1973). However, it is still the case that the asymptotic distribution of the test statistic would be unknown. As a result, diagnostic statistics based on general spectral shape tests do not appear to be viable alternatives.

Rather than a global distance measure, a researcher may be more interested in a specific range of frequencies. In business cycle analysis for example, researchers often concentrate on 2-4 year cycles. In this instance, for fixed s > t,

$$U_T(s) - U_T(t) \Rightarrow_w N(0, V) \qquad (2.2.17)$$

for some constant V that will typically depending on F(λ). In the white noise example, V depends only on the length of the interval and not on F, while in the general case examined here, the variability of the region of spectral deviations under examination depends on F.

As a basis for frequency interval tests, the model can be simulated and the sampling variability in the simulated spectral distribution can be used to evaluate the distance between sample and theoretical spectral distribution functions. Let $F^I(\omega)$ represent the spectral distribution function of the historical time series, and let $F^M(\omega)$ be the spectral distribution function for the model. Consider basing a test of the null hypothesis that these functions are equal on the sample periodogram estimate of the normalized spectral distribution function from the historical time series, $U_T(t)$, and an estimate constructed from a simulated sample of length T from the model, $U_T^M(t)$. For R separate simulations of length T, a test procedure that evaluates the difference and that is implicit in many standard business cycle studies would evaluate the average frequency deviation

$$\overline{D}(s,t) = \frac{1}{R}\sum_{i=1}^{R}(U_T(s) - U_T(t)) - (U_{T,i}^M(s) - U_{T,i}^M(t)) \tag{2.2.18}$$

using the standard asymptotic distribution

$$\overline{D}(s,t)/S.E.(\overline{D}(s,t)) \sim N(0,1). \tag{2.2.19}$$

An alternative procedure that has been suggested by Gregory and Smith (1992) is to focus on p-values rather than a t-test. To illustrate, for any two-sample frequency interval test, a diagnostic statistic $D_{T_R}$ constructed from a historical sample of length T, and R simulated samples of length T, the probability (under the null hypothesis that $F^1 = F^M$) that the model with spectral density $F^M$ generates a spectral distribution that is different from the sample spectral distribution function $F^1$ is the relative frequency of observing an absolute difference in excess of the critical value $x_\alpha$, *i.e.*

$$\frac{1}{R}\sum_{i=1}^{R} I(D_{T_i} > x_\alpha) \tag{2.2.20}$$

where the indicator I(.) equals 1 when the argument is positive and zero otherwise.

### 2.2.2 Multivariate Test Statistics

The multivariate analogs of these univariate test statistics are more interesting for the analysis of business cycles, as they provide a description of comovements between various time series within the frequency domain. As in the univariate case, we will consider the periodogram estimate of the normalized cross-spectral distribution function.

For the cospectrum portion we have,

$$C_T(t) = \frac{1}{\pi}\sum_{k=1}^{T-1}[\hat{\rho}_{yx}(k) + \hat{\rho}_{xy}(k)]\frac{sin(\pi t)}{k}], \qquad (2.2.21)$$

and where we again normalized to remove the contemporaneous covariance term and focus on dynamics. For the quadrature portion we have

$$Q_T(t) = -\frac{1}{\pi}\sum_{k=1}^{T-1}[\hat{\rho}_{yx}(k) - \hat{\rho}_{xy}(k)]\frac{cos(\pi t)}{k}]. \qquad (2.2.22)$$

Frequency interval tests that detect cross-spectral deviations from the null hypothesis can once again be based on differences between the cross-spectral distribution functions implied by the model, and those estimated from the historical time series. For the cospectrum we have

$$\overline{D}_c(s,t) = \frac{1}{R}\sum_{i=1}^{R}(C_T(s) - C_T(t)) - (C_{T,i}^M(s) - C_{T,i}^M(t)). \qquad (2.2.23)$$

For the quadrature spectrum we have

$$\overline{D}_q(s,t) = \frac{1}{R}\sum_{i=1}^{R}(Q_T(s) - Q_T(t)) - (Q_{T,i}^M(s) - Q_{T,i}^M(t)). \qquad (2.2.24)$$

## 2.3 Some Additional Issues

It is important to discuss the practical implementation of these tests both from the perspective of classical frequency domain econometric methods, and within the context of the calibration and simulation literature.

One important characteristic of the test statistics presented here is derived directly from our focus on spectral distribution rather than density estimates. As mentioned previously, periodogram estimates of the spectral and cross-spectral density functions do not provide consistent estimates of the true density functions. For frequency domain estimation that focuses on these estimates, it is the case that neighbouring periodogram estimates must be "averaged" to provide consistent estimates of the density function. The selection of the number of periodogram ordinates and their weights in the average, called "windowing" in general, necessarily introduces a degree of arbitrariness in the analysis. While there are a variety of procedures for determining optimal window widths and weights with reference to the particular parametric specification of the time series, the true parametric time domain representation is never known. As a result, the nonparametric estimation of spectral density functions is an "art", and different degrees of smoothing can produce significantly different results. Spectral and cross-spectral density functions for several international macroeconomic time series are graphed in Chapter 3. The graphs indicate that spectra and cross-spectra for many time series can appear extremely erratic with conventionally chosen windows, while the spectral and cross-spectral distribution functions are much less variable.[6]

A useful perspective on the relationship between spectral density and distribution function estimation can be gained from a comparison with the estimation of probability density and distribution functions. The empirical distribution function is

[6]Durlauf (1993) makes this point with respect to U.S. real GNP.

a consistent estimate of the true distribution function under very general conditions, yet the empirical probability density function (*i.e.* the density with mass $\frac{1}{N}$ centered at each data point) is not. To obtain a consistent density estimate, neighbouring data points must be averaged by some method; the histogram, nearest-neighbour, kernel density estimates, and their variants can all be viewed in this fashion, Ullah (1988). Conditions on window-widths and kernels for consistent probability density estimates typically imply that the rate of convergence for the estimates is generally less than $N^{\frac{1}{2}}$, while the distribution function estimates have an $N^{\frac{1}{2}}$ rate of convergence. As a result, one would expect the asymptotic distribution for the density function estimates to be less reliable approximations to the true finite sample properties of the estimates than the corresponding distribution function estimates. From the perspective of spectral distribution function estimation, an analogous argument implies that the asymptotic distribution theory for the spectral distribution function estimates should be more reliable than the corresponding theory for density estimates. Note that this does not imply that the periodogram-based spectral distribution function estimates have more attractive finite sample properties, when compared with distribution function estimates based on windowed density estimates. In fact, while the first order asymptotic properties of distribution function estimates based on windowed density estimates will be identical to the periodogram-based estimates (as the effects of standard windows will wash out under the integral operator), higher order asymptotic properties and finite sample performance will differ.

A second characteristic of our test statistics is the absence of information concerning contemporaneous variances and covariances. Our focus on the dynamic aspects of the economic theory under consideration is important, because it is these aspects that have traditionally been ignored in calibration and simulation studies. Moreover, the contemporaneous characteristics of economic time series are often employed as the basis for baseline calibrations, which suggests that including these elements would make the tests less powerful as diagnostics. However, including these elements within the current framework is conceptually straightforward (see Priestley, (1981), Ch. 8). From the perspective of the empirical work examined in Chapter 3, recent research examining the relationship between the terms of trade and the trade balance has focused specifically on the dynamic predictions of the theory and the cross-correlation function; the diagnostics presented here are the appropriate frequency domain statistics for this situation.

A third characteristic of the test statistics proposed here is derived from their application within current calibration and simulation practises. It is now well known that many economic time series are nonstationary, although they can be rendered stationary through differencing or the application of other filters. Following the seminal works of Hodrick and Prescott (1980) and Kydland and Prescott (1982), the Hodrick and Prescott (henceforth HP) filter has been the method of choice for most business cycle studies within the calibration and simulation framework. Standard empirical practise is to apply the filter to the historical time series as well as the output from the simulated model, and then to compare the properties of the model. Not surprisingly, there is a growing theoretical and empirical literature that questions the blind aplication of the HP filter in these problems, arguing that the filter implies a particular definition of business cycle frequency that essentially excludes growth cycles, and that conclusions are often not robust to the application of alternative filters with similar *a priori* justification. For example, King and Rebelo (1993) provide a striking illustration of dramatic effects of the filter on persistence, comovements and variability, for U.S. consumption, investment, and output time series within a particular calibration study.

For several reasons, we view the filtering issue as somewhat peripheral to a discussion of the properties of the test statistics. First, under the null hypothesis that the model and the historical time series have the same data generating process, the filter should affect both historical and simulated data identically, and should leave the results of the tests unaffected. Of course, this may not necessarily be true under alternative hypotheses. Additionally, it is certainly the case that the HP filter substantially modifies the dynamic properties of time series in certain instances, and this has certainly shaped the specific empirical regularities that are characterized as interesting by business cycle theorists. As a result, it is important that both the classification of empirical regularities and the calibration and simulation exercise be investigated for alternative filters. For the empirical work presented in Chapter 3 and Chapter 4, we consider the HP and first-difference filters.

# Chapter 3

# Fitting the J-curve

## 3.1 Introduction

Standard international macroeconomics textbooks typically contain a section examining the dynamic behavior of the terms of trade, aggregate output, and the trade balance. Typical renditions of the empirical regularities describe the relationship between the terms of trade and the trade balance as a "J-curve", where the trade balance initially worsens in response to a real depreciation, although the long-run response is positive. In general, the initial perverse reaction is attributed to an increase in spending on imports denominated in domestic currency, relative to the increase in exports.

The conventional view of the joint relationships between output, terms of trade, and trade balance dynamics has recently received a large amount of attention from both theoretical and empirical researchers in the United States. This increased interest can be attributed to the extremely large fluctuations in the real exchange rate that have occurred over the last decade, and the failure of existing theoretical models to provide adequate empirical explanations of trade dynamics over the same period. Although the consensus view appears to be that the J-curve exists and that the initial perverse effect lasts about one year (Dornbusch and Krugman (1976), Krugman and Baldwin (1987)), recent empirical work by Rose and Yellen (1989) fails to find statistically reliable evidence supporting the J-curve using U.S. data. In addition, Meade (1987) argues that the J-curve relationship is quantitatively unimportant for explaining the behavior of the trade balance. However, it should be emphasized that the conventional J-curve perspective is inherently partial equilibrium, as it is essentially concerned with the short- and long-run elasticity of the real trade balance with respect to changes in the real exchange rate. From a general equilibrium perspective, foreign and domestic output, real exchange rates, and trade balances are all endogenous; the empirical evidence provided by Rose and Yellen (1989) and Meade (1987) is based on the estimation of short and long-run trade balance elasticities from a partial equilibrium perspective, and this does not provide evidence concerning the true dynamic relationships.

Backus, Kehoe, and Kydland (1991) develop a general equilibrium model for the evaluation of the dynamic relationships between output, the terms of trade, and the trade balance that is based on a two-country extension of the Kydland and Prescott closed-economy real business-cycle model. Empirically, they find striking support for two empirical regularities concerning the dynamic behavior of output, the terms of trade, and the trade balance that combine to produce a standard J curve. First, across most industrialized economies, the trade balance is negatively correlated with future movements in the terms-of-trade, and positively correlated with past movements. As a result, when the cross-correlation function for the terms-of-trade and output is plotted, the shape resembles a horizontal "S-Curve", a term the authors employ to describe the relationship. Second, the trade balance is countercyclical in most countries. Empirically, the authors also point out that the U.S. historical experience may also be atypical, in that the relationships appear to be supported by the majority of industrialized countries, although the trade balance is procyclical in the U.S. data they examine and the "S-curve" only appears in the pre-1972 sample. Overall, their evidence provides strong support for a general equilibrium perspective on the dynamic relationships that underlie the J-curve.

This chapter re-examines the Backus, Kehoe, and Kydland (1994) (henceforth BKK (1994)) model using the frequency domain diagnostic statistics developed in Chapter 2. The frequency domain approach is advantageous for several reasons. First, the authors emphasize the model's ability to replicate the "S-curve": as the null hypothesis specifically concerns the shape of the cross-correlation function between net exports and the terms-of-trade, the cross-spectral frequency domain diagnostics represent convenient summary measures of the dynamic properties of the model and the data, and the distance between them. In addition, the frequency domain perspective can potentially provide new insight into the dynamic relationships between output, the trade balance, and the terms of trade. While we also investigate the time domain properties of the data, we attempt to document the frequency domain regularities in the data and provide a frequency domain interpretation of the J-curve effect. It is also important to emphasize that the BKK (1994) model is a dynamic non-linear rational expectations model that is ideally suited to analysis by calibration and simulation techniques. As a result, the results reported here are the frequency domain analogs of the time domain evidence reported by BKK.

The remainder of the chapter is organized as follows: Section 2 presents a preliminary data analysis describing the time and frequency domain properties of the historical time series. Section 3 develops the theoretical model. Section 4 discusses the selection of the parameters and calculation of steady state values for the endogenous variables in the calibration portion of the empirical analysis. Section 5 outlines the method for computing the equilibrium time paths for the endogenous variables. Section 6 replicates the time domain analysis of BKK and presents results from the diagnostic tests developed in Chapter 2 for a representative set of simulations. Section 7 concludes and suggests avenues for future research.

## 3.2 Preliminary Data Analysis

The data used in this chapter are postwar quarterly trade statistics for eleven countries taken from the Organization for Economic Co-operation and Development's *Quarterly National Accounts*. The countries and samples under investigation are summarized in Table 3.2.1 and were chosen to maintain comparability with BKK (1994). This chapter will focus on real output (y), the trade balance (nx), and the terms of trade (p). Real output is defined as either G.N.P. or G.D.P. (depending on the country) in base year prices. The trade balance is represented by the ratio of net exports (exports minus imports) to output, with both measured in *current* prices.

The terms of trade (p), is defined as the ratio of the price of imports to the price of exports, where both prices have been calculated using implicit price deflators from the national income and product accounts. Note that for real G.D.P. and the terms of trade the analysis is concerned with the logarithm of each series. All series have been filtered with the Hodrick and Prescott (1980) filter. While use of the HP filter is somewhat controversial (see the discussion in Ch 2, Section 3), we will focus on the results from HP filtered data to maintain comparability with BKK. Most of the *general* results presented here are robust to the choice between HP and first-difference filters, although results in specific cases sometimes change substantially.

### 3.2.1 Time Domain Properties of the Data

Tables 3.2.2 and 3.2.3 report some time domain summary statistics for the sample; these are identical to those presented in BKK (1991), and will only be discussed briefly. Regarding variability, these tables illustrate that the standard deviation of terms-of-trade typically exceeds the standard deviation of output, and that there is a fair bit of heterogeneity across countries. Regarding covariability, net exports are countercyclical and the contemporaneous correlation between net exports and the terms of trade is generally negative, although the latter varies much more across countries and is significantly positive for the United States. Dynamically, the autocorrelation coefficients presented in Table 3.2.2 and the autocorrelation functions for the trade balance presented in Table 3.2.3 indicate that both the trade balance and the terms-of-trade are highly persistent.

Regarding the "S-curve", Figure 3.2.1 presents the cross-correlation function estimates for all countries, for leads and lags of at most two years. In most cases, the graphs depict an asymmetric pattern resembling a horizontal S, with negative cross-correlations for leads of the terms of trade relative to the trade balance, and positive cross-correlations at lags. BKK (1994) demonstrate that the asymmetric shape of the cross-correlation function is independent of the filter being used, and also independent of the sample period for most countries.

### 3.2.2 Frequency Domain Properties of the Data

This section presents some descriptive evidence concerning the univariate and multivariate frequency domain properties of the data. Estimates of the univariate spectral density functions for output, the terms of trade, and the trade balance are presented in Figures 3.2.2-3.2.4. The univariate spectral density estimates are constructed using a tent shaped window of the Bartlett type and a window width which considers $M = 11$ autocorrelations. For interpretation, the graphs are normalized as sixths of $\pi$, *i.e.* a 1 on the horizontal axis corresponds to $\frac{\pi}{6}$, 2 corresponds to $\frac{2\pi}{6}$, *etc...* The frequency shows the number of cycles per unit time, *i.e.* $f = \frac{\omega}{2\pi}$ where $\omega$ is the angular frequency. As a result, the wavelength is clearly $\frac{1}{f}$ and an angular frequency of $\omega = \frac{\pi}{6}$ corresponds to a cycle which occurs every twelve observations, or three years with quarterly data. It is interesting that for all three variables, univariate spectral power is concentrated primarily in the low frequencies (*i.e.* $0 - \frac{\pi}{6}$ or $\frac{\pi}{6} - \frac{2\pi}{6}$), even after filtering, and the concentration in low frequencies is greatest for the terms-of-trade. Of course, substantial power at low frequencies indicates persistence and this is a well-known characteristic of trade variables, real exchange rates, and international aggregate output. Overall, the graphs are representative of the "typical spectral shape" of most economic time series.

Figures 3.2.11-3.2.13 contain analogous graphs of the cumulative normalized periodogram which will be the basis for the univariate diagnostic tests. Note that the conclusions regarding concentrations of power at low frequencies are similar, although the distribution functions appear much less erratic. As argued in Chapter 2, this is indicative of superior finite sample performance for the distribution function estimates, relative to the density function estimates, and is one of the primary motivating factors for the choice of diagnostic procedures.

For the multivariate analysis, attention was concentrated on cross-spectra for the terms-of-trade and the trade balance, and for real output and the trade balance: these should provide evidence concerning the S-curve and the countercyclical behavior of trade balance, respectively. Figures 3.2.5 , 3.2.7 and 3.2.9 graph the cospectrum for each pair for each country, while the quadrature spectrum for each pair is graphed in Figures 3.2.6, 3.2.8 and 3.2.10.

When interpreting the graphs for the co- and quadrature spectra it is important to realize that they may take negative values, and it is the absolute magnitude of the spectra that determines the presence or absence of power. Figure 3.2.5 contains the estimated cospectrum for the trade balance and terms of trade for each of the countries. What is important to notice for all countries is the predominance of absolute power in the low frequencies, although it is clear that the estimated densities are quite erratic. In addition, there appears to be some additional spectral power at higher frequencies, particularly around $\frac{2\pi}{3}$, corresponding to cycles of nine to twelve months. Figure 3.2.6 contains similar graphs for the quadrature spectra. Once again, the graphs exhibit substantial power at low frequencies, with additional spikes of spectral power occurring at frequencies in the neighborhood of $\frac{2\pi}{3}$.

Across countries, it is clear that the cross-spectral characteristics for the relationship between $nx$ and $p$ appear much more variable than the simple time domain S-curve reported by BKK. As the cross-spectral properties of the model will be fixed for a given set of parameters, it is obvious that no particular simulation will be able to replicate this cross-sectional variability; in this sense, the cross spectral frequency domain measures appear to be much finer diagnostic tools than the cross correlation function. From the perspective of evaluating the model, the simulations will provide evidence concerning the cross-spectral properties of the model, and which countries are similar.

Figure 3.2.7 presents estimated cospectra for $y$ and $nx$ and reveals a uniform pattern of substantial negative cospectral power at low frequencies for all countries. This is the frequency domain representation of the countercyclical behavior of the trade balance, and indicates that it is a low frequency phenomenon. The quadrature spectra presented in Figure 3.2.8 are somewhat more variable, although the power is also concentrated in low frequencies. Finally Figures 3.2.9 and 3.2.10 graph estimated co- and quadrature spectra for $y$ and $p$.

## 3.3 A Theoretical World Economy

The model that we use is a two country stochastic growth model, in which each country is inhabited by a large number of identical consumers. This model was initially developed by BKK (1991) as a streamlined version of their international real business cycle model (Backus, Kehoe and Kydland, (1992)), which in turn is a two-country extension of Kydland and Prescott's (1982) closed economy model. In this model, each country produces its own good with its own technology, and labor, which is internationally immobile. Economic fluctuations arise in response to stochastic movements in technological change and government spending.

A representative consumer for each country maximizes a utility function of the form

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_{i,t}, \ell_{i,t}), \quad for \quad i = h, f \tag{3.3.1}$$

where $U$ has the form

$$U(c, \ell) = \frac{[c^{\mu} \ell^{1-\mu}]^{\gamma}}{\gamma}. \tag{3.3.2}$$

$c_{i,t}$ is consumption, $\ell_{i,t}$ is leisure, and it is assumed that $0 < \mu < 1$, and $\gamma < 1$.

Each country specializes in the production of a single good ($a$ for country 1 and $b$ for country 2). The production functions use constant returns to scale technologies and are the same for each country. The inputs of production are capital $k$ and labor $n$. The resource constraints, for each country are

$$a_{1t} + a_{2t} = y_{1t} = z_{1t}F(k_{1t}, n_{1t}), \tag{3.3.3}$$

$$b_{1t} + b_{2t} = y_{2t} = z_{2t}F(k_{2t}, n_{2t}), \tag{3.3.4}$$

with $F(k, n) = k^{\theta}n^{1-\theta}$. Where $y_{1t}$ represents output for country 1 measured in units of the domestic good, and $a_{1t}$ and $b_{1t}$ denote uses of the two goods in country 1. Note that $a_{2t}$ will be exports from country 1 to country 2 (imports for country 2). The vector $z_t = (z_{1t}, z_{2t})$ represents a stochastic shock to productivity.

Consumption (c), investment (x) and government spending (g) in each country are composites of the foreign and domestic goods, *i.e.*

$$c_{1t} + x_{1t} + g_{1t} = G(a_{1t}, b_{1t}), \tag{3.3.5}$$

$$c_{2t} + x_{2t} + g_{2t} = G(a_{2t}, b_{2t}), \tag{3.3.6}$$

where $G(a, b) = [\omega_1 a^{-\rho} + \omega_2 b^{-\rho}]^{\frac{-1}{\rho}}$ is homogeneous of degree one and $\rho \geq -1$. The elasticity of substitution between foreign and domestic goods is $\sigma = \frac{1}{1+\rho}$. This device for aggregating domestic and foreign goods was suggested by Armington (1969). The treatment of domestic and foreign goods as different is a standard feature of computable static general equilibrium trade models, and has been employed by Deardoff and Stern (1990) and Whalley (1985), among others. The weights $\omega_i$ in the aggregator function are equal across countries and denote the relative domestic and foreign content of aggregate domestic spending.

Capital formation follows the time-to-build structure of Kydland and Prescott (1982), *i.e.*

$$k_{i,t+1} = (1-\delta)k_{i,t} + s_{i,t-J+1}, \tag{3.3.7}$$

where $\delta$ is the rate of depreciation and $s_{i,t-J+1}$ is the number of investment projects in country $i$ at date $t$ that are $j$ periods from completion. At period $t$, total expenditure on gross capital formation is

$$x_{i,t} = \sum_{j=1}^{J} \phi_j s_{i,t-j}, \tag{3.3.8}$$

where $\phi_j$, $j = 1, \ldots, J$, is the fraction of total value added to an investment project in the $j^{th}$ period before completion. In order to emphasize trade dynamics, J will initially be set to 1, although time-to-build is considered as one of the simulation scenarios later on in this chapter. With J=1, (3.3.7) and (3.3.8) imply that

$$k_{i,t+1} = (1-\delta)k_{i,t} + s_{i,1t}. \tag{3.3.9}$$

From this it is obvious that an investment project starting at period $t$ increases the capital stock at period $t+1$, *i.e.* there are no costs of adjustment in this economy.

The underlying shocks in the economy are assumed to be independent bivariate autoregressions. The technology shocks follow

$$z_{t+1} = Az_t + \epsilon^z_{t+1}, \tag{3.3.10}$$

where $\{\epsilon^z_t\} = \{(\epsilon^h_t, \epsilon^f_t)\}$ is a sequence of i.i.d., mean-zero, multivariate-normal innovations with nondiagonal covariance matrix $V_z$. The off-diagonal elements of $A$ and

$V_z$ determine the correlation between the technology shocks. In the same way, shocks to government spending are governed by

$$g_{t+1} = Bg_t + \epsilon^g_{t+1}, \tag{3.3.11}$$

where $g_t = (g_{1t}, g_{2t})$ and $\{\epsilon^g\}$ is a sequence of i.i.d., mean-zero, multivariate-normal innovations with *diagonal* covariance matrix $V_g$. Technology shocks $\{\epsilon^z_t\}$, and government spending shocks $\{\epsilon^g_t\}$ are assumed to be independent.

The trade variables of interest can be constructed for each country using the production resource constraint and the Armington aggregator relationship. Note that the Armington aggregator is homogeneous of degree one. As a result, in equilibrium we have

$$c_{1t} + x_{1t} + g_{1t} = q_{1t}a_{1t} + q_{2t}b_{1t}, \tag{3.3.12}$$

where $q_{1t}$ is the price of the domestic good and $q_{2t}$ is the price of the imported good. Note that both prices are measured in units of the composite good. Solving these equation for $a_{1t}$ and substituting into the resource constraint yields

$$y_{1t} = \frac{c_{1t} + x_{1t} + g_{1t}}{q_{1t}} + a_{2t} - p_t b_{1t}, \tag{3.3.13}$$

where $p_t = \frac{q_{2t}}{q_{1t}}$ is the terms-of-trade. Note that the terms-of-trade for country 1 will be equal to the equilibrium marginal rate of transformation between the two goods in country 1, *i.e.*

$$p_t = \frac{q_{2t}}{q_{1t}} = \frac{\frac{\partial G(a_{1t},b_{1t})}{\partial b_{1t}}}{\frac{\partial G(a_{1t},b_{1t})}{\partial a_{1t}}}. \quad (3.3.14)$$

Equation 3.3.13 represents output as the sum of absorption, $\frac{c_{1t}+x_{1t}+g_{1t}}{q_{1t}}$, and net exports, $a_{2t} - p_t b_{1t}$. To be consistent with the rest of the analysis, net exports will be defined as

$$nx_t = \frac{a_{2t} - p_t b_{1t}}{y_{1t}}. \quad (3.3.15)$$

## 3.4 Calibration

In order to calibrate the model properly, the long-run steady state values for the endogenous variables must be calculated as functions of the *deep* technology and preference parameters of the model. In the calibration portion of the empirical analysis, initial parameter values are chosen based on information from a wide variety of sources. The approach taken here follows Kydland and Prescott (1982).

To begin, note that the steady state real interest rate is $r = \frac{1-\beta}{\beta}$. By equating the marginal rate of return on capital to the rental rate, we have $f_k = (\sum_{j=1}^{J}(1+r)^{j-1}\sigma_j)(r+\delta)$. With J=1, $\sum_{j=1}^{J}(1+r)^{j-1}\sigma_j$=1, and $f_k = r+\delta$. From the production function and this equation, it is possible to determine the relationship for the steady state value of capital in terms of labor, *i.e*

$$k = \pi^{\frac{1}{1-\theta}} n. \quad (3.4.16)$$

where $\pi = \frac{z\theta}{q(\delta+r)}$.

Output in terms of labor is found to be $y = z\pi^{\frac{\theta}{1-\theta}}n$. In the steady state, net investment is zero, implying that $y = c + \delta k$. The steady state value of consumption can then be obtained from the steady state values of capital and output, *i.e.*

$$c = \pi^{\frac{1}{1-\theta}}(z\pi^{-1} - \delta)n. \tag{3.1.17}$$

The utility function is of the form $U(c_t, l_t) = \frac{(c_t^{\frac{1}{3}} l_t^{\frac{2}{3}})^{\gamma}}{\gamma}$, and the budget constraint is $n_t w_t = c_t$, or $(1 - l_t)w_t = c_t$, where $n_t$ is labor market activity, $w_t$ is the real wage, and $l_t$ is leisure. The individual faces the following problem:

$$\sum_{t=0}^{\infty} \beta^t \frac{c_t^{\frac{\gamma}{3}} l_t^{\frac{2\gamma}{3}}}{\gamma} + \mu(n_t w_t - c_t) \tag{3.1.18}$$

where $\mu$ is the Lagrange multiplier.

The first order conditions for this problem can be written as

$$\partial R/\partial c_t = \frac{1}{3}\beta^t c_t^{\frac{\gamma-3}{3}} l_t^{\frac{2\gamma}{3}} = \mu. \tag{3.1.19}$$

and

$$\partial R/\partial l_t = \frac{2}{3}\beta^t l_t^{\frac{2\gamma-3}{3}} c_t^{\frac{\gamma}{3}} = \mu w_t. \tag{3.4.20}$$

In the steady state, $c_t = c$, $l_t = l$ and $w_t = w$. Making these substitutions and eliminating $\mu$ from these equations yields the following relationship between the steady state value of the real wage and the steady state values for consumption and leisure (labor),

$$w = \frac{2c}{l}, \quad or \quad w = \frac{2c}{1-n}. \tag{3.4.21}$$

In equilibrium, the marginal product of labor is equal to the wage rate, *i.e.* $w = (1-\theta)zk^{\theta}n^{1-\theta}$. As a result, the steady state value of labor is

$$n = \frac{z(1-\theta)}{3z - 2\delta\pi - z\theta}. \tag{3.4.22}$$

and the steady state values of output, capital and consumption are therefore

$$k = \frac{\pi^{\frac{1}{1-\theta}} z(1-\theta)}{3z - 2\delta\pi - z\theta}. \tag{3.4.23}$$

$$y = \frac{z^2 \pi^{\frac{\theta}{1-\theta}}(1-\theta)}{3z - 2\delta\pi - z\theta}. \tag{3.1.24}$$

and

$$c = \pi^{\frac{1}{1-\theta}}(z\frac{1}{\pi} - \delta)\frac{z(1-\theta)}{3z - 2\delta\pi - z\theta}. \tag{3.4.25}$$

It is also obvious from the capital formation constraint, $k_{i,t+1} = (1-\delta)k_{it} + s_{it}$ for i=1,2, so that at the steady state value for new investment, $k_{t+1} = k_t = k$, and $s_t = s = \delta k$. As a result,

$$s = \frac{\delta\pi^{\frac{1}{1-\theta}} z(1-\theta)}{3z - 2\delta\pi - z\theta}. \tag{3.4.26}$$

Additional variables of interest in this application are $a_{1t}$, $b_{1t}$, $a_{2t}$ and $b_{2t}$. In equilibrium, it is the case that $a_1 + a_2 = y_1$ and $b_1 + b_2 = y_2$, where variables with no time subscript denote steady state values. It will initially be assumed that the world economy is symmetric, so that the import shares for both countries are identical. As a result, $\frac{a_2}{y_2} = \frac{b_1}{y_1}$. We will also normalize the steady state value of output to 1 in both economies, *i.e.* $y_1 = y_2 = 1$. As a result, $a_1 = b_2$ and $a_2 = b_1$. Using these relationships, it is possible to show that $p = \frac{w_2}{w_1}(\frac{a_1}{b_1})^{\frac{1}{\sigma}}$ where $w_1$ and $w_2$ the Armington aggregator weights. If the steady state value for the terms of trade is normalized to 1 (p=1), some manipulation yields the following for steady state values of for $\{a_1, a_2, b_1, b_2\}$, *i.e.*

$$a_1 = b_2 = \frac{w_1^\sigma}{w_1^\sigma + w_2^\sigma} \tag{3.4.27}$$

$$a_2 = b_1 = \frac{w_2^\sigma}{w_1^\sigma + w_2^\sigma}. \tag{3.4.28}$$

Numerical values for these endogenous variables are obviously determined by the weights in the Armington aggregator function. Following BKK (1991), one can solve for the two weights, given the chosen normalizations for p and y and assuming an import share of 0.15.[1] Under these assumptions, $w_1 = 0.76$ and $w_2 = 0.24$.

The steady state variables for the endogenous variables have now been expressed as functions of the technology and preference parameters in the theoretical model. Numerical values for these steady states can be calculated once we assign numerical values to these parameters. The initial benchmark simulations presented

[1] It should be noted that this value for the import share is slightly larger than the average value for the majority of countries in the sample.

here will be based on the technology and preference parameter values from the closed-economy model of Kydland and Prescott (1982). Briefly, the choice of a Cobb-Douglas production function replicates the property that labor's share of output is constant across most economies. In the model, the shares for capital and labor are $\theta$ and $1-\theta$; from postwar U.S. data, Kydland and Prescott estimate labor's share as 0.64, which implies that $\theta = 0.36$. They also assumed a value for the depreciation rate ($\delta$) of 0.025. Finally, the benchmark simulation will set the time-to-build parameter J at 1, although some additional simulations will be based on higher values of J.[2]

For preference parameters, Kydland and Prescott also employ a Cobb-Douglas specification between consumption and leisure; this replicates the property that the fraction of time a household spends on labor market activities appears to be constant across most countries in the postwar period. The share parameter $\mu$ is set equal to 0.34, which corresponds to a share of 30 percent of the endowment of non-sleeping time to labor market activities. Eichenbaum, Hansen and Singleton (1988), suggest that the curvature parameter can take values between -2 and 0.5. Following Backus et al. (1992) and (1991), it is assumed that $\gamma = -1$. The elasticity of substitution between foreign and domestic goods, $\sigma$ will initially be set to 1.5, although additional simulations will be based on other choices. Note that $\sigma = \frac{1}{1+\rho}$ and consequently, $\rho = -.33$ initially. Also, for the benchmark economy, we will assume that there is no government expenditure, *i.e.* $g_t = 0$. The only parameter we have not reported is the discount factor $\beta$, which is set equal to 0.99 and corresponds to a real interest rate, r, of 1 percent per quarter, a value which is close to the average rate of return

[2] BKK argue that time-to-build plays a small role in the international aspects of the model.

on capital over the past century (Prescott (1986)).

For the interaction between foreign and domestic technology shocks, we employ the same values as BKK (1991), *i.e.*

$$A = \begin{bmatrix} 0.906 & 0.088 \\ 0.088 & 0.906 \end{bmatrix}.$$

We also assume that Var $\epsilon_1^z$ = Var $\epsilon_1^z$ = $0.00852^2$ and Corr $(\epsilon_1, \epsilon_2) = 0.258$. These values are consistent with the properties of Solow residuals for the postwar U.S. economy.

## 3.5 Computation of Equilibrium Time Paths

There is a wide variety of techniques that can be employed to obtain an approximate solution to this nonlinear rational expectations model. We will employ a Taylor-series expansion of the objective function in the neighborhood of the steady state to obtain a linear quadratic approximation that can be solved using standard recursive methods for optimal regulator problems. This method has gained considerable popularity in recent years, due primarily to the availability of convenient numerical solution procedures that have been integrated with the Hansen and Sargent (1990) text on the subject. It is important to point out that both the approximation and the solution technique chosen here are *different* from the ones employed by BKK (1991), who based their results on slightly revised versions of the Kydland and Prescott (1982) approach. More specifically, the BKK (1991) approach adjusts the location of the approximation to account for approximation error, and while the adjustment is arbitrary, their approach will be more sensitive to the curvature of the objective function.

Following BKK (1991), we will approximate and then solve a social planner's problem that weighs equally the utility of consumers in the two countries. The corresponding objective function can be written as

$$\frac{1}{2}E[\sum_{t=0}^{\infty}\beta^t(U^h(c_t,l_t)+U^f(c_t,l_t))]. \tag{3.5.1}$$

subject to constraints 3.3.3 to 3.3.11. The choice of equal weights has been proposed by Negishi (1960) and Mantel (1971).

To simplify the numerical analysis, we solve equation (3.3.6) for $c_{2t}$ and substitute this value into equation 3.5.1, which represents the world economy. Next we solve the resource constraint equations (3.3.3) and (3.3.4) for $a_{2t}$ and $b_{2t}$ respectively and substitute these values into the objective function. In the resulting function we substitute the fixed investment constraint for each country (equation 3.3.8). After all these substitutions, the objective function can be written as

$$\sum_{t=0}^{\infty}\beta^t(U^h(a_{1t},b_{1t},g_{1t},s_{1t},n_{1t})+U^f(a_{1t},b_{1t},z_{1t},z_{2t},k_{1t},k_{2t},g_{2t},s_{2t},n_{1t},n_{2t})) \tag{3.5.2}$$

After the expansion around the steady state, the resulting objective function can be written as

$$\sum_{t=0}^{\infty}\beta^t(x_t'Qx_t+u_t'Ru_t+2u_t'Nx_t). \tag{3.5.3}$$

subject to the constraint equations

$$x_{t+1} = Ax_t + Bu_t + C\epsilon_{t+1}, \tag{3.5.4}$$

where

$$x_t = \begin{bmatrix} z_{1t} - z_1 \\ z_{2t} - z_2 \\ k_{1t} - k_1 \\ k_{2t} - k_2 \\ 1 \end{bmatrix},$$

$$u_t = \begin{bmatrix} a_{1t} - a_1 \\ b_{1t} - b_1 \\ n_{1t} - n_1 \\ n_{2t} - n_2 \\ s_{1t} - s_1 \\ s_{2t} - s_2 \end{bmatrix},$$

and where $x_t$ is a vector of state variables and $u_t$ is a vector of control variables. Variables with no time subscripts denote steady state values.[3] Sufficient conditions for the existence of a solution to the approximate problem without cross terms in the objective function are given in Sargent (1987). More general sufficient conditions that are applicable to the problem at hand are described below.

[3]The variables are defined as deviations from their mean values. This choice has no effect on the solution of the problem.

With the chosen parameter values, we have the following matrices:[4]

$$Q = \begin{bmatrix} 1.8537 & 0.5012 & 0.0507 & 0.0164 & -0.2839 \\ 0.5012 & 0.7399 & 0.0164 & 0.0143 & -0.2839 \\ 0.0507 & 0.0164 & 0.0026 & 0.0005 & -0.0093 \\ 0.0164 & 0.0143 & 0.0005 & 0.0014 & -0.0093 \\ -0.2839 & -0.2839 & -0.0093 & -0.0093 & 0.9024 \end{bmatrix};$$

$$R = \begin{bmatrix} 2.2578 & 0.8727 & -3.6826 & -0.6781 & -1.0061 & 1.0061 \\ 0.8727 & 2.2578 & -1.1782 & -1.1201 & -1.0061 & 1.0061 \\ -3.6826 & -1.1782 & 8.9873 & 1.4125 & 0.1105 & -2.1402 \\ -0.6781 & -1.1201 & 1.4125 & 3.5362 & 0 & -1.7297 \\ -1.0061 & -1.0061 & 0.1105 & 0 & 1.6515 & 0 \\ 1.0061 & 1.0061 & -2.1402 & -1.7297 & 0 & 1.6515 \end{bmatrix};$$

$$N = \begin{bmatrix} -1.7295 & -0.1677 & 3.0751 & 0.7268 & 0 & -1.0784 \\ -0.1677 & -0.6903 & 0.9918 & 0.5965 & 0 & -1.0784 \\ -0.0566 & -0.0153 & 0.1007 & 0.0238 & 0 & -0.0353 \\ -0.0153 & -0.0226 & 0.0326 & 0.0195 & 0 & -0.0353 \\ 0.0000 & 0.0000 & -0.1375 & -0.1375 & 0.1348 & 0.4348 \end{bmatrix};$$

[4]Please note that the elements are truncated to four decimal places for presentation. All actual calculations were performed in double precision.

$$A = \begin{bmatrix} .906 & .088 & 0 & 0 & 0 \\ .088 & .906 & 0 & 0 & 0 \\ 0 & 0 & 0 & .975 & 0 \\ 0 & 0 & 0 & .975 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix};$$

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Following Sargent (1987), the solution to the problem with no discount factor and no cross terms between states and control variables is of the form $u = -Fx$ where $F = (Q + B'PB)^{-1}B'A$ and $P$ is the solution to the algebraic Ricatti equation

$$P = Q + A'PA - A'PA - A'PB(Q + B'PB)^{-1}B'PA. \qquad (3.5.5)$$

As this problem has a more general form that involves cross-products and a discount factor, we follow the steps in Hansen and Sargent (1990) to transform it back to the simpler problem. This is also convenient computationally, as algorithms for iterating on the Ricatti equations are available for the simpler problem. For the cross product term, we replace $Q$ and $A$ with the following matrices:

$$\hat{Q} = Q - NR^{-1}N'; \tag{3.5.6}$$

$$\hat{A} = A - BR^{-1}N'. \tag{3.5.7}$$

To convert the discounted problem to an undiscounted one, define

$$\grave{B} = \beta^{\frac{1}{2}} B. \tag{3.5.8}$$

$$\grave{A} = \beta^{\frac{1}{2}} \hat{A} \tag{3.5.9}$$

After these transformations, the optimal policy function is the time-invariant linear rule $u_t = -FX_t$, where

$$F = (R + \beta B'PB)^{-1}(\beta B'PA + N'). \tag{3.5.10}$$

Alternatively, in terms of the transformed variables, we have

$$F = (R + \grave{B}'P\grave{B})^{-1}(\grave{B}'P\grave{A} + R^{-1}N') \tag{3.5.11}$$

In terms of the transformed problem, the solution to the matrix Ricatti equations is

$$P = \hat{Q} + \grave{A}'P\grave{A} - \grave{A}'P\grave{B} - \grave{A}'P\grave{B}(R + \grave{B}'P\grave{B})^{-1}\grave{B}'P\grave{A}. \tag{3.5.12}$$

Sufficient conditions for the more general problem to have a unique non-negative solution have been stated by Whittle (1982). We require that $R$ be symmetric and positive definite. In addition, the matrix $W$ should be non-negative definite, where

$$W = \begin{bmatrix} R & N \\ N' & Q \end{bmatrix}.$$

Solutions to the Ricatti equations were obtained using the following iterative technique.

1. Choose an initial symmetric Ricatti matrix $P^0 \leq 0$. Set $n = 0$.
2. Compute $P^{n+1}$ and $\hat{F}^n$ where

$$P^{n+1} = \hat{Q} + \hat{A}'P^n\hat{A} - \hat{A}'P^n\hat{B} - \hat{A}'P^n\hat{B}(R + \hat{B}'P^n\hat{B})^{-1}\hat{B}'P^n\hat{A}, \tag{3.5.13}$$

and

$$\hat{F}^n = (R + \hat{B}'P^n\hat{B})^{-1}\hat{B}'P^n\hat{A}. \tag{3.5.14}$$

3. If $\| P^{n+1} - P^n \| \leq \gamma_1 \| P^n \|$ and $\| \hat{F}^{n+1} - \hat{F}^n \| \leq \gamma_2 \| F^n \|$ go to step 3; otherwise, repeat step 2.
4. Set $F = \hat{F}^n + R^{-1}N'$ and $P = P^n$.[5]

[5]Note that $\gamma_1$ and $\gamma_2$ are tolerance criteria and $\|.\|$ is the matrix norm. We also experimented with Vaughan's (1970) eigenvector algorithm and the results were the same.

For the initial problem, the solution to the matrix Ricatti equation is

$$P = \begin{bmatrix} -0.7476 & -0.2992 & 0.0590 & 0.0390 & -7.3312 \\ -0.2992 & -0.7476 & 0.0390 & 0.0590 & -7.3312 \\ 0.0590 & 0.0390 & 0.0175 & 0.0026 & -0.2055 \\ 0.0390 & 0.0590 & 0.0026 & 0.0175 & -0.2055 \\ -7.3312 & -7.3312 & -0.2055 & -0.2055 & 82.5434 \end{bmatrix},$$

and the corresponding F matrix is

$$F = \begin{bmatrix} -0.1191 & 0.0123 & -0.0130 & 0.0196 & 0.2182 \\ -0.0531 & 0.0292 & -0.0181 & 0.0110 & 0.0138 \\ -0.0563 & -0.0193 & -0.0137 & 0.0007 & 0.1373 \\ -0.0193 & -0.0563 & 0.0007 & -0.0137 & 0.1373 \\ -0.1550 & 0.0066 & -0.0141 & 0.0186 & 0.2691 \\ 0.0066 & -0.1550 & 0.0186 & -0.0441 & 0.2691 \end{bmatrix}.$$

The closed system solution is completed by substituting the optimal feedback rule $u = -Fx_t$ into the constraint to get $x_{t+1} = A0x_t + C\epsilon_{t+1}$ where $A0 = A - BF$. As noted by Hansen and Sargent (1990), this system is stable for all $x_0 \in R^n$ if the eigenvalues of A0 in absolute value are less than $\frac{1}{\beta^{1/2}}$.

## 3.6 Simulations Results

Simulation results are reported for seven different economies that were included in the BKK (1991) study. Results for the benchmark economy are associated with parameters values from Section 4 of this chapter. In order to investigate the effects of changing the elasticity of substitution between foreign and domestic goods, we considered a small elasticity simulation ($\sigma$=0.5) and a large elasticity simulation ($\sigma$=2.5). For the fourth economy (labelled "Two Shocks"), government expenditure shocks are added to the domestic and foreign economies. The fifth economy considers time-to build, with J=2. The sixth economy (labelled "Government Shocks") considers only government expenditure shocks. For the seventh economy (labelled "No Capital"), we set $\theta$=0.001, and consider the effects of an extremely small capital share.

The empirical results will be discussed in two subsections. First, the time domain properties of the solution will be examined. This will provide some intuition for the behavior of the model and will also provide some evidence of the sensitivity of the model to alternative solution procedures, as the BKK results were obtained with a different procedure. The second section presents the frequency domain diagnostic test results. In both cases, all statistics have been computed as averages over 20 simulations of 100 quarters each.

### 3.6.1 Time Domain Properties of Simulated Data

Table 3.6.1 reports standard summary statistics for all of the experiments that are comparable with Table 3.2.2 for the historical data. As in BKK (1991), we find that output, the terms of trade, and net exports are all highly autocorrelated in the benchmark economy and the majority of the other experiments. More specifically, for the benchmark economy, the first-order autocorrelation coefficient for net exports is 0.59 (the median is 0.80 for the historical data): for the terms of trade, the model value is 0.57 versus 0.74 in the data; for output, the model yields 0.67 while the median value in the historical data is 0.74. Overall, this is not surprising as most of the endogenous variables inherit the persistence properties of the technology shocks.

The purpose of this thesis is to investigate the complete dynamic behavior of the variables in the model. With this in mind we report the autocorrelation function for the variables of interest for all experiments up to twenty lags. These can be seen in figures 3.6.1 to 3.6.3 . In figure 3.6.1 we graph the autocorrelation function for output for different experiments. One can see that for all experiments the autocorrelation function does not exhibit any peaks. It is also evident from the graph that the autocorrelation function dies exponentially as the number of lags increases. In Figure 3.6.2 we report the autocorrelation function for net exports. Interesting to note is the fact that in the time to build experiment the autocorrelation function appears to have a high value for the first lag and also that the degree of dissipation for this experiment is not as high as in the other experiments. For the terms of trade the autocorrelation function behaves in a similar way to the other variables (Figure 3.6.3). Note the very low values that the autocorrelation function exhibits for the no capital experiment and for the government expenditure experiment.

Turning to the contemporaneous variances and covariances, note that the benchmark economy replicates the variability of output in the sample of countries, but does not do as well as mimicking the variability of $nx$ and $p$. Although the standard deviations of the terms of trade and net exports are somewhat smaller than those reported in BKK (1991), it is also true that their results are nevertheless similar. BKK (1991) report values of 0.3 and 0.48 for the standard deviations of net exports and terms of trade respectively. For the historical time series, the variability of net exports is similar to the variability of output, and the variability of the terms of trade exceeds both. We will discuss the differences in model variability at some length below; for now, it is clear that both solutions behave similarly relative to the historical time series.

In the benchmark economy, the contemporaneous cross correlation between net exports and output is -0.6, while the median value for the sample is 0.23. However, the model value is within the range observed in the historical sample. As a result, the model definitely captures the countercyclical nature of net exports. BKK point out that the countercyclical nature of the trade balance is also associated with strong procyclical changes in investment. Consumption smoothing implies that output net of consumption must be procyclical; as output net of consumption equals investment plus net exports, countercyclical net exports are also associated with more strongly procyclical investment flows.

We also find a strong negative correlation between net exports and the terms of trade. For our benchmark economy we obtain -0.98, while BKK obtain 0.43; the values in the historical sample are generally negative (except for the United States). We also observe a strong positive correlation between the terms of trade and output in the model (as do Backus, Kehoe, and Kydland), although there is no pattern in the data.

A key characteristic of the time domain properties of this model is the asymmetric shape of the cross-correlation function between net exports and the terms of trade. In Figure 3.6.4, Panel (a), we graph the cross-correlation function for these two variables for the benchmark economy. While the smoothly varying "S" shape that is exhibited by the BKK solution is not present in our solution, the basic relationship of these two variables in the data holds in the sense that the cross-correlation function decreases and then increases as we move from right to left. BKK point out that the dynamic relationship between net exports and the terms of trade are derived from the effects of investment and capital formation. In response to a domestic technology shock, domestic output increases and the relative price of domestic versus foreign output decreases, which implies that $p$ rises. At the same time, net exports decrease as investment increases so that the contemporaneous correlation between $p$ and $nx$ is negative. As the investment response diminishes over time, the trade balance moves into surplus, and the correlation between $p_t$ and $nx_{t+k}$ increases with k in the short term.

Panels (b) and (c) in Figure 3.6.4 graph the cross-correlation function for different values of $\sigma$. An interesting feature of the solution presented here, in comparison with BKK (1991), is the insensitivity of the results concerning the cross-correlation function to changes in $\sigma$. BKK argue that the timing of the cross-correlation relationship between net exports and the terms of trade is sensitive to the value of $\sigma$, in the sense that the location of the point at which the cross-correlations shift from negative to positive decreases as $\sigma$ increases, although the shape of the basic relationship is not. We find that neither the location nor the shape of the cross-correlation function is sensitive to changes in $\sigma$. We attribute the differences in simulation properties to a difference in model solution techniques. Basically, the Kydland and Prescott (1982) approximation procedures allow for greater sensitivity to curvature in the objective function, and in this model the degree of curvature is determined by $\sigma$.

We now turn to the time domain properties for some of the other experiments. The experiment labelled "Two Shocks" adds independent government expenditure shocks to the domestic and foreign economies. We will assume that B=diag(0.95,0.95) and that the standard deviation of each of the innovations is 0.001. These estimates are based on V. V. Chari et al.'s (1991) estimates for the U.S economy and are identical to those employed by BKK (1994). The mean value of $g$ in each country is assumed to be 0.20, so that government purchases represent 20 percent of the steady state value of output. Table 3.6.1 illustrates that our economy is somewhat more sensitive to government expenditure shocks than BKK (1994), although the properties of economy with respect to the time domain properties of net exports, the terms of trade, and output remain qualitatively unaffected. Most notable are the increase in the magnitude of the contemporaneous correlations between output and net exports (-0.60 to -0.73) and the change in sign of the contomporaneous correlations between output and the terms of trade and net exports and the terms of trade, -0.98 and 0.59 the corresponding numbers in the benchmark economy versus 0.80 and -0.79 in the Two Shocks experiment.

For the next experiment, we introduced time-to-build and increased the value of J from 1 to 2. In this scenario, two quarters are required before capital is ready to be used in the production process. Time-to-build will influence investment dynamics and could potentially have an important effect on the dynamic relationship between the terms of trade and the trade balance.[6] Following BKK (1994), we choose J=2 which is half of the J=4 suggested by Kydland and Prescott (1982). Table 3.4 indicates that the introduction of time-to build significantly increases the variability of the terms of trade which brings the model closer to the historical data. This effect is somewhat larger than in BKK (1994). In addition, the net exports become more strongly countercyclical, with a contemporaneous correlation of -0.93 relative to -0.60 in the benchmark. However, as illustrated in Figure 3.6.5, the introduction of time-to-build has a negligible effect on the cross-correlation function for net exports and the terms of trade.

Finally, we consider two extreme experiments. For the results labeled "Government Shocks", we exclude productivity shocks from the economy and set $z$=1 in both countries. The parameters of this experiment are the same as those used in the "Two Shocks" experiment for the government purchases process. This experiment is designed to illustrate the importance of the nature of the shocks for trade and price dynamics. Our results here mimic those in BKK (1994), in that the cross-correlation function between the terms of trade and net exports develops a maximum at zero and becomes tent shaped. As a result, the dynamic relationship between the terms-of-trade and net exports is totally changed. This result illustrates the importance of identifying the source of fluctuations before predicting comovements between the trade balance and the terms of trade.

In the experiment labelled "No Capital", we set $\theta$=0.001. This experiment

[6] An alternative approach to modifying the dynamics of capital formation is to posit adjustment costs. Mendoza (1991) and Baxter and Crucini (1993) consider convex costs of changing the capital stock.

is designed to illustrate the crucial relationship between investment and the trade balance. In the no-capital experiment, the most striking change in the trade statistics appears to be the reduction in the first order autocorrelation for the terms of trade, from 0.57 in the benchmark economy down to 0.01. In addition, the terms of trade becomes countercyclical and the magnitude of the correlation between net exports and output is dramatically reduced. Once again, the cross-correlation function for the terms of trade and net exports looks nothing like the cross-correlation function for the benchmark economy or the historical data. These results are also broadly consistent with the results in BKK (1991).

While there are some specific differences between the results presented here and those in BKK (1991), the model generates predictions that are qualitatively similar in the time domain. We now turn to an analysis of the model's properties in the frequency domain.

### 3.6.2 Frequency Domain Properties of The Simulated Data

This section reports the results from the frequency domain diagnostic tests developed in Chapter 2. We report two types of diagnostics: 1) accumulated variance and covariance contributions which represent frequency interval tests defined relative to frequency zero, and which depict spectral deviations between the model and the historical data measured from the origin to the frequency of interest; 2) incremental variance and covariance contributions which represent spectral frequency interval tests defined over intervals of width $\frac{\pi}{6}$, and which measure spectral deviations between the model and the data over a particular interval. Standard errors for the test statistics measured over replications of the simulations are reported at the bottom of each table.

For the univariate spectra in the benchmark economy, the incremental and accumulated variance contributions presented in Tables 3.6.2-3.6.7 clearly indicate that the model does well at fitting the low frequency behavior of output, the terms of trade, and net exports. As mentioned previously, this no doubt reflects the rather simple univariate patterns of persistence exhibited by the data for the majority of countries in the sample.

For the multivariate spectra, the patterns in the data are quite diverse, and it is probably most useful to characterize first the co-and quadrature spectra for the benchmark economy, and then to consider which economies are similar. Tables 3.6.8-3.6.13 present incremental and accumulated cospectral deviations for the benchmark economy for net-exports and the terms of trade, net exports and output, output and the terms of trade. At least for the cospectrum between net exports and the terms of trade, it is clear that the model produces results that are similar to a wide variety of countries, with negative cospectral power at low frequencies and not much activity at higher frequencies. For the cospectral relationship between $nx$ and $y$ presented in Tables 3.6.10-3.6.11, the model does fairly well at predicting the strong negative cospectral power at low frequencies that is exhibited by the data. For the cospectral relationship between $y$ and $p$, the benchmark model predicts positive cospectral power at low frequencies (like Italy and Switzerland), although the majority of countries have negative cospectral power at these frequencies.

Tables 3.6.14-3.6.19 present accumulated and incremental quadrature spectral deviations for the same time series in the benchmark economy. Recall that the spectral deviations are measured as the historical frequency increments, relative to those implied by the model. Tables 3.6.14 and 3.6.15 clearly indicate that quadrature spectra between net exports and the terms of trade for the model is considerably lower (*i.e.* more negative) than the historical quadrature spectra at low frequencies for all countries, and that this deficiency persists for accumulated covariance contributions across almost all frequencies. It is clear that this is one feature of the model that clearly overestimates the properties of the historical time series. Moreover, as the cross-spectral distribution function estimates are normalized, it is clear that the quadrature spectral deviations correspond to important dynamic aspects of the relationship between net exports and the terms of trade that are simply missed by our application of the theory.

Tables 3.6.16-3.6.19 present accumulated and incremental quadrature spectral deviations for the benchmark economy for $nx$ and $y$, and $y$ and $p$ respectively. The model predicts very little activity at low frequencies for these variables, and the test statistics essentially mimic the properties of the historical spectra.

Overall, it is clear that the benchmark economy replicates some of the frequency domain properties of the endogenous variables, although there are prominent areas where the model fails.

For the remainder of the simulations, we will focus on the multivariate frequency domain statistics for $nx$ and $p$, as these are the primary statistics of interest for examining the J-curve relationship. Tables 3.6.20-3.6.27 report cross spectral statistics for the small- and large- elasticity economies. In general, all of the results are similar to those reported for the benchmark economy. This is not surprising, as the time domain simulations revealed that the solution was primarily insensitive to changes in $\sigma$.

Turning to the "Two Shocks Economy", Tables 3.6.28-3.6.31 reveal that the introduction of government expenditure into the model causes a serious deterioration in the multivariate frequency domain fit of the model, relative to the historical data. This is also supported by the results in Tables 3.6.36-3.6.39 which report results from the "Government Expenditures" economy. Basically, government expenditure shocks induce a large positive contemporaneous cross-correlation between $nx$ and $p$, and this also occurs at moderate leads and lags. For cospectral deviations measured as historical minus model-based frequencies, this translates into large negative additions to the test statistics at low frequencies, and this swamps any satisfactory frequency domain fits in the benchmark case. Deviations for the quadrature-based statistics are also made worse, when compared to the benchmark case. Overall, it is clear that government expenditure shocks do not provide a satisfactory basis for J-curve dynamics within the context of this model.

Turning to the time-to-build results in Tables 3.6.32-3.6.35, it is clear that introduction of time-to-build magnifies the negative cospectral power of the model at low frequencies, although the qualitative features of the economy are similar to the benchmark economy from this perspective, and there are countries where the cospectral diagnostics are within two standard deviations of zero across a majority of frequencies. For the quadrature spectrum between $nx$ and $p$, it is clear that the introduction of time-to-build makes things worse.

For a final simulation, we consider the "No Capital" economy where $\theta$ is assumed to be 0.001. Tables 3.6.40-3.6.43 indicate that the frequency domain properties of the relationship between the terms of trade and net exports is altered substantially, relative to the previous scenarios. In almost all of the previous simulations, the cospectrum for the model has exceeded the cospectrum for the historical time series (in absolute value), at low frequencies. In this economy, this is reversed and the cospectral deviations are therefore primarily negative at low frequencies. For the quadrature spectrum, the signs of the test statistics are also reversed relative to the benchmark economy, and the degree of fit is much worse.

## 3.7 Conclusion

This chapter developed a general equilibrium international business cycle model in order to investigate the dynamic relationship between output, the terms of trade, and the trade balance. The nonlinear rational expectations model was linearized and then solved and simulated using standard computational algorithms available for recursive optimal linear regulator problems. Both time and frequency domain diagnostic tests were used to evaluate the properties of the model.

In general, the results illustrate that models of this sort are capable of replicating simple univariate dynamics in both the time and the frequency domain, but that the multivariate dynamics exhibited by the data are too varied to be represented accurately by one specific parameterization. In addition, it is clear that the cross spectral frequency domain diagnostics provide a very different and detailed evaluation of the model, relative to the cross-correlation function. Finally, it is also clear that the numerical properties of these types of models vary substantially across approximation procedures; this has been illustrated in a much simpler dynamic economy by Taylor and Uhlig (1989).

As the frequency domain properties of the data vary considerably across countries, it is difficult to suggest a single theoretical modification to the model that would satisfactorily replicate the properties of the data for all countries. As trade statistics are notorious for containing substantial measurement error, the next chapter considers the addition of stochastic measurement error to the data as a potential for reconciling the theory with the data.

Figure 3.2.1: Sample Cross-Correlations, Historical Data, HP-Filtered

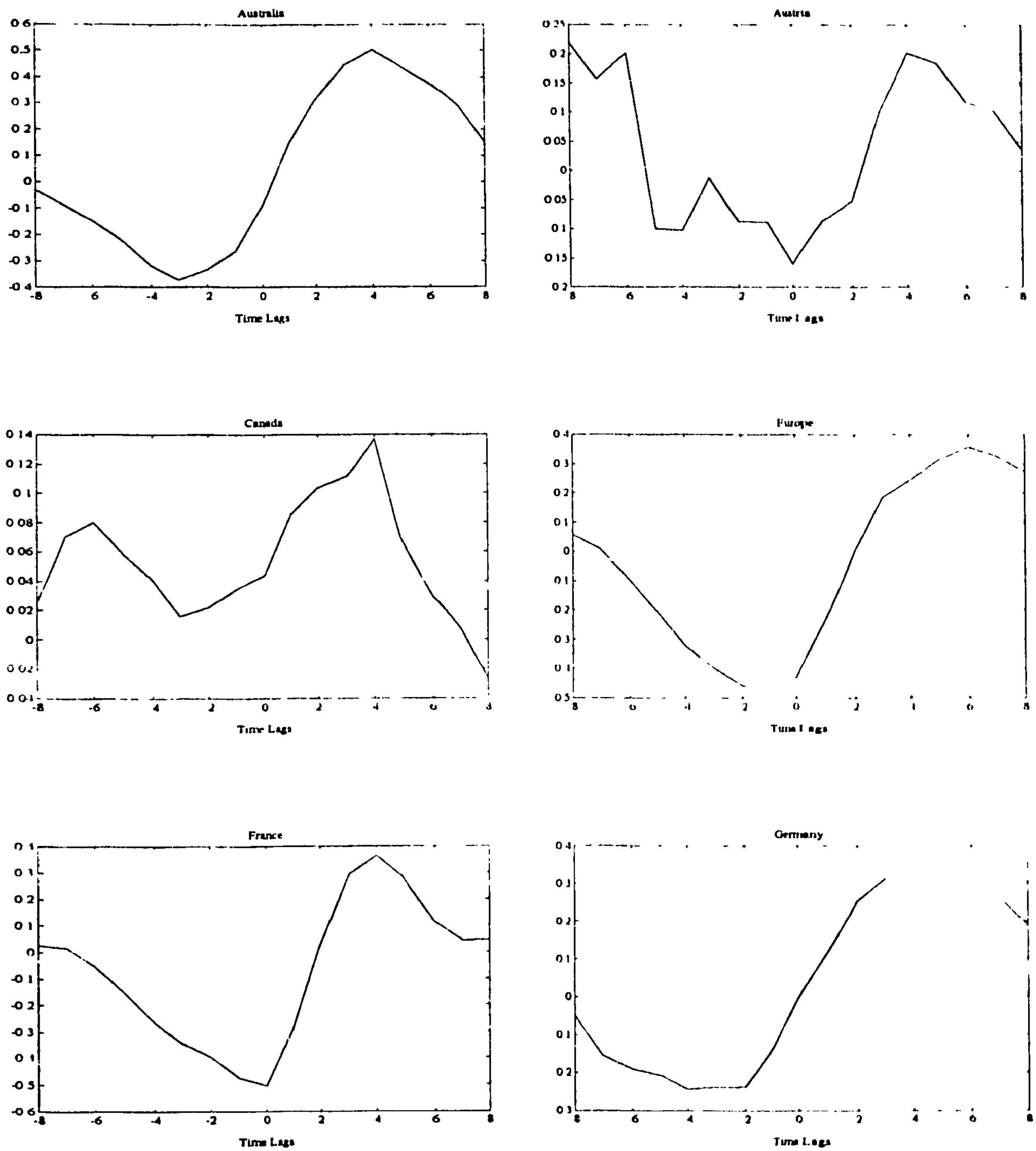

Figure 3.2.1: Continued

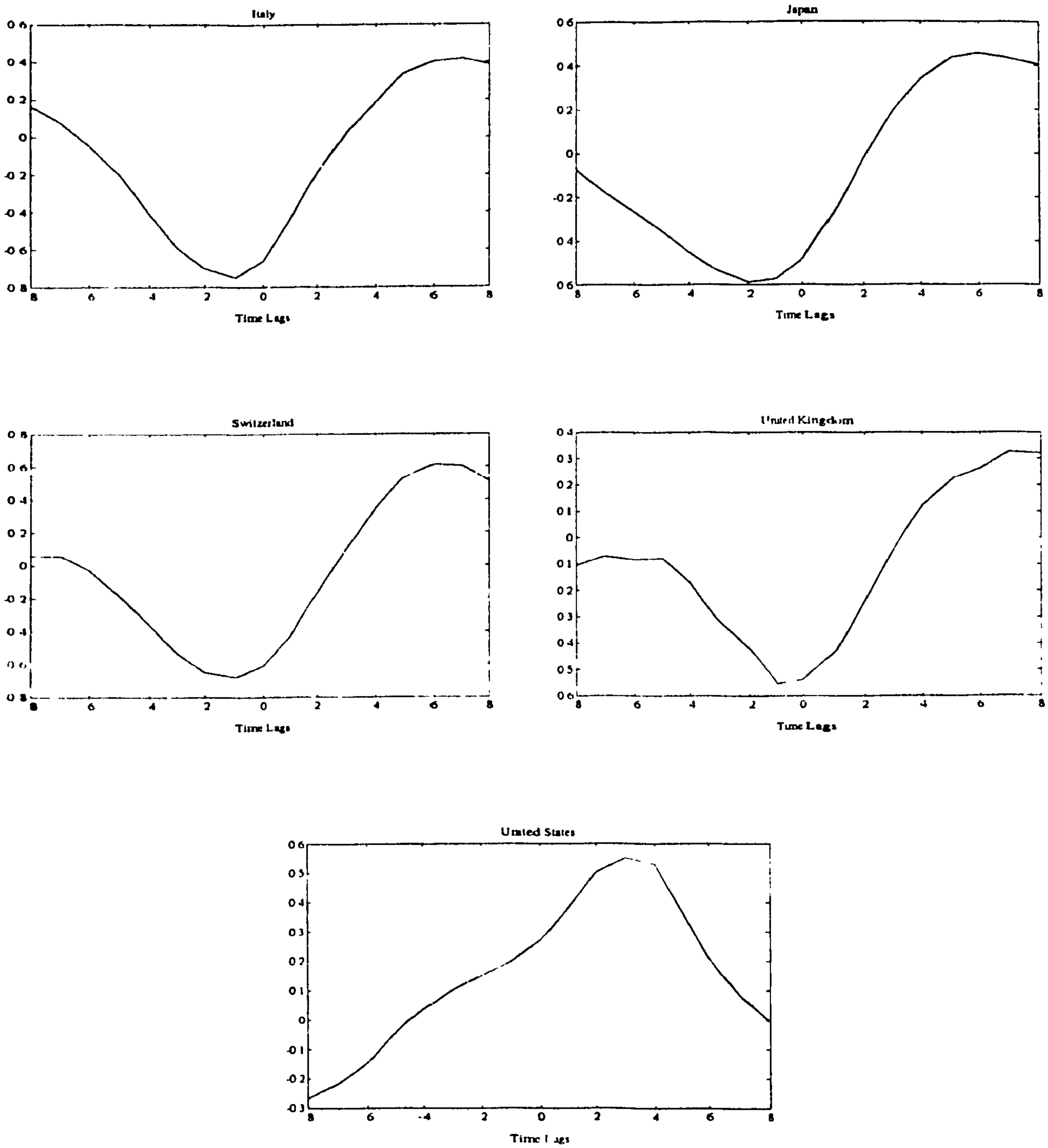

Figure 3.2.2: Estimated Spectral Density Functions, Trade Balance (NX)

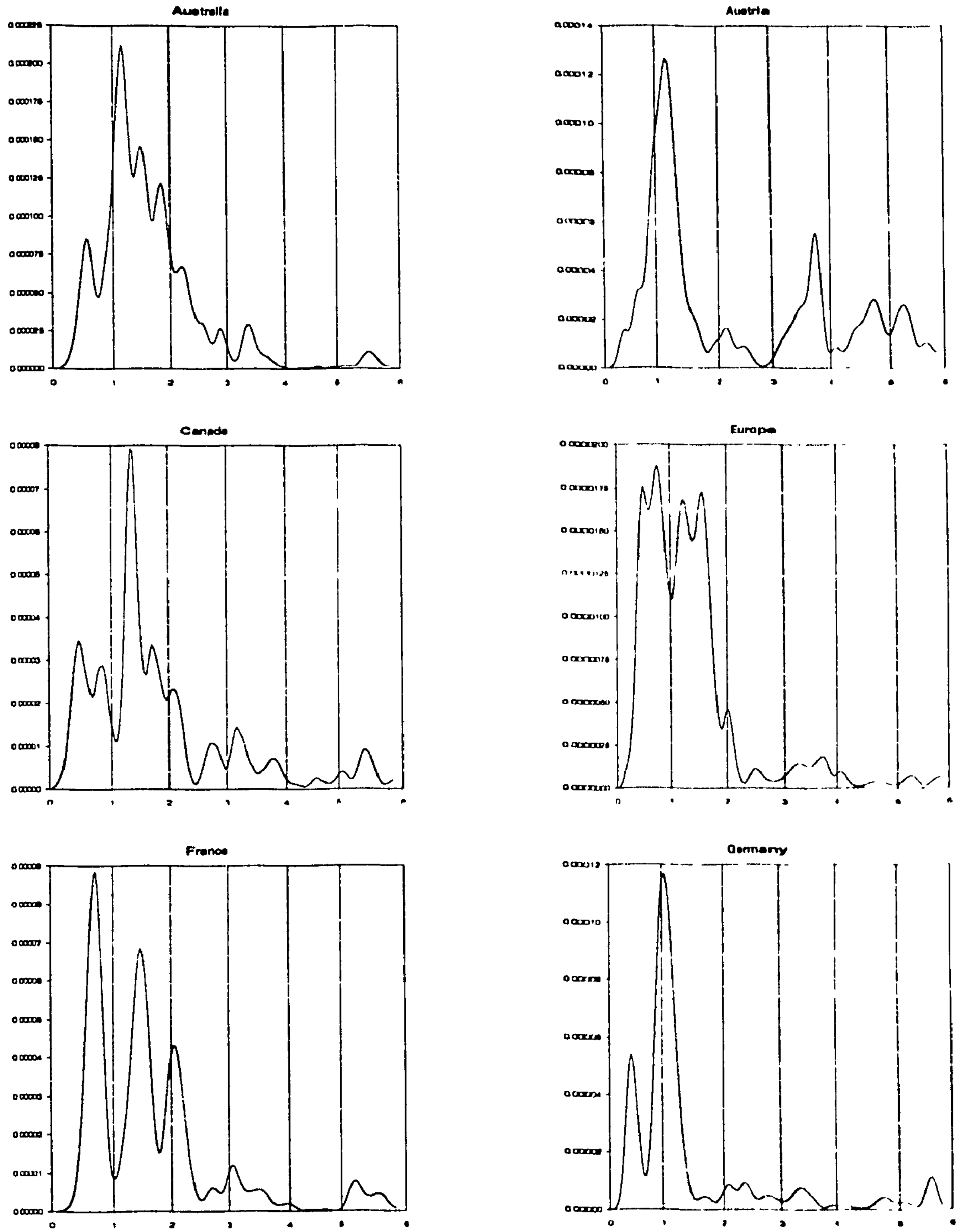

Figure 3.2.2: Continued

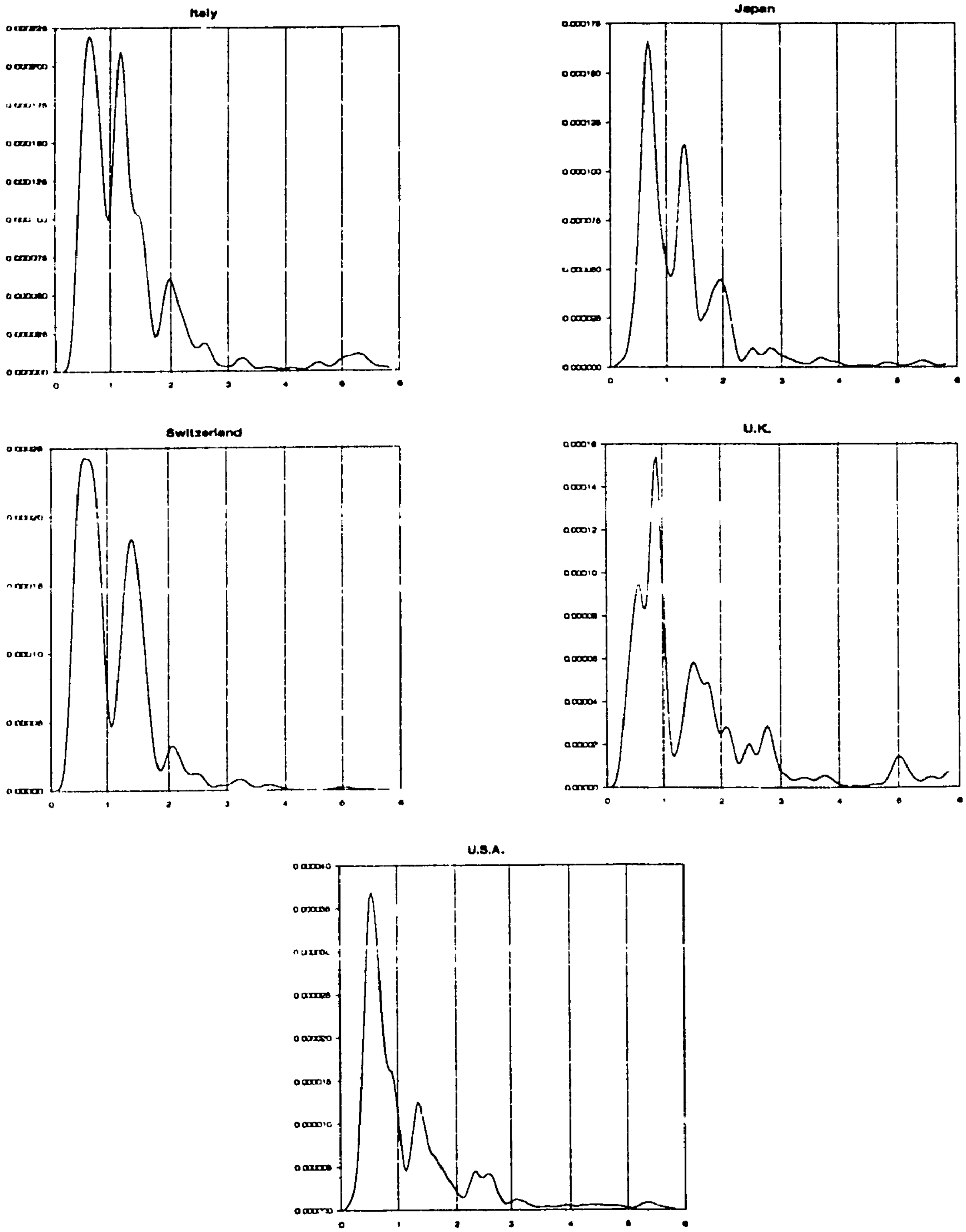

Figure 3.2.3: Estimated Spectral Density Functions, Terms of Trade (P)

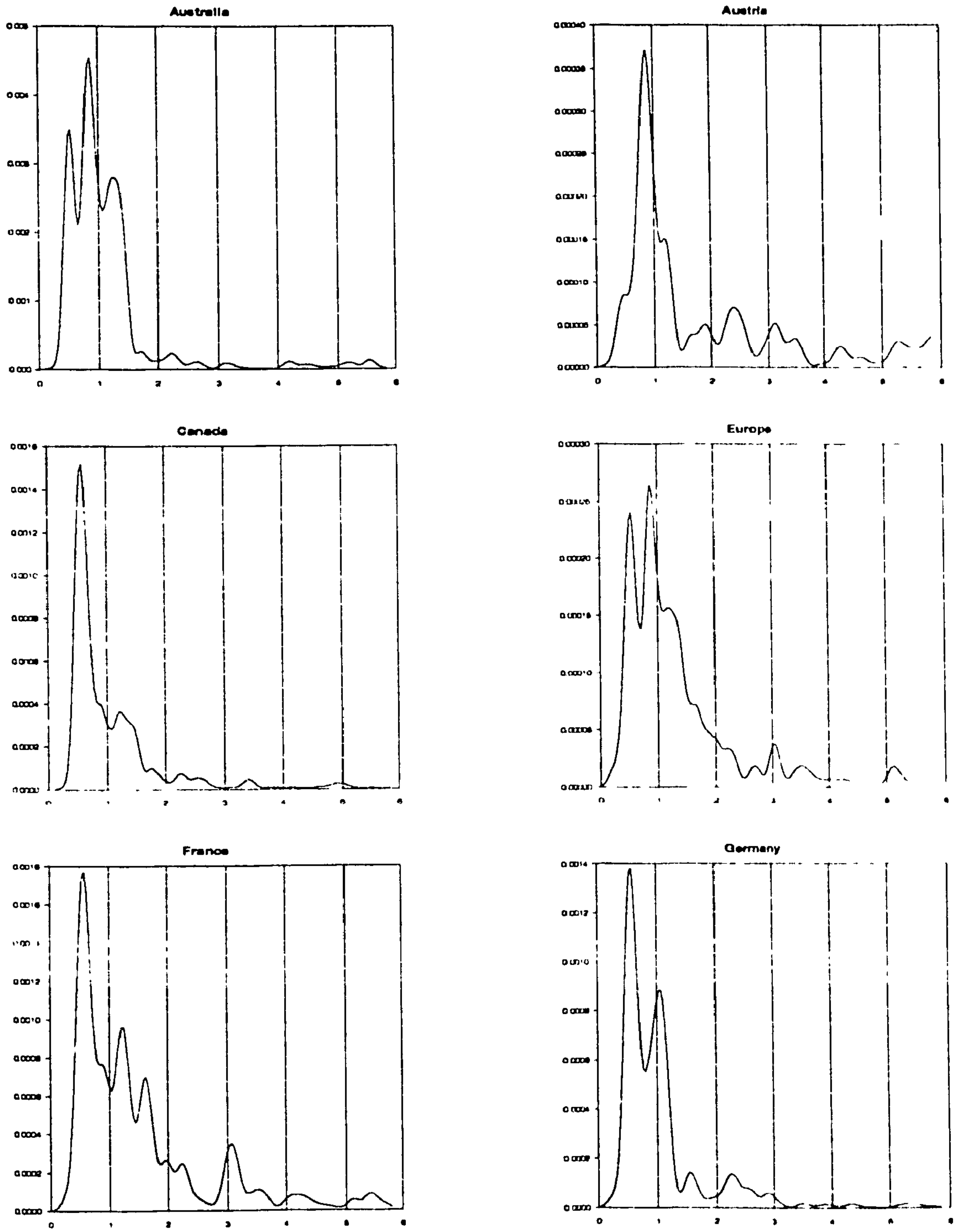

Figure 3.2.3: Continued

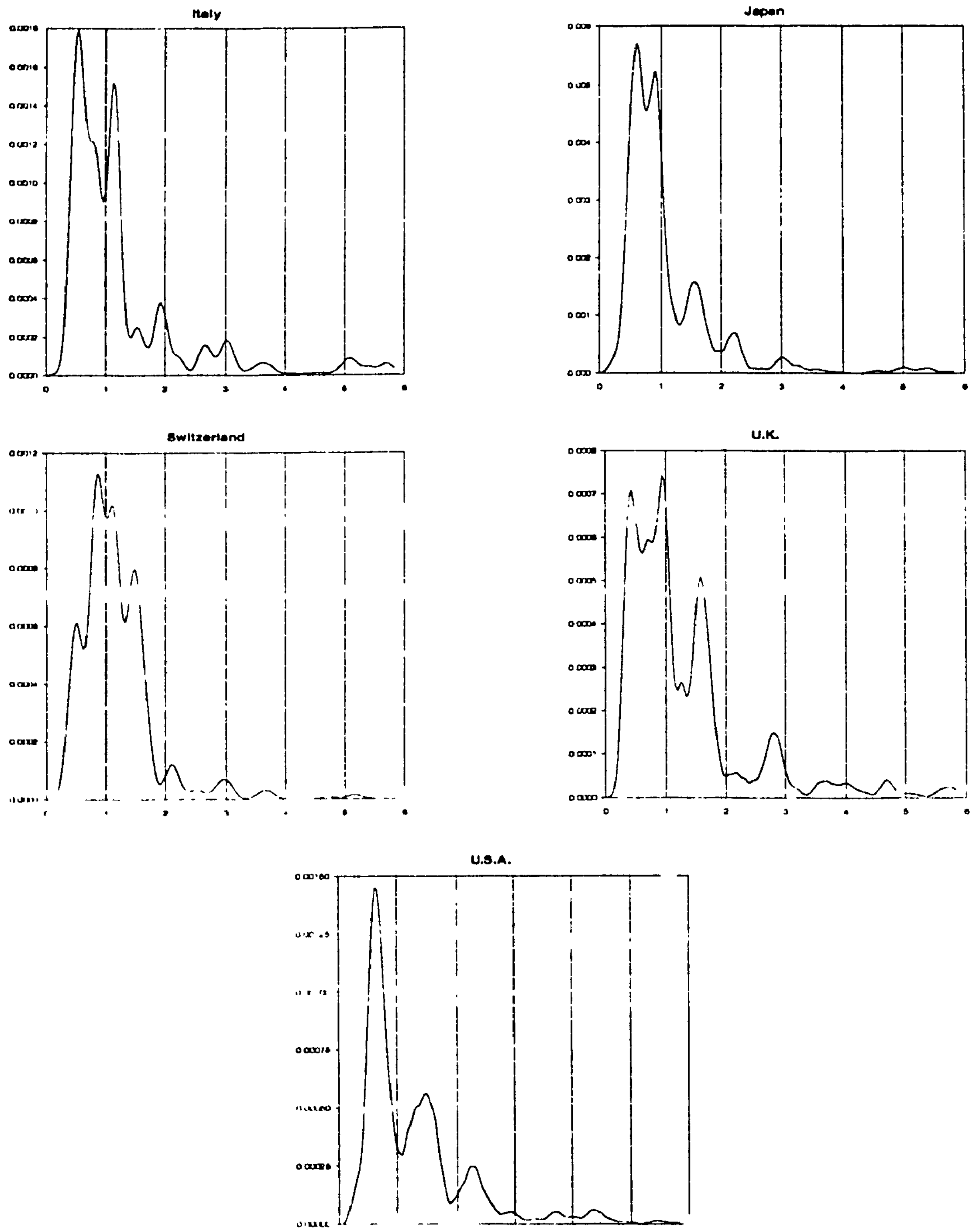

Figure 3.2.4: Estimated Spectral Density Functions, Real G.D.P. (Y)

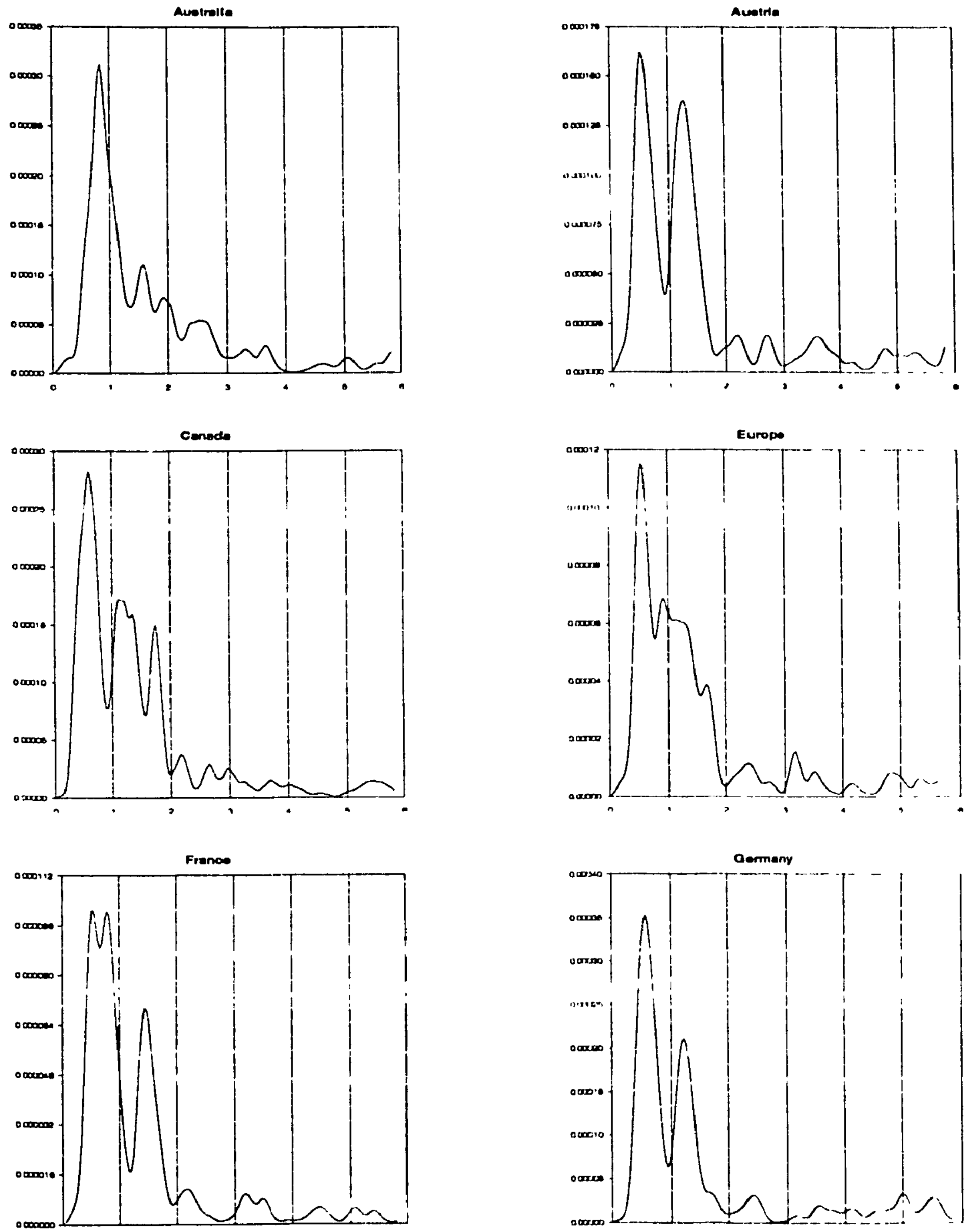

Figure 3.2.4: Continued

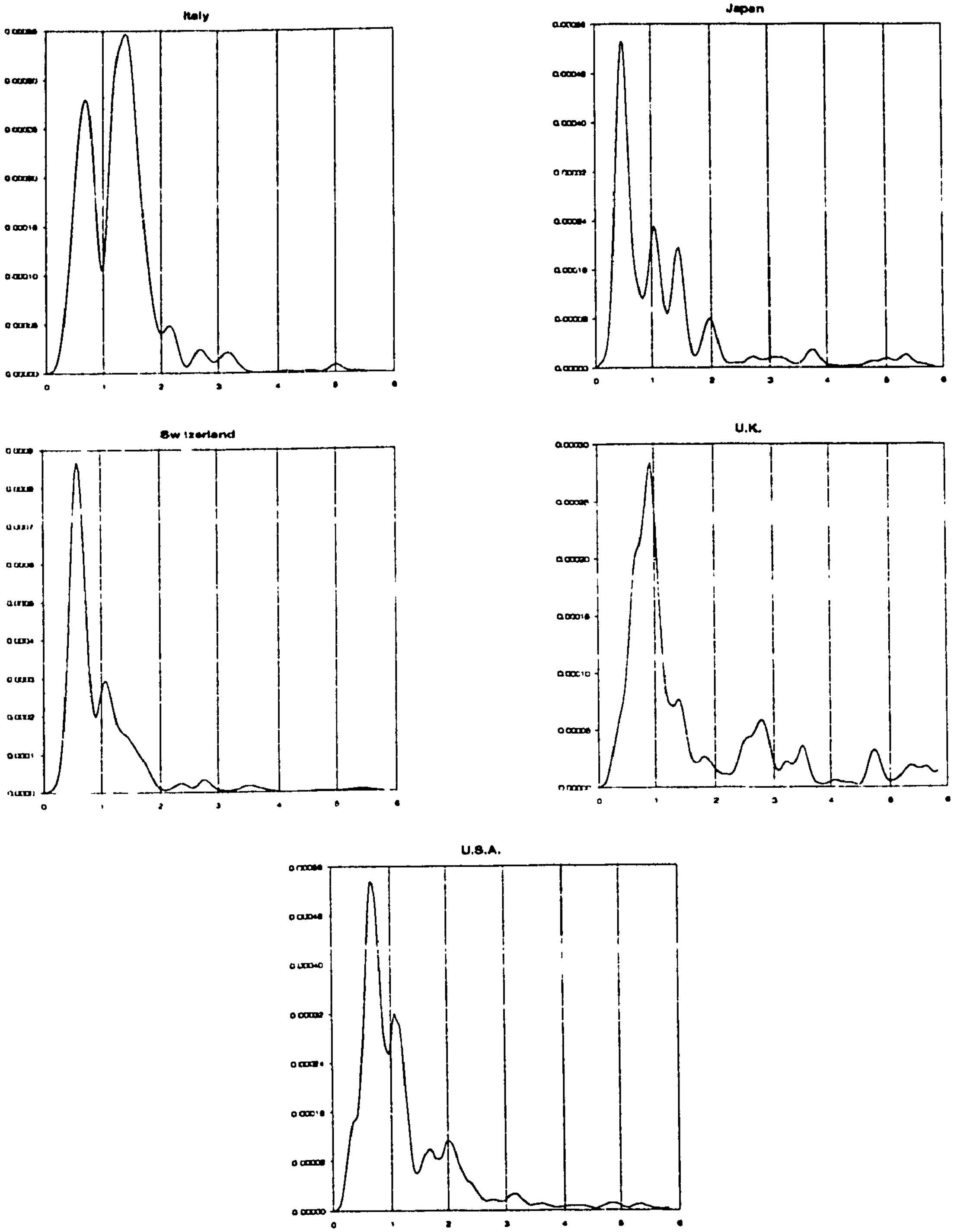

Figure 3.2.5: Estimated Cospectrum (NX,P)

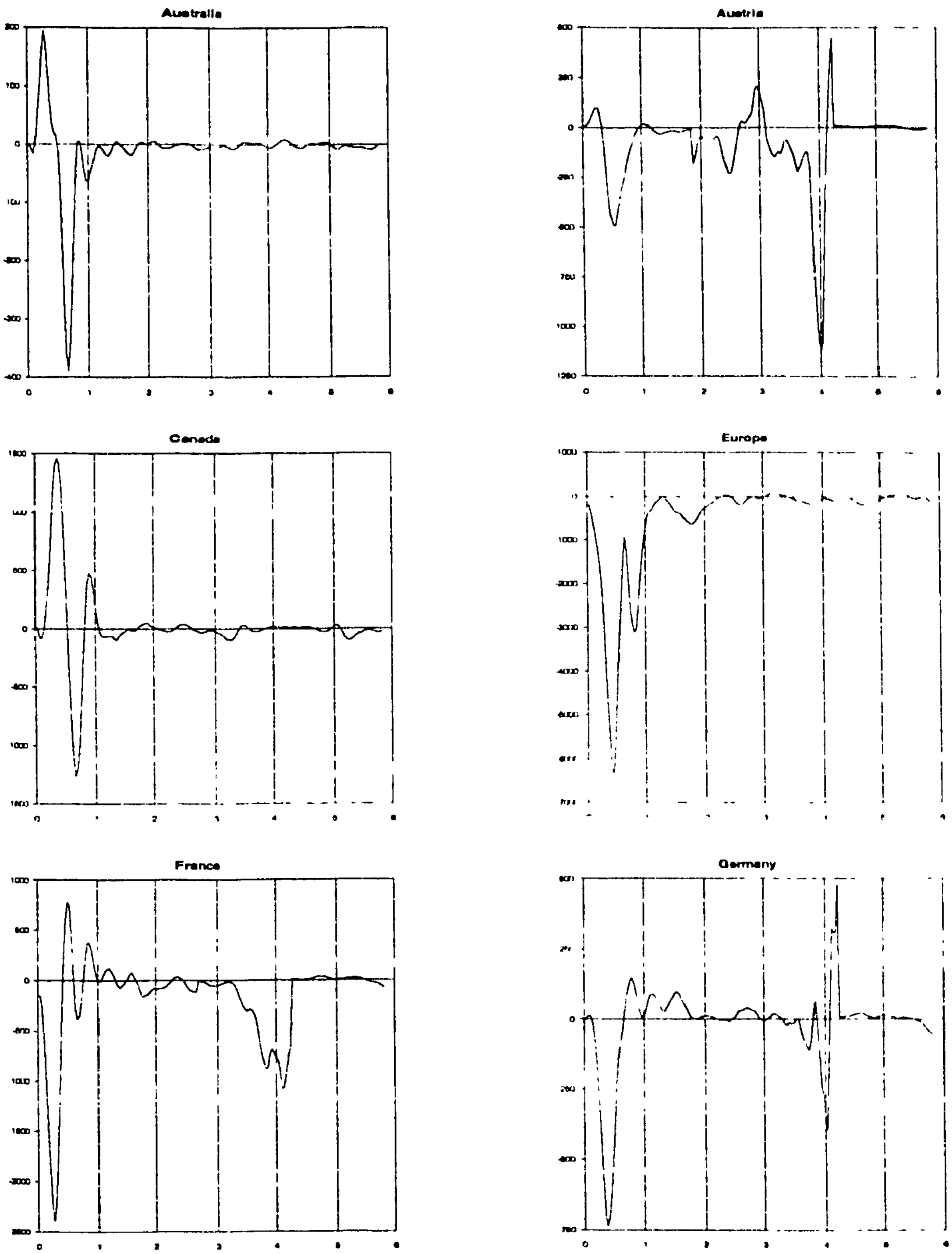

Figure 3.2.5: Continued

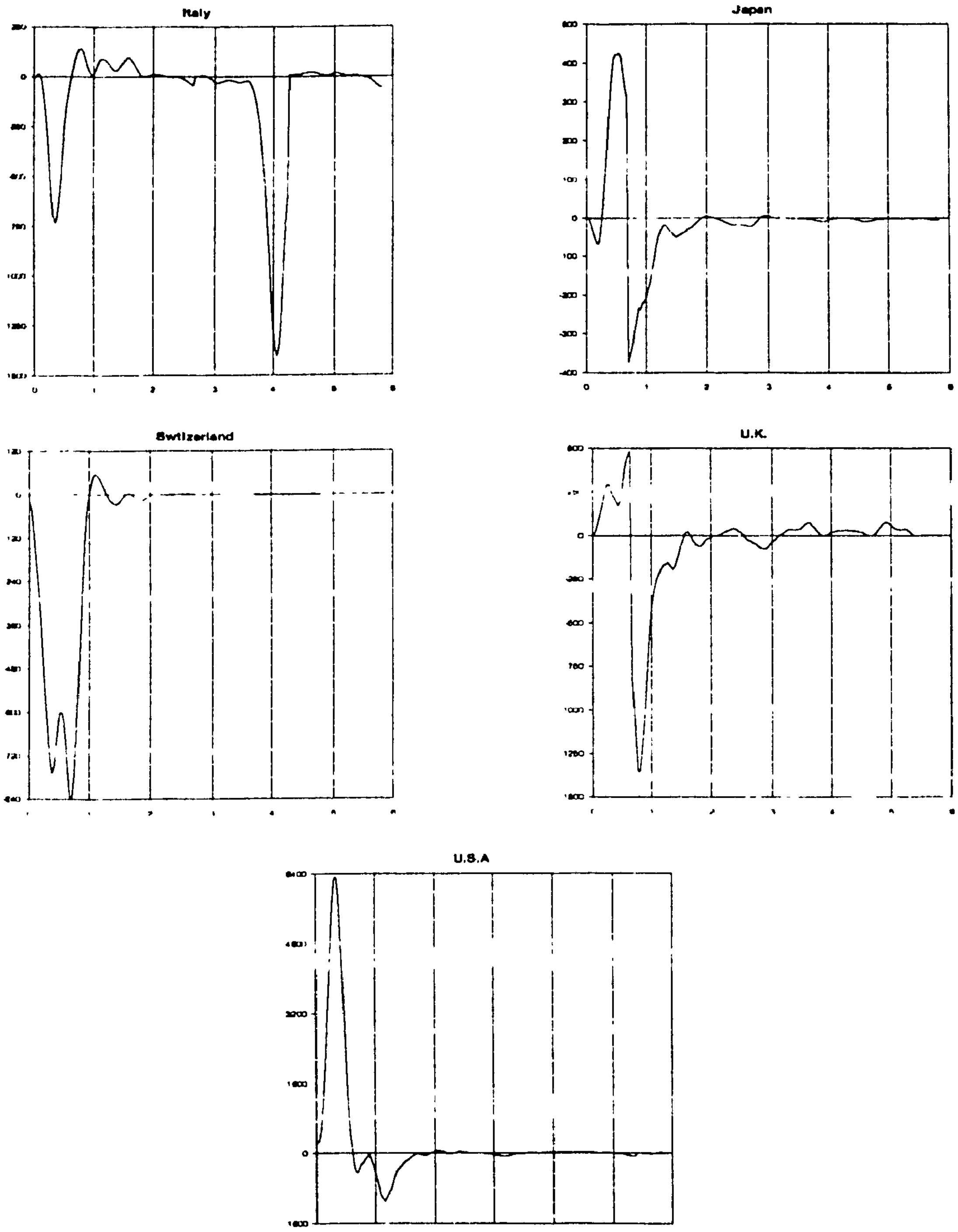

Figure 3.2.6: Estimated Quadrature Spectrum (NX,P)

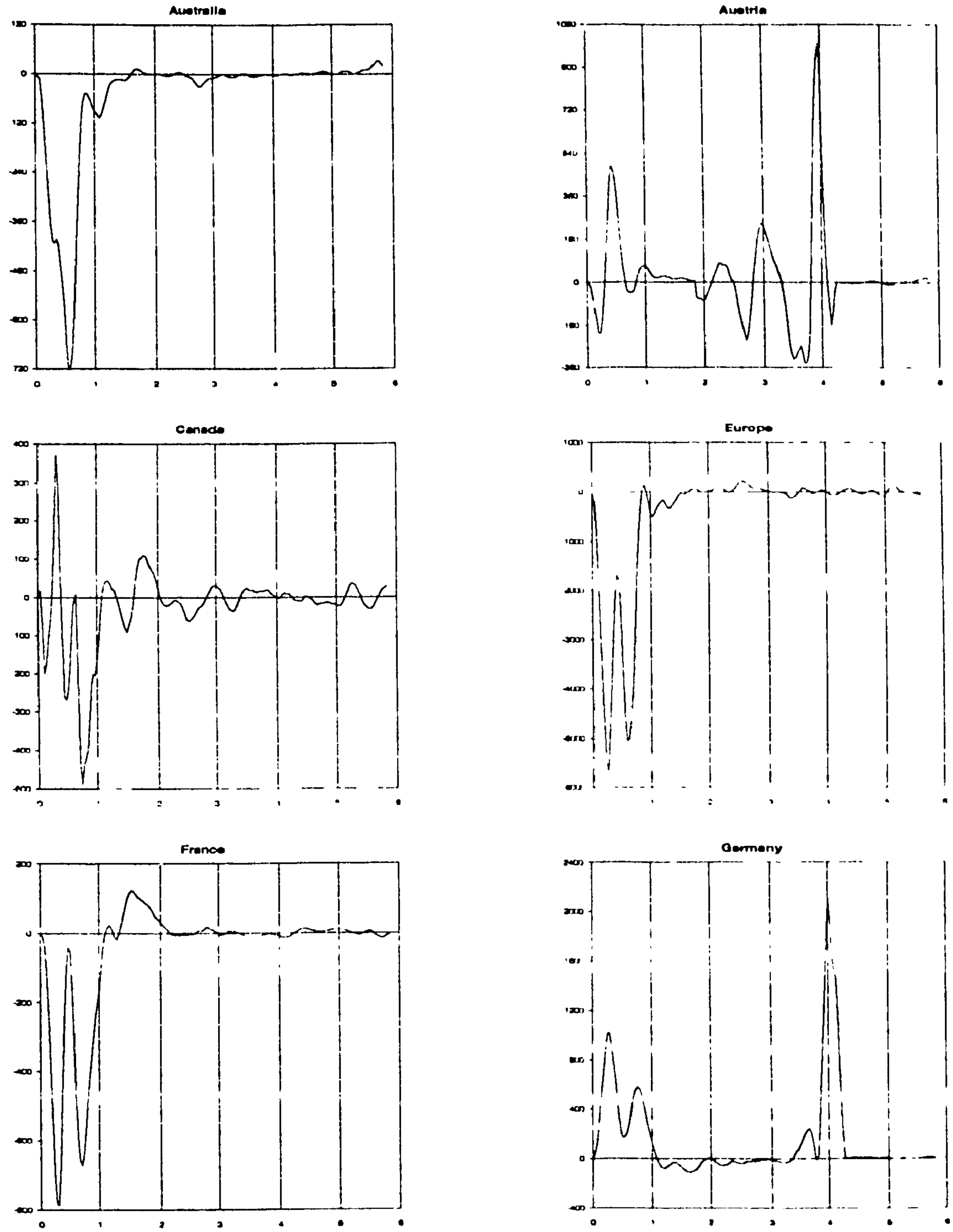

Figure 3.2.6: Continued

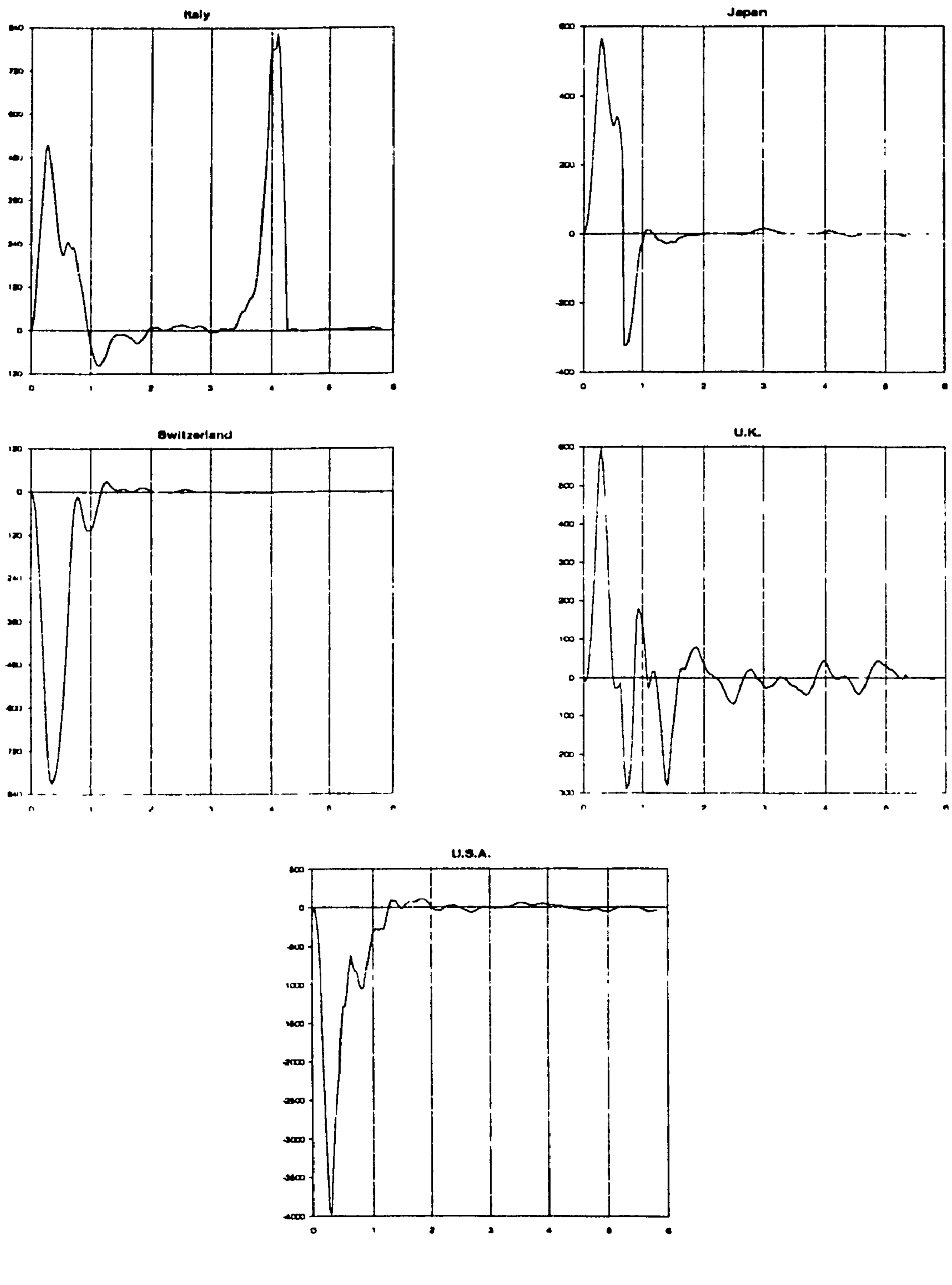

Figure 3.2.7: Estimated Cospectrum (NX,Y)

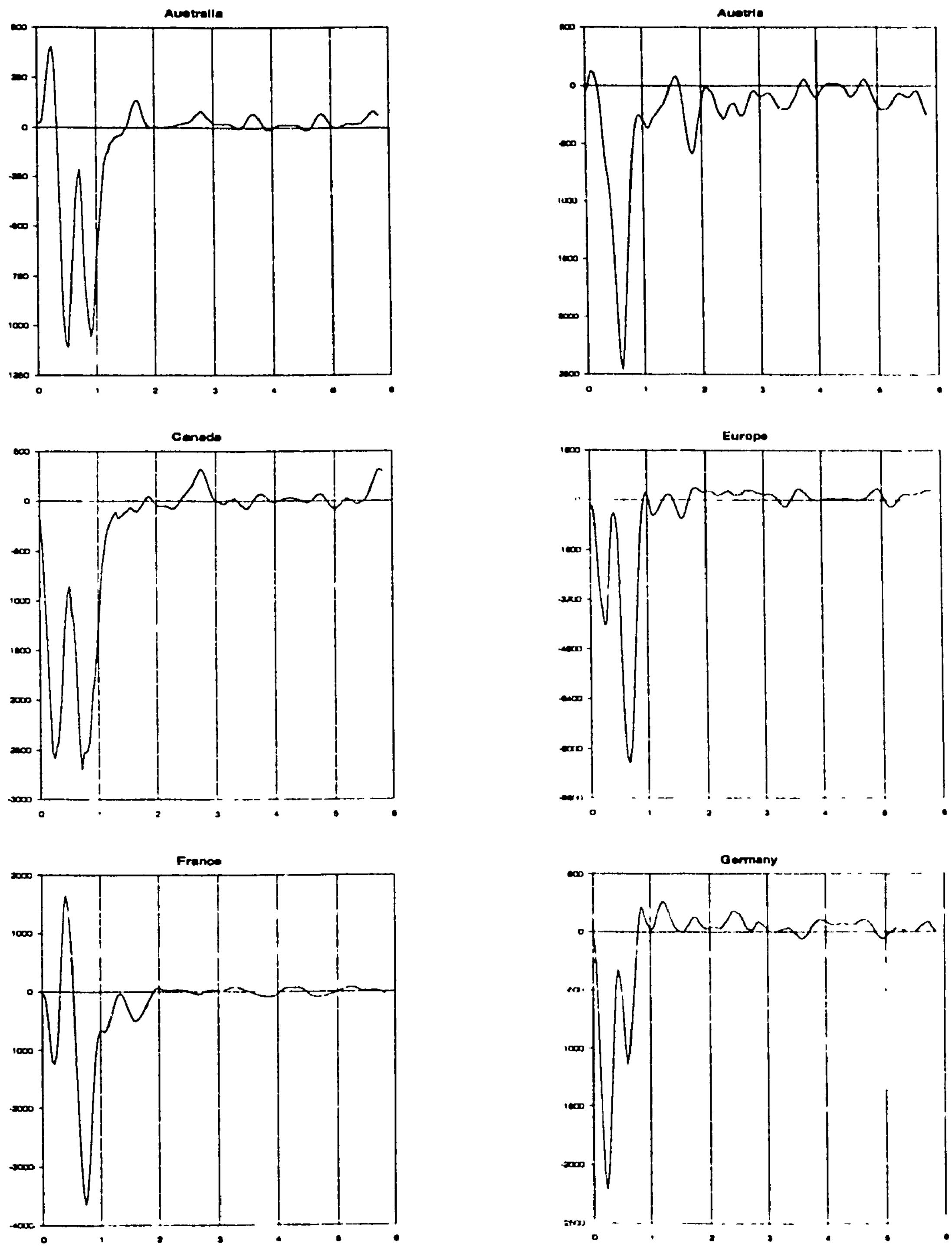

Figure 3.2.7: Continued

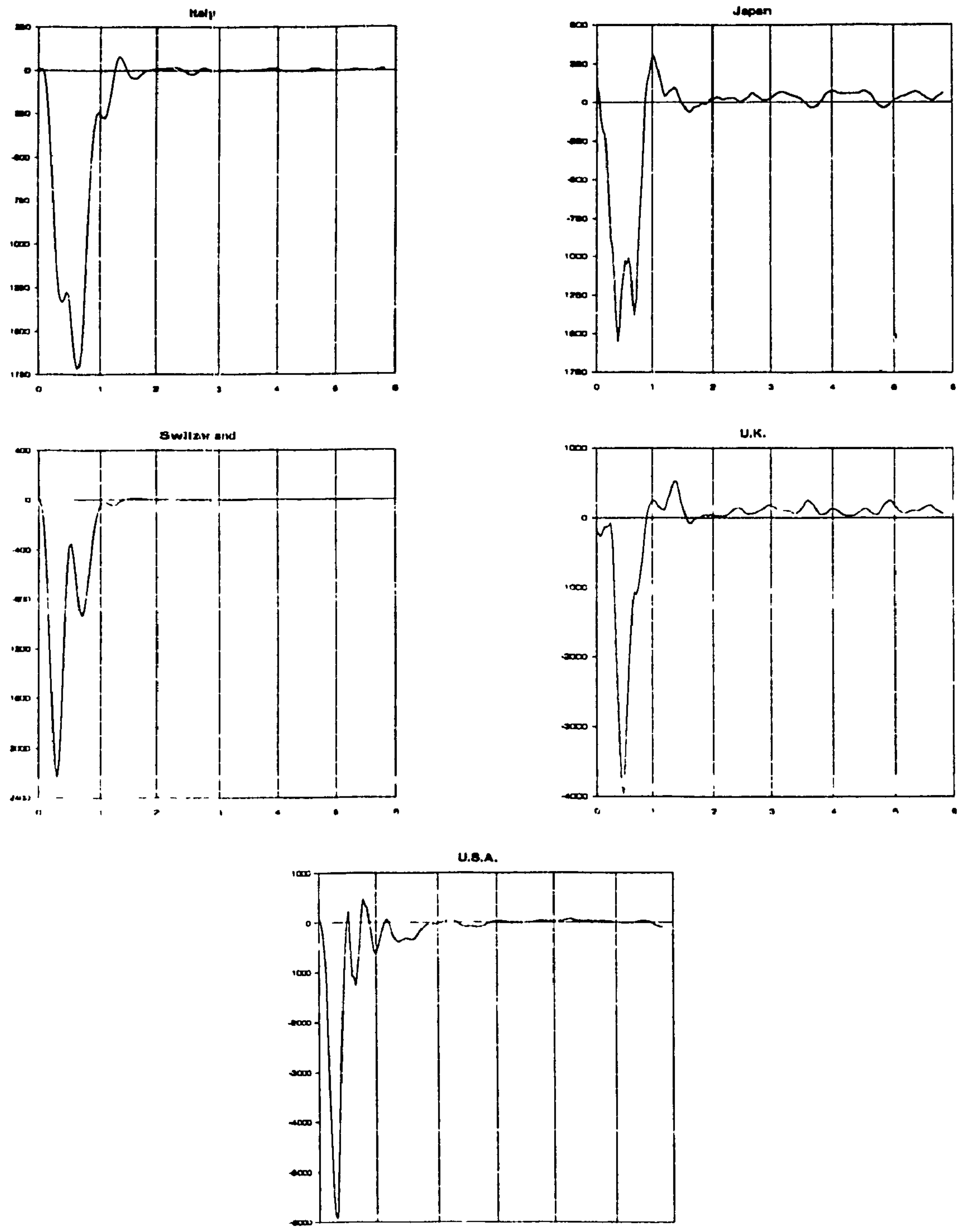

Figure 3.2.8: Estimated Quadrature Spectrum (NX,Y)

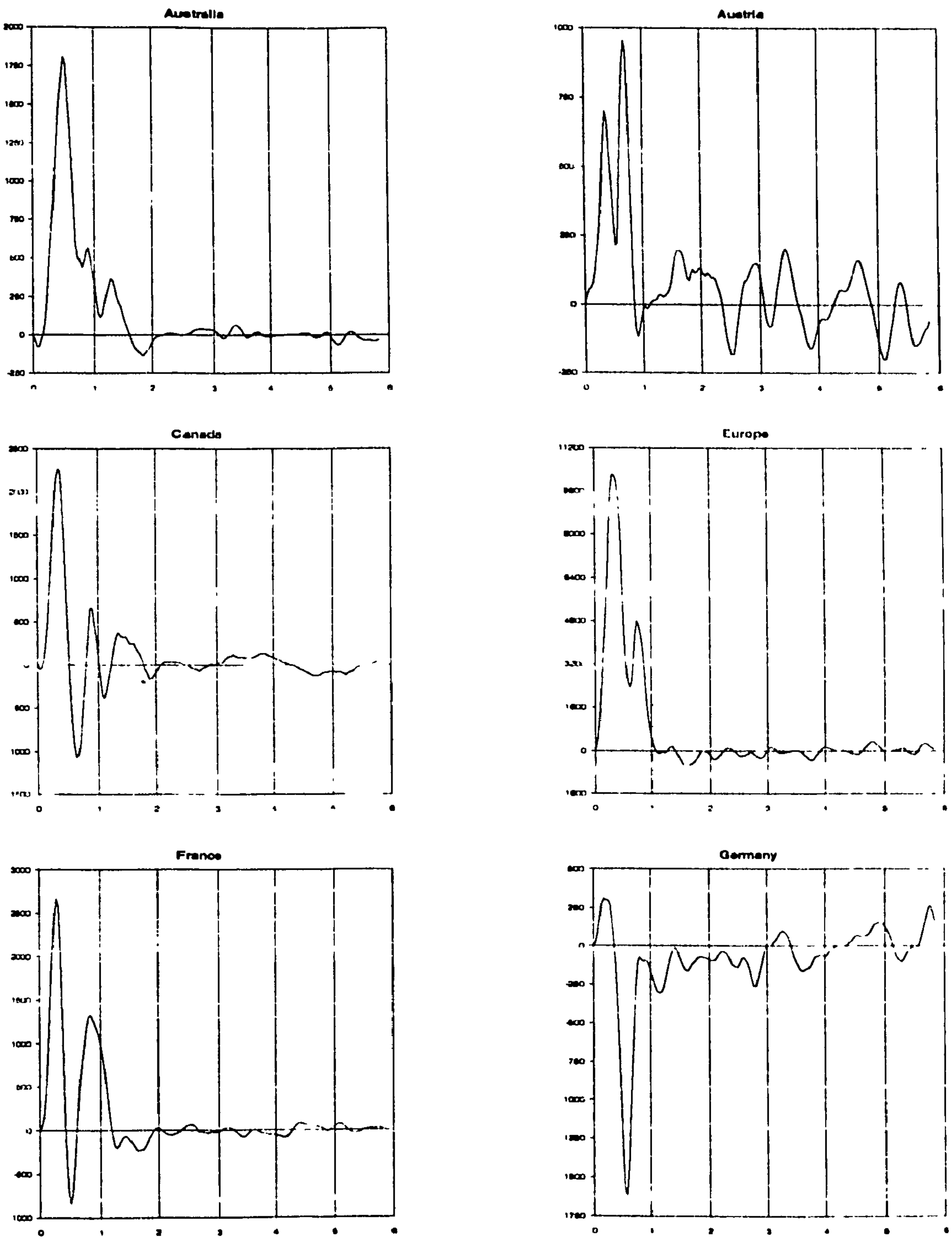

Figure 3.2.8: Continued

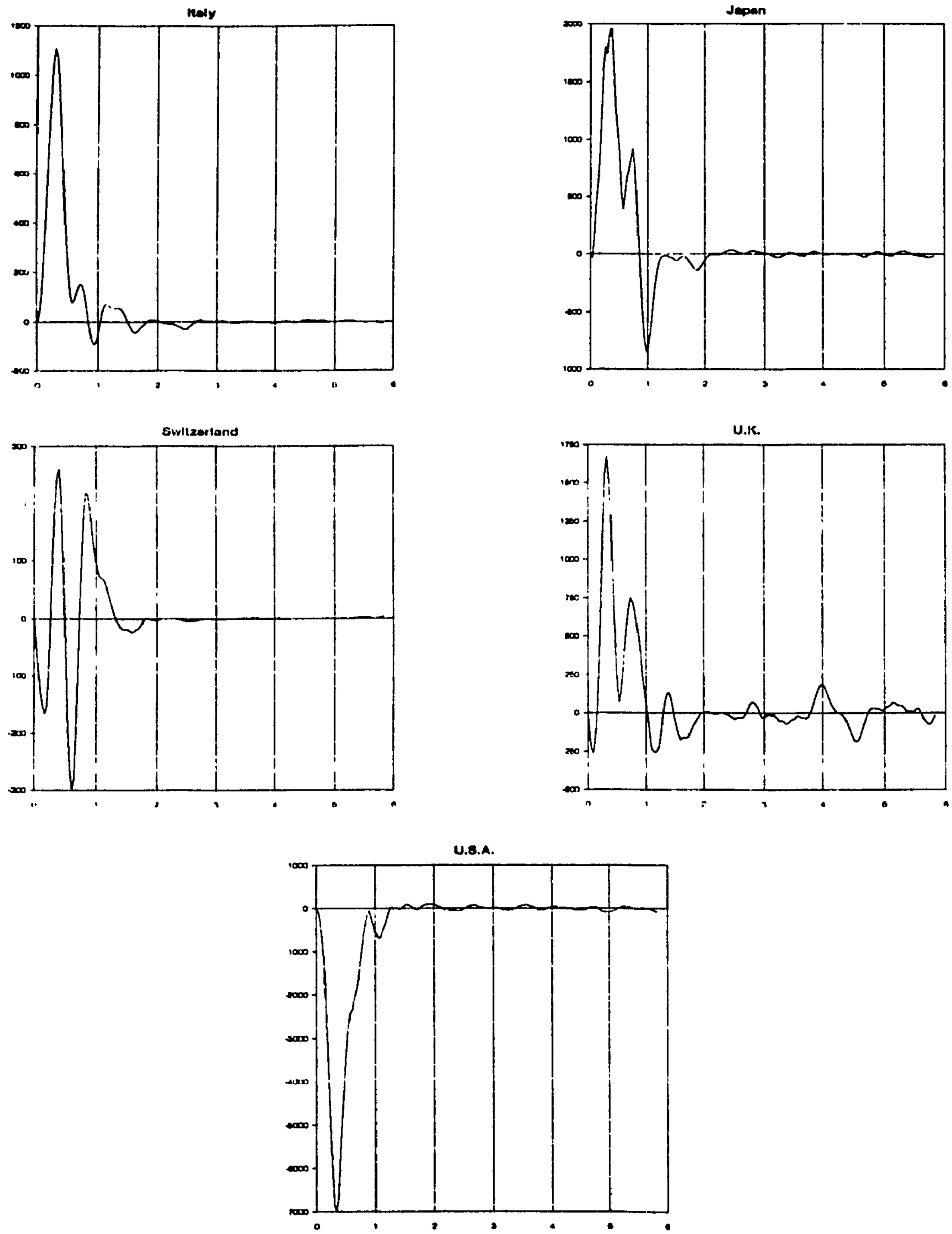

Figure 3.2.9: Estimated Cospectrum (P,Y)

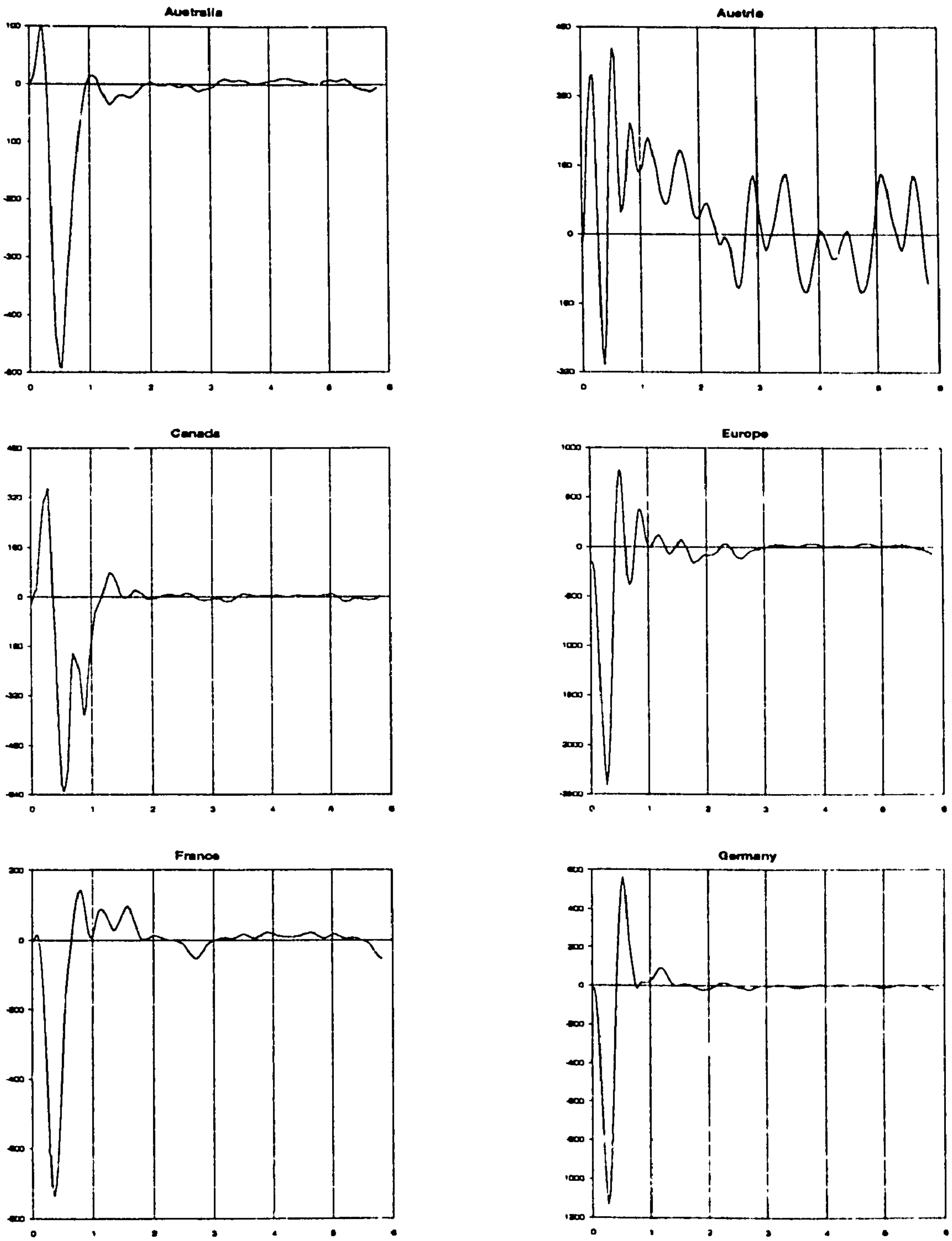

Figure 3.2.9: Continued

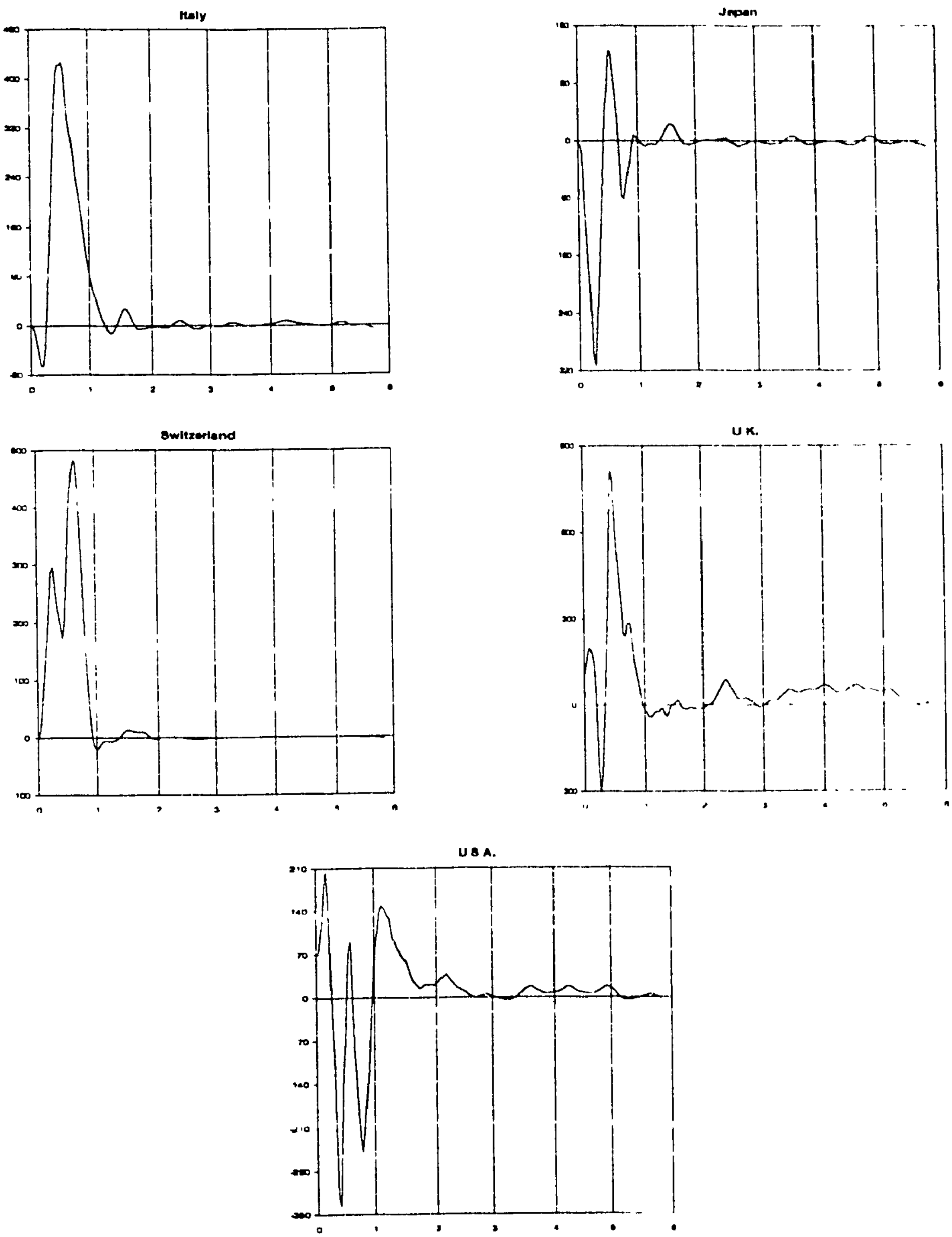

Figure 3.2.10: Estimated Quadrature Spectrum (P,Y)

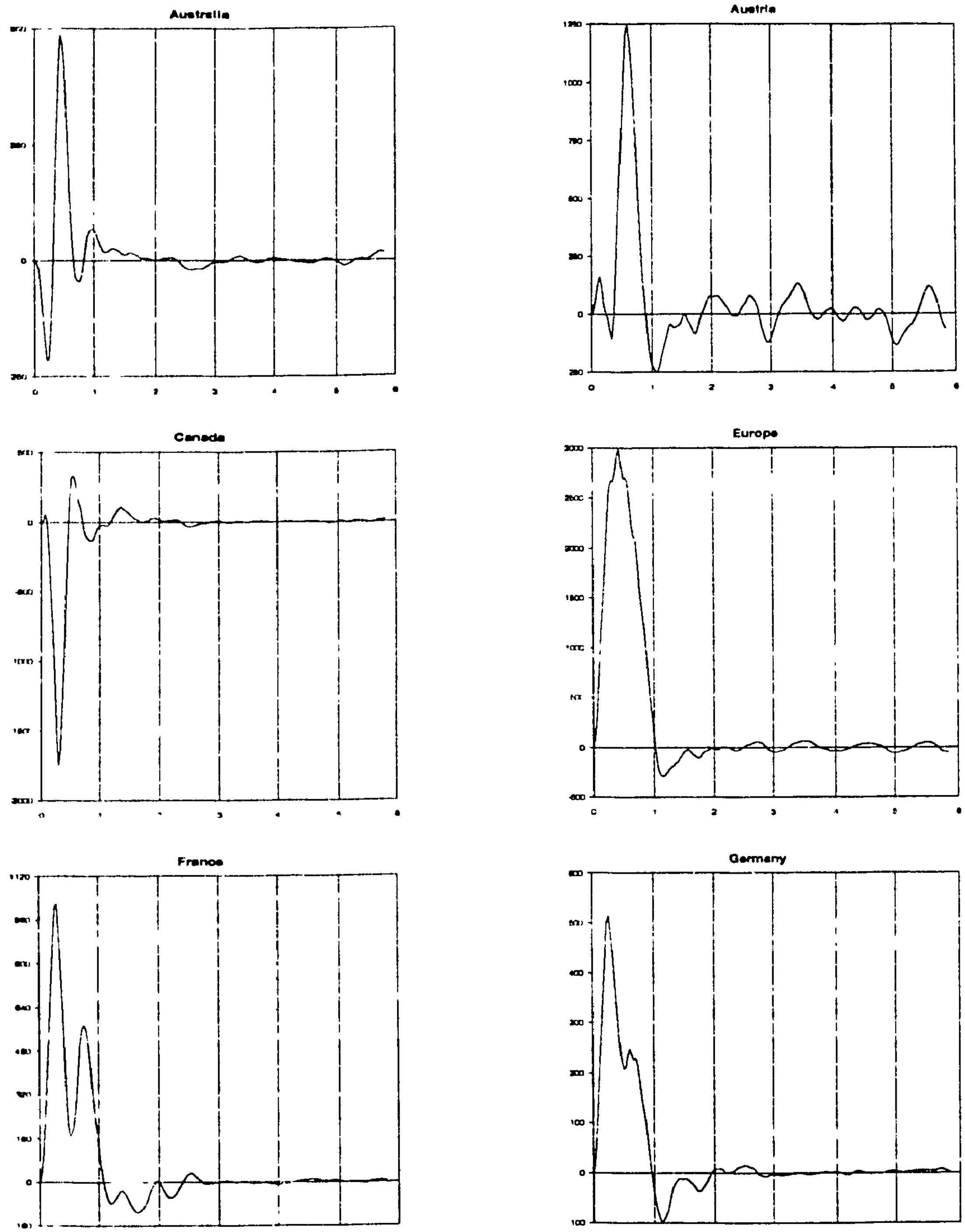

Figure 3.2.10: Continued

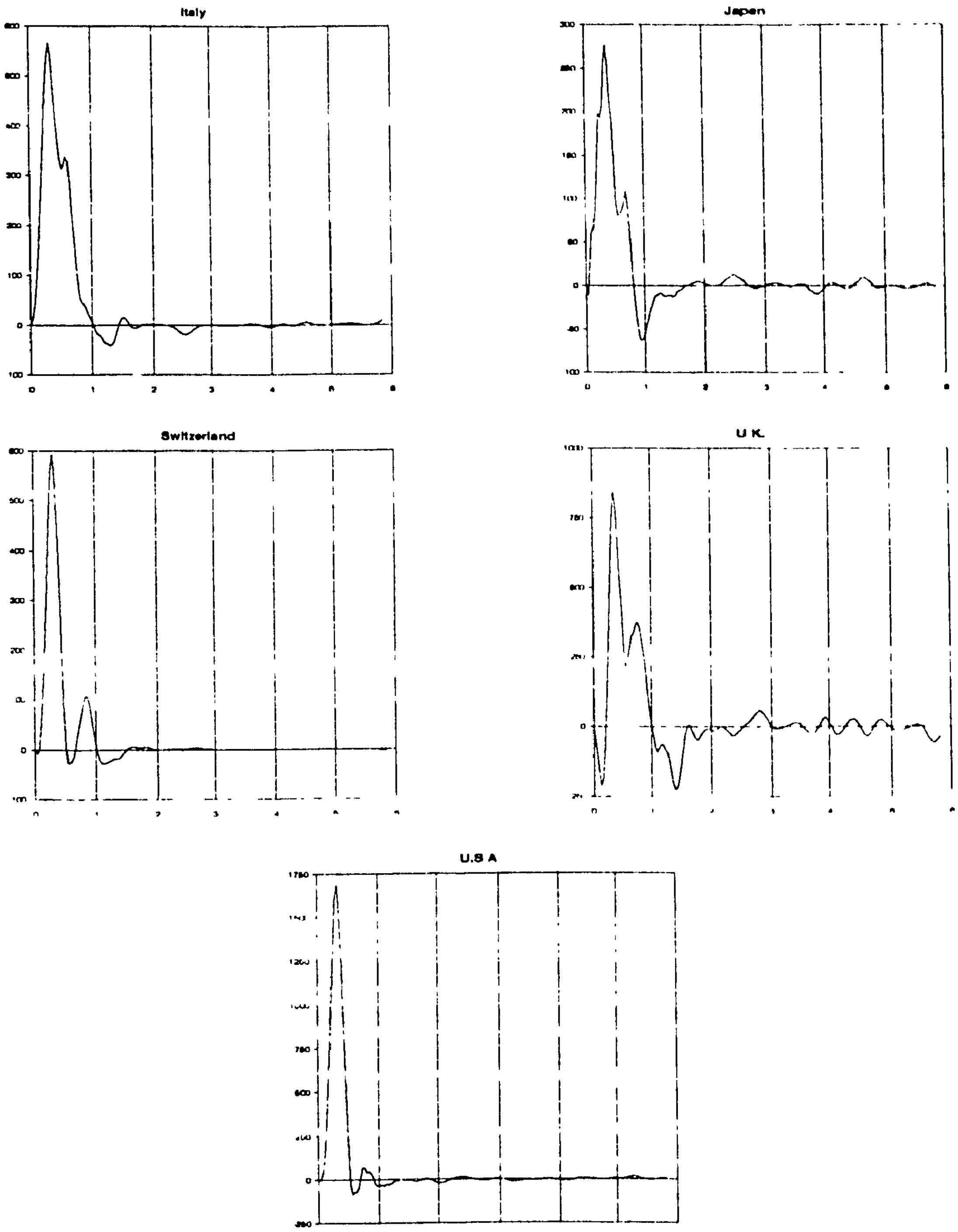

Figure 3.2.11: Estimated Spectral Distribution Functions, Trade Balance (NX)

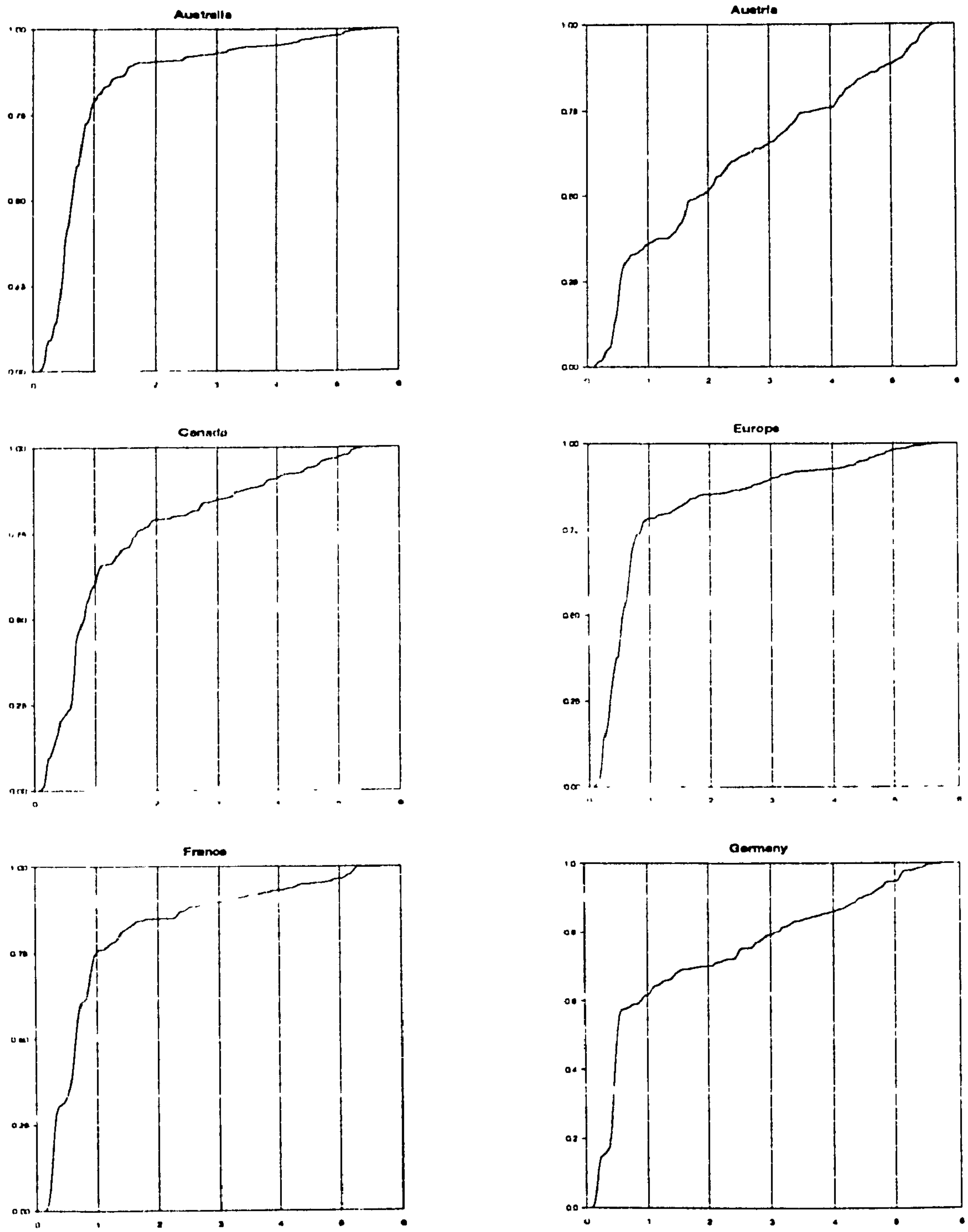

Figure 3.2.11: Continued

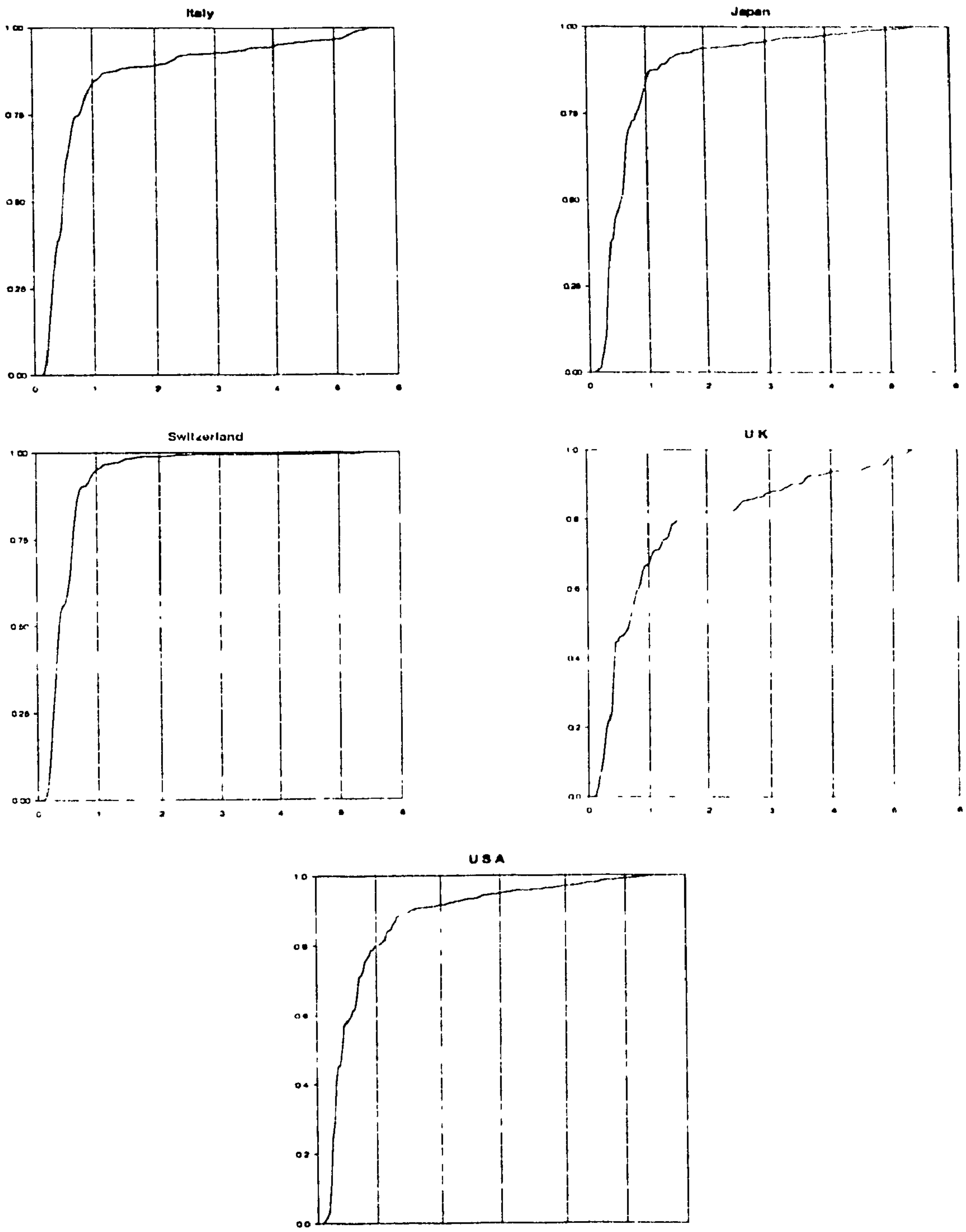

Figure 3.2.12: Estimated Spectral Distribution Functions, Terms of Trade (P)

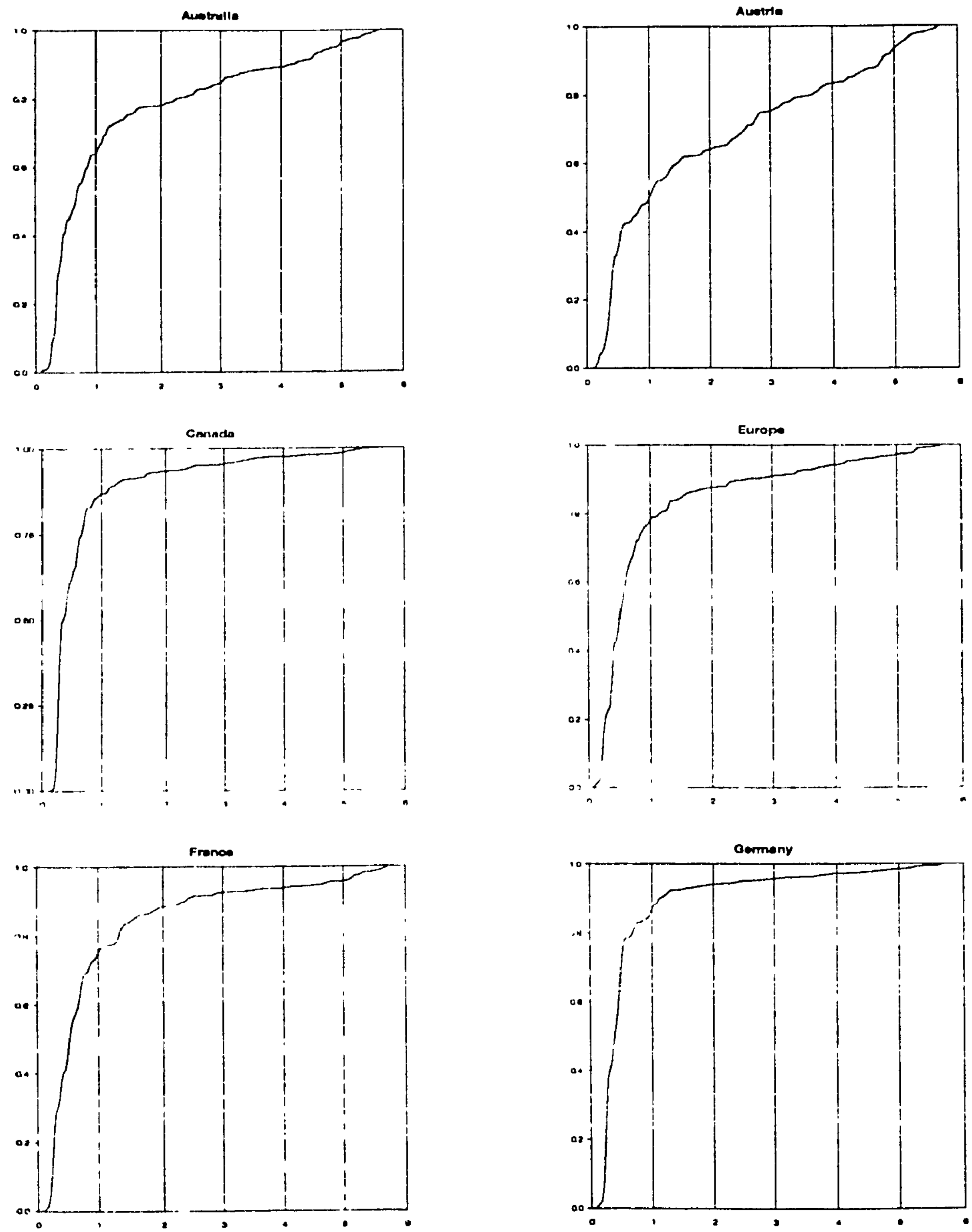

Figure 3.2.12: Continued

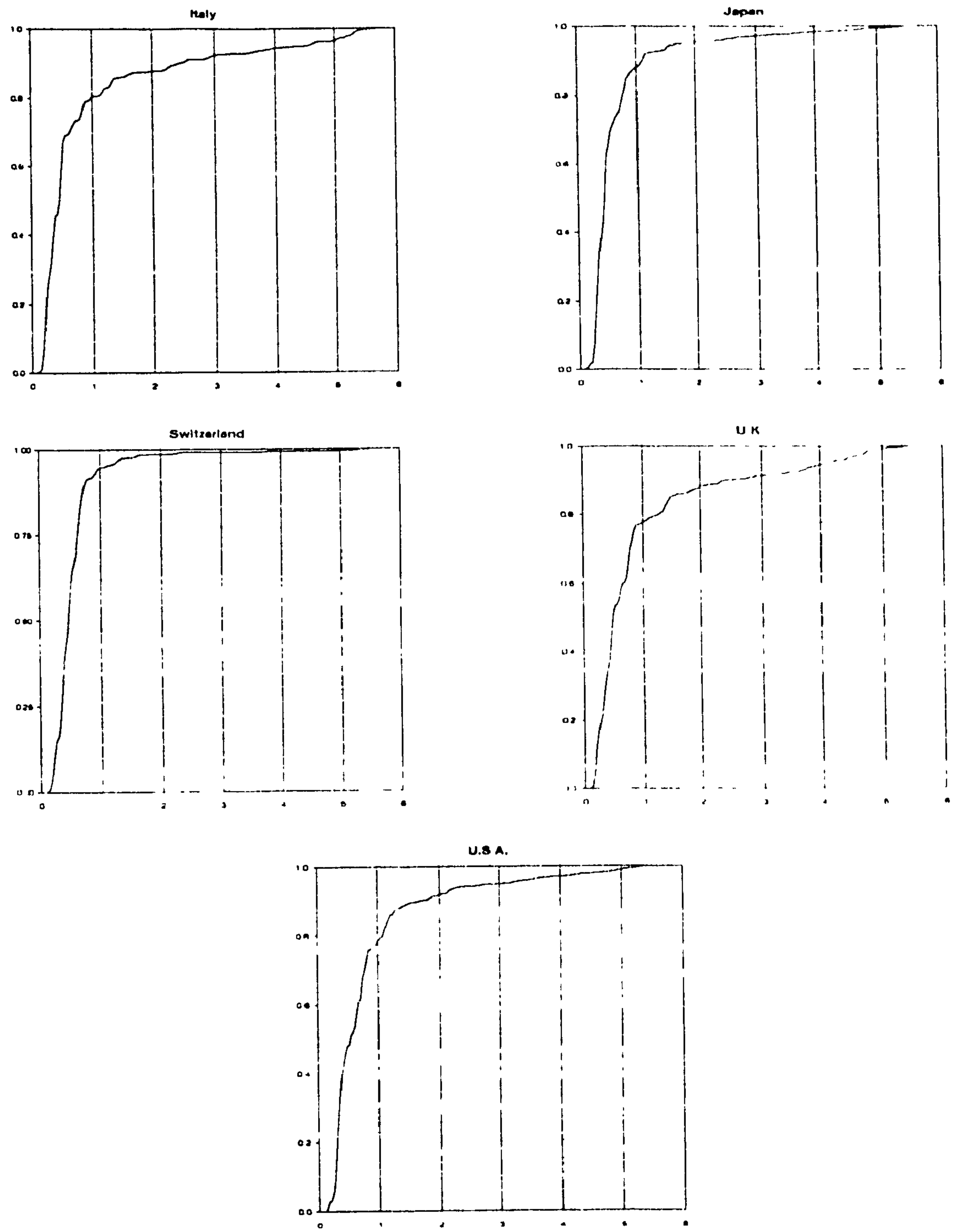

Figure 3.2.13: Estimated Spectral Distribution Functions, Real G.D.P. (Y)

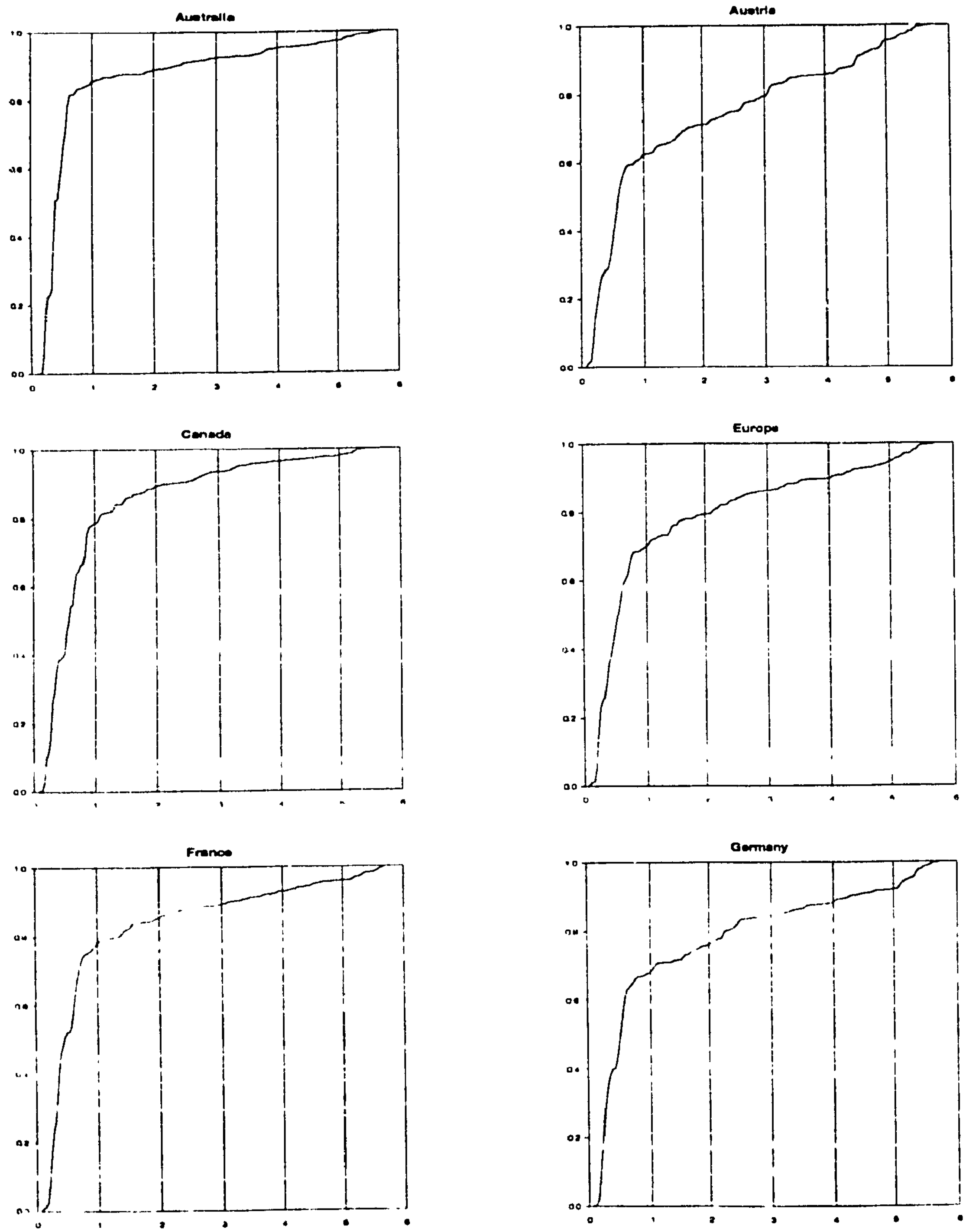

Figure 3.2.13: Continued

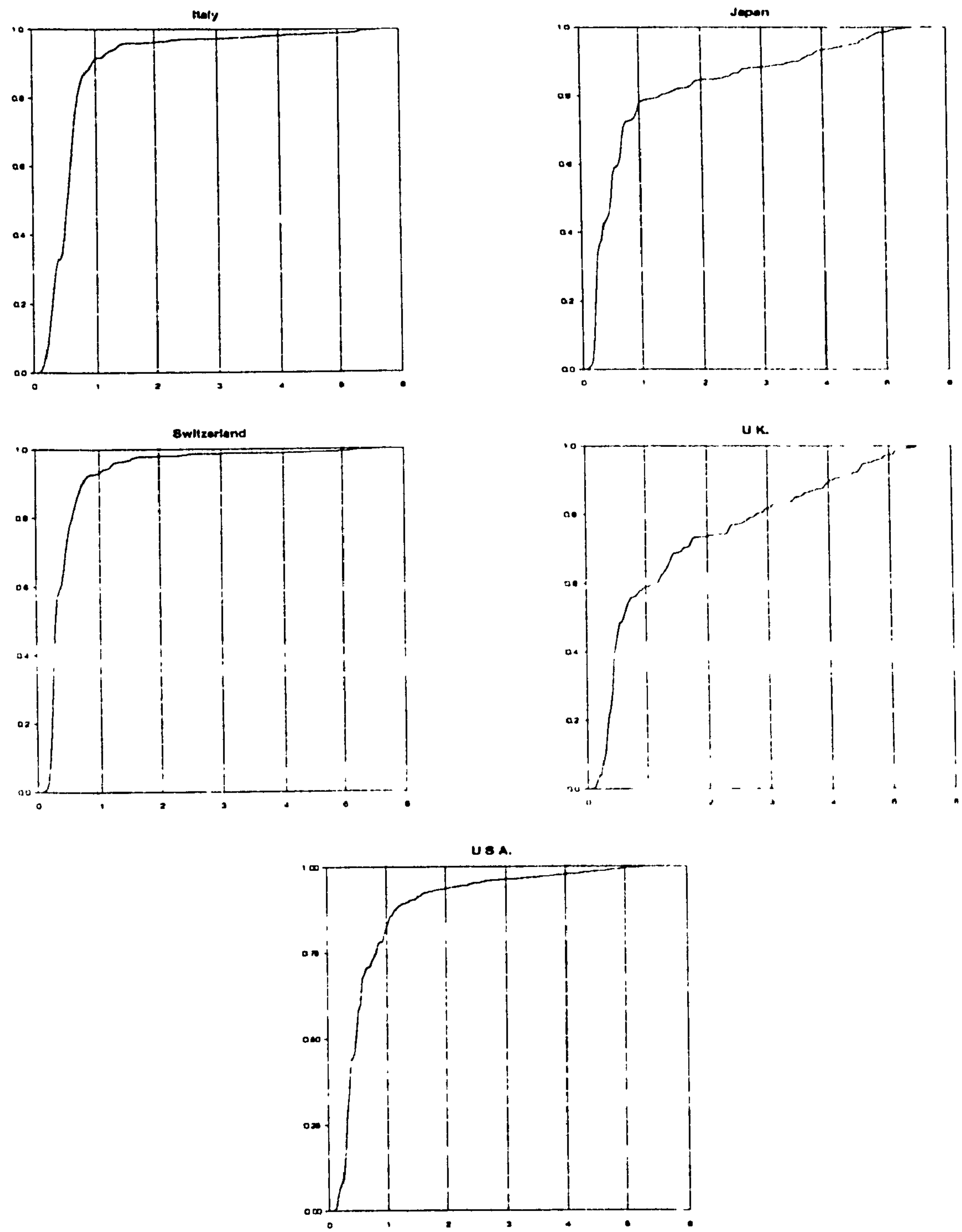

Figure 3.6.1: Autocorrelation Functions, Simulated Data, Real G.D.P. (Y)

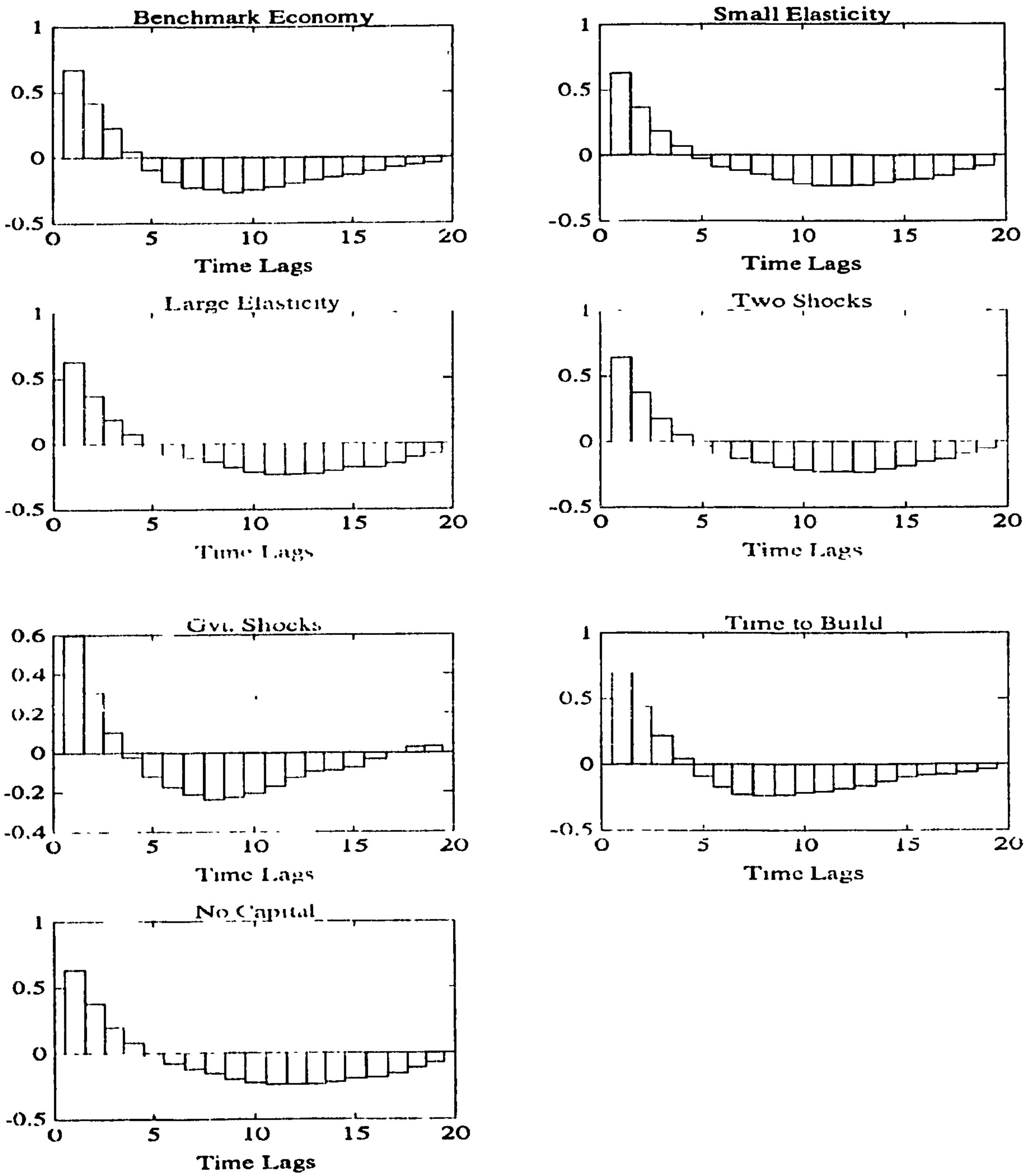

Figure 3.6.2: Autocorrelation Functions, Simulated Data, Terms of Trade (P)

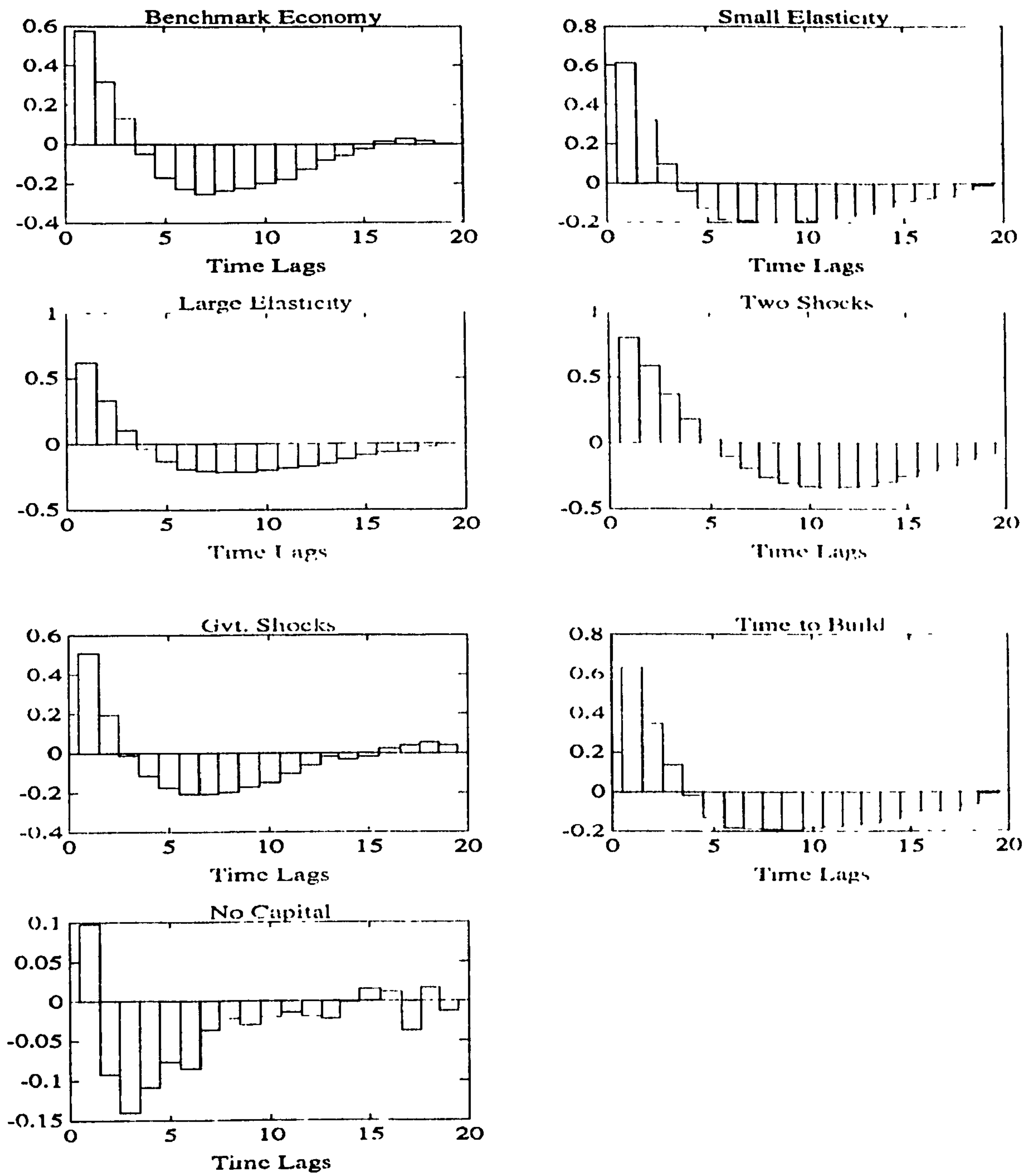

Figure 3.6.3: Autocorrelation Functions, Simulated Data, Trade Balance (NX)

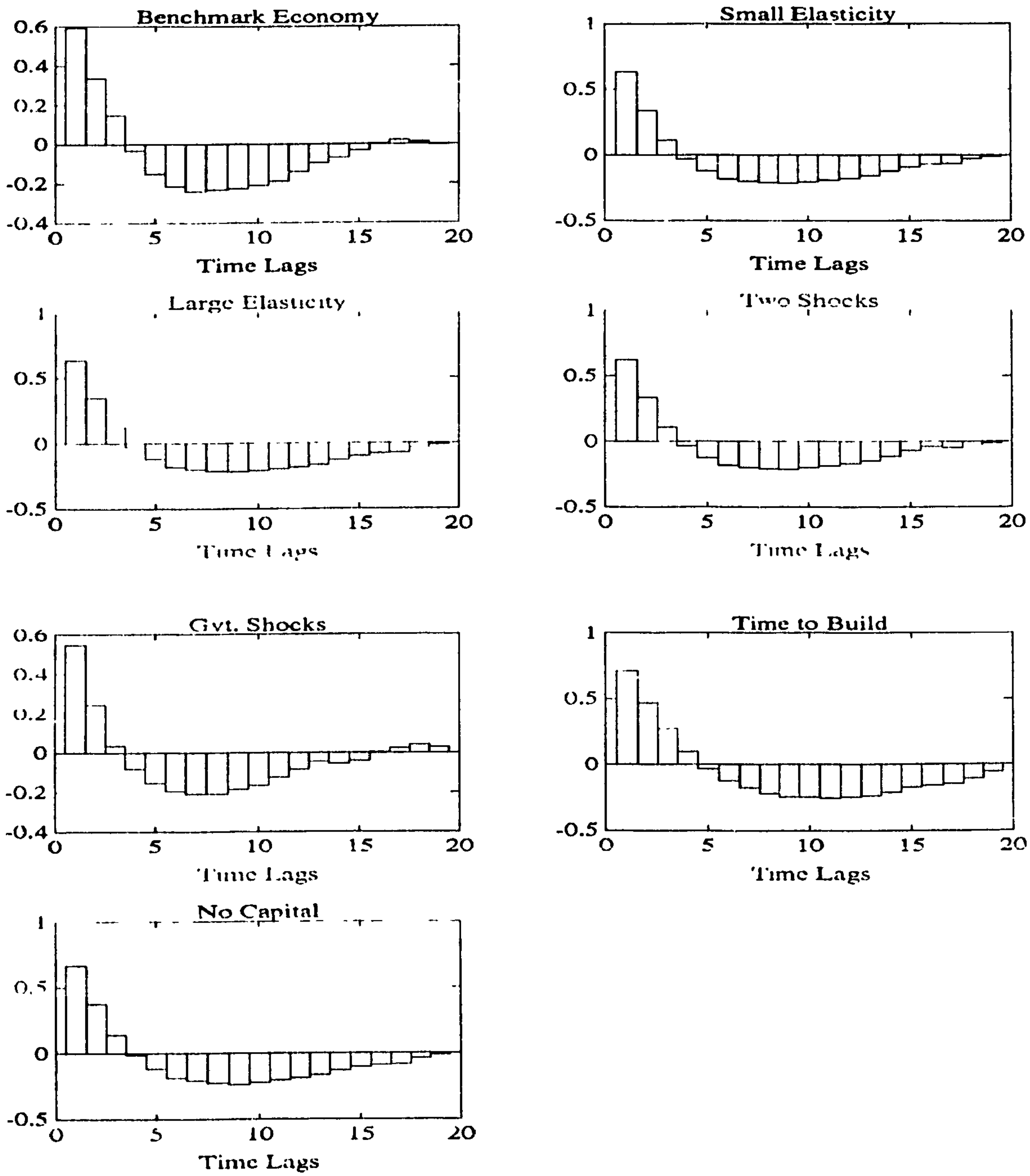

Figure 3.6.4: Cross-Correlation Functions, Simulated Data, Simulations 1-3 (NX,P)

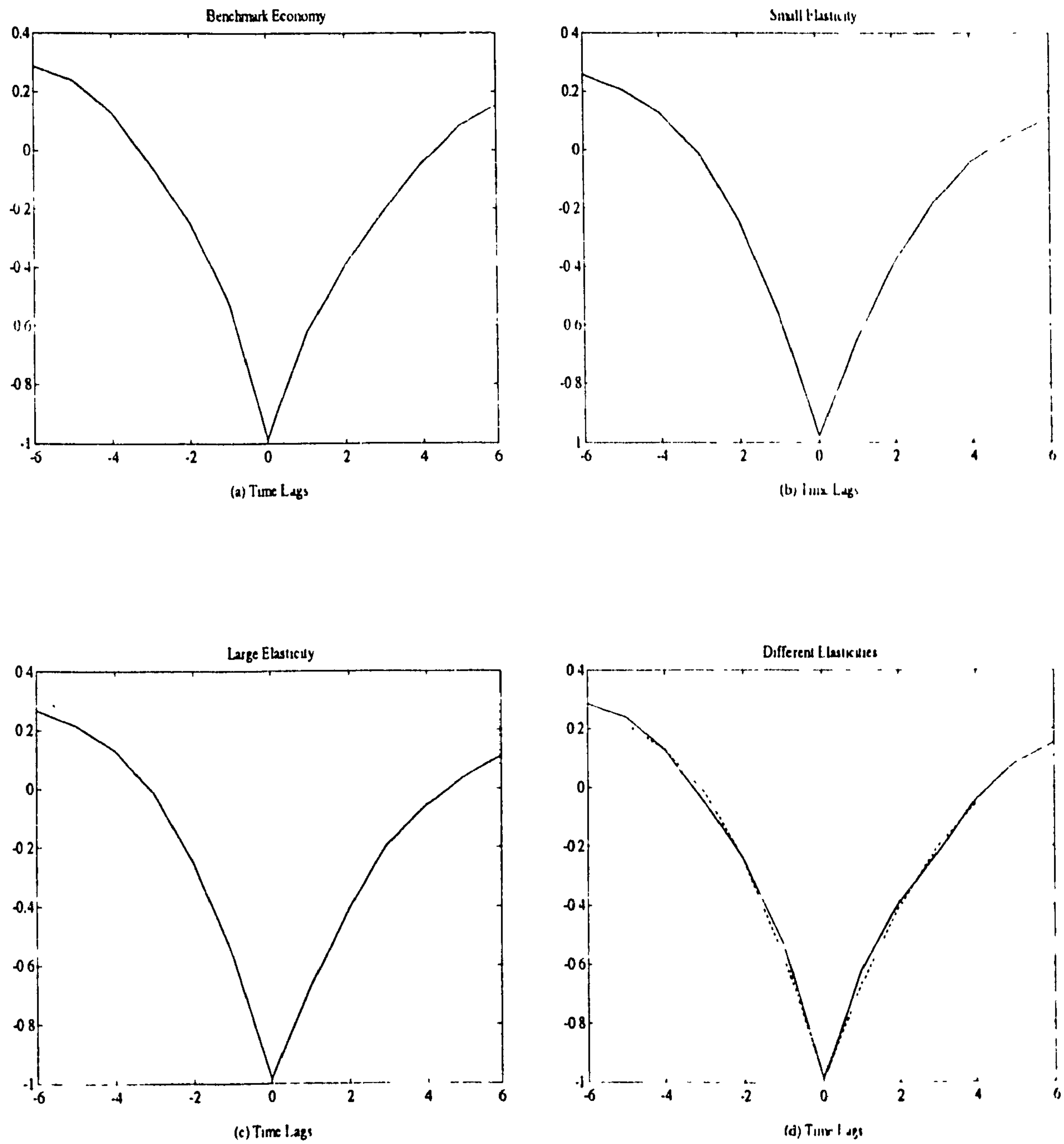

Figure 3.6.5: Cross-Correlation Functions, Simulated Data Simulations 4-7 (NX.P)

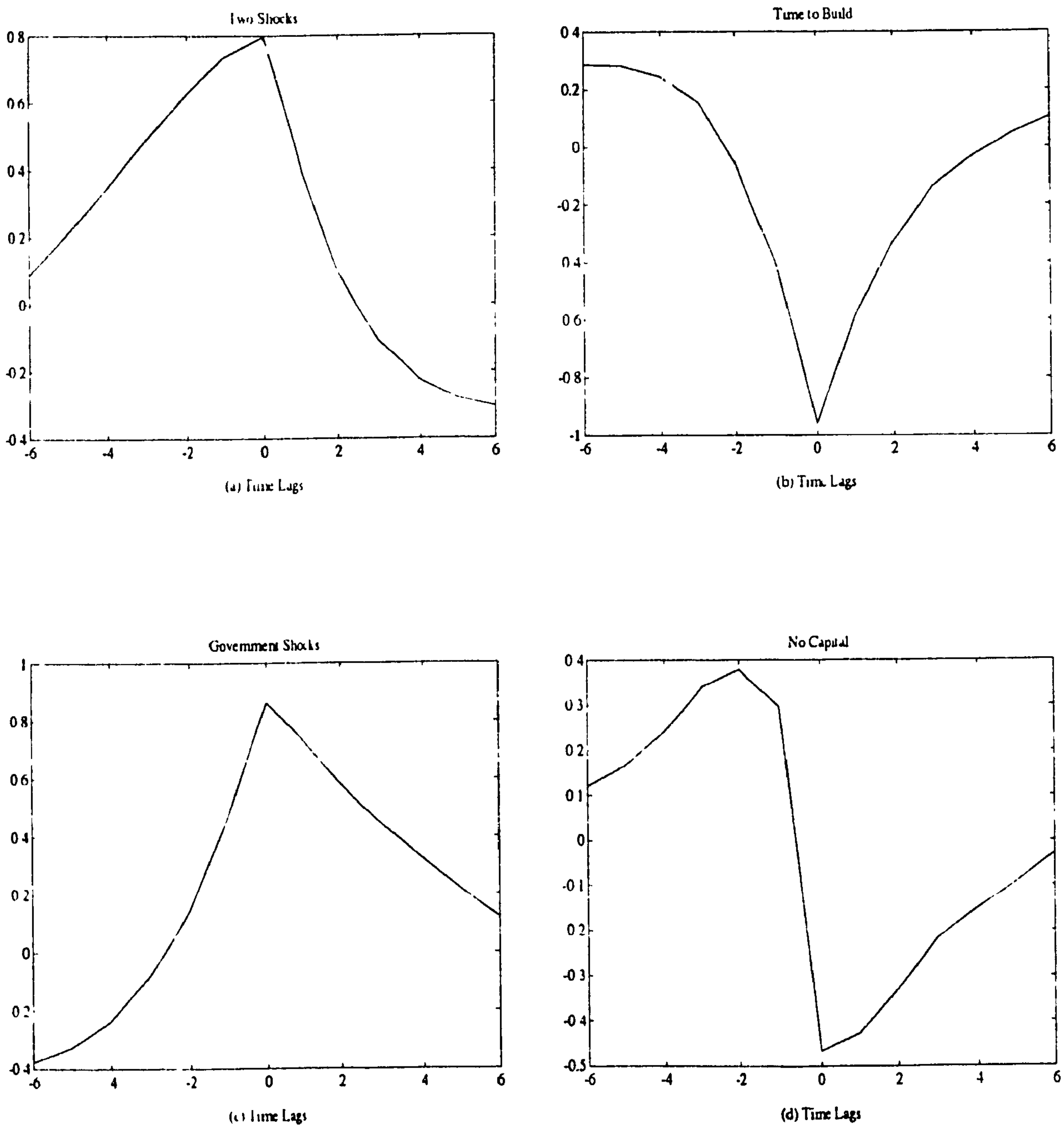

Figure 3.6.6: Cross-Correlation Functions, Simulated Data, Simulations 1, 6 and 7 (NX,P)

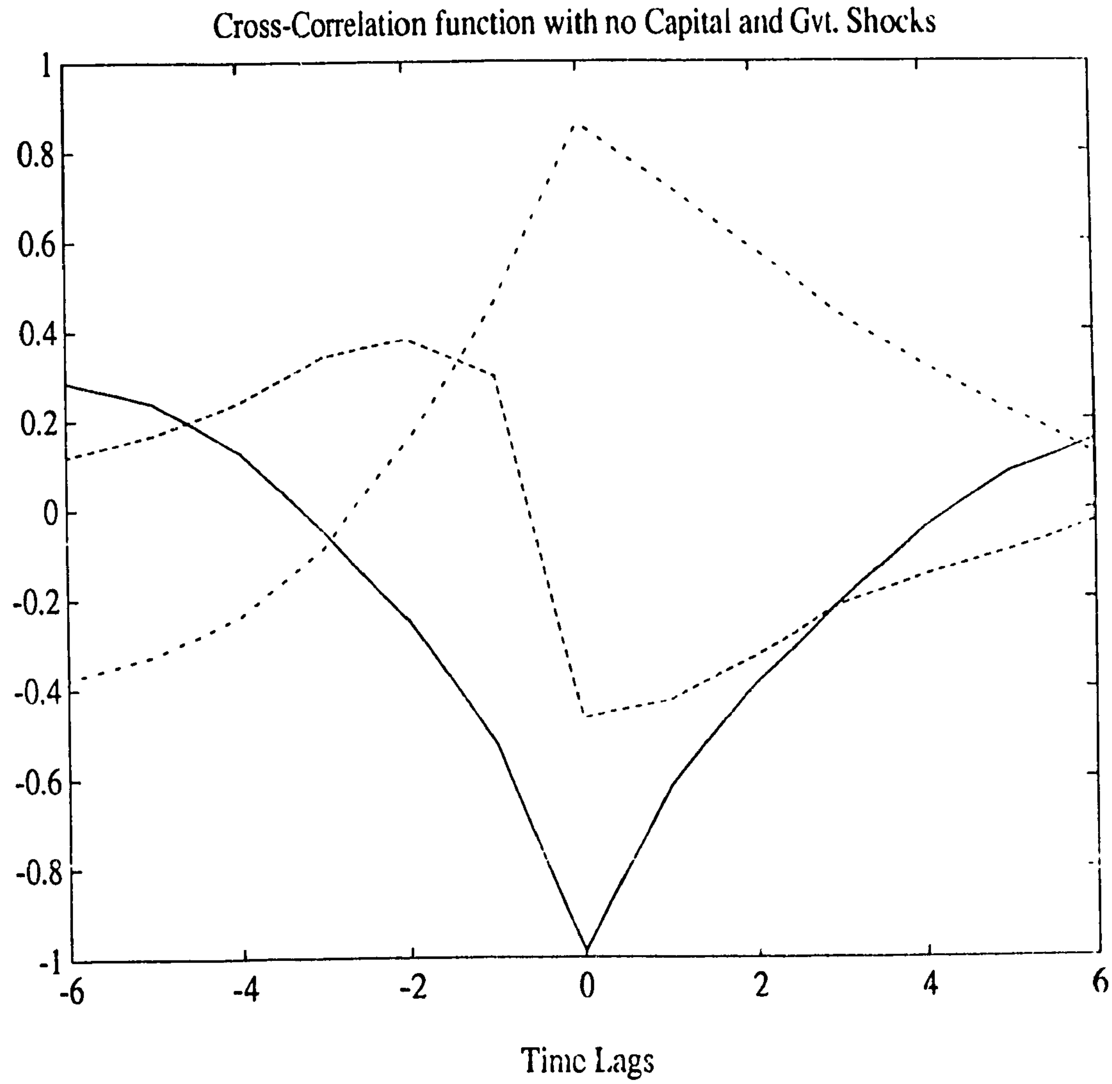

Table 3.2.1 Sample Size For Each Country

| Country | Sample | Number of Observations |
|---|---|---|
| Australia | 1960:1-1990:1 | 121 |
| Austria | 1964:1-1990:1 | 109 |
| Canada | 1955:1-1990:1 | 151 |
| Europe | 1960:1-1990:1 | 121 |
| France | 1970:1-1990:1 | 81 |
| Germany | 1968:1-1990:1 | 97 |
| Italy | 1970:1-1990:1 | 81 |
| Japan | 1955:2-1990:1 | 150 |
| Switzerland | 1970:1-1990:1 | 81 |
| UK | 1955:1-1990:1 | 151 |
| US | 1950:1-1990:1 | 161 |

Table 3.2.2. Summary Statistics, Historical Data, HP-Filtered.

| | Standard deviation | | | Autocorrelation | | | Correlation | | |
|---|---|---|---|---|---|---|---|---|---|
| Country | y | p | nx | y | p | nx | (nx,y) | (nx,p) | (y,p) |
| Australia | 1.53 | 5.25 | 1.36 | 0.65 | 0.82 | 0.74 | -0.19 | -0.09 | -0.27 |
| | (0.16) | (0.70) | (0.15) | (0.19) | (0.23) | (0.18) | (0.17) | (0.11) | (0.11) |
| Austria | 1.20 | 1.63 | 1.11 | 0.60 | 0.50 | 0.29 | -0.44 | -0.16 | 0.13 |
| | (0.13) | (0.20) | (0.09) | (0.18) | (0.15) | (0.12) | (0.12) | (0.12) | (0.11) |
| Canada | 1.52 | 2.44 | 0.79 | 0.76 | 0.85 | 0.59 | -0.42 | 0.04 | -0.10 |
| | (0.18) | (0.35) | (0.06) | (0.22) | (0.25) | (0.13) | (0.19) | (0.08) | ((0.10) |
| Europe | 0.94 | 1.48 | 0.45 | 0.70 | 0.76 | 0.74 | -0.25 | -0.43 | -0.14 |
| | (0.12) | (0.14) | (0.09) | (0.11) | (0.17) | (0.16) | (0.13) | (0.19) | (0.07) |
| France | 0.91 | 3.54 | 0.83 | 0.76 | 0.75 | 0.71 | -0.15 | -0.50 | -0.12 |
| | (0.14) | (0.54) | (0.10) | (0.27) | (0.21) | (0.19) | (0.21) | (0.22) | (0.15) |
| Germany | 1.50 | 2.64 | 0.80 | 0.69 | 0.86 | 0.60 | -0.17 | -0.00 | 0.13 |
| | (0.19) | (0.26) | (0.08) | (0.23) | (0.18) | (0.19) | (0.13) | (0.16) | (0.10) |
| Italy | 1.69 | 3.52 | 1.34 | 0.85 | 0.79 | 0.80 | -0.68 | -0.66 | 0.38 |
| | (0.28) | (0.40) | (0.19) | (0.29) | (0.19) | (0.26) | (0.28) | (0.21) | (0.21) |
| Japan | 1.67 | 5.89 | 1.01 | 0.74 | 0.86 | 0.81 | -0.18 | -0.48 | -0.10 |
| | (0.16) | (0.86) | (0.10) | (0.17) | (0.27) | (0.17) | (0.12) | (0.13) | (0.16) |
| Switzerland | 1.93 | 2.92 | 1.33 | 0.90 | 0.88 | 0.90 | -0.68 | -0.61 | 0.40 |
| | (0.38) | (0.32) | (0.23) | (0.36) | (0.20) | (0.32) | (0.29) | (0.19) | (0.19) |
| United | 1.47 | 2.66 | 1.06 | 0.56 | 0.75 | 0.67 | -0.23 | -0.54 | 0.19 |
| Kingdom | (0.15) | (0.47) | (0.13) | (0.15) | (0.32) | (0.21) | (0.08) | (0.27) | (0.07) |
| United | 1.83 | 2.92 | 0.45 | 0.82 | 0.80 | 0.80 | -0.22 | 0.27 | 0.03 |
| States | (0.17) | (0.42) | (0.04) | (0.16) | (0.24) | (0.14) | (0.14) | (0.11) | (0.15) |
| Median | 1.52 | 2.92 | 1.01 | 0.74 | 0.74 | 0.80 | -0.23 | -0.43 | -0.10 |

Table 3.2.3. Autocorrelation functions for eleven developed countries

| Country | Number of Lags 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Australia | 0.74 | 0.37 | -0.0[illegible] | -0.29 | -0.37 | -0.38 | -0.30 | -0.21 | -0.06 | 0.01 | 0.02 | 0.03 | 0.2 | 0.05 | 0.06 | 0.03 | -0.01 | -0.02 | -0.04 |
| Austria | 0.29 | 0.12 | 0.09 | 0.06 | -0.16 | -0.30 | -0.28 | -0.38 | -0.15 | -0.10 | -0.08 | -0.12 | 0.10 | 0.25 | 0.22 | 0.11 | 0.05 | 0.06 | 0.01 |
| Canada | 0.78 | 0.30 | 0.0[illegible] | -0.16 | -0.23 | -0.24 | -0.2[illegible] | -0.17 | 0.00 | 0.11 | 0.11 | 0.05 | 0.3 | -0.09 | 0.09 | -0.12 | -0.17 | -0.15 | -0.17 |
| Europe | 0.[illegible] | 0.31 | 0.19 | -0.06 | -0.01 | -0.18 | -0.21 | -0.20 | -0.08 | -0.10 | -0.10 | -0.10 | -0.05 | -0.15 | -0.24 | -0.16 | -0.19 | -0.04 | -0.00 |
| France | 0.70 | 0.38 | 0.07 | -0.15 | -0.24 | -0.21 | -0.19 | -0.15 | -0.01 | 0.00 | -0.07 | -0.16 | -0.23 | -0.28 | -0.28 | -0.20 | -0.22 | -0.21 | -0.15 |
| Germany | 0.69 | 0.48 | 0.3[illegible] | 0.06 | -0.03 | -0.17 | -0.31 | -0.41 | -0.39 | -0.34 | -0.26 | -0.13 | -0.01 | 0.13 | 0.23 | 0.26 | 0.18 | 0.14 | 0.04 |
| Italy | 0.8[illegible] | 0.55 | 0.27 | 0.00 | -0.21 | -0.28 | -0.30 | -0.34 | -0.30 | -0.26 | -0.23 | -0.19 | -0.11 | -0.08 | -0.05 | -0.05 | -0.07 | -0.13 | -0.12 |
| Japan | 0.81 | 0.54 | 0.2[illegible] | -0.02 | -0.19 | -0.28 | -0.33 | -0.31 | -0.28 | -0.25 | -0.22 | -0.22 | -0.19 | -0.18 | -0.15 | -0.11 | -0.06 | -0.04 | -0.03 |
| Switzerland | 0.8[illegible] | 0.64 | 0.3[illegible] | 0.08 | -0.11 | -0.24 | -0.30 | -0.30 | -0.25 | -0.20 | -0.18 | -0.18 | -0.19 | -0.23 | -0.27 | -0.31 | -0.32 | -0.30 | -0.22 |
| UK | 0.63 | 0.43 | 0.20 | -0.03 | -0.10 | -0.13 | -0.13 | -0.21 | -0.22 | -0.19 | -0.22 | -0.22 | -0.21 | -0.20 | -0.23 | -0.18 | -0.02 | 0.04 | 0.11 |
| USA | 0.89 | 0.57 | 0.34 | 0.15 | 0.01 | -0.04 | -0.12 | -0.19 | -0.26 | -0.30 | -0.30 | -0.28 | -0.27 | -0.30 | -0.33 | -0.34 | -0.33 | -0.27 | -0.16 |

Table 3.6.1. Summary Statistics, Simulated Data, HP-Filtered

| | Standard deviation | | | Autocorrelation | | | Correlation | | |
|---|---|---|---|---|---|---|---|---|---|
| Economy | y | p | nx | y | p | nx | (nx,y) | (nx,p) | (y,p) |
| Benchmark | 1.03 | 0.14 | 0.20 | 0.67 | 0.57 | 0.59 | -0.60 | -0.98 | 0.59 |
| | (0.09) | (0.01) | (0.03) | (0.05) | (0.08) | (0.07) | (0.12) | (0.01) | (0.12) |
| Small elasticity | 0.80 | 0.41 | 0.23 | 0.63 | 0.61 | 0.63 | -0.64 | -0.97 | 0.71 |
| | (0.01) | (0.03) | (0.02) | (0.10) | (0.06) | (0.05) | (0.09) | (0.01) | (0.08) |
| Large elasticity | 1.01 | 0.06 | 0.21 | 0.63 | 0.62 | 0.63 | -0.59 | -0.98 | 0.57 |
| | (0.17) | (0.01) | (0.02) | (0.10) | (0.05) | (0.06) | (0.10) | (0.01) | (0.11) |
| Two shocks | 0.92 | 0.16 | 0.21 | 0.64 | 0.80 | 0.62 | -0.74 | 0.80 | -0.79 |
| | (0.13) | (0.03) | (0.02) | (0.09) | (0.07) | (0.06) | (0.08) | (0.03) | (0.07) |
| Time to build | 0.52 | 0.46 | 0.21 | 0.60 | 0.50 | 0.54 | -0.93 | -0.95 | 0.85 |
| | (0.06) | (0.05) | (0.02) | (0.09) | (0.10) | (0.01) | (0.04) | (0.01) | (0.05) |
| Government shocks | 0.28 | 0.19 | 0.02 | 0.69 | 0.63 | 0.71 | -0.57 | 0.86 | -0.49 |
| | (0.03) | (0.02) | (0.00) | (0.06) | (0.08) | (0.08) | (0.13) | (0.03) | (0.17) |
| No capital | 0.30 | 0.33 | 0.06 | 0.65 | 0.01 | 0.67 | -0.32 | -0.47 | -0.25 |
| | (0.05) | (0.03) | (0.00) | (0.12) | (0.07) | (0.06) | (0.16) | (0.07) | (0.10) |

Table 3.6.2. Univariate Spectral Shape Tests
Accumulated Variance Contributions
$\hat{D}(t,0)$
Benchmark Economy (Y)

| Country | Frequencies | | | | |
|---|---|---|---|---|---|
| | $t=\frac{\pi}{6}$ | $t=\frac{2\pi}{6}$ | $t=\frac{3\pi}{6}$ | $t=\frac{4\pi}{6}$ | $t=\frac{5\pi}{6}$ |
| Australia | -0.0323 | 0.0033 | -0.0009 | -0.0096 | 0.0001 |
| Austria | 0.0063 | -0.0205 | 0.0245 | 0.0258 | -0.0018 |
| Canada | 0.0304 | 0.0325 | 0.0259 | 0.0147 | 0.0091 |
| Europe | 0.0218 | 0.0062 | 0.0030 | 0.0044 | -0.0019 |
| France | 0.0260 | 0.0295 | 0.0195 | 0.0137 | 0.0029 |
| Germany | 0.0550 | 0.0011 | -0.0080 | 0.0097 | -0.0020 |
| Italy | 0.0221 | 0.0735 | 0.0515 | 0.0298 | 0.0112 |
| Japan | 0.0195 | 0.0245 | 0.0114 | 0.0027 | -0.0079 |
| Switzerland | 0.1210 | 0.0821 | 0.0558 | 0.0327 | 0.0137 |
| United Kingdom | 0.0063 | -0.0374 | -0.0330 | -0.0254 | -0.0189 |
| United States | 0.0611 | 0.0574 | 0.0393 | 0.0235 | 0.0076 |
| Standard error | (0.0460) | (0.0206) | (0.0137) | (0.0092) | (0.0040) |

Table 3.6.3. Univariate Spectral Shape Tests
Incremental Variance Contributions
$\hat{D}(t,t-\frac{\pi}{6})$
Benchmark Economy (Y)

| Country | Frequencies | | | | | |
|---|---|---|---|---|---|---|
| | $t=\frac{\pi}{6}$ | $t=\frac{2\pi}{6}$ | $t=\frac{3\pi}{6}$ | $t=\frac{4\pi}{6}$ | $t=\frac{5\pi}{6}$ | $t=\pi$ |
| Australia | -0.0323 | 0.0356 | -0.0042 | -0.0086 | 0.0097 | -0.0001 |
| Austria | 0.0063 | -0.0268 | -0.0039 | -0.0014 | 0.0241 | 0.0018 |
| Canada | 0.0301 | 0.0021 | -0.0067 | -0.0111 | -0.0057 | -0.0091 |
| Europe | 0.0218 | 0.0156 | 0.0031 | 0.0013 | 0.0062 | 0.0019 |
| France | 0.0260 | 0.0036 | -0.0101 | -0.0057 | -0.0109 | -0.0029 |
| Germany | 0.0550 | -0.0539 | -0.0090 | 0.0177 | -0.0117 | 0.0020 |
| Italy | 0.0221 | 0.0534 | -0.0239 | -0.0217 | -0.0186 | -0.0112 |
| Japan | 0.0195 | -0.0250 | -0.0131 | -0.0087 | -0.0107 | 0.0079 |
| Switzerland | 0.1210 | -0.0389 | 0.0263 | 0.0231 | -0.0190 | -0.0137 |
| United Kingdom | -0.0063 | -0.0311 | 0.0044 | 0.0076 | 0.0065 | 0.0189 |
| United States | 0.0611 | -0.0037 | -0.0180 | -0.0159 | -0.0159 | -0.0076 |
| Standard error | (0.0460) | (0.0370) | (0.0140) | (0.0107) | (0.0082) | (0.0040) |

Table 3.6.4. Univariate Spectral Shape Tests
Accumulated Variance Contributions
$\hat{D}(t,0)$
Benchmark Economy (P)

| Country | Frequencies | | | | |
|---|---|---|---|---|---|
| | $t = \frac{\pi}{6}$ | $t = \frac{2\pi}{6}$ | $t = \frac{3\pi}{6}$ | $t = \frac{4\pi}{6}$ | $t = \frac{5\pi}{6}$ |
| Australia | 0.1732 | 0.0894 | 0.0526 | 0.0330 | 0.0133 |
| Austria | 0.0065 | -0.0323 | -0.0296 | -0.0285 | 0.0071 |
| Canada | 0.1390 | 0.0947 | 0.0611 | 0.0362 | 0.0173 |
| Europe | 0.0858 | 0.0598 | 0.0485 | 0.0293 | 0.0092 |
| France | 0.0492 | 0.0429 | 0.0502 | 0.0370 | 0.0168 |
| Germany | 0.1519 | 0.0987 | 0.0666 | 0.0404 | 0.0172 |
| Italy | 0.1156 | 0.0708 | 0.0496 | 0.0321 | 0.0150 |
| Japan | 0.1254 | 0.0995 | 0.0669 | 0.0411 | 0.0171 |
| Switzerland | 0.1082 | 0.1111 | 0.0788 | 0.0486 | 0.0209 |
| United Kingdom | 0.0599 | 0.0643 | 0.0450 | 0.0236 | 0.0031 |
| United States | 0.0604 | 0.0822 | 0.0555 | 0.0351 | 0.0139 |
| Standard error | (0.0509) | (0.0304) | (0.0219) | (0.0174) | (0.0067) |

Table 3.6.5 Univariate Spectral Shape Tests
Incremental Variance Contributions
$\hat{D}(t, t - \frac{\pi}{6})$
Benchmark Economy (P)

| Country | Frequencies | | | | | |
|---|---|---|---|---|---|---|
| | $t = \frac{\pi}{6}$ | $t = \frac{2\pi}{6}$ | $t = \frac{3\pi}{6}$ | $t = \frac{4\pi}{6}$ | $t = \frac{5\pi}{6}$ | $t = \pi$ |
| Australia | 0.1732 | -0.0838 | -0.0368 | -0.0196 | -0.0196 | -0.0133 |
| Austria | 0.0065 | -0.0388 | 0.0027 | 0.0011 | 0.0211 | 0.0074 |
| Canada | 0.1390 | -0.0444 | -0.0335 | -0.0249 | -0.0189 | 0.0173 |
| Europe | 0.0858 | -0.0260 | -0.0112 | -0.0192 | 0.0202 | 0.0092 |
| France | 0.0492 | -0.0063 | 0.0073 | 0.0131 | 0.0203 | 0.0168 |
| Germany | 0.1519 | -0.0532 | -0.0321 | -0.0262 | -0.0232 | -0.0172 |
| Italy | 0.1156 | -0.0148 | -0.0213 | -0.0175 | 0.0171 | 0.0150 |
| Japan | 0.1254 | -0.0259 | -0.0326 | -0.0258 | -0.0240 | 0.0171 |
| Switzerland | 0.1082 | 0.0030 | -0.0323 | -0.0302 | 0.0277 | 0.0209 |
| United Kingdom | 0.0599 | 0.0044 | -0.0192 | -0.0215 | -0.0205 | -0.0031 |
| United States | 0.0601 | 0.0218 | -0.0267 | -0.0204 | -0.0213 | -0.0139 |
| Standard error | (0.0509) | (0.0375) | (0.0133) | (0.0110) | (0.0144) | (0.0067) |

Table 3.6.6. Univariate Spectral Shape Tests
Accumulated Variance Contributions
$D(t,0)$
Benchmark Economy (NX)

| Country | Frequencies $t=\frac{\pi}{6}$ | $t=\frac{2\pi}{6}$ | $t=\frac{3\pi}{6}$ | $t=\frac{4\pi}{6}$ | $t=\frac{5\pi}{6}$ |
|---|---|---|---|---|---|
| Australia | -0.0259 | 0.0662 | 0.0542 | 0.0308 | 0.0110 |
| Austria | -0.0575 | -0.1258 | 0.0863 | -0.0450 | -0.0323 |
| Canada | -0.0367 | 0.0017 | 0.0077 | 0.0019 | -0.0033 |
| Europe | .0434 | 0.0532 | 0.0391 | 0.0154 | 0.0093 |
| France | -0.0366 | 0.0458 | 0.0383 | 0.0252 | 0.0073 |
| Germany | 0.0764 | 0.0085 | -0.0085 | -0 0112 | -0.0073 |
| Italy | 0.0854 | 0.0851 | 0.0505 | 0.0352 | 0.0127 |
| Japan | 0.0927 | 0.0861 | 0.0582 | 0.0340 | 0.0149 |
| Switzerland | 0.1114 | 0.1101 | 0.0769 | 0.0465 | 0.0203 |
| United Kingdom | 0.0075 | 0.0410 | 0.0155 | 0.0063 | 0.0017 |
| United States | 0.0979 | 0 0800 | 0.0508 | 0 0315 | 0.0123 |
| Standard error | (0.0483) | (0.0281) | (0.0204) | (0.0158) | (0.0064) |

Table 3.6.7. Univariate Spectral Shape Tests
Incremental Variance Contributions
$D(t, t-\frac{\pi}{6})$
Benchmark Economy (NX)

| Country | Frequencies $t=\frac{\pi}{6}$ | $t=\frac{2\pi}{6}$ | $t=\frac{3\pi}{6}$ | $t=\frac{4\pi}{6}$ | $t=\frac{5\pi}{6}$ | $t=\pi$ |
|---|---|---|---|---|---|---|
| Australia | -0.0259 | 0.0920 | -0.0119 | -0.0235 | -0.0198 | -0.0110 |
| Austria | -0.0575 | -0.0683 | 0.0395 | 0.0412 | 0.0128 | 0.0323 |
| Canada | -0 0367 | 0.0384 | 0.0059 | -0.0057 | -0.0052 | 0.0033 |
| Europe | 0.0434 | 0.0098 | -0.0142 | -0.0237 | -0.0061 | -0.0093 |
| France | 0 0366 | 0.0824 | -0.0074 | -0.0132 | 0.0179 | -0.0073 |
| Germany | 0.0764 | -0.0679 | -0.0170 | -0.0027 | 0.0039 | 0.0073 |
| Italy | 0.0854 | -0.0002 | -0.0346 | -0.0153 | -0.0224 | -0.0127 |
| Japan | 0 0927 | -0.0066 | -0.0279 | -0.0242 | -0.0191 | -0.0149 |
| Switzerland | 0.1114 | -0.0009 | -0.0336 | -0.0301 | -0.0263 | -0.0203 |
| United Kingdom | 0.0075 | 0.0335 | -0.0255 | -0.0092 | -0.0047 | -0.0017 |
| United States | 0.0979 | -0.0179 | -0.0292 | -0.0193 | -0.0192 | -0.0123 |
| Standard error | (0.0483) | (0.0362) | (0.0127) | (0.0105) | (0.0131) | (0.0064) |

Table 3.6.8. Multivariate Spectral Shape Tests
Accumulated Covariance Contributions
$\hat{D}_c(t,0)$
Benchmark Economy (NX,P)

| Country | Frequencies $t=\frac{\pi}{6}$ | $t=\frac{2\pi}{6}$ | $t=\frac{3\pi}{6}$ | $t=\frac{4\pi}{6}$ | $t=\frac{5\pi}{6}$ |
|---|---|---|---|---|---|
| Australia | 0.3012 | 0.3596 | 0.2961 | 0.2076 | 0.1070 |
| Austria | 0.2688 | 0.3330 | 0.2678 | 0.2068 | 0.1173 |
| Canada | 0.3824 | 0.4420 | 0.3629 | 0.2550 | 0.1303 |
| Europe | 0.1621 | 0.1733 | 0.1300 | 0.0984 | 0.0626 |
| France | 0.2178 | 0.1527 | 0.1035 | 0.0718 | 0.0450 |
| Germany | 0.3357 | 0.4014 | 0.3237 | 0.2271 | 0.1251 |
| Italy | -0.0860 | -0.0087 | 0.0122 | 0.0116 | 0.0140 |
| Japan | 0.0535 | 0.1085 | 0.1016 | 0.0739 | 0.0448 |
| Switzerland | -0.0024 | 0.0095 | 0.0278 | 0.0303 | 0.0218 |
| United Kingdom | 0.0689 | 0.0493 | 0.0577 | 0.0451 | 0.0238 |
| United States | 0.6409 | 0.5927 | 0.4662 | 0.3238 | 0.1644 |
| Standard error | (0.0982) | (0.0581) | (0.0420) | (0.0330) | (0.0131) |

Table 3.6.9. Multivariate Spectral Shape Tests
Incremental Covariance Contributions
$D_c(t,t-\frac{\pi}{6})$
Benchmark Economy (NX,P)

| Country | Frequencies $t=\frac{\pi}{6}$ | $t=\frac{2\pi}{6}$ | $t=\frac{3\pi}{6}$ | $t=\frac{4\pi}{6}$ | $t=\frac{5\pi}{6}$ | $t=\pi$ |
|---|---|---|---|---|---|---|
| Australia | 0.3012 | 0.0584 | -0.0635 | -0.0885 | -0.1006 | -0.1070 |
| Austria | 0.2688 | 0.0642 | -0.0652 | -0.0610 | -0.0895 | -0.1173 |
| Canada | 0.3824 | 0.0596 | -0.0791 | -0.1078 | -0.1247 | -0.1303 |
| Europe | 0.1621 | 0.0112 | -0.0433 | -0.0316 | -0.0358 | -0.0626 |
| France | 0.2178 | -0.0652 | 0.0492 | -0.0317 | -0.0268 | 0.0450 |
| Germany | 0.3357 | 0.0657 | -0.0777 | -0.0966 | -0.1020 | 0.1251 |
| Italy | -0.0860 | 0.0773 | 0.0209 | -0.0006 | 0.0024 | -0.0140 |
| Japan | 0.0535 | 0.0550 | -0.0070 | -0.0277 | -0.0291 | -0.0418 |
| Switzerland | -0.0021 | 0.0118 | 0.0183 | 0.0025 | 0.0085 | 0.0218 |
| United Kingdom | 0.0689 | -0.0195 | 0.0084 | -0.0126 | -0.0213 | -0.0238 |
| United States | 0.6409 | -0.0483 | -0.1264 | -0.1424 | -0.1594 | -0.1644 |
| Standard error | (0.0982) | (0.0731) | (0.0258) | (0.0214) | (0.0272) | (0.0131) |

Table 3.6.10. Multivariate Spectral Shape Tests
Accumulated Covariance Contributions
$\hat{D}_c(t,0)$
Benchmark Economy (NX,Y)

| Country | Frequencies | | | | |
|---|---|---|---|---|---|
| | $t=\frac{\pi}{6}$ | $t=\frac{2\pi}{6}$ | $t=\frac{3\pi}{6}$ | $t=\frac{4\pi}{6}$ | $t=\frac{5\pi}{6}$ |
| Australia | 0.1955 | 0.0956 | 0.0938 | 0.0737 | 0.0400 |
| Austria | 0.0758 | 0.0883 | 0.0730 | 0.0352 | 0.0092 |
| Canada | 0.0119 | -0.0348 | -0.0173 | 0.0067 | 0.0062 |
| Europe | 0.0571 | 0.0604 | 0.0575 | 0.0520 | 0.0284 |
| France | 0.2195 | 0.0945 | 0.0591 | 0.0474 | 0.0343 |
| Germany | 0.0023 | 0.0907 | 0.0851 | 0.0753 | 0.0386 |
| Italy | -0.1598 | -0.1901 | -0.1327 | -0.0814 | -0.0358 |
| Japan | 0.0395 | 0.1201 | 0.0978 | 0.0724 | 0.0435 |
| Switzerland | -0.2232 | -0.1959 | -0.1278 | -0.0777 | -0.0335 |
| United Kingdom | -0.0277 | 0.0479 | 0.0341 | 0.0250 | 0.0151 |
| United States | 0.0645 | 0.1207 | 0.0975 | 0.0691 | 0.0390 |
| Standard error | (0.1072) | (0.0869) | (0.0664) | (0.0440) | (0.0211) |

Table 3.6.11. Multivariate Spectral Shape Tests
Incremental Covariance Contributions
$\hat{D}_c(t,t-\frac{\pi}{6})$
Benchmark Economy (NX,Y)

| Country | Frequencies | | | | | |
|---|---|---|---|---|---|---|
| | $t=\frac{\pi}{6}$ | $t=\frac{2\pi}{6}$ | $t=\frac{3\pi}{6}$ | $t=\frac{4\pi}{6}$ | $t=\frac{5\pi}{6}$ | $t=\pi$ |
| Australia | 0.1955 | -0.0999 | -0.0018 | -0.0201 | -0.0337 | -0.0400 |
| Austria | 0.0758 | 0.0125 | -0.0153 | -0.0378 | -0.0259 | -0.0092 |
| Canada | 0.0119 | -0.0467 | 0.0174 | 0.0240 | -0.0004 | -0.0062 |
| Europe | 0.0571 | 0.0032 | -0.0029 | 0.0055 | 0.0236 | -0.0284 |
| France | 0.2195 | 0.1250 | -0.0351 | -0.0117 | -0.0131 | -0.0343 |
| Germany | 0.0023 | 0.0884 | -0.0056 | -0.0098 | -0.0367 | -0.0386 |
| Italy | -0.1598 | -0.0303 | 0.0574 | 0.0512 | 0.0456 | 0.0358 |
| Japan | 0.0395 | 0.0806 | -0.0224 | -0.0253 | -0.0289 | -0.0435 |
| Switzerland | 0.2232 | 0.0273 | 0.0681 | 0.0502 | 0.0442 | 0.0335 |
| United Kingdom | -0.0277 | 0.0756 | -0.0138 | -0.0091 | -0.0100 | -0.0151 |
| United States | 0.0645 | 0.0562 | -0.0232 | -0.0285 | -0.0301 | -0.0390 |
| Standard error | (0.1072) | (0.0488) | (0.0268) | (0.0291) | (0.0271) | (0.0211) |

Table 3.6.12. Multivariate Spectral Shape Tests
Accumulated Covariance Contributions
$D_c(t,0)$
Benchmark Economy (Y,P)

| Country | Frequencies $t=\frac{\pi}{6}$ | $t=\frac{2\pi}{6}$ | $t=\frac{3\pi}{6}$ | $t=\frac{4\pi}{6}$ | $t=\frac{5\pi}{6}$ |
|---|---|---|---|---|---|
| Australia | -0.4047 | -0.4190 | 0.3416 | 0.2124 | -0.1282 |
| Austria | -0.2136 | -0.1901 | -0.1305 | -0.1029 | -0.0322 |
| Canada | -0.2440 | -0.3158 | -0.2534 | -0.1752 | -0.0954 |
| Europe | -0.3608 | -0.3223 | -0.2672 | -0.1917 | -0.1028 |
| France | -0.4190 | -0.3538 | -0.2616 | -0.1966 | -0.1064 |
| Germany | 0.3551 | -0.3262 | 0.2663 | 0.1862 | -0.1000 |
| Italy | -0.0403 | -0.0043 | -0.0149 | -0.0190 | -0.0158 |
| Japan | -0.2738 | -0.3217 | -0.2517 | -0.1754 | -0.0947 |
| Switzerland | 0.0311 | 0.0132 | -0.0001 | -0.0086 | 0.0103 |
| United Kingdom | -0.1395 | -0.1806 | -0.1637 | -0.1149 | -0.0725 |
| United States | -0.2424 | -0.2663 | -0.2086 | -0.1136 | -0.0807 |
| Standard error | (0.1103) | (0.0867) | (0.0661) | (0.0435) | (0.0206) |

Table 3.6.13. Multivariate Spectral Shape Tests
Incremental Covariance Contributions
$D_c(t,t-\frac{\pi}{6})$
Benchmark Economy (Y,P)

| Country | Frequencies $t=\frac{\pi}{6}$ | $t=\frac{2\pi}{6}$ | $t=\frac{3\pi}{6}$ | $t=\frac{4\pi}{6}$ | $t=\frac{5\pi}{6}$ | $t=\pi$ |
|---|---|---|---|---|---|---|
| Australia | -0.4047 | -0.0143 | 0.0774 | 0.0992 | 0.1142 | 0.1282 |
| Austria | -0.2136 | 0.0234 | 0.0597 | 0.0275 | 0.0707 | 0.0322 |
| Canada | -0.2440 | -0.0718 | 0.0624 | 0.0782 | 0.0798 | 0.0954 |
| Europe | 0.3608 | 0.0385 | 0.0550 | 0.0755 | 0.0889 | 0.1028 |
| France | 0.4190 | 0.0652 | 0.0923 | 0.0650 | 0.0902 | 0.1064 |
| Germany | -0.3551 | 0.0289 | 0.0598 | 0.0801 | 0.0862 | 0.1000 |
| Italy | -0.0403 | 0.0361 | -0.0107 | -0.0041 | 0.0032 | 0.0158 |
| Japan | -0.2738 | -0.0479 | 0.0700 | 0.0763 | 0.0807 | 0.0947 |
| Switzerland | 0.0341 | 0.0269 | 0.0136 | 0.0082 | 0.0016 | 0.0103 |
| United Kingdom | 0.1395 | -0.0412 | 0.0169 | 0.0488 | 0.0424 | 0.0725 |
| United States | -0.2424 | -0.0238 | 0.0577 | 0.0650 | 0.0628 | 0.0807 |
| Standard error | (0.1103) | (0.0508) | (0.0272) | (0.0295) | (0.0274) | (0.0206) |

Table 3.6.14. Multivariate Spectral Shape Tests
Accumulated Covariance Contributions
$\hat{D}_q(t, 0)$
Benchmark Economy (NX,P)

| Country | Frequencies $t = \frac{\pi}{6}$ | $t = \frac{2\pi}{6}$ | $t = \frac{3\pi}{6}$ | $t = \frac{4\pi}{6}$ | $t = \frac{5\pi}{6}$ |
|---|---|---|---|---|---|
| Australia | 0.3552 | 0.1602 | 0.0921 | 0.0640 | 0.0283 |
| Austria | 0.1343 | 0.0106 | 0.0171 | 0.0190 | 0.0041 |
| Canada | 0.1437 | 0.0494 | 0.0272 | 0.0175 | 0.0066 |
| Europe | 0.2861 | 0.1085 | 0.0530 | 0.0269 | 0.0122 |
| France | 0.2970 | 0.0766 | 0.0380 | 0.0214 | 0.0141 |
| Germany | 0 2441 | 0 1277 | 0.0612 | 0.0329 | 0.0081 |
| Italy | 0.2624 | 0.1364 | 0.0735 | 0.0462 | 0.0218 |
| Japan | 0.2980 | 0.1265 | 0.0693 | 0.0374 | 0.0165 |
| Switzerland | 0.2213 | 0 1039 | 0.0571 | 0.0326 | 0.0152 |
| United Kingdom | 0.1415 | 0.0854 | 0.0387 | 0 0258 | 0.0153 |
| United States | 0.2071 | 0.0902 | 0.0492 | 0.0296 | 0.0098 |
| Standard error | (0.0118) | (0.0030) | (0.0014) | (0.0010) | (0.0006) |

Table 3.6.15. Multivariate Spectral Shape Tests
Incremental Covariance Contributions
$\hat{D}_q(t, t - \frac{\pi}{6})$
Benchmark Economy (NX,P)

| Country | Frequencies $t = \frac{\pi}{6}$ | $t = \frac{2\pi}{6}$ | $t = \frac{3\pi}{6}$ | $t = \frac{4\pi}{6}$ | $t = \frac{5\pi}{6}$ | $t = \pi$ |
|---|---|---|---|---|---|---|
| Australia | 0.3552 | -0.1950 | -0.0681 | -0.0281 | -0.0357 | -0.0283 |
| Austria | 0.1343 | -0 1237 | 0.0064 | 0 0019 | -0.0149 | -0.0041 |
| Canada | 0 1437 | -0 0911 | -0.0222 | -0.0097 | -0 0109 | -0.0066 |
| Europe | 0 2861 | 0.1777 | -0.0555 | -0.0261 | 0.0117 | 0.0122 |
| France | 0 2970 | -0 2204 | -0.0386 | -0 0166 | -0.0073 | -0.0141 |
| Germany | 0 2441 | -0 1164 | -0.0665 | -0.0282 | -0.0248 | -0 0081 |
| Italy | 0.2624 | -0.1260 | -0.0628 | -0 0273 | -0.0214 | -0.0218 |
| Japan | 0.2980 | -0.1715 | -0.0572 | -0.0319 | -0.0209 | -0.0165 |
| Switzerland | 0.2213 | 0.1174 | 0.0468 | 0.0245 | 0.0174 | 0.0152 |
| United Kingdom | 0.1415 | -0.0561 | -0.0467 | -0 0129 | -0.0105 | -0.0153 |
| United States | 0.2071 | -0.1168 | -0.0410 | -0.0196 | -0.0199 | -0.0098 |
| Standard error | (0.0118) | (0.096) | (0.0023) | (0.0010) | (0.0008) | (0.0006) |

Table 3.6.16. Multivariate Spectral Shape Tests
Accumulated Covariance Contributions
$D_q(t, 0)$
Benchmark Economy (NX,Y)

| Country | Frequencies | | | | |
|---|---|---|---|---|---|
| | $t = \frac{\pi}{6}$ | $t = \frac{2\pi}{6}$ | $t = \frac{3\pi}{6}$ | $t = \frac{4\pi}{6}$ | $t = \frac{5\pi}{6}$ |
| Australia | 0.2063 | 0.1204 | 0.0543 | 0.0368 | 0.0169 |
| Austria | 0.1064 | 0.0167 | 0.0423 | 0.0266 | 0.0169 |
| Canada | 0.0084 | 0.0346 | 0.0158 | 0.0109 | 0.0139 |
| Europe | 0.2291 | 0.0652 | 0.0263 | 0.0081 | 0.0033 |
| France | 0.1033 | 0.0370 | 0.0155 | 0.0067 | -0.0006 |
| Germany | 0.1089 | -0.0786 | -0.0660 | -0.0579 | -0.0364 |
| Italy | 0.0852 | 0.0376 | 0.0147 | 0.0068 | 0.0022 |
| Japan | 0.1064 | 0.0050 | 0.0019 | 0.0030 | -0.0004 |
| Switzerland | 0.0070 | 0.0065 | 0.0037 | 0.0003 | -0.0007 |
| United Kingdom | 0.0926 | 0.0192 | 0.0073 | 0.0068 | 0.0005 |
| United States | -0.1632 | -0.0682 | -0.0337 | -0.0199 | -0.0084 |
| Standard error | (0.0660) | (0.0408) | (0.0233) | (0.0144) | (0.0066) |

Table 3.6.17. Multivariate Spectral Shape Tests
Incremental Covariance Contributions
$D_q(t, t - \frac{\pi}{6})$
Benchmark Economy (NX,Y)

| Country | Frequencies | | | | | |
|---|---|---|---|---|---|---|
| | $t = \frac{\pi}{6}$ | $t = \frac{2\pi}{6}$ | $t = \frac{3\pi}{6}$ | $t = \frac{4\pi}{6}$ | $t = \frac{5\pi}{6}$ | $t = \pi$ |
| Australia | 0.2063 | -0.0859 | -0.0660 | -0.0175 | -0.0199 | -0.0169 |
| Austria | 0.1064 | -0.0597 | -0.0045 | -0.0157 | -0.0097 | -0.0169 |
| Canada | 0.0084 | 0.0262 | -0.0188 | -0.0049 | 0.0030 | -0.0139 |
| Europe | 0.2291 | -0.1639 | 0.0389 | -0.0182 | 0.0049 | 0.0033 |
| France | 0.1033 | -0.0663 | -0.0215 | -0.0088 | 0.0073 | 0.0006 |
| Germany | -0.1089 | 0.0304 | 0.0125 | 0.0081 | 0.0214 | 0.0364 |
| Italy | 0.0852 | -0.0176 | 0.0229 | -0.0079 | -0.0046 | 0.0022 |
| Japan | 0.1064 | -0.1013 | -0.0031 | 0.0010 | -0.0034 | 0.0004 |
| Switzerland | 0.0070 | 0.0006 | 0.0028 | 0.0034 | 0.0010 | 0.0007 |
| United Kingdom | 0.0926 | -0.0735 | -0.0118 | -0.0005 | -0.0063 | -0.0005 |
| United States | -0.1632 | 0.0950 | 0.0345 | 0.0138 | 0.0115 | 0.0084 |
| Standard error | (0.0660) | (0.0472) | (0.0217) | (0.0120) | (0.0100) | (0.0066) |

Table 3.6.18. Multivariate Spectral Shape Tests
Accumulated Covariance Contributions
$\bar{D}_q(t,0)$
Benchmark Economy (Y,P)

| Country | Frequencies $t = \frac{\pi}{6}$ | $t = \frac{2\pi}{6}$ | $t = \frac{3\pi}{6}$ | $t = \frac{4\pi}{6}$ | $t = \frac{5\pi}{6}$ |
|---|---|---|---|---|---|
| Australia | 0.1008 | 0.0697 | 0.0370 | 0.0102 | 0.0068 |
| Austria | 0.1688 | 0.0369 | 0.0398 | 0.0172 | 0.0223 |
| Canada | 0.0066 | 0.0109 | 0.0050 | -0.0022 | -0.0006 |
| Europe | 0.3443 | 0.0983 | 0.0593 | 0.0359 | 0.0189 |
| France | 0.2904 | 0.0669 | 0.0233 | 0.0181 | 0.0060 |
| Germany | 0.1540 | 0.0345 | 0.0249 | 0.0118 | 0.0031 |
| Italy | 0.2303 | 0.0780 | 0.0432 | 0.0191 | 0.0081 |
| Japan | 0.1282 | 0.0297 | 0.0241 | 0.0132 | 0.0050 |
| Switzerland | 0.1173 | 0.0158 | 0.0329 | 0.0187 | 0.0073 |
| United Kingdom | 0.1543 | -0.0006 | 0.0170 | 0.0197 | 0.0053 |
| United States | 0.1327 | 0.0525 | 0.0297 | 0.0152 | 0.0040 |
| Standard error | (0.0674) | (0.0413) | (0.0239) | (0.0147) | (0.0068) |

Table 3.6.19. Multivariate Spectral Shape Tests
Incremental Covariance Contributions
$\bar{D}_q(t, t - \frac{\pi}{6})$
Benchmark Economy (Y,P)

| Country | Frequencies $t = \frac{\pi}{6}$ | $t = \frac{2\pi}{6}$ | $t = \frac{3\pi}{6}$ | $t = \frac{4\pi}{6}$ | $t = \frac{5\pi}{6}$ | $t = \pi$ |
|---|---|---|---|---|---|---|
| Australia | 0.1008 | -0.0311 | -0.0327 | -0.0269 | -0.0033 | -0.0068 |
| Austria | 0.1688 | -0.1319 | 0.0029 | -0.0226 | 0.0051 | -0.0223 |
| Canada | 0.0066 | 0.0013 | -0.0059 | -0.0073 | 0.0017 | 0.0006 |
| Europe | 0.3443 | -0.2460 | -0.0390 | -0.0234 | -0.0170 | -0.0189 |
| France | 0.2904 | 0.2235 | 0.0137 | -0.0052 | 0.0121 | -0.0060 |
| Germany | 0.1540 | 0.1195 | -0.0096 | -0.0131 | -0.0087 | -0.0031 |
| Italy | 0.2303 | -0.1523 | -0.0318 | -0.0241 | -0.0111 | -0.0081 |
| Japan | 0.1282 | -0.0986 | -0.0056 | -0.0109 | -0.0082 | -0.0050 |
| Switzerland | 0.1173 | 0.0715 | 0.0128 | 0.0113 | 0.0113 | -0.0073 |
| United Kingdom | 0.1543 | -0.1549 | 0.0177 | 0.0027 | -0.0145 | -0.0053 |
| United States | 0.1327 | 0.0801 | -0.0228 | -0.0115 | -0.0113 | -0.0040 |
| Standard error | (0.0674) | (0.0479) | (0.0221) | (0.0124) | (0.0102) | (0.0068) |

Table 3.6.20. Multivariate Spectral Shape Tests
Accumulated Covariance Contributions
$\hat{D}_c(t,0)$
Small Elasticity Economy (NX,P)

| Country | Frequencies $t=\frac{\pi}{6}$ | $t=\frac{2\pi}{6}$ | $t=\frac{3\pi}{6}$ | $t=\frac{4\pi}{6}$ | $t=\frac{5\pi}{6}$ |
|---|---|---|---|---|---|
| Australia | 0.2811 | 0.3751 | 0.3215 | 0.2245 | 0.1103 |
| Austria | 0.2487 | 0.3485 | 0.2932 | 0.2236 | 0.1206 |
| Canada | 0.3623 | 0.4574 | 0.3883 | 0.2719 | 0.1336 |
| Europe | 0.1420 | 0.1888 | 0.1554 | 0.1153 | 0.0659 |
| France | 0.1977 | 0.1681 | 0.1289 | 0.0887 | 0.0482 |
| Germany | 0.3156 | 0.4168 | 0.3491 | 0.2440 | 0.1284 |
| Italy | -0.1061 | 0.0068 | 0.0376 | 0.0284 | 0.0173 |
| Japan | 0.0334 | 0.1240 | 0.1269 | 0.0907 | 0.0481 |
| Switzerland | -0.0225 | 0.0249 | 0.0532 | 0.0472 | 0.0251 |
| United Kingdom | 0.0487 | 0.0618 | 0.0831 | 0.0620 | 0.0271 |
| United States | 0.6208 | 0.6081 | 0.4916 | 0.3406 | 0.1677 |
| Standard error | (0.1061) | (0.0460) | (0.0298) | (0.0177) | (0.0100) |

Table 3.6.21. Multivariate Spectral Shape Tests
Incremental Covariance Contributions
$\hat{D}_c(t, t-\frac{\pi}{6})$
Small Elasticity Economy (NX,P)

| Country | Frequencies $t=\frac{\pi}{6}$ | $t=\frac{2\pi}{6}$ | $t=\frac{3\pi}{6}$ | $t=\frac{4\pi}{6}$ | $t=\frac{5\pi}{6}$ | $t=\pi$ |
|---|---|---|---|---|---|---|
| Australia | 0.2811 | 0.0940 | -0.0536 | -0.0970 | -0.1142 | -0.1103 |
| Austria | 0.2487 | 0.0998 | -0.0553 | -0.0695 | -0.1030 | -0.1206 |
| Canada | 0.3623 | 0.0952 | -0.0692 | -0.1164 | -0.1383 | -0.1336 |
| Europe | 0.1420 | 0.0468 | -0.0334 | -0.0401 | -0.0493 | -0.0659 |
| France | 0.1977 | 0.0296 | 0.0392 | -0.0402 | -0.0104 | -0.0182 |
| Germany | 0.3156 | 0.1012 | -0.0677 | -0.1051 | -0.1155 | -0.1284 |
| Italy | -0.1061 | 0.1129 | 0.0308 | -0.0092 | -0.0112 | -0.0173 |
| Japan | 0.0334 | 0.0906 | 0.0030 | -0.0362 | -0.0427 | -0.0481 |
| Switzerland | 0.0225 | 0.0471 | 0.0283 | 0.0060 | 0.0220 | 0.0251 |
| United Kingdom | 0.0487 | 0.0160 | 0.0183 | -0.0211 | -0.0349 | -0.0271 |
| United States | 0.6208 | -0.0127 | -0.1165 | -0.1510 | -0.1729 | -0.1677 |
| Standard error | (0.1061) | (0.0946) | (0.0284) | (0.0212) | (0.0129) | (0.0100) |

Table 3.6.22. Multivariate Spectral Shape Tests
Accumulated Covariance Contributions
$\hat{D}_q(t,0)$
Small Elasticity Economy (NX,P)

| Country | Frequencies $t=\frac{\pi}{6}$ | $t=\frac{2\pi}{6}$ | $t=\frac{3\pi}{6}$ | $t=\frac{4\pi}{6}$ | $t=\frac{5\pi}{6}$ |
|---|---|---|---|---|---|
| Australia | 0.3515 | 0.1600 | 0.0918 | 0.0631 | 0.0282 |
| Austria | 0.1306 | 0.0104 | 0.0167 | 0.0181 | 0.0039 |
| Canada | 0.1400 | 0.0491 | 0.0268 | 0.0166 | 0.0065 |
| Europe | 0.2824 | 0.1083 | 0.0526 | 0.0260 | 0.0120 |
| France | 0.2933 | 0.0764 | 0.0377 | 0.0205 | 0.0140 |
| Germany | 0.2404 | 0.1274 | 0.0608 | 0.0321 | 0.0080 |
| Italy | 0.2587 | 0.1362 | 0.0732 | 0.0453 | 0.0217 |
| Japan | 0.2942 | 0.1262 | 0.0689 | 0.0365 | 0.0164 |
| Switzerland | 0.2176 | 0.1037 | 0.0567 | 0.0318 | 0.0151 |
| United Kingdom | 0.1377 | 0.0851 | 0.0383 | 0.0249 | 0.0152 |
| United States | 0.2033 | 0.0900 | 0.0488 | 0.0288 | 0.0096 |
| Standard error | (0.0158) | (0.0048) | (0.0033) | (0.0021) | (0.0013) |

Table 3.6.23 Multivariate Spectral Shape Tests
Incremental Covariance Contributions
$D_q(t,t-\frac{\pi}{6})$
Small Elasticity Economy (NX,P)

| Country | Frequencies $t=\frac{\pi}{6}$ | $t=\frac{2\pi}{6}$ | $t=\frac{3\pi}{6}$ | $t=\frac{4\pi}{6}$ | $t=\frac{5\pi}{6}$ | $t=\pi$ |
|---|---|---|---|---|---|---|
| Australia | 0.3515 | -0.1915 | -0.0683 | -0.0286 | -0.0349 | -0.0282 |
| Austria | 0.1306 | -0.1202 | 0.0063 | 0.0014 | -0.0112 | -0.0039 |
| Canada | 0.1400 | -0.0908 | -0.0223 | -0.0102 | -0.0101 | -0.0065 |
| Europe | 0.2824 | -0.1712 | -0.0556 | -0.0266 | -0.0140 | -0.0120 |
| France | 0.2933 | 0.2169 | -0.0388 | 0.0171 | 0.0065 | -0.0140 |
| Germany | 0.2404 | -0.1129 | -0.0666 | -0.0287 | -0.0240 | -0.0080 |
| Italy | 0.2587 | -0.1225 | -0.0630 | -0.0278 | -0.0237 | -0.0217 |
| Japan | 0.2942 | -0.1680 | -0.0573 | -0.0324 | -0.0202 | -0.0164 |
| Switzerland | 0.2176 | 0.1139 | -0.0170 | -0.0250 | 0.0166 | 0.0151 |
| United Kingdom | 0.1377 | -0.0526 | -0.0468 | -0.0134 | -0.0097 | -0.0152 |
| United States | 0.2033 | -0.1133 | -0.0412 | -0.0201 | -0.0191 | -0.0096 |
| Standard error | (0.0158) | (0.0131) | (0.0034) | (0.0020) | (0.0015) | (0.0013) |

Table 3.6.24. Multivariate Spectral Shape Tests
Accumulated Covariance Contributions
$\hat{D}_c(t,0)$
Large Elasticity Economy (NX,P)

| Country | Frequencies $t = \frac{\pi}{6}$ | $t = \frac{2\pi}{6}$ | $t = \frac{3\pi}{6}$ | $t = \frac{4\pi}{6}$ | $t = \frac{5\pi}{6}$ |
|---|---|---|---|---|---|
| Australia | 0.2931 | 0.3835 | 0.3271 | 0.2283 | 0.1122 |
| Austria | 0.2607 | 0.3569 | 0.2988 | 0.2274 | 0.1226 |
| Canada | 0.3743 | 0.4659 | 0.3939 | 0.2757 | 0.1356 |
| Europe | 0.1540 | 0.1972 | 0.1610 | 0.1191 | 0.0679 |
| France | 0.2097 | 0.1766 | 0.1345 | 0.0925 | 0.0502 |
| Germany | 0.3276 | 0.4253 | 0.3547 | 0.2178 | 0.1304 |
| Italy | -0.0941 | 0.0152 | 0.0432 | 0.0322 | 0.0192 |
| Japan | 0.0454 | 0.1324 | 0.1326 | 0.0945 | 0.0500 |
| Switzerland | -0.0105 | 0.0334 | 0.0588 | 0.0509 | 0.0270 |
| United Kingdom | 0.0607 | 0.0732 | 0.0888 | 0.0658 | 0.0290 |
| United States | 0.6328 | 0.6166 | 0.4972 | 0.3444 | 0.1696 |
| Standard error | (0.1058) | (0.0501) | (0.0289) | (0.0176) | (0.0099) |

Table 3.6.25. Multivariate Spectral Shape Tests
Incremental Covariance Contributions
$\hat{D}_c(t, t - \frac{\pi}{6})$
Large Elasticity Economy (NX,P)

| Country | Frequencies $t = \frac{\pi}{6}$ | $t = \frac{2\pi}{6}$ | $t = \frac{3\pi}{6}$ | $t = \frac{4\pi}{6}$ | $t = \frac{5\pi}{6}$ | $t = \pi$ |
|---|---|---|---|---|---|---|
| Australia | 0.2931 | 0.0904 | -0.0564 | -0.0989 | -0.1160 | -0.1122 |
| Austria | 0.2607 | 0.0963 | -0.0581 | -0.0714 | -0.1049 | -0.1226 |
| Canada | 0.3743 | 0.0916 | -0.0720 | -0.1182 | -0.1401 | -0.1356 |
| Europe | 0.1540 | 0.0432 | -0.0362 | -0.0420 | -0.0512 | -0.0679 |
| France | 0.2097 | 0.0332 | -0.0421 | -0.0420 | 0.0423 | 0.0502 |
| Germany | 0.3276 | 0.0977 | -0.0706 | -0.1069 | -0.1174 | -0.1304 |
| Italy | -0.0941 | 0.1091 | 0.0280 | -0.0110 | 0.0130 | 0.0192 |
| Japan | 0.0454 | 0.0870 | 0.0001 | -0.0380 | -0.0445 | -0.0500 |
| Switzerland | 0.0105 | 0.0438 | 0.0255 | 0.0079 | 0.0239 | 0.0270 |
| United Kingdom | 0.0607 | 0.0125 | 0.0155 | -0.0230 | -0.0368 | -0.0290 |
| United States | 0.6328 | -0.0162 | -0.1193 | -0.1523 | -0.1718 | -0.1696 |
| Standard error | (0.1058) | (0.0936) | (0.0317) | (0.0204) | (0.0129) | (0.0099) |

Table 3.6.26. Multivariate Spectral Shape Tests
Accumulated Covariance Contributions
$D_q(t,0)$
Large Elasticity Economy (NX,P)

| Country | Frequencies $t=\frac{\pi}{6}$ | $t=\frac{2\pi}{6}$ | $t=\frac{3\pi}{6}$ | $t=\frac{4\pi}{6}$ | $t=\frac{5\pi}{6}$ |
|---|---|---|---|---|---|
| Australia | 0.3569 | 0.1622 | 0.0931 | 0.0642 | 0.0286 |
| Austria | 0.1360 | 0.0126 | 0.0181 | 0.0191 | 0.0044 |
| Canada | 0.1454 | 0.0513 | 0.0282 | 0.0176 | 0.0069 |
| Europe | 0.2878 | 0.1105 | 0.0540 | 0.0270 | 0.0125 |
| France | 0.2987 | 0.0786 | 0.0390 | 0.0216 | 0.0145 |
| Germany | 0.2458 | 0.1297 | 0.0622 | 0.0331 | 0.0085 |
| Italy | 0.2641 | 0.1384 | 0.0745 | 0.0464 | 0.0221 |
| Japan | 0.2996 | 0.1285 | 0.0703 | 0.0375 | 0.0168 |
| Switzerland | 0.2230 | 0.1059 | 0.0581 | 0.0328 | 0.0155 |
| United Kingdom | 0.1431 | 0.0874 | 0.0397 | 0.0259 | 0.0156 |
| United States | 0.2087 | 0.0922 | 0.0502 | 0.0298 | 0.0101 |
| Standard error | (0.0094) | (0.0013) | (0.0011) | (0.0006) | (0.0004) |

Table 3.6.27. Multivariate Spectral Shape Tests
Incremental Covariance Contributions
$D_q(t,t-\frac{\pi}{6})$
Large Elasticity Economy (NX,P)

| Country | Frequencies $t=\frac{\pi}{6}$ | $t=\frac{2\pi}{6}$ | $t=\frac{3\pi}{6}$ | $t=\frac{4\pi}{6}$ | $t=\frac{5\pi}{6}$ | $t=\pi$ |
|---|---|---|---|---|---|---|
| Australia | 0.3569 | -0.1946 | -0.0691 | -0.0290 | -0.0355 | -0.0286 |
| Austria | 0.1360 | -0.1234 | 0.0054 | 0.0011 | -0.0148 | -0.0044 |
| Canada | 0.1454 | -0.0940 | -0.0231 | -0.0106 | -0.0107 | -0.0069 |
| Europe | 0.2878 | -0.1773 | -0.0565 | -0.0270 | -0.0146 | -0.0125 |
| France | 0.2987 | 0.2200 | -0.0396 | -0.0175 | 0.0071 | -0.0145 |
| Germany | 0.2458 | -0.1161 | -0.0675 | -0.0291 | 0.0246 | -0.0085 |
| Italy | 0.2641 | -0.1257 | -0.0638 | -0.0282 | -0.0243 | -0.0221 |
| Japan | 0.2996 | -0.1712 | -0.0582 | -0.0327 | -0.0208 | -0.0168 |
| Switzerland | 0.2230 | -0.1171 | -0.0478 | -0.0253 | -0.0172 | -0.0155 |
| United Kingdom | 0.1431 | -0.0558 | -0.0477 | -0.0137 | -0.0103 | -0.0156 |
| United States | 0.2087 | -0.1165 | -0.0420 | -0.0204 | -0.0197 | -0.0101 |
| Standard error | (0.0094) | (0.0093) | (0.0010) | (0.0008) | (0.0007) | (0.0004) |

Table 3.6.28. Multivariate Spectral Shape Tests
Accumulated Covariance Contributions
$\hat{D}_c(t,0)$
Two Shocks Economy (NX,P)

| Country | Frequencies $t=\frac{\pi}{6}$ | $t=\frac{2\pi}{6}$ | $t=\frac{3\pi}{6}$ | $t=\frac{4\pi}{6}$ | $t=\frac{5\pi}{6}$ |
|---|---|---|---|---|---|
| Australia | -0.3971 | -0.4396 | 0.3566 | -0.2152 | -0.1267 |
| Austria | -0.4295 | -0.4662 | 0.3849 | -0.2460 | -0.1164 |
| Canada | -0.3159 | -0.3572 | -0.2898 | -0.1977 | -0.1034 |
| Europe | -0.5362 | -0.6259 | -0.5227 | -0.3543 | -0.1711 |
| France | -0.4804 | -0.6466 | -0.5492 | -0.3810 | -0.1887 |
| Germany | 0 3626 | 0 3978 | -0.3290 | 0.2256 | -0.1085 |
| Italy | -0.7843 | -0.8079 | -0 6405 | -0.4412 | -0.2197 |
| Japan | -0.6448 | -0.6907 | -0.5512 | -0.3789 | -0.1889 |
| Switzerland | -0.7006 | -0.7897 | -0.6249 | -0.4225 | -0.2119 |
| United Kingdom | -0.6291 | -0 7499 | -0 5950 | 0.4076 | -0.2099 |
| United States | -0.0574 | -0.2066 | -0.1865 | -0.1290 | -0.0693 |
| Standard error | (0.1068) | (0.0477) | (0.0264) | (0.0173) | (0.0112) |

Table 3.6.29 Multivariate Spectral Shape Tests
Incremantal Covariance Contributions
$\hat{D}_c(t,t-\frac{\pi}{6})$
Two Shocks Economy (NX,P)

| Country | Frequencies $t=\frac{\pi}{6}$ | $t=\frac{2\pi}{6}$ | $t=\frac{3\pi}{6}$ | $t=\frac{4\pi}{6}$ | $t=\frac{5\pi}{6}$ | $t=\pi$ |
|---|---|---|---|---|---|---|
| Australia | -0.3971 | -0.0425 | 0.0829 | 0.1115 | 0.1184 | 0.1267 |
| Austria | -0.4295 | -0.0367 | 0.0812 | 0.1390 | 0 1296 | 0 1164 |
| Canada | -0.3159 | -0 0413 | 0.0671 | [illegible] | 0 0944 | 0 1034 |
| Europe | -0.5362 | -0.0897 | 0 1032 | 0.1684 | 0.1833 | 0.1711 |
| France | 0 4801 | -0.1661 | 0.0973 | 0.1683 | 0 1922 | 0 1887 |
| Germany | -0.3626 | -0 0353 | 0 0688 | 0 1034 | 0 1171 | 0 1085 |
| Italy | -0.7843 | -0.0235 | 0 1674 | 0.1993 | 0.2215 | 0 2197 |
| Japan | -0.6448 | -0 0159 | 0.1395 | 0.1723 | 0.1900 | 0 1889 |
| Switzerland | -0.7006 | -0.0891 | 0.1648 | 0.2025 | 0.2106 | 0.2119 |
| United Kingdom | -0.6294 | -0.1201 | 0 1549 | 0.1873 | 0 1977 | 0 2099 |
| United States | -0.0574 | -0.1492 | 0.0200 | 0.0575 | 0.0597 | 0 0693 |
| Standard error | (0.1068) | (0 0688) | (0 0302) | (0 0138) | (0 0096) | (0.0112) |

Table 3.6.30. Multivariate Spectral Shape Tests
Accumulated Covariance Contributions
$D_q(t,0)$
Two Shocks Economy (NX,P)

| Country | Frequencies | | | | |
|---|---|---|---|---|---|
| | $t=\frac{\pi}{6}$ | $t=\frac{2\pi}{6}$ | $t=\frac{3\pi}{6}$ | $t=\frac{4\pi}{6}$ | $t=\frac{5\pi}{6}$ |
| Australia | 0.5171 | 0.2468 | 0.1422 | 0.0929 | 0.0421 |
| Austria | 0.2962 | 0.0971 | 0.0671 | 0.0478 | 0.0178 |
| Canada | 0.3056 | 0.1359 | 0.0772 | 0.0463 | 0.0203 |
| Europe | 0.4480 | 0.1950 | 0.1030 | 0.0557 | 0.0259 |
| France | 0.4589 | 0.1632 | 0.0881 | 0.0503 | 0.0279 |
| Germany | 0.4060 | 0.2142 | 0.1112 | 0.0618 | 0.0219 |
| Italy | 0.4243 | 0.2229 | 0.1236 | 0.0751 | 0.0355 |
| Japan | 0.4598 | 0.2130 | 0.1193 | 0.0662 | 0.0302 |
| Switzerland | 0.3832 | 0.1904 | 0.1071 | 0.0615 | 0.0290 |
| United Kingdom | 0.3033 | 0.1719 | 0.0887 | 0.0546 | 0.0290 |
| United States | 0.3689 | 0.1768 | 0.0992 | 0.0585 | 0.0235 |
| Standard error | (0.0552) | (0.0208) | (0.0127) | (0.0082) | (0.0040) |

Table 3.6.31. Multivariate Spectral Shape Tests
Incremental Covariance Contributions
$D_q(t,t-\frac{\pi}{6})$
Two Shocks Economy (NX,P)

| Country | Frequencies | | | | | |
|---|---|---|---|---|---|---|
| | $t=\frac{\pi}{6}$ | $t=\frac{2\pi}{6}$ | $t=\frac{3\pi}{6}$ | $t=\frac{4\pi}{6}$ | $t=\frac{5\pi}{6}$ | $t=\pi$ |
| Australia | 0.5171 | -0.2703 | -0.1046 | -0.0493 | -0.0508 | -0.0421 |
| Austria | 0.2962 | -0.1990 | -0.0300 | -0.0193 | -0.0300 | -0.0178 |
| Canada | 0.3056 | -0.1697 | -0.0586 | -0.0309 | -0.0260 | -0.0203 |
| Europe | 0.4480 | 0.2530 | -0.0920 | 0.0173 | -0.0298 | -0.0259 |
| France | 0.4589 | -0.2957 | 0.0751 | -0.0378 | -0.0221 | 0.0279 |
| Germany | 0.4060 | -0.1918 | 0.1030 | -0.0494 | -0.0399 | -0.0219 |
| Italy | 0.4243 | -0.2014 | 0.0993 | -0.0185 | -0.0395 | -0.0355 |
| Japan | 0.4598 | -0.2468 | -0.0937 | -0.0731 | -0.0360 | -0.0302 |
| Switzerland | 0.3832 | -0.1927 | -0.0833 | -0.0457 | -0.0325 | -0.0290 |
| United Kingdom | 0.3033 | -0.1311 | 0.0832 | -0.0341 | -0.0256 | -0.0290 |
| United States | 0.3689 | -0.1922 | 0.0775 | -0.0108 | -0.0350 | 0.0235 |
| Standard error | (0.0552) | (0.0411) | (0.0108) | (0.0064) | (0.0047) | (0.0040) |

Table 3.6.32. Multivariate Spectral Shape Tests
Accumulated Covariance Contributions
$\hat{D}_c(t,0)$
Time to Build Economy (NX,P)

| Country | Frequencies $t = \frac{\pi}{6}$ | $t = \frac{2\pi}{6}$ | $t = \frac{3\pi}{6}$ | $t = \frac{4\pi}{6}$ | $t = \frac{5\pi}{6}$ |
|---|---|---|---|---|---|
| Australia | 0.1839 | 0.2880 | 0.2684 | 0.1919 | 0.0922 |
| Austria | 0.1515 | 0.2614 | 0.2401 | 0.1910 | 0.1026 |
| Canada | 0.2651 | 0.3704 | 0.3352 | 0.2393 | 0.1156 |
| Europe | 0.0448 | 0.1017 | 0.1023 | 0.0827 | 0.0479 |
| France | 0.1006 | 0.0811 | 0.0758 | 0.0561 | 0.0302 |
| Germany | 0.2184 | 0.3298 | 0.2960 | 0.2114 | 0.1104 |
| Italy | -0.2033 | -0.0803 | -0.0155 | -0.0042 | -0.0008 |
| Japan | -0.0637 | 0.0369 | 0.0739 | 0.0581 | 0.0300 |
| Switzerland | -0.1196 | -0.0621 | 0.0001 | 0.0146 | 0.0071 |
| United Kingdom | -0.0484 | -0.0223 | 0.0300 | 0.0294 | 0.0091 |
| United States | 0.5236 | 0.5211 | 0.4385 | 0.3080 | 0.1497 |
| Standard error | (0.0968) | (0.0759) | (0.0532) | (0.0339) | (0.0176) |

Table 3.6.33 Multivariate Spectral Shape Tests
Incremental Covariance Contributions
$D_c(t, t - \frac{\pi}{6})$
Time to Build Economy (NX,P)

| Country | Frequencies $t = \frac{\pi}{6}$ | $t = \frac{2\pi}{6}$ | $t = \frac{3\pi}{6}$ | $t = \frac{4\pi}{6}$ | $t = \frac{5\pi}{6}$ | $t = \pi$ |
|---|---|---|---|---|---|---|
| Australia | 0.1839 | 0.1041 | -0.0196 | -0.0765 | -0.0996 | -0.0922 |
| Austria | 0.1515 | 0.1099 | -0.0213 | -0.0490 | -0.0885 | -0.1026 |
| Canada | 0.2651 | 0.1053 | -0.0352 | -0.0959 | -0.1237 | -0.1156 |
| Europe | 0.0448 | 0.0569 | 0.0006 | -0.0196 | -0.0348 | 0.0179 |
| France | 0.1006 | -0.0195 | -0.0053 | -0.0197 | -0.0258 | -0.0302 |
| Germany | 0.2184 | 0.1113 | -0.0338 | -0.0846 | -0.1010 | -0.1104 |
| Italy | -0.2033 | 0.1230 | 0.0648 | 0.0113 | 0.0034 | 0.0008 |
| Japan | -0.0637 | 0.1007 | 0.0369 | -0.0157 | -0.0281 | -0.0300 |
| Switzerland | -0.1196 | 0.0575 | 0.0622 | 0.0144 | 0.0075 | 0.0071 |
| United Kingdom | -0.0484 | 0.0262 | 0.0523 | -0.0007 | -0.0203 | -0.0091 |
| United States | 0.5236 | -0.0026 | -0.0825 | -0.1305 | -0.1581 | 0.1497 |
| Standard error | (0.0968) | (0.0710) | (0.0445) | (0.0277) | (0.0225) | (0.0176) |

Table 3.6.34. Multivariate Spectral Shape Tests
Accumulated Covariance Contributions
$D_q(t,0)$
Time to Build Economy (NX,P)

| Country | Frequencies $t=\frac{\pi}{6}$ | $t=\frac{2\pi}{6}$ | $t=\frac{3\pi}{6}$ | $t=\frac{4\pi}{6}$ | $t=\frac{5\pi}{6}$ |
|---|---|---|---|---|---|
| Australia | 0.4070 | 0.1947 | 0.1126 | 0.0758 | 0.0339 |
| Austria | 0.1861 | 0.0451 | 0.0376 | 0.0307 | 0.0096 |
| Canada | 0.1955 | 0.0838 | 0.0477 | 0.0292 | 0.0122 |
| Europe | 0.3379 | 0.1430 | 0.0735 | 0.0386 | 0.0177 |
| France | 0.3488 | 0.1111 | 0.0585 | 0.0332 | 0.0197 |
| Germany | 0.2959 | 0.1621 | 0.0817 | 0.0447 | 0.0137 |
| Italy | 0.3142 | 0.1708 | 0.0940 | 0.0580 | 0.0274 |
| Japan | 0.3497 | 0.1609 | 0.0898 | 0.0491 | 0.0221 |
| Switzerland | 0.2731 | 0.1384 | 0.0776 | 0.0444 | 0.0208 |
| United Kingdom | 0.1932 | 0.1198 | 0.0592 | 0.0375 | 0.0209 |
| United States | 0.2588 | 0.1247 | 0.0697 | 0.0414 | 0 0153 |
| Standard error | (0.0215) | (0.0068) | (0.0034) | (0.0014) | (0.0008) |

Table 3.6.35. Multivariate Spectral Shape Tests
Incremental Covariance Contributions
$D_q(t,t-\frac{\pi}{6})$
Time to Build Economy (NX,P)

| Country | Frequencies $t=\frac{\pi}{6}$ | $t=\frac{2\pi}{6}$ | $t=\frac{3\pi}{6}$ | $t=\frac{4\pi}{6}$ | $t=\frac{5\pi}{6}$ | $t=\pi$ |
|---|---|---|---|---|---|---|
| Australia | 0.4070 | -0.2123 | -0.0821 | -0.0369 | -0.0419 | -0.0339 |
| Austria | 0.1861 | -0.1410 | -0.0075 | -0 0068 | -0.0211 | -0.0096 |
| Canada | 0.1955 | -0.1116 | -0 0361 | -0.0185 | -0.0171 | -0 0122 |
| Europe | 0.3379 | -0 1950 | -0.0695 | -0.0349 | -0.0209 | -0.0177 |
| France | 0.3488 | -0.2377 | -0.0526 | -0.0253 | -0.0134 | -0.0197 |
| Germany | 0 2959 | -0 1337 | -0.0805 | -0.0370 | -0 0310 | -0.0137 |
| Italy | 0.3142 | -0.1433 | -0.0768 | -0.0361 | -0.0306 | -0.0274 |
| Japan | 0.3497 | -0.1888 | -0.0712 | -0.0406 | -0.0271 | -0.0221 |
| Switzerland | 0.2731 | 0.1347 | -0.0608 | 0.0332 | -0.0236 | 0.0208 |
| United Kingdom | 0.1932 | -0 0734 | -0.0607 | -0.0216 | -0.0166 | -0 0209 |
| United States | 0.2588 | -0.1341 | -0.0550 | -0.0283 | -0.0261 | -0.0153 |
| Standard error | (0.0215) | (0.0200) | (0.0059) | (0.0024) | (0.0012) | (0.0008) |

Table 3.6.36. Multivariate Spectral Shape Tests
Accumulated Covariance Contributions
$D_c(t,0)$
Government Expenditures Economy (NX,P)

| Country | Frequencies | | | | |
|---|---|---|---|---|---|
| | $t=\frac{\pi}{6}$ | $t=\frac{2\pi}{6}$ | $t=\frac{3\pi}{6}$ | $t=\frac{4\pi}{6}$ | $t=\frac{5\pi}{6}$ |
| Australia | -0.3616 | -0.4461 | -0.3590 | -0.2563 | -0.1333 |
| Austria | -0.3971 | -0.4727 | -0.3873 | -0.2571 | -0.1229 |
| Canada | -0.2835 | -0.3638 | -0.2922 | -0.2089 | -0.1099 |
| Europe | -0.5038 | -0.6324 | -0.5251 | -0.3655 | -0.1776 |
| France | -0.4480 | -0.6531 | -0.5516 | -0.3921 | -0.1953 |
| Germany | -0.3301 | -0.4044 | -0.3314 | -0.2368 | -0.1151 |
| Italy | -0.7519 | -0.8144 | -0.6429 | -0.4524 | -0.2263 |
| Japan | -0.6123 | -0.6972 | -0.5535 | -0.3901 | -0.1955 |
| Switzerland | -0.6682 | -0.7963 | -0.6273 | -0.4336 | -0.2184 |
| United Kingdom | -0.5970 | -0.7564 | -0.5973 | -0.4188 | -0.2165 |
| United States | -0.0249 | -0.2131 | -0.1889 | -0.1402 | -0.0759 |
| Standard error | (0.0993) | (0.0502) | (0.0337) | (0.0239) | (0.0116) |

Table 3.6.37. Multivariate Spectral Shape Tests
Incremental Covariance Contributions
$D_c(t,t-\frac{\pi}{6})$
Government Expenditures Economy (NX,P)

| Country | Frequencies | | | | | |
|---|---|---|---|---|---|---|
| | $t=\frac{\pi}{6}$ | $t=\frac{2\pi}{6}$ | $t=\frac{3\pi}{6}$ | $t=\frac{4\pi}{6}$ | $t=\frac{5\pi}{6}$ | $t=\pi$ |
| Australia | -0.3646 | -0.0815 | 0.0872 | 0.1026 | 0.1231 | 0.1333 |
| Austria | -0.3971 | -0.0757 | 0.0855 | 0.1301 | 0.1342 | 0.1229 |
| Canada | -0.2835 | -0.0803 | 0.0716 | 0.0833 | 0.0990 | 0.1099 |
| Europe | -0.5038 | -0.1287 | 0.1074 | 0.1595 | 0.1879 | 0.1776 |
| France | -0.4480 | -0.2051 | 0.1015 | 0.1594 | 0.1968 | 0.1953 |
| Germany | -0.3301 | -0.0742 | 0.0730 | 0.0946 | 0.1217 | 0.1151 |
| Italy | -0.7519 | -0.0626 | 0.1716 | 0.1905 | 0.2261 | 0.2263 |
| Japan | -0.6123 | -0.0849 | 0.1437 | 0.1635 | 0.1946 | 0.1955 |
| Switzerland | -0.6682 | -0.1281 | 0.1690 | 0.1936 | 0.2152 | 0.2184 |
| United Kingdom | -0.5970 | -0.1594 | 0.1591 | 0.1785 | 0.2024 | 0.2165 |
| United States | -0.0249 | -0.1882 | 0.0243 | 0.0487 | 0.0643 | 0.0759 |
| Standard error | (0.0993) | (0.0721) | (0.0239) | (0.0191) | (0.0163) | (0.0116) |

Table 3.6.38 Multivariate Spectral Shape Tests
Accumulated Covariance Contributions
$D_q(t,0)$
Government Expenditures Economy (NX,P)

| Country | Frequencies | | | | |
|---|---|---|---|---|---|
| | $t=\frac{\pi}{6}$ | $t=\frac{2\pi}{6}$ | $t=\frac{3\pi}{6}$ | $t=\frac{4\pi}{6}$ | $t=\frac{5\pi}{6}$ |
| Australia | 0.1355 | 0.0467 | 0.0252 | 0.0243 | 0.0101 |
| Austria | -0.0853 | -0.1029 | -0.0499 | -0.0207 | -0.0142 |
| Canada | -0.0760 | -0.0642 | -0.0398 | -0.0222 | -0.0117 |
| Europe | 0.0665 | -0.0051 | -0.0140 | -0.0128 | -0.0061 |
| France | 0.0773 | -0.0369 | -0.0290 | -0.0182 | -0.0041 |
| Germany | 0.0244 | 0.0141 | -0.0058 | 0.0067 | -0.0101 |
| Italy | 0.0427 | 0.0228 | 0.0066 | 0.0065 | 0.0036 |
| Japan | 0.0783 | 0.0129 | 0.0023 | -0.0023 | -0.0018 |
| Switzerland | 0.0016 | -0.0097 | -0.0099 | -0.0070 | -0.0030 |
| United Kingdom | -0.0782 | -0.0282 | -0.0283 | -0.0139 | -0.0029 |
| United States | -0.0126 | -0.0233 | -0.0178 | -0.0100 | -0.0085 |
| Standard error | (0.0248) | (0.0110) | (0.0058) | (0.0031) | (0.0018) |

Table 3.6.39. Multivariate Spectral Shape Tests
Incremental Covariance Contributions
$D_q(t,t-\frac{\pi}{6})$
Government Expenditures Economy (NX,P)

| Country | Frequencies | | | | | |
|---|---|---|---|---|---|---|
| | $t=\frac{\pi}{6}$ | $t=\frac{2\pi}{6}$ | $t=\frac{3\pi}{6}$ | $t=\frac{4\pi}{6}$ | $t=\frac{5\pi}{6}$ | $t=\pi$ |
| Australia | 0.1355 | -0.0888 | -0.0215 | -0.0008 | -0.0143 | -0.0101 |
| Austria | -0.0853 | -0.0176 | 0.0530 | 0.0292 | 0.0065 | 0.0142 |
| Canada | -0.0760 | 0.0118 | 0.0244 | 0.0176 | 0.0105 | 0.0117 |
| Europe | 0.0665 | -0.0715 | -0.0089 | 0.0012 | 0.0067 | 0.0061 |
| France | 0.0773 | -0.1142 | 0.0079 | 0.0107 | 0.0142 | 0.0041 |
| Germany | 0.0244 | -0.0103 | -0.0199 | -0.0009 | -0.0034 | 0.0101 |
| Italy | 0.0427 | -0.0199 | -0.0163 | -0.0001 | -0.0030 | -0.0036 |
| Japan | 0.0783 | -0.0654 | -0.0106 | -0.0046 | 0.0005 | 0.0018 |
| Switzerland | 0.0016 | -0.0113 | 0.0002 | 0.0029 | 0.0040 | 0.0030 |
| United Kingdom | -0.0782 | 0.0500 | -0.0001 | 0.0144 | 0.0109 | 0.0029 |
| United States | -0.0126 | -0.0107 | 0.0055 | 0.0078 | 0.0015 | 0.0085 |
| Standard error | (0.0248) | (0.0213) | (0.0064) | (0.0035) | (0.0021) | (0.0018) |

Table 3.6.40. Multivariate Spectral Shape Tests
Accumulated Covariance Contributions
$D_c(t,0)$
No Capital Economy (NX,P)

| Country | Frequencies $t = \frac{\pi}{6}$ | $t = \frac{2\pi}{6}$ | $t = \frac{3\pi}{6}$ | $t = \frac{4\pi}{6}$ | $t = \frac{5\pi}{6}$ |
|---|---|---|---|---|---|
| Australia | -0.0456 | -0.0104 | 0.0138 | 0.0154 | 0.0034 |
| Austria | -0.0780 | -0.0371 | 0.0145 | 0.0146 | 0.0137 |
| Canada | 0.0356 | 0.0719 | 0.0806 | 0.0629 | 0.0267 |
| Europe | -0.1847 | -0.1968 | -0.1523 | -0.0938 | -0.0410 |
| France | -0.1289 | -0.2174 | -0.1788 | -0.1204 | -0.0586 |
| Germany | -0.0111 | 0.0313 | 0.0114 | 0.0349 | 0.0216 |
| Italy | -0.4328 | -0.3788 | 0.2701 | 0.1806 | -0.0896 |
| Japan | -0.2932 | -0.2616 | -0.1808 | -0.1183 | -0.0588 |
| Switzerland | -0.3491 | -0.3606 | -0.2545 | -0.1619 | -0.0818 |
| United Kingdom | -0.2779 | -0.3208 | -0.2246 | -0.1471 | -0.0798 |
| United States | 0.2941 | 0.2226 | 0.1839 | 0.1316 | 0.0608 |
| Standard error | (0.0263) | (0.0333) | (0.0257) | (0.0171) | (0.0147) |

Table 3.6.41. Multivariate Spectral Shape Tests
Incremental Covariance Contributions
$D_c(t, t - \frac{\pi}{6})$
No Capital Economy (NX,P)

| Country | Frequencies $t = \frac{\pi}{6}$ | $t = \frac{2\pi}{6}$ | $t = \frac{3\pi}{6}$ | $t = \frac{4\pi}{6}$ | $t = \frac{5\pi}{6}$ | $t = \pi$ |
|---|---|---|---|---|---|---|
| Australia | -0.0456 | 0.0351 | 0.0242 | 0.0016 | -0.0120 | -0.0034 |
| Austria | -0.0780 | 0.0409 | 0.0225 | 0.0291 | -0.0009 | -0.0137 |
| Canada | 0.0356 | 0.0363 | 0.0087 | -0.0177 | -0.0361 | -0.0267 |
| Europe | -0.1847 | 0.0121 | 0.0444 | 0.0585 | 0.0528 | 0.0110 |
| France | 0.1289 | 0.0885 | 0.0386 | 0.0585 | 0.0617 | 0.0586 |
| Germany | -0.0111 | 0.0423 | 0.0101 | -0.0064 | -0.0134 | -0.0216 |
| Italy | -0.4328 | 0.0540 | 0.1086 | 0.0895 | 0.0910 | 0.0896 |
| Japan | -0.2932 | -0.0316 | 0.0808 | 0.0625 | 0.0595 | 0.0588 |
| Switzerland | 0.3491 | 0.0115 | 0.1061 | 0.0926 | 0.0801 | 0.0818 |
| United Kingdom | -0.2779 | -0.0428 | 0.0962 | 0.0775 | 0.0673 | 0.0798 |
| United States | 0.2941 | -0.0716 | -0.0387 | -0.0523 | -0.0708 | -0.0608 |
| Standard error | (0.0263) | (0.0367) | (0.0250) | (0.0257) | (0.0159) | (0.0147) |

Table 3.6.42. Multivariate Spectral Shape Tests
Accumulated Covariance Contributions
$D_q(t,0)$
No Capital Economy (NX,P)

| Country | Frequencies | | | | |
|---|---|---|---|---|---|
| | $t=\frac{\pi}{6}$ | $t=\frac{2\pi}{6}$ | $t=\frac{3\pi}{6}$ | $t=\frac{4\pi}{6}$ | $t=\frac{5\pi}{6}$ |
| Australia | 0.5971 | 0.3872 | 0.2585 | 0.1690 | 0.0807 |
| Austria | 0.3762 | 0.2375 | 0.1834 | 0.1240 | 0.0565 |
| Canada | 0.3855 | 0.2763 | 0.1936 | 0.1225 | 0.0590 |
| Europe | 0.5280 | 0.3354 | 0.2194 | 0.1319 | 0.0645 |
| France | 0.5389 | 0.3036 | 0.2044 | 0.1264 | 0.0665 |
| Germany | 0.4859 | 0.3546 | 0.2275 | 0.1379 | 0.0605 |
| Italy | 0.5042 | 0.3633 | 0.2399 | 0.1512 | 0.0742 |
| Japan | 0.5398 | 0.3534 | 0.2356 | 0.1424 | 0.0689 |
| Switzerland | 0.4632 | 0.3308 | 0.2234 | 0.1376 | 0.0676 |
| United Kingdom | 0.3833 | 0.3123 | 0.2050 | 0.1308 | 0.0677 |
| United States | 0.4489 | 0.3172 | 0.2156 | 0.1346 | 0.0621 |
| Standard error | (0.0709) | (0.0333) | (0.0182) | (0.0130) | (0.0068) |

Table 3.6.43. Multivariate Spectral Shape Tests
Incremental Covariance Contributions
$D_q(t,t-\frac{\pi}{6})$
No Capital Economy (NX,P)

| Country | Frequencies | | | | | |
|---|---|---|---|---|---|---|
| | $t=\frac{\pi}{6}$ | $t=\frac{2\pi}{6}$ | $t=\frac{3\pi}{6}$ | $t=\frac{4\pi}{6}$ | $t=\frac{5\pi}{6}$ | $t=\pi$ |
| Australia | 0.5971 | -0.2099 | -0.1287 | -0.0895 | -0.0883 | -0.0807 |
| Austria | 0.3762 | -0.1386 | -0.0541 | -0.0595 | -0.0675 | -0.0565 |
| Canada | 0.3855 | -0.1093 | -0.0827 | -0.0711 | -0.0635 | -0.0590 |
| Europe | 0.5280 | -0.1926 | -0.1161 | -0.0875 | -0.0673 | -0.0645 |
| France | 0.5389 | 0.2353 | -0.0992 | 0.0780 | -0.0599 | -0.0665 |
| Germany | 0.4859 | -0.1314 | -0.1270 | -0.0896 | -0.0774 | -0.0605 |
| Italy | 0.5042 | -0.1410 | -0.1231 | -0.0887 | -0.0770 | -0.0712 |
| Japan | 0.5398 | -0.1864 | -0.1178 | -0.0932 | -0.0735 | -0.0689 |
| Switzerland | 0.4632 | -0.1323 | -0.1074 | -0.0858 | -0.0700 | -0.0676 |
| United Kingdom | 0.3833 | -0.0710 | -0.1073 | -0.0743 | -0.0631 | -0.0677 |
| United States | 0.4489 | -0.1318 | -0.1016 | -0.0809 | -0.0725 | -0.0621 |
| Standard error | (0.0709) | (0.0708) | (0.0300) | (0.0125) | (0.0080) | (0.0068) |

# Chapter 4

# Modelling Measurement Error in Trade Statistics

## 4.1 Introduction.

The empirical results from Chapter 3 illustrated a striking characteristic of the relationship between net exports and the terms of trade that has extremely important implications for econometric modelling. While the majority of activity in the cross-spectrum between the terms of trade and the trade balance is concentrated at low frequencies, the location and magnitude of co- and quadrature spectral power differs substantially across countries. The wide cross-sectional variability that is evident in the frequency domain contrasts sharply with the generic S curve relationship for the time domain cross-correlation function between net exports and the terms of trade that was highlighted by Backus, Kehoe, and Kydland (1994) (henceforth, BKK). Moreover, the multivariate dynamics appear to be much more complicated than the univariate dynamics for the same time series. Univariate time series for the trade balance and the terms of trade exhibit a large amount of persistence; univariate spectra for the trade balance and the terms of trade have power concentrated at low frequencies and are reminiscent of the "typical spectral shape" of economic time series depicted by Granger (1967) and Nerlove (1964).

Within a traditional calibration and simulation econometric framework, statistically significant cross-sectional variability in key statistics of interest presents an enormously difficult and time-consuming challenge. A key feature of the methodology is that the mapping between parameter choices in the calibration stage and statistical output at the simulation stage is not immediate. In particular, the complicated non-linear objective function must be approximated. Ricatti equations for the approximate model must be iteratively solved, and the model must be simulated repeatedly, and each of these steps occurs for each separate set of parameters. For the two-sector international business cycle model developed by BKK (1994) and re-examined in Chapter 3, it is clear that the choice of parameters for calibration is not sufficient to generate the variety in cross-spectral characteristics between the terms of trade and net exports that is exhibited by the data. While it would be possible to attempt further calibrations, or evaluate the model for each country by more formal econometric procedures such as GMM, it seems that this approach would be unsuccessful, given the tightly restricted and relatively simple dynamics exhibited by the model.

An alternative approach that has intuitive appeal in this particular situation and that has potential for replicating a wide range of cross-spectral dynamics is to augment the model with stochastic time-varying measurement errors. Within the context of the model, it is possible to maintain baseline calibration parameters and vary the properties of the measurement error process to mimic cross-sectional variability in statistics of interest. From a theoretical perspective, the model of economic behavior remains unchanged across, while the measurement error processes may vary substantially. As data collection procedures and the definitions of variables in national accounts vary significantly across countries, this approach appears intuitively plausible as well as practical.

Moreover, there is a growing statistical literature arguing that trade statistics are more error-ridden than any other statistics of an economy. For example, Alterman (1991) employed improved indexes of U.S. import and export prices to reduce the variability in the historical time series for the terms of trade by approximately 30 percent. A recent study by the National Research Council in the U.S. has also found that there exists a statistical discrepancy in the U.S. balance-of-payments accounts which amounted to approximately $63.5 billion in 1990 and accounted for as much as 70 percent of the U.S. current account deficit. Table 4.1.1 presents average statistical discrepancies and discrepancy percentages (as a percent of output) for the last 10 years for some of the countries in our sample. [1] It is clear that average and relative magnitude of the errors varies considerably across countries. The empirical evidence concerning measurement error in trade statistics is particularly important, as the addition of stochastic measurement error to a theoretical model can potentially reconcile any theory with the data.

[1]Source: National Research Council, "Behind the numbers U.S trade in the world Economy (1992).

This chapter augments the theoretical model from Chapter 3 with a vector of time-varying measurment errors. The model is a version of the classical "errors-in-variables" model and has been proposed by Sargent (1989) in a similar context. While it is theoretically possible to augment all of the endogenous variables with measurement errors, we will focus on the relationship between net exports and the terms of trade, and will consider the case where the measurement error occurs in exports. Empirically, we ask the following question: "For a given set of calibration parameters, what are the properties of measurement error in exports that will reconcile the cross-spectral properties of net exports and the terms of trade in the model with the historical data? Within the calibration and simulation framework, our approach is similar to Watson (1993), although the rationale for the measurement error is different.[2]

This chapter will proceed as follows. Section 2 presents some motivation for the measurement error model in the frequency domain. In addition, the theoretical impact of time-varying measurement error on the spectral properties of the model developed in Chapter 3 is examined. Section 3 presents the empirical results. Section 4 concludes.

[2]Watson (1993) argues that the model is not a null hypothesis, and that the measurement error is actually an inevitable approximation error that will be present in any econometric modelling effort.

## 4.2 Measurement Error in the BKK Model

### 4.2.1 Motivation

In developing a potential model for the measurement error, it is useful to re-examine the empirical results from Chapter 3 regarding the cross-spectral properties of the terms-of-trade and net exports. For the benchmark economy, and the majority of other cases examined, it was argued that the key difference between the model and the data was the extreme concentration of negative quadrature power at low frequencies for the model relative to the data

Consider a situation where the true economic variables of interest are $y$ and $x$, but where we only observe an error-corrupted version of $x$ called $x^*$, where $x^* = x + \epsilon$. The measurement error $\epsilon$ has a zero mean and is assumed to be independent of $x$ and $y$. The cross-spectrum between $x^*$ and $y$ is defined as

$$f_{yx^*}(\omega) = \sum_{k=-\infty}^{k=\infty} \gamma(k) e^{i\omega k} \tag{4.2.1}$$

where $\gamma(k)=Cov(Y_t, X^*_{t+k})=Cov(Y_t, X_{t+k})$. By definition, the non-normalized cross spectrum will remain unaffected by the addition of the measurement error, although this is not true for the normalized cross spectrum. Define $\alpha$ as the variance of $\epsilon$ relative to $x$, *i.e.* $\alpha=\frac{Var(\epsilon)}{Var(x)}$. It follows that $\mathrm{Var}(x^*)=(1+\alpha)\mathrm{Var}(x)$ and in terms of the normalized cross-spectrum

$$f_{yx}(\omega) = \frac{1}{\sqrt{1+\alpha}} f_{yx^*}(\omega). \tag{4.2.2}$$

As a result, the addition of the measurement error modifies the cross-spectral properties of the "true" time series at all frequencies. In this sense, accounting for measurement errors in net exports will certainly diminish the concentration of power in the quadrature spectrum between net exports and the terms of trade at low frequencies, although this will come at the expense of reducing the cospectral power and thus the model fit in this regard is much better. Overall, the trade-off between these two effects will determine the success of the measurement error exercise.

It is also important to emphasize that we have not yet assumed anything about the univariate dynamics of $\epsilon$. In terms of the univariate spectrum for $x^*$, note that

$$f_{x^*}(\omega) = f_x(\omega) + f_\epsilon(\omega). \tag{4.2.3}$$

The univariate spectrum for the observed time series reflects both the dynamic properties of the original time series and the dynamic properties of the measurement error.

### 4.2.2 The Classical Errors-In-Variables Model

This sub-section reviews the classical formulation of the measurement error model. We will assume that economic theory provides a stationary stochastic process for some true variables which are of interest to the public. The data are made available through a reporting agency that observes only error-corrupted versions of the data.

More precisely, we will assume that the data being observed by the agency are the sum of the true economic variables and a measurement error that may be serially correlated but which is uncorrelated with the true variables. Note that we have assumed that the measurement errors process is independent of the internal workings of the economy as characterized by the economic theory, *i.e.* there is no feedback between the measurement errors and the actual behavior of the economy. As pointed out by Sargent (1989), this is crucial for allowing the problem to remain tractable.

In an infinite horizon framework, the equlibrium of the economic model can be represented as a weak stationary stochastic process with a Wold moving average representation, *i.e.*

$$u_t = C_u(L)\epsilon_{ut}, \tag{1.2.4}$$

where $u_t$ is an $n \times 1$ vector of variables,

$$C_u(L) = \sum_{j=0}^{\infty} c_{uj} L^j, \tag{1.2.5}$$

and $E[\epsilon_{ut}] = 0$ and $E\epsilon'_{ut}\epsilon_{us} = I$ for all $t$ and $s$. The data collecting agency observes an error-ridden version of $u_t$, *i.e.*

$$Y_t = u_t + v_t \tag{1.2.6}$$

where $v_t$ is orthogonal to the true variables, *i.e.* $E\epsilon_{ut}v'_s = 0$ for all $t$ and $s$. The measurement error also has a Wold moving average representation given by

$$v_t = C_v(L)\epsilon_{vt}, \tag{4.2.7}$$

with $C_v(L) = \sum_{j=0}^{\infty} \psi_{vj} L^j$ and where the innovations have zero mean and are serially uncorrelated. As a result, the error-corrupted data have a Wold moving average representation given by

$$V_t = C_V(L)\epsilon_{Vt}, \tag{4.2.8}$$

where

$$C_V(L) = \sum_{j=0}^{\infty} \psi_{Vj} L^j. \tag{4.2.9}$$

### 4.2.3 A Recursive Formulation

In order to make the classical framework more useful for the problem at hand, we drop the infinite horizon assumption and represent the model as one with finite dimensional state vectors. The advantage of this formulation is that recursive methods can be applied through the Kalman filter.

The economic model in state space form can be represented by the following system of equations:

$$x_{t+1} = Ax_t + \epsilon_t, \qquad (4.2.10)$$

$$z_t = Cx_t, \qquad (4.2.11)$$

and where

$$E(\epsilon_t \epsilon_s') = \begin{cases} Q, & for\ s = t \\ 0, & for\ s \neq t, \end{cases}$$

In this equation, $x_t$ is an $n \times 1$ state vector, $\epsilon_t$ is an $n \times 1$ vector white noise, and $z_t$ is an $m \times 1$ vector of variables related to $x_t$ . The public is interested in measuring the variables in $z_t$. The $n \times n$ transition matrix A and the $m \times n$ matrix C have elements are known and are functions of the *deep* parameters of the model. We further assume that the eigenvalues of A are less than unity in modulus.

The model is augmented with an $m \times 1$ vector of measurement errors $v_t$ that obeys

$$v_{t+1} = Dv_t + \nu_t. \qquad (4.2.12)$$

where

$$E\nu_t\nu_s' = \begin{cases} R, & for\ s = t \\ 0, & for\ s \neq t \end{cases}$$

and where $E\nu_t = 0$ for all t, and $E\epsilon_t\nu_s' = 0$ for all t and s. The eigenvalues of D are also required to be less than unity in modulus. The agency is assumed to observe an error-corrupted version of $Z_t$, namely $z_t$ which can be represented by

$$z_t = Cx_t + v_t. \tag{4.2.13}$$

This system of equations can be represented in a state space system as

$$x_{t+1} = Ax_t + \epsilon_t. \tag{4.2.14}$$

$$y_t = Cx_t + w_t. \tag{4.2.15}$$

where the dimensions of the matrices is as before. Both $\epsilon_t$ and $w_t$ are each vector white noise with

$$E\epsilon_t\epsilon_t' = \Sigma \tag{4.2.16}$$

$$Ew_tw_t' = G. \tag{4.2.17}$$

$$E\epsilon_tw_t' = W. \tag{4.2.18}$$

The matrices $\Sigma$ and $G$ are each positive semidefinite. Define $S$ as the unique positive semidefinite matrix that satisfies the algebraic Ricatti matrix equation

$$S = G + ASA' - (ASC' + W)(G + CSC')^{-1}(CSA' + W'). \tag{4.2.19}$$

Define $K$ in terms of $S$ as

$$K = (ASC' + W)(CSC' + G)^{-1} \tag{4.2.20}$$

Methods of solving for these two equations have been described in Chapter 3, Section 5.

As pointed out by Sargent(1989) the Wold vector moving average representation for the $w$ process is given by

$$y_{t+1} = (I - \hat{C}L)^{-1}[\hat{C}(I - AL)^{-1}KL + I]\epsilon_t \tag{4.2.21}$$

where $\hat{C} = (CA - DC)$, $K$ is as defined in equation (4.2.20) and $L$ is a vector lag operator.

### 4.2.4 The BKK Model

This subsection uses the previous results to examine the effects of multivariate measurement error on the spectral matrix of the endogenous variables in the BKK model. From Chapter 3, Section 5, the solution to the BKK model can be represented as

$$x_{t+1} = A0x_t + \epsilon_{t+1} \tag{4.2.22}$$

$$u_t = -Fx_t \tag{4.2.23}$$

where $x_t$ is a $5 \times 1$ vector of state variables, $u_t$ is a $6 \times 1$ vector of control variables, $A0 = A - BF$ is a $5 \times 5$ transition matrix with elements being estimated by solving the discrete linear quadratic regulator problem, and $F = (R + \hat{B}'P\hat{B})^{-1}(\hat{B}'P\hat{A} + R^{-1}N')$. If we augment the control variables with a vector of measurement errors, the model can be represented as

$$x_{t+1} = A0x_t + \epsilon_{t+1}, \tag{4.2.24}$$

$$u_t = -Fx_t + w_t \tag{4.2.25}$$

where the dimensions of the matrices are as before, and

$$E\epsilon_t\epsilon_t' = \Sigma, \tag{4.2.26}$$

$$Ew_tw_t' = G, \tag{4.2.27}$$

$$E\epsilon_tw_t' = W. \tag{4.2.28}$$

The matrices $\Sigma$ and $G$ are each positive semidefinite. $P$ is the unique positive semidefinite matrix that satisfies the algebraic Ricatti equation

$$P = G + A0PA0' + (A0PF' + W)(G + FPF')^{-1}(FPA0' + W'). \tag{4.2.29}$$

As in the previous case, the matrix $K$ (in terms of $F$) can be written as

$$K = -(A0SF' + W)(FSF' + G)^{-1} \tag{4.2.30}$$

Sargent (1987) derives the mutlivariate spectral density for the stationary distribution of the endogenous variables in a model within the linear regulator class as

$$S_x(\omega) = [I - (A - BF)\epsilon^{-i\omega}]^{-1}\Sigma[I - (A - BF)'\epsilon^{+i\omega}]^{-1} \tag{4.2.31}$$

with $F = (R + \dot{B}'PB)^{-1}(\dot{B}'P\dot{A} + R^{-1}X')$ and $P$ is the solution to the matrix Ricatti equation which (in terms of the transformed variables) can be represented by

$$P = \hat{Q} + \tilde{A}'P\dot{A} - \dot{A}'P\dot{B} - \dot{A}'P\dot{B}(R + \dot{B}'P\dot{B})^{-1}\dot{B}'P\dot{A}. \qquad (4.2.32)$$

For the model with the measurement errors included, the spectral density function for the stationary probability distribution can be written as

$$S_x^{me}(\omega) = [I - (A - BF - FK)e^{-i\omega}]^{-1}\Sigma[I - (A - BF_fK)'e^{+i\omega}]^{-1} \qquad (4.2.33)$$

with $F$ as denoted above, $K = (A0SF' + W)(FSF' + R)^{-1}$, and with $S$ being the solution to the algebraic Ricatti equation

$$S = G + A0SA0' - (A0SF' + W)(G + FSF')^{-1}(FSA0' + W') \qquad (4.2.34)$$

From this last equation it is apparent that the spectral density function for the augmented model with the measurement errors depends also on the properties of $G$ and $W$ which are the matrices which summarize the behavior of the measurement error.

## 4.3 Empirical Results

Although simulation results will be reported here for all of the experiments from Chapter 3, the discussion will focus primarily on the properties of net exports and the terms of trade in the benchmark economy. As in Chapter 3, we will consider the time and frequency domain properties separately.

For each of the simulations, the measurement error process followes an AR(1) process with $\rho = 0.7$ and $\mathrm{Var}(\epsilon) = 0.0418^2$; this implies that the variance of the measurement error represents roughly 35 percent of the overall variability of exports. We experimented with a variety of different measurment error processes and we found that the level of persistence in the process, as well as the magnitude of the variance of the process, can yield different results for the frequency domain test statistics.

### 4.3.1 Time Domain Properties Of The Simulated Series

Table 4.3.2 reports the summary statistics for our simulated series for each of the experiments. In the benchmark economy, although we succesfully increase the variability of net exports (0.28 versus 0.20), we are still far away from the numbers observed in historical data. The contemporaneous correlation between net exports and output also improves in this economy; it is -0.51, relative to -0.60 in the benchmark, and it is closer to the mean value in the data. Also, the contemporaneous cross-correlation between net exports and the terms of trade decreases from -0.98 to -0.80, and this is closer to -0.46 which is the mean for the historical data.

Figure 4.3.1 graphs the cross-correlation functions between net exports and the terms of trade for the benchmark and large and small elasticity economies. It is apparent from this figure that the basic V-shape between the two variables remains unaffected in each of these economies, and the level of these curves has decreased.

Overall, introducing the measurement error does not appear to affect the time domain properties of the cross-correlation much.

### 4.3.2 Frequency Domain Properties Of The Simulated Series

This section reports results from the univariate and multivariate frequency domain diagnostics. Note that since we have assumed only one measurement error in exports (net exports), there is no need to reproduce the test statistics for the other variables.

Tables 4.3.3 and 4.3.4 present the univariate spectral shape tests for net exports for the benchmark economy for the measurement error model. In comparison with the univariate test statistics from Chapter 3, it is clear that the measurement error does not have a substantial impact on the results, and that the model still tracks the simple univariate dynamics fairly well.

Turning to the accumulated and incremental cospectral deviations in Tables 4.3.5 and 4.3.6, we observe that for the frequency $\frac{\pi}{6}$ there are four significant deviations. At the $\frac{2\pi}{6}$ frequency, there are five deviations. In the benchmark economy in Chapter 3, the corresponding numbers were six and seven, so the measurement error model represents an improvement. For the quadrature spectrum results presented in Tables 4.3.6 and 4.3.7, significant low frequency deviations are reduced from eleven in the benchmark economy in Chapter 3, to nine here. As a result, it is clear that the measurement error model moves the cross-spectral properties of the model in the right direction, although the required variability in the error appears large relative to the effects that are gained.[3]

## 4.4 Conclusion

This chapter investigated the effects of time-varying measurement error on the time and frequency domain properties of the BKK model. The theoretical effects of measurement error on the spectral density matrix of the endogenous variables was derived and related to the spectral density matrix for the endogenous variables in the model with no measurement error. In comparison with the results from the frequency domain diagnostic tests in Chapter 3, introducing measurement error in net exports makes the model predictions more compatible with the data, although the magnitude of the variability in the error appears large, relative to the gain in cross-spectral performance at the model level.

It is possible to modify a wide variety of spectral properties through the introduction of additional measurement errors that are cross-sectionally correlated. As the dimensionality of this problem grows rapidly, it is important independently to investigate the specific nature of the measurement errors in each dataset prior to formulating a measurement error model. Otherwise, the theoretical exercise becomes vacuous as the measurement error model can be tailored to fix any theoretical inadequacies.

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[3]The results for the other experiments are similar to those presented for the benchmark economy and we are not to going to discuss them separately

Figure 4.3.1: Cross-Correlation Functions, Simulated Data, Simulations 1-3 (NX,P)

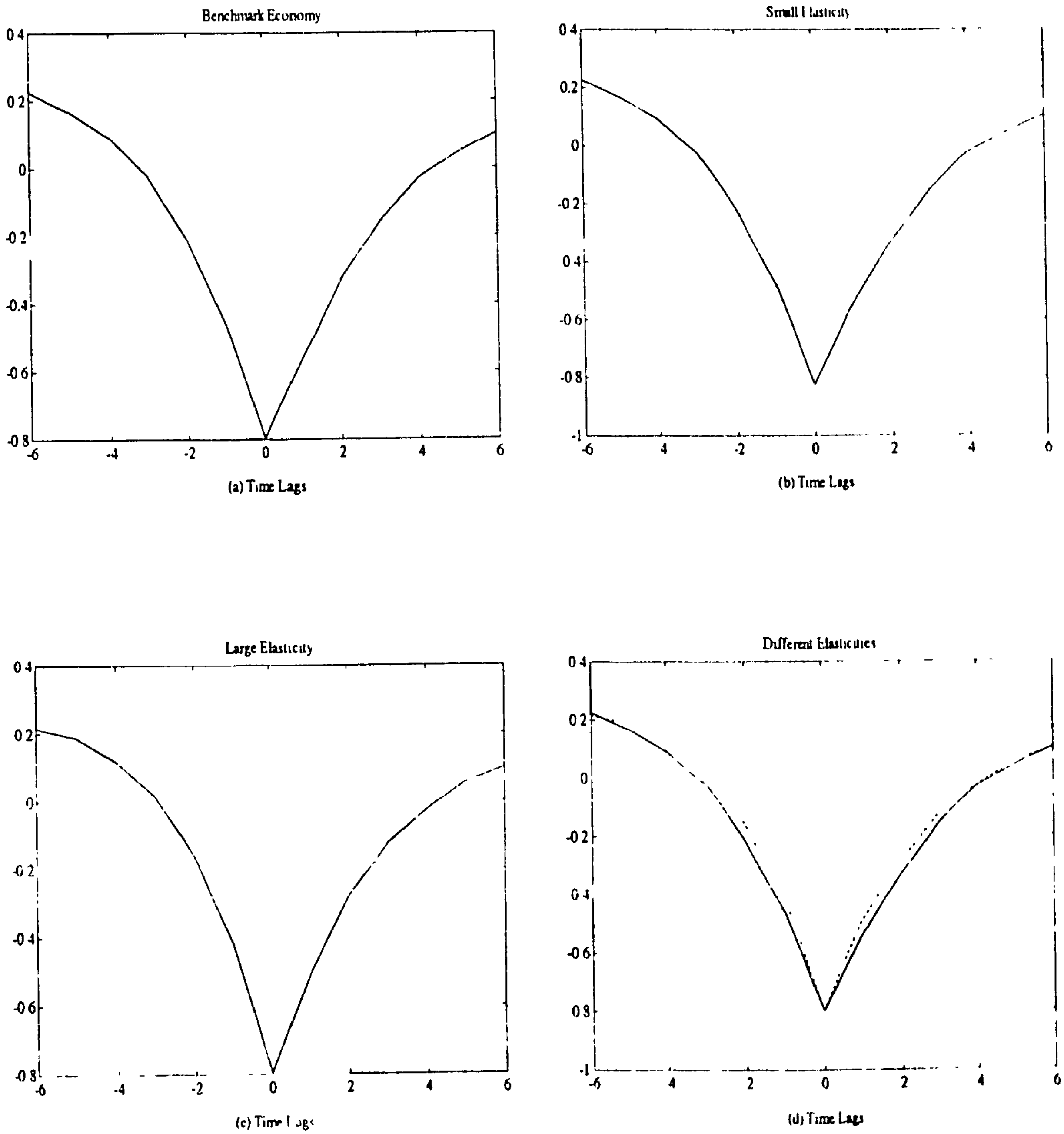

Figure 4.3.2: Cross-Correlation Functions, Simulated Data, Simulations 4-7 (NX,P)

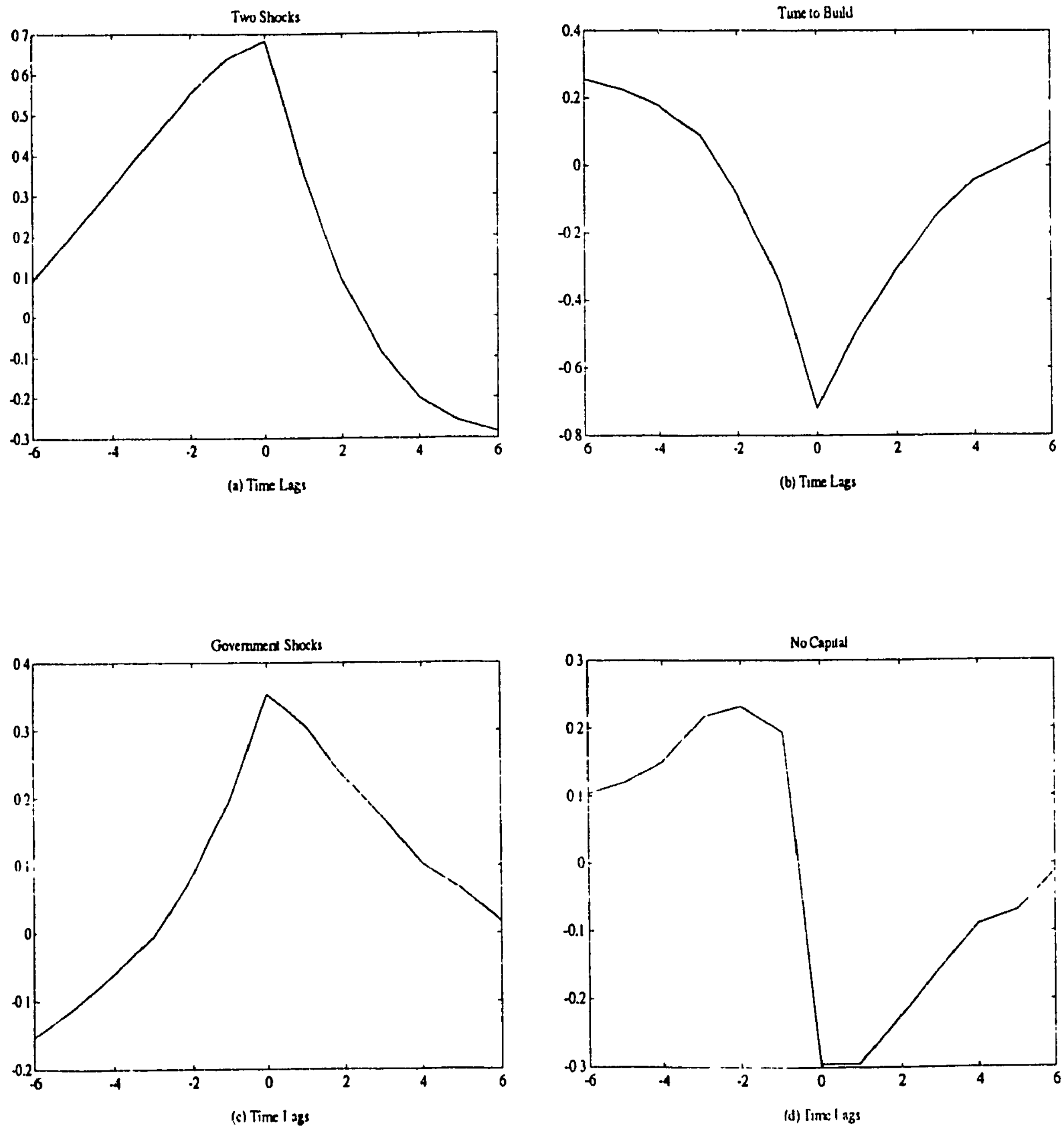

Figure 4.3.3: Cross-Correlation Functions, Simulated Data, Simulations 1, 6 and 7 (NX,P)

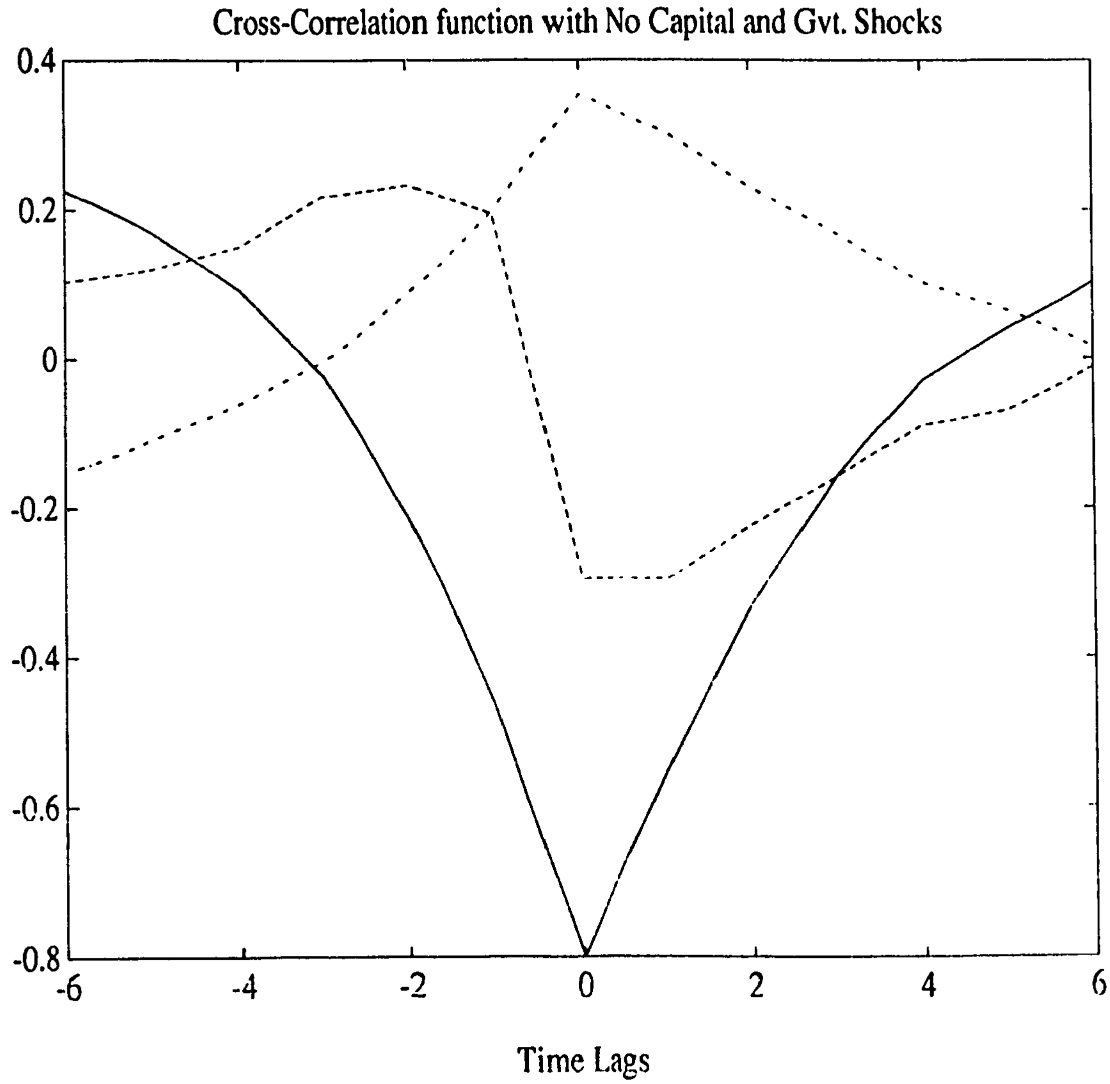

Table 4.1.1. Statistical Discrepancies and Percentages to Real G.D.P.

| Country | $Millions | Percentages |
|---|---|---|
| Canada | 689 | 1.13 |
| Germany | 30 | 0.00 |
| Japan | 92 | 0.36 |
| United Kingdom | -1,163 | -1.13 |
| United States | -7,734 | -7.96 |

Table 4.3.1. Summary Statistics Simulated Economy HP Filtered Data

| | Standard deviation | | | Autocorrelation | | | Correlation | | |
|---|---|---|---|---|---|---|---|---|---|
| Economy | y | p | nx | y | p | nx | (nx,y) | (nx,p) | (y,p) |
| Benchmark | 0.96 | 0.15 | 0.28 | 0.63 | 0.62 | 0.61 | -0.51 | 0.80 | 0.59 |
| | (0.16) | (0.01) | (0.04) | (0.10) | (0.06) | (0.07) | (0.12) | (0.07) | 0.11) |
| Small elasticity | 0.80 | 0.44 | 0.29 | 0.63 | 0.61 | 0.61 | -0.56 | -0.82 | 0.71 |
| | (0.14) | (0.04) | (0.04) | (0.10) | (0.06) | (0.07) | (0.11) | (0.06) | (0.08) |
| Large elasticity | 1.03 | 0.06 | 0.26 | 0.65 | 0.56 | 0.57 | -0.49 | -0.79 | 0.58 |
| | (0.12) | (0.00) | (0.04) | (0.08) | (0.09) | (0.08) | (0.11) | (0.06) | (0.13) |
| Two shocks | 0.91 | 0.17 | 0.25 | 0.64 | 0.81 | 0.62 | -0.65 | 0.68 | -0.79 |
| | (0.14) | (0.02) | (0.03) | (0.10) | (0.04) | (0.07) | (0.09) | (0.07) | (0.07) |
| Time to build | 0.55 | 0.46 | 0.31 | 0.65 | 0.57 | 0.58 | -0.71 | -0.72 | 0.84 |
| | (0.07) | (0.04) | (0.04) | (0.07) | (0.06) | (0.07) | (0.08) | (0.07) | (0.05) |
| Government shocks | 0.27 | 0.20 | 0.05 | 0.65 | 0.68 | 0.57 | -0.26 | 0.35 | -0.47 |
| | (0.05) | (0.02) | (0.01) | (0.12) | (0.06) | (0.05) | (0.11) | (0.13) | (0.12) |
| No capital | 0.30 | 0.33 | 0.09 | 0.65 | 0.10 | 0.60 | -0.21 | -0.30 | -0.25 |
| | (0.05) | (0.03) | (0.01) | (0.12) | (0.07) | (0.06) | (0.15) | (0.09) | (0.10) |

Table 4.3.2. Univariate Spectral Shape Tests
Accumulated Variance Contributions
$\hat{D}(t,0)$
Benchmark Economy (NX)

| Country | Frequencies | | | | |
|---|---|---|---|---|---|
| | $t = \frac{\pi}{6}$ | $t = \frac{2\pi}{6}$ | $t = \frac{3\pi}{6}$ | $t = \frac{4\pi}{6}$ | $t = \frac{5\pi}{6}$ |
| Australia | -0.0089 | 0.0656 | 0.0436 | 0.0248 | 0.0107 |
| Austria | -0.0405 | -0.1263 | -0.0969 | -0.0510 | 0.0325 |
| Canada | -0.0196 | 0.0012 | -0.0030 | -0.0040 | -0.0035 |
| Europe | 0.0605 | 0.0527 | 0.0284 | 0.0094 | 0.0090 |
| France | -0.0196 | 0.0453 | 0.0277 | 0.0192 | 0.0070 |
| Germany | 0.0935 | 0.0080 | -0.0191 | -0.0171 | -0.0076 |
| Italy | 0.1024 | 0.0846 | 0.0399 | 0.0292 | 0.0125 |
| Japan | 0.1098 | 0.0856 | 0.0476 | 0.0281 | 0.0146 |
| Switzerland | 0.1284 | 0.1099 | 0.0662 | 0.0406 | 0.0200 |
| United Kingdom | 0.0245 | 0.0405 | 0.0049 | 0.0004 | 0.0014 |
| United States | 0.1149 | 0.0795 | 0.0402 | 0.0255 | 0.0120 |
| Standard error | (0.0578) | (0.0288) | (0.0141) | (0.0126) | (0.0077) |

Table 4.3.3. Univariate Spectral Shape Tests
Incremental Variance Contributions
$\hat{D}(t, t - \frac{\pi}{6})$
Benchmark Economy (NX)

| Country | Frequencies | | | | | |
|---|---|---|---|---|---|---|
| | $t = \frac{\pi}{6}$ | $t = \frac{2\pi}{6}$ | $t = \frac{3\pi}{6}$ | $t = \frac{4\pi}{6}$ | $t = \frac{5\pi}{6}$ | $t = \pi$ |
| Australia | -0.0089 | 0.0745 | -0.0220 | -0.0188 | -0.0141 | -0.0107 |
| Austria | -0.0405 | -0.0858 | 0.0294 | 0.0459 | 0.0184 | 0.0325 |
| Canada | -0.0196 | 0.0209 | -0.0042 | -0.0011 | 0.0005 | 0.0035 |
| Europe | 0.0605 | -0.0078 | 0.0243 | -0.0190 | 0.0004 | 0.0090 |
| France | -0.0196 | 0.0619 | -0.0175 | 0.0085 | 0.0122 | -0.0070 |
| Germany | 0.0935 | -0.0855 | 0.0271 | 0.0020 | 0.0096 | 0.0076 |
| Italy | 0.1024 | -0.0178 | -0.0448 | -0.0107 | -0.0168 | -0.0125 |
| Japan | 0.1098 | -0.0212 | -0.0380 | -0.0195 | -0.0135 | -0.0146 |
| Switzerland | 0.1284 | 0.0185 | 0.0437 | 0.0257 | -0.0206 | -0.0200 |
| United Kingdom | 0.0245 | 0.0160 | -0.0356 | -0.0045 | 0.0010 | -0.0014 |
| United States | 0.1149 | -0.0355 | -0.0393 | -0.0147 | -0.0135 | -0.0120 |
| Standard error | (0.0578) | (0.0419) | (0.0204) | (0.0094) | (0.0083) | (0.0077) |

Table 4.3.4. Multivariate Spectral Shape Tests
Accumulated Covariance Contributions
$\hat{D}_c(t,0)$
Benchmark Economy (NX,P)

| Country | Frequencies | | | | |
|---|---|---|---|---|---|
| | $t=\frac{\pi}{6}$ | $t=\frac{2\pi}{6}$ | $t=\frac{3\pi}{6}$ | $t=\frac{4\pi}{6}$ | $t=\frac{5\pi}{6}$ |
| Australia | 0.2351 | 0.3046 | 0.2625 | 0.1809 | 0.0880 |
| Austria | 0.2027 | 0.2780 | 0.2342 | 0.1800 | 0.0984 |
| Canada | 0.3163 | 0.3870 | 0.3293 | 0.2283 | 0.1114 |
| Europe | 0.0960 | 0.1183 | 0.0964 | 0.0717 | 0.0437 |
| France | 0.1517 | 0.0976 | 0.0699 | 0.0451 | 0.0260 |
| Germany | 0.2696 | 0.3464 | 0.2901 | 0.2004 | 0.1062 |
| Italy | -0.1521 | -0.0637 | -0.0214 | -0.0152 | -0.0050 |
| Japan | -0.0126 | 0.0535 | 0.0680 | 0.0471 | 0.0258 |
| Switzerland | -0.0685 | -0.0455 | -0.0058 | 0.0036 | 0.0028 |
| United Kingdom | 0.0028 | -0.0057 | 0.0212 | 0.0181 | 0.0048 |
| United States | 0.5748 | 0.5376 | 0.4326 | 0.2970 | 0.1454 |
| Standard error | (0.1072) | (0.0611) | (0.0412) | (0.0306) | (0.0161) |

Table 4.3.5 Multivariate Spectral Shape Tests
Incremental Covariance Contributions
$\hat{D}_c(t,t-\frac{\pi}{6})$
Benchmark Economy (NX,P)

| Country | Frequencies | | | | | |
|---|---|---|---|---|---|---|
| | $t=\frac{\pi}{6}$ | $t=\frac{2\pi}{6}$ | $t=\frac{3\pi}{6}$ | $t=\frac{4\pi}{6}$ | $t=\frac{5\pi}{6}$ | $t=\pi$ |
| Australia | 0.2351 | 0.0695 | -0.0421 | -0.0817 | -0.0929 | -0.0880 |
| Austria | 0.2027 | 0.0753 | -0.0438 | -0.0542 | -0.0817 | -0.0981 |
| Canada | 0.3163 | 0.0707 | -0.0577 | -0.1010 | -0.1169 | -0.1114 |
| Europe | 0.0960 | 0.0223 | 0.0219 | 0.0217 | 0.0280 | -0.0137 |
| France | 0.1517 | -0.0541 | 0.0278 | 0.0218 | -0.0191 | -0.0260 |
| Germany | 0.2696 | 0.0768 | 0.0563 | -0.0897 | -0.0942 | -0.1062 |
| Italy | -0.1521 | 0.0884 | 0.0423 | 0.0062 | 0.0102 | 0.0050 |
| Japan | -0.0126 | 0.0661 | 0.0145 | -0.0208 | -0.0213 | -0.0258 |
| Switzerland | 0.0685 | 0.0229 | 0.0398 | 0.0093 | -0.0007 | 0.0028 |
| United Kingdom | 0.0028 | -0.0084 | 0.0298 | -0.0058 | -0.0136 | -0.0048 |
| United States | 0.5748 | -0.0372 | -0.1050 | -0.1356 | -0.1516 | -0.1454 |
| Standard error | (0.1072) | (0.0792) | (0.0316) | (0.0181) | (0.0174) | (0.0161) |

Table 4.3.6. Multivariate Spectral Shape Tests
Accumulated Covariance Contributions
$\hat{D}_q(t,0)$
Benchmark Economy (NX,P)

| Country | Frequencies $t=\frac{\pi}{6}$ | $t=\frac{2\pi}{6}$ | $t=\frac{3\pi}{6}$ | $t=\frac{4\pi}{6}$ | $t=\frac{5\pi}{6}$ |
|---|---|---|---|---|---|
| Australia | 0.3471 | 0.1605 | 0.0911 | 0.0638 | 0.0306 |
| Austria | 0.1262 | 0.0109 | 0.0160 | 0.0188 | 0.0063 |
| Canada | 0.1356 | 0.0496 | 0.0261 | 0.0173 | 0.0088 |
| Europe | 0.2780 | 0.1088 | 0.0519 | 0.0267 | 0.0144 |
| France | 0.2889 | 0.0769 | 0.0370 | 0.0212 | 0.0164 |
| Germany | 0.2360 | 0.1279 | 0.0601 | 0.0327 | 0.0104 |
| Italy | 0.2543 | 0.1367 | 0.0725 | 0.0460 | 0.0241 |
| Japan | 0.2898 | 0.1267 | 0.0682 | 0.0372 | 0.0187 |
| Switzerland | 0.2132 | 0.1042 | 0.0560 | 0.0324 | 0.0175 |
| United Kingdom | 0.1333 | 0.0857 | 0.0376 | 0.0256 | 0.0176 |
| United States | 0.1989 | 0.0905 | 0.0481 | 0.0294 | 0.0120 |
| Standard error | (0.0501) | (0.0352) | (0.0214) | (0.0141) | (0.0064) |

Table 4.3.7. Multivariate Spectral Shape Tests
Incremental Covariance Contributions
$\hat{D}_q(t,t-\frac{\pi}{6})$
Benchmark Economy (NX,P)

| Country | Frequencies $t=\frac{\pi}{6}$ | $t=\frac{2\pi}{6}$ | $t=\frac{3\pi}{6}$ | $t=\frac{4\pi}{6}$ | $t=\frac{5\pi}{6}$ | $t=\pi$ |
|---|---|---|---|---|---|---|
| Australia | 0.3471 | -0.1866 | -0.0695 | -0.0273 | -0.0332 | -0.0306 |
| Austria | 0.1262 | 0.1153 | 0.0051 | 0.0028 | -0.0121 | -0.0063 |
| Canada | 0.1356 | -0.0859 | -0.0235 | -0.0089 | -0.0084 | -0.0088 |
| Europe | 0.2780 | -0.1692 | -0.0569 | 0.0253 | -0.0122 | 0.0144 |
| France | 0.2889 | -0.2120 | -0.0400 | -0.0158 | -0.0048 | -0.0164 |
| Germany | 0.2360 | -0.1080 | -0.0678 | -0.0274 | -0.0223 | 0.0104 |
| Italy | 0.2543 | -0.1176 | -0.0642 | -0.0265 | -0.0219 | -0.0241 |
| Japan | 0.2898 | -0.1631 | -0.0586 | -0.0310 | -0.0184 | -0.0187 |
| Switzerland | 0.2132 | 0.1090 | 0.0182 | 0.0236 | 0.0149 | 0.0175 |
| United Kingdom | 0.1333 | -0.0477 | -0.0481 | -0.0120 | -0.0080 | -0.0176 |
| United States | 0.1989 | -0.1084 | -0.0424 | -0.0187 | -0.0174 | -0.0120 |
| Standard error | (0.0501) | (0.0409) | (0.0220) | (0.0109) | (0.0100) | (0.0064) |

Table 4.3.8. Multivariate Spectral Shape Tests
Accumulated Covariance Contributions
$\hat{D}_c(t,0)$
Small Elasticity Economy (NX,P)

| Country | Frequencies $t = \frac{\pi}{6}$ | $t = \frac{2\pi}{6}$ | $t = \frac{3\pi}{6}$ | $t = \frac{4\pi}{6}$ | $t = \frac{5\pi}{6}$ |
|---|---|---|---|---|---|
| Australia | 0.2374 | 0.3118 | 0.2691 | 0.1855 | 0.0901 |
| Austria | 0.2050 | 0.2851 | 0.2407 | 0.1847 | 0.1005 |
| Canada | 0.3186 | 0.3941 | 0.3358 | 0.2330 | 0.1135 |
| Europe | 0.0983 | 0.1254 | 0.1030 | 0.0763 | 0.0458 |
| France | 0.1540 | 0.1048 | 0.0764 | 0.0497 | 0.0281 |
| Germany | 0.2719 | 0.3535 | 0.2967 | 0.2051 | 0.1083 |
| Italy | -0.1498 | -0.0566 | -0.0148 | -0.0105 | -0.0029 |
| Japan | -0.0103 | 0.0606 | 0.0745 | 0.0518 | 0.0279 |
| Switzerland | -0.0662 | -0.0384 | 0.0008 | 0.0082 | 0.0050 |
| United Kingdom | 0.0050 | 0.0015 | 0.0307 | 0.0236 | 0.0069 |
| United States | 0.5771 | 0.5448 | 0.4392 | 0.3017 | 0.1475 |
| Standard error | (0.1074) | (0.0588) | (0.0393) | (0.0285) | (0.0152) |

Table 4.3.9. Multivariate Spectral Shape Tests
Incremental Covariance Contributions
$\hat{D}_c(t, t - \frac{\pi}{6})$
Small Elasticity Economy (NX,P)

| Country | Frequencies $t = \frac{\pi}{6}$ | $t = \frac{2\pi}{6}$ | $t = \frac{3\pi}{6}$ | $t = \frac{4\pi}{6}$ | $t = \frac{5\pi}{6}$ | $t = \pi$ |
|---|---|---|---|---|---|---|
| Australia | 0.2374 | 0.0744 | -0.0427 | -0.0835 | -0.0954 | -0.0901 |
| Austria | 0.2050 | 0.0802 | -0.0411 | -0.0560 | -0.0812 | 0.1005 |
| Canada | 0.3186 | 0.0755 | -0.0582 | -0.1029 | -0.1195 | -0.1135 |
| Europe | 0.0983 | 0.0272 | 0.0225 | -0.0266 | 0.0305 | 0.0458 |
| France | 0.1540 | 0.0193 | -0.0283 | -0.0267 | -0.0216 | 0.0281 |
| Germany | 0.2719 | 0.0816 | -0.0568 | -0.0916 | 0.0968 | -0.1083 |
| Italy | -0.1498 | 0.0933 | 0.0417 | 0.0043 | 0.0076 | 0.0029 |
| Japan | -0.0103 | 0.0709 | 0.0139 | -0.0227 | -0.0239 | -0.0279 |
| Switzerland | 0.0662 | 0.0277 | 0.0392 | 0.0071 | 0.0033 | 0.0050 |
| United Kingdom | 0.0050 | -0.0036 | 0.0292 | -0.0077 | -0.0161 | -0.0069 |
| United States | 0.5771 | -0.0323 | -0.1056 | 0.1375 | -0.1512 | -0.1475 |
| Standard error | (0.1071) | (0.0812) | (0.0320) | (0.0191) | (0.0166) | (0.0152) |

Table 4.3.10. Multivariate Spectral Shape Tests
Accumulated Covariance Contributions
$\hat{D}_q(t,0)$
Small Elasticity Economy (NX,P)

| Country | Frequencies $t=\frac{\pi}{6}$ | $t=\frac{2\pi}{6}$ | $t=\frac{3\pi}{6}$ | $t=\frac{4\pi}{6}$ | $t=\frac{5\pi}{6}$ |
|---|---|---|---|---|---|
| Australia | 0.3451 | 0.1599 | 0.0902 | 0.0636 | 0.0301 |
| Austria | 0.1243 | 0.0103 | 0.0152 | 0.0185 | 0.0058 |
| Canada | 0.1336 | 0.0490 | 0.0253 | 0.0170 | 0.0084 |
| Europe | 0.2761 | 0.1082 | 0.0511 | 0.0264 | 0.0139 |
| France | 0.2869 | 0.0763 | 0.0361 | 0.0210 | 0.0159 |
| Germany | 0.2340 | 0.1273 | 0.0593 | 0.0325 | 0.0099 |
| Italy | 0.2523 | 0.1361 | 0.0716 | 0.0458 | 0.0236 |
| Japan | 0.2879 | 0.1261 | 0.0674 | 0.0369 | 0.0183 |
| Switzerland | 0.2112 | 0.1036 | 0.0552 | 0.0322 | 0.0170 |
| United Kingdom | 0.1314 | 0.0850 | 0.0368 | 0.0253 | 0.0171 |
| United States | 0.1970 | 0.0899 | 0.0473 | 0.0292 | 0.0115 |
| Standard error | (0.0493) | (0.0329) | (0.0204) | (0.0134) | (0.0061) |

Table 4.3.11. Multivariate Spectral Shape Tests
Incremental Covariance Contributions
$\hat{D}_q(t,t-\frac{\pi}{6})$
Small Elasticity Economy (NX,P)

| Country | Frequencies $t=\frac{\pi}{6}$ | $t=\frac{2\pi}{6}$ | $t=\frac{3\pi}{6}$ | $t=\frac{4\pi}{6}$ | $t=\frac{5\pi}{6}$ | $t=\pi$ |
|---|---|---|---|---|---|---|
| Australia | 0.3451 | -0.1852 | -0.0697 | -0.0267 | -0.0335 | -0.0301 |
| Austria | 0.1243 | -0.1110 | 0.0049 | 0.0031 | -0.0127 | -0.0058 |
| Canada | 0.1336 | -0.0816 | -0.0237 | -0.0083 | -0.0087 | -0.0084 |
| Europe | 0.2761 | -0.1679 | -0.0571 | -0.0247 | -0.0125 | -0.0139 |
| France | 0.2869 | -0.2106 | -0.0102 | 0.0152 | 0.0050 | -0.0159 |
| Germany | 0.2340 | -0.1067 | -0.0681 | -0.0268 | -0.0226 | -0.0099 |
| Italy | 0.2523 | -0.1163 | -0.0644 | -0.0259 | -0.0222 | -0.0236 |
| Japan | 0.2879 | -0.1618 | -0.0588 | -0.0304 | -0.0187 | -0.0183 |
| Switzerland | 0.2112 | 0.1077 | -0.0484 | 0.0230 | 0.0152 | 0.0170 |
| United Kingdom | 0.1314 | -0.0463 | -0.0483 | -0.0144 | -0.0083 | -0.0171 |
| United States | 0.1970 | -0.1071 | -0.0426 | -0.0181 | 0.0177 | -0.0115 |
| Standard error | (0.0193) | (0.0393) | (0.0205) | (0.0106) | (0.0095) | (0.0061) |

Table 4.3.12. Multivariate Spectral Shape Tests
Accumulated Covariance Contributions
$\hat{D}_c(t,0)$
Large Elasticity Economy (NX,P)

| Country | Frequencies | | | | |
|---|---|---|---|---|---|
| | $t=\frac{\pi}{6}$ | $t=\frac{2\pi}{6}$ | $t=\frac{3\pi}{6}$ | $t=\frac{4\pi}{6}$ | $t=\frac{5\pi}{6}$ |
| Australia | 0.2031 | 0.2701 | 0.2376 | 0.1683 | 0.0801 |
| Austria | 0.1706 | 0.2435 | 0.2092 | 0.1675 | 0.0905 |
| Canada | 0.2842 | 0.3525 | 0.3043 | 0.2158 | 0.1035 |
| Europe | 0.0639 | 0.0838 | 0.0714 | 0.0591 | 0.0358 |
| France | 0.1197 | 0.0632 | 0.0449 | 0.0325 | 0.0181 |
| Germany | 0.2376 | 0.3119 | 0.2651 | 0.1879 | 0.0983 |
| Italy | 0.1842 | -0.0982 | -0.0463 | -0.0277 | 0.0129 |
| Japan | -0.0446 | 0.0190 | 0.0430 | 0.0346 | 0.0179 |
| Switzerland | -0.1005 | -0.0800 | -0.0307 | -0.0090 | -0.0051 |
| United Kingdom | 0.0293 | 0.0402 | -0.0008 | 0.0059 | -0.0031 |
| United States | 0.5428 | 0.5032 | 0.4077 | 0.2845 | 0.1375 |
| Standard error | (0.1058) | (0.0782) | (0.0520) | (0.0323) | (0.0185) |

Table 4.3.13. Multivariate Spectral Shape Tests
Incremental Covariance Contributions
$\hat{D}_c(t,t-\frac{\pi}{6})$
Large Elasticity Economy (NX,P)

| Country | Frequencies | | | | | |
|---|---|---|---|---|---|---|
| | $t=\frac{\pi}{6}$ | $t=\frac{2\pi}{6}$ | $t=\frac{3\pi}{6}$ | $t=\frac{4\pi}{6}$ | $t=\frac{5\pi}{6}$ | $t=\pi$ |
| Australia | 0.2031 | 0.0671 | -0.0326 | -0.0692 | -0.0882 | -0.0801 |
| Austria | 0.1706 | 0.0729 | -0.0343 | -0.0417 | -0.0771 | -0.0905 |
| Canada | 0.2842 | 0.0683 | -0.0181 | -0.0885 | -0.1123 | -0.1035 |
| Europe | 0.0639 | 0.0199 | -0.0124 | -0.0123 | -0.0234 | -0.0358 |
| France | 0.1197 | -0.0565 | -0.0182 | -0.0124 | -0.0145 | -0.0181 |
| Germany | 0.2376 | 0.0743 | -0.0467 | -0.0773 | -0.0896 | -0.0983 |
| Italy | 0.1842 | 0.0860 | 0.0518 | 0.0187 | 0.0148 | 0.0129 |
| Japan | -0.0446 | 0.0637 | 0.0240 | -0.0084 | -0.0167 | -0.0179 |
| Switzerland | 0.1005 | 0.0205 | 0.0493 | 0.0218 | 0.0039 | 0.0051 |
| United Kingdom | -0.0293 | -0.0108 | 0.0393 | 0.0067 | -0.0089 | 0.0031 |
| United States | 0.5428 | -0.0396 | -0.0955 | -0.1232 | -0.1470 | -0.1375 |
| Standard error | (0.1058) | (0.0730) | (0.0422) | (0.0251) | (0.0177) | (0.0185) |

Table 4.3.14. Multivariate Spectral Shape Tests
Accumulated Covariance Contributions
$\hat{D}_q(t,0)$
Large Elasticity Economy (NX,P)

| Country | Frequencies | | | | |
|---|---|---|---|---|---|
| | $t=\frac{\pi}{6}$ | $t=\frac{2\pi}{6}$ | $t=\frac{3\pi}{6}$ | $t=\frac{4\pi}{6}$ | $t=\frac{5\pi}{6}$ |
| Australia | 0.3439 | 0.1560 | 0.0899 | 0.0610 | 0.0282 |
| Austria | 0.1230 | 0.0064 | 0.0149 | 0.0159 | 0.0039 |
| Canada | 0.1324 | 0.0451 | 0.0250 | 0.0144 | 0.0065 |
| Europe | 0.2748 | 0.1042 | 0.0508 | 0.0238 | 0.0120 |
| France | 0.2857 | 0.0724 | 0.0358 | 0.0184 | 0.0140 |
| Germany | 0.2328 | 0.1234 | 0.0590 | 0.0299 | 0.0080 |
| Italy | 0.2511 | 0.1321 | 0.0713 | 0.0432 | 0.0217 |
| Japan | 0.2867 | 0.1222 | 0.0671 | 0.0344 | 0.0164 |
| Switzerland | 0.2100 | 0.0997 | 0.0549 | 0.0296 | 0.0151 |
| United Kingdom | 0.1302 | 0.0811 | 0.0365 | 0.0227 | 0.0152 |
| United States | 0.1958 | 0.0860 | 0.0470 | 0.0266 | 0.0096 |
| Standard error | (0.0446) | (0.0362) | (0.0216) | (0.0117) | (0.0060) |

Table 4.3.15. Multivariate Spectral Shape Tests
Incremental Covariance Contributions
$\hat{D}_q(t,t-\frac{\pi}{6})$
Large Elasticity Economy (NX,P)

| Country | Frequencies | | | | | |
|---|---|---|---|---|---|---|
| | $t=\frac{\pi}{6}$ | $t=\frac{2\pi}{6}$ | $t=\frac{3\pi}{6}$ | $t=\frac{4\pi}{6}$ | $t=\frac{5\pi}{6}$ | $t=\pi$ |
| Australia | 0.3439 | -0.1879 | -0.0661 | -0.0290 | -0.0328 | -0.0282 |
| Austria | 0.1230 | 0.1167 | 0.0085 | 0.0011 | -0.0120 | -0.0039 |
| Canada | 0.1324 | 0.0873 | -0.0201 | -0.0106 | -0.0080 | 0.0065 |
| Europe | 0.2748 | -0.1706 | -0.0534 | -0.0270 | -0.0118 | -0.0120 |
| France | 0.2857 | -0.2133 | 0.0366 | 0.0175 | 0.0044 | -0.0140 |
| Germany | 0.2328 | -0.1094 | -0.0644 | 0.0291 | -0.0219 | -0.0080 |
| Italy | 0.2511 | -0.1190 | -0.0608 | -0.0282 | -0.0215 | -0.0217 |
| Japan | 0.2867 | -0.1644 | -0.0551 | -0.0327 | -0.0180 | -0.0164 |
| Switzerland | 0.2100 | 0.1104 | -0.0448 | -0.0253 | -0.0145 | -0.0151 |
| United Kingdom | 0.1302 | -0.0490 | -0.0446 | -0.0137 | -0.0076 | -0.0152 |
| United States | 0.1958 | -0.1098 | -0.0390 | -0.0204 | -0.0170 | -0.0096 |
| Standard error | (0.0446) | (0.0327) | (0.0184) | (0.0124) | (0.0090) | (0.0060) |

Table 4.3.16. Multivariate Spectral Shape Tests
Accumulated Covariance Contributions
$\bar{D}_c(t,0)$
Two Shocks Economy (NX,P)

| Country | Frequencies | | | | |
|---|---|---|---|---|---|
| | $t=\frac{\pi}{6}$ | $t=\frac{2\pi}{6}$ | $t=\frac{3\pi}{6}$ | $t=\frac{4\pi}{6}$ | $t=\frac{5\pi}{6}$ |
| Australia | -0 3533 | -0.3898 | -0.3147 | 0.2155 | -0.1114 |
| Austria | -0.3857 | -0.4164 | -0.3430 | -0.2163 | -0.1010 |
| Canada | -0.2721 | -0.3075 | -0.2479 | -0.1680 | -0.0880 |
| Europe | -0.4924 | -0.5761 | -0.4808 | -0.3246 | -0.1557 |
| France | -0.4367 | -0.5968 | -0.5073 | -0.3513 | -0.1734 |
| Germany | -0.3188 | -0.3480 | -0.2871 | -0.1959 | -0.0932 |
| Italy | -0.7405 | -0 7581 | -0 5986 | -0 4115 | -0.2044 |
| Japan | -0.6010 | -0.6409 | -0.5092 | -0.3492 | -0.1736 |
| Switzerland | -0.6569 | -0.7400 | -0.5830 | -0.3928 | -0.1965 |
| United Kingdom | -0.5857 | -0.7001 | -0.5531 | 0.3779 | -0.1916 |
| United States | -0.0136 | -0.1568 | -0.1446 | -0.0993 | -0.0540 |
| Standard error | (0.1095) | (0.0617) | (0.0444) | (0.0283) | (0.0125) |

Table 4.3.17. Multivariate Spectral Shape Tests
Incremental Covariance Contributions
$\bar{D}_c(t,t-\frac{\pi}{6})$
Two Shocks Economy (NX,P)

| Country | Frequencies | | | | | |
|---|---|---|---|---|---|---|
| | $t=\frac{\pi}{6}$ | $t=\frac{2\pi}{6}$ | $t=\frac{3\pi}{6}$ | $t=\frac{4\pi}{6}$ | $t=\frac{5\pi}{6}$ | $t=\pi$ |
| Australia | -0 3533 | -0.0365 | 0.0751 | 0.0993 | 0.1041 | 0.1114 |
| Austria | 0 3857 | -0.0307 | 0.0734 | 0 1268 | 0.1152 | 0.1010 |
| Canada | -0.2721 | -0 0353 | 0.0595 | 0 0799 | 0 0800 | 0.0880 |
| Europe | -0.4924 | -0.0837 | 0.0953 | 0.1562 | 0.1689 | 0.1557 |
| France | 0.1367 | 0.1601 | 0 0891 | 0 1561 | 0.1778 | 0 1734 |
| Germany | 0 3188 | -0 0292 | 0 0609 | 0.0912 | 0.1027 | 0.0932 |
| Italy | -0.7405 | -0.0176 | 0.1595 | 0.1871 | 0.2071 | 0.2044 |
| Japan | -0.6010 | -0.0399 | 0.1316 | 0.1601 | 0.1756 | 0.1736 |
| Switzerland | -0.6569 | -0.0831 | 0.1570 | 0.1902 | 0.1962 | 0.1965 |
| United Kingdom | -0 5857 | -0.1144 | 0 1470 | 0.1751 | 0.1834 | 0.1946 |
| United States | -0.0136 | -0.1432 | 0.0122 | 0.0453 | 0.0453 | 0.0540 |
| Standard error | (0.1095) | (0.0725) | (0.0241) | (0.0191) | (0.0170) | (0.0125) |

Table 4.3.18. Multivariate Spectral Shape Tests
Accumulated Covariance Contributions
$D_q(t,0)$
Two Shocks Economy (NX,P)

| Country | Frequencies | | | | |
|---|---|---|---|---|---|
| | $t=\frac{\pi}{6}$ | $t=\frac{2\pi}{6}$ | $t=\frac{3\pi}{6}$ | $t=\frac{4\pi}{6}$ | $t=\frac{5\pi}{6}$ |
| Australia | 0.4832 | 0.2257 | 0.1324 | 0.0861 | 0.0385 |
| Austria | 0.2623 | 0.0761 | 0.0573 | 0.0411 | 0.0143 |
| Canada | 0.2716 | 0.1148 | 0.0674 | 0.0396 | 0.0168 |
| Europe | 0.4141 | 0.1740 | 0.0932 | 0.0490 | 0.0224 |
| France | 0.4250 | 0.1421 | 0.0782 | 0.0436 | 0.0244 |
| Germany | 0.3720 | 0.1931 | 0.1014 | 0.0551 | 0.0184 |
| Italy | 0.3903 | 0.2019 | 0.1138 | 0.0683 | 0.0320 |
| Japan | 0.4259 | 0.1919 | 0.1095 | 0.0595 | 0.0267 |
| Switzerland | 0.3493 | 0.1694 | 0.0973 | 0.0548 | 0.0255 |
| United Kingdom | 0.2691 | 0.1508 | 0.0789 | 0.0479 | 0.0255 |
| United States | 0.3350 | 0.1557 | 0.0894 | 0.0518 | 0.0200 |
| Standard error | (0.0599) | (0.0320) | (0.0189) | (0.0119) | (0.0057) |

Table 4.3.19. Multivariate Spectral Shape Tests
Incremental Covariance Contributions
$D_q(t,t-\frac{\pi}{6})$
Two Shocks Economy (NX,P)

| Country | Frequencies | | | | | |
|---|---|---|---|---|---|---|
| | $t=\frac{\pi}{6}$ | $t=\frac{2\pi}{6}$ | $t=\frac{3\pi}{6}$ | $t=\frac{4\pi}{6}$ | $t=\frac{5\pi}{6}$ | $t=\pi$ |
| Australia | 0.4832 | -0.2574 | -0.0934 | -0.0462 | -0.0476 | -0.0385 |
| Austria | 0.2623 | -0.1862 | -0.0188 | -0.0162 | -0.0268 | -0.0143 |
| Canada | 0.2716 | -0.1568 | 0.0174 | -0.0278 | 0.0228 | 0.0168 |
| Europe | 0.4141 | -0.2401 | -0.0808 | -0.0442 | 0.0266 | -0.0224 |
| France | 0.4250 | 0.2828 | 0.0639 | -0.0317 | 0.0192 | 0.0244 |
| Germany | 0.3720 | 0.1789 | -0.0917 | -0.0463 | 0.0367 | -0.0184 |
| Italy | 0.3903 | -0.1885 | -0.0881 | 0.0451 | -0.0363 | -0.0320 |
| Japan | 0.4259 | -0.2340 | -0.0825 | -0.0500 | -0.0328 | -0.0267 |
| Switzerland | 0.3493 | -0.1799 | -0.0721 | -0.0425 | -0.0293 | -0.0255 |
| United Kingdom | 0.2694 | -0.1186 | -0.0720 | -0.0310 | -0.0224 | -0.0255 |
| United States | 0.3350 | -0.1793 | -0.0663 | -0.0376 | 0.0318 | 0.0200 |
| Standard error | (0.0599) | (0.0275) | (0.0178) | (0.0092) | (0.0073) | (0.0057) |

Table 4.3.20. Multivariate Spectral Shape Tests
Accumulated Covariance Contributions
$\hat{D}_c(t,0)$
Time To Build Economy (NX,P)

| Country | Frequencies $t=\frac{\pi}{6}$ | $t=\frac{2\pi}{6}$ | $t=\frac{3\pi}{6}$ | $t=\frac{4\pi}{6}$ | $t=\frac{5\pi}{6}$ |
|---|---|---|---|---|---|
| Australia | 0.1646 | 0.2458 | 0.2192 | 0.1519 | 0.0734 |
| Austria | 0.1321 | 0.2192 | 0.1909 | 0.1511 | 0.0837 |
| Canada | 0.2457 | 0.3281 | 0.2860 | 0.1994 | 0.0968 |
| Europe | 0.0255 | 0.0595 | 0.0531 | 0.0427 | 0.0291 |
| France | 0.0812 | 0.0388 | 0.0266 | 0.0161 | 0.0114 |
| Germany | 0.1991 | 0.2876 | 0.2468 | 0.1715 | 0.0916 |
| Italy | -0.2227 | -0.1225 | -0.0647 | -0.0441 | -0.0196 |
| Japan | -0.0831 | -0.0053 | 0.0247 | 0.0182 | 0.0112 |
| Switzerland | -0.1390 | -0.1044 | -0.0491 | -0.0254 | -0.0118 |
| United Kingdom | -0.0678 | -0.0615 | -0.0191 | -0.0106 | -0.0098 |
| United States | 0.5043 | 0.4788 | 0.3894 | 0.2681 | 0.1308 |
| Standard error | (0.0977) | (0.0647) | (0.0450) | (0.0345) | (0.0188) |

Table 4.3.21. Multivariate Spectral Shape Tests
Incremental Covariance Contributions
$\hat{D}_c(t,t-\frac{\pi}{6})$
Time To Build Economy (NX,P)

| Country | Frequencies $t=\frac{\pi}{6}$ | $t=\frac{2\pi}{6}$ | $t=\frac{3\pi}{6}$ | $t=\frac{4\pi}{6}$ | $t=\frac{5\pi}{6}$ | $t=\pi$ |
|---|---|---|---|---|---|---|
| Australia | 0.1646 | 0.0813 | -0.0266 | -0.0673 | -0.0785 | -0.0734 |
| Austria | 0.1321 | 0.0871 | -0.0283 | -0.0398 | -0.0674 | -0.0837 |
| Canada | 0.2457 | 0.0824 | -0.0421 | -0.0866 | -0.1026 | -0.0968 |
| Europe | 0.0255 | 0.0341 | -0.0064 | -0.0104 | -0.0137 | 0.0291 |
| France | 0.0812 | 0.0421 | -0.0122 | -0.0105 | 0.0018 | -0.0114 |
| Germany | 0.1991 | 0.0885 | -0.0407 | -0.0754 | -0.0799 | -0.0916 |
| Italy | -0.2227 | 0.1002 | 0.0578 | 0.0206 | 0.0245 | 0.0196 |
| Japan | -0.0831 | 0.0778 | 0.0300 | -0.0065 | -0.0070 | -0.0112 |
| Switzerland | -0.1390 | 0.0346 | 0.0553 | 0.0237 | 0.0136 | 0.0118 |
| United Kingdom | -0.0678 | 0.0033 | 0.0454 | 0.0086 | 0.0008 | 0.0098 |
| United States | 0.5043 | -0.0254 | -0.0895 | -0.1212 | -0.1373 | -0.1308 |
| Standard error | (0.0977) | (0.0721) | (0.0338) | (0.0185) | (0.0190) | (0.0188) |

Table 4.3.22. Multivariate Spectral Shape Tests
Accumulated Covariance Contributions
$\hat{D}_q(t,0)$
Time To Build Economy (NX,P)

| | Frequencies | | | | |
|---|---|---|---|---|---|
| Country | $t = \frac{\pi}{6}$ | $t = \frac{2\pi}{6}$ | $t = \frac{3\pi}{6}$ | $t = \frac{4\pi}{6}$ | $t = \frac{5\pi}{6}$ |
| Australia | 0.3923 | 0.1862 | 0.1064 | 0.0727 | 0.0352 |
| Austria | 0.1714 | 0.0365 | 0.0313 | 0.0276 | 0.0109 |
| Canada | 0.1808 | 0.0753 | 0.0415 | 0.0261 | 0.0135 |
| Europe | 0.3232 | 0.1344 | 0.0673 | 0.0355 | 0.0190 |
| France | 0.3341 | 0.1026 | 0.0523 | 0.0301 | 0.0210 |
| Germany | 0.2812 | 0.1536 | 0.0754 | 0.0416 | 0.0150 |
| Italy | 0.2995 | 0.1623 | 0.0878 | 0.0549 | 0.0287 |
| Japan | 0.3351 | 0.1524 | 0.0835 | 0.0460 | 0.0231 |
| Switzerland | 0.2584 | 0.1298 | 0.0714 | 0.0413 | 0.0221 |
| United Kingdom | 0.1786 | 0.1113 | 0.0529 | 0.0311 | 0.0222 |
| United States | 0.2442 | 0.1161 | 0.0635 | 0.0383 | 0.0166 |
| Standard error | (0.0588) | (0.0387) | (0.0251) | (0.0167) | (0.0075) |

Table 4.3.23. Multivariate Spectral Shape Tests
Incremental Covariance Contributions
$\hat{D}_q(t, t - \frac{\pi}{6})$
Time To Build Economy (NX,P)

| | Frequencies | | | | | |
|---|---|---|---|---|---|---|
| Country | $t = \frac{\pi}{6}$ | $t = \frac{2\pi}{6}$ | $t = \frac{3\pi}{6}$ | $t = \frac{4\pi}{6}$ | $t = \frac{5\pi}{6}$ | $t = \pi$ |
| Australia | 0.3923 | -0.2062 | -0.0798 | -0.0337 | -0.0375 | -0.0352 |
| Austria | 0.1714 | -0.1349 | -0.0052 | -0.0037 | -0.0167 | -0.0109 |
| Canada | 0.1808 | -0.1056 | 0.0338 | 0.0153 | 0.0127 | -0.0135 |
| Europe | 0.3232 | -0.1889 | -0.0671 | -0.0317 | -0.0165 | -0.0190 |
| France | 0.3341 | -0.2316 | -0.0503 | -0.0222 | 0.0091 | -0.0210 |
| Germany | 0.2812 | 0.1276 | -0.0781 | -0.0338 | -0.0266 | 0.0150 |
| Italy | 0.2995 | -0.1372 | -0.0745 | -0.0329 | -0.0262 | -0.0287 |
| Japan | 0.3351 | -0.1827 | -0.0688 | 0.0375 | -0.0297 | -0.0234 |
| Switzerland | 0.2584 | -0.1286 | -0.0585 | -0.0301 | -0.0192 | -0.0221 |
| United Kingdom | 0.1786 | -0.0673 | -0.0583 | -0.0185 | -0.0123 | -0.0222 |
| United States | 0.2442 | -0.1280 | -0.0527 | -0.0252 | -0.0217 | -0.0166 |
| Standard error | (0.0588) | (0.0514) | (0.0251) | (0.0132) | (0.0119) | (0.0075) |

Table 4.3.24. Multivariate Spectral Shape Tests
Accumulated Covariance Contributions
$\bar{D}_c(t,0)$
Government Expenditures Economy (XX,P)

| Country | Frequencies | | | | |
|---|---|---|---|---|---|
| | $t=\frac{\pi}{6}$ | $t=\frac{2\pi}{6}$ | $t=\frac{3\pi}{6}$ | $t=\frac{4\pi}{6}$ | $t=\frac{5\pi}{6}$ |
| Australia | -0.1962 | -0.2197 | -0.1810 | -0.1226 | -0.0647 |
| Austria | -0.2286 | -0.2463 | -0.2033 | -0.1234 | -0.0543 |
| Canada | -0.1150 | -0.1374 | -0.1142 | -0.0752 | -0.0413 |
| Europe | -0.3353 | -0.4060 | -0.3471 | -0.2318 | -0.1090 |
| France | -0.2796 | -0.4267 | -0.3736 | -0.2584 | -0.1267 |
| Germany | -0.1617 | -0.1779 | -0.1534 | -0.1031 | -0.0465 |
| Italy | -0.5834 | -0.5880 | -0.4649 | 0.3186 | -0.1577 |
| Japan | -0.4439 | -0.4708 | -0.3755 | -0.2563 | -0.1269 |
| Switzerland | -0.4998 | -0.5699 | -0.4493 | -0.2999 | -0.1499 |
| United Kingdom | 0.4285 | -0.5300 | -0.4193 | 0.2851 | -0.1479 |
| United States | 0.1435 | 0.0133 | -0.0109 | -0.0064 | -0.0073 |
| Standard error | (0.1183) | (0.0950) | (0.0662) | (0.0474) | (0.0238) |

Table 4.3.25. Multivariate Spectral Shape Tests
Incremental Covariance Contributions
$\bar{D}_c(t, t-\frac{\pi}{6})$
Government Expenditures Economy (XX,P)

| Country | Frequencies | | | | | |
|---|---|---|---|---|---|---|
| | $t=\frac{\pi}{6}$ | $t=\frac{2\pi}{6}$ | $t=\frac{3\pi}{6}$ | $t=\frac{4\pi}{6}$ | $t=\frac{5\pi}{6}$ | $t=\pi$ |
| Australia | -0.1962 | -0.0235 | 0.0387 | 0.0584 | 0.0579 | 0.0647 |
| Austria | -0.2286 | -0.0177 | 0.0370 | 0.0859 | 0.0691 | 0.0513 |
| Canada | -0.1150 | -0.0223 | 0.0232 | 0.0390 | 0.0338 | 0.0413 |
| Europe | -0.3353 | 0.0707 | 0.0589 | 0.1153 | 0.1223 | 0.1090 |
| France | 0.2796 | -0.1471 | 0.0531 | 0.1152 | 0.1317 | 0.1267 |
| Germany | -0.1617 | 0.0163 | 0.0246 | 0.0503 | 0.0566 | 0.0465 |
| Italy | -0.5834 | -0.0016 | 0.1231 | 0.1162 | 0.1610 | 0.1577 |
| Japan | -0.4439 | -0.0269 | 0.0953 | 0.1192 | 0.1294 | 0.1269 |
| Switzerland | 0.4998 | 0.0701 | 0.1206 | 0.1493 | 0.1501 | 0.1499 |
| United Kingdom | -0.4285 | -0.1014 | 0.1107 | 0.1342 | 0.1572 | 0.1479 |
| United States | 0.1135 | -0.1302 | -0.0242 | 0.0044 | -0.0008 | 0.0073 |
| Standard error | (0.1183) | (0.0509) | (0.0393) | (0.0219) | (0.0258) | (0.0238) |

Table 4.3.26. Multivariate Spectral Shape Tests
Accumulated Covariance Contributions
$\hat{D}_q(t,0)$
Government Expenditures Economy (NX,P)

| Country | Frequencies $t = \frac{\pi}{6}$ | $t = \frac{2\pi}{6}$ | $t = \frac{3\pi}{6}$ | $t = \frac{4\pi}{6}$ | $t = \frac{5\pi}{6}$ |
|---|---|---|---|---|---|
| Australia | 0.2258 | 0.0926 | 0.0536 | 0.0400 | 0.0144 |
| Austria | 0.0049 | -0.0570 | -0.0214 | -0.0050 | -0.0099 |
| Canada | 0.0143 | -0.0183 | -0.0113 | -0.0065 | -0.0074 |
| Europe | 0.1567 | 0.0408 | 0.0145 | 0.0029 | -0.0018 |
| France | 0.1676 | 0.0090 | -0.0005 | -0.0026 | 0.0002 |
| Germany | 0.1147 | 0.0600 | 0.0227 | 0.0090 | -0.0058 |
| Italy | 0.1330 | 0.0687 | 0.0350 | 0.0222 | 0.0078 |
| Japan | 0.1685 | 0.0588 | 0.0308 | 0.0134 | 0.0025 |
| Switzerland | 0.0919 | 0.0362 | 0.0186 | 0.0087 | 0.0013 |
| United Kingdom | 0.0120 | 0.0177 | 0.0002 | 0.0018 | 0.0013 |
| United States | 0.0776 | 0.0226 | 0.0107 | 0.0057 | -0.0042 |
| Standard error | (0.0848) | (0.0492) | (0.0292) | (0.0187) | (0.0095) |

Table 4.3.27. Multivariate Spectral Shape Tests
Incremental Covariance Contributions
$\hat{D}_q(t, t - \frac{\pi}{6})$
Government Expenditures Economy (NX,P)

| Country | Frequencies $t = \frac{\pi}{6}$ | $t = \frac{2\pi}{6}$ | $t = \frac{3\pi}{6}$ | $t = \frac{4\pi}{6}$ | $t = \frac{5\pi}{6}$ | $t = \pi$ |
|---|---|---|---|---|---|---|
| Australia | 0.2258 | -0.1332 | -0.0390 | -0.0136 | -0.0257 | -0.0144 |
| Austria | 0.0049 | -0.0619 | 0.0356 | 0.0164 | -0.0049 | 0.0099 |
| Canada | 0.0143 | -0.0326 | 0.0070 | 0.0048 | -0.0009 | 0.0074 |
| Europe | 0.1567 | -0.1159 | -0.0263 | -0.0116 | -0.0017 | 0.0018 |
| France | 0.1676 | -0.1586 | 0.0095 | 0.0021 | 0.0027 | 0.0002 |
| Germany | 0.1147 | -0.0517 | -0.0373 | -0.0137 | -0.0148 | 0.0058 |
| Italy | 0.1330 | -0.0643 | -0.0337 | -0.0128 | 0.0144 | 0.0078 |
| Japan | 0.1685 | -0.1097 | -0.0280 | -0.0173 | -0.0109 | -0.0025 |
| Switzerland | 0.0919 | -0.0556 | 0.0177 | -0.0099 | 0.0074 | 0.0013 |
| United Kingdom | 0.0120 | 0.0057 | -0.0175 | 0.0016 | -0.0005 | -0.0013 |
| United States | 0.0776 | -0.0551 | -0.0119 | -0.0050 | -0.0099 | 0.0042 |
| Standard error | (0.0848) | (0.0652) | (0.0326) | (0.0157) | (0.0126) | (0.0095) |

Table 4.3.28. Multivariate Spectral Shape Tests
Accumulated Covariance Contributions
$\hat{D}_c(t,0)$
No Capital Economy (NX,P)

| Country | Frequencies $t=\frac{\pi}{6}$ | $t=\frac{2\pi}{6}$ | $t=\frac{3\pi}{6}$ | $t=\frac{4\pi}{6}$ | $t=\frac{5\pi}{6}$ |
|---|---|---|---|---|---|
| Australia | -0.0360 | -0.0164 | 0.0036 | 0.0020 | -0.0035 |
| Austria | -0.0684 | -0.0430 | -0.0247 | 0.0012 | 0.0068 |
| Canada | 0.0452 | 0.0659 | 0.0704 | 0.0494 | 0.0198 |
| Europe | -0.1751 | -0.2027 | -0.1625 | -0.1072 | -0.0479 |
| France | -0.1193 | -0.2234 | -0.1890 | -0.1338 | -0.0656 |
| Germany | -0.0015 | 0.0253 | 0.0312 | 0.0215 | 0.0146 |
| Italy | -0.4232 | -0.3847 | -0.2803 | -0.1940 | -0.0965 |
| Japan | -0.2836 | -0.2675 | -0.1910 | -0.1317 | -0.0657 |
| Switzerland | -0.3395 | -0.3666 | -0.2647 | -0.1753 | -0.0877 |
| United Kingdom | -0.2683 | -0.3267 | -0.2348 | -0.1605 | -0.0867 |
| United States | 0.3037 | 0.2166 | 0.1737 | 0.1182 | 0.0539 |
| Standard error | (0.0475) | (0.0524) | (0.0411) | (0.0349) | (0.0218) |

Table 4.3.29. Multivariate Spectral Shape Tests
Incremental Covariance Contributions
$D_c(t, t-\frac{\pi}{6})$
No Capital Economy (NX,P)

| Country | Frequencies $t=\frac{\pi}{6}$ | $t=\frac{2\pi}{6}$ | $t=\frac{3\pi}{6}$ | $t=\frac{4\pi}{6}$ | $t=\frac{5\pi}{6}$ | $t=\pi$ |
|---|---|---|---|---|---|---|
| Australia | -0.0360 | 0.0195 | 0.0200 | -0.0016 | -0.0055 | 0.0035 |
| Austria | -0.0684 | 0.0254 | 0.0183 | 0.0259 | 0.0056 | -0.0068 |
| Canada | 0.0452 | 0.0207 | 0.0044 | -0.0209 | -0.0296 | -0.0198 |
| Europe | -0.1751 | -0.0277 | 0.0402 | 0.0553 | 0.0593 | 0.0179 |
| France | -0.1193 | -0.1011 | 0.0344 | 0.0552 | 0.0682 | 0.0656 |
| Germany | -0.0015 | 0.0268 | 0.0059 | -0.0097 | -0.0069 | -0.0116 |
| Italy | -0.4232 | 0.0385 | 0.1041 | 0.0863 | 0.0975 | 0.0965 |
| Japan | -0.2836 | 0.0161 | 0.0766 | 0.0592 | 0.0660 | 0.0657 |
| Switzerland | -0.3395 | -0.0271 | 0.1019 | 0.0894 | 0.0566 | 0.0887 |
| United Kingdom | -0.2683 | -0.0584 | 0.0919 | 0.0743 | 0.0738 | 0.0867 |
| United States | 0.3037 | -0.0871 | -0.0429 | -0.0555 | -0.0643 | -0.0539 |
| Standard error | (0.0475) | (0.0366) | (0.0345) | (0.0182) | (0.0203) | (0.0218) |

Table 4.3.30. Multivariate Spectral Shape Tests
Accumulated Covariance Contributions
$D_q(\omega, 0)$
No Capital Economy (NX,P)

| Country | Frequencies | | | | |
|---|---|---|---|---|---|
| | $\omega = \frac{\pi}{6}$ | $\omega = \frac{2\pi}{6}$ | $\omega = \frac{3\pi}{6}$ | $\omega = \frac{4\pi}{6}$ | $\omega = \frac{5\pi}{6}$ |
| Australia | 0.4978 | 0.3033 | 0.1970 | 0.1322 | 0.0678 |
| Austria | 0.2769 | 0.1537 | 0.1219 | 0.0872 | 0.0435 |
| Canada | 0.2863 | 0.1924 | 0.1321 | 0.0857 | 0.0460 |
| Europe | 0.4287 | 0.2516 | 0.1579 | 0.0951 | 0.0516 |
| France | 0.4396 | 0.2197 | 0.1429 | 0.0896 | 0.0536 |
| Germany | 0.3867 | 0.2708 | 0.1661 | 0.1012 | 0.0476 |
| Italy | 0.4050 | 0.2795 | 0.1781 | 0.1144 | 0.0613 |
| Japan | 0.4406 | 0.2696 | 0.1741 | 0.1056 | 0.0560 |
| Switzerland | 0.3639 | 0.2470 | 0.1620 | 0.1008 | 0.0547 |
| United Kingdom | 0.2841 | 0.2285 | 0.1435 | 0.0940 | 0.0548 |
| United States | 0.3497 | 0.2333 | 0.1511 | 0.0979 | 0.0492 |
| Standard error | (0.0711) | (0.0498) | (0.0416) | (0.0288) | (0.0143) |

Table 4.3.31. Multivariate Spectral Shape Tests
Incremental Covariance Contributions
$D_q(\omega, \omega - \frac{\pi}{6})$
No Capital Economy (NX,P)

| Country | Frequencies | | | | | |
|---|---|---|---|---|---|---|
| | $\omega = \frac{\pi}{6}$ | $\omega = \frac{2\pi}{6}$ | $\omega = \frac{3\pi}{6}$ | $\omega = \frac{4\pi}{6}$ | $\omega = \frac{5\pi}{6}$ | $\omega = \pi$ |
| Australia | 0.4978 | -0.1945 | -0.1063 | -0.0648 | -0.0644 | -0.0678 |
| Austria | 0.2769 | -0.1232 | -0.0318 | -0.0348 | -0.0437 | -0.0435 |
| Canada | 0.2863 | -0.0938 | -0.0604 | -0.0464 | -0.0396 | -0.0460 |
| Europe | 0.4287 | -0.1772 | 0.0937 | 0.0628 | 0.0435 | 0.0516 |
| France | 0.4396 | 0.2199 | 0.0768 | 0.0533 | 0.0360 | -0.0536 |
| Germany | 0.3867 | 0.1159 | 0.1017 | -0.0649 | -0.0535 | -0.0476 |
| Italy | 0.4050 | -0.1255 | -0.1011 | -0.0640 | -0.0532 | -0.0613 |
| Japan | 0.4406 | -0.1710 | -0.0954 | -0.0685 | -0.0496 | -0.0560 |
| Switzerland | 0.3639 | 0.1169 | 0.0850 | 0.0611 | 0.0161 | 0.0547 |
| United Kingdom | 0.2841 | -0.0556 | -0.0849 | -0.0496 | -0.0392 | -0.0548 |
| United States | 0.3497 | -0.1163 | -0.0792 | -0.0562 | -0.0486 | -0.0492 |
| Standard error | (0.0711) | (0.0701) | (0.0368) | (0.0234) | (0.0212) | (0.0113) |

# Chapter 5

# Conclusion

This thesis has developed a set of frequency domain diagnostic tests for evaluating the dynamic properties of nonlinear general equilibrium rational expectations models that are commonly employed in business cycle research. The diagnostic tests measure the distance between spectral and cross-spectral distribution functions estimated from historical time series, and spectral and cross-spectral distribution functions estimated from simulated data. The tests are advantageous because it is possible to examine the distance between simulated and historical spectral distribution functions over a particular region of frequencies, because they provide a convenient summary of the ability of the model to replicate the complete temporal behaviour of the endogenous variables in the model.

In Chapter 3, the diagnostic test statistics were employed to evaluate the frequency domain properties of the relationship between the trade balance, output, and the terms of trade, using a model originally developed by Backus, Kehoe and Kydland (1994). While the correlation function between the trade balance and the terms of trade for the majority of industrialized countries is shaped like an "S", we find that the cross-sepctral properties of the data are much more varied. Not surprisingly, the frequency domain properties of the model differ significantly from the historical data in a wide variety of cases. In addition, while the qualitative results from the approximate solution to the model are similar to Backus, Kehoe, and Kydland (1991), there are significant differences in several cases that may result from differences in approximation and solution procedures.

In order to bring the theory closer to the data, Chapter 4 considered measurement error in trade statistics as a possible reason for the empirical rejection of the model in Chapter 3. The frequency domain properties of the model under a variety of plausible measurement error specifications were considered. While the addition of measurement error improved the properties of the model, the results were still found to be unsatisfatory.

It is clear that the results presented here are not really that surprising, as the replication of a wide variety of business cycle properties across a wide variety of countries is a daunting task for any model. Moreover, it appears that the frequency domain diagnostics represent a detailed diagnostic tool, and that the rather simple dynamics exhibited by what might be considered to be a fairly complicated model are correctly depicted as inadequate by the tests. While some researchers may view such a specific approach as somewhat contradictory to the general thrust of empirical research in the calibration and simulation framework, it is also clear that the diagnostics provide useful information that would remain obscured in the time domain. Future research examining the frequency domain properties of other models within the calibration and simulation framework should therefore be considered. In addition, further research concerning the properties of models across solution procedures should be investigated.

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