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A Novel Approach for the Generation of Two-Variable Very Strict Hurwitz Polynomials and Applications in the Design of Stable Two-Dimensional Recursive Digital Filters

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ABSTRACT

A NOVEL APPROACH FOR THE GENERATION OF TWO-VARIABLE VERY STRICT HURWITZ POLYNOMIALS AND APPLICATIONS IN THE DESIGN OF STABLE TWO-DIMENSIONAL RECURSIVE DIGITAL FILTERS

Mohammad A. Abiri, Ph. D.,
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A systematic procedure is proposed for the generation of two-variable Very Strict Hurwitz Polynomials (VSHPs). Methods and procedures for the design of stable two-dimensional (2-D) (1-D as special case) filters satisfying prescribed specifications are described next, and are then applied to the design of several filters such as lowpass, highpass, bandpass, fan, Laplacian, and homomorphic filters with low sensitivity.

It is well known that in the area of signal processing, 2-D and m-D filters which are used are of little value unless they are stable and also satisfy prescribed specifications. 2-D and m-D filtering are concerned with the extension of 1-D filtering techniques to two and more dimensions. Recursive filtering is computationally advantageous over most convolutional methods of filtering. However, one major stumbling block in the design of 2-D and m-D recursive filters is to ensure their stability. Unlike 1-D recursive filters, 2-D recursive filters could become unstable in the presence of non-essential singularities of the second kind. Another difficulty is the lack of a Fundamental Theorem of Algebra for polynomials of two or more variables. This implies non-factorability of the polynomials in two or more variables, which complicates the checking of stability. This thesis discusses the design of 2-D (1-D as special case) recursive digital filters with guaranteed stability.

Part of the work reported in this thesis is devoted to the study of different methods for generating stable two-variable Hurwitz polynomials. Then, a novel approach which

uses the properties of positive definite matrices and their application in generating two-variable VSHPs is proposed. The method considers a passive n-port network N terminated in n-variable reactances as starting point Based on realizability of this as an n-variable reactance network we were able to generate two-variable VSHPs This method generalizes and extends the previous techniques, in which only a sub-class of matrices could be used to generate two-variable VSHPs Also, for the same order of the VSHP, a greater number of variables is made available, which results in low sensitivities for the designed filters.

The generated two-variable VSHP is then assigned to the denominator of a 2-D analog reference filter with a properly designated numerator polynomial The resulting analog transfer function is discretized by the application of bilinear transformations Non-linear programming techniques are employed to design recursive digital filters This is an efficient method for the design of recursive digital filters to approximate the desired magnitude frequency response or the desired magnitude and phase frequency responses simultaneously. The latter is an important requirement for many applications, especially image processing. Using this approach, several stable 2-D (1-D as special case) quarter plane recursive digital filters are designed

As the coefficient word-length has an effect on the cost as well as the speed of a filter, a practical algorithm based on an optimization procedure for the design of recursive digital filters with integer coefficients is described It has the advantage of being easy to program. The filters so designed are free from roundoff errors caused by quantization of the real coefficients with full precision and accumulation of these errors in arithmetic operations The numerical performance of the algorithm has been illustrated by examples

Finally, a detailed sensitivity analysis is undertaken whereby the designed filters are compared with respect to different coefficient word-lengths This study is felt to be of importance as the required coefficient word-length of the digital filter derived from the aforementioned structure has a strong effect on both the cost and speed of the filter when it is implemented. First, a sensitivity in terms of the structure of a filter is defined. Then, we concentrate on variations of the parameters pertaining to the 2-D recursive digital filter transfer functions derived previously. This can serve as a sensitivity measure of the digital filters. The results show that reduction in the coefficient word-length has little effect on the response of the filter, indicating the low sensitivity of the suggested technique for the generation of VSHPs.

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کس مشکل اسرار اجل را نگشاد — کس یک قدم از نهاد بیرون ننهاد

من می‌نگرم ز مبتدی تا استاد — عجز است بدست هر که از مادر زاد

حکیم عمر خیام نیشابوری.

I dedicate this work

to

my father

whose boundless love

and encouragement

I deeply miss

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LIST OF IMPORTANT SYMBOLS AND ABBREVIATIONS

Unless stated otherwise, we label the following notations and will make frequent references to them later:

$\cup$	union
$\mid$	such that
$\forall$	for all
$\sum_{i=1}^{N} d_i$	defined as $d_1+d_2+\cdots+d_N$
$\triangleq$	defined as
$\in$	member of
$i=1,n$	defined as $i=1,2,3,\ldots,n$
$\boldsymbol{a}_i,\ i=1,n$	defined as column matrix
$\boldsymbol{I}_n$	an $n \times n$ identity matrix
$\boldsymbol{A}=\left\{a_{ij}\right\}$ or $\left[a_{ij}\right]$	defined as matrix $\boldsymbol{A}$
$\boldsymbol{A}^T$	transpose of $\boldsymbol{A}$
$\boldsymbol{A}^{-1}$	inverse of $\boldsymbol{A}$
$\det(A)$ or $\lvert A \rvert$	determinant of A
$\lVert (\cdot) \rVert_p$	p-th norm of ( )
$\dfrac{\partial}{\partial \mu_i},\ i=1,n$	partial derivative w r t μ_i
μ_i (or s_i) $= j\omega_i,\ i=1,n$	complex frequency variable in analog domain for n-variables
ω_1,ω_2	analog domain frequencies in two-variables

ω_p	passband edge frequency
ω_s	stopband edge frequency
T_1, T_2	sampling periods in two-dimensions
τ_1, τ_2	group delays in two-variables
$z_i = e^{j\omega_i T_i}$, $i = 1,2$	complex frequency variable in digital domain for two-variables
$\bigcap_{i=1}^{n} \lvert z_i \rvert = 1$	$(\lvert z_1\rvert = 1) \cap (\lvert z_2\rvert = 1) \ldots \cap (\lvert z_n \rvert = 1)$
H^*	complex conjugate of H
Re	real part
Im	imaginary part

The following abbreviations are used frequently in the thesis:

$1\text{-}D$	one-dimensional
(BP)	bandpass
(BS)	bandstop
e(o)	defined as even or odd
FIR	finite impulse response
(HP)	highpass
HP	Hurwitz polynomial
SHP	strict HP
VSHP	very SHP
$H(s_1, s_2)$	two-variable analog transfer function
$H(z_1, z_2)$	two-dimensional digital transfer function
IIR	infinite impulse response

(LP)	lowpass
LSI	linear shift invariant
PLSI	planar least square inverse
N-D ; $N \geq 2$	n-dimensional
PRF	positive real function
RF	reactance function
$\boldsymbol{R}$ (axis,θ)	rotation about axis by θ degree
CW	clockwise
CCW	counterclockwise
$S_{c_i}^{H}(z_1,z_2)$	sensitivity of $H(z_1,z_2)$ w.r t. c_i

Throughout the text, upper case bold italic letters will denote real as well as complex matrices, vectors that correspond to a column or row of a matrix will be denoted by lower case bold italic letters with a single subscript. The subscript will correspond to the column or the row of the matrix to which the vector corresponds $e.g$, $\boldsymbol{a}_i$ denotes the i-th column of $\boldsymbol{A}$. Lower case Greek and roman letters will denote scalars Elements of a vector will be identified by a single subscript and elements of a matrix by a double subscript $e.g$, the element in the i-th row and j-th column of matrix $\boldsymbol{A}$ is denoted by $a_{i,j}$ and the i-th element of a vector b is denoted by b_i . All vectors are assumed to be column vectors, the row vectors are denoted with a transposition sign

CHAPTER I

INTRODUCTION

1.1. GENERAL

For many years, it is known that it is possible to reconstruct a continuous signal from its sampled version, if the rate of sampling is at least twice the highest frequency component of the continuous signal. This is the basis of the PAM and PCM communication systems [1]. It has led to wide spread use of sampled data for transmission and processing of analog signals. With recent advances in the digital integrated circuit technology, digital signal processing has become a powerful tool for the simulation of analog systems.

In general, the most important component in most digital signal processing systems is the digital filter. The term 'digital filter' refers to the computational process or algorithm by which a digital signal or sequence of numbers is transformed into a second sequence of numbers. In practice, the digital filter is implemented in hardware or using software on a general-purpose digital computer.

Two-dimensional (2-D) digital filters have found numerous applications in areas such as image processing, seismic data processing, aerial photo, biomedical engineering, sonar, radar to name just a few [2-7]. For example, lowpass filtering is employed as a smoothing operation to reduce high spatial frequency components that may be present in an image. On the other hand, highpass filters are being used in the enhancement of edges and other high-frequency components within an image. Images that do not appear clear may be sharpened by highpass filtering. Also, bandstop filters can be used for reduction of noises in an image, while the Laplacian filter is used in the extraction of object edges, or boundaries. Pattern recognition algorithms for use in robotics control often begin processing with a Laplacian operation [8-9].

1.2. CHARACTERIZATION OF 2-D DIGITAL FILTERS

A 2-D digital filter consists of arithmetic operations which transform a 2-D array of real numbers to another 2-D array of real numbers according to a prescribed specification. 2-D filtering problems can be classified as linear and nonlinear. In the linear filtering problem, it is assumed that the spectrum of the signal and noise are non-overlapping. Therefore, with the use of a linear filter (e.g., lowpass, highpass, bandpass, etc.) the signal can be recovered, eliminating signal contamination such as noise. On the other hand, nonlinear filtering is used for a specific application, where the spectrum of noise and signal are not necessarily non-overlapping and generally no specific models for noise or signals are considered. Removal of the impulsive noise by median filtering, contrast enhancement through histogram modification are few examples of nonlinear filtering. It should be noted that linear filtering plays an important role in the area of image processing, in general, for preprocessing and postprocessing of the signal through lowpass filtering and highpass filtering which are smoothing and sharpening operations, respectively. This thesis will be mainly concerned with linear 2-D digital filters that are shift-invariant, where the input, x, and the output, y, satisfy a 2-D linear constant-coefficient *difference equation* of the form [2,3,7,10]

$$\sum_{(k_1,k_2)\in R_a}\sum a(k_1,k_2)x(n_1-k_1,n_2-k_2) - \sum_{(l_1,l_2)\in R_b}\sum b(l_1,l_2)y(n_1-l_1,n_2-l_2) = 0, \tag{1.1}$$

where R_a and R_b are finite sets of spatial grid points, called the input and output masks, respectively. Note that (1.1) applies to sequences with uniform as well as nonuniform time spacings. However, for convenience, we have assumed exclusively digital signals with uniform time spacings, i.e., $T_1 = T_2 = 1$. Linear shift-invariant (LSI) filters are classified either as finite-extent impulse response (FIR), or as infinite-extent impulse response (IIR) as defined below.

1.3. NONRECURSIVE FILTERS

An FIR, or nonrecursive, filter is one whose impulse response possesses only a finite number of nonzero samples. The output of such filters can be determined from its input by means of the convolution sum. This can be seen by writing (1.1) as

$$y(n_1,n_2) = \sum\sum_{(k_1,k_2)\in R_a} a(k_1,k_2)x(n_1-k_1,n_2-k_2), \tag{1.2}$$

which is a special case of the difference equation (1.1) with $b(0,0)=1$. Comparing the above difference equation with the convolution sum, we see that the output array $y(n_1,n_2)$ is the convolution of the input array with the array of coefficients $a(k_1,k_2)$, where $a(k_1,k_2)$ can be identified as the impulse response of the filter.

For any FIR filter, the limits of summation in (1.2) are finite. In this case the convolution sum serves as a computational algorithm for realizing such filters. Taking the Z-transform of both sides of (1.2) with finite limits on summations yields the z-domain transfer function of a 2-D nonrecursive digital filter which is of the form

$$\begin{aligned} H(z_1,z_2) &= \frac{Y(z_1,z_2)}{X(z_1,z_2)} \\ &= \sum\sum_{(k_1,k_2)\in R_a} h(k_1,k_2)z_1^{-k_1}z_2^{-k_2}. \end{aligned} \tag{1.3}$$

For FIR filters, the impulse response is always absolutely summable, and therefore, these filters are always stable.

1.4. RECURSIVE FILTERS

An IIR, or recursive, filter is one whose input and output satisfy a finite-order difference equation. This can be seen by rewriting (1.1) as

$$\begin{aligned} y(n_1,n_2) = &\sum\sum_{(k_1,k_2)\in R_a} a(k_1,k_2)x(n_1-k_1,n_2-k_2) - \\ &\sum\sum_{(l_1,l_2)\in R_b} b(l_1,l_2)y(n_1-l_1,n_2-l_2), \end{aligned} \tag{1.4}$$

where $l_1, l_2 \neq 0$ simultaneously. Eq. (1.4) implies that the present value of the output of a recursive filter is not only dependent on present and past inputs but also on the past output samples. Therefore, these filters can produce a large output independent of the size of input signal and therefore can become unstable. Taking the Z-transform of both sides of (1.4) yields the z-domain transfer function of a 2-D recursive digital filter which is of the form,

$$H(z_1,z_2) = \frac{Y(z_1,z_2)}{X(z_1,z_2)}$$

$$= \frac{\sum\sum_{(k_1,k_2)\in R_a} a(k_1,k_2) z_1^{k_1} z_2^{k_2}}{\sum\sum_{(l_1,l_2)\in R_b} b(l_1,l_2) z_1^{l_1} z_2^{l_2}}$$

$$= \frac{N(z_1,z_2)}{D(z_1,z_2)}. \qquad (1.5)$$

It has been shown [11-12] that these filters are stable if and only if,

$$D(z_1,z_2) \neq 0 \text{ for } \bigcap_{i=0}^{2} |z_i| \geq 1. \qquad (1.6)$$

1.5. ADVANTAGES AND DISADVANTAGES OF FIR FILTERS OVER IIR FILTERS

There are three fundamental differences suggesting that designing nonrecursive filters are advantageous to their recursive counterpart. The first can be viewed as linearity. In most of the 2-D filtering problems, it is required for the phase to be either linear or zero. For example, linearity of the phase has been shown to be of importance in the processing of images [13]. Nonrecursive filters can readily be designed to have linear phase (constant group delay) as well as prescribed magnitude specifications. In contrast, designing a recursive filter to achieve constant group delay and prescribed magnitude specifications simultaneously is difficult. Another advantage of nonrecursive filters is that they are free from stability problems (since the transfer function of these filters is simply a polynomial in two unit delay variables z_1 and z_2, and therefore possesses only zeros). On the other hand, recursive filter design requires stability constraints to be incorporated in the design procedure; otherwise they may be unstable. Finally, the design of nonrecursive filters is easier than their recursive counterpart as they do not have the stability problem and their approximations are linear in nature.

In the design of 2-D filters for the same specifications, an FIR filter requires more coefficients than IIR filter by a factor of about ten [7]. This is because, concerning the pole locations, the transfer function of a recursive filter, which is the ratio of two polynomials in two unit-delay variables z_1 and z_2 as shown in (1.5), has more degree of freedom than a nonrecursive filter whose poles are fixed at the origin. As a result, recursive filters are capable of approximating a given specification with lower order compared to their nonrecursive counterpart. Unlike most of the methods published to solve the problem of realizing 2-D arrays of data nonrecursively [14-20], recursive realization, however, offers greater speed of filtering and smaller memory requirement [7]. Thus, IIR filters are more attractive because of less complexity (based on the number of multiplications and additions) and easier implementation than the FIR filters.

1.6. DESIGN OF 2-D DIGITAL FILTERS

2-D digital filters are initially specified by either the impulse response or the frequency characteristics. The aim in either the space-domain or the frequency domain design, in general, is to calculate the coefficient arrays $\boldsymbol{a}$ and $\boldsymbol{b}$ of a 2-D recursive filter in (1.4) or (1.5) (or the coefficient array $\boldsymbol{a}$ of a 2-D nonrecursive filter, (1.2) or (1.3)), in such a way that the impulse response or frequency characteristics (amplitude, phase, or both), respectively, of the designed filter approximates to those of the desired one. In addition, the stability of the filter should be maintained when designing recursive filters

There are several approaches for the design of 2-D filters. The use of a particular method would depend on whether an FIR or IIR filter is called for, and on whether the design specification is given in the space-domain or in the frequency domain. For example 2-D nonrecursive filters can be designed using windows introduced by Huang [21] (a straightforward extension of the 1-D technique), or by using transformation of 1-D prototype filters to 2-D filters [5,22]. They can also be designed using straightforward application of linear programming due to Hu and Rabiner [23] and refined later by Harris and Mersereau [24]. However, the latter approach is computationally slow as compared to the first two methods [25].

Similarly, several techniques have been proposed for the design of 2-D recursive filters. One of the earliest methods used to design stable 2-D recursive filters was the use of a transformation suggested by Shanks [11] starting with a 1-D prototype in analog domain [26] or in discrete domain [27] both resulting in the same coefficients. Ahmadi et al [28] presented another transformation for the design of 2-D recursive filters. Chang and Aggarwal [29] introduced design technique entirely in digital domain where the spectral transformation carries a stable 1-D transfer function into a 2-D transfer function Chang and Aggarwal considered only the separable filters. However, their method is also applicable for the case of nonseparable 2-D filters [30] McClellan Transformation [22] has also been used to design 2-D recursive filters [31] Several direct 2-D recursive filter designs based on optimization techniques have similarly been proposed [32-38]. This approach corresponds to the direct approximation of amplitude and/or group delay (linear phase shift) characteristics of ratio of two polynomials If the problem is directly attacked by minimizing some measure of the difference between the actual response of the filter and the desired response, it leads to a nonlinear problem in parameters which cannot be solved exactly and requires iterative methods for its solution This is due to the fact that it is difficult to obtain a polynomial in closed form, that is, as a function of the elements of reference matrix This method is essentially the same as the iterative scheme being used for the 1-D case. It starts from an initial set of parameters and iterates towards a solution by minimizing the l_p-norm ($p = 1, 2, \ldots$) taken on a dense grid of points in the 2-D frequency plane by a nonlinear optimization algorithm. This approach was first used by Maria and Fahmy [32]. It involves minimization of the sum of squares of the differences between the desired and the actual response by a nonlinear optimization algorithm which also incorporates stability constraints. The stability checking at each iteration in the process of minimization is, however, facilitated by choosing cascaded sections of first and second order sections from the beginning. The technique of Maria and Fahmy, however, lacks the computational efficiency as the stability checking must be applied repeatedly. Based on this direction, Karivaratharajan and Swamy [36-37] have also proposed methods to approximate quadrantal symmetric filters in which the denominator polynomial of the transfer function is separable in z_1 and z_2. The advantage of these methods is that once the filter characteristic has been determined, a single check is required to determine whether or not stability requirements are met. Also, minimax technique ($p \rightarrow \infty$) rather than least-squares technique ($p = 2$) has been used by Charalambous [38] to approximate prespecified amplitude response. If, however, a general rational filter of preselected order (the higher the order of the filter, i.e., the more coefficients to be computed, the better is the approximation) is to be computed to approximate some prespecified response, the repeated stability checking will computationally be involved in this approach. Yet there is another approach which simplifies the stability problem. It consists of deriving 2-D discrete transfer function from a 2-D analog transfer function by means of the bilinear transformations in conjunction with an optimization technique This approach is essentially based on the properties of 2-variable positive real functions [39]. Design algorithms to choose the coefficients for a 2-D recursive filter, that are based on Koga's result [39], can be found in [40-41]. Although many of the very first design algorithms proposed for 2-D recursive filters were successful, unfortunately general 2-D IIR filters designed with these methods have several complications as given in the following section.

1.7. PROBLEMS OF 2-D RECURSIVE DIGITAL FILTERS

1.7.1. Stability

One major difficulty encountered in the design of 2-D recursive filter is the stability. The stability requirement has prohibited the wide spread use of these filters inspite of their computational advantages over the nonrecursive counterpart. In case of 1-D filter the stability is determined by finding the locations of the roots of the denominator polynomial of the transfer function. However, this stability testing process is considerably more complicated for multidimensional case as it is not possible to factorize a multivariable polynomial into products of lower order polynomials [42-43]. The stability of 2-D recursive digital filters was first studied by Shanks et al. [11]. Shanks presented a theorem, based on direct extension of the conditions for stability in the 1-D case, which can be used for testing the stability of 2-D recursive filters. At about the same time, using a theorem due to Ansell [44], Huang [12] greatly simplified the stability conditions of Shanks' theorem that reduced the amount of computations required to check the stability of a 2-D recursive filter. While Anderson and Jury [45-47], and Maria and Fahmy [48] simplified these results even further by putting the stability tests in terms of the root locations or positivity of a set of polynomials in one variable, the stability testing is still computationally difficult task except for certain special cases [46,49].

1.7.2. Stabilization

Another problem encountered in the design of 2-D recursive filters, and not found with FIR filters, is stabilization of an unstable filter. In the special case of a separable filter, the 2-D filter becomes two 1-D filters in cascade. In that case, an unstable filter can be made stable by means of pole removal using mirror image technique. However, it is not always possible to realize the designed 2-D IIR filter as a cascade connection of 1-D filters because of the problem of factorization. In such a case, stabilization techniques are needed to convert an unstable design into a useful, stable design. Double Planar Least Square Inverse (DPLSI) [11] and discrete Hilbert transform (DHT) [50] are two different techniques that have been suggested for stabilization of an unstable 2-D IIR filter. However, the method of [11] is either a subclass of polynomials [51] or costly [52], while the technique of [50] faces a counter example [53].

1.7.3. Effect of the Numerator Polynomial on Stability

Unlike 1-D recursive filters, the numerator of a 2-D recursive filter may influence the stability of the filter. In the 1-D case, if the numerator and denominator polynomials of the filter transfer function are relatively prime, the stability of the filter depends only on the locations of the poles. However, in 2-D this is not the case. Goodman in his award-winning paper [54] has shown that given the denominator polynomial of a transfer function is not zero at a point in the unit bidisk, and there exists a non-essential singularity of the second kind (a point at which both the relatively prime numerator and denominator polynomials become zero) on the unit bidisk then the recursive filter may or may not be stable. As an example, consider the following three filter transfer functions [54]:

$$H_1(z_1,z_2)=\frac{1}{1-\frac{1}{2}z_1^{-1}-\frac{1}{2}z_2^{-1}}$$

$$H_2(z_1,z_2)=\frac{(1-z_1^{-1})^8(1-z_2^{-1})^8}{1-\frac{1}{2}z_1^{-1}-\frac{1}{2}z_2^{-1}}$$

$$H_3(z_1,z_2)=\frac{(1-z_1^{-1})(1-z_2^{-1})}{1-\frac{1}{2}z_1^{-1}-\frac{1}{2}z_2^{-1}}$$

We see that the only point on or inside the unit bidisk where the denominator polynomials are zero is at the point $z_1=z_2=1$. The numerator polynomials of both $H_2(z_1,z_2)$

and $H_3(z_1,z_2)$ are also zero at this point. Thus $H_2(z_1,z_2)$ and $H_3(z_1,z_2)$ both possess a non-essential singularity of the second kind on the unit bidisk. Goodman established the stability of $H_2(z_1,z_2)$ and the instability of $H_1(z_1,z_2)$ and $H_3(z_1,z_2)$. It can be seen that the numerator polynomial of $H_2(z_1,z_2)$ has played a role in the stability of the filter transfer function in spite of the fact that the poles and zeros of $H_2(z_1,z_2)$ and $H_3(z_1,z_2)$ are the same. This is due to the special nature of bivariate polynomials, which could be mutually prime and still have common zeros [55]. Goodman's stability condition with second kind singularities is a necessary condition which does not lead to a simple standard procedure to test for stability. However, in a recent paper Roytman et al. [56] have derived simple necessary and sufficient conditions for the stability of a transfer function in the presence of simple non-essential singularities of the second kind on the distinguished boundary of the unit bidisk.

1.8. HOW TO OVERCOME THESE PROBLEMS ?

One approach to the design of 2-D recursive filters, which simplifies the stability problem, is to derive 2-D discrete transfer function from a stable 2-variable analog transfer function (a function with Strict Hurwitz Polynomial (SHP) in its denominator) by means of the bilinear transformations, in conjunction with an optimization technique.

Based on Koga's result [39] that an n x n multivariable positive real matrix can be realized as multivariable finite lumped passive networks, Dubois and Blostein [40] proposed a method for the design of 2-D recursive digital filters by defining the transfer function of an analog 2-variable, passive 2-port network N as a starting point. Given such a function, the transfer function of a 2-D digital filter is obtained by performing double bilinear transformation. The response of the resulting filter is approximated using near minimax optimization technique. In this method, the generation of SHP is guaranteed if the 2-variable 2-port network N is chosen to be a lossy one. Prasad and Reddy [57] have modified the approach of Ramamoorthy and Bruton [58] for generation of SHP by considering a passive and lossy n-port network N (containing resistors and ideal gyrators) terminated in n_1-ports and n_2-ports by unit s_1-plane and unit s_2-plane capacitors, respectively. In another article [59], the same authors made use of the properties of bilinear transformations of SHP to obtain 2-D digital filters, in conjunction with the unconstrained minimization technique of Fletcher-Powell [60], to approximate a given response. Ramamoorthy and Bruton [41] also devised a technique which is based on Koga's result and the decomposition of an n x n multivariable positive real admittance matrix for the frequency domain design of 2-D analog and digital filters starting with a two-variable passive analog network. These techniques guarantee the stability of the digital filter except for non-essential singularities of the second kind [61], since the reference network is passive and absolutely stable.

Goodman [61] showed that not all predetermined 2-variable transfer functions possessing SHP denominators in the analog domain will yield stable 2-D digital transfer functions upon application of double bilinear transformation. A sufficient condition for the resulting 2-D digital transfer function to be stable is that it should be devoid of non-essential singularities of the second kind [61]. The test for the existance of such singularities, however, can be carried out directly in the analog domain [62]. To overcome this problem, Rajan et al. [62] refined the strictness of Hurwitz polynomials and defined the resulting polynomial as Very Strictly Hurwitz Polynomial (VSHP). Further information on multidimensional VSHP is available in [63].

Using the technique of 1-D wave digital filter design [64], extension has been made to the design of 2-D wave digital filters of the recursive type [65]. In this method doubly terminated LC ladder networks in two variables are considered where different series and shunt arm elements are replaced by the corresponding wave digital two-ports. Such ladder structures are particularly of interest because they possess low sensitivity characteristics both as analog and as digital filters [66]. Another method of designing 2-D digital filters is by applying a 1-D to 2-D analog transformation [of the form $s \rightarrow g(s_1, s_2)$]

on existing 1-D LC ladder transfer functions, and then the bilinear transformations. Erfani and Peikari [67] have used the concept of generalized delay unit [68] to design 2-D digital filters. By using such an element one can digitally realize any reactance function placed in the series and/or the shunt arm of a general ladder structure. However, in order to avoid the difficulties pointed out by Goodman [61] and to get a stable realization, the transformation g has been chosen to be a driving point reactance function. The design technique developed in [67] lends itself to generate 2-D filters with low sensitivity with respect to parameter perturbations, and also with tunable characteristics [69]. In a recent paper [70], however, the same authors have given a simplified algorithm, based on cascade connections of two-ports, for the digital simulation of RLC structures which do not require the concept of generalized delay units.

In view of VSHPs, we will review several methods of generating 2-variable strict Hurwitz polynomials devoid of non-essential singularities of the second kind. First let us explain certain preliminaries.

1.9. PRELIMINARIES [62]

1.9.1. Value of a Two-Variable Function at Infinity

The 2-dimensional biplane, (s_1, s_2), consists of two complex planes s_1 and s_2 and an infinite distant point can have infinite coordinates in any one or both of these planes and so there exist an infinite number of points at infinity. They can be classified into three categories as follows.

Category 1: $s_1 = \infty$, and $s_2 =$ finite

Category 2: $s_1 =$ finite, and $s_2 = \infty$

Category 3: $s_1 = \infty$, and $s_2 = \infty$

Applying the transformation $s \rightarrow 1/s$, which maps the infinity to the origin, to each variable the value of the function $H(s_1,s_2)$ at each of the above points, is defined as

$$H(\infty,s_2) = \underset{s_1 \rightarrow 0}{\text{Limit}}\, H(1/s_1,s_2) \qquad \text{(category 1)...} \qquad (1.7a)$$

$$H(s_1,\infty) = \underset{s_2 \rightarrow 0}{\text{Limit}}\, H(s_1,1/s_2) \qquad \text{(category 2)...} \qquad (1.7b)$$

$$H(\infty,\infty) = \underset{s_{1,2} \rightarrow 0}{\text{Limit}}\, H(1/s_1,1/s_2) \qquad \text{(category 3)...} \qquad (1.7c)$$

1.9.2. Singularities

It is well known that a 2-variable rational function

$$H(s_1,s_2) = P(s_1,s_2)/Q(s_1,s_2)$$

where P and Q are relatively prime, may possess two types of singularities and they may be defined as follows:

(i) Non-essential singularity of the first kind: $H(s_1,s_2)$ is said to possess non-essential singularity of the first kind at a point (s_1^*,s_2^*) if

$$P(s_1^*,s_2^*) \neq 0 \text{ and } Q(s_1^*,s_2^*) = 0.$$

(ii) Non-essential singularity of the second kind: $H(s_1,s_2)$ is said to possess non-essential singularity of the second kind at a point (s_1^*,s_2^*) if

$$P(s_1^*,s_2^*) = Q(s_1^*,s_2^*) = 0.$$

1.9.3. Two-Variable Hurwitz Polynomials

There are four types of *Hurwitz polynomials* and their definitions in terms of singularities are given below. In the following definitions $Q(s_1,s_2)$ is a polynomial in s_1 and s_2 and $\text{Re}(s)$ refers to the real part of s.

Definitions 1 & 2

$Q(s_1,s_2)$ is a *broad / narrow sense Hurwitz polynomial* (BHP / NHP) if $1/Q(s_1,s_2)$ does not possess any singularities in the region D_1 / $D_1 \cup D_2 \cup D_3$, where

$$D_1 \triangleq \left\{ (s_1,s_2) | \ \mathrm{Re}(s_1)>0, \ \mathrm{Re}(s_2)>0, \ |s_1| < \infty, \text{ and } |s_2| < \infty \right\},$$

$$D_2 \triangleq \left\{ (s_1,s_2) | \ \mathrm{Re}(s_1)=0, \ \mathrm{Re}(s_2)>0, \ |s_1| \leq \infty, \text{ and } |s_2| < \infty \right\},$$

and

$$D_3 \triangleq \left\{ (s_1,s_2) | \ \mathrm{Re}(s_1)>0, \ \mathrm{Re}(s_2)=0, \ |s_1| < \infty, \text{ and } |s_2| \leq \infty \right\}.$$

Definition 3

$Q(s_1,s_2)$ is a *strict Hurwitz polynomial* (SHP) if $1/Q(s_1,s_2)$ does not possess any singularities in the region

$$\left\{ (s_1,s_2) | \ \mathrm{Re}(s_1)\geq 0, \ \mathrm{Re}(s_2)\geq 0, \ |s_1| < \infty, \text{ and } |s_2| < \infty \right\}.$$

Definition 4

$Q(s_1,s_2)$ is a *very strict Hurwitz polynomial* (VSHP) if $1/Q(s_1,s_2)$ does not possess any singularities in the region

$$\left\{ (s_1,s_2) | \ \mathrm{Re}(s_1)\geq 0, \ \mathrm{Re}(s_2)\geq 0, \ |s_1| \leq \infty, \text{ and } |s_2| \leq \infty \right\}.$$

1.10. SURVEY OF PREVIOUS WORKS

As far as the stability of a 2-D IIR digital filter is concerned, it has been shown that when we start with a passive 2-variable analog network transfer function having a VSHP denominator and obtain a corresponding digital transfer function by using bilinear transformation in two variables, the stability of the resulting 2-D digital transfer function is guaranteed [62]. Following this approach, it is the purpose of this section to review different methods of generating 2-variable VSHPs. These methods essentially start with either (i) a reactance transformation, (ii) a resistively terminated network, or (iii) a purely reactive network of 2-variables.

Ahmadi et al. [28] introduced a class of 2-variable reactance function, whose sum of the numerator and the denominator constitutes a 2-variable VSHP, as a transformation applied to a 1-D analog lowpass filter in order to obtain a stable 2-variable transfer function.

Ramachandran and Ahmadi [71] have recently shown how a 2-variable VSHP could directly be generated. This method essentially considers a structure reported in [41]. That is, a structure consisting of an $(n+1)$-port lossless frequency independent (gyrator) network terminated by a unit s_1-plane capacitor in m-ports, by a unit s_2-plane capacitor in the next m-ports, and by resistors in the $(n-2m)$-ports. The determinant of the admittance matrix of such a network yields a 2-variable VSHP. In this technique, however, certain conditions involving principal sub-determinants of the admittance matrix of the gyrator network have to be satisfied.

Properties of derivatives of polynomials have also been used [72] to generate VSHPs. The method starts with a polynomial given by

$$P_n = \det[\boldsymbol{\mu}+G] \tag{1.8a}$$

where $\boldsymbol{\mu}$ is a diagonal matrix of order n given by

$$\boldsymbol{\mu} = \text{diag}\,[\mu_1, \mu_2, \mu_3, \ldots, \mu_n], \tag{1.8b}$$

and G is a gyrator (skew-symmetric) matrix of order n given by

$$G = \begin{bmatrix} 0 & g_{12} & g_{13} & \cdots & g_{1n} \\ -g_{12} & 0 & g_{23} & \cdots & g_{2n} \\ -g_{13} & -g_{23} & 0 & \cdots & g_{3n} \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ -g_{1n} & -g_{2n} & -g_{3n} & \cdots & 0 \end{bmatrix}. \tag{1.8c}$$

Since the matrix $[\mu+G]$ is physically realizable (any positive definite or positive semi-definite matrix is always physically realizable [39]), P_n represents the even part, M_n, or the odd part, N_n, of an n-variable HP, depending on whether n is even or odd, respectively. Therefore $(\partial P_n/\partial \mu_i)/P_n$ is a reactance function. By associating the corresponding partial derivatives with respect to each variable, which represents the odd part (n being even) or the even part (n being odd) of an n-variable HP, to P_n an n-variable HP could be produced. That is

$$Q_n = P_n + \sum_{i=1}^{n} K_i \frac{\partial P_n}{\partial \mu_i} \tag{1.9}$$

is an n-variable HP.

From (1.9), a 2-variable VSHP can be generated by putting some of the μ's equal to s_1 and the rest of the μ's equal to s_2 and also ensuring that the conditions of a 2-variable HP without non-essential singularities of the second kind are satisfied.

Based on realizability of positive definite (or positive semi-definite) matrices [39] and also on decomposability of any positive definite matrix into a product of the form PDP^T, where P is an upper or a lower triangular matrix and D is a diagonal matrix with non-negative elements [73], Ahmadi and Ramachandran [74] were able to generate 2-variable VSHPs. In [74] it has been shown that the matrix

$$Q = A\Gamma A^T s_1 + B\Delta B^T s_2 + G \tag{1.10a}$$

with A and B being upper-triangular matrices, respectively, of the form

$$
A = \begin{bmatrix} 1 & a_{12} & a_{13} & \cdots & a_{1n} \\ 0 & 1 & a_{23} & \cdots & a_{2n} \\ 0 & 0 & 1 & \cdots & a_{3n} \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix} \tag{1.10b}
$$

and

$$
B = \begin{bmatrix} 1 & b_{12} & b_{13} & \cdots & b_{1n} \\ 0 & 1 & b_{23} & \cdots & b_{2n} \\ 0 & 0 & 1 & \cdots & b_{3n} \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix} \tag{1.10c}
$$

and Γ and Δ being diagonal matrices with non-negative elements, respectively, of the form

$$
\Gamma = \begin{bmatrix} \gamma_1^2 & 0 & 0 & \cdots & 0 \\ 0 & \gamma_2^2 & 0 & \cdots & 0 \\ 0 & 0 & \gamma_3^2 & \cdots & 0 \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ 0 & 0 & 0 & \cdots & \gamma_n^2 \end{bmatrix} \tag{1.10d}
$$

and

$$
\Delta = \begin{bmatrix} \delta_1^2 & 0 & 0 & \cdots & 0 \\ 0 & \delta_2^2 & 0 & \cdots & 0 \\ 0 & 0 & \delta_3^2 & \cdots & 0 \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ 0 & 0 & 0 & \cdots & \delta_n^2 \end{bmatrix} \tag{1.10e}
$$

and G being a skew-symmetric matrix of the form given in (1.8c), is realizable as a 2-variable reactance network. Therefore, $\det Q$ constitutes either the even part or the odd part of a 2-variable HP, depending on whether the order is even or odd. As a consequence, a 2-variable HP can be obtained by associating the corresponding partial polynomial derivatives of $\det Q$ with respect to each variable [72], i.e.,

$$Q(s_1,s_2) = \det Q + K_1 \frac{\partial(\det Q)}{\partial s_1} + K_2 \frac{\partial(\det Q)}{\partial s_2} \tag{1.11}$$

with K_1 and K_2 being non-negative constants is a HP. The required VSHP is generated using higher-order partial polynomial derivatives as given in [72]. This choice of reactance matrix of 2-variables, however, required conditions to be imposed in order to meet one of the requirements [62] that determinant of the 2-variable reactance matrix associated with its partial derivatives becomes a VSHP. This condition was reported to be the semi-positive definiteness of the starting matrices A and B. It has been suggested in [75] that the concept of resistance matrix can be introduced in (1.10) to avoid the lengthy process of calculating the partial polynomial derivatives. That is, if we consider the matrix

$$Q = A \Gamma A^T s_1 + B \Delta B^T s_2 + G + C E C^T \tag{1.12a}$$

where A , B , Γ , Δ and G are the same as defined in (1.10b-e) and (1.8c), respectively, and matrices C and E are, respectively,

$$C = \begin{bmatrix} 1 & c_{12} & c_{13} & . & . & . & c_{1n} \\ 0 & 1 & c_{23} & . & . & . & c_{2n} \\ 0 & 0 & 1 & . & . & . & c_{3n} \\ . & & . & . & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ 0 & 0 & 0 & . & . & . & 1 \end{bmatrix} \tag{1.12b}$$

and

$$E = \begin{bmatrix} \epsilon_1^2 & 0 & 0 & . & . & . & 0 \\ 0 & \epsilon_2^2 & 0 & . & . & . & 0 \\ 0 & 0 & \epsilon_3^2 & . & . & . & 0 \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ 0 & 0 & 0 & . & . & . & \epsilon_n^2 \end{bmatrix} \tag{1.12c}$$

then under certain conditions $\det Q$ would yield a 2-variable VSHP directly.

1.11. SCOPE OF THE THESIS

In Chapter II, an alternative approach to the generation of VSHPs will be given. The emphasis in this work is on a new method for the generation of VSHPs which simplifies, generalizes, and extends the earlier approaches. Some examples are presented to illustrate the proposed method. Also, this will provide more number of variables for the optimization required in the design of 2-D IIR filters.

Chapter III describes a systematic procedure to formulate the design problem of 2-D recursive digital filters which approximate either the magnitude or simultaneously, both the magnitude and constant group delay (linear phase) specifications by applying the technique presented in Chapter II. A number of design examples with amplitude and/or both amplitude and constant group delay characteristics are reported in this chapter. Also the design of 1-D recursive digital filters with both amplitude and constant group delay specifications are worked out as special case. Further, a procedure for the design of IIR filters with integer coefficients is introduced. This utilizes a scheme of successive integerization and reoptimization. Design example in 2-D IIR filter (1-D as special case) to show the usefulness of the scheme is presented.

Chapter IV studies the sensitivity performance. The effect of coefficient quantization is treated here by computing the actual responses under floating-point finite arithmetic in several third-order filter designs of previous chapter. Several comparisons of examples of Chapter III are then undertaken on the basis of different coefficient lengths.

The final chapter, Chapter V, presents the conclusions drawn from the work of the thesis and suggests some further research work which may be conducted.

CHAPTER II

GENERATION OF TWO-VARIABLE VERY STRICTLY HURWITZ POLYNOMIALS

2.1. INTRODUCTION

As indicated in Chapter I, it has been shown in [41] how a 2-D stable analog filter could be designed by using the properties of the immitance function of a lossless frequency independent N-port network. The method, however, has failed to deal with the possibility of generating functions with non-essential singularities of the second kind which could result in an unstable discrete filter if double bilinear transformation is applied. In [72, 74] methods have been given to ensure that the denominator of the filter is always VSHP. This avoids the uncertainty of the method of [41]. Based on the aforementioned results [72, 74] a new approach for generating VSHPs, i.e., polynomials whose roots are in the left half plane, and their inverses are devoid of non-essential singularities of the second kind, will be dealt with in this chapter. The method consists of the following steps:

(i) A suitable even or odd part of an n-variable Hurwitz polynomial is generated.

(ii) The odd or even part of the n-variable Hurwitz polynomial is obtained by the corresponding derivatives, associated with it.

(iii) The resulting n-variable HP is converted to a 2-variable VSHP.

2.2. A NEW APPROACH FOR GENERATING VSHPs [76-77]

Based on the aforementioned results [72, 74], the proposed method makes use of a square matrix C_n of order n defined as

$$C_n \triangleq \sum_{i=1}^{n} \mu_i \, a_i \, a_i^T + G \tag{2.1a}$$

where each μ_i is a complex variable and each $\boldsymbol{a}_i$ is a column matrix of the form

$$\boldsymbol{a}_i = \begin{bmatrix} a_{1i} \\ a_{2i} \\ \cdot \\ \cdot \\ \cdot \\ a_{ni} \end{bmatrix}, \quad i = 1, 2, 3, \ldots, n \tag{2.1b}$$

and where G is a real skew-symmetric matrix given by

$$\mathbf{G} = \begin{bmatrix} 0 & g_{12} & g_{13} & \cdots & g_{1n} \\ -g_{12} & 0 & g_{23} & \cdots & g_{2n} \\ -g_{13} & -g_{23} & 0 & \cdots & g_{3n} \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ -g_{1n} & -g_{2n} & -g_{3n} & \cdots & 0 \end{bmatrix}. \tag{2.1c}$$

Eq. (2.1) can alternatively be written in the form

$$C_n = \sum_{i=1}^{n} \boldsymbol{a}_i \mu_i \boldsymbol{a}_i^T + G \tag{2.2}$$

or

$$C_n = \begin{bmatrix} \boldsymbol{a}_1 & \boldsymbol{a}_2 & \boldsymbol{a}_3 & \cdots & \boldsymbol{a}_n \end{bmatrix} \cdot \begin{bmatrix} \mu_1 & 0 & 0 & \cdots & 0 \\ 0 & \mu_2 & 0 & \cdots & 0 \\ 0 & 0 & \mu_3 & \cdots & 0 \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ 0 & 0 & 0 & \cdots & \mu_n \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{a}_1^T \\ \boldsymbol{a}_2^T \\ \boldsymbol{a}_3^T \\ \cdot \\ \cdot \\ \cdot \\ \boldsymbol{a}_n^T \end{bmatrix} + G. \tag{2.3}$$

We may write (2.3) in its compact form as

$$C_n = A \, \mu A^T + G \tag{2.4a}$$

where $\boldsymbol{A}$ is a square matrix of the form

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3n} \\ \vdots & \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \cdots & a_{nn} \end{bmatrix} \tag{2.4b}$$

and μ is a diagonal matrix of order n given by

$$\mu = \text{diag}\begin{bmatrix}\mu_1 & \mu_2 & \mu_3 & \cdots & \mu_n\end{bmatrix}. \tag{2.4c}$$

The square matrix A has an inverse if and only if A is nonsingular. Here we assume that the matrix A is nonsingular, i.e., det $A = |A| \neq 0$. This implies that the rank of the matrix A is n and there are n linearly independent rows or columns with n being the order of the matrix. Pre- and post-multiplying both sides of Eq (2.4a) by A^{-1} and $(A^T)^{-1}$, respectively, gives

$$\begin{aligned} A^{-1}C_n(A^T)^{-1} &= A^{-1}(A\mu A^T + G)(A^T)^{-1} \\ &= \mu + A^{-1}G(A^T)^{-1} \\ &= \mu + G' \end{aligned} \tag{2.5a}$$

where

$$G' \triangleq A^{-1}G(A^T)^{-1} \tag{2.5b}$$

But

$$\begin{aligned} (G')^T &= \left[A^{-1}G(A^T)^{-1}\right]^T \\ &= A^{-1}G^T(A^{-1})^T \\ &= -\left[A^{-1}G(A^{-1})^T\right] \\ &= -G' \end{aligned} \tag{2.6}$$

since G is a skew-symmetric matrix, i.e. $G^T = -G$.

Eq (2.6) shows that G' is also a skew-symmetric matrix. We note that any skew-symmetric matrix G' can be decomposed into $A^{-1}G(A^T)^{-1}$. It should be noted here that the higher the order of the matrix A, the greater the number of variables are in the matrix G'. This will enable one to obtain a polynomial of higher order, if a good approximation is to be achieved. Therefore the number of variables in the matrix C_n can be increased by increasing the order of the matrix A. Also, for the same order of the matrix C_n, a larger number of variables are obtained, since A is, in general, a non-symmetric matrix.

Note: Since G is skew-symmetric the maximum number of linearly independent elements is a function of the order of the matrix, n, and is given by $\frac{n(n-1)}{2}$. Now, when A is symmetric the number of elements in A is $\frac{n(n+1)}{2}$, in which case the total number of variables in C_n is

$$\frac{n(n+1)}{2}+\frac{n(n-1)}{2}=\frac{n^2}{2}.$$

On the other hand, if A is non-symmetric the number of elements in A is n^2, in which case the total number of variables in C_n is

$$n^2+\frac{n(n-1)}{2}=\frac{n(3n-1)}{2}.$$

Now consider

$$\det\left[A^{-1}C_n(A^T)^{-1}\right] = \det(\mu+G') \tag{2.7}$$

or

$$|A^{-1}||C_n||(A^T)^{-1}| = \det(\mu+G'). \tag{2.8}$$

Since $\det(A^{-1})=\frac{1}{\det A}$, we have

$$\begin{aligned} M_{e(o)} &\triangleq |C_n| \\ &=|A|^2\det(\mu+G') \\ &=|A|^2\det(W) \end{aligned} \tag{2.9a}$$

where subscript $e\,(o\,)$ stands for even (odd) part of an n-variable HP when n is even (odd), and

$$W \triangleq \mu + G' \ . \tag{2.9b}$$

Eq. (2.9b) is a reactance matrix. Any such reactance matrix may always be realized as an $(n+1)$-port gyrator network terminated in n-variable single reactance function. The determinant of W represents the denominator polynomial of the input admittance function of the $(n+1)$-port passive network (lossless, frequency independent), terminated at port i of its ports with unit μ_i-plane capacitor for $i = 1,2,\ldots,n$ which is an n-variable SHP. This excludes the degenerate case when some coefficients of the polynomial are absent, in which case the determinant becomes a modified Hurwitz polynomial. It may also be noted that Eq. (2.9b) can be written as $A^{-1} C_n (A^T)^{-1}$ with A being nonsingular.

Since W is already known to be a reactance matrix, it follows that the matrix C_n is realizable as an n-variable reactance network [30]. Thus $M_{e(o)}$ constitutes the denominator polynomial of the driving function of a lossless passive network. That is, the driving point admittance function of an $(n+1)$-port passive (lossless, frequency independent) network whose i-th port is terminated in unit μ_i-plane capacitor, $i = 1, 2, 3, \ldots, n$, and the last port is the driving port. We may equivalently say that $M_{e(o)}$ represents either the even polynomial or the odd polynomial of an n-variable reactance function (i.e. for $\mu_i = j\omega_{i0}$, $i = 2, 3, \ldots, n$ the resulting polynomial in μ_1 will have simple zeros on the imaginary axis of μ_1-plane) depending on whether the order of the matrix A is even or odd with the condition that $\det A \neq 0$. The corresponding derivative with respect to the μ_1 variable on the other hand gives the odd polynomial or the even polynomial of the driving point reactance function. Therefore $\dfrac{(\partial M_e / \partial \mu_1)}{M_e}$ being the ratio of odd to even (or vice versa) parts of Hurwitz polynomials (or the ratio of odd to even (or even to odd) parts of a SHP) with respect to the variable μ_1 has the reactance property [39] and has no singularity of the second kind. In view of HP, generally, all coefficients should be present, and they should be real and positive. However, all odd terms or all even terms may be missing. Also, HP should contain simple imaginary axis zeros. On the other hand, for SHP all the coefficients must be present, and they must be real and positive. Also, it must not contain zeros on the imaginary axis. For the ratio

$$\frac{(\partial M_{\epsilon_0} / \partial \mu_1)}{M_{\epsilon_0}}$$

being either the even to odd or the odd to even polynomials, means that it is an odd positive real function (PRF). In view of positive real property, the function must contain its zeros and poles alternately on the $j\omega$-axis [68] and they shall be simple. Consequently,

$$\frac{(\partial M_{\epsilon_0} / \partial \mu_1)}{M_{\epsilon_0}} + \frac{1}{K_1}$$

with K_1 being a positive constant, is also PRF. This leads to a point that

$$M_\mu = M_{\epsilon_0} + K_1 \frac{\partial M_{\epsilon_0}}{\partial \mu_1} \tag{2.10}$$

must be a SHP in μ_1 variable. This is true since M_{ϵ_0} is a HP; it becomes zero for $\mu_1 = j\omega$ and therefore $\mu_1^2 + \omega^2$ can be a factor of M_{ϵ_0} but not of $\dfrac{\partial M_{\epsilon_0}}{\partial \mu_1}$. In fact the presence of a resistor is to shift the $j\omega$-axis zeros of M_{ϵ_0} (a HP) to the left half of μ_1-plane and forces HP to become a SHP. As a consequence

$$M = M_{\epsilon_0} + \sum_{j=1}^{n} K_j \frac{\partial M_{\epsilon_0}}{\partial \mu_j} \tag{2.11}$$

is an n-variable SHP with K's being positive constants. In this case all the coefficients shall be positive and no coefficients shall be missing. It shall be noted that as $\det(\mathbf{a}_i . \mathbf{a}_i^T) = 0$, (2.9) (and hence (2.11)) does not contain terms like μ_i^n. In fact, the strictness of HP of (2.11) and the absence of terms like μ_i^n are two of the conditions required that M shall become a VSHP.

Therefore, (2.9) shows the method of generating the even part or the odd part of an n-variable SHP, whereas (2.11) shows that of an n-variable SHP from an n-variable HP.

Eq. (2.11) can be made a 2-variable SHP, simply by putting some of the μ's equal to s_1 and the rest of the μ's equal to s_2. That is

$$M_1 = M_{e(o)} + \sum_{j=1}^{2} K_{j1} \frac{\partial M_{e(o)}}{\partial s_j} \tag{2.12}$$

There will be a large number of choices to make.

The required VSHP is generated by using higher order partial polynomial derivatives as follows.

Form

$$M_2 = M_1 + \sum_{j=1}^{2} K_{j2} \frac{\partial M_1}{\partial s_j} \tag{2.13}$$

which involves higher order partial derivatives of $M_{e(o)}$. The process can be continued until we form

$$M_r = M_{r-1} + \sum_{j=1}^{2} K_{jr} \frac{\partial M_{r-1}}{\partial s_j} \tag{2.14}$$

and finally

$$M_v = M_{v-1} + \sum_{j} K_{jv} \frac{\partial M_{v-1}}{\partial s_j} \tag{2.15}$$

[illegible] K [illegible] (2.12)-(2.15) [illegible] constants.

However, in the special case when some of μ's are set equal to constants, the derivatives are not needed for the polynomial $M_{e(o)}$ of (2.9) to become a VSHP. This, however, requires certain conditions involving principal sub-determinants of the gyrator matrix G'.

From the diagonal expansion of the determinant of a matrix [73], det C_n given by (2.9) can be written as

$$\det C_n = |A|^2 \left[\det G' + \sum_{i=1}^{n} \mu_i G'_{i,i} + \sum_{1 \le i < j \le n} \mu_i \mu_j G'_{ij,ij} \right.$$

$$+ \sum_{1 \le i < j < k \le n} \mu_i \mu_j \mu_k G'_{ijk,ijk}$$

$$\left. + \sum_{1 \le i < j < k < l \le n} \mu_i \mu_j \mu_k \mu_l G_{ijkl,ijkl} + \ldots + \mu_1 \mu_2 \mu_3 \cdots \mu_n \right]. \tag{2.16}$$

where $G'_{i,i}$ is the determinant of the submatrix of G' obtained by deleting the i th row and column and is of order (n-1), $G_{ij,ij}$ is the determinant of the submatrix of G' obtained by deleting both the i th and j th rows and columns, and is of order (n-2), and so on. In the foregoing procedure two cases could be studied:

1) The even part of a HP as the starting point, in which case Eq (2.16) reduces to

$$\det C_n = |A|^2 \left[\det G' + \sum_{1 \le i < j \le n} \mu_i \mu_j G_{ij,ij} \right.$$

$$\left. + \sum_{1 \le i < j < k < l \le n} \mu_i \mu_j \mu_k \mu_l G_{ijkl,ijkl} + \ldots + \mu_1 \mu_2 \mu_3 \cdots \mu_n \right] \tag{2.17}$$

2) The odd part of a HP as the starting point, in which case Eq (2.16) reduces to

$$\det C_n = |A|^2 \left[\sum_{i=1}^{n} \mu_i G_{i,i} \right.$$

$$\left. + \sum_{1 \le i < j < k \le n} \mu_i \mu_j \mu_k G_{ijk,ijk} + \ldots + \mu_1 \mu_2 \mu_3 \cdots \mu_n \right] \tag{2.18}$$

From the diagonal expansion of the determinant of the matrix C_n, following properties are noted:

In both case (1) and (2) the degree of any μ_i ($i = 1, n$) will be unity, i.e., μ_i^2, μ_i^3, etc cannot exist

In case (1) the following additional properties are noted

i) All odd order terms of the type μ_i, $\mu_i \mu_j \mu_k$, etc in Eq (2.17) are absent, since the coefficients of these terms are determinants of odd order skew-symmetric matrices, which are zero

ii) The determinant and sub-determinants of G present are all non-negative since the determinant of an even order skew-symmetric matrix is a perfect square

In case (2) the following additional properties are noted

i) All even order terms of the type $\mu_i \mu_j$, $\mu_i \mu_j \mu_k \mu_l$, etc in Eq (2.18) are absent since their coefficients are determinants of odd order skew-symmetric matrices

ii) The sub-determinants of G present are all non-negative numbers

2.3. EXAMPLES

To illustrate the proposed method some examples are given below

2.3.1. Example 1

In this example the order of the matrices A and G in (2.1) is considered to be two, i.e.

$$C_2 = A \mu A^t + G \tag{2.19a}$$

where

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, \tag{2.19b}$$

$$G = \begin{bmatrix} 0 & g_{12} \\ g_{21} & 0 \end{bmatrix} \tag{2.19c}$$

and

$$\mu = \text{dlag} \begin{bmatrix} \mu_1 & \mu_2 \end{bmatrix}. \tag{2.19d}$$

By using (2.9), we can write

$$M_2 = \det C_2 = d_{11}\mu_1\mu_2 + g_{12}^2 \tag{2.20a}$$

where

$$d_{11} = (a_{11}a_{22} - a_{12}a_{21})^2. \tag{2.20b}$$

Now using (2.11) gives

$$\begin{aligned} M_2' &= M_2 + \sum_{j=1}^{2} \frac{\partial M_2}{\partial \mu_j} \\ &= (d_{11}\mu_1\mu_2 + g_{12}^2) + (d_{11}\mu_2 + d_{11}\mu_1). \end{aligned} \tag{2.21}$$

This is a VSHP in $\mu_1 = s_1$ and $\mu_2 = s_2$ variables.

2.3.2. Example 2

In this second example, we choose the order of the matrices $\boldsymbol{A}$ and $\boldsymbol{G}$ of (2.4) to be three, i.e.,

$$C_3 = A\mu A^T + G \tag{2.22a}$$

where

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}, \tag{2.22b}$$

$$G = \begin{bmatrix} 0 & g_{12} & g_{13} \\ -g_{12} & 0 & g_{23} \\ -g_{13} & -g_{23} & 0 \end{bmatrix}, \tag{2.22c}$$

and

$$\mu = \mathrm{diag}\left[\mu_1 \ \ \mu_2 \ \ \mu_3\right]. \tag{2.22d}$$

By using (2.9) we may write

$$\begin{aligned} N_3 &= \det \boldsymbol{C}_3 \\ &= d_{11}\mu_1+d_{21}\mu_2+d_{31}\mu_3+d_{41}\mu_1\mu_2\mu_3 \end{aligned} \tag{2.23a}$$

where

$$\begin{aligned} d_{11} &= (a_{11}g_{23}-a_{12}g_{13}+a_{13}g_{12})^2, \\ d_{21} &= (a_{21}g_{23}-a_{22}g_{13}+a_{23}g_{12})^2, \\ d_{31} &= (a_{31}g_{23}-a_{32}g_{13}+a_{33}g_{12})^2, \end{aligned}$$

and

$$\begin{aligned} d_{41} = (&a_{11}a_{22}a_{33}-a_{11}a_{23}a_{32}-a_{12}a_{21}a_{33}+ \\ &a_{13}a_{21}a_{32}+a_{12}a_{23}a_{31}-a_{13}a_{22}a_{31})^2 \end{aligned} \tag{2.23b}$$

As stated earlier, if any one of the μ's in (2.23a) is set equal to some constant value the result is already a VSHP and thus we need not to make use of the derivatives.

Now the use of (2.11) leads to

$$\begin{aligned} N_3 &= N_3 + \sum_{j=1}^{3} \frac{\partial N_3}{\partial \mu_j} \\ &= (d_{11}\mu_1+d_{21}\mu_2+d_{31}\mu_3+d_{41}\mu_1\mu_2\mu_3)+ \\ &\quad \left[(d_{11}+d_{41}\mu_2\mu_3)+(d_{21}+d_{41}\mu_1\mu_3)+(d_{31}+d_{41}\mu_1\mu_2)\right] \\ &= (d_{11}\mu_1+d_{21}\mu_2+d_{31}\mu_3+d_{41}\mu_1\mu_2\mu_3)+ \\ &\quad \left[(d_{11}+d_{21}+d_{31})+d_{41}(\mu_2\mu_3+\mu_1\mu_3+\mu_1\mu_2)\right] \end{aligned} \tag{2.24}$$

The last equality can be made a 2-variable HP simply by putting two of the μ's equal to s_1 and the rest [illegible] or vice versa. There will be many choices. For example making [illegible] μ [illegible] μ [illegible] s_1 and μ [illegible] s_2 will lead to

$$Q_1(s_1,s_2,\boldsymbol{a},\boldsymbol{g}) \triangleq N_3'$$
$$= d_{41}s_1^2(s_2+1)+s_1\left[2d_{41}s_2+(d_{11}+d_{21})\right]$$
$$+\left[d_{31}s_2+(d_{11}+d_{21}+d_{31})\right]. \tag{2.25}$$

By Theorem [62, 78] (2.25) can be shown to be a VSHP. To show this, (2.25) can be written as

$$Q_1(s_1,s_2,\boldsymbol{a},\boldsymbol{g}) = \sum_{i=0}^{m} Q_{1i}(s_2,\boldsymbol{a},\boldsymbol{g})s_1^i$$
$$= \sum_{i=0}^{n} Q_{2i}(s_1,\boldsymbol{a},\boldsymbol{g})s_2^i$$

(here, it is assumed that s_1 and s_2 are of degrees m and n, respectively.) The polynomial Q_1 is a VSHP if and only if the highest degree coefficient in both the variables, $s_1^m s_2^n$, is not zero, $Q_{1i}(s_2,\boldsymbol{a},\boldsymbol{g})$ or $Q_{2i}(s_1,\boldsymbol{a},\boldsymbol{g})$ are single variable SHPs and of the same order, and $Q_1(j\omega_1,j\omega_2,\boldsymbol{a},\boldsymbol{g})$ for all finite values of ω_1 and ω_2 is not zero. Let us express $Q_1(s_1,s_2,\boldsymbol{a},\boldsymbol{g})$ as a polynomial in s_2

$$Q_1(s_1,s_2,\boldsymbol{a},\boldsymbol{g}) = Q_{21}(s_1,\boldsymbol{a},\boldsymbol{g})s_2+Q_{20}(s_1,\boldsymbol{a},\boldsymbol{g}).$$

We have the following observation to make:

1) $Q_{20}(s_1,\boldsymbol{a},\boldsymbol{g})$ and $Q_{21}(s_1,\boldsymbol{a},\boldsymbol{g})$ are SHPs of the same order without any missing coefficients.

2) The highest terms in s_1 ,i.e., the term s_1^2 , is present in $Q_{21}(s_1,\boldsymbol{a},\boldsymbol{g})$. So the coefficient of $s_1^2 s_2$ in $Q_1(s_1,s_2,\boldsymbol{a},\boldsymbol{g})$ is not zero. We now write $Q_1(s_1,s_2,\boldsymbol{a},\boldsymbol{g})$ as a polynomial in s_1

$$Q_1(s_1,s_2,\boldsymbol{a},\boldsymbol{g}) = Q_{12}(s_2,\boldsymbol{a},\boldsymbol{g})s_1^2+Q_{11}(s_2,\boldsymbol{a},\boldsymbol{g})s_1+Q_{10}(s_2,\boldsymbol{a},\boldsymbol{g}).$$

3) Following similar steps as above, we can show that $Q_{12}(s_2,\boldsymbol{a},\boldsymbol{g})$ is a SHP.

4) As $Q_{20}(s_1,\boldsymbol{a},\boldsymbol{g})$ and $Q_{21}(s_1,\boldsymbol{a},\boldsymbol{g})$ are SHPs

$$Q_1(j\omega_1, j\omega_2, \boldsymbol{a}, \boldsymbol{g}) = Q_{21}(j\omega_1, \boldsymbol{a}, \boldsymbol{g})\, j\omega_2 + Q_{20}(j\omega_1, \boldsymbol{a}, \boldsymbol{g})$$

$Q_{21}(j\omega_1, \boldsymbol{a}, \boldsymbol{g}) \neq 0$ since $Q_{21}(s_1, \boldsymbol{a}, \boldsymbol{g})$ is an SHP.

and

$$j\omega_2 + \frac{Q_{20}(j\omega_1, \boldsymbol{a}, \boldsymbol{g})}{Q_{21}(j\omega_1, \boldsymbol{a}, \boldsymbol{g})} \neq 0$$

since $\mathrm{Re}(Q_{20}(j\omega_1, \boldsymbol{a}, \boldsymbol{g})/Q_{21}(j\omega_1, \boldsymbol{a}, \boldsymbol{g})) > 0$ for all finite values of ω_1. Thus $Q_1(j\omega_1, j\omega_2, \boldsymbol{a}, \boldsymbol{g}) \neq 0 \ \forall (\omega_1, \omega_2)$.

This, along with the property 1) given above, namely, $Q_{20}(s_1, \boldsymbol{a}, \boldsymbol{g})$ is a SHP, ensures that $Q_1(s_1, s_2, \boldsymbol{a}, \boldsymbol{g})$ is a SHP. Properties 1), 2), and 3) establish that $Q_1(s_1, s_2, \boldsymbol{a}, \boldsymbol{g})$ satisfies the necessary conditions for VSHP [62]. In addition, property 4) above shows that $Q_1(s_1, s_2, \boldsymbol{a}, \boldsymbol{g})$ is also a SHP. Thus, we say that $Q_1(s_1, s_2, \boldsymbol{a}, \boldsymbol{g})$ is a VSHP

2.4. SUMMARY AND DISCUSSION

In this chapter a different approach in generation of VSHPs is studied. It is shown how properties of matrices could be utilized to generate 2-variable VSHPs. This new technique is based on the extension and generalization of the earlier methods [72, 74]. Polynomials generated by this technique have a larger number of variables associated with any coefficient and hence the filters designed are expected to be of lower sensitivities as compared with the previous ones. Also the presented method is simple to implement. When these polynomials are used in the design of 2-D recursive digital filters the stability of the designed filter is guaranteed with nonsingularity of the matrix $\boldsymbol{A}$ as precondition. Some examples are included to illustrate the method. This approach can also be extended to N-D case (1-D as special case).

CHAPTER III

DESIGN OF STABLE TWO-DIMENSIONAL RECURSIVE DIGITAL FILTERS

3.1. INTRODUCTION

In this chapter, we present a method of designing stable 2-D digital filters of the recursive type (1-D as special case) starting with a two-variable continuous time transfer function with VSHP denominator and then using the bilinear z-transformations, so that the filter will have the desired behavior in magnitude or both the magnitude and phase. Stability of the recursive digital filters obtained by such a technique is guaranteed since the original prototype analog filter is absolutely stable. It should be noted that the approximation is carried out in discrete domain (after the application of the double bilinear transformation) to avoid any distortion of the phase characteristics. The proposed approach of Chapter II was used to design a large number of 2-D recursive digital filters, the results of which are reported in this chapter.

Consider the 2-D recursive digital filter transfer function

$$H(z_1,z_2) = \frac{\sum\limits_m \sum\limits_n a(m,n) z_1^{-m} z_2^{-n}}{\sum\limits_m \sum\limits_n b(m,n) z_1^{-m} z_2^{-n}}$$

$$= M(\theta_{1m},\theta_{2n}) e^{j\phi(\theta_{1m},\theta_{2n})} \tag{3.1}$$

where $z_i = \exp(j\theta_i)$, $\theta_i = \omega_i T_i$ for $i = 1,2$ while $M(\theta_{1m},\theta_{2n}) = |H(e^{j\theta_{1m}}, e^{j\theta_{2n}})|$ and $\phi(\theta_{1m},\theta_{2n}) = \arg H(e^{j\theta_{1m}}, e^{j\theta_{2n}})$ are the magnitude and phase frequency responses of the 2-D filter, respectively.

Group-delay response is defined as

$$\tau_{\theta_i}(\theta_{1m}, \theta_{2n}) \triangleq \frac{-\partial\phi(\theta_{1m}, \theta_{2n})}{\partial\theta_i}, \qquad \text{for } i = 1,2. \tag{3.2}$$

It gives the envelope delay of a signal in the vicinity of a particular frequency, provided that the magnitude within that narrow frequency band remains rather constant.

In data transmission it is of importance that the wave shape of a signal be preserved as it passes from a transmitting source to some receiver [1]. This requires that the frequencies which constitute the signal at the transmitting end arrive at the receiving end with about the same time delay; then τ_{θ_i} must remain constant over the essential portion of the frequency spectrum. We must also require an essentially constant magnitude over this band, although this is often less important.

Different specifications for approximating such responses that may be considered are the following:

(i) Approximation of magnitude or squared magnitude of the frequency response

(ii) Approximation of the group-delay response.

(iii) Approximation of both the magnitude and group delay responses.

The cases (i) and (iii) will be considered in this thesis. Methods based on optimization techniques, linear [34, 79-80] or non-linear [32-33, 35-38, 40-41] programming, may be used for designing 2-D filters to meet a given frequency response specification.

3.2. FORMULATION OF THE DESIGN PROBLEM

3.2.1. Magnitude Function Approximation

A frequency domain l_p approximation to the design of 2-D IIR filters has been given for the magnitude frequency specifications which are defined on a discrete set of frequency points. It is given by

$$F_M = \sum_{m,n \in I_{ps}} \sum \left[\mid H_D(e^{j\theta_{1m}}, e^{j\theta_{2n}}) \mid - H_I(e^{j\theta_{1m}}, e^{j\theta_{2n}}) \right]^p$$

$$\text{for} \quad m = 1,2,\ldots,M; \qquad n = 1,2,\ldots,N \tag{3.3}$$

where $\theta_i = \omega_i T_i$, $i = 1,2$; $H_I(.)$ is the ideal or desired magnitude frequency response, $H_D(.)$ represents the designed filter transfer function, and I_{ps} is the discrete set of $M \times N$ frequency points along the ω_1 and ω_2 axes in the passband and stopband of the 2-D filter in which the error is computed. The magnitude error function F_M is minimized using a nonlinear optimization technique The optimization problem becomes simple for $p = 2$ and the l_p error norm reduces to the l_2 error norm which is called minimum mean square error (least mean square error) minimization.

The design method employs a 2-D analog transfer function which is of the form

$$H_a(s_1, s_2) = \frac{\sum_{k=0}^{M_1} \sum_{l=0}^{N_1} p_{kl} s_1^k s_2^l}{\sum_{k=0}^{M_1} \sum_{l=0}^{N_1} q_{kl} s_1^k s_2^l}$$

$$\equiv \frac{P(s_1, s_2, p_{kl})}{Q(s_1, s_2, q_{kl})} \tag{3.4}$$

Assuming that $P(s_1, s_2, p_{kl})$ and $Q(s_1, s_2, q_{kl})$ are non-separable, $H_a(s_1, s_2)$ represents the transfer function of a general class of 2-D analog filter $Q(s_1, s_2, q_{kl})$, the denominator polynomial of $H_a(s_1, s_2)$, should satisfy stability conditions In addition to meeting the degree requirements between Q and P, a sufficient condition for stability is that Q be a VSHP [62, 78], that is,

$$Q(s_1, s_2, q_{kl}) \neq 0 \qquad \forall (s_1, s_2) \in S_{\bar{2}} \, \bar{c}$$

where

$$S_{\oplus\oplus} \triangleq \left\{ (s_1, s_2) \mid \mathrm{Re}(s_1) \geq 0,\ \mathrm{Re}(s_2) \geq 0,\ |s_1| \leq \infty, \text{ and } |s_2| \leq \infty \right\}.$$

An established necessary condition for stability is that Q be a SHP [47], that is,

$$Q(s_1, s_2, q_{kl}) \neq 0 \qquad \forall (s_1, s_2) \in S_{(+(+}$$

where

$$S_{(+(+} \triangleq \left\{ (s_1, s_2) \mid \mathrm{Re}(s_1) \geq 0,\ \mathrm{Re}(s_2) \geq 0,\ |s_1| < \infty, \text{ and } |s_2| < \infty \right\}.$$

In a special case that the 2-variable analog transfer function, $H_a(s_1, s_2)$, is denominator separable or product separable certain symmetry restrictions are imposed on $H_a(s_1, s_2)$ In the case of a separable denominator, we can write

$$H(s_1, s_2) = \frac{P(s_1, s_2)}{Q_{1s}(s_1) Q_{2s}(s_2)}$$

in which case the filter is stable if and only if $Q_{1s}(s_1)$ and $Q_{2s}(s_2)$ are single variable SHPs Thus the stability test is equivalent to two 1-D tests With this scheme only quadrantal and fourfold rotational symmetry is possible, depending on the nature of the numerator and denominator polynomials For example, $H_a(s_1, s_2)$ possesses quadrantal magnitude symmetry if and only if the numerator polynomial is expressible as [81] $P_1(s_1, s_2^2) P_2(s_1^2, s_2)$ where P_1 and P_2 are 2-variable polynomials and one of the two may be a constant However, if one requires to design a filter with non-symmetric cutoff boundaries this method fails

On the other hand when the transfer function is product separable, i.e.,

$$H_a(s_1, s_2) = \frac{P_1(s_1) P_2(s_2)}{Q_1(s_1) Q_2(s_2)} = H_1(s_1) H_2(s_2)$$

only octagonal symmetry and rectangular symmetry is possible For example, when the numerator is expressible as $P_{1s}(s_1)P_{1s}(s_2)$ and the denominator is expressible as $Q_{1s}(s_1)Q_{1s}(s_2)$, with $Q_{1s}(s)$ being a single variable SHP [81], then $H_a(s_1,s_2)$ satisfies the conditions for octagonal symmetry. This class of transfer function is a sub-class of the latter one whereas the latter case is a sub-class of the general form of the transfer function of (3.4).

In this chapter, the design method employs the 2-variable VSHP given by (2.25) of Chapter II, to be assigned to the denominator of (3.4) while the numerator is left unchanged. This possesses a general frequency response which can be designed with any given boundaries.

As can be seen from (2.25) the degrees of s_1 and s_2 variables in $Q_1(s_1,s_2,\boldsymbol{a},\boldsymbol{g})$ are not equal. It is preferred to keep them equal, as this will influence the filter response, particularly regarding symmetry. The degrees of s_1 and s_2 can be made equal when $Q_1(.)$ is properly multiplied with another 2-variable polynomial.

Following the same procedure outlined in Chapter II, and making the substitution $\mu_1=s_1$ and $\mu_2=\mu_3=s_2$, another 2-variable VSHP can be generated which will be of the form

$$\begin{aligned} Q_2(s_1,s_2,\boldsymbol{a}',\boldsymbol{g}') = \; & d_{42}s_2^2(s_1+1)+s_2\left[2d_{42}s_1+(d_{22}+d_{32})\right] \\ & +\left[d_{12}s_1+(d_{12}+d_{22}+d_{32})\right] \end{aligned} \tag{3.5a}$$

where

$$d_{12}=(a_{112}g_{232}-a_{122}g_{132}+a_{132}g_{122})^2,$$
$$d_{22}=(a_{212}g_{232}-a_{222}g_{132}+a_{232}g_{122})^2,$$
$$d_{32}=(a_{312}g_{232}-a_{322}g_{132}+a_{332}g_{122})^2,$$

and

$$\begin{aligned} d_{42}=(&a_{112}a_{222}a_{332}-a_{112}a_{232}a_{322}-a_{122}a_{212}a_{332}+ \\ &a_{132}a_{212}a_{322}+a_{122}a_{232}a_{312}-a_{132}a_{222}a_{312})^2. \end{aligned} \tag{3.5b}$$

By multiplying $Q_1(.)$ and $Q_2(.)$ we can write.

$$Q(s_1,s_2,\boldsymbol{a},\boldsymbol{g},\boldsymbol{a}',\boldsymbol{g}') \triangleq Q_1(s_1,s_2,\boldsymbol{a},\boldsymbol{g}) \cdot Q_2(s_1,s_2,\boldsymbol{a}',\boldsymbol{g}') \tag{3.6}$$

which by Theorem [62, 76] is a 2-variable VSHP as $Q_1(.)$ and $Q_2(.)$ are both 2-variable VSHPs.

Now the polynomial $Q(.)$ is employed as the denominator of the 2-D prototype analog transfer function of (3.4) and the numerator is left unchanged. Using this technique, a stable 2-D recursive digital filter with the system function $H_d(z_1,z_2)$ could be designed. The approach consists of applying bilinear z-transformations to the resulting analog transfer function to obtain the digital transfer function $H_d(z_1,z_2)$, and then to carry out the optimization, with the coefficients p_{kl}, $\boldsymbol{a}$, $\boldsymbol{g}$, $\boldsymbol{a}'$, and $\boldsymbol{g}'$ as variables, in the digital domain. The digital transfer function is given by

$$H_d(z_1,z_2) = \frac{N(z_1,z_2)}{D(z_1,z_2)}$$

$$= \left. \frac{P(s_1,s_2,p_{kl})}{Q(s_1,s_2,\boldsymbol{a},\boldsymbol{g},\boldsymbol{a}',\boldsymbol{g}')} \right|_{s_r = \frac{2}{T}\frac{z_r - 1}{z_r + 1} \quad \text{for } r = 1,2} \tag{3.7}$$

Since the analog transfer function has a VSHP denominator, the digital transfer function $H_d(z_1,z_2)$ obtained by using the double bilinear transformation preserves the stability. That is

$$D(z_1,z_2) \neq 0 \text{ for } \bigcap_{r=1}^{2} |z_r| > 1 \tag{3.8}$$

In this technique the relationship (3.3) with p = 2 was chosen as the cost function which is

$$F_M(\mathbf{c}) = \sum_{m}\sum_{n} E_{M}^{p}(\mathbf{c}) \qquad \text{for } m = 0,1,2,\ldots,M,$$

$$n = 0,1,2,\ldots,N \tag{3.9a}$$

where

$$E_M(\mathbf{c}) = |H_D(e^{j\omega_{1m}}, e^{j\omega_{2n}}, \mathbf{c})| - |H_I(e^{j\omega_{1m}}, e^{j\omega_{2n}})| \tag{3.9b}$$

with $\theta_i = \omega_i T_i$, $i = 1,2$, is the error of the magnitude response, and the vector $\boldsymbol{c}$ is chosen as the vector of the coefficients of the filter

$$\boldsymbol{c} = (p_{kl}, \boldsymbol{a}, \boldsymbol{g}, \boldsymbol{a}', \boldsymbol{g}'), \text{ for } k = 0,1,...,M_1,\ l = 0,1,...,N_1. \tag{3.9c}$$

$|H_D|$ can be calculated as

$$|H_D| = |H_d(z_1, z_2)|\ z_r = \exp(j\theta_r),\ (r = 1,2). \tag{3.9d}$$

After we have obtained a measure of error for initial parameters, we want to know how to change the parameters so as to minimize the error. The error function given by (3.9) is a nonlinear function of the coefficients, since the objective function to be minimized contains products of unknowns. So a nonlinear technique must be used for minimizing the unconstrained function $F_M(\boldsymbol{c})$.

Many different computer-aided optimization algorithms that can be utilized to minimize the error function exist; non-derivative, first derivative, and second derivative methods. In this thesis, we use the second derivative (second-order) technique, i.e., the Fletcher and Powell algorithm [60]. This algorithm requires function values and the first derivative of the cost function with respect to optimization parameters, the filter coefficients. Here, the first derivatives of the error function $(\partial F_M(\boldsymbol{c})/\partial c_i)$ are evaluated and used as information to indicate how the parameters should be changed in order to minimize the error in the optimization procedure. The first derivatives determine the gradient of the error function. The gradient is the direction of maximum local increase of error function. This can be seen by using the first order Taylor's series expansion of the objective function. Thus, to minimize the error one proceeds in the direction opposite to the gradient. This is the basis of the steepest-decent method, which uses the gradient to predict parameter changes for error minimization. The gradient of the error function $F_M(\boldsymbol{c})$ with respect to variables p_{kl}, $\boldsymbol{a}$, $\boldsymbol{g}$, $\boldsymbol{a}'$, and $\boldsymbol{g}'$ in $\boldsymbol{c}$ is calculated as, [32]

$$\frac{\partial F_M(\mathbf{c})}{\partial c_i} = 2 \sum_m \sum_{n \in I_p} \left[|H_D(e^{j\theta_{1m}}, e^{j\theta_{2n}}, \mathbf{c})| - |H_I(e^{j\theta_{1m}}, e^{j\theta_{2n}})| \right]$$

$$\frac{1}{|H_D(e^{j\theta_{1m}}, e^{j\theta_{2n}}, \mathbf{c})|}$$

$$\mathrm{Re}\left[H_D^*(e^{j\theta_{1m}}, e^{j\theta_{2n}}, \mathbf{c}) \frac{\partial H_D(e^{j\theta_{1m}}, e^{j\theta_{2n}}, \mathbf{c})}{\partial c_i} \right] \qquad (3.10)$$

where $H_D^*(\ )$ is conjugate of $H_D(\ )$ and Re[ ] is real part of the complex number inside the bracket

The flow diagram for the optimization algorithm is shown in Fig. 3.1, where the inputs to be specified are the number of independent variables N, starting values of the independent vector $\mathbf{c}$ containing the coefficients of the filter, the estimate of the minimum value of the objective function EST which is a constant, a prescribed tolerance EPS, number of iterations $LIMIT$, sampling frequency ω_s, passband edge ω_1 and stopband edge ω_2. The algorithm gives the optimized parameters, the final minimum value of the cost function and the maximum error in the design as its outputs. The digital computers used were *Cyber 830D* and *Vax 11/780*.

3.2.2. Design Examples

In this subsection we shall consider design of simple linear 2-D recursive digital filters which have application in image enhancement. Here 2-D recursive digital filters such as lowpass, highpass, bandpass are sought. 2-D recursive digital filters such as homomorphic and Laplacian filters are also designed as they are an effective means of enhancing and restoring image. Further they may be used to separate signals which are [illegible]. With the help of the procedure [illegible] in the previous subsection, we [illegible] 2-D recursive digital filters satisfying a prescribed [illegible]

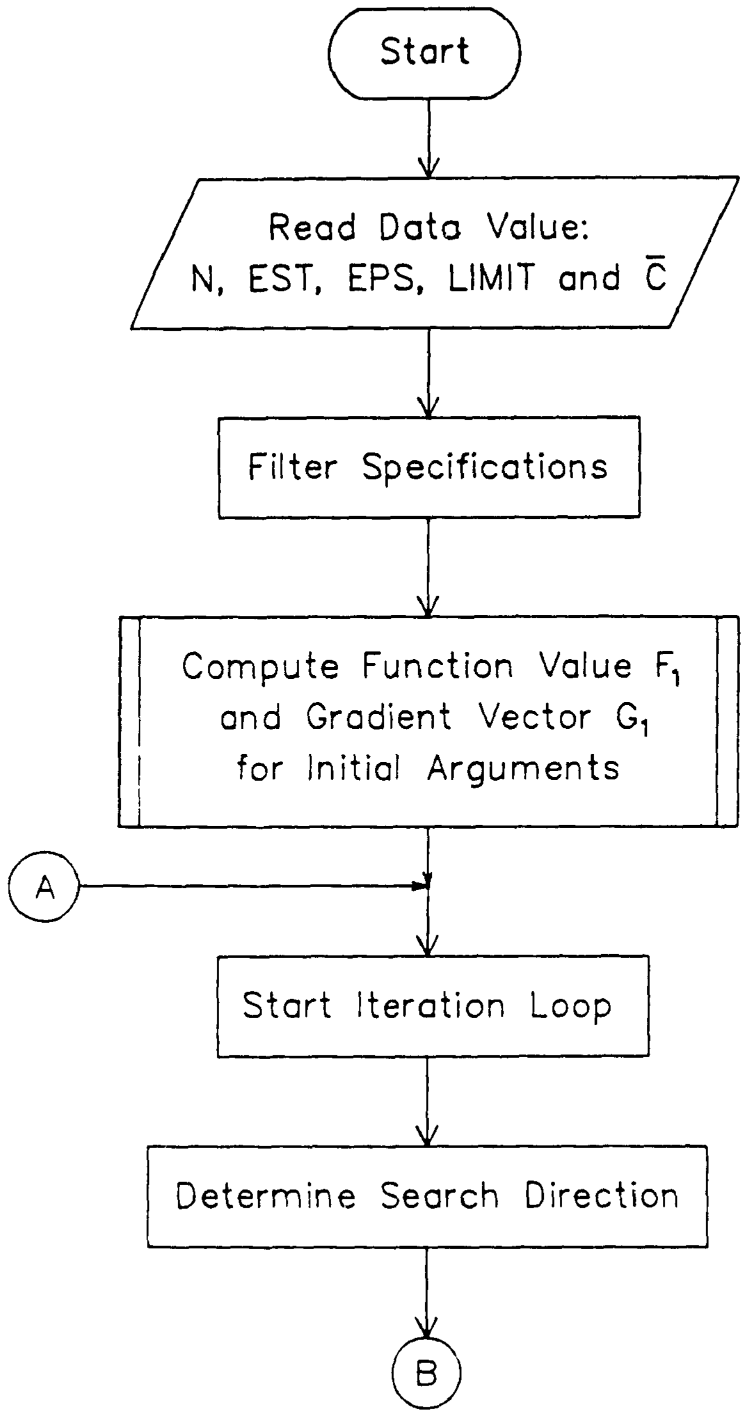
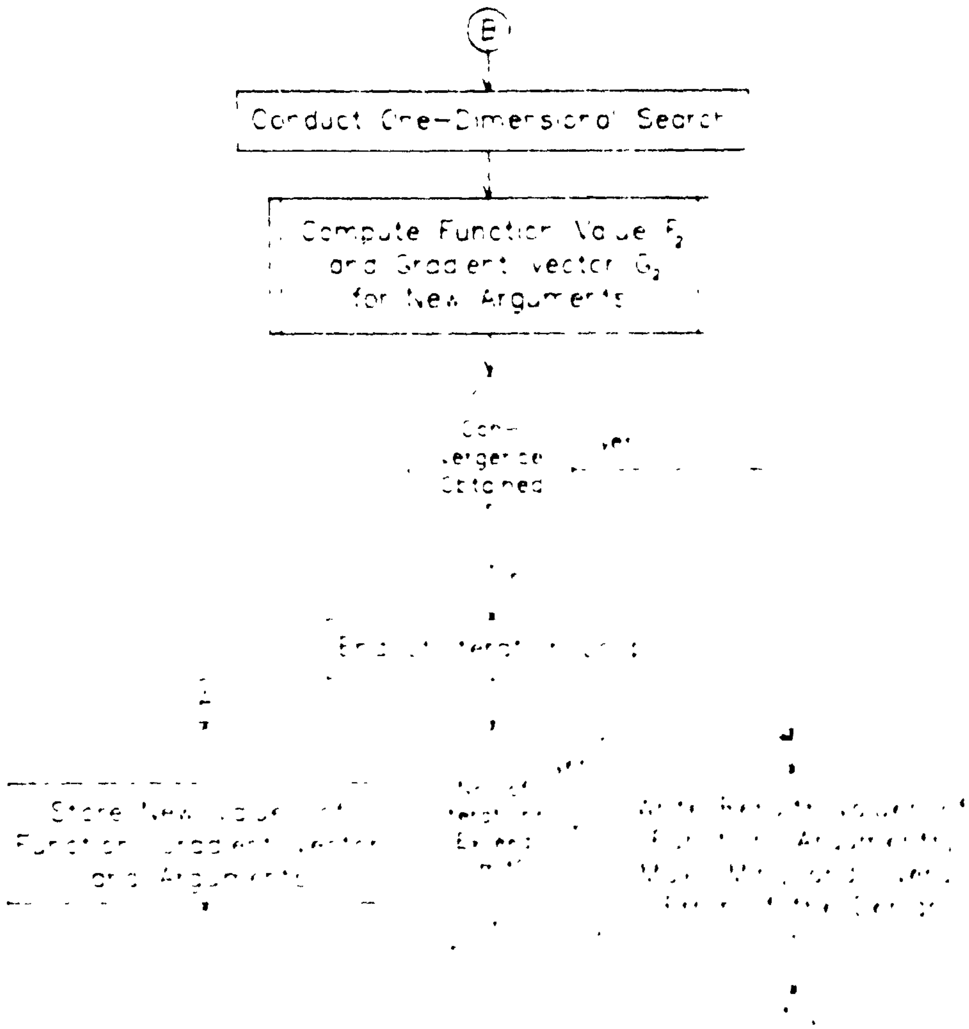


Fig. 3.1. Fletcher and Powell (FMFP Algorithm) Logic Diagram


Conduct One-Dimensional Search
Compute Function Value F_2 and Gradient Vector G_2 for New Arguments

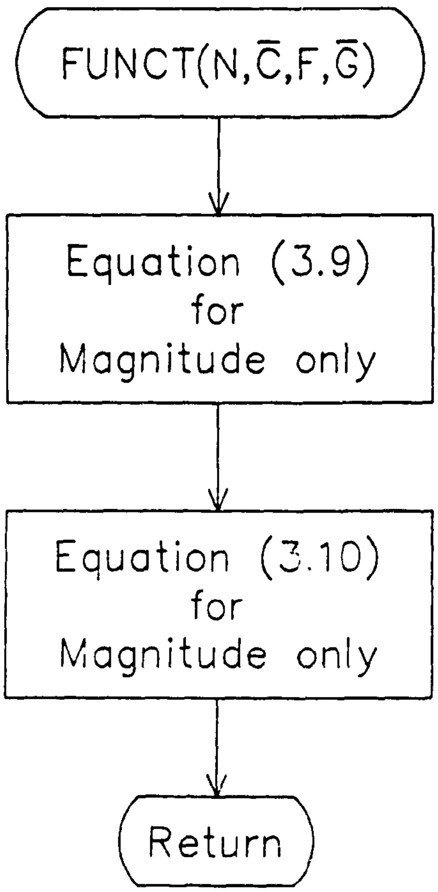


Fig. 3.1. Fletcher and Powell (FMFP Algorithm) Logic Diagram (continued).

filter was chosen to be of order 3×3 so that the 2-variable VSHP, $Q(s_1, s_2, c)$, of (3.6) could be used.

3.2.2.1. Design Example 1

In this example, the problem of designing 2-D recursive digital filters of lowpass type is considered. As stated earlier they have applications in capturing the majority of the low-energy components of an image. To apply (3.9) to the design problem, consider contour plot of Fig. 3.2 in which the filter response regions of a lowpass type are plotted in the (ω_1, ω_2)-plane. The passband is the region where

$$\rho(\omega_1, \omega_2) \triangleq (\omega_1^2 + \omega_2^2)^{1/2} \leq R_1 .$$

The transition band is the region where

$$R_1 < \rho(\omega_1, \omega_2) < R_2$$

and the stopband is the region where

$$R_2 \leq \rho(\omega_1, \omega_2) \leq \frac{\omega_s}{2}$$

With respect to Fig. 3.2, the frequency samples are located on a matrix of M columns and N rows in the (ω_1, ω_2)-plane as shown in Fig. 3.3.

In the right-half of the (ω_1, ω_2)-plane $M \times N$ frequency points (also called grid points) are chosen. The radius R_1 defines the passband edge, ω_p, whereas the radius R_2 defines the stopband edge, ω_s. The samples in the region*

$$\rho(m, n) \triangleq (\omega_{1m}^2 + \omega_{2n}^2)^{1/2} \leq R_1 \quad \text{for } m = 0,1,2,\ldots,M, \quad n = 0,1,2,\ldots,N$$

are fixed to be of unity value in magnitude, while the samples in the region

$$R_1 < \rho(m, n) < R_2$$

* The [illegible] $\rho(m,n)$ [illegible] $\rho(\omega_{1m}, \omega_{2n})$ [illegible]

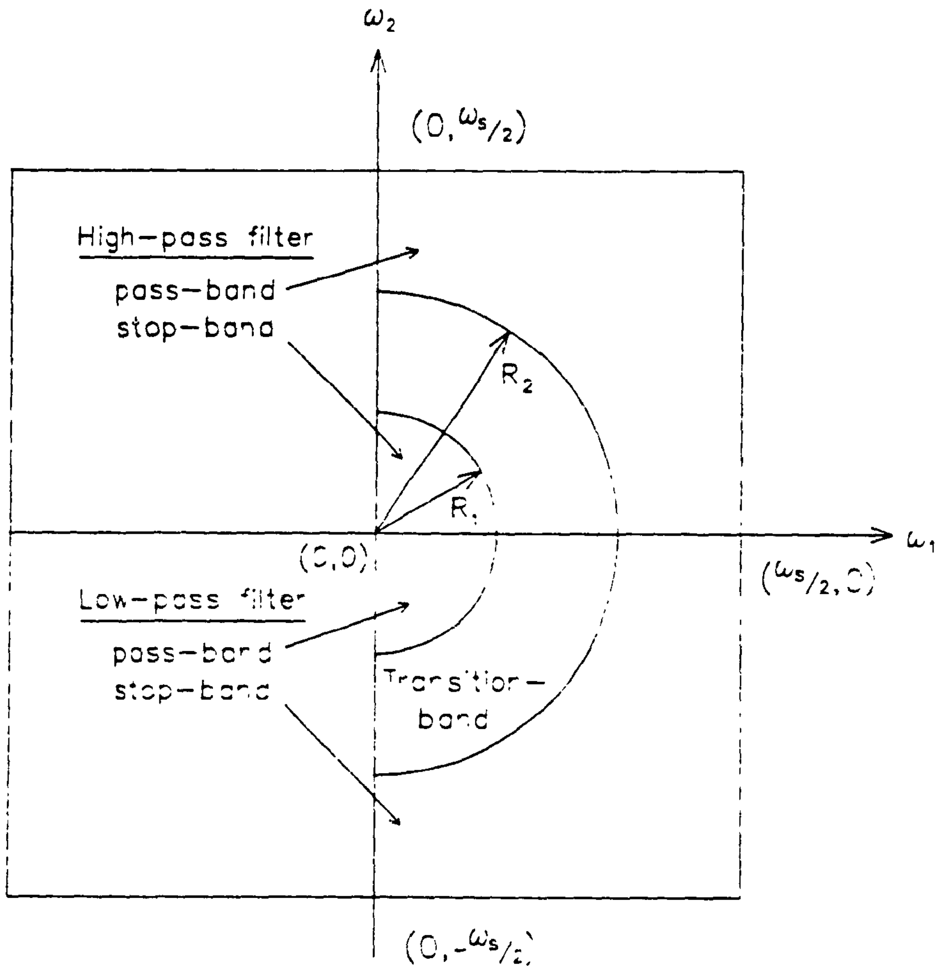


Fig. 3.2. Regions in the (ω_1,ω_2)-Plane over which the Filter Response is Specified for a Low- and High- Pass Filter.

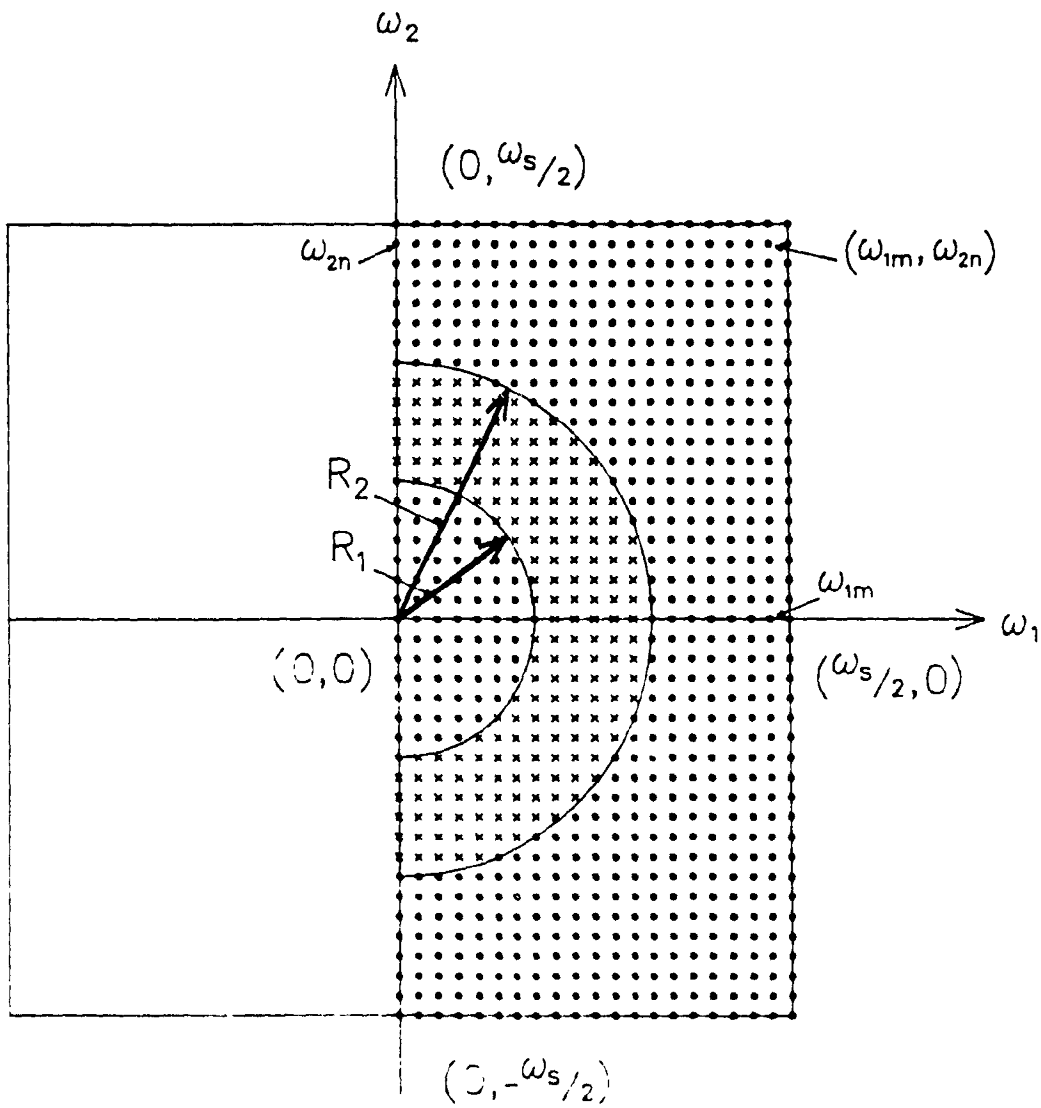


Fig. 3.3 Locations of Frequency Samples (Optimization Points)

for a 21 X 41 Grid

are restricted to vary exponentially, and the samples in the region

$$R_2 \le \rho(m,n) \le \frac{\omega_s}{2}$$

are fixed equal to zero. Formally, we consider a frequency sampling approximation to the following ideal response:

$$|H_I(e^{j\theta_{1m}}, e^{j\theta_{2n}})| = \begin{cases} 1.0 & \text{for } 0.0 \le \rho(m,n) \le 1.0 \text{ rad/sec.} \\ 0.0 & \text{for } 2.0 \le \rho(m,n) \le \frac{\omega_s}{2} = 5.0 \text{ rad/sec.} \end{cases}$$

where

$$\theta_i = \omega_i T_i,\ i = 1,2$$

The optimization yielded the coefficients given in Table 3.1(a) and the error performance given Table 3.7(p. 66).

It should be noted that the frequency grid for all 3-D plots was chosen to be 51$\times$51 points whereas the number of prescribed samples in the design process were 21$\times$41 in the right half of the frequency plane as was shown in Fig. 3.3.

Note that the larger the value of R_1, the more spatial frequencies are passed through the filter, and therefore the sharper is the image. With this in mind, we considered a second lowpass filter with double passband edge. The results of the optimization for the filter with R_1=2.0 rad/sec and R_2=3.0 rad/sec were with the coefficients of the filter given in Table 3.1(b) and the error performance given Table 3.7(p. 66). Figs. 3.4(a-b) show the perspective and the contour plots of the final design.

3.2.2.2. Design Example 2

Bandpass filters having similar characteristics as lowpass filters may sometimes be more important in practice. As is well known, in 1-D case the transfer function of a

Table 3.1(a) Values of the Parameters of the Designed 2-D Filter in Example 3.1

Parameters of the Numerator			
p_{00}=−6.001664*e* +00	p_{01}= 1.133959*e* +00	p_{02}=−3.956801*e* −01	p_{03}= 3.062932*e* −02
p_{10}= 1.349127*e* +00	p_{11}=−1.335317*e* +01	p_{12}=−7.310864*e* −02	p_{13}=−7.812736*e* −01
p_{20}=−3.857310*e* −01	p_{21}=−6.137185*e* −02	p_{22}= 5.023150*e* −01	p_{23}=−2.906945*e* −02
p_{30}= 3.585195*e* −02	p_{31}=−7.756838*e* −01	p_{32}=−3.619504*e* −02	p_{33}=−4.416538*e* −02

Parameters of the Denominator	
Section 1	Section 2
a_{111}= 1.34555101*e* +00	a_{112}= 1.08933354*e* +00
a_{121}= 9.37348944*e* −01	a_{122}=−1.22449045*e* +00
a_{131}=−1.17374068*e* −01	a_{132}= 1.33152943*e* +00
a_{211}=−6.42962296*e* −01	a_{212}= 1.13097808*e* +00
a_{221}=−1.75236527*e* −01	a_{222}=−2.82203532*e* −01
a_{231}=−6.11067739*e* −01	a_{232}= 9.47208471*e* −01
a_{311}= 4.37249657*e* −01	a_{312}=−4.48302773*e* −01
a_{321}=−1.26160383*e* +00	a_{322}=−1.06202024*e* +00
a_{331}=−6.71265052*e* −01	a_{332}= 1 33458512*e* +00
g_{121}= 1.55814711*e* +00	g_{122}=−8.02652470*e* −01
g_{131}= 1.25529513*e* +00	g_{132}= 6.62765298*e* −01
g_{231}= 1.16822653*e* +00	g_{232}= −5.37783650*e* −01

Table 3.1(b) Values of the Parameters of the Designed 2-D Filter in Example 3.1

Parameters of the Numerator			
p_{00}=−1.458960e +01	p_{01}=−5 416078e +00	p_{02}=−2.113927e −01	p_{03}=−1.018117e −01
p_{10}= 5.889783e +00	p_{11}=−8.942051e −01	p_{12}= 3.033497e −01	p_{13}=−2.829903e −02
p_{20}=−1.908147e −01	p_{21}=−2.514233e −01	p_{22}= 1.084123e −01	p_{23}= 6.876815e −03
p_{30}= 1.107897e −01	p_{31}=−2.637277e −02	p_{32}=−5.942138e −03	p_{33}=−5.098582e −04

Parameters of the Denominator	
Section 1	Section 2
a_{111}= 1.25403272e +00	a_{112}= 1.42015616e +00
a_{121}= 8.31945627e −01	a_{122}=−1.26382461e +00
a_{131}= 4.42693380e −02	a_{132}= 1.68587132e +00
a_{211}=−4.52512222e −01	a_{212}= 9.69903171e −01
a_{221}=−3.09896087e −01	a_{222}=−2.32830633e −01
a_{231}=−4.06170676e −01	a_{232}= 6 13605927e −01
a_{311}= 4.37820288e −01	a_{312}=−6 49571460e −01
a_{321}=−1.16120421e +00	a_{322}=−7.47372370e −01
a_{331}=−4.10008752e −01	a_{332}= 1.31133225e +00
g_{121}= 1.53400033e +00	g_{122}=−7.97248086e −01
g_{131}= 1.33431227e +00	g_{132}= 4.85500163e −01
g_{231}= 1.25517334e +00	g_{232}=−7.55281876e −01

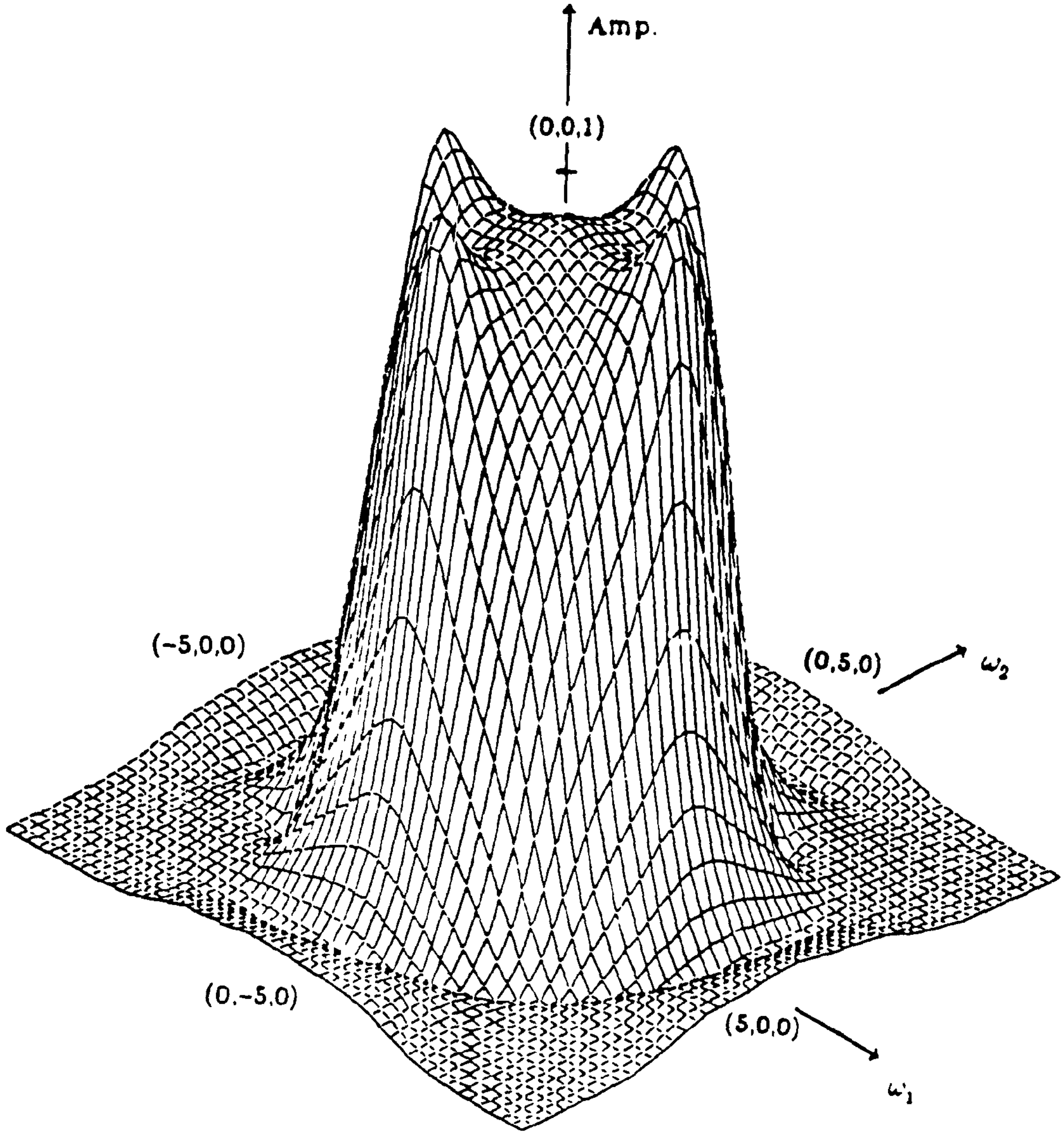


Fig 3.4(a) Magnitude-Frequency Response of the 2-D Lowpass Filter of Example 3.1 as Viewed after CCW R(amp.45)R(ω_1,30) of the Object

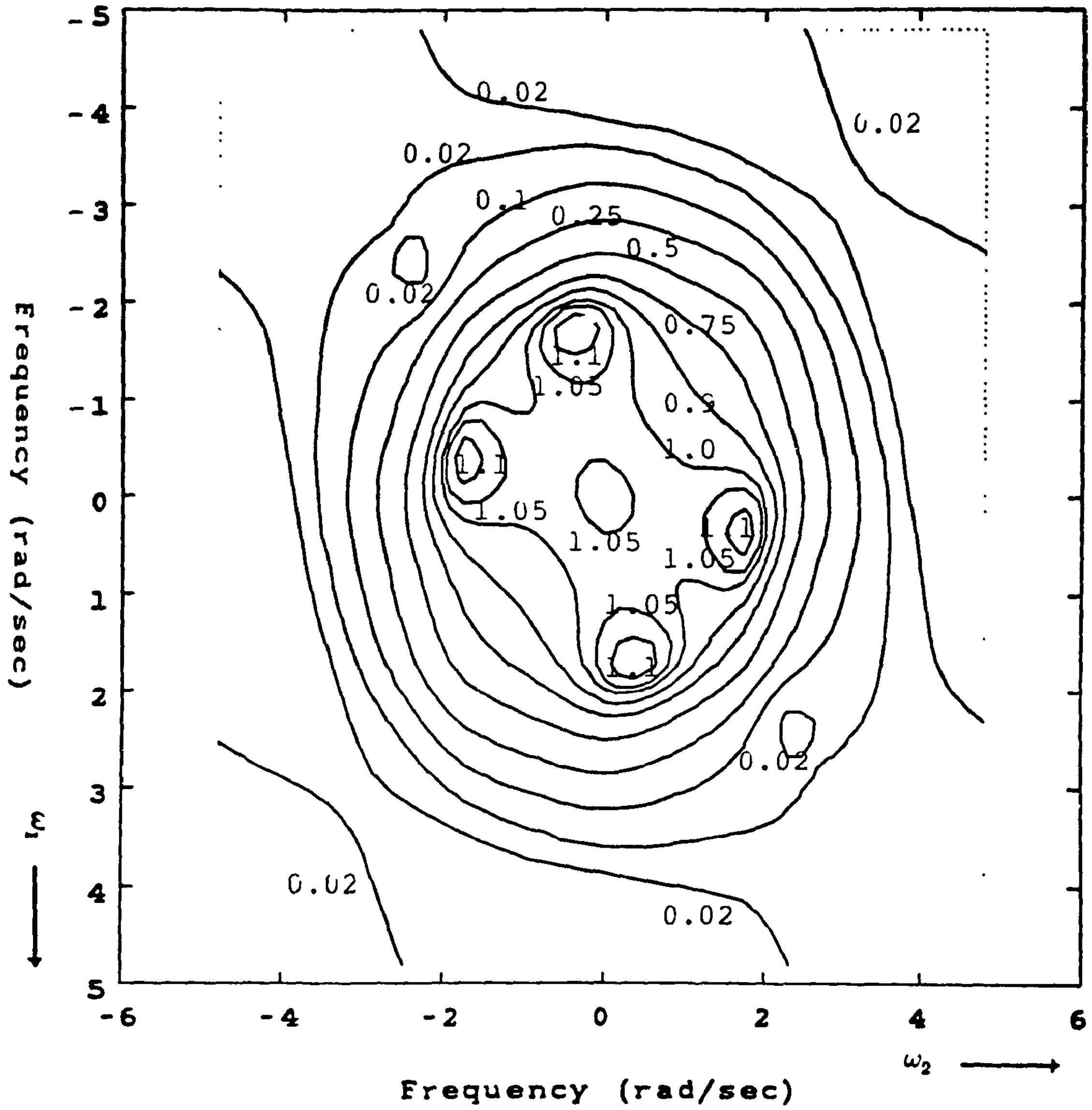


Fig. 3.4(b). Contour Plot of the Magnitude-Frequency Response of the Designed 2-D Lowpass Filter of Example 3 1.

bandpass filter can be obtained by applying the so-called lowpass to bandpass transformation to the prototype lowpass filter. On the other hand, if only the magnitude characteristics are of concern, the transfer function of the digital filter can be derived from that of the analog filter by the use of a simple bilinear transformation. Lowpass to bandpass transformation is, however, not applicable in 2-D case. This example, therefore, considers a direct design of 2-D recursive bandpass digital filter in which only the magnitude characteristics are of concern. That is,

$$|H_I(e^{j\theta_{1m}}, e^{j\theta_{2n}})| = \begin{cases} 0.0 & \text{for } 0.0 \leq \rho(m,n) \leq 0.5 \text{ rad/sec.} \\ 1.0 & \text{for } 1.5 \leq \rho(m,n) \leq 2.5 \text{ rad/sec.} \\ 0.0 & \text{for } 3.5 \leq \rho(m,n) \leq \frac{\omega_s}{2} = 5.0 \text{ rad/sec.} \end{cases}$$

The optimization starts with 21×41 sampling points using squared error criterion. The algorithm converged to a value of coefficients as shown in Table 3.2(a) and the error performance given Table 3 7(p. 66).

We considered a second bandpass filter with double passband edge. That is,

$$|H_I(e^{j\theta_{1m}}, e^{j\theta_{2n}})| = \begin{cases} 0.0 & \text{for } 0.0 \leq \rho(m,n) \leq 0.5 \text{ rad/sec.} \\ 1.0 & \text{for } 1.5 \leq \rho(m,n) \leq 3.5 \text{ rad/sec.} \\ 0.0 & \text{for } 4.5 \leq \rho(m,n) \leq \frac{\omega_s}{2} = 5.0 \text{ rad/sec.} \end{cases}$$

The results of the optimization for the filter with $R_1 = 1.5$ rad/sec and $R_2 = 3.5$ rad/sec were with the coefficients given in Table 3 2(b) and the error performance given Table 3 7(p. 66).

Table 3.2(a) Values of the Parameters of the Designed 2-D Filter in Example 3.2

Parameters of the Numerator			
p_{00}=−2.293084e +01	p_{01}= 4.641385e +01	p_{02}= 1.987760e +01	p_{03}= 8.527046e −03
p_{10}= 4.322579e +01	p_{11}=−5.549869e +01	p_{12}= 1.793286e −01	p_{13}=−1.153886e +00
p_{20}= 2.206771e +01	p_{21}=−4.479763e −01	p_{22}= 4.468196e +00	p_{23}=−2.140576e −01
p_{30}=−5.361302e −02	p_{31}=−1.184249e +00	p_{32}=−1.819978e −01	p_{33}=−5.415111e −03

Parameters of the Denominator	
Section 1	Section 2
a_{111}=−1.24729471e +00	a_{112}=−4.64810383e −01
a_{121}=−1.25926168e −02	a_{122}= 1.97461771e +00
a_{131}= 1.20074329e +00	a_{132}=−7.11654643e −01
a_{211}= 1.03247064e +00	a_{212}=−1.01923858e +00
a_{221}= 4.71150997e −01	a_{222}=−1.18940865e +00
a_{231}=−1.71229922e −01	a_{232}=−1.07289307e +00
a_{311}= 3.80932058e −01	a_{312}= 2.22982250e −01
a_{321}= 1.97604374e −01	a_{322}= 6.81935161e −01
a_{331}= 1.96746833e +00	a_{332}=−4.17582961e −01
g_{121}= 9.16873272e −01	g_{122}=−9.89043102e −01
g_{131}=−6.67321434e −0!	g_{132}=−1.48179001e +00
g_{231}= 1.20481638e +00	g_{232}= 5.76970474e −01

Table 3.2(b) Values of the Parameters of the Designed 2-D Filter in Example 3.2

Parameters of the Numerator			
p_{00}=−1.462428e +01	p_{01}= 6.510615e +01	p_{02}= 2.057257e +01	p_{03}= 3.401620e −02
p_{10}= 6.765675e +01	p_{11}=−1.246397e +01	p_{12}= 3.014989e +00	p_{13}= 5.126194e −02
p_{20}= 2.069120e +01	p_{21}= 3.022838e +00	p_{22}= 1.948324e +00	p_{23}= 5.058117e −03
p_{30}= 3.333198e −02	p_{31}= 5.610374e −02	p_{32}= 2.767369e −03	p_{33}= 1.231896e −02

Parameters of the Denominator	
Section 1	Section 2
a_{111}=−9.09836960e −01	a_{112}=−5.46202922e −01
a_{121}= 3.70803181e −02	a_{122}= 2.05680810e +00
a_{131}= 6.63005100e −01	a_{132}=−4.02716029e −01
a_{211}= 7.25648393e −01	a_{212}=−7.01500934e −01
a_{221}= 2.98601886e −01	a_{222}=−1.27585196e +00
a_{231}= 1.56720394e −01	a_{232}=−8.82196919e −01
a_{311}= 5.49829269e −01	a_{312}=−2.40744238e −02
a_{321}=−1.92182765e −01	a_{322}= 7.01027502e −01
a_{331}= 1.56184029e +00	a_{332}=−4.03956926e −01
g_{121}= 9.67914870e −01	g_{122}=−1.02722483e +00
g_{131}=−6.24083673e −01	g_{132}=−1.52374697e +00
g_{231}= 1.53201499e +00	g_{232}= 8.66564145e −01

3.2.2.3. Design Example 3

The high spatial frequency components of an image, related to the sharp brightness transitions, have an effect on our appreciation of the edge detail in a picture. Bearing in mind that these frequencies could be isolated by the application of highpass filters, we now consider the problem of designing a 2-D highpass recursive digital filter with the following specifications:

$$|H_I(e^{j\theta_{1m}}, e^{j\theta_{2n}})| = \begin{cases} 0.0 & \text{for } 0.0 \leq \rho(m,n) \leq 2.0 \text{ rad/sec.} \\ 1.0 & \text{for } 3.0 \leq \rho(m,n) \leq \frac{\omega_s}{2} = 5.0 \text{ rad/sec.} \end{cases}$$

In this example, when random initial values were given for the optimization parameters, the convergence was obtained with the coefficients shown in Table 3.3 and the error performance shown in Table 3.7(p. 66).

3.2.2.4. Design Example 4

Digital fan or velocity filters are well known as practical 2-D digital filters in geophysical industry. These filters are of the type as shown in Fig. 3.5 and can be used for the processing of seismic-data that contains valuable information about layers. The technique for generating stable filters discussed earlier is used to produce fan filters of order 3×3 in the following. In this example, two fan filters are considered:

i) First a fan filter with a total angular passband width of $\theta = 45^o$ with the following specifications is considered,

$$|H_I(e^{j\theta_{1m}}, e^{j\theta_{2n}})| = \begin{cases} 1.0 & \text{for } 0.0 \leq |\theta| \leq \frac{\pi}{8} \text{ rad/sec.} \\ 0.0 & \text{for } \frac{\pi}{4} \leq |\theta| \leq \frac{\pi}{2} \text{ rad/sec.} \end{cases}$$

Table 3.3 Values of the Parameters of the Designed 2-D Filter in Example 3.3

Parameters of the Numerator			
p_{00}= 5.152210e +01	p_{01}=−1.063662e +02	p_{02}= 1.451416e +01	p_{03}=−5.332539e +01
p_{10}=−1.119699e +02	p_{11}=−1.106526e +02	p_{12}= 6.113085e +01	p_{13}=−2.477476e +01
p_{20}= 7.862729e +00	p_{21}= 7.123247e +01	p_{22}=−4.678191e +01	p_{23}= 7.632517e +00
p_{30}=−6.104445e +01	p_{31}=−3.067774e +01	p_{32}= 7.695174e +00	p_{33}=−8.553854e +00

Parameters of the Denominator	
Section 1	Section 2
a_{111}= 1.66974190e +00	a_{112}= 1.72448553e +00
a_{121}=−1.54945542e +00	a_{122}=−1 70887345e +00
a_{131}=−9.06602817e -01	a_{132}= 1.00308205e +00
a_{211}=−1.38740988e +00	a_{212}= 2.32759846e +00
a_{221}=−8.86922270e -01	a_{222}=−2.45504272e +00
a_{231}=−1.18089931e +00	a_{232}=−2.10505132e -01
a_{311}=−1.97851539e +00	a_{312}= 1.78740843e +00
a_{321}= 9.98068048e -01	a_{322}=−2.12443456e +00
a_{331}= 1.12041977e +00	a_{332}= 1.36010209e +00
g_{121}=−2.30094491e +00	g_{122}=−2.20534576e +00
g_{131}=−5.24549129e -02	g_{132}= 4.21013609e +00
g_{231}= 2.21815819e +00	g_{232}=−4.41929483e +00

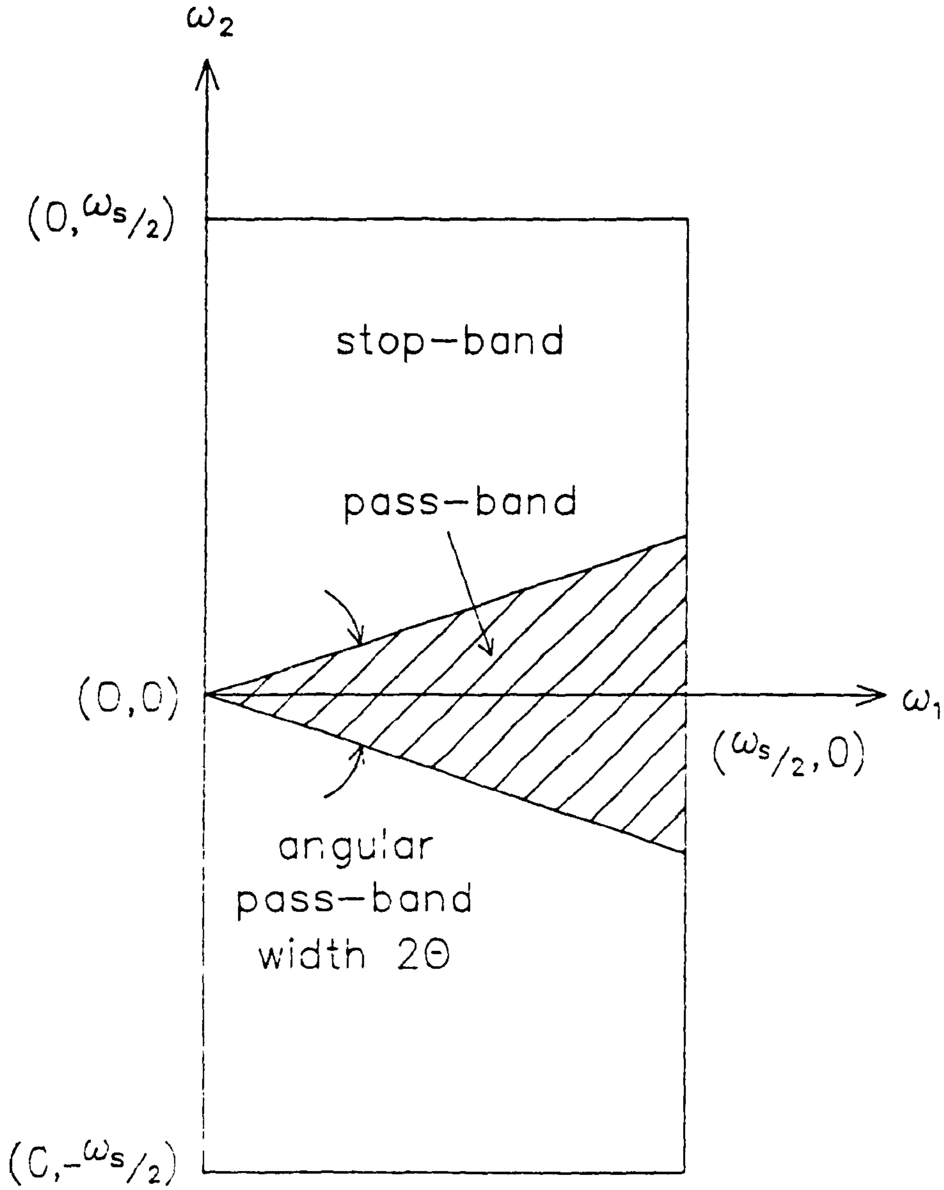


Fig. 3.5. Fan Filter Response.

with $\omega_s = 2\pi$ rad/sec.

The final design is reached with the coefficients given in Table 3.4(a), and the error performance shown in Table 3.7(p. 66).

ii) A 90-degree fan filter with the following specifications is considered,

$$|H_I(e^{j\theta_{1m}}, e^{j\theta_{2n}})| = \begin{cases} 0.0 & \text{for } 0.0 \leq |\theta| \leq \frac{\pi}{8} \text{ rad /sec.} \\ 1.0 & \text{for } \frac{\pi}{4} \leq |\theta| \leq \frac{\pi}{2} \text{ rad /sec.} \end{cases}$$

with $\omega_s = 2\pi$ rad/sec.

The final design yielded the coefficients given in Table 3.4(b), and the error performance shown in Table 3.7(p. 66).

3.2.2.5. Design Example 5 [8, 82-83]

The Laplacian edge enhancement produces an output where high spatial frequency components such as edges are highly accentuated, and low spatial frequency components are attenuated sharply. Therefore, they can be generated to be used in the extraction of object edges, or boundaries. As a design example, consider the following specifications

$$|H_I(e^{j\theta_{1m}}, e^{j\theta_{2n}})| = \sqrt{\frac{m^2+(20-n)^2}{800}}, \text{ for } m = 0,20, \ (n = 0,40)$$

In this example, the convergence was obtained with the coefficients shown in Table 3.5 and the error performance shown in Table 3.7(p. 66).

3.2.2.6. Design Example 6 [9, 14]

A scene, referred to as an image, is a 2-D light intensity function whose magnitude at spatial coordinates (x, y) gives the intensity (brightness) of the image at that point. We can express an image, denoted by $f(x, y)$, as the product of two components.

Table 3.4(a) Values of the Parameters of the Designed 2-D Filter in Example 3.4

Parameters of the Numerator			
p_{00}= 9.973097e −01	p_{01}= 2.108767e −01	p_{02}=−2.444763e −01	p_{03}= 3.193882e −04
p_{10}=−2.321696e +00	p_{11}= 2.301058e +01	p_{12}= 1.331884e +00	p_{13}= 3.220717e −01
p_{20}=−7.277292e +01	p_{21}=−9.721841e −01	p_{22}=−1.388008e +01	p_{23}= 1.931532e −02
p_{30}=−4.640635e +01	p_{31}= 3.335079e +01	p_{32}=−3.723885e +00	p_{33}= 3.936199e −01

Parameters of the Denominator	
Section 1	Section 2
a_{111}=−4.71512047e +00	a_{112}=−3.99348616e +00
a_{121}= 6.30271125e +00	a_{122}=−3.17635745e −01
a_{131}=−3.95295848e +00	a_{132}= 1.38853726e +00
a_{211}=−1.27869595e +01	a_{212}= 5.05170118e +00
a_{221}=−3.29558904e +00	a_{222}=−3.01577110e +00
a_{231}=−1.35038049e +00	a_{232}= 1.88135457e +00
a_{311}=−1.26750828e +01	a_{312}=−8.85024423e −01
a_{321}= 4.58088161e +00	a_{322}=−1.48446166e +00
a_{331}=−4.95589529e +00	a_{332}= 2.01670183e +00
g_{121}= 4.45364462e +00	g_{122}=−6.40477019e +00
g_{131}=−1.96564491e +00	g_{132}= 7.01373816e +00
g_{231}=−1.01769391e +00	g_{232}=−3.21977438e +00

Table 3.4(b) Values of the Parameters of the Designed 2-D Filter in Example 3.4

Parameters of the Numerator			
p_{00}=−7.446010e +01	p_{01}= 1.050644e +02	p_{02}= 3.590641e +01	p_{03}= 1.346326e +01
p_{10}= 8.503278e −01	p_{11}=−1.372691e +01	p_{12}= 2.292969e +01	p_{13}= 3.446465e +00
p_{20}=−1.534959e +01	p_{21}= 7.400303e +00	p_{22}=−1.176962e +01	p_{23}=−7.560180e −01
p_{30}=−2.969921e −01	p_{31}=−4.363617e +00	p_{32}= 1.043172e −02	p_{33}=−2.985303e +00

Parameters of the Denominator	
Section 1	Section 2
a_{111}=−7.54304826e −01	a_{112}= 2.53382151e +00
a_{121}=−4.18409634e +00	a_{122}= 5.49097694e +00
a_{131}=−5.17420741e −01	a_{132}=−4.26956408e −01
a_{211}= 6.44551086e −01	a_{212}=−1.19965966e +00
a_{221}= 3.38531529e +00	a_{222}= 9.48253083e −01
a_{231}=−5.91097304e −01	a_{232}=−2.31834714e +00
a_{311}= 6.81642955e −01	a_{312}=−1.47496018e +00
a_{321}= 5.32478820e +00	a_{322}=−2.15836893e +00
a_{331}=−2.79900850e −01	a_{332}=−3.28526279e −01
g_{121}=−3.04888060e +00	g_{122}=− 2.40751909e −01
g_{131}=−3.33900991e −01	g_{132}=−3.40457437e −01
g_{231}= −2.87217787e −01	g_{232}=− 1.63814961e +00

Table 3.5 Values of the Parameters of the Designed 2-D Filter in Example 3.5

Parameters of the Numerator			
p_{00}= 5.384142e +00	p_{01}= 1.999999e +01	p_{02}= 1.715091e +00	p_{03}=–9.904415e -01
p_{10}= 1.542249e +01	p_{11}= 1.279347e +01	p_{12}= 7.303106e +00	p_{13}= 2.563506e -01
p_{20}= 2.745601e +00	p_{21}= 6.961423e +00	p_{22}= 8.628288e +00	p_{23}= 6.894803e -01
p_{30}=–1.562525e +00	p_{31}= 9.880482e -01	p_{32}= 7.057480e -01	p_{33}= 7.853305e -01

Parameters of the Denominator	
Section 1	Section 2
a_{111}= 5.96732081e -01	a_{112}= 7.15166255e -01
a_{121}= 6.54507396e -01	a_{122}=-2.02025377e -01
a_{131}= 1.62559456e +00	a_{132}= 2.81597262e -01
a_{211}= 6.37155976e -03	a_{212}= 1.06158411e +00
a_{221}= 8.33129379e -01	a_{222}= 5.31816011e -01
a_{231}=-1.33437218e +00	a_{232}= 8.77104025e -01
a_{311}= 4.14772711e -01	a_{312}=-4.96632719e -01
a_{321}=-5.92286218e -01	a_{322}= 3.20416791e -01
a_{331}=-2.61889405e -01	a_{332}= 9.80094063e -01
g_{121}= 1.11914659e +00	g_{122}=-1.07869180e +00
g_{131}= 2.14999721e +00	g_{132}=-3.56080950e -02
g_{231}= 2.08025686e +00	g_{232}=-9.67980553e -01

These are (i) an illumination function (the amount of source light incident on the scene being viewed), denoted by $i(x,y)$, and (ii) a reflectance function (the amount of light reflected by the objects in the scene), denoted by $r(x,y)$. Formally

$$f(x,y)=i(x,y)\cdot r(x,y). \tag{3.11}$$

Two frequently occurring problems in processing $f(x,y)$ are dynamic-range compression and contrast enhancement. The dynamic range is treated as a problem concerning the illumination function while the contrast enhancement is treated as a problem concerning the reflectance function. That is, the illumination is directly responsible for the dynamic range encountered in an image whereas the edges of objects in an image contribute only to the reflectance component. Thus we can separate the illumination and reflectance functions, modifying each with different parameters, and then recombining to form the modified image. With this approach, the modified illumination-reflectance function is given as

$$g(x,y)=[i(x,y)]^{\gamma_i}[r(x,y)]^{\gamma_r} \tag{3.12}$$

where γs are real constants. When $\gamma_i<1$ dynamic range in the brightness is reduced, and when $\gamma_r>1$ contrast is increased, lending more sharpness to the edges of the image $f(x,y)$.

With this objective in mind, an indirect frequency domain procedure can be applied to (3.12) in order to operate separately on the frequency components that $i(x,y)$ and $r(x,y)$ possess. The illumination and reflectance components of an image are characterized by slow and rapid spatial coordinate variations, respectively. Thus it is assumed that the low spatial frequencies of the Fourier transform of an image, in the logarithm, are associated with illumination, and the high spatial frequencies with reflectance. With this assumption, and with the desire to process the image according to (3.12), a filter can be used to multiply the low frequencies by γ_i and the high frequencies by γ_r. A simultaneous control over the dynamic range and contrast can be achieved by using a homomorphic filter whose cross sectional frequency characteristic is shown in Fig. (3.6). With a low frequency gain of $\gamma_l < 1$, and a high frequency gain of $\gamma_r > 1$, the filter function will, respectively, tend to compress the dynamic range and enhance the contrast. In the design example to be presented here, the low frequency gain of the filter is chosen to be 0.5, and the high frequency gain is chosen to be 2.0, corresponding to a choice of

$$| H_I(e^{j\theta_{1m}}, e^{j\theta_{2n}})| = \begin{cases} \gamma_l = 0.5 & \text{for } \rho(m,n) = 0.0 \text{ rad/sec.} \\ \gamma_r = 2.0 & \text{for } 2.0 \le \rho(m,n) \le \frac{\omega_s}{2} = 5.0 \text{ rad/sec.} \end{cases}$$

The final design is reached with the coefficients given in Table 3.6, and the error performance shown in Table 3.7(p. 66).

3.2.3. Magnitude and Group Delay Functions Approximation

This section proposes an iterative method of designing stable 2-D (1-D as a special case) recursive digital filters satisfying prespecified magnitude frequency response with constant group delay. This has proven [13] to be of better use in image processing applications than the case when only the magnitude specifications are considered. We assume that the filter is in the form of (3 7). The filter coefficients are calculated to approximate both the magnitude and group delay characteristics simultaneously.

The frequency response of the filter may be expressed as

$$H_D(e^{j\theta_{1m}}, e^{j\theta_{2n}}, \mathbf{c}) = | H_D(e^{j\theta_{1m}}, e^{j\theta_{2n}}, \mathbf{c})| \, e^{j\phi(\theta_{1m}, \theta_{2n}, \mathbf{c})},$$

and the group delay functions as

$$\tau_{D_i}(\omega_{1m}, \omega_{2n}, \mathbf{c}) = \frac{-\partial\phi(\theta_{1m}, \theta_{2n}, \mathbf{c})}{\partial\theta_i} \cdot \frac{\partial\theta_i}{\partial\omega_i}, \quad i = 1, 2,$$

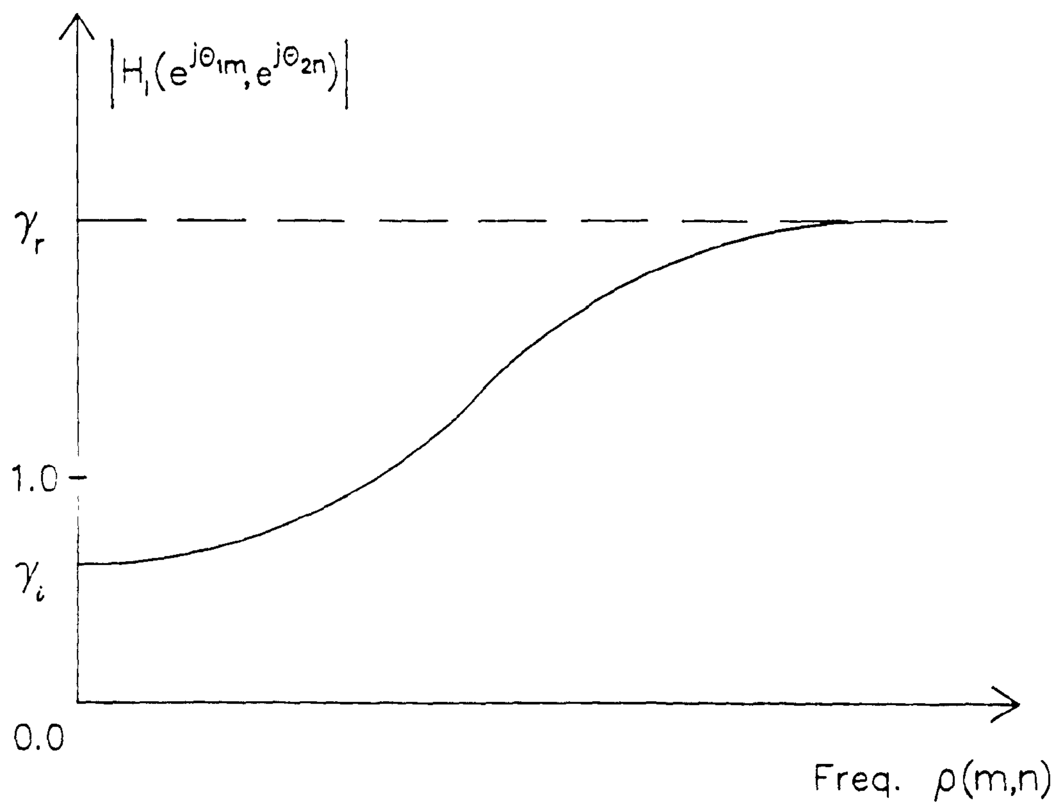


Fig. 3.6. Frequency Characteristics used for Homomorphic Filter when Simultaneous Dynamic-Range Compression and Contrast Enhancement are to be Achieved.

Table 3.6 Values of the Parameters of the Designed 2-D Filter in Example 3.6

Parameters of the Numerator			
p_{00}=−1.033655e +02	p_{01}= 1.887656e +01	p_{02}= 6.876364e +01	p_{03}=−2.004793e +01
p_{10}= 6.358050e +01	p_{11}=−1.559820e +02	p_{12}= 5.791468e +01	p_{13}= 2.828292e +01
p_{20}= 5.579354e +01	p_{21}= 9.091095e +01	p_{22}=−7.273340e +01	p_{23}= 4.147347e +01
p_{30}=−2.464102e +01	p_{31}= 1.376450e +01	p_{32}= 4.680384e +01	p_{33}=−2.963128e +01

Parameters of the Denominator	
Section 1	Section 2
a_{111}=−1.67439022e +00	a_{112}= 1.52082699e −01
a_{121}= 7.65991421e −01	a_{122}= 4.90399743e −02
a_{131}=−2.55919860e +00	a_{132}= 1.25425754e −01
a_{211}=−9.50162195e −01	a_{212}= 2.94203754e +00
a_{221}= 3.44489352e +00	a_{222}=−3.55097155e +00
a_{231}=−1.70968554e +00	a_{232}=−3.19021697e −01
a_{311}=−4.36444393e −01	a_{312}=−1.01510490e +00
a_{321}= 1.53467806e +00	a_{322}=−2.04407262e −01
a_{331}=−3.62526366e −01	a_{332}= 1.90789104e +00
g_{121}=−2.16690658e +00	g_{122}=−4.40214290e +00
g_{131}= 3.03771610e −01	g_{132}= 4.95720244e +00
g_{231}= 4.68487331e +00	g_{232}=−5.70527112e +00

Table 3.7

Magnitude Response Performance for Different Design Examples

Type of 2-D Filter	Value of Overall Error Function, F (Sum of Squared Errors) 5 Iterations	10 Iterations	After Convergence	Maximum Error in Passband (dB)	Maximum Error in Stopband (dB)	Total CPU Time (Sec.)
Lowpass (1)	0.138159e +03	0.557121e +02	0.326567e +01	-0.419106e +01	-0.834124e +01	219
Lowpass (2)	0.256957e +02	0.197990e +02	0.320573e +01	-0.215388e +00	-0.133357e +02	963
Bandpass (1)	0.124925e +03	0.598731e +02	0.133302e +02	-0.474118e +01	-0.752158e +01	1345
Bandpass (2)	0.389457e +02	0.235438e +02	0.157343e +02	-0.570307e +01	-0.635018e +01	2127
Highpass	0.809451e +02	0.411245e +02	0.638156e +01	-0.340969e +01	-0.977113e +01	896
45-degree Fan	0.558935e +02	0.353719e +02	0.183419e +02	-0.436493e +01	-0.806800e +01	1011
90-degree Fan	0.125251e +03	0.759299e +02	0.203571e +02	-0.627490e +01	-0.577353e +01	1265
Laplacian	0.118714e +02	0.770973e +01	0.218207e +01	-0.174705e +01	-0.147891e +02	205
Homomorphic	0.472529e +02	0.365098e +02	0.616935e +00	-0.310744e +01	-0.104357e +02	606

where

$$\phi(\theta_{1m}, \theta_{2n}, c) = \arg H_D(.) \text{ and } \frac{\partial \theta_i}{\partial \omega_i} = T_i .$$

Let the cost function, $F(c)$, be defined as

$$F(c) = F_M(c) + F_{\tau_1}(c) + F_{\tau_2}(c)$$

where $F_M(c)$ is the error function of the magnitude frequency response given by eqn(3.9) and F_{τ_i} defined as

$$F_{\tau_i}(c) \triangleq \sum_{m \in M_\tau} \sum_{n \in N_\tau} E_{\tau_i}{}^2(c) \quad \text{for } i=1,2$$

with

$$E_{\tau_i}(c) = T_i \cdot \tau_{I_i} - \tau_{D_i}(\omega_{1m}, \omega_{2n}, c) \quad \text{for } i=1,2 \tag{3.13}$$

is the error function for the group delay. $H_I(.)$ and $\tau_{I_i}(.)$ are the desired magnitude and group delay filter characteristics, respectively, defined as

$$H_I(e^{j\theta_{1m}}, e^{j\theta_{2n}}) = \begin{cases} e^{j\phi(\theta_{1m}, \theta_{2n})} & \text{for passband} \\ 0 & \text{otherwise} \end{cases}$$

and

$$\phi(\theta_{1m}, \theta_{2n}) = \arg H_I(.).$$

The value of the ideal group-delay response of the filter is chosen to be equal to the order of the filter ± 1 [34, 84]. We choose $\tau_{I_i}(\theta_{1m}, \theta_{2n}) = 3$. Hence $\tau_{I_i}(\omega_{1m}, \omega_{2n}) = \tau_{I_i}(\theta_{1m}, \theta_{2n}) \cdot T_i = 3T_i$. In this case, the Fletcher-Powell algorithm is used for minimizing the unconstrained nonlinear general squared error, $F(c)$. Gradient of the error function, $F(c)$, with respect to parameters of the vector c is calculated in the same manner as mentioned in Section 3.2.1.

3.2.4. Design Examples

The above procedure provides the facility to design stable 2-D recursive digital filters for numerous image processing applications.

3.2.4.1. Design Example 7

A 2-D lowpass filter with the following specifications is designed,

$$|H_I(e^{j\theta_{1m}}, e^{j\theta_{2n}})| = \begin{cases} 1.0 & \text{for } 0.0 \leq \rho(m,n) \leq 2.0 \text{ rad/sec.} \\ 0.0 & \text{for } 3.0 \leq \rho(m,n) \leq \dfrac{\omega_s}{2} = 5.0 \text{ rad/sec.} \end{cases}$$

The final design is reached with the coefficients given in Table 3.8, and the error performance shown in Table 3.10(p. 71).

3.2.4.2. Design Example 8

In this example, we consider the design of a 2-D bandpass recursive digital filter with the following specifications,

$$|H_I(e^{j\theta_{1m}}, e^{j\theta_{2n}})| = \begin{cases} 0.0 & \text{for } 0.0 \leq \rho(m,n) \leq 0.5 \text{ rad/sec.} \\ 1.0 & \text{for } 1.5 \leq \rho(m,n) \leq 3.5 \text{ rad/sec.} \\ 0.0 & \text{for } 4.5 \leq \rho(m,n) \leq \dfrac{\omega_s}{2} = 5.0 \text{ rad/sec.} \end{cases}$$

The results obtained are the coefficients as shown in Table 3.9, and the error performance shown in Table 3.10(p. 71). The perspective and the contour plots of the magnitude and group delay responses are shown in Figs. 3.7(a-f).

Table 3.8 Values of the Parameters of the Designed 2-D Filter in Example 3.7

Parameters of the Numerator			
p_{00}=-2.484172e +01	p_{01}= 1.835473e +00	p_{02}=-2.022767e +00	p_{03}= 1.085432e -01
p_{10}= 2.352315e +00	p_{11}=-5.150997e +00	p_{12}= 1.241865e +00	p_{13}=-1.880569e -01
p_{20}=-1.986758e +00	p_{21}= 1.227040e +00	p_{22}=-2.666711e -01	p_{23}= 1.122101e -01
p_{30}= 1.265100e -01	p_{31}=-1.798422e -01	p_{32}= 1.196374e -01	p_{33}=-4.078500e -03

Parameters of the Denominator	
Section 1	Section 2
a_{111}= 2.04131571e +00	a_{112}= 2.84374842e +00
a_{121}= 2.68858935e +00	a_{122}=-7.59469059e -01
a_{131}= 1.43597720e +00	a_{132}= 1.18868153e +00
a_{211}= 4.75478418e -01	a_{212}= 1.53917343e +00
a_{221}= 1.66899726e +00	a_{222}=-2.90072912e -01
a_{231}= 1.18413314e -01	a_{232}= 4.91517752e -01
a_{311}=-3.61613314e -01	a_{312}= 1.47409322e +00
a_{321}=-2.16404385e +00	a_{322}=-1.71842180e +00
a_{331}=-5.11550352e -01	a_{332}= 4.25870229e +00
g_{121}= 2.77925181e +00	g_{122}=-1.81040200e +00
g_{131}= 1.66548432e +00	g_{132}= 6.08442601e +00
g_{231}= 1.19245244e +00	g_{232}=-1.17973709e +00

Table 3.9 Values of the Parameters of the Designed 2-D Filter in Example 3.8

Parameters of the Numerator			
p_{00}=-2.698526e +01	p_{01}= 5.737273e +01	p_{02}= 3.085087e +01	p_{03}=-7.060143e -01
p_{10}= 5.974223e +01	p_{11}=-2.114953e +01	p_{12}= 4.574978e +00	p_{13}= 4.803433e -01
p_{20}= 2.874610e +01	p_{21}= 2.631738e +00	p_{22}= 1.304461e +00	p_{23}=-3.800815e -01
p_{30}=-1.003458e +00	p_{31}= 3.818076e -01	p_{32}=-3.902030e -01	p_{33}= 1.914990e -01

Parameters of the Denominator	
Section 1	Section 2
a_{111}=-8.89894466e -01	a_{112}=-3.98490031e -01
a_{121}= 1.73029556e -01	a_{122}= 2.08345813e +00
a_{131}= 7.70392301e -01	a_{132}=-1.04526495e +00
a_{211}= 1.12200385e +00	a_{212}=-5.97528453e -01
a_{221}= 5.01174282e -01	a_{222}=-2.08663127e +00
a_{231}=-4.36602631e -01	a_{232}= 3.98375179e -02
a_{311}= 1.32208488e +00	a_{312}=-1.53059768e -01
a_{321}=-3.27285774e -01	a_{322}= 8.78252657e -01
a_{331}= 8.03951574e -01	a_{332}=-8.80797477e -01
g_{121}= 8.32836965e -01	g_{122}=-2.29208269e +00
g_{131}=-1.28497682e +00	g_{132}= 2.17033504e -02
g_{231}= 2.01994708e +00	g_{232}= 4.43334369e -01

Table 3.10

Magnitude and Group-Delay Responses Performance for Different Design Examples

Type of 2-D Filter	Value of Overall Error Function, F (Sum of Squared Errors)			Max Amp Err in Passband (dB)	Max. Amp Err in Stopband (dB)	Total CPU-Time (Sec.)
	5 Iterations	10 Iterations	After Convergence			
Lowpass: (Amp & Gdly)	Amplitude. 0.135779e +02	Amplitude 0.111457e +02	Amplitude: 0.962936e +01	-0.331190e +01	-0.997815e +01	1895
	Group delay 1 0.209425e +04	Group delay 1: 0.198002e +04	Group delay 1· 0.107747e +01			
	Group delay 2: 0.133905e +04	Group delay 2· 0.139046e +04	Group delay 2: 0.129513e +01			
Bandpass: (Amp. & Gdly.)	Amplitude: 0.184000e +02	Amplitude. 0.192802e +02	Amplitude: 0.116390e +02	-0.386761e +01	-0.888960e +01	3034
	Group delay 1: 0.568265e +03	Group delay 1 0.330335e +03	Group delay 1. 0.163205e +02			
	Group delay 2· 0.574545e +03	Group delay 2: 0.320099e +03	Group delay 2: 0.192124e +02			

Amp. & Gdly. - Amplitude & Group delay; Max. Amp. Err. - Maximum Amplitude Error.

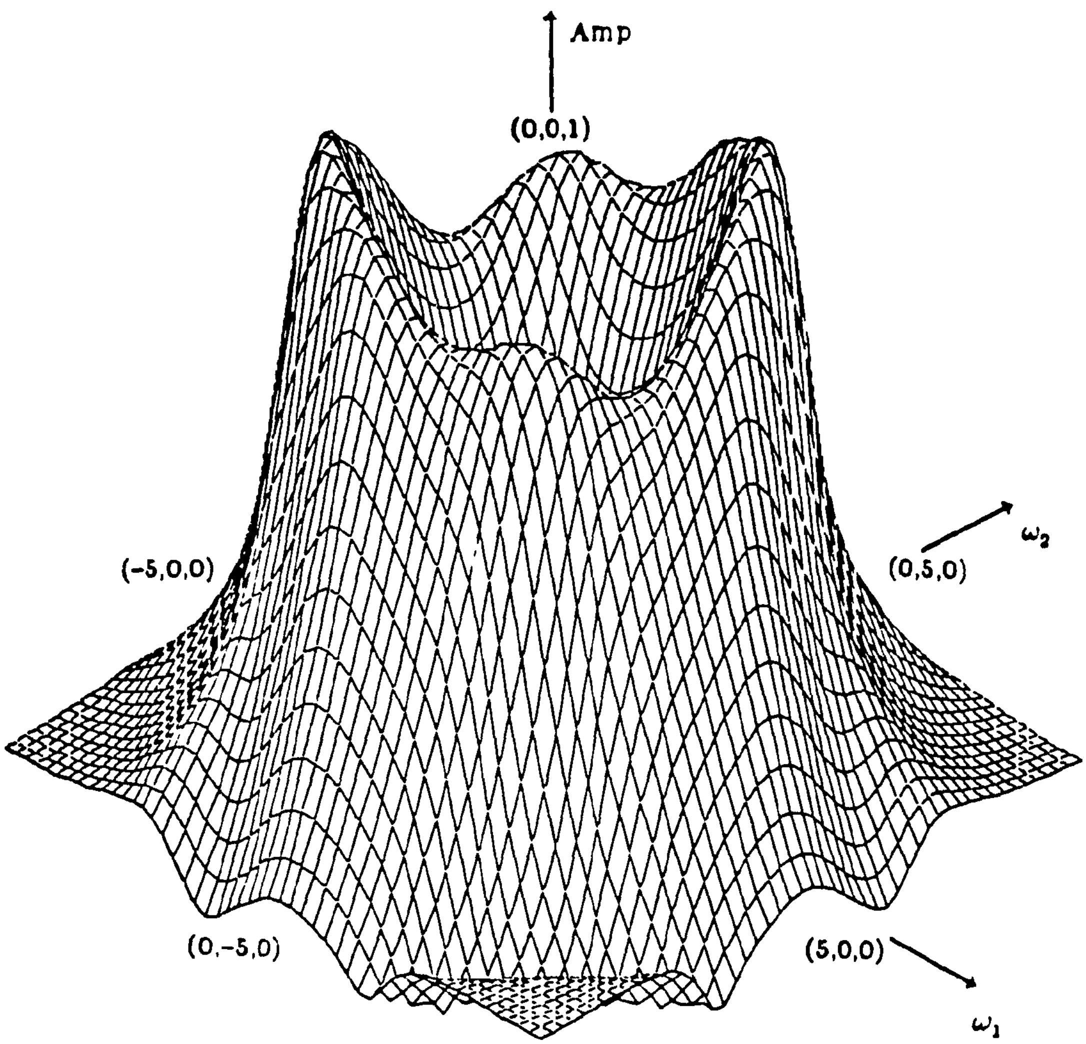


Fig. 3.7(a). Magnitude-Frequency Response of the 2-D Bandpass Filter of Example 3.8 as Viewed after CCW $R(\text{amp},45)R(\omega_1,30)$ of the Object.

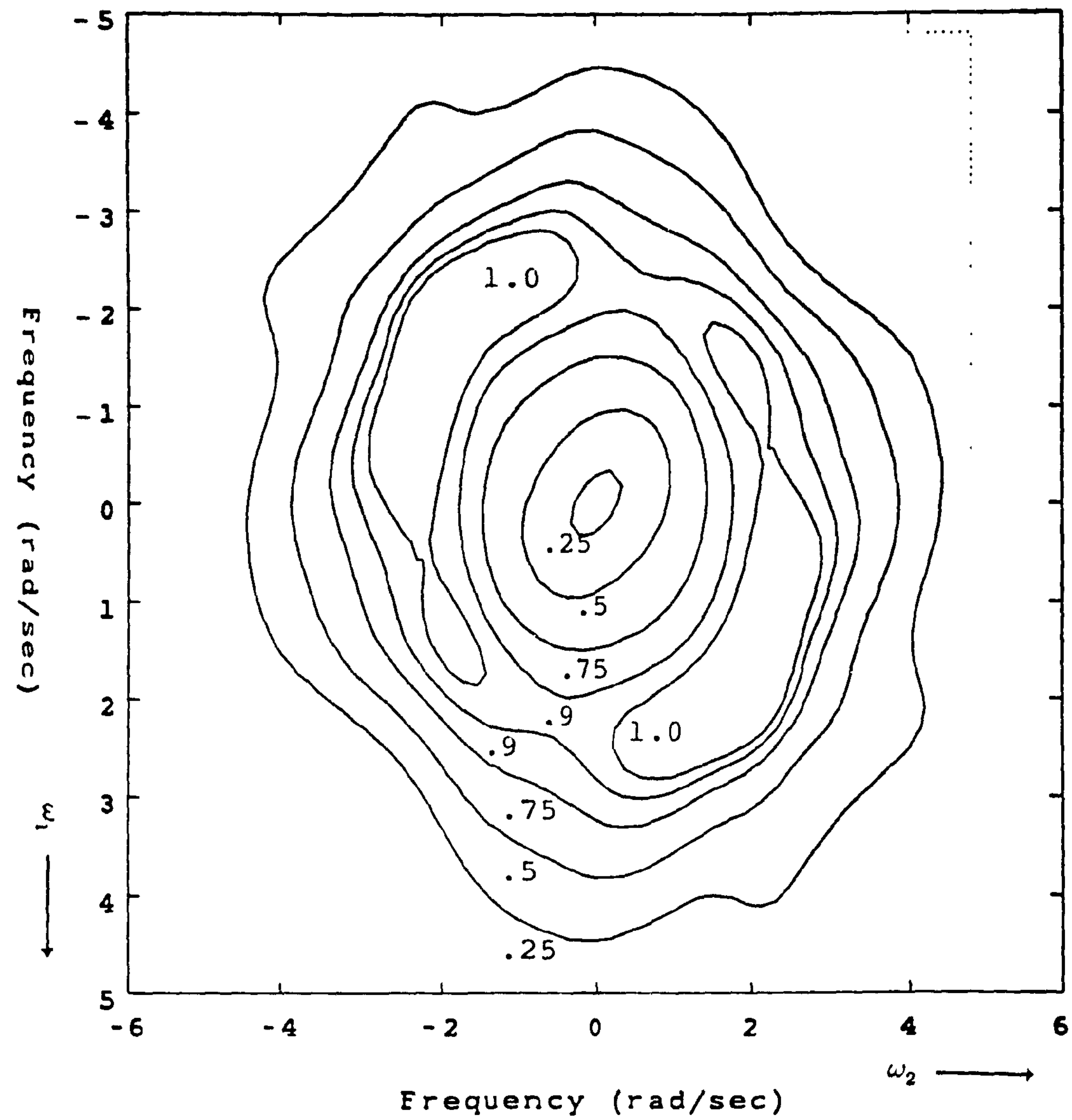


Fig. 3.7(b). Contour Plot of the Magnitude-Frequency Response of the Designed 2-D Bandpass Filter of Example 3.8.

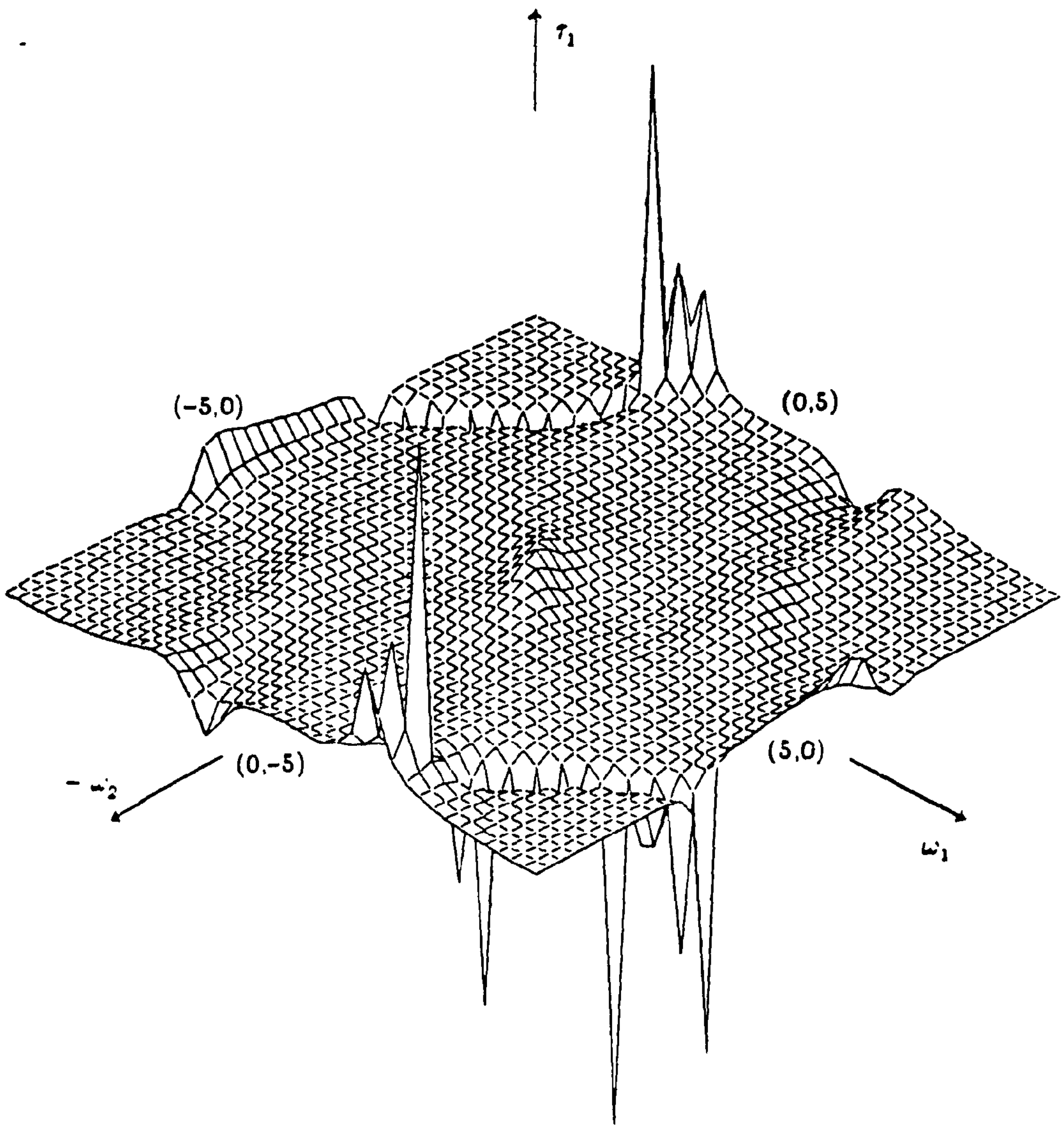


Fig. 3.7(c). Group-Delay, τ_1, Frequency Response of the 2-D Bandpass Filter of Example 3.8 with respect to ω_1 as Viewed after CCW $R\,(\text{amp},45)R\,(\omega_1,30)$ of the Object.

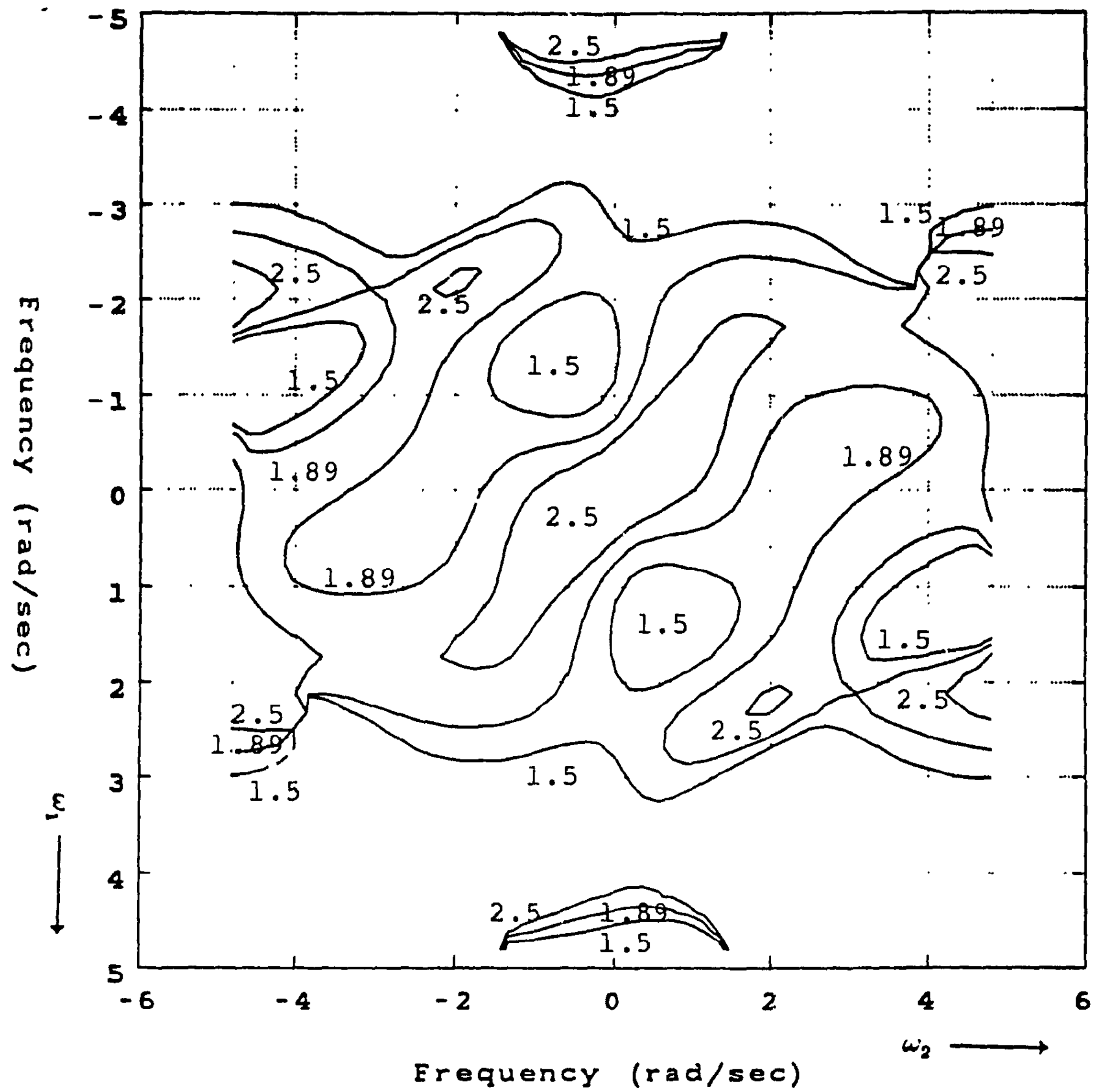


Fig. 3.7(d). Contour Plot of the Group-Delay, τ_1, Frequency Response of the Designed 2-D Bandpass Filter of Example 3.8.

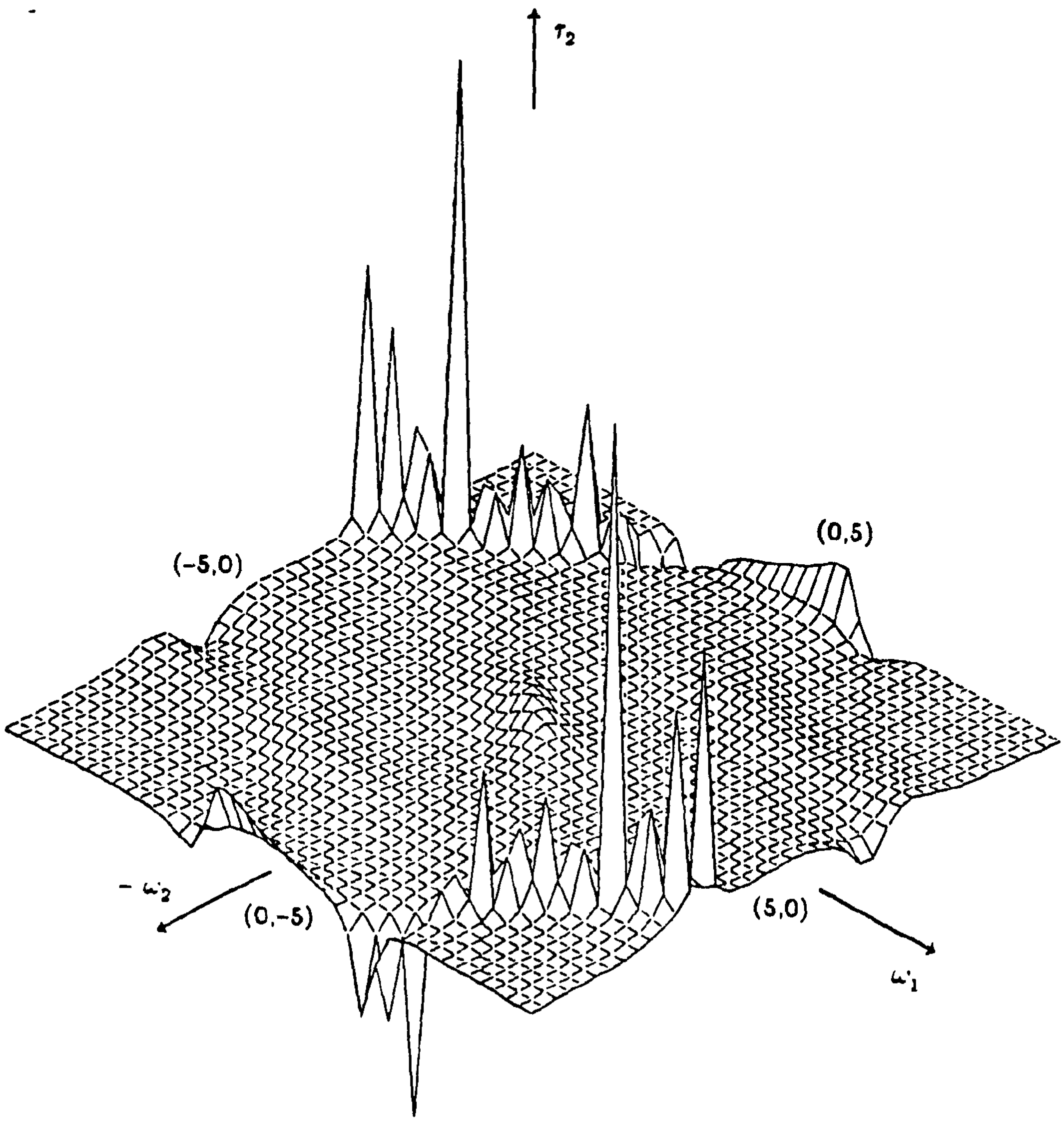


Fig. 3.7(e). Group-Delay, τ_2, Frequency Response of the 2-D Bandpass Filter of Example 3.8 with respect to ω_2 as Viewed after CCW R (amp ,45)R (ω_1,30) of the Object.

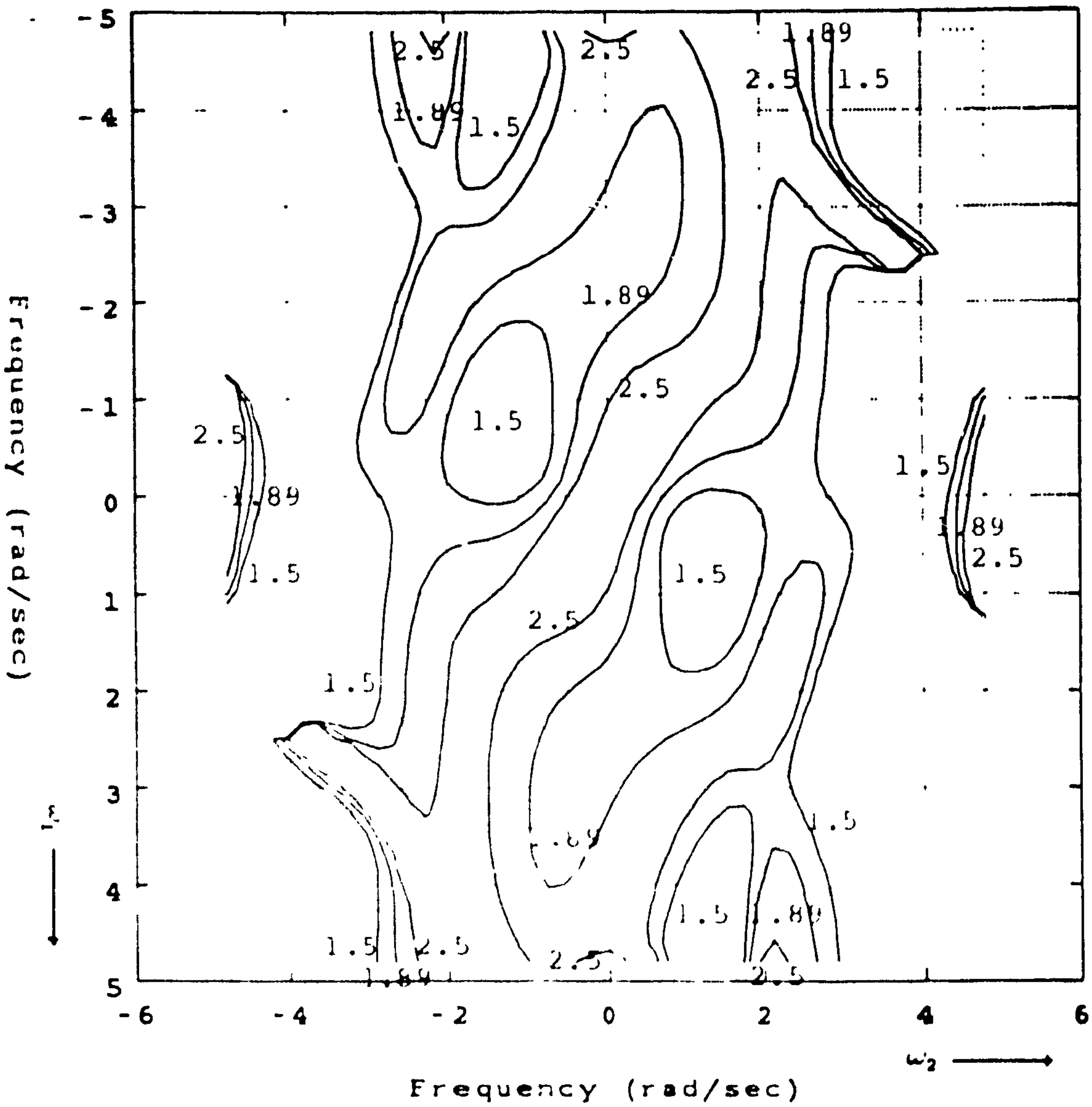


Fig. 3.7(f) Contour Plot of the Group-Delay τ_2 Frequency Response of the Designed 2-D Bandpass Filter of Example 3.8

3.3. SPECIAL CASE

In this section, generation of 1-variable SHP from an n-variable HP may be considered as a special case. Use is made of analog functions in conjunction with an iterative technique based on nonlinear programming in developing 1-D recursive digital filters to approximate prescribed magnitude specifications with constant group delay responses without the need to impose any stability test on the filters.

Consider the 3-variable HP of (2.24) of Chapter II which may be reproduced for convenience as

$$M = (d_{11}\mu_1+d_{21}\mu_2+d_{31}\mu_3+d_{41}\mu_1\mu_2\mu_3)+ \\ [(d_{11}+d_{21}+d_{31})+d_{41}(\mu_2\mu_3+\mu_1\mu_3+\mu_1\mu_2)].$$

By setting $\mu_1=\mu_2=\mu_3=s$ we get,

$$Q(s,a,g) \triangleq M = d_{41}s^3+3d_{41}s^2+(d_{11}+d_{21}+d_{31})s+(d_{11}+d_{21}+d_{31}).$$

where all d's are the same as was given by (2.23b) of Chapter II This is a 1-variable SHP, i.e., a 1-D stable polynomial in variable s which has no zeros in Re $s \geq 0$ The procedure outlined here provides the facility to the design of stable 1-D recursive digital filters satisfying a given magnitude frequency response with constant group delay characteristic.

A frequency domain approximation to the design of a 1-D digital filter, using an L_2-norm (the least mean-square-error), for the magnitude function is given by

$$E_2 \triangleq \| L_2 \| = \sum_{m=1}^{M} \left[| H_D(e^{j\omega_m})| - |H_I(e^{j\omega_m})| \right]^2$$

in conjunction with a nonlinear optimization technique. Here $H_I(e^{j\omega_m})$ is the desired frequency response while $H_D(e^{j\omega_m})$ represents the approximation function The coefficients of the latter are to be calculated to yield minimum L_2-norm.

The design method employs a 1-D analog filter with the overall system function

$$H_a(s) = \frac{\sum_{l=0}^{N} p_l s^l}{\sum_{l=0}^{N} q_l s^l}$$

$$\equiv \frac{P(s, p_l)}{Q(s, q_l)}$$

In the following examples, we assign a third order 1-variable SHP to the denominator of the 1-D prototype analog transfer function while the numerator is left unchanged. We then cascade the resulting transfer function with another section of the same type before the bilinear transformation is applied. Here τ_l is chosen equal to six.

3.3.1. Design Example 9

In this example, we consider the design of a 1-D lowpass recursive digital filter with cutoff frequency equal to one fifth of the Nyquist frequency. The specifications of this filter are as follows

$$H(e^{j\omega_m T}) = e^{j6\omega_m T} \quad \text{for } 0.0 \le \omega_m \le 1.0 \ \text{rad/sec}$$

$$= 0.0 \qquad \text{for } 2.0 \le \omega_m \le \frac{\omega_s}{2} = 5.0 \ \text{rad/sec}.$$

The optimization for the approximation of this filter was started with 21 sampling points using squared error criterion. However, some spikes were noticed in the frequency response after the filter was designed. To eliminate these spikes, the number of sampling points was increased to 31. Now, the algorithm had converged to $F_M = 2.46112905e+00$ and $F_2 = 2.34734120e+00$ with the coefficients as given in Table 3.11 and the filter responses as shown in Figs. 3.8(a-b). It should be noted that the 1-D responses were plotted with 64 frequency points. In this example, the total CPU time was observed to be 163 seconds.

Table 3.11 Values of the Parameters of the Designed 1-D Filter in Example 3.9

Parameters of the Transfer Function	
Section 1	Section 2
a_{01}= −2 27343096e +01	a_{02}= 9.15125126e +00
a_{11}= −2.64210100e −02	a_{12}= −3 37822581e +00
a_{21}= −2.95915506e +00	a_{22}= 1.26974641e −01
a_{31}= 1.05726470e −01	a_{32}= −1.89470390e +01
a_{111}= 2 01682818e +00	a_{112}= 1.55751861e +00
a_{121}=−2.13967930e +00	a_{122}=−1.06829388e +00
a_{131}= 1.84074804e +00	a_{132}=−3.99546490e −01
a_{211}=− 2.34272334e +00	a_{212}=−2.01767886e +00
a_{221}=−1.99589749e +00	a_{222}=−4.59985700e −01
a_{231}= 2.11587757e +00	a_{232}= 2.46090610e +00
a_{311}=−8.20290760e −01	a_{312}=−1.52827310e +00
a_{321}= 1.18765660e +00	a_{322}= 2.43705635e +00
a_{331}= 7.45041000e −03	a_{332}= 2 00568650e +00
g_{121}= 5.56221590e −01	g_{122}=−1.16789891e +00
g_{131}= 2 00576380e −01	g_{132}= 3.06097040e −01
g_{231}=−1.36153780e −01	g_{232}= 2.07270343e +00

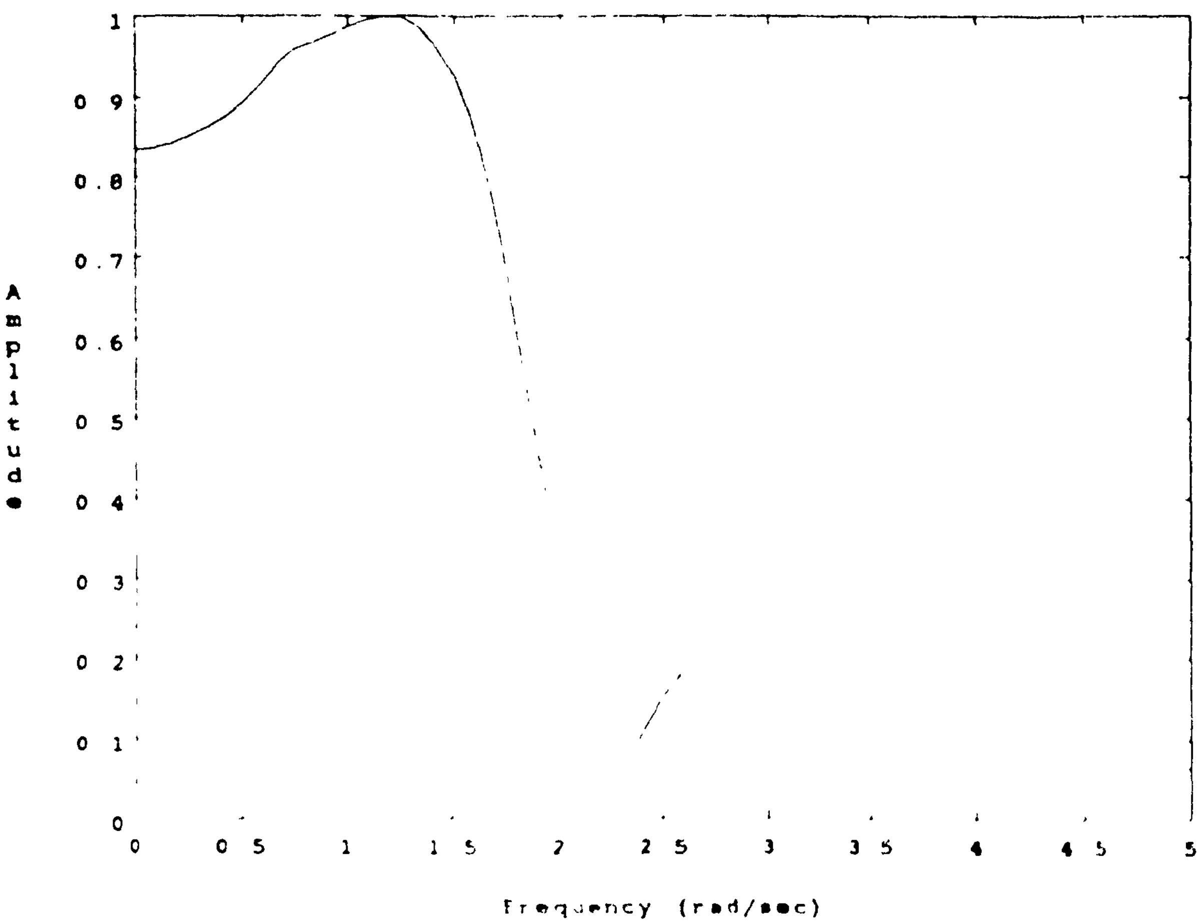


Fig. 3.8(a) Magnitude Frequency Response of the 1-D Lowpass Filter of Example 3.9

Max. Error of the Design = 0.3224654e+00

Max. Error from the Top(DB) = 0.3481371e+01

Max. Error from the Bottom(DB) = 0.0830337e+01

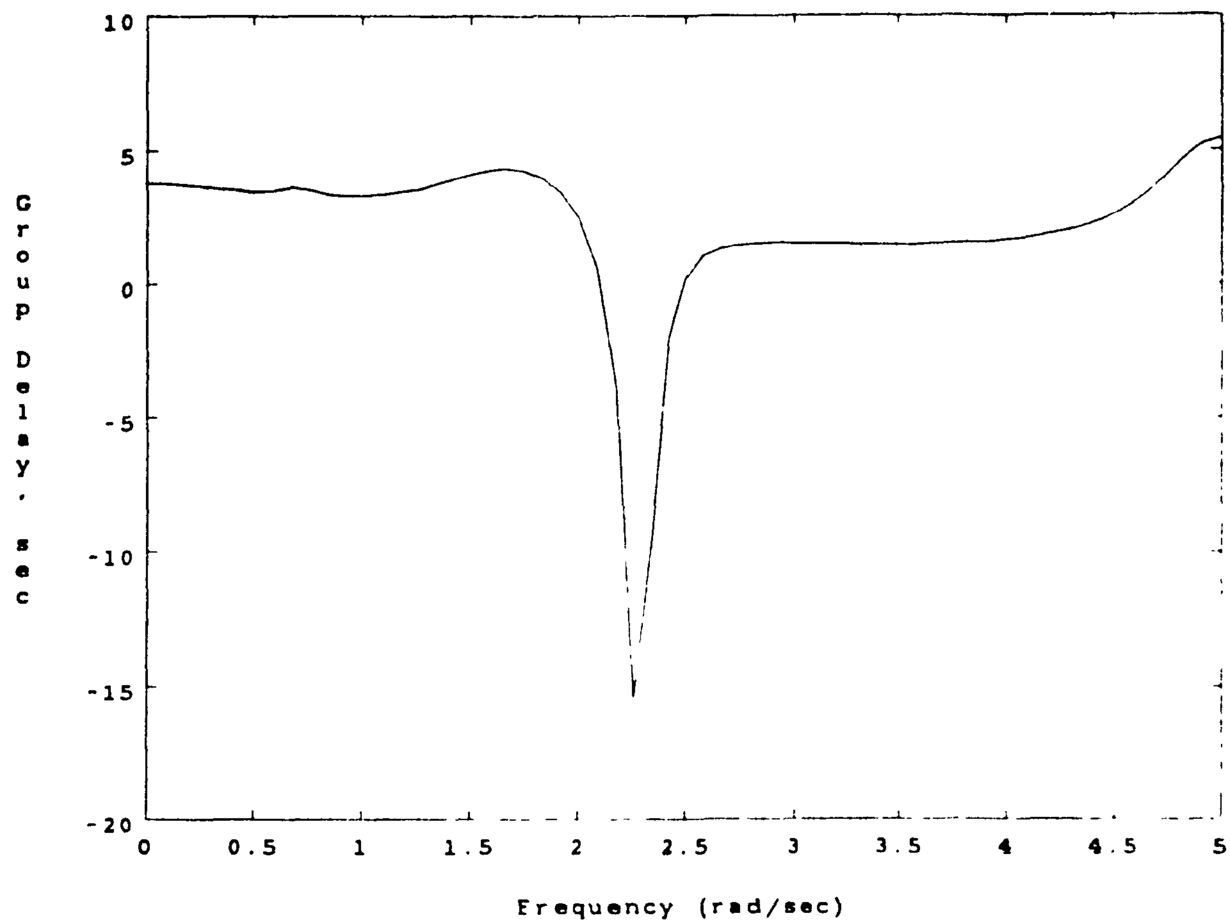


Fig. 3.8(b). Group Delay, τ, Frequency Response of the 1-D Lowpass Filter of Example 3.9

This is a lowpass filter, whereas bandpass filters having similar characteristics may be more important in practice. As is well known, the transfer function of a 1-D bandpass filter can be obtained by applying the lowpass to bandpass transformation to the lowpass prototype filter if only the magnitude characteristics is of concern. On the other hand, the transfer function of the corresponding digital filter can be derived from that of the analog filter by the use of a simple bilinear transformation. These transformations are, however, not applicable to the delay characteristics. Therefore, it is difficult to derive bandpass filters in which both the magnitude and delay characteristics are of interest. The following example considers a direct design of 1-D bandpass filter satisfying both the magnitude and delay characteristics in digital domain.

3.3.2. Design Example 10

Consider a 1-D bandpass digital filter with the following specifications

$$H_I(e^{j\omega T}) = \begin{cases} 0.0 & \text{for } 0.0 \le \omega_m \le 0.5 \text{ rad/sec.} \\ e^{-j\omega_m T} & \text{for } 2.0 \le \omega_m \le 3.0 \text{ rad/sec.} \\ 0.0 & \text{for } 1.5 \le \omega_m \le \frac{\omega_s}{2} = 5.0 \text{ rad/sec.} \end{cases}$$

The algorithm converged to a pre-specified value of $f_M = 2.01085865e+00$ and $f = 4.39578962e-01$ with the coefficients as given in Table 3.12 and the filter responses as shown in Figs. 3.9(a-b). In this example the total CPU time was observed to be 198 seconds.

3.4. FILTERS WITH INTEGER COEFFICIENTS

Implementation of the digital filters in hardware makes it necessary to limit the number of bits representing various coefficients of the transfer function. This is because the coefficient word length has an effect on the cost as well as speed of the filter [65].

Table 3.12 Values of the Parameters of the Designed 1-D Filter in Example 3.10

Parameters of the Transfer Function	
Section 1	Section 2
$a_{01}=$ 7.83346605e +00	$a_{02}=$ 2.66604648e +00
$a_{11}=$ -2.20104554e +00	$a_{12}=$ 1.64832911e -01
$a_{21}=$ -3.16957562e -01	$a_{22}=$ -3.11197161e +00
$a_{31}=$ -1.73756889e -01	$a_{32}=$ -7 80589469e -01
$a_{111}=$-1.91354605e +00	$a_{112}=$-6.60962295e -01
$a_{121}=$-1.36949913e +00	$a_{122}=$ 1.83864195e +00
$a_{131}=$-6 15670090e -01	$a_{132}=$-4.91490354e -01
$a_{211}=$ 3.77687670e -01	$a_{212}=$ 1 03476090e +00
$a_{221}=$-9.73127876e -01	$a_{222}=$-2 79292816e +00
$a_{231}=$ 2.01054413e -01	$a_{232}=$ 3 02226494e -01
$a_{311}=$ 8.87418540e 01	$a_{312}=$ 1 45911901e +00
$a_{321}=$ 2.83766805e +00	$a_{322}=$ 8 92975794e 01
$a_{331}=$ 3.80299973e -01	$a_{332}=$-3 39374699e 01
$g_{121}=$ 5.62774681e -01	$g_{122}=$-2 28717201e +00
$g_{131}=$ 1.02553292e +00	$g_{132}=$-9 15998963e -01
g_{231}— 8.37882158e -01	g_{232}— 1 41731873e +00

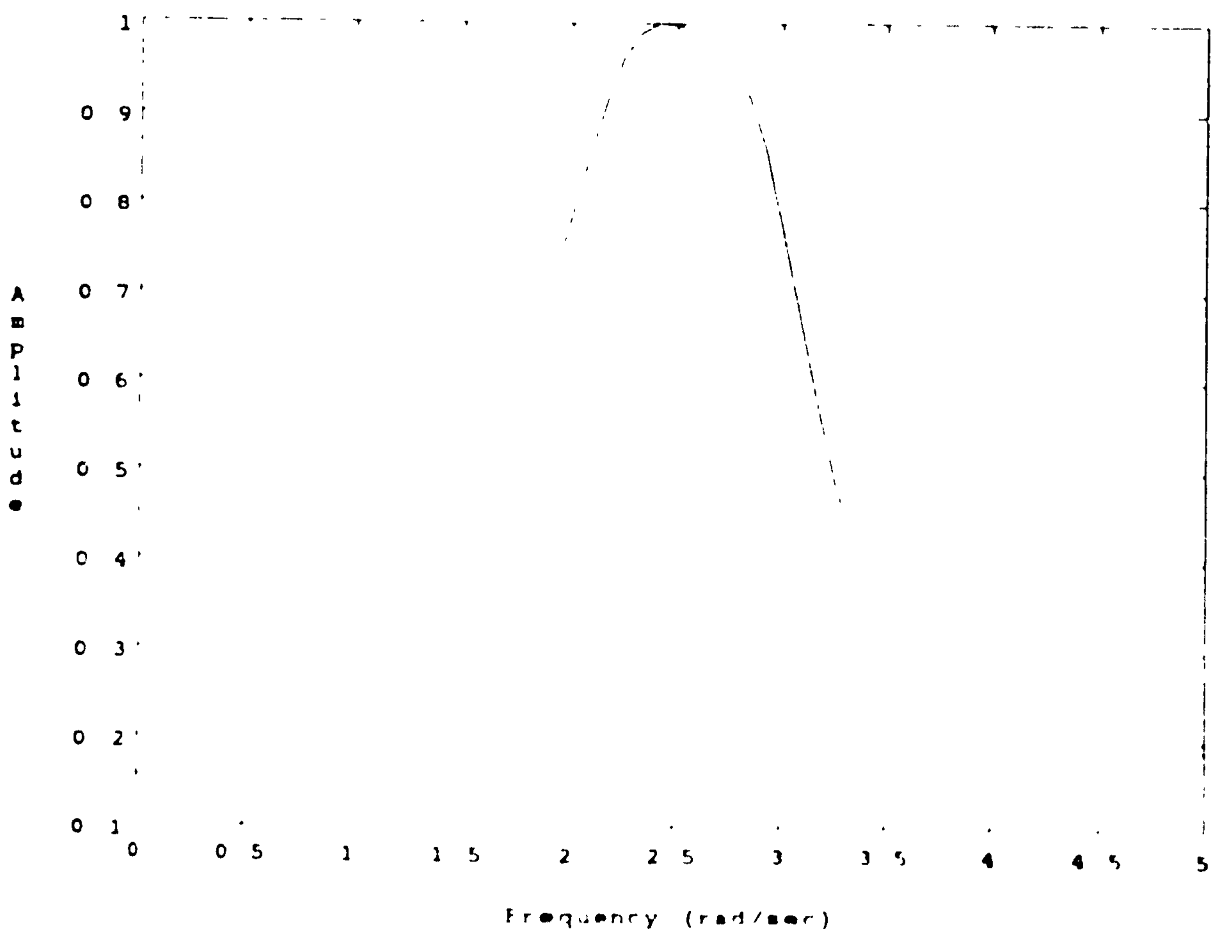


Fig. 3.9(a) Magnitude Frequency Response of the 1-D Bandpass Filter of Example 3.10

Max. Error of the Design = 0.2013916e+00

Max. Error from the Top(DB) = 0.2001874e+01

Max. Error from the Bottom(DB) = 0.1071046e+02

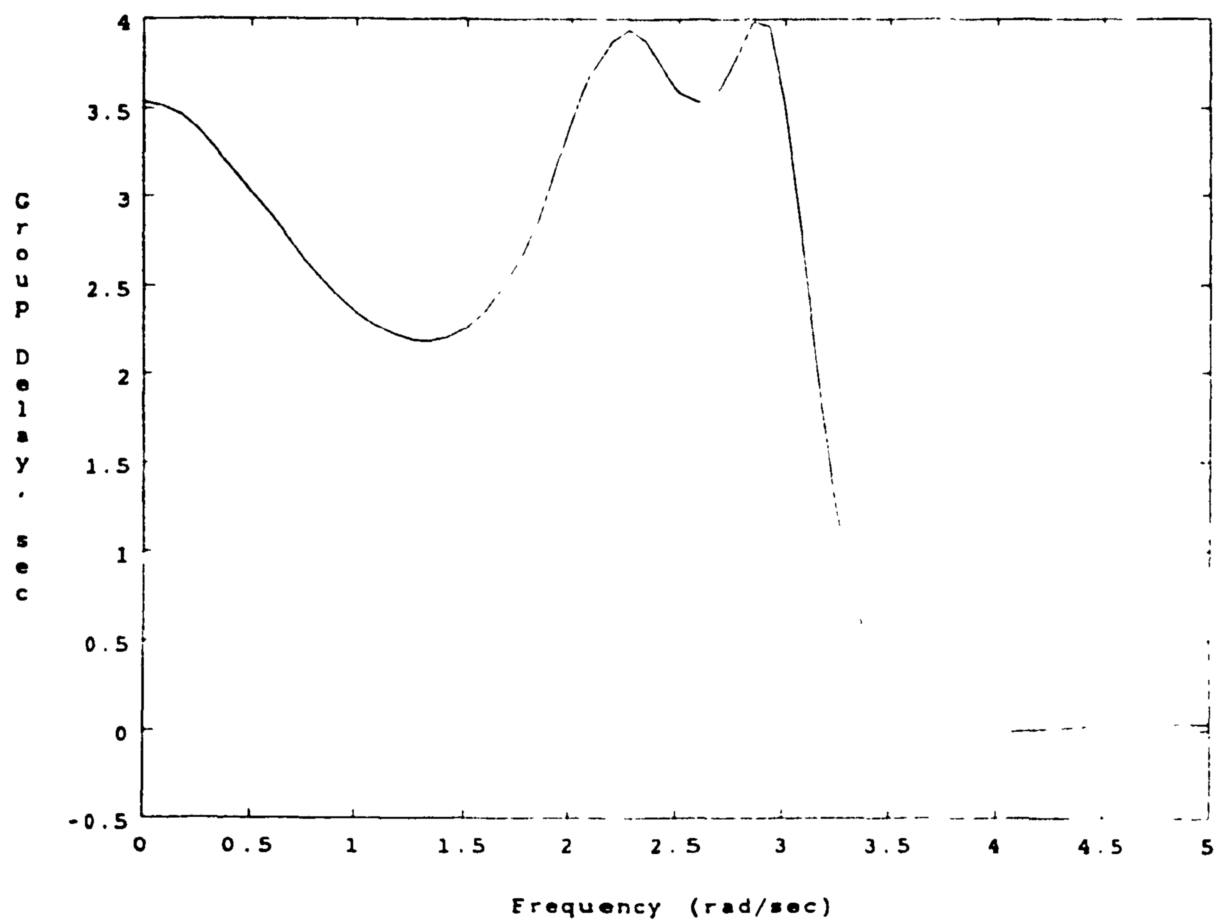


Fig. 3.9(b). Group Delay, τ, Frequency Response of the 1-D Bandpass Filter of Example 3.10

This leads to the desirable property that the necessary coefficient word-length be kept as small as possible.

Several attempts to estimate the coefficient word-length necessary in order to keep the system function response within a prescribed error have been made by different authors. For example, 86-96 have reported results on the problem of designing digital filters with limited word-length coefficients to meet magnitude specifications in frequency domain. In this section, we shall describe an algorithm based on an optimization procedure for the design of recursive digital filters with integer coefficients. It has the advantage of being easy to program. When these filters are implemented they are free from roundoff errors caused by quantization of the real coefficients with full precision and accumulation of these errors in arithmetic operations in the system.

3.4.1. Formulation of the Design Problem

This section deals with the problem of designing digital filters with integer coefficients to meet magnitude specifications in the frequency domain.

A frequency domain approximation to the design of a 2-D recursive digital filter using an L_2 norm, for the magnitude function is considered here. The design method employs a 2-D all pole analog filter with the overall system function

$$H_2(s_1,s_2) = \frac{K}{\sum_{k=0}^{M_2}\sum_{l=0}^{N_2} q_{kl}\, s_1^k s_2^l}$$

$$= \frac{K}{Q(s_1,s_2,q_{kl})}$$

Now the polynomial $Q(.)$ of (3.6) is assigned to the denominator of the 2-D prototype analog transfer function. This will enable us to obtain more number of variables thereby introducing the flexibility in optimization beside guaranteeing the stability of the resulting transfer function. The design approach is to find the coefficients

$\boldsymbol{a}, \boldsymbol{g}, \boldsymbol{a}'$, and $\boldsymbol{g}'$ of the resulting analog transfer function subject to the constraint

$$H_d(z_1,z_2) = \frac{K}{D(z_1,z_2)}$$

$$= \frac{K}{Q(s_1,s_2,\boldsymbol{a},\boldsymbol{g},\boldsymbol{a}',\boldsymbol{g}')}\Bigg|_{s_r = \frac{2}{T}\frac{z_r - 1}{z_r + 1}} \quad \text{for } r = 1, 2$$

3.4.1.1. Special Case

A frequency domain approximation to the design of a 1-D recursive digital filter for the magnitude function is to employ a 1-D all pole analog filter with the overall system function

$$H_a(s) = \frac{K}{\sum_{l=0}^{N} q_l s^l}$$

$$\equiv \frac{K}{Q(s, q_l)}$$

Based on the procedure outlined in the previous section, let $Q_1(s, \boldsymbol{a}, \boldsymbol{g})$ be a third order 1-D polynomial, generated by the use of Eq. (2.24) of Chapter II. Then, multiplying this polynomial with another polynomial of the same type results in a sixth order 1-D polynomial, $Q(s, \boldsymbol{a}, \boldsymbol{g}, \boldsymbol{a}', \boldsymbol{g}')$. We assign this polynomial to the denominator of the all pole prototype analog transfer function. The design approach is to find the coefficients $\boldsymbol{a}, \boldsymbol{g}, \boldsymbol{a}'$, and $\boldsymbol{g}'$ of the resulting analog transfer function subject to the constraint

$$H_d(z) = \frac{K}{D(z)}$$

$$= \frac{K}{Q(s, \boldsymbol{a}, \boldsymbol{g}, \boldsymbol{a}', \boldsymbol{g}')}\Bigg|_{s = \frac{2}{T}\frac{z - 1}{z + 1}}$$

3.4.2. Optimization, Integerization, and Reoptimization Technique

In this subsection, we propose an algorithm for designing a recursive digital filter with integer coefficients.

3.4.2.1. Algorithm [INTCOEF]

__Step 0__

i) Set $M = N$, where N is the number of parameters

__Step 1__

i) Initialize the coefficients, $X(I)$, $I = 1, N$

ii) Perform optimization using FMFP routine

iii) If convergence, go to step 2, otherwise go to (i) of step 1, with the results to serve as a new initial starting point

__Step 2__

i) If all the coefficients have an absolute value less than unity, go to step 3, otherwise go to step 4

__Step 3__

i) Use some weighting function to bring the value of all the coefficients to greater than unity

__Step 4__

i) Take the coefficient which has the maximum absolute value

ii) Round up and round down that coefficient

iii) Calculate the error function, E_2 for the rounded up and rounded down coefficient while the remaining coefficients are left unchanged

iv) Assign the value of the coefficient to the integer value which results in the minimum error function

v) Set $M = M-1$.

Step 5

i) If $N-M \neq N$, assume that $(N-M)$ coefficient(s) to be constant integer(s), go to (i) of step 1. Otherwise go to step 6.

Remark: A coefficient can be kept as a constant integer, in optimization process, simply by setting the derivative of the error function with respect to that parameter equal to zero.

Step 6

i) Round up and round down the last coefficient.

ii) Calculate the error function for the last rounded up and rounded down coefficient.

iii) Assign the value of the coefficient to the integer value which results in the minimum error function.

iv) Print results.

3.4.2.2. Design Example 11

To demonstrate the method, an all-pole lowpass recursive digital filter in 2-D is considered. The filter is of the type given in the previous section. The design problem is to find the coefficients of the filter transfer function to meet a magnitude specification in the frequency domain as follows

$$|H_I(e^{j\omega_{1m}}, e^{j\omega_{2n}})| = \begin{cases} 1.0 & \text{for } 0.0 \leq \rho(m,n) \leq 1.0 \text{ rad/sec.} \\ 0.0 & \text{for } 2.0 \leq \rho(m,n) \leq \dfrac{\omega_s'}{2} = \pi \text{ rad/sec.} \end{cases}$$

The algorithm converged to a prespecified value of $F_M = 6.94197$ before integerization and to a value of $F_M = 7.13925$ after integerization with the coefficients shown in

Table 3.13(a-b) and the filter responses in Figs. 3.10(a-d). In this example, the total CPU time was observed to be 253.22 seconds.

3.4.2.3. Design Example 12

In this example, we deal with the problem of designing a 1-D recursive digital filter with integer coefficients to meet a magnitude specification in the frequency domain as follows.

$$|H_I(e^{j\omega_m})| = \begin{cases} 1.0 & \text{for } 0.0 \leq \omega_m \leq 1.0 \ \text{rad/sec.} \\ 0.0 & \text{for } 2.0 \leq \omega_m \leq \dfrac{\omega_s}{2} = \pi \ \text{rad/sec.} \end{cases}$$

The results obtained were $F_M = 2.18684578e\ 02$ before integerization, $F_M = 4.00796026e\ 02$ with 23 of the coefficients being integerized, and $F_M = 1.13266869e+00$, after all the coefficients were integerized. The coefficients are shown in Tables 3.14(a-c), and the magnitudes of the frequency responses are shown in Fig. 3.11. In this example, the total CPU time was observed to be 189.13 seconds.

Table 3.13(a) Values of the Parameters of the Designed 2-D Filter in Example 3.11

Parameters of the Denominator	
Section 1	Section 2
a_{111}= 3.71723030e -01	a_{112}= 3.08144705e -01
a_{121}=-2.18852682e +00	a_{122}= 2.11926881e -01
a_{131}= 5.41110042e -01	a_{132}=-3.55363591e -02
a_{211}= 7.19419611e -01	a_{212}=-1.69844556e -01
a_{221}=-2.61332848e +00	a_{222}=-1.10363665e +00
a_{231}= 1.17856090e +00	a_{232}= 8.68094889e -02
a_{311}=-2.70670114e -01	a_{312}=-4.04722643e -01
a_{321}= 2.81652988e +00	a_{322}=-1.97390019e +00
a_{331}=-2.95037965e -01	a_{332}= 1.62168797e -01
g_{121}=-1.20102312e +00	g_{122}=-2.31357035e -01
g_{131}= 1.45847095e +00	g_{132}= 1.68532157e -01
g_{231}= 5.21832489e -01	g_{232}= 4.98652656e -01

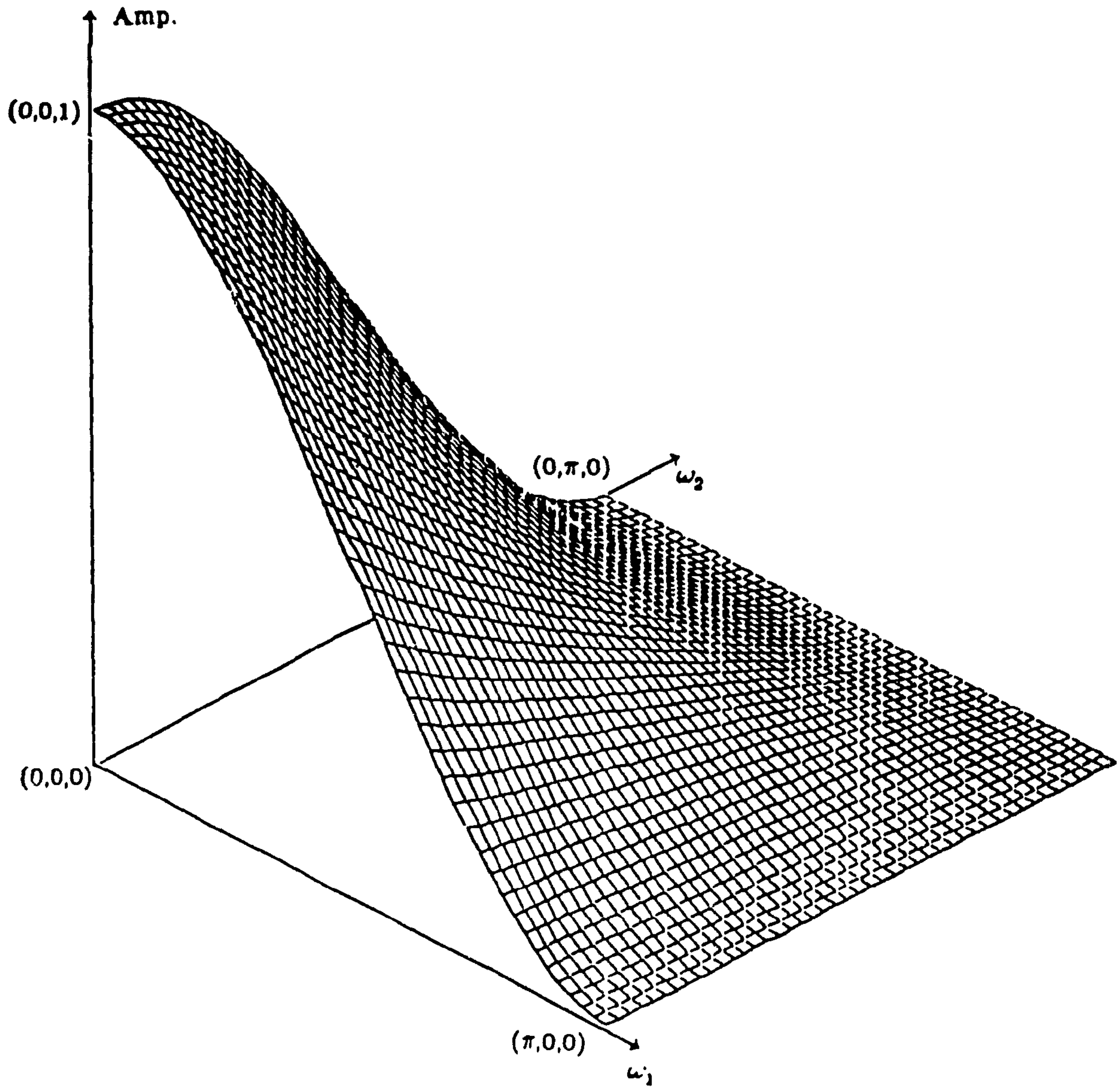


Fig. 3.10(a). Magnitude-Frequency Response of the 2-D Allpole Lowpass Filter with Real Coefficients in Example 3.11 in the First Quadrant of (ω_1,ω_2)-Plane as Viewed after CCW $R(\text{amp},45)R(\omega_1,30)$ of the Object.

Max. Error of the Design = 0.3032101e +00

Max. Error from the Top(DB) = -0.3137963e +01

Max. Error from the Bottom(DB) = -0.1030513e +02

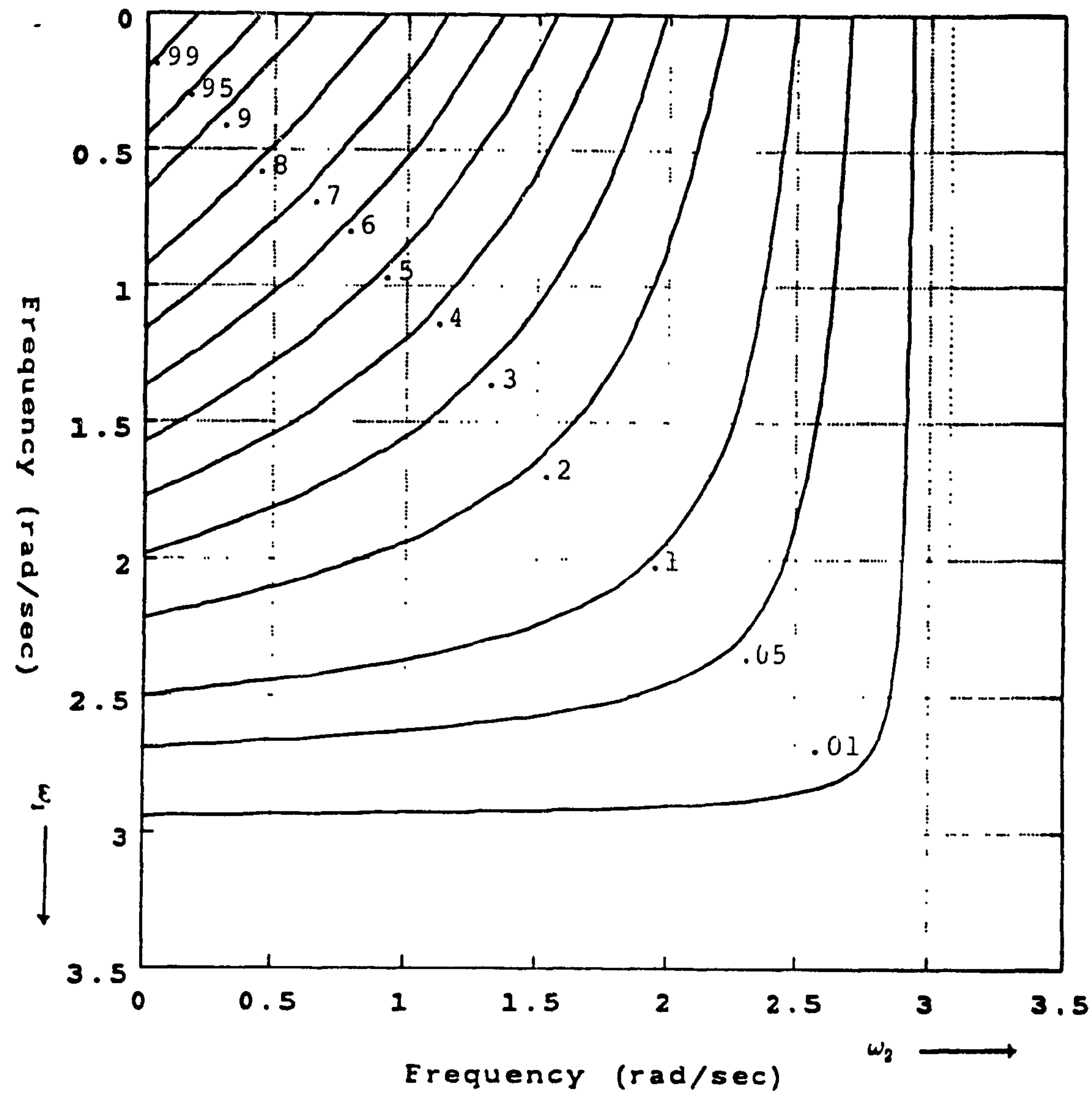


Fig. 3.10(b). Contour Plot of the Magnitude-Frequency Response of the 2-D Allpole Lowpass Filter with Real Coefficients in Example 3.11.

Table 3.13(b) Values of the Parameters of the Designed 2-D Filter in Example 3.11

Section 1		
$a_{111} = 1$	$a_{121} = -8$	$a_{131} = 2$
$a_{211} = 2$	$a_{221} = -12$	$a_{231} = 4$
$a_{311} = -2$	$a_{321} = 12$	$a_{331} = -2$
$g_{121} = -16$	$g_{131} = 32$	$g_{231} = 4$

Section 2		
$a_{112} = 2$	$a_{122} = 3$	$a_{132} = 0$
$a_{212} = -4$	$a_{222} = 18$	$a_{232} = 1$
$a_{312} = -3$	$a_{322} = 36$	$a_{332} = 2$
$g_{122} = -36$	$g_{132} = 54$	$g_{232} = 486$

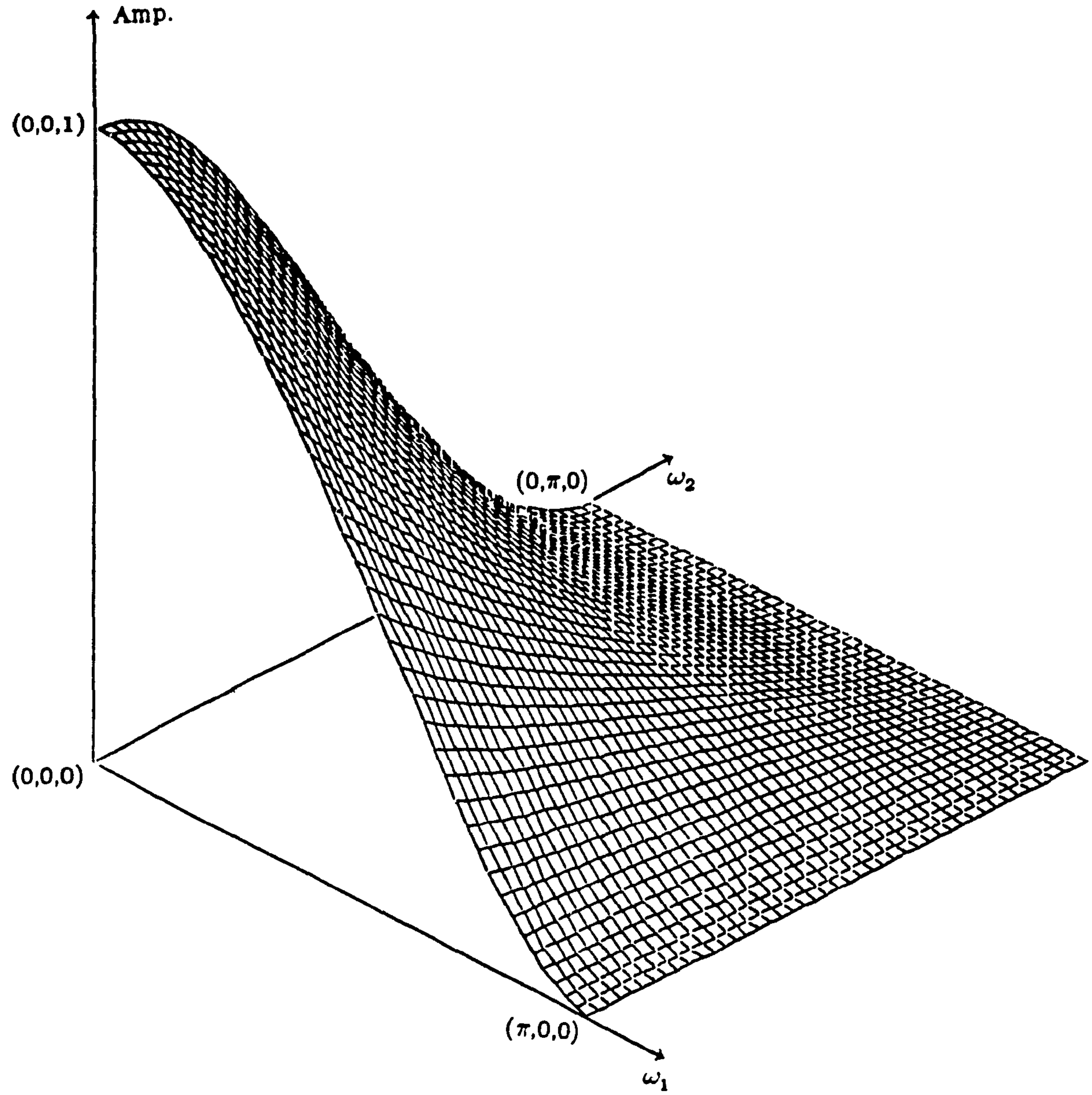


Fig. 3.10(c). Magnitude-Frequency Response of the 2-D Allpole Lowpass Filter with Integer Coefficients in Example 3.11 in the First Quadrant of (ω_1,ω_2)-Plane as Viewed after CCW $R\,(\text{amp}\ ,45)R\,(\omega_1,30)$ of the Object.

Max. Error of the Design = 0.3155704e +00

Max. Error from the Top(DB) =-0.3293424e +01

Max. Error from the Bottom(DB) =-0.1001807e +02

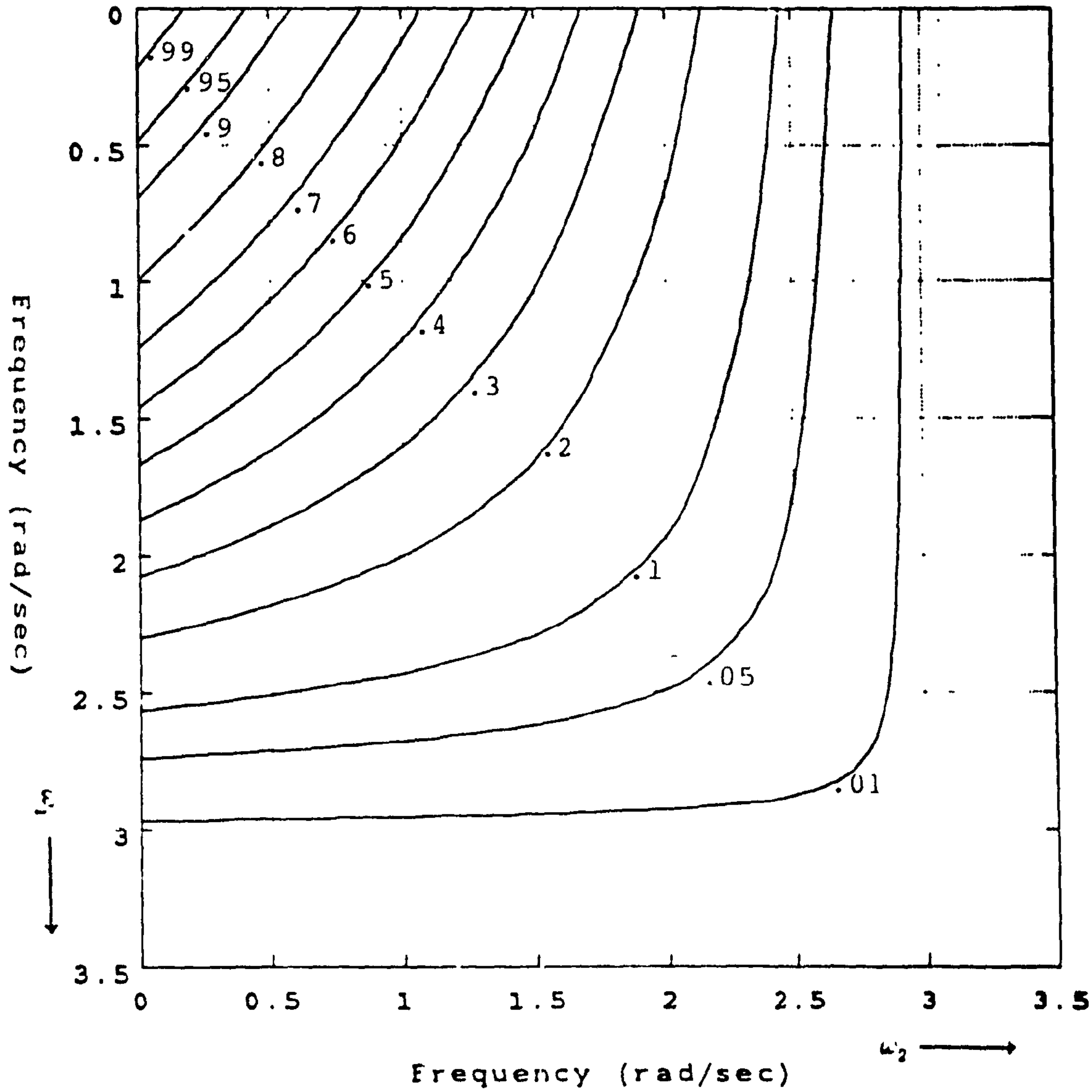


Fig 3 10(d) Contour Plot of the Magnitude-Frequency Response of the 2-D Allpole Lowpass Filter with Integer Coefficients in Example 3 11

Table 3.14(a) Values of the Parameters of the Designed 1-D Filter in Example 3.12

Parameters of the Denominator	
Section 1	Section 2
a_{111}= 1.96976369e +00	a_{112}= 9.18883198e +00
a_{121}=−2.48957899e +00	a_{122}=−2.99055596e +00
a_{131}= 2.08329841e +00	a_{132}= 1.19499135e +01
a_{211}= 1.84395472e +00	a_{212}=−1.88344247e +01
a_{221}=−2.41608263e +00	a_{222}= 1.82687799e −02
a_{231}= 1.75817999e +00	a_{232}= 2.30636790e +00
a_{311}=−4.41538380e −01	a_{312}=−1.57420179e +00
a_{321}= 6.82410330e −01	a_{322}=−1.48657179e −01
a_{331}=−5.01321660e −01	a_{332}= 1.29992271e +00
g_{121}=−3.22340968e −01	g_{122}=−1.01931456e +00
g_{131}= 2.20473181e −01	g_{132}= 4.00225770e −01
g_{231}= 8.01105085e −03	g_{232}= 4.46936476e +00

Max. Error of the Design = 0.7906495e -01

Max. Error from the Top(DB) =−0.7154200e +00

Max. Error from the Bottom(DB) =−0.2204032e +02

Table 3.14(b) Values of the Parameters of the Designed 1-D Filter in Example 3.12

Parameters of the Denominator	
Section 1	Section 2
a_{111}= 1.2000e +01	a_{112}= 1.1700e +02
a_{121}=-1.2000e +01	a_{122}=-3.9000e +01
a_{131}= 6.0000e +00	a_{132}= 1.5600e +02
a_{211}= 6.0000e +00	a_{212}=-2.4700e +02
a_{221}=-1.8000e +01	a_{222}= 1.3000e +01
a_{231}= 6.0000e +00	a_{232}= 2.6000e +01
a_{311}=-8.0000e +00	a_{312}=-1.3000e +01
a_{321}= 1.0000e +01	a_{322}= 1.0000e +00
a_{331}=-4.5451e +00	a_{332}= 1.3000e +01
g_{121}=-2.2500e +02	g_{122}=-3.3800e +02
g_{131}= 5.8000e +01	g_{132}=-1.6900e +02
g_{231}= 5.4000e +01	g_{232}= 8.4500e +02

Max. Error of the Design = 0 6356526e -01

Max Error from the Top(DB) = 0.5704496e +00

Max. Error from the Bottom(DB) = -0 2393560e +02

Table 3.14(c) Values of the Parameters of the Designed 1-D Filter in Example 3.12

Section 1		
$a_{111}=24$	$a_{121}=-24$	$a_{131}=12$
$a_{211}=12$	$a_{221}=-36$	$a_{231}=12$
$a_{311}=-16$	$a_{321}=20$	$a_{331}=-9$
$g_{121}=-900$	$g_{131}=232$	$g_{231}=216$

Section 2		
$a_{112}=117$	$a_{122}=-39$	$a_{132}=156$
$a_{212}=-247$	$a_{222}=13$	$a_{232}=26$
$a_{312}=-13$	$a_{322}=1$	$a_{332}=13$
$g_{122}=-338$	$g_{132}=-169$	$g_{232}=845$

Max. Error of the Design $=0.2982256e+00$

Max. Error from the Top(DB) $=-0.3076049e+01$

Max. Error from the Bottom(DB) $=-0.1050910e+02$

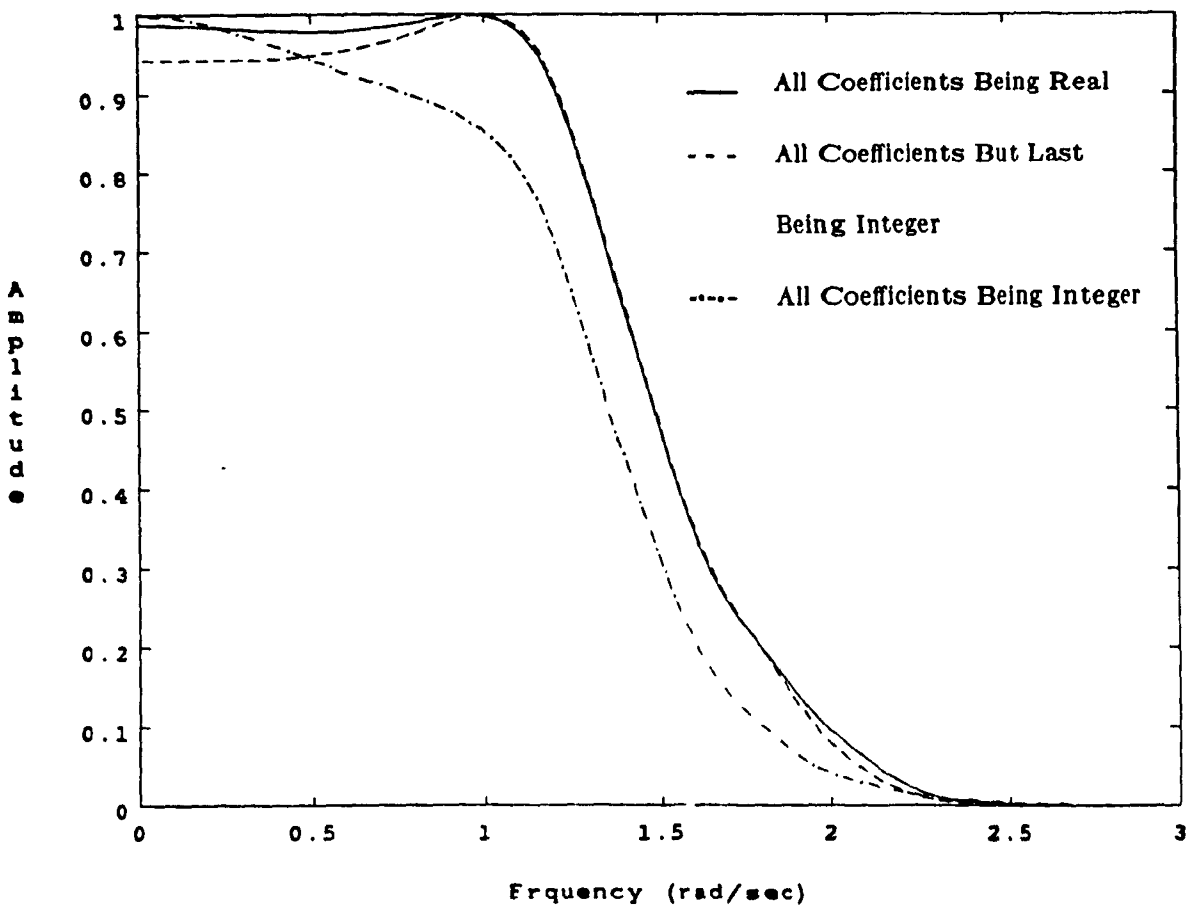


Fig. 3.11. Magnitude-Frequency Response of the 1-D Allpole Lowpass Filter with Real and Integer Coefficients in Example 3.12.

3.5. SUMMARY AND DISCUSSION

In this chapter, a minimum p-error criterion with $p=2$ (minimum squared error) and a nonlinear optimization technique is usea ::: the design of 2-D IIR digital filters. In this procedure, the approximation function to be miiii.'ized was magnitude or both magnitude and group delay functions, using the coefficients of the analog reference filter as optimization parameters, which makes the control of the stability of the filter easier. Some modifications to the Fletcher and Powell algorithm was made to facilitate a stable filter derivation as the output of the optimization procedure. This was done by constraining the det A of Chapter II to be as non-zero. Thus the entire procedure consisted in the minimization of the error. Design of stable 1-D recursive digital filters is also considered to be as a special case to which the method of Chapter II is applicable.

A practical algorithm for the design of recursive digital filters, with integer coefficients, with reasonable computation time is also given. The numerical performance of the algorithm is illustrated through examples. From the examples it can be seen that actual coefficient rounding and reoptimizing will lead to the actual maximum performance function (or minimum cost function). The closeness of the last coefficient to an integer before rounding, however, produced a variety of final solutions, the best of which is selected, using weighting function, as the final solution.

CHAPTER IV

COEFFICIENT SENSITIVITY AND EFFECT OF FINITE WORD-LENGTH

4.1. INTRODUCTION

Despite the many attractive advantages of digital filters over their analog counterparts, there are some practical limitations associated with their actual implementation. The most important limitation is caused by quantization. The three major sources of quantization errors are: (i) input signal quantization errors, (ii) arithmetic quantization (round-off or truncation noise at the output of multipliers) errors, and (iii) filter coefficient quantization errors (sensitivity). These errors have been discussed in detail in the literature [14, 16, 86-96].

It is well known that computer word-length is limited, and that small word-length is desired in special-purpose hardware systems. Hence, when digital filters (algorithms) are implemented their performance is degenerated. That is to say, in the implementation of digital filters account must be taken of effects due to the finite word-length used to represent signal values, coefficient values, and the arithmetic operations performed as they may alter the response of the digital filters.

The coefficient quantization results in a deviation of the resulting filter frequency response from the ideal one. This effect is known as the sensitivity characteristic of the filter. Sensitivity that is independent of the form of the input signal is defined in terms of the structure of a filter. For example, if the filter is represented by a transfer function, the sensitivity is defined on the basis of the parameter induced change of the transfer function [97].

The coefficients of the transfer function of 2-D recursive digital filters (1-D as special case) of Chapter III are evaluated to a high degree of accuracy during the approximation step. However, when these parameters are quantized by means of rounding or truncating the frequency response of the digital filter with new coefficients may fail to meet the desired specifications [85, 96]. In this chapter we will concentrate on variations in the parameters pertaining to the 2-D recursive filter transfer functions derived from the structure of Chapter II (skew-symmetric matrix). This can serve as a sensitivity measure of the digital filters. This study is felt to be important as the required coefficient word-length of the digital filter derived from this structure has a strong effect on both the cost and speed of the filter when it is implemented [85, 94]. If the word-length is large, then the cost of the filter will be high and the operating speed low. Here, the maximum sensitivity is used as a sensitivity measure.

4.2. SENSITIVITY MEASURE

Consider a linear time-invariant 2-D recursive digital filter whose transfer function $H(z_1,z_2,\boldsymbol{c})$ is a discrete function not only of the complex frequency $z_i = \exp(j\,\omega_i\,T_i)$, $i=1, 2$ but also of the $n \times 1$ parameter vector $\boldsymbol{c} = [c_1, c_2, \ldots, c_n]^T$.

Let $H = H(z_1,z_2,\boldsymbol{c})$ and $H_o = H(z_1,z_2,\boldsymbol{c}_o)$ be the actual and nominal transfer functions, respectively, and $\boldsymbol{c}_o$ the nominal parameter vector which is assumed to vary to $\boldsymbol{c} = \boldsymbol{c}_o + \Delta\boldsymbol{c}$. $\boldsymbol{c}$ is called as the actual (or perturbed) parameter value. In this term the sensitivity function of the transfer function

$$H(z_1,z_2\boldsymbol{c}) = \frac{N(z_1,z_{2,}\boldsymbol{c}_1)}{D(z_1,z_{2,}\boldsymbol{c}_2)}$$

$$\equiv \left.\frac{P(s_1,s_2,p_{kl})}{Q(s_1,s_2,q_{kl})}\right|_{s_r = \frac{2}{T}\cdot\frac{z_r - 1}{z_r + 1}} \quad \text{for } r=1,\,2 \tag{4.1}$$

with the vector $\boldsymbol{c}$

$$\boldsymbol{c} = (p_{kl}\,, q_{kl}), \qquad \text{for } k=0,1,\ldots,M_1,\ l=0,1,\ldots,N_1$$

chosen as the vector of the coefficients of the filter, of a 2-D recursive digital structure, with respect to variations in parameters c_j is defined as [97]

$$S_{c_j}^{H}(z_1,z_2,\mathbf{c}) \triangleq \left.\frac{\partial \ln H}{\partial \ln c_j}\right|_{c_{jo}}$$

$$= \left.\frac{\partial H/H}{\partial c_j/c_j}\right|_{c_{jo}}$$

$$= \left.\frac{\partial H}{\partial c_j}\right|_{c_{jo}} \frac{c_{jo}}{H_o}. \tag{4.2}$$

With $z_i = e^{j\omega_i T_i}$, $i=1,2$, we can write

$$S_{c_j}^{H}(e^{j\omega_1 T_1}, e^{j\omega_2 T_2}, \mathbf{c}) = \mathrm{Re}\left[S_{c_j}^{H}(e^{j\omega_1 T_1}, e^{j\omega_2 T_2}, \mathbf{c})\right]$$

$$+ j\,\mathrm{Im}\left[S_{c_j}^{H}(e^{j\omega_1 T_1}, e^{j\omega_2 T_2}, \mathbf{c})\right]. \tag{4.3}$$

If

$$H(e^{j\omega_1 T_1}, e^{j\omega_2 T_2}, \mathbf{c}) = |H(e^{j\omega_1 T_1}, e^{j\omega_2 T_2}, \mathbf{c})|\, e^{j\phi(\omega_1,\omega_2,\mathbf{c})}$$

with $\phi(\omega_1,\omega_2,\mathbf{c}) = \mathrm{Arg}\, H(.)$, then

$$S_{c_j}^{H}(e^{j\omega_1 T_1}, e^{j\omega_2 T_2}, \mathbf{c}) = S_{c_j}^{|H|}(\omega_1,\omega_2,\mathbf{c}) + j\,\phi(\omega_1,\omega_2,\mathbf{c})\, S_{c_j}^{\phi(\omega_1,\omega_2,\mathbf{c})}(\omega_1,\omega_2,\mathbf{c}). \tag{4.4}$$

Comparing (4.3) with (4.4), we see that

$$S_{c_j}^{|H|}(\omega_1,\omega_2,\mathbf{c}) = \mathrm{Re}\left[S_{c_j}^{H}(e^{j\omega_1 T_1}, e^{j\omega_2 T_2}, \mathbf{c})\right] \tag{4.5}$$

This is called the sensitivity function of the magnitude of H. Since the transfer function of the filter under consideration is a rational function comprising two polynomials, N and D, the sensitivity can be written as

$$S_{c_j}^{H}(e^{j\omega_1 T_1},e^{j\omega_2 T_2},\mathbf{c}) = \left.\frac{\partial ln\,(\frac{N}{D})}{\partial \ln c_j}\right|_{c_{jo}}$$

$$= \left.\frac{\partial \ln N}{\partial \ln c_j}\right|_{c_{jo}} - \left.\frac{\partial \ln D}{\partial \ln c_j}\right|_{c_{jo}}$$

$$= S_{c_j}^{N}(e^{j\omega_1 T_1},e^{j\omega_2 T_2},\mathbf{c}) - S_{c_j}^{D}(e^{j\omega_1 T_1},e^{j\omega_2 T_2},\mathbf{c}). \tag{4.6}$$

Therefore, we may write the sensitivity as

$$S_{c_j}^{H}(e^{j\omega_1 T_1},e^{j\omega_2 T_2},\mathbf{c}) = \left.\frac{\partial N(e^{j\omega_1 T_1},e^{j\omega_2 T_2},\mathbf{c})}{\partial c_j}\right|_{c_{jo}} \cdot \frac{c_{jo}}{N(e^{j\omega_1 T_1},e^{j\omega_2 T_2},\mathbf{c}_o)}$$

$$- \left.\frac{\partial D(e^{j\omega_1 T_1},e^{j\omega_2 T_2},\mathbf{c})}{\partial c_j}\right|_{c_{jo}} \cdot \frac{c_{jo}}{D(e^{j\omega_1 T_1},e^{j\omega_2 T_2},\mathbf{c}_o)}. \tag{4.7}$$

In terms of the sensitivity function of the magnitude of H, the function

$$S = \sum_{j=1}^{n} | S_{c_j}^{|H|} | \tag{4.8}$$

can be formed where n is the number of the parameters in the transfer function. This quantity can serve as a sensitivity measure which can be used for the comparison of results obtained of rounding or truncating the coefficients of a transfer function.

4.3. EXAMPLE

Given a 2-D recursive digital filter described by the transfer function

$$H(z_1,z_2,\mathbf{c}) \equiv \left.\frac{P(s_1,s_2,p_{kl})}{Q(s_1,s_2,q_{kl})}\right|_{s_r = \frac{2}{T}\cdot\frac{z_r - 1}{z_r + 1}} \tag{4.9a}$$

where

$$P(s_1,s_2,p_{kl}) = \sum_{k=0}^{3} \sum_{l=0}^{3} p_{kl}\, s_1^k s_2^l$$

$$= p_{00}+p_{01}s_2+p_{02}s_2^2+p_{03}s_2^3$$

$$+ p_{10}s_1+p_{11}s_1s_2+p_{12}s_1s_2^2+p_{13}s_1s_2^3$$

$$+ p_{20}s_1^2+p_{21}s_1^2s_2+p_{22}s_1^2s_2^2+p_{23}s_1^2s_2^3$$

$$+ p_{30}s_1^3+p_{31}s_1^3s_2+p_{32}s_1^3s_2^2+p_{33}s_1^3s_2^3. \tag{4.9b}$$

and

$$Q(s_1,s_2,q_{kl}) = \sum_{k=0}^{3} \sum_{l=0}^{3} q_{kl}\, s_1^k s_2^l$$

$$= q_{00}+q_{01}s_2+q_{02}s_2^2+q_{03}s_2^3$$

$$+ q_{10}s_1+q_{11}s_1s_2+q_{12}s_1s_2^2+q_{13}s_1s_2^3$$

$$+ q_{20}s_1^2+q_{21}s_1^2s_2+q_{22}s_1^2s_2^2+q_{23}s_1^2s_2^3$$

$$+ q_{30}s_1^3+q_{31}s_1^3s_2+q_{32}s_1^3s_2^2+q_{33}s_1^3s_2^3.$$

$$\triangleq Q(s_1,s_2,\boldsymbol{a},\boldsymbol{g},\boldsymbol{a}',\boldsymbol{g}'). \tag{4.9c}$$

The polynomial $Q(.)$ is defined as

$$Q(s_1,s_2,\boldsymbol{a},\boldsymbol{g},\boldsymbol{a}',\boldsymbol{g}') \triangleq Q_1(s_1,s_2,\boldsymbol{a},\boldsymbol{g})\, Q_2(s_1,s_2,\boldsymbol{a}',\boldsymbol{g}') \tag{4.9d}$$

where

$$Q_1(s_1,s_2,\boldsymbol{a},\boldsymbol{g}) = d_{41}(s_2+1)s_1^2 + \left[2d_{41}s_2+(d_{11}+d_{21})\right]s_1$$

$$+ \left[d_{31}s_2+(d_{11}+d_{21}+d_{31})\right] \tag{4.9e}$$

with

$$d_{11}=(a_{111}g_{231} \;\; a_{121}g_{131}+a_{131}g_{121})^2,$$
$$d_{21}=(a_{211}g_{231} \;\; a_{221}g_{131}+a_{231}g_{121})^2,$$

$$d_{31}=(a_{311}g_{231}-a_{321}g_{131}+a_{331}g_{121})^2,$$

and

$$d_{41} = (a_{111}a_{221}a_{331}-a_{111}a_{231}a_{321}-a_{121}a_{211}a_{331}+ \\ a_{131}a_{211}a_{321}+a_{121}a_{231}a_{311}-a_{131}a_{221}a_{311})^2$$

and

$$Q_2(s_1,s_2,a',g') = d_{42}(s_1+1)s_2^2 + \left[2d_{42}s_1+(d_{22}+d_{32})\right]s_2 \\ + \left[d_{12}s_1+(d_{12}+d_{22}+d_{32})\right] \quad (4.9f)$$

with

$$d_{12}=(a_{112}g_{232}-a_{122}g_{132}+a_{132}g_{122})^2,$$
$$d_{22}=(a_{212}g_{232}-a_{222}g_{132}+a_{232}g_{122})^2,$$
$$d_{32}=(a_{312}g_{232}-a_{322}g_{132}+a_{332}g_{122})^2,$$

and

$$d_{42} = (a_{112}a_{222}a_{332}-a_{112}a_{232}a_{322}-a_{122}a_{212}a_{332} \\ + a_{132}a_{212}a_{322}+a_{122}a_{232}a_{312}-a_{132}a_{222}a_{312})^2.$$

With

$$\alpha_1 \triangleq d_{11}+d_{21},$$

$$\alpha_2 \triangleq d_{11}+d_{21}+d_{31},$$

$$\beta_1 \triangleq d_{22}+d_{32},$$

and

$$\beta_2 \triangleq d_{12}+d_{22}+d_{32}$$

q_{kl}'s, $k = 0,1,2,3,\ l = 0,1,2,3$ are given by

$$q_{00} = \alpha_2 \times \beta_2$$

$$q_{01} = \alpha_2 \times \beta_1 + d_{31} \times \beta_2$$

$$q_{02} = \alpha_2 \times d_{42} + \beta_1 \times d_{31}$$

$$q_{03} = d_{31} \times d_{42}$$

$$q_{10} = \alpha_2 \times d_{12} + \alpha_1 \times \beta_2$$

$$q_{11} = 2 \times \alpha_2 \times d_{42} + d_{31} \times d_{12} + \alpha_1 \times \beta_1 + 2 \times d_{41} \times \beta 2$$

$$q_{12} = \alpha_2 \times d_{42} + 2 \times d_{31} \times d_{42} + \alpha_1 \times d_{42} + 2 \times d_{41} \times \beta_1$$

$$q_{13} = d_{31} \times d_{42} + 2 \times d_{41} \times d_{42}$$

$$q_{20} = \alpha_1 \times d_{12} + d_{41} \times \beta_2$$

$$q_{21} = 2 \times \alpha_1 \times d_{42} + 2 \times d_{41} \times d_{12} + (\beta_1 + \beta_2) \times d_{41}$$

$$q_{22} = \alpha_1 \times d_{42} + 5 \times d_{41} \times d_{42} + d_{41} \times \beta_1$$

$$q_{23} = 3 \times d_{41} \times d_{42}$$

$$q_{30} = d_{41} \times d_{12}$$

$$q_{31} = 2 \times d_{41} \times d_{42} + d_{41} \times d_{12}$$

$$q_{32} = 3 \times d_{41} \times d_{42}$$

$$q_{33} = d_{41} \times d_{42}.$$

We wish to consider the effect of coefficient quantization of various 2-D recursive digital filters. For the sake of comparison, coefficient quantization has been applied to the lowpass, bandpass, highpass, fan, Laplacian and homomorphic filters of Chapter III with the transfer function given by (4.9). In (4.9), theoretical values of the parameters "p_{kl}", "$\boldsymbol{a}$", "$\boldsymbol{g}$", "$\boldsymbol{a}'$", and "$\boldsymbol{g}'$" are the same as given in Tables 3.1-3.6 of Chapter III. The coefficients were assumed to be in floating-point format and the quantization by means of rounding.

The sensitivity performance $S = \sum_{j=1}^{n} | S_{c_j}^{|H(z_1,z_2,c)|} |$, $z_i = \exp(j\omega_i T_i)$, $i=1,2$ for ω_i such that $0.0 \leq \omega_i \leq \omega_s/2 = 5.0$ rad/sec for different unsigned mantissa of the coefficients of the above filters ($0.0 \leq \omega_i \leq \pi$ rad/sec for the case of fan filters) is compared in Table 4.1. In calculating the sensitivity for different design examples, the fractional part of the value of each coefficient, expressed in the decimal form, is rounded to an accuracy of 8, 6, 4, 3, and 2 decimal places. As can be seen from the Table 4.1, the rounding of the coefficients up to 4 decimal places does not affect the sensitivity of the filters. In most of the cases except for the Laplacian and homomorphic filters, a very poor performance has been noticed when the coefficients are taken with an accuracy of 2 decimal places. For the exceptional cases of filters, the accuracy can be taken up to 1 decimal place without affecting their frequency responses.

Figures 4.1(a-d) to 4.5(a-d) show variation of peak value of the sensitivity, S, with angles $0.0 \leq \omega_i \leq \omega_s/2 = 5.0$ rad/sec, $i=1,2$ and the actual amplitude responses of the bandpass filter discussed in Example 3.2(b) of Chapter III. In these figures, respectively, the coefficients have been assumed to be of either with full precision or with an accuracy of six, four, three, and two decimal places. The plots showing the sensitivity performances are quite similar and so do the plots of the magnitude frequency responses. However, in the case of one decimal place, the magnitude frequency response is not quite good as can be seen from Fig. 4.6(c,d).

Table 4.1

Sensitivity Performance for Different Design Examples

2-D Filter	Full Precision	Six decimal places	Four decimal places	Three decimal places
Lowpass (1)	$.20e+01 \leq S \leq .129483e+03$	$.20e+01 \leq S \leq .129476e+03$	$.20e+01 \leq S \leq .130450e+03$	$.20e+01 \leq S \leq .126220e+03$
Lowpass (2)	$.20e+01 \leq S \leq .812833e+02$	$.20e+01 \leq S \leq .812921e+02$	$.20e+01 \leq S \leq .810001e+02$	$.20e+01 \leq S \leq .142394e+03$
Bandpass (1)	$.20e+01 \leq S \leq .130526e+03$	$.20e+01 \leq S \leq .130527e+03$	$.20e+01 \leq S \leq .130408e+03$	$.20e+01 \leq S \leq .125450e+03$
Bandpass (2)	$.20e+01 \leq S \leq .630206e+02$	$.20e+01 \leq S \leq .630203e+02$	$.20e+01 \leq S \leq .632400e+02$	$.20e+01 \leq S \leq .661376e+02$
Highpass	$.20e+01 \leq S \leq .529553e+02$	$.20e+01 \leq S \leq .529553e+02$	$.20e+01 \leq S \leq .529537e+02$	$.20e+01 \leq S \leq .529585e+02$
45-degree fan	$.20e+01 \leq S \leq .114205e+03$	$20e+01 \leq S \leq .114204e+03$	$.20e+01 \leq S \leq .114165e+03$	$.20e+01 \leq S \leq .114368e+03$
90-degree fan	$.20e+01 \leq S \leq .217549e+03$	$.20e+01 \leq S \leq .217552e+03$	$.20e+01 \leq S \leq .217409e+03$	$.20e+01 \leq S \leq .212363e+03$
Laplacian	$.20e+01 \leq S \leq .283453e+02$	$.20e+01 \leq S \leq .283453e+02$	$.20e+01 \leq S \leq .283414e+02$	$20e+01 \leq S \leq .283267e+02$
Homomorphic	$.20e+01 \leq S \leq .280778e+02$	$.20e+01 \leq S \leq .280779e+02$	$.20e+01 \leq S \leq .280696e+02$	$.20e+01 \leq S \leq .279862e+02$

Note: Sensitivity performance for two decimal places of unsigned mantissa in some of the examples are as follows:

Lowpass (2): $0.20e+01 \leq S \leq 0.138348e+03$

Bandpass (2): $0.20e+01 \leq S \leq 0.129367e+03$

Highpass: $0.20e+01 \leq S \leq 0.530755e+02$

Laplacian: $0.20e+01 \leq S \leq 0.279006e+02$

Homomorphic: $0.20e+01 \leq S \leq 0.286071e+02$

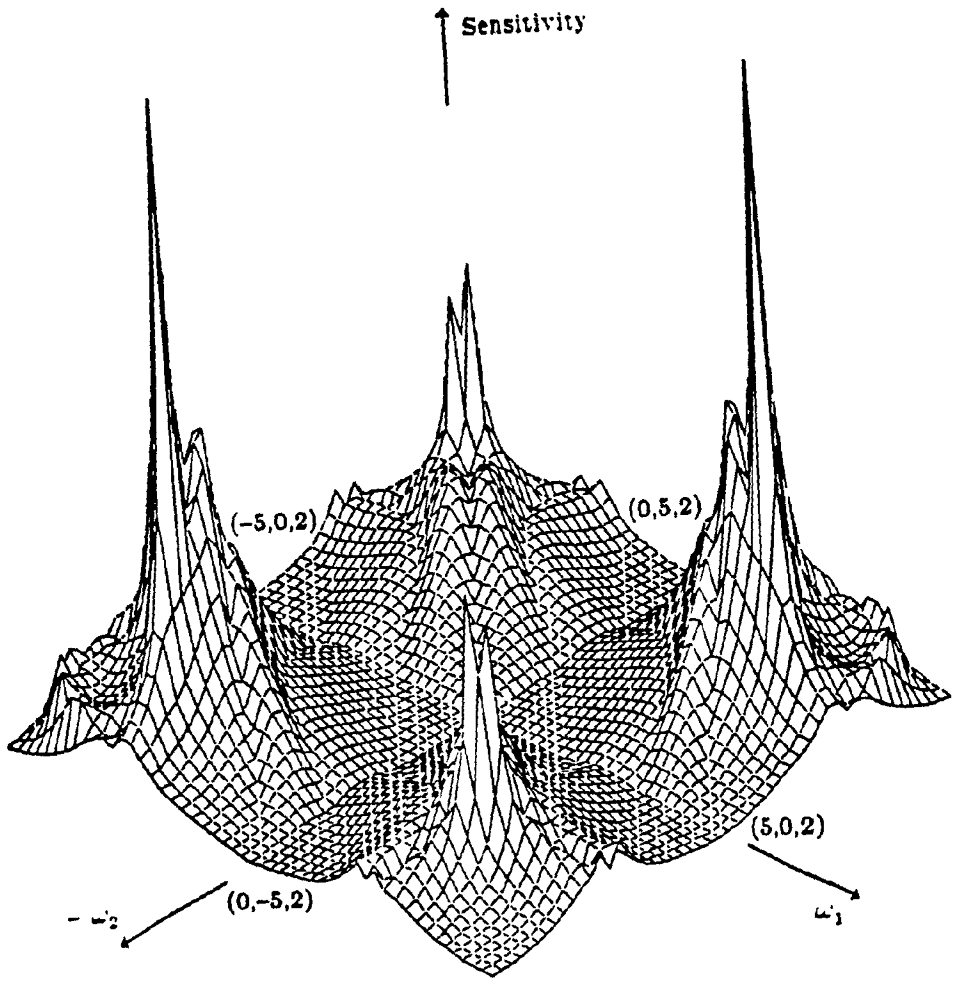


Fig. 4.1(a). Sensitivity-Frequency Response of the 2-D Bandpass Filter in Example 3.2(b) with Full-Precision Coefficients.

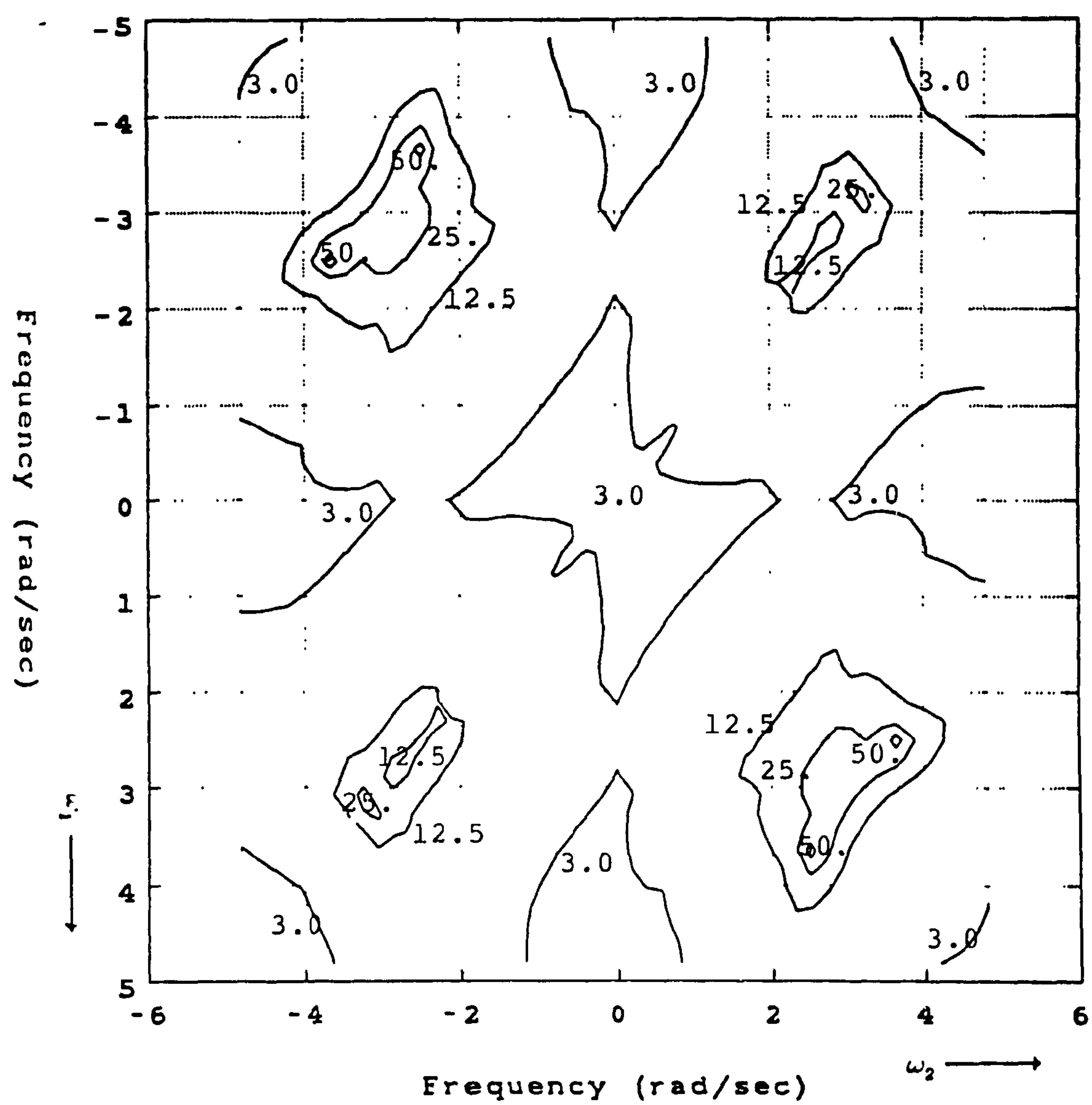


Fig. 4.1(b). Contour Plot of the Sensitivity-Frequency Respone of the Designed Bandpass Filter of Example 3.2(b) with Full-Precision Coefficients

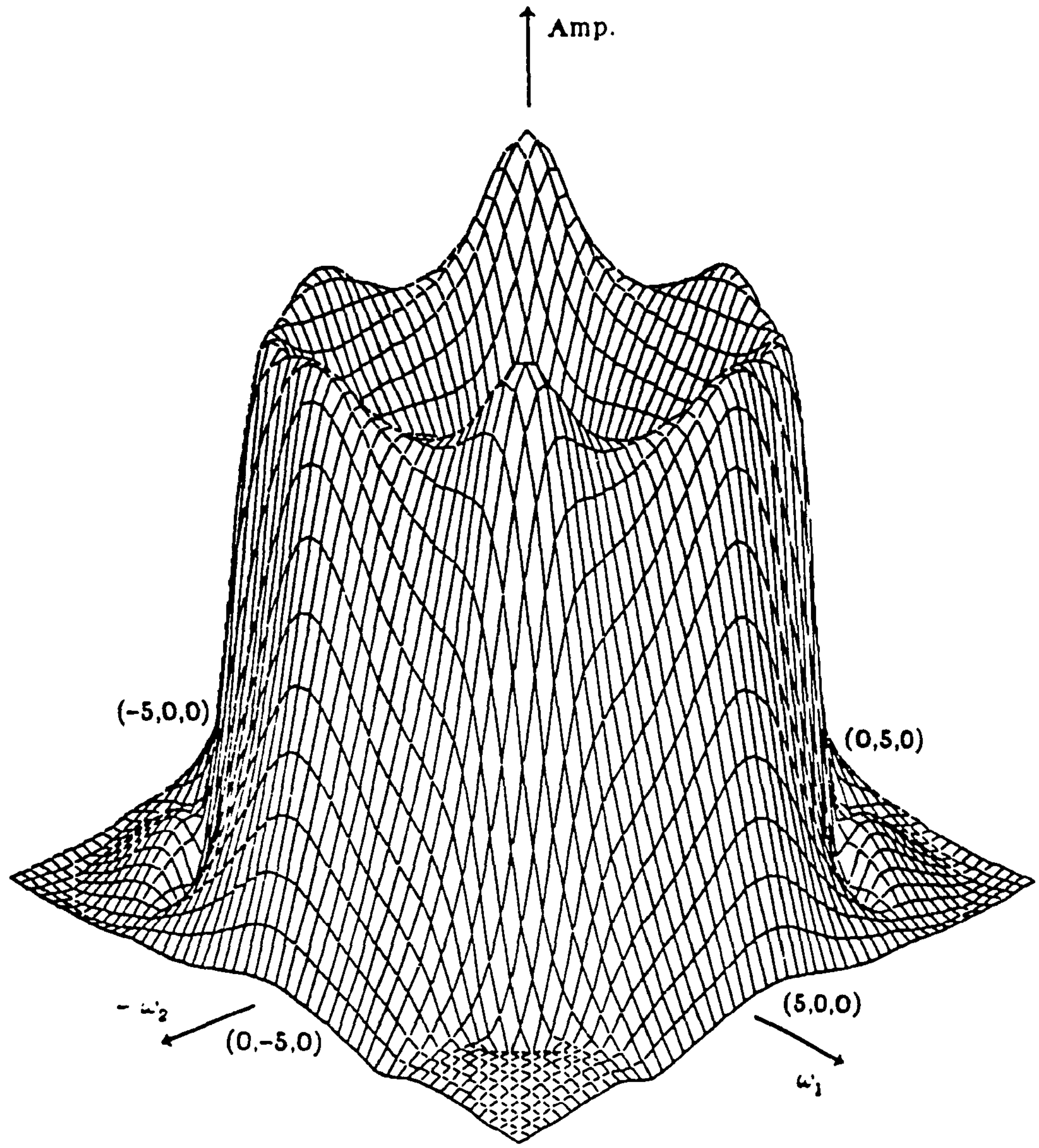


Fig. 4.1(c). Magnitude-Frequency Response of the 2-D Bandpass Filter in Example 3.2(b) with Full-Precision Coefficients.

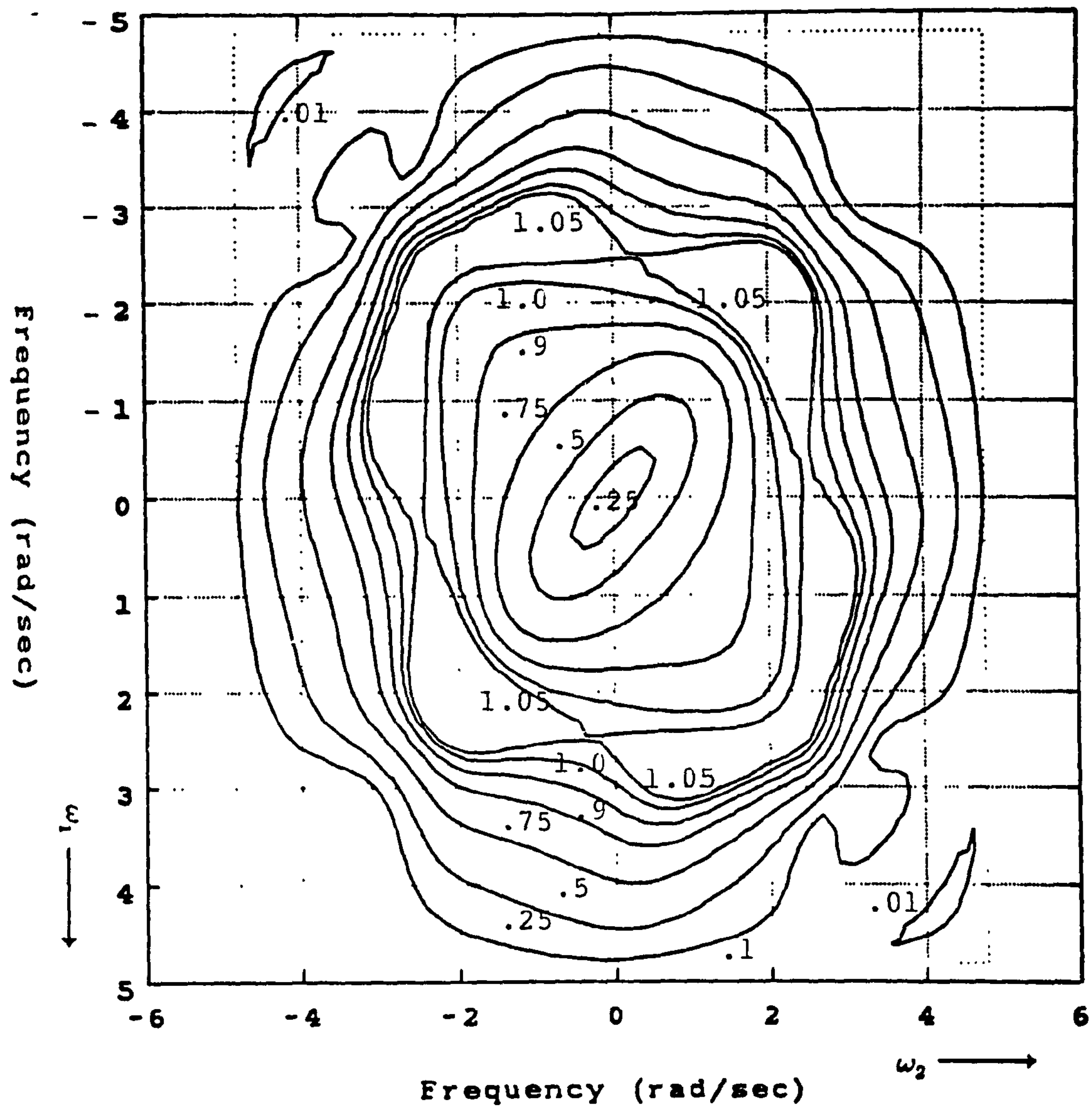


Fig. 4.1(d). Contour Plot of the Magnitude-Frequency Respone of the Designed Bandpass Filter of Example 3.2(b) with Full-Precision Coefficients.

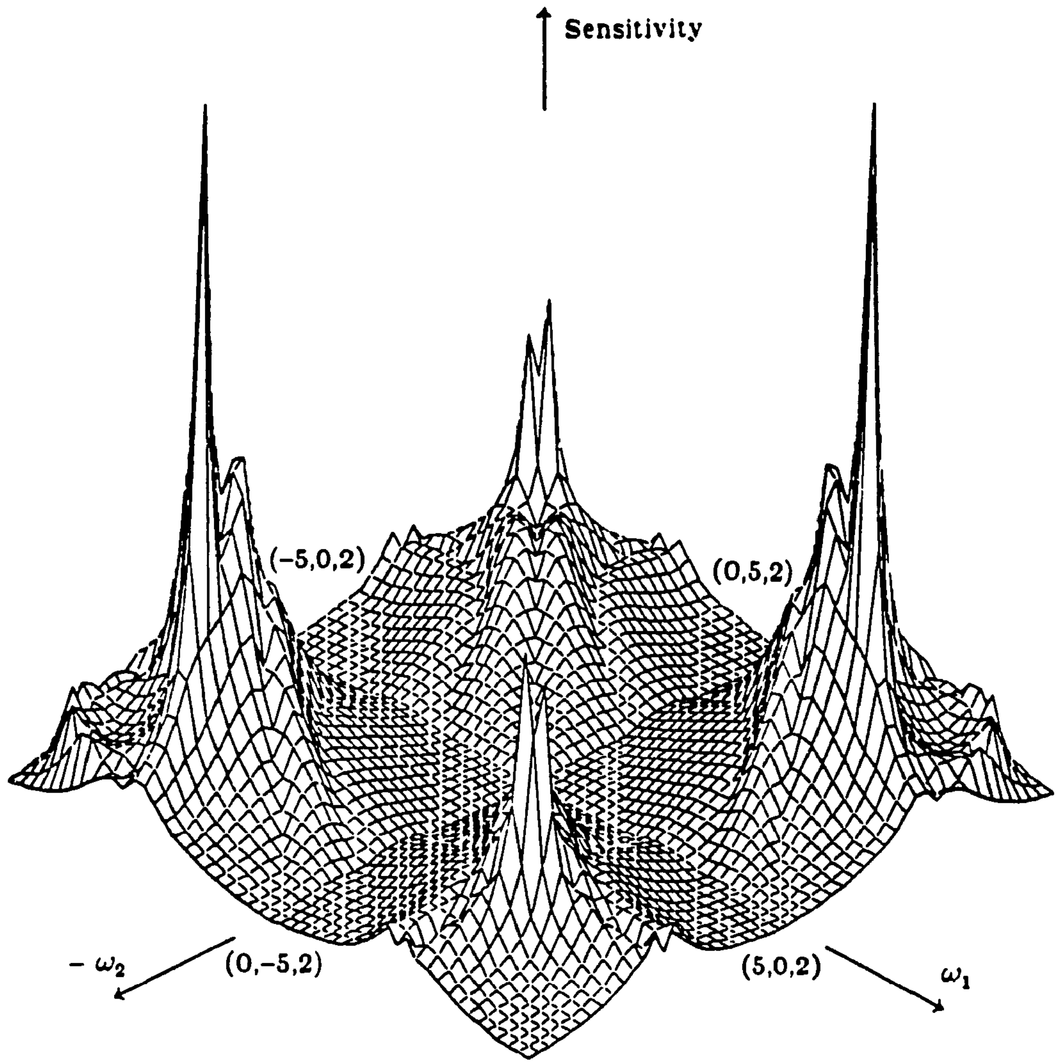


Fig. 4.2(a). Sensitivity-Frequency Response of the 2-D Bandpass Filter in Example 3.2(b) with Six-Decimal Places of Coefficients.

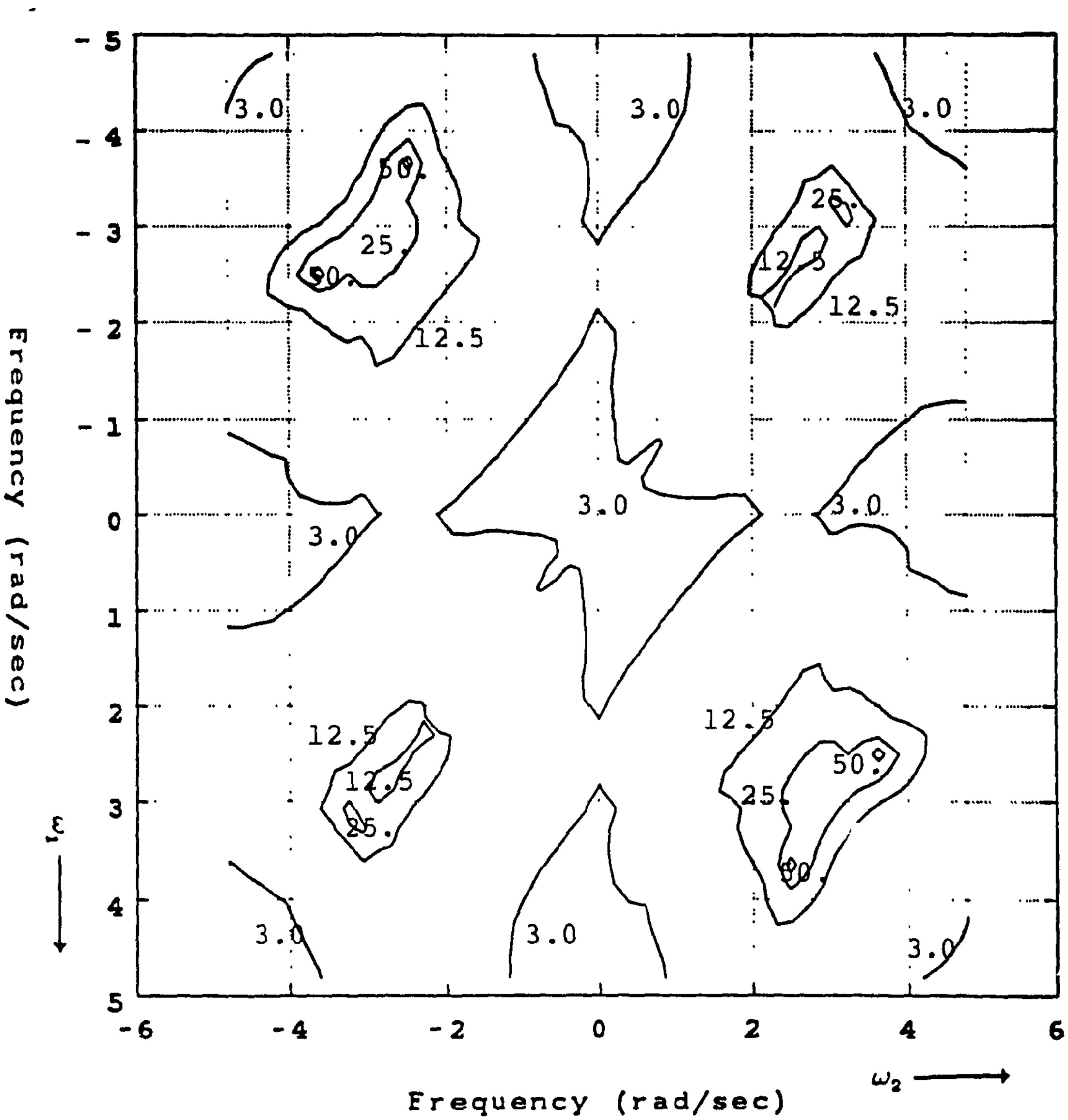


Fig. 4.2(b). Contour Plot of the Sensitivity-Frequency Respone of the Designed Bandpass Filter of Example 3.2(b) with Six-Decimal Places of Coefficients.

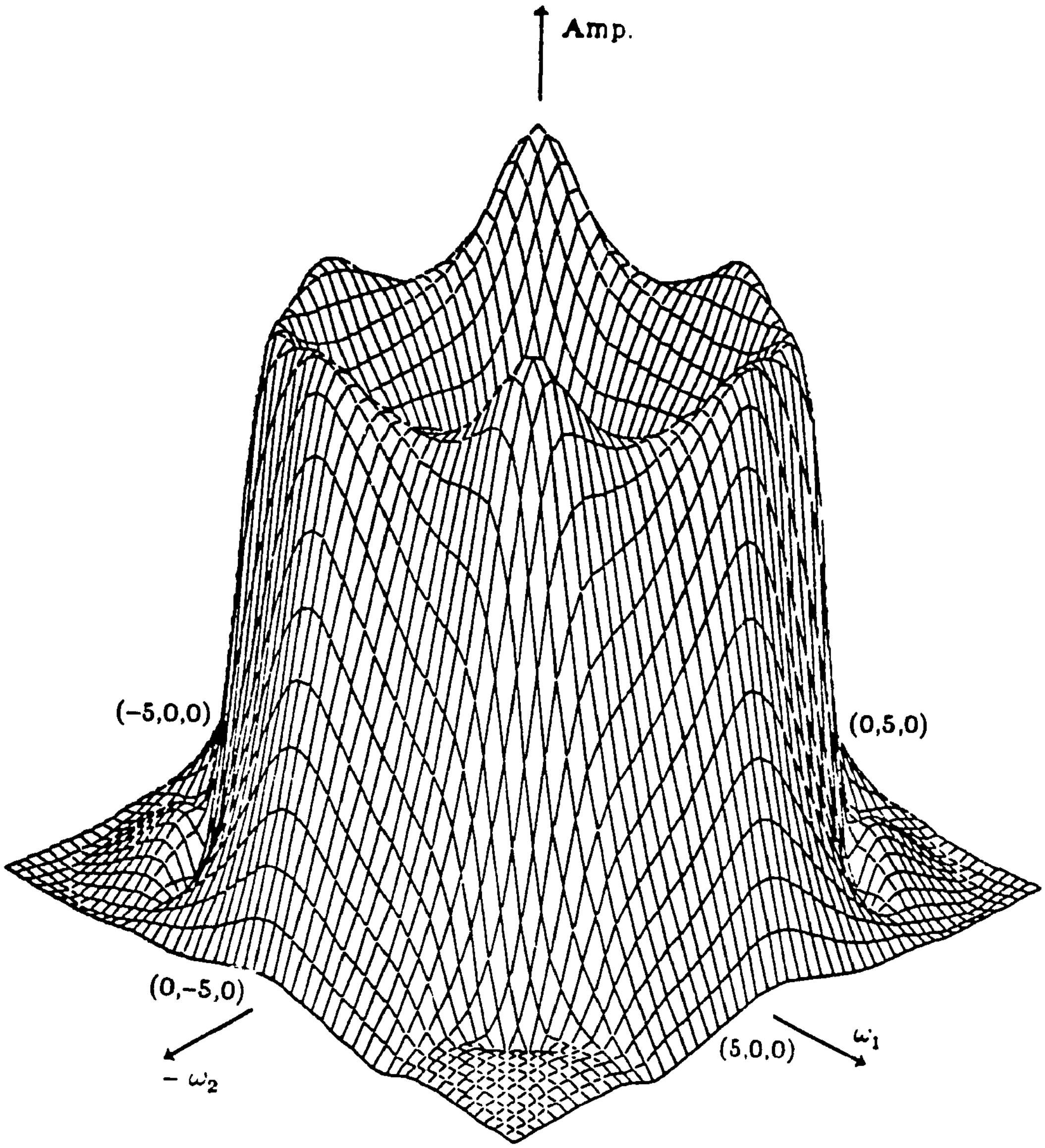


Fig. 4.2(c). Magnitude-Frequency Response of the 2-D Bandpass Filter in Example 3 2(b) with Six-Decimal Places of Coefficients.

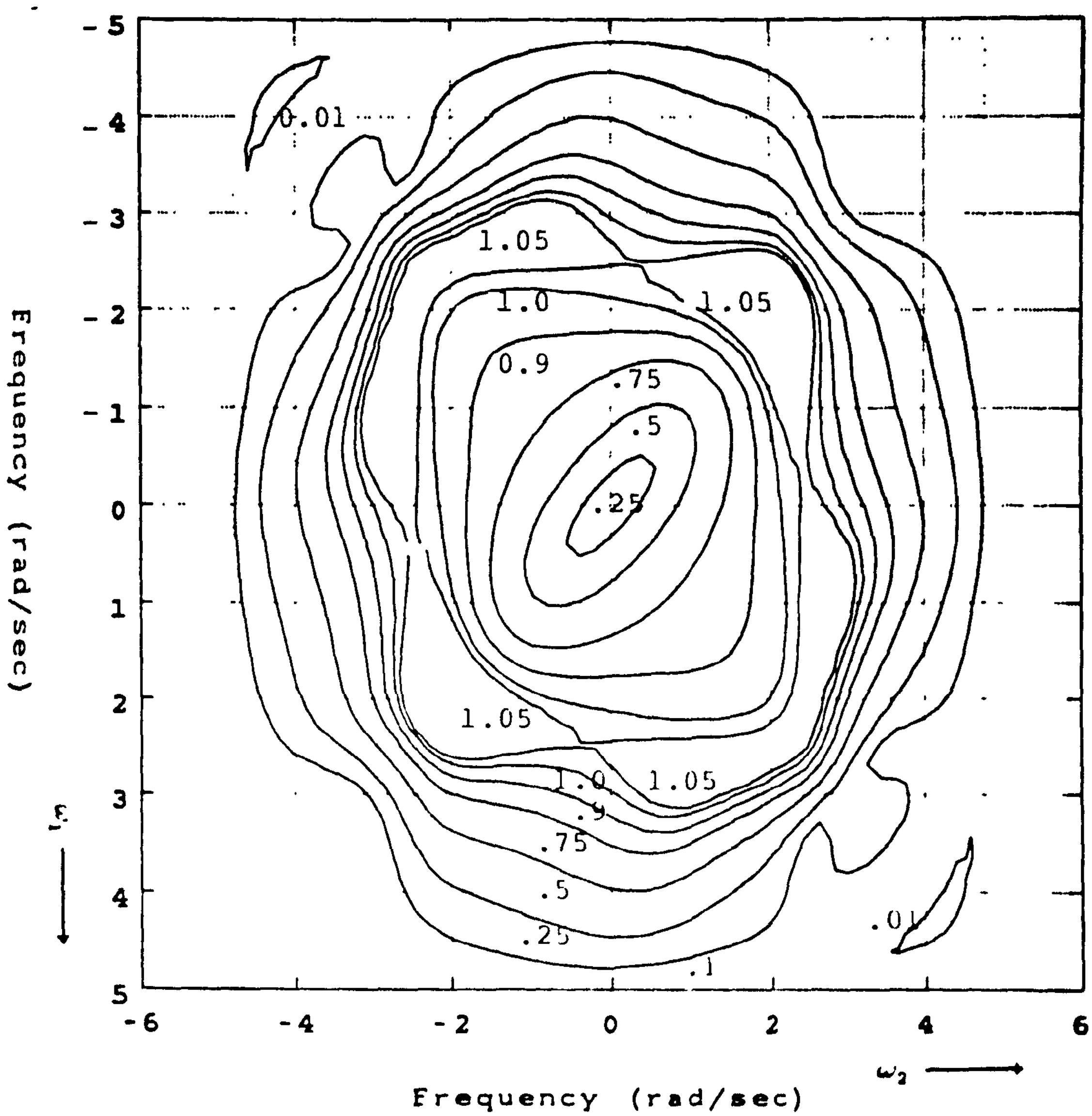


Fig. 4.2(d) Contour Plot of the Magnitude Frequency Response of the Designed Bandpass Filter of Example 3.2(b) with Six-Decimal Places of Coefficients

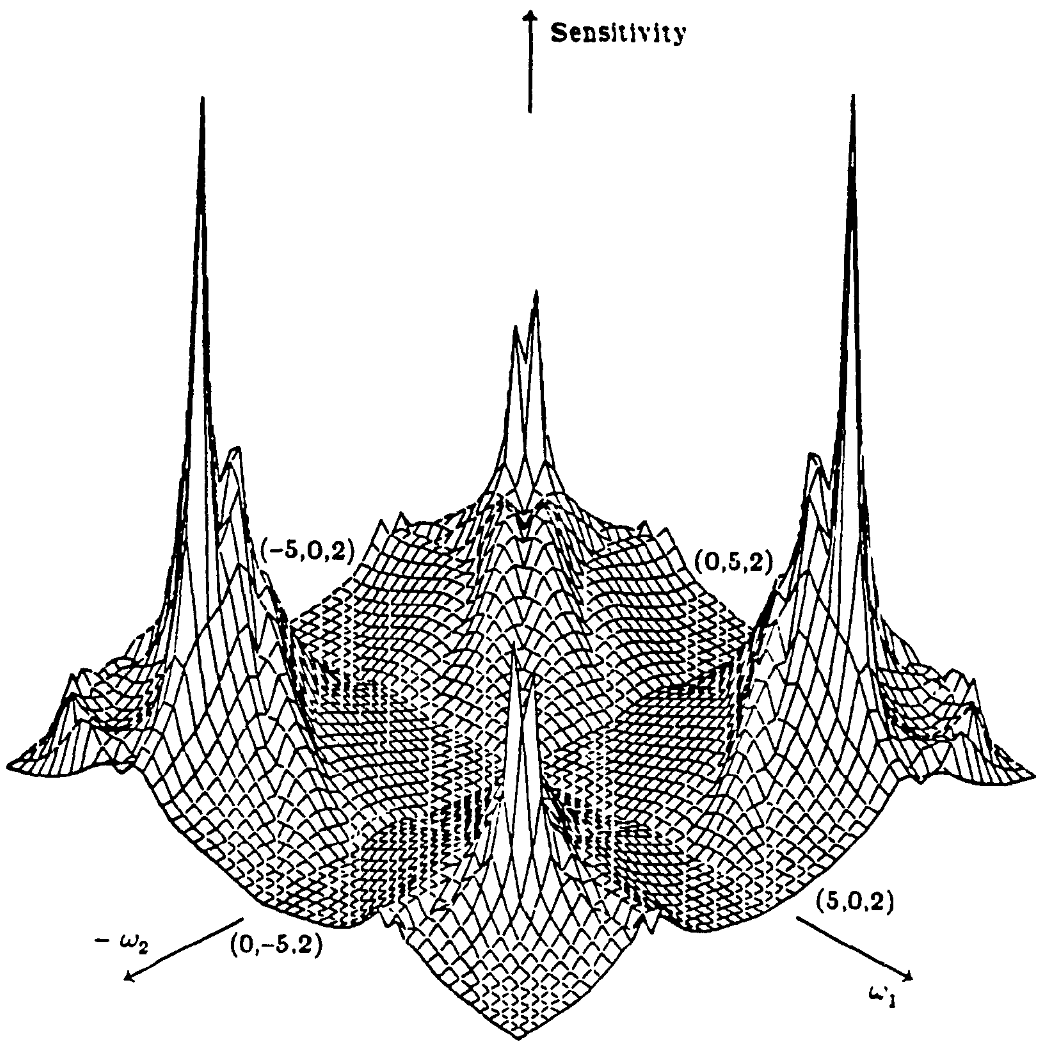


Fig. 4.3(a). Sensitivity-Frequency Response of the 2-D Bandpass Filter in Example 3.2(b) with Four-Decimal Places of Coefficients.

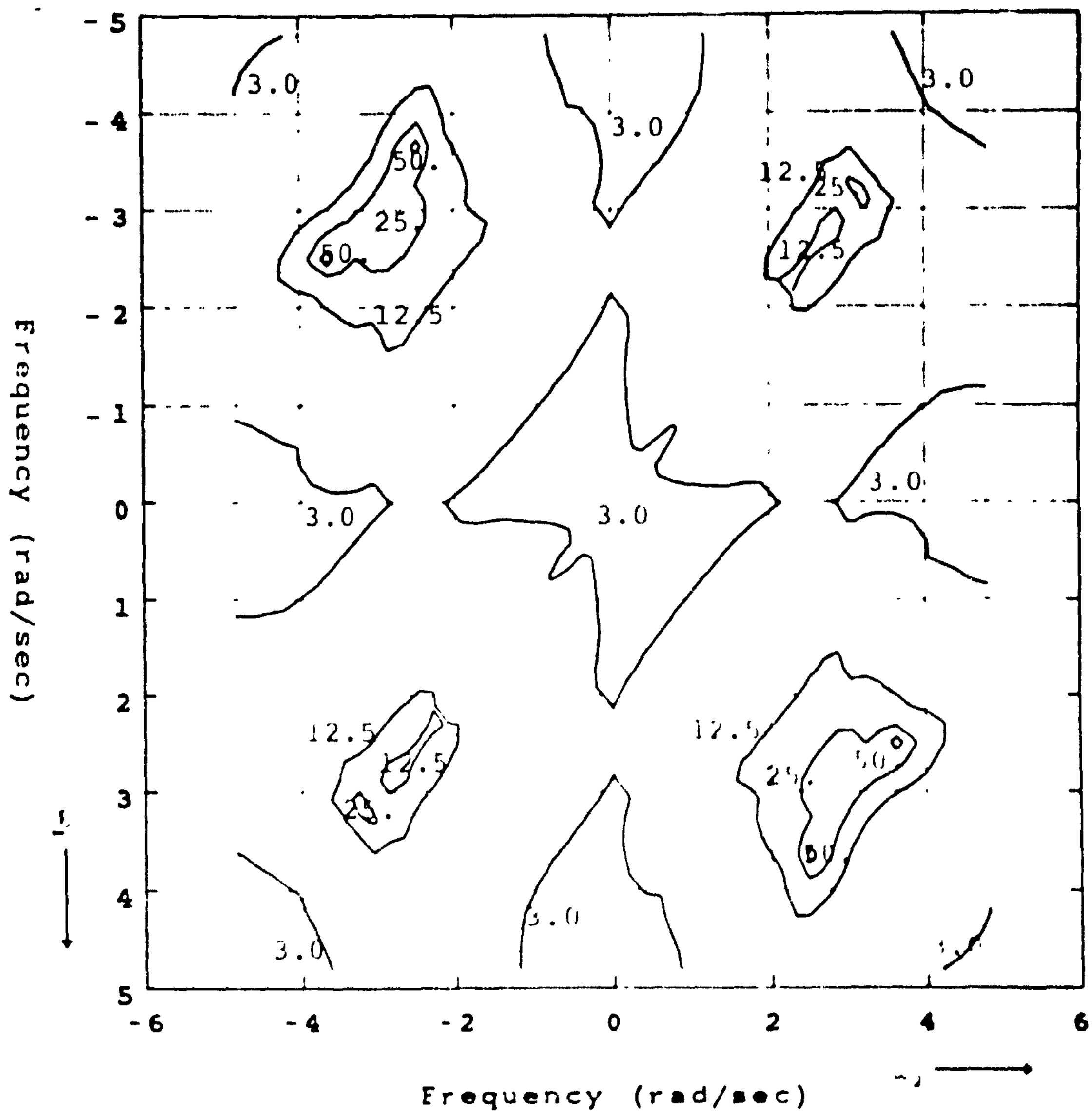


Fig. 4.3(b) Contour Plot of the Sensitivity Frequency Response of the Designed Bandpass Filter of Example 4.2(b) with Four Decimal Places of Coefficients

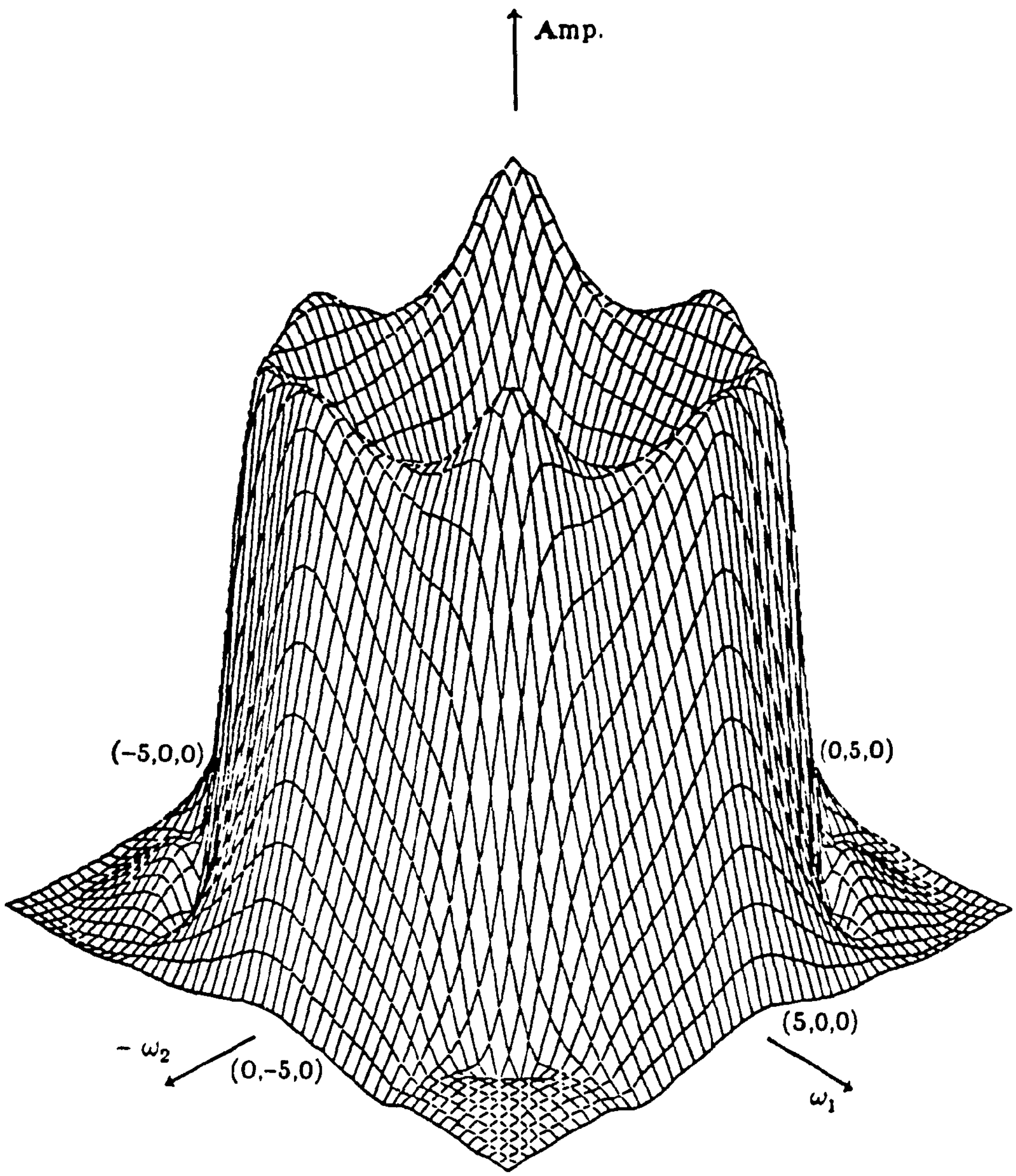


Fig. 4.3(c). Magnitude-Frequency Response of the 2-D Bandpass Filter in Example 3.2(b) with Four-Decimal Places of Coefficients

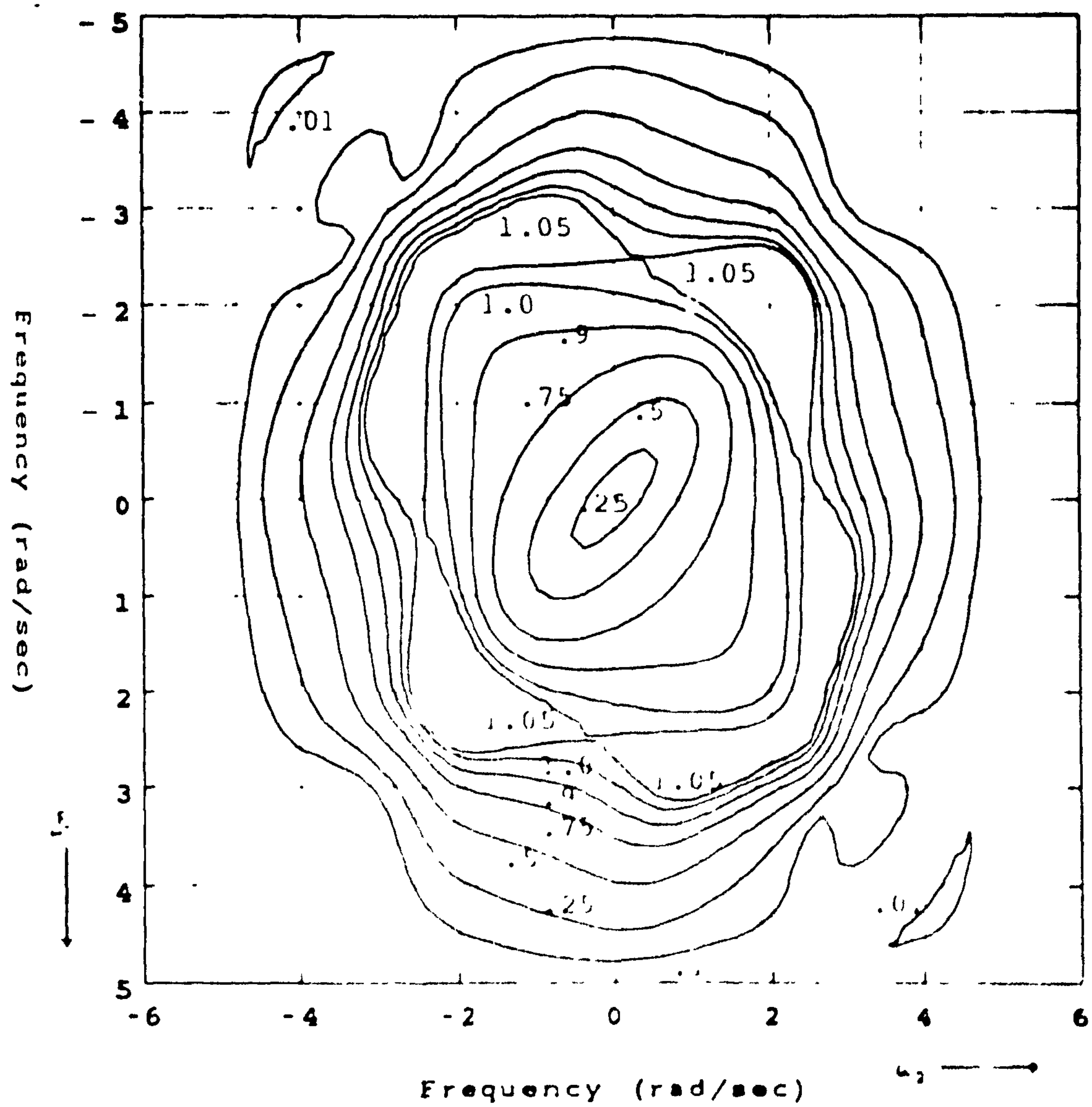


Fig. 4.3(d) Contour Plot of the Magnitude Frequency Response

of the Designed Bandpass Filter of Example 4.2

[illegible]

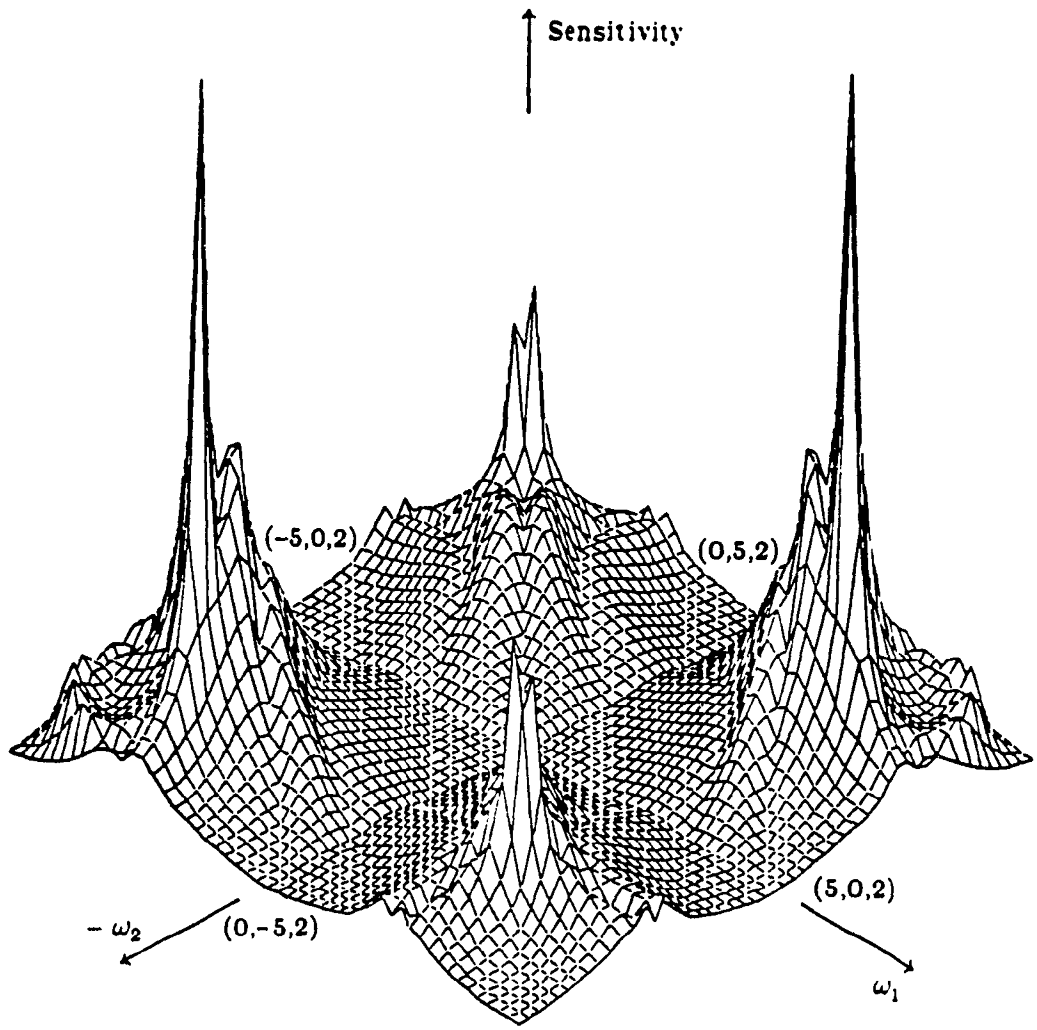


Fig. 4.4(a). Sensitivity-Frequency Response of the 2-D Bandpass Filter in Example 3.2(b) with Three-Decimal Places of Coefficients

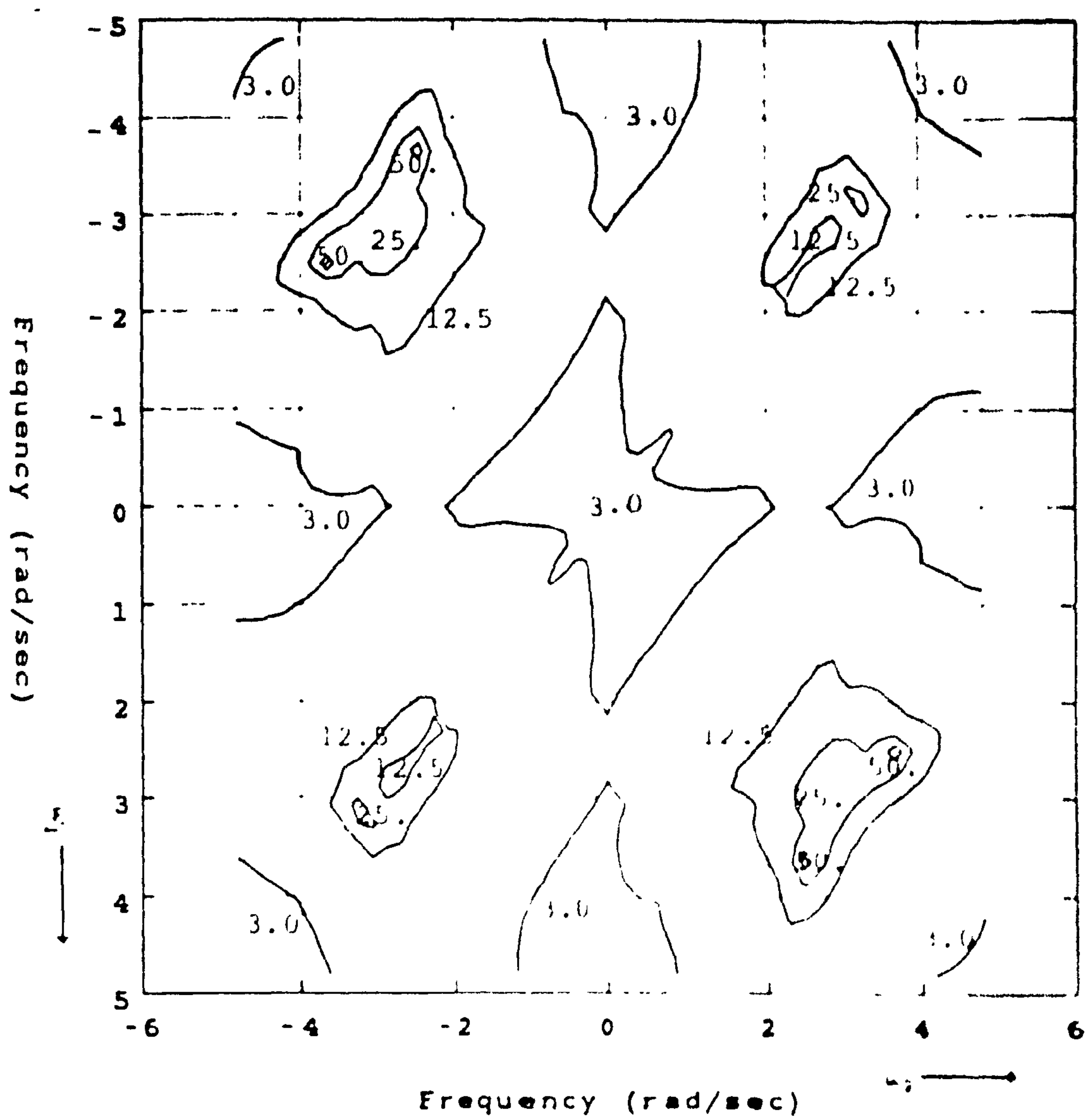


Fig. [illegible] Contour Plot of the Sensitivity Frequency Response of the Designed Bandpass Filter of Example [illegible] with Three Decimal Places of Coefficients

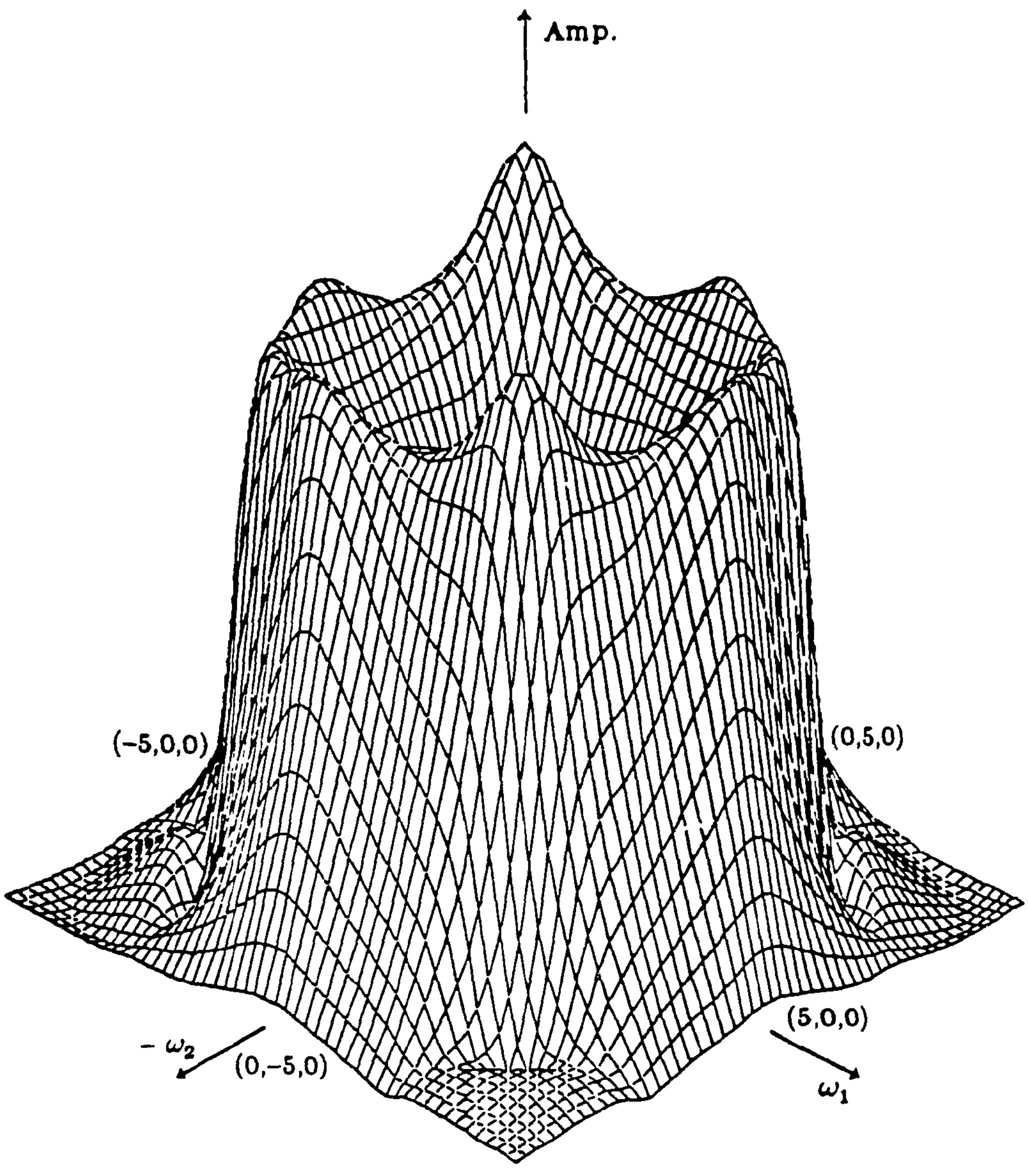


Fig. 4.4(c). Magnitude-Frequency Response of the 2-D Bandpass Filter in Example 3.2(b) with Three-Decimal Places of Coefficients.

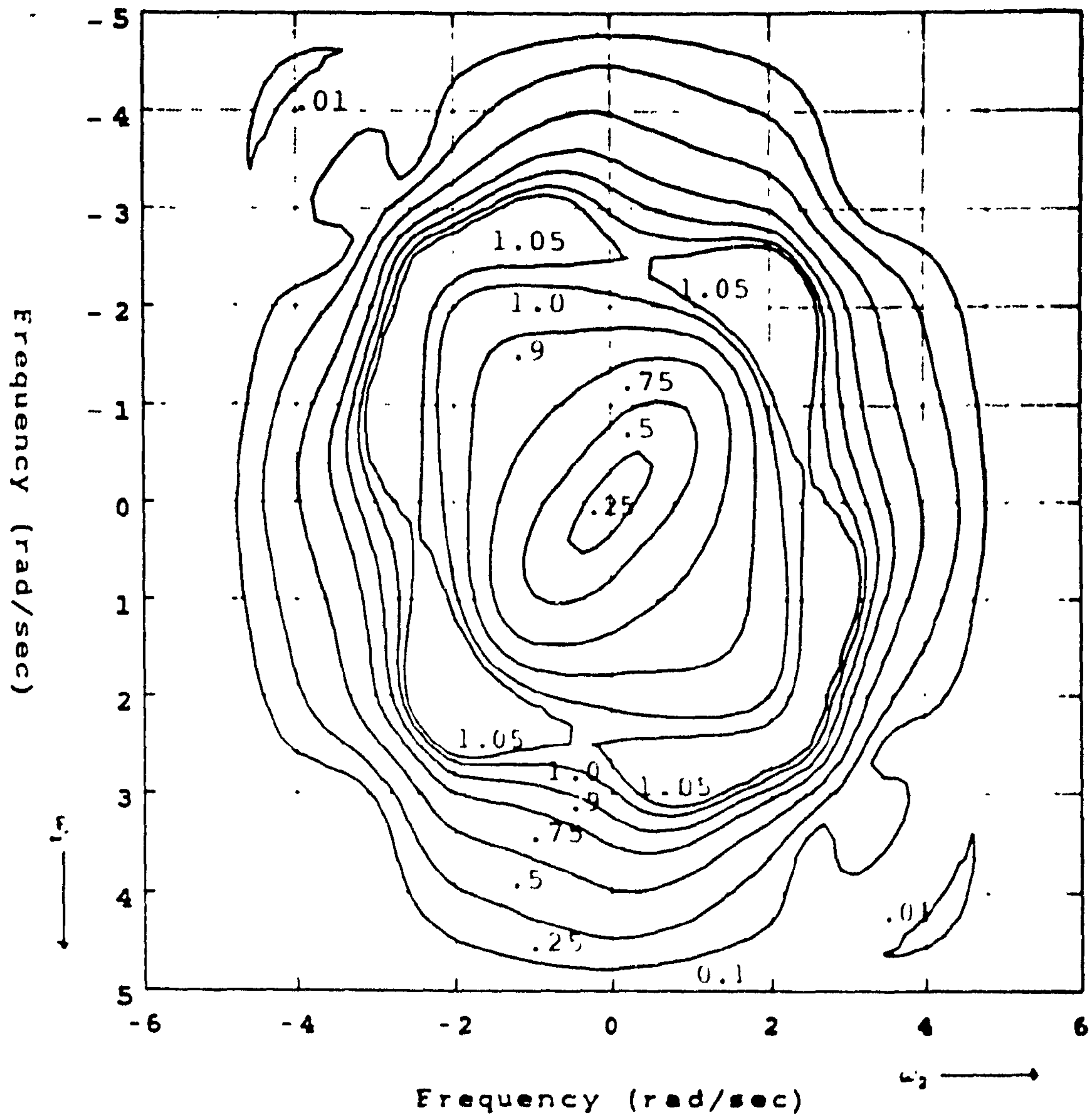


Fig. 4.4(d) Contour Plot of the Magnitude Frequency Response of the Designed Bandpass Filter of Example 4.2(b) with Three Decimal Places of Coefficients

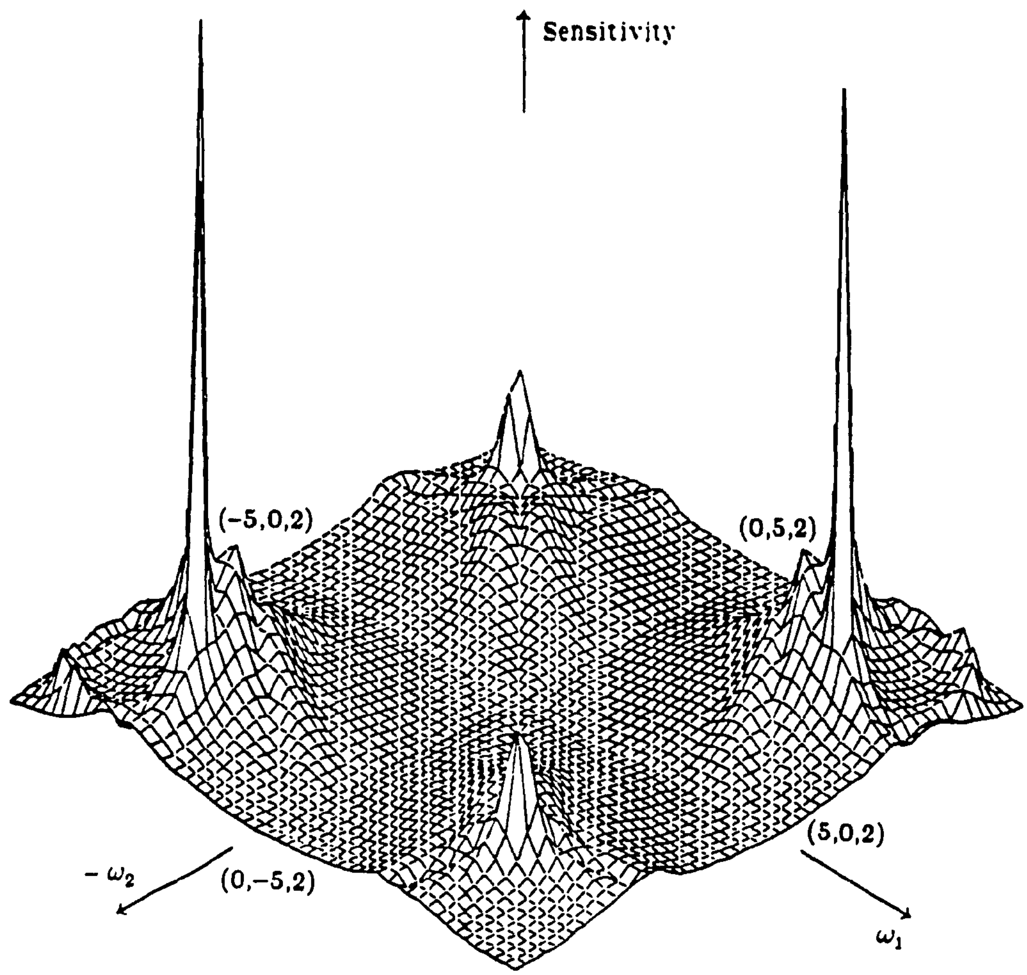


Fig. 4.5(a). Sensitivity-Frequency Response of the 2-D Bandpass Filter in Example 3.2(b) with Two-Decimal Places of Coefficients.

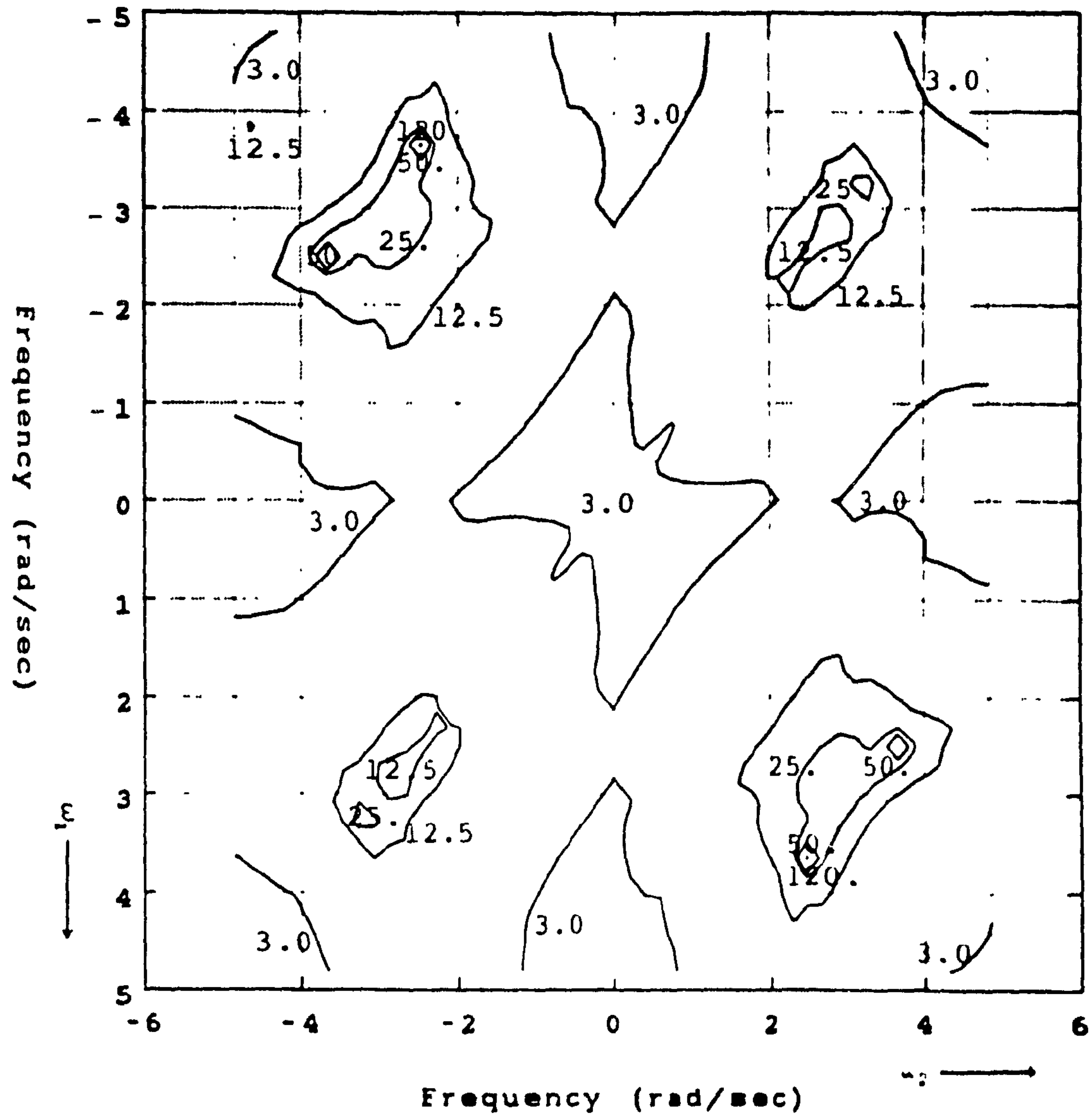


Fig 4.5(b) Contour Plot of the Sensitivity-Frequency Respone of the Designed Bandpass Filter of Example 3.2(b) with Two-Decimal Places of Coefficients

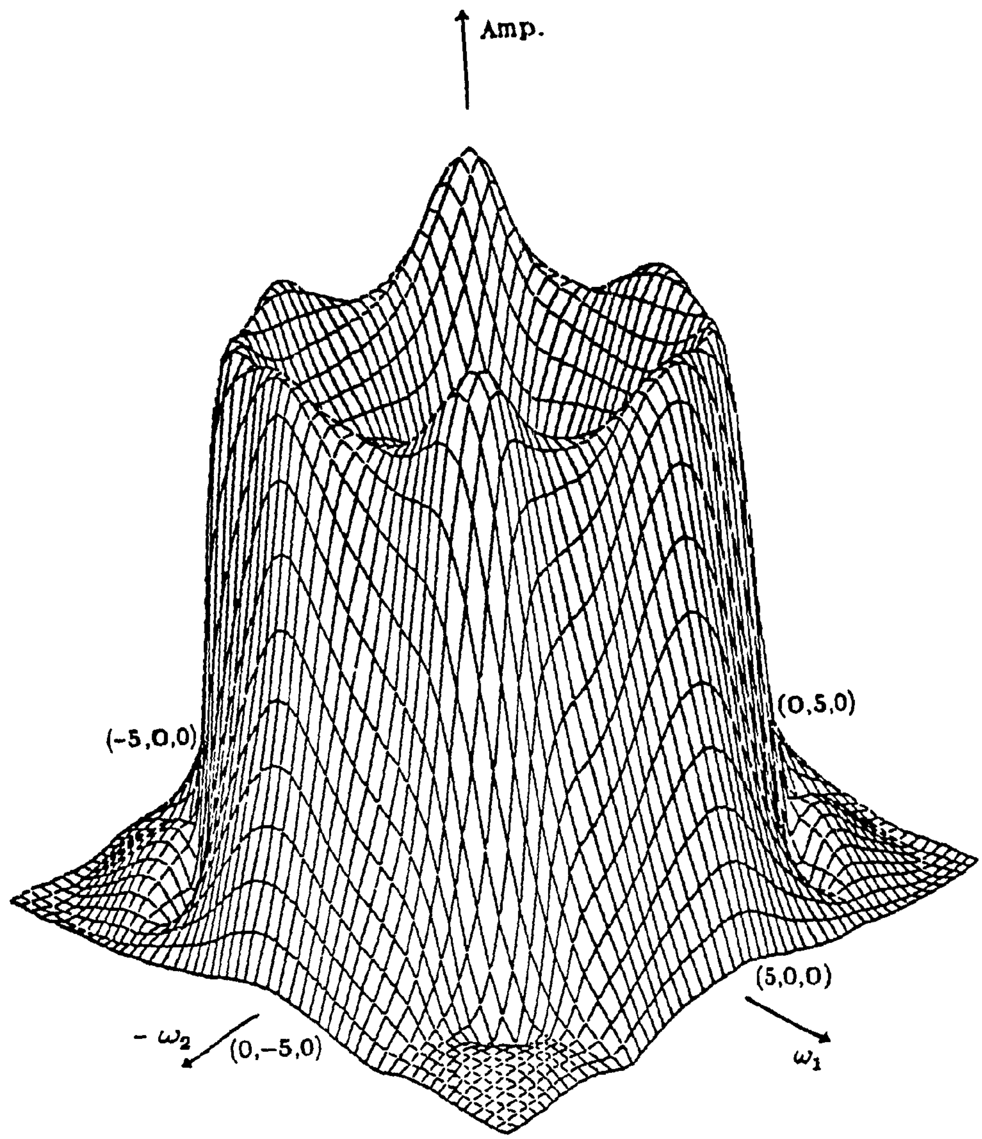


Fig. 4.5(c). Magnitude-Frequency Response of the 2-D Bandpass Filter in Example 3.2(b) with Two-Decimal Places of Coefficients.

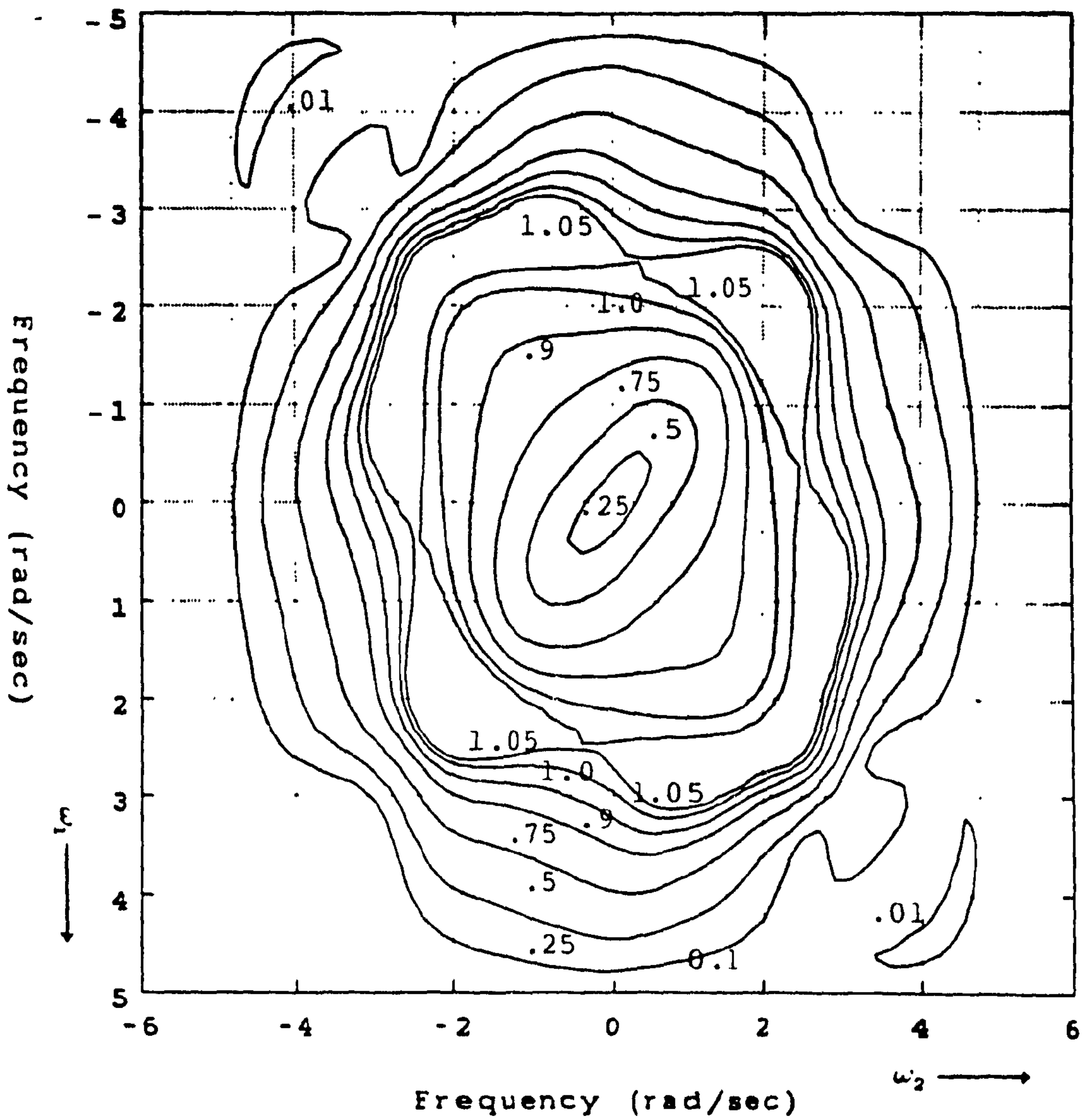


Fig. 4.5(d) Contour Plot of the Magnitude-Frequency Respone of the Designed Bandpass Filter of Example 3.2(b) with Two-Decimal Places of Coefficients

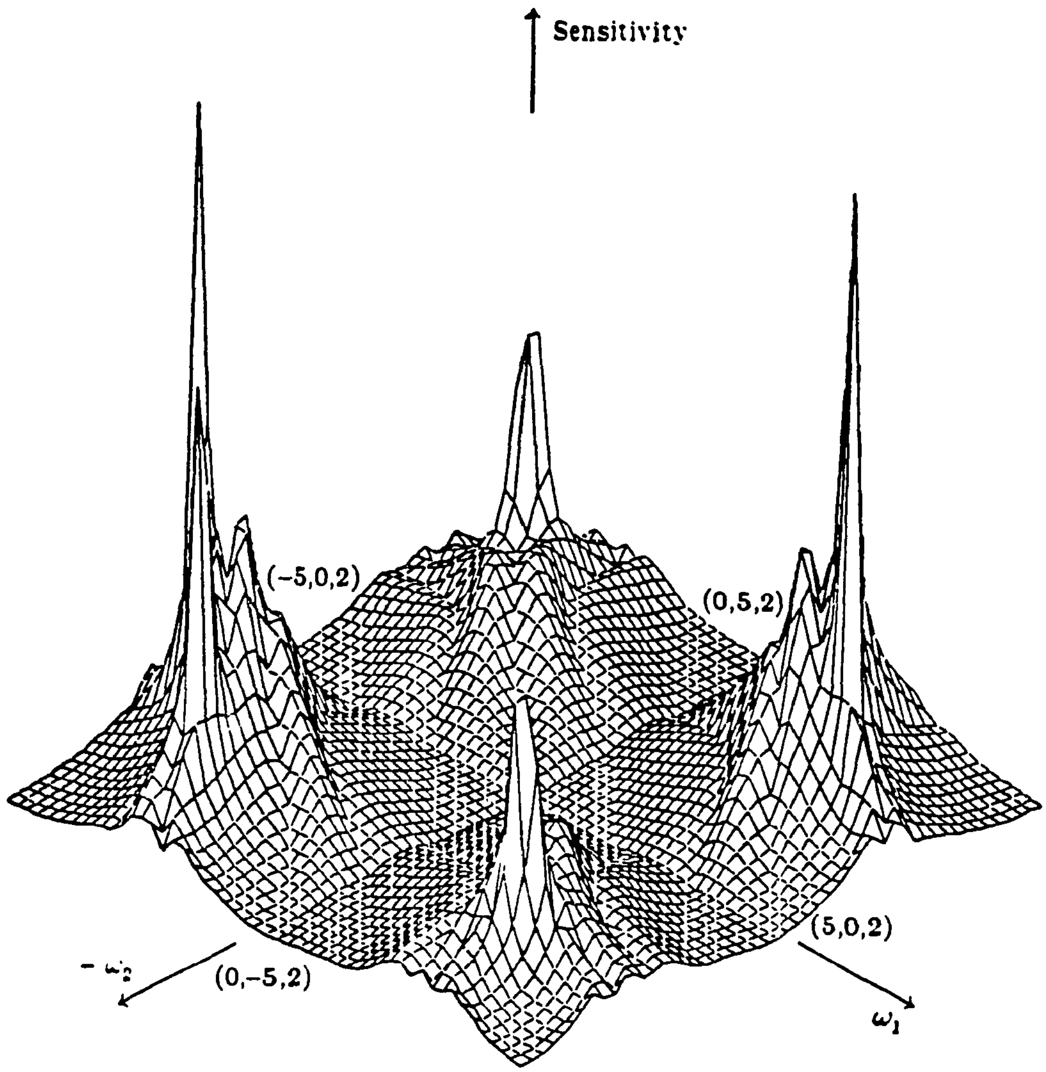


Fig. 4.6(a). Sensitivity-Frequency Response of the 2-D Bandpass Filter in Example 3.2(b) with One-Decimal Place of Coefficients.

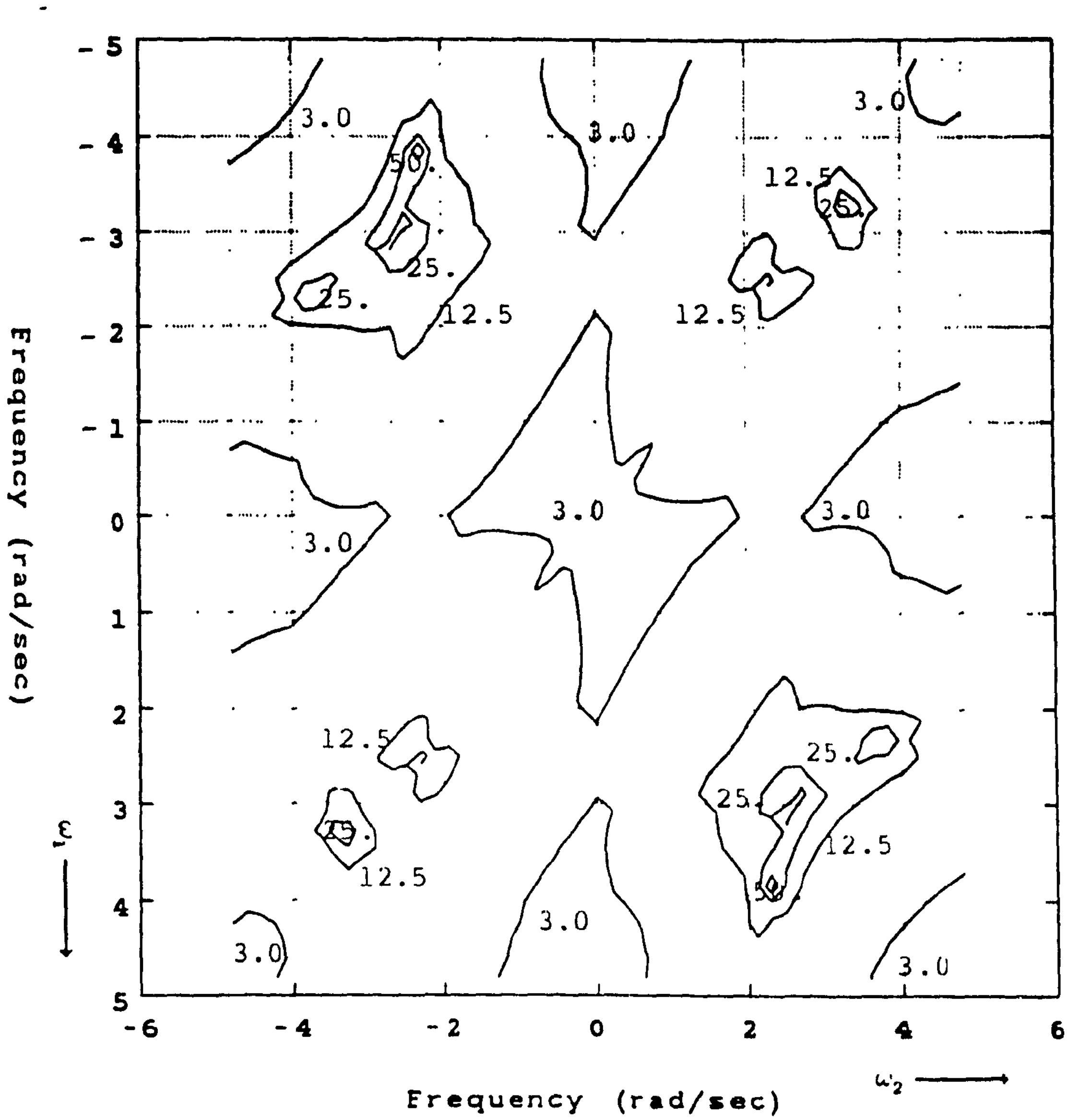


Fig 4.6(b). Contour Plot of the Sensitivity-Frequency Respone of the Designed Bandpass Filter of Example 3.2(b) with One-Decimal Place of Coefficients

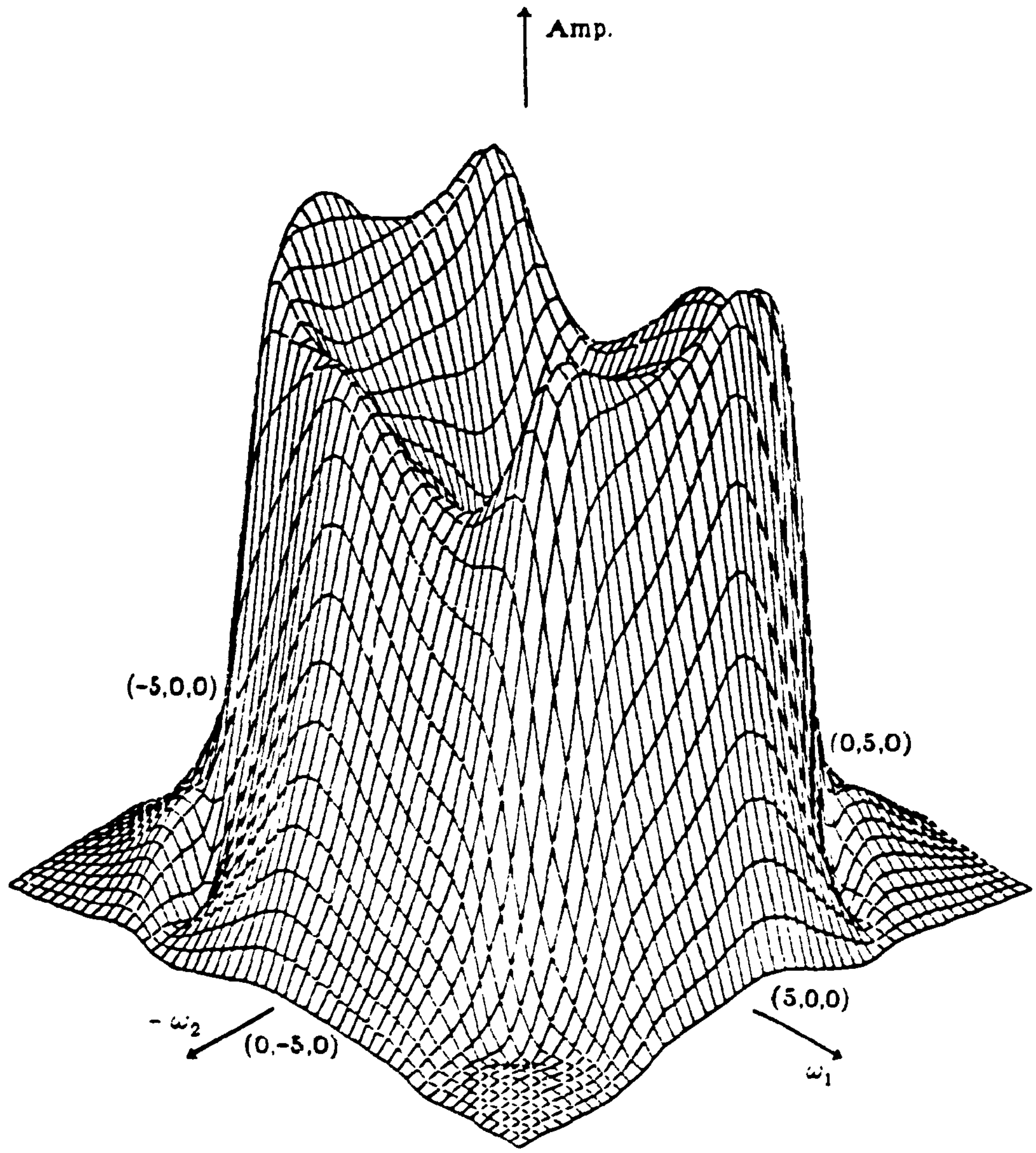
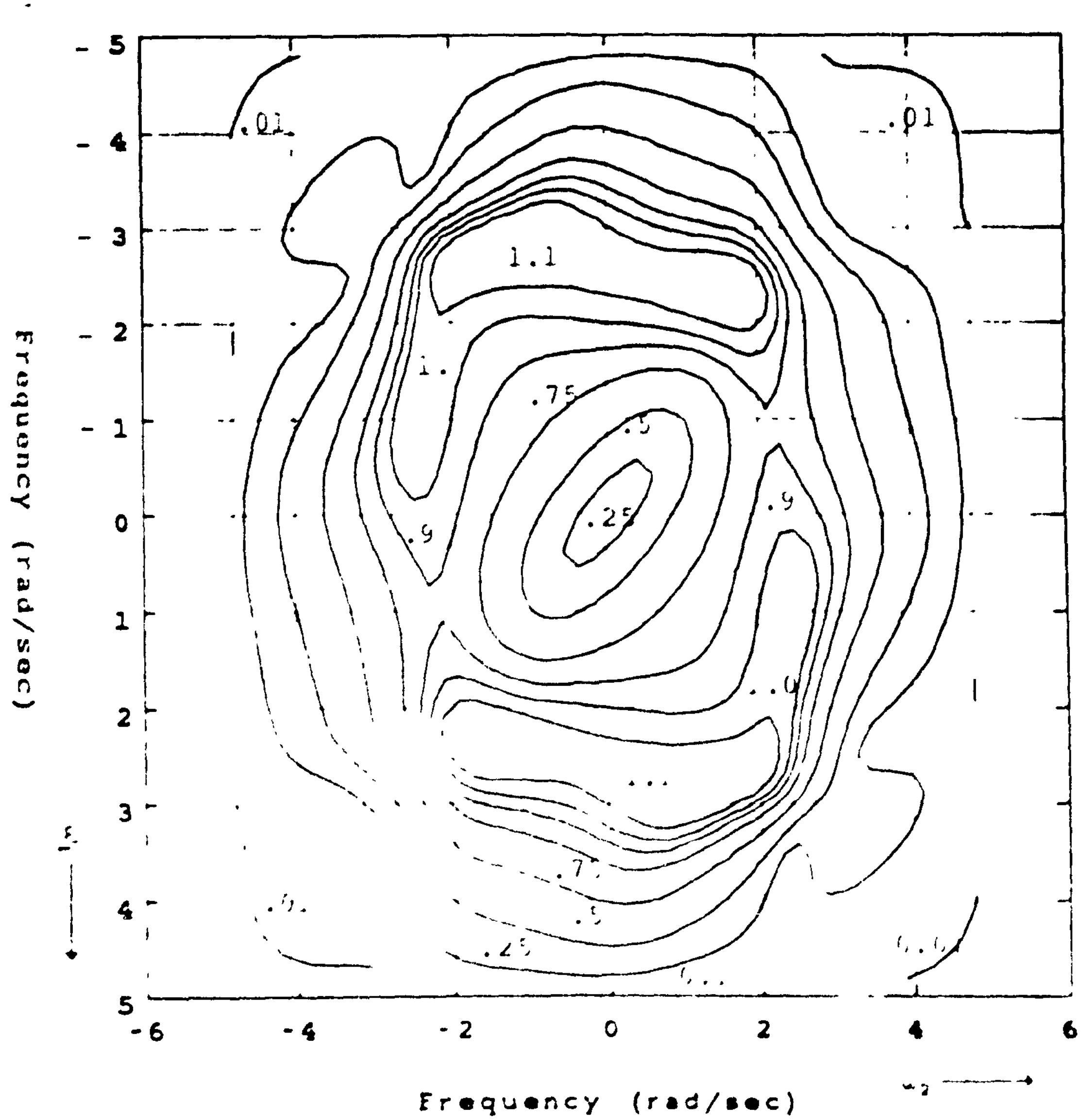


Fig 4 6(c) Magnitude-Frequency Response of the 2-D Bandpass Filter in Example 3.2(b) with One-Decimal Place of Coefficients



[illegible] Contour Plot of the Magnitude Frequency Response [illegible]

[illegible]

[illegible]

4.4. SUMMARY AND DISCUSSION

In this chapter, we have considered the effect of coefficient quantization of 2-D recursive digital filters designed in Chapter III, under floating point arithmetic. By using the maximum sensitivity S as a sensitivity measure, a sensitivity comparison of the parameters of various designs with different word-lengths has been undertaken. The results show that the suggested technique in Chapter II (skew-symmetric matrix structures), requires very low precision up to 3 (in some cases only up to 2) decimal places of unsigned mantissa.

CHAPTER V

CONCLUSIONS AND FUTURE WORK

5.1. CONCLUSIONS

In Chapter II, a new technique for the generation of two variable VSHPs has been presented. The technique is based on the extension and generalization of the previous methods. This has been possible because of the recognition of some useful properties of the matrices encountered in the development of the technique. Polynomials generated by this technique have more number of variables associated with any coefficient and hence the designed filters are expected to be of lower sensitivity as compared with the previous ones. Also, the number of variables in the polynomials can be increased by increasing the order of the matrix A. Further, the presented method is simple to implement.

The application of such polynomials for the design of stable quarter plane two-dimensional recursive digital filters were considered in Chapter III. The design techniques make use of nonlinear optimization for the approximation of desired magnitude frequency response. The design examples presented in Chapter III have shown that VSHPs in conjunction with nonlinear optimization can successfully be used to design 2-D recursive digital filters. The stability of the designed filters is guaranteed with nonsingularity of the matrix A as a precondition. Since in image processing applications the filter phase linearity is important, it is demonstrated in Chapter III that the method of Chapter II and nonlinear optimization design can successfully be employed for the simultaneous approximation of both magnitude and constant group delay (linear phase) characteristics. In this work, with 21 × 11 points of frequency [illegible] specifications, the order of the filters was chosen [illegible] 16 coefficients in the numerator and 2 × 12 in the denominator. However, because [illegible] large [illegible] the range of the order of the filters that can be designed using this technique is not limited.

The Chapter III of the thesis, also considers the design of 1-D recursive digital filters to be as special case to which the method of Chapter II is applicable. This will enable one to obtain a large number of variables, thereby introducing the flexibility in optimization.

The coefficients of these filters were initially based on full precision approximation. A practical algorithm is described in Chapter III for the design of 2-D as well as 1-D recursive digital filters with integer coefficients to meet arbitrary response specifications of the magnitude characteristic. Unlike most of the other techniques of discretization of coefficients of a filter having full precision word-length (called continuous problem) the proposed algorithm is very efficient although it might be suboptimal. For example the application of the "branch and bound" technique for nonlinear discrete optimization due to Dakin [86], for choosing the coefficients of a digital filter with finite word-length is very much time consuming as one might have to search its entire tree structure. The numerical performance of the algorithm is illustrated through examples both in 2-D as well as 1-D. In our design examples the value of $p=2$ was used. However, any value of p in the range $1<p<\infty$ can be used, but in general it will require much more time to reach the optimum.

A sensitivity analysis given in Chapter IV has shown that another advantage of the proposed design technique for the desired filters is that the filters are less sensitive to the parameter perturbations. This implies that rounding of the coefficients of the filters in Chapter III does not alter the performance of the filters appreciably

5.2. EXTENSIONS

This thesis has presented a technique of generating 2-variable VSHP to be used in the design of stable two-dimensional (1-D as special case) quarter plane recursive digital filters. As an extension, one can examine the use of this technique to the design of 2-D half plane recursive digital filters. The change in the frequency grid points for these filters should be taken into account. One might also consider the application of 2-D recursive filters in the area of digital image processing. The Fletcher-Powell optimization technique [60] used in this thesis is one of the best known second-order methods. However, the main drawback of this method is that it requires a large amount of core memory if the number of optimization parameters is large [98]. This deficiency can be overcome by adopting the *conjugate gradient method* developed by Fletcher and Reeves [99]. In all the steepest-descent methods, the gradient of the error function plays an essential role in the optimization process. Since at each iteration the gradient must be computed, the computation time could be quite large, even for moderate number of parameters. To circumvent this problem, one can implement the properties of the adjoint networks [98,100] to compute the gradient vector of the error function. It is worthwhile to consider the design of separable denominator 2-D filters with circularly symmetric responses where the numerator coefficients are $a(i,j)=a(N-i,j)=a(i,N-j)=a(N-i,N-j)$, and octagonally symmetric responses where the coefficients are $a(i,j)=a(j,i)$. Three term separable denominator (3-TSD) $D(z_1,z_2)=D_1(z_1)D_2(z_2)D_3(z_1z_2)$ 2-D filters can also be designed. Further, the extension of the technique presented here can be explored to the design of 3-D filters.

References

[1] *Transmission Systems for Communications.* 4th edition, North Carolina: Bell Telephone Laboratories, Western Electric Technical Publications, Winston-Salem, 1970.

[2] L.R. Rabiner and B. Gold, *Theory and Application of Digital Signal Processing.* Englewood Cliffs, New Jersey: Prentice-Hall, 1975.

[3] D.E. Dudgeon and R.M. Mersereau, *Multidimensional Digital Signal Processing.* Englewood Cliffs, New jersey: Prentice Hall, 1984.

[4] K. Hirano, M. Sabene, and M.Z. Mulk, "Design of Three-Dimensional Recursive Digital Filters," *IEEE Trans. Circuits Syst.*, vol. CAS-31, pp. 550-61, June 1984.

[5] R.M. Mersereau, W.F.G. Mecklenbräuker, and T.F. Quatieri, Jr., "McClellan Transformations for Two-Dimensional Digital Filtering : I- Design," *IEEE Trans. Circuits Syst.*, vol. CAS-23, no. 7, July 1976.

[6] R.M. Mersereau and D.E. Dudgeon, "Two-Dimensional Digital Filtering," *Proc. IEEE,* vol. 63, no. 4, pp. 610-623, April 1975.

[7] R.E. Twogood and M.P. Ekstrom, "Why Filter Recursively in Two-Dimensions ?," *Proc. Int. Conf. Acoust., Speech, Sig. Proc.*, pp. 20-23, April 1979.

[8] G.A. Baxes, *Digital Image Processing.* Englewood Cliffs, New Jersey: Prentice-Hall, 1984.

[9] R.C. Gonzalez and P. Wintz, *Digital Image Processing.* Reading, Massachusetts: Addison-Wesley, 1977.

[10] J.K. Aggarwal, *Digital Signal Processing.* North Hollywood, California: Western Periodicals, 1979.

[11] J.L. Shanks, S. Treitel, and J.H. Justice, "Stability and Synthesis of Two-Dimensional Recursive Filters," *IEEE Trans. Audio Electroacoust.*, vol. AU-20, no. 2, pp. 115-128, June 1972.

[12] T.S. Huang, "Stability of Two-Dimensional Recursive Filters," *IEEE Trans. Audio Electroacoust.*, vol. AU-20, no. 2, pp. 158-163, June 1972.

[13] T.S. Huang, J.W. Burnett, and A.G. Deczky, "The Importance of Phase in Image Processing Filters," *IEEE Trans. Acoust., Speech, Sig. Proc.*, vol. ASSP-23, no. 6, pp. 529-542, Dec. 1975.

[14] B. Gold and C.M. Rader, *Digital Processing of Signals*. New York: McGraw-Hill, 1969.

[15] R.C. Agarwal and C.S. Burrus, "Fast Convolution Using Fermat Number Transforms with Applications to Digital Filtering," *IEEE Trans. Acoust., Speech, Sig. Proc.*, vol. ASSP-22, no. 2, April 1974.

[16] A.V. Oppenheim and C. Weinstein, "Effects of Finite Register Length in Digital Filtering and the Fast Fourier Transform," *Proc. IEEE*, vol. 60, pp. 957-976, Aug. 1972.

[17] W.F.G. Mecklenbrauker and R.M. Mersereau, "McClellan Transformations for Two-Dimensional Digital Filtering: II. Implementation," *IEEE Trans. Circuits Syst.*, vol. CAS-23, pp. 414-422, July 1976.

[18] S. Winograd, "On Computing the Discrete Fourier Transform," *Mathematics of Computation*, vol. 32, no. 141, pp. 175-199, Jan. 1978.

[19] J.H. McClellan and C.M. Rader, *Number Theory in Digital Signal Processing*. Englewood Cliffs, New Jersey: Prentice-Hall, 1979.

[20] H. Nussbaumer, "Digital Filtering Using Complex Mersenne Transforms," *IBM J. Res. Develop.*, Dec. 197[illegible].

[21] T.S. Huang, "Two-Dimensional Windows," *IEEE Trans. Audio Electroacoust.*, vol. AU-20, no. 1, pp. 88-90, Mar. 1972.

[22] J.H. McClellan, "The Design of Two-Dimensional Digital Filters by Transformations," *Proc. 7th Annual Princeton Conf. Info. Sci. Syst.*, pp. 247-251, 1973.

[23] J.V. Hu and L.R. Rabiner, "Design Techniques for Two-Dimensional Digital Filters," *IEEE Trans. Audio Electroacoust.*, vol. AU-20, no. 4, pp. 249-257, Oct. 1972.

[24] D.B. Harris and R.M. Mersereau, "A Comparison of Iterative Methods for Optimal Two-Dimensional Filter Design," *Proc. Int. Conf. Acoust., Speech, Sig. Proc.*, pp. 527-530, May 1977.

[25] T.S. Huang (editor), *Two-Dimensional Digital Signal Processing I*. Springer-Verlag, 1981.

[26] J.M. Costa and A.N. Venetsanopoulos, "Design of Circularly Symmetric Two-Dimensional Recursive Filters," *IEEE Trans. Acoust., Speech, Sig. Proc.*, vol. ASSP-22, pp. 432-443, Dec. 1974.

[27] D.M. Goodman, "A Design Technique for Circularly Symmetric Low-Pass Filters," *IEEE Trans. Acoust., Speech, Sig. Proc.*, vol. ASSP-26, no. 4, pp. 290-304, Aug. 1978.

[28] M. Ahmadi, A.G. Constantinides, and R.A. King, "Design Technique for a Class of Stable Two-Dimensional Recursive Digital Filters," *Proc. IEEE Int. Conf. Acoust., Speech, Sig. Proc.*, April 1976.

[29] H. Chang and J.K. Aggarwal, "Design of Two-Dimensional Recursive Filters by Interpolation," *IEEE Trans. Circuits Syst.*, vol. CAS-24, pp. 281-291, June 1977

[30] S. Chakrabarti and S.K. Mitra, "Design of Two-Dimensional Digital Filters via Spectral Transformations," *Proc. IEEE*, June 1977.

[31] F. Bernabo, P.L. Emiliani, and V. Cappellini, "Design of 2-Dimensional Recursive Digital Filters," *Electron. Lett.*, vol. 12, no. 11, pp. 288-289, May 1976.

[32] G.A. Maria and M.M. Fahmy, "An l_p-Design Technique for Two-Dimensional Digital Recursive Filters," *IEEE Trans. Acoust., Speech, Sig. Proc.*, vol. ASSP-22, no. 1, pp. 15-21, Feb. 1974

[33] ______________, "l_p Approximation of the Group Delay Response of One- and Two-Dimensional Filters," *IEEE Trans. Circuits Syst.*, vol. CAS-21, no. 3, pp. 431-436, May 1974.

[34] A. Chottera and G.A. Jullien, "Designing Near Linear Phase Recursive Filters Using Linear Programming," *Proc. IEEE Int. Conf. Acoust., Speech, Sig. Proc.*, pp. 88-92, May 1977.

[35] S.A.H. Aly and M.M. Fahmy, "Design of Two-Dimensional Recursive Digital Filters with Specified Magnitude and Group Delay Characteristics," *IEEE Trans. Circuits Syst.*, vol. CAS-25, no. 11, pp. 908-915, Nov. 1978

[36] P. Karivaratharajan and M.N.S. Swamy, "Quadrantal Symmetry Associated with Two-Dimensional Digital Transfer Functions," *IEEE Trans. Circuits Syst.*, vol. CAS-25, no. 6, pp. 340-343, June 1978

[37] P. Karivaratharajan and M.N.S. Swamy, "Design of Separable Denominator 2-Dimensional Digital Filters Possessing Real Circularly Symmetric Frequency Response," *IEE Proc.*, vol. 129, Pt. G, no. 5, pp. 235-240, Oct. 1982

[38] C. Charalambous, "Design of 2-Dimensional Circularly Symmetric Digital Filters," *IEE Proc.*, vol. 129, Pt. G, no. 2, pp. 47-54, April 1982

[39] T. Koga, "Synthesis of Finite Passive N-ports with Prescribed Positive Real Matrices of Several Variables," *IEEE Trans. Circuit Theory*, vol. CT-15, no. 1, pp. 2-23, Mar. 1968

[40] E. Dubois and M.L. Blostein, "A Circuit Analysis Method for the Design of Recursive Two-Dimensional Digital Filters," *Proc. IEEE Int. Symp. Circuits Syst.*, pp. 451-454, 1975.

[41] P.A. Ramamoorthy and L.T. Bruton, "Design of Stable Two-Dimensional Analogue and Digital Filters with Applications in Image Processing," *Int. J. Circuit Theory Appl.*, vol. 7, no. 2, pp. 229-245, April 1979.

[42] T.S. Huang, W.F. Schreiber, and O. Tretiak, "Image Processing," *Proc. IEEE*, vol. 59, pp. 1586-1609, November 1971.

[43] M. Morf, B.C. Levy, and S.Y. Kung, "2-D Polynomial Matrices, Factorization, and Coprimeness," *Proc. IEEE*, vol. 65, pp. 861-872, 1977.

[44] H.G. Ansell, "On Certain Two-Variable Generalizations of Circuit Theory, with Applications to Networks of Transmission Lines and Lumped Reactances," *IEEE Trans. Circuit Theory*, vol. CT-11, pp. 214-223, June 1964.

[45] B.D.O. Anderson and E.I. Jury, "Stability Test for Two-Dimensional Recursive Filters," *IEEE Trans. Audio Electroacoust.*, vol. AU-21, pp. 366--372, Aug. 1973.

[46] E.I. Jury, "Theory and Application of Inners," *Proc. IEEE*, vol. 63, pp. 1044-1068, July 1975.

[47] E.I. Jury, "Stability of Multidimensional Scalar and Matrix Polynomials," *Proc. IEEE*, vol. 66, no. 9, pp. 1018-1047, Sept. 1978.

[48] G.A. Maria and M.M. Fahmy, "On the Stability of Two-Dimensional Digital Filters," *IEEE Trans. Audio Electroacoust.*, vol. AU-21, pp. 470-472, Oct. 1973.

[49] M.P. Ekstrom and R.E. Twogood, "A Stability Test for 2-D Recursive Digital Filters Using the Complex Cepstrum," *Proc. IEEE*, vol. 65, pp. 535-538, May 1977.

[50] R.R. Read and S. Treitel, "The Stabilization of Two-Dimensional Recursive Filters via the Discrete Hilbert Transform," *IEEE Trans. Geosci. Electron.*, vol. GE-11,

no. 3, pp. 153-160, July 1973 and no. 4, pp. 205-207, Oct. 1973.

[51] E.I. Jury, V.R. Kolavennu, and B.D.O. Anderson, "Stabilization of Certain Two-Dimensional Recursive Digital Filters," *Proc. IEEE,* vol. 65, no. 6, pp. 887-892, June 1977.

[52] N. Nagamuthu, *Transformation Design of 2-D Recursive Digital Filters Using Spectral Factorization.* A Masters Thesis, Univ. of Windsor, Canada, 1982.

[53] J.W. Woods, "A Counter Example to Read and Treitel's Two-Dimensional Recursive Filter Stabilization Procedure," *IEEE Trans. Geosci. Electron.*, vol. GE-11, pp. 104-105, July 1974.

[54] D. Goodman, "Some Stability Properties of Two-Dimensional Linear Shift-Invariant Digital Filters," *IEEE Trans. Circuits Syst.*, vol. CAS-24, no 4, pp. 201-208, April 1977.

[55] M.N.S. Swamy, L.M. Roytman, and E.I. Plotkin, "On Stability Properties of Three- and Higher-Dimensional Linear Shift-Invariant Digital Filters," *IEEE Trans. Circuits Syst.*, vol. CAS-32, no. 9, pp 888-892, Sept 1985

[56] L.M Roytman, M N.S. Swamy, and G. Eichmann, "BIBO Stability in the Presence of Nonessential Singularities of the Second Kind in 2-D Digital Filters," *IEEE Trans. Circuits Syst* , vol. CAS-34, no. 1, pp 60-72, Jan 1987

[57] K.P. Prasad and P S Reddy, "Method for Designing Stable 2-D Digital Filters," *Electron Lett* , vol 13, no 13, pp 385-386, June 1977

[58] P.A. Ramamoorthy and L.T Bruton, "Design of Stable 2-Dimensional Discrete Recursive Filters," *Electron Lett* , vol 12, pp 659-660, 1976

[59] K P Prasad and P S Reddy, "Computer-Aided Design of Stable Recursive Digital Filters from Magnitude Specifications" *Int J Circuit Theory Appl* , vol 7, pp 121-126, 1979

[60] R. Fletcher and M.J.D. Powell, "A Rapidly Convergent Descent Method of Minimization," *Comp. J.*, vol. 6, no. 4, pp. 163-168, 1963.

[61] D. Goodman, "Some Difficulties with the Double Bilinear Transformation in 2-D Recursive Filter Design," *Proc. IEEE*, vol. 66, no. 7, pp. 796-797, July 1978.

[62] P.K. Rajan, H.C. Reddy, M.N.S. Swamy, and V. Ramachandran, "Generation of Two-Dimensional Digital Functions Without Nonessential Singularities of the Second Kind," *IEEE Trans. Acoust., Speech, Sig. Proc.*, vol. ASSP-28, no. 2, pp. 216-223, April 1980.

[63] C.H. Reddy, P.K. Rajan, and M.N.S. Swamy, "Studies on N-Dimensional Filter Transfer Functions Without Second Kind Singularities," *Proc. IEEE Int. Conf. Acoust., Speech, Sig. Proc.*, pp. 753-757, 1980.

[64] M.N.S. Swamy and K.S. Thyagarajan, "A New Type of Wave Digital Filter," *J. Franklin Inst.*, vol. 300, no. 1, pp. 41-58, July 1975.

[65] M.N.S. Swamy, K.S. Thyagarajan, and V. Ramachandran, "Two-Dimensional Wave Digital Filters Using Doubly Terminated Two-Variable LC-Ladder Configurations," *J. Franklin Inst.*, vol. 304, no. 4/5, pp. 201-215, Oct./Nov. 1977.

[66] A. Fettweis, "Digital Filter Structures Related to Classical Filter Networks," *Arch. Elektr. Übertr.*, vol. 25, pp. 79-89, Sept. 1971.

[67] S. Erfani and B. Peikari, "Digital Design of Two-Dimensional LC Structures," *IEEE Trans. Circuits Syst.*, vol. CAS-28, no. 1, pp. 75-77, Jan. 1981.

[68] S. Erfani and B. Peikari, "Digital Design of General LC Structures," *IEEE Trans. Circuits Syst.*, vol. CAS-25, no. 5, pp. 269-273, May 1978.

[69] S. Erfani and B. Peikari, "Variable Cut-Off Digital Ladder Filters," *Int. J. Electron.* vol. 45, no. 5, pp. 535-549, Nov. 1978.

[70] S. Erfani and B. Peikari, "A Note on Digital Simulation of RLC Structures," *IEEE*

Trans. Circuits Syst., vol. CAS-32, no. 6, pp. 600-603, June 1985.

[71] V. Ramachandran and M. Ahmadi, "Design of Stable 2-D Recursive Filters by Generation of VSHP Using Terminated N-Port Gyrator Networks," *J. Franklin Inst.*, vol. 316, no. 5, pp. 373-380, Nov. 1983.

[72] V. Ramachandran and M. Ahmadi, "Design of 2-D Stable Analog and Recursive Digital Filters Using Properties of the Derivative of Even or Odd Parts of Hurwitz Polynomials," *J. Franklin Inst.*, vol. 315, no. 4, pp. 259-267, April 1983.

[73] F.E. Hohn, *Elementary Matrix Algebra.* McMillan Co., 1964.

[74] M. Ahmadi and V. Ramachandran, "New Method of Generating Two-Variable VSHP and Its Application in the Design of Two-Dimensional Recursive Digital Filters with Prescribed Magnitude and Constant Group Delay Responses," *IEE Proc.*, vol. 131, Pt. G, no. 4, pp. 151-155, Aug. 1984.

[75] M. Ahmadi, M.T. Boraie, and V. Ramachandran, "Generation of 2-Variable VSHP Using Properties of the Resistance Matrices and Its Application in Recursive 2-D Filter Design," *Proc. IEEE* 24th *Int. Conf. on Decision and Control*, pp. 1590-1593, Dec. 1985.

[76] M.A. Abiri, M. Ahmadi, and V. Ramachandran, "An Alternative Approach in Generation of Two-Variable VSHP and Its Application in 2-D Recursive Digital Filter Design Satisfying a Prescribed Magnitude With or Without Constant Group Delay Specifications," *Proc. IEEE Int. Conf. Circuits Syst.*, pp. 1093-1096, June 1985.

[77] M.A. Abiri, V. Ramachandran, and M Ahmadi, "An Alternative Approach in Generation of 2-Variable Very Strictly Hurwitz Polynomial (VSHP) and Its Application," *J Franklin Inst*, pp. 187-203, Aug. 1987.

[78] C.H. Reddy, P.K. Rajan, M.N.S Swamy, and C P. Reddy, "Simplified Testing Procedure for Very Strict Hurwitz Polynomials- Some Applications," *Proc. IEEE* 14th *ASILOMAR Conf. Circuits, Syst., Comp*, pp. 124-126, Nov 1980.

[79] L.R. Rabiner, N.Y. Graham, and H.D. Helms, "Linear Programming Design of IIR Digital Filters with Arbitrary Magnitude Function," *IEEE Trans. Acoust., Speech, Sig. Proc.*, vol. ASSP-22, no. 2, pp. 117-123, April 1974.

[80] A. Chottera and G.A. Jullien, "Design of Two-Dimensional Recursive Digital Filters Using Linear Programming," *IEEE Trans. Circuits Syst.*, vol. CAS-29, no. 12, pp. 817-826, Dec. 1982.

[81] M.N.S. Swamy and P.K. Rajan, "Symmetry in Two-Dimensional Filters and Its Application," an article in *Multidimensional Systems: Techniques and Applications*, edited by S.G. Tzafestas, Marcel Dekker, Inc., pp. 401-468, 1986.

[82] A. Rozenfeld, *Picture Processing By Computer.* New York, NY: Academic Press, Inc., 1969.

[83] G. A. Jullien, W. C. Miller, and A. Chottera, "Design Procedures for a Class of Stable Two-Dimensional Recursive Digital Filters," *Proc.* 18th *Midwest Symp. Circuits Syst.*, pp. 309-313, Aug. 1975.

[84] A. Chottera, *Recursive Digital Filters: Design and Applications to Image Processing.* A PhD Dissertation, Univ. of Windsor, Canada, 1979.

[85] A. Antoniou, *Digital Filters: Analysis and Design.* New York: McGraw-Hill, 1979.

[86] R. J. Dakin, "A tree-search algorithm for mixed integer programming problems," *Comp. J.*, vol. 8, pp. 250-255, 1966.

[87] K. Steiglitz, "Designing Short-Word Recursive Digital Filters," *Proc.* 9th *Annu. Allerton Conf. Circuits Syst. Theory*, pp. 778-788, Oct. 1971.

[88] E. Avenhaus, "On the Design of Digital Filters with Coefficients of Limited Word Length," *IEEE Trans. Audio Electroacoust.*, vol. AU-20, pp. 206-212, Aug. 1972.

[89] M. Suk and S.K. Mitra, "Computer-Aided Design of Digital Filters with Finite Word Lengths," *IEEE Trans Audio Electroacoust.*, vol AU-20, no 5, pp. 356-363,

Dec. 1972.

[90] D.S.K. Chan and L.R. Rabiner, "Analysis of Quantization Errors in the Direct Form for Finite Impulse Response Digital Filters," *IEEE Trans. Audio Electroacoust.*, vol. AU-21, pp. 354-366, Aug. 1973.

[91] C. Charalambous and M.J. Best, "Optimization of Recursive Digital Filters with Finite Word Lengths," *IEEE Trans. Acoust., Speech, Sig. Proc.*, vol. ASSP-22, pp. 424-431, Dec. 1974.

[92] R.E. Crochiere, "A New Statistical Approach to the Coefficient Word Length Problem for Digital Filters," *IEEE Trans. Circuits Syst.*, vol. CAS-22, pp. 190-196, Mar. 1975.

[93] R.C. Agarwal and C.S. Burrus, "New Recursive Digital Filter Structures Having Very Low Sensitivity and Roundoff Noise," *IEEE Trans. Circuits Syst.*, vol. CAS-22, no. 12, Dec. 1975.

[94] H.K. Kwan, "On the Problem of Designing IIR Digital Filters with Short Coefficient Word Lengths," *IEEE Trans. Acoust , Speech, Sig. Proc.*, vol. ASSP-27, no. 6, pp. 620-624, Dec. 1979.

[95] Z. Jing and A.T. Fam, "Design of Finite Wordlength IIR Filters by Successive Discretization and Reoptimization," *Proc. IEEE Int. Elect., Electro. Conf. and Exposition,* (ELECTRONICOM'83), pp. 516-519, 1983.

[96] J.B. Knowles and E.M. Olcayto, "Coefficient Accuracy and Digital Filter Response," *IEEE Trans. Circuit Theory,* vol CT-15, pp. 31-41, Mar. 1980.

[97] P. M. Frank, *Introduction to System Sensitivity Theory*. New York, NY: Academic Press, Inc., 1978.

[98] B. Peikari, *Fundamentals of Network Analysis and Synthesis*. Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1974.

[99] R.M. Fletcher and C.M. Reeves, "Function Minimization by Conjugate Gradient," *Comp. J.*, vol. 7, pp. 149-154, July 1964.

[100] S.W. Director and R.A. Rohrer, "Automated Network Design-The Frequency Domain Case," *IEEE Trans. Circuit Theory*, vol. CT-16, no. 3, pp. 330-337, Aug. 1969.