VIBRATION CONTROL OF PIPING SYSTEMS

by

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ABSTRACT

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In piping systems, when the frequency of the exciting force is equal to one of the natural frequencies of the system, severe pressure head and discharge oscillations develop. In such a case, the system is said to be in resonance and serious damage or complete failure may ensue. Periodic oscillations of pressure and discharge in fluid systems have been observed by many investigators in the field of fluid transients. In view of the fact that a fluid conduit is a continuous system, most of the work done in the analysis of piping system resonance phenomena is based on multi-degree-of-freedom models [1,2,3][1]. However, in many cases the multi-degree-of-freedom approach is not practical and simplified methods based on one degree of freedom models are employed to find the critical natural frequencies of piping systems.

In this paper, an overview of the methods used in industrial applications to compute piping system resonant conditions is presented. Moreover, several of the more sophisticated analytical approaches reported in the current literature are reviewed and formulae and charts for finding the natural frequencies of different configurations are given. The water-hammer phenomenon and the potential equipment damage associated with its occurrence is also discussed. Finally, a numerical example is considered, illustrating the various frequency calculation methods, the effects of rotating machines on elastic foundations and the pressure rise caused by water-hammer.

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[1] [ ] Brackets indicate the number of reference.

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NOMENCLATURE

SYMBOL	DESCRIPTION	BRITISH UNITS	S.I. UNITS
a	Water-hammer velocity	ft./sec.	m/sec.
a_c	Water-hammer velocity in concrete pipes	ft./sec.	m/sec.
a_i	Water-hammer velocity for cast iron pipes	ft./sec.	m/sec.
b	Thickness of pipe wall	ft.	m
C	Coefficient of viscous damping	lb_f-sec/ft	N-sec/m
C_c	Critical damping = $2mf_n$	lb_f-sec/ft	N-sec/m
c	Velocity of propagation (wave velocity)	ft./sec.	m/sec.
Di	I.D. of pipe	ft.	m
Do	O.D. of pipe	ft.	m
D_s	O.D. of a vessel or stack	in.	m
d	Pipe bore	in.	m
E	Modulus of elasticity	lb_f/in^2	N/m^2
F	Impulse force	lb_f	N
F^*	Centrifugal force	lb_f	N
F_{max}	Maximum Dynamic force in a spring	lb_f	N
f_d	Forcing or disturbing frequency	H_z	H_z
f_n	Natural frequency	H_z	H_z
f_w	Frequency of wind gusts	H_z	H_z
g	Acceleration due to gravity	$32.2 \frac{ft}{sec^2}$	$9.8 \frac{m}{sec^2}$
H	Pressure rise due to water-hammer	ft	m
I	Moment of inertia	in^4	m^4
J_a	Polar moment of inertia about the center of gravity of a circular section	in^4	m^4
K	Spring constant	lb_f/in	N/m
K_1	Volume of compression of water	ft	m
K_2	Ratio of elastic modulus of liquid and elastic modulus of pipe material, or bulk modulus at the liquid. For water 300,000.		
K_3	Radius of gyratio	in^2	m^2
L_s	Length of span	ft	m
ℓ	Length of pipe	ft	m

NOMENCLATURE - continued

SYMBOL	DESCRIPTION	BRITISH UNITS	S.I. UNITS
m	mass = W/g	$\frac{lb_f}{32.2\ ft/sec^2}$	$\frac{N}{9.8\ m/sec^2}$
P	Load	lb_f	N
P_1	Normal flow pressure of fluid	lb_f/in^2	N/m^2
P_2	Maximum allowable pressure; if no information is given then 1.5 P_1 must be used	lb_f/in^2	N/m^2
P_3	Pressure increased caused by water-hammer	lb_f/in^2	N/m^2
Q	Flow rate	USGPM	liters/sec.
q	Density of fluid	lb_m/ft^3	kg/m^3
S	Maximum bending stress	lb_f/in^2	N/m^2
S_1	Strouhal number for pipes	$S_1 = 0.18$	-
t	Time	sec.	sec.
t_s	Thickness of vessel or stack	in	m
U	Wind velocity	ft/sec	m/sec
V	Flow velocity	ft/sec	m/sec
V_a	Accumulator capacity	US Gallons	liters
W	Weight	lb_f	N
W_c	Weight of covering (insulation) of pipe	lb_f/ft	N/m
W_f	Weight of concentrated load at center of span	lb_f	N
W_p	Weight of pipe	lb_f/ft	N/m
W_t	Total weight of piping $W_t = W_p - W_w - W_c$	lb_f/ft	N/m
W_w	Weight of water, or fluid being piped, whichever is heavier	lb_f/ft	N/m
W_y	Weight per ft. of pipe (including pipe contents and insulation)	lb_f/ft	N/m
w	Total weight of pipe	lb_f/ft	N/m
w_1	Weight of unit volume of water	lb_f/ft^3	N/m^3
w_2	Angular frequency	rad/sec	rad/sec

GREEK ALPHABET

SYMBOL	DESCRIPTION	BRITISH UNITS	S.I. UNITS
α	Constant, depending on the type of supports (Table 3)		
δ	Static deflection	in	m
δ_{st}	Static deflection of the spring under force F*	in	m
δ_{max},	Maximum dynamic deflection in the absence of damping	in	m
δ^*_{st}	Static deflection been independent of natural frequency together with F*; case when rotational speed is equal to undamped natural frequency.	in	m
θ	Angle of bend-change	rad	rad

CHAPTER 1 - INTRODUCTION

Pipe properties are characterized with respect to the axial coordinate of the system and are divided in two categories. Pipes with properties such as diameter, wall thickness and velocity of water-hammer wave which remain unchanged with the axial coordinate are said to have constant characteristics. On the other hand, pipes with properties which vary with length are said to have variable characteristics. Piping systems used in commercial refinery networks are typical examples of constant characteristic piping installations. Variable characteristic systems are usually found in hydropower plants.

Resonance phenomena in pipes having constant characteristics, have been studied by a great number of investigators [1, 3 to 10]. The literature on resonance in pipe systems begins with Allievi's work first published in 1903 [3]. He analyzed the increase of the pressure head caused by periodically opening and closing a valve situated at the downstream end of a pipeline and having a constant head reservoir at the upstream end.

A few years later Camichel, Eydoux and Gariel together with Neeser and Boucher [4], undertook experimental studies using laboratory pipes. They discovered that a pipe consisting of several sections with different diameters and wall thicknesses exhibited two distinct periods. These were designated as the Theoretical and Apparent periods.

The θth "Theoretical Period" [5] is equal to the sum of the periods of the pipe having sections L_1, L_2, L_3 ....

$$\theta = \frac{4L}{a_m} = \frac{4L_1}{a_1} + \frac{4L_2}{a_2} + \frac{4L_3}{a_3} + \dots\dots = \sum_i \frac{4L_i}{a_i}$$

where a_1, a_2, a_3 .... are the wave velocities in sections 1, 2, 3 ... $L = L_1 + L_2 + L_3$ ...., and a_m an average velocity. The "Apparent period" is the period corresponding to the lowest natural frequency of the complex piping system [3]. When a pipeline has a nonuniform diameter, then the period of the fundamental pressure variation is the apparent period [5].

Another result of this work, is the observation that harmonics are related to the theoretical period and that the odd harmonics lead to resonant conditions, regardless of whether the pipeline has a constant or varying diameter or wall thickness.

Using graphical techniques, Bergeron [11] showed that the maximum pressure head in a single pipeline during resonating conditions was twice the static head. Schnyder [12] demonstrated graphically the amplification of the pressure head oscillations on a leaking valve. Favre [2, 13] developed expressions for the period of the fundamental and the higher odd harmonics of a pipe having variable characteristics, with a reservoir of constant head at the upstream end, and a rhythmically opening and closing valve located at the downstream end. Papers written by de Sparre, Foch, Schnyder, Bergeron, et al, [5] dealt with specific problems related to resonance. A study by Jaeger [5] resulted in a method for calculating the fundamental frequency in a pipe with non-uniform diameter.

Despite the progress made in the analytical aspects of resonance in piping systems, the majority of industrial installations continue to be designed on the basis of a single degree of freedom system. The main reason for this is that this approach appears to be adequate in practice and very seldom does it become necessary to resort to the more accurate but extremely cumbersome techniques associated with the use of more complicated models. In addition, it has been found that it is more expedient to identify and correct resonance problems after installation through on-site measurements on the actual system, rather than attempt to eliminate them at the design stage. An experimental technique frequently used to identify resonance condition is the impedance method and this is discussed in some detail in the next section.

1.1 Impedance method [3]

In general, the term impedance is used to describe the ratio of output/input. For mechanical systems, impedance defines the ratio of the velocity response to the forcing function. Because of existing phase differences in these parameters, impedance is a complex quantity and for a parallel one degree of freedom damped system it is given by

$$Z = C + j\left[2\pi f_d m - \frac{K}{2\pi f_d}\right]$$

where

C = Damping Coefficient

f_d = Excitation frequency (Hz)

It is apparent from its definition that impedance varies with frequency and reaches a maximum when the system is in resonance. Resonance conditions arise at all natural frequencies and for a piping system with a constant head reservoir, it has been shown that these correspond to the fundamental and all odd harmonics [2, 13].

Impedance values decrease to almost zero in between the resonance zones. These areas are called antiresonances. Thus the behavior of Z with respect to the excitation frequency is a very useful tool in the analysis of piping system resonances, because the critical frequencies arising under operating conditions are clearly identified. Such diagrams can be obtained analytically [3] , or experimentally after the system has been installed, or from a scale model. Reference [3] gives typical impedance plots for a series system, a branch system and a parallel piping system. These diagrams, show the resonances and antiresonances as points of high impedance and low impedance respectively. The impedance method for studying resonance or natural frequencies in pressurized piping system as outlined in [3], consists of analyzing each harmonic at its own particular frequency. The results are then superimposed yielding the complete solution.

1.2 The Lac Blanc-Lac Noir Case

The importance of avoiding resonances within the operating range cannot be overemphasized. The ramifications of neglecting this phenomenon were illustrated in the Lac Blanc-Lac Noir installation. The layout of this development which is located in Vosges, France, is shown in Figure 1. It consists of a facility where a volume of 2,000,000 m^3 of water was drawn from Lac Blanc by four 40,000hp vertical shaft Francis turbines during the day and pumped back again during the night by three turbine driven pumps. While the plant was still being tested, a pipe section near the pumping station burst causing extensive damage to the installation and loss of life in the subsequent flood.

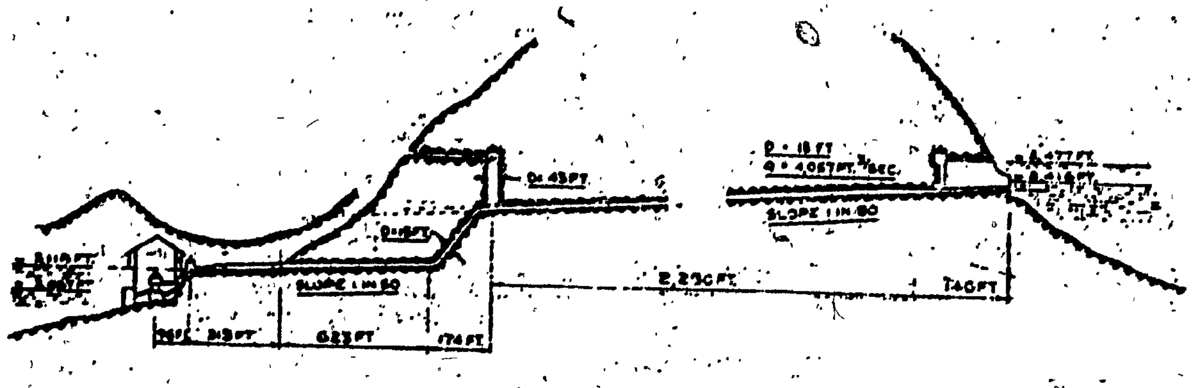

Fig. 1 The Lac Blanc-Lac Noir pumped storage scheme [5].

The rupture of the pipe was attributed to a resonance caused by a guide vane in the pump, which was assumed to have started vibrating [5] . Initially, it was suspected that the resonance was caused at the fundamental frequency of the pumps. However, after similar accidents occured at different power stations, it became apparent that resonance conditions were excited at the higher odd harmonics. These resonances were maintained through the mechanism of self excitation. Finally, a detailed impedance study of this installation [1, 14] confirmed this conclusion and further identified that chatter of gate valves, air valves, aerodynamic effects of guide vanes and pressure fluctuations in the flow head as other possible causes of resonance.

CHAPTER 2 - FUNDAMENTALS OF VIBRATION FOR PIPING SYSTEMS

2.1 Terminology

The field of piping vibrations is a specialized area of mechanical vibrations. It deals with the effect of various excitation forces due to operating machinery, fluid flows, wind, etc., on the piping network. It is concerned primarily with the design and installation of piping sections so that resonant conditions are eliminated altogether or wherever this is not practical, their effects are minimized to assure that the system functions satisfactorily. The definitions of terms and parameters used in such studies can be found in any standard reference on linear vibrations [15]. However, it would be useful at this point to review some of these terms as they apply to the design of piping system.

a) Forcing frequency

This refers to the frequency of any external force which may be present and hence influence the response of the piping system. Typically, such periodic forces have their origin in machinery elements, fluid flow pulsation and action of wind gusts. All these components possess frequencies in the 5-2000 Hz range.

b) Forced vibration

A system is said to undergo forced vibration if it exhibits a dynamic response as a result of an imposed dynamic excitation. Such excitation might arise from the imbalance of rotating machines, misalignment of shafts, defective antifriction bearings, worn gears or the acceleration of reciprocating masses.

In piping systems, forced vibration may also be caused by the periodic variation of fluid pressure and the effects of winds and pressure fluctuations.

A common cause of vibration in machines is due to the imbalance of the rotating parts. If the rotating speed is close or equal to the natural frequency of the structure (foundation, or connected piping) resonance will occur causing failure of the piping or other components.

Reciprocating compressors are another source of pressure excitation at a frequency (cps) equal to the speed (rps) of the compressor times the number of cylinders for simple action and twice the number of cylinders for double action compressors for each stage. When this frequency is close or equal to the acoustic frequency of the connected piping, vibration will occur in the piping system due to the acoustic resonance, which appears in the form of pressure pulsations. Vibration is then transmitted directly to the foundations, and buildings or indirectly through connecting piping systems to vessels and structures.

If again the natural frequency of the piping system is equal to the speed of the rotating machine or equal to the stroke frequency, as in the case of reciprocating devices, large amplitudes should be expected. This phenomena can be corrected by various methods such as by balancing of rotors, use of pulsation snubbers, etc.

The action of the wind is another source of periodic excitation of exposed piping systems. It is customary to consider that air hits the pipe at an angle of 90^o to the axis of the pipe. The frequency of the piping vibration due to wind effect can be found by the following equation [16] :

$f_w = S_1 U/Do = 0.18\ U/Do$ (1)[2] or

$f_w = 2.16\ U/Do$ (1[1])[3]

Insulation should be considered as part of the outside diameter of the pipe. Forces on piping are due to vortex motion around the cylinder and perpendicular to the direction of the wind. When f_w is in the neighborhood of a natural frequency of a piping system, resonance will occur resulting in large amplitudes of vibration, depending of course on the amount of damping available in the system.

Another source of piping vibration is water-hammer. This is defined as the pressure change in a closed conduit. When a liquid flows through a pipe there is a certain amount of energy in the liquid. By restricting the flow of the fluid, as in the case of a valve, the energy of the liquid is used to compress the liquid and stretch the pipe. Therefore, changes in pressure appear to produce a hammering effect. Mechanical equipment, such as centrifugal pumps or turbines are subjected to water-hammer. Water-hammer is very important since it can damage machines and piping systems and it will be discussed in more details in a later section.

c) Self-excited vibration

Self-excited vibration is self-governed vibration which gains its energy through its own periodic motions from external forces. In other words, the exciting force is a function of the velocity, the displacement or the acceleration of the mass of the system. In piping systems self-excited vibrations are caused as a result of the instabilities of flow, vortices, surging of compressors, etc.

Analysis of self-excited vibration is very difficult and therefore it is not usually included in the design stage. If such a problem is suspected, it is customarily dealt experimentally by carrying out measurements on a scale model or on the installed system itself.

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2 ( ) indicates number of equation in British units.

3 ([1]) indicates number of equation in SI units

2.2 Fluid Vibration in a rigid tube without friction

Fluid flow pulsations have been identified as one of the external energy sources which contribute to the vibration of piping networks. The salient points of the analysis of such vibration will be reviewed in this section. Figure 2 shows a constant diameter section open at both ends, containing a compressible fluid and having negligible wall friction.

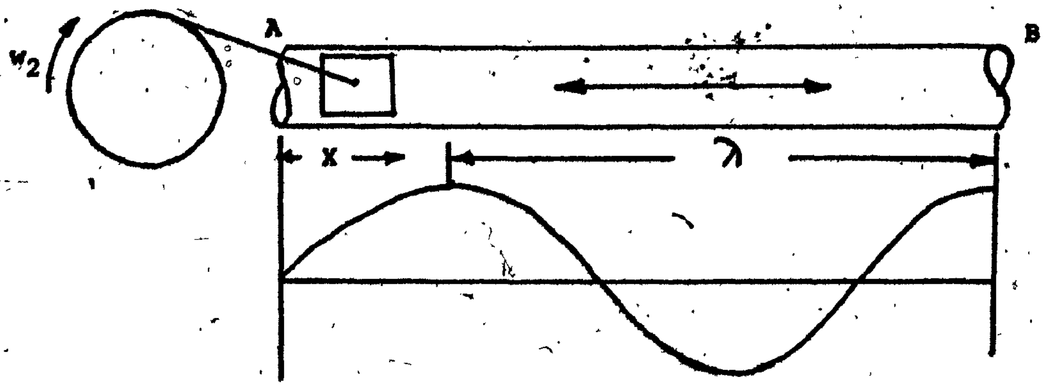


Fig. 2 Fluid Vibration in a rigid tube [17, 18].

Imagine a massless disk at one end A moving back and forth vibrating at a constant frequency w_2 radians per unit time. In this case, the vibrations are caused by the pulsation of the flow. For a continuous vibration of a disc at the same point A, a series of waves is generated at constant angular frequency w_2. At the other end of the pipe point B, the fluid vibrates and waves are reflected backwards and towards point A.

ΔP, the instantaneous excess pressure at any section of the tube is found to be equal to [17] :

$$\Delta P = qc \left[E\, e^{iw_2 (t - \frac{x}{c})} + G\, e^{iw_2 (t + \frac{x}{c})} \right]$$

where E and G are constants.

If the lowest angular frequency is w_o and the lowest natural frequency of the pipe is f_o then [19]

$$w_o = c\pi/L \qquad (2),\ (2^1)$$

$$f_o = c/2L \qquad (3),\ (3^1)$$

Therefore, for a given pipe length L and a given wave velocity c, (we consider only compressible fluids), a pipe has a certain characteristic f_o.

When the forcing or disturbing frequency f_d is equal to the natural frequency f_o, resonance occurs with large velocity amplitudes. When both ends are open, pipe sections with lengths given below should be avoided [18].

$$L = \frac{\pi c}{2w_2},\ \frac{3\pi c}{2w_2},\ \frac{5\pi c}{2w_2} \qquad (4),\ (4^1)$$

which corresponds to the series of values

$$L = \frac{1\lambda}{4},\ \frac{3\lambda}{4},\ \frac{5\lambda}{4} \qquad (5),\ (5^1)$$

where λ = wave length

w_2 = angular velocity = constant.

Finally, if in the same tube, point B is closed then the lowest natural frequency occurs when $w_2 L/c$ equals $\pi/2$ radians or when [17].

$$f_o = \frac{c}{4L} \qquad (6),\ (6^1)$$

The presence of friction complicates the equations considerably. For most practical applications, fluid friction can be ignored without incurring a significant error. However, if more accurate results are required, as may be the case for some piping runs where the length is restricted by the physical constraints of the layout, then the complete equations should be used [17].

CHAPTER 3 - VIBRATION PREVENTION AND CONTROL

Vibration control can be divided in two parts, vibration isolation and vibration damping.

1. Vibration isolation

Vibration isolation is defined as the reduction of the transmission of vibration from a machine to its foundation or supporting structure. In essence, vibration isolation is the protection of machinery from vibratory external forces transmitted to the equipment. The dynamic amplitude of the undamped single-degree-of-freedom mass-spring system subjected to a periodic force $F\cos ft$ can be found by the following equation [19] :

$$x = \frac{F/K}{1 - (f_d/f_n)^2} \cos ft \qquad (7) \quad \text{or}$$

$$x = \frac{0.0254\ (F/K)}{1 - (f_d/f_n)} \cos ft \qquad (7^1)$$

where F = External force amplitude,

K = Spring constant

f_d = Forcing frequency, rad/sec

f_n = Natural frequency, rad/sec

The corresponding force transmissibility equation is [15] .

$$\text{Force transmissibility, } F_t = \frac{1}{\left(\frac{f_d}{f_n}\right)^2 - 1} \qquad (8),\ (8^1)$$

The force at the base of a machine increases as the exciting frequency (f_d) approaches the resonant frequency (f_n), and reaches a maximum value when $f_d = f_n$ as shown in Fig. 3 [19] . By increasing the forcing frequency beyond this point, the force (or displacement) decreases up to the value of $f_d = \sqrt{2}f_n$. If the exciting frequency is larger than $\sqrt{2}f_n$, the force (or displacement) is less than that of the exciting force (or displacement).

Therefore, for an isolator to be effective in reducing the transmitted force, $f_d > \sqrt{2}\, f_n$ must apply.

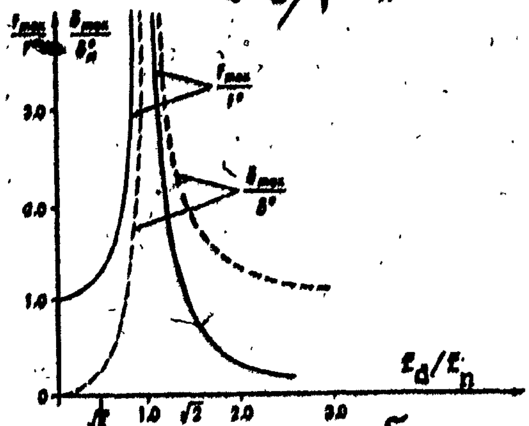


Fig. 3 F_{max}/F^* and δ_{max}/δ^* v.s f_d/f_n no damping [19].

Transmissibility can be defined in terms of the forcing frequency (f_d) and the static deflection (δ) by the following equation (9) or Fig. 4 [15].

$$T = \frac{1}{\frac{(2\pi f_d)^2}{g} - 1} \qquad (9) \quad \text{or}$$

$$T = \frac{0.0254\,(2\pi f_d)^2}{g} - 1 \qquad (9^1)$$

The above equation is applicable only to cases having vertical motion. For movements in other directions, the appropriate acceleration components must be used in deriving the corresponding equations.

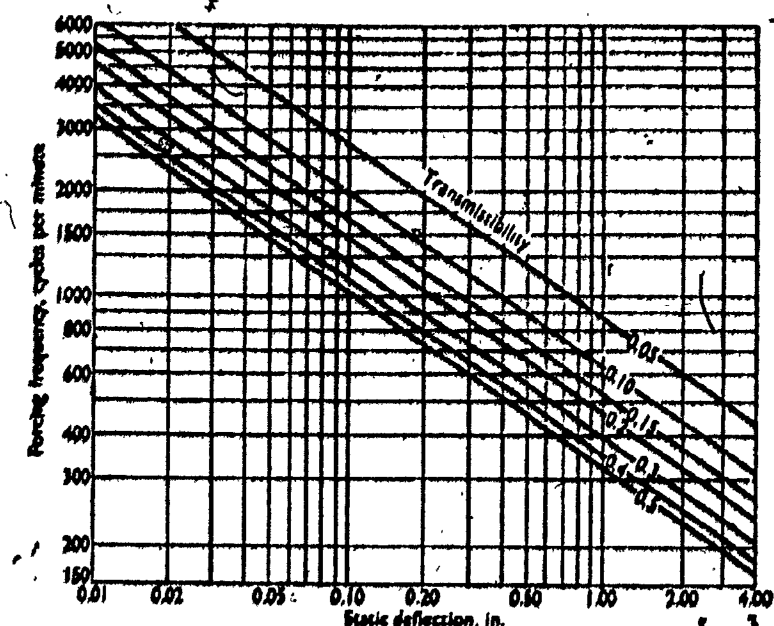


Fig. 4 Static deflection vs. Forcing frequency [19].

2. Vibration damping

Vibration damping is defined as the reduction of the air-borne noise radiated from a vibrating structure. In the case of solids, such as metals, the radiated energy is very high due to the fact that internal damping in metals is low. Therefore, the reduction of the radiated sound and hence the reduction of vibration, can be achieved by dissipation of the vibration energy in the form of heat in a damping device incorporated in the system. The procedure for selecting the proper damper is given in [19] and it will not be mentioned in this report.

The most desirable solution to a piping vibration problem is the isolation or elimination of the sources of vibration. The design engineer must be certain that the fundamental natural frequency of the piping system will not be close or equal to the forcing frequency. Gas pulsation smoothing and spacing of supports are the main elements available to the designer in eliminating resonant conditions. In order to avoid resonance, the forcing frequencies and the natural frequencies must be known. The forcing frequencies can be found from the characteristics of the rotating and reciprocating equipment, analysis of the fluid flow and an assessment of wind gusts. Methods of calculating piping natural frequencies are given in Section 3.4.

If the fundamental natural frequency is at the neighborhood of a forcing frequency, the designer should attempt to shift the natural frequencies of the piping system. This can be accomplished by changing the spacing of supports and guides, or, by using elastic supports which are quite common in refinery piping. The later method requires thermal expansion calculations; it is very popular and it will be considered in more detail later. If the natural frequency of the piping system still remains close or equal to the exciting force frequency, then other means for reducing vibration must be considered. These consist of elastic foundations, snubbers and shock absorbers.

There is no general analytical way of dealing with vibration problems in piping systems. However, some typical solutions for simple designs may be found in [16] . The most important aspects of Vibration prevention and control, will be outlined in the following sections.

3.1 Elastic Foundation for Centrifugal Machinery [16]

In practice, the majority of machinery foundation pads are not rigid enough to isolate completely the equipment from external vibration sources. The pad acts as an elastic body possessing certain stiffness and damping characteristics as shown in Fig. 5. If f_d (rad/sec) is the rotating speed of the machine then the centrifugal force F* (lb) which arises from the unbalanced rotor mass m(lb-sec^2/ft) acting at a radius r (ft), is given by

$$F^* = mrf_d^2 \qquad (10)$$

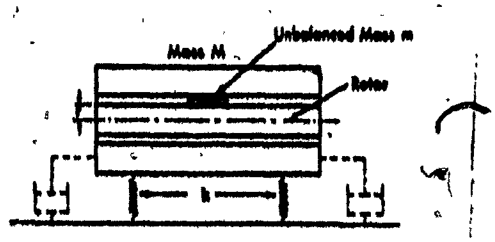


Fig. 5 Rotating machinery on elastic foundation [16] .

The corresponding static deflection (δ_{st}) of the spring with an equivalent striffness K (lb/ft) is

$$\delta_{st} = m\, r f_d^{\,2} / K \tag{11}$$

and the undamped natural frequency f_n (rad/sec) of the mass m_o (lb-sec^2/ft) is

$$f_n = \sqrt{K/m_o} \tag{12}$$

where $K = mf_n^{\,2}$ and $\delta_{st} = \left(\frac{m}{m_o}\right) r \left(\frac{f_d}{f_n}\right)^2 \quad (13)$

If the rotating speed is equal to the undamped natural frequency, then the static deflection and the force F* will be independent of the natural frequency.

$$\delta^*_{st} = \left(\frac{m}{m_o}\right) r \tag{14}$$

With no damping the maximum dynamic deflection is

$$\delta_{max} = \frac{\delta_{st}}{\left| 1 - \left(\frac{f_d}{f_n}\right)^2 \right|} \tag{15}$$

and the maximum force in the spring is

$$F_{max} = K\,\delta_{max} = m_o f_n^{\,2}\,\delta_{max} = \frac{m_o f_d^{\,2}\,\delta_{st}}{\left| 1 - \left(\frac{f_d}{f_n}\right)^2 \right|} \tag{16}$$

By substituting eq. 13 in eq. 15 and eq. 16 we get

$$\delta_{max} = \frac{m}{m_o}\, r\, \frac{\left(\frac{f_d}{f_n}\right)^2}{1 - \left(\frac{f_d}{f_n}\right)^2} \tag{17}$$

$$F_{max} = \frac{mrf_d^2}{\left|1 - \frac{f_d}{f_n}\right|} \tag{18}$$

and from eq. 14

$$\frac{\delta_{max}}{\delta^*_{st}} = \left|1 - \left(\frac{f_n}{f_d}\right)^2\right|^{-1} \tag{19}$$

$$\frac{F_{max}}{F^*} = \left|1 - \left(\frac{f_d}{f_n}\right)^2\right|^{-1} \tag{20}$$

The ratio F_{max}/F^* is defined as TRANSMISSIBILITY and it is shown in Fig. 3. It is apparent from this figure that minimizing the deflection, (f_d/f_n) becomes small and F_{max} tends to F^*.

If we require to reduce to a minimum the maximum force transmitted to the foundation through the spring, then (f_d/f_n) should be as high as possible, in which case $\delta_{max.}$ tends to δ^*_{st}. Further, $\frac{F_{max}}{F^*} < 1$ only when $\frac{f_d}{f_n} \geqslant 2$. It should be noted that the damping effects of the foundation pad have not been considered in this analysis. If the rotating speed of the machine is in the neighborhood of the frequency of the system and the ratio f_d/f_n cannot be changed, or if a range of rotational speeds is required while resonance must be avoided, then viscous damping (hydraulic shock absorbers etc.) must be used as shown by the dotted line in Fig. 5. The following equation gives the value of the ratio $\delta_{max}/\delta^*_{st}$ [16].

$$\frac{\delta_{max}}{\delta^*_{st}} = \sqrt{\left[1 - \left(\frac{f_n}{f_d}\right)\right]^2 + \left(2\frac{C}{C_c}\ \frac{f_n}{f_d}\right)^2} \tag{21}$$

Fig. 16 of Appendix C shows a plot of F_{max}/F^* (transmissibility) vs. f_d/f_n with C/C_c as a parameter. From this figure we can observe that for $f_d/f_n < \sqrt{2}$, the maximum force transmitted to the foundation (F_{max}) is greater than the centrifugal force F^*. In other words the magnification is greater than unity, which is the range where damping is beneficial.

In Fig. 17 of Appendix C, it can be seen that damping is always beneficial. The above equations and figures are based on a one degree spring - mass - dashpot system. They may also be applied to more complicated systems provided that the effective inertia and effective spring constant can be properly estimated.

3.2 Piping Supports and Guides

Supports and hangers must be designed to meet static and operational conditions to which the piping system is subjected. Their location is dependent on pipe size, pipe configuration, position of valves, fittings and the type of structure which will be used to support the piping.

The primary aspects in the design of hangers or supports is the selection of the span distance. This involves consideration of the pipe weight, magnitude of resulting bending moments and required rigidity to assure that no resonance arises and that the pipe does not rupture under static loads. In such calculations, each pipe span is considered as a uniformly loaded beam.

Chart 1 [16] of Appendix B is based on a maximum moment $M = \frac{1}{10} w\ell^2$, and represents a moment equal to $M = \frac{1}{12} w\ell^2$ for a beam having fixed ends, and a moment equal to $M = \frac{1}{8} w\ell^2$ for a beam having free ends. For pipes having end conditions different from the above, Table 2 [16] of Appendix A can be used with appropriate correction factors.

The following formulae give the maximum bending stress (S) and maximum deflection (δ) for two types of supports: (1) single span, free ends and (2) continuous beam based on the assumption that all concentrated loads (valves, etc) are located at the center of the span [20].

By knowing δ, the natural frequency (f_n) of the system can be calculated.

In British Units

	S	δ
Single span	$\frac{(0.75W_tL_s^2 + 1.5W_fL_s)D_o}{I}$	$\frac{22W_tL_s^4 + 36W_fL_s^3}{EI}$
Continuous straight line	$\frac{(0.5W_tL_s^2 + 0.75\ W_fL_s)Do}{I}$	$\frac{4.5W_tL_s^4 + 9W_fL_s^3}{EI}$

Or in SI Units

	S	δ
Single span	$\frac{(W_tL_s^2 + 1.99W_fL_s)Do}{1.61 \times 10^{-5}I}$	$\frac{W_tL_s^4 + 1.64\ W_fL_s^3}{1.035 \times 10^{-3}EI}$
Continuous straight line	$\frac{(W_tL_s^2 + 1.5\ W_fL_s)Do}{2.41 \times 10^{-5}I}$	$\frac{W_tL_s^4 + 1.99\ W_fL_s^3}{5.06 \times 10^{-2}\ EI}$

The symbols used in the above table are defined in the Nomenclature.

3.3 Water-Hammer

Water-hammer occurs when the velocity or pressure in the contents of a closed conduit is suddenly changed. As the liquid flows in such a conduit, the flow energy compresses the fluid and stretches the confining walls everytime there is a sudden restriction to the flow. Such restrictions or fluctuations may arise from the action of valves or the start up of compressors, pumps and other related equipment.

Water-hammer requires detailed analysis because it can easily give rise to large stresses which may cause damage to mechanical equipment and to structures associated with piping.

Water-hammer waves travel (theoretically) in water at a speed equal to 4660 fps (1420.37 m/s) [21]. The water-hammer wave velocity in systems filled with water can be calculated using the following general equations [21]:

$$a = \frac{1}{\frac{W_1}{g}\frac{1}{K} + \frac{D_i}{bE}} \qquad (22) \text{ or}$$

$$a = \frac{1}{\frac{.515\ W^1}{gK_1} + \frac{D_i}{6894Eb}} \qquad (22^1)$$

Similarly from [19]:

$$a = \frac{4720}{\sqrt{1 + \frac{K_2 D_i}{Eb}}} \qquad (23)$$

Where K_2 is a constant which takes different values, depending on the material of the piping.

Equation (22) can be modified to reflect the usage of steel pipes, cast iron and concrete pipes as follows:

For steel pipes [21] :

$$a_s = \frac{4660}{1 + \frac{D_i}{100b}} \qquad (24)$$

For cast iron-pipes [21] :

$$a_i = \frac{3290}{0.51 + \frac{D_i}{100b}} \qquad (25)$$

and for concrete pipes [21] :

$$a_c = \frac{1340}{0.085 + \frac{D_i}{100b}} \qquad (26)$$

Once the water hammer velocity (a) is found, the pressure rise (H) in feet or meters can be calculated using the following equation [19] .

$$H = \frac{V_a}{32.2} \qquad (27) \text{ or}$$

$$H = 2.88 \times 10^{-3}\ V_a \qquad (27^1)$$

And the time required for the pressure wave to travel a length ℓ (i.e., back to the pump from the valve where the water-hammer phenomenon was instigated) can be found by [19]

$$t = \frac{2\ell}{a} \quad (28),\ (28^1)$$

Finally, the velocity of the pressure wave can be found using chart 3 of Appendix B [21] . Note that the pressure rise increases with increasing flow velocity. Therefore, by changing the flow rate, that is, lowering the flow velocity, or increasing the pipe size, water-hammer velocity (a) decreases, and the corresponding head (H) decreases. Thus, in the design of a new piping system, a piping diameter giving low velocities should be selected. Location of the first valve after the discharge of the pump should be chosen in such a manner, so that, the frequency of the pressure wave (2t per sec.) do not occur at the neighborhood of the frequency of the pump or compressor.

For an existing piping system where changes to the size of pipes or relocation of the valves is difficult, other solutions must be found. In such cases, corrective action may consist of (1) installation of air-relief valves and (2) the introduction of flexible elements in the system. Air-relief valves are usually employed in large pipelines and they are intended to relieve air and water during a surge. Typical flexible elements are accumulators and shock absorbers. An empirical equation for sizing accumulators is [19] .

$$V_a = \frac{0.004QP_1 (0.005\ell - t)}{P_2 - P_1} \quad (29) \text{ or}$$

$$V_a = \frac{0.00025QP_1 (0.005\ell - t)}{P_2 - P_1} \quad (29^1)$$

3.4 Structural Natural Frequency Calculations

In order to control and identify vibration, the designer must know the natural frequencies of the system. Natural frequencies of straight lengths of piping with various end conditions can be easily computed [16, 22 & 23] . Some of the most common arrangements are given below:

For a simple spring-mass system, Fig. 6 the natural frequency is given by the following equation [16] .

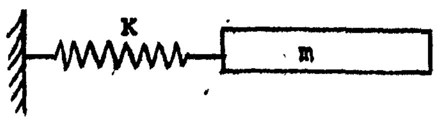


Fig. 6 Mass-Spring System [16] .

$$f_n = \frac{1}{2\pi}\sqrt{\frac{K}{m}} \qquad (30) \text{ or}$$

$$f_n = 3.127\sqrt{\frac{K}{m}} \qquad (30^1)$$

In piping systems m represents the mass of the section of the pipe between supports and K is the spring stiffness of the pipe hanger or support.

By using the static deflection δ, frequency can also be found from [16] .

$$f_n = \frac{1}{2\pi}\sqrt{\frac{g}{\delta}} \qquad (31) \text{ or}$$

$$f_n = \frac{1}{2\pi}\sqrt{\frac{g}{0.0254\delta}} \qquad (31^1)$$

In the case of a cantilever beam as in Fig. 7

$$\delta = 48 \frac{w\ell^2}{EI} \qquad (32) \text{ or}$$

$$\delta = 2107.27 \sqrt{\frac{w\ell^3}{EI}} \qquad (32^1)$$

and $$f_n = 0.906 \sqrt{\frac{EI}{48W\ell^3}} = 0.130 \sqrt{\frac{EI}{W\ell^3}} = 0.13 \sqrt{\frac{EI}{WL^3}} \qquad (33) \text{ or}$$

$$f_n = 0.0197 \sqrt{\frac{EI}{W\ell^3}} = 0.00283 \sqrt{\frac{EI}{WL^3}} \qquad (33^1)$$

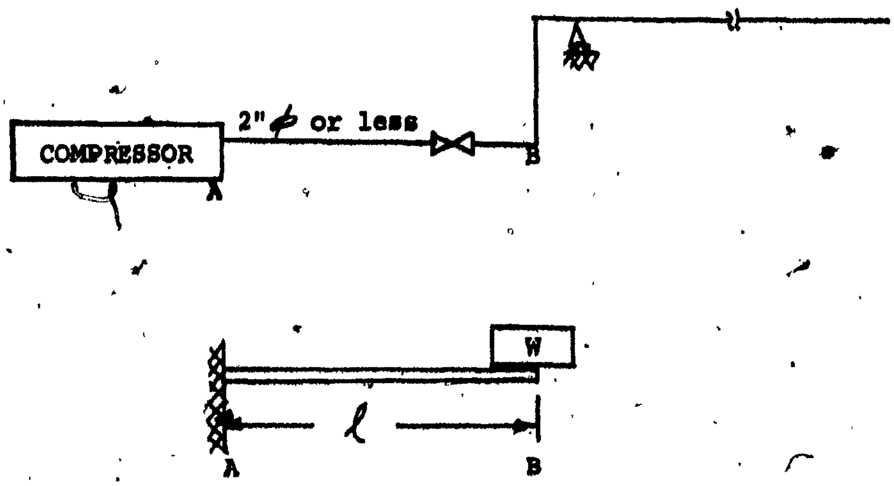


Fig. 7. Cantilever beam with a concentrated weight [16].

If the weight of the pipe is significant (i.e., large diameter pipes) then the weight is considered to be uniformly distributed as shown in Fig. 8 and the following equation can be used [16].

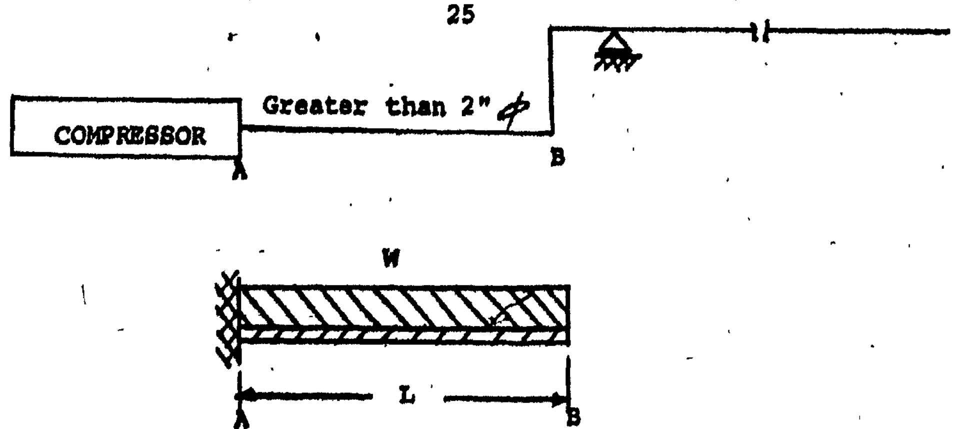


Fig. 8. Cantilever beam with distributed weight [16].

$$f_n = \alpha\sqrt{\frac{EI}{WL^3}} = \frac{\alpha}{L^2}\sqrt{\frac{EI}{W_y}} \quad (34) \text{ or}$$

$$f_n = 0.151\,\alpha\sqrt{\frac{EI}{WL^3}} = \frac{0.151\,\alpha}{L^2}\sqrt{\frac{EI}{W_y}} \quad (34^1)$$

Values of W_y, I and E can be found in Table 1 of Appendix A. Table 3 of Appendix A gives values for α for different types of supports.

Therefore, for a massless cantilever beam having a concentrated weight, the natural frequency is given by equation (36), while for the same beam having similar length but distributed load equation (35) holds.

Table 3 of Appendix A is valid for distributed loads and thus it is applicable to all piping systems neglecting concentrated loads as valves, etc.

Figure 9 shows a cantilever having a total weight W uniformly distributed and a concentrated end load [16].

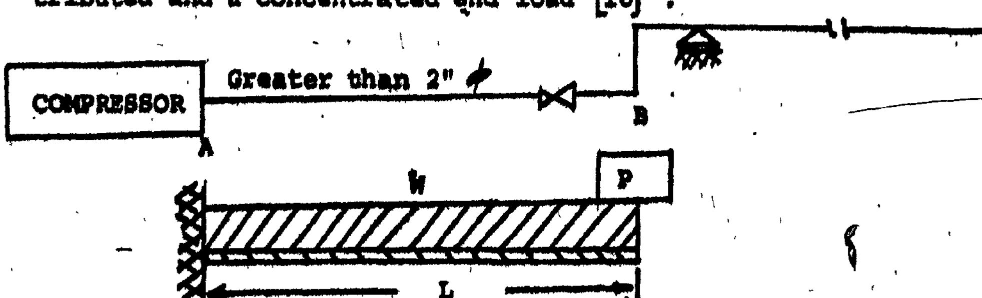


Fig. 9 Cantilever beam with concentrated and distributed weight [16].

Then

$$f_n = 0.13\sqrt{\frac{EI}{(\frac{1}{4}W + P)L^3}} = \frac{0.13}{L^2}\sqrt{\frac{EI}{\frac{1}{4}W_y + \frac{P}{L}}} \qquad (35) \text{ or}$$

$$f_n = 0.0196\sqrt{\frac{EI}{(\frac{1}{4}W + P)L^3}} = \frac{0.0392}{L^2}\sqrt{\frac{EI}{W_y + \frac{3.99P}{L}}} \qquad (35^1)$$

The natural frequency for the cantilever shown in Fig. 10 can be found by the equation below [16].

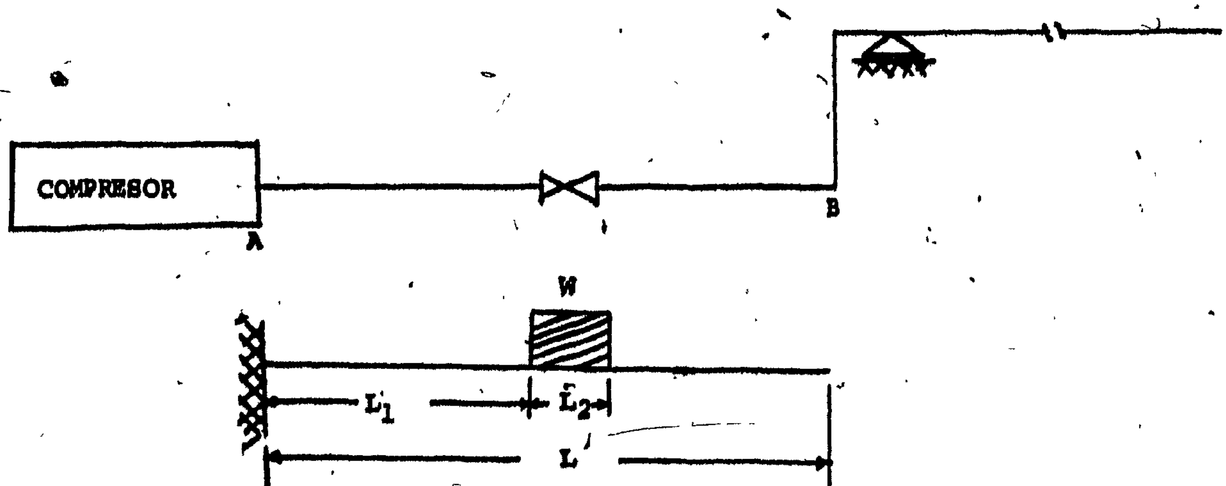


Fig. 10 Cantilever beam with concentrated weight [16].

$$f_n = 0.13\sqrt{\frac{EI}{W_{eff}\,L^3}} \qquad (36) \text{ or}$$

$$f_n = 0.0196\sqrt{\frac{EI}{W_{eff}L^3}} \qquad (36^1)$$

where $W_{eff} = 4.448W\left(\frac{L_1}{L}\right)^3$

The following equations summarize the results for the first mode natural frequency calculations [16].

<u>Cantilever</u>

$$f_n = 0.13\sqrt{\frac{EI}{(\frac{1}{4}W + P)L^3}} = \frac{0.13}{L^2}\sqrt{\frac{EI}{\frac{1}{4}W_y + \frac{P}{L}}} \qquad (37) \text{ or}$$

$$f_n = 0.0196\sqrt{\frac{EI}{(\frac{1}{4}W + P)L^3}} \quad \frac{0.0392}{L^2}\sqrt{\frac{EI}{W_y + \frac{3.99P}{L}}} \qquad (37^1)$$

<u>Simply Supported Beam</u> (pipe between two supports)

$$f_n = 0.525\sqrt{\frac{EI}{(\frac{1}{2}W + P)L^3}} = \frac{0.525}{L^2}\sqrt{\frac{EI}{\frac{1}{2}W_y + \frac{P}{2}}} \qquad (38) \text{ or}$$

$$f_n = 0.0792\sqrt{\frac{EI}{(\frac{1}{2}W + 4.448P)L^3}} = \frac{0.11189}{L^2}\sqrt{\frac{EI}{W_y + \frac{2P}{L}}} \qquad (38^1)$$

<u>Fixed end beam</u> (pipe between rigid anchors)

$$f_n = 1.03\sqrt{\frac{EI}{(\frac{3}{8}W + P)L^3}} = \frac{1.03}{L^2}\sqrt{\frac{EI}{\frac{3}{8}W_y + \frac{P}{L}}} \qquad (39) \text{ or}$$

$$f_n = 0.155\sqrt{\frac{EI}{(\frac{3}{8}W + P)L^3}} = \frac{0.155}{L^2}\sqrt{\frac{EI}{\frac{3}{8}W_y + \frac{P}{L}}} \qquad (39^1)$$

For a piping system similar to Fig. 11, having both pipe segments with the same diameter, material and schedule, the fundamental natural frequency is equal to [16] :

$$f_n = \alpha\sqrt{\frac{EI}{W_t L^3}} \qquad (40) \text{ or}$$

$$f_n = 0.0266\,\alpha\sqrt{\frac{EI}{W_t L^3}} \qquad (40^1)$$

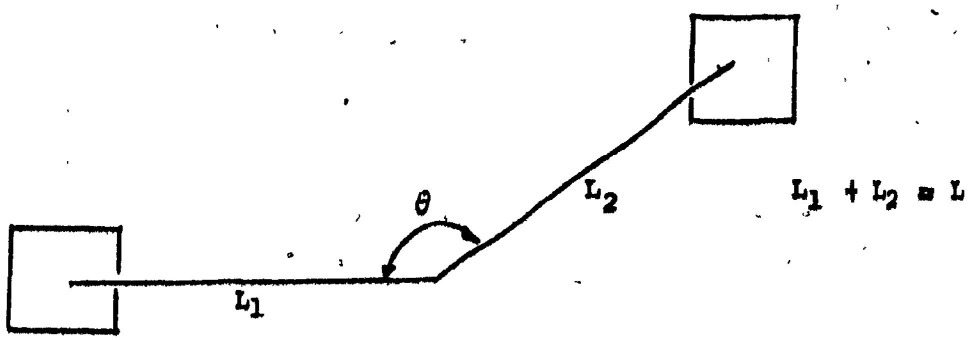


Fig. 11 Piping system with equal pipe lengths [16].

The following example will illustrate the use of eq. (40).

(1) $L_2 = 0$ or $L_1 = L$

then $\alpha = 1.69$

(2) $L_1 = L_2 = \frac{1}{2} L$

In such a case $L_1 = L_2$ if $\theta = 0$ and bend reduces to two parallel cantilever, then $\alpha = 1.06$

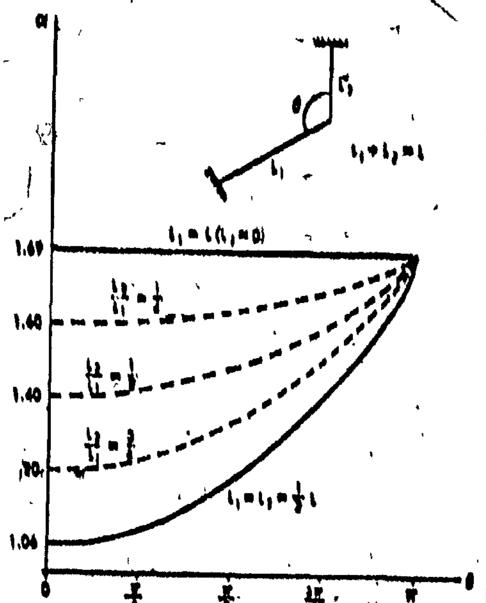
(3) $\theta = \pi$ then $\alpha = 1.69$

(4) $\theta = \frac{\pi}{2}$ then $\alpha = 1.2$

Therefore, for bends of equal lengths, the relation between θ and α is as follows [16].

θ	0	$\frac{\pi}{2}$	π
η	1.06	1.2	1.69

For unequal lengths of piping having the same material and thickness, the following Fig. 12 [16] can be used, where, depending on θ and the ratio L_2/L_1, the value of α can be found. Equation (40) gives the natural frequency of such a piping system with acceptable accuracy.



$$f(\text{cps}) = \alpha\sqrt{\frac{EI}{WL^3}}$$

Fig. 12 Relationship between the frequency coefficient (α) and the angle of bend-change θ [16].

Finally, it is interesting to note another type of frequency occuring on large stacks at certain wind speeds. This phenomenon is known as ovalling frequency and it does not arise in ordinary piping systems since ovalling frequencies are very high. Therefore, for a large steel vessel or a stack, the natural frequency is equal to [16].

$$f_n = \frac{50,000\ t_s}{D_s} \qquad (41) \text{ or}$$

$$f_n = \frac{1,968,504\ t_s}{D_s} \qquad (41^1)$$

Considering the cantilever beam of Fig. 13 and

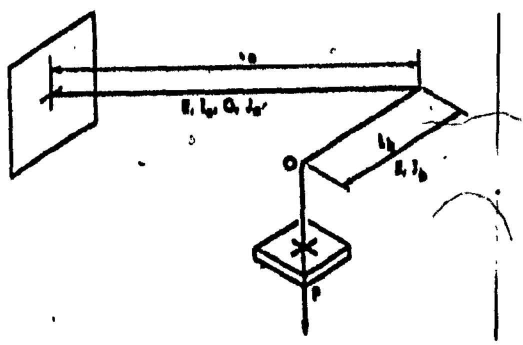


Fig. 13 Configuration of bending tortion [16].

neglecting the mass of the pipe the natural frequency can be found by [16].

$$f_n = 0.13 \sqrt{\frac{EI_b/W_{eff}\ L_b^3}{1 + \frac{I_b}{I_a}\left(\frac{L_a}{L_b}\right)^3 + 3\ \frac{EI_b}{GJ_a}\ \frac{L_a}{l_b}}} \qquad (42) \text{ or}$$

$$f_n = 0.0196 \sqrt{\frac{EI_b/W_{eff}\ L_b^3}{1 + \frac{I_b}{I_a}\left(\frac{L_a}{L_b}\right)^3 + 3\ \frac{EI_b}{GJ_a}\ \frac{L_a}{L_b}}} \qquad (42^1)$$

where $W_{eff} = \frac{1}{4} W_a + \frac{1}{2} W_b + P$

W_a and W_b are the weights of lengths L_a and L_b.

CHAPTER 4 - ILLUSTRATIVE EXAMPLES

4.1 Vibration Analysis of Piping System

The configurations of Figs. 14 and 15 are selected to illustrate a possible practical application of the analysis described previously.

Given: Compressor characteristics

Single stage, double acting, steam turbine driven.

Total weight		=	10,000 lb
Pressure:	Inlet	=	500 to 600 psia
	Discharge	=	1300 psia
Temperature:	Inlet	=	75°F
	Discharge	=	300°F

Load	50%	75%	100%
Speed	100	160	220
Piston displacement c.f.m.	17	30	42

Required: Natural frequencies of piping system.

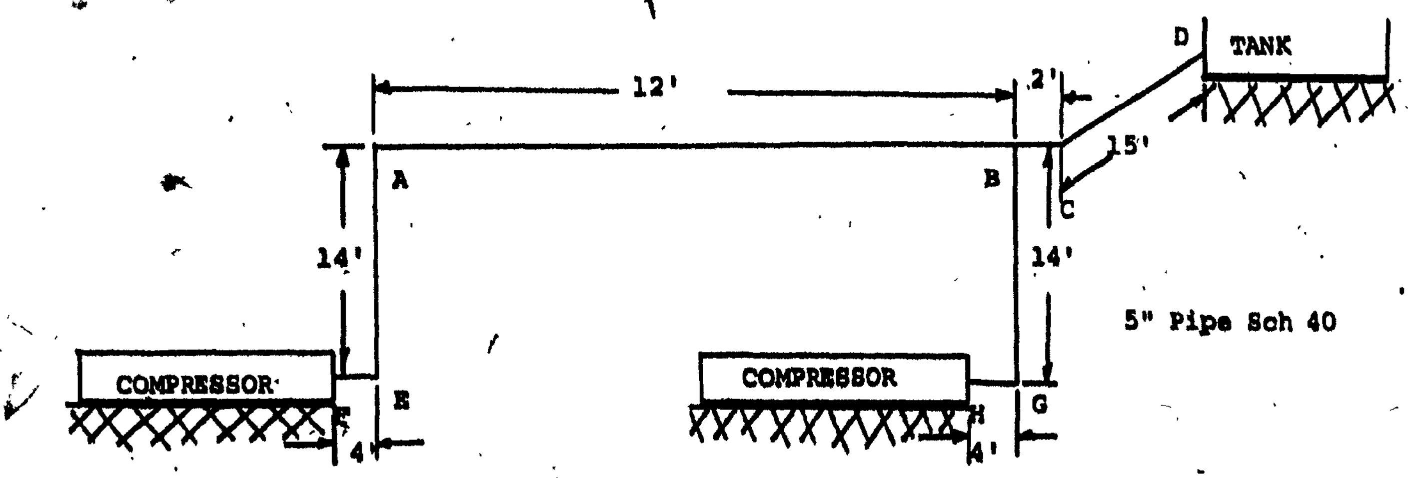


Fig. 14 Piping System with two compressors and a storage tank [16].

Solution

Moment of inertia = $15.2\ in^4$ (Table 1 of Appendix A for 5" pipe SCH 40)

$EI = 15.2 \times 29 \times 10^6 = 441 \times 10^6\ lb - in^2$

Weight of pipe = 14.6 lb/ft (Table 1)

Natural frequencies of piping system

Pipe A-B

$W = 12 \times 14.6 = 175.2$ lb

$L = 12$ ft

Case 1: Fixed ends A and B

$$f_n = \alpha \sqrt{\frac{EI}{WL^3}} \qquad \text{(eq. 34)}$$

Where: $\alpha = 1.69$ (Table 3)

Therefore:

$$f_n = 1.69 \sqrt{\frac{441 \times 10^6}{175.2 \times 12^3}} = 64.5 \text{ cps or } 3870 \text{ cpm.}$$

If $f_n \leqslant 30$ cps then f_n can be found from Chart 1 of App. B

Case 2: Simply supported ends at A and B

$$f_n = \alpha \sqrt{\frac{EI}{WL^3}}$$

Where: $\alpha = 0.743$

Therefore:

$$f_n = 0.743 \sqrt{\frac{441 \times 10^6}{175.2 \times 12^3}} = 28.35 \text{ cps or } 1701 \text{ cpm}$$

Pipe B - C - D (90° bend)

$L = 2 + 15 = 17$ ft

$\frac{L_1}{L} = \frac{15}{17} = 0.882$

$W = 17 \times 14.6 = 248.2$ lb

$\alpha = 1.25$ (Fig. 12 for $\theta = \pi/2$ and $\frac{L_1}{L} = 0.882$).

$f_n = 1.25 \sqrt{\frac{441 \times 10^6}{248.2\ (17)^3}} = 23.77$ cps or 1426 cpm

Pipe A - E - F or B - G - H (90° bend)

$L = 14 + 4 = 18$ ft

$W = 18 \times 14.6 = 262.8$ lb

$L_1/L = 14/18 = 0.778$

$\alpha = 1.3$ (Fig. 12)

$f_n = 1.3 \sqrt{\frac{441 \times 10^6}{262.8 \times (18)^3}} = 22.1$ cps or 1326 cpm

E (or G) Fixed and A - B - C Flexible

Flexibility could arise if A-E were very long and the diameter of the pipe was 10" or greater. In such a case the pipe should be anchored to the ground. Pipe A-E acts as cantilever with a uniform mass distribution of 14.6 lb/ft and a concentrated weight at the free end equals to 313.9 lb.

$W = (12 + 2 + \frac{15}{2}) \times 14.6 = 313.9$ lb

then

$f_n = 0.13 \sqrt{\frac{EI}{(\frac{1}{4} W + P) L^3}}$ (eg. 35)

$= 0.13 \sqrt{\frac{441 \times 10^6}{(\frac{1}{4} (14 \times 14.6) + 319.9)\ 14^3}} = 2.7$ cps or 162 cpm

E - F (or G - H) to be considered flexible

Then

$$f_n = 0.13 \sqrt{\frac{EI_b / W_{eff} \; L_b^3}{1 + \frac{I_b}{I_a}\left(\frac{L_a}{L_b}\right)^3 + 3 \frac{EI_b}{GJ_a} \; \frac{L_a}{L_b}}} \qquad \text{(eg. 42)}$$

$$W_{eff} = \frac{1}{4} W_a + \frac{1}{2} W_b + P$$

$$= \frac{1}{4} \times 4 \times 14.6 + \frac{1}{2} \times 8 \times 14.6 + 313.9 = 386.9$$

$$f_n = 0.13 \sqrt{\frac{441 \times 10^6 / 386.9 \times (14)^3}{1 + \left(\frac{4}{14}\right)^3 + 3 \times \frac{4}{14}}} = 1.93 \text{ cps or } 115.8 \text{ cpm}$$

Thus, in all cases the natural frequencies of the pipe or bends are higher or lower than the speed of the compressor, and the only possibility for resonance is when the compressor is running at 160 rpm (75% load), and point E (or G) is fixed while A - B - C are flexible. Therefore, if point E (or G) is to be fixed then points A - B - C should not be flexible.

A summary of the natural frequencies of individual points are shown in the following table.

	f_n (cps)	f_n (cpm)
A - B fixed	64.5	3870
B - B simply supported	28.35	1701
A - E - F (or B - G - H) 90° bend	21.1	1326
E (or G) fixed; A - B - C flexible	2.7	162
E - F (or G - H) are considered flexible	1.93	115.8

Compressor's speed 100 rpm

160 rpm

220 rpm

4.2 Effect of Elasticity of Machine Foundation

From the compressor characteristics we see that the lowest rpm of the compressor is 100 rpm or 1.67 rps. From eq. 20 or Fig. 3 it is seen that in order that F* (periodic force on the foundation due to unbalanced mass) be reduced to a minimum, say 0.2 of the F_{max}. (centrifugal force itself or dynamic force in the spring), with no presence of damping, the ratio of the forcing frequency to natural frequency (of the machine on its foundation = f_d / f_n) must be at least 3 to 1. Fig. 16 shows that if $\frac{f_d}{f_m} > \sqrt{2}$, then by increasing damping ratio $\frac{C}{C_c}$ (C can vary only, C_c is defined as critical damping and belongs to a particular system) F* increases rather than decreasing. By knowing the frequency ratio $\frac{f_d}{f_n}$ say 3, and f_d = 1.67 cps (100 cpm of the rotating machinery), the frequency of the machine and elastic foundation can be found to be equal to

$$f_n = \frac{f_d}{3} = \frac{1.67}{3} = 0.557 \text{ cps.}$$

Then from eq. 31 the static deflection can be found

$$\delta = \frac{g}{(2\pi)^2} \frac{1}{f_n^2} = \frac{32.2}{(6.28)^2} \frac{1}{(0.557)^2} = 2.63 \text{ ft}$$

This shows that such a soft foundation is impractical. Fig. 18 of Appendix C can also be used to get δ when f_n is known. Looking at Fig. 5, $\frac{f_d}{f_n}$ = 1 should be avoided since resonance will occur, unless damping is used.

At 100% load the speed of the compressor is 220 rpm or 3.67 rps. Then, if $\frac{f_d}{f_n} = \frac{1}{3}$ the natural frequency of the machine and foundation should be equal or greater to

$$f_n = 3 \times f_d = 3 \times 3.67 = 11 \text{ cps.}$$

and $$\delta = \frac{g}{(2\pi)^2} \frac{1}{f_n^2} = \frac{32.2}{(6.28)^2} \frac{1}{(11)^2} = 0.00674 \text{ ft or } 0.08 \text{ in.}$$

From this example it can be concluded that for low rpm, elastic foundations are not practical. Therefore, foundation should be as rigid as possible (when considering $\frac{f_d}{f_n} \gg 3$) while at the same time any unbalance of rotating parts must be reduced to a minimum.

4.3 Resonance Effects due to Wind Velocity

The frequency of the system due to wind velocity can be found by using eg. 1

$$f_w = 0.18 \frac{U}{D_o} = 0.18 \frac{U}{5/12} = 0.432 \; U$$

or

$$U = \frac{f_w}{0.432} = 2.31 \; f_w$$

From the calculated natural frequencies of individual points, a range of probable fundamental frequencies is chosen out of the neighborhood of the natural frequencies of the system.

Lower natural frequency = 22 cps
Higher natural frequency = 64 cps

Therefore, the corresponding wind velocities are:

Minimum U = 2.31 x 22 = 50.82 ft/sec or 34.65 miles/hr
Maximum U = 2.31 x 64 = 147.84 ft/sec or 100.79 miles/hr

Thus, at a range of wind velocity of 35 to 100 miles per hour the piping possibly will vibrate but not at a resonant frequency.

4.4 Water-Hammer Effects in Liquid Pipelines

Given: Water at 250 psi

10" SCH 40 steel pipe

$Q = 3{,}000$ gpm

$\ell_{total} = 1{,}000$ ft

Gate valve closes at 5 sec.

Required:

1. Velocity of the pressure wave

Eq. 23 gives

$$a = \frac{4720}{L + \frac{K_2 D_i}{E_b}} = \frac{4{,}720}{1 + \frac{300{,}000 \times \frac{10.020}{12}}{30 \times 10^6 \times \frac{0.365}{12}}} = 3703.4 \text{ fps.}$$

Chart 3 of Appendix B gives $a = 3700$ fps

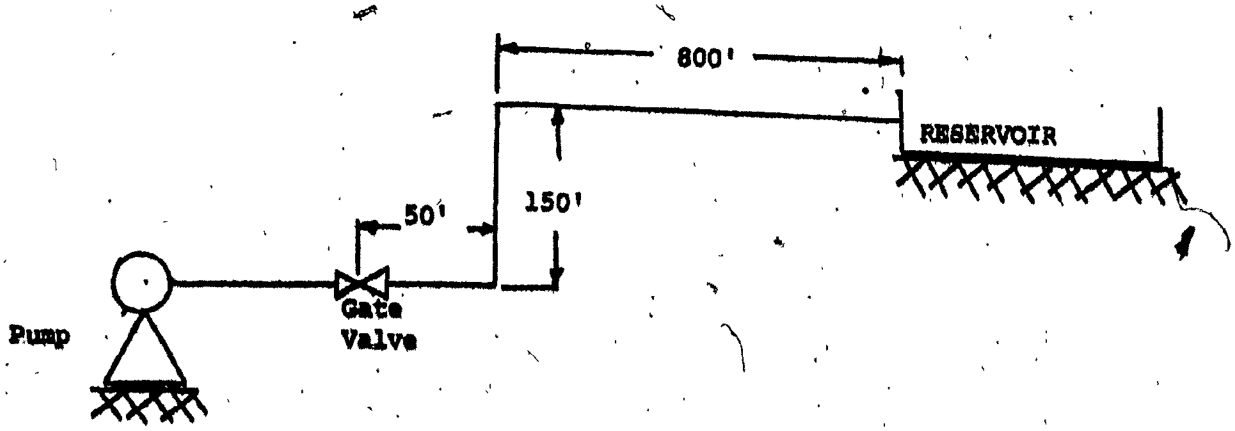


Fig. 15. Piping system with pump and reservoir.

2. Pressure increase caused by water-hammer

$$H = \frac{V_a}{32.2} \qquad \text{(eq. 27)}$$

or

$$P_3 = \frac{V_a}{32.2 \times 2.31} \qquad \text{where}$$

$$V = \frac{0.4085\ (\text{gpm})}{(D_i/12)^2} = \frac{0.4085 \times 3{,}000}{\left(\frac{10.020 \times 12}{12}\right)^2} = 12.21 \text{ fps}$$

then

$$P_3 = \frac{3703.4 \times 12.21}{32.2 \times 2.31} = 608 \text{ psi}$$

Therefore, the maximum pressure developed in the pipes is 250 + 608 = 858 psi

To check the pipe wall thickness for 858 psi the following equation is used [24]

$$b = \left[\frac{\frac{D_o P_1}{12}}{2(S - YP)} + C \right] /12$$

where, valves for S, Y and C are given in the code for Pressure Piping in the section on Power Piping

$$b = \left[\frac{\frac{10.750 \times 858}{12}}{2(13{,}750 + (0.4 \times 858)} + 0.065 \right] /12 =$$

$$= 0.007 < 0.365$$

In this case the pressure build up of 608 psi is below the maximum value for this particular pipe.

3. Water-hammer pressure rise caused by valve closure

The rise of water hammer pressure P_u (psi) caused by the closure of the valve is found by [24]

$$P_u = \frac{2 P_3 \ell}{a t} = \frac{2 \times 608 \times 1000}{3703.4 \times 5} = 66 \text{ psi}$$

The same results can be found by using Chart 2 of Appendix B. From this chart P_u is found to be 60 psi. Therefore, the maximum pressure in the pipe is 250 + 66 = 316 psi.

CONCLUSIONS

The primary objective of this paper is to review and assess current industrial techniques of controlling vibration in piping systems. The majority of these methods involve the evaluation of resonant frequencies by modelling the piping layouts as linear single-degree-of-freedom systems. Based on the one-degree-of-freedom approximation, a series of tables and charts in both British and SI units have been developed to facilitate the calculations. Because of the complexity of calculating exact natural frequencies and deflections for the actual configurations which are used in practice, the major effort in industrial vibration control is expended in undertaking corrective measures after the installation is completed.

At the design stage, consideration is given only to the known excitation factors, such as machinery, foundation characteristics, nominal pipe length and their interactions with the expected or known natural frequencies.

The magnitude of the resonance amplitudes attained is directly related to the amount of energy developed by the exciting element. Because of inherent damping, a minimum of excitation energy level is required to instigate resonance. The amount of damping available in piping networks to suppress resonant amplitudes is directly proportional to the installed piping length.

Another method of minimising resonant amplitudes is direct reduction of energy levels generated by all associated equipment present in piping systems, such as, operating machinery, control valves, guide vanes, water-hammer and wind effects. This is accomplished through the use of dampers.

In the design of piping systems not only the fundamental frequency should be considered but the higher harmonics as well. Detailed analytical studies utilising more reliable models clearly demonstrate the high probability of a particular group of harmonics which is capable of exciting resonance conditions. Experience shows that even the eleventh harmonic possesses adequate energy to excite resonance as reported in the kandergrund tunnel accident [5] and the Lac Blanc - Lac Noir case [5]. The use of charts and tables contained in the Appendices in the solution of common piping vibration problems is further illustrated in several examples discussed in Section 4.

APPENDIX A

TABLE 1

Properties and Weights of Pipe [16]

Nominal Size Outside Diameter inches D	Weight Designation and/or Schedule Number			Average Wall Thickness inches t	Minimum Wall Thickness ($= \frac{7}{8}t$) inches t_m	Inside Diameter inches d	Cross-Sectional Metal Area square inches A	Moment of Inertia inches^4 I	Section Modulus inches^3 Z	Bend Characteristic per Unit Bend Radius 1/ft h/R	Radius of Gyration inches r_g	Weight of Pipe w_p lb per ft	Weight of Water w_w lb per ft
1/8″ 0.405			10S	0.049	0.043	0.307	0.055	0.0009	0.0043	18.0	0.127	0.186	0.032
	Std.	40	40S	0.068	0.060	0.269	0.072	0.0011	0.0053	28.7	0.122	0.245	0.025
	XS	80	80S	0.095	0.083	0.215	0.093	0.0012	0.0060	47.5	0.115	0.315	0.016
1/4″ 0.540			10S	0.065	0.057	0.410	0.097	0.0028	0.0103	13.8	0.169	0.330	0.057
	Std.	40	40S	0.088	0.077	0.364	0.125	0.0033	0.0123	20.7	0.163	0.425	0.045
	XS	80	80S	0.119	0.104	0.302	0.157	0.0038	0.0140	33.2	0.155	0.535	0.031
3/8″ 0.675			10S	0.065	0.057	0.545	0.124	0.0059	0.0174	8.38	0.217	0.423	0.101
	Std.	40	40S	0.091	0.080	0.493	0.167	0.0073	0.0216	12.81	0.209	0.568	0.083
	XS	80	80S	0.126	0.110	0.423	0.217	0.0086	0.0255	20.1	0.199	0.739	0.061
1/2″ 0.840			10S	0.083	0.073	0.674	0.197	0.0143	0.0341	6.95	0.269	0.671	0.154
	Std.	40	40S	0.109	0.095	0.622	0.250	0.0171	0.0407	9.79	0.261	0.851	0.132
	XS	80	80S	0.147	0.129	0.546	0.320	0.0201	0.0478	14.7	0.250	1.09	0.101
		160		0.187	0.164	0.466	0.384	0.0221	0.0527	21.1	0.240	1.30	0.074
	XXS			0.294	0.258	0.252	0.504	0.0243	0.0577	47.3	0.219	1.72	0.022
3/4″ 1.050			5S	0.065	0.057	0.920	0.201	0.0245	0.0467	3.92	0.349	0.684	0.288
			10S	0.083	0.073	0.884	0.252	0.0297	0.0566	4.96	0.343	0.857	0.266
	Std.	40	40S	0.113	0.099	0.824	0.333	0.0370	0.0706	6.18	0.334	1.13	0.231
	XS	80	80S	0.154	0.135	0.742	0.434	0.0448	0.0853	9.21	0.321	1.47	0.187
		160		0.218	0.191	0.614	0.570	0.0527	0.100	15.1	0.304	1.94	0.128
	XXS			0.308	0.270	0.434	0.718	0.0579	0.110	26.0	0.284	2.44	0.064
1″ 1.315			5S	0.065	0.057	1.185	0.255	0.0500	0.076	2.00	0.443	0.868	0.478
			10S	0.109	0.095	1.097	0.413	0.0757	0.115	3.60	0.428	1.40	0.409
	Std.	40	40S	0.133	0.116	1.049	0.494	0.0874	0.133	4.57	0.420	1.68	0.374
	XS	80	80S	0.179	0.157	0.957	0.639	0.106	0.161	6.66	0.407	2.17	0.311
		160		0.250	0.219	0.815	0.836	0.125	0.190	10.38	0.387	2.84	0.226
	XXS			0.358	0.313	0.599	1.08	0.141	0.214	18.76	0.361	3.66	0.122

Properties and Weights of Pipe — Continued

Nominal Size Outside Diameter, inches D	Weight Designation and/or Schedule Number			Average Wall Thickness, inches t	Minimum Wall Thickness (= 7/8 t), inches t_m	Inside Diameter, inches d	Cross-Sectional Metal Area, square inches A	Moment of Inertia, inches⁴ I	Section Modulus, inches³ Z	Bend Characteristic per Unit Bend Radius, 1/ft h/R	Radius of Gyration, inches r_g	Weight of Pipe, w_p lb per ft	Weight of Water, w_w lb per ft
1¼″ 1.660			5S	0.065	0.057	1.530	0.33	0.104	0.125	1.93	0.56	1.11	0.80
			10S	0.109	[illegible]	1.442	0.53	0.161	0.193	2.17	0.55	1.81	0.71
	Std.	40	40S	0.140	0.123	1.380	0.67	0.195	0.235	2.91	0.54	2.27	0.65
	XS	80	80S	0.191	0.167	1.278	0.88	0.242	0.291	4.25	0.52	3.00	0.56
		160		0.250	0.219	1.160	1.11	0.284	0.342	6.04	0.51	3.76	0.46
	XXS			0.382	0.334	0.896	1.53	0.341	0.411	11.2	0.47	5.22	0.27
1½″ 1.900			5S	0.065	0.057	1.770	0.38	0.158	0.166	0.927	0.65	1.27	1.07
			10S	0.109	0.095	1.682	0.61	0.247	0.260	1.63	0.64	2.09	0.96
	Std.	40	40S	0.145	0.127	1.610	0.80	0.310	0.326	2.26	0.62	2.72	0.88
	XS	80	80S	0.200	0.175	1.500	1.07	0.391	0.412	3.32	0.61	3.63	0.77
		160		0.281	0.246	1.338	1.43	0.483	0.508	5.15	0.58	4.87	0.61
	XXS			0.400	0.350	1.100	1.89	0.568	0.598	8.53	0.55	6.41	0.41
2″ 2.375			5S	0.065	0.057	2.245	0.47	0.315	0.265	0.585	0.82	1.60	1.72
			10S	0.109	0.095	2.157	0.78	0.499	0.420	1.02	0.80	2.64	1.58
	Std.	40	40S	0.154	0.135	2.067	1.07	0.666	0.561	1.50	0.79	3.65	1.45
	XS	80	80S	0.218	0.191	1.939	1.48	0.868	0.731	2.25	0.77	5.02	1.28
		160		0.343	0.300	1.689	2.19	1.16	0.979	3.90	0.73	7.46	0.97
	XXS			0.436	0.382	1.503	2.66	1.31	1.10	5.57	0.70	9.03	0.77
2½″ 2.875			5S	0.083	0.073	2.709	0.73	0.710	0.494	0.577	0.99	2.48	2.50
			10S	0.120	0.105	2.635	1.04	0.988	0.687	0.750	0.98	3.53	2.36
	Std.	40	40S	0.203	0.178	2.469	1.70	1.53	1.06	1.37	0.95	5.79	2.08
	XS	80	80S	0.276	0.242	2.323	2.25	1.92	1.34	1.90	0.92	7.66	1.84
		160		0.375	0.328	2.125	2.95	2.35	1.64	2.88	0.89	10.0	1.54
	XXS			0.552	0.483	1.771	4.03	2.87	2.00	4.61	0.84	13.7	1.07
3″ 3.500			5S	0.083	0.073	3.334	0.89	1.30	0.744	0.341	1.21	3.03	3.78
			10S	0.120	0.105	3.260	1.27	1.82	1.04	0.501	1.20	4.33	3.61
	Std.	40	40S	0.216	0.189	3.068	2.23	3.02	1.72	0.981	1.16	7.58	3.20
	XS	80	80S	0.300	0.263	2.900	3.02	3.89	2.23	1.41	1.14	10.3	2.86
		160		0.438	0.383	2.624	4.21	5.04	2.88	2.24	1.09	14.3	2.34
	XXS			0.600	0.525	2.300	5.47	5.99	3.43	3.42	1.05	18.6	1.80
3½″ 4.000			5S	0.083	0.073	3.834	1.02	1.96	0.980	0.266	1.39	3.47	5.00
			10S	0.120	0.105	3.760	1.46	2.76	1.38	0.383	1.37	4.97	4.81
	Std.	40	40S	0.226	0.198	3.548	2.68	4.79	2.39	0.702	1.34	9.11	4.28
	XS	80	80S	0.318	0.278	3.364	3.68	6.28	3.14	1.13	1.31	12.5	3.85
	XXS			0.636	0.557	2.728	6.72	9.85	4.93	2.70	1.21	22.9	2.53
4″ 4.500			5S	0.083	0.073	4.334	1.15	2.81	1.25	0.204	1.56	3.92	6.40
			10S	0.120	0.105	4.260	1.65	3.96	1.76	0.300	1.55	5.61	6.17
	Std.	40	40S	0.237	0.207	4.026	3.17	7.23	3.21	0.621	1.51	10.8	5.51
	XS	80	80S	0.337	0.295	3.826	4.41	9.61	4.27	0.933	1.48	15.0	4.98
		120		0.438	0.383	3.624	5.59	11.7	5.18	1.27	1.45	19.0	4.47
		160		0.531	0.465	3.438	6.62	13.3	5.90	1.62	1.42	22.5	4.02
	XXS			0.674	0.590	3.152	8.10	15.3	6.79	2.21	1.37	27.5	3.38

Properties and Weights of Pipe — Continued

Nominal Size Outside Diameter, inches D	Weight Designation and/or Schedule Number			Average Wall Thickness, inches t	Minimum Wall Thickness ($= \frac{7}{8}t$), inches t_m	Inside Diameter, inches d	Cross-Sectional Metal Area, square inches A	Moment of Inertia, inches⁴ I	Section Modulus, inches³ Z	Bend Characteristic per Unit Bend Radius t/R, h/R	Radius of Gyration, inches r_g	Weight of Pipe w_p lb per ft	Weight of Water w_w lb per ft
			5S	0.109	0.095	5.345	1.87	6.95	2.50	0.176	1.93	6.36	9.73
			10S	0.134	0.117	5.295	2.29	8.43	3.03	0.218	1.92	7.77	9.53
	Std.	40	40S	0.258	0.226	5.047	4.30	15.2	5.45	0.440	1.88	14.6	8.66
5″ 5.563	XS	80	80S	0.375	0.328	4.813	6.11	20.7	7.43	0.666	1.84	20.8	7.88
		120		0.500	0.438	4.563	7.95	25.7	9.25	0.929	1.80	27.0	7.09
		160		0.625	0.547	4.313	9.70	30.0	10.8	1.23	1.76	33.0	6.33
	XXS			0.750	0.656	4.063	11.3	33.6	12.1	1.55	1.72	38.6	5.62
			5S	0.109	0.095	6.407	2.23	11.9	3.58	0.123	2.30	7.57	14.0
			10S	0.134	0.117	6.357	2.73	14.4	4.35	0.153	2.30	9.29	13.7
	Std.	40	40S	0.280	0.245	6.065	5.58	28.1	8.50	0.334	2.25	19.0	12.5
6″ 6.625	XS	80	80S	0.432	0.378	5.761	8.40	40.5	12.2	0.541	2.20	28.6	11.3
		120		0.562	0.492	5.501	10.7	49.6	15.0	0.735	2.15	36.4	10.3
		160		0.718	0.628	5.189	13.3	59.0	17.8	0.988	2.10	45.3	9.16
	XXS			0.864	0.756	4.897	15.6	66.3	20.0	1.25	2.06	53.2	8.14
			5S	0.109	0.095	8.407	2.92	26.5	6.13	0.072	3.01	9.91	24.1
			10S	0.148	0.130	8.329	3.94	35.4	8.21	0.099	3.00	13.4	23.6
		20		0.250	0.219	8.125	6.58	57.7	13.4	0.171	2.96	22.4	22.5
8″ 8.625		30		0.277	0.242	8.071	7.26	63.4	14.7	0.191	2.95	24.7	22.2
	Std	40	40S	0.322	0.282	7.981	8.40	72.5	16.8	0.224	2.94	28.6	21.7
		60		0.406	0.355	7.813	10.5	88.8	20.6	0.289	2.91	35.6	20.8
	XS	80	80S	0.500	0.438	7.625	12.8	106	24.5	0.364	2.88	43.4	19.8
		100		0.593	0.519	7.439	15.0	121	28.1	0.441	2.85	50.9	18.8
		120		0.718	0.628	7.189	17.8	141	32.6	0.551	2.81	60.6	17.6
		140		0.812	0.711	7.001	19.9	154	35.6	0.639	2.78	67.8	16.7
	XXS			0.875	0.766	6.875	21.3	162	37.6	0.699	2.76	72.4	16.1
		160		0.906	0.793	6.813	22.0	166	38.5	0.730	2.75	74.7	15.8
			5S	0.134	0.117	10.482	4.52	63.7	11.9	0.057	3.76	15.2	37.4
			10S	0.165	0.144	10.420	5.49	76.9	14.3	0.071	3.74	18.7	36.9
		20		0.250	0.219	10.250	8.24	114	21.2	0.109	3.71	28.0	35.7
		30		0.307	0.269	10.136	10.1	138	25.6	0.135	3.69	34.2	34.9
	Std.	40	40S	0.365	0.319	10.020	11.9	161	29.9	0.163	3.67	40.5	34.1
	XS	60	80S	0.500	0.438	9.750	16.1	212	39.4	0.228	3.63	54.7	32.3
10″		80		0.593	0.519	9.564	18.9	245	45.5	0.276	3.60	64.3	31.1
10.750				0.625	0.547	9.500	19.9	256	47.6	0.293	3.59	67.5	30.7
		100		0.718	0.628	9.314	22.6	286	53.2	0.342	3.56	76.9	29.5
				0.750	0.656	9.250	23.6	296	55.1	0.360	3.55	80.1	29.1
		120		0.843	0.738	9.064	26.2	324	60.3	0.412	3.52	89.2	27.9
				0.875	0.766	9.000	27.1	333	62.0	0.431	3.51	92.3	27.5
		140		1.000	0.875	8.750	30.6	368	68.4	0.505	3.47	104	26.0
		160		1.125	0.984	8.500	34.0	399	74.3	0.583	3.43	116	24.6

Properties and Weights of Pipe — Continued

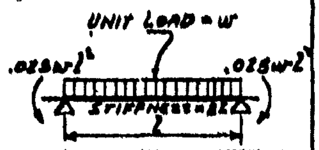
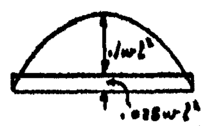
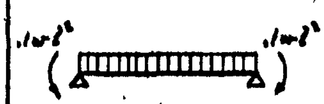
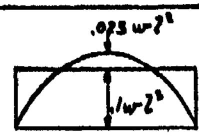
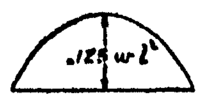
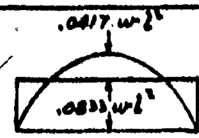
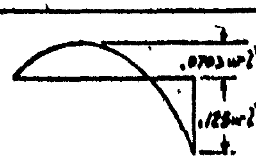
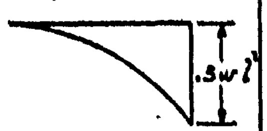
Nominal Size Outside Diameter, inches D	Weight Designation and/or Schedule Number	Average Wall Thickness, inches t	Minimum Wall Thickness (= 7/8 t), inches t_m	Inside Diameter, inches d	Cross-Sectional Metal Area, square inches A	Moment of Inertia, inches[4] I	Section Modulus, inches[3] Z	Bend Characteristic per Unit Bend Radius 1/ft h/R	Radius of Gyration, inches r_g	Weight of Pipe w_p lb per ft	Weight of Water w_w lb per ft
	5S	0.165	0.144	12.420	6.52	120	20.3	0.030	4.45	19.0	52.5
	10S	0.180	0.158	12.390	7.11	141	22.0	0.035	4.44	24.2	52.2
	20	0.250	0.219	12.250	9.82	192	30.0	0.077	4.42	33.4	51.1
	30	0.330	0.289	12.090	12.9	219	39.0	0.103	4.39	43.8	49.7
	Std. 40S	0.375	0.328	12.000	14.6	279	43.8	0.118	4.38	49.6	49.0
	40	0.406	0.355	11.938	15.7	300	47.1	0.128	4.37	53.5	48.5
12" 12.750	XS 80S	0.500	0.438	11.750	19.2	362	56.7	0.160	4.33	65.4	47.0
	60	0.562	0.492	11.626	21.5	401	62.8	0.182	4.31	73.2	46.0
		0.625	0.547	11.500	23.8	439	68.8	0.204	4.29	80.9	45.0
	80	0.687	0.601	11.376	26.0	475	74.5	0.227	4.27	88.5	44.0
		0.750	0.656	11.250	28.3	511	80.2	0.250	4.25	96.2	43.0
	100	0.843	0.738	11.064	31.5	562	88.1	0.285	4.22	107	41.6
		0.875	0.766	11.000	32.6	579	90.8	0.298	4.21	111	41.1
	120	1.000	0.875	10.750	36.9	642	101	0.348	4.17	125	39.3
	140	1.125	0.984	10.500	41.1	701	110	0.400	4.13	140	37.5
	160	1.312	1.148	10.126	47.1	781	123	0.481	4.07	160	34.9
	10	0.250	0.219	13.500	10.8	255	36.5	0.064	4.86	36.7	62.0
	20	0.312	0.273	13.376	13.4	315	45.0	0.080	4.84	45.7	60.8
	Std. 30	0.375	0.328	13.250	16.1	373	53.3	0.097	4.82	54.6	59.7
	40	0.438	0.383	13.125	18.7	429	61.4	0.114	4.80	63.4	58.6
	XS	0.500	0.438	13.000	21.2	484	69.1	0.132	4.78	72.1	57.5
	60	0.593	0.519	12.814	25.0	562	80.3	0.158	4.74	84.9	55.9
14" 14.000		0.625	0.547	12.750	26.3	589	84.1	0.168	4.73	89.3	55.3
	80	0.750	0.656	12.500	31.2	687	98.2	0.205	4.69	106	53.1
		0.875	0.766	12.250	36.1	781	112	0.244	4.65	123	51.1
	100	0.937	0.820	12.126	38.4	825	118	0.264	4.63	131	50.0
	120	1.093	0.956	11.814	44.3	930	133	0.315	4.58	151	47.5
	140	1.250	1.094	11.500	50.1	1030	147	0.369	4.53	170	45.0
	160	1.406	1.230	11.188	55.6	1120	160	0.426	4.48	189	42.6
	10	0.250	0.219	15.500	12.4	384	48.0	0.048	5.57	42.1	81.7
	20	0.312	0.273	15.376	15.4	474	59.3	0.061	5.55	52.3	80.5
	Std. 30	0.375	0.328	15.250	18.4	562	70.3	0.074	5.53	62.6	79.1
	XS 40	0.500	0.438	15.000	24.4	732	91.5	0.100	5.48	82.8	76.5
		0.625	0.547	14.750	30.2	894	112	0.127	5.44	103	74.1
	60	0.656	0.574	14.688	31.6	933	117	0.134	5.43	108	73.4
16" 16.000		0.750	0.656	14.500	35.9	1030	131	0.155	5.40	122	71.5
	80	0.843	0.738	14.314	40.1	1160	145	0.176	5.37	136	69.7
		0.875	0.766	14.250	41.6	1170	149	0.184	5.36	141	69.1
	100	1.031	0.902	13.938	48.5	1370	171	0.221	5.29	165	66.1
	120	1.218	1.066	13.564	56.6	1560	195	0.268	5.23	192	62.6
	140	1.438	1.258	13.124	65.8	1760	220	0.325	5.17	224	58.6
	160	1.593	1.394	12.814	72.1	1890	237	0.368	5.12	245	55.9

Properties and Weights of Pipe — Continued

Nominal Size Outside Diameter inches D	Weight Designation and/or Schedule Number	Average Wall Thickness inches t	Minimum Wall Thickness (= ⅞ t) inches t_m	Inside Diameter inches d	Cross-Sectional Metal Area square inches A	Moment of Inertia inches⁴ I	Section Modulus inches³ Z	Bend Characteristic per Unit Bend Radius 1/ft h/R	Radius of Gyration inches r_g	Weight of Pipe w_p lb per ft	Weight of Water w_w lb per ft
	10	0.250	0.219	17.500	13.9	549	61.0	0.038	6.28	47.4	104
	20	0.312	0.273	17.376	17.3	679	75.5	0.048	6.25	59.0	103
	Std.	0.375	0.328	17.250	20.8	807	89.6	0.058	6.23	70.6	101
	30	0.438	0.382	17.124	24.2	932	104	0.068	6.21	82.2	99.7
	XS	0.500	0.438	17.000	27.5	1050	117	0.078	6.19	93.5	98.3
	40	0.562	0.492	16.876	30.8	1170	130	0.089	6.17	105	96.9
		0.625	0.547	16.750	34.1	1290	143	0.099	6.15	116	95.4
18″	60	0.750	0.656	16.500	40.6	1520	168	0.121	6.10	138	92.6
18.000		0.875	0.766	16.250	47.1	1730	192	0.143	6.06	160	89.9
	80	0.937	0.820	16.126	50.2	1830	204	0.155	6.04	171	88.5
	100	1.156	1.012	15.688	61.2	2180	242	0.196	5.97	208	83.7
	120	1.375	1.203	15.250	71.8	2500	278	0.239	5.90	244	79.1
	140	1.562	1.367	14.876	80.7	2750	306	0.278	5.84	274	75.3
	160	1.781	1.558	14.438	90.8	3020	336	0.325	5.77	309	70.9
	10	0.250	0.219	19.500	15.5	757	75.7	0.031	6.98	52.7	129
	Std. 20	0.375	0.328	19.250	23.1	1110	111	0.047	6.94	78.6	126
	XS 30	0.500	0.438	19.000	30.6	1460	146	0.063	6.90	104	123
	40	0.593	0.519	18.814	36.2	1700	170	0.076	6.86	123	120
		0.625	0.547	18.750	38.0	1770	179	0.080	6.85	129	120
		0.750	0.656	18.500	45.4	2100	210	0.097	6.81	154	117
20″	60	0.812	0.711	18.376	48.9	2260	226	0.106	6.79	166	115
20.000		0.875	0.766	18.250	52.6	2410	241	0.115	6.77	179	113
	80	1.031	0.902	17.938	61.4	2770	277	0.138	6.72	209	109
	100	1.281	1.121	17.438	75.3	3320	332	0.175	6.63	256	103
	120	1.500	1.313	17.000	87.2	3760	376	0.210	6.56	296	98.3
	140	1.750	1.531	16.500	100.	4220	422	0.252	6.48	341	92.6
	160	1.968	1.722	16.064	112	4590	459	0.291	6.41	379	87.8
	10	0.250	0.219	23.500	18.7	1320	110	0.021	8.40	63.4	188
	Std. 20	0.375	0.328	23.250	27.8	1940	162	0.032	8.35	94.6	184
	XS	0.500	0.438	23.000	36.9	2550	213	0.043	8.31	125	180
	30	0.562	0.492	22.876	41.4	2840	237	0.049	8.29	141	178
		0.625	0.547	22.750	45.9	3140	261	0.055	8.27	156	176
24″	40	0.687	0.601	22.626	50.3	3420	285	0.061	8.25	171	174
24.000		0.750	0.656	22.500	54.8	3710	309	0.067	8.22	186	172
	60	0.968	0.847	22.064	70.0	4650	388	0.089	8.15	238	166
	80	1.218	1.066	21.564	87.2	5670	473	0.113	8.07	296	158
	100	1.531	1.340	20.938	108	6850	571	0.146	7.96	367	149
	120	[illegible]	1.581	20.376	126	7820	652	0.177	7.87	429	141
	140	2.062	1.804	19.876	142	8630	719	0.206	7.79	483	134
	160	2.343	2.050	19.314	159	9460	788	0.240	7.70	542	127
	10	0.312	0.273	29.376	29.1	3210	214	0.017	10.5	98.9	294
		0.375	0.328	29.250	34.9	3830	255	0.021	10.0	119	291
	20	0.500	0.438	29.000	46.3	5040	336	0.028	10.4	157	286
30″											
30.000		0.562	0.492	28.876	52.0	5640	376	0.031	10.4	177	284
	30	0.625	0.547	28.750	57.6	6220	415	0.035	10.4	196	281
		0.750	0.656	28.500	68.9	7380	492	0.042	10.3	234	277

TABLE 2

Correction Factors for Use with Chart 1 [16]

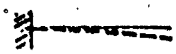
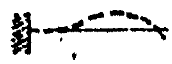
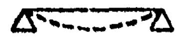
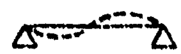
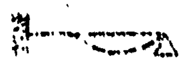
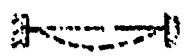
CASE	END FIXATION	MOMENT DIAG.	MAXIMUM MOMENT	MAXIMUM DEFLECTION	F_s	F_δ
1			$\frac{wl^2}{10}$	$\frac{19}{1920}\frac{wl^4}{EI}$	1	1
2			$\frac{wl^2}{10}$	$\frac{1}{1920}\frac{wl^4}{EI}$	1	.0526
3			$\frac{wl^2}{8}$	$\frac{5}{384}\frac{wl^4}{EI}$	1.25	1.316
4			$\frac{wl^2}{12}$	$\frac{1}{384}\frac{wl^4}{EI}$	.833	.263
5			$\frac{wl^2}{8}$	$\frac{1}{185}\frac{wl^4}{EI}$	1.25	.547
6			$\frac{wl^2}{2}$	$\frac{1}{8}\frac{wl^4}{EI}$	5	12.63

For Chart 1 Multiply δ value from chart by F_δ to obtain maximum deflection for case shown.

TABLE 3

Frequency coefficient (α) depending on the pipe support and mode of vibration [16].

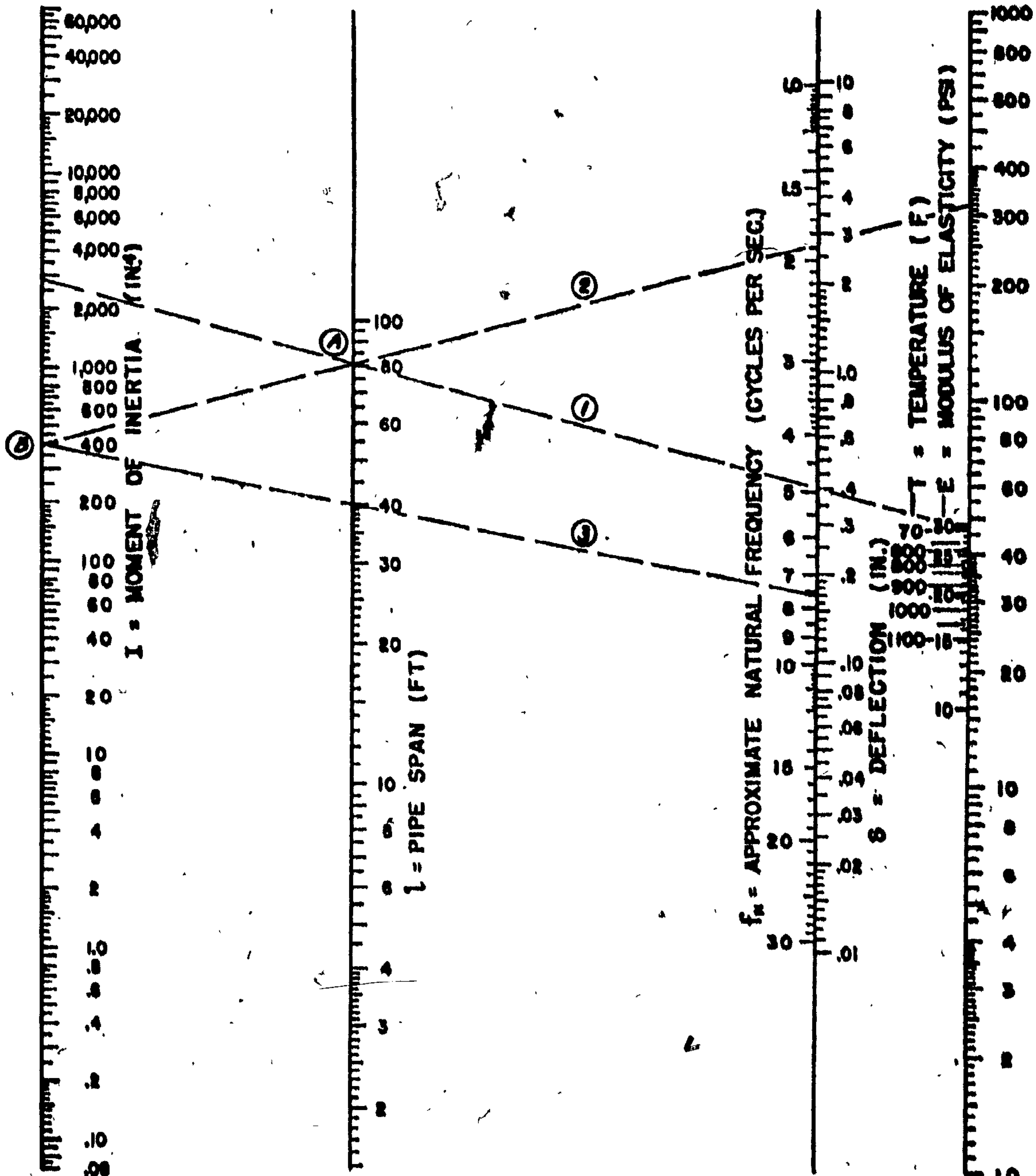
Table

Support	Mode of Vibration		Frequency Coefficient α for Pipe or Uniform Bar
Cantilever	Fundamental (1st)		0.265
	Second Mode		1.66
Both ends simply supported	Fundamental (1st)		0.743
	Second Mode		2.97
One end simply supported, other end fixed	Fundamental (1st)		1.16
	Second Mode		3.76
Both ends fixed	Fundamental (1st)		1.69
	Second Mode		4.64

APPENDIX B

Chart 1

Span vs. Natural Frequency and vs. Deflection
Horizontal Pipe Lines, Uniform Load [16]

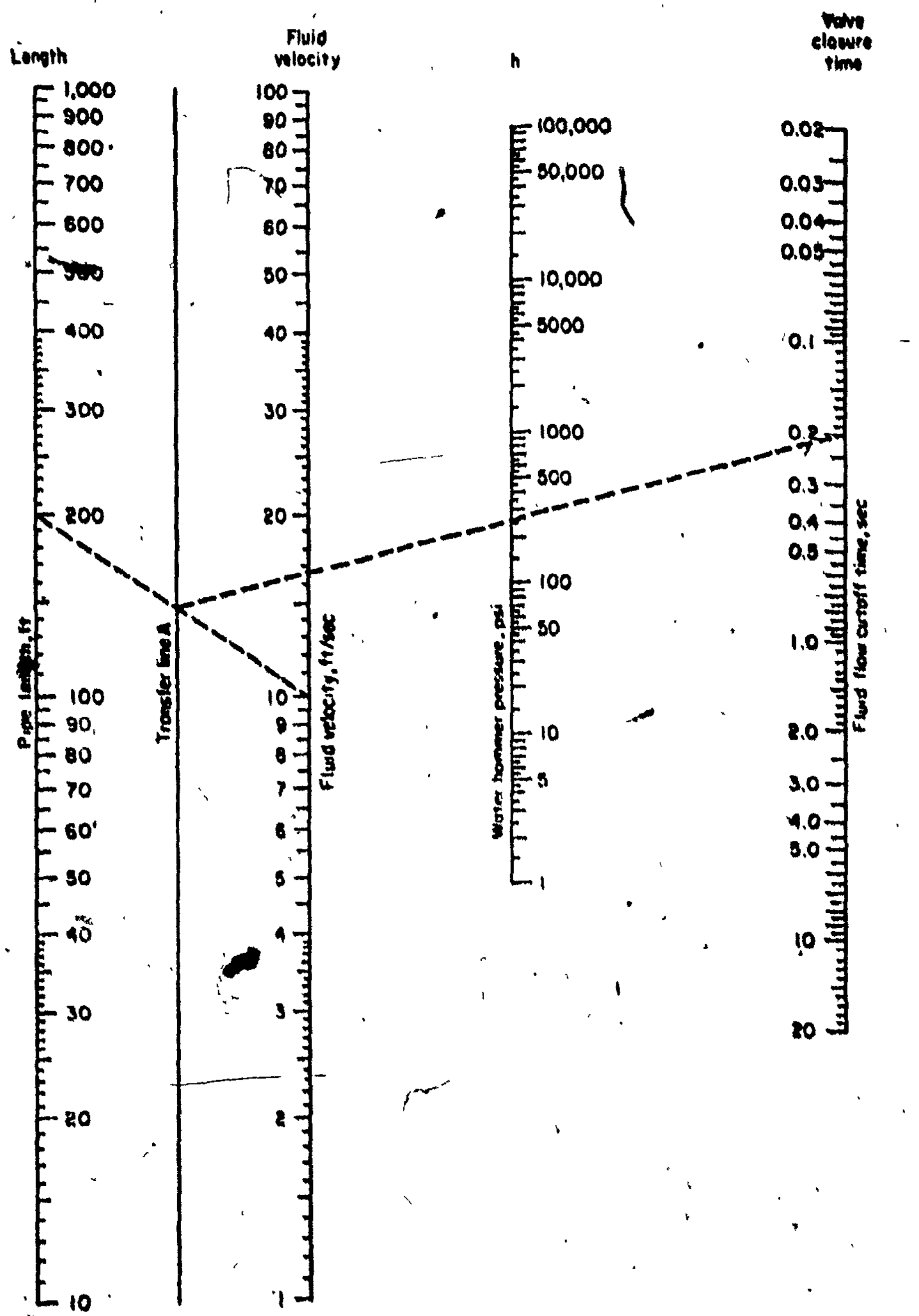


Formulas: $\delta = 17.1wl^4/EI$ $\quad f_n = 3.13/\sqrt{\delta}$

Key: (1) Connect E (or T) with I locating turning point (A) at intersection on l.
(2) Connect (A) with w locating turning point (B) at intersection on I.
(3) Connect (B) with l locating δ (or f_n).
Connect (B) with δ (or f_n) locating l.

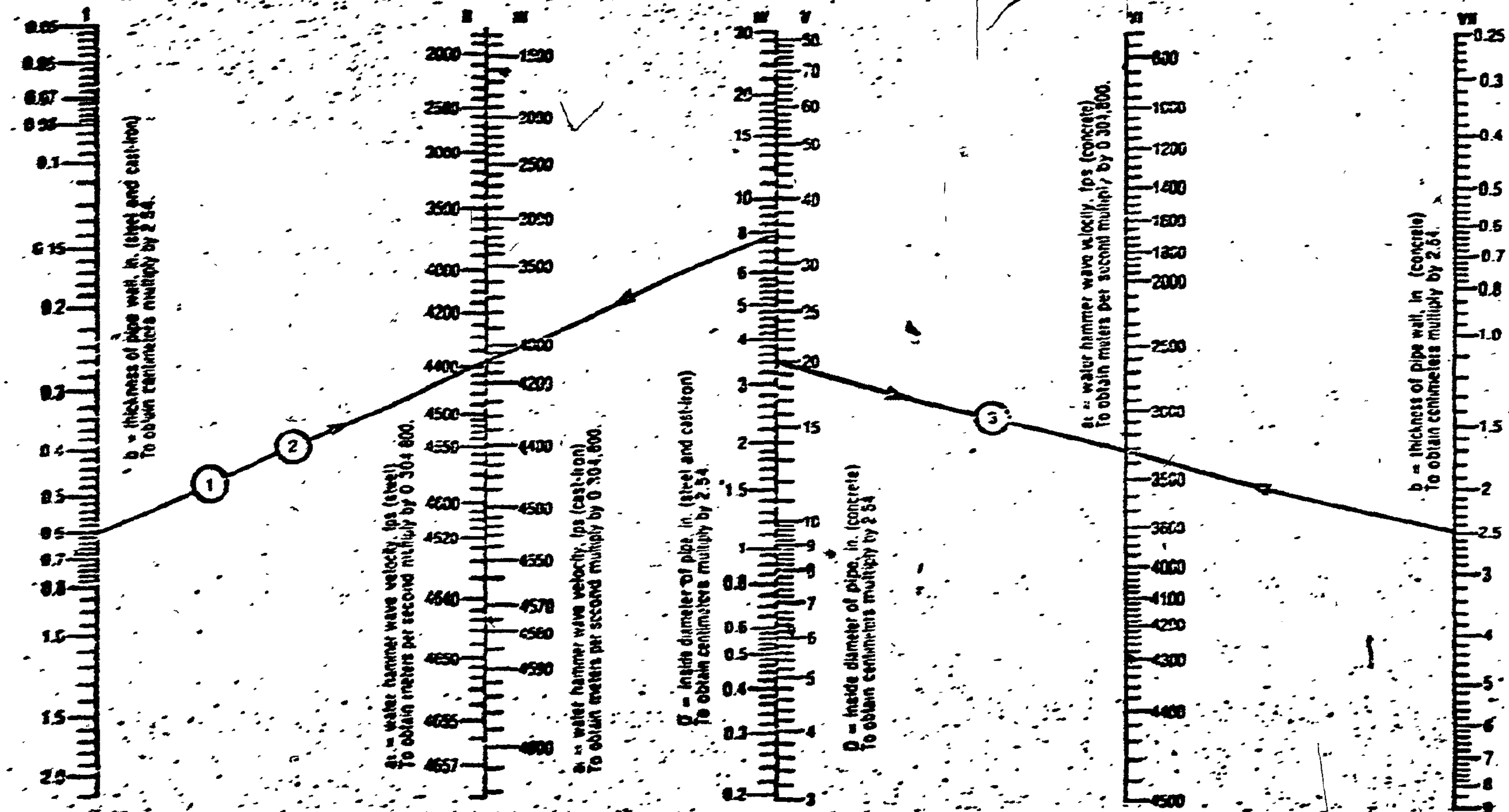
Example: Given: $E = 29 \times 10^6$ psi, $I = 2840$ in.^{4}, $w = 320$ lb/ft.
Result: $f_n = 0.76$ cy/sec, $\delta = 0.17$ in.

Chart 2



Pressure rise due to water hammer. *h* = pressure rise due to water hammer which should be added to normal operating pressure. [21]

Chart 3



Nomograph calculates water hammer wave velocity in steel, cast-iron and concrete pipe. Numbers on broken lines indicate answers to problems presented in the text. The nomograph scales are graduated in inches for greater convenience. [20]

APPENDIX C

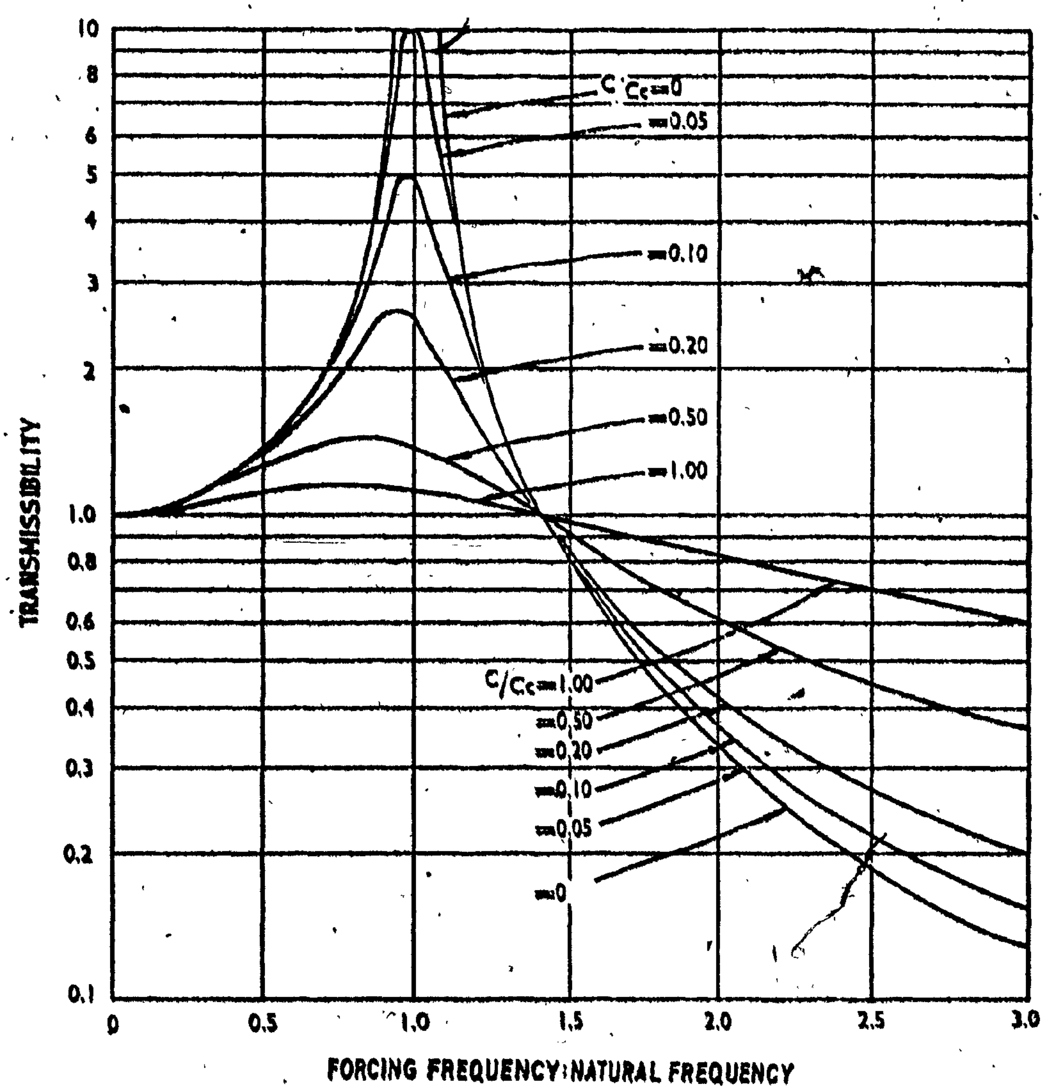


Fig. 16 Transmissibility in terms of the ratio of forcing frequency to natural frequency for a viscously damped system. [1?]

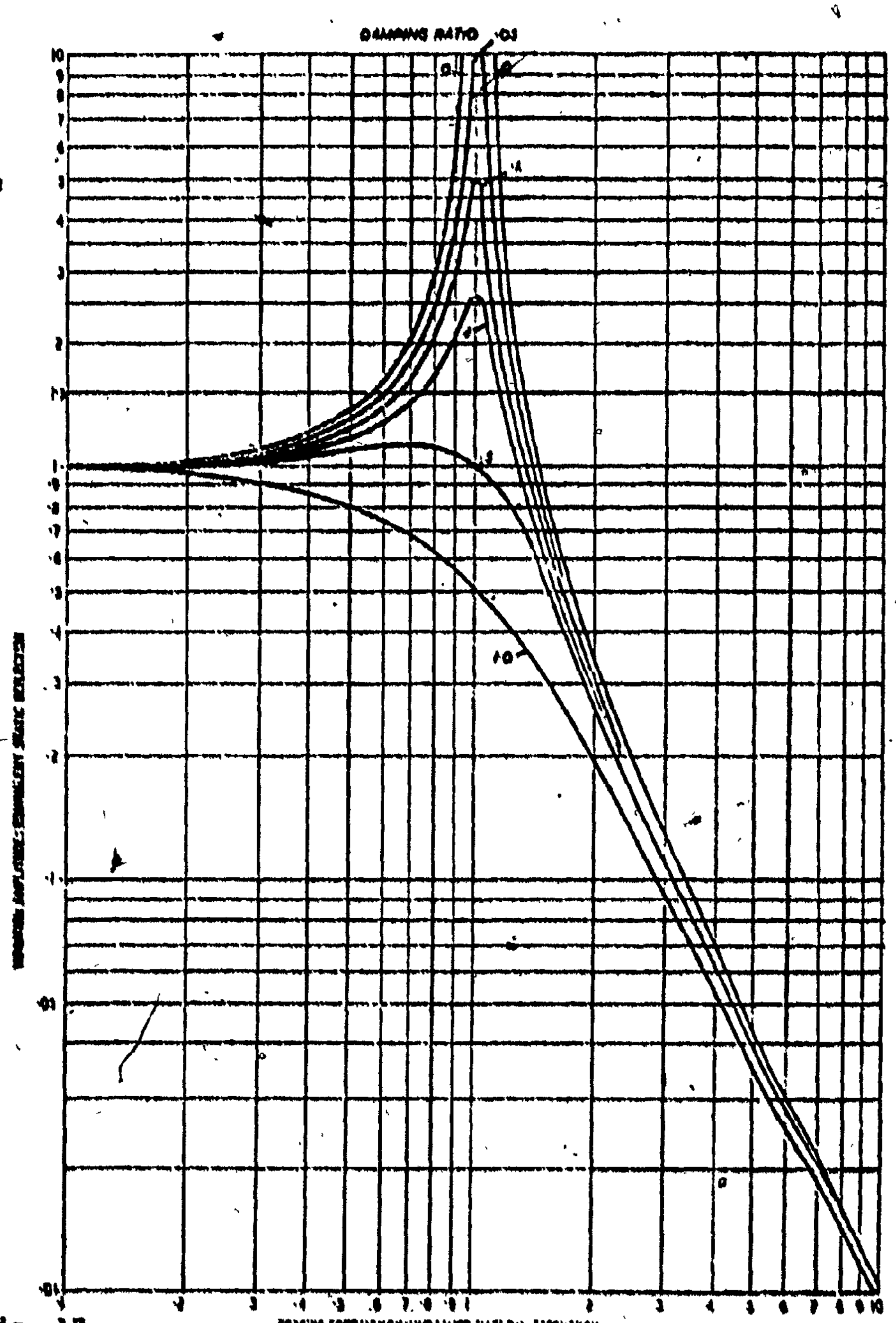


Fig. 17.

Vibration Amplitude

These curves are appropriate to a single-degree-of-freedom system with viscous damping. Vibration amplitude is given in terms of the ratio relative to equivalent static deflection. In the case of an undamped system the damping ratio equals 0 [19]

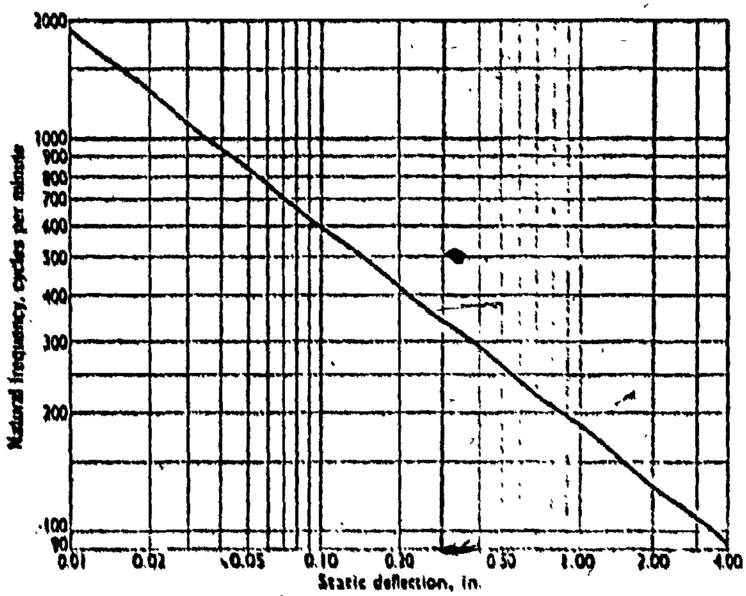


Fig. 18 Natural frequency vs. Static deflection [19].

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