COMPUTER-AIDED STUDY OF A MIXED LUMPED-DISTRIBUTED FILTER

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ABSTRACT

COMPUTER-AIDED STUDY OF A MIXED LUMPED
DISTRIBUTED FILTER

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Four different combinations of lumped passive lossless Butterworth filter arrangements together with distributed passive lossless transmission line arrangements are considered for applications in low pass, high pass, band pass and band stop filters. Since it has long been established that analytical investigation of the behaviour of such combinations are very long and time consuming as well as very complicated, in order to provide a meaningful insight of each of the mentioned cases, resolution to the computer-aided designs was a necessity.

This report reviews the analytical basis for the basic building block of these various combinations, discusses the results of each combination in terms of the order and the length of the transmission line, and finally reaches the conclusions for optimum combinations. The report includes samples of computer plots as well as the main program.

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LIST OF ABBREVIATIONS

L.P.	Low-pass
H.P.	High-pass
B.P.	Band-pass
B.S.	Band-stop
T.L.	Transmission Line
UHF	Ultra High Frequency
UTL	Uniform Lossless Transmission Line
TEM	Transverse Electromagnetic Mode
NUTL	Non Uniform Transmission Line
τ	The Time Delay of the Transmission Line
T	The Time Delay of the Transmission Line as shown on the computer program and curves
ω	Frequency in radians per second
W	Frequency in radians per second
γ	Propagation constant of the unit element of a Transmission Line
ℓ	The electrical length of the unit element of a Transmission Line
P	Butterworth Filter Segment
TW	Network's transfer function
TWDB	TW in decibles

LIST OF FIGURES

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CHAPTER I

INTRODUCTION

1.1. General

Filters have been a corner-stone in communication and electronic circuits ever since the first Morse code signal was transmitted. Transmission line behaviour, band width requirement, multiplexing techniques, frequency generation, amplification and propagation, etc., have all been dependent to one degree or another on frequency selecting elements and devices. Subsequently a continuing interest in filters and in their design has resulted, due to the demand that they meet, in more and more stringent specifications.

Several established design techniques have been adopted where a rather relaxed frequency cut-off (or separation) is available, such as for K-type, m-derived and Image-parameter design methods. For more economic (or cost-effective) units, the modern synthesis techniques to realize filters of particular specifications with minimum amount of drawbacks (e.g. band-pass attenuation, phase distortion, group delay, etc.) have been extensively used since the 1930's. Due to the great amount of computation required, very few combinations were tried. With the aid of computers this problem does not exist anymore.

An outcome of the employment of synthesis procedures to the filters was that certain functions are readily available for use, such as Butterworth, Chebychev and Campbell (Hamilton). However, there are certain inherent drawbacks in these readily available functions, e.g. cut-off frequency, selectivity, etc. The alternative is either to seek a totally new set of analytical functions without (or at least with less) drawbacks, or, to use a set of combinations that yield that better response.

This report addresses this last proposition by using the lossless transmission line as a frequency selecting element of variable characteristic, in conjunction with one of these readily available synthesizable functions in various arrangements. The report investigates the interactive effects of sectionalizing, and of the transmission line length on the resulting frequency response.

1.2 Lossless Transmission Lines

The lossless lines are of particular interest since it is desirable to transmit energy from source to load with a minimum loss. It has been found, in fact, that most U.H.F. and microwave transmission systems in existence do approximate the lossless condition. The Uniform Lossless Transmission line (UTL) has been extensively studied and its properties are well established. These lines support Transverse Electromagnetic Mode waves (TEM) and are widely used as transformers, resonators, filters, phase equalizers and matching stubs. However, in these applications, the UTL's can be used only in a limited range of frequencies. On the other hand, many microwave communication systems require components to operate over a wide band of frequencies so that they can handle signals at various frequencies without degrading the system performance. In order to achieve wider band widths, cascaded sections of UTL having different characteristic impedances have been employed. Though this extends the frequency band of operation to some extent, the resulting networks are larger in weight and size. To obtain superior characteristics with reduction in size and weight, attention has been given in recent years to the possibility of using lossless Non Uniform Transmission Lines (NUTL) as various acoustic and microwave components.

A lossless NUTL is a lossless transmission line, whose nominal characteristic impedance varies continuously along the length of the line according to a prescribed law and is described by the following distribution:-

$$L(x) = L_o \; f(x) \qquad\qquad o \leq x \leq \ell$$

$$C(x) = C_o \; g(x)$$

where:

$L(x)$ = series impedance per unit length at a distance x from the transmitting end.

$C(x)$ = shunt capacitance per unit length at a distance x from the transmitting end.

L_o, C_o = arbitrary scaling factors.

1.3 Filter Application

The most obvious application of filter structure, of course, is for the rejection of unwanted signal frequency while permitting good transmission of wanted frequencies. The most common filters of this sort are designed for either Low-Pass, High-Pass, Band-Pass or Band-Stop attenuation characteristics, such as those shown in figure 1.3.1.

Of course in this case of practical filters, for the microwave or any other frequency range, these characteristics are only achieved approximately, since there is a high frequency limit for any given practical filter structure above which characteristics will deteriorate due to junction effects, resonance within the elements, etc.

Filters are used in one capacity or another over the entire frequency band up to below coherent light waves.

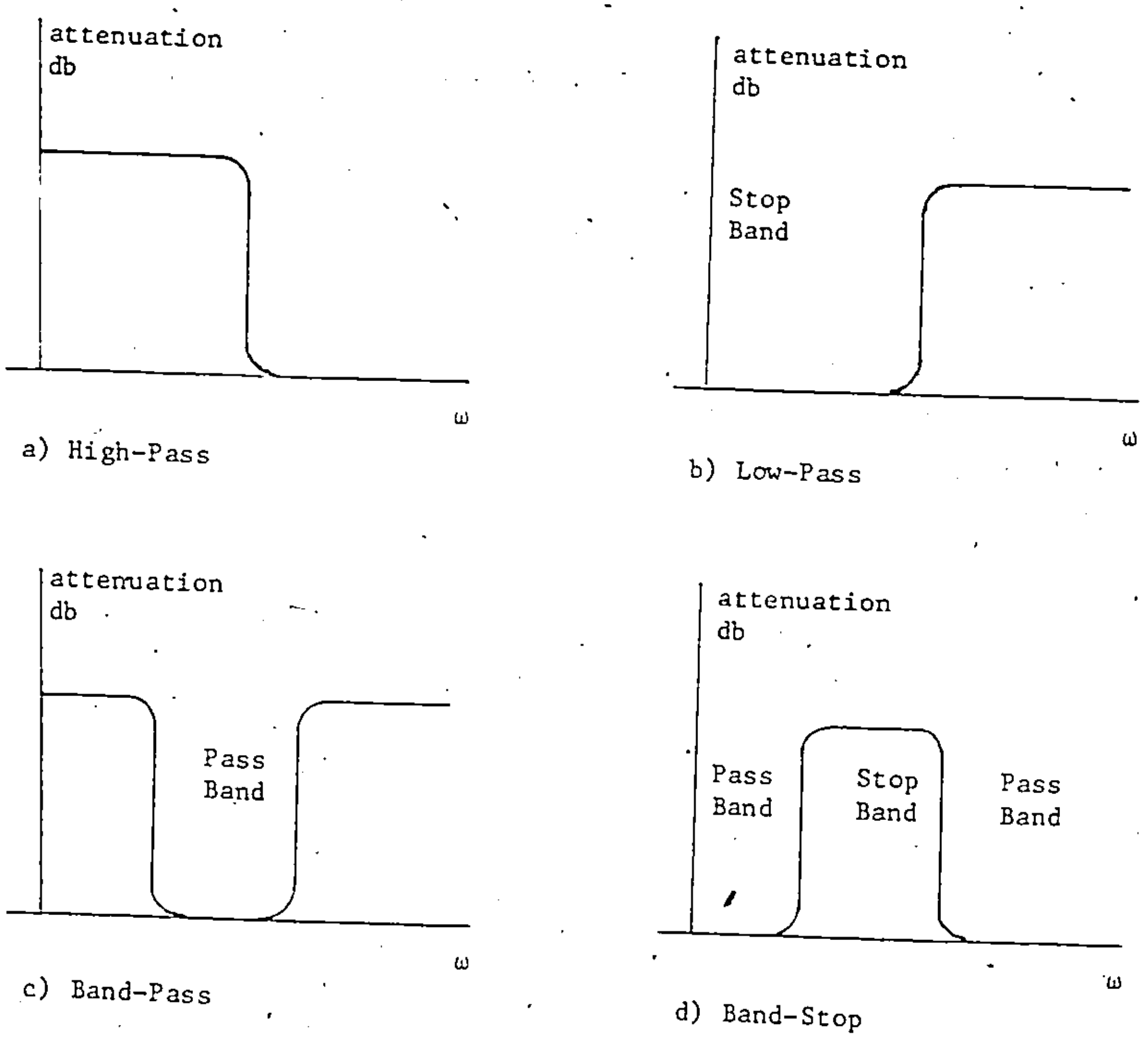


Fig. 1.3.1 Various filter configurations.

1.4 Scope of the Report

Several attempts have been made to use a combination of mixed lumped-distributed filters in various applications that necessitate having the transfer characteristics in order to fulfil certain requirements. This report examines certain combinations of an Nth order Butterworth filter in series with a transmission line. Four cases of these mixed filters were considered.

1. Transmission Line in series with an Nth order non-segmented Butterworth Filter.

2. Transmission Line in series with an Nth order Butterworth Filter repeated twice.

3. Transmission Line in series with an Nth order Butterworth Filter repeated three times.
4. Transmission Line in series with an Nth order segmented Butterworth Filter.

In all of the aforementioned cases, the transfer characteristics were calculated for L.P., H.P., B.P. and B.S. Filters, at various values of " τ " where " τ " = the time delay of the transmission line.

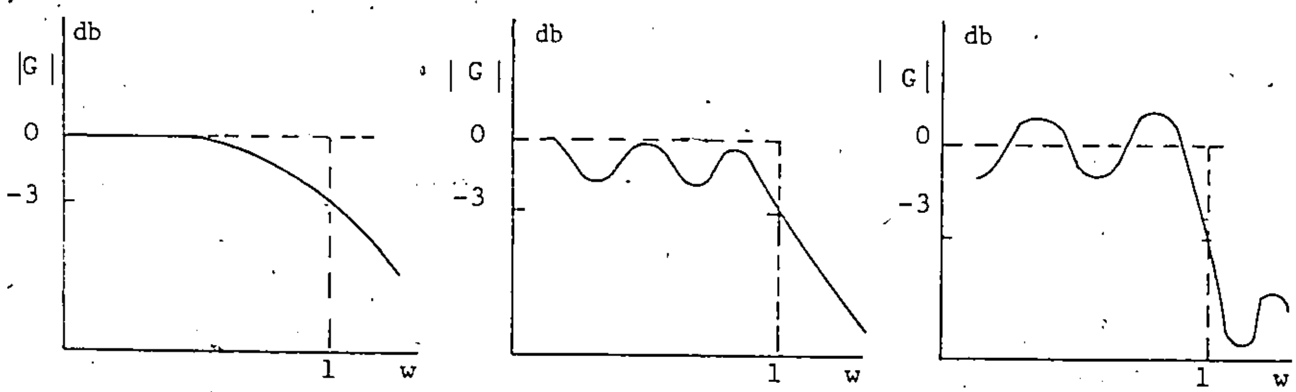
CHAPTER II

REVIEW OF BASIC ELEMENTS

2.1 The Filter Response Approximation Functions

The approximation response functions for physically realizable lumped passive filters are the Butterworth, Chebyshev and the maximally flat delay (or equal ripple) functions.

The typical behaviour of these functions are given in figure 2.1.



(a) Butterworth filter (b) Chebyshev filter (c) Maximally flat delay filter

Figure 2.1 Various Filters response

Where the transfer functions are as follows:-

a) $$|G_{12}(jw)| = \frac{1}{(1 + w^{2N})^{\frac{1}{2}}}$$ Butterworth ... (2.1)

b) $$|G_{12}(jw)| = \frac{1}{(1 + \varepsilon^2 C_N^2(w))^{\frac{1}{2}}}$$ Chebyshev ... (2.2)

c) $$G_{12}(jw) = e^{-jwT}$$ (2.3)

and $|G_{12}(jw)| = 1$, Arg $G_{12}(jw) = -wT$

or $$G_{12}(s) = \frac{1}{\text{Cosh } sT + \text{Sinh } sT}$$ Maximally flat delay (2.4)

The most important feature of these response functions is that they all meet the realizability conditions for lumped passive network synthesis.

In this report, only the passive lumped lossless element filter of a Butterworth response function is considered. No attempt is made to check realizability conditions since that network function has long been proven to be ~~fully~~ realizable.

2.2 The characteristics of the Butterworth Response Filter

From the Binomial theorem, the right hand side of equation 2.1 can be expanded as follows:

$$(1 + w^{2N})^{-\frac{1}{2}} = 1 - \frac{1}{2} w^{2N} + \frac{3}{8} w^{4N} - \frac{5}{10} w^{6N} \quad \cdots\cdots \qquad (2.5)$$

It also follows that for the first (2N - 1) derivatives, their coefficients will be zero for w = 0. Similarly.

$$|G_{12} (j1)| = 0.707 \quad \text{for all } N \qquad (2.6)$$

As an example for calculating the coefficients consider the case of N = 6 .

2.2.1 A Study of the Butterworth filter; for N = 6 terminated by 1 Ω and driven from an ideal source

The Butterworth form of response is as follows:-

$$|G_{12} (jw)| \doteq \frac{1}{\sqrt{1 + w^{2N}}} = \frac{1}{\sqrt{1 + w^{12}}} \qquad (2.7)$$

2.2.2 Pole location

To determine the pole locations corresponding to the Butterworth response, we study the analytic continuation of $|G_{12}|^2$ of the equation:-

$$|G_{12} (jw)| = \frac{1}{\sqrt{1 + w^{2N}}} \qquad (2.8)$$

The poles of this function are determined by the equation: $1 + (-s^2)^N = 0$

Then the pole locations are:-

$$S_K = e^{j \frac{2K - 1}{N} \frac{\pi}{2}} \qquad N : \text{even} \qquad (2.9)$$

$$S_K = e^{j \frac{2K}{N} \frac{\pi}{2}} \qquad N : \text{odd} \qquad (2.10)$$

or $$S_K = e^{j \frac{2K + N - 1}{N} \frac{\pi}{2}} \qquad K = 1, 2, \ldots. 2N \qquad (2.11)$$

the poles are located on a unit circle in the S plane and have symmetry with respect to both the real and the imaginary axes. For N odd, a pair of poles are located on the real axis, but the poles are not located on the imaginary axis for either N even or N odd. These properties follow because the poles are separated by $\frac{\pi}{N}$ radians, and are located $\frac{\pi}{2N}$ radians from the real axis for N even and on the real axis for N odd. Poles locations for the N = 6 are indicated below.

N	angles from Real axis in degrees	$\lvert\text{Cos } \theta\rvert$	$\lvert\text{Sin } \theta\rvert$
6	± 15	.96563	.25882
	± 45	.70711	.70711
	± 75	.25882	.96563

2.2.3 Formation of the function G_{12} (s)

To form the function G_{12} (s) from the given $|G_{12}|^2$, we reject the right half plane poles, and from the left half plane poles form the all-pole function:

$$G_{12}(s) = \frac{1}{1 + a_1 s + a_2 s^2 + a_3 s^3 + a_4 s^4 + a_5 s^5 + a_6 s^6} \qquad (2.12)$$

which we know to have the $|G_{12}(jw)|$ characteristic expressed by equation 2.7.

The coefficients of the denominator polynomials of $G_{12}(s)$, some times called Butterworth polynomials are shown on page 376 of "Modern Network Synthesis" by Van Valkenburg. $(N = 6)$

$a_1 = 3.8637$
$a_2 = 7.4641$
$a_3 = 9.1416$
$a_4 = 7.4641$
$a_5 = 3.8637$
$a_6 = 1$

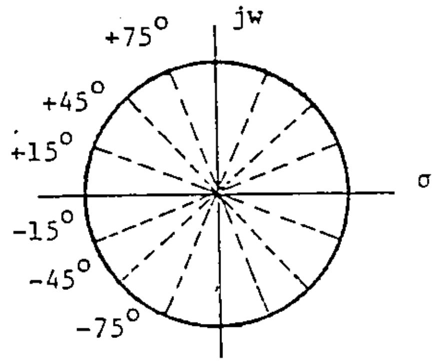


Figure 2.2 Pole location for the Butterworth response function for N = 6

$$\therefore \quad G_{12}(s) = \frac{1}{1 + 3.8637s + 7.4641s^2 + 9.1416s^3 + 7.4641s^4 + 3.8637s^5 + s^6} \tag{2.13}$$

$$= -Y_{12}(s) \qquad -y_{12}(s) = \frac{p(s)}{q_2(s)} \quad \ldots \tag{2.14}$$

$$= \frac{p(s)}{q_1(s) + q_2(s)} \qquad y_{22}(s) = \frac{q_1(s)}{q_2(s)} \quad \ldots \tag{2.15}$$

where $p(s) = 1$

$q_1(s) =$ even part of polynomial.

$q_2(s) =$ odd part of polynomial.

then $q_1(s)/q_2(s)$ is a Hurwitz Ψ function (a reactance)

then y_{22} of equation (2.14) , (2.15) represents an LC network

$$\therefore \; y_{22} = \frac{q_1}{q_2}$$

$$= \frac{\text{even part}}{\text{odd part}}$$

$$= \frac{s^6 + 7.4641s^4 + 7.4641s^2 + 1}{3.8637s^5 + 9.1416s^3 + 3.8637s} \qquad (2.16)$$

2.2.4 Realization

By performing a simple division (eqn. 2.16) all the parameters can be found. Figure 2.3 shows the actual filter circuit values, in the normalized form.

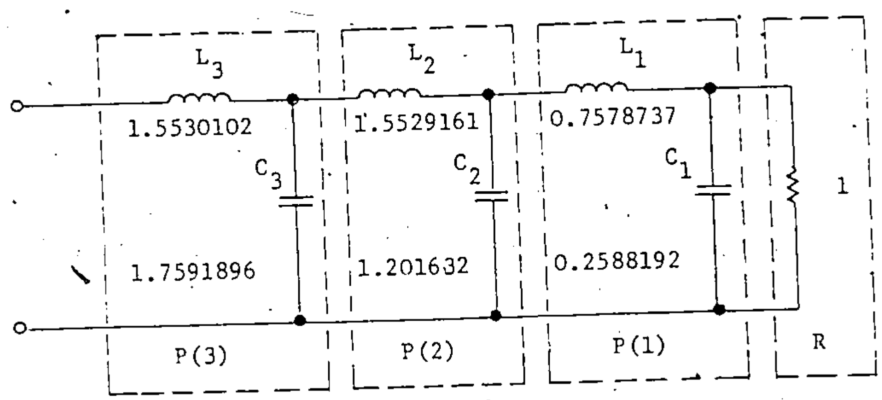


Figure 2.3 Network realization of equation 2.16

2.3 Derivation of a unit segment transfer function

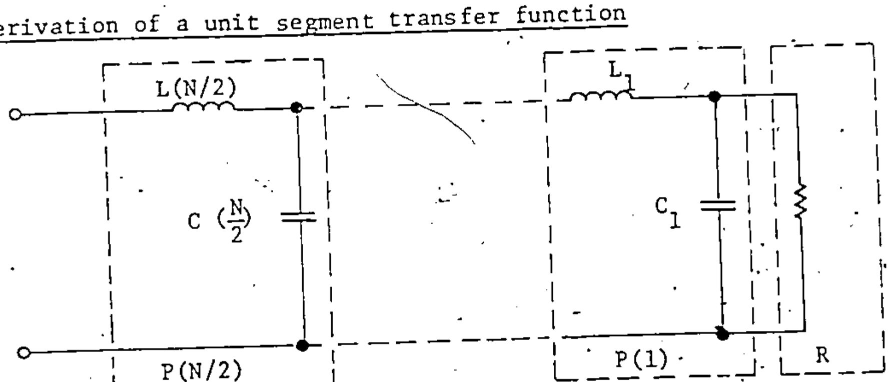


Figure 2.4 Butterworth Filter Segments

Consider the Nth order Butterworth filter shown in Figure 2.4. It can be divided into M segments, where $M = 1, 2, \ldots \frac{N}{2}$

Thus the sub section will be as shown in Figure 2.5 using the short circuit current transfer function

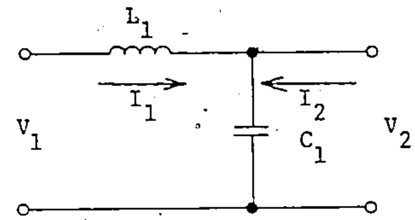


Figure 2.5 Butterworth filter unit segment P(1)

$$\therefore \quad a = \frac{V_1}{V_2} \Big|_{I_2 = 0} \qquad c = \frac{I_1}{V_2} \Big|_{I_2 = 0}$$

$$b = \frac{V_1}{-I_2} \Big|_{V_2 = 0} \qquad d = \frac{I_1}{-I_2} \Big|_{V_2 = 0}$$

$$\therefore \quad V_2 = I_1 / S C_1$$

$$V_1 = V_2 + S L_1 I_1$$

$$= V_2 + S C_1 S L_1 V_2$$

$$= V_2 \{ 1 + S^2 L_1 C_1 \}$$

$$\therefore \quad a = \frac{V_1}{V_2} \Big|_{I_2 = 0}$$

$$= 1 + S^2 L_1 C_1$$

$$C = \frac{I_1}{V_2} \Big|_{I_2 = 0}$$

$$= S C_1$$

$$V_2 = 0$$

$$\therefore \quad -I_2 = I_1$$

$$\therefore \quad d = 1$$

$$b = \frac{V_1}{I_1} = S\,L_1$$

A short circuit transfer function of a unit segment P(1) (Fig. 2.5) can be expressed as follows:-

$$\begin{bmatrix} A \end{bmatrix}_{P(1)} \doteq \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix}_{P(1)} = \begin{bmatrix} 1 + S^2 L_1 C_1 & S\,L_1 \\ S\,C_1 & 1 \end{bmatrix}_{P(1)}$$

2.4 Derivation of a Termination R Transfer Function

Using the same procedure used in section 2.3 the Transfer function of a Resistor R as shown in figure 2.6 can be represented as follows:-

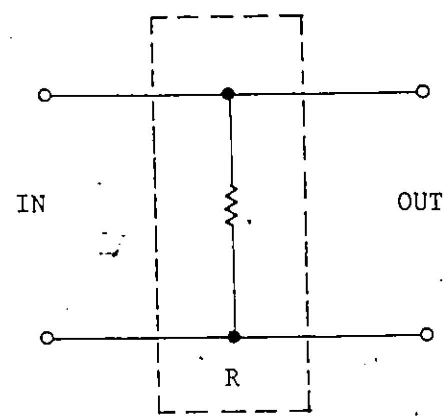


Figure 2.6 Termination Network R

$$\begin{bmatrix} A \end{bmatrix}_{R} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}_{R} = \begin{bmatrix} 1 & 0 \\ \frac{1}{R} & 1 \end{bmatrix}_{R}$$

2.5 The Transmission Lines Characteristics

The Transmission Line has a uniformly distributed parameters. For a unit length $\Delta \ell$, a series impedance $(R + jwL)\,\Delta \ell = [Z_0\,(\alpha + j\beta)]\,\Delta\ell$

$$= Z\,\Delta\,\ell$$

$$\text{Shunt Admittance} = (G + jwc)\,\Delta\,\ell$$

$$= \left[\frac{\alpha + j\beta}{Z_0}\right]\Delta\,\ell$$

$$= Y\,\Delta\,\ell$$

The Transmission Line can be modelled as shown in Figure 2.7

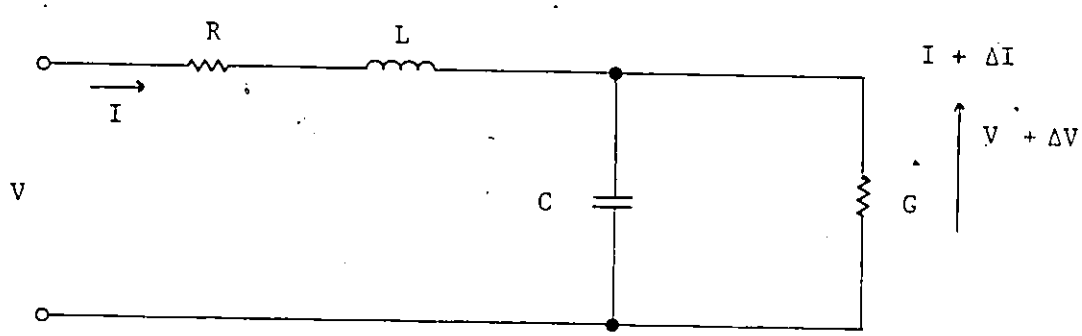


Figure 2.7 Transmission line equivalent circuit

where R = series resistance per loop unit length.

L = series inductance per loop unit length.

G = Leakage conductance per loop unit length.

C = Mutual capacitance per loop unit length.

Z_0 = Characteristic Impedance = $R_0 + j\,X_0$

α = Attenuation in nepers/unit length.

β = Phase change in radians/unit length.

From these equations,

$$Z_0 = \{\frac{R + jwL}{G + jwc}\}^{\frac{1}{2}} = \{\frac{Z}{Y}\}^{\frac{1}{2}}$$

From Kirchoff low, the current and voltage equations are:-

$$V + \Delta V = V - I.Z.\Delta\ell \longrightarrow \Delta V = - I.Z.\Delta\ell$$

$$I + \Delta I = I - V.Y.\Delta\ell \longrightarrow \Delta I = - V.Y.\Delta\ell.$$

In the limit

$$\frac{dV}{d\ell} = - I Z \quad \text{and} \quad \frac{dI}{d\ell} = - V Y$$

$$\therefore \quad \frac{d^2V}{d\ell^2} = -Z\frac{dI}{d\ell} = -Z (- V.Y)$$

i.e. $$\frac{d^2V}{d\ell^2} = Z.Y.V$$

let $Z.Y = \gamma^2 = (\alpha + j\beta)^2$

$$\therefore \quad \frac{d^2V}{d\ell^2} - \gamma^2 V = 0$$

i.e. $$V = A e^{-\gamma\ell} + B e^{+\gamma\ell}$$

$$I = -\frac{1}{Z}\frac{dV}{d\ell} = -\frac{1}{Z}(- \gamma A e^{-\gamma\ell} + \gamma B e^{+\gamma\ell})$$

i.e. $$I = \frac{\gamma}{Z} (A e^{-\gamma\ell} - B e^{\gamma\ell})$$

now $$\frac{\gamma}{Z} = \frac{\sqrt{ZY}}{Z} = \{\frac{Y}{Z}\}^{\frac{1}{2}} = \frac{1}{Z_0}$$

$$\therefore \quad I = \frac{1}{Z_0} (A e^{-\gamma\ell} - B e^{\gamma\ell})$$

& $$V = A e^{-\gamma\ell} + B e^{\gamma\ell}$$

at $\ell = 0$, $I = I_i$, $V = V_i$

$$\therefore \quad V_i = A + B \quad \& \quad I_i = \frac{A - B}{Z_0}$$

$$A = \frac{1}{2} (V_i + I_i Z_0) \quad \& \quad B = \frac{1}{2} (V_i - I_i Z_0)$$

$$2V = (V_i + I_i Z_0)\, e^{-\gamma\ell} + (V_i - I_i Z_0)\, e^{\gamma\ell}$$

and $$2 I Z_0 = (V_i + I_i Z_0)\, e^{-\gamma\ell} - (V_i - I_i Z_0)\, e^{\gamma\ell}$$

$$\therefore \quad (V + I Z_0) = (V_i + I_i Z_0)\, e^{-\gamma\ell}$$

$$\therefore \quad V_i + I_i Z_0 = (V + I Z_0)\, e^{\gamma\ell}$$

also $$(V - I Z_0) = (V_i - I_i Z_0)\, e^{\gamma\ell}$$

or $$V_i - I_i Z_0 = (V - I Z_0)\, e^{-\gamma\ell}$$

$$\therefore \quad 2V_i = V\,(e^{\gamma\ell} + e^{-\gamma\ell}) + I Z_0\,(e^{\gamma\ell} - e^{-\gamma\ell})$$

$$\& \quad 2 I_i Z_0 = V\,(e^{\gamma\ell} - e^{-\gamma\ell}) + I Z_0\,(e^{\gamma\ell} + e^{-\gamma\ell})$$

$$\therefore \quad V_i = V\,\frac{e^{\gamma\ell} + e^{-\gamma\ell}}{2} + I Z_0\,\frac{e^{\gamma\ell} - e^{-\gamma\ell}}{2}$$

$$I_i = \frac{1}{Z_0}\, V\,\frac{e^{\gamma\ell} - e^{-\gamma\ell}}{2} + I\,\frac{e^{\gamma\ell} + e^{-\gamma\ell}}{2}$$

$$V_i = V \cosh \gamma\ell + I Z_0 \sinh \gamma\ell$$

$$I_i = \frac{V}{Z_0} \sinh \gamma\ell + I_i \cosh \gamma\ell$$

$$\begin{bmatrix} V_i \\ I_i \end{bmatrix} = \begin{bmatrix} \text{Cosh } \gamma\ell & Z_0 \sinh \gamma\ell \\ \dfrac{\sinh \gamma\ell}{Z_0} & \cosh \gamma\ell \end{bmatrix} \begin{bmatrix} V \\ I \end{bmatrix}$$

Thus A transmission line can be represented by a chain matrix given by

$$\begin{bmatrix} A \end{bmatrix}_{T.L.} \equiv \begin{bmatrix} a & b \\ c & d \end{bmatrix}_{T.L.} \equiv \begin{bmatrix} \cosh \gamma\ell & Z_0 \sinh \gamma\ell \\ \dfrac{\sinh \gamma\ell}{Z_0} & \cosh \gamma\ell \end{bmatrix}$$

The frequency response of a transmission line is typically shown in Figure 2.8

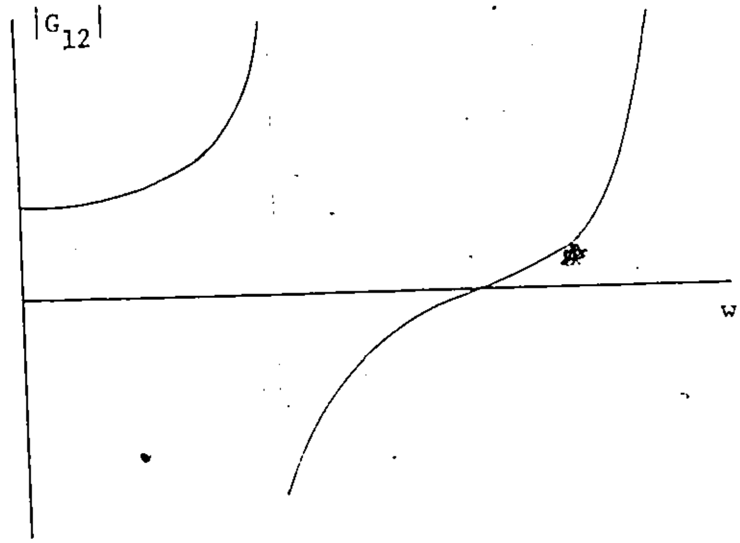


Figure 2.8 Transmission line frequency response

2.6 Frequency Transformation

In all of the foregoing analysis, the low-pass case was considered. If the technique of frequency transformation is applied, the characteristics of High-Pass, Band-Pass, Band-Stop filters can be easily realized. The principle of frequency transformation asks simply to replace the complex frequency S with a new complex frequency corresponding to the filter type under consideration.

From Low-Pass to:

a) High-Pass $\quad S \longrightarrow w_o/s$

b) Band-Pass $\quad S \longrightarrow (w_o/B)\left(\frac{s}{w_o} + \frac{w_o}{s}\right)$

c) Band-Stop $\quad S \longrightarrow \dfrac{1}{(w_o/B)\left(\frac{s}{w_o} + \frac{w_o}{s}\right)}$

where $B \equiv$ Normalized band width

$w_o \equiv$ Center frequency.

This Transformation is used throughout the coming analysis and computations.

2.7 Discussion

A review of approximation response functions for physically realizable lumped passive filters was discussed. The characteristics of a Butterworth filter of order 6 was also examined with respect to poles location, $G_{12}(s)$ function formation as well as its network realization.

Using the chain matrix method, a sectionalized modeling of the Butterworth filter, the Transmission line and the termination resistor R were calculated.

The overall chain matrix is obtained by multiplying all the successive matrices involved. The transfer function of the network is the inverse of the overall a parameter of the resultant chain matrix.

$$\left[A \right]_{overall} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}_{overall}$$

Thus the Transfer function $\dfrac{1}{a_{overall}}$

Since the frequency transformation technique was used throughout this report, its principles were also included.

CHAPTER III

TRANSMISSION LINE IN SERIES WITH AN Nth ORDER NON-SEGMENTED BUTTERWORTH FILTER

3.1 Introduction

In this chapter we shall consider the circuit shown in Figure 3.1 in which a lossless transmission line consists of a variable element (ℓ) with characteristic impedance $Z_0 = 1$, is connected in series with a passive lumped element Butterworth filter or order N. The entire network is terminated in a 1 ohm resistance. The objective of this chapter is to analyse the effect of varying the electrical length of the Transmission line on the overall transfer function of the whole network.

3.2 The Lumped Section is a Low-Pass B.W. Filter

The considered network is shown in Figure 3.1. In order to find the transfer function, the chain matrix method is used. We have for the lumped elements:-

$$\begin{bmatrix} A \end{bmatrix}_{P(M)} \begin{bmatrix} a_M & b_M \\ c_M & d_M \end{bmatrix}_{P(M)} = \begin{bmatrix} 1 - S^2 L_M C_M & S L_M \\ S C_m & 1 \end{bmatrix}_{P(M)} \qquad \ldots \quad 3.1$$

where $M = 1, 2, \ldots \frac{N}{2}$

N = Butterworth filter order

and for the distributed element:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}_{T.L.} = \begin{bmatrix} \cosh \gamma\ell & \sinh \gamma\ell \\ \sinh \gamma\ell & \cosh \gamma\ell \end{bmatrix}_{T.L.} \qquad \ldots \quad 3.2$$

where γ is the propagation constant of the unit element (U.E.)

and ℓ is the electrical length of the unit element.

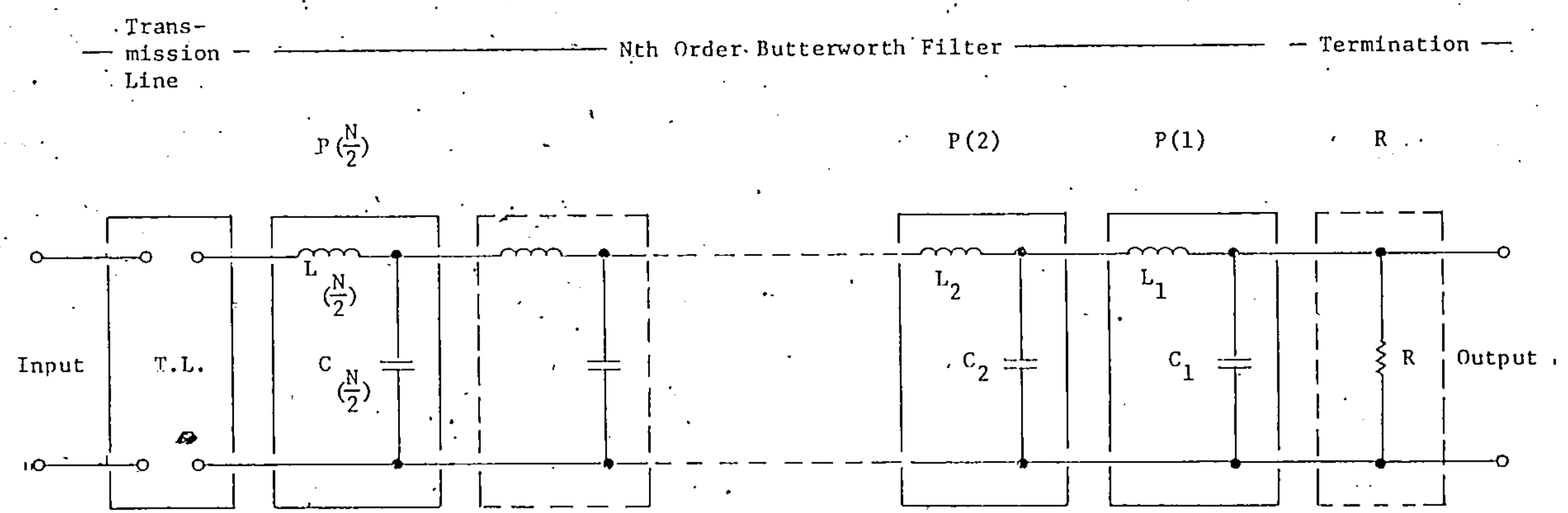


Figure 3.1 - Transmission Line in series with an Nth Order non-segmented Butterworth Filter terminated in a Resistor R

and for the termination R; R = 1

$$\begin{bmatrix} A \end{bmatrix}_R \quad \begin{bmatrix} a & b \\ c & d \end{bmatrix}_R = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}_R \qquad \ldots 3.3$$

Since the sub networks are individually reciprocal (ac - bd) = 1 for each of these sub networks, then the resultant network is also a reciprocal network. Thus by multiplying the corresponding matrices in their corresponding order, the transfer function of the network is the inverse of "a" of the overall A. matrix. To simplify the equations the lumped section considered will be of order 2. Thus the chain matrix shall be as follows:-

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}_{overall} \equiv \begin{bmatrix} a & b \\ c & d \end{bmatrix}_{T.L.} \cdot \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix}_{P(1)} \cdot \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}_R \qquad \ldots 3.4$$

or:-
$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}_{overall} \equiv \begin{bmatrix} \cosh \gamma\ell & \sinh \gamma\ell \\ \sinh \gamma\ell & \cosh \gamma\ell \end{bmatrix}_{T.L.} \cdot \begin{bmatrix} 1 - S^2L_1C_1 & SL_1 \\ SC_1 & 1 \end{bmatrix}_{P(1)} \cdot \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}_R \qquad \ldots 3.5$$

$$a_{overall} = a_{T.L.} \, (a_1 + b_1)_{P(1)} + b_{T.L.} \, (c_1 + d_1)_{P(1)} \qquad \ldots 3.6$$

$$\text{Transfer function} = \frac{1}{a_{overall}} = \frac{1}{(a_1 + b_1)_{P(1)} \cosh \gamma\ell + (c_1 + d_1)_{P(1)} \sinh \gamma\ell} \qquad \ldots 3.7$$

substituting each parameter in the above equation (3.7) and considering

$\gamma\ell = S\tau$ where:

τ = the time delay

$$\text{Transfer function} = \frac{1}{(1 + S^2L_1C_1 + SL_1) \, \text{Cosh} \, S\tau + (SC_1 + 1) \, \text{Sinh} \, S\tau}$$

$$= \frac{1}{(1 - w^2L_1C_1 + jwL_1) \cos w\tau + (1 + jwC_1) \, j \sin w\tau}$$

$$T(jw) = \frac{1}{\left[(1 - w^2 L_1 C_1)\cos w\tau - wC_1 \sin w\tau\right] + j(wL_1 \cos w\tau + \sin w\tau)}$$

$$|T(jw)|^2 = \left| \frac{1}{\left[(1 - w^2 L_1 C_1)\cos w\tau - wC_1 \sin w\tau\right]^2 + |wL_1 \cos w\tau + \sin w\tau|^2} \right|$$

$$|T_V| = 1 / \left[(K_1 \cos w\tau - wC_1 \sin w\tau)^2 + (wL_1 \cos w\tau + \sin w\tau)^2\right]^{\frac{1}{2}} \quad \ldots 3.8$$

where $K_1 = (1 - w^2 L_1 C_1)$

It is obviously seen that if we increase the order of the filter, then the problem will become more and more complicated.

A computer program was written to calculate the transfer function for the circuit shown in Figure 3.1 for the various combinations of the two following cases:

1. Filter order $N = 2, 4, 6, 8$ and 10
2. Filter modes L.P., H.P., B.P. and B.S.

All the above curves were calculated at the values of " τ " that will render a frequency response with a reasonable response variation in the pass band or the stop band depending on the filter mode being calculated.

3.2.1 L.P. Filter (N = 2)

- Figure 3.2.1.1 shows the frequency response of the circuit in question for a L.P. filter of order 2 calculated at " τ " $= 0.0$. This shows also the L.P. response of the same filter only since the transmission line will give no effect at " τ " $= 0.0$.

- Figure 3.2.1.2 gives the frequency response of the same circuit calculated at "τ" $=0.3$. A peak in the curve at $w = 5.4$ appears due to the rising effect of the transmission line at that frequency. However it should be expected that as τ increased, the peak will move towards the left. The response at $w = 1$ is now -4.4 db compared to -3 db at $\tau = 0.0$ as shown in Figure 3.2.1.1.

3.2.2 L.P. Filter ($N = 4$)

- Figure 3.2.2.1 shows the frequency response of the same circuit for a L.P. Filter of order 4 calculated at $\tau = 0.0$. It can be seen that the curve has more selectivity than the case for $N = 2$.
- Figure 3.2.2.2 gives the frequency response of the same circuit calculated at $\tau = 0.3$. A lower peak appears at the same frequency ($w - 5.4$) but at a lower magnitude than in the case of $N = 2$. Also an attenuation of 4.86 dbs is obtained at $w = 1$.

3.2.3 L.P. Filter ($N = 6$)

- Figure 3.2.3.1 shows the frequency response of the same circuit for a L.P. Filter of order 6 calculated at $\tau = 0.0$. More selectivity is obtained than in previous cases.
- Figure 3.2.3.2 gives the frequency response calculated at $\tau = 0.5$. A peak (-40.08 dbs) of magnitude appears at $w = 3$ is now present indicating that as the filter order increases a higher value of can be used without having high amplitude peaks in the stop band. Also at $w = 1$ the response goes down to -5.95 dbs.

3.2.4 L.P. Filter ($N = 8$)

- Figure 3.2.4.1 shows the frequency response of the same circuit for a L.P. Filter of order 8 calculated at $\tau = 0.0$. More selectivity is shown.

- Figure 3.2.4.2 gives the frequency response calculated at $\tau = 0.5$. A peak of -62.85dbs of amplitude appears at $w = 3.4$. The response at $w = 1$ is -6.55dbs.

3.2.5. L.P. Filter (N = 10)

- Figure 3.2.5.1 shows the frequency response of the same circuit for a L.P. Filter of order 10 calculated at $\tau = 0.0$. Better selectivity is clear.
- Figure 3.2.5.2 gives the frequency response calculated at $\tau = 0.9$. A peak of -116.42dbs of amplitude appears at $w = 5.4$. The response at $w = 1$ is now -6.83dbs which shows an improvement of -3.82 dbs over the curve at ($\tau = 0.0$).

3.3 The Lumped Section is a high-Pass Filter

Using the frequency transformation mentioned in section 2.6, the following High-Pass Filter characteristics were obtained by replacing S by wo/s

3.3.1 High Pass Filter (N = 2)

- Figure 3.3.1.1 shows the frequency response of the circuit shown in Figure 3.1 where the B.W. Filter is a H.P. Filter of order 2, calculated at $\tau = 0.0$.
- Figure 3.3.1.2 shows the frequency response of the same circuit but calculated at $\tau = 0.3$. The frequency response varies within 2.14 db between $w = 2$ and $w = 10$.

T=0.00

W	TW	TWDB
0.0	1.000000	0.00
.1	.999950	-.00
.2	.999202	-.01
.3	.995976	-.04
.4	.987444	-.11
.5	.970147	-.26
.6	.940893	-.53
.7	.897997	-.93
.8	.842279	-1.49
.9	.777071	-2.19
1.0	.707114	-3.01
1.1	.637052	-3.92
1.2	.570401	-4.88
1.3	.509248	-5.86
1.4	.454474	-6.85
1.5	.406141	-7.83
1.6	.363853	-8.78
1.7	.327000	-9.71
1.8	.294916	-10.61
1.9	.266957	-11.47
2.0	.242537	-12.30
2.4	.171053	-15.34
3.0	.110432	-19.14
3.4	.086183	-21.29
4.0	.062378	-24.10
4.4	.051584	-25.75
5.0	.039968	-27.97
5.4	.034273	-29.30
6.0	.027767	-31.13
6.4	.024407	-32.25
7.0	.020404	-33.81
8.0	.015623	-36.12
9.0	.012345	-38.17
10.0	.010000	-40.00

TW IN DB

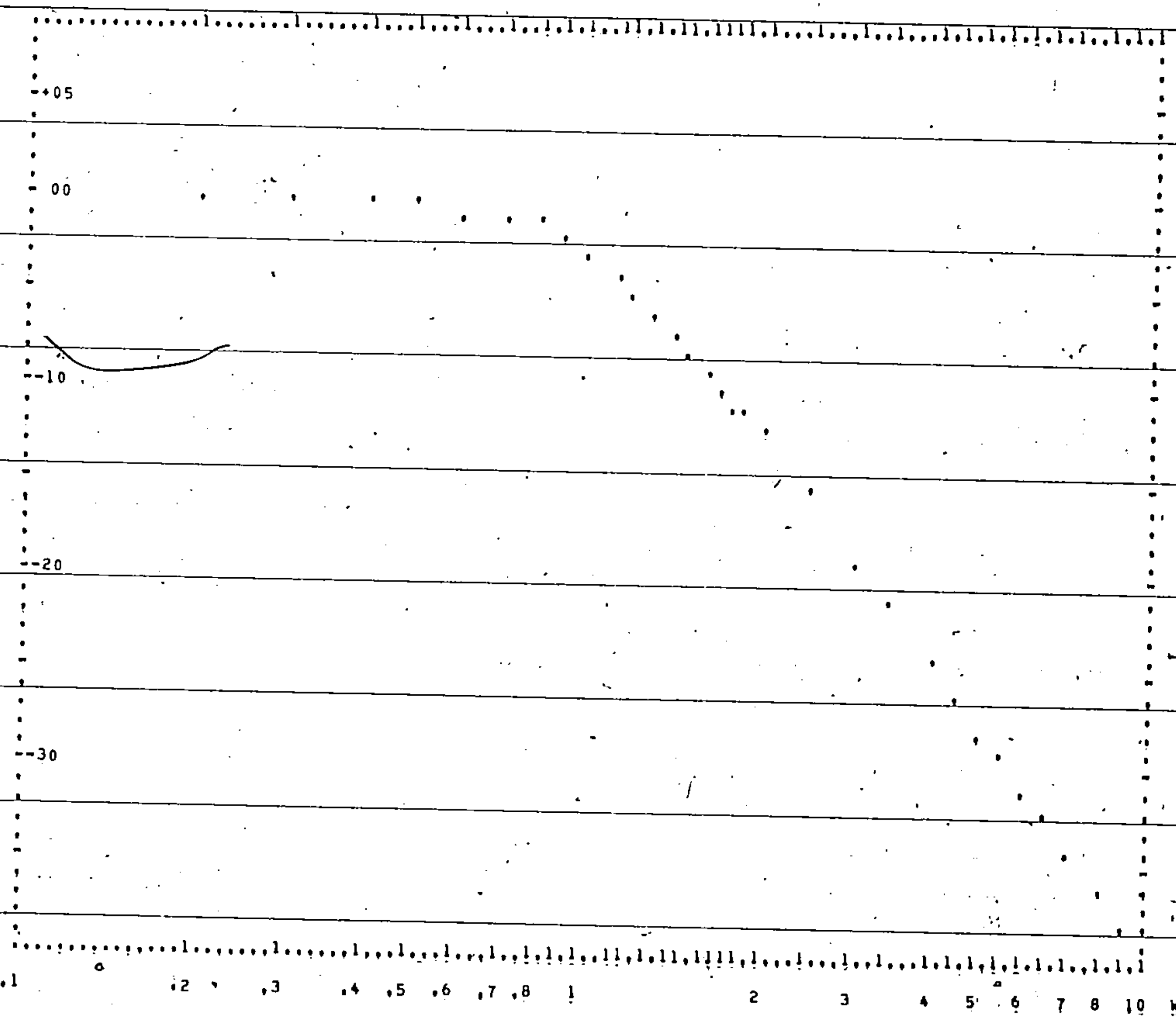


FIGURE 3.2.1.1 RESPONSE OF A T.L. + A L.P. BUTTERWORTH FILTER OF ORDER 2 (REFER TO FIG.3.1)

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T= .30

W	TW	TWDB
0.0	1.000000	0.00
.1	.997814	-.02
.2	.990501	-.08
.3	.975998	-.21
.4	.951578	-.43
.5	.914975	-.77
.6	.865660	-1.25
.7	.805557	-1.88
.8	.730634	-2.63
.9	.669586	-3.48
1.0	.602495	-4.40
1.1	.540124	-5.35
1.2	.483891	-6.31
1.3	.434193	-7.25
1.4	.390800	-8.16
1.5	.353163	-9.04
1.6	.320611	-9.88
1.7	.292472	-10.68
1.8	.260120	-11.43
1.9	.247005	-12.15
2.0	.228653	-12.82
2.4	.175740	-15.10
3.0	.133968	-17.46
3.4	.120619	-18.37
4.0	.116215	-18.69
4.4	.125366	-18.04
5.0	.183998	-14.70
5.4	.398811	-7.98
6.0	.253958	-11.90
6.4	.103480	-19.70
7.0	.049059	-26.19
8.0	.023085	-32.73
9.0	.014188	-36.96
10.0	.010204	-39.82

TW IN DB

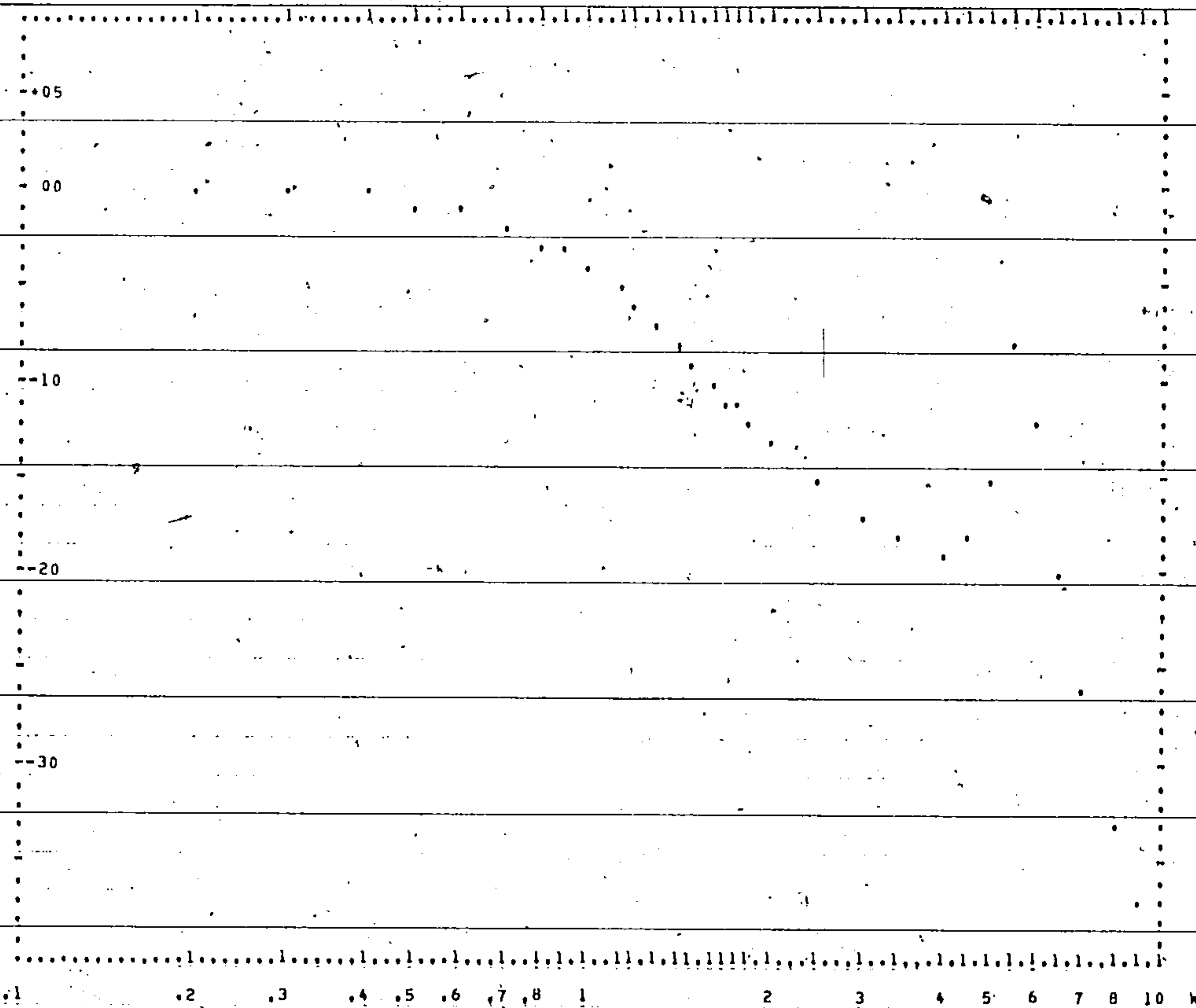

FIGURE 3.2.1.2 RESPONSE OF A T.L. + A L.P. BUTTERWORTH FILTER OF ORDER 2 (REFER TO FIG.3.1)

T=0,00

W	TW	TWDB
0.0	1.000000	0.00
.1	1.000001	.00
.2	1.000001	.00
.3	.999971	-.00
.4	.999679	-.00
.5	.998061	-.02
.6	.991716	-.07
.7	.972376	-.24
.8	.925392	-.67
.9	.836114	-1.55
1.0	.707114	-3.01
1.1	.564016	-4.97
1.2	.434384	-7.24
1.3	.330461	-9.62
1.4	.251916	-11.97
1.5	.193789	-14.25
1.6	.150844	-16.43
1.7	.118883	-18.50
1.8	.094832	-20.46
1.9	.076509	-22.33
2.0	.062379	-24.10
2.4	.030127	-30.42
3.0	.012345	-38.17
3.4	.007483	-42.52
4.0	.003906	-48.16
4.4	.002668	-51.48
5.0	.001600	-55.92
5.4	.001176	-58.59
6.0	.000772	-62.25
6.4	.000596	-64.49
7.0	.000416	-67.61
8.0	.000244	-72.25
9.0	.000152	-76.34
10.0	.000100	-80.00

TW IN DB

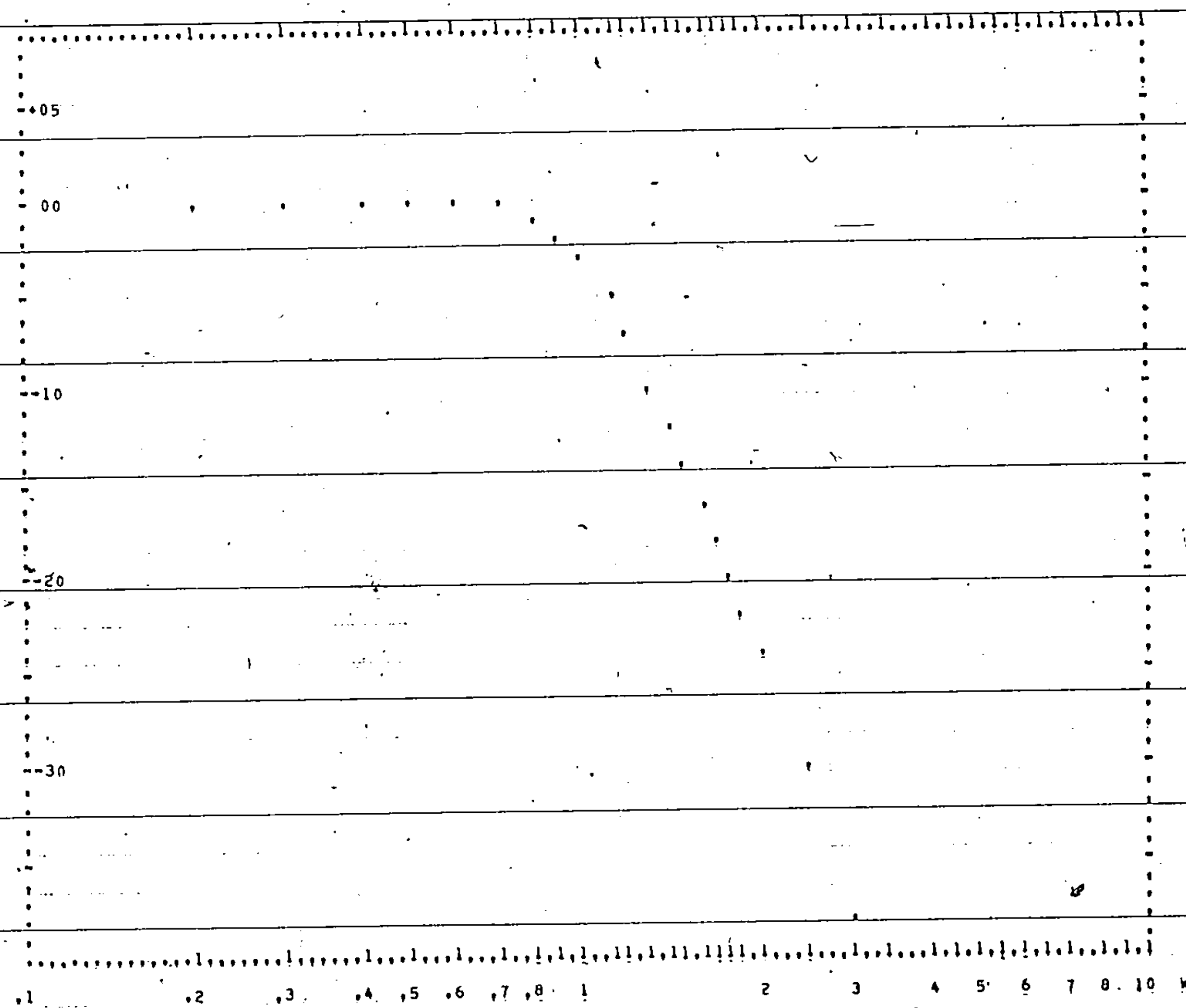

FIGURE 3.2.2.1 RESPONSE OF A T.L. + A L.P. BUTTERWORTH FILTER OF ORDER 4 (REFER TO FIG.3.1)

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T= .30

W	TW	TWDB
0.0	1.000000	0.00
.1	.998038	-.02
.2	.992108	-.07
.3	.982032	-.16
.4	.967230	-.29
.5	.946029	-.48
.6	.914476	-.78
.7	.865526	-1.25
.8	.791131	-2.04
.9	.689482	-3.23
1.0	.571534	-4.86
1.1	.455697	-6.83
1.2	.355689	-8.98
1.3	.275964	-11.18
1.4	.214924	-13.35
1.5	.168897	-15.45
1.6	.134233	-17.44
1.7	.107971	-19.33
1.8	.087887	-21.12
1.9	.072362	-22.81
2.0	.060228	-24.40
2.4	.031807	-29.95
3.0	.015409	-36.24
3.4	.010789	-39.34
4.0	.007529	-42.46
4.4	.006750	-43.41
5.0	.007869	-42.08
5.4	.016016	-35.91
6.0	.006435	-43.83
6.4	.002430	-52.29
7.0	.000983	-60.15
8.0	.000358	-68.92
9.0	.000175	-75.16
10.0	.000102	-79.83

TW IN DB

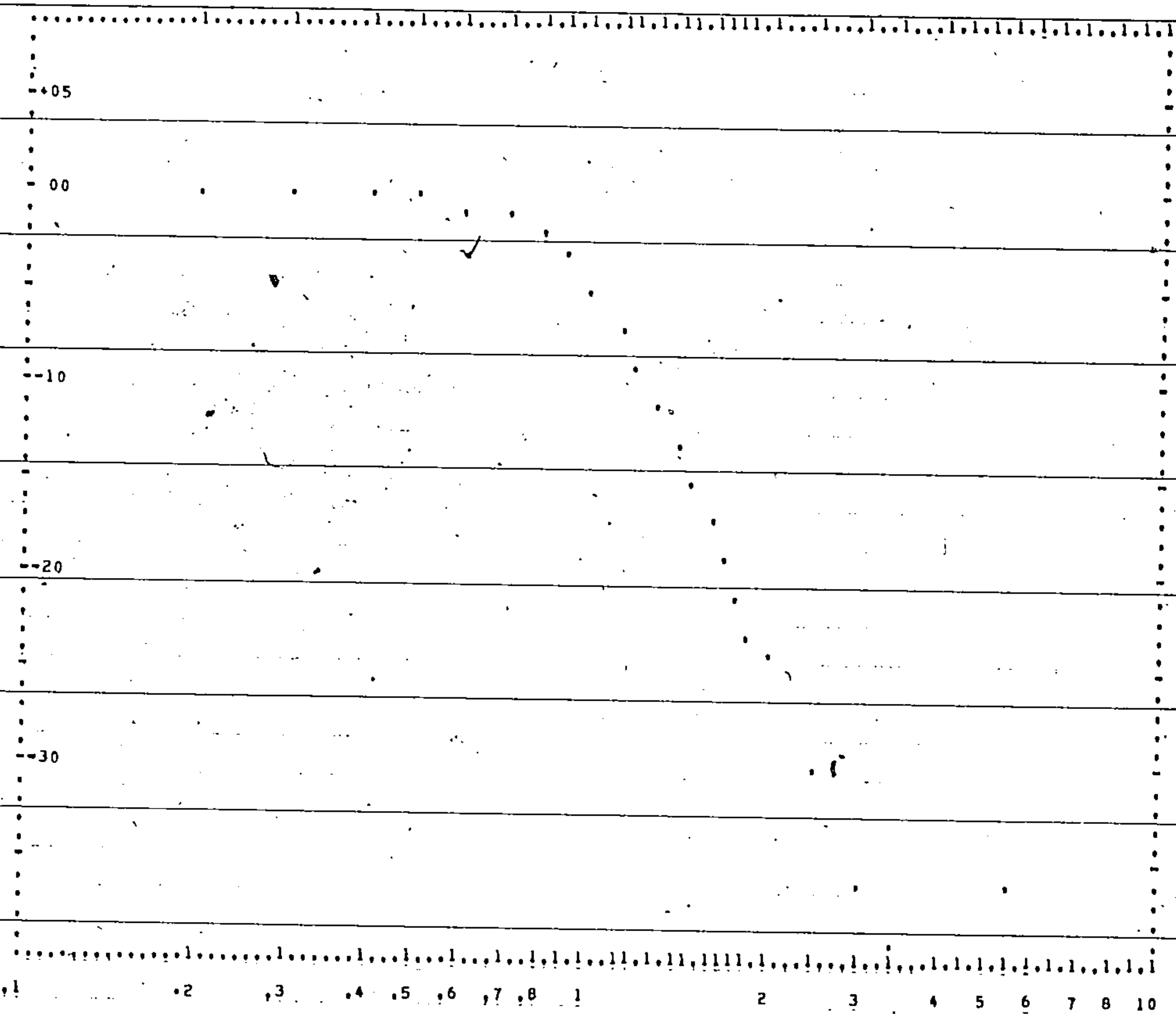

FIGURE 3.2.2.2 RESPONSE OF A T.L. + A L.P. BUTTERWORTH FILTER OF ORDER 4 (REFER TO FIG.3.1)

T=0,00

W	TW	TWDB
0,0	1,000000	0,00
,1	,999998	-,00
,2	,999991	-,00
,3	,999980	-,00
,4	,999957	-,00
,5	,999828	-,00
,6	,998849	-,01
,7	,993078	-,06
,8	,967253	-,29
,9	,883019	-1,08
1,0	,707118	-3,01
1,1	,491587	-6,17
1,2	,317576	-9,96
1,3	,202875	-13,86
1,4	,131658	-17,61
1,5	,087457	-21,16
1,6	,059500	-24,51
1,7	,041394	-27,66
1,8	,029389	-30,64
1,9	,021251	-33,45
2,0	,015623	-36,12
2,4	,005233	-45,63
3,0	,001372	-57,25
3,4	,000647	-63,78
4,0	,000244	-72,25
4,4	,000138	-77,21
5,0	,000064	-83,88
5,4	,000040	-87,89
6,0	,000021	-93,38
6,4	,000015	-96,74
7,0	,000008	-101,41
8,0	,000004	-108,37
9,0	,000002	-114,51
10,0	,000001	-120,00

TW IN DB

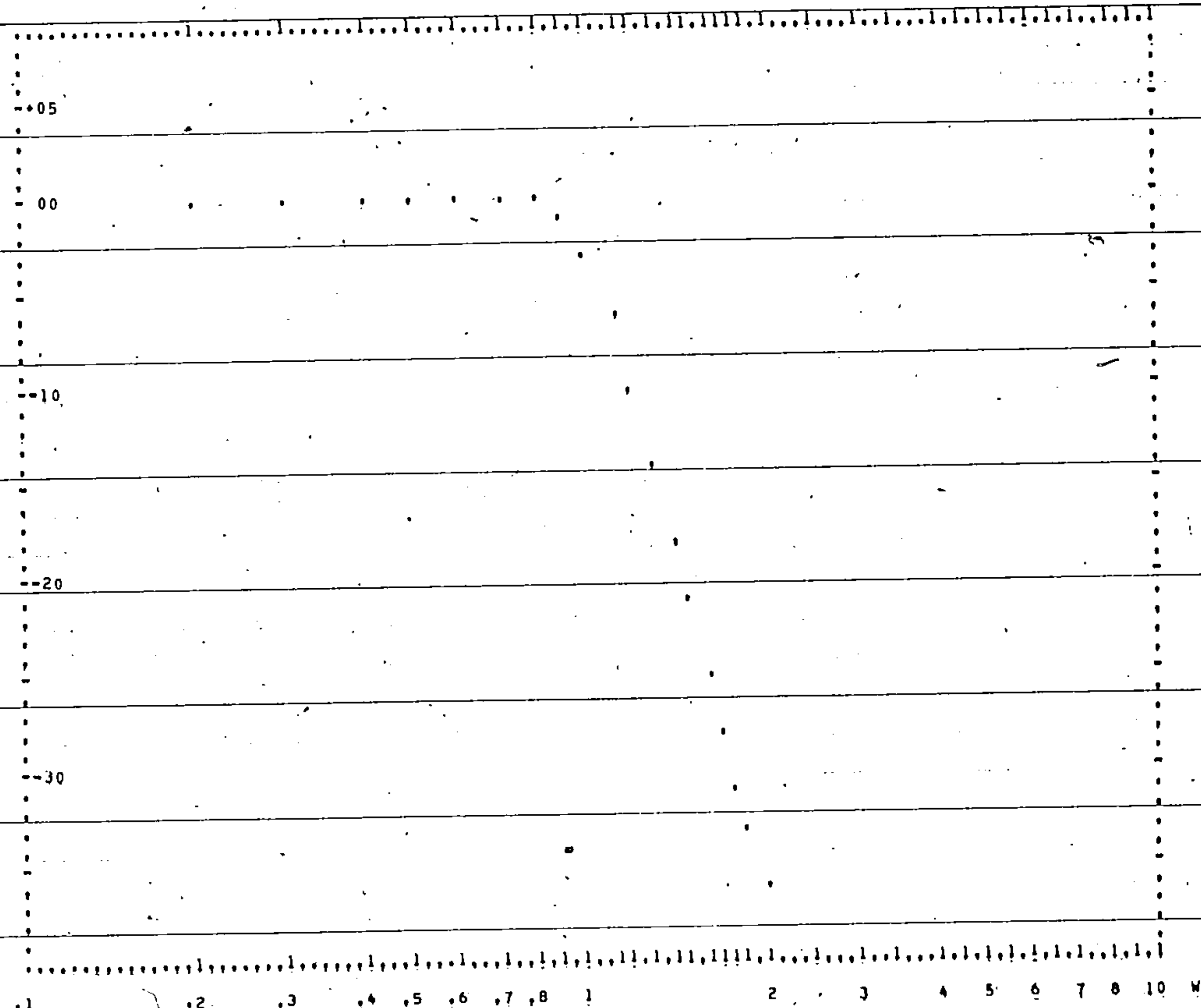

FIGURE 3.2.3.1 RESPONSE OF A T.L. + A L.P. BUTTERWORTH FILTER OF ORDER 6 (REFER TO FIG.3.1)

T= .50

W	TW	TWDB
0.0	1.000000	0.00
.1	.996781	-.03
.2	.987167	-.11
.3	.971260	-.25
.4	.949115	-.45
.5	.920451	-.72
.6	.883788	-1.07
.7	.834280	-1.57
.8	.760723	-2.38
.9	.649157	-3.75
1.0	.504140	-5.95
1.1	.359547	-8.88
1.2	.245212	-12.21
1.3	.165927	-15.60
1.4	.113656	-18.89
1.5	.079456	-22.00
1.6	.056832	-24.91
1.7	.041594	-27.62
1.8	.031123	-30.14
1.9	.023786	-32.47
2.0	.018549	-34.63
2.4	.008295	-41.62
3.0	.004660	-46.63
3.4	.009910	-40.08
4.0	.000917	-60.75
4.4	.000295	-70.62
5.0	.000089	-81.06
5.4	.000047	-86.50
6.0	.000022	-93.16
6.4	.000014	-96.78
7.0	.000009	-101.14
8.0	.000005	-105.46
9.0	.000007	-103.48
10.0	.000005	-106.91

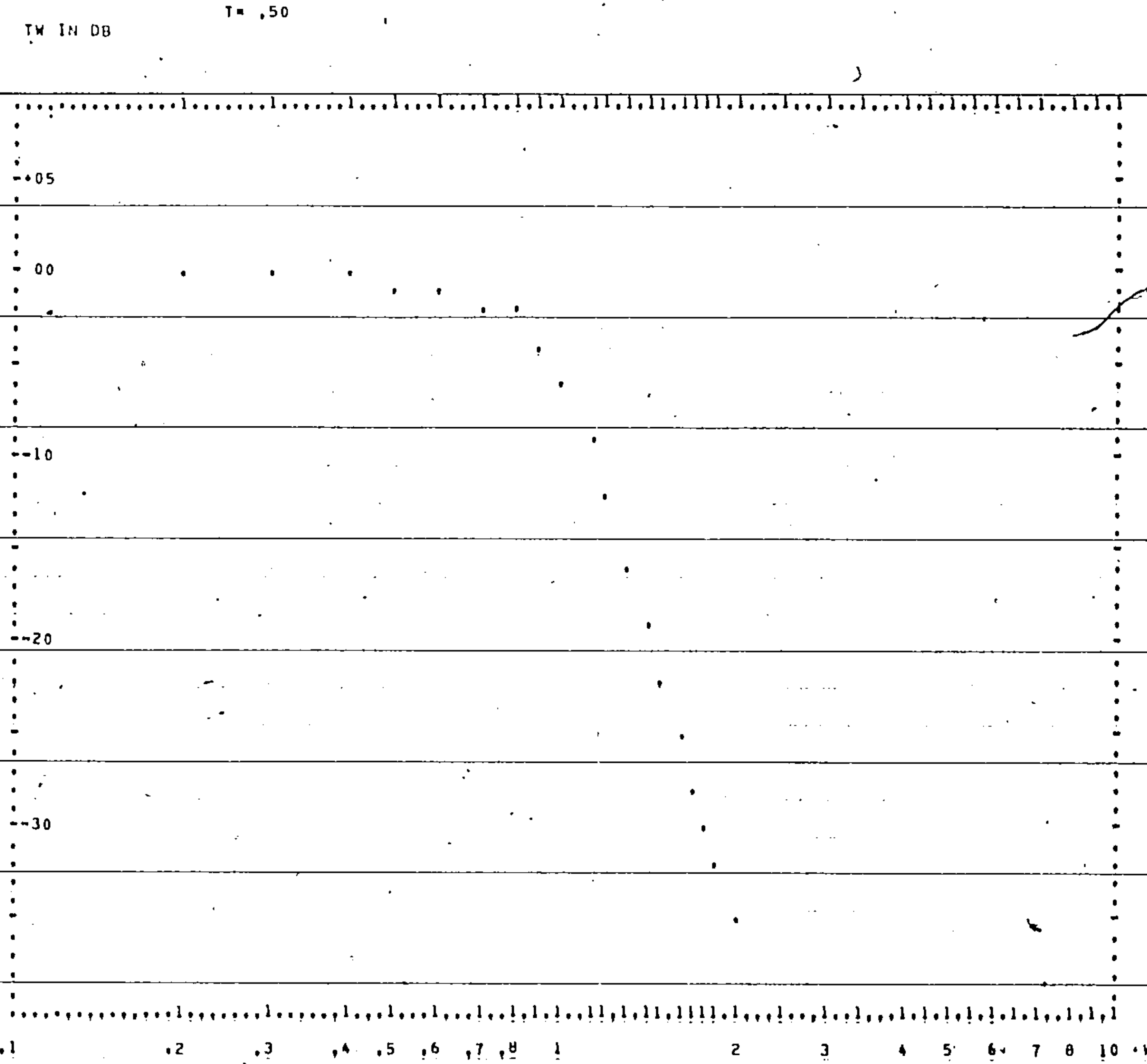

FIGURE 3.2.3.2 RESPONSE OF A T.L. + A L.P. BUTTERWORTH FILTER OF ORDER 6 (REFER TO FIG.3.1)

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T=0,00

W	TW	TWDB	TW IN DB
0,0	1,000000	0,00	
,1	,999587	-,00	
,2	,994426	-,05	
,3	,979535	-,18	
,4	,961622	-,34	
,5	,961325	-,34	
,6	1,003311	,03	
,7	1,097383	,81	
,8	1,163335	1,31	
,9	1,002231	,02	
1,0	,707107	-3,01	
1,1	,455642	-6,83	
1,2	,265150	-11,53	
1,3	,142042	-16,95	
1,4	,075396	-22,45	
1,5	,041428	-27,65	
1,6	,023807	-32,47	
1,7	,014271	-36,91	
1,8	,008875	-41,04	
1,9	,005696	-44,89	
2,0	,003755	-48,51	
2,4	,000872	-61,19	
3,0	,000148	-76,58	
3,4	,000055	-85,21	
4,0	,000015	-96,43	
4,4	,000007	-103,03	
5,0	,000003	-111,88	
5,4	,000001	-117,22	
6,0	,000001	-124,53	
6,4	,000000	-129,01	
7,0	,000000	-135,23	
8,0	,000000	-144,50	
9,0	,000000	-152,68	
10,0	,000000	-160,00	

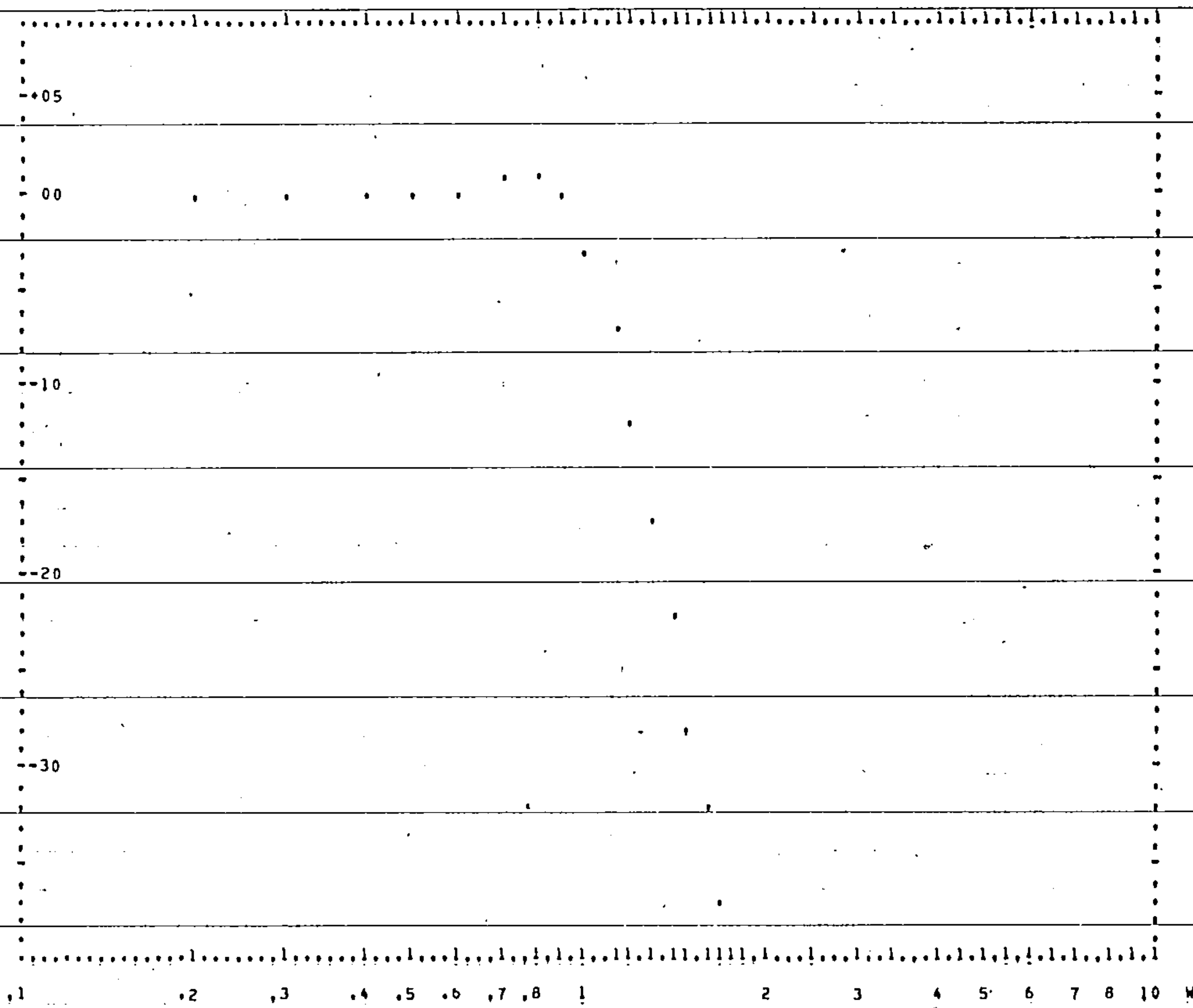

FIGURE 3,2,4,1 RESPONSE OF A T,L, + A L,P, BUTTERWORTH FILTER OF ORDER 8 (REFER TO FIG,3,1)

T= .50

W	TW	TWDB
0.0	1.000000	0.00
.1	.996221	-.03
.2	.980848	-.17
.3	.951557	-.43
.4	.921533	-.71
.5	.913874	-.78
.6	.941610	-.52
.7	.969616	-.27
.8	.879182	-1.12
.9	.666539	-3.52
1.0	.470597	-6.55
1.1	.319840	-9.90
1.2	.198929	-14.03
1.3	.113722	-18.88
1.4	.063918	-23.89
1.5	.036988	-28.64
1.6	.022342	-33.02
1.7	.014082	-37.03
1.8	.009225	-40.70
1.9	.006253	-44.08
2.0	.004370	-47.19
2.4	.001350	-57.40
3.0	.000483	-66.32
3.4	.000720	-62.85
4.0	.000058	-84.66
4.4	.000015	-96.30
5.0	.000004	-109.01
5.4	.000002	-115.80
6.0	.000001	-124.30
6.4	.000000	-129.05
7.0	.000000	-134.97
8.0	.000000	-141.63
9.0	.000000	-141.78
10.0	.000000	-146.78

TW IN DB

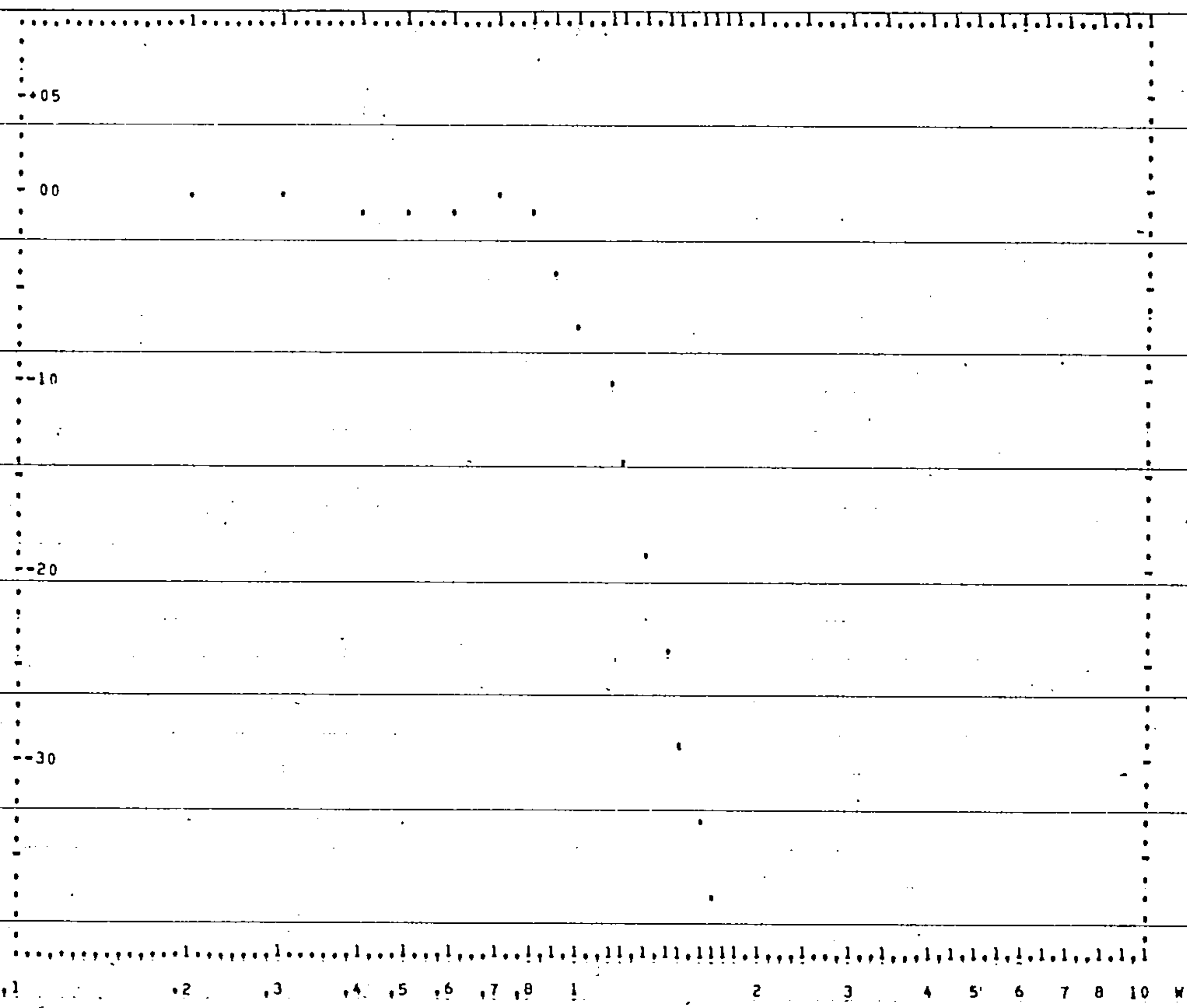

FIGURE 3.2.4.2 RESPONSE OF A T.L. + A L.P. BUTTERWORTH FILTER OF ORDER 8 (REFER TO FIG.3.1)

[POOR PRINT]

T=0,00

W	TW	TWDB
0,0	0,000000	-200,00
,1	,999888	-,00
,2	,999385	-,01
,3	1,003711	,03
,4	1,026744	,23
,5	1,070000	,59
,6	1,075562	,63
,7	,971505	-,25
,8	,847134	-1,44
,9	,816510	-1,76
1,0	,707107	-3,01
1,1	,315243	-10,03
1,2	,133181	-17,51
1,3	,063724	-23,91
1,4	,032887	-29,66
1,5	,017623	-35,08
1,6	,009637	-40,32
1,7	,005358	-45,42
1,8	,003037	-50,35
1,9	,001760	-55,09
2,0	,001045	-59,62
2,4	,000163	-75,76
3,0	,000017	-95,35
3,4	,000005	-106,27
4,0	,000001	-120,41
4,4	,000000	-128,69
5,0	,000000	-139,80
5,4	,000000	-146,48
6,0	,000000	-155,63
6,4	,000000	-161,24
7,0	,000000	-169,02
8,0	,000000	-180,62
9,0	,000000	-190,85
10,0	,000000	-200,00

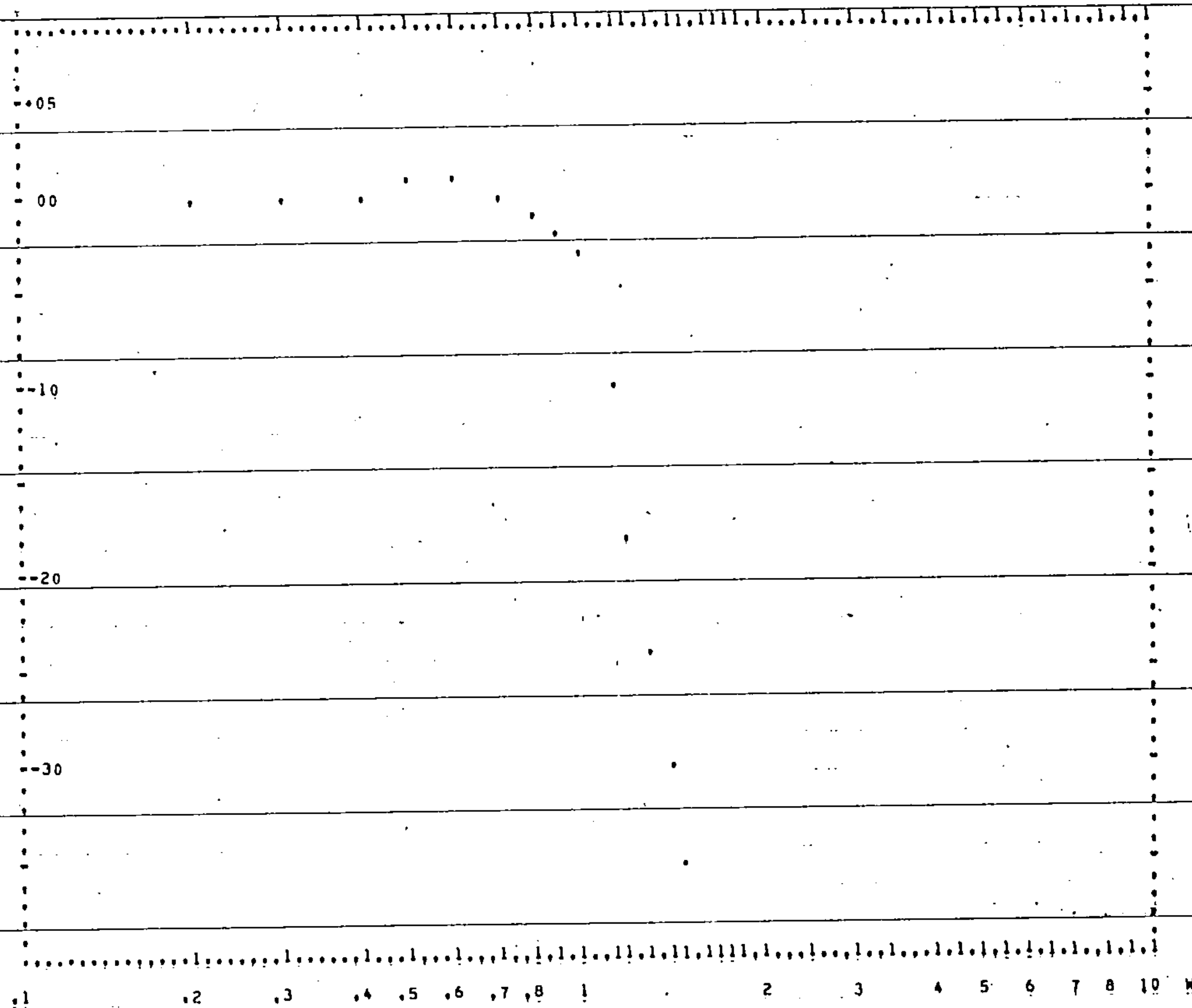

FIGURE 3.2.5.1 RESPONSE OF A T.L. + A L.P. BUTTERWORTH FILTER OF ORDER 10 (REFER TO FIG.3.1)

T= .90

W	TW	TWDB
0.0	1.000000	0.00
.1	.994503	-.05
.2	.979039	-.18
.3	.962022	-.34
.4	.951350	-.43
.5	.919443	-.73
.6	.016114	-1.76
.7	.682512	-3.32
.8	.601656	-4.41
.9	.581841	-4.70
1.0	.455281	-6.83
1.1	.230734	-12.74
1.2	.116481	-18.67
1.3	.065116	-23.73
1.4	.039238	-28.13
1.5	.024893	-32.08
1.6	.016537	-35.63
1.7	.011640	-38.68
1.8	.008931	-40.98
1.9	.007937	-42.01
2.0	.009878	-40.11
2.4	.000500	-66.02
3.0	.000021	-93.54
3.4	.000005	-106.10
4.0	.000001	-120.12
4.4	.000000	-126.63
5.0	.000000	-130.29
5.4	.000002	-116.42
6.0	.000000	-150.50
6.4	.000000	-159.49
7.0	.000000	-169.03
8.0	.000000	-177.15
9.0	.000000	-175.76
10.0	.000000	-198.94

TW IN DB

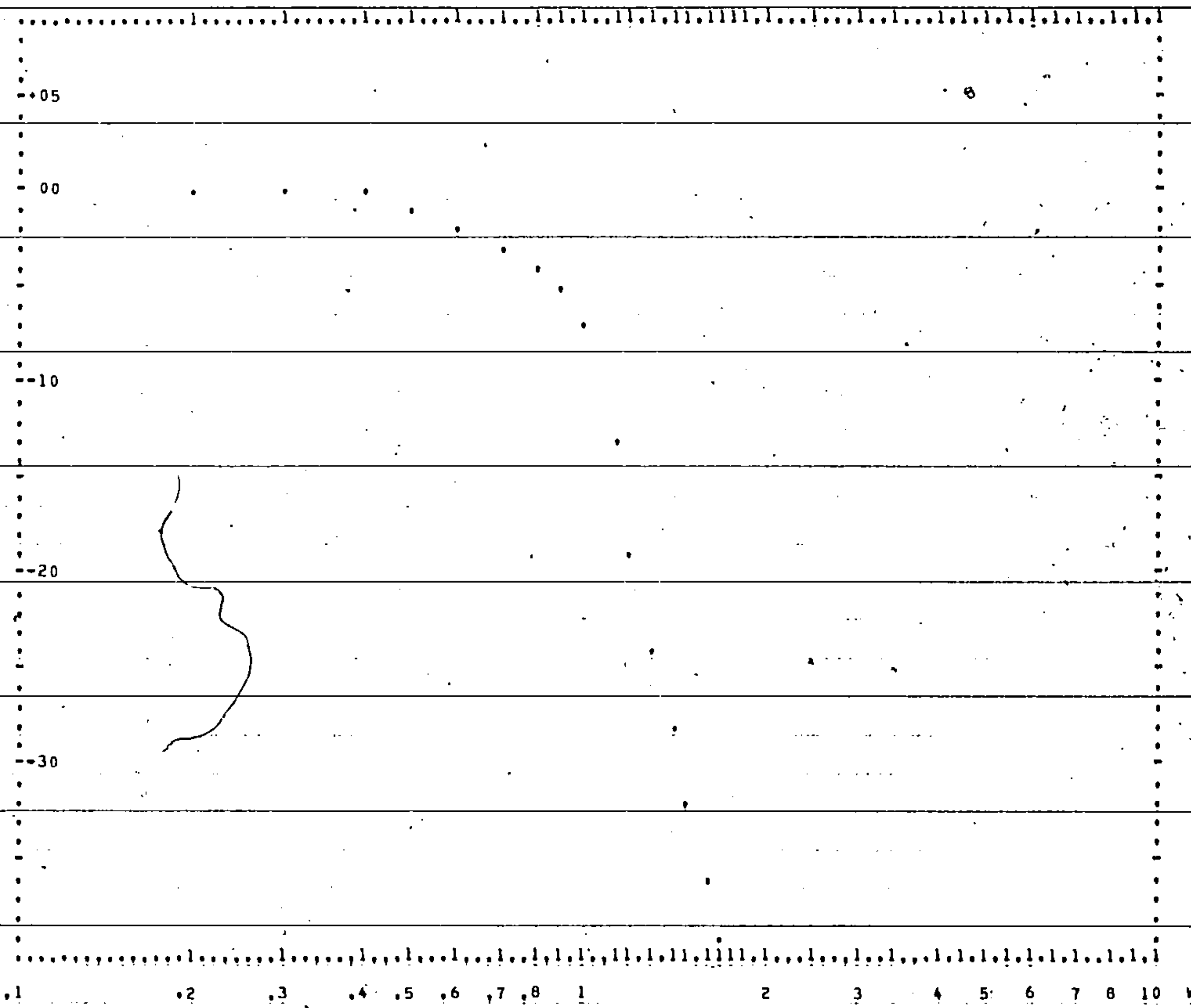

FIGURE 3.2.5.2 RESPONSE OF A T.L. + A L.P. BUTTERWORTH FILTER OF ORDER 10 (REFER TO FIG.3.1)

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3.3.2 High Pass Filter (N = 4)

- Figure 3.3.2.1 shows the frequency response of the same circuit for a High-pass filter of order 4 calculated at $\tau = 0.0$.
- Figure 3.3.2.2 shows the frequency response of the same circuit calculated at $\tau = 0.3$. The frequency response varies within 1.94 db between w = 2 and w = 10.

3.3.3 High-Pass Filter (N = 6)

- Figure 3.3.3.1 shows the frequency response for a High-Pass filter of order 6 calculated at $\tau = 0.0$.
- Figure 3.3.3.2 was calculated at $\tau = 0.5$. The frequency response varies within 4.68 dbs.

3.3.4 High-Pass Filter (N = 8)

- Figure 3.3.4.1 shows the frequency response for a High-Pass filter of order 8 calculated at $\tau = 0.0$.
- Figure 3.3.4.2 was calculated at $\tau = 0.5$. It is clear in that figure that the frequency response varies within 6.48 db.

In both cases of N = 6, N = 8, a higher value of τ will introduce a peak in the response closer to the L.H.S. of curve, approaching to the cut-off band.

3.3.5 High-Pass Filter (N = 10)

- Figure 3.3.5.1 shows the frequency response for a High-Pass filter of order 10 calculated at $\tau = 0.0$.
- Figure 3.3.5.2 shows the frequency response for a High-Pass filter of order 10 calculated at $\tau = 0.9$. Although a figure of $\tau = 0.9$ was to yield a good frequency response of L.P. filter yet it is too high to be useful for a H.P. filter at the same setup.

POOR PRINT

T=0.00

W	TW	TWDB
0.0	0.000000	-200.00
.1	.010000	-40.00
.2	.039968	-27.97
.3	.089638	-20.95
.4	.157991	-16.03
.5	.242537	-12.30
.6	.338722	-9.40
.7	.440019	-7.13
.8	.539058	-5.37
.9	.629428	-4.02
1.0	.707114	-3.01
1.1	.770833	-2.26
1.2	.821377	-1.71
1.3	.860629	-1.30
1.4	.890769	-1.00
1.5	.913818	-.78
1.6	.931463	-.62
1.7	.945031	-.49
1.8	.955529	-.40
1.9	.963714	-.32
2.0	.970147	-.26
2.4	.985265	-.13
3.0	.993886	-.05
3.4	.996281	-.03
4.0	.998054	-.02
4.4	.998670	-.01
5.0	.999202	-.01
5.4	.999413	-.01
6.0	.999615	-.00
6.4	.999703	-.00
7.0	.999792	-.00
8.0	.999878	-.00
9.0	.999924	-.00
10.0	.999950	-.00

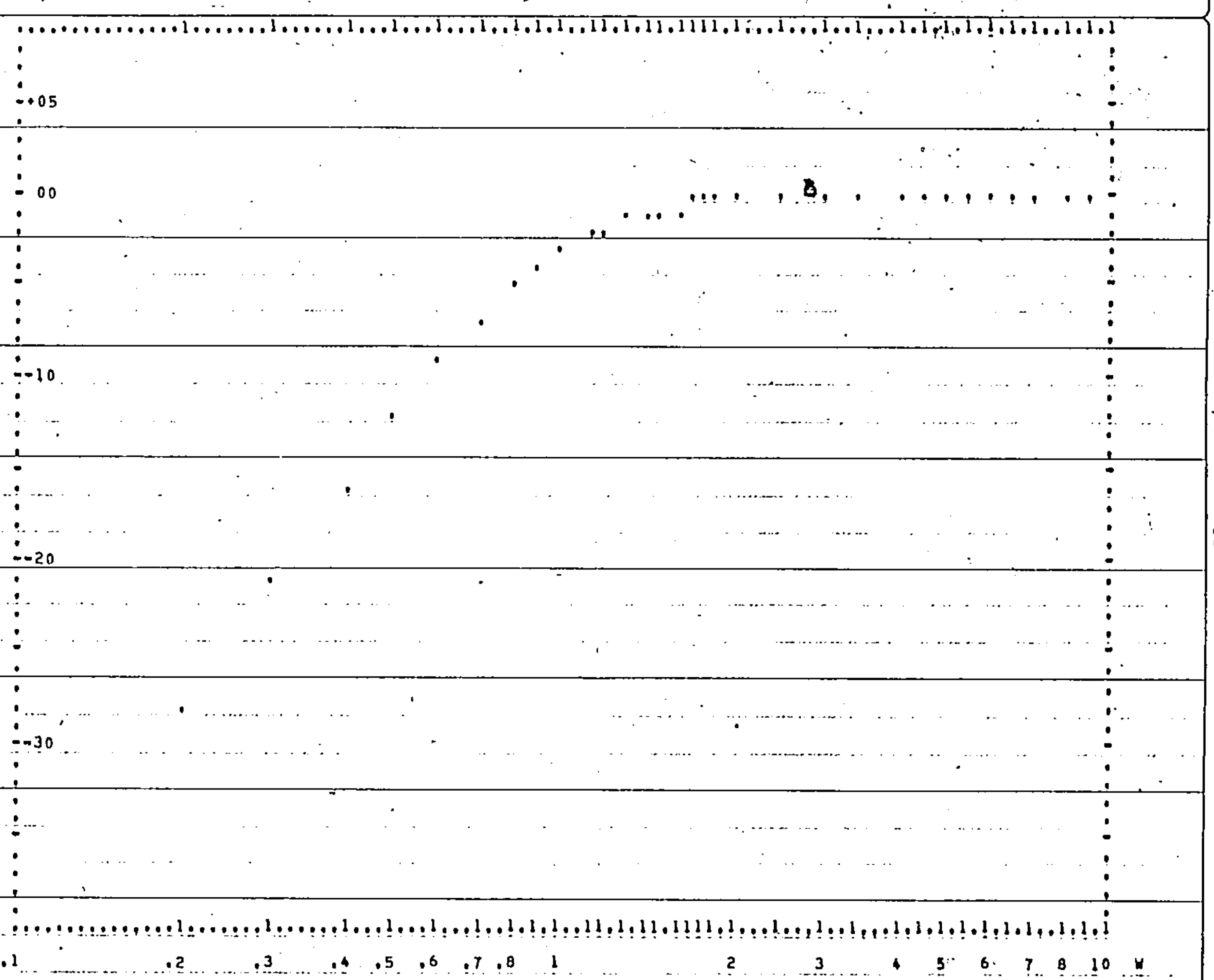

FIGURE 3.3.1.1 RESPONSE OF A T.L. + A H.P. BUTTERWORTH FILTER OF ORDER 2 (REFER TO FIG.3.1)

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[POOR PRINT]

T= .30

W	TW	TWDB
0.0	0.000000	-200.00
.1	.010025	-39.98
.2	.040396	-27.87
.3	.091904	-20.73
.4	.165519	-15.62
.5	.261734	-11.64
.6	.379464	-8.42
.7	.514537	-5.77
.8	.659145	-3.62
.9	.801421	-1.92
1.0	.929365	-.64
1.1	1.034278	.29
1.2	1.113097	.93
1.3	1.167704	1.35
1.4	1.202599	1.60
1.5	1.222774	1.75
1.6	1.232558	1.82
1.7	1.235267	1.84
1.8	1.233278	1.82
1.9	1.228227	1.79
2.0	1.221219	1.74
2.4	1.185149	1.48
3.0	1.129513	1.06
3.4	1.096377	.80
4.0	1.053628	.45
4.4	1.029703	.25
5.0	1.000424	.00
5.4	.985131	-.13
6.0	.968180	-.28
6.4	.960641	-.35
7.0	.954528	-.40
8.0	.956459	-.39
9.0	.970045	-.26
10.0	.990074	-.09

TW IN DB

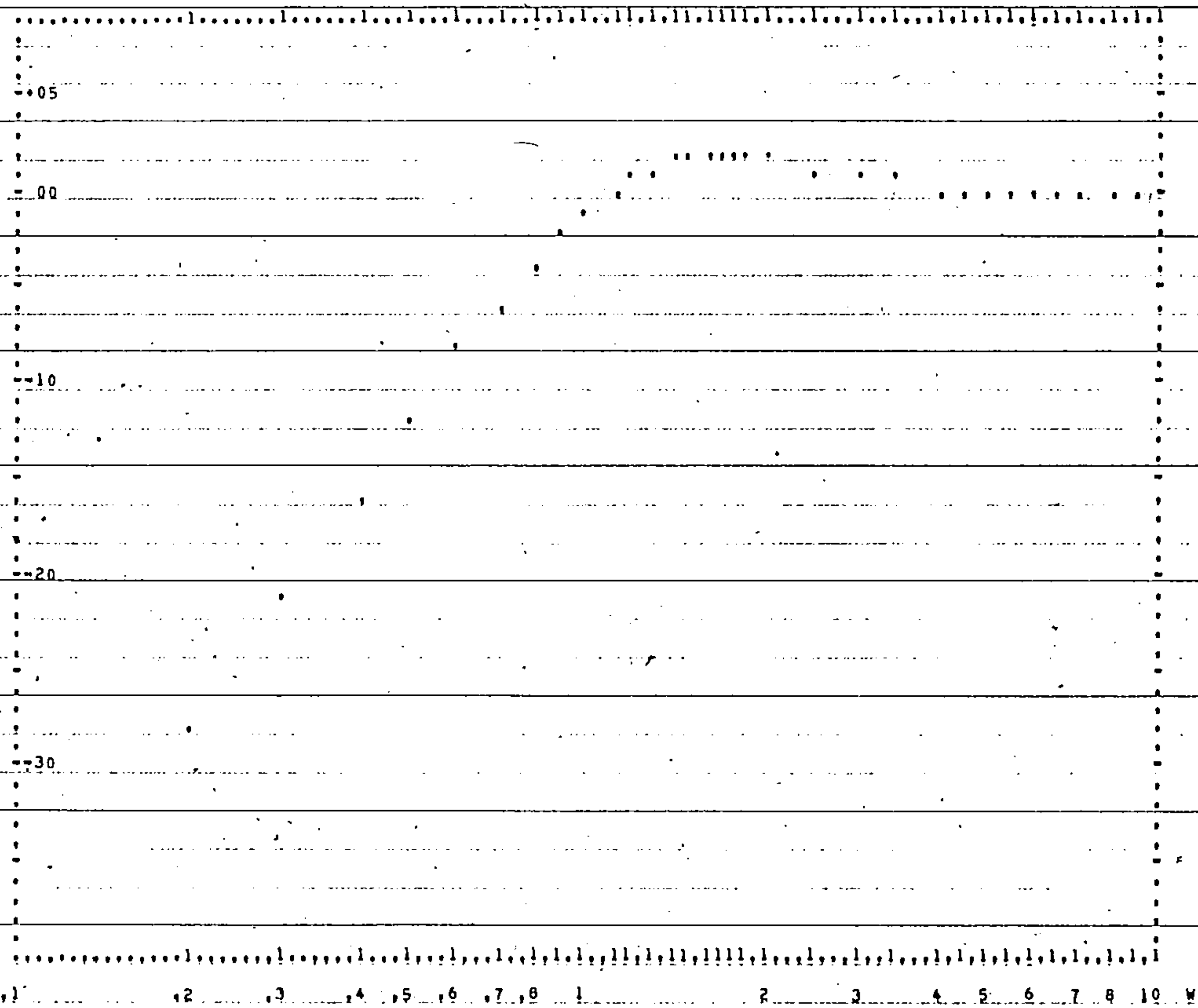

FIGURE 3.3.1.2 RESPONSE OF A T.L. + A H.P. BUTTERWORTH FILTER OF ORDER 2 (REFER TO FIG.3.1)

T=0,00

W	Tw	TWDB
0.0	0.000000	-200,00
.1	.000100	-80,00
.2	.001600	-55,92
.3	.008100	-41,83
.4	.025592	-31,84
.5	.062379	-24,10
.6	.128526	-17,82
.7	.233467	-12,64
.8	.379040	-8,43
.9	.548574	-5,22
1.0	.707114	-3.01
1.1	.825776	-1,66
1.2	.900739	-.91
1.3	.943831	-.50
1.4	.967760	-.28
1.5	.981054	-.17
1.6	.988568	-.10
1.7	.992919	-.06
1.8	.995503	-.04
1.9	.997078	-.03
2.0	.998061	-.02
2.4	.999553	-.00
3.0	.999929	-.00
3.4	.999976	-.00
4.0	.999995	-.00
4.4	.999999	-.00
5.0	1.000001	.00
5.4	1.000001	.00
6.0	1.000001	.00
6.4	1.000001	.00
7.0	1.000001	.00
8.0	1.000001	.00
9.0	1.000001	.00
10.0	1.000001	.00

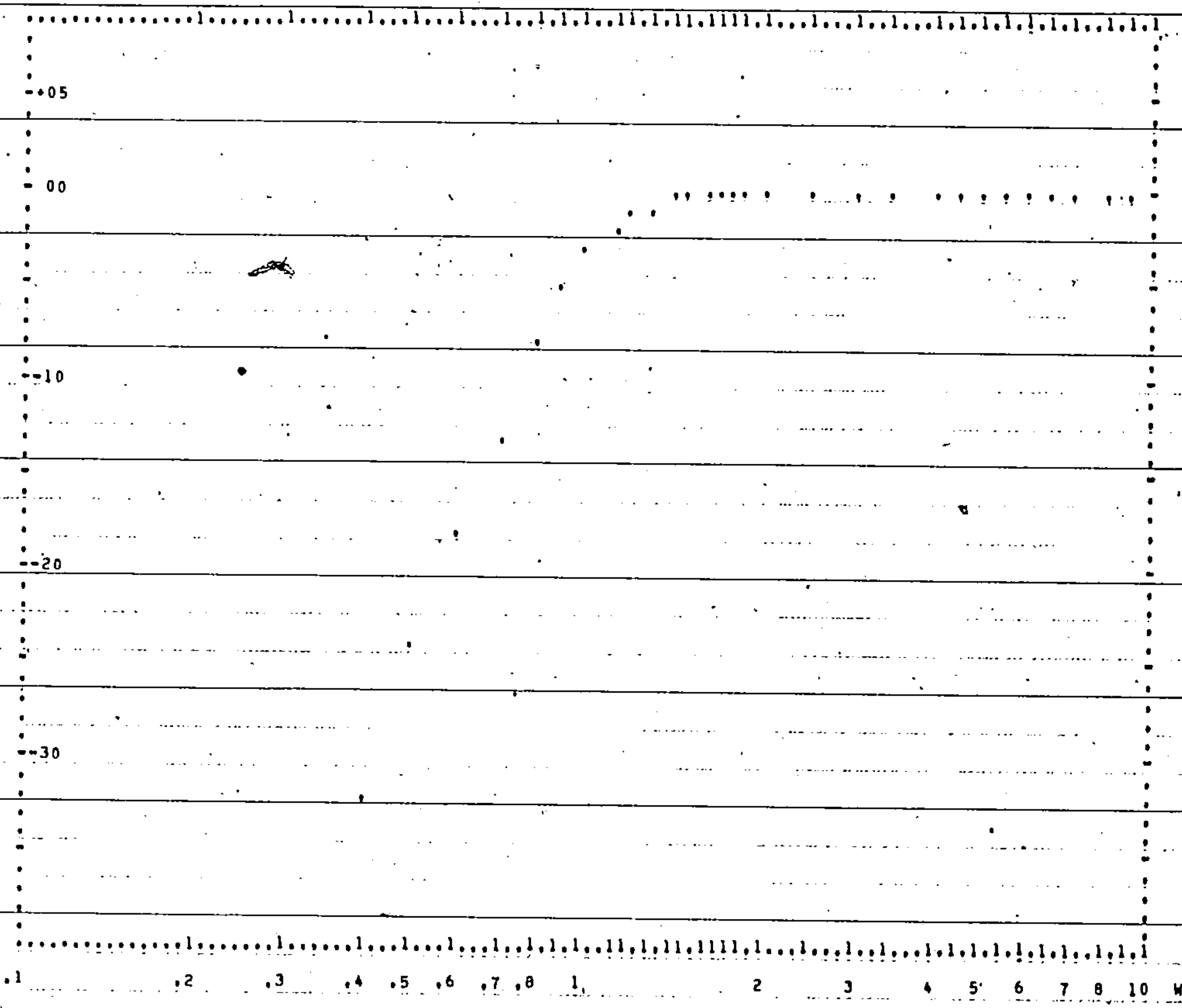

FIGURE 3.3.2.1 RESPONSE OF A T.L. + A H.P. BUTTERWORTH FILTER OF ORDER 4 (REFER TO FIG.3.1)

[POOR PRINT]

T= .30

W	TW	TWDB
0.0	0.000000	-200.00
.1	.000100	-79.98
.2	.001616	-55.83
.3	.008285	-41.63
.4	.026685	-31.47
.5	.066852	-23.50
.6	.143186	-16.88
.7	.274632	-11.22
.8	.478857	-6.40
.9	.747684	-2.53
1.0	1.012877	.11
1.1	1.188574	1.50
1.2	1.264694	2.04
1.3	1.282977	2.16
1.4	1.277341	2.13
1.5	1.263908	2.03
1.6	1.248910	1.93
1.7	1.234453	1.83
1.8	1.221096	1.73
1.9	1.208856	1.65
2.0	1.197588	1.57
2.4	1.159135	1.28
3.0	1.111796	.92
3.4	1.084146	.70
4.0	1.047656	.40
4.4	1.026720	.23
5.0	1.000590	.01
5.4	.986722	-.12
6.0	.971166	-.25
6.4	.964170	-.32
7.0	.958424	-.37
8.0	.960049	-.35
9.0	.972502	-.24
10.0	.990918	-.08

TW IN DB

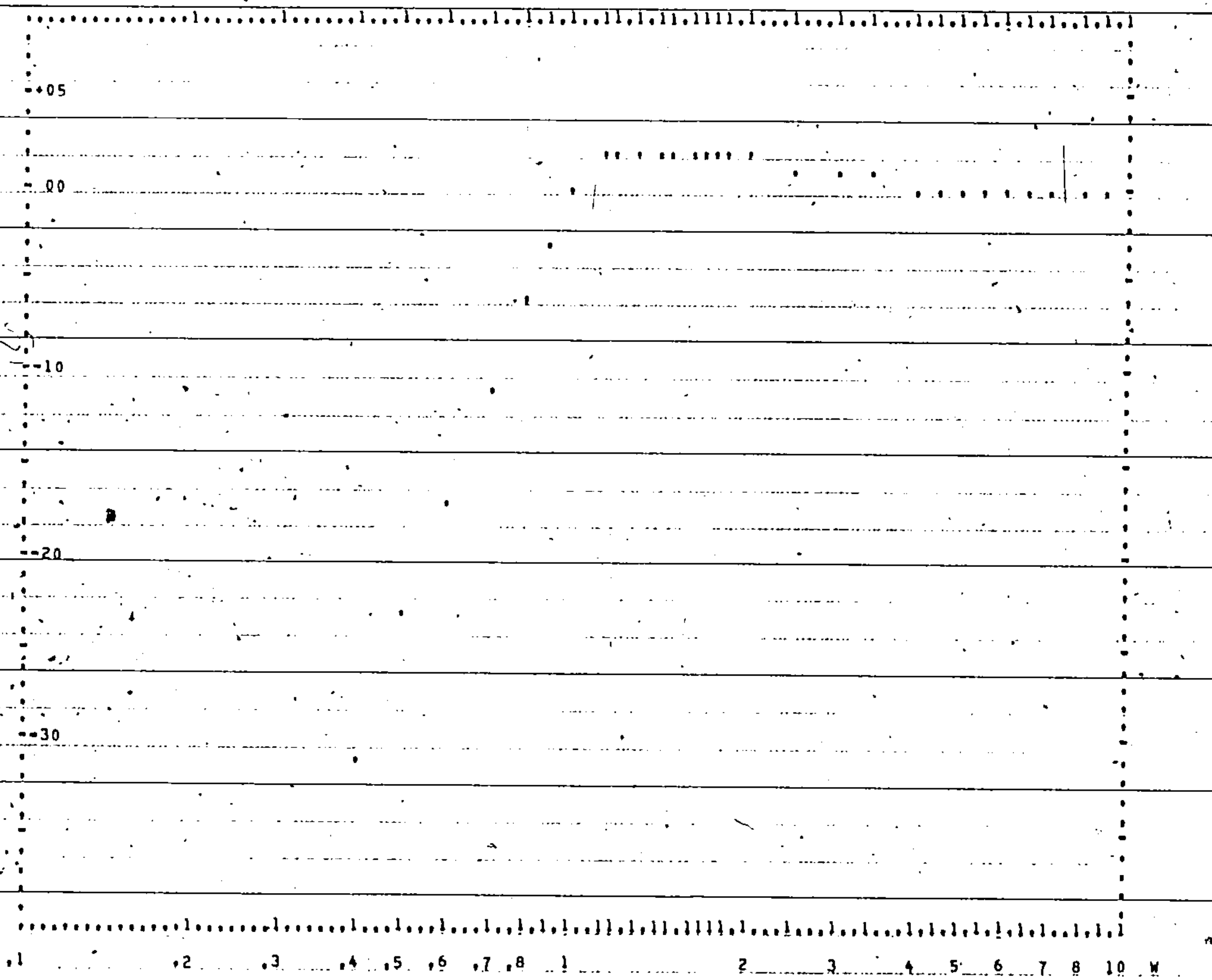

FIGURE 3.3.2.2 RESPONSE OF A T.L. + A H.P. BUTTERWORTH FILTER OF ORDER 4 (REFER TO FIG.3.1)

[POOR PRINT]

W	TW	TWDB
0.0	0.000000	-200.00
.1	.000001	-120.00
.2	.000064	-83.88
.3	.000729	-62.75
.4	.004096	-47.75
.5	.015623	-36.12
.6	.046606	-26.63
.7	.116846	-18.65
.8	.253586	-11.92
.9	.469306	-6.57
1.0	.707118	-3.01
1.1	.870817	-1.20
1.2	.948184	-.46
1.3	.979138	-.18
1.4	.991224	-.08
1.5	.996098	-.03
1.6	.998161	-.02
1.7	.999080	-.01
1.8	.999509	-.00
1.9	.999720	-.00
2.0	.999828	-.00
2.4	.999949	-.00
3.0	.999975	-.00
3.4	.999981	-.00
4.0	.999986	-.00
4.4	.999988	-.00
5.0	.999991	-.00
5.4	.999992	-.00
6.0	.999994	-.00
6.4	.999994	-.00
7.0	.999995	-.00
8.0	.999996	-.00
9.0	.999997	-.00
10.0	.999998	-.00

TW IN DB T=0.00

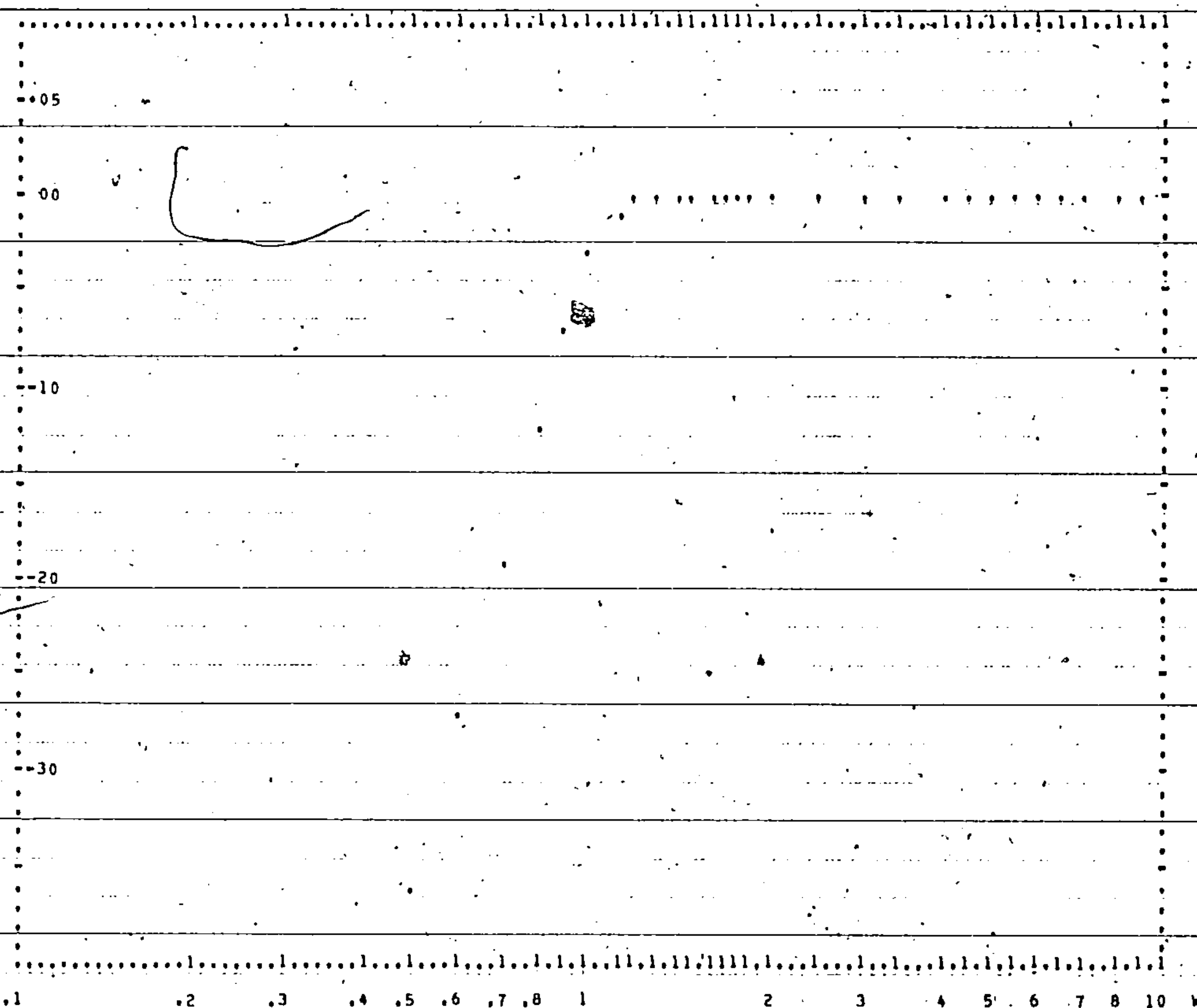

FIGURE 3.3.3.1 RESPONSE OF A T.L. + A H.P. BUTTERWORTH FILTER OF ORDER 6 (REFER TO FIG.3.1)

POOR PRINT

T= .50

W	TW	TWDB
0.0	0.000000	-200.00
.1	.000001	-119.96
.2	.000065	-83.72
.3	.000760	-62.38
.4	.004426	-47.08
.5	.017751	-35.02
.6	.056862	-24.90
.7	.158444	-16.00
.8	.406113	-7.83
.9	.952016	-.43
1.0	1.594250	4.05
1.1	1.659870	4.40
1.2	1.528032	3.68
1.3	1.423139	3.06
1.4	1.350708	2.61
1.5	1.297947	2.27
1.6	1.256710	1.98
1.7	1.222581	1.75
1.8	1.193161	1.53
1.9	1.167099	1.34
2.0	1.143598	1.17
2.4	1.067570	.57
3.0	.990956	-.08
3.4	.958821	-.37
4.0	.933618	-.60
4.4	.929837	-.63
5.0	.940084	-.54
5.4	.955182	-.40
6.0	.985064	-.13
6.4	1.005948	.05
7.0	1.031339	.27
8.0	1.040520	.35
9.0	1.012582	.11
10.0	.981047	-.17

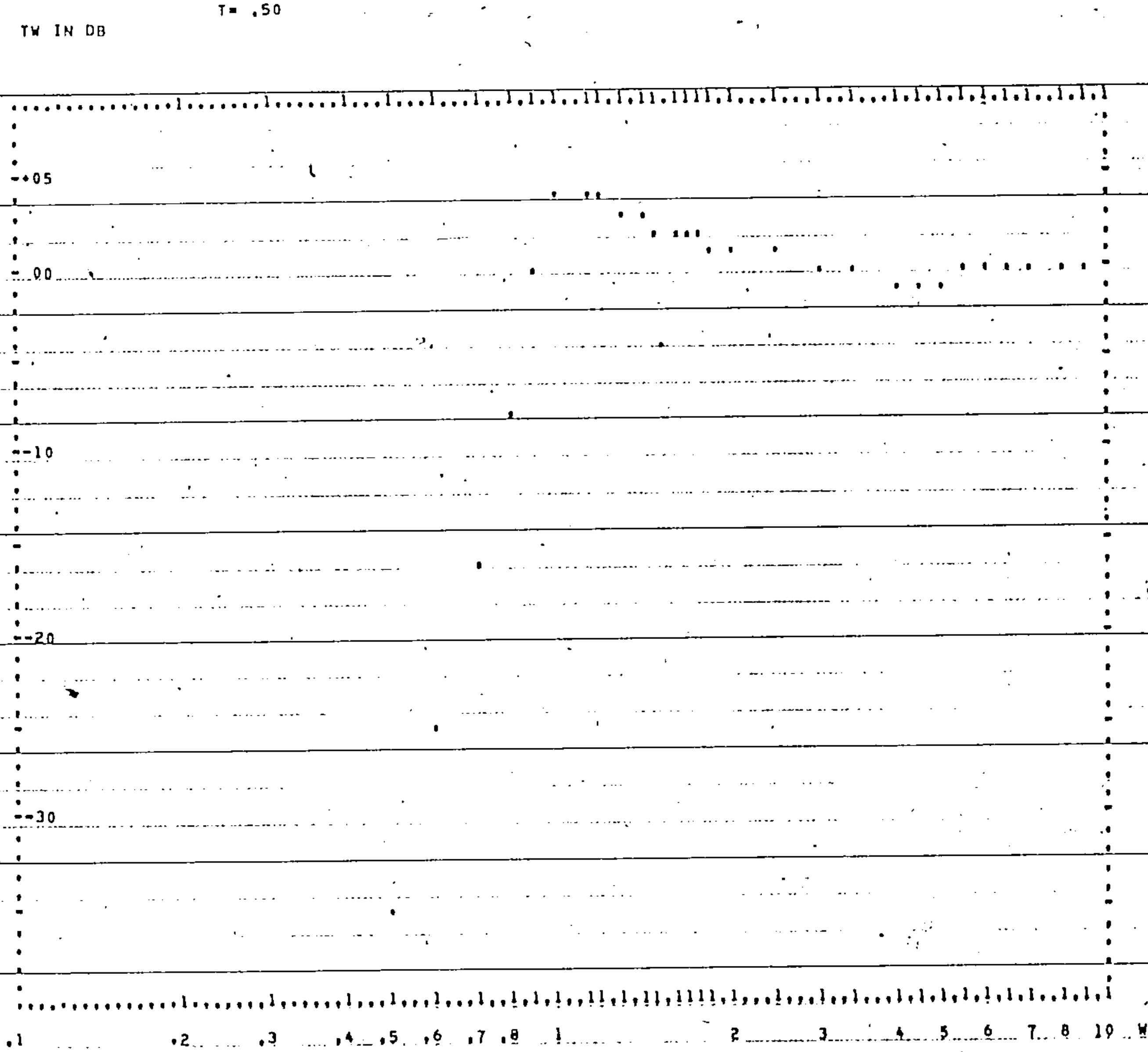

FIGURE 3.3.3.2 RESPONSE OF A T.L. + A H.P. BUTTERWORTH FILTER OF ORDER 6 (REFER TO FIG.3.1)

T=0.00

W	TW	TWDB
0.0	0.000000	-200.00
.1	.000000	-160.00
.2	.000003	-111.88
.3	.000064	-83.84
.4	.000630	-64.01
.5	.003755	-48.51
.6	.016852	-35.47
.7	.063262	-23.98
.8	.195175	-14.19
.9	.431428	-7.30
1.0	.707107	-3.01
1.1	.976709	-.20
1.2	1.140096	1.14
1.3	1.158679	1.28
1.4	1.112692	.93
1.5	1.061759	.52
1.6	1.022502	.19
1.7	.995525	-.04
1.8	.978060	-.19
1.9	.967362	-.29
2.0	.961325	-.34
2.4	.959667	-.36
3.0	.973099	-.24
3.4	.980639	-.17
4.0	.988163	-.10
4.4	.991343	-.08
5.0	.994426	-.05
5.4	.995762	-.04
6.0	.997112	-.03
6.4	.997725	-.02
7.0	.998374	-.01
8.0	.999021	-.01
9.0	.999378	-.01
10.0	.999587	-.00

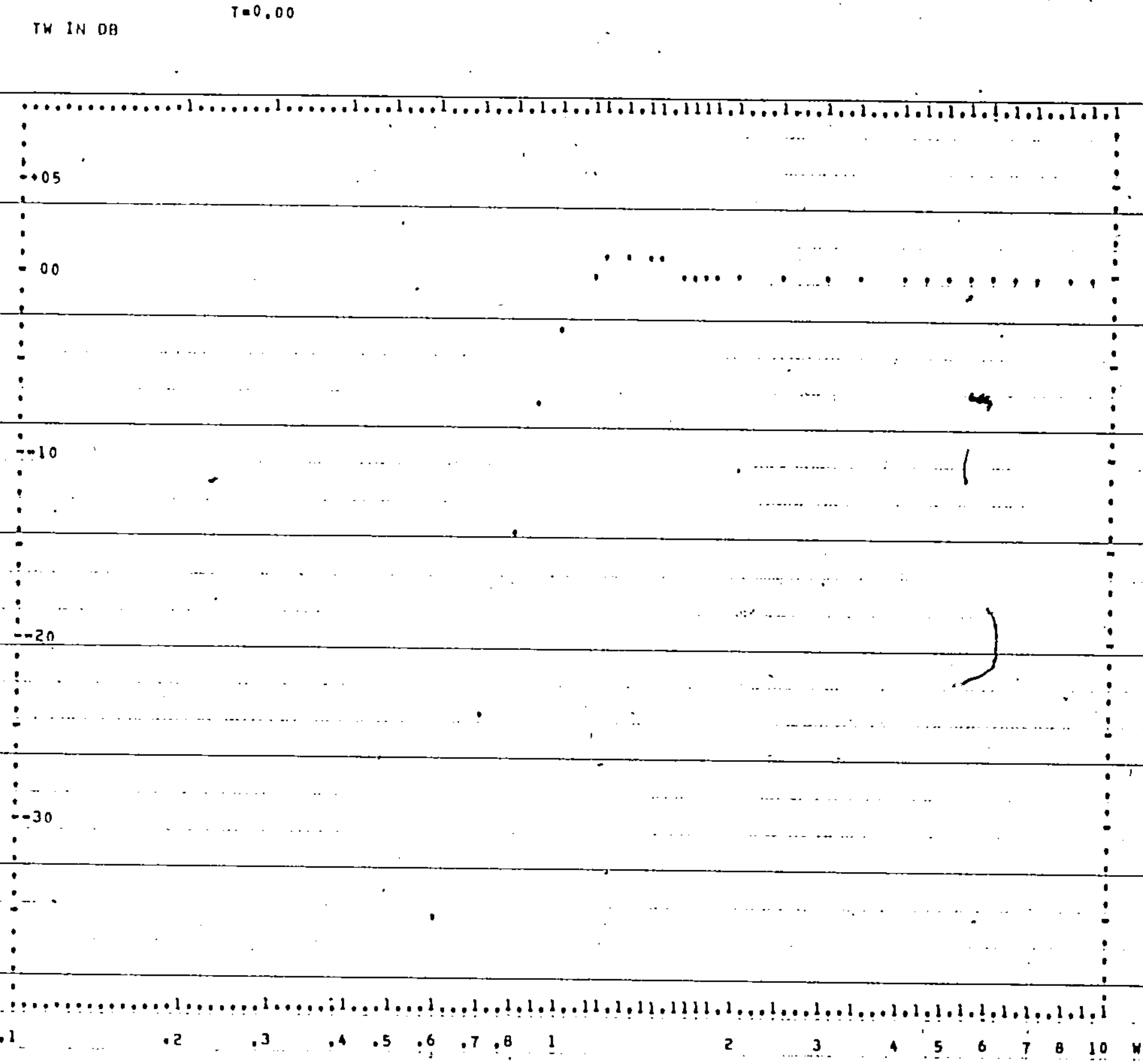

FIGURE 3.3.4.1 RESPONSE OF A T.L. + A H.P. BUTTERWORTH FILTER OF ORDER 8 (REFER TO FIG.3.1)

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POOR PRINT

T = .50

W	TW	TWDB
0.0	0.000000	-200.00
.1	.000000	-159.96
.2	.000003	-111.72
.3	.000067	-83.46
.4	.000683	-63.31
.5	.004291	-47.35
.6	.020745	-33.66
.7	.087051	-21.20
.8	.323615	-9.80
.9	.970054	-.26
1.0	1.951002	5.81
1.1	1.824727	5.22
1.2	1.444247	3.19
1.3	1.252251	1.95
1.4	1.164198	1.32
1.5	1.127392	1.04
1.6	1.116368	.96
1.7	1.117564	.97
1.8	1.123114	1.01
1.9	1.128369	1.05
2.0	1.130798	1.07
2.4	1.103473	.86
3.0	1.017694	.15
3.4	.972148	-.25
4.0	.933924	-.59
4.4	.925852	-.67
5.0	.933897	-.59
5.4	.949343	-.45
6.0	.981311	-.16
6.4	1.004082	.04
7.0	1.032137	.27
8.0	1.043089	.37
9.0	1.013707	.12
10.0	.980395	-.17

TW IN DB

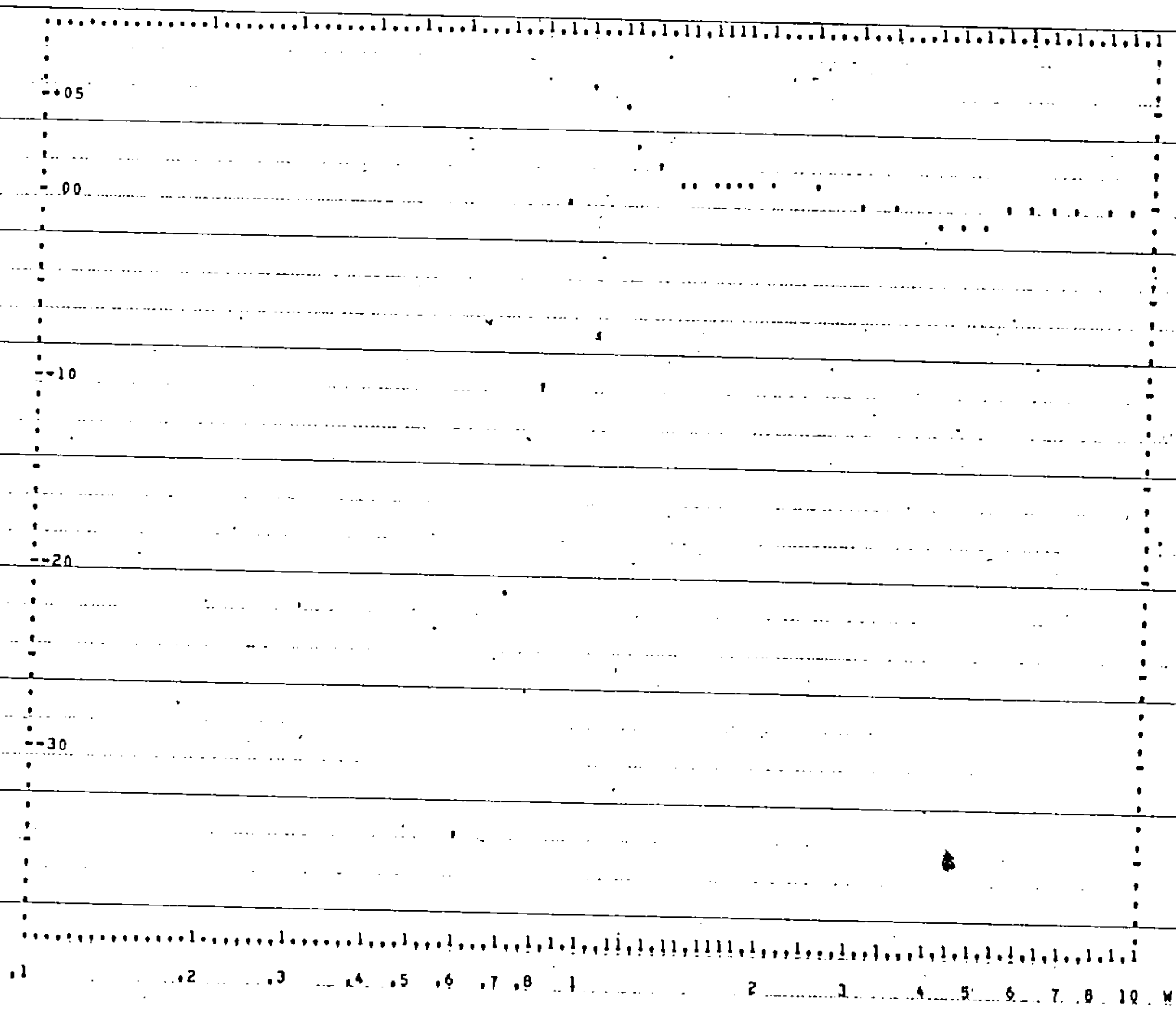

FIGURE 3.3.4.2 RESPONSE OF A T.L. + A H.P. BUTTERWORTH FILTER OF ORDER 8 (REFER TO FIG.3.1)

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POOR PRINT

T=0,00

W	TW	TWDB
0.0	0.000000	-200.00
.1	.000000	-200.00
.2	.000000	-139.80
.3	.000006	-104.54
.4	.000188	-79.36
.5	.001045	-59.62
.6	.006504	-43.74
.7	.027443	-31.23
.8	.090960	-20.82
.9	.284699	-10.91
1.0	.707107	-3.01
1.1	.817659	-1.75
1.2	.824621	-1.67
1.3	.878485	-1.13
1.4	.951270	-.43
1.5	1.016245	.14
1.6	1.059564	.50
1.7	1.080260	.67
1.8	1.084480	.70
1.9	1.079323	.66
2.0	1.070000	.59
2.4	1.033103	.28
3.0	1.008604	.07
3.4	1.003088	.03
4.0	1.000189	.00
4.4	.999604	-.00
5.0	.999385	-.01
5.4	.999400	-.01
6.0	.999489	-.00
6.4	.999557	-.00
7.0	.999648	-.00
8.0	.999763	-.00
9.0	.999839	-.00
10.0	.999888	-.00

TW IN DB

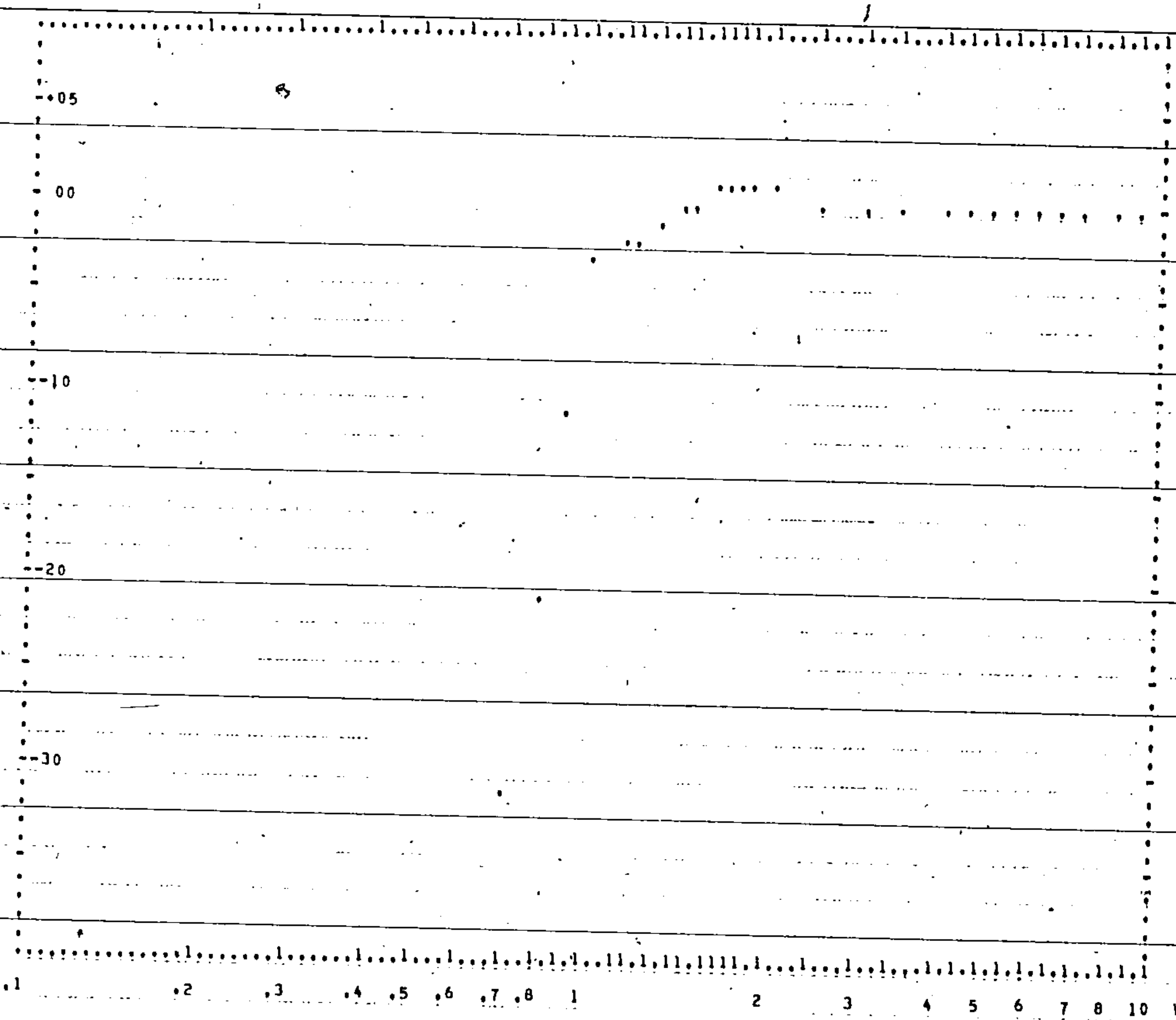

FIGURE 3.3.5.1 RESPONSE OF A T.L. + A H.P. BUTTERWORTH FILTER OF ORDER 10 (REFER TO FIG.3.1)

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[POOR PRINT]

T= .90

W	TW	TWDB
0.0	0.000000	-200.00
.1	.000000	-199.92
.2	.000000	-139.46
.3	.000006	-103.75
.4	.000128	-77.87
.5	.001390	-57.14
.6	.010226	-39.81
.7	.056073	-25.02
.8	.308413	-10.22
.9	4.833191	13.68
1.0	1.504659	3.55
1.1	1.388108	2.85
1.2	1.242561	1.89
1.3	1.063503	.53
1.4	.945507	-.49
1.5	.881428	-1.10
1.6	.850986	-1.40
1.7	.840280	-1.51
1.8	.840725	-1.51
1.9	.846971	-1.44
2.0	.855699	-1.35
2.4	.889377	-1.02
3.0	.939459	-.54
3.4	.989883	-.09
4.0	1.060015	.51
4.4	1.067740	.57
5.0	1.018776	.16
5.4	.979260	-.18
6.0	.951866	-.43
6.4	.960521	-.35
7.0	1.001096	.01
8.0	1.036641	.31
9.0	.982611	-.15
10.0	.977663	-.20

TW IN DB

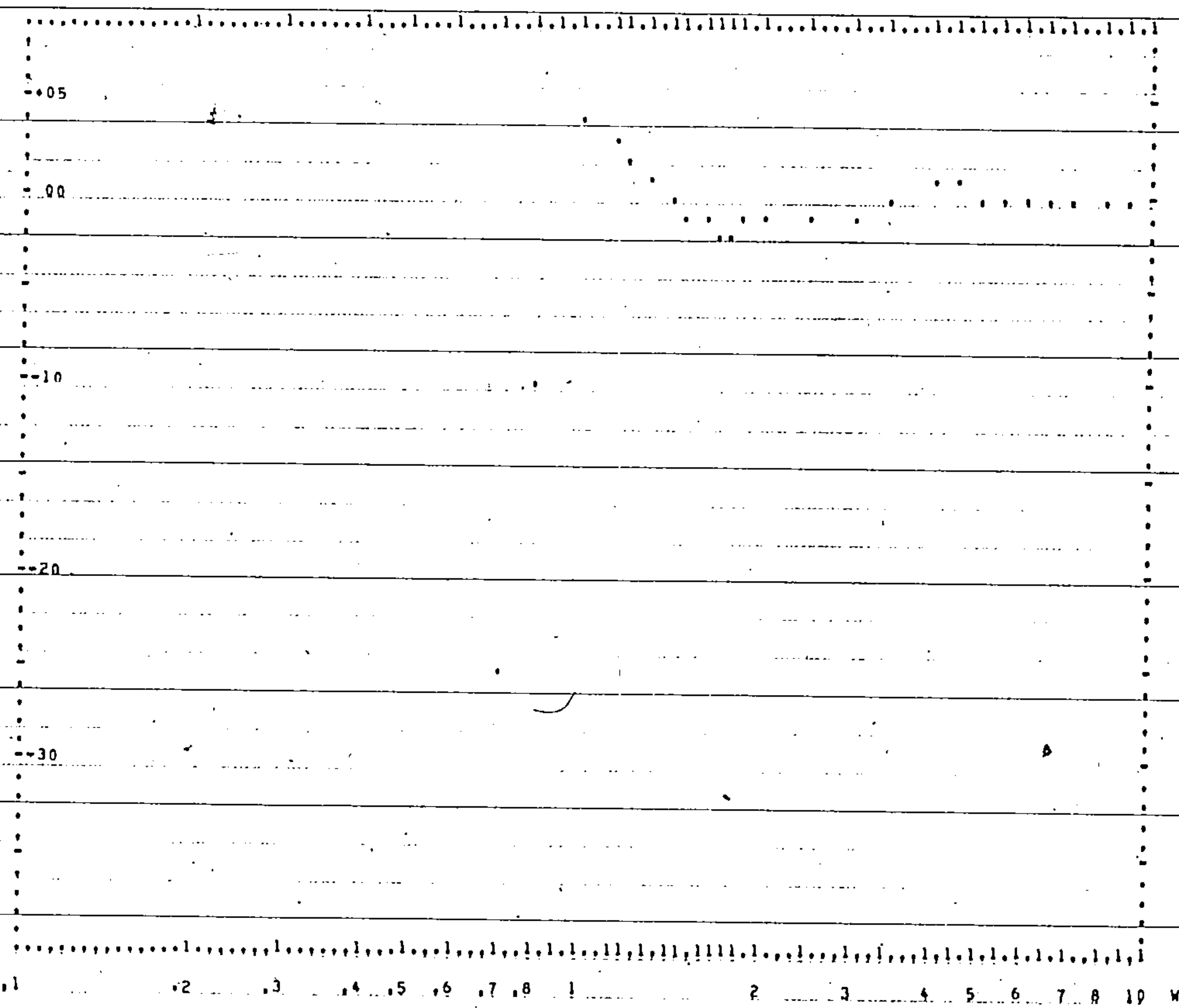

FIGURE 3.3.5.2 RESPONSE OF A T.L. + A H.P. BUTTERWORTH FILTER OF ORDER 10 (REFER TO FIG.3.1)

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[POOR PRINT]

3.4 The Lumped Section is a Band-Pass Filter

Replacing S by $\frac{wo}{B}\left(\frac{S}{wo} + \frac{wo}{S}\right)$ where

B — Normalized band width

wo — Centre frequency

in the previous L.P. equations transforms the L.P. to a Band-Pass filter.

3.4.1 Band-Pass Filter (N = 2)

Figures 3.4.1.1, 3.4.1.2 show the frequency response of the same filter (N = 2) calculated at $\tau = 0.0$ and $\tau = 0.3$ respectively.

We notice that in Figure 3.4.1.2 there is a peak appearing at w = 6 indicating that τ should be reduced in order to avoid having a peak signal with such magnitude (4.46 dbs).

3.4.2 Band Pass Filter (N = 4)

Figures 3.4.2.1 and 3.4.2.2 show the frequency response of the same filter (N = 4) calculated at $\tau = 0.0$ and at $\tau = 0.3$ respectively. A peak is present at w = 6.0 (-22.57 dbs) in this case.

3.4.3 Band-Pass Filter (N = 6)

Figures 3.4.3.1 and 3.4.3.2 show the frequency response of the B.P. filter for N = 6 calculated at $\tau = 0.0$ and at $\tau = 0.5$ respectively. A peak with small magnitude is clear at w = 3.4 for a value of $\tau = 0.5$.

3.4.4 Band-Pass Filter (N = 8)

Figures 3.4.4.1 and 3.4.4.2 show the frequency response of the B.P. filter for N = 8 calculated at $\tau = 0.0$ and at $\tau = 0.5$. In Figure 3.4.4.2 the frequency response falls down to - 7.4 db from - 3 db which indicates an improvement in the selectivity.

[POOR PRINT]

3.4.5 Band-Pass Filter (N = 10)

Figures 3.4.5.1 and 3.4.5.2 show the frequency response of the B.P. filter for N = 10 calculated at τ = 0.0 and at τ = 0.9. It should be noted here that as the filter order increases it is possible to increase τ higher than at low order filter.

T=0.00

W	TW	TWDB
0.0	0.000000	-200.00
.1	.022951	-32.78
.2	.097194	-20.25
.3	.237538	-12.49
.4	.454474	-6.85
.5	.707114	-3.01
.6	.892398	-.99
.7	.973286	-.24
.8	.995976	-.04
.9	.999804	-.00
1.0	1.000000	.00
1.1	.999869	-.00
1.2	.998221	-.02
1.3	.992255	-.07
1.4	.978858	-.19
1.5	.955529	-.40
1.6	.921164	-.71
1.7	.876466	-1.15
1.8	.823747	-1.68
1.9	.766214	-2.31
2.0	.707114	-3.01
2.4	.496513	-6.08
3.0	.301668	-10.41
3.4	.227149	-12.87
4.0	.157991	-16.03
4.4	.128159	-17.85
5.0	.097194	-20.25
5.4	.082456	-21.68
6.0	.065978	-23.61
6.4	.057620	-24.79
7.0	.047797	-26.41
8.0	.036257	-28.81
9.0	.028465	-30.91
10.0	.022951	-32.78

TW IN DB

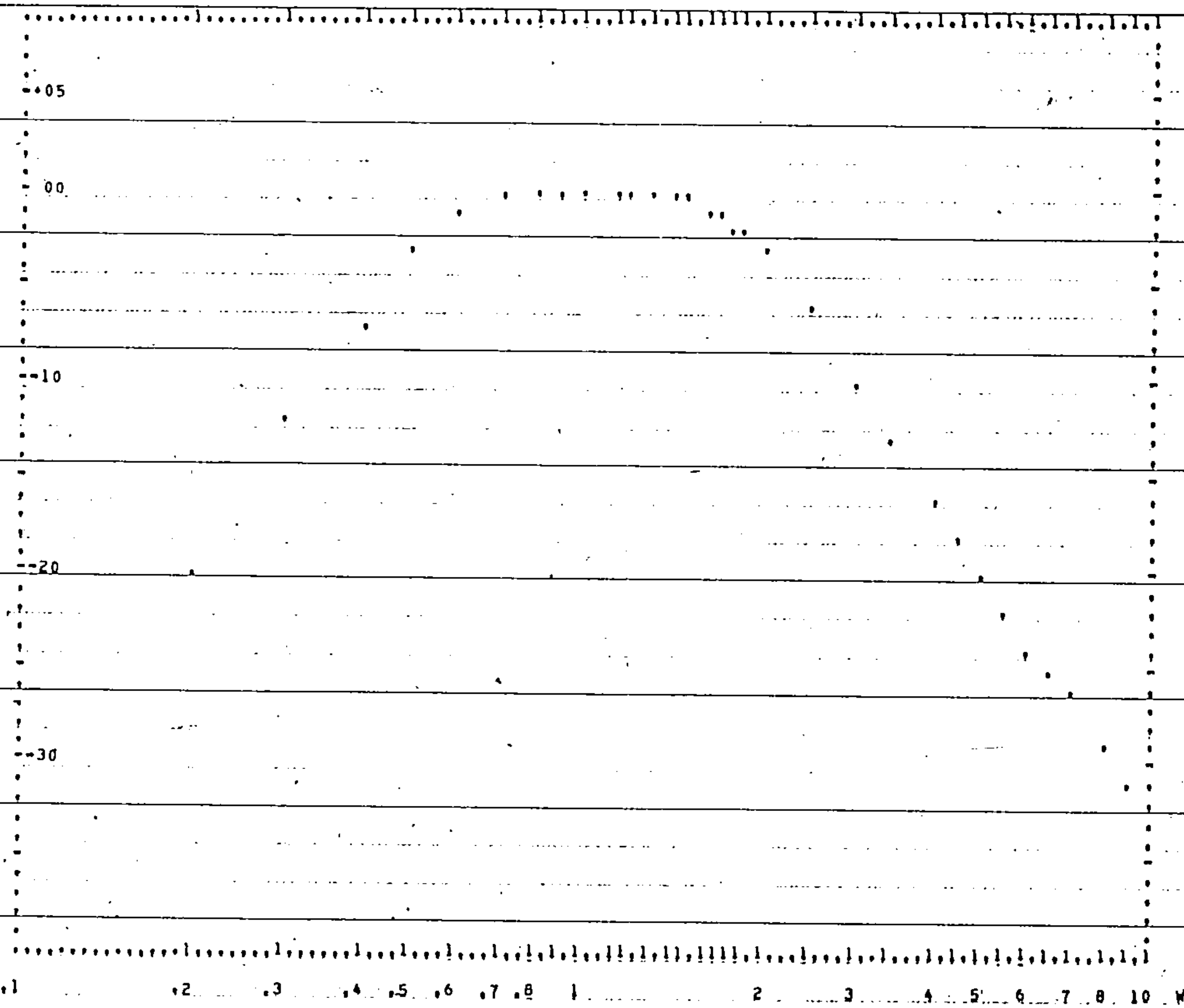

FIGURE 3.4.1.1 RESPONSE OF A T.L. + A B.P. BUTTERWORTH FILTER OF ORDER 2 (REFER TO FIG.3.1)

[POOR PRINT]

T= .30

W	TW	TWDB	TW IN DB
0.0	0.000000	-200.00	
.1	.023037	-32.75	
.2	.098795	-20.11	
.3	.247678	-12.12	
.4	.493625	-6.13	
.5	.797865	-1.96	
.6	1.005598	.05	
.7	1.062823	.53	
.8	1.052101	.44	
.9	1.026578	.23	
1.0	1.000000	.00	
1.1	.972592	-.24	
1.2	.941716	-.52	
1.3	.905191	-.87	
1.4	.862332	-1.29	
1.5	.814028	-1.79	
1.6	.762258	-2.36	
1.7	.709380	-2.98	
1.8	.657532	-3.64	
1.9	.608300	-4.32	
2.0	.562664	-5.00	
2.4	.420513	-7.52	
3.0	.302579	-10.38	
3.4	.262649	-11.61	
4.0	.239041	-12.43	
4.4	.245557	-12.20	
5.0	.312985	-10.09	
5.4	.486714	-6.25	
6.0	1.671623	4.46	
6.4	.331335	-9.59	
7.0	.130857	-17.66	
8.0	.056367	-24.98	
9.0	.033423	-29.52	
10.0	.023551	-32.56	

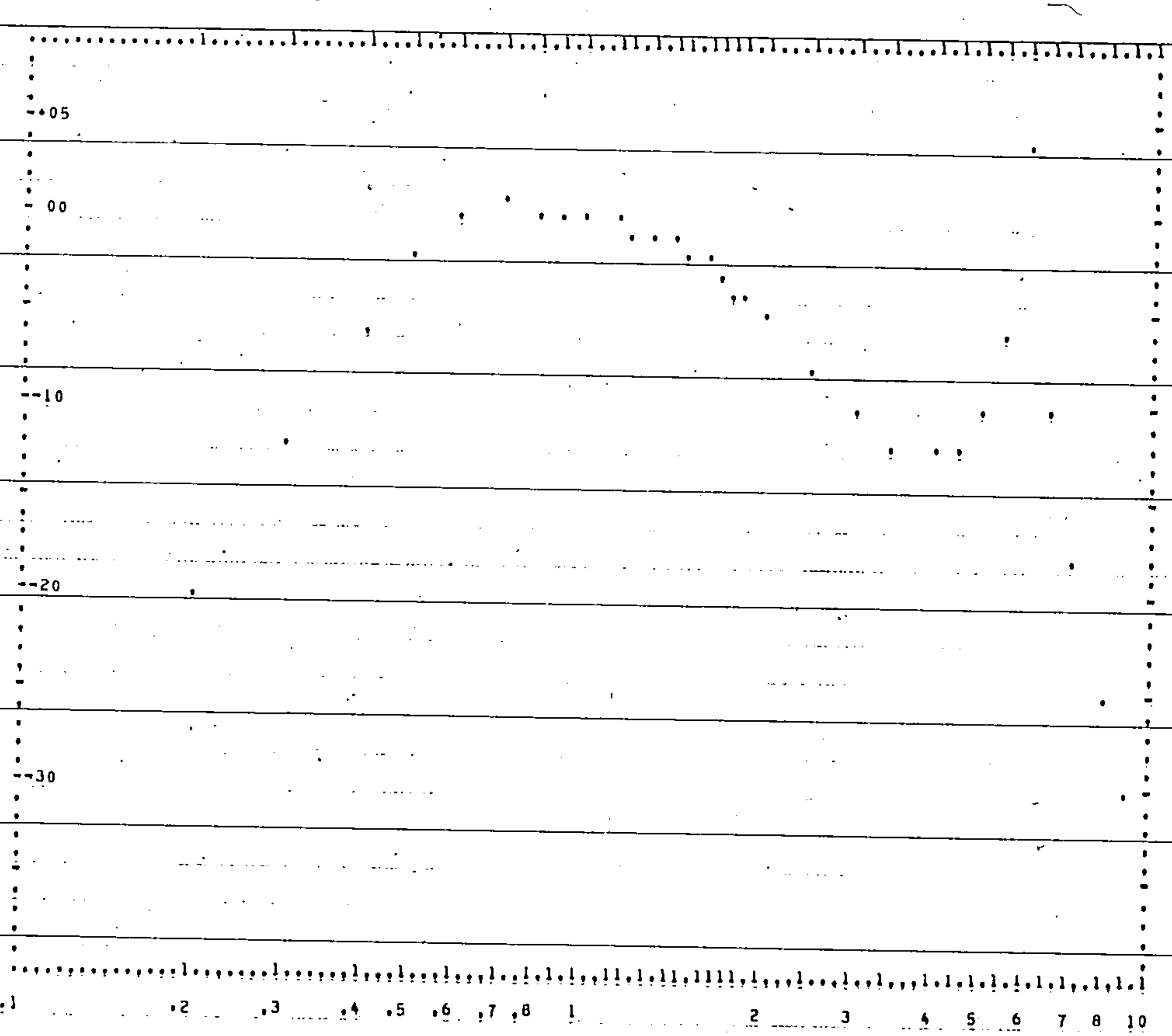

FIGURE 3.4.1.2 RESPONSE OF A T.L. + A B.P. BUTTERWORTH FILTER OF ORDER 2 (REFER TO FIG.3.1)

T=0.00

W	TW	TWDB
0.0	0.000000	-200.00
.1	.000527	-65.56
.2	.009536	-40.41
.3	.059692	-24.48
.4	.251916	-11.97
.5	.707114	-3.01
.6	.968837	-.27
.7	.998463	-.01
.8	.999971	-.00
.9	1.000001	.00
1.0	1.000000	.00
1.1	1.000001	.00
1.2	.999997	-.00
1.3	.999883	-.00
1.4	.999056	-.01
1.5	.995503	-.04
1.6	.984449	-.14
1.7	.957367	-.38
1.8	.903729	-.88
1.9	.817944	-1.75
2.0	.707114	-3.01
2.4	.310960	-10.15
3.0	.099616	-20.03
3.4	.054323	-25.30
4.0	.025592	-31.84
4.4	.016697	-35.55
5.0	.009536	-40.41
5.4	.006845	-43.29
6.0	.004372	-47.19
6.4	.003331	-49.55
7.0	.002290	-52.80
8.0	.001316	-57.61
9.0	.000811	-61.82
10.0	.000527	-65.56

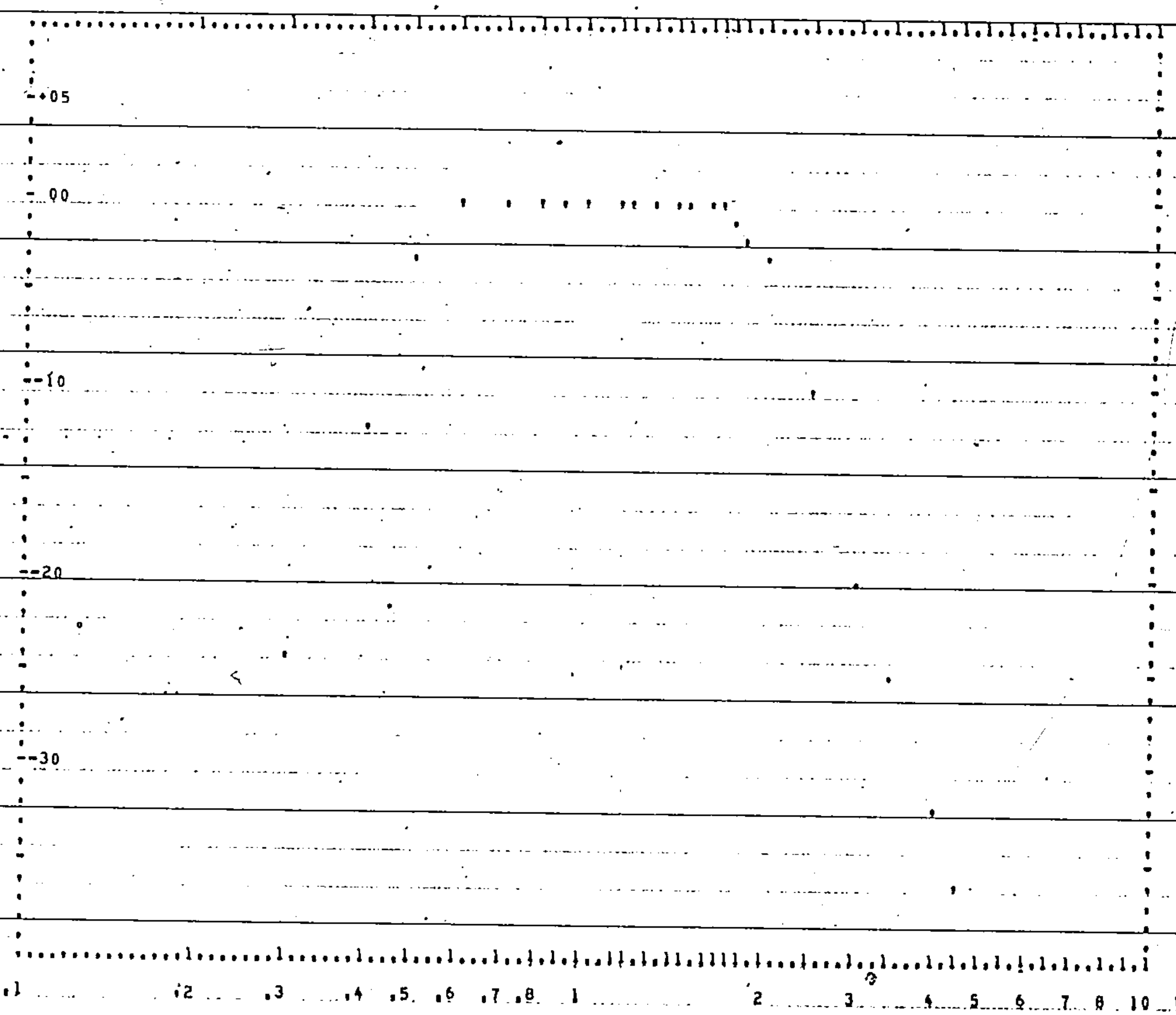

FIGURE 3.4.2.1 RESPONSE OF A T.L. + A B.P. BUTTERWORTH FILTER OF ORDER 4 (REFER TO FIG.3.1)

[POOR PRINT]

T= .30

W	TW	TWDB
0.0	0.000000	-200.00
.1	.000529	-65.53
.2	.009678	-40.28
.3	.061988	-24.15
.4	.274754	-11.22
.5	.828020	-1.64
.6	1.092109	.77
.7	1.077254	.65
.8	1.049344	.42
.9	1.024391	.21
1.0	1.000000	.00
1.1	.974946	-.22
1.2	.948554	-.46
1.3	.920100	-.72
1.4	.888302	-1.03
1.5	.850880	-1.40
1.6	.804646	-1.89
1.7	.746612	-2.54
1.8	.676088	-3.40
1.9	.596328	-4.49
2.0	.513788	-5.78
2.4	.256471	-11.82
3.0	.103246	-19.72
3.4	.065706	-23.65
4.0	.040916	-27.76
4.4	.034085	-29.35
5.0	.033627	-29.47
5.4	.047027	-26.55
6.0	.074375	-22.57
6.4	.017447	-35.17
7.0	.006043	-44.37
8.0	.002019	-53.90
9.0	.000947	-60.47
10.0	.000540	-65.35

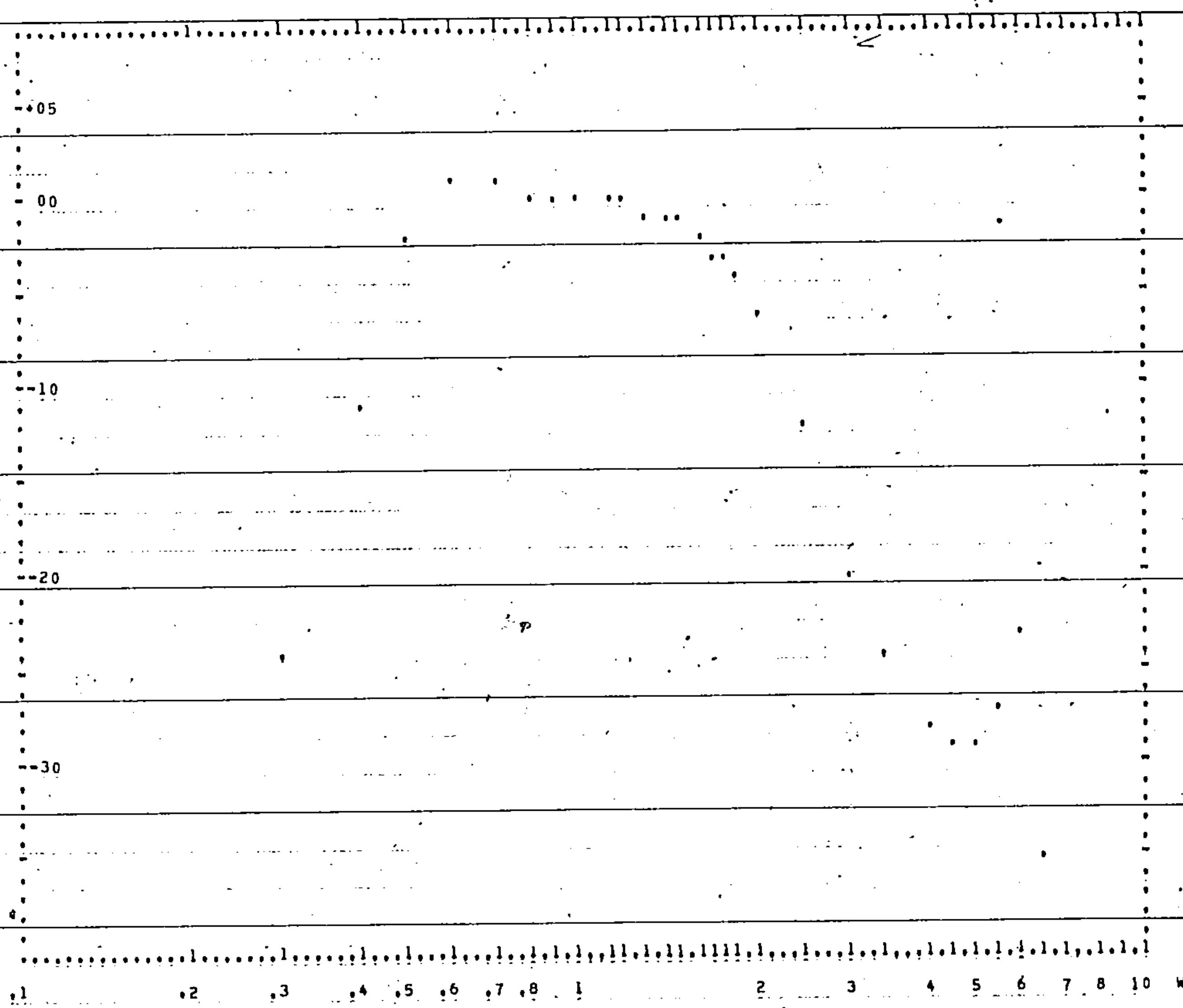

FIGURE 3.4.2.2 RESPONSE OF A T.L. + A B.P. BUTTERWORTH FILTER OF ORDER 4 (REFER TO FIG.3.1)

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[POOR PRINT]

T=0.00

W	TW	TWDB
0.0	0.000000	-200.00
.1	.000012	-98.35
.2	.000931	-60.62
.3	.014621	-36.70
.4	.131658	-17.61
.5	.707118	-3.01
.6	.991671	-.07
.7	.999866	-.00
.8	.999980	-.00
.9	.999995	-.00
1.0	1.000000	.00
1.1	.999996	-.00
1.2	.999987	-.00
1.3	.999971	-.00
1.4	.999915	-.00
1.5	.999509	-.00
1.6	.997098	-.03
1.7	.986465	-.12
1.8	.950681	-.44
1.9	.861289	-1.30
2.0	.707118	-3.01
2.4	.183955	-14.71
3.0	.031661	-29.99
3.4	.012688	-37.93
4.0	.004096	-47.75
4.4	.002158	-53.32
5.0	.000931	-60.62
5.4	.000566	-64.94
6.0	.000289	-70.78
6.4	.000192	-74.32
7.0	.000110	-79.21
8.0	.000048	-86.42
9.0	.000023	-92.73
10.0	.000012	-98.35

TW IN DB

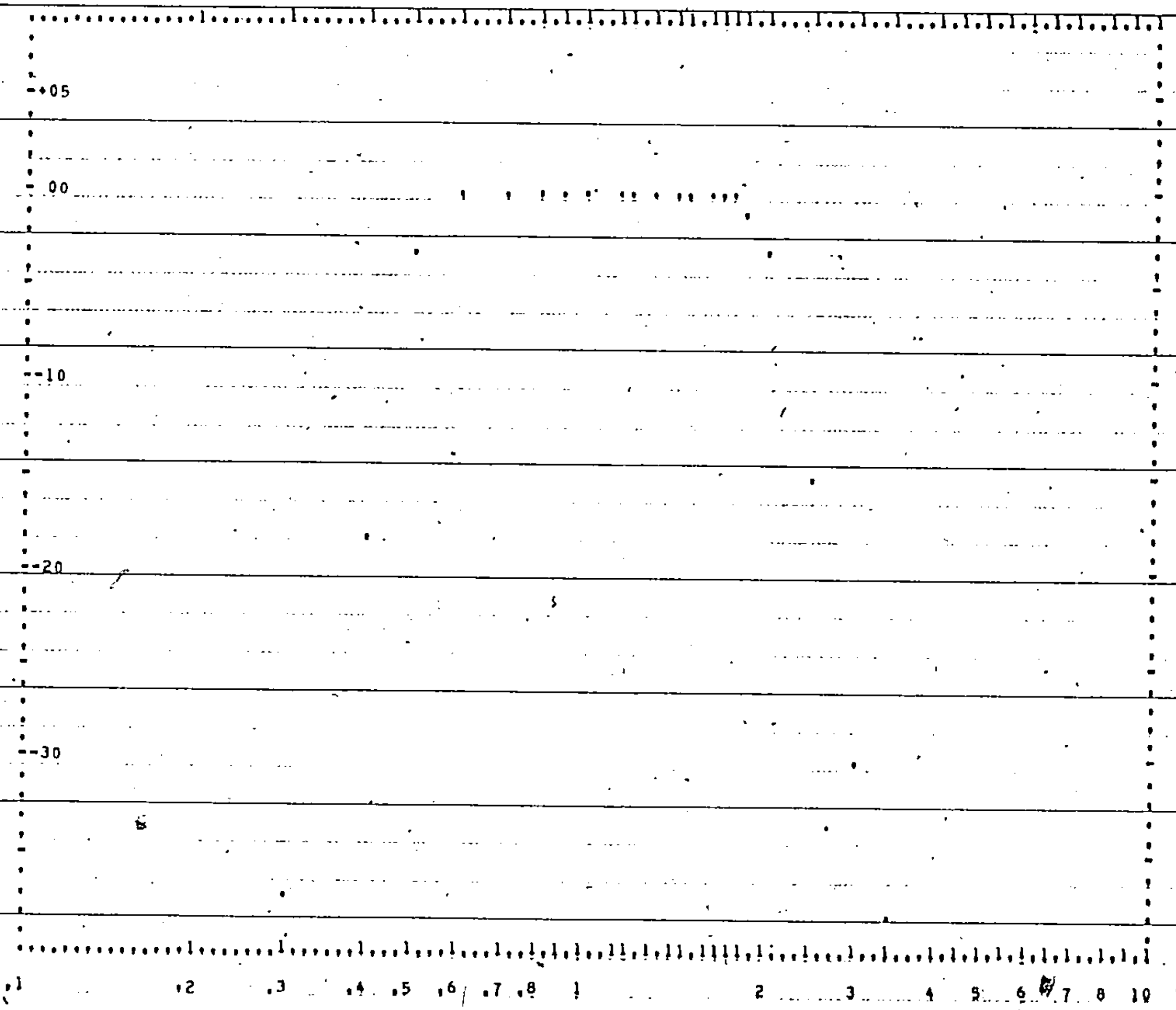

FIGURE 3.4.3.1 RESPONSE OF A T.L. + A B.P. BUTTERWORTH FILTER OF ORDER 6 (REFER TO FIG.3.1)

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POOR PRINT

T= .50

W	TW	TWDB
0.0	0.000000	-200.00
.1	.000012	-98.29
.2	.000956	-60.39
.3	.015623	-36.12
.4	.153318	-16.29
.5	.983014	-.15
.6	1.196350	1.56
.7	1.124191	1.02
.8	1.076716	.64
.9	1.036926	.31
1.0	1.000000	.00
1.1	.964322	-.32
1.2	.929119	-.64
1.3	.893797	-.98
1.4	.857592	-1.33
1.5	.819135	-1.73
1.6	.775783	-2.21
1.7	.723050	-2.82
1.8	.655552	-3.67
1.9	.571125	-4.87
2.0	.475724	-6.45
2.4	.184383	-14.69
3.0	.064788	-23.77
3.4	.059877	-24.45
4.0	.024632	-32.17
4.4	.005511	-45.17
5.0	.001378	-57.22
5.4	.000689	-63.24
6.0	.000299	-70.48
6.4	.000191	-74.39
7.0	.000111	-79.09
8.0	.000064	-83.89
9.0	.000073	-82.78
10.0	.000064	-83.89

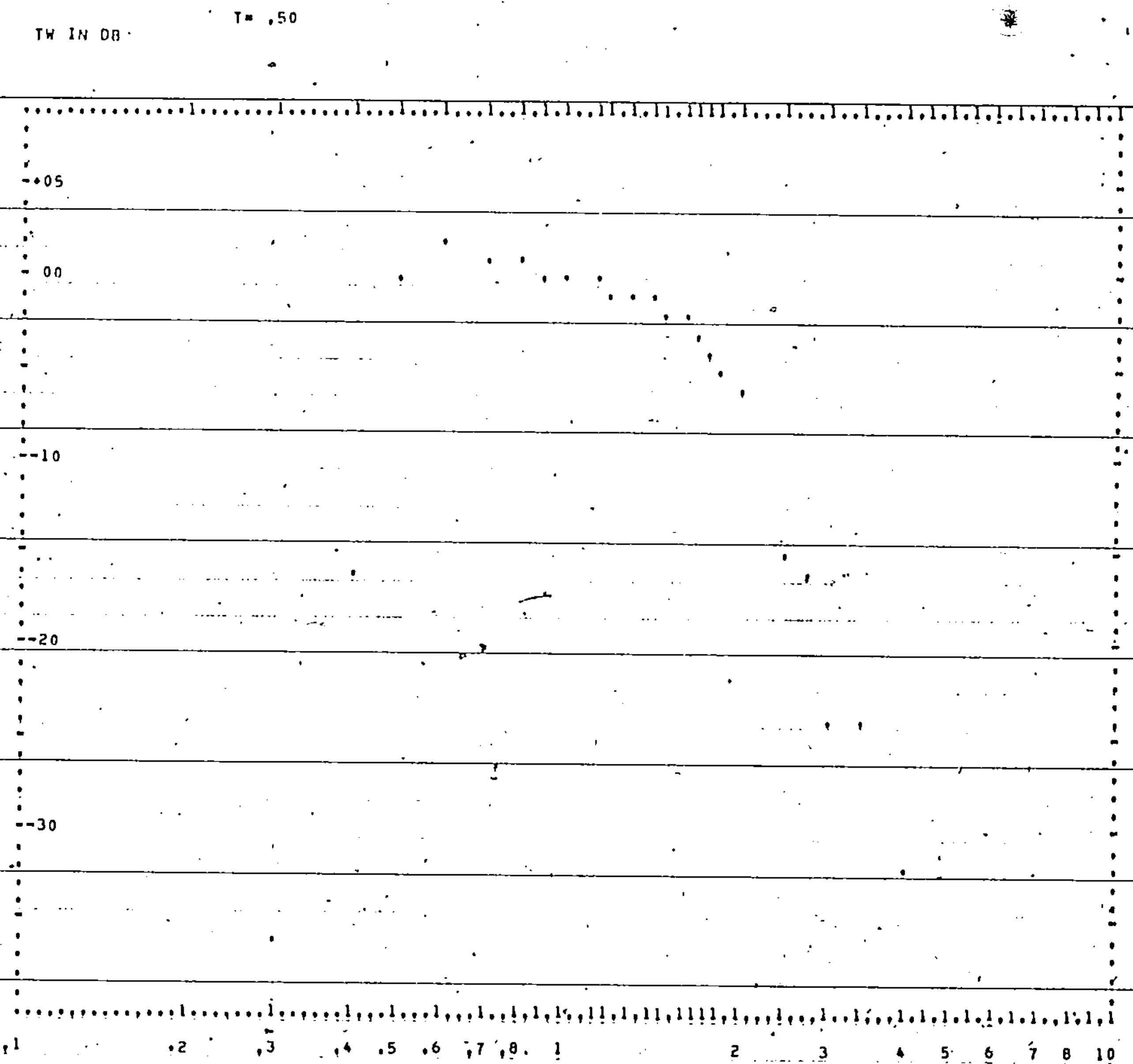

FIGURE 3.4.3.2 RESPONSE OF A T.L. + A B.P. BUTTERWORTH FILTER OF ORDER 6 (REFER TO FIG.3.1)

CONCORDIA UNIVERSITY COMPUTER CENTER

T=0,00

W	TW.	TWDB
0,0	0,000000	-200,00
,1	,000000	-131,14
,2	,000089	-81,02
,3	,003434	-49,28
,4	,075396	-22,45
,5	,707107	-3,01
,6	1,109330	,90
,7	,959322	-,36
,8	,979535	-,18
,9	,998463	-,01
1,0	1,000000	,00
1,1	,998951	-,01
1,2	,988990	-,10
1,3	,969185	-,27
1,4	,957662	-,38
1,5	,978060	-,19
1,6	1,045092	,38
1,7	1,139064	1,13
1,8	1,144300	1,17
1,9	,957347	-,36
2,0	,707107	-3,01
2,4	,123212	-18,19
3,0	,009835	-40,14
3,4	,002838	-50,94
4,0	,000630	-64,01
4,4	,000270	-71,37
5,0	,000089	-81,02
5,4	,000046	-86,74
6,0	,000019	-94,49
6,4	,000011	-99,19
7,0	,000005	-105,67
8,0	,000002	-115,27
9,0	,000001	-123,67
10,0	,000000	-131,14

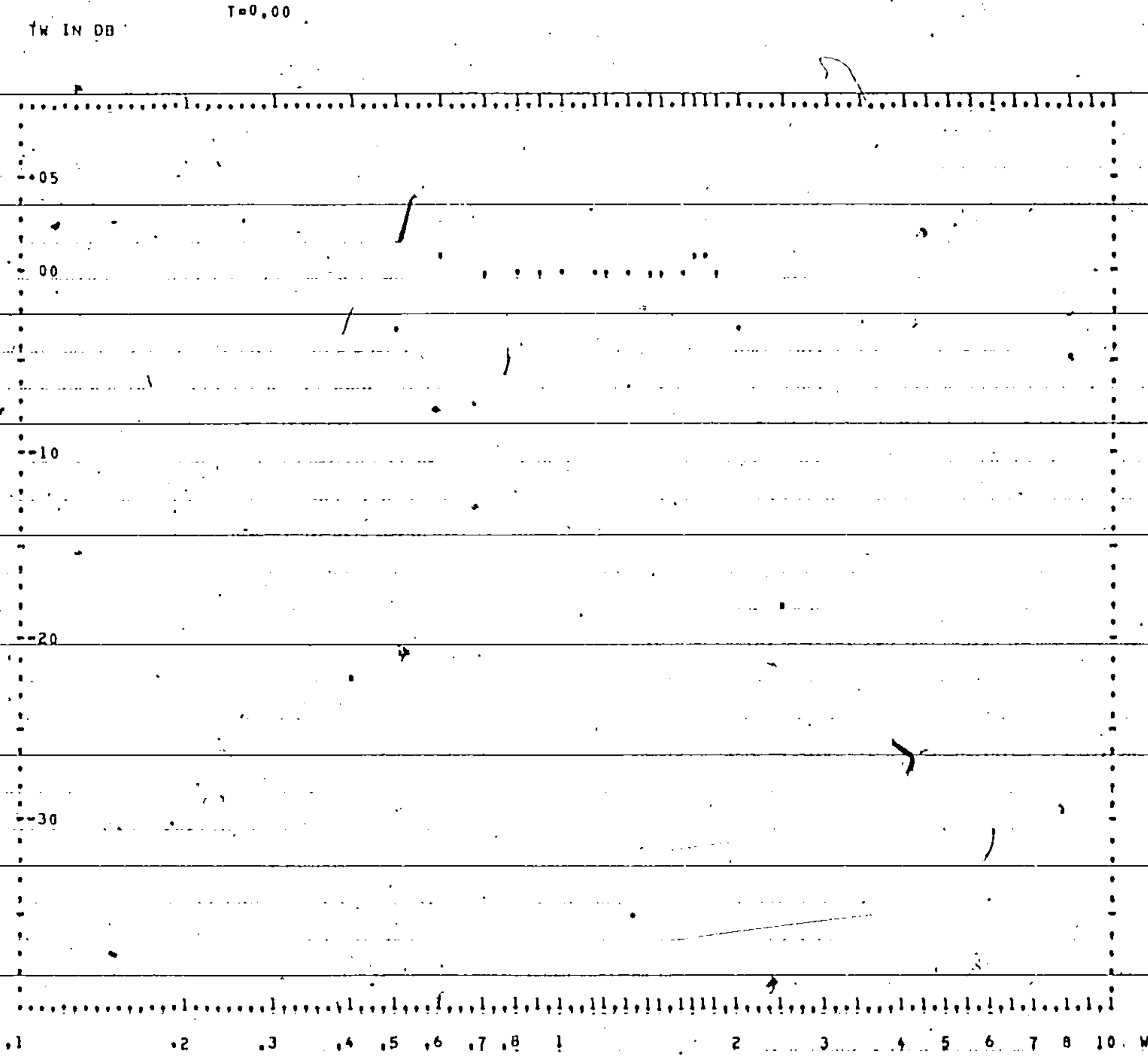

FIGURE 3.4.4.1 RESPONSE OF A T.L. + A B.P. BUTTERWORTH FILTER OF ORDER 8 (REFER TO FIG.3.1)

POOR PRINT

T= .50

W	TW	TWDB
0.0	0.000000	-200.00
.1	.000000	-131.09
.2	.000091	-80.79
.3	.003681	-48.68
.4	.088476	-21.06
.5	1.058737	.50
.6	1.201532	1.59
.7	1.048795	.41
.8	1.064057	.54
.9	1.038361	.33
1.0	1.000000	.00
1.1	.961836	-.34
1.2	.920056	-.72
1.3	.886684	-1.04
1.4	.876869	-1.14
1.5	.888759	-1.02
1.6	.886755	-1.04
1.7	.813560	-1.79
1.8	.675991	-3.40
1.9	.538020	-5.38
2.0	.426542	-7.40
2.4	.119037	-18.49
3.0	.019221	-34.32
3.4	.012280	-38.22
4.0	.004143	-47.65
4.4	.000710	-62.98
5.0	.000133	-77.53
5.4	.000056	-85.00
6.0	.000020	-94.17
6.4	.000011	-99.26
7.0	.000005	-105.58
8.0	.000002	-112.80
9.0	.000002	-113.88
10.0	.000002	-116.44

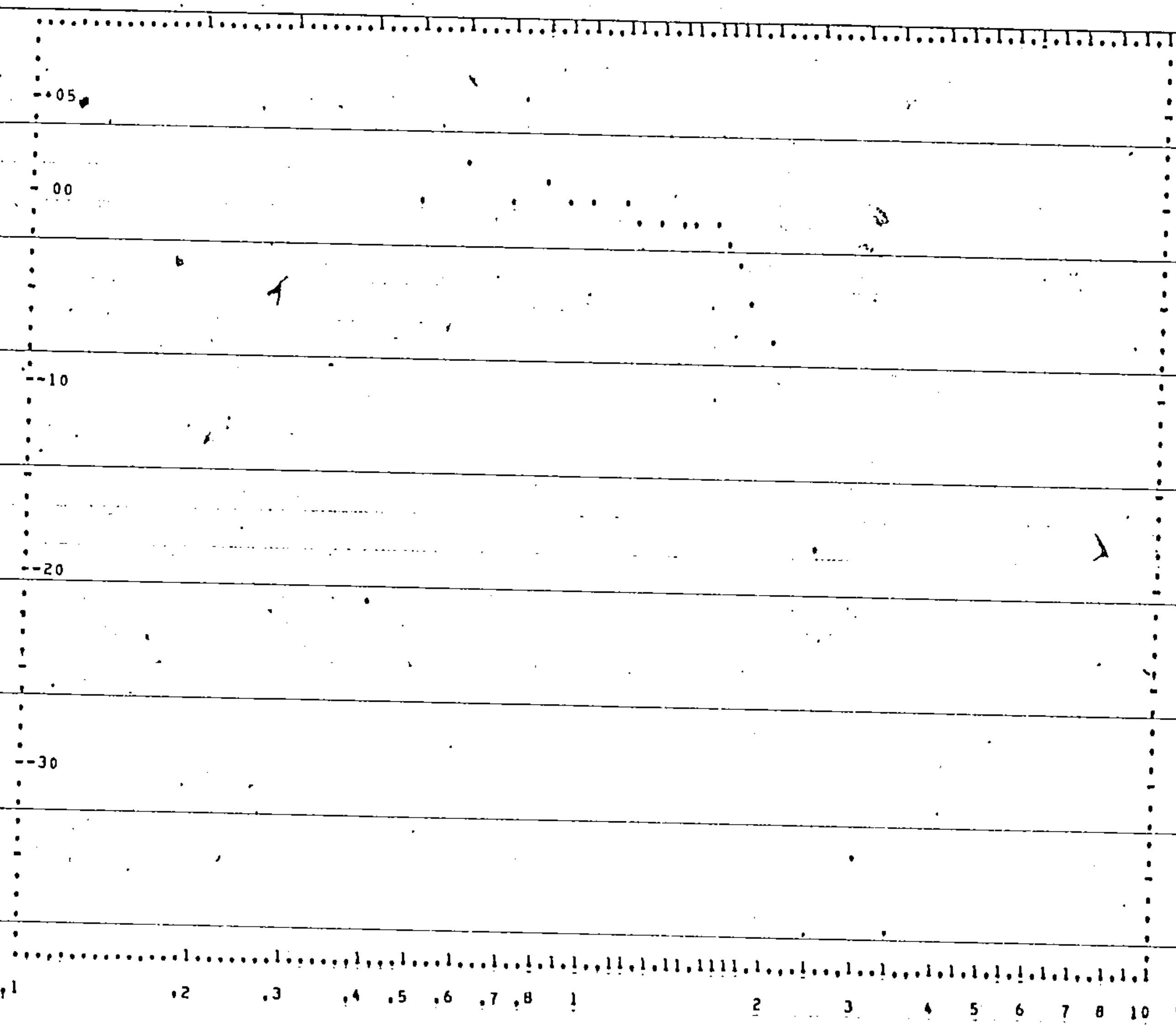

FIGURE 3.4.4.2 RESPONSE OF A T.L. + A B.P. BUTTERWORTH FILTER OF ORDER 8 (REFER TO FIG.3.1)

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[POOR PRINT]

T=0,00

W	TW	TWDB
0,0	0,000000	-200,00
,1	,000000	-163,91
,2	,000009	-100,98
,3	,000934	-60,60
,4	,032887	-29,66
,5	,707107	-3,01
,6	,955775	-,39
,7	1,063875	,54
,8	1,003711	,03
,9	,999663	-,00
1,0	1,000000	,00
1,1	,999750	-,00
1,2	1,000002	,00
1,3	1,012925	,11
1,4	1,050787	,43
1,5	1,084480	,70
1,6	1,035763	,31
1,7	,913856	-,78
1,8	,826480	-1,66
1,9	,818342	-1,74
2,0	,707107	-3,01
2,4	,054747	-25,23
3,0	,003439	-49,27
3,4	,000734	-62,69
4,0	,000108	-79,36
4,4	,000037	-88,74
5,0	,000009	-100,98
5,4	,000004	-108,21
6,0	,000001	-117,96
6,4	,000001	-123,87
7,0	,000000	-132,01
8,0	,000000	-144,04
9,0	,000000	-154,56
10,0	,000000	-163,91

TW IN DB

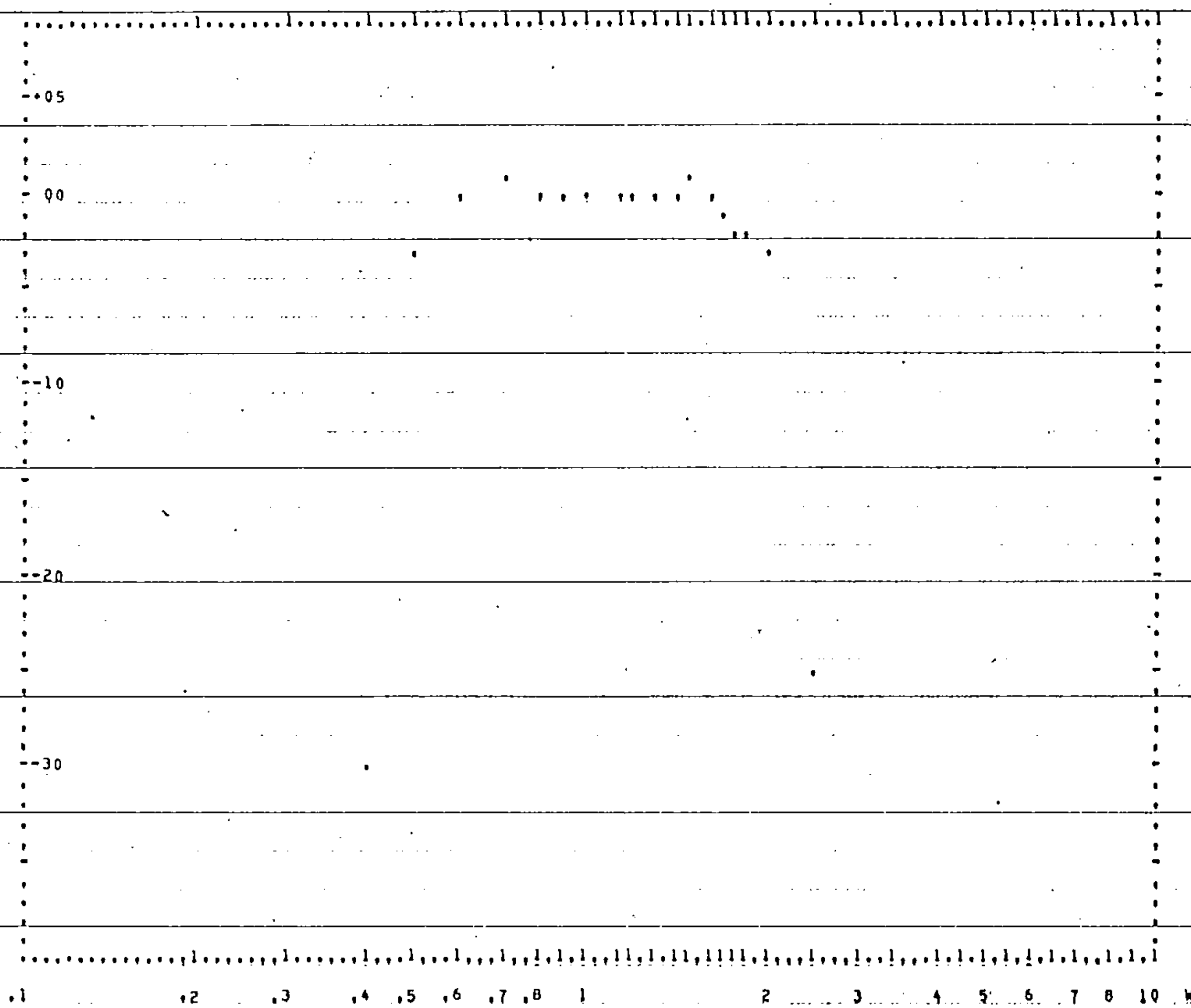

FIGURE 3.4.5.1 RESPONSE OF A T.L. + A B.P. BUTTERWORTH FILTER OF ORDER 10 (REFER TO FIG.3.1)

[POOR PRINT]

T= .90

W	TW	TWDB
0.0	0.000000	-200.00
.1	.000000	-163.80
.2	.000009	-100.52
.3	.001068	-59.42
.4	.044506	-27.03
.5	1.520478	3.64
.6	1.527541	3.68
.7	1.169476	1.36
.8	1.086375	.72
.9	1.043126	.37
1.0	1.000000	.00
1.1	.964739	-.31
1.2	.939688	-.54
1.3	.919543	-.73
1.4	.878782	-1.12
1.5	.809399	-1.84
1.6	.748521	-2.52
1.7	.735328	-2.67
1.8	.781639	-2.14
1.9	.822653	-1.70
2.0	.710191	-2.97
2.4	1.321786	2.42
3.0	.004681	-46.59
3.4	.000756	-62.43
4.0	.000106	-79.48
4.4	.000043	-87.38
5.0	.000022	-93.20
5.4	.000109	-79.23
6.0	.000002	-112.08
6.4	.000001	-121.82
7.0	.000000	-132.03
8.0	.000000	-140.99
9.0	.000000	-137.50
10.0	.000000	-162.72

TW IN DB

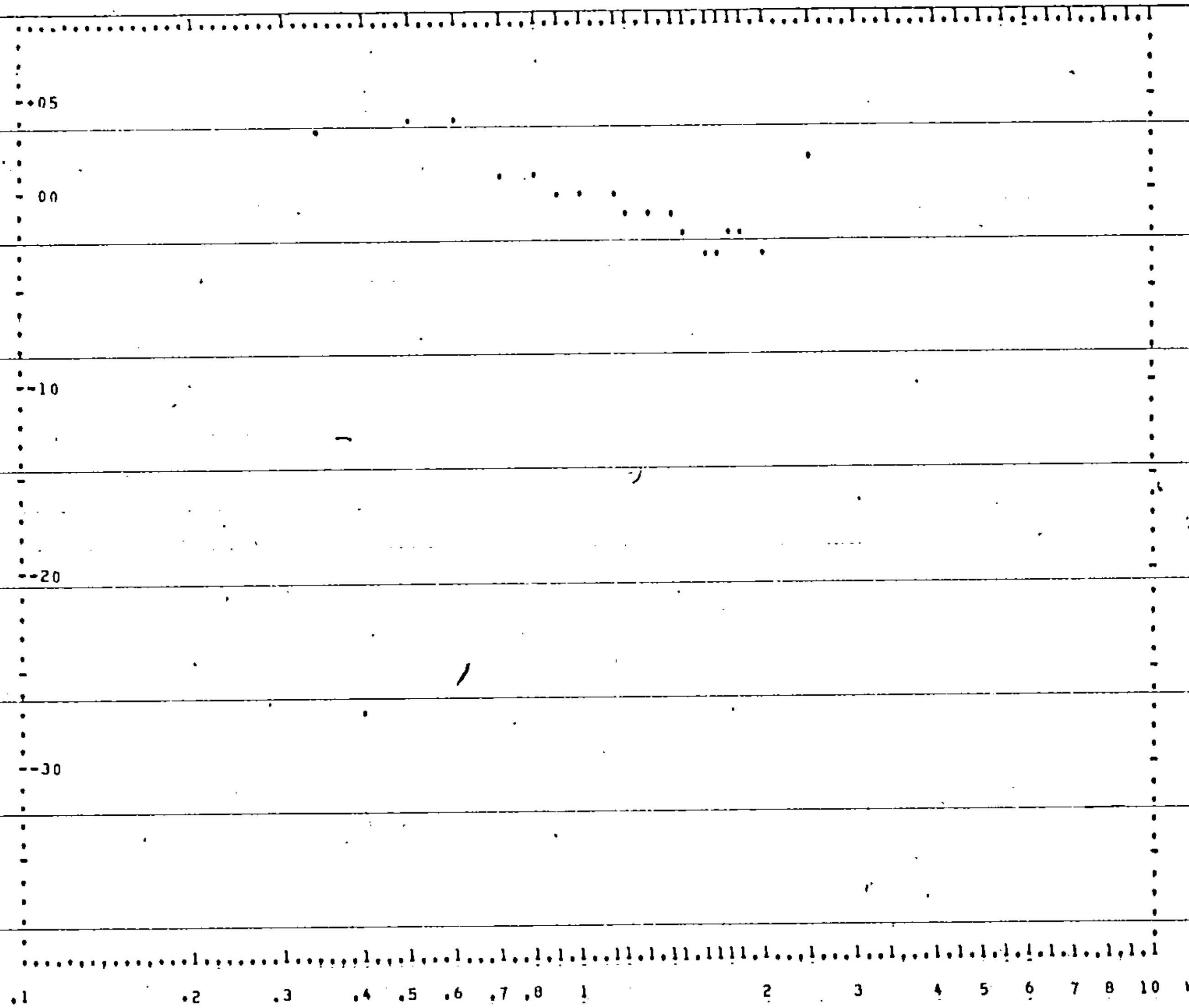

FIGURE 3.4.5.2 RESPONSE OF A T.L. + A B.P. BUTTERWORTH FILTER OF ORDER 10 (REFER TO FIG.3.1)

[POOR PRINT]

3.5 The Lumped Section is a Band Stop Filter

Replacing S by $1/\left(\frac{wo}{B}\right)\left(\frac{s}{wo} + \frac{wo}{s}\right)$ where

B = Normalized band width

wo = Centre frequency

in the previous Low-Pass equations, transforms the L.P. to a Band-Stop filter.

3.5.1 Band Stop Filter (N = 2)

Figures 3.5.1.1 and 3.5.1.2 show the frequency response curve of the filter (N = 2) calculated at τ = 0.0 and τ = 0.3 respectively.

3.5.2 Band Stop Filter (N = 4)

Figures 3.5.2.1 and 3.5.2.2 show the frequency response curve of the filter (N = 4) calculated at τ = 0.0 and τ = 0.3 respectively.

3.5.3 Band Stop Filter (N = 6)

Figures 3.5.3.1 and 3.5.3.2 show the frequency response curve of the filter (N = 8) calculated at τ = 0.0 and τ = 0.5 respectively.

3.5.4 Band Stop Filter (N = 8)

Figures 3.5.4.1 and 3.5.4.2 show the frequency response curve of the filter (N = 8) calculated at τ = 0.0 and at τ = 0.5 respectively.

3.5.5 Band Stop Filter (N = 10)

Figures 3.5.5.1 and 3.5.5.2 show the frequency response curve of the filter (N = 10) calculated at τ = 0.0 and at τ = 0.9 respectively.

T=0,00

W	TW	TWDB
0,0	1,000000	0,00
,1	,999737	-,00
,2	,995267	-,04
,3	,971383	-,25
,4	,890769	-1,00
,5	,707114	-3,01
,6	,451267	-6,91
,7	,229616	-12,78
,8	,089638	-20,95
,9	,019804	-34,06
1,0	0,000000	-200,00
1,1	,016196	-35,81
1,2	,059647	-24,49
1,3	,124237	-18,11
1,4	,204561	-13,78
1,5	,294916	-10,61
1,6	,389192	-8,20
1,7	,481480	-6,35
1,8	,566973	-4,93
1,9	,642601	-3,84
2,0	,707114	-3,01
2,4	,868039	-1,23
3,0	,953419	-,41
3,4	,973864	-,23
4,0	,987444	-,11
4,4	,991756	-,07
5,0	,995267	-,04
5,4	,996596	-,03
6,0	,997822	-,02
6,4	,998340	-,01
7,0	,998858	-,01
8,0	,999343	-,01
9,0	,999595	-,00
10,0	,999737	-,00

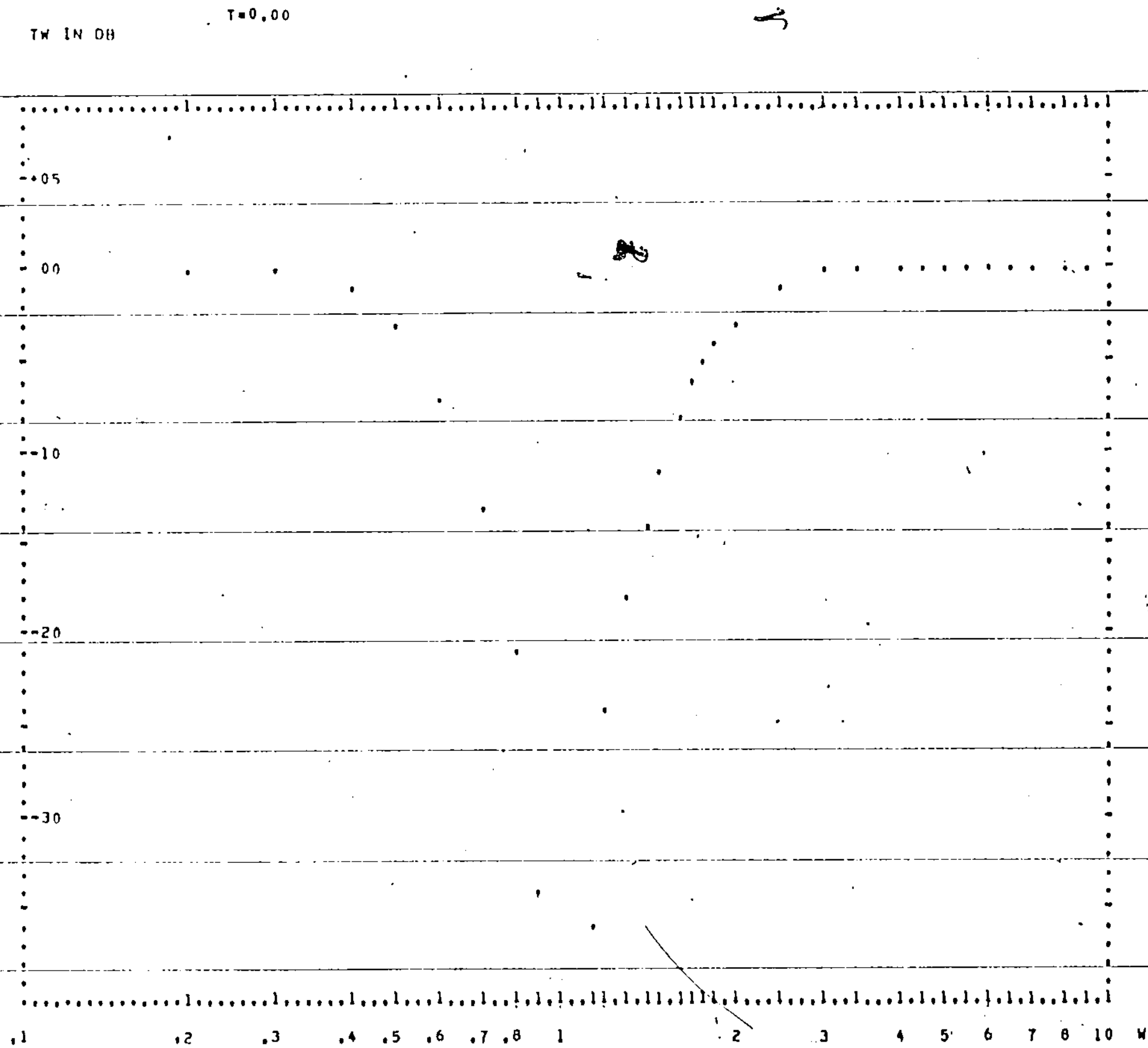

FIGURE 3.5.1.1 RESPONSE OF A T.L. + A B.S. BUTTERWORTH FILTER OF ORDER 2 (REFER TO FIG.3.1)

[POOR PRINT]

T= .30

W	TW	TWDB
0.0	1.000000	0.00
.1	.996465	-.03
.2	.981189	-.16
.3	.937326	-.56
.4	.832893	-1.59
.5	.644595	-3.81
.6	.413098	-7.68
.7	.216229	-13.30
.8	.087378	-21.17
.9	.019987	-33.98
1.0	0.000000	-200.00
1.1	.017673	-35.05
1.2	.068434	-23.29
1.3	.151593	-16.39
1.4	.268950	-11.41
1.5	.422933	-7.47
1.6	.613216	-4.25
1.7	.831004	-1.61
1.8	1.053482	.45
1.9	1.246896	1.92
2.0	1.383460	2.82
2.4	1.449772	3.23
3.0	1.264042	2.04
3.4	1.175293	1.40
4.0	1.083859	.70
4.4	1.040509	.34
5.0	.992875	-.06
5.4	.969950	-.27
6.0	.946026	-.48
6.4	.936011	-.57
7.0	.928607	-.64
8.0	.933306	-.60
9.0	.954289	-.41
10.0	.984676	-.13

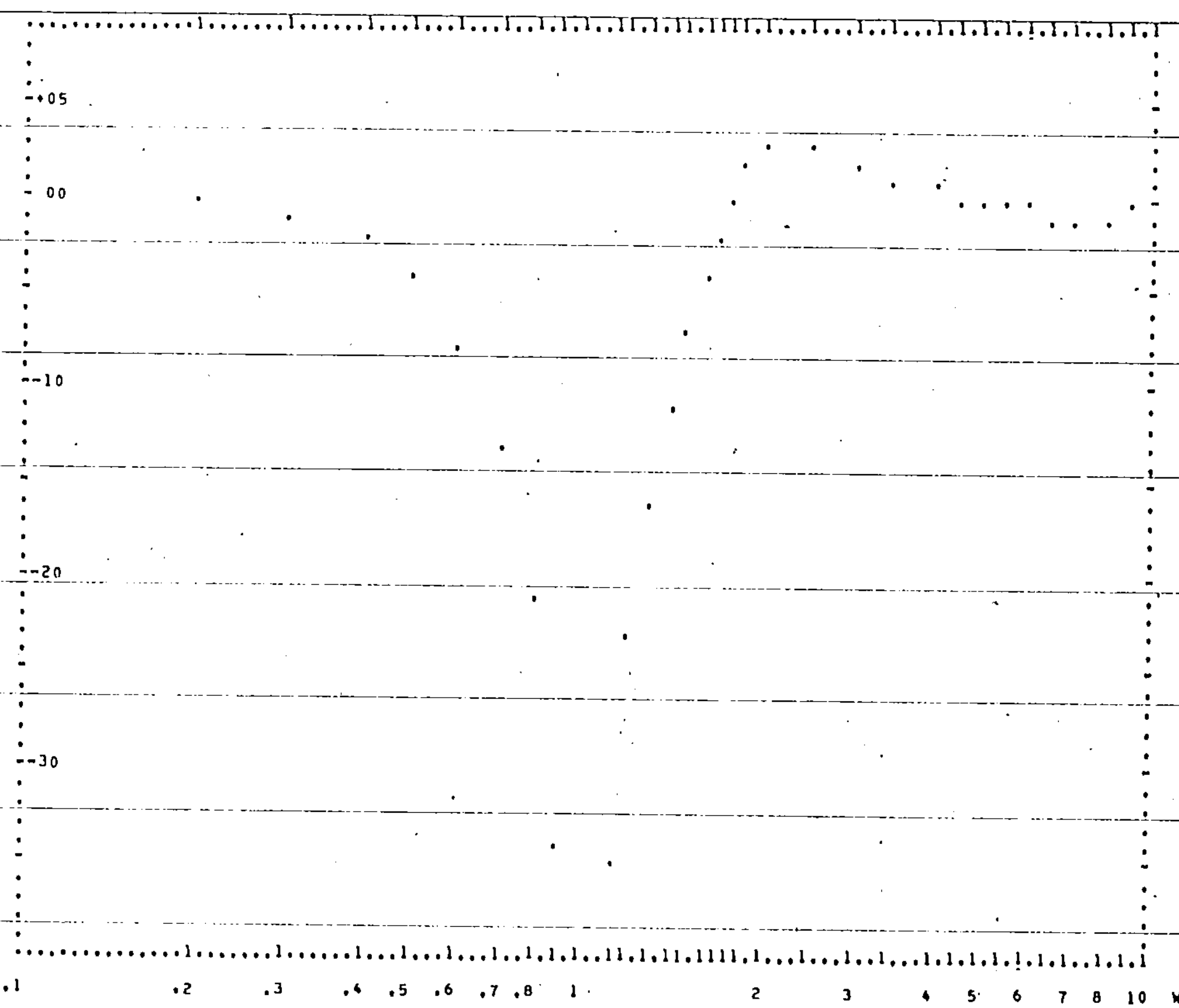

FIGURE 3.5.1.2 RESPONSE OF A T.L. + A B.S. BUTTERWORTH FILTER OF ORDER 2 (REFER TO FIG.3.1)

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[POOR PRINT]

T=0.00

W	TW	TWDB
0.0	1.000000	0.00
.1	1.000001	.00
.2	.999959	-.00
.3	.998226	-.02
.4	.967760	-.28
.5	.707114	-3.01
.6	.247743	-12.12
.7	.055572	-25.10
.8	.008100	-41.83
.9	.000392	-68.13
1.0	0.000000	-200.00
1.1	.000262	-71.62
1.2	.003570	-48.95
1.3	.015675	-36.10
1.4	.043631	-27.20
1.5	.094832	-20.46
1.6	.175730	-15.10
1.7	.288912	-10.78
1.8	.428130	-7.37
1.9	.575315	-4.80
2.0	.707114	-3.01
2.4	.950434	-.44
3.0	.995036	-.04
3.4	.998532	-.01
4.0	.999679	-.00
4.4	.999866	-.00
5.0	.999959	-.00
5.4	.999980	-.00
6.0	.999994	-.00
6.4	.999997	-.00
7.0	1.000000	-.00
8.0	1.000001	.00
9.0	1.000001	.00
10.0	1.000001	.00

TW IN DB

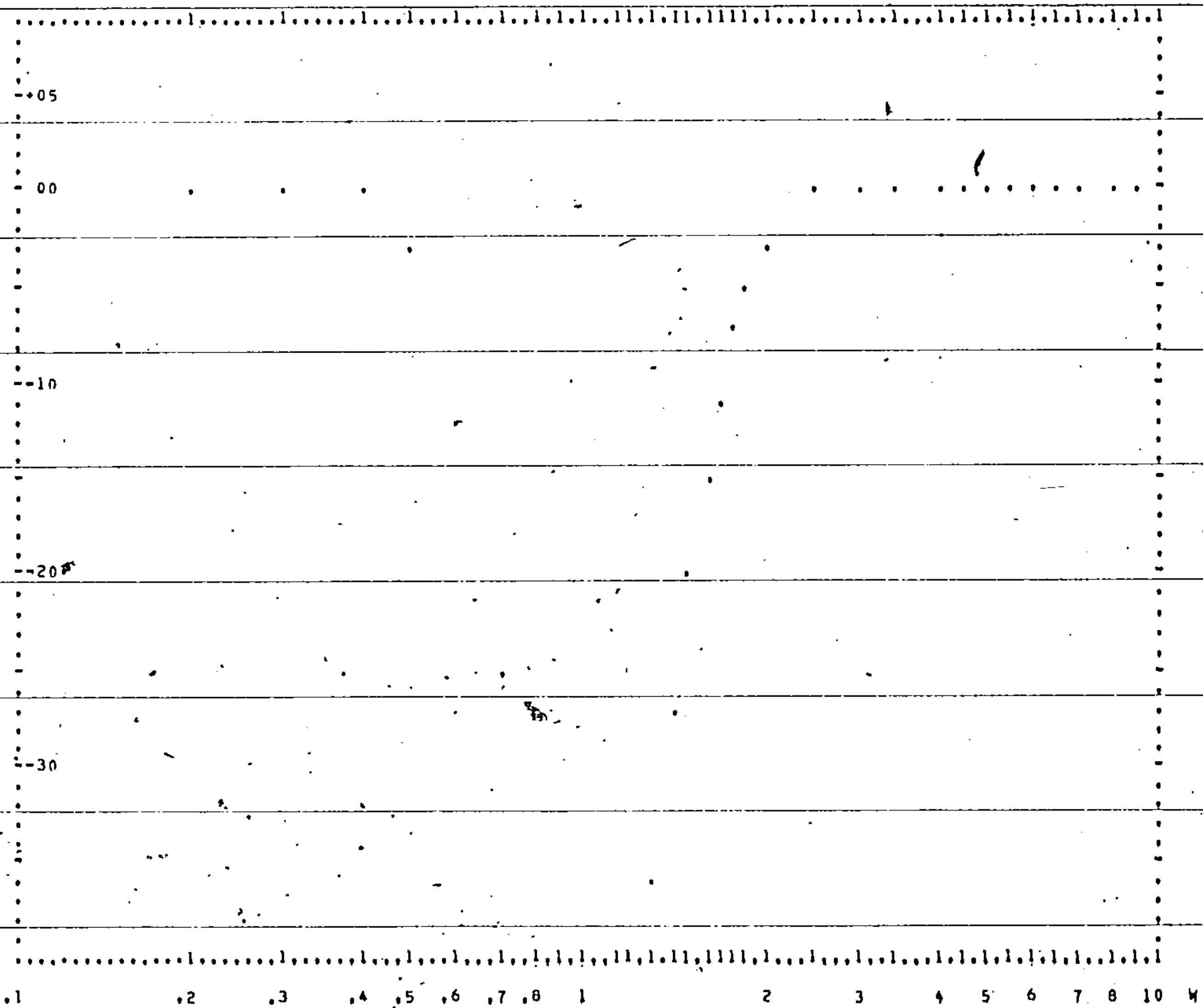

FIGURE 3.5.2.1 RESPONSE OF A T.L. + A B.S. BUTTERWORTH FILTER OF ORDER 4 (REFER TO FIG.3.1)

[POOR PRINT]

W	TW	TWDB
0.0	1.000000	0.00
.1	.997014	-.03
.2	.987349	-.11
.3	.966482	-.30
.4	.900547	-.91
.5	.626146	-4.07
.6	.225902	-12.92
.7	.052785	-25.55
.8	.007942	-42.00
.9	.000397	-68.03
1.0	0.000000	-200.00
1.1	.000286	-70.89
1.2	.004066	-47.82
1.3	.018846	-34.50
1.4	.056102	-25.02
1.5	.132959	-17.53
1.6	.276532	-11.17
1.7	.529794	-5.52
1.8	.938429	-.55
1.9	1.429881	3.11
2.0	1.705392	4.64
2.4	1.431959	3.12
3.0	1.207523	1.64
3.4	1.137767	1.12
4.0	1.067218	.57
4.4	1.032410	.28
5.0	.992529	-.07
5.4	.972644	-.24
6.0	.951357	-.43
6.4	.942246	-.52
7.0	.935355	-.58
8.0	.939403	-.54
9.0	.958489	-.37
10.0	.986229	-.12

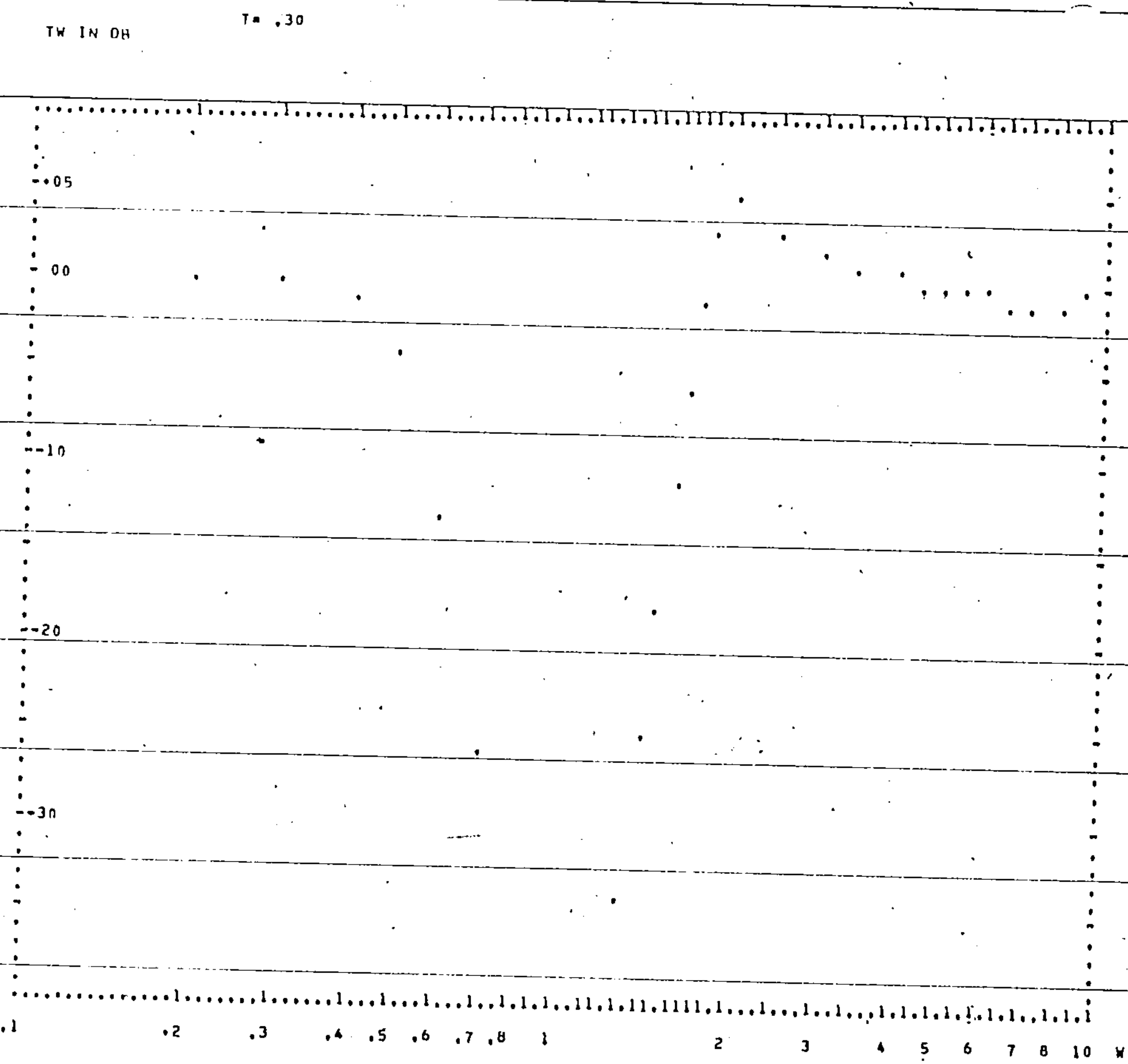

FIGURE 3.5.2.2 RESPONSE OF A T.L. + A B.S. BUTTERWORTH FILTER OF ORDER 4 (REFER TO FIG.3.1)

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[POOR PRINT]

T=0.00

W	TW	TWDB
0.0	1.000000	0.00
.1	.999995	-.00
.2	.999978	-.00
.3	.999844	-.00
.4	.991224	-.08
.5	.707118	-3.01
.6	.128244	-17.84
.7	.013130	-37.63
.8	.000729	-62.75
.9	.000008	-102.19
1.0	0.000000	-200.00
1.1	.000004	-107.43
1.2	.000213	-73.42
1.3	.001963	-54.14
1.4	.009126	-40.79
1.5	.029389	-30.64
1.6	.075207	-22.47
1.7	.163552	-15.73
1.8	.310016	-10.17
1.9	.508101	-5.88
2.0	.707118	-3.01
2.4	.982866	-.15
3.0	.999439	-.00
3.4	.999872	-.00
4.0	.999957	-.00
4.4	.999970	-.00
5.0	.999978	-.00
5.4	.999981	-.00
6.0	.999985	-.00
6.4	.999987	-.00
7.0	.999989	-.00
8.0	.999992	-.00
9.0	.999994	-.00
10.0	.999995	-.00

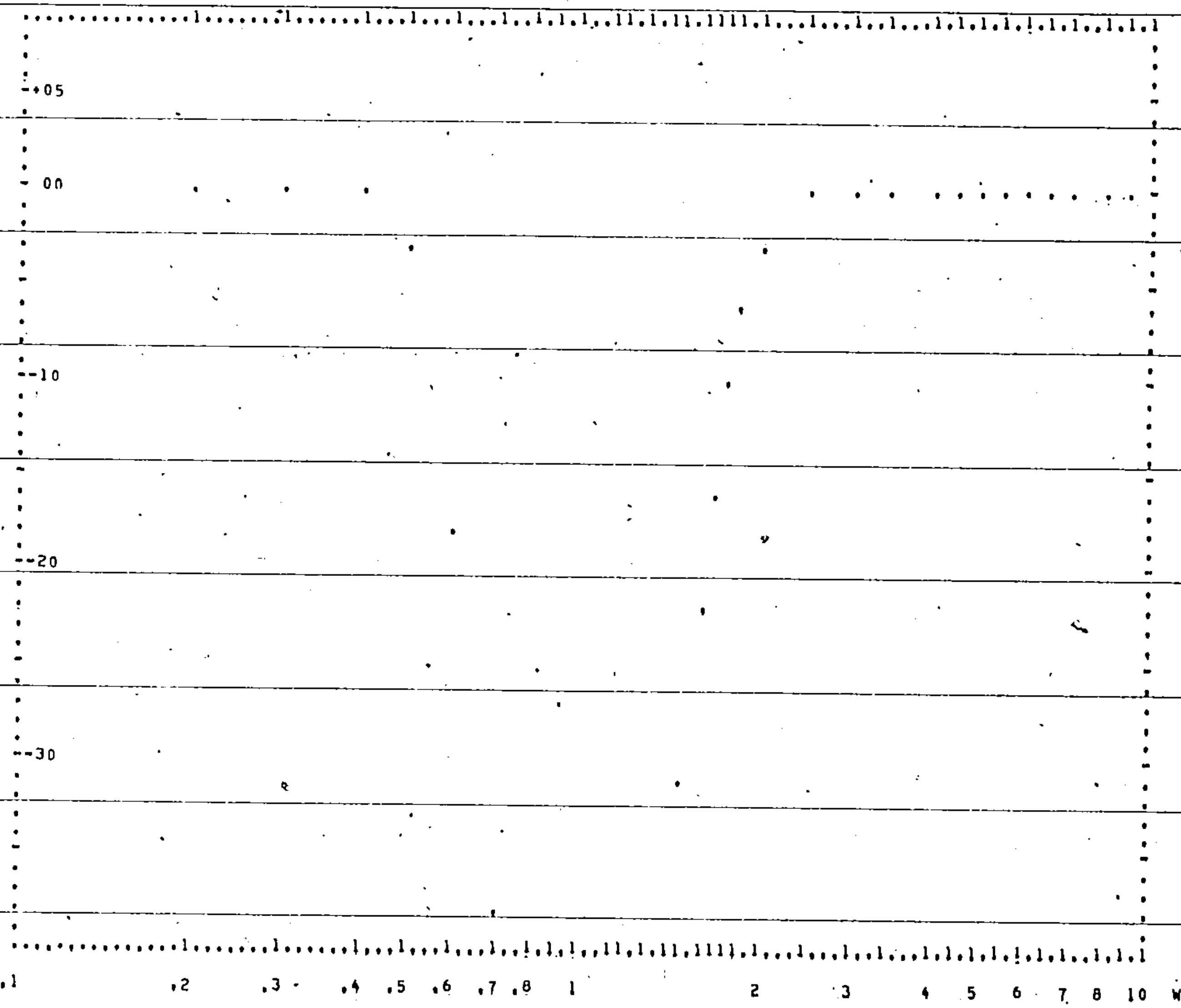

FIGURE 3.5.3.1 RESPONSE OF A T.L. + A B.S. BUTTERWORTH FILTER OF ORDER 6 (REFER TO FIG.3.1)

CONCORDIA UNIVERSITY COMPUTER CENTER

[POOR PRINT]

T= .50

W	TW	TWDB
0.0	1.000000	0.00
.1	.995105	-.04
.2	.979633	-.18
.3	.950131	-.44
.4	.886148	-1.05
.5	.572373	-4.85
.6	.113051	-18.93
.7	.012409	-38.12
.8	.000730	-62.74
.9	.000008	-101.65
1.0	0.000000	-200.00
1.1	.000005	-105.60
1.2	.000290	-70.74
1.3	.003014	-50.42
1.4	.016360	-35.72
1.5	.065177	-23.72
1.6	.231975	-12.69
1.7	.966379	-.30
1.8	3.945554	11.92
1.9	1.675518	4.48
2.0	1.299581	2.28
2.4	1.044840	.38
3.0	.947197	-.47
3.4	.911435	-.81
4.0	.887205	-1.04
4.4	.887109	-1.04
5.0	.906794	-.85
5.4	.930683	-.62
6.0	.976802	-.20
6.4	1.009289	.08
7.0	1.049269	.42
8.0	1.062502	.53
9.0	1.017317	.15
10.0	.970165	-.26

TW IN DB

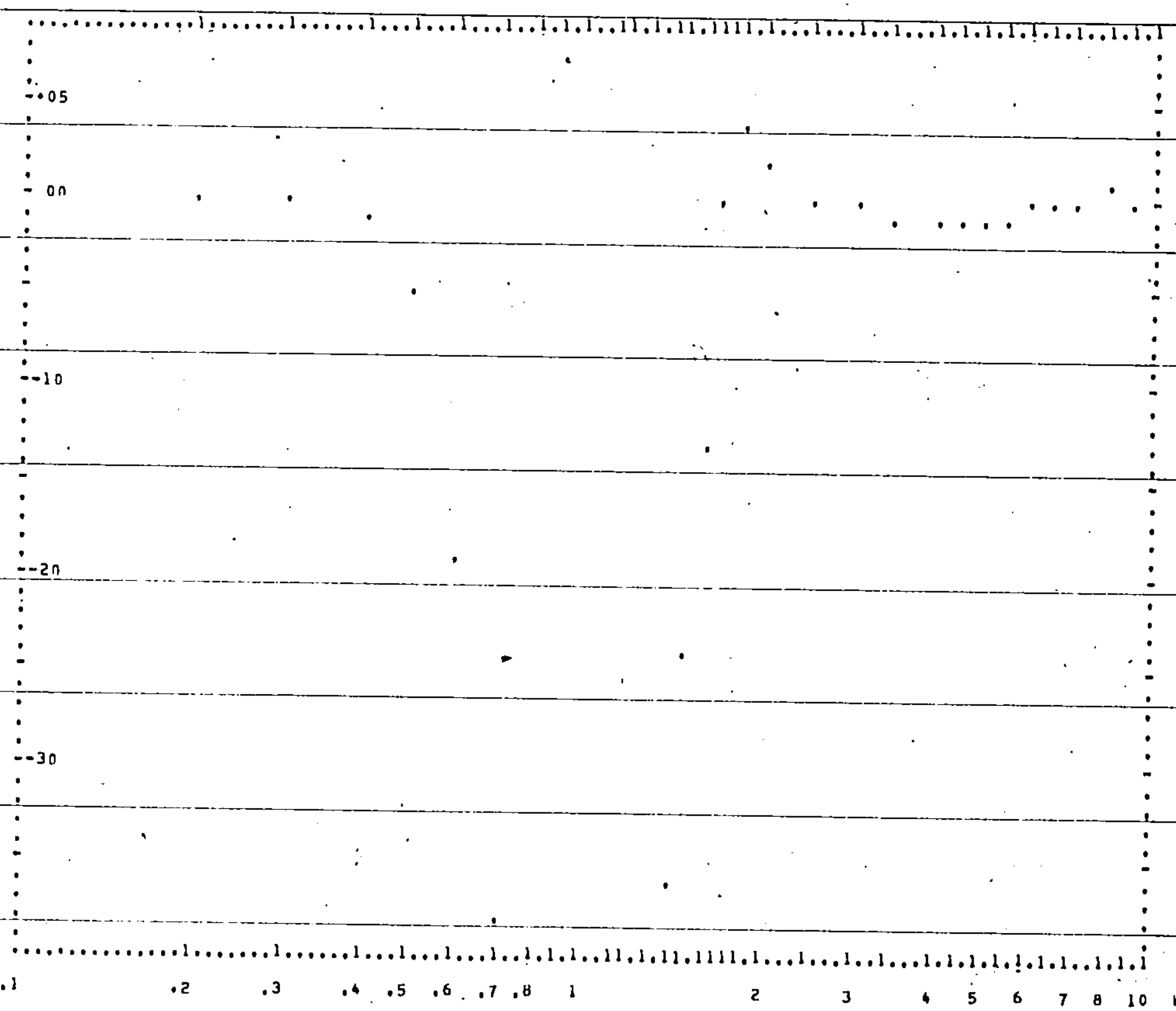

FIGURE 3.5.3.2 RESPONSE OF A T.L. + A B.S. BUTTERWORTH FILTER OF ORDER 6 (REFER TO FIG.3.1)

CONCORDIA UNIVERSITY COMPUTER CENTER

[POOR PRINT]

T=0.00

W	TW	TWDB
0.0	1.000000	0.00
.1	.997972	-.02
.2	.977145	-.20
.3	.960455	-.35
.4	1.112692	.93
.5	.707107	-3.01
.6	.072537	-22.79
.7	.002972	-50.54
.8	.000064	-83.84
.9	.000000	-136.27
1.0	0.000000	-200.00
1.1	.000000	-143.25
1.2	.000013	-97.99
1.3	.000238	-72.46
1.4	.001827	-54.77
1.5	.008875	-41.04
1.6	.033301	-29.55
1.7	.103734	-19.68
1.8	.256811	-11.81
1.9	.473625	-6.49
2.0	.707107	-3.01
2.4	1.151032	1.22
3.0	.981268	-.16
3.4	.959030	-.36
4.0	.961622	-.34
4.4	.968147	-.28
5.0	.977145	-.20
5.4	.981832	-.16
6.0	.987055	-.11
6.4	.989594	-.09
7.0	.992394	-.07
8.0	.995312	-.04
9.0	.996977	-.03
10.0	.997972	-.02

TW IN DB

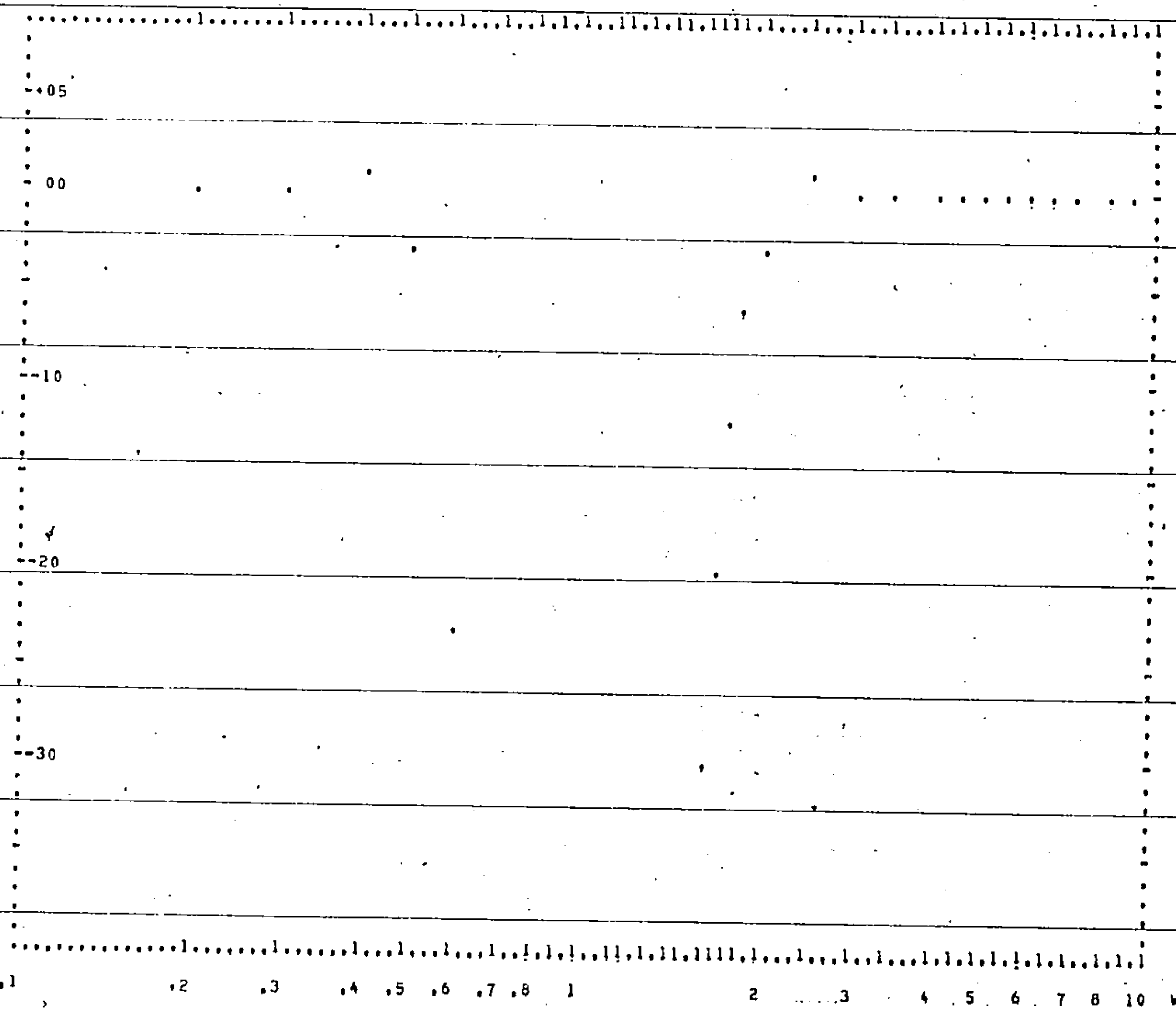

FIGURE 3.5.4.1 RESPONSE OF A T.L. + A B.S. BUTTERWORTH FILTER OF ORDER 8 (REFER TO FIG.3.1)

T= .50

W	TW	TWDB
0.0	1.000000	0.00
.1	.992784	-.06
.2	.957493	-.38
.3	.929707	-.63
.4	1.028492	.24
.5	.549226	-5.20
.6	.063406	-23.96
.7	.002791	-51.08
.8	.000064	-83.87
.9	.000000	-135.75
1.0	0.000000	-200.00
1.1	.000000	-141.39
1.2	.000017	-95.25
1.3	.000370	-68.63
1.4	.003345	-49.51
1.5	.020384	-33.81
1.6	.109335	-19.22
1.7	.739603	-2.62
1.8	2.833472	9.05
1.9	1.301174	2.29
2.0	1.053597	.45
2.4	.927417	-.65
3.0	1.008526	.07
3.4	.976962	-.20
4.0	.914256	-.78
4.4	.894024	-.97
5.0	.897043	-.94
5.4	.916619	-.76
6.0	.963322	-.32
6.4	.999492	-.00
7.0	1.047172	.40
8.0	1.068656	.58
9.0	1.021130	.18
10.0	.969431	-.27

TW IN DB

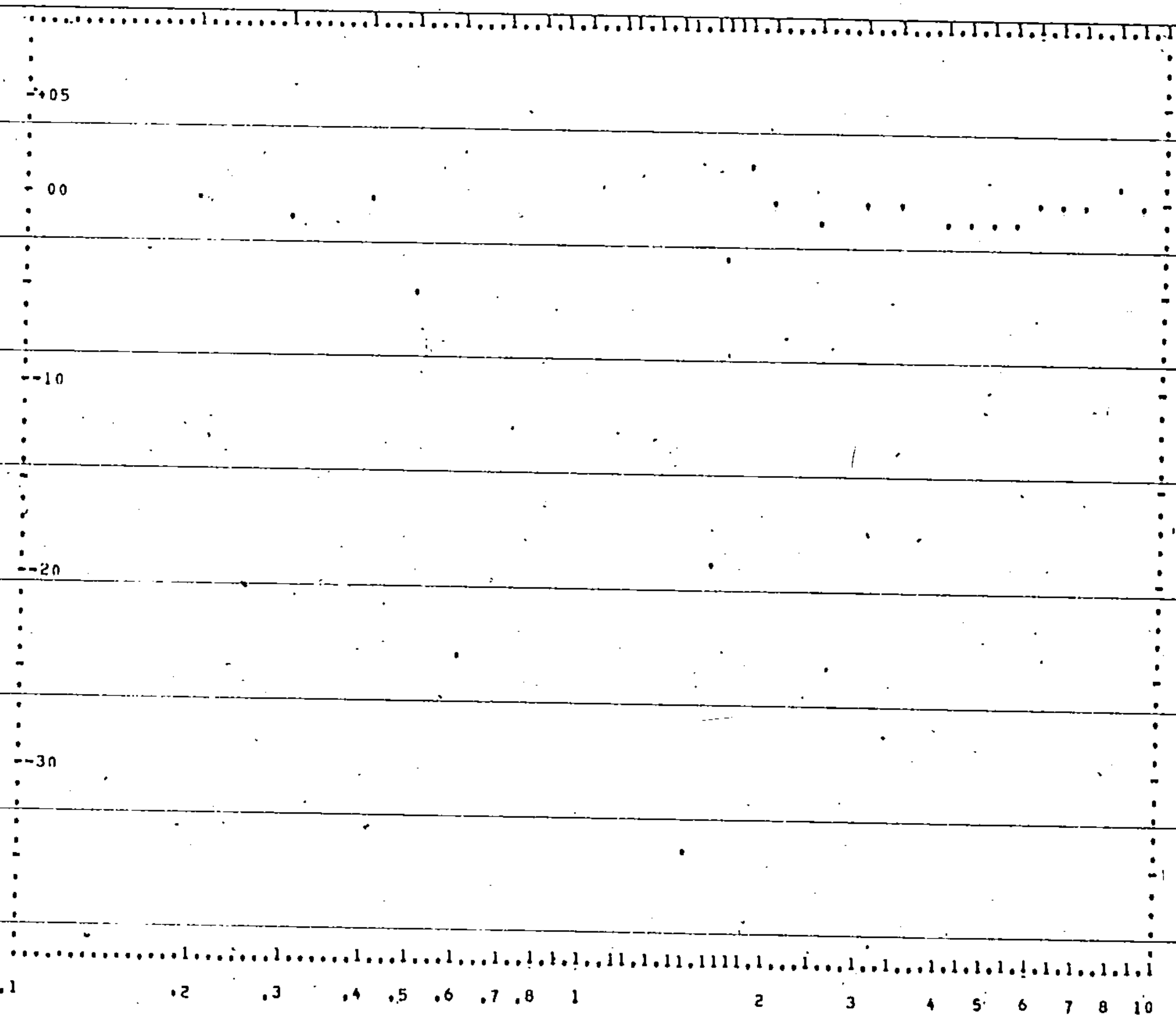

FIGURE 3.5.4.2 RESPONSE OF A T.L. + A B.S. BUTTERWORTH FILTER OF ORDER 8 (REFER TO FIG.3.1)

POOR PRINT

T=0.00

W	TW	TWDB
0.0	1.000000	0.00
.1	.999589	-.00
.2	1.005262	.05
.3	1.067705	.57
.4	.951270	-.43
.5	.707107	-3.01
.6	.031603	-30.01
.7	.000777	-62.19
.8	.000006	-104.54
.9	.000000	-170.32
1.0	0.000000	-200.00
1.1	.000000	-179.06
1.2	.000001	-122.36
1.3	.000031	-90.13
1.4	.000419	-67.56
1.5	.003037	-50.35
1.6	.013944	-37.11
1.7	.045719	-26.80
1.8	.127666	-17.88
1.9	.339540	-9.38
2.0	.707107	-3.01
2.4	.894134	-.97
3.0	1.084581	.71
3.4	1.062642	.53
4.0	1.026744	.23
4.4	1.014303	.12
5.0	1.005262	.05
5.4	1.002475	.02
6.0	1.000478	.00
6.4	.999881	-.00
7.0	.999490	-.00
8.0	.999387	-.01
9.0	.999477	-.00
10.0	.999589	-.00

TW IN DB

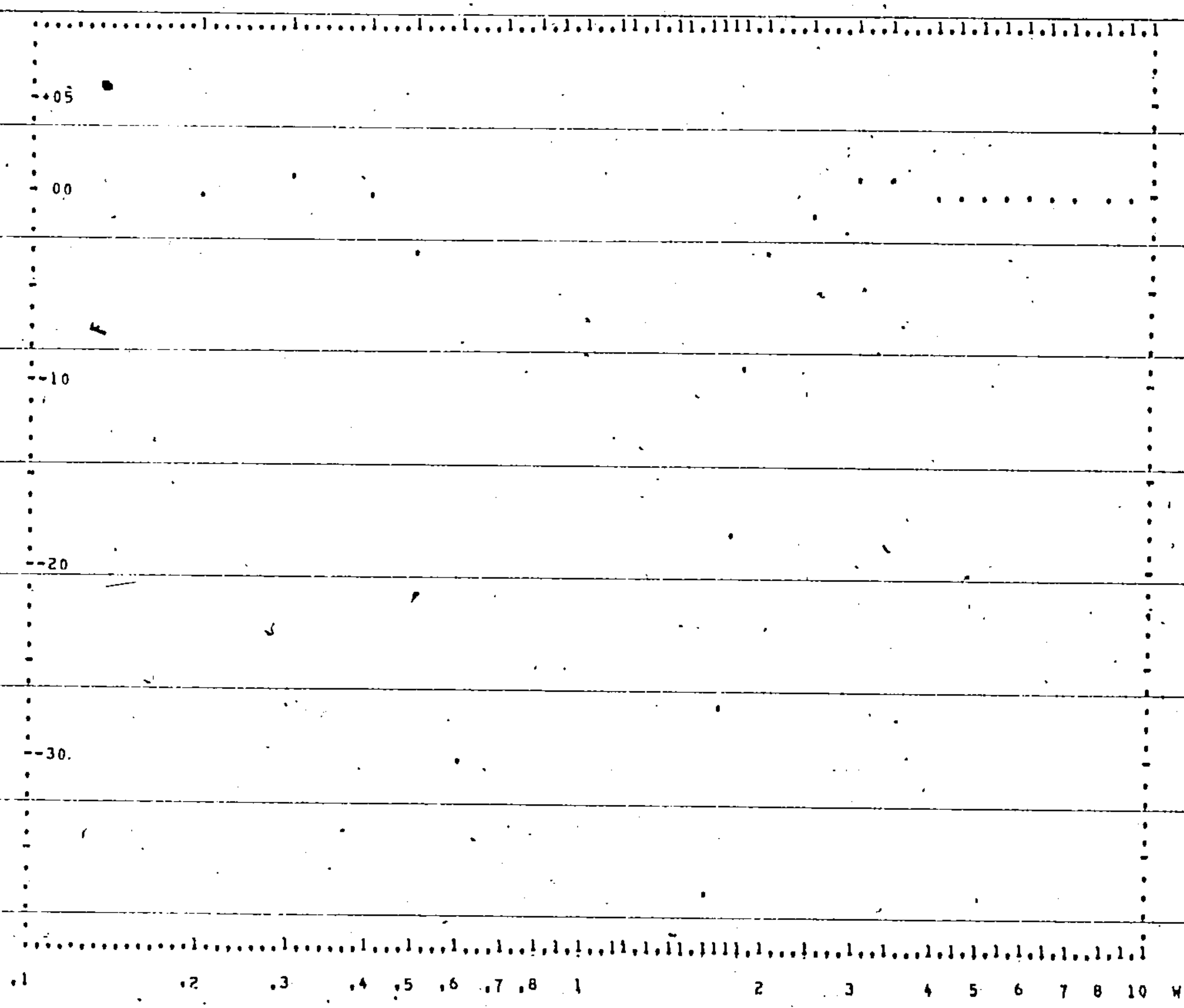

FIGURE 3.5.5.1 RESPONSE OF A T.L. + A B.S. BUTTERWORTH FILTER OF ORDER 10 (REFER TO FIG.3.1)

W | TW | TWDB | TW IN DB | T= .90

W	TW	TWDB
0.0	1.000000	0.00
.1	.991515	-.07
.2	.975389	-.22
.3	.973535	-.23
.4	.742272	-2.59
.5	.502657	-5.97
.6	.027648	-31.17
.7	.000775	-62.21
.8	.000007	-103.42
.9	.000000	-167.86
1.0	0.000000	-200.00
1.1	.000000	-172.71
1.2	.000002	-112.84
1.3	.000175	-75.13
1.4	.031968	-29.91
1.5	.018835	-34.50
1.6	.040063	-27.95
1.7	.082398	-21.68
1.8	.159185	-15.96
1.9	.299979	-10.46
2.0	.501622	-5.99
2.4	.616352	-4.20
3.0	.883644	-1.07
3.4	1.035115	.30
4.0	1.109244	.90
4.4	1.095516	.79
5.0	1.016680	.14
5.4	.963864	-.32
6.0	.929690	-.63
6.4	.942780	-.51
7.0	1.001646	.01
8.0	1.054990	.46
9.0	.972939	.24
10.0	.966564	.30

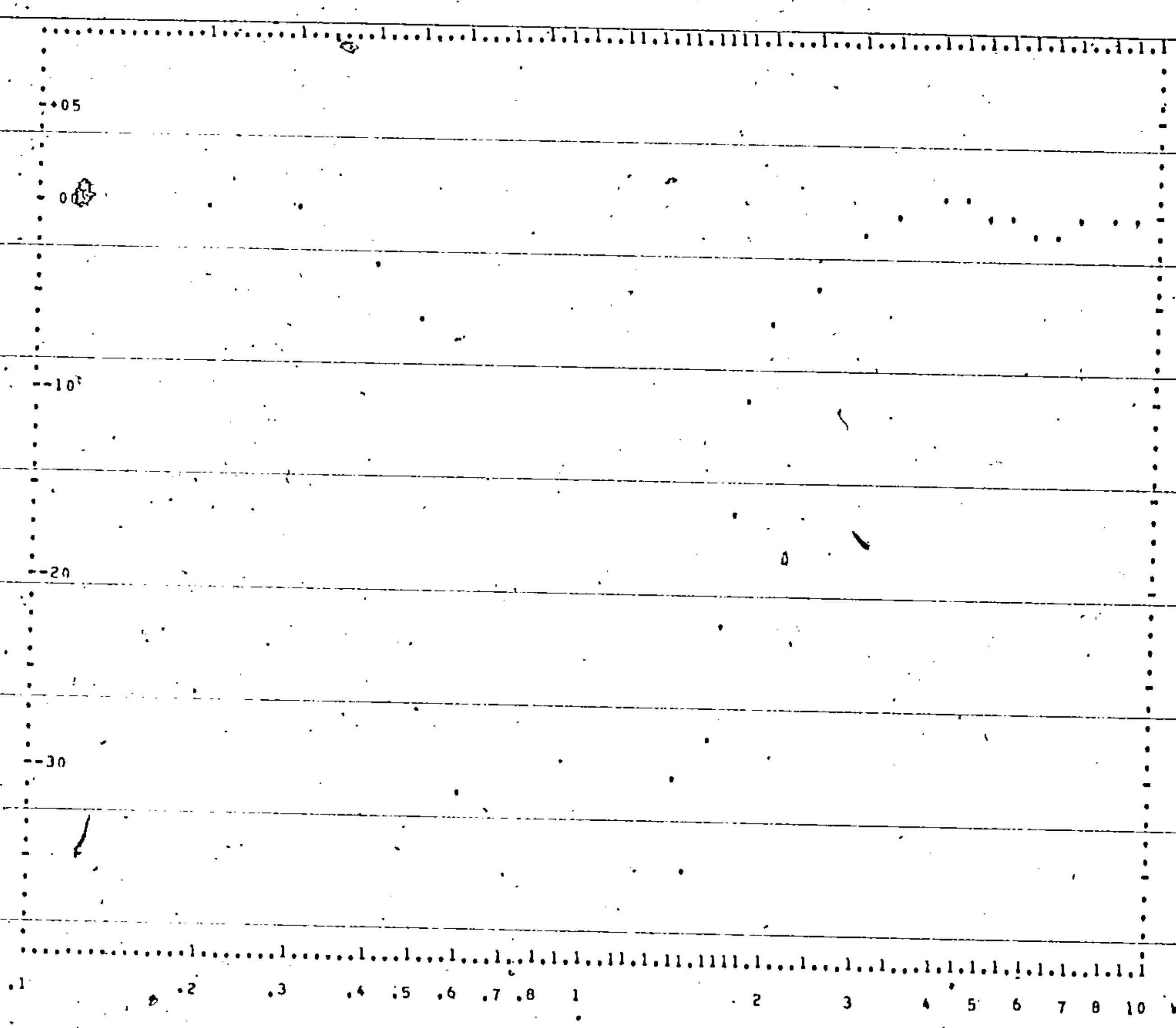

FIGURE 3.5.5.2 RESPONSE OF A T.L. + A B.S. BUTTERWORTH FILTER OF ORDER 10 (REFER TO FIG.3.1)

[POOR PRINT]

3.6 Discussion

3.6.1 Low-pass filter

For the low-pass filter case and for attenuation values up to 60 db, the addition of a lossless transmission line is seen to have insignificant effect on either the selectivity or the smoothness of the curve. The only case that shows a direct effect is that of the second order Butterworth filter in series with the transmission line (Fig. 3.2.1.2). Here one finds a sag with its valley at $w = 4.0$ (around 18 db), followed by a steep attenuation occurring at $w = 5.4$ (around 7 db). Most of the disturbing effect of the transmission line occurs outside the bandwidth of interest and at attenuation levels well below the 60 db figure.

From this investigation it is concluded that such a combination in the low frequency case is of little or no relevance.

3.6.2 High-pass filter

For high pass applications, the cascading of the lossless transmission line has a marked effect in the passband. As can be visualized in all curves with $\tau > 0$ and for various values of N, there is always a net gain in at least one frequency or another in the pass band. This is attribured to the net resonance effect of the combined networks. There are 3 three particular cases of interest:-

a) Case of $N = 6$, $\tau = 0.5$ Figure 3.3.3.2

The pass-band response is identical to the equal ripple case with the exception of the existance of a sagging mean value. There is also a hump shaped effect at the cut off frequency.

b) Case of $N = 8$, $\tau = 0.5$ Figure 3.3.4.2

The pass band response is characterized by a 6 db gain starting hump followed by non uniform oscillations around a steep median value.

c) Case of N = 10, τ = 0.9   Figure 3.3.5.2

The pass band shows a spike of 13.68 db gain at w = 0.9. This is followed by a sharp drop to the zero db mean value with some oscillations around that line.

From these cases, it can be calculated that the addition of the lossless transmission line results in a sharper corner frequency response with the additional penalities of adding some ripple to the rest of the pass band.

Apart from that, the transmission line contributed nothing to the selectivity of the filter response.

3.6.3 Band-Pass Filter

Only in the cases of N > 8 and $\tau > 0.5$ is a marked improvement to the filter response noticed. Again, the improvement is with respect to the degree of linearity of the pass-band response and the corner cut-off frequency. As far as the selectivity is concerned, there is no marked effect due to the transmission line presence.

3.6.4 Band-Stop Filter

In the high frequency cut-off corner, the transmission line addition has shown an improved sharpness over the original network. For example, the case N = 8 shows an (0.45 + 3.01 = 3.46 db) improvement at w = 2 for τ = 0.5. There are similar peaking effects for cases of N = 4 and for N = 6 and values of $\tau > 0.4$. However, the response deteriorates for N = 10 and $\tau > 0.5$, particularly for the high frequency attenuation side.

Again on the high frequency pass-band side, the ripple effect appears as soon as the transmission line is introduced. Whether such ripples can be tolerated is a question of design trade-off against the sharper corner to be obtained at the high frequency end of the stop-band.

CHAPTER IV

TRANSMISSION LINE IN SERIES WITH AN Nth ORDER BUTTERWORTH FILTER REPEATED TWICE

4.1 Introduction

This chapter deals with a combination of similar pairs. Each pair consists of a passive lossless transmission line connected in series with a lumped passive element Butterworth filter. Each transmission line has a variable length " ℓ" and a characteristic impedance $Z_0 = 1$ ohm, while each Butterworth filter is of order N. Figure 4.1 shows a block diagram of the entire network.

4.2 The Lumped section is a Low-Pass B.B. Filter

As it was demonstrated in section 3.2 the overall transfer function of a similar network was too complicated to analyse and hence a computer program was written to calculate the transfer function for the circuit shown in Figure 4.1 for the various combinations of the following cases:

1. Filter order $N = 2, 4, 6, 8$ and 10
2. Filter modes: L.P., H.P., B.P. and B.S.

All the above curves were calculated at the same values of τ selected in section 3.2. For the presentation in this chapter, the low pass results are the only ones given. However, the discussion covers all the 4 cases.

4.2.1 L.P. Filter (N = 2)

Figures 4.2.1.1 and 4.2.1.2 show the response of the circuit shown in Figure 4.1 for a L.P. filter of order 2, calculated at $\tau = 0.2$ and 0.4 respectively.

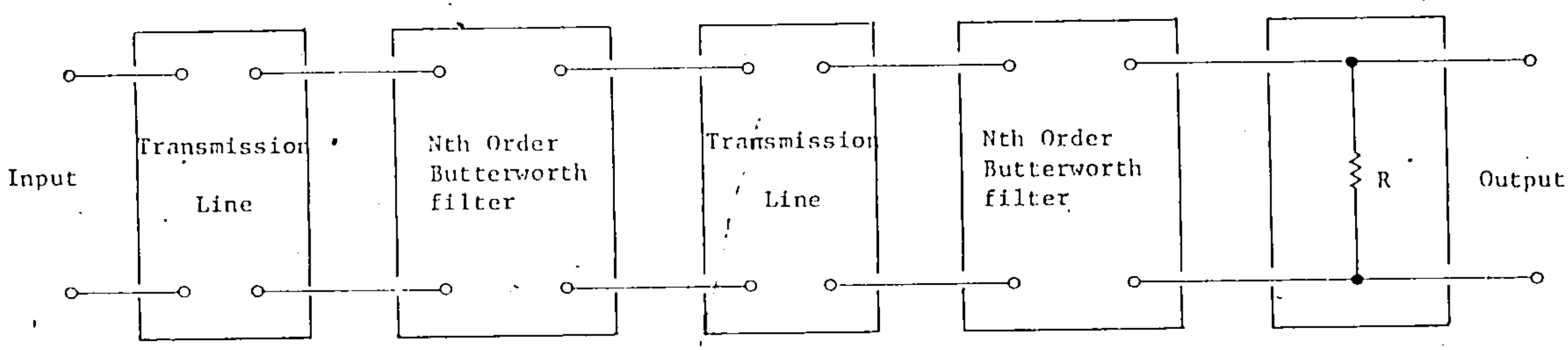

Figure 4.1 - A combination of a Transmission Line and an Nth order Butterworth filter repeated twice, terminated in a resistor R.

4.2.2 L.P. Filter (N = 4)

Figure 4.2.2.1 and 4.2.2.2 show the response of the network for a L.P. filter of order 4 calculated at $\tau = 0.2$ and 0.4 respectively.

4.2.3 L.P. Filter (N = 6)

Figures 4.2.3.1 and 4.2.3.2 show the response of the network for a L.P. filter of order 6 calculated at $\tau = 0.2$ and 0.4 respectively.

4.2.4 L.P. Filter (N = 8)

Figures 4.2.4.1 and 4.2.4.2 show the response of the network for a L.P. filter of order 8 calculated at $\tau = 0.4$ and 0.6 respectively.

4.2.5 L.P. Filter (N = 10)

Figures 4.2.5.1 and 4.2.5.2 show the response of the network for a L.P. filter of order 10 calculated at $\tau = 0.4$ and 0.6 respectively.

[POOR PRINT]

T= .20

W	TW	TWDB
0.0	1.000000	0.00
.1	.985952	-.12
.2	.946156	-.48
.3	.887551	-1.04
.4	.820392	-1.72
.5	.755350	-2.44
.6	.700986	-3.09
.7	.663349	-3.57
.8	.647570	-3.77
.9	.660770	-3.60
1.0	.716709	-2.89
1.1	.841471	-1.50
1.2	1.032602	.28
1.3	.957644	-.38
1.4	.586194	-4.64
1.5	.351317	-9.09
1.6	.227855	-12.85
1.7	.157867	-16.03
1.8	.114818	-18.80
1.9	.086606	-21.25
2.0	.067203	-23.45
2.4	.029565	-30.58
3.0	.012263	-38.23
3.4	.007895	-42.05
4.0	.004760	-46.45
4.4	.003697	-48.64
5.0	.002827	-50.97
5.4	.002540	-51.90
6.0	.002422	-52.32
6.4	.002562	-51.83
7.0	.003333	-49.54
8.0	.017329	-35.22
9.0	.113688	-18.89
10.0	.001326	-57.55

TW IN DB

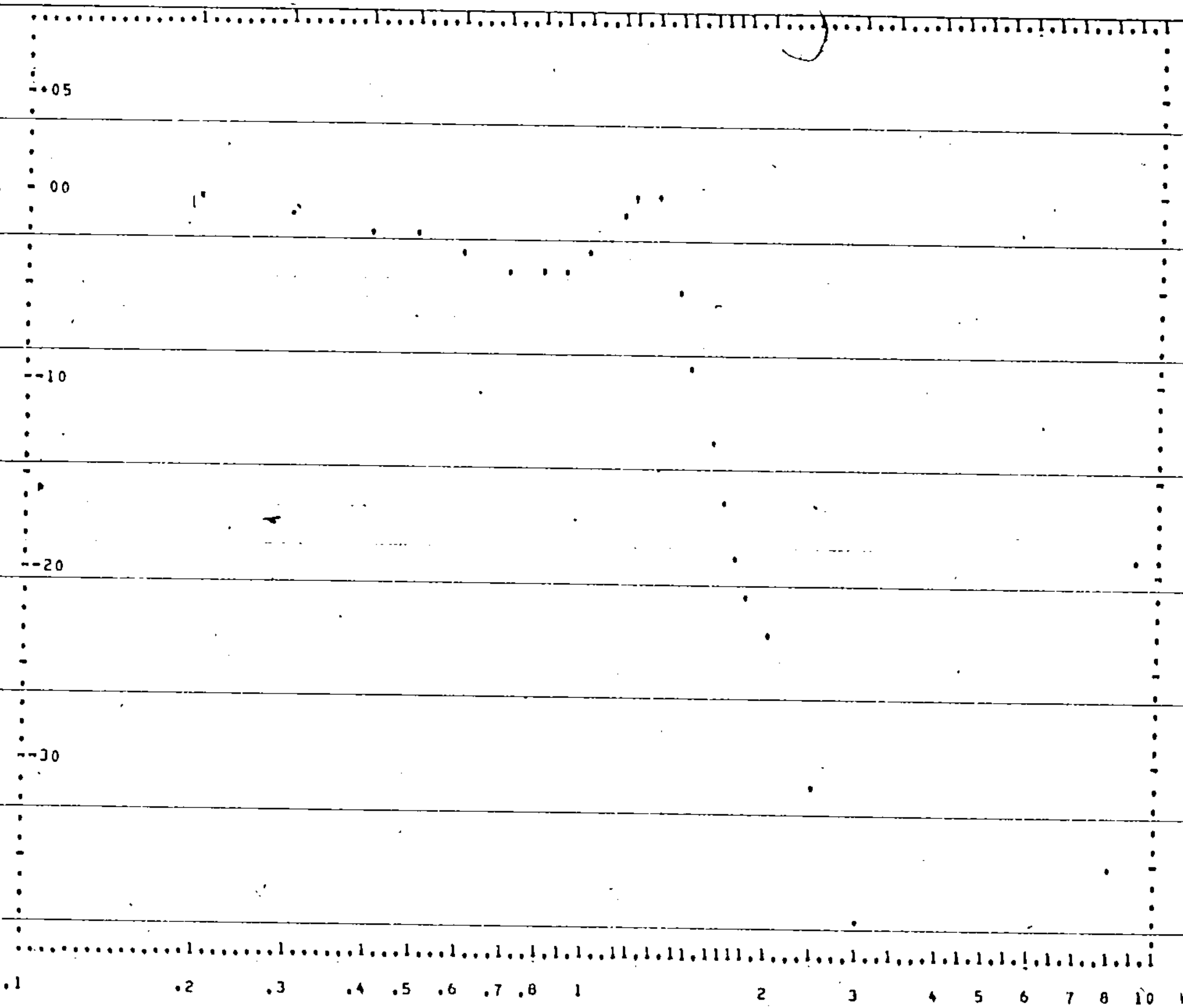

FIGURE 4.2.1.1 RESPONSE OF A T.L. + A L.P. BUTTERWORTH FILTER OF ORDER 2 (REFER TO FIG.4.1)

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POOR PRINT

T = .40

W	TW	TWDB
0.0	1.000000	0.00
.1	.982008	-.16
.2	.933505	-.60
.3	.868072	-1.23
.4	.801203	-1.93
.5	.745472	-2.55
.6	.709712	-2.98
.7	.701403	-3.08
.8	.730778	-2.72
.9	.815145	-1.78
1.0	.962776	-.33
1.1	1.003992	.03
1.2	.706614	-3.02
1.3	.432929	-7.27
1.4	.279914	-11.06
1.5	.193730	-14.26
1.6	.141569	-16.98
1.7	.107911	-19.34
1.8	.085070	-21.40
1.9	.068950	-23.23
2.0	.057220	-24.85
2.4	.032709	-29.71
3.0	.022030	-33.14
3.4	.022671	-32.89
4.0	.051311	-25.80
4.4	.283725	-10.94
5.0	.665431	-3.54
5.4	.014073	-37.03
6.0	.002577	-51.78
6.4	.001259	-58.00
7.0	.000572	-64.85
8.0	.000248	-72.12
9.0	.000167	-75.54
10.0	.000176	-75.07

TW IN DB

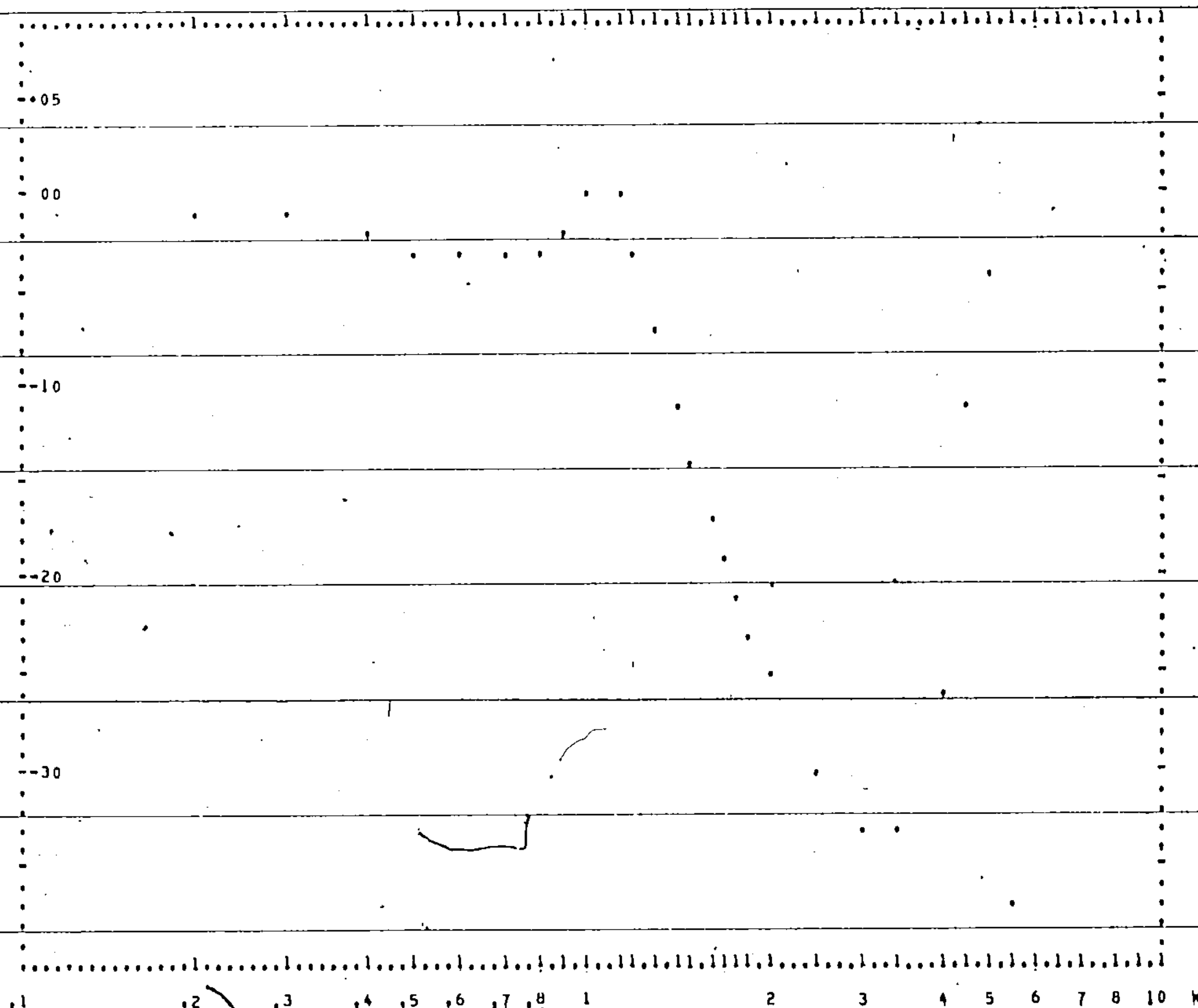

FIGURE 4.2.1.2 RESPONSE OF A T.L. + A L.P. BUTTERWORTH FILTER OF ORDER 2 (REFER TO FIG.4.1)

CONCORDIA UNIVERSITY COMPUTER CENTER

POOR PRINT

T= .20

W	TW	TWDB
0.0	1.000000	0.00
.1	.980431	-.17
.2	.935708	-.58
.3	.893126	-.98
.4	.874979	-1.16
.5	.899095	-.92
.6	.990360	-.08
.7	1.170707	1.37
.8	1.157248	1.27
.9	.664956	-3.54
1.0	.348765	-9.15
1.1	.202186	-13.88
1.2	.129236	-17.77
1.3	.090134	-20.90
1.4	.068604	-23.27
1.5	.057854	-24.75
1.6	.056624	-24.94
1.7	.075529	-22.44
1.8	.959811	-.36
1.9	.037999	-28.40
2.0	.013564	-37.35
2.4	.001414	-56.99
3.0	.000178	-74.98
3.4	.000064	-83.94
4.0	.000018	-94.69
4.4	.000009	-100.53
5.0	.000004	-107.68
5.4	.000003	-111.51
6.0	.000002	-115.91
6.4	.000001	-117.90
7.0	.000001	-119.16
8.0	.000003	-110.39
9.0	.000001	-119.12
10.0	.000000	-131.22

TW IN DB

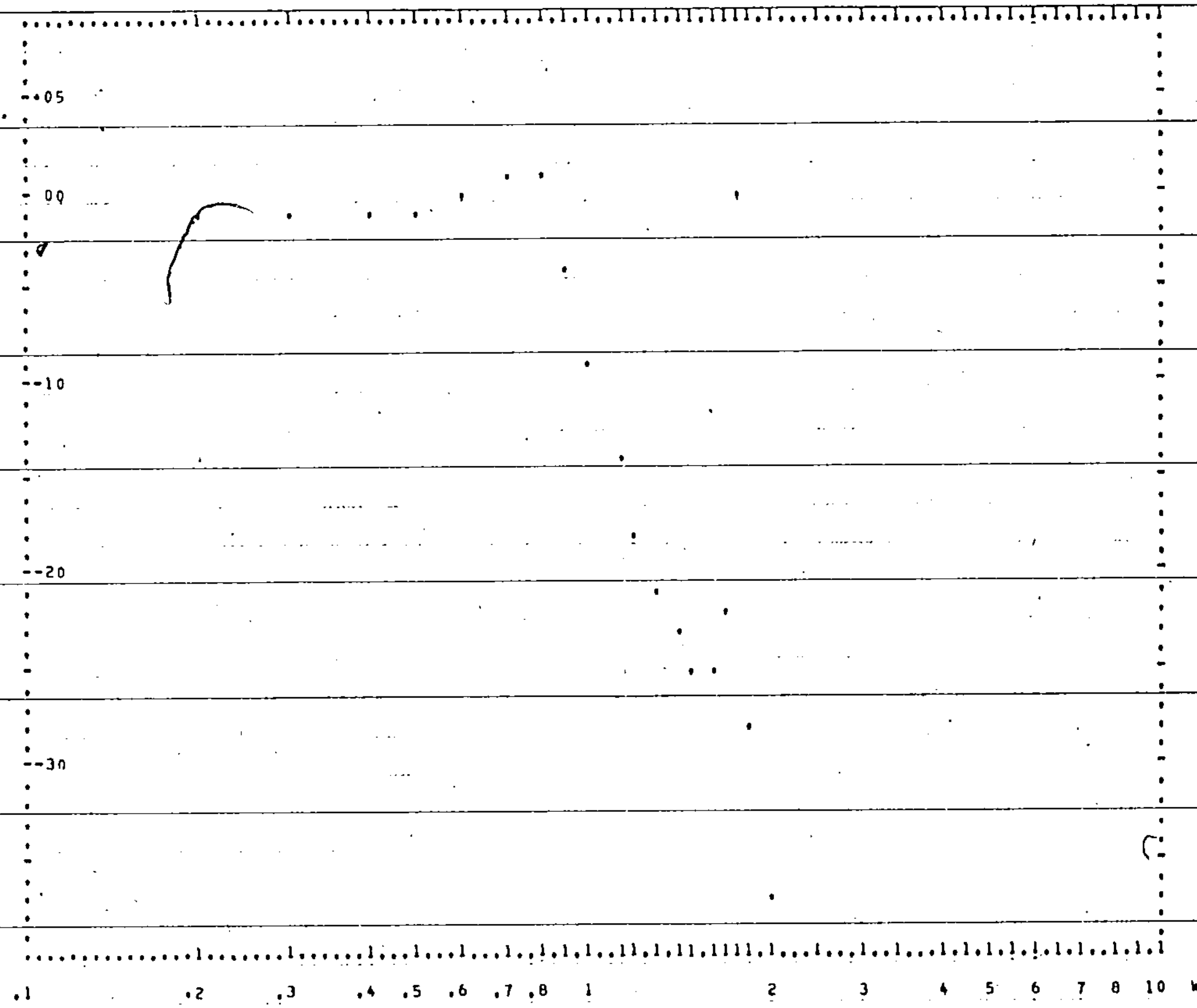

FIGURE 4.2.2.1 RESPONSE OF A T.L. + A L.P. BUTTERWORTH FILTER OF ORDER 4 (REFER TO FIG.4.1)

POOR PRINT

T= .40

W	TW	TWDB
0.0	1.000000	0.00
.1	.977131	-.20
.2	.927878	-.65
.3	.887351	-1.04
.4	.881370	-1.10
.5	.929265	-.64
.6	1.047171	.40
.7	1.141328	1.15
.8	.851100	-1.40
.9	.482044	-6.34
1.0	.283560	-10.95
1.1	.183949	-14.71
1.2	.132180	-17.58
1.3	.106564	-19.45
1.4	.100775	-19.93
1.5	.131102	-17.65
1.6	1.010225	.09
1.7	.063288	-23.97
1.8	.022603	-32.92
1.9	.011035	-39.14
2.0	.006199	-44.15
2.4	.001181	-58.55
3.0	.000259	-71.74
3.4	.000146	-76.69
4.0	.000152	-76.37
4.4	.000340	-69.37
5.0	.000038	-88.42
5.4	.000158	-76.05
6.0	.000003	-109.88
6.4	.000001	-120.10
7.0	.000000	-131.21
8.0	.000000	-144.00
9.0	.000000	-152.02
10.0	.000000	-155.67

TW IN DB

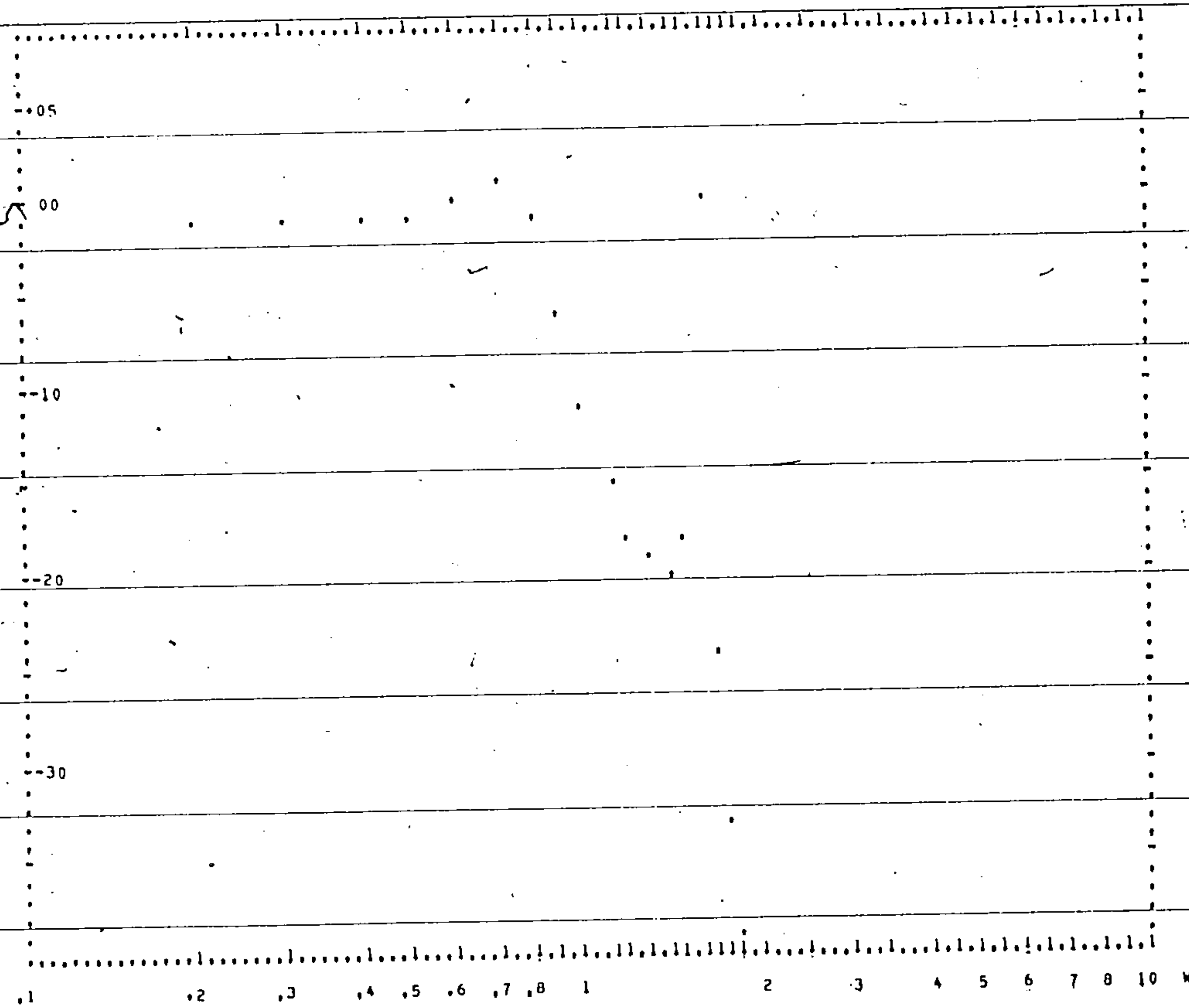

FIGURE 4.2.2.2 RESPONSE OF A T.L. + A L.P. BUTTERWORTH FILTER OF ORDER 4 (REFER TO FIG.4.1)

[POOR PRINT]

T= .20

W	TW	TWDB
0.0	1.000000	0.00
.1	.974894	-.22
.2	.932030	-.61
.3	.924299	-.68
.4	.986431	-.12
.5	1.119187	.98
.6	1.153067	1.24
.7	.886652	-1.04
.8	.611806	-4.27
.9	.465075	-6.65
1.0	.479298	-6.39
1.1	1.051348	.43
1.2	.113365	-18.91
1.3	.033235	-29.57
1.4	.013184	-37.60
1.5	.006141	-44.24
1.6	.003203	-49.89
1.7	.001834	-54.73
1.8	.001145	-58.83
1.9	.000782	-62.14
2.0	.000595	-64.51
2.4	.000210	-73.57
3.0	.000003	-109.86
3.4	.000001	-124.75
4.0	.000000	-142.33
4.4	.000000	-151.95
5.0	.000000	-164.05
5.4	.000000	-170.82
6.0	.000000	-179.23
6.4	.000000	-183.70
7.0	.000000	-188.46
8.0	.000000	-185.20
9.0	.000000	-200.00
10.0	.000000	-198.00

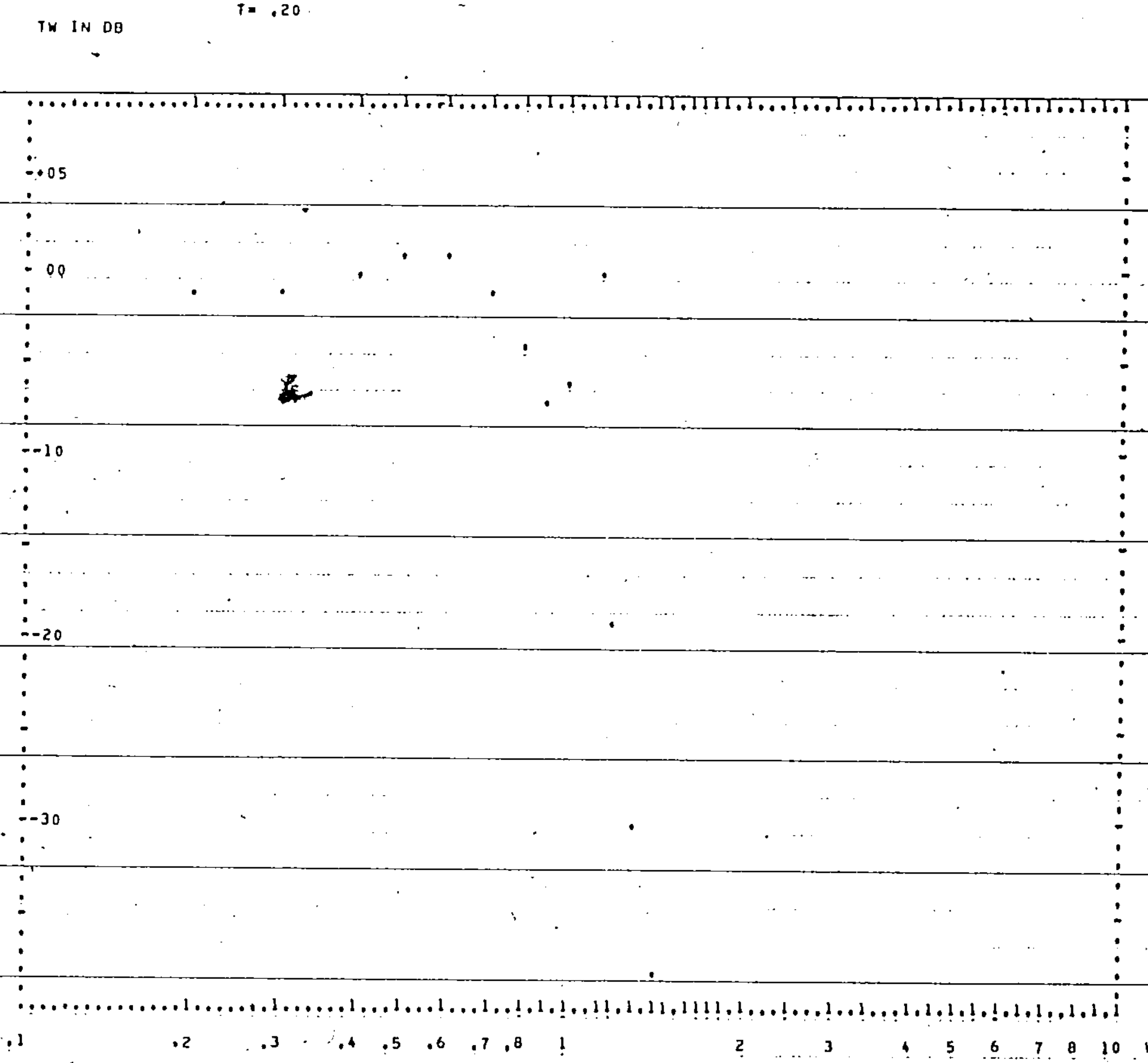

FIGURE 4.2.3.1 RESPONSE OF A T.L. + A L.P. BUTTERWORTH FILTER OF ORDER 6 (REFER TO FIG.4.1)

[POOR PRINT]

W	TW	TWDB
0.0	1.000000	0.00
.1	.972083	-.25
.2	.928559	-.64
.3	.930304	-.63
.4	1.008355	.07
.5	1.115085	.95
.6	1.009899	.09
.7	.731430	-2.72
.8	.537152	-5.40
.9	.464380	-6.66
1.0	.649229	-3.75
1.1	.390605	-8.17
1.2	.079128	-22.03
1.3	.028321	-30.96
1.4	.012799	-37.86
1.5	.006746	-43.42
1.6	.004054	-47.84
1.7	.002798	-51.06
1.8	.002331	-52.65
1.9	.002894	-50.77
2.0	.008723	-41.19
2.4	.000057	-84.84
3.0	.000003	-109.39
3.4	.000001	-119.77
4.0	.000000	-126.29
4.4	.000001	-124.35
5.0	.000000	-150.53
5.4	.000000	-156.86
6.0	.000000	-160.75
6.4	.000000	-180.27
7.0	.000000	-196.87
8.0	.000000	-200.00
9.0	.000000	-200.00
10.0	.000000	-200.00

TW IN DB   T= .40

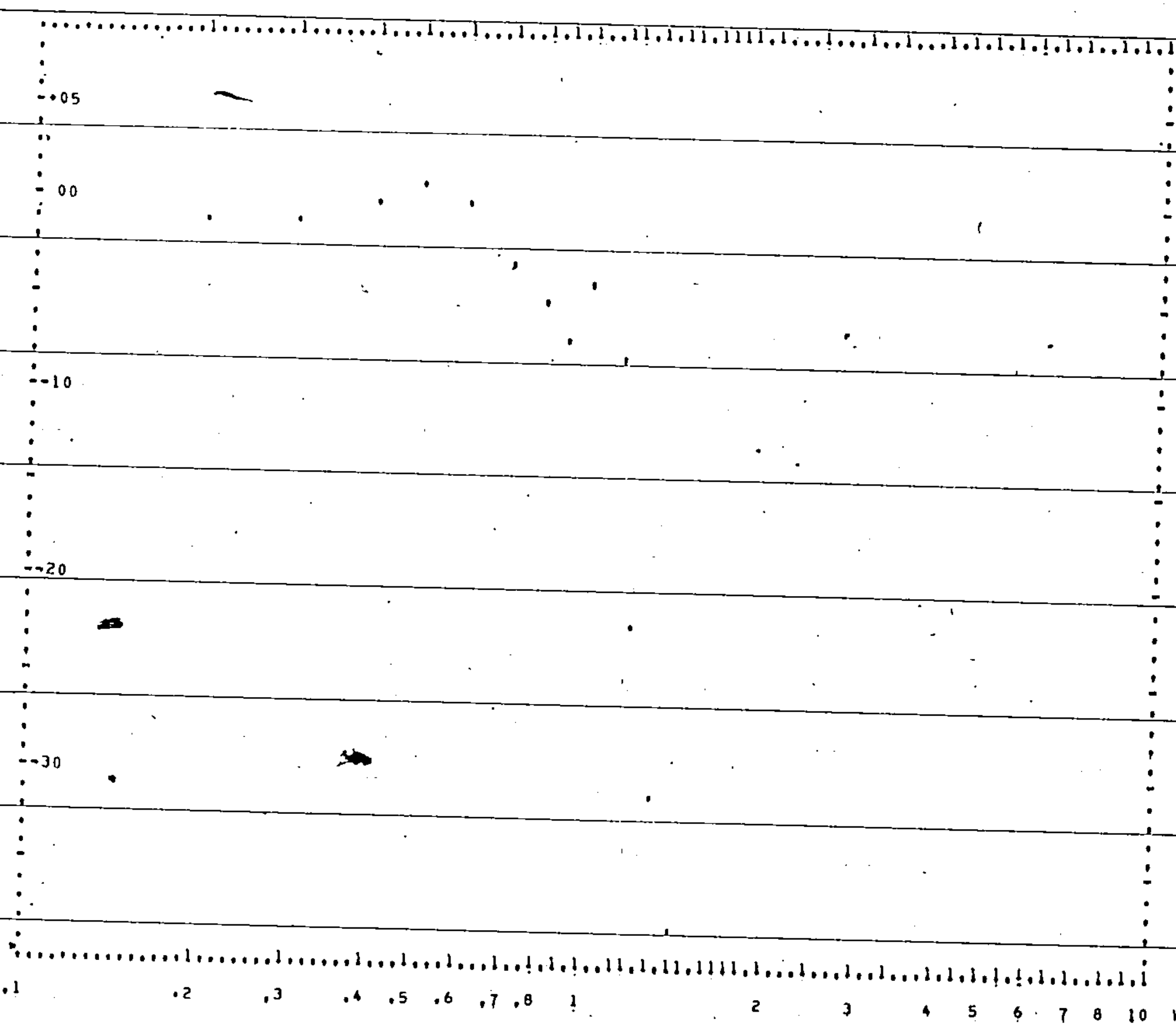

FIGURE 4.2.3.2 RESPONSE OF A T.L. + A.L.P. BUTTERWORTH FILTER OF ORDER 6 (REFER TO FIG.4.1)

T= .40

W	TW	TWDB
0.0	1.000000	0.00
.1	.966172	-.30
.2	.934821	-.59
.3	.999998	-.00
.4	1.052374	.44
.5	.927856	-.65
.6	.876466	-1.15
.7	.832448	-1.59
.8	.879431	-1.12
.9	.977392	-.20
1.0	.229533	-12.78
1.1	.112245	-19.00
1.2	.119563	-18.45
1.3	.039126	-28.15
1.4	.005256	-45.59
1.5	.001355	-57.36
1.6	.000455	-66.84
1.7	.000182	-74.81
1.8	.000083	-81.59
1.9	.000043	-87.31
2.0	.000025	-91.97
2.4	.000011	-99.22
3.0	.000000	-145.91
3.4	.000000	-162.28
4.0	.000000	-176.13
4.4	.000000	-175.80
5.0	.000000	-200.00
5.4	.000000	-200.00
6.0	.000000	-200.00
6.4	.000000	-200.00
7.0	.000000	-200.00
8.0	.000000	-200.00
9.0	.000000	-200.00
10.0	.000000	-200.00

TW IN DB

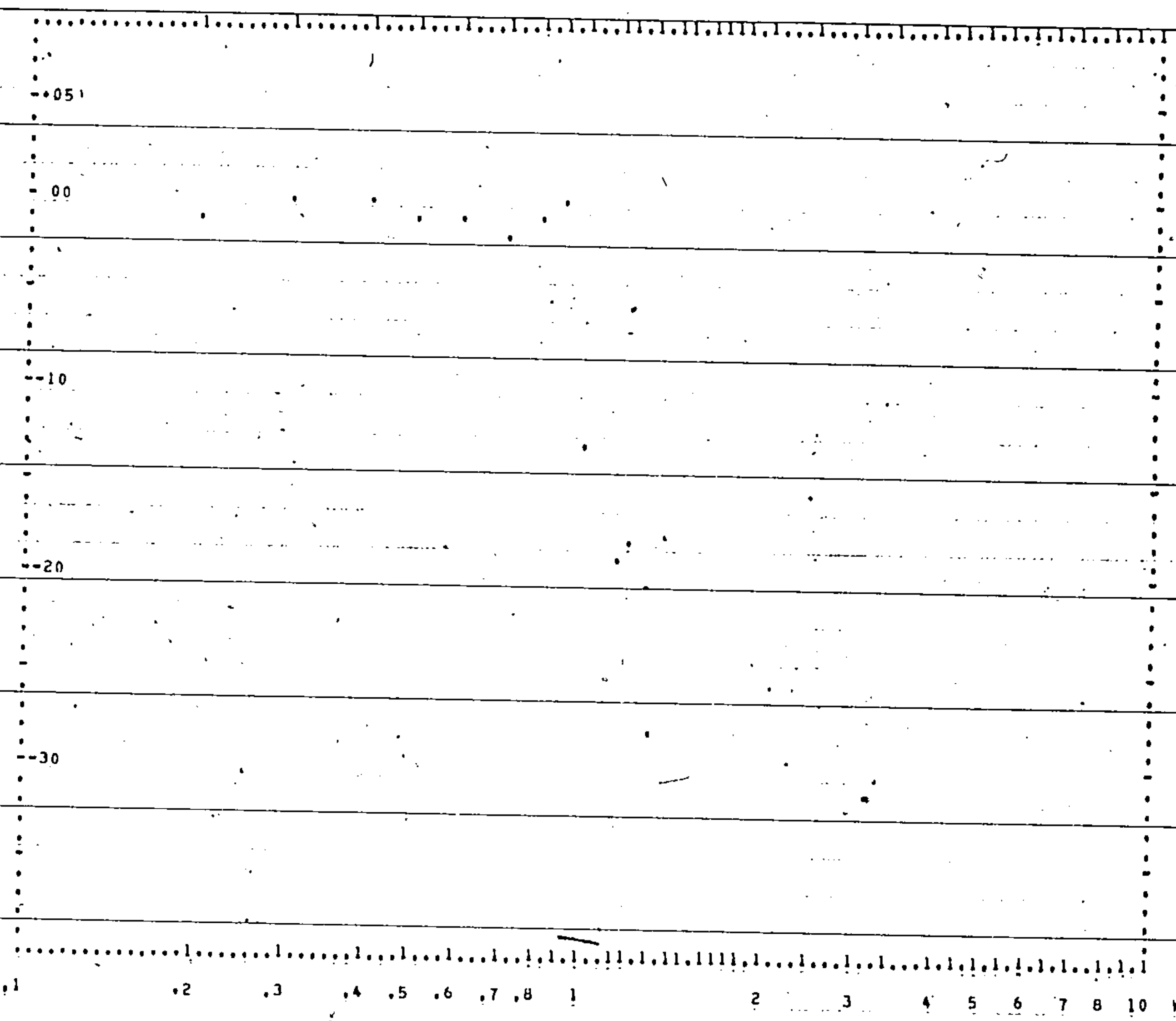

FIGURE 4.2.4.1 RESPONSE OF A T.L. + A L.P. BUTTERWORTH FILTER OF ORDER 8 (REFER TO FIG.4.1)

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POOR PRINT

T= .60

W	TW	TWDB
0.0	1.000000	0.00
.1	.964004	-.32
.2	.937249	-.56
.3	1.011218	.10
.4	1.019121	.16
.5	.871995	-1.19
.6	.836754	-1.55
.7	.813216	-1.80
.8	.986335	-.12
.9	.614562	-4.23
1.0	.195109	-14.19
1.1	.122131	-18.26
1.2	.394262	-8.08
1.3	.023028	-32.76
1.4	.004563	-46.82
1.5	.001397	-57.10
1.6	.000544	-65.29
1.7	.000256	-71.85
1.8	.000144	-76.85
1.9	.000101	-79.92
2.0	.000108	-79.30
2.4	.000003	-110.00
3.0	.000006	-103.95
3.4	.000000	-158.72
4.0	.000000	-184.34
4.4	.000000	-192.53
5.0	.000000	-200.00
5.4	.000000	-200.00
6.0	.000000	-200.00
6.4	.000000	-200.00
7.0	.000000	-200.00
8.0	.000000	-200.00
9.0	.000000	-200.00
10.0	.000000	-200.00

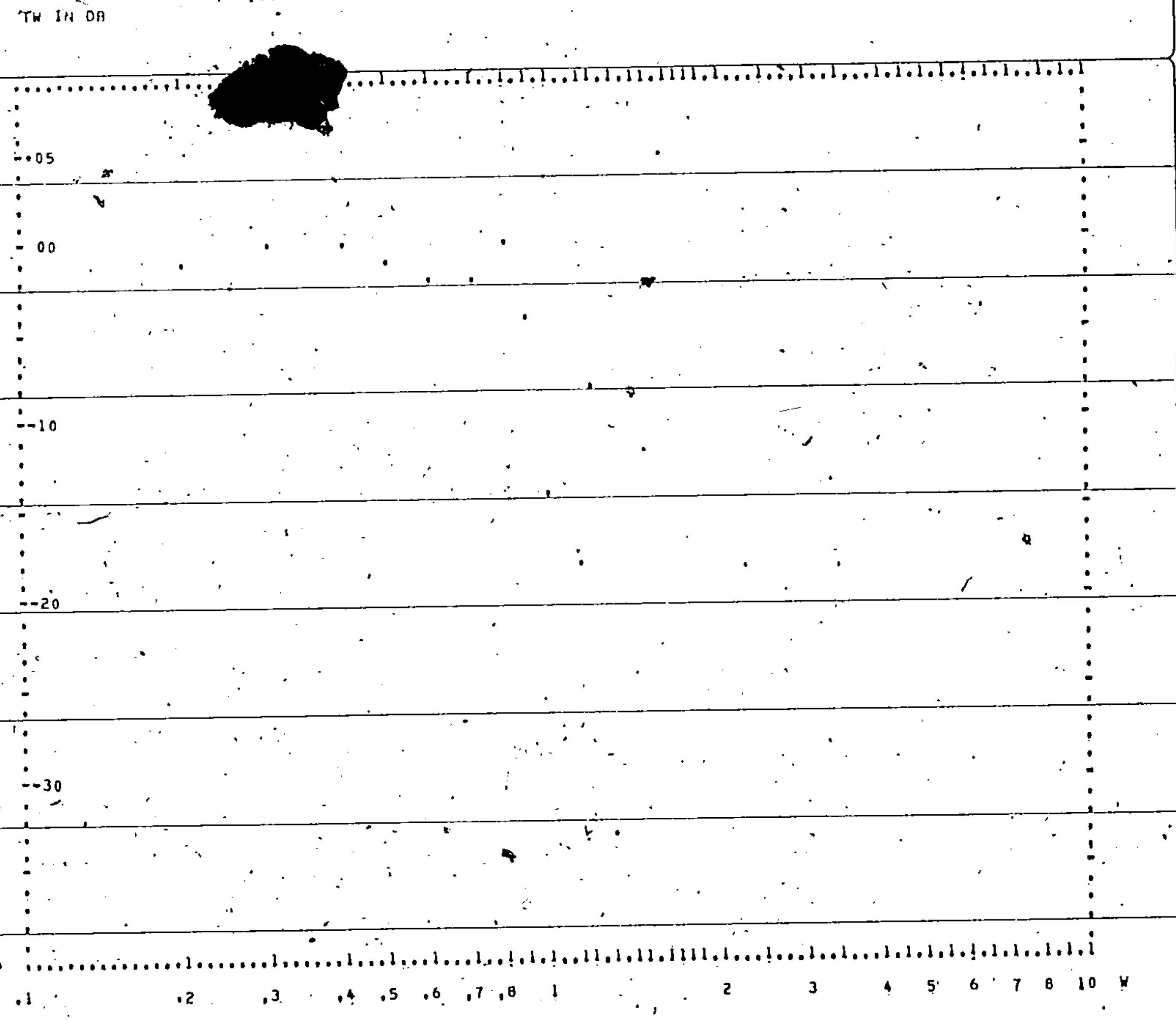

FIGURE 4.2.4.2 RESPONSE OF A T.L. + A L.P. BUTTERWORTH FILTER OF ORDER 8 (REFER TO FIG.4.1)

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POOR PRINT !

TW = .40

W	TW	TWDB
0.0	1.000000	0.00
.1	.968097	-.28
.2	.969539	-.27
.3	1.056253	.48
.4	1.065862	.55
.5	.885729	-1.05
.6	.738435	-2.63
.7	1.108647	.90
.8	.589837	-4.59
.9	.375991	-8.50
1.0	.587837	-4.61
1.1	.071411	-22.92
1.2	.011445	-38.83
1.3	.003839	-48.31
1.4	.002644	-51.56
1.5	.001557	-56.15
1.6	.000141	-77.02
1.7	.000031	-90.25
1.8	.000009	-101.10
1.9	.000003	-110.50
2.0	.000001	-118.79
2.4	.000000	-142.57
3.0	.000000	-177.97
3.4	.000000	-200.00
4.0	.000000	-200.00
4.4	.000000	-200.00
5.0	.000000	-200.00
5.4	.000000	-200.00
6.0	.000000	-200.00
6.4	.000000	-200.00
7.0	.000000	-200.00
8.0	.000000	-200.00
9.0	.000000	-200.00
10.0	.000000	-200.00

TW IN DB

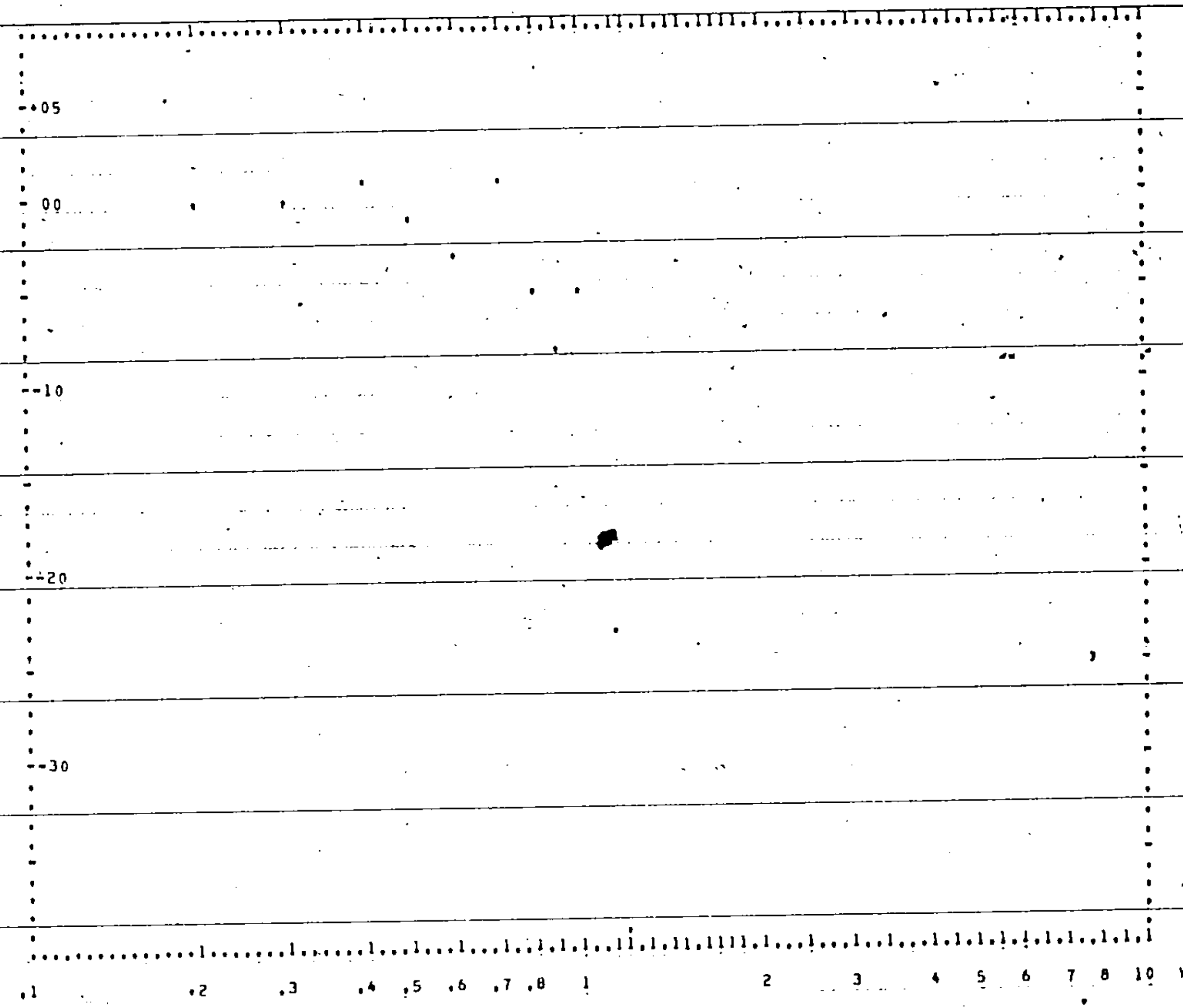

FIGURE 4.2.5.1 RESPONSE OF A T.L. + A L.P. BUTTERWORTH FILTER OF ORDER 10 (REFER TO FIG.4.1)

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T= .60

W	TW	TWDB
0.0	1.000000	0.00
.1	.966678	-.29
.2	.973662	-.23
.3	1.055355	.47
.4	1.015711	.14
.5	.824996	-1.67
.6	.751431	-2.48
.7	1.129033	1.05
.8	.462796	-6.69
.9	.364726	-8.76
1.0	.932043	-.61
1.1	.057863	-24.75
1.2	.011940	-38.46
1.3	.005300	-45.52
1.4	.023468	-32.59
1.5	.000615	-64.23
1.6	.000113	-78.95
1.7	.000031	-90.16
1.8	.000011	-99.49
1.9	.000004	-107.41
2.0	.000002	-114.03
2.4	.000002	-114.96
3.0	.000000	-153.94
3.4	.000000	-200.00
4.0	.000000	-200.00
4.4	.000000	-200.00
5.0	.000000	-200.00
5.4	.000000	-200.00
6.0	.000000	-200.00
6.4	.000000	-200.00
7.0	.000000	-200.00
8.0	.000000	-200.00
9.0	.000000	-200.00
10.0	.000000	-200.00

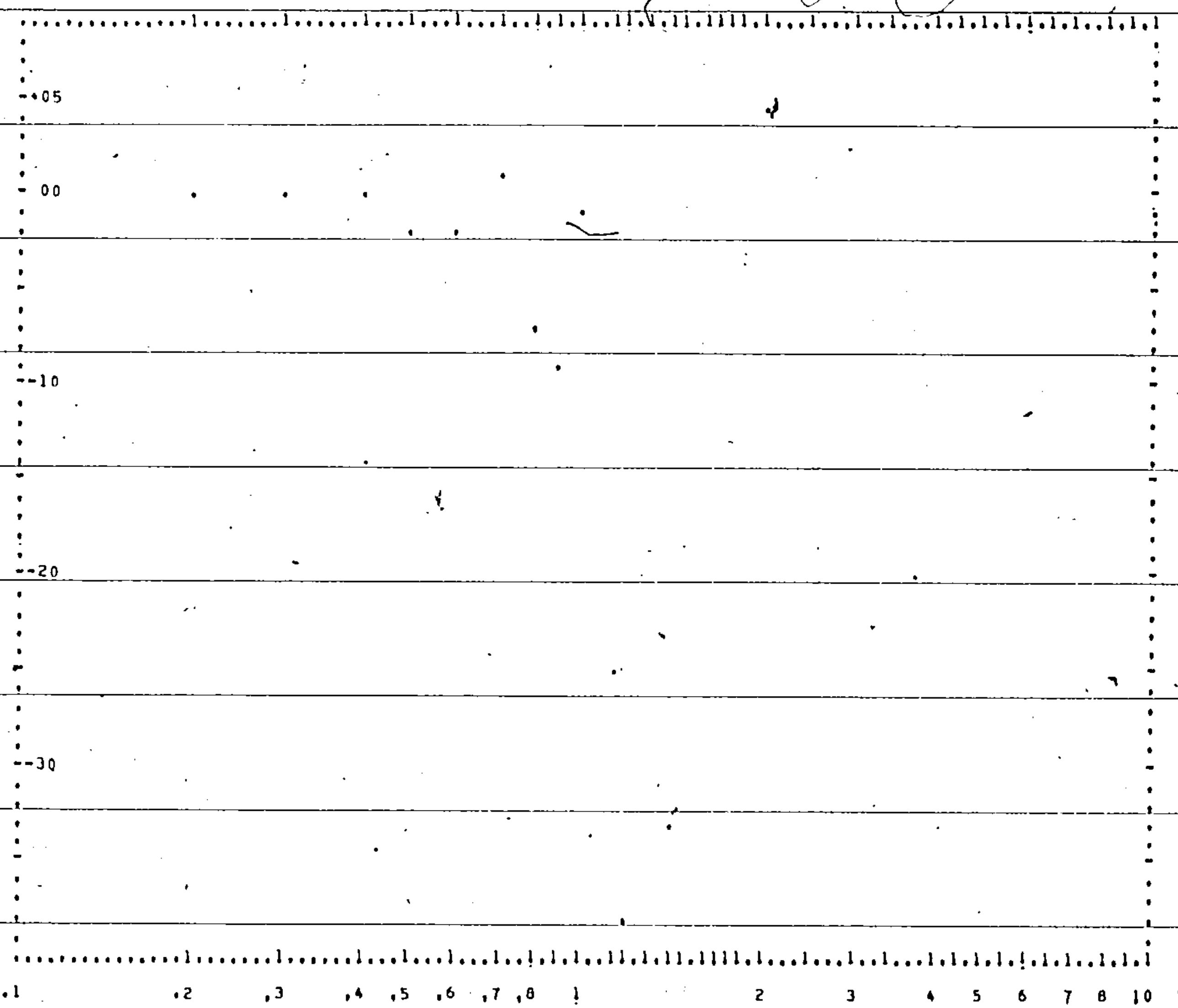

FIGURE 4.2.5.2 RESPONSE OF A T.L. + A L.P. BUTTERWORTH FILTER OF ORDER 10 (REFER TO FIG.4.1)

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POOR PRINT !

T= .40

W	TW	TWDB
0.0	0.000000	-200.00
.1	.000000	-200.00
.2	.000000	-200.00
.3	.000000	-157.33
.4	.000001	-126.00
.5	.000015	-96.73
.6	.000378	-[illegible].45
.7	.029906	-30.48
.8	.057542	-24.80
.9	.250034	-12.04
1.0	2.383509	7.54
1.1	.971503	-.25
1.2	1.364627	2.70
1.3	1.716638	4.69
1.4	1.415705	3.02
1.5	1.225163	1.76
1.6	1.146022	1.18
1.7	1.100577	.83
1.8	1.060903	.51
1.9	1.026982	.23
2.0	1.005200	.05
2.4	1.069681	.59
3.0	1.218849	1.72
3.4	1.079937	.67
4.0	.918059	-.74
4.4	.893902	-.97
5.0	.933180	-.60
5.4	.974527	-.22
6.0	.999590	-.00
6.4	.984000	-.14
7.0	.950557	-.44
8.0	.967341	-.29
9.0	1.056255	.48
10.0	1.054089	.46

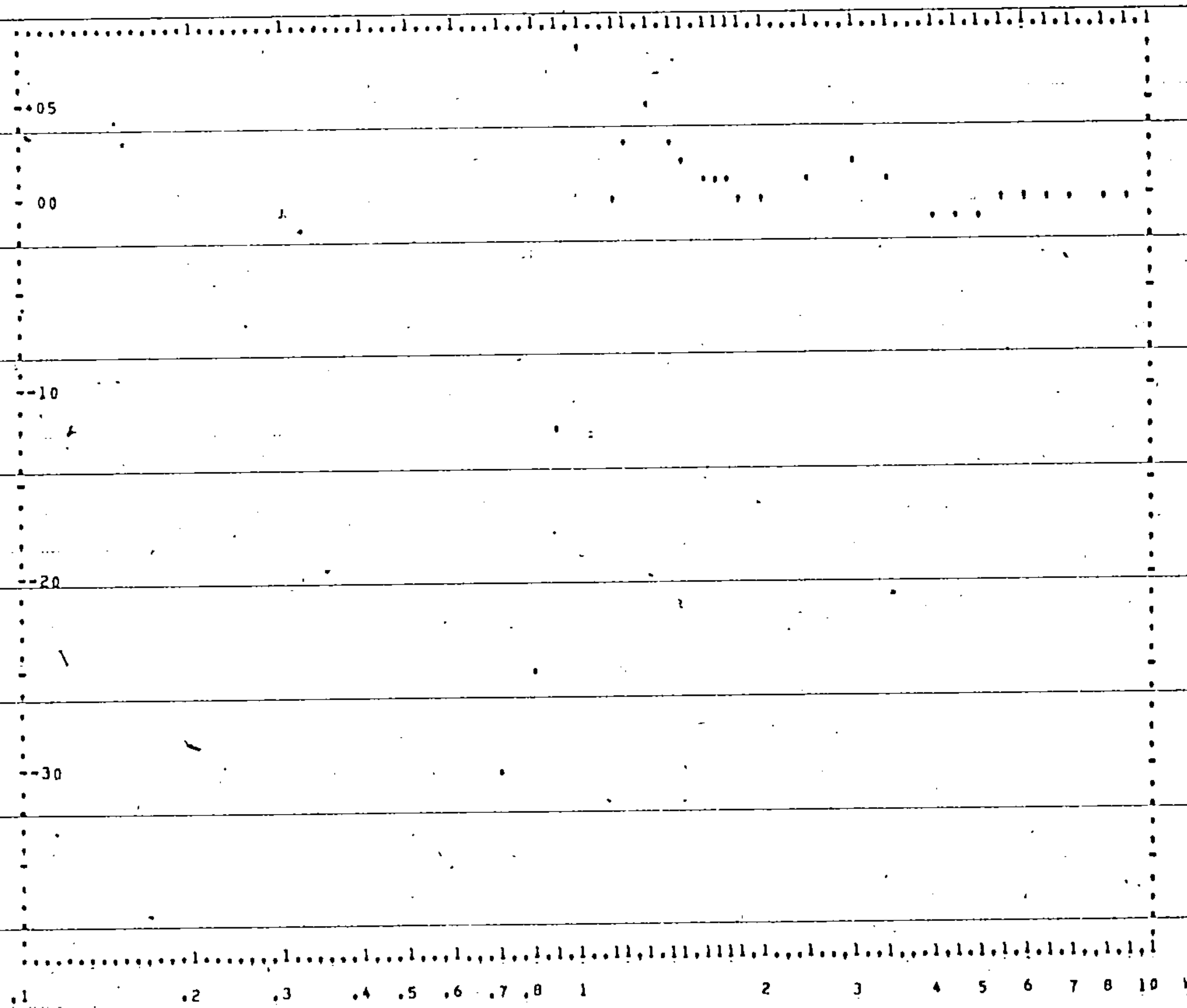

FIGURE 4.3.4.1 RESPONSE OF A T.L. + A H.P. BUTTERWORTH FILTER OF ORDER 8 (REFER TO FIG.4.1)

POOR PRINT

T= .20

W	TW	TWDB
0.0	0.000000	-200.00
.1	.000000	-129.94
.2	.000190	-74.43
.3	.018555	-34.63
.4	.062379	-24.10
.5	.651640	-3.72
.6	1.025042	.21
.7	.884532	-1.07
.8	.953860	-.41
.9	1.016054	.14
1.0	1.000000	.00
1.1	.942317	-.52
1.2	.898010	-.93
1.3	.890851	-1.00
1.4	.929783	-.63
1.5	1.018242	.16
1.6	1.112520	.93
1.7	1.009276	.08
1.8	.688924	-3.24
1.9	.433092	-7.27
2.0	.283560	-10.95
2.4	.097946	-20.18
3.0	.040238	-27.91
3.4	.004913	-46.17
4.0	.000820	-61.72
4.4	.000344	-69.27
5.0	.000124	-78.16
5.4	.000072	-82.84
6.0	.000038	-88.33
6.4	.000028	-91.02
7.0	.000021	-93.45
8.0	.000031	-90.04
9.0	.000017	-95.25
10.0	.000013	-98.01

TW IN DB

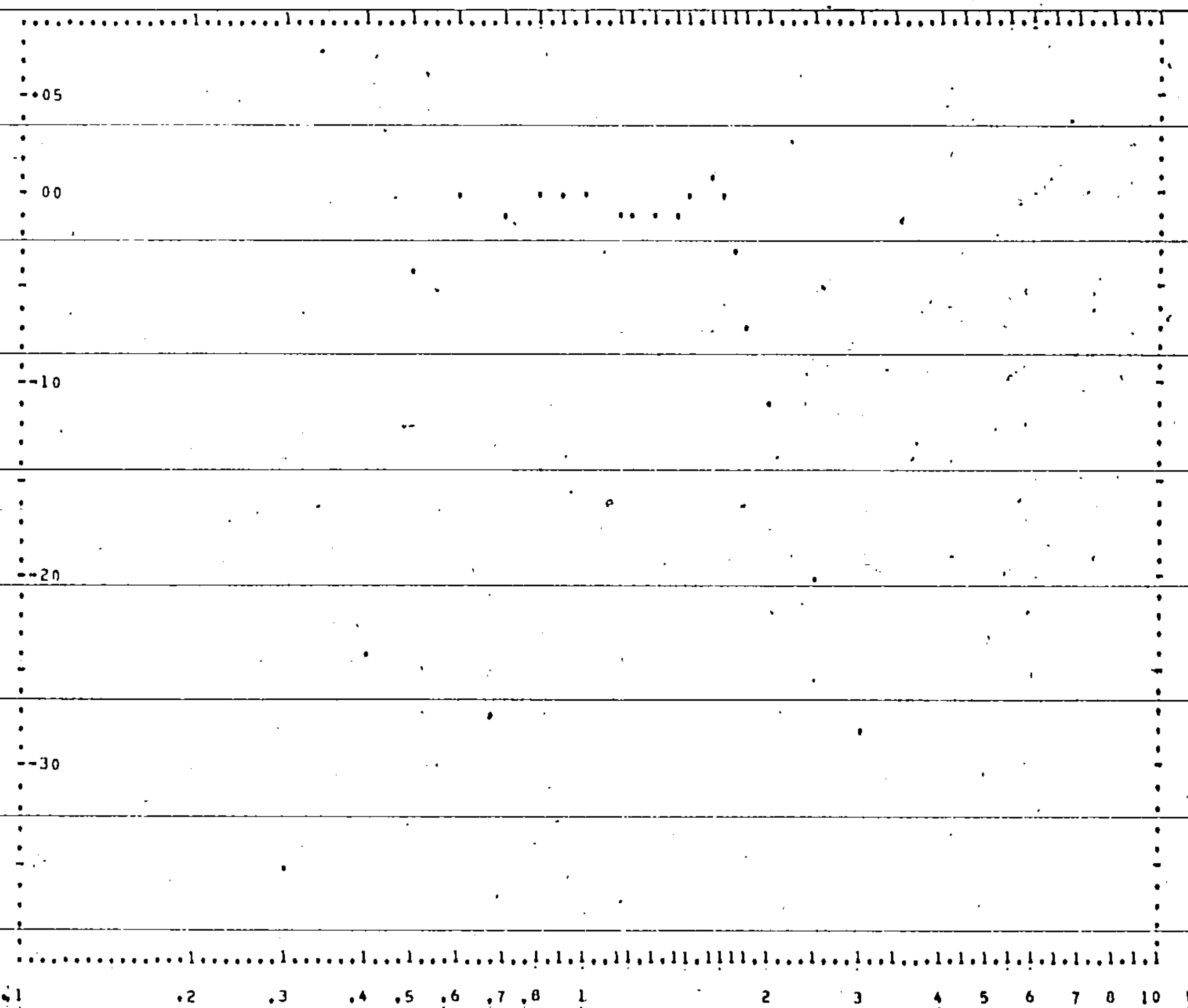

FIGURE 4.4.2.1 RESPONSE OF A T.L. + A B.P. BUTTERWORTH FILTER OF ORDER 4 (REFER TO FIG.4.1)

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POOR PRINT !

T= .40

W	TW	TWDB
0.0	1.000000	0.00
.1	.943331	-.51
.2	1.004583	.04
.3	.984577	-.14
.4	.861998	-1.29
.5	.307156	-10.25
.6	.007253	-42.79
.7	.000009	-100.68
.8	.000000	-150.11
.9	.000000	-200.00
1.0	0.000000	-200.00
1.1	.000000	-200.00
1.2	.000000	-184.79
1.3	.000000	-141.27
1.4	.000005	-106.44
1.5	.000168	-75.48
1.6	.012415	-38.12
1.7	.044087	-27.11
1.8	.293904	-10.64
1.9	1.049059	.42
2.0	.940890	-.53
2.4	.900467	-.91
3.0	.996650	-.03
3.4	1.148948	1.21
4.0	1.057120	.48
4.4	.896321	-.95
5.0	.829241	-1.63
5.4	.866852	-1.24
6.0	.968472	-.28
6.4	1.004933	.04
7.0	.976539	-.21
8.0	.938725	-.55
9.0	1.052073	.44
10.0	1.103965	.86

TW IN DB

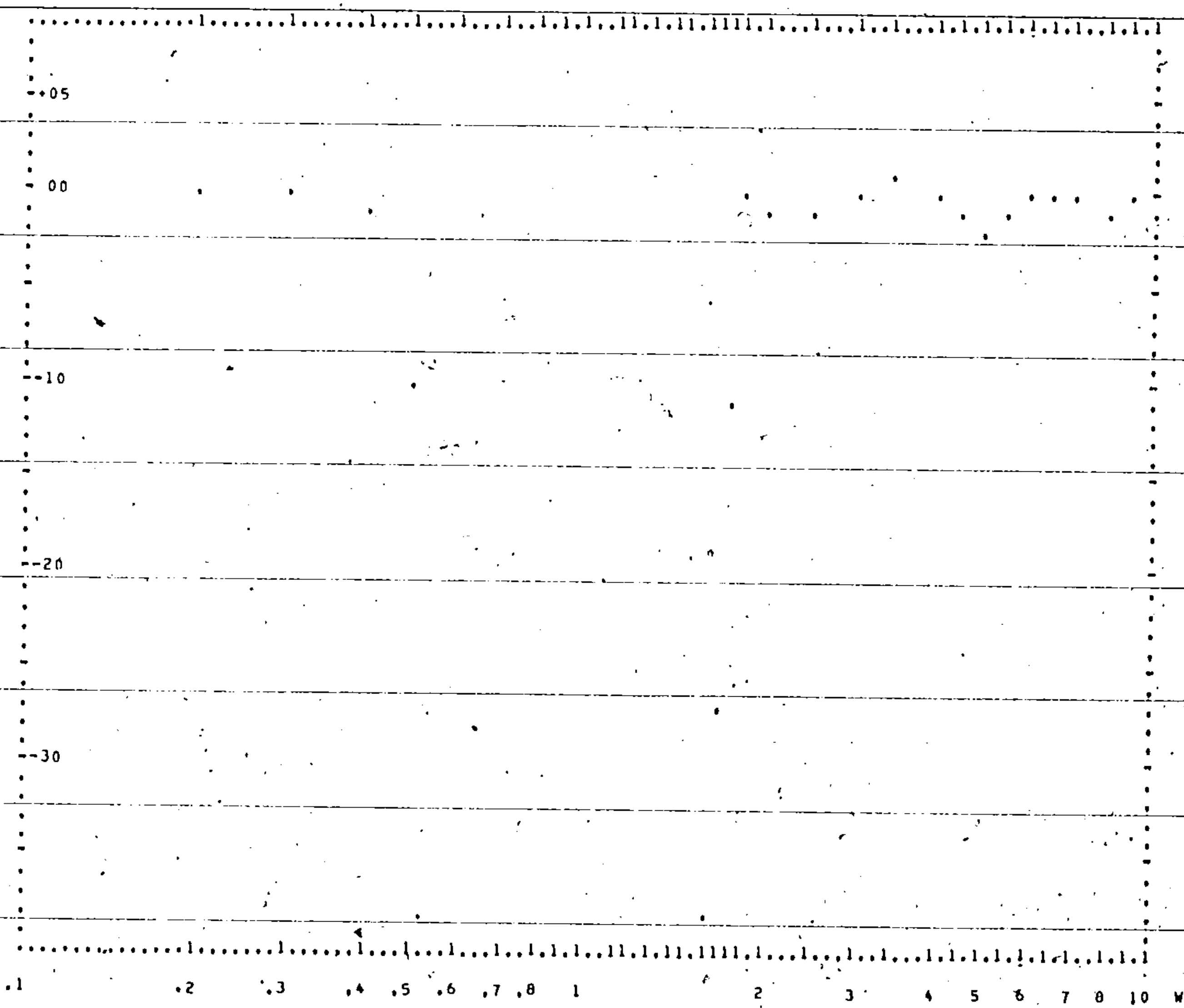

FIGURE 4.5.4.1 RESPONSE OF A T.L. + A B.S. BUTTERWORTH FILTER OF ORDER 8 (REFER TO FIG.4.1)

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4.3 Discussion

By examining all the figures shown in this chapter, the following remarks are mentioned:

a) In the pass-band of the filter, the response varied more than a single Butterworth filter due to the missmatch existing between various sections of the network in question.

b) In the stop band of the filter, the peaks appearing are having higher magnitudes that exceed 0 db at times. These peaks were due to the fact that there are two transmission lines peaking at the same frequency in the circuit.

c) Higher values of τ can be achieved at higher filter orders.

d) It is hard to judge if there was an improvement in the response using this combination as was obvious from the shown cases.

e) It is worth mentioning in this discussion the most relevant responses detailed in the following three cases:

1) High-Pass Filter ($N = 8$, $\tau = 0.4$) Figure 4.3.4.1

The case of High-Pass Filter ($N = 8$, $\tau = 0.4$) shows a slowly damped ripple response over the pass-band. There are two marked peaks of + 7.54 db at $w = 1$ and 4.69 db at $w = 1.3$. That arrangement shows a very sharp cut-off response.

2) Band-Pass Filter ($N = 4$, $\tau = 0.2$) Figure 4.4.2.1

The most interesting Band-Pass response, from the uniformity point of view, is the case for $N = 4$, $\tau = 0.2$. The Band-Pass shows a ripple behaviour with a maximum peak-to-peak value of 2 db (+ 0.93 to − 1.07 db) and a mean value of 0.0 db. The pass band responses show a sharp selectivity of 20.38 db/w and 20.56 db/w. In other words, this arrangement is highly symmetrical with less than ± 1.07 db ripples in the pass-band.

POOR PRINT

3) Band-Stop Filter ($N = 8$, $\tau = 0.4$) Figure 4.5.4.1

The last relevant case is the band-stop with $N = 8$ and $\tau = 0.4$. Here there is a marked corner sharpness in the high frequency side at $w = 1.9$. In the high frequency pass band side there is a peak-to-peak ripple of 2.84 db.

Apart from these three cases, a generally distorted response resulted in the three cases of High-Pass, Band-Pass and Band-Stop filters.

CHAPTER V

TRANSMISSION LINE IN SERIES WITH AN Nth ORDER BUTTERWORTH FILTER REPEATED THREE TIMES

5.1 Introduction

In this chapter we shall consider the circuit shown in Figure 5.1 in which a combination of a lossless transmission line consists of a variable element (ℓ) having a characteristic impedance $Z_0 = 1$, is connected in series with a passive lumped element B.W. Filter of order N. This network combination is connected in series with 2 similar networks. The resultant network is then terminated in a resistor $R = 1$.

5.2 The Lumped Section is a Low-Pass B.W. Filter

Similar to section 4.2, A computer program was written to calculate the transfer function for the circuit shown in Figure 5.1 for the various combinations of the following cases:

1. Filter order N = 2, 4, 6, 8 and 10
2. Filter modes L.P., H.P., B.P. and B.S.

All the above curves were calculated at the same values of τ selected in section 3.2.

Although the filter response was obtained for all 4 modes the L.P. mode is the basic one dealt with. However, the discussions cover all modes.

5.2.1. L.P. Filter (N = 2)

Figures 5.2.1.1 and 5.2.1.2 show the response of the circuit shown in Figure 5.1 for a L.P. Filter of order 2, calculated at $\tau = 0.2$ and 0.4 respectively.

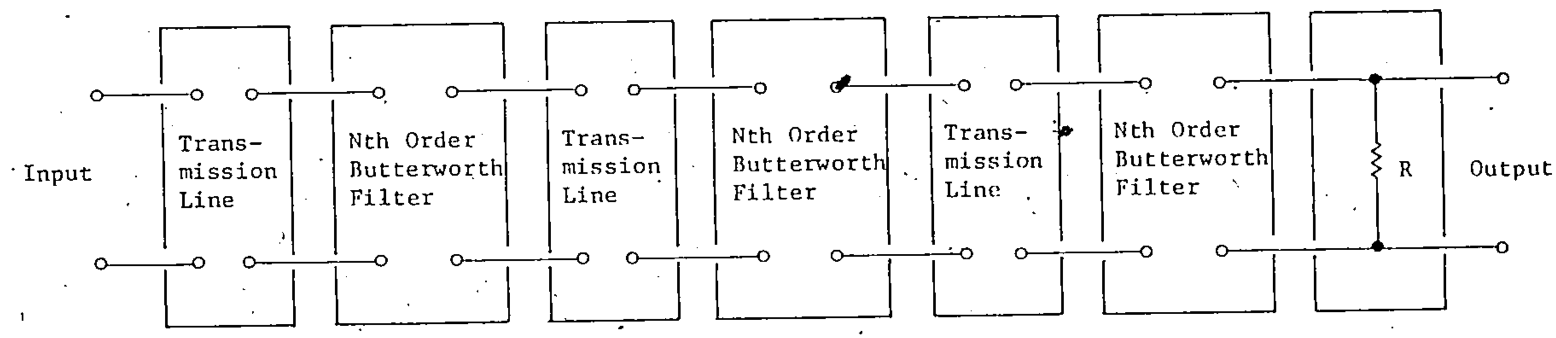

Figure 5.1 - A Combination of a Transmission Line and an Nth Order Butterworth Filter repeated three times, terminated in a resistor R.

5.2.2 L.P. Filter (N = 4)

Figures 5.2.2.1 and 5.2.2.2 show the response of the network for a L.P. filter of order 4 calculated at $\tau = 0.2$ and 0.4 respectively.

5.2.3 L.P. Filter (N = 6)

Figures 5.2.3.2 and 5.2.3.2 show the response of the network for a L.P. filter of order 6 calculated at $\tau = 0.2$ and 0.4 respectively.

5.2.4 L.P. Filter (N = 8)

Figures 5.2.4.1 and 5.2.4.2 show the response of the network for a L.P. filter of order 8 calculated at $\tau = 0.4$ and 0.6 respectively.

5.2.5 L.P. Filter (N = 10)

Figures 5.2.5.1 and 5.2.5.2 show the response of the network for a L.P. filter of order 10 calculated at $\tau = 0.4$ and 0.6 respectively.

5.3 Discussion

In the low pass case, the introduction of the transmission line has always shown a disturbing effect on both the pass-band, the corner frequency and the cut-off edge. The manner in which one or two of these disturbances occur is always out of phase, i.e. if the amplitude of the ripples is reduced, the cut-off edge shows severe deviation from smoothness. Therefore it is concluded that this combination has no merits in the low-pass case.

The high pass combinations show a rippling effect in the pass band. The disturbance occurring on the edge is almost non-existant in all cases. For frequency values $0.8 < \hat{w} < 1.3$ the ripple amplitude is appreciable between + 7 db and - 10 db. For the rest of the high frequency pass-band (i.e. $w > 2$), the ripple amplitude is $< \pm 1.4$ db. The most favourable response is that with $N = 4$ and $\tau = 0.2$ which shows a sharp corner effect combined with a ripple amplitude between - 1.11 db and 2.57 db. (page 102 to 107 inclusive)

The only band pass case of acceptable behaviour is when $N = 4$ and $\tau = 0.2$. In that case, the high frequency corner is sharper than the low frequency corner. For the pass-band frequencies $.5 < w < 1.8$, the ripple values are within + 1.09 db and - 1.29 db. (Figure 5.4.2.1)

For the band-stop case, the ripple disturbance on the high frequency side due to the transmission line is always present on the high frequency pass band side only. The most acceptable behaviour occurs when $w = 8$ and $\tau = 0.6$. In that case, the rippling effect becomes more or less an equi-ripple behaviour within $< \pm 0.6$ db. It also shows a very sharp corner high cut-off frequency at $w = 2$. (Figure 5.5.4.2)

[POOR PRINT]

T= .20

W	TW	TWDB
0.0	1.000000	0.00
.1	.964161	-.32
.2	.881441	-1.10
.3	.795126	-1.99
.4	.732449	-2.70
.5	.705763	-3.03
.6	.724651	-2.80
.7	.804253	-1.89
.8	.952316	-.42
.9	1.039184	.33
1.0	.844589	-1.47
1.1	.617375	-4.19
1.2	.499750	-6.02
1.3	.487124	-6.25
1.4	.663932	-3.56
1.5	.924216	-.68
1.6	.247674	-12.12
1.7	.104472	-19.62
1.8	.055240	-25.16
1.9	.032850	-29.67
2.0	.021038	-33.54
2.4	.005370	-45.40
3.0	.001324	-57.56
3.4	.000664	-63.55
4.0	.000302	-70.41
4.4	.000203	-73.84
5.0	.000133	-77.54
5.4	.000111	-79.09
6.0	.000100	-79.97
6.4	.000106	-79.48
7.0	.000148	-76.62
8.0	.001149	-58.80
9.0	.154509	-16.22
10.0	.000062	-84.17

TW IN DB

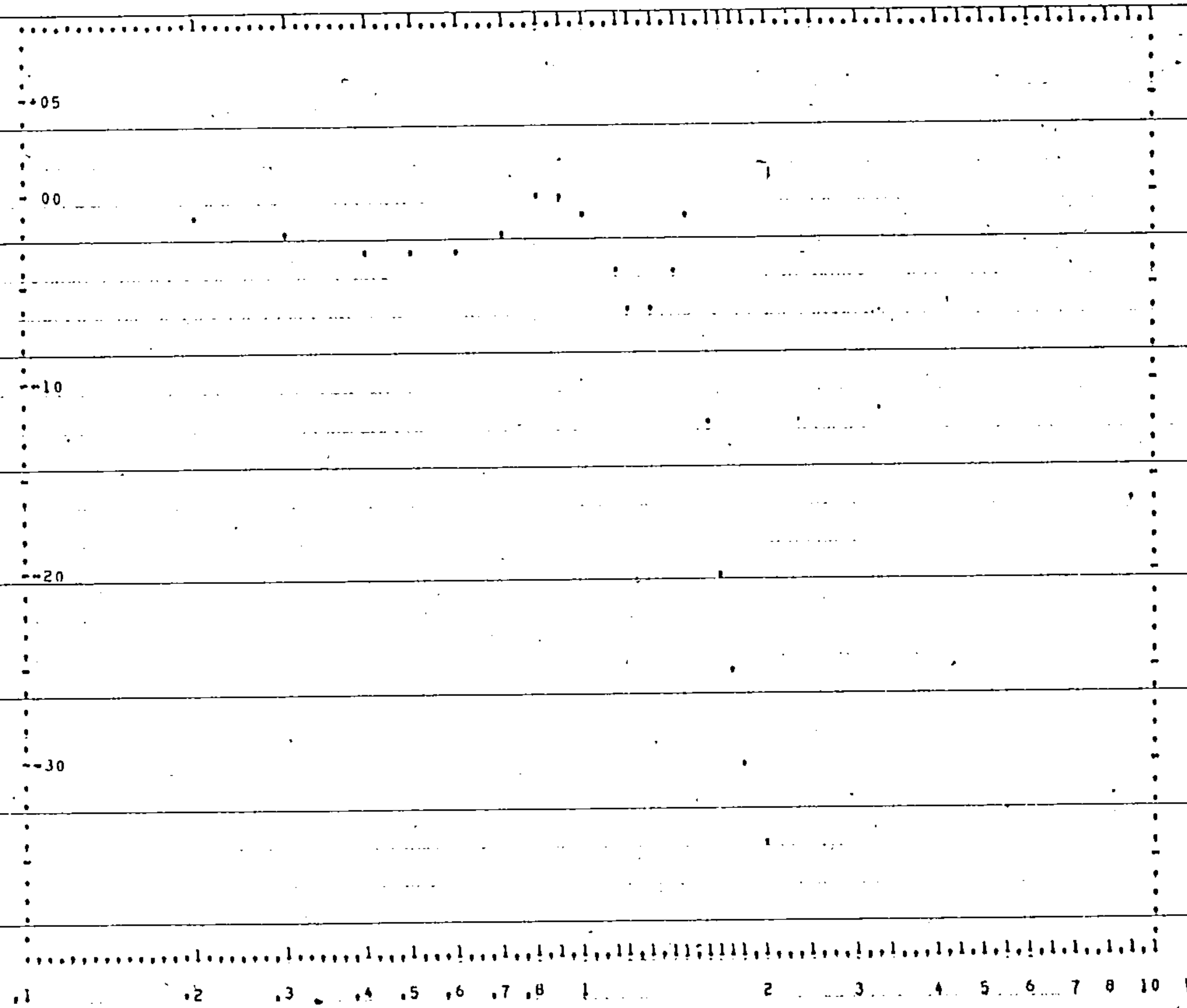

FIGURE 5.2.1.1 RESPONSE OF A T.L. + A L.P. BUTTERWORTH FILTER OF ORDER 2 (REFER TO FIG.5.1)

POOR PRINT

T= .40

W	TW	TWDB
0.0	1.000000	0.00
.1	.957386	-.38
.2	.867531	-1.23
.3	.788278	-2.07
.4	.749280	-2.51
.5	.764627	-2.33
.6	.848143	-1.43
.7	.983723	-.14
.8	.982907	-.15
.9	.765101	-2.33
1.0	.593074	-4.54
1.1	.539540	-5.36
1.2	.654226	-3.69
1.3	1.017088	.15
1.4	.325760	-9.74
1.5	.132340	-17.57
1.6	.069319	-23.18
1.7	.041505	-27.64
1.8	.027057	-31.35
1.9	.018747	-34.54
2.0	.013617	-37.32
2.4	.005306	-45.50
3.0	.002639	-51.57
3.4	.002536	-51.92
4.0	.006460	-43.80
4.4	.049101	-26.18
5.0	.143608	-16.86
5.4	.002612	-51.66
6.0	.000157	-76.08
6.4	.000050	-85.98
7.0	.000015	-96.73
8.0	.000004	-108.09
9.0	.000002	-113.52
10.0	.000002	-113.14

TW IN DB)

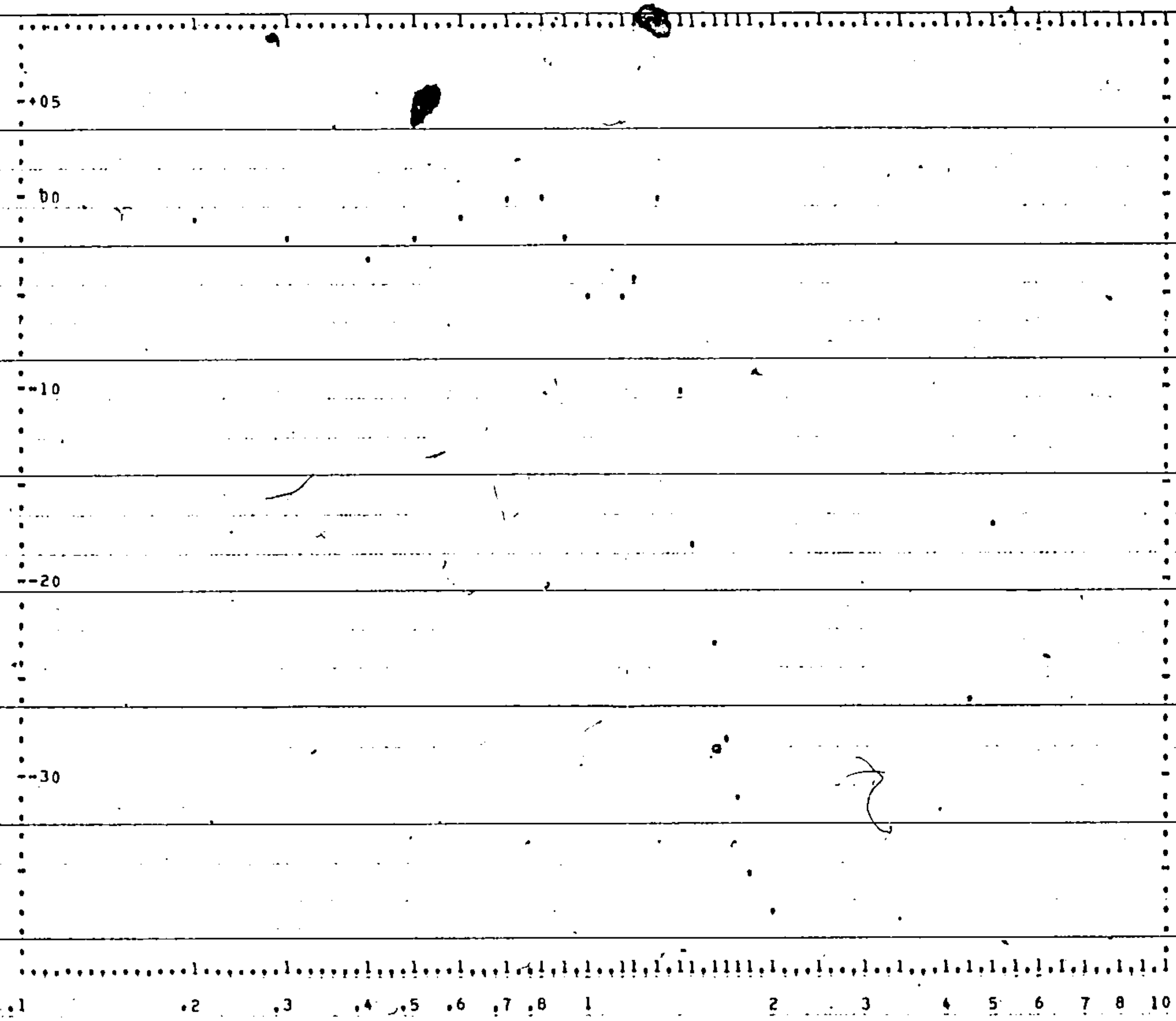

FIGURE 5.2.1.2 RESPONSE OF A T.L. + A L.P. BUTTERWORTH FILTER OF ORDER 2 (REFER TO FIG.5.1)

.T= .20

W	TW	TWDB
0.0	1.000000	0.00
.1	.951696	-.43
.2	.885367	-1.06
.3	.889417	-1.02
.4	.980924	-.17
.5	1.055598	.47
.6	.946457	-.48
.7	.859443	-1.32
.8	1.168176	1.35
.9	.742510	-2.59
1.0	.197833	-14.07
1.1	.084334	-21.48
1.2	.045758	-26.79
1.3	.029633	-30.56
1.4	.022745	-32.86
1.5	.021461	-33.37
1.6	.028276	-30.97
1.7	.102002	-19.83
1.8	.087248	-21.18
1.9	.028277	-30.97
2.0	.003314	-49.59
2.4	.000068	-83.34
3.0	.000002	-112.18
3.4	.000000	-126.24
4.0	.000000	-142.98
4.4	.000000	-152.03
5.0	.000000	-163.15
5.4	.000000	-169.16
6.0	.000000	-176.21
6.4	.000000	-179.58
7.0	.000000	-182.31
8.0	.000000	-174.06
9.0	.000000	-178.03
10.0	.000000	-191.40

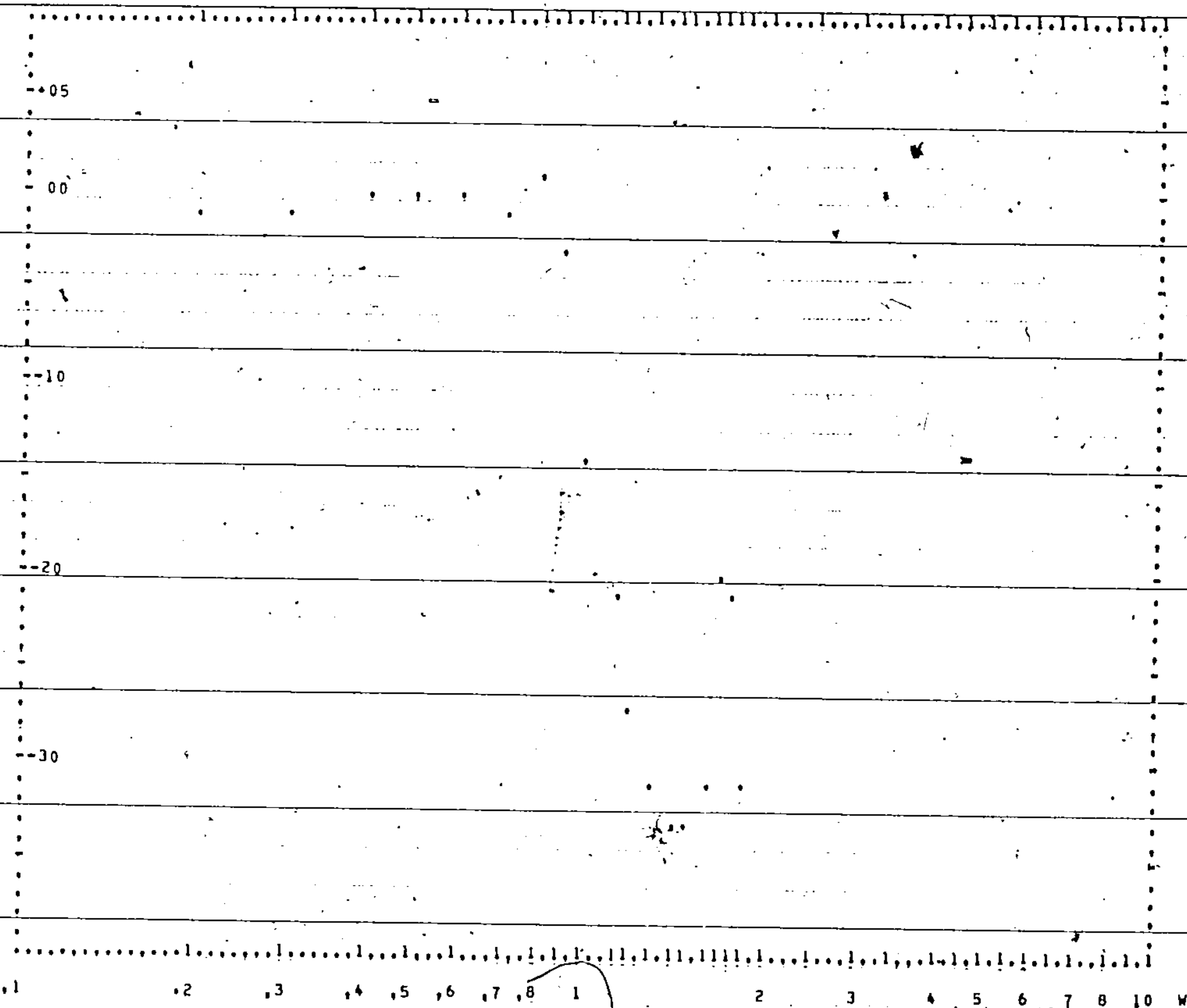

FIGURE 5.2.2.1 RESPONSE OF A T.L. + A L.P. BUTTERWORTH FILTER OF ORDER 4 (REFER TO FIG.5.1)

[POOR PRINT]

T= .40

W	TW	TWDB
0.0	1.000000	0.00
.1	.947194	-.47
.2	.885757	-1.05
.3	.912417	-.80
.4	1.012990	.11
.5	1.001399	.01
.6	.867829	-1.23
.7	.912834	-.79
.8	1.277757	2.13
.9	.376546	-8.48
1.0	.145884	-16.72
1.1	.078470	-22.11
1.2	.053441	-25.44
1.3	.046498	-26.65
1.4	.058472	-24.66
1.5	.230391	-12.75
1.6	.136160	-17.32
1.7	.053880	-25.37
1.8	.005936	-44.53
1.9	.001623	-55.79
2.0	.000598	-64.46
2.4	.000038	-88.44
3.0	.000003	-110.09
3.4	.000001	-118.72
4.0	.000001	-121.73
4.4	.000002	-115.44
5.0	.000000	-131.52
5.4	.000010	-100.33
6.0	.000000	-161.09
6.4	.000000	-177.87
7.0	.000000	-195.63
8.0	.000000	-200.00
9.0	.000000	-200.00
10.0	.000000	-200.00

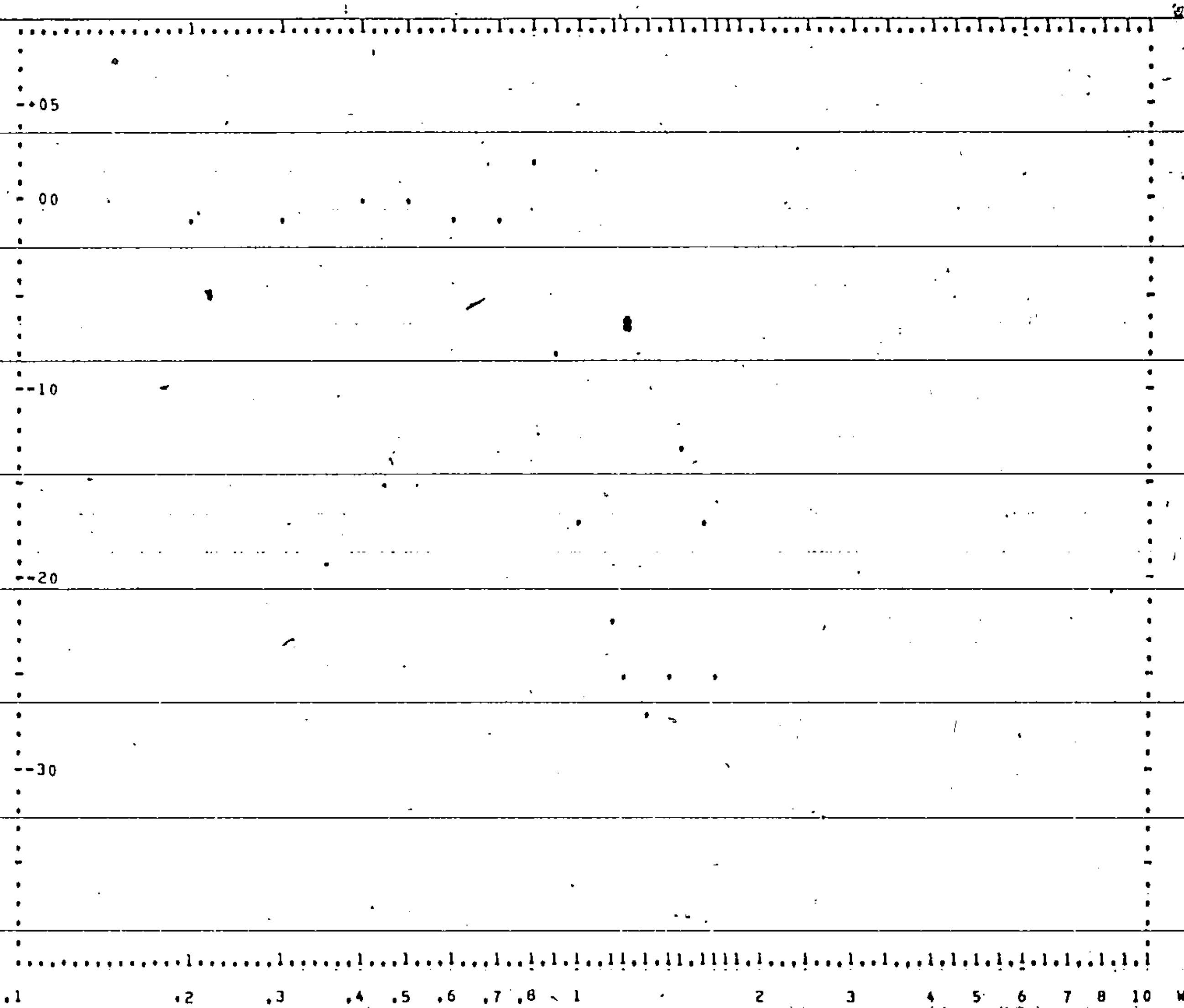

FIGURE 5.2.2.2 RESPONSE OF A T.L. + A L.P. BUTTERWORTH FILTER OF ORDER 4 (REFER TO FIG.5.1)

[POOR PRINT]

T= .20

W	TW	TWDB
0.0	1.000000	0.00
.1	.943915	-.50
.2	.928024	-.65
.3	1.020596	.18
.4	.991854	-.07
.5	.936197	-.57
.6	1.261056	2.01
.7	.897272	-.94
.8	.416220	-7.61
.9	.348382	-9.16
1.0	.947716	-.47
1.1	.449770	-6.94
1.2	.051154	-25.82
1.3	.006263	-44.06
1.4	.001491	-56.53
1.5	.000481	-66.35
1.6	.000190	-74.41
1.7	.000089	-81.03
1.8	.000048	-86.33
1.9	.000031	-90.22
2.0	.000024	-92.37
2.4	.000009	-101.34
3.0	.000000	-162.90
3.4	.000000	-186.63
4.0	.000000	-200.00
4.4	.000000	-200.00
5.0	.000000	-200.00
5.4	.000000	-200.00
6.0	.000000	-200.00
6.4	.000000	-200.00
7.0	.000000	-200.00
8.0	.000000	-200.00
9.0	.000000	-200.00
10.0	.000000	-200.00

TW IN DB

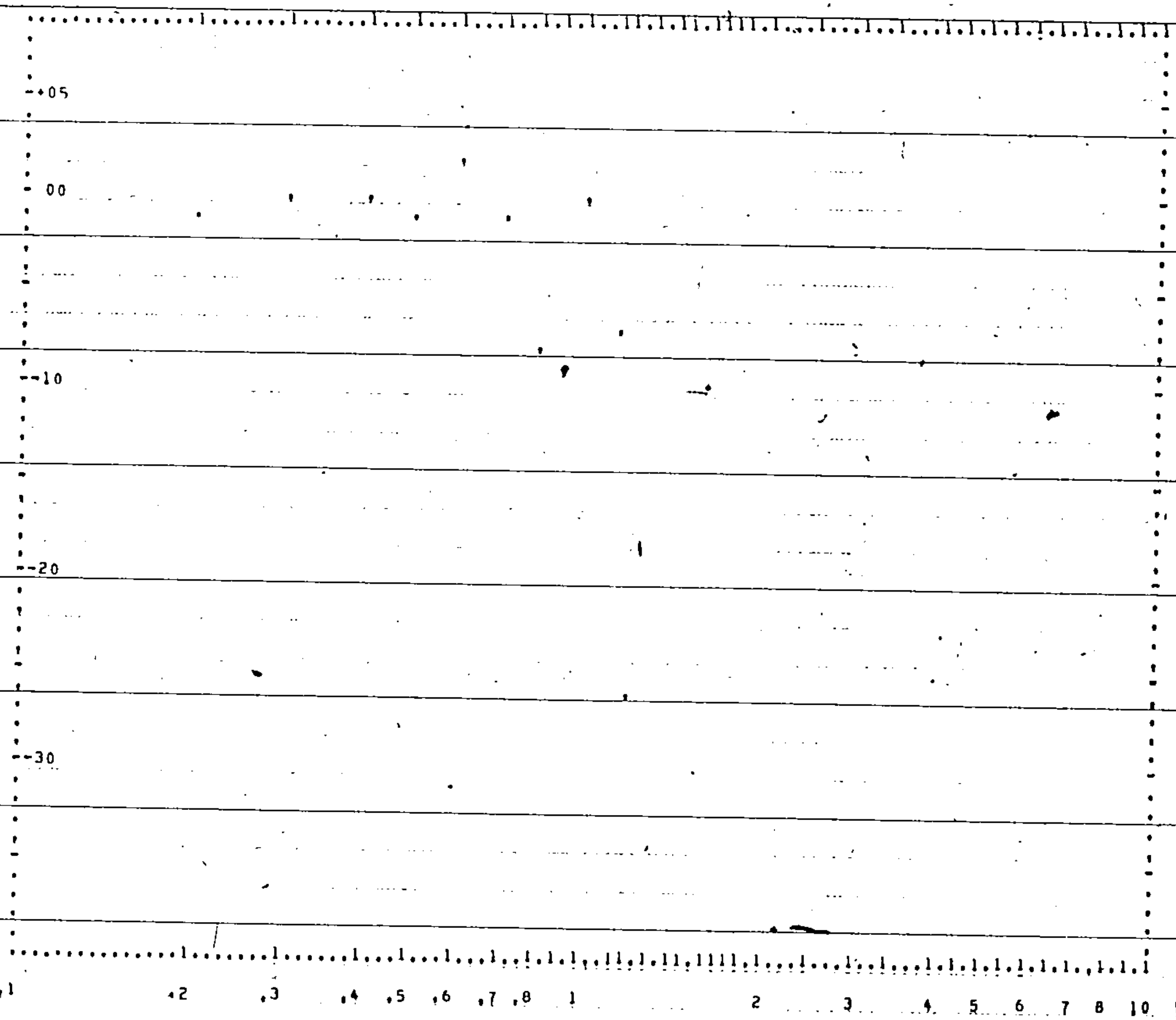

FIGURE 5.2.3.1 RESPONSE OF A T.L. + A L.P. BUTTERWORTH FILTER OF ORDER 6 (REFER TO FIG.5.1)

CONCORDIA UNIVERSITY COMPUTER CENTER

T= .40

W	TW	TWDB
0.0	1.000000	0.00
.1	.941525	-.52
.2	.937934	-.56
.3	1.025016	.21
.4	.951491	-.43
.5	.964888	-.31
.6	1.265469	2.05
.7	.619286	-4.16
.8	.373449	-8.56
.9	.451994	-6.90
1.0	.730126	-2.73
1.1	1.116518	.96
1.2	.025849	-31.75
1.3	.004820	-46.34
1.4	.001458	-56.72
1.5	.000589	-64.59
1.6	.000303	-70.38
1.7	.000201	-73.95
1.8	.000190	-74.41
1.9	.000396	-68.04
2.0	.006522	-43.71
2.4	.000001	-125.89
3.0	.000000	-166.38
3.4	.000000	-183.71
4.0	.000000	-197.68
4.4	.000000	-199.12
5.0	.000000	-200.00
5.4	.000000	-200.00
6.0	.000000	-200.00
6.4	.000000	-200.00
7.0	.000000	-200.00
8.0	.000000	-200.00
9.0	.000000	-200.00
10.0	.000000	-200.00

TW IN DB

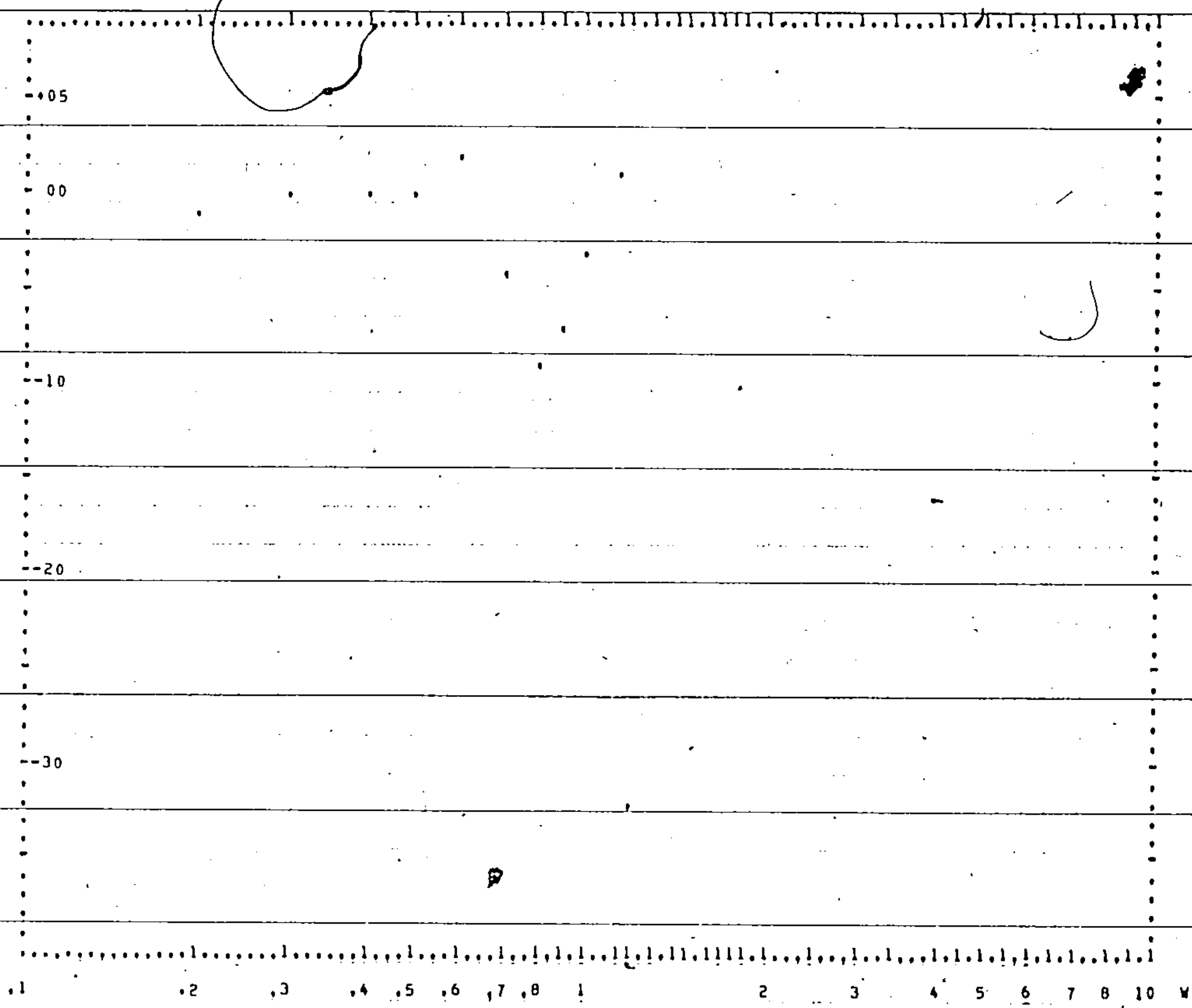

FIGURE 5.2.3.2 RESPONSE OF A T.L. + A L.P. BUTTERWORTH FILTER OF ORDER 6 (REFER TO FIG.5.1)

POOR PRINT

T= .40

W	TW	TWDB
0.0	1.000000	0.00
.1	.940294	-.53
.2	.998894	-.01
.3	.956781	-.38
.4	1.007959	.07
.5	1.017034	.15
.6	.790315	-2.04
.7	.797164	-1.97
.8	1.056005	.47
.9	1.293815	2.24
1.0	.104956	-19.58
1.1	.039517	-28.06
1.2	.105980	-19.50
1.3	.014840	-36.57
1.4	.000437	-67.19
1.5	.000051	-85.88
1.6	.000010	-100.31
1.7	.000002	-112.09
1.8	.000001	-121.81
1.9	.000000	-129.71
2.0	.000000	-135.72
2.4	.000000	-138.93
3.0	.000000	-200.00
3.4	.000000	-200.00
4.0	.000000	-200.00
4.4	.000000	-200.00
5.0	.000000	-200.00
5.4	.000000	-200.00
6.0	.000000	-200.00
6.4	.000000	-200.00
7.0	.000000	-200.00
8.0	.000000	-200.00
9.0	.000000	-200.00
10.0	.000000	-200.00

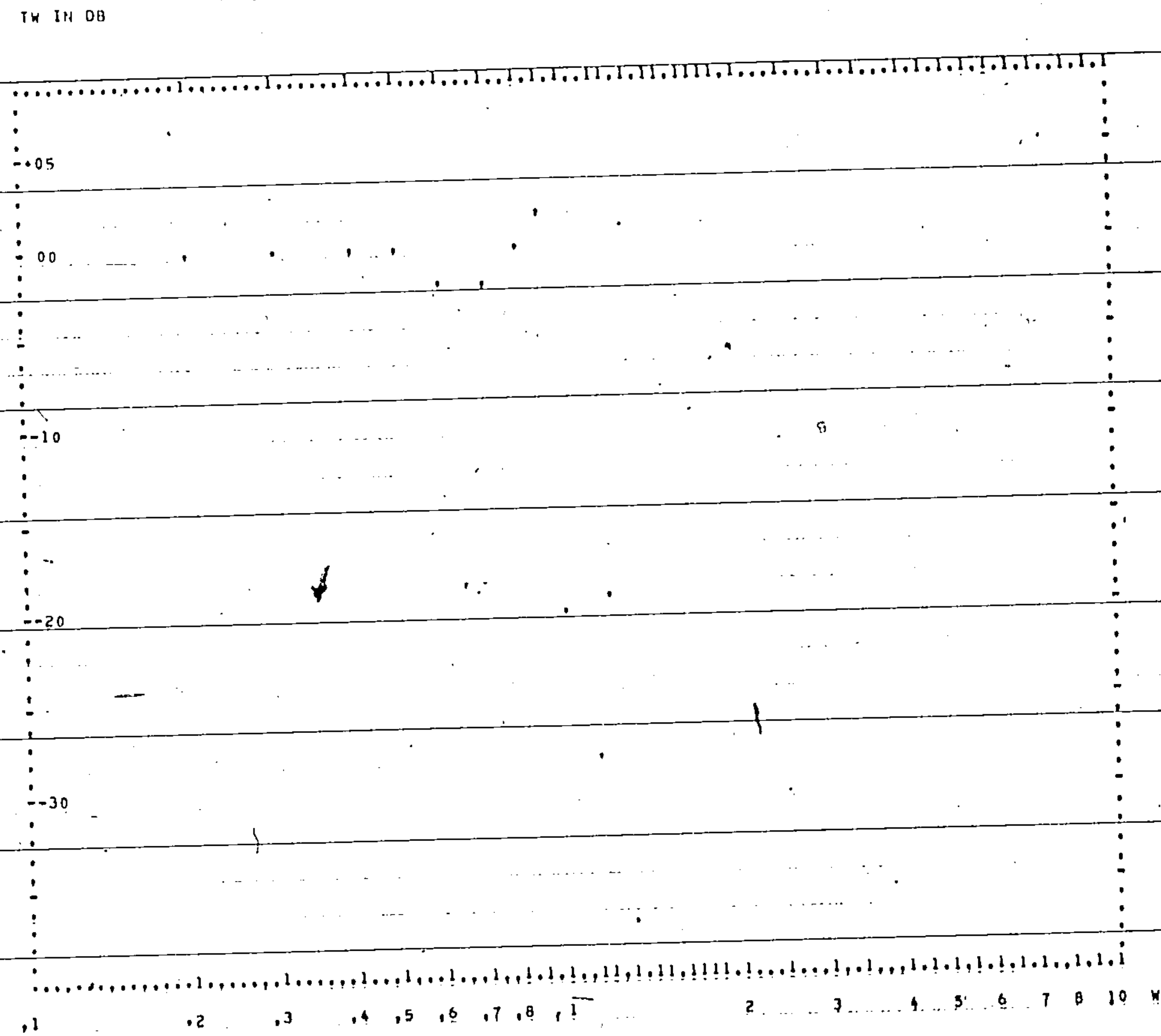

FIGURE 5.2.4.1 RESPONSE OF A T.L. + A L.P. BUTTERWORTH FILTER OF ORDER 8 (REFER TO FIG.5.1)

CONCORDIA UNIVERSITY COMPUTER CENTER

[POOR PRINT]

T= .60

W	TW	TWDB
0.0	1.000000	0.00
.1	.940467	-.53
.2	1.005490	.05
.3	.938152	-.55
.4	1.044268	.38
.5	.895614	-.96
.6	.747031	-2.53
.7	.876452	-1.15
.8	.849617	-1.42
.9	.906209	-.86
1.0	.083796	-21.54
1.1	.051814	-25.71
1.2	.250558	-12.02
1.3	.004762	-46.44
1.4	.000317	-69.98
1.5	.000050	-85.97
1.6	.000012	-98.16
1.7	.000004	-107.47
1.8	.000002	-114.02
1.9	.000001	-117.05
2.0	.000002	-113.00
2.4	.000000	-166.09
3.0	.000000	-180.65
3.4	.000000	-200.00
4.0	.000000	-200.00
4.4	.000000	-200.00
5.0	.000000	-200.00
5.4	.000000	-200.00
6.0	.000000	-200.00
6.4	.000000	-200.00
7.0	.000000	-200.00
8.0	.000000	-200.00
9.0	.000000	-200.00
10.0	.000000	-200.00

TW IN DB

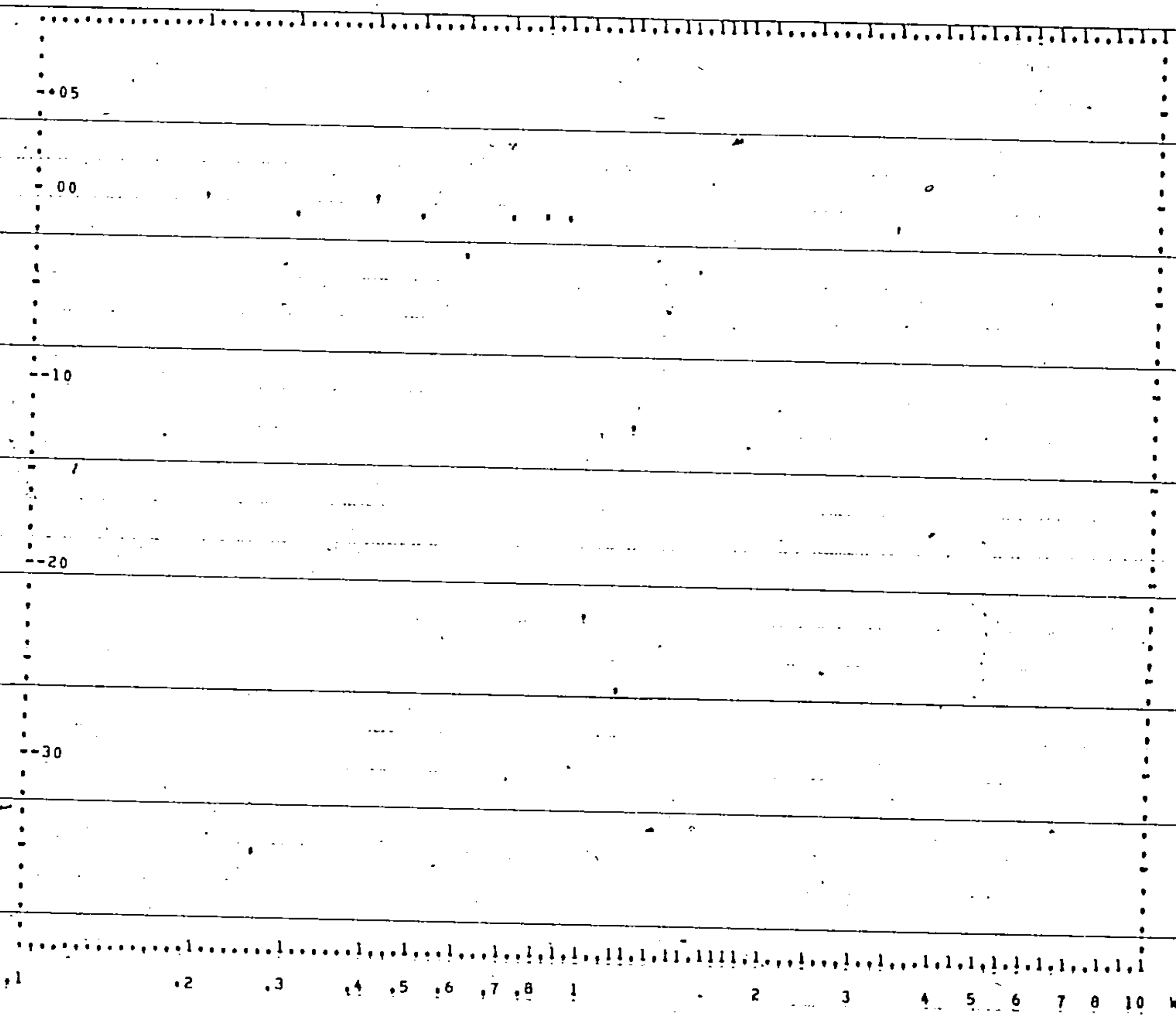

FIGURE 5.2.4.2 RESPONSE OF A T.L. + A L.P. BUTTERWORTH FILTER OF ORDER 8 (REFER TO FIG.5.1)

POOR PRINT

T= .40

W	TW	TWDB
0.0	1.000000	0.00
.1	.955512	-.40
.2	1.016093	.14
.3	.973617	-.23
.4	1.189541	1.51
.5	.747331	-2.53
.6	.858278	-1.33
.7	.761542	-2.37
.8	.627678	-4.05
.9	.223134	-13.03
1.0	.811577	-1.81
1.1	.021880	-33.20
1.2	.001219	-58.28
1.3	.000275	-71.22
1.4	.000245	-72.22
1.5	.000154	-76.25
1.6	.000002	-113.10
1.7	.000000	-134.73
1.8	.000000	-151.77
1.9	.000000	-166.12
2.0	.000000	-178.49
2.4	.000000	-200.00
3.0	.000000	-200.00
3.4	.000000	-200.00
4.0	.000000	-200.00
4.4	.000000	-200.00
5.0	.000000	-200.00
5.4	.000000	-200.00
6.0	.000000	-200.00
6.4	.000000	-200.00
7.0	.000000	-200.00
8.0	.000000	-200.00
9.0	.000000	-200.00
10.0	.000000	-200.00

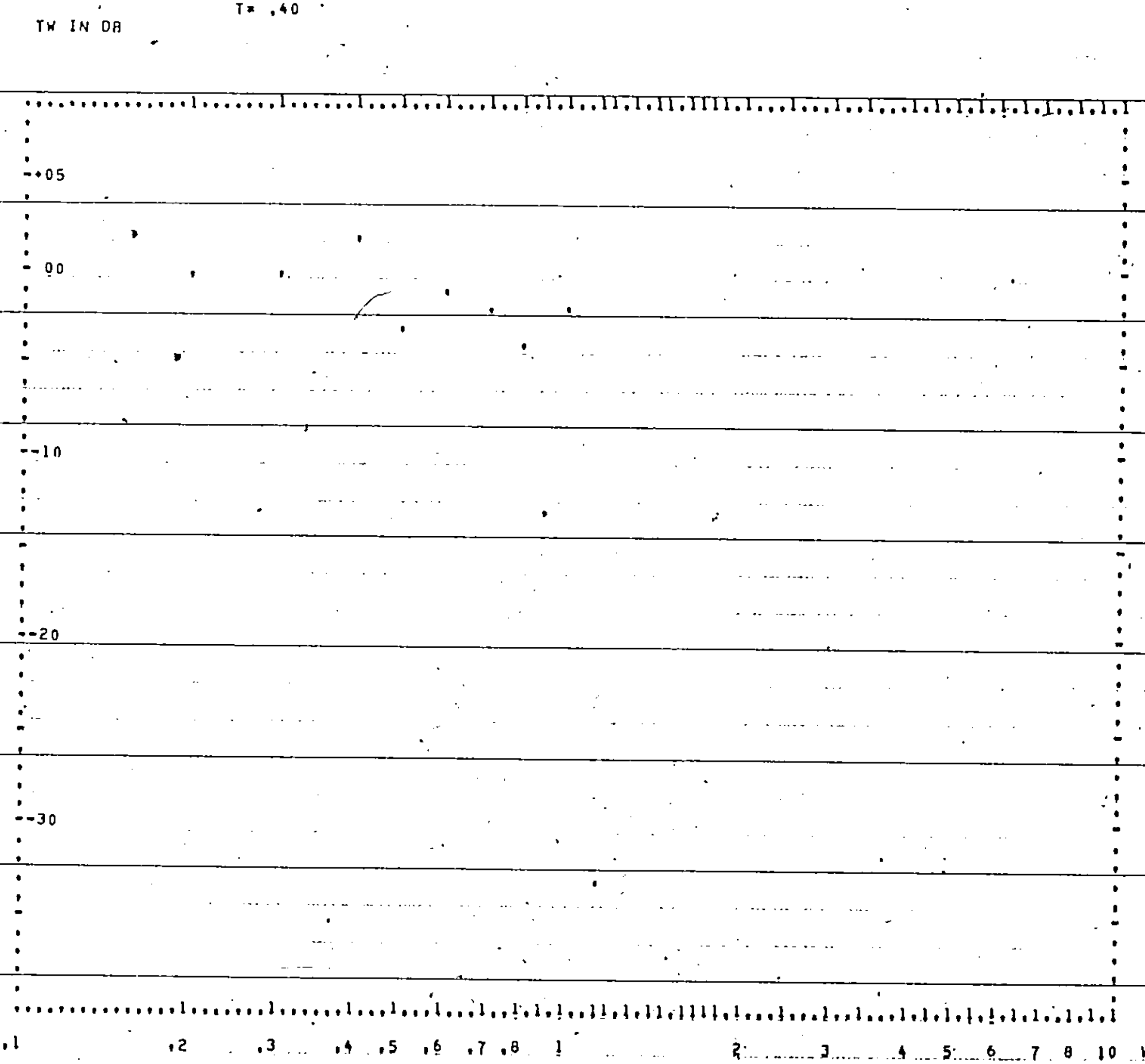

FIGURE 5.2.5.1 RESPONSE OF A T.L. + A L.P. BUTTERWORTH FILTER OF ORDER 10 (REFER TO FIG.5.1)

[POOR PRINT]

T= .60

W	TW	TWDB
0.0	1.000000	0.00
.1	.957152	-.38
.2	1.010228	.09
.3	.983301	-.15
.4	1.118486	.97
.5	.694225	-3.17
.6	.989247	-.09
.7	.760759	-2.38
.8	.367735	-8.69
.9	.235513	-12.56
1.0	.486135	-6.26
1.1	.014921	-36.52
1.2	.001344	-57.43
1.3	.000514	-65.78
1.4	.044227	-27.09
1.5	.000022	-93.24
1.6	.000001	-118.06
1.7	.000000	-135.99
1.8	.000000	-150.41
1.9	.000000	-162.26
2.0	.000000	-171.78
2.4	.000000	-162.02
3.0	.000000	-200.00
3.4	.000000	-200.00
4.0	.000000	-200.00
4.4	.000000	-200.00
5.0	.000000	-200.00
5.4	.000000	-200.00
6.0	.000000	-200.00
6.4	.000000	-200.00
7.0	.000000	-200.00
8.0	.000000	-200.00
9.0	.000000	-200.00
10.0	.000000	-200.00

TW IN DB

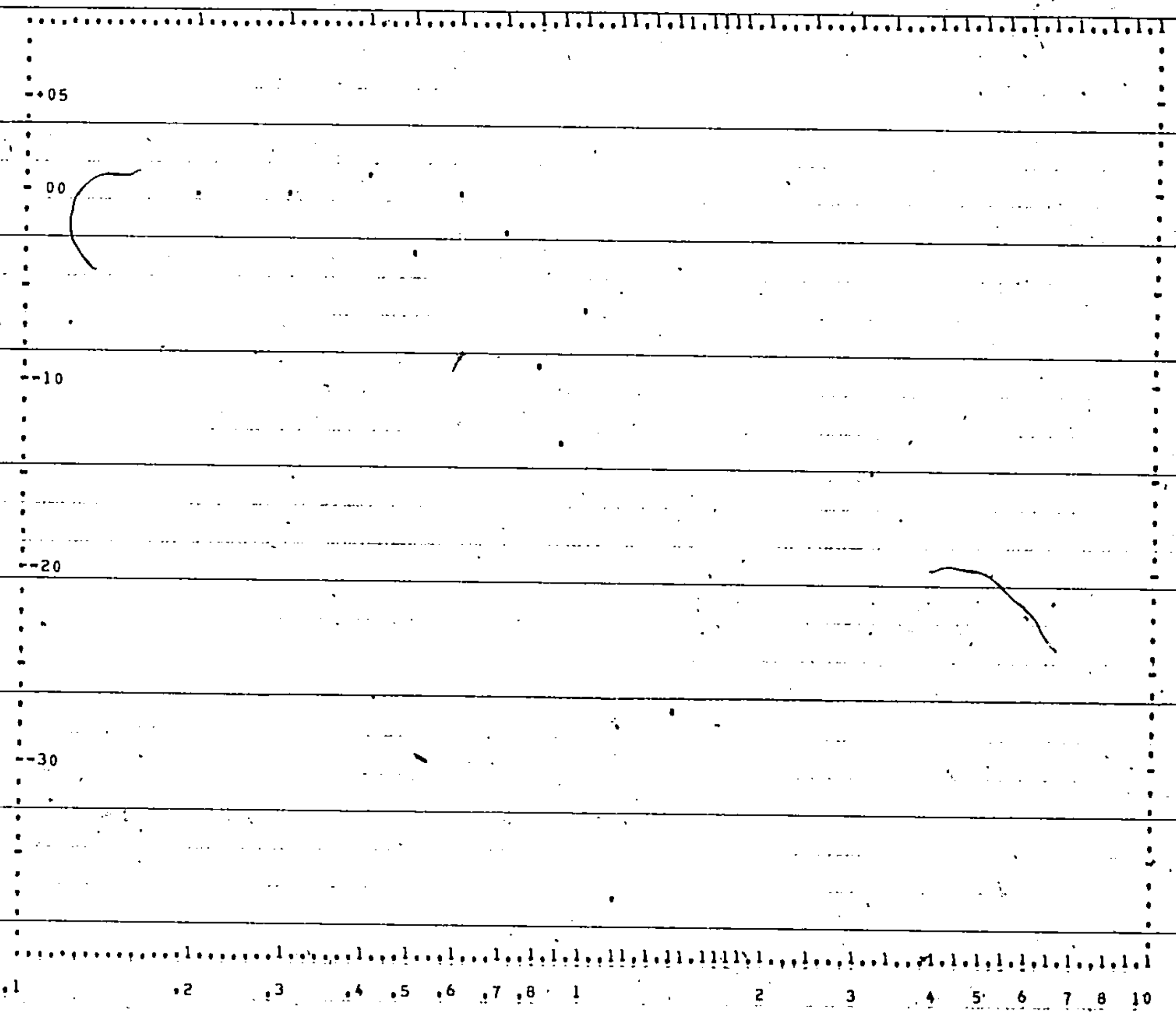

FIGURE 5.2.5.2 RESPONSE OF A T.L. + A L.P. BUTTERWORTH FILTER OF ORDER 10 (REFER TO FIG.5.1)

POOR PRINT

T= .20

W	TW	TWDB
0.0	0.000000	-200.00
.1	.000000	-200.00
.2	.000000	-163.23
.3	.000002	-113.43
.4	.000790	-62.05
.5	.003963	-48.04
.6	.004939	-46.13
.7	.012747	-37.89
.8	.039774	-28.01
.9	.151578	-16.39
1.0	1.179702	1.44
1.1	1.107094	.88
1.2	.944450	-.50
1.3	1.099672	.83
1.4	1.245616	1.91
1.5	1.217077	1.71
1.6	1.100560	.83
1.7	.998639	-.01
1.8	.931959	-.61
1.9	.894979	-.96
2.0	.880194	-1.11
2.4	.958106	-.37
3.0	1.255811	1.98
3.4	1.344042	2.57
4.0	1.197888	1.57
4.4	1.083601	.70
5.0	.984203	-.14
5.4	.958979	-.36
6.0	.960968	-.35
6.4	.977070	-.20
7.0	1.003846	.03
8.0	1.011549	.10
9.0	.983439	-.15
10.0	.973766	-.23

TW IN DB

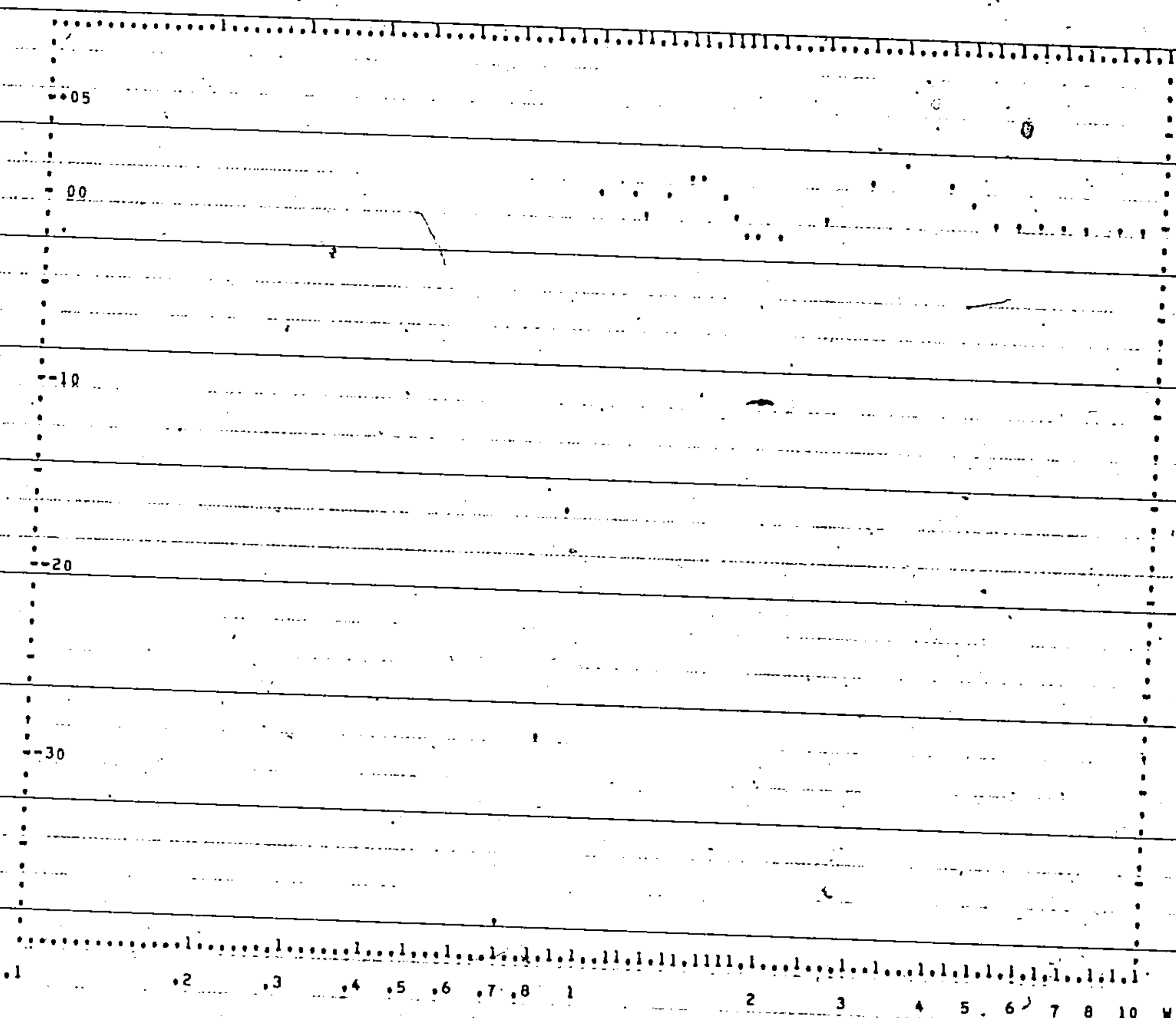

FIGURE 5.3.2.1 RESPONSE OF A T.L. + A H.P. BUTTERWORTH FILTER OF ORDER 4 (REFER TO FIG.5.1)

POOR PRINT

T= .20

W	TW	TWDB
0.0	0.000000	-200.00
.1	.000000	-200.00
.2	.000000	-200.00
.3	.000000	-154.37
.4	.000001	-125.33
.5	.000006	-104.43
.6	.000091	-80.80
.7	.001615	-55.83
.8	.076306	-22.35
.9	1.228392	1.79
1.0	.316960	-9.98
1.1	.398682	-7.99
1.2	.871890	-1.19
1.3	2.241971	7.01
1.4	1.357820	2.66
1.5	1.046479	.39
1.6	.994657	-.05
1.7	1.035236	.30
1.8	1.105682	.87
1.9	1.158510	1.28
2.0	1.164853	1.33
2.4	.988020	-.10
3.0	.991464	-.07
3.4	1.131694	1.07
4.0	1.275676	2.11
4.4	1.214690	1.69
5.0	1.061040	.51
5.4	.992463	-.07
6.0	.949418	-.45
6.4	.949841	-.45
7.0	.973721	-.23
8.0	1.009412	.08
9.0	.993773	-.05
10.0	.972875	-.24

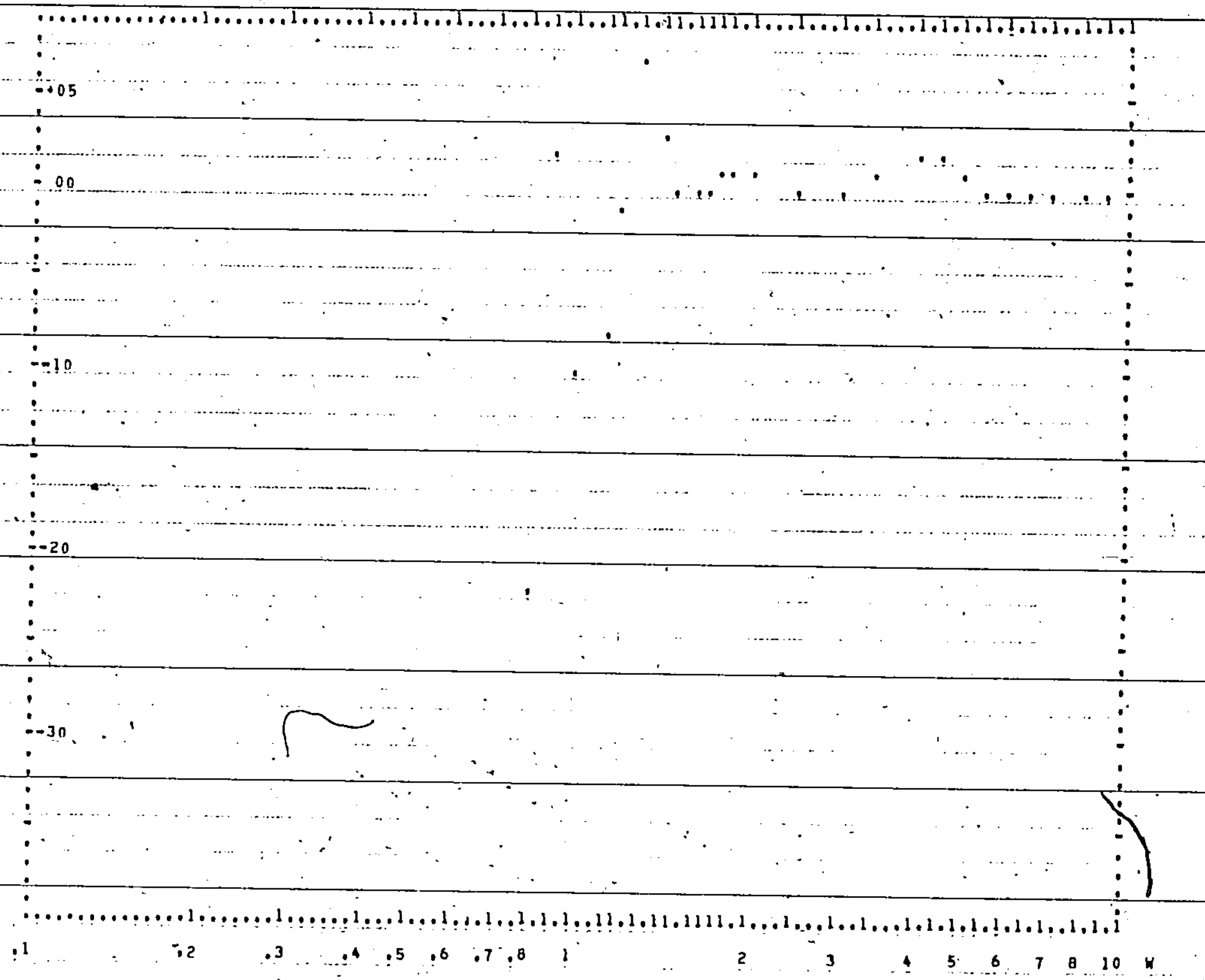

FIGURE 5.3.3.1 RESPONSE OF A T.L. + A H.P. BUTTERWORTH FILTER OF ORDER 6 (REFER TO FIG.5.1)

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[POOR PRINT]

T= .40

W	TW	TWDB
0.0	0.000000	-200.00
.1	.000000	-200.00
.2	.000000	-200.00
.3	.000000	-144.63
.4	.000000	-128.39
.5	.000005	-105.28
.6	.000097	-80.26
.7	.002168	-53.28
.8	.365324	-8.75
.9	.601910	-4.41
1.0	.334121	-9.52
1.1	.834497	-1.57
1.2	2.586842	8.26
1.3	1.083599	.70
1.4	.981734	-.16
1.5	1.100866	.83
1.6	1.241671	1.88
1.7	1.234448	1.83
1.8	1.126337	1.03
1.9	1.035336	.30
2.0	.994277	-.05
2.4	1.236561	1.84
3.0	1.108443	.89
3.4	.883150	-1.08
4.0	.897023	-.94
4.4	.980765	-.17
5.0	.982614	-.15
5.4	.947937	-.46
6.0	.965596	-.30
6.4	.996274	-.03
7.0	.974518	-.22
8.0	.939395	-.54
9.0	1.096056	.80
10.0	1.021548	.19

TW IN DB

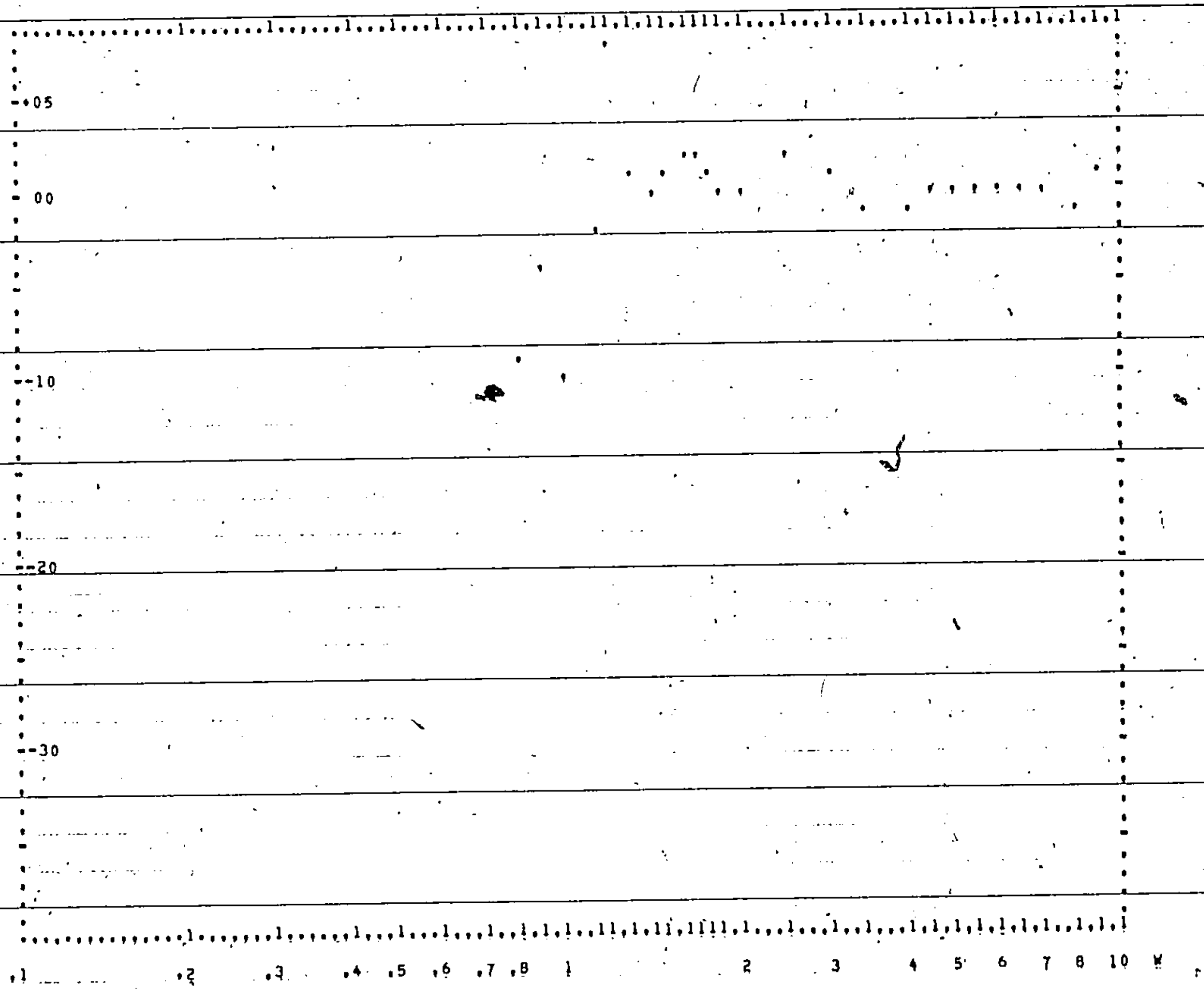

FIGURE 5.3.3.2 RESPONSE OF A T.L. + A H.P. BUTTERWORTH FILTER OF ORDER 6 (REFER TO FIG.5.1)

CONCORDIA UNIVERSITY COMPUTER CENTER

POOR PRINT

T= .60

W	TW	-TWDB
0.0	0.000000	-200.00
.1	.000000	-200.00
.2	.000000	-200.00
.3	.000000	-200.00
.4	.000000	-189.08
.5	.000000	-145.12
.6	.000010	-99.96
.7	.096702	-20.29
.8	.012743	-37.89
.9	.192567	-14.31
1.0	1.201020	1.59
1.1	1.003603	.03
1.2	2.119267	6.52
1.3	1.205675	1.62
1.4	.851082	-1.40
1.5	.873457	-1.18
1.6	.967832	-.28
1.7	1.063479	.53
1.8	1.111986	.92
1.9	1.078099	.65
2.0	1.018066	.16
2.4	1.264181	2.04
3.0	.770644	-2.26
3.4	.851795	-1.39
4.0	.969130	-.27
4.4	.929930	-.63
5.0	1.007889	.07
5.4	.939112	-.55
6.0	1.014902	.13
6.4	1.166988	1.34
7.0	1.018448	.16
8.0	1.001121	.01
9.0	.966604	-.30
10.0	.967774	-.28

TW IN DB

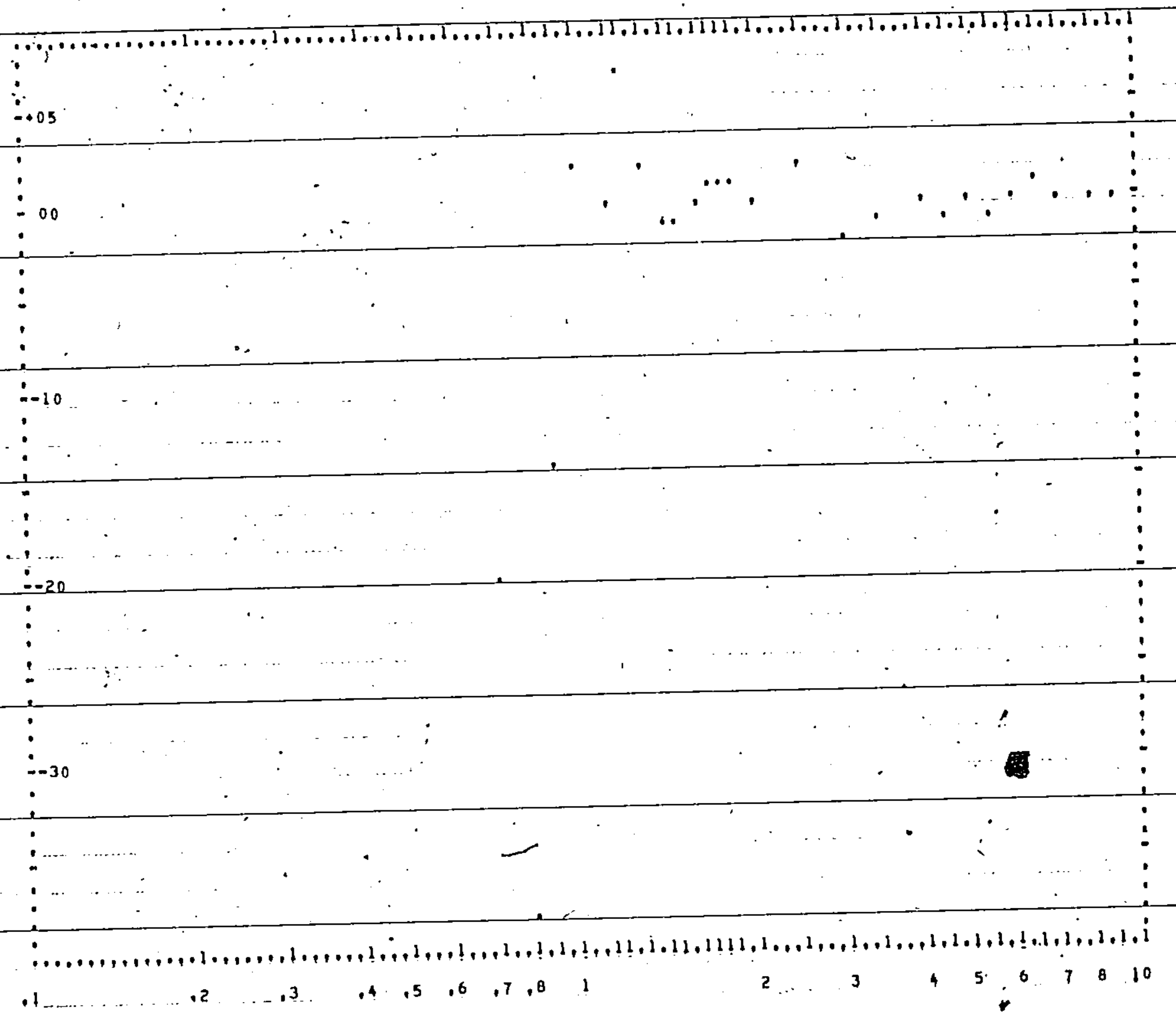

FIGURE 5.3.4.2 RESPONSE OF A T.L. + A H.P. BUTTERWORTH FILTER OF ORDER 8 (REFER TO FIG.5.1)

[POOR PRINT]

T= .40

W	TW	TWDB
0.0	0.000000	-200.00
.1	.000000	-200.00
.2	.000000	-200.00
.3	.000000	-200.00
.4	.000000	-200.00
.5	.000000	-174.40
.6	.000091	-80.78
.7	.000027	-91.40
.8	.000819	-61.73
.9	.705009	-3.04
1.0	.321291	-9.86
1.1	3.477913	10.83
1.2	1.010997	.09
1.3	1.525140	3.67
1.4	.886062	-1.05
1.5	1.066810	.56
1.6	2.332812	7.36
1.7	1.544861	3.78
1.8	.995895	-.04
1.9	.874833	-1.16
2.0	.892547	-.99
2.4	1.093113	.77
3.0	1.050440	.43
3.4	1.178708	1.43
4.0	.909982	-.82
4.4	.856706	-1.34
5.0	.959586	-.36
5.4	1.000504	.00
6.0	.959249	-.36
6.4	.959726	-.36
7.0	1.000985	.01
8.0	.953289	-.42
9.0	1.039588	.34
10.0	1.064562	.54

TW IN DB

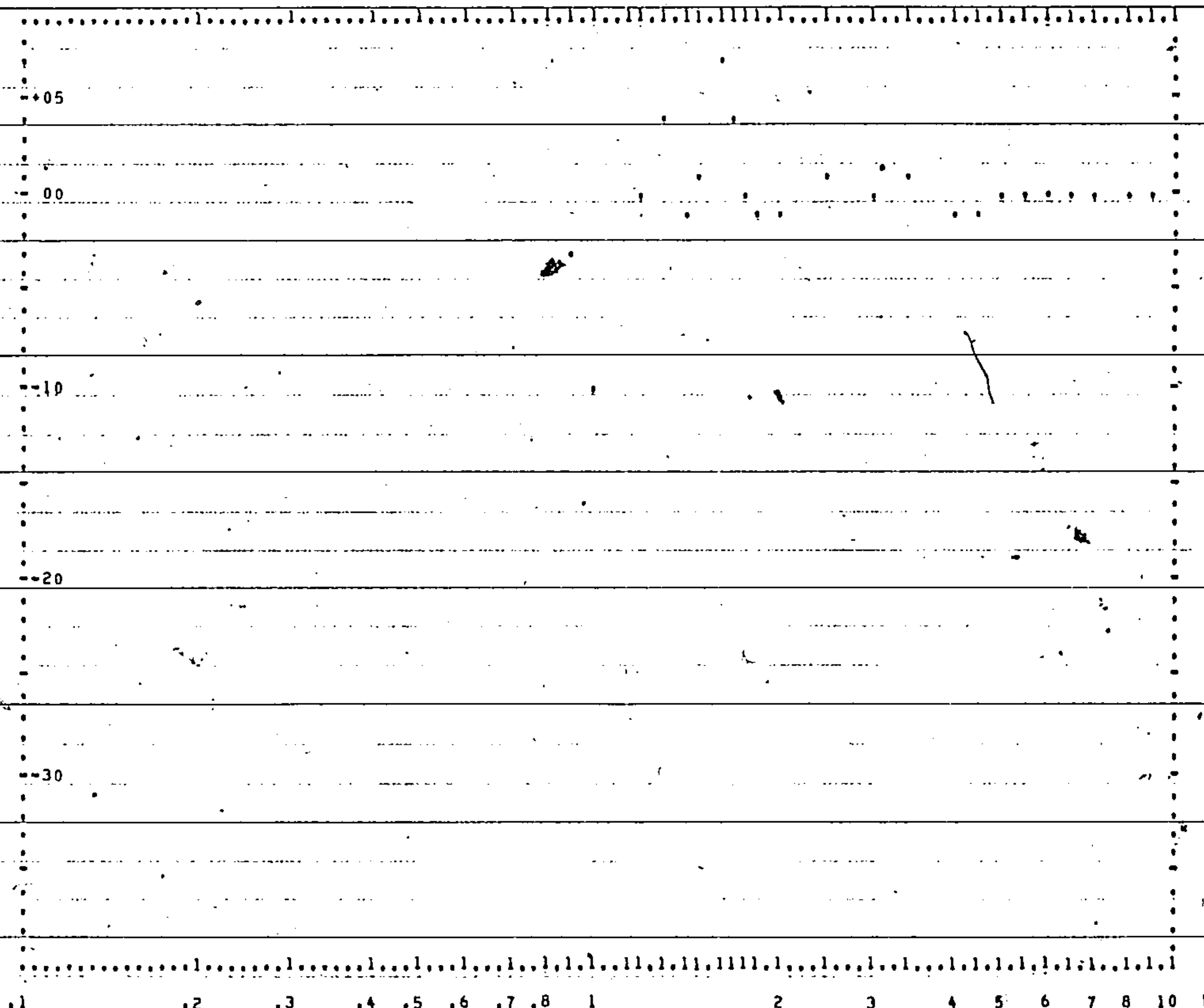

FIGURE 5.3.5.1 RESPONSE OF A T.L. + A H.P. BUTTERWORTH FILTER OF ORDER 10 (REFER TO FIG.5.1)

POOR PRINT

T= .60

W	TW	TWDB
0.0	0.000000	-200.00
.1	.000000	-200.00
.2	.000000	-200.00
.3	.000000	-200.00
.4	.000000	-200.00
.5	.000000	-172.22
.6	.000010	-100.14
.7	.000027	-91.28
.8	.001252	-58.05
.9	.793856	-2.01
1.0	.676105	-3.40
1.1	.986987	-.11
1.2	1.566100	3.90
1.3	1.013043	.11
1.4	1.431415	3.12
1.5	1.811853	5.16
1.6	.789774	-2.05
1.7	.664445	-3.55
1.8	.740581	-2.61
1.9	.922836	-.70
2.0	1.062682	.53
2.4	.967685	-.29
3.0	.912841	-.79
3.4	.796411	-1.98
4.0	1.013128	.11
4.4	.961346	-.34
5.0	1.006334	.05
5.4	.997298	-.02
6.0	.977317	-.20
6.4	1.100904	.83
7.0	1.062374	.53
8.0	.980729	-.17
9.0	.967310	-.29
10.0	.987254	-.11

TW IN DB

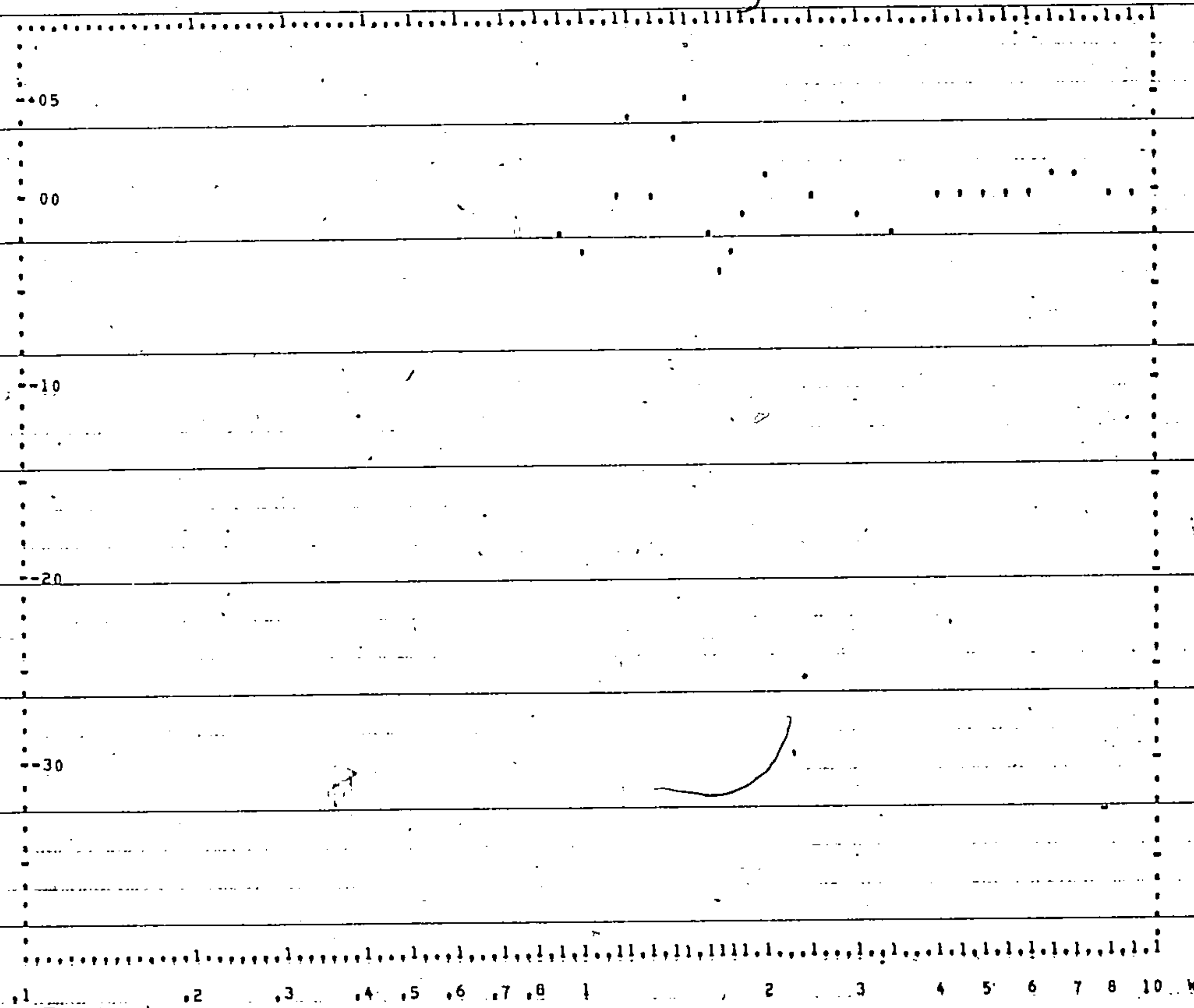

FIGURE 5.3.5.2 RESPONSE OF A T.L. + A H.P. BUTTERWORTH FILTER OF ORDER 10 (REFER TO FIG.5.1)

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W	TW.	TWDB	TW IN DB
0.0	0.000000	-200.00	
.1	.000000	-194.33	
.2	.000004	-108.52	
.3	.006231	-44.11	
.4	.015018	-36.47	
.5	.567635	-4.92	
.6	1.021350	.18	
.7	.964226	-.32	
.8	.868982	-1.22	
.9	.998491	-.01	
1.0	1.000000	.00	
1.1	.918781	-.74	
1.2	.930955	-.62	
1.3	1.006671	.06	
1.4	.988182	-.10	
1.5	.876426	-1.15	
1.6	.862079	-1.29	
1.7	1.134264	1.09	
1.8	.865296	-1.26	
1.9	.304399	-10.33	
2.0	.145884	-16.72	
2.4	.041109	-27.72	
3.0	.021455	-33.37	
3.4	.000448	-66.97	
4.0	.000024	-92.5[illegible]	
4.4	.000006	-104.[illegible]	
5.0	.000001	-118.[illegible]	
5.4	.000000	-126.23	
6.0	.000000	-135.10	
6.4	.000000	-139.60	
7.0	.000000	-144.16	
8.0	.000000	-142.71	
9.0	.000000	-147.20	
10.0	.000000	-140.21	

T= .20

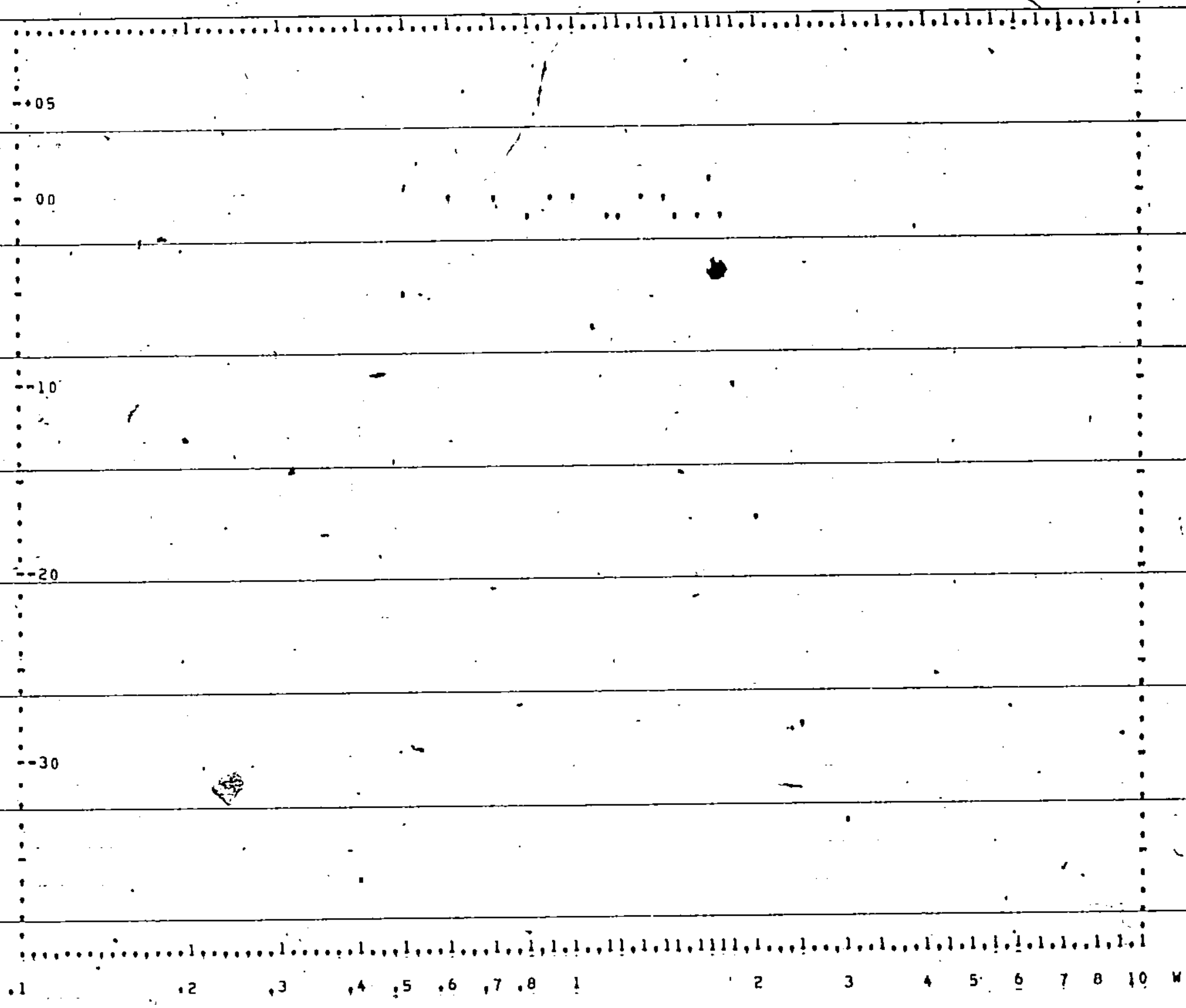

FIGURE 5.4.2.1 RESPONSE OF A T.L. + A B.P. BUTTERWORTH FILTER OF ORDER 4 (REFER TO FIG.5.1)

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T= .60

W	TW	TWDB
0.0	1.000000	0.00
.1	.949343	-.45
.2	.944077	-.50
.3	1.057217	.48
.4	.805841	-1.88
.5	.126356	-17.97
.6	.000571	-64.87
.7	.000000	-148.50
.8	.000000	-200.00
.9	.000000	-200.00
1.0	0.000000	-200.00
1.1	.000000	-200.00
1.2	.000000	-200.00
1.3	.000000	-200.00
1.4	.000000	-156.27
1.5	.000008	-101.83
1.6	.009653	-40.31
1.7	.019351	-34.27
1.8	.031288	-30.09
1.9	.464527	-6.66
2.0	.884978	-1.06
2.4	.641808	-3.85
3.0	1.025565	.22
3.4	.872059	-1.19
4.0	.785759	-2.09
4.4	1.038554	.33
5.0	.938075	-.56
5.4	1.042139	.36
6.0	.937473	-.56
6.4	1.053642	.45
7.0	1.200113	1.58
8.0	.923702	-.69
9.0	.950543	-.44
10.0	.996310	-.03

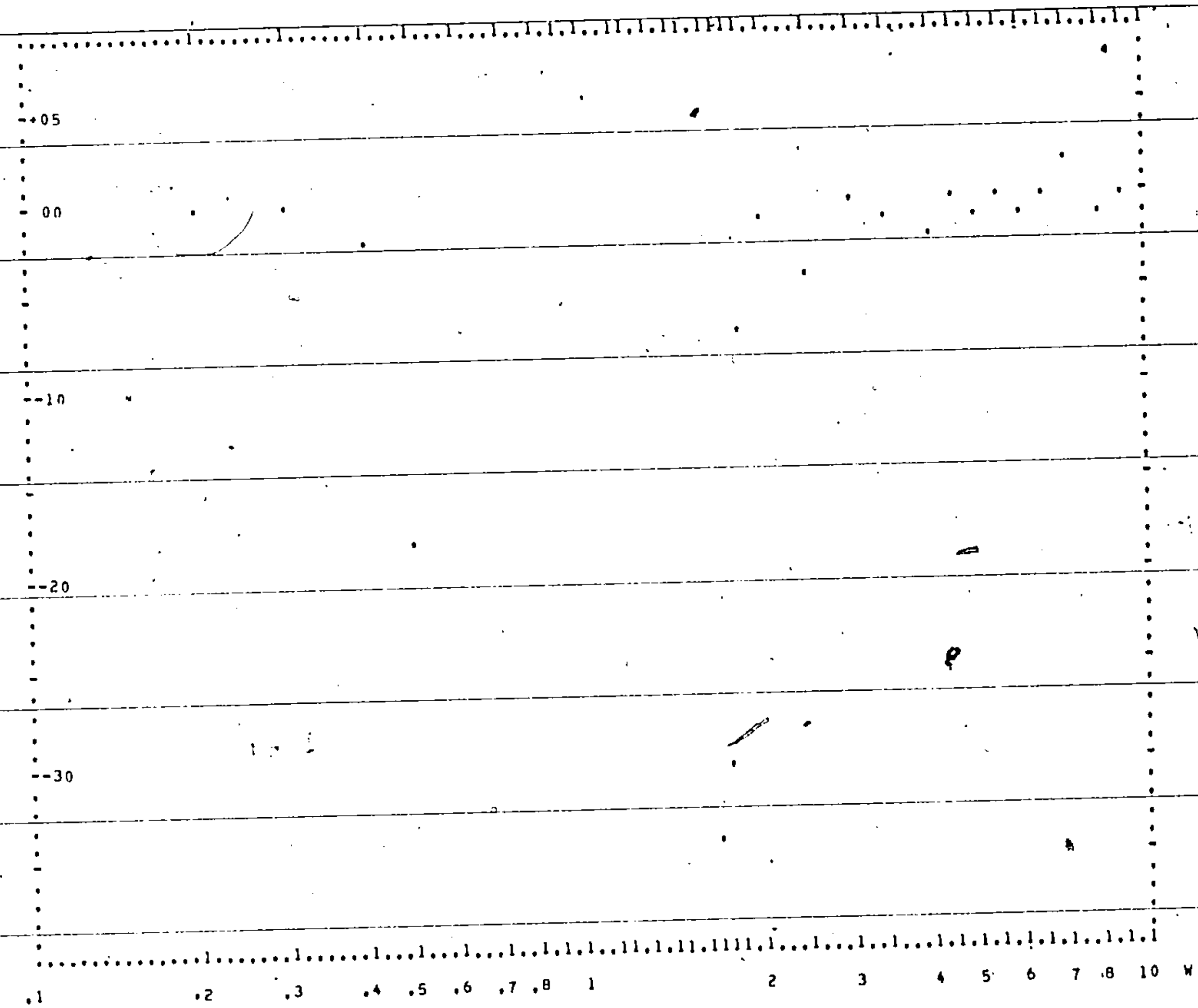

FIGURE 5.5.4.2 RESPONSE OF A T.L. + A B.S. BUTTERWORTH FILTER OF ORDER 8 (REFER TO FIG.5.1)

[POOR PRINT]

CHAPTER VI

TRANSMISSION LINE IN SERIES WITH AN Nth ORDER SEGMENTED BUTTERWORTH FILTER

6.1 Introduction

In this chapter, we shall consider the circuit shown in Figure 6.1 in which a lossless transmission line consists of a variable element (ℓ) with a characteristic impedance $Z_0 = 1$, is series with a passive lumped element Butterworth filter of order N. The lumped element is split into $\frac{N}{2}$ sections and in between each section a transmission line of the same characteristics as above is also introduced. The entire network is terminated in a 1 Ω resistor R.

6.2 The Lumped Section is a L.P. Butterworth Filter

Similar to section 4.2, A special subprogram was written, and when called by the main computer program - the transfer function of the network shown in Figure 6.1 would be calculated. The program was written to calculate the various combinations similar to those mentioned in section 3.2.

6.2.1 L.P. Filter (N = 2)

Figures 6.2.1.1 and 6.2.1.2 show the frequency response of the circuit shown in Figure 6.1 for a L.P. filter of order 2 calculated at $\tau = 0.2$ and $\tau = 0.4$ respectively. There are 2 points noticed in the first curve:-

1. At $w = 1$ the frequency response is - 4.03 db which is an improvement compared to a filter response alone as shown in Figure 3.2.1.1.
2. There is a rise in the response peaking at $w = 8.0$ which is the effect of introducing the transmission lines in the network. As τ increases this peak is expected to move to the left.

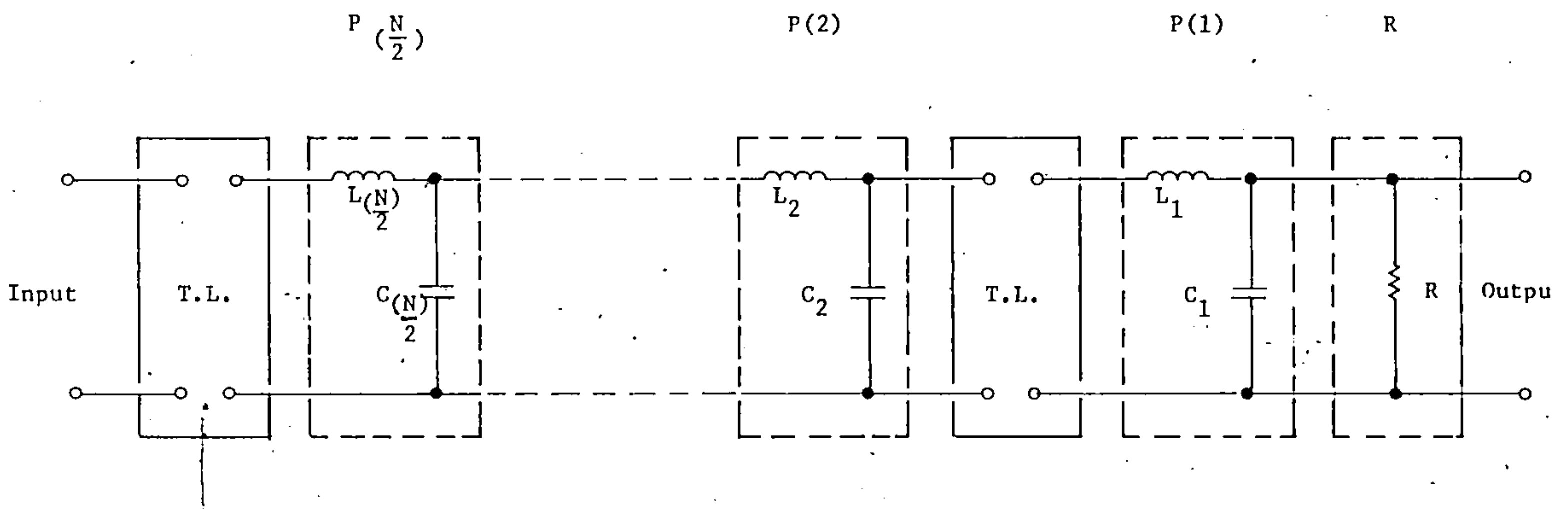

Figure 6.1 - Transmission Line and an Nth Order segmented Butterworth Filter terminated in a resistor R.

6.2.2 L.P. Filter (N = 4)

Figures 6.2.2.1 and 6.2.2.2 show the frequency response for a L.P. filter of order 4 calculated at $\tau = 0.2$ and $\tau = 0.4$ respectively.

As the filter order increases, the response becomes more and more selective, thus the peak signal that appears on the R.H.S. of the curve goes down in magnitude but still present at the same frequency for the same value of τ. For Figure 6.2.2.1, the 3 db point of the filter alone is now down to 5.48 which is an improvement.

6.2.3 L.P. Filter (N = 6)

Figures 6.2.3.1 and 6.2.3.2 show the frequency response for a L.P. filter of order 6 calculated at $\tau = 0.2$ and $\tau = 0.4$ respectively. As it was mentioned in section 6.2.2 the frequency response becomes more selective as the order of filter increases. In the first curve, the response at $w = 1$ now is - 6.07 dbs.

6.2.4 L.P. Filter (N = 8)

Figures 6.2.4.1 and 6.2.4.2 show the frequency response for a L.P. filter of order 8 calculated at $\tau = 0.2$ and 0.4 respectively.

6.2.5 L.P. Filter (N = 10)

Figures 6.2.5.1 and 6.2.5.2 show the frequency response for a L.P. filter of order 10 calculated at $\tau = 0.2$ and 0.4 respectively.

In both figures at $w = 1$ the corresponding attenuation is 10.11 db and 16.95 db respectively. A peak signal appears at $w = 9$ at -88.16 db for the first curve.

T= ,20

W	TW	TWDB
0,0	1,000000	0,00
,1	,998525	-,01
,2	,993376	-,06
,3	,982515	-,15
,4	,963048	-,33
,5	,932135	-,61
,6	,888245	-1,03
,7	,832181	-1,60
,8	,767162	-2,30
,9	,697785	-3,13
1,0	,628577	-4,03
1,1	,562961	-4,99
1,2	,502955	-5,97
1,3	,449386	-6,95
1,4	,402276	-7,91
1,5	,361198	-8,85
1,6	,325524	-9,75
1,7	,294575	-10,62
1,8	,267699	-11,45
1,9	,244308	-12,24
2,0	,223891	-13,00
2,4	,164145	-15,70
3,0	,113679	-18,89
3,4	,093819	-20,55
4,0	,075071	-22,49
4,4	,067248	-23,45
5,0	,060205	-24,41
5,4	,058028	-24,73
6,0	,058437	-24,67
6,4	,061776	-24,18
7,0	,075152	-22,48
8,0	,258186	-11,76
9,0	,082432	-21,68
10,0	,028471	-30,91

TW IN DB

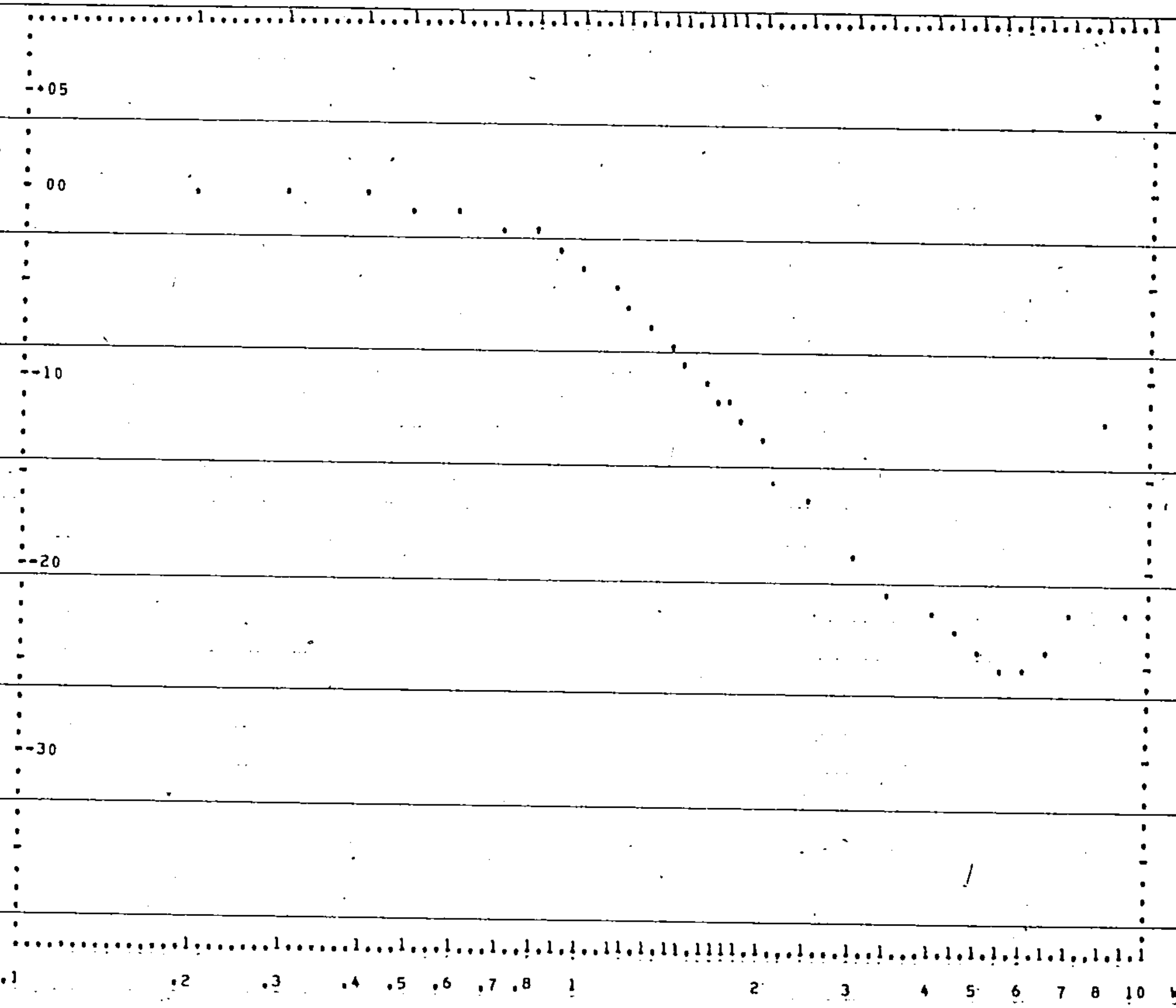

FIGURE 6.2.1.1 RESPONSE OF A T.L. + A L.P. BUTTERWORTH FILTER OF ORDER 2 (REFER TO FIG.6.1)

T= .40

W	TW	TWDB
0.0	1.000000	0.00
.1	.997105	-.03
.2	.987658	-.11
.3	.969647	-.27
.4	.940646	-.53
.5	.899095	-.92
.6	.845494	-1.46
.7	.782731	-2.13
.8	.715247	-2.91
.9	.647601	-3.77
1.0	.583348	-4.68
1.1	.524642	-5.60
1.2	.472405	-6.51
1.3	.426710	-7.40
1.4	.387154	-8.24
1.5	.353113	-9.04
1.6	.323909	-9.79
1.7	.298891	-10.49
1.8	.277476	-11.14
1.9	.259159	-11.73
2.0	.243514	-12.27
2.4	.201516	-13.91
3.0	.182960	-14.75
3.4	.201071	-13.93
4.0	.395138	-8.07
4.4	2.274731	7.14
5.0	.141417	-16.99
5.4	.077326	-22.23
6.0	.042351	-27.46
6.4	.031556	-30.02
7.0	.022479	-32.96
8.0	.015568	-36.16
9.0	.013246	-37.56
10.0	.014130	-37.00

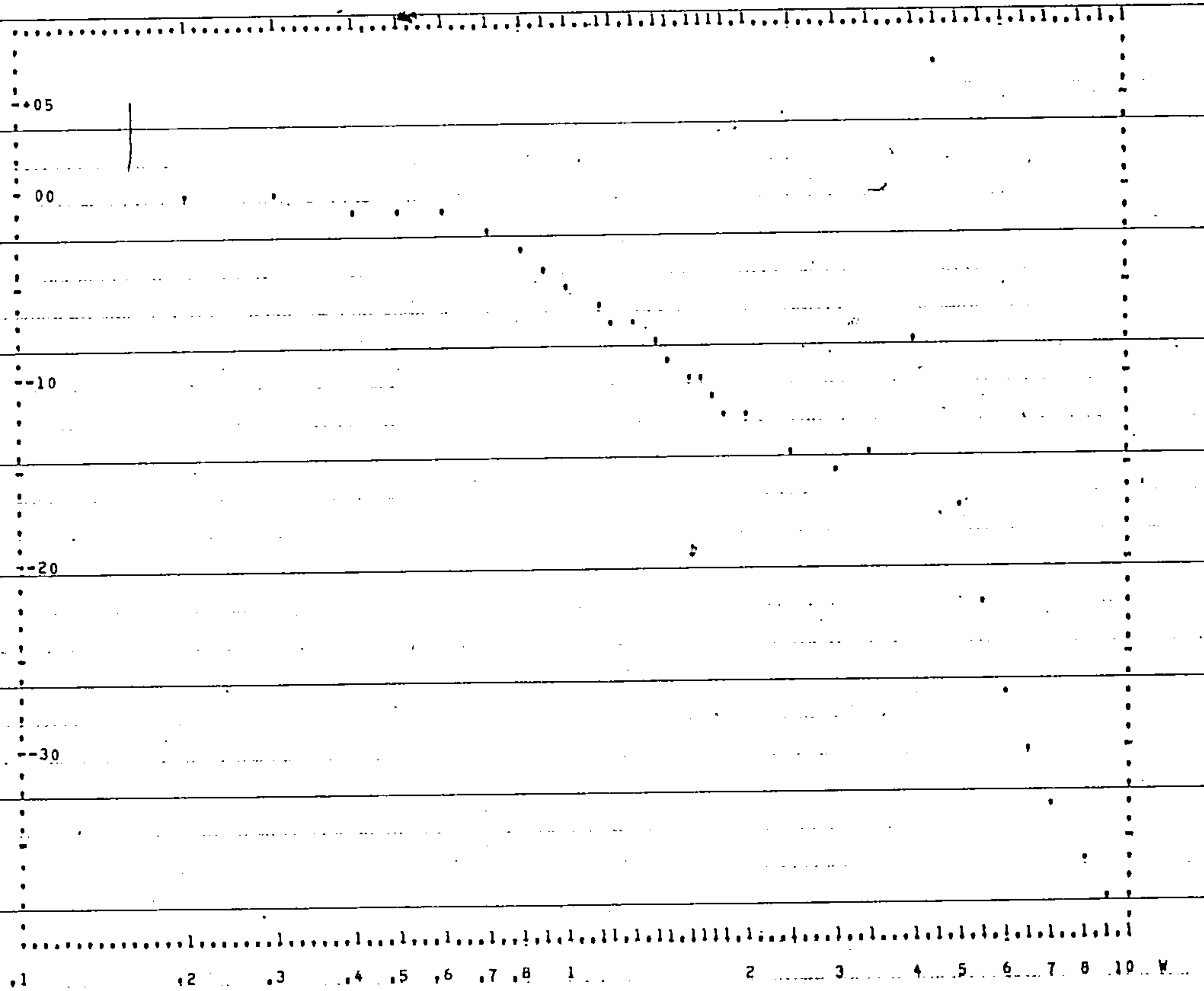

FIGURE 6.2.1.2 RESPONSE OF A T.L. + A L.P. BUTTERWORTH FILTER OF ORDER 2 (REFER TO FIG.6.1)

T= .20

W	TW	TWDB
0.0	1.000000	0.00
.1	.997430	-.02
.2	.991241	-.08
.3	.985248	-.13
.4	.983591	-.14
.5	.987500	-.11
.6	.989243	-.09
.7	.962815	-.33
.8	.869260	-1.22
.9	.707046	-3.01
1.0	.531992	-5.48
1.1	.389015	-8.20
1.2	.285427	-10.89
1.3	.212834	-13.44
1.4	.161808	-15.82
1.5	.125366	-18.04
1.6	.098836	-20.10
1.7	.079148	-22.03
1.8	.064277	-23.84
1.9	.052862	-25.54
2.0	.043971	-27.14
2.4	.023147	-32.71
3.0	.010981	-39.19
3.4	.007446	-42.56
4.0	.004732	-46.50
4.4	.003771	-48.47
5.0	.002978	-50.52
5.4	.002729	-51.28
6.0	.002684	-51.42
6.4	.002908	-50.73
7.0	.003978	-48.01
8.0	.026936	-31.39
9.0	.018075	-34.86
10.0	.001034	-59.71

TW IN DB

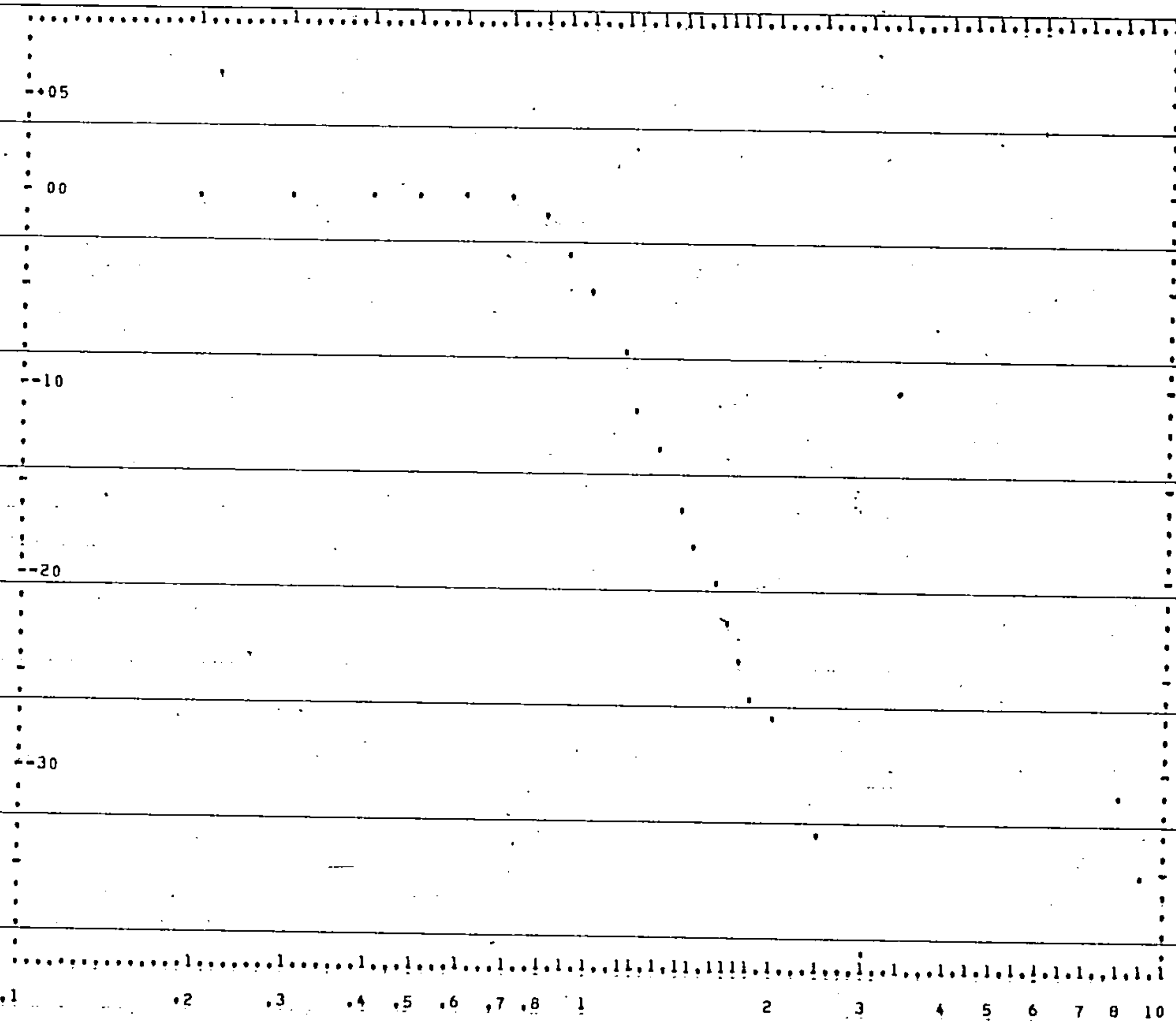

FIGURE 6.2.2.1 RESPONSE OF A T.L. + A L.P. BUTTERWORTH FILTER OF ORDER 4 (REFER TO FIG.6.1)

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T= .40

W	TW	TWDB
0.0	1.000000	0.00
.1	.994947	-.04
.2	.983658	-.14
.3	.975114	-.22
.4	.977697	-.20
.5	.991647	-.07
.6	.994307	-.05
.7	.926870	-.66
.8	.756991	-2.42
.9	.557324	-5.08
1.0	.398168	-8.00
1.1	.288153	-10.81
1.2	.214083	-13.39
1.3	.163512	-15.73
1.4	.128119	-17.85
1.5	.102709	-19.77
1.6	.084035	-21.51
1.7	.070028	-23.09
1.8	.059337	-24.53
1.9	.051056	-25.84
2.0	.044562	-27.02
2.4	.029431	-30.62
3.0	.022757	-32.86
3.4	.025402	-31.90
4.0	.069943	-23.11
4.4	.326502	-9.72
5.0	.049715	-26.07
5.4	.008685	-41.22
6.0	.002110	-53.52
6.4	.001101	-59.16
7.0	.000529	-65.53
8.0	.000241	-72.36
9.0	.0001[illegible]	-75.48
10.0	.000184	-74.72

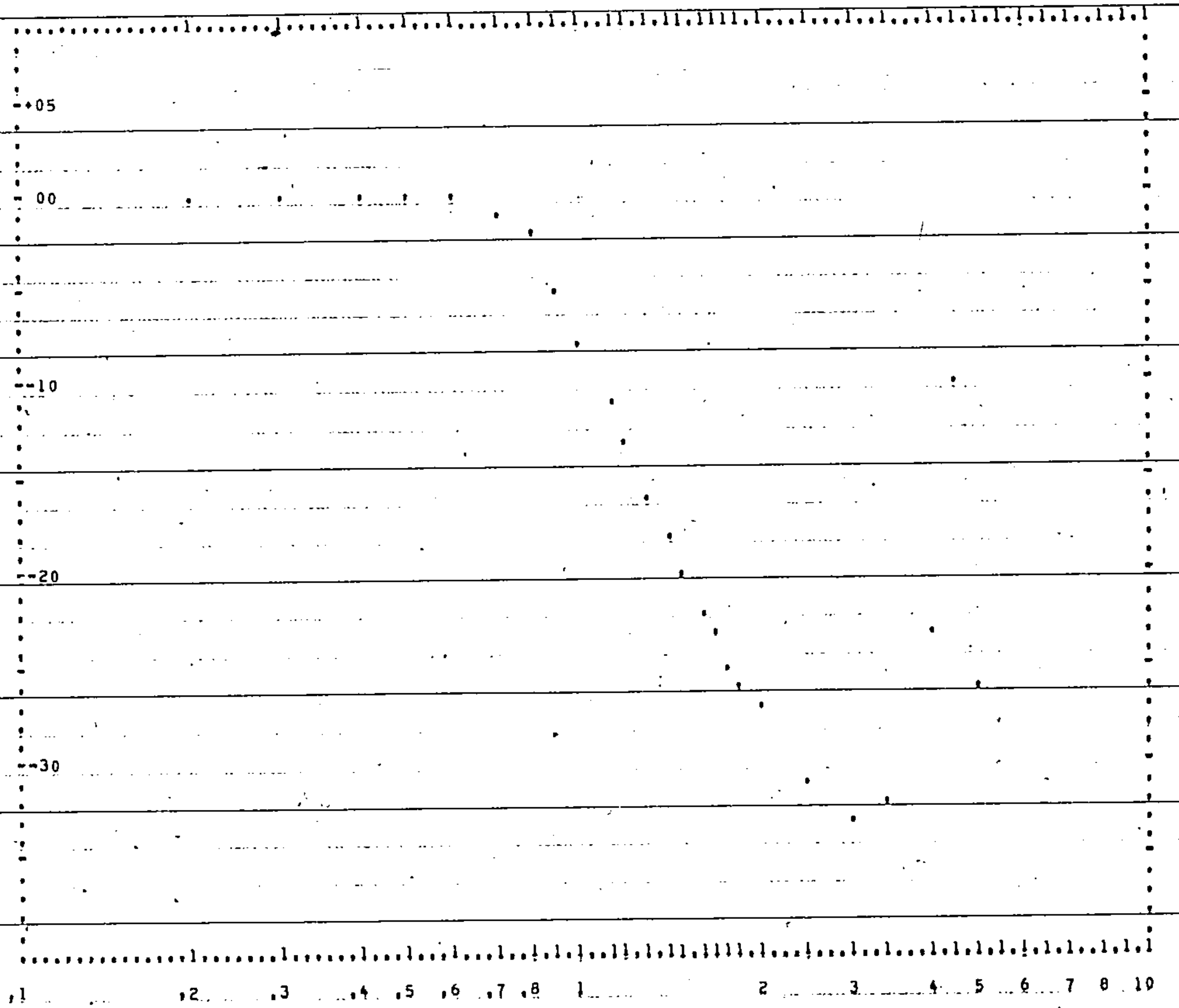

FIGURE 6.2.2.2 RESPONSE OF A T.L. • A L.P. BUTTERWORTH FILTER OF ORDER 4 (REFER TO FIG.6.1)

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[POOR PRINT]

T= .20

W	TW	TWDB
0.0	1.000000	0.00
.1	.996474	-.03
.2	.990614	-.08
.3	.991588	-.07
.4	1.003156	.03
.5	1.015563	.13
.6	1.005651	.05
.7	.955042	-.40
.8	.861868	-1.29
.9	.711038	-2.96
1.0	.497290	-6.07
1.1	.301796	-10.41
1.2	.177046	-15.04
1.3	.106722	-19.43
1.4	.067085	-23.47
1.5	.043917	-27.15
1.6	.029796	-30.52
1.7	.020846	-33.62
1.8	.014977	-36.49
1.9	.011010	-39.16
2.0	.008260	-41.66
2.4	.003075	-50.24
3.0	.000992	-60.07
3.4	.000552	-65.16
4.0	.000278	-71.11
4.4	.000197	-74.10
5.0	.000137	-77.26
5.4	.000119	-78.48
6.0	.000114	-78.88
6.4	.000126	-78.02
7.0	.000190	-74.41
8.0	.002326	-52.67
9.0	.006830	-43.31
10.0	.000043	-87.28

TW IN DB

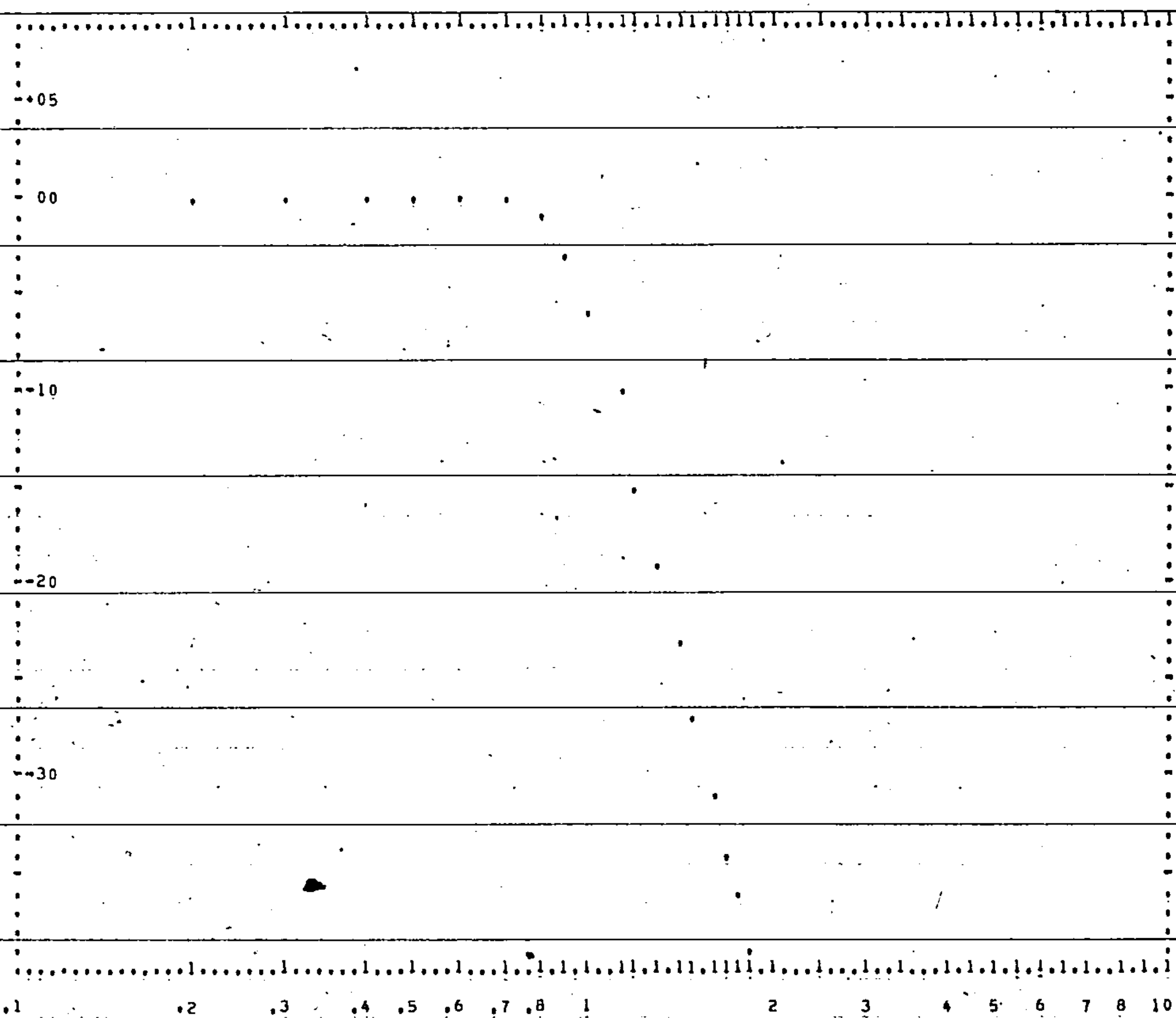

FIGURE 6.2.3.1 RESPONSE OF A T.L. + A L.P. BUTTERWORTH FILTER OF ORDER 6 (REFER TO FIG.6.1)

T= .40

W	TW	TWDB
0.0	1.000000	0.00
.1	.993205	-.06
.2	.984101	-.14
.3	.991510	-.07
.4	1.016258	.14
.5	1.024771	.21
.6	.974723	-.22
.7	.880311	-1.11
.8	.762047	-2.36
.9	.565402	-4.95
1.0	.334849	-9.50
1.1	.185712	-14.62
1.2	.107675	-19.36
1.3	.066508	-23.54
1.4	.043497	-27.23
1.5	.029854	-30.50
1.6	.021341	-33.42
1.7	.015796	-36.03
1.8	.012053	-38.38
1.9	.009448	-40.49
2.0	.007590	-42.40
2.4	.003901	-48.18
3.0	.002510	-52.01
3.4	.002799	-51.06
4.0	.010145	-39.87
4.4	.085352	-21.38
5.0	.023015	-32.76
5.4	.001339	-57.47
6.0	.000115	-78.77
6.4	.000040	-87.87
7.0	.000013	-97.91
8.0	.000004	-108.59
9.0	.000002	-113.55
10.0	.000002	-112.70

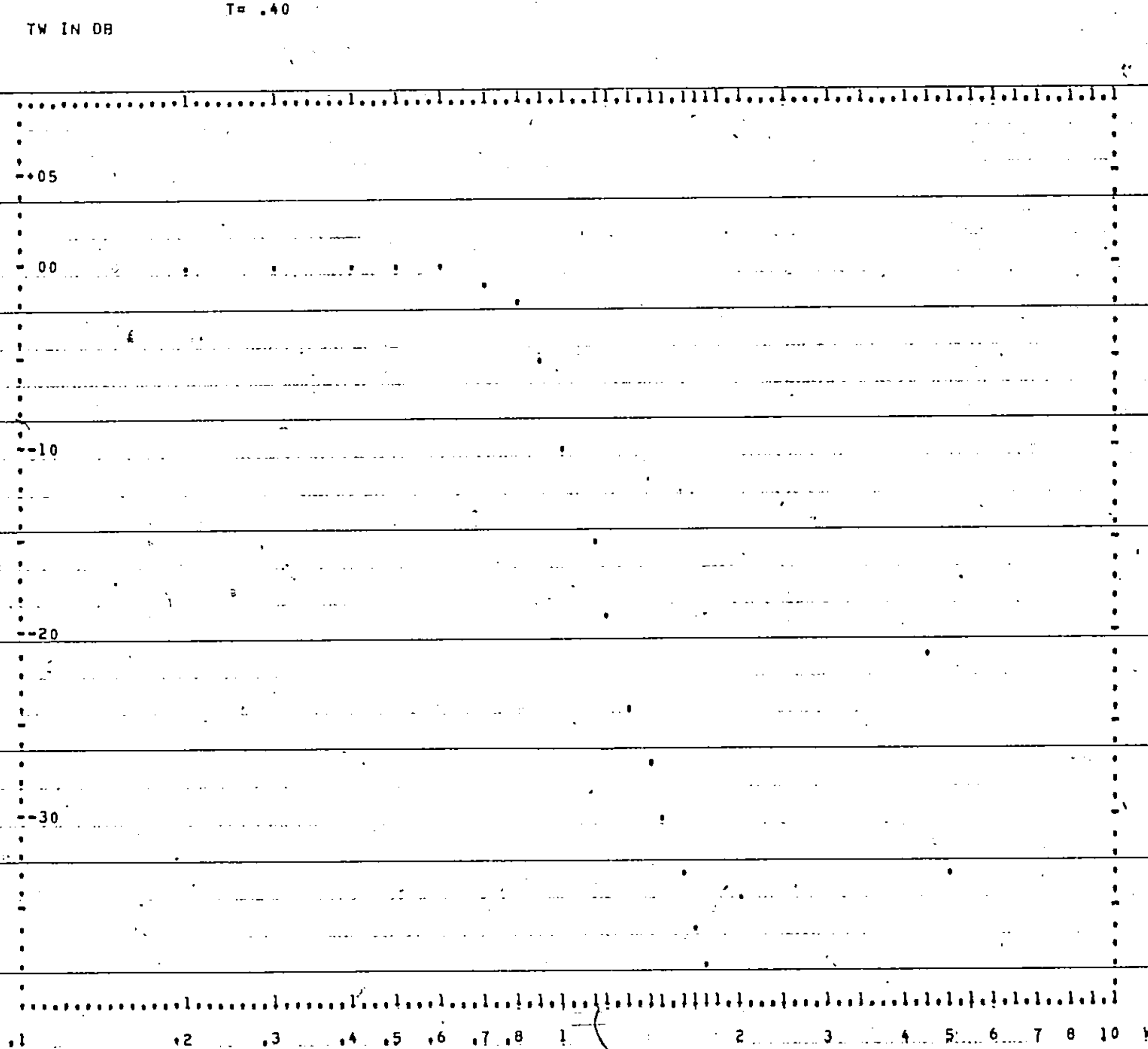

FIGURE 6.2.3.2 RESPONSE OF A T.L. + A L.P. BUTTERWORTH FILTER OF ORDER 6 (REFER TO FIG.6.1)

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T= ,20

W	TW	TWDB
0.0	1.000000	0.00
.1	.995268	-.04
.2	.985416	-.13
.3	.977588	-.20
.4	.975981	-.21
.5	.988818	-.10
.6	1.035524	.30
.7	1.110466	.91
.8	1.036153	.31
.9	.713859	-2.93
1.0	.430306	-7.32
1.1	.239487	-12.41
1.2	.120784	-18.36
1.3	.059393	-24.53
1.4	.030393	-30.34
1.5	.016484	-35.66
1.6	.009453	-40.49
1.7	.005691	-44.90
1.8	.003572	-48.94
1.9	.002323	-52.68
2.0	.001558	-56.15
2.4	.000401	-67.94
3.0	.000086	-81.27
3.4	.000039	-88.12
4.0	.000016	-96.12
4.4	.000010	-100.14
5.0	.000006	-104.42
5.4	.000005	-106.12
6.0	.000005	-106.81
6.4	.000005	-105.81
7.0	.000009	-101.38
8.0	.000174	-75.20
9.0	.000233	-72.64
10.0	.000002	-113.39

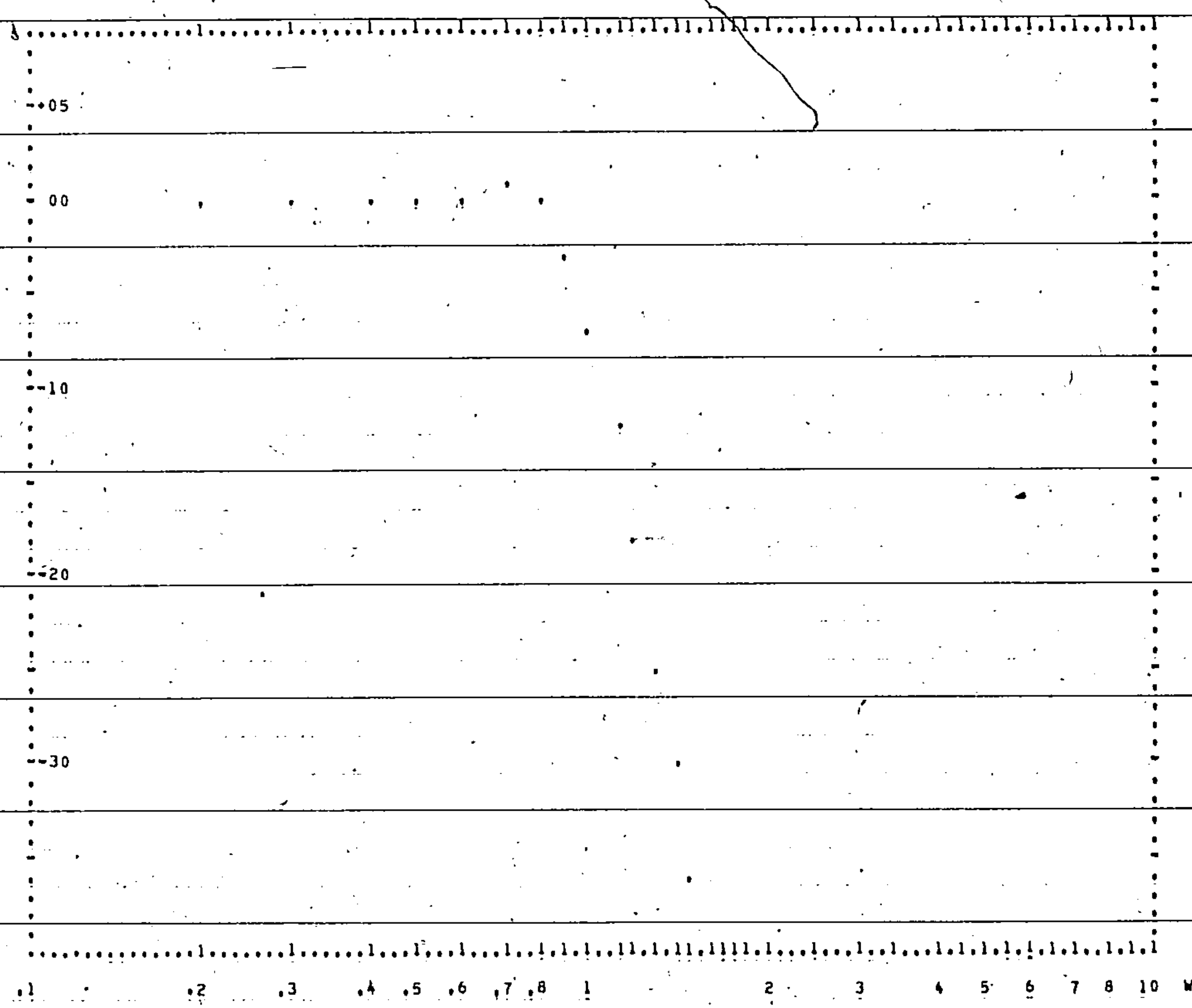

FIGURE 6.2.4.1 RESPONSE OF A T.L. + A L.P. BUTTERWORTH FILTER OF ORDER 8 (REFER TO FIG.6.1)

POOR PRINT

TW = .40

W	TW	TWDB
0.0	1.000000	0.00
.1	.991430	-.07
.2	.980819	-.17
.3	.983866	-.14
.4	.990783	-.08
.5	1.001552	.01
.6	1.054973	.46
.7	1.086633	.72
.8	.807091	-1.86
.9	.485260	-6.28
1.0	.274070	-11.24
1.1	.133210	-17.51
1.2	.061398	-24.24
1.3	.030087	-30.43
1.4	.016044	-35.89
1.5	.009235	-40.69
1.6	.005668	-44.93
1.7	.003673	-48.70
1.8	.002493	-52.07
1.9	.001762	-55.08
2.0	.001290	-57.78
2.4	.000502	-65.98
3.0	.000262	-71.63
3.4	.000288	-70.83
4.0	.001300	-57.72
4.4	.028744	-30.83
5.0	.001468	-56.66
5.4	.000292	-70.70
6.0	.000007	-103.26
6.4	.000002	-116.19
7.0	.000000	-130.16
8.0	.000000	-144.84
9.0	.000000	-151.73
10.0	.000000	-150.87

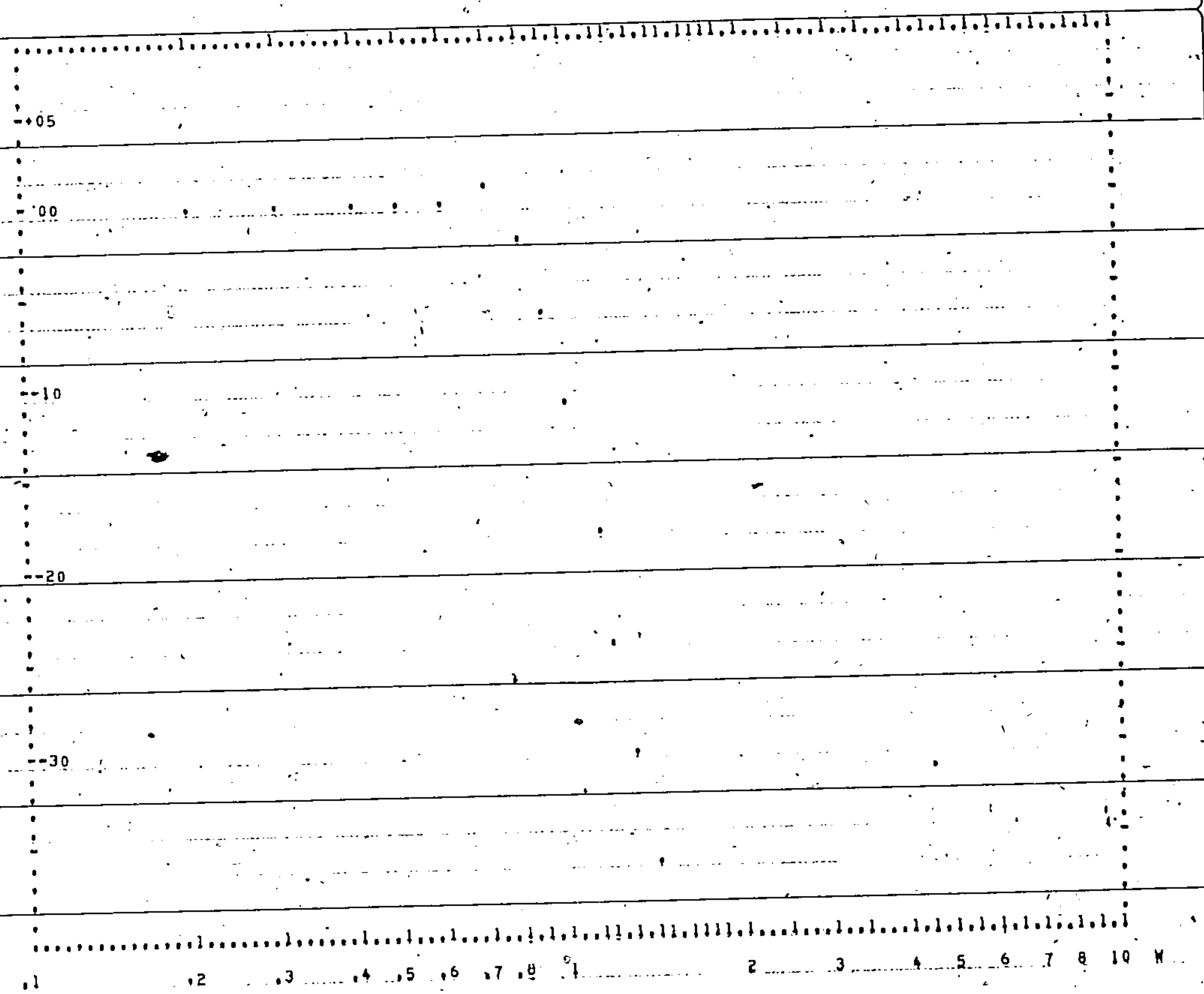

FIGURE 6.2.4.2 RESPONSE OF A T.L. + A L.P. BUTTERWORTH FILTER OF ORDER 8 (REFER TO FIG.6.1)

POOR PRINT

T= .20

W	TW	TWDB
0.0	1.000000	0.00
.1	.995317	-.04
.2	.996253	-.03
.3	1.018040	.16
.4	1.045606	.39
.5	1.050837	.43
.6	1.004429	.04
.7	.890733	-1.01
.8	.805350	-1.88
.9	.727804	-2.76
1.0	.312360	-10.11
1.1	.118035	-18.56
1.2	.053284	-25.47
1.3	.026573	-31.51
1.4	.013638	-37.31
1.5	.006976	-43.13
1.6	.003582	-48.92
1.7	.001882	-54.51
1.8	.001025	-59.79
1.9	.000581	-64.72
2.0	.000342	-69.31
2.4	.000057	-84.85
3.0	.000008	-102.15
3.4	.000003	-110.94
4.0	.000001	-121.14
4.4	.000000	-126.26
5.0	.000000	-131.73
5.4	.000000	-133.92
6.0	.000000	-134.90
6.4	.000000	-133.78
7.0	.000000	-128.47
8.0	.000015	-96.63
9.0	.000039	-88.16
10.0	.000000	-136.07

TW IN DB

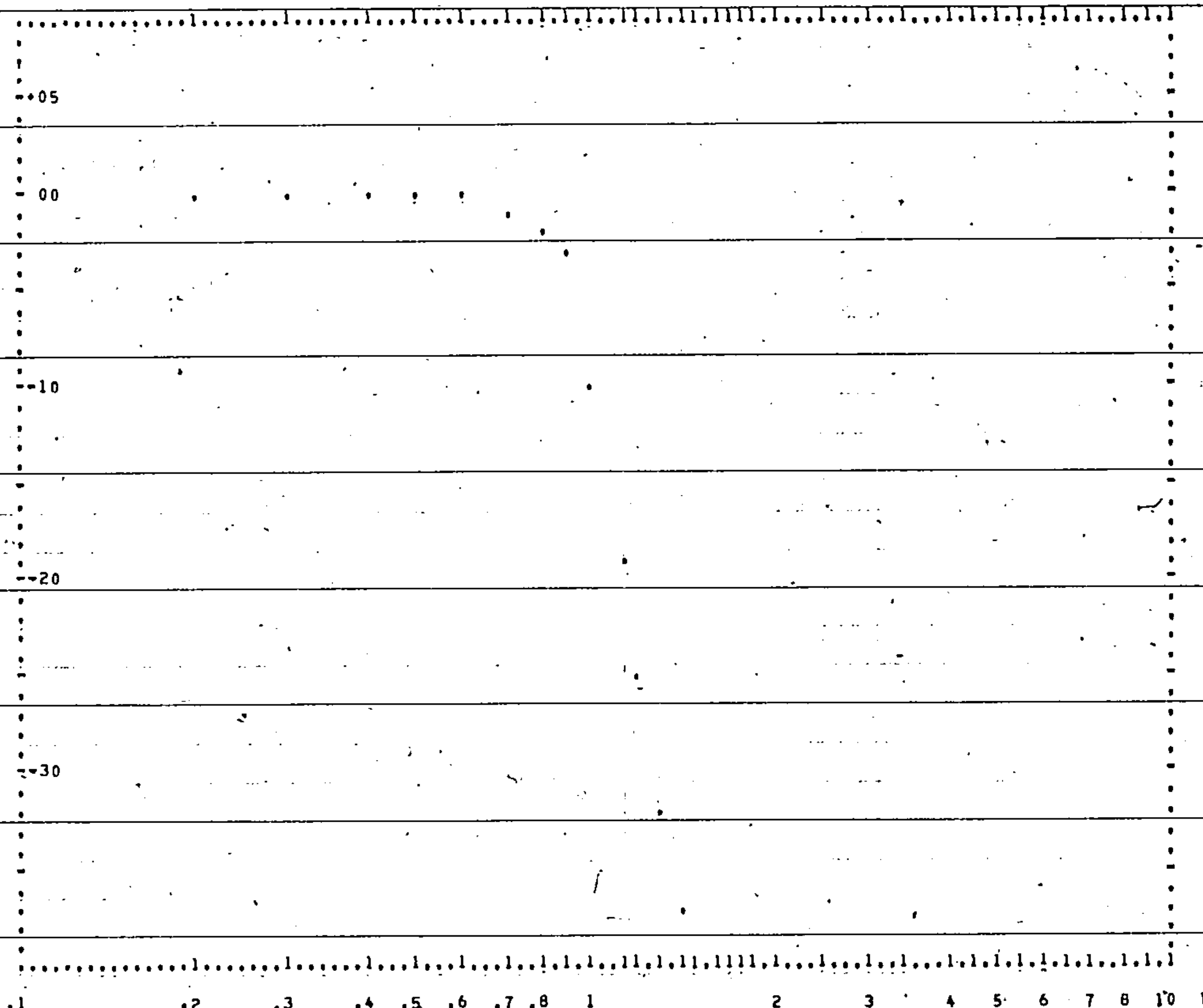

FIGURE 6.2.5.1 RESPONSE OF A T.L. + A L.P. BUTTERWORTH FILTER OF ORDER 10 (REFER TO FIG.6.1)

[POOR PRINT]

T= .40

W	TW	TWDB
0.0	1.000000	0.00
.1	.991604	-.07
.2	.998819	-.01
.3	1.033211	.28
.4	1.041244	.35
.5	1.021147	.18
.6	.954016	-.41
.7	.824066	-1.68
.8	.787938	-2.07
.9	.411510	-7.71
1.0	.142135	-16.95
1.1	.061960	-24.16
1.2	.030219	-30.39
1.3	.014689	-36.66
1.4	.007016	-43.08
1.5	.003449	-49.25
1.6	.001798	-54.90
1.7	.001001	-59.99
1.8	.000592	-64.55
1.9	.000371	-68.62
2.0	.000244	-72.27
2.4	.000069	-83.27
3.0	.000028	-91.02
3.4	.000030	-90.39
4.0	.000184	-74.71
4.4	.005944	-44.52
5.0	.000341	-69.35
5.4	.000016	-95.81
6.0	.000000	-126.60
6.4	.000000	-143.98
7.0	.000000	-162.21
8.0	.000000	-181.10
9.0	.000000	-190.00
10.0	.000000	-189.19

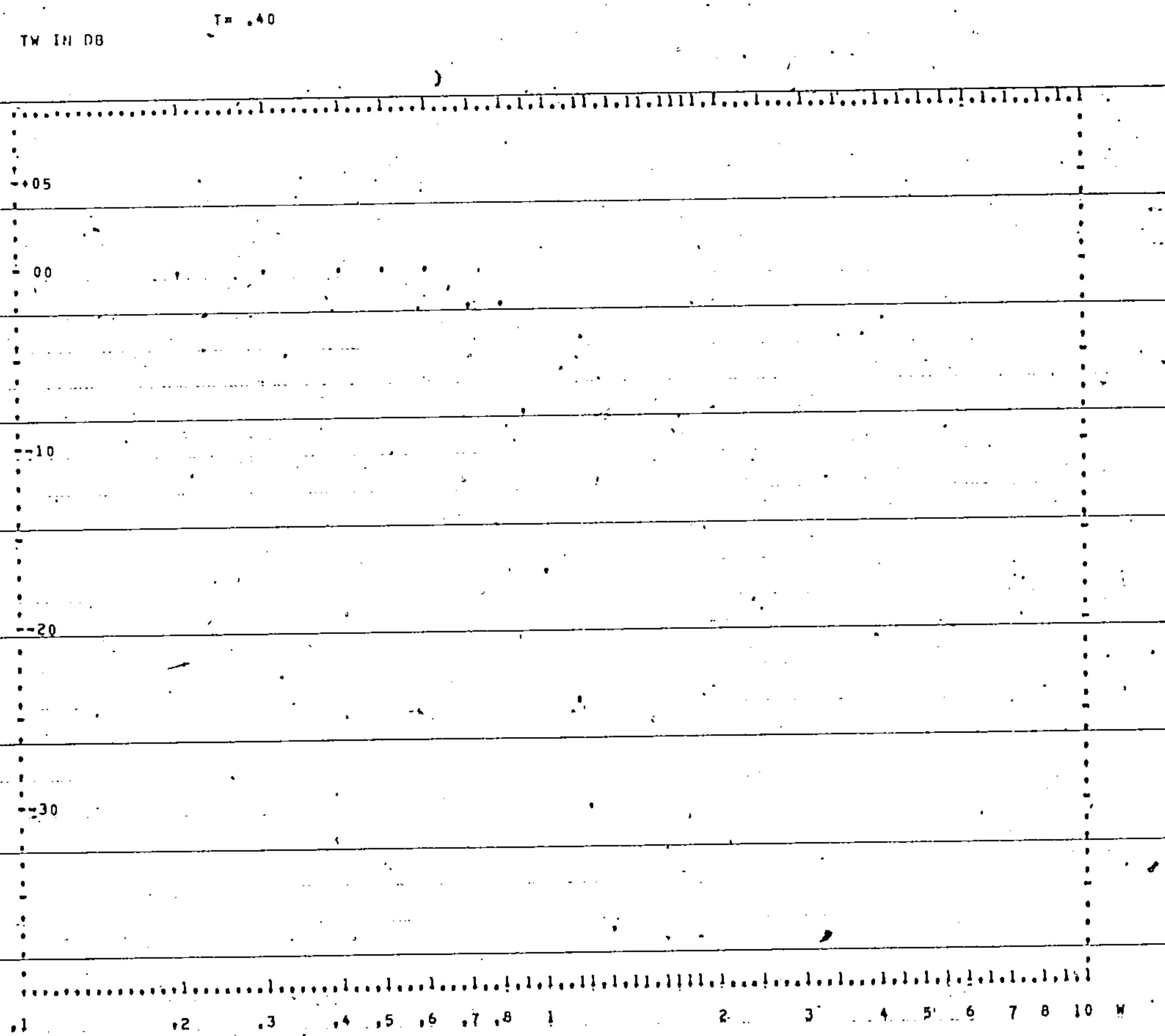

FIGURE 6.2.5.2 RESPONSE OF A T.L. + A L.P. BUTTERWORTH FILTER OF ORDER 10. (REFER TO FIG.6.1)

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6.3 The Lumped Section is a H.P. Filter

Applying the same transformation mentioned in section 2.6 to the main computer program the following set of curves was developed.

6.3.1 H.P. Filter (N = 2)

Figure 6.3.1.1 and Figure 6.3.1.2 show the frequency response for a H.P. filter of order 2 calculated at $\tau = 0.2$ and 0.4 respectively. The frequency response varies within 1.47 db in the pass band for the first curve and 2.85 dbs for the second curve.

6.3.2 H.P. Filter (N = 4)

Figure 6.3.2.1 shows the frequency response curve for a H.P. filter of order 4 calculated at $\tau = 0.2$. The frequency response varies within 2.99 db in the pass-band.

6.3.3 H.P. Filter (N = 6)

Figure 6.3.3.1 shows the frequency response curve for a H.P. filter of order 6 calculated at $\tau = 0.2$. The frequency response varies within 4.36 dbs in the pass-band.

6.3.4 High Pass Filter (N = 8)

Figure 6.3.4.1 shows the frequency response curve for a H.P. filter of order 8, calculated at $\tau = 0.2$. The frequency response varies within 5.3 dbs within the pass-band.

6.3.5 High Pass Filter (N = 10)

Figure 6.3.5.1 shows the frequency response curve for a H.P. filter of order 10, calculated at $\tau = 0.2$. The frequency response varies within 5.62 db within the pass-band.

[POOR PRINT]

T= .20

W	TW	TWDB
0.0	0.000000	-200.00
.1	.010016	-39.99
.2	.040237	-27.91
.3	.091054	-20.81
.4	.162670	-15.77
.5	.254366	-11.89
.6	.363529	-8.79
.7	.484801	-6.29
.8	.610081	-4.29
.9	.729989	-2.73
1.0	.836384	-1.55
1.1	.924480	-.68
1.2	.993300	-.06
1.3	1.044653	.38
1.4	1.081649	.68
1.5	1.107559	.89
1.6	1.125239	1.02
1.7	1.136945	1.11
1.8	1.144366	1.17
1.9	1.148729	1.20
2.0	1.150911	1.22
2.4	1.147859	1.20
3.0	1.130805	1.07
3.4	1.117567	.97
4.0	1.097486	.81
4.4	1.084373	.70
5.0	1.065462	.55
5.4	1.053506	.45
6.0	1.036755	.31
6.4	1.026477	.23
7.0	1.012535	.11
8.0	.993540	-.06
9.0	.980082	-.17
10.0	.972112	-.25

TW IN DB

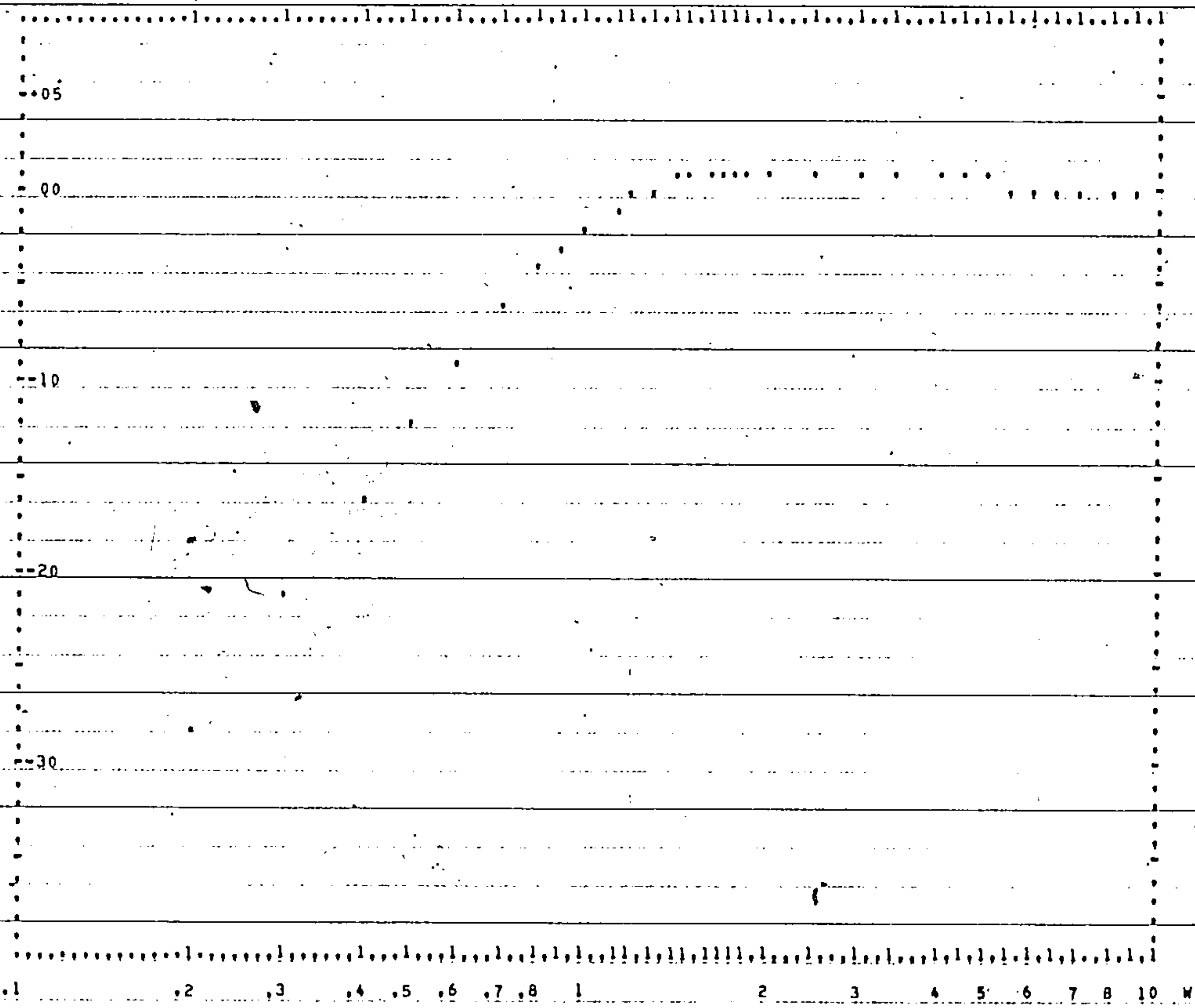

FIGURE 6.3.1.1 RESPONSE OF A T.L. + A H.P. BUTTERWORTH FILTER OF ORDER 2 (REFER TO FIG.6.1)

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[POOR PRINT]

T= .40

W	TW	TWDB
0.0	0.000000	-200.00
.1	.010036	-39.97
.2	.040574	-27.84
.3	.092855	-20.64
.4	.168745	-15.46
.5	.270234	-11.37
.6	.398329	-8.00
.7	.551061	-5.18
.8	.720850	-2.84
.9	.893020	-.98
1.0	1.048480	.41
1.1	1.171101	1.37
1.2	1.254399	1.97
1.3	1.301809	2.29
1.4	1.321903	2.42
1.5	1.323648	2.44
1.6	1.314136	2.37
1.7	1.298239	2.27
1.8	1.279057	2.14
1.9	1.258468	2.00
2.0	1.237575	1.85
2.4	1.159921	1.29
3.0	1.070276	.59
3.4	1.026399	.23
4.0	.979549	-.18
4.4	.958951	-.36
5.0	.941505	-.52
5.4	.937675	-.56
6.0	.941800	-.52
6.4	.950811	-.45
7.0	.968285	-.28
8.0	1.005136	.04
9.0	1.032407	.28
10.0	1.035699	.30

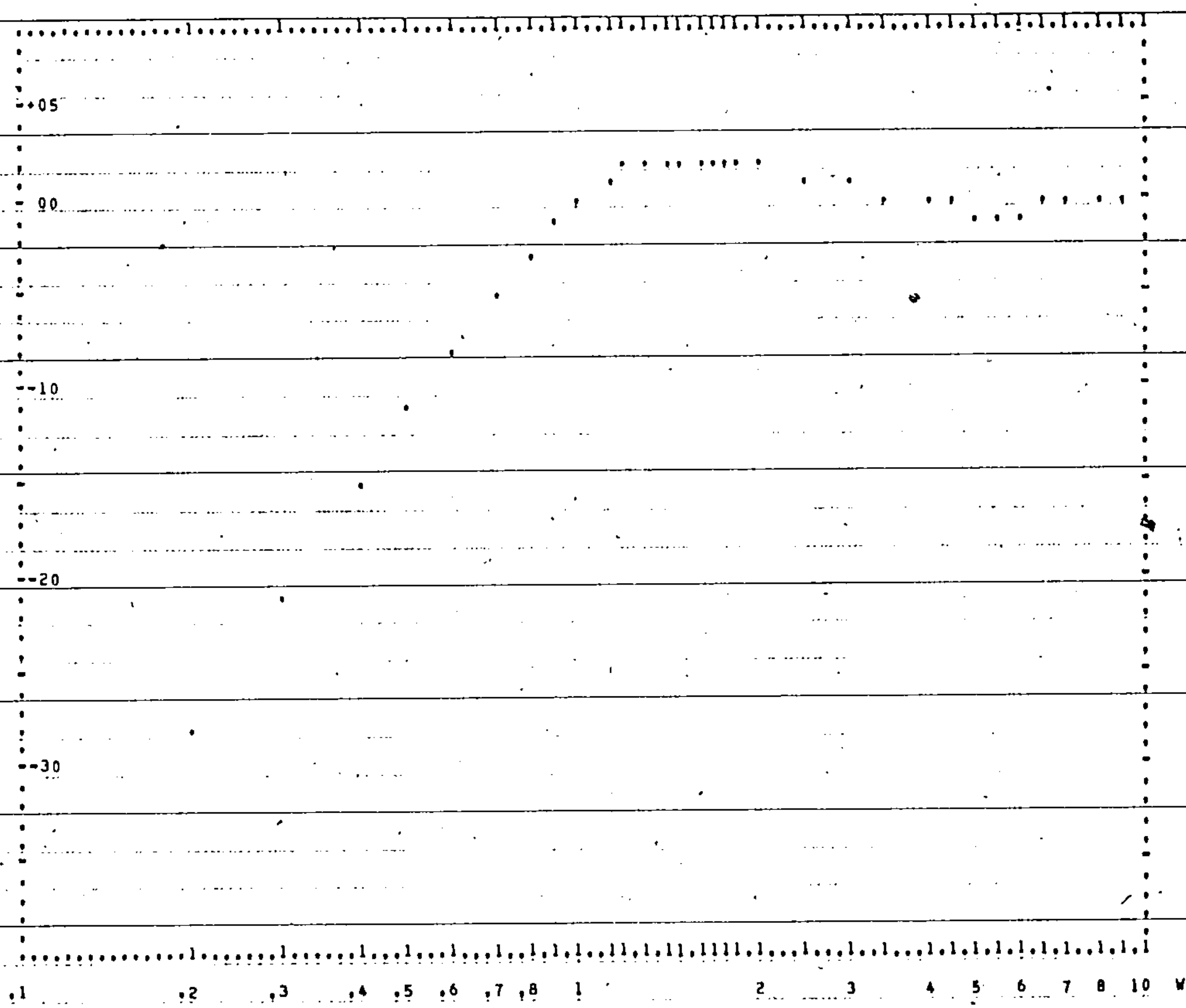

FIGURE 6.3.1.2 RESPONSE OF A T.L. + A H.P. BUTTERWORTH FILTER OF ORDER 2 (REFER TO FIG.6.1)

CONCORDIA UNIVERSITY COMPUTER CENTER

[POOR PRINT]

T= .20

W	TW	TWDB
0.0	0.000000	-200.00
.1	.000100	-79.96
.2	.001633	-55.74
.3	.008509	-41.40
.4	.028131	-31.02
.5	.073089	-22.72
.6	.162875	-15.76
.7	.317626	-9.96
.8	.523574	-5.62
.9	.707847	-3.00
1.0	.825620	-1.66
1.1	.898509	-.93
1.2	.953548	-.41
1.3	1.002994	.03
1.4	1.050532	.43
1.5	1.096501	.80
1.6	1.140081	1.14
1.7	1.180146	1.44
1.8	1.215610	1.70
1.9	1.245599	1.91
2.0	1.269537	2.07
2.4	1.304298	2.31
3.0	1.233311	1.82
3.4	1.161196	1.30
4.0	1.064651	.54
4.4	1.015288	.13
5.0	.969692	-.32
5.4	.942378	-.52
6.0	.926641	-.66
6.4	.925202	-.68
7.0	.933820	-.59
8.0	.967106	-.29
9.0	1.005304	.05
10.0	1.026847	.23

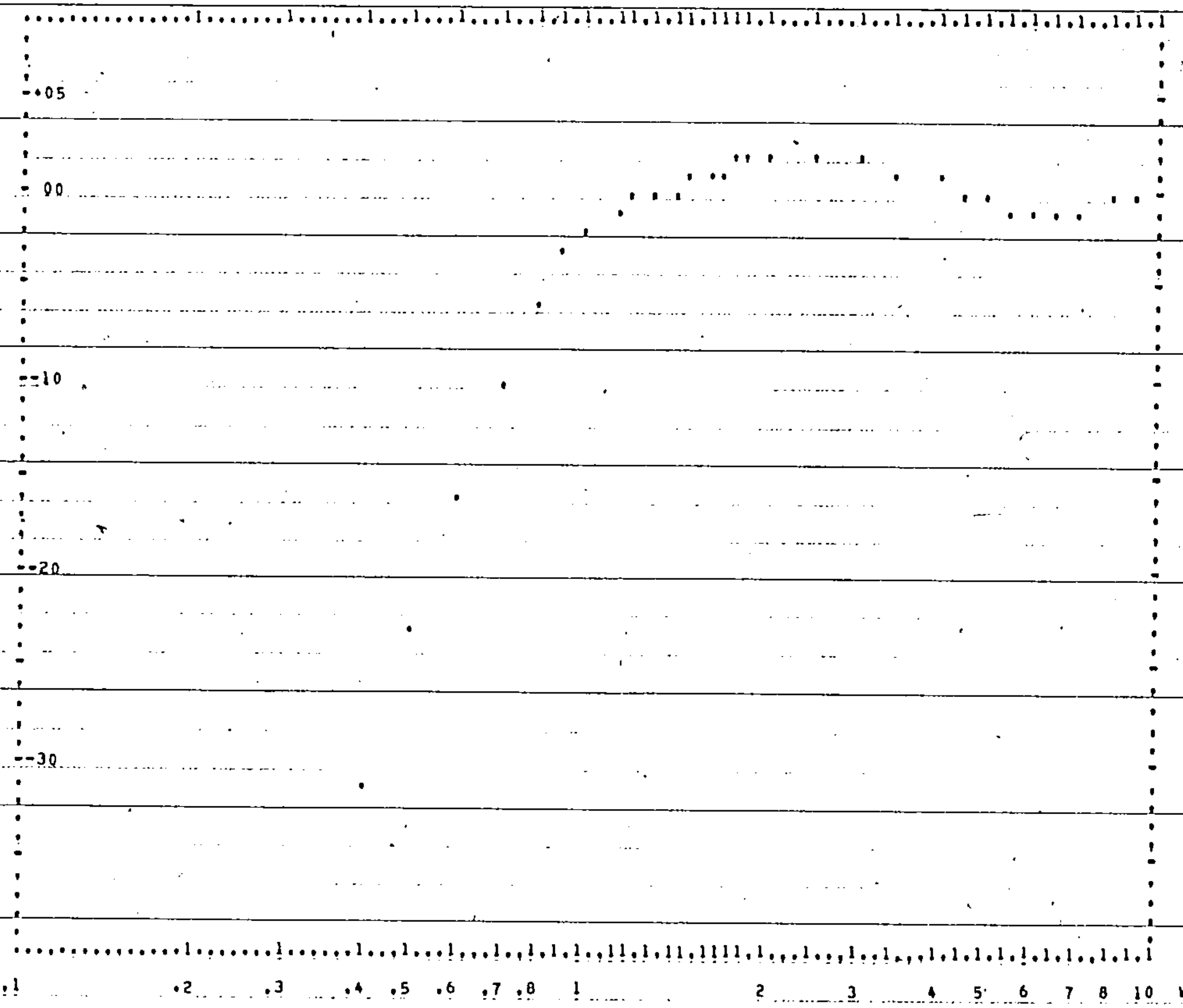

FIGURE 6.3.2.1 RESPONSE OF A T.L. + A H.P. BUTTERWORTH FILTER OF ORDER 4 (REFER TO FIG.6.1)

CONCORDIA UNIVERSITY COMPUTER CENTER.

[POOR PRINT]

T= .20

W	TW	TWDB
0.0	0.000000	-200.00
.1	.000001	-119.92
.2	.000066	-83.55
.3	.000799	-61.95
.4	.004858	-46.27
.5	.020334	-33.84
.6	.065626	-23.66
.7	.171896	-15.29
.8	.384419	-8.30
.9	.727731	-2.76
1.0	.990870	-.08
1.1	1.006253	.05
1.2	.960487	-.35
1.3	.936365	-.57
1.4	.939128	-.55
1.5	.963260	-.33
1.6	1.003986	.03
1.7	1.057772	.49
1.8	1.121553	1.00
1.9	1.191975	1.53
2.0	1.264846	2.04
2.4	1.470051	3.35
3.0	1.306167	2.32
3.4	1.132691	1.08
4.0	.971485	-.25
4.4	.918201	-.74
5.0	.890429	-1.01
5.4	.895474	-.96
6.0	.923848	-.69
6.4	.948544	-.46
7.0	.980750	-.17
8.0	.997540	-.02
9.0	.987436	-.11
10.0	.992051	-.07

TW IN DB

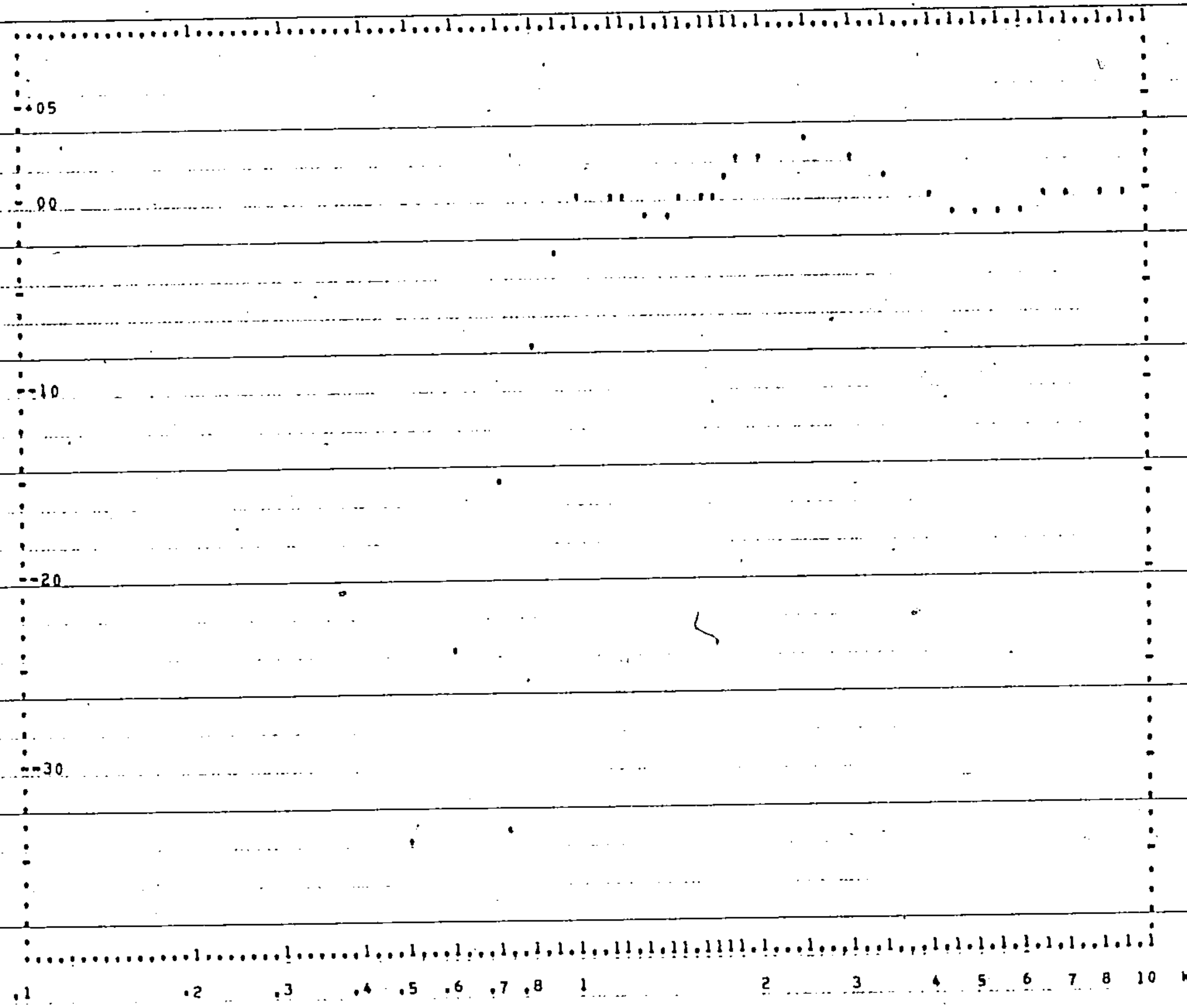

FIGURE 6.3.3.1 RESPONSE OF A T.L. + A H.P. BUTTERWORTH FILTER OF ORDER 6 (REFER TO FIG.6.1)

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[POOR PRINT]

T= .20

W	TW	TWDB
0.0	0.000000	-200.00
.1	.000000	-159.89
.2	.000003	-111.39
.3	.000074	-82.67
.4	.000796	-61.99
.5	.005225	-45.64
.6	.025900	-31.73
.7	.111320	-19.07
.8	.372207	-8.58
.9	.699159	-3.11
1.0	1.068490	.58
1.1	1.313121	2.37
1.2	1.167521	1.35
1.3	.992973	-.06
1.4	.897860	-.94
1.5	.860573	-1.30
1.6	.861748	-1.29
1.7	.891509	-1.00
1.8	.945433	-.49
1.9	1.021783	.19
2.0	1.119578	.98
2.4	1.598122	4.07
3.0	1.325522	2.45
3.4	1.055977	.47
4.0	.892870	-.98
4.4	.868039	-1.23
5.0	.889904	-1.01
5.4	.919049	-.73
6.0	.950581	-.44
6.4	.955651	-.39
7.0	.951127	-.44
8.0	.966268	-.30
9.0	1.013044	.11
10.0	1.024746	.21

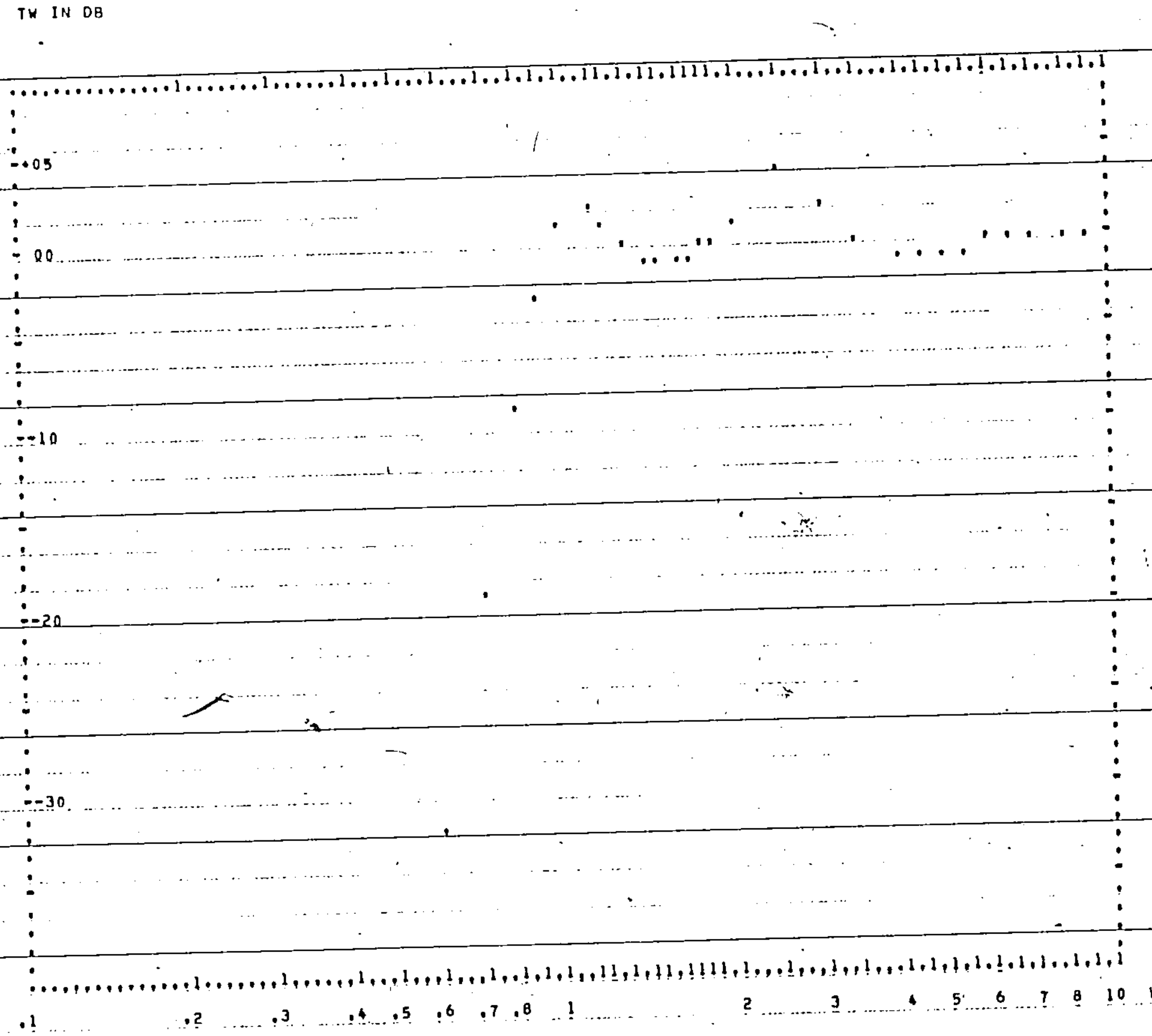

FIGURE 6.3.4.1 RESPONSE OF A T.L. + A H.P. BUTTERWORTH FILTER OF ORDER 8 (REFER TO FIG.6.1)

POOR PRINT

T= .20

W	TW	TWDB
0.0	0.000000	-200.00
.1	.000000	-199.84
.2	.000000	-139.11
.3	.000007	-102.98
.4	.000143	-76.87
.5	.001516	-56.39
.6	.010522	-39.56
.7	.050753	-25.89
.8	.194555	-14.22
.9	.724076	-2.80
1.0	.805521	-1.88
1.1	.820553	-1.72
1.2	1.020726	.18
1.3	1.241207	1.88
1.4	1.215145	1.69
1.5	1.061458	.52
1.6	.949322	-.45
1.7	.896269	-.95
1.8	.889450	-1.02
1.9	.920139	-.72
2.0	.985563	-.13
2.4	1.613167	4.15
3.0	1.353757	2.63
3.4	.990475	-.08
4.0	.843997	-1.47
4.4	.847710	-1.44
5.0	.895072	-.96
5.4	.917669	-.75
6.0	.930201	-.63
6.4	.940888	-.53
7.0	.973424	-.23
8.0	1.007767	.07
9.0	.980072	-.17
10.0	.993237	-.06

TW IN DB

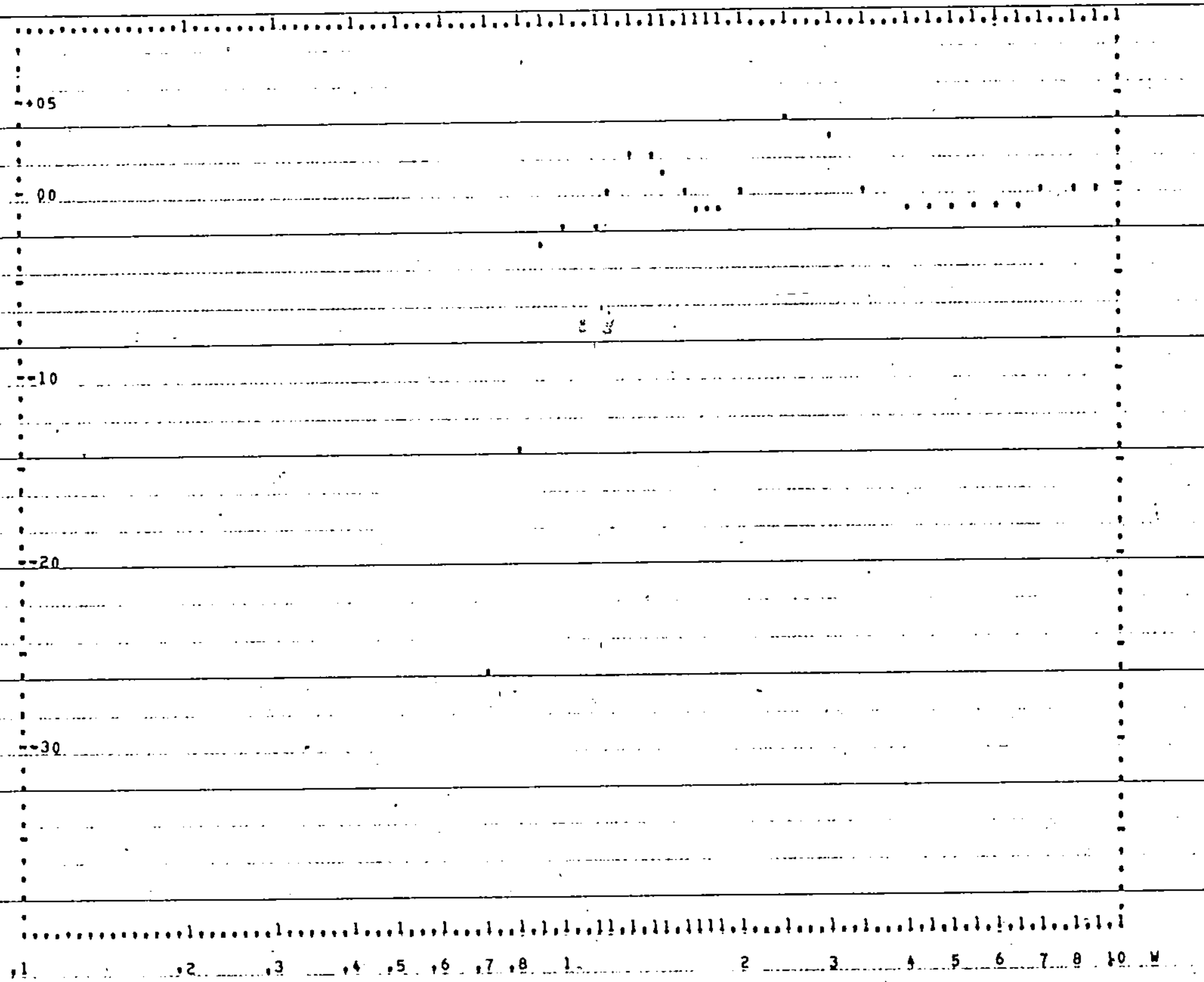

FIGURE 6.3.5.1 RESPONSE OF A T.L. + A H.P. BUTTERWORTH FILTER OF ORDER 10 (REFER TO FIG.6.1)

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6.4 The Lumped Section is a Band-Pass Filter

Applying the before-mentioned transformation (section 2.6) to the main computer program, the following set of curves was developed.

6.4.1 B.P. Filter (N = 2)

Figure 6.4.1.1 shows the frequency response curve for a Band-Pass filter of order 2 calculated at $\tau = 0.2$. There is a peak signal in the stop band of - 9.65 db at $w = 8.0$.

6.4.2 B.P. Filter (N = 4)

Figure 6.4.2.1 shows the frequency response curve for a Band Pass filter of order 4 calculated at $\tau = 0.2$. A peak signal appears at the R.H.S. of the curve at $w = 9$ with a magnitude of - 16.46 db.

6.4.3 B.P. Filter (N = 6)

Figure 6.4.3.1 shows the frequency response curve for a Band-Pass filter of order 6 calculated at $\tau = 0.2$. There is a peak signal of -26.3 db at $w = 9.0$.

6.4.4 B.P. Filter (N = 8)

Figures 6.4.4.1 and 6.4.4.2 show the frequency response curve for a Band-Pass filter of order 8 calculated at $\tau = 0.2$ and 0.4 respectively. There is a signal in the stop band of -35.52 db at $w = 9.0$ shown in Figure 6.4.4.1.

6.4.5 B.B. Filter (N = 10)

Figure 6.4.5.1 shows the frequency response curve for a Band-Pass filter of order 10 calculated at $\tau = 0.2$. There is a peak signal in the stop band of -48.65 db at $w = 9.0$. The frequency response varies $< \pm 0.37$ db within the pass band of the filter.

[POOR PRINT]

T= ,20

W	TW	TWDB
0.0	0.000000	-200.00
.1	.023006	-32.76
.2	.098216	-20.16
.3	.243983	-12.25
.4	.479110	-6.39
.5	.763822	-2.34
.6	.964177	-.32
.7	1.031664	.27
.8	1.033457	.29
.9	1.018002	.15
1.0	1.000000	.00
1.1	.980775	-.17
1.2	.957985	-.37
1.3	.929014	-.64
1.4	.892326	-.99
1.5	.847983	-1.43
1.6	.797534	-1.97
1.7	.743435	-2.58
1.8	.688317	-3.24
1.9	.634423	-3.95
2.0	.583348	-4.68
2.4	.419003	-7.56
3.0	.275424	-11.20
3.4	.220696	-13.12
4.0	.170548	-15.36
4.4	.149920	-16.48
5.0	.130898	-17.66
5.4	.124132	-18.12
6.0	.121672	-18.30
6.4	.125727	-18.01
7.0	.145132	-16.76
8.0	.329192	-9.65
9.0	.264084	-11.57
10.0	.072505	-22.79

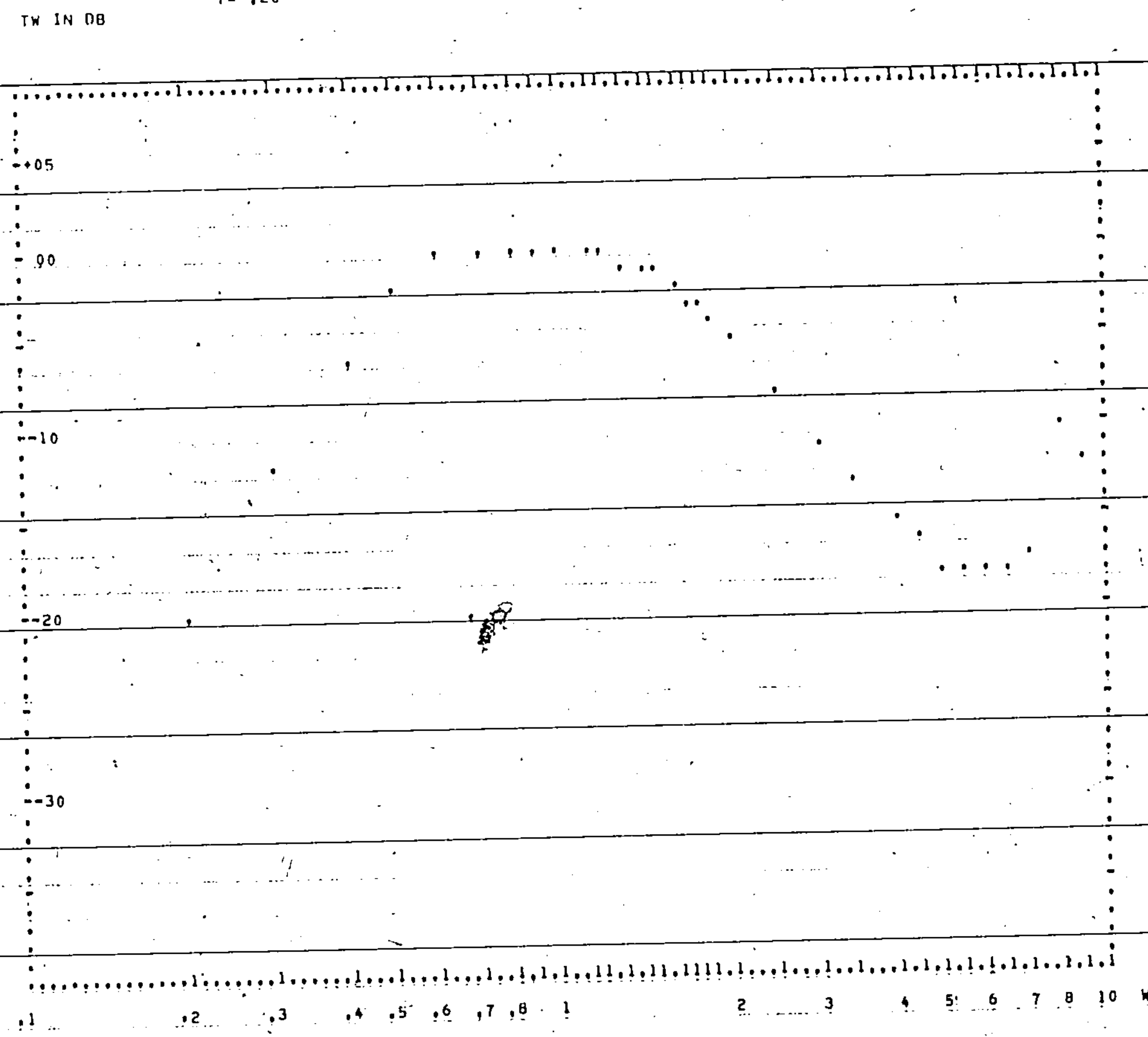

FIGURE 6.4.1.1 RESPONSE OF A T.L. + A B.P. BUTTERWORTH FILTER OF ORDER 2 (REFER TO FIG.6.1)

CONCORDIA UNIVERSITY COMPUTER CENTER

POOR PRINT

T= .20

W	TW	TWDB
0.0	0.000000	-200.00
.1	.000531	-65.50
.2	.009861	-40.12
.3	.065295	-23.70
.4	.298499	-10.50
.5	.779335	-2.17
.6	.984110	-.14
.7	1.048703	.41
.8	1.059972	.51
.9	1.034763	.30
1.0	1.000000	.00
1.1	.974395	-.23
1.2	.965843	-.30
1.3	.975152	-.22
1.4	.995270	-.04
1.5	1.003864	.03
1.6	.959905	-.36
1.7	.838004	-1.54
1.8	.673067	-3.44
1.9	.519021	-5.70
2.0	.398168	-8.00
2.4	.160044	-15.92
3.0	.061402	-24.24
3.4	.038443	-28.30
4.0	.022547	-32.94
4.4	.017263	-35.26
5.0	.012945	-37.76
5.4	.011468	-38.81
6.0	.010663	-39.44
6.4	.011023	-39.15
7.0	.013587	-37.34
8.0	.047196	-26.52
9.0	.150294	-16.46
10.0	.008488	-41.42

TW IN DB

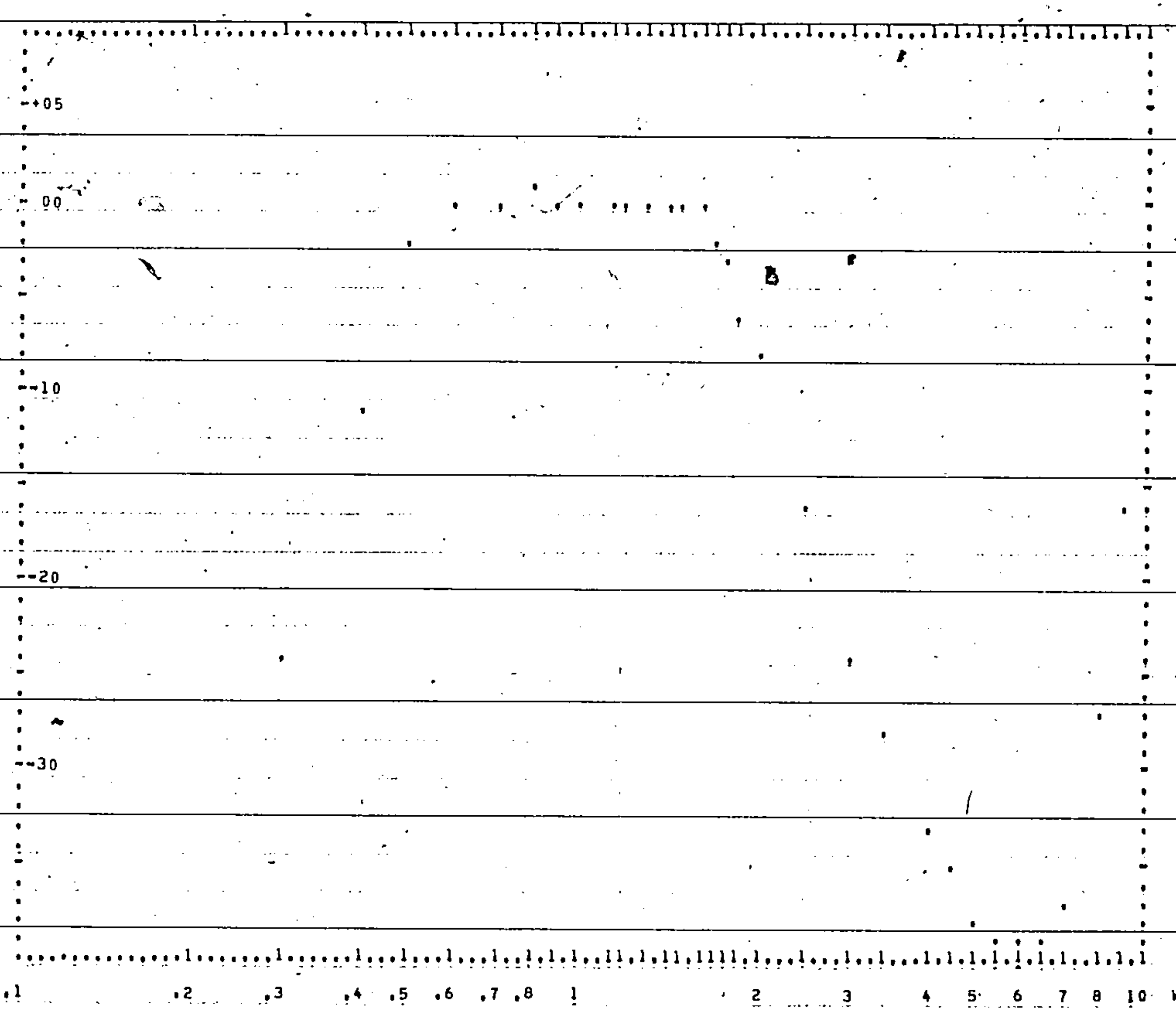

FIGURE 6.4.2.1 RESPONSE OF A T.L. + A B.P. BUTTERWORTH FILTER OF ORDER 4 (REFER TO FIG.6.1)

[POOR PRINT]

T= .20

W	TW	TWDB
0.0	0.000000	-200.00
.1	.000012	-98.23
.2	.000991	-60.08
.3	.017011	-35.39
.4	.163678	-15.72
.5	.848218	-1.43
.6	.976150	-.21
.7	1.009843	.09
.8	1.070511	.59
.9	1.051659	.44
1.0	1.000000	.00
1.1	.977621	-.20
1.2	.994605	-.05
1.3	1.027253	.23
1.4	1.030728	.26
1.5	.979923	-.18
1.6	.900113	-.91
1.7	.815032	-1.78
1.8	.697080	-3.13
1.9	.515581	-5.75
2.0	.334849	-9.50
2.4	.064529	-23.80
3.0	.013133	-37.63
3.4	.006263	-44.06
4.0	.002737	-51.26
4.4	.001813	-54.83
5.0	.001160	-58.71
5.4	.000956	-60.39
6.0	.000838	-61.53
6.4	.000862	-61.29
7.0	.001119	-59.02
8.0	.005581	-45.07
9.0	.048420	-26.30
10.0	.001626	-55.78

TW IN DB

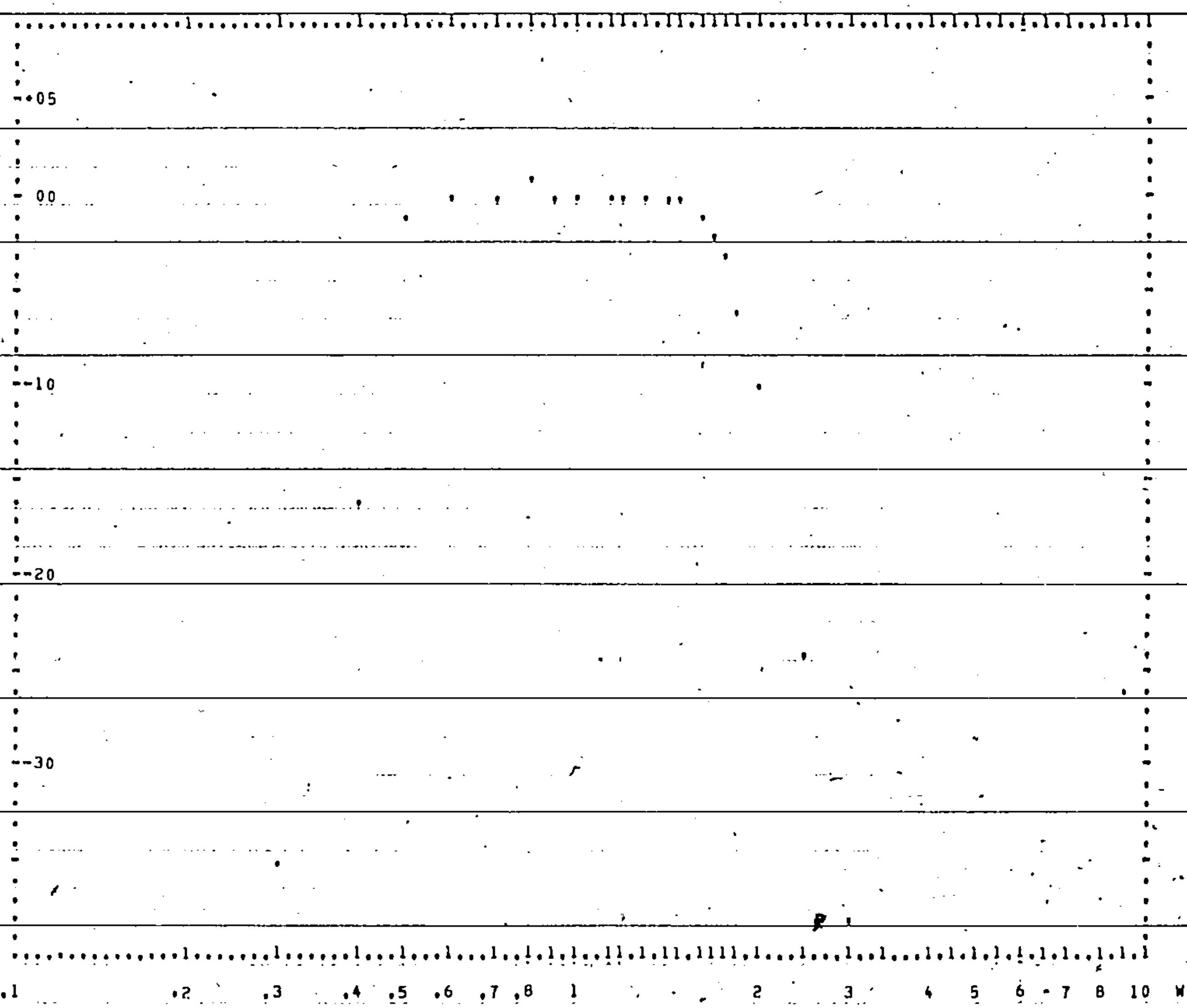

FIGURE 6.4.3.1 RESPONSE OF A T.L. + A B.P. BUTTERWORTH FILTER OF ORDER 6 (REFER TO FIG.6.1)

[POOR PRINT]

T= .20

W	TW	TWDB
0.0	0.000000	-200.00
.1	.000000	-130.97
.2	.000097	-80.23
.3	.004163	-47.61
.4	.103063	-19.74
.5	.871725	-1.19
.6	1.051285	.43
.7	.928925	-.64
.8	1.053165	.45
.9	1.067724	.57
1.0	1.000000	.00
1.1	.983024	-.15
1.2	1.000744	.01
1.3	1.002822	.02
1.4	.998084	-.02
1.5	1.036773	.31
1.6	1.093493	.78
1.7	.954608	-.40
1.8	.660167	-3.61
1.9	.434787	-7.23
2.0	.274070	-11.24
2.4	.028865	-30.79
3.0	.002878	-50.82
3.4	.001017	-59.85
4.0	.000323	-69.82
4.4	.000183	-74.74
5.0	.000099	-80.09
5.4	.000076	-82.44
6.0	.000062	-84.16
6.4	.000063	-84.01
7.0	.000085	-81.37
8.0	.000578	-64.77
9.0	.016741	-35.52
10.0	.000410	-67.73

TW IN DB

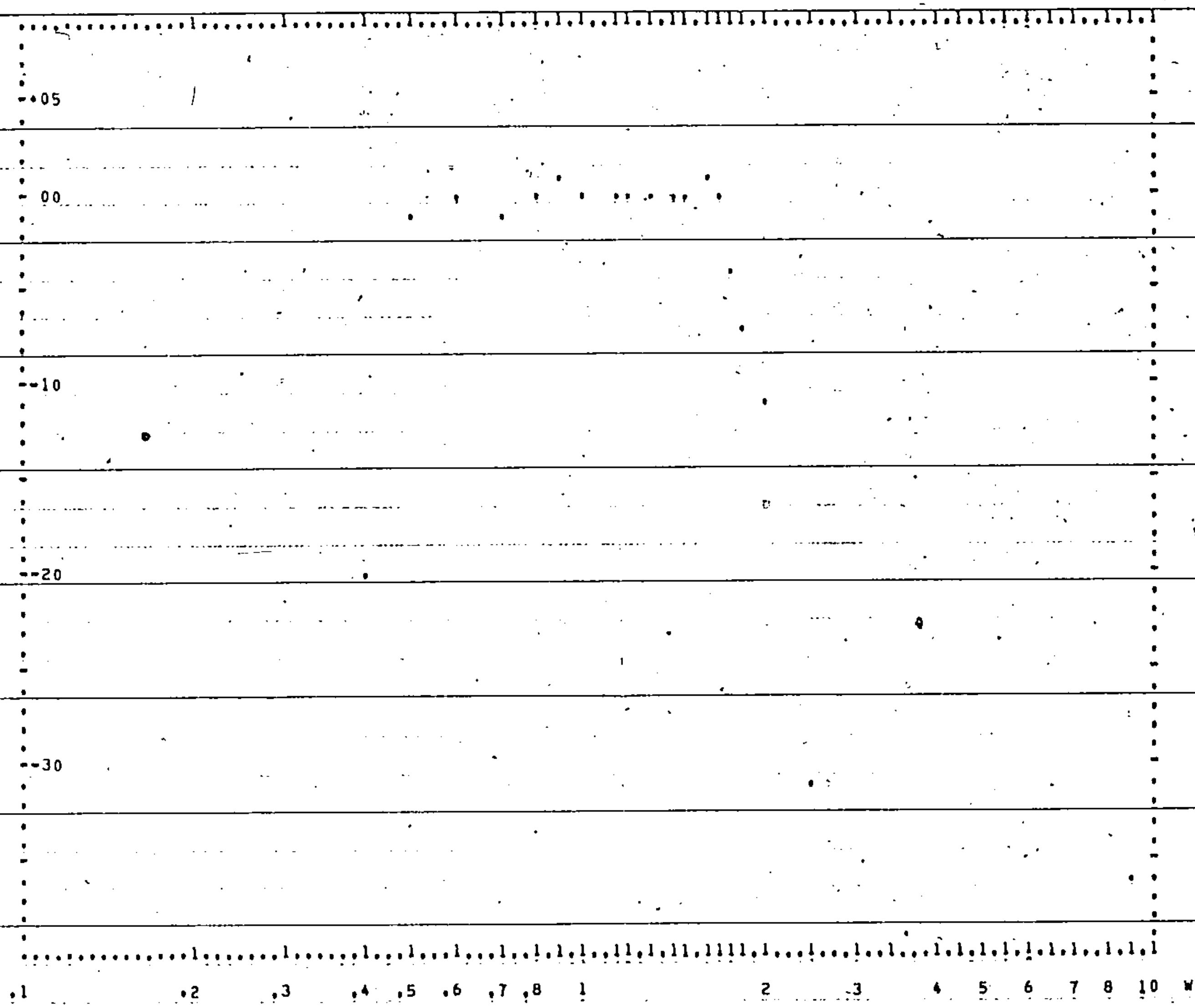

FIGURE 6.4.4.1 RESPONSE OF A T.L. + A.B.P. BUTTERWORTH FILTER OF ORDER 8. (REFER TO FIG.6.1)

[POOR PRINT]

T= .40

W	TW	TWDB
0.0	0.000000	-200.00
.1	.000000	-130.77
.2	.000108	-79.36
.3	.005121	-45.81
.4	.146918	-16.66
.5	1.068490	.58
.6	.934618	-.59
.7	.975239	-.22
.8	1.253230	1.96
.9	1.099305	.82
1.0	1.000000	.00
1.1	1.015777	.14
1.2	1.013382	.12
1.3	.996401	-.03
1.4	1.047037	.40
1.5	1.052285	.44
1.6	.805991	-1.87
1.7	.561992	-5.01
1.8	.379652	-8.41
1.9	.215545	-13.33
2.0	.111804	-19.03
2.4	.015686	-36.09
3.0	.003881	-48.22
3.4	.002911	-50.72
4.0	.005323	-45.48
4.4	.044999	-26.94
5.0	.109943	-19.18
5.4	.006203	-44.15
6.0	.000549	-65.21
6.4	.000086	-81.31
7.0	.000012	-98.57
8.0	.000002	-115.84
9.0	.000001	-124.24
10.0	.000001	-124.78

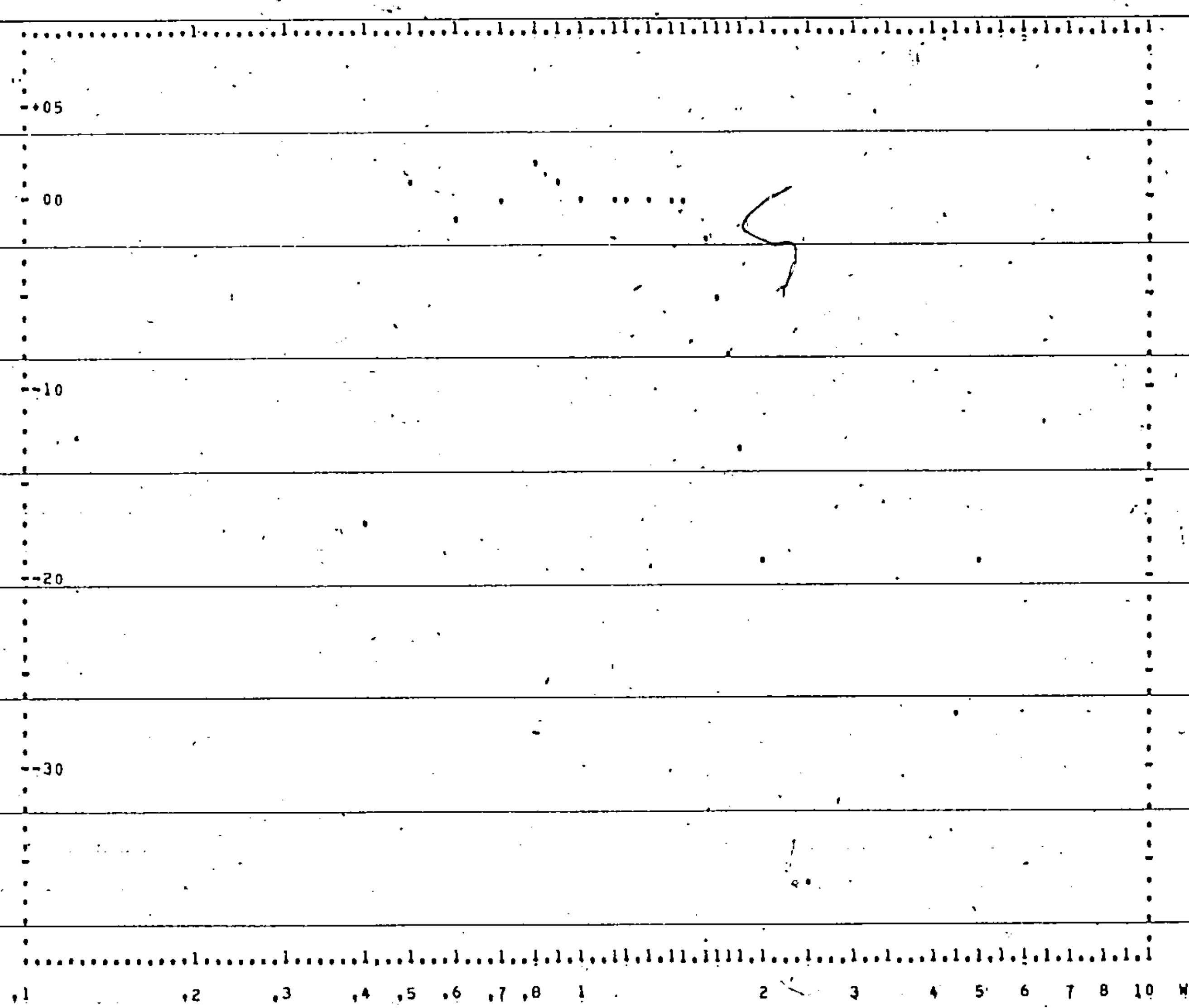

FIGURE 6.4.4.2 RESPONSE OF A T.L. + A B.P. BUTTERWORTH FILTER OF ORDER 8 (REFER TO FIG.6.1)

[POOR PRINT]

T= .20

W	TW	TWDB
0.0	0.000000	-200.00
.1	.000000	-163.67
.2	.000010	-99.93
.3	.001162	-58.70
.4	.046194	-26.71
.5	.839374	-1.52
.6	1.088421	.74
.7	.997344	-.02
.8	1.037138	.32
.9	1.084778	.71
1.0	1.000000	.00
1.1	1.003981	.03
1.2	1.040482	.34
1.3	1.027727	.24
1.4	1.013574	.12
1.5	.970894	-.26
1.6	.853416	-1.38
1.7	.794857	-1.99
1.8	.709228	-2.98
1.9	.325253	-9.76
2.0	.142135	-16.95
2.4	.013866	-37.16
3.0	.000740	-62.62
3.4	.000184	-74.70
4.0	.000040	-87.86
4.4	.000019	-94.31
5.0	.000009	-101.32
5.4	.000006	-104.41
6.0	.000005	-106.78
6.4	.000005	-106.73
7.0	.000007	-103.68
8.0	.000066	-83.59
9.0	.003695	-48.65
10.0	.000012	-98.67

TW IN DB

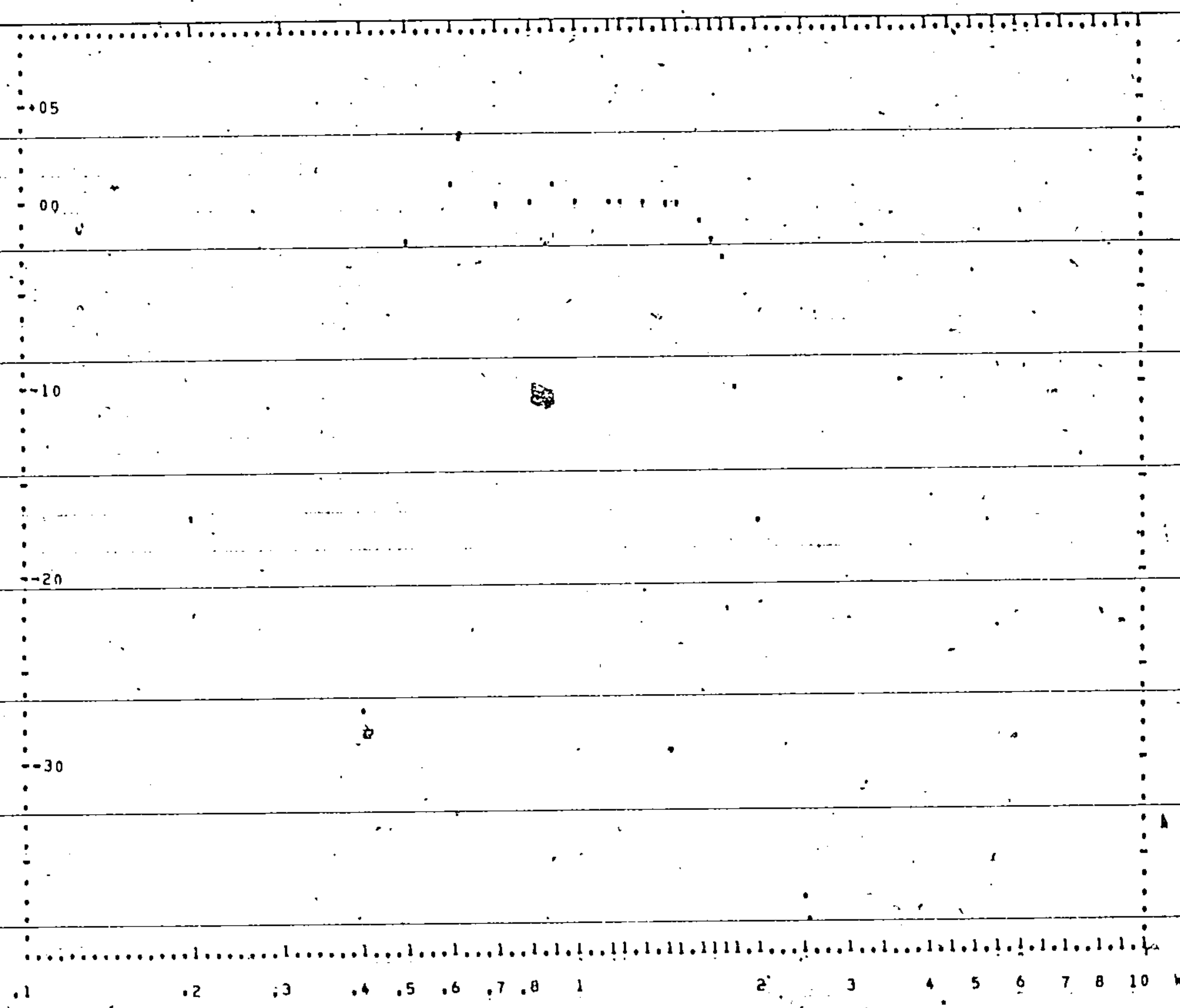

FIGURE 6.4.5.1 RESPONSE OF A T.L. + A.B.P. BUTTERWORTH FILTER OF ORDER 10 (REFER TO FIG.6.1)

6.5 The Lumped Section is a Band-Stop Filter

Applying the before-mentioned transformation (section 2.6) to the main computer program, the following set of curves was developed.

6.5.1 B.S. Filter (N = 2)

Figures 6.5.1.1 and 6.5.1.2 show the frequency response curves for a Band-Stop filter of order 2 calculated at $\tau = 0.2$ and 0.4 respectively. The frequency response varies in the R.H.S. of the first curve within 2.39 db and within 5.74 db for the second curve.

6.5.2 B.S. Filter (N = 4)

Figure 6.5.2.1 shows the frequency response curve for a Band-Stop filter of order 4 calculated at $\tau = 0.2$. The frequency response varies in the R.H.S. of the curve within 4.70 db.

6.5.3 B.S. Filter (N = 6)

Figure 6.5.3.1 shows the frequency response curve for a Band-Stop filter of order 6 calculated at $\tau = 0.2$. The frequency response varies in the R.H.S. of the curve within 6.92 db.

6.5.4 B.S. Filter (N = 8)

Figure 6.5.4.1 shows the frequency response curve of a Band-Stop filter of order 8 calculated at $\tau = 0.2$ The frequency response varies in the R.H.S. of the curve within 7.86 db.

6.5.5 B.S. Filter (N = 10)

Figure 6.5.5.1. shows the frequency response curve of a Band-Stop filter of order 10 calculated at $\tau = 0.2$. The frequency response varies in the R.H.S. of the curve within 10.01 db.

[POOR PRINT]

T= .20

W	TW	TWDB
0.0	1.000000	0.00
.1	.997552	-.02
.2	.985818	-.12
.3	.948258	-.46
.4	.850709	-1.40
.5	.662857	-3.57
.6	.423619	-7.46
.7	.219444	-13.17
.8	.087557	-21.15
.9	.019764	-34.08
1.0	0.000000	-200.00
1.1	.016943	-35.42
1.2	.064278	-23.84
1.3	.138791	-17.15
1.4	.238513	-12.45
1.5	.360961	-8.85
1.6	.501418	-6.00
1.7	.651712	-3.72
1.8	.800512	-1.93
1.9	.935796	-.58
2.0	1.048480	.41
2.4	1.260171	2.01
3.0	1.259260	2.00
3.4	1.224277	1.76
4.0	1.173921	1.39
4.4	1.144632	1.17
5.0	1.106468	.88
5.4	1.084260	.70
6.0	1.055055	.47
6.4	1.038038	.32
7.0	1.015866	.14
8.0	.987116	-.11
9.0	.967709	-.29
10.0	.956720	-.38

TW IN DB

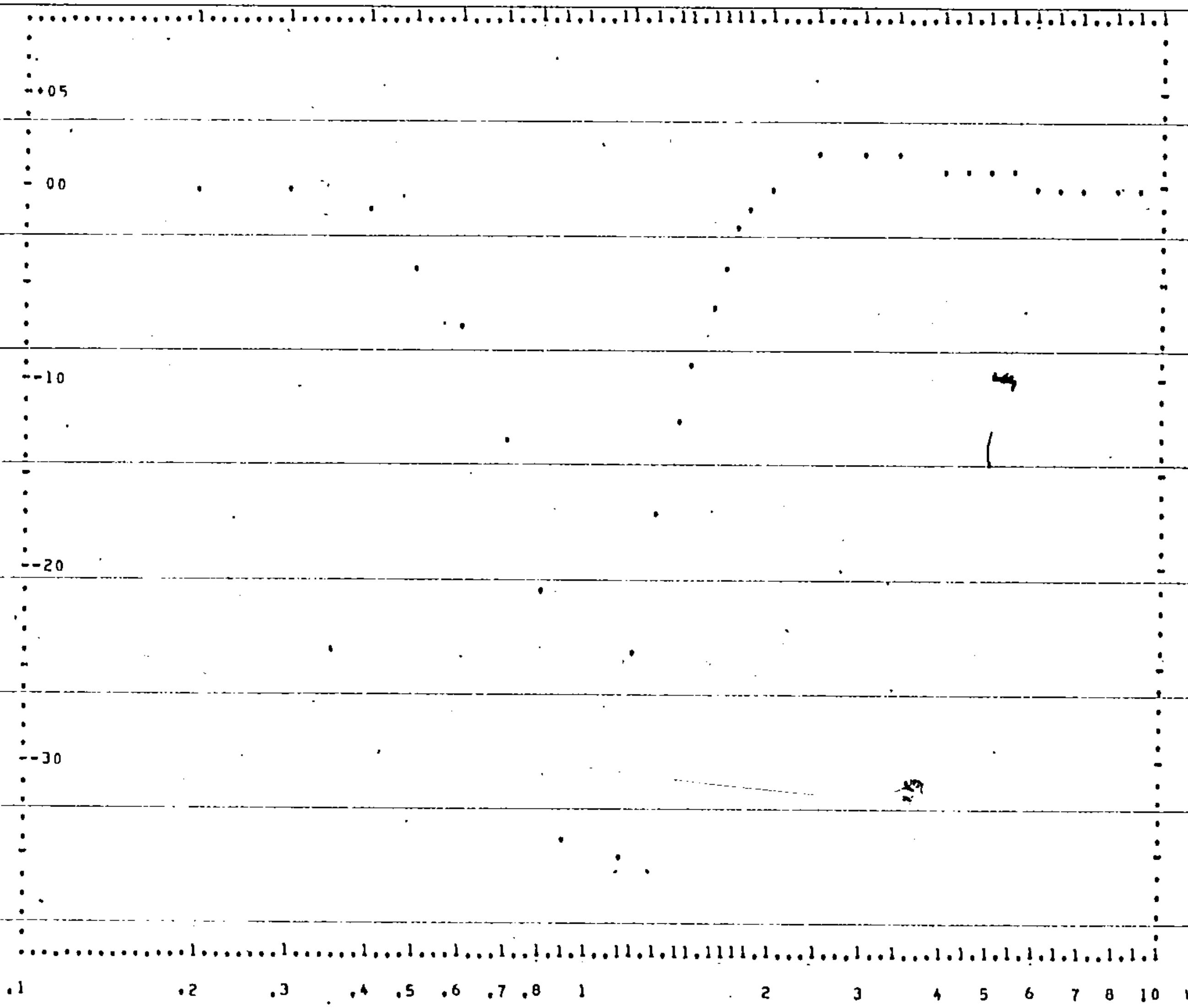

FIGURE 6.5.1.1 RESPONSE OF A T.L. + A B.S. BUTTERWORTH FILTER OF ORDER 2 (REFER TO FIG.6.1)

[POOR PRINT]

T= .40

W	TW	TWDB
0.0	1.000000	0.00
.1	.995381	-.04
.2	.976633	-.21
.3	.926831	-.66
.4	.816471	-1.76
.5	.628577	-4.03
.6	.404467	-7.86
.7	.214139	-13.39
.8	.087761	-21.13
.9	.020382	-33.81
1.0	0.000000	-200.00
1.1	.018796	-34.56
1.2	.074307	-22.58
1.3	.170158	-15.38
1.4	.315325	-10.02
1.5	.524339	-5.61
1.6	.811843	-1.81
1.7	1.168639	1.35
1.8	1.514193	3.60
1.9	1.716735	4.69
2.0	1.743183	4.83
2.4	1.400819	2.93
3.0	1.120197	.99
3.4	1.030565	.26
4.0	.953850	-.41
4.4	.924912	-.68
5.0	.903293	-.88
5.4	.900139	-.91
6.0	.908880	-.83
6.4	.922144	-.70
7.0	.950430	-.44
8.0	1.007531	.07
9.0	1.050572	.43
10.0	1.055047	.47

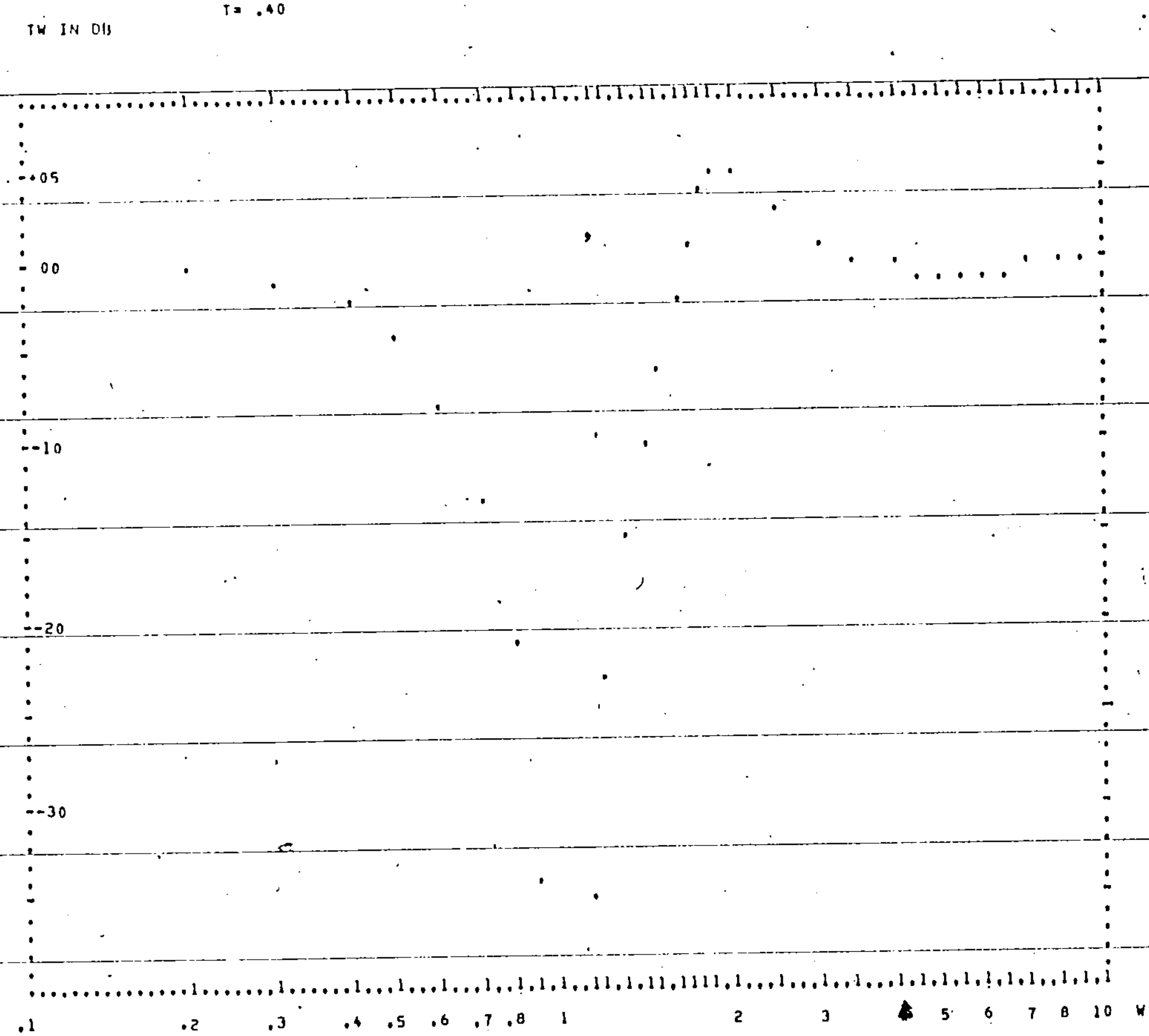

FIGURE 6.5.1.2 RESPONSE OF A T.L. + A B.S. BUTTERWORTH FILTER OF ORDER 2 (REFER TO FIG.6.1)

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[POOR PRINT]

T= .20

W	TW	TWDB
0.0	1.000000	0.00
.1	.996318	-.03
.2	.989615	-.09
.3	.989524	-.09
.4	.962079	-.34
.5	.618895	-4.17
.6	.199306	-14.01
.7	.047219	-26.52
.8	.007393	-42.62
.9	.000383	-68.34
1.0	0.000000	-200.00
1.1	.000294	-70.63
1.2	.004401	-47.13
1.3	.021876	-33.20
1.4	.071277	-22.94
1.5	.184477	-14.68
1.6	.383450	-8.33
1.7	.593492	-4.53
1.8	.725915	-2.78
1.9	.811346	-1.82
2.0	.888388	-1.03
2.4	1.244182	1.90
3.0	1.557493	3.85
3.4	1.447232	3.21
4.0	1.204311	1.61
4.4	1.[illegible]	.72
5.0	.971633	-.25
5.4	.925770	-.67
6.0	.888290	-1.03
6.4	.879067	-1.12
7.0	.883170	-1.08
8.0	.924052	-.69
9.0	.983749	-.14
10.0	1.028364	.24

TW IN DB

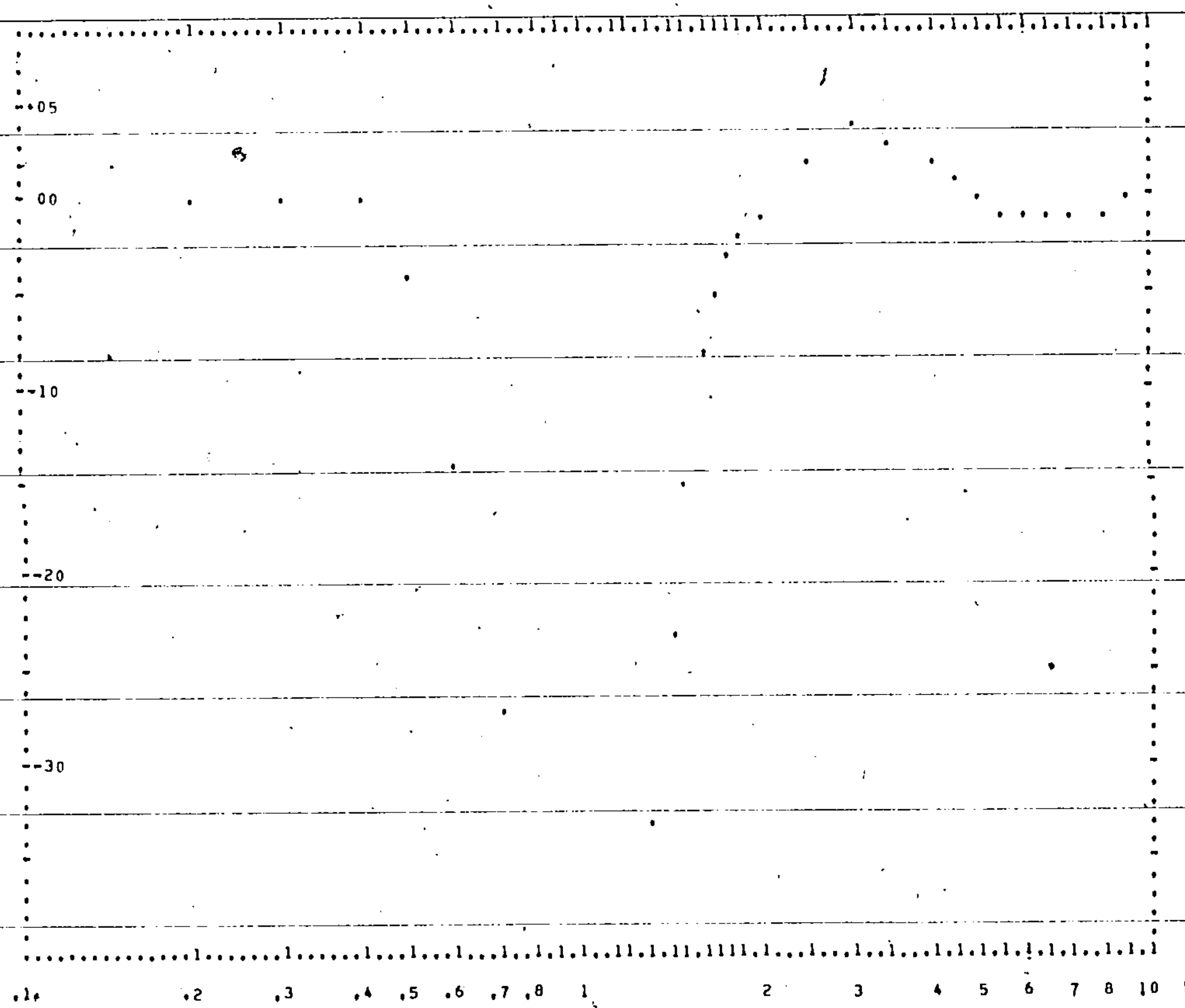

FIGURE 6.5.2.1 RESPONSE OF A T.L. + A B.S. BUTTERWORTH FILTER OF ORDER 4 (REFER TO FIG.6.1)

[POOR PRINT]

T= .20

W	TW	TWDB
0.0	1.000000	0.00
.1	.995359	-.04
.2	.994067	-.05
.3	1.008677	.08
.4	.972875	-.24
.5	.593276	-4.53
.6	.093838	-20.55
.7	.009862	-40.12
.8	.000610	-64.29
.9	.000007	-102.68
1.0	0.000000	-200.00
1.1	.000005	-105.74
1.2	.000310	-70.17
1.3	.003584	-48.91
1.4	.020732	-33.67
1.5	.073617	-22.66
1.6	.189054	-14.47
1.7	.437127	-7.19
1.8	.908510	-.83
1.9	1.083777	.70
2.0	.955120	-.40
2.4	.974015	-.23
3.0	1.812668	5.17
3.4	1.693320	4.57
4.0	1.111715	.92
4.4	.940379	-.53
5.0	.834959	-1.57
5.4	.817797	-1.75
6.0	.837565	-1.54
6.4	.869666	-1.21
7.0	.927389	-.65
8.0	.986939	-.11
9.0	.981366	-.16
10.0	.978654	-.19

TW IN DB

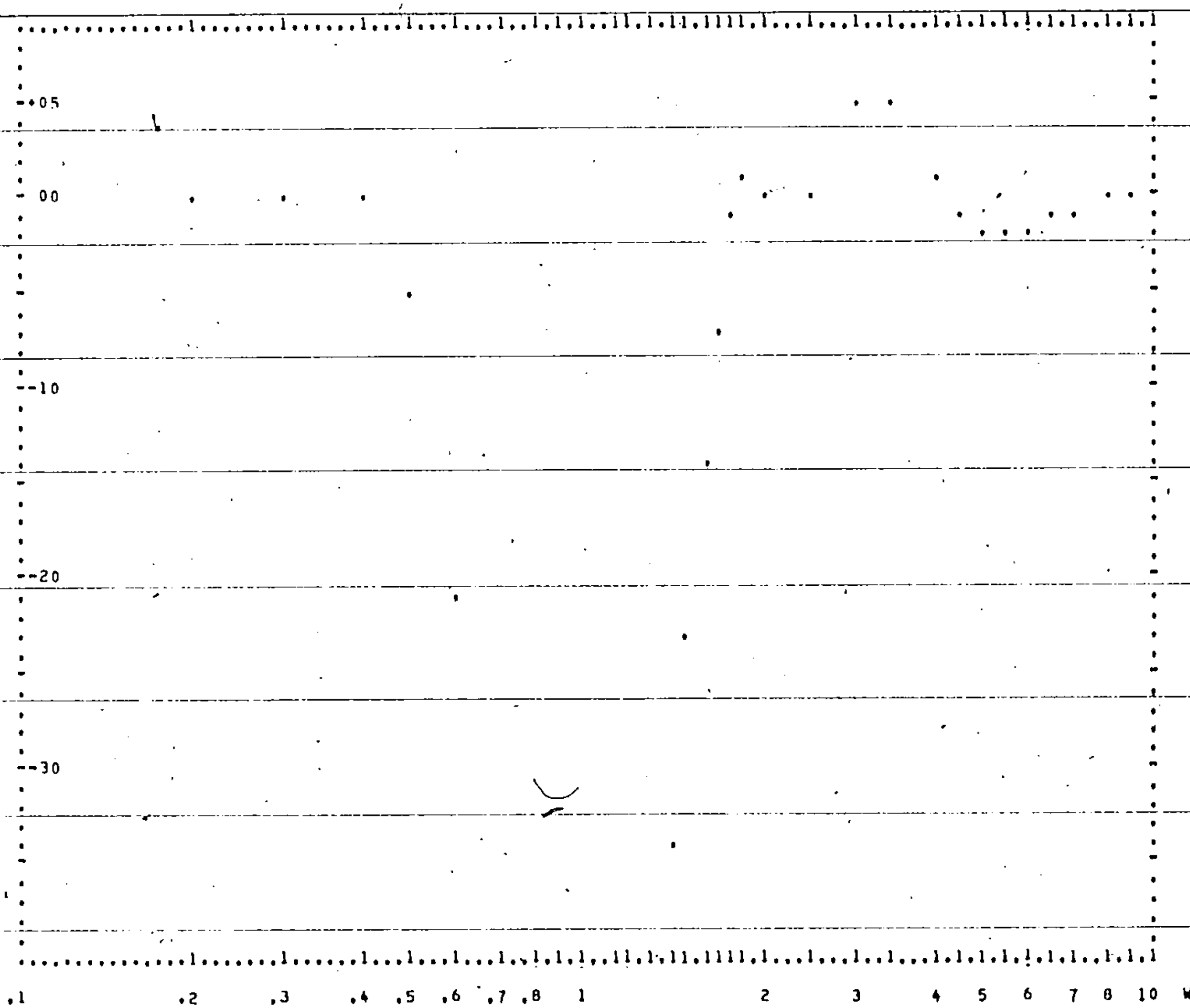

FIGURE 6.5.3.1 RESPONSE OF A T.L. + A.B.S. BUTTERWORTH FILTER OF ORDER 6 (REFER TO FIG.6.1)

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[POOR PRINT]

T= .20

W	TW	TWDB
0.0	1.000000	0.00
.1	.992706	-.06
.2	.975843	-.21
.3	.978256	-.19
.4	1.119015	.98
.5	.553624	-5.14
.6	.047749	-26.42
.7	.002028	-53.86
.8	.000049	-86.18
.9	.000000	-137.09
1.0	0.000000	-200.00
1.1	.000000	-140.79
1.2	.000022	-93.15
1.3	.000549	-65.21
1.4	.004953	-46.10
1.5	.027676	-31.16
1.6	.139684	-17.10
1.7	.478072	-6.41
1.8	.753061	-2.46
1.9	1.298215	2.27
2.0	1.430423	3.11
2.4	.784516	-2.11
3.0	1.772761	4.97
3.4	1.906333	5.60
4.0	.965905	-.30
4.4	.812934	-1.80
5.0	.770820	-2.26
5.4	.799467	-1.94
6.0	.870489	-1.20
6.4	.907043	-.85
7.0	.923731	-.69
8.0	.929558	-.63
9.0	.992670	-.06
10.0	1.036717	.31

TW IN DB

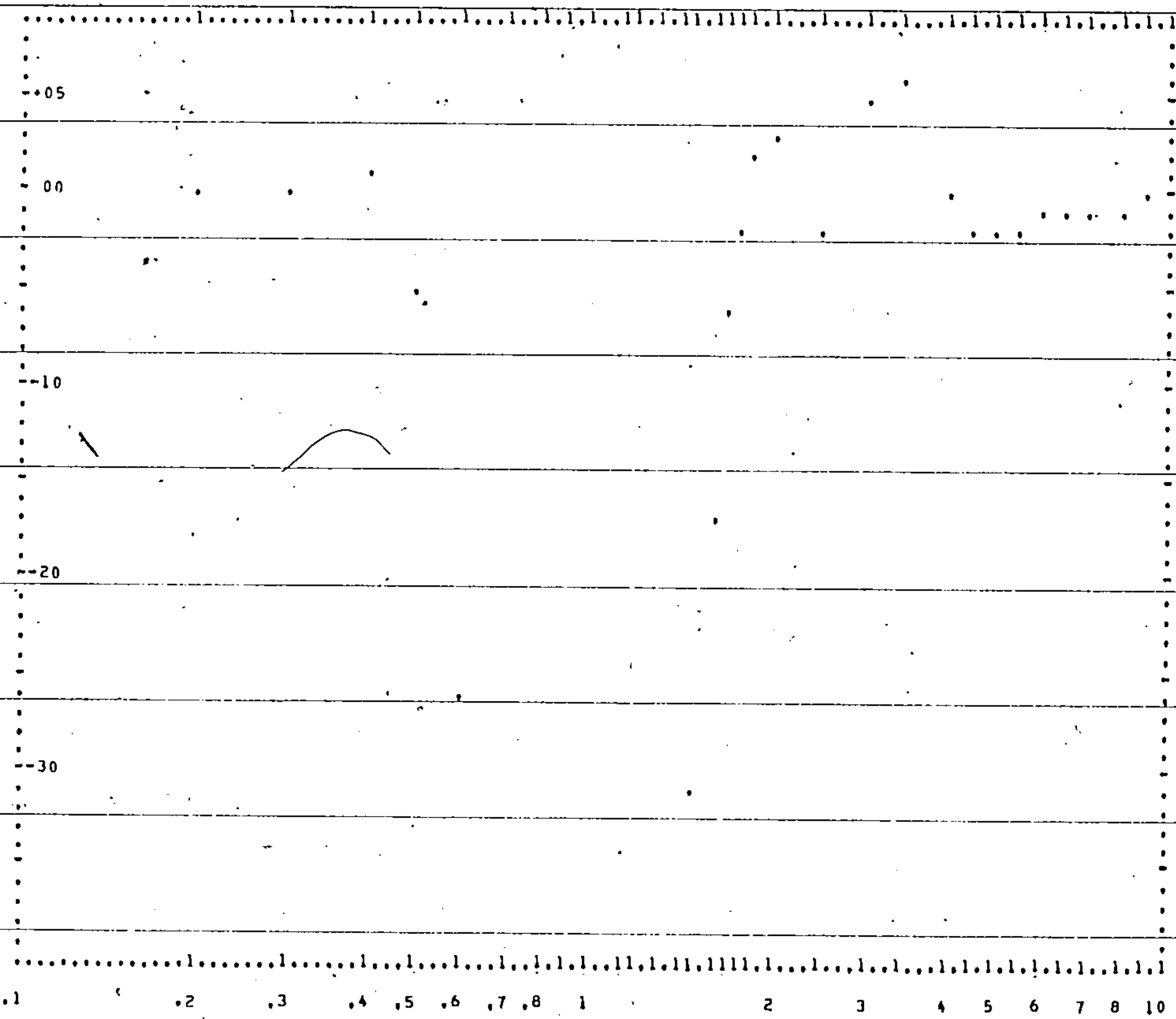

FIGURE 6.5.4.1 RESPONSE OF A T.L. + A B.S. BUTTERWORTH FILTER OF ORDER 8 (REFER TO FIG.6.1)

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POOR PRINT

T= .20

W	TW	TWDB
0.0	1.000000	0.00
.1	.995253	-.04
.2	1.015477	.13
.3	1.062073	.52
.4	.903661	-.88
.5	.490050	-6.20
.6	.020637	-33.71
.7	.000490	-66.20
.8	.000004	-107.74
.9	.000000	-171.53
1.0	0.000000	-200.00
1.1	.000000	-175.74
1.2	.000002	-115.81
1.3	.000086	-81.30
1.4	.001293	-57.77
1.5	.012047	-38.38
1.6	.064195	-23.85
1.7	.241910	-12.33
1.8	.945476	-.49
1.9	.718725	-2.87
2.0	.862096	-1.29
2.4	.863275	-1.28
3.0	1.396994	2.90
3.4	2.334896	7.37
4.0	.871879	-1.19
4.4	.737661	-2.64
5.0	.745512	-2.55
5.4	.797334	-1.97
6.0	.857722	-1.33
6.4	.876396	-1.15
7.0	.911238	-.81
8.0	.999167	-.01
9.0	.978805	-.19
10.0	.972083	-.25

TW IN DB

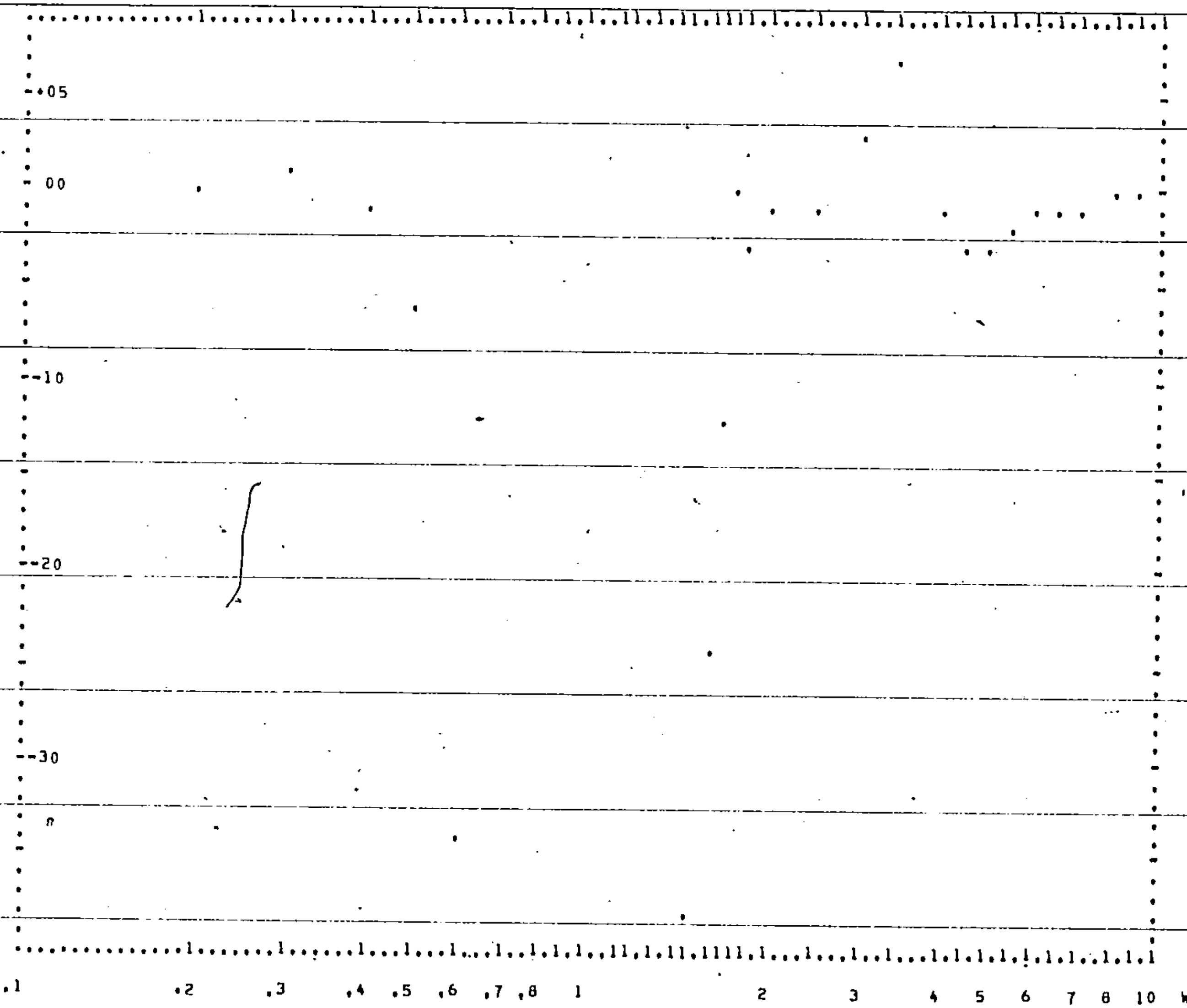

FIGURE 6.5.5.1 RESPONSE OF A T.L. + A B.S. BUTTERWORTH FILTER OF ORDER 10 (REFER TO FIG.6.1)

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6.6 Discussion

The study of the transmission line effects on the Low pass case revealed that there is always an improvement in the selectivity combined with a reduction of the band width. Under certain combination ($N = 8$, $\tau = 0.4$), there is < 1.0 db peaking effect at the corner frequency. For $N = 10$, $\tau = 0.4$, there is a net reduction in the band width. Finally, the overall behaviour of this case is very much similar to the one given in section 3.2.

The high pass case showed the ripple effect in the pass band. The band width is always increased whenever the effect of the transmission line has a $\tau > 0.2$. The acceptable Butterworth order for a smooth high frequency response is $N < 6$. For values $N \geq 6$, the ripple peak-to-peak amplitude is < 7 db. It is concluded that the addition of the transmission line into this case increases the band width, and for $N \leq 4$, the ripple effect can be tolerated.

The band-pass case shows a marked improvement in all cases where $N > 4$. The band-pass ripple effect is very limited in appearance with a peak-to-peak swing of < 1.0 db. However there is a minor selectivity curve appearing at $w = 9$, but with a highly attenuated effect (attenuation at $w = 9$ is -26.3 db (for $\tau = 0.2$, $N = 6$) and better for the rest of cases). This particular case is the most significant of all cases tested.

The band-stop case shows the rippling effect in the high frequency pass-band. There is a marked improvement of the corner effect, though at the expense of the untolerable ripple swings. The ripple peak-to-peak swing ≈ 10 db.

From this it is concluded that this example is only of relevance for High-Pass and Band-Pass cases. It is not only the selection of the "τ" value of the transmission line but also the order of the Butterworth filter that determines the suitability of the combination to the application at hand.

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CHAPTER VII

SUMMARY AND DISCUSSION

- In general, the addition of a lossless transmission line introduced:

  a) Rippling effect mainly in the $w > 1$ region.

  b) A shift of the 3 db point towards the lower frequency side.

  c) A sharper corner frequency response than that without the transmission line.

- The selectivity is determined mainly by the Butterworth filter order. Little or no effect is contributed to the transmission line in that respect.

- The band width is affected as follows:-

  a) Reduction for the Low-pass case.

  b) Increase for the High-pass case.

- From the cases considered in chapters (5) and (6) for the band-pass filter, there are certain examples of very good filter responses where the pass band ripples are within engineering design criteria. For other combinations, there is a marked disturbance in the response symmetry.

- Except for one particular case the Band Stop filter shows disturbance ripples in the high frequency pass-band upon the introduction of the transmission line. The particular case occurs with the circuit studied in chapter (5) (when $N = 8$, $\tau = 0.6$) and which yields an equi-ripple behaviour within $< \pm 0.6$ db. (Figure 5.5.4.2)

- At selected frequencies, it is possible to increase the band width.

- Optimization to the promising cases pointed out throughout the various foregoing discussions is recommended. This is particularly important for the band pass cases.

[POOR PRINT]

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[POOR PRINT]

APPENDIX

COMPUTER PROGRAM

Response calculation and Plotting for a filter consisting of a lossless transmission line in series with passive lumped element. Nth order Butterworth filter.

```
                    73/172  OPT=1                     FTN 4.6+446        78/02/14. 18.33.04        PAGE    1

      PROGRAM MAURICE(INPUT,OUTPUT,TAPE5=INPUT)
C
C
C     TITLE
C             RESPONSE CALCULATION AND PLOTTING FOR A
C
C                    FILTER CONSISTING OF
C
C             A LOSSLESS TRANSMISSION LINE IN SERIES
C
C             WITH PASSIVE LUMPED ELEMENT N-ORDER
C
C                    BUTTERWORTH FILTER.
C
C
C
C     PURPOSE
C         FOR VARIOUS VALUES OF TRANSMISSION LINE DELAY,
C         AND
C         FOR VARIOUS ORDERS OF BUTTERWORTH PASSIVE LUMPED FILTER,
C         A-TO CALCULATE THE OVERALL FREQUENCY/ATTENUATION
C           CHARACTERISTICS OF THE COMBINED ARRANGEMENT.
C           THESE ARRANGEMENTS COMPRISE
C             1-LOSSLESS TRANSMISSION LINE IN SERIES WITH
C               BUTTERWORTH FILTER OF THE ORDER  N.
C             2-LOSSLESS TRANSMISSION LINE IN SERIES WITH
C               BUTTERWORTH FILTER OF THE SECOND ORDER TWICE.
C             3-LOSSLESS TRANSMISSION LINE IN SERIES WITH,
C               BUTTERWORTH FILTER OF THE NTH ORDER 3 TIMES.
C             4-LOSSLESS TRANSMISSION LINE IN SERIES WITH
C               A UNIT LC BUTTERWORTH SECTION REPEATED N TIMES,
C         B-TO PLOT THE CALCULATED FREQUENCY/ATTENUATION
C           CHARACTERISTICS AS CALCULATED ABOVE FOR THE FOLLOWING
C             1-LOW-PASS FILTER
C             2-HIGH-PASS FILTER,
C             3-BAND-PASS FILTER,
C             4-BAND STOP FILTER.
C         IN ALL THESE CASES , THE FREQUENCY IS NORMALIZED , AND
C         THE ATTENUATION IS GIVEN IN DECIBELS.
C
C
C     LANGUAGE
C         STANDARD FORTRAN IV
C
C
C     DATA
C         THE NECESSARY DATA TO RUN THIS PROGRAM ARE
C             1-  N
C               THIS IS THE ORDER OF THE BUTTERWORTH FUNCTION
C             2-L1,L2,L3,L4,L5,C1,C2,C3,C4,C5
C               THESE ARE THE BUTTERWORTH COEFFICIENTS
C         NOTE THAT EACH VALUE IS GIVEN ON A SEPERATE CARD.
C
C
C     REFERENCE
C         VAN VALKENBURG,M.E.,
C             INTRODUCTION TO NETWORK SYNTHESIS
C         JOHN WILY AND SONS,NEW YORK,1959
```

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[POOR PRINT]


```
C
C
      COMPLEX B1,B2,B3,BK,TH,X
      DIMENSION B1(2,2),B2(2,2),B3(2,2),BK(5,2,2),TH(2,2),X(2,2)
      DIMENSION TW(34),OM(34),TWDB(34)
      DIMENSION AL(5),AK(5),C(5)
      DIMENSION AH(34)
      DIMENSION TAW(5,4,3)
      CALL OMEGA(OM,AH,34)
      DO 149 J=1,5
      DO 150 K=1,4
      READ 151,(TAW(J,K,L),L=1,3)
  151 FORMAT(3F4.2)
  150 CONTINUE
  149 CONTINUE
      Z=1.0
      R=1.0
      DO 555 LL=4,4
      IF(LL.EQ.1)GO TO  191
      IF(LL.EQ.2)GO TO 192
      IF(LL.EQ.3)GO TO 192
      IF(LL.EQ.4)GO TO 191
  192 TW(1)=0.0
      GO TO 193
  191 TW(1)=1.0
C
C                     READING OF THE BUTTERWORTH FUNCTION ORDER.
  193 READ 111,N
  111 FORMAT(I2)
C
C                     READING OF THE BUTTERWORTH FUNCTION COEFFICIENTS
C
C                     NOTE THAT COEFFICIENTS L1 TO L5 ARE IN AL(I).
C
C                     NOTE THAT COEFFICIENTS C1 TO C5 ARE IN C(I)
C
   78 READ 113,(AL(I),I=1,N)
  113 FORMAT(F11.9)
      READ 114,(C(I),I=1,N)
  114 FORMAT(F11.9)
      DO 199 I=1,N
      AK(I)=AL(I)*C(I)
  199 CONTINUE
      DO 8 MMM=1,4
      DO 50 NK=1,3
      T=TAW(N,MMM,NK)
      PRINT 17,T
   17 FORMAT(1H1,/,50X,* T=*,F4.2)
      PRINT 11
   11 FORMAT(3X,*W*,8X,*TW*,8X,*TWDB*,7X,*TW IN DB*)
      DO 20 I=2,34
      W=OM(I)
      X(1,1)=(1.0,0.0)*COS(W*T)
      X(1,2)=(0.0,1.0)*Z*SIN(W*T)
      X(2,1)=(0.0,1.0)*(SIN(W*T)/Z)
      X(2,2)=(1.0,0.0)*COS(W*T)
      TH(1,I)=(1.0,0.0)
```

```
PROGRAM MAURICE     73/172  OPT=1                    FTN 4.6+446        78/02/14. 18.33.04        PAGE     3

          TM(1,2)=(0.0,0.0)
          TM(2,1)=(1.0,0.0)*(1.0/R).
          TM(2,2)=(1.0,0.0)
          CALL BAND(W,S,LL)
          IF(S.GT.10.0**(8.0))GO TO 21
          DO 95 J=1,N
          BK(J,1,1)=(1.0,0.0)*(1.0-S*S*AK(J))
          BK(J,1,2)=(0.0,1.0)*(S*AL(J))
          BK(J,2,1)=(0.0,1.0)*S*C(J)
          BK(J,2,2)=(1.0,0.0)
       95 CONTINUE
          DO 96 J=1,2
          DO 100 K=1,2
      100 B2(J,K)=TM(J,K)
       96 CONTINUE
          IF(MMM.EQ.1)GO TO 81
          IF(MMM.EQ.2)GO TO 82
          IF(MMM.EQ.3)GO TO 83
          IF(MMM.EQ.4)GO TO 84
       81 CALL AFIG1(BK,B2,X,N)
          GO TO 89
       82 CALL AFIG2(BK,B2,X,N)
          GO TO 89
       83 CALL AFIG3(BK,B2,X,N)
          GO TO 89
       84 CALL AFIG4(BK,B2,X,N)
       89 ABC=CABS(B2(1,1))
          TW(I)=1.0/ABC
          IF (TW(I).GT.5.0)GO TO 50
          GO TO 20
       21 TW(I)=0.0
       20 CONTINUE
          CALL SCALE(TW,TWDB,34)
          CALL PLOT(AM,TWDB,34,OM,TW)
          CALL TITLE(N,MMM,LL,NK)
       50 CONTINUE
        8 CONTINUE
          N=N-1
          IF(N.GT.0)GO TO 78
          DO 7 I=1,31
        7 BACKSPACE5
      555 CONTINUE
          STOP
          END
```

```
SUBROUTINE OMEGA       73/172  OPT=1                FTN 4.6+446       78/02/14. 18.33.04       PAGE    1

             SUBROUTINE OMEGA(OM,AH,N)
      C
      C                THIS SUBROUTINE CALCULATES THE VALUES OF W NEEDED
      C                OM IS THE LINEAR VALUE OF W
      C                AH IS THE LOGARETHMIC VALUE OF,W
      C                N IS THE NUMBER OF POINTS REQUIRED
      C
             DIMENSION OM(N),AH(N)
             OM(1)=0.0
             AH(1)=10**(-12)
             BB=0.0
             DO 31 I=2,N
             IF(OM(I-1).GT.1.9)GO TO 34
             OM(I)=OM(I-1)+0.1
             GO TO 31
          34 IF(OM(I-1).GT.6.4)GO TO 37
             IF(BB.EQ.0.0)GO TO 2
             IF(BB.EQ.1.0)GO TO 3
           2 OM(I)=OM(I-1)+0.4
             BB=1.0
             GO TO 31
           3 OM(I)=OM(I-1)+0.6
             BB=0.0
             GO TO 31
          37 OM(I)=OM(I-1)+1.0
          31 CONTINUE
             DO 32 I=2,N
          32 AH(I)=ALOG10(OM(I))+1.0
             RETURN
             END

SUBROUTINE BAND        73/172  OPT=1                FTN 4.6+446       78/02/14. 18.33.04       PAGE    1

             SUBROUTINE BAND(W,S,LL)
             W0=1.0
             B=1.5
             IF(LL.EQ.1)GO TO 1
             IF(LL.EQ.2)GO TO 2
             IF(LL.EQ.3)GO TO 3
             IF(LL.EQ.4)GO TO 4
           1 S=W
             GO TO 10
           2 S=-W0/W
             GO TO 10
           3 S=(W0/B)*((W/W0)-(W0/W))
             GO TO 10
           4 IF(W.EQ.1.0)GO TO 5
             S=1.0/((W0/B)*((W0/W)-(W/W0)))
             GO TO 10
           5 S=10.0**(10.0)
          10 RETURN
             END
```

```
SUBROUTINE TITLE       73/172  OPT=1                       FTN 4.6+446        78/02/14. 18.33.04        PAGE    1

          SUBROUTINE TITLE(N,MMM,LL,NK)
          K1=MMM+2
          K2=LL+1
          K3=2*N
          IF(LL.EQ.1)GO TO 1
          IF(LL.EQ.2)GO TO 2
          IF(LL.EQ.3)GO TO 3
          IF(LL.EQ.4)GO TO 4
        1 PRINT 11,K1,K2,N,NK,K3,K1
       11 FORMAT(//,22X,*FIGURE *,I1,*.*,I1,*.*,I1,*.*,I1,
         1* RESPONSE OF A T.L. +*,
         1* A L.P. BUTTERWORTH FILTER OF ORDER *,I2,5X,*( REFER TO FIG.*,
         1I1,*.1 )*)
          GO TO 20
        2 PRINT 12,K1,K2,N,NK,K3,K1
       12 FORMAT(//,22X,*FIGURE *,I1,*.*,I1,*.*,I1,*.*,I1,
         1* RESPONSE OF A T.L. +*,
         1* A H.P. BUTTERWORTH FILTER OF ORDER *,I2,5X,*( REFER TO FIG.*,
         1I1,*.1 )*)
          GO TO 20
        3 PRINT 13,K1,K2,N,NK,K3,K1
       13 FORMAT(//,22X,*FIGURE *,I1,*.*,I1,*.*,I1,*.*,I1,
         1* RESPONSE OF A T.L. +*,
         1* A B.P. BUTTERWORTH FILTER OF ORDER *,I2,5X,*( REFER TO FIG.*,
         1I1,*.1 )*)
          GO TO 20
        4 PRINT 14,K1,K2,N,NK,K3,K1
       14 FORMAT(//,22X,*FIGURE *,I1,*.*,I1,*.*,I1,*.*,I1,
         1* RESPONSE OF A T.L. +*,
         1* A B.S. BUTTERWORTH FILTER OF ORDER *,I2,5X,*( REFER TO FIG.*,
         1I1,*.1 )*)
       20 RETURN
          END
```

```
SUBROUTINE AMIR        73/172  OPT=1                       FTN 4.6+446        78/02/14. 18.33.04        PAGE    1

          SUBROUTINE AMIR(A,B,R,N,M,L)
C
C                   THIS SUBROUTINE MULTIPLIES TWO COMPLEX MATRICES
C                   MATRIX A IS MULTIPLIED BY MATRIX B AND THE
C                   RESULT IS STORED IN MATRIX R
C                   ALL MATRICES ARE OF DIMENSION 2X2
C
          COMPLEX A,B,R
          DIMENSION A(2,2),B(2,2),R(2,2)
          DO 10 I=1,M
          DO 11 K=1,L
          R(I,K)=(0.0,0.0)
          DO 12 J=1,M
       12 R(I,K)=R(I,K)+A(I,J)*B(J,K)
       11 CONTINUE
       10 CONTINUE
          RETURN
          END
```

[POOR PRINT]


```
SUBROUTINE AFIG1       73/172  OPT=1                    FTN 4.6+446        78/02/14. 18.33.04        PAGE    1

      SUBROUTINE AFIG1(BK,B2,X,N)
C
C                     THIS SUBROUTINE CALCULATES THE REQUIRMENT NUMBER 1
C
      COMPLEX BK,B1,B2,B3,X
      DIMENSION BK(5,2,2),X(2,2),B3(2,2),B2(2,2),B1(2,2)
      DO 1 L=1,N
      DO 2 J=1,2
      DO 3 K=1,2
      B1(J,K)=BK(L,J,K)
    3 CONTINUE
    2 CONTINUE
      CALL AMIR(B1,B2,B3,2,2,2)
      DO 4 J=1,2
      DO 5 K=1,2
    5 B2(J,K)=B3(J,K)
    4 CONTINUE
    1 CONTINUE
      CALL AMIR(X,B3,B2,2,2,2)
      RETURN
      END
```

```
SUBROUTINE AFIG2       73/172  OPT=1                    FTN 4.6+446        78/02/14. 18.33.04        PAGE    1

      SUBROUTINE AFIG2(BK,B2,X,N)
C
C                     THIS SUBROUTINE CALCULATES THE REQUIRMENT NUMBER 2
C
      COMPLEX BK,B1,B2,B3,X
      DIMENSION BK(5,2,2),X(2,2),B3(2,2),B2(2,2),B1(2,2)
      DO 7 I=1,2
      DO 1 L=1,N
      DO 2 J=1,2
      DO 3 K=1,2
      B1(J,K)=BK(L,J,K)
    3 CONTINUE
    2 CONTINUE
      CALL AMIR(B1,B2,B3,2,2,2)
      DO 4 J=1,2
      DO 5 K=1,2
    5 B2(J,K)=B3(J,K)
    4 CONTINUE
    1 CONTINUE
      CALL AMIR(X,B3,B2,2,2,2)
    7 CONTINUE
      RETURN
      END
```

```
SUBROUTINE AFIG3       73/172  OPT=1                    FTN 4.6+446        78/02/14. 18.33.04        PAGE     1

              SUBROUTINE AFIG3(BK,B2,X,N)
        C
        C                 THIS SUBROUTINE CALCULATES THE REQUIRMENT NUMBER 3
        C
              COMPLEX BK,B1,B2,B3,X
              DIMENSION BK(5,2,2),X(2,2),B3(2,2),B2(2,2),B1(2,2)
              DO 7 I=1,3
              DO 1 L=1,N
              DO 2 J=1,2
              DO 3 K=1,2
              B1(J,K)=BK(L,J,K)
            3 CONTINUE
            2 CONTINUE
              CALL AMIR(B1,B2,B3,2,2,2)
              DO 4 J=1,2
              DO 5 K=1,2
            5 B2(J,K)=B3(J,K)
            4 CONTINUE
            1 CONTINUE
              CALL AMIR(X,B3,B2,2,2,2)
            7 CONTINUE
              RETURN
              END
```

```
SUBROUTINE AFIG4       73/172  OPT=1                    FTN 4.6+446        78/02/14. 18.33.04        PAGE     1

              SUBROUTINE AFIG4(BK,B2,X,N)
        C
        C                 THIS SUBROUTINE CALCULATES THE REQUIRMENT NUMBER 4
        C
              COMPLEX BK,B1,B2,B3,X
              DIMENSION BK(5,2,2),X(2,2),B3(2,2),B2(2,2),B1(2,2)
              DO 4 J=1,2
              DO 5 K=1,2
              B1(J,K)=(0.0,0.0)
            5 B3(J,K)=(0.0,0.0)
            4 CONTINUE
              DO 1 L=1,N
              DO 2 J=1,2
              DO 3 K=1,2
              B1(J,K)=BK(L,J,K)
            3 CONTINUE
            2 CONTINUE
              CALL AMIR(B1,B2,B3,2,2,2)
              CALL AMIR(X,B3,B2,2,2,2)
            1 CONTINUE
              RETURN
              END
```

[POOR PRINT]


```
SUBROUTINE PLOT        73/172  OPT=1                          FTN 4.6+446        78/02/14. 18.33.04        PAGE    1

              SUBROUTINE PLOT(X,Y,N,Z,W)
      C
      C
      C              THIS SUBROUTINE PLOTS ALLL POINTS IN ONE PAGE.
      C
      C              X  IS ONE DIMENTIONAL ARRAY OF N-ELEMENTS FOR X- VALUES,
      C              Y  IS ONE DIMENSIONAL ARRAY OF N-ELEMENTS FOR Y-VALUES.
      C              Z  IS THE UNSCALED X VALUE,
      C              W  IS THE UNSCALED Y VALUE.
      C              N  IS THE TOTAL NUMBER TO BE PLOTTED.
      C
      C              NOTE
      C                   ALL POINTS MUST BE MADE OF POSITIVE VALUE BEFORE
      C                   CALLING THIS SUBROUTINE.
      C
              DIMENSION W(N),X(N),Y(N),Z(N),A(50,120)
              DO 2 I=1,50
              DO 1 J=1,120
            1 A(I,J)=1H
            2 CONTINUE
              D1=0.0
              DO 3 I=1,N
              D2=ABS(Y(I))
              IF(D2.LT.D1)GO TO 3
              D1=D2
            3 CONTINUE
              D1=50.0
              D2=1.0
              DO 5 J=1,120
              A(50,J)=1H.
            5 A(1,J)=1H.
              DO 4 I=1,N
              J=IFIX(X(I)*D1)+1
              IF(Y(I).GT.9.0)GO TO 199
              IF(Y(I)+40.0)199,199,299
          199 K=1
              GOTO 399
          299 K=IFIX((Y(I)+40.5)*D2)+1
          399 A(K,J)=1H.
              A(1,J)=1H1
              A(50,J)=1H1
            4 CONTINUE
              DO 7 I=1,50
              A(I,100)=1H.
            7 A(I,1)=1H.
              DO 8 I=1,9
              J=I*5+1
              A(J,1)=1H-
              A(J,100)=1H-
            8 CONTINUE
              A(1,100)=1H1
              A(50,100)=1H1
              A(46,2)=1H+
              A(46,3)=1H0
              A(46,4)=1H5
              A(41,2)=1H
              A(41,3)=1H0
```

```
SUBROUTINE PLOT        73/172  OPT=1                FTN 4.6+446.       78/02/14. 18.33.04        PAGE    2

      A(4),4)=1H0
      A(3),2)=1H-
      A(3),3)=1H1
      A(3),4)=1H0
      A(2),2)=1H-
      A(2),3)=1H2
      A(2),4)=1H0
      A(1),2)=1H-
      A(1),3)=1H3
      A(1),4)=1H0
      PRINT 13
   13 FORMAT(//)
      DO 10 I=1,50
      L=51-I
      IF(I.GT.N)GO TO 27
      PRINT 9,Z(I),W(I),Y(I),(A(L,K),K=1,100)
    9 FORMAT(1X,F5.1,2X,F10.6,2X,F7.2,5X,100A1)
      GOTO10
   27 PRINT 17,(A(L,K),K=1,100)
   17 FORMAT(32X,100A1)
   10 CONTINUE
      PRINT 12
   12 FORMAT(/,31X,*,1*,13X,*,2*,6X,*,3*,5X,*,4*,2X,*,5*,2X,*,6*,
     12X,*,7*,1X,*,8*,3X,*1*,15X,*2*,7X,*3*,6X,*4*,3X,*5*,3X,*6*,3X,*7*,
     12X,*8*,2X,*10*,2X,*W*)
      RETURN
      END
```

```
SUBROUTINE SCALE       73/172  OPT=1                FTN 4.6+446        78/02/14. 18.33.04        PAGE    1

      SUBROUTINE SCALE(TW,TWDB,N)
      DIMENSION TW(N),TWDB(N)
      DO 1 I=1,N
      TWDB(I)=0.0
      IF(TW(I).LT.(10.0**(-10.)))GO TO 5
      TWDB(I)=20*(ALOG10(TW(I)))
      GO TO 1
    5 TWDB(I)=-200.0
    1 CONTINUE
      RETURN
      END
```