YIELD MANGEMENT IN THE HOTEL INDUSTRY

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ABSTRACT

Yield Management in the Hotel Industry

by

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Yield management offers an integrated and dynamic approach to maximizing revenue through appropriate pricing and inventory control. Based on demand patterns and the price sensitivity of different market segments, yield management helps determine the number of rooms that should be offered at a certain price. As an operating strategy, yield management is best suited to industries such as hotels and airlines.

The research on yield management applications in the hotel industry, however, is scarce. Since the airline industry is well versed in the development and application of yield management techniques, the first part of this thesis reviews the literature on yield management models in the airline industry. In an effort to learn more about yield management as it is practiced in the hotel industry, a survey of yield management at Montreal hotels was conducted. Seven hotels were visited, and interviews were conducted.

Based on the findings from the literature and the industry survey, the airline inventory control model developed by Belobaba (1987) is adapted to a hotel setting. An application of the model is then performed using actual hotel reservations data.

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To Mama and Papa,

for their inspiration and their Love

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1. INTRODUCTION

As Orkin (1988) put it, hotels are in the business of generating revenue from *space*. This space includes guest rooms, restaurants, convention centers, etc.. All facilities in a hotel are inter-related in terms of revenue generation. When guests rent a room, they will, most likely, use the restaurant and/or room service. Very often, they will also spend money at the hotel boutiques and/or souvenir stores. Renting a hotel's convention center often implies renting out rooms. It also implies meals at the hotel restaurant(s). The main function of a hotel, therefore, is to utilize its scarce resource, space, as wisely and efficiently as possible.

Traditional performance criteria, such as occupancy rate and average room rate, used by the hotel industry, however, do not paint a complete picture (Orkin, 1988). They may actually lead to bad decisions sometimes. A sales department that is measured by room-night productivity, i.e. occupancy rate, for example, will tend to accept group business as often as possible (groups will raise occupancy). This forces the hotel to turn away higher paying business customers who tend to be late bookers (Relihan, 1989). On the other hand, if performance is measured by average room rate, a hotel will try to sell at the highest rate possible, thus foregoing leisure demand which tends to be quite price sensitive.

In 1990 the hotel industry stock prices went down by almost 68%, the steepest downturn for the industry as a whole since the Great Depression and steeper than any other US industry group during the 1990s (Dahl & Carlton, 1990). The hard times faced by the industry are due to several factors: over-building and excess capacity generated in the eighties, cut-throat competition, declining occupancy rates, a drop-off in both, business and leisure travel, and lastly, severe short term liquidity problems for many hotels. To make the situation even more bleak, inflation rose, lenders became more reluctant as well as demanding, equity capital became more expensive due to stock market price declines, and foreign investment funds seemed to be drying off (Arbel & Woods, 1991).

Wasn't the hotel industry supposed to be *inflation-proof* ? What are the reasons behind all its woes then? Contrary to general belief, the hotel industry is quite vulnerable to inflation and to unsound business practices, just like any other industry. The misleading performance criteria used by the industry, however, were not the only culprits. Traditional pricing policies used by the industry were just as responsible. Hotels use a cost-plus pricing policy (Relihan, 1989; Arbel & Woods, 1991; Jean-Richard, 1995). The goal behind this policy is to find a room rate that will cover all costs including that of providing a reasonable return on investment capital. This simplistic approach, however, ignores important demand- and supply-side variables. In terms of basic economic theory, it may result in prices that do not clear the market; i.e., a hotel will be left with a large inventory of unsold rooms (Arbel & Woods, 1991).

The hotel industry committed business *hubris*, as it were. They believed that hotels were shielded against inflation as well as all other economic downturns. All they had to do was raise room rates more than the inflation rate; this will insure against the dilution of revenue and will keep profits at the desired level they thought. This mind set persisted for over two decades. As long as demand was greater than supply, the faults of this strategy were not obvious. In the eighties a construction boom erupted and by the mid to late eighties the supply of hotel rooms far exceeded the demand. It was then that hoteliers started to realize the flaws of their operating strategies

OVERVIEW

This thesis will discuss yield management in the hotel industry, what it is, what are its different components, and how it can be improved. Since there is not a lot of published research on yield management as it pertains to the hotel industry, this thesis will rely heavily on airline yield management literature. Section 2 will define yield management and discuss its different components. Sections 3 and 4 will review the literature on the components of the yield management problem. Section 3 will address the overbooking component, while section 4 will address that of discount allocation.

In section 5, the results of an industry survey on yield management practices at Montreal hotels, will be presented and discussed. Based on the literature review and the industry survey, an airline discount allocation model is adapted to a hotel setting in section 6. The model is applied to real hotel reservations data and the results are presented. Finally, section 7 will conclude with a discussion about the results of the application in section 6, and some directions for future research.

2. YIELD MANAGEMENT

By 1988 things were starting to look a bit better: the room rate to inflation ratio was less than one for the first time ever (Arbel & Woods, 1991). Hotels were embracing yield management. Born and nurtured in the airline industry, yield management offers an integrated and dynamic approach to increasing (maximizing, if possible) revenue through effective pricing and inventory control (Belobaba, 1987b; Kimes, 1989a,b; Lieberman, 1993). Yield management concepts are based on economic principles, while its techniques are grounded in mathematical optimization.

Cross (1989) defines yield management as "using price incentives and inventory controls to maximize the value of existing processes." Relihan (1989) says of yield management as "applying basic economic principles of pricing and controlling room inventory for the purpose of maximizing revenue".

According to Kimes (1989a) and Smith, Leimkuhler, and Darrow (1992), the purpose of yield management is maximizing revenue or yield. This is achieved through allocating the right type of resource to the right customer at the right price at the appropriate time. Smith et al. (1992) continue to define yield management as "the control and management of reservations inventory in a way that increases (maximizes, if possible) company profitability, given [the fare structure]". When is yield management the most appropriate strategy to use? Where would yield management make a difference in terms of revenue? The next section will address these issues.

2.1 ENVIRONMENT

An environment most conducive to yield management is characterized by a fixed capacity, low marginal sales costs / high marginal production costs, advance selling, perishable inventory, the ability to segment the market, and fluctuating demand (Kimes, 1989a; Orkin, 1988,1990; Lieberman, 1993; Relihan, 1988; Weatherford & Bodily, 1992).

2.1.1 MARGINAL PRODUCTION COSTS

Hotels face very high marginal production costs. Once a hotel is built, it is very hard, indeed costly, to increase its capacity. The marginal cost of an extra room is far too big. Adding rooms to an existing hotel is a long term planning decision. Hotels usually add capacity only in large chunks, and after careful deliberation. Airlines face a similar situation. Once a certain aircraft is scheduled for a flight, it is impossible to increase the number of seats available on that flight.

2.1.2 MARGINAL SALES COSTS

On the other hand, hotels have relatively low marginal sales costs. Once a hotel is in operation, it automatically implies that rooms are ready to be rented; that is, they have been cleaned and prepared for guests. These operational costs can be considered a sunk cost, since they have

to be incurred for the hotel to be in business. The cost of actually renting the room : check-in, porters, etc., is negligible. In other words, the marginal cost of selling one more room is quite low. From this perspective, every rented room is additional revenue.

2.1.3 Advance Selling

Although some hotels sell their rooms a day or two in advance, most hotels operate under a long-frame reservation system; that is, reservations are made well in advance - sometimes years in advance. Selling in advance, however, creates a trade off. Should a group that wants to pay a low rate, but is willing to reserve well ahead of time, be accepted? If the group is accepted, there is a possibility that a higher paying customer, who is not going to reserve until later, will be displaced.

How many rooms should be protected for this type of customer? How many rooms should be sold at the discounted prices? When should a group be accepted? All of these are questions addressed by yield management.

2.1.4 Perishable Inventory

A hotel room, just like an airplane seat on a departing flight, is a perishable item. If it is not sold tonight, it is lost forever. A hotel cannot store its room in inventory for later use. Since a hotel or an airline cannot keep inventory, they must maximize daily revenue in order to maximize operating profits (Kimes, 1988; Orkin, 1988; Belobaba, 1989; Smith et al., 1992).

2.1.5 MARKET SEGMENTATION

For yield management to be most effective, a firm has to be able to segment its market into different types of customers (Kimes, 1989a; Weatherford & Bodily, 1992). In terms of certain characteristics of demand, the airline and hotel industries are similar (Relihan, 1989). The two broadest categories of customer types are business vs. leisure travelers. Each segment has distinct price elasticity and time sensitivity characteristics (Relihan, 1989).

The airline industry has noticed that "leisure traffic normally books early, as holidays are planned well ahead. They may be time sensitive to some degree, but price is the overriding factor in the decision. Business bookings, on the other hand, tend to be concentrated in the days immediately before [a trip]" (Relihan, 1989).

The motive behind this segmentation is to have different marketing plans for the different types of customers (Relihan, 1989; Gilbert, 1993). In the case of leisure customers, lower priced rooms that must be booked in advance may prove to be the best policy. For business customers, on the other hand, a higher rate with no time penalty would probably serve the hotel better. The basic idea is identifying a hotel's different market segments, or types of customers, and capitalizing on their different characteristics (Federer, 1995; Richardson, 1995).

2.1.6 FLUCTUATING DEMAND

Hotels and airlines face highly fluctuating demand patterns. Demand varies by season, by month, and by day of the week. Each hotel will observe different demand patterns, depending on its geographical location and client orientation.

Typically, people take their vacations during the summer, and consequently want to travel. Around Christmas time, a similar pattern is observed. In January and February, however, demand drops off because most people return to work or school. In cold countries, such as Canada, however, a lot of people would rather travel during the cold months of January or February. Canadians flock to the Caribbean Islands and South America seeking the warm weather. A hotel in the Caribbean will probably witness a heightened demand during those two months.

2.2. THE YIELD MANAGEMENT PROBLEM

The ultimate goal of yield management is the optimal allocation of resources between revenue-generating options. The yield management problem is defined by Smith et al. (1992), of American Airlines Decision Technologies (AADT), as "a non-linear, stochastic, mixed-integer mathematical program that requires data such as [customer] demand and cancellation, as well as, other estimates of [customer] behavior that are subject to frequent changes". It is estimated that, to solve the system-wide problem, would require around 250 million decision variables.

The mere size of the problem makes it practically, unsolvable. Consequently, the large problem is usually divided into three much smaller and manageable subproblems, or components: Overbooking, Discount Allocation, and Traffic Management. The combined results of the subproblems determine the final inventory levels (Smith et al., 1992).

2.2.1 OVERBOOKING

Overbooking is the practice of accepting more reservations than actual capacity. This action is spurred on by the high rates of cancellations and no-shows. Both airlines and hotels have been practicing overbooking for a very long time (Rothstein, 1971; Jean-Richard, 1995). AADT estimates that around half of all reservations made for a particular flight are either canceled or become no-shows (Smith et al., 1992). As a result, airlines set reservation levels higher than plane capacity in order to compensate for those cancellations and/or no-shows.

Hotels face the exact same situation. Not all reservations translate into an occupied room. Consequently, hotels overbook in order to make up for the cancellations and no-shows (Federer, 1995; Richardson. 1995). On the one hand, overbooking compensates for some of the uncertainties; on the other hand, however, it introduces a new cost, the cost of oversales. If a hotel overbooks too much, for example, and more people show up than there are rooms, the hotel must incur the costs of compensation: finding alternate accommodations, transportation to other hotel, a free night's stay at another time, etc., and the intangible cost of lost goodwill. The oversale cost is not constant; the more oversales incurred, the higher the cost.

2.2.2 DISCOUNT ALLOCATION

Discount allocation, is the process of determining the number of discount fares or rates to offer on a certain date (this could be a flight, in the case of airlines, or a certain night in the case of hotels). Discount rates are usually offered to stimulate demand or to fill capacity that would otherwise be empty.

Airlines started offering discount fares in the mid-seventies. The simplest case was when there were two fares to choose from: full and discount. The decision to accept or reject a discount request depends on the probability of getting a full fare when the discount request is rejected.

After deregulation, airlines started offering all kinds of discount fares: super saver, maxi saver, etc.. With multiple fares the problem becomes more complicated, but the approach is still the same: weigh the marginal value of a full fare request against the marginal value of all other fares.

These days most airlines use a *nested* fare class structure (Belobaba, 1987b, 1989; Smith et al., 1992). A nested reservation system is in contrast with independent or distinct, fare class inventories. In an independent system, once a seat is assigned to a fare class inventory, it can only be booked in that class. If it is not sold in that class, it remains unsold.

In a nested reservations system, on the other hand, a high fare request will not be denied as long as *any* seats remain available in lower fare classes. Nesting makes subsets of seats available to various levels of distinct fares. Lower classes have smaller subsets. If classes are controlled independently, it is possible to sell a reservation in a lower class while, at the same time, turning away a request in a higher class. Nesting ensures that a low value seat is never available when higher value fares are closed (Belobaba, 1987a; Smith et al., 1992). The concept of nesting started in the airlines. Its merits, however, have been recognized by the hotel industry and it is being applied by some hotels (Beachamp, 1995; Richardson, 1995; Federer, 1995).

2.2.3 Traffic Management

The term Traffic Management was coined by the airline yield management researchers. It refers to the control of reservations by passenger-origin and destination (O/D), to provide the desired mix of multiple-flight connecting vs. single-flight markets, that will maximize revenue (Smith et al., 1992). To maximize revenue across the system, reservations inventory control for a flight should consider passenger demand on all connecting flights and not only demand on that particular flight.

A full fare passenger going from Montreal to Paris is usually more desirable than the same passenger paying a discount fare. The discount passenger may be more desirable - in terms of revenue - if they are connecting in Paris to go to Beirut, for example. The two discount sectors, Montreal-Paris and Paris-Beirut, may generate more revenue than one full fare from Montreal to Paris. If inventory control is done on a flight by flight basis, the available seat will be given to the full fare passenger, when in terms of total system revenues, the discount connecting passenger is the more profitable one. Discount allocation should then be performed in view of the whole system and not just on a flight by flight basis.

This scenario is analogous to a hotel which is allocating its rooms between discount and full paying customers. There are no connecting flights, but there are single- vs. multiple-night stays. Some guests stay for one night, while some others stay for a whole week. On a night by night basis, when forced to choose, a hotel would probably sell a room at the higher rate, because this will maximize revenue. If all guests were staying one night only, this would be the more profitable thing to do. But since some guests stay longer than others, allocation decisions should consider length of stay as an important contributing factor. A guest staying for one day at a high rate, may actually be less desirable than a guest who is paying less, but is staying for three days.

Another factor to consider when allocating reservations inventory, is displacement. That customer who is staying for three days at $80 may generate more revenue than a guest staying one day at $100. But that same eighty-dollar guest may be displacing a $100-paying guest who is checking in only the next day, and who will be staying for three days. A certain horizon has to be chosen and allocations done based on demand forecasts for *all* days falling within that horizon.

Because the problem of Traffic Management is new and complex, there is not a lot of literature on the topic. As a result, this thesis will focus on the first two components of the yield management problem: Overbooking and Discount Allocation.

As was mentioned earlier, yield management is both, pricing and inventory control. Everything that has been discussed till now was on the inventory control aspect of yield management; that is, given a certain rate structure, what are the optimal inventory allocation decisions. Determining the right prices is not a simple task. The next section will briefly address this issue. The main focus of the thesis, however, will be the inventory control aspect of yield management.

2.3 Pricing

Traditionally, hotels increased room rates whenever inflation went up. Room rates were actually rising at a higher rate than inflation from 1946 to 1988-89. While the CPI increased by 6.4 times, average room rate rose by 11.9 times (Arbel & Woods, 1991). Raising prices, however, does not always translate into increased profits. The impact of price changes on profitability depends on price elasticity, income elasticity, and the substitution effect (Relihan, 1989; Orkin, 1988; Arbel & Woods, 1991).

When price elasticity is high, for example, customers will buy less when prices go up. When prices go down, on the other hand, they tend to buy more. This can mean that a room rate increase can make a negative incremental contribution to revenues. A high income elasticity, on the other hand, means that consumers with declining incomes will tend to buy less. Furthermore, the price of substitutes affects demand. Consumers will evaluate the price of domestic vs. foreign travel, for example, before deciding to buy a domestic hotel room. Arbel and Woods (1991) conducted a study on the effect of these variables (as well as others) on the occupancy and average room rates. They used data on macro-economic variables from 1975 to 1989. A multiple regression analysis showed that occupancy percentage, relative to other variables for the period 75-89 as a whole, decreased by about 1.8% for every 1% increase in inflation (significant Beta coefficient of -1.84).

Hotels have been using a cost plus approach to pricing. The most widely used cost plus approach is known as the Hubbart formula (Coltman, 1987). The Hubbart approach is endorsed by the American Hotel & Motel Association and considered worldwide as an industry standard and a textbook approach to hotel room pricing (Arbel & Woods, 1991). The Hubbart approach is sometimes referred to as the bottoms-up method, because the first item that is considered, profit, is at the bottom of the income statement, then come taxes, which are second from the bottom, etc..

The goal is to find a room rate that will cover all costs, including that of providing a reasonable return on investment capital. To establish room rates using the Hubbart formula, one first chooses the desired return on capital, then one adds taxes, operating expenses, and so on (Coltman, 1987). Roughly, a 30 to 35% markup over costs is not uncommon (Jean-Richard, 1995).

Up until the early nineties, the majority of hotels were still using this approach to pricing. The industry was attempting to cover all costs during unfavorable economic conditions by raising room rates. But given the typical price and income elasticities for the hotel services, the correct pricing approach would be to lower room rates when consumers' disposable income decreased, or when competition increased (Orkin, 1990; Arbel & Woods, 1991). While demand may not be the sole determinant of rates, it has to be one of the most salient factors. The most widely used pricing policies in the hotel industry seem to leave out one very important consideration: the customer's willingness to pay (Relihan, 1989).

The hotel industry has not only been setting prices based on the wrong inputs, but they have also been managing their rates inappropriately (Relihan, 1989; Orkin, 1990; Lieberman, Tatzin, & Buchin, 1992). Managing the rates offered is just as important as setting correct rates. Using a yield management pricing strategy means incorporating relevant demand and supply variables, building a multiple price structure and managing it prudently.

3. OVERBOOKING

3.1 INTRODUCTION

Maximizing the number of rooms rented per night is not an easy task. Not so much because of a hotel's marketing strategy or because of a lack of commitment on the part of the hotel's employees; but because of the fact that human nature is quite fickle. People call in for a reservation and specify their intended length of stay; but then, some will cancel, some will just not show up (no-shows) and some will alter their planned length of stay. A reservation does not always translate into a sale. People reserve, but have the prerogative of cancelling their reservation, or of even just not showing up. This means that a hotel or an airline cannot rely on the number of reservations as an indicator of profit.

Airlines reserve more seats than they actually have; that is, they overbook, hoping that the fraction of those passengers who do actually show up will equal the aircraft capacity. No-shows vary considerably among different airlines and different markets. The typical range in Europe is five to twenty percent (Alstrup, Andersson, Boas, Madsen, & Vidal, 1989). American Airlines estimates that, without overbooking, 15% of seats on sold out flights would be *spoiled*, that is, will remain empty (Smith et al., 1992).

It is estimated that the average net yield per passenger in Europe is around US$100. With around 100 departures every day, this translates into a direct loss of around US$50 million a year (Alstrup et al., 1989). This shows the gravity of the situation and the potential gains from even a slight improvement. The airline industry is aware of the situation, and they have not been standing by hoping that people will begin to behave more consistently. On the contrary, they have been aggressively analysing the problem and its potential solutions for around three decades now (Richter, 1989). The gains accrued to the airline industry from overbooking are quite substantial (Rothstein, 1975,1985; Shlifer & Vardi, 1975; Richter, 1989). American Airlines estimates a $225 million increase in revenue due to overbooking (Smith et al., 1992).

The airlines are not alone in using overbooking as a strategic weapon against the fickleness of human nature. Hoteliers recognize the gains to be made from overbooking and readily apply the process (Lambert, Lambert, & Cullen, 1989; Kimes, 1989; Lefever, 1988; Williams, 1977; Ladany, 1976; Rothstein, 1974; Federer, 1995; Richardson, 1995).

The demand for hotel rooms is made up of three categories: reservations, stayovers, and walk-ins. Reservations are those people who have already reserved a room. Stayovers are those people who are already registered in the hotel and plan to extend their stay. Walk-ins are those guests who check in without any reservation. Reservations are held till 6 p.m., after that rooms are freed for renting.

Reservations have the highest priority. It is here that the dilemma lies. Ideally, all reservations should be honored by the hotel. This could be achieved, if a hotel consistently underbooks. That way each guest could be guaranteed a room and no guest is turned away. This sounds plausible, but given the fact that a certain percentage of the reservations will be cancelled, the number of rooms left un-rented will be considerable. Some hotels rely on walk-ins to fill up the empty rooms. But walk-ins are not a very reliable way of securing a rental; besides, the number of walk-ins is usually not enough to compensate for cancellations and no-shows (Federer, 1995; Folkersma, 1995).

Assuming that there is a demand for rooms, a vacant room represents a lost opportunity. Consequently, any room left un-rented is foregone profit. Cancelled reservations can also be viewed as foregone profit. Therefore, there is a cost associated with leaving rooms un-rented. When a hotel overbooks, on the other hand, it incurs the risk of walking a guest (that is, letting a guest with a confirmed reservation go because there are no rooms available) This includes the cost of finding alternate accommodations, and more important, the loss of good will generated by turning away customers who already have reservations. The question, therefore, is what reservation policy should a hotel adopt in order to strike a balance between the two costs, of walking customers and of un-rented rooms.

Since the airline industry is more mature, compared to the hotel industry, in terms of research on overbooking, the following section will review literature on airline overbooking models followed by the literature on hotel overbooking. A summary (in table format) of all airline overbooking models is included in Appendix 1; while that of hotel overbooking models is included in Appendix 2.

3.2 AIRLINE OVERBOOKING

3.2.1 EARLIER MODELS

Research into airline overbooking dates back to 1958 when Beckman first published an article containing a mathematical model to determine the booking level (i.e. the upper bounds for reservations) that minimizes the lost revenue due to empty seats and the cost of oversales (that is, selling more seats than actual capacity). Beckman's model ignored cancellations and the derived booking level was a single number for a flight.

Kosten (1960: in Rothstein, 1985) expanded Beckman's model taking into account reservations and cancellations. Kosten's booking level is a function of the number of days yet to transpire before departure. Both models, Beckman's and Kosten's, require the cost of an oversold passenger and the probability distribution of reservations demand. Kosten's requires that of cancellations.

In the early sixties, estimating the probability distributions was not a simple task - nor is it now for that matter, but that will be left for a later discussion. The statistical data needed to derive the distributions was just not available. Consequently, Thompson (1961) developed a more implementable model which completely ignores the probability distribution of demand as well as overbooking costs. It only requires data on the cancellation proportions out of any fixed number of reserved passengers. The result is a set of conditional oversale probabilities. If a certain number, n, of passengers hold reservations at time period t before departure, the model provides the probability distribution of oversales conditioned upon accepting no more reservations from period t and on.

Thompson's model was more implementable in the sense that for each day t, one could determine an n which constrains the probability of overselling. In other words, this approach could be used to maintain a *standard* regarding oversales; for example, no more than 1 in 5000. This appealed to the airlines since they were already used to working with standards in so many of their operations (punctuality standards, mishandled baggage, etc.) (Rothstein, 1985).

More precisely, Thompson (1961) postulated that, if on day t prior to flight, there are n_t recorded reservations, then the number of cancellations, x, occurring prior to flight (including no-shows) would follow a binomial probability distribution, $b(x;n_t,p_t)$; where p_t is the probability of an individual cancellation between day t and flight time. Thompson estimated this probability by considering the different number of people already holding reservations at various dates before flight. Thompson made two assumptions in his model:

1. the probability of cancellations does *not* depend on whether the reservation is part of a group or not; i.e. groups may split up.

2. The probability of cancellation on day t is *independent* of the time the reservation was made; i.e., all reservations already recorded on day t have the same probability, p_t, of cancelling by flight time.

Assumption 2 would later receive more attention. Two separate studies, Ladany and Bedi (1977) of Iberia, and Richter (1982) of El-Al, based on two distinct data sets, supported the hypothesis. Subsequently, the phenomenon was termed the *property of forgetfulness*. Moreover, Shlifer and Vardi (1975) calculated the cancellation probability to vary from 0.8 for reservations made a few months in advance, to around 0.3 when made one or two weeks in advance.

Taylor (1962), an operations researcher at British European Airways then, adopted Thompson's approach and developed a model similar to that of Thompson's, but with a much more exact treatment of cancellations, no-shows, and group sizes. He presented no numerical examples, however. Deetman (1964) at KLM improved on Taylor's model; and subsequently, based on Deetman's and Taylor's work, Rothstein and Stone (1967) developed a model to determine booking levels which was implemented at American Airlines. It enabled the reservations department to operate with higher booking levels than before and achieve higher *load factors* (number of boardings divided by the seat capacity of departing plane) without any increase in the number of passengers denied boarding.

3.2.2 SINGLE LEG, SINGLE FARE CLASS MODELS

Rothstein and Stone (1967) modelled the overbooking process as a dynamic decision process with one fare class on a non-stop fixed flight. In short, their approach works as follows: At a given time *t* before departure and for a given number of reservations in the system, the consequences of taking an extra reservation (economic and/or service level) are evaluated. Then the dynamic decision process is solved using a sequence of independent static optimization models and the optimal booking limits are calculated.

In 1971, Rothstein expanded the model from Rothstein and Stone (1967) to a more comprehensive and exact treatment of the reservation process for a fixed non-stop flight with one fare class. The model was not implemented , either at American Airlines, or at any other airline, however (Rothstein, 1985).

Rothstein's model takes into account reservation demand, cancellations, and no-shows. The reservation procedure is viewed as Markovian sequential decision process. A discrete time approach is used and the underlying transition probabilities are non-homogeneous; i.e., time dependent. Rothstein uses dynamic programming to reach an optimal solution; i.e., an optimal booking policy.

Rothstein models two objective functions. One maximizes expected *gain* (passenger revenue minus costs of denied boardings), while the other maximizes expected revenue subject to a constraint on the *probability* of denied boardings, or the *proportion* of reserved passengers denied boarding. That is, given a proportion of denied boardings or overbooking proportion (proportion of reserved passengers who are denied boarding), the model finds an optimal booking policy. For this second optimization criterion, the cost of denied boarding need not be calculated as such, but rather a desired *proportion* of denied passengers will be set. The model does then give an imputed value for the cost of denied boardings depending on the proportion chosen. This second approach is more implementable - in the same sense as was Thompson's model in 1961. Users need only to specify a desired proportion of overbooking/oversales.

The *state* of the system is defined as the number of reservations already recorded at any time prior to flight. The length of the time period was chosen as one day; however, it is not essential that periods be of equal length, or that the day be the unit of time. The system changes states according to transition probabilities, which are a function of the demand for reservations, cancellations, and no-shows. Rothstein ignores the possibility that a passenger makes a reservation and cancels the same day. The model determines a booking policy for the span from $t=0$ back to day T, where $t=0$ corresponds to flight or departure time. Prior to day T, all reservations will be accepted.

Other than confirmed reservations, there are two additional sources of demand for seats at flight time: *standbys* and *no-records*. Standbys are people without reservations who want to board. The airline is not obliged to give service to these people and there is no revenue lost if they don't board. But, if there are empty seats on the plane, standbys are the perfect way to fill them.

No-records are people who show up with valid tickets but are not recorded in the system. These passengers are accorded the same priority as reserved customers. No-records still occur these days and are treated as reserved customers. Accordingly, Rothstein defined discrete random variables for the number of standbys and no-records and included them in his model:

<u>Optimal Booking Policy with respect to maximizing expected Gain:</u>

<u>Booking Policy:</u>

Let $k_n(t)$ = the number of additional reservations to accept on day t when n reservations are already recorded on the morning of day t, n=0,1,...., t= 1,2,...,T. A set of values, $k_n(t)$, for all relevant n and t consists a *booking policy*.

<u>Maximum Expected Gain:</u>

Rothstein assumes that the cost of passengers denied boarding is estimable and proportional to the number of such passengers. The revenue from fares minus the costs of denied boarding, b, is the *gain*, G, which Rothstein defines as a random variable. The distribution of G depends on the booking policy chosen. Let

$V_n(t)$ = the maximum expected gain achievable through any booking policy, given that n passengers are already booked on the morning of day $t \geq 1$,

$V_n(0)$ = the expected gain when n passengers with recorded reservations, plus any no-records and standbys, show up for flight.

Rothstein defines an optimal policy as one which gives $V_n(t)$ for all n and t and then he uses dynamic programming to determine this policy. By the "principle of optimality" of dynamic programming,

$$V_n(t) = \max_k V_{n,k}(t).$$

where,

$V_{n,k}(t)$ = expected gain when :

(1) the state on the morning of day t is n passengers,

(2) the policy prescribes that k additional reservations may be accepted ($k \geq 0$),

(3) an optimal booking policy is followed thereafter

The optimal policy maximizes expected gain conditioned upon having n passengers already booked on day t, for every n and t. This policy also maximizes the unconditional expected gain, defined as,

$$E(G) = \sum_n \Psi_n(T)V_n(T).$$

where $\Psi_n(T)$ is the probability that n passengers will already be booked on the morning of day t.

Maximizing Gain given a proportion of denied boardings:

To find an optimal booking policy in the previous section, one had to know the cost of denied boardings, b. Estimating the cost of denied boardings is quite difficult, however, because it depends on so many factors which are neither easily quantifiable nor identifiable. As a result, Rothstein developed another method of finding an optimal booking policy based, not on the trade off between overbooking costs and the costs of an unfilled plane, but based on a specified proportion of denied boardings. To that end, he defines the *denied boardings ratio,* ***R***, where:

$$R=0 \qquad \text{if } W \leq c,$$

$$R=(W-c)/W \qquad \text{if } W > c,$$

and where,

W is the number of passengers with recorded reservations, plus the number of no-records, arriving for flight,

and c is the total capacity of the plane.

Rothstein defines **E(R)** as the expected proportion of passengers with denied boardings. In order to maximize revenue, first a value, $\mathbf{r_o}$, for **R** is chosen. Then we maximize revenue by finding a booking policy for which **E(R)** is less than or equal to $\mathbf{r_o}$

$$E(R) = \sum_n \Psi_n (T)\, E_n(R,T)$$

$\mathbf{E_n(R,}t\mathbf{)}$ as the conditional expectation of **R**, given n passengers booked on the morning of $t \geq 1$.

Smith et al. (1992) of American Airlines Decision Technologies, developed an optimization model that maximizes net revenue associated with overbooking, by balancing the additional revenue that can be gained from selling a reservation against the cost of incurring additional oversale risk. The total cost of oversales is non-linear with a positive slope (Smith et al.,1992). As the overbooking level increases, net revenue increases to a maximum and then decreases, because at that stage, the incremental cost of an additional oversale exceeds the value of an additional reservation. The optimal overbooking level is the point where the marginal revenue gained from taking an additional reservation is equal to the marginal cost of the next oversale.

Because this model might allow too high a number of oversales, a constraint is added to limit the expected number of oversales per flight. This constraint was incorporated into the model using Lagrangian relaxation (Smith et al.,1992). In addition to accounting for the expected number of reservations, cancellations, no-shows, the model takes into consideration the probability that a passenger refused a reservation request on a certain flight, will choose another flight with the same airline (Smith et al.,1992).

3.2.3 Multiple Fare Class Models

3.2.3.1 Deterministic Models

The approach used by Rothstein and Stone (1967) was extended to flights carrying two types of passengers and to two-leg flights by Shlifer and Vardi (1975) and Richter (1982), respectively. In their model, Shlifer and Vardi (1975) first consider a single-leg flight carrying two types of passengers differing in cancellation probability but generating the same amount of revenue. Demand for the two types of passengers is assumed to be independent. Then they consider a two-leg flight carrying one type of passenger, where passengers on the long leg generate more revenue than those on the short leg.

The single-leg, single-class model in Shlifer and Vardi (1975) maximizes the expected revenue from a flight subject to a maximum "allowable" probability, or ratio, of overbooking (total number of show-ups exceeding plane capacity), the ratio of the loss from rejecting a customer over the gain from carrying one, and a maximum allowable ratio of expected rejections over expected show-ups. The model does not take into account changes in demand or cancellations, however.

Based on the same criteria used in the single-leg, single-class model mentioned above, Shlifer and Vardi (1975) determine a booking policy for two types of passengers on a single leg, given a certain number of reservations already booked at time *t*. A booking policy gives the additional number of bookings of both types to accept (Shlifer and Vardi, 1975). The same strategy is used to reach a booking policy for customers on a two-leg flight. The decision rules developed according to these criteria were implemented at EL-AL airlines in 1972-73 (Shlifer and Vardi, 1975).

3.2.3.2 Dynamic Models

Alstrup, Boas, Madsen, & Vidal (1986) expanded Rothstein's (1971) dynamic model into one for a fixed non-stop flight with *two* types of passengers (instead of one type). Moreover, they modelled an aircraft with a flexible cabin. This is a cabin with an easily adjustable dividing curtain, which permits a flexible allocation of cabin space between EURO (C), and tourist (M) classes. The existence of the flexible cabin divider implies that all practical optimization can now be based on the simplified assumption of one aircraft that can be split into two classes at a late stage in the computation (Alstrup et al., 1989). The rear part of the cabin may be used for M-class (tourist) only, whereas the front part may be divided between C-(EURO) and M-classes. The cabin divider is moved to separate the two sections after passenger check-in is complete.

In Rothstein (1971), each fare class would have to be considered independently, and a separate calculation done for each class; in other words, an optimal booking policy is found for one class with the capacity of the plane taken as the number of seats allocated to that particular class, then another separate calculation is done for the other class where capacity in this case would be the number of seats allocated to these other fare class. In addition to the situations considered by Rothstein (1971), the model in Alstrup et al. (1986) takes into account the following:

- C-class passengers have priority
- Denied boardings of both, C- and M-class passengers.
- Upgrading of M-class passengers; that is, an M passenger sits in front part of the cabin, behind the divider, has more leg room, but receives M-class service.
- Downgrading of C-class passengers; that is, C passenger sits behind the divider and receives M-class service.
- Empty seats in both, the rear and front of the cabin.

They use the same time convention, the same booking policy definition, and the same decision variables as Rothstein (1971). Moreover, the basic assumptions - about cancellations and demand - are the same as Rothstein's. Whereas Rothstein has an objective function of maximizing expected gain or revenue, the objective function in Alstrup et al. (1986) is the minimization of total losses. They use a two dimensional generalisation of the newsboy problem to find the expected cost at departure. This cost is a function of the number of C- and M-class passengers booked before flight (Alstrup et al., 1986). More precisely,

total loss = maximum possible profit, given a C and M seat allocation, minus actual profit.

and **maximum profit** = profit when departing C and M passengers exactly equal the number of seats allocated to C and M, respectively.

The state of the system is defined by (BC,BM) the number of passengers already booked on C- and M-classes respectively. The decision variables at time period *t* are UC and UM, the number of additional reservations that can be accepted for C- and M-class passengers respectively. A booking policy is the set of values for UC and UM for all relevant BC, BM, and *t*. According to the data from SAS airlines, Alstrup et al.(1986) estimated the probability distributions of reservations, cancellations, and no-shows as Poisson, binomial, and normal, respectively. However, they used normal approximations for their calculations.

The model developed by Alstrup et al. (1986) was tested at SAS airlines. It was compared to the heuristic already being used by the airline at that time. The results showed the newly developed model to be superior to the heuristic in terms of net revenue. The difference in potential net revenue increase by using the new model would be around US$2 million per year. Moreover, the comparison showed that the heuristic allowed too low an acceptance level for C-class (Alstrup et al., 1989). At the time (1986), however, SAS did not invest in implementation of the new model, since the potential gains from the new model did not offset the required investment. However, based on the comparisons with Alstrup et al.'s optimal model, corrections were made to the heuristic and SAS reaped part of the potential revenue gains (Alstrup et al., 1989).

The estimated running time for the optimal dynamic programming model of Alstrup et al. was around 100 hours (Alstrup et al., 1989). This was deemed too long; so two main adjustments were made to the model in order to reduce the running time without threatening the optimality of the results (Alstrup et al., 1989). The adjustments Alstrup et al. (1986) make to their model are similar to the suggestions made a few years earlier by Hersh and Ladany (1978). After the adjustments, the running time for a 110 seat airplane was reduced to around one minute. The two main changes were as follows.

The allowed range of each state and decision variables were analysed and made as small as possible. To calculate the losses at day *t*, the losses at day *(t-1)* for all the possible states should be known. It is not necessary, however, to analyse combinations of booking levels where more reservations will increase the loss. This lead to the following rule: given a certain state, (BC,BM), if it is known that the optimal values of the decision variables are zero, then the values of the decision variables for any state ($BC' \geq BC$, $BM' \geq BM$) will also be equal to zero. According to other similar strategies, Alstrup et al. (1986) developed rules that decreased the number of feasible values by a factor of one hundred.

The second adjustment Alstrup et al. (1986) made was to assume that customers book or cancel as clusters of individuals instead of single individuals. Consequently, the state variables are aggregated to multiples of the chosen cluster size, STR. Then linear interpolation is used to fill in the decisions for the missing values. An STR of five was used initially, and then sensitivity analysis was used to see how critical the cluster size would be. Alstrup et al. (1986) stress that this was just a measure to decrease the effect of dimentionality of the problem and has nothing to do with passenger behavior or policies.

3.3 HOTEL OVERBOOKING

3.3.1 DYNAMIC MODELS

Rothstein (1974) was probably the first to address the problem of hotel reservations. He based his hotel reservations work on his model for the airline overbooking problem (Rothstein, 1971) discussed in the previous section. Although the airline and hotel industries are similar, they are not identical. To accurately portray the hotel environment, Rothstein's (1974) model would have to account for two main situations:

(1) Single vs. double occupancy. Hotel room rates depend on the number of occupants. An airline seat can be used by exactly one person.

(2) Multiple-day stays. Hotel reservations are made for one or more consecutive nights. A daily airline flight would be analogous to a hotel where guests stay only one night.

Rothstein (1974), however, ignored the distinction between single and double occupancy, and used a unique room rate based on a weighted average of rates. Although most hotels these days only have double rooms, the rates still differ. The room per se does not change, but if it is one person, then they will pay single rate, and if it is two people, they pay the double rate.

Changing the model to account for point two is analogous to the multiple-flight-leg situation in the case of airline overbooking. This would imply fundamental changes to the airline model. New types of state definitions and probability distributions would be required. What Rothstein did was assume that a request for a multiple-day reservation is equivalent to several independent requests for single-day reservations - with similar assumptions for cancellations and no-shows.

This means that the model is applied independently for each night and booking policies appropriate for each day of a multiple-day stay, considered simultaneously, would determine whether the entire reservation is accepted. A more exact model of the hotel system would include the probabilities of reservation requests made for stays of varying durations and the probabilities of partial or complete cancellations of those stays.

Ladany (1976) expanded Rothstein's model to include different types of rooms (double and single) and different allocations; that is, a double room could be rented to someone who wants a single room. However, he did not alter the second assumption. Ladany's model assumes that a guest rents for one period only. Similarly to Rothstein's, the allocation policy was derived as a sequential decision process.

The objective of the model is determining the operating policy for assigning guests to single- and double-bed rooms in order to maximize each day's profit. The booking period prior to the rental date is divided into t decision periods , not necessarily of equal length. At the start of each period, t, a decision must be made about the maximum number of additional bookings, k_t and h_t, that can be allowed during period t for double and single bed rooms respectively. This decision is based on:

(1) The number of double- and single-bed rooms in the hotel.

(2)The contribution to profit per day from rental of double and single rooms.

(3) The overbooking penalty for double single rooms.

(4) The probability distribution of future demand, cancellation and no-shows for the two kinds of rooms.

(5) The probability distribution of walk-in customers.

(6) The number of double and single rooms reserved.

3.3.2 Other Approaches

Williams (1977) examined the question of determining a booking policy in a somewhat different way. He divided hotel room demand into three sources: stayovers, reservations, and walk-ins and assessed reservation policies in a peak demand period, *D*. The general approach was to regard room demand for each source as a random variable and to build probabilistic models of the number of rooms demanded from each source.

The show rate for reservations is the percentage of those who have a reservation and do actually show up. Therefore, taking the reservation show rate as a random variable (since reservations are themselves random) with a known probability distribution, we can find the law governing reservation room demand.

Williams assumed that stayover demand depended, if only implicitly, on occupancy on $D - 1$; with the assumption that $D - 1$ is known. This becomes quite unrealistic with long lead times , however.

Then the show rate for scheduled stayovers is those guests who actually stay, while the show rate for unscheduled stayovers is those scheduled check-outs who stay. Therefore, given the number of occupied rooms on $D - 1$, we can derive the probability law for stayover demand on *D*.

Walk-ins do not affect the model since there is no cost associated with turning away a walk-in. Combining the probability laws governing reservation and stayover demand, we get a probability law for total demand. The criteria Williams (1977) used for determining the number of reservations to accept are: (1) total expected cost, (2) expected cost of underbooking and expected number of walks, (3) expected cost of overbooking and expected number of walks.

Since the model assumes peak demand, any underbooking means a lost opportunity cost. When the number of rooms reserved plus stayovers, plus walk-ins is less than capacity, an opportunity cost is incurred. If it is greater than capacity, then an overbooking cost is incurred. Therefore calculate the total expected cost of each reservation policy, and chose the policy with the lowest total expected cost. For cases 2 and 3, depending on the unit cost of under/over booking, we will get different expected costs and different number of walks.

4. DISCOUNT ALLOCATION

Belobaba (1989) defines seat inventory control as the "practice of balancing the number of discount and full fare reservations accepted for a flight, so as to maximize total passenger revenues and/or load factors". Load factors can increase if too many seats are sold at a discount price. This will cause revenue per passenger to decrease, however. Delta airlines estimated that selling just one seat per flight at full fare can increase annual revenues by over $50 million (Llyod's Aviation Economist, 1985). Seat inventory control is viewed as more important than the actual prices charged, for it enables an airline to influence revenue on a flight-by-flight basis.

The airline seat inventory management problem is both, probabilistic and dynamic in nature (Belobaba, 1987a). It is probabilistic because there exists uncertainty about the ultimate number of requests that an airline will receive for seats on a future flight. More specifically, there is uncertainty about the number of requests that will be received for the different fare classes offered on that flight. The dynamic element stems from the fact that the number of reservation requests accepted for a flight will change from day to day (due to cancellations), potentially affecting estimates of requests still to come and in turn, the optimal allocation of the remaining seats among the fare classes. A summary (in table format) of all discount allocation models reviewed in this section is included in Appendix 3.

4.1 DYNAMIC PROGRAMMING

Hersh and Ladany (1978) developed a dynamic model for optimal seat allocation for two leg flights. Passengers booking for different legs are treated as two separate types of customers, each generating a different revenue. Passengers going on the long leg generate more revenue than those on the shorter leg. At the begging of each decision period t, prior to departure, a decision must be made on the maximum number of additional bookings to be taken for each type of passenger for that period. The decision is based on:

- the capacity of the plane
- the contribution to profit of each type of customer
- the overbooking penalties for each type of customer
- the probability distributions of demand for reservations, cancellations, and no-shows for each type of customer
- the probability distribution of stand-by and wait-list passengers
- the number of reservations already received for both types of passengers[1]
- the number of passengers already booked on the two legs
- preference in allocating seats is accorded to passengers with the higher contribution to profit

The optimal allocation policy is derived as a sequential decision process. Based on the above variables, the objective is to maximize the total expected contribution to profit at the end of period t. Similar to Rothstein (1971), Hersh and Ladany's (1978) model ignores the possibility of a person booking and then cancelling in the same period. Although the model was not applied at any airline, it was validated using a hypothetical six-seat aircraft. A computer program written in FORTRAN was used to find the optimal allocation procedure. The

[1] this is the number of requests already received (which are either booked or refused); and may be used as an indication of the changes in future demand patterns. The authors then use a Bayesian approach to revise the demand distribution

running time was approximately 7 seconds on an IBM 370. Based on this, the running time for a one-hundred-seat airplane would be around 3600 seconds (Hersh and Ladany, 1978). Considering this run time, the authors suggest that policies be grouped in sets of 5 to 10 seat intervals, in order to reduce computation time. Further suggestions to reduce computation time were: grouping of demand data and the reduction of decision periods (Hersh and Ladany, 1978).

Based on Hersh and Ladany's (1978) model, Ladany and Bedi (1977) restructured the model and decreased the number of demand variables from four to two, thus rendering it a non-Bayesian model. The authors argue that the gain in feasibility and computation time offsets the potential loss in revenue due to the reduction in the number of variables (Ladany and Bedi, 1977). The authors validate their model for using a six-seat hypothetical aircraft. Their results showed that, in comparison with Hersh and Ladany's (1978) original model, this revised model yields a decrease in total expected profit of only 4.83%. Computation time was less than 0.1 seconds and the computer memory requirements were drastically reduced (Ladany and Bedi, 1977).

4.2 Marginal Revenue Analysis

A very common approach to solving the seat allocation problem is marginal revenue analysis. Seats are allocated between fare classes such that the marginal expected total revenue with respect to additional seats in each class is equal to zero. At optimality, total expected flight revenue cannot be increased by taking a seat from class 1 and allocating it to class 2. The expected marginal seat revenue for each class i will be equal across all relevant classes, but not necessarily equal to zero because of the capacity constraint (total number of seats allocated among classes cannot exceed capacity of plane).

The very simple model is where allocations are made only once at the beginning of the reservation period; this is referred to as the static model. Due to the dynamic nature of the reservations process, however, a static model is not sufficient. Furthermore, in most cases demand for each fare class is assumed to be independent; and this may not always be the case.

Littlewood (1972) developed what is probably the simplest, yet most applied, model to determine the optimal allocation of seats between two independent fare classes. He equated the marginal revenue in each of the two fare classes to find the revenue-maximizing seat allotments for a one leg flight applied to a dynamic context. Total flight revenue would be maximized by closing down the low fare class to additional bookings when the *certain* revenue from selling another low fare seat is exceeded by the *expected* revenue of selling that same seat at a higher fare (Littlewood, 1972). Littelwood based his model on three assumptions:

1. low fare passengers book first
2. no cancellations of bookings (this means that a reservation request implies certain revenue).
3. rejected requests represent revenue lost by the airline

4.3 Variations On Littlewood

Bhatia and Parekh (1973) of Trans World Airlines, as well as Richter (1982) of Lufthansa, extended Littlewood's model. Both extensions, however, proved to be essentially equivalent to the original model (Belobaba, 1987). Mayer (1976) of El-Al performed a sensitivity analysis of the simple model which showed that the greater the difference between rates of the two fare classes, the more sensitive the total expected flight revenue is to a non-optimal allocation of seats. It was found that the decrease in expected revenue is smaller when too many seats are allocated to low fare, than when too few seats are offered. Mayer (1976), as well as Titzer and Griesshaber (1983) addressed assumption 1, that low fare customers book first. Both studies showed that the booking behavior assumption should not have significant impact on the optimal seat allotments determined by the model - as long as demand for each fare class is *independent*.

Buhr (1982) of Lufthansa applied the expected marginal revenue approach to a two leg flight with only one fare class. He assumed independent demand for each origin-destination (OD) market. Wang (1983) of Cathay Pacific extended Buhr's formulation to include multiple fare classes. Wang (1983) also assumed independence of OD markets and of fare class demand. When the problem is expanded to multiple classes and multiple markets, however, it becomes more difficult to find the optimal points analytically (Belobaba, 1987a).

4.4 NETWORK FLOW APPROACH

Most of the attempts at finding an optimal seat allocation have modelled the system as either a sequential decision process and then used dynamic programming to solve for the optimal solution, or as a deterministic problem where booking limits are derived subject to several constraints. Glover, Glover, Lorenzo, and McMillan (1982), on the other hand, modelled the system as a minimum cost (maximum profit) network flow problem with special side constraints. One set of arcs (forward arcs) in the network corresponds to segments of flight; the other (back arcs) corresponds to different passenger itineraries (PI), or routes, based on fare classes.

Flow on the forward arcs represents the number of passengers on a flight segment, while flow on the back arcs represents the number of passengers on each passenger itinerary at each of the different fare classes. Flow on each forward arc is limited by the capacity of the plane; and flow on each back arc is limited by the demand for the different passenger itineraries and the fare class that the arc represents. A unit flow on each back arc, is an increase in revenue which is equal to the price of a ticket on the class which that arc represents (Glover et al., 1982). One short coming of the model is the deterministic estimates of demand for each PI/fare class combination used as inputs. The model was implemented at Frontier Airlines. It accommodated up to 600 daily flights and 30,000 PI in five fare classes (Glover et al., 1982).

D'Sylva (1982) of Boeing expanded Glover et al.'s (1982) algorithm to include stochastic demand. He used a piece-wise linear approximation of the expected revenue curve in an integer programming formulation. A comparison of the solutions, to the probabilistic and the deterministic formulations, showed that the deterministic approach (that is, Glover et al.)

overestimated revenue by around 12%, and that the probabilistic solution produced a 5% higher expected revenue (D'Sylva, 1982).

Wollmer (1985) of McDonnell Douglas, as well as analysts at Boeing[2] have developed mathematical programming techniques that take into account probabilistic demand for each fare class. Given a constant fare and stochastic demand, the expected- revenue objective function for each class is non-linear. Therefore, a simple linear programming approach is inadequate. McDonnell Douglas analysts derived an integer programming formulation for the single-leg seat allocation problem by using binary decision variables. Each variable, X_{ik}, represents a combination of fare class i and seat k on a flight leg. A 150-seat plane, four fare class combination, would thus, require 600 such variables. Associated with each is the marginal expected revenue of selling the k^{th} seat in class i, derived by multiplying the average fare level in class i by the probability of selling k or more seats in that class.

Wollmer (1986) developed a network flow model to include multiple fare classes and multiple flight legs (multiple passenger itineraries). The number of binary decision variables increased rapidly, however. Wollmer (1986) suggests that only a few of the arcs need to be included at any one time for consideration under the longest path criterion. At most, two arcs: the shortest (lowest revenue) arc with an existing flow of one, and the longest (highest revenue) arc with a flow of zero. This would mean that a solution algorithm need only solve a set of longest path problems for a relatively small network. At each iteration, the expected marginal revenue of each fare class/PI combination is calculated, the largest value found, then a

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[2] The optimal partitioning of an airplane's seating capacity (1982)
Unpublished internal report, Boeing Commercial Aircraft Co , Seattle,WA in Belobaba (1987)

seat allocated to that path. The expected marginal revenue of each arc (length) are then revised and the procedure repeated.

4.5 The Expected Marginal Seat Revenue Model

4.5.1 The Basics

The assumed independence of fare class inventories is a major shortcoming of all the models mentioned above. The models discussed so far do not take into account the relationship between overbooking and seat inventory control. Given the wide use and revenue enhancements of overbooking, a comprehensive seat allocation model must take overbooking into account. Moreover, the assumption that a rejected request is lost by the airline may not necessarily be valid. The refused passenger might accept a reservation at a higher fare, or on another flight by the same airline. A last potential shortcoming is the assumed independence of fare-class demand.

Today, most airlines use a *nested* fare class structure; that is, fare classes share a common inventory of seats, making it possible to take seats from lower fare classes and use in higher classes. The optimal seat allotments derived by the methods discussed so far will not necessarily maximize revenue in a nested fare class structure (Belobaba, 1989). The advantage of a nested fare structure is that the airline will not have to limit itself with a certain number of seats for each class. A nested structure is more flexible and more profitable.

The more accurately a model reflects the real situation, the more precise and reliable its results. In this day and age technology is advancing very rapidly. Competition is fierce, and companies can no longer afford to make do with what they have. They have to strive for excellence as well as be the first to achieve it, or they will be left behind. This means that an optimal seat allocation model has to portray the system as realistically as possible, and ultimately, replicate it exactly.

Belobaba (1987a) fills the gap with respect to several of the aforementioned issues. Belobaba (1987a) does not, however, address the issues of dependent fare class demands or seat allocation on multiple leg flights. He used the expected marginal revenue approach to determine the allocation of seats in a nested fare class structure on a single leg flight. He named the model the Expected Marginal Seat Revenue (EMSR) model. The model was developed for implementation at Western Airlines. It takes into account overbooking, cancellations, future demand. The model also considers the recapture probability, the probability that a rejected request will buy a seat in the next higher fare class.

The static problem is establishing fare class booking limits at the beginning of the booking process. The dynamic problem is revising those initial limits based on additional information provided by the actual bookings as departure day approaches and re-applying the static model. The static model assumes:

1. lower fare classes book first
2. nested fare class structure
3. independent demand for different fare classes
4. no cancellations and/or no-shows
5. rejected requests are lost by the airline
6. no overbooking

The problem, therefore, is to determine how many seats *not* to sell in the lowest fare classes and retain for possible sale in higher classes closer to the day of departure (Belobaba, 1989). This follows from assumption one, since it is assumed that the lower fares book first. The decision model determines the *protection levels* for higher fare classes, which can be converted into booking limits on the lower classes. In other words, it determines how many seats should be protected for sale to higher fare classes only. In a nested fare class system the booking limit is the maximum number of seats that may be sold at that price or to that class. This includes *all* lower fare classes and their own booking limits. The booking limit on the highest fare class is the capacity of the plane. The protection for a fare class is that class' booking limit minus the booking limit of the next lower class.

Let $p_i(r_i)$ be the probability density function of the total number of reservation requests , r_i, received for seats in class *i* by the close of the booking process for a scheduled leg departure. Let S_i be the number of seats allocated to a particular class *i*. This might not exceed the number of actual requests for that class, and thus resulting in rejected demand, or *spill*.

The cumulative probability that all requests for a class will be accepted, therefore, is a continuous function of Si:

$$P_i(S_i) = P[r_i \leq S_i] = \int_0^{S_i} p_i(r_i)\, dr_i$$

Conversely,

$$P[r_i > S_i] = \int_{S_i}^{\infty} p_i(r_i)\, dr_i = 1 - P_i(S_i) = \bar{P}(S_i)$$

$\bar{P}(S_i)$ is the probability of spill, that is, receiving more requests than S_i. $EMSR_i$ is the expected marginal seat revenue for class *i* when the number of seats available to that class is increased by one. This implies that the EMSR of the Si^{th} seat in class *i* is the average fare level, in class *i*, f_i, multiplied by the probability of selling S_i or more seats: An average fare level is used since there will be some variation in fares on similar flights.

$$EMSR_i(S_i) = f_i \,.\, \bar{P}(S_i)$$

The value of the $EMSR_i(S_i)$ depends directly on $\bar{P}(S_i)$, the probability that the Si^{th} seat will be sold.

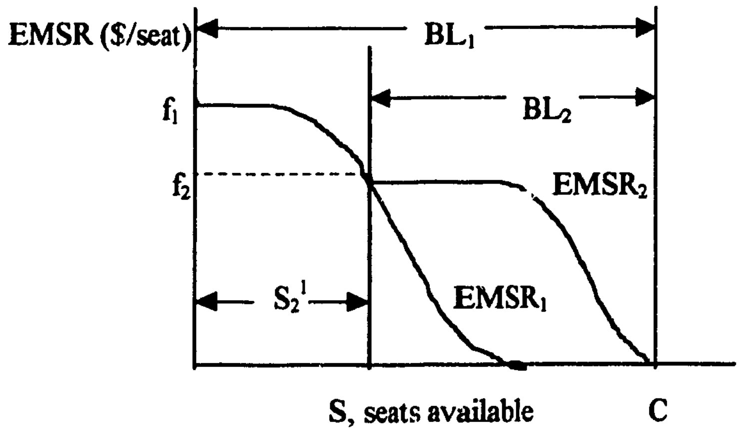
4.5.2 Two Fare Classes

There are two classes, 1 and 2. The fare in class 1 is higher; consequently, priority is given to class 1 passengers. Let S_2^1 be the number of seats protected from class 2, and available exclusively to class 1. The optimal value of S_2^1 is that which satisfies the condition:

$$EMSR_1(S_2^1) = f_2$$

Graphically, the optimal value of S_2^1 is the point at which the $EMSR_1(S_1)$ curve intersects f_2. Since class 1 is the highest fare class, then its booking limit, BL_1, is the capacity, C, of the plane. Consequently, the booking limit on class 2, BL_2, is : $BL_2 = C - S_2^1$

FIGURE 1.



This solution will maximize the expected revenue in cases where booking limits are set at the beginning of the reservations process; this is known as static inventory control (Belobaba, 1989). A class 2 request will be rejected when BL_2 is reached. At that point, the expected revenue for all remaining seats will be greater than f_2, the average fare for class 2.

4.5.3 Multiple Fare Classes

For *k* fare classes, the optimal values of S_i^j must satisfy:

$$EMSR_i\,(S_j^i) = f_j, \qquad i<j,\ j=1,\ldots,k.$$

The total number of comparisons required will be $\frac{k(k-1)}{2}$.

These protection levels determine booking limits for each class *j*,

$$BL_j = C - \Sigma_{i<j}\, S_j^i\,.$$

This means that all seats with an expected marginal revenue greater than f_j should be held back from sale to class *j*. If BL_j is negative, it means that class *j* should not be offered at all. Therefore,

$$BL_j = \max\,[0,\ C - \Sigma_{i<j}\, S_j^i].$$

The nested protection level for class j, NP_j , is the incremental number of seats protected for class j,

$$NP_j = BL_j - BL_{j+1}$$

$$C = \sum_{j<k} NP_j + BL_k.$$

4.5.4 DYNAMIC APPLICATION OF THE EMSR MODEL

The dynamic application of the EMSR model is the repetitive use of the static model, but with revised input data. The objective is to determine the optimal fare class limits for the time *remaining*, irrespective of the optimality or non-optimality of the booking decisions already made (Belobaba, 1989).

Booking limits are revised on a regular basis as departure day approaches. The additional information used is the actual bookings already accepted. Because of the assumption of no cancellations, a booking implies revenue. Therefore, incorporating actual bookings, as they occur, into the decision framework reduces the uncertainty associated with estimates of expected demand (Belobaba, 1989).

Let r_i^t be the number of requests for class i between days t and 0 before departure. Then, let $P_i(r_i^t)$ be the probability density of requests from day t onward; and $\bar{P}_i^t(S)$ be the probability of receiving S or more requests for class i in the time remaining.

Estimates of $\bar{P}_i^t(S)$, derived from $P_i(r_i^t)$, would be required for all relevant values of S. The optimal seat protection level for class 1 relative to class 2, for the period remaining, for example, would be $S_2^1(t)$, such that,

$$EMSR_1^t[S_2^1(t)] = f_1 \cdot \bar{P}_1^t(S_2^1) = f_2.$$

This would be used to find the revised booking limit for class 2, BL_2:

$$BL_2(t) = C - b_1^t - S_2^1(t),$$

where b_1^t is the number of bookings already accepted for class 1, at time t,

$C - b_1^t$ is the maximum number of seats available,

and S_2^1 is the protection level for class 1.

If class i is higher than class j, then in general terms:

$$EMSR_i\,[S_j^i(t)] = f_i \,.\, \bar{P}_i^t(S_j^i) = f_j$$

$$BL_j(t) = C - \sum_{i<j} b_i^t - \sum_{i<j} S_j^i(t)$$

$$NP_j(t) = BL_j(t) - BL_{j+1}(t)$$

$$= \sum_{i\leq j} S_{j+1}^i(t) - \sum_{i<j} S_j^i(t) + b_j^t\,.$$

4.5.5 CANCELLATIONS & OVERBOOKING

Belobaba (1987a) looks at the cost of a cancellation as the opportunity cost of having removed a seat from the available inventory and then not having received any revenue for it. The adjustments made to the EMSR model in order to account for cancellations are as follows.

Demand inputs are the estimates of the densities of requests for reservations. However, since the possibility of cancellation exists, the revenue associated with a booking cannot be treated as though it will always be realised. This means that there is a probability that not all revenue will be achieved. The expected revenue associated with accepting a request in class i is less than the actual fare level in that class. Therefore, the overbooking percentage for each class determines the extent to which the expected revenue from a booking is reduced by this uncertainty (Belobaba, 1987a). In other words, revenue levels are deflated by the overbooking factors.

Let OV_i be the overbooking factor for class i, where $OV_i \geq 1$. The optimal protection level for class 1 from 2, S_2^1, must satisfy

$$\bar{P}_1(S) \cdot f_1 \cdot \frac{1}{OV_1} = f_2 \cdot \frac{1}{OV_2}$$

The generalised decision rule is expressed as:

$$EMSR_i(S_j^i) \cdot \frac{1}{OV_i} = \bar{P}_i(S_j^i) \cdot f_i \cdot \frac{1}{OV_i} = f_j \cdot \frac{1}{OV_j}$$

When overbooking is introduced, the protection levels, S_j^i, no longer represent physical seats; on the contrary, they now represent *reservation spaces.* This complicates the derivation of the optimal booking limits because optimal protection levels are expressed in reservation spaces, while capacity is expressed in actual seats.

The simplest case is when all overbooking factors are equal across all classes, thus dropping out of the equation. The second case is when one single overbooking factor is applied to the whole aircraft. Booking limits for each class may be derived after the overbooking target is established for total capacity:

$$C^* = BL_1^* = OV.(C)$$

$$BL_j^* = C^* - \Sigma_{i<j} S_j^i.$$

Each fare class may be overbooked by the same percentage, and C^* will be the same regardless of the class mix actually booked. Although incorporating different overbooking factors for the different classes would make the model more responsive to changes in the fare class mix bookings accepted, it would also make the model much more complicated.

4.5.6 ALTERED ASSUMPTIONS

The original model assumes that a rejected request is lost by the airlines. Belobaba (1987) revisits this assumption and proposes other outcomes to a rejected request. That a rejected request in class i is lost, is not the only possibility. A rejected request may result in :

1. A vertical shift to a higher class on the same flight.
2. A horizontal shift, that is, same class but different flight.
3. A booking loss (which is the first case).

Since Belobaba (1987a) was addressing the problem of managing inventory for a single flight leg, possibility number 1 is of most interest to an airline. Let $P_i(v)$ be the *upsell probability*, or the probability of a vertical shift, v, in other words, the probability that a passenger refused a request for class i, will accept a booking in the next highest class ($i-1$).

Assuming no overbooking or no-shows, if a class 2 request is received and accepted, the revenue realised is f_2. If, on the other hand, the request is refused, the expected revenue associated with the denied passenger accepting a vertical shift is: $P_2(v) \cdot f_1$. Therefore, the incremental protection level for class 1 has to take into account this potential revenue when class 2 is closed. This translates into additional seats protected for class 1, V_2^1, which can be taken by class 1 passengers or by refused class 2 passengers. That is,

$$EMSR_1(S_2^1 + V_2^1) = f_1 \cdot \bar{P}_1(S_2^1 + V_2^1).$$

If the upsell probability, $P_2(v)$, is greater than zero, then the incremental expected revenue associated with a potential vertical shift from class 2 to class 1 may be realised only if the seat is not purchased by a class 1 passenger. Therefore the optimal value of V_2^1 must satisfy

$$EMSR_1(S_2^1 + V_2^1) \cdot [1 - P_2(v)] + P_2(v) \cdot f_1 = f_2,$$

where $EMSR_1(S_2^1 + V_2^1) \cdot [1 - P_2(v)]$ is the probability that a class 1 request will be received, and $P_2(v)$ is the probability that a vertical choice shift is accepted. Given that BL_2 is reached, the combined expected revenue from each additional seat protected for class 1 will be greater or equal to f_2. If BL_2 is not reached, the additional protection, V_2^1, has no impact in a nested system.

$$BL_2 = C - S_2^1 - V_2^1$$

Incorporating overbooking factors implies that $(S_2^1 + V_2^1)$ is treated as S_2^1 in the previous case. The impact of including more than one upsell probability, $P_i(v)$, is the increase in protection levels for the higher classes. Each lower class booking limit decreases by the incremental protection level required to account for the possibility of vertical choice shifts to the next highest class.

5. Current Trends in the Hotel Industry

5.1 General Findings

In an effort to learn more about the role of yield management in the hotel industry, different hotels in Montreal were studied. Eight hotels were visited and interviews with the general manager, rooms division manager, or both, were conducted. Among other questions, the interviewees were asked about their yield management systems, how they defined yield management, about their forecasting systems and overbooking policies. The first part of this section will discuss the general trends and views gathered from the interviews, the second will elaborate on the individual hotels.

5.1.1 Yield Management

All the hoteliers agreed that the goal of yield management is maximizing revenue. Each hotel may have a slightly different approach in achieving this goal, but they all agreed that increasing revenue was the ultimate goal. Most use yield management as the main operating strategy.

5.1.2 Overbooking

The second most noticeable thing was the lack of consistent and structured overbooking policies. Most all hotels base their overbooking decisions on experience and personal judgement. It was noticed that upscale and luxury hotels tend to more cautious about their overbooking compared to other hotels. It should be mentioned, however, that some of the hotels which do overbook more readily are the ones with a more structured overbooking systems. Having more structured overbooking systems probably allows them to overbook more confidently.

5.1.3 Pricing

Rates in most hotels are set based on market analysis. Hotels are no longer using a cost-plus approach (as discussed at the beginning of the thesis). More and more, hotels are looking at real economic indicators, and more importantly, how much the customer is willing to pay.

5.1.4 Forecasting and Data Gathering

From what this author learned, none of the hotels visited uses a mathematical forecasting model such as time series analysis. It is not that they do not use relevant inputs, but rather the way they analyse those inputs is not based on any mathematical forecasting model. All hotels build up their forecasts in teams. Top executives meet and discuss the different inputs, from the previous year's trends to the current year's economic situation and city wide events. The forecast is based on the previous year's data. The influence of each of the other factors is weighed and the forecast is adjusted accordingly. The potential flaw in this approach is that the previous year's forecast may not be a good base to start with. If the hotel's policies, whether overbooking, or allocation or pricing, are faulty in any way, last year's results will not be as good as they could be. Consequently, basing the current year's forecast on the results of previous years may perpetuate the error.

In order to develop more sophisticated forecasting methods, however, a lot of relevant data on demand variables is needed. This is an area where most hotels are in need. Most, if not all, hotels have a property management system (PMS) which is probably capable of collecting this kind of data. Currently, however, these systems are not geared towards collecting all this demand data.

5.1.5 LOOKING AHEAD

Most hoteliers agreed that there is a need for developing more sophisticated yield management systems. In most cases, there is no documentation; everything is confined in the minds of those few executives who know what they are doing. If any of those people leaves, it will take a long time for their successor, no matter how talented he/she is, to reach his/her predecessor's level of expertise. There is a need also in the area of systematic demand analysis.

5.2 Hotel du Parc[3]

5.2.1 Background

Hotel du Parc originally opened in 1976. It has 445 rooms and 70,000 sq. feet of conference space. As a result, conferences make up around 50% of its business; the rest is transient and some business travel. It is a very computerised hotel. Its central property management system (PMS) is interfaced with its sales and catering system.

5.2.2 Yield Management

Yield management was introduced at Hotel du Parc four years ago when a new management team took over. Yield management is a tactic, rather than a strategy for Hotel du Parc. The global, more encompassing, strategy is Revenue Management. Because the hotel deals with such a big volume of conference and tour and travel business, it allocates a certain number of rooms exclusively for that segment. This leaves only a few rooms per day to practice yield management on.

Marketing, Sales, Rooms Division, the Financial comptroller, and the general manager are all directly involved in yield management. There is a *yield* meeting every ten days where an outlook for a period of 40 days is prepared.

Yield management can only be practised around one hundred days a year for Hotel du Parc, since it is useless with less than a 60% occupancy level. Under 60% occupancy, the city wide demand is so low that a hotel cannot sell a room at a higher rate. A potential customer sitting in Toronto, for example, decides to come to Montreal over Easter (when city wide

[3] A Federer and E Dager, (1995)

demand is low). She calls up Hotel du Parc to find out the rates. If they quote $125, when the average rate across town is $75, that customer will go to another hotel.

During those slow months, instead of lowering their rates, most hotels usually layoff half of their employees, in the hope of loosing as little money as possible. Hotel du Parc, on the other hand, has a different strategy. It would rather drop its rates by up to 60% and drive its volume and revenue-per-available-room (REVPAR) up, and not lay off employees.

Its prices are based on a monthly "market capture analysis", which in turn is based on volume (expressed in occupancy level), average rate, total revenue, and REVPAR. Furthermore, information on other hotels in the market is gathered and used in the analysis.

5.2.3 Market Segmentation and Forecasting

Hotel du Parc divides its market into two main segments: Group and Transient, each of which is further subdivided into six different subgroups. Forecasting for the Transient segment is done by the Rooms Division department, while that for the Group segment is done by Sales and Marketing. Daily forecasts for a period of 90 days, as well as one year, are prepared and updated daily for both segments.

Forecasts are built from information on the same month the previous year, the previous month of the *current* year, as well as other inputs such as the economic situation, city wide trends, the competition, etc.. All this information is entered into a program on LOTUS which then gives the forecast in terms of occupancy levels.

A booking horizon of 90 days is preferred to a shorter one, say of 60 days, because if the hotel is far from achieving its forecasted budget (in terms of occupancy) for a month, it is very hard to make up demand in such a short period of time. Let's suppose we are in February and the budget for March is 68% occupancy, but the books show an actual occupancy of 20%. This means that the hotel has to make up the remaining 48% in only two months. This is not always feasible; consequently, the 90-day forecast.

5.2.4 OVERBOOKING

For Hotel du Parc, overbooking is a very important part of revenue management. There is a set overbooking limit of 50 rooms per day, and management are the only ones who can override that limit. Incorporated into their reservation system are *triggers* which signal when to overbook for a particular market segment. These triggers are based on daily occupancy levels per market segment, cancellations and no-shows, as well as the percentage of non-guaranteed reservations per segment. Reservation agents are trained on the triggers; that is, when they can overbook. The actual overbooking level is decided on by either the general manger or the rooms division manger. In the end, overbooking is still a matter of style. A lot of it is based on experience and personal judgement. Some people are bigger risk takers than others.

Suppose half the hotel is full with a convention whose specific pick-up rate (those who do actually show up) is known 20 days in advance. The rest of the hotel is booked with transient guests, 17 of which have non-guaranteed 6 p.m. reservations. One person might overbook 15-16 rooms, even if that meant walking three guests, while another would only overbook five or six, even if that meant leaving one room empty.

5.2.5 SPECIFIC PRACTICES

Hotel du Parc is one of the few hotels interviewed which uses a nested reservations structure. Its overbooking system is one of the most developed. In terms of their pricing, its rates are lower than the average for its category, because it follows the strategy of dropping prices to stimulate demand. This is not done haphazardly, though; a lot of careful and accurate analysis is performed to ensure correct pricing per market segment.

5.3 THE QUEEN ELIZABETH HOTEL[4]

5.3.1 BACKGROUND

The Queen Elizabeth Hotel opened in 1958. With more than 1000 rooms, it s currently the largest hotel in Montreal. Its orientation is mainly as a convention/meeting hotel, catering for groups ranging from 10 to 800 rooms. Secondly, it caters to corporate individual clientele, and thirdly, leisure travellers The Queen Elizabeth Hotel is part of the Canadian Pacific (CP) Hotels chain.

5.3.2 YIELD MANAGEMENT

Yield management, as a concept, was introduced at the Queen E around seven or eight years ago. At the early stages of its inception, yield management was used as an operational tool, to manage reservations and rates. Now, it is becoming more of an operating strategy, under the global, more encompassing, strategy of Revenue Management. The ultimate goal is maximizing revenue. The terms Revenue management and Yield Management, however, are used interchangeably at the Queen E.

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[4] J Richardson, (1995)

The Rooms Division, Front Desk, Sales and Marketing, Reservations, and sometimes the Catering departments, as well as the General Manager, are all involved in yield management. The computer systems at the Queen E are quite sophisticated. In 1989, CP Hotels launched a chain wide PMS. By 1995, 90% of all CP hotels carry the system. In the long run, the goal is to have a central reservation system (CRS) with all properties interfaced with each other; that is, the Queen E will be able to access the reservation system of another CP hotel in Alberta or Vancouver, for example, and vice versa.

At the present time, however, there is no separate yield management computer system. Just like most other hotels, everything is in the minds of those people working at the Queen Elizabeth (e.g., general manager, rooms division manager). Under the strategy of Revenue Management, the Queen E is embarking on a huge project of systematising and structuring its yield management. Under the guidance of CP Hotels and in co-operation with other CP hotels, the Queen E is documenting all information on yield management, and working on developing a yield management computer system.

5.3.3 MARKET SEGMENTATION AND FORECASTING

The total number of market segments at the Queen Elizabeth is 12. These could be aggregated into three major segments : Transient, Corporate, and Group. The Transient and Group segments are each made up of three different subgroups, while the Corporate segment consists of four (including government). Probably the most important segment for the hotel is the Corporate segment. The second is the tour series, because of the chain wide commitment to several tour companies.

Monthly as well as yearly forecasts are prepared. Information on the same month in the previous year, the *current* year's previous month, and current trends in the city and per market segments is used. The hotel has two separate computer systems which help with the forecast. The Delphi, sales and catering system, gives data on the different segments. There is another separate program, which draws out data from the Delphi system as of the date and time desired. Data from those two sources are used as a base for the forecast.

One important factor that management has to account for in the forecasts, is the changing booking and stayover trends. The typical stay of a corporate individual, for example, used to be between Tuesday and Thursday, never over the weekend. These days, the trend is shifting more and more towards staying over the weekend. This is due to two main reasons. First, corporate individuals are blending business and leisure trips; something never done before. Second, they feel they can take advantage of some lower rates over the weekend, if they extend their stay, rather than book a separate stay for the weekend.

5.3.4 OVERBOOKING

There is no stringent overbooking policy. It is left up to the personal judgement of the person in charge. The typical range can vary between 10 and 20 percent, depending on the season, city wide demand, the mix of business on a specific day, and the number of non-guaranteed reservations.

5.3.5 Specific Practices

Once a hotel is 100% full, what is next? The Queen Elizabeth is turning towards more accurate demand analysis. It is gathering data on enquiries and refused requests in the aim of estimating the true demand for rooms. In the same aim, it is also working on developing more sophisticated and accurate forecasting models, ones which can paint a more realistic picture. The Queen E is also refining its operating systems, by shifting to a nested reservations structure.

5.4 The Inter-Continental Hotel[5]

5.4.1 Background

The Inter-Continental hotel opened in 1991 as part of the Inter-Continental chain of hotels. It has 357 rooms, including 23 suites. Its client orientation is mostly corporate/business. The hotel is hooked up with the Inter-Continental central reservation offices, which operate on the CRS *Global 2,* situated in eight places around the world. Properties are not interfaced with each other, however. The Inter-Continental in Montreal is currently interfaced with HIS (Hospitality Information Systems) reservation system, based in Houston.

[5] J. Lageaux and C. Steckar (1995)

5.4.2 YIELD MANAGEMENT

The practice of yield management is not a chain wide directive; it is specific to this property. Because Inter-Continental encourages new ideas, yield management was introduced at the hotel by the new rooms division manager one year after it opened. The purpose of yield management, as it pertains to the Inter-Continental, is maximizing sales, in terms of both, revenue and room nights. Reservations, Rooms Division, and Sales departments are directly involved in yield management decisions.

5.4.3 MARKET SEGMENTATION AND FORECASTING

The hotel has three main market segments : Group, Individual, and Packages. Each of those main segments is further devided into subgroups. The Group segment, for example, has eight sub-segments, the individual segment has nine, and so on.

A forecast is prepared for the year as a whole, and then there is a forecast for each individual month, which is updated on the 20th of the previous month. There is a short term, 14-day forecast which is updated every day, and finaly, there is a 30-day forecast which is updated weekly. The forecasts are mainly based on booking trends per market segment, and current events in the city which might affect demand (e.g., the Referendum).

5.4.4 Pricing

For each market segment there are three different rates which are offered based on occupancy level, as well as other factors. Rates are set based on the competition (prices at competing hotels), economic situation, city wide occupancy, and how much the customer is willing to pay.

The Inter-Continental is a luxury hotel; and, consequently, has to maintain high standards with respect to quality of service and product. People are willing to pay more, but only in return for something better.

5.4.5 Overbooking

On the whole, the Inter-Continental does not overbook a lot (around 10% overbooking level). They try to minimize overbooking by getting "quality" reservations; that is, guaranteed and/or confirmed reservations. On sold-out days, for example, the reservations office tries to confirm all non-confirmed reservations, either by telephone or fax. Furthermore, they send out confirmations by mail or fax a the same day a booking is registered, so as to decrease the no-show rate. According to the reservations manager, receiving a confirmation makes people remember that they have a reservation and have to either show up or cancel.

5.4.6 SPECIFIC PRACTICES

Reservation agents at the Inter-Continental are trained in a rather particular way. All agents, in groups of five or six, are shown all the different rooms in the hotel. They spend around two hours examining every little detail, from the way the bed is made to the color of the towels in the bathroom. The reservation agents are actually sales people; hence they should know the product they are selling. The strategy motivating this practice is referred to as "selling on the basis of features and benefits."

Let us take an example. Someone calls in for a room. The reservation agent tells her that this kind of room costs this much *but* has a king-size bed and a large window overlooking the mountain. The king-size bed and the large window are the features. What are the benefits? The benefit of the king-size bed is that it is so much more comfortable; and that of the large window is that it brings in much more light into the room and the view it provides is enchanting.

On the lines of getting "quality" reservations, the Inter-Continental chain is considering moving towards a policy of taking only guaranteed reservations. In terms of serving the customer, the Inter-Continental chain is developing a database called *Global 2000* which collects information on customer preferences. It will provide information on how much money guests are spending at a property, which credit cards they use most, who booked their reservations and where they were booked from. If most bookings at the Montreal Inter-Continental are coming from Chicago, then why spend promotion dollars in Montreal, when they will serve better spent in Chicago.

5.5 Le Westin Mont-Royal[6]

5.5.1 Background

The Westin Mont-Royal was originally the Four Seasons hotel which opened in 1976. In April 1994, it changed names and became the Westin Mont-Royal, part of the Westin hotel chain. It is an upscale hotel with 300 rooms. Its reservation system is interfaced with all Westin properties as well as with airline reservation systems. The Montreal Westin, for example, can access the reservation system of the Westin in Boston.

5.5.2 Yield Management

Yield management is viewed as a basic operating strategy whose objective is to maximize revenue. Rooms Division, Reservations, Sales and Marketing departments, as well as the hotel manager are all involved in yield management decisions. They meet every week to look ahead for the entire year.

There are six main market segments: Transient, Corporate, Convention, Government, Packages, and Special, each of which is further sub-divided. Demand is tracked by market segment. Every week the following 60 days are forecast, and is updated every day.

For each market segment there are four different rates in increments of $15. The occupancy rate on a given day determines the rate that will be offered for that segment. If occupancy is high, one of the higher rates is offered; if it is low, a lower rate is offered. Once a rate has been determined at the beginning of a day, it is not changed during that day. The four rates per market segment are revised twice a year based on market conditions (city wide demand, economic conditions, prices at other hotels, etc.).

[6] R Beauchamp (1995)

The Westin is very cautious when it comes to overbooking. They would rather keep a room empty than walk a guest. When they do overbook, it is based on the percentage of cancellations and non-guaranteed reservations.

5.6 The Howard Johnson Hotel Plaza[7]

The Howard Johnson opened as a hotel in 1976. In 1986 it was renovated, yielding a total of 194 rooms, 50% of which are suites. The hotel mainly serves group tours.

Yield management was introduced at this property in 1994 and has become an operating strategy. To the Howard Johnson, yield management is managing availability and rates, with the goal of maximizing revenue. Sales and Marketing, Reservations, and the Rooms Division departments are directly involved in yield management. The Howard Johnson has no separate yield management computer system, but it does have a CRS which is not interfaced with its PMS.

There are a total of nine market segments at the Howard Johnson. Ninety-day forecasts per market segment are prepared and updated every month. Forecasting is based on the booking trends and what is happening in the city.

Rates at the Howard Johnson are based on a cost-plus approach. The non-room revenue component is taken into account, however. The restaurant contribution per guest is $20, for example. It is also estimated that around 60% of guests will have breakfast at the hotel, while almost none will have dinner there.

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[7] M Alfiero, (1995)

There is no set overbooking policy. Overbooking rates depend on the day of the week, on what is happening in the city, percentage of cancellations, etc.. Overbooking rates are determined for the hotel in general, and not per market segment.

5.7 The Ritz-Carlton Kempinski Hotel[8]

The Ritz-Carlton Hotel is one of the oldest hotels in Montreal. It has 230 rooms and is mainly a business/luxury hotel. Yield management is relatively new at the Ritz, since it was introduced by the general manger when he joined a few years ago. The Ritz is looking to structuring its yield management systems and developing computer models to help in the decision making. There are three types of forecasts at the Ritz: 18 month, 3 month, and one month forecasts. The 3 month forecast is updated once a week, while the one month forecast is updated everyday.

The name Ritz-Carlton is associated with high quality rooms and service. The Ritz-Carlton Hotel Company won the Malcom Baldridge Award for TQM in 1992 (Partlow, 1993). As a result, just like the Inter-Continental and the Westin, they are very cautious in their overbooking and would rather keep a room empty than annoy a guest.

[8] C. Folkersma and L. Remington (1995)

5.8 Holiday Inn Centre-Ville[9]

The Holiday Inn Centre-Ville opened four years ago. It has 235 rooms and is mainly a business/ convention hotel. Holiday Inn as a hotel chain is well versed in the development of yield management (concepts and computer systems). Holiday Inn hotels have a property management system named HOLIDEX and a newly developed (2 years) reservation system named HIRO (Holiday Inn Reservation Optimization). While HOLIDEX is a standard attribute to all Holiday Inn hotels, HIRO is not. It has to be purchased by the individual property. HIRO would be interfaced with HOLIDEX and would deal with setting overbooking limits and room allocation decisions, as well as other features. Holiday Inn is quite reluctant, though, in sharing information regarding its new HIRO system, and understandably so. The Holiday Inn Centre-Ville will be installing HIRO in early 1996.

At Holiday Inn, yield management is used as a strategy, the goal of which is to maximize revenue. The hotel has 21 market segments; each of which has its own three month forecast which is updated daily. Then, depending on the forecast, the rates are decided upon.

Two interesting aspects about the yield management system at Holiday Inn are that there are different overbooking policies for each market segment, and that the hotel collects data on denied requests (refused requests when the hotel is sold out).

[9] J Lavoie (1995)

6. APPLICATION OF THE EMSR MODEL TO A HOTEL SETTING

More and more hotels are moving towards a nested room inventory system (e.g., Federer, 1995; Richardson, 1995). Given a certain rate structure, a hotel who prudently manages its reservations inventory will posses a competitive advantage. The actual rooms that are being reserved are quite similar; the only difference is the rate at which they are being offered. The question, therefore, is : from this *shared* inventory of rooms, how many should be allocated to each of the respective classes or rates. The Expected Marginal Seat Revenue model (Belobaba, 1987) discussed in section four, can be adapted to a hotel setting to determine the number of rooms that should be protected for sale at higher rates only. A protection level will be set for each class, except the lowest one. Analogously, this can be expressed as finding the *booking limits* for each class; i.e., the maximum number of rooms that should be sold at a particular rate.

The problem has two stages. The first is deriving booking limits for each class at the *beginning* of the reservation process. Belobaba (1987) labels these as *initial* booking limits. The second stage is revising these initial limits based on new information provided by the actual bookings, as the day in question approaches. Reservations will be sold at a lower rate until the *expected* revenue from selling a room at a higher rate exceeds the known revenue from selling that same room at the lower rate.

6.1 WHY EMSR?

Most of the dynamic programming or network flow models presented in the previous sections are either too complex, in terms of the number of decision variables, or practically infeasible because of computational complexity. Given the dynamic nature and the complexity of the yield management problem, any model that is going to be used should be practical enough in terms of computation time and ease of application.

Belobaba's (1987) EMSR model is the most suited for adaptation to a hotel setting, for it can easily be applied on a spreadsheet program, its data requirements are relatively straight forward, its computation time is quite negligible, and finally, its results are straight forward and easily understood by potential users. Furthermore, it incorporates probabilistic demand, and can be applied to multiple rate classes in a nested reservations system.

Revenue maximizing room inventory decisions are first made on the basis of historical demand. Then, as the day in question approaches, actual accepted bookings are taken into account, thus making it a dynamic inventory control model. In order to account for more realistic demand behavior, overbooking and upsell probabilities are also incorporated into the model.

6.2 THE EXPECTED MARGINAL ROOM REVENUE MODEL[10]

In this section only the static EMSR model, described in section four, will be adapted and applied to actual hotel reservations data. The static EMSR determines the optimal allocation of seats from a shared inventory, for a one leg flight at the beginning of the booking process. Analogously, the Expected Marginal Room Revenue model will determine the

[10] Kimes (1990) devised this name in an unpublished working paper.

allocation of rooms between different rate classes, in a nested reservations system, at the *beginning* of the booking process, for a *one night* stay.

Let $P_i(R_i)$ be the probability that all requests, r_i, for rooms in class i, on a particular night are accepted.

$$P_i(R_i) = P[r_i \leq R_i] = \int_0^{R_i} p_i(r_i)\, dr_i$$

$$\Rightarrow \qquad P[r_i > R_i] = \int_{R_i}^{\infty} p_i(r_i)\, dr_i$$

$$= 1 - P_i(R_i) = \bar{P}_i(R_i),$$

where $\bar{P}_i(R_i)$ is the probability of receiving more than R_i requests in rate class i.

Let $EMRR_i$ be the expected marginal revenue for class i, when the number of rooms available to that class is increased by one. The expected marginal revenue of the R_i^{th} room, $EMRR_i(R_i)$, therefore, is the average rate level in that class, m_i, multiplied by the probability of selling R_i or more rooms : (Refer to Figure 2, page 74 for this discussion.)

$$EMRR_i(R_i) = m_i \bullet \bar{P}_i(R_i). \qquad \text{Equation (1)}$$

Suppose there are two rate classes, 1 and 2. The optimal protection level, $R_2^{\,1}$, for class 1 is the value of $R_2^{\,1}$ that satisfies the condition:

$$EMRR_1(R_2^{\,1}) = m_1 \bullet \bar{P}(R_2^{\,1}) = m_2. \qquad \text{Equation (2)}$$

$$\Rightarrow \qquad P[r_1 > R_2^{\,1}] = m_2/m_1. \qquad \text{Equation (3)}$$

6.3 DEMAND

Demand for airline seats is shown to follow a normal distribution (Belobaba, 1985). Not a lot of research has been done on the behavior of hotel-room demand, however it has also been assumed to follow a normal distribution (Kimes, 1990; Orkin, 1989). Although the EMSR model can be used with any demand distribution, deriving probabilities of selling S_i or more seats is much easier and straight forward, with normally distributed demand. In the following application, demand will be assumed to follow a normal distribution.

Two types of input data are required to calculate booking limits for different rate classes on a particular night. Estimates of expected demand and average revenue per class are needed. For the initial application of the EMRR model, demand inputs are estimates of the total number of requests expected for a future arrival date, by rate class. Because the model takes into account the stochastic variation in demand, the estimate of the variance for total requests is also required.

For dynamic applications of the EMRR, estimates of partial demand by rate class for requests still to come *t* days before the date in question would be required. The estimate of the variance of these requests would also be needed. Furthermore, the number of actual reservations already on the books *t* days before the date would also be required.

6.4 BOOKING LIMITS

The value of the protection level for class 1 from class 2, R_2^1, that maximizes *expected* revenue in class 1 is determined by the ratio m_2/m_1, ($m_2 < m_1$), as well as by the parameters of demand (mean and standard deviation) for reservation requests in class 1. The ratio m_2/m_1

gives the probability of receiving more than $R_2{}^1$ req ests (Eq. 3). Since we are assuming normally distributed demand, we can find the standard Z value for which the probability is equal to m_2/m_1. If, for example, m_2/m_1 is 0.65, then we find the value of Z which has a probability of 0.65 of being exceeded. Having obtained Z, and knowing the mean, $\bar{r}$, and standard deviation, σ, of reservation requests for a certain class i, we can calculate the protection level R_i:

$$Z = \frac{R_i - \bar{r}}{\sigma}$$

$$\Rightarrow \qquad R_i = \bar{r} + Z\sigma \qquad \text{Equation (4)}$$

The booking limit for the highest rate class is nothing but the total of the shared capacity, C. The booking limits for the lower fare classes, j, are calculated by:

$$BL_j = C - \Sigma_{i<j} R_j'.$$

BL_j may be less than zero. In that case that rate should not be offered. Therefore, the booking limit becomes:

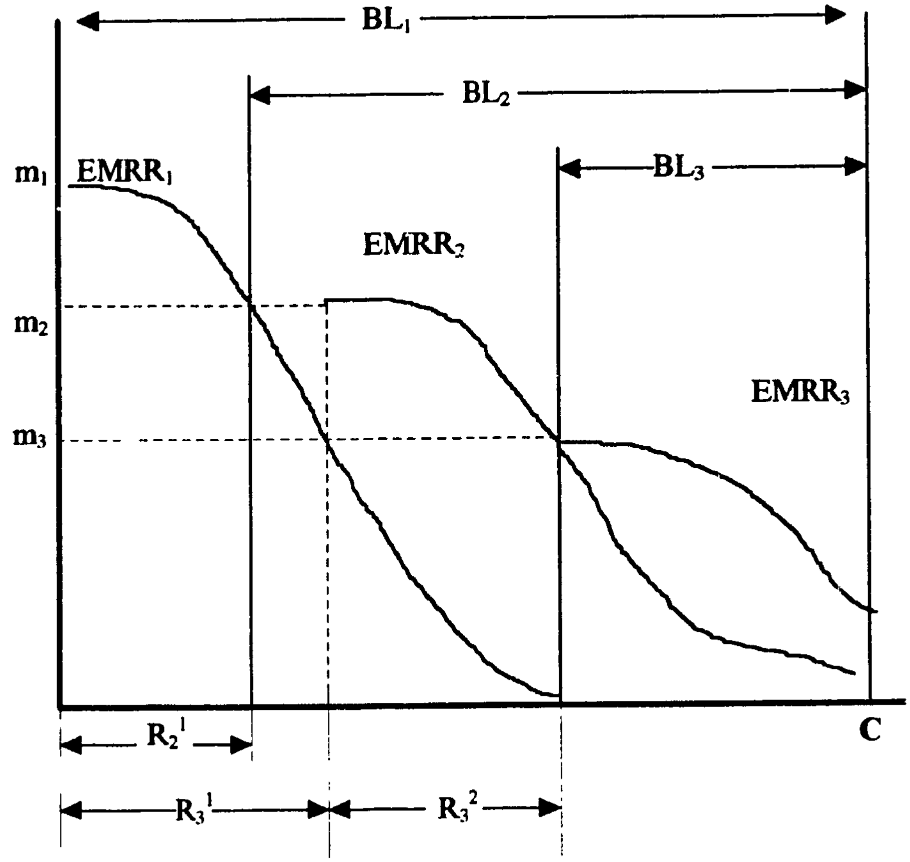
$$BL_j = \max[0, C - \Sigma_{i<j} R_j']. \qquad \text{Equation (5)}$$

The incremental number of rooms protected for class j is the nested protection level for that class:

$$NP_j = BL_j - BL_{j+1}. \qquad \text{Equation (6)}$$

The nested protection level for class 2, NP_2, for example is equal to $BL_2 - BL_3$. There is no nested protection level for the lowest class, but rather a booking limit, while the nested protection level for the highest class is the total shared capacity.

FIGURE 2.



6.5 OVERBOOKING

Overbooking is a very important component of yield management. Furthermore, most hotels do apply the process. Consequently, it should be included in room allocation decisions. As was described in section four, Belobaba (1987) does include overbooking into the expected marginal revenue model. For this application, one single overbooking factor will be applied to the overall capacity. This means that the overall capacity, C, will be increased by the assumed overbooking factor, OV, such that,

$$C^{*} = OV \bullet C. \qquad \text{Equation (7)}$$

Since the booking limit of the highest rate class, BL_1, is equal to the available capacity, then

$$BL_1^* = C^* = OV \bullet C.$$

The overbooking limits on each subsequent class j, BL_j, are given by:

$$BL_j^* = C^* - \Sigma_{i<j} R_j^i. \qquad \text{Equation (8)}$$

6.6 The Data

The data that will be used was given to the author by one of the hotels here in Montreal. The data are on monthly realised reservations per market segment, per day. For example, on Monday, February 2nd, there were 75 realised reservations for the Corporate segment which showed up, on the 3rd there were 98, etc.. The data are for five months, and are aggregated into three market segments, ONE, TWO, and THREE. These three segments will represent three different classes, each with its respective rate. The data on the number of reservations, refused requests, the total shared capacity, as well as the average rates per class were multiplied by a factor, in order to preserve the confidentiality of the hotel.

Since the model requires data on *requests* for rooms, data on actual realised reservations may not necessarily reflect the true demand for rooms. If the hotel was sold out one night, then the reservations for that night will be equal to the hotel capacity, and the denied requests (which are part of the real demand) would not be accounted for.

In order to have a more accurate representation of the requests per rate class, data on the actual reservations observed on specific nights, as well as data on the refused requests for that class on that night are needed. Data on refused requests was obtained, but are not per market segment. The data on refused requests are aggregated over all the different segments.

To derive the number of refused requests per segment, two steps were taken. The percentage of actual reservations per market segment, or class, is calculated for each month. This percentage is then multiplied by the total number of refused requests for that month, in order to get the number of refused requests per class. The number of refused requests per class was then added to the actual reservations, giving the total demand for each class per month.

Let us take an example. (See Appendix 4 for detailed calculations.) The total number of refused requests (SOUT) for Month 1 is 852. The total number of realised reservations for the class 1 (ONE) for that same month is 10017, for class 2 (TWO) it is 6581, and for THREE it is 29836. Thus yielding a total of 46434 reservations across all three classes.

The percentage of reservations for class 1 (%DEM) is 0.216 (10017/46434). To obtain the number of refused requests for class 1, the total number of refusals, is multiplied by the percentage of class 1 reservations: $0.216 \times 852 = 184$ refused requests for class 1 in Month 1. Similarly, we get 121 refused requests for TWO and 547 for THREE.

Adding the number of reservations and the number of refused requests per class gives the final estimate of the number of *requests for reservations* per class (TOTDEM). For example, the estimate for the total number of reservation requests for class 1 in Month 1 is 10201 requests (10071+184).

As was mentioned in section 6.3, it is assumed that the data on hotel room demand follows a normal distribution. Since the number of observations obtained were very few (one set of data for the first three months and two sets for the last two months), it was not possible to perform sophisticated tests concerning the normality assumption. Frequency histograms were plotted for each segment for each of the five months (See Appendix 5). Data used in the plots is that on the actual number of realised reservations per class only. Data on the refused requests is not included, for the number of refusals per day per class could not be calculated. As can be seen in Appendix 5, most of the histograms resemble a skewed normal distribution.

6.7 Calculation Of Means and Variances

Including the number of refused requests in the estimate for demand, implies that the mean and variance are going to change. Recalculating the mean (MEAN) is straight forward: calculate total demand (which is equal to actual number of reservations plus the number of refused requests per class), then divide this number by the days of the month (Appendix 4). To recalculate the variance for requests to come: first the variance of the refused requests is calculated (TVAROUT); second, per class percentage of demand is multiplied by the variance of the refused requests. This number is then added to the variance of the reservations data (REGVAR), obtaining a final estimate of the variance for *requests* for rooms (TOTVAR). Since the standard deviations (STDEV) are needed for calculation of booking limits, the square root of the total variance (calculated in previous step) is taken. (See Appendix 4 for detailed calculations)

6.8 RESULTS

Given the mean, standard deviation, and the average rate per class, booking limits can be calculated based on the EMRR framework. Since the data is monthly, the protection levels and the booking limits derived will be static and for a whole month. In other words, these initial booking limits would not be recalculated as actual reservations are accepted. The only adjustment that would be made is subtracting the number of accepted reservations from the initial booking limits.

Protection levels and booking limits were calculated for each of the five months. The results can be found in Table 5. Let us take Month 1 as an example. The input data (means, standard deviations, and average rates) are listed in Table 1.

TABLE. 1

C = 1869	ONE	TWO	THREE
MEAN	329	216	980
STDEV	136.944	70.433	343.539
RATE, m_i	$205	$192	$181

To calculate the protection levels R_2^1 and R_3^1, for class 1, the probability of receiving more than R_2^1 requests, $P[r_1 > R_2^1]$, and the probability of receiving more than R_3^1 requests, $P[r_1 > R_3^1]$, have to be calculated. From Equation (3) this is equal to the ratio of the respective rates: Therefore,

$$P[r_1 > R_2^1] = m_2/m_1 = 192/205 = 0.9366$$

$$P[r_1 > R_3^1] = m_3/m_1 = 181/205 = 0.8839$$

Having obtained the probabilities of selling Ri of more rooms, the corresponding Z_j^1 values can be found:

$$Z_2^1 = -1.53$$

$$Z_3^1 = -1.19$$

TABLE 2

C = 1869	ONE	TWO	THREE
MEAN	329	216	980
STDEV	136.944	70.433	343.539
RATE, m_i	\$205	\$192	\$181
m_i/m_1, i=2,3		0.9366	0.8839
m_2/m_3			0.9427
Z_j^1, j=2,3		-1.53	-1.19
Z_3^1			-1.58

Now that we have Z, we can find the protection levels, R_i^j, for classes, 1 and 2.

From Equation (4):

$$R_2^1 = 329 + (-1.53 \times 136.944)$$

$$R_2^1 = 120$$

$$R_3^1 = 329 + (-1.19 \times 136.944)$$

$$R_3^1 = 166$$

Similarly, $R_3^2 = 105$

Given the protection level for classes 1 and 2, and the total shared capacity, C, we can calculate the booking limits on classes 2 and 3, BL_2 and BL_3, respectively from Equation (5).

$$BL_1 = C = 1869$$

$$BL_2 = 1869 - 120$$

$$BL_2 = 1749$$

$BL_3 = 1869 - 166 - 105$

$BL_3 = 1598$

From Equation (6), the nested protection level for class 2, NP_2, is

$NP_{2,} = 1749 - 1598$

$NP_{2,} = 151$

For class 3 there is no nested protection level, and for class 1, the nested protection level is equal to the booking limit which is equal to total capacity, C.

TABLE 3

C = 1869	ONE	TWO	THREE
MEAN	329	216	980
STDEV	136.944	70.433	343.539
RATE, m_i	\$205	\$192	\$181
m_i/m_1, i=2,3		0.9366	0.8839
m_2/m_3			0.9427
Z_j^1, j=2,3		-1.53	-1.19
Z_3^1			-1.58
R_2^1, R_3^2	120	105	
R_3^1	166		
BL_j, j=1,2,3	1869	1749	1598
BL_j^* j=1,2,3	2150	2030	1879

Incorporating an overbooking factor, OV, of 15%, yields an increased capacity of 2150 rooms (1869 × 1.15), which is now the new booking limit for class 1. The adjusted booking limits, BL_j^*, for classes 2 and 3 will calculated using Equation (8).

Table 4 includes the protection levels and booking limits for all the five months. The ratios, rates, and Z values are not included since they the same as in Tables 1 to 3 used for all the five months.

TABLE 4. Protection Levels & Booking Limits: Months 1 to 5

C = 1869	MONTH 1			MONTH 2			MONTH 3		
OV = 1.15	ONE	TWO	THREE	ONE	TWO	THREE	ONE	TWO	THREE
MEAN	329	216	980	276	283	567	172	299	151
STDEV.	136.944	70.433	343.539	115.385	68.564	309.881	153.639	103.13	194.822
RATE, m_i	$205	$192	$181	$205	$192	$181	$205	$192	$181
m_i/m_1		0.9366	0.8839		0.9366	0.8839		0.9366	0.8839
m_3/m_2			0.9427			0.9427			0.9427
Z_4^1		-1.53	-1.19		-1.53	-1.19		-1.53	-1.19
Z_3^1			-1.58			-1.58			-1.58
R_2^1, R_3^2	120	105		100	175		0*	136	
R_3^1	166			139			0*		
BL_4	1869	1749	1598	1869	1769	1555	0*	1869	1733
BL_4^*	2050	2030	1879	2050	1950	1736	0*	2050	1914

Table 4 - Continued

C = 1869	MONTH 4			MONTH 5		
OV =1.15	ONE	TWO	THREE	ONE	TWO	THREE
MEAN	199	227	306	269	271	423
STDEV.	126.244	93.35	268.26	124.859	117.259	337.61
RATE, m_i	$205	$192	$181	$205	$192	$181
m_i/m_1		0.9366	0.8839		0.9366	0.8839
m_3/m_2			0.9427			0.9427
Z_4^1		-1.53	-1.19		-1.53	-1.19
Z_3^1			-1.58			-1.58
R_2^1, R_3^2	6	80		78	86	
R_3^1	49			120		
BL_4	1869	1863	1741	1869	1791	1663
BL_4^*	2050	2044	1921	2050	1972	1844

7. DISCUSSION AND CONCLUSION

7.1 DISCUSSION

The mean of class One for Month 3 is too small compared to its variance (172 < 153.6). Furthermore, the ratio m_2/m_1 as well m_3/m_1 are too big. Thus yielding negative protection levels, hence the 0 booking limits and protection levels. This means that the hotel might be better off not offering class One in Month 3 and only offering classes Two and Three. Realistically, though, maybe the hotel is bound to offer this class (contracts, repeat customers, etc.). In this case, it might be advisable to give class One less priority than class Two. Class Two seems to be the one doing best in Month 3 in terms of average demand (mean is 299 compared to 172 and 151 for classes One and Three respectively). The hotel could capitalise on this by concentrating on selling class Two.

If we compare the results for Month 2 with those for Month 1, we can see that, although we are using the same rates, and hence Z values, protection levels for Month 1 are higher than those for Month 2. The reason behind this is the difference in the means and standard deviations. Let us look at class One. The mean for class One in Month 1 is 329 requests, while its standard deviation is 136.9, for Month 2, the corresponding figures are 276 and 115.4. The difference between the mean and the standard deviation in Month 1 is greater than that for Month 2 (192 > 160). Furthermore, the actual mean for Month 1 is greater than the mean for Month 2 (329>276). All these factors contribute to make the protection levels in Month 1 greater than those in Month 2.

Everything else held constant, the greater the standard deviation and the difference between the mean and the standard deviation for a certain class, the larger the protection limits. Another factor which affects the protection levels is the ratio of the rates between two classes. The larger the ratio, the smaller the protection level. This is true because the ratios are nothing but the probability of receiving R_i or more requests (R_i being the protection level for class *i*. The higher this probability, the less rooms we have to protect.

When the ratio is greater than 0.5 but less than 1, the corresponding Z-value will be negative, and hence the protection level will be less than the mean. When the ratio is greater than zero but less than 0.5, Z will be positive, and the protection level will be greater than the mean. The case when the ratio, m_{i+1}/m_i, is equal to 0.5, gives a Z-value of zero, and the protection level will be equal to the mean. When the ratio is greater than 0.5 *and* the standard deviation is large, then we might get negative protection levels, as was the case class One in Month 3.

7.2 LIMITATIONS

The application of Belobaba's (1987) EMSR model to a hotel setting was a preliminary effort. It can and should be expanded to incorporate more realistic conditions. The protection levels calculated in section six are on a monthly basis. A more accurate representation would be calculation of protection levels and booking limits on a daily or weekly basis. For this, however, more data would be needed. Hotels accept reservations well in advance. This implies that an inventory allocation model should give allocation decisions for a certain number of *days before arrival*.

Let us take an example. We are June first and are accepting reservations for October first. We are 60 days before arrival. As the date in question approaches, allocation decisions need to change in order to take advantage of the booking patterns of the different classes.

The other limitation of this application is the lack of sufficient data. Ideally, data should be grouped into day of the week and month of the year. Hotels witness a wide variation in demand patterns among the different days of the week. The data should also be grouped into appropriate seasons. The data available to this author was not enough to make these groupings.

The other aspect with respect to demand estimates is that of refused requests. The data on refused requests used in section 6 is not per market segment, but aggregated over all segments. What this author did to reach the number of refused requests per segment is mostly an approximation. Data on refused requests should be collected per segment and classified by day of the week and month of the year.

I can go on about limitations, however, this thesis contributes to both, the research on yield management in the hotel industry, as well as to hoteliers alike. Most hotels are moving towards a yield management approach, and thus the timeliness of the study. Adapting mathematical models developed for the airlines is a first step towards developing more accurate and custom-made models for the hotel industry per se. In terms of general yield management concepts, the study sheds some light on the common trends observed in the industry.

7.3 Directions For Future Research

As was mentioned in section five, most hoteliers agree that yield management for the hotel industry needs to be refined. This means more research. There are several avenues to follow in this respect. Most importantly, the need is greatest in the area of demand analysis and forecasting. More sophisticated demand models need to be generated. In the end, a model is as good as its inputs; and demand estimates are the biggest drivers of the hotel industry. Modelling demand accurately should be coupled by forecasting models that can make use of this information.

Another area which needs to be addressed is that of overbooking. As was seen in section 3, most overbooking models are not applied in any real situation, for they are just too complicated and/or do not accurately portray the situation. The potential gains from overbooking are quite substantial, and the hotel which has a structured approach to follow, will overbook more confidently and the benefits accrued will be substantial.

An area which was addressed briefly at the beginning of the thesis is that of Traffic Management. Taking into account the number of nights a guest is staying is of utmost importance. Single night room allocation decisions will not maximize revenue over the whole week or month or year. Room allocation models which take into account multiple night stays would make up an integral part of a yield management system.

Last but not least, all the issues mentioned above are very important, yet without an efficient information system, none of it can be of much use. This is an area of tremendous possibilities, in terms of both, improvements and innovation.

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APPENDIX 1. **AIRLINE OVERBOOKING MODELS**

REFERENCE	MODELING APPROACH	MODELED SITUATION	INCORPORATED CHARACTERISTICS	APPLICATION
Beckman (1958); Kosten (1964)	Minimize lost revenue	• Single leg • single fare class	Reservations demand; Cancellations (in Kosten)	———
Thompson (1961) Taylor (1962); Deetman (1964)	Conditional probability depending on number of days before departure	• Single leg • single fare class	Reservations demand; cancellations; number of days before departure	———
Rothstein & Stone (1967)	Dynamic decision process	• Single leg • single fare class	Reservations demand; cancellations; no-shows	American Airlines
Rothstein (1971)	Markovian sequential decision process	• Single leg • single fare class	Reservations demand; cancellations; no-shows	———
Shlifer & Vardi (1975)	Dynamic decision process	•Single leg, single class; •single leg, two fare class •multiple leg	Reservations demand; cancellations; no-shows	EL-AL airlines (1972-73):
Alstrup, Boas, Madsen & Vidal (1986, 1989)	Markovian sequential decision process	• single leg • two fare classes • flexible cabin	reservations demand, cancellations, no-shows for each type of customer; upgrading and/or downgrading	Tested at SAS airlines, but not implemented although results proved better than existing heuristic (too big an investment)
Smith, Leimkuhler & Darrow (1992)	Marginal Cost of Oversale (Lagrangian relaxation)	• multiple leg • multiple fare class	revenue of additional reservation; cancellation/ no-shows; oversale cost max # of oversales allowed; recapture prob.	American Airlines

APPENDIX 2. HOTEL OVERBOOKING MODELS

REFERENCES	MODELING APPROACH	MODELED SITUATION	INCORPORATED CHARACTERISTICS	APPLICATION
Rothstein (1974)	Markovian sequential decision process	•Single night stay, •single rate	• reservations demand, • cancellations, no-shows	——
Ladany (1976)	Markovian sequential decision process	•Single night stay, •two rates	• reservations demand, • cancellations / no-shows • upgrading of guests	——
Williams (1977)	Probabilistic decision model: minimize expected cost	•Single night stay	• reservations • stay-overs • walk-ins	——
Lieberman & Yechiali (1978)	*n*-stage dynamic programming problem	•Single night stay •single rate	• reservations demand, • cancellations	——

APPENDIX 3. DISCOUNT ALLOCATION MODELS

REFERENCE	MODELING APPROACH	MODELED SITUATION	INCORPORATED CHARACTERISTICS	APPLICATION
Littlewood (1972); Bhatia & Parekh (1973); Richter (1982);	Marginal Revenue Analysis	•Single leg flight; •two fare classes; •low fare customers book first	• independent fare class demand • no cancellations • rejected requests represent lost revenue to airline	•Bhatia & Parekh: TWA •Richter: Lufthansa
Mayer (1976); Titzer & Griesshaber (1983)	*ibid.*	•Single leg flight; •two fare classes;	• independent fare class demand • no cancellations • rejected requests represent lost revenue to airline	•Mayer: El-Al
Buhr (1982)	*ibid.*	•Two leg flight •one fare class	•Independent fare class demand	Lufthansa
Wang (1983)	*ibid.*	•Two leg flight •multiple fare classes	•Independent fare class demand	Cathay Pacific
Hersh & Ladany (1978)	Dynamic decision process (Bayesian)	•two leg flight with passengers on each leg as separate type of customer	•Reservations demand, •cancellations ,no-shows	——
Ladany & Bedi (1977)	non-Bayesian dynamic decision process	*ibid.*	*ibid.*	——

Glover, Glover, Lorenzo, McMiillan (1982)	Minimun cost Network flow, with side constraints	•multiple leg •multiple class	•Deterministic, independent fare class demand	Frontier Airlines: accomodated up to 600 daily flights & 5 fare classes
D'Sylva (1982)	*ibid.*	*ibid.*	Stochastic demand	Boeing
Wollmer (1986)	Integer programming: binary decision variables	•multiple leg •multiple class	• Stochastic demand • constant fare	McDonnell Douglas
Belobaba (1987;1989)	Marginal Revenue Analysis	• single leg • multiple class	• independent, fare class demand •cancellations •overbooking •recapture probabilities • nested fare classes	Western Airlines
Pfeifer (1989)	Single period newsboy problem	• single leg • two class	•demand devided into shoppers & nonshoppers	———
Brumelle, McGill, Oum, Sawaki, & Trethway (1990)	Monotone optimal stopping problem	• single leg • two class	•nested fare classes; lower fare class books first; no cancellations; dependent fare class	———
Curry (1990)	Mathematical programming: peicewise linear approximations ==> linear programming algorithms	• multiple leg • two class	*ibid.*	———
Smith et al. (1992)	Marginal Revenue Analysis	• multiple leg • multiple class	nested fare classes; recapture prob.; virtual nesting (different O/D)	American Airlines

APPENDIX 4
CALCULATION OF MEANS AND VARIANCES

	MONTH1		
REFUSED REQUESTS (SOUT)			
TOTAL	851.88		
MEAN	27.48		
VAROUT	1424.93		
	ONE	TWO	THREE	
SUBTOT.	10017	6581	29836	46434
% DEM.	0.215734	0.141719	0.642547	
# SOUT.	184	121	547	
TOTDEM	10201	6701	30383	
MEAN	329	216	980	
REGVAR	18446.18	4758.844	117103.7	
VAROUT	307.4061	201.939	915.5844	
TOTVAR	18753.59	4960.783	118019.3	
STDEV.	136.944	70.433	343.539	

	MONTH 2			
REFUSED REQUESTS (SOUT)				
TOTAL	51			
MEAN	2			
VAROUT	12.24703			
	ONE	TWO	THREE	
SUBTOT.	8255	8477	16990	33722
% DEM.	0.244798	0.251372	0.50383	
# SOUT.	13	13	26	
TOTDEM	8268	8490	17016	
MEAN	276	283	567	
REGVAR	13310.65	4697.946	96020.12	
VAROUT	2.998049	3.078556	6.17042	
TOTVAR	13313.64	4701.024	96026.29	
STDEV.	115.385	68.564	309.881	

APPENDIX 4
CALCULATION OF MEANS AND VARIANCES

MONTH 3

REFUSED REQUESTS (SOUT)				
TOTAL	48			
MEAN	2			
VAROUT	6.1185			
	ONE	**TWO**	**THREE**	
SUBTOT.	5324	9248	4679	19251
% DEM.	0.276551	0.480396	0.243053	
# SOUT.	13	23	12	
TOTDEM	5337	9271	4691	
MEAN	172	299	151	
REGVAR	23603.2	10632.76	37954.12	
VAROUT	1.692075	2.939296	1.487116	
TOTVAR	23604.89	10635.7	37955.61	
STDEV.	153.639	103.130	194.822	

MONTH 4

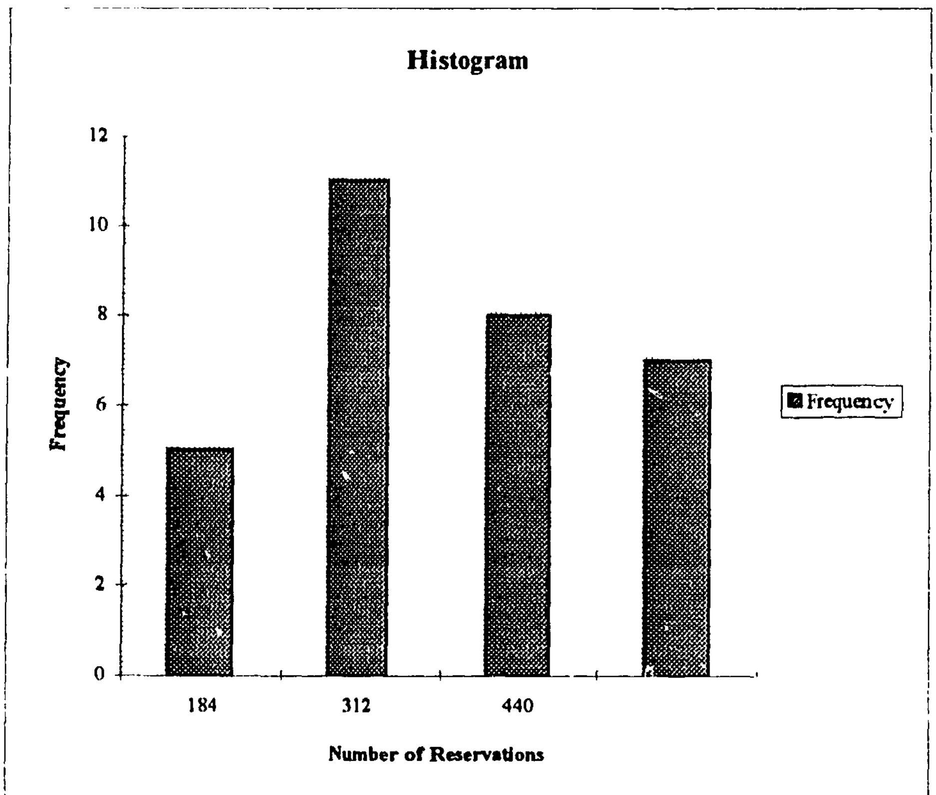
REFUSED REQUESTS (SOUT)				
TOTAL	130			
MEAN	4			
VAROUT	109.2108			
	ONE	**TWO**	**THREE**	
SUBTOT.	6142	6987	9416	22546
% DEM.	0.272417	0.309918	0.417665	
# SOUT.	35	40	54	
TOTDEM	6177	7028	9471	
MEAN	199	227	306	
REGVAR	15907.79	8680.31	71917.82	
VAROUT	29.75089	33.84635	45.6136	
TOTVAR	15937.54	8714.157	71963.43	
STDEV.	126.244	93.350	268.26	

APPENDIX 4
CALCULATION OF MEANS AND VARIANCES

	MONTH 5			
REFUSED REQUESTS (SOUT)				
TOTAL	112			
MEAN	4			
VAROUT	47.81323			
	ONE	**TWO**	**THREE**	
SUBTOT.	7499	7562	11804	26866
% DEM.	0.279134	0.281487	0.439379	
# SOUT.	31	31	49	
TOTDEM	7530	7594	11854	
MEAN	269	271	423	
REGVAR	15576.51	13736.26	113959 4	
VAROUT	13.3463	13.45878	21.00815	
TOTVAR	15589.85	13749.72	113980.5	
STDEV.	124.859	117.259	337.610	

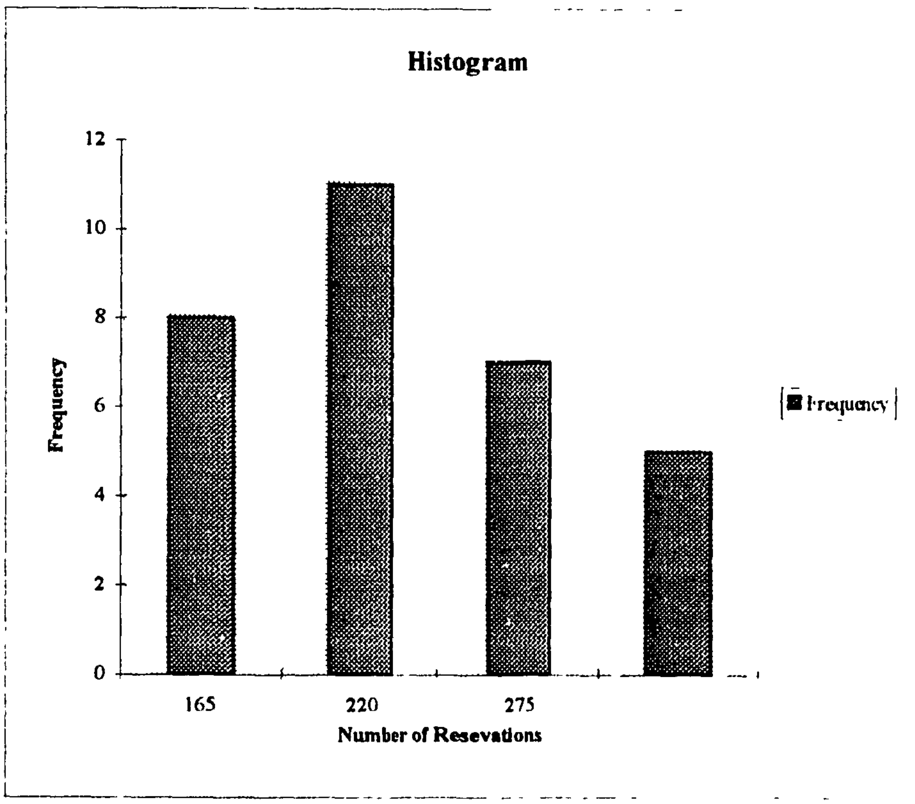
APPENDIX 5
FREQUENCY HISTOGRAMS

MONTH 1: ONE



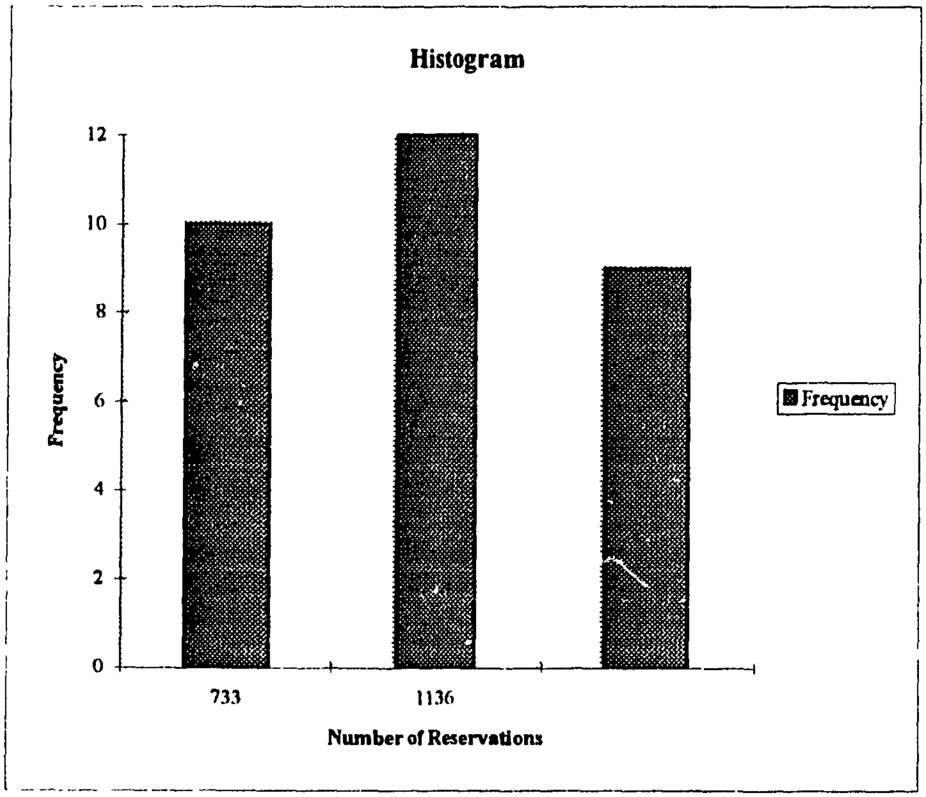
APPENDIX 5

MONTH :1 TWO



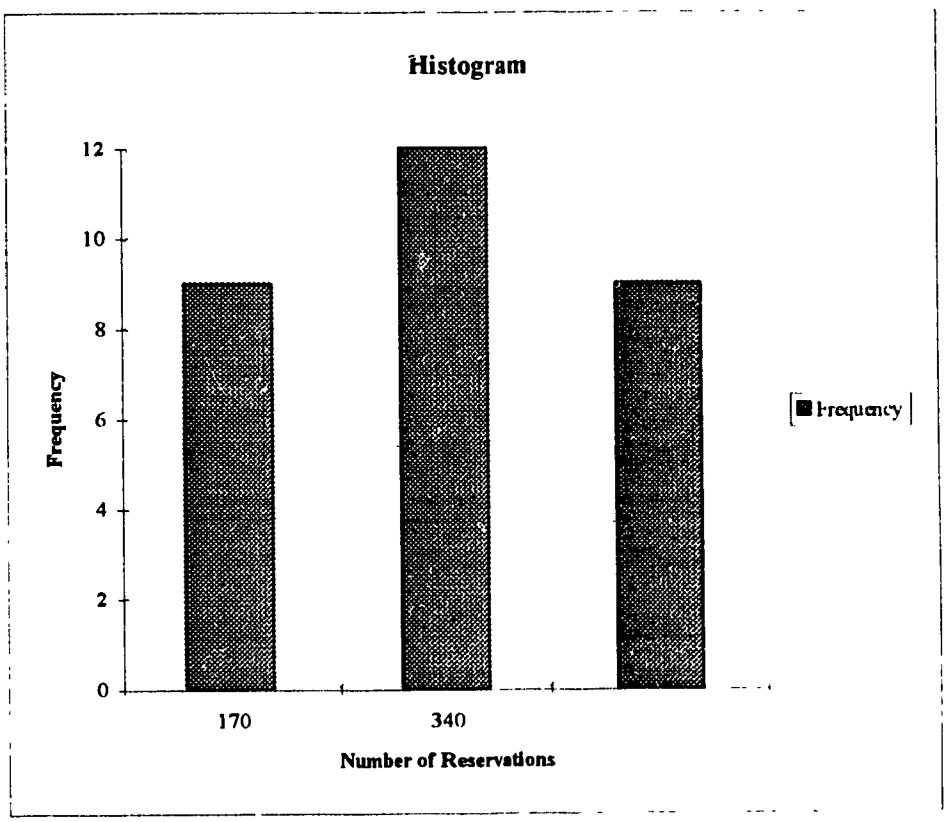
APPENDIX 5

MONTH 1: THREE



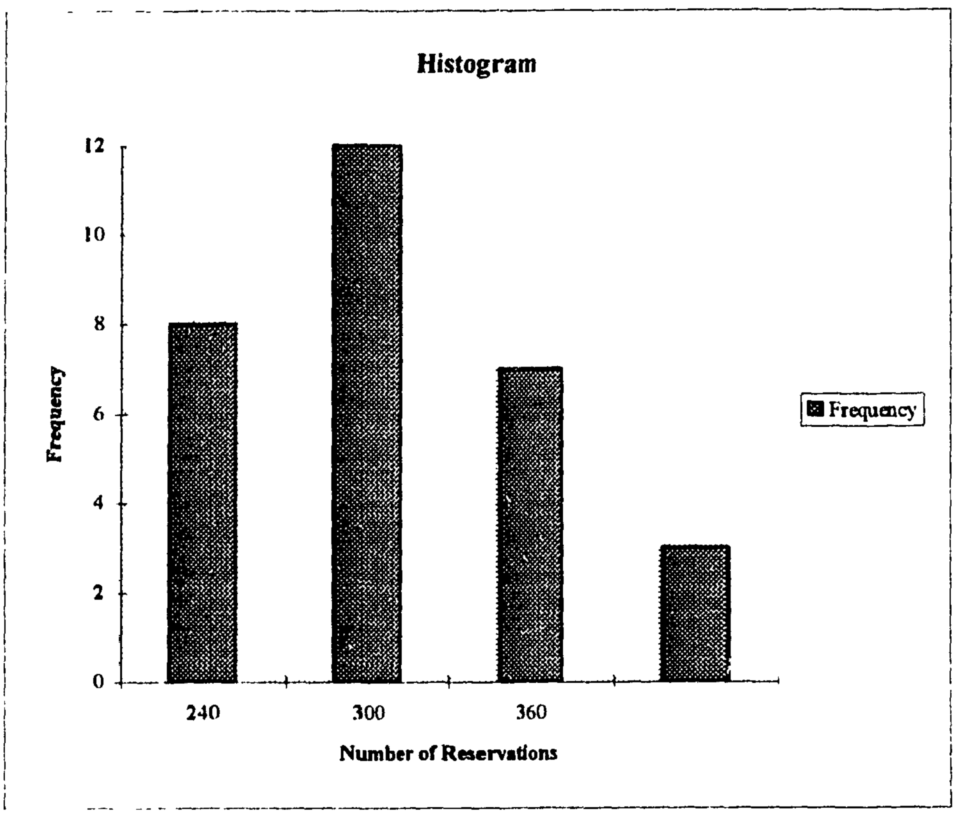
APPENDIX 5

MONTH 2: ONE



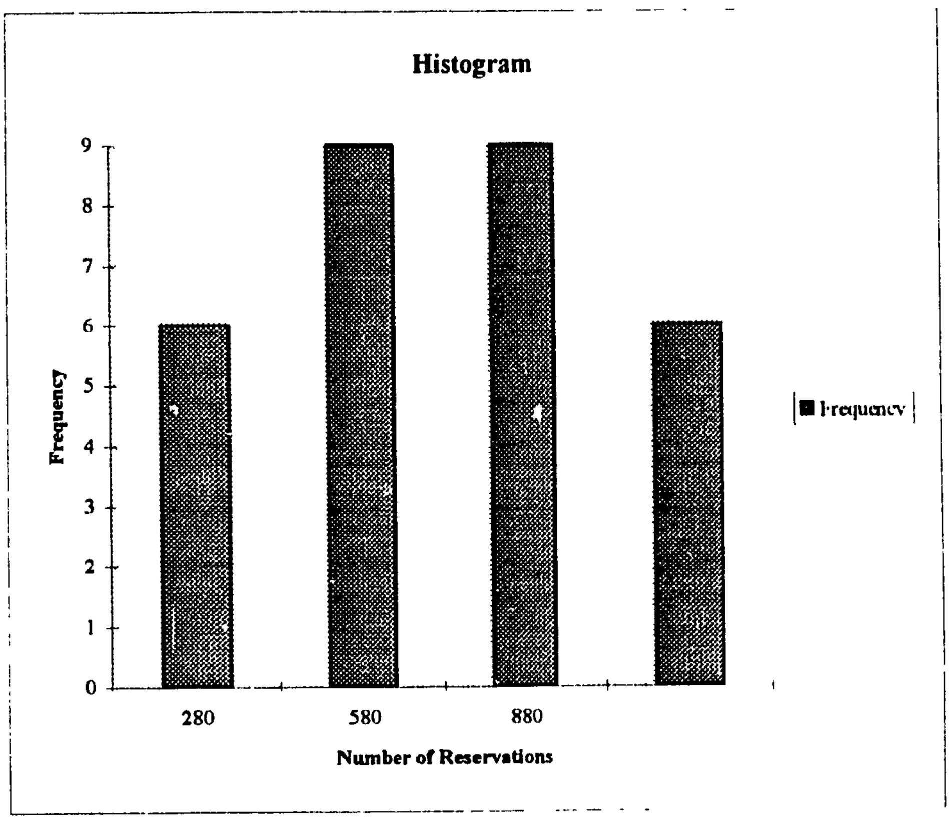
APPENDIX 5

MONTH 2: TWO



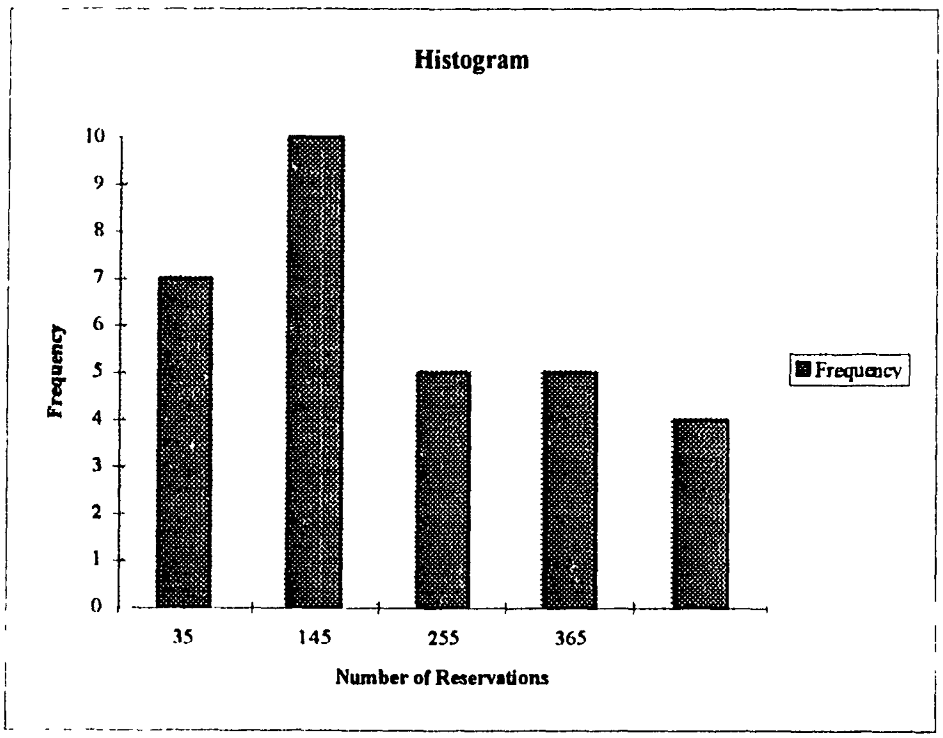
APPENDIX 5

MONTH 2: THREE



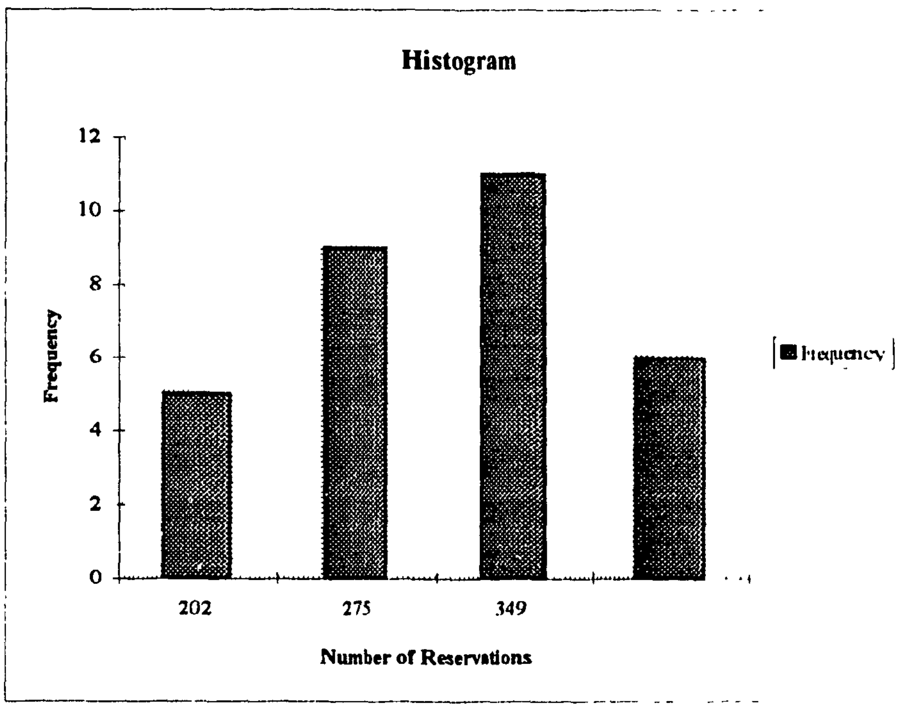
APPENDIX 5

MONTH 3: ONE



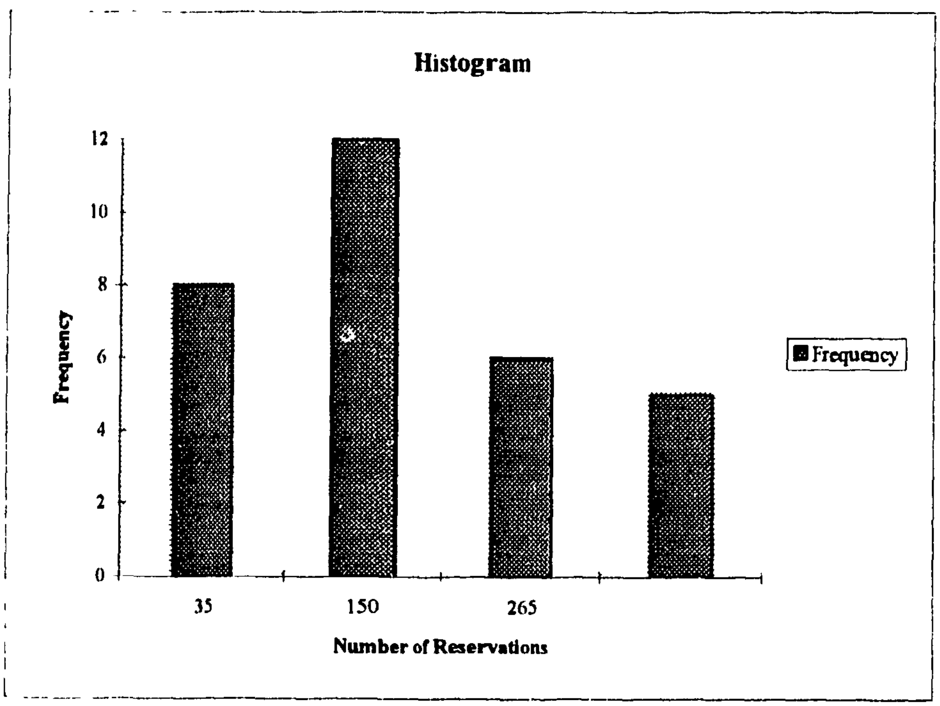
APPENDIX 5

MONTH 3: TWO



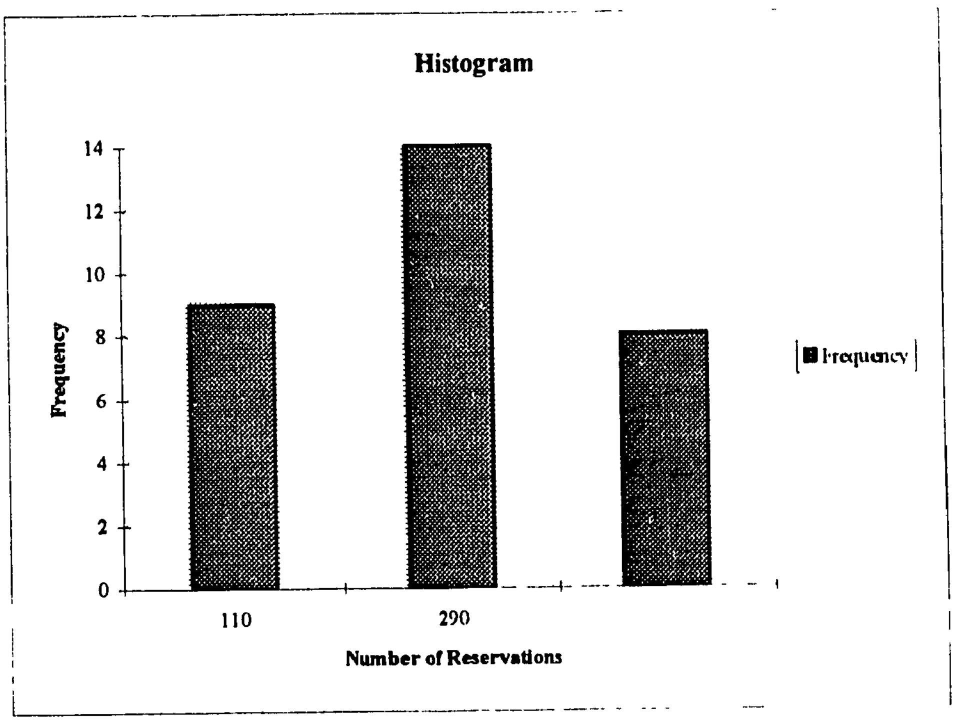
APPENDIX 5

MONTH 3: THREE



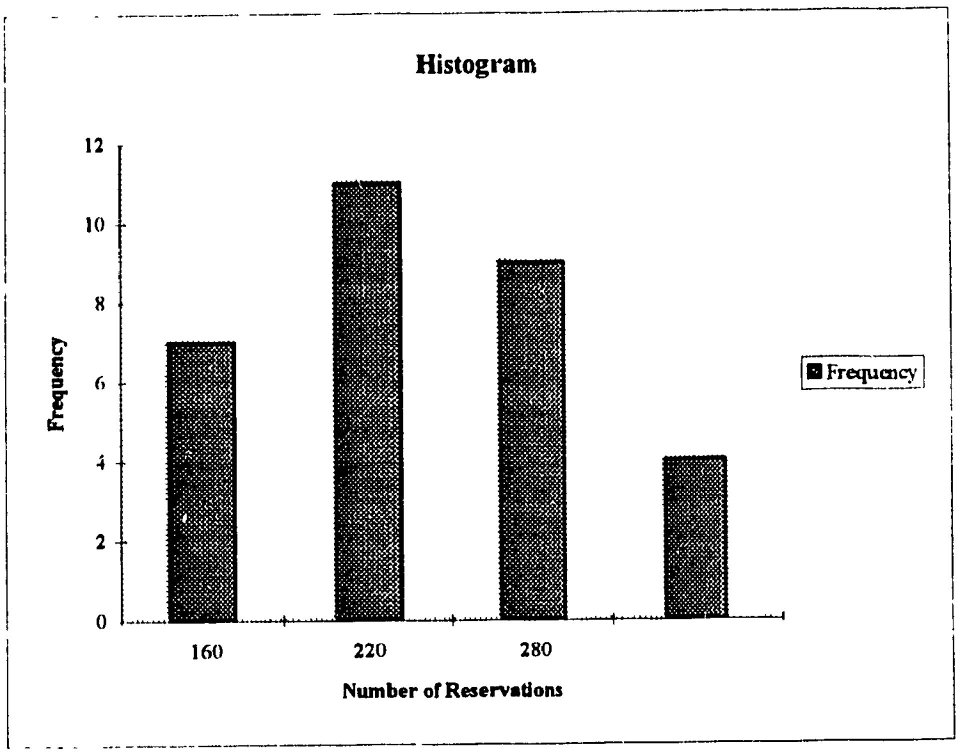
APPENDIX 5

MONTH 4: ONE



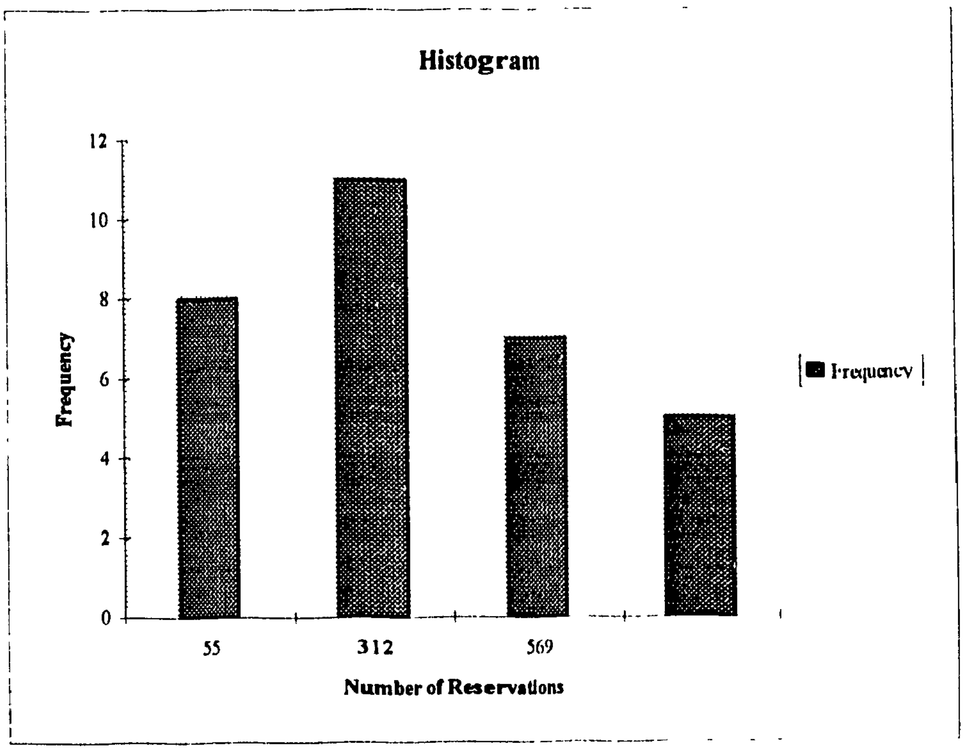
APPENDIX 5

MONTH 4: TWO



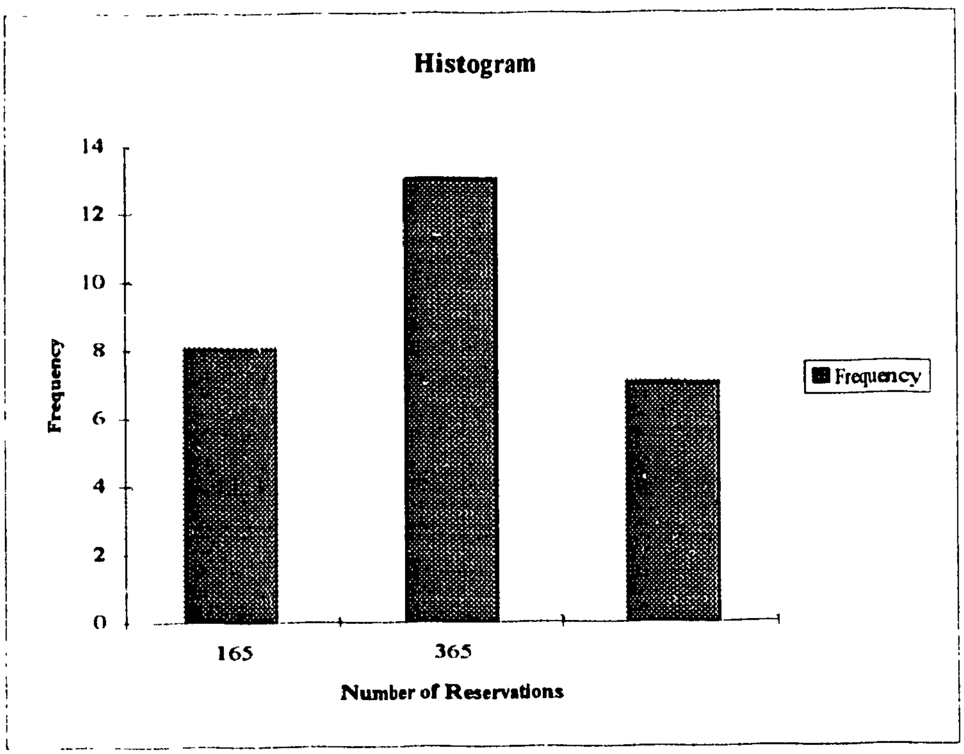
APPENDIX 5

MONTH 4: THREE



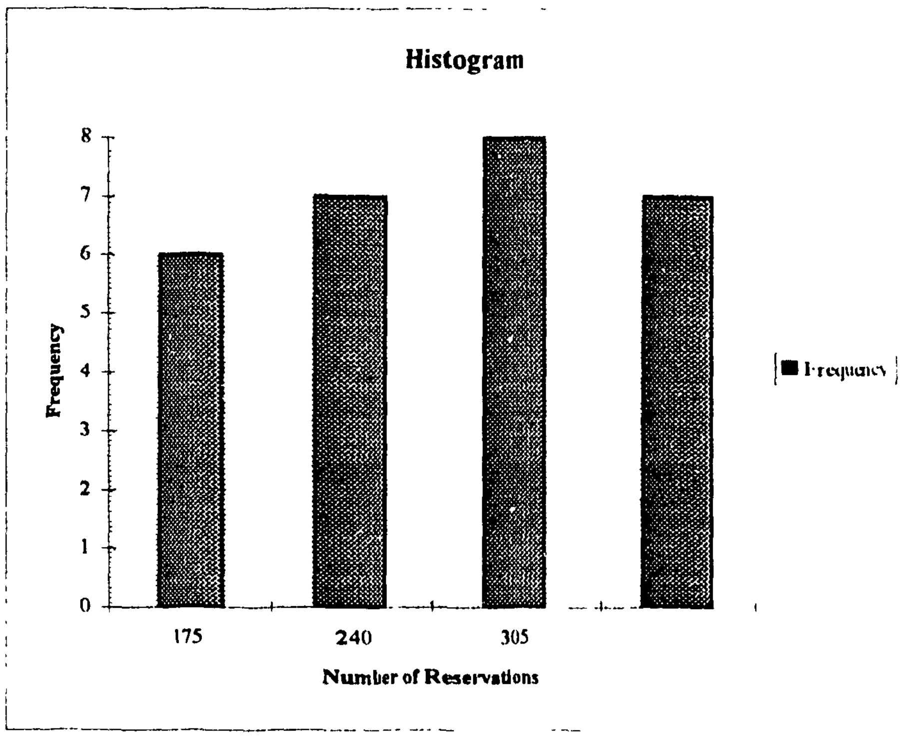
APPENDIX 5

MONTH 5: ONE



APPENDIX 5

MONTH 5: TWO



APPENDIX 5

MONTH 5: THREE