NOTICE

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Using Logo as a Tool to Develop the Concept of Angle in Secondary I

Deanna Mendelson

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ABSTRACT

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This research study describes the acquisition of the concept of angle in a specially created Logo environment. Eight Secondary I (grade 7) students, aged 12-13, participated in the study implemented during the second semester (January-May) of the school year. The study was conducted by the teacher who also acted as the researcher, in the natural setting of the school's computer lab. Teacher-devised activities formed the basis of the analysis attempting to identify whether the created Logo environment was an effective tool in developing the specific concepts of angle therein concurrently with the mathematical symbolism.

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CHAPTER I

INTRODUCTION

The study of the concept of angle has for years been a challenge to student and teacher alike. One of the main problems in the teaching of angle in the traditional school curriculum is the use of definitions and stereotypical models of angles in standard position. Freudenthal (1973) stresses the need for both the static and dynamic definitions of angle to be introduced at the same time as a means of developing the notion of angle. To this end, the Logo environment offers an attractive tool to aid the teacher in presenting the concept of angle to the student.

This study undertakes to examine the effect of the use of Logo as a tool in the acquisition of the concept of angle. The concepts developed in the study involve, (a) the right angle, (b) angles less than/greater than 90°, (c) a complete rotation of 360°, (d) complementary/supplementary angles, (e) interior/exterior angle rotations - all in standard and non-standard position. These concepts were taken from the Secondary I curriculum.

The research review in this study begins with the findings in the acquisition of angle concepts in the traditional curriculum. It also includes an inspection of the Secondary I (grade seven) curriculum and the treatment of the concept of angle development in the current textbooks in Quebec. This part of the research review reveals the protractor as the only tool offering manipulative experience in the development of angle, and its general application is to angles less than 180° in standard position and rotated from right to left.

The study continues with a review of the Logo research (Hillel, Kieran, Taurisson, Hoyles, Sutherland, Noss) which points out several trends in the students' acquisition of the angle concept in a Logo environment. Although the research generally points to the usefulness of Logo as a tool for the enhancement of the concept of angle, some problems have also been noted by the aforementioned researchers.

The research project was undertaken as a teaching experiment where the researcher is the actual teacher of the students in the study. A Logo environment was created especially for this study. The structure of the project lay in the teacher devised activities which developed the concept of angle. The teacher's role was to motivate the students' interest in the mathematical concepts embedded in the Logo activities, and thus to promote the acquisition of the mathematical symbols concurrently with those of the Logo Turtle Geometry.

The analysis undertaken in this study is twofold. First there is a task analysis within each activity. This section outlines the concept being developed in each activity through the various structured tasks, and looks at how each task was interpreted by the students. Secondly there are the student profiles which examine the behaviour of the students in carrying out the various tasks of the seven activities. The findings of the research study reveal the effect of Logo as a tool from the observation of the students in their acquisition of the angle concept developed in the teacher/researcher devised activities. Some problems which were observed are also outlined in this section. A further evaluation of the concepts was carried out in the items incorporated into the students' final June mathematics examination.

CHAPTER II

THE TRADITIONAL CONCEPT OF ANGLE

The Traditional Definition of Angle

According to Webster's Ninth Collegiate Dictionary an angle may be defined as 1) a corner whether constituting a projecting part or a partially enclosed space, 2a) the figure formed by two lines extending from the same point, 2b) a measure of an angle or of the amount of turning necessary to bring one line or plane into coincidence with or parallel to another...etc.

A look at a popular encyclopedia (World Book, Vol. A, 1, 1973) used by most upper elementary to secondary students in Montreal reveals the definition of an angle as "the opening between two lines that meet at a point. The point is the vertex and the two lines that form the angle are its sides. Angle comes from the Latin word angulus, meaning corner."

Both definitions stress the static angle. The amount of turning in the first is related to the measure of an angle.

From earliest times mathematicians widely accepted Euclid's definition of angles in a plane. He defined the angle in terms of the relationship between two lines rather than in terms of the rotation of one of the lines. The modern definition of angle, whether popular (as above) or mathematical (as in textbooks) generally defines this static angle of Euclid as follows: a figure is an angle if and only if consists of two noncollinear rays with a common endpoint. This definition generally is accompanied by an explanation, usually pictorial, representing the sides and the common vertex of the angle. The measurement of angles is generally dealt with next by illustrating the use of the protractor. This explanation of angle measurement, often unrelated to the actual definition of an angle, is usually developed with an imaginary rotation of a ray extending from the common endpoint. Thus the definition of the angle rotation is usually associated with the "measure" of an angle and learned as another concept of angle.

The need to include both the static and dynamic aspects of an angle in its definition has been stressed by many researchers. According to Heath (1956) the essential nature of an angle is that of rotation, and therefore should be included in developing the definition of angle. Freudenthal (1973) cites the need for both static and dynamic definitions of angle. He suggests introducing the measurement of angles at the same time as angle concepts, as a means of developing the understanding of the latter. Thus it may be concluded that the visual aspect of the static angle and the notion involved in the dynamic approach would seem to be of equal importance in developing the concept of angle.

The General Approach to Teaching Angle

Most geometry in the school curriculum seems to be largely taught for rote memorization. A survey of textbooks usually yields pages of definitions, classifications, and theorems presented in an attractive array of colours and accompanied by diagrams which often replace any active manipulative experience on the part of the student. (see Appendix A)

Children learn from a variety of sources - from drill, lectures, tests, observing models, manipulating real things, and from contemplating ideas. Following the geometry curriculum as a strictly page by page textbook course often eliminates any manipulative learning experiences. This goes against what most psychologists suggest. According to Piaget (1969) the mental images children create and move about in their minds are a product of their experiences. Therefore, children who see and manipulate an abundance of objects, shapes and symbols should have clearer mental images than those without such experiences. While the theories of Piaget and others like Brunet, Gagné, and Skemp provide a theoretical basis for organizing children's experiences, they offer little value unless the teacher is systematic in planning and deciding how to apply the psychological principles to what she/he is teaching. Thus the student may be led to memorize a model of a concept for which he cannot appreciate all the aspects thereof, simply because he cannot appreciate the organization which is inherent to the material.

Angles and other geometric shapes are generally represented in textbooks in standard form and from one perspective. This often leads to the students' retention of a stereotypical model of a given angle. Gillian Close's (1982) research found that primary children possessed a static concept of ray pair angles due to the fact that teachers and/or textbooks emphasize this concept of angle. This, she continues, formed the basis for the pupils in the secondary level whose concept of angle was not fully developed, and who were not able to recognize the right angle in unusual orientations.

The traditional classroom method of teaching the concept of angle generally employs one basic tool - the protractor, used for the measurement of a contained angle. Most children master the use of this tool without any real knowledge of what it is they are measuring, as is often exemplified in the common error of reading the "number" on a protractor from an incorrect side, i.e., left to right, instead of right to left. Close (1982) states that the secondary students failed to recognize that the use of a different scale or a different position with the same protractor can give different, yet equally correct answers. This indicates the lack of the basic understanding of the rotation of an angle.

The Traditional Curriculum in the Teaching of Angle (In Quebec)

After the skill of using a protractor is considered to be mastered, in Secondary I, the curriculum extends the notion of angle measure with an array of classifications:

acute angle $< 90^\circ$

right angle $= 90^\circ$

obtuse angle $> 90^\circ$ and $< 180^\circ$

straight angle $= 180^\circ$

reflex angle between 180° and 360°

This is generally the first set of classifications to be memorized by the students in order to facilitate a rapid identification of isolated diagrams of angles. This leads to the acquisition of vocabulary of an abstract nature, which when not developed with the concrete manipulations of rotations of angle rays, is generally not associated with angle rotation.

It must be noted here, that in an attempt to introduce the concept of angle rotation, more recent textbooks have included, in diagram form, the concept of a 360° rotation and'/the partial rotations possible. However, the student is once again led to a pictorial representation where the angle may be perceived as two straight lines and their measure, rather than the amount of angle rotation. An extension of the concept of angle measure includes the relationship of special pairs of angles, such as complementary/supplementary angles. The student is again led to acquire new vocabulary, introduced with accompanying diagrams. Suddenly classifications previously learned are no longer discussed, only to be replaced by new and more complex ones. The student generally learns each of these classifications as a separate entity, with little reason or support for the various mathematical calculations and rarely any need for estimation skills.

Geometry tests often include definitions as part of the evaluation. Other items require identification of figures which are based on models presented in the texts.

In Secondary I, the students are also introduced to constructions, with ruler and compass, of congruent angles following the learning of the previously mentioned classification of angles. The problems encountered at this stage involve the requirement of reproducing a figure of an angle, which until now has been pictorially represented without any real emphasis on rotations. The sole means of measuring an angle until this stage, has been with the protractor, and often only in standard position. Also, the student most likely has not fully mastered drawing an angle of a given measure using the protractor due to the difficulties arising in the measurement of obtuse and reflex angles with the scale of the protractor. Thus with new tools in hand (compass and ruler), the student is led through another bout of frustration as she/he is given the steps to complete each formal construction; another set of rules to be memorized, in most cases! A lack of understanding by the students is often observed in a construction with compass and ruler where a few intersections have been added here and there just to make it "look good"!

In Secondary II and III the formal constructions build upon those supposedly acquired in Secondary I; the construction of bisectors of angles, perpendicular lines and perpendicular bisectors, and their applications to constructing triangles using the three theorems of congruency: SSS, SAS, ASA. The problems that were encountered in Secondary I are usually repeated in Secondary II, since memorization of steps and definitions of congruency seem to be the basis of these concepts rather than any manipulative experience. Angle applications are then extended to all regular polygons, and designs featuring inscribed squares and triangles in circles, are undertaken.

In Secondary II, the curriculum also calls for the "application of skills". Here the properties of angles in a triangle or a quadrilateral are explored once again by means of diagrams and definitions! Angle relations for parallel lines follow and are soon combined with the properties of polygons that preceded these concepts of angle relations. One can see that the concepts of angle at this level build up at a tremendously fast rate, since this is only the geometry aspect of the mathematics curriculum! One cannot help but wonder whether the designers of the curriculum believe that the students can master these concepts; the identical ones may be found in the standard text for Secondary III in the public school system.

The problem with angle concepts is inherent even in the Secondary III curriculum where the student is required to apply the learned concepts to discover (once again) the relationship of equality of angles or congruency of triangles. Without a basic knowledge of geometry concepts developed through manipulative and visual activities, the student's visual perception is now called into play. For now she/he has to apply what she/he has acquired, mainly in the form of rotely memorized abstract rules for symbols on a page in a text book. (see Appendix A) It is not surprising that the National Assessment (1978) found that about one quarter of the 17-year old could not produce the correct number of degrees in a right angle or in a circle. In like manner the APU (1978-82) found that at age 15, only 58% could accurately measure an angle of 106°.

Van Hiele's analysis (1959) of a typical geometry class presents the teacher as one who reasons by a network of relations which his students do not necessarily comprehend. On the basis of this network the teacher presents the mathematical relations which the students learn to apply out of habit, but which they do not understand. Van Hiele maintains that since this network of relations is not formed upon the sensory experience of the students, it is imposed and therefore not understood, and thus risks being forgotten in a short time.

Wirszup (1969) claims that the majority of high school students are at the first level (recognition) of Van Hiele's development in formal geometry. At this level the students can learn names of figures and recognize shapes as a whole, i.e., squares and rectangles seem to be different. Yet, the course that the high school students take, demands the fourth level of Van Hiele's development. He describes this level as "deduction", where the student understands the significance of deduction and the roles of postulates, theorems and proof; and where the student can write proofs with understanding. This would seem to indicate that the high school student is still perceiving geometric figures in their totality as entities, and is hence analysing figures according to their appearance.

One can easily relate the plight of the student to the Van Hiele's analysis of the different levels in understanding geometry, and the claim that a person must go through the levels in order. Usiskin (1982) exemplifies this plight with the often heard remark, "I can follow a proof when you do it in class, but I can't do it at home." Usiskin points out that, "This student may be at level 3, while the teacher is operating at level 4." At level 3 (order) simple deduction can be followed but proof is not understood, whereas level 4 demands proofs to be written with understanding. Usiskin (1982) further explains that each level has its own linguistic symbols and its own network of relationships connecting these symbols. Therefore, two persons who reason at different levels cannot understand each other. Thus "the student cannot understand the teacher who is using objects (propositions, in case of proof) and a network of relationships (proof itself) which the student does not yet understand used in this way" (Usiskin 1982).

Children's Understanding of Angle in the Traditional Curriculum

I. RESULTS OF ASSESSEMENTS

The study of the concept of angle has for years been a challenge not only to the student, but to the teacher, as well. In a recent report of the National Assessment of Educational Progress (1978) 17-year old students were asked the following item (RK04): "What is the measure in degrees of the angle formed by the hands of the clock when the time is 3 o'clock?"

The results reported for the 17-year olds are slightly less than satisfactory. If one can assume that a 17-year old should be able to visualize the hands of a clock at 3 o'clock, then it appears that about one quarter of the 17-year olds either do not know that there are 90° in a right angle, or that there are 360° in a circle. The report further suggests that the relatively low performances in the geometry section of the National Assessment appear to be the result of a lack of appropriate instruction or inadequate attention given to the initial development of the idea during the early secondary school level.

In an unreleased exercise of the same Assessment (K11004) only 55% of 13-year olds responded correctly as to the number of degrees in a right angle. 20% indicated that they "did not know". It was also shown that 13-year old respondants had less difficulty telling how many degrees there are in a right angle than they did in visualizing the right angle from the position of the hands on the clock.

In the exercise (RE01) the students were tested in their recognition of vocabulary of measurement of angles:

"An angle may be measured in units called: centimetres; degrees; grams; inches; I don't know" - 15% of the 13-year olds selected "inches" as units of measure for angles.

A further examination of the literature was undertaken in an attempt to review the research where the major misconceptions of the concept of angle were studied. There is only a small amount of literature relating to children's acquisition of the concept of angle. The literature review is of children's concept of angle, first in the traditional curriculum, using traditional methods of instruction such as the protractor and then using the computer and the programming language Logo.

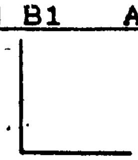
In a study of mathematics performance in the United Kingdom, the <u>Assessment Performance Unit</u> (APU 1978-1982) examined the students' concept of angle, (ages 11 and 15). Direct testing of the definition of angle was undertaken in the practical survey only with the 15-year olds. Of these, only 4% defined an angle as an amount of turns or rotations. Most of the pupils spoke of angles in terms of "distance" and "area". (29%). In the items which compared equal angles (drawn on graph paper), over half of the 11-year olds in the bottom or lower middle attainment levels gave answers based on the comparison of arm lengths.

It was also noted that the combination of long lines and a long arc suggested a large angle to 11-year olds far more strongly than it did to 15-year olds.

The results of this study on the estimation of angles showed a preference of certain "round numbers" as responses, e.g., over 40% of 15-year olds responded 45° when estimating an angle of 33°, around 10% of 11-year olds responded 100° when estimating an angle of 120°. Only 14% of 11-year olds in the study correctly estimated the measure of a 30° angle. When asked to estimate obtuse angles, around 10-15% of the pupils, ages 11-15, gave responses outside the appropriate quadrant; 69% of 15-year olds correctly estimated an angle of 106°.

The items in the APU survey which dealt with right angles required the pupils to estimate or to supply the special name given to an angle which actually measured 90°, e.g.

<u>ITEM B1 AGE 11</u>



"This particular angle has a special name. Do you know what it is?"

65% of 11-year olds responded with "right angle", 5%

with "90° angle". In a subsequent item where the question was, "Estimate the size of this angle." Only 60-70% of 11-year olds recognized a right angle or a 90° angle in this item.

Drawing and measuring angles using a protractor was also investigated in the APU Assessment. It was found that measuring acute angles with the protractor posed no problem. When measuring obtuse angles, however, between 15% and 20% of 15-year olds and up to 25% of 11-year olds used the wrong scale and gave an acute angle as their response, e.g., at age 11, 45% could measure a 53° angle, at age 15, 58% could measure an angle of 106° within a margin of 1°.

ITEM B7 - AGE 15

"Measure the size of this angle"

(Actual size 106°)

A total of 16% used the wrong scale on the protractor and gave the supplementary acute angle as their answer; the rest also misread the scale.

Very few pupils used the wrong scale when drawing angles, even for obtuse angles, in contrast with the results for measuring. The report concludes that the high success rates obtained for the practical items was probably due to the fact that they were the final questions in a topic and all pupils had already estimated and measured various other angles prior to being asked to draw an angle.

II. G. CLOSE - CHILDREN'S UNDERSTANDING OF ANGLE AT THE PRIMARY/SECONDARY TRANSFER STAGE (1982)

Gillian Close lists similar findings of how children understood angles in her review of research. G. Holloway's work with 7-year olds stresses children's reliance on intuition rather than measurement in the comparison of angles as well as their confusion regarding which aspect of angle, such as arm length should be measured. A study by Geoffrey Giles also found that children made errors such as measuring arm lengths in items involving the listing of five angles in order of size. Many of his pupils demonstrated difficulty in identifying all the angles in diagrams with more than two rays emanating from a vertex, and over half, in examining such diagrams demonstrated uncertainty as to whether an angle had exactly two arms or not.

Close cites the findings of David Fielker, who attributes the difficulties children have in recognizing 45 degree angles to the strong feelings for perpendicularity that all children seem to have, from the "teaching of square corners" before being introduced to a more general idea of angle. (This was also observed in the APU survey.) Over 80% of the pupils were able to fold an irregular sheet of paper twice to make a right angle, but fewer were able to identify the position of the right angle on the folded paper. Consequently some folded the paper in half again in a variety of incorrect ways when asked to halve their right angle. They seemed to have perceived a right angle as the whole piece of paper and perhaps as an area.

Close's research into children's understanding of angle using a protractor as a tool, reveals many misconceptions in children's acquisition of angle:

A. PRIMARY CHILDREN'S CONCEPT OF ANGLE

The primary children's definition of acute and obtuse angles was found to be one of perception rather than measurement. Furthermore, they possessed a static concept of ray pair angles. This she attributes to the fact that most primary work emphasizes the static angle either by the teacher, and/or the amount of work in the textbooks, which is not related to ray pairs.

B. SECONDARY STUDENTS' CONCEPT OF ANGLE

Many Secondary level pupils possessed a narrow concept of angle. Their predominantly static definitions of angle related to their primary school experiences of rotations to drawings of static ray pairs.

C. ANGLE MEASUREMENT

1. Primary Level

Degree measure was introduced via the use of the protractor, and thus the children had a grasp of the 90° angle. However, they were unable to use multiples of 90° due to the fact that a 90° angle had not been introduced as a rotation of a quarter turn.

2. Secondary Level

Close found that a lack of understanding of the relationship between intuitive angle size and angle measurement was widespread; a problem she feels could be overcome with the use of a full protractor. This would especially assist in the measurement of the reflex angle. Close also found misconceptions of an angle in the students' confusion of the measurement of a given angle and the classification or name (acute, obtuse) of the angle.

D. THE RIGHT ANGLE IN NON-STANDARD POSITION

Another difficulty that was exhibited by children was the inability to recognize a right angle in unusual orientations. According to Close, "the inability to recognize right angles in all orientations precludes advancement in work on both properties of shapes and angles".

E. THE NOTION OF THE 90° ROTATION

Some pupils were not aware that 90° was a quantity which could be subdivided while others could not relate the particular concept of a right angle to a slightly smaller angle. Although they were able to conceptualize rotations through multiples of right angles, the students were unsure of the meaning of a full turn.

F. ANGLE ESTIMATION

The item of lowest facility in Close's research was the one in which angle estimation was required.

Pupils found it most difficult to estimate angles which were not near 0° or 90°, or whose non-reflex parts were not near to 0° or 90°. Close emphasizes that measurement with a protractor can only have meaning after "a long and very gradual development ... to create an understanding of the approximate size of an angle from its drawing... and the relationship of this to its degree measure". Another serious misconception that Close reports is the effect of arm length on the angle size. A number of pupils felt that arc length, irrespective of its radius, indicated angle size, while others considered the position of angles to be important in determining their size.

G. THE PROTRACTOR AS A TOOL FOR MEASURING ANGLE

An effect of the use of different protractors was seen in the fact that some pupils were unaware that there is a unique measure of an angle. They failed to recognize that the use of a different scale or a different position with the same protactor can give different, yet equally correct answers.

CHAPTER III

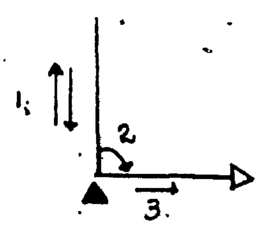
LOGO AND ANGLE

The Concept of Angle in Logo

Before reviewing the literature in the children's conception of angle in Logo, we will first discuss how angle is used in Turtle Geometry.

The Logo Turtle can be commanded to rotate and move about the computer screen, creating graphic images. These images are built from descriptions based on what the turtle knows how to do. The commands FD, BK immediately output a line. The commands to pivot or rotate the turtle are RT, LT. In order to construct an angle, one must give a sequence of commands: FD, BK to produce the initial ray and to bring the turtle back to its initial state. This has to be followed by a rotation indicated by the commands RT or LT to orient the turtle so that the final ray of the angle may be produced. This is followed by FD, which will allow the turtle to complete the angle.

Any constructed angle may be viewed as commands to execute three basic steps:

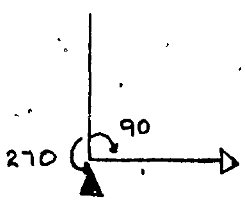


1. FD :SIZE
   BK :SIZE
2. RT :ANGLE
3. FD :SIZE

In this way the concept of angle in Logo is dependent on the amount of rotation required to begin the construction of the second ray, rather than on the visualization of the ray pairs of the traditional static angle.

The description given to the Turtle will cause it to produce an angle no matter where it is initially positioned or headed on the computer screen. Thus the child can be led to acquire the concept of angle in non-standard position since a constructed angle is readily reproduced especially once it has been defined as a procedure.

An angle in Logo may be perceived as the exterior or inverse rotation as well. For example, for right angles the commands can involve a rotation of 270° in order to construct the interior right angle, i.e.,



FD 50 BK 50 RT 90 FD 50

OR

FD 50 BK 50 LT 270 FD 50

In this instance the construction of angle in Logo necessitates the coordination of the relationship between an angle of rotation and the adjacent constructed angle.

The Computer and the Logo Language

I. THE FINAL REPORT OF THE BROOKLINE LOGO PROJECT (1979)

According to the research of The Brookline Report, the Logo language sets forth as "subject matter" of Turtle Geometry the concept of "The use of numbers to measure lengths and angles ... and formal and intuitive understanding of special angles: 90, 360, 180." The Report points out that after 20-40 hours of hands-on experience with the computers, the grade 6 children's ability to make size comparisons between grossly different angles was augmented by the Logo experience. As to the concept of angle, the Report states that although the students may have had only a vague idea or none at all of the meaning of degree as a measure of an angle, they did have an intuitive sense of the relative sizes of angles as a result of their Logo experience.

The Report also states that it was difficult to discover that a rotation of more than 360° turns the Turtle around more than once because the Logo Turtle moves instantaneously to its new orientation without visibly rotating. With regard to direction of rotations, the Report concludes that many students never achieved the steps of combining rotations to the right and to the left into a single rotation.

Although the Brookline Report deals with the subject matter of Turtle Geometry, it does not make a clear distinction between the mathematics acquired and the programming skills exhibited by the children.

II. REVIEW OF STUDIES RELATING TO LOGO AND ANGLE

A. J. HILLEL - MATHEMATICAL AND PROGRAMMING CONCEPTS ACQUIRED BY CHILDREN AGED 8-9 IN A RESTRICTED LOGO ENVIRONMENT (1985)

Hillel reports the results of close observations of five pairs of children, aged 8-9, over twelve weeks. While his research looked at general programming and mathematical aspects of the children's work, he does report some specific results on the concept of angles. Using a Logo environment in which the Turtle's movements were substantially slowed down, Hillel reports that all groups, by the end of the 12th session, were consistent in (1) using 90° and 180°, (2) realizing the inverse relation between right and left rotations, and (3) being able to add two successive rotations of 90°.

Furthermore, three of the five pairs were able to consistently differentiate between acute and obtuse angles. Hillel also reports that lateralization errors were the most frequent ones associated with rotation.

B) KIERAN AND TAURISSON - <u>LOGO ET LA NOTION D'ANGLE EN QUATRIEME ANNEE (1983)</u>

Kieran and Taurisson's work with fourth grade students points to children's difficulty in defining angle as rotation. They found that while rotation associated with a change in direction was generally well perceived, it was not the criteria used to classify angles. Both the Logo group and the regular math group in their research seemed to base angle measurement on the length of the sides or rays. They stress that more experience is necessary for children to incorporate the idea of space contained between two lines that extend from a common vertex with the idea of rotation. Yet this notion of angle was greatly developed during the year of Logo. Kieran and Taurisson examined the students' understanding of interior and exterior rotations. Their observations seem to indicate that students isolate the exterior rotation and then perceive the interior rotation. This difficulty to grasp the notion of the interior angle may be explained as its being more abstract for children at this level. The exterior turn may be likened to a person who leaves tracks in the snow as he turns to go the other way. The Turtle does not leave a trace as it turns to orient itself to produce the final ray of a given interior angle.

C) C. KIERAN - <u>LOGO AND THE NOTION OF ANGLE AMONG FOURTH AND SIXTH GRADE CHILDREN (1986)</u>

In another study of Logo and angle among fourth and sixth grade children, Kieran's research findings of the grade 6 students' acquisition of the dynamic concept of angle at mid-year demonstrated a lack of recognition between the input to turtle turn and the size of an angle. The students were still inclined to use the visual measure of the arm length to classify the size of the angle. By the second half of the year of the same study, Kieran reports that the reliance of perceptual clues was still present in the students' first attempt at determining input to turtle turns. She points out that the students looked at the relationship of the input to the angle formed only when the initial perceptual attempts failed. It was also evident in Kieran's grade 6 study that the students were not able to perceive the supplementary relationship inherent in many Logo situations.

D. HOYLES AND SUTHERLAND - WHEN 45 EQUALS 60 (1985)

Hoyles and Sutherland's 3-year longitudinal study of 11 and 12-year olds examines many facets of children's mathematical work in Logo including the build up of mathematical meaning, the synthesis and extension of concepts, as well as the social interaction and affective aspects. However, in tracing out the use of inputs to rotations of one pair of children, they noted that the children may use inputs to rotations of RT and LT without reflection on what turn has already been made or without synthesis with angle used in other angle constructions. They point out that children may be adding or subtracting numbers, or adding and subtracting actions but not angles. They have found that children have a tendency to estimate the size of any acute angle as 45°, even when the size of the angle adjacent to it on a straight line or the reflex angle completing a turn is given. Hoyles and Sutherland noted that some children used a restricted subset of inputs to RT/LT such as repeated equal digits and multiples of 30 and 45. The use of these subsets, they point out, facilitated the production of a range of projects in Turtle Geometry but it is not clear that pupils were aware of the process by which their goals were reached.

E. R. NOSS - CREATING A MATHEMATICAL ENVIRONMENT THROUGH PROGRAMMING: A STUDY OF YOUNG CHILDREN LEARNING LOGO (1985)

Noss's study of children aged 8-12 years also reveals the choice of inputs to RT/LT were often randomly selected, or consisted of conveniently positioned keys (e.g., 44, 77, 56). In the introduction phase of his research, Noss's results of students combining inputs to RT/LT show 10% never combined, 80% combined similar commands (RT 20 RT 10 into RT 30), and two-thirds of the students combined inverses of angles (RT 30 LT 10 into RT 20). To the younger children in particular, the Turtle effectively served as an introduction to the concept of angle. The younger students were restricted to inputs of 10 or 20. The special effect of 90 was quickly grasped by all but the lowest attainers. As a teacher in Noss's study points out, "The concept of angles is understood so much better than if I had stood at the blackboard with a protractor because the children are making the angles themselves for a specific purpose..." A few children "discovered" turns of 180° often in the context of turning 90 the wrong way, and then turning 180° to compensate. 360° turns were, in general, encountered in the context of circle constructions.

Noss, in the final phase of his research, examines the effects of Logo on aspects of angle acquisition in the geometry study. These aspects may be categorized as follows:

1. Right angle conservation - using varying positions and arm length. The effect of Logo on this aspect of angle was strongest in the school with the youngest children (8-9 years old). It was suggested that the Logo children in this category may have been introduced to the concept of right angle only in the context of their Logo work, while the older children (10-11 years old) had already been introduced to the concept in a conventional manner. (There was also indication that Logo work narrowed the gap between the sexes, since the Logo boys and girls obtained comparable scores on this section, while in the controlled group the boys outscored the girls by 20%.)

2. Angle conservation - using varying positions, arm length and angle measure. A significant effect was found in favour of the Logo children on an item comparing two unequal angles. This finding did not extend, however, to items involving an angle embedded in a triangle, or in the children's ability to conserve angle under rotation.

3. Angle measurement - the ability to recognize or estimate relative sizes of angles from a rotational (exterior) angle point of view.

   In this aspect, a significant effect was found in favour of the Logo children's ability to order (exterior) angles into relative size, i.e., greatest and least. This ability may be related, according to Noss, to the concept of an angle as a rotation, which could be supposed to result from the Logo environment.

III. SUMMARY OF RESEARCH IN THE ACQUISITION OF ANGLE IN A LOGO ENVIRONMENT

The research findings dealt with in this review of the literature seem to point to several trends in the children's acquisition of angle concept in a Logo environment. They may be summarized as follows:

A. RIGHT ANGLE

Most children in all age groups were able to grasp the concept of right angle without difficulty.

B. ANGLE COMPARISON

Logo effectively enhanced the children's ability to make sized comparison between grossly different right angles.

C. ANGLE MEASURE

Inputs to RT/LT were more difficult to estimate than those to FD/BK. This was in part due to the speed with which the Logo turtle rotates, making it difficult for children to "see" the turtle turn. Rotations of 360° or more were particularly difficult to perceive due to this fact. Often choices of inputs were selected based on conveniently located keys or as multiples of 30, or 45, which produced desired effects on the screen.

D. ANGLE CONSERVATION

Although Logo seemed to provide an enhancement for recognizing angles in varying positions, it was difficult to ascertain whether the children understood the angle measure as a sum of degrees of rotation or as a collection of actions to produce a given angle.

E. SUPPLEMENTARITY RELATIONSHIP

Inherent in Logo work is the necessity to distinguish between an angle of rotation and the adjacent constructed angle. Children often used the input to the rotation necessary to position the turtle as the measure of the constructed interior angle.

Pedagogical Implications of Using Logo

"The Logo turtle can be related to things people know because it is not stripped totally of all properties..." Papert (1980). "Since a turtle has direction, it is like a person and allows children to identify with it by bringing their knowledge about their bodies and how they move into the work of learning formal geometry." The goal of children's first experience in the turtle geometry learning environment is not to learn formal rules, but to develop insights into the way they move in space, and to try to establish a firm connection between the personal activity and the creation of formal knowledge. By using the turtle to draw simple geometric shapes, concepts such as angle may be introduced to the child. Papert further purports that turtle geometry is an effective way to carry out Polya's suggestions for the need to instruct children in the use of problem-solving and mathematical strategies. He also claims that Logo encourages students to plan their work, develop logical sequences, and then to test it directly on the computer.

According to O'Shea (1978) Logo provides an aide to one's own thought process by allowing one to trace the components of each problem.. Also, abstract thinking is encouraged since children have to consider new possibilities and learn what to anticipate from a given experience.

Papert (1980) describes the effect of turtle geometry on the use of numbers to measure angles as primarily "relational" or "affective". His view is that in the turtle geometry context, children pick this ability up almost unconsciously. In <u>Mindstorms</u> (1980), Papert stresses the fact that everyone emerges from the Logo experience with a much better sense of what is meant by 45° or 10° or 360° than the majority of high school students ever acquire. "In this way the student can be better prepared for all the formal topics such as geometry, trigonometry, etc., in which the concept of angle is central." Furthermore, he stresses the "body syntonocity" of Logo, as well as a "cultural syntonocity", where the turtle connects the idea of angle to the extra school culture of many children. For example, <u>The Total Turtle Trip Theorem</u> (Papert) of Logo which states, "If a turtle takes a trip around the boundary of any area, and ends up in the state in which it started, then the sum of all its turns will be 360°" becomes more intelligible since it can be related to personal knowledge. Therefore the concept of a 360° rotation becomes an idea that can be used as a tool to think within everyday situations.

In Logo, procedures are manipulable entities (Papert). "They can be named, stored away, retrieved, changed, used as building blocks for superprocedures and analysed into subprocedures. In this process they are assimilated to schematic or frames of more familiar entities. Thus they acquire the quality of 'being entities'. They inherit 'concreteness'." This aspect of Logo permits the use of procedures to be incorporated into the teaching of the concepts of angle, where more than one rotation is involved.

Perhaps one of the most powerful pedagogical implications of Logo concerns the child as learner. Logo allows children to correct errors right on the screen without erasing or starting over. This is especially important for the reluctant learner whose revision strategies will be vastly simplified using Logo.

"Mathematics deals with abstraction. Its symbolism is an essential and powerful tool in effecting that abstraction - of decontextualizing knowledge and relationships, and yet it is just this necessity for decontextualization and formality which many children find so difficult." (Noss, 1985) Social interactions between pairs of children or children working independently within Logo environment contribute to the development of the programming strategies. The feedback from the graphics spontaneously encourages discussion of strategy and outcome. This discussion inevitably involves the verbalization of the mathematical problem at hand. This verbalization would seem, then, to promote the acquisition of the abstract symbolism of the mathematics, concurrently with the more concrete visualization offered as a result of manipulating the Logo turtle on the computer screen.

CHAPTER IV

THE RESEARCH STUDY

Creating a Logo Angle Environment

It is claimed by its designers that the structure of the Logo language provides a concrete model of specific and powerful mathematical ideas. The research literature (Hillel, Kieran, Hoyles and Sutherland, etc.) supports the claim that a Logo environment is rich and potentially well suited to learning a wide range of mathematical concepts, such as angle. This is due in particular to the Turtle Geometry aspect of Logo in which the turtle responds to simple commands such as FD; BK; RT; LT. These commands may be extended into more complicated instructions such as REPEAT. In Logo, basic commands form the backbone of more complex ones. In this sense Logo is consistent, e.g., REPEAT 4 [FD 50 RT 90]. This command will produce a "square" as obviously to the student as the repetitive commands of FD 50 RT 90 FD 50 RT 90 FD 50 RT 90 FD 50 RT 90.

The above commands may be programmed into a procedure. This results in teaching the turtle how to perform a new command. Thus in order to have the turtle produce a square on the screen, the command may simply be given as "square".

This study attempts to employ Logo's Turtle Geometry as a means of providing the student with a concrete model with which to investigate the concepts of angle. An intention of this study is also to examine the extent to which Logo may be used as a tool to model the concepts of angle such as complementarity and supplementarity.

I. SLOW TURTLE

The designers of Logo stress the fact that the properties of displacement and rotation and their associated length and degree measures underlie the most basic turtle geometry activities in the basic commands of RT, LT, FD, BK. In order to produce the turtle graphics, the student must use these basic geometric concepts of angle and line.

As the review of the research literature indicates, the students often had difficulty perceiving the rotation of the turtle, especially those of more than 360°, due to the speed with which the turtle turned. To this end, as in the Hillel study reported above, the turtle's rotation was slowed down. The students' commands to turn the turtle were now called TRT (turn right) and TLT (turn left). This facilitated the students' visualization of the turtle's rotation on the screen.

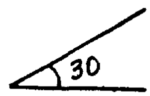
II. NEED FOR STRUCTURE

As the research literature purports, Logo users left entirely to their own resources, become enraptured with the graphics (of Logo) without actually being aware of the geometry and mathematics involved. For example, the Brookline Report (Papert, et al, 1978) identified a range of mathematical concepts used by 11-12-year olds that included the quantitative and qualitative notion of length and angle, but noted that in several cases the children were unaware of the embedded mathematical concepts used in their programming activity.

Subsequently, the researchers seemed to find a need to introduce some structure into the Logo classes in order to make the student more aware of the mathematics of the Turtle Geometry. Pea (1984) states that the guidance and support of the teacher are necessary if Logo is to be used as the designers hoped. Hoyles (1984) points out that the research studies led researchers to decide that it was essential that a balance of pupil initiated exploration and teacher initiated structure should be obtained within the learning of the mathematical ideas. Cathcart (1985) suggests that if students were required to do some pre-planning during their off-computer time, they would likely benefit in two ways: (1) it would result in more carefully structured procedures and (2) time would be used more efficiently for entering commands.

With the recommendations in mind, this study was set up with teacher-initiated structured activities involving specific concepts of angle as objective goals. These angle concepts include complementarity and supplementarity. Furthermore, off-computer planning time was included.

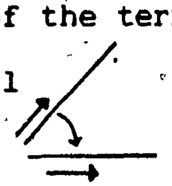
III. DEFINED ANGLES RTA/LTA

In order to link the turtle's rotations with that of the (constructed) angle, special procedures RTA (right angle) and LTA (left angle) were used in the environment. For example, RTA30 and LTA30 both produced 

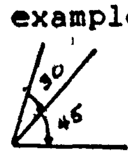
with the turtle rotating 30° to its right in the first case, and 30° to the left in the second case. (The actual orientation of the initial side depends, of course, on the turtle's initial orientation on the screen.) The underlying procedure for RTA was as follows:

```
TO RTA :ANGLE
FD 30 BK 30
TRT :ANGLE
FD 30  BK 30
END
```

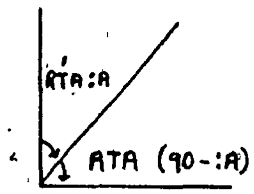
Thus the turtle's initial state is at the vertex, oriented in the direction of the initial ray, and its final state is at the vertex, oriented in the direction of the terminal ray, i.e.,



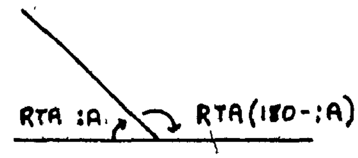
This allowed for producing larger constructed angles as the sum of two angles.

For example, the sequence RTA30 RTA45 would produce: 

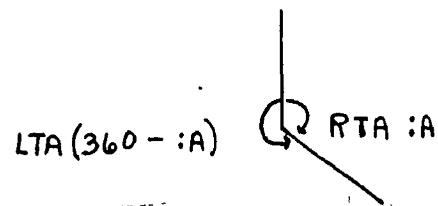
which is equivalent to RTA75. In this way, these angle procedures could be used as subprocedures in producing several angle relations such as:



complementary



supplementary



exterior rotation   interior rotation

Initially, the students were asked to define procedures such as RTA90, RTA45, RTA60, RTA135, RTA120, RTA110 AND RTA150. These procedures were then discussed with them, and modified so that they would be the same for each student, (i.e., same length of arms, same initial and final turtle states).

IV. PRE-PLANNING

The Logo environment also included time for pre-planning tasks, using paper and pencil, during off-computer time. Teacher-initiated discussions served to motivate the students' interest in the mathematical concepts embedded in the Logo activities. As the students worked through the tasks, knowledge was generated independently as well as via their interaction with one another. The students' explorations were shared upon completion of each activity sheet, thus reinforcing the mathematical concepts of the Logo tasks.

V. VERIFICATION WITH COMPUTER

The computer output may be considered the concrete model to develop the child's notion of angle, since as the child constructs the angle on the computer, he is actually manipulating the figure on the screen via the turtle. The visual image of the computer offers the necessary reinforcement to the acquisition of the concept. Composing the special angles using the angle components provides the student with a visually manipulative experience in constructing angles in varying positions and orientation and would thereby promote a better understanding of angles and their relationships.

VI. SOCIAL INTERACTION

In interacting with other students and the teacher, the student is verbalizing the concepts and is assigning the mathematical symbols to the Logo figures concurrently.

VII. SUMMARY OF COMPONENTS OF THE LOGO ENVIRONMENT

The environment created may be outlined as follows:

A. Stencilled activities

B. Teacher motivated instruction and discussion

C. Pre-planning on paper

D. Using slow turtle and defined angles RTA/LTA to produce the required figures

E. Verification with the computer

F. Interaction of students with one another

G. Interaction of students with teacher

H. Verbalization in Logo syntax concurrently with mathematical symbols of the ideas generated by the Logo tasks

I. Formulation of the angle concepts into mathematical terms.

From the visual feedback offered by the Logo environment, logical connections among the angles produced on the screen may be deduced and their properties can be reinforced by the mathematical symbols developed through the teacher - guided class discussions, as well as the structured written exercises. This would seem to provide a sound base on which the student should be able to build the geometry concepts offered to him at the secondary school levels beyond grade 7.

Physical Setting of the Study

The activities for this research were carried out in the computer lab of the high school. This lab was an actual classroom, outfitted with fourteen computers (10 Apple, 4 compatibles) and four printers. The computers lined two adjacent walls, as well as either side of a center island created specifically for the electrical outlets for the equipment. Against the wall beneath the windows was a work area, consisting of tables and chairs. The front of the lab was equipped with a blackboard, as in any standard classroom.

Students in the Study

The class of three girls and six boys (one boy left the school before the project was completed) was selected for this study. These eight students of average to above average intelligence formed a Secondary I (grade 7) remedial mathematics class in a private high school. Some of the students were placed in this class due to their inability to cope with the larger regular classes. Their ages ranged from 12 to 13 years, and their level of mathematics achievement was varied. Most of the students were able to perform at their grade level in the computational aspects of the mathematics curriculum where whole numbers were involved. Decimal fractions and common fractions presented more difficulty to these children as they attempted to memorize algorithms exclusively, as a method of learning these concepts. This style of learning was one which they were comfortable with since it placed less of a demand on them to listen attentively in the classroom as the mathematical concepts were being introduced to them.

Most of the students were low to average achievers in other areas of the school curriculum as well. Their inability to concentrate due to a short attention span hindered their development in conceptual thinking as it applied to languages as well as social studies. Some of the students also demonstrated learning difficulties in the area of memory and sequencing.

For two of the students in the study, English was not a first language, although they had been in the school system for all or most of their elementary education. They still spoke a different language at home, and were experiencing difficulties with vocabulary attainment. The language development of some of the students may have contributed to their weakness in solving word problems, as well as in reading and interpreting the instructions in the activities of this study.

The weaknesses manifested as a result of the students' reliance on rote memorization were especially evident in the aspects of the Secondary I mathematics curriculum which involved the application of basic concepts to percent, ratio and proportion, as well as geometry and measurement concepts. Another difficulty exhibited by those students who were above average in their ability to grasp new math concepts was their lack of attention to details and organization. These students were often quick and careless workers or they were unable to write down the mathematics that they were using mentally.

For these reasons the implementation of a concrete tool was thought to be beneficial to this group of students by the teacher.

Logo Experience of the Students

The students chosen for this study were part of a larger remedial class which was split for remediation in the areas of language and mathematics (as outlined). This class of twenty-one students met together for a weekly session of forty-five minutes of computer science which was part of the regular curriculum. The reseacher/teacher instructed this weekly class in the Apple Logo language. The experimental sessions were carried out in the math sub-group of this class, during their regular mathematics periods.

The background in computer experience of this class was diversified, since the school was fed by three other main schools, two of which belonged to the same private school system. In these two schools, the children had been exposed to Logo as a free exploration with the turtle, within the grade six program. These sessions of approximately 20-30 minutes were blocked into one semester of the school year. One school had used the Atari version of Logo, the other Apple Logo. Some ideas of programming may have been introduced, but were generally mastered only by those who had computer access outside the school (home or camp). In the third school, Atari computers had been used exclusively as a means of introducing the children to computers via games, computer-aided instruction, and some students thought they had also played with "turtles". Although they seemed to have had more access to their computers in their grade 6 school year, they had less experience with Logo than their counterparts. Thus it was the teacher's task in the first semester of Secondary I to find the level of the Logo experience of each student, and to generally develop the basic rudiments of the Logo language.

The weekly classes were set up according to the philosophy outlined by Papert (<u>Mindstorms</u>), the designer of

Logo. The students were introduced to the commands and basic ideas of programming, and through various graphic exploration activities, they acquired the skills which would enable them to use procedures in building other superprocedures. They kept a log of their activities, and worked with partners randomly chosen. By the end of the first semester all the students had acquired the basic knowledge of how to employ the following commands: RIGHT, LEFT, FORWARD, BACK, PENUP, PENDOWN, REPEAT, WINDOW, WRAP, FENCE, PENERASE.

They could also work in DRAW MODE or PROCEDURE MODE, and they were able to use EDIT MODE to debug their procedures. The students were also eager to learn how to save their procedures. This led to the learning of LOAD, SAVE, and CATALOG as well as the understanding of manipulating files using ERASE and ERASEFILE.

The students ended the first semester with the introduction to building superprocedures with already defined procedures.

All the aforementioned commands were introduced and developed through activities which did not emphasize any of the mathematical ideas to be developed later on in the study. The (computer) Logo periods throughout the year were totally different in approach (little structure) as well as in goal, which was solely to learn the programming skills via graphic projects in Logo. This is the experience the students, selected for the research study, brought with them to the Logo/math classes of the study.

Methodology of the Research Study

I. CONCEPTS

The following concepts were introduced to the students via a set of structured activities produced by the teacher/researcher.

A. RIGHT ANGLE

B. ANGLE ROTATIONS OF LESS THAN/GREATER THAN 90°

C. ONE COMPLETE ROTATION EQUALS 360°

D. COMPLEMENTARY ANGLES

E. SUPPLEMENTARY ANGLES

F. INTERIOR/EXTERIOR ROTATIONS OF ANGLES

II. ACTIVITIES

Stencils were distributed to the students at the beginning of each new concept. The concept was developed through the Logo tasks outlined in each activity sheet.

A. INTRODUCTORY SHEET

A general instruction sheet (see Introduction to Logo Activities in Appendix B) was introduced to the class at the onset of the project. It contained information that would be used as a reference to interpret any symbols used on the activity sheets. Amongst the items listed with interpretations were arrows of rotation, a shaded triangle to represent the turtle's initial heading, and a non-shaded triangle to represent the turtle's final heading.

Also presented in this introductory sheet was the general way to proceed for all the tasks in the activities. The children were encouraged to plan all the tasks on paper before checking with the computer.

B. GENERAL CHARACTERISTICS: ACTIVITY SHEETS 1-7

Each new concept was developed through the structured worksheets (see Analysis of Activities) which were stenciled with enough space for the students to write and plan their tasks. The teacher introduced each worksheet orally, reading through all the instructions for each task in order to reinforce the meanings of the instructions, and to ensure that the students could properly interpret the symbols. This also allowed for students to clarify any directions necessary.

Each activity sheet was composed of two or more pages and was broken into several tasks. These tasks were presented either as figures to reproduce using Logo, or as a set of commands that the students had to visualize and sketch on their own. In other tasks the students were presented with a problem which they had to sketch and plan on paper, and then verify their strategy with the computer.

Other tasks were strictly mathematical in nature and served to check the concepts, using appropriate mathematical language. This involved calculating angles, filling in charts, and listing reasons for solving a problem in a given manner.

C. STRUCTURE IN ACTIVITIES

The stencils provided exercises to help the students develop the basic notion of angle rotation. This included estimation and calculation of angle measure, and the relation of an angle to a complete rotation of 360° using the REPEAT command.

The next part of the activities centered on defining angle procedures for acute angles of 90°, 45°, 60°, 30° as well as obtuse angles of 135°, 120°, 110°, 150° (RTA/LTA). These angles were defined by the students and discussed with them by the teacher in order to ensure the proper positioning of the turtle. This was essential to the activities developing the complementary/supplementary angles, as RTA/LTA were employed as subprocedures to produce the new procedure or angle. (see Analysis of Activities)

The final phase of the activities required the students to produce a given angle using the inverse rotation. Since this was completed at the end of the school year, less structure was built into this concept as time did not allow for more classes. The items in this section were comprised of figures where two arrows of rotation were given - interior and exterior turns. The angle measure for either the interior or exterior rotation was provided. The students' task was to supply the missing measure of rotation using the computer as verification.

D. TIME FRAME

Each activity sheet was completed over two or more forty-five minute periods. This time was separate from the regular Logo periods of forty-five minutes per week.

The activities began in the second semester, after an initial period of introduction to Logo programming The activities were not necessarily completed in consecutive weeks due to a break for holidays, exams, etc. within the school calendar. However, each concept was completed within two or three consecutive lessons to enable the students to carry forth the ideas with which they were working.

III. TEACHER/CLASS DISCUSSIONS

The teacher acted as motivator in introducing the concepts dealt with in each activity sheet. The teacher also clarified general directives in the initial presentation of each activity. During the off-computer planning time, the teacher motivated discussions which acted as reinforcement to the concepts being developed at the time. This allowed the pupils to share their findings and to discuss any questions about the activities that they might have forgotten from the previous lesson. At the completion of each activity (concept) an in-class discussion was held where the concepts formulated through the tasks were summarized and verbalized mathematically, in order to ensure a basic understanding before proceeding to the subsequent activity.

IV. METHODS OF EVALUATION

Observations during the activities were noted. Interesting problems that arose from discussions amongst students were noted, along with the written work on the stenciled sheets. Since no tape recorder or video camera was employed, the teacher transcribed all the notes at the end of each class.

To ensure that the activities were being developed by each student, their folders were inspected at the end of each class as well. The class discussions outlined previously also served as a formative evaluation on an ongoing basis. A summative evaluation was given in the form of four questions involving the concepts of angle on the final June exam.

V. OUTSIDE INTERVENTION

Intervention by the teacher, once the students began their work, was minimal and was usually reserved for settling disagreements amongst students or encouraging a deeply frustrated student to continue. The teacher did not supply ideas "to try" but, rather, nudged the children to think back about something they had already learned that might be useful in the situation at hand.

CHAPTER V

ANALYSIS OF THE STUDENTS' WORK

The Logo Activities

I. ANALYSIS OF ACTIVITIES PER TASK

The analysis that follows includes the tasks as they were presented in each activity, as well as examples of the students' interpretations of each task. The students' commands were typed horizontally. Any commands that were corrected by the students are underlined with an arrow pointing to the correction. Examples of the students' sketches have been reproduced where possible.

Each activity contains a specific objective in the development of the concept of angle:

Activity 1 -	RIGHT ANGLE; ANGLES LESS/GREATER THAN 90°.
Activity 2 -	A COMPLETE ROTATION OF 360°.
Activities 3 - 4	COMPLEMENTARY ANGLES.
Activities 5 - 6	SUPPLEMENTARY ANGLES.
Activity 7 -	INTERIOR AND EXTERIOR ANGLE ROTATIONS.

All tasks in the activities are structured to provide the students with experience in observing and manipulating the angles in varying positions (standard, rotated and/or reflected). The tasks include written questions which require the student to reflect on what she/he has constructed in a previous task, and to incorporate the mathematical language with the syntax of the Logo construction.

Upon completion of each activity, the concept being developed was formulated into mathematical terms in order to bridge the Logo ideas with the formal mathematical language of the curriculum.

II. INTERPRETING THE FIGURES

The figures in the activities have specific indications which are necessary in the interpretation of each task. The students were given an explanatory sheet outlining the inherent instructions of these figures. The following is a guide to reading the figures.

A. The solid lines show the first part of the figure to be reproduced.

B. The shaded turtle indicates the initial turtle state.

C. The dotted or dashed lines show the part of the figure to be completed by the student.

D. The unshaded turtle indicates the final position of the turtle.

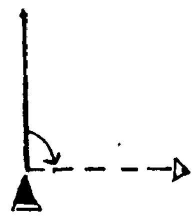
F. The arrow indicates the direction of the rotation.

LOGO ACTIVITY SHEET 1

TASK 1

WRITE THE COMMANDS TO COMPLETE THE FIGURE.

Fig. (a)



FD 50

BK 50

Activity 1 intends to reinforce the students' concept of a 90° angle as a rotation of 90° to the right. The figure is presented in standard position. The student is required to write the commands to draw the dashed line, given the turtle's position after the completion of one ray. No difficulties were encountered with this task. All the students demonstrated an implicit assumption of HOME as the starting position.

TASK 2

WRITE THE COMMANDS TO COMPLETE THE FOLLOWING FIGURES.

Use 50 for your INPUT in FD and BK

Fig. (b)

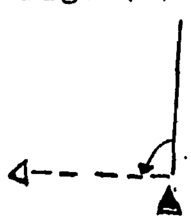


Fig. (c)

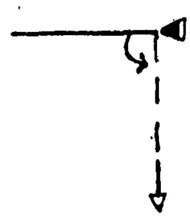


Fig. (d)

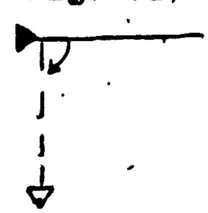


Figure (b) is presented in standard position with a rotation of 90° to the left. The activity is to be completed the same manner as figure (a), with the exception that (i), all the commands must be written to form both rays of the angle, and (ii) the turtle rotation is to the left.

This task presented no problem. The opposite rotation was executed correctly by all.

Fig. (c) and (d)

These tasks present a right angle drawn in an inverted position. The students' initial task is to position the turtle to begin to draw the solid ray of each angle. The second step is to rotate the turtle to the RT or LT to complete the 90° angle.

Most students were successful with both of the above steps to execute the angle in figures (c) and (d)

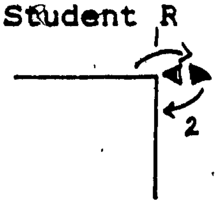
Some work that is of interest is noted below.

Student S and A

Fig. (c)

PU
FD 50
TLT 90 } initial positioning of turtle
PD
FD 50
BK 50
TRT 270 ← ignores the rotation arrow, uses
FD50 exterior or inverse turn of 270°, to complete the right angle.

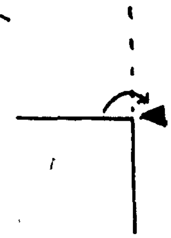
Student R



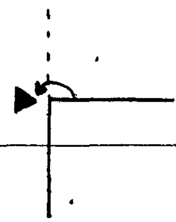
TLT 90, FD 50 BK 50 <u>TRT 180</u> FD 50
↓
TLT 90

R used the round about turn of TRT 180 and then either corrected or added TLT 90 to complete the 90° interior angle. R demonstrated difficulty with the perception of 90° from an upside-down position.

Students T and L

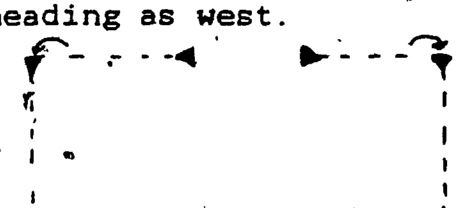
(c) 

FD 50 BK 50 TRT 90 FD 50
↓
FD 50 ______ TLT 90 FD 50

(d) 

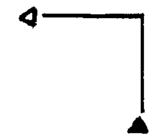
FD 50 BK 50 TLT 90 FD 50
↓
FD 50 ______ TRT 90 FD 50

In the preplanning stage of paper and pencil T and L assumed the default heading of the turtle to be as shown by the shaded turtle of the diagram. Therefore the TRT 90 in figure (c) demonstrates a lateralization problem, since it would bring the turtle to HOME position. The TLT 90 in figure (d) would produce the identical effect in reverse. They corrected the lateralization problem upon feedback from the computer. T & L demonstrated a preoccupation with the angle rotation to complete the figure and thus did not correct the errors that their new commands would still produce when executed on the computer. They assumed the default heading as west.



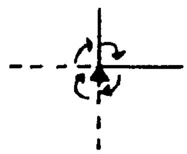
It is difficult to know whether the students verified their commands with the computer which would produce the above figures. T & L demonstrated a lateralization problem which the computer feedback should have helped them visualize.

Starting in default position completely changed the location of the figure on the screen.



TASK 3

WRITE THE COMMANDS TO COMPLETE THE FIGURE. BRING THE TURTLE BACK TO THE STARTING POSITION AFTER THE LAST TURN.



HOW MANY 90 DEGREE TURNS DID THE TURTLE MAKE? ________

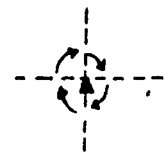
WHAT IS THE TOTAL NUMBER OF DEGREES OF THESE TURNS?______

THE TURTLE WENT ALL THE WAY AROUND ______ TIME.

The arrows indicate the direction of rotation.

The students readily recognized the need to reproduce the given right angle before the completion of the figure was possible. However, there was confusion for some in interpreting the three questions that followed. Some students calculated only the three right angles needed to complete the dotted angles of the figure. Subsequently they answered that a complete rotation was 270°. When actually working on the computer they did go through four right angles. Other difficulties were encountered when the rotation arrows were not followed.

Student T:

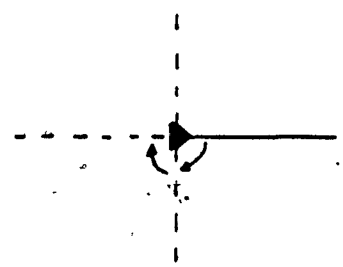


FD 50 BK 50 RT 90 FD 50 BK 50 BK 50
FD 50 RT 90 FD 50 BK 50 TRT 180

T is using the visual cues of the ⊥ pattern before executing the second right angle. T then completes the figure correctly by turning 180° to the right. Judging by the commands that were written and then erased, T seemed to work in direct mode in order to figure the angles.

In answer to the first question, T responded two turns (90° & 180°), yet in response to question 2, T states the total number of degrees as 180°. The last question T answered as the turtle went all the way around <u>four time</u>. Since the word <u>time</u> was given in the singular T showed a possible lack of understanding the written instructions as well as of the arrow notation used to indicate the direction of rotation. Student A is an example of the completion of the three dotted angles while assuming that the first 90° is already drawn. His commands begin with

FD 50
BK 50
TRT 90
FD 50



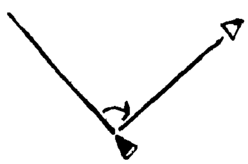
A answered all the questions correctly. A's interpretation of the instructions "to complete the figure" were taken literally as the completion. It is difficult to know whether or not A verified the commands with the computer.

TASK 4

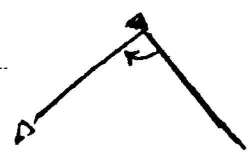
HOW MUCH DO YOU TURN THE TURTLE TO THE LEFT TO PRODUCE FIGURE (a)? ________

Fig. (a). Task 4(a) introduces the student to the 45° angle concept. The students are required to estimate the amount of rotation to the left.

All the students readily recognized the angle measure as half of 90° right angle and correctly answered 45°.



(b)



(c)

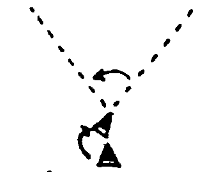
Tasks (b) and (c) present the right angle in two non-standard positions: (b) is a right angle rotated 45°, while (c) is the reflected image of (b). Both figures require the student to recognize the contained angle to be a right angle and to recognize that the drawing of the figures cannot be executed until the turtle has completed an initial turn of 45° in (b) and of 90° + 45° (135°) in (c).

Thus the difficulties of the students fell into two main categories: (i) those students who were able to orient the turtle correctly to its initial state but failed to recognize the right angle in the non-standard position and (ii) those students who had difficulty with the initial state of the turtle.

Student M

TRT 45 FD 50 BK 50 TLT 90 FD 50 BK 50

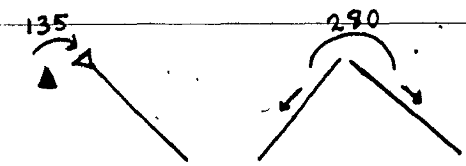
(b)



M produced the correct figure with paper and pencil. However the computer feedback did not point out to him that he had ignored the rotation arrow in the diagram.

M wrote the following to produce figure (c)

TRT 135 FD 50 BK 50 TLT 280 FD 50

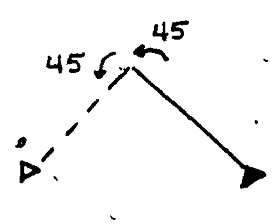


M positioned the turtle correctly for the initial state. To complete the interior angle of 90° M rotated the turtle back around the exterior 280°. The feedback from the computer would give the impression that the inputs were correct. It is not clear whether 280° is the result of incorrect addition or just a guess or estimate.

Student A

A's work demonstrates an incorrect interpretation of the task.

Fig. (c)



TLT 45 FD 50 TLT 45 FD 50

adds TLT 45

A successfully produces the figure, but he ignores all the conventions explicit in the task. A begins the angle from the ray rather than the vertex of the angle. He ends his figure in the given position, but the angle has not been completed from the inverted position although A does demonstrate recognition of the 90° angle.

Student L

Fig. (b) TLT 50 FD 50 BK 50 TRT 50 FD 50 BK 50

inserts TRT 50

Fig. (c) PU FD 50 TRT 90 TRT 45 →

TRT 50 BK 50 BK 50 TRT 50 FD 50

FD 50

L had difficulty estimating the 45° angle correctly. However, the visual feedback for the rotation of TLT 50 presented L with an acceptable reproduction of the initial heading of the turtle in figure (b).

L seems to be launched on an automatic input number of 50, as she applies it to FD, BK, and TRT. The completed figure was corrected by L to include an additional rotation of (TRT) 50° (10° more than 90° given).

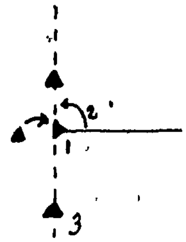
The work for figure (c) again makes use of 50 as input for both moves as well as rotations. L has difficulty with the inverted figures' initial heading. The scribblings to the left of the worksheet indicate some attempt to correct the heading, but it is difficult to tell whether someone was dictating the commands to her quickly, thereby causing spelling errors, or whether L was actually able to calculate the turn of 135° independently.

Student T

Fig. (b)

TRT 90 FD 50 BK 50 TLT 90 FD 50 BK 50 BK 270 FD 50

TLT50



T disregards the rotation arrow in the given figure. This results in a lateralization error. T attempts to fix up the lateralization error after verification with the computer but becomes confused.

Fig. (c)

TLT 90 TLT 50 TRT 50 FD 50 TRT 90 TLT 180 FD 50

T has erased the first commands that were planned on paper and produces the above commands using direct mode. Again it is difficult to know whether T was beginning at the end position and working backwards.

Student CH

Fig. (c) TRT 90 FD 45 BK 45 TLT 90 FD 45

TRT 135 TLT 135

CH corrected very well with the computer feedback. CH was able to correct the initial turn to 135°. However, CH assumed that the interior angle must then also be 135°, without actually checking the whole figure with the computer.

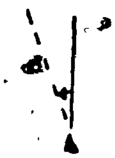
TASK 5

HOW MUCH DO YOU HAVE TO TURN THE TURTLE TO COMPLETE EACH OF THE FOLLOWING FIGURES?

Fig. (a)

TLT ____

Fig. (b)



TLT ____

Fig. (c)

TRT ____

WHY DID YOU CHOOSE THIS NUMBER OF DEGREES FOR EACH ANSWER?

____

This task checks the estimation skills needed to identify angle rotations of less than 90°. The direction of rotation is important to achieve the correct figure.

Most students cited visual cues as the reason for the estimation without making explicit reference to 90° as a basis for comparison.

Only one student was able to express the need to calculate the number of degrees to be less than a right angle.

TASK 6

HOW MUCH DO YOU HAVE TO TURN THE TURTLE TO COMPLETE EACH OF THE FOLLOWING?

Fig. (a)

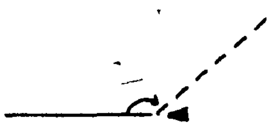
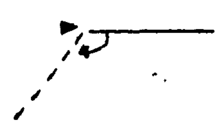


Fig. (b)



This task elicits the estimation of angle greater than 90° from both a standard and inverted position. All students recognized the contained angle to be greater than 90° using visual cues, "it looks right", as well as being able to express the fact that, "because 90° is straight, so we add 20° to make the angle".

This shows good estimation skills which have been fostered by their (use of) Logo skills.

LOGO ACTIVITY SHEET 2

TASK 1

WHAT FIGURE WILL THE FOLLOWING COMMANDS PRODUCE?

SKETCH HERE

FD 50

BK 50

TRT 90

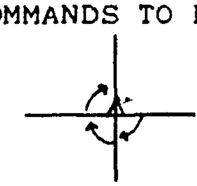
FD 50

BK 50

CHECK YOUR ANSWER BY TYPING THE COMMANDS INTO THE COMPUTER.

DO NOT CLEAR SCREEN

NOW WRITE THE COMMANDS TO PRODUCE THIS FIGURE:



BRING TURTLE BACK TO THE END POSITION SHOWN.

HOW MANY 90 DEGREES TURNS DID THE TURTLE MAKE IN ALL?

______

WHAT IS THE TOTAL NUMBER OF DEGREES THE TURTLE TURNED?____

This task reviews the concept of the right angle by checking the pupils' skill to visualize the figure from the written commands. Recognition of 90° turn to form a right angle is necessary to complete this task

Completion of this figure posed no problem to the students.

The second part of this task deals with the concept of a complete rotation being equal to 360°. This is developed by asking the students to complete three more rotations of 90°

each, which would then bring the turtle to the same state at which it began. Thus the total rotation is completed. The students are then asked to

(a) calculate the number of right angles the figure has in all

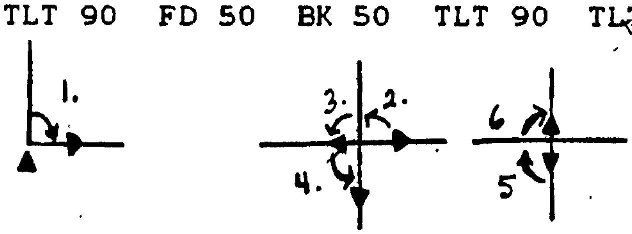
(b) calculate the number of degrees in all the right angles, or 4 x 90° = 360°.

The major problem in analysing this task stemmed from the fact that many of the students did not interpret the task as was intended. Although all the students completed the first right angle correctly; many did not interpret the next task as a continuation of the first, and thus did not include the first rotation in calculating the sum of the angles in the complete rotation of 360°.

The following examples of the students' work illustrate how the figure was reproduced correctly visually, but the task objective could not be fulfilled because of the individual interpretations.

Student T

FD 50 BK 50 TRT 90 FD 50 BK 50 TLT 90 TLT 90 FD 50 BK 50 TLT 90 FD 50 BK 50 TLT 90 TLT 90

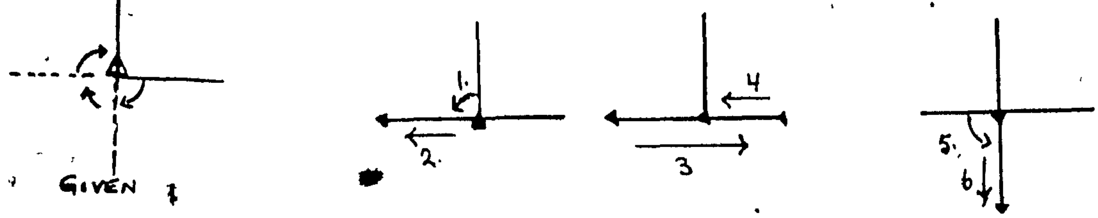


T successfully completes the figure given, disregarding the rotational arrows and thus will have difficulty in answering the follow-up questions that deal with the 360° rotation.

Student M

TLT 90 FD 50 BK 100 FD 50 TRT 90 FD 50

BK 50 X TLT 90



1. 2. 3.

Original commands show completion of the figure with no regard to inherent instructions. M perceives the figure in the above manner and proceeds directly from his initial perception.

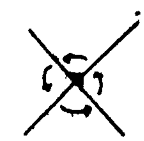
M begins to correct his commands but does not verify them with the computer, and therefore does not carry through the correct number of rotations which will lead to the correct answers to the questions that follow.

Once again the objective of this task will not be fulfilled because of the individual's interpretation of the task.

TASK 2

WRITE THE COMMANDS TO PRODUCE THE FOLLOWING FIGURE.

THE FINAL POSITION OF THE TURTLE IS SHOWN



HOW MANY DEGREES DID THE TURTLE TURN EACH TIME? ________

WHAT IS THE TOTAL NUMBER OF DEGREES THE TURTLE TURNED? __

SINCE THE TURTLE CAME BACK TO ITS STARTING POSITION, THE TURTLE MADE ______ COMPLETE ROTATION.

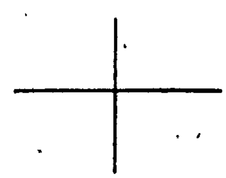
This task deals with the 90° angle in a rotated position as well as with the concept of 4-90° angles are equal to 360°. The rotational arrows indicate the progression in which the angles are to be completed, and the final position shown is a repetition of the previous task. This time four angles had to be drawn in succession, yet, some students assumed the first angle was already formed or else they did not bring the TURTLE back to the final position, and therefore did not complete the fourth right angle. Again, if either of these omissions was incurred, the answers to the questions which followed the task would not be answered correctly, and the complete rotation of 360° concept would not be reinforced as intended.

Another difficulty that was encountered in this task was the recognition of the 90° angle in rotated position. This led to some students estimating the angular measure of each rotation which, of course did not lead to four equal angles. Thus the computer feedback was visually misleading.

The following students' work illustrates some of the aforementioned difficulties.

Student A

TLT 90 FD 50 BK 50 TLT 90 FD 50 BK 50 TLT 90 FD 50 BK 50 TLT 90 FD 50 BK 50



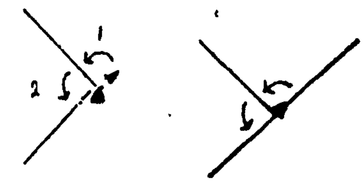
omits initial turn of 45°

A completely ignores the rotated position of this figure, and writes the identical commands as in Task 1. It is doubtful that A verified his commands with the computer.

Student Y

RT 45 TLT 90 TLT 90 BK 50 FD 50 FD 50 FD 50 BK 50 BK 50

FD 50 BK 50

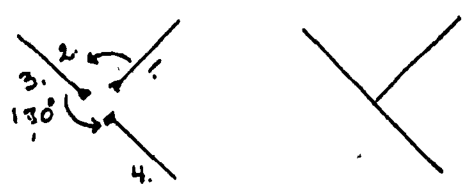


Y initially writes commands which result in only two right angles being drawn. Y then corrects the first commands to FD and BK, but verification with the computer would have shown Y an incomplete diagram. Y obviously used the figure on the stencil to answer the questions that followed, because they were all answered correctly (although Task 1 was incorrect). Perhaps the four rotational arrows aided in identifying 4 angles.

Student L

TLT 45 FD 50 BK 50 TLT 90 FD 50 BK 50 FD 50 BK 50

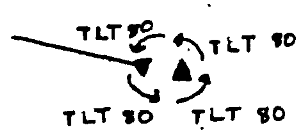
TLT 180 FD 50 BK 50



L recognizes the initial positioning of the TURTLE to be 45° to the right. L completes the first right angle correctly but omits the rotation of 90°, before writing the next set of move commands L then compensates by rotating the turtle 2 x 90° or 180° to complete the figure. Thus L has written commands for four turtle turns, but L has notcalculated the total correctly, nor has L verified the commands with the computer. . Since the feedback from the computer will not duplicate the figure, one can assume that L either did not check the feedback, or that L did check with the computer, but did not know how to proceed to correct the commands.

Student T

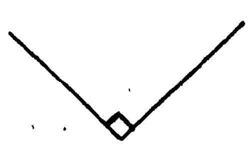
TLT 80 FD 50 BK 50 TLT 80 FD 50 BK 50 TLT 80 FD 50 BK 50 TLT 80 FD 50 BK 50



T uses visual cues to help her complete the figure. By following the rotational arrows, T rotates the turtle four times. Each rotation is 80° to the left. T does not position the turtle to begin as shown, which may lead one to conclude that T does not recognize:

a) that the beginning and ending state of a complete rotation are the same

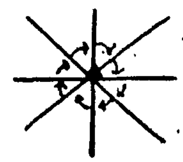
b) the right angle in a rotated position.



Since there are erasures behind the commands given above, T probably wrote these commands using the computer feedback as a visual aide. The fact that the four angles are not equal does not seem to bother T.

TASK 3 - TRY THIS!

WRITE THE COMMANDS TO PRODUCE THE FIGURE BELOW. POSITION THE TURTLE AT THE END, SO THAT IT IS IN THE SAME POSITION AS IT STARTED. (as shown).



HOW MANY ANGLES ARE THERE IN ALL? ______

HOW MANY DEGREES DID THE TURTLE TURN EACH TIME? ______

WHAT IS THE TOTAL NUMBER OF DEGREES THE TURTLE TURNED? __

THE TURTLE HAS MADE ______ COMPLETE ROTATION.

TASK 4

FROM THE ABOVE ACTIVITY, ANSWER THE FOLLOWING QUESTIONS.

a) WHAT DO YOU NOTICE ABOUT THE COMMANDS FD, BK, TRT? _____

b) DO THE INPUTS CHANGE EACH TIME? ____________________

c) HOW MANY TIMES IS EACH COMMAND REPEATED? ______________

d) SINCE THE COMMANDS ARE REPEATED - THAT IS - THE COMMAND AND THE INPUT REMAIN THE SAME -

WE CAN WRITE THESE COMMANDS AS:

REPEAT INPUT [COMMANDS IN ORDER TO BE CARRIED OUT] square bracket

How many times?

REPEAT IS A VERY USEFUL COMMAND WHEN YOU ALREADY KNOW HOW MANY TIMES TO REPEAT SOMETHING!

Task 3 asks the students to produce a figure that is the result of rotating the turtle 45° to the right, eight times. This results in one complete rotation of 360°. The rotational arrows in the figure once again provide the cue to the number of angles the TURTLE must draw to complete the figure. The instructions again emphasize that the starting position of the TURTLE is the same as the ending position, thus offering the cue to the complete rotation of the turtle.

The major difficulty in completing this task was the task of recognition of the two angles that must now make up one right angle. Therefore the simple calculation of 90 ÷ 2 = 45 was not part of the initial planning of this task for some children. This led some students to use visual cues to estimate the amount of rotation in each angle, and to therefore not rotate each angle equally. The computer feedback for these unequal angles proved to generate much frustration for those students.

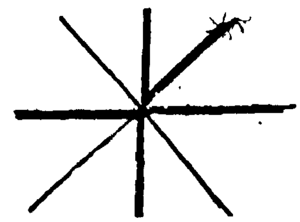
As the students went on to Task 4 some were able to use the questions about the repetition of the commands as cues to producing the figure in Task 3. Task 4 introduces the students to the REPEAT COMMAND of the Logo language. It also serves to reinforce the concept that the complete rotation of 360° may be broken up into "X" number of equal angles.

Here are some examples of how the students perceived Task 3.

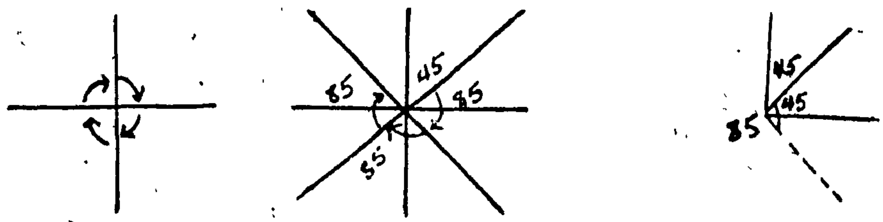
Student T

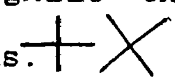
T chooses turns of 90, 90, 90, 85, 85, 85 as the sequence of inputs.

T has heavily pencilled in the following lines in the figure on the worksheet.



T first completes figure by making four rotations of 90°. T recognizes the 45° turn to make the two angles of 45° each equal to the 90° angle T has already formed.



However T is aware that to get to the next angle is going to be a rotation greater than 45°, but T fails to recognize that this figure is made up of the previous two tasks.  T estimates 85°. There are erasures under the writing, which leads one to assume it was the computer feedback that led to that estimation. In answering the questions, T correctly answers the total number of angles to be 8 - the number of degrees is calculated in several ways

90 x 4

360° + 45

85 x 3

In answer to the question: What do you notice about the commands FD BK TRT?

T answers, "...that you always have to use those commands".

T answers the question: Do the inputs change each time? with, "No".

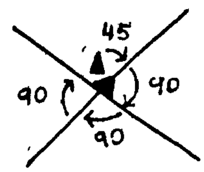
T's answers show a lack of recognition of any mathematical calculations associated with the concept of one complete rotation is equal to 360°.

The visual cues are what T uses to produce the geometric figure. The computer aids T with the visual aspect of this task.

Student A

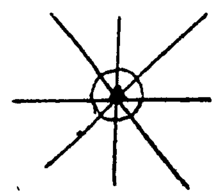
First set of commands looks like this:

TRT 45 FD 50 BK 50 TRT 90 FD 50 BK 50 TRT 90 FD 50 BK 50 TRT 90 FD 50 BK 50



A realizes his mistake and crosses out the above commands and rewrites the commands correctly, rotating the turtle to the right 45° each time.

A's stencil figure now looks like this:

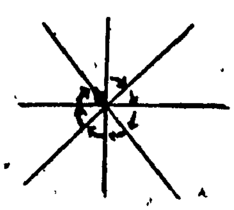


A has drawn a circle around the figure. A demonstrates that the rotation of 360° applies here by drawing the rotational circle.

Student R

R repeats seven times:

FD 50 BK 50 TRT 45



R's commands are an example of completing the figure without returning the turtle to the end position. Therefore, the feedback of the computer will be correct, but the amount of angle rotation does not equal 360°. R misinterprets the question which asks, "How many angles in all?" and answers with TRT 45. R answers the number of degrees the turtle turned each time correctly as 45.

R calculates the answer to the total number of degrees the turtle turned as 315 or 7 x 45.

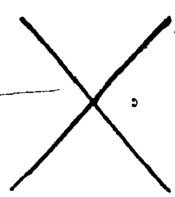
Task 4 also checks the students' comprehension of the mathematical terms **angle** and **degrees**.

It is interesting to note that R interpreted the question asking, <u>how</u> <u>many</u> angles are there, with the Logo command TRT 45. The term angle and the turtle rotation seem to be synonymous for R.

TASK 5

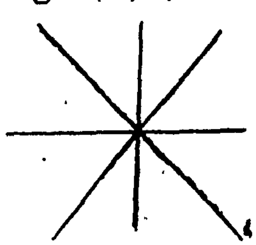
TRY TO WRITE A REPEAT COMMAND TO PRODUCE THE FOLLOWING FIGURES.

Fig. (a) COMMAND



____

Fig. (b)



____

A) DID YOUR GRAPHIC DISPLAY LOOK LIKE FIG. (a), (b)? IF YES, GO ON

B) NOW WRITE THE REPEAT COMMAND, CHANGING THE ORDER IN THE SQUARE BRACKETS.

C) WHAT HAPPENS?________ WHY?________

This task was completed by the students after a class discussion about Task 3 and 4. Thus the task serves as reinforcement of the use of the REPEAT command of Logo as well as a reinforcement of the concept of one complete rotation is equal to 360°. Generally, the students performed well as they recognized the figures from the previous tasks. The fact that the repeat input times the angle rotation is to equal 360° in one complete rotation, stressed the concept of the equal angles in each figure.

LOGO ACTIVITY SHEET 3

TASK 1

WRITE THE COMMANDS TO PRODUCE THE FIGURES.

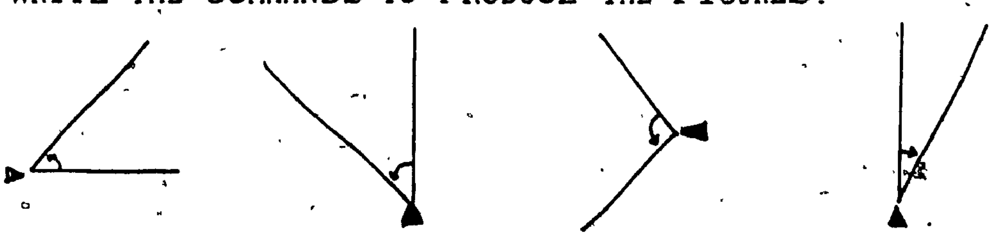


Fig. (a) Fig. (b) Fig. (c) Fig. (d)

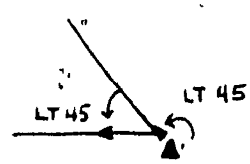
THE NUMBER OF DEGREES IN EACH FIGURE IS __________ DEGREES OR LESS.

Task 1 serves to check the students' estimation skills for angles 90° or less. The figures were positioned in standard and non-standard positions; the rotations were given either to the left or right. The initial heading was given (in each figure).

Some of the main difficulties with this task were not in the angle estimation, but in the interpretation of each task, from the visual cues given; i.e., rotation arrows and turtle position.

Many students omitted the initial turn of the turtle to the position given in the figure. An example of this may be seen in the work of A for figure c).

TLT 45 FD 30 BK 30 TLT 45 FD 30 BK 30 TRT 30

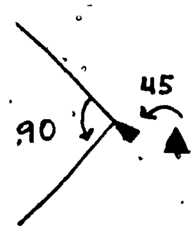


A does not recognize the right angle in figure (c) and so completes this figure by rotating the turtle left 45°. At the end, the turtle is returned to HOME position. No verification with the computer has been effected since the feedback would have been different from the original figure.

Student CH

TLT 45 FD 50 BK 50 TLT 90 FD 50 BK 50

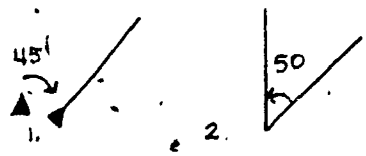
Fig. (c)



CH recognizes the 90° angle.

Student T

TRT 45 FD 50 BK 50 TLT 55 FD 50

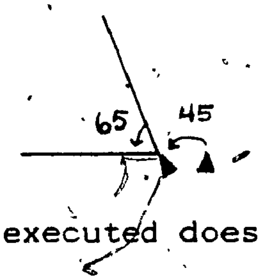
Fig. (b) 

T writes commands for this figure from the visual cue. Lateralization seems to be a problem here.

T checks with the computer but only corrected the TLT to 55 (original input is erased).

In figure (c) T estimates that angle to be 65°.

TLT 45 FD 50 BK 50 TLT 65 FD 50 BK 50



The figure T executed does not produce the figure given on the worksheet. T does not feel the need to correct it further.

It is interesting to note that T has no difficulty in completing figure (d).

TASK 2

WRITE A <u>PROCEDURE</u> FOR EACH OF THE FOLLOWING FIGURES.

USE THE STARTING POSITION, ▲

POSITION THE TURTLE AT THE END OF EACH FIGURE AS SHOWN.

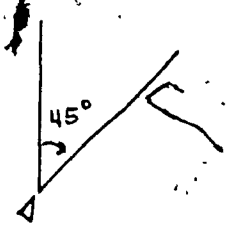
To RTA90

______

______

______

END



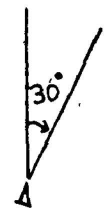
To RTA45

______

______

______

END



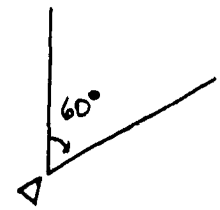
To RTA30

______

______

______

END



To RTA60

______

______

______

END

Task 2 is structured to include the idea of a procedure so that the resulting angles may be saved on the students' personal diskettes. Here the specific angles of 90°, 45°, 30°, 60° will be formed so that they may be used later as components to building larger angles. Therefore, the students were instructed to begin and end in the same manner and to use 30 as the input to FD and BK. For example: TO RTA90 FD 30 BK 30 TRT 90 FD 30 BK 30 END.

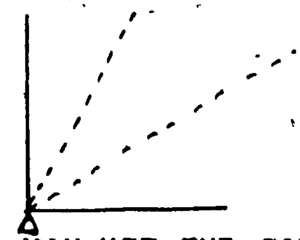
The greatest problem encountered with this activity lay in the change of the input to FD and BK to 30 from 50 that the students had been using up to this point.

T and L defined RTA30 using 45 as the angle rotation but were able to correct the error satisfactorily, after checking with the computer.

The procedures for each of these angles was undertaken independently by each student but the teacher/researcher did correct each one in order to ensure the workability of these modules in later tasks.

TASK 3

USE THE COMMAND RTA30 TO PRODUCE THE FOLLOWING FIGURE.



a) HOW MANY TIMES DID YOU USE THE COMMAND RTA30?______

b) HOW MANY DEGREES DID THE TURTLE TURN EACH TIME?______

c) THE TOTAL ROTATION IS __________DEGREES.

This task once again incorporates the features of the Logo commands to build one angle of 90° by putting together three angles of 30°. Simply by commanding the turtle to RTA30 three times, the turtle will carry out the procedure of completing a 30° angle, stopping after each angle in the correct position to "stack" another 30° angle alongside the first, and so on, until a 90° angle is produced.

Some of the students did not realize this capability and so included an extra turning command to reposition the turtle to begin another 30° angle.

This error was readily corrected from the visual cues offered by the computer screen.

Another difficulty encountered was in the use of the Logo command RTA30. For example, student A wrote out the commands to form a 30° angle until he realized that this was not necessary.

Student A

FD 30 BK 30 TRT 30 FD 30 BK 30

RTA30 RTA30 RTA30

Student R

RTA30 }
RTA30 } was written above an erasure of the commands to form a 30° angle.

Student T

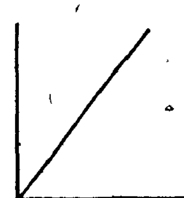
RTA30 TRT 20 RTA30
↓ ↓ ↓
RTA30 RTA30 RTA30

The computer check reinforces the use of the command.

All the students were able to answer the three questions that were based on calculating three rotations of 30° to result in 1 right angle or 90°. Teacher/pupil discussion reinforced the idea of building one large angle from a number of smaller ones.

TASK 4

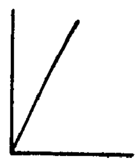
WRITE A PROCEDURE, USING THE COMMAND RTA45 AS A SUBPROCEDURE, TO PRODUCE THE FOLLOWING FIGURE.



This task employs the Logo syntax for building a complex figure from components. In this task the complex figure is the 90° angle, which is composed of 2, 45° angles. The command RTA45 thus becomes a subprocedure of the overall procedure for 90°. This idea was discussed orally with the students who developed the idea of putting together the "pieces of a puzzle". The verbalization of the Logo commands as well as the mathematical calculations which result in 2 x 45° = 90°, helped any students who were still writing out all the commands instead of using subprocedures to complete the figure.

TASK 5

WHICH OTHER 2 SUBPROCEDURES MAY BE USED TO PRODUCE THE FOLLOWING FIGURE? WRITE THE PROCEDURE.



This task offers the students verbal as well as visual cues. By now the students have grasped the idea of using the subprocedures as components. The difficulties encountered here by two students are illustrated below.

Student L

~~RTA60~~ → RTA30 RTA60

a) First the turtle turned ~~60~~ → 30 degrees.

b) The second time the turtle turned ~~90~~ → 60 degrees.

c) The turtle turned ~~150°~~ → 60 in all.

L demonstrates an almost automatic response to this task in writing two commands of equal rotation. L ignores the visual cues of two different-sized angles, as well as the verbal cues of "which <u>other 2</u> ...". In answering the questions L calculates 60° for the first turn. Interpretation of the second turn is 90°, and the total is added correctly as 150°. However, L does not seem to make any connection between actual figure of 90°, and (a) the total of 150° › 90°; (b) the visual cues of 60 + 60 that the computer screen offers.

R on the other hand writes the commands correctly

'RTA30 RTA60'

but R answers the questions in the same way as L. There does not seem to be any association between the visual and the mathematical calculations. R seems to be doing a cumulative addition 60 ⟶ 90 ⟶ 150, rather than adding the components included in the one large angle of 90°.

TASK 6

CAN YOU FIND OTHER PAIRS OF TURTLE TURNS THAT WILL PRODUCE A FIGURE WITH A TOTAL OF 90 DEGREES? 

WRITE THE COMMANDS FOR EACH TURN.

FIRST TURN							
SECOND TURN							
TOTAL NUMBER OF DEGREES IN 1st & 2nd TURN.							

Many students had difficulty interpreting the task. Teacher explanation clarified the 90° was the figure produced by the sum of the degrees used in the frist and second turns.

The chart in this task serves to draw attention to the fact that other components of 90° may be used. This also helps the students to drill the mathematical computations

of _____ + _____ = 90° or 90 - _____ = _______

and the association with the visual output of the computer.

The answers were discussed orally in order to reinforce the concept (of complementarity).

On the whole the students were reluctant to experiment and correct with the computer.

TASK 7

SKETCH THE FIGURE THAT WILL BE PRODUCED AS A RESULT OF THE FOLLOWING COMMANDS.

a)	b)	c)	d)
RTA45	PU	TRT 90	PU
RTA45	FD 50		FD 50
	TRT 90	RTA60	TRT 180
	PD	RTA30	PD
	RTA30		RTA45
	RTA60		RTA45

CHECK EACH SKETCH WITH THE COMPUTER SCREEN.

This task checks the students' perception of a 90° angle as being composed of two smaller angles. It asks the students to estimate or project what they think the computer turtle will produce as a result of each set of commands. Only (a) is in standard position. For the other figures the students have to go through the moves of the turtle with their pencil on paper. The angles may be estimated.

The students as a whole seemed to pay particular attention to the turtle state in carrying out each activity. The angles tended to be drawn less carefully and sometimes resulted in the initial angle being drawn too large or too small.

Student L

This is L's sketch for (b)  . L positions the turtle correctly but her estimate of RTA30 is way too large, and then L is confronted with drawing the second angle of RTA60 even larger. This results in an angle greater than 90°. L does not seem to have checked with the computer. The sum of the rotations 30 + 60 = 90° do not seem to be associated with her sketch.

Student A

A's sketch for (c)  also shows no recognition of the sum of the rotations equalling 90° or a right angle. A also seems to have drawn the first angle less than the second in size.

Student R

R represents (c) as  .

R also does not quite complete a figure of 90°. R seems to have taken great care to draw the second angle smaller than the first. In doing so R has neglected to form a right angle even though R has been successful in figure (b)  , there does not seem to be any association with the two components used in each.

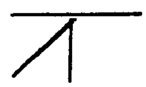
Student T

T has difficulty with the adjacent side of the two components. T draws

a) 

b)  which is erased → 

c)  which is corrected → 

T does not seem to check figure (a) since the 45° is one that T is more confident with. In figure (b) T does not seem to be sure of how to proceed and so T checks with the computer. The drawing exhibits some perceptual difficulty. T proceeds in the same way for figure (c). In both figures there is no real differentiation between the angle of 30° and the angle of 60°. Figure (d) is correct after the initial position was realized  ← erased.

<u>TASK 8</u>

THE PAIRS OF ANGLES THAT ADD UP TO 90° ARE CALLED COMPLEMENTARY ANGLES.

USE YOUR CHART ON PG 2 TO FIND DIFFERENT PAIRS OF COMPLEMENTARY ANGLES. LIST THEM BELOW.

____ + ____ = 90°	____ + ____ = ____

In this activity the mathematical vocabulary for complementary angles is developed. Again the students are asked to reinforce the concept by calculating (without the computer, this time) pairs of angles that equal 90°.

Many students omitted (the second column) to write the sum of the angles = <u>90°</u>. Some computational errors were made. e.g., A → 5 + 82 = 90°. Y used one pair as 90° + 0 = 90°, demonstrating a need to fill in the blanks with any numbers to total 90°. The measurement of the angles seems to be lacking in the interpretation of the task in the following example as well: 1 + 89 = 90

2 + 88 = 90

none of the students used decimals as complements.

TASK 9

a) CAN YOU MAKE AN 8 STAR PATTERN? 

USE YOUR ANGLES TO HELP YOU WRITE A REPEAT COMMAND.

This task is repeated once again in the context of using the RTA45 to build the star or to complete the rotation of 360°.

The students were all able to put the idea of the subprocedure into a repeat command REPEAT 8 [RTA45].

L and T wrote REPEAT 8 [RTA30] which they easily corrected after the computer feedback.

TASK 9 b)

CAN YOU MAKE A 12 STAR PATTERN?

SKETCH	COMMANDS
______	______
	______
	______

Before the students tackled this problem, the second session of this group of activities had taken place.

The activities leading up to the development of the COMPLEMENTARY ANGLES concept were discussed. The pairs of angles were seen by the students as:

a) concrete pieces of a puzzle that fit together to form one larger piece, as represented visually on the monitor.

b) mathematically as two angles whose sum is 90°.

After this discussion the students were able to use their calculations to help them in estimating the angle measure needed to complete the star patterns. The need to use only visual cues was diminished considerably.

L and T still find the need to use visual cues to complete

9 (b) REPEAT 12 [RTA60]

12 [RTA30]

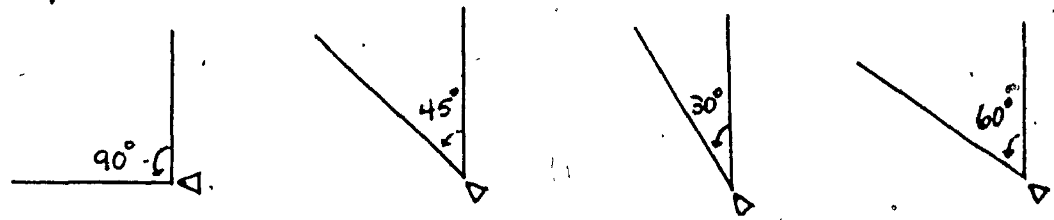
LOGO ACTIVITY SHEET 4

This activity has as its aim to further develop the concept of complementary angles. To practice this concept this activity uses angles with rotations to the left, which are defined as LTA90, LTA45, LTA30, LTA60. The tasks are structured similar to those in Activity 3, except for the inverse rotation (to the left) of the angle.

TASK 1

WRITE A PROCEDURE FOR EACH OF THE FOLLOWING FIGURES. USE THE STARTING POSITION ▲ .

POSITION THE TURTLE AT THE END OF EACH FIGURE AS SHOWN.

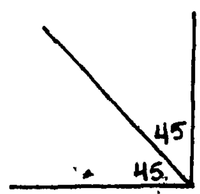


TO LTA90	TO LTA45	TO LTA30	TO LTA60
____	____	____	____
____	____	____	____
END	END	END	END

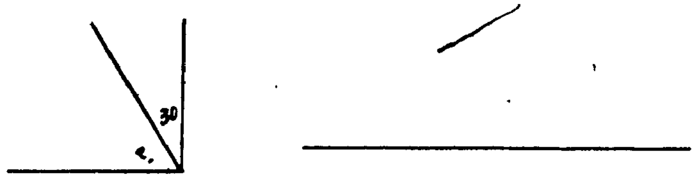
This task asks the students to write a procedure to form angles of 90°, 45°, 30° and 60° which would appear graphically as the opposite of those programmed in activity 3. The students did not encounter any problems with lateralization. As in the previous activity all the procedures were checked by the teacher to ensure the uniformity of the construction of the angles, since they would be used as components to form one angle of 90°.

TASK 2

WHICH TWO SUBPROCEDURES MAY BE USED TO PRODUCE THE FOLLOWING FIGURES? WRITE THE PROCEDURE.

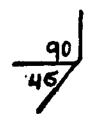
(a) 

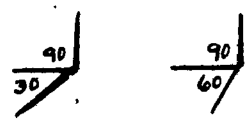
____

(b) 

This task uses the Logo syntax of subprocedures to form another PROCEDURE. In this way the students are required to "build " a 90° angle using two smaller angles whose sum is 90°. One difficulty encountered was in the interpretation of the figures.

Students K and Y

a) LTA90 LTA45 a) 

b) LTA90 LTA30 or LTA60 b) 

These students looked first at the whole angle of 90° and at the contained angle of 45°; or 30° or 60°. The concept of putting together adjacent angles to build a larger one of 90° is not recognized. A procedure has not been written.

The fact that the commands were not corrected indicates that no verification with the computer was carried out as the graphic display would not look like the given diagram.

Another difficulty was in the interpretation of the Logo instructions.

Student CH

a) FD 30 BK 30 TRT 45 FD 30 BK 30 TRT 45 FD 30 BK 30
TLT 45 TLT 45

to 2LT A45 end.

CH initially writes all the commands to form the given figure but with the angle rotation to the right instead of to the left. Verification with the computer has led CH to change the angle turn to TLT to correct the lateralization problem. When CH was asked to use a procedure composed of the subprocedure in Task 1, CH produced: To 2LT A45 End. This shows a lack of understanding of the Logo syntax for combining procedures.

The interpretation of the instructions to write a "procedure" was not realized by the majority of the students. Most wrote the commands as demonstrated by K and Y, that is, using the defined angles of Task 1. This may be due to the fact that the instructions in the previous Activity 3 were given in that form and the students automatically performed in like manner.

TASK 3

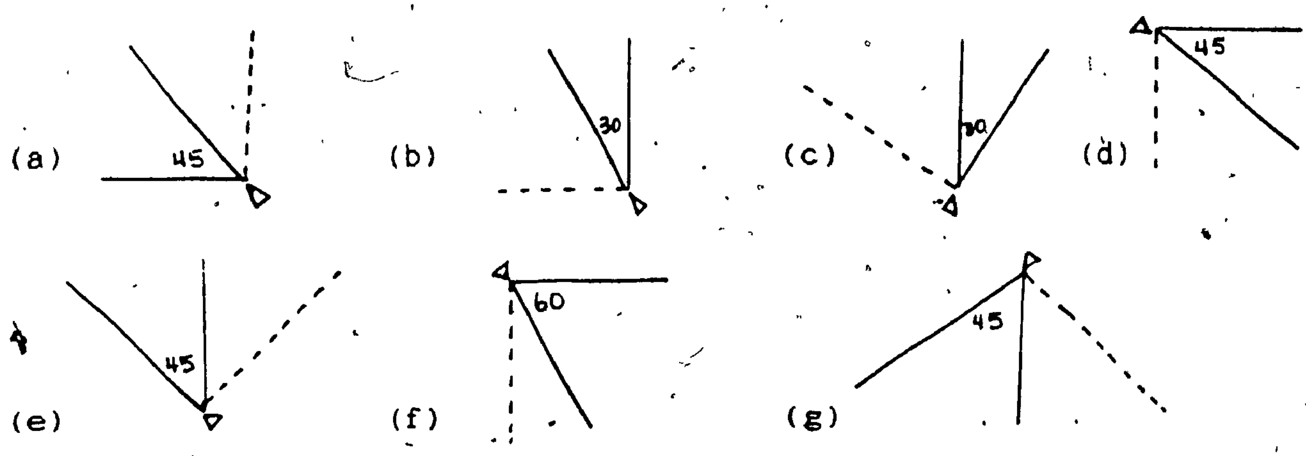
FIND OTHER PAIRS OF COMPLEMENTARY ANGLES TO MAKE OTHER FIGURES. USE THE COMMAND TLT.

FIRST TURN						
SECOND TURN						
TOTAL NUMBER OF DEGREES						

This task checks the pupils' ability to use the calculations of two angle rotations that equal 90° or to compute 90° - ____ = ____. All the students were able to write the number representing the angle measure in each case correctly. However, all the students completely neglected the instruction to "use the command TLT", which would enable them to visualize the rotation of the turtle to the left and to verify the rotational measures with the computer. Since no commands were given, it is difficult to know whether they were verified with the computer. The students seemed more confident in carrying out this task than its counterpart in the previous Activity 3.

TASK 4

WRITE THE COMMANDS TO COMPLETE THE 90-DEGREE ANGLE IN EACH FIGURE. USE YOUR PROCEDURES AS COMMANDS WHERE POSSIBLE.



The figures in this task represent complementary angles in standard and non-standard positions. In each figure the angle measure is given of one complement of the pair of complementary angles. The position of the turtle shown in the diagram is the position of the turtle after completion of the given complement. The students' task is to first of all, calculate the measure of the complement needed to complete the complementary angle, and secondly to reproduce the given figure using the angle procedures defined in Task 1 as the "pieces" to compose the angle of 90°. This task also incorporates the angle procedures from Activity 3, i.e., rotations to the right as well as angle rotations to the left (in this activity).

Figure (a) was reproduced without any difficulty. However, most students wrote commands:

LTA45 LTA45 or REPEAT 2 [LTA45]

This would result in the correct figure but the dotted angle would be drawn first. Thus the position of the turtle in the given diagram was largely ignored.

Figure (b) was easily completed using two angles with rotation to the left. LTA30 was completed first in most cases. Student Y wrote commands for (a) and (b) as a right angle and a contained angle of a smaller measure:

figure (b) LTA90 LTA30 was corrected to LTA30 LTA60

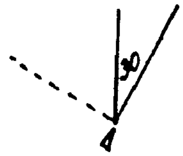
Y's first attempt ignores the visual cues presented in the figure on the stencil. There is no complementarity in the initial commands. Feedback from the computer helped Y correct the commands.

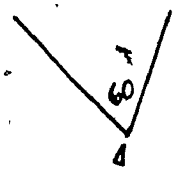
Figure (c) presented some difficulty as this figure of a 90° angle is not presented in standard position. Most of the children readily calculated the "missing" number of degrees in the figure to be 60. However, the rotated position of the figure presented problems with writing the

Logo commands.

Student S

RTA 30 LTA30

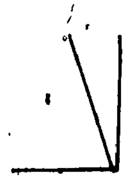




S observes the position of the turtle in the diagram and correctly forms the 30° angle by rotating to the right. However, S shows an erasure beneath her next command. Since it is not clear, it is difficult to analyse except to note that S has some difficulty with executing the next angle. The computer feedback has told S that the commands are not correct as they are written. The problem here is the positioning of the turtle correctly before producing the LTA60.

Another difficulty encountered was in recognizing the 30° rotation to the right first and an automatic response of

LTA30 LTA60 was given

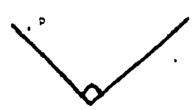


A has a problem with the rotated position.

Another interpretation of figure (c) completely ignored complementarity but produced the correct figure.

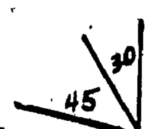
Students K, Y, T and L

RTA30 LTA90



Student R

R shows no complementarity in his commands LTA30 LTA45



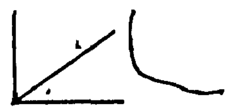
Here the calculations as well as the figure is incorrect.

This has not been verified with the computer.

Figure (d) was completed by many students without giving the command to position the turtle to begin the figure, which is in inverted position.

REPEAT 2 [RTA45]

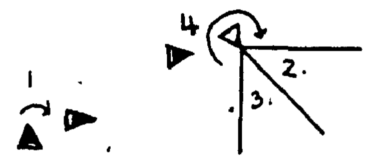
Verification with the computer would present this figure in standard position.



The heading of the turtle may have been assumed by these students.

Student K

TRT90 RTA45 RTA45 LTA45



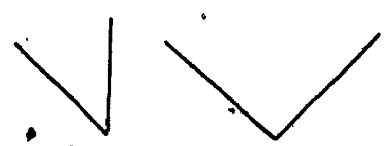
K has written commands to complete a 90° angle. The last command places the turtle back in the position given in the diagram on paper. There seems to be confusion with the position given, which is where the turtle will be at the end of the first RTA45.

Figure (e) again deals with a complementary angle which is made up of two 45° angles. It is presented in a rotated position.

Again, the commands to complete this figure were more difficult to write for some students.

Students R and T

LTA45 RTA90



The above commands will produce the correct graphic without the concept of complementarity being fulfilled.

Student S LTA45 RTA45 RTA45

Student CH RTA45 LTA45

S and CH both recognize the complementarity and the need to form two angles of 45°. They are experiencing difficulty with the interfacing of the Logo angle procedures.

S correctly rotates left 45° first, but for some reason crosses it out. The new figure is not rotated. It is difficult to know whether CH has verified his commands with the computer. CH does not recognize that he has written the inverse commands. The turtle state at the end of each rotation has not been observed.

Figure (f) is in inverted form. Most of the students were able to complete this correctly. Some students who had assumed the turtle state in figure (d) did not omit the initial commands to TRT90 before beginning the two angles of 60 and 30. Only two students omitted the initial turn in their commands.

Figure (g) is inverted and rotated. T, L, K and Y correctly positioned the turtle before beginning the figure. Their commands are:

TLT 135 LTA45 LTA45

The following students assumed the default position.

Student A REPEAT 2 [LTA45]

Student R RTA45 LTA90

R's commands also show no recognition of complementarity.

Student S

S has difficulty with the inverted and rotated position

TRT 90 RTA45 RTA30 

S has not calculated the angle measure correctly. S may be using visual cues only.

The above students have not taken the time to correct the commands with the computer. This may have been due to time constraints in completing this activity. CH totally ignored this item.

TASK 5

SKETCH THE FIGURE THAT WILL BE PRODUCED AS A RESULT OF THE FOLLOWING COMMANDS. CHECK YOUR ANSWERS WITH THE COMPUTER.

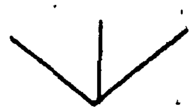
a)	(b)	(c)
TRT 45	TLT 30	TRT 90
LTA45	RTA60	RTA45
LTA45	LTA	TRT 90
		RTA45
SKETCH	SKETCH	SKETCH

(d) REPEAT 8 [LTA45]

(e) FIND THE MISSING ANGLE TO COMPLETE THE STAR COMMAND:

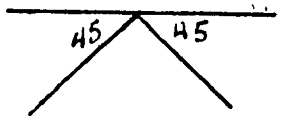
REPEAT 6 [LTA___] SKETCH ______

This task provides the students with the opportunity to use their estimation skills and visual perceptual skills in drawing the figure that each set of commands will produce. Included in the figures are:

(a)  an example of a pair of complementary angles where the turtle is first rotated to produce the non-standard form,

(b) 

a reproduction of a typical error in interfacing, which will not result in complementary angles

(c) 

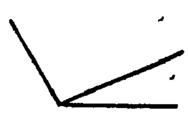
an example of a figure where an interfacing rotation was not needed in this position.

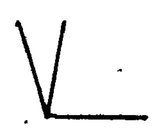
The students' previous experience in writing commands and verifying them with the computer should be an aide in completing this task.

Figure (d) checks the students' ability to calculate one rotation of 360° that will be completed as a result of this command, while (e) solicits the number of degrees needed to rotate to make the star pattern with 6 equal angles.

All the students successfully sketched figure (a) although the right angle was not always evident in their final figure.

Figure (b) presented more difficulty. Some students could not correctly portray the rotated position since they were intent on showing a difference in angle measure.

Student S 

Student T 

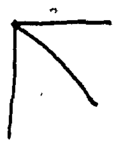
Student R 

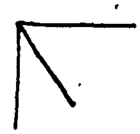
Student A 

Student CH 

R's figure shows a complete error in rotation. The other figures seem to have more difficulty with the commands RTA60 and LTA30, which will result in a bisected 60° angle rather than a 90° angle. K and Y were successful in sketching. L omitted the entire page of tasks.

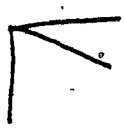
Figure (c) was completed correctly by three students. The other students were successful with the initial turn of TRT90 but they ignored the second turn of TRT90 between the two 45° angles.

Student K 

Student Y 

Student CH 

corrected to 

Student A 

Of the above students only CH seems to have verified and attempted to correct his sketch from the computer feedback.

Figure (d) was completed correctly by all the students.

Figure (e) was completed correctly by all the students except S who used 25° as the rotation. S calculated 25x 6 = 150 but did not seem to remember the number of degrees in a complete rotation nor did she try to work it out with the computer.

TASK 6

THERE ARE ______ DEGREES IN ONE ROTATION.

COMPLEMENTARY ANGLES ADD UP TO ______ DEGREES.

a) WHICH FIGURES ABOVE ARE EXAMPLES OF A ROTATION OF 360 DEGREES? ____________

b) WHICH FIGURES ARE EXAMPLES OF COMPLEMENTARY ANGLES? ______

This task is presented in the form of statements and questions in order to reinforce the language associated with the mathematical concepts acquired in the activities thus far.

Two students confused the rotation with the degrees of a right angle and corrected 90° to 360°. R and A, whose first language is not English, confused the number of degrees in complementary angles with those of one rotation. It is difficult to know if perhaps the sentence should have read "one complete rotation".

A was unable to answer questions (a) and (b) correctly. R answered (a) correctly but used the figure in 5(e) to answer (b).

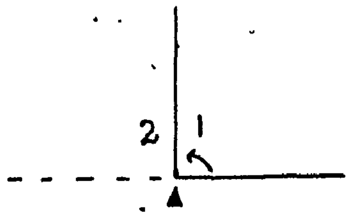
LOGO ACTIVITY SHEET 5

TASK 1

WRITE THE COMMANDS TO PRODUCE FIGURES:

THE STARTING POSITION IS GIVEN (ALL ANGLES ARE EQUAL.)

Fig. (a)

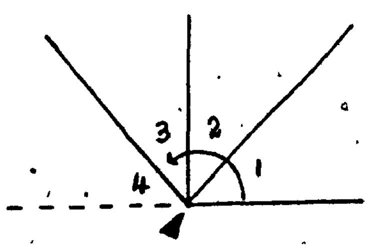


HOW MAY DEGREES DID THE TURTLE TURN IN THE FIRST ANGLE?

HOW MANY DEGREES DID THE TURTLE TURN IN ANGLE #2? ____

HOW MANY DEGREES DID THE TURTLE TURN IN ANGLE #1? ____

Fig. (b)



HOW MANY DEGREES DID THE TURTLE TURN IN EACH ANGLE?

ANGLE #1 ____

ANGLE #2 ____

ANGLE #3 ____

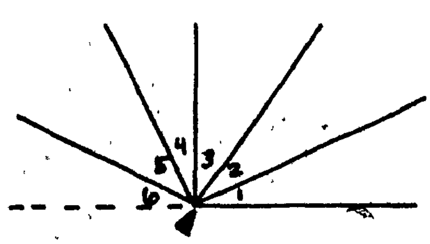
TOTAL DEGREES #1 + #2 = #3 = ____

HOW MANY DEGREES DID THE TURTLE TURN IN ANGLE #4? ____

HOW MANY DEGREES DID THE TURTLE TURN IN ALL 4 ANGLES?

TOTAL #3 + #4 = ____

Fig. (c)



HOW MANY DEGREES DID THE TURTLE TURN IN ANGLE #1 ____

ANGLE #2 ____

ANGLE #3 ____

ANGLE #4 ____

ANGLE #5 ____

HOW MANY DEGREES IN ALL 5 ANGLES? ⟶ ____

HOW MANY DEGREES DID THE TURTLE TURN IN #6? ____

THE TOTAL NUMBER OF DEGREES THE TURTLE TURNED IS ____

Task 1 requires the students to interpret the angle rotations according to the numbers in ascending order (in the written diagram) as well as to interpret the questions following the mathematical terms for angles and measurement rather than the Logo terms. The questions develop the concept of a number of equal angles which add up to a total of 180° or a half rotation. The students, therefore, had to calculate the measurement of each angle before proceeding. The students were expected to use the angles that had been defined in the previous activities as components to complete the half rotation of 180°.

Only one student interpreted the instructions correctly. This student used the initial position of the turtle as given in the diagram. The angles were correctly rotated although she did not use the defined angles as components of the figures.

All the students were able to answer the questions to figure (a) without difficulty, even though two students omitted the commands to produce the figure.

One student used the defined angles as components correctly, but disregarded the initial state given in the diagram and assumed the initial state to be HOME position.

Student K

Fig. (b) RT 90 LTA45 LTA45 LTA45 LTA45

Fig. (c) RT 90 REPEAT 6 [LTA30]

Another manner in which the task was interpreted, was to assume the initial state was HOME position and to complete a) the angles to the right, and b) the angles to the left

Student A

Fig. (b) RTA45 RTA45 TLT 90 LTA45 LTA45

Fig (c) RTA30 RTA30 RTA30 LTL 90 LTA30 LTA30 LTA30

Two students had the most difficulty understanding this task and wrote the following commands:

Students T and L

Fig. (b) TRT 45 TRA45 LTA90 LTA45 LTA45 angle #1 45°

#2 45°

#3 90°

#1 + 2 + 3 = #4 45°

Total: #1 + 2 + 3 + 4 = 135°

Fig. (c) TRT 30 RTA60 LTA30 LTA60 LTA30 LTA30 LTA30

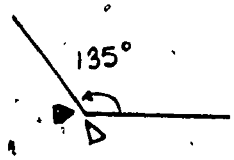
Both these figures' commands will produce the correct figure on the Logo screen. It is difficult to know whether the girls worked with the computer initially or corrected with it. (There are erasures on the paper.) There may have been confusion with the initial state given in the diagram, as an additional rotation would be needed to begin a repeat of LTA45 or LTA30. Using the turtle state in the diagram would necessitate the LTA90 or RTA60, but would confuse the calculations of the number of degrees to complete the half rotation of 180°. Therefore, if this exercise was completed using only the visual cues that the diagram offers, the calculation of the measure of the angles could be sacrificed.

TASK 2

WRITE A PROCEDURE TO FORM THE FOLLOWING FIGURES.

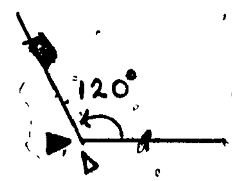
USE 30 AS INPUT FOR FD AND BK

Fig. (a)



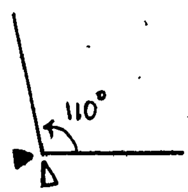
TO RTA135

Fig. (b)



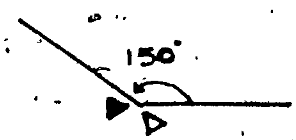
TO RTA120

Fig. (c)



TO RTA110

Fig. (d)



TO RTA150

This task asks the students to define angles of more than 90° in order that they may be used as components for building supplementary angles. The procedures were planned on paper and verified by the students with the computer. The procedures were orally discussed by the teacher and class in order to ensure that all the defined procedures were uniform. The turtle is rotated 90° to the right in order to begin each angle. The ending position as is shown on the sheet by the unshaded turtle will enable the students to begin the supplement to the already defined angle.

For example., TO RTA110

TRT 90 FD 30 BK 30 TLT 110 FD 30 BK 30

END

The inclusion of the initial turn in this procedure differs from the procedure for acute angles. Consequently an interfacing command may be necessary before the command of the defined acute angle can be implanted. This allows for a "right and left" component to be used in the construction of a given supplementary angle.

TASK 3

WRITE THE COMMANDS TO PRODUCE THE FOLLOWING FIGURES.

THE MEASURE OF THE FIRST ANGLE IS GIVEN.

USE THE PROCEDURES FROM TASK 2 (ABOVE) WHERE POSSIBLE

Fig. (a)

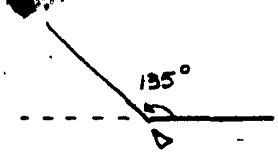
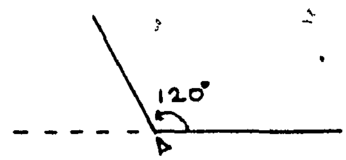


Fig. (b)



Since this was the beginning of a new concept the students seemed to be confused with the interpretation of the instructions in the diagram. Many began by writing the commands for the 135° angle and not completing the dashed angle. There may also have been a need to review orally the written instructions, to use the procedures from Task 2 since many, in fact, rewrote the commands.

After brief teacher intervention, in order to clarify the instruction, most students successfully interpreted the task.

Only two students were unable to rewrite the procedure correctly. One other student estimated the measure of the missing angle.

Student CH

Fig. (a) RTA135 LTL 46 FD 30

Fig. (b) RTA120 LTL 61 FD 30

Since the estimate was so close, the verification with the computer would "look" correct. CH did not use the angles defined for the activities for complementary angles to complete the supplement. Two other students also failed to use the defined angles but wrote correct commands to complete the supplement.

Fig. (c)

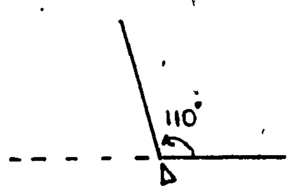


Fig. (d)

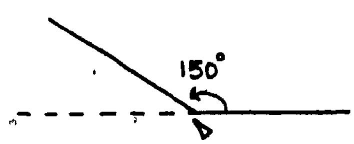


Fig. (e)

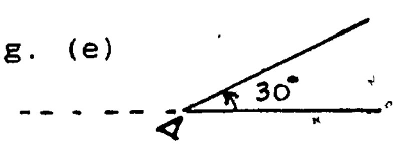
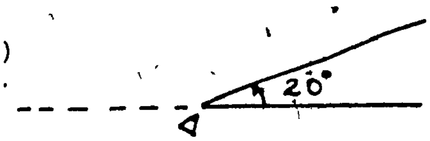


Fig. (f)



Figures (c) and (d) continue the objective set out in figures (a) and (b). These two figures were completed in the same way by the aforementioned students. The two students who were not successful in completing figures (a) and (b) wrote the following commands which illustrate the commands for only the first angle.

Students T and L

Fig. (c) TRT 90 FD 30 BK 60 FD 30 TLT 110 FD 30 BK 30

Fig. (d) TRT 90 FD 30 BK 60 FD 30 TLT 150 FD 30 BK 30

Figures (e) and (f) were completed in like manner. The figures will "look" correct when verified with the computer. However, the figure was completed by drawing the base line "BK 60", and then drawing the diagonal line for the given angle in the diagram. Thus the concept of a pair of angles that equal 180° has not been observed here. The same method was used by Y and M but with different commands.

Fig. (f) RT 90 FD 30 BK 30 TLT 20 FD 30 BK 30 RT 20
BK 30 FD 30

CH completed figure (e) correctly but used the same commands as Y and M to complete figure (f)

Student R

Fig.(e) <u>TRT 90 FD 30 BK 30 TLT 10 FD 30 BK 30 TRT 45</u>
<u>RTA30</u>

TRT 55 RTA30 TLT 180 FD 30

R corrected this figure with the computer. The line was drawn to complete the base. The base will not appear perfectly straight on the Logo screen.

<u>TASK 4</u>

HOW MANY DEGREES IN THE FIRST TURN? ______

HOW MANY DEGREES IN THE SECOND TURN? ______

THE TOTAL NUMBER OF DEGREES IN 1 AND 2? ______

	Turn #1	Turn #2	Sum #1 & 2	
Fig. a b c d e f				
Other pairs of angles that produce 180°				Write commands for these pairs of angles (on paper)

<u>THESE PAIRS OF ANGLES ARE CALLED SUPPLEMENTARY ANGLES.</u>

<u>BECAUSE THEIR SUM IS EQUAL TO 180 DEGREES</u>.

This task summarizes the work in Task 3. The students are asked to fill in the chart with the degrees of each angle in each figure. They are then to observe the sum of 180° in each completed figure, and they are to learn the mathematical terms and theorem for supplementary angles.

The students are asked to form other pairs of supplementary angles and to write the commands on the sheet of paper provided. Most students omitted the commands but did the mathematical calculations needed to form pairs of supplementary angles.

The students who had failed to complete 2 angle rotations in each figure, but had only extended the base line to complete the figure, had to read the definition of supplementary angles before filling the chart.

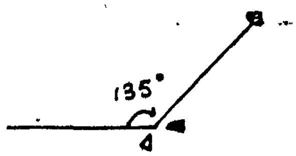
These students did not take the time to go back and correct those figures.

LOGO ACTIVITY 6

TASK 1

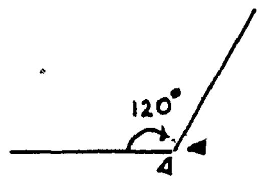
WRITE A PROCEDURE TO FORM THE FOLLOWING FIGURES.

a)



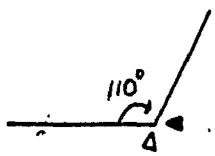
TO LTA135

b)



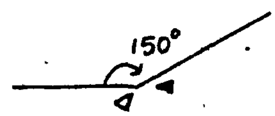
TO LTA120

c)



TO LTA110

d)



TO LTA150

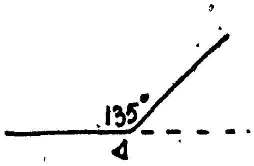
This activity begins with the task to define angles which could be used as a "left" component to build a pair of supplementary angles. Since the students have already defined procedures for these angles but with the inverse rotation they had no difficulty with this task. Again, the procedures were discussed orally by the teacher and the class in order to make sure that all the commands within the procedures were uniform.

TASK 2

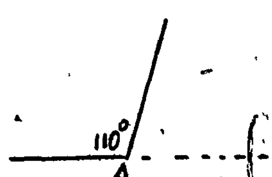
USE THE PROCEDURES YOU HAVE JUST DEFINED TO COMPLETE THE FOLLOWING PAIRS OF SUPPLEMENTARY ANGLES.

WRITE THE COMMANDS.

a)



b)



The written instructions for this task use the mathematical term "supplementary angles". Before beginning the set of tasks in this activity a general discussion and review of the mathematical vocabulary was held by the class and teacher.

On the whole the students had no difficulty writing the commands for the figures.

L and R were successful in using the defined angles for components to complete each figure. S and T wrote the commands to successfully complete the figures rather than using the procedures for the angles. Two students wrote out the commands for one figure and switched to the defined angle procedure. Students K and Y who worked together completed the figures by extending the base line and using the procedure for the given angle.

Students Y and K

a) RT 90 FD 30 BK 30 LT 90 LTA135

b) RT 90 FD 30 BK 30 LT 90 LTA110

The figure appeared correct on the monitor. These students had completed the tasks in Activity 5 in the same manner. Two angle rotations to form two actual angles have not been completed here. The first rotations are necessary in Logo to orient the turtle in order to draw the lines as shown in the figure. Thus the instructions to form supplementary angles have not been observed by these students.

TASK 3

SKETCH OTHER PAIRS OF ANGLES THAT ADD UP TO 180°. WRITE THE COMMANDS FOR EACH SKETCH. CHECK YOUR COMMANDS WITH THE COMPUTER!

a) SKETCH

COMMANDS

b) SKETCH

COMMANDS

c) SKETCH

COMMANDS

d) SKETCH

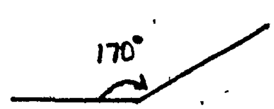
COMMANDS

The instructions for this task employ the properties of supplementary angles. The students are asked to first sketch the pair of supplementary angles in order to check their estimation skills for the angle measure, as well as their comprehension of the visual concept of supplementary angles. The students are then required to write the Logo commands that would produce the sketches on the computer screen, and to verify them with the computer.

The students' work for this task produced very varied and interesting results. All the students chose to work independently as each one sketched his/her own figures. Two students constantly drew sketches for only one angle. These were Y and M who may have copied each others' sketches, but the commands were different. The following is an example:

Student M

SKETCH



LT 90 FD 30 BK 30 RT 170

FD 30 BK 30

Student Y

SKETCH

LT 90 FD 30 BK 30 RT 135
FD 30 BK 30

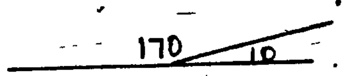
In this figure Y has attempted to sketch supplementary angles, but M has sketched one angle of 170°. M's commands are correct for his sketch. Y chooses RT 135 for his angle rotation where his sketch shows 170°.

The remaining sketches for both Y and M are identical but their angle rotations differ (as above). M seems to have confused obtuse angles with supplementary angles. Y does not seem to have grasped supplementarity either. He also is not relating the angle measure with the rotational input in the Logo command to turn.

Another common error was found in writing the commands for sketches which were well estimated and sketched. In this instance the commands were either for drawing the base line first and the arm for one of the angles; or the adjacent ray or angle, followed by the extended base line. This is illustrated by the following:

Student A

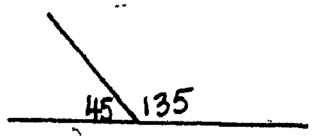
SKETCH



RT 90 FD 30 BK 30 LT 10
FD 30 BK 30 RT 10 BK 30
FD 30

R has correctly oriented the turtle to draw an angle of 10°, and then completes the figure employing the visual cue to extend the base line to the left. This figure will appear correct on the Logo screen.

Student R

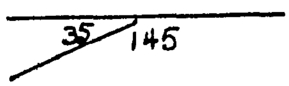
SKETCH 

RT 90 FD 30 BK 30 LT 180
FD 30 BK 30 RT 45 FD 30
BK 30

R begins his figure by drawing the base line to the right. He then rotates the turtle 180° to the left in order to draw the base arm of the 45° angle, and then he completes the angle by drawing the adjacent ray.

Two students attempted to sketch supplementary angles in rotated or inverted position.

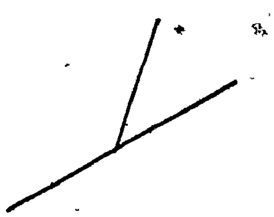
Student A

SKETCH 

RT 90 FD 30 BK 60 FD 20
<u>RT 135</u> FD 30 BK 30
↓
uses incorrect input

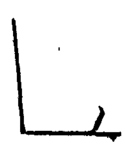
A has reproduced his sketch with an input of RT 135 instead of RT 145. Since he has only drawn one angle and completed the figure with an extension of the base, this figure will appear correct on the computer screen.

STUDENT S

SKETCH 

<u>TLT 45</u> FD 30 BK 60 <u>TRT 60</u>
↓ TRT 45 ↓ TLT 60
FD 30 BK 30 <u>TRT 120</u> FD 30
BK 30 ↓ TLT 120

These commands produce the following figure.

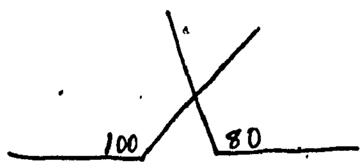


S seems to have confused the right and left rotations of the turtle, and so the rotations should be the opposite of what she has written in order to produce her original sketch. Lateralization seems to be the problem here.

One student sketched a pair of supplementary angles as follows:

Student T

SKETCH



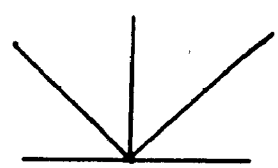
TLT 90 FD 30 BK 30 TRT 180 FD 30 BK 30

T sketches two distinct angles whose sum is 180°. The concept of the adjacent ray is completely ignored. The commands produce only a straight line or one rotation of 180°.

In another sketch T shows supplementarity as follows:

TRT 45 FD 30 BK 30 TLT 45 FD 30 BK 30 TLT 45 FD 30 BK 30 TLT 45 FD 30 BK 60

These commands produce the following:

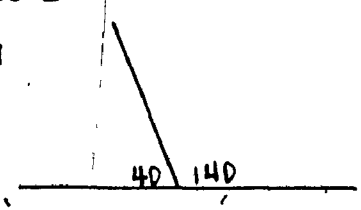


Since there are many erasures on T's paper, it is difficult to know if the commands were verified with the computer, or if the sketch was done in draw mode with the computer, and copied incorrectly onto paper. T's last sketch was completed correctly using the computer in draw mode. Supplementarity is a problem here.

One student completed only two sketches.

Student L

SKETCH



TRT 90 FD 30 BK 30 TLT 140 FD 30 BK 30 TLT 40 FD 30 BK 30

L has produced a figure of supplementary angles for which she wrote the commands for two angles whose sum is 180°.

It is interesting to note that none of the students used the defined angles as components for any of their sketches. They readily understood the need to calculate other (sums) supplements to equal 180°.

TASK 4

LIST OTHER PAIRS OF ANGLES THAT ADD UP TO 180°.

____ + ____ = ____

PAIRS OF ANGLES WHOSE MEASURES ADD UP TO 180° ARE CALLED

____

This task checks the students' comprehension of the concept of supplementarity. It did not present any problem.

TASK 5

WRITE THE COMMANDS TO PRODUCE THE 180-DEGREE ANGLE IN EACH FIGURE.

Fig. (a)

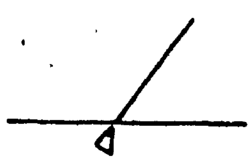


Fig. (b)

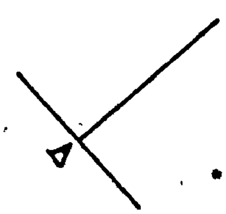
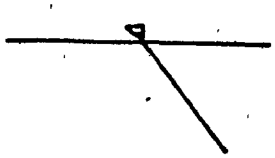


Fig. (c)



This task requires the students to estimate the measure of each supplement and to write the commands to produce the figures on the Logo screen. The concept of supplementary angles is reinforced by the instructions and the given visual cues. Figures (b) and (c) are in non-standard positions. The orientation of the turtle given in the diagram is the ending state after completion of one supplement. Since no beginning state was given the students were given the freedom to begin their figures wherever they chose.

One student did not complete this activity and omitted Tasks 5 and 6. Another student omitted figure (c).

The difficulties in this task were in estimating the angle measure in figure (b) and (c) which are in non-standard position. Three students did not recognize the two right angles in this figure. These students drew angles of 60° and 120°.

Student S

Fig (b) TLT 45 FD 30 BK 60 TRT 60 FD 30 BK 30 TRT 120
FD 30 BK 30

S does not use the angle procedures to produce her figure. S does not use the turtle's position as a cue to construct an angle supplement. S also has not perceived the right angles in the rotated position of this figure.

One student only attempted to use the defined angles to form figures (a) and (b).

Student L

Fig. (a) LTA120 TRT 60 FD 30 Fig. (b) LTA110 TRT 70

In both figures L assumes the initial turtle state is as shown in the figure, and proceeds to write the commands to form the angles omitting the commands to position the turtle to begin these angle formations.

In figure b) L has difficulty recognizing the two right angles even though her erasures indicate that the figure has been verified with the computer.

Many students wrote commands to reproduce the supplementary angles by either drawing one angle and extending the base (ray) or by drawing the lines of the diagram without paying attention to the angle rotation.

Student A

Fig. (b) RT 45 FD 30 BK 30 RT 90 FD 30 BK 60 FD 30 LT 90

The HOME position is assumed.

A positions the turtle at a 45° angle to begin his figure. He completes the vertical ray to form the 90° angle and then turns RT 90 to complete the base to form the two rays of the adjacent right angles. A brings the turtle back to the position given in the diagram at the end.

Student R

Fig. (b) RT 135 FD 60 LT 180 FD 30 RT 60 FD 30 BK 30

R assumes the default position of the turtle and positions the turtle by rotating RT 135. R completes the base and rotates the turtle LT 180 and moves the turtle so as to position it in the centre of the base. He then forms a 60° angle. The turtle ends in the position given in the diagram.

This figure will be incorrect when verified with the computer. R has crossed out commands and has shown that corrections have been made. R does not seem to notice the incorrect angle from this rotated position.

All the students who attempted this figure (c) were successful in reproducing the figure on the Logo screen. Most assumed the initial state as HOME, drew the base line, and then formed an angle of either 30° or 45°.

Student M

Fig. (c) RT 90 BK 60 RT 45 BK 30 FD 30 FD 30 FD 30

It is difficult to know if R wrote his commands first or used draw mode.

Student R

Fig. (c) RT 90 FD 30 BK 30 LT 180 FD 30 BK 30 RT 225
FD 30 BK 30

R has used an exterior turn of 225° to form his angle.

Figure (b) was completed correctly by most of the students who attempted it. None used the angle components but preferred to write single line or angle rotation commands. Some corrections were evident in their work.

Student A

Fig. (b) RT 45 FD 30 BK 30 FD 30 RT 45 FD 30 BK 30
FD 30 FD 30 RT 140 FD 30 BK 30 LT 30 FD 30
BK 30 LT 90

A has not estimated his angle rotations accurately. A proceeds from HOME to form the 45° ray but is unsuccessful with the next rotation.

A has not corrected the figure. It is difficult to ascertain whether A wrote his commands using draw mode and did not recognize the angle discrepancy.

Student S

Fig. (b) TRT 90 FD 30 BK 60 FD 30 TLT 45 FD 30 BK 60
FD 30 TRT 135 FD 30

Shows calculation of → 90 -45 135

TASK 6

WRITE THE COMMANDS TO FORM EACH OF THE FOLLOWING FIGURES.

Fig. (a)

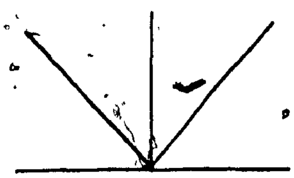
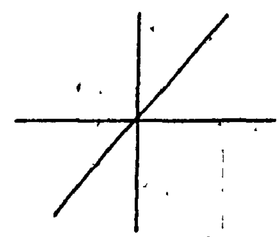


Fig. (b)



The task incorporates the angle concepts that the students have acquired thus far: complementarity and supplementarity.

The figures in the diagram do not give the students any constraint as to beginning and ending states. Nor do the written instructions guide them to use any particular angles to complete the figures. Therefore, the students demonstrated varied approaches to reproducing the figures.

Three students completed figure (a) by drawing four successive angles of 45°. They did not use the repeat command.

One student formed the base first and then filled in the four angles of 45°

Students' M and Y attempted to use the defined angles to form Fig. a)

RTA45 LT 90 RT 90 FD 30 LT 45 RTA45 RT 45 BK 30

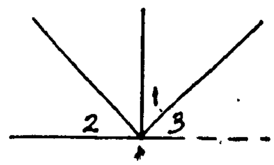


Fig. b)

RT 45 FD 30 FD 30 FD 30 FD 30 FD 30 BK 60 RT 45

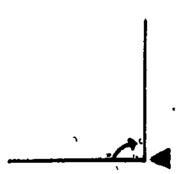
BK 60 RT 90 BK 30

The above two examples are typical of the commands chosen by the students. They seem to be looking at the lines in the figure rather than the angles. To that end they have taken great care to position the turtle correctly in order to reproduce the lines in the pattern of the figure, correctly.

LOGO ACTIVITY SHEET 7

TASK 1

(a) HOW MANY DEGREES TO THE RIGHT DO YOU HAVE TO TURN THE TURTLE TO COMPLETE THE FIGURE?



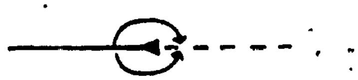
FD__ BK __ RT __ FD __ BK _

(b) HOW MANY DEGREES TO THE LEFT DO YOU HAVE TO TURN THE TURTLE TO COMPLETE THE FIGURE? ____

(c) HOW MANY DEGREES DID THE TURTLE TURN IN FIG (A) + FIG (B)

_____ + _____ = _____

Task 1 introduces the students to the exterior rotation of the familiar right angle. Some students were immediately able to identify the exterior rotation as being 360° less the interior angle measure. For those who did not, the assumed answer was 90° for each of the interior and exterior rotations. Interestingly the total arrived at in question (c) of 180° did not seem to indicate to these students that this sum was the sum of the angles they had used for supplementarity. The four students who answered incorrectly did not choose to use the computer to verify their answers.



(a) HOW MANY DEGREES TO THE RIGHT DO YOU HAVE TO TURN THE TURTLE TO COMPLETE THE FIGURE?_____

(b) HOW MANY DEGREES TO THE LEFT TO COMPLETE THE FIGURE?_____

<u>CHECK YOUR COMMANDS</u> WITH THE COMPUTER.

_____	_____
_____	_____
a) _____	b) _____

This task deals with a straight angle or 180° rotation to the right or left. This task also instructs the students to check their angle rotations (turn LT or RT) with the computer. Rotational arrows are given in the diagram.

Four students filled the blanks with 90. They did not verify with the computer.

Two students answered (a) with 180 and (b) with 270. These students have check marks under the verification columns.

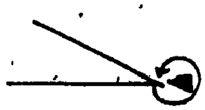
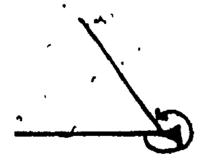
Two students answered (a) and (b) correctly.

Since the students were not instructed to write commands for the figures in Tasks 1 and 2, many simply filled in the blanks without bothering to use the Logo commands to verify their answers. This led to a failure of 6/8 students'responses.

<u>TASK 3</u>

FOR EACH FIGURE WRITE THE COMMANDS TO PRODUCE THE FIGURE WHEN (A) YOU TURN THE TURTLE TO THE RIGHT

(B) YOU TURN THE TURTLE TO THE LEFT.

(A)  (B)  (C)  (D) 

LT 90	LT 90	LT 90	LT 90
FD ___	___	___	___
BK ___	___	___	___
RT ___			
FD ___	___	___	___
BK ___	___	___	___

TURN RT ______°	TURN RT ______°
TURN LT ______°	TURN LT ______°
TOTAL ______°	TOTAL ______°

Task 3 instructs the students to write the commands to reproduce the figures by using two different rotations: (a) RT; (b) LT.

The initial state is HOME. The orientation of the turtle to begin the rotation has been given as LT 90. The diagrams have rotational arrows to guide the students in rotating the turtle. The figures are all in standard position.

The students were required to fill in the chart with the angle rotations and to calculate the total number of degrees for both rotations.

Only two students were able to successfully complete the task and to calculate the total as 360°. They had grasped the concept and had subtractions on their paper to show for it.

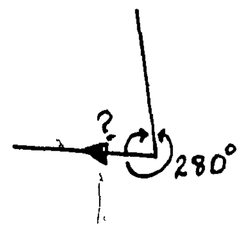
This task was partially correct for the rest of the students. The rotations to the right were always correct. In most cases the rotation to the left was not observed and the command was written as a rotation RT usually with the identical input. Since no teacher/class discussion as to the total number of degrees the right and left turn must equal had taken place, the students were not upset with their errors. The exploration with the computer yielded the correct figure on the screen, so how it was achieved did not seem important.

TASK 4

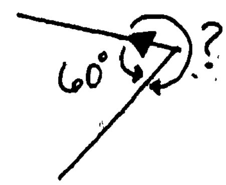
HOW MANY DEGREES ARE NEEDED TO COMPLETE THE ANGLE TURN AND PRODUCE THE FIGURE? CHECK WITH THE COMPUTER.

(A)

(B)



(C)



This task asks the students to calculate the number of degrees needed to rotate the turtle in the direction of the arrow with the "?" One angle measure is given - figure (a) and (c) interior angle rotation is given, (b) exterior rotation is given.

If the students are not able to calculate mathematically the number of degrees, they are to reproduce the figure using both the given rotation and the estimated one. The two figures can be compared for accuracy.

Two students answered all three "rotations" correctly as they had already grasped the concept of one complete rotation.

Three other students completed figure (c) correctly. One can speculate that perhaps the visual cue of the circle formed by the two rotational arrows as well as the given angle of 60° prompted them to recall the complete rotation as being equal to 360°.

Since this activity took place at the end of May, time did not allow for the class to discuss this activity and to formulate the concept of the interior + exterior angle rotations being equal to one complete rotation. It is evident from the students' work that the computer exploration without the teacher's guidance and instruction in the interpretation of the task is not necessarily enough to learn a geometric concept such as this one.

CHAPTER VI

STUDENT PROFILES

Each student in the research project completed the set of Logo activities as was outlined in the previous section (Analysis of Activities). The activity sheets were kept in individual student files, along with the pencil and paper pre-planning of the tasks. The following is an analysis of the work of each student, compiled from the individual files, and the researcher/teacher observations and anecdotal notes. For description of students, see Appendix C.

The introduction to each student profile provides some insight to the general characteristics of the student in her/his regular classroom programmes and in the Logo classes. Any differences or similarities have been noted here.

The analysis of each student's specific Logo behaviour is discussed under the following headings:

- Planning
- Debugging
- Concepts
    - 90° angle
    - 360° rotation
    - complementary/supplementary angles
    - standard/non-standard position
    - interior/exterior angles
- Performance on final exam

STUDENT A

GENERAL CHARACTERISTICS

GENERAL CLASS CHARACTERISTICS

A was an average student whose work pattern in the classroom was inconsistent. He was prone to playing quietly, thus losing concentration while instruction was being given. This would result in the need for individual instruction both in the classroom and at home. A's first language was not English, yet he had no difficulty verbalizing his thoughts. However, his vocabulary recognition was poor and, this fact, coupled with a high frustration level, often led to difficulties in problem solving.

With one to one aid, A demonstrated mastery of arithmetic concepts but their application to problems involving ratio percent was poor.

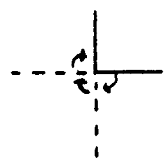
BEHAVIOUR IN LOGO CLASS

In the computer lab, A had no problem with concentration. He usually worked alone and his attention never turned from the computer and his activities. A was very proud of his work and often offered to help those around him (even if he was not always correct). A exhibited a sense of self-confidence in the computer lab that was not evident in the classroom. Perhaps it was the feedback from the computer that boosted his self-esteem. He was not afraid to make a mistake since he could correct it quite easily and show the edited product. At times he was challenged by one of the other students (with whom he would talk or play in class). He responded by working at the computer task to produce the figure for the other student to see.

SPECIFIC LOGO BEHAVIOUR

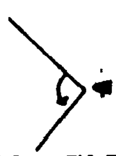
PLANNING

A began the research project by writing down commands to produce the figures but without following the explicit directions in the tasks. One example of this was in Activity 1, Task 3 where A assumed the first 90° angle to be drawn and continued to write commands for the remaining rotations.



A worked from the given figure but he often missed all the cues in the diagram. This included disregarding rotational arrows as well as the turtle state shown as either shaded (for starting position) or clear (for ending position).

A used estimation skills in planning the angle rotations. These were not always verified and therefore there is evidence of an angle rotation being assumed from previous tasks. An example of this is found in angles in non-standard position.



Act. 3, task 1 (c)

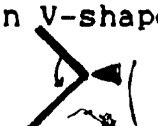
TLT 45 FD 30 BK 30 <u>TLT 45</u> FD 30 BK 30 TRT 30

The V-shape was assumed to be 45° (as in figure (a), (b)). Often in sketching complementary angles, A was so intent on the size of each angle, that the objective of the 90° angle was overlooked.

SKETCH  TRT 90 RTA60 RTA30

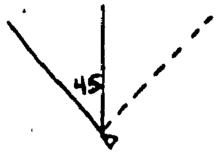
A began Activity 5 tasks dealing with supplementarity by interpreting the figure as a set of Logo draw commands. He had not planned to use the defined angles. A seemed to be more comfortable using the Logo draw commands that would execute the correct figures without incorporating the defined angle components.

DEBUGGING

A has demonstrated that he was confident of his planning of tasks, especially in Activities 1-3, that he often did not verify with the computer and, therefore, debugging in these activities was very limited. The major errors that needed attention in this section included the assumption about initial turtle state, following rotational arrows, and the interpretation of written instructions. The assumption that angles in V-shape were 45° was evident in Activity 3, Task 1 (c)  which was not verified.

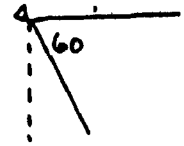
At first A wrote commands to draw the angles to form either complementary or supplementary angles. However, A demonstrated that he was able to go back and successfully use the components made of the defined angles to compose the larger angles. In Activity 4 he employed the components throughout. However in Task 4, A again assumed the initial turtle state as given in the figures (c), (d), (e), (f) and (g), which are all in non-standard position. The angle components were correct but the Logo commands were incomplete to produce the figures, since the initial commands to position the turtle were omitted.

e.g. (e)

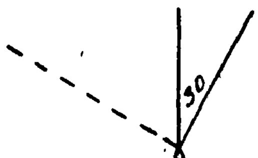


REPEAT 2 [LTA45]

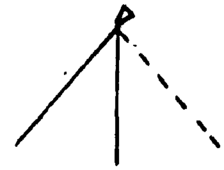
(f)



LTA60 LTA30



(c) LTA30 LTA60



(g) REPEAT 2 [LTA45]

None of these commands was verified with the computer and, therefore, not debugged.

In A's sketches of complementary angles in Task 5, A has not paid attention to details of angle sizes, e.g.,

Fig. (a) TRT 45 LTA45 LTA45 SKETCH

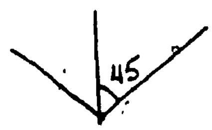
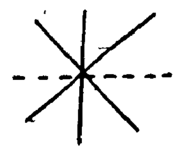


Fig. (e) FIND THE MISSING ANGLE TO COMPLETE THE STAR

COMMAND:

REPEAT 6 [LTA 60]



Originally he sketched eight angles and then erased the dotted line shown to produce six angles, but of unequal size.

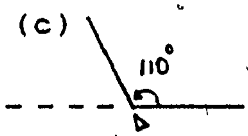
Fig. (c)

TRT 90 RTA45 TRT 90 RTA45

The middle command was not considered in sketching the figure.. A has not verified his sketches with the computer.

A's work in Activity 5 exhibited much uncertainty at the onset. In Task (e) A first wrote commands to complete each given angle, completely disregarding the second angle to be completed. However, after filling in the table at the end of the task and following the class discussion of supplementary angles, A returned to the task and was successful in correctly writing the commands using the defined angles for most of the figures.

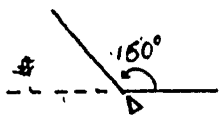
e.g.,

Fig. (c) 

RT 90 FD 30 BK 30 LT 110 F4 30 BK 30

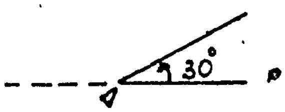
was corrected to: RTA110 LTA60

This would still be incorrect as the sum is 170°.

Fig. (d) 

RT 90 FD 30 BK 30 LT 150 FD 30 BK 30

was corrected to: RTA150 LTA30

Fig. (e) 

RT 90 FD 30 BK 30 LT 30 FD 30 BK 30

was corrected to: TRT 90 LTA30 .

No attention was given to the completion of the angle of 150°

Fig. (f) 

RT 90 FD 30 BK 30 LT 20 FD 30 BK 30

was corrected to:

TRT 60 FD 30 BK 30 TRT 30 FD 30 BK 30

A seemed to have reverted back to writing commands to form the angle using visual cues only.

In Activity 6, A once again failed to incorporate the defined angles to build a supplementary angle, but resorted to producing the figures in Task 2 by forming one of the angles and then extending the base arm to complete the 180°. This was illustrated in figure (c) RT 90 FD 30 BK 30 LT 10 FD 30 BK 30 RT 10 BK 30 FD 30. Verification with the computer produced the correct figure.

CONCEPTS

90° ANGLE

A readily recognized the 90° angle in standard and inverted positions. A experienced some difficulty with rotated positions.

360° ROTATION

A's grasp of the complete rotation concept was strong. Although he omitted the commands for the first angle in the star patterns, he was able to answer the written questions correctly and to calculate the measure of the angles in the star patterns using the repeat command.

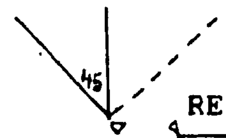
A seemed to have difficulty interpreting the instructions in Activity 4, task 6 where questions asked to list examples (from the given figures) of a) a 360° rotation and b) complementary angles.

A answered figure (a), (b), (c) to both questions. Since A was not interviewed about these responses it is difficult to know whether A could not associate the vocabulary with the concepts, or if the concepts had not yet been fully grasped.

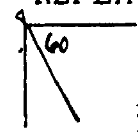
Note: figure (a), (b), (c) were complementary angles
figure (d), (e) were star patterns, using the repeat command

COMPLEMENTARY/SUPPLEMENTARY ANGLES

A has demonstrated a solid understanding of the concept of complementarity. His work showed an implicit grasp of the sum of 90° over and above the use of the Logo commands needed to interface the angle components in certain figures (rotated positions).

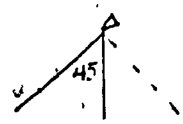


REPEAT 2 [LTA45]



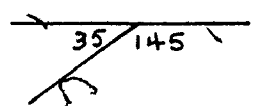
(inverted position) LTA60 LTA30

(rotated and inverted)



REPEAT 2 [LTA45]

A did not demonstrate a firm grasp of supplementary angles, although he did attempt to employ the defined angles in his commands. He omitted the second component necessary to complete the sum of 180° in some of his tasks (see debugging). In Activity 6 A drew sketches of supplementary angles in standard as well as inverted positions.

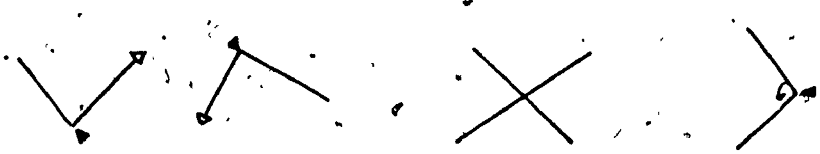


He also wrote commands for the sketches that did not incorporate the concept of supplementarity but simply drew the figure in Logo.

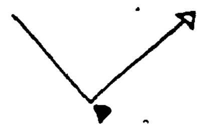
A correctly calculated the supplementary angles in all the written questions.

STANDARD/NON-STANDARD POSITION

A had difficulty recognizing the right angle in a rotated position.



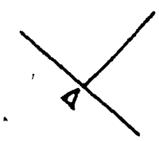
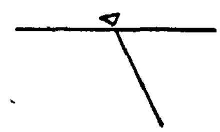
He recognized the initial rotation of 45° in each of these figures but then supplied 45 as the input to the angle rotation. With the computer, A was able to correct the error by adding another rotation of 45°.

e.g.  TLT 45 FD 50 BK 50 TRT 45 FD 50

TRT 45 (added)

In writing commands for complementary angles in the rotated positions (shown above), A omitted the initial positioning of the turtle, although the angle components were correctly written.

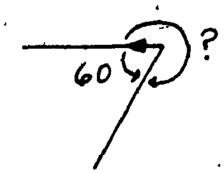
A demonstrated excellent recognition of supplementary angles in standard as well as non-standard position and sketched these angles from both perspectives.

Act. 6, task 5 (b).  (c) 

A was successful in identifying the right angle in (b) and in estimating the angles in (c). The concept of supplementarity was not achieved by using the desired angles RTA/LTA, but by writing commands in draw mode

INTERIOR/EXTERIOR ANGLES

A successfully identified the interior angles in Activity 7 but was unable to produce the angle by rotating around the exterior In task 4, (c) A identified the exterior angle as the completion of a 360° circular rotation.



RT 300 FD 30 BK 30

PERFORMANCE ON FINAL EXAM

A answered three out of four questions correctly. In item (d) he calculated the sum of the angles in the star pattern incorrectly as 45° x 8 = 280°. He estimated the angles correctly at 45°. He still seemed to rely on visual cues of the diagram.

STUDENT L

GENERAL CHARACTERISTICS

GENERAL CLASSROOM CHARACTERISTICS

L was a hard-working student who relied on rules and rote memorization in order to learn her mathematical concepts. Her achievement in mathematics was below the grade level, with her computation skills the weaker of the other two skills of concepts and applications. She still relied on her fingers to count up or down, and her multiplication tables had to be calculated very often. Once L mastered a concept by forming an association or by employing some other memory device, she could complete the exercise of drill successfully, However, any cumulative exercise presented great difficulty as the underlying understanding of the concepts was often lacking. Problem solving presented a challenge for L since her vocabulary skills were also weak and she often guessed on the procedure from given word cues.

L generally needed a partner to work with. This helped her to drill the concepts she had just been taught. Often she would assume the role of instructor, guiding her partner, as she orally repeated the procedures she was in the process of memorizing (herself). Since L was a very warm, outgoing person, she was usually able to mask the many frustrations she experienced when her memory failed her. Individual help from the teacher or a classmate always helped L retrieve the information she needed to go on with her task. The operations with fractions and related topics such as ratio and percent posed the greatest challenge in the class curriculum. Here she had no concrete model to rely on and pictorial representations were sometimes difficult for her to interpret.

<u>BEHAVIOUR IN LOGO</u>

Since much of L's math (concepts) acquisition was still at the concrete or pictorial stage, L was very at ease with her work in the computer lab. Since the initial tasks were relatively easy for her to perform, L's self-confidence was augmented and she approached her work with enthusiasm. All through the project she would ask the teacher during regular math class if this would be the day to work in the computer lab.

Logo revealed problems with lateralization which sometimes led to periods of frustration and anger for L. Relying heavily on visual cues, L discovered that this was not often reliable in proceeding with the tasks The frustration this caused for her often prevented her from continuing in a disciplined manner. She did not feel she had to be polite to a computer!

As the project progressed, so did L's perseverance. She learned to discuss her problems with her partner (who was not always the same person) and to attempt different avenues of approach. She also became less inhibited with the computer and was often seen "playing turtle" to help with turtle state problems. Her new-found self-confidence allowed her to help other students, as she was prone to do in the classroom. Perhaps using Logo as a learning tool injected L's learning of math with fun and pleasure, since she never complained, "I can't remember!", but instead she would say, "Oh, I see!"

SPECIFIC LOGO BEHAVIOUR

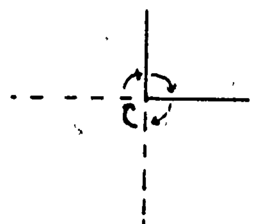
<u>PLANNING</u>

L usually planned her tasks with the computer in draw mode. She was able to plan activities dealing with 90° angles in standard form without the computer. L relied on visual cues as she used draw mode. She learned to recognize angles associated with 90° or 180°, but her estimation skills for 90° and 45° were the easiest to plan with paper and pencil.

After the concepts of complementary and supplementary angles were discussed and a theorem was formulated, L was able to calculate angle rotations with paper and pencil. Sometimes L planned a task based on a previous one she had already completed, but without paying attention to whether the figures were in the identical position. Often she wrote commands which assumed the HOME position or she omitted commands to orient the turtle to the position given in the diagram.

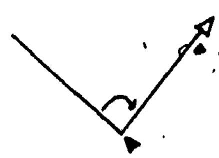
DEBUGGING

L worked seriously at debugging her commands. One difficulty she encountered were omissions in her commands due to the fact that she worked in draw mode and neglected to write everything down. This was illustrated in Activity 1, Task 3 where she counted three right angles instead of four.



She unnecessarily crossed out the commands for the last rotation.

In planning task 4 figure (b) L gave inputs for moves and rotations as 50°.



TLT 50 FD 50 BK 50 TRT 50 TRT 50 FD 50 BK 50

She did not recognize the right angle in non-standard position. It is difficult to know whether L has been able to distinguish the difference between FD, BK and TRT.

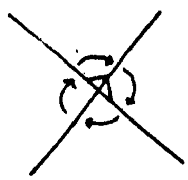
Although this was verified with the computer, L failed to perceive the incorrect angle.

When interviewed about her replies to Task 5, "Why did you choose the number of degrees for each answer?" (which was less than 90°), L said, "because these are the degrees you need to make the shape!!".

In the interview for her choice of answer to the next question which concerned obtuse angles, L observed that, "Figure (a) looks bigger than (c) on paper, but on the computer they look equal".

In Activity 2, Task 1, L used TLT and TRT interchangeably without considering the difference to the final outcome. As in Activity 1, she has calculated the total number of angles from the ones she has written commands for. In Task 2, she has calculated the total number of degrees as 270 due to two errors in her commands.

Activity 2, Task 2



TLT 45 FD 50 BK 50 TLT 90 FD 50 BK 50 ___ FD 50 BK 50 TLT 180 FD 50 BK 50

Having omitted the second rotation of 90°, L compensated by rotating the turtle 180° to complete the figure. L has not verified these commands with the computer or she may have tried to correct it and become frustrated.

An interview with L and her partner at the onset of this task resulted in the teacher trying to get L to become aware of the turtle's orientation.

T "Where is the turtle heading?"

L "To the left."

T "Make the turtle go to the left in a circle."

L turned the turtle left 40°

This results in a wavy line when the turtle moves FD.

T "Turn the turtle to make a straight line."

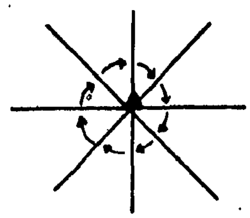
L's partner "Turn 95°. Turn 60°."

L "No! Turn 45°."

L and her partner were uncertain of the number of degrees in each rotation. They used visual trial and error methods to get the turtle to reproduce the angle. They did not recognize the right angle in rotated position.

As we can see, L and her partner were able to arrive at the correct solution using draw mode. The total figure was not completed correctly however.

Activity 2, Task 3



L used trial and error in Task 3 as well, beginning with 40 as the input to the rotations and 50 to FD and BK. Then she tried 50 as the input to the turns. Since L was using visual cues only, she then tried input 40 or 50 interchangeably. When the teacher intervened, L was at the point of frustration where she was ready to give up.

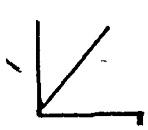
T "Check to see if the same input is needed for each rotation."

L "That's why it's not working!"

L's partner discovered 45° as the input.

L then proceeded to change all her inputs to 45° and was ecstatic that it worked. L computed 8 x 45 = 320° incorrectly.

Usually L could incorporate what she had learned during a class discussion to aid her to debug a set of commands. This was exemplified in Activity 3, Task 5.



RTA60 RTA60

The commands show a repeat of two RTA60. L wrote these quite automatically since the previous figures had all been figures of equal angles. She had assumed this figure to follow the same pattern. She also could not fill in the subsequent chart where pairs of complementary angles had to be calculated. It was only after the class discussions that she grasped the concept that the two angles did not have to be equal but that their sum must equal 90. She then went back and debugged correctly, and then proceeded to fill in the chart.

L's reliance on visual cues did not allow her to debug figures where equal angles had to be calculated. The time that lapsed between complementary and supplementary angles seemed to be too long for her to remember what was discussed in the previous figure (In fact it was only about one and half weeks).

L's lack of ability in calculating equal angles resulted in much frustration in Task 1 of Activity 5. L interpreted the inherent instructions of these tasks so as to begin with the second angle. This resulted in the incorrect number of angles being drawn and she compensated by rotating a greater number of degrees to complete the figure, e.g.,

(b) TRT 45 RTA45 <u>LTA90</u> LTA45 LTA45

Only three angles of 45° have actually been formed.

(c) TRT 30 <u>RTA60</u> LTA30 <u>LTA60</u> LTA30 LTA30 LTA30.

This figure was completed by drawing a larger angle (60°) and then a smaller (30°) contained angle within it.

Fig (b)

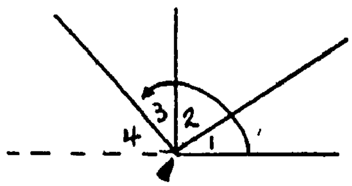
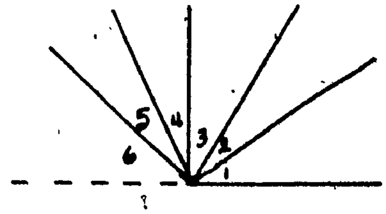
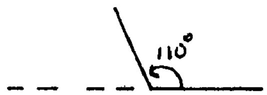


Fig (c)



The remaining tasks in Activity 5 were completed with commands which produced a straight line and the rotation of the given angle.

e.g. Task 5 (c) TRT 90 FD 30 BK 60 FD 30 TLT 110
FD 30 BK 30



The figure, although appearing correct on the Logo screen, does not show supplementarity. No attempt has been made to debug the tasks in this activity.

Thus one can see that L's debugging usually depended on her being able to "see" the incorrect figure rather than on her knowledge of the concepts of angles.

CONCEPTS

90°ANGLE

L readily recognized the right angle and was able to use it in star patterns.

360° DEGREE ROTATION

L has not demonstrated that she has fully grasped the concept of a complete rotation of 360° or parts thereof. L did not demonstrate the ability to calculate equal angles to complete a rotation as was seen in the star patterns where an angle of less than 90° or 45° was needed, e.g.

"CAN YOU MAKE A 12 STAR PATTERN?"

REPEAT 12 [RTA60].

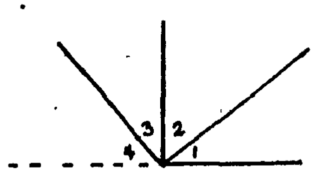
In activity 2, she was able to use draw mode to write commands for an eight star pattern (see debugging) but answered the question:

WHAT IS THE TOTAL NUMBER OF DEGREES THE TURTLE TURNED? 320°

The same question for a four star pattern where she recognized the 90° angle, was 270°.

In the introductory tasks to Activity 5 on supplementarity, L had difficulty calculating and recognizing the equal angles needed to compose an angle of 180° or half of 360°.

(b)



HOW MANY DEGREES IN: ANGLE #1 45

#2 45

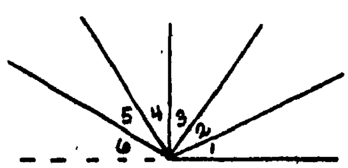
#3 90

#4 45

TOTAL DEGREES # 1 + 2 + 3 = ____

HOW MANY DEGREES IN ALL 4 ANGLES? TOTAL #3 + #4? 135

(c)



TRT 30 RTA60 LTA45 LTA60

was corrected to: TRT 30 RTA30 RTA60 LTA30 LTA60 LTA30 LTA30 LTA30

HOW MANY DEGREES DID THE TURTLE TURN IN: ANGLE #1 30

#2 60

#3 30

#4 60

#5 30

HOW MANY DEGREES IN ALL 5 ANGLES? 240

180

HOW MANY DEGREES IN #6? 180

THE TOTAL NUMBER OF DEGREES THE TURTLE TURNED IS 180

These tasks demonstrated once again L's reliance on visual cues. In Task (C), L knew that 180 was the number she needed for a correct answer, but her work exhibited her lack of understanding.

COMPLEMENTARY/SUPPLEMENTARY ANGLES

L demonstrated the recognition of complementary angles in her written charts and in the figures she reproduced using the angle components. L was able to calculate supplementary angles using the theorem, but was unable to use the two-angle components to produce a supplementary angle.

She has written commands to complete the figures of supplementary angles by drawing the straight line at the base, and then forming the arm of the given angle by rotating the given amount of rotation in the figure.

e.g. Task 5 (c)   TRT 90   FD 30   BK 60   FD 30   TLT 40

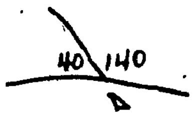
FD 30   BK 30

This does not show two angles whose sum is 180°. L completed the arithmetic calculations of the chart correctly.

L did not complete Activity 6 due to illness. She began Task 3 and successfully sketched two supplementary angles for (a) and (b) and wrote correct commands. This time both angles were formed.

Activity 6, Task 3

SKETCH TRT 90 FD 30 BK 30 TLT 140



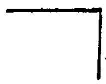
FD 30 BK 30 TLT 40 FD 30 BK 30

SKETCH TRT 90 FD 30 BK 30 TLT 90

FD 30 BK 30 TLT 90 FD 30 BK 30

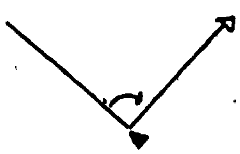
STANDARD/NON-STANDARD POSITION

L was most successful in recognizing angles of 90° or 45° in standard position. Her recognition of a right angle in inverted position was also solid.



L had difficulty recognizing the right angle in the non-standard rotated positions of Activity 1, Task 4, figures (b) and (c). L relied on her visual estimation of 50° which she repeated twice to complete figure (b). She has also used 50° for the input to FD and BK. It is difficult to know whether L has distinguished the rotations from the moves in completing these figures.

e.g., Fig. (b)



TLT 50 FD 50 BK 50 TRT 50

TRT 50 FD 50 BK 50

Fig. (c)

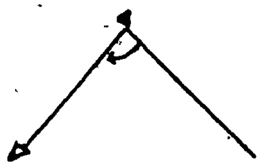


Figure (c) began with: TRT 50 FD 50 BK 50 TRT 50 FD

to which she added the initial commands of:

PU FD 50 TRT 90 TRT 45 which were hastily written on the side.

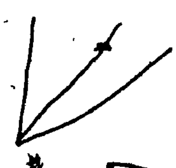
The reflected version of figure (b) seems to have been difficult for L to perceive. Her original commands would have resulted in the identical figure to (b). L has also neglected to follow the turtle states given in the diagram (c) as well as the rotational arrow.

In Activity 3, L was successful in the review Task 1. Her recognition of the right angle in standard and non-standard position was correct as was the 45° angle. L encountered difficulty with Task 3 since she did not recognize that the right angle was now broken into three equal angles of 30°. After teacher/class discussion of the tasks, L was able to incorporate the defined angles into the composition of the complementary angles in the subsequent tasks of Activity 3.

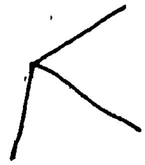
Task 1, Fig (c)

Task 3

Task 7 checked L's estimation and visualization skills. Only the sketch for (d) resulted in a near perfect right angle. Figure (a) was begun correctly but the second angle did not complete a right angle.

SKETCH	COMMANDS
(a) 	RTA45 RTA45
(b) 	TRT 90 RTA60 RTA30

Since the 30° angle was rather large, L drew the 60° angle even greater resulting in a total greater than 90°.

SKETCH	COMMANDS
(c) 	TRT 90 RTA60 RTA30

This sketch demonstrated L's inability to visualize the non-standard position of the complementary angle.

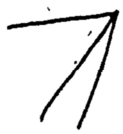
(d) 	PU FD 50 TRT 180 PD RTA45 RTA45

Figure (b) and (c) offered the most challenge. None of these sketches were corrected with the computer. Perhaps L felt more confident since she did not have to write any commands for this task.

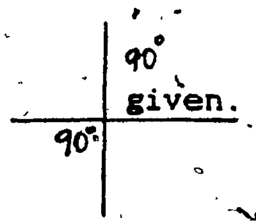
INTERIOR/EXTERIOR ANGLES

L was not successful in identifying the exterior rotation to produce the figures in Activity 7. She was able to estimate the interior angles of 45° and 60° where no measures were given. However, where the interior angle of 110°, 60° or the exterior angle of 280° were given, L was unable to calculate the measure to produce the opposite angle rotation.

PERFORMANCE ON FINAL EXAM

L scored two out of four on the final exam. She was unable to identify the supplementary angles in (a). She was also unable to calculate the sum of the angles in the eight-star pattern (d). This performance concurred with the findings of the research project.

In item (c) she wrote 90° in one angle only.



STUDENT CH

GENERAL CHARACTERISTICS

GENERAL CLASSROOM CHARACTERISTICS

CH was a student who often neglected to work in the classroom mathematics period unless he was individually motivated. Easily distracted CH was seldom enthusiastic about any written assignments at school. His math achievement was below average for his grade level as was his reading comprehension level. CH' verbal skills were very strong and when interviewed on a one to one basis he was generally able to verbalize his thinking patterns very well. Often he was able to write a mathematical statement to illustrate his thinking process. With individual help CH was able to grasp the concepts he was lacking and to apply them to this work.

BEHAVIOUR IN LOGO CLASS

CH usually worked with a partner during the research project. Since he was not very aggressive he often allowed his partner to influence his thinking. However, as the project progressed, CH became frustrated when his partner was inattentive. Teacher intervention was often necessary to get the pair back to work.

As a result CH resorted to working independently. This was in total contrast to his classroom performance, where he would more readily be distracted and "join in the fun" than work alone seriously. CH's insight to problem-solving seemed to be highlighted as he worked at the computer. The reinforcement he received from the computer seemed to be enough to motivate him to persevere.

SPECIFIC LOGO BEHAVIOUR

PLANNING

CH worked very carefully in the paper and pencil planning of each activity. He calculated the angle measures on his worksheets and then used the computer to verify his work. Often he used the computer in draw mode to write commands to an activity where he needed the visual reinforcement as he went along.

DEBUGGING

In debugging, CH was inconsistent in his performance. As the activities became more complex CH would debug his commands for the positioning of the turtle at the beginning of a task, but would then assume the same input for the remaining commands without verifying them with the computer.

At times CH would use visual cues to reproduce a figure when the angles should have been equal in measure. Frustration set in when the computer did not reproduce what he interpreted from the visual cues. Intervention was often necessary to remind him that perhaps there was a way to calculate the angles before proceeding.

Other times CH could produce a perfect sketch but would write commands for it without verifying them with the computer. Often when time did not allow for CH to debug each task completely with computer verification, he omitted tasks randomly returning to them later.

CONCEPTS

360° ROTATION

CH was successful in rotating a 90° angle to complete one rotation of 360°. In Activity 2, Task 3 where a complete rotation of 360° was made up of eight - 45° angle, CH used visual cues to reproduce the figure, and upon verification with the computer became totally frustrated. Teacher intervention helped him observe the irregular line drawn by the turtle. This helped him to realize that the turtle's rotation had to be 90° or a factor of 90 to produce a straight line by the turtle.

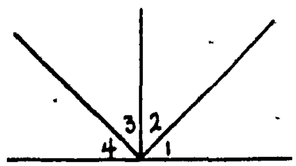
In Task 5, figure (b) he was able to use a REPEAT command to produce the same figure. In Activity 3, Task 8 he was successful in writing a REPEAT command for both an eight-star and twelve-star pattern. In Activity 4, Task 5 he was correct in answering all the questions and performing the tasks dealing with one rotation of 360°. On the final test he did not respond correctly to the item based on this concept.

COMPLEMENTARY/SUPPLEMENTARY ANGLES

CH demonstrated that he was able to use the components he had defined to build complementary angles. He was able to sketch complementary angles correctly and to write the commands to produce the angle.

The activities dealing with supplementarity were more difficult to analyse since CH worked more inconsistently and omitted the last page of Activity 6 completely.

In Activity 5, Task 1, CH was able to calculate the measures of the angles correctly as 45°, but his addition was incorrect at 185



∠ 1 = 45 °

∠ 2 = 45

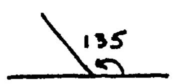
∠ 3 = 45

∠ 4 = <u>45</u>

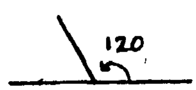
TOTAL 185

In Task 3, figure (c), (d) and (e) were correct while figure (b) was incorrect as he has omitted the supplement (of 160°). CH used the components he had defined in the first task of Activity 5 to reproduce the supplementary angles in figure (a), (b), (c) and (d).

In completing the chart which asked CH to record the measures of the two angles in the activity (Task 5) and to find the sum, CH's calculations of his estimates in figure (b) - 46° and 61° respectively did not motivate him to go back and correct so that the sum would be 180°.

(a) 

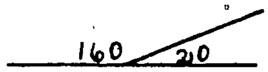
RTA135 TLT <u>46</u> FD 30

(b) 

RTA120 TLT <u>61</u> FD 30

In Activity 6, CH showed an understanding of supplementarity in the written question in Task 4, but he was not successful in using the angle components to form the supplementary angles. He instead wrote out all the commands correctly, calculating the angle rotations. In Task 3 his sketches were all correct but the commands for (c) and (d) were the only ones to produce the figure he had drawn. Here, too, he did not employ the angle components.

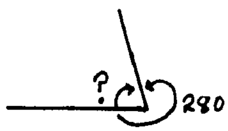
Task 3 (b)



TRT 90 FD 30 BK 30 TLT 60 FD 30
BK 30 TRT 160 FD 30

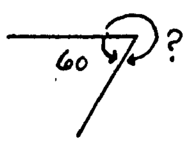
INTERIOR/EXTERIOR ANGLES

CH did not demonstrate that he had grasped the concept of interior and exterior angle rotations in Activity 7. He did recognize the need to rotate a greater number of degrees when making the exterior turn in each of the figures. It was only in the last two figures that CH was correct. However, in Task 4 (b) he used the input for the rotation with the command FD.



FD 80 LT 280

In figure (c) he recognized the total rotation of 360°



LT 60 RT 300

STANDARD/NON-STANDARD POSITION

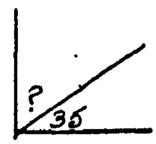
CH was able to readily recognize the angles dealt with in the research project when they were given in standard position. At times, CH had difficulty with commands for angles in non-standard position. This assumption is based partly on the fact that these are the figures that CH most often omitted and never returned to. In Activity 1, Task 4, (c). CH demonstrated the difficulty in writing the commands to produce an inverted, rotated 90° angle. Although he initially recognized the 90° angle, he changed his command inputs after having difficulty with the initial positioning of the turtle in the figure.

Task 4, Fig. (c)

TRT 135 FD 45 BK 45 TLT 135 FD 45

PERFORMANCE ON FINAL EXAM

CH answered two out of four questions correctly. The question that dealt with complementarity was not correctly calculated.



CH supplied 85 as the missing measure. Although he had no difficulty recognizing the 90° angles in item (c) he was incorrect in his response to the sum of the angles in the star pattern of figure (d). His answer was 280°.

STUDENT M

GENERAL CHARACTERISTICS

GENERAL CLASSROOM CHARACTERISTICS

M was a confident and able student who exhibited great potential in the mathematics program. His class achievement in math was in the above average range. M was always quick to pick up a new concept and to attempt to relate it to something he already knew. In his self-confident manner, however, M often neglected to listen to the methodology required to implement the new idea, and would begin to work independently. This allowed him to finish ahead of the class and the other students often solicited his help in completing their assignments. Usually M's results were perfect, but every so often errors in calculations due to incorrect procedure would appear. This resulted in M needing individualized instruction to catch up on what he had neglected to listen to. Another cause for M's errors was carelessness caused by the speedy way in which he worked.

BEHAVIOUR IN LOGO CLASS

M's classroom work habits were also evident in the computer lab. M was adept at using the computer and the Logo language. Therefore, he was often impatient with the introduction to each activity and often began the tasks ahead of the class. The teacher had to intervene on many occasions since M's interpretations of the activities were not always the correct ones.

As M worked through the activities, he displayed a sense of self-confidence and much enthusiasm. At first he worked alone and later, a new student entered the class and became his partner. The commands for the research tasks were generally characterized by hasty rotations, with a lack of attention to the details of the visual cues in the given diagrams. Therefore, rotation arrows were ignored; diagrams were perceived differently when more than one angle was involved. What seemed to be most important to M, was to reproduce the figure on the screen (which he almost always did) and to finish each activity in enough time to be able to extend what he had acquired to reproduce more complex figures on the computer.

SPECIFIC LOGO BEHAVIOUR

PLANNING

M was very quick to write down the commands for the task at hand. He rarely used the draw mode to assist him. His estimation skills as well as his visual perception enabled him to produce his plan quickly and usually efficiently. Where his original plan was incorrect, the figure would be reproduced correctly but would not necessarily demonstrate the concept intended by the researcher. M always sought the shortest, most efficient way to plan his commands.

DEBUGGING

M demonstrated complete ease when at work debugging a set of commands. His work clearly showed where he crossed out an incorrect command or where he applied liquid paper to a line. M was always cheerful when the teacher/ researcher had to intervene to get his attention back to the class to listen to the explanation of the objective for the day.

Debugging for M often included attention to the rotational arrows or the correct use of the components to form complementary or supplementary angles. Many of M's commands assumed the HOME position of the turtle. M demonstrated no difficulty in using the repeat command.

CONCEPTS

90°angle

M experienced no difficulty with this concept. His haste often led him to rotate the turtle in the wrong direction, but the result was always correct.

360°rotation

M was quick to grasp the concept of the total rotation and to apply it to the repeat command. He successfully completed all the tasks in Activity 2, and went on to form other star patterns using decimal numbers (calculated on the computer) as the input to the rotations.

COMPLEMENTARY/SUPPLEMENTARY ANGLES

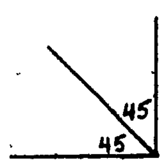
Although M was successful in reproducing the given figure, M did not always employ the defined components correctly to demonstrate complementarity or supplementarity.

a) COMPLEMENTARY ANGLES

M demonstrated the acquisition of the concept in all the verbal questions as well as in all of Activity 3.

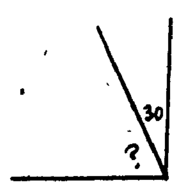
In Activity 4 he used the components incorrectly in several tasks.

e.g. Task 3 (a)



LTA 90 LTA 45

(b)



LTA 90 LTA 30

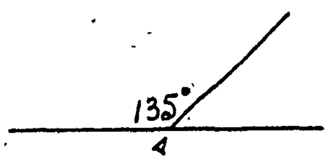
In each figure he produced first a 90° angle and then a smaller angle within the right angle; no complementarity is demonstrated.

(b) SUPPLEMENTARITY

M correctly answered the written questions and filled in the chart with pairs of supplementary angles. However, in writing commands to reproduce the supplementary angles, M used the obtuse angle component and then completed the figure by extending the base line. The figure appears correct on the Logo screen, but it does not demonstrate supplementarity using two defined angles.

e.g.

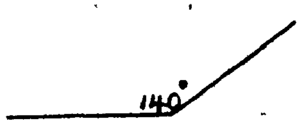
Activity 6 Task 2 (a) RT 90 FD 30 BK 30 LT 90 LTA135



In Activity 6, Task 3, M sketched only one angle, the obtuse angle, and completely omitted the supplement. One can assume that in writing the commands for the previous activity in the aforementioned manner, M came to visualize a supplementary angle as the obtuse angle, but neglected to extend the base line.

e.g. SKETCH COMMANDS

LT 90 FD 30 BK 30 RT140 FD30 BK 30



In this task he has also omitted the use of the defined angles.

In the set of complex figures in Task 6, M successfully reproduced the figure using simple Commands to draw it.

INTERIOR/EXTERIOR ANGLES

M immediately grasped the concept that the sum of the interior and exterior angle rotations must equal 360°. Although it was the end of the school year, he worked diligently to master the concept effectively.

STANDARD/NON-STANDARD POSITION

M had no difficulty in recognizing or reproducing angles in non-standard position. He readily used exterior rotations in order to correctly orient the turtle.

PERFORMANCE ON FINAL EXAM

M responded correctly to all four questions. His only error was in the subtraction of 180 - 45 = 35. This demonstrates his inattentiveness to detail once again.

STUDENT S

GENERAL CHARACTERISTICS

GENERAL CLASSROOM CHARACTERISTICS

S was a student who did not approach her work at school with any enthusiasm or interest. She expended a lot of energy in attracting attention to herself with her attitude of indifference, and, by involving other students into distraction. S's performance in mathematics ranged from average to below average, while her reading comprehension and vocabulary skills were even lower for her grade level. However, there were periods of concentration in class where S was able to follow the concepts and apply them to the exercises at hand. This accounted for the uneven acquisition of math concepts in S's all over performance level.

BEHAVIOUR IN LOGO CLASS

In the computer lab S also showed inconsistency in her behaviour. She often chose to work with one particular partner, but since she insisted on being the aggressive participant of the partnership, there was much time lost to arguing and then hasty completion of an activity. This also led to the partner's departure at times. As in the classroom, S demonstrated the ability to carry out the activities with precision once she decided to be serious. When S worked alone, she was able to use the Logo language effectively, and the positive reinforcement from the computer served to bolster her pride in her work.

SPECIFIC LOGO BEHAVIOUR

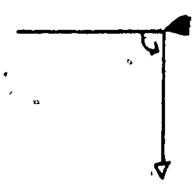
PLANNING

S began the project by writing all the commands for each task on paper and then verifying these with the computer. She was usually successful in estimating the amount of rotation in each figure. As the tasks became more involved, S often used draw mode to assist her planning. This was particularly evident in the figures where a repeat command could be used. However, S did use the skills she acquired in a task to help plan similar subsequent tasks. This was demonstrated again in the figures where a shorter plan could involve the repeat command of the previous activity.

DEBUGGING

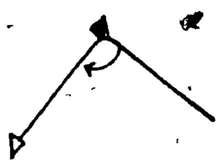
S demonstrated a good level of perseverance in debugging the errors in her commands. Often the source of an incorrect reproduction was due to her neglect in following the rotation arrows given in the figures. S demonstrated an excellent use of arithmetic in calculating inputs for rotations when the figure on the computer showed an incorrect angle or when an exterior turn was necessary.

Activity 1:
Task 2, Fig. (c)



PU FD 50 TLT 90 PD
BK 50 BK 50, TRT 90
FD 50 TRT 270

Activity 1:
Task 4, Fig. (c)



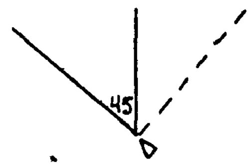
TLT 45 FD 50 TLT 45
FD 50 TLT 45 * 2

In completing the star pattern in Activity 3, S initially wrote the commands in draw mode before she was reminded that she could use the repeat command since all her angles were equal in measure. She successfully wrote a repeat command for the twelve star pattern.

Another source of error was found in the initial turtle state. This was evident when two defined angles were used to compose one complementary angle.

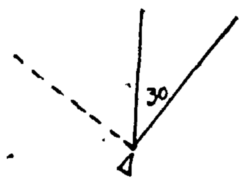
Activity 4:

Task 4, Fig. (e)



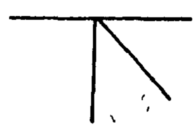
RTA45 RTA45

S has omitted the command to orient the turtle to execute the commands correctly. S has not verified the commands with the computer. The above figure is the inverse of figure (c) in which S exhibited similar difficulty.



RTA30 LTA60

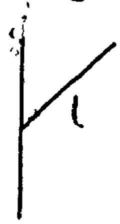
In the task that followed (Act. 4, Task 5) S has not attempted to verify her sketches. Thus the command to position the turtle in figure (c) was interpreted as angle formation.



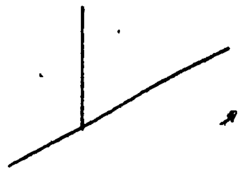
TRT 90 RTA45 TRT 90 RTA45

S encountered the most frustration and difficulty in Activity 6, Task 3 where she was required to sketch a pair of supplementary angles and then to write the commands to produce them. Her last two sketches were in non-standard position and orienting the turtle presented many difficulties for her. Although there are many erasures supporting her debugging efforts, she has not been successful in carrying through her corrections.

(c)



FD 30 BK 30 TRT 50 FD 30 BK 30
TRT 130 FD 30 BK 30

(d)  TLT 45 FD 30 BK 60 TRT 60 FD 30 BK 30 TRT 120 FD 30 BK 30

These are her final entries for the two sketches. They exhibit a lack of perseverance in debugging completely. S has demonstrated throughout the project good estimation skills especially with 45° rotations. She had more difficulty with the rotation of 30°, but the paper and pencil activities helped her clarify the visual 30° rotation since she was easily able to calculate the measure of an angle using the geometry concept of a 360° rotation.

CONCEPTS

90° ANGLE

S readily recognized the 90° angle in the activities introducing the project. She was able to identify her error when given the rotated and inverse position  and corrected the initial rotation of 45° to 45 x 2. She had difficulty verbalizing her ideas about rotation and responded to a question about angle measure simply as "Because it's an angle".

360° ROTATION

S was successful in identifying the complete rotation of 360° as being made up of 4-90° angles or 8-45° angles. She was able to use this concept later on in calculating angles with a sum of 360° to produce a six or twelve-star figure. S successfully employed this concept when using the repeat command.

However, S demonstrated difficulty in verbalizing what happens in a repeat command when the order of the inputs is reversed. Her response was: "It comes in a different shape". Her reason - "Because we mixed them up. So they won't come out the same and different order".

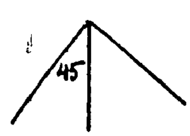
COMPLEMENTARY/SUPPLEMENTARY ANGLES

Although S showed accuracy in Activity 3, Task 1-5 by writing the angle measures on the diagrams and doing the arithmetic calculations on her worksheet, she was uncertain in interpreting Task 6 where a chart with complementary angles of different measures was required. After the class discussion of complementarity she was able to proceed successfully. The sketches in this activity were omitted by S.

In Activity 4 S exhibited that she had grasped the concept of complementarity although her Logo commands were often incomplete. Her angle procedures were correct but she often assumed the default position and gave only the angle commands. She did not verify her work with the computer. e.g. figure (a) LTA45 LTA45 would not produce the figure in the sequence given.

e.g. TRT 90 RTA45/RTA30

Fig. (g)



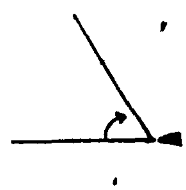
She has incorrectly calculated the complement to 45°. Her responses to the verbal tasks were all correct. The introductory tasks to the concept of supplementary angles support S's grasp of the concept. So do the written charts where pairs of angles with sums of 180° were recorded correctly.

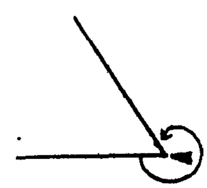
The activities where two defined angles were to be used as components to form a supplementary angle do not demonstrate supplementarity. In these tasks S wrote all the commands to produce the figures correctly in Logo. She did not employ the two components in any of the tasks. Her sketches of supplementary angles were also correct.

INTERIOR/EXTERIOR ANGLES

S was not able to demonstrate any understanding of the exterior angle rotation. She did not grasp the concept of opposite rotational direction. In the activities where an angle was to be formed by using a right rotation and then by using the opposite rotation, S could complete only the interior rotation to the right.

e.g. , Activity 7, Task 3

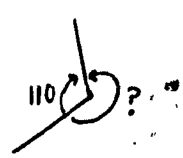




LT 90 FD 50 BK 50 LT 90 FD 50 BK 50

TRT 60 FD 50 BK 50 TRT 90 FD 50 BK 50

When asked how many degrees were needed to complete the interior and exterior rotations only the interior angle rotation was calculated correctly.



TLT 45 FD 50 BK 50 TRT 90 FD 50 BK 50

No attempt was made to solve this problem with the computer.

STANDARD/NON-STANDARD POSITION

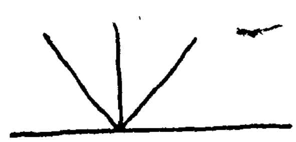
Although S initially did not seem to recognize a 90° angle in non-standard position, she readily picked up the skill and was successful throughout the remainder of the project.

She has also demonstrated the ability to recognize a figure rotated 45°.

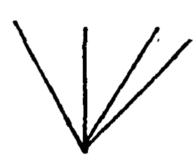
S seemed to experience more difficulty with turtle orientation in the activities with complementary angles, especially in inverted position. (see complementary/supplementary angles). This presented a problem for S again in Activity 6, Task 3.

In figure (c) and (d) S attempted to produce figures in rotated or non-standard position.

However, she was unsuccessful in writing commands to produce these figures since she had to deal first with the initial state of the turtle and then with the angles (for which she did not use the defined procedures). Her first attempt at (c) provided her with four angles.



original sketch



computer feedback

The erasures show inputs only of 30 or 45.

The commands for this were erased and corrected to:

FD 30 BK 30 TRT 50 FD 30

BK 30 TRT 120 FD 30 BK 30

For sketch (d):

LT 45 FD 30 BK 60 TRT 60 FD 30

BK 30 TRT 120 FD 30 BK 30

A lateralization problem seems to be in evidence here as S would have to rotate the turtle opposite to what she has written in order to complete her figure.

In Task 5, figure (a) S omitted the commands for the second angle.

In Task 5, figure (b) S failed to recognize the 90° angles in the rotated position of the figure. This was her perception: 60° + 120° and she did not verify with the computer.

In figure (c) S showed recognition of the 45° rotation by beginning her figure at default position: TLT 45, FD 30, TLT 45, FD 30, BK 30, but she neglected to complete the second angle. She had no difficulty with Task 6.

Thus one can conclude that S was able to work with the concept of supplementarity when the figures were in standard position, but became confused when the figures were rotated. Her lack of perseverance did not enable her to complete those figures successfully.

PERFORMANCE ON FINAL EXAM

S's score was three out of four. She correctly calculated the complement and supplement and she was able to recognize the sum of the angles in (d) as 360°. She also correctly placed the measures of each angle of 45° in the star pattern. Her interpretation of item (c) was to supply 180° as the missing measures when one 90° angle was given. She could not identify the four rotations of 90° each.

STUDENT R

GENERAL CHARACTERISTICS

GENERAL CLASSROOM CHARACTERISTICS

R was a student whose performance in the language arts program was below grade average. Although he had been in Canada for several years, he still exhibited difficulties with cursive writing and his reading comprehension level remained far below the average for his grade level. In mathematics class he was performing at or above the grade level. Yet (in math class) R displayed an insecure self-image and constantly sought individual reinforcement and verification, especially when a cumulative exercise was being completed. Often R preferred to work with a partner when solving word problems or when working through an exercise involving a new concept in the mathematics curriculum. In this way R was able to verbalize with someone the thinking process involved in acquiring the new concept, and thereby gained the reinforcement he required to continue to develop strategies that would culminate in a solution.

BEHAVIOUR IN LOGO CLASS

In the computer lab R rarely worked with a partner. The computer seemed to be an effective aide which offered him the instant verification he needed to persevere. His self-confidence was boosted so that he would be seen comparing a set of commands he had written with a friend close by, while insisting his were the correct ones.

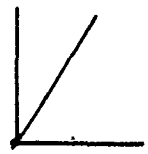
Further into the project, R displayed less attention to the class introductions and proceeded to work independently on the tasks. This often resulted in misinterpretation of tasks, especially where it involved following cues inherent to the given diagrams and/or written instructions.

SPECIFIC LOGO BEHAVIOUR

PLANNING

At the beginning of the project, R worked diligently, planning on paper, and then verifying with the computer. As the project progressed it became more difficult to observe planning and/or corrections that had been made due to erasures. At times R experienced difficulty in interpreting a task where the instructions had to be read and then carried out in a specific manner. This was seen in Activity 3, Task 5:

WHICH OTHER 2 SUBPROCEDURES MAY BE USED TO PRODUCE THE FOLLOWING FIGURE? WRITE THE PROCEDURE



RTA30 RTA60,

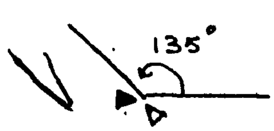
R was correct in writing the two-angle commands, even though he did not write a procedure. He answered the following questions incorrectly.

FIRST THE TURTLE TURNED 60 DEGREES. THE SECOND TIME THE TURTLE TURNED 90 DEGREES. THE TURTLE TURNED 150 DEGREES IN ALL.

R has interpreted the figure as an angle of 60° + 90° whose total is 150°. The two components in his commands were not observed here. After class discussion R was successful in completing the chart that followed, where he had to mathematically calculate other pairs of complementary angles. The clarification of the written instructions seemed to be necessary for him.

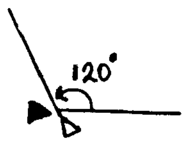
R also demonstrated a lack of understanding in the interpretation of visual cues such as angle rotation arrows. In defining the procedures for the angles greater than 90° in Activity 5, Task 2, R began figure (a) and figure (b) by using 30 as the input for the initial rotation, as well as FD, BK.

Fig. (a)




```
TO RTA135
TRT 30  FD 30  BK 30  RT 120
FD 30  BK 30
END
```

Fig. (b)



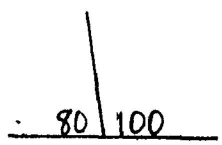

```
TO  RTA120
TRT 30  FD 30  BK 30  TRT 120
FD 30  BK 30
END
```

R has ignored the rotations given in the diagrams. He has confused the instruction to use 30 as input for FD and BK. He has also ignored the turtle positions which would require an initial rotation of TRT 90. R corrected these errors after the class discussion. Other areas where R did not follow instructions were seen in tasks where defined angles were to be used to write procedures, but where R used the commands of draw mode to plan instead. As in the aforementioned examples, R was capable of carrying out the required tasks, but needed help in interpreting the instructions. R also displayed a tendency to assume the default position in planning tasks, e.g. Activity 1, Task 4 (c): PU FD 50 PD TRT 45 FD 50 TLT 90 FD 50

DEBUGGING

When R was not able to perform a task readily, he would use draw mode to assist him in his corrections. If this strategy did not work, R resorted to discussing the problem with a classmate., but he would promptly return to his computer to attempt to employ those techniques. Under the circumstances, in contrast to the classroom, he did not elicit help from the teacher. Although his corrections at the beginning of the project were quite clear, they deteriorated as the project progressed. Often R erased his original commands, which made analysis difficult for the observer. An example of this was Activity 6, Task 3 where figure (a) had been incorrectly completed, then erased and corrected. Remnants of erasures with Liquid Paper™ seemed to indicate a sketch of complementary angles. His corrected sketch did show supplementary angles, but his commands did not. They were simple draw mode commands.

SKETCH COMMANDS



RT 90 FD 30 BK 30 LT 180 FD 30 BK 30

RT 80 FD 30 BK 30

In Task 5 of the same activity, R did not verify his commands and so did not correct them. The same was true of Activity 7.

CONCEPTS

<u>90° ANGLE</u>

R was able to recognize the right angle in standard position. He experienced difficulty with the recognition of the 90° angle in non-standard position at the beginning of the project, but this was ameliorated as the project progressed.

<u>360° ROTATION</u>

R demonstrated a grasp of the concept of a complete rotation in the tasks involving star patterns and in his ability to use the repeat command correctly. As previously discussed, R's main difficulty seemed to be in the area of the written exercises where he was required to use the commands to supply answers. He often misinterpreted these by omitting a given angle and, therefore, not arriving at the correct total number of degrees (360°).

COMPLEMENTARY/SUPPLEMENTARY ANGLES

R has demonstrated skill in calculating complementary and supplementary angles. However, his commands do not show consistent understanding of these concepts.

R was successful in using the angle components to build complementary angles. In a few tasks, however, his commands produced a right angle with a contained angle of less than 90°.

e.g. Activity 4, Task 4

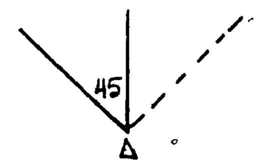
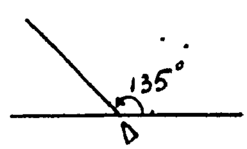


Fig. (c) RTA45 RTA90 (default position is assumed)

In Activity 5 and 6, R was not successful in incorporating the angle components into his commands. He did not demonstrate supplementarity since he either produced only one component (Activity 5, Task 3) or else used simple draw mode commands to reproduce the figure graphically (Activity 6, Task 3).

e.g. Activity 5, Task 3

Fig. (a) TRT 90 FD 30 BK 30 TLT 135 FD 30



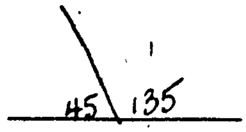
BK 30 was corrected to

RTA135 TLT 45 FD 30

R assumed that the angle to be produced was the given angle. The supplement necessary to complete the angle was totally ignored. The remainder of this task was also incorrectly completed, but corrected after class discussion.

Activity 6, Task 3

SKETCH | COMMANDS



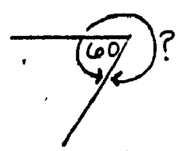
RT 90 FD 30 BK 30 LT 180 FD 30

BK 30 RT 45 FD 30 BK 30

R's style of programming in this activity did not indicate a grasp of supplementarity. R did not employ the defined angles. He used draw mode commands to reproduce the figure graphically.

INTERIOR/EXTERIOR ANGLES

R correctly identified the interior rotation of the given figures. He was unsuccessful in calculating the amount of exterior rotation to produce the same figure. In Activity 7, Task 4, figure (c) was calculated correctly as 300°. This may have been due to the circle formed by the arrows.

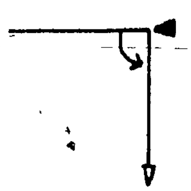


The tasks were not verified with the computer.

STANDARD/NON-STANDARD POSITION

R exhibited some difficulty in recognizing the right angle in non-standard position in Activity 1, Task 2.

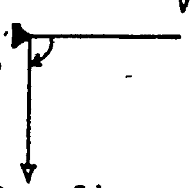
~~fig (c)~~



TLT 90 FD 50 BK 50 TRT 180 FD 50

TLT 90

fig (d)

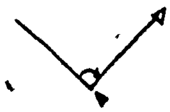


TRT 90 FD 50 BK 50 TRT 90 FD 50

TLT 90 TLT 90

In figure (c) R initially turned RT 180 and corrected it to LT 90; while in figure (d) R assumed the angle to be 90°, but he did not take into account the flip position of the angle. The computer helped him correct this.

R also had difficulty with Task 4 (b)

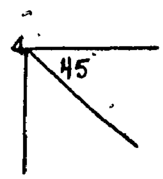


TRT45 FD 50 BK 50 TRT 90 FD 50 BK 50
TLT 45 TLT 35

R has assumed the angle to be greater than 90° in the position. He was successful in debugging to: TLT 45 FD 50 BK 50 TRT 90 FD 50

In the activities concerning complementary angles, R was successful in recognizing the non-standard position of the right angle, although his commands assumed the default position.

e.g. Activity 4, Task 4 (d)



REPEAT 2 [RTA45]

PERFORMANCE ON FINAL EXAM

R scored perfectly (four out of four) on the final exam items. He was able to recognize the concepts of supplementarity, complementarity, as well as the sum of the angles of a complete rotation. The recognition of the right angle posed no problem. In (b) R computed the complement to be 45°, but corrected it to 55°.

STUDENT Y

GENERAL CHARACTERISTICS

GENERAL CLASSROOM CHARACTERISTICS

Y was an impetuous student prone to calling out and attracting attention to himself especially when he felt he could master a situation. Most of his teachers felt that Y was a disruptive student who often came to class without his assignments done. In math class Y was always prepared, his assignments completed on time. Perhaps his achievement in math served as motivation for him to listen and learn. An average student, Y had potential which he used in math class but which he rarely demonstrated in the other disciplines.

BEHAVIOUR IN LOGO CLASS

Y joined the class during Activity 2. His knowledge of Logo was very solid and he had no difficulty catching up to the rest of the class. He chose to work with a partner (M).

For Y the computer was a sour[illegible] of instant reinforcement and motivation combined. He felt very powerful whenever the graphic display was correct and he would always be self-motivated to pursue a more challenging task. He worked seriously and diligently every time he entered the computer lab. Y's self-confidence in the lab often led to his work being com[illegible] too quickly with careless errors. Lack of attention to specific detail and visual cues as well as verbal instructions accounted for most of his errors in interpretation of the tasks.

SPECIFIC LOGO BEHAVIOUR

PLANNING

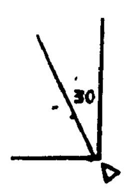
Y planned his tasks on paper almost impetuously. He often disregarded visual cues such as rotation arrows. He also did not pay too much attention to verification of the commands with the computer. Often he corrected his work from his partner's verifications at the end of the class. So, although he worked with a partner, he often worked ahead independently. In the tasks requiring sketches he often sketched only one angle of the pair required.

In planning his tasks, Y often calculated angle measures or made sketches to help him visualize more clearly the commands he was writing. At times, Y used draw mode to assist him in his plan. Like his partner M, he was more concerned with what he perceived as the "correct answer" rather than incorporating the concepts intended by the researcher.

DEBUGGING

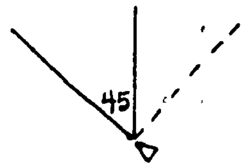
Often Y did not verify his commands with the computer. Therefore it is difficult to analyse his debugging methods. This is evident in Activity 4, Task 4

Fig. c)



RTA30 LTA90

Fig. (e), LTA45 RTA90



The computer display would have been an angle greater than 90°. In writing the commands for the figures, Y assumed the HOME position. He did not display any problem with turtle orientation.

CONCEPTS

90° ANGLE

Y had no difficulty identifying the right angle except in the rotated star position where he estimated 45°

360° ROTATION

Y was able to use the concept of one complete rotation to effectively calculate equal angles and to apply it to the repeat command.

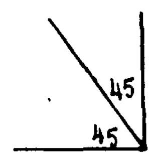
COMPLEMENTARY/SUPPLEMENTARY ANGLES

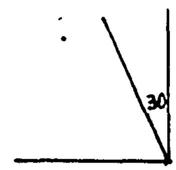
Y successfully completed the written charts and questions dealing with the concepts of complementary and supplementary angles. His commands to reproduce the figures of complementary angles were inconsistently written. When these were incorrect, Y did not always verify with the computer, and he made assumptions which did not always execute the commands correctly.

This was evident in Activity 4, task 4 (c) where he wrote the commands RTA30 LTA90, and in 4 (e) - LTA45 RTA90. Verification with the computer would have displayed angles with a sum total greater than 90°. Y was able to recognize the 90° angle and the contained angle of 30° or 45°, but was negligent in observing that the turtle's rotation of 30° + 90° or 45°+ 90° would be greater than 90°. (see debugging)

In Task 2 (a) and 2 (b), Y wrote the following to complete the complementary angles.

Task 2



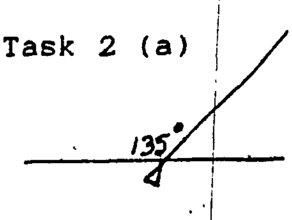


(a) LTA 90 LTA45 (b) LTA90 LTA30 or LTA60

In this case the figure was correctly reproduced since the right angle was formed first, bringing the turtle to the correct position to form the contained angle of less than 90°. Perhaps the verification of these items led Y to complete task 4 (above) as he had without realizing that the proper sequencing of the Logo commands was necessary to result in the figure to be reproduced.

In his commands to reproduce supplementary angles, Y quickly realized he could produce the correct figure in Logo by using a procedure for one angle, and then extending the base arm to draw a straight line or straight angle. Usually the angle he chose was the obtuse angle that had been defined earlier in the activity. Y did not recall that he could also employ the acute angles that he had defined in Activity 3.

e.g. Act. 6, Task 2 (a)



RT 90 FD 30 BK 30 LT 90

LTA135

Y's hasty work showed more inconsistency when in Activity 6, Task 3 he was asked to sketch other pairs of angles that add up to 180° and to write the commands and verify them with the computer.

Only 3 (a) had a sketch of supplementary angles. 3 (b), (c) and (d) were sketches of only the obtuse angle. The commands for the sketches would form the sketched angle.

	SKETCH	COMMANDS
Task 3 (d)		LT 90 FD 30 BK 30 RT 130 FD 30 BK 30

The commands for (a), (b) and (c) all had angle inputs that were different from those sketched. It is difficult to know if Y actually did not grasp the concept of supplementarity or if the method he chose to reproduce the figures using Logo led him to disregard the need for a pair of angles in the later tasks.

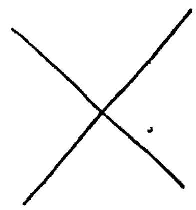
INTERIOR/EXTERIOR ANGLES

Y was able to identify and calculate correctly the interior and exterior rotations in Activity 7.

STANDARD/NON-STANDARD POSITION

Y has demonstrated a recognition of angles in both standard and non-standard position. However, in writing commands for the figures in non-standard position he experienced the most difficulty with the repeat of 4-90°

angles in a 45° rotated position.



REPEAT 4 [TRT 45 FD 50 BK 50 TRT 45]

This commahd was corrected several times and is still incorrect. Recognition of the right angle is lacking here.

PERFORMANCE ON FINAL EXAM

Y responded correctly to all four questions.

STUDENT T

GENERAL CHARACTERISTICS

GENERAL CLASSROOM CHARACTERISTICS

T was a cheerful student who worked slowly and persistently until she was able to solve a problem. Her achievement in mathematics was below grade level. Her class performance fluctuated from high to low, depending on how well she understood the concept so as to be able to retain it for future application. Her reading comprehension and vocabulary attainment were also below average for the grade level. This often was part of the cause of T's hesitancy in beginning an assignment. She often approached the teacher for individual assurance that she understood how to proceed. Many times T relied on rote rules in recognizing the procedure of a specific task. She especially enjoyed working at the blackboard were she would repeat the teacher's methodology almost verbatum as she wrote the solution. Looking for patterns in the written solutions was one of T's favourite ways of remembering a certain procedure in mathematics.

T relied on her fingers to count "up or down", as well as any other form of concrete or pictorial aide to assist her in arithmetic calculations. Her approach to solving word problems usually consisted of singling out word cues to signal a specific operation. As the problems became more involved with multi-steps or with applications to ratio and percent, T encountered greater difficulty and frustration in sequencing the steps to the final solution.

BEHAVIOUR IN LOGO

T's lack of self-confidence was also evident in the computer lab. At the beginning of the project, T preferred to work alone, fearful of anyone's criticism should she make a mistake. Even if she consulted with a classmate at the next computer, she returned to her independent work. T always worked in a slow, steady manner, cautiously hiding any feelings of frustration she might have experienced.

As the project progressed, T chose to work with a partner she felt she could trust and be comfortable with. She did not usually elicit help from the teacher but worked persistently in draw mode, erasing and changing commands to fit the monitor display with the stencil diagram. By the third activity T was exhibiting enthusiasm about going to the computer lab to work on the project. She tackled the assignments with more self-confidence and was less anxious about making mistakes. The fact that the work was not "marked", as well as the user friendly language of Logo had relaxed T sufficiently to allow her to enjoy the problem solving at hand.

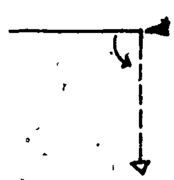
SPECIFIC LOGO BEHAVIOUR

PLANNING

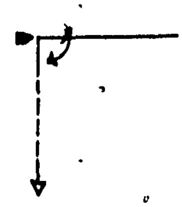
T usually worked in draw mode using visual cues from the diagrams to guide her. Once she felt she had mastered a concept, she attempted to plan with paper and pencil and then verify with the computer. T's visual cues were often incorrect as a lateralization problem emerged from the tasks. Often she perceived a figure on paper in a totally different way from the one she drew on the computer and vice versa.

e.g. Act. 1, Task 2

(c)



(d)

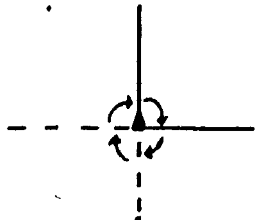


FD 50 BK 50 TRT 90 FD 50 FD 50 BK 50 TLT 90 FD 50

As the figures demonstrate, T often assumed the default position of the turtle in planning her tasks.

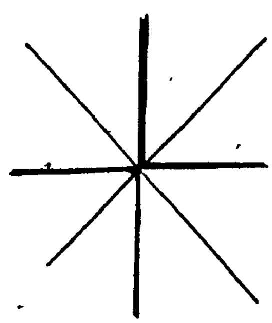
Instead of calculating the angle measure, T relied on visual cues. This often resulted in incorrect estimations, especially where equal angles were involved. As well, T often disregarded rotational arrows, as she became intent on reproducing the figure only, with no attention to procedure.

Act. 2, Task 3



FD 50 BK 50 RT 90 FD 50 BK 50
BK 50 FD 50 RT 90

As in the classroom mathematics, T searched for patterns of something she knew to apply to another problem. This was evident in Activity 2, Task 3 where T identified the right angles in an attempt to reproduce the eight-star pattern. T has coloured over the parts of the figure, isolating the right angles. She then wrote commands to complete that figure first. She then proceeded to backtrack in an effort to form the remaining angles, but became inconsistent with the input for the rotation (85°).

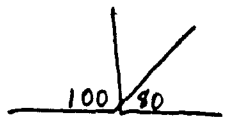


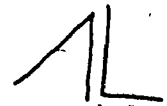
TRT 90 FD 50 BK 50 TRT 90
FD 50 BK 50 TRT 90 FD 50 BK 50
TRT 90 FD 50 BK 50 TRT 45 FD 50
BK 50 TRT <u>85</u> FD 50 BK 50 TRT <u>85</u>
FD 50 BK 50 TRT <u>85</u> FD 50 BK 50

Many erasures in the last part attest to T's use of draw mode to plan this task.

What is evident throughout T's activities is the planning style which might successfully reproduce the figure using draw mode, but which does not necessarily fulfil the concept to be acquired via the structured tasks.

In planning her tasks which involved drawing a sketch for a set of commands or drawing a sketch for a particular angle concept, T often did not incorporate the skills she had practised in forming the angles in previous tasks, e.g., T's sketches of supplementary angles omitted the components of two adjacent angles.





This is demonstrated in a sketch of complementary angles where the commands were given

RTA45 RTA45

DEBUGGING

Throughout the project T relied heavily on visual cues to help her debug commands that did not produce correct figures. The following are examples of this strategy.

Activity 1 ACUTE AND OBTUSE ANGLES

T's answers to the questions in Tasks 5 and 6 lend some insight to her approach. In Task 5 her reply to her choice of degree estimates for each angle was: "You try out different numbers and then you get the approximate answer".

For Task 6, her answer was: "By looking how the form is the shape".

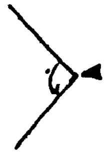
T has stated her strategy as using visual cues to help determine the measure of the angle rotation. She has not based any calculations on the quarter rotation of 90°.

Activity 3

In Task 3, T did not compute 90 ÷ 3, but used visual cues again to write : RTA30 TRT 20 RTA30 which she corrected successfully with the computer.

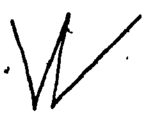
Task 3

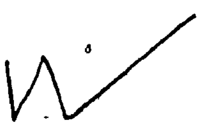
Figures in non-standard position posed problems for T even in debugging with the computer

Task 1  TLT 45 FD 50 BK 50 TLT 65 FD 50 BK 50

The right angle has not been recognized. Confusion with the input to FD, BK and RT, LT was also demonstrated in this task with the following reply: THE NUMBER OF DEGREES IN EACH FIGURE IS 50° OR LESS.

The sketches of Task 7 demonstrated T's perceptual difficulties as well as her lateralization ones. Her original sketches for (a), (b) and (c) actually showed two separate angles almost in a W pattern.

a)  not corrected

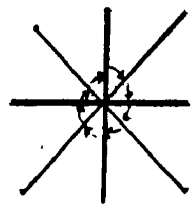
b)  corrected to 

c)  corrected to 

d)  corrected to 

The concept of complementary angles formed by two adjacent angles has not been observed here. T's corrections demonstrated the effectiveness of using the computer as a tool to aid T in correctly visualizing complementary angles. It is difficult to know if T did not bother to correct sketch (a) or if it actually appeared correct to her.

Activity 2



Using visual cues T debugged the eight-star pattern - TRT 90 FD 50 BK 50 TRT 90 FD 50 BK 50 TRT 90 FD 50 BK 50 TRT 90 FD 50 BK 50 TRT 45 FD BK 50 TRT 85 FD 50 BK 50 TRT 85 FD 50 B 50 TRT 85 FD 50 BK 50

The erasures that accompanied these commands indicate T's perseverance in attempting to reproduce the figure using draw mode. The inputs to the rotations indicated the lack of recognition of equal angles as well as the concept of 360° rotation.

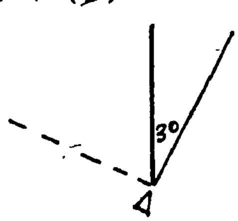
Following this task were questions relating to the concept that demonstrated her confusion with the number of angles and the measure of angle rotation.

Both were answered with "8".

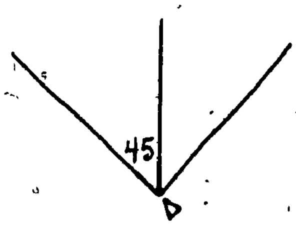
Activity 4

T wrote commands to produce a 90° angle and a contained angle of 30° OT 45°.

Task 4 (c)



a)

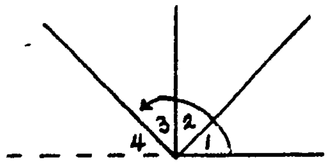


RTA30 LTA90

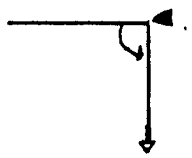
LTA45 RTA90

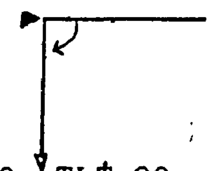
The figures would appear correct.

This style was observed again in Activity 5.

Task 1 (b)  TRT 45 RTA45 LTA90 LTA45 LTA45

The problem T encountered with lateralization in Activity 1 was debugged effectively.





FD 50 BK 50 TRT 90 FD 50 || FD 50 BK 50 TLT 90 FD 50

TLT 90 TRT 90

T used "play turtle" techniques to help her in later tasks. This seemed to resolve the problem for her in subsequent tasks.

CONCEPTS

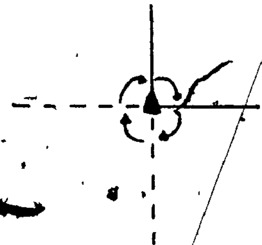
90° ANGLE

T grasped the concept of a 90° angle and used it to guide her in constructing more complex angles. She experienced some difficulty in recognizing the angle in non-standard position especially since she relied so heavily on visual cues.

360° ANGLE

T did not demonstrate a grasp of the concept of one complete rotation of 360°. This was observed in the commands written to reproduce the figures as well as in the written questions that checked the understanding of the concept. An example of this was the four-star pattern in

Activity 1, Task 3.



FD 50 BK 50 RT 90 FD 50

BK 50 BK 50 FD 50 RT 90

FD 50 BK TRT 180

Having relied on visual cues, T did not produce four consecutive right angles to produce a 360° rotation.

The commands produced the correct figure but the rotational arrows were not followed. T was therefore not able to extract the information from the commands to correctly answer the questions that followed:

HOW MANY 90° TURNS DID THE TURTLE MAKE? 2

WHAT IS THE TOTAL NUMBER OF DEGREES OF THESE TURNS? 180°

The turtle went all the way around 4 time.

In the last question T missed the cue given in the singular word TIME. She had clearly shown a lack of understanding of a complete rotation.

In her responses to the eight-star pattern of Activity 2, Task 3, T was able to identify the eight angles but her response to the number of degrees in each rotation was also 8 (see debugging). In calculating the sum of the degrees the turtle turned, T wrote:

90 x 4

+45

85 x 3

Somewhere in the space is the number 360°. One can only assume that this was filled in later during the class discussion.

In response to the questions concerning the commands for the pattern.

WHAT DO YOU NOTICE ABOUT THE COMMANDS FD BK TRT?

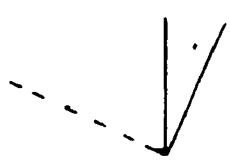
T replied, "That you always have to use those commands."

There has been no interpretation as to input. Perhaps this question was not specific enough for Task 7.

T did not attempt the task where the repeat command was experimented with. T has placed more emphasis on the visual representation of the tasks in the activity and has therefore shown little understanding of the concept of a complete rotation of 360°. T did recognize the 90° angle in standard position in all figures.

COMPLEMENTARY/SUPPLEMENTARY ANGLES

T demonstrated that she had understood the concept of complementarity by using the defined angles as components and by filling in her charts where the complementary angles had to be calculated. In two figure of Task 4, Activity 4, T wrote commands which were incorrect as to complementarity since they produced a right angle with another contained angle.

e.g.  RTA30 LTA90

T was able to correctly sketch one out of the three figures in Task 5, Act. 4

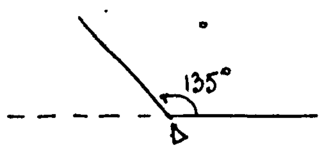
b)  TLT 30 RTA60 LTA30

c)  TRT 90 RTA45 TRT 90 RTA45

These figures were not debugged.

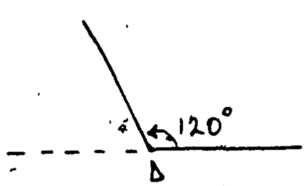
T did not demonstrate any understanding of the concept of supplementarity. This was observed in Activity 5.

In producing the commands to Task 3, where supplementary angles were to be formed by using the defined angles as components, T wrote commands for only one angle (where the measure was given) as in figure (a) and (b). In figure (c) and (f), T drew the figure in draw mode, beginning with a straight line and then rotating the turtle the amount indicated to produce the diagonal line or adjacent ray of the supplementary angle.

a) 

TRT 90 FD 30 BK 100 FD 30

TLT 135 FD 30 BK 30

b) 

TRT 90 FD 30 BK 60 FD 30

TLT 120 FD 30 BK 30

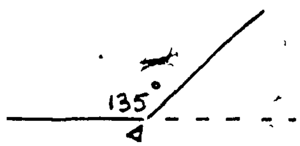
Confusion in the completion of the chart was due to the fact that T did not have a first and second turn in her figures and was thus unable to calculate the total 180°.

The chart was completed correctly after the class discussion of the concept which led the observer to assume that the calculations were performed as a rote exercise.

ORIGINAL CHART	TURN 1	TURN 2	SUM # 1 & 2
	180°	90°	180°
	180°	130°	180°
	180°	150°	180°
	180°	160°	180°

T's style of producing supplementary angles was carried into Activity 6. Only in Task 2 has T written commands for the turtle to draw two angles with a sum of 180°. She has not used the components to produce one angle of 180°.

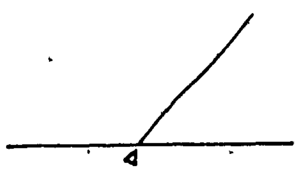
a)



TLT 90 FD 30 BK 30 TRT 135 FD 30 BK 30 TRT 45 FD 30 BK 30

Task 5

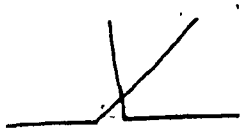
Fig (a)



TRT 90 FD 30 BK 60 FD 30 TLT 45 FD 30 BK 30

The sketches in Activity 6, Task 3 exhibit a lack of understanding of the adjacent property of supplementary angles.

a)



d)

In sketch (c)

TRT 45 FD 30 BK 30 TLT 45 FD 30 BK 30 TLT 45 FD 30 BK 30 TLT 45 FD 30 BK 30

T has produced four angles with a sum of 180°. The concept of the straight angle is missing even though the actual figure produced by her commands is correct.

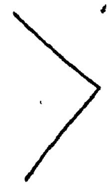
INTERIOR/EXTERIOR ANGLES

T did not demonstrate any understanding of the inverse rotation to produce a given angle. She was able to write commands for the interior angles given in the figures. She was unsuccessful in writing commands for the exterior angle rotation even where the rotations were given.

STANDARD/NON-STANDARD POSITION

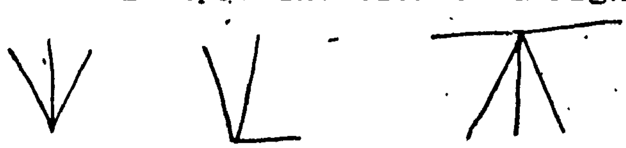
T had difficulty with recognition of the right angle in rotated and inverse positions. This was particularly demonstrated in Activity 3.

Task 1 (c)

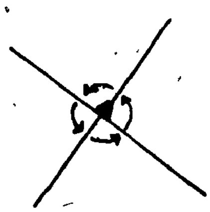


TLT 45 FD 50 BK 50 TLT 65 FD 50 BK 50

In Activity 4 her sketches of a complementary angle did not present pictorial representation of a right angle.

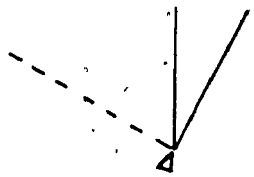


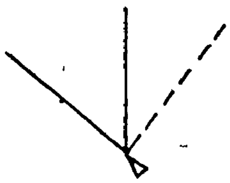
In writing commands for a rotation of 360° (Act. 2, Task 2) where the four-star pattern was rotated, T did not recognize the four right angles.

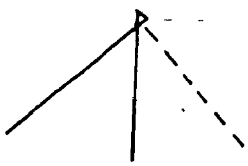


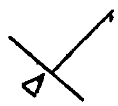
TLT 80 FD 50 BK 50 TLT 80 FD 50 BK 50 TLT 80 FD 50 BK 50 TLT 80 FD 50 BK 50

T showed improvement in her recognition of the 90° angle in rotated position in Activity 4, Task 3, figure (c), (e), (g).

c)  RTA30 LTA90

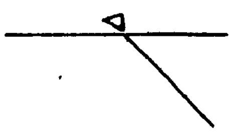
e)  LTA45 RTA90

g)  TLT 135 LTA45 LTA45

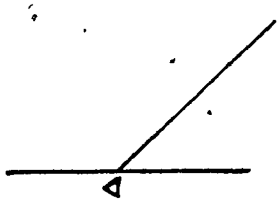
Activity 6, Task 5, Fig. (b) 

TLT 45 FD 30 BK 60 TRT 60 FD 30 BK 30 TRT 120 FD 30 BK 30

In figure (b) T has not recognized the two right angles in the non-standard position.

c)  TLT 45 FD 30 TLT 45 FD 30 BK 60

The commands (above) were written over erasures. One can conclude that this figure was completed in draw mode. Since T has begun with the adjacent ray, she has used the same visual cues style as previously, although there is no recognition of the reflected version of figure (a).

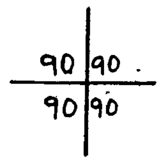
a) 

PERFORMANCE ON FINAL EXAM

T scored two out of four. Both errors were in the figures where supplementary or complementary angles had to be calculated. Since T's answers seem to be computational errors, a) 125° b) 45°

(180-45) (90-35)

one can assume that she identified the angles correctly. Item (c) was filled in with 90



and the sum was correctly calculated as 360° although it was not required. The sum was required in (d), an eight-star pattern. Characteristically T added the words "whole rotation" under the answer.

CHAPTER VII

RESULTS OF RESEARCH STUDY

The Effect of Logo as a Tool Observed in Angle Development

The concepts of angle that were developed in the research project included the recognition of the right angle in standard and non-standard position, the complete rotation of 360°, complementary angles, supplementary angles and interior and exterior angles. The Logo referred to in the following analysis are the structured activities, formulated by the researcher/teacher, to be carried out by the students using paper and pencil planning concurrently with the verification on the computer by Logo graphics.

The concepts of complementary and supplementary angles involved the use of angle procedures (RTA, LTA) which were defined at the onset of each concept. These procedures were to be used as subprocedures to form the required angles. In order to assure uniformity in programming these procedures, the commands were discussed orally by the students and teacher. This provided for the turtle state to be uniform for all students. The activity tasks where complementary/supplementary angles were to be produced were executed by the students independently. Details of the analysis may be found in the student profiles.

I. ESTIMATION OF ANGLES

The majority of the students experienced no difficulty with the estimation skills required in the paper and pencil planning stage of each task. For the few who met with difficulties due to a lack of recognition of an angle in non-standard position, the computer provided the visual reinforcement at the beginning of the project. This led to the development of the intrinsic understanding necessary to approximate the angle measure. Those who were weak in estimation skills at the beginning of the project were adept by the end of the project.

II. DEVELOPMENT OF MATHEMATICAL TERMS

Early in the activities, the amount of rotation of the turtle became synonymous with the degrees of angle rotation. This developed concurrently with the written exercises and the computer commands. The Logo activities provided for a visual image of the abstract symbols for the measurement of angles.

III. CONCEPTS OF THE CONSTRUCTED ANGLE

The construction of an angle and its attributes became systematic as the students wrote commands for the turtle to draw the angles. The procedures of RTA and LTA necessitated the use of rays and a vertex.

IV. THE DYNAMIC ANGLE

Logo emphasized the dynamic angle. Logo must have a rotation command in order to draw an angle. The students always measured the angle by verifying the amount of the turtle rotation rather than the arm lengths characterized by the movements FD and BK of the turtle. Thus the Logo activities reinforced the notion of the unique angle measure for each angle. The perception of an angle as static was observed in Close's study of secondary students using the protractor as a tool. As a consequence of Logo, the students were aware of the angle rotation rather than the measurement of the arms.

V. PERCEPTION OF RIGHT ANGLE

A. STANDARD POSITION

From the introductory activities of the research and throughout the development of more complex angles, the students had no difficulty recognizing the right angle in standard position. They were successful in identifying the right angle in a standard position when it was used as a subprocedure in a complete rotation or in complementary/supplementary angles.

B. NON-STANDARD POSITION

The non-standard position of the right angle posed problems for a few students, especially at the beginning of the project. The Logo activities developed the students perception of the right angle in non-standard positions in the early activities. These activities included executing figures of right angles where an initial positioning and direction of the turtle was required before the rotation of 90°. Thus the visual image of the right angle in a reflected and/or rotated position was developed via the Logo activities. The identification of these figures was particularly evident in the activities related to complementary angles. The verification of figures in non-standard position using RTA and LTA aided the students in predicting the outcome of the use of the procedures and the interfacing necessary to execute the given figure.

VI. 360 DEGREE ROTATION

The complete rotation was readily identified when composed of four 90° angles in standard position. Later on, in the activities, the students were successful in recognizing factors of 90 (45,30) as well as other components of 360° necessary to complete a rotation of 360° by employing equal angles. The star patterns in non-standard position were recognized as well, as the angles discussed in the previous section (Perception of Right Angle) were developed. For most of the students the complete rotation posed no problem as they employed the concept in writing the Logo repeat command.

VII. 45 DEGREE ANGLE

Students acquired the recognition of a 45° rotation which they identified as half of 90° from the activities.

VIII. COMPLEMENTARY ANGLES

The students were able to identify complementary angles. They demonstrated the ability to construct a complementary angle when given a pair of angles. This was evident in the paper and pencil activities, where the use of the angle procedures of RTA and LTA were used as commands to compose the complementary angle.

The students were also successful in calculating the complement necessary to complete a given angle, using different numbers (decimal numerals).

IX. SUPPLEMENTARY ANGLES

Although the students were able to calculate supplementary angles in the paper and pencil activities, the majority was not successful in using the procedures, RTA and LTA in composing supplementary angles. They misconstrued the tasks, using the draw mode to produce the angles. Some students used one procedure (LTA/RTA) and then a simple turn to effectively construct a supplementary angle. Two students were able to use RTA/LTA for both angles in some of the tasks.

X. ROTATION & DIRECTION

Students were able to use the inverse relation between RT and LT to execute a given angle and the direction relation $\longrightarrow$ RTA a = LTA(360 - a).

XI. CALCULATIONS OF ANGLES

The completion of the charts of complementary angles was enhanced by the computer. Students were able to "see" the addition of two complements to form one angle of 90° on the monitor concurrently with the abstract symbols being "written" on the blackboard by the teacher. Similarly, the students were led to interpret the chart dealing with supplementary angles. The need to rely solely on visual cues for angle measure was greatly diminished.

XII. NON-STANDARD POSITION

After acquiring the skill in the first two activities, the students readily used the non-standard position to sketch angles in subsequent activities.

Problems Observed

I: PROBLEMS WITH RESEARCH ACTIVITIES

A. TIME LIMITATIONS:

Time constraints affected the successful completion of an activity. Often the 45 minutes period was not adequate to complete an activity, and therefore the development of the ideas inherent to each activity did not flow as had been intended. Since several days might have elapsed between periods, some students required a discussion review before proceeding with the remainder of an incomplete activity.

B. INTERPRETATION OF INSTRUCTIONS

Inherent instructions in the tasks were often disregarded by the students.

1. ROTATIONAL ARROWS:

Confusion in interpretation of a task ensued when arrows were not followed. This was particularly observed in activities of 180 and 360° rotations where figures were completed independent of instructions. This resulted in incorrect answers to the written summations that followed a particular task. (Act. 2, Task 3; Act. 5, Task 1; Act. 7)

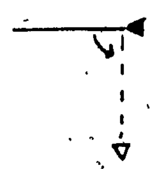
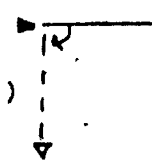
2. WRITTEN INSTRUCTIONS:

Written instructions may need to have been more specific. Often the instructions were not interpreted as the researcher had intended. The problem may have been due to reading difficulties or the prevalent attitude of "getting the figure right" without enough attention to the written instructions. This was particularly observed in the activities dealing with supplementarity and interior/exterior angles, where the written instructions were not given as much oral discussion as in the previous activities.

Another example of misinterpretation was noticed in questions where the students were required to express their reasons for their choice of input (Act. 1, Task 5 and 6) or when they were asked to describe an observation (Act. 1, Task 5).

3. ASSUMPTION OF SIMILAR TASKS:

Assumption of tasks being similar or alike was observed. e.g., Activity 1, Task 2.

Fig. (b)  Fig. (c)  Fig. (d) 

CH wrote the correct commands for each figure, without paying attention to the different positions of the right angle in each case.

Fig. (b & c) FD 50, BK 50, TLT 90, FD 50

Fig. (d) FD 50, BK 50, TRT 90, FD 50

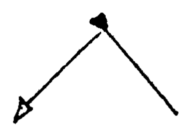
4. TURTLE POSITION:

Students ignored the turtle positions given in the figures. The shaded turtle indicated the starting position, while the clear turtle indicated the end position. Students who did not follow these positions may have produced correct figures, but in different positions on the screen.

e.g, In order to position the right angle in reflected and rotated position, different interpretations were observed as to (a) initial position of turtle, and (b) position of figure on screen.

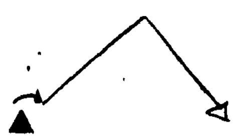
Therefore to produce the following figure, the Logo commands may vary, but the figure will appear correct on the screen although in a different quadrant of the screen.

Given Fig.:



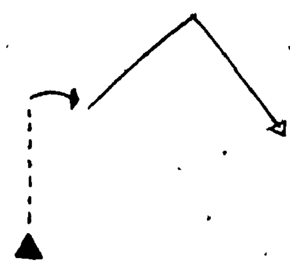
Reproductions using default position:

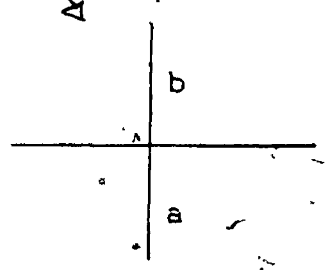
a)



or

b)





screen quadrant

C. VERIFICATION WITH THE COMPUTER

Not all the students verified each task's commands with the computer. This resulted in difficulty for the researcher in analysing the student's work in some cases.

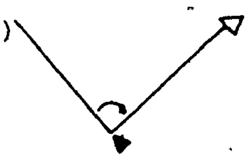
D. RESEARCH INTENTION OF TASK

The tendency to use FD, ~~BK,~~ RT, LT instead of the procedures RTA/LTA, given as tools, was a source of misinterpretation of the researcher's tasks. This was particularly true with the supplementary angle tasks, where at most one procedure was used by a number of children.

II. CONCEPTUAL PROBLEMS

A. EARLY ESTIMATION OF ANGLE PROBLEM

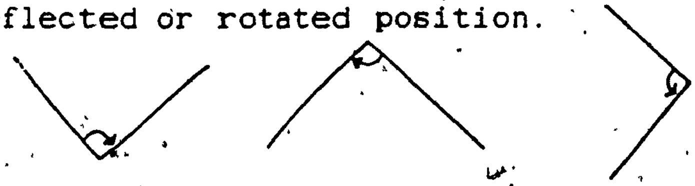
Some students had difficulty estimating the angle rotation in Activities 1 and 2, especially when the figure was in non-standard position. Angle estimation of an initial rotation to position the turtle to produce an angle in rotated position also presented difficulty at this stage.

e.g. Act. 1, Task 4 (b)  c) 

While the computer activities reinforced this skill for most students, students T and L continued to experience difficulty with angle estimation throughout the project.

B. NON-STANDARD POSITION

In the early activities (1 & 2) of the research the students could not always recognize the 90° angle in reflected or rotated position.



These difficulties became less prevalent in the activities with complementary and supplementary angles.

Difficulties were observed with non-standard position in complementary angles, related to predicting the outcome of the defined angle component in the non-standard position. This was particularly evident in the task where the students were asked to draw sketches for given commands (Act. 4, Task 5).

C. LINEAR AND ANGULAR MEASURE

A few students were confused with inputs to FD, BK and TRT, TLT. Instead of calculating the angle rotation, they supplied the same number as the input to FD, BK. This confusion was most apparent in debugging in the early activities of the star patterns of the 360° rotation concept. (Act. 2, Task 3). This problem was not observed in subsequent activities.

D. TURTLE STATE

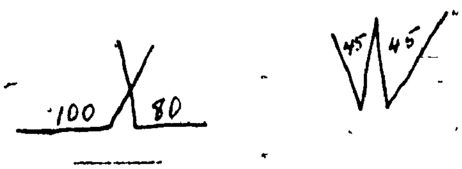
Confusion ensued when students ignored turtle positions given in the figures. Many students assumed the default position and omitted commands to position the turtle. Turtle state especially affected results in non-standard position figures.

E. 360 DEGREE ROTATION

Some students used visual cues exclusively in producing figures involving star patterns. This inhibited their processing of the information necessary to calculate the number of equal rotations needed to complete a rotation of 360°.

F. CRITERION OF ACCEPTABILITY OF SOLUTION

Students T and L consistently exhibited problems where recognition of differences in the given figure and the monitor's figure occurred. T sketched pairs of complementary and supplementary angles without an adjacent ray.



T accepted the sketches even with computer verification. The difficulty demonstrated may have been perceptual or the diagrams may have been accepted as the solution.

G. LATERALIZATION

Difficulties with lateralization resulted in angles being formed in inverse positions (student T).

H. COMPUTATIONAL ERRORS

Often complementary angles were incorrect due to mathematical computation errors. Calculating equal angles to complete one rotation also involved errors.

I. ROTATIONAL SYMMETRY

Difficulties in using the repeat command of Logo may have impeded the learning of the concept where a number of equal angles with a sum of 360° constitutes one complete rotation of the turtle. This also led to difficulties in the recognition of iterative pattern.

J. ROTATIONS GREATER THAN 90 DEGREES

An input for a rotation greater than 90° was often not accomplished with one number, but rather by many, whose sum was greater than 90° or greater than 180°. Students seemed hesitant in the final activity (7) to explore with larger inputs to rotations, since most items in previous activities did not have angles greater than 180°.

K. LINKS BETWEEN ACTIVITIES

Links between activities dealing with right angles and 360° rotations, or complementary angles were made early in the project. Links between right angles and 180° rotation were also grasped. However, links between other activities or concepts were not as readily made. This was demonstrated in the lack of application of the defined angles RTA/LTA, which were successfully employed to construct complementary angles, to the construction of supplementary angles. In like manner, there was no link evident in Activity #2 to the concept of interior/exterior angle rotations in Activity #7. As the student profiles indicate, two students, M and Y, were successful in making these links.

CHAPTER VIII

CONCLUSION

The results of the study seem to indicate that Logo as a tool enhances the acquisition of the concept of angle at the grade seven level. As was stated in the review of the research, the concept of angle as acquired in the traditional mathematics curriculum often involves the concept of static ray pair angles. This is a result of the emphasis of this notion presented in textbooks and/or by the teacher at both the elementary and secondary levels. The stereotypical model of a given angle, as presented pictorially in the traditional curriculum, especially hinders the secondary level student in his/her perception of figures of angles in rotated or non-standard positions. As well, the use of the protractor as a tool for measuring an angle presented problems to students when dealing with exterior rotations or reflex angles, or angles greater than 90 degrees (obtuse angles). This difficulty often leads to poor estimation skills at the secondary level, since the basic understanding of the rotation of an angle is lacking. As the research indicates, the students had less difficulty in telling how many degrees there were in a right angle, than in visualizing the right angle on the clock (National Assessment 1978).

In an effort to provide concrete manipulative experience in rotating angle rays, Turtle Geometry (Logo computer language) was employed in this study. In this way, the concept of angle was dependent on the amount of rotation (see Concept of Angle in Logo) required to begin the construction of the second ray, rather than on ray pairs of the traditional static angle.

Midway through the research project, the students seemed to have a good grasp of the dynamic angle. They had formed a visual image of an angle. This was particularly enhanced by the use of the constructed angles RTA/LTA. The students demonstrated this ability to visualize a constructed angle in their sketches of both the complementary and supplementary angles. Several students did not hesitate to sketch angles in non-standard position, an indication that they had become comfortable with the angles in varying orientations - a concept that had presented difficulty at the very beginning of the project.

The meaning of rotation in the concept of angle was further enhanced by employing the slow turtle to enable the visualization of the turtle rotating or pivoting in a given position before moving to produce a line (ray). This was evident even in the students' language as they described the process of construction. Unlike the students in a regular Logo class, the students of the research project never described the turtle as going F-D-50 or R-T-30, but rather as moving forward 50 and turning or rotating 30 degrees to the right.

The tools RTA/LTA and the indicators presented in the tasks enabled the students to clearly identify the constructed interior angle. The computer enabled the students to "see" the adjacent angle pairs. In this way the addition of the angles was seemingly concretized as opposed to the abstract mathematics of adding two degree measures to form one complementary/supplementary angle. Calculations of angles with paper and pencil was considerably enhanced as a result of the angle concepts having been developed concurrently with the Logo activities. This was revealed in the items on the final June exam where the students were required to provide the missing measure in the given figures. The three errors obserbed in calculating a complement or supplement to a given angle were computational rather than conceptual in nature. The strategy to calculate the missing measure was readily recognized.

The implicit understanding of a sum of 90°/180° over and above the Logo commands was evident in constructions of complementary/supplementary angles in rotated or inverse positions. The computer seemed to enhance the development of this as the students were able to project a sketch of the completed figure on paper.

Perhaps of prime importance is the significance of the reduction of the frustration level often permeating a classroom geometry lesson. The students were significantly more at ease with their problems, since the problem was not perceived as a geometry problem, where the links to former material acquired were unattainable, but rather as a graphic problem where the correct commands had to be given to the turtle in order for it to produce the correct figure. If a strategy was not working, it was readily "seen" on the computer screen. Interaction with other students rather than relying on "the teacher for the solution on the blackboard" resulted in a more relaxed and, therefore, less competitive problem solving atmosphere. Speed was not the important factor. Everyone was always busy with some aspect of the angle concept at hand.

The need for teacher guidance was observed throughout the research project. This guidance was exemplified in the discussions prior to each new activity as well as before each lesson where written instructions needed clarification or links between concepts had to be emphasized.

Guidance also was present in the settling of an argument or in the encouragement to persevere. Never did guidance mean "telling". This was quickly grasped by the students and respected throughout the project. The students were prepared to use the computer for feedback and, where necessary, to interact with each other.

The results of this study seem to point to the fact that parallel development of mathematical concepts, such as angle, and programming knowledge of Logo is possible with structured activities initiated and carried out by the teacher. What is not known from this study, however, is if the relations developed as a result of using Logo as a tool are so well learned that they do not risk being forgotten in a short time. More longitudinal study is required to answer this question. A recommendation would be to begin to develop the concept of angle via Turtle Geometry at the elementary level so as to build a solid base for the understanding of the dynamic angle, and to continue the study into the secondary level where the concept could be developed further. The Logo experience with the angle concept could be extended to triangles and to congruency theorems. What is of importance is that the conceptual development begin with the Logo experience and then be linked to the abstract mathematics, instead of beginning with the traditional curriculum and enriching with the computer. In this way, Logo can truly be used as a tool for acquiring concepts in mathematics.

BIBLIOGRAPHY

Assessment Performance Unit, (APU 1985) Mathematical Development Retrospective Report: A Review of Performance in Mathematics. Department of Education and Science, Welsh Office Education Department, Department of Education, Northern Ireland, Ireland: (1978-1982).

Carpenter, Coburn, Reys Wilson, The Results from the first Mathematics Assessment of the National Assessment of Educational Progress. NCTM Publication (1978).

Cathcart, M.G. Logo Programming and Debugging Stategies of Grade 6 Students. M.ED Thesis University of Alberta (1985).

Close, G.S., Children's Understanding of Angle at the Primary/Secondary Transfer Stage. (Unpublished Masters Thesis) Polytechnic of South Bank, London: (1982)

Ebos, F. Robinson, B., Tuck B. MATH is/1, Nelson Canada: (1982)

Ebos, F. Robinson, B., Tuck B. MATH is/2, Nelson Canada: (1984).

Freudenthal, H., Mathematics as an Educational Task, Reidel, Dordecht, Holland: (1973)

Heath, T., The Thirteen Books of Euclids'Elements, Vol. 1, Dover, New York: (1956).

Hillel, J., Mathematical and Programming Concepts Acquired by Children, Aged 8-9, In a Restricted Logo Environment, Report N° 1. Recherches en Didactique des Mathématiques, Vol. 6 n°2, 3, pp. 215-268 : (1985).

Hoyles, C., and Sutherland, R, When 45 Equals 60. Proceedings of Second Logo and Mathematcs Education Conference, London: (1986).

Kieran, C., Logo and The Notion of Angle Among Fourth and Sixth Grade Children. Proceedings of Tenth Psychology of Mathematics Education Conference, London: (1986).

Kieran, C., and Taurisson, A., with the collaboration of J. Boutin, Logo et la Notion d'Angle en Quatrième Année. Commission scolaire, Baldwin-Cartier and l'Université de Québec à Montréal, Montréal: (1985).

Noss, R., Creating a Mathematical Environment Through Programming: A Study of Young Children Learning Logo. (Doctoral Thesis) Chelsea College, University of London: (1985).

Papert, Watt, Disessa, Weir, Final Report of the Brookline Project, Part II, Project Summary and Data Analysis. MIT, Massachusettes: (1978).

Papert, Mindstorms: Children, Computers & Powerful Ideas Basic Books, Inc. New York: (1980).

Pea, R., Logo Programming and Problem Solving, Bank Street College of Education, N.Y., Paper presented at annual meeting of the American Educational Research Association, Montréal, Québec: (1983).

Piaget, J., and Inhelder, B. The Psychology of the Child. Basic Books, New York: (1969).

Usiskin, Z., Van Hiele Levels and Achievement in Secondary School Geometry, University of Chicago, Ilinois: (1982).

Webster's Ninth New Collegiate Dictionary, Merriam-Webster Inc., Springfield, Mass.: (1985).

Wirszup, I., Breakthroughs in the Psychology of Learning and Teaching of Geometry. Soviet Studies in the Psychology of Learning and Teaching Mathematics, University of Chicago. Chicago: (1969).

Worldbook Encyclopedia, Vol. 1, Field Enterprises Educatiional Corporation, Chicago, Ilinois: (1974).

APPENDIX

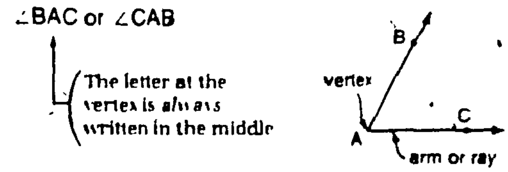
A. Samples from Mathematics Textbooks

MATH IS/1, Secondary I Level

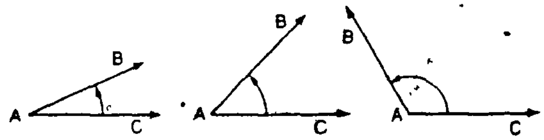
6.3 angles and their measurement

Angles occur in many familiar and unfamiliar places. Angles are formed if two lines, or line segments, intersect.

Various names are used to refer to the parts of an angle. To name an angle, capital letters and the symbol ∠ are used as shown.



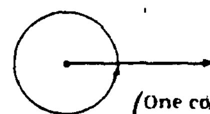
An angle is also formed by rotating a ray. If the ray AB is rotated, the size of the angle changes.



Measuring an angle is based on a procedure similar to measuring length, area, or volume. A **standard** or **basic** unit of measurement is chosen. The basic unit for angles is a **degree** (symbol °).

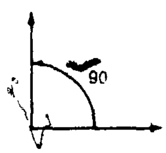
A complete rotation or turn is given a measure of 360°.

A degree is defined as $\frac{1}{360}$ of a complete rotation.

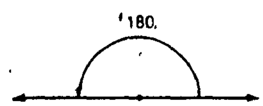


Certain rotations are given special names.

$\frac{1}{4}$ rotation or **right angle**

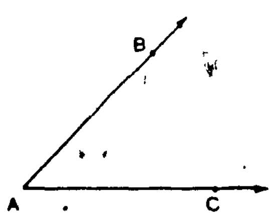


$\frac{1}{2}$ rotation or **straight angle**

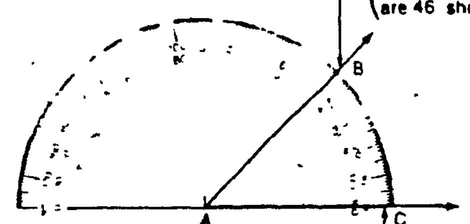


A **protractor** is used to measure angles. The centre of the protractor is placed at the vertex of the angle. The base line of the protractor lies along one of the rays of the angle.

Measure ∠BAC



Use the protractor



The measure of ∠BAC is 46°.

There are two scales on the protractor to measure angles drawn in different positions. To measure the angle, always start from 0 on the scale.

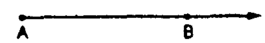
(Two angles are congruent if they have equal measures.)

You can also use the protractor to construct an angle.

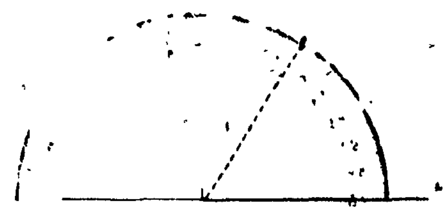
Example Draw an angle that has a measure of 57°.

Solution

- ▶ Draw AB, one ray of the angle

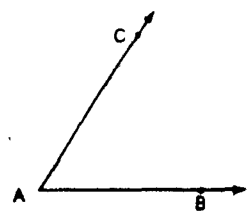


- ▶ Find AC, the other ray of the angle. Use the protractor as shown. The scale on the protractor shows the point C.



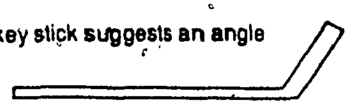
- ▶ Construct the ray AC.

(Then check the measure of ∠BAC. (Is it 57°?))



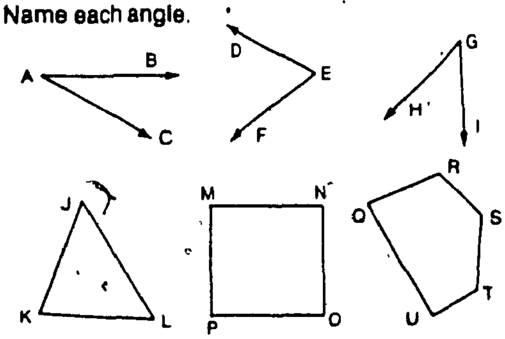
6.3 exercise

1 A hockey stick suggests an angle



List five other items that suggest angles.

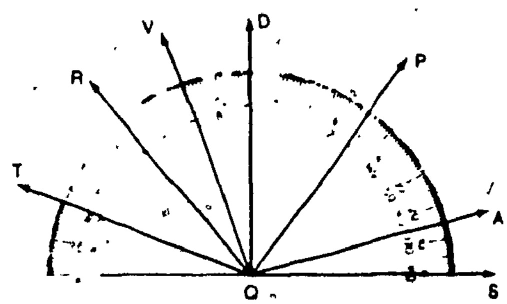
2 Name each angle.



3 Use the diagram. What is the measure of each angle?

(a) ∠AQS (b) ∠SQP (c) ∠RQS

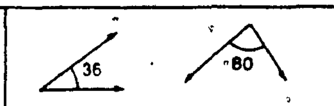
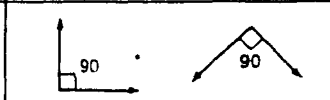
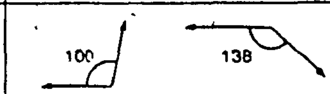
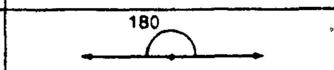
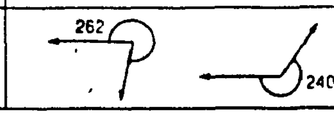
(d) ∠TQS (e) ∠SQV (f) ∠DQS



6.4 vocabulary for angles

Angles belong to certain families of angles, based on their measures Each angle is classified and has a special name, as shown in the chart.

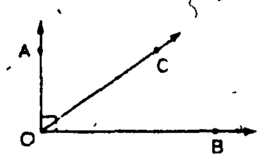
(We often use *angles are equal* to mean the measure of angles are equal)

Family of angles	Measures of the angles	Example
Acute angles	Between 0° and 90°	
Right angles	90°	
Obtuse angles	Between 90° and 180°	
Straight angles	180°	
Reflex angles	Between 180° and 360°	

Other names are used to refer to special pairs of angles.

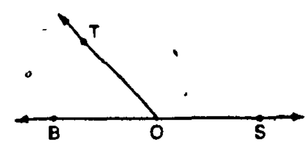
- **Complementary angles** are two angles whose measures add to 90°.

  ∠AOC and ∠BOC are complementary
  ∠BOC is called the **complement** of ∠AOC.
  Also, ∠AOC is called the complement of ∠BOC.



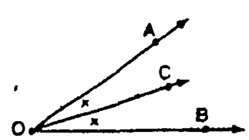
- **Supplementary angles** are two angles whose measures add to 180°.

  ∠BOT and ∠SOT are supplementary.
  ∠BOT is the **supplement** of ∠SOT.
  Also, ∠SOT is the supplement of ∠BOT.



Another word that is related to angles is **bisector**.

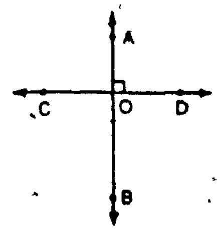
- The bisector of an angle divides the angle in half. $\overrightarrow{OC}$ is the bisector of ∠AOB since ∠AOC= ∠COB.



(These symbols are used to show that angles have equal measures.)

The property of **perpendicular lines** is described in terms of right angles.

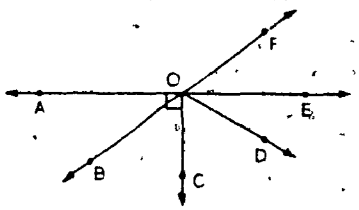
▶ Two lines are perpendicular if they intersect at right angles
$\overleftrightarrow{AB}$ is perpendicular to $\overleftrightarrow{CD}$.
$\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$. (This symbol means "is perpendicular to.")



6.4 exercise

A

1 Write the names of two angles that are
(a) complementary (b) supplementary.

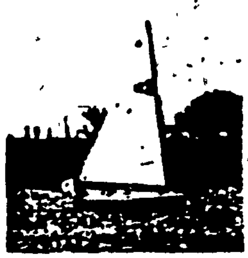


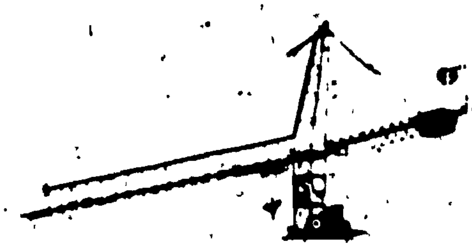
2 Name two angles in the previous diagram that are
(a) straight angles. (b) right angles.
(c) acute angles. (d) obtuse angles.

3 Classify each angle

(a) 146°	(b) 29°	(c) 90°	(d) 45°
(e) 30°	(f) 135°	(g) 320°	(h) 60°
(i) 180°	(j) 150°	(k) 3°	(l) 203°

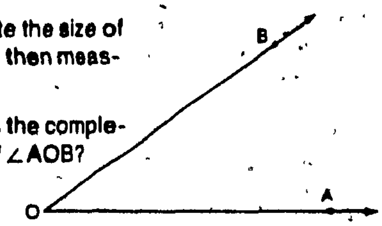
4 For each angle marked,
▶ estimate its size, then measure.
▶ classify the angle.





B

5 (a) Estimate the size of ∠AOB, then measure.
(b) What is the complement of ∠AOB?



6 (a) An angle measures 83°. What is the measure of its complement?
(b) An angle measures 39°. What is the measure of its supplement?
(c) An angle measures 146°. What is the measure of its supplement?
(d) An angle measures 46°.
(i) What is the measure of its complement?
(ii) What is the measure of its supplement?

7 (a) Construct an angle of 50°. Draw the bisector of the angle.
(b) Construct an angle of 130°. Draw the bisector of the angle.

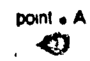
5.1 language of geometry

To study geometry you need to learn the language of geometry.

(To work with geometry, you need to learn the vocabulary and the meaning of symbols)

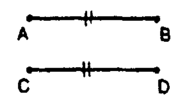
Words to know

▶ A **point** shows position and is a "building block" of geometry. **Capital letters** are used to name points.



▶ Two points, A and B, name a **straight line**. The arrows show that the line extends in each direction. (You will use *line* to mean *straight line*.)

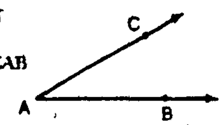
▶ **Line segment** AB is part of a line. It has end-points A and B. Its length can be measured. Line segment AB has the same length as line segment CD. Line segment AB is said to be **congruent** to line segment CD.



▶ AB is also part of a line. It is called a **ray**. It starts at an end point A and extends indefinitely in one direction.

▶ An **angle** BAC is constructed from two rays as shown. These rays are often referred to as the **arms** of the angle. When naming an angle the letter at the vertex of the angle is always written in the middle. Two angles are *congruent* if they have equal measures.

The name of the angle is ∠BAC or ∠CAB



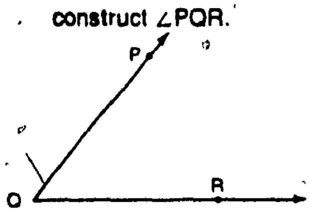
A is the vertex

Tools to use

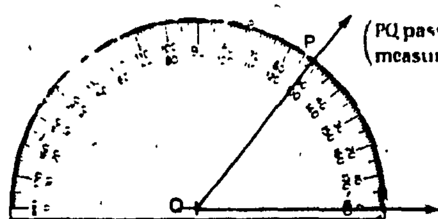
Certain tools for geometry are used to help in your study. For example, a protractor can be used to measure and construct angles. The centre of the protractor is placed at the vertex of the angle. The base line of the protractor lies along one of the rays of the angle.

(There are two scales on the protractor to measure or construct angles in different positions. To measure or construct the angle, always start from 0 on the scale.)

To measure ∠PQR or construct ∠PQR.



The measure of ∠PQR = 52°.



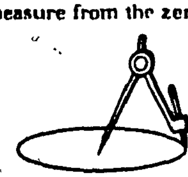
(PQ passes through measure showing 52°.)

Always measure from the zero.

Compasses can be used to draw circles.

▶ The **centre** of the circle is at the metal point.

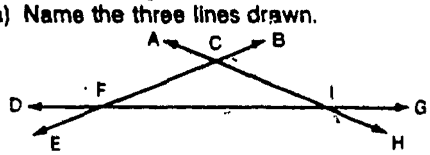
▶ The pencil point is on the circle. The **radius** is the distance between the pencil point and metal point.



5.1 exercise

A

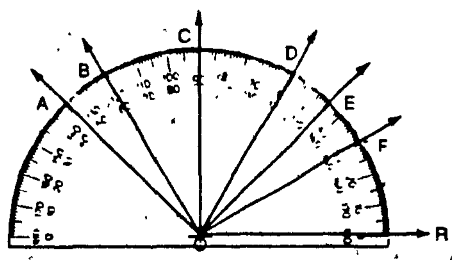
1 Refer to the diagram.
(a) Name the three lines drawn.



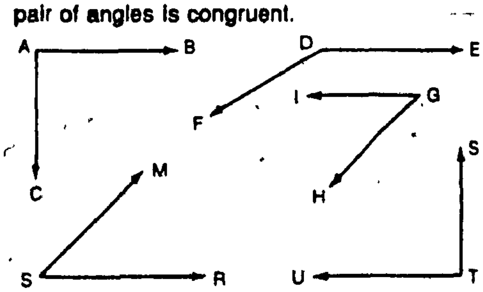
(b) Name three different angles.
(c) Name three different line segments and rays.

2 Refer to the diagram. What is the measure of each angle?

(a) ∠ROF (b) ∠ROE (c) ∠ROC
(d) ∠AOR (e) ∠BOR (f) ∠DOR



3 Measure the following angles and state which pair of angles is congruent.

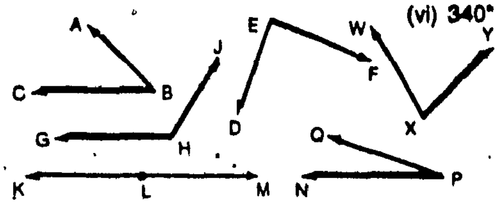


4 Construct each circle
(a) with radius 6 cm (b) with radius 4.5 cm
(c) with diameter 10 cm (Remember: the radius is $\frac{1}{2}$ of the diameter.)

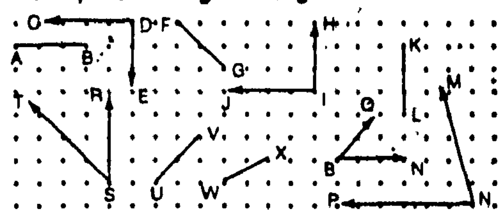
B

5 For each angle drawn below:
▶ Choose the best estimated measure from those given to the right.
▶ Then measure.

(i) 75°
(ii) 180°
(iii) 45°
(iv) 120°
(v) 90°
(vi) 340°



6 Refer to the diagram. Name
▶ two pairs of congruent line segments.
▶ two pairs of congruent angles.



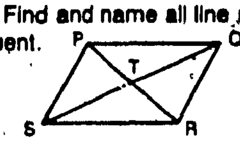
7 Construct an angle for each measure.

(a) 30° (b) 90° (c) 128° (d) 83°
(e) 36° (f) 150° (g) 210° (h) 100°

8 (a) Draw an angle that you think measures 90°. Do not use a protractor.
(b) Measure the angle you drew in (a). How accurate is your estimated angle?
(c) Repeat the steps in (a) and (b) for 60° and 45° angles.

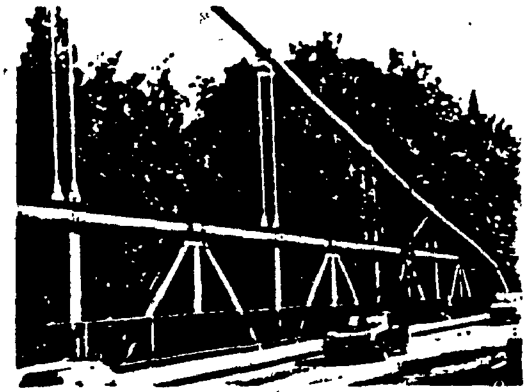
C

9 For the figure, find and name all the angles that are congruent. Find and name all line segments that are congruent.



5.3 angles: constructions and language

You can see angles used wherever you look. How are angles used in the photograph?

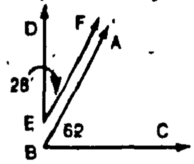


Angles are classified according to their measures, as shown in the table.

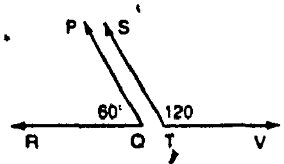
Angle name	Measure of angle
Acute	between 0° and 90°
Right	90°
Obtuse	between 90° and 180°
Straight	180°
Reflex	between 180° and 360°

There are other important words you need to learn to work with angles.

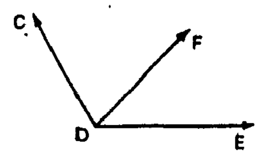
Complementary. The sum of ∠ABC and ∠DEF is 90°. The angles are **complementary**.



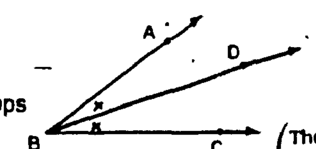
Supplementary The sum of ∠PQR and ∠STV is 180°. The angles are **supplementary**.



∠CDF and ∠FDE are called **adjacent** angles. Adjacent angles share a common vertex and ray and their interiors do not intersect



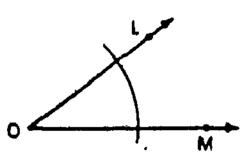
Angles can be **bisected**. The **bisector** of an angle divides an angle in half. The ray BD is called the bisector of ∠ABC since ∠ABD and ∠DBC are congruent. Each is half of ∠ABC.



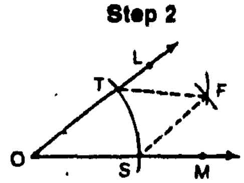
(These marks show that the angles are equal. x x)

You can bisect ∠LOM using a straight edge and compasses, as shown in the following diagrams. Can you describe the steps in words?

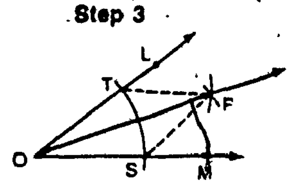
Step 1



Step 2

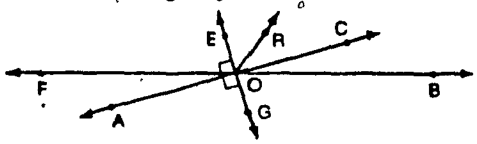


Step 3



5.3 exercise

A Use the following diagram for Questions 1 to 3.



1 Write the names of two angles that are
(a) complementary. (b) supplementary.

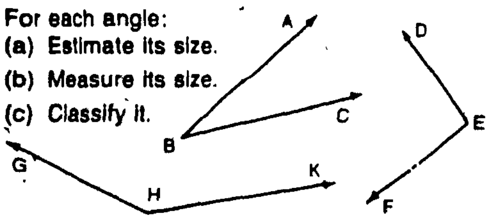
2 Name two angles that are
(a) acute (b) obtuse (c) right
(d) straight (e) reflex

3 Name two angles that are
(a) adjacent (b) vertically opposite

4 Classify each of the following angles.

(a) 123°	(b) 45°	(c) 90°	(d) 94°
(e) 180°	(f) 300°	(g) 10°	(h) 115°
(i) 198°	(j) 7°	(k) 70°	(l) 185°

5 For each angle:
(a) Estimate its size.
(b) Measure its size.
(c) Classify it.



6 (a) Trace each angle in the previous question.
(b) Construct the bisector of each angle.

7 Find the complement of each angle.

(a) 20°	(b) 50°	(c) 15°	(d) 35°
(e) 42°	(f) 67°	(g) 10°	(h) 80°

8 Find the supplement of each angle.

(a) 20°	(b) 150°	(c) 115°	(d) 35°
(e) 72°	(f) 167°	(g) 112°	(h) 73°

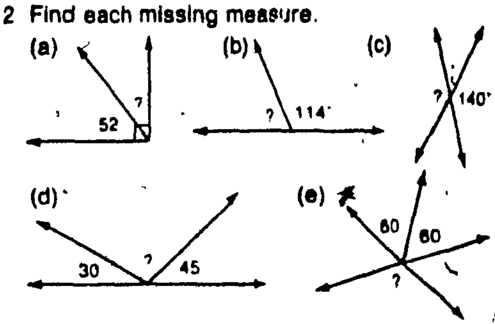
9 (a) Draw an acute angle on a sheet of paper.
(b) Fold the paper to find the bisector of the angle.
(c) Repeat steps (a) and (b) for a right angle; obtuse angle; straight angle.

B

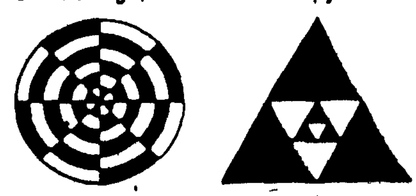
10 (a) Draw an acute angle. Construct its bisector.
(b) Check your results in (a) by measuring.
(c) Repeat the steps for an obtuse angle.

11 (a) Draw a straight angle and construct its bisector. Check your work.
(b) Construct an angle whose measure is $\frac{1}{4}$ of the measure of the angle in (a). What is the measure of this angle?

12 Find each missing measure.



13 To draw each design, angle bisectors were used. Determine the steps used to construct each design; then make a copy of it.



C

14 Make a design of your own based on constructing bisectors of angles.

B. Samples of Students' Logo Activity Worksheets

INTRODUCTION TO LOGO ACTIVITIES NAME

MATERIALS NEEDED:

1) FOLDER
2) PENCIL & ERASER
3) GRAPH PAPER
4) RULER

HOW TO READ THE FIGURES:

1) The SOLID LINES in each figure show you the first part of the task to be completed.
2) The COLOURED TURTLE ▲ shows you the HEADING of the TURTLE at the BEGINNING of the figure.
3) The DOTTED LINES show you the part of the figure to be completed by you.
4) The CLEAR TURTLE △ shows you the HEADING of the TURTLE at the END of the figure.
5) The ARROW ↶ INDICATES in WHICH DIRECTION THE TURTLE is to be TURNED in the figure.

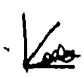
PROCEDURE:

1) All COMMANDS are to be written ON THE SHEET in the SPACE PROVIDED.
2) PRE-PLANNING is to be done on GRAPH PAPER.
3) ANSWER ANY QUESTIONS IN THE SPACE PROVIDED.

LOGO ACTIVITIES # 3 pg 3 Name:

SKETCH the FIGURE THAT WILL BE PRODUCED AS A RESULT OF THE FOLLOWING COMMANDS.

RTA 45
RTA 45.



(b) PU
FD 50
TRT 90
PD
RTA30
RTA60

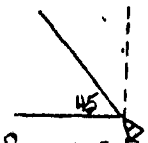
(c) TRT 90
RTA60
RTA30

(d) PU
FD 50
TRT 180
RTA45
RTA 45

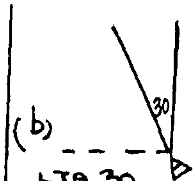
CHECK each sketch on the COMPUTER GRAPHIC SCREEN.

Logo Activities #4 pg 2 Name

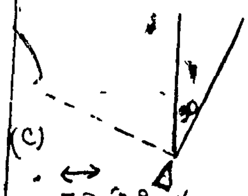
b) WRITE THE COMMANDS to complete the 90degree angle in each figure
Use your procedures as commands wherever possible

a) 

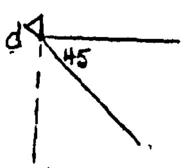
Repeat 2 [LTA45]

(b) 

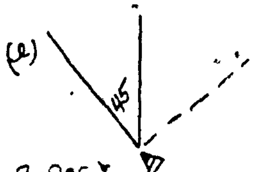
LTA 30
LTA 60

(c) 

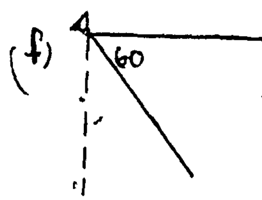
LTA 30
LTA 60

d 

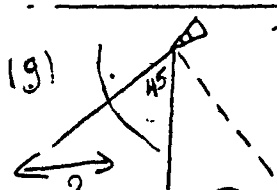
Repeat 2(LTA45)

(e) 

Repeat 2 [LTA45]

(f) 

LTA 60
LTA 30.

(g) 

Repeat 2 [LTA45]

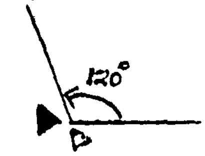
LOGO ACTIVITIES # 5 pg 2 Name

Write a PROCEDURE to form the following figures
Use [30] as [input] for FD and BK

fig (a)

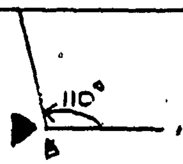
TO RTA135
RT 90 FD30 BK30
LT 135 FD30 BK30
HT

fig (b)



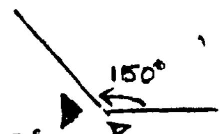
TO RTA120
RT 90 FD30
BK 30 LT 120
FD 30 BK 30
HT

fig (c)



TO RTA110
RT 90 FD 30°
BK30° LT 110°
FD30 BK30
HT

fig (d)



TO RTA150°
RT 90 FD30
BK 30 LT 150
FD 30 BK 30

Write the commands to produce the following figures. The measure of the first angle is given
Use the procedures from #2 (above) where possible.

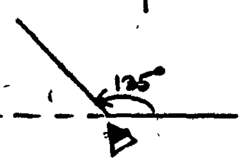


fig (a)

RTA 135
LTA 45

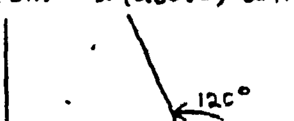


fig (b)

RTA 120
LTA 60

LOGO ACTIVITY #9. p.2 . Name

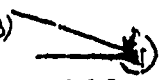
③ For each figure write the commands to produce the figure when
a) you turn the turtle to the right
b) you turn the turtle to the left.

2̶8̶6̶0
  75
 285

(A)

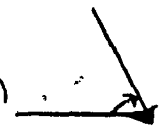
LT 90
FD 30
BK 30
RT (45)
FD 30
BK 30

(B)



LT 90
FD 30
BK 30
LT 335
FD 30
BK 30

(C)



LT90
FD 30
BK 30
RT 75
FD 30
BK 30

(D)

LT 90
FD 30
BK 30
LT 285
FD 30
BK 30

Turn RT 25 degrees
Turn LT 335 degrees
Total 360 degrees

Turn RT 75 degrees
Turn LT 285 degrees.
Total 360 degrees

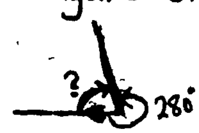
2̶3̶6̶0
280
40

④ How many degrees are needed to complete the angle? turn and produce the figure. Check with the computer

(a)

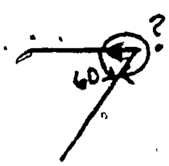
LT 45
FD 30
BK 30
RT 250
FD 30
BK 30

(b)



LT 90
FD 30 BK 30
RT 80
30 BK 30

(c)



LT 90 FD 30
FD 30
BK 30 RT 300
FD 30 BK 30

C. Description of Students

Students	Sex	Age Years	Months
A	M	12	9
L	F	12	8
CH	M	12	8
M	M	13	1
S	F	13	1
R	M	13	4
Y	M	13	3
T	F	13	1

The eight children in the study, members of a remedial class, required, specifically, mathematics remediation. Their poor performance was due to gaps in their mathemathics background which were in some instances related to classroom behaviour as described in the individual profiles.