SPECIFIC DECAY WIDTHS IN THE LEFT-RIGHT SUPERSYMMETRIC MODEL

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Abstract

We review briefly the Standard Model (SM), Left-Right Model (L-R) and Minimal Supersymmetric Standard Model (MSSM). We study their specific symmetry breaking mechanisms and particle contents. Finally, we calculate specific interaction vertices of doubly charged Higgsinos in the Left-Right Supersymmetric model (L-R SUSY) and their decay widths. A possible extension of this work is proposed.

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Chapter 1

Introduction

Ever since the time of Democritus, and perhaps earlier, people have tried to understand the basic building blocs of matter. Things have changed dramatically with the discovery of Quantum Mechanics and Special Relativity at the beginning of the century.

Later on, Dirac combined both theories to derive his wave equation for the electron which can be considered the basis for a new era dominated by Quantum Electrodynamics (QED) and its extension, Quantum Field Theory (QFT).

We have also gone a long way trying to understand the forces acting on the particles. The first step, arising from the experiments of Oërsted and Faraday, were Maxwell's famous equations which put under the same roof, so to speak, electricity and magnetism. The main characteristic of these equations is that they satisfy the so-called Lorentz space-time transformations. Historically this was the main reason that led Einstein to formulate special relativity, which ever since has been the cornerstone of all theories that attempt to describe any aspect of the physical world. In the 1960's Glashow, Weinberg and Salam published a series of papers [1, 2, 3] which successfully unified electromagnetism with the weak force (electroweak) and received the Nobel prize for their work. The strong force is included with electroweak in an

$SU(3)_C \times SU(2)_L \times U(1)_Y$ model but there still remain a lot of things to learn about strong interactions since most of our results are based on approximations. The above gauge theory is the so-called Standard Model (SM). We will clarify some of the terminology later on.

The SM is undoubtedly a very successful theory. It has been able to accurately explain almost all experimental results, especially in the fermionic sector. For example the calculations of the theoretical value of the anomalous magnetic moment of the electron agrees with the experimentally obtained result to one part in 10^9 [4]! The predictive power of the SM has also been proved, the gauge bosons being the best example, their existence (and their masses) were postulated as an integral part of the theory in the early 1970's almost 10 years before their experimental detection at CERN in 1983.

Despite its phenomenological successes the SM has theoretical problems and this is the main reason why theorists believe that it is merely a low energy limit of a larger unified theory. The SM contains at least 19 arbitrary parameters so it is quite far from every theoretician's dream: a single unified theory with at most one undetermined coupling constant.

1.1 Lagrangian Formulation

At present the most widely used field theory formulation is the so called Lagrangian which, as is classical physics, is based on the principle of least action [5, 6]. We make the transition from a point particle to a field through the replacement : $x(t) \rightarrow \phi(x^\mu)$

and proceed to find the equation of motion by minimizing the *action*:

$$S = \int \mathcal{L}(\phi, \partial_\mu \phi)\, d^4x.$$

$\mathcal{L}$ is called the Lagrangian density but we usually refer to it as the *Lagrangian*. The minimization of the action is expressed mathematically through the familiar Euler-Lagrange equations:

$$\frac{\partial \mathcal{L}}{\partial \phi} - \frac{\partial}{\partial x^\mu}\left[\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)}\right] = 0. \tag{1.1}$$

As an illustrative example let us apply the above ideas to the Lagrangian:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{m^2}{2}\phi^2. \tag{1.2}$$

We then have:

$$\frac{\partial \mathcal{L}}{\partial \phi} = -m^2\phi,\ \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} = \partial^\mu \phi$$

and replacing in eq. (1.1) we get:

$$m^2\phi + \partial_\mu(\partial^\mu \phi) = 0$$

$$\Rightarrow (\Box + m^2)\phi = 0.$$

Therefore, starting with eq. (1.2) we recovered the Klein-Gordon equation. In a similar fashion

$$\mathcal{L} = i\,\bar{\psi}\gamma_\mu \partial^\mu \psi - m\bar{\psi}\psi$$

gives Dirac's equation (where we treat ψ and $\overline{\psi}$ independently) and

$$\mathcal{L} = \frac{-1}{4}F^{\mu\nu}F_{\mu\nu} - j^\mu A_\mu,$$

$$F^{\mu\nu} \equiv \partial^\mu A^\nu - \partial^\nu A^\mu$$

gives Maxwell's equations with a source j^μ.

Without formally deriving this assertion we can state the two basic rules to follow in order to make the connection between the terms in the Lagrangian and the Feynman rules of perturbation theory [7]:

1. The terms in the Lagrangian that are quadratic in the fields, e.g.: $\phi^2, \bar{\psi}\psi, \bar{\psi}(i\gamma_\mu\partial^\mu - m)\psi, \ldots$, determine the relevant propagators[1].

2. The other terms give the interaction vertices. In particular after taking away the fields from a specific term of $i\mathcal{L}$ what is left is the interaction vertex. For example, if $\mathcal{L} = \cdots + ie\bar{\psi}\gamma^\mu\psi A_\mu$ the vertex factor for the interacting fields $\bar{\psi}\psi A_\mu$ is: $ie\gamma^\mu$.

By carefully inspecting any Lagrangian we notice that it is invariant under a phase transformation:

$$\psi(x) \rightarrow e^{i\alpha}\psi(x),\ \alpha\epsilon\mathcal{R} \tag{1.3}$$

since, $\partial_\mu\psi \rightarrow e^{i\alpha}\partial_\mu\psi$ and $\bar{\psi} \rightarrow e^{-i\alpha}\bar{\psi}$.

If we extend this invariance requirement to any Lagrangian we can show that it implies the existence of a conserved current and in particular a conserved total charge Q. Incidentally this is an example of Noether's theorem which states that invariance under translations, time displacements, rotations etc. leads to conservation of momentum, energy and angular momentum respectively. The above invariance is called ***global gauge invariance***[2].

[1]For example, taking the inverse and multiplying by $-i$ of the term $(i\gamma_\mu\partial^\mu - m)\psi = (\not{p} - m)\psi$ gives $i/(\not{p}-m) = i(\not{p}+m)/(p^2-m^2)$ which is one of the known forms of an $s = \frac{1}{2}$ field propagator.

[2]It is actually a phase invariance, but the word gauge has been established due to historical reasons.

It would be interesting if we could generalize the above transformation (1.3) so that the parameter α is a function of every space-time point, i.e. $\alpha \equiv \alpha(x)$. The problem arises with the transformation of the derivative: $\partial_\mu\psi \rightarrow e^{i\alpha(x)}\partial_\mu\psi + ie^{i\alpha(x)}\psi\partial_\mu\alpha$. We can clearly see that the last term breaks the invariance of $\mathcal{L}$. We could remedy this situation with the construction of a *covariant* derivative D_μ which has an added vector field term $-ieA_\mu$: $D_\mu \equiv \partial_\mu - ieA_\mu$. We further demand that A_μ transforms as follows:

$$A_\mu \rightarrow A_\mu + \frac{1}{e}\partial_\mu\alpha \tag{1.4}$$

where the last term of eq. (1.4) will cancel the unwanted term of the derivative. Therefore the new Lagrangian containing the covariant derivative is invariant under the $U(1)$ phase transformation (1.3), and takes the form:

$$\mathcal{L} = \bar{\psi}(i\gamma_\mu\partial^\mu - m)\psi + e\bar{\psi}\gamma^\mu\psi A_\mu. \tag{1.5}$$

Further on, regarding A_μ as the photon field we can add a (gauge invariant) term corresponding to its kinetic energy: $-\frac{1}{4}F^{\mu\nu}F_{\mu\nu}$. Therefore the final form of the QED Lagrangian, which describes the interactions of electrons with photons, is[3]:

$$\mathcal{L} = \bar{\psi}(i\gamma_\mu\partial^\mu - m)\psi + e\bar{\psi}\gamma^\mu\psi A_\mu - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}. \tag{1.6}$$

This is indeed a remarkable result since the simple requirement of a $U(1)$ gauge invariance led us to an interacting-field Lagrangian. The most general form of the $U(1)$ transformation is: $e^{i\alpha(x)Q}$, where Q is the charge operator with eigenvalues: $Q_e = -1, Q_u = +\frac{2}{3}$, etc., and this gives rise to the general form of the electromagnetic current: $j_\mu^{em} = \bar{\psi}\gamma_\mu Q\psi$, which applies to all charged matter particles. The

[3]Note that, as expected, the photon field is massless since the addition of a mass term $\sim m^2A_\mu^2$ would break the gauge invariance

electromagnetic character of the above group is emphasized by using the subscript *em*: $U(1)_{em}$.

1.2 GWS model and symmetry breaking

In 1956, in an attempt to put some order in a vast collection of experimental observations and results, Lee and Yang [8] proposed that weak interactions violate *parity*[4]. Soon after, their work was confirmed independently by Wu [9] and Garwin [10] in a series of experiments.

Parity violation implies the non-existence of 'right-handed' neutrinos. With this in mind we try to find internal symmetries of the Lagrangian that apply between particles that have the same space-time properties. The first step is to arrange the leptons in left and right-handed families as follows, the left-handed fields:

$$e_L \equiv \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \ \mu_L \equiv \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L, \ \tau_L \equiv \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$$

and the right-handed ones:

$$e_R, \mu_R, \tau_R.$$

Then, in a similar to the previous section analysis we have the following groups of transformations :

$$U(1)_Y : \quad \psi_L \to e^{i\beta(x)Y}\psi_L, \ \psi_R \to e^{i\beta(x)Y}\psi_R,$$

$$SU(2)_L : \ \psi_L \to e^{i\,\alpha(x)\mathbf{T}}\psi_L, \ \psi_R \to \psi_R$$

where $\mathbf{T} \equiv \frac{1}{2}\sigma$ and $\psi_{L/R}$ can be any member of the above families[5]. Next, we introduce the following vector fields: $\mathbf{W}^\mu \equiv (W^1, W^2, W^3)$ and B^μ, that couple to the

[4] Parity (left-right symmetry) was always thought to be one of the conserved symmetries of nature.

[5] The reader should be warned that in the literature there are small variations in the coefficients of the exponents. Here we adopt the conventions of ref. [7].

corresponding currents in a similar way that A^μ couples to the electromagnetic current:

$$-ig\mathbf{J}_\mu \mathbf{W}^\mu = -ig\bar{\psi}_L\gamma_\mu \mathbf{T}\mathbf{W}^\mu \psi_L,$$

$$-i\frac{g'}{2}j_\mu^Y B^\mu = -ig'\bar{\psi}_R\gamma_\mu \frac{Y}{2}\psi_R B^\mu.$$

The generators of the two groups satisfy the Gell-Mann–Nishijima relation:

$$Q = T^3 + \frac{Y}{2} \tag{1.7}$$

which translated into the current form implies: $j_\mu^{em} = J_\mu^3 + \frac{1}{2}j_\mu^Y$. Later we will show in a more transparent way how the electromagnetic current is embedded in the two currents. The new covariant derivatives needed due to the introduction of the four massless fields $\mathbf{W}^\mu, B^\mu$ can be shown to be:

$$\psi_R: \quad D_\mu = (i\partial_\mu - g'\tfrac{Y}{2}B_\mu),$$

$$\psi_L: \quad D_\mu = (i\partial_\mu - \tfrac{g}{2}\sigma\mathbf{W}_\mu - g'\tfrac{Y}{2}B_\mu).$$

Therefore the new Lagrangian is:

$$\mathcal{L} = i\bar{\psi}\gamma_\mu D^\mu\psi - \frac{1}{4}\mathbf{W}_{\mu\nu}\mathbf{W}^{\mu\nu} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu},$$

where :

$$\mathbf{W}_{\mu\nu} \equiv \partial_\mu \mathbf{W}_\nu - \partial_\nu \mathbf{W}_\mu - g\mathbf{W}_\mu \times \mathbf{W}_\nu.$$

1.3 The Higgs mechanism

The Higgs mechanism *or* spontaneous symmetry breaking mechanism 'gives' masses to the massless fields introduced above without breaking gauge invariance [4, 6, 7]. This idea is not new to physics. For example, a ferromagnetic crystal extended to infinity is described by a rotationaly invariant Lagrangian. In the ground state though, all spins allign in a particular (random) direction therefore breaking the original rotational symmetry. In the electroweak $SU(2)_L \times U(1)_Y$ model the spontaneous breaking of symmetry is induced by a complex isospin doublet, the so called Higgs field:

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2) \\ \frac{1}{\sqrt{2}}(\phi_3 + i\phi_4) \end{pmatrix} \tag{1.8}$$

with $Q = 1, 0$, $T^3 = \pm\frac{1}{2}$ and $Y = 1$.

The next step is to add the appropriate kinetic and potential terms in the Lagrangian corresponding to the scalar field introduced :

$$\mathcal{L}_{Higgs} = (D_\mu\phi)^\dagger(D^\mu\phi) - V(\phi^\dagger\phi) \tag{1.9}$$

where :

$$V = -\mu^2(\phi^\dagger\phi) + \lambda(\phi^\dagger\phi)^2 \quad \mu^2 > 0, \lambda < 0.$$

We choose the vacuum expectation value (vev) of the Higgs field to be :

$$\langle\phi\rangle \equiv \phi_0 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}, \; v \equiv \frac{\mu}{\sqrt{\lambda}}. \tag{1.10}$$

This is the most natural and economical choice since we want the vacuum (ϕ^0) to be invariant under $U(1)_{em}$ transformations and thus the photon to remain massless. For this particular choice, since $T_0^3 = -\frac{1}{2}$ and $Y = 1$, replacing in eq. (1.7) we get the following:

$$Q = -\frac{1}{2} + \frac{1}{2} = 0 \Rightarrow Q\phi_0 = 0,$$

and so:

$$\phi_0{}' = e^{i\alpha(x)Q}\phi_0 = \phi_0$$

therefore the transformed field remains the same as the original.

Before going further, it is a good idea to gather all the fields mentioned so far and write down the full Lagrangian of the Standard Model :

$$\begin{aligned}
\mathcal{L}_{SM} &= \bar{\psi}_L\gamma^\mu(i\partial_\mu - \tfrac{1}{2}g\boldsymbol{\sigma}\mathbf{W}_\mu - g'\tfrac{Y}{2}B_\mu)\psi_L \\
&+\bar{\psi}_R\gamma^\mu(i\partial_\mu - g'\tfrac{Y}{2}B_\mu)\psi_R \\
&-\tfrac{1}{4}\mathbf{W}^{\mu\nu}\mathbf{W}_{\mu\nu} - \tfrac{1}{4}B^{\mu\nu}B_{\mu\nu} \\
&+\left|(i\partial_\mu - \tfrac{1}{2}g\boldsymbol{\sigma}\mathbf{W}_\mu - g'\tfrac{Y}{2}B_\mu)\phi\right|^2 - V(\phi) \\
&-(G_1\bar{\psi}_L\phi\,\psi_R + G_2\bar{\psi}_L\phi_c\psi_R + \text{h.c.}),
\end{aligned} \tag{1.11}$$

where $|\ |^2 \equiv (\ )^\dagger(\ )$. The first two terms represent quark and lepton kinetic energies (depending on the contents of the ψ fields) and their interactions with the gauge bosons. The third term is associated with the gauge boson kinetic energies and self interactions while the fourth represents the masses and couplings of the gauge bosons to the Higgs field. Finally, the last term is similar to the previous one since it is related to the fermion masses and their couplings to Higgs. G_1 and G_2 are coupling constants similar to g and g', whose values are *not* predicted by the model. Later we will see the importance of these terms in determining the various masses. Let us now substitute the vev of the Higgs field ϕ_0 into the relevant term of $\mathcal{L}_{SM}$:

$$\begin{aligned}
&\left|\left(-\tfrac{i}{2}g\boldsymbol{\sigma}\mathbf{W} - \tfrac{i}{2}g'B_\mu\right)\phi\right|^2 \\
= \ &\tfrac{1}{8}\left|\begin{pmatrix} gW_\mu^3 + g'B_\mu & g(W_\mu^1 - iW_\mu^2) \\ g(W_\mu^1 + iW_\mu^2) & -gW_\mu^3 + g'B_\mu \end{pmatrix}\begin{pmatrix} 0 \\ v \end{pmatrix}\right|^2.
\end{aligned} \tag{1.12}$$

If we define :

$$W_\mu^\pm \equiv \frac{1}{\sqrt{2}}(W_\mu^1 \mp iW_\mu^2) \tag{1.13}$$

then eq. (1.12) becomes:

$$\begin{aligned}
&= (\frac{1}{2}vg)^2 W_\mu^+ W^{-\mu} + \frac{1}{8}v^2[g^2(W_\mu^3)^2 - 2gg'W_\mu^3 B^\mu + g'^2(B_\mu)^2] \\
&= (\frac{1}{2}vg)^2 W_\mu^+ W^{-\mu} + \frac{1}{8}v^2(W_\mu^3, B_\mu)\begin{pmatrix} g^2 & -gg' \\ -gg' & g'^2 \end{pmatrix}\begin{pmatrix} W^{3\mu} \\ B^\mu \end{pmatrix} \\
&= (\frac{1}{2}vg)^2 W_\mu^+ W_\mu^- + \frac{1}{8}v^2(Z_\mu, A_\mu)\begin{pmatrix} g^2+g'^2 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix}.
\end{aligned} \tag{1.14}$$

Notice that by diagonalizing the mixing matrix we get two new neutral fields. Therefore, after symmetry breaking, we end up with four fields, three massive and one massless: W_μ^+, W_μ^-, Z^μ and A^μ. Assuming normalization of the original fields we have to normalize the new ones:

$$\begin{aligned}
A^\mu &\equiv \frac{g'W^{3\mu} + gB^\mu}{\sqrt{g^2+g'^2}}, \\
Z^\mu &\equiv \frac{gW^{3\mu} - g'B^\mu}{\sqrt{g^2+g'^2}}.
\end{aligned} \tag{1.15}$$

Comparing the mass terms in the above part of the Lagrangian with the ones expected for charged and neutral bosons we have :

$$\begin{aligned}
M_{W^\pm} &= \tfrac{1}{2}vg, \\
M_Z &= \tfrac{1}{2}v\sqrt{g^2+g'^2}, \\
M_A &= 0.
\end{aligned} \tag{1.16}$$

And we define the *Weinberg angle* through the relation :

$$\cos\theta_W \equiv \frac{M_W}{M_Z}. \tag{1.17}$$

It is interesting to note that the result $M_A = 0$ is *not* a prediction of the model since the original Lagrangian had a massless photon. The GWS model though fixes the parameter $\rho \equiv M_W^2/M_Z^2 cos^2\theta_W$, which specifies the relative strength of the neutral and charged currents in weak interactions to be equal to unity. This result has been confirmed by experiment (within a small error).

The above mechanism gives masses to the gauge bosons; let us now see how fermions acquire mass in the SM. It can be shown that terms like $-m\bar{\psi}\psi$ or $\frac{1}{2}M^2B^\mu B_\mu$ are not gauge invariant, we can however use the same Higgs doublet as before and the last term of eq. (1.11) in order to generate quark and lepton masses. In particular, in order to get a mass term for the electron field we have:

$$\begin{aligned} \mathcal{L}_e &= -G_e \bar{e_L}\phi e_R + \text{h.c.} \\ &= -G_e \left[(\bar{\nu}_e, \bar{e})_L \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} e_R + \bar{e}_R \, (\bar{\phi^-}, \bar{\phi}^0) \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \right]. \end{aligned} \tag{1.18}$$

To break the symmetry we use the Higgs doublet with the following choice of its vacuum expectation value:

$$\sqrt{\frac{1}{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}, \tag{1.19}$$

where $h(x)$ is the neutral Higgs field and we have gauged away the three other fields[6]. Replacing in $\mathcal{L}_e$ we get :

$$\mathcal{L}_e = -\frac{G_e}{\sqrt{2}} v \, (\bar{e}_L e_R + \bar{e}_R e_L) - \frac{G_e}{\sqrt{2}} (\bar{e}_L e_R + \bar{e}_R e_L) \, h. \tag{1.20}$$

[6]For details see Ch. 14 of ref. [7].

Incidentally, this is one example of the free parameters that have to be put 'by hand' in the SM. We know, for example, that the mass term is of the form: $-m\bar{\psi}\psi$, which means that $G_e v/2 = m_e$. Therefore we have to experimentally measure m_e in order to determine the value of G_e. Notice that the coupling of the second term is $\sim m_e/v \simeq 10^{-2}$ so it is quite small to produce a detectable signal and therefore evidence for the Higgs boson.

We proceed in a similar way to get the quark masses; the only difference is that we have to have an $SU(2)$ transformed Higgs doublet in order to give mass to the upper member of the quark doublet:

$$\phi_c \equiv -i\sigma_2\phi^* = \begin{pmatrix} -\bar{\phi}^0 \\ \phi^- \end{pmatrix} \longrightarrow \langle\phi_C\rangle = \sqrt{\frac{1}{2}}\begin{pmatrix} v+h \\ 0 \end{pmatrix} \tag{1.21}$$

Note that the Lagrangian can be written in diagonal form in order to include linear combinations of flavor eigenstates.

As a final remark, we should emphasize that the Higgs sector is the least investigated part of the SM mainly because, as we saw above, the Higgs couplings are proportional to the fermionic masses and the only "easily" produced fermions are the light ones: e^-, u, etc.

1.4 Beyond the Standard Model

As we mentioned in the introduction, the SM cannot be the end of the story since it builds on many assumptions and leaves many fundamental questions unanswered. We do not attempt here to give a detailed list of all the drawbacks that the SM has but merely overview some of them to justify the search for new models equipped with solutions to some (or possibly all) of its problems.

One of the most obscure aspects of the SM is the Higgs sector. As we saw in the previous section, the choice and the vacuum expectation value of the Higgs boson is totally *ad hoc*, its mass diverges quadratically if one includes higher order corrections, and finally there is not even a trace of a possible experimental detection that would support its existence. Secondly, the SM is based on the three gauge groups that we mentioned before which are basically postulated on a purely phenomenological basis. In order for a theory to be complete it would have to provide an understanding of the origin of these groups along with the values and correlations of its coupling constants. Associated with this is the (arbitrary) choice of assigning left-handed fermions to doublets and right-handed ones to singlets which is done to fit experimental observations such as beta decay, the value of electric charge, etc. The above is related to the so-called *generation problem* which basically asks the fundamental questions of why there are only three (?) families and why their masses and mixings follow a hierarchical pattern. Including to the above the possibility of a non-zero neutrino mass, CP violation, quantization of electric charge, etc. then a new model or at least an extended one becomes a neccessity.

Finally, if we are to believe that a unified theory exists then we have to look for ways to incorporate gravity into the SM or look for a totally different theory. In contrast to all other interactions, gravity is totally outside the framework of gauge theories since General Relativity does not follow from the principle of local gauge invariance and at the same time General Relativity incorporates, contrary to all other interactions, the principle of equivalence[7].

[7]For different models that are suggested as a remedy for the various problems, together with their own drawbacks, the interested reader is referred to Table I on ref. [11].

Chapter 2

Left-Right Symmetry

According to the standard model all interactions except the weak, respect all space-time symmetries. It would appear therefore natural to try and extend the SM to make it left-right symmetric. But symmetry, although intuitive, is not the only concern. There are three main reasons [11, 12, 13] for trying to incorporate right handed bidoublets in order to extend the SM in the $SU(2)_L \times SU(2)_R \times U(1)$ group; the so-called *Left-Right Model* (L-R).

The first reason is trying to understand parity violation, one approach would be to assume that nature is intrinsically left-right symmetric and the observed asymmetry takes place after some breakdown at low energies, that is the vacuum is non-invariant under parity. This is why left-right symmetries involve some sort of parity breakdown mechanism that takes effect at some energy scale M_P.

Another reason, closely associated with the first, is the neutrino mass which has initiated many controversial experiments [14] and debates in the last decade. Astrophysical problems involving the missing mass of the universe, galaxy formation, etc., would be easily resolved if the neutrino indeed has a mass in the eV range [15]. If $m_\nu \neq 0$ then the left-right symmetric model is the most natural framework to incorporate it. It has been shown [16] that such a particle in a left-right symmetric model is a Majorana[1] particle with mass:

$$m_{\nu_e} \sim O\left(\frac{1}{M_{W_R}}\right).$$

Lastly, another reason is the lack of any physical interpretation of the $U(1)_Y$ symmetry [17], mainly due to the multiplicity of values of Y in the SM: $Y(\nu_L) = -1, Y(e_R^-) = -2, Y(u_L) = \frac{1}{3}$, etc. In a left-right model $U(1)$ becomes the $B-L$ generator which is the only anomaly-free quantum number left ungauged. With this inclusion the weak gauge group becomes: $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ and a similar to the Gell-Mann–Nishijima formula holds:

$$Q = I_L^3 + I_R^3 + \frac{B-L}{2}. \tag{2.1}$$

Then $Y_{(B-L)} = -1$ for all leptons and $+\frac{1}{3}$ for quarks of all generations and helicities.

2.1 Description of the model

In defining the components of our model, we follow the same path as in the SM with the addition, of course, of right handed isospinors. For one generation we have :

$$Q_{L/R} \equiv \begin{pmatrix} u \\ d \end{pmatrix}_{L/R}, L_{L/R} \equiv \begin{pmatrix} \nu_e \\ e \end{pmatrix}_{L/R} \tag{2.2}$$

with quantum numbers

$$Q_L : \left(\tfrac{1}{2}, 0, \tfrac{1}{3}\right),\ Q_R : \left(0, \tfrac{1}{2}, \tfrac{1}{3}\right),$$

$$L_L : \left(\tfrac{1}{2}, 0, -1\right),\ L_R : \left(0, \tfrac{1}{2}, -1\right).$$

[1] A *Majorana* particle is identical to its own antiparticle [7].

The third number in parenthesis is $(B-L)$ and, as we saw above, corresponds to the $U(1)$ generator.

The fermion-gauge boson interaction part of the Lagrangian is given by :

$$\begin{aligned} \mathcal{L}_g &= \frac{ig_L}{2}\left[\bar{Q}_L\gamma_\mu\sigma Q_L + \bar{L}_L\gamma_\mu\sigma L_L\right]\mathbf{W}_L^\mu \\ &+ \frac{ig_R}{2}\left[\bar{Q}_R\gamma_\mu\sigma Q_R + \bar{L}_R\gamma_\mu\sigma L_R\right]\mathbf{W}_R^\mu \\ &+ \frac{ig'}{2}\left[\frac{1}{3}\bar{Q}\gamma_\mu Q - \bar{L}\gamma_\mu L\right]B^\mu, \end{aligned} \tag{2.3}$$

where in the last term of eq. (2.3) we imply both left and right contributions. Similarly, the fermion kinetic energy terms are:

$$\begin{aligned} \mathcal{L}_{kin} &= -\bar{Q}_L\gamma_\mu(\partial^\mu - \tfrac{ig_L}{2}\sigma\mathbf{W}_L^\mu - \tfrac{ig'}{6}B^\mu)L_L + \text{R.h.p.} \\ &\quad -\bar{L}_L\gamma_\mu(\partial^\mu - \tfrac{ig_L}{2}\sigma\mathbf{W}_L^\mu - \tfrac{ig'}{6}B^\mu)L_L + \text{R.h.p.} \end{aligned} \tag{2.4}$$

Notice that if we want the Lagrangian to be invariant under a left-right interchange, it immediately follows that: $g_L = g_R \equiv g$.

One of the possible minimal Higgs sets required to break the symmetry 'down to' $U(1)_{em}$ is the following: two triplets

$$\Delta_{L,R} \equiv \sigma\boldsymbol{\delta}_{L,R} = \begin{pmatrix} \delta^+/\sqrt{2} & \delta^{++} \\ \delta^0 & -\delta^+/\sqrt{2} \end{pmatrix}_{L,R}, \tag{2.5}$$

with quantum numbers $(1,0,2)$ and $(0,1,2)$ respectively, and one bi-doublet

$$\Phi \equiv \begin{pmatrix} \phi_1^0 & \phi_1^+ \\ \phi_2^- & \phi_2^0 \end{pmatrix} \tag{2.6}$$

with $(1/2, 1/2, 0)$. Using the above fields we can break the symmetry in the following three stages:

$$
\begin{aligned}
& SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times P \\
\xrightarrow{M_P}\ & SU(2)_L \times SU(2)_R \times U(1)_{B-L} \\
\xrightarrow{M_{W_R}}\ & SU(2)_L \times U(1)_Y \\
\xrightarrow{M_{W_L}}\ & U(1)_{em}.
\end{aligned}
$$

With our particular choise of Higgs multiplets the parity P and $SU(2)$ symmetry can be broken at the same energy scale: $M_P = M_{W_R}$.

The vev's of the Higgs fields are chosen to be:

$$\langle \Delta_{L,R} \rangle = \begin{pmatrix} 0 & 0 \\ v_{L,R} & 0 \end{pmatrix}$$

and

$$\langle \Phi \rangle = \begin{pmatrix} \kappa & 0 \\ 0 & \kappa' e^{i\alpha} \end{pmatrix},$$

where $e^{i\alpha}$ is a CP violating phase.

With this background information we can go on and derive the charged and neutral gauge boson masses in a similar way as we did for the SM. Also, certain arguments [11] can show that $\kappa \ll v_R$ and $v_L \ll \kappa$ which is essential in understanding why certain physical parameters like, for example, the mass of the neutrino are small compared to others.

Next, we will consider the Supersymmetric model (SUSY) where we will apply many of the ideas introduced here to specific calculations. It should be understood that the left-right model introduced here is a complete model of its own, but in our case it will be used as a basis for a more extensive model based on Supersymmetry, the so-called ***Left-Right Supersymmetric model*** (L-R SUSY).

Chapter 3

Supersymmetry

3.1 Introduction

We will attempt hereby to introduce the basic concepts of supersymmetry and try to explain and analyze some of its predictions. This short introduction can by no means be considered either an extensive or in depth approach to the subject. It is based mainly on references [18, 19, 20] and the reader is encouraged to consult them.

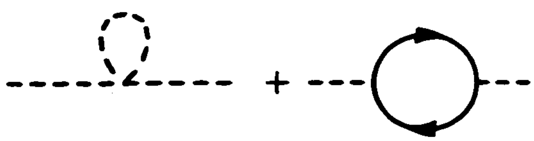
According to our present understanding of subatomic physics all particles have fermions as their basic constituents (quarks) or they themselves are considered elementary (leptons). On the other hand all known forces are mediated by bosonic carriers ($\gamma, W^{\pm}, Z^0, g$).[1] Therefore, if we ever hope to unify particles and interactions into a single self-consistent theory we will probably have to somehow unify bosons and fermions. The basic idea of supersymmetry is that it transforms bosons into fermions and vice versa; we will later discuss the basic features of supersymmetry generators and some of the implications of their algebra.

Supersymmetry (SUSY) predicts the existence of a *superpartner* for all known par-

[1] Gravitons have not yet been observed, possibly due to their weak strength, but it is believed that they exist and have spin 2.

ticles as we will show below. The total angular momentum (j) of the superpartner differs by half a unit from the original field. Therefore the superpartners of bosons are fermions and vice versa. All the other quantum numbers are the same as those of the original partner (color, lepton number, etc.). SUSY too, incorporates a symmetry breaking mechanism, similar to the SM and L-R Model, which breaks the mass degeneracy between supersymmetric partners. After symmetry breaking the fields are mass eigenstates and we can further proceed with possible generation mixings (super CKM matrix) and even supersymmetric particle mixing between scalar leptons, quarks, etc.

One of the most serious problems of the SM which is inherited in the L-R model is the so-called *gauge hierarchy problem* (GHP). It manifests itself when we try to calculate the masses of the Higgs particles. By doing so, using perturbation series beyond tree level, we get quadratic divergences which would push the masses to the order of Plank scale ($M_P \sim 10^{19}$ GeV) unless the perturbation terms cancel to 26 decimal places(!) [13, 31]. Supersymmetric models on the other hand do not have such a problem since, as we saw above, for every fermionic(bosonic) loop there is a corresponding bosonic(fermionic) with the same mass and coupling strength but opposite sign. Therefore, as shown in eq. (3.1) below, they cancel each other out[2]:



$= 0$ (3.1)

All particles in a supersymmetric theory obey a continuous global symmetry called

[2]In *broken* supersymmetry, due to non-equality of the masses, the divergences do appear but are not quadratic.

R-symmetry. Certain conditions can break the continuity but nevertheless a *discrete* R-symmetry almost always applies which in turn gives rise to a conserved quantum number called ***R-parity***, defined to be:

$$R \equiv (-1)^{2j+3B+L}.$$

All 'ordinary' particles have $R = +1$ and all supersymmetric partners have $R = -1$. R is a multiplicative quantum number which, combined with the above observation, gives rise to two fundamental consequences on any supersymmetric model:

Firstly, all supersymmetric particles must be produced in pairs since at any present experimental situation incoming states consist only of ordinary particles, therefore the final state has to contain an even number of supersymmetric particles.

Secondly, there must be a *lightest* and stable SUSY particle since any decaying super-particle would not be able to decay only to non-supersymmetric particles due to R-parity conservation[3].

It is remarkable that supersymmetry has occupied so many theorists over the years without a hint of experimental evidence. The two main reasons for this is the potential resolution of the gauge hierarchy problem and a possible inclusion of gravity through local supersymmetry. But there is an additional reason for hoping that one day we will detect some sign of supersymmetry: according to our models almost all supersymmetric partners, after symmetry breaking, acquire masses which are, in general, much heavier than ordinary particles.

[3]The photino ($\tilde{\gamma}$), which is the photon's superpartner, is *one* of the possible candidates.

3.2 Supersymmetric Algebra

The generator(s) of supersymmetric transformations interchange fermions into bosons and vice versa, according to the defining relations [4, 19]:

$$\begin{aligned} Q\,|\mathrm{fermion}\rangle &= |\mathrm{boson}\rangle \\ Q\,|\mathrm{boson}\rangle &= |\mathrm{fermion}\rangle \end{aligned} \tag{3.2}$$

Applying the unitary operator U, which represents a 2π rotation, on the first relation above we have:

$$UQ\,|\mathrm{fermion}\rangle = U\,|\mathrm{boson}\rangle = UQU^{-1}U\,|\mathrm{fermion}\rangle\,.$$

But we know that for fermionic states:

$$U\,|\mathrm{fermion}\rangle = -\,|\mathrm{fermion}\rangle\,,$$

therefore

$$\begin{aligned} UQU^{-1}U\,|\mathrm{fermion}\rangle &= U\,|\mathrm{boson}\rangle = |\mathrm{boson}\rangle \\ &= Q\,|\mathrm{fermion}\rangle = -QU\,|\mathrm{fermion}\rangle \\ \Rightarrow \qquad UQU^{-1} &= -Q. \end{aligned}$$

And similarly for the second equation.

We see that a 2π rotation of Q is accompanied by a change in sign; it can be shown that for *any* Lorentz transformation the behavior of the Q's is that of a spinor operator. On the other hand, it can also be shown that Q operators are invariant under translations which implies that:

$$[Q, E] = [Q, \mathbf{P}] = 0 \tag{3.3}$$

and the *anticommutator* of two symmetry generators is also a symmetry generator. Let us for example consider the eigenvalues of $\{Q, Q^\dagger\}$ applied to any state $|s\rangle$:

$$\begin{aligned}
&\langle s| \{Q, Q^\dagger\} |s\rangle \\
&= \langle s| QQ^\dagger |s\rangle + \langle s| Q^\dagger Q |s\rangle \\
&= |Q^\dagger |s\rangle|^2 + |Q |s\rangle|^2 \geq 0
\end{aligned} \tag{3.4}$$

and since $|s\rangle$ represents any state, the above Hermitian operator will have positive definite eigenvalues, unless $Q \equiv 0$.

The anticommutator of Q and $Q^\dagger$ is a linear combination of the energy and momentum operators:

$$\{Q, Q^\dagger\} = \alpha E + \boldsymbol{\beta} \cdot \mathbf{P}, \tag{3.5}$$

which suggests the connection of supersymmetry with space-time transformations and therefore supergravity. We can also show that by summing over all generators of our model the momentum terms cancel and we are left with one term proportional to the energy:

$$\sum_Q \{Q, Q^\dagger\} = \gamma E$$

For a sensible theory, we want our energy eigenvalues to be bounded from above and therefore (3.4) implies that: $\gamma > 0$. As usual, we denote by $|0\rangle$ the state with the lowest eigenvalue (vacuum). According to eq. (3.4), if $Q\,|0\rangle = Q^\dagger\,|0\rangle = 0$ then $E\,|0\rangle = 0$ and vice versa. This means that any state $|s\rangle$, except the vacuum, will have at least one 'superpartner' state: $Q\,|s\rangle$ or $Q^\dagger\,|s\rangle$ with a spin number that differs by $\frac{1}{2}$ from the original state and they are all members of the same supermultiplet. Combining this result with eq. (3.3) we see that supersymmetric partners must have the same mass. We know that this result is *not* supported by experiments, which leads us to the conclusion that if supersymmetry is to be a realizable theory we must provide it with a symmetry breaking mechanism, which although lifts the mass degeneracy it does not affect the structure of the multiplets .

Different models have different numbers of generators (N) but all of the Q's have four components since, as we showed above, they are spinor operators. Therefore, $N = 1$ supersymmetry is the simplest example of such a theory and it is called the *Minimal Supersymmetric Standard Model* (MSSM). Other candidates are $N = 4$ and $N = 8$ (supergravity) which have advantages and disadvantages: finiteness, particle spectra etc. For example, for theories with $N > 1$ we would get an expression analogous to (3.5):

$$\{Q_i, Q_j^\dagger\} = \delta_{ij}\,(\alpha E + \beta \mathbf{P})$$

In this work we are exclusively working with $N = 1$ supersymmetry.

3.3 Particle content and notation

Supersymmetric partners of ordinary particles are denoted by an overhead (~). Often, weak eigenstates will mix in a similar to the SM fashion, giving mass eigenstates which in Table 1 we put on a separate column. Fermionic superpartners of bosons are named with a suffix *-ino* and bosonic superpartners of fermions are named with a prefix *scalar* (or simply *s-*). Table 1 lists the most common superpartners and their names[4].

A word of caution is appropriate here: the following table is only *representative* of the particle content in the MSSM. The reader should be warned of the existence of different notations and multiple mixing mechanisms (an example of which we shall see in a following section) which give rise to different physical particles (mass eigenstates).

[4]We will not include mass eigenstates with specific couplings but the reader should be aware of their existence.

Ordinary particles	Supersymmetric partners			
	Weak eigenstates		*Mass eigenstates*	
q	$\tilde{q}_L, \tilde{q}_R$	s-quark	$\tilde{q}_1, \tilde{q}_2$	s-quark
l	$\tilde{l}_L, \tilde{l}_R$	s-lepton	$\tilde{l}_1, \tilde{l}_2$	s-lepton
ν	$\tilde{\nu}$	s-neutrino	$\tilde{\nu}$	s-neutrino
g	$\tilde{g}$	gluino	$\tilde{g}$	gluino
$W^\pm$	$\tilde{W}^\pm$	wino		
H_1^+	$\tilde{H}_1^+$	higgsino	$\tilde{\chi}_{1,2}^\pm$	charginos
H_2^-	$\tilde{H}_2^-$	higgsino		
γ	$\tilde{\gamma}$	photino		
Z^0	$\tilde{Z}^0$	zino	$\tilde{\chi}_i^0$	neutralinos
H_1^0	$\tilde{H}_1^0$	higgsino		
H_2^0	$\tilde{H}_2^0$	higgsino		

Table 3.1: Most common supersymmetric partners.

Incidentally, assuming the photino ($\tilde{\gamma}$) is the lightest SUSY-particle, experimental searches look for photino production which would show up as substantial fractions of the total missing energy and momentum since, unlike neutrinos, its production is not associated with any lepton. (e.g. for the reaction: $\tilde{l}^\pm \to l^\pm \tilde{\gamma}$ in the $\tilde{l}^\pm$ rest frame and neglecting the mass of the lepton, half the energy of the event escapes).

3.4 Structure of the MSSM Lagrangian

Below we give the Lagrangian terms needed for an $SU(2) \times U(1)$ model of broken supersymmetry [18, 21]. Supersymmetric Yang-Mills theories contain two basic types of fields: *gauge multiplets* (λ^a, V_μ^a) in the adjoint representation of a gauge group G and *matter multiplets* (A_i, ψ_i) in some representation R of G. The A_i's are *complex* scalar fields, the V_μ^a's spin-1 real vector fields and finally λ and ψ are two-component fermions (see Appendix A). Employing the summation convention throughout we have the following four main terms:

1. **Kinetic energy terms**

2. **Self interactions of gauge multiplets** similar to the SM giving the familiar three and four-gauge-boson vertices [7] :

$$\mathcal{L}_s = -\frac{1}{4} F_{\mu\nu}^a F_a^{\mu\nu} \tag{3.6}$$

where we borrowed the notation from the SM non-Abelian field strength tensor: $F_{\mu\nu}^a \equiv \partial_\mu V_\nu^a - \partial_\nu V_\mu^a - g f_{abc} V_\mu^b V_\nu^c$. And the **gaugino-gauge field interactions**:

$$\mathcal{L}_{gg} = i g f_{abc} \lambda^a \sigma^\mu \bar{\lambda}^b V_\mu^c \tag{3.7}$$

where f_{abc} are the structure constants of G.

We should note that there is no interaction between the $U(1)$ singlets λ' and V'_μ, therefore in this particular case we set $f_{abc} = 0$.

3. **Gauge and matter multiplets interaction** term which is important for this work:

$$\begin{aligned} \mathcal{L}_{gm} = \; & -g T_{ij}^a V_\mu^a (\bar{\psi}_i \bar{\sigma}^\mu \psi_j + i A_i^* \overleftrightarrow{\partial}_\mu A_j) + i g \sqrt{2} T_{ij}^a (\lambda^a \psi_j A_i^* - \bar{\lambda}^a \bar{\psi}^i A_j) \\ & + g^2 (T^a T^b)_{ij} V_\mu^a V^{\mu b} A_i^* A_j \end{aligned} \tag{3.8}$$

where T^a is the *Hermitian* group generator in the representation R of G. Note that for $U(1)$ multiplets we have to perform the following replacements:

$$\begin{aligned} V_\mu^a &\to V'_\mu, \\ gT_{ij}^a &\to \tfrac{1}{2}g'Y_i\delta_{ij}, \end{aligned} \tag{3.9}$$

where Y_i is the $U(1)$ quantum number of the matter field (hypercharge).

4. Finally, the **fermion mass terms** and **Yukawa interactions** are given by:

$$-\frac{1}{2}\left(\frac{\partial^2 W}{\partial A_i \partial A_j}\right)\psi_i\psi_j + \text{h.c.} \tag{3.10}$$

The ordinary *scalar potential* is :

$$V = \frac{1}{2}D^a D^a + F_i^* F_i. \tag{3.11}$$

Where the superpotential W is some cubic function of the scalar fields and we define:

$$\begin{aligned} F_i &\equiv \partial W/\partial A_i, \\ D^a &= gA_i^* T_{ij}^a A_j \end{aligned} \tag{3.12}$$

Another important piece needed is the *soft-supersymmetry breaking* terms which do not introduce quadratic divergences to the unrenormalized theory. The possible explicit terms are [22]:

$$\tilde{M}_1\Re[A^2] + \tilde{M}_2\Im[A^2] + c(A^3 + \text{h.c.}) + \tilde{M}_3(\lambda^a\lambda^a + \bar{\lambda}^a\bar{\lambda}^a) + \tilde{M}_4(\lambda'\lambda' + \bar{\lambda}'\bar{\lambda}') \tag{3.13}$$

With A^2 and A^3 being (gauge invariant) quadratic and cubic combinations of the scalar fields[5]. The terms $\tilde{M}_3$ and $\tilde{M}_4$ are Majorana mass terms for the gauginos, corresponding to the groups G and $U(1)$ respectively. We shall see the importance of the last two terms in (3.13) later in our discussion of neutralinos.

[5]Note that $\Re[\,]$ and $\Im[\,]$ are the real and imaginary parts respectively.

3.5 Chargino Mixing

3.5.1 Introduction

Before showing explicitly the Lagrangian terms responsible for the mixing of charged gauginos and higgsinos, it is appropriate to say a few words about the model:

As we mentioned before, we deal with possibly the simplest model of broken $SU(2)\times U(1)$ supersymmetry with the minimal Higgs set required, namely two Higgs doublets H_1 and H_2 with vev's:

$$\langle H_1\rangle = \frac{v_1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \end{pmatrix}, \ \langle H_2\rangle = \frac{v_2}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{3.14}$$

where $v_1 \neq v_2$.

In order to obtain the Feynman rules we list the appropriate interaction terms and then shift the scalar fields: $H_i \to H_i + \langle H_i\rangle$, $i = 1,2$ (unitary gauge). Finally, we switch from two to four-component notation and we use the definitions of the mass eigenstates (physical particles).

3.5.2 The Chargino Lagrangian

The Lagrangian term for chargino mixing is the following:

$$\mathcal{L}_C = \frac{ig}{\sqrt{2}}\left[v_1\lambda^+\psi^2_{H_1} + v_2\lambda^-\psi^1_{H_2}\right] + M\lambda^+\lambda^- - \mu\psi^2_{H_1}\psi^1_{H_2} + \text{h.c.}. \tag{3.15}$$

The first term in eq. (3.15) comes from (3.8) keeping only the $a = 1,2$ contributions and taking $\sigma/2$ as the $SU(2)$ group generator for the representation R, where σ are the Pauli matrices. We also used the relation: $\lambda^\pm \equiv \frac{1}{\sqrt{2}}(\lambda^1 \mp i\lambda^2)$.

The second term comes from the soft-SUSY breaking part of the Lagrangian (3.13) where again we kept only the $a = 1,2$ contributions and defined $M \equiv 2\tilde{M}_3$. The last term comes from an additional term in the superpotential of the form: $\mu\epsilon_{ij}H_1^iH_2^j$.

Let us now try and rewrite the above part in a more compact form, with the aid of the following definitions:

$$\begin{aligned}
&\psi_j^+ \equiv (-i\lambda^+, \psi_{H_2}^1),\ \psi_j^- \equiv (-i\lambda^-, \psi_{H_1}^2),\\
&\sin\theta_v = \tfrac{v_1}{\sqrt{v_1^2+v_2^2}},\ \cos\theta_v = \tfrac{v_2}{\sqrt{v_1^2+v_2^2}}, \qquad (3.16)\\
&m_W^2 \equiv \tfrac{1}{4}g^2(v_1^2+v_2^2).
\end{aligned}$$

Further, we can easily verify that:

$$\mathcal{L}_C = -\frac{1}{2}(\psi^+, \psi^-)\begin{pmatrix} 0 & X^T \\ X & 0 \end{pmatrix}\begin{pmatrix} \psi^{+T} \\ \psi^{-T} \end{pmatrix} + \text{h.c.}. \qquad (3.17)$$

Where:

$$X = \begin{pmatrix} M & m_W\sqrt{2}\cos\theta_v \\ m_W\sqrt{2}\cos\theta_v & \mu \end{pmatrix}. \qquad (3.18)$$

We then *define* the mass eigenstates as:

$$(\chi^+)^T \equiv V(\psi^+)^T,\ (\chi^-)^T \equiv U(\psi^-)^T \qquad (3.19)$$

where U and V are *unitary* and obey the following relation:

$$U^* X V^{-1} = M_D. \qquad (3.20)$$

M_D being *diagonal* with nonnegative entries: $\tilde{M}_+, \tilde{M}_-$, where by convention we choose $\tilde{M}_+ > \tilde{M}_-$. From the above definition we obtain a mass term form:

$$\begin{aligned}
\mathcal{L}_C &= -\tfrac{1}{2}\left[\psi^- X\psi^{+T} + \psi^+ X^T\psi^{-T}\right] + \text{h.c.}\\
&= -\tfrac{1}{2}\left[\psi^-(U^*)^{-1}U^* X V^{-1} V\psi^{+T}\right. \qquad (3.21)\\
&\qquad \left. + \psi^+(V^*)^{-1}V^* X^T U^{-1} U\psi^{-T}\right] + \text{h.c.}.
\end{aligned}$$

Now, due to eq. (3.20) and the unitarity condition we have:

$$M_D = M_D^T = (V^{-1})^T X^T (U^*)^T = V^* X^T U^{-1} \tag{3.22}$$

and so eq. (3.21) becomes:

$$\begin{aligned}
\mathcal{L}_C &= -\tfrac{1}{2}\left[\psi^-(U^*)^{-1}M_D\chi^{+T} + \psi^+(V^*)^{-1}M_D\chi^{-T}\right] + \text{h.c.} \\
&= -\tfrac{1}{2}\left[\psi^- U^T M_D\chi^{+T} + \psi^+ V^T M_D\chi^{-T}\right] + \text{h.c.} \\
&= -\tfrac{1}{2}\left[\chi^- M_D\chi^{+T} + \chi^+ M_D\chi^{-T}\right] + \text{h.c.} \\
&= -\left[\chi^- M_D\chi^{+T}\right] + \text{h.c.} \\
&= -\left[\chi_i^-(M_D)_{ij}\chi_j^+\right] + \text{h.c.}
\end{aligned} \tag{3.23}$$

where we used the first relation of (A.12).

If we now define the *four-component* Dirac spinors:

$$\tilde{\chi}_1 = \begin{pmatrix} \chi_1^+ \\ \bar{\chi}_1^- \end{pmatrix}, \; \tilde{\chi}_2 = \begin{pmatrix} \chi_2^+ \\ \bar{\chi}_2^- \end{pmatrix} \tag{3.24}$$

and use eq. (A.13) we can write (3.23) as follows:

$$\mathcal{L}_C = -(\tilde{M}_+\bar{\tilde{\chi}}_1\tilde{\chi}_1 + \tilde{M}_-\bar{\tilde{\chi}}_2\tilde{\chi}_2). \tag{3.25}$$

From (3.20) we can easily derive that:

$$M_D^2 = VX^\dagger XV^{-1} = U^* XX^\dagger (U^*)^{-1} \tag{3.26}$$

which, by computing the eigenvectors of $X^\dagger X$ and $XX^\dagger$, can give us the elements of V and U^* respectively.

We list here some of the formulas given in refs. [18, 23] for the resulting values (under the assumption that M and μ are real), the interested reader can find a more involved treatment in the references above.

$$\begin{aligned}\tilde{M}^2_{\pm} &= \frac{1}{2}\Big\{M^2+\mu^2+2m^2_W \pm \Big[(M^2-\mu^2)^2+4m^4_W\cos^2 2\theta_v \\ &+ 4m^4_W\left(M^2+\mu^2+2M\mu\sin 2\theta_v\right)\Big]^{\frac{1}{2}}\Big\}. \end{aligned} \tag{3.27}$$

Further, parametrizing U and V with respect to a parameter ϕ we get $U = O_-$ and $V = O_+$ (or $V = \sigma_3 O_+$ if $detX < 0$ [6]), where:

$$O_{\pm} = \begin{pmatrix} \cos\phi_{\pm} & \sin\phi_{\pm} \\ -\sin\phi_{\pm} & \cos\phi_{\pm} \end{pmatrix}$$

The above formulas attack the general case. In order to get explicit and simple expressions for the charginos it would be instructive to consider the special case: $\mu = 0$ and $v_1 = v_2$. With this choice $\phi_- = \phi_+$, collectively denoted as ϕ and

$$\tilde{M}_{\pm} = \left[m^2_W + \frac{1}{4}M^2\right]^{1/2} \pm \frac{1}{2}M \ , \ \cos\phi = \left(\frac{\tilde{M}_+}{\tilde{M}_+ + \tilde{M}_-}\right)^{1/2}.$$

The defining relations (3.19) and (3.24) will give:

$$\tilde{\chi}_1 = \begin{pmatrix} -i\lambda^+\cos\phi + \psi^1_{H_2}\sin\phi \\ i\bar{\lambda}^-\cos\phi + \bar{\psi}^2_{H_1}\sin\phi \end{pmatrix}, \ \tilde{\chi}_2 = \begin{pmatrix} -i\lambda^+\sin\phi - \psi^1_{H_2}\cos\phi \\ -i\bar{\lambda}^-\sin\phi + \bar{\psi}^2_{H_1}\cos\phi \end{pmatrix}. \tag{3.28}$$

3.5.3 Feynman rules

We will now try to show the connection between the relevant Lagrangian terms, the four-component spinors and the derivation of Feynman rules. We will take as an example the following $\lambda - \psi - A$ term (see (3.8)) which is a supersymmetric version of the familiar $W^{\pm}e\nu$ term in the SM:

$$\mathcal{L}_{\lambda\psi A} = \frac{i}{\sqrt{2}}\left[g\sigma^a_{ij}\lambda^a\right]\psi^j_L L^*_i + \text{h.c.}. \tag{3.29}$$

[6]Under this condition both resulting masses $\tilde{M}_{\pm}$ are positive.

Keeping only the $a = 1, 2$ contributions we have

$$\begin{aligned}
\mathcal{L}_{\lambda\psi A} &= \frac{i}{\sqrt{2}} g \left[\sigma^1_{ij}\lambda^1 + \sigma^2_{ij}\lambda^2\right] \psi^j_L L^*_i + \text{h.c.} \\
&= \frac{i}{\sqrt{2}} g \left[(\lambda^1 - i\lambda^2)\psi^2_L L^*_1 + (\lambda^1 + i\lambda^2)\psi^1_L L^*_2\right] + \text{h.c.} \qquad (3.30) \\
&= ig\left[\lambda^+ e_L \tilde{\nu}^* + \lambda^- \nu_L \tilde{e}^*_L - \bar{\lambda}^+ \bar{e}_L \tilde{\nu} - \bar{\lambda}^- \bar{\nu}_L \tilde{e}_L\right],
\end{aligned}$$

where Table 14 of ref. [18] has been used and the notation conventions of Appendix A. Following our usual strategy, we will try to convert eq. (3.30) into four-component notation with the aid of the following definitions:

$$\tilde{W} = \begin{pmatrix} -i\lambda^+ \\ i\bar{\lambda}^- \end{pmatrix}, \ \tilde{H} = \begin{pmatrix} \psi^1_{H_2} \\ \bar{\psi}^2_{H_1} \end{pmatrix}. \qquad (3.31)$$

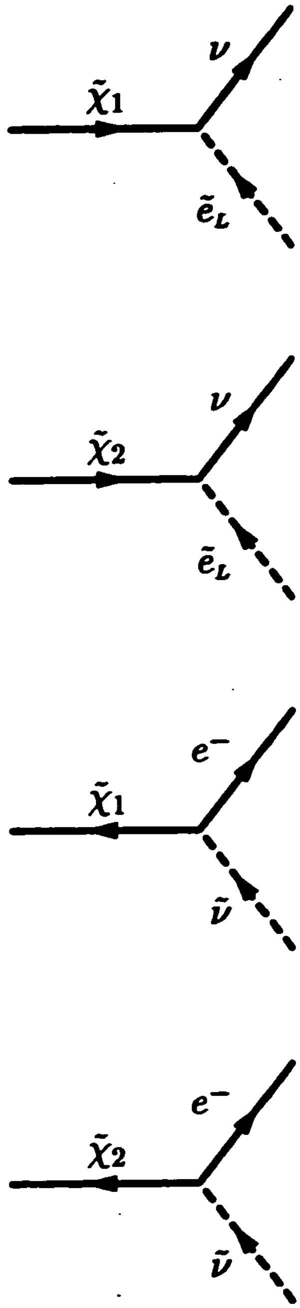
In particular, using (A.14), eq. (3.30) becomes:

$$\mathcal{L}_{int} = -g\left[\bar{\tilde{W}} P_L \nu \tilde{e}^*_L + \bar{\nu} P_R \tilde{W} \tilde{e}_L + \bar{\tilde{W}}^c P_L e \tilde{\nu}^* + \bar{e} P_R \tilde{W}^c \tilde{\nu}\right]. \qquad (3.32)$$

Expressing the above in terms of the chargino mass eigenstates we get:

$$\begin{aligned}
\mathcal{L}_{int} = & -g\{(U^*_{11}\bar{\tilde{\chi}}_1 + U^*_{21}\bar{\tilde{\chi}}_2) P_L \nu \tilde{e}^*_L + \bar{\nu} P_R (U_{11}\tilde{\chi}_1 + U_{21}\tilde{\chi}_2)\tilde{e}_L \\
& + \left(V^*_{11}\bar{\tilde{\chi}}^c_1 + V^*_{21}\bar{\tilde{\chi}}^c_2\right) P_L e \tilde{\nu}^* + \bar{e} P_R (V_{11}\tilde{\chi}^c_1 + V_{21}\tilde{\chi}^c_2)\tilde{\nu}\}.
\end{aligned} \qquad (3.33)$$

We can then easily read the vertex factors for these specific interactions in the MSSM which appear in 3.33, they are shown explicitly in Figure 3.1. Note that the arrows indicate lepton and flavour quantum-number flow for leptons and quarks respectively. Finally, for the last two diagrams the charge conjugation matrix C appears explicitly in the vertex elements instead of the chargino spinors: $\tilde{\chi}^c_i$.



$$-igU_{11}\left(\frac{1+\gamma^5}{2}\right)$$

$$-igU_{21}\left(\frac{1+\gamma^5}{2}\right)$$

$$-igV_{11}\left(\frac{1+\gamma^5}{2}\right)C$$

$$-igV_{11}\left(\frac{1+\gamma^5}{2}\right)C$$

Figure 3.1: Feynman rules for some characteristic chargino interactions in the MSSM.

3.6 Neutralino Mixing

Similarly to the chargino case, the contributions for the neutralino-mixing Lagrangian piece come from the $\lambda - \psi - A$ terms of (3.8) with $a = 3$, the additional superpotential term and the relevant soft-SUSY breaking terms, where we have defined $M' \equiv 2\tilde{M}_4$. We should note here that some supersymmetric models, incorporated into GUT scenarios, predict that M and M' are proportional [18, 23]:

$$M' = \frac{5g'^2}{3g^2}M, \tag{3.34}$$

similar to the θ_W prediction. These models also predict that $M = (g^2/g_s^2)\tilde{M}_g$, where $\tilde{M}_g$ stands for the gluino mass and g_s the $SU(3)_c$ coupling constant. Let us show explicitly the derivation of the $\lambda - \psi - A$ terms ($a = 3$):

$$\begin{aligned}
\mathcal{L}_N &= ig\sqrt{2}(\sigma_{ij}/\sqrt{2})(\lambda^3\psi_j H_i^*) + \sqrt{2}\tfrac{1}{2}g' y_i\delta_{ij}(\lambda'\psi_j H_i^*) + \text{h.c.} \\
&= \tfrac{i}{\sqrt{2}}g\lambda^3[\sigma^3_{1j}\tfrac{v_1}{\sqrt{2}}\psi^j_{H_1} + \sigma^3_{2j}\tfrac{v_2}{\sqrt{2}}\psi^j_{H_2}] + \tfrac{1}{\sqrt{2}}g'\lambda'[-\psi^1_{H_1}\tfrac{v_1}{\sqrt{2}} + \psi^2_{H_2}\tfrac{v_2}{\sqrt{2}}] \\
&= \tfrac{i}{\sqrt{2}}g\lambda^3[\tfrac{v_1}{\sqrt{2}}\psi^1_{H_1} - \tfrac{v_2}{\sqrt{2}}\psi^2_{H_2}] + \tfrac{i}{\sqrt{2}}g'\lambda'[\psi^2_{H_2}\tfrac{v_2}{\sqrt{2}} - \psi^1_{H_1}\tfrac{v_1}{\sqrt{2}}].
\end{aligned}$$

Therefore, adding the other two terms, the neutralino piece becomes:

$$\begin{aligned}
\mathcal{L}_N &= \tfrac{i}{2}g\lambda^3(v_1\psi^1_{H_1} - v_2\psi^2_{H_2}) - \tfrac{i}{2}g'\lambda'(v_1\psi^1_{H_1} - v_2\psi^2_{H_2}) \\
&+ \tfrac{1}{2}M\lambda^3\lambda^3 + \tfrac{1}{2}M'\lambda'\lambda' + \mu\psi^1_{H_1}\psi^2_{H_2} + \text{h.c.}
\end{aligned} \tag{3.35}$$

Again, using the same approach as in the chargino case we define:

$$\psi^0_j = (-i\lambda', -i\lambda^3, \psi^1_{H_1}, \psi^2_{H_2}). \tag{3.36}$$

With this definition $\mathcal{L}_N$ becomes:

$$\mathcal{L}_N = -\frac{1}{2}\left(\psi^0\right)^T Y\psi^0 + \text{h.c.} \tag{3.37}$$

where

$$Y = \begin{pmatrix} M' & 0 & -m_z \sin\theta_v \sin\theta_W & m_z \cos\theta_v \sin\theta_W \\ 0 & M & m_z \sin\theta_v \cos\theta_W & -m_z \cos\theta_v \cos\theta_W \\ -m_z \sin\theta_v \sin\theta_W & m_z \sin\theta_v \cos\theta_W & 0 & -\mu \\ m_z \cos\theta_v \sin\theta_W & -m_z \cos\theta_v \cos\theta_W & -\mu & 0 \end{pmatrix}. \tag{3.38}$$

The neutralino mass eigenstates are defined by the equation:

$$(\chi^0)^T = N(\psi^0)^T, \tag{3.39}$$

where N is a unitary matrix satisfying the relation :

$$N^* Y N^{-1} = N_D\,,\ \ (N_D)_{ij} \equiv \tilde{M}_i \geq 0. \tag{3.40}$$

Notice that Y is a symmetric matrix which is due to the Majorana nature of the particles[7], therefore we only need one diagonalizing matrix. Squaring the neutralino mixing matrix N_D we get (compare with eq. (3.20)):

$$N_D^2 = N Y^\dagger Y N^{-1}. \tag{3.41}$$

We can then rewrite (3.37) as:

$$\mathcal{L}_N = -\frac{1}{2}\tilde{M}_i \bar{\tilde{\chi}}_i^0 \tilde{\chi}_i^0.$$

Summation over repeated indices is implied. This is clearly a mass term for the $\tilde{\chi}_i^0$ *Majorana* spinors (see (A.17)):

$$\tilde{\chi}_i^0 \equiv \begin{pmatrix} \chi^0 \\ \bar{\chi}^0 \end{pmatrix}. \tag{3.42}$$

In an attempt to get a feeling for these results we consider the following simplifying assumptions which yield a massive photino (λ_γ) whose mass satisfies: $\frac{1}{2}M = \tilde{M}$. We

[7]It can be shown that for four-component Majorana spinors the following holds: $\bar{\tilde{\chi}}_j^0(1 \pm \gamma^5)\tilde{\chi}_k^0 = \bar{\tilde{\chi}}_k^0(1 \pm \gamma^5)\tilde{\chi}_j^0$

then redefine our basis as:

$$\psi_j^{0\prime} = \left(-i\lambda_\gamma, -i\lambda_z, \frac{v_1\psi_{H_1}^1 - v_2\psi_{H_2}^2}{(v_1^2+v_2^2)^{1/2}}, \frac{v_2\psi_{H_1}^1 + v_1\psi_{H_2}^2}{(v_1^2+v_2^2)^{1/2}}\right).$$

In this basis the new mixing matrix simplifies to:

$$Y' = \begin{pmatrix} M & 0 & 0 & 0 \\ 0 & M & m_z & 0 \\ 0 & m_z & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{3.43}$$

and the mass eigenstates are defined by:

$$\begin{aligned} (\chi^0)^T &= N'(\psi^{0\prime})^T, \\ N'^{*}Y'N'^{-1} &= N_D. \end{aligned} \tag{3.44}$$

From eq. (3.43) we see that we only need to diagonalize the non-trivial 2×2 submatrix of the matrix Y, and by doing so we get the diagonalizing matrix:

$$N' = \begin{pmatrix} \cos\phi & \sin\phi \\ -i\sin\phi & i\cos\phi \end{pmatrix}$$

where $\phi = \arccos\left[\tilde{M}_+/\left(\tilde{M}_+ + \tilde{M}_-\right)\right]^{1/2}$.

The fact that we have four neutralinos in the MSSM means that we can go on and make further simplifying assumptions or try and see the effect of different limits on spinor components and the corresponding Feynman rules [18].

3.6.1 Analytic results

We will not show any numerical results for the diagonalizing matrices U,V and Y since it is beyond the scope of this work. The interested reader is referred to ref. [24] for an extensive numerical analysis. We will however, for the sake of completeness, show some approximate analytic results below.

With the assumption that $|M \pm \mu|, |M' \pm \mu| \gg m_z$, which is a rather reasonable one remembering that the origins of the masses M and M' are from the soft-SUSY breaking terms and μ from the additional superpotential, then the following approximate relations hold [24, 25, 26]:

$$U \simeq \begin{pmatrix} 1 & m_W\sqrt{2}\frac{(M\sin\theta_v+\mu\cos\theta_v)}{M^2-\mu^2} \\ -m_W\sqrt{2}\frac{(M\sin\theta_v+\mu\cos\theta_v)}{M^2-\mu^2} & 1 \end{pmatrix} \tag{3.45}$$

and

$$V \simeq \begin{pmatrix} 1 & m_W\frac{(\sqrt{2}M\cos\theta_v+\mu\sin\theta_v)}{M^2-\mu^2} \\ -m_W\frac{(\sqrt{2}M\cos\theta_v+\mu\sin\theta_v)}{M^2-\mu^2} & \mathrm{sgn}(\mu) \end{pmatrix}. \tag{3.46}$$

For the above we used the assumption that $|M\mu| > m_W^2 \sin 2\beta$, where

$$tan\beta \equiv \frac{v_2}{v_1}$$

or equivalently that $\det X > 0$, since $\sin 2\beta = 2\sin\beta\cos\beta = 2\cos\theta_v\sin\theta_v$. Using the above matrices to diagonalize X, we finally get the following elements of M_D:

$$\begin{aligned} m_{\tilde{\chi}_1} &\simeq M + m_W^2\left(\frac{M+\mu\sin 2\beta}{M^2-\mu^2}\right), \\ m_{\tilde{\chi}_2} &\simeq |\mu| + m_W^2\mathrm{sgn}(\mu)\left(\frac{\mu+M\sin 2\beta}{\mu^2-M^2}\right). \end{aligned} \tag{3.47}$$

Note that if $\det X < 0$ then we have to replace $\mathrm{sgn}(\mu)$ with -1.

Finally, the neutralino mixing matrix is:

$$Y \simeq \begin{bmatrix} 1 & \frac{m_Z^2 \sin 2\theta_W (M'+\mu \sin 2\theta_v)}{2(M'-M)(\mu^2-M'^2)} & \frac{-m_Z \sin\theta_W (M' \sin\theta_v + \mu \cos\theta_v)}{M'^2-\mu^2} & \frac{m_Z \cos\theta_W (M' \cos\theta_v + \mu \sin\theta_v)}{M'^2-\mu^2} \\ \frac{m_Z^2 \sin 2\theta_W (M+\mu \sin 2\theta_v)}{2(M-M')(\mu^2-M^2)} & 1 & \frac{m_Z \cos\theta_W (M \sin\theta_v + \mu \cos\theta_v)}{M^2-\mu^2} & \frac{-m_Z \cos\theta_W (M \cos\theta_v + \mu \sin\theta_v)}{M^2-\mu^2} \\ \frac{-m_Z \sin\theta_W (\cos\theta_v - \sin\theta_v)}{\sqrt{2}(\mu+M')} & \frac{m_Z \cos\theta_W (\cos\theta_v - \sin\theta_v)}{\sqrt{2}(\mu+M)} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{-m_Z \sin\theta_W (\cos\theta_v + \sin\theta_v)}{\sqrt{2}(\mu-M')} & \frac{m_Z \cos\theta_W (\cos\theta_v + \sin\theta_v)}{\sqrt{2}(\mu-M)} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \tag{3.48}$$

Similarly to the chargino case we use the above result to obtain the (approximate) neutralino masses:

$$\begin{aligned} m_{\tilde\chi_1^0} &\simeq M' + \frac{m_Z^2 (M'+\mu \sin 2\theta_v) \sin^2\theta_W}{M'^2-\mu^2}, \\ m_{\tilde\chi_2^0} &\simeq M + \frac{m_Z^2 (M'+\mu \sin 2\theta_v) \cos^2\theta_W}{M^2-\mu^2}, \\ m_{\tilde\chi_3^0} &\simeq |\mu| + \frac{m_Z^2 (1-\sin 2\theta_v)(\mu + M \sin^2\theta_W + M' \cos^2\theta_W)\mathrm{sgn}(\mu)}{2(M+\mu)(M'+\mu)}, \\ m_{\tilde\chi_4^0} &\simeq |\mu| + \frac{m_Z^2 (1+\sin 2\theta_v)(\mu + M \sin^2\theta_W + M' \cos^2\theta_W)\mathrm{sgn}(\mu)}{2(M+\mu)(M'+\mu)}. \end{aligned} \tag{3.49}$$

Before we close our short introduction to the chargino and neutralino mass eigenstates, we should mention that, except from (3.34) which is true *only* if we impose SUSY-GUT, there are no other 'tight' constraints on the different parameters associated with the mixing matrix. Obviously different models put different constraints and make assumptions about the ordering of the states. We should emphasize that the column

$M' < M < \|\mu\|$	$M' < \|\mu\| < M$	$\|\mu\| < M' < M$
$m_{\tilde{\chi}_1^+} < m_{\tilde{\chi}_2^+}$	$m_{\tilde{\chi}_2^+} < m_{\tilde{\chi}_1^+}$	$m_{\tilde{\chi}_2^+} < m_{\tilde{\chi}_1^+}$
$m_{\tilde{\chi}_1^0} < m_{\tilde{\chi}_2^0} < m_{\tilde{\chi}_{3,4}^0}$	$m_{\tilde{\chi}_1^0} < m_{\tilde{\chi}_{3,4}^0} < m_{\tilde{\chi}_2^0}$	$m_{\tilde{\chi}_{3,4}^0} < m_{\tilde{\chi}_1^0} < m_{\tilde{\chi}_2^0}$

Table 3.2: Ordering of positive mass eigenvalues of charginos and neutralinos in three possible μ regions.

arrangement of U,V and Y is directly related to the mass ordering of the states (ordered with increasing mass e.g. $m_{\tilde{\chi}_2^0} > m_{\tilde{\chi}_1^0}$), simply because they are constructed this way. So, for example, if any chargino or neutralino: $\tilde{\chi}_i^0$ ($i = 2, 3, 4$) is produced it will cascade till the lightest-supersymmetric-particle[8] (LSP) $\tilde{\chi}_1^0$ is produced.

Finally, from (3.49) we clearly see that the relative size of $m_{\tilde{\chi}_3^0}$ and $m_{\tilde{\chi}_4^0}$ is a rather sensitive function of μ. It is therefore instructive to show the mass ordering in the three distinct regions as shown in Table 3.2, always under the assumption that $M' < M$. Ref. [26] gives an extensive numerical analysis of the mass hierarchy and different potential scenarios that may apply. Here we just skimmed the surface by borrowing their table and mentioning the three different regions. As a last remark on this topic we should say that in the limiting cases: $M' \simeq M$, $|\mu| \simeq M$ and $|\mu| \simeq M'$, we get degenerate eigenvalues and therefore we should use degenerate perturbation theory.

[8]As we have seen before $\tilde{\chi}_1^0$ is *one* of the possible candidates for the LSP.

Chapter 4

Left-Right Supersymmetry

We saw in the previous chapter that SUSY solves some of the problems of the MSSM but unfortunately not all. One possible scenario is SUSY GUTs which are quite succesful in predicting unification of the coupling constants and give relationships between parameters of the SM. After having studied the basic characteristics of both the L-R and MSSM it would therefore be natural to try and combine the two in a fully left-right supersymmetric model (L-R SUSY). The proposed gauge group is the $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ [13, 24, 27]. L-R SUSY could be a SUSY GUT but it could also be incorporated in SUSY GUTs among with other attractive features. As with all newly proposed models the problem that arises is simply how to distinguish L-R SUSY from the MSSM. The main direction should be to look for signals of particles that are absent from the MSSM. Later we will see that the model predicts the existence of doubly charged scalars and higgsinos which, as we saw in the previous chapter, they do not exist in the Standard Supersymmetric model.

As before, every field has a superpartner denoted by a ($\sim$) over it and the same name conventions as in the $N = 1$ supersymmetric model apply. The model includes the following types of fields:

1. Left and right *matter fields* are assigned to doublets similarly to the L-R model and their bosonic superpartners to singlets as shown on Table 4.1.

2. The $SU(2)_{L,R}$ *gauge fields* $\mathbf{W}^{\mu}_{L,R}$ are the vector boson triplets with $\lambda_{L,R}$ being their superpartners. Similarly, the V^{μ} is the $U(1)_{B-L}$ singlet vector boson and λ_V its superpartner[1].

3. Finally, the Higgs sector of the theory consists of two bidoublets: $\Phi_{u,d}$, responsible for both the up and down quark masses, two triplets $\Delta_{L,R}$, and two additional triplets $\delta_{L,R}$ which do not play any role in spontaneous symmetry breaking but are introduced to cancel triangle anomalies which would occur without their presence[28].

In Table LR-fields we list the particle content of the model and their quantum numbers. A complete and detailed description of the Lagrangian can be found in refs. [13, 24, 27]. We will not include it in this work since it is quite involved and we will only use the required parts for specific mechanisms. Let us however list the individual parts of the Lagrangian with a brief description of their function [27]:

$$\mathcal{L} = \mathcal{L}_{gauge} + \mathcal{L}_{matter} + \mathcal{L}_Y - V + \mathcal{L}_{soft}$$

where the first and second terms deal with the gauge and matter fields (kinetic energy terms, self interactions, etc.), the third is the Yukawa piece involving self interactions of the matter and Higgs multiplets. V is the scalar potential and $\mathcal{L}_{soft}$ is the soft supersymmetry-breaking part which, among other things, gives Majorana mass to the gauginos. We have reviewed all these terms when describing the MSSM in the previous

[1]The gauge bosons are the only exception to the notation rule.

Fields	$SU(2)_L \times$	$SU(2)_R \times$	$U(1)_{B-L}$
Matter			
$Q_{L,R} = \begin{pmatrix} u \\ d \end{pmatrix}_{L,R}$	$\frac{1}{2}(0)$	$0(\frac{1}{2})$	$\frac{1}{3}$
$\tilde{Q}_{L,R} = \begin{pmatrix} \tilde{u} \\ \tilde{d} \end{pmatrix}_{L,R}$	$\frac{1}{2}(0)$	$0(\frac{1}{2})$	$\frac{1}{3}$
$L_{L,R} = \begin{pmatrix} \nu \\ e \end{pmatrix}_{L,R}$	$\frac{1}{2}(0)$	$0(\frac{1}{2})$	-1
$\tilde{L}_{L,R} = \begin{pmatrix} \tilde{\nu} \\ \tilde{e} \end{pmatrix}_{L,R}$	$\frac{1}{2}(0)$	$0(\frac{1}{2})$	-1
Higgs			
$\Phi_{u,d} = \begin{pmatrix} \phi_1^0 & \phi_1^+ \\ \phi_2^- & \phi_2^0 \end{pmatrix}_{u,d}$	$\frac{1}{2}$	$\frac{1}{2}$	0
$\tilde{\Phi}_{u,d} = \begin{pmatrix} \tilde{\phi}_1^0 & \tilde{\phi}_1^+ \\ \tilde{\phi}_2^- & \tilde{\phi}_2^0 \end{pmatrix}_{u,d}$	$\frac{1}{2}$	$\frac{1}{2}$	0
$\Delta_{L,R} = \begin{pmatrix} \frac{1}{\sqrt{2}}\Delta^+ & \Delta^{++} \\ \Delta^0 & -\frac{1}{\sqrt{2}}\Delta^+ \end{pmatrix}_{L,R}$	$1(0)$	$0(1)$	2
$\tilde{\Delta}_{L,R} = \begin{pmatrix} \frac{1}{\sqrt{2}}\tilde{\Delta}^+ & \tilde{\Delta}^{++} \\ \tilde{\Delta}^0 & -\frac{1}{\sqrt{2}}\tilde{\Delta}^+ \end{pmatrix}_{L,R}$	$1(0)$	$0(1)$	2
$\delta_{L,R} = \begin{pmatrix} \frac{1}{\sqrt{2}}\delta^- & \delta^0 \\ \delta^{--} & -\frac{1}{\sqrt{2}}\delta^- \end{pmatrix}_{L,R}$	$1(0)$	$0(1)$	-2
$\tilde{\delta}_{L,R} = \begin{pmatrix} \frac{1}{\sqrt{2}}\tilde{\delta}^- & \tilde{\delta}^0 \\ \tilde{\delta}^{--} & -\frac{1}{\sqrt{2}}\tilde{\delta}^- \end{pmatrix}_{L,R}$	$1(0)$	$0(1)$	-2

Table 4.1: Matter and Higgs field content of the L-R model.

chapter. The main difference is that the left-right supersymmetric model contains both left *and* right parts along with the extra fields (Higgs sector, etc.). For illustrative purposes let us write down the *gauge* terms of the Lagrangian:

$$\begin{aligned}\mathcal{L}_{\text{gauge}} &= -\tfrac{1}{4}W^L_{\mu\nu}W^{\mu\nu}_L + \tfrac{1}{2}\bar{\lambda}_L\bar{\sigma}_\mu(\partial_\mu - igT_aG^a_\mu)\lambda_L \\ &\quad -\tfrac{1}{4}W^R_{\mu\nu}W^{\mu\nu}_R + \tfrac{1}{2}\bar{\lambda}_R\bar{\sigma}_\mu(\partial_\mu - igT_aG^a_\mu)\lambda_R \\ &\quad -\tfrac{1}{4}V_{\mu\nu}V^{\mu\nu} + \tfrac{1}{2}\bar{\lambda}_V\bar{\sigma}_\mu\partial_\mu\lambda_V\end{aligned}$$

and the *soft SUSY-breaking* ones:

$$\mathcal{L}_{\text{soft}} = m_L(\lambda^a_L\lambda^a_L + \bar{\lambda}^a_L\bar{\lambda}^a_L) + m_R(\lambda^a_R\lambda^a_R + \bar{\lambda}^a_R\bar{\lambda}^a_R) + m_V(\lambda_V\lambda_V + \bar{\lambda}_V\bar{\lambda}_V).$$

(Compare the above equation with (3.13)).

Below we will examine the mechanisms of symmetry breaking, which are quite similar to the L-R model.

4.1 Symmetry Breaking

We clearly see from the group structure of the model that it contains three gauge symmetries and therefore three coupling constants: g_L, g_R and g_V. By requiring the first stage of symmetry breaking to involve Parity breaking at a mass scale M_P, we end up with $g_L = g_R$ and the gauge fields $\mathbf{W}^\mu_{L,R}$ massless. The second step is to break the 'right-symmetric' part, i.e. $SU(2)_R \times U(1)_{B-L}$ to $U(1)_Y$, which is accomplished through the vev of Δ_R. It should be noted that by appropriately choosing the Higgs multiplets both symmetries can be broken at the same scale: $M_P = M_{W_R}$. Finally, since certain arguments indicate that the breaking scale of supersymmetry is close to the weak scale [13] in this model we break the 'left-symmetric' part, i.e., $SU(2)_R \times U(1)_Y$

to $U(1)_{em}$ *together* with SUSY through the non-zero vev. of Φ:

$$\begin{aligned}
& \text{(SUSY) } SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times P \\
\xrightarrow{M_P} \; & \text{(SUSY) } SU(2)_L \times SU(2)_R \times U(1)_{B-L} \\
\xrightarrow{\langle\Delta_R\rangle\neq 0} \; & \text{(SUSY) } SU(2)_L \times U(1)_Y \\
\xrightarrow{\langle\Phi_{u,d}\rangle\neq 0} \; & U(1)_{em}
\end{aligned} \tag{4.1}$$

In an analogous to the standard model fashion, we allow non-zero vacuum expectation values only for the neutral Higgs fields. In particular the vev's of the Higgs multiplets are:

$$\begin{aligned}
\langle\Delta_R\rangle &= \begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix}, \quad \langle\Delta_L\rangle = \langle\delta_{L,R}\rangle = 0, \\
\langle\Phi_u\rangle &= \begin{pmatrix} \kappa_u & 0 \\ 0 & 0 \end{pmatrix}, \quad \langle\Phi_d\rangle = \begin{pmatrix} 0 & 0 \\ 0 & \kappa_d \end{pmatrix}.
\end{aligned} \tag{4.2}$$

4.2 Mass eigenstates of Vector Bosons

In the first stage of symmetry breaking we generate masses for $W_R^{\pm}, W_R^0$ and V. The relevant term in the Lagrangian [28] is:

$$\mathrm{Tr}\left|\left(-\frac{i}{2}g_R\boldsymbol{\sigma}\mathbf{W}_\mu^R - ig_V V_\mu\right)\Delta_R\right|^2. \tag{4.3}$$

If we now substitute the vev of Δ_R and simplify, we get:

$$\begin{aligned}
&\tfrac{1}{4}\mathrm{Tr}\left|\begin{pmatrix} -ig_R W_R^0 - 2ig_V V & -i\sqrt{2}g_R W_R^+ \\ -i\sqrt{2}g_R W_R^- & ig_R W_R^0 - 2ig_V V \end{pmatrix}\begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix}\right|^2 \\
&= \tfrac{1}{2}v_R^2 g_R^2 W_R^+ W_R^- + v_R^2\left(\tfrac{1}{2}g_R W_R^0 - g_V V\right)^2
\end{aligned} \tag{4.4}$$

where, as usual, $|\;\;|^2 \equiv (\;\;)^\dagger(\;\;)$. Notice that we have defined :

$$W_{\mu R}^{\pm} \equiv \frac{1}{\sqrt{2}}(W_{\mu R}^1 \mp iW_{\mu R}^2). \tag{4.5}$$

From the first term of eq. (4.4) above, we clearly see that the mass of the charged right-handed gauge boson, which is expected to be of the form $M_{W_R}^2 W_{\mu R}^+ W_{\mu R}^-$, is:

$$M_{W_R} = \frac{g_R v_R}{\sqrt{2}}. \tag{4.6}$$

Also, notice that we can write the second term of (4.4) in the matrix form:

$$\frac{1}{4} v_R^2 (W_{\mu R}^0, V_\mu) \begin{pmatrix} g_R^2 & -2g_V g_R \\ -2g_V g_R & 4g_V^2 \end{pmatrix} \begin{pmatrix} W_{\mu R}^0 \\ V^\mu \end{pmatrix},$$

which is off-diagonal and therefore suggests mixing of the $W_{\mu R}^0$ and V_μ fields. By diagonalizing the matrix we see that it has the eigenvalues: $(g_R^2 + 4g_V^2)$ and 0. Proceeding further by identifing the linear combinations of the original fields $W_{\mu R}^0$ and V_μ with the physical fields $Z_{\mu R}$ and B_μ we expect mass terms of the form: $\frac{1}{2} M_Z^2 Z_{\mu R}^2$ and $\frac{1}{2} M_B^2 B_\mu^2$. Following similar steps as we did in the standard model (and normalizing the fields) we finally get:

$$Z_{\mu R} = \frac{g_R W_{\mu R}^0 - 2g_V V_\mu}{(g_R^2 + 4g_V^2)^{\frac{1}{2}}}, \tag{4.7}$$

with mass:

$$M_{Z_R} = \frac{1}{\sqrt{2}} v_R (g_R^2 + 4g_V^2)^{\frac{1}{2}}$$

and similarly for the massless field B_μ we get:

$$B_\mu = \frac{2g_V W_{\mu R}^0 + g_R V_\mu}{(g_R^2 + 4g_V^2)^{\frac{1}{2}}}, \quad M_B = 0. \tag{4.8}$$

For the second stage of symmetry breaking the relevant term in the Lagrangian [13, 28] is:

$$\frac{1}{4} \mathrm{Tr} \left| \left(i g_L \sigma \mathbf{W}_\mu^L + i g_R \sigma \mathbf{W}_\mu^R \right) \Phi_u \right|^2 + \frac{1}{4} \mathrm{Tr} \left| \left(i g_L \sigma \mathbf{W}_\mu^L + i g_R \sigma \mathbf{W}_\mu^R \right) \Phi_d \right|^2. \tag{4.9}$$

We would normally follow exactly the same procedure as above and replace the vev's of the two Higgs fields but there is an important observation to be made before. The first stage is energetically much higher than the second and therefore the right-handed vector bosons $W^{\pm}_{\mu R}$ and $Z_{\mu R}$, decouple from the second stage leaving only the massless B_μ contribution. With this in mind, eq. (4.9) simplifies to:

$$\frac{1}{4}\mathrm{Tr}\left|\left(ig_L\sigma^3 W^{0L}_\mu + ig_R\sigma \mathbf{W}^R_\mu\right)\Phi_u\right|^2 + \frac{1}{4}\mathrm{Tr}\left|\left(ig_L\sigma^3 W^{0L}_\mu + ig_R\sigma \mathbf{W}^R_\mu\right)\Phi_d\right|^2$$

$$= \frac{1}{4}\mathrm{Tr}\left|\begin{pmatrix} ig_L W^{0L}_\mu + ig_R W^{0R}_\mu & ig_L W^{+L}_\mu \\ ig_L W^{-L}_\mu & -ig_L W^{0L}_\mu - ig_R W^{0R}_\mu \end{pmatrix}\begin{pmatrix} \kappa_u & 0 \\ 0 & 0 \end{pmatrix}\right|^2$$

$$+\frac{1}{4}\mathrm{Tr}\left|\begin{pmatrix} ig_L W^{0L}_\mu + ig_R W^{0R}_\mu & ig_L W^{+L}_\mu \\ ig_L W^{-L}_\mu & -ig_L W^{0L}_\mu - ig_R W^{0R}_\mu \end{pmatrix}\begin{pmatrix} 0 & 0 \\ 0 & \kappa_d \end{pmatrix}\right|^2 \tag{4.10}$$

where, again, we have defined:

$$W^{\pm}_{\mu L} \equiv \frac{1}{\sqrt{2}}(W^1_{\mu L} \mp iW^2_{\mu L}).$$

From (4.7) and (4.8) we can write W^{0R}_μ in terms of the two neutral fields:

$$W^0_{\mu R} = \frac{g_R Z_{\mu R} + 2g_V B_\mu}{(g_R^2 + 4g_V^2)^{\frac{1}{2}}}.$$

Using the above we get the approximate result:

$$\left[\frac{1}{2}g_L^2(\kappa_u^2+\kappa_d^2)\right] W^+_{\mu L} W^{-\mu}_L$$
$$+\frac{1}{4}(\kappa_u^2+\kappa_d^2)(W^0_{\mu L}, B_\mu)\begin{pmatrix} g_L^2 & -2g_L g' \\ -2g_L g' & 4g'^2 \end{pmatrix}\begin{pmatrix} W^0_{\mu L} \\ B_\mu \end{pmatrix} \tag{4.11}$$

where :

$$g' \equiv \frac{g_R g_V}{(g_R^2 + 4g_V^2)^{\frac{1}{2}}}.$$

Notice that this is not a strict equality since we ignored all terms containing the 'heavy' $Z_{\mu R}$ field. Comparing the first term of eq. (4.11) with that of eq. (4.4) we immediately recognize that it is a mass term for the charged vector bosons, similarly to the right-handed case above. Therefore:

$$M_{W_L} = \frac{1}{\sqrt{2}} g_L (\kappa_u^2 + \kappa_d^2)^{\frac{1}{2}}. \tag{4.12}$$

Finally, we see that the mixing matrix of (4.11) is off diagonal which again suggests that $W^0_{\mu L}$ and B_μ combine to give physical mass eigenstates. Either by diagonalizing the matrix or by simply rewriting the second term we get:

$$\begin{aligned} & \tfrac{1}{4}(\kappa_u^2 + \kappa_d^2)\left[g_L^2 (W^0_{\mu L})^2 - 2 g_L g' W^0_{\mu L} B_\mu + 4 g'^2 B_\mu^2\right] \\ = \; & \tfrac{1}{4}(\kappa_u^2 + \kappa_d^2)\left[g_L W^0_{\mu L} - 2 g' B_\mu\right]^2 + 0\left[2 g' W^0_{\mu L} + g_L B_\mu\right]^2. \end{aligned} \tag{4.13}$$

Identifying the above with the expected mass terms for two neutral physical fields: $\frac{1}{2} M_Z^2 Z_{\mu L}^2 + \frac{1}{2} M_A^2 A_\mu^2$, we get the equivalent expression to relations (4.7) and (4.8):

$$Z_{\mu L} = \frac{g_L W^0_{\mu L} - 2 g' B_\mu}{(g_L^2 + 4 g'^2)^{\frac{1}{2}}}, \tag{4.14}$$

which has mass :

$$M_{Z_L} = \frac{1}{\sqrt{2}} (\kappa_u^2 + \kappa_d^2)^{\frac{1}{2}} (g_L^2 + 4 g'^2)^{\frac{1}{2}} .$$

Finally, the second term of (4.13) gives the familiar photon field:

$$A_\mu = \frac{2 g' W^0_{\mu L} + g_L B_\mu}{(g_L^2 + 4 g'^2)^{\frac{1}{2}}}; \quad M_A = 0. \tag{4.15}$$

4.3 Gaugino-Higgsino Mixing

Let us now introduce the part which consists of the gaugino and higgsino mixing terms which come from the familiar $\lambda - \psi - A$ term of the Lagrangian. The terms involved are the soft supersymmetry-breaking term and the scalar potential [13, 24, 27]:

$$
\begin{aligned}
\mathcal{L}_{GH} &= i\sqrt{2}\mathrm{Tr}\left[(\boldsymbol{\sigma}\cdot\boldsymbol{\Delta}_L)^\dagger(g_L\boldsymbol{\sigma}\cdot\boldsymbol{\lambda}_L + 2g_V\lambda_V)\boldsymbol{\sigma}\cdot\tilde{\boldsymbol{\Delta}}_L\right] + \text{h.c.} \\
&+ i\sqrt{2}\mathrm{Tr}\left[(\boldsymbol{\sigma}\cdot\boldsymbol{\Delta}_R)^\dagger(g_R\boldsymbol{\sigma}\cdot\boldsymbol{\lambda}_R + 2g_V\lambda_V)\boldsymbol{\sigma}\cdot\tilde{\boldsymbol{\Delta}}_R\right] + \text{h.c.} \\
&+ \frac{i}{\sqrt{2}}\mathrm{Tr}\left[\boldsymbol{\Phi}_u^\dagger(g_L\boldsymbol{\sigma}\cdot\boldsymbol{\lambda}_L + g_R\boldsymbol{\sigma}\cdot\boldsymbol{\lambda}_R)\tilde{\boldsymbol{\Phi}}_u\right] + \text{h.c.} \\
&+ \frac{i}{\sqrt{2}}\mathrm{Tr}\left[\boldsymbol{\Phi}_d^\dagger(g_L\boldsymbol{\sigma}\cdot\boldsymbol{\lambda}_L + g_R\boldsymbol{\sigma}\cdot\boldsymbol{\lambda}_R)\tilde{\boldsymbol{\Phi}}_d\right] + \text{h.c.} \\
&+ \mathrm{Tr}\left[\mu_2(\boldsymbol{\sigma}\cdot\tilde{\boldsymbol{\Delta}}_L)(\boldsymbol{\sigma}\cdot\tilde{\boldsymbol{\delta}}_L)\right] + \mathrm{Tr}\left[\mu_3(\boldsymbol{\sigma}\cdot\tilde{\boldsymbol{\Delta}}_R)(\boldsymbol{\sigma}\cdot\tilde{\boldsymbol{\delta}}_R)\right] \\
&+ m_L(\lambda_L^\alpha\lambda_L^\alpha + \bar{\lambda}_L^\alpha\bar{\lambda}_L^\alpha) + m_R(\lambda_R^\alpha\lambda_R^\alpha + \bar{\lambda}_R^\alpha\bar{\lambda}_R^\alpha) + m_V(\lambda_V\lambda_V + \bar{\lambda}_V\bar{\lambda}_V) \\
&+ \mathrm{Tr}\left[\mu_1(\sigma_1\tilde{\boldsymbol{\Phi}}_u\sigma_1)^T\tilde{\boldsymbol{\Phi}}_d\right],
\end{aligned}
\tag{4.16}
$$

where the fields are given in Table 4.1.

4.3.1 Chargino Mixing

Next, we substitute the vacuum expectation values of the Higgs fields from eq. (4.2) into eq. (4.16). Keeping as usual the terms involving charged fields, we get:

$$
\begin{aligned}
\mathcal{L}_C &= \Big\{i\lambda_R^-\left(\sqrt{2}g_Rv_R\tilde{\Delta}_R^+ + g_R\kappa_d\tilde{\phi}_d^+\right) \\
&+ i\lambda_L^-g_L\kappa_d\tilde{\phi}_d^+ + i\lambda_R^+g_R\kappa_u\tilde{\phi}_u^- + i\lambda_L^+g_L\kappa_u\tilde{\phi}_u^- \\
&+ 4m_L\lambda_L^+\lambda_L^- + 4m_R\lambda_R^+\lambda_R^- \\
&+ \mu_1\tilde{\phi}_u^+\tilde{\phi}_d^- + \mu_1\tilde{\phi}_u^-\tilde{\phi}_d^+\Big\} + \text{h.c.}
\end{aligned}
\tag{4.17}
$$

Following the same principles as in the MSSM we rewrite the rest of the *charged* terms in the following way:

$$\mathcal{L}_C = -\frac{1}{2}(\psi^+, \psi^-) \begin{pmatrix} 0 & X^T \\ X & 0 \end{pmatrix} \begin{pmatrix} \psi^{+T} \\ \psi^{-T} \end{pmatrix} + \text{h.c.} \tag{4.18}$$

In this model we *define* the following fields:

$$\begin{aligned} \psi^+ &\equiv (-i\lambda_L^+, -i\lambda_R^+, \tilde{\phi}_u^+, \tilde{\phi}_d^+, \tilde{\Delta}_L^+, \tilde{\Delta}_R^+), \\ \psi^- &\equiv (-i\lambda_L^-, -i\lambda_R^-, \tilde{\phi}_u^-, \tilde{\phi}_d^-, \tilde{\Delta}_L^-, \tilde{\Delta}_R^-). \end{aligned} \tag{4.19}$$

With these definitions the mixing matrix is:

$$X \equiv \begin{pmatrix} 4m_L & 0 & 0 & g_L\kappa_d & 0 & 0 \\ 0 & 4m_R & 0 & g_R\kappa_d & 0 & \sqrt{2}g_R v_R \\ g_L\kappa_u & g_R\kappa_u & 0 & -\mu_1 & 0 & 0 \\ 0 & 0 & -\mu_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}. \tag{4.20}$$

Notice that in the L-R SUSY we have six charginos, instead of two. The mass eigenstates are *defined* as follows:

$$\chi^{+T} \equiv V\psi^{+T}, \; \chi^{-T} \equiv U\psi^{-T}. \tag{4.21}$$

With U and V being unitary matrices such that

$$U^* X V^{-1} = M_D, \tag{4.22}$$

where M_D is a diagonal matrix with non-negative entries. With the above definitions we can rewrite eq. (4.18) as:

$$\mathcal{L}_C = -\left[\chi_i^-(M_D)_{ij}\chi_j^+\right] \tag{4.23}$$

or using the first equation of (A.13) and define the following four component Dirac spinors

$$\tilde{\chi}_i \equiv \begin{pmatrix} \chi_i^+ \\ \bar{\chi}_i^- \end{pmatrix}, i = 1, \ldots, 4 \tag{4.24}$$

eq. (4.23) becomes:

$$\mathcal{L}_C = -M_i \bar{\tilde{\chi}}_i \tilde{\chi}_i, \tag{4.25}$$

where summation over i is implied.

Since M_D is the mass matrix it is required to contain only nonnegative entries. We follow the same path as in the minimal SUSY case and consider the following relations:

$$M_D^2 = VX^\dagger XV^{-1} = U^* XX^\dagger (U^*)^{-1}. \tag{4.26}$$

Therefore, we have to consider the eigenvalue problem for $X^\dagger X$ and $XX^\dagger$ in order to compute the elements of the diagonalizing matrices U^* and V.

The eigenvalue problem is complicated and must be solved numerically. We will not quote any results since they are not particularly illuminating, Saif [24] has done an extensive work on both analytic and numerical solutions to the masses and elements of the diagonalizing matrices and the interested reader is encouraged to consult this work.

4.3.2 Neutralino mixing

In order to obtain the neutralino part of the Lagrangian we once more replace the vev's of the Higgs fields into (4.16), but this time we keep only the neutral terms:

$$\begin{aligned}
\mathcal{L}_N = & - i\sqrt{2}(\lambda_R^0 g_R - 2\lambda_V^0 g_V) v_R \tilde{\Delta}_R^0 + \text{h.c.} \\
& + i\frac{1}{\sqrt{2}}(\lambda_R^0 g_R - \lambda_L^0 g_L)\kappa_u \tilde{\phi}_{1u}^0 + \text{h.c.} \\
& - i\frac{1}{\sqrt{2}}(\lambda_R^0 g_R - \lambda_L^0 g_L)\kappa_d \tilde{\phi}_{2d}^0 + \text{h.c.} \\
& + m_L(\lambda_L^0\lambda_L^0 + \bar{\lambda}_L^0\bar{\lambda}_L^0) + m_R(\lambda_R^0\lambda_R^0 + \bar{\lambda}_R^0\bar{\lambda}_R^0) + m_V(\lambda_V\lambda_V + \bar{\lambda}_V\bar{\lambda}_V) \\
& + \mu_1(\tilde{\phi}_{1u}^0\tilde{\phi}_{2d}^0 + \tilde{\phi}_{2u}^0\tilde{\phi}_{1d}^0) + \text{h.c.}
\end{aligned} \tag{4.27}$$

As usual we define:

$$\psi^0 \equiv (-i\lambda_L^0, -i\lambda_R^0, -i\lambda_V^0, \tilde{\phi}_{1u}^0, \tilde{\phi}_{2u}^0, \tilde{\phi}_{1d}^0, \tilde{\phi}_{2d}^0, \tilde{\Delta}_R^0). \tag{4.28}$$

Then eq. (4.27) can be written as:

$$\mathcal{L}_N = -\frac{1}{2}\psi^0 Y \psi^{0T} + \text{h.c.} \tag{4.29}$$

where

$$Y \equiv \begin{pmatrix}
m_L & 0 & 0 & \frac{-1}{\sqrt{2}}g_L\kappa_u & 0 & 0 & \frac{1}{\sqrt{2}}g_L\kappa_d & 0 \\
0 & m_R & 0 & \frac{1}{\sqrt{2}}g_R\kappa_u & 0 & 0 & \frac{-1}{\sqrt{2}}g_R\kappa_d & -\sqrt{2}g_R v_R \\
0 & 0 & m_V & 0 & 0 & 0 & 0 & 2\sqrt{2}g_V v_R \\
\frac{-1}{\sqrt{2}}g_L\kappa_u & \frac{1}{\sqrt{2}}g_R\kappa_u & 0 & 0 & 0 & 0 & -\mu_1 & 0 \\
0 & 0 & 0 & 0 & 0 & -\mu_1 & 0 & 0 \\
0 & 0 & 0 & 0 & -\mu_1 & 0 & 0 & 0 \\
\frac{1}{\sqrt{2}}g_L\kappa_d & \frac{-1}{\sqrt{2}}g_R\kappa_d & 0 & -\mu_1 & 0 & 0 & 0 & 0 \\
0 & -\sqrt{2}g_R v_R & 2\sqrt{2}g_V v_R & 0 & 0 & 0 & 0 & 0
\end{pmatrix}. \tag{4.30}$$

The neutralino mass eigenstates are defined by the usual form:

$$\chi^{0^T} \equiv N\xi^{0^T}, \tag{4.31}$$

with N being a unitary matrix which satisfies the relation:

$$N^{*}YN^{-1} = N_D. \tag{4.32}$$

N_D is a diagonal matrix with positive elements. Notice that since we are dealing with neutral fields we only need one diagonalization matrix N, which obeys the relation:

$$N_D^2 = NY^{\dagger}YN^{-1}. \tag{4.33}$$

Chapter 5

Chargino Decay Widths

After having introduced the basic mechanisms of the Left-Right model and its particle content we can now proceed and calculate specific decay widths of singly or doubly charged charginos. As usual, we first have to isolate the relevant terms of the L-R Lagrangian and calculate the vertex factors of the interactions. It is then straightforward to calculate the different decay widths.

5.1 Chargino-lepton-slepton vertices

Let us focus on the *Yukawa* piece of the Lagrangian and particularly on the term:

$$\mathcal{L} = h_{LR}\left(\tilde{L}_L^T \sigma_1(\sigma \cdot \tilde{\boldsymbol{\Delta}}_L) L_L + \tilde{L}_R^T \sigma_1(\sigma \cdot \tilde{\boldsymbol{\Delta}}_R) L_R\right) + \text{h.c.} \tag{5.1}$$

Replacing the explicit field structure on the above we get :

$$\begin{aligned}\mathcal{L} = h_{LR} \ & \left\{\left(\begin{pmatrix} \tilde{\nu} \\ \tilde{e} \end{pmatrix}_L^T \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}}\tilde{\Delta}^+ & \tilde{\Delta}^{++} \\ \tilde{\Delta}^0 & -\frac{1}{\sqrt{2}}\tilde{\Delta}^+ \end{pmatrix}_L \begin{pmatrix} \nu \\ e \end{pmatrix}_L\right)\right. \\ + & \left.\left(\begin{pmatrix} \tilde{\nu} \\ \tilde{e} \end{pmatrix}_R^T \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}}\tilde{\Delta}^+ & \tilde{\Delta}^{++} \\ \tilde{\Delta}^0 & -\frac{1}{\sqrt{2}}\tilde{\Delta}^+ \end{pmatrix}_R \begin{pmatrix} \nu \\ e \end{pmatrix}_R\right)\right\} + \text{h.c.}\end{aligned} \tag{5.2}$$

Now, using the fact that $\tilde{e}_{L/R}$ are scalars, the identity $(\eta\xi)^\dagger = \bar{\eta}\bar{\xi}$ and some trivial algebra we end up with the following form:

$$\mathcal{L} = h_{LR}\left(\tilde{e}_L\tilde{\Delta}_L^{++}e_L + \tilde{e}_R\tilde{\Delta}_R^{++}e_R + \bar{\tilde{\Delta}}_L^{++}\bar{e}_L\tilde{e}_L^* + \bar{\tilde{\Delta}}_R^{++}\bar{e}_R\tilde{e}_R^*\right). \tag{5.3}$$

The next step will be to convert eq. (5.3) into 4-component notation. In order to do this we *define* the following fields[1]:

$$\begin{aligned} \tilde{\chi}^{++}_{L/R} &\equiv \begin{pmatrix} -i\tilde{\Delta}^{++} \\ i\bar{\tilde{\Delta}}^{--} \end{pmatrix}_{L/R}, \\ e &\equiv \begin{pmatrix} e_L \\ \bar{e}_R \end{pmatrix}. \end{aligned} \tag{5.4}$$

With this at hand and using eqs. (A.14) and (A.16), we have :

$$\begin{aligned} (\bar{\tilde{\chi}}_L^{++})^C P_L e\tilde{e}_L &= \cdots = i\tilde{\Delta}_L^{++}e_L\tilde{e}_L, \\ \bar{e}P_L\tilde{\chi}_R^{++}\tilde{e}_R &= \cdots = -ie_R\tilde{\Delta}_R^{++}\tilde{e}_R, \\ \bar{e}P_R(\tilde{\chi}_L^{++})^C\tilde{e}_L^* &= \cdots = i\bar{e}_L\bar{\tilde{\Delta}}_L^{++}\tilde{e}_L^*, \\ \bar{\tilde{\chi}}_R^{++}P_R e\tilde{e}_R^* &= \cdots = -i\bar{\tilde{\Delta}}_R^{++}\bar{e}_R\tilde{e}_R^*. \end{aligned} \tag{5.5}$$

Using the above definitions eq. (5.3) becomes :

$$\mathcal{L} = ih_{LR}\left(-(\bar{\tilde{\chi}}_L^{++})^C P_L e\tilde{e}_L + \bar{e}P_L\tilde{\chi}_R^{++}\tilde{e}_R - \bar{e}P_R(\tilde{\chi}_L^{++})^C\tilde{e}_L^* + \bar{\tilde{\chi}}_R^{++}P_R e\tilde{e}_R^*\right) \tag{5.6}$$

From the second and third terms of (5.6) we can easily read off the interaction vertices which are shown in Figure 5.1. Notice that the charge conjugation matrix C is written explicitly at the vertex factor. In Appendix B we give all the details of the decay-width calculation.

[1]See eq. (C27) in [18].

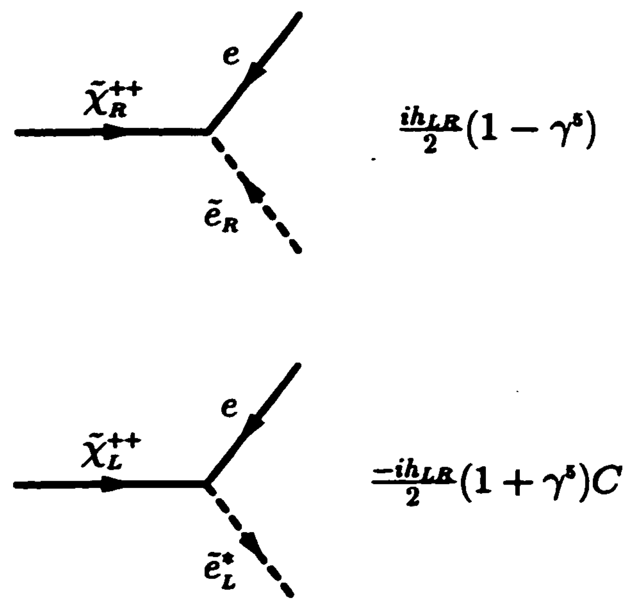


Figure 5.1: Chargino-lepton-slepton vertices.

5.2 Chargino-neutralino-Higgs boson vertices

For the vertices involving the Higgs bosons we focus on the following terms of the Lagrangian:

$$\mathcal{L} = i\sqrt{2}\mathrm{Tr}\left\{(\boldsymbol{\sigma}\cdot\Delta_L)^\dagger(g_L\boldsymbol{\sigma}\cdot\lambda_L + 2g_V\lambda_V)(\boldsymbol{\sigma}\cdot\tilde{\Delta}_L)\right\} + \text{R.h.p.} + \text{h.c.} \tag{5.7}$$

Expanding the above using the explicit field structure we get:

$$\begin{aligned}\mathcal{L} &= i\sqrt{2}\mathrm{Tr}\left\{g_L\begin{pmatrix}\frac{1}{\sqrt{2}}\tilde{\Delta}^+\Delta^+\lambda^0 + \tilde{\Delta}^+\Delta^0\lambda^- + \tilde{\Delta}^0\Delta^+\lambda^+ - \tilde{\Delta}^0\Delta^0\lambda^0 & \cdots \\ \cdots & \tilde{\Delta}^{++}\Delta^{++}\lambda^0 - \tilde{\Delta}^{++}\Delta^+\lambda^- - \tilde{\Delta}^+\Delta^{++}\lambda^+ - \frac{1}{2}\tilde{\Delta}^+\Delta^+\lambda^0\end{pmatrix}_L\right. \\ &\left. +2g_V\lambda_V\begin{pmatrix}\frac{1}{2}\Delta^+\tilde{\Delta}^+ + \Delta^{++}\tilde{\Delta}^0 & \cdots \\ \cdots & \Delta^0\tilde{\Delta}^{++} + \frac{1}{2}\Delta^+\tilde{\Delta}^+\end{pmatrix}_L\right\} + \text{R.h.p.} + \text{h.c.}\end{aligned} \tag{5.8}$$

If we now expand further our expression and keep only the *charged* λ terms, we end up with the following:

$$\begin{aligned}\mathcal{L}_C = i\sqrt{2}g_L &\left(\tilde{\Delta}^+\Delta^0\lambda^- + \tilde{\Delta}^0\Delta^+\lambda^+ - \tilde{\Delta}^{++}\Delta^+\lambda^- - \tilde{\Delta}^+\Delta^{++}\lambda^+\right)_L \\ &+\text{R.h.p.} + \text{h.c.}\end{aligned} \tag{5.9}$$

Defining the 4 component spinors:

$$\begin{aligned}\tilde{W}_{L/R} &\equiv \begin{pmatrix}-i\lambda^+ \\ i\bar{\lambda}^-\end{pmatrix}_{L/R}, \\ \tilde{\chi}_i^+ &\equiv \begin{pmatrix}\chi_i^+ \\ \bar{\chi}_i^-\end{pmatrix}, \\ \tilde{D}^0 &\equiv \begin{pmatrix}\tilde{\Delta}_L^0 \\ \bar{\tilde{\Delta}}_L^0\end{pmatrix},\end{aligned} \tag{5.10}$$

and following the diagonalization conventions of Huitu *et al.* [29], we have:

$$\chi_i^+ \equiv V_{ij}\psi_j^+ \Rightarrow \tilde{\Delta}_L^+ = V_{15}^*\chi_1^+ + \cdots + V_{65}^*\chi_5^+ = V_{i5}^*\chi_i^+. \tag{5.11}$$

In a similar fashion:

$$\tilde{\Delta}_R^+ = V_{i6}^* \chi_i^+$$
$$\tilde{\Delta}_R^0 = N_{i8}^* \chi_i^0 \tag{5.12}$$

Before continuing, there are two points worth noting in the above formulas:

1. For compactness, we imply summation over repeated indices.
2. Notice that there is no expression similar to (5.12) for $\tilde{\Delta}_L^0$ since it is not included in our neutralino basis ψ^0.

Therefore, replacing the above definitions in the expression (5.9) we get:

$$\begin{aligned}
\mathcal{L}_C = & \Big\{ i\sqrt{2} g_L \left(V_{i5}^* \chi_i^+ \Delta^0 \lambda^- + \tilde{\Delta}^0 \Delta^+ \lambda^+ - \tilde{\Delta}^{++} \Delta^+ \lambda^- - V_{i5}^* \chi_i^+ \Delta^{++} \lambda^+ \right)_L \\
& + i\sqrt{2} g_R \left(V_{i6}^* \chi_i^+ \Delta^0 \lambda^- + N_{i8}^* \chi_i^0 \Delta^+ \lambda^+ - \tilde{\Delta}^{++} \Delta^+ \lambda^- - V_{i6}^* \chi_i^+ \Delta^{++} \lambda^+ \right)_R \Big\} \\
& + \text{h.c.}
\end{aligned} \tag{5.13}$$

Using (A.14) and (A.16), we have :

$$\begin{aligned}
\bar{\tilde{W}}_{L(R)} P_L \tilde{\chi}_i^+ &= \cdots = -i\lambda_{L(R)}^- \chi_i^+, \\
\bar{\tilde{\chi}}_i^0 P_L \tilde{W}_{L(R)} &= \cdots = -i\chi_i^0 \lambda_{L(R)}^+, \\
\bar{\tilde{W}}_{L(R)} P_L \tilde{\chi}^{++} &= \cdots = -\lambda_{L(R)}^- \tilde{\Delta}^{++}, \\
\bar{\tilde{\chi}}_i^+ P_L \tilde{W}_{L(R)}^c &= \cdots = -\chi_i^+ \lambda_{L(R)}^+, \\
\bar{\tilde{D}}^0 P_L \tilde{W}_L &= \cdots = -i\lambda_L^+ \tilde{\Delta}_L^0.
\end{aligned} \tag{5.14}$$

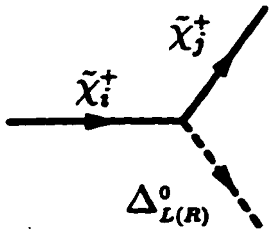
Now, due to the *unitarity* of the mixing matrices we can easily prove that:

$$\begin{aligned}
\bar{\tilde{W}}_{L(R)} P_L &= (U_{j1(2)}^* \bar{\tilde{\chi}}_j^+) P_L, \\
P_L \tilde{W}_{L(R)} &= P_L (V_{j1(2)}^* \tilde{\chi}_j^+), \\
P_L \tilde{W}_{L(R)}^c &= P_L (U_{j1(2)}^* (\tilde{\chi}_j^+)^c) \left(\equiv (U_{j1(2)}^* \tilde{\chi}_j^-) \right).
\end{aligned} \tag{5.15}$$

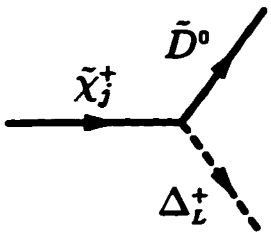
Notice that we *defined*: $(\tilde{\chi}_i^+)^c \equiv \tilde{\chi}_i^-$. Lastly, using (5.15) and (5.14) the final form of the Lagrangian is:

$$
\begin{aligned}
\mathcal{L}_C = \ & \Big\{ i\sqrt{2} g_L \Big(iV_{i5}^* U_{j1}^* \bar{\tilde{\chi}}_j^+ P_L \tilde{\chi}_i^+ \Delta_L^0 + iV_{j1}^* \bar{\tilde{D}}^0 P_L \tilde{\chi}_j^+ \Delta_L^+ \\
& + U_{j1}^* \bar{\tilde{\chi}}_j^+ P_L \tilde{\chi}_L^{++} \Delta_L^+ - iV_{i5}^* U_{j1}^* \bar{\tilde{\chi}}_i^+ P_L \tilde{\chi}_j^- \Delta_L^{++} \Big) \\
& + i\sqrt{2} g_R \Big(iV_{i6}^* U_{j2}^* \bar{\tilde{\chi}}_j^+ P_L \tilde{\chi}_i^+ \Delta_R^0 + iN_{i8}^* V_{j2}^* \bar{\tilde{\chi}}_i^0 P_L \tilde{\chi}_j^+ \Delta_R^+ \\
& + U_{j2}^* \bar{\tilde{\chi}}_j^+ P_L \tilde{\chi}_R^{++} \Delta_R^+ - iV_{i6}^* U_{j2}^* \bar{\tilde{\chi}}_i^+ P_L \tilde{\chi}_j^- \Delta_R^{++} \Big) \Big\} \\
& + \text{h.c.}
\end{aligned}
\tag{5.16}
$$

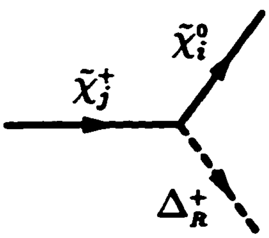
From (5.16) we can easily read off the interaction vertices which are shown in Figure 5.2.



$$\frac{-1}{\sqrt{2}} g_{L(R)} V^*_{i5(6)} U^*_{j1(2)} (1-\gamma^5), \quad i > j$$

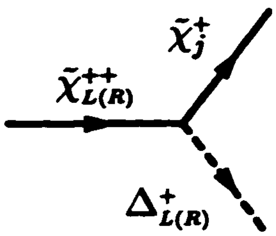


$$\frac{-1}{\sqrt{2}} g_L V^*_{j1} (1-\gamma^5)$$

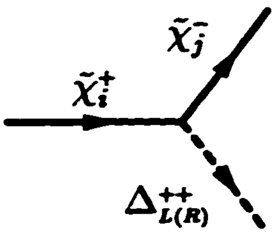


$$\frac{-1}{\sqrt{2}} g_R N^*_{i8} V^*_{j2} (1-\gamma^5), \quad j > i$$

Figure 5.2: Chargino-Neutralino-Higgs boson vertices in the L-R-SUSY model.



$$\frac{i}{\sqrt{2}} g_L U_{j1(2)}^{*} (1 - \gamma^5)$$



$$\frac{i}{\sqrt{2}} g_L V_{i5(6)}^{*} U_{j1(2)}^{*} (1 - \gamma^5)$$

Figure 5.3: Chargino-Neutralino-Higgs boson vertices in the L-R-SUSY model (cont'd).

5.3 Chargino-neutralino-Vector boson vertices

Finally, let us look at terms of the Lagrangian that give us the Chargino-neutralino-Vector boson interaction:

$$\mathcal{L} = \mathrm{Tr}\left\{\left(\sigma\cdot\tilde{\Delta}_L\right)^\dagger \bar{\sigma}^\mu \left(\partial_\mu - \frac{i}{2}g_L\sigma\cdot\mathbf{W}^L_\mu - ig_V V_\mu\right)\left(\sigma\cdot\tilde{\Delta}_L\right)\right\} + \mathrm{R.h.p.} \tag{5.17}$$

Again, we keep only the charged terms of (5.17). After some algebraic steps we get:

$$\begin{aligned}\mathcal{L}_C = \left(-\tfrac{i}{2}g_L\right) & \left[\bar{\tilde{\Delta}}^0\bar{\sigma}_\mu W^{-\mu}\tilde{\Delta}^+ + \bar{\tilde{\Delta}}^+\bar{\sigma}_\mu W^{+\mu}\tilde{\Delta}^0\right. \\ & \left. -\bar{\tilde{\Delta}}^+\bar{\sigma}_\mu W^{-\mu}\tilde{\Delta}^{++} - \bar{\tilde{\Delta}}^{++}\bar{\sigma}_\mu W^{+\mu}\tilde{\Delta}^+\right]_L + \mathrm{R.h.p.}\end{aligned} \tag{5.18}$$

Making the usual replacements with the charginos and neutralinos we have:

$$\begin{aligned}\mathcal{L}_C &= \left(-\tfrac{i}{2}g_L\right) \left[\bar{\tilde{\Delta}}^0_L\bar{\sigma}_\mu W_L^{-\mu}(V^*_{j5}\chi^+_j) + (V_{i5}\bar{\chi}^+_i)\bar{\sigma}_\mu W_L^{+\mu}\tilde{\Delta}^0_L\right. \\ &\qquad \left. -(V_{i5}\bar{\chi}^+_i)\bar{\sigma}_\mu W_L^{-\mu}\tilde{\Delta}^{++}_L - \bar{\tilde{\Delta}}^{++}_L\bar{\sigma}_\mu W_L^{+\mu}(V^*_{i5}\chi^+_i)\right] \\ &+ \left(-\tfrac{i}{2}g_R\right) \left[(N_{i8}\bar{\chi}^0_i)\bar{\sigma}_\mu W_R^{-\mu}(V^*_{j6}\chi^+_j) + (V_{i6}\bar{\chi}^+_i)\bar{\sigma}_\mu W_R^{+\mu}(N^*_{j8}\chi^0_j)\right. \\ &\qquad \left. -(V_{i6}\bar{\chi}^+_i)\bar{\sigma}_\mu W_R^{-\mu}\tilde{\Delta}^{++}_R - \bar{\tilde{\Delta}}^{++}_R\bar{\sigma}_\mu W_R^{+\mu}(V^*_{i6}\chi^+_i)\right] \\ &= \left(-\tfrac{i}{2}g_L\right) \left[V^*_{j5}\bar{\tilde{\Delta}}^0_L\bar{\sigma}_\mu\chi^+_j W_L^{-\mu} + V_{i5}\bar{\chi}^+_i\bar{\sigma}_\mu W_L^{+\mu}\tilde{\Delta}^0_L\right. \\ &\qquad \left. -V_{i5}\bar{\chi}^+_i\bar{\sigma}_\mu\tilde{\Delta}^{++}_L W_L^{-\mu} - V^*_{i5}\bar{\tilde{\Delta}}^{++}_L\bar{\sigma}_\mu\chi^+_i W_L^{+\mu}\right] \\ &+ \left(-\tfrac{i}{2}g_R\right) \left[N_{i8}V^*_{j6}\bar{\chi}^0_i\bar{\sigma}_\mu\chi^+_j W_R^{-\mu} + V_{i6}N^*_{j8}\bar{\chi}^+_i\bar{\sigma}_\mu\chi^0_j W_R^{+\mu}\right. \\ &\qquad \left. -V_{i6}\bar{\chi}^+_i\bar{\sigma}_\mu\tilde{\Delta}^{++}_R W_R^{-\mu} - V^*_{i6}\bar{\tilde{\Delta}}^{++}_R\bar{\sigma}_\mu\chi^+_i W_R^{+\mu}\right].\end{aligned} \tag{5.19}$$

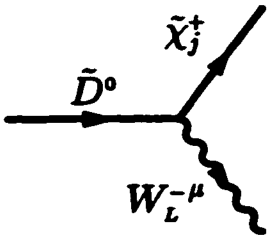
Note that according to our original definitions we can show that:

$$
\begin{aligned}
\bar{\tilde{\chi}}_i^0\gamma^\mu P_L\tilde{\chi}_j^+ &= \bar{\chi}_i^0\bar{\sigma}_\mu\chi_j^+,\\
\bar{\tilde{\chi}}_i^+\gamma^\mu P_L\tilde{\chi}_j^0 &= \bar{\chi}_i^+\bar{\sigma}_\mu\chi_j^0,\\
\bar{\tilde{\chi}}_i^+\gamma^\mu P_L\tilde{\chi}_{L(R)}^{++} &= -i\bar{\chi}_i^+\bar{\sigma}_\mu\tilde{\Delta}_{L(R)}^{++},\\
\bar{\tilde{\chi}}_{L(R)}^{++}\gamma^\mu P_L\tilde{\chi}_i^+ &= -i\bar{\tilde{\Delta}}_{L(R)}^{++}\bar{\sigma}_\mu\chi_i^+,\\
\bar{\tilde{D}}^0\gamma^\mu P_L\tilde{\chi}_j^+ &= \bar{\tilde{\Delta}}^0\bar{\sigma}_\mu\chi_j^+,\\
\bar{\tilde{\chi}}_i^+\gamma^\mu P_L\tilde{D}^0 &= \bar{\chi}_i^+\bar{\sigma}_\mu\tilde{\Delta}_L^0.
\end{aligned}
\tag{5.20}
$$

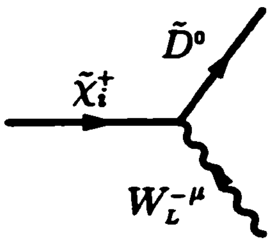
Therefore we can rewrite $\mathcal{L}_C$ in four component notation as follows:

$$
\begin{aligned}
\mathcal{L}_W \;=\; & \left(-\tfrac{i}{2}g_L\right) \Big[V_{j5}^*\bar{\tilde{D}}^0\gamma^\mu P_L\tilde{\chi}_j^+W_L^{-\mu} + V_{i5}\bar{\tilde{\chi}}_i^+\gamma^\mu P_L\tilde{D}^0W_L^{+\mu}\\
& -iV_{i5}\bar{\tilde{\chi}}_i^+\gamma^\mu P_L\tilde{\chi}_L^{++}W_L^{-\mu} - iV_{i5}^*\bar{\tilde{\chi}}_L^{++}\gamma^\mu P_L\tilde{\chi}_i^+W_L^{+\mu}\Big]\\
+ & \left(-\tfrac{i}{2}g_R\right) \Big[N_{i8}V_{j6}^*\bar{\tilde{\chi}}_i^0\gamma^\mu P_L\tilde{\chi}_j^+W_R^{-\mu} + V_{i6}N_{j8}^*\bar{\tilde{\chi}}_i^+\gamma^\mu P_L\tilde{\chi}_j^0W_R^{+\mu}\\
& -iV_{i6}\bar{\tilde{\chi}}_i^+\gamma^\mu P_L\tilde{\chi}_R^{++}W_R^{-\mu} - iV_{i6}^*\bar{\tilde{\chi}}_R^{++}\gamma^\mu P_L\tilde{\chi}_i^+W_R^{+\mu}\Big].
\end{aligned}
\tag{5.21}
$$

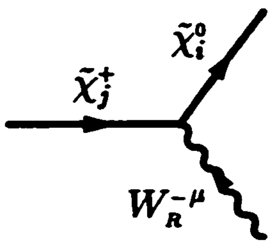
Finally, from expression (5.21) we can easily read off the interaction vertices which are shown in Figures 5.4 and 5.5.



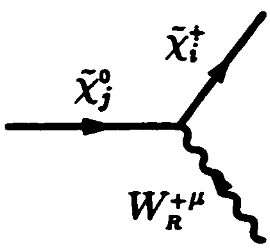
$$\frac{\mp i}{4} g_L V_{j5}^* \gamma^\mu (1-\gamma^5)$$



$$\frac{\mp i}{4} g_L V_{i5} \gamma^\mu (1-\gamma^5)$$

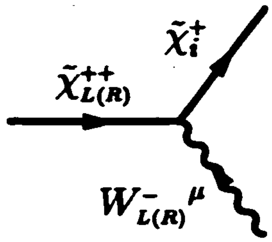


$$\frac{\mp i}{4} g_R N_{i8} V_{j6}^* \gamma^\mu (1-\gamma^5)$$

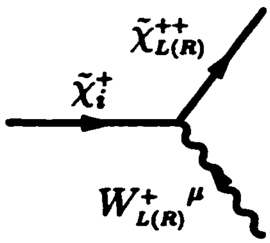


$$\frac{\mp i}{4} g_R V_{i6} N_{j8}^* \gamma^\mu (1-\gamma^5)$$

Figure 5.4: Chargino-Neutralino-Vector boson vertices in the L-R-SUSY model.



$$\frac{1}{4} g_{L(R)} V_{i5(6)} \gamma^{\mu} (1 - \gamma^5)$$



$$\frac{1}{4} g_{L(R)} V^{*}_{i5(6)} \gamma^{\mu} (1 - \gamma^5)$$

Figure 5.5: Chargino-Neutralino-Vector boson vertices in the L-R-SUSY model (cont'd).

5.4 Decay Widths

We are now in the position to calculate the lowest order decay widths for the different supersymmetric interactions. The details of a sample calculation are given in Appendix B, in Tables 5.1 and 5.2 we will simply quote the results which are expressed with respect to the initial particle's rest frame.

Process	Decay Width
$\tilde{\chi}^{++}_{L(R)} \rightarrow e^{+}\tilde{e}^{+}_{L(R)}$	$\Gamma = \frac{h^2_{LR}}{8\pi M_{++}} \left(M^2_{++} + m^2_{e} - m^2_{\tilde{e}}\right) \lambda^{1/2}\left(1, \frac{m^2_{e}}{M^2_{++}}, \frac{m^2_{\tilde{e}}}{M^2_{++}}\right)$
$\tilde{\chi}^{+}_{i} \rightarrow \tilde{\chi}^{+}_{j}\Delta^{0}_{L(R)}$	$\Gamma = \frac{g^2_{L(R)}\lvert V_{i5(6)}\rvert^2 \lvert U_{j1(2)}\rvert^2}{16\pi m_i} \left(m^2_{i} + m^2_{j} - M^2_{\Delta}\right) \lambda^{1/2}\left(1, \frac{m^2_{j}}{m^2_{i}}, \frac{M^2_{\Delta}}{m^2_{i}}\right)$
$\tilde{\chi}^{+}_{j} \rightarrow \tilde{\chi}^{0}_{i}\Delta^{+}_{R}$	$\Gamma = \frac{g^2_{R}\lvert N_{i8}\rvert^2 \lvert V_{j2}\rvert^2}{16\pi m_j} \left(m^2_{j} + m^2_{i} - M^2_{\Delta}\right) \lambda^{1/2}\left(1, \frac{m^2_{i}}{m^2_{j}}, \frac{M^2_{\Delta}}{m^2_{j}}\right)$
$\tilde{\chi}^{+}_{j} \rightarrow \tilde{D}^{0}\Delta^{+}_{L}$	$\Gamma = \frac{g^2_{L}\lvert V_{j1}\rvert^2}{16\pi m_j} \left(m^2_{j} + M^2_{D} - M^2_{\Delta}\right) \lambda^{1/2}\left(1, \frac{M^2_{D}}{m^2_{j}}, \frac{M^2_{\Delta}}{m^2_{j}}\right)$
$\tilde{\chi}^{++}_{L(R)} \rightarrow \tilde{\chi}^{+}_{j}\Delta^{+}_{L(R)}$	$\Gamma = \frac{g^2_{L(R)}\lvert U_{j1(2)}\rvert^2}{16\pi M_{++}} \left(M^2_{++} + m^2_{j} - M^2_{\Delta}\right) \lambda^{1/2}\left(1, \frac{m^2_{j}}{M^2_{++}}, \frac{M^2_{\Delta}}{M^2_{++}}\right)$
$\tilde{\chi}^{+}_{i} \rightarrow \tilde{\chi}^{-}_{j}\Delta^{++}_{L(R)}$	$\Gamma = \frac{g^2_{L(R)}\lvert U_{j1(2)}\rvert^2 \lvert V_{i5(6)}\rvert^2}{16\pi m_i} \left(m^2_{i} + m^2_{j} - M^2_{\Delta}\right) \lambda^{1/2}\left(1, \frac{m^2_{j}}{m^2_{i}}, \frac{M^2_{\Delta}}{m^2_{i}}\right)$

Table 5.1: Total decay widths for chargino decays into lepton, s-lepton and Higgs.

Process	Decay Width
$\tilde{\chi}_j^+ \to \tilde{\chi}_i^0 W_R^+$	$\Gamma = \frac{1}{16\pi m_j}\frac{g_R^2}{8M_W^2}\lvert N_{i8}\rvert^2\lvert V_{j6}\rvert^2\left[M_W^2(m_j^2+m_i^2)+(m_j^2-m_i^2)^2-2M_W^4\right]$ $\times\lambda^{1/2}\left(1,\frac{m_i^2}{m_j^2},\frac{M_W^2}{m_j^2}\right)$
$\tilde{\chi}_j^0 \to \tilde{\chi}_i^+ W_R^-$	$\Gamma = \frac{1}{16\pi m_j}\frac{g_R^2}{8M_W^2}\lvert V_{i6}\rvert^2\lvert N_{j8}\rvert^2\left[M_W^2(m_j^2+m_i^2)+(m_j^2-m_i^2)^2-2M_W^4\right]$ $\times\lambda^{1/2}\left(1,\frac{m_i^2}{m_j^2},\frac{M_W^2}{m_j^2}\right)$
$\tilde{\chi}_i^+ \to \tilde{D}^0 W_L^+$	$\Gamma = \frac{1}{16\pi m_i}\frac{g_L^2}{8M_W^2}\lvert V_{i5}\rvert^2[M_W^2(m_i^2+M_D^2)+(m_i^2-M_D^2)^2-2M_W^4]$ $\times\lambda^{1/2}\left(1,\frac{M_D^2}{m_i^2},\frac{M_W^2}{m_i^2}\right)$
$\tilde{D}^0 \to \tilde{\chi}_j^+ W^-$	$\Gamma = \frac{1}{16\pi M_D}\frac{g_L^2}{8M_W^2}\lvert V_{i5}\rvert^2\left[M_W^2(M_D^2+m_j^2)+(M_D^2-m_j^2)^2-2M_W^4\right]$ $\times\lambda^{1/2}\left(1,\frac{m_j^2}{M_D^2},\frac{M_W^2}{M_D^2}\right)$
$\tilde{\chi}_{L(R)}^{++} \to \tilde{\chi}_i^+ W_{L(R)}^+$	$\Gamma = \frac{1}{16\pi M_{++}}\frac{g_{L(R)}^2}{8M_W^2}\lvert V_{i5(6)}\rvert^2[M_W^2(M_{++}+m_i^2)+(M_{++}-m_i^2)^2-2M_W^4]$ $\times\lambda^{1/2}\left(1,\frac{m_i^2}{M_{++}^2},\frac{M_W^2}{M_{++}^2}\right)$
$\tilde{\chi}_i^+ \to \tilde{\chi}_{L(R)}^{++} W_{L(R)}^-$	$\Gamma = \frac{1}{16\pi m_i}\frac{g_{L(R)}^2}{8M_W^2}\lvert V_{i5(6)}\rvert^2\left[M_W^2(m_i^2+M_+^2)+(m_i^2-M_+^2)^2-2M_W^4\right]$ $\times\lambda^{1/2}\left(1,\frac{M_{++}^2}{m_i^2},\frac{M_W^2}{m_i^2}\right)$

Table 5.2: Total decay widths for chargino and neutralino decays into Vector bosons.

5.5 Conclusion

As we mentioned in the introduction, the Standard Model, successful as it is, is most probably not the final answer of theoretical physics for explaining and unifying the different manifestations of matter and fundamental forces of Nature. The gauge hierarchy problem (GHP), CP and Parity violation, seemingly unrelated structure constants, etc. are *some* of the problems the SM is faced with. Different models have been proposed in an attempt to remedy some or possibly all of the above drawbacks. We reviewed some of these models and saw their differences and their similarities. We then introduced the L-R SUSY model which, although retains all of the SM's desirable features, yet it provides solutions to many of the problems mentioned above (e.g.: GHP, Parity violation.).

After explaining the basic structure of the model we have calculated interaction vertices among various doubly and singly charged charginos and neutralinos. Further, we used these interaction vertices to calculate specific decay widths.

In order to extend and complete this work the decays would have to be evaluated numerically. This requires that certain values be given to some of the free constants that would come from different scenarios and restrictions found in the literature. By doing so we would be able to comment on what to expect for realistic decay rates. Since the doubly charged Higgses and Higgsinos are *not* restricted by the theory, there is a chance that they are quite light and therefore we can hope that they may provide a sign for L-R SUSY and enhance production rates. We believe and hope that further study will concentrate on this direction.

Appendix A

Spinor notation and conventions

Let $M \epsilon SL(2,C)$, then the self-representation is defined as:

$$D(M) \equiv M, \ \forall M \epsilon SL(2,C).$$

The elements of the representation space transform as[1]:

$$\xi'_\alpha = M_\alpha{}^\beta \psi_\beta \ \ \alpha, \beta = 1, 2.$$

Similarly, the representation $D(M) \equiv (M^{*-1})^T$ is *equivalent* to the complex conjugate self representation M^* and $\dot{F}^*$ is its representation space. The $\dot{F}^*$ elements are dotted (right-handed) Weyl spinors, whereas the elements of F are undotted (left-handed) Weyl spinors. We now define the direct sum of the two spaces F and $\dot{F}^*$ as:

$$E \equiv F \oplus \dot{F}^*,$$

where E is the four-dimensional complex representation space of Dirac spinors. Therefore, if $\xi \epsilon F$ and $\bar{\eta} \epsilon \dot{F}^*$ then:

$$\psi \equiv \begin{pmatrix} \xi \\ \bar{\eta} \end{pmatrix} \epsilon E$$

[1]The spinor notation and subscript-superscript conventions in this work are based mainly on refs.[18, 30].

is a Dirac spinor. Further, we define a representation of $SL(2,C)$ on E by the map:

$$M \,\epsilon\, SL(2,C) \rightarrow S(M) \equiv \begin{pmatrix} M & 0 \\ 0 & M^{*-1} \end{pmatrix}. \tag{A.1}$$

Therefore,

$$\psi' = S(M)\psi = \begin{pmatrix} M\xi \\ M^{*-1}\bar{\eta} \end{pmatrix}. \tag{A.2}$$

Notice that the dotted and undotted indices refer to different (non-equivalent) representations and should not be confused. Writing the index structure of the Dirac spinor explicitly we get:

$$\psi = \begin{pmatrix} \xi_\alpha \\ \bar{\eta}^{\dot{\alpha}} \end{pmatrix}, \text{ where } \alpha = 1,2 \text{ and } \dot{\alpha} = \dot{1},\dot{2}\,. \tag{A.3}$$

In a similar fashion, the spinors $\bar{\xi}_{\dot{\alpha}}$ and ξ^α transform under M^* and M^{-1} respectively.

We now *define* the following conventions:

$$\begin{aligned} g_{\mu\nu} &\equiv \text{diag}(1,-1,-1,-1), \\ p^\mu &\equiv (E,\boldsymbol{p}), \\ \sigma^\mu &\equiv (1,\boldsymbol{\sigma}),\ \bar{\sigma}^\mu \equiv (1,-\boldsymbol{\sigma}). \end{aligned} \tag{A.4}$$

In the so-called Weyl basis or *chiral* representation the γ matrices are as follows:

$$\gamma^\mu \equiv \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix} \tag{A.5}$$

and furthermore:

$$\begin{aligned} \gamma_5 &\equiv i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \\ \sigma^{\mu\nu} &\equiv \tfrac{i}{2}[\gamma^\mu,\gamma^\nu] = 2i\begin{pmatrix} \sigma_\alpha^{\mu\nu\beta} & 0 \\ 0 & \bar{\sigma}^{\mu\nu\dot{\alpha}}{}_{\dot{\beta}} \end{pmatrix}, \end{aligned} \tag{A.6}$$

where

$$\sigma_\alpha^{\mu\nu\beta} = \frac{1}{4}(\sigma^\mu_{\alpha\dot{\alpha}}\bar{\sigma}^{\nu\dot{\alpha}\beta} - \sigma^\nu_{\alpha\dot{\alpha}}\bar{\sigma}^{\mu\dot{\alpha}\beta}), \tag{A.7}$$

$$\bar{\sigma}^{\mu\nu\dot{\alpha}}{}_{\dot{\beta}} = \frac{1}{4}(\bar{\sigma}^{\mu\dot{\alpha}\alpha}\sigma^\nu_{\alpha\dot{\beta}} - \bar{\sigma}^{\nu\dot{\alpha}\alpha}\sigma^\mu_{\alpha\dot{\beta}}). \tag{A.8}$$

Writing the complete index structure of expression (A.1) we see that any 4×4 matrix Γ acting on Dirac four-spinors must have the following index structure:

$$\Gamma_{AB} = \begin{pmatrix} A_\alpha{}^\beta & B_{\alpha\dot{\beta}} \\ C^{\dot{\alpha}\beta} & D^{\dot{\alpha}}{}_{\dot{\beta}} \end{pmatrix} \text{ where } \alpha, \beta = 1, 2; \; \dot{\alpha}, \dot{\beta} = \dot{1}, \dot{2} \text{ and } A, B = 1, 2, 3, 4. \tag{A.9}$$

With this background material at hand let us rewrite the Dirac equation as follows:

$$(\gamma^\mu p_\mu - m)\psi = 0$$

$$\Rightarrow \; (\gamma^\mu p_\mu)_{AB}\psi_B = m\psi_A$$

$$\Rightarrow \; \begin{pmatrix} 0 & (\sigma_\mu p^\mu)_{\alpha\dot{\beta}} \\ (\bar{\sigma}_\mu p^\mu)^{\dot{\alpha}\beta} & 0 \end{pmatrix} \begin{pmatrix} \xi_\beta \\ \bar{\eta}^{\dot{\beta}} \end{pmatrix} = m \begin{pmatrix} \xi_\alpha \\ \bar{\eta}^{\dot{\alpha}} \end{pmatrix}.$$

Therefore the Dirac equation in *two-component* notation becomes:

$$(\bar{\sigma}_\mu p^\mu)^{\dot{\alpha}\beta}\xi_\beta = m\bar{\eta}^{\dot{\alpha}}, \quad (\sigma_\mu p^\mu)_{\alpha\dot{\beta}}\bar{\eta}^{\dot{\beta}} = m\xi_\alpha \,. \tag{A.10}$$

In order to raise and lower the spinor indices we define the antisymmetric tensor $\varepsilon^{\alpha\beta}$ which plays the role of a metric[2] in the spinor space F:

$$\varepsilon^{\alpha\beta} = -\varepsilon^{\beta\alpha} = -\varepsilon_{\alpha\beta} = i\sigma^2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \tag{A.11}$$

So, for example, $\xi^\alpha = \varepsilon^{\alpha\beta}\xi_\beta$ and similarly for the dotted indices: $\bar{\eta}_{\dot{\alpha}} = \varepsilon_{\dot{\alpha}\dot{\beta}}\bar{\eta}^{\dot{\beta}}$. Finally we define the familiar left and right projection operators:

$$P_{L,R} \equiv \frac{1}{2}(1 \mp \gamma_5)$$

and so if $\psi_{L,R} \equiv P_{L,R}\psi$ we have $\psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}$.

[2]Actually, $\varepsilon M \varepsilon^{-1} = M^{-1T}$, $\varepsilon \in GL(2, C)$. For further details see ref.[30].

We will now state some characteristic and useful identities of the component spinors:

$$\begin{aligned}
\eta\xi &\equiv \eta^\alpha \xi_\alpha = \xi\eta, \\
\bar{\eta}\bar{\xi} &\equiv \bar{\eta}_{\dot{\alpha}} \bar{\xi}^{\dot{\alpha}} = \bar{\xi}\bar{\eta}, \\
\bar{\eta}_2 \bar{\sigma}^\mu \eta_1 &\equiv \bar{\eta}_{2\dot{\alpha}} \bar{\sigma}^{\mu\dot{\alpha}\beta} \eta_{1\beta} = -\eta_1 \bar{\sigma}^\mu \bar{\eta}_2, \\
\bar{\eta}\bar{\sigma}^{\mu\nu}\bar{\xi} &\equiv \bar{\eta}_{\dot{\alpha}} \bar{\sigma}^{\mu\nu\dot{\alpha}}{}_{\dot{\beta}} \bar{\xi}^{\dot{\beta}} = -\bar{\xi}\bar{\sigma}^{\mu\nu}\bar{\eta}, \\
\eta\sigma^{\mu\nu}\xi &\equiv \eta^\alpha \sigma^{\mu\nu}_\alpha{}^\beta \xi_\beta = -\xi\sigma^{\mu\nu}\eta.
\end{aligned} \tag{A.12}$$

Let us for the sake of clarity prove the second formula above:

$$\begin{aligned}
(\bar{\eta}\bar{\xi}) &\equiv \bar{\eta}_{\dot{\alpha}}\bar{\xi}^{\dot{\alpha}} = -\bar{\xi}^{\dot{\alpha}}\bar{\eta}_{\dot{\alpha}} = -\varepsilon^{\dot{\alpha}\dot{\beta}}\bar{\xi}_{\dot{\beta}}\varepsilon_{\dot{\alpha}\dot{\gamma}}\bar{\eta}^{\dot{\gamma}} = -\bar{\xi}_{\dot{\beta}}(\varepsilon^T)^{\dot{\beta}\dot{\alpha}}\varepsilon_{\dot{\alpha}\dot{\gamma}}\bar{\eta}^{\dot{\gamma}} \\
&= -\bar{\xi}_{\dot{\beta}}(-\varepsilon)^{\dot{\beta}\dot{\alpha}}\varepsilon_{\dot{\alpha}\dot{\gamma}}\bar{\eta}^{\dot{\gamma}} = \bar{\xi}_{\dot{\beta}}\delta^{\dot{\beta}}_{\dot{\gamma}}\bar{\eta}^{\dot{\gamma}} = (\bar{\xi}\bar{\eta}).
\end{aligned}$$

Using the above relations we can prove the following set of equations which are used in 'translating' the four-component into two-component notation:

$$\begin{aligned}
\bar{\psi}_1\psi_2 &= \eta_1\xi_2 + \bar{\eta}_2\bar{\xi}_1, \\
\bar{\psi}_1\gamma_5\psi_2 &= -\eta_1\xi_2 + \bar{\eta}_2\bar{\xi}_1, \\
\bar{\psi}_1\gamma^\mu\psi_2 &= \bar{\xi}_1\bar{\sigma}^\mu\xi_2 - \bar{\eta}_2\bar{\sigma}^\mu\eta_1, \\
\bar{\psi}_1\gamma^\mu\gamma_5\psi_2 &= -\bar{\xi}_1\bar{\sigma}^\mu\xi_2 - \bar{\eta}_2\bar{\sigma}^\mu\eta_1, \\
-\tfrac{1}{2}i\bar{\psi}_1\sigma^{\mu\nu}\psi_2 &= \eta_1\sigma^{\mu\nu}\xi_2 - \bar{\eta}_2\bar{\sigma}^{\mu\nu}\bar{\xi}_1.
\end{aligned} \tag{A.13}$$

We clearly see that the indices 1 and 2 label different four-component spinors and their two-component subspinors. Using specific combinations of the above equations we get the following relations which are useful in building four-component spinors from Lagrangians expressed in two-component notation:

$$\begin{aligned}
\bar{\psi}_1 P_L \psi_2 &= \eta_1\xi_2, \\
\bar{\psi}_1 P_R \psi_2 &= \bar{\eta}_2\bar{\xi}_1, \\
\bar{\psi}_1\gamma^\mu P_L \psi_2 &= \bar{\xi}_1\bar{\sigma}^\mu\xi_2, \\
\bar{\psi}_1\gamma^\mu P_R \psi_2 &= -\bar{\eta}_2\bar{\sigma}^\mu\eta_1.
\end{aligned} \tag{A.14}$$

The charge conjugated spinor ψ^C is given by $\psi^C = C\,\bar{\psi}^T$ where (in the chiral representation):

$$C = -i\gamma^2\gamma^0 = \begin{pmatrix} \varepsilon_{\beta\alpha} & 0 \\ 0 & \varepsilon_{\dot{\beta}\dot{\alpha}} \end{pmatrix}. \tag{A.15}$$

If $\psi = \begin{pmatrix} \xi_\alpha \\ \bar{\eta}^{\dot{\alpha}} \end{pmatrix}$ then

$$\psi^C = \ldots = \begin{pmatrix} i\sigma^{2\,\beta}_{\alpha} & 0 \\ 0 & i\bar{\sigma}^{2\dot{\alpha}}{}_{\dot{\beta}} \end{pmatrix} \begin{pmatrix} \eta_\beta \\ \bar{\xi}^\beta \end{pmatrix} = \begin{pmatrix} \eta_\alpha \\ \bar{\xi}^{\dot{\alpha}} \end{pmatrix}. \tag{A.16}$$

A *Majorana* spinor has by definition the property $\psi^C = \psi$, and so, according to (A.16), $\eta = \xi$. Therefore a Majorana spinor has the general form:

$$\psi_M = \begin{pmatrix} \xi_\alpha \\ \bar{\xi}^{\dot{\alpha}} \end{pmatrix} = \begin{pmatrix} \psi_L \\ i\sigma^2\psi_L^* \end{pmatrix}. \tag{A.17}$$

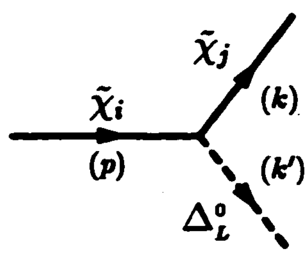
If now ψ_1 and ψ_2 are anticommuting four-component Majorana spinors, then from eq. (A.13) and the defining relation (A.17) we can prove that:

$$\begin{aligned}
&\bar{\psi}_1\psi_2 = \bar{\psi}_2\psi_1, \\
&\bar{\psi}_1\gamma_5\psi_2 = \bar{\psi}_2\gamma_5\psi_1, \\
&\bar{\psi}_1\gamma_\mu\psi_2 = -\bar{\psi}_2\gamma_\mu\psi_1, \\
&\bar{\psi}_1\gamma_\mu\gamma_5\psi_2 = \bar{\psi}_2\gamma_\mu\gamma_5\psi_1, \\
&\bar{\psi}_1\sigma_{\mu\nu}\psi_2 = -\bar{\psi}_2\sigma_{\mu\nu}\psi_1.
\end{aligned} \tag{A.18}$$

Appendix B

Decay width sample calculation

Let us now, for illustrative purposes, have a closer look at a specific decay. We will use as an example the process: $\tilde{\chi}_i \to \tilde{\chi}_j \Delta^0, i > j$. The tree-level diagram for this process is shown in Figure B below together with the vertex factor. The letters in parentheses denote the particle momenta.



$$\frac{-1}{\sqrt{2}} g_L V^*_{i5} U^*_{j1} (1 - \gamma^5), \quad i > j$$

Figure B.1: Decay diagram with momenta and vertex factor.

The amplitude for this process is clearly:

$$\mathcal{M} = -i\bar{u}(k) \cdot \left(-\frac{g_L}{\sqrt{2}} V^*_{i5} U^*_{j1} (1 - \gamma^5) \right) \cdot u(p). \tag{B.1}$$

Squaring the amplitude we get:

$$\overline{|\mathcal{M}|^2} = \frac{g_L^2}{4} |V_{i5}|^2 |U_{j1}|^2 \ \mathrm{Tr}\{(\not{k} + m_j)(1 - \gamma^5)(\not{p} + m_i)(1 + \gamma^5)\}. \tag{B.2}$$

Evaluating the trace we have:

$$
\begin{aligned}
&\mathrm{Tr}\{\cdots\} \\
&= k_\mu p_\nu \mathrm{Tr}\{\gamma^\mu(\gamma^\nu + m_i) + m_j(\gamma^\nu + m_i) \\
&-\gamma^\mu\gamma^5(\gamma^\nu + m_i) - m_j\gamma^5(\gamma^\nu + m_i) \\
&+\gamma^\mu(\gamma^\nu + m_i)\gamma^5 + m_j(\gamma^\nu + m_i)\gamma^5 \\
&-\gamma^\mu\gamma^5(\gamma^\nu + m_i)\gamma^5 - m_j\gamma^5(\gamma^\nu + m_i)\gamma^5\} \qquad \text{(B.3)} \\
&= \cdots \\
&= k_\mu p_\nu \mathrm{Tr}\{\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu\} \\
&= k_\mu p_\nu 8 g^{\mu\nu} \\
&= 8\, k \cdot p.
\end{aligned}
$$

Here we used the standard trace theorems, see for example ref [7]. Now, according to our momenta definitions we know that: $p = k + k'$. Therefore,

$$
\begin{aligned}
&(p-k)^2 = k'^2 \Rightarrow p^2 + k^2 - 2p\cdot k = k'^2 \\
&\Rightarrow p \cdot k = \tfrac{1}{2}(p^2 + k^2 - k'^2) = \tfrac{1}{2}(m_i^2 + m_j^2 - M_\Delta^2).
\end{aligned}
$$

Working in the initial particle's rest frame (center of mass frame) and using the appropriate formula [31], we get the following decay width:

$$
\begin{aligned}
\Gamma &= \tfrac{1}{2m_i}\tfrac{1}{4\pi^2}\overline{|\mathcal{M}|^2}\,\tfrac{1}{2}\pi\lambda^{1/2}\Big(1, \tfrac{m_j^2}{m_i^2}, \tfrac{M_\Delta^2}{m_i^2}\Big) \\
&= \tfrac{1}{16\pi m_i^2} g_L |V_{i5}|^2 |U_{j1}|^2 \left(m_i^2 + m_j^2 - M_\Delta^2\right)\ \lambda^{1/2}\Big(1, \tfrac{m_j^2}{m_i^2}, \tfrac{M_\Delta^2}{m_i^2}\Big)
\end{aligned} \qquad \text{(B.4)}
$$

where λ is the kinematic factor [26, 31].

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