cription of the grid being used, together with possible methods of calculation for the various parts of the grounding network, taking into account the existence of multi-layer grounding paths. The methods described are general and can be applied to other types of plants as well with suitable adaptation.

Steady state analysis is also extended to develop a new analytical method to determine the ground fault current distribution in grounding network of the plant forming a parallel path with the substation grounding grid and with the ladder type network constituted by tower ground, overhead ground wire and counterprise of the associated transmission complex, taking into consideration the various system parameters detailed in the first chapter.

Under transient analysis, apart from dealing with basic concepts involved in the study of impulse impedance of grounding grids, their equivalent circuit is formulated and analysed for their response to pratical impulse inputs, developing expressions for their impulse impedance so as to gauge not only the initial potential rise of a power plant, substation or tower under ground fault conditions, but also to study their impact in controlling insulator flashover potential. Inductance of grounding grids is shown to be the governing factor contributing towards their impulse impedance, and the necessary formulae to evaluate the inductance of a few typical grids are also developed.

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LIST OF ABBREVIATIONS AND SYMBOLS

a	Radius of a conductor, m
A	Area covered by a grid, m^2
C	Capacitance of a ground electrode to remote earth
C'	Distributed capacitance of counterpoise per meter, F
D	Dielectric flux density
E	Electric field strength
E_x	Electric field strength at a distance x
E'	dE/dt
F	Farad
g	Distributed conductance per meter, mho
G	Green Function
H	Henry

I_f	System fault current, ampere
I_k	Fault current through substation grid, ampere
I_n	Fault current through electrode n, ampere
J	Current density, ampere/m^2
J_x	Current density at distance x, ampere/m^2
k	Permittivity of earth, F/m
kcmil	Thousand circular mils
k_o	Permittivity of evacuated (free) space, F/m
k_r	Relative permittivity of earth to free space
ℓ	Length of electrode, meters
L	Inductance of an electrode, H
L'	Distributed inductance of counterpoise per meter, H
L_{ii}	Self inductance of electrode i, H
L_{ij}	Mutual inductance between electrodes i and j, H

n	Number of electrodes
p	Laplace operator
Q	Charge on a conductor
r	Radius of a circular plate occupying the same area as the grid (or a sphere or cylinder), m
r'	Metallic resistance of a wire per meter, Ω
R	Ground resistance of an electrode to remote earth, Ω
R'	Distributed ground resistance of counterpoise, ohm-meter
R_{ii}	Ground resistance of electrode i, Ω
R_{ij}	Mutual resistance between electrodes i and j, Ω
R_k	Ground resistance of a substation, Ω
R_t	Ground resistance of a tower, Ω

$R_{t(p.u.)}$	Ground resistance of a tower per unit length of the transmission line, Ω
t	Time, seconds
$2T$	Time for a wave to travel down a tower and back to the tower top after reflection at the junction of tower base ground grid, seconds
T_L	Inductive time constant of a grid
T_C	Capacitive time constant of a grid
V	Potential rise of an electrode or a grid, volts
V_k	Potential rise of a substation grid, volts
V_i	Potential rise of electrode i, volts
x_{ij}	Distance between two electrodes, m
y_c	Admittance of counterpoise per unit length, mho
z_c	Impedance of counterpoise per unit length, Ω
Z_c	Impedance of counterpoise, Ω

Z_g	Impedance of overhead ground wire between two towers, Ω
$Z_{g(p.u.)}$	Impedance of ground wire per unit length , Ω
Z_s	System impedance per phase upto the fault , Ω
$Z_{(\tau)}$	Impulse impedance of a grid , Ω
Z_∞	Network earth impedance looking back from the fault , Ω
μ	Coupling factor between the overhead phase and ground wire
ρ	Soil resistivity, Ω-m
ρ_i	Soil resistivity of layer n, Ω-m
σ	Conductivity of soil, $\mho$-m
γ	Propagation constant per unit length of transmission line
ω	Angular frequency, rad./second

CHAPTER I

INTRODUCTION

CHAPTER I

INTRODUCTION

1.1 General Background

Grounding has come to play a greater role in recent times on power system operation. As a result, there has been a growing need for further analysis of several aspects of grounding problems in order to keep pace with the growth and advances in power system operation.

The subject of grounding requires two predominant design criteria to be satisfied in design and analysis of grounding grids, viz:

i) to ensure safety of personnel and equipment.

ii) to ensure reliable operation of the relaying system.

To meet these requirements, it is necessary to attain a low value of the total resistance to ground of the overall power plant (e.g. in the order of $0.5\,\Omega$), as well as safe values of touch and step potentials.

There have been several papers [1-7] written on theoretical methods to determine steady state (i.e. at power frequency) grounding resistance of various electrode configurations, and necessity had been felt to develop a technique to deal with a specific application and related calculation for the design of a low resistance grounding system for a complete installation e.g. hydro-electric generation and transmission complex situated on highly resistive soils. Without this, it becomes difficult, at the design stage, to assess the efficiency or performance of the grounding network of such a vast complex. On the face of it, one gets an apparent impression that since such plants are surrounded by sufficient volumes of water having low resistivity, it should be fairly simple to realise a low resistance to remote earth. The basic fact that the ground-fault current must traverse down to earth (e.g. rock) after traversing through water is often set aside. On same basis, electrodes are buried in some nearby confined soft deposits (e.g. clay), if available, and ground resistance calculated accounting alone for low resistivity of top soft medium. The lower hard medium of higher resistivity, adds to the otherwise low resistance afforded by the top soft medium and thus changes the design completely. Only, if the upper medium is of vast dimensions, will the effect of rock below be minimal.

The first design criterion, viz., verification of touch and step potentials at various locations of these grids obviates a need to correctly evaluate the amount of ground-fault current traversing through the grounding network of the powerhouse, in relation to that diverted through the substation grid, towers, ground wire etc. The practice hitherto has been either to assume total ground fault current traversing in the substation grounding grid or assume an approximate value of about 60% or so of the total value, resulting in either a wasteful expenditure or unsafe design.

Above are the problems calling for an analysis under steady state domain. But, what happens under transient portion of the ground fault current or under discharge of impulse currents through the grids? This area too calls for a detailed investigation of the problems involved. There have been some theoretical investigations resulting in empirical approximations of test results and demand development of the basic relation to fundamental constants such as ρ, μ and k, needed for such an analysis, except dealt by Sunde [8], Bellaschi and al [9] for a linear electrode. The important requirement under transient state domain is to determine the impulse impedance of grounding grids of the usual shape and type (e.g. tower base type, substation grid, etc.) so as to accurately analyse problems concerning lightning protection and transients under ground fault conditions, impulse impedance being the ratio of peak value of the voltage developed at the feeding point of the grid to the peak value of the current under impulse inputs.

Ground behaves essentially as pure resistance [5,10] at commercial frequencies and with usual values of resistivities. Very high frequencies or steep wave fronts, with very high soil resistivities, tend to make charging current flowing in ground capacitance relatively more important. Thus, in addition to conduction currents, there will be displacement currents in case the electrode voltage changes with time and the sum of these currents flows through the self inductance of the grounding grid, resulting in equivalent circuit based on its ground resistance (R), capacitance (C) and the self inductance (L), requiring analysis so as to determine the impulse impedance for impulse inputs.

1.2 Scope of the Thesis

Based on the above considerations, the present work is divided into three phases as follows.

1.2.1 Analysis for Design of a Low Resistance Grounding System for a Hydro-electric Plant

Phase I gives a description of the initial design as well as the principle guide lines [11] employed in the analysis carried out for the grounding grid design of a hydro-electric plant by taking a real live plant, viz. La Grande - 2, the first of several plants being built by the Société d'Energie de la Baie James in the Northern part of the Province of Quebec, in Canada. The site is located on the Canadian Shield, and consists mainly of precambrian rock of very high resistivity (up to 35,000 Ω-m as measured), with scattered deposits of glacial and post-glacial over-burden materials. This value of resistivity is the highest possible expected for a plant site and is considered representative for a similar situation or problem. Since the powerhouse, surge chamber and switchyard are being built in or on the rock, it was not possible to obtain a low resistance to ground. Some useful elements, mainly the intake structure and the water basin, and a nearby deposit of low resistivity material of sufficient dimensions have been analysed and found effective in reducing the overall ground resistance of the plant. Some other elements, for instance the penstock lining in contact with the water, have also been analysed and are not always as effective grounding paths as they are often believed to be.

For the purpose of analysing different areas of the plant, several methods have been compiled and applied in this phase of the work to calculate the resistance of the above and other elements of the underground power plant . This proved valuable as a way to select the most efficient grounding paths at the design stage. This phase of the work also highlights the fundamental concepts behind the evaluation of grounding resistance of a ground grid installed in the forebay of a dam or in an isolated low resistivity medium e.g. clay or the like, duly developing an exact expression for evaluation of the resistance involved.

1.2.2 Ground-Fault Current Distribution

It is often difficult, at the design stage, to assess the magnitude of fault current flowing in the grounding grid of a substation or a plant. On the other hand, an assumption of the total ground-fault current flowing in such a grid requires use of extra buried copper which results in a considerable waste of expenditure.

There are several papers written on theoretical considerations for the calculations of ground resistance of a substation grid and counterpoise and for the coupling effect between the phase and ground wire. These call for a complete analysis to calculate the distribution of ground fault current duly taking into account all the system impedances, and other relevant parameters or effects. A recent paper by Dawalibi and Mukhedhar [12] has offered some useful contribution in this direction, excluding, however, the effect of coupling and the counterpoise and does not develop any specific formulae for evaluation. This phase of the work is a further step in pinpointing the various values of the network impedances or their evaluation procedure and develops an analytical method [13] to determine the ground fault current distribution in substation grid (connected to the grounding network of the power plant), towers and ground wire, taking into consideration all these parameters and effects.

1.2.3 Impulse Impedance of Grounding Grids

This phase of the work [14] outlines the basic concepts relating to the impulse impedance, taking into consideration all the parameters involved and establishes the following results for a grounding grid:

i) Relation between its power frequency ground resistance (R) and capacitance (C)

ii) Formulation of its equivalent circuit

iii) Expression for its impulse impedance as a function of time (following analysis for response of the equivalent circuit to impulse inputs) and reduction of the same to steady-state (power frequency grounding resistance).

iv) Effect of impulse impedance of the tower grid on insulator flashover potential

v) Effect of grid's self inductance (L) on the impulse impedance and its evaluation.

Only those values of current which do not cause breakdown of the soil are considered as they result in a higher value of impulse impedance and thus in a safe design. The equivalent circuit considered refers to the usual size and type of grids e.g. the tower base or the substation types i.e. the concentrated ones. The effect of leakage of impulse current as it flows along the grid is ignored in their basic analysis, and this effect accounted for subsequently in the final expression. This holds good for these usual types of grids, but in the analysis for a long buried wire i.e. counterpoise, on account of its length involved, it is essential to take leakage per unit length of the conductor into account right from the beginning and accordingly analyse the equivalent ladder type network for impulse inputs. An analysis [15] , duly covering this aspect is included as a separate section in the Chapter IV.

CHAPTER II

ANALYSIS FOR

DESIGN OF A LOW RESISTANCE GROUNDING SYSTEM

FOR A HYDRO-ELECTRIC PLANT

LOCATED ON HIGHLY RESISTIVE SOILS

CHAPTER II

ANALYSIS FOR

DESIGN OF A LOW RESISTANCE GROUNDING SYSTEM

FOR A HYDRO-ELECTRIC PLANT

LOCATED ON HIGHLY RESISTIVE SOILS

2.1 General

Hydro-electric plants are generally situated on rocky terrains having high resistivity, and thus it is often difficult to achieve a low resistance path to remote earth. This chapter affords a complete analysis as how to achieve the desired low resistance in order to reduce the value of the plant potential rise above remote earth. Since such an analysis would require assumption of a power plant layout with various dimensions, the most appropriate way would be to take a specific real live problem and then peform the analysis. Such an analysis, in this chapter, is based on the LG-2 hydro-electric plant of the Société d'Energie de la Baie James in the Northern part of the Province of Quebec, in Canada. The author himself was responsible for its design. This leads to establishing the main guidelines and a desin procedure [11] as outlined in this chapter.

The project LG-2, referred to above, includes essentially a 16 unit, underground, 5328 MW powerhouse, a 735 kV switchyard installed above the powerhouse, an intake structure, 16 underground penstocks and an underground surge chamber. The site is located on the Canadian Shield, and consists mainly of outcropping precambrian rock of very high resistivity (up to 35000 Ω-m as measured), with scattered deposits of glacial and post-glacial over-burden materials. Among these, an extensive clay zone (resistivity 40 to 70Ω-m) is situated nearby. Water resistivity in the area is also very high (up to 500 Ω-m as measured).

2.2 Description of the Grounding Grids

2.2.1 Description

As pointed out in section 1.2.1, since the powerhouse, surge chamber and swithyard are built in or on the rock, it was not possible to obtain a resistance to ground less than an order of a multiple of ten ohms for any of these areas by using buried copper conductor. Even the fact that the powerhouse and surge chamber are in contact with water was of little help due to the high resistivity and relatively small volumes of water involved. For the same reason, reinforcing steel used in the floor areas is of no help to reduce the grounding resistance although it does contribute to reduce the step and touch potentials to safe levels.

The intake structure, however, is in direct contact with the large volume of water of the forebay. Its resistance to ground could, therefore, be brought to a value closer to what we were aiming for.

Consequently, the grounding grid was designed to take advantage of this situation, by interconnecting, through heavy copper conductors, the powerhouse, surge chamber, switchyard and intake structure. It would then appear as several electrodes to ground connected in parallel, the overall resistance to ground being the equivalent resistance of this network.

Advantage could also be taken of the nearby deposit of low resistivity clay, where ground rods could be buried and connected to the above grounding grid to further decrease the overall ground resistance, should this become necessary after the resistance measurements.

An isometric view (Fig. 2.1) illustrates these various grounding grids. All metallic enclosures are connected to the relevant grid through conductors of a size depending on the location of the associated equipment.

2.2.2 Calculated Values

Table 2.1 gives a summary of the ground resistance for each area. The calculations are detailed in subsequent sections. For maximum ground-fault current of 16000 amperes, the value of 0.5 ohm gives a maximum voltage rise of the mesh of 8000 volts which was considered acceptable. The risk of transfer of such a high potential beyond the safe limits of the plant is reduced by insulating any outgoing rails or pipes at the exit locations. The various grids at different locations were designed in order to keep the step and mesh potentials within safe limits. The usual calculations that were performed to check these potentials are not included in this thesis.

2.3 Methods Used in Calculating the Resistances of Grids at Various Locations

2.3.1 General Procedure

The ground resistance for each area described above was calculated by one or several methods, depending upon the geometrical structure of the grounding grid and on the nature of the surrounding media. Values obtained by different methods for each area were, in general, reasonably close to one another and the most conservative value was usually taken as the design value.

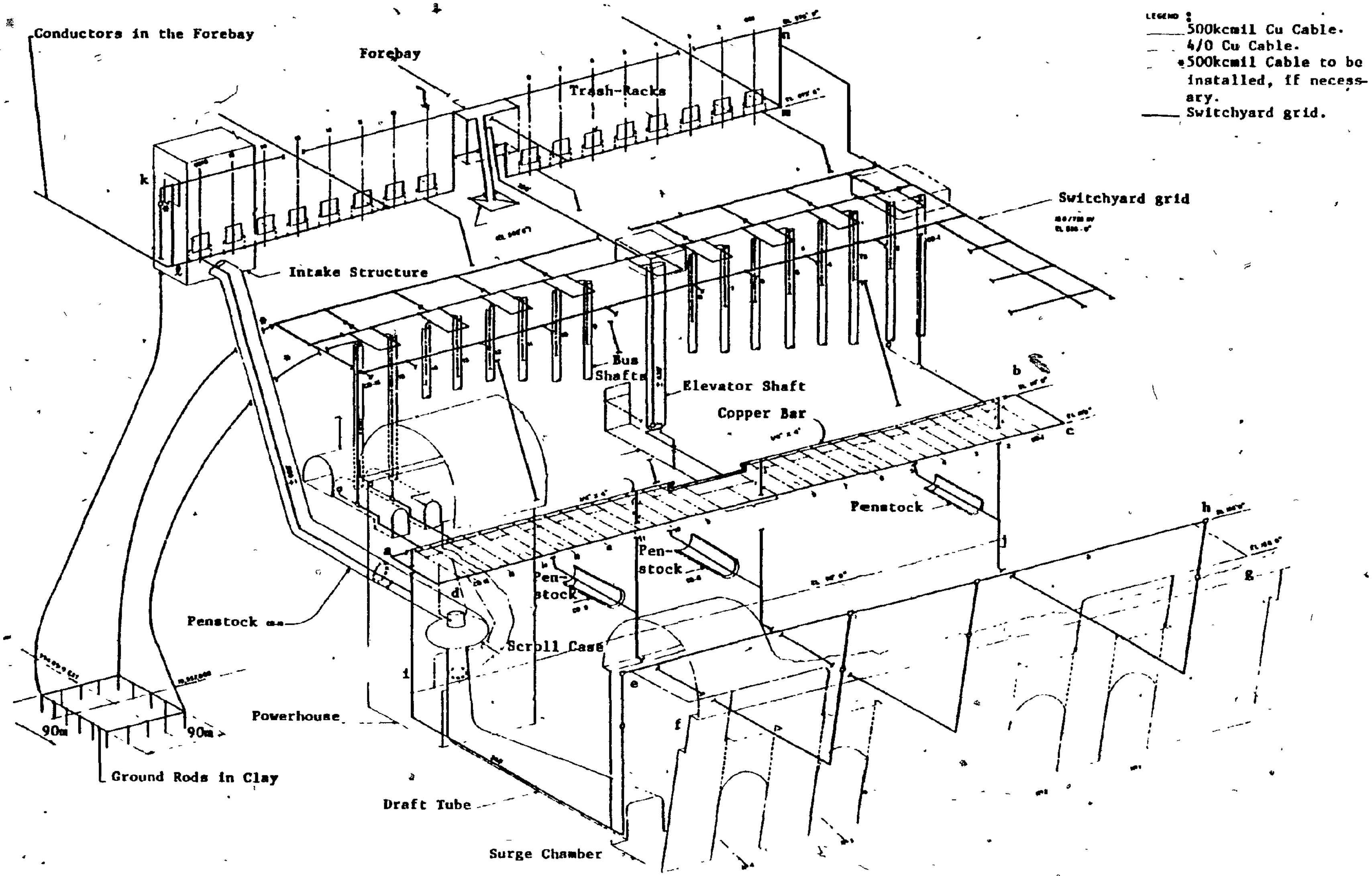


FIGURE 2.1: AN ISOMETRIC VIEW OF THE GROUNDING NETWORK.

TABLE 2.1

GROUND RESISTANCE OF VARIOUS GRIDS

OF THE

HYDRO-ELECTRIC PLANT

Location	Ground Resistance (Ω)
Surge chamber	192
Draft tube	204
Powerhouse	178
Intake	0.6
Ground rods	4
Switchyard	43
Overall value including ground rods	0.5
Overall value without ground rods	0.6

The main guidelines for the calculations are given in the following paragraphs. More detailed references can be found in Appendix I. In all the calculations, the basic definition of the ground resistance of a body was kept in mind, it being the resistance in the path of the current flowing from the body to remote earth.

2.3.2 Powerhouse

The following elements were considered to contribute to the grounding of the powerhouse:

- Grounding conductors buried in concerte.
- Rock bolts installed to support the arch roof of the powerhouse.
- Steel linings in the lower portion of the penstock, the scroll case and the draft tube of each unit and directly in contact with the water.

2.3.2.1 Conductors buried in concrete:

Copper conductors in the form of a grid (abcd, Fig. 2.1) are buried at the turbine floor. The ground resistance, calculated using equations (I.1), (I.2), and (I.3) of Appendix I, appears in Table 2.2. A design value of 178 Ω was adopted.

2.3.2.2 Rock Bolts in the Arch Roof:

Equation (I.9) of Appendix I was applied to calculate the ground resistance of this multi-electrode network. The calculated ground resistance for

TABLE 2.2

GROUND RESISTANCE OF THE POWERHOUSE GRIDS

LOCATION	MESH AREA		BURIED COPPER			RESISTANCE (ohms)		
	Length (m)	Width (m)	Length (m)	Diam. (cm)	Depth (cm)	Laurent [1]	Schwarz [2]	Dwight [3]
Turbine floor (grid abcd)	320	23	1285	1.61	45.7	178	112	261
Surge chamber (grid efgh)	455	20.6	1047	1.61	45.7	192	117	275
Gallery at draft tube (conductor ij)	457	(single conductor)	457	1.61	45.7	-	203	204

the powerhouse was 89.7 Ω. This value was considered too high for the bolts to be of any use in the grounding, especially considering the fact that there is no electrical apparatus nearby.

n = number of bolts = 3519

$2b$ = diameter of bolt = 5 cm

k_1 = 1.0, factor for ratio of length (440 m) to width (34 m) of the mesh

ℓ_1 = length of bolt = 5.25 m

ρ = soil resistivity = 35000 Ω-m for rock

2.3.2.3 Steel Linings in Contact with Water:

Because of the high rock resistivity, the lowest resistance path to remote earth is through the water in the penstock to the forebay. Equation (2.1) below gives the resistance of this column of water simply regarding it as a cylindrical conductor, not taking into account the non-uniform current density across the section:

$$R = \frac{\rho \ell}{\pi r^2} \tag{2.1}$$

Where, ℓ = length (m)

r = radius (m)

ρ = resistivity of water (Ω-m)

For, ℓ = 177 m, r = 3 m and ρ = 400 Ω-m (an average value), R amounts to 2504 Ω and for 16 penstocks in parallel, the equivalent resistance is 156.6 Ω (without mutual effect). This is obviously too high a resistive path to be effective for passage of ground fault currents. Taking into account the variable current density would further increase this value.

2.3.3 Surge Chamber

Copper conductors in the form of a grid (efgh, Fig. 2.1) are buried in the surge chamber. The ground resistance, calculated by the same methods as in section 2.3.2.1 appears in Table 2.2. A design value of 192 Ω was adopted.

2.3.4 Draft Tube

Copper conductors (ij), Fig. 2.1, are embedded in the concrete of each draft tube. Table 2.2 shows ground resistance obtained by equations (I.8) and (I.7). A design value of 204 Ω was taken.

2.3.5 Intake Structure

The following elements were considered to contribute to the grounding of the intake structure:

- Grounding conductors embedded in the concrete.
- Steel trash-racks in permanent contact with the forebay water.
- A number of conductors laid on the floor of the forebay and connected to the intake grounding mesh.

2.3.5.1 Grounding Conductors Embedded in Concrete

(Mesh klmn, Fig. 2.1). Equation (I.5) (considering the mesh equivalent to a vertical plate) or equation (I.3) (considering the mesh as a ring of wire), of Appendix I were used to calculate the ground resistance.

mesh length = 402.7 m

height = 30.75 m

depth = 1/2 (height)

diameter of wire = 2 cm

The resistance is 1.3 Ω and 2.17 Ω using the first and second methods respectively and assuming ρ (concrete) is 400 Ω-m. This calculation is valid only for an infinite concrete medium. The effects of underlying rock and forebay water were taken into account by the following relation.

Resistance of mesh to ground = resistance of mesh to concrete (considering concrete as an infinite volume) - resistance of an electrode formed by the intake structure into infinite concrete + resistance of concrete bound by water and rock. (2.2)

The basic concept behind this equation is analysed in a separate section 2.4

The first term is already calculated above.

Equation (I.6) was used to determine the second term, regarding the intake as an equivalent horizontal cylinder shown in Fig. 2.2.

The cylinder cross-section is taken equal to the intake cross-section,

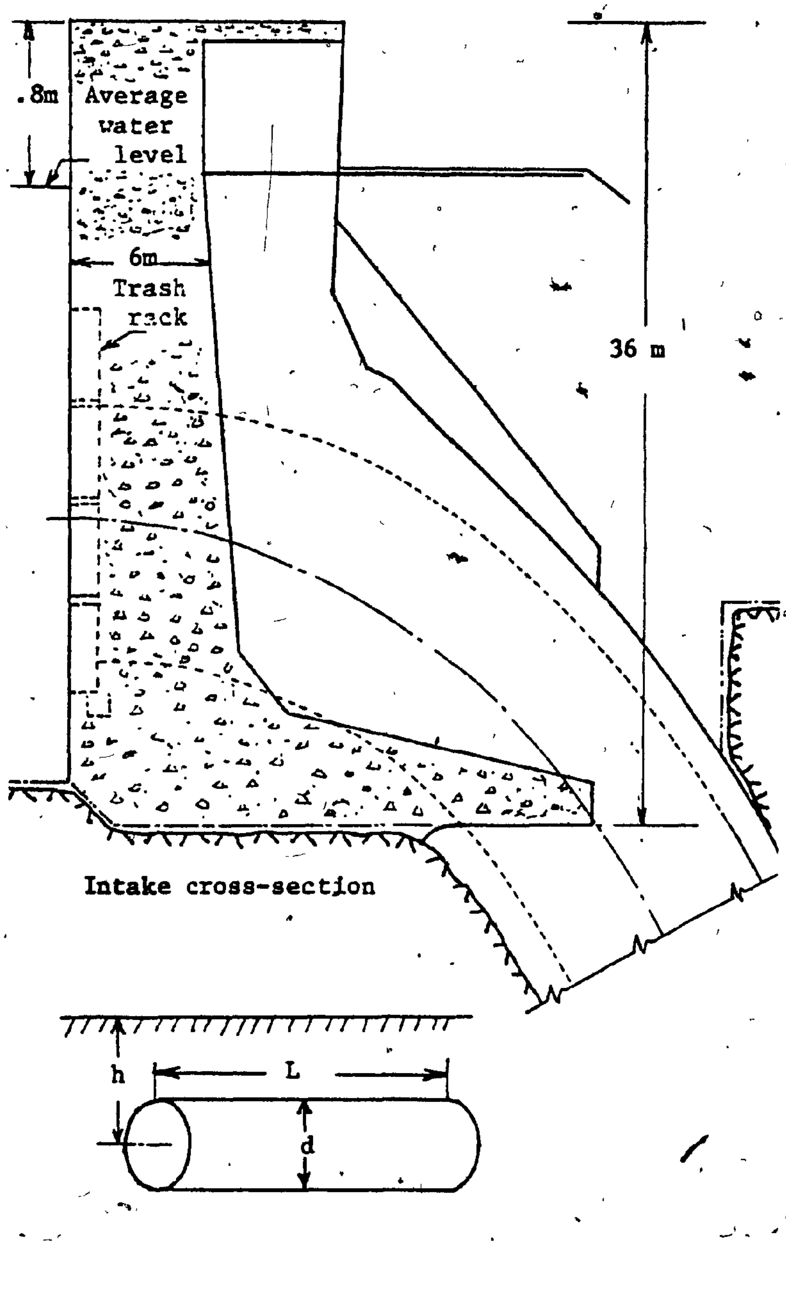


FIGURE 2.2

INTAKE CROSS-SECTION AND ITS EQUIVALENT CYLINDER and its length is the same as that of the intake. For an average width of 6 m and a depth of 36 m, the diameter of this cylinder would be 16.6 m.

For: $\ell = 400$ m

$d = 16.6$ m

$h = 18$ m

$\rho = 400\ \Omega$-m (concrete),

we get: $R = 0.94\ \Omega$

The third term of (2.2) requires the apparent resistivity ρ_a due to the dual medium (water and rock). This was calculated using equation (I.14). The circle equivalent to the intake structure (400 m long, 6 m wide) has a radius of 27.6 m, with its center at mid depth of water. This yielded a value $\rho_a = 1.39\ \rho$ (water).

Substituting ρ_a for ρ in equation (I.6) the resistance of the concrete in contact with water and rock was calculated to be 1.42 Ω.

Equation (2.2) then becomes:

$$R = 2.17 - 0.94 + 1.42 = 2.65\ \Omega$$

2.3.5.2. <u>Trash-Racks</u>

Each trash-rack can be regarded as a vertical electrode in water, and since all of them are interconnected through the buried copper conductor, as shown in Fig. 2.1, they act as parallel electrodes. For calculation of ground resistance, the apparent resistivity due to the combined effect of water and rock has to be considered, as well as the mutual effect of parallel currents flowing to water via these electrodes.

Calculation considering water only:

All electrodes are at the same potential, given by equation (2.3) below:

$$\begin{array}{l} V_1 = I_1 R_{11} + I_2 R_{22} + \ldots + I_n R_{1n} \\ V_2 = I_2 R_{21} + I_2 R_{22} + \ldots + I_n R_{2n} \\ \vdots \qquad \vdots \qquad \vdots \qquad \vdots \\ V_n = I_1 R_{n1} + I_2 R_{n2} + \ldots + I_n R_{nn} \end{array} \tag{2.3}$$

Putting in matrix form:

$$\left[\bar{V}\right] = \left[R\right] \left[\bar{I}\right] \tag{2.3a}$$

or, $\left[\bar{I}\right] = \left[R\right]^{-1} \left[\bar{V}\right]$

n = number of electrodes

$V_1 = V_2 = \ldots V_n = V$, being potential rise of each electrode

R_{ii} = electrode resistance to ground

R_{ij} = mutual resistance between electrodes i and j

$I_1, I_2, \ldots I_n$ are currents in the electrodes $1, 2 \ldots n$, respectively.

Since $V_1 = V_2 = \ldots V_n = V$, the values of $I_1, I_2 \ldots I_n$ can be obtained in terms of V through inversion of $[R]$. Since the copper conductor connecting the electrodes is connected to the system grid at both ends and at the center, the symmetrical distribution of current allows the order of the matrix to be reduced to n/2.

The values of $[R]$ can be obtained as follows:

R_{ii} (where i = 1,2 ... n) can be calculated by equation (I.5) or (I.4), considering it in either case as a vertical plate in water.

R_{ij} [1,2] can be calculated with a reasonable approximation, using equation (2.4) below:

$$R_{ij} \doteq \frac{\rho}{2\pi x_{ij}} \tag{2.4}$$

x_{ij} = distance between electrodes i and j, it should be greater than the linear dimension of each electrode.

ρ = resistivity of water

From these, knowing matrix $[R]$, $[R]^{-1}$ can be found. The total ground resistance R_{tot} would then be:

$$R_{tot} = \frac{V}{\sum_{i=1}^{n} I_i}$$

The trash-racks are equally spaced at 22.5 meters. For a trash-rack 13.3 m x 13.6 m, $R_{ii} = 8.23\,\Omega$ by the first method and $8.33\,\Omega$ by the second method. The value of $R_{tot} \doteq 1.32\,\Omega$.

Calculation considering the effect of rock:

To take this into account, an additional term R_a must be added, which can be calculated from equation (I.13b).

For, h = 22.5 m (for minimum water level)

ℓ = 13.5 m

ρ_1 (water) = 400 Ω-m

ρ_2 (rock) = 35000 Ω-m

R_a = 9.52 Ω

This modifies the earlier value of R_{ii} to 17.75 Ω. On account of this, the diagonal elements of the [R] matrix get modified, changing the total resistance to 1.93 Ω.

The matrix [R] (reduced to size n/2) is shown in Table 2.3, with and without the effect of rock; the current distribution in each trash-rack is also shown in Table 2.4.

2.3.5.3 Conductors in the Forebay

The problem involves a 2-layer medium (water, ρ_1, on rock, ρ_2). Since the electrode layout is horizontal, equation (I.14) was used to determine the apparent resistivity ρ_a. To optimize the resistance with respect to the length, number of conductors, area covered by these conductors and depth of immersion, computer was used to evaluate both the apparent resistivity and the grid resistance to ground.

TABLE 2.3

MATRIX [R] FOR THE TRASH-RACKS

$$[R]^{*} = \begin{bmatrix} 8.41 & 3 & 1.60 & 1.16 & .95 & .84 & .77 & .75 \\ 3.00 & 8.44 & 3.03 & 1.65 & 1.21 & 1.01 & .91 & .86 \\ 1.60 & 3.03 & 8.48 & 3.08 & 1.71 & 1.28 & 1.10 & 1.02 \\ 1.16 & 1.65 & 3.08 & 8.54 & 3.15 & 1.80 & 1.39 & 1.26 \\ .95 & 1.21 & 1.71 & 3.15 & 8.63 & 3.26 & 1.96 & 1.63 \\ .84 & 1.01 & 1.28 & 1.80 & 3.26 & 8.79 & 3.50 & 2.33 \\ .77 & .91 & 1.10 & 1.39 & 1.96 & 3.50 & 9.16 & 4.20 \\ .75 & .86 & 1.02 & 1.26 & 1.63 & 2.33 & 4.20 & 11.03 \end{bmatrix}$$

* For 2-layer medium of water on rock, the diagonal elements getting modified to 17.93, 17.96, 18, 18.06, 18.15, 18.31, 18.68, 20.55.

TABLE 2.4

CURRENT DISTRIBUTION IN TRASH-RACKS

Current	I_1 $=I_{16}$	I_2 $=I_{15}$	I_3 $=I_{14}$	I_4 $=I_{13}$	I_5 $=I_{12}$	I_6 $=I_{11}$	I_7 $=I_{10}$	I_8 $=I_9$	
Considering water as infinite volume	7.02	4.85	4.59	4.39	4.29	4.21	4.18	4.16	$\times 10^{-2}$ V
Considering effect of rock below water	3.95	3.30	3.23	3.14	3.08	3.04	3.03	3.02	$\times 10^{-2}$ V

To calculate the ground resistance of a single conductor (500 kcmil), equation (I.8) was used, replacing ρ by ρ_a. The mutual resistance between conductors and the overall resistance were determined utilizing the same procedure used for the trash-racks.

Values obtained are shown in Table 2.5 and are as well depicted graphically in Fig. 2.3. A design value of 1.71 Ω using 4 conductors (each 1000 m long) was adopted.

2.3.6 Switchyard

The grounding grid of the switchyard covers an area of 503 m x 305 m and involves 11000 m of copper conductors buried 0.5 m below ground surface. Applying Laurent [1] equation (I.1) and taking ρ as 35000 Ω-m, the ground resistance is 43 Ω.

2.3.7 Ground Rods in Nearby Clay

2.3.7.1 General

As stated above, the site offers the possibility of installing ground rods in clay of relatively low resistivity. As will be shown in the following analysis (Table 2.6), to ensure a reasonably low path to ground, this deposit should be of sufficient extent and depth.

Fig. 2.4 shows a grid made of 36 rods arranged in a hollow square. Though different number of rods may be tried, this arrangment affords an optimum value [6] .

TABLE 2.5

GROUND RESISTANCE OF CONDUCTORS IN THE FOREBAY

Length (m)	120	300	450	600	800	1000	1250	1500	2000	2500	3000	3500	4000
$\frac{\rho_a}{\rho_1}$ for $\frac{h}{d} = 1$	3.22	4.45	5.2	5.28	6.54	7.15	7.85	8.5	9.5	10.5	11.36	12.12	12.84
R (1 cable) (Ω)	16.65	10.96	9.12	8.02	7.05	6.37	5.77	5.32	4.67	4.23	3.9	3.64	3.43
R (2 cables) (Ω)	8.32	5.53	4.62	4.08	3.6	3.26	2.96	2.73	2.41	2.19	2.03	1.89	1.79
R (3 cables) (Ω)	5.75	3.86	3.25	2.88	2.56	2.33	2.13	1.98	1.76	1.62	1.51	1.42	1.35
R (4 cables) (Ω)	4.16	2.83	2.39	2.12	1.87	1.71	1.56	1.45	1.28	1.20	1.09	1.03	.98

$\rho_1 = 400\ \Omega$-m, $\rho_2 = 35000\ \Omega$-m, width of forebay = 400 m, conductors are equally spaced.

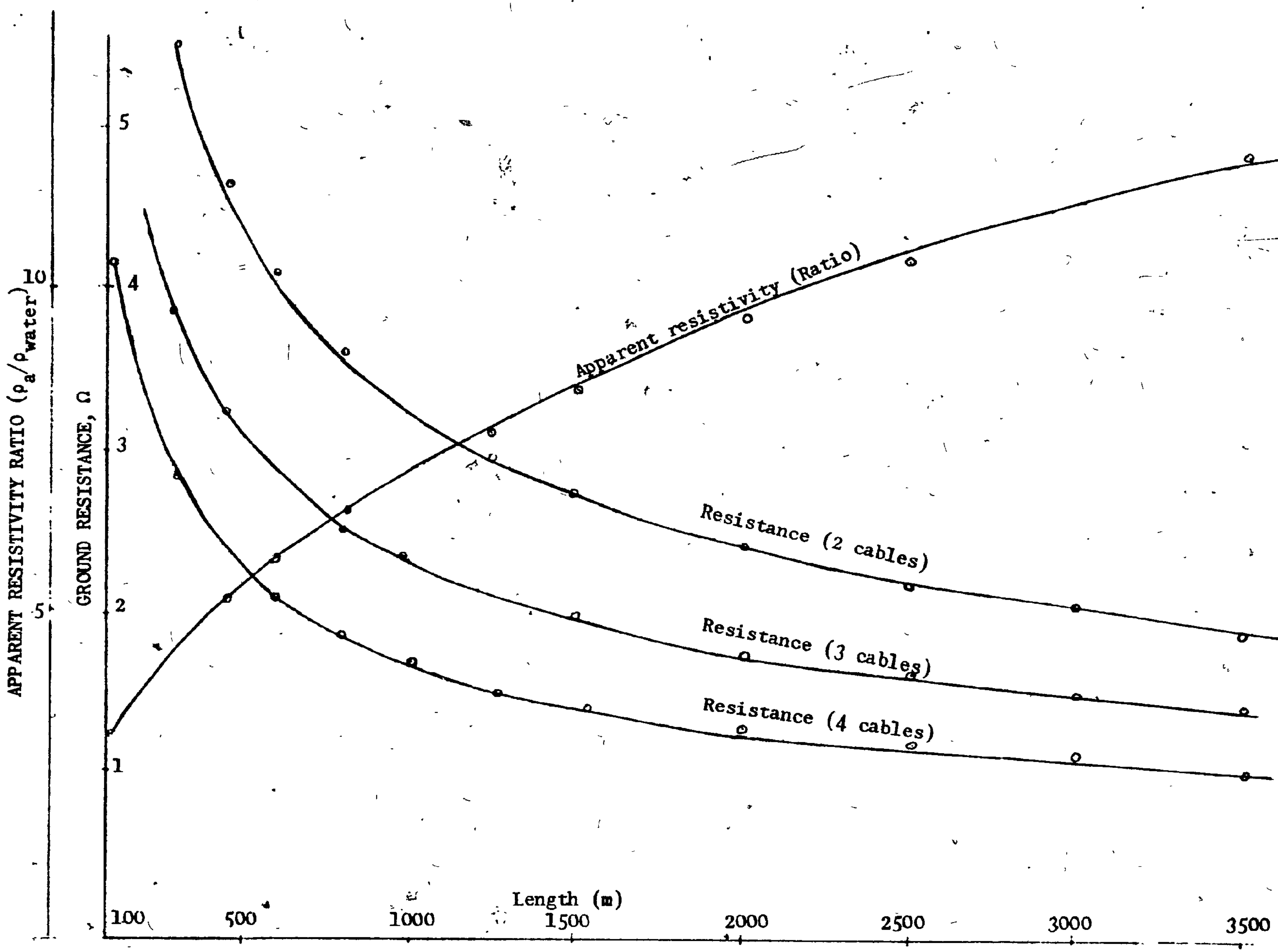


FIGURE 2.3: APPARENT RESISTIVITY OF FOREBAY, AND GROUND RESISTANCE OF CONDUCTORS

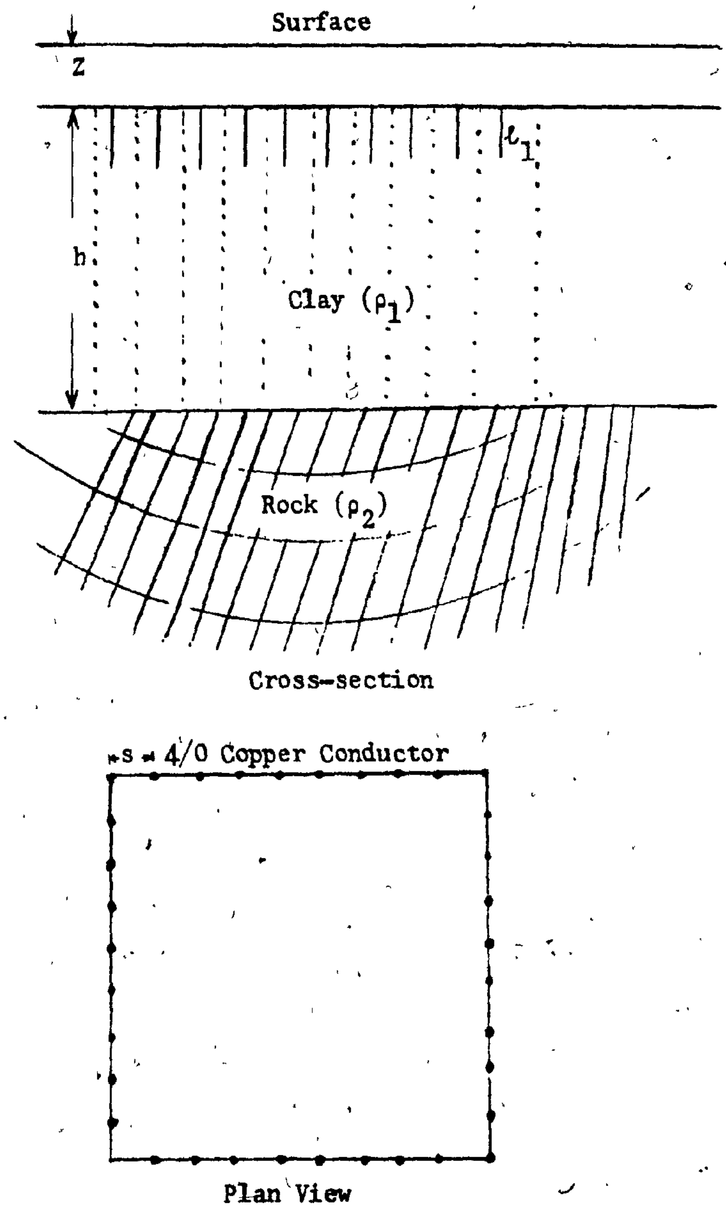


FIGURE 2.4

GROUND RODS (36) IN HOLLOW SQUARE

2.3.7.2 Design of the Rod-bed

Since the resistivity of rock (ρ_2) is much higher than that of clay (ρ_1), the rod separation s has to be less than h. For optimum design, one could use the ratio $\frac{h}{s}$ as two [6]. Also, since the rods must be buried deep enough to minimize the influence of the high resistivity frozen surface layer in winter, the ratio [6] between frozen layer thickness and rod separation was taken as 0.2. Minimum resistance is achieved by using a long rod, buried below the frozen layer. However, the rod should be kept well above the underlying rock.

2.3.7.3 Ground Resistance

As for the intake structure, the effect of the underlying rock can be taken into account by using the following relation:

Resistance of the grid to remote earth = resistance of grid to clay (considering upper medium, i.e. clay as having an infinite volume) - resistance of an electrode formed by the volume of upper medium clay, considered as half ellipsoid, into infinite clay + resistance of this half ellipsoid clay in rock. (2.5)

The basic concept behind this equation is analysed in section 2.4.

The resistance R of a single rod in clay is calculated by equation (I.11):

$R = 4.3\,\Omega$ (using $h = 20$ m, $z = 2$m, $s = 10$ m, $\ell_1 = 10$ m)

The resistance of 36 rods forming a hollow square may be calculated by equation (I.12). Equation (2.6) below gives the equivalent hemisphere radius r_e to be used in this equation:

$$R = \frac{\rho}{2\pi r_e} \tag{2.6}$$

Hence $r_e \doteq 1.47$ m,

$$\alpha = \frac{r_e}{\text{distance between rods}} = 0.147$$

From equation (I.12), the total ground resistance R_{22} for 36 rods is $0.26\,\Omega$.

2.3.7.4 Interconnection

The resistance R_{11} of the buried copper conductor grid connecting the rods is calculated by using equations (I.1) or (I.2).

For, $z = 2$ m

$\ell = 360$ m

$2a = 1.32$ cm, (diameter of conductor)

$R_{11} = 0.31\,\Omega$ from equation (I.1)

$R_{11} = 0.29\,\Omega$ from equation (I.2)

2.3.7.5 Mutual Resistance

The mutual resistance R_{12} between the copper mesh and electrodes is calculated by equation (I.10).

For, $\ell = 360$ m

$\ell_1 = 10$ m

$A = 90\ m \times 90\ m$

$z = 2\ m$

$\rho = 40\ \Omega\text{-m}$

we get: $R_{12} = R_{21} = 0.18\ \Omega$

2.3.7.6 Ground Resistance of Clay

The clay overburden is regarded as a half of an ellipsoid (Fig. 2.5). To calculate its resistance, equation (2.7) is used [6].

$$\text{Resistance of clay} = \frac{\rho_{(rock)}}{2\pi C_1} \tag{2.7}$$

C_1 = capacitance of the clay combined with its image over the surface of the earth, the combined electrode being considered as in air, in meters

ρ = resistivity of the underlying rock in Ω-m

Case 1:

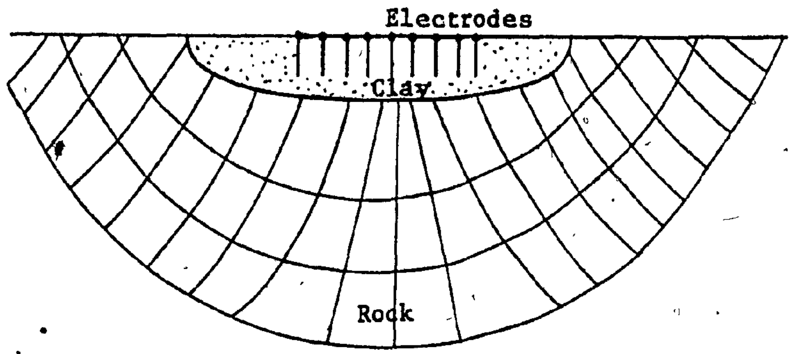
$a_1 = b > c$

$$C_1 = \frac{\sqrt{a_1^2 - c^2}}{\arcsin \frac{\sqrt{a_1^2 - c^2}}{a_1}} \qquad \text{(from reference [16])} \tag{2.8}$$

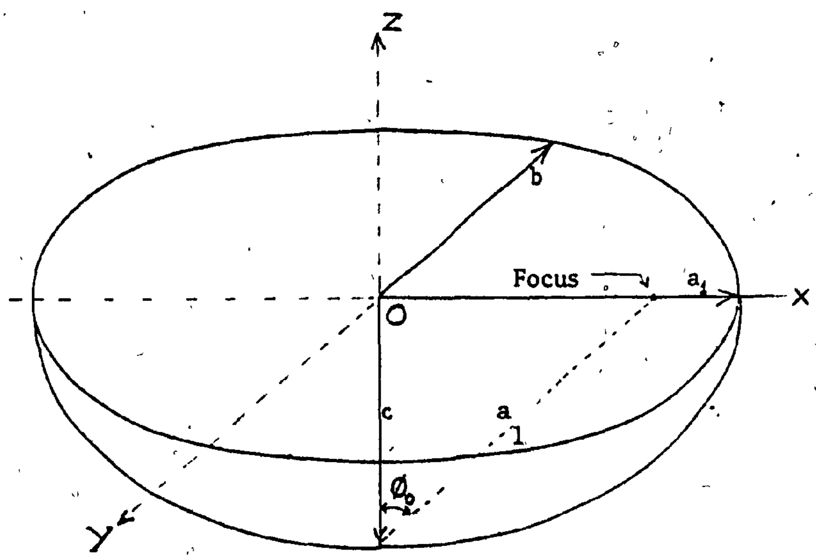
All dimensions are in meters and refer to Fig. 2.5 shown in the next page.

Case 2:

$a_1 \neq b$



Ground Rods in Clay



Clay Ellipsoid

FIGURE 2.5

GROUND RODS AND GRID IN CLAY

$$\text{Capacitance} = \frac{\sqrt{a_1^2 - c^2}}{F(\phi_o, q)} \qquad \text{(reference [16])} \qquad (2.9)$$

where, $q^2 = \dfrac{a_1^2 - b^2}{a_1^2 - c^2} < 1$

$$\phi_o = \text{arc sin} \sqrt{1 - \frac{c^2}{a_1^2}} < 1$$

$$F(\phi_o, q) = \int_o^{\phi_o} \frac{d\,\phi}{\sqrt{1-q^2 \sin^2 \phi}}$$

For ϕ_o very close to $\pi/2$, this integral can be approximated to:

$$= \frac{\pi}{2}\left[1 + (\tfrac{1}{2})^2 q^2 + (\tfrac{1.3}{2.4})^2 q^4 + (\tfrac{1.3.5}{2.4.6})^2 q^6\right], \; q^2 < 1$$

Table 2.6 shows values of capacitance and resistance based on ρ (rock) as 35000 Ω-m.

The R values given in table 2.6 correspond to the third term in relation (2.5). The second term is obtained by dividing by ρ (rock) as 35000 and multiplying by ρ clay as 40.

This clearly shows that the dimensions of the clay medium play an important role in determining the value of the ground resistance for the grid. The dimensions of the clay deposit at the site permit us to use a ground resistance of 3.7 Ω.

2.3.7.7 Total Ground Resistance of the Grid

The total grid resistance to ground as obtained from expression (2.5) is:

TABLE 2.6

GROUND RESISTANCE OF CLAY

a_1 (m)	b (m)	c (m)	Capacitance (m)	R' (Ω)
121	121	25	86.67	64.3
2500	800	30	1134.0	4.9
3340	1060	30	1522.4	3.7

$$\frac{R_{11}R_{22} - R_{12}R_{21}}{R_{11} + R_{22} - R_{12} - R_{21}} - \frac{3.7 \times 40}{35000} + 3.7 \qquad (2.10)$$

With $R_{11} = 0.31\,\Omega$, $R_{22} = 0.26\,\Omega$ and $R_{12} = R_{21} = 0.18\,\Omega$, total grid resistance = $3.9\,\Omega$. The first term is the resistance [2] of grid and rod bed in infinite clay.

2.4 Ground Resistance of Grids in Confined Soft Soils

The expressions (2.2) and (2.5) have been used in earlier sections to calculate the grounding resistance of such grids and the concept behind these equations is outlined below:

2.4.1 Concept for Formulation of Equations (2.2) and (2.5)

Assume an electrode M buried in medium A, bound by surface S (see Fig. 2.6). The body A is in turn buried in medium B considered as bound by remote earth R. The ground resistance to a current flowing from electrode M is constituted of the resistance of A and B in series [1], i.e.,

$$R = R_A + R_B$$

If the body (medium B) bound by surfaces S and R was of the same material as A, there would be a single medium path. If the surface S is close enough to an equipotential of this new medium, the value of R_A remains the same, and the total ground resistance becomes:

$$R' = R_A + R'_A$$

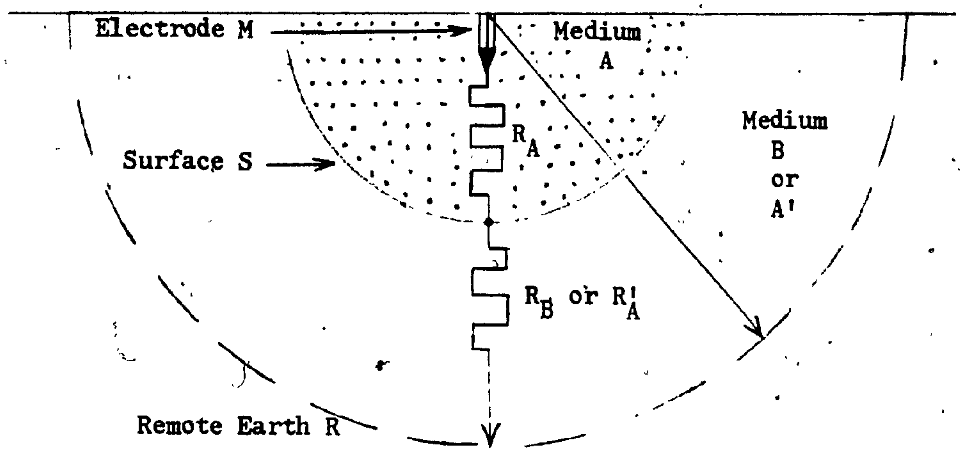


FIGURE 2.6

ELECTRODE BURIED IN TOP CONFINED MEDIUM A

Where, R_A = resistance through body A (bound by surface S), same as above

R'_A = resistance through body A' (bound by surfaces S and R)

R' is obviously the ground resistance of the electrode M in an infinite medium A and can be easily calculated. R'_A is the resistance of an electrode having the shape S in an infinite medium A. This can be calculated if the shape S of the electrode being considered is simple.

Hence, $R_A = R' - R'_A$

R_B is the resistance of an electrode having the shape S in an infinite medium B. The formula is the same as for R'_A, replacing ρ_A by ρ_B.

Hence, $R = R_A + R_B$

or, $R = R' - R'_A + R_B$, which is the same as equations (2.2) and (2.5). This is found to be identical to the one pointed out [4], as applied to a concrete encased electrode. The relation arrived at above, however, is general and can be applied to any similar situation.

2.5 Summary

A design procedure to achieve a low grounding resistance for an underground power plant located on highly resistive soils has been developed. Contributions of different elements of the powerhouse structure to the total resistance of its grounding network have been analysed by applying suitable methods depending upon geometrical configurations. Some of the grounding paths, viz. the intake structure and water are shown to be more efficient in reducing the overall grounding resistance, whereas some other elements, for instance the penstock linings in contact with water are not as effective grounding paths as they are often believed to be. It is also shown that installation of a grid in a confined low resistivity area is no guarantee of a low resistance. In such cases, new expressions (2.2) or (2.5) developed should be used to calculate the grounding resistance.

CHAPTER III

GROUND FAULT CURRENT DISTRIBUTION

IN

POWERHOUSE, SUBSTATION, TOWERS AND GROUND WIRE

CHAPTER III

GROUND FAULT CURRENT DISTRIBUTION

IN

POWERHOUSE, SUBSTATION, TOWERS AND GROUND WIRE

3.1 General

It is often difficult, at the design stage to assess the magnitude of fault current flowing in the grounding grid of a plant or a substation. The portion of the current flowing in such grids is important for verification of touch and step potentials in order to ensure safety of personnel and as well to determine the potential rise of the plant to remote earth.

The substation grid in parallel with the powerhouse grounding network are connected to the overhead grounding wire, which runs along the transmission line, strectching from tower to tower and connected at each tower to tower ground through the metallic structure of the tower. A ladder type of network gets formulated, with terminations by the substation and plant grounds. The next sections deal with the analysis of the resulting network so as to determine the desired ground fault current distribution, taking into consideration the effect of counterpoise (buried in the ground and connected to each tower footing), coupling between the phase and ground wire and that of whether the line is short or long. Since maximum current through the substation or plant will traverse in case the ground fault takes place at close proximity to the substation, the analysis below covers faults at the terminal towers in order to ensure a safe design.

3.2 Current Distribution Due to a Fault at the Terminal Tower of a Line

Following Fig. 3.1 shows the connection of a ground wire connected to earth through the transmission towers, each tower having its own grounding electrode or grid. Ground current due to fault at any tower, apart from traversing through it, will also get diverted in part to the ground wire and other towers. The current I_n flowing to ground through the nth tower, counted from the terminal tower where the fault is assumed to take place, is equal to $i_n - i_{n+1}$.

With reference to Fig. 3.1, R_t is assumed equal for all towers and any mutual interference for their own ground currents is neglected. The node and loop equations for the nth mesh give us the following relations:

$$\left.\begin{aligned} I_n &= i_n - i_{n+1} \\ I_{n-1} &= i_{n-1} - i_n \end{aligned}\right\} \qquad (3.1)$$

and, $i_n Z_g - \mu I_f Z_g + I_n R_t - I_{n-1} R_1 = 0$ (3.2)

where, I_n, I_{n-1}, i_n, i_{n+1}, I_f etc. are as shown in Figure 3.1 and μ, the coupling factor is given by the expression [17] below:

$$\mu = \frac{\log\ (b/m)}{\log\ (\frac{2h}{a})} \qquad (3.3)$$

This coupling factor is the ratio of the mutual impedance (Z_m) between the faulted phase conductor and the ground wire and the self impedance of the ground wire with common earth return.

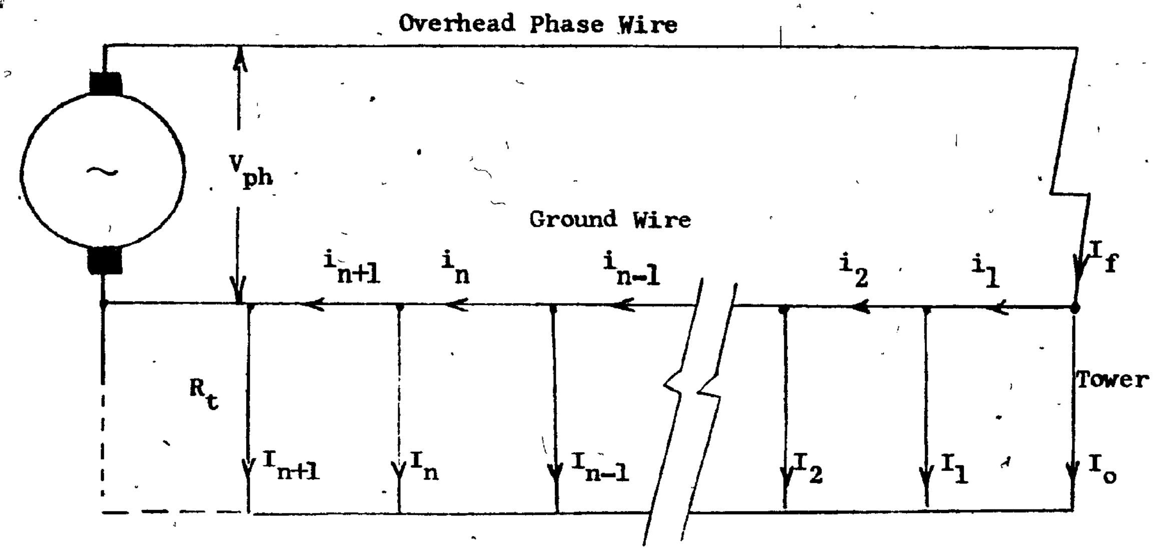


FIGURE 3.1

FAULT CURRENT DISTRIBUTION AT TERMINAL TOWER where, b = distance for the conductor to image of the ground wire, the latter being the same distance below the ground surface as the wire is above it (for infinite conductivity earth).

m = distance from conductor to ground wire

h = height of ground wire above earth

a = radius of the ground wire

The evaluation of the factor μ given above is applicable only for a perfectly conducting earth. In practice, earth's surface is not a perfect conductor and a perfectly conducting plane electrically equivalent to the earth would, in general, be situated some distance below the earth's actual surface and accordingly the distance between the conductor and its image (the latter situated at the same distance below this fictitious plane as the conductor is above it) will increase, thus increasing both 2h and b of (3.3). The exact value of this factor is Z_m/self impedance of the ground wire, these being obtainable from reference [16].

Re-writing the equation (3.2) yields us:

$$i_n = \frac{(I_{n-1} - I_n)R_t}{Z_g} + \mu I_f$$

Similarly, $$i_{n+1} = \frac{(I_n - I_{n+1})R_t}{Z_g} + \mu I_f$$

Substituting these in (3.1), we get:

$$I_n = \frac{(I_{n-1} - I_n)R_t}{Z_g} + \mu I_f - \frac{(I_n - I_{n+1})R_t}{Z_g} - \mu I_f$$

or, $\frac{I_n Z_g}{R_t} = (I_{n+1} - I_n) - (I_n - I_{n-1})$

$= \Delta^2 I_n$, a difference equation which has the following solution [18]

$$I_n = Ae^{\alpha n} + Be^{-\alpha n}, \text{ where } \alpha = \sqrt{Z_g/R_t} \qquad (3.4)$$

Also, by substituting for the values of I_n and I_{n-1} from (3.1)in (3.2), we get:

$$i_n Z_g - \mu I_f Z_g + (i_n - i_{n+1}) R_t - (i_{n-1} - i_n) R_t = 0$$

or, $i_n Z_g/R_t = \Delta^2 i_n + \mu I_f Z_g/R_t$, a difference equation with a constant term and has the following solution:

$$i_n = ae^{\alpha n} + be^{-\alpha n} + \mu I_f \qquad (3.5)$$

The first two terms refer to similar solution (3.4) if $\mu I_f Z_g/R_t$ were not there and the third term refers to particular solution of the non-homogeneous equation obtained through variation of parameters [19] method.

On substituting the values of i_n and i_{n+1} from (3.5) into (3.1), we get:

$$A = a(1 - e^{\alpha}) \text{ and } B = b(1 - e^{-\alpha})$$

$$\therefore i_n = Ae^{\alpha n}/(1 - e^{\alpha}) + Be^{-\alpha n}/(1 - e^{-\alpha}) + \mu I_f \qquad (3.6)$$

If the line is sufficiently long so that, after some distance, the varying portion of the current exponentially decays to zero, $A \Rightarrow 0$

$$\therefore I_n = Be^{-\alpha n} \tag{3.7}$$

$$\text{and,}\quad i_n = Be^{-\alpha n}/(1 - e^{-\alpha}) + \mu I_f \tag{3.8}$$

In order to determine the current in the terminal tower, $n = 0$.

$$\therefore I_0 = B \tag{3.9}$$

$$\text{and,}\quad i_1 = Be^{-\alpha}/(1 - e^{-\alpha}) + \mu I_f \tag{3.10}$$

The boundary condition at the terminal tower of Fig. 3.1 is:

$$I_f = I_0 + i_1 \tag{3.11}$$

Substituting the values from (3.9) and (3.10) in (3.11) and simplifying, we get:

$$I_f = B/(1 - e^{-\alpha}) + \mu I_f$$

$$\therefore B = I_f\,(1 - e^{-\alpha})\,(1 - \mu) = I_0 \tag{3.12}$$

I_0 is the current that traverses to the ground through the terminal tower of Fig. 3.1 and it is evident that its magnitude gets decreased on account of the coupling factor μ.

Substituting for B in (3.10) affords us with the following result:

$$i_1 = I_f\left[e^{-\alpha} + \mu(1 - e^{-\alpha})\right] \tag{3.13}$$

Expression (3.13) shows the increase in the ground wire current due to μ. The voltage rise V of the terminal tower is:

$$V = I_0 R_t$$

$$= I_f\,(1 - e^{-\alpha})\,(1 - \mu)\,R_t$$

$$= I_f Z_\infty$$

where Z_∞ is the network earth impedance looking back from the fault and is:

$$Z_\infty = (1 - e^{-\alpha})\,(1 - \mu)\,R_t \tag{3.14}$$

$$= \left[\frac{2\tanh\alpha/2}{1+\tanh\alpha/2}\right](1 - \mu)\,R_t$$

$$= \frac{\sqrt{\dfrac{Z_g/R_t}{1 + Z_g/4R_t}}}{1 + \frac{1}{2}\sqrt{\dfrac{Z_g/R_t}{1 + Z_g/4R_t}}}\cdot(1 - \mu)\,R_t \tag{3.15}$$

$$= \sqrt{Z_g R_t}\,(1 - \mu) \text{ if } Z_g \ll R_t \tag{3.16}$$

Thus if the line is sufficiently long, it behaves towards the source like an equivalent impedance Z_∞ independent of its length.

The above expressions (3.14) to (3.16) prove the reduction of the equivalent earth impedance due to μ. These results are identical to results [5, 20] obtained without the effect of μ (to verify, put $\mu = 0$).

3.3 Current Distribution Due to a Fault at Terminal Tower Connected to a Substation Grounding Grid

In section 3.2 above, we have analysed the distribution of ground fault current based on fault at the terminal tower, without its connection to a substation grounding grid. In actual practice, the latter is connected to the earth wire at the terminal tower. In this case, only a portion I_f' of the total ground fault current I_f will flow through Z_∞ as the rest will get diverted through the substation ground resistance R_k. If one bases all the calculations on the basis of I_f' flowing through earth network with equivalent impedance Z_∞, still one will get the same difference equations and thus the previous results will be valid except for replacement of I_f by I_f' and thus the value of I_0 will be:

$$I_0 = I_f' (1 - e^{-\alpha}) (1 - \mu) \tag{3.17}$$

Let I_k be the current through R_k, i.e. the substation grounding grid resistance,

$$\therefore I_k = I_f \cdot \frac{Z_\infty}{R_k + Z_\infty} \text{ (refer to Figure 3.2)} \tag{3.18}$$

In case the values of R_k and Z_∞ are known, I_k can be found out from (3.18) above.

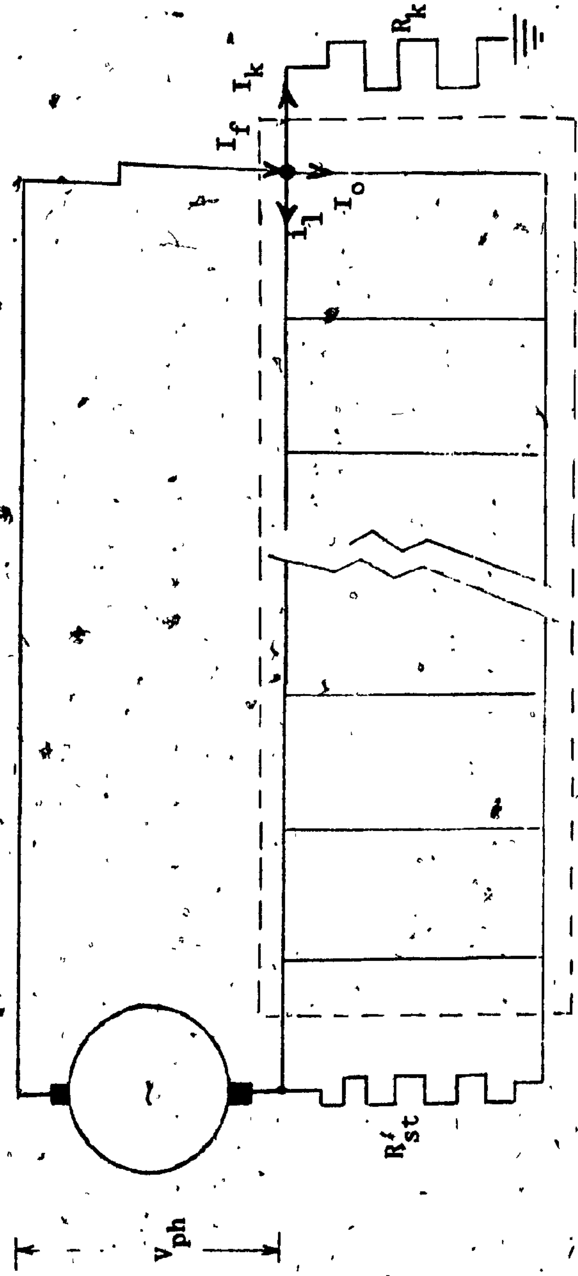


FIGURE 3.2

FAULT CURRENT DISTRIBUTION AT SUBSTATION

$$\therefore I_f' = I_f - I_k = I_0 + i_1 . \tag{3.19}$$

From (3.17), (3.18) and (3.19), one can find out the current distribution I_k, I_0, i_1 etc. However, this analysis is valid so long as A in equation (3.4) can be neglected. If not, Z_∞ of equation (3.18) will have to be replaced by Z_{eq} given by the following [20] expression (3.20) considering the ground wire and tower network as a distributed ladder:

$$Z_{eq} = \frac{Z_{eq}'(1-\mu).R_t}{Z_{eq}'(1-\mu)+R_t} . \tag{3.20}$$

$$\text{where, } Z_{eq}' = Z_0 \left[\frac{Z_R \cosh \gamma\ell - R_{st} + Z_0 \sinh \gamma\ell}{Z_R \sinh \gamma\ell + Z_0 \cosh \gamma\ell} \right] \tag{3.21}$$

Here, $Z_0 = \sqrt{Z_{g(p.u.)} \cdot R_{t(p.u.)}}$, $Z_{g(p.u.)}$ and $R_{t(p.u.)}$ being the per unit length impedances, and Z_R being the receiving end impedance and is equal to R_{st} of Fig. 3.2 if one neglects the impedance of a small feeder between the alternator ground to earth wire.

If the line in the dotted rectangle is infinite (refer to Appendix II), $\gamma\ell \geq 2$ where $\gamma = \ell \sqrt{Z_{g(p.u.)}/R_{t(p.u.)}}$ and in that case $Z_{eq}' = Z_0$. If $R_t \gg Z_0$, $Z_{eq} = Z_0(I-\mu)$, a result identical to (3.16).

If several lines terminating at a station have earth wires connected to structural frameworks, the conductance of the earth wires will add up more or less arithmetically.

To give an idea as to how long a line should be to be regarded as infinite, the following Table 3.1 indicates the necessary length based on $\gamma\ell \geq 2$. In this Table, n is the nth tower as counted from the terminal tower where the current gets reduced to 1 percent than that traversing through the terminal tower as applied to Fig. 3.1. The ground wire is assumed to have an impedance of (2.27 + j0.31) ohm per km and has 4 ground connections per kilometer via the towers.

3.4 Effect of Counterpoise

If, in addition to ground wire and tower grounding, there exists counterpoise, it will reduce not only the surge impedance of the ground connection, but also increase the coupling between the overhead ground wire and the earth. In case of a counterpoise serving just like an additional ground to the tower instead of stretching from one grounded tower to another, it is enough only to modify the value of R_t as follows:

$$\text{Total tower ground resistance} = \frac{R_t Z_c}{R_t + Z_c} \qquad (3.22)$$

where, Z_c is given [21] by the following expression (3.23):

$$Z_c = \sqrt{\frac{z_c}{y_c}} \coth \left(\ell \sqrt{z_c \cdot y_c}\right) \qquad (3.23)$$

Where z_c and y_c are the p.u. parameters of the counterpoise wire of length ℓ, the counterpoise being regarded as a ladder network [18,20] with distributed parameters.

TABLE 3.1

LENGTH REQUIRED FOR A SUFFICIENTLY LONG LINE

R_t (ohm)	1000	100	50	25
γ (per span)	0.0238	0.0753	0.1065	0.1507
n	193	61	43	30
ℓ (km)	84	26.5	18.8	13.2

In case of a counterpoise running parallel to the ground wire throughout its length, its effect can be taken into account by modifying the value of Z_{eq}' in (3.21) as follows:

The resistance to ground of counterpoise is as given by the expression (3.23) and is also given by [20,22] :

$$R_2 = \frac{\rho}{\pi \ell} \cdot \ln \frac{\ell}{a} \tag{3.24}$$

where, ℓ = length of the wire

a = radius of the conductor

ρ = resistivity of the earth soil

and as explained [20] , y_c can be determined by equating

$$\frac{dR_2}{dI} = \frac{dZ_c}{dI} \tag{3.25}$$

which, on simplification (see Appendix III), yields the following results for y_c:

$$y_c = \left(\frac{\pi \ell}{\rho \ \ln \ \ell/a}\right)^2 \left[z_c - \frac{\rho}{\pi \ell^2} \left(1 - \ln \frac{\ell}{a}\right) \right] \tag{3.26}$$

Now, $1/y_c$ can be combined in parallel with $R_{t(p.u.)}$ to get $R_t'(p.u.)$ and z_c combined in parallel with $Z_{g(p.u.)}$ to get $Z_g'(p.u.)$. Then for Z_0 of the equivalent impedance Z_{eq}' of equation (3.21), simply replace $R_{t(p.u.)}$ by $R_t'(p.u.)$ and $Z_{g(p.u.)}$ by $Z_g'(p.u.)$.

3.5 Voltage Rise of the Substation

With reference to Fig. 3.2, we have:

$$I_f = Vph/\left[Z_s + Z_{eq}\, R_k/(Z_{eq} + R_k)\right] \tag{3.27}$$

where, R_k can be calculated by any of the methods outlined in the author's paper [11].

Now, $I_k = I_f \left[\dfrac{Z_{eq}}{Z_{eq} + R_k} \right]$

$$= \frac{Vph}{\left[Z_s(1 + R_k/Z_{eq}) + R_k\right]} \tag{3.28}$$

∴ Voltage rise of the substation is:

$V_k = R_k I_k$

$$= \frac{Vph R_k}{\left[Z_s(1 + R_k/Z_{eq}) + R_k\right]} \tag{3.29}$$

Special Cases:

(i) If $Z_s = 0$, $V_k = Vph$

(ii) $Z_{eq} = \infty$, $V_k = Vph \left[R_k/(Z_s + R_k)\right]$

(iii) $Z_s = R_k$, $V_k = Vph/2$

Also, $$I_k = \frac{Vph}{\left[R_s + A\rho + A\rho\,\alpha\right] + j(X_s + A\rho\beta)} \tag{3.30}$$

where, $R_k = A\rho$, and $\alpha + j\beta = \dfrac{R_s + jX_s}{R_{eq} + jX_{eq}}$

$Z_{eq} = R_{eq} + jX_{eq}$

ρ = resistivity of the substation soil.

Expression (3.30) shows that in order to change the value of I_k, the parameter A is required to be changed considering the other parameters to be already fixed.

3.6 Example

In order to illustrate the theoretical approach outlined in earlier sections, consider a generating station delivering power to a distribution substation via a grounded transmission line. Let there be 4 earth connections per kilometer via towers and assume the line to be sufficiently long.

Let $Z_g = (2.27 + j0.31)\ \Omega/\text{km}$

$I_f = 10{,}000$ Amperes

$R_k = 0.5\ \Omega$

$R_t = 50\ \Omega$

$$\left.\begin{aligned} \text{and } m &= 3.048 \text{ m} \\ b &= 45.72 \text{ m} \\ h &= 24.384 \text{ m} \\ a &= 0.473 \text{ cm} \end{aligned}\right\} \text{ refer to equation (3.3)}$$

(corresponds to 70 mm^2 cross-section)

$\therefore \mu = 0.292$ [from equation (3.3)]

$Z_\infty = 3.8\ \Omega$ [from equation (3.16)]

$I_k = 8837$ Amperes [from equation (3.18)]

$I_f^1 = 1163$ amperes [from equation (3.19)]

α per span = .107

$I_0 = 82.5$ amperes

$V_k = 4418.4$ volts

<u>If $\mu = 0$,</u>

$Z_\infty = 5.35\ \Omega$

$I_k = 9145$ amperes

$I_f^1 = 855$ amperes

$I_0 = 85.5$ amperes

$V_k = 4572.5$ volts

<u>Effect of Counterpcise</u>

Using #2 Cu wire, having diameter of 6.54mm, resistance of 0.5247 Ω /km, and soil resistivity ρ of 1000 Ω-m and applying equation (3.26), we get:

$$y_c = 0.3246 \text{ mho/km}$$

$$\therefore 1/y_c = 30.80\ \Omega/\text{km}$$

$$\therefore R_t'(\text{p.u.}) = \frac{12.5 \times 30.80}{12.5 + 20.80} = 8.89\ \Omega/\text{km}$$

$$Z_g'(\text{p.u.}) = \frac{2.27 \times .5247}{2.7947} = .4262\ \Omega/\text{km}$$

$$\therefore Z_{eq} = \sqrt{8.89 \times .4262}\,(1 - .29)$$

$$= 1.38\ \Omega$$

$\therefore I_k = 7340$ amperes

$I_f' = 2660$ amperes

$$\alpha \text{ per span} = \sqrt{\frac{0.4262}{4} \times \frac{1}{8.89 \times 4}} = .054$$

$I_0 = 139.8$ amperes

$V_k = 3670$ volts

The example above clearly shows the reduction of fault current traversing through the substation grid to 8837 Amps. due to coupling alone and further to 7340 Amps. due to coupling and counterpoise both, a substantial difference from the total fault current of 10,000 Amperes had it been assumed to pass fully through the substation grounding grid.

3.7 <u>Summary</u>

An analytical method to determine ground fault current traversing through the substation grid has been developed. Effects of tower, overhead wires and counterpoises have been presented in terms of system parameters. This simplifies the calculations of a specified system. It is shown that a substantial portion of the ground fault current gets diverted via overhead wires and counterpoises. This leads to a significant reduction of the voltage rise of this grid, resulting in economy of copper of the grid required to cater for safe touch and step potentials.

CHAPTER IV

IMPULSE IMPEDANCE OF GROUNDING GRIDS

CHAPTER IV

IMPULSE IMPEDANCE OF GROUNDING GRIDS

4.1 General

The previous chapters deal with the analysis of grounding grids under steady state domain. When grounding grids are subjected to discharge of impulse currents to ground as is the case with substation grids, it is essential to know their impulse impedance (defined on page 3), as it is this value which will determine the initial potential rise of such grids. The duration involved is in microseconds and the human body seems able to tolerate very high currents under lightning surges [5]; however, too high a potential rise can damage the insulation of equipment or cables and can also lead to a high voltage build up on tower top connected to the substation grid resulting in an insulator flashover and thus in a transmission outage.

This chapter deals with performance of such grids while discharging impulse currents to ground, developing expressions for their impulse impedance to enable problems concerning lightning protection and transients under ground faults to be accurately analysed.

Ground behaves as pure resistance at power frequencies. At very high frequencies or steep wave fronts, the self inductance of the grounding grid as well as the displacement currents become predominant. These effects are required to be taken into account in order to determine the impulse impedance of grounding grids. Based on this background, the following sections deal with the basic concepts for such an analysis, followed by development of the equivalent circuit [5,23] for usual types of grounding grids and their subsequent response for impulse inputs. The work is extended to cover the analysis for long buried wires (i.e. counterpoises). Only those values of current inputs which do not cause breakdown of the soil are considered, as these result in higher value of impulse impedance and thus a safer design.

4.1.1 Soil Ionisation and Arcing Discharge

In case of localised grounds (tubes, plates, short bars, etc), the soil surrounding the electrode breaks down in case the voltage gradient at the surface of the conductor exceeds the critical voltage gradient [24,25]. This higher gradient cannot exist in the soil in the neighbourhood of the electrode [1] without giving rise to internal discharges. This effectively extends the electrode radius through this discharge [1,24,25,26] until the gradient at the boundary of this discharge space is decreased to the critical value. This increase in effective dimensions of the electrode results in decrease of ground resistance. Inductance of such electrodes, on account of small length involved, is negligible and has little affect on the impulse impedance.

For grounds not localised, as is the case with usual size and shape of practical grids (e.g. substation type, counterpoises, etc.) dealt with in chapters II and III, the current dissipated per unit length is not very large so that soil ionisation plays very little part in affecting the impulse impedance [24,27]. On the other hand, as pointed above, the self inductance of such grids becomes important on account of their considerable length involved and is the main factor in controlling the impulse impedance.

4.2 Capacitance of Grounding Grids

Like any other substance, earth is not a perfect conductor and has both resistivity and dielectric constant, thus possessing both resistance and capacitance. To evaluate the capacitance, a little reflection on the background of expression for grounding resistance of grids reveals that it is basically obtained [3,6] by finding out the capacitance of the electrode combined with its image above the surface of the earth, the combined electrode being considered as in air. For finding out electrode's capacitance in the earth, the dielectric constant of the earth shall have to be considered.

To evaluate, consider a charged sphere of radius r with charge Q_o, embedded in earth of resistivity ρ and permittivity k. Regarding the earth as a homogeneous medium, equipotential surfaces will be concentric spheres. Therefore D and E at a distance x are:

$$D = Q_o/(4\pi x^2)$$

Since $D = kE$,

$$E = Q_o/(4\pi k x^2)$$

$\therefore$ V i.e., Voltage between the sphere and remote earth

$$= \int_r^\infty \frac{Q_o}{4\pi k x^2}\, dx$$

$$= Q_o/(4\pi k r)$$

and C i.e. Capacitance between the sphere and remote earth

$$= Q_o/V$$

$$= 4\pi k r \qquad (4.1)$$

where, $k = k_o\, k_r$

k_o = Permittivity of evacuated (free) space

$$= \frac{1}{36\pi} \times \frac{1}{10^9} \text{ F/m}$$

(k_r = Variable between 4 and 70, being 9 for ordinary moist soil, 4 for dry soil and 70 for distilled water [28]).

If I_o is the current radiating from this sphere, J_x and E_x at a distance x from its center are:

$$J_x = I_o/(4\pi x^2)$$

$$= \sigma E$$

$$\therefore, \; E_x = \frac{J_x}{\sigma} = \frac{I_o}{4\pi\sigma x^2}$$

where, $\sigma = \frac{1}{\rho}$

$\therefore$, V i.e. Voltage between the sphere and remote earth

$$= \int_r^\infty \frac{I_o}{4\pi\sigma x^2}\, dx = I_o \left(\frac{\rho}{4\pi r}\right)$$

$\therefore$, R i.e. Ground resistance between sphere and remote earth

$$= V/I_o = \frac{\rho}{4\pi r} \qquad (4.2)$$

In above, we have imagined the current entering through a sphere, which is not the real case. This is just for simplicity as we can determine the equivalent diameter of the sphere for any shape of the electrode.

From (4.1) and (4.2), we get:

$$RC = \rho k \qquad (4.3)$$

This is exactly what one would get on considering a small medium of earth limited by two concentric surfaces of spherical equipotential shells, assumed parallel and removed by l, between which flows a current or an electrostatic field develops, as shown [1] in Appendix IV. The resistance and capacitance form a parallel circuit due to the displacement currents being in parallel with the conduction currents in case the electrode voltage changes with time, this being otherwise obvious from the Maxwell equation (4.4) in the next section.

4.3 Equivalent Circuit of Grounding Grids

Here, we will analyse the equivalent circuit [5,23] of the usual size and type of grounding grids (e.g. substation or tower base type) based on lumped circuit parameters.

4.3.1 Basic Considerations

Let us, to begin with, ignore the inductance of the grounding grids and analyse the Maxwell equation below for sinusoidal and exponentially decaying inputs. For this, consider an hemisphere electrode of radius r embedded in earth of conductivity σ and permittivity k and discharging current I_g into the ground. At any point of the medium, J and E for a time varying field are related by (4.4) below:

$$J = \sigma E + k\dot{E}, \text{ where } \dot{E} = dE/dt \qquad (4.4)$$

4.3.1.1 Performance Under Sinusoidal Inputs

If the input current is of the form $e^{i\omega t}$, the electric field strength would also be of the form $e^{i\omega t}$.

$$\therefore,\ J = \sigma E + ki\omega E$$

$$= E\ (\sigma + ki\omega)$$

$$\text{or, } E = \frac{J}{\sigma + ki\omega}$$

$$J = I_o/(2\pi x^2), \text{ at a distance } x$$

Therefore, E_x at a distance x and the potential V of the electrode to remote earth are as follows:

$$E_x = \frac{I_o}{2\pi x^2\ (\sigma + ki\omega)}$$

$$\therefore, V = \int_r^\infty E_x . dx$$

$$= \frac{I_o}{2\pi r\ (\sigma + ki\omega)} \tag{4.5}$$

$$\therefore,\ \text{Admittance} = I_o/V = 2\pi r\sigma + i\omega 2\pi k r \tag{4.6}$$

$$= \text{Conductance} + \text{Susceptance}$$

4.3.1.2 Performance Under Exponential Inputs

If the waveshape of the input current is of the form $e^{-t/T}$, the electric field strength would also be of the same form.

$$\therefore, \; J = \sigma E - \frac{kE}{T}$$

Therefore E_x at a distance x and the potential V of the electrode to remote earth are:

$$E_x = \frac{J_x}{(\sigma - k/T)}$$

$$= \frac{I_o}{2\pi x^2(\sigma - k/T)}$$

$$V = \int_r^\infty E_x . dx$$

$$= \frac{I}{2\pi r(\sigma - k/T)}$$

$$\therefore \; \frac{V}{I} = \frac{1}{2\pi r(\sigma - k/T)} \qquad (4.7)$$

From equation (4.5), it is evident that for commercial power frequencies and with usual values of resistivities, earth will behave as a pure dc resistance, as the product $k\omega$ is infinitesimal ($k = \frac{1}{36\pi \times 10^9}$ x dielectric constant of earth relative to free space). It is also clear from equation (4.7) that the impulse impedance will be higher for exponential inputs due to transient portion of short circuit currents.

In above, we have excluded the effect of inductance so far. As a matter of fact, the currents in electrode and ground form a magnetic field and it is the strongest where the current is most concentrated [23] namely around the electrode. Therefore, the inductance of the electrode or grid is mainly governed by the distribution of the current in the electrode or grid.

Thus, for an extremely rapid discharge to the ground, as is evident from equation (4.4) and (4.6), a considerable displacement current in the capacitance of the ground will parallel with the conduction current in the resistance and together with the self inductance of both of these currents through the grid will effect and equivalent circuit [5,23] as shown in Fig. 4.1:

4.4 Response of Equivalent Circuit to Impulse Inputs

4.4.1 Idealised Lightning Wave Input

Let us apply an idealised lightning current $i_{(t)} = I.t$ (i.e. ramp type) as input to the equivalent circuit, shown in Fig. 4.2 below:

Case I: Neglecting Capacitance

The voltage $V_{(t)}$ across the circuit to remote earth and the impulse impedance $Z_{(t)}$ are obtained as follows:

$$V_{(t)} = Ri + L\frac{di}{dt} \quad (i_R = i, \text{ as } C \text{ is ignored})$$

$$= R\,(I.t) + LI$$

$$\therefore Z_{(t)} = R + \frac{L}{t}, \quad t > 0 \tag{4.8}$$

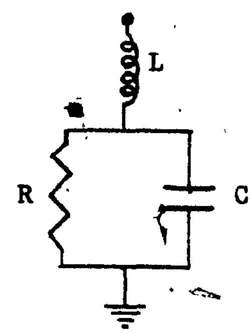


FIGURE 4.1

EQUIVALENT CIRCUIT OF GROUNDING GRID

FOR IMPULSE CURRENT INPUTS

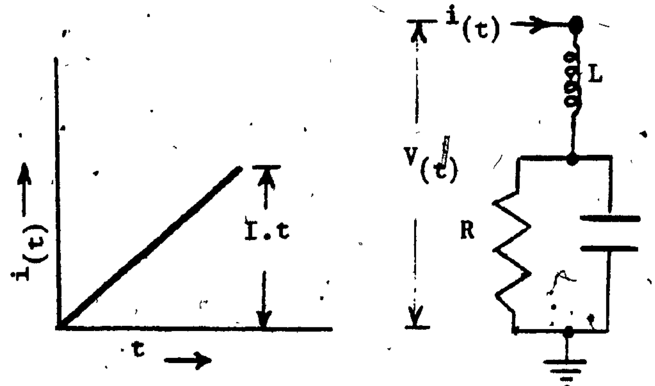


FIGURE 4.2

RAMP INPUT

It may be noted that the dimension of L/t is ohm-sec./sec. i.e. ohms, and the result (4.8) is identical to the one obtained by Vanier and al [29] neglecting leakage for earthings which are not long.

Case II: Including Capacitance

The voltage $V_{(t)}$ to remote earth and $Z_{(t)}$ are obtained as follows. From Figure 4.2,

$$i = i_R + i_C, \text{ and}$$

$$\frac{1}{C}\int i_C \, dt = i_R R$$

$$\text{or, } i_C = RC \cdot \frac{di_R}{dt}$$

$$\text{or, } i - i_R = RC \cdot \frac{di_R}{dt}$$

Taking Laplace transform, we get:

$$i_R(p) = \frac{i_{(p)}}{pRC + 1} = \frac{i_{(p)}}{RC(p+1/RC)}$$

$$\text{Also, } V_{(t)} = Ri_R + L\frac{di}{dt}, \quad t>0$$

$$\text{or, } V_{(p)} = Ri_R(p) + Lpi_{(p)}$$

$$= i_{(p)} \left[\frac{1}{C(p+1/RC)} + Lp\right] \tag{4.9}$$

$$= I \left[\frac{1}{p^2 C(p+1/RC)} + \frac{L}{p}\right]$$

Taking inverse Laplace transform [30], we get:

$$V_{(t)} = I\left[\frac{1}{C.1/(R^2C^2)}\left(e^{-t/RC} + \frac{t}{RC} - 1\right) + L\right]$$

$$\therefore, Z_{(t)} = \frac{V_{(t)}}{I.t}$$

$$= R + \frac{L}{t} + \frac{R^2C}{t}\left(e^{-t/RC} - 1\right) \quad (4.10)$$

If $t \to \infty$, $Z_{(t)} \to R$

If $C=0$, $Z_{(t)} = R + \frac{L}{t}$, a result identical to (4.8), $t>0$

4.4.2 Double Exponential (Lightning Waveshape) Input

Let us apply a double exponential wave $i_{(t)} = I\,(e^{-\alpha t} - e^{-\beta t})$ i.e. the practical form of the lightning impulse waveshape as input to the aquivalent circuit, shown in Fig. 4.3 below:

Case I: Neglecting Capacitance

The voltage $V_{(t)}$ and $Z_{(t)}$ are as follows:

$$V_{(t)} = Ri + L\frac{di}{dt} \; (i_R = i, \text{ as C is ignored})$$

$$= RI\,(e^{-\alpha t} - e^{-\beta t}) + LI\,(\beta e^{-\beta t} - \alpha e^{-\alpha t}), \quad t>0$$

$$\therefore Z_{(t)} = R + \frac{L\,(\beta e^{-\beta t} - \alpha e^{-\alpha t})}{e^{-\alpha t} - e^{-\beta t}}$$

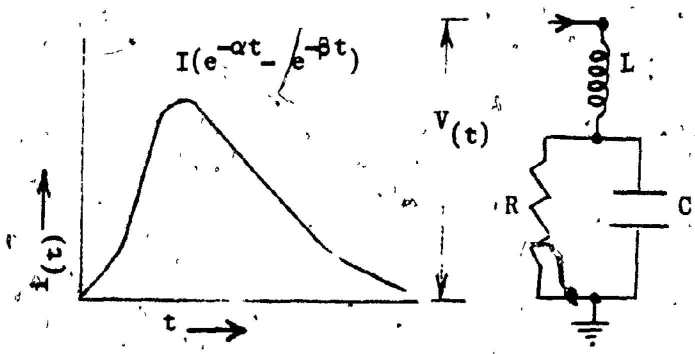


FIGURE 4.4

DOUBLE EXPONENTIAL WAVE INPUT

$$= R + \frac{L\left[\beta(1-\beta t) - \alpha(1-\alpha t)\right]}{(1-\alpha t)-(1-\beta t)} \quad \text{(for small } t>0)$$

$$= R + \frac{L\left[(\beta-\alpha)-t(\beta+\alpha)(\beta-\alpha)\right]}{t(\beta-\alpha)}$$

$$= R + \frac{L}{t}\left[1 - (\beta+\alpha)\, t\right], \quad t>0 \qquad (4.11)$$

Case II: Including Capacitance

From equation (4.9), we have:

$$V_{(p)} = i_{(p)}\left[\frac{1}{C(p+1/RC)} + Lp\right]$$

$$= I\left(\frac{1}{p+\alpha} - \frac{1}{p+\beta}\right)\left[\frac{1}{C(p+1/RC)} + Lp\right]$$

$$= I\left[\frac{1}{C(p+\alpha)(p+1/RC)} - \frac{1}{C(p+\beta)(p+1/RC)} + \left(\frac{Lp}{p+\alpha} - \frac{Lp}{p+\beta}\right)\right]$$

Taking inverse Laplace transform [30], we get:

$$\therefore V_{(t)} = I\left[\frac{e^{-\alpha t}-e^{-t/RC}}{C(1/RC-\alpha)} - \frac{e^{-\beta t}-e^{-t/RC}}{C(1/RC-\beta)} + L(\beta-\alpha)\left\{\frac{\beta e^{-\beta t}-\alpha e^{-\alpha t}}{\beta-\alpha}\right\}\right]$$

$$= I\left[\frac{R(e^{-\alpha t}-e^{-t/RC})}{1-\alpha RC} - \frac{R(e^{-\beta t}-e^{-t/RC})}{1-\beta RC} + L\left(\beta e^{-\beta t} - \alpha e^{-\alpha t}\right)\right]$$

$$= I\left[\frac{R\,(e^{-\alpha t}-e^{-\beta t})+R^2C(\alpha e^{-\beta t}-\beta e^{-\alpha t})+R^2Ce^{-\frac{t}{RC}}(\beta-\alpha)}{(1-\alpha RC)\,(1-\beta RC)}+\right.$$

$$\left. L(\beta e^{-\beta t}-\alpha e^{-\alpha t})\right]$$

$$\therefore Z_{(t)}=\frac{V_{(t)}}{I_{(t)}}\simeq\left[\frac{R}{(1-\alpha RC)(1-\beta RC)}+\frac{R^2C\{\alpha(1-\beta t)-\beta(1-\alpha t)+e^{-\frac{t}{RC}}(\beta-\alpha)\}}{(1-\alpha RC)(1-\beta RC)(1-\alpha t-1+\beta t)}+\right.$$

$$\left.\frac{L\{\beta(1-\beta t)-\alpha(1-\alpha t)\}}{1-\alpha t-1+\beta t}\right],\ \text{for small } t>0$$

$$=\frac{R}{(1-\alpha RC)(1-\beta RC)}+\frac{R^2C(e^{-t/RC}-1)}{t(1-\alpha RC)(1-\beta RC)}+\frac{L}{t}\left[1-(\beta+\alpha)\,t\right] \qquad (4.12)$$

If $t=\infty$, $Z_{(t)}\rightarrow\frac{R}{(1-\alpha RC)(1-\beta RC)}-L\,(\beta+\alpha)$

If $C=0$; $Z_{(t)}=R+\frac{L}{t}\left[(1-(\beta+\alpha)t\right]$, a result identical to (4.11), $t>0$

4.4.3 Impulse Impedance Accounting for Off-Flow of Impulse Current

In the expressions for $Z_{(t)}$ as shown in sections 4.4.1 and 4.4.2 above, leakage on account of off-flow of impulse currents along the length of the conductor has been ignored. To account for this, as shown in references [29,31], the effective inductance of long earthings is one third of the total inductance of the earthing, thus modifying L as appearing in expressions (4.8), (4.10), (4.11) and (4.12) to L/3, e.g., expressions (4.10) and (4.12) become:

$$Z_{(t)} = R\left[1 + \frac{L}{3Rt} + \frac{R\,C}{t}\,(e^{-t/RC} - 1)\right] \tag{4.13}$$

$$Z_{(t)} = R\left[\frac{1}{(1-\alpha RC)(1-\beta RC)} + \frac{L}{3Rt}\,[1-(\beta+\alpha)t] + \frac{R\,C(e^{-t/RC}-1}{t(1-\alpha RC)(1-\beta RC)}\right] \tag{4.14}$$

The value of R can be calculated by the usual methods, as indexed in paper [11]. The inductance L can be evaluated as described in the next section. Also a little thought to the middle term in expressions (4.13) and (4.14) indicates that $Z_{(t)}$ would be significantly higher if t is $< \frac{1}{3} \cdot \frac{L}{R}$ (i.e. the effective inductive time constant) which would be the case if length (and hence inductance) involved is not too small and for higher values of t, $Z_{(t)}$ starts decaying to R. It may also be noted that time to the wavefront of the lightning impulses is generally far greater than R.C i.e. the capacitive time constant (= ρk) for resistivity of soils usually encountered in practice.

4.5 Inductance of Electrodes and Grids

Inductance L, appearing in expressions for $Z_{(t)}$ can be calculated as follows:

4.5.1 Inductance of a Rod

Inductance of a ground rod for practical length ℓ (meters) and radius a (meter) can be calculated [5,23] through equation (4.15) as shown below:

$$L = 2\ell \quad \ln\left(2\frac{\ell}{a}\right) \times 10^{-7},\ H \tag{4.15}$$

The voltage developed at the feeding point based on his operational analysis on transmission line formula is given by expression (VII.9). But it is a slowly convergent series especially for very short time, for which we are presently interested, and computation is rather time-consuming. This has also been dealt by Rudenberg [23], excluding effect of wire resistance.

The present section summarises and indexes the important results and is a step forward in studying this problem further, duly taking all the parameters into account and establish expressions for the following:

i) Current distribution along the wire.

ii) General expression for surge impedance.

iii) Impulse impedance at time corresponding to the beginning of the steep ascent of the impulse input.

iv) Reduction of surge impedance to steady-state terminal impedance.

This sending-end surge impedance, with passage of time, gets reduced to the usual steady-state value for the grounding resistance.

4.7.2 Performance of Buried Ground Wire

One is quite familiar with the usual telephone and telegraph equations involving current and voltage distribution along an overhead transmission line. These will be equally valid for a ground wire so long as we replace the leakage conductance per unit length by that equal to reciprocal of distributed ground resistance. Consider a buried wire of length ℓ meters with the parameters shown on the Fig. 4.6, where:

$R' = R^1 . \ell$ For R^1, see expression (VII.2)

Fig. 4.6 shows the remaining data, i.e., the wire current I in amperes and the ground current i in amperes per meter.

Applying Kirchhoff's laws to section Δx,

$$\Delta e = -(r' \Delta x)\, I - L' (\Delta x) \frac{\partial I}{\partial t}$$

$$\text{or,}\quad \frac{\partial e}{\partial x} = -r'I - L' \frac{\partial I}{\partial t},\ \Delta x \to 0 \qquad (4.25)$$

$$\text{and,}\quad \Delta I = -(g\, \Delta x)\, e - (C' \Delta x) \frac{\partial e}{\partial t}$$

$$\text{or}\quad \frac{\partial I}{\partial x} = -ge - C' \frac{\partial e}{\partial t},\ \Delta x \to 0 \qquad (4.26)$$

These equations (4.25 and 4.26) give rise to the usual telephone and telegraph equation below [40]:

$$\frac{\partial^2 I}{\partial x^2} = L'C' \frac{\partial^2 I}{\partial t^2} + (r'C' + L'g) \frac{\partial I}{\partial t} + r'gI \qquad (4.27)$$

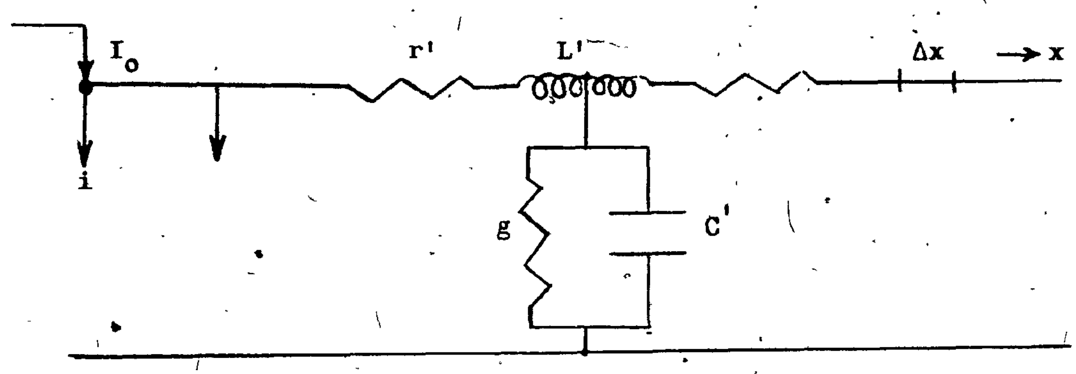


FIGURE 4.6

BURIED GROUND WIRE CURRENT DISTRIBUTION

Neglecting capacitance, this reduces to:

$$\frac{R'}{L'}\;\frac{\partial^2 I}{\partial x^2} - \frac{\partial I}{\partial t} - \frac{r'}{L'}\;I = 0 \tag{4.28}$$

Let, $u = I\,e^{\frac{r'}{L'}.t}$ and on substituting this in (4.28), we get:

$$\frac{\partial^2 u}{\partial x^2} - \frac{L'}{R'}\;\frac{\partial u}{\partial t} = 0 \tag{4.29}$$

This is a well-known equation of heat diffusion. If we solve it through separation of variables, it gives rise to solution involving Fourier series and the solution converges slowly with respect to orthogonal functions, particularly at the terminal feed point of the stroke and for the short time involved. The boundary conditions of present interest are simply that the total current is zero until zero time and, thereafter, a sudden short impulse defined by expression below is applied:

$$\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \delta\,(t-0)\;\delta\,(x-0)\;dt\;dx = 1 \tag{4.30}$$

Applying this unit impulse to the circuit governed by equation (4.29), we get:

$$\frac{\partial^2 u}{\partial x^2} - \frac{1}{m}\cdot\frac{\partial u}{\partial t} = \delta t.\delta x, \text{ where } m = R'/L' \tag{4.31}$$

Let G be the Green Function of equation (4.29) which is also the fundamental solution of equation (4.31). Laplace transform on (4.31) gives:

$$\frac{d^2 G}{dx^2} - \frac{p}{m}\;G = \delta x$$

Solution of this equation, subject to the condition that $G\,(x, t)$ is bounded for infinite x is given by:

$$G\,(x, p) = \frac{1}{2} \cdot \frac{1}{\sqrt{p/m}}\; e^{-\sqrt{\frac{p}{m}}\cdot x},\; 0 \leqslant x \leqslant \infty,\; t > 0 \qquad (4.32)$$

or $$G\,(x, t) = \frac{1}{2} \cdot \frac{\sqrt{m}}{\sqrt{\pi t}}\; e^{-\frac{x^2}{4mt}} \text{ (see reference [41])} \qquad (4.33)$$

On substitution of this in equation (4.29), it can be easily verified to be its solution.

$$\therefore\; I\,(x, t) = \frac{1}{2} \cdot \sqrt{\frac{m}{\pi t}}\; e^{-\left(\frac{x^2}{4mt} + \frac{r't}{L'}\right)} \qquad (4.34)$$

$$\begin{aligned} I\,(x, t) &= 0 \text{ for } t = 0,\; x \neq 0 \\ &= \infty \text{ for } t = 0,\; x = 0 \end{aligned}$$

4.7.3 Impulse Impedance

The ground current i (being equal to g . e) is related to the current traversing through the wire as per equation (4.26), i.e.:

$$\frac{\partial I}{\partial x} + i = 0 \qquad (4.35)$$

The voltage E at every point of the wire is determined by the ohmic drop of the ground current i in the ground resistance R', and is:

$$E = R'i = R' \left(-\frac{\partial I}{\partial x}\right)$$

$$= \frac{L'x}{2t} \quad I$$

$$\therefore \text{Surge Impedance } Z_{(x,\ t)} = \frac{L'x}{2t} \tag{4.36}$$

From this, it must be noted that the hyperbolically diminishing transient impedance for short impulse current is infinite only for zero length of time - a mathematically indeterminate condition. Actually, an exactly perfect impulse current application at zero time can never be realized experimentally; and in practice too, lightning impulse will traverse some distance down the tower to strike the terminal point of the buried wire, but the mathematical convenience of considering the impulse is justified as a very close approximation to such practical cases and is very instructive from a physical viewpoint in understanding.

Figure 4.7 below shows the variation of current I in the wire in relation to time t from the terminal for several fixed distances, based on equation (4.34).

This characteristic, based upon report of test results, can be approximated to the time characteristic of a lightning discharge. If t_o is the time corresponding to the beginning (horizontal slope) of the steep ascent of the current, put $\frac{dI}{dt} = 0$ for equation (4.34) and this gives:

$$\frac{x^2}{t_o} = \frac{4R'}{L'} \left[\frac{1}{2} + \frac{r't_o}{L'}\right] \tag{4.37}$$

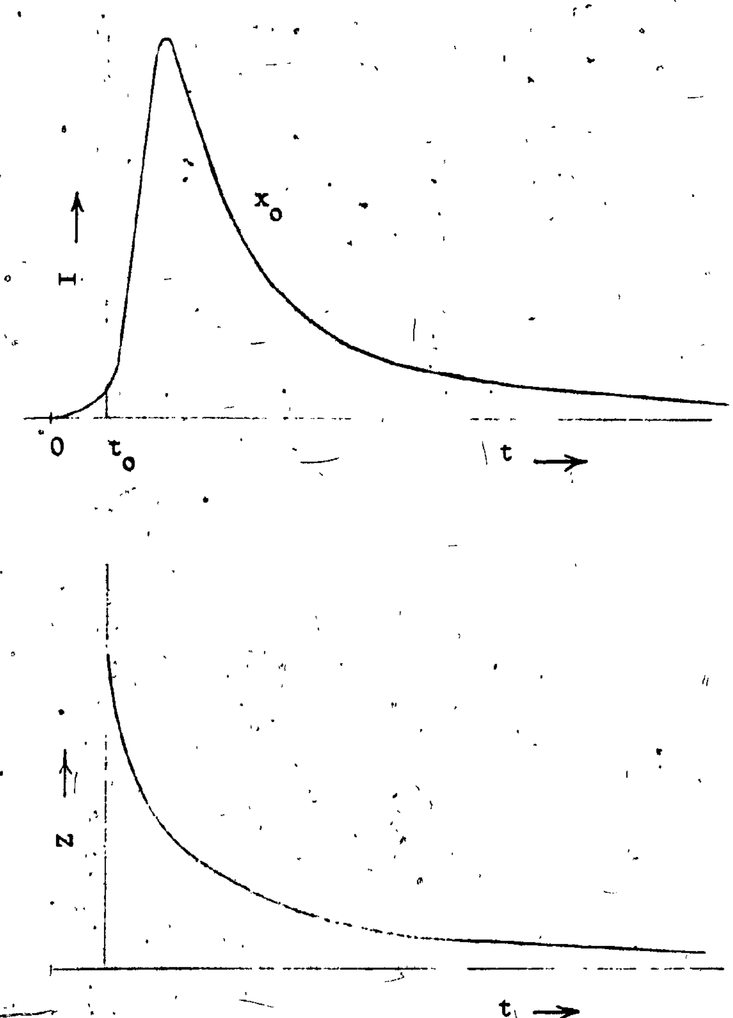


FIGURE 4.7

i) VARIATION OF CURRENT I WITH TIME t

ii) VARIATION OF IMPEDANCE $Z_{(x,t)}$ WITH TIME t

This equation also represents the location of wave-front points of curves [23] between I and x, which can be plotted for fixed instants of time. The curves are all of the same shape and in fact are similar to the half of the hump of the normal error or probability curves. These points travel with velocity v given below:

$$v = \frac{dx}{dt} = \frac{2R'}{x_o L'} \cdot \left[\frac{1}{2} + \frac{2r't_o}{L'}\right] \tag{4.38}$$

where, x_o is the corresponding distance to time t_o.

Distance x_o travelled in time t_o will be:

$$x_o = v \cdot t_o$$

$$= \frac{1}{x_o} \left[\frac{R't_o}{L'} + \frac{4R'r't_o^2}{L'^2}\right] \tag{4.39}$$

If r is neglected, these wave-front points travel with velocity proportional to $1/\sqrt{t}$ and penetrate distance x proportional to $\sqrt{t}$, as is obvious from (4.38) and (4.39).

The surge impedance Z_o at time t_o, as obtained from equation (4.36) is:

$$Z(x_o, t_o) = \sqrt{\frac{L'R'}{4t_o} + R'r'} \tag{4.40}$$

It may be noted that this result is for a short unit impulse, but since it is a ratio, it is equally applicable for magnitudes of a similar wave-shape.

Steady-State Value for Grounding Resistance of a Long Buried Electrode

From equation (4.34), one can write the following operational expression [42]:

$$I_{(x,\ p)} = \frac{e^{-\sqrt{\frac{x^2}{m}\left(p+\frac{r'}{L'}\right)}}}{\sqrt{p+\frac{r'}{L'}}} \cdot \frac{\sqrt{m}}{2}\,, \quad t > 0, \quad 0 \leq x \leq \infty \tag{4.41}$$

$$\therefore\ E_{(x,\ p)} = R'\,\frac{\partial I_{(x,\ p)}}{\partial x}$$

$$= \frac{R'}{2}\, e^{-\sqrt{\frac{x^2}{m}\left(p+\frac{r'}{L'}\right)}}$$

$$= \frac{R'}{\sqrt{m}}\, I_{(x,\ p)} \cdot \sqrt{p+\frac{r'}{L'}} \tag{4.42}$$

$$\therefore\ Z_{(x,\ p)} = \frac{E_{(x,\ p)}}{I_{(x,\ p)}} = \sqrt{R'L'p + R'r'} \tag{4.43}$$

For steady-state condition, operation p and L' both will be non-existant. This is also obvious from equation (4.28), the solution of which for an infinite wire gives value for the terminal impedance [5,18] as $\sqrt{R'r'}$, while its value for a finite electrode is [5,18,21] $Z = \sqrt{R'r'}\ \coth\left(\sqrt{\frac{r'}{R'}}\cdot \ell\right)$. For long electrode, this reduces to $\sqrt{R'r'}$. If ℓ is small, it reduces to R'/ℓ, i.e. $1/g\ell$, as is evident from expression (VII.9) in the Appendix VII.

4.8 Example

In order to illustrate the above, the following example is worked out.

4.8.1 Impulse Impedance of a Buried Wire

For a wire of length $\ell = 75$ m, radius $a = 0.5$ cm, buried in ground of $\rho = 1000\ \Omega$-m and $k_r = 9$, the various values are:

$$L = 1545 \times 10^{-7}\ \text{H.}$$

$$R = \frac{\rho}{\pi\ell}\ \ln\ \left(\frac{2\ell}{a}\right)\ \text{ohms} = 43.75\ \text{ohms}$$

$$RC = .08 \times 10^{-6}\ \text{sec.}$$

$$L/R = 3.53 \times 10^{-6}\ \text{sec.}$$

For wave peak time t of 1.2 μsec.

$$Z_{(t)} = 83.75\ \text{ohms. from (4.13)}$$

This value of $Z_{(t)}$ is 92 Ω as obtained from (4.40) and is of the same order as that obtained above by a different approach.

4.8.2 Impulse Impedance of a Hollow Square Mesh

For a mesh of 30m x 30m with radius of wire as 0.5 cm, buried in ground of $\rho = 1000\ \Omega$-m and $k_r = 9$, the various values are:

$$L = 605 \times 10^{-7}\ \text{H, from (4.19)}$$

$R = \frac{\rho}{4r}$ (r = radius of equivalent circle for the area occupied by the grid)

$= 14.77$ ohms

$\therefore RC = 0.08 \times 10^{-6}$ sec.*

For a wave peak time of 1.2 μsec.,

$Z_{(t)} = 30.57$ ohms, from (4.13)

Repeating the calculation for the same size of grid and wave peak time as above but divided into several square meshes, the values of L are 535×10^{-7}, 450×10^{-7}, 401.3×10^{-7} and 392×10^{-7} H respectively for 4, 16, 36 and 64 meshes, affording values of $Z_{(t)}$ as 28.6, 26.3, 24.9 and 24.67 ohms respectively.

4.8.3 Impulse Impedance of Hollow Rectangular Mesh

For a 30m x 20m mesh, L is 500×10^{-7} H and $Z_{(t)}$ would be 26.7 Ω.

4.9 Summary

In previous chapters, the analysis of grounding grids dealt with ground-fault currents at power frequency, for which ground behaves as pure resistance. In this chapter, the analysis deals with performance of grids, of the types provided at high voltage substations, when subjected to discharge of impulse inputs similar to lightning waveshape. Their equivalent circuit is formulated and analysed to develop expressions for their impulse impedance, which is shown to be greater than their power frequency grounding resistance and decreasing with passage of time to the latter value. It is the impulse impedance which must be used to determine the potential at the top of the tower, connected to such grids, in order to determine occurrence of any insulator flashover under discharge of a lightning stroke. A suitable design so as to attain a low value for the impulse impedance of a substation grid will reduce thransmission line outages resulting from such a flashover.

CHAPTER V

CONCLUSIONS

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CONCLUSIONS

5.1 Summary

Some useful elements, mainly the intake structure and water, and a nearby deposit of low resistivity material of sufficient dimensions have been shown to be effective in reducing the overall ground resistance of a hydro-electric plant located on soils with extremely high resistivity. Different methods have been compiled and suitably applied to calculate the resistance of the above and other elements of such plants and have proved valuable in choice of the most efficient grounding paths. A new method for calculation of ground resistance of grids in confined soft soils is also developed. These methods are general and can be applied to identical situations. Realistic results are obtained if careful measurements to determine the resistivities are made. The resistivity of concrete for instance, will vary with the amount of water it contains; resistivity of the concrete of a dam in contact with water is not the same as that for a powerhouse built on rock.

The design of a grid is essentially based on the amount of ground fault current traversing through the same and an analytical procedure, as to how to determine the ground fault current distribution in case of a fault at the terminal tower at which the grounding grid of a substation is connected to the earth wire, is developed. The work outlines the effects of coupling and counterpoise on the current distribution and shows a quantitative reduction of the fault current traversing through the substation grid and of the grid potential rise to remote earth.

The present work, besides dealing with the steady-state analysis outlined above, deals with fundamental concepts involved in studying the impulse impedance of the practical types of grids and establishes expressions for its evaluation as well. The impulse impedance is shown to have higher value to begin with and diminishes finally to the steady-state (i.e. power frequency) grounding resistance. Its initially higher value, apart from raising the substation ground potential, has impact on the tower insulator flashover and the best way to avoid this is to keep the impulse impedance down through suitable design of the grid or electrode. It is also shown that the governing factor in controlling the impulse impedance is the inductance of the grid. An expression each for evaluation of the inductance of a hollow square and rectangular grid is developed and it is shown that the inductance for the usual size of a substation square grid upto 64 meshes reduces to a value $\geq 67\%$ of that for the equivalent hollow square grid. This is expected to hold even for a rectangular grid. The grids of the usual types viz substation or tower base types i.e. the concentrated types have been analysed through their equivalent circuit and the grid in the form of a counterpoise has been dealt with through analysis of the resulting ladder type network, duly taking into account the leakage along the wire. The impulse impedance for counterpoise is shown to have the highest value at the beginning of the ascent of the wave-front and is directly dependant upon its parameters.

5.2 Scope for Future Work

Further investigation can be carried out in the area of ground-fault current distribution in transmission line towers when a ground fault takes place somewhere on the line, considering the sections on each side of the fault as combinations of infinite or finite sections. The effect of varying transmission tower grounds can also be included in case the line is located along continuous blocks of soil with varying resistivity.

Under impulse inputs to grids, inductance is shown to be an important factor in controlling their impulse impedance. A systematic study to evaluate the same for different geometrical configurations possible for all types of grids can be conducted for inputs at different locations on the grid.

REFERENCES

[1] Laurent, P.G., "Les bases générales de la technique des mises à la terre dans les installations électriques", Bulletin de la Société Française des Electriciens, 7ième série, tome 1, no. 7, July 1951, pp. 368-402.

[2] Schwarz, S.J., "Analytical Expressions for the Resistance of Grounding Systems", AIEE Transactions, Vol. 73, part III-B, 1954, pp. 1011-1016.

[3] Dwight, H.B., "Calculations of Resistances to Ground", AIEE Transactions, Vol. 55, Dec. 1936, pp. 1319-1328.

[4] Fagan, E.J. and Lee, R.H., "The Use of Concrete-Enclosed Reinforcing Rods as Grounding Electrodes", I.E.E. Transactions on Industry and General Applications, Vol. IGA-6, No. 4, July/August 1970, pp. 337-348.

[5] "Guide for Safety in A.C. Substations Grounding", IEEE Standard #80, 1961.

[6] Tagg, G.F., Earth Resistances, George Newnes Limited, London, Great Britain, 1964.

[7] Endreyni, J., "Evaluation of Resistivity Tests for Design of Station Grounds in Nonuniform Soil", AIEE Transactions, Dec. 1963, pp. 966-970.

[8] Sunde, E.D., "Surge Characteristics of a Buried Bare Wire", AIEE Transactions, Vol. 59, pp. 987-991.

[9] Bellaschi, P.L. and Armington, R.E., "Impulse and 60-cycle Characteristics of Driven Grounds-III, Effect of Lead in Ground Installation", AIEE Trans., Vol. 62, 1943, pp. 334-45.

[10] AIEE Committee Report, "Voltage Gradients Through the Ground Under Fault Conditions", AIEE Transactions, Oct. 1958, pp. 669-692.

[11] Verma, R., Mérand, A. and Barbeau, P., "Design of a Low Resistance Grounding System for a Hydro-Electric Plant Located on Highly Resistive Soils", IEEE Trans. PA&S, Vol. PAS-97, #5, Sept. /Oct. 78, pp. 1760-68.

[12] Dawalibi, Farid and Mukhedkar, Dinkar, "Ground Fault Current Distribution in Power Systems",The Necessary Link", Paper # A 77 754-5 presented at IEEE Summer Meeting, 1977, Mexico City.

[13] Verma, R. and Mukhedkar, D., "Ground Fault Current Distribution

in Substation, Towers and Ground Wire", IEEE Transactions, PA&S, Vol. PAS-98, #3, May/June 79, pp. 724-730.

[14] Verma, R. and Mukhedkar, D., "Fundamental Considerations and Impulse Impedance of Grounding Grids", Paper #F 79-630-5 presented at IEEE Summer Meeting, 1979, Vancouver.

[15] Verma, R. and Mukhedkar, D., "Impulse Impedance of Buried Ground Wire", IEEE Transactions PAS Paper #F 79-201-5, presented at IEEE Winter Meeting, 1979, N.Y.

[16] Felici, N., "Electrostatique, Etude de Champ Electrique et Applications", Guathier-Villars & Cie, Editeurs-Imprimeur, Paris, 1962.

[17] Electrical Transmission and Distribution Reference Book, East Pittsburgh, Pa.: Westinghouse Electric Co., 1950, p. 41.

[18] Rudenberg, R., "Transient Performance of Electric Power Systems", N.Y. McGraw-Hill, 1950, pp. 357-366.

[19] Kaplan, W., "Ordinary Differential Equations", Addison Wesley Publishing Co. Inc., pp. 145-147.

[20] Endrenyi, Janos, "Analysis of Tower Potentials During Ground Faults", IEEE Transactions in PAS, Vol. PAS-86, No. 10, Oct. 1967.

[21] Alger, Philip L., "Mathematics for Science and Engineering", McGraw Hill Book Co., Inc. 1957, N.Y.

[22] Sunde, E.D., "Earth Conduction Effects in Transmission Systems", N.Y. Van Nostrand, 1949, pp. 66-73.

[23] Rudenberg, Reinhold, "Electric Shock Waves in Power Systems", Harvard University Press, Cambridge, Massachussetts 1968, pp. 273-277.

[24] Berger, K., "Les Comportement des Prises de terre sous courants de Choc de Grande Intensité", CIGRE Report 215, 1946.

[25] Bellaschi, P.L. and Armington, R.E., "Impulse and 60-Cycle Characteristics of driven Grounds-II", AIEE. Trans. Vol. 61, 1942, pp. 349-63.

[26] Norinder and Petropoulos, "Characteristics of Pointed Electrodes", CIGRE Paper No. 310, 1948 Session.

[27] Gupta, B.R. and Thaper, B., "Impulse Impedance of Grounding Grids", IEEE PES. Winter Meeting, Paper No. F-79-700-6, N.Y.

[28] Fortesque, Charles L.G., "Counterpoises for Transmission lines", AIEE Lightning Reference Book, p. 1087-1095.

[29] Vainer, A.L, and Floru, V.N., "Experimental Study and Method of Calculation of the Impulse Characteristics of Deep Earthings", Elektrichestvo (USSR), No. 5, 18-22, 1971, pp. 79-90.

[30] Greenwood, Allan, "Electrical Transients on Power Systems", Wiley-Interscience - a Division of John Wiley & Sons, Inc., N.Y., 1971, pp. 524-525.

[31] Annenkov, V.Z., "Calculating the Impulse Impedance of Long Earthing in Poor-Conducting Ground", Elektrichestvo (USSR), No. 11, 59-65, 1974, pp. 62-78.

[32] Grover, F.W., "Inductance Calculations", D Van Nostrand Co., N.Y., 1946.

[33] Gupta, B.R. and Thapar, B., "Impulse Impedance of Grounding Grids", IEEE Transactions Paper #F 79-700-6 for presentation at the PES Winter Meeting 1980, N.Y.

[34] Davis, R. and Johnston, J.E.M., "The Surge Characteristics of Tower and Tower-footing Impedances", IEE Proceedings, 88, Part II, 1941, pp. 453-465.

[35] Bewley, L.V., "Travelling Waves on Transmission Systems", John Wiley & Sons, Inc., N.Y. 1951, pp. 198, 211-212.

[36] Wagner, C.F. and Hileman, A.R., "A New Approach to the Calculation of the Lightning Performance of Transmission Lines III-A Simplified Method: Stroke to Tower", AIEE Trans. PA&S, Vol. 79, Oct. 1960, pp. 589-603.

[37] Sargent, M.A. and Darveniza, M., "Tower Surge Impedance", IEEE Trans. PA&S, Vol. PAS-88, No. 5, May 1969, pp. 680-687.

[38] Kawai, M., "Studies of the Surge Response on a Transmission Line Tower", IEEE Trans., PA&S, Vol. 83, Jan. 1964, pp. 30-34.

[39] Gupta, B.R. and Thapar, B., "Impulse Impedance of Grounding Systems", IEEE PES Summer Meeting, Paper No. A-78-563-9, L.A., Ca.

[40] Reddick, H.W. and Miller, F.H., "Advanced Mathematics for Engineers", John Wiley & Sons, Inc., N.Y., 1938, pp. 263-264.

[41] Carslaw, H.S. and Jaeger, J.C., "Conduction of Heat in Solids", Oxford University Press, 1959, p. 494.

[42] Bohn, E.V., "The Transform Analysis of Linear Systems", Addison-Wesley Publishing Co., Inc., Reading, Mass. Pal Alto. London, 1963, p. 339.

APPENDIX I

ANALYTICAL EXPRESSIONS FOR RESISTANCE OF GROUNDING SYSTEMS

ρ = resistivity of homogeneous medium, Ω-m

ρa = apparent resistivity due to two layers of different resistivity, Ω-m

R = resistance to remote earth of a grid, Ω

A. Resistance of Grounding Grids

1. Resistance of a ground grid occupying the same area as an equivalent circular plate from Laurent [1]:

$$R = \frac{\rho}{4r} + \frac{\rho}{\ell} \tag{I.1}$$

ℓ = total length of the buried conductor, m

r = radius in meters of a circular plate occupying the same area as the grid, m

2. Resistance of an intermeshed grounding network from Schwarz [2]:

$$R = \frac{\rho}{\pi\ell}\left[\ln\frac{2\ell}{a'} + k_1\frac{\ell}{\sqrt{A}} - k_2\right] \tag{I.2}$$

ℓ = total length of all conductors in meters

$a' = \sqrt{2az}$ for conductors buried at a depth of z in meters, or

a' = a for conductors on earth surface

2a = diameter of conductor, m

A = area covered by conductors, m^2

k_1 and k_2 are co-efficients depending upon the dimensions of the area (refer to paper [2]).

B. Ground Resistance of a Loop of Wire, a Circular Plate or a Rectilinear Electrode

1. Ground resistance of a ring of wire from Dwight [3]:

$$R = \frac{\rho}{2\pi^2 D}\left[\ln\frac{8D}{d} + \ln\frac{4D}{S}\right] \qquad (I.3)$$

D = diameter of the ring, m

d = diameter of the wire, m

S/2 = depth of the ring, m

2. Ground resistance of a vertical circular plate from Dwight [3]:

$$R = \frac{\rho}{8a} + \frac{\rho}{4\pi S}\left[1 + \frac{7}{24}\frac{r^2}{S^2} + \frac{99}{320}\frac{r^4}{S^4} + \ldots\right] \qquad (I.4)$$

r = radius of the plate, m

S/2 = depth, m

3. Ground resistance of a vertical or horizontal circular plate from AIEE [5] Guide 80:

$$R = \frac{\rho}{8r}\left[1 + \frac{r}{2.5h + r}\right] \qquad (I.5)$$

r = radius of the plate, m

h = depth of the center of the plate, m

4. Ground resistance of a rectilinear electrode from AIEE [5] Guide 80:

$$R = \frac{0.366\,\rho}{\ell}\left[\log\frac{3\ell}{2d} + \log\frac{3\ell}{8h}\right] \qquad (I.6)$$

ℓ = length of electrode, m

h = depth below surface, m

d = diameter of the electrode, m

C. <u>Ground Resistance of a Buried Horizontal Wire</u>

1. Ground resistance of a buried horizontal wire from Dwight [3]:

$$R = \frac{\rho}{4\pi\ell}\left[\ln\frac{4\ell}{a} + \ln\frac{4\ell}{S} - 2 + \frac{S}{2\ell} - \frac{S^2}{16\ell^2} + \frac{S^4}{512\ell^4} \cdots\right] \qquad (I.7)$$

2ℓ = length, m

S/2 = depth, m

a = radius, m

2. Ground resistance of a straight horizontal wire from Schwarz [2]:

$$R = \frac{\rho}{\pi\ell}\left[\ln\frac{2\ell}{a'} - 1\right] \qquad (I.8)$$

For, $a \ll \ell$ and $2Z \ll \ell$

ℓ = conductor length, m

Z = depth (m) to which conductor is buried

2a = conductor diameter, m

$a' = \sqrt{2az}$

$a' = a$ if conductor is on surface

D. Resistance of a Rod-Bed

1. Combined resistance of several closely spaced rods from Schwarz [2]:

$$R = \frac{\rho}{2\pi n \ell_1}\left[\ln \frac{4\ell_1}{b} - 1 + \frac{2K_1\ell_1}{\sqrt{A}}(\sqrt{n} - 1)^2\right] \qquad (I.9)$$

ℓ_1 = length of each rod, m

$2b$ = diameter of each rod, m

n = number of rods in area A

K_1 = a co-efficient, function of the ratio between length and width of the area (refer to paper [2])

2. Mutual ground resistance R_{12} between rod-bed and grid to which it is attached from Schwarz [2]:

$$R_{12} = \frac{\rho}{\pi\ell}\left[\ln \frac{2\ell}{\ell_1} + K_1\frac{\ell}{\sqrt{A}} - K_2 + 1\right] \qquad (I.10)$$

ℓ_1 = length of rod, m

L = length of buried conductor of the grid, m

A = area covered by conductors, square metres

K_1, K_2 = co-efficients (refer to paper [2])

3. Ground resistance of a vertical rod from Schwarz [2]:

$$R = \frac{\rho}{2\pi \ell_1} \left[\ln \frac{4\ell_1}{b} - 1 + \ln \frac{1 + Z/\ell_1}{1 + 2Z/\ell_1} + \frac{Z}{\ell_1} \ln \frac{4Z/\ell_1 + 4\,(Z/\ell_1)^2}{1 + 4Z/\ell_1 + 4\,(Z/\ell_1)^2} \right] \qquad \text{(I.11)}$$

$b \ll \ell_1$

ℓ_1 = length of rod, m

2b = diameter of rod, m

Z = depth of earth fill over top of rod, m

4. Ground resistance of a number of equally spaced rods forming a hollow square from Tagg [6]:

$$R = \frac{1 + K\alpha}{N} \text{ (Resistance of one rod)} \qquad \text{(I.12)}$$

$$\alpha = \frac{\text{equivalent hemisphere radius representing resistance of a single rod}}{\text{distance between rods}}$$

N = number of rods

K = a factor depending upon the number of rods and their spacing (refer to book [6]).

5. Ground resistance R' of a rod driven in an upper layer (ρ_1), taking into account the effect of a second layer (ρ_2) below from Tagg [6]:

$$R' = R + R_a \qquad \text{(I.13a)}$$

R = resistance of rod considering layer ρ_1 above as infinite

R_a = additional resistance due to effect of layer (ρ_2) below

$$= \frac{\rho_1}{2\pi l} \sum_{n=1}^{\infty} \frac{K^n}{2} \ln \frac{(nh/l + 1)}{(nh/l - 1)} \qquad (I.13b)$$

$$K = \frac{\rho_2 - \rho_1}{\rho_2 + \rho_1}$$

l = length of rod, m

h = height of layer ρ_1, m

E. Apparent Resistivity of Station Ground in Non-Uniform Soils From Endreyni [7]

Ratio N between apparent resistivity (ρa) and top layer resistivity (ρ_1) for electrode having dominant dimention as horizontal:

$$N = \frac{\rho a}{\rho_1} = 1 + \frac{\sum_{m=1}^{\infty} \mu^m \left[\frac{Km+}{Cm+} + \frac{2Km}{Cm} + \frac{Km-}{Cm-}\right]}{\ln \frac{16a}{do} + \frac{Ko+}{Co+}} \qquad (I.14)$$

$$\mu = \frac{\rho_2 - \rho_1}{\rho_2 + \rho_1}$$

$$Km = \frac{\alpha}{\sqrt{\alpha^2 + m^2}}, \quad Cm = \sqrt{1 + (\frac{m}{\alpha})^2}$$

$$Km+ = \frac{\alpha}{\sqrt{\alpha^2 + (m + \phi)^2}}, \quad Cm+ = \sqrt{1 + (\frac{m + \phi}{\alpha})^2}$$

$$Km- = \frac{\alpha}{\sqrt{\alpha^2 + (m - \phi)^2}}, \quad cm- = \sqrt{1 + (\frac{m - \phi}{\alpha})^2}$$

$$\phi = \frac{Z}{h} = \frac{\text{depth of wire}}{\text{depth of layer } \rho_1}$$

$$\alpha = \frac{a}{h} = \frac{\text{Equivalent radius of the electrode in metres}}{\text{h (in meters)}}$$

ρ_2 = resistivity of the bottom layer

do = diameter of the conductor, m

APPENDIX II

IMPEDANCE OF AN INFINITE LINE

In case of a line with distributed ladder network, if r and s refer to the receiving and sending end respectively, Z_0 is the characteristic impedance, γ the propagation constant, then:

$$V_s = V_r \cosh \gamma l + I_r Z_0 \sinh \gamma l$$

$$I_s = I_r \cosh \gamma l + \frac{V_r}{Z_0} \sinh \gamma l$$

If the circuit is open at the receiving side,

$$I_r = 0$$

$$\therefore \frac{V_s}{I_s} = Z_o \coth \gamma l$$

If the line length is infinite,

$$\frac{V_s}{I_s} = Z_0 \Rightarrow \coth \gamma l = 1 \Rightarrow \gamma l \geqslant 2 .$$

APPENDIX III

ADMITTANCE OF COUNTERPOISE

$$Z_c = \sqrt{\frac{z_c}{y_c}} \; \coth\left(\ell \, . \, \sqrt{z_c \, y_c}\right)$$

$$= \sqrt{\frac{z_c}{y_c}} \; . \; \frac{\cosh\left(\ell \, . \, \sqrt{z_c \, y_c}\right)}{\sinh\left(\ell \, . \, \sqrt{z_c \, y_c}\right)}$$

$$\therefore \frac{dZ_c}{d\ell} = z_c \; . \; \frac{\sinh^2\left(\ell \sqrt{z_c \, y_c}\right) - \cosh^2\left(\ell \sqrt{z_c \, y_c}\right)}{\sinh^2\left(\ell \, . \, \sqrt{z_c \, y_c}\right)}$$

$$= Z_c \left[1 - \coth^2\left(\ell \sqrt{z_c \, y_c}\right)\right]$$

$$R_2 = \frac{\rho}{\pi\ell} \; \ln \; \frac{\ell}{a} \quad \text{[from equation (3.24)]}$$

$$\therefore \frac{dR_2}{d\ell} = \frac{\rho}{\pi\ell^2} \left(1 - \ln \ell/a\right)$$

Since $\frac{dR_2}{d\ell} = \frac{dZ_c}{d\ell}$,

$$z_c \left[1 - \coth^2\left(\ell \, . \, \sqrt{z_c \, y_c}\right)\right] = \frac{\rho}{\pi\ell^2} \left[1 - \ln \; \frac{\ell}{a}\right]$$

$$\therefore \coth\left(\ell \, . \, \sqrt{z_c \, y_c}\right) = \sqrt{1 - \frac{\rho}{\pi\ell^2 . z_c} \left[1 - \ln \; \ell/a\right]}$$

$$\text{since } Z_c = R_2, \; \coth\left(\ell \, . \, \sqrt{z_c \, y_c}\right) = Z_c \sqrt{\frac{y_c}{z_c}} = R_2 \sqrt{\frac{y_c}{z_c}}$$

$$\therefore \frac{\rho}{\pi\ell} \ln \; \ell/a = \sqrt{\frac{z_c}{y_c}} \sqrt{1 - \frac{\rho}{\pi\ell^2 . z_c} \left(1 - \ln \ell/a\right)}$$

$$\therefore y_c = \left(\frac{\pi\ell}{\rho \; \ln \; \ell/a}\right)^2 \left[z_c - \frac{\rho}{\pi\ell^2} \left(1 - \ln \ell/a\right)\right]$$

APPENDIX IV

RELATION BETWEEN GROUNDING RESISTANCE AND CAPACITANCE OF A GRID

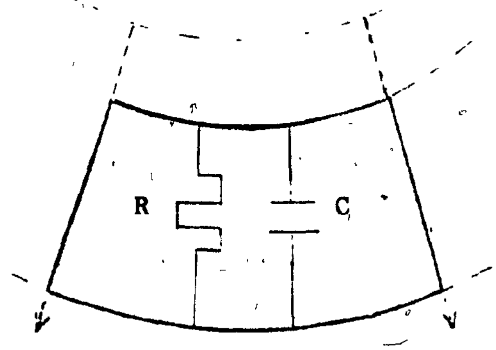

Fig. IV.1: A Small Unit of Ground between two Equipotential Surfaces

Let ρ = resistivity of the conductive medium (earth) Ω-m

k_r = relative permittivity of the insulating medium (earth) to free space

A = area between equipotential surfaces separated by a distance ℓ

With reference to Fig. IV.1, the resistance and capacitance of area A are:

Resistance $R = \frac{\rho\ell}{A}\ \Omega$

Capacitance $C = \frac{k_r A}{4\pi\ell}$ meters

$$= \frac{k_r A}{4\pi\ell \times 9000}\ \mu F,\ (9000\text{ m} = 1\ \mu F)$$

$$= \frac{k_r A}{36\pi\ell \times 10^9}\ \text{Farads}$$

$$\therefore,\ RC = \frac{\rho k_r}{36\pi \times 10^9} = \rho k_r k_o,\ (k_o = \text{permittivity of free space}) \quad \text{(IV.1)}$$

APPENDIX V

INDUCTANCE OF SQUARE GRIDS

1. Inductance L of a square grid upto 64 meshes [33] with current being fed at corner is:

$$L = (\text{Self inductance of one element}) \times K' \tag{V.1}$$

$K' = 2.04$ for 4 meshes

$= 3.75$ for 16 meshes

$= 5.28$ for 36 meshes

$= 7.15$ for 64 meshes

2. Inductance of a square plate [33] is approximated to be the same as that for a 64 meshes square grid

APPENDIX VI

SURGE IMPEDANCE OF A TOWER

1. Surge Impedance Z of a Tower from Bewley [35]:

$$Z = 60 \left[\log \frac{h(h+2a)}{r(h+a)} + \frac{a}{h} \log \frac{(h+2a)^2}{(h+a)(2a+\sqrt{4a^2+r^2})} \right] + \frac{30}{h} \left[\sqrt{4a^2 + r^2} - 2(h+a-r) \right] \text{ ohms} \qquad \text{(VI.1)}$$

h = height of the tower

r = equivalent radius of the tower (mean periphery divided by 2 π)

a = depth of the ground plane below earth's surface

2. Surge impedance Z of a Cylindrical Tower for a Unit step current from Wagner and Hileman [36]: (VI.2)

$$Z = 60 \ln [\sqrt{2} \, (c \cdot t/r)]$$

c = velocity of light

t = time

h = height of cylinder

r = radius of cylinder

3. Surge Impedance Z of a Cylindrical Tower for a Ramp Current from Sargent and al. [37]:

$$Z = 60 \left[\ln \left(\sqrt{2} \cdot \frac{2h}{r} \right) - 1 \right], \quad h \gg r$$

h = height of the tower (VI.3)

r = radius of the tower

4. Surge Impedance Z of a Conical Tower for a Unit Impulse (rectangular) from Sargent and al. [37]:

$$Z \doteq 60 \ln \left(\frac{\sqrt{2}}{S}\right) \qquad \text{(VI.4)}$$

S = Sine of the half-angle of the cone

5. Surge Impedance of a Transmission line Tower by Kawai [38]: This is a "Direct Method" for measurement of the surge impedance by field testing.

COMPARISON OF THE ABOVE METHODS

Wagner and Hileman [36] evaluate the surge impedance of a tower by applying unit step current to a cylindrical tower. Sargent and Darveniza [37] improved upon the work and developed expressions of impulse impedance for practical shapes of towers when subjected to practical waveshapes of current.

Surge impedance of a tower is not constant as it is dependant upon velocity of the wave traversing down the tower, which depends upon the tower's inductance and capacitance, which are not constant due to non-uniform cross section and presence of cross-arms. Bewley [35] ignores the effect of this change in wave velocity. Kawai [38] solves the problem by physically measuring it. The application of field theory methods to both conical and cylindrical approximations to a steel tower and their comparison of results with various measured values by Sargent and Darveniza [37] establishes that the conical approximation is more suitable for the usual conventional double-circuit towers.

APPENDIX VII

ANALYTICAL EXPRESSIONS INVOLVING SURGE PHENOMENA

A. Parameters of a buried ground wire from Rudenberg [18,23]:

1. Inductance $L = 2\ell \;\ln\left(\frac{2\ell}{a}\right) \times 10^{-7}$ Henry (VII.1)

2. Ground Resistance $R^1 = \frac{\rho}{2\pi\ell} \;\ln\left(\frac{2\ell}{a}\right)$ ohm (VII.2a)

(for deeply buried wire)

$= \frac{\rho}{\pi\ell} \;\ln\left(\frac{2\ell}{a}\right)$ ohm (VII.2b)

(for wire near the surface)

3. Inductive Time Constant T_L:

$$T_L = \frac{L}{R^1} = 4\pi \frac{\ell^2}{\rho} \times 10^{-7} \qquad \text{(VII.3)}$$

4. Capacitive Time Constant T_C:

$$T_C = \frac{\rho k_r}{4\pi} \times \frac{1}{9 \times 10^9} \qquad \text{(VII.4)}$$

where, ℓ = length of wire, m

k_r = dielectric constant

ρ = soil resistivity, Ω-m

a = radius of the wire, m

B. Impulse Impedance of a Buried Ground Wire to a Cosine-front impulse from Gupta and al [39]:

Case I: $\ell \leq \ell_e$

Impulse Impedance Z = A x R (VII.5)

where $A = e^{.333\,(\ell/\ell_e)^{2.3}}$

$$\ell_e = 1.4\,(\rho T)^{.5} \qquad \text{(VII.6)}$$

R = Power frequency grounding resistance for length ℓ, Ω

ℓ = Length of the wire, m

T = Wave-front time (μsec)

<u>Case II</u>: $\ell > \ell_e$

$$\text{Impulse Impedance } Z = \text{Impulse impedance for length } \ell_e \qquad \text{(VII.7)}$$

C. Sending-end impedance for unit-function applied voltage to a counterpoise from Bewley [35]:

$$Z_c(t) = \frac{1}{G\ell\left\{1-\sum_1^\infty \frac{8e^{-\alpha t}}{(2k-1)^2\pi^2}\left[\cos\omega_k t + \left(\frac{G}{4\omega_k C} - \frac{\omega_k C}{G}\right)\sin\omega_k t\right]\right\}} \qquad \text{(VII.8)}$$

ℓ = length

G = leakage conductance per unit length

L = Inductance per unit length

C = Capacitance per unit length

Wire resistance (neglected)

D. Voltage developped at the sending-end for unit step current applied, from Bellaschi [9]: