STARTING OF SYNCHRONOUS MOTORS

- DIGITAL SIMULATION -

© Boutros Albert NADER

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"LIST OF SYMBOLS"

a,b,c,	Letters used to refer to the three phases in a three phase system
d,q	To refer to direct and quadrature axis respectively
E	Magnitude of the voltage quantity
f	Frequency in Hz
G	Torque tensor
H	Inertia value
I	Current
J	Inertia quantity
j	Complex number such as $j^2 = -1$
K	Stiffness factor related to power systems
L	Inductance (self)
M	Inductance (mutual)
P	Real power in Watts
p	Derivative operator
Q	Reactive power in Vars
R	Resistance
s	Slip
T	Torque
V	Instantaneous value of voltage

W	Frequency in rad/sec; or speed in rad/sec.
Z	Impedance
Ø	Flux
Θ	Angle between direct-axis and axis of phase "a"

ABSTRACT

Starting of Synchronous Motors.

- Digital Simulation -

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There is an increasing tendancy for industrial plants located in remote areas to expand and increase their productivity. As an immediate consequence, large electrical systems will be installed; and an important part of these new systems will consist of large horsepower electrical motors. Among large motors, the synchronous type has almost no competitor; this is the reason why these expanding plants will see big synchronous motors installed on their electrical power system which is not always capable of handling such loads.

The objective of this work is to produce a mathematical equivalent for the synchronous motor that will enable an easy, accurate and precise analysis of any system containing such machines. Very few assumtions have been made in order to have as close representation as possible; however it will be found that most of the emphasis has been placed on the starting and synchronizing sequence of these motors,

since most of the problems encountered are related to this specific sequence of the operation.

Therefore the first three sections give the various representations of a synchronous motor; the fourth section is an explanation and a qualitative analysis of the synchronization mechanism. The fifth section introduces the equations that will lead to a digital computer representation. The electrical power system mathematical description is given in section F. After the implementation on the digital computer (section G) of the motor which parameters are given in section H; appropriate computer runs are presented followed by a discussion in section K, which shows the adequacy of the descriptions and implementation.

Due to the largeness of the subject treated, a deep investigation of the interaction that exists between two or more synchronous motors and the electrical system associated with was not possible, however a brief introduction to this interesting subject was mentioned.

A) MATHEMATICAL DESCRIPTION OF A SYNCHRONOUS MACHINE

1) GENERAL

Synchronous machines generally consists of rotor and stator in relative motion and rather different in structure. The following mathematical theory is developped from the fundamental starting point that the machine consists of several inductively coupled circuits, the self and mutual inductances of which vary periodically with the angular position of the rotor.

A three phase salient pole machine without amortisseur winding will be first considered; after developing the theory addition of the amortisseur windings will come to complete the description of the machine. The magnetic surfaces are assumed to be smooth so that slot effects could be neglected, and iron loss and saturation could be neglected as well. Such a machine has then four windings; the field winding and three armature phase windings.

The instantaneous terminal voltage of any one of these windings may be written in the form:

$$V = R.I + \frac{d\emptyset}{dt} \qquad (1)$$

Where: R is the resistance of the winding

I is the current in the winding

$\emptyset$ is the flux linkage of the winding, which depends upon the self-inductance of the winding, the mutual inductances between it and other windings and the currents in all the coupled windings; ie: $\emptyset = \sum L.I$

V is the voltage across the winding.

If we give to the three armature windings the subscripts a, b, c; and to the field winding the subscript f; equation (1) yields to the following set of equations:

$$V_a = R.I_a + \frac{d\emptyset a}{dt} \quad (2a)$$

$$V_b = R.I_b + \frac{d\emptyset b}{dt} \quad (2b)$$

$$V_c = R.I_c + \frac{d\emptyset c}{dt} \quad (2c)$$

$$V_f = R_f I_f + \frac{d\emptyset f}{dt} \quad (2d)$$

Where R represents the resistance of each of the armature phase and R_f the field resistance.

The four fluxes mentioned in equations (2) have the following expressions:

$$\emptyset_a = L_{aa}.I_a + L_{ab}.I_b + L_{ac}.I_c + L_{af}.I_f \quad (3a)$$

$$\emptyset_b = L_{ba}.I_a + L_{bb}.I_b + L_{bc}.I_c + L_{bf}.I_f \quad (3b)$$

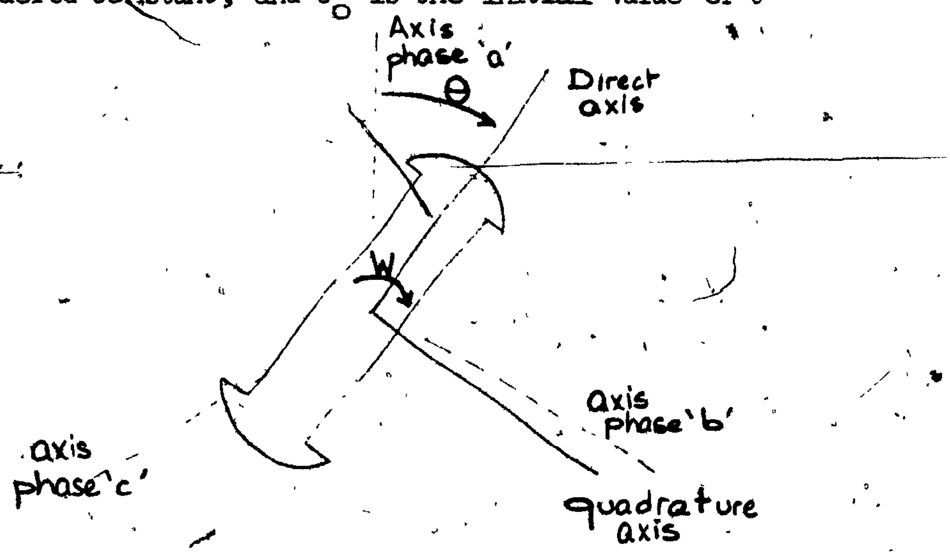
$$\emptyset_c = L_{ca}.I_a + L_{cb}.I_b + L_{cc}.I_c + L_{cf}.I_f \quad (3c)$$

$$\emptyset_f = L_{fa}.I_a + L_{fb}.I_b + L_{fc}.I_c + L_{ff}.I_{ff} \quad (3d)$$

It is however possible to solve the differential equations (2) to find the currents for any given terminal voltages by expressing the inductances in terms of the angular position of the rotor defined as θ, which changes with time, thus

$$\theta = W.t + \theta_0$$

Where W is the angular speed, which for present purposes may be considered constant, and θ_0 is the initial value of θ



Convention for phase and rotor rotations
Fig-1-

From Fig-1- we can then define θ as being the angle by which the direct axis of the field has turned beyond the axis of armature phase a.

The self inductance of each armature phase is always positive but varies with the position of the rotor. It is greatest when the d-axis of the field coincides with the axis of the armature phase; and least when q-axis of the field coincides with it.

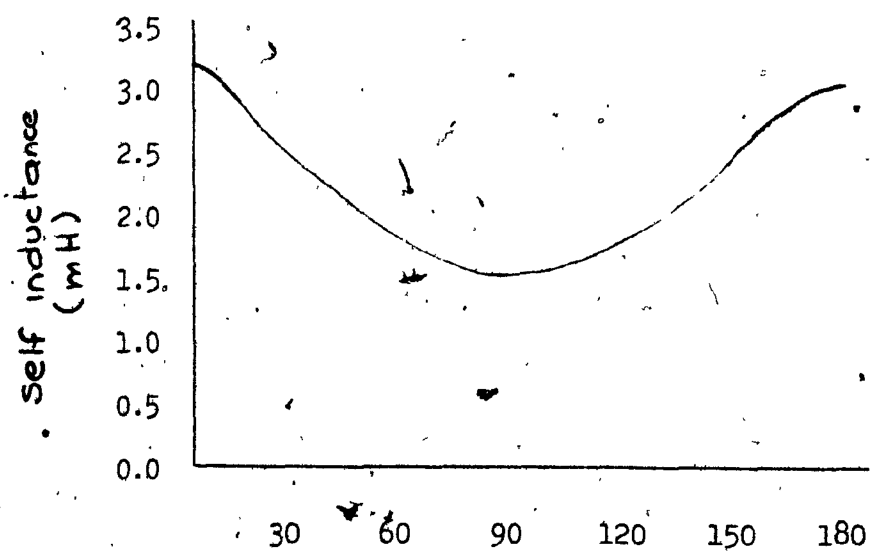
We can write the expression of the inductance for phase a for example:

$$L_{aa} = L_s + L_m.\cos 2\theta$$

The mutual inductance between two armature phases varies with the position of the field, is always negative and is greatest (in absolute value) when d-axis of the field lies midway between the axis of one phase and the reversed axis of the other phase. The equation could then be written for example for the mutual between phases a and b:

$$L_{ab} = -M_s - L_m.\cos 2(\theta + 30^{\circ})$$

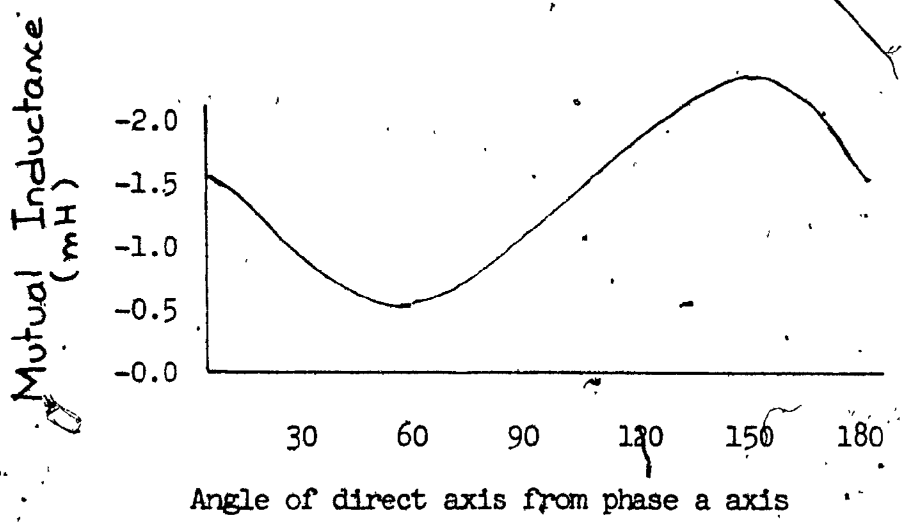
Figures 2 & 2a show the variation of the armature self and mutual inductances as a function of the rotor position.



Angle of direct axis from phase a axis

Variation of armature self-inductance

Fig-2-



Variation of armature mutual-inductance

Fig -2a-

The self-inductance of the field winding is constant and is denoted by L_{ff}.

The mutual inductance between the field winding and the three armature windings is greatest when the d-axis of the field coincides with the axis of the armature winding phase.

$$L_{af} = L_{fa} - M_f.\cos\theta$$

We are now able to write the expressions of the self and mutual inductances of the four windings under consideration.

$$L_{aa} = L_s + L_m.\cos 2\theta$$
$$L_{bb} = L_s + L_m.\cos(2\theta + 120^\circ)$$
$$L_{cc} = L_s + L_m.\cos(2\theta - 120^\circ)$$
$$L_{ff} = L_{ff}$$

$$L_{ab} = L_{ba} = -M_s - L_m \cos(2\theta - 120^o)$$
$$L_{bc} = L_{cb} = -M_s - L_m \cos(2\theta)$$
$$L_{ca} = L_{ac} = -M_s - L_m \cos(2\theta + 120^o)$$
$$L_{af} = L_{fa} = M_f \cos\theta$$
$$L_{bf} = L_{fb} = M_f \cos(\theta - 120^o)$$
$$L_{cf} = L_{fc} = M_f \cos(\theta + 120^o)$$

We can now substitute the values of the inductances into equations (3) to obtain the following expressions for the flux linkages:

$$\phi_a = (L_s + L_m \cos 2\theta).I_a + \left[-M_s + L_m \cos(2\theta - 120^o)\right].I_b + \left[-M_s + L_m \cos(2\theta + 120^o)\right].I_c + \left[M_f \cos\theta\right].I_f \quad (4a)$$

$$\phi_b = \left[-M_s + L_m \cos(2\theta - 120)\right].I_a + \left[L_s + L_m \cos(2\theta + 120)\right].I_b + \left[-M_s + L_m \cos 2\theta\right].I_c + \left[M_f \cos(\theta - 120)\right].I_f \quad (4b)$$

$$\phi_c = \left[-M_s + L_m \cos(2\theta + 120)\right].I_a + \left[-M_s + L_m \cos 2\theta\right].I_b + \left[L_s + L_m \cos(20 - 120)\right].I_c + \left[M_f \cos(\theta + 120)\right].I_f \quad (4c)$$

$$\phi_f = \left[M_f \cos\theta\right].I_a + \left[M_f \cos(\theta - 120)\right].I_b + \left[M_f \cos(\theta + 120)\right].I_c + L_{ff}.I_f \quad (4d)$$

If we now assume that the voltages impressed on the four windings are known as function of time and that expressions are wanted for currents as functions of time; four differential equations are then needed which have no unknowns except the four currents and their time derivatives. To obtain such equations, we can assume that the speed W can be considered constant. The resulting equations are then very complicated as can be noticed from the expression of the voltage V_a.

$$
\begin{aligned}
V_a = & \left[R - 2WL_m \sin 2(Wt + \theta_o) \right] . I_a - \left[2WL_m \sin(2Wt + 2\theta_o - 120) \right] . I_b \\
& - \left[2WL_m \sin(2Wt + 2\theta_o + 120) \right] . I_c - \left[WM_f \sin(Wt + \theta_o) \right] . I_f \\
& + \left[L_s + L_m \cos 2(Wt + \theta_o) \right] . \frac{dIa}{dt} + \left[M_s + L_m \cos(2Wt + 2\theta_o - 120) \right] . \frac{dIb}{dt} \\
& + \left[M_s + L_m \cos(2Wt + 2\theta_o + 120) \right] . \frac{dIc}{dt} + \left[M_f \cos(Wt + \theta_o) \right] . \frac{dIf}{dt}
\end{aligned}
$$

2) PARK'S TRANSFORMATION

To simplify the equations of the synchronous machines; a set of fictitious currents, voltages and flux linkages could be considered. The new variables will be a function of actual currents, voltages and fluxes; and may then be solved as functions of time.

The substitution could be regarded as purely mathematical; but the particular substitution in this case is based on physical reasoning and the substituted variables could be considered as having a physical interpretation.

Let us replace the actual currents I_a, I_b and I_c by the following set of currents I^d, I^q and I^o; such that:

$$I_a = I^d \cos\theta - I^q \sin\theta + I^o \qquad (5a)$$

$$I_b = I^d \cos(\theta - 120) - I^q \sin(\theta - 120) + I^o \qquad (5b)$$

$$I_c = I^d \cos(\theta + 120) - I^q \sin(\theta + 120) + I^o \qquad (5c)$$

I_f stays unchanged but can be noted as I^f (5d)

The transformation matrix could then be written:

$$C = \frac{2}{3} \begin{bmatrix} \cos\theta & \sin\theta & 1/2 \\ \cos(\theta - 120) & \sin(\theta - 120) & 1/2 \\ \cos(\theta + 120) & \sin(\theta + 120) & 1/2 \end{bmatrix}$$

It is found that the matrix C is orthogonal; $\det(C)=1$ and C^{-1} is identical to the transpose of C .

Physical interpretation of Park's transformation

The mmf of each armature phase, being sinusoidally distributed in space, may be represented by a vector having the phase axis for direction and a magnitude proportional to instantaneous phase current.

The combined mmf of the three phases may be represented by a vector sum of the phase mmf vectors, the projection of which on the field direct and quadrature axis are equal to the sum of the projections of the phase mmf vectors on the respective axis.

At this stage we would accept the fact that 2/3 is nothing but an arbitrary constant.

We may therefore conclude by saying that:

* I^d may be interpreted as the instanteneous current in a fictitious armature winding which rotates at the same speed as the field winding and remains in such a position that this winding axis always coincides with the direct axis of the field. The value of I^d being such that it gives the same mmf on this axis as do the three actual instantaneous phase currents flowing in the actual armature windings.
* I^q may be defined as I^d but referring to the field quadrature axis.
* I^o is identical to the zero sequence current; except that it is an instantaneous value, and is defined as a function of the instantaneous phase currents. It is worthwile to note that I^o gives no

space-fundamental air gap flux.

We will see that the flux linkages of the fictitious armature windings in which I^d and I^q flow are ϕ_d and ϕ_q respectively.

It is important to note at this stage that the mmf's created by the two currents I^d and I^q are stationary with respect to the rotor position and therefore act on paths of constant permeance ; we may then deduce that the corresponding inductances L_d and L_q are independent of the rotor position.

3) CIRCUIT EQUATIONS IN TERMS OF PARK'S VARIABLES

Although the reference systems are not the same, the form of the equations will remain unchanged; and we will still have the following form for the voltage relationship:

$$V = R.I + \frac{d\emptyset}{dt}$$

The voltages in the new reference frame will be given by the following relation

$$V_{d,q,o} = \left(V_{a,b,c}\right) \cdot \left(C\right)$$

We then obtain:

$$V_d = 2/3 \left[V_a \cos\theta + V_b \cos(\theta - 120) + V_c \cos(\theta + 120) \right]$$
$$V_q = -2/3 \left[V_a \sin\theta + V_b \sin(\theta - 120) + V_c \sin(\theta + 120) \right]$$
$$V_o = 1/3 \left[V_a + V_b + V_c \right]$$

Substituting in equations (2), we obtain;

$$V_d = R.I^d + \frac{d\emptyset d}{dt} - W.\emptyset_q \qquad (6a)$$

$$V_q = R.I^q + \frac{d\emptyset q}{dt} + W.\emptyset_d \qquad (6b)$$

$$V_o = R.I^o + \frac{d\emptyset o}{dt} \qquad (6c)$$

The expressions of the currents are given in equations (5).

DISCUSSION

The presence of the two terms $W.\emptyset_d$ and $W.\emptyset_q$ in the expressions of V_q and V_d respectively demands a physical interpretation.

As was seen before the currents I^d and I^q were the currents in the fictitious stator windings that are rotating at the same speed as the rotor, that give the same mmf's as the armature currents in the armature windings.

The same effect can be obtained by modifying physically the machine; let the armature winding be stationary and a closed circuit with a commutator on which rest brushes that rotate with the field, ie: the rotor. The magnetic axis of the stator will coincide with the brush axis. I^d may then be interpreted as the current entering and leaving the armature through a pair of brushes which are aligned with the direct axis of the field; similarly I^q will flow through another pair of brushes aligned with the quadrature axis of the field. This physical interpretation will still be true as far as the voltages are concerned. The two terms $W.\emptyset_q$ and $W.\emptyset_d$ may be regarded as components of applied voltages required to balance the speed voltages; the speed voltages across each pair of brushes is proportional to the flux on the axis 90^o ahead on the brush axis.

B) DIFFERENT MODELS OF THE ACTUAL SYNCHRONOUS MACHINE

1) THE MATHEMATICAL MODEL

We will assume that the damper windings can be lumped into two windings, one in the direct axis of the field and the second in the quadrature axis. Therefore we are able to write the flux linkages relations for a synchronous machine having damper windings.

$$V_d = R_a . I^d + \frac{d\emptyset d}{dt} - W.\emptyset_q \qquad (7a)$$

$$V_q = R_a . I^q + \frac{d\emptyset q}{dt} + W.\emptyset_d \qquad (7b)$$

$$V_{kd} = R_{kd} I^{kd} + \frac{d\emptyset kd}{dt} \qquad (7c)$$

$$V_{kq} = R_{kq} I^{kq} + \frac{d\emptyset kq}{dt} \qquad (7d)$$

$$V_f = R_f . I^f + \frac{d\emptyset f}{dt} \qquad (7e)$$

V_{kd} and V_{kq} are the voltages across the direct axis damper windings and the quadrature axis damper winding respectively. It is agreed that these two voltages are equal to zero since the damper windings are normally short-circuited.

As we will consider the supply to the synchronous machine to have balanced phase voltages, it is convinient to assume that the zero-sequence voltage component V_o will be also equal to 0.

At this stage, it is considered that the saturation of the field iron could be neglected and that there is no remaneent flux in the armature iron.

2) TENSOR REPRESENTATION OF SYNCHRONOUS MACHINES

Without going through all the algebraic and trigonometric work required to proove the correcteness of the following equalities, we will just note the results.

We will first define the quantities below.

L_d : Direct-axis synchronous inductance.

L_q : Quadrature-axis synchronous inductance.

L_{kd}: Direct-axis damper winding synchronous inductance.

L_{kq}: Quadrature-axis damper winding synchronous inductance.

L_f : Field winding synchronous inductance.

Note: The field coil is assumed to be in the direct-axis, and therefore it does not have any linkage flux with either the quadrature axis stator winding or damper winding.

M_{ad}: Synchronous mutual inductance between direct-axis windings, the mutual inductance between direct-axis stator winding and field or direct-axis damper windings. Both the mutual inductances are assumed to be equal; the assumtion will not introduce at this stage any loss of accuracy.

M_{aq}: Synchronous mutual inductance between quadrature-axis windings, the mutual inductance between quadrature-axis stator winding and the quadrature-axis damper winding.

After definition of the synchronous inductances in the Park's system of variables, we may now proceed to the writing of the flux expressions.

$$
\begin{aligned}
\phi_d &= M_{ad}.I^f + M_{ad}.I^{kd} + L_d.I^d \\
\phi_q &= M_{aq}.I^{kq} + L_q.I^q \\
\phi_{kd} &= M_{ad}.I^f + M_{ad}.I^d + L_{kd}.I^{kd} \\
\phi_{kq} &= M_{aq}.I^q + L_{kq}.I^{kq} \\
\phi_f &= M_{ad}.I^d + M_{ad}.I^{kd} + L_f.I^f
\end{aligned}
$$

By replacing the above expressions into equations (7), we obtain:

$$
V_d = R_a.I^d + \frac{d}{dt}\left[M_{ad}.I^f + M_{ad}.I^{kd} + L_d.I^d\right] - W.\left[M_{aq}.I^{kq} + L_q.I^q\right] \qquad (8a)
$$

$$
V_q = R_a.I^q + \frac{d}{dt}\left[M_{aq}.I^{kq} + L_q.I^q\right] + W.\left[M_{ad}.I^f + M_{ad}.I^{kd} + L_d.I^d\right] \qquad (8b)
$$

$$
V_{kd} = R_{kd}.I^{kd} + \frac{d}{dt}\left[M_{ad}.I^f + M_{ad}.I^d + L_{kd}.I^{kd}\right] \qquad (8c)
$$

$$
V_{kq} = R_{kq}.I^{kq} + \frac{d}{dt}\left[M_{aq}.I^q + L_{kq}.I^{kq}\right] \qquad (8d)
$$

$$
V_f = R_f.I^f + \frac{d}{dt}\left[M_{ad}.I^d + M_{ad}.I^{kd} + L_f.I^f\right] \qquad (8e)
$$

Since the direct and quadrature axis are fixed with respect to the rotor, the different values of the synchronous inductances do not change with the position of the rotor, and therefore do not change as a function of time.

In this case, equations (8) could be rewritten in the following form:

$$V_d = R_a.I^d + M_{ad}pI^f + M_{ad}pI^{kd} + L_d.pI^d - W.\left[M_{aq}.I^{kq} + L_q.I^q\right] \quad (9a)$$

$$V_q = R_a.I^q + M_{aq}.pI^{kq} + L_q.pI^q + W.\left[M_{ad}.I^f + M_{ad}.I^{kd} + L_d.I^d\right] \quad (9b)$$

$$V_{kd} = R_{kd}.I^{kd} + M_{ad}.pI^f + M_{ad}.pI^d + L_{kd}.pI^{kd} \quad (9c)$$

$$V_{kq} = R_{kq}.I^{kq} + M_{aq}.pI^q + L_{kq}.pI^{kq} \quad (9d)$$

$$V_f = R_f.I^f + M_{ad}.pI^d + M_{ad}.pI^{kd} + L_f.pI^f \quad (9e)$$

The letter p indicates the derivative operator $\frac{d}{dt}$.

It is possible now to put equations (9) in matricial form, but it would be more correct to talk about tensors rathe than matrices, since matrices do not have inherent law of transformations, while tensors do have such a law. (SEE APPENDIX I)

Before doing so let us first introduce the expressions of the electro-magnetic torque developped by the machine:

$$T = \emptyset_d.I^q - \emptyset_q.I^d \quad (10)$$

Replacing $\emptyset_d$ and $\emptyset_q$ by their respective values, the torque expression becomes:

$$T = M_{ad}.I^f.I^q + M_{ad}.I^q.I^{kd} + L_d.I^d.I^q$$

$$- M_{aq}.I^{kq}.I^d - L_q.I^q.I^d \qquad (11)$$

We can at this stage represent in tensor form a synchronous machine, it is only required to introduce some of the terms needed.

If the voltage tensor is defined as being (V) and the current tensor as being (I) where:

$$(V) = \begin{bmatrix} V_d \\ V_q \\ V_{kd} \\ V_{kq} \\ V_f \end{bmatrix} \quad \text{and} \quad (I) = \begin{bmatrix} I^d \\ I^q \\ I^{kd} \\ I^{kq} \\ I^f \end{bmatrix}$$

Then by properly applying Ohm's law we can write:

$$(V) = (Z) \cdot (I)$$

Where (Z) could be defined as being the impedance tensor and the equation could then be expanded in the following way.

$$\begin{pmatrix} V_d \\ V_q \\ V_{kd} \\ V_{kq} \\ V_f \end{pmatrix} = \begin{pmatrix} R_a + L.p & -W.L_q & M_{ad}.p & -W.M_{aq} & M_{ad}.p \\ W.L_d & R_a + L_q.p & W.M_{ad} & M_{aq}.p & W.M_{ad} \\ M_{ad}.p & 0 & R_{kd}+L_{kd}.p & 0 & M_{ad}.p \\ 0 & M_{aq}.p & 0 & R_{kq}+L_{kq}.p & 0 \\ M_{ad}.p & 0 & M_{ad}.p & 0 & R_f + L_f.p \end{pmatrix} \times \begin{pmatrix} I^d \\ I^q \\ I^{kd} \\ I^{kq} \\ I^f \end{pmatrix}$$

By the same talking we can introduce the tensor G, such that the torque expression could be given in the form:

$$T = (I)_t \times (G) \times (I)$$

Therefore:

$$(T) = \begin{pmatrix} I^d & I^q & I^{kd} & I^{kq} & I^f \end{pmatrix} \times \begin{pmatrix} 0 & -L_q & 0 & -M_{aq} & 0 \\ L_d & 0 & M_{ad} & 0 & M_{ad} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \times \begin{pmatrix} I^d \\ I^q \\ I^{kd} \\ I^{kq} \\ I^f \end{pmatrix}$$

C) EQUIVALENT CIRCUIT OF THE SALIENT POLE SYNCHRONOUS MACHINE

In setting up equivalent circuits for the various types of machines, it has been found that the physical reconstruction of the equations was only possible if each symbol in this equation is a tensor. Geometric objects and other non tensor invariants could not be physically represented. Kron stated as an engineering principle: A set of equations expressing the performance of a physical system (be it electrical, mechanical, thermal or any other system) may be represented by a model (equivalent circuit) only if the equations are tensor equations.

In the previous section, we have been able to write transient impedance tensor (Z) as well as the torque tensor that we defined as (G).

By just observing the tensor equations and applying Khirchoff's laws, it will be possible to draw the equivalent circuit of a salient pole synchronous machine with damper windings.

We can restate here that the voltages across the damper windings are equal to zero, since these windings are normally short-circuited.

One should note at this stage the following computation: Wb being the supply frequency, it will be possible to calculate the impedances of each of the windings in the machines for steady state.

Therefore we define:

X_f as being $Wb.L_f$

X_{kd} as being $Wb.L_{kd}$

X_{kq} as being $Wb.L_{kq}$

X_d as being $Wb.L_d$

X_q as being $Wb.L_q$

X_{aq} as being $Wb.M_{aq}$

X_{ad} as being $Wb.M_{ad}$.

Hence we can define:

$X_f = X_{ad} + x_{fl}$

$X_{kd} = X_{ad} + x_{kdl}$ x's: leakage reactance

$X_{kq} = X_{aq} + x_{kql}$

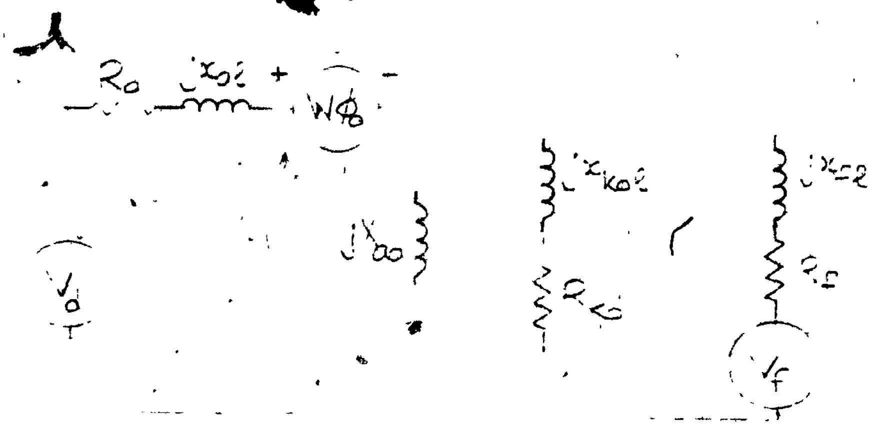
$X_d = X_{ad} + x_{al}$ x_{al}: leakage reactance of d and q axis stator windings

$X_q = X_{aq} + x_{al}$

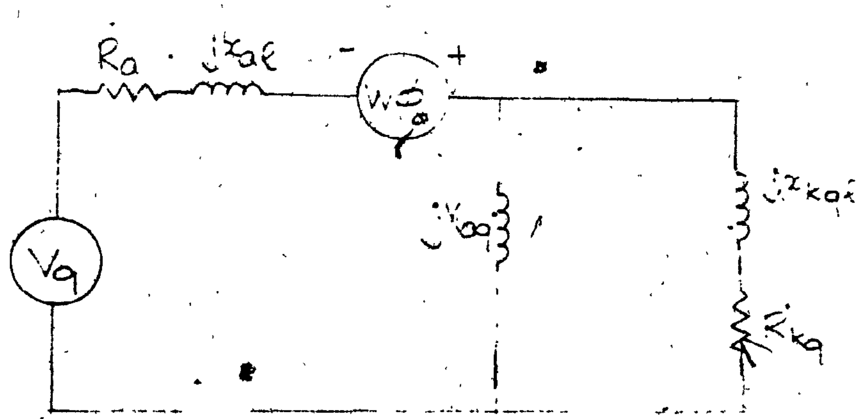
Also it should be remembered that the expression of the impedance is:

$$Z = R + j.X$$

The drawing of the equivalent circuit is then easy, and the two separate circuits, one for the direct-axis and one for the quadrature-axis will look like:



DIRECT-AXIS EQUIVALENT CIRCUIT



QUADRATURE-AXIS EQUIVALENT CIRCUIT

It is obvious that the two equivalent circuits along the direct and quadrature axis are interdependent, however it is possible to obtain a transient equivalent circuit of a salient pole synchronous machine by considering a change in reference frames. This subject has been treated by G. Kron in his book "Equivalent circuits for electric machinery".

In his book, Kron considers a revolving field theory in opposition to a cross field theory (ie: d an q axis). In the revolving field theory, he considers forward and backward axis after introducing the concept of frequency tensor. Since the result of this treatment was not of any help for our purpose we find it adequate just to mention it.

As a matter of fact, we considered seriously using the equivalent circuit for digital computer simulation, but the programs available would have made it a rather hard task, plus the fact that the mechanical aspect of the problem had to be given an electrical equivalent.

D) STARTING AND SYNCHRONIZING OF SYNCHRONOUS MOTORS

1) GENERAL

Synchronous motors start as induction motors because the damper windings that are built-in the machine do have the same effect as the squirrel cage bars in an induction motor ; it should be noted that although the primary purpose of these windings is to develop torque at start and during acceleration, it also serves to dampen power oscillations while running at synchronous speed. It should be remembered that this torque developped is proportional to slip; therefore the damper windings do not contribute to the torque produced at synchronous speed. Torque at synchronous speed is largely derived from the magnetic field produced by the field coils on the rotor linking the rotating magnetic field produced by the currents in the armature windings on the stator.

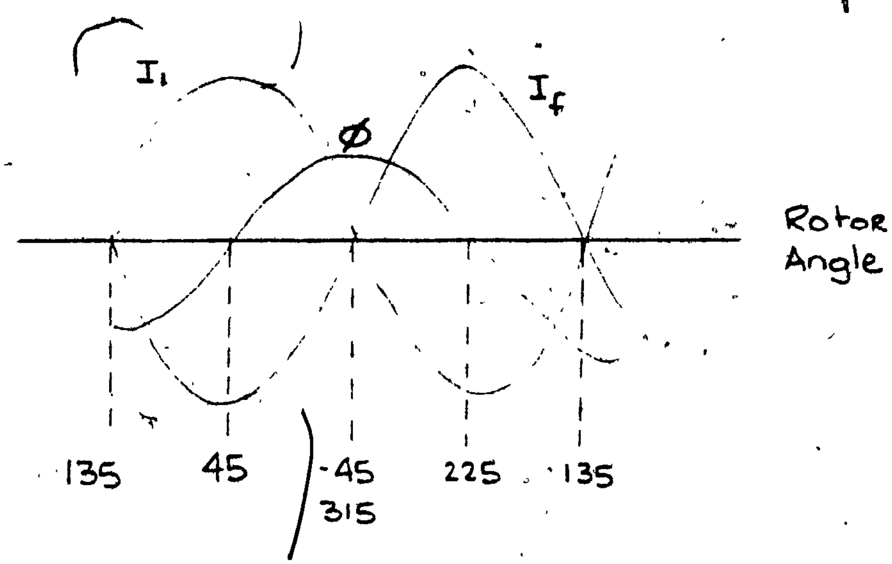
Synchronous motors possess two general categories of torque characteristics. The first being determined by the damper winding design, the second being determined by the flux in the salient field poles on the rotor. The first is the starting torque, the second is the synchronous torque.

In the starting mode, the synchronous motor salient poles are not excited by their external dc source. If they were, there would be no useful torque developed by them. The reason for saying this is that the average torque due to field excitation during slip would be a negative or breaking torque and hence would act to reduce the total amount of accelerating torque. In addition there is a very large amount of oscillating component at slip frequency produced by field excitation which could result in damage to the motor if full field current was applied during the whole starting sequence.

At synchronous speed, the unsymmetrical nature of the rotor (ie: saliency) produces a small torque (reluctance torque) which enables the motor to run at very light loads in synchronism without external excitation. Reluctance torque can also pull the motor into step if it is very lightly loaded and coupled to low inertia.

It is convenient to make an analogy of a synchronous motor to a current transformer for the purpose of demonstrating angular relationships of field current and fluxes with rotor position.

If I_1 is an imaginary current in the stator causing the transformer action (due to stator flux cutting the rotor winding) then I_1 will be about 180° from I_f (induced current in field coils) and the flux Ø will be 90° behind I_f.



Maximum induced flux occurs when I_f passes through zero going from negative value to positive (maximum rate of change of induced current). The rotor angle at which I_1 and I_f go through zero depends on the ratio (inductance:resistance) of the field circuit. Inductance is high at low speed, since the slip frequency is then high, the reactance decreases with increasing speed, and the angle arc tan $\frac{L}{R}$ will shift towards zero if the circuit contains high resistance (like discharge resistors) but if field is short circuited (as if brushless excitation is used) very little shift will occur.

As the stator goes beyond -45°, torque increases (essentially due to increased stator flux). I_f then gives a very convenient indicator of maximum flux and increasing torque from which we may apply excitation for maximum effectiveness.

If the field discharge loop is opened at the point of maximum flux, then this flux is "trapped". Applying external amperes to the current path in correct polarity to increase this trapped flux at this instant then makes maximum use of its existence. Furthermore, the rotor has just moved by and is in the position to pull the rotor forward into synchronous alignment.

Before going into a more detailed investigation of the pulling into step operation, we can make the following note. In addition to permitting closer matching of motor load, optimum application of excitation also reduces power system disturbance which occurs when the motor goes through a complete slip cycle with field energized. If the motor is large relatively to the power system, surges transmitted to the system will be at a minimum if field is applied to prevent slip at pull-in.

2) ASYNCHRONOUS STARTING AND SYNCHRONIZATION OF SYNCHRONOUS MOTORS

We can start by evaluating the torques acting on the rotor at the instant the field coils are excited; they are:

- T_o shaft torque considered constant.
- T_{ae} Asynchronous electrical torque, this torque being proportional to slip as long as the later has a small value.
- θ Being the angle between the rotor direct-axis and the axis of mmf produced by armature currents of rated frequency.

  The angular velocity is $\frac{d\theta}{dt}$ and θ decreases continuously if running below synchronous speed.
- s The slip; which is equal to $\frac{\text{sync. speed - effect. speed}}{\text{sync. speed}}$.

The expression for the asynchronous electrical torque can be given as:

$$T_{ae} = K_a . s . w_{sync}.$$

or

$$T_{ae} = -K_a . \frac{d\theta}{dt}.$$

Once the asynchronous start has been accomplished, and before exciting the field, the asynchronous torque will be balanced by T_s and a constant slip operation is established such that:

$$T_s = -K_a . s_0 . w_{sync.}$$

were s_0 is the value of the constant slip.

Synchronous electrical torque: When field is excited, a synchronous torque T_{sync}, proportional to $\sin\theta$, is applied on the rotor. The average value of $T_{sync.}$ is zero as long as θ is not constant. $T_{sync.}$ will be added to the T_{ae} when it is assuming a motoring value accelerating the machine to bring it in synchronism When $T_{sync.}$ is assuming a non motoring value it will be substracted from T_{ae}; decelerating the machine.

$T_{sync.} > 0$ motoring

$T_{sync.} < 0$ non motoring

$$T_{sync.} = -K_{sync.} . \sin\theta$$

$$K_{sync.} = \frac{P_m}{\Omega}$$

Where Ω is the instantaneous speed, which could be assumed the synchronizing speed.

The mechanical torque T_m (mainly due to inertia) which opposes any variation of speed of rotor is expressed as:

$$T_m = -K_m \frac{d}{dt} \left(\frac{d\theta}{dt}\right)$$

acceleration is: $\frac{d}{dt}(\frac{d\theta}{dt})$

T_m is positive if the acceleration is negative.

The general equation of motion could then be written:

$$T_m + T_{ae} + T_{sync.} = T_0$$

$$-K_m \cdot \frac{d^2\theta}{dt^2} \quad -K_a \cdot \frac{d\theta}{dt} \quad -K_{sync.} \cdot \sin\theta = K_a \cdot s_0 \cdot w_{sync.}$$

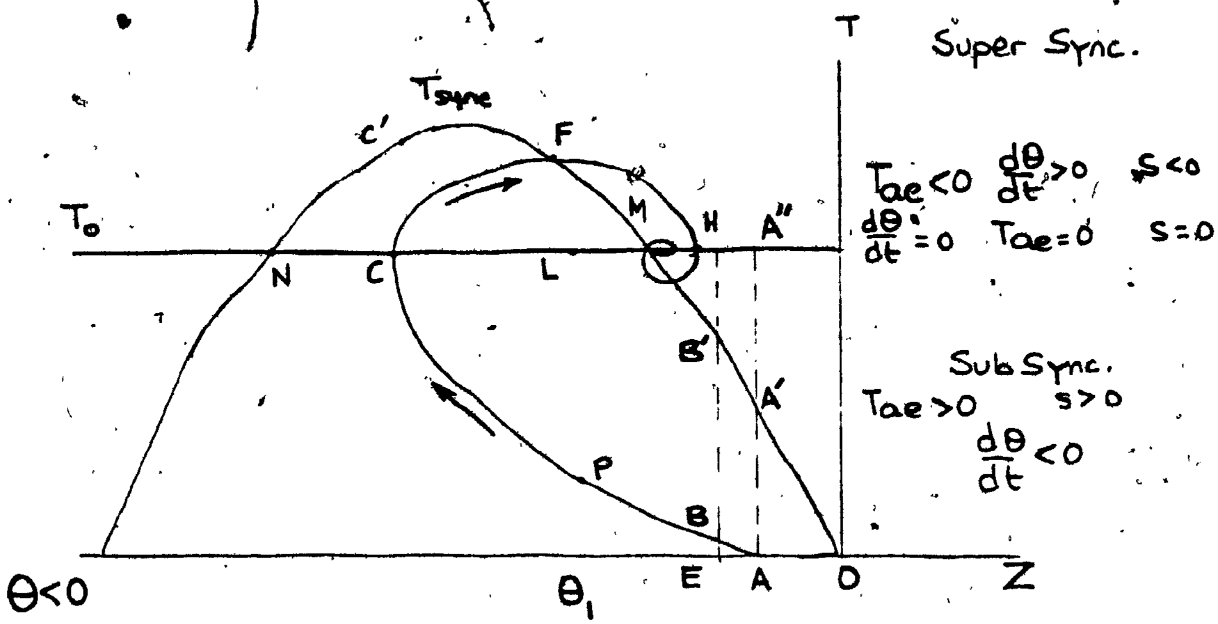
The particular solutions of this equation show that θ approaches a constant value (corresponding to pull-in angle) if:

$$s_0 \cdot w_{sync.} = a \sqrt{\frac{K_{sync.}}{K_m}}$$

Where "a" is a constant very close to 1 if the angles are assumed in radian.

A qualitative analysis of pull-in mechanism is done referring to the following diagram.

- Abscissa axis: Angular displacement
- Ordonate axis: Torques
- Origin of abscissa axis is a displacement such that $T_{sync.} = 0$, and going from negative to positive:



DISCUSSION:

The following paragraph will explain the graphical representation of the pull-in mechanism.

As long as the machine is not excited, T_{ae} balances T_0 as can be seen for example at point A. $T_0 = AA''$.

When the machine runs in synchronism after being excited $T_{sync.}$ balances T_0 and $T_{ae} = 0$. This is observed at point M intersection of curves of $T_{sync.}$ and T_0.

Before excitation; the operating point of asynchronous operation moves along ZOA. Let us assume that field is applied at θ_0;OA. We have $T_{sync.}(\theta_0) = AA'$ which adds to $T_{ae}(\theta_0) = AA''$. Torque equilibrium is then broken; the resultant torque is:

$$T(A) = AA' + AA'' - T_0 = AA' \quad \text{Positive.}$$

The machine will accelerate with an acceleration of $\frac{d^2\theta}{dt^2}$.

Therefore the difference in velocity $\frac{d\theta}{dt}$ decreases and so does T_{ae}. At this stage the operating point leaves the axis of abscissa going in an upward direction.

At point B; the different acting torques are proportional to:

$$T_{ae}(B) = BD$$

$$T_{sync.}(B) = EB'$$

[illegible]$(B) = ED$.

The resultant torque will be $T(B) = BB'$. Positive, therefore the operating point keeps moving upwards following ABC.

If:
- s is small enough
- the inertia is not too high
- I_f excitation current strong enough
- and application point A not far from 0.

Then the curve ABC... will reach T_0 before point N; ie: synchronism is attained while rotor is submitted to an accelerating torque CC' > 0. In this case rotor speed will exceed the synchronous speed and T_{ae} starts to become negative as well as $\frac{d\theta}{dt}$. θ will then start to increase in value and the operating point will follow CFH.

The resultant torque T(F) will be zero; $\frac{d\theta}{dt}$ is maximum; the speed is getting close to synchronous and the resultant torque becomes then resistive or decelerating torque untill point H is reached; then the speed equals the synchronous speed. This type of hunting operation, if it can be called this way, will go on untill point M is reached.

At point M: Speed = Sync. Speed = Constant

θ = Constant

$$\frac{d\theta}{dt} = 0$$

θ is then the machine phase angle, and we can see from the graph that for a stable operation to take place, θ has to be always less than 90°.

E) DIGITAL COMPUTER REPRESENTATION OF THE SYNCHRONOUS MACHINE

1) STATEMENT OF THE EQUATIONS

Rewriting equations (7) we obtain:

$$V_d = R_a \cdot I^d + \frac{d\phi d}{dt} - W.\phi q$$
$$V_q = R_a \cdot I^q + \frac{d\phi q}{dt} + W.\phi d$$
$$V_{kd} = R_{kd} \cdot I^{kd} + \frac{d\phi kd}{dt}$$
$$V_{kq} = R_{kq} \cdot I^{kq} + \frac{d\phi kq}{dt}$$
$$V_f = R_f \cdot I^f + \frac{d\phi f}{dt}$$

And redifining the reactive component of the impedances of the machine, ie: the reactances;

$$X_f = Wb.L_f = X_{ad} + x_{f1}$$
$$X_{kd} = Wb.L_{kd} = X_{ad} + x_{kd1}$$
$$X_{kq} = Wb.L_{kq} = X_{aq} + x_{kq1}$$
$$X_d = Wb.L_d = X_{ad} + x_{a1}$$
$$X_q = Wb.L_q = X_{aq} + x_{a1}$$
$$X_{ad} = Wb.M_{ad}$$
$$X_{aq} = Wb.M_{aq}$$

Where W is the speed of the rotor, and Wb the base frequency (the supply frequency).

We can also rewrite the flux expressions:

$$\emptyset_d = \emptyset_{ad} + \emptyset_{dl}$$

$$\frac{1}{Wb}\left[X_{ad}.I^f + X_{ad}.I^{kd} + (X_{ad} + x_{al}).I^d\right]$$

$$\emptyset_{kd} = \emptyset_{ad} + \emptyset_{kdl}$$

$$\frac{1}{Wb}\left[X_{ad}.I^f + X_{ad}.I^d + (X_{ad} + x_{kdl}).I^{kd}\right]$$

$$\emptyset_q = \emptyset_{aq} + \emptyset_{ql}$$

$$\frac{1}{Wb}\left[X_{aq}.I^{kq} + (X_{aq} + x_{al}).I^q\right]$$

$$\emptyset_{kq} = \emptyset_{aq} + \emptyset_{kql}$$

$$\frac{1}{Wb}\left[X_{aq}.I^q + (X_{aq} + x_{kql}).I^{kq}\right]$$

$$\emptyset_f = \emptyset_{ad} + \emptyset_{fl}$$

$$\frac{1}{Wb}\left[X_{ad}.I^{kd} + X_{ad}.I^d + (X_{ad} + x_{fl}).I^f\right]$$

AND

$$\emptyset_{ad} = \frac{1}{Wb}\left[I^f + I^d + I^{kd}\right].X_{ad}$$

$$\emptyset_{aq} = \frac{1}{Wb}\left[I^{kq} + I^q\right].X_{aq}$$

We should now express the two fluxes $\emptyset_{ad}$ and $\emptyset_{aq}$ in function of the fluxes $\emptyset_d$, $\emptyset_q$, $\emptyset_{kd}$, $\emptyset_{kq}$, $\emptyset_f$. For that the following calculations should be undertaken. (The reason for doing this, is because the expressions of $\emptyset_{ad}$ and $\emptyset_{aq}$ shold not be implicit, ie: it should not require the calculations of the currents to determine their values, which will make the calculation of these fluxes need a multiple closed loop computation; therefore an explicit definition of these fluxes is required).

Calculations:

$$\emptyset_{d1} = \frac{1}{Wb} \; x_{a1} . I^{d} = \emptyset_d - \emptyset_{ad}$$

$$\emptyset_{q1} = \frac{1}{Wb} \; x_{a1} . I^{q} = \emptyset_q - \emptyset_{aq}$$

$$\emptyset_{kd1} = \frac{1}{Wb} \; x_{kd1} . I^{kd} = \emptyset_{kd} - \emptyset_{ad}$$

$$\emptyset_{kq1} = \frac{1}{Wb} \; x_{kq1} . I^{kq} = \emptyset_{kq} - \emptyset_{aq}$$

$$\emptyset_{f1} = \frac{1}{Wb} \; x_{f1} . I^{f} = \emptyset_f - \emptyset_{ad}$$

And the currents can then be expressed as :

$$I^{d} = \frac{Wb.(\emptyset d - \emptyset ad)}{x_{al}}$$

$$I^{q} = \frac{Wb.(\emptyset q - \emptyset aq)}{x_{al}}$$

$$I^{kd} = \frac{Wb.(\emptyset kd - \emptyset ad)}{x_{kdl}}$$

$$I^{kq} = \frac{Wb.(\emptyset kq - \emptyset aq)}{x_{kql}}$$

$$I^{f} = \frac{Wb.(\emptyset f - \emptyset ad)}{x_{fl}}$$

We can rewrite:

$$\emptyset_{ad} = \frac{X_{ad}}{Wb}\left[I^{f} + I^{kd} + I^{d}\right]$$

$$\emptyset_{aq} = \frac{X_{aq}}{Wb}\left[I^{kq} + I^{q}\right]$$

And replacing in the expressions of $\emptyset_{ad}$ and $\emptyset_{aq}$ the currents by the form obtained above, we get:

$$\emptyset_{ad} = \frac{X_{ad}}{Wb}\left[\frac{Wb.(\emptyset f - \emptyset ad)}{x_{fl}} + \frac{Wb.(\emptyset kd - \emptyset ad)}{x_{kdl}} + \frac{Wb.(\emptyset d - \emptyset ad)}{x_{al}}\right]$$

$$\emptyset_{ad} = X_{ad}\left[\frac{\emptyset f}{x_{fl}} + \frac{\emptyset kd}{x_{kdl}} + \frac{\emptyset d}{x_{al}} - \emptyset_{ad}\left(\frac{1}{x_{fl}} + \frac{1}{x_{al}} + \frac{1}{x_{kdl}}\right)\right]$$

$$\emptyset_{ad}\left[1 + X_{ad}\left(\frac{1}{x_{fl}} + \frac{1}{x_{kdl}} + \frac{1}{x_{al}}\right)\right] = X_{ad}\left[\frac{\emptyset f}{x_{fl}} + \frac{\emptyset kd}{x_{kdl}} + \frac{\emptyset d}{x_{al}}\right]$$

$$\emptyset_{ad} = \frac{X_{ad}}{1 + X_{ad}\left(\frac{1}{x_{fl}} + \frac{1}{x_{kdl}} + \frac{1}{x_{al}}\right)}\left[\frac{\emptyset f}{x_{fl}} + \frac{\emptyset kd}{x_{kdl}} + \frac{\emptyset d}{x_{al}}\right]$$

We could express the above $\emptyset_{ad}$ expression as:

$$\emptyset_{ad} = X_{md}\left[\frac{\emptyset f}{x_{fl}} + \frac{\emptyset kd}{x_{kdl}} + \frac{\emptyset d}{x_{al}}\right]$$

Where:

$$X_{md} = \frac{X_{ad}}{1 + X_{ad}\left(\frac{1}{x_{fl}} + \frac{1}{x_{kdl}} + \frac{1}{x_{al}}\right)}$$

$$X_{md} = \frac{1}{\frac{1}{X_{ad}} + \frac{1}{x_{fl}} + \frac{1}{x_{kdl}} + \frac{1}{x_{al}}}$$

Repeating for $\emptyset_{aq}$, we would obtain:

$$\emptyset_{aq} = X_{mq}\left[\frac{\emptyset kq}{x_{kql}} + \frac{\emptyset q}{x_{al}}\right]$$

Where;

$$X_{mq} = \frac{1}{\frac{1}{X_{aq}} + \frac{1}{x_{kq1}} + \frac{1}{x_{al}}}$$

Rearranging and completing the equations; we obtain:

$$V_d = R_a . I^d + \frac{d\emptyset d}{dt} - W.\emptyset_q$$

$$V_q = R_a . I^q + \frac{d\emptyset q}{dt} + W.\emptyset_d$$

$$V_{kd} = R_{kd} . I^{kd} + \frac{d\emptyset kd}{dt}$$

$$V_{kq} = R_{kq} . I^{kq} + \frac{d\emptyset kq}{dt}$$

$$V_f = R_f . I^f + \frac{d\emptyset f}{dt}$$

$$X_{md} = \frac{1}{1/X_{ad} + 1/x_{f1} + 1/x_{kd1} + 1/x_{al}}$$

$$X_{mq} = \frac{1}{1/X_{aq} + 1/x_{kq1} + 1/x_{al}}$$

$$I^d = -\frac{Wb}{x_{al}} \left[\frac{X_{md}}{x_{f1}} \emptyset_f + \frac{X_{md}}{x_{kd1}} \emptyset_{kd} + \left(\frac{X_{md}}{x_{al}} - 1 \right) \emptyset_d \right]$$

$$I^q = -\frac{Wb}{x_{al}} \left[\frac{X_{mq}}{x_{kq1}} \emptyset_{kq} + \left(\frac{X_{mq}}{x_{al}} - 1 \right) \emptyset_q \right]$$

$$I^{kd} = -\frac{Wb}{x_{kdl}}\left[\frac{X_{md}}{x_{fl}}\phi_f + \frac{X_{md}}{x_{al}}\phi_d + \left(\frac{X_{md}}{x_{kdl}} - 1\right)\phi_{kd}\right]$$

$$I^{kq} = -\frac{Wb}{x_{kql}}\left[\frac{X_{mq}}{x_{al}}\phi_q + \left(\frac{X_{mq}}{x_{kql}} - 1\right)\phi_{kq}\right]$$

$$I^{f} = -\frac{Wb}{x_{fl}}\left[\frac{X_{md}}{x_{kdl}}\phi_{kd} + \frac{X_{md}}{x_{al}}\phi_d + \left(\frac{X_{md}}{x_{fl}} - 1\right)\phi_f\right]$$

Also the air gap torque developped by the motor is given by:

$$T_{ag} = \phi_d . I^q + \phi_q . I^d$$

We should recall here the definition of the direct and quadrature axis voltages in function of the three phase voltages.

Θ being the angle between the phase 'a' axis and the direct-axis, and W being the actual speed of the rotor, we can simply state:

$$W = \frac{d\Theta}{dt}$$

assuming that at time 0 , the value of Θ is 0.

Therefore;

$$V_d = 2/3 \left[V_a.\cos\theta + V_b.\cos(\theta - 120) + V_c.\cos(\theta + 120) \right]$$

$$V_q = -2/3 \left[V_a.\sin\theta + V_b.\sin(\theta - 120) \quad V_c.\sin(\theta + 120) \right]$$

And:

$$V_a = E.\sin(Wb.t)$$

$$V_b = E.\sin(Wb.t - 120)$$

$$V_c = E.\sin(Wb.t + 120)$$

The value of the voltage V_f that will be applied to the field winding is considered to be supplied by a direct-current power supply, and having the appropriate value to circulate the required fiel current.

2) THE MECHANICAL SYSTEM EQUATIONS

Looking at the mechanical system formed by the rotor of the machine, its shaft and the load connected to it; one can make the following observations.

The electrical torque developped by the motor, or what has been defined so far as the air gap torque, is the drivinf force of the mechanical system. The inertia of the rotating bodies around the machine shaft multiplied by their acceleration forms one portion of the load torque, while the other portion is mainly a function of the product of the speed and the friction factor which is due to the friction in the bearings and also a function of the work required to produce a mechanical force to drive the load.

Summarizing:

$$T_{ag} = T_{inertia} + T_{friction}$$

Where we define;

$$T_{inertia} = J_{(total\ inertia)} \cdot \text{Acceleration}$$

$$T_{friction} = \text{Friction factor} \cdot \text{Speed}$$

F) MATHEMATICAL REPRESENTATION OF ELECTRICAL POWER SYSTEMS

1) GENERAL

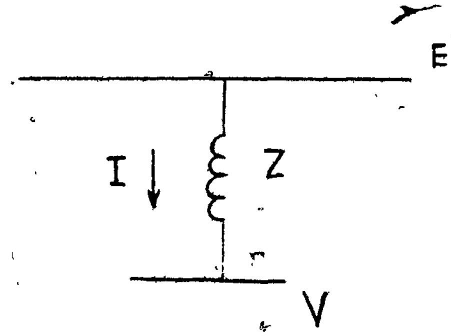
In most of the theoretical studies, power systems are considered to be ideal; it means that the system is assumed to be equivalent to a constant voltage supply that is not affected at all by the nature of the load. However we know that this approach is effectively far from the reality, especially in cases where relatively large loads are involved.

Therefore to be able to define a realistic electrical power system, one can assume the system to be ideal and then add the various parameters that would then transform this system into a more practical one.

A power system is fairly well defined by its capacity to supply the required current to the load; and to be able to have a loading situation common to every power system, one can consider a short-circuit condition. Hence, by knowing the performance of the power supply having a short-circuit across it, we may then compare it with other systems. It has been a common practice to give this figure in MVA since it could be compared with actual power.

Without going into the details of the calculations involved, an electrical power system could then be represented by an ideal power system, which delivers infinite MVA under short-circuit, having in series with it an impedance that would have no other effect than to limit the value of the MVA supplied into a short-circuit to the actual value the real system supplies. The impedance under consideration will be a function of the internal impedances of the generators, the transmission lines and the apparatus used like transformers. The distributed capacitance that exists between the current carrying conductors and the ground will be neglected in our discussion; and the resistive component of the impedance will also be neglected since it does represent a negligible part of it at rated frequency.

Therfore we can represent an electrical power system by means of a diagram in the following way.



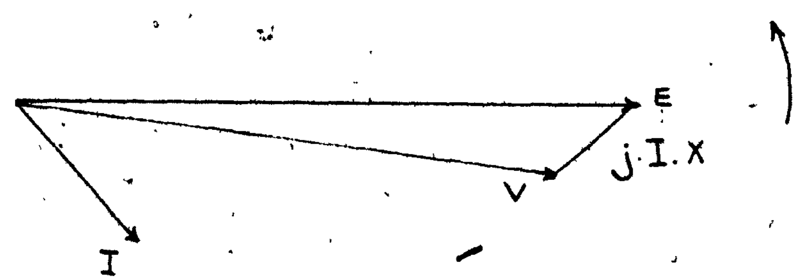
Where:

E is the voltage under no load; or the voltage of the ideal power system.

Z is the impedance representing the distributed differnt impedances of the components of the system.

V is the voltage at the point of consideration. We can note that this voltage will be equal to E if there is no load current flowing through the impedance Z.

The same system can be represented by using vectors that are assumed to be rotating at the rated frequency.



In the previous vector giagram, E, V and I have the same definitions; whereas jIX is nothing else than the voltage drop component. We have assumed that the impedance Z of the power system is purely inductive; therefore:

$$Z = R + j.X$$

becomes

$$Z = j.X$$

And the voltage drop being expressed as Z.I will be:

$$j.I.X$$

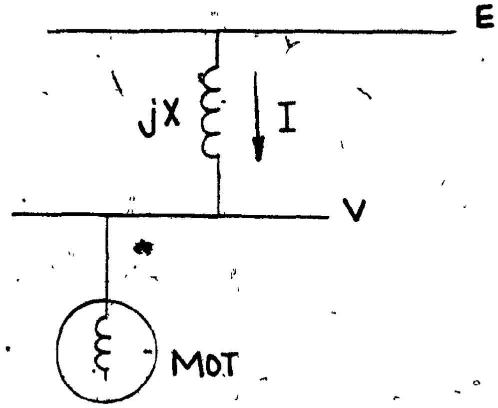
2) CASE OF A MOTOR CONNECTED TO A POWER SYSTEM

We will discuss the case when one electrical motor is connected on a power system. The motor under consideration could be either an induction motor which draws a lagging line current, or a synchronous motor that draws either a leading current or a current in phase with the voltage.

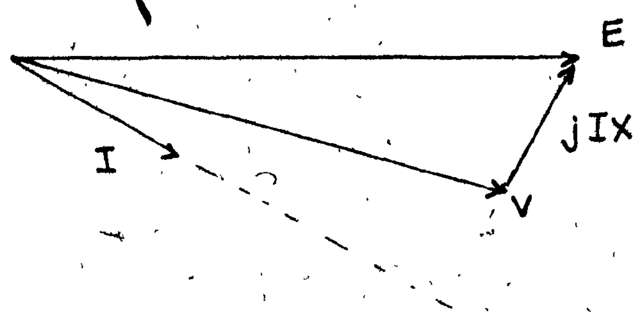
a) Case of a motor with lagging current.

As mentioned before, this could represent an induction motor ; we will investigate the relationships between the different electrical quantities involved.

The one line diagram of the system will look as :



And the vectorial representation will be:

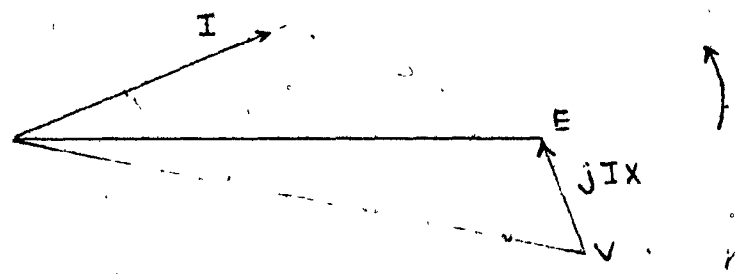


From the above figures, we can deduce that a load drawing a lagging current from the system will introduce a drop in the voltage at the point where it is connected. The magnitude of this drop is a function of the system impedance, the motor size and its power factor.

b) Case of a motor with a leading current.

In this case we shall assume a synchronous motor already synchronized with the system and drawing a leading current.

The vector diagram will then be:



We can, by looking at the vector diagram, deduce that a motor drawing a leading current will have an effect of rising the voltage at the point where it is connected. The voltage rise is a function of the power system impedance, the machine size and its power factor.

3) MATHEMATICAL REPRESENTATION

From the previous vector representations we are now capable of writing the equations that define the system. The general form of these equations will be:

$$\vec{E} = \vec{V} + j.\vec{I}.X$$

(all quantities are vectors)

Or analytically the equation could be written:

(for phase 'a' for example)

$$E_a = V_a + X. \frac{dIa}{dt}$$

where:

$$E_a = E_s \sin (W.t)$$

E_s being the magnitude of the system's voltage, and W a function of the frequency of the system.

I_a is the instantaneous value of the line current.

We can then write the expression of the voltage at the motor terminals. (for phase 'a')

$$V_a = E_s \sin(W.t) - X. \frac{dIa}{dt}$$

If the motor KVA is considered as being the base KVA, then the system KVA would be equal to :

$$KVA_{sys} = K. KVA_{mot}$$

K is called the stiffness factor.

Since we are dealing with the per unit system, we will find that the ratio of the motor impedance to the system impedance is a function of the stiffness factor K.

$$\frac{X_{mot}}{X_{sys}} = K$$

Therefore the expression of the voltage at the motor terminal could be rewritten as:

$$V_a = E_s \sin(W.t) - K \frac{dIa}{dt}$$

G) IMPLEMENTATION ON THE DIGITAL COMPUTER

From the mathematical model of a synchronous machine one can see that the exact behaviour of the machine can be determined by solving the flux linkages differential equations. In solving the flux equations, it would then be possible to know the currents into every winding as well as the electro-magnetic torque that the machine develops. With a known torque, the mechanical equation could then be solved, therefore a solution for the speed of the rotor.

To solve these differential equations using a digital computer, numerical methods have to be used. The most common, and very efficient technique is the Runge-Kutta method of the fourth order. However since the resolution of the machine equations requires several differentiations, the MIMIC language was then considered.

The programming and debugging time for MIMIC is estimated to be the tenth of the equivalent work done in FORTRAN. The writing of the instructions is done in a very natural way, since it does have an automatic recording of the instructions. The integration scheme uses a fourth-order Runge-Kutta method. The program and the results will be the best justifications for the use of this computer languge.

1) THE EQUATIONS, IDENTIFICATION OF THE VARIABLES.

From equations 10, we can express the flux linkages as:

$$\phi_f = Wb \int (V_f - R_f I^f) \quad dt$$

$$\phi_{kd} = Wb \int (R_{kd} . I^{kd}) \quad dt$$

$$\phi_{kq} = Wb \int (R_{kq} . I^{kq}) \quad dt$$

$$\phi_d = Wb \int (V_d - R_a . I^d + W . \phi_q) \quad dt$$

$$\phi_q = Wb \int (V_q - R_a . I^q - W . \phi_d) \quad dt$$

The only differnce that there is between the above expressions and the equations 10 is the introduction of the integral operator.

We can recall the expressions of the currents in the machine windings:

$$I^f = \frac{-1}{x_{f1}}\left[\frac{X_{md}}{x_{kd1}}\phi_{kd} + \frac{X_{md}}{x_{a1}}\phi_d + \left(\frac{X_{md}}{x_{f1}} - 1\right)\phi_f\right]$$

$$I^{kd} = \frac{-1}{x_{kd1}}\left[\frac{X_{md}}{x_{f1}}\phi_f + \frac{X_{md}}{x_{a1}}\phi_d + \left(\frac{X_{md}}{x_{kd1}} - 1\right)\phi_{kd}\right]$$

$$I^{kq} = \frac{-1}{x_{kq1}}\left[\frac{X_{mq}}{x_{a1}}\phi_q + \left(\frac{X_{mq}}{x_{kq1}} - 1\right)\phi_{kq}\right]$$

$$I^d = \frac{-1}{x_{a1}}\left[\frac{X_{md}}{x_{f1}}\phi_f + \frac{X_{md}}{x_{kd1}}\phi_{kd} + \left(\frac{X_{md}}{x_{a1}} - 1\right)\phi_d\right]$$

$$I^q = \frac{-1}{x_{a1}}\left[\frac{X_{mq}}{x_{kq1}}\phi_{kq} + \left(\frac{X_{mq}}{x_{a1}} - 1\right)\phi_q\right]$$

All quantities used in the programs are in per unit values. The base value for the per unit calculations being the motor KVA and the motor rated voltage and frequency. The speed of the motor will be given in per unit and to calculate the angle θ we assume that the speed is in electrical radians per seconds. Therefore the number of pair of poles will not have any effect on the calculations done.

For the inertia calculations, the per unit value of the inertia of the rotating bodies will be considered. The calculations involved to find the per unit value are explained in Appendix II; and in this case too the motor KVA will be the base value.

Also one can notice that since we are dealing with more than one machine, the first one will always have a 1 following each variable, while the second will have a 2 .

Identification of the variables:

RA	The armature winding resistance
RF	The field winding resistance
RKD	The direct-axis amortisseur winding resistance
RKQ	The quadrature-axis amortisseur winding resistance
XAL	The armature winding leakage inductance
XFL	The field winding leakage inductance
XKDL	The direct-axis amortisseur winding leakage inductance
XKQL	The quadrature-axis amortisseur winding leakage inductance
XAD	The mutual inductance along the direct-axis
XAQ	The mutual inductance along the quadrature-axis
VD	The direct-axis armature winding voltage
VQ	The quatrature-axis armature winding voltage
VF	The field winding voltage
ID	Current into the direct-axis armature winding
IQ	Current into the quadrature-axis armature winding
IF	Current into the field winding
IKD	Current into the direct-axis amortisseur winding
IKQ	Current into the quadrature-axis amortisseur winding
VA,VB & VC	The three phase voltages
EA	The system voltage, considered an infinite bus
K	The ratio of the system MVA to the machine MVA

FID	The direct-axis flux linkage
DFID	Time derivative of FID
FIQ	The quadrature-axis flux linkage
DFIQ	Time derivative of FIQ
FIF	The field winding flux linkage
DFIF	Time derivative of FIF
FIKD	The direct-axis amortisseur winding flux linkage
DFIKD	Time derivative of FIKD
FIKQ	The quadrature-axis amortisseur flux linkage
DFIKQ	Time derivative of FIKQ
TA	The air gap torque
J	The mechanical inertia of the rotating system
FRIC	The friction factor
WB	The supply frequency
W	The rotor speed in electrical radian per second
TETA	The angle the rotor has turned
IAN	The phase a armature current
IBN	The phase b armature current
ICN	The phase c armature current
P	The active power (electrical) into the machine
Q	The reactive power (electrical) into the machine
PF	The power factor
PFA	The power factor angle
IARMS	The root mean square value of the current flowing in phase a.

H) THE EXPERIMENTAL MOTOR CONSIDERED

The machine that has been considered in the discussion is a practical machine used in industry. The motor is driving grinding stones of a wook grinder. The technical characteristics of the system are given below:

MOTOR HP	7000
MOTOR KVA	5750
RATED VOLTAGE (Volts)	6600
RATED LINE CURRENT (Amps)	476.8
RATED FIELD CURRENT (Amps)	188.7
RATED FREQUENCY (hz)	60.0

The per unit resistances and inductances are:

RA = 0.0071	RF = 0.0015
RKD = 0.129	RKQ = 0.062
XD = 1.42	XQ = 0.835
XAL = 0.183	XFL = 0.221
XKDL = 0.177	XKQL = 0.083

The machine has 22 poles; therefore its synchronous speed when connected to a rated frequency supply, is:

60 X 60/(22:2) = 327 RPM

INERTIA OF THE MOTOR ALONE	160 000 $lb.ft^2$
INERTIA OF MOTOR WITH DRIVEN LOAD	200 000 $lb.ft^2$.

K) RESULTS AND DISCUSSION

This section intends to present the results of the implementation on the digital computer and to highlight some of the major points that are of concern. There are seven computer studies, and each one of them is a proof of the adequacy of the representation done. Each study is different from the other, and its respective computer program is given in Appendix III.

1). FIRST STUDY

In this study the motor considered is started as an induction motor, the voltage applied to its terminals is the rated voltage, and the power system is assumed to be ideal, therefore we will not consider any drop in the voltage magnitude. The purpose of this study is to show the different instantaneous values of the armature current in phase 'a', the speed, the torque and the field current (actually induced in the field winding) referenced to the phase 'a' voltage.

The time interval selected is 2 msec. Looking at the printed graph, the letter A gives the trace of phase 'a' voltage

B the trace of the line current in phase 'a' or IA1

C the trace of the current induced in the field winding or IF1

D the trace of the air-gap torque or TA1

E the trace of the speed of the rotor or W1

We can make the following remarks:

- The value of IA1 continues to increase during the first half cycle, to start to decrease and assume the system frequency after the first 3/4 of the first cycle has elapsed. Also we note that IA1 is lagging the voltage by almost 90°, since during this time the motor draws mostly magnetizing current. An offset of IA1 trace can be seen for the first few cycles.
- Also if we look at the enveloppe of this waveform and compare it to the trace available from field measurements we can deduce that they very closely match. (See graph 1 for the experimental trace)
- Looking at the trace of IF1, we can see that this quantity varies with a decreasing frequency at the same rate than the motor acceleration. Around the time 2sec. the frequency starts to be quite small, the speed being around 95% of full speed. It is appropriate to note that the frequency of the

current induced in the field winding as well as the amortisseur windings is inversely proportional to the speed of the motor and directly proportional to the slip value.

- The air-gap torque waveform also is proportional to the slip value, and while the motor is accelerating, the average value of this torque is positive, but the instantaneous values can assume both positive and negative values depending on the position of the rotor.

After this first study we are now in a position to decide what is the best time to apply the field supply, and this is what the second study shows.

2) SECOND STUDY

In this study, the field supply will be equal to 0 Volts until time 2.28 sec. when it will have to supply a field current of 1.3 pu Amps. The reason we will have a 1.3 pu Amps rather than 1.0 puAmps is because in order to create a 1 pu field flux, we need to have more than 1pu field Amps to take care of the saturation in the field. This subject has not been treated but the saturation effect was only taken into account based on the results of works done on this matter.

The output graph does use the same letters to represent the same value as in study 1 .

We can then make the following remarks:

- The second study does not differ from the first one for the first 2.28 sec. .
- At time 2,28 sec., the field supply is connected to the field winding, and we can notice the higher magnitude of IF1, as well as TA1. After time 2.6 sec., the air-gap torque waveform starts to be in phase with the field current waveform; whereas in the previous study these two quantities were almost out of phase.
- The important fact to underline is the waveform of IA1 between times 3.5 sec and 4.7 sec.. During this interval the value of IA1 is small, and its waveform shape is not exactly sinusoidal. After this period has elapsed, the current IA1 does not lag the voltage as it was before the synchronization of the motor. On the contrary this current now leads the voltage by almost 90°, and the

reason being the very light load the motor is driving. We can note also that the period we were observing this strange waveform of the line current corresponds to the slipping of poles that the motor will have to perform in order to synchronize with the system.

- The air gap torque final value is approximately equal to 0.2pu since the friction factor is only 0.2pu.
- We can also note the variation of the speed while the motor was trying to pull-in, the acceleration and deceleration are very easily noticed.
- After the motor has synchronized and come to a fairly stable operation we see that the field current assumes a constant value of 1.3 pu Amps

3) THIRD STUDY

Although the first two studies were very detailed, it is more practical to reduce the length of the output by increasing the time interval. Using MIMIC, a larger time interval does not upset the accuracy of the calculations, because the MIMIC language does iterate the calculations untill the error gets below a determined value. Hence increasing the time interval will only give us a shorter computer output.

On the other hand, we have to be able to represent the sinusoidal quantities by a value that does not vary with the frequency of the system; therefore we must compute the amplitude of these sinusoidal quantites.

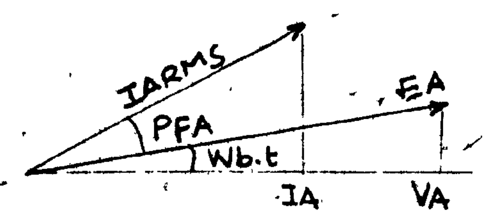
If IA1 is the instantaneous value of the line current in phase 'a', the instantaneous value of this quantity that we name IARMS1 will be:

$$IARMS1 = \frac{IA1}{\sin(Wb.t + PFA)}$$

wher: $PFA = \cos^{-1}(PF)$ Power factor angle.

PF is the instantaneous power factor.

The illustration below will prove the above reasoning.



Also it would be advantageous to represent the power angle which is the angle that exists between the direct-axis and the axis of phase 'a'. This angle is no different than θ, If S1 is the slip of the motor,

W1 its speed in per unit, and

Wb the supply frequency in per unit

Then S1 = 1.0 - W1

And the power angle PANG1 will be equal to S1.Wb Modulo π

The third study gives therefore the graphical representation of:

The magnitude of the current in phase 'a' noted by A

The power angle variations and values, noted by B

The reactive power component noted by C

The air-gap torque noted by D

and the speed of the machine noted by E

The field supply is still applied at 2.28 sec. after the motor has been started. If we look at the output of this third study we can make the following remarks :

- The line current for the beginning of the starting period does vary widely, but this is due to the oscillations that this quantity has while the motor slip is high.

- What is really interesting to see is the variation of the power angle. Especially at the time the motor is trying to pull-in; ie: between times 2.5 sec. and 3.5 sec.. The angle starts to increase, then decreases again and continue to oscillate with decreasing magnitude untill the motor stabilizes and then the power angle assumes an angle which will be proportional to the torque developped by the motor.
- Untill the motor synchronizes, the reactive power component is very negative, which means that the motor is drawing lagging current. But after the time the motor pulls-in, the reactive power starts to increase in value and gets to a final positive value at time about 4.7 sec. The final value of this reactive power component is a function of the load the motor is driving as well as a function of the field current or the excitation.

4) FOURTH STUDY

In this fourth study we will modify the previous study by applying an additional load to the motor after its synchronization sequence has terminated. Therefore, the load is applied to the motor at time 6.0 sec..

The symbols on the output graph represent the same quantities as in the third study. Looking at the graph the following remarks can be made:

- We can see that the motor is well stabilized before the application of the load. At time 6.0sec. we change the value of the friction factor from 0.2 to 1.0. This would be equivalent to loading the motor to almost its rated load.
- After application of this load we see that the speed trace went through a small perturbation, but the motor was capable of keeping this synchronous speed after about 0.2sec..
- The current jumped to a higher value and then after oscillating for almost 1.0 sec. stabilized at about 1.0pu.
- The power angle also had to go through an oscillation

during almost 1.0 sec. before stabilizing at a higher value. Obviously the torque also oscillated for about the same duration, to assume a higher final value.

- The reactive power component was also affected, but its oscillations did not have an as important magnitude as the current, the torque or the power angle. However we see that this quantity decreased in magnitude. In other words the reactive power that the motor was generating is now less. The current the motor draws is less leading .

The above are the interesting results that we could deduce from this fourth study.

5) FIFTH STUDY

In this study we are going to have a system with two motors. But the electrical power system is still assumed ideal, the first motor will be started, its field applied, load it and then start the second motor.

The graph output that we have shows the two currents, IARMS1 and IARMS2 denoted respectively by A and B; and the two speeds W1 and W2 denoted by C and D.

There is not really any comment that we can do on this output except that it gives us an idea of how the program looks like when we deal with more than one motor.

6) SIXTH STUDY

If we assume now that the electrical power system is a real one, and that it does not have an infinite stiffness, we would have to change the description of the power system. The equations of the three phase voltages VA, VB and VC will be:

VA = VRA - VAD

VB = VRB - VBD

VC = VRC - VCD

wher; VRA, VRB and VRC are the phase voltages of an ideal system and VAD, VBD, and VCD are the voltage drops that are a function of the load current and the stiffaness of the power system.

Due to the fact that we have introduced a derivation in order to compute each voltage drop, the time to run the program was increased by a great deal, and to be able to show the representation of the system, runs that compute the motor quantities for short times will be performed.

In the present run we will just calculate the motor quantities for the first two seconds of the starting sequence,

Looking at this graphical output, we can immediately conclude that the effect of having a drop in the system's voltage due to the motor current influences the time required for the motor to reach its synchronous speed. In other terms the acceleration is much slower, and the average value of the line current is less than the average value it has if there is no system disturbance. At the end of this 2.0 sec. period the motor is just at half speed.

It is really unfortunate that we could not see the whole starting sequence, but it would have been a complicated programming exercise to get to this stage.

7) SEVENTH STUDY

Since I was curious to see what effect a start of a second motor has on a first motor already running and loaded, I performed this study. Due to the fact that we are limited by the amount of computer time, the following method has been followed.

The first motor is started, synchronized and loaded considering and ideal system. After the motor had stabilized, I no longer assume an ideal system and then the voltage drop component is introduced. When the transient period following the introduction of the voltage drop is over, the second motor is started.

Referring to the graphical output available, we should make the following comments:

- The shape of the different waveforms shown are not unknown to us untill we get to the time 8.0sec. when the electrical power system is no more assumed ideal.
- The instant we introduce the drop in the voltages so that the system is close to a real system, we can note a perturbation in the line current, the power angle,

the torque and the reactive power. The speed was not affected a bit.

- When the second motor is started at time 8.46 sec., we note a serious perturbation of the line current, the power angle and the torque. The speed was not affected, but the reactive power component had a tendancy of increasing.
- The increment in the reactive power has been noticed in practice, the power factor of the motor running goes leading while another motor connected to the same bus is being started. This is mainly due to the fact that the excitation of the field is constant.

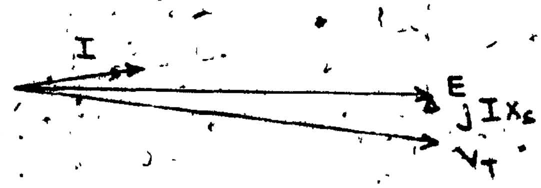
The stiffness factor in this run has been given a value of 20.0 . I think that on such systems we should not expect problems of this kind, because the effect of another motor being started does not upset very much the electrical power system. Runs with smaller stiffness factor were performed, but there was no possibility to reach any conlusion .

L) CONCLUSION

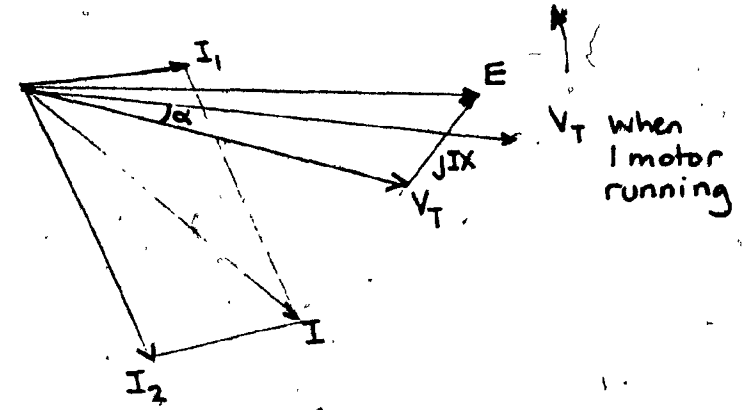
In this work, the objective of modelling a synchronous motor and to implement it on the digital computer was reached. The correcteness of the modelling could be considered satisfactory enough, so that in depth analysis of such electrical apparatus could be possible.

After being involved in so many details that the modelling invokes, my attention was turned towards a subject that may be of importance, but due to the scope of this work could not be investigated. The statement of this matter will come to complete present work.

We have seen that when a synchronous motor is running stabilized, the current it draws is either in phase or leading the voltage. If we assume that we are dealing with a 0.8 PF motor, the vector representaion of the system will be:



The instant the second motor is being started, the current that the system delivers will be the sum of the two currents through each of the motors. One current will be leading and the other will be higher in magnitude and lagging. Then the vector representation will be:



We can note that the voltage at the bus where the two motors are connected decreases in magnitude but also shifts by an angle α with respect to the voltage behind the power system. At this stage the motor already synchronized will have its internal voltage leading its terminal voltage, and in order to satisfy the power transfer equation, this motor will have to slow down. The shift that was introduced in this paragraph could be noticed on graph -2-, which is a fiel recording on a motor where the voltage drop was as high as 30%.

Therefore while the second motor is accelerating, the first one will stabilize after resynchronizing with the perturbated system. But the main concern is when the second motor starts to be near synchronous speed, the shift in the terminal voltage will be in the other direction, and the first motor will have to accelerate to keep the synchronism. And the moment the second motor synchronizes with the system and starts drawing leading current, the shift will then be more important.

Hence, if we happen to have the above situation, we will end up with a problem where both motors will try to synchronize at the same instant. Since motor 1 could be loaded at that time, chances are that it will loose its synchronism.

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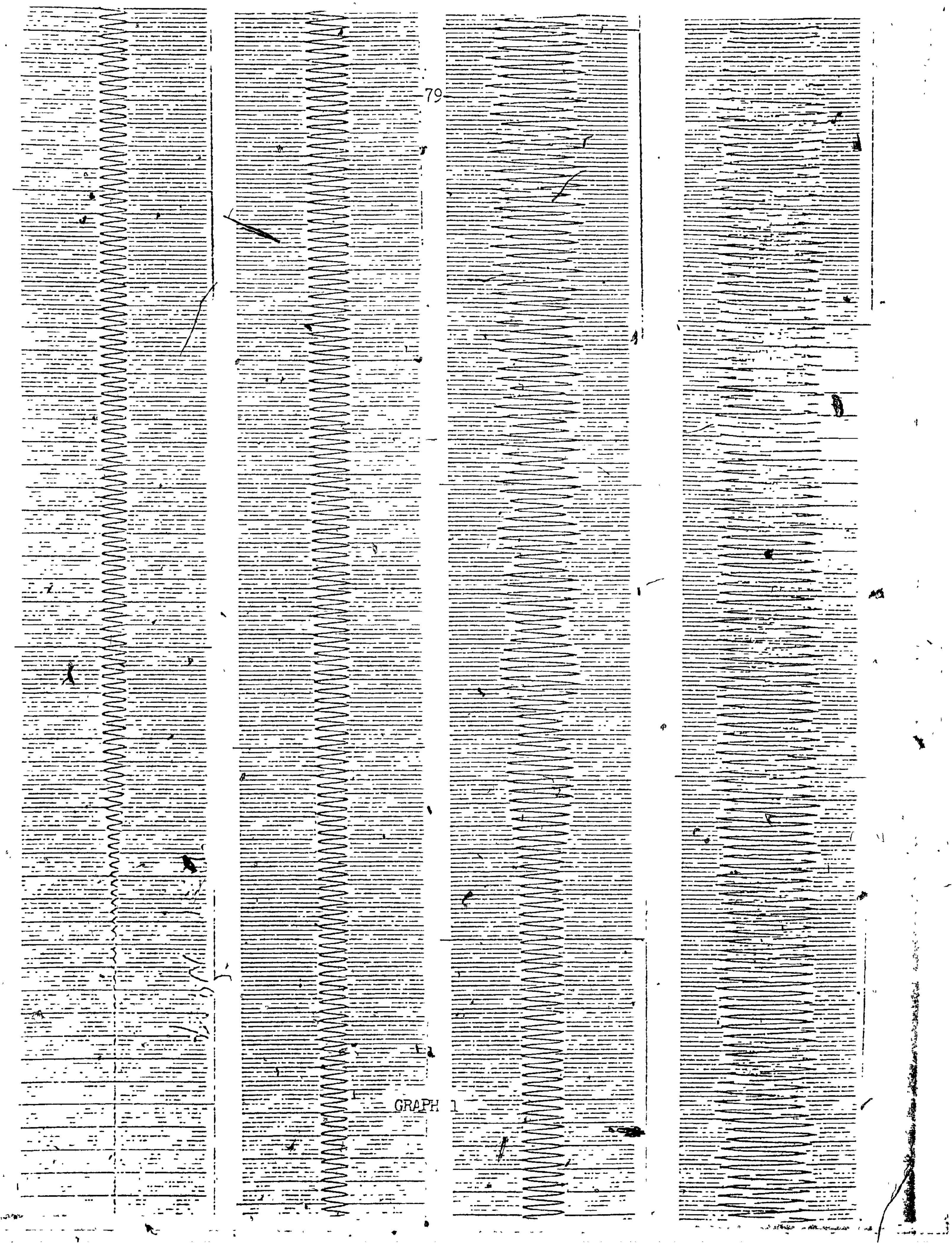
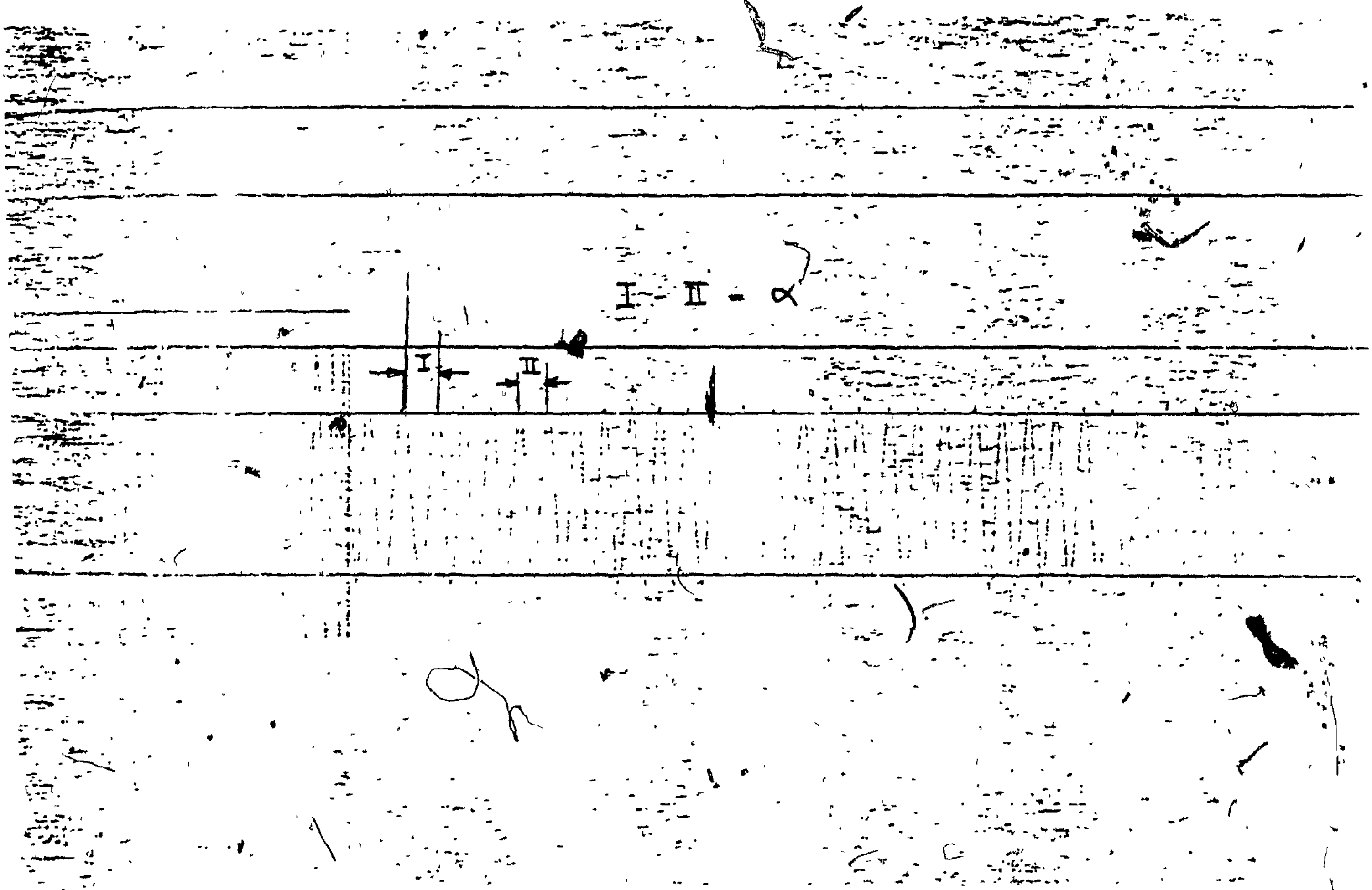


GRAPH 1



Graph II

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APPENDIX I

INTRODUCTION TO TENSORS

1) DEFINITION

Given a network with n-meshes; instead of saying that the network has n-currents; $i^a, i^b, \ldots, i^n$

n-voltages; $v_a, v_b, \ldots, v_n$

n.n Impedances; $z_{aa}, z_{bb}, \ldots, z_{nn}$

one can say that the network has

one current i

one voltage v

one impedance z

while the individual currents, voltages and impedances are simply elements of the matrices i,v and z.

Also for each n-mesh network containing the same coils but interconnected in the same manner one associates at least one i, v, and z matrix.

By the same thinking instead of saying that there are current matrices as there are networks; it is said that one current vector i, a Physical Entity whose projections along the various reference frames exists.

The key to this definition is the fact that it is possible to find the components of i in any reference frame from the components on another frame with the aid of a group of transformation matrices C by a definite formula.

If C is not available, the different n-matrices do not form the projections of a single physical entity.

Hence a collection of n-way matrices forms a physical entity, or "a tensor of valence n" if with the aid of a group of transformation matrices C they can be changed into one another.

A tensor of valence one is called a vector; a tensor of valence zero is called a scalar; tensors of other valences have no special names. A tensor is transformed with the aid of as many transformation matrices as its valence.

The advantage of considering i, v and z as tensors rather than simply matrices could be explained by an example:

Given $$v = ri + Lpi + \frac{1}{pC} i$$

if the symbols are tensors, it automatically follows that the equation is the same in any other analogous system. However if the symbols are matrices every analogous system may have an entirely different equation.

Therefore:

- By selecting one system whose analysis is simple, establish all the Tensors in this system (the so-called "primitive system") and the desired equation of performance in terms of tensors.
- To find the tensors of any particular system it is then only required to find the particular transformation matrix C.
- By routine laws of transformation the tensors of the given system could be established.
- Then the sought equation of performance is a copy of that of the primitive system.

It would be wrong to talk just about matrices; because the method of reasoning is that of tensor analysis. A matrix has no inherent law of transformation; a tensor has such a law.

2) LAW OF TRANSFORMATION

From one reference frame to another the instantaneous power input is invariant.

$$P = P'$$

$$i = Ci'$$

$$vi = v'i'$$

it follows that $$v = C_t^{-1} v'$$

and $$z' = C_t . z . C$$

APPENDIX II

INERTIA CONSTANT

The following is a derivation of the inertia constant term used in the acceleration equation in per unit of a synchronous machines.

The equation of motion of a rotating body is:

$$T = \frac{WR^2}{32.2} \cdot a \qquad \text{A.II.1}$$

Where:

- a is the angular acceleration in radians per second per second
- T the acceleration torque in pound-feet
- WR^2 the inertia of the rotating body in pound-feet^2

The mechanical angular acceleration is equal to the electrical angular acceleration divided by the number of pair of poles, therefore;

$$a = \frac{2}{P} \cdot \frac{d^2\theta}{dt^2} \qquad \text{A.II.2}$$

Where:

- $\frac{d^2\theta}{dt^2}$ is the electrical angular acceleration in electrical radians per second per second
- P number of poles $\frac{2 X 60.f}{rpm}$
- f normal frequency
- rpm normal revolutions per minute (rated speed).

In electrical terms. Eq. A.II.2 then becomes;

$$a = \frac{rpm}{60.f} \cdot \frac{d^2\theta}{dt^2} \qquad \text{A.II.3}$$

It is desirable to put equation A.II.1 in per unit terms by dividing the acceleration torque in pound-feet bu unit torque of the machine under consideration. Unit torque can be defined as the torque required to produce unit KW at rated speed or:

$$T(unit) = \frac{KW(unit)}{1.42 X rpm X 10^{-4}} \qquad \text{A.II.4}$$

Unit KW is assumed to be equal to unit KVA in the per unit system, and Eq. A.II.4 therefore becomes;

$$T(unit) = \frac{KVA(base)}{1.42 X rpm X 10^{-4}} \qquad \text{A.II.5}$$

We can then give the expression of the per unit equation of torque in terms of the per unit acceleration and the per unit inertia;

$$T_a = J.a \qquad \text{A.II.6}$$

Where:

- T_a is the per unit value of the accelerating torque
- a is the acceleration in per unit
- J the per unit value of the inertia

And the calculations show that J equals 2H where;

$$H = \frac{0.231 \text{ X } WR^2 \text{ X } rpm^{-2} \text{ X } 10^{-6}}{KVA \text{ (base)}}.$$

APPENDIX III

This Appendix contains the computer program for each of the seven studies performed throughout the dissertation.

```
          CON(RA,XAL,XKDL,XKQL,XFL,RF)
          CON(RKQ,RKD,XAQ,XAD,WB,EAS)
DT        0.002
PI        377.0/120.0
XMD       1.0/((1.0/XAD)+(1.0/XAL)+(1.0/XFL)+(1.0/XKDL))
XMQ       1.0/((1.0/XAQ)+(1.0/XAL)+(1.0/XKQL))
AF        XMD/XFL
AKD       XMD/XKDL
AKQ       XMQ/XKQL
AD        XMD/XAL
AQ        XMQ/XAL
VF1       0.0
ONE1      (AD*FID1)+((AF-1.0)*FIF1)+(AKD*FIKD1)
IF1       NEG(ONE1/XFL)
DFIF1     ADD(VF1,NEG(RF*IF1))
TWO1      (AD*FID1)+(AF*FIF1)+((AKD-1.0)*FIKD1)
IKD1      NEG(TWO1/XKDL)
DFIKD1    NEG(RKD*IKD1)
THREE1    (AQ*FIQ1)+((AKQ-1.0)*FIKQ1)
IKQ1      NEG(THREE1/XKQL)
DFIKQ1    NEG(RKQ*IKQ1)
FOUR1     ((AD-1.0)*FID1)+(AF*FIF1)+(AKD*FIKD1)
ID1       NEG(FOUR1/XAL)
DFID1     ADD(VD1,NEG(RA*ID1),W1*FIQ1)
FIVE1     ((AQ-1.0)*FIQ1)+(AKQ*FIKQ1)
IQ1       NEG(FIVE1/XAL)
DFIQ1     ADD(VQ1,NEG(RA*IQ1),NEG(W1*FID1))
TA1       (FID1*IQ1)-(FIQ1*ID1)
J1        0.7
FRIC1     0.2
DW1       ((TA1)-(FRIC1*W1))/(2.0*J1)
W1        INT(DW1,0.0)
FIQ1      INT((DFIQ1*WB),0.0)
FID1      INT((DFID1*WB),0.0)
FIF1      INT((DFIF1*WB),0.0)
FIKD1     INT((DFIKD1*WB),0.0)
FIKQ1     INT((DFIKQ1*WB),0.0)
WE1       W1*WB
TETA1     INT(WE1,0.0)
1AN1      TETA1-(2.0*PI/3.0)
2AN1      TETA1+(2.0*PI/3.0)
EA        EAS
VA        EA*SIN(WB*T)
VB        EA*SIN((WB*T)-(2.0*PI/3.0))
VC        EA*SIN((WB*T)-(4.0*PI/3.0))
VDS1      ADD(VA*COS(TETA1),VB*COS(1AN1),VC*COS(2AN1))
VD1       (2.0/3.0)*VDS1
VQS1      ADD(VA*SIN(TETA1),VB*SIN(1AN1),VC*SIN(2AN1))
VQ1       (-2.0/3.0)*VQS1
IA1       (ID1*COS(TETA1))-(IQ1*SIN(TETA1))
IB1       (ID1*COS(1AN1))-(IQ1*SIN(1AN1))
IC1       (ID1*COS(2AN1))-(IQ1*SIN(2AN1))
P1        (VD1*ID1)+(VQ1*IQ1)
Q1        (VD1*IQ1)-(VQ1*ID1)
PF1       P1/SQRT((P1*P1)+(Q1*Q1))
PFA1      (ABS(PF1))*(Q1/ABS(Q1))
SNN1      SIN((WB*T)+PFA1)
IARMS1    IA1/SNN1
S1        1.0-W1
ANG1      INT((S1*WB),0.0)
```

```
FANG1     MOD(ANG1,PI)
          PLO(T,VA,IA1,IF1,TA1,W1)
          SCA(0.01,0.1,0.2,0.1,0.1,0.02)
          ZER(0.0,10.0,50.0,70.0,50.0,30.0)
          OFT(5.0,1.0,1.0,1.0,1.0,1.0)
          FIN(T,6.0)
          END
```

```
                      CON(RA,XAL,XKDL,XKQL,XFL,RF)
                      CON(RKQ,RKD,XAQ,XAD,WB,EAS)
          DT          0.002
          PI          377.0/120.0
          XMD         1.0/((1.0/XAD)+(1.0/XAL)+(1.0/XFL)+(1.0/XKDL))
          XMQ         1.0/((1.0/XAQ)+(1.0/XAL)+(1.0/XKQL))
          AF          XMD/XFL
          AKD         XMD/XKDL
          AKQ         XMQ/XKQL
          AD          XMD/XAL
          AQ          XMQ/XAL
          TS2         FSW(T-2.25,TRUE,TRUE,FALSE)
          TB2         NOT(TS2)
TS2       VF1         0.0
TB2       VF1         1.3*RF
          ONE1        (AD*FID1)+((AF-1.0)*FIF1)+(AKD*FIKD1)
          IF1         NEG(ONE1/XFL)
          DFIF1       ADD(VF1,NEG(RF*IF1))
          TWO1        (AD*FID1)+(AF*FIF1)+((AKD-1.0)*FIKD1)
          IKD1        NEG(TWO1/XKDL)
          DFIKD1      NEG(RKD*IKD1)
          THREE1      (AQ*FIQ1)+((AKQ-1.0)*FIKQ1)
          IKQ1        NEG(THREE1/XKQL)
          DFIKQ1      NEG(RKQ*IKQ1)
          FOUR1       ((AD-1.0)*FID1)+(AF*FIF1)+(AKD*FIKD1)
          ID1         NEG(FOUR1/XAL)
          DFID1       ADD(VD1,NEG(RA*ID1),W1*FIQ1)
          FIVE1       ((AQ-1.0)*FIQ1)+(AKQ*FIKQ1)
          IQ1         NEG(FIVE1/XAL)
          DFIQ1       ADD(VQ1,NEG(RA*IQ1),NEG(W1*FID1))
          TA1         (FID1*IQ1)-(FIQ1*ID1)
          J1          0.7
          FRIC1       0.2
          DW1         ((TA1)-(FRIC1*W1))/(2.0*J1)
          W1          INT(DW1,0.0)
          FIQ1        INT((DFIQ1*WB),0.0)
          FID1        INT((DFID1*WB),0.0)
          FIF1        INT((DFIF1*WB),0.0)
          FIKD1       INT((DFIKD1*WB),0.0)
          FIKQ1       INT((DFIKQ1*WB),0.0)
          WE1         W1*WB
          TETA1       INT(WE1,0.0)
          1AN1        TETA1-(2.0*PI/3.0)
          2AN1        TETA1+(2.0*PI/3.0)
          EA          EAS
          VA          EA*SIN(WB*T)
          VB          EA*SIN((WB*T)-(2.0*PI/3.0))
          VC          EA*SIN((WB*T)-(4.0*PI/3.0))
          VDS1        ADD(VA*COS(TETA1),VB*COS(1AN1),VC*COS(2AN1))
          VD1         (2.0/3.0)*VDS1
          VQS1        ADD(VA*SIN(TETA1),VB*SIN(1AN1),VC*SIN(2AN1))
          VQ1         (-2.0/3.0)*VQS1
          IA1         (ID1*COS(TETA1))-(IQ1*SIN(TETA1))
          IB1         (ID1*COS(1AN1))-(IQ1*SIN(1AN1))
          IC1         (ID1*COS(2AN1))-(IQ1*SIN(2AN1))
          P1          (VD1*ID1)+(VQ1*IQ1)
          Q1          (VD1*IQ1)-(VQ1*ID1)
          PF1         P1/SQRT((P1*P1)+(Q1*Q1))
          PFA1        (ACS(PF1))*(Q1/ABS(Q1))
          SNN1        SIN((WB*T)+PFA1)
```

```
IARMS1    IA1/SNN1
S1        1.0-W1
ANG1      INT((S1*WB),0.0)
PANG1     MOD(ANG1,PI)
          PLO(T,VA,IA1,IF1,TA1,W1)
          SCA(0.01,0.1,0.2,0.1,0.1,0.02)
          ZER(0.0,10.0,50.0,70.0,50.0,30.0)
          OFT(5.0,1.0,1.0,1.0,1.0,1.0)
          FIN(T,6.0)
          END
```

```
                 ***MIMIC SOURCE-LANGUAGE PROGRAM***
                 VERSION 1  10/01/68  MOD LEVEL 0000

                 CON(RA,XAL,XKDL,XKQL,XFL,RF)
                 CON(RKQ,RKD,XAQ,XAD,WB,EAS)
       DT        0.02
       PI        377.0/120.0
       XMD       1.0/((1.0/XAD)+(1.0/XAL)+(1.0/XFL)+(1.0/XKDL))
       XMQ       1.0/((1.0/XAQ)+(1.0/XAL)+(1.0/XKQL))
       AF        XMD/XFL
       AKD       XMD/XKDL
       AKQ       XMQ/XKQL
       AD        XMD/XAL
       AQ        XMQ/XAL
       TS2       FSW(T-2.26,TRUE,TRUE,FALSE)
       TB2       NOT(TS2)
TS2    VF1       0.0
TB2    VF1       1.3*RF
       ONE1      (AD*FID1)+((AF-1.0)*FIF1)+(AKD*FIKD1)
       IF1       NEG(ONE1/XFL)
       DFIF1     ADD(VF1,NEG(RF*IF1))
       TWO1      (AD*FID1)+(AF*FIF1)+((AKD-1.0)*FIKD1)
       IKD1      NEG(TWO1/XKDL)
       DFIKD1    NEG(RKD*IKD1)
       THREE1    (AQ*FIQ1)+((AKQ-1.0)*FIKQ1)
       IKQ1      NEG(THREE1/XKQL)
       DFIKQ1    NEG(RKQ*IKQ1)
       FOUR1     ((AD-1.0)*FID1)+(AF*FIF1)+(AKD*FIKD1)
       ID1       NEG(FOUR1/XAL)
       DFID1     ADD(VD1,NEG(RA*ID1),W1*FIQ1)
       FIVE1     ((AQ-1.0)*FIQ1)+(AKQ*FIKQ1)
       IQ1       NEG(FIVE1/XAL)
       DFIQ1     ADD(VQ1,NEG(RA*IQ1),NEG(W1*FID1))
       TA1       (FID1*IQ1)-(FIQ1*ID1)
       J1        0.7
       FRIC1     0.2
       DW1       ((TA1)-(FRIC1*W1))/(2.0*J1)
       W1        INT(DW1,0.0)
       FIQ1      INT((DFIQ1*WB),0.0)
       FID1      INT((DFID1*WB),0.0)
       FIF1      INT((DFIF1*WB),0.0)
       FIKD1     INT((DFIKD1*WB),0.0)
       FIKQ1     INT((DFIKQ1*WB),0.0)
       WE1       W1*WB
       TETA1     INT(WE1,0.0)
       1AN1      TETA1-(2.0*PI/3.0)
       2AN1      TETA1+(2.0*PI/3.0)
       EA        EAS
       VA        EA*SIN(WB*T)
       VB        EA*SIN((WB*T)-(2.0*PI/3.0))
       VC        EA*SIN((WB*T)-(4.0*PI/3.0))
       VDS1      ADD(VA*COS(TETA1),VB*COS(1AN1),VC*COS(2AN1))
       VD1       (2.0/3.0)*VDS1
       VQS1      ADD(VA*SIN(TETA1),VB*SIN(1AN1),VC*SIN(2AN1))
       VQ1       (-2.0/3.0)*VQS1
       IA1       (ID1*COS(TETA1))-(IQ1*SIN(TETA1))
       IB1       (ID1*COS(1AN1))-(IQ1*SIN(1AN1))
       IC1       (ID1*COS(2AN1))-(IQ1*SIN(2AN1))
       P1        (VD1*ID1)+(VQ1*IQ1)
       Q1        (VD1*IQ1)-(VQ1*ID1)
       PF1       P1/SQRT((P1*P1)+(Q1*Q1))
       PFA1      (ACOS(PF1))*(Q1/ABS(Q1))
       SNA1      SIN((WB*T)+PFA1)
```

```
IARMS1   TA1/SNN1
S1       1.0-W1
ANG1     INT((S1*WB),0.0)
PANG1    MOD(ANG1,PI)
         PLO(7,IARMS1,PANG1,Q1,TA1,W1)
         SCA(0.1,0.05,0.1,0.1,0.1,0.02)
         ZER(0.0,0.0,50.0,30.0,30.0,0.0)
         OPT(5.0,0.1,0.1,0.1,0.1,0.1)
         FIN(T,12.0)
         END
```

```
                CON(RA,XAL,XKDL,XKQL,XFL,RF)
                CON(RKQ,RKD,XAQ,XAD,WB,EAS)
        DT      0.02
        PI      377.0/120.0
        XMD     1.0/((1.0/XAD)+(1.0/XAL)+(1.0/XFL)+(1.0/XKDL))
        XMQ     1.0/((1.0/XAQ)+(1.0/XAL)+(1.0/XKQL))
        AF      XMD/XFL
        AKD     XMD/XKDL
        AKQ     XMQ/XKQL
        AD      XMD/XAL
        AQ      XMQ/XAL
        TS2     FSW(T-2.26,TRUE,TRUE,FALSE)
        TB2     NOT(TS2)
TS2     VF1     8.0
TB2     VF1     1.3*RF
        ONE1    (AD*FID1)+((AF-1.0)*FIF1)+(AKD*FIKD1)
        IF1     NEG(ONE1/XFL)
        DFIF1   ADD(VF1,NEG(RF*IF1))
        TWO1    (AD*FID1)+(AF*FIF1)+((AKD-1.0)*FIKD1)
        IKD1    NEG(TWO1/XKDL)
        DFIKD1  NEG(RKD*IKD1)
        THREE1  (AQ*FIQ1)+((AKQ-1.0)*FIKQ1)
        IKQ1    NEG(THREE1/XKQL)
        DFIKQ1  NEG(RKQ*IKQ1)
        FOUR1   ((AD-1.0)*FID1)+(AF*FIF1)+(AKD*FIKD1)
        ID1     NEG(FOUR1/XAL)
        DFID1   ADD(VD1,NEG(RA*ID1),W1*FIQ1)
        FIVE1   ((AQ-1.0)*FIQ1)+(AKQ*FIKQ1)
        IQ1     NEG(FIVE1/XAL)
        DFIQ1   ADD(VQ1,NEG(RA*IQ1),NEG(W1*FID1))
        TA1     (FID1*IQ1)-(FIQ1*ID1)
        J1      0.7
        TS6     FSW(T-6.0,TRUE,TRUE,FALSE)
        TB6     NOT(TS6)
TS6     FRIC1   0.2
TB6     FRIC1   1.0
        DW1     ((TA1)-(FRIC1*W1))/(2.0*J1)
        W1      INT(DW1,0.0)
        FIQ1    INT((DFIQ1*WB),0.0)
        FID1    INT((DFID1*WB),0.0)
        FIF1    INT((DFIF1*WB),1.0)
        FIKD1   INT((DFIKD1*WB),0.0)
        FIKQ1   INT((DFIKQ1*WB),0.0)
        WE1     W1*WB
        TETA1   INT(WE1,0.0)
        1AN1    TETA1-(2.0*PI/3.0)
        2AN1    TETA1+(2.0*PI/3.0)
        EA      EAS
        VA      EA*SIN(WB*T)
        VB      EA*SIN((WB*T)-(2.0*PI/3.0))
        VC      EA*SIN((WB*T)-(4.0*PI/3.0))
        VDS1    ADD(VA*COS(TETA1),VB*COS(1AN1),VC*COS(2AN1))
        VD1     (2.0/3.0)*VDS1
        VQS1    ADD(VA*SIN(TETA1),VB*SIN(1AN1),VC*SIN(2AN1))
        VQ1     (-2.0/3.0)*VQS1
        IA1     (ID1*COS(TETA1))-(IQ1*SIN(TETA1))
        IB1     (ID1*COS(1AN1))-(IQ1*SIN(1AN1))
        IC1     (ID1*COS(2AN1))-(IQ1*SIN(2AN1))
        P1      (VD1*ID1)+(VQ1*IQ1)
        Q1      (VD1*IQ1)-(VQ1*ID1)
```

```
PF1       P1/SQRT((P1*P1+[illegible]*Q1))
PFA1      (ACS(PF1))*(Q1/A[illegible]Q1))
SNN1      SIN((WB*T)+PFA1)
IARMS1    IA1/SNN1
S1        1.0-W1
ANG1      INT((S1*WB),0.0)
PANG1     MOD(ANG1,PI)
          PLO(T,IARMS1,PANG1,Q1,TA1,W1)
          SCA(0.1,0.05,0.1,0.1,0.1,0.02)
          ZER(0.0,0.0,50.0,30.0,30.0,0.0)
          OPT(5.0,0.1,0.1,0.1,0.1,0.1)
          FIN(T,12.0)
          END
```

```
                        CON(RA,XAL,XKDL,XKQL,XFL,RF)
                        CON(RKQ,RKD,XAQ,XAD,WB,EAS)
              DT        0.02
              PI        377.0/120.0
              XMD       1.0/((1.0/XAD)+(1.0/XAL)+(1.0/XFL)+(1.0/XKDL))
              XMQ       1.0/((1.0/XAQ)+(1.0/XAL)+(1.0/XKQL))
              AF        XMD/XFL
              AKD       XMD/XKDL
              AKQ       XMQ/XKQL
              AD        XMD/XAL
              AQ        XMQ/XAL
              TS2       FSW(T-2.28,TRUE,TRUE,FALSE)
              TB2       NOT(TS2)
     TS2      VF1       0.0
     TB2      VF1       1.3*RF
              ONE1      (AD*FID1)+((AF-1.0)*FIF1)+(AKD*FIKD1)
              IF1       NEG(ONE1/XFL)
              DFIF1     ADD(VF1,NEG(RF*IF1))
              TWO1      (AD*FID1)+(AF*FIF1)+((AKD-1.0)*FIKD1)
              IKD1      NEG(TWO1/XKDL)
              DFIKD1    NEG(RKD*IKD1)
              THREE1    (AQ*FIQ1)+((AKQ-1.0)*FIKQ1)
              IKQ1      NEG(THREE1/XKQL)
              DFIKQ1    NEG(RKQ*IKQ1)
              FOUR1     ((AD-1.0)*FID1)+(AF*FIF1)+(AKD*FIKD1)
              ID1       NEG(FOUR1/XAL)
              DFID1     ADD(VD1,NEG(RA*ID1),W1*FIQ1)
              FIVE1     ((AQ-1.0)*FIQ1)+(AKQ*FIKQ1)
              IQ1       NEG(FIVE1/XAL)
              DFIQ1     ADD(VQ1,NEG(RA*IQ1),NEG(W1*FID1))
              TA1       (FID1*IQ1)-(FIQ1*ID1)
              J1        0.7
              TS6       FSW(T-6.0,TRUE,TRUE,FALSE)
              TB6       NOT(TS6)
     TS6      FRIC1     0.2
     TB6      FRIC1     1.0
              DW1       ((TA1)-(FRIC1*W1))/(2.0*J1)
              W1        INT(DW1,0.0)
              FIQ1      INT((DFIQ1*WB),0.0)
              FID1      INT((DFID1*WB),0.0)
              FIF1      INT((DFIF1*WB),0.0)
              FIKD1     INT((DFIKD1*WB),0.0)
              FIKQ1     INT((DFIKQ1*WB),0.0)
              WE1       W1*WB
              TETA1     INT(WE1,0.0)
              1AN1      TETA1-(2.0*PI/3.0)
              2AN1      TETA1+(2.0*PI/3.0)
              EA        EAS
              VA        EA*SIN(WB*T)
              VB        EA*SIN((WB*T)-(2.0*PI/3.0))
              VC        EA*SIN((WB*T)-(4.0*PI/3.0))
              VF2       0.0
              ONE2      (AD*FID2)+((AF-1.0)*FIF2)+(AKD*FIKD2)
              IF2       NEG(ONE2/XFL)
              DFIF2     ADD(VF2,NEG(RF*IF2))
              TWO2      (AD*FID2)+(AF*FIF2)+((AKD-1.0)*FIKD2)
              IKD2      NEG(TWO2/XKDL)
              DFIKD2    NEG(RKD*IKD2)
              THREE2    (AQ*FIQ2)+((AKQ-1.0)*FIKQ2)
              IKQ2      NEG(THREE2/XKQL)
```

```
          DFIKQ2    NEG(KKQ*IKQ2)
          FOUR2     ((AD-1.0)*FID2)+(AF*FIF2)+(AKD*FIKD2)
          ID2       NEG(FOUR2/XAL)
          DFID2     ADD(VD2,NEG(RA*ID2),W2*FIQ2)
          FIVE2     ((AQ-1.0)*FIQ2)+(AKQ*FIKQ2)
          IQ2       NEG(FIVE2/XAL)
          DFIQ2     ADD(VQ2,NEG(RA*IQ2),NEG(W2*FID2))
          TA2       (FID2*IQ2)-(FIQ2*ID2)
          J2        0.7
          FRIC2     0.2
          DW2       ((TA2)-(FRIC2*W2))/(2.0*J2)
          W2        INT(DW2,0.0)
          FIQ2      INT((DFIQ2*WB),0.0)
          FID2      INT((DFID2*WB),0.0)
          FIF2      INT((DFIF2*WB),0.0)
          FIKD2     INT((DFIKD2*WB),0.0)
          FIKQ2     INT((DFIKQ2*WB),0.0)
          WE2       W2*WB
          TETA2     INT(WE2,0.0)
          1AN2      TETA2-(2.0*PI/3.0)
          2AN2      TETA2+(2.0*PI/3.0)
          TS10      FSW(T-8.0,TRUE,TRUE,FALSE)
          TB10      NOT(TS10)
TS10      VDS2      0.0
TS10      VQS2      0.0
TB10      VQS2      ADD(VA*SIN(TETA2),VB*SIN(1AN2),VC*SIN(2AN2))
TB10      VDS2      ADD(VA*COS(TETA2),VB*COS(1AN2),VC*COS(2AN2))
          VD2       (2.0/3.0)*VDS2
          VQ2       (-2.0/3.0)*VQS2
          IA2       (ID2*COS(TETA2))-(IQ2*SIN(TETA2))
          IB2       (ID2*COS(1AN2))-(IQ2*SIN(1AN2))
          IC2       (ID2*COS(2AN2))-(IQ2*SIN(2AN2))
          P2        (VD2*ID2)+(VQ2*IQ2)
          Q2        (VD2*IQ2)-(VQ2*ID2)
          PF2       P2/SQRT((P2*P2)+(Q2*Q2))
          PFA2      (ACS(PF2))*(Q2/(ABS(Q2)))
          SNN2      SIN((WB*T)+PFA2)
          IARMS2    IA2/SNN2
          VDS1      ADD(VA*COS(TETA1),VB*COS(1AN1),VC*COS(2AN1))
          VD1       (2.0/3.0)*VDS1
          VQS1      ADD(VA*SIN(TETA1),VB*SIN(1AN1),VC*SIN(2AN1))
          VQ1       (-2.0/3.0)*VQS1
          IA1       (ID1*COS(TETA1))-(IQ1*SIN(TETA1))
          IB1       (ID1*COS(1AN1))-(IQ1*SIN(1AN1))
          IC1       (ID1*COS(2AN1))-(IQ1*SIN(2AN1))
          P1        (VD1*ID1)+(VQ1*IQ1)
          Q1        (VD1*IQ1)-(VQ1*ID1)
          PF1       P1/SQRT((P1*P1)+(Q1*Q1))
          PFA1      (ACS(PF1))*(Q1/ABS(Q1))
          SNN1      SIN((WB*T)+PFA1)
          IARMS1    IA1/SNN1
          S1        1.0-W1
          ANG1      INT((S1*WB),0.0)
          PANG1     MOD(ANG1,PI)
                    PLO(T,IARMS1,IARMS2,W1,W2)
                    SCA(0.1,0.05,0.05,0.02,0.02)
                    ZER(0.0,10.0,10.0,20.0,20.0)
                    OPT(5.0,0.1,0.1,0.1,0.1,0.1)
                    FIN(T,12.0)
                    END
```

```
                    ***MIMIC SOURCE-LANGUAGE PROGRAM***
                    VERSION 1  10/01/68  MOD LEVEL 0000

            CON(RA,XAL,XKDL,XKQL,XFL,RF)
            CON(RKQ,RKD,XAD,XAD,WB,EAS)
DT          0.02
PI          377.0/120.0
XMD         1.0/((1.0/XAD)+(1.0/XAL)+(1.0/XFL)+(1.0/XKDL))
XMQ         1.0/((1.0/XAQ)+(1.0/XAL)+(1.0/XKQL))
AF          XMD/XFL
AKD         XMD/XKDL
AKQ         XMQ/XKQL
AD          XMD/XAL
AQ          XMQ/XAL
VF1         0.0
ONE1        (AD*FID1)+((AF-1.0)*FIF1)+(AKD*FIKD1)
IF1         NEG(ONE1/XFL)
DFIF1       ADD(VF1,NEG(RF*IF1))
TWO1        (AD*FID1)+(AF*FIF1)+((AKD-1.0)*FIKD1)
IKD1        NEG(TWO1/XKDL)
DFIKD1      NEG(RKD*IKD1)
THREE1      (AQ*FIQ1)+((AKQ-1.0)*FIKQ1)
IKQ1        NEG(THREE1/XKQL)
DFIKQ1      NEG(RKQ*IKQ1)
FOUR1       ((AD-1.0)*FID1)+(AF*FIF1)+(AKD*FIKD1)
ID1         NEG(FOUR1/XAL)
DFID1       ADD(VD1,NEG(RA*ID1),W1*FIQ1)
FIVE1       ((AQ-1.0)*FIQ1)+(AKQ*FIKQ1)
IQ1         NEG(FIVE1/XAL)
DFIQ1       ADD(VQ1,NEG(RA*IQ1),NEG(W1*FID1))
TA1         (FID1*IQ1)-(FIQ1*ID1)
J1          0.7
FRIC1       0.2
DW1         ((TA1)-(FRIC1*W1))/(2.0*J1)
W1          INT(DW1,0.0)
FIQ1        INT((DFIQ1*WB),0.0)
FID1        INT((DFID1*WB),0.0)
FIF1        INT((DFIF1*WB),0.0)
FIKD1       INT((DFIKD1*WB),0.0)
FIKQ1       INT((DFIKQ1*WB),0.0)
WE1         W1*WB
TETA1       INT(WE1,0.0)
1AN1        TETA1-(2.0*PI/3.0)
2AN1        TETA1+(2.0*PI/3.0)
EA          EAS
VRA         EA*SIN(WB*T)
VRB         EA*SIN((WB*T)-(2.0*PI/3.0))
VRC         EA*SIN((WB*T)-(4.0*PI/3.0))
DIC1W       DER(T,IC1,0.0)
DIB1W       DER(T,IB1,0.0)
DIA1W       DER(T,IA1,0.0)
DIA1        DIA1W/WB
DIB1        DIB1W/WB
DIC1        DIC1W/WB
K           20.0
VAD         DIA1/K
VBD         DIB1/K
VCD         DIC1/K
VA          VRA-VAD
VB          VRB-VBD
VC          VRC-VCD
VDS1        ADD(VA*COS(TETA1),VB*COS(1AN1),VC*COS(2AN1))
VD1         (2.0/3.0)*VDS1
```

```
VOS1      ADD(VA*SIN(TETA1),VB*SIN(1AN1),VC*SIN(2AN1))
VQ1       (-2.0/3.0)*VQS1
IA1       (ID1*COS(TETA1))-(IQ1*SIN(TETA1))
IB1       (ID1*COS(1AN1))-(IQ1*SIN(1AN1))
IC1       (ID1*COS(2AN1))-(IQ1*SIN(2AN1))
P1        (VD1*ID1)+(VQ1*IQ1)
Q1        (VD1*IQ1)-(VQ1*ID1)
PF1       P1/SQRT((P1*P1)+(Q1*Q1))
PFA1      (ACS(PF1))*(Q1/ABS(Q1))
SNN1      SIN((W3*T)+PFA1)
IARMS1    IA1/SNN1
S1        1.0-W1
ANG1      INT((S1*W3),0.0)
PANG1     MOD(ANG1,PI)
          PLOT,IARMS1,PANG1,Q1,TA1,W1)
          SCA(0.1,0.05,0.1,0.1,0.1,0.02)
          ZEF(0.0,0.0,50.0,30.0,30.0,0.0)
          OPT(5.0,0.1,0.1,0.1,0.1,0.1)
          FIN(T,2.0)
          END
```

```
                    CON(RA,YAL,XKDL,XKQL,XFL,RF)
                    CON(RKQ,RKD,XAQ,XAD,WB,EAS)
          DT        0.02
          PI        377.0/120.0
          XMD       1.0/((1.0/XAD)+(1.0/XAL)+(1.0/XFL)+(1.0/XKDL))
          XMQ       1.0/((1.0/XAQ)+(1.0/XAL)+(1.0/XKQL))
          AF        XMD/XFL
          AKD       XMD/XKD
          AKQ       XMQ/XKQL
          AD        XMD/XAL
          AQ        XMQ/XAL
          TS2       FSW(T-2.25,TRUE,TRUE,FALSE)
          TB2       NOT(TS2)
TS2       VF        0.0
TB2       VF1       1.0*RF
          ONE1      (AD*FID1)+((AF-1.0)*FIF1)+(AKD*FIKD1)
          IF1       NEG(ONE1/XFL)
          DFIF1     ADD(VF1,NEG(RF*IF1))
          TWO1      (AD*FID1)+(AF*FIF1)+((AKD-1.0)*FIKD1)
          IKD1      NEG(TWO1/XKDL)
          DFIKD1    NEG(RKD*IKD1)
          THREE1    (AQ*FIQ1)+((AKQ-1.0)*FIKQ1)
          IKQ1      NEG(THREE1/XKQL)
          DFIKQ1    NEG(RKQ*IKQ1)
          FOUR1     ((AD-1.0)*FID1)+(AF*FIF1)+(AKD*FIKD1)
          ID1       NEG(FOUR1/XAL)
          DFID1     ADD(VD1,NEG(RA*ID1),W1*FIQ1)
          FIVE1     ((AQ-1.0)*FIQ1)+(AKQ*FIKQ1)
          IQ1       NEG(FIVE1/XAL)
          DFIQ1     ADD(VQ1,NEG(RA*IQ1),NEG(W1*FID1))
          TA1       (FID1*IQ1)-(FIQ1*ID1)
          J1        0.7
          TS6       FSW(T-6.0,TRUE,TRUE,FALSE)
          TB6       NOT(TS6)
TS6       FRID1     0.2
TB6       FRID1     1.0
          DW1       ((TA1)-(FRID1*W1))/(2.0*J1)
          W1        INT(DW1,1.0)
          FID1      INT((DFID1*WB),0.0)
          FIQ1      INT((DFIQ1*WB),0.0)
          FIF1      INT((DFIF1*WB),0.0)
          FIKD1     INT((DFIKD1*WB),0.0)
          FIKQ1     INT((DFIKQ1*WB),0.0)
          WE1       W1*WB
          TETA1     INT(WE1,0.0)
          1AN1      TETA1-(2.0*PI/3.0)
          2AN1      TETA1+(2.0*PI/3.0)
          K         2.0
          EA        EAS
          VRA       EA*SIN(WB*T)
          VRB       EA*SIN((WB*T)-(2.0*PI/3.0))
          VRC       EA*SIN((WB*T)-(4.0*PI/3.0))
          VF2       0.0
          ONE2      (AD*FID2)+((AF-1.0)*FIF2)+(AKD*FIKD2)
          IF2       NEG(ONE2/XFL)
          DFIF2     ADD(VF2,NEG(RF*IF2))
          TWO2      (AD*FID2)+(AF*FIF2)+((AKD-1.0)*FIKD2)
          IKD2      NEG(TWO2/XKDL)
          DFIKD2    NEG(RKD*IKD2)
          THREE2    (AQ*FIQ2)+((AKQ-1.0)*FIKQ2)
```

```
            IKQ2      NEG(THREE2/XKQL)
            DFIKQ2    NEG(RKQ*IKQ2)
            FOUR2     ((AD-1.0)*FID2)+(AF*FIF2)+(AKD*FIKD2)
            ID2       NEG(FOUR2/XAL)
            DFID2     ADD(VD2,NEG(RA*ID2),W2*FIQ2)
            FIVE2     ((AQ-1.0)*FIQ2)+(AKQ*FIKQ2)
            IQ2       NEG(FIVE2/XAL)
            DFIQ2     ADD(VQ2,NEG(RA*IQ2),NEG(W2*FID2))
            TA2       (FID2*IQ2)-(FIQ2*ID2)
            J2        0.7
            FRIC2     0.2
            DW2       ((TA2)-(FRIC2*W2))/(2.0*J2)
            W2        INT(DW2,0.0)
            FIQ2      INT((DFIQ2*WB),0.0)
            FID2      INT((DFID2*WB),0.0)
            FIF2      INT((DFIF2*WB),0.0)
            FIKD2     INT((DFIKD2*WB),0.0)
            FIKQ2     INT((DFIKQ2*WB),0.0)
            WE2       W2*WB
            TETA2     INT(WE2,0.0)
            1AN2      TETA2-(2.0*PI/3.0)
            2AN2      TETA2+(2.0*PI/3.0)
            TS10      FSW(T-8.0,TRUE,TRUE,FALSE)
            TB10      NOT(TS10)
   TS10     VDS2      0.0
   TS10     VQS2      0.0
   TB10     VQS2      ADD(VA*SIN(TETA2),VB*SIN(1AN2),VC*SIN(2AN2))
   TB10     VDS2      ADD(VA*COS(TETA2),VB*COS(1AN2),VC*COS(2AN2))
            VD2       (2.0/3.0)*VDS2
            VQ2       (-2.0/3.0)*VQS2
            IA2       (ID2*COS(TETA2))-(IQ2*SIN(TETA2))
            IB2       (ID2*COS(1AN2))-(IQ2*SIN(1AN2))
            IC2       (ID2*COS(2AN2))-(IQ2*SIN(2AN2))
            P2        (VD2*ID2)+(VQ2*IQ2)
            Q2        (VD2*IQ2)-(VQ2*ID2)
            PF2       P2/SQRT((P2*P2)+(Q2*Q2))
            PFA2      (ABS(PF2))*(Q2/(ABS(Q2)))
            SNN2      SIN((WB*T)+PFA2)
            IAAMS2    IA2/SNN2
            IA        IA1+IA2
            IB        IB1+IB2
            IC        IC1+IC2
            TS8       FSW(T-8.0,TRUE,TRUE,FALSE)
            TB8       NOT(TS8)
   TS8      DIA1W     0.0
   TS8      DIB1W     0.0
   TS8      DIC1W     0.0
   TB8      DIA1W     DER(T,IA,0.0)
   TB8      DIB1W     DER(T,IB,0.0)
   TB8      DIC1W     DER(T,IC,0.0)
            DIA1      DIA1W/WB
            DIB1      DIB1W/WB
            DIC1      DIC1W/WB
            VAD       DIA1/K
            VBD       DIB1/K
            VCD       DIC1/K
            VA        VRA-VAD
            VB        VRB-VBD
            VC        VRC-VCD
            VDS1      ADD(VA*COS(TETA1),VB*COS(1AN1),VC*COS(2AN1))
            VD1       (2.0/3.0)*VDS1
            VQS1      ADD(VA*SIN(TETA1),VB*SIN(1AN1),VC*SIN(2AN1))
            VQ1       (-2.0/3.0)*VQS1
            IA1       (ID1*COS(TETA1))-(IQ1*SIN(TETA1))
```

```
IB1      (ID1*COS(1AN1))-(IQ1*SIN(1AN1))
IC1      (ID1*COS(2AN1))-(IQ1*SIN(2AN1))
P1       (VD1*ID1)+(VQ1*IQ1)
Q1       (VD1*IQ1)-(VQ1*ID1)
PF1      P1/SQRT((P1*P1)+(Q1*Q1))
PFA1     (ACS(PF1))*(Q1/ABS(Q1))
SNN1     SIN((WB*T)+PFA1)
IARMS1   IA1/SNN1
S1       [illegible]-W1
ANG1     INT((S1*WB),0.0)
PANG1    MOD(ANG1,PI)
         PLO(T,IARMS1,PANG1,Q1,TA1,W1)
         SCA(0.1,0.05,0.1,0.1,[illegible].1, [illegible]2)
         ZER([illegible],0.0,50.0,30.[illegible],3[illegible],0.0)
         OPT(5.[illegible],0.1,[illegible])
         FIN(T,9.0)
         END
```