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**Torque Pulsations and Currents
in
Induction Motors
with
Unbalanced Voltages**

Asghar Khan

**A Thesis
in
The Department
of
Electrical Engineering**

**Presented in Partial Fulfillment of the Requirements
for the Degree of Master of Engineering at
Concordia University
Montréal, Québec, Canada**

October 1987

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ABSTRACT

Torque Pulsations and Currents in Induction Motors with Unbalanced Voltages

Asghar Khan

A dynamic circuit model of the induction motor is developed for unbalanced supply voltages using the conventional circuit model for balanced supply conditions. The equations governing the various motor performance variables such as currents, average torque and pulsating torque are derived, and it is ascertained that the frequency of the pulsating torque is twice that of the supply. Transient analysis of the motor with this model is provided. Graphical plots are shown for the currents and pulsating torque against average torque for different kinds of unbalance.

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LIST OF SYMBOLS

| Symbol | Definition |
|------------------|--|
| s | slip |
| N | Negative sequence components |
| P | Positive sequence components |
| R _s | Stator resistance |
| R _r | Rotor resistance referred to stator |
| X _{ls} | Stator leakage reactance |
| X _{lr} | Rotor leakage reactance referred to stator |
| X _s | Stator reactance |
| X _r | Rotor reactance referred to stator |
| X _m | Magnetizing reactance referred to stator |
| X _{ms} | Mutual reactance between stator coils |
| X _{sr} | Mutual reactance between stator and rotor |
| T _p | Amplitude of pulsating torque |
| T _{ave} | Average torque |
| ω_m | rotor speed |
| α | position of stator winding ab |
| β | position of stator winding bc |
| γ | position of stator winding ca |

CHAPTER 1

INTRODUCTION

Unbalanced three-phase voltages are a common occurrence in an electric supply system. A major cause of this is the non-uniform application of single-phase loads on the three phases. Also, unbalanced impedances in three-phase transmission and distribution equipment used by utilities can lead to such conditions. This may detrimentally affect the operation and integrity of three-phase apparatus in both customer and utility installations.

Unbalanced voltages and currents can be resolved into components of symmetrical balanced three-phase quantities known as positive, negative and zero sequence components. For a balanced system, the positive sequence voltage or current is the magnitude and displacement as each phase voltage or current, and the negative and zero sequence components are nil. Therefore, the negative and zero sequence components represent a measure of unbalance.

The two definitions [1] currently used to describe voltage unbalance are the 'NEMA unbalance', which is mostly used in North America, and the 'Voltage Unbalance Factor (VUF)', most often used in Europe. These can be stated as follows :

$$\text{NEMA} = \frac{\text{Max. deviation from. average voltage}}{\text{Average voltage}} \quad (1.1)$$

$$\text{VUF} = \frac{\text{Negative sequence voltage}}{\text{Positive sequence voltage}} \quad (1.2)$$

These definitions result in different values of unbalance for a given set of phase voltages. Generally, the VUF is higher than than the NEMA depending on the phase displacement of the negative sequence component with respect to the positive sequence.

1.1 Significance of Negative Sequence

Polyphase induction motors are widely used for industrial, commercial and domestic purposes. It is well known that voltage unbalance plays a serious role in three-phase induction motor performance, in terms of losses and currents, both line and phase, and torque. The negative sequence impedance, which corresponds to slips of approximately two [2], is much lower than the positive sequence impedance. Hence for a given negative sequence voltage, a disproportionately high negative sequence current will flow. This negative sequence impedance is comparable to the motor locked rotor impedance. Moreover, rotor losses will increase more than the stator losses. In addition, the negative sequence also brings about a negative rotating field in the air-gap, thus producing a negative torque. This not only reduces the average torque which is a well known phenomenon, but also introduces torque pulsations, a cause that has not been studied to its full extent, in the mechanical drive system.

1.2 Sources of Negative Sequence

The negative sequence voltages can be created in several ways. One such method is shown in Fig.1.1. A single-phase, highly reactive load produces a well recognized voltage unbalance. The load unbalance in this illustration could also be the result of one blown fuse on a power factor correcting

capacitor. The resulting negative sequence voltage would be the same as that due to a comparable lagging load connected to that phase.

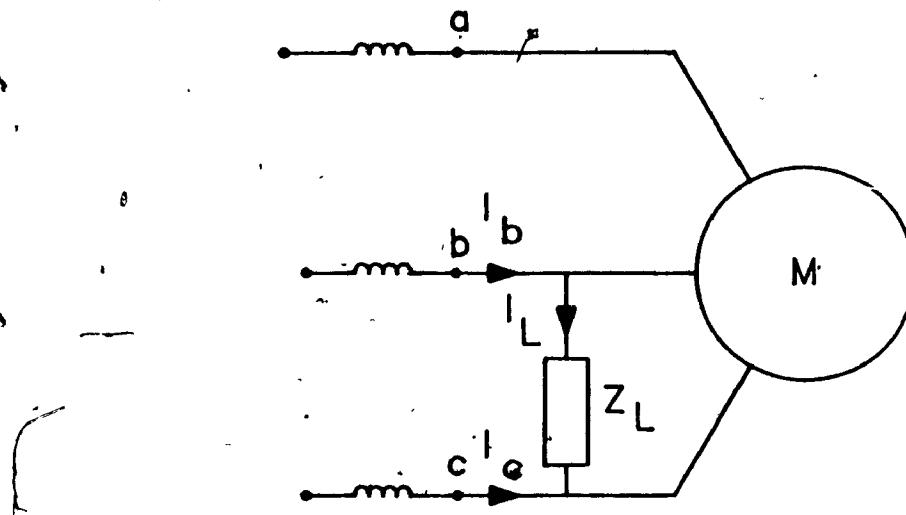


Fig. 1.1. Negative Sequence unbalance due to single-phase loads.

The second method of producing the negative sequence voltage is through unequal impedances in each phase of the supply system. This occurs due to widely spaced untransposed conductors, such as low voltage high impedance bus runs. Severe negative sequence conditions, which can rapidly lead to motor failure, happen when one phase is completely lost. This may be due to a fuse blowing or any other opening of just one of the three motor phases.

1.3 Review of Earlier Work

Earlier work [3,4] was concerned with the evaluation of the positive, negative and zero sequence components of voltages, currents and impedances of the motor at a particular point of operation. Motor losses were determined and graphs presented in an attempt to provide easy access to the respective voltage sequence components from line-to-line voltages. The protection of induction

motors against excessive line currents and overheating [5,6,7] has been another area of interest. Graphs were obtained to suggest critical points of operation.

Unbalanced voltages also occur due to system faults such as outage of a phase, phase-to-phase faults or phase-to-ground faults. Motor performance in terms of heat loss and rotor overheating [8,9,10] has been investigated when subjected to such disturbances. With the advent of power electronic converting equipment, non-sinusoidal voltages are fed to the induction motors. The harmonics in the supply have led to a greater concern over torque pulsation [11]. Recent literature has mentioned torque pulsations [12] to a small extent, but no simple method for their prediction has been included.

1.4 Scope of this Thesis

A quantitative analysis of motor currents and torque pulsations for various unbalances is outlined. To provide a suitable comparison, the open-line characteristics will be presented. Graphs of line and phase currents, and amplitude of pulsating torque against the average torque are shown. Transient motor behaviour when subjected to unbalance or vice versa is provided. The calculations of the currents is based on a dynamic circuit model of the motor having three axes on the stator and two on the rotor. This enables the direct modelling of currents and voltages of the actual stator coils. Details of the derivation of the required parameters values from those of the conventional model for balanced operation are included.

CHAPTER 2

THE TRANSFORMED CIRCUIT MODEL

2.1 Introduction

The induction motor can be represented by the conventional model for balanced operation as shown in Fig.2.1.

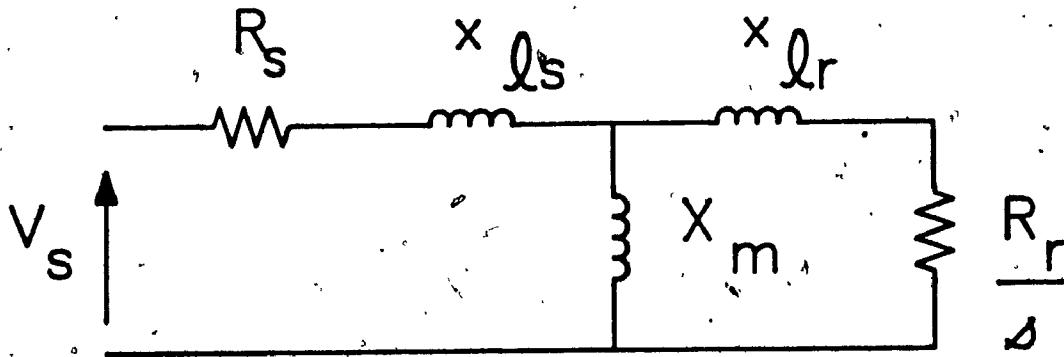


Fig. 2.1. Model for balanced motor operation.

For unbalanced operation, the circuit model of Fig.2.2 is used. When working with this model the 3-phase voltage variables are transformed to their symmetrical components in order to obtain the symmetrical current components and then transformed back to the actual currents. However, a dynamic circuit model having three axes or phases on the stator as shown in Fig.2.3 [13] will provide the solutions directly. The various equations for this model will be formulated in this chapter.

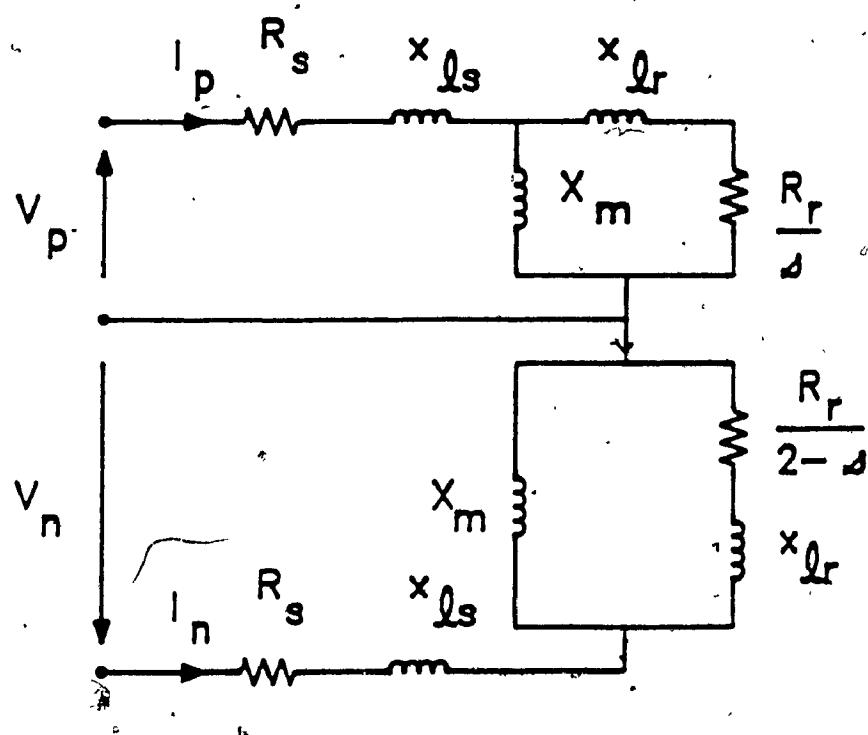


Fig. 2.2. Model for unbalanced operation of motor.

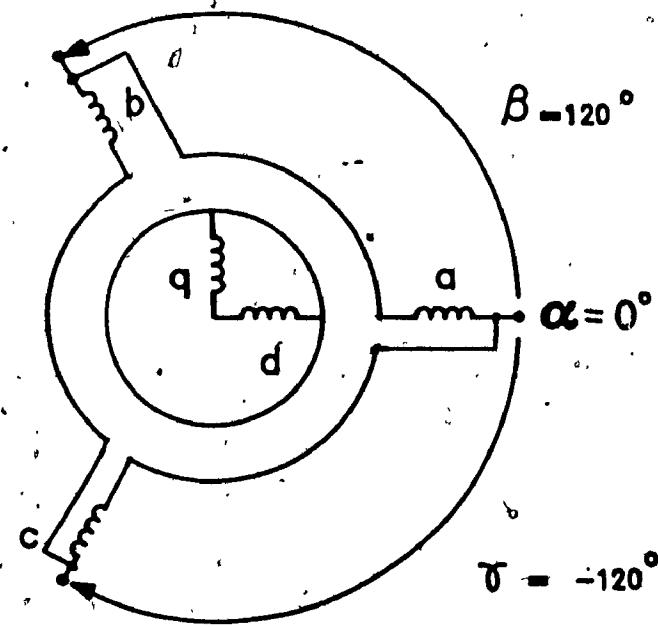


Fig. 2.3. Transformed circuit model of motor.

2.2 Analysis

For sinusoidal steady-state operation the parameters of the transformed model are related to those of the conventional model as follows :

$$X_{ms} = \frac{2X_m}{3} \quad (2.1)$$

$$X_s = \omega L_s = x_{ls} + X_{ms} \quad (2.2)$$

$$X_r = \omega L_r = x_{lr} + X_m \quad (2.3)$$

$$X_{sr} = X_m \sqrt{\frac{2}{3}} \quad (2.4)$$

The equation for steady-state operation of the machine is given as

$$V = R I + p\omega_m G I + jX I = Z I \quad (2.5)$$

where 'p' is the number of pole-pairs and the resistance, reactance and rotational coefficient matrices can be given as follows:

$$R = \begin{bmatrix} R_s & 0 & 0 & 0 & 0 \\ 0 & R_s & 0 & 0 & 0 \\ 0 & 0 & R_s & 0 & 0 \\ 0 & 0 & 0 & R_r & 0 \\ 0 & 0 & 0 & 0 & R_r \end{bmatrix} \quad (2.6)$$

$$X = \begin{bmatrix} x_{ls} + X_{ms} & X_{ms}\cos(\beta-\alpha) & X_{ms}\cos(\gamma-\alpha) & X_{sr}\cos\alpha & X_{sr}\sin\alpha \\ X_{ms}\cos(\alpha-\beta) & x_{ls} + X_{ms} & X_{ms}\cos(\gamma-\beta) & X_{sr}\cos\beta & X_{sr}\sin\beta \\ X_{ms}\cos(\alpha-\gamma) & X_{ms}\cos(\beta-\gamma) & x_{ls} + X_{ms} & X_{sr}\cos\gamma & X_{sr}\sin\gamma \\ X_{sr}\cos\alpha & X_{sr}\cos\beta & X_{sr}\cos\gamma & x_{lr} + X_m & 0 \\ X_{sr}\sin\alpha & X_{sr}\sin\beta & X_{sr}\sin\gamma & 0 & x_{lr} + X_m \end{bmatrix} \quad (2.7)$$

$$G = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ M_{sr}\sin\alpha & M_{sr}\sin\beta & M_{sr}\sin\gamma & 0 & L_{r2} \\ -M_{sr}\cos\alpha & -M_{sr}\cos\beta & -M_{sr}\cos\gamma & -L_{r2} & 0 \end{bmatrix} \quad (2.8)$$

The steady-state solution of (2.5) is as follows:

$$I = Z^{-1}V$$

If only the average value of the torque is required, it is obtained from

$$T_{ave} = I^t G I \quad (2.9)$$

provided the vector I contains the rms values of the currents. The instantaneous torque (2.10) is obtained by first deriving the instantaneous currents corresponding to the magnitudes and phase angles in the vector I ,

$$T = I^t G I \quad (2.10)$$

After manipulating and simplifying (2.10), the result is

$$T = \sum_{n=1}^{n=3} A_n [-\cos(2\omega t + \phi_d + \phi_n) + \cos(\phi_d - \phi_n)] \\ + \sum_{n=1}^{n=3} B_n [-\cos(2\omega t + \phi_q + \phi_n) + \cos(\phi_q - \phi_n)] \quad (2.11)$$

$$= T_p(2\omega t) + T_{ave} \quad (2.12)$$

where 1,2 and 3 represent phases ab, bc and ca respectively, and

$$A_1 = 0$$

$$A_2 = G_{42} I_d I_{bc}$$

$$A_3 = G_{43} I_d I_{ca}$$

$$B_1 = G_{51} I_q I_{ab}$$

$$B_2 = G_{52} I_q I_{bc}$$

$$B_3 = G_{53} I_q I_{ca}$$

The magnitude of the pulsating torque is obtained by adding the oscillatory terms only, which have both magnitude and phase angle, and the average torque by adding the constant terms only. This is presented in (2.13) and (2.14).

$$T_p = \left[\sum_{n=2}^{n=3} A_n / (\phi_d + \phi_n)^0 + \sum_{n=1}^{n=3} B_n / (\phi_q + \phi_n)^0 \right] \quad (2.13)$$

and

$$T_{ave} = \left[\sum_{n=2}^{n=3} A_n \cos(\phi_d - \phi_n) + \sum_{n=1}^{n=3} B_n \cos(\phi_q - \phi_n) \right] \quad (2.14)$$

Although it is not evident, (2.14) gives the same values for the average torque as (2.9).

2.3 Numerical Parameters

$$[R] + [G] = \begin{bmatrix} .2 & 0 & 0 & 0 & 0 \\ 0 & .2 & 0 & 0 & 0 \\ 0 & 0 & .2 & 0 & 0 \\ 0 & .03 & -.03 & .3 & .0439 \\ -.0347 & .0173 & .0173 & -.0439 & .3 \end{bmatrix} \quad (2.15)$$

$$[X] = \begin{bmatrix} 11.22 & -5.33 & -5.33 & 13.06 & 0 \\ -5.33 & 11.22 & -5.33 & -6.5 & 11.31 \\ -5.33 & -5.33 & 11.22 & -6.5 & -11.31 \\ 13.06 & -6.5 & -6.5 & 16.55 & 0 \\ 0 & 11.31 & -11.31 & 0 & 16.55 \end{bmatrix} \quad (2.16)$$

CHAPTER 3

OPEN-LINE AND TRANSIENT CONDITIONS

3.1 Introduction

This chapter deals with the analysis of the induction motor behaviour when one line is open circuited, and the transient performance of the motor when unbalance occurs and when the balance is restored. The equations governing such cases are developed from the ones obtained in Chapter 2.

3.2 Analysis

The balanced and open-line circuit diagrams are shown in Fig.3.1. The various voltage and current relations that can be derived from these figures are as follows:

$$v_{a'b'} = v_{ab} \quad (3.1)$$

$$v_{a'c'b'} = -v_{bc} - v_{ca} \quad (3.2)$$

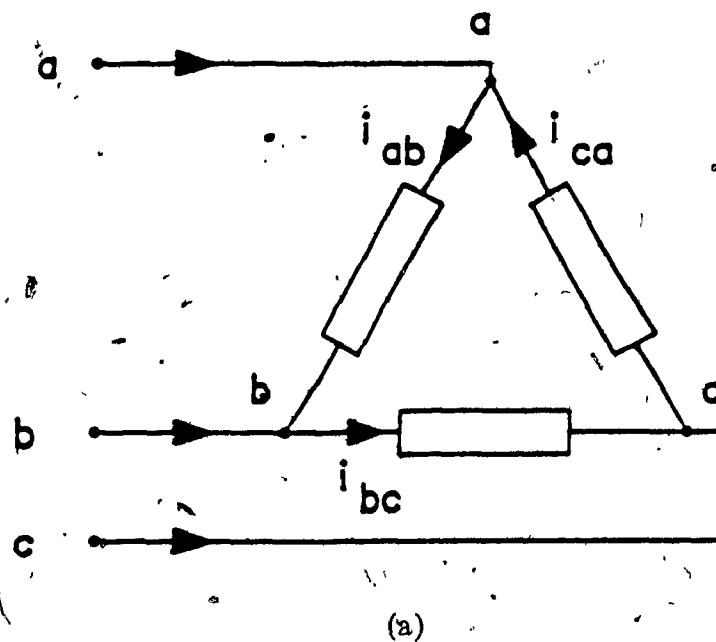
$$v_{d'} = v_d \quad (3.3)$$

$$v_{q'} = v_q \quad (3.4)$$

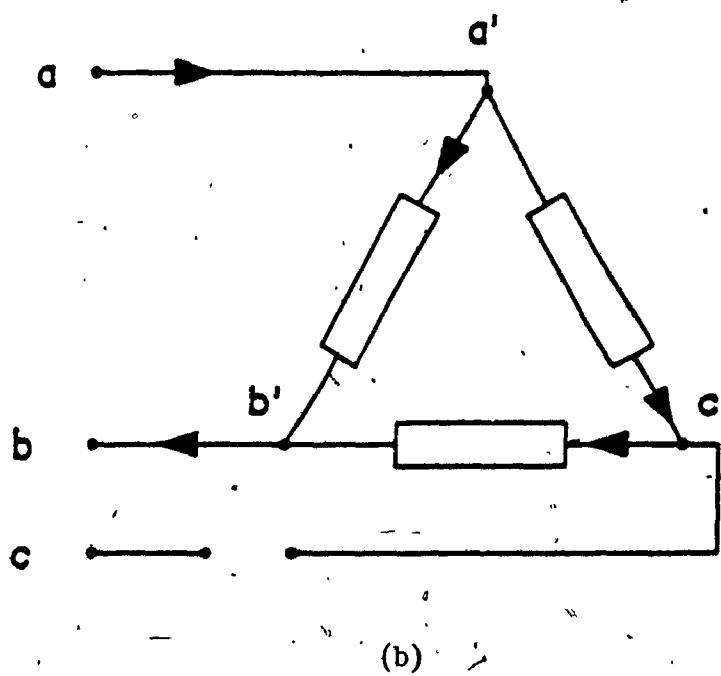
$$i_{a'b'} = i_{ab} \quad (3.5)$$

$$i_{a'c'b'} = -i_{ca} = -i_{bc} \quad (3.6)$$

$$i_{d'} = i_d \quad (3.7)$$



(a)



(b)

Fig. 3.1. (a) Schematic of balanced motor.

(b) Schematic of an open-line at C.

$$I_{q'} = I_q \quad (3.8)$$

The above equations(3.1-3.8) can be presented as follows:

$$v_n = C^t v_o \quad (3.9)$$

where

$$v_n = \begin{bmatrix} v_{a'} & b' \\ v_{a'} & c' & b' \\ v_d \\ v_q \end{bmatrix}$$

$$v_o = \begin{bmatrix} v_{ab} \\ v_{bc} \\ v_{ca} \\ v_d \\ v_q \end{bmatrix}$$

$$C^t = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (3.10)$$

and

$$I_o = C I_n \quad (3.11)$$

where

$$I_o = \begin{bmatrix} I_{ab} \\ I_{bc} \\ I_{ca} \\ I_d \\ I_q \end{bmatrix}$$

$$I_n = \begin{bmatrix} I_a' & b' \\ I_a' & c' & b' \\ I_d' \\ I_q' \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (3.12)$$

The new resistance, reactance and rotational coefficient matrices can be obtained from their old matrices as follows:

$$[\text{new parameters}] = C^t [\text{old parameters}] C \quad (3.13)$$

Hence, the new matrices will be as shown:

$$R_n = \begin{bmatrix} R_s & 0 & 0 & 0 \\ 0 & 2R_s & 0 & 0 \\ 0 & 0 & R_t & 0 \\ 0 & 0 & 0 & R_t \end{bmatrix} \quad (3.14)$$

$$X_n = \begin{bmatrix} X_{ls} + X_{ms} & X_{12} & X_{sr}\cos\alpha & X_{sr}\sin\alpha \\ X_{21} & 2(X_{ls} + X_{ms}) & X_{23} & X_{24} \\ X_{sr}\cos\alpha & X_{32} & X_{lr} + X_m & 0 \\ X_{sr}\sin\alpha & X_{42} & 0 & X_{lr} + X_m \end{bmatrix} \quad (3.15)$$

where

$$X_{12} = X_{ms}[\cos(\alpha-\beta) + \cos(\alpha-\gamma)]$$

$$X_{21} = X_{ms}[\cos(\beta-\alpha) + \cos(\gamma-\alpha)]$$

$$X_{23} = X_{32} = -X_{sr}[\cos\beta + \cos\gamma]$$

$$X_{24} = X_{42} = -X_{sr}[\sin\beta + \sin\gamma]$$

$$G_n = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ M_{sr}\sin\alpha & -M_{sr}(\sin\beta + \sin\gamma) & 0 & L_{r2} \\ -M_{sr}\cos\alpha & M_{sr}(\cos\beta + \cos\gamma) & -L_{r2} & 0 \end{bmatrix} \quad (3.16)$$

Thus, the new equations for steady-state operation of the induction motor with an open-line at C can be written as in (3.17).

$$V = [R_n + p\omega_m G_n + jX_n] I = Z_n I \quad (3.17)$$

The solution of (3.17) will give the new currents. Equations [2.9 - 2.14] can be then be used in a similar procedure with the new parameters to obtain the average torque and pulsating torque.

In the transient situation, the steady-state equation (2.5) can be rewritten as

$$v = [R + p\omega_m G] i + L i' \quad (3.18)$$

To solve for i , the above equation is rearranged as follows

$$i' = L^{-1}[-(R + p\omega_m G) i + v] \quad (3.19)$$

The torque can then be attained from (2.10), which is

$$T = i^t G i$$

The mechanical system representation can be given as follows:

$$T = B\omega_m + J\omega_m' \quad (3.20)$$

or

$$\omega_m' = \frac{1}{J} [T - \omega_m] \quad (3.21)$$

Equations (3.19) and (3.21) can be combined and solved using a Fourth Order Runge Kutta numerical integration.

3.3 Numerical parameters

$$[R_n] + [G_n] = \begin{bmatrix} 0.2 & 0. & 0. & 0. \\ 0. & 0.4 & 0. & 0. \\ 0. & 0. & 0.3 & 0.0439 \\ -0.0347 & 0.0347 & -0.0439 & 0.3 \end{bmatrix}$$

$$[X_n] = \begin{bmatrix} 11.22 & 10.67 & 13.06 & 0. \\ 10.67 & 22.00 & 13.06 & 0. \\ 13.06 & 13.06 & 16.55 & 0. \\ 0. & 0. & 0. & 16.55 \end{bmatrix}$$

CHAPTER 4

RESULTS

4.1 Introduction

This chapter presents the results attained from the equations determined in the previous chapters. Plots of steady-state performance of the induction motor for the two unbalanced cases (line-to-line and line-to-neutral loads) with regards to current and pulsating torque are given. The transient situation is shown by comparison of speed and average torque for the various cases.

4.2 Steady-state case

The steady-state unbalanced situation is presented in two ways; the line-to-neutral load and line-to-line load. Figures 4.1-4.7 represent the first case while the latter is shown in figures 4.8-4.14. The various degrees of voltage unbalance and the per unit values of torque that were used in the above mentioned figures are as follows :

1 = balanced supply

2 = 1% unbalance

3 = 5% unbalance

4 = 10% unbalance

5 = open-line c

1 p.u. Torque = 20 N.m

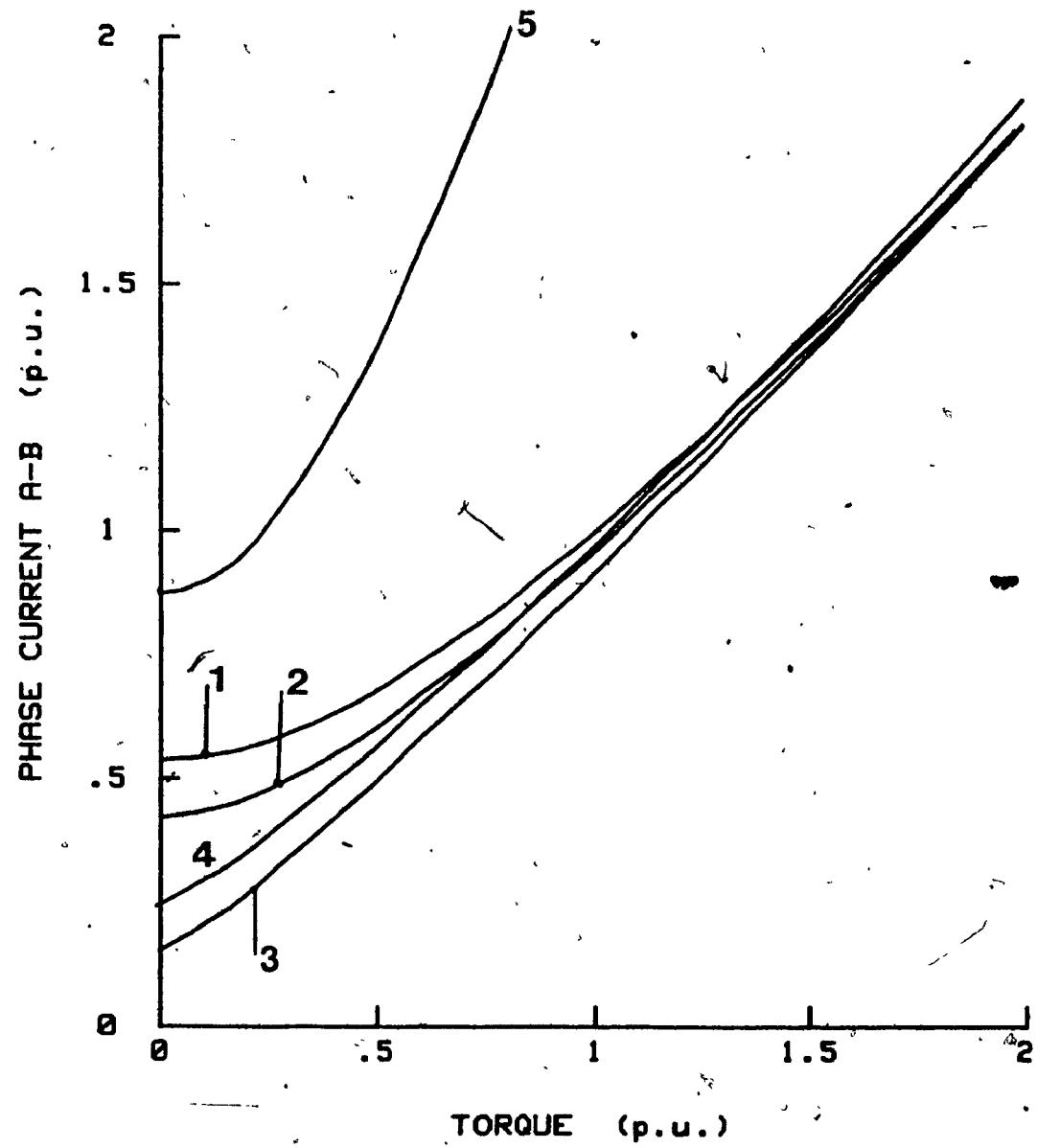


Fig. 4.1. I_{ab} versus average torque for line-to-neutral load unbalance.

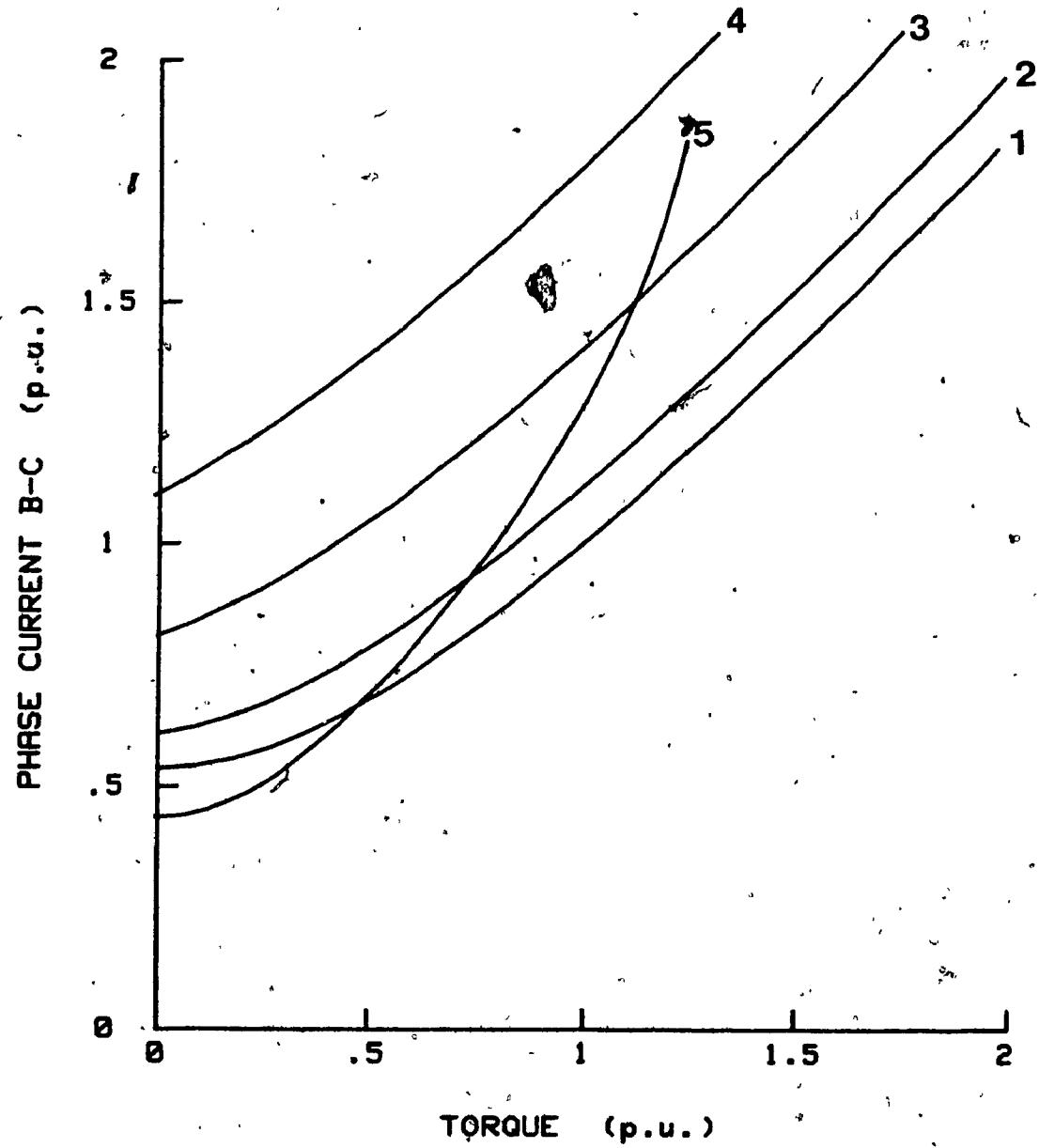


Fig. 4.2. I_{bc} versus average torque for line-to-neutral load unbalance.

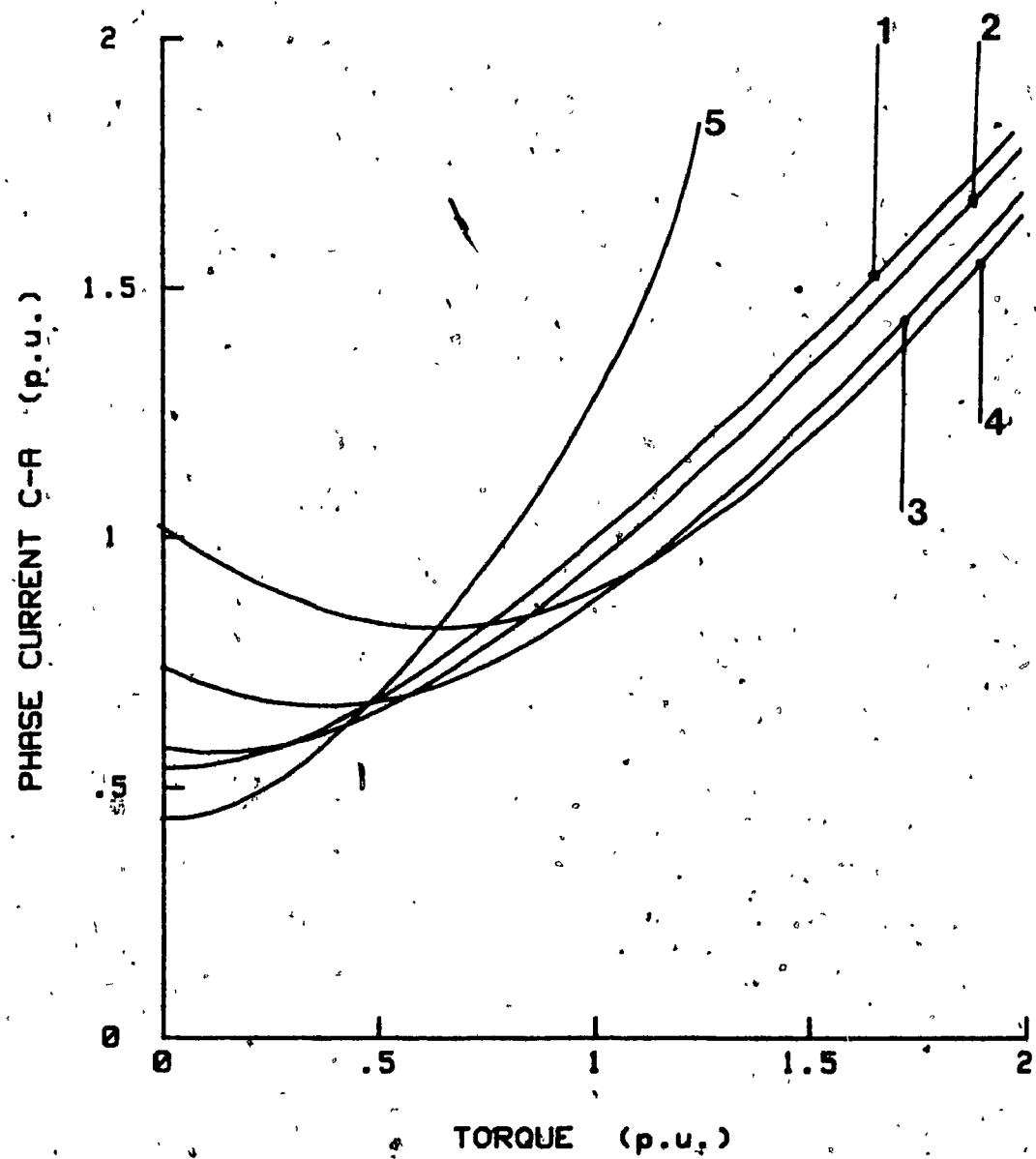


Fig. 4.3. I_{ca} versus average torque for line-to-neutral-load unbalance.

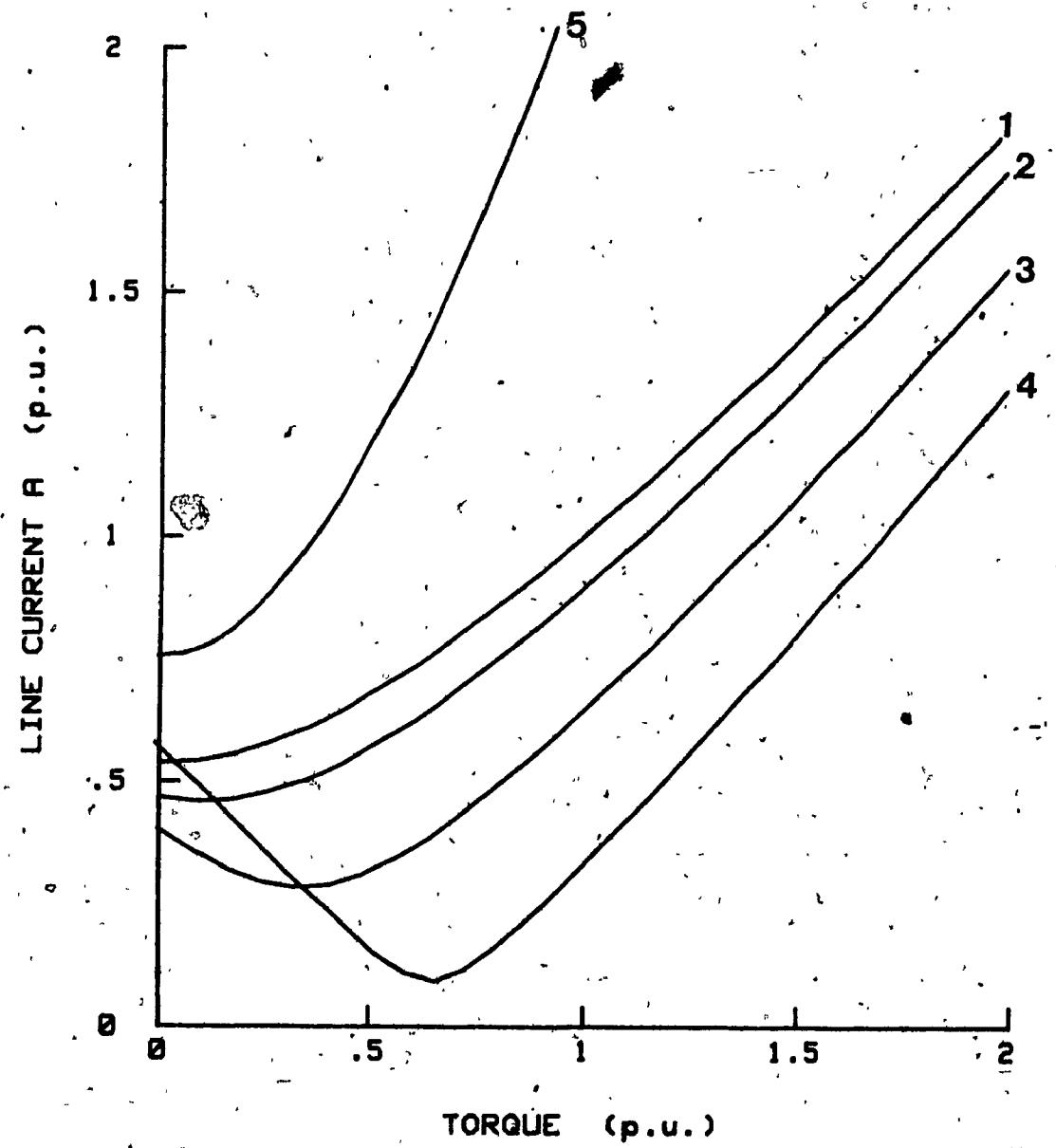


Fig. 4.4. I_a versus average torque for line-to-neutral load unbalance.

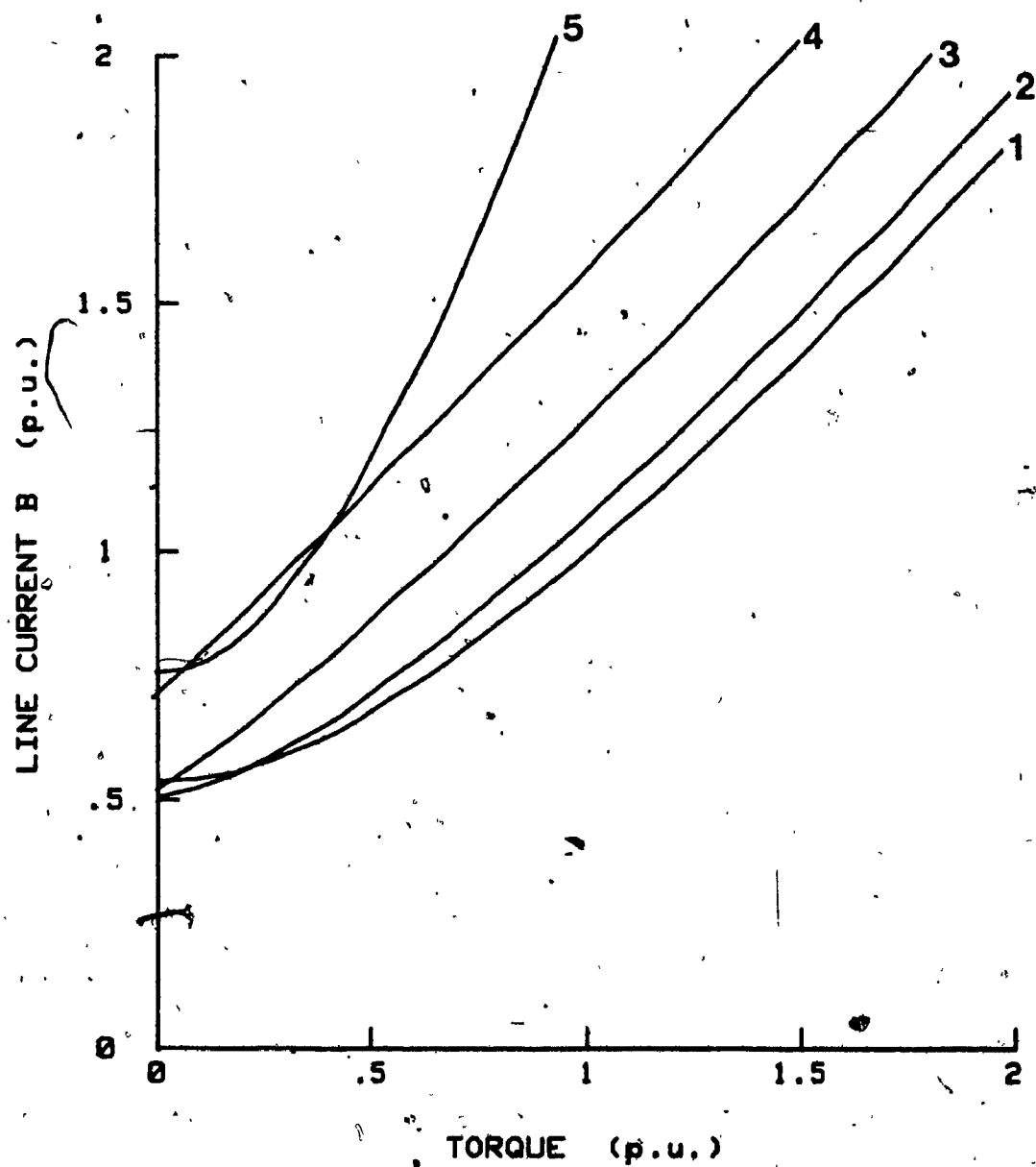


Fig. 4.5. I_b versus average torque for line-to-neutral load unbalance.

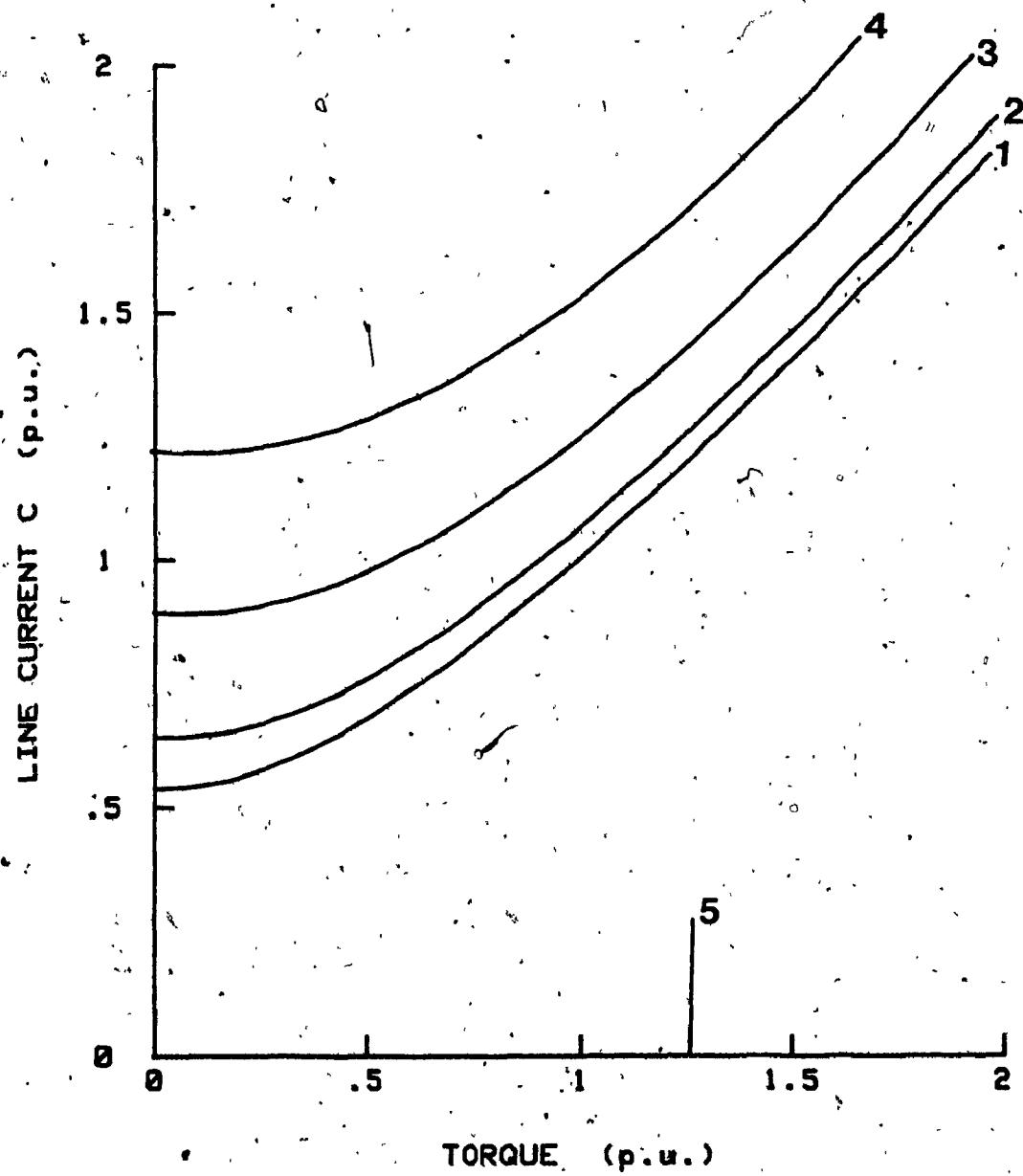


Fig. 4.6. I_c versus average torque for line-to-neutral load unbalance.

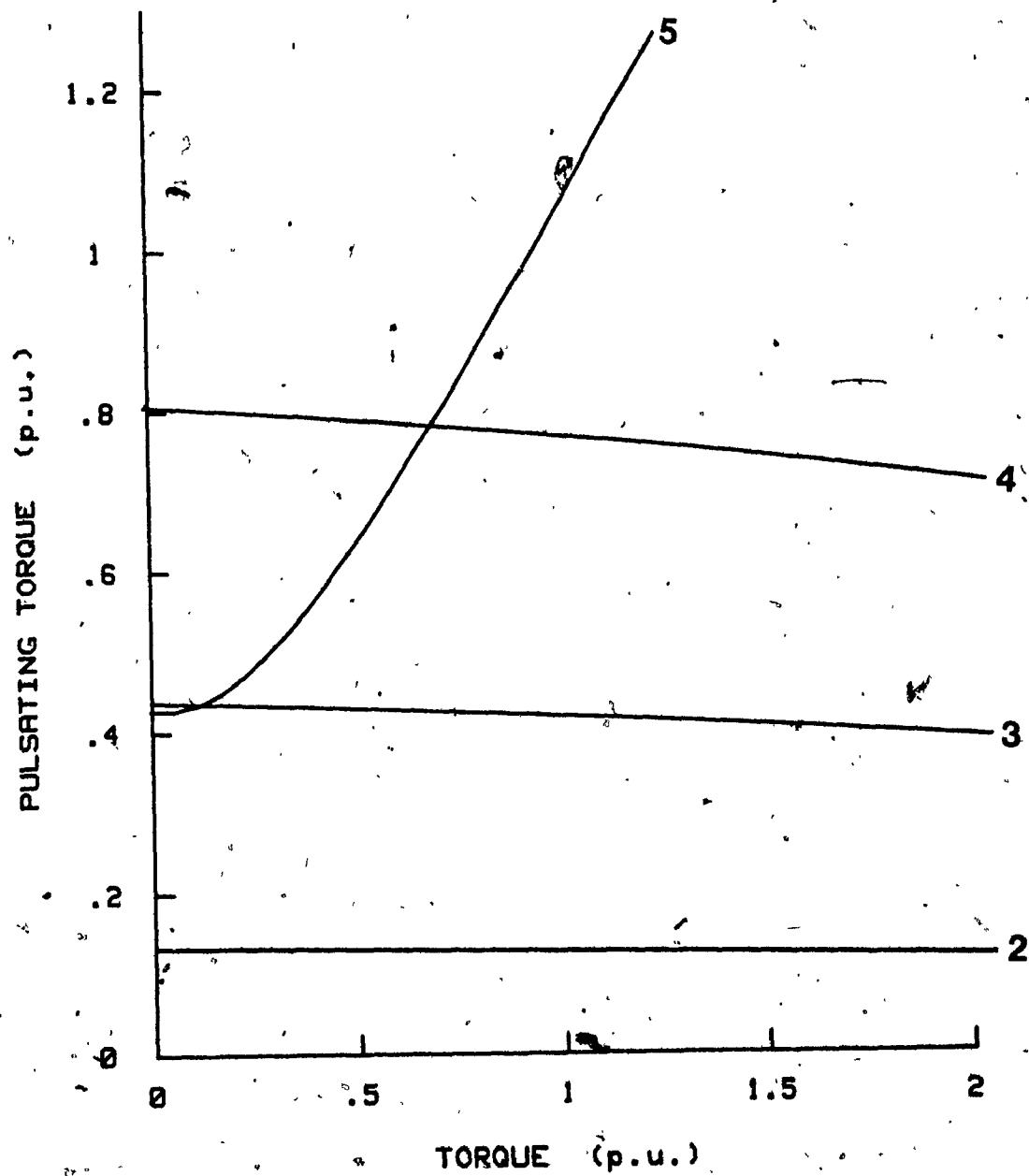


Fig. 4.7. Pulsating torque versus average torque for line-to-neutral load unbalance.

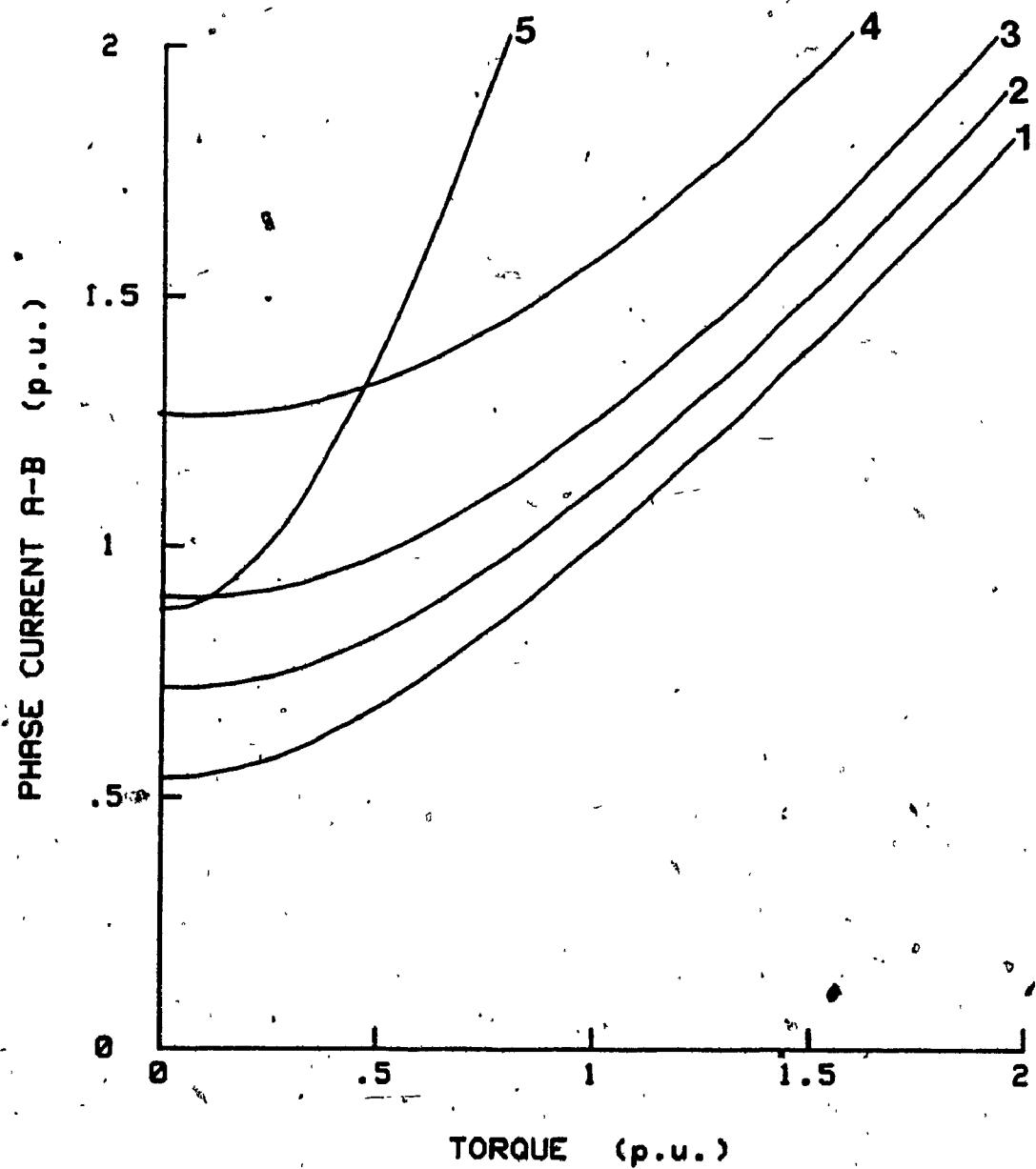


Fig. 4.8. I_{ab} versus average torque for line-to-line load unbalance.

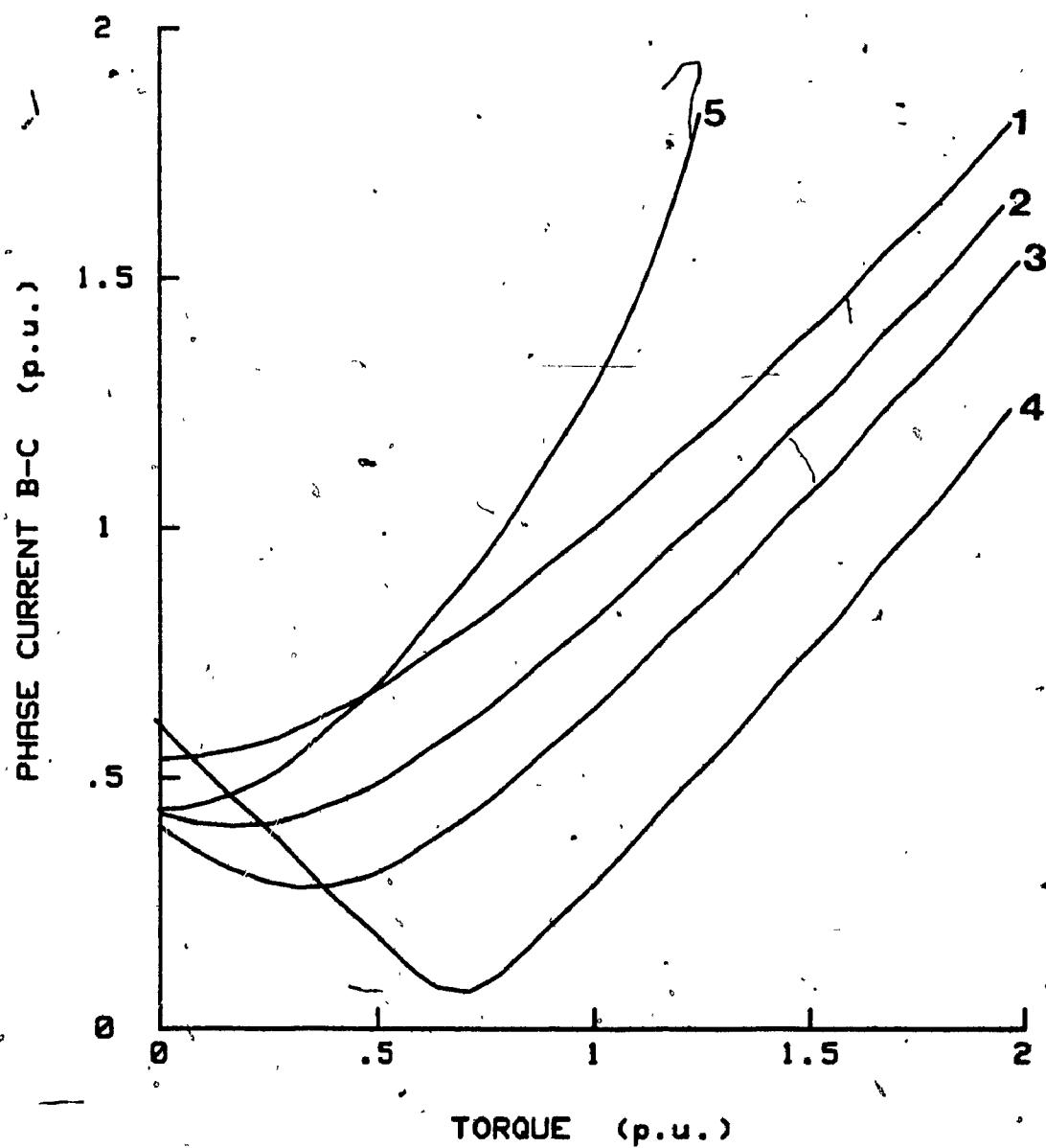


Fig. 4.9. I_{bc} versus average torque for line-to-line load unbalance.

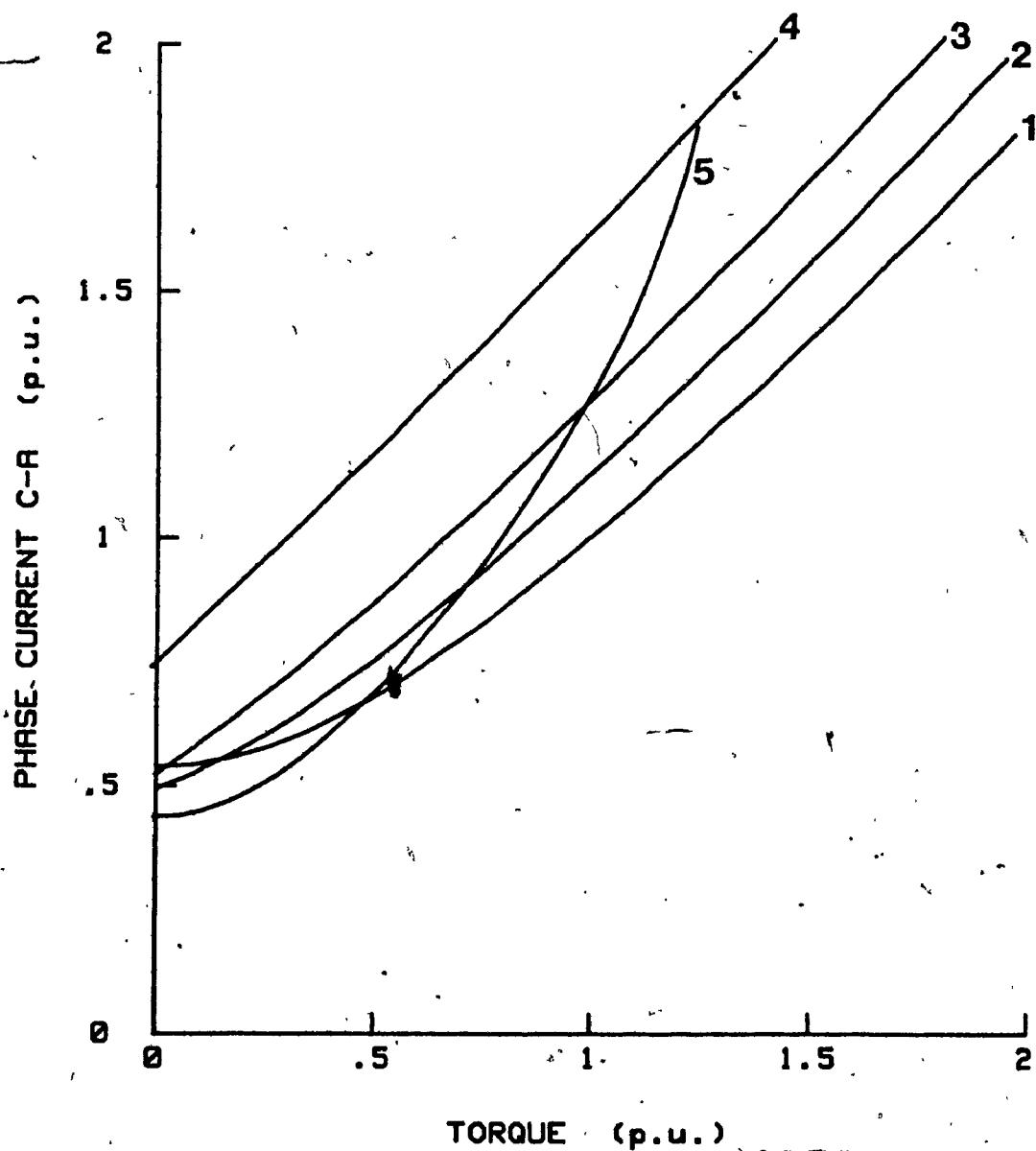


Fig. 4.10. I_{ca} versus average torque for line-to-line load unbalance.

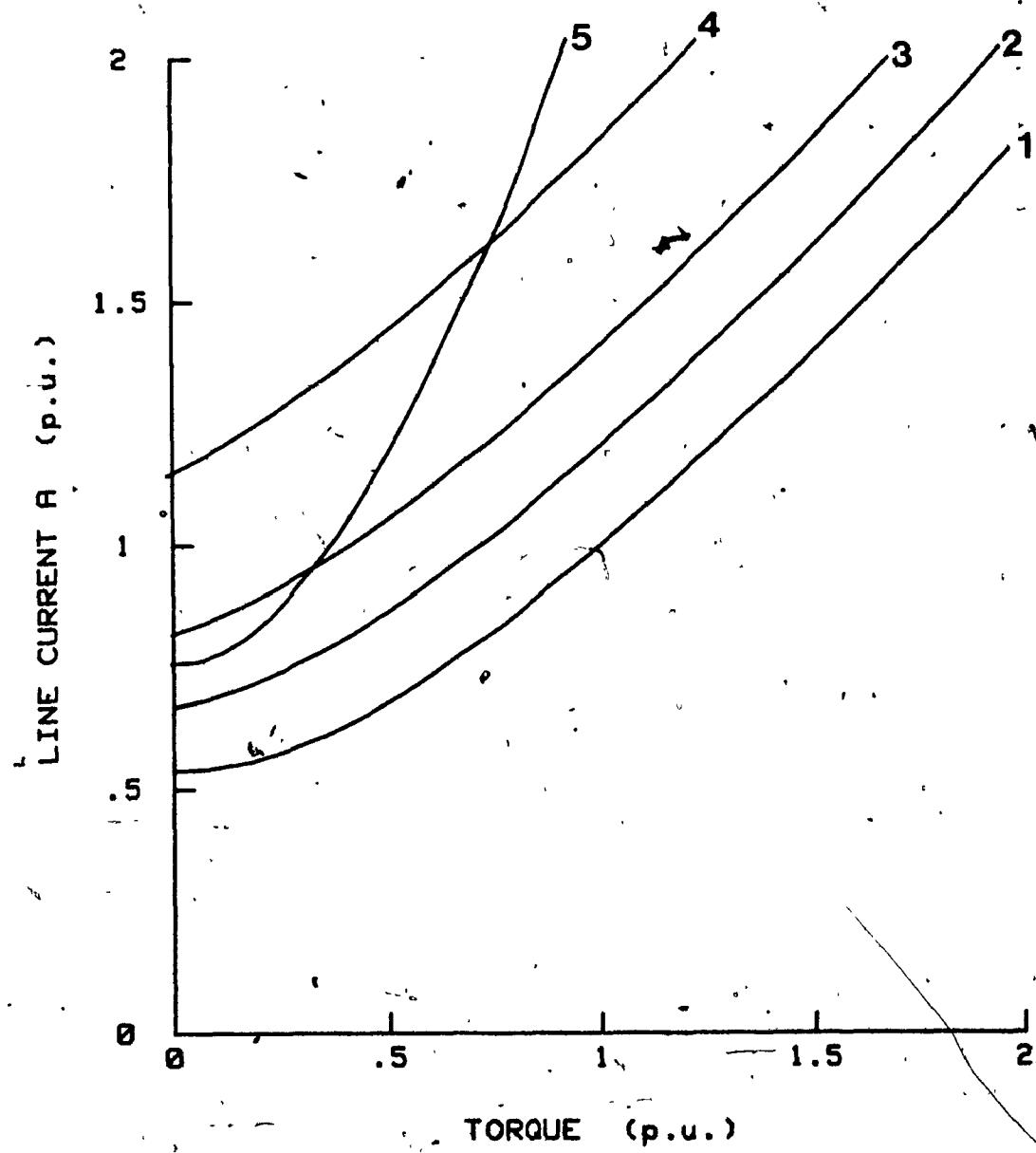


Fig. 4.11. I_a versus average torque for line-to-line load unbalance.

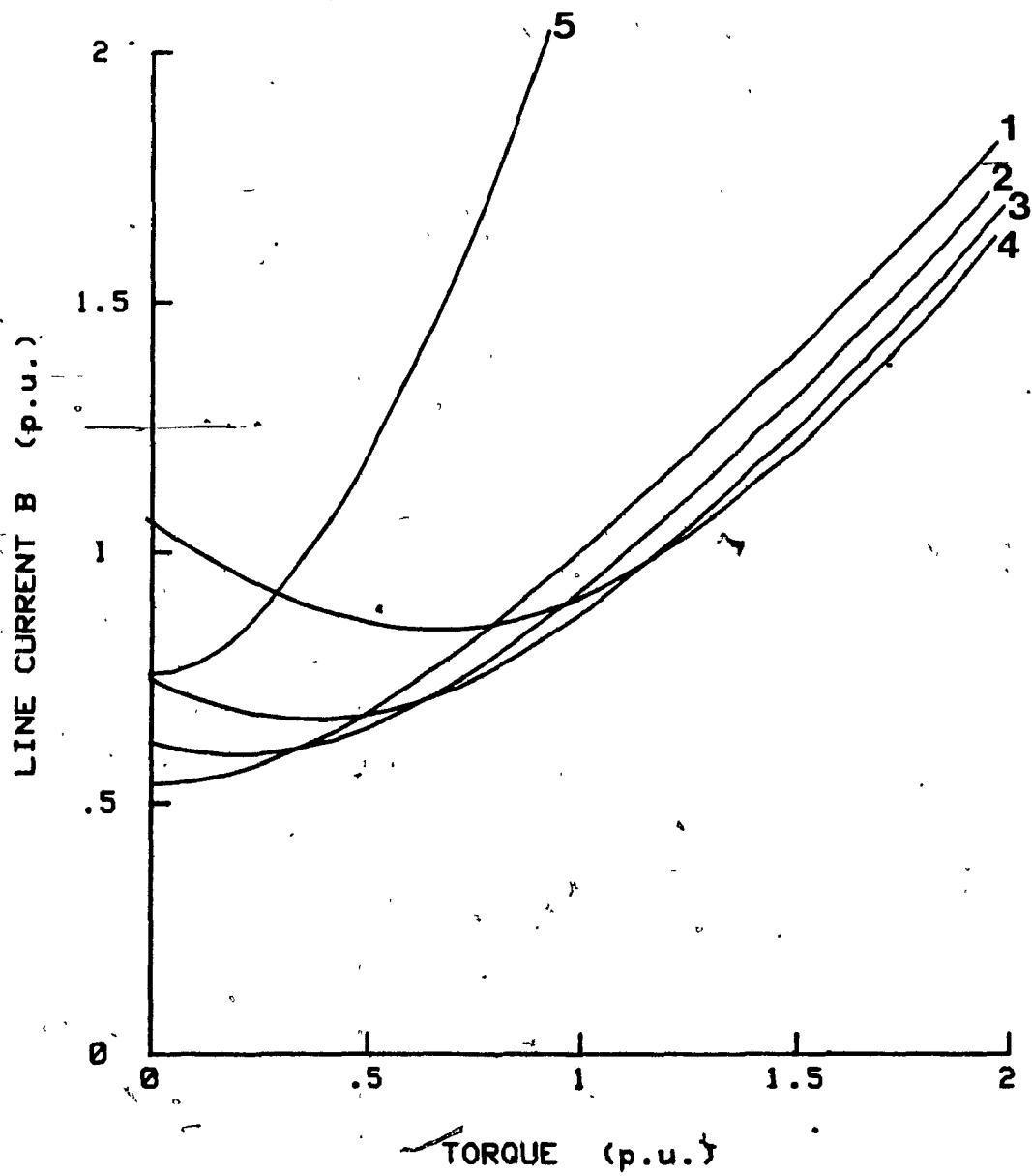


Fig. 4.12. I_b versus average torque for line-to-line load unbalance.

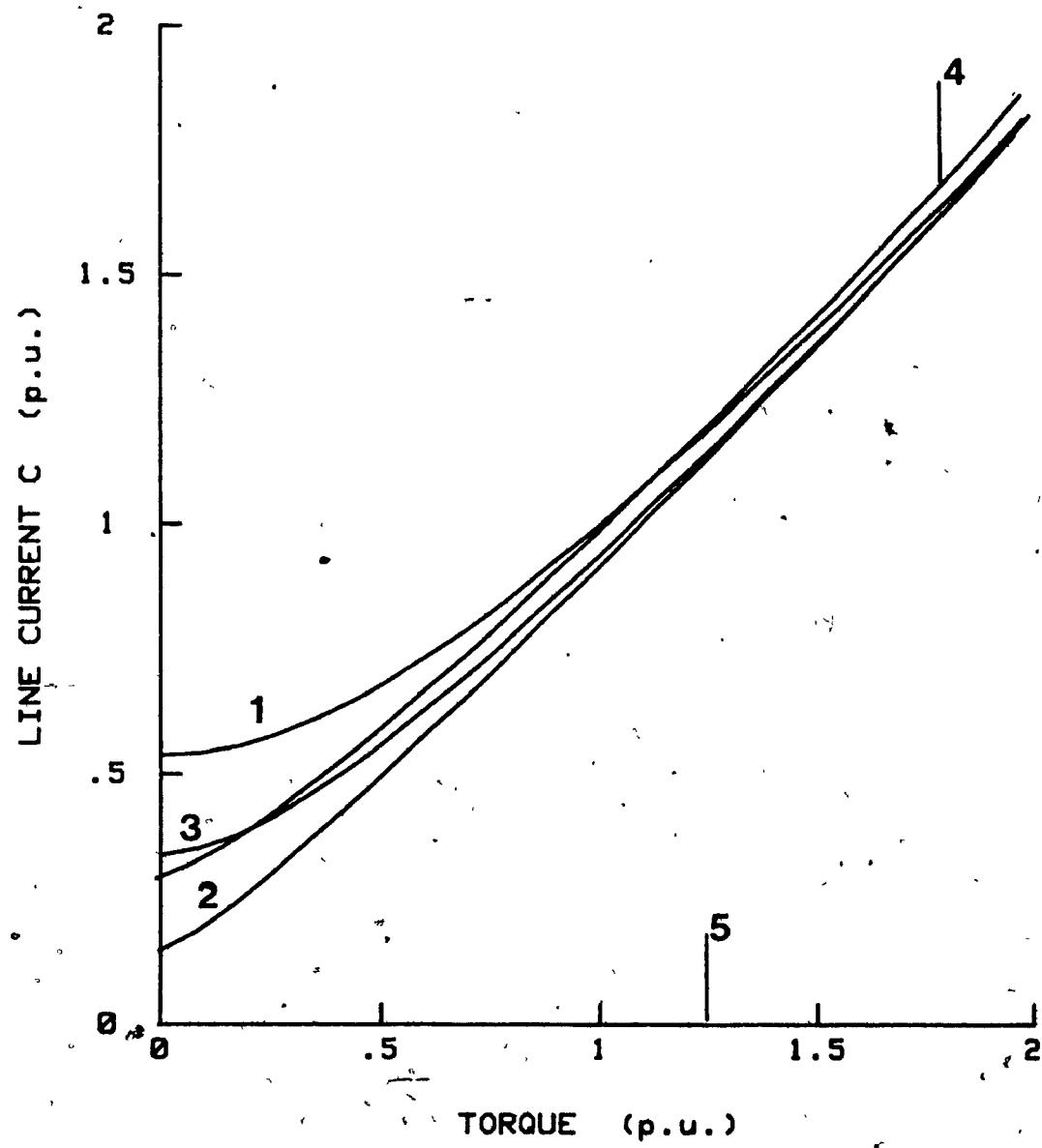


Fig. 4.13. I_c versus average torque for line-to-line load unbalance.

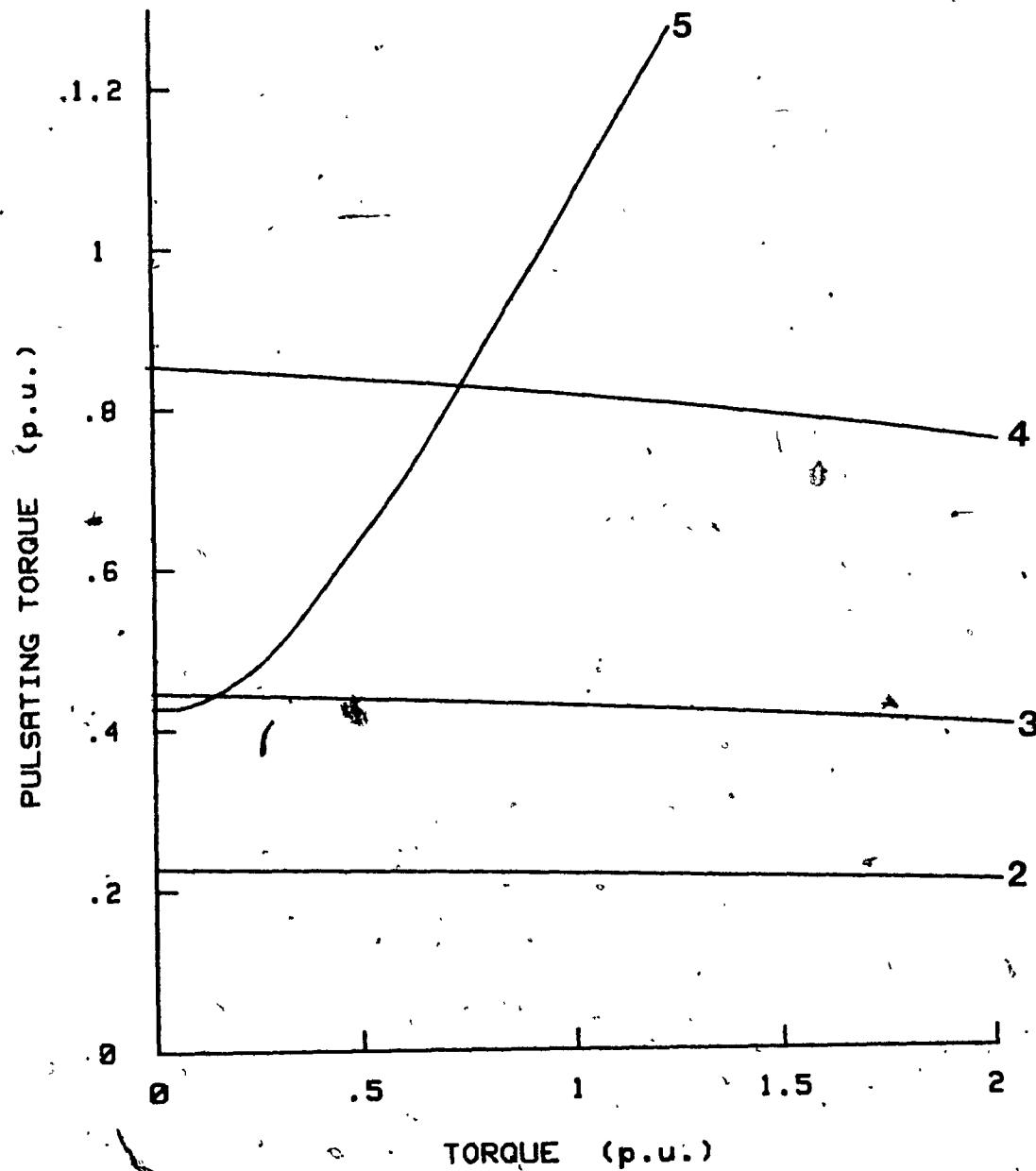


Fig. 4.14. Pulsating torque versus average torque for line-to-line load unbalance.

4.3 Transient case

The transient performance of the induction motor is presented for the following cases:

- (I) Balanced direct on-line start of motor.
- (II) Unbalanced direct on-line start of motor.
- (III) The supply becomes unbalanced at slip of 0.03 and switching angle of 0^0 .
- (IV) The supply becomes unbalanced at slip of 0.03 and switching angle of 90^0 .
- (V) Balanced is restored.

The above mentioned situations are presented in Figures 4.15-4.19.

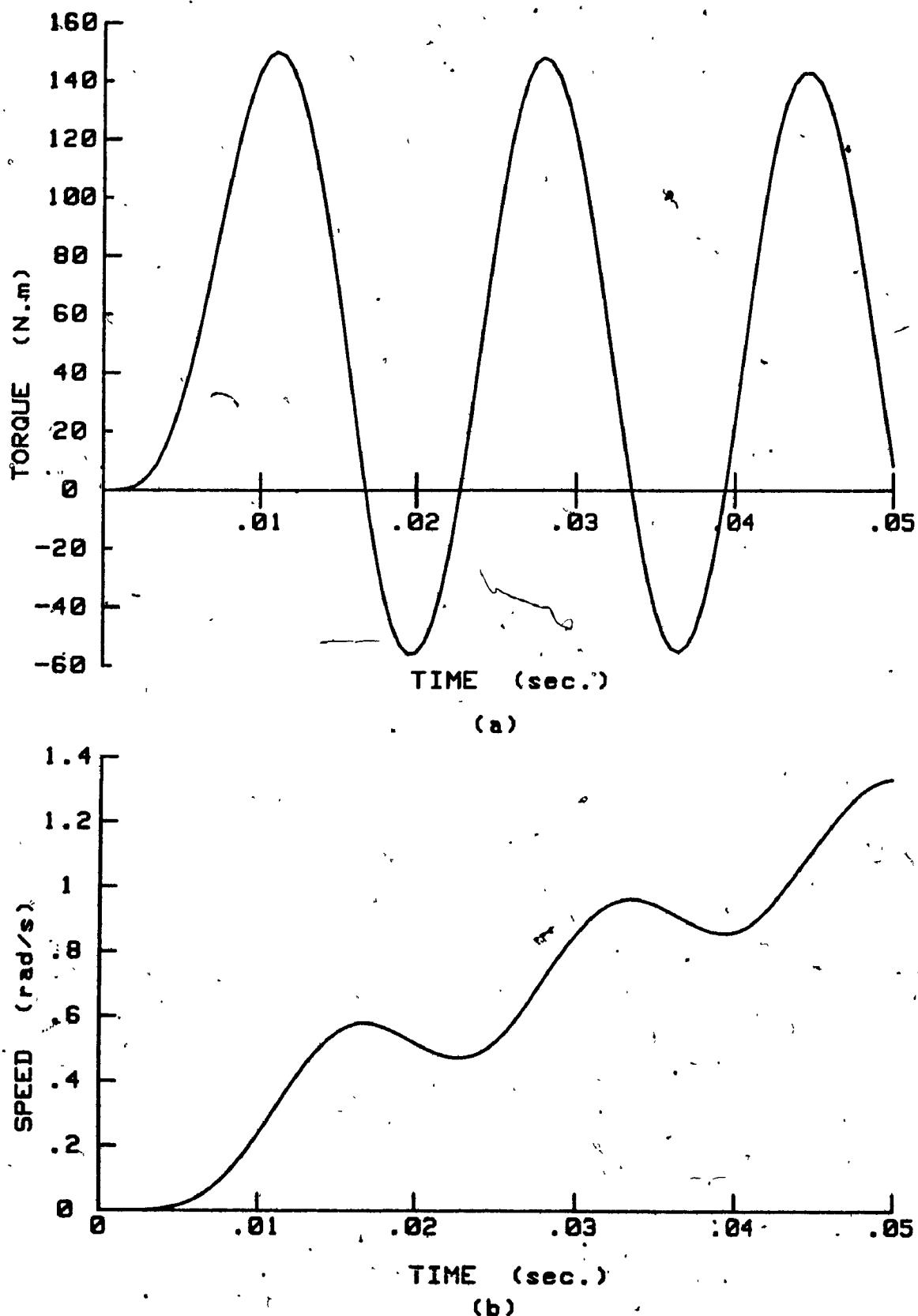


Fig. 4.15. Starting torque and speed characteristics for a balanced supply.

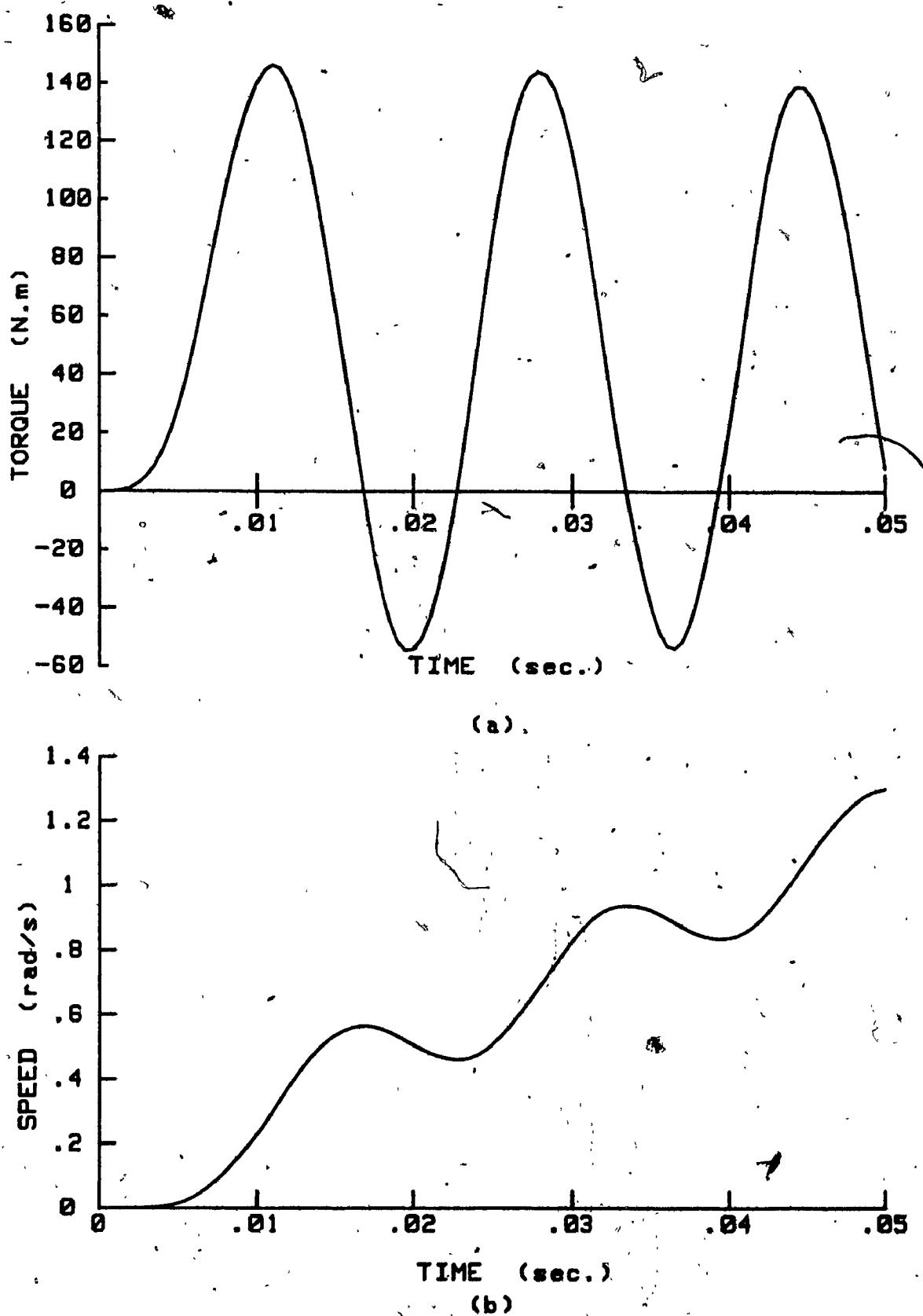
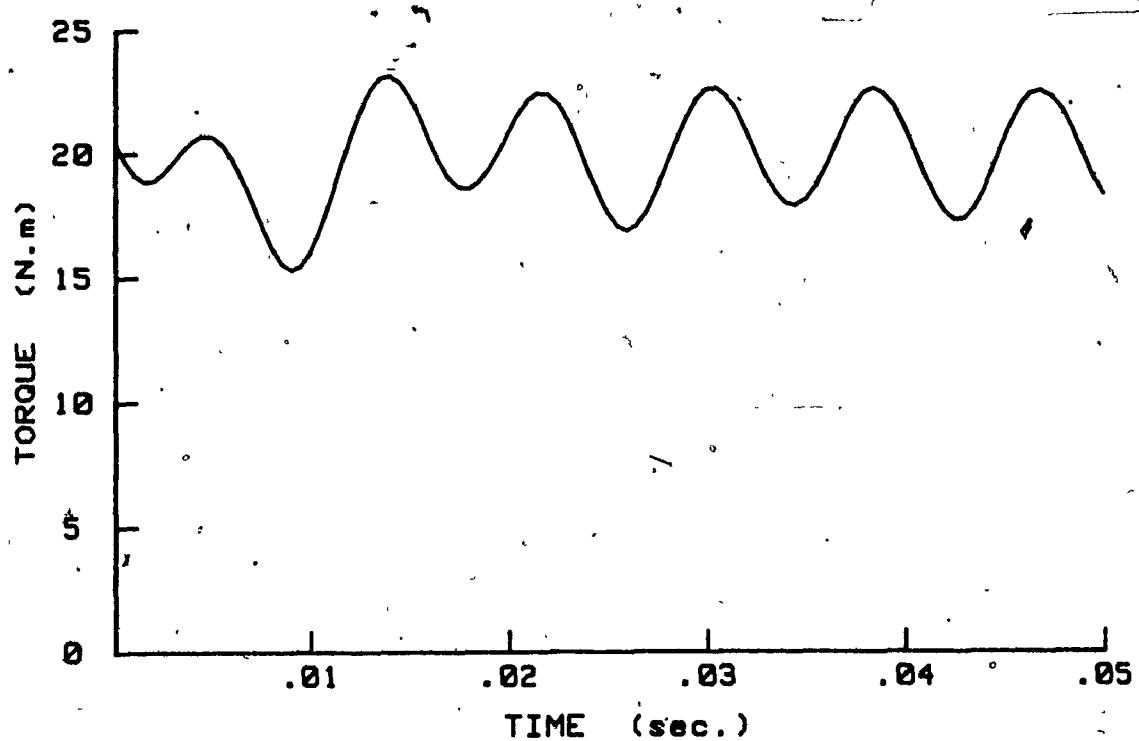
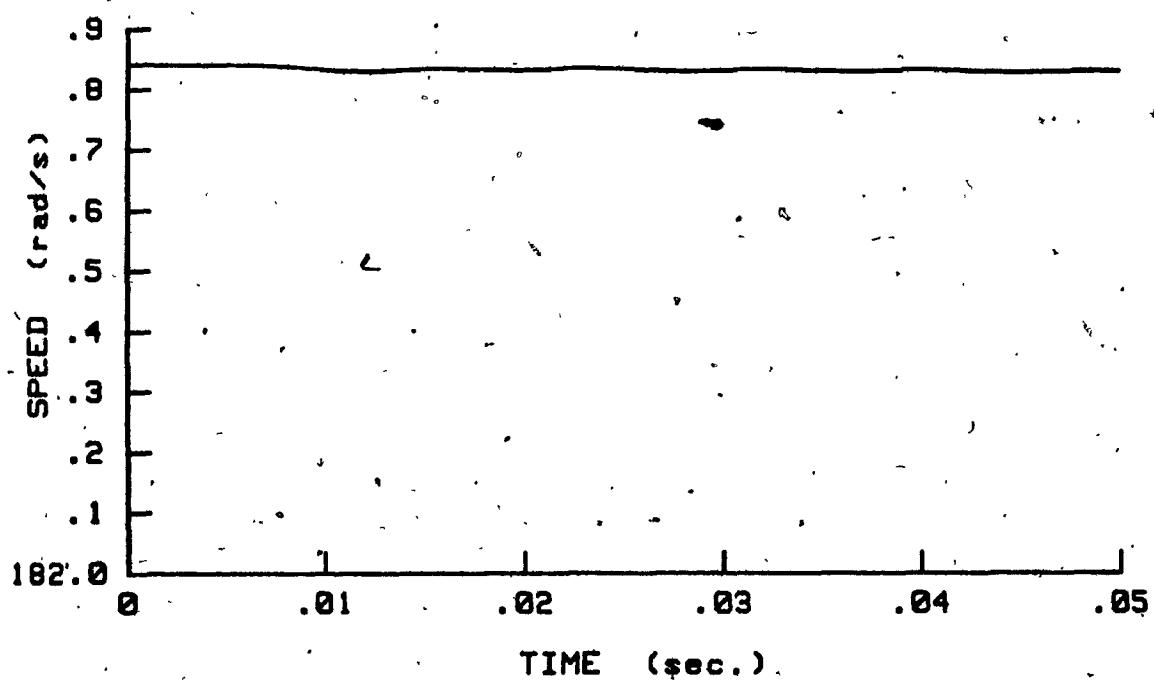


Fig. 4.16. Starting torque and speed characteristics for unbalanced supply.

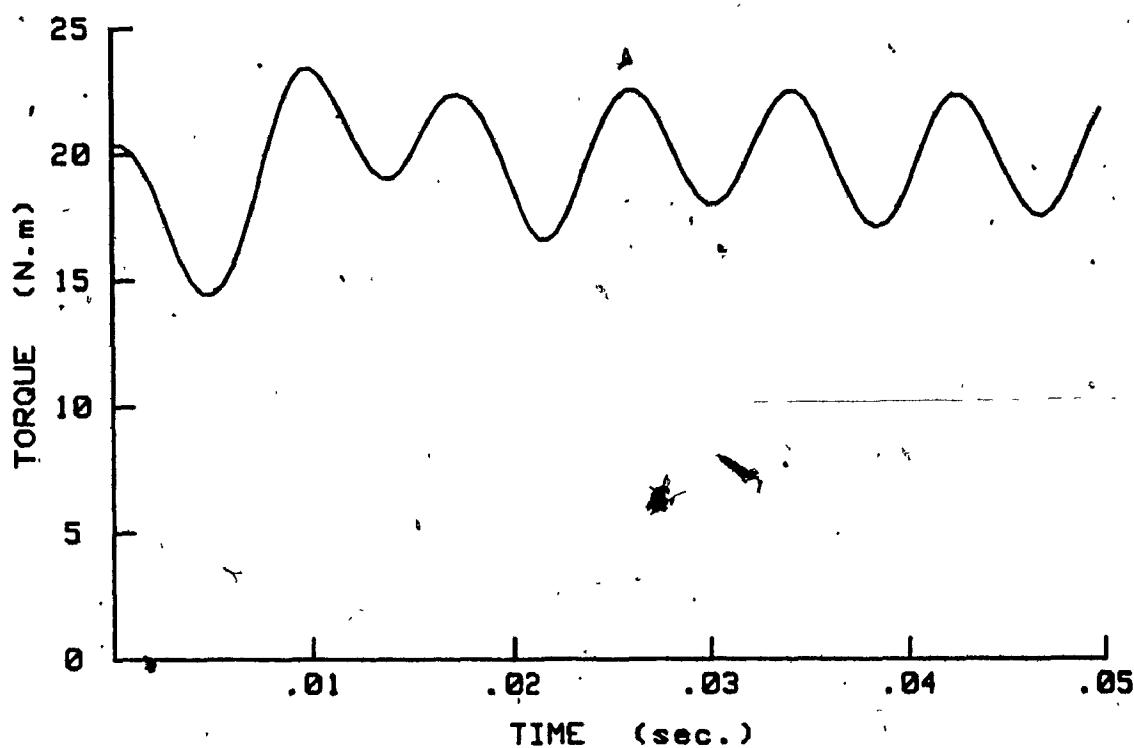


(a)

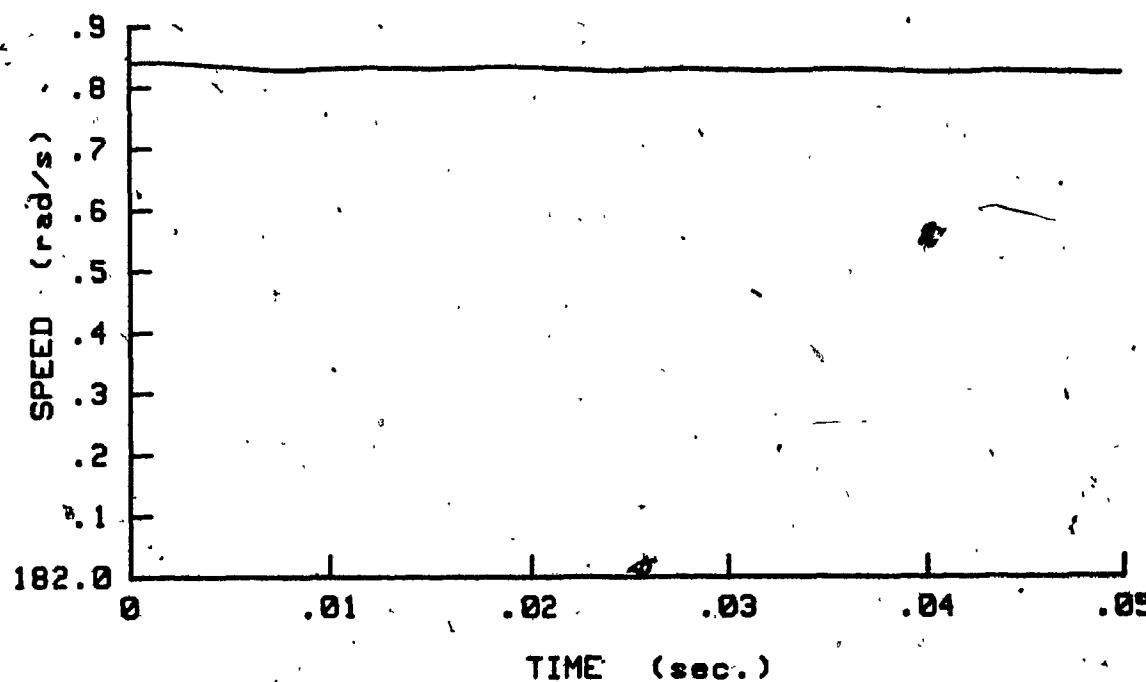


(b)

Fig. 4.17. Transient characteristics when the supply gets unbalanced
at a slip of .03 and switching angle of 0° .

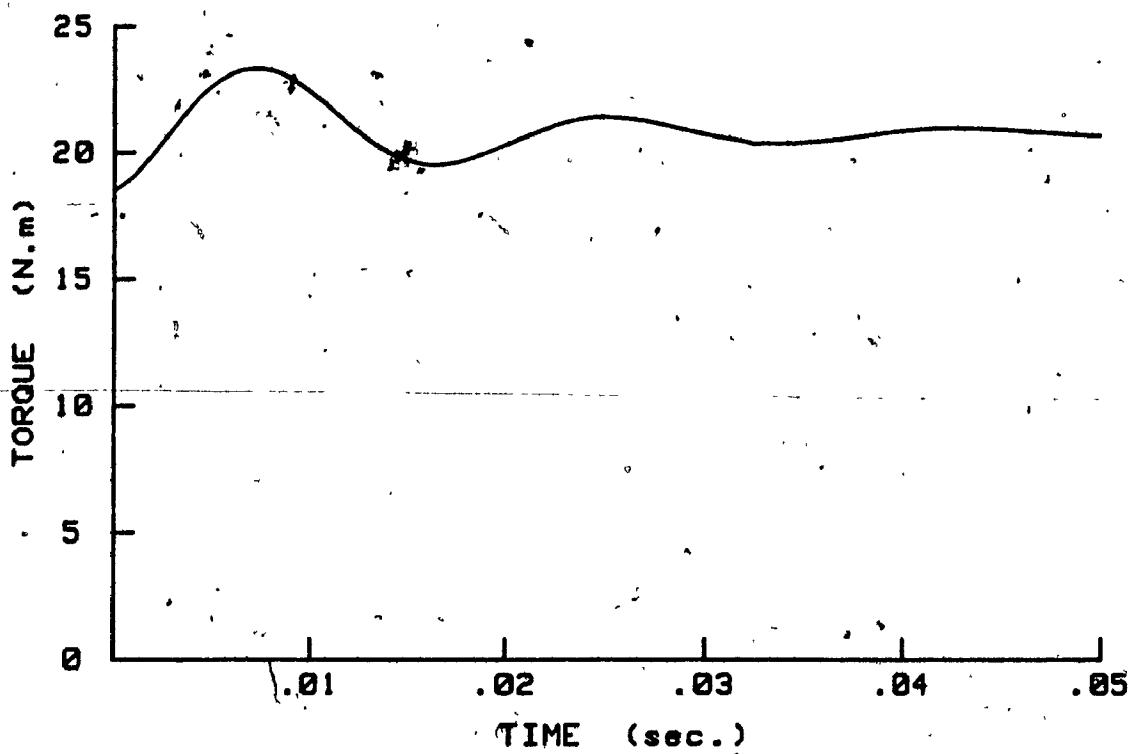


(a)



(b)

Fig. 4.18. Transient characteristics when the supply gets unbalanced
at a slip of .03 and switching angle of 90° ,



(a)

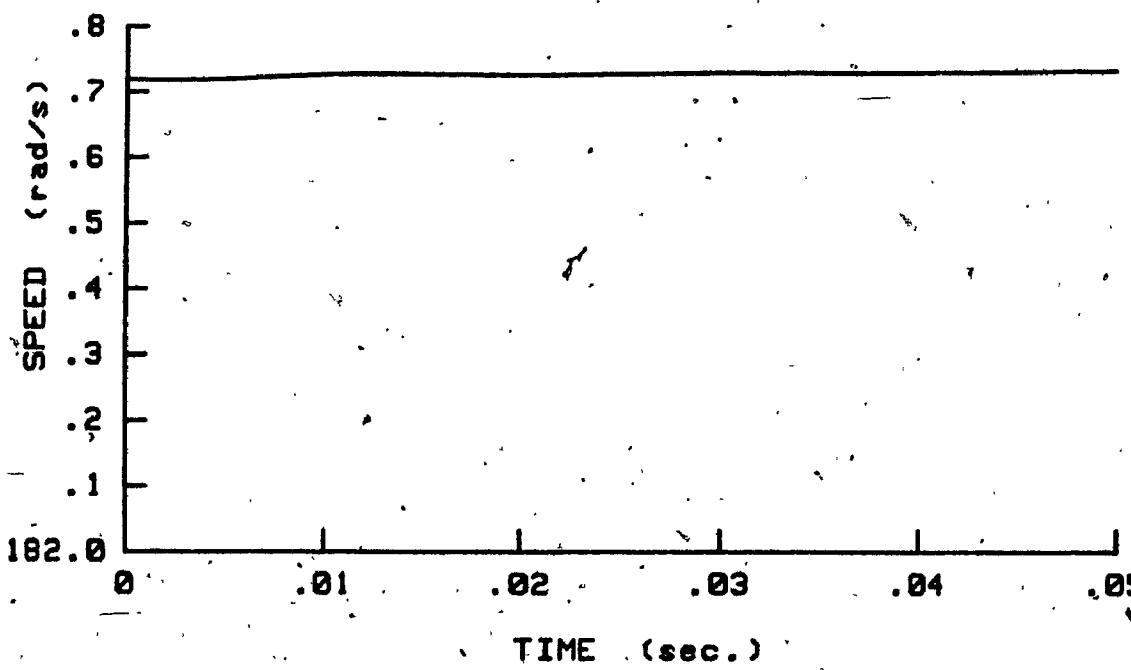


Fig. 4.19. Transient analysis when balance is restored.

CHAPTER 5

CONCLUSIONS

The discussion and results of the previous chapters lead to the following points:-

1. The dynamic circuit model gives a convenient method for detailed calculations of currents, average torque, pulsating torque and motor losses.
2. The phase-angle of the negative sequence voltage depends on the way the single-phase load is connected, as well as the phase to which it is connected. Hence, there is no simpler procedure to calculate the currents based on the magnitude of the negative sequence current only.
3. The superposition of the positive and negative sequence currents in the simplified model is not feasible.
4. The initial unbalanced transient behaviour of the motor for a direct line start has a slight difference from the balanced one. The amplitude of the oscillating torque is reduced by a small amount because of the negative developed torque.
5. The inertia of the motor keeps the transient speed fairly similar to that with a balanced source.
6. The changeover from balanced to unbalanced conditions and vice versa under standard operating conditions presents a very stable performance of the motor and is not likely to cause any problems.

7. The pulsating torque is virtually independent of the load on the motor and the amplitude of this torque approaches half rated torque when the negative sequence is 5%. There may be some applications where this would make a simple induction motor unsuitable.
8. The open-line curve shows dependence on motor load since the negative phase sequence on open line is dependent on motor load.
9. Some characteristics for seemingly small unbalance are surprisingly close to those for the open-line situation.

Nowadays, many applications of induction motors are being employed with power converting equipment. Due to the non-linearity of the switching devices used in converters/inverters, harmonics are produced even when the supply is balanced. Therefore, with an unbalanced three-phase source, the generation of such harmonics will be greatly increased. Further studies that are being conducted presently, will investigate and analyse the harmonics generated for a three-phase, six-pulse, AC-DC converter under unbalanced supply conditions, where both the magnitude of the supply voltage and the phase angle are unbalanced.

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APPENDIX A

STATOR VOLTAGES

| Line-to-Neutral Load Unbalance | | | |
|--------------------------------|-----------------------------------|-------------|-------------------|
| Condition | Phase Voltage (Magnitude / Angle) | | |
| | V_{ab} | V_{bc} | V_{ca} |
| Balanced | 120 / 30 ° | 120 / 270 ° | 120 / 150 ° |
| 1% unbalance | 117.04 / 29.93 ° | 120 / 270 ° | 118.67 / 148.73 ° |
| 5% unbalance | 109.94 / 29.76 ° | 120 / 270 ° | 115.71 / 145.57 ° |
| 10% unbalance | 100.78 / 29.51 ° | 120 / 270 ° | 112.44 / 141.26 ° |

Table A.1. Stator voltages for line-to-neutral load unbalance.

| Line-to-Line Load Unbalance | | | |
|-----------------------------|-----------------------------------|-------------------|-------------------|
| Condition | Phase Voltage (Magnitude / Angle) | | |
| | V_{ab} | V_{bc} | V_{ca} |
| balanced | 120 / 30 ° | 120 / 270 ° | 120 / 150 ° |
| 1% unbalance | 120.17 / 28.59 ° | 115.05 / 268.41 ° | 117.38 / 150.86 ° |
| 5% unbalance | 120.41 / 27.18 ° | 110.21 / 266.68 ° | 114.78 / 151.36 ° |
| 10% unbalance | 121.11 / 24.40 ° | 109.85 / 262.74 ° | 109.63 / 152.85 ° |

Table A.2. Stator voltages for line-to-line load unbalance.

APPENDIX B**MOTOR PARAMETERS:**

Stator resistance, $R_s = 0.2 \Omega$

Stator reactance, $X_s = 0.55 \Omega$

Rotor resistance, $R_r = 0.3 \Omega$

Rotor reactance, $X_r = 0.55 \Omega$

Magnetizing reactance, $X_m = 16.0 \Omega$

APPENDIX C

COMPUTER PROGRAM

C THIS PROGRAM DETERMINES THE VALUES OF CURRENT,
C TORQUE, AND COPPER LOSSES FOR ANY SET
C OF VOLTAGES APPLIED TO AN INDUCTION MOTOR
C FOR SPEEDS RANGING FROM 1620-1800 RPM.

C PROGRAM CAL(INPUT,OUTPUT)
C LEGEND OF VARIABLES TO BE USED

C RS = STATOR RESISTANCE --
C RR = ROTOR RESISTANCE
C XM2 = MAGNETIZING REACTANCE
C XS2 = STATOR LEAKAGE REACTANCE
C XR2 = ROTOR LEAKAGE REACTANCE
C XM3 = MUTUAL REACTANCE BETWEEN STATOR PHASES
C XS3 = STATOR SELF-REACTANCE
C XRT = ROTOR SELF-REACTANCE
C XMSR = MUTUAL REACTANCE BETWEEN STATOR-ROTOR
C CASE = CHARACTER THAT SPECIFIES THE CONDITION
C WHETHER THE MOTOR IS OPERATING UNDER
C NORMAL OR OPEN-LINE CASES.

COMPLEX A(5,5),A3(5,5),B(5),B1(5),C(5),CC(5),CL(3),SUM
COMPLEX AO(4,4),A4(4,4),BO(4),B2(4),CO(4),CCO(4)
REAL ANGB(5),ANGC(5),ANGCC(5),WA(5),RESIS(5,5),REAC(5,5),G(5,5)
REAL RESIO(4,4),REA0(4,4),GO(4,4),XY(5,4)
INTEGER PP,CA(5,4),CT(4,5)
CHARACTER CASE*2

PI=ACOS(-1.)
READ*,PP
C PP= POLE PAIRS

CALL MATSET(RESIS,REAC,G,5)
* SET UP WORKING MATRICES *

3 READ*,CASE
IF(CASE.EQ.'C') THEN
* NORMAL OPERATION *

READ*,(B(I),I=1,5)
* READ IN STATOR VOLTAGES *

PRINT1
FORMAT('1')
CALL COND(B,5)
C * CHECK IF SUPPLY BALANCED OR NOT *

CALL ANGLE(B,ANGB,5,PI,1)
* FIND PHASE ANGLES OF VOLTAGES *

PRINT99
99 FORMAT(/' SPEED',5X,'CUR(AB)',5X,'CUR(BC)',5X,'CUR(CA)',5X,
*'CUR(A)',6X,'CUR(B)',6X,'CUR(C)',6X,'TORQUE',6X,'PULSE TOR')
C * SET UP OUTPUT TABLE *

D055,SP=1620,1800,4,5
C * RUN FOR SPEEDS FROM 1620 TO 1800 RPM *

W=2.*SP*PI/60.
C * CHANGE FROM RPM TO RAD/SEC *

```

      CALL ASET(RESIS,REAC,G,A,W,5,PP)
C   * FORM IMPEDANCE MATRIX *
      CALL SOLUA(A,A3,B,B1,C,WA,5,SUM)
C   * DETERMINE THE PHASE CURRENTS *
      CALL ANGLE(C,ANGC,5,PI,0)
C   * FIND PHASE ANGLE OF CURRENTS *
      CALL LINEC(C,CL,5)
C   * GET LINE CURRENTS *
      CALL CCONJ(C,CC,5)
C   * DETERMINE CONJUGATE OF CURRENTS *
      CALL ANGLE(CC,ANGCC,5,PI,0)
C   * FIND ANGLE OF CONJUGATE *
      CALL TORQ(G,C,CC,PP,TRQ,5)
C   * GET AVERAGE TORQUE *
      CALL PTRQ(C,G,ANGC,PP,PULT,5)
C   * GET PULSATING TORQUE *
      CALL LOSS(C,RESIS,COPLOS,5)
C   * GET COPPER LOSSES *
      PRINT115,SP,(ABS(C(L)),L=1,3),(ABS(CL(L)),L=1,3),TRQ,PULT
115 FORMAT(2X,F6.1,4X,3(F7.4,5X),3(F7.4,5X),F7.4,5X,F7.4,)
55  CONTINUE
      GO TO 3
      ELSE IF(CASE.EQ.'0') THEN
      * OPEN-LINE CONDITION *

      READ*,(B(L),L=1,5)

C   * READ IN THE STATOR VOLTAGES AND
C   * FORM THE RESISTANCE, REACTANCE AND ROTATIONAL
C   * COEFFICIENT MATRICES FOR THE OPEN-LINE CASE.

      PRINT116
116  FORMAT('1'/4X,'OPEN LINE CASE'/4X,1('-')/6X,'RESISTANCE MATRIX'
      /)
      CALL CTZC(RESIS,CA,CT,RESIO,5,4,XY)
      PRINT117
117  FORMAT(/6X,'REACTANCE MATRIX')
      CALL CTZC(REAC,CA,CT,REAO,5,4,XY)
      PRINT118
118  FORMAT(/6X,'G MATRIX')
      CALL CTZC(G,CA,CT,GO,5,4,XY)

C   * TRANSFORM THE INPUT VOLTAGES FOR OPEN-LINE CASE.

      D072,I=1,4
      SUMM=0.
      D070,J=1,5
      SUMM=SUMM+CT(I,J)*B(J)
70   CONTINUE
      B0(I)=SUMM
72   CONTINUE
      PRINT99
      D075,SP=1620,1800,4.5
      SP=1620
      W=2.*SP*PI/60.
      CALL ASET(RESIO,REAO,GO,AO,W,4,PP)
      CALL SOLUA(AO,A4,B0,B2,CO,WA,4,SUM)
      CALL PHCUR(CO,C,CA,4,5)
      CALL LINEC(C,CL,5)

      CALL CCONJ(C,CC,5)
      CALL ANGLE(C,ANGC,5,PI,0)
      CALL ANGLE(CC,ANGCC,5,PI,0)
      CALL TORQ(G,C,CC,PP,TRQ,5)
      CALL PTRQ(C,G,ANGC,PP,PULT,5)
      CALL LOSS(C,RESIS,COPLOS,5)
      PRINT115,SP,(ABS(C(L)),L=1,3),(ABS(CL(L)),L=1,3),TRQ,PULT
75   CONTINUE
      ENDIF

999 STOP
END

```

SUBROUTINE MATSET (RESIS, REAC, G, K)

```

C * THIS SUBROUTINE USES THE D-Q PARAMETERS OF THE INDUCTION MOTOR
C * AND TRANSFORMS THEM TO 3-PHASE STATOR, 2-PHASE ROTOR VALUES.
C * THESE VALUES ARE THEN USED TO FORM THE RESISTANCE, REACTANCE AND
C * G MATRICES.

REAL RESIS(K,K),REAC(K,K),G(K,K)
T=2./3.
PI= ACOS(-1.)
TH1=PI*T
TH2=2.*TH1
READ*,RS,RR,XS2,XR2,XM2
PRINT100,RS,RR,XS2,XR2,XM2
100 FORMAT('1',/, ' STATOR RESISTANCE',7X,F4.2,/,'
     '' ROTOR RESISTANCE',BX,F4.2,/, ' STATOR REACTANCE',BX,F4.2,/,'
     '' ROTOR REACTANCE',9X,F4.2,/, ' MAGNETIZING REACTANCE',2X,
     *F5.2,/)
XM3 = XM2*T
XS3 = XS2+(XM2*T)
XRT = XR2+XM2
XMSR= XM2*SQRT(T)

D07,I=1,3
D05,J=1,5
IF(I.EQ.J) THEN
  RESIS(I,J)=RS
  REAC (I,J)=XS3
  G    (I,J)=0.
ELSE
  IF(J.LE.3)THEN
    RESIS(I,J)=0.
    REAC (I,J)=XM3*COS(TH1)
    G    (I,J)=0.
  ELSE
    RESIS(I,J)=0.
    G    (I,J)=0.
  ENDIF
ENDIF
5 CONTINUE
7 CONTINUE

.D09,I=4,5
D08J=4,5
IF(I.EQ.J) THEN
  RESIS(I,J)=RR
  REAC (I,J)=XRT
  G    (I,J)=0.
ELSE
  RESIS(I,J)=0.
  REAC (I,J)=0.
  G    (I,J)=XRT/(120*PI)
ENDIF
8 CONTINUE

9 CONTINUE
G(5,4)=-(G(4,5))
REAC(1,4)=XMSR
REAC(4,1)=XMSR
REAC(1,5)=0.
REAC(5,1)=0.
REAC(2,4)=XMSR*COS(TH1)
REAC(4,2)=REAC(2,4)
REAC(3,4)=XMSR*COS(TH2)
REAC(4,3)=REAC(3,4)
REAC(2,5)=XMSR*SIN(TH1)
REAC(5,2)=REAC(2,5)
REAC(3,5)=XMSR*SIN(TH2)
REAC(5,3)=REAC(3,5)
G(4,1)=0.
G(4,2)=XMSR*SIN(TH1)/(120*PI)
G(4,3)=XMSR*SIN(TH2)/(120*PI)
G(5,1)=-(XMSR/(120*PI))
G(5,2)=-(XMSR*COS(TH1)/(120*PI))
G(5,3)=-(XMSR*COS(TH2)/(120*PI))

```

```

PRINT105
105 FORMAT(6X,'RESISTANCE MATRIX')
PRINT110,((RESIS(I,J),J=1,K),I=1,K)
110 FORMAT(5(4X,F8.4))
PRINT106
106 FORMAT(6X,'REACTANCE MATRIX')
PRINT110,((REAC(I,J),J=1,K),I=1,K)
PRINT107
107 FORMAT(6X,'G MATRIX')
PRINT110,((G(I,J),J=1,K),I=1,K)
RETURN
END

```

SUBROUTINE COND(B,K)

C * THIS SUBROUTINE FINDS OUT IF THE SUPPLY VOLTAGE IS BALANCED OR
C UNBALANCED AND PRINTS VOLTAGE UNBALANCE FACTORS (BOTH NEMA AND VUF).

COMPLEX B(K),H1,H2,VP,VN

```

H=SQRT(3.)
H1=CMPLX(-.5,H/2.)
H2=CMPLX(-.5,-H/2.)
D=.001
D1=ABS(B(1))-ABS(B(2))
D2=ABS(B(2))-ABS(B(3))
D3=ABS(B(1))-ABS(B(3))
IF((D1.GE.D).OR.(D2.GE.D).OR.(D3.GE.D))THEN
PRINT125
125 FORMAT(56X,'UNBALANCED SUPPLY',/,55X,19('''))
VP=(B(1)+(H1*B(2))+(H2*B(3)))/H
VN=(B(1)+(H2*B(2))+(H1*B(3)))/H
AVG=(ABS(B(1))+ABS(B(2))+ABS(B(3)))/3.
AMX=0.
DO95,I=1,3
    AX=AVG-ABS(B(I))
    IF(AX.GT.AMX) AMX=AX
95 CONTINUE
PRINT130,ABS(VN/VP),AMX/AVG
130 FORMAT(10X,'VOLTAGE UNBALANCE FACTOR = ',20X,F5.3,/,10X,
'NEMA UNBALANCE FACTOR = ',23X,F5.3//)
ELSE
PRINT126
126 FORMAT(58X,'BALANCED SUPPLY'/57X,17('''))
ENDIF
RETURN
END

```

SUBROUTINE ASET(RESIS,REAC,G,A,W,K,PP)

C * FORM THE IMPEDANCE (Z) MATRIX.

COMPLEX A(K,K)
REAL RESIS(K,K),REAC(K,K),G(K,K)
INTEGER PP
C PP=POLE PAIRS

```

DO20,I=1,5
DO15,J=1,5
    A1=RESIS(I,J)+(G(I,J)*W*PP)
    A2=REAC(I,J)
    A(I,J)=CMPLX(A1,A2)
15 CONTINUE
20 CONTINUE
RETURN
END

```

SUBROUTINE SOLUA(A,A3,B,B1,C,WA,K,SUM).

C * SOLVES FOR THE CURRENT IN THE STATOR AND ROTOR.

COMPLEX A(K,K),B(K),A3(K,K),B1(K),C(K),SUM
REAL WA(K)

PI=ACOS(-1.)

N=K

IA=K

M=1

IB=K

IJOB=0

DO25,K1=1,K

B1(K1)=B(K1)

DO23,K2=1,K

A3(K1,K2)=A(K1,K2)

23 CONTINUE

25 CONTINUE

CALL LEQT1C(A,N,IA,B,M,IB,IJOB,WA,IER)

C * THIS SUBROUTINE IS FROM IMSL. THE RESULT IS THE
C * SOLUTION OF A COMPLEX MATRIX IN VECTOR B.
C * VECTOR B IS CHANGED TO C AND THE ORIGINAL
C * CONTENTS RESTORED, I.E., STATOR VOLTAGES.

SUM=CMPLX(0.,0.)

DO30,K1=1,K

C(K1)=B(K1)

B(K1)=B1(K1)

SUM=SUM+(A3(1,K1)*C(K1))

DO27,K2=1,K

A(K1,K2)=A3(K1,K2)

27 CONTINUE

30 CONTINUE

RETURN

END

SUBROUTINE ANGLE(B,ANG,K,PI,N)

C * FIND PHASE ANGLE OF CURRENT OR VOLTAGE.

COMPLEX B(K)

REAL ANG(K)

RTD=180./PI

DO66,I=1,K

X1=REAL(B(I))

X2=AIMAG(B(I))

IF((X1.GT.0.).AND.(X2.GT.0.))THEN

ANG(I)=ATAN(X2/X1)*RTD

ELSE IF((X1.GT.0.).AND.(X2.LT.0.))THEN

ANG(I)=360-(ABS(ATAN(X2/X1))*RTD)

ELSE IF((X1.LT.0.).AND.(X2.LT.0.))THEN

ANG(I)=180+(ATAN(X2/X1)*RTD)

ELSE IF((X1.LT.0.).AND.(X2.GT.0.))THEN

ANG(I)=180-(ABS(ATAN(X2/X1))*RTD)

ELSE IF((X1.EQ.0.).AND.(X2.GT.0.))THEN

ANG(I)= .90.

ELSE IF((X1.EQ.0.).AND.(X2.LT.0.))THEN

ANG(I)= 270.

ELSE IF((X1.GT.0.).AND.(X2.EQ.0.))THEN

ANG(I)= 0.

ELSE IF((X1.LT.0.).AND.(X2.EQ.0.))THEN

ANG(I)= 180.

ENDIF

IF(N.EQ.1) PRINT135,B(I),ABS(B(I)),ANG(I)

135 FORMAT(5X,F10.5,' +J ',F10.5,10X,'OR',BX,F9.5,' AT ',F9.5,
' DEGREES')

66 CONTINUE

RETURN

END

SUBROUTINE CTZC(X,CA,CT,Y,K,NK,XV)

C * HERE THE [C]TRANS.*[Z]*[C] IS OBTAINED FOR THE OPEN-LINE CASE.

```

REAL X(K,K),Y(NK,NK),XY(K,NK)
INTEGER CA(K,NK),CT(NK,K)
DO48,I=1,K
  DO47,J=1,NK
    CA(I,J)=0
    CT(J,I)=0
47  CONTINUE
48  CONTINUE
  CA(1,1)=1
  CA(2,2)=-1
  CA(3,2)=1
  CA(4,3)=1
  CA(5,4)=1
  CT(1,1)=1
  CT(2,2)=-1
  CT(2,3)=1
  CT(3,4)=1
  CT(4,5)=1
DO54,I=1,K
  DO52,J=1,NK
    XX=0.
    DO50,L=1,K
      XX=XX+X(I,L)*CA(L,J)
50  CONTINUE
  XY(I,J)=XX
52  CONTINUE
54  CONTINUE
DO64,I=1,NK
  DO62,J=1,NK
    XX=0.
    DO60,L=1,K
      XX=XX+CT(I,L)*XY(L,J)
60  CONTINUE
  Y(I,J)=XX
62  CONTINUE
64  CONTINUE
PRINT120,((Y(II,JJ),JJ=1,NK),II=1,NK)
120 FORMAT(4(4X,F8.4))
RETURN
END

```

SUBROUTINE TORQ(G,C,CC,PP,TRQ,K)

C * DETERMINES AVERAGE TORQUE USING [I] TRANSPOSE CONJUGATE TIMES THE
C * G-MATRIX TIMES [I].

```

COMPLEX C(K),CC(K),TR
REAL G(K,K)
INTEGER PP
TR=CMPXL(0.,0.)
DO37,I=1,K
  DO 35,J=1,K
    TR=TR+(G(I,J)*C(J)*CC(I))
35  CONTINUE
37  CONTINUE
  TRQ=TR*PP
RETURN
END

```

```

SUBROUTINE PTRQ(C,G,ANGC,PP,PULT,K)
C * PULSATIING TORQUE AS WELL AS AVERAGE TORQUE ARE DETERMINED.
IMPLICIT COMPLEX(Z)
COMPLEX C(K)
REAL G(K,K),ANGC(K)
INTEGER PP
PI=ACOS(-1.)
DTR=PI/180.
DO90,I=1,K
    ANGC(I)=ANGC(I)*DTR
90 CONTINUE
A1=G(4,2)*ABS(C(2))*ABS(C(4))
A2=G(4,3)*ABS(C(3))*ABS(C(4))
A3=G(5,1)*ABS(C(1))*ABS(C(5))
A4=G(5,2)*ABS(C(2))*ABS(C(5))
A5=G(5,3)*ABS(C(3))*ABS(C(5))
Z1=CMPLX(A1*COS(ANGC(2)+ANGC(4)),A1*SIN(ANGC(2)+ANGC(4)))
Z2=CMPLX(A2*COS(ANGC(3)+ANGC(4)),A2*SIN(ANGC(3)+ANGC(4)))
Z3=CMPLX(A3*COS(ANGC(1)+ANGC(5)),A3*SIN(ANGC(1)+ANGC(5)))
Z4=CMPLX(A4*COS(ANGC(2)+ANGC(5)),A4*SIN(ANGC(2)+ANGC(5)))
Z5=CMPLX(A5*COS(ANGC(3)+ANGC(5)),A5*SIN(ANGC(3)+ANGC(5)))
ZZ=-(Z1+Z2+Z3+Z4+Z5)
ZP=ZZ*PP
PULT=ABS(ZP)
C X1=A1*COS(ANGC(4)-ANGC(2))
C X2=A2*COS(ANGC(4)-ANGC(3))
C X3=A3*COS(ANGC(5)-ANGC(1))
C X4=A4*COS(ANGC(5)-ANGC(2))
C X5=A5*COS(ANGC(5)-ANGC(3))
C TORDC=(X1+X2+X3+X4+X5)*PP
RETURN
END

```

```

SUBROUTINE PHCUR(C,C1,CA,NK,K)
C * GET THE PHASE CURRENTS FOR THE OPEN-LINE SITUATION.
COMPLEX C(NK),C1(K),X
INTEGER CA(K,NK)
DO80,I=1,K
    X=0.
    DO78,J=1,NK
        X=X+CA(I,J)*C(J)
78 CONTINUE
    C1(I)=X
80 CONTINUE
RETURN
END

```

SUBROUTINE LINEC(C,CL,K)

C * GETS THE LINE CURRENTS FROM PHASE CURRENTS
 C * OBTAINED IN SUBROUTINE SOLUA.

```
COMPLEX C(1),CL(K)
CL(1)=C(1)-C(3)
CL(2)=C(2)-C(1)
CL(3)=C(3)-C(2)
RETURN
END
```

SUBROUTINE CCONJ(C,CC,K)

C * GETS THE CONJUGATE OF THE CURRENT.

```
COMPLEX C(K),CC(K)
DO32,I=1,K
  X1=REAL(C(I))
  X2=-AIMAG(C(I))
  CC(I)=CMPLX(X1,X2)
32 CONTINUE
RETURN
END
```

SUBROUTINE LOSS(C,RESIS,COPLOS,K)

C * OBTAIN POWER LOSSES.

```
COMPLEX C(K)
REAL RESIS(K,K)
COPLOS=0.
DO45,I=1,K
  Z=ABS(C(I))**2
  COPLOS=COPLOS+Z*RESIS(I,I)
45 CONTINUE
RETURN
END
```