The Formulation and Numerical Solution of the Coupler Curve Equations for a Multi-Link Planar Mechanism with Multiple Design Parameters

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ABSTRACT

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The objective of this thesis is to develop the coupler curve equation of an eight-bar mechanism consisting of binary and ternary links with two fixed pivots. The coupler curve equations consist of three formulated parametric equations, each of which relates the (x,y) coordinates of the coupler point with the input crank angle, output rocker angle and a total of eighteen constant design parameters which identify the mechanism.

By means of a digital computer, these simultaneous equations are numerically solved by the implementation of Newton's iterative technique whereby the coupler point and output rocker angle are determined for the complete cycle of the input crank. Subsequent computation of all corresponding linkage orientations are then solved. A number of test cases are selected to satisfy the mobility criteria where their one-degree of freedom motion corresponds to that of a crank-rocker mechanism.

An experimental model is built and tested to simulate the eight-bar mechanism. Its experimental coupler curve solution is compared with that obtained from the analytical model and the correlation between the two is verified.

With the presented eight-bar mechanism, coupler curves of significantly larger variations can be generated due to the availability of more design parameters and the improvement of the coupler link's freedom of motion. This makes the mechanism more "flexible" and especially attractive in the field of synthesis.

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NOMENCLATURE

r_{i1}	length of link number (i) in direction (1)
r_{i2}	length of link number (i) in direction (2)
r_{i3}	length of link number (i) in direction (3)
α_i	angle between link number (i) in direction (1) and the same link number (i) in direction (2) measured in counterclockwise direction
γ	angle between the fixed and rotated frames of references
X_j, Y_j	coupler point coordinate with respect to the fixed frame
x_i, y_i	coupler point coordinate with respect to the rotated frame
θ_i	angle between the + ve fixed X axis and the direction of link number (i) in the direction (1) measured in counterclockwise direction
$\vec{R}_{st}$	vector description of link number (s) in direction (t)
$\vec{R}_j$	vector description of coupler point

Subscripts

x	partial differentiation with respect to variable x_j
y	partial differentiation with respect to variable y_j
m	partial differentiation with respect to variable m

CHAPTER 1

INTRODUCTION

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The analysis of linkage mechanisms has often posed as an intriguing problem to the kinematicians. Their investigations have included mechanisms of all shapes and sizes, ranging from a simple four-bar planar mechanism to a highly complex spatial multi-linkage system. But regardless of its type, the analysis generally follows a sequential and systematic order. The first stage in the kinematic study, and perhaps the most difficult part, generally begins with an analysis of the displacement of the mechanism. An investigation on its corresponding velocity and acceleration can then be determined by straightforward and successive differentiation.

The study of the displacement of linkage mechanisms generally serves as a crucial starting point to a complete kinematic analysis. This will involve, for example, of inducing a rotational motion to an input crank and observing the output motion of other links. This output motion may be defined by the orientation or angular position of a link or a path followed by a point on a link and is usually depicted graphically or by a governing mathematical equation. The graphical interpretation of the linkage motion is useful for complex mechanisms where the analytical approach becomes difficult. However, its impracticality becomes evident where a rapid solution is required.

In the analytical approach, formulating the displacement equations can be of a difficult task depending on the complexity of the mechanism. Here, mathematical expressions are developed to describe the motion whereby analytic geometry such as vectors or complex variables is applied to the mechanisms configuration. When the equations derived relate the output displacement of the mechanism to its corresponding input in an explicit form, a closed form solution is represented. These equations are usually of high order and require extensive and elaborate algebraic manipulations to formulate.

Other methods include the iterative or numerical approach where, for example, a set of mathematical constraints, depicting the mechanism, are iteratively solved to determine the displacements. It is not uncommon that the numerical manipulations are carried out by means of an analog or a digital computer.

1.1 Survey of Previous Work

For over decades, the study of mechanisms have been pursued in all directions. Whether it be of a kinematic or synthesis point of view, mechanisms of all types, large or small, simple or complex, have been analyzed. One of the simplest and well known mechanism is the basic four-bar linkage which, during the past years, has served as the subject of many analytical investigations, focussing on the area of kinematics and synthesis. Freudenstein, for example, stood in the forefront in performing extensive research on the related topics of linkage mechanisms. Some of his earliest works [1,2,3]* include the design of four-link mechanisms using an analytical approach or a proposed method of approximation on its synthesis as a function generator.

Much of Freudenstein's work has encouraged widespread interests and has spurred further studies on linkages particularly in the field

*Numbers in brackets [ ] designate references at end of text.

of kinematic analysis. In the texts of Beyer [4] and Hartenberg and Denavit [5], for instance, the coupler curve equation is explicitly illustrated for a four-bar mechanism with six design parameters. Such works illustrate the complexity in formulating the high order equation for a simple mechanism.

With the limitation of the four-bar mechanism, kinematicians have turned towards higher numbered linkages. This, in effect, makes the finding of the coupler curve equations very difficult if not almost impossible and the implementation of numerical techniques becomes a necessity. With this in mind, Molian [6] published a paper illustrating a technique of solving kinematic equations with Newton's method where its application is oriented towards planar mechanisms. Examples of works using this approach include Chi-Yeh [7], Osman and Mansour [8] and Murata and Harada [9] where planar as well as spatial mechanisms are subjects of their investigations.

Belletrutti [10] and Jones and Rooney [11] described another approach in kinematic analysis whereby an analog computer is used to simulate the mechanism. Examples used to illustrate this technique include the simulation of a crank-rocker, a crank-slider and a two-pivot Stephenson linkage. Townsend [12,13] and Smith [14], on the other hand, demonstrated a non-iterative technique by which one can solve for the coupler curve of all planar mechanisms. The scheme of their investigation involves the development of a closed-form solution. Their methodology includes the manipulation of algebraic equations or the implementation of the complex conjugate exponential method. Their work, though, is general and only provide the initial steps in formulating the loop equations or constraints and do not explicitly illustrate the closed-form solution in their example which would be highly extensive in its derivation.

Funabashi, Ogawa and Hara [15] proposed a relatively unique approach in solving for the displacements of planar multi-link mechanisms. Here, transformation functions are used to define the relative displacements of links in each basic open chain as well as its connecting conditions. These transformation functions are arranged in block diagram form to simulate the overall mechanism. This method, however, does not provide the closed-form equation directly, but can solve for the displacements numerically by the use of a computer programming that manipulates the transformation functions.

A.H. Soni played a prominent role in the field of mechanisms whereby he conducted numerous research in the areas of kinematics and synthesis. His text [16] provides a concise overview of all topics relating to mechanism synthesis and analysis. His studies also included notable works on multi-linkage mechanisms where, for example, in his two-part series paper [17,18] an eight-link mechanism is geometrically analyzed with the establishment of its coupler cognates. In his later works, Soni teamed with Hamid [19] to release a paper on the synthesis of an eight-bar mechanism using the iterative matrix approach. The mechanism presented has five links in each of its three loops with two pivot points, and is similar to the proposed mechanism of this present work.

Other multi-linkage mechanisms studied include that of nine-linkages by Ramaiyan, Lakshminarayana and Narayanamurthi [20,21]. The different types of nine-link planar mechanisms for two-variable function generations are illustrated in part I with a proposed method to its synthesis in part II.

Past works not only delved in the kinematic analysis but also on the investigation of the coupler curve profile. For instance, Dijksman [22] concentrated on the design criteria of the Watt-1 linkage mechanisms which would produce symmetrical coupler curves, whereas, Davies [23] proposed his own finite 5-dimensional atlas of crank-rocker coupler curves. Here, he discusses a method of defining a circumscribed region where all coupler points should be confined to avoid duplication and omission. Wunderlich [24], on the other hand, investigated on the design requirements of a planar four-bar linkage to generate coupler curves possessing the peculiarity of self-osculation.

Researchers, though, did not restrict themselves to planar mechanisms only but had extended their analysis to spatial ones. To name a few, Dukkipati [25] illustrated his proposed closed-form displacement analysis of a five-link R-C-R-C-P spatial mechanism and teaming up with Osman [26] and Bahgat [27], he developed a unique approach to kinematically analyze spatial mechanisms with the former paper implementing matrices with dual-number elements and the latter using the train component technique.

1.2 Purpose and Scope of Research Work

The generation of the coupler curve is characteristic of its corresponding mechanism wherein the diversity of its profile depends very much on the number of design parameters available within the system. The four-bar mechanism, for instance, has on an average of six or seven design parameters. Though its analysis will be simpler, the variation in coupler curve profiles will be much more limited than that of a five or six-bar mechanism with a greater number of design parameters. It is, thus, the author's intention to develop and analyze a linkage mechanism with sufficient number of design parameters that allow the user to have the capability of generating wider and more diverse varieties of coupler curve shapes.

It has been found that with much of the research work done in the past only a few, if any, have actually derived and explicity illustrated the coupler curve equation of its corresponding linkage mechanism. Moreover, many of these works dealt with simple mechanisms only, namely, the four-bar linkage. Where mechanisms involved higher number of links, its authors resorted to purely iterative or numerical techniques to generate the coupler curve, otherwise they would simply describe the method and assume the reader to perform the rest.

In this present research, the author proposes to develop the coupler curve equations of an eight-linkage mechanism with eighteen design parameters. These coupler curve equations consist of three formulated parametric equations, each of which relates the (x,y) coordinates of the coupler point with the input crank angle, output rocker angle and a total of eighteen constant design parameters which identify the mechanism. Furthermore, a numerical iterative technique is implemented for the purpose of solving the three simultaneous equations.

The content of this thesis consists of a detailed description of the eight-bar mechanism with an explanation on the basis of its configuration in the second chapter. In the third chapter, the coupler curve equations for the eight-link mechanism are formulated and presented along with a study on its mobility criteria and in the fourth chapter, a complete description is given to illustrate the computational as well as experimental procedure used to numerically evaluate the coupler curve.

CHAPTER 2

FORMULATION OF MECHANISM

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2.1 Description of Eight-Link Mechanism

Figure 2.1 illustrates the eight-link mechanism chosen to be the subject of this present investigation. The mechanism consists of one coupler link, two binary links and four ternary links, held by revolute joints and stationed by two pivot points. Its configuration is arranged so as to form three five-link loops with its mobility confined to a one-degree of freedom planar motion. A total of eighteen parameters define the mechanism with thirteen of them being the linear dimension of the linkages and five being the angular specifications. The definition of the three loops along with the classification of the eighteen design parameters are depicted in Table 2.1.

In the notation of parameters r_{ij} and α_i, subscript i represents the link number which the dimension corresponds to, whereas j designates a specific component of that link. Thus, j will assume a sole value of 1 for binary links and a value of 1,2, and 3 for ternary links. With respect to the frame, r_{11} represents the linear distance between the two fixed pivots and γ is the angle with which the cartesian coordinate axes are rotated. In the parametric description of the ternary link, α_i represents the fixed angle between links r_{i1} and r_{i2}. Hence, with the designation of α_i, r_{i3} becomes redundant and is not included as one of the design parameters. The eighteen parameters described are constant during the motion of the coupler point but can be changed to obtain a different coupler curve test case.

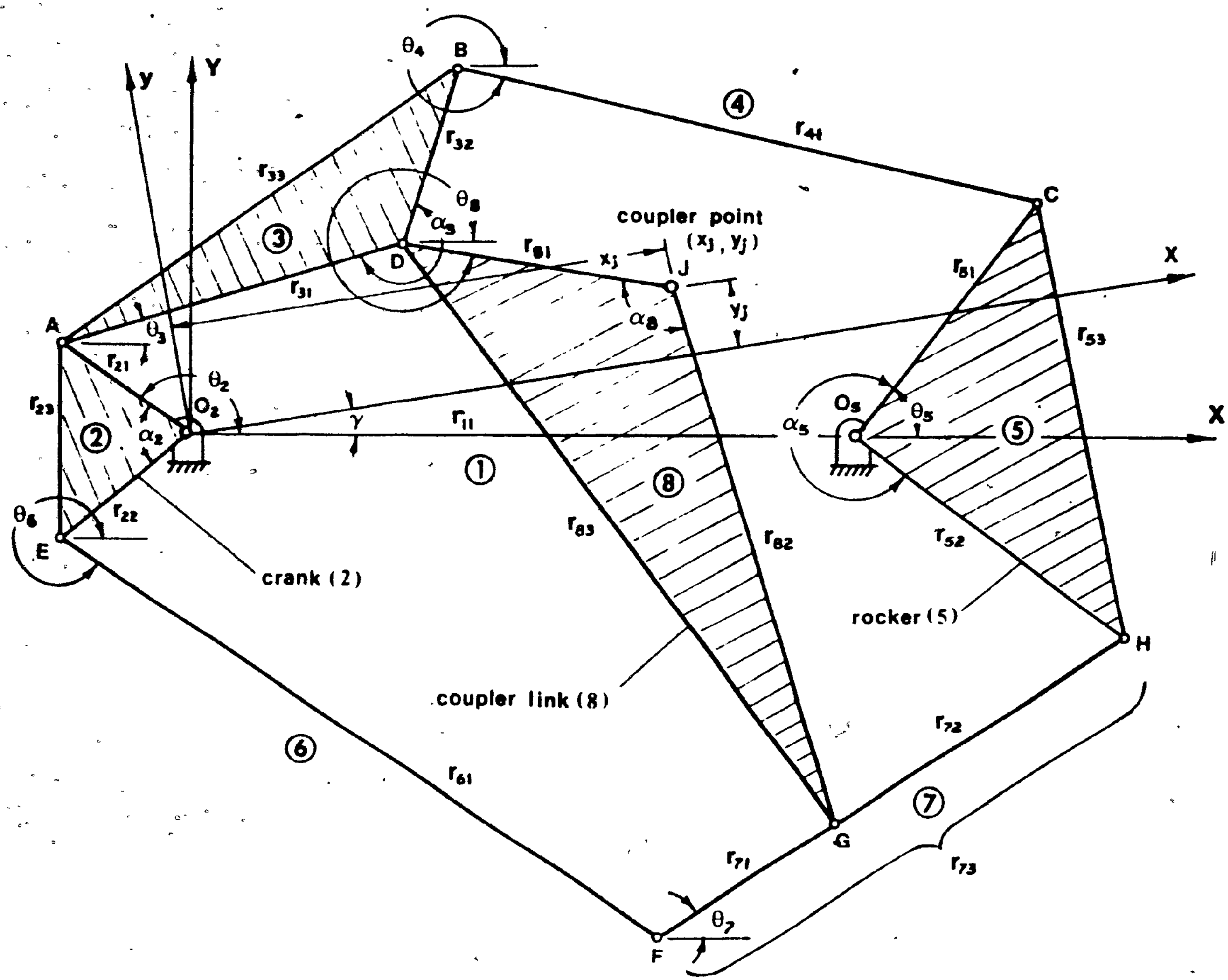

Fig. 2.1: Schematic Diagram of the Eight-Link Mechanism

Table 2.1: Classification of Parameters, Variables and Independent Loops of the Eight-Link Mechanism

LINK NO	PARAMETERS		VARIABLES	
	Linear	Angular	Linear	Angular
1	r_{11}	γ	-	-
2	r_{21} r_{22}	α_2	-	θ_2
3	r_{31} r_{32}	α_3	-	θ_3
4	r_{41}	-	-	θ_4
5	r_{51} r_{52}	α_5	-	θ_5
6	r_{61}	-	-	θ_6
7	r_{71} r_{72}	α_7	-	θ_7
8	r_{81} r_{82}	α_8	x_j, y_j	θ_8
Loop I: $O_2ADBCO_5O_2$ Loop II: $O_2O_5HGFEO_2$ Loop III: $O_2ADJGFEO_2$				

Having described the parameters, it is only appropriate to define the variables of the mechanism. Two variables define the position of the coupler point j (Fig. 2.1) with respect to the cartesian frame and are designated by their coordinates (x_j, y_j). All other variables describe the angular orientation of each link and are denoted by θ_i where i represents the link to which the angle corresponds to. θ_2 is the angular position of the crank (link 2) and represents the input of the system. For the sake of completeness, the variables are included and classified in Table 2.1.

In this study, the eight-bar mechanism is selected to conform to that of a crank-rocker system where link 2 (crank) is capable of making a full revolution and link 5 (rocker) moves with a back and forth rocking motion. Link 8 represents the coupler link with the coupler point located at j (Fig. 2.1).

2.2 Development of the Eight-Link Model

The proposed eight-link mechanism is intricate in its configuration. Its development is not a product of random selection but through a logical formulation. The basis of its configuration evolves from the simple four-bar mechanism as illustrated in Fig. 2.2. This mechanism consists of an input crank b, an output rocker d and a coupler link c-e-f where point c is the coupler point that defines the coupler curve. Assuming a fixed reference frame, this four-bar mechnaism has a total of six design parameters, namely a,b,e,f,d and γ.

One of the disadvantages of the four-bar mechanism is that its

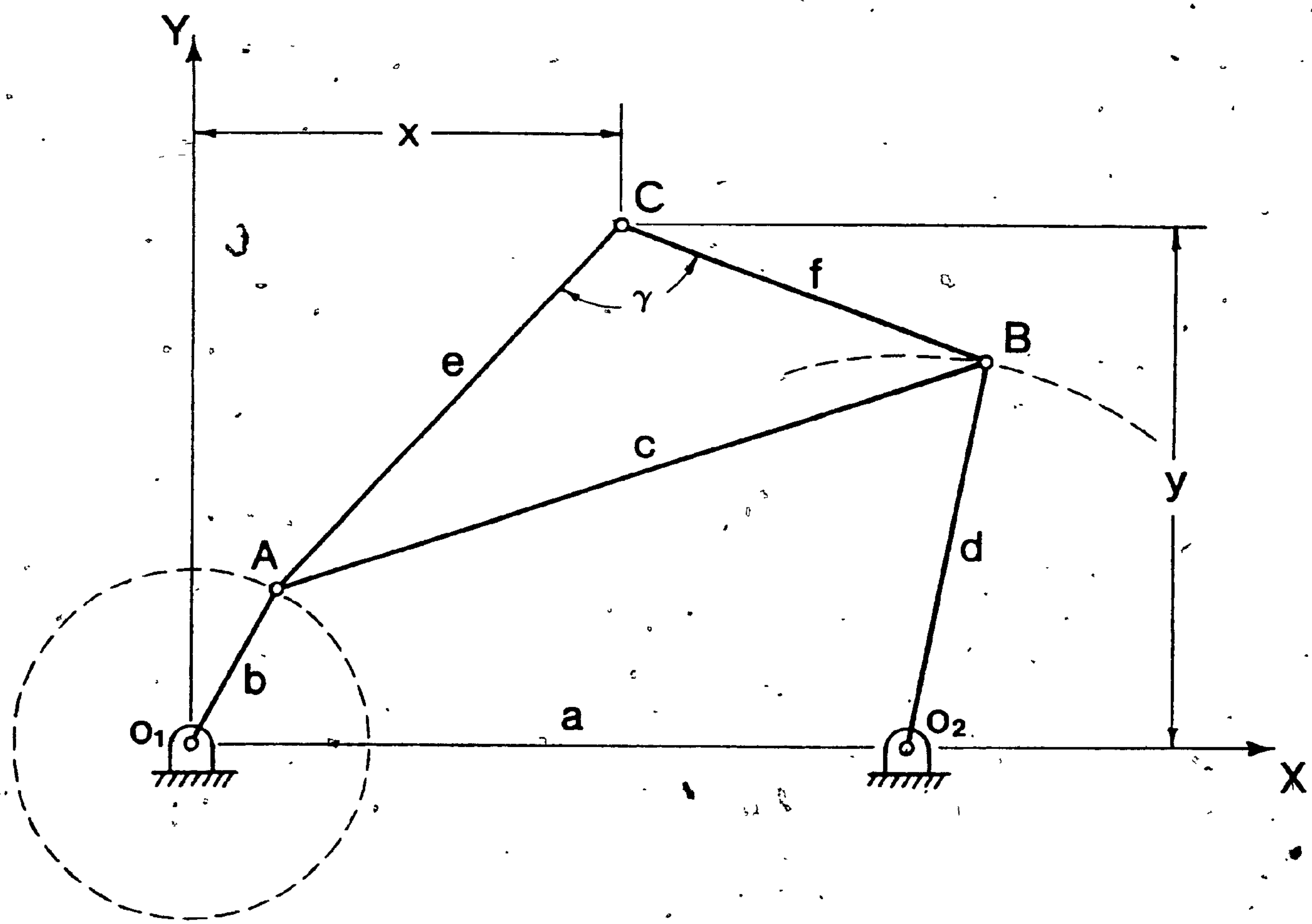

Fig. 2.2: Schematic Diagram of the Four-Link Mechanism

coupler curve shape is very limited. Two factors contribute to this limitation with the first being the limited number of design parameters available. The mechanism depicted in Fig. 2.2 has only six design parameters. Furthermore, the coupler link c-e-f is restricted in mobility in that its two end points A and B are confined to circular motions only. This, in effect, limits the motion of the coupler point C, and reduces the variation in coupler curve profiles.

With this in mind, the author proposes to design a mechanism whose coupler link do not have their two end points restricted to circular motions. Thus, it is suggested that point C of the four-bar mechanism (Fig. 2.2) be integrated as the end point of a coupler link since its motion is general and not restricted to a circle. This concept is shown in Fig. 2.3 where links ABC and EFG are analogous to the coupler link of the four-bar mechanism. It can be said that this mechanism (Fig. 2.3) is comprised of two four-bar mechanisms fixed opposite to each other with the added links of BD and FH to provide additional design parameters as well as a one-degree of freedom mobility. The angle between links O_1A and O_1H is fixed to function as a single input crank and similarly the angle between links O_2D and O_2E is fixed to function as a single output rocker. Link CGJ is the new coupler link whose end points C and G are no longer confined to a circular path but are capable of moving along general plane curves.

As a final step in formulating the eight-link model, a simplification is made whereby the angular description, α_7, of the ternary link EFG (Fig. 2.3), is permanently assigned a value of 180°. The grounds for such a simplification is to greatly reduce the difficulty and

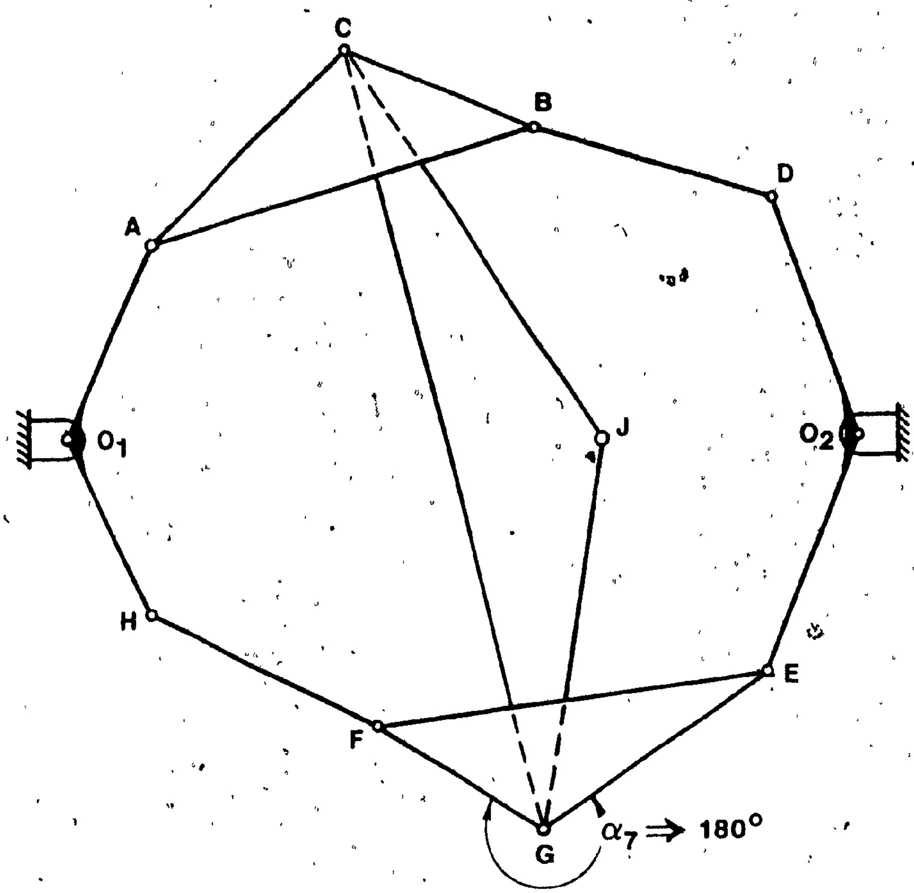

Fig. 2.3: Mid-Stage Development of the Eight-Link Mechanism

lengthiness in deriving the coupler curve equations particularly in the application of Gaussian Elimination (Chapter 3). The inclusion of α_7 as a design parameter can be suggested as a future expansion of this research work.

With the mentioned simplification, the model depicted in Fig. 2.1 results.

2.3 Application of Grubler's Criterion

The application of Grubler's criterion is an examination on the degree of freedom of a planar linkage system. This investigation serves the purpose of determining the number of inputs that must be given to the linkage so that the linkage has a constrained motion.

The determination of the number of degrees of freedom of a linkage is accomplished by the use of a formula proposed by Grubler [16]:

$$F = 3(N-1) - 2P_1 - P_2 \tag{2.1}$$

where

F = degree of freedom of the linkage

N = total number of links

P_1 = total number of kinematic pairs having one degree of freedom

P_2 = total number of kinematic pairs having two degrees of freedom

Note: Grubler's mobility criterion is valid for linkages that move in parallel planes.

Kinematic pairs having one degree of freedom include all revolute joints whereas kinematic pairs having two degrees of freedom include, for example, cam followers or mating teeth of gears.

Applying Grubler's criterion to our eight-bar linkage, the following results:

N = 8 links total

P_1 = 10 revolute joints

P_2 = 0

∴ $F = 3(8-1)-2(10)-0 = 1$ degree of freedom.

Thus, the proposed eight-bar linkage has one degree of freedom (one input motion is required to provide a constrained motion) and is classified as a "mechanism".

CHAPTER 3

MATHEMATICAL MODELLING OF MECHANISM

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3.1 Review of the Four-Bar Mechanism's Coupler Curve Equation

Figure 2.2 illustrates a four-bar mechanism having six design parameters, assuming a fixed coordinate frame. Formulating the coupler curve equation of point C (Fig. 2.2) requires extensive and elaborate algebraic manipulations, with its resulting equation being of high order. Very few works done in the past have actually formulated explicitly this coupler curve equation and, moreover, attempts of this nature were performed mainly on simple mechanisms such as this four-bar linkage. In the text of Beyer [4], an outlining procedure is presented in the formulation and derivation of the coupler curve equation for the four-bar mechanism depicted in Fig. 2.2. The final equation is:

$$f^2[(x-a)^2 + y^2](x^2+y^2+e^2-b^2)^2 - 2ef[(x^2+y^2-ax)\cos\gamma + ay\ \sin\gamma]$$

$$(x^2+y^2+e^2-b^2)[(x-a)^2 + y^2 + f^2 - d^2] + e^2(x^2 + y^2)[(x-a)^2 +$$

$$y^2 + f^2 - d^2]^2 - 4f^2e^2[(x^2 + y^2 - ax)\sin\gamma - ay\ \cos\gamma]^2 = 0 \qquad (3.1)$$

...whose variables and parameters correspond to the notation of that figure.

The above equation is of a general and indistinct form in that it represents the solution of two coupler curves simultaneously. This concept is illustrated in Fig. 3.1 where for identical link dimensions and input crank angle, two possible configurations exist with each providing a different coupler curve profile. No distinction between the

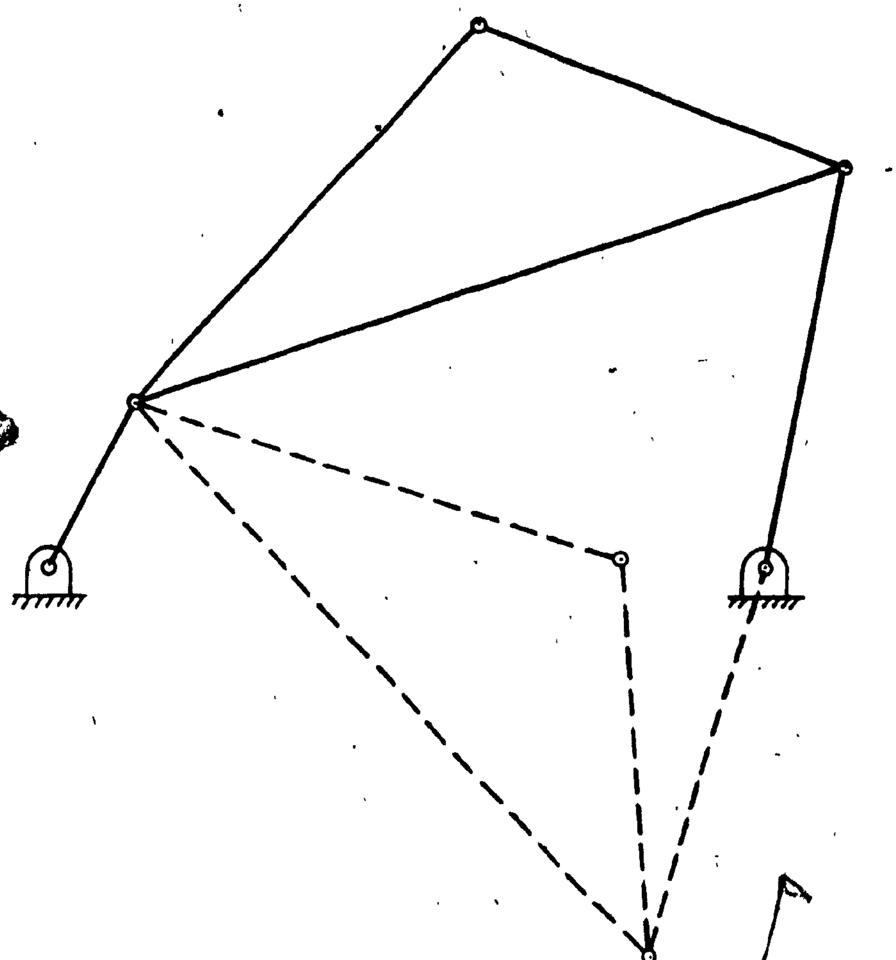


Fig. 3.1: Double Configuration of the Four-Link
Mechanism

two is made with the said coupler curve equation. Also, the equation provides no informaton whatsoever on the relation of the coupler point coordinate (x,y) or output rocker angle with its corresponding input crank angle, which can be useful. In order to explicitly solve for the coupler curve coordinate (x,y), a second equation of similar nature is required.

With the above equation being of sixth order, an insight is given to the reader on the complexity of the coupler curve equations for an eight-link mechanism. These equations are illustrated in the next section.

3.2 The Coupler Curve Equations for an Eight-Link Mechanism

The coupler curve equations formulated in this thesis are based on the notation defined in Fig.2.1. These equations relate the (x,y) coordinate of the coupler point with the 18 constant design parameters and the input crank and output rocker angles. Similar to the four-bar mechanism, the eight-link mechanism has two possible configurations for a single input crank angle, with each configuration corresponding to its own unique output rocker angle. Thus, with the existence of the output rocker angle in the coupler curve equation, a unique configuration can be pinpointed whereby in the numerical stage an initial approximate output angle corresponding to this configuration is assigned. This technique pinpoints and converges the solution to the desired configuration.

In general, the parametric coupler curve equations for the eight-link mechanism depicted in Fig. 2.1 is described as:

$$f_n(x_j, y_j, \theta_2, \theta_5, r_{i1}, r_{i2}, \alpha_i, \gamma) = 0 \qquad n = 1,2,3 \qquad (3.2)$$

and $r_{i1}(i = 1,2,\ldots 8)$, $r_{i2}(i = 2,3,5,7,8)$, $\alpha_i(i = 2,3,5,8)$

With the unknown variables of x_j, y_j and θ_5, three simultaneous equations f_1, f_2, f_3 are needed for the solution.

Explicitly, the coupler curve equations at coupler point J are:

$$f_n = \sum_{p=1}^{3} a^n_{(p-1)} m^{p-1} = 0 \qquad n = 1,2,3 \qquad (3.2)$$

where: $m = \tan(\theta_5/2)$

$$a^n_0 = \sin\theta_3(G_{1n} + G_{3n}) - (L_{1n} + L_{3n})$$

$$a^n_1 = 2(G_{2n}\sin\theta_3 - L_{2n})$$

$$a^n_2 = \sin\theta_3(G_{1n} - G_{3n}) - (L_{1n} - L_{3n})$$

$$\sin\theta_3 = -(B_0/2) \pm \sqrt{(B_0^2/4) - A_0}$$

$$A_0 = x_0^2[((x_0^2 + y_0^2 + d_3^2 - d_8^2)/2x_0)^2 - d_3^2]/[d_3^2(x_0^2 + y_0^2)]$$

$$B_0 = -y_0(x_0^2 + y_0^2 + d_3^2 - d_8^2)/[d_3(x_0^2 + y_0^2)]$$

$$L_{11} = 1+d_2^2+d_3^2-d_4^2+d_5^2 - 2d_2\cos\theta_2 - 2d_3^2\ell_3\cos\alpha_3 + d_3^2\ell_3^2$$

$$-x_0^{-1}(x_0^2+y_0^2+d_3^2-d_8^2)(1+d_2\ell_3\cos(\theta_2-\alpha_3) - d_2\cos\theta_2 - \ell_3\cos\alpha_3)$$

$$L_{12} = \ell_7^{-2}\{1-\ell_7^2 d_6^2+\ell_5^2 d_5^2+\ell_8^2(\ell_7+1)^2 d_3^2 + (\ell_7+1)^2 d_2^2+\ell_2^2\ell_7^2 d_2^2$$

$$-2\ell_2\ell_7(\ell_7+1)^2 d_2^2\cos\alpha_2 - 2(\ell_7+1)d_2\cos\theta_2 + 2\ell_2\ell_7 d_2\cos(\theta_2+\alpha_2)$$

$$+(\ell_7+1)^2(x_0^2+y_0^2)(\ell_8^2 - 2\ell_8\cos\alpha_8+1)$$

$$-\ell_8(\ell_7+1)x_0^{-1}(x_0^2+y_0^2+d_3^2-d_8^2)[\cos\alpha_8 - (\ell_7+1)d_2\cos(\theta_2-\alpha_8)$$

$$+\ell_2\ell_7 d_2\cos(\theta_2+\alpha_2-\alpha_8) + (\ell_7+1)(\ell_8 x_0-x_0\cos\alpha_8-y_0\sin\alpha_8)]$$

$$+2\ell_8(\ell_7+1)[x_0\cos\alpha_8 - y_0\sin\alpha_8 - \ell_8^{-1}x_0$$

$$+\ell_8^{-1}(\ell_7+1)d_2(x_0\cos\theta_2+y_0\sin\theta_2)$$

$$-(\ell_7+1)d_2(x_0\cos(\theta_2-\alpha_8) + y_0\sin(\theta_2-\alpha_8))$$

$$+\ell_2\ell_7\ell_8^{-1}d_2(x_0\cos(\theta_2+\alpha_2) + y_0\sin(\theta_2+\alpha_2))$$

$$+\ell_2\ell_7 d_2(x_0\cos(\theta_2+\alpha_2-\alpha_8) + y_0\sin(\theta_2+\alpha_2-\alpha_8))]\}$$

$$L_{13} = \ell_7^{-2}\{1-\ell_7^2 d_7^2 + \ell_5^2 d_5^2 + \ell_8 d_3^2 + d_2^2 - 2d_2\cos\theta_2$$

$$+(x_0^2+y_0^2)(\ell_8^2-2\ell_8\cos\alpha_8+1)$$

$$+\ell_8 x_0^{-1}(x_0^2+y_0^2+d_3^2-d_8^2)[d_2\cos(\theta_2-\alpha_8) - \cos\alpha_8-\ell_8 x_0$$

$$+x_0\cos\alpha_8 + y_0\sin\alpha_8] - 2[\ell_8 d_2 x_0\cos(\theta_2-\alpha_8) + \ell_8 d_2 y_0\sin(\theta_2-\alpha_8)$$

$$-d_2 x_0\cos\theta_2 - d_2 y_0\sin\theta_2 - \ell_8 x_0\cos\alpha_8 + \ell_8 y_0\sin\alpha_8 + x_0]\}$$

$$L_{21} = d_5[x_0^{-1}(x_0^2 + y_0^2 + d_3^2 - d_8^2)\ell_3\sin\alpha_3 - 2d_2\sin\theta_2]$$

$$L_{22} = 2\ell_5\ell_7^{-2}d_5[\tfrac{1}{2}\,x_0^{-1}(x_0^2 + y_0^2 + d_3^2 - d_8^2)(\ell_7+1)\ell_8\sin(\alpha_5-\alpha_8)$$

$$-d_2(\ell_7+1)\sin(\theta_2-\alpha_5) + d_2\ell_2\ell_7\sin(\theta_2+\alpha_2-\alpha_5)$$

$$-(\ell_7+1)\ell_8(x_0\sin(\alpha_5-\alpha_8) - y_0\cos(\alpha_5-\alpha_8))$$

$$+(\ell_7+1)(x_0\sin\alpha_5 - y_0\cos\alpha_5) - \sin\alpha_5]$$

$$L_{23} = 2\ell_5\ell_7^{-2}d_5[\tfrac{1}{2}\,x_0^{-1}(x_0^2 + y_0^2 + d_3^2 - d_8^2)\ell_8\sin(\alpha_5-\alpha_8)$$

$$-d_2\sin(\theta_2-\alpha_5) - \ell_8x_0\sin(\alpha_5-\alpha_8) + \ell_8y_0\cos(\alpha_5-\alpha_8)$$

$$+x_0\sin\alpha_5 - y_0\cos\alpha_5 - \sin\alpha_5]$$

$$L_{31} = d_5[x_0^{-1}(x_0^2+y_0^2+d_3^2-d_8^2)(\ell_3\cos\alpha_3-1) - 2d_2\cos\theta_2 + 2]$$

$$L_{32} = -2\ell_5\ell_7^{-2}d_5[\tfrac{1}{2}\,x_0^{-1}(x_0^2+y_0^2+d_3^2-d_8^2)(\ell_7+1)\ell_8\cos(\alpha_5-\alpha_8)$$

$$+d_2(\ell_7+1)\cos(\theta_2-\alpha_5) - d_2\ell_2\ell_7\cos(\theta_2+\alpha_2-\alpha_5)$$

$$-(\ell_7+1)\ell_8(x_0\cos(\alpha_5-\alpha_8) + y_0\sin(\alpha_5-\alpha_8))$$

$$+(\ell_7+1)(x_0\cos\alpha_5 + y_0\sin\alpha_5) - \cos\alpha_5]$$

$$L_{33} = 2\ell_5\ell_7^{-2}d_5[-\tfrac{1}{2}\,x_0^{-1}(x_0^2+y_0^2+d_3^2-d_8^2)\ell_8\cos(\alpha_5-\alpha_8)$$

$$-d_2\cos(\theta_2-\alpha_5) + \ell_8x_0\cos(\alpha_5-\alpha_8) + \ell_8y_0\sin(\alpha_5-\alpha_8)$$

$$-x_0\cos\alpha_5 - y_0\sin\alpha_5 + \cos\alpha_5]$$

$$G_{11} = 2d_3\{y_0x_0^{-1}[-1 + \ell_3\cos\alpha_3 + d_2\cos\theta_2 - d_2\ell_3\cos(\theta_2-\alpha_3)]$$
$$-d_2\ell_3\sin(\theta_2+\alpha_3) + \ell_3\sin\alpha_3 + d_2\sin\theta_2\}$$

$$G_{12} = 2\ell_7^{-2}(\ell_7+1)\ell_8d_3\{(\ell_7+1)d_2[y_0x_0^{-1}\cos(\theta_2-\alpha_8) - \sin(\theta_2-\alpha_8)]$$
$$-d_2\ell_2\ell_7[y_0x_0^{-1}\cos(\theta_2+\alpha_2-\alpha_8) - \sin(\theta_2+\alpha_2-\alpha_8)]$$
$$+[x_0(\ell_7+1)-1](y_0x_0^{-1}\cos\alpha_8 + \sin\alpha_8)$$
$$+y_0(\ell_7+1)(y_0x_0^{-1}\sin\alpha_8 - \cos\alpha_8)$$

$$G_{13} = 2\ell_7^{-1}\ell_8d_3[y_0x_0^{-1}d_2\cos(\theta_2-\alpha_8) - d_2\sin(\theta_2-\alpha_8)$$
$$-y_0x_0^{-1}\cos\alpha_8 + (y_0^2x_0^{-1} + x_0-1)\sin\alpha_8]$$

$$G_{21} = d_5[\ell_3x_0^{-1}(x_0^2+y_0^2+d_3^2-d_8^2)\sin\alpha_3 - 2d_2\sin\theta_2]$$

$$G_{22} = 2d_3d_5\ell_5\ell_8\ell_7^{-1}(\ell_7+1)[y_0x_0^{-1}\sin(\alpha_5-\alpha_8) + \cos(\alpha_5-\alpha_8)$$

$$G_{23} = 2d_3d_5\ell_5\ell_8\ell_7^{-1}[y_0x_0^{-1}\sin(\alpha_5-\alpha_8) + \cos(\alpha_5-\alpha_8)]$$

$$G_{31} = 2d_3d_5[y_0x_0^{-1}(\ell_3\cos\alpha_3-1) + \ell_3\sin\alpha_3]$$

$$G_{32} = -2d_3d_5\ell_5\ell_8\ell_7^{-2}(\ell_7+1)[y_0x_0^{-1}\cos(\alpha_5-\alpha_8) - \sin(\alpha_5-\alpha_8)]$$

$$G_{33} = -2d_3d_5\ell_5\ell_8\ell_7^{-2}[y_0x_0^{-1}\cos(\alpha_5-\alpha_8) - \sin(\alpha_5-\alpha_8)]$$

$$x_0 = (X_j/r_{11}) - d_2\cos\theta_2$$

$$y_0 = (Y_j/r_{11}) - d_2\sin\theta_2$$

$$X_j = x_j\cos\gamma - y_j\sin\gamma$$

$$Y_j = y_j\cos\gamma + x_j\sin\gamma$$

$$\ell_i = (r_{i2}/r_{i1}) \qquad i = 2,3,5,7,8$$

$$d_i = (r_{i1}/r_{11}) \qquad i = 2,3,4,5,6,7,8$$

3.3 Formulation of the Eight-Link Mechanism's Coupler Curve Equations

The application of vector analysis to the mechanism's configuration serves as the foundation for the derivation. This involves the determination of all independent link loops within the mechanism and to describe each by a vector equation. In the eight-link mechanism, three independent link loops exist each of which is shown in Figs. 3.2, 3.3 and 3.4. Consider Loop I (Fig. 3.2), whose vector equation is described as follows:

$$\vec{R}_{21} + \vec{R}_{31} + \vec{R}_{32} + \vec{R}_{41} - \vec{R}_{51} = \vec{R}_{11} \tag{3.3}$$

Similarly, the vector equation of Loop II (Fig. 3.3) is:

$$\vec{R}_{22} + \vec{R}_{61} + \vec{R}_{71} + \vec{R}_{72} - \vec{R}_{52} = \vec{R}_{11} \tag{3.4}$$

To fully characterize the mechanism, a third loop (Loop III) (Fig. 3.4) is needed. By defining the position coordinate of the coupler point (j), two more vector equations are obtained by using two paths, namely:

$$\vec{R}_j = \vec{R}_{21} + \vec{R}_{31} + \vec{R}_{81} \tag{3.5}$$

and

$$\vec{R}_j = \vec{R}_{22} + \vec{R}_{61} + \vec{R}_{71} + \vec{R}_{82} \tag{3.6}$$

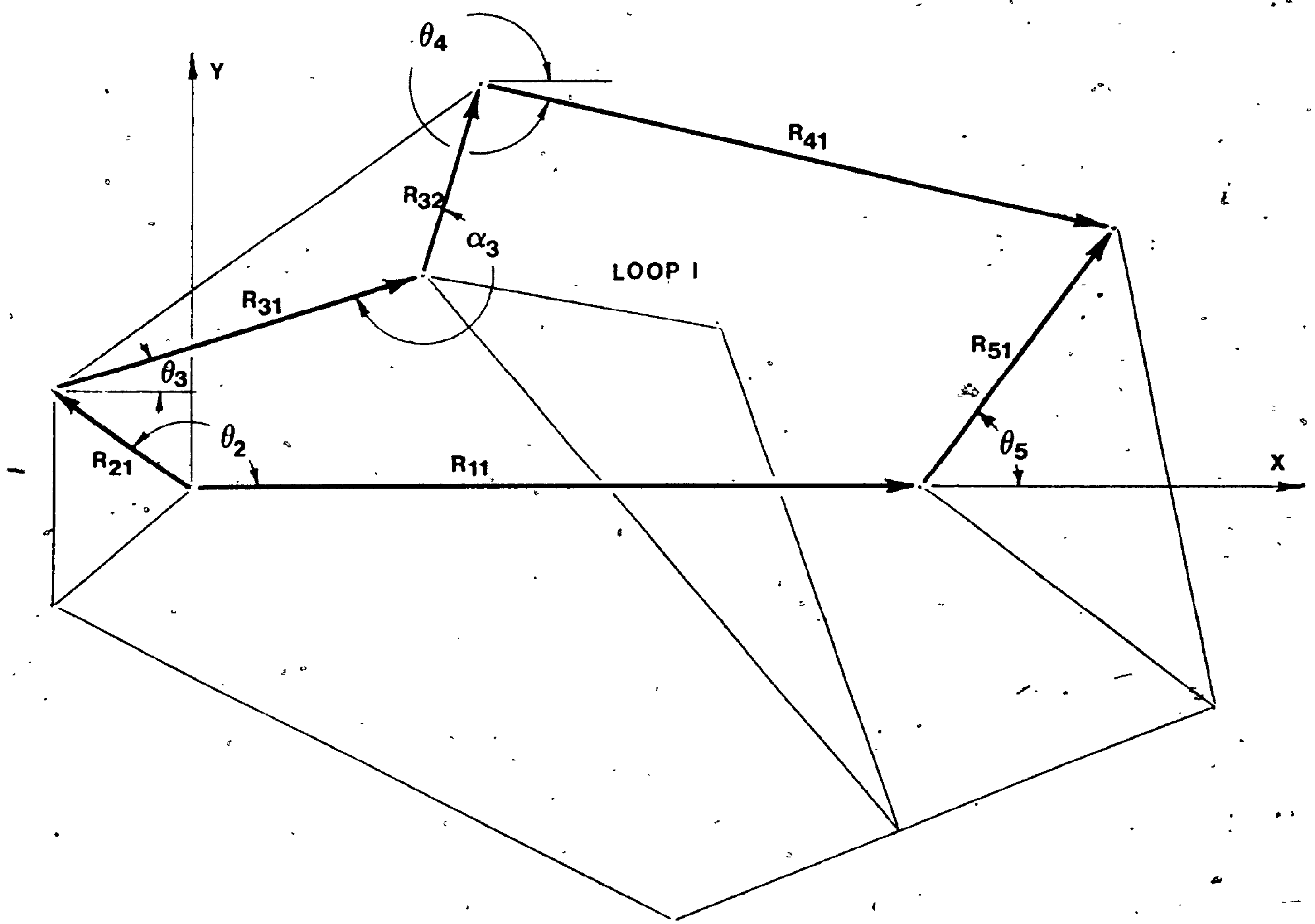


Fig. 3.2: Vector Loop I of the Eight-Link Mechanism

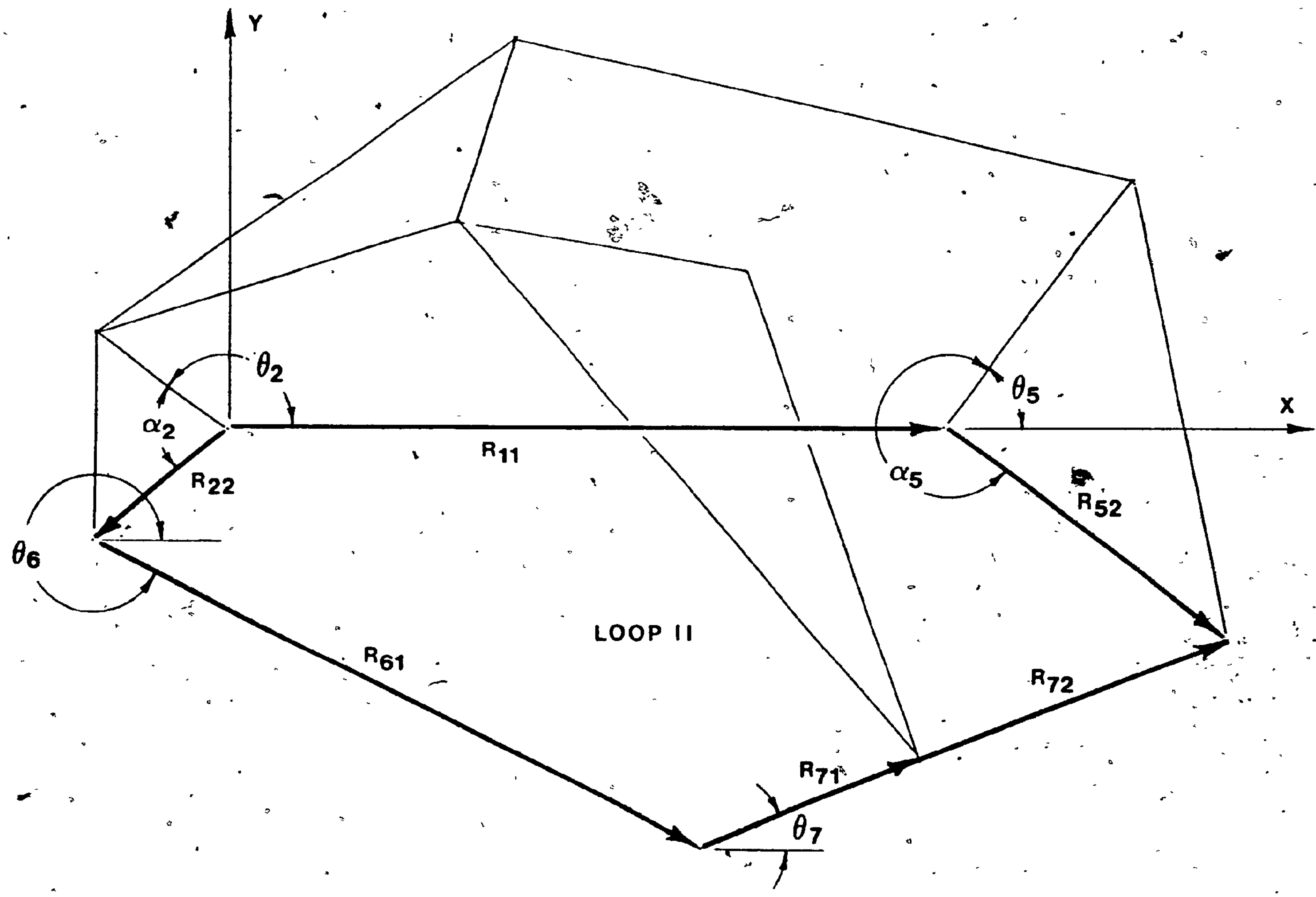


Fig. 3.3: Vector Loop II of the Eight-Link Mechanism

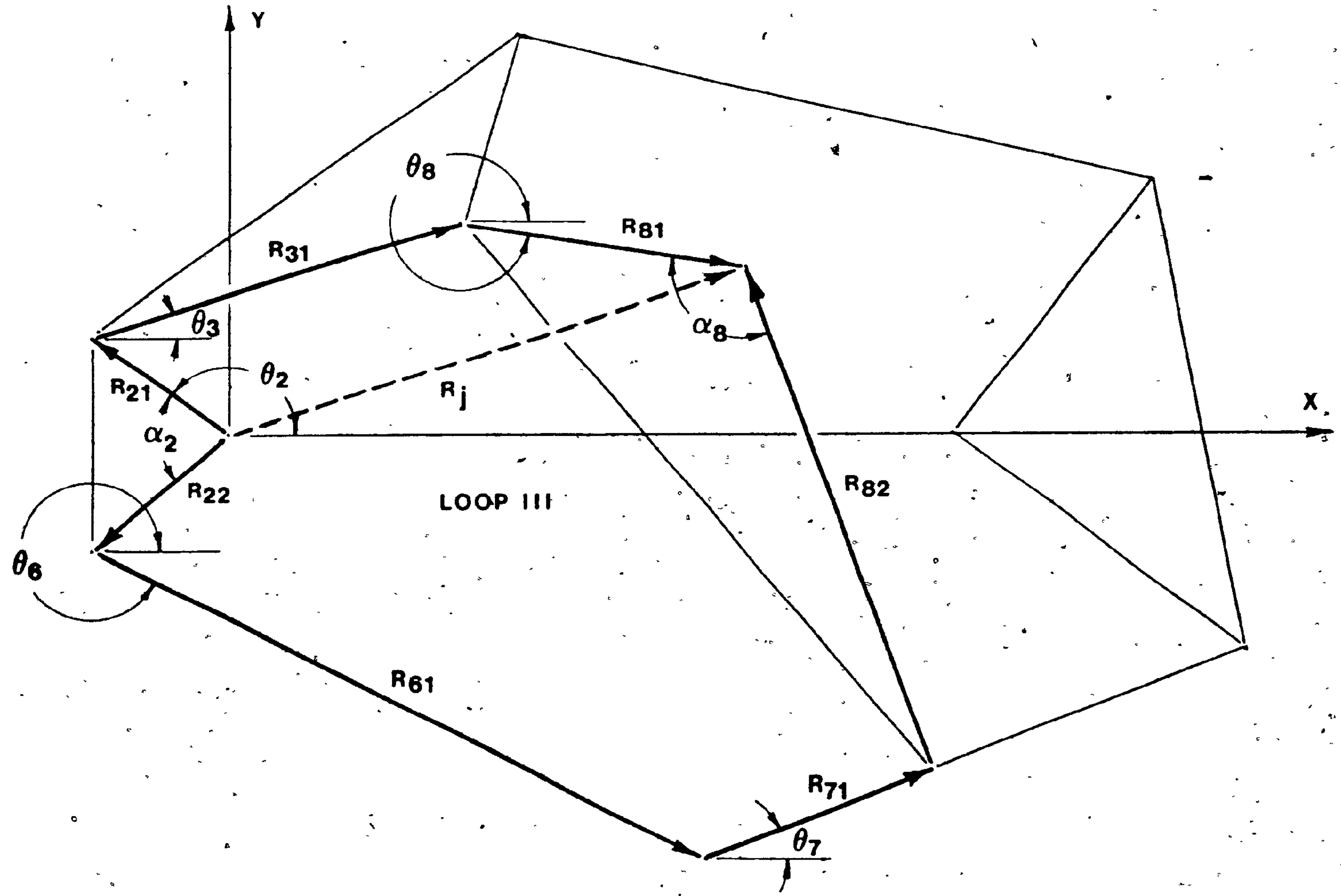


Fig. 3.4: Vector Loop III of the Eight-Link Mechanism

The above four vector equations completely define the eight-link mechanism with which the displacement equations can be derived.

In equations (3.5) and (3.6), $\vec{R}_j$ represents the coupler point coordinate (X_j,Y_j). Expressing this in terms of the coordinate (x_j,y_j) with reference to the rotated frame, the following coordinate transformation is implemented:

$$\begin{aligned} X_j &= x_j\cos\gamma - y_j\sin\gamma \\ Y_j &= x_j\sin\gamma + y_j\cos\gamma \end{aligned} \tag{3.7}$$

where γ is the angle between the fixed and rotated frames of reference.

Rewriting each of the vector equations into scalar form, the following results:

$$r_{21}\cos\theta_2 + r_{31}\cos\theta_3 - r_{32}\cos(\theta_3+\alpha_3) + r_{41}\cos\theta_4 - r_{51}\cos\theta_5 = r_{11} \tag{3.8}$$

$$r_{21}\sin\theta_2 + r_{31}\sin\theta_3 - r_{32}\sin(\theta_3+\alpha_3) + r_{41}\sin\theta_4 - r_{51}\sin\theta_5 = 0 \tag{3.9}$$

$$r_{22}\cos(\theta_2+\alpha_2) + r_{61}\cos\theta_6 + r_{71}\cos\theta_7 - r_{72}\cos(\theta_7+\alpha_7) - r_{52}\cos(\theta_5+\alpha_5) = r_{11} \tag{3.10}$$

$$r_{22}\sin(\theta_2+\alpha_2) + r_{61}\sin\theta_6 + r_{71}\sin\theta_7 - r_{72}\sin(\theta_7+\alpha_7) - r_{52}\sin(\theta_5+\alpha_5) = 0 \tag{3.11}$$

$$x_j\cos\gamma - y_j\sin\gamma = r_{21}\cos\theta_2 + r_{31}\cos\theta_3 + r_{81}\cos\theta_8 \tag{3.12}$$

$$x_j\sin\gamma + y_j\cos\gamma = r_{21}\sin\theta_2 + r_{31}\sin\theta_3 + r_{81}\sin\theta_8 \tag{3.13}$$

$$x_j\cos\gamma - y_j\sin\gamma = r_{22}\cos(\theta_2+\alpha_2) + r_{61}\cos\theta_6 + r_{71}\cos\theta_7 + r_{82}\cos(\theta_8+\alpha_8) \tag{3.14}$$

$$x_j\sin\gamma + y_j\cos\gamma = r_{22}\sin(\theta_2+\alpha_2) + r_{61}\sin\theta_6 + r_{71}\sin\theta_7 + r_{82}\sin(\theta_8+\alpha_8) \tag{3.15}$$

where for each vector equation, two scalar equations expressing the x and y components are obtained.

Expressing the linear design parameters in non-dimensional form, the following are defined:

$$\ell_i = r_{i2}/r_{i1} \quad ; \quad i = 2,3,5,7,8$$

$$d_i = r_{i1}/r_{11} \quad ; \quad i = 2,3,4,\ldots 8$$

with the following non-dimensional coordinate (x_0,y_0) as:

$$x_0 = [(x_j\cos\gamma - y_j\sin\gamma)/r_{11}] - d_2\cos\theta_2$$

$$y_0 = [(x_j\sin\gamma + y_j\cos\gamma)/r_{11}] - d_2\sin\theta_2$$

With the rearrangement and substitution of the non-dimensional parameters and coordinate into the scalar equations, the following are obtained:

$$-d_4\cos\theta_4 + d_5\cos\theta_5 - d_3(1-\ell_3\cos\alpha_3)\cos\theta_3 - d_3(\ell_3\sin\alpha_3)\sin\theta_3 = -1 + d_2\cos\theta_2 \tag{3.16}$$

$$-d_4\sin\theta_4 + d_5\sin\theta_5 - d_3(\ell_3\sin\alpha_3)\cos\theta_3 - d_3(1-\ell_3\cos\alpha_3)\sin\theta_3 = d_2\sin\theta_2 \tag{3.17}$$

$$-d_5(\ell_5\cos\alpha_5)\cos\theta_5 + d_5(\ell_5\sin\alpha_5)\sin\theta_5$$
$$+d_7(1-\ell_7\cos\alpha_7)\cos\theta_7 + d_7(\ell_7\sin\alpha_7)\sin\theta_7 + d_6\cos\theta_6$$
$$= 1-d_2(\ell_2\cos\alpha_2)\cos\theta_2 + d_2(\ell_2\sin\alpha_2)\sin\theta_2 \qquad (3.18)$$

$$-d_5(\ell_5\sin\alpha_5)\cos\theta_5 - d_5(\ell_5\cos\alpha_5)\sin\theta_5$$
$$-d_7(\ell_7\sin\alpha_7)\cos\theta_7 + d_7(1-\ell_7\cos\alpha_7)\sin\theta_7 + d_6\sin\theta_6$$
$$= -d_2(\ell_2\sin\alpha_2)\cos\theta_2 - d_2(\ell_2\cos\alpha_2)\sin\theta_2 \qquad (3.19)$$

$$-d_7\cos\theta_7 - d_8(\ell_8\cos\alpha_8)\cos\theta_8 + d_8(\ell_8\sin\alpha_8)\sin\theta_8$$
$$-d_6\cos\theta_6 + x_0 = -d_2(1-\ell_2\cos\alpha_2)\cos\theta_2$$
$$-d_2(\ell_2\sin\alpha_2)\sin\theta_2 \qquad (3.20)$$

$$-d_7\sin\theta_7 - d_8(\ell_8\sin\alpha_8)\cos\theta_8 - d_8(\ell_8\cos\alpha_8)\sin\theta_8$$
$$-d_6\sin\theta_6 + y_0 = d_2(\ell_2\sin\alpha_2)\cos\theta_2$$
$$-d_2(1-\ell_2\cos\alpha_2)\sin\theta_2 \qquad (3.21)$$

$$-d_8\cos\theta_8 - d_3\cos\theta_3 + x_0 = 0 \qquad (3.22)$$

$$-d_8\sin\theta_8 - d_3\sin\theta_3 + y_0 = 0 \qquad (3.23)$$

Another useful equation to obtain is by the squaring and adding of the two scalar equations (3.22) and (3.23). Performing this gives the following equation:

$$d_3(\cos\theta_3 + s_0\sin\theta_3) = z_0 \qquad (3.24)$$

where

$$s_0 = y_0/x_0$$

and $z_0 = [(x_0 + s_0y_0)/2] + [(d_3^2 - d_8^2)/2x_0]$

Thus, with eight independent non-dimensional equations (eqs. (3.16) to (3.23)), it is possible to solve for the trigonometric functions $\cos\theta_i$ and $\sin\theta_i$ for i = 4,6,7 and 8. The redundant equation (3.24) serves the purpose of eliminating the $\cos\theta_3$ term from all other equations. All these are accomplished by the implementation of Gaussian elimination whereby equations (3.16) to (3.24) are put into matrix form as an initial step. This matrix is illustrated in Fig. 3.5, whose first column indicates the coefficient of the '$d_4\cos\theta_4$' term, etcetera, and whose last column represents the remaining terms placed on the other side of the equality sign. The following figure after, Fig. 3.6, illustrates the final matrix obtained after performing Gaussian elimination. Here, α_7 was assigned a value of 180° before hand to greatly ease the algebraic manipulations involved in the Gaussian process.

Execution of the Gaussian elimination results in the following solution for the trigonometric terms:

$$\left.\begin{aligned}\cos\theta_4 &= (-U_1d_3\sin\theta_3 + d_5\cos\theta_5 + Q_1)/d_4\\ \sin\theta_4 &= (-V_1d_3\sin\theta_3 + d_5\sin\theta_5 + Q_2)/d_4\end{aligned}\right\} \qquad (3.25)$$

$$\left.\begin{aligned}\cos\theta_6 &= (-U_2d_3\sin\theta_3 - C_5d_5\cos\theta_5 + S_5d_5\sin\theta_5 + Q_3)/d_6\\ \sin\theta_6 &= (-V_2d_3\sin\theta_3 - S_5d_5\cos\theta_5 - C_5d_5\sin\theta_5 + Q_4)/d_6\end{aligned}\right\} \qquad (3.26)$$

$$\left.\begin{aligned}\cos\theta_7 &= (U_3d_3\sin\theta_3 + C_5d_5\cos\theta_5 - S_5d_5\sin\theta_5 + Q_5)/d_7\\ \sin\theta_7 &= (V_3d_3\sin\theta_3 + S_5d_5\cos\theta_5 + C_5d_5\sin\theta_5 + Q_6)/d_7\end{aligned}\right\} \qquad (3.27)$$

$$\left.\begin{aligned}\cos\theta_8 &= (s_0d_3\sin\theta_3 + Q_7)/d_8\\ \sin\theta_8 &= (-d_3\sin\theta_3 + Q_8)/d_8\end{aligned}\right\} \qquad (3.28)$$

$d_4\cos\theta_4$	$d_4\sin\theta_4$	$d_6\cos\theta_6$	$d_6\sin\theta_6$	$d_7\cos\theta_7$	$d_7\sin\theta_7$	$d_8\cos\theta_8$	$d_8\sin\theta_8$	$d_3\cos\theta_3$	$d_3\sin\theta_3$	$d_5\cos\theta_5$	$d_5\sin\theta_5$	=	C
-1	0	0	0	0	0	0	0	$l_3\cos\alpha_3-1$	$-l_3\sin\alpha_3$	1	0		$d_2\cos\theta_2-1$
0	-1	0	0	0	0	0	0	$l_3\sin\alpha_3$	$l_3\cos\alpha_3-1$	0	1		$d_2\sin\theta_2$
0	0	-1	0	$-l_7-1$	0	0	0	0	0	$l_5\cos\alpha_5$	$-l_5\sin\alpha_5$		$d_2l_2\cos(\alpha_2+\theta_2)$ -1
0	0	0	-1	0	$-l_7-1$	0	0	0	0	$l_5\sin\alpha_5$	$l_5\cos\alpha_5$		$d_2l_2\sin(\alpha_2+\theta_2)$
0	0	-1	0	-1	0	$-l_8\cos\alpha_8$	$l_8\sin\alpha_8$	0	0	0	0		$-d_2\cos\theta_2-x_o$ $+d_2l_2\cos(\alpha_2+\theta_2)$
0	0	0	-1	0	-1	$-l_8\sin\alpha_8$	$-l_8\cos\alpha_8$	0	0	0	0		$-d_2\sin\theta_2-y_o$ $+d_2l_2\sin(\alpha_2+\theta_2)$
0	0	0	0	0	0	-1	0	-1	0	0	0		x_o
0	0	0	0	0	0	0	-1	0	-1	0	0		y_o
0	0	0	0	0	0	0	0	-1	$-S_o$	0	0		$-Z_o$

Fig. 3.5: Pre-Gaussian Elimination Matrix

$d_4\cos\theta_4$	$d_4\sin\theta_4$	$d_6\cos\theta_6$	$d_6\sin\theta_6$	$d_7\cos\theta_7$	$d_7\sin\theta_7$	$d_8\cos\theta_8$	$d_8\sin\theta_8$	$d_3\cos\theta_3$	$d_3\sin\theta_3$	$d_5\cos\theta_5$	$d_5\sin\theta_5$	=	C
-1	0	0	0	0	0	0	0	0	$-U_1$	1	0		$-Q_1$
0	-1	0	0	0	0	0	0	0	$-V_1$	0	1		$-Q_2$
0	0	-1	0	0	0	0	0	0	$-U_2$	$-C_5$	S_5		$-Q_3$
0	0	0	-1	0	0	0	0	0	$-V_2$	$-S_5$	$-C_5$		$-Q_4$
0	0	0	0	-1	0	0	0	0	U_3	C_5	$-S_5$		$-Q_5$
0	0	0	0	0	-1	0	0	0	V_3	S_5	C_5		$-Q_6$
0	0	0	0	0	0	-1	0	0	S_o	0	0		$-Q_7$
0	0	0	0	0	0	0	-1	0	-1	0	0		$-Q_8$
0	0	0	0	0	0	0	0	-1	$-S_o$	0	0		$-Z_o$

Fig. 3.6: Post-Gaussian Elimination Matrix

$$\cos\theta_3 = (-s_0 d_3 \sin\theta_3 + z_0)/d_3 \tag{3.29}$$

where:

$Q_1 = C_3 z_0 - q_1$

$Q_2 = S_3 z_0 - q_2$

$Q_3 = C_8' z_0 - q_3 + h_1$

$Q_4 = S_8' z_0 - q_4 + h_2$

$Q_5 = -C_8 z_0 - q_5 + h_3$

$Q_6 = -S_8 z_0 - q_6 + h_4$

$Q_7 = -z_0 + x_0$

$Q_8 = y_0$

$h_1 = -a\ x_0 + S_8' y_0$

$h_2 = -S_8' x_0 - a y_0$

$h_3 = b x_0 - S_8 y_0$

$h_4 = S_8 x_0 + b y_0$

$a = C_8' - (\ell_7 + 1)/\ell_7$

$b = C_8 - 1/\ell_7$

$U_1 = (C_3 S_0 + S_3)$

$V_1 = (S_3 S_0 - C_3)$

$$U_2 = (C_8' S_0 + S_8')$$

$$V_2 = (S_8' S_0 - C_8')$$

$$U_3 = U_2/(\ell_7 + 1)$$

$$V_3 = V_2/(\ell_7 + 1)$$

$$q_1 = d_2\cos\theta_2 - 1$$

$$q_2 = d_2\sin\theta_2$$

$$q_3 = \ell_7^{-1} - (\ell_7+1)\ell_7^{-1} d_2\cos\theta_2 + d_2\ell_2\cos(\theta_2+\alpha_2)$$

$$q_4 = -(\ell_7+1)\ell_7^{-1} d_2\sin\theta_2 + d_2\ell_2\sin(\theta_2+\alpha_2)$$

$$q_5 = q_1/\ell_7$$

$$q_6 = q_2/\ell_7$$

$$C_3 = \ell_3\cos\alpha_3 - 1$$

$$S_5 = \ell_3\sin\alpha_3$$

$$C_5 = \ell_5\cos\alpha_5/\ell_7$$

$$S_5 = \ell_5\sin\alpha_5/\ell_7$$

$$C_8 = \ell_8\cos\alpha_8/\ell_7$$

$$S_8 = \ell_8\sin\alpha_8/\ell_7$$

$$C_8' = C_8(\ell_7 + 1)$$

$$S_8' = S_8(\ell_7 + 1)$$

...with all other parameters and variables being as defined earlier.

The next procedure in our formulation is to eliminate the angles θ_4, θ_6, θ_7 and θ_8. This is accomplished by squaring and adding the set of equations (3.25), (3.26), (3.27) and (3.28) respectively, giving the following:

$$\begin{aligned} d_4^2 - d_5^2 - Q_1^2 - Q_2^2 = {} & (U_1^2+V_1^2)d_3^2\sin^2\theta_3 - 2U_1d_3d_5\sin\theta_3\cos\theta_5 \\ & -2V_1d_3d_5\sin\theta_3\sin\theta_5 - 2d_3(U_1Q_1+V_1Q_2)\sin\theta_3 \\ & +2d_5Q_1\cos\theta_5 + 2d_5Q_2\sin\theta_5 \end{aligned} \tag{3.30}$$

$$\begin{aligned} d_6^2 - (C_5^2+S_5^2)d_5^2 - Q_3^2 - Q_4^2 = {} & (U_2^2+V_2^2)d_3^2\sin^2\theta_3 \\ & +2(U_2C_5+V_2S_5)d_3d_5\sin\theta_3\cos\theta_5 \\ & +2(V_2C_5-U_2S_5)d_3d_5\sin\theta_3\sin\theta_5 \\ & -2(U_2Q_3+V_2Q_4)d_3\sin\theta_3 - 2(C_5Q_3+S_5Q_4)d_5\cos\theta_5 \\ & -2(C_5Q_4-S_5Q_3)d_5\sin\theta_5 \end{aligned} \tag{3.31}$$

$$\begin{aligned} d_7^2 - (C_5^2+S_5^2)d_5^2 - Q_5^2 - Q_6^2 = {} & (U_3^2+V_3^2)d_3^2\sin^2\theta_3 \\ & +2(U_3C_5+V_3S_5)d_3d_5\sin\theta_3\cos\theta_5 \\ & +2(V_3C_5-U_3S_5)d_3d_5\sin\theta_3\sin\theta_5 \\ & +2(U_3Q_5+V_3Q_6)d_3\sin\theta_3 + 2(C_5Q_5+S_5Q_6)d_5\cos\theta_5 \\ & +2(C_5Q_6- S_5Q_5)d_5\sin\theta_5 \end{aligned} \tag{3.32}$$

$$-Q = (S_0^2+1)d_3^2\sin^2\theta_3 + 2d_3Q_0\sin\theta_3 \tag{3.33}$$

where: $Q_0 = (S_0Q_7 - Q_8)$

$Q = (Q_7^2 + Q_8^2 - d_8^2)$

Implementing equation (3.33) in conjunction with equations (3.30), (3.31) and (3.32) to eliminate the '$\sin^2\theta_3$' terms leads to our three coupler curve equations:

$$\begin{aligned} a_1Q + (d_4^2-d_5^2-Q_1^2-Q_2^2) = &-2U_1d_3d_5\sin\theta_3\cos\theta_5 \\ &-2V_1d_3d_5\sin\theta_3\sin\theta_5 \\ &-2d_3(U_1Q_1+V_1Q_2+a_1Q_0)\sin\theta_3 \\ &+2d_5Q_1\cos\theta_5 + 2d_5Q_2\sin\theta_5 \end{aligned} \tag{3.34}$$

$$\begin{aligned} a_2Q + [d_6^2-(C_5^2+S_5^2)d_5^2-Q_3^2-Q_4^2] = &\ 2d_3d_5(U_2C_5+V_2S_5)\sin\theta_3\cos\theta_5 \\ &+2d_3d_5(V_2C_5-U_2S_5)\sin\theta_3\sin\theta_5 \\ &-2d_3(U_2Q_3+V_2Q_4+a_2Q_0)\sin\theta_3 \\ &-2d_5(C_5Q_3+S_5Q_4)\cos\theta_5 - 2d_5(C_5Q_4-S_5Q_3)\sin\theta_5 \end{aligned} \tag{3.35}$$

$$\begin{aligned} a_3Q + [d_7^2-(C_5^2+S_5^2)d_5^2 - Q_5^2 - Q_6^2] = &\ 2d_3d_5(U_3C_5+V_3S_5)\sin\theta_3\cos\theta_5 \\ &+2d_3d_5(V_3C_5-U_3S_5)\sin\theta_3\sin\theta_5 \\ &+2d_3(U_3Q_5+V_3Q_6-a_3Q_0)\sin\theta_3 \\ &+2d_5(C_5Q_5+S_5Q_6)\cos\theta_5 + 2d_5(C_5Q_6-S_5Q_5)\sin\theta_5 \end{aligned} \tag{3.36}$$

where: $a_1 = (C_3^2 + S_3^2)$

$a_2 = (C_8'^2 + S_8'^2)$

$a_3 = a_2/(\ell_7 + 1)^2$

Rearranging, the above three coupler curve equations can be re-written in tensor notation:

$$\sin\theta_3 = \frac{(L_{1n} + L_{2n}\sin\theta_5 + L_{3n}\cos\theta_5)}{(G_{1n} + G_{2n}\sin\theta_5 + G_{3n}\cos\theta_5)} \qquad n = 1,2,3 \qquad (3.37)$$

where:

$L_{11} = d_5^2 + Q_1^2 + Q_2^2 - d_4^2 - a_1 Q$

$L_{12} = (C_5^2 + S_5^2)d_5^2 + Q_3^2 + Q_4^2 - d_6^2 - a_2 Q$

$L_{13} = (C_5^2 + S_5^2)d_5^2 + Q_5^2 + Q_6^2 - d_7^2 - a_3 Q$

$L_{21} = 2d_5 Q_2$

$L_{22} = 2d_5(S_5 Q_3 - C_5 Q_4)$

$L_{23} = 2d_5(C_5 Q_6 - S_5 Q_5)$

$L_{31} = 2d_5 Q_1$

$L_{32} = -2d_5(C_5 Q_3 + S_5 Q_4)$

$L_{33} = 2d_5(C_5 Q_5 + S_5 Q_6)$

$$G_{11} = 2d_3(U_1Q_1 + V_1Q_2 + a_1Q_0)$$

$$G_{12} = 2d_3(U_2Q_3 + V_2Q_4 + a_2Q_0)$$

$$G_{13} = 2d_3(a_3Q_0 - U_3Q_5 - V_3Q_6)$$

$$G_{21} = 2d_3d_5V_1$$

$$G_{22} = 2d_3d_5(U_2S_5 - V_2C_5)$$

$$G_{23} = 2d_3d_5(U_3S_5 - V_3C_5)$$

$$G_{31} = 2d_3d_5U_1$$

$$G_{32} = -2d_3d_5(U_2C_5 + V_2S_5)$$

$$G_{33} = -2d_3d_5(U_3C_5 + V_3S_5)$$

By defining:

$$\tan(\theta_5/2) = m$$

then, it follows that:

$$\sin\theta_5 = 2m/(1+m^2)$$

and

$$\cos\theta_5 = (1-m^2)/(1+m^2)$$

Substituting the above identities into equation (3.37) and rearranging, the final coupler curve equations are obtained:

$$f_n = \sum_{p=1}^{3} a^n_{(p-1)} m^{p-1} = 0 \qquad n = 1,2,3 \tag{3.38}$$

where:

$$a^n_0 = \sin\theta_3(G_{1n} + G_{3n}) - (L_{1n} + L_{3n})$$

$$a^n_1 = 2(G_{2n}\sin\theta_3 - L_{2n})$$

$$a^n_2 = \sin\theta_3(G_{1n} - G_{3n}) - (L_{1n} - L_{3n})$$

The value for '$\sin\theta_3$' is obtained directly by solving equation (3.33). Applying the quadratic formula, the following results:

$$\sin\theta_3 = -(B_0/2) \pm \sqrt{(B_0^2/4)-A_0} \tag{3.39}$$

where:

$$A_0 = Q/[d_3^2(S_0^2 + 1)]$$

$$B_0 = 2Q_0/[d_3(S_0^2 + 1)]$$

According to equation (3.39), two possible solutions exist depending on the nature of its sign, + or - ; that is for a given set of linkage dimensions and input crank angle, two possible angles exist for θ_3. This corresponds to the two possible configurations which the mechanism may have as mentioned earlier. Another important factor to note is that for a particular configuration, the sign of equation (3.39) may change during the course of its motion. As an example, a given mechanism with a specific configuration may exhibit a + sign in equation (3.39) for the range of $0 < \theta_2 < 180°$ and a - sign for $180° < \theta_2 < 360°$. With the configuration of its counterpart, a - sign will govern in the range of $0 < \theta_2 < 180°$ and a + sign for $180° < \theta_2 < 360°$. The exact prediction of when to use a + or - sign, however, is not known, but in this present application, its determination is accurately found by a 'trial and error' procedure which will be illustrated in the numerical solution section.

3.4 Investigation on the Mobility Criteria

With reference to the mobility analysis for nonjamming, many past studies have dealt with only four-bar mechanisms. With mechanisms of higher linkage numbers, the analysis of mobility becomes very difficult depending on its configuration.

In this section, an attempt is made to define a set of criteria for which the eight-link mechanism should satisfy in order to have a crank-rocker motion with no interference or jamming. Unlike that done for the four-link mechanism, though, the mobility criteria will not be of a definite form but will serve as a starting point in determining the requirements of the link lengths, even though leeways may exist which still satisfy the complete motion. The finding of the mobility criteria in a definite form for the eight-link mechanism is very difficult and is recommended as a separate topic for investigation.

To begin, recall the mobility criteria for the four-bar mechanism based on Grashof's condition. Grashof states that for the condition of nonjamming for a four-bar crank-rocker mechanism [16]:

1. The shortest link is the input link.
2. The sum of the lengths of its shortest and longest links is less than or equal to the sum of the length of its other two links.

In a more explicit form, Wilson et al. describe the criteria for the four-bar crank-rocker mechanism (Fig. 2.2) as [28]:

$$b < a$$

$$b < c$$

$$b < d$$

$$a+b < c+d$$

$$b+c < a+d$$

To obtain the mobility equations for the eight-link crank-rocker mechanism, a similar procedure is applied. Consider Figs. 3.2, 3.3 and 3.4, whose three independent loops, Loop I, Loop II and Loop III are defined. We can simulate each of these loops as a four-bar mechanism with a coupler link whose length varies with the input crank angle. Analagous to the four-bar linkage, if we take the first loop, for example, r_{21} represents the input crank, r_{33} and r_{41} combined is the coupler link, and r_{51} is the output rocker. Hence, length 'c' of the four-bar linkage is equivalent to:

$$c \rightarrow |\xi r_{33} + r_{41}| \qquad \text{where } -1\leq\xi\leq 1$$

and similarly for Loop II and III:

$$c \rightarrow |\xi r_{73} + r_{61}|$$

$$c \rightarrow |\xi r_{31} + r_{83}|$$

...since the angle between links r_{33} and r_{41}, for example, is not constant but changes during the course of motion. ξ is, thus, a function of the input crank angle. The given range of ξ, though, is of an extreme case whereas in actual application, ξ may be of a very small range depending on the choice of parameters.

Thus, the application of Grashof's condition to the eight-link mechanism results in the following mobility criteria:

$$r_{21} < r_{11}$$

$$r_{22} < r_{11}$$

$$r_{23} < r_{61}$$

$$r_{21} < |\xi r_{33} + r_{41}|$$

$$r_{22} < |\xi r_{73} + r_{61}|$$

$$r_{23} < |\xi r_{31} + r_{83}|$$

$$r_{21} < r_{51}$$

$$r_{22} < r_{52}$$

$$r_{23} < r_{71}$$

$$r_{11} + r_{21} < |\xi r_{33} + r_{41}| + r_{51}$$

$$r_{11} + r_{22} < |\xi r_{73} + r_{61}| + r_{52}$$

$$r_{61} + r_{23} < |\xi r_{31} + r_{83}| + r_{71}$$

$$r_{21} + |-\xi r_{33} + r_{41}| < r_{11} + r_{51}$$

$$r_{22} + |-\xi r_{73} + r_{61}| < r_{11} + r_{52}$$

$$r_{23} + |-\xi r_{31} + r_{83}| < r_{61} + r_{71}$$

where: $-1 < \xi < 1$

Satisfaction of these mobility criteria, however, will not provide a definite nonjamming situation, but will give a foundation and starting point in the selection of the link lengths. In actual fact, more leeway may be given to the selection of the link lengths since during the course of the entire motion of the input crank, ξ may not necessarily cover the entire range of -1 to +1, but only a portion. On the other hand, further constraints may be required due to the restrictions of link angles α_2, α_3, α_5 and α_7.

In the present application, approximate linkage dimensions are selected based on the use of the mentioned mobility criteria. If, however, these values do not fully satisfy the condition of a crank-rocker mechanism, then final adjustments are made by a method of 'trial and error' with the experimental model or by the condition of convergence in the numerical formulation.

CHAPTER 4

NUMERICAL SOLUTION OF THE MECHANISMS MOTION

CHAPTER 4

NUMERICAL SOLUTION OF THE MECHANISM'S MOTION

4.1 Computational Procedure

With the formulation of the coupler curve equations accomplished, the next procedure is its application and verification. Solving for the coupler curve equations requires extensive calculations and, thus, a digital computer is implemented. The final coupler curve equations consist of three nonlinear equations with three unknowns being x_j, y_j and m. The key tool used to solve these equations is the application of Newton's method for a system of nonlinear equations [6,29,30]. Given a set of three coupler curve equations of the form:

$$f_1(x_j,\ y_j,\ m) = 0$$

$$f_2(x_j,\ y_j,\ m) = 0$$

$$f_3(x_j,\ y_j,\ m) = 0$$

...then each of these equations can be expanded in Taylor's series form as:

$$f_i(x_j+\delta x_j, y_j+\delta y_j, m+\delta m) = f_i(x_j, y_j, m) + \frac{\partial f_i}{\partial x_j}\delta x_j + \frac{\partial f_i}{\partial y_j}\delta y_i + \frac{\partial f_i}{\partial m}\delta m \qquad i = 1,2,3 \qquad (4.1)$$

where, at a solution,

$$f_i(x_j+\delta x_j,\ y_j+\delta y_j,\ m+\delta m) \rightarrow 0$$

Arranging equations (4.1) into matrix form:

$$\begin{bmatrix} \frac{\partial f_1}{\partial x_j} & \frac{\partial f_1}{\partial y_j} & \frac{\partial f_1}{\partial m} \\ \frac{\partial f_2}{\partial x_j} & \frac{\partial f_2}{\partial y_j} & \frac{\partial f_2}{\partial m} \\ \frac{\partial f_3}{\partial x_j} & \frac{\partial f_3}{\partial y_j} & \frac{\partial f_3}{\partial m} \end{bmatrix} \begin{Bmatrix} \delta x_j \\ \delta y_j \\ \delta m \end{Bmatrix} = \begin{Bmatrix} -f_1 \\ -f_2 \\ -f_3 \end{Bmatrix} \qquad (4.2)$$

...the correction vector δ can be solved to obtain the next approximation to the unknown vector; that is:

$$[x_j,\ y_j,\ m]^T_{k+1} = [x_j,\ y_j,\ m]_k + [\delta x_j,\ \delta y_j,\ \delta m]_k \qquad (4.3)$$

where its correct solution is obtained when convergence is reached ($\delta \to 0$) [29].

The said iterative technique is utilized to solve equations (3.2) for the coupler point coordinate (x_j, y_j) and the output rocker angle θ_5 (via'm'), whereby the solution obtained at step number 'k' is considered as an initial approximate solution at the next step number 'k+1'. A good approximation on the initial point can be achieved by drawing and measurement or directly from the experimental model itself.

The implementation of Newton's method for a system of nonlinear equations, though, requires that the first derivatives of the function f with respect to x_j, y_j and m be calculated to form the Jacobian matrix of equation (4.2). The following is a list of the derivatives where subscripts x, y and m indicate what variable the function is partially differentiated with respect to:

$$f_x^n = (a_{0_x}^n + a_{1_x}^n m + a_{2_x}^n m^2) \qquad n = 1,2,3$$

$$f_y^n = (a_{0_y}^n + a_{1_y}^n m + a_{2_y}^n m^2)$$

$$f_m^n = (a_1^n + 2a_2^n m)$$

$$a_{0_x}^n = \sin\theta_{3_x}(G_{1n} + G_{3n}) + \sin\theta_3(G_{1n_x} + G_{3n_x}) - (L_{1n_x} + L_{3n_x})$$

$$a_{0_y}^n = \sin\theta_{3_y}(G_{1n} + G_{3n}) + \sin\theta_3(G_{1n_y} + G_{3n_y}) - (L_{1n_y} + L_{3n_y})$$

$$a_{1_x}^n = 2(\sin\theta_{3_x} G_{2n} + \sin\theta_3 G_{2n_x} - L_{2n_x})$$

$$a_{1_y}^n = 2(\sin\theta_{3_y} G_{2n} + \sin\theta_3 G_{2n_y} - L_{2n_y})$$

$$a_{2_x}^n = \sin\theta_{3_x}(G_{1n}-G_{3n}) + \sin\theta_3(G_{1n_x}-G_{3n_x}) - (L_{1n_x}-L_{3n_x})$$

$$a_{2_y}^n = \sin\theta_{3_y}(G_{1n}-G_{3n}) + \sin\theta_3(G_{1n_y}-G_{3n_y}) - (L_{1n_y}-L_{3n_y})$$

$$\sin\theta_{3_x} = [Q_{0_x}/d_3(S_0^2+1)] - [Q_0/d_3(S_0^2+1)^2]2S_0S_{0_x}$$

$$\sin\theta_{3_y} = [Q_{0_y}/d_3(S_0^2+1)] - [Q_0/d_3(S_0^2+1)^2]2S_0S_{0_y}$$

$$L_{11_x} = 2Q_1Q_{1_x} + 2Q_2Q_{2_x} - a_1Q_x$$

$$L_{11_y} = 2Q_1Q_{1_y} + 2Q_2Q_{2_y} - a_1Q_y$$

$$L_{12_x} = 2Q_3Q_{3_x} + 2Q_4Q_{4_x} - a_2Q_x$$

$$L_{12_y} = 2Q_3Q_{3_y} + 2Q_4Q_{4_y} - a_2Q_y$$

$$L_{13_x} = 2Q_5Q_{5_x} + 2Q_6Q_{6_x} - a_3Q_x$$

$$L_{13_y} = 2Q_5Q_{5_y} + 2Q_6Q_{6_y} - a_3Q_y$$

$$L_{21_x} = 2d_5Q_{2_x}$$

$$L_{21_y} = 2d_5Q_{2_y}$$

$$L_{22_x} = 2d_5(S_5Q_{3_x} - C_5Q_{4_x})$$

$$L_{22_y} = 2d_5(S_5Q_{3_y} - C_5Q_{4_y})$$

$$L_{23_x} = 2d_5(C_5Q_{6_x} - S_5Q_{5_x})$$

$$L_{23_y} = 2d_5(C_5Q_{6_y} - S_5Q_{5_y})$$

$$L_{31_x} = 2d_5Q_{1_x}$$

$$L_{31_y} = 2d_5Q_{1_y}$$

$$L_{32_x} = -2d_5(C_5Q_{3_x} + S_5Q_{4_x})$$

$$L_{32_y} = -2d_5(C_5Q_{3_y} + S_5Q_{4_y})$$

$$L_{33_x} = 2d_5(C_5Q_{5_x} + S_5Q_{6_x})$$

$$L_{33_y} = 2d_5(C_5Q_{5_y} + S_5Q_{6_y})$$

$$G_{11_x} = 2d_3(U_{1_x}Q_1 + U_1Q_{1_x} + V_{1_x}Q_2 + V_1Q_{2_x} + a_1Q_{0_x})$$

$$G_{11_y} = 2d_3(U_{1_y}Q_1 + U_1Q_{1_y} + V_{1_y}Q_2 + V_1Q_{2_y} + a_1Q_{0_y})$$

$$G_{12_x} = 2d_3(U_{2_x}Q_3 + U_2Q_{3_x} + V_{2_x}Q_4 + V_2Q_{4_x} + a_2Q_{0_x})$$

$$G_{12_y} = 2d_3(U_{2_y}Q_3 + U_2Q_{3_y} + V_{2_y}Q_4 + V_2Q_{4_y} + a_2Q_{0_y})$$

$$G_{13_x} = 2d_3(a_3Q_{0_x} - U_{3_x}Q_5 - U_{3_x}Q_{5_x} - V_{3_x}Q_6 - V_3Q_{6_x})$$

$$G_{13_y} = 2d_3(a_3Q_{0_y} - U_{3_y}Q_5 - U_3Q_{5_y} - V_{3_y}Q_6 - V_3Q_{6_y})$$

$$G_{21_x} = 2d_3d_5V_{1_x}$$

$$G_{21_y} = 2d_3d_5V_{1_y}$$

$$G_{22_x} = 2d_3d_5(U_{2_x}S_5 - V_{2_x}C_5)$$

$$G_{22_y} = 2d_3d_5(U_{2_y}S_5 - V_{2_y}C_5)$$

$$G_{23_x} = 2d_3d_5(U_{3_x}S_5 - V_{3_x}C_5)$$

$$G_{23_y} = 2d_3d_5(U_{3_y}S_5 - V_{3_y}C_5)$$

$$G_{31_x} = 2d_3d_5\,U_{1_x}$$

$$G_{31_y} = 2d_3d_5U_{1_y}$$

$$G_{32_x} = -2d_3d_5(U_{2_x}C_5 + V_{2_x}S_5)$$

$$G_{32_y} = -2d_3d_5(U_{2_y}C_5 + V_{2_y}S_5)$$

$$G_{33_x} = -2d_3d_5(U_{3_x}C_5 + V_{3_x}S_5)$$

$$G_{33_y} = -2d_3d_5(U_{3_y}C_5 + V_{3_y}S_5)$$

$$Q_{0_x} = S_{0_x}Z_0 + S_0Z_{0_x} \pm \frac{(d_3^2S_0S_{0_x} - Z_0Z_{0_x})}{\sqrt{d_3^2(S_0^2+1) - Z_0^2}}$$

$$Q_{0_y} = S_{0_y}Z_0 + S_0Z_{0_y} \pm \frac{(d_3^2S_0S_{0_y} - Z_0Z_{0_y})}{\sqrt{d_3^2(S_0^2+1) - Z_0^2}}$$

$$Q_x = 2Q_7Q_{7_x} + 2Q_8Q_{8_x}$$

$$Q_y = 2Q_7Q_{7_y} + 2Q_8Q_{8_y}$$

$$Q_{1_x} = C_3Z_{0_x}$$

$$Q_{1_y} = C_3Z_{0_y}$$

$$Q_{2_x} = S_3Z_{0_x}$$

$$Q_{2_y} = S_3Z_{0_y}$$

$$Q_{3_x} = C_8'Z_{0_x} + h_{1_x}$$

$$Q_{3_y} = C_8' Z_{0_y} + h_{1_y}$$

$$Q_{4_x} = S_8' Z_{0_x} + h_{2_x}$$

$$Q_{4_y} = S_8' Z_{0_y} + h_{2_y}$$

$$Q_{5_x} = -C_8 Z_{o_x} + h_{3_x}$$

$$Q_{5_y} = -C_8 Z_{0_y} + h_{3_y}$$

$$Q_{6_x} = -S_8 Z_{0_x} + h_{4_x}$$

$$Q_{6_y} = -S_8 Z_{0_y} + h_{4_y}$$

$$Q_{7_x} = -Z_{0_x} + X_{0_x}$$

$$Q_{7_y} = -Z_{0_y} + X_{0_y}$$

$$Q_{8_x} = y_{0_x}$$

$$Q_{8_y} = y_{0_y}.$$

$$U_{1_x} = C_3 S_{0_x}$$

$$U_{1_y} = C_3 S_{0_y}$$

$$V_{1_x} = S_3 S_{0_x}$$

$$V_{1_y} = S_3 S_{0_y}$$

$$U_{2_x} = C_8' S_{0_x}$$

$$U_{2_y} = C_8' S_{0_y}$$

$$V_{2_x} = S_8' S_{0_x}$$

$$V_{2_y} = S_8' S_{0_y}$$

$$U_{3_x} = U_{2_x} / (\ell_7 + 1)$$

$$U_{3_y} = U_{2_y} / (\ell_7 + 1)$$

$$V_{3_x} = V_{2_x} / (\ell_7 + 1)$$

$$V_{3_y} = V_{2_y} / (\ell_7 + 1)$$

$$h_{1_x} = (-a x_{0_x} + S_8' y_{0_x})$$

$$h_{1_y} = (-a x_{0_y} + S_8' y_{0_y})$$

$$h_{2_x} = (-S_8' x_{0_x} - a y_{0_x})$$

$$h_{2_y} = (-S_8' x_{0_y} - a y_{0_y})$$

$$h_{3_x} = (b x_{0_x} - S_8 y_{0_x})$$

$$h_{3_y} = (b x_{0_y} - S_8 y_{0_y})$$

$$h_{4_x} = (S_8 x_{0_x} + b y_{0_x})$$

$$h_{4_y} = (S_8 x_{0_y} + b y_{0_y})$$

$$Z_{0_x} = \frac{(x_{0_x} + S_{0_x} y_0 + S_0 y_{0_x})}{2} - \frac{(d_3^2 - d_8^2)}{2x_0^2} X_{0_x}$$

$$Z_{0_x} = \frac{(x_{0_y} + S_{0_y} y_0 + S_0 y_{0_y})}{2} - \frac{(d_3^2 - d_8^2)}{2x_0^2} X_{0_y}$$

$$S_{0_x} = \frac{y_{0_x}}{x_0} - \frac{y_0}{x_0^2} X_{0_x}$$

$$S_{0_y} = \frac{y_{0_y}}{x_0} - \frac{y_0}{X_0^2} X_{0_y}$$

$$X_{0_x} = \cos\gamma / r_{11}$$

$$X_{0_y} = -\sin\gamma / r_{11}$$

$$y_{0_x} = \sin\gamma / r_{11}$$

$$y_{0_y} = \cos\gamma / r_{11}$$

$$X_{j_x} = \cos\gamma$$

$$X_{j_y} = -\sin\gamma$$

$Y_{j_x} = \sin\gamma$

$Y_{j_y} = \cos\gamma$

The complete numerical calculation of the coupler point coordinate and angular position of each link is performed by a computer program called "LINKAGE". The complete listing of this program package is given in the Appendix at the end of this thesis. All that is needed to do is to input the link dimensions and an approximate initial value for the coupler point coordinate (x_j, y_j) and output rocker angle θ_5 corresponding to zero crank angle. The rest is done by the computer whereby the coupler point coordinate and angular orientation of each link is computed for the entire cycle of the input crank. The angular orientation of links 3,4,6,7,8 are computed by the use of equations (3.25) to (3.29). Other useful information is provided such as the number of iterations (IC) taken for convergence, the residual error (ERROR) remaining when convergence is attained. Moreover, the program is incorporated with the capability of sensing any jamming situation, where, when the number of iterations exceed the limit, then the solution does not converge due to an improper selection of the linkage dimensions, resulting in a locking condition.

In the computer program, a major problem was encountered which led to the addition of an alternative coupler curve equation. This involves equation (3.39), particularly with the term under the square root:

$$\sin\theta_3 = -(B_0/2) \pm \sqrt{(B_0^2/4) - A_0}$$

This term can never have a negative value for a correct value of x_j, y_j and m. However, it is possible that before convergence to a correct solution is reached, the term under the square root may temporarily attain a negtive value during one of the iteration cycles. The instant that this term reaches negative, a complete stoppage of the computer execution occurs due to a default by an arithmetic indefinite. Thus, an alternate means is required when encountering this situation. This is achieved by rearranging the coupler curve equations, whereby equation (3.37) is directly substituted into equation (3.33). Upon rearranging, a higher order coupler curve equation is attained of following form:

$$f_n = \sum_{p=1}^{5} a^n_{2(p-1)} m^{2(p-1)} = 0 \qquad n = 1,2,3 \qquad (4.4)$$

where:

$$m = \tan(\theta_5/2)$$

$$a^n_0 = (E^n_{11} + 2E^n_{13} + E^n_{33})$$

$$a^n_2 = 4(E^n_{12} + E^n_{23})$$

$$a^n_4 = 2(E^n_{11} + 2E^n_{22} - E^n_{33})$$

$$a^n_6 = 4(E^n_{12} - E^n_{23})$$

$$a^n_8 = (E^n_{11} - 2E^n_{13} + E^n_{33})$$

$$E^n_{st} = 2L_{sn}L_{tn} + B_0(L_{sn}G_{tn} + L_{tn}G_{sn}) + 2A_0G_{sn}G_{tn}$$

$$s = 1,2,3 \quad ; \quad t = 1,2,3$$

with its corresponding first derivatives being:

$$f^n_x = a^n_{8_x} m^8 + a^n_{6_x} m^6 + a^n_{4_x} m^4 + a^n_{2_x} m^2 + a^n_{0_x} \qquad n = 1,2,3$$

$$f^n_y = a^n_{8_y} m^8 + a^n_{6_y} m^6 + a^n_{4_y} m^4 + a^n_{2_y} m^2 + a^n_{0_y}$$

$$f^n_m = 8a^n_8 m^7 + 6a^n_6 m^5 + 4a^n_4 m^3 + 2a^n_2 m$$

$$a^n_{0_x} = E^n_{11_x} + 2E^n_{13_x} + E^n_{33_x}$$

$$a^n_{0_y} = E^n_{11_y} + 2E^n_{13_y} + E^n_{33_y}$$

$$a^n_{2_x} = 4(E^n_{12_x} + E^n_{23_x})$$

$$a^n_{2_y} = 4(E^n_{12_y} + E^n_{23_y})$$

$$a^n_{4_x} = 2(E^n_{11_x} + 2E^n_{22_x} - E^n_{33_x})$$

$$a^n_{4_y} = 2(E^n_{11_y} + 2E^n_{22_y} - E^n_{33_y})$$

$$a^n_{6_x} = 4(E^n_{12_x} - E^n_{23_x})$$

$$a^n_{6_y} = 4(E^n_{12_y} - E^n_{23_y})$$

$$a^n_{8_x} = (E^n_{11_x} - 2E^n_{13_x} + E^n_{33_x})$$

$$a^n_{8_y} = (E^n_{11_y} - 2E^n_{13_y} + E^n_{33_y})$$

$$E^n_{jk_x} = 2L_{jn_x}L_{kn} + 2L_{jn}L_{kn_x} + B_{0_x}(L_{jn}G_{kn} + L_{kn}G_{jn})$$
$$+ 2A_{0_x}G_{jn}G_{kn} + B_0(L_{jn_x}G_{kn} + L_{jn}G_{kn_x} + L_{kn_x}G_{jn} + L_{kn}G_{jn_x})$$
$$+ 2A_0(G_{jn_x}G_{kn} + G_{jn}G_{kn_x})$$

$$E^n_{jk_y} = 2L_{jn_y}L_{kn} + 2L_{jn}L_{kn_y} + B_{0_y}(L_{jn}G_{kn} + L_{kn}G_{jn})$$
$$+ 2A_{0_y}G_{jn}G_{kn} + B_0(L_{jn_y}G_{kn} + L_{jn}G_{kn_y} + L_{kn_y}G_{jn} + L_{kn}G_{jn_y})$$
$$+ 2A_0(G_{jn_y}G_{kn} + G_{jn}G_{kn_y})$$

$$Q_{0_x} = S_{0_x}Q_7 + S_0Q_{7_x} - Q_{8_x}$$

$$Q_{0_y} = S_{0_y}Q_7 + S_0Q_{7_y} - Q_{8_y}$$

$$A_{0_x} = [Q_x/d_3^2(S_0^2 + 1)] - [2QS_0S_{0_x}/d_3^2(S_0^2 + 1)^2]$$

$$A_{0_y} = [Q_y/d_3^2(S_0^2 + 1)] - [2QS_0S_{0_y}/d_3^2(S_0^2 + 1)^2]$$

$$B_{0_x} = [2Q_{0_x}/d_3^2(S_0^2 + 1)] - [4S_0S_{0_x}Q_0/d_3(S_0^2 + 1)^2]$$

$$B_{0_y} = [2Q_{0_y}/d_3^2(S_0^2 + 1)] - [4S_0S_{0_y}Q_0/d_3^2(S_0^2 + 1)^2]$$

All remaining terms are as defined earlier.

Equation (4.4) is identical to equation (3.38) with the only difference being its arrangement. Both will give the same solution. Although, the modified equation eliminates any problem encountered with the square root of a negative number, its implementation requires considerably much more computer time and is thus used only when equation (3.39) fails.

The accurate prediction of whether equation (3.39) attains a + or - sign is also considered within the program. Its correspondence to the right configuration is based on the good approximate selection of the initial values (θ_5, x_j and y_j) which match with this particular configuration. If the wrong sign is assigned, convergence will not occur and once the set sublimit of number of iterations is reached, the sign is changed to the correct one. Also, in the case where the sign changes during the course of motion, this is taken care of by monitoring its assemblage error (ER1). To explain this, suppose the sign is not changed where it is supposed to. Then the computation may converge but to an incorrect solution. This results in a solution where the linkage mechanism will not assemble. Hence, a change in sign is triggered when this is sensed. This assemblage error is tested by calculating the linear distance between points D and G (Fig. 2.1) based on the computer results and comparing it with the true value of r_{83}.

A generalized flowchart depicting the computer program is illustrated in Fig. 4.1 with the folloiwng notations defined as:

IC : number of iterations

SS : sign (+ or -) of equation (3.39)

AX1 : value of term under square root in equation (3.39)

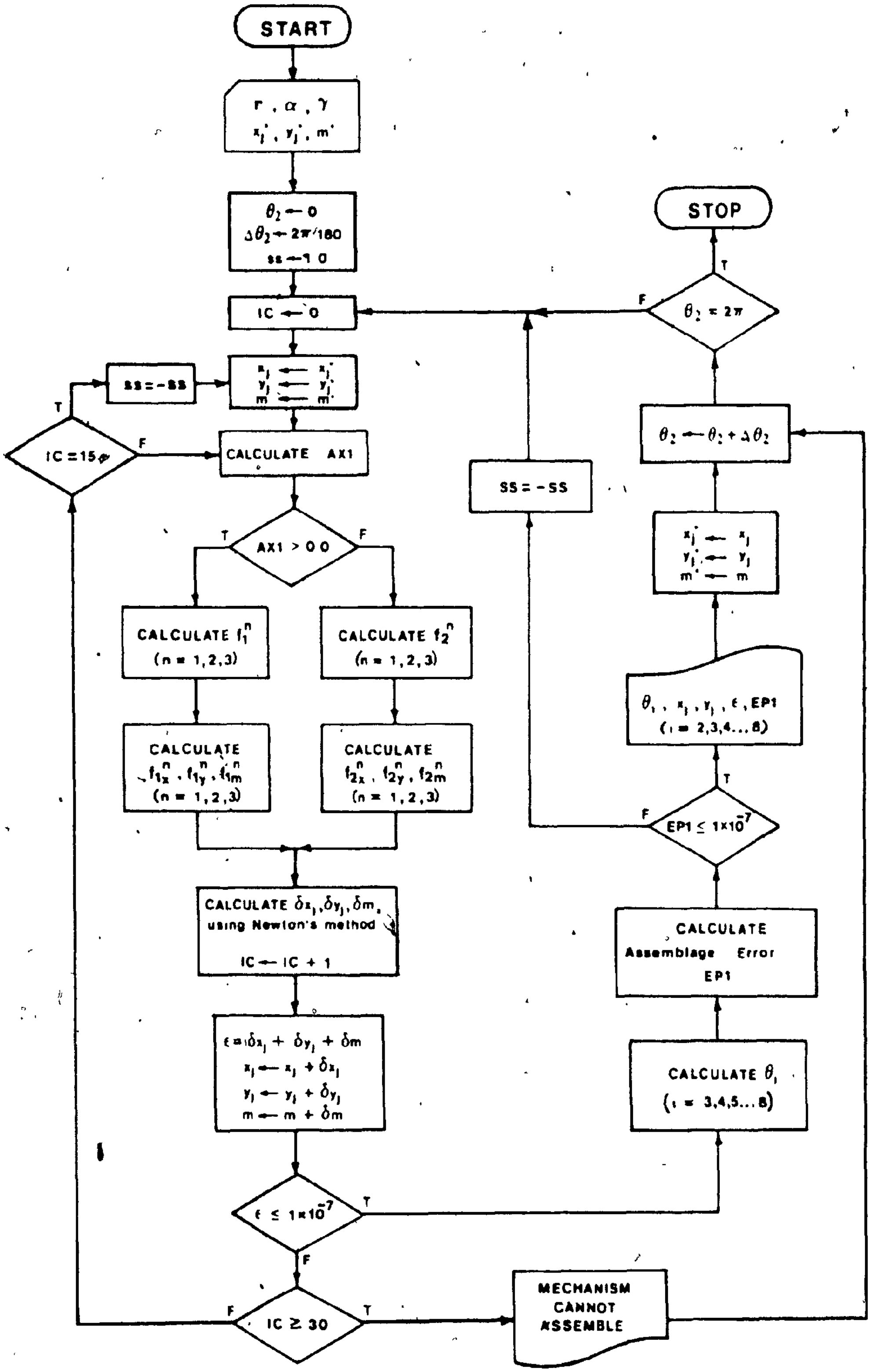


Fig. 4.1: Generalized Flow Chart of the Computational Procedure

ε : residual error from Newton's method

EP1 : assemblage error

f_1 : coupler curve equation (3.38) (Approach 1)

f_2 : coupler curve equation (4.4) (Approach 2)

AP : inidication of whether coupler curve equation (3.38) or (4.4) was used (AP=1 or AP=2, respectively).

4.2 Experimental Procedure

Figure 4.2 illustrates the experimental model constructed to simulate the eight-link mechanism. Its feature consists of fully adjustable links whose lengths and angles can be varied and fixed to the user's taste by a simple turn of the screws. A total of eighteen possible adjustments are available to simulate the design parameters. A writing implement is attached to the coupler link to scribe the coupler curve.

The entire model is constructed out of aluminum for its durability and lightweight whereas all revolute joints are comprised of lubricated brass journal bearings. Although, play and backlash may exist in the joints due to manufacturing errors, its resulting deviation is very small and negligible. Moreover, with its motion still retaining a planar characteristic, the mechanism is built in a multi-levelled fashion with extended ground supports so as to avoid any problems of interference or interaction between the linkages.

The key purpose of this experimental model is to verify the coupler curve equations where its result serves as the basis of comparison with that obtained from the computer. The linkages, however, are not

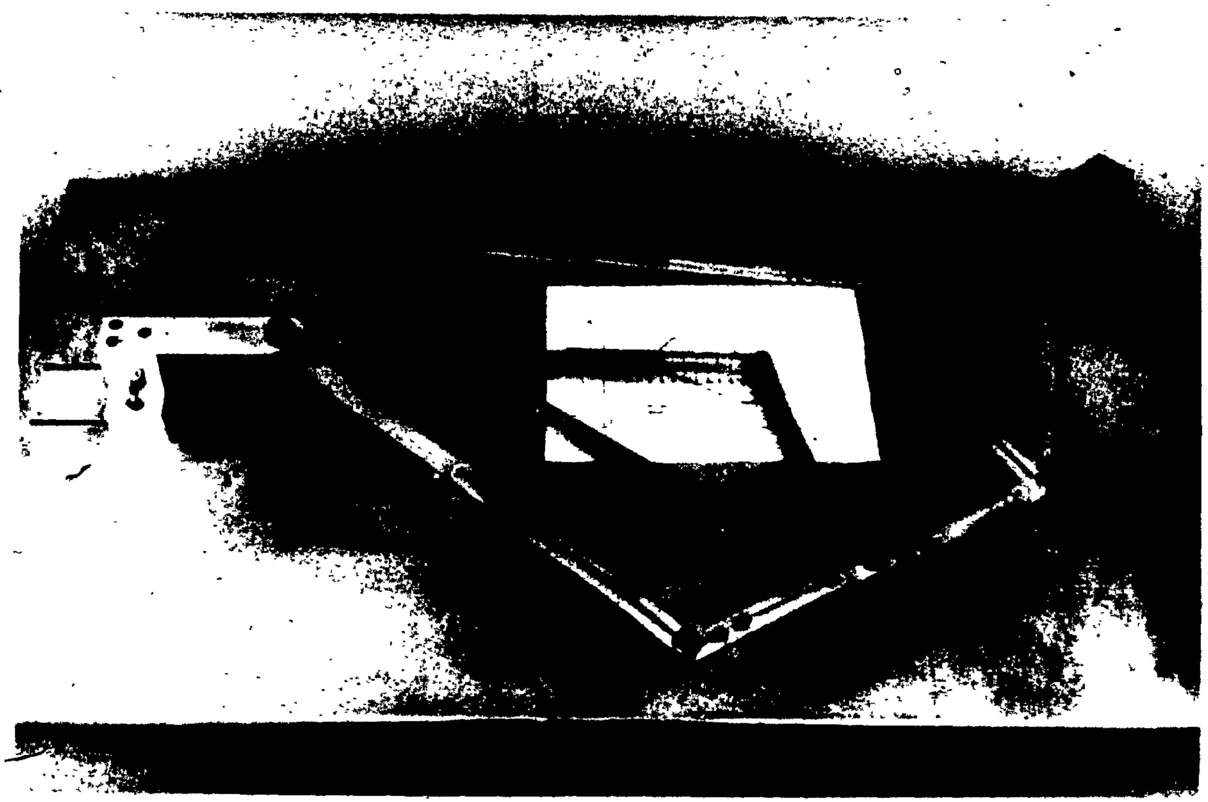


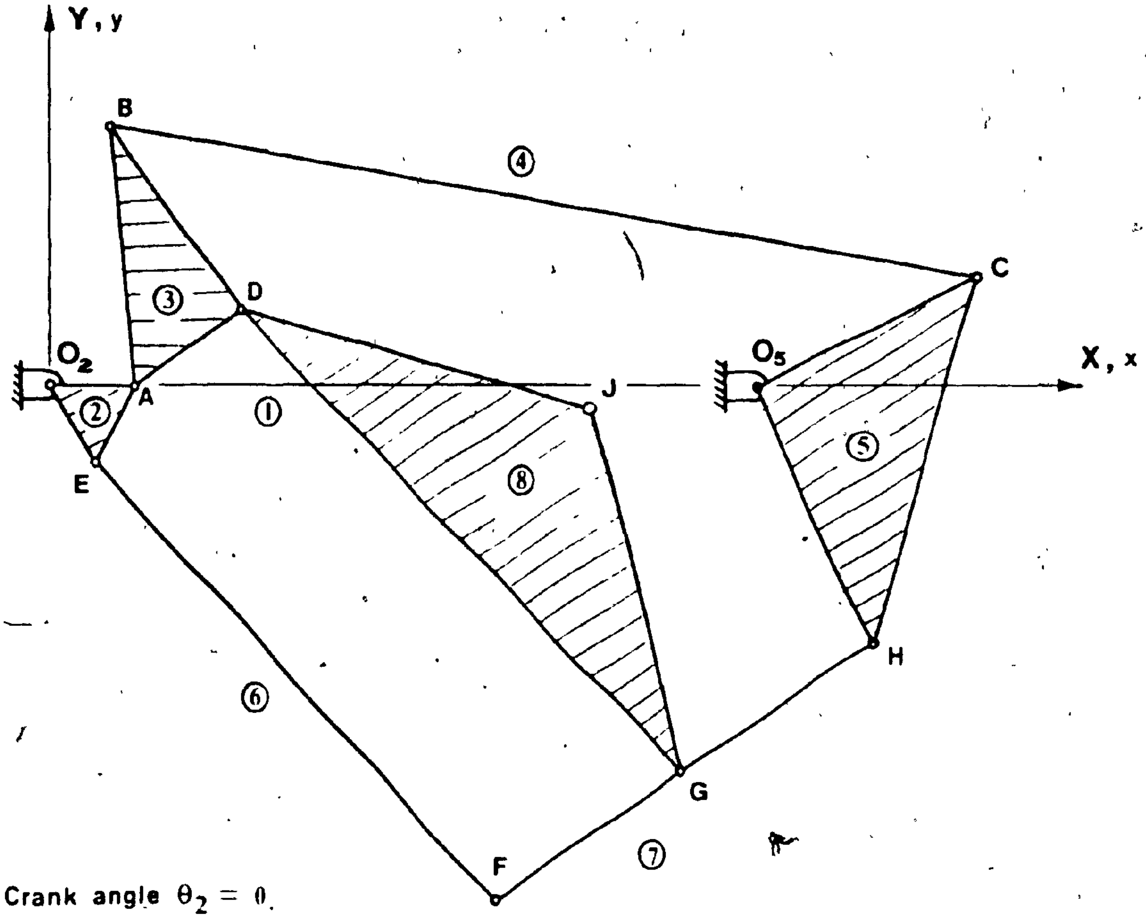
Fig. 4.2 The experimental Model of the Eight-Link Mechanism

infinitely variable (though it is mathematically feasible) where their lengths and angles can only be adjusted to within their physical capacity. As a consequence, the selection of the samples cases must be made with discretion and in keeping within the limits and bounds of the mechanism.

4.3 Results and Dicusssion

Figure 4.3 illustrates a schematic of the sample mechanism selected as one of our test cases. Having inputed the necessary data needed to describe the mechanism into the computer program, results indicate that the input crank is capable of making a full revolution without any locking or jamming occurring. These results are included with the computer listing in the Appendix. This sample case is further applied and tested on the experimental model. The coupler point coordinates obtained from the computer results are then plotted simultaneously with that obtained from the experimental model as shown in Fig. 4.4. An important conclusion can be deduced from this graph and that is, the coupler curve equations are indeed correct. Both, the theoretical and experimental coupler curves show an exact match. Furthermore, the experimental model showed that the angular position of each link as a function of the input crank angle corresponded exactly as that indicated by the computer results.

To further confirm the coupler curve equations, more sample cases were selected and tested. Figure 4.5 to Fig. 4.16 illustrate the resulting coupler curves for each test case with the corresponding linkage dimensions listed in Table 4.1. Each case shows an exact match between



Crank angle $\theta_2 = 0$.

$\gamma = 0°$

Mechanism Dimensions

link no. (i)	r_{i1} (mm)	r_{i2} (mm)	r_{i3} (mm)	α_i
1	500	—	—	—
2	60	60	60	300°
3	90	160	184	270°
4	630	—	—	—
5	180	200	269	270°
6	420	—	—	—
7	160	160	320	180°
8	260	260	450	120°

Fig. 4.3: Illustrated Sample Mechanism of Test Case (1)

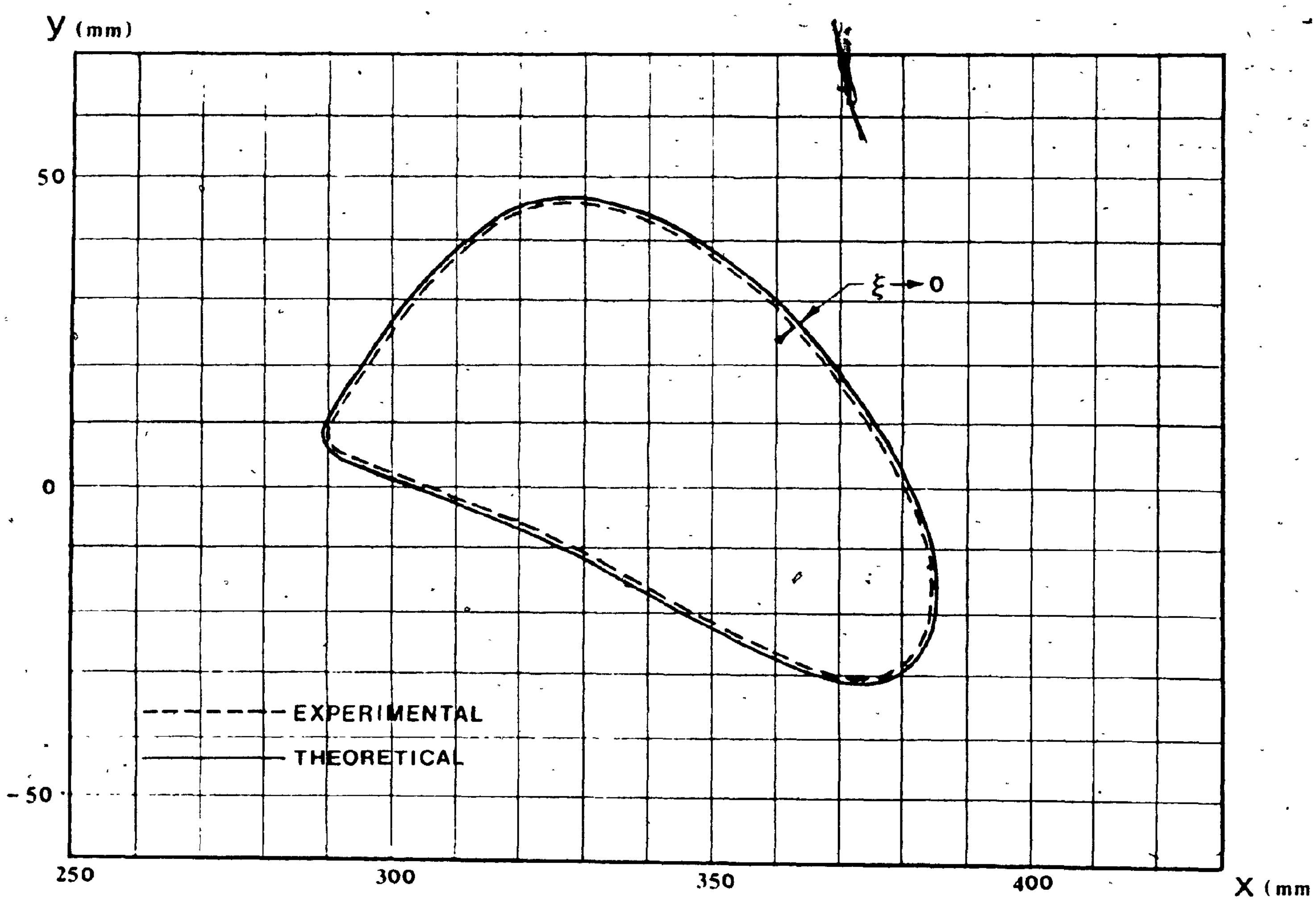


Fig. 4.4: Coupler Curve Result : Test Case (1)

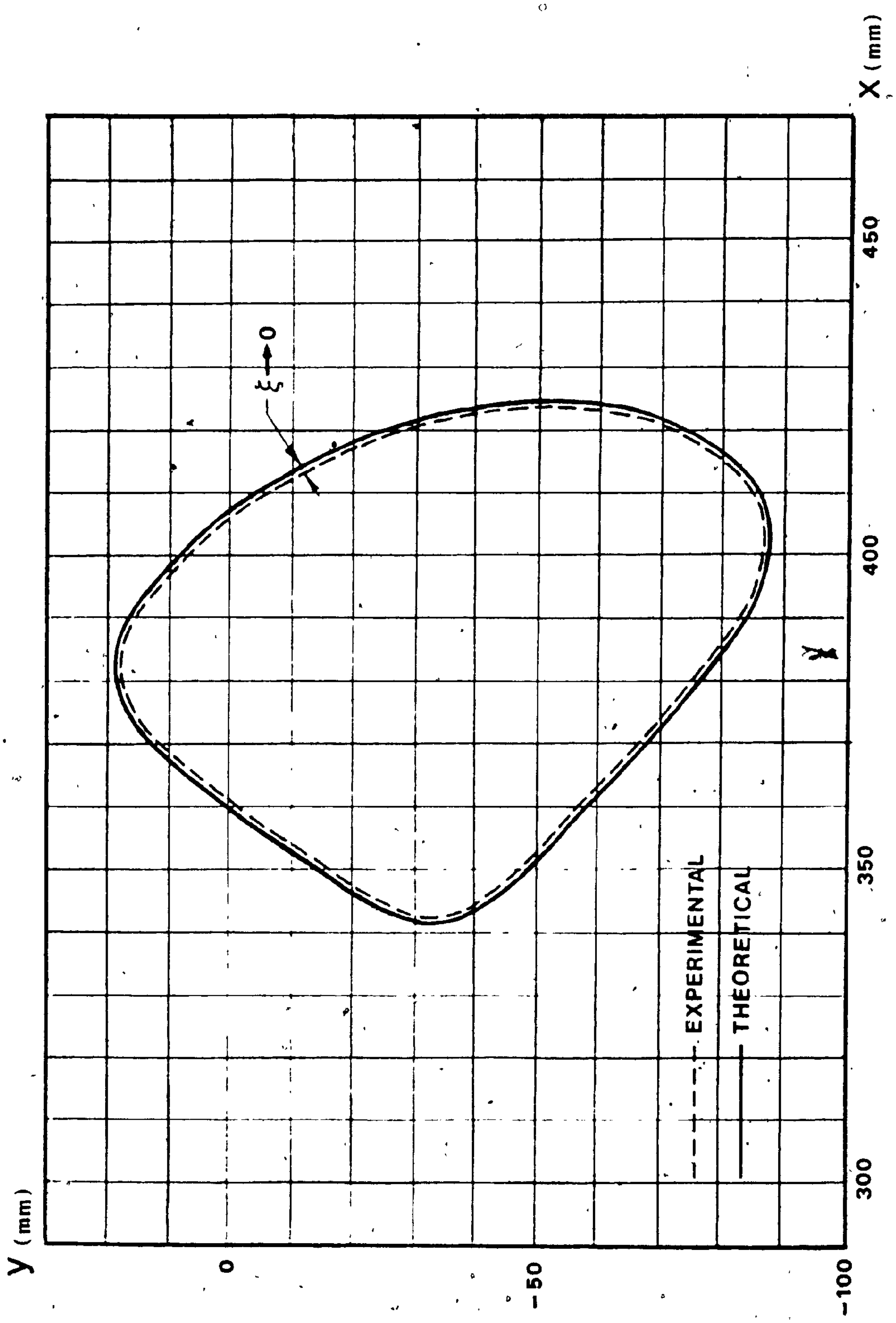


Fig. 4.5: Coupler Curve Result : Test Case (2)

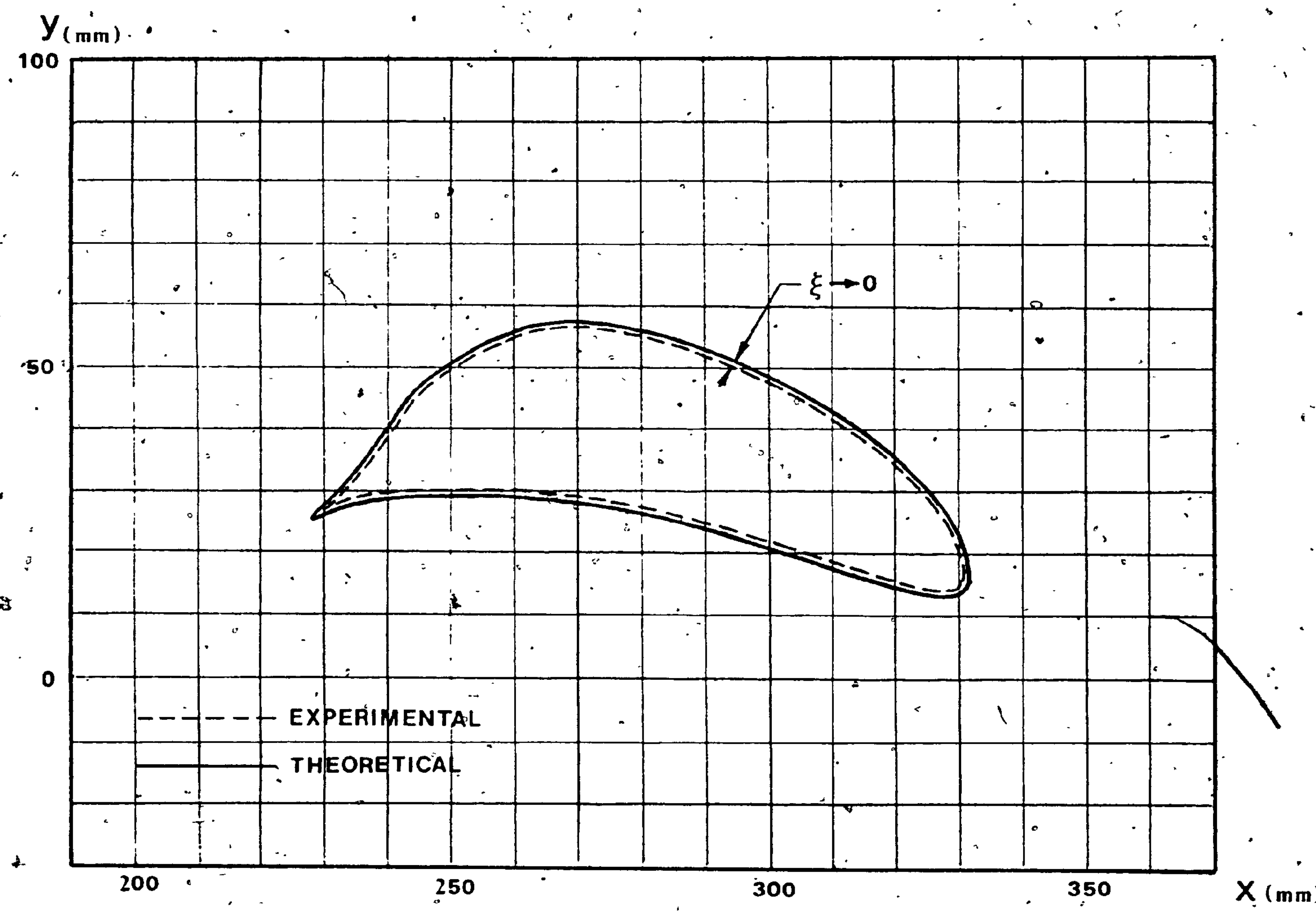


Fig. 4.6: Coupler Curve Result : Test Case (3)

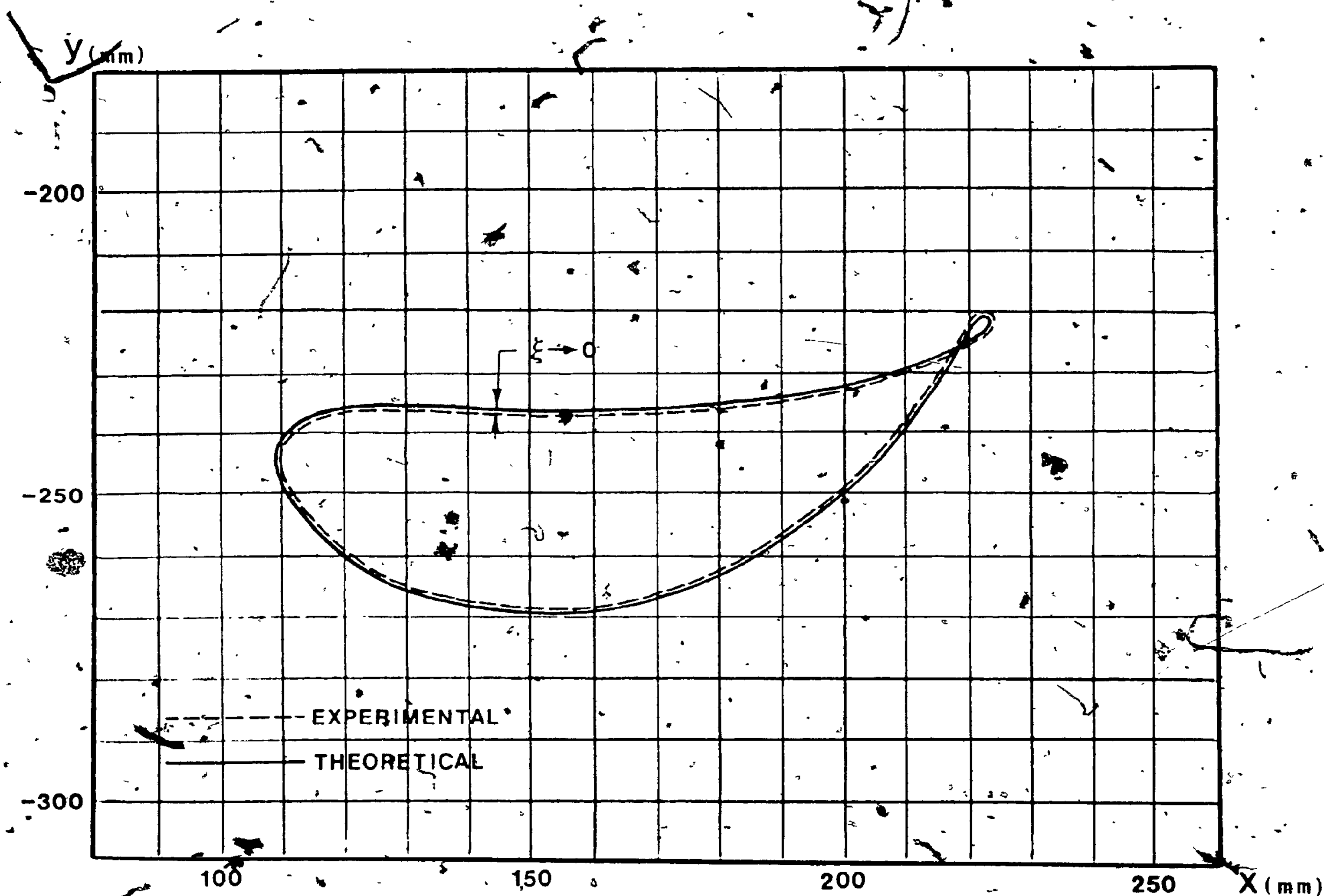


Fig. 4.7: Coupler Curve Results : Test Case (4)

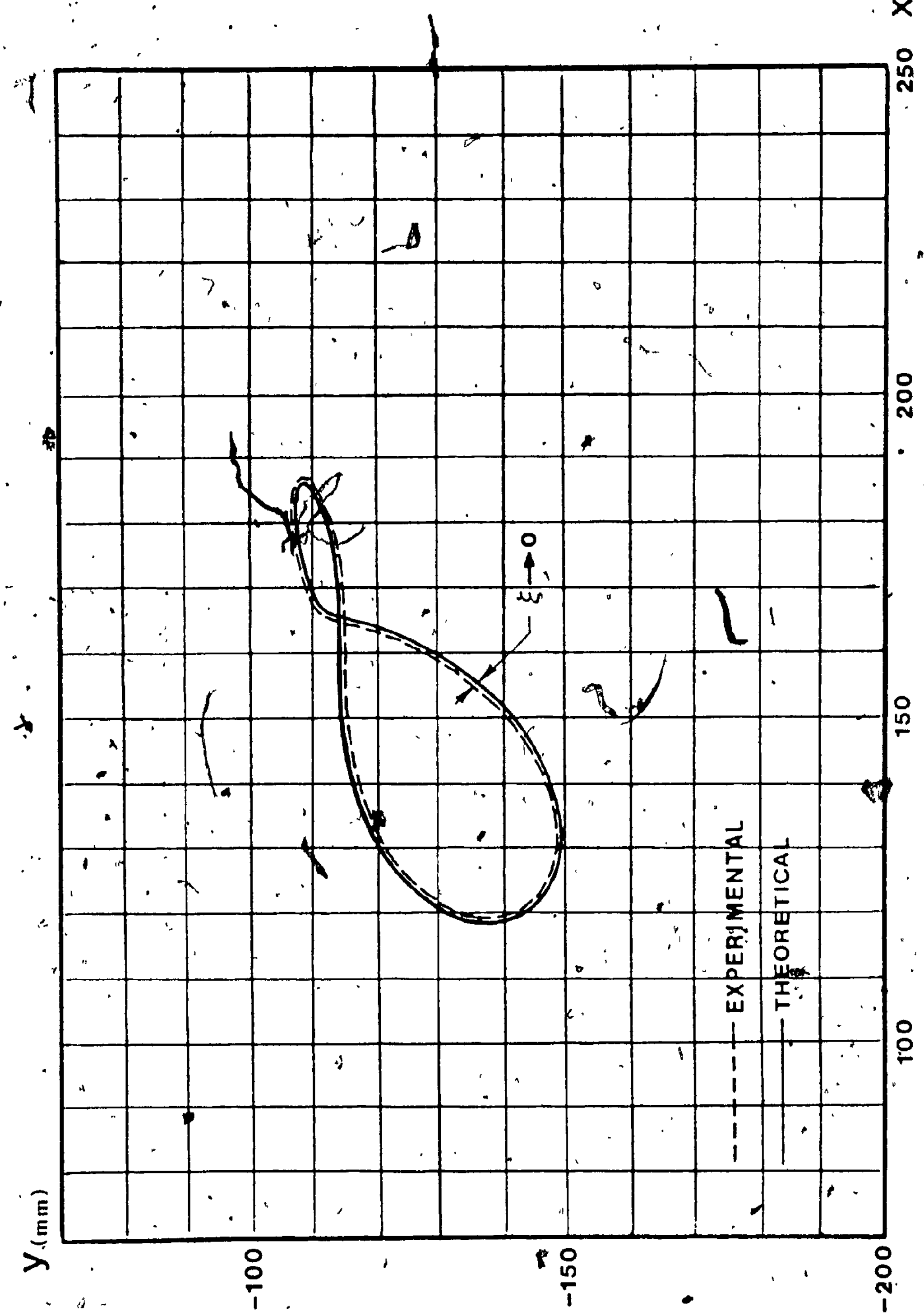


Fig. 4.8: Coupler Curve Results : Test Case (5)

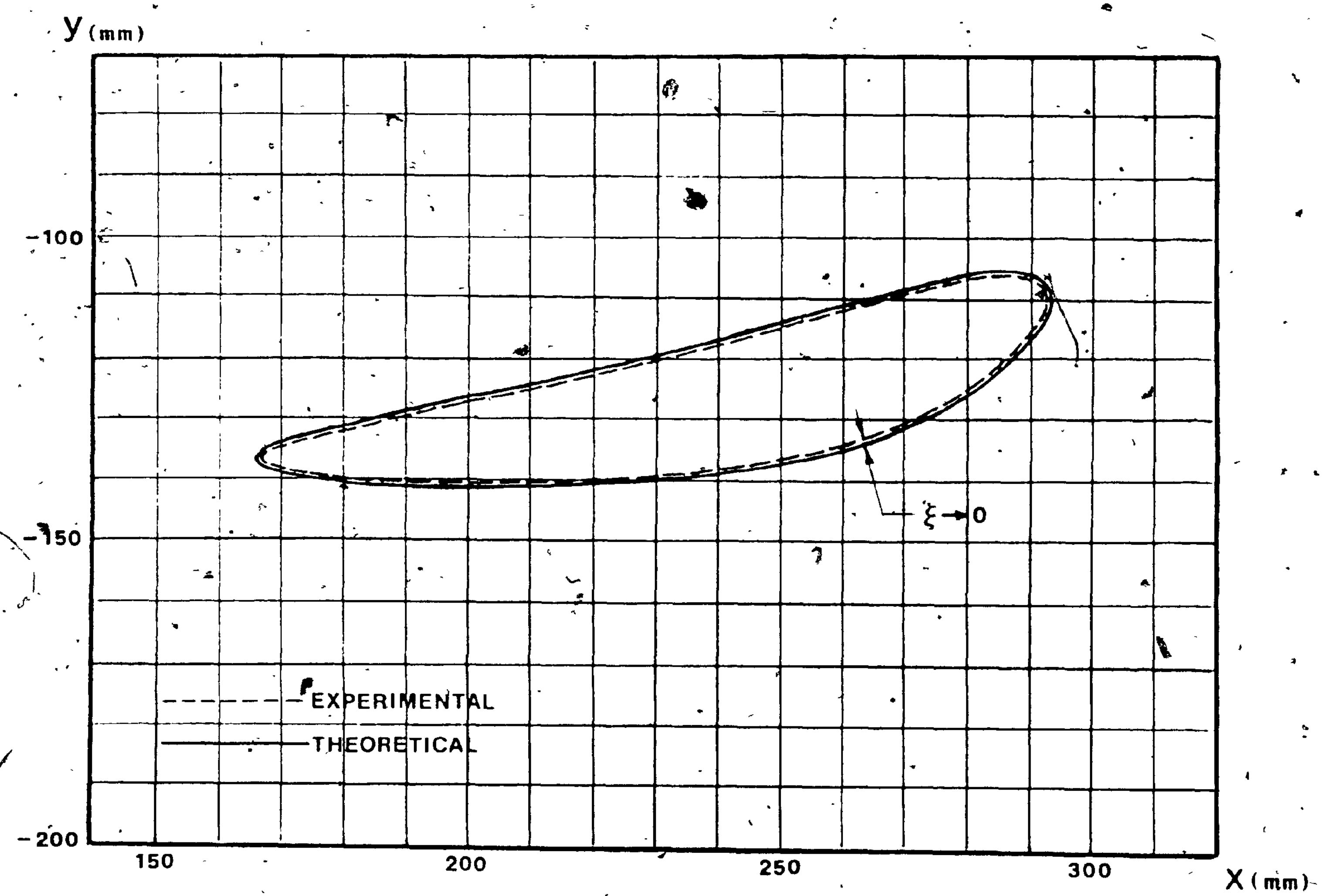


Fig. 4.9: Coupler Curve Result : Test Case (6)

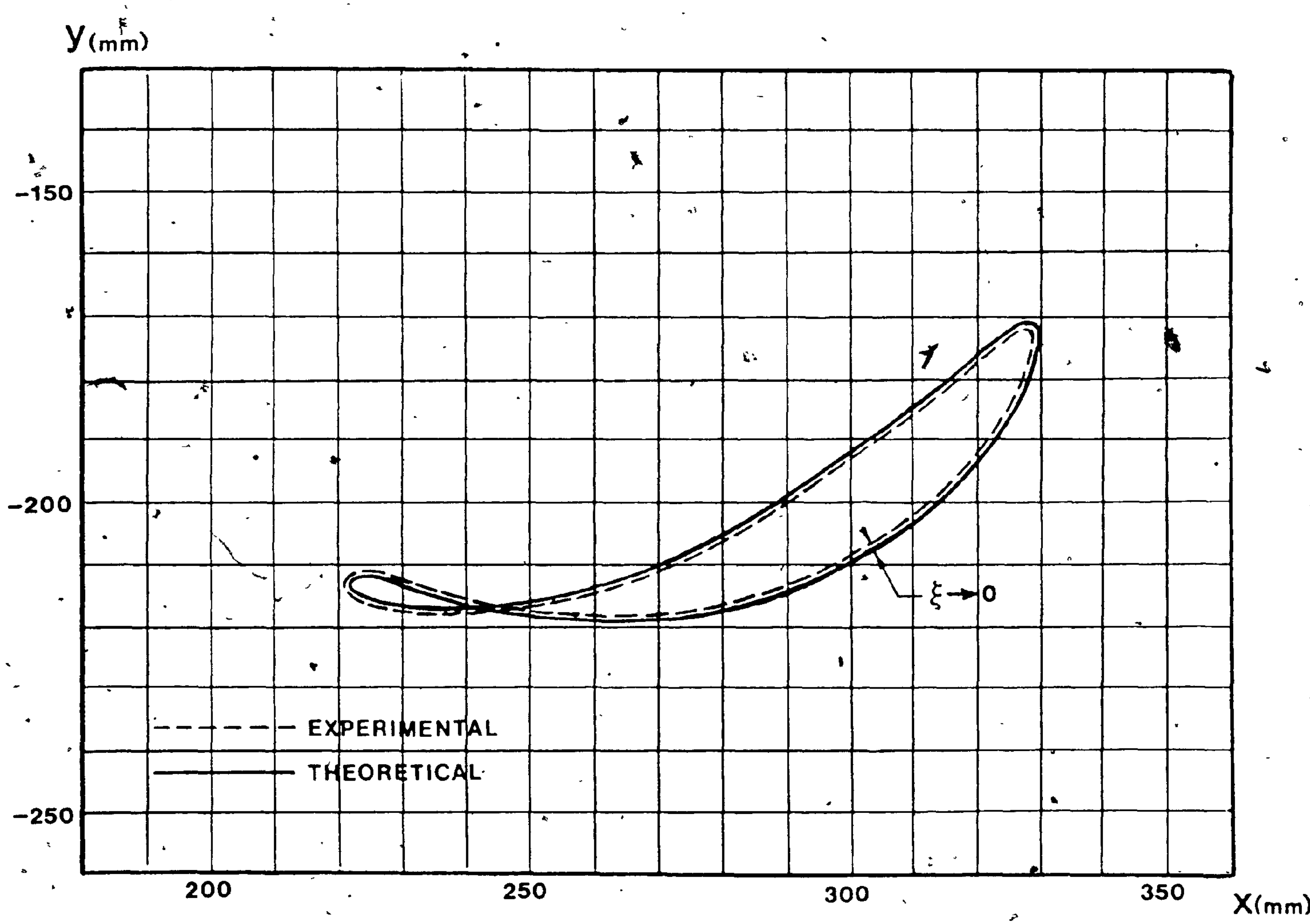


Fig. 4.10: Coupler Curve Result : Test Case (7)

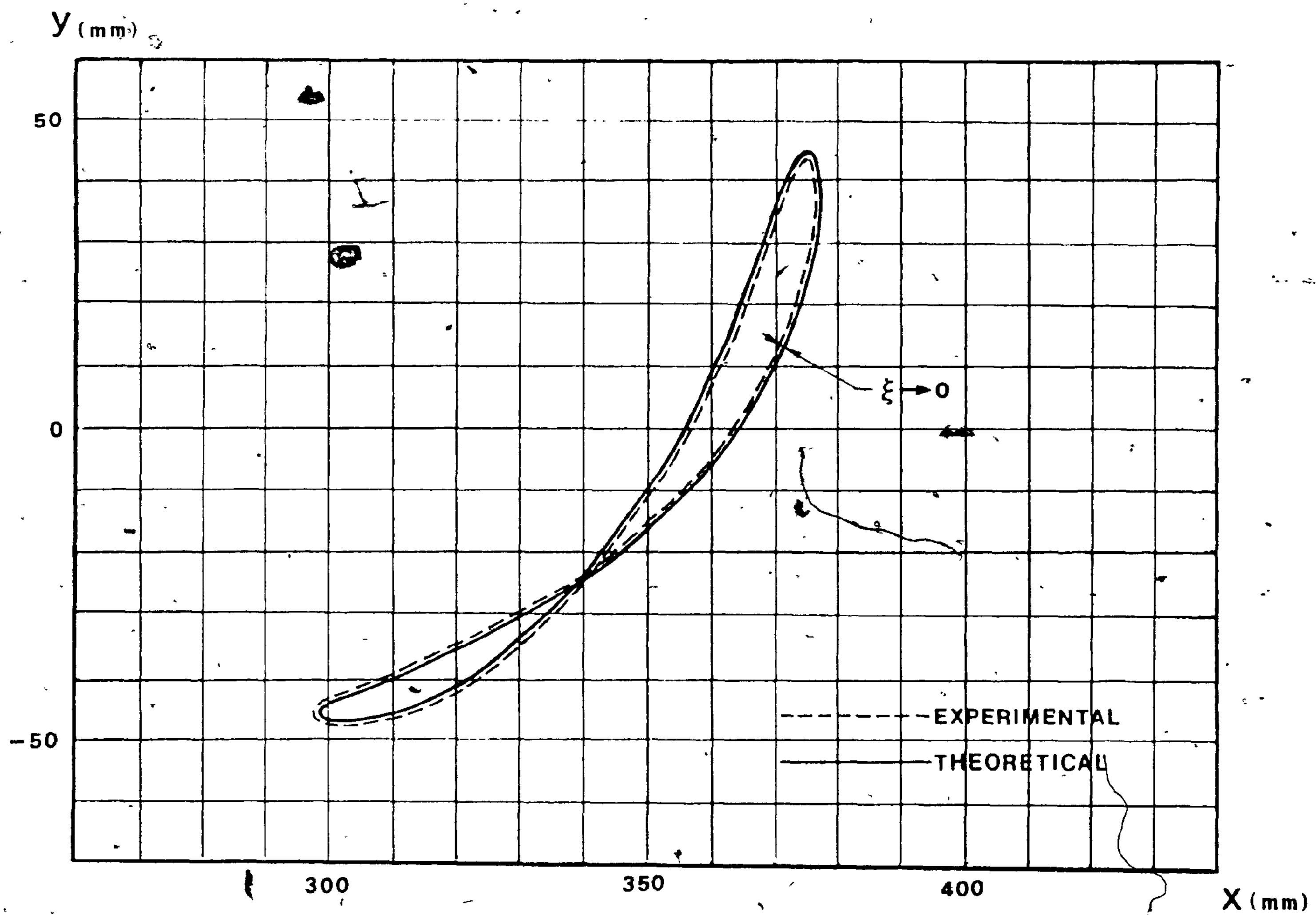


Fig. 4.11: Coupler Curve Result : Test Case (3)

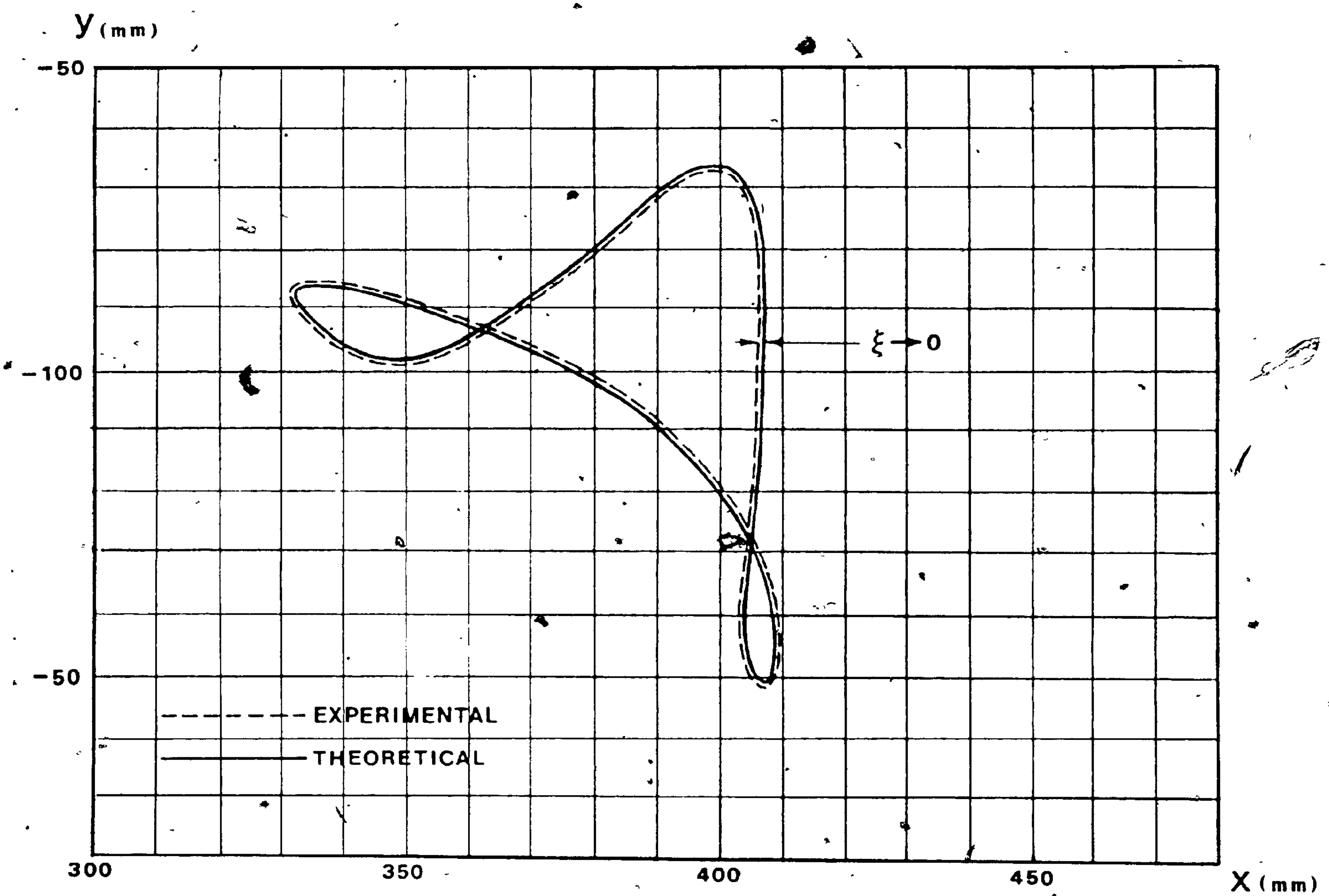


Fig. 4.12: Coupler Curve Result : Test Case (9)

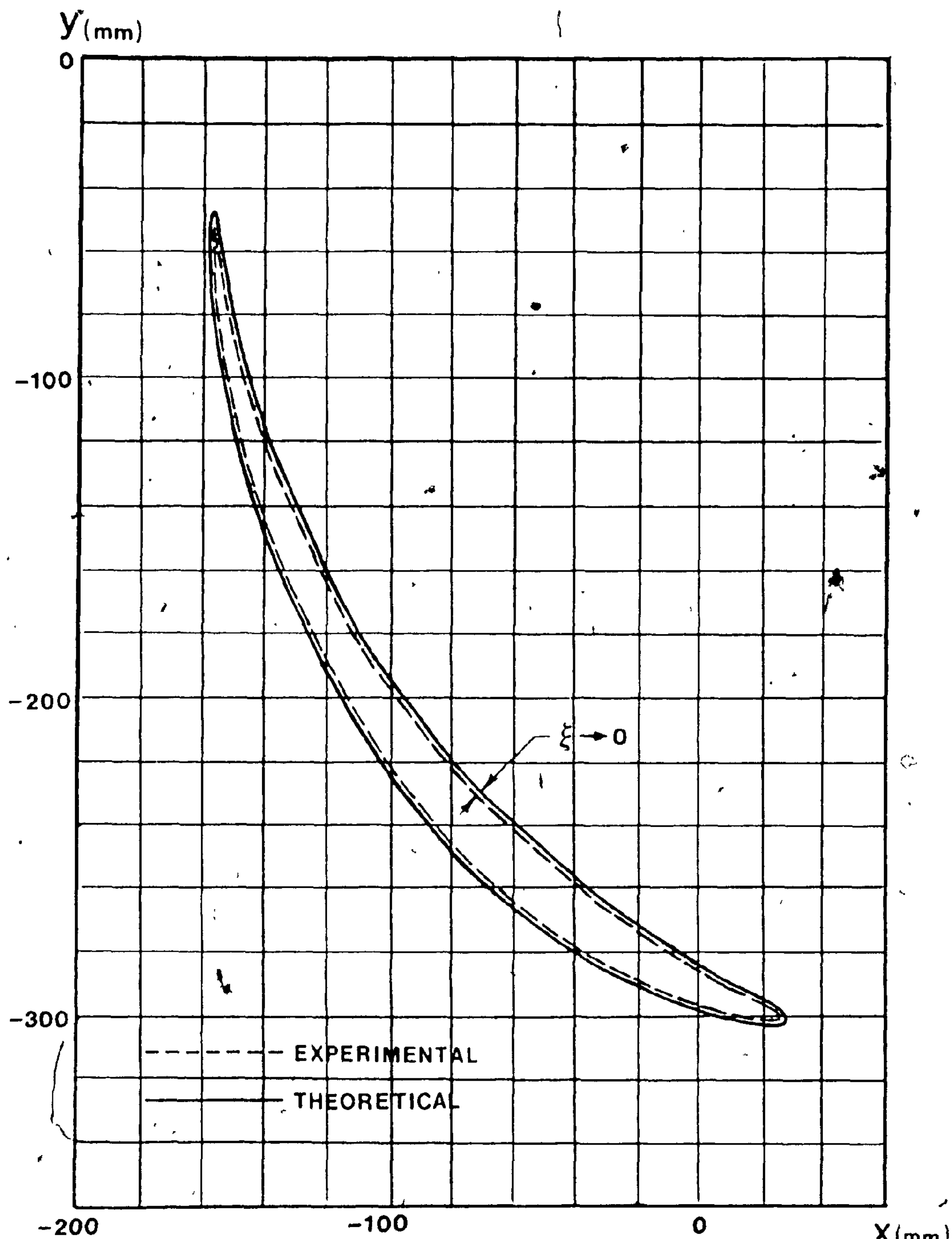


Fig. 4.13: Coupler Curve Result : Test Case (10)

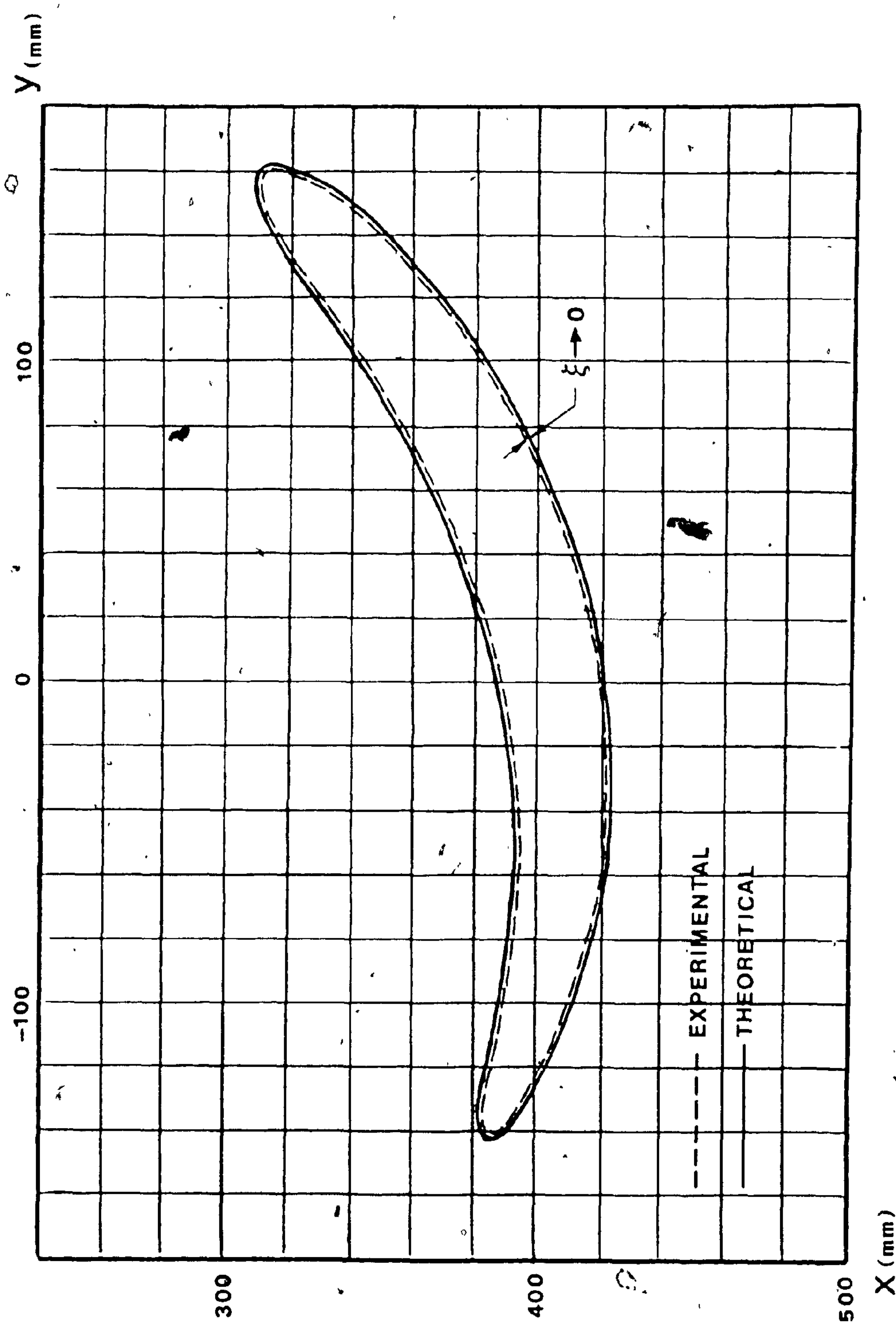


Fig. 4.14: Coupler Curve Result : Test Case (11)

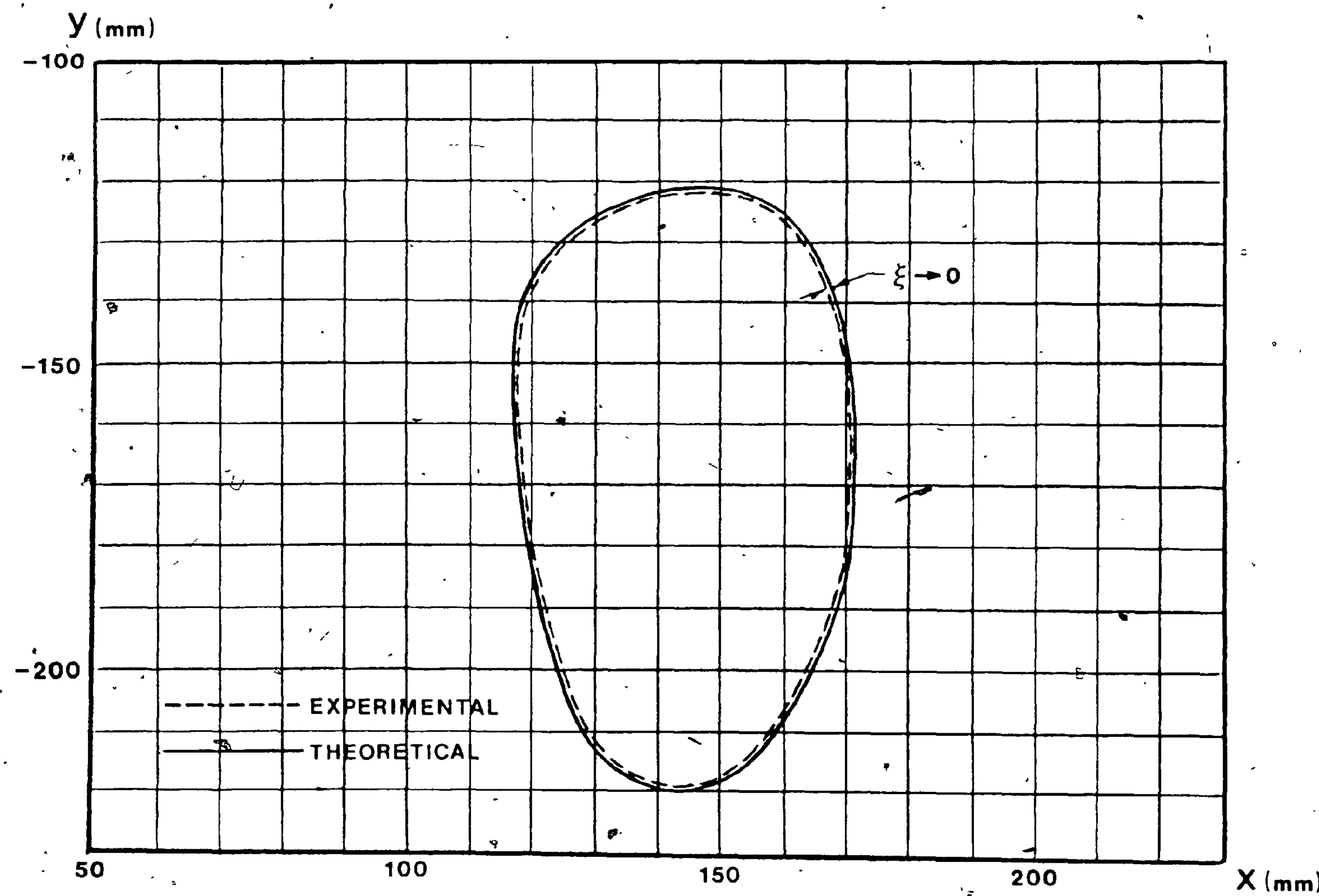


Fig. 4.15: Coupler Curve Result : Test Case (12).

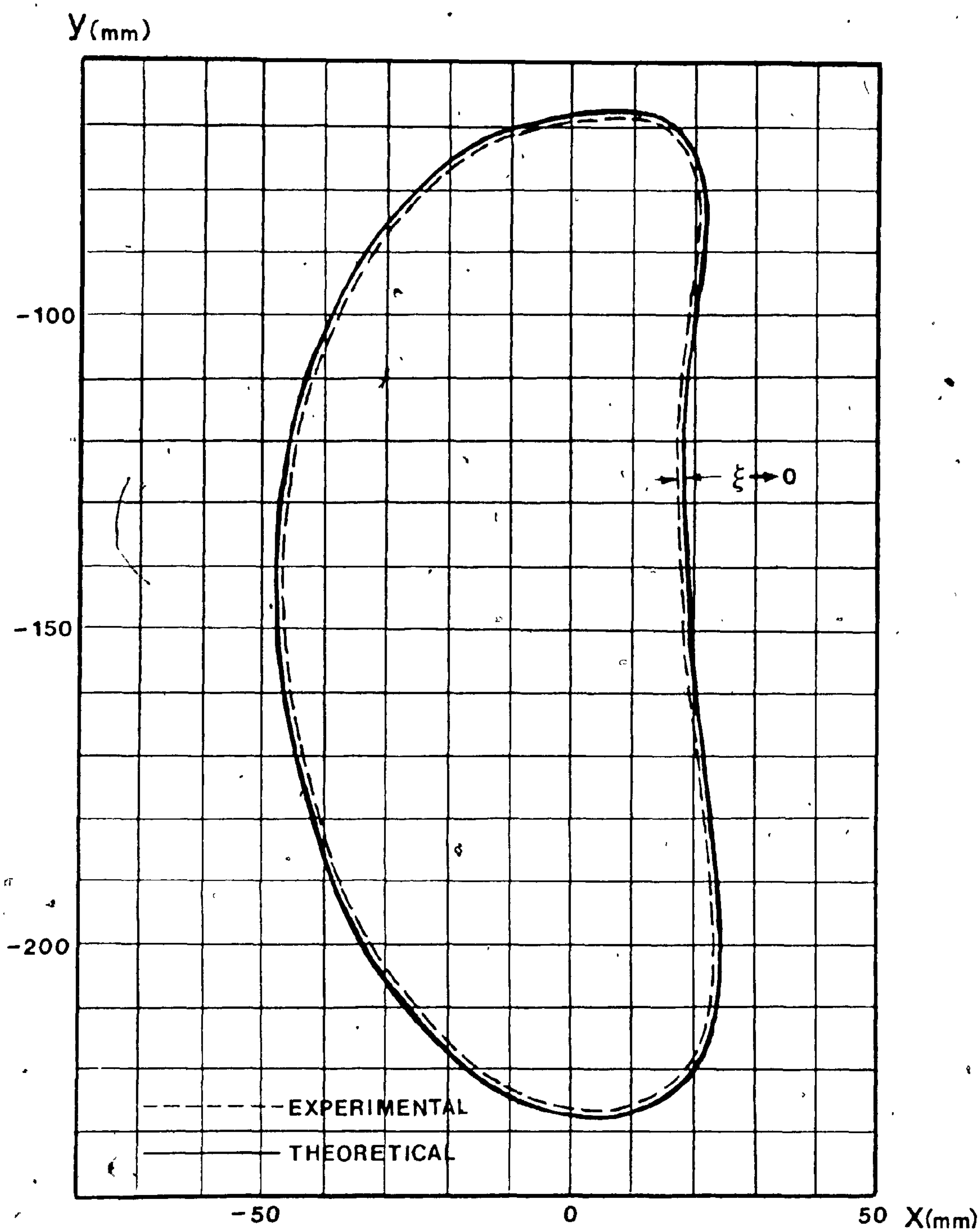


Fig. 4.16: Coupler Curve Result : Test Case (13)

Table 4.1: Specifications of Test Cases' Design Parameters

Test Case No	r_{11}	r_{21}	r_{22}	r_{31}	r_{32}	r_{41}	r_{51}	r_{52}	r_{61}	r_{71}	r_{72}	r_{81}	r_{82}	α_2	α_3	α_5	α_8	γ
1	500	60	60	90	160	630	180	200	420	160	160	260	260	300	270	270	120	0
2	500	60	60	90	160	630	180	200	420	160	160	313	200	300	270	270	122	0
3	500	60	60	90	160	630	180	200	420	160	160	200	313	300	270	270	122	0
4	400	60	60	90	160	457	180	200	430	160	160	313	313	300	270	270	269	0
5	400	60	60	90	160	457	180	200	430	160	160	195	313	300	270	270	238	0
6	400	60	60	90	160	457	180	200	430	160	160	195	313	300	270	270	238	-40
7	500	60	60	90	160	630	180	200	420	160	160	313	195	300	270	270	230	0
8	500	60	60	90	160	630	180	200	420	160	160	313	195	300	270	270	237	-40
9	500	60	60	90	160	630	180	200	420	160	160	313	183	300	270	270	81	0
10	278	60	60	90	160	455	180	200	280	156	149	313	313	300	270	270	301	0
11	278	60	60	90	160	455	180	200	280	156	149	313	313	300	270	270	59	0
12	500	60	60	90	160	630	180	200	420	160	160	183	313	300	270	270	279	0
13	500	60	60	90	160	630	180	200	420	160	160	183	313	300	270	270	279	0

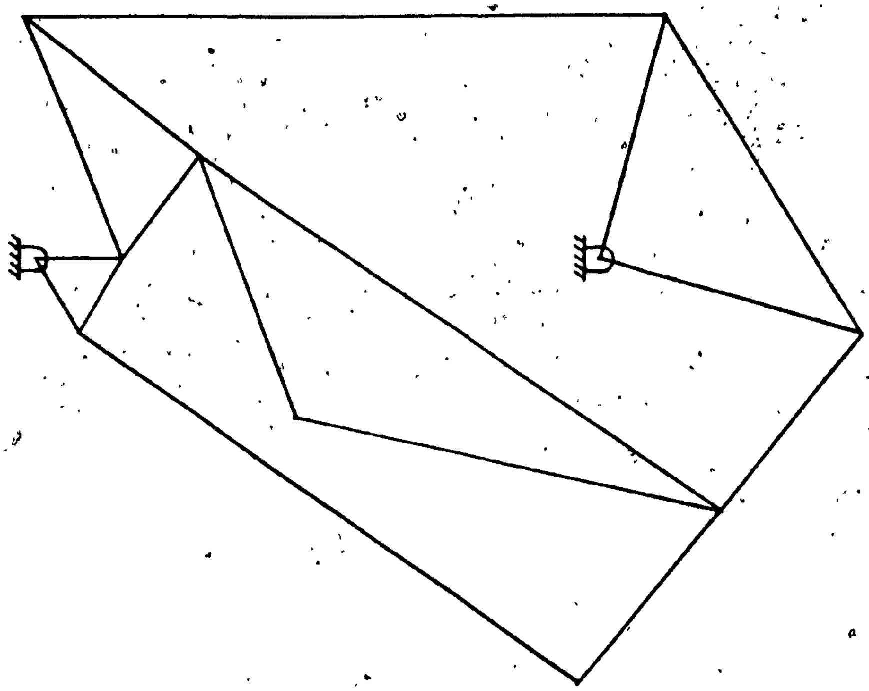
*Units are in (mm) for link lengths r and (degrees) for angles α and γ
All values are rounded to the nearest digit

the theoretical and experimental results. Note that the selection of the sample cases were bounded by the limitations of the experimental model. The selection of these numerous cases further illustrate the wide variations of coupler curve profiles capable of being produced by the eight-link mechanism especially where very little adjustments are made to the mechanism. This demonstrates its versatility. In avoiding cases with locking conditions, the moility criteria were employed in conjunction with a 'trial and error' technique whereby the linkages were randomly adjusted until a jam-free state was found.

In seach of the different test cases a few examples were found with double configurations. Figure 4.17 and Fig. 4.18 illustrate two samples depicting the two possible configurations for a given set of linkage dimensions. Their correspoinding coupler curve profiles are very much different for the two possible configurations inspite of the fact that they both carry identical values for the design parameters. Moreover computer results verified that the two configurations exhibited opposite signs in equation (3.39) for each of the cases.

Furthermore, upon observation of the computer output, the modified coupler curve equation served its purpose whereby its implementation is evident (ie. AP=2) when the value of the term under the square root (AX1) is close to zero. It has been proven, though, that before the incorporation of the modified coupler curve equation, the computer program terminated altogether at the instant AX1 touched negative, resulting in an incomplete run.

With reference to the computer time, execution of the presented



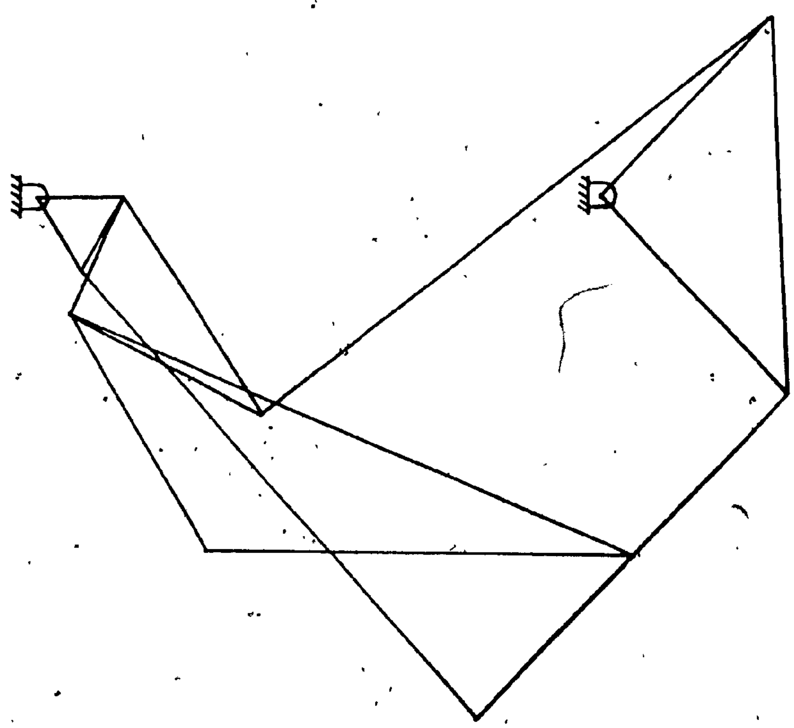


Fig. 4.17: Double Configuration of Test Case (5) and (6)

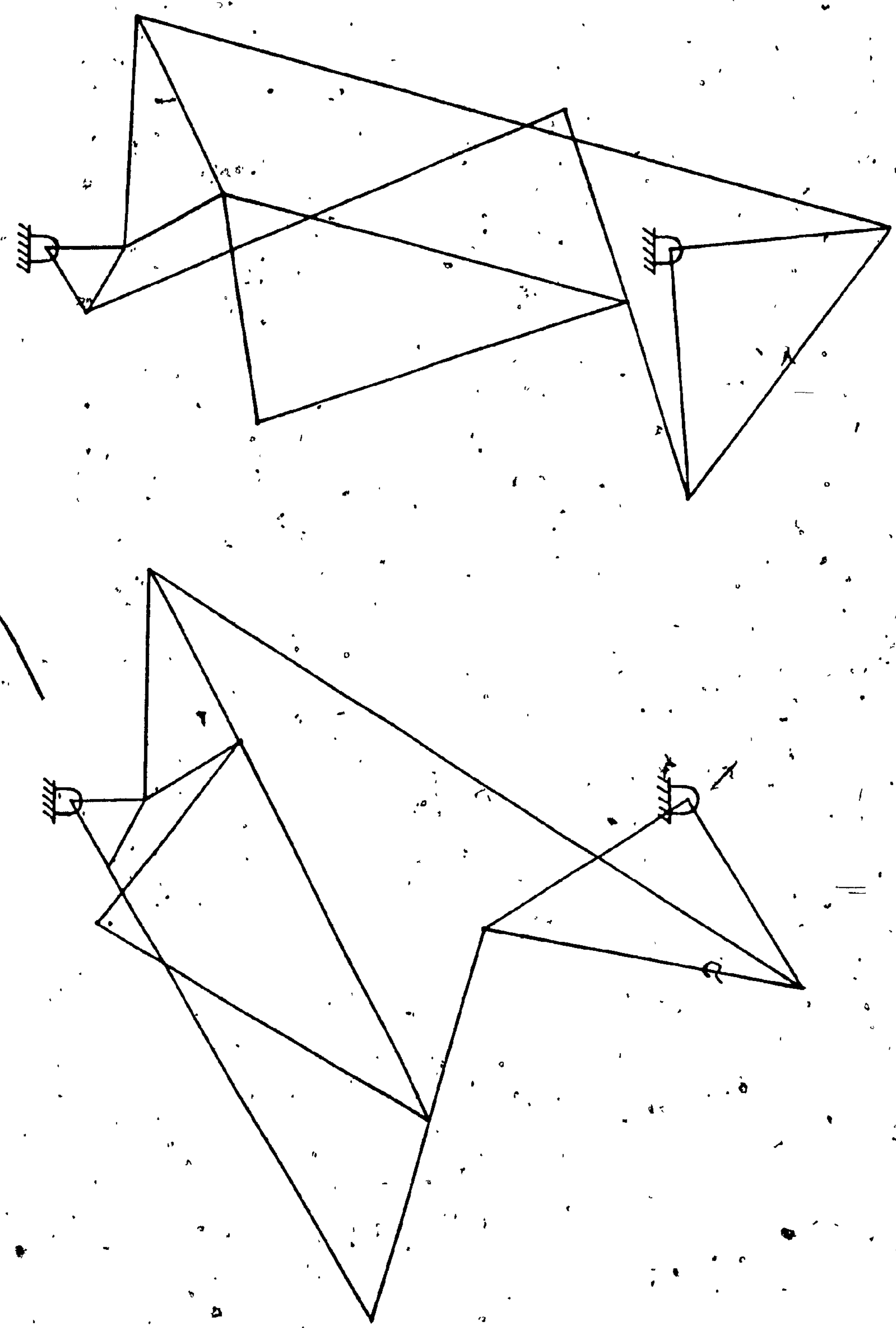


Fig. 4.18: Double Configuration of Test Case (12) and (13)

computer program proved to be very fast and efficient. A single run required an average of about 3 to 5 seconds of execution time. It has been tested where the modified coupler curve equation was solely used which resulted in considerably longer execution time of about 15 to 20 seconds, proving it to be less efficient and more costly.

To obtain a successful run, though, one must make the proper selections of the computer parameters. The interval size for the input crank angle is one example. The run presented in the Appendix is based on a 2° interval. However, the results are only printed at every 4° of the input crank angle to avoid any over-stuffing of material. The selection of a small interval size will result in better accuracy and convergence, but one must pay, as a consequence, for more computer time. An interval too large, however, might increase the number of iterations or even cause the solution to diverge. Another factor to consider is the proper selection of the initial values for x_j, y_j and θ_5 at zero crank angle. As mentioned earlier, this can be best achieved by drawing or from the experimental model directly. An erroneous selection of a certain degree may result in the solution of its alternate configuration. How much error can be given to these initial guesses still remains as a question. But once the first point is successfully solved, successive points can then be easily found by using the previous result as an initial guess for the iteration process of the next step interval. This is sometimes known as the 'proximity perturbation' technique. In a majority of the test cases, the maximum number of iterations taken occurred at the first point corresponding to zero crank angle. But once this was established, all other successive intervals required only about four iterations when a two degree interval step size was chosen. Obviously, there exists an optimum range in the selection of all these factors. For now, this can only be best chosen from experience.

CHAPTER 5

CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE WORK

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CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE WORK

The derivation of the coupler curve equations for an eight-link planar mechanism with eighteen design parameters is presented and numerically solved through a computer program, giving a complete motion analysis of the mechanism. The coupler curve equations consist of three simultaneous equations, each of which relates the coupler curve coordinates and output rocker angle as a function of the mechanism's eighteen design parameters and input crank angle. Its verification is based on the correlation of its solution with the experimental results.

In the past, many works have been written on the kinematic analysis of linkage mechanisms but to the best of the author's knowledge, none have explicitly formulated and solved the coupler curve equations for the eight-link mechanism presented here. This is due to the extensive work needed for this analysis. The coupler curve equations derived are of high order in nature and require the final solution to be determined by the implementation of a digital computer. Solution of the three simultaneous equations is achieved by the use of Newton's method for a system of nonlinear equations. This method proves to be efficient and effective in that a fast convergence is yielded. However, the selection of the initial points at zero crank angle must be made with discretion in order to avoid any problems of divergence or convergence to a solution of another configuration other than that of interest.

The derivation of the mobility criteria for the eight-link mechanism is also presented. Its function serves as a foundation and starting point in the selection of the linkage dimensions to give a nonlocking condition. Its use, however, should not be solely based on, but should be implemented in conjunction with the experimental model until further development is made in defining more definite mobility criteria.

The presented method in deriving the coupler curve equations whereby Gaussian elimination is incorporated provides a simple and straightforward technique in formulation. Unlike many past research works, its methodology does not require a knowledge of advanced mathematical analysis such as those required when using screw matrices, dual number quaternion algebra or tensors, to name a few, especially when dealing with mechanisms of complicated geometrical configurations. Moreover, the presented method provides the coupler curve equations that pinpoints to a unique solution when the initial values are properly selected.

With the presented eight-bar mechanism, coupler curves of significantly larger variations can be generated due to the availability of more design parameters and the improvement of the coupler link's freedom of motion. This makes the mechanism more "flexible" and especially attractive in the field of synthesis. This application is best suited for computer-aided design.

The following areas are recommended for further investigation:

1. The presented method of formulation can be further extended to the application of different or more complex configurations, such as the consideration of α_7, the incorporation of sliders or its application to spatial or multi-degree of freedom mechanisms.

2. An extended kinematic analysis on the velocity and acceleration of the eight-link mechanism.

3. Further investigation on the development of a definite and explicit form for the mobility criteria of the eight-link mechanism.

4. Further studies on the domain of the initial guess to assure convergence.

5. Further extension of this work to the synthesis of the eight-link mechanism; that is given a particular displacement (ie. coupler curve shape; angular motion), to specify the corresponding design parameters of the mechanism.

6. Studies on the possibility of its application to profile machining as well as an alternative to robotics.

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APPENDIX I

Computer Program

```
      PROGRAM LINKAGE(INPUT,TAPE1=INPUT,OUTPUT,TAPE3=OUTPUT)
C
C     ****************************************************************
C     *                                                              *
C     *                                                              *
C     *     STEPHEN MAH                                              *
C     *                                                              *
C     *     DEPARTMENT OF MECHANICAL ENGINEERING                     *
C     *     CONCORDIA UNIVERSITY                                     *
C     *                                                              *
C     *     PROGRAM LINKAGE PERFORMS A KINEMATIC ANALYSIS            *
C     *     OF AN 8-LINK MECHANISM WITH EIGHTEEN DESIGN              *
C     *     PARAMETERS WHEREBY THE COUPLER CURVE POINTS              *
C     *     AND THE ANGULAR POSITION OF EACH LINK ARE                *
C     *     CALCULATED.                                              *
C     *                                                              *
C     *                                                              *
C     ****************************************************************
C
      DIMENSION D(8),AM(8),AL(8),XO(3),YO(3),SO(3),ZO(3),
     *E(4,3),QI(6),Q(8,3),U(3,3),V(3,3),AO(3),BO(3),QO(3),
     *QQ(3),BL(3,3,3),GL(3,3,3),AR(3,3,3,3),CCO(3,3),CC2(3,3),
     *CC4(3,3),CC6(3,3),CC8(3,3),FF(3),AF(3,3),DD(3)
C
C   ----------------------------------------------------------
C   REGISTERING AND COMPILING OF DATA (LINKAGE DIMENSIONS)
C   ----------------------------------------------------------
C
      READ*,XX,YY,THETA5
      READ(1,2)(D(I),I=1,8),(AM(I),I=1,8),(AL(I),I=1,8),GM
2     FORMAT(2X,8F8.2/2X,8F8.2/2X,8F8.2/2X,F8.2)
      WRITE(3,4)(D(I),I=1,8),(AM(I),I=1,8),(AL(I),I=1,8),GM
4     FORMAT(//2X,'THE DATA ARE (MM) '//2X,'RI1=',8F7.2//2X,
     *       'RI2=',8F7.2//2X,'AFI=',8F7.2//2X,
     *       'GA(DEGREES)=',F7.2)
      PRINT 6,XX,YY,THETA5
6     FORMAT(/2X,'INITIAL VALUE ',2X,'XX=',F7.2,2X,'YY=',
     *          F7.2,2X,'THETA5=',F7.2)
      PI=4.*(ATAN(1.00))
      DO 8  I=1,8
      AM(I)=AM(I)/D(I)
8     AL(I)=AL(I)*(PI/180.)
      DO 10  I=2,8
10    D(I)=D(I)/D(1)
      GM=GM*(PI/180.)
      EP=0.0000001
      DH2=(2.*PI)/180.
      TH2=0.0
      THETA5=THETA5*(PI/180.)
      CMM=TAN(THETA5/2.)
      SS=1.0
```

```
C
C      ---------------------------------------------------------
C      NUMERICAL CALCULATION OF THE COUPLER CURVE EQUATIONS
C      ---------------------------------------------------------
C
12      IC=0
        XI=XX
        YI=YY
        CM=CMM
        III=1
        MM=1
14      CS=COS(GM)
        SN=SIN(GM)
        X=(XI*CS)-(YI*SN)
        Y=(YI*CS)+(XI*SN)
        C5=(AM(5)/AM(7))*(COS(AL(5)))
        S5=(AM(5)/AM(7))*(SIN(AL(5)))
        C8=(AM(8)/AM(7))*(COS(AL(8)))
        S8=(AM(8)/AM(7))*(SIN(AL(8)))
        CB8=C8*(AM(7)+1.0)
        SB8=S8*(AM(7)+1.0)
        C3=(AM(3)*(COS(AL(3))))-1.0
        S3=AM(3)*(SIN(AL(3)))
        QI(1)=(D(2)*(COS(TH2)))-1.0
        QI(2)=D(2)*(SIN(TH2))
        QI(3)=(1./AM(7))-(((AM(7)+1.)*D(2)*(COS(TH2)))/AM(7))
        QI(3)=QI(3)+(D(2)*AM(2)*(COS(TH2+AL(2))))
        QI(4)=-(((AM(7)+1.)*D(2)*(SIN(TH2)))/AM(7))
        QI(4)=QI(4)+(D(2)*AM(2)*(SIN(TH2+AL(2))))
        QI(5)=QI(1)/AM(7)
        QI(6)=QI(2)/AM(7)
        A1=(C3*C3)+(S3*S3)
        A2=(CB8*CB8)+(SB8*SB8)
        A3=A2/((AM(7)+1.)*(AM(7)+1.))
        XO(1)=(X/D(1))-(D(2)*(COS(TH2)))
        YO(1)=(Y/D(1))-(D(2)*(SIN(TH2)))
        XO(2)=(CS/D(1))
        XO(3)=-(SN/D(1))
        YO(2)=(SN/D(1))
        YO(3)=(CS/D(1))
        SO(1)=(YO(1)/XO(1))
        SO(2)=(YO(2)/XO(1))-((SO(1)*XO(2))/XO(1))
        SO(3)=(YO(3)/XO(1))-((SO(1)*XO(3))/XO(1))
        ZO(1)=(XO(1)+(SO(1)*YO(1)))/2.
        ZO(1)=ZO(1)+(((D(3)*D(3))-(D(8)*D(8)))/(2.*XO(1)))
        ZO(2)=(XO(2)+(SO(2)*YO(1))+(SO(1)*YO(2)))/2.
        ZO(2)=ZO(2)-(((D(3)*D(3))-(D(8)*D(8)))*(XO(2)/
     *        (2.*XO(1)*XO(1))))
        ZO(3)=(XO(3)+(SO(3)*YO(1))+(SO(1)*YO(3)))/2.
        ZO(3)=ZO(3)-(((D(3)*D(3))-(D(8)*D(8)))*(XO(3)/
     *        (2.*XO(1)*XO(1))))
        U(1,1)=(C3*SO(1))+S3
        U(2,1)=(CB8*SO(1))+SB8
        U(3,1)=U(2,1)/(AM(7)+1.)
```

```
      V(1,1)=(S3*SO(1))-C3
      V(2,1)=(SB8*SO(1))-CB8
      V(3,1)=V(2,1)/(AM(7)+1.)
      DO 16  J=2,3
      U(1,J)=C3*SO(J)
      V(1,J)=S3*SO(J)
      U(2,J)=CB8*SO(J)
      V(2,J)=SB8*SO(J)
      U(3,J)=U(2,J)/(AM(7)+1.)
16    V(3,J)=V(2,J)/(AM(7)+1.)
      A=CB8-((AM(7)+1.)/AM(7))
      B=C8-(1./AM(7))
      D4=D(4)*D(4)
      D5=D(5)*D(5)
      D6=D(6)*D(6)
      D7=D(7)*D(7)
      B2=(C5*C5)+(S5*S5)
      DO 18  J=1,3
      E(1,J)=-(A*XO(J))+(SB8*YO(J))
      E(2,J)=-(SB8*XO(J))-(A*YO(J))
      E(3,J)=(B*XO(J))-(S8*YO(J))
18    E(4,J)=(S8*XO(J))+(B*YO(J))
      Q(1,1)=(C3*ZO(1))-QI(1)
      Q(2,1)=(S3*ZO(1))-QI(2)
      Q(3,1)=(CB8*ZO(1))-QI(3)+E(1,1)
      Q(4,1)=(SB8*ZO(1))-QI(4)+E(2,1)
      Q(5,1)=-(C8*ZO(1))-QI(5)+E(3,1)
      Q(6,1)=-(S8*ZO(1))-QI(6)+E(4,1)
      Q(7,1)=-ZO(1)+XO(1)
      Q(8,1)=YO(1)
      DO 20  J=2,3
      Q(1,J)=C3*ZO(J)
      Q(2,J)=S3*ZO(J)
      Q(3,J)=(CB8*ZO(J))+E(1,J)
      Q(4,J)=(SB8*ZO(J))+E(2,J)
      Q(5,J)=-(C8*ZO(J))+E(3,J)
      Q(6,J)=-(S8*ZO(J))+E(4,J)
      Q(7,J)=-ZO(J)+XO(J)
20    Q(8,J)=YO(J)
      QO(1)=(SO(1)*Q(7,1))-Q(8,1)
      QQ(1)=(Q(7,1)*Q(7,1))+(Q(8,1)*Q(8,1))-(D(8)*D(8))
      DO 22  J=2,3
      QO(J)=(SO(J)*Q(7,1))+(SO(1)*Q(7,J))-Q(8,J)
22    QQ(J)=(2.*Q(7,1)*Q(7,J))+(2.*Q(8,1)*Q(8,J))
      BL(1,1,1)=D5+(Q(1,1)*Q(1,1))+(Q(2,1)*Q(2,1))-D4-(A1*QQ(1))
      BL(1,2,1)=(B2*D5)+(Q(3,1)*Q(3,1))+(Q(4,1)*Q(4,1))
     *          -D6-(A2*QQ(1))
      BL(1,3,1)=(B2*D5)+(Q(5,1)*Q(5,1))+(Q(6,1)*Q(6,1))
     *          -D7-(A3*QQ(1))
      BL(2,1,1)=2.*D(5)*Q(2,1)
      BL(2,2,1)=2.*D(5)*((S5*Q(3,1))-(C5*Q(4,1)))
      BL(2,3,1)=2.*D(5)*((C5*Q(6,1))-(S5*Q(5,1)))
      BL(3,1,1)=2.*D(5)*Q(1,1)
      BL(3,2,1)=-2.*D(5)*((C5*Q(3,1))+(S5*Q(4,1)))
```

```
      BL(3,3,1)=2.*D(5)*((C5*Q(5,1))+(S5*Q(6,1)))
      GL(1,1,1)=2.*D(3)*((U(1,1)*Q(1,1))
     *          +(V(1,1)*Q(2,1))+(A1*QO(1)))
      GL(1,2,1)=2.*D(3)*((U(2,1)*Q(3,1))
     *          +(V(2,1)*Q(4,1))+(A2*QO(1)))
      GL(1,3,1)=2.*D(3)*((A3*QO(1))-(U(3,1)*Q(5,1))
     *          -(V(3,1)*Q(6,1)))
      GL(2,1,1)=2.*D(3)*D(5)*V(1,1)
      GL(2,2,1)=2.*D(3)*D(5)*((U(2,1)*S5)-(V(2,1)*C5))
      GL(2,3,1)=2.*D(3)*D(5)*((U(3,1)*S5)-(V(3,1)*C5))
      GL(3,1,1)=2.*D(3)*D(5)*U(1,1)
      GL(3,2,1)=-2.*D(3)*D(5)*((U(2,1)*C5)+(V(2,1)*S5))
      GL(3,3,1)=-2.*D(3)*D(5)*((U(3,1)*C5)+(V(3,1)*S5))
      DO 24  J=2,3
      BL(1,1,J)=(2.*Q(1,1)*Q(1,J))+(2.*Q(2,1)*Q(2,J))-(A1*QQ(J))
      BL(1,2,J)=(2.*Q(3,1)*Q(3,J))+(2.*Q(4,1)*Q(4,J))-(A2*QQ(J))
      BL(1,3,J)=(2.*Q(5,1)*Q(5,J))+(2.*Q(6,1)*Q(6,J))-(A3*QQ(J))
      BL(2,1,J)=2.*D(5)*Q(2,J)
      BL(2,2,J)=2.*D(5)*((S5*Q(3,J))-(C5*Q(4,J)))
      BL(2,3,J)=2.*D(5)*((C5*Q(6,J))-(S5*Q(5,J)))
      BL(3,1,J)=2.*D(5)*Q(1,J)
      BL(3,2,J)=-2.*D(5)*((C5*Q(3,J))+(S5*Q(4,J)))
      BL(3,3,J)=2.*D(5)*((C5*Q(5,J))+(S5*Q(6,J)))
      GL(1,1,J)=2.*D(3)*((U(1,J)*Q(1,1))+(U(1,1)*Q(1,J))+
     *          (V(1,J)*Q(2,1))+(V(1,1)*Q(2,J))+(A1*QO(J)))
      GL(1,2,J)=2.*D(3)*((U(2,J)*Q(3,1))+(U(2,1)*Q(3,J))+
     *          (V(2,J)*Q(4,1))+(V(2,1)*Q(4,J))+(A2*QO(J)))
      GL(1,3,J)=2.*D(3)*((A3*QO(J))-(U(3,J)*Q(5,1))-
     *          (U(3,1)*Q(5,J))-(V(3,J)*Q(6,1))-(V(3,1)*Q(6,J)))
      GL(2,1,J)=2.*D(3)*D(5)*V(1,J)
      GL(2,2,J)=2.*D(3)*D(5)*((U(2,J)*S5)-(V(2,J)*C5))
      GL(2,3,J)=2.*D(3)*D(5)*((U(3,J)*S5)-(V(3,J)*C5))
      GL(3,1,J)=2.*D(3)*D(5)*U(1,J)
      GL(3,2,J)=-2.*D(3)*D(5)*((U(2,J)*C5)+(V(2,J)*S5))
 24   GL(3,3,J)=-2.*D(3)*D(5)*((U(3,J)*C5)+(V(3,J)*S5))
      AX1=((SO(1)*SO(1))+1.0)*D(3)
      AX2=(D(3)*AX1)-(ZO(1)*ZO(1))
C
C    ------------------------------------------
C    CHECKS IF TERM UNDER SQRT APPROCHES NEGATIVE
C    ------------------------------------------
C    ------------------------------------------
C       IF SO  USE 2ND APPROACH
C       IF NOT  USE 1ST APPROACH
C    ------------------------------------------
C
      IF(MM.EQ.0) GO TO 38
      IF(AX2.GT.1.0E-5) THEN
C
C    ----------------
C    1ST APPROACH...
C    ----------------
C
      AO(1)=(SS*(SQRT(AX2)))+(SO(1)*ZO(1))
```

```
      BO(1)=AO(1)/AX1
      AX3=SQRT(AX2)
      DO 26  J=2,3
      AO(J)=(D(3)*D(3)*SO(1)*SO(J))-(ZO(1)*ZO(J))
      AO(J)=((AO(J)*SS)/AX3)+(SO(J)*ZO(1))+(SO(1)*ZO(J))
      BO(J)=(BO(1)*AO(J))/AO(1)
26    BO(J)=BO(J)-((2.*SO(1)*SO(J)*BO(1))/((SO(1)*SO(1))+1.))
      DO 30  I=1,3
      DO 28  J=1,3
      CCO(I,J)=BO(1)*(GL(1,I,J)+GL(3,I,J))
      CCO(I,J)=CCO(I,J)-(BL(1,I,J)+BL(3,I,J))
      CC2(I,J)=2.*((BO(1)*GL(2,I,J))-BL(2,I,J))
      CC4(I,J)=BO(1)*(GL(1,I,J)-GL(3,I,J))
      CC4(I,J)=CC4(I,J)-(BL(1,I,J)-BL(3,I,J))
      IF(J.LT.2) GO TO 28
      CCO(I,J)=CCO(I,J)+(BO(J)*(GL(1,I,1)+GL(3,I,1)))
      CC2(I,J)=CC2(I,J)+(2.*BO(J)*GL(2,I,1))
      CC4(I,J)=CC4(I,J)+(BO(J)*(GL(1,I,1)-GL(3,I,1)))
28    CONTINUE
30    CONTINUE
      AA=0.0
      CM2=CM*CM
      DO 32  I=1,3
C
C    ---------------------------------------------
C    FINAL COUPLER CURVE EQUATION (1ST APPROACH)
C    ---------------------------------------------
C
      AA=(CC4(I,1)*CM2)+(CC2(I,1)*CM)+CCO(I,1)
      FF(I)=-AA
32    AA=0.0
      AA=0.0
      BB=0.0
      DO 36  I=1,3
      DO 34  J=1,2
      J1=J+1
      AA=(CC4(I,J1)*CM2)+(CC2(I,J1)*CM)+CCO(I,J1)
      AF(I,J)=AA
34    AA=0.0
      BB=(2.*CC4(I,1)*CM)+CC2(I,1)
      AF(I,3)=BB
36    BB=0.0
      NA=1
      GO TO 66
      ELSE
      END IF
C
C    ----------------
C    2ND APPROACH...
C    ----------------
C
38    MM=0
      AO(1)=QQ(1)/(D(3)*AX1)
      BO(1)=(2.*QO(1))/AX1
```

```
      DO 40  J=2,3
      AO(J)=(QQ(J)/(D(3)*AX1))-((2.*QQ(1)*SO(1)*SO(J))/
     *       (AX1*AX1))
40    BO(J)=((2.*QO(J))/AX1)-((4.*QO(1)*SO(1)*SO(J)*D(3))/
     *       (AX1*AX1))
      DO 46  I=1,3
      DO 44  J=1,3
      KO=J
      DO 42, K=KO,3
      AR(J,K,I,1)=(2.*BL(J,I,1)*BL(K,I,1))+(2.*AO(1)*GL(J,I,1)*
     *             GL(K,I,1))
42    AR(J,K,I,1)=AR(J,K,I,1)+(BO(1)*((BL(J,I,1)*GL(K,I,1))
     *             +(BL(K,I,1)*GL(J,I,1))))
44    CONTINUE
46    CONTINUE
      DO 54  J1=2,3
      DO 52  I=1,3
      DO 50  J=1,3
      KO=J
      DO 48  K=KO,3
      AA=(2.*BL(J,I,J1)*BL(K,I,1))+(2.*BL(J,I,1)*BL(K,I,J1))
      AA=AA+(BO(J1)*((BL(J,I,1)*GL(K,I,1))
     *   +(BL(K,I,1)*GL(J,I,1))))
      AA=AA+(2.*AO(J1)*GL(J,I,1)*GL(K,I,1))
      AA=AA+(BO(1)*((BL(J,I,J1)*GL(K,I,1))
     *   +(BL(J,I,1)*GL(K,I,J1))))
      AA=AA+(BO(1)*((BL(K,I,J1)*GL(J,I,1))
     *   +(BL(K,I,1)*GL(J,I,J1))))
      AA=AA+(2.*AO(1)*((GL(J,I,J1)*GL(K,I,1))
     *   +(GL(J,I,1)*GL(K,I,J1))))
      AR(J,K,I,J1)=AA
48    AA=0.0
50    CONTINUE
52    CONTINUE
54    CONTINUE
      DO 58  I=1,3
      DO 56  J=1,3
      CCO(I,J)=AR(1,1,I,J)+(2.*AR(1,3,I,J))+AR(3,3,I,J)
      CC2(I,J)=4.*(AR(1,2,I,J)+AR(2,3,I,J))
      CC4(I,J)=2.*(AR(1,1,I,J)+(2.*AR(2,2,I,J))-AR(3,3,I,J))
      CC6(I,J)=4.*(AR(1,2,I,J)-AR(2,3,I,J))
56    CC8(I,J)=AR(1,1,I,J)-(2.*AR(1,3,I,J))+AR(3,3,I,J)
58    CONTINUE
      AA=0.0
      CM2=CM*CM
      CM3=CM*CM2
      CM4=CM2*CM2
      DO 60  I=1,3
C
C    ----------------------------------------------
C    FINAL COUPLER CURVE EQUATION (2ND APPROACH)
C    ----------------------------------------------
C
      AA=(CC8(I,1)*CM4)+(CC6(I,1)*CM3)+(CC4(I,1)*CM2)
```

```
      AA=AA+(CC2(I,1)*CM)+CCO(I,1)
      FF(I)=-AA
60    AA=0.0
      AA=0.0
      BB=0.0
      DO 64  I=1,3
      DO 62  J=1,2
      J1=J+1
      AA=(CC8(I,J1)*CM4)+(CC6(I,J1)*CM3)+(CC4(I,J1)*CM2)
      AA=AA+(CC2(I,J1)*CM)+CCO(I,J1)
      AF(I,J)=AA
62    AA=0.0
      BB=(4.*CC8(I,1)*CM3)+(3.*CC6(I,1)*CM2)
      BB=BB+(2.*CC4(I,1)*CM)+CC2(I,1)
      AF(I,3)=BB
64    BB=0.0
      NA=2
C
C   -----------------------------------------------------------
C   SOLVING THE COUPLER CURVE EQUATIONS (FOR X,Y,THETA5)
C   USING NEWTON'S METHOD
C   -----------------------------------------------------------
C
66    CALL INVERS(AF,3,KRR,IRANK)
      DO 70  I=1,3
      CC=0.0
      DO 68  J=1,3
68    CC=CC+AF(I,J)*FF(J)
70    DD(I)=CC
      IC=IC+1
      ER2=(ABS(DD(1)))+(ABS(DD(2)))+(ABS(DD(3)))
      IF(ER2.LE.EP) GO TO 76
      IF(IC.EQ.15) GO TO 77
      IF(IC.GE.30) GO TO 72
C
C   ---------------------
C   INCREMENT AND RETURN
C   ---------------------
C
      XI=XI+DD(1)
      YI=YI+DD(2)
      CM=CM+DD(3)
      GO TO 14
72    T2=TH2*(180./PI)
      WRITE(3,74) T2
74    FORMAT(//2X,'MECHANISM CAN NOT ASSEMBLE AT TH2=',F6.2)
      GO TO 82
C
C   -----------------------------------------------------
C   CALCULATION OF ANGULAR POSITION FOR EACH LINK
C   -----------------------------------------------------
C
76    TH5=2.*(ATAN(CM))
      SN5=SIN(TH5)
```

```
      CS5=COS(TH5)
      T2=TH2*(180./PI)
      T5=TH5*(180./PI)
      SN3=BL(1,1,1)+(BL(2,1,1)*SN5)+(BL(3,1,1)*CS5)
      SN3=SN3/(GL(1,1,1)+(GL(2,1,1)*SN5)+(GL(3,1,1)*CS5))
      CSO=(-(SO(1)*D(3)*SN3)+ZO(1))/D(3)
      CALL ANGLE(SN3,CSO,PI,T3)
      SNO=(-(D(3)*SN3)+Q(8,1))/D(8)
      CSO=((SO(1)*D(3)*SN3)+Q(7,1))/D(8)
      CALL ANGLE(SNO,CSO,PI,T8)
      SNO=((V(3,1)*D(3)*SN3)+(S5*D(5)*CS5)
     *     +(C5*D(5)*SN5)+Q(6,1))/D(7)
      CSO=((U(3,1)*D(3)*SN3)+(C5*D(5)*CS5)
     *     -(S5*D(5)*SN5)+Q(5,1))/D(7)
      CALL ANGLE(SNO,CSO,PI,T7)
      SNO=(-(V(2,1)*D(3)*SN3)-(S5*D(5)*CS5)
     *     -(C5*D(5)*SN5)+Q(4,1))/D(6)
      CSO=(-(U(2,1)*D(3)*SN3)-(C5*D(5)*CS5)
     *     +(S5*D(5)*SN5)+Q(3,1))/D(6)
      CALL ANGLE(SNO,CSO,PI,T6)
      SNO=(-(V(1,1)*D(3)*SN3)+(D(5)*SN5)+Q(2,1))/D(4)
      CSO=(-(U(1,1)*D(3)*SN3)+(D(5)*CS5)+Q(1,1))/D(4)
      CALL ANGLE(SNO,CSO,PI,T4)
C
C     ------------------------------
C     CHECKING FOR ASSEMBLAGE ERROR
C     ------------------------------
C
      TB2=T2*(PI/180.)
      TB3=T3*(PI/180.)
      TB6=T6*(PI/180.)
      TB7=T7*(PI/180.)
      XBB=(D(2)*(COS(TB2)))+(D(3)*(COS(TB3)))
      YBB=(D(2)*(SIN(TB2)))+(D(3)*(SIN(TB3)))
      XGG=(D(6)*(COS(TB6)))+(D(7)*(COS(TB7)))
      YGG=(D(6)*(SIN(TB6)))+(D(7)*(SIN(TB7)))
      XGG=XGG+(D(2)*AM(2)*(COS(TB2+AL(2))))
      YGG=YGG+(D(2)*AM(2)*(SIN(TB2+AL(2))))
      EP1=((XBB-XGG)**2)+((YBB-YGG)**2)
      EP2=1.+(AM(8)*AM(8))-(2.*AM(8)*(COS(AL(8))))
      EP1=(D(8)*(SQRT(EP2)))-(SQRT(EP1))
      IF(ABS(EP1).LE.EP) GO TO 78
77    IF(III.EQ.2) GO TO 78
      SS=-SS
      XI=XX
      YI=YY
      CM=CMM
      III=III+1
      GO TO 14
78    WRITE(3,80) T2,XI,YI,T3,T4,T5,T6,T7,T8,
     *            ER2,IC,AX2,EP1,NA,SS
80    FORMAT(//2X,'CRANK ANGLE(DEGREES)=',F7.2,//,2X,
     *       'COORDINATE X,Y (MM)=',E12.6,2X,E12.6,//,2X,
     *       'ANGLES T3,T4,T5,T6,T7,T8 (DEGREES) = ',/,2X,
```

```
     *         6(F7.2,2X),//,2X,'ERROR=',E12.6,2X,'IC=',I3,
     *         2X,'AX1=',E12.6,/,2X,'ASSEMBLY ERROR=',E12.6,
     *         2X,'AP=',I2,2X,'SS=',F4.1)
      ELSE
      END IF
      XX=XI
      YY=YI
      CMM=CM
82    TH2=TH2+DH2
      IF(TH2.GT.(2.*PI+.001)) GO TO 84
      GO TO 12
84    STOP
      END
```

```
      SUBROUTINE INVERS(EM,NN,KRR,IRANK)
C
C     ------------------------------------------
C     THIS SUBROUTINE FINDS THE INVERSE OF A MATRIX
C     TO USE IN NEWTON'S METHOD
C     ------------------------------------------
C
      DIMENSION EM(3,3)
      IRANK=NN
      KRR=1
      DO 150  N=1,NN
      DV=EM(N,N)
      IF(DV.EQ.0.0) GO TO 160
      DO 100  J=1,NN
100   EM(N,J)=-EM(N,J)/DV
      DO 140  I=1,NN
      IF(N-I)  110,140,110
110   DO 130  J=1,NN
      IF(N-J)  120,130,120
120   EM(I,J)=EM(I,J)+EM(I,N)*EM(N,J)
130   CONTINUE
140   EM(I,N)=EM(I,N)/DV
      EM(N,N)=1.0/DV
150   CONTINUE
      RETURN
160   KRR=0
      IRANK=N-1
      RETURN
      END
```

```
      SUBROUTINE ANGLE(SNO,CSO,PI,TT)
C
C    ----------------------------------------------
C    THIS SUBROUTINE CALCULATES THE TRUE ANGLE GIVEN
C    ITS CORRESPONDING SINE AND COSINE VALUE.
C    ----------------------------------------------
C
      AXC=ABS(CSO)
      AXS=ABS(SNO)
      IF(AXC.LE.0.00001) GO TO 200
      GO TO 220
200   IF(SNO.GT.0.0) GO TO 210
      TT=270.
      GO TO 260
210   TT=90.
      GO TO 260
220   TO=ATAN(AXS/AXC)
      TO=TO*(180./PI)
      IF(SNO.GT.0.0) GO TO 240
      IF(CSO.GT.0.0) GO TO 230
      TT=180.+TO
      GO TO 260
230   TT=360.-TO
      GO TO 260
240   IF(CSO.GT.0.0) GO TO 250
      TT=180.-TO
      GO TO 260
250   TT=TO
260   RETURN
      END
```

APPENDIX II

Computer Results

THE DATA ARE (MM)

RI1= 500.00 60.00 90.00 630.00 180.00 420.00 160.00 260.00

RI2= .00 60.00 160.00 .00 200.00 .00 160.00 260.00

AFI= .00 300.00 270.00 .00 270.00 .00 180.00 120.00

GA(DEGREES)= .00

INITIAL VALUE XX= 385.00 YY= -10.00 THETA5= 25.00

CRANK ANGLE(DEGREES)= .00

COORDINATE X,Y (MM)= .384550E+03 -.191195E+02

ANGLES T3,T4,T5,T6,T7,T8 (DEGREES) =
34.50 350.05 24.27 312.89 33.66 344.36

ERROR= .937548E-08 IC= 4 AX1= .122502E-01
ASSEMBLY ERROR= .154046E-10 AP= 1 SS= 1.0

CRANK ANGLE(DEGREES)= 4.00

COORDINATE X,Y (MM)= .384158E+03 -.218461E+02

ANGLES T3,T4,T5,T6,T7,T8 (DEGREES) =
33.09 349.10 22.29 311.38 34.20 343.20

ERROR= .391942E-11 IC= 4 AX1= .121816E-01
ASSEMBLY ERROR= .355271E-14 AP= 1 SS= 1.0

CRANK ANGLE(DEGREES)= 8.00

COORDINATE X,Y (MM)= .383478E+03 -.242236E+02

ANGLES T3,T4,T5,T6,T7,T8 (DEGREES) =
31.65 348.18 20.40 309.92 34.65 342.13

ERROR= .874862E-11 IC= 4 AX1= .120691E-01
ASSEMBLY ERROR=-.710543E-14 AP= 1 SS= 1.0

CRANK ANGLE(DEGREES)= 12.00

COORDINATE X,Y (MM)= .382537E+03 -.262415E+02

ANGLES T3,T4,T5,T6,T7,T8 (DEGREES) =
30.20 347.30 18.63 308.53 35.01 341.15

ERROR= .601480E-11 IC= 4 AX1= .119111E-01
ASSEMBLY ERROR=-.355271E-14 AP= 1 SS= 1.0

CRANK ANGLE(DEGREES)= 16.00

COORDINATE X,Y (MM)= .381358E+03 -.278975E+02

ANGLES T3,T4,T5,T6,T7,T8 (DEGREES) =
28.73 346.45 16.97 307.22 35.29 340.29

ERROR= .228545E-11 IC= 4 AX1= .117064E-01
ASSEMBLY ERROR= .355271E-14 AP= 1 SS= 1.0

CRANK ANGLE(DEGREES)= 20.00

COORDINATE X,Y (MM)= .379966E+03 -.291962E+02

ANGLES T3,T4,T5,T6,T7,T8 (DEGREES) =
27.26 345.66 15.46 305.98 35.47 339.53

ERROR= .367355E-11 IC= 4 AX1= .114543E-01
ASSEMBLY ERROR= .000000E+00 AP= 1 SS= 1.0

CRANK ANGLE(DEGREES)= 24.00

COORDINATE X,Y (MM)= .378384E+03 -.301486E+02

ANGLES T3,T4,T5,T6,T7,T8 (DEGREES) =
25.78 344.93 14.08 304.82 35.56 338.88

ERROR= .299575E-11 IC= 4 AX1= .111549E-01
ASSEMBLY ERROR= .710543E-14 AP= 1 SS= 1.0

CRANK ANGLE(DEGREES)= 28.00

COORDINATE X,Y (MM)= .376632E+03 -.307701E+02

ANGLES T3,T4,T5,T6,T7,T8 (DEGREES) =
24.31 344.25 12.85 303.74 35.55 338.34

ERROR= .553924E-11 IC= 4 AX1= .108091E-01
ASSEMBLY ERROR= .106581E-13 AP= 1 SS= 1.0

```
CRANK ANGLE(DEGREES)=  32.00

COORDINATE X,Y (MM)= .374729E+03  -.310801E+02

ANGLES T3,T4,T5,T6,T7,T8 (DEGREES) =
  22.84   343.64    11.77   302.74    35.46   337.90

ERROR= .510330E-11  IC=  4  AX1= .104187E-01
ASSEMBLY ERROR=-.106581E-13  AP= 1  SS= 1.0


CRANK ANGLE(DEGREES)=  36.00

COORDINATE X,Y (MM)= .372690E+03  -.311004E+02

ANGLES T3,T4,T5,T6,T7,T8 (DEGREES) =
  21.38   343.09    10.84   301.83    35.27   337.58

ERROR= .126325E-11  IC=  4  AX1= .998663E-02
ASSEMBLY ERROR=-.355271E-14  AP= 1  SS= 1.0


CRANK ANGLE(DEGREES)=  40.00

COORDINATE X,Y (MM)= .370530E+03  -.308544E+02

ANGLES T3,T4,T5,T6,T7,T8 (DEGREES) =
  19.93   342.61    10.04   301.01    35.00   337.36

ERROR= .122631E-11  IC=  4  AX1= .951648E-02
ASSEMBLY ERROR=-.710543E-14  AP= 1  SS= 1.0


CRANK ANGLE(DEGREES)=  44.00

COORDINATE X,Y (MM)= .368260E+03  -.303667E+02

ANGLES T3,T4,T5,T6,T7,T8 (DEGREES) =
  18.50   342.20     9.39   300.27    34.65   337.24

ERROR= .341772E-11  IC=  4  AX1= .901282E-02
ASSEMBLY ERROR= .710543E-14  AP= 1  SS= 1.0


CRANK ANGLE(DEGREES)=  48.00

COORDINATE X,Y (MM)= .365890E+03  -.296621E+02

ANGLES T3,T4,T5,T6,T7,T8 (DEGREES) =
  17.08   341.85     8.87   299.62    34.23   337.22

ERROR= .391390E-11  IC=  4  AX1= .848088E-02
ASSEMBLY ERROR= .710543E-14  AP= 1  SS= 1.0
```

```
CRANK ANGLE(DEGREES)=  52.00

COORDINATE X,Y (MM)= .363429E+03  -.287656E+02

ANGLES T3,T4,T5,T6,T7,T8 (DEGREES) =
  15.69   341.57     8.48   299.05     33.73   337.29

ERROR= .559309E-11  IC=  4  AX1= .792645E-02
ASSEMBLY ERROR= .142109E-13  AP= 1  SS= 1.0


CRANK ANGLE(DEGREES)=  56.00

COORDINATE X,Y (MM)= .360885E+03  -.277016E+02

ANGLES T3,T4,T5,T6,T7,T8 (DEGREES) =
  14.31   341.36     8.20   298.57     33.17   337.45

ERROR= .241858E-11  IC=  4  AX1= .735576E-02
ASSEMBLY ERROR= .710543E-14  AP= 1  SS= 1.0


CRANK ANGLE(DEGREES)=  60.00

COORDINATE X,Y (MM)= .358263E+03  -.264944E+02

ANGLES T3,T4,T5,T6,T7,T8 (DEGREES) =
  12.97   341.21     8.04   298.17     32.56   337.70

ERROR= .276704E-11  IC=  4  AX1= .677525E-02
ASSEMBLY ERROR=-.355271E-14  AP= 1  SS= 1.0


CRANK ANGLE(DEGREES)=  64.00

COORDINATE X,Y (MM)= .355571E+03  -.251672E+02

ANGLES T3,T4,T5,T6,T7,T8 (DEGREES) =
  11.65   341.12     7.97   297.85     31.90   338.03

ERROR= .127597E-10  IC=  4  AX1= .619146E-02
ASSEMBLY ERROR= .142109E-13  AP= 1  SS= 1.0


CRANK ANGLE(DEGREES)=  68.00

COORDINATE X,Y (MM)= .352815E+03  -.237427E+02

ANGLES T3,T4,T5,T6,T7,T8 (DEGREES) =
  10.36   341.09     8.00   297.62     31.20   338.44

ERROR= .355269E-10  IC=  4  AX1= .561085E-02
ASSEMBLY ERROR= .284217E-13  AP= 1  SS= 1.0
```

```
CRANK ANGLE(DEGREES)=  72.00

COORDINATE X,Y (MM)= .350000E+03  -.222423E+02

ANGLES T3,T4,T5,T6,T7,T8 (DEGREES) =
   9.10   341.12     8.11   297.46    30.46   338.91

ERROR= .101073E-09  IC=  4  AX1= .503962E-02
ASSEMBLY ERROR= .994760E-13  AP= 1  SS= 1.0


CRANK ANGLE(DEGREES)=  76.00

COORDINATE X,Y (MM)= .347133E+03  -.206865E+02

ANGLES T3,T4,T5,T6,T7,T8 (DEGREES) =
   7.88   341.20     8.29   297.38    29.69   339.46

ERROR= .305061E-09  IC=  4  AX1= .448365E-02
ASSEMBLY ERROR= .245137E-12  AP= 1  SS= 1.0


CRANK ANGLE(DEGREES)=  80.00

COORDINATE X,Y (MM)= .344221E+03  -.190948E+02

ANGLES T3,T4,T5,T6,T7,T8 (DEGREES) =
   6.69   341.32     8.55   297.38    28.90   340.06

ERROR= .993221E-09  IC=  4  AX1= .394831E-02
ASSEMBLY ERROR= .699885E-12  AP= 1  SS= 1.0


CRANK ANGLE(DEGREES)=  84.00

COORDINATE X,Y (MM)= .341271E+03  -.174850E+02

ANGLES T3,T4,T5,T6,T7,T8 (DEGREES) =
   5.54   341.50     8.86   297.46    28.09   340.72

ERROR= .345343E-08  IC=  4  AX1= .343842E-02
ASSEMBLY ERROR= .216005E-11  AP= 1  SS= 1.0


CRANK ANGLE(DEGREES)=  88.00

COORDINATE X,Y (MM)= .338291E+03  -.158738E+02

ANGLES T3,T4,T5,T6,T7,T8 (DEGREES) =
   4.43   341.72     9.22   297.61    27.27   341.43

ERROR= .130526E-07  IC=  4  AX1= .295813E-02
ASSEMBLY ERROR= .688161E-11  AP= 1  SS= 1.0
```

```
CRANK ANGLE(DEGREES)=  92.00

COORDINATE X,Y (MM)= .335290E+03  -.142762E+02

ANGLES T3,T4,T5,T6,T7,T8 (DEGREES) =
   3.37   341.98     9.63   297.82     26.44   342.19

ERROR= .548619E-07  IC=  4  AX1= .251093E-02
ASSEMBLY ERROR= .236575E-10  AP= 1  SS= 1.0


CRANK ANGLE(DEGREES)=  96.00

COORDINATE X,Y (MM)= .332278E+03  -.127055E+02

ANGLES T3,T4,T5,T6,T7,T8 (DEGREES) =
   2.35   342.28    10.08   298.11     25.60   342.99

ERROR= .624099E-11  IC=  5  AX1= .209955E-02
ASSEMBLY ERROR= .142109E-13  AP= 1  SS= 1.0


CRANK ANGLE(DEGREES)= 100.00

COORDINATE X,Y (MM)= .329265E+03  -.111737E+02

ANGLES T3,T4,T5,T6,T7,T8 (DEGREES) =
   1.38   342.62    10.57   298.46     24.77   343.83

ERROR= .689398E-11  IC=  5  AX1= .172600E-02
ASSEMBLY ERROR=-.142109E-13  AP= 1  SS= 1.0


CRANK ANGLE(DEGREES)= 104.00

COORDINATE X,Y (MM)= .326262E+03  -.969050E+01

ANGLES T3,T4,T5,T6,T7,T8 (DEGREES) =
    .46   342.99    11.09   298.88     23.94   344.70

ERROR= .147940E-10  IC=  5  AX1= .139152E-02
ASSEMBLY ERROR= .710543E-14  AP= 1  SS= 1.0


CRANK ANGLE(DEGREES)= 108.00

COORDINATE X,Y (MM)= .323282E+03  -.826413E+01

ANGLES T3,T4,T5,T6,T7,T8 (DEGREES) =
 359.59   343.39    11.63   299.35     23.11   345.59

ERROR= .393106E-08  IC=  5  AX1= .109665E-02
ASSEMBLY ERROR= .120792E-12  AP= 1  SS= 1.0
```

```
CRANK ANGLE(DEGREES)= 112.00

COORDINATE X,Y (MM)= .320338E+03  -.690079E+01

ANGLES T3,T4,T5,T6,T7,T8 (DEGREES) =
 358.78   343.81    12.21   299.89    22.29   346.52

ERROR= .733415E-10  IC=  5  AX1= .841208E-03
ASSEMBLY ERROR= .241585E-12  AP= 2  SS= 1.0


CRANK ANGLE(DEGREES)= 116.00

COORDINATE X,Y (MM)= .317443E+03  -.560476E+01

ANGLES T3,T4,T5,T6,T7,T8 (DEGREES) =
 358.04   344.27    12.81   300.48    21.49   347.46

ERROR= .413625E-09  IC=  5  AX1= .624341E-03
ASSEMBLY ERROR= .106226E-11  AP= 2  SS= 1.0


CRANK ANGLE(DEGREES)= 120.00

COORDINATE X,Y (MM)= .314613E+03  -.437844E+01

ANGLES T3,T4,T5,T6,T7,T8 (DEGREES) =
 357.35   344.74    13.43   301.12    20.69   348.42

ERROR= .275830E-08  IC=  5  AX1= .444589E-03
ASSEMBLY ERROR= .727951E-11  AP= 2  SS= 1.0


CRANK ANGLE(DEGREES)= 124.00

COORDINATE X,Y (MM)= .311862E+03  -.322224E+01

ANGLES T3,T4,T5,T6,T7,T8 (DEGREES) =
 356.74   345.23    14.08   301.82    19.90   349.40

ERROR= .201428E-07  IC=  5  AX1= .299924E-03
ASSEMBLY ERROR= .536886E-10  AP= 2  SS= 1.0


CRANK ANGLE(DEGREES)= 128.00

COORDINATE X,Y (MM)= .309206E+03  -.213471E+01

ANGLES T3,T4,T5,T6,T7,T8 (DEGREES) =
 356.20   345.74    14.75   302.57    19.14   350.38

ERROR= .138243E-10  IC=  6  AX1= .187823E-03
ASSEMBLY ERROR= .710543E-14  AP= 2  SS= 1.0
```

```
CRANK ANGLE(DEGREES)= 132.00

COORDINATE X,Y (MM)= .306660E+03  -.111251E+01

ANGLES T3,T4,T5,T6,T7,T8 (DEGREES) =
 355.73   346.27   15.44   303.37   18.38   351.37

ERROR= .450829E-11  IC=  6  AX1= .105333E-03
ASSEMBLY ERROR=-.603961E-13  AP= 2  SS= 1.0


CRANK ANGLE(DEGREES)= 136.00

COORDINATE X,Y (MM)= .304240E+03  -.150511E+00

ANGLES T3,T4,T5,T6,T7,T8 (DEGREES) =
 355.35   346.81   16.16   304.22   17.64   352.37

ERROR= .676364E-09  IC=  6  AX1= .491552E-04
ASSEMBLY ERROR= .131095E-11  AP= 2  SS= 1.0


CRANK ANGLE(DEGREES)= 140.00

COORDINATE X,Y (MM)= .301960E+03   .758163E+00

ANGLES T3,T4,T5,T6,T7,T8 (DEGREES) =
 355.05   347.36   16.90   305.11   16.92   353.36

ERROR= .135296E-09  IC=  7  AX1= .157255E-04
ASSEMBLY ERROR=-.994760E-13  AP= 2  SS= 1.0


CRANK ANGLE(DEGREES)= 144.00

COORDINATE X,Y (MM)= .299836E+03   .162199E+01

ANGLES T3,T4,T5,T6,T7,T8 (DEGREES) =
 354.85   347.92   17.67   306.05   16.21   354.36

ERROR= .102628E-08  IC=  8  AX1= .130606E-05
ASSEMBLY ERROR=-.238742E-11  AP= 2  SS= 1.0


CRANK ANGLE(DEGREES)= 148.00

COORDINATE X,Y (MM)= .297865E+03   .233655E+01

ANGLES T3,T4,T5,T6,T7,T8 (DEGREES) =
 354.43   348.37   17.90   307.36   15.50   355.43

ERROR= .125253E-08  IC= 14  AX1= .644024E-05
ASSEMBLY ERROR=-.113166E-02  AP= 2  SS= 1.0
```

CRANK ANGLE(DEGREES)= 152.00

COORDINATE X,Y (MM)= .296043E+03 .294133E+01

ANGLES T3,T4,T5,T6,T7,T8 (DEGREES) =
353.92 348.77 17.80 308.90 14.76 356.54

ERROR= .460985E-10 IC= 12 AX1= .373791E-04
ASSEMBLY ERROR=-.311735E-02 AP= 2 SS= 1.0

CRANK ANGLE(DEGREES)= 156.00

COORDINATE X,Y (MM)= .294397E+03 .353206E+01

ANGLES T3,T4,T5,T6,T7,T8 (DEGREES) =
353.57 349.19 17.83 310.41 13.99 357.62

ERROR= .321510E-08 IC= 10 AX1= .898109E-04
ASSEMBLY ERROR=-.502688E-02 AP= 2 SS= 1.0

CRANK ANGLE(DEGREES)= 160.00

COORDINATE X,Y (MM)= .292946E+03 .411926E+01

ANGLES T3,T4,T5,T6,T7,T8 (DEGREES) =
353.36 349.62 17.99 311.88 13.19 358.68

ERROR= .108751E-09 IC= 10 AX1= .153837E-03
ASSEMBLY ERROR=-.677757E-02 AP= 2 SS= 1.0

CRANK ANGLE(DEGREES)= 164.00

COORDINATE X,Y (MM)= .291710E+03 .471564E+01

ANGLES T3,T4,T5,T6,T7,T8 (DEGREES) =
353.32 350.08 18.29 313.31 12.38 359.70

ERROR= .769531E-10 IC= 10 AX1= .221305E-03
ASSEMBLY ERROR=-.829402E-02 AP= 2 SS= 1.0

CRANK ANGLE(DEGREES)= 168.00

COORDINATE X,Y (MM)= .291061E+03 .650186E+01

ANGLES T3,T4,T5,T6,T7,T8 (DEGREES) =
355.73 351.32 22.96 312.53 12.32 .16

ERROR= .606141E-09 IC= 5 AX1= .106721E-03
ASSEMBLY ERROR=-.181188E-12 AP= 1 SS=-1.0

```
CRANK ANGLE(DEGREES)= 172.00

COORDINATE X,Y (MM)= .290347E+03   .738971E+01

ANGLES T3,T4,T5,T6,T7,T8 (DEGREES) =
 356.27   351.87    23.96   313.74     11.73      1.08

ERROR= .466557E-10  IC=  6  AX1= .126057E-03
ASSEMBLY ERROR= .103029E-12  AP= 2  SS=-1.0


CRANK ANGLE(DEGREES)= 176.00

COORDINATE X,Y (MM)= .289860E+03   .833676E+01

ANGLES T3,T4,T5,T6,T7,T8 (DEGREES) =
 356.92   352.43    25.01   314.98     11.15      1.98

ERROR= .797338E-08  IC=  5  AX1= .139545E-03
ASSEMBLY ERROR=-.253308E-11  AP= 1  SS=-1.0


CRANK ANGLE(DEGREES)= 180.00

COORDINATE X,Y (MM)= .289602E+03   .935670E+01

ANGLES T3,T4,T5,T6,T7,T8 (DEGREES) =
 357.69   352.97    26.09   316.25     10.59      2.86

ERROR= .978569E-08  IC=  4  AX1= .145982E-03
ASSEMBLY ERROR=-.300560E-11  AP= 1  SS=-1.0


CRANK ANGLE(DEGREES)= 184.00

COORDINATE X,Y (MM)= .289569E+03   .104623E+02

ANGLES T3,T4,T5,T6,T7,T8 (DEGREES) =
 358.57   353.51    27.22   317.56     10.05      3.73

ERROR= .744715E-11  IC=  4  AX1= .144785E-03
ASSEMBLY ERROR= .142109E-13  AP= 1  SS=-1.0


CRANK ANGLE(DEGREES)= 188.00

COORDINATE X,Y (MM)= .289757E+03   .116650E+02

ANGLES T3,T4,T5,T6,T7,T8 (DEGREES) =
 359.57   354.04    28.38   318.88      9.52      4.56

ERROR= .438572E-11  IC=  5  AX1= .136015E-03
ASSEMBLY ERROR= .142109E-13  AP= 1  SS=-1.0
```

CRANK ANGLE(DEGREES)= 192.00

COORDINATE X,Y (MM)= .290160E+03 .129747E+02

ANGLES T3,T4,T5,T6,T7,T8 (DEGREES) =
.69 354.56 29.58 320.23 9.02 5.38

ERROR= .188791E-08 IC= 5 AX1= .120383E-03
ASSEMBLY ERROR=-.504485E-12 AP= 1 SS=-1.0

CRANK ANGLE(DEGREES)= 196.00

COORDINATE X,Y (MM)= .290769E+03 .143993E+02

ANGLES T3,T4,T5,T6,T7,T8 (DEGREES) =
1.91 355.07 30.81 321.60 8.53 6.17

ERROR= .498295E-10 IC= 6 AX1= .992419E-04
ASSEMBLY ERROR= .710543E-14 AP= 1 SS=-1.0

CRANK ANGLE(DEGREES)= 200.00

COORDINATE X,Y (MM)= .291575E+03 .159442E+02

ANGLES T3,T4,T5,T6,T7,T8 (DEGREES) =
3.25 355.56 32.08 322.98 8.06 6.93

ERROR= .116106E-09 IC= 5 AX1= .745693E-04
ASSEMBLY ERROR= .344613E-12 AP= 2 SS=-1.0

CRANK ANGLE(DEGREES)= 204.00

COORDINATE X,Y (MM)= .292565E+03 .176120E+02

ANGLES T3,T4,T5,T6,T7,T8 (DEGREES) =
4.69 356.05 33.37 324.37 7.61 7.66

ERROR= .315240E-08 IC= 5 AX1= .489346E-04
ASSEMBLY ERROR=-.652634E-11 AP= 2 SS=-1.0

CRANK ANGLE(DEGREES)= 208.00

COORDINATE X,Y (MM)= .293728E+03 .194025E+02

ANGLES T3,T4,T5,T6,T7,T8 (DEGREES) =
6.22 356.52 34.69 325.77 7.19 8.36

ERROR= .193166E-09 IC= 6 AX1= .254647E-04
ASSEMBLY ERROR=-.568434E-13 AP= 2 SS=-1.0

```
CRANK ANGLE(DEGREES)= 212.00

COORDINATE X,Y (MM)= .295050E+03   .213114E+02

ANGLES T3,T4,T5,T6,T7,T8 (DEGREES) =
    7.85   356.99    36.04   327.16     6.79     9.03

ERROR= .498379E-09  IC=  7  AX1= .780181E-05
ASSEMBLY ERROR=-.301981E-12  AP= 2  SS=-1.0


CRANK ANGLE(DEGREES)= 216.00

COORDINATE X,Y (MM)= .296534E+03   .232417E+02

ANGLES T3,T4,T5,T6,T7,T8 (DEGREES) =
    9.49   357.42    37.29   328.58     6.39     9.67

ERROR= .858971E-05  IC= 15  AX1= .178639E-06
ASSEMBLY ERROR=-.930326E-04  AP= 2  SS= 1.0


CRANK ANGLE(DEGREES)= 220.00

COORDINATE X,Y (MM)= .298121E+03   .254482E+02

ANGLES T3,T4,T5,T6,T7,T8 (DEGREES) =
   11.35   357.88    38.77   329.92     6.11    10.26

ERROR= .147958E-08  IC= 13  AX1= .677062E-05
ASSEMBLY ERROR= .228439E-11  AP= 2  SS= 1.0


CRANK ANGLE(DEGREES)= 224.00

COORDINATE X,Y (MM)= .299845E+03   .276458E+02

ANGLES T3,T4,T5,T6,T7,T8 (DEGREES) =
   13.21   358.31    40.14   331.26     5.83    10.81

ERROR= .376708E-09  IC=  6  AX1= .328481E-04
ASSEMBLY ERROR=-.341061E-12  AP= 2  SS= 1.0


CRANK ANGLE(DEGREES)= 228.00

COORDINATE X,Y (MM)= .301681E+03   .299005E+02

ANGLES T3,T4,T5,T6,T7,T8 (DEGREES) =
   15.14   358.72    41.51   332.57     5.61    11.31

ERROR= .190711E-07  IC=  5  AX1= .835180E-04
ASSEMBLY ERROR= .293525E-10  AP= 2  SS= 1.0
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CRANK ANGLE(DEGREES)= 232.00

COORDINATE X,Y (MM)= .303621E+03   .321830E+02

ANGLES T3,T4,T5,T6,T7,T8 (DEGREES) =
  17.11    359.12    42.87   333.82      5.44     11.76

ERROR= .480839E-09  IC=  5  AX1= .164257E-03
ASSEMBLY ERROR= .650147E-12  AP= 2  SS= 1.0


CRANK ANGLE(DEGREES)= 236.00

COORDINATE X,Y (MM)= .305659E+03   .344575E+02

ANGLES T3,T4,T5,T6,T7,T8 (DEGREES) =
  19.12    359.51    44.20   335.01      5.35     12.15

ERROR= .588754E-10  IC=  6  AX1= .280700E-03
ASSEMBLY ERROR=-.142109E-13  AP= 1  SS= 1.0


CRANK ANGLE(DEGREES)= 240.00

COORDINATE X,Y (MM)= .307794E+03   .366820E+02

ANGLES T3,T4,T5,T6,T7,T8 (DEGREES) =
  21.16    359.88    45.50   336.12      5.35     12.47

ERROR= .256592E-07  IC=  5  AX1= .438506E-03
ASSEMBLY ERROR= .219558E-11  AP= 1  SS= 1.0


CRANK ANGLE(DEGREES)= 244.00

COORDINATE X,Y (MM)= .310025E+03   .388081E+02

ANGLES T3,T4,T5,T6,T7,T8 (DEGREES) =
  23.22       .23    46.76   337.13      5.45     12.72

ERROR= .195679E-09  IC=  5  AX1= .643164E-03
ASSEMBLY ERROR= .390799E-13  AP= 1  SS= 1.0


CRANK ANGLE(DEGREES)= 248.00

COORDINATE X,Y (MM)= .312358E+03   .407831E+02

ANGLES T3,T4,T5,T6,T7,T8 (DEGREES) =
  25.28       .56    47.95   338.03      5.66     12.89

ERROR= .112694E-10  IC=  5  AX1= .899707E-03
ASSEMBLY ERROR= .710543E-14  AP= 1  SS= 1.0
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```
CRANK ANGLE(DEGREES)= 252.00

COORDINATE X,Y (MM)= .314798E+03   .425519E+02

ANGLES T3,T4,T5,T6,T7,T8 (DEGREES) =
  27.32      .87    49.07   338.79     5.99    12.96

ERROR= .865117E-11  IC=  5  AX1= .121232E-02
ASSEMBLY ERROR= .106581E-13  AP= 1  SS= 1.0


CRANK ANGLE(DEGREES)= 256.00

COORDINATE X,Y (MM)= .317353E+03   .440608E+02

ANGLES T3,T4,T5,T6,T7,T8 (DEGREES) =
  29.33     1.16    50.10   339.40     6.46    12.93

ERROR= .105075E-10  IC=  5  AX1= .158384E-02
ASSEMBLY ERROR=-.142109E-13  AP= 1  SS= 1.0


CRANK ANGLE(DEGREES)= 260.00

COORDINATE X,Y (MM)= .320032E+03   .452610E+02

ANGLES T3,T4,T5,T6,T7,T8 (DEGREES) =
  31.28     1.41    51.02   339.85     7.07    12.80

ERROR= .289967E-07  IC=  4  AX1= .201525E-02
ASSEMBLY ERROR= .138769E-10  AP= 1  SS= 1.0


CRANK ANGLE(DEGREES)= 264.00

COORDINATE X,Y (MM)= .322841E+03   .461129E+02

ANGLES T3,T4,T5,T6,T7,T8 (DEGREES) =
  33.17     1.64    51.82   340.13     7.82    12.56

ERROR= .637427E-08  IC=  4  AX1= .250516E-02
ASSEMBLY ERROR= .396128E-11  AP= 1  SS= 1.0


CRANK ANGLE(DEGREES)= 268.00

COORDINATE X,Y (MM)= .325782E+03   .465877E+02

ANGLES T3,T4,T5,T6,T7,T8 (DEGREES) =
  34.97     1.82    52.47   340.24     8.69    12.21

ERROR= .158089E-08  IC=  4  AX1= .304955E-02
ASSEMBLY ERROR= .122213E-11  AP= 1  SS= 1.0
```

CRANK ANGLE(DEGREES)= 272.00

COORDINATE X,Y (MM)= .328855E+03 .466684E+02

ANGLES T3,T4,T5,T6,T7,T8 (DEGREES) =
36.66 1.96 52.97 340.17 9.68 11.74

ERROR= .463670E-09 IC= 4 AX1= .364173E-02
ASSEMBLY ERROR= .397904E-12 AP= 1 SS= 1.0

CRANK ANGLE(DEGREES)= 276.00

COORDINATE X,Y (MM)= .332054E+03 .463482E+02

ANGLES T3,T4,T5,T6,T7,T8 (DEGREES) =
38.23 2.06 53.31 339.95 10.76 11.16

ERROR= .156750E-09 IC= 4 AX1= .427269E-02
ASSEMBLY ERROR= .152767E-12 AP= 1 SS= 1.0

CRANK ANGLE(DEGREES)= 280.00

COORDINATE X,Y (MM)= .335371E+03 .456285E+02

ANGLES T3,T4,T5,T6,T7,T8 (DEGREES) =
39.66 2.10 53.46 339.56 11.93 10.48

ERROR= .516679E-10 IC= 4 AX1= .493156E-02
ASSEMBLY ERROR= .675016E-13 AP= 1 SS= 1.0

CRANK ANGLE(DEGREES)= 284.00

COORDINATE X,Y (MM)= .338791E+03 .445163E+02

ANGLES T3,T4,T5,T6,T7,T8 (DEGREES) =
40.94 2.09 53.43 339.03 13.15 9.69

ERROR= .227692E-10 IC= 4 AX1= .560639E-02
ASSEMBLY ERROR= .426326E-13 AP= 1 SS= 1.0

CRANK ANGLE(DEGREES)= 288.00

COORDINATE X,Y (MM)= .342296E+03 .430222E+02

ANGLES T3,T4,T5,T6,T7,T8 (DEGREES) =
42.06 2.01 53.22 338.37 14.42 8.80

ERROR= .918425E-11 IC= 4 AX1= .628487E-02
ASSEMBLY ERROR= .248690E-13 AP= 1 SS= 1.0

```
CRANK ANGLE(DEGREES)= 292.00

COORDINATE X,Y (MM)= .345864E+03   .411595E+02

ANGLES T3,T4,T5,T6,T7,T8 (DEGREES) =
  43.01      1.88      52.81    337.57     15.71      7.82

ERROR= .945796E-11  IC=  4  AX1= .695501E-02
ASSEMBLY ERROR= .710543E-14  AP= 1  SS= 1.0


CRANK ANGLE(DEGREES)= 296.00

COORDINATE X,Y (MM)= .349469E+03   .389433E+02

ANGLES T3,T4,T5,T6,T7,T8 (DEGREES) =
  43.79      1.67      52.21    336.65     17.01      6.76

ERROR= .377864E-11  IC=  4  AX1= .760579E-02
ASSEMBLY ERROR= .106581E-13  AP= 1  SS= 1.0


CRANK ANGLE(DEGREES)= 300.00

COORDINATE X,Y (MM)= .353083E+03   .363909E+02

ANGLES T3,T4,T5,T6,T7,T8 (DEGREES) =
  44.38      1.40      51.42    335.63     18.32      5.61

ERROR= .225457E-11  IC=  4  AX1= .822761E-02
ASSEMBLY ERROR= .142109E-13  AP= 1  SS= 1.0


CRANK ANGLE(DEGREES)= 304.00

COORDINATE X,Y (MM)= .356671E+03   .335219E+02

ANGLES T3,T4,T5,T6,T7,T8 (DEGREES) =
  44.78      1.05      50.45    334.50     19.61      4.38

ERROR= .106954E-10  IC=  4  AX1= .881262E-02
ASSEMBLY ERROR= .710543E-14  AP= 1  SS= 1.0


CRANK ANGLE(DEGREES)= 308.00

COORDINATE X,Y (MM)= .360198E+03   .303594E+02

ANGLES T3,T4,T5,T6,T7,T8 (DEGREES) =
  45.00       .63      49.29    333.27     20.89      3.09

ERROR= .123454E-10  IC=  4  AX1= .935489E-02
ASSEMBLY ERROR= .710543E-14  AP= 1  SS= 1.0
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CRANK ANGLE(DEGREES)= 312.00

COORDINATE X,Y (MM)= .363624E+03   .269304E+02

ANGLES T3,T4,T5,T6,T7,T8 (DEGREES) =
  45.04      .14    47.97   331.96    22.14      1.73

ERROR= .121334E-10  IC=  4  AX1= .985044E-02
ASSEMBLY ERROR= .710543E-14  AP= 1  SS= 1.0

CRANK ANGLE(DEGREES)= 316.00

COORDINATE X,Y (MM)= .366909E+03   .232664E+02

ANGLES T3,T4,T5,T6,T7,T8 (DEGREES) =
  44.90   359.58    46.48   330.56    23.37       .31

ERROR= .138983E-10  IC=  4  AX1= .102971E-01
ASSEMBLY ERROR=-.106581E-13  AP= 1  SS= 1.0

CRANK ANGLE(DEGREES)= 320.00

COORDINATE X,Y (MM)= .370012E+03   .194044E+02

ANGLES T3,T4,T5,T6,T7,T8 (DEGREES) =
  44.58   358.94    44.83   329.10    24.55   358.85

ERROR= .243383E-11  IC=  4  AX1= .106943E-01
ASSEMBLY ERROR= .355271E-14  AP= 1  SS= 1.0

CRANK ANGLE(DEGREES)= 324.00

COORDINATE X,Y (MM)= .372893E+03   .153867E+02

ANGLES T3,T4,T5,T6,T7,T8 (DEGREES) =
  44.10   358.24    43.06   327.57    25.70   357.36

ERROR= .174765E-11  IC=  4  AX1= .110426E-01
ASSEMBLY ERROR=-.355271E-14  AP= 1  SS= 1.0

CRANK ANGLE(DEGREES)= 328.00

COORDINATE X,Y (MM)= .375515E+03   .112607E+02

ANGLES T3,T4,T5,T6,T7,T8 (DEGREES) =
  43.47   357.47    41.16   325.99    26.81   355.84

ERROR= .681530E-11  IC=  4  AX1= .118437E-01
ASSEMBLY ERROR= .213163E-13  AP= 1  SS= 1.0
```

CRANK ANGLE(DEGREES)= 332.00

COORDINATE X,Y (MM)= .377844E+03 .707871E+01

ANGLES T3,T4,T5,T6,T7,T8 (DEGREES) =
42.69 356.65 39.17 324.36 27.87 354.31

ERROR= .419962E-11 IC= 4 AX1= .115996E-01
ASSEMBLY ERROR= .106581E-13 AP= 1 SS= 1.0

CRANK ANGLE(DEGREES)= 336.00

COORDINATE X,Y (MM)= .379853E+03 .289617E+01

ANGLES T3,T4,T5,T6,T7,T8 (DEGREES) =
41.79 355.78 37.09 322.71 28.88 352.78

ERROR= .177213E-11 IC= 4 AX1= .118123E-01
ASSEMBLY ERROR= .710543E-14 AP= 1 SS= 1.0

CRANK ANGLE(DEGREES)= 340.00

COORDINATE X,Y (MM)= .381520E+03 -.122949E+01

ANGLES T3,T4,T5,T6,T7,T8 (DEGREES) =
40.78 354.87 34.96 321.05 29.84 351.26

ERROR= .589689E-11 IC= 4 AX1= .119838E-01
ASSEMBLY ERROR= .213163E-13 AP= 1 SS= 1.0

CRANK ANGLE(DEGREES)= 344.00

COORDINATE X,Y (MM)= .382832E+03 -.524097E+01

ANGLES T3,T4,T5,T6,T7,T8 (DEGREES) =
39.66 353.93 32.79 319.37 30.74 349.78

ERROR= .580569E-12 IC= 4 AX1= .121156E-01
ASSEMBLY ERROR= .710543E-14 AP= 1 SS= 1.0

CRANK ANGLE(DEGREES)= 348.00

COORDINATE X,Y (MM)= .383786E+03 -.908321E+01

ANGLES T3,T4,T5,T6,T7,T8 (DEGREES) =
38.46 352.97 30.62 317.71 31.57 348.33

ERROR= .293314E-11 IC= 4 AX1= .122084E-01
ASSEMBLY ERROR= .248690E-13 AP= 1 SS= 1.0

```
CRANK ANGLE(DEGREES)= 352.00

COORDINATE X,Y (MM)= .384382E+03  -.127055E+02

ANGLES T3,T4,T5,T6,T7,T8 (DEGREES) =
  37.19   351.99    28.46   316.07    32.34   346.94

ERROR= .399182E-11  IC=  4  AX1= .122622E-01
ASSEMBLY ERROR= .355271E-14  AP= 1  SS= 1.0


CRANK ANGLE(DEGREES)= 356.00

COORDINATE X,Y (MM)= .384632E+03  -.160632E+02

ANGLES T3,T4,T5,T6,T7,T8 (DEGREES) =
  35.87   351.02    26.34   314.46    33.04   345.61

ERROR= .139107E-11  IC=  4  AX1= .122765E-01
ASSEMBLY ERROR= .000000E+00  AP= 1  SS= 1.0


CRANK ANGLE(DEGREES)= 360.00

COORDINATE X,Y (MM)= .384550E+03  -.191195E+02

ANGLES T3,T4,T5,T6,T7,T8 (DEGREES) =
  34.50   350.05    24.27   312.89    33.66   344.36

ERROR= .135309E-10  IC=  4  AX1= .122502E-01
ASSEMBLY ERROR=-.710543E-14  AP= 1  SS= 1.0
```