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TOWARDS AN IMPLEMENTATION OF *SchemaLog* – A  
DATABASE PROGRAMMING LANGUAGE

ALANOLY JOSEPH ANDREWS

A THESIS  
IN  
THE DEPARTMENT  
OF  
COMPUTER SCIENCE

PRESENTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS  
FOR THE DEGREE OF MASTER OF COMPUTER SCIENCE  
CONCORDIA UNIVERSITY  
MONTRÉAL, QUÉBEC, CANADA

DECEMBER 1997

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0-612-39984-2

# Abstract

## Towards An Implementation of *SchemaLog* – A Database Programming Language

Alanoly Joseph Andrews

The efficient implementation of advanced database programming languages calls for investigating novel architectures and algorithms. In this thesis, we discuss our implementation of *SchemaLog*, a logic-based database programming language, capable of offering a powerful platform for a variety of database applications involving data/meta-data querying and restructuring. Our architecture for the implementation is based on compiling *SchemaLog* constructs into an extended version of relational algebra called Schema Algebra ( $\mathcal{SA}$ ). We modify the new operators in this algebra to suit our implementation, and we illustrate how a *SchemaLog* program can be evaluated by conversion into an expression in  $\mathcal{SA}$ . Based on this algebra, we develop a top-down algorithm, using the Rule/Goal Tree method, for evaluating *SchemaLog* programs. We then discuss three alternative storage structures for the implementation and study their effect on the efficiency of implementation. For each storage structure, we propose strategies for implementing our algebraic operators. We have implemented all these algorithms, using MicroSoft Access DBMS running on Windows 3.1, and have run an extensive set of experiments for evaluating the efficiency of alternative strategies under a varied mix of querying and restructuring operations and varying parameters on the type and size of data. We discuss the results of our experiments and make recommendations on the type of storage structures to be used.

# Acknowledgments

It should come as no surprise that I start by acknowledging my debt of gratitude to Prof. Laks V.S. Lakshmanan, my thesis advisor and supervisor. He has played a major role throughout the work I have done towards this presentation of my thesis and has seen to it that it has finally seen the light of day. His erudition in the area of logic and databases is well-known. The long list of publications in the subject in the last few years bears ample testimony to that assertion. It is my hope that some of that expertise, as well as enthusiasm for continued research has rubbed off on his students, including me.

My work in this thesis is based in part on the work done by Iyer Subramanian, alias Subbu, for his doctoral dissertation. During the course of my work of implementation, I naturally had to spend long hours with Subbu to share ideas on such “inane” topics as data structures, algorithms, etc. (not to mention the more interesting hours we had discussing more “pagan” subjects). I am grateful to Subbu for the valuable contributions he has made to making this thesis a reality.

Another fellow-student with whom I have had close contact through the years in Concordia is Nematollaah Shiri. Though not directly involved with my area of research, Shiri was a constant presence, a friend to whom I could turn in any circumstance.

Financial support for this thesis was provided in part by the Natural Sciences and Engineering Research Council of Canada and the Fonds Pour Formation De Chercheurs Et L'Aide À La Recherche of Quebec.

I would also like to thank the support staff in the Computer Science Department: the analysts (Stan Swiercz deserves special mention here), and the secretaries (in particular, Edwina, Halina and Stephanie) for their quick response to questions and problems.

Finally, I would like to thank the members of my own family (my wife Mary, and

my children Deepti, Preeti and Jyoti) for their love and support for me during some difficult years. The early years of my stay in Montreal were spent with my brother Jose. I would like to express here my gratitude to him, his wife Ruby and their children. Thanks also to my brother James who, too, was a pillar of support during the past years.

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# Chapter 1

## Introduction

### 1.1 Opening Remarks

We are living in the middle of an electronic “Information Revolution”. The printed medium which has so far dominated the storage and dissemination of information is being supplanted by the electronic medium. Our store of knowledge itself has not grown that dramatically; what has in fact happened is that the numerous repositories of information have been linked to one another and been made accessible to anyone possessing the basic hardware and software.

The most common type of store in the electronic medium is a database system. The information in this type of store is well-managed and easy to get to in a systematic manner. The great advance that we have made in networking various data repositories has brought with it several thorny issues which need to be resolved if we are to take full advantage of all the information that has been suddenly made available to us.

A basic problem with our current network of information sources is that of *incompatibility*. Tools and methods of interacting with the data in one store need not work in another store. The pressing need is for “interoperable” systems which would be able to manage multiple information sources in a seamless manner and enable users to pose search queries which are applicable over the entire network of information sites.

Even within the same information source, within a well-structured environment such as a database management system (DBMS), a query against one database may be “incompatible” against another. This is because what is considered as “data”

in one database could be seen as part of the “schema” in another. Existing query languages do not allow the user to ask for elements of the schema itself to form part of the output.

It is in this context that we introduce *SchemaLog*, a powerful language for advanced database logic programming first proposed by Lakshmanan et al., in [LSS93, LSS96] and later expatiated upon by Subramanian in [Sub97]. It was established there that *SchemaLog* can offer a powerful platform in a variety of settings including multi-database *interoperability*, *database programming* with *schema browsing*, *cooperative query answering*, computing forms of *aggregate queries* which are beyond the scope of conventional database query languages, *database restructuring*, and, in the context of the World Wide Web (WWW), *querying and restructuring Web pages*. *SchemaLog* has a higher order syntax, but has a first-order semantics. Indeed, it has a sound and complete proof theory as shown in [LSS96], [Sub97].

A prototype platform for interoperability among a number of INGRES databases was completed recently [LSPS95], based on a fragment of *SchemaLog*.

The theoretical basis for the use of *SchemaLog* as a powerful database programming language even in the context of a *single database* was established in the papers we have already referred to earlier. The main goal of this thesis, is to establish the *practical basis* for this claim by describing efficient architectures and algorithms for the implementation of *SchemaLog* as a database programming language, in the context of a *single database*.

In the efficient implementation of a language like *SchemaLog*, we can distinguish two types of optimization issues:

- problems peculiar to the querying of a federation of homogeneous/heterogeneous databases.
- problems that arise even within the context of a single database because of requirements such as data/meta-data interaction and the dynamic restructuring of data and schema

These two types of issues are clearly orthogonal and can be tackled independently of each other. The first area has been partially studied recently in [LSPS95] which provided a prototype platform for interoperability among a number of INGRES databases. The focus in *this thesis* is on the second area, that of implementation issues in a single database context.

## 1.2 Outline of the thesis

The major goal of this thesis has been to work towards an implementation of *SchemaLog* as a database programming language. Although *SchemaLog* as a language has the capability of addressing a *federation* of databases, we have restricted ourselves in this study, as mentioned earlier, to the context of a single database.

1. Since our implementation is based on the *SchemaLog* language, which is a recently-developed language and not that widely-known, we have gone to some detail in Chapter 2 to give the reader sufficient information on the syntax and semantics of the language to understand what follows in the rest of the thesis. Later in the chapter, we present the new operators in Schema Algebra ( $SA$ ), with examples to illustrate their use. The basic theoretical notions in this chapter are derived from [LSS96], and [Sub97]. We have made several modifications to the syntax and semantics of the operators in order to suit a single database context. New and detailed examples have been provided to illustrate the working of the operators as well as the translation of a sample *SchemaLog* program into the procedural level of algebraic operators.
2. For database applications, it is important that querying and restructuring be implemented in a set-oriented manner, as opposed to the “tuple-at-a-time” paradigm of Prolog. In chapter 3, we present the theoretical basis for a top-down implementation of *SchemaLog* following the set-oriented Rule/Goal Tree (RGT) evaluation method of *Datalog*. Detailed algorithms for the RGT evaluation are set forth, and an example of a Rule-Goal Tree for a specific *SchemaLog* program has also been worked out.
3. Any implementation, if it is to be efficient, needs to have suitable storage structures. Since *SchemaLog* treats schema elements on the same level as data elements, traditional storage structures may prove unsuitable. With this in mind, we present, in chapter 4, three different physical storage architectures. For each of these architectures we have developed new algorithms to implement the various operators in  $SA$ . For each algorithm, a theoretical cost estimate has also been done.



4. To test the practical efficacy of the storage strategies introduced in chapter 4, we implemented a few chosen  $\mathcal{SA}$  operators (according to the algorithms presented in Chapter 4) in all the three strategies using the MS-Access DBMS. We ran a number of experiments, varying such parameters as table size, join density and selection density. The results of these experiments were tabulated and also plotted on graphs. Details of these experiments form the contents of chapter 5.
5. Chapter 6 concludes our study with a summary, a comparison of our work to similar other practical implementations, and some pointers to future research in the area.

### 1.3 Contributions of this thesis

We have just seen an overview of the contents of this thesis. We conclude this introductory chapter with a listing of what we consider to be the significant contributions of this thesis.

1. *SchemaLog* had been designed by its authors to suit a variety of needs in database programming in such diverse environments as multi-database interoperability and world-wide-web querying. In order to study in detail the issues peculiar to a single database setting, we have, in this thesis, modified the *SchemaLog* syntax and semantics to suit such a context.
2. Schema Algebra as proposed in [Sub97] contains complexities of syntax which are not required in a single database setting. We have, therefore, in this thesis developed a simplified version of the Algebra needed for a procedural view of the high-level *SchemaLog* language.
3. Being a database programming language, it is important that an evaluation of a *SchemaLog* program yield a set of tuples in the output. To achieve this we have proposed a top-down evaluation based on the well-known Rule-Goal Tree method. We have devised the algorithms needed to implement this method in the case of *SchemaLog* programs. We have also defined the role that the Schema Algebra operators play within the top-down evaluation process.

4. To improve the efficiency of the evaluation process, we have proposed three different ways of storing and accessing the data used in a *SchemaLog* program. One of them is an adaptation of the conventional storage strategy, and the other two have been specially devised to have relevance in the context of the implementation of the “restructuring” operators of *SchemaLog* .
5. In each of the three strategies mentioned, above we have evolved algorithms to implement some of the key operators in Schema Algebra. A theoretical estimate of the cost involved in an implementation of each of the algorithms is also worked out.
6. Finally, we ran a number of experiments to test the performances of the various storage strategies under varying conditions of data size and data contents. Based on our detailed tabulations of the results, we were able to provide valuable pointers as to the best implementation strategies to be followed.

# Chapter 2

## Basics of *SchemaLog* and Schema Algebra

### 2.1 Introduction

This chapter provides a condensed view of those aspects of *SchemaLog* which are needed for a proper understanding of this thesis. Section 2.2 gives an abbreviated view of the syntax of *SchemaLog*. The semantics of the language is illustrated by means of an example later in the section. Section 2.3 is devoted to a more detailed discussion of the Algebraic Operators in *Schema Algebra*, an algebra that has been developed in order to implement the *SchemaLog* language. The chapter concludes with a section containing a further examples to illustrate *SchemaLog* rules and Schema Algebra operators.

### 2.2 Syntax of *SchemaLog*

We introduce here the syntax of *SchemaLog* and illustrate the semantics informally via examples. The syntax we will use is an adaptation of the full *SchemaLog* given in [Sub97] and [LSS96] to a single database context. For a full and formal account, the reader is referred to [Sub97].

*SchemaLog* features three<sup>1</sup> kinds of basic expressions, called *atoms*. Their notation as well as an informal meaning is given below.

---

<sup>1</sup>Actually, full *SchemaLog* features four. In a single database context, these reduce to three.

- $\langle rel \rangle$  – there is a relation named  $\langle rel \rangle$  in the database.
- $\langle rel \rangle[\langle attr \rangle]$  – there is a relation named  $\langle rel \rangle$  in the database, and the schema of  $\langle rel \rangle$  includes an attribute named  $\langle attr \rangle$ .
- $\langle rel \rangle[\langle tid \rangle : \langle attr \rangle \rightarrow \langle val \rangle]$  – there is a relation named  $\langle rel \rangle$  in the database, and the schema of  $\langle rel \rangle$  includes an attribute named  $\langle attr \rangle$ , and furthermore  $\langle rel \rangle$  contains a tuple  $\langle tid \rangle$  which has value  $\langle val \rangle$  under column  $\langle attr \rangle$ .

In the above,  $\langle rel \rangle$ ,  $\langle tid \rangle$ ,  $\langle attr \rangle$ , and  $\langle val \rangle$  are arbitrary first-order terms, built up as usual from a vocabulary including constants, function symbols, and variables. In this thesis, for simplicity, we shall deal only with function-free *SchemaLog*.

*Molecules* are expressions of the form  $\langle rel \rangle[\langle tid \rangle : \langle attr \rangle_1 \rightarrow \langle val \rangle_1, \dots, \langle attr \rangle_n \rightarrow \langle val \rangle_n]$  and are a syntactic sugar for the conjunction  $\langle rel \rangle[\langle tid \rangle : \langle attr \rangle_1 \rightarrow \langle val \rangle_1] \wedge \dots \wedge \langle rel \rangle[\langle tid \rangle : \langle attr \rangle_n \rightarrow \langle val \rangle_n]$ . The  $\langle tid \rangle$  acts as a “glue” to combine the pieces from different atoms together. Existential “don’t care” tid variables appearing in rule bodies can be omitted (see Example 2.1).

Besides these, *SchemaLog* also uses *programming predicates* of the form  $p(t_1, \dots, t_n)$  where  $p$  is an  $n$ -ary predicate symbol and  $t_i$  are terms. While programming predicates can be simulated using *SchemaLog* molecules, they add to programming convenience. We use the generic term *database predicates* to refer to *SchemaLog* atoms and molecules. The main difference between database predicates and programming predicates is that the former are used as an “interface” to relations — either existing in a database (i.e. the so-called EDB relations) or created by database programs written in *SchemaLog* (i.e. the so-called IDB relations) – whose schematic information (i.e. names of relations and their attributes) is regarded just as important as data, and needs to be queried and manipulated. On the other hand, programming predicates are used as a convenient device for storing intermediate results in computations, or sometimes for storing results of queries, where no particular attention is paid to the schema under which such results are stored. The term *database relations* refers to relations corresponding to database predicates, while *programming relations* refer to relations corresponding to programming predicates. When no confusion arises, we refer to database relations, simply as relations. It should be pointed out that in classical deductive database query languages such as Datalog, and actual prototypes which are based on them, the only kind of predicates supported are programming predicates.

As a result, the schematic information of *all* relations is essentially ignored in such languages.

[LSS96] provides a number of examples to illustrate the semantics of *SchemaLog* programs, and the various terminologies and conventions introduced above. In this section, we provide one example. A few more examples will follow later on in the Chapter, in Section 2.4

**Example 2.1** Consider a stock exchange database that stores quarterly information on the companies traded in the exchange. The database also contains overall information on how the various industries are performing in each quarter. The following is the schema of this database.

```

stocks(comp, ticker, industry)
ind_review(industry, quarter, asp, aeps)
ibm(quarter, asp, aeps)
msft(quarter, asp, aeps)
xon(quarter, asp, aeps)

```

The *stocks* relation contains information on each company, its ticker name and its type of business. The *ind\_review* relation maintains information on the average share price (*asp*) and the average earnings per share (*aeps*) of each business category, on a quarterly basis. The other relations in the database are named after the companies traded in the exchange and carry information related to the quarterly performance of the relevant company.

The following is a *SchemaLog* program that restructures the above database such that the restructured database presents a summary of all the information present in the original database.

```

A[T : quarter → Q, ticker → T, A' → V', comp_perf → V] ← stocks[ticker → T,
    A' → V'], A' ≠ ticker, T[quarter → Q, A → V], A ≠ quarter
A[X : quarter → Q, ind_perf → V] ← A[X : industry → B],
    ind_review[quarter → Q, industry → B, A → V], A ≠ quarter,
    A ≠ industry

```

The tables generated by the above rules are shown in Figure 2.1. Such a representation allows one to compare the quarterly performance of each company with the overall performance of the particular industry (e.g., computers) as a whole. A key point to note here is that the *schema* (and hence tuples) of the output tables is being assembled “piecemeal” by the above rules. ■

	quarter	ticker	company	industry	comp-perf	ind-perf
aeps	1	xon	Exxon Corp	oil	0.14	0.16
	1	ibm	IBM Corp	computer	0.26	0.24
	2	msft	Microsoft Corp	computer	0.28	0.25
	...	...	...	...	...	...
	...	...	...	...	...	...

	quarter	ticker	company	industry	comp-perf	ind-perf
asp	...	...	...	...	...	...
	...	...	...	...	...	...

Figure 2.1: Example Illustrating Database Restructuring

## 2.3 Operators in Schema Algebra

Our approach to the implementation of *SchemaLog* is based on compiling constructs in *SchemaLog* into corresponding operations in an extended version of relational algebra, called *Schema Algebra*, or *SA* for short. This algebra, described in detail in [Sub97] includes conventional relational algebra as a proper subset and features extensions that facilitate meta-data querying and restructuring. Specifically, *SA* consists of the following kinds of operations.

1. Classical RA operators: these are capable of querying all relations (*programming* as well as *database*).
2. Operators which take a programming relation as input parameter along with some additional parameters, query the data and schema of (database) relations

relative to the parameters and present the output as programming relations. They map relations in a database to programming relations.

3. Operators which take a programming relation as input, along with some parameters and restructure the information in the input relation according to the supplied parameters . Thus, these operators map programming relations into database relations.

Operators of type (2) and (3) are new and unique to  $\mathcal{SA}$  . In principle,  $\mathcal{SA}$  can well be defined in the context of a federation of databases. Indeed, operations of type 1 and 2 were defined in such a context in [LSS96] and it was proved that they have an expressive power equivalent to that of *SchemaLog* programs containing only programming predicates in rule heads.

The main difference between classical RA operators and the new  $\mathcal{SA}$  operators is that the latter treat both the schema of relations and the data in them in a uniform manner. In other words, the schema is given first class status. Classical RA has no means of retrieving or restructuring the schema of relations.

Throughout the thesis, we shall use the database pertaining to the New York Stock Exchange, shown in Figure 2.2, as a running example. The data is based on the *actual* data maintained at the URL <http://www.ai.mit.edu/stocks.html>. For brevity, only small parts of the actual database are shown.

### 2.3.1 Definitions of Operators

The classical relational algebra operators — selection, projection, join, union, intersection, difference, cartesian product – are defined in the usual manner.

We give below the definitions of the operators of type (2) and (3). The definitions below, although based on those given in [Sub97], have been modified to fit into a single database context. The algorithms and the implementations described in Chapters 4 and 5 also assume a single database environment.

We start with a descriptive definition of what a *database* is, in the *SchemaLog* context. The *Herbrand Base*  $HB$  is the set of all facts that we can express in the language of *SchemaLog* , i.e, all literals of the form  $r[t : a \rightarrow v]$  such that  $r, t, a$  and  $v$  are constants. Then, all literals of  $HB$  in which the value of  $r$  is the name of a relation in the “Extensional Database” (i.e., the physically stored database) form

	Name	Ticker	Type
stocks	Exxon Corp	xon	oil
	IBM Corp	ibm	computer
	Microsoft Corp	msft	computer

	Date	High	Low	Close	Volume
ibm	95/06/16	93.62	92.62	92.62	2696.8
	95/06/19	94.62	93.25	94.62	2078.7
	95/06/20	98.00	93.87	97.75	3568.9
	95/06/21	98.62	97.12	97.12	3580.5

	Date	High	Low	Close	Volume
xon	95/06/16	72.25	71.37	72.12	3420.6
	95/06/19	72.25	71.50	71.50	826.1
	95/06/20	71.37	70.00	70.12	1267.3
	95/06/21	70.00	68.75	69.25	1792.2

	Date	High	Low	Close	Volume
msft	95/06/16	87.50	84.87	87.00	5767.5
	95/06/19	89.87	86.87	89.62	4410.1
	95/06/20	91.37	89.75	91.37	3541.9
	95/06/21	92.37	90.00	90.50	3583.8

Figure 2.2: The NYSE Database



what can be called *EHB* (or, the extensional part of the Herbrand Base). It is this set of ground atoms that we call the “database”. Note that in a *SchemaLog* atom, the schema information about a relation is also available since the position of the constants in the atom denote whether it is a relation name, an attribute name or a value (the usual “data”). For a formal definition, please refer to [Sub97]. The idea is to view a database as a function, say  $d$ , which when given a symbol, maps it to a function, say  $s$ , which when given a symbol, maps it to another function, say  $v$ , which when given a *tid*, maps it to a value.

**Definition 2.1 (Fetching relation names)**  $\rho() = \{r \mid r \text{ is the name of a relation in the database}\}$ .

Operator,  $\rho$ , is a 0-ary operator. It returns as output the *names* of all relations in the database.

E.g.,  $\rho()$  would return the set  $\{\text{stocks}, \text{ibm}, \text{msft}, \text{xon}\}$ , w.r.t. the stock market database of Figure 2.2. Notice that the output of  $\rho()$  includes both base and derived (database) relations, in general.

**Definition 2.2 (Fetching relations and their schemas)** *Let  $s$  be a programming relation of arity  $k$  and  $i \leq k$  a positive number. Then  $\alpha_i(s) = \{(r, a) \mid r \in \pi_i(s), r \text{ is the name of a relation in the database, and } r \text{ has an attribute with name } a\}$ .*

This operator,  $\alpha$ , takes a programming relation as input and a column number as parameter. It then interprets the values appearing in that column as possible relation names in the database, and retrieves the names of their attributes. Note that  $\pi_i(s)$  denotes the classical projection.

As an example, suppose  $s = \{(\text{msft}, \text{microsoft}, 100), (\text{hp}, \text{hewlett-packard}, 200)\}$ . Then  $\alpha_1(s) = \{(\text{msft}, \text{date}), (\text{msft}, \text{high}), (\text{msft}, \text{low}), (\text{msft}, \text{close}), (\text{msft}, \text{volume})\}$ . Notice that no output corresponding to *hp* is produced since *hp* does not correspond to any relation name in the database.

Before presenting the definition of the next operator, we need the notion of a *pattern*, introduced in [LSS96] in a different context and further extended in [Sub97]. A *pattern* is of one of the following forms: ‘ $a \rightarrow v$ ’, ‘ $a \rightarrow$ ’, ‘ $\rightarrow v$ ’, or ‘ $\rightarrow$ ’. Intuitively, a *pattern* may be viewed as an attribute value pair, where either the attribute or the

value component (or both) of the pair could be missing. The next two operators defined here use a pattern to query data and meta-data in a database. This is achieved using a notion of *satisfaction*.

Let  $r$  be a relation,  $t$  a tuple in  $r$ , and  $p$  a pattern. Then we say  $t$  satisfies  $p$  provided one of the following conditions holds:

- $p$  is of the form  $a \rightarrow v$ , and  $r$  has  $a$  as one of its attributes and  $t[a] = v$ . In this case, the attribute value pair  $(a, v)$  is said to be a *witness pair*.
- $p$  is of the form  $a \rightarrow \_$ , and  $r$  has  $a$  as one of its attributes. In this case, the attribute value pair  $(a, v)$  where  $t[a] = v$ , is said to be a *witness pair*.
- $p$  is of the form  $\_ \rightarrow v$ , and there is some attribute  $b$  in the schema of  $r$ , such that  $t[b] = v$ . In this case, for every attribute  $b$  for which  $t[b] = v$ ,  $(b, v)$  is said to be a *witness pair*.
- $p$  is of the form  $\_ \rightarrow \_$ . In this case, for every attribute  $a$  of  $r$ , and every value  $v$  such that  $t[a] = v$ ,  $(a, v)$  is a *witness pair*.

**Definition 2.3 (Querying data and meta-data)** *Let  $s$  be any programming relation of arity  $k$ ,  $i \leq k$  any number, and let  $p$  be any pattern. Then  $\gamma_{i;p}(s) = \{(r, t, a, v) \mid r \in \pi_i(s), t \text{ is the id of a tuple that satisfies } p, \text{ and } (a, v) \text{ is an associated witness pair}\}$ .*

This operator,  $\gamma$ , allows us to relate data to meta-data. It takes as input a programming relation, a column number and a pattern as parameters, and returns as output the details of all tuples in the database which satisfy the pattern.

E.g., let  $s = \{xon, ibm\}$ . Then  $\gamma_{1;high \rightarrow}(s) = \{(xon, t4, high, 72.25), \dots, (xon, t7, high, 70.00), (ibm, t8, high, 93.62), \dots, (ibm, t11, high, 98.62)\}$ .

Next we introduce an operator, which can be derived from the previous operators.

The main motivation for this operator is query processing efficiency. The analogy is that in classical relational algebra, the *join* is a derived operator which is helpful in efficient query processing, compared with the *Cartesian product*. This operator,  $\gamma^\wedge$ , behaves essentially the same as  $\gamma$  except that it does not explicitly extract tuple ids, and it deals with a conjunction of patterns in one shot. We denote a conjunction of patterns as  $\langle p_1, \dots, p_n \rangle$ , where each  $p_i$  is a pattern. Satisfaction of conjunctions of

patterns is defined in the obvious manner: a tuple satisfies a conjunctive pattern if it satisfies all patterns in the conjunction.

**Definition 2.4 (Querying conjunctive patterns)** *Let  $s$  be a programming relation,  $i$  a column number, and  $\langle p_1, \dots, p_n \rangle$  a conjunctive pattern, where each  $p_j$  is of one of the forms — ‘ $a \rightarrow v$ ’, ‘ $a \rightarrow$ ’, ‘ $\rightarrow v$ ’, or ‘ $\rightarrow$ ’. Then  $\gamma_{i;\langle p_1, \dots, p_n \rangle}^\wedge(s) = \{(r, a_1, v_1, \dots, a_n, v_n) \mid r \in \pi_i(s), r \text{ is the name of a relation in the database, } a_j \text{'s are attributes in the schema of } r, \text{ and there is a tuple } t \in r \text{ such that } t[a_1, \dots, a_n] = v_1, \dots, v_n, \text{ and } t \text{ satisfies } \langle p_1, \dots, p_n \rangle\}$ .*

This operator takes a programming relation of arity  $k$  as input, a column number  $i \leq k$  and a conjunctive pattern as parameters, and returns as output the details of all parts of the database queried about using the conjunctive pattern.

E.g., let  $s = \{stock\}$ . Then for the stock market database,  $\gamma_{1;\langle ticker \rightarrow, \rightarrow computer \rangle}^\wedge(s) = \{(stock, ticker, ibm, type, computer), (stock, ticker, msft, type, computer)\}$ .

The relation between  $\gamma$  and  $\gamma^\wedge$  can be expressed formally as:

$$\gamma_{i;\langle p_1, \dots, p_n \rangle}^\wedge(s) = \sigma_{\$2=\$6 \wedge, \dots, \wedge \$4(n-1)-2=\$4n-2} (\gamma_{i;p_1}(s) \times \dots \times \gamma_{i;p_n}(s))$$

This completes the definition of operators that extract the information in relations and convert it into programming relations.

We next turn to the type (3) (restructuring) operators.

**Definition 2.5 (Creating relations)** *For a programming relation  $s$  and a column number  $i$ ,  $\kappa_i(s)$  creates a relation named  $r$  for each  $r \in \pi_i(s)$ , if such a relation does not already exist.*

In other words, the operator,  $\kappa$ , takes a programming relation of arity  $k$  as input and a column number  $i \leq k$  as a parameter and creates relations with names corresponding to the entries appearing in column  $i$  of the input relation.

E.g.,  $\kappa_1(\{(close), (high), (low), (volume)\})$  creates the relations *close*, *high*, *low*, *volume* (whose schema is not yet defined).

**Definition 2.6 (Creating relations with schemas)** *For a programming relation  $s$  and column numbers  $i, j$ ,  $\varsigma_{i,j}(s)$  creates a relation  $r$  with attributes  $a_1, \dots, a_n$  exactly when  $\sigma_{\$1=r}(\pi_{i,j}(s)) = \{(r, a_1), \dots, (r, a_n)\}$ , whenever such a relation does not already exist.*

This operator,  $\varsigma$ , takes a programming relation of arity  $k$  as input and two column numbers  $i, j \leq k$  as parameters. It creates relations with names corresponding to the entries in column  $i$  whose schemas are determined by interpreting the entries appearing column  $j$  as the attributes associated with the relation names in column  $i$ .

For example, let  $s = \{(close, date), (close, ibm), (close, msft), (close, xon), (volume, date), (volume, ibm), (volume, msft), (volume, xon)\}$ . Then  $\varsigma_{1,2}(r)$  will create two relations  $close$  and  $volume$  both with the schema  $\{date, ibm, msft, xon\}$ , and with no data, as shown in Figure 2.3.

<b>close</b>	date	ibm	msft	xon

<b>volume</b>	date	ibm	msft	xon

Figure 2.3: Example for creating relation schemas

**Definition 2.7 (Creating and populating relations with schemas)** *The operator  $\rho_{i,j,k;g_1,\dots,g_m}(s)$  creates a relation  $r$  with attributes  $a_1, \dots, a_n$  exactly when  $\sigma_{\$1='r'}(\pi_{i,j}(s)) = \{(r, a_1), \dots, (r, a_n)\}$ , whenever such a relation does not already exist. Furthermore, it populates the relation  $r$  with a tuple  $t$  such that  $t[a_1, \dots, a_n] = \langle v_1, \dots, v_n \rangle$  exactly when  $\exists t_1, \dots, t_n \in r$  such that  $t_i[i, j, k] = (s, a_i, v_i)$ ,  $\pi_{i,j,k}(\{t_1, \dots, t_n\}) = \{(s, a_1, v_1), \dots, (s, a_n, v_n)\}$ , and finally,  $t_1[g_1, \dots, g_m] = \dots = t_n[g_1, \dots, g_m]$ . When the relation  $s$  already exists, the tuples generated above are appended to this relation.*

In less formal language,  $\rho$  takes a programming relation of arity  $k$  as input, three column numbers  $i, j, k$  and another list of column numbers  $g_1, \dots, g_m$  as parameters and returns as output several relations structured according to the interpretation of column  $i$  entries as relation names, column  $j$  entries as attribute names, and column  $k$  entries as values. The facts thus produced form pieces of larger tuples which are put together based on equality on the columns  $g_1, \dots, g_m$ .

For example, let  $r =$   
 $\{(close, ibm, 95/06/21, 97.12),$   
 $(close, msft, 95/06/21, 90.50),$   
 $(close, xon, 95/06/21, 69.25),$   
 $(close, ibm, 95/06/20, 97.75),$   
 $(close, msft, 95/06/20, 91.37),$   
 $(close, xon, 95/06/20, 69.25),$   
 $(close, date, 95/06/21, 95/06/21),$   
 $(close, date, 95/06/20, 95/06/20)\}.$

Then  $\rho_{1,2,4;3}(r)$  will create the relation shown in Figure 2.4.

	date	ibm	msft	xon
close	95/06/20	97.75	91.37	69.25
	95/06/21	97.12	90.50	69.25

Figure 2.4: Example for creating and populating relations

It must be noted that this restructuring operator also has some limited update capabilities. Whenever the relation corresponding to a newly generated tuple already exists, the tuple is appended to such a relation (as a typical tuple *insert*).

For example, suppose that the relation *close* shown in Figure 2.4 already exists. Let  $s =$

$(close, msft, 95/06/19, 90.50),$   
 $(close, xon, 95/06/19, 70.25),$   
 $(close, ibm, 95/06/19, 99.75),$   
 $(close, date, 95/06/19, 95/06/19)\}.$

Then  $\rho_{1,2,4;3}(s)$  will have the effect of *appending* the tuple (95/06/19, 99.75, 90.50, 70.25) to the *existing* relation *close*.

## 2.4 Further Examples

We conclude this chapter with further examples which illustrate some of the features of *SchemaLog* programs and also show how the various operators defined above are actually used in the processing of a *SchemaLog* program to achieve desired results.

**Example 2.2** The first example shows how queries involving data and meta-data can be expressed in *SchemaLog*.

Consider the query “Determine companies such that the *low* of Microsoft stock on some day is more than the *high* of the company’s stock on the same day.” In *SchemaLog*, this would be expressed as

$$\begin{aligned} p(C_1, C_2) \leftarrow & \text{stocks}[name \rightarrow C_1, ticker \rightarrow S_1], \text{stocks}[name \rightarrow C_2, ticker \rightarrow S_2], \\ & S_1[date \rightarrow D, low \rightarrow L], S_2[date \rightarrow D, high \rightarrow H], L > H. \\ ?-p(\text{microsoft}, C_2). \end{aligned} \quad \blacksquare$$

**Example 2.3** This example is a detailed demonstration of the working of the *restructuring* operator,  $\rho$ , with regard to a given input.

Consider the stock market database of Figure 2.2. The database stores information related to individual stocks in different (database) relations, organised by the stock ticker symbol. Figure 2.4 shows a restructured view of the information in the original database. It permits a quick comparison of the closing prices of different stocks on a given day. Such a restructuring is often referred to as *cross-tabulation* (*crosstabs*) [GBLP96]. This restructuring can be done readily by writing the one-line *SchemaLog* program

$$\text{close}[D : date \rightarrow D, S \rightarrow P] \leftarrow S[date \rightarrow D, close \rightarrow P], \text{stocks}[ticker \rightarrow S].$$

In this program, both retrieving information from the input EDB, and presenting the output are done through *SchemaLog molecules*. The rule computes attribute value pairs forming any *one* tuple in the output relation *close* in a piecemeal manner. The tuple id *D* acts as a “glue” and makes sure that all pieces related to one tuple in the output relation *close* are correctly grouped together into one tuple.

For example, for the stock market data of Figure 2.2, the facts computed for *close* are:

$$\begin{aligned} & \text{close}[95/06/21 : date \rightarrow 95/06/21, \text{ibm} \rightarrow 97.125], \\ & \text{close}[95/06/21 : date \rightarrow 95/06/21, \text{msft} \rightarrow 90.500], \text{ and} \end{aligned}$$

$close[95/06/21 : date \rightarrow 95/06/21, xon \rightarrow 69.250]$ .

Since all three facts have the same tuple id 95/06/21, they are correctly grouped into one tuple

$close[95/06/21 : date \rightarrow 95/06/21, ibm \rightarrow 97.125, msft \rightarrow 90.500, xon \rightarrow 69.250]$ . ■

**Example 2.4** This example takes a generalized version of the program in Example 2.3 and illustrates how a *SchemaLog* program can be rewritten in terms of the *SA* operators we have described in this chapter.

Suppose it is required to restructure the information in the stock market database such that it is readily possible to compare the relative performance of the various stocks with respect to each of their properties – *low*, *high*, and *closing* prices and *volume* of trading. The intended effect is that there would be separate relations organized around each of the above properties which list the performances of various stocks in each row, on a per day basis. It should be pointed out that such a query/restructuring is not a whim. Indeed in stock market applications, such restructuring arises naturally. In fact, the traditional approach to restructuring information is to write down special code for this purpose. The disadvantage of this approach is that such code tends to be specialized, *ad hoc*, and rather complex. Thus it does not permit general forms of restructuring and does not admit easy modification. (See the URL <http://www.ai.mit.edu/stocks.html> for details.)

In order to create separate relations corresponding to each of the properties *close*, *high*, *low* and *volume* we need to make only a minor modification to the *SchemaLog* program of Example 2.3.

$A[D : date \rightarrow D, S \rightarrow P] \leftarrow S[date \rightarrow D, A \rightarrow P], A \neq date, stocks[ticker \rightarrow S]$ .

This program can be rewritten in the form of the following algebraic expression, using the *SA* operators defined earlier:

$$\rho_{4,1,5,3}((\sigma_{\$4 \neq 'date'}(\gamma_{1; \langle date \rightarrow, \rightarrow \rangle}^{\wedge}(\pi_{ticker}(stocks)))) \cup (\pi_{2,2,3,4,3}(\gamma_{1; \langle date \rightarrow, \rightarrow \rangle}^{\wedge}(\pi_{ticker}(stocks))))))$$

The computation of the rather complex algebraic expression above can be easily understood if broken down into a sequence of simpler computations storing intermediate results. In the steps given below,  $r_1$  to  $r_5$  are temporary relations.

1.  $r_1 := \pi_{ticker}(stocks)$
2.  $r_2 := \gamma_{1; \langle date \rightarrow, \rightarrow \rangle}^{\wedge}(r_1)$ ;
3.  $r_3 := \sigma_{\$4 \neq 'date'}(r_2)$ ;
4.  $r_4 := \pi_{2,2,3,4,3}(r_2)$ ;
5.  $r_5 := r_3 \cup r_4$ ;
6.  $\rho_{4,1,5;3}(r_5)$ ;

A detailed study of the result of each step above will help us to understand the working of the  $\mathcal{SA}$  operators, especially  $\gamma^{\wedge}$  and  $\rho$ .

The result of **Step 1** (above) is the relation  $r_1$  which is given in Figure 2.5.

**stocks**

ticker
xon
ibm
msft

Figure 2.5:  $r_1$ : the result of Step 1

In **Step 2**, every relation mentioned in  $r_1$  of figure 2.5 is taken up and every row of these relations is processed to fit the given pattern. For the sake of brevity, we have processed only one row from each of the relations. The result is  $r_2$  as given in Figure 2.6.

**Step 3** removes those rows from  $r_2$  where the fourth column contains the string “date”. The result is  $r_3$  as in Figure 2.7.

**Step 4** eliminates some of the columns in  $r_2$  and re-orders the other columns, and produces the relation  $r_4$  in Figure 2.8.

**Step 5** does a regular union of the tables  $r_3$  and  $r_4$  and writes the result in  $r_5$  as in Figure 2.9.

Finally, in **Step 6** the  $\rho$  operator is applied to  $r_5$  according the interpretation of the input parameters. The result is the set of tables given in Figure 2.10. Please note that processing only the rows of  $r_5$  will produce merely the first row of each of the tables in figure 2.10. This is because in **Step 2** we did not process all the rows in the



xon	date	95/06/16	date	95/06/16
xon	date	95/06/16	high	72.25
xon	date	95/06/16	low	71.37
xon	date	95/06/16	close	72.12
xon	date	95/06/16	volume	3420.6
ibm	date	95/06/16	date	95/06/16
ibm	date	95/06/16	high	93.62
ibm	date	95/06/16	low	92.62
ibm	date	95/06/16	close	92.62
ibm	date	95/06/16	volume	2696.8
msft	date	95/06/16	date	95/06/16
msft	date	95/06/16	high	87.50
msft	date	95/06/16	low	84.87
msft	date	95/06/16	close	87.00
msft	date	95/06/16	volume	5767.5

Figure 2.6:  $r_2$ : the result of Step 2

xon	date	95/06/16	high	72.25
xon	date	95/06/16	low	71.37
xon	date	95/06/16	close	72.12
xon	date	95/06/16	volume	3420.6
ibm	date	95/06/16	high	93.62
ibm	date	95/06/16	low	92.62
ibm	date	95/06/16	close	92.62
ibm	date	95/06/16	volume	2696.8
msft	date	95/06/16	high	87.50
msft	date	95/06/16	low	84.87
msft	date	95/06/16	close	87.00
msft	date	95/06/16	volume	5767.5

Figure 2.7:  $r_3$ : the result of Step 3

date	date	95/06/16	high	96/06/16
date	date	95/06/16	low	96/06/16
date	date	95/06/16	close	96/06/16
date	date	95/06/16	volume	96/06/16

Figure 2.8:  $r_4$ : the result of Step 4

xon	date	95/06/16	high	72.25
xon	date	95/06/16	low	71.37
xon	date	95/06/16	close	72.12
xon	date	95/06/16	volume	3420.6
ibm	date	95/06/16	high	93.62
ibm	date	95/06/16	low	92.62
ibm	date	95/06/16	close	92.62
ibm	date	95/06/16	volume	2696.8
msft	date	95/06/16	high	87.50
msft	date	95/06/16	low	84.87
msft	date	95/06/16	close	87.00
msft	date	95/06/16	volume	5767.5
date	date	95/06/16	high	96/06/16
date	date	95/06/16	low	96/06/16
date	date	95/06/16	close	96/06/16
date	date	95/06/16	volume	96/06/16

Figure 2.9:  $r_5$ : the result of Step 5

input. The other rows in figure 2.10 are the result of the complete processing of the input database of Figure 2.2.

A comparison of the database of Figure 2.2 with the one obtained in Figure 2.10 is a good illustration of the power and usefulness of the  $\rho$  operator. It can be seen that the **columns** of figure 2.2 have become **tables** in figure 2.10 and the **tables** of figure 2.2 have become **columns** in figure 2.10, with the data suitably reallocated. ■

## 2.5 Concluding Remarks

This chapter provided a basic introduction to the high-level database programming language called *SchemaLog*. The syntax of the language was summarised and its semantics illustrated by some examples (in Sections 2.2 and 2.4). Section 2.3 was a rather detailed treatment of the algebraic operators needed to translate a *SchemaLog* program into an equivalent algebraic expression. Both traditional Relational Algebraic Operators as well as some new ones specific to the needs of *SchemaLog* were defined and their semantics illustrated with examples where necessary. Finally in

	Date	xon	ibm	msft
<b>high</b>	95/06/16	72.25	93.62	87.50
	95/06/19	72.25	94.62	89.87
	95/06/20	71.37	98.00	91.37
	95/06/21	70.00	98.62	92.37

	Date	xon	ibm	msft
<b>low</b>	95/06/16	71.37	92.62	84.87
	95/06/19	71.50	93.25	86.87
	95/06/20	70.00	93.87	89.75
	95/06/21	68.75	97.12	90.00

	Date	xon	ibm	msft
<b>close</b>	95/06/16	72.12	92.62	87.00
	95/06/19	71.50	94.62	89.62
	95/06/20	70.12	97.75	91.37
	95/06/21	69.25	97.12	90.50

	Date	xon	ibm	msft
<b>volume</b>	95/06/16	3420.6	2698.8	5767.5
	95/06/19	826.1	2078.7	4410.1
	95/06/20	1267.3	3568.9	3541.9
	95/06/21	1792.2	3580.5	3583.8

Figure 2.10: The result of Step 6

Section 2.4, Example 2.4 we showed how a *SchemaLog* program can be expressed in an equivalent algebraic expression. We concluded with a detailed example of the step-by-step evaluation of a *SchemaLog* rule.

In a subsequent chapter, we shall provide algorithms for implementing the *SA* operators in different storage strategies. A program and its equivalent algebraic expression, such as the one in Example 2.4, may then be readily implemented in the required storage format.

# Chapter 3

## Top-Down Processing of *SchemaLog* Programs

### 3.1 Introduction

In this chapter we discuss the top-down processing of *SchemaLog* programs. In particular, we investigate how the set oriented Rule/Goal Tree (RGT) evaluation method [Ull89] proposed for classical logic can be extended to the *SchemaLog* setting. Our choice of this methodology is due to the fact that set-oriented query processing techniques are more suitable for database applications as opposed to the tuple-at-a-time paradigm of Prolog. At the guts of the algorithm we discuss here, lie the *SA* operators defined in the previous chapter.

The rule/goal tree(RGT) is a representation of the sequence of exploration of the goals, their rules, and their subgoals required to answer a query. The RGT evaluation method is based on a depth-first search of the rule/goal tree in which a set of tuples of bindings is obtained at each node of the tree. Details of the rule/goal tree as well as the attendant query processing algorithm for classical logic can be found in [Ull89].

The notion of unification plays an important role in the construction of RGTs. Unification in *SchemaLog* is different from its classical counterpart mainly because *SchemaLog* requires unification of “literals” of unequal depth. [LSS96] discusses this issue at depth and presents an algorithm for computing the most general unifier (MGU)

of two *SchemaLog* atoms. Based on the *SchemaLog* notion of unification, the conventional algorithm for constructing the RGT of a program can be adapted to serve our purpose of evaluating *SchemaLog* programs. An important consequence of the fact that *SchemaLog* unification is performed on atoms is that we need to ‘atomize’ a *SchemaLog* program (which in general might contain molecules) before applying the top-down algorithm. Example 3.2 illustrates this point. We give an algorithm to “atomize” any given *SchemaLog* program.

The major strength of *SchemaLog* lies in its ability to express novel querying as well as powerful restructuring operations. Our adaptation of the classical RGT evaluation algorithm accounts for these unique features of *SchemaLog*. In the following, we sketch the major issues that arise in the development of such an algorithm. After that, we present a comprehensive algorithm for the RGT evaluation of a *SchemaLog* program. We conclude with an example of a RGT expansion of a *SchemaLog* program.

### 3.2 Atomizing a *SchemaLog* program

The use of “molecules” in a *SchemaLog* program makes for user convenience in reading and writing programs. But when the program is to be evaluated in a top-down paradigm, we need to use the program as input to such algorithms as *unification* (discussed in [Sub97]) and *Rule-Goal Tree Processing* (discussed later in this Chapter). Since these algorithms accept only *SchemaLog* atoms as input, all molecules in a given *SchemaLog* program need to be atomized as a necessary pre-processing step to top-down evaluation. In this section we first present an algorithm for “atomization” and then illustrate its use with an example.

---

**Algorithm 3.1** *Atomization of a SchemaLog program*

**Input:** A *SchemaLog* program

**Output:** A *SchemaLog* program that contains no “molecules”

**begin**

*for every rule in the given program*

*if the body of the rule contains a molecule*

rewrite the body as a new body,  $\mathbf{B}$ , such that  
every molecule of the form  $r[t : a_1 \rightarrow v_1, \dots, a_n \rightarrow v_n]$   
is written as  $r[t : a_1 \rightarrow v_1] \wedge, \dots, \wedge r[t : a_n \rightarrow v_n]$   
if the molecule does not have a tuple-id,

supply a tuple-id, distinct from the ones used for other molecules.

if the head of the rule contains a molecule of the form

$$r[t : a_1 \rightarrow v_1, \dots, a_m \rightarrow v_m]$$

construct  $m$  rules of the form:

$$r[t : a_1 \rightarrow v_1] \leftarrow \mathbf{B}$$

... ..

$$r[t : a_m \rightarrow v_m] \leftarrow \mathbf{B}$$

end

---

We illustrate the working of the above algorithm with the following example:

**Example 3.1** Let us revisit the one-rule *SchemaLog* program of Example 2.3:

$$close[D : date \rightarrow D, S \rightarrow P] \leftarrow S[date \rightarrow D, close \rightarrow P], stocks[ticker \rightarrow S].$$

This rule contains molecules both in the head and in the body. Applying the *Atomization Algorithm* to this rule yields the following two rules:

$$close[D : date \rightarrow D] \leftarrow S[T : date \rightarrow D], S[T : close \rightarrow P], stocks[ticker \rightarrow S]$$

$$close[D : S \rightarrow P] \leftarrow S[T : date \rightarrow D], S[T : close \rightarrow P], stocks[ticker \rightarrow S]$$

As we can see, the two rules above do not contain any molecules.

### 3.3 Outline of the Top-Down Processing procedure

At the very outset we should note that the area of top-down processing of logic programs is a well-studied one. The relevant algorithms have been discussed and analyzed in detail in logic programming literature. We shall, therefore, model our

algorithms on the classical ones and make modifications and additions to them for the specific requirements of *SchemaLog*.

There are two operations which are often invoked during the top-down evaluation of *SchemaLog* programs — one for converting the database relations corresponding to (programming) subgoals to relations over variables mentioned in that subgoal, and the other for converting a (programming) relation for the body to a relation for the head by translating from the viewpoint of variables to the viewpoint of arguments. For Datalog (which is the classical programming language for deductive databases)<sup>1</sup>, this switching between argument and variable viewpoints is accomplished by means of procedures called `atov()` and `vtoa()` [Ull89]. As *SchemaLog* atoms are syntactically different from their classical counterparts, the `atov()` and `vtoa()` procedures are somewhat different for our setting. Our approach efficiently realizes these same operations by using the technique of first reducing the database predicate arguments to a template that corresponds to a conventional predicate, and then applying the classical version of the operations.

For instance, in our `a2v()` procedure, in order to convert a relation  $M$  into a relation whose attributes correspond to variables appearing in a *SchemaLog* atom  $\mathcal{A}$  of the form  $\alpha_1[\alpha_2 : \alpha_3 \rightarrow \alpha_4]$ ,  $\mathcal{A}$  is reduced to a template  $temp(\alpha_1, \alpha_2, \alpha_3, \alpha_4)$  and the conventional `atov()` algorithm with the template and  $M$  as arguments is applied. The adaptation of the `vtoa()` algorithm is similar in nature.

At the heart of the RGT evaluation algorithm lies two mutually recursive procedures — `EXPAND_GOAL()` and `EXPAND_RULE()`. Given a *SchemaLog* goal  $G$  and a relation  $M$  that provides bindings for variables in  $G$ , `EXPAND_GOAL()` returns a relation  $R$  that is the set of tuples (bound by  $M$ ) that can be inferred from the database using the program, and match  $G$ . `EXPAND_RULE()` on the other hand, takes a rule  $r$  and the initial bindings for the variables in this rule and generates a relation  $R$  that is the set of tuples inferred from the database and the rule. Further, this procedure performs the restructuring dictated by  $R$  and the head predicate of rule  $r$ . Thus the querying and restructuring facets of *SchemaLog* query processing are neatly decoupled in procedures `EXPAND_GOAL()` and `EXPAND_RULE()` respectively.

`EXPAND_GOAL()` invokes the querying operations in  $\mathcal{SA}$  via a procedure called

---

<sup>1</sup>For an introduction to *Datalog*, refer to [CGT89]



QUERY\_GOAL(). Corresponding to each type of *SchemaLog* atom, this procedure invokes an appropriate *SA* expression involving type (2) operations. For instance, the call QUERY\_GOAL( $X[T : a \rightarrow V]$ ) invokes the operation  $\sigma_{s_3=a} \gamma_{s_1} \rightarrow (\rho)$ . Procedure RESTRUCTURE\_HEAD(), called from EXPAND\_RULE(), invokes an *SA* expression that includes a type (3) operation corresponding to the head predicate of the *SchemaLog* rule under expansion.

As in the classical case, a queue-based version of this algorithm based on a breadth-first search of the RGT can be realized by queueing the calls to EXPAND\_GOAL() and EXPAND\_RULE() rather than stacking them.

### 3.4 Top-Down Algorithm for *SchemaLog*

In this section, we present the top-down query processing algorithm for *SchemaLog*.

#### Algorithm 3.2 RULE/GOAL TREE EVALUATION

**Input:** A *SchemaLog* program, a database  $D$ , a query  $G_0$ , and the relation  $M_0$  that provides bindings for zero or more arguments of  $G_0$ .

**Output:** A set of tuples of bindings for variables in  $G_0$ , that satisfy the query against  $D$ .

**Body:** The main components of the algorithm are procedures QUERY\_GOAL(), RESTRUCTURE\_HEAD(), and two mutually recursive procedures EXPAND\_GOAL() and EXPAND\_RULE(). To account for the higher-order syntax of *SchemaLog*, procedures A2V() and V2A() are implemented differently from their classical counterpart.

**procedure** A2V( $A, M$ )

**Input:** A *SchemaLog* atom  $A$  of the form  $\beta_1[\beta_2 : \beta_3 \rightarrow \beta_4]$  or an atom of a lesser depth, and a relation  $M$ .

**Output:** A relation whose attributes are variables in  $A$ .

**begin**

    construct a template  $temp(\beta_i, \dots, \beta_k)$ ,  $1 \leq i \leq k \leq 4$ ,  $\beta_j$  is a variable

        corresponding to an attribute of  $M$  and appears in  $A$ ;

    let  $X_1, \dots, X_n$  be the distinct variables among  $\beta_i, \dots, \beta_k$ ;

    let  $Q$ , the output relation have scheme  $X_1, \dots, X_n$  and be empty;

```

    for each tuple  $t$  in  $M$ 
    begin
        if there is a term matching  $\tau$  for  $\text{temp}(\beta_1, \dots, \beta_k)$  and tuple  $t$ ,
        add to  $Q$ , the tuple  $(\tau(X_1), \dots, \tau(X_n))$ 
    end
return  $Q$ 
end

```

---

**procedure**  $v2A(A, R)$

**Input:** A SchemaLog atom  $A$  of the form  $\beta_1[\beta_2 : \beta_3 \rightarrow \beta_4]$  or an atom of a lesser depth, and a relation  $R$  with scheme  $(X_1, \dots, X_n)$ .

**begin**

construct a predicate  $\text{temp}(\beta_1, \dots, \beta_k)$ ,  $1 \leq i \leq k \leq 4$ , where  $\beta_1, \dots, \beta_k$   
are all the variable occurrences in  $A$  and  $\beta_j$  is an attribute of  $R$ ;

let  $S$ , the output relation be empty;

for each tuple  $t$  of relation  $R$

**begin**

for each variable  $X$  appearing in  $\text{temp}$ , replace all occurrences of  $X$   
in  $\text{temp}$  by  $t[X]$ ;

add the resulting tuple  $(t_1, \dots, t_m)$  to  $S$

**end**

**return**  $S$

**end**

---

**procedure**  $\text{EXPAND\_GOAL}(M, G, R)$

**begin**

if  $M = \phi$  then

**begin**

$R = \phi$ ; **return**;

**end**

$R = \phi$ ;

for each rule  $r$  with head  $H$  such that  $G$  is unifiable to  $H$

```

begin
  let  $\tau$  be the MGU from  $G$  to  $H$ ;
  compute  $S_0 = \text{A2V}(\tau(H), M)$ ;
  EXPAND_RULE( $S_0, \tau(\tau)$ );
end
 $R = \text{QUERY\_GOAL}(G) \bowtie M$ ;
end

```

---

```

procedure EXPAND_RULE( $S_0, r$ )

```

```

begin

```

```

  let  $r = H \leftarrow G_1, \dots, G_k$ ;

```

```

  for  $i = 1$  to  $k$  do

```

```

    begin

```

```

       $M_i = \text{V2A}(G_i, S_{i-1})$ ;

```

```

      EXPAND_GOAL( $M_i, G_i, R_i$ );

```

```

       $Q_i = \text{A2V}(G_i, R_i)$ ;

```

```

       $S_i = \prod_T(S_{i-1} \bowtie Q_i)$ ; /*  $T$  is the set of variables that appear in the scheme
      of  $S_{i-1}$  or  $Q_i$ , and also appear in one of  $H, G_{i+1}, \dots, G_k$  */

```

```

    end

```

```

    RESTRUCTURE_HEAD( $H, S_k$ )

```

```

end

```

---

```

procedure QUERY_GOAL( $A$ )

```

**Input:** A SchemaLog atom of the form  $\beta_1[\beta_2 : \beta_3 \rightarrow \beta_4]$ , or an atom of a lesser depth.

```

begin

```

```

  case  $A$  is of depth:

```

```

    1 : if  $\beta_1$  is a constant, return  $\sigma_{s_1=\beta_1}(\rho)$ 

```

```

      else return  $\rho$ ;

```

```

    2: return  $\sigma_{\wedge_i s_i=\beta_i} \alpha_{s_1}(\rho)$ ,  $\beta_i$  is a constant;

```

```

    3: return  $\sigma_{\wedge_i s_i=\beta_i} \gamma_{s_1} \rightarrow (\rho)$ ,  $\beta_i$  is a constant

```

```

end

```

---

**procedure** RESTRUCTURE\_HEAD( $A, P$ )

**Input:** A SchemaLog atom of the form  $\beta_1[\beta_2 : \beta_3 \rightarrow \beta_4]$ , or an atom of a lesser depth, and a relation  $P$  whose attributes correspond to variables in  $A$ .

**Note:** The unary function  $\nu$  used below, takes as argument a variable appearing in  $A$  and returns the position of its corresponding attribute in  $P$ .

**begin**

case  $A$  is of depth

1: if  $\beta_1$  is a constant, return  $\kappa_1\{\beta_1\}$

else return  $\kappa_{\nu(\beta_1)}(P)$ ;

2: form a tuple  $t = \langle \beta_i, \dots, \beta_k \rangle, 1 \leq i \leq k \leq 2, \beta_i, \dots, \beta_k$  are all the constants in  $A$ ;

compute  $Q = P \times \{t\}$ ;

return  $\varsigma_{\nu(\beta_1), \nu(\beta_2)}(Q)$ ;

3: form a tuple  $t = \langle \beta_i, \dots, \beta_k \rangle, 1 \leq i \leq k \leq 4, \beta_i, \dots, \beta_k$  are all the constants in  $A$ ;

compute  $Q = P \times \{t\}$ ;

return  $\rho_{\nu(\beta_1), \nu(\beta_3), \nu(\beta_4); \nu(\beta_2)}(Q)$

**end**

The algorithm starts off with an initial call to *expand\_goal* with the two input parameters of a *SchemaLog* goal  $G$  and a relation  $M$  that provides the initial bindings for the variables of  $G$ .

### 3.5 Example of Rule-Goal Tree Expansion in *SchemaLog*

We now give an illustration of what a Rule-Goal Tree looks like when applied to a *SchemaLog* program during top-down computation.

**Example 3.2** Let us consider the following *SchemaLog* program: (from example 2.1)

$$A[T : \text{quarter} \rightarrow Q, \text{ticker} \rightarrow T, A' \rightarrow V', \text{comp\_perf} \rightarrow V] \leftarrow \text{stocks}[\text{ticker} \rightarrow T, A' \rightarrow V'], A' \neq \text{ticker}, T[\text{quarter} \rightarrow Q, A \rightarrow V], A \neq \text{quarter}$$

$$\begin{aligned}
A[X : quarter \rightarrow Q, ind\_perf \rightarrow V] \leftarrow & A[X : industry \rightarrow B], \\
& ind\_review[quarter \rightarrow Q, industry \rightarrow B, A \rightarrow V], A \neq quarter, \\
& A \neq industry
\end{aligned}$$

with the query:

$$?-aeps[T : X \rightarrow Y].$$

The following is the atomized version of this program:

$$\begin{aligned}
A[T : quarter \rightarrow Q] &\leftarrow body_1 \\
A[T : ticker \rightarrow T] &\leftarrow body_1 \\
A[T : A' \rightarrow V'] &\leftarrow body_1 \\
A[T : comp\_perf \rightarrow V] &\leftarrow body_1 \\
A[X : quarter \rightarrow Q] &\leftarrow body_2 \\
A[X : ind\_perf \rightarrow V] &\leftarrow body_2
\end{aligned}$$

where,

$$\begin{aligned}
body_1 \equiv & stocks[T_1 : ticker \rightarrow T], stocks[T_1 : A' \rightarrow V'], A' \neq ticker, \\
& T[T_2 : quarter \rightarrow Q], T[T_2 : A \rightarrow V], A \neq quarter
\end{aligned}$$

and,

$$\begin{aligned}
body_2 \equiv & A[X : industry \rightarrow B], ind\_review[T_1 : quarter \rightarrow Q], ind\_review[T_1 : \\
& industry \rightarrow B], ind\_review[T_1 : A \rightarrow V], A \neq quarter, A \neq industry
\end{aligned}$$

By factoring out rule bodies into temporary predicates  $body_i$ , re-computation is avoided. Figure 3.1 shows the rule/goal tree for this example. For lack of space, only two representative branches are shown in the figure.

Execution of our top-down algorithm invokes the  $\gamma$  operation of the algebra for the EXPAND\_GOAL() call corresponding to the subgoal  $stocks[T_1 : A' \rightarrow V_1]$  in Level 1. Call to EXPAND\_RULE() for this rule invokes the  $\rho$  operation. In fact, the non-classical algebraic operations (of type 2 or 3) are invoked at all the nodes depicted in this example tree. ■

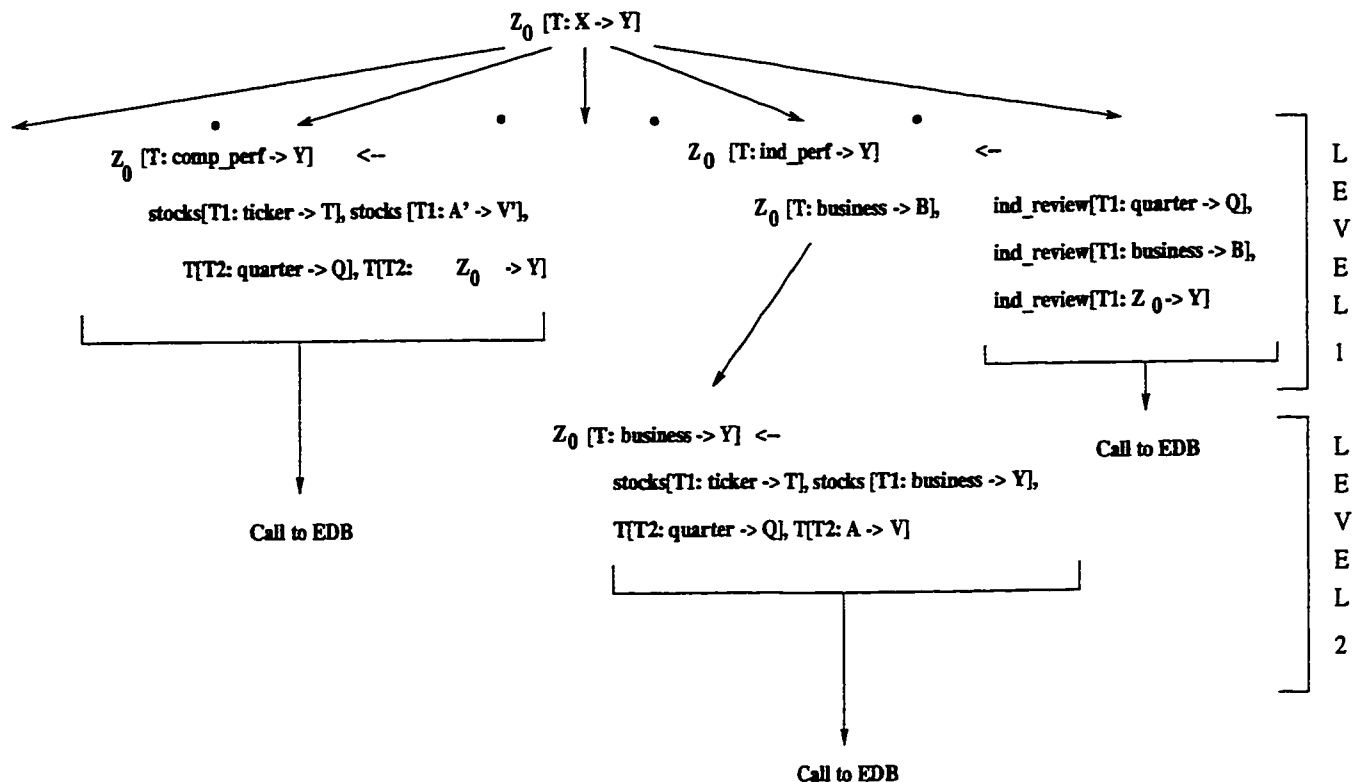


Figure 3.1: Example of Rule/Goal Tree

### 3.6 Conclusion

In this chapter, we saw how a given *SchemaLog* program can be processed in a top-down manner in procedures similar to the classical top-down processing of first order logic programs. To accommodate the higher order of the *SchemaLog* syntax, we need to modify such well-known algorithms such as “unification”, “atomization”, “atov”, “vtoa”, etc. The *SchemaLog* version of “unification” was addressed in [Sub97]. We presented here the other algorithms mentioned above. The Rule-Goal Tree structure, which is basic to the top-down processing paradigm, was also illustrated with an example of a *SchemaLog* program.

# Chapter 4

## Physical Storage Architectures

### 4.1 Introduction

As illustrated in the preceding chapters, *SchemaLog* possesses powerful capabilities for querying data and meta-data of relations as well as for restructuring them. Supporting these features in an implementation requires efficient storage structures. Recall that *SchemaLog* is implemented by compiling its constructs into appropriate operations in Schema Algebra ( $\mathcal{SA}$ ). Clearly, depending on the chosen storage structures, the underlying implementation strategy for the algebraic operations would differ.

In this chapter, we outline three alternative storage structures at the level of physical schemas. We also discuss the implementation of  $\mathcal{SA}$  operators (both conventional and additional) corresponding to each of them. After each algorithm we discuss the cost of implementing the algorithm in terms of tuple reads (or writes). We use tuple reads as a theoretical way of comparing the costs among the various strategies. The actual time taken for the various operations would be platform-dependent and will vary according to such factors as disk space, main memory size, block size, clock speed, etc. In a later chapter we shall also discuss a practical implementation of these algorithms in MS-Access and compare the actual costs of the operations under the various strategies.

Before we present the alternative storage structures, we remark that for existing (i.e. base) database relations, it is unrealistic to suppose that they can be converted into any form other than their existing form. For one thing, such a conversion would incur a massive overhead. For another, this would disrupt applications running on

the existing database. [LSS96] discusses these issues in detail and argues that from a practical perspective, the base relations should be preserved in their existing form. Thus, we are really considering alternative storage structures for database relations which are *created* or *derived* by *SchemaLog* programs.

## 4.2 Conventional Storage

The idea is to use the same schema as for conventional database relations. In other words, a relation  $r$  with attributes  $A, B, C$  would be implemented as a file of records with those fields. Thus, derived database relations would be stored and accessed the same way as base relations are.

We now present algorithms to implement some chosen operators.

### 4.2.1 Selection

We consider two principal cases: one in which the selection condition(s) involve(s) only key attribute(s), and the other in which non-key attributes are also involved. In the first case, we consider two possibilities: one in which there exists an index on the key attribute(s), and the other in which no index exists. Assume that the selection is done on a relation called  $r$ .

---

**Algorithm 4.1** *Selection on indexed key attribute(s)*

**begin**

$result = \phi$

*use the index on the key attribute to read the relevant tuple(s)*

$result = result \cup tuple(s)$

*write result*

**end**

---

### Cost

In the worst case, the whole relation may need to be read and written once (ie, if all tuples are selected). For uniformity of comparison, we take the most common case of selection, with the *equality* operator. Here, we read and write at most one tuple.



Next, we take up the case when there is no index on the key attribute. We make the assumption that the file is ordered on the key attribute.

---

**Algorithm 4.2** *Selection on non-indexed key attribute(s)*

**begin**

*result* =  $\phi$

*use binary search to locate the tuple(s) satisfying the given condition*

*if such tuples are found, result* = *result*  $\cup$  *tuple(s)*

*write result*

**end**

---

**Cost**

The reading costs  $\log_2 N_t$ . Here (and in the following pages),  $N_t$  stands for the “average number of tuples in a relation”. The expression stands for the number of tuples read during a binary search of the file containing the relation. The writing involves at most one tuple (assuming *equality* selection).

The final case of selection we consider involves non-key attributes for which no indexes exist.

---

**Algorithm 4.3** *Selection on non-key attribute(s)*

**begin**

*result* =  $\phi$

**while** *there are tuples in r*

*read a tuple*

*check whether it satisfies the given condition*

*if it does, result* = *result*  $\cup$  *tuple*

**endwhile**

*write result*

**end**

---

**Cost**

The reading costs  $N_t$  (linear read of all tuples in the relation); the writing costs  $S_d$ .  $S_d$  stands for “selection density”. Selection Density for a specified column is

defined as the average number of times a specific value occurs in that column. The number of distinct values in that column would then be  $N_t/S_d$ .

Note that if secondary indexes are available, selection on non-key attributes can be further optimized.

## 4.2.2 Projection

Projection involves two cases: one in which duplicates are not eliminated and the other in which they are.

---

**Algorithm 4.4** *Projection without elimination of duplicates*

**begin**

*result =  $\phi$*

**while** *there are tuples in r*

*read a tuple*

*remove unwanted attributes from the tuple*

*result = result  $\cup$  tuple*

**endwhile**

*write result*

**end**

---

**Cost**

The reading costs  $N_t$ ; and the writing, too, costs  $N_t$ .

---

**Algorithm 4.5** *Projection with elimination of duplicates*

**begin**

*read in the relation r*

*sort r using the desired attribute(s) as key*

*write back r' without duplicates and without the unwanted attributes*

**end**

---

**Cost**

If the file is small enough to fit into main memory, the cost can be approximated as  $(N_t + N'_t)$  where  $N'_t$  is the number of tuples in a relation after duplicate elimination. It can be considered equal to  $N_t/S_d$ .

For files larger than main memory, an external sort is required; the cost approximates to  $4N_t$ .

### 4.2.3 Join

We consider two-way joins only, of relations  $r_1$  and  $r_2$ . And in the most general case, we assume that no indexes are available. In the first case considered, the join attribute(s) are a key of both  $r_1$  and  $r_2$ . Both relations are ordered on the key.

---

**Algorithm 4.6** *Join on Key attributes*

**begin**

*result =  $\phi$*

*Let  $P_1$  point to the first tuple of  $r_1$*

*Let  $P_2$  point to the first tuple of  $r_2$*

**while** *there remain tuples in  $r_1$  and  $r_2$*

*if the join attribute values in the two tuples match,*

*make the new join tuple  $t$*

*result = result  $\cup$   $t$*

*move  $P_1$  and  $P_2$  down their respective relations*

**elseif**  *$P_1$  value <  $P_2$*

*move  $P_1$  down by a row*

**elseif**  *$P_1$  value >  $P_2$*

*move  $P_2$  down by a row*

**endif**

**endwhile**

**end**

---

#### Cost

The reading costs  $2 \times N_t$  since each relation is read once. The writing costs  $N_t \times J_d$ , where  $J_d$  stands for "Join Density". Join Density is defined here as the ratio of the number of tuples produced by the join to the number of tuples in a relation. It follows that the number of tuples produced by the join is  $N_t \times J_d$ .

When the join attributes are not keys (and no indexes are available), the most efficient method for the join, in most cases, is to first sort the two relations on the join attribute(s) and then to make the join. This is traditionally known as a *Sort Join*.

---

**Algorithm 4.7** *Sort Join*

**begin**  
    *sort  $r_1$  on the join attribute(s)*  
    *sort  $r_2$  on the join attribute(s)*  
    *make the join as in Algorithm 4.6 above*  
**end**

---

**Cost**

We assume that each *sort* of a relation involves at least one read and one write of all the tuples in the relation, making up a cost of  $2 \times N_t$ . The total cost would, then, be  $2N_t + 2N_t + (2N_t + N_t \times J_d)$ .

Let us now consider the case when the join attributes have secondary indexes. We look at the join  $r_1 \bowtie_{r_1.A = r_2.B} r_2$ .

---

**Algorithm 4.8** *Indexed Join*

**begin**  
    **repeat**  
        *read a block of  $r_1$  tuples with the same values on A*  
        *read the corresponding tuples in  $r_2$  (ie, with the same values for B*  
        *make the join and add the tuples to the result*  
    **until** *there are no more tuples in  $r_1$*   
**end**

---

**Cost**

The cost comes to  $2N_t + N_t \times J_d$ .

For more detailed discussions of Join strategies and cost computations, one can refer to any standard Text on Databases such as [Des90], [KS91], [Ram97], [Ull89].

#### 4.2.4 Fetching Relation Names

We assume that the DBMS maintains the names of all its relations in its system tables. The application of the  $\rho$  operator, then, involves one read through the corresponding system table and a write of all relation names.

The cost would be  $N_r \times N_a + N_r$ , where  $N_r$  is defined as the number of relations in the database and  $N_a$  as the average number of attributes in a relation.

#### 4.2.5 Fetching Relations and their Schemas

Here again, we assume that the information on relations and their attributes is available in the form of system tables. The schema of a system table is generally of the form  $(relation, attribute)$ . The application of the  $\alpha$  operator involves using the relation name (supplied as an argument to the operator) to search the system tables and write out tuples of the form  $(relation, attribute)$  for each attribute in the relation.

The cost of this operation would be  $N_r \times N_a$  for reading and  $N_r \times N_a$  for writing.

#### 4.2.6 Querying Data and Meta-data

---

**Algorithm 4.9**  $\gamma_{i;p}(s)$  *Querying with a single pattern*

**begin**

*Identify the relations involved by inspecting the  $i^{\text{th}}$  column of relation  $s$ .*

**for each relation  $r$  thus identified**

**for each tuple  $t$  in  $r$**

*if  $t$  matches the pattern  $p$ , add to the result a tuple*

*of the form  $\langle r, tid, a, v \rangle$  where  $tid$  is the tuple-id of*

*$t$  and  $(a, v)$  is an associated witness pair*

**endfor**

**endfor**

**end**

---

## Cost

The reading cost can be written as  $N_t \times N'_r$ , where  $N'_r$  denotes the number of relations involved in the operation.

Note that this is the worst-case scenario. The cost can be considerably reduced if the pattern is of the form  $\langle a \rightarrow \rangle$  or  $\langle a \rightarrow v \rangle$ . In such cases, during the first step of the algorithm above, we choose only relations with the attribute  $a$  in their schemas. Note also that if there exists an index or an ordering on  $a$ , the algorithm can be further optimized.

In the worst case, i.e., if the pattern is  $\langle \rightarrow \rangle$ , we have the cost as  $N_t \times N_a \times N'_r$ . Writing costs for the other patterns will vary widely depending on whether  $a$  and/or  $v$  are known and what their actual values are. For a rough estimate, we use a factor called "Pattern Density (Single)" or  $P_{ds}$ . Pattern Density, in general, is defined as the number of tuples that actually fit the pattern (whether single or multiple) to the maximum number of tuples that could be formed. With this factor, then, we can summarise the writing cost as  $N_t \times N_a \times N'_r \times P_{ds}$ .

---

**Algorithm 4.10**  $\gamma_{i:(p_1, \dots, p_n)}^\wedge(s)$  *Querying with a multiple pattern*

**begin**

*Identify the relations involved by inspecting the  $i^{\text{th}}$  column of relation  $s$ .*

*for each relation  $r$  thus identified*

*for each tuple  $t$  in  $r$*

*if  $t$  matches the pattern  $\langle p_1, \dots, p_n \rangle$ ,*

*add to the result as many tuples as possible of the form*

*$\langle r, a_1, v_1, \dots, a_n, v_n \rangle$  using  $(a, v)$  combinations of tuple  $t$*

*in all ways that fit the pattern*

**endfor**

**endfor**

**end**

---

## Cost

The reading cost is the same as that for the single pattern query, i.e.,  $N_t \times N'_r$ .

Here, too, the writing cost will vary widely according the number of patterns in the condition, the number of attributes and other such factors. In cases where the patterns do not contain any (or few) attribute names, each original tuple could be

used to compose a number of tuples in the output. We use the factor “Pattern Density (Multiple)” or  $P_{dm}$  to help us arrive at an expression for the cost. “Pattern Density” has already been defined earlier. If each original tuple is used to make at most one tuple in the output, the number of tuples written would be  $N_t \times N'_r \times P_{dm}$ . If we assume on the average that a tuple could be used to make  $N_a$  tuples in the output, the writing cost would be  $N_t \times N'_r \times N_a \times P_{dm}$ .

## 4.2.7 Creating Relations

---

**Algorithm 4.11**  $\kappa_i(s)$

```

begin
  while there are rows to be considered in  $s$  (the given schemaless relation)
    read a row
    store the relation name from the  $i^{\text{th}}$  column of  $s$ 
  endwhile
  for each relation name  $r$  stored
    create a table called  $r$ 
  endfor
end

```

---

### Cost

The reading cost is  $N_t$  (if the “schemaless” relation is also considered to have the average  $N_t$  number of rows). This cost is negligible compared to the cost of creating the tables which is obviously much larger.

The writing cost can be estimated to be  $N_r \times T_c$ , where  $N_r$  is the number of relations identified by  $\kappa$  and  $T_c$  is the cost of creating a table.

## 4.2.8 Creating Relations with Schemas

---

**Algorithm 4.12**  $\varsigma_{i,j}(s)$

```

begin
  while there are rows to be considered in  $s$  (the given schemaless relation)
    read a row

```

```

        store the relation name from the  $i^{\text{th}}$  column and the attribute name from
            the  $j^{\text{th}}$  column of  $s$  in an appropriate data structure
    endwhile
    for each relation name  $r$  stored
        create a table called  $r$  with the corresponding stored attributes
    endfor
end

```

---

### Cost

The reading and writing costs can be considered identical to that of the previous operator,  $\kappa$ .

## 4.2.9 Creating Relations with Schema and Data

In this operation, information about the schema of a table as well as about the data contained in it arrive in a piecemeal fashion. Data can be added piecemeal to the conventional relational storage. But the modification of schema is an expensive operation and impractical to do “on the fly” as soon as a new column name appears as input. In our algorithm below, all information about tables (old as well as new), schema and data available in the “schemaless relation” (output by the computation of a *SchemaLog* program) is stored in temporary data structures and used to make temporary tables. When all expected information is in, these temporary tables are joined (wherever possible) to existing database tables.

---

### Algorithm 4.13 $\rho_{i,j,k;l}(s)$

```

begin
    while there are rows to be considered in  $s$  (the given schemaless relation)
        read a row
        store the relation name, attribute name, value and grouping
            attribute value in an appropriate data structure
    endwhile
    for each relation name  $rel$  in the stored relation
        create a table called  $temp\_rel$  with the appropriate (stored) attributes
    endfor
end

```



```

        append tuples to the table (using the stored values)
    endfor
    for each temp_rel created
        if  $\exists$  a relation named rel in the database
            rel = rel  $\bowtie$  temp_rel
        else create a relation rel = temp_rel
    endfor
end

```

---

### Cost

The cost (reading as well as writing) can be separated into the following parts:

reading in the Schemaless relation =  $N_r' \times N_a \times N_t$   
 creating temporary data structures =  $N_r' \times N_a \times N_t$   
 creating temporary tables =  $N_r' \times T_c$   
 writing to the temporary tables =  $N_r' \times N_t$   
 making joins with existing tables =  $N_r' \times \text{'joincost'}$

We have made use of previously defined terms.  $N_r'$  stands for the average number of tables that would be mentioned in the input.

## 4.3 Reduced Storage

The term *reduced* refers to the fact that *SchemaLog* admits a faithful first-order reduction, as established in [LSS96]. The idea is that each relation in the database can be “flattened” and all the information in the database can be compiled into three relations –  $call_4(R, T, A, V)$  (corresponding to  $R[T : A \rightarrow V]$ ),  $call_2(R, A)$  (corresponding to  $R[A]$ ), and  $call_1(R)$  (corresponding to  $R[]$ ).

For example, consider Figure 4.1 which shows two tables of conventional data storage.

In the “Reduced Storage”, this information would take the form of the tables given in Figure 4.2

As pointed out earlier, in a practical setting, this flattening can only be applied to “derived” database relations. Under reduced storage, meta-data querying is essentially reduced to conventional data querying, and restructuring is reduced to updating the  $call_i$  relations. In other words, the extended operations of type (2) and (3) in

	name	age	city
person	John	35	Montreal
	Peter	42	Toronto
	Andrew	31	Montreal

	name	company
works_in	John	CIBC
	Peter	IBM

Figure 4.1: Conventional Storage Tables

$\mathcal{SA}$  are reduced to classical RA operations, under this storage. In particular, the piecemeal computation of the operator  $\rho$  under conventional storage reduces to normal computation (where entire tuples are computed at a time, rather than in parts). By contrast, simple classical operations like selection and join translate into complex operations in this storage method.

We now provide algorithms to implement the various Relational Algebraic Operators we had mentioned with regard to the Conventional Strategy. In the algorithms and in the cost analysis which follow, we shall assume the presence of three indexes: a primary key index on  $(relation, tid)$  and a secondary key indexes on  $attribute$  and  $value$ . Considering that the whole database in this strategy consists practically of one relation, the  $call_4$  relation, a total of three indexes is a very reasonable overhead.

### 4.3.1 Selection

---

**Algorithm 4.14**  $\sigma_{a=v}r$

**begin**

*scan the  $call_4$  relation using the index on value*

*and write out the tuple set,  $S$ , with the value  $v$ , attribute  $a$  and relation  $r$*

*result =  $\phi$*

**for** *each tuple in the set  $S$*

*scan the  $call_4$  relation for all tuples,  $P$ , in  $r$*

*with the same  $tid$  (using the  $(relation, tid)$  index)*

*result = result  $\cup$   $P$*

**endfor**

**end**

<i>call</i> <sub>1</sub>	relation
	person
	works_in

<i>call</i> <sub>2</sub>	relation	attribute
	person	name
	person	age
	person	city
	works_in	name
	works_in	company

<i>call</i> <sub>4</sub>	relation	tuple_id	attribute	value
	person	<i>t</i> <sub>1</sub>	name	John
	person	<i>t</i> <sub>1</sub>	age	35
	person	<i>t</i> <sub>1</sub>	city	Montreal
	person	<i>t</i> <sub>2</sub>	name	Peter
	...	...	...	...
	works_in	<i>t</i> <sub>1</sub>	name	John
	works_in	<i>t</i> <sub>1</sub>	company	CIBC
	works_in	<i>t</i> <sub>2</sub>	name	Peter
	works_in	<i>t</i> <sub>2</sub>	company	IBM

Figure 4.2: Reduced Storage Tables

### Cost

Producing the tuple set  $S$  involves  $S_d \times N_r$  tuple reads and  $S_d$  tuple writes. There is a further read of  $S_d$  tuples in the **for** loop and the final write of  $S_d \times N_a$  tuples in the result.

### 4.3.2 Projection

**Algorithm 4.15**  $\Pi_{a_1, \dots, a_n} r$

**begin**

*result* =  $\phi$

*scan the call*<sub>4</sub> *relation using the index on r*

```

    for each tuple  $t$  thus retrieved
        if the attribute is in the set  $\{a_1, \dots, a_n\}$ 
            then,  $result = result \cup t$ 
        endfor
    end

```

---

### Cost

The algorithm involves a read of  $N_a \times N_t$  tuples (i.e., the subset of the  $call_4$  relation corresponding to the relation  $r$ , and a write of  $n \times N_t$  result tuples, where  $n$  is the number of attributes projected.

### 4.3.3 Join

When relation tuples are stored in the *Reduced Storage* strategy, the joinable tuples have to be first located, and their *tid*'s used to locate the other *reduced* tuples that form part of the *conventional* joined tuple. The basic outline of the algorithm is given below.

---

#### Algorithm 4.16 $r_1 \bowtie r_2$

```

begin
    result =  $\phi$ 
    create a temporary relation temp_rel with the scheme  $\langle r_1, tid_1, r_2, tid_2 \rangle$ 
    use a conventional join algorithm on two copies of the  $call_4$  relation
        to match tuples where the relation names and the attribute names
        correspond to the join requirement and the values are the same.
    write out tuples of the form  $\langle r_1, tid_1, r_2, tid_2 \rangle$  into temp_rel
    for each row  $t$  in temp_rel
        retrieve all rows in  $call_4$  with the same values for  $r_1$  and  $tid_1$ 
        for every row  $r$  thus retrieved
            assign the new relation name (ie, the name of the joined table)
                to the 1st column
            assign a tuple id
            assign a new column name (if desired)

```

```

        /* the contents of the value column are left the same*/
        result = result  $\cup$  r
    endfor
    retrieve all rows in call4 with the same values for r2 and tid2
    for every row s thus retrieved
        assign the new relation name (ie, the name of the joined table)
            to the 1st column
        assign the tuple id /* same as for r above */
        assign a new column name (if desired)
        /* the contents of the value column are left the same*/
        result = result  $\cup$  s
    endfor
endfor
end

```

---

## Cost

First we consider the cost of the join on the two copies of *call*<sub>4</sub>. Using the Indexed Join, this would be  $2 \times N_a \times N_t + N_t \times N_a \times J_d$ . At this stage the *temp\_rel* mentioned in the algorithm is estimated to contain about  $N_t \times N_a \times J_d$  tuples. In the remaining part of the algorithm, for each row in *temp\_rel*, the two *for* loops read and write exactly  $N_a$  tuples each. The cost for this part would then be  $2 \times N_t \times N_a \times J_d \times N_a$  for the read and  $2 \times N_t \times N_a \times J_d \times N_a$  for the write.

### 4.3.4 Fetching Relation Names

In the *Reduced Storage* Strategy, the output of the  $\rho$  operator is simply the stored *call*<sub>1</sub> relation.

The number of rows in *call*<sub>1</sub> usually very small and the cost the operation can be considered negligible.

### 4.3.5 Fetching Relations and their Schemas

This operation, too, is somewhat trivial under this Storage scheme. An implementation of the operation  $\alpha_i(r)$  would simply produce a subset of the tuples in the *call*<sub>2</sub>

relation.

The cost can be estimated as  $N_r \times N_a$  for reading and  $N_r \times N_a$  for writing.

### 4.3.6 Querying Data and Meta-data

Querying with a single pattern is fairly straightforward. We only have to read out tuples from  $call_4$  corresponding to the relations identified by the operator and check whether the tuples match the pattern.

---

**Algorithm 4.17**  $\gamma_{i;p}(s)$  *Querying with a single pattern*

**begin**

*Identify the relations involved by inspecting the  $i^{th}$  column of relation  $s$ .*

*for each relation  $r$  thus identified*

*for each tuple  $t$  in  $r$*

*if  $t$  matches the pattern  $p$ , add  $t$  to the result*

**endfor**

**endfor**

**end**

---

#### Cost

The reading cost can be written as  $N_t \times N_a \times N'_r$ , where  $N'_r$  denotes the number of relations identified by the operator. This, again, is a basic and general algorithm. It can be optimized according to the nature of the pattern. If both *attribute* and *value* are known, for example, the indexes on *attribute* and *value* can be used to go straight to the tuples matching the pattern and then the *relation* names can be checked against the required relation names.

Making use of the  $P_{ds}$  factor (defined earlier), we can summarise the writing cost as  $N_t \times N_a \times N'_r \times P_{ds}$ .

Querying for a multiple pattern is somewhat complex in this Strategy because of the very nature of the storage method. Each row in  $call_4$  contains information on a

single pattern only; hence, rows with the same *tid* have to be put together before we can decide on the occurrence or otherwise of the given multiple pattern.

Another point to note here is that a single *conventional* row can produce a number of rows in the output in cases where one or more of the *attribute* positions are left blank in the pattern. For example, if  $\langle \text{John}, 35, \text{Montreal} \rangle$  is a tuple in the relation  $\text{person}(\text{name}, \text{age}, \text{city})$  and the given pattern is  $\langle \rightarrow, \rightarrow \rangle$ , then the  $\gamma^\wedge$  operator would produce the following three tuples:  $\langle \text{person}, \text{name}, \text{John}, \text{age}, 35 \rangle$ ,  $\langle \text{person}, \text{name}, \text{John}, \text{city}, \text{Montreal} \rangle$  and  $\langle \text{person}, \text{age}, 35, \text{city}, \text{Montreal} \rangle$ . In general, if there are  $N_a$  attributes and  $p$  patterns, a single *conventional* tuple can produce  $N_a C_p$  tuples in the output.

The algorithm below does not go into the details of implementation for each type of pattern. It outlines a general procedure applicable in all cases.

**Algorithm 4.18**  $\gamma_{i; \langle p_1, \dots, p_n \rangle}^\wedge(s)$       *Querying with a multiple pattern*

**begin**

*Identify the relations involved by inspecting the  $i^{\text{th}}$  column of relation  $s$ .*

**for** *each relation  $r$  thus identified*

**for** *each set  $R$  of tuples with the same tid*

*if the tuples in  $R$  match the pattern  $\langle p_1, \dots, p_n \rangle$ ,*

*add to the result as many tuples as possible of the form*

$\langle r, a_1, v_1, \dots, a_n, v_n \rangle$  *using  $(a, v)$  combinations of the tuple set  $R$*   
                *in all ways that fit the pattern*

**endfor**

**endfor**

**end**

### Cost

The reading cost is the same as that for the single pattern query, i.e.,  $N_t \times N_a \times N_r'$ .

If each tuple set  $R$  (see algorithm above) is used to make at most one tuple in the output, the number of tuples written would be  $N_t \times N_r' \times P_{dm}$ , where  $P_{dm}$  is the "Pattern Density" factor for a multiple pattern. If we assume on the average that set  $R$  could be used to make  $N_a$  tuples in the output, the writing cost would be  $N_t \times N_r' \times N_a \times P_{dm}$ .

### 4.3.7 Creating Relations

The effect of the  $\kappa$  operator in this Strategy is to add as many new rows to the  $call_1$  relation as there are relations to be created. The cost is negligible.

### 4.3.8 Creating Relations with Schemas

---

**Algorithm 4.19**  $\varsigma_{i,j}(s)$

**begin**

*while there are rows to be considered in  $s$  (the given schemaless relation)*

*read a row*

*store the relation name from the  $i^{\text{th}}$  column and the attribute name from the  $j^{\text{th}}$  column of  $s$  in an appropriate data structure*

**endwhile**

**for** *each relation name  $r$  stored*

**for** *each attribute  $a$  in  $r$*

*insert a row  $\langle r, a \rangle$  in the  $call_2$  relation*

**endfor**

*insert a row  $\langle r \rangle$  in the  $call_1$  relation*

**endfor**

**end**

---

**Cost**

$N_r' \times N_a$  rows are written to the  $call_2$  relation; and  $N_r'$  rows are written to the  $call_1$  relation.

### 4.3.9 Creating Relations with Schema and Data

This operation which was quite complex in the Conventional Strategy is quite straightforward under this Strategy. The given "Schemaless relation" is already in the same format as the  $call_4$  relation. And so we only need to take each row in the input, rearrange the column values as specified in the operator and insert the row into  $call_4$ .



---

**Algorithm 4.20**  $q_{i,j,k;l}(r)$

**begin**

**while** *tuples remain in r*

*read tuple  $\tau$  from r.*

*form a tuple  $\langle r, t, a, v \rangle$  such that  $r, t, a,$  and  $v$  are the  $i^{\text{th}}, l^{\text{th}}, j^{\text{th}},$  and  $k^{\text{th}}$  components respectively of  $\tau.$*

*write the new tuple.*

**endwhile**

**end**

---

**Cost**

Reading in the input relation needs  $N_r' \times N_a \times N_t$  tuple reads; and the same number of tuples are written to the  $call_4$  relation.

## 4.4 Reduced, Atomized Storage

This is really a refinement on the previous storage scheme. Since information pertaining to several different database relations is lumped into one relation  $call_4$  (and also  $call_2, call_1$ ) there is some attendant redundancy e.g., in the repetition of relation names. One way to minimize this redundancy is to (i) separate information in different database relations in their flattened representations, and (ii) split the information corresponding to different attributes of the same relation, using the tuple id's as a glue.

A derived database relation of the form  $r(a_1, \dots, a_n)$  is physically stored in relations  $physrel(r, a_1)(tid, val), \dots, physrel(r, a_n)(tid, val)$ . Note that in this storage scheme,  $physrel(r, a_j)$  is the *name* of a relation used for physical storage while  $\{tid, val\}$  is its schema. The first column in a physical relation corresponds to the tuple-ids of tuples in the database relation  $r$ , and the second column contains the values. The tuple  $\langle i, v \rangle$  in a physical relation  $physrel(r, a_j)$  represents the fact that a tuple  $i$  in relation  $r$  has value  $v$  on attribute  $a_j$ .

To illustrate the above, let us consider the data in the conventional tables of Figure 4.1. In the “Reduced, Atomized Storage”, this information would take the form of the tables given in Figure 4.3

<i>physrel(person, name)</i>	tid	value
	$t_1$	John
	$t_2$	Peter
	$t_3$	Andrew

<i>physrel(person, age)</i>	tid	value
	$t_1$	35
	$t_2$	42
	$t_3$	31

<i>physrel(person, city)</i>	tid	value
	$t_1$	Montreal
	$t_2$	Toronto
	$t_3$	Montreal

<i>physrel(works_in, name)</i>	tid	value
	$t_1$	John
	$t_2$	Peter

<i>physrel(works_in, company)</i>	tid	value
	$t_1$	CIBC
	$t_2$	IBM

Figure 4.3: Reduced, Atomized Storage Tables

Thus, in this strategy, a derived database relation is stored using as many physical relations as there are attributes in it – each such relation storing one column of the database relation. The relation name and the attribute name corresponding to the column are ‘encoded’ in the name of the physical relation. The *SA* operations are interpreted against such a representation; for instance, operations that add attributes to an existing relation translate in this strategy to operations that add new relations. Many of the comments made for reduced storage also apply to reduced atomized storage. Thus, we expect that this scheme will suit meta-data querying and restructuring better than conventional data querying.

We now proceed to the algorithms for implementing Algebraic Operators under this Strategy. Here we shall assume that each *physrel* relation is indexed on *tid* and on *value*.

#### 4.4.1 Selection

---

**Algorithm 4.21**  $\sigma_{a=v}r$

**begin**

**for** *each tuple in* *physrel*(*r*,*a*)

*read tuple t*

*if t.val = v, write t to the output relation* *physrel*(*r<sub>out</sub>*, *a*)

**for** *each* *physrel*(*r*, *a<sub>i</sub>*), *a<sub>i</sub> ≠ a*

*use the index on tid to read from* *physrel*(*r*, *a<sub>i</sub>*) *the tuple with tid t.tid*

*write this tuple to the output relation* *physrel*(*r<sub>out</sub>*, *a<sub>i</sub>*)

**endfor**

**endfor**

**end**

---

#### Cost

If  $S_d$  is the “Selection Density” (defined earlier), the outer *for* loop executes  $S_d$  times. And the inner loop executes  $N_a$  times (actually  $N_a - 1$  times; but remember that  $N_a$  is considered an average value). There are therefore  $S_d \times N_a$  tuple reads and the same number of tuple writes.

#### 4.4.2 Projection

If the projected attributes contain key attributes or if duplicate elimination is not required, the projection operation is very simple in this strategy as it would involve merely marking the appropriate *physrel*'s.

The algorithm below assumes that duplicate elimination is required in the result of the projection.

---

**Algorithm 4.22**  $\Pi_{a_1, \dots, a_n}(r)$ **begin**

*make a join on tid among all physrel( $r, a_i$ ) where  $i = 1, \dots, n$ , retaining only the value columns.*

*sort the resultant relation*

*write out the same relation eliminating duplicates*

*write out the  $n$  columns from this relation into  $n$  physrel relations assigning new tid's for each row*

**end**

---

**Cost**

The cost of the Join (“indexed join”) is  $2N_t \times J_d \times N_t$ . Sorting and rewriting involves approximately  $2N_t + N'_t$  tuple reads and writes. Writing the result into *physrel* schemes costs  $n \times N'_t$  where  $N'_t$  stands for the number of tuples after duplicate elimination and  $n$  represents the number of attributes projected.

### 4.4.3 Join

The complexity of a Join operation in this Strategy arises from the fact that information on a single “conventional tuple” is scattered across several physical relations. For a simple join such as  $r \bowtie_{m=n} s$ , first *physrel*( $r, m$ ) and *physrel*( $s, n$ ) are joined in the conventional way. Thereafter the other columns in the “join” tuple will have to be assembled one by one using the *tid*'s of  $r$  and  $s$  as glue. The assembled “join” tuple will also have to be assigned a new tuple id. In the algorithm below a function *newtid*( $i, j$ ) that generates a unique tid based on tid's  $i$  and  $j$  is used.

---

**Algorithm 4.23**  $r \bowtie_{m=n} s$ **begin**

*make a join on value between physrel( $r, m$ ) and physrel( $s, n$ )*

```

        producing tuples of the form  $\langle r[tid], r[val], s[tid], s[val] \rangle$ 
    for each row obtained above
        for each relation  $\text{physrel}(r, a_i)$ 
            read tuple  $t_r$  with tid  $r[tid]$ 
            write  $\langle \text{newtid}(r[tid], s[tid]), t_r[\text{value}] \rangle$  in  $\text{physrel}(\text{join}, a_i)$ 
        endfor
        for each relation  $\text{physrel}(s, a_j)$ 
            read tuple  $t_s$  with tid  $s[tid]$ 
            write  $\langle \text{newtid}(r[tid], s[tid]), t_s[\text{value}] \rangle$  in  $\text{physrel}(\text{join}, a_j)$ 
        endfor
    endfor
end

```

---

#### Cost

The cost of the first join is  $2N_t + J_d \times N_t$ . For each row produced by join, we have two *for* loops, each loop consisting of one tuple read and one tuple write, and iterating on the average  $N_a$  times. The cost for this part, then, would be  $J_d \times N_t \times 4N_a$ .

#### 4.4.4 Fetching Relation Names

Here we assume the existence of system tables (as in the “Conventional” Strategy). The cost would be  $N_r \times N_a$  tuple reads and  $N_r$  tuple writes

#### 4.4.5 Fetching Relations and their Schemas

The implementation of the operation  $\alpha_i(r)$  would also refer to the system tables. The cost can be estimated as  $N_r \times N_a$  for reading and  $N_r \times N_a$  for writing.

#### 4.4.6 Querying Data and Meta-data

In the Single Pattern query, if the attribute name is given in the pattern, we go straight to only those *physrel* relations relevant to the given relation and attribute,  $\text{physrel}(r, a)$ . Otherwise, every *physrel* relation with the given relation name has to

be queried against the pattern. And if the *value* is given in the pattern, the index on *value* can be used to speed up the operation.

---

**Algorithm 4.24**  $\gamma_{i;p}(s)$  *Querying with a single pattern*

**begin**

*Identify the relations involved by inspecting the  $i^{\text{th}}$  column of relation  $s$ .*

**for** *each relation  $r$  thus identified*

**for** *each physical relation  $\text{physrel}(r, a_i)$*

*read each tuple  $t$*

*if  $t$  matches the pattern  $p$ , form an output tuple of the form*

*$\langle r, tid, a_i, value \rangle$ , where  $r$  and  $a_i$  are taken*

*from the current  $\text{physrel}$  and  $tid, value$  are taken*

*from the current tuple  $t$*

**endfor**

**endfor**

**end**

---

### Cost

In the most general case, i.e, if the pattern is  $\langle \Rightarrow \rangle$ , the reading cost can be written as  $N_t \times N_a \times N'_r$ , where  $N'_r$  denotes the number of relations identified by the operator.

Making use of the  $P_{ds}$  factor, we can summarise the writing cost as  $N_t \times N_a \times N'_r \times P_{ds}$ .

Querying for a multiple pattern introduces similar complexities here as in the *Reduced Strategy*. Each *physrel* contains information on a single pattern only; hence, rows with the same *tid* from different *physrel*'s have to be put together before we can decide on the occurrence or otherwise of the given multiple pattern. The algorithm below outlines a general procedure applicable in all types of patterns.

---

**Algorithm 4.25**  $\gamma_{i;\langle p_1, \dots, p_n \rangle}^\wedge(s)$       *Querying with a multiple pattern*

**begin**

*Identify the relations involved by inspecting the  $i^{\text{th}}$  column of relation  $s$ .*

**for each relation  $r$  thus identified**

*select a physrel of the relation*

**for each tuple  $t$  in the physrel**

*select tuples with the same tid from the*

*remaining physrel's of the relation*

*if the  $\langle a, v \rangle$  combinations of this set of tuples*

*match the pattern  $\langle p_1, \dots, p_n \rangle$ ,*

*add to the result as many tuples as possible of the form*

*$\langle r, a_1, v_1, \dots, a_n, v_n \rangle$  using  $(a, v)$  combinations from the*

*tuple set under consideration in all ways that fit the pattern*

**endfor**

**endfor**

**end**

---

#### Cost

The reading cost is  $N_t \times N_a \times N_r'$  (same as that for the single pattern query).

If each “conventional” tuple (obtained by placing tuples with the same *tid* from *physrel*'s of the same relation side by side) is used to make at most one tuple in the output, the number of tuples written would be  $N_t \times N_r' \times P_{dm}$ , where  $P_{dm}$  is the “Pattern Density” factor for a multiple pattern. If we assume on the average that such a “conventional” tuple can be used to make  $N_a$  tuples in the output, the writing cost would be  $N_t \times N_r' \times N_a \times P_{dm}$ .

#### 4.4.7 Creating Relations

The  $\kappa$  operator in this Strategy would merely add relation names to the System Table. The cost is negligible.

## 4.4.8 Creating Relations with Schemas

---

### Algorithm 4.26 $\varsigma_{i,j}(s)$

```
begin
  while there are rows to be considered in  $s$  (the given schemaless relation)
    read a row
    store the relation name from the  $i^{\text{th}}$  column and the attribute name from
      the  $j^{\text{th}}$  column of  $s$  in an appropriate data structure
  endwhile
  for each relation name  $r$  stored
    for each attribute  $a$  in  $r$ 
      create a physical relation  $\text{physrel}(r,a)$ 
        with attributes  $\text{tid}$  and  $\text{value}$ 
    endfor
  endfor
end
```

---

### Cost

$N_r' \times N_a$  physical relations are created. If  $T_c$  is the cost of creating a table, the total cost is  $N_r' \times N_a \times T_c$ .

## 4.4.9 Creating Relations with Schema and Data

In this Strategy, too, the  $\rho$  operator has a simple implementation. We look at each row input by the given “Schemaless” relation, check whether the *physrel* corresponding to the relation and attribute exists and add the relevant tuple to the table.

---

### Algorithm 4.27 $\rho_{i,j,k;\ell}(r)$

```
begin
  for each tuple in the input relation  $r$ 
    read tuple  $t$ .
    if the table  $\text{physrel}(t[i], t[j])$  does not exist
      create the table  $\text{physrel}(t[i], t[j])$ 
      add tuple  $\langle t[\ell], t[k] \rangle$  to the relation  $\text{physrel}(t[i], t[j])$ 
    endfor
end
```



## Cost

Reading in the input relation needs  $N_r' \times N_a \times N_t$  tuple reads; and the same number of tuples are written to the various *physrel* relations. If we assume that on the average half of the  $N_r'$  relations are to be created, the cost of creating these tables would be  $\frac{1}{2}N_r' \times N_a \times T_c$ .

## 4.5 Conclusion

From long theoretical study as well as long practical usage, it is clear that the storage of data in the conventional method of tables with well-defined schemes is well-suited to the efficient implementation of the traditional Relational Algebraic Operators. Since  $\mathcal{SA}$  contains new operators both for querying as well as restructuring the database, the question is whether the conventional storage methods are still the best for the purpose of storing the data on which these operators are applied. We therefore presented in this chapter two new storage strategies – the “reduced” storage, and the “reduced, atomized” storage – which we consider might be more suitable for certain operations, especially the restructuring operations introduced in  $\mathcal{SA}$ . For all three strategies, we presented algorithms for the implementation of  $\mathcal{SA}$  operators and estimated the theoretical cost.

If we consider the *Join* operator as representative of all querying operators and the  $\rho$  operator as representative of the restructuring operators and compare their theoretical costs across the three strategies, it is immediately evident that querying costs are low in the Conventional Strategy and very high in the other two strategies. As regards restructuring, the position is reversed: costs are high in the Conventional Strategy and much lower in the other two. Comparing the theoretical costs for the other querying and restructuring operations leads to similar conclusions. In applications which have a high proportion of queries and relatively few restructuring operations, the conventional storage method would therefore appear to be the better option. Where restructuring is frequent, one of the other two methods would appear to be the better choice.

In the next chapter, we shall analyse the experimental results from the implementation of the three strategies discussed here. We shall see there whether the

theoretical results agree with the experimental results. And in the case of restructuring operations in particular, the experimental results will help us to decide which of the three strategies is the most efficient.

# Chapter 5

## Experimental Results

### 5.1 Introduction

This chapter presents a report on the practical implementation of the  $SA$  operators under the various storage strategies presented in the previous chapter. In that Chapter we also estimated the theoretical cost of the implementation of the  $SA$  operators. To supplement those theoretical estimates, we study in this chapter the actual running time of various querying and restructuring operations on databases of varying sizes and other parameters such as *Join Density* and *Selection Density*. For each set of parameters, we present a table of readings taken for the various operations under the three strategies mentioned in the preceding chapter. Graphs on the table values illustrate the comparative suitability of the strategies for specific operations. We conclude the chapter with an analysis of the results obtained.

The implementation was done in the MS-Access Database Management System on the PC/Windows platform. All source code for the creation of tables for the test bed, the working of the  $SA$  operators, etc. was written in Visual Basic.

### 5.2 Preparation of the Test Bed

The first task at the experimental stage was to prepare a set of tables (according to the three Strategies under consideration) with adjustable parameters such as the number of tuples per relation, the number of attributes per relation, the selection density of various columns and the join density between two columns. On the implementation

interface the user is prompted to supply the number of tables to be created, the number of columns in each table, the number of rows per table, the selection density of a selected column in a table and the join density between two selected columns in two tables. Making use of these parameters, the specified number of *conventional* tables are created and data generated for each table.

Once the *conventional* database is ready, the user is asked to initiate the creation of a *Reduced* database (the second storage Strategy discussed earlier). The data already generated for the *conventional* database is used to create and populate the three *call* tables in this Strategy.

The *Reduced, Atomized* Database (the third Strategy), too, is generated in like manner from the conventionally stored data.

### 5.3 Implementation of Schema Algebra operators

Six operators from each of the three Strategies under consideration were taken up for detailed coding and experimental study: three “traditional” relational algebra operators and three operators specific only to *SA*. They are:

1. Selection
2. Projection
3. Join
4. Querying with Single Pattern (this will also be referred to later in an abbreviated form as *gamsing*)
5. Querying with Multiple Pattern (sometimes abbreviated to *gamult*)
6. Creating and populating relations with schemas (abbreviated to *var-rho*)

Of these, the first five are merely querying operators which do not in any way modify the database, whereas the sixth is a restructuring operator which can create new tables, add new columns to existing tables and also insert rows(full or partial) to the tables.

The implementation interface allows the user to choose any operation under any strategy and specify the parameters for the operation, such as the table(s) to be used,

the selection or join condition to be applied, etc. Once all parameters (if required) for the operation are filled in, the user initiates the operation and the program displays the time taken (in milliseconds) for the operation to complete.

### 5.3.1 Experimental Results and Graphs

We now present the actual results of some experiments conducted on the test bed described earlier. Execution time for the various operators were recorded under varying conditions. As has been already mentioned,  $N_t$  refers to the number of rows in the table,  $J_d$  to the Join Density and  $S_d$  to the Selection Density.

$N_t = 500; J_d = 0.25; S_d = 3$			
Operation	Time(S1)	Time(S2)	Time(S3)
Selection	934	1263	2307
Projection	879	1043	1318
Join	1043	10239	13193
Gamsing	1813	1263	2142
Gamult	2142	3186	1648
Var-rho	16066	7307	14499

Table 5.1: Execution time with table size of 500, join density 0.25

Table 5.1 shows the results of running the six operations on two tables of 500 tuples and 3 columns each, with a selection density of 3 for the column referred to in the selection condition and a join density of 0.25 for the columns referred to in the join condition.  $S1$  refers to the “conventional” strategy,  $S2$  to the “reduced” strategy and  $S3$  to the “reduced, atomized” strategy. All times are given in milliseconds.

The tables which follow, ie, Tables 5.2 to 5.10 show the results when tuple numbers and join densities are varied.

After each table, we present two graphs to illustrate the time variations involved in the operations. The first graph, titled *Individual Operations* plots the six operations against the time taken, for each of the three storage strategies studied. The second graph is titled *Mix of Operations*, and the motivation behind offering this view is as follows. Processing a *SchemaLog* program typically consists of a mixture of various operations from  $\mathcal{SA}$ . To simulate this scenario, we have manually combined

the results of our timing experiments in varying proportions. The mixes we have investigated are:

1. 100% querying operations
2. 75% querying and 25% restructuring operations
3. 50% querying and 50% restructuring operations
4. 25% querying and 75% restructuring operations
5. 100% restructuring operations

Within each of the above combinations, the load for querying is shared equally among the different querying operations of  $\mathcal{SA}$  and the load for restructuring is distributed likewise. Among the six operators we have studied, the first five are strictly querying operators and the last alone is a restructuring operator. Then, a mixture of *75% querying and 25% restructuring operations* would mean that each of *Selection*, *Projection*, *Join*, *Gamsing* and *Gamult* would be allocated 15% of the work load and *Var-Rho* would be given 25%. It is our belief that such a study would better reveal the performances of the various Strategies than specifically chosen *SchemaLog* programs would.

### **Graph Labels**

In all the graphs that follow:

the values on the Y-axis denote the time, in milliseconds

the values on Z-axis denote the three storage Strategies, viz:

$S1 =$  Conventional Strategy

$S2 =$  Reduced Strategy

$S3 =$  Reduced, Atomized Strategy

In graphs titled “Individual Operations”, the values on the X-axis refer to the six operators, viz:

$Op1 =$  Selection

$Op2 =$  Projection

$Op3 =$  Join

- Op4* = Gamsing (Single Pattern Querying)
- Op5* = Gamult (Multiple Pattern Querying)
- Op6* = Var-rho (Creating relations, schemas; adding data)

In graphs titled “Mix of Operations”, the values on the X-axis refer to the five different mixes of operations, viz;

- M1: 100% querying operations
- M2: 75% querying and 25% restructuring operations
- M3: 50% querying and 50% restructuring operations
- M4: 25% querying and 75% restructuring operations
- M5: 100% restructuring operations

The two graphs for Table 5.1 are in figures 5.1 and 5.2.

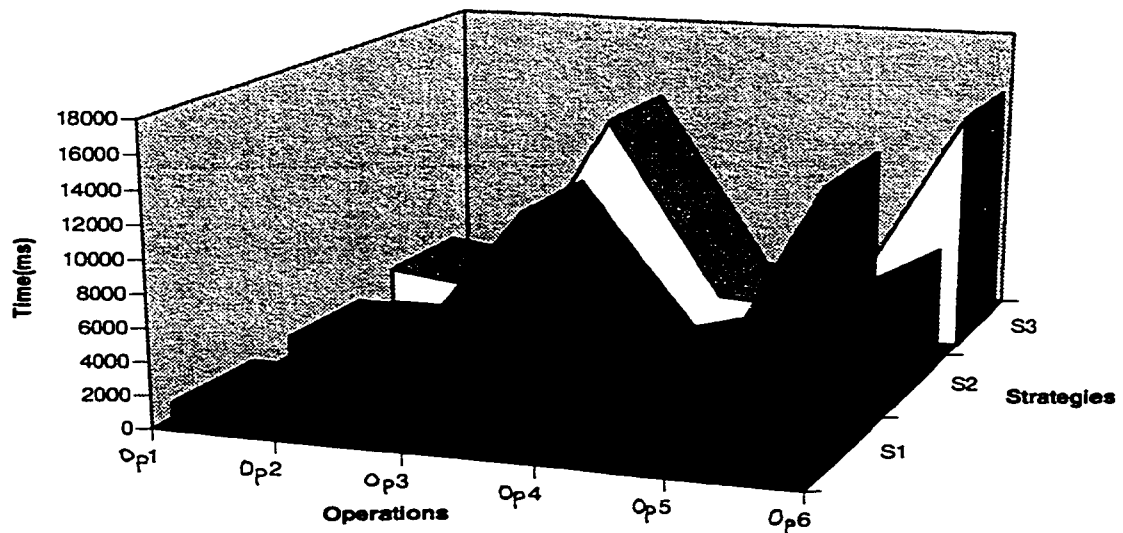


Figure 5.1: Individual Operations (from Table 5.1)

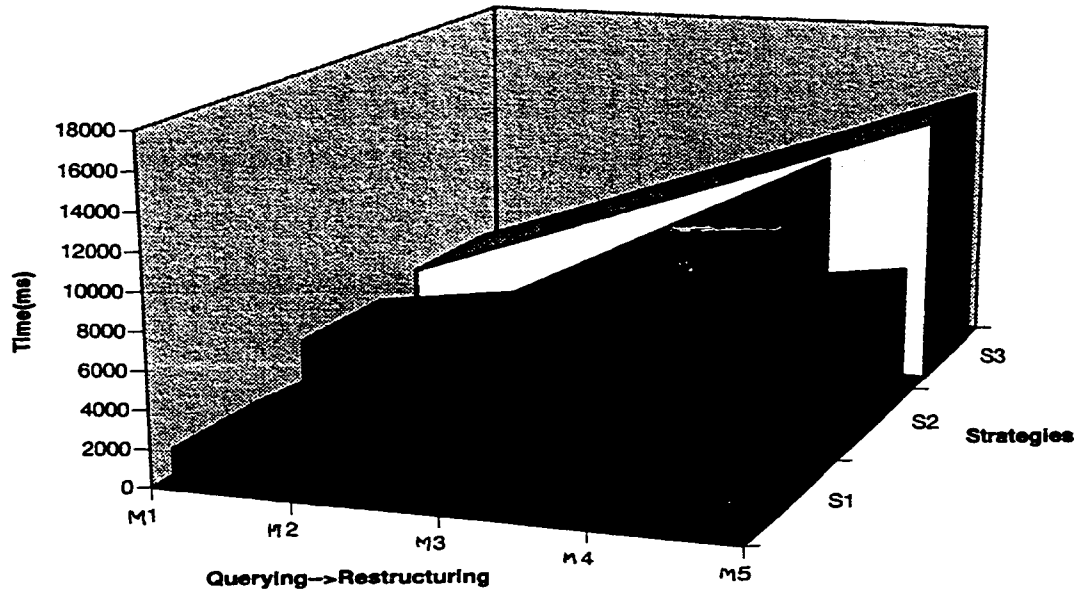


Figure 5.2: Mix of Operations (from Table 5.1)

$N_t = 1000; J_d = 0.25; S_d = 3$			
Operation	Time(S1)	Time(S2)	Time(S3)
Selection	934	1208	2142
Projection	989	1099	1648
Join	1154	19528	26711
Gamsing	1812	1043	2307
Gamult	2527	3405	1648
Var-rho	29330	13021	27684

Table 5.2: Execution time with table size of 1000, join density 0.25



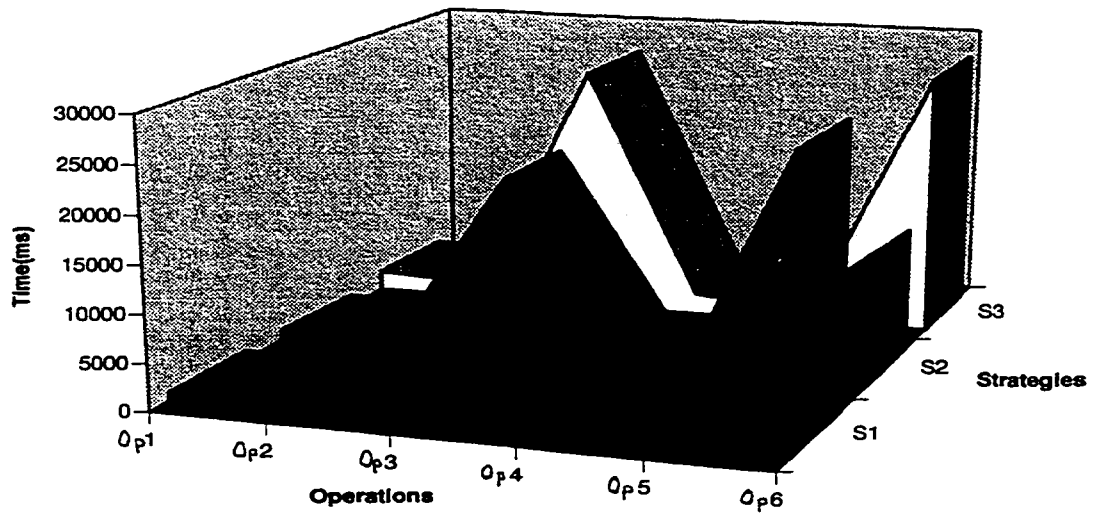


Figure 5.3: Individual Operations (from Table 5.2)

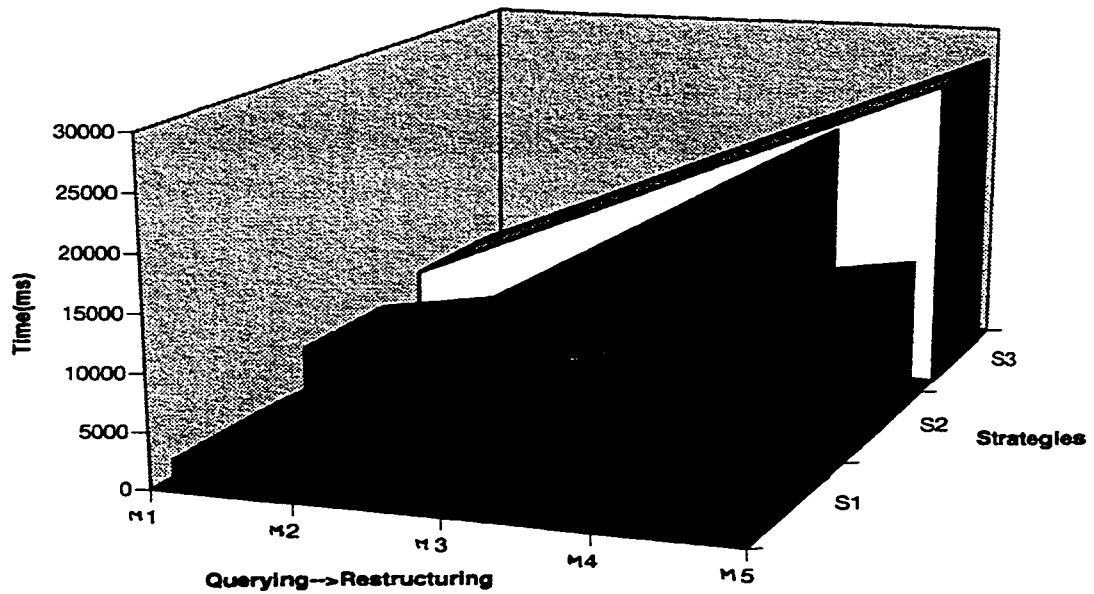


Figure 5.4: Mix of Operations (from Table 5.2)

$N_t = 2500; J_d = 0.25; S_d = 3$			
Operation	Time(S1)	Time(S2)	Time(S3)
Selection	934	1209	2417
Projection	1318	1373	3594
Join	1538	60642	63741
Gamsing	2802	1098	2032
Gamult	3515	3845	1813
Var-rho	67887	40893	80395

Table 5.3: Execution time with table size of 2500, join density 0.25

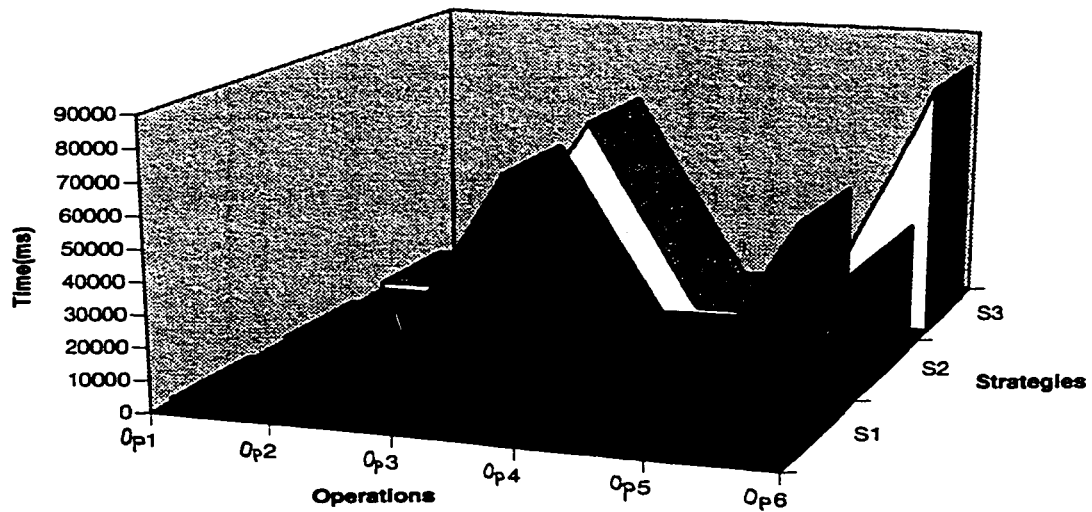


Figure 5.5: Individual Operations (from Table 5.3)

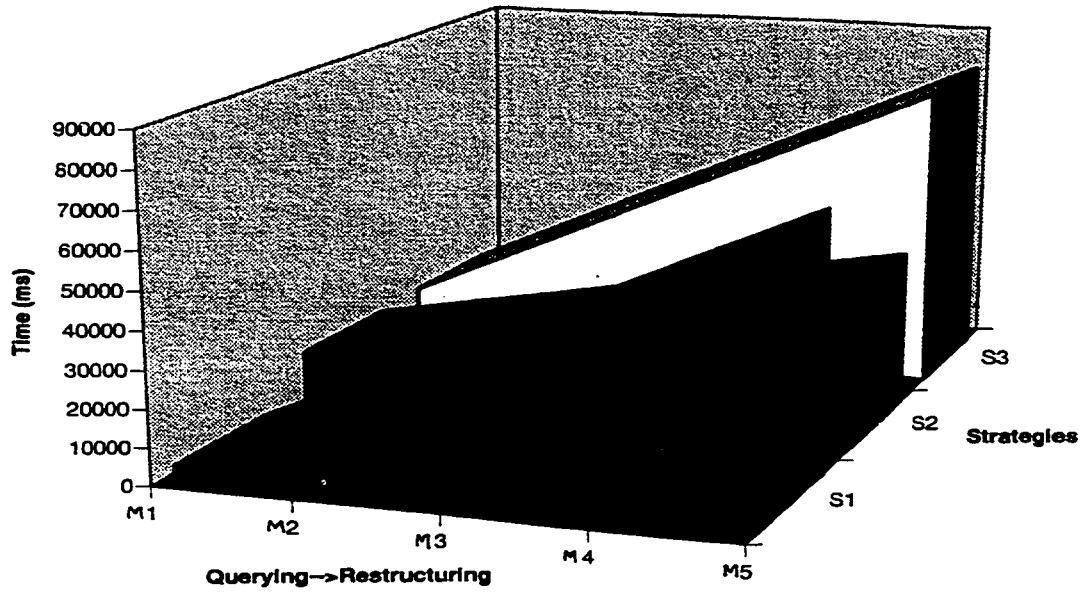


Figure 5.6: Mix of Operations (from Table 5.3)

$N_t = 5000; J_d = 0.25; S_d = 3$			
Operation	Time(S1)	Time(S2)	Time(S3)
Selection	934	1374	2197
Projection	2417	1483	6956
Join	1978	111752	115703
Gamsing	4339	1153	1978
Gamult	5273	3570	1757
Var-rho	120767	69208	164790

Table 5.4: Execution time with table size of 5000, join density 0.25

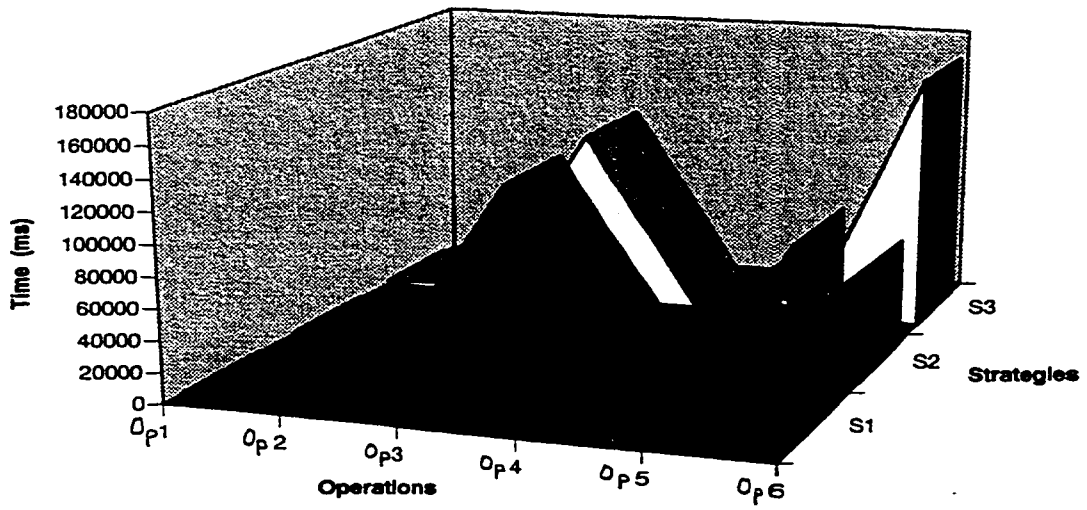


Figure 5.7: Individual Operations (from Table 5.4)

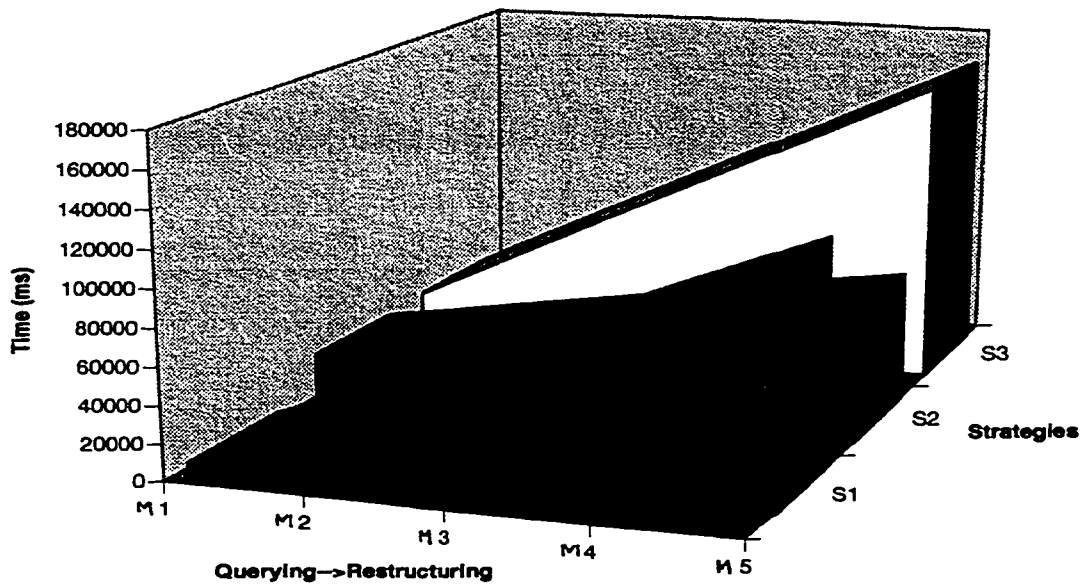


Figure 5.8: Mix of Operations (from Table 5.4)

$N_t = 10000; J_d = 0.25; S_d = 3$			
Operation	Time(S1)	Time(S2)	Time(S3)
Selection	1977	1977	2526
Projection	6316	3515	25114
Join	7634	249817	236131
Gamsing	7634	1208	2033
Gamult	8238	4064	2142
Var-rho	271672	182963	411960

Table 5.5: Execution time with table size of 10000, join density 0.25

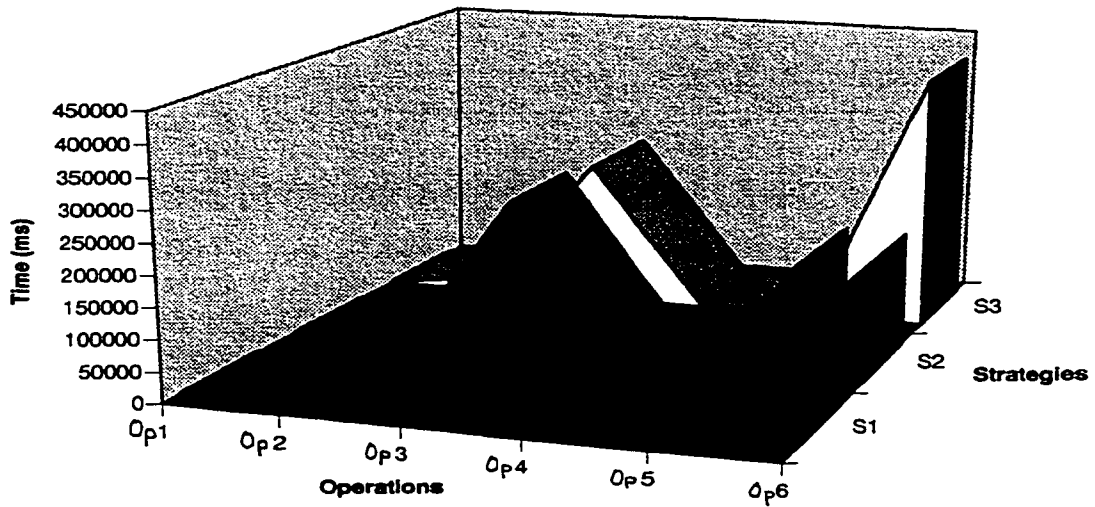


Figure 5.9: Individual Operations (from Table 5.5)

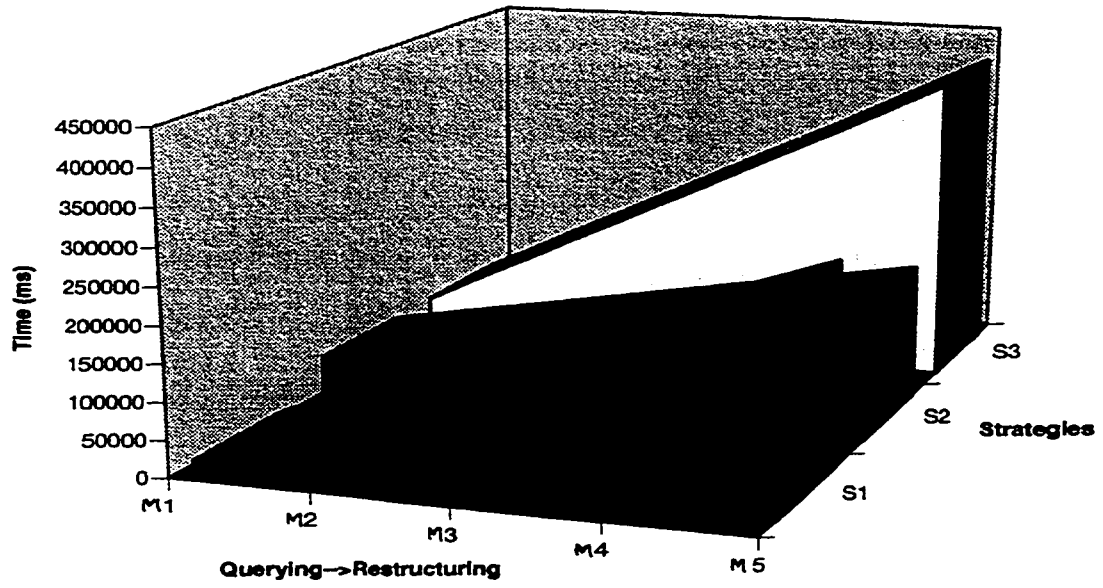


Figure 5.10: Mix of Operations (from Table 5.5)

$N_t = 500; J_d = 0.5; S_d = 3$			
Operation	Time(S1)	Time(S2)	Time(S3)
Selection	933	1208	2252
Projection	879	1154	1373
Join	1044	18222	25680
Gamsing	1647	1098	1977
Gamult	2362	3185	1868
Var-rho	15626	7469	14172

Table 5.6: Execution time with table size of 500, join density 0.5

$N_t = 1000; J_d = 0.5; S_d = 3$			
Operation	Time(S1)	Time(S2)	Time(S3)
Selection	989	1373	2526
Projection	1098	1209	1758
Join	1153	38117	50258
Gamsing	2252	1098	2362
Gamult	2856	3295	1867
Var-rho	27464	12906	28338

Table 5.7: Execution time with table size of 1000, join density 0.5

$N_t = 2500; J_d = 0.5; S_d = 3$			
Operation	Time(S1)	Time(S2)	Time(S3)
Selection	879	1373	2527
Projection	1373	1538	3805
Join	1868	110357	118990
Gamsing	3240	1099	2307
Gamult	3954	3680	2087
Var-rho	67035	29504	79095

Table 5.8: Execution time with table size of 2500, join density 0.5

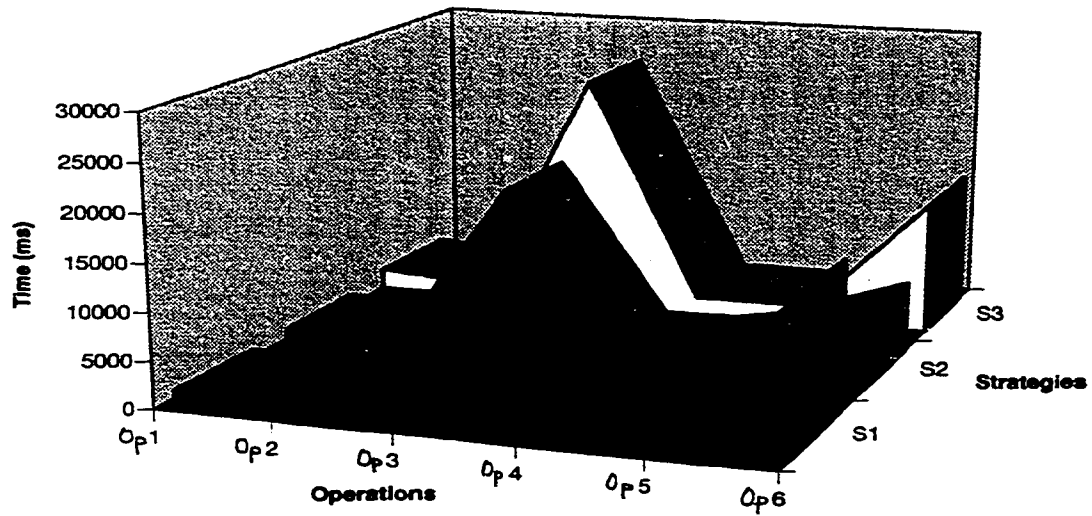


Figure 5.11: Individual Operations (from Table 5.6)

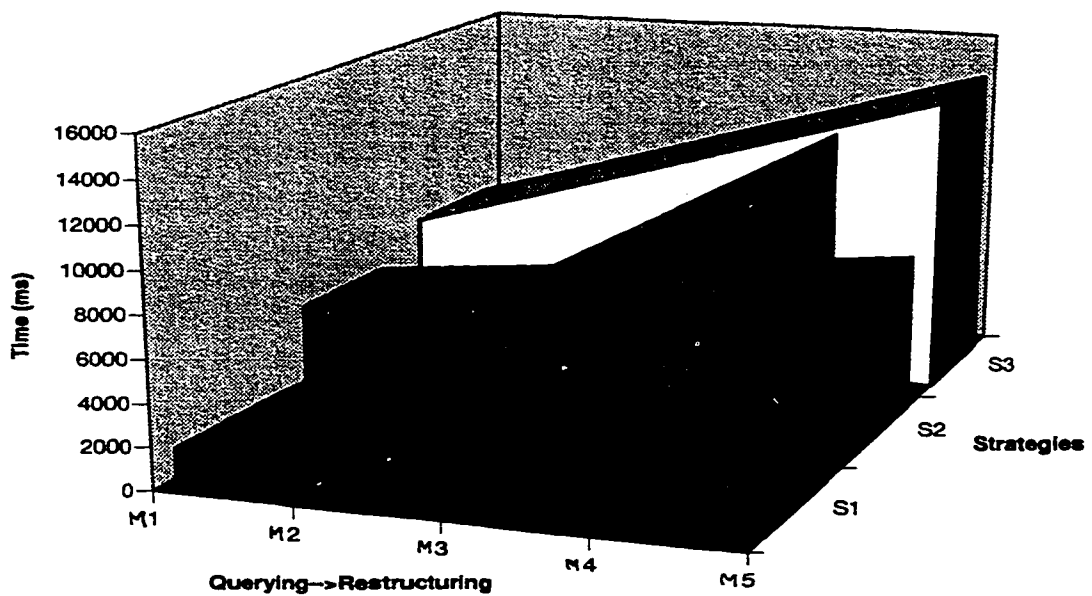


Figure 5.12: Mix of Operations (from Table 5.6)



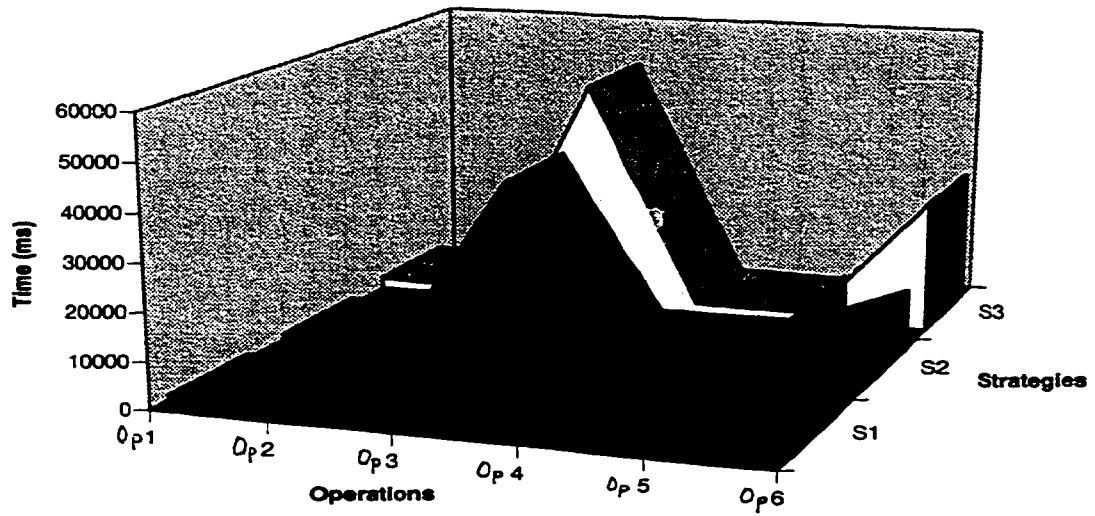


Figure 5.13: Individual Operations (from Table 5.7)

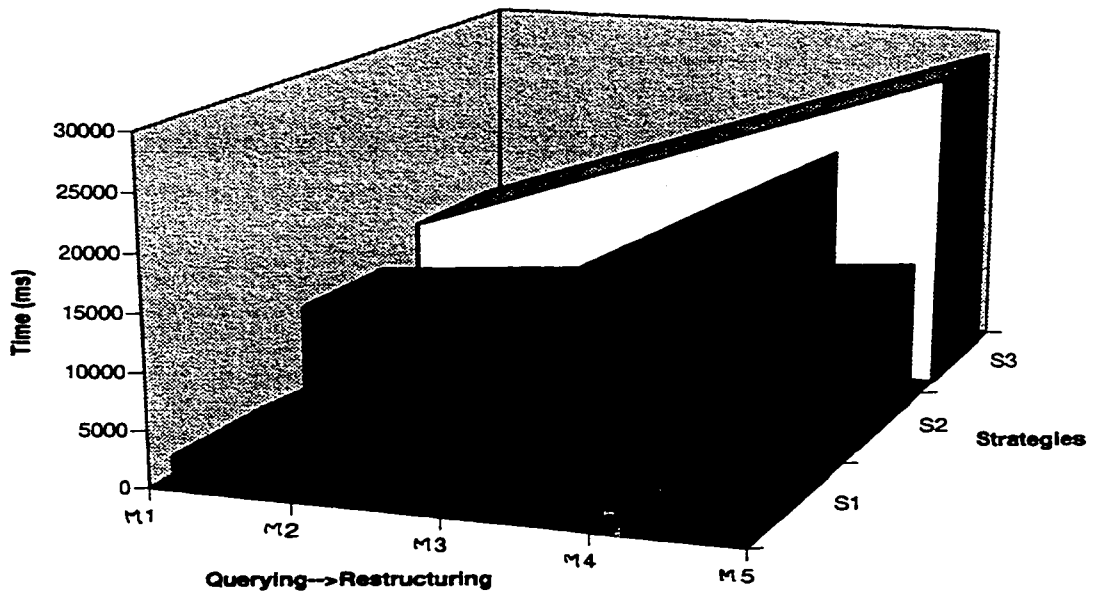


Figure 5.14: Mix of Operations (from Table 5.7)

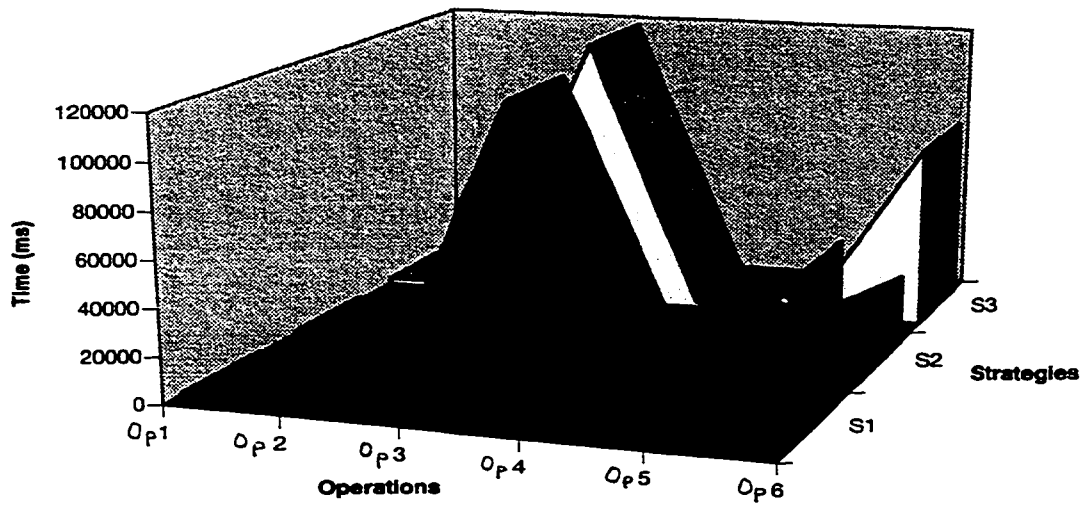


Figure 5.15: Individual Operations (from Table 5.8)

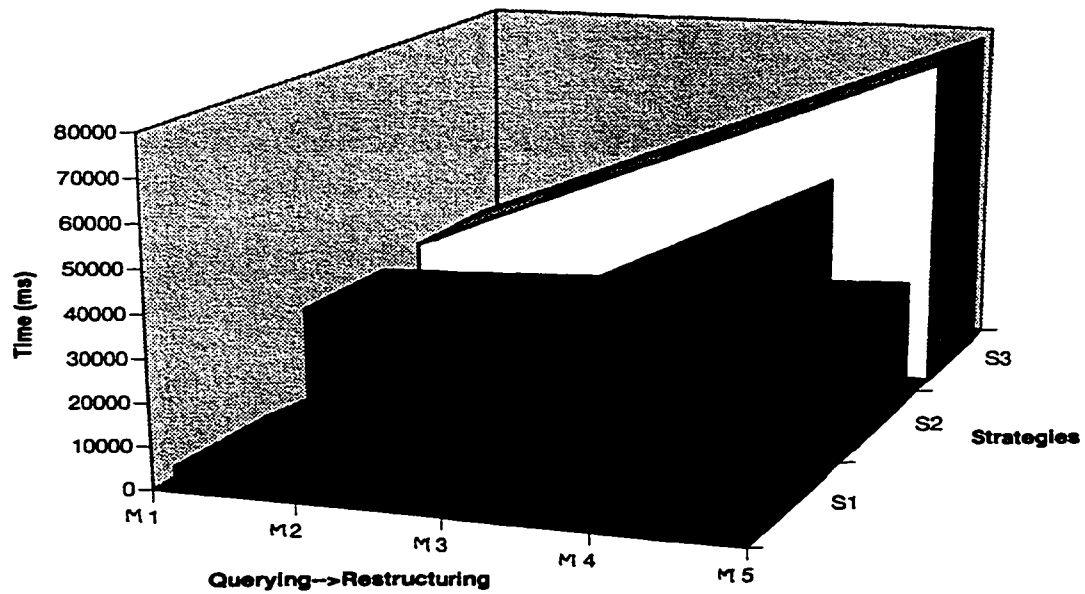


Figure 5.16: Mix of Operations (from Table 5.8)

$N_t = 5000; J_d = 0.5; S_d = 3$			
Operation	Time(S1)	Time(S2)	Time(S3)
Selection	988	1483	2362
Projection	1813	1702	6840
Join	2033	229932	234200
Gamsing	5053	1044	2472
Gamult	5657	4099	2087
Var-rho	125956	73993	173010

Table 5.9: Execution time with table size of 5000, join density 0.5

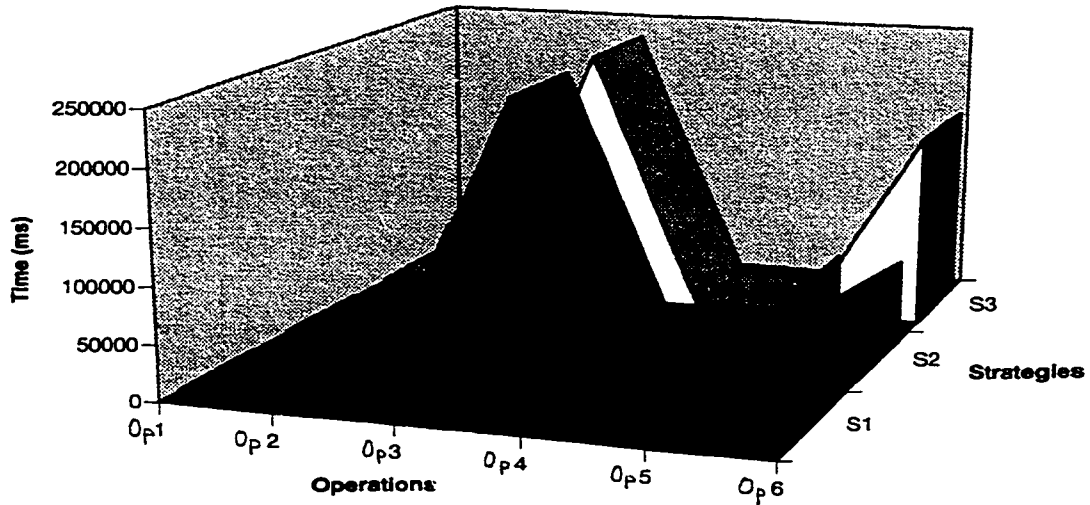


Figure 5.17: Individual Operations (from Table 5.9)

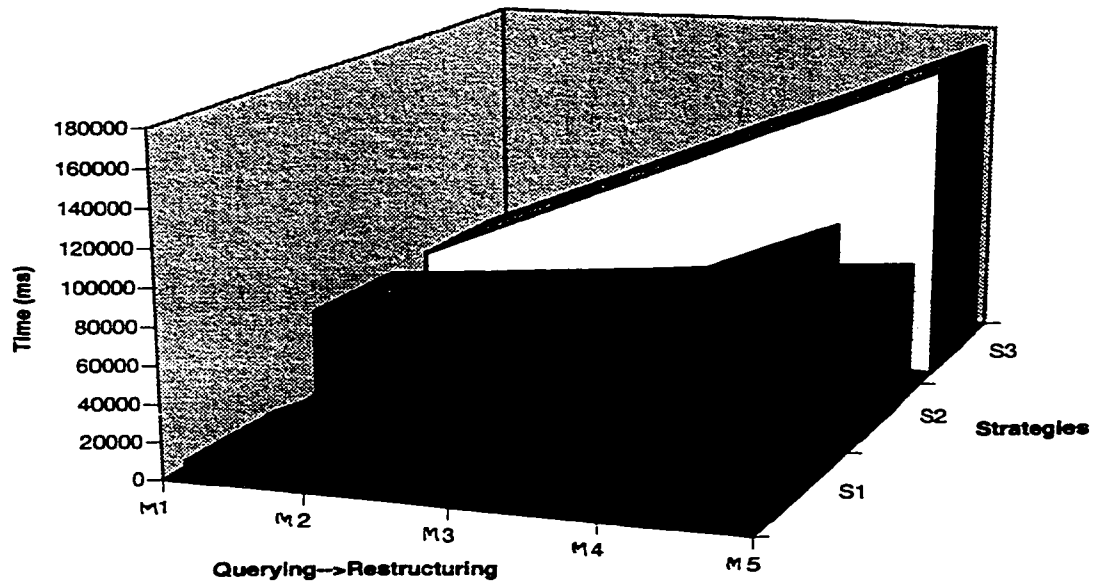


Figure 5.18: Mix of Operations (from Table 5.9)

$N_t = 10000; J_d = 0.5; S_d = 3$			
Operation	Time(S1)	Time(S2)	Time(S3)
Selection	988	1923	2362
Projection	6811	3240	25749
Join	8624	471871	494273
Gamsing	8184	1154	2581
Gamult	8513	4339	1923
Var-rho	262709	212662	418500

Table 5.10: Execution time with table size of 10000, join density 0.5

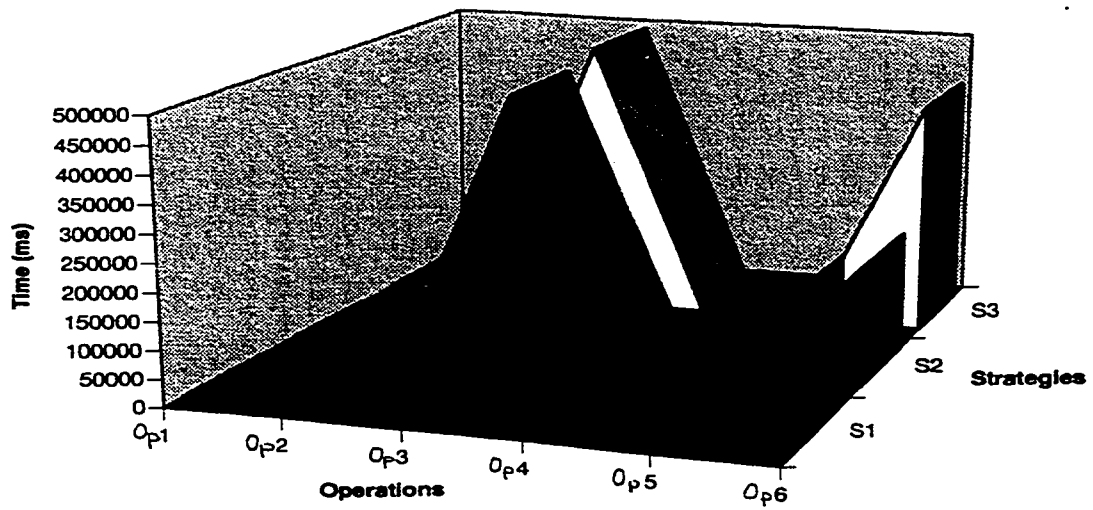


Figure 5.19: Individual Operations (from Table 5.10)

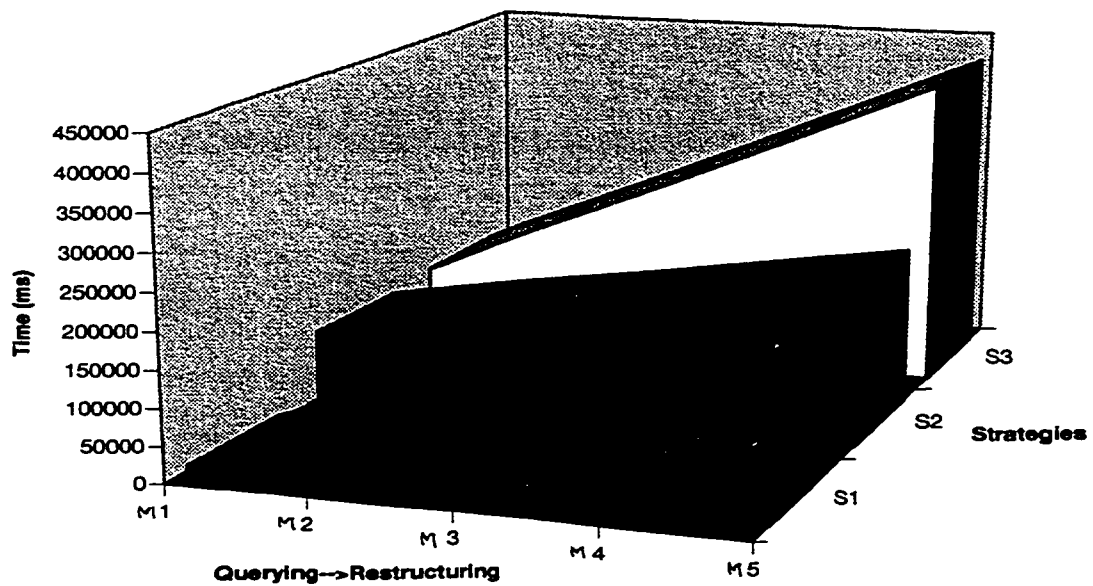


Figure 5.20: Mix of Operations (from Table 5.10)

## 5.4 Analysis of Results

### 5.4.1 Individual Operations

The various graphs where the performance of the various operators is plotted against time (under the three Strategies) show great similarities. Refer to graphs in Figures 5.1 , 5.3 , 5.5 , 5.7 , 5.9, 5.11 , 5.13 , 5.15 , 5.17 and 5.19. It is quite clear that for the purely querying operations (operators 1 to 5 in the graphs), the *Conventional* storage strategy is decidedly superior. The other two Strategies show comparable performances for all the querying operators but the *Join*. For the *Reduced* Strategy, the peak in the graph for the *Join* can be attributed to the fact that the join is made only on a fragment of the original *conventional* tuple and the full tuples have to be reassembled once the results of the join are known.

To illustrate, consider the data in the conventional tables given in Figure 4.1. The first table, *person* contains the tuple  $\langle \text{John}, 35, \text{Montreal} \rangle$  and the second, *works\_in*, contains the tuple  $\langle \text{John}, \text{CIBC} \rangle$ . In the *Reduced* form these two “conventional” tuples would produce the following five tuples:

```
<person,t1,name,John>
<person,t1,age,35>
<person,t1,city,Montreal>
<works_in,t2,name,John>
<works_in,t2,company,CIBC>
```

Now if a join of *person* and *works\_on* on the column *name* were required, the “conventional” join would produce the single tuple  $\langle \text{John}, 35, \text{Montreal}, \text{CIBC} \rangle$ . In the *Reduced* strategy, the joinability of  $t_1$  and  $t_2$  would have to be determined by looking for a match on *name* (here,  $\langle \text{person}, t_1, \text{name}, \text{John} \rangle$  and  $\langle \text{works\_in}, t_2, \text{name}, \text{John} \rangle$ ). A new tuple id (as well as a new table name) has then to be assigned to all tuples with the existing id’s of  $t_1$  and  $t_2$  (to indicate that they can be reconstituted into just one tuple in “conventional” form). In our case, the *reduced* tuples after the join would be:

```
<newtable,t3,name,John>
<newtable,t3,age,35>
```

<newtable,t<sub>3</sub>,city,Montreal>

<newtable,t<sub>3</sub>,company,CIBC>

A similar explanation can be given for the peak in the join plot for the *Reduced, atomized* storage. Here, too, the joinable tuples are determined, their *tid*'s established, and the joined tuples assembled using the *tid*.

As regards restructuring operations (operator 6 in the graphs), the cost in the *Conventional* Strategy is expectedly high. The operation involves such costly elements as changing table schema, creating new tables, etc. And again, as expected, the *Reduced* Strategy has a low cost for restructuring. There are no new tables created, nor is the existing schema changed. New tuples are merely added to existing tables. The peak for restructuring in the plot for the *Reduced, atomized* storage may be surprising, at first sight. True, some new tables may need to be created when attributes are added to existing schema, but on the whole here the operation involves adding rows to existing tables. On closer examination of the practical effects of the restructuring, we, however, see that the various tables being created/modified have to be successively opened and closed as tuples from the input "schemaless" relation are being processed. Opening/closing of data sets is an operation that does incur some overhead. This cost is essentially responsible for the observed peak for this Strategy.

## 5.4.2 Mix of Operations

Figures 5.2 , 5.4 , 5.6 , 5.8 , 5.10, 5.12 , 5.14 , 5.16 , 5.18 and 5.20 show the cost of a program that has a mix of operations (as explained earlier in this chapter). We see that in an application used predominantly for querying operations, the *conventional* storage strategy is superior. Querying costs for the reduced storage strategy are somewhat higher. But as we move towards a greater mix of restructuring operations, we see that *the cost for conventional storage strategy (as also for the Reduced, atomized strategy) keeps on increasing sharply whereas the cost for the reduced strategy remains fairly constant*. In a *SchemaLog* application that has typically more restructuring operations than querying operations, the preferred strategy should then be the *Reduced* strategy. A mixture of the two strategies, in which the *Conventional* storage is used for base relations and the *Reduced* strategy for derived relations would appear to be appropriate. In this context it is worth noting that many emerging database applications such as "Online Analytical Processing" (OLAP) technology ([CCS95])

have to deal with a considerable amount of restructuring operations.

### 5.4.3 Effect of table size on cost of SA operations

Yet another interesting use of the time data in the experimental tables given earlier is to study the cost of the individual operations (in the various Strategies) with regard to the size of the database tables. In our experiments, the size of the tables was varied from 500 tuples to 10,000 tuples. There follow now the graphs corresponding to the six operators studied.

#### Graph Labels

The labels on the Y-axis refer to time in milliseconds.

The labels on the X-axis refer to the size of a table

1 = 500 tuples

2 = 1000 tuples

3 = 2500 tuples

4 = 5000 tuples

5 = 10000 tuples

S1 refers to the *Conventional* Strategy

S2 refers to the *Reduced* Strategy

S3 refers to the *Reduced, Atomized* Strategy



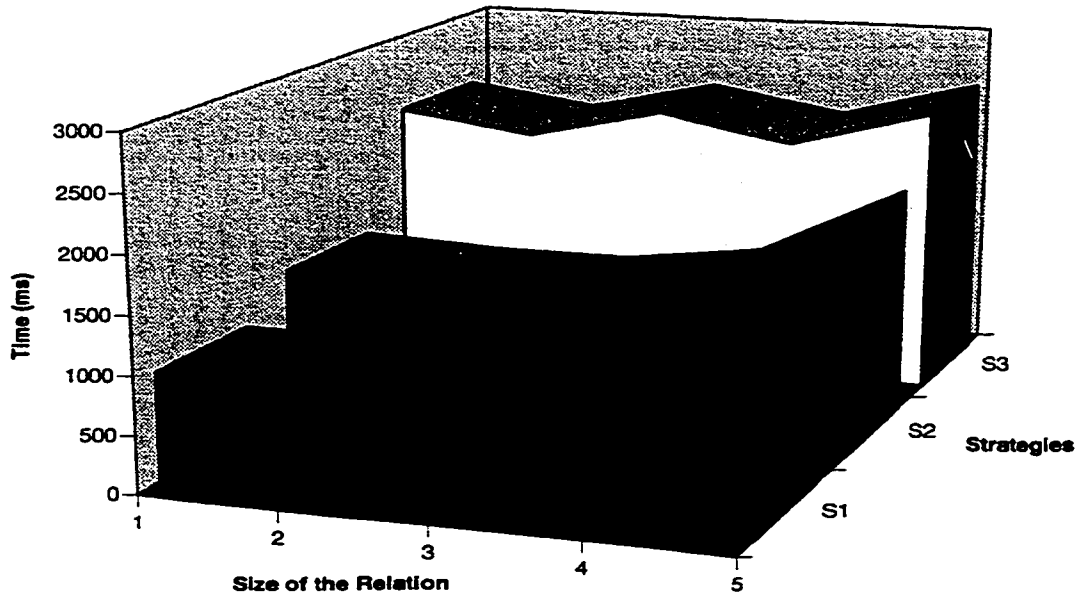


Figure 5.21: Selection

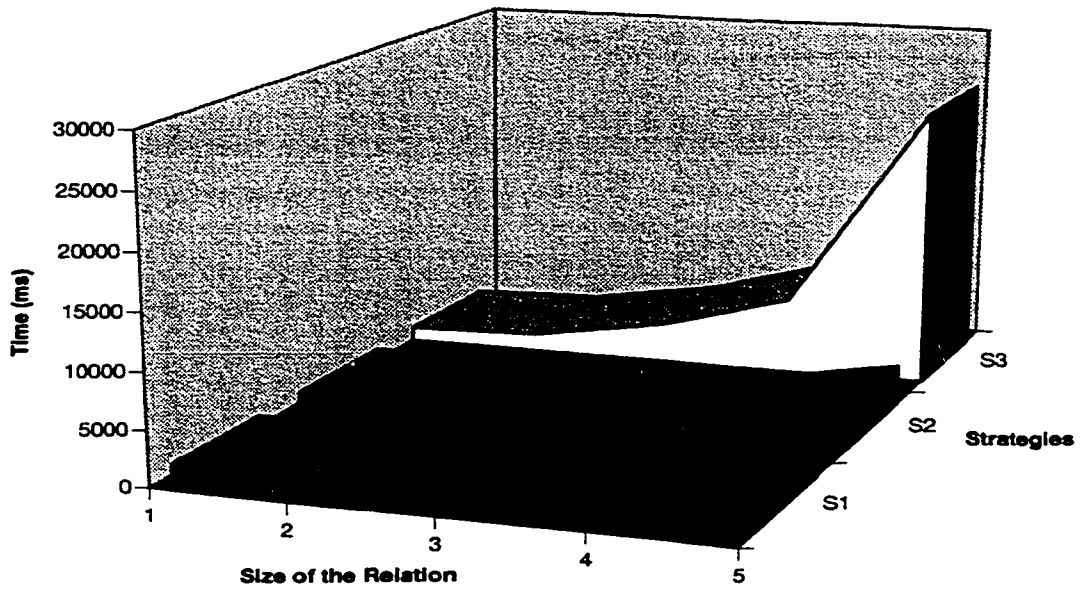


Figure 5.22: Projection

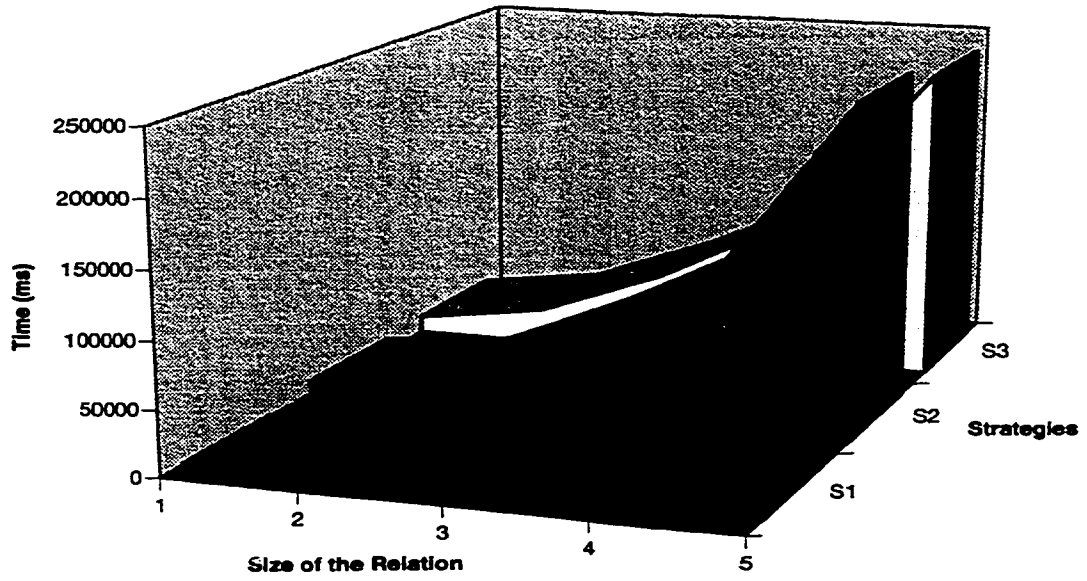


Figure 5.23: Join

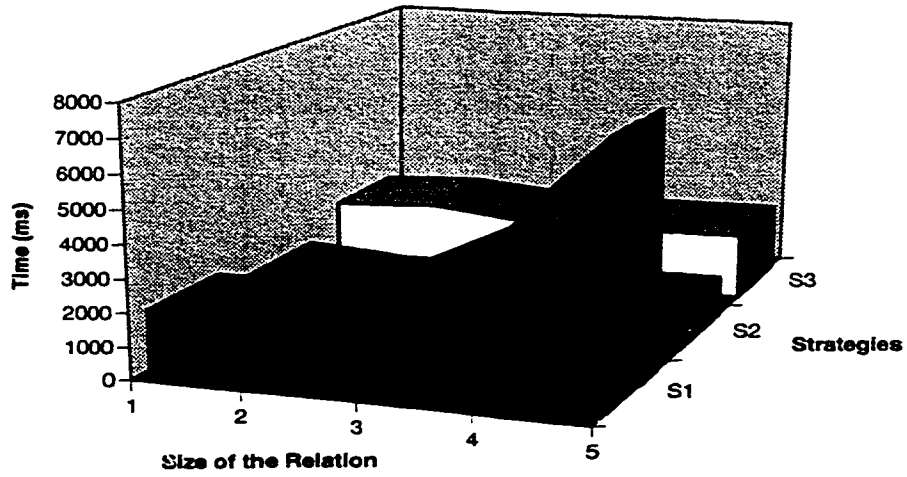


Figure 5.24: Single Pattern Querying (gamsing)

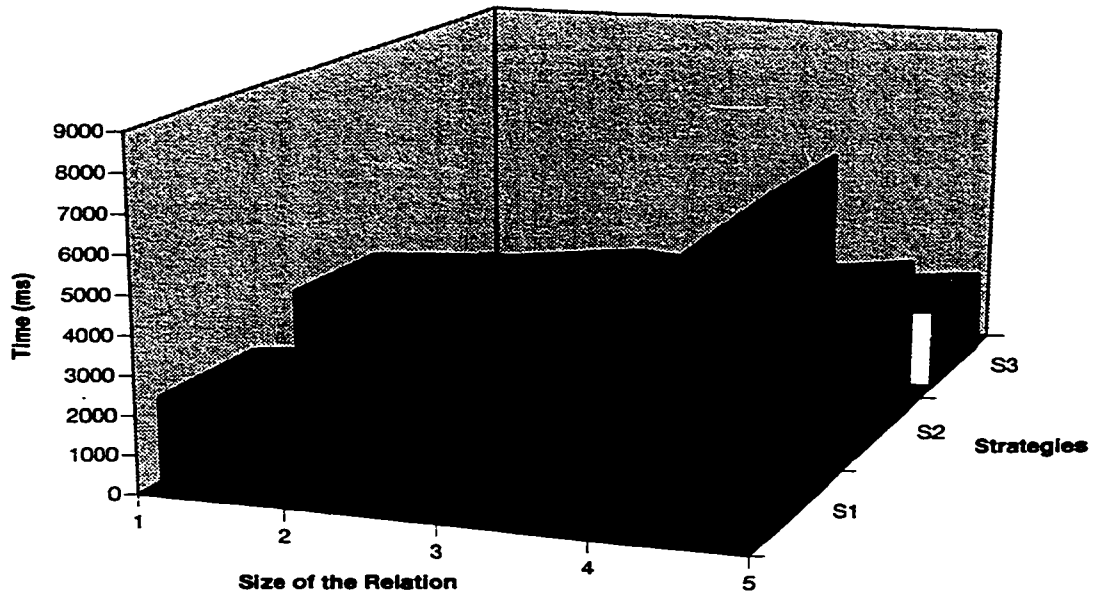


Figure 5.25: Multiple Pattern Querying (gamult)

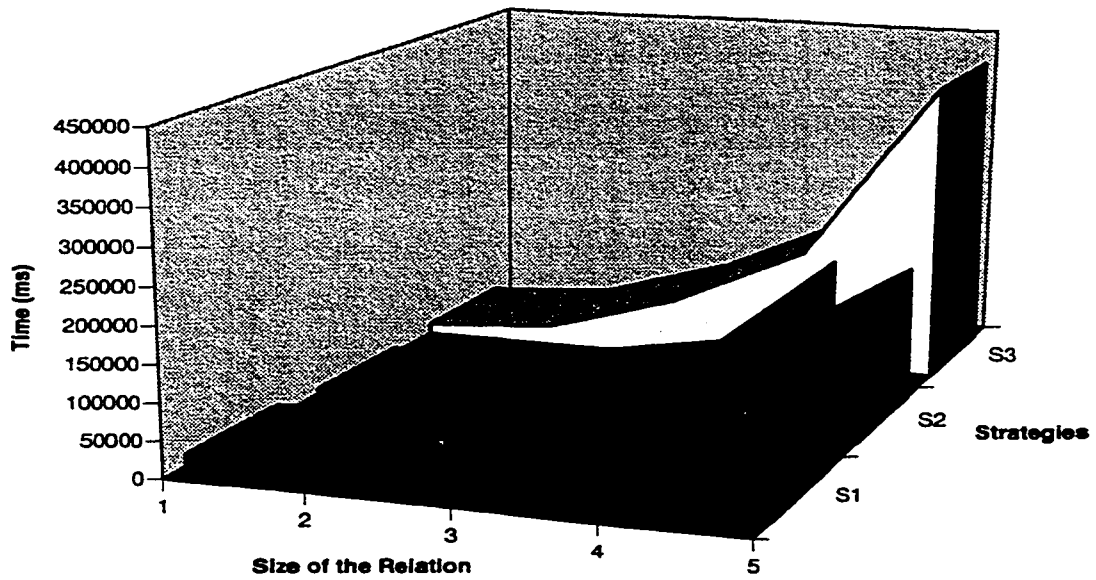


Figure 5.26: Creating Tables, Schema; Adding Rows (Var-Rho)

Some points worth noting after a study of the graphs are:

1. In the *Conventional* Strategy, the cost of *Selection* and *Join* remain fairly constant, irrespective of the size of the tables. This can be explained by the fact that these operations work with only the “selected” and the “joinable” tuples which are directly available from indexes, and exhaustive reads through the tables are not required.
2. In the other two strategies, *selection* costs are fairly constant, but at a considerably higher level than for the *Conventional* strategy. But *Join* costs rise quite dramatically as the table size is increased. For any considerably-sized database a join in the two non-conventional strategies is an expensive operation.
3. Restructuring costs rise with increase in table size, in all three strategies. But the rise is least steep in the *Reduced Strategy*.

## 5.5 Conclusion

This Chapter presented the results of the experiments we conducted in implementing *SA* operators under the three storage strategies in Chapter 4. The results comparing the performances were tabulated in detail for varying conditions such as table size, join density and selection density. Performances of the various strategies for a mix of different operations (as likely to occur in a real-life scenario) were also illustrated graphically. Another angle studied was the comparative performance of various operations when table size was varied. The chapter concluded with a summary the findings of the experiments. The most significant of these is that in conventional querying operations, the “conventional” storage strategy is decidedly superior and must be retained as the strategy of choice. But when restructuring operations are involved, the “reduced” strategy has a clear edge over the others. And since modern applications tend to have an increasing proportion of restructuring operations, this strategy must find a place in an efficiency-oriented implementation.

# Chapter 6

## Conclusion

In this Chapter, we compare our work to some similar ones done in the field. We then summarise the principal contributions of this thesis and conclude with mentioning the possibilities for future work in the area.

### 6.1 Comparison

Here, we shall compare our work with similar work related to the implementation of other higher-order logic database languages. We specifically consider four such implementations:

1. Implementation of F-logic [KLW95], one of the most comprehensive logical accounts for the object-oriented data model.
2. Implementation of Gulog, an object-oriented logic developed at Griffith University, Australia [Dob95].
3. Implementation of HiLog, a higher-order database logic programming language developed at SUNY, Stony Brook [CKW89].
4. Implementation of F-logic in FLORID, developed at the University of Freiburg, Germany.

Our comparison will deal with the implementations rather than the language features. For a comparison of the languages, interested readers are referred to [LSS96].

In [Law93], Lawley describes the implementation of an interpreter for F-logic. This is achieved by translating the F-logic syntax into an appropriate representation in NU-Prolog, Quintus, Eclipse or XSB Prolog. As described by the author, the goal of this implementation was simplicity – indeed the interpreter is just a few lines of meta-programming code – rather than efficiency. While this is useful for running small F-logic programs, it is not clear how this can be used in a real database context.

In [Lef93] Lefebvre describes an implementation of Gulog, as a declarative query language for a deductive object-oriented database, with F-logic acting as an “application programming language”. This implementation bootstraps on the implementation of the F-logic interpreter above and inherits its limitations.

Sagonas and Warren [SW95] describe an efficient implementation of HiLog within the WAM (Warren Abstract Machine) framework. Their idea is based on using a first-order translation of HiLog different from the one used for the proof of first-order semantics by the authors of HiLog. HiLog runs on top of XSB Prolog and fully exploits the run time optimization of XSB Prolog. HiLog can, in fact, be implemented thus in any Prolog system simply by changing its input/output predicates to support terms that are expressed using higher-order syntax. The authors show experimentally that Hilog programs that do not use any higher-order features execute at the same speed as Prolog programs and that generic Hilog predicates, when compiled using their special compilation scheme, execute at least an order of magnitude faster than generic Prolog predicates.

This implementation again, though efficient, runs a HiLog program ultimately as a Prolog program. The authors, too, view their work as a compile time program specialisation pre-processing step. There is no attempt at a straight-forward translation of the higher-order Hilog syntax into a correspondingly expressive procedural language.

FLORID, [FHKS97], developed at the Universities of Mannheim and Freiburg, is described by its authors as a prototype environment in which the practical programming aspects of F-logic can be tested. It is claimed that, in contrast to the Gulog implementation, nearly all the distinctive features of F-logic have been realized in FLORID. The prototype supports such aspects of the object-oriented model as multiple, non-monotonic inheritance. The evaluation strategy is bottom-up, using an extension of *Datalog* methods. The authors themselves state that efficiency was not

the goal of this implementation, but rather the demonstration of all the features of the language of F-logic.

We remark here that, while useful as a testing medium for the extensive features of the F-logic language, FLORID does not seem, in its present form, to be suitable in the area of large databases.

In contrast with all the above implementations, our implementation of *SchemaLog* has the following unique features:

1. It is not based on translation into any other language like Prolog or encoding into a lower-order syntax. Rather, our implementation is direct and follows a straightforward translation of *SchemaLog* rules into a corresponding sequence of procedural *SA* operations which, we have shown, can be implemented under various physical storage architectures.
2. Schema Algebra is at the core of our implementation. This is especially suited for set-oriented processing which is more appropriate for a database context as opposed to a purely logic programming context.
3. To our knowledge, issues like meta-data querying and piecemeal computation have not been dealt with in previous implementations.
4. One of our main objectives was efficiency of implementation in a large database context. To this end, we proposed alternative strategies for storage and handling of data, and evaluated their effectiveness with a series of experiments.

## 6.2 Summary

This thesis forms part of an on-going project on the full implementation of *SchemaLog*, an advanced database programming language. *SchemaLog*, with its higher order logical syntax, has the capability of performing a variety of tasks that are essential in the current state of information technology. We therefore began this dissertation with an introduction to the basic theory of *SchemaLog*, its syntax and semantics. We followed it up with detailed definitions and illustrations of the operators of Schema Algebra. Since our implementation of *SchemaLog* was restricted to the single database context,

relevant modifications were made to the original definitions of *SchemaLog* syntax and of  $\mathcal{SA}$  operators. It is upon these  $\mathcal{SA}$  operators that our implementations are based.

The rules in a *SchemaLog* program can, like all logic language programs, be evaluated top-down or bottom-up. We have chosen a top-down implementation. And since we are working with databases expecting a set of tuples as answers to queries, we chose the Rule/Goal Tree evaluation Method for the *SchemaLog* rules. We presented a complete set of algorithms to deal with the RGT evaluation process as applied to *SchemaLog* programs.

Realising that some of *SchemaLog*'s novel constructs, which led to the definition of some novel operators in the extended relational algebra called  $\mathcal{SA}$ , would need some novel methods of physical storage of data, we proposed three different storage strategies. One of these is an extension of the conventional method, but the other two were devised with particular attention to *SchemaLog* requirements such as restructuring and piecemeal computation of tuples. We described the three strategies and presented detailed algorithms for implementing the  $\mathcal{SA}$  operators in all strategies, as well as theoretical cost estimates for the implementation.

We followed this up with the presentation of the results of some extensive experimental tests conducted on the  $\mathcal{SA}$  operators under the various strategies. From practical considerations and from the results of the experiments, we can recommend that the most efficient way to implement *SchemaLog* programs would be to use the "conventional" storage for existing database relations, and the "reduced" strategy for the derived database relations.

### 6.3 Future Work

As we have stated on several occasions during the course of this thesis, this implementation of *SchemaLog* has been restricted to the single database context. Hence much work remains to be done in order to achieve a full implementation of the many aspects of this powerful programming language. An implementation of *SchemaLog* for multi-database interoperability among a federation of INGRES databases has been done and is described in [LSPS95].

As noted in [Sub97],  $\mathcal{SA}$  is not a sufficiently powerful language to express every program in the full-fledged *SchemaLog* language. As it stands, not all *SchemaLog*



programs can be translated into an equivalent  $SA$  expression. When  $SA$  has been sufficiently developed to this purpose, work needs to be done on an implementation that converts any given *SchemaLog* program to an equivalent  $SA$  expression and sends it on for evaluation. In our work, we have supplied algorithms for a top-down implementation and recommended physical storage structures that can efficiently hold permanent and temporary tables during evaluation. There is as yet no single implemented system that is able to achieve all of the above at the same time.

We have proposed a top-down method of evaluating *SchemaLog* programs by adapting the classical RGT approach to the special needs of *SchemaLog*. Logic programs can also be efficiently evaluated bottom-up. Research into how the bottom-up evaluation methods of “Semi-Naive” coupled with “Magic Sets” can be tailored to the requirements of *SchemaLog* can provide an interesting alternative to evaluating *SchemaLog* programs.

Implementations of other aspects of *SchemaLog* and of SchemaSQL – a systematic extension of standard SQL with the capabilities of *SchemaLog* features – are already in progress. We feel confident that the contributions of this thesis will form part of an integrated system that could fully realize in practice the extensive theoretical possibilities thrown up by *SchemaLog*.

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