SO_2 DISTRIBUTION IN THE MONTREAL CITY CENTER

Jean-Pierre Basire

A Major Technical Report
In The
Faculty of Engineering

Presented in partial fulfilment of the requirements for
the Degree of Master of Engineering at
CONCORDIA UNIVERSITY
Montreal, Canada

August, 1978

ABSTRACT

SO_2 DISTRIBUTIONS IN THE MONTREAL CITY CENTER

JEAN-PIERRE BASIRE

Background concentrations of sulphur dioxide in ambient air are estimated by applying the K.W. Ragland "Multiple box model" to the Montreal city center. To utilize the model, four input parameters are necessary. They are: the mean concentration per box, the source strength, the advection and the diffusion. Routine local measurements of SO_2 concentrations, temperatures and wind velocities are used to calculate the four input parameters.

Initially, a method to estimate the surface wind velocities at city center in a neutral atmosphere is developed. Based on the estimated wind velocities a vertical wind velocity profile is developed. It compares well with that deduced from measurements in other North American cities in neutral atmosphere.

As a second step, three box model set ups are developed, however, low background concentrations are obtained below the mean roof level. They are mainly due to the inaccuracy of the model below the mean roof level.

Above the mean roof level, good results are obtained for box set ups having equal height. Smaller box height provide a better accuracy of estimation.

ACKNOWLEDGMENTS

The cooperation of:

Mr. Mathew of the Meteorology Department of McGill University, Mr. J. Marier, Ing. Manager, O. Jadah, Eng., Bouleris, Chemist, Allard, Chemist, of the Air Purification Branch of the M.U.C. Dept. of Health.

Mr. P. Bergeret of the Environmental Department of the Provincial Government.

Dr. C. East of the Air Pollution Department of the Université du Québec à Montreal,

is gratefully acknowledged.

This report was prepared under the supervision of Dr. C. Kwok and Dr. R. Cheng, and the author is deeply indebted for their guidance and continuous encouragement.

TABLE OF CONTENTS

CHAPTER

LIST OF TABLES

CALCULATION OF THE BACKGROUND CONCENTRATIONS

LIST OF FIGURES

FIGURE PAGE

LIST OF APPENDICES

LIST OF SYMBOLS

$\bar{A}$	Average lot area	m^2
a	Silhquette area	m^2
$a_1 \ldots a_i$	Advection	m^3/s
$B_1 \ldots B_i$	Background concentration	mg/m^3
$C_1 \ldots C_i$	Concentration in box	mg/m^3
$C_{po} \ldots C_p$	Specific heat	cal/gm-s (J/gm-s)
d	Downwind distance	m
E	Total SO_2 emission rate	gm/s
$e_o\ e_1 \ldots e_i$	Atmospheric diffusion	m^3/s
H	Building height	m
H_1	Evenly distributed heat gain	cal/cm^2-s
H_h	Daily heat production for heating of building	cal/day
H_s	Evenly distributed heat source strength	cal/m^3-s
h_d	Rate of heat production per degree day	$cal/^{o}C$-day
h_e	Heating value	cal/gm
h_s	Mixing height	m
K_z	Diffusivity	m^2/s
k	Thermal conductivity	cal/s-m-^{o}C
L	Downwind distance from the background city edge to city center	m

LIST OF SYMBOLS (CONT'D)

m	Index	
Q	Source strength	$\mu g/m^2$-s
r_k	Distance from the station k to the grid analysis mn	m
r_{mn}	Distance from the grid ij to the grid analysis mn	m
T	Temperature	^oC
T_b	Surface temperature at the meteorological tower at the Botanical Garden	^oC
T_d	Mean outdoor temperature of the day	^oC
T_m	Annual mean daily temperature	^oC
T_o	Background surface temperature	^oC
T_x	Surface temperature at downwind distance x in the city	^oC
U	Wind velocity	m/s
U_B	Surface wind velocity at Botanical Garden	m/s
U_o	Surface wind velocity at background city edge.	m/s
U_s	Surface wind velocity at sampling station 1	m/s
U_x	Surface wind velocity at downwind distance x in the city	m/s
U_{ij}	Horizontal component of the wind velocity in the x direction at grid ij.	m/s

LIST OF SYMBOLS (CONT'D)

U_*	Friction velocity	m/s
V_{ij}	Horizontal component of the wind velocity in the y direction at grid ij	m/s
W_h	Daily amount of oil for heating of building	gm/day
W_i	Daily amount of oil for industrial use	gm/day
W_o	Yearly amount of oil for heating of building	gm/year
W_T	Daily total amount of oil consumption	gm/day
x	Downwind distance	m
z	Height above ground	m
z_o	Reference height above ground	m
Z_o	Aerodynamic Roughness Length	m
α	Difference between the background country lapse rate and the adiabatic lapse rate	^{o}C/m
Γ	Lapse rate	^{o}C/m
Γ_a	Actual lapse rate	^{o}C/m
Γ_d	Adiabatic lapse rate	^{o}C/m
Δx	Box length	m
Δz	Box height	m
ω	Humidity ratio	gm/gm

CHAPTER I

INTRODUCTION

Air pollution, simply stated, is the presence of foreign matter in the air. The many classifications of pollutants include almost any natural or artificial composition of matter capable of being airborne. The classes overlap, but two broad categories are particulate and gaseous emissions.

Air pollution can arise from a multiplicity of sources, by no means all of which are industrial. Most commonly, of course, it is man-made, a by-product of our growing population and increasingly urbanized, industrialized and complex society. By far, one of the most common air pollutants in an urban environment is sulphur dioxide (SO_2). The primary source of SO_2 is the combination of atmospheric oxygen with the sulphur in certain fossil fuels during their combustion. The total emitted quantity of SO_2 is, therefore, directly related to the sulphur content and total quantities of the principal fuels used in a community. SO_2 pollution problem is thus more severe in cities in cold climate such as Montreal, due to increased use of fossil fuels as a heat source.

SO_2 is a colorless gas with a suffocating odor and causes damages to vegetation and human health such as bronchitis and cardio-respiratory diseases. To limit these damages air pollution is controlled and air quality standards are established. The SO_2 source emission rates are controlled to meet the air quality standards. Since spacial air quality measurements are expensive, ambient air quality models have been developed as a means to relate air pollutant emission rate to atmospheric concentration. Most of these ambient air quality models simulate the complex urban atmospheric air pollution processes. Here, the pollutant concentration at any location is influenced by the two following pollutant sources:

1. Local emmission.
2. The pollutant concentration of the air advected from the background area.

Basically, there are two types of models available in the literature:

a) Statistical models
b) Deterministic models

Statistical Models

They are diffusion models in which all pollutants emitted by a source are within a single plume. The central axis of that plume is the direction of the wind. The pollutant concentration is assumed to follow a three-dimensional (x, y, z) Gaussian distribution. The three dimensions are:

1) The wind direction (x)
2) The crosswind direction (y)
3) The vertical direction (z)

The emission source can be idealized either by a point or a line source. A line source is an emission source which is spread out uniformly over a finite straight line. However, the multiple urban emission sources are spread out over a region which is called an area source. Modelling of area sources by statistical models is carried out by subdividing the area source into subareas. The emission sources of each subarea are converted into a virtual point source (1) or a line source (2) of a certain total emission rate (mg/s) and the pollutant concentration at a given location is determined by applying the principle of superposition.

In the statistical approach, two untenable assumptions are used:

1) The plume spread observed for a point source is assumed to hold for area sources.
2) Constant mean wind velocity is assumed in the mixing layer.

The mixing layer is the ground based atmospheric layer above which the pollutants do not rise due to a change in the vertical temperature profile of the atmosphere. The height of this layer is generally termed as the mixing height.

Deterministic Models

There are two types of deterministic models:

1) The trajectory models.
2) The box models.

Trajectory models: Any trajectory model attempts to describe the change in concentration in a coordinate system which moves along a surface wind trajectory. It is a time dependent model. The Erschenroeder and Martinez model (3) is a typical trajectory model. This model is best suited for the description of atmospheric transport of pollutants of moderate reaction rate.

Box models; A box model is the simplest method to simulate an area source. Here, the mixing layer extending over a given area source is divided into an array of control boxes. This array of control boxes may extend to the mixing height. The numerous emission sources are assumed to be uniformly spread out over the area. Box models may be subdivided into two types:

1) The box models based on the atmospheric diffusion equation. The model developed by Shir and Shieh (4) is a typical model of this category. Here it may be noted that the temperature variation within the area source is ignored.
2) The box models based on the law of conservation of mass. The Reiquam model (5) and the Ragland model (6) are the two important examples of the models of this type.

The Reiquam model which makes use of a row of boxes extending from the ground to the mixing height is not accurate enough. On the other hand, in the Ragland model the effects of the mixing height are also incorporated. This model is used in the present investigation for evaluating the ambient air background concentration of SO_2 a few hundred meters upwind of the Montreal urban community sampling station 1. Location of this sampling station is indicated in Chapter III. Emphasis is put on surface background concentration.

CHAPTER II

DESCRIPTION OF THE RAGLAND MODEL

The Ragland multiple box model (7) allows the estimation of the steady state distribution of the ambient air concentration of pollutants in a narrow atmospheric corridor. This atmospheric corridor is located within the mixing layer and is oriented in the direction of the wind. This atmospheric corridor is divided into an array of control boxes.

The model is based on the application of the law of conservation of mass to each control box in steady state conditions. Pollutants are assumed to undergo no chemical reaction in the atmosphere.

The basic assumptions associated with box modelling may be described as follows:

1. The pollutant concentration in each box is assumed to be uniformly distributed.
2. The pollutant concentration of the air advected into each control box from the background area is assumed to be uniformly distributed both in the vertical and crosswind directions.
3. The vertical wind velocity profile at the centre of the control box is assumed to be constant through the entire control box.

As shown in Fig. 1, the model involves five parameters:

1. The pollutant source strength Q ($\mu g/s\text{-}m^2$). This parameter is found only in box 1 and is the mean pollutant emission rate per

unit area of ground surface sources..

2. The advection in and out of box i: a_i (both in m^3/S).
3. The diffusion in (e_{i-1}) and out (e_i) of box i (both in m^3/s).
4. The pollutant concentration C_i within i box (mg/m^3).
5. The pollutant background concentration B_i (mg/m^3) of the air entering the box i.

In a particular situation the model could be used to evaluate any one of the above five parameters with the knowledge of the others. The parameters under items 1 through 3 depend on numerous environmental variables such as: wind velocities and their respective vertical gradient, height above ground, local surface temperatures, fossil fuel consumption in box 1 and the box dimensions. The values of these variables may be obtained from the local environmental data files. The pollutant concentration at any point in the locality of investigation may be deduced by a single or finite number of measurements within the area of interest (C_i) or in the immediate background area (B_i).

It may be worthwhile to briefly review the effects of some of the above important environmental variables on advection and diffusion.

1. Advection

Advection is the horizontal component of atmospheric ventilation. It is governed by the mean wind velocity (8). It is responsible for carrying the pollutants from the background of concentration (B_i) into the box (of concentration C_i), and also remove pollutant from the box (9). Thus advection helps to increase or decrease the pollutant concentration inside the box.

2. Diffusion

Diffusion is the mean of the fluctuation component of air velocity. The resultant of pollutant transport by diffusion from one box to the next is obtained by multiplying the difference in concentrations within the boxes by the diffusion parameter between these boxes. Diffusion is governed by turbulence which is high frequency fluctuations of wind velocity. It is a three dimensional phenomenon of mechanical or thermal origin (10). Let us first analyze its vertical component. In a built up area such as the Montreal city center, just above ground, diffusion is due to the aerodynamic roughness of buildings and structures. At a level beyond the top of the buildings and structures, diffusion is mainly of thermal origin. The thermal diffusion is controlled by the temperature profile at the particular situation. There are two horizontal components of diffusion:

1. The longitudinal component in the wind direction which is generally neglected.

2. The lateral component in the crosswind direction. Walters (11) considered the importance of retaining the lateral component in the computation of the pollutant concentration. He found that the lateral component may generally be neglected except under conditions of very light winds (< 0.5 m/s approximately).

2.1 INFLUENCE OF THE ATMOSPHERIC STABILITY ON VERTICAL DIFFUSION

In a given atmosphere, tendency for vertical motion of air is called atmospheric stability. In a stable atmosphere, vertical diffusion is opposed. Conversely, in an unstable atmosphere, vertical diffusion is enhanced. At low wind velocities, the atmospheric stability is governed by the lapse rate $\Gamma(^{o}C/m)$, which is the vertical gradient of the temperature profile. It is given by (12):

$$\Gamma = - dT/dz \quad \ldots\ldots\ldots\ldots\ldots \quad (2.1.)$$

Generally, the wind velocities are considered to be low when they are below 11 m/s (13). The stability of a given atmosphere may be determined by comparing its lapse rate to that of a reference atmosphere.

The reference lapse rate is the lapse rate of an ideal atmosphere where any mass of air, once set in motion raises adiabatically in an atmosphere which is in hydrostatic equilibrium. It is known as the reference lapse rate or adiabatic lapse rate (13). As such, a unit mass of air rises slowly in the atmosphere and expands slowly to accomodate the reduced pressure at higher altitude. As the unit mass of air expands slowly upon rising, its temperature decreases. Such an atmosphere which is in thermal equilibrium is also called neutral atmosphere. Its lapse rate is called adiabatic lapse rate $\Gamma_d(^{o}C/m)$.

The relation of the lapse rate Γ_a of a given atmosphere to the adiabatic lapse rate Γ_d governs the stability of the atmosphere (14). If: $\Gamma_a > \Gamma_d$: the atmosphere is considered unstable and vertical diffusion is enhanced, whereas $\Gamma_a < \Gamma_d$ the atmosphere is considered stable and vertical diffusion is opposed. It shall be noted that a given atmospheric layer has only one lapse rate. There are as many layers as there are lapse rate. Generally, for an urban atmosphere at night, different lapse rates can be measured at various altitudes (figure 2). They generally may be explained as follows:

Since the space between the sun and the earth is near vacuum, the sun heats the atmosphere by radiation (15). In turn, right after sun-set, the earth radiates heat back to the atmosphere. A large amount of this radiated heat is absorbed by water vapour and carbon dioxide in the lower atmosphere. On account of the heat lost by the ground, the air right above it is cooled. The temperature increases with height causing a negative lapse rate. Such a situation in which the lapse rate has a negative sign is called inversion. Experiments have shown that the height of an inversion may reach up to 500 m.

Detailed observations of the spacial distribution of temperature have been carried out in many cities and their surroundings. Results have shown that, on an annual basis, the city center surface temperature is higher than the surface temperature of the corresponding background country (16). This phenomena is called Urban Heat Island. The main contribution to the heat island is the vertical convection of heat leaking out of the buildings or released by industrial activities. This, in turn, modifies the temperature profile of the stable air advected from the background country. Modifications of the temperature profile occurs up to a height where convective heat transfer is completed. This height is the mixing height h_s(m).

Successive supply of convective heat, as the air flows over the city, results in the formation of a mixing layer of increasing height (fig. 3). This mixing layer is topped by the unaltered portion of the background country inversion layer which forces the emitted pollutants from the urban sources to remain in the mixing layer. This situation results in the following boundary conditions for model analysis, i.e. at the mixing height h_s the gradient of pollutant concentration with respect to the height above ground is zero (17).

$$(\delta C/\delta z = 0 \quad \text{for} \quad z = h_s) \qquad (2.1)$$

In such cases, the height of the columns of control boxes in Ragland model should not exceed the mixing height h_s.

2.2 EQUATIONS OF CONSERVATION OF MASS

As mentioned in the introduction of this chapter, the diffusion in the crosswind direction (y) is negligible. Thus, the model may be fully described in a two-dimensional (x,z) Eulerian system in which as shown in Fig. 4, the x axis is oriented along the wind and the z axis is the vertical direction. Moreover, for each column a minimum of three boxes are necessary to carry out further calculations. The procedure is as follows: consider a column of three boxes of unit width and equal length Δx(m). In the model, the steady state mean background concentrations B_1, B_2, B_3 are related to the corresponding concentrations within the boxes C_1, C_2, C_3 by applying the law of conservation of mass (18):

$$a_1 B_1 = a_1 C_1 - Q \Delta x + e_1 (C_1 - C_2) - e_0 (C_0 - C_1) \qquad (2.2)$$

$$e_0 = 0 \quad \text{for a ground based box} \qquad (2.3)$$

$$a_2 B_2 = a_2 C_2 - e_1 (C_1 - C_2) + e_2 (C_2 - C_3) \qquad (2.4)$$

$$a_3 B_3 = a_3 C_3 - e_2 (C_2 - C_3) + e_3 (C_3 - C_4) \qquad (2.5)$$

where the suffixes identify the box where the parameters are referred to and C_4 is the concentration of pollutant above box 3. This set of equations may be represented by the following general equation:

$$a_i B_i = a_i C_i - Q\Delta x - e_{i-1}(C_{i-1} - C_i) + e_i(C_i - C_{i+1}) \qquad (2.6)$$

where:

a_iB_i is the mass flow of pollutant entering the box by advection (a_i being the advection parameter).

a_iC_i is the mass flow of pollutant leaving the box by advection.

$e_{i-1}(C_i - C_{i-1})$ is the mass flow of pollutant entering the box by diffusion (e_{i-1} being the diffusion parameter).

$e_i(C_i - C_{i+1})$ is the mass flow of pollutant leaving the box by diffusion (e_i being the diffusion parameter).

$Q\Delta x$ is the mass of pollutants emitted per unit of time within the box 1 by the source of strength Q.

CHAPTER III

APPLICATION OF THE RAGLAND MODEL TO THE MONTREAL CITY CENTER

3.1 GEOGRAPHICAL MODEL SET UP

The Ragland model is applied to a small part of the Montreal city center as shown on Fig.5. This area is located between Mount Royal St. (north), Ontario St. (south), Amherst St. (east) and St. Denis St. (west). In that part most of the dwellings are two storage buildings (duplexes). The Montreal urban community SO_2 sampling station 1 is located at the corner of Amherst and Ontario St. (Fig.5). Here the vertical SO_2 concentration profile has been measured (19). The other meteorological stations which are not located within the area under investigation are:

1) The meteorological tower of the Botanical Garden where wind velocity is measured at the higher and lower measuring stations and temperature difference between these two stations is also measured. (Appendices I and II).

2) The Mount Royal CBC station:where wind velocity is measured at the lowest station and temperature difference is measured between the higher and lower station (Appendices I and II),

3) McGill observatory:where city center surface temperature and humidity level are measured (Appendices I and II).

4) Dorval airport:where the background area surface temperature and surface wind velocity are measured (Appendices I and II).

The knowledge of the vertical wind velocity profile is necessary to estimate the diffusion and advection parameters. Since no wind measurements are available at sampling Station 1, the author has proposed an indirect method of determination of the wind velocity profile. It is based on:

1) A set of two reference wind velocities taken at different elevations of a location in the city.

2) The respective downwind distance between the background city edge and:

a) The location where the vertical wind velocity profile is to be determined.

b) The location of the reference wind velocities.

In the present case, only the northwest wind direction is considered, as this is the major wind direction during the investigated period (February 16, 1975). Thus, the orientation of the downwind coordinate is taken in the northwest direction for the present study. Further, the corner of the Laurentide Bld. and the Concorde Bld. in the City of Laval has been chosen as the background city edge, where densely built up area begins. The atmospheric corridor extends from this location to the sampling Station 1.

The respective configuration of the control box set ups is carried out based on the following points:

1) Height to which measurements are valid. It may be noted that Ragland assumed a monotonic increasing function for the diffusion parameter within the mixing layer. However, measurements in various urban atmospheres have shown (20) that diffusion increases up to a certain height and then decreases with height within the mixing layer. To avoid these

uncertainties the maximum height of the column of boxes for the present investigation is limited to 40 meters.

2. Minimizing of errors usually arising in box modelling.

A general guideline for choosing appropriate grid sizes is that they must be small whenever the gradient of either the wind velocity or the concentration of pollutants is large. Based on this guideline, Hsien Ta Liu and allied (21) conducted various investigations and suggested that a box height Δz in the range of 8m to 15m may be chosen for box model analysis. Accordingly, a box height (Δz) of 10m is arbitrarily chosen for the first set up of the present analysis. Based on similar reasoning an arbitrary box length (Δx) of 500m is taken for the analysis. The width of the box is chosen as 1m.

The lowest point at which the SO_2 concentrations are measured is the local roof level (approximately 7m). Below this point (box 1) the SO_2 concentration is linearly extrapolated. However, complex concentration distribution occur on account of the street canyon effect, which is present when wind blows across a built up urban street (22). A second model box set up is chosen to minimize the uncertainties of linear extrapolations in box 1. Accordingly box 1 is taken 7m above ground and the box height is reduced to 3m, the height of the boxes 2 and 3 is chosen to 3 and 7m arbitrarily. A third set up (set up III) is introduced to explore the effect of uniform box height on background concentrations. Therefore, a uniform box height of 3m is chosen for all boxes under this set up.

3. Source strength Q.

4. SO_2 concentration in the box C_i.

The remaining unknown parameter is the background concentration. This is evaluated using the model.

Before computing the diffusion and advection parameters the maximum height of the corridor of control boxes (mixing height) has to be checked.

DETERMINATION OF THE MIXING HEIGHT

Since the vertical temperature profile is not available within the investigated area, the mixing height is evaluated using measurements from other measuring stations located outside of the investigated area. The following measurements are available:

1) Surface temperature measurement at Dorval airport (Appendices I & II).
2) Temperature difference between the higher and lower measuring station at the CBC tower (Appendices I & II).
3) Difference in temperature between the higher and lower measuring stations at the meteorological tower of the Botanical Garden (Appendices I & II).
4) Surface temperature at McGill University (Appendices I & II).

From the measurements under items 1 and 2, the background temperature profile is estimated (Appendix II). From the measurements under Item 3, the slope of the vertical temperature profile at the meteorological tower of the Botanical Garden is estimated (Appendix II). The surface temperature at the Botanical Garden is obtained from the surface temperature measurement under Item 4, using the Summers model (23), (Appendix II). From the above, the mixing height at the meteorological station of the Botanical Garden is obtained by the intersection of the two vertical profiles, namely:

The background vertical temperature profile and the vertical temperature profile at the meteorological tower of the Botanical Garden (Fig. 6).

The mixing height at sampling Station 1 is deduced from that at the meteorological tower of the Botanical Garden using the Summers model (24) (Appendix II).

In the present case, the neutral lapse rate of the mixing layer is obtained from the following equation (25).

$$\Gamma_d = \frac{9}{C_{pa}(1-\omega) + \omega C_{pw}} \qquad \ldots\ldots\ldots\ldots (3.1)$$

$$= 0.00962\,^{\circ}C/m \text{ (see Appendix II)}$$

where:

C_{pa} = the specific heat at constant pressure of the dry air in cal/gm-$^{\circ}$C

C_{pw} = the specific heat at constant pressure of the water vapour at constant pressure in Cal/gm - $^{\circ}$C.

ω = the humidity ratio in gm of water vapour per gm of dry air (50% measured at McGill sampling station).

Following Summers (26), the lapse rate in the box is assumed to be approximately the same as that at the meteorological tower of the Botanical Garden (Γ_a = 0.0104 $^{\circ}$C/m - Appendix II). Since the lapse rate of the mixing layer is approximately equal to the neutral lapse rate, the investigated atmosphere is taken as neutral.

3.2 INPUT PARAMETERS

3.2.1 Diffusion (e_i)

The diffusion parameter controls the vertical transport of air. It is given by: (27)

$$e_i = K_z \frac{\Delta x}{\Delta z} \quad \ldots\ldots\ldots\ldots\ldots\ldots (3.2)$$

where:

K_z is the diffusivity parameter in m^2/s or eddy diffusivity momentum of the vertical air motion.

Δx and Δz respectively are the length and the height of the box at the top of which diffusion is estimated.

The Monin Obukov similarity theory allows a simple evaluation of the diffusivity K_z (28).

$$K_z = kU_* z \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.3)$$

where:

k is the von Karman constant = 0.4.

U_* (m/s) is the characteristic velocity which reflects the aerodynamic drag. z(m) is the height above ground at which the diffusivity is estimated.

As discussed in the Introduction of Chapter II, diffusion, hence diffusivity is governed by the turbulence. Near the ground, the aerodynamic roughness of buildings influences the atmospheric turbulence. Aloft, in low wind conditions, diffusion is governed by thermal turbulence. Accordingly, the characteristic velocity U_* is evaluated for mechanical and/or thermal turbulence as the case may be.

MECHANICAL DIFFUSION

Mechanical diffusion is confined in the lower portion of the mixing layer. The characteristic velocity U_* is given by (29):

3.2 INPUT PARAMETERS

3.2.1 Diffusion (e_i)

The diffusion parameter controls the vertical transport of air. It is given by:(27)

$$e_i = K_z \frac{\Delta x}{\Delta z} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.2)$$

where:

K_z is the diffusivity parameter in m^2/s or eddy diffusivity momentum of the vertical air motion.

Δx and Δz respectively are the length and the height of the box at the top of which diffusion is estimated.

The Monin Obukov similarity theory allows a simple evaluation of the diffusivity K_z (28).

$$K_z = kU_*z \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.3)$$

where:

k is the von Karman constant = 0.4.

U_* (m/s) is the characteristic velocity which reflects the aerodynamic drag. z(m) is the height above ground at which the diffusivity is estimated.

As discussed in the Introduction of Chapter II, diffusion, hence diffusivity is governed by the turbulence. Near the ground, the aerodynamic roughness of buildings influences the atmospheric turbulence. Aloft, in low wind conditions, diffusion is governed by thermal turbulence. Accordingly, the characteristic velocity U_* is evaluated for mechanical and/or thermal turbulence as the case may be.

MECHANICAL DIFFUSION

Mechanical diffusion is confined in the lower portion of the mixing layer. The characteristic velocity U_* is given by (29):

3.2 INPUT PARAMETERS

3.2.1 Diffusion (e_i)

The diffusion parameter controls the vertical transport of air. It is given by:(27)

$$e_i = K_z \frac{\Delta x}{\Delta z} \quad \dots\dots\dots\dots\dots\dots (3.2)$$

where:

K_z is the diffusivity parameter in m^2/s or eddy diffusivity momentum of the vertical air motion.

Δx and Δz respectively are the length and the height of the box at the top of which diffusion is estimated.

The Monin Obukov similarity theory allows a simple evaluation of the diffusivity K_z (28).

$$K_z = kU_* z \quad \dots\dots\dots\dots\dots\dots\dots\dots (3.3)$$

where:

k is the von Karman constant = 0.4.

U_* (m/s) is the characteristic velocity which reflects the aerodynamic drag. z(m) is the height above ground at which the diffusivity is estimated.

As discussed in the Introduction of Chapter II, diffusion, hence diffusivity is governed by the turbulence. Near the ground, the aerodynamic roughness of buildings influences the atmospheric turbulence. Aloft, in low wind conditions, diffusion is governed by thermal turbulence. Accordingly, the characteristic velocity U_* is evaluated for mechanical and/or thermal turbulence as the case may be.

MECHANICAL DIFFUSION

Mechanical diffusion is confined in the lower portion of the mixing layer. The characteristic velocity U_* is given by (29):

$$U_*/U = \mathrm{Ln}\,\frac{z + Z_o}{Z_o} \quad \ldots\ldots \quad (3.4)$$

where:

U = the wind velocity.

k = the von Karman constant = 0.4.

z = the height at which the diffusion is estimated.

Z_o = the aerodynamic roughness of the typical building. In Appendix III, Z_o is estimated according to the H.H. Lettau Method (30) as 1.5m.

THERMAL DIFFUSION

In neutral condition, the friction velocity for thermal diffusion is given by (31):

$$U_* = kz\,\frac{du}{dz} \quad \ldots\ldots \quad (3.5)$$

where:

k = the von Karman constant 0.4.

z = the height at which the diffusion is estimated.

$\frac{du}{dz}$ = the value of the vertical gradient of the wind velocity where the diffusion is estimated (1/s).

VERTICAL WIND VELOCITY PROFILE

It has been found that at a given location, the mean wind velocity in neutral conditions is best described by a power law. (32).

$$U/U_o = (z/z_o)^m \quad \ldots\ldots \quad (3.6)$$

where:

U = the value of the wind velocity at height z.

U_o = the reference value of the wind velocity at height z_o.

m = a positive number <1.

The surface wind velocity has been chosen as the reference wind velocity U_0 in the power law. At most of the North American weather stations the surface wind velocity is measured at a standard height of 10m above ground. However, the surface wind velocity may be measured at a convenient height above ground. In order to evaluate the index m of the power law at least two measurements are necessary at a particular location and atmospheric condition. At the meteorological tower of the Botanical Garden, wind velocity measurements are carried out at a height of 6.1m and 59.7m above ground. Based on these measurements, the power law index "m" is evaluated (m = 0.255). Since no wind velocity measurements are available at the Montreal Urban Community sampling Station 1, the wind velocities at this location are evaluated using the measurements at the Botanical Garden (see Section 4.4). Using the velocity profile (3.6) and equations (3.4) and (3.5) the friction velocities U_* for mechanical and thermal diffusion are deduced respectively. Further, diffusion and diffusivity parameters are evaluated using equation (3.2) and (3.3) in Tables II, III and IV for the three cases of model set up.

3.2.2. ADVECTION (a_i)

The advection governs the horizontal transport of SO_2. Advection parameters are obtained by summing the elementary flows over the front area ($\Delta z \times 1m^2$) of the box under investigation. The advection parameters of the boxes 1, 2, 3 are given as:(33).

$$a_i = \int_{z_i \text{ bottom}}^{z_i \text{ top}} U_i(z)dz \quad \ldots\ldots\ldots\ldots\ldots (3.7)$$

where the index i = 1, 2, 3 refers to the box 1, 2, 3. The computed values of the advection parameters using equation (3.7) and the power law

(3.6) are included in Tables II, III and IV for the three cases of model set up.

3.2.3 SOURCE STRENGTH (Q)

In the absence of an accurate inventory of the numerous emission sources, the current approach to evaluate the total source strength of the city is based on an assumption that the emission rate is uniformly distributed over the entire urban area. This method which has been developed by Summers (34) is summarized in Fig.7. Emitted SO_2 is assumed to be mainly the result of combustion of oil for industrial purposes and for heating of buildings. Estimation of the SO_2 emission rate is based on the following assumptions: All consumed oil has the same sulphur content of 0.7% by weight and have the same heating value $h_e = 11{,}000$ cal/gm (35) and all the sulphur in the oil is converted to SO_2 after combustion.

INDUSTRIAL OIL CONSUMPTION RATE

The industrial oil consumption rate W_i is independent of the temperature of the ambient air and is nearly constant throughout the year. Therefore, the daily amount of oil consumed in the entire city may be obtained strictly from Statistics Canada (36), (37). For instance, the figure for 1974 was 28.8×10^6 barrels or 5.23×10^{12} gm.

OIL CONSUMPTION RATE FOR HEATING OF BUILDINGS

On the other hand, oil consumption for heating of domestic and office buildings depends on the outside temperature. It is commonly assumed that the heating of buildings is required to overcome the temperature difference existing between the indoor comfort temperature of 18.33°C and the outside temperature. Since the outside temperature varies, the mean daily outside temperature T_d is used. The amount of heat per day H_h required to compensate the amount of heat lost to the atmosphere is given by (38).

$$H_h = h_d\ (18.33 - T_d) \quad \ldots \quad (3.8)$$

where:

h_d (Cal/°C day) is the rate of heat production per degree day.

In a given city, it is assumed that the yearly amount of heat produced for heating of buildings is proportional to the difference between the comfort temperature of 18.33°C and the annual mean daily temperature T_m. In Montreal, the annual mean daily temperature has been measured as $T_m = 5.964\,^{o}C$ (39). The heat production per degree day may be deduced from the yearly amount of oil W_o (gm/year) consumed for the heating of buildings in the investigated area. It is given by:

$$h_d = \frac{1}{365}\ \frac{W_o\ h_e}{18.33 - T_m} \quad \ldots \quad (3.9)$$

where:

h_e = the mean oil heating value = 11,000 cal/gm.

W_o = the yearly amount of oil consumed for heating of buildings,
= $4.62 10^{12}$ gm/y. (Statistics 1974).

The rate of heat required to heat the buildings of the investigated city (heat lost to the atmosphere) during a given period of time is deduced from equations (3.8) and (3.9).

The corresponding oil consumption rate is given by:

$$W_h = H_h/h_e \quad \ldots \quad (3.10)$$

where:

h_e = 11,000 cal/gm.

Letting W_T be the total amount of oil consumption :

$$W_T = W_i + W_h \quad \ldots \quad (3.11)$$

where:

W_i = oil consumption for industrial use.

The total SO_2 emission rate E(g/s) is given by:

$$E = W_T \times 1.4\% \times \frac{1}{24 \times 3600} \quad \ldots\ldots\ldots\ldots (3.12)$$

Finally, the SO_2 source strength Q (mg/m^2-s) is obtained by envenly distributing the total SO_2 emission rate E over the densely built up area of the Urban area under investigation. (The densely built up area of the Greater Montreal is estimated as: $3.66 10^8 m^2$). (40).

The computed maximum value of the source strength is found to be 14.7μg/m^2-s. (Appendix IV).

Since in the investigated area there are no industrial activities, a reduced source strength based on oil consumption for heating of building only is estimated as 8.63μg/m^2-s in (Appendix IV).

3.2.4 VERTICAL CONCENTRATION PROFILE (C)

The estimation of the vertical concentration profile in box is based on a linear interpolation of traverse measurements carried out by T. Oke & C. East (41). Atmospheric conditions were similar. The traverse measurements of ambient air SO_2 concentrations involved the monitoring of air samples from a helicopter. Sampling and monitoring were carried out using the electro-conductivity technique. The intake of the SO_2 sampler was located in a white metal shield 60 cm ahead of the helicopter. At given sites on the traverse, the helicopter took measurements during a 70.0 m spiral ascents. Simultaneously, height above ground was measured using a pressure transducer type measuring device. Finally, wind velocity was monitored using a cup anemometer device. Based on these measurements, the linear vertical concentration profile is estimated in Appendix V as:

$$C = 0.00258z + 1.514 \quad \ldots\ldots\ldots\ldots (3.13)$$

CHAPTER IV

DETERMINATION OF THE VERTICAL WIND PROFILE

In Chapter III, the necessity for predicting the vertical wind profile within the investigated area based on measurements at a reference location within the urban area is underlined. It may be of interest to briefly discuss the available methods for predicting wind velocity.

4.1 SURVEY OF THE CONVENTIONAL METHODS USED TO DETERMINE THE WIND VELOCITY

Few comprehensive studies have been done on this subject. The conventional methods can be classified into two categories:

a) The experimental extrapolation.

b) The Wendell method.

a) Experimental extrapolation

Some investigators make use of an empirical linear relationship between the respective surface wind velocities at the area of investigation and at the nearby airport (42). The intercept and the slope of the linear relationship is obtained by simultaneous measurements at both the locations. Unfortunately, the only one wind measurement at city center is carried out at the meteorological laboratory of the McGill University. Even this measurement is not reliable on account of the effect of the nearby Mount-Royal hill.

b) Wendell's Method

The Wendell equations (43) are applicable to a grid system. They provide relationships between the respective horizontal components of the wind velocity in different elements of the grid. These equations are:

$$U_{ij} = \Sigma_{m,n} (U_{mn}/r^2{}_{mn})/\Sigma_{mn} \ 1/r^2{}_{mn} \quad \ldots\ldots\ldots\ldots\ldots \quad (4.1a)$$

$$V_{ij} = \Sigma_{m,n} (V_{mn}/r^2{}_{mn})/\Sigma_{mn} \ 1/r^2{}_{m} \quad \ldots\ldots\ldots\ldots\ldots \quad (4.1b)$$

where:

U_{ij} and V_{ij} are the horizontal components of the wind velocity at analysis grid (ij) in the x and y direction respectively.

U_{mn} and V_{mn} are the guessfield analysis at grid mn in the x and y direction respectively.

r_{mn} is the distance from the grid ij to the grid mn.

The initial guess figure is obtained for $U_{mn} = U_k$ $V_{mn} = V_k$ and $r_{mn} = r_k$, where U_k and V_k are the horizontal components of the wind measured at station k, r_k is the distance from the station k to the grid analysis ij. This method which is valid for long distance r_{mn} requires the knowledge of two reference vertical wind profiles. It cannot be utilized in the present case because of erroneous measurements at the CBC tower measuring stations and the McGill meteorological laboratory. As a result of the inconveniences of the conventional methods presently described, the author has developed the following method.

4.2 THE AUTHOR'S METHOD

The method is based on the application of the energy equation to a shallow portion of the neutral atmospheric corridor including the surface level. In the case of neutral (adiabatic) conditions the energy equation take the same form of equation of the heat conduction, as follows:

4.2.1 Application of Energy Equation (44)

$$\rho_o C_{po} U_x \frac{\delta T_x}{\delta x} = k \frac{\delta^2 T_x}{\delta x^2} + H_s \quad \ldots\ldots\ldots\ldots\ldots \quad (4.2)$$

where:

ρ_o = the density of the surface air (gm/m^3).

C_{po} = the specific heat at constant pressure (cal/gm-^oC) of the urban surface air.

U_x = the surface wind velocity (m/s) at downwind distance x from the background city edge.

T_x = the surface temperature of the urban air (^oC) at downwind distance x from the background city edge.

x = the downwind distance from the background city edge (m).

k = the heat conductivity (cal/s-m-^oC) of the surface urban air.

H_s = the source strength of the heat emission of the urban area per unit volume of the surface layer (cal/s-m^3). It is assumed constant over the surface area. It may be noted that ρ_o, C_{po} and k are almost constant in the shallow layer.

Simplification of the energy equation is carried out by applying the Summer's heat island model to the surface temperature gain $T_x - T_o$. Here T_o is the surface temperature at the background city edge.

4.2.2 Application of the Summers Model

The surface temperature gain $T_x - T_o$ has been estimated by P.W. Summers for a neutral urban atmosphere. For a constant area heat source it is given by (Eq. 12 of ref. 45):

$$T_x - T_o = \left(\frac{2H_1 \alpha x}{U_o \rho_o C_{po}} \right)^{1/2} 10^2 \quad \ldots\ldots\ldots\ldots \quad (4.3)$$

where:

x = downwind distance from the background city edge (m)(as above).

α = the amount by which the lapse rate at the background city edge exceeds the adiabatic lapse rate Γ_d ($^\circ$C/m).

H_1 = the evenly distributed heat gain as the air moves into the city (cal/cm^2 -s) or (ley/s). It is constant for an evenly distributed SO_2 emission source.

U_o = the surface wind velocity at the background city edge (m/s).

ρ_o = the density of the surface urban air (gm/m^3).

C_{po} = the specific heat at constant pressure.

It may be noted that α, H_1, U_o, ρ_o, C_{po}, are almost constant for a given steady state condition. This equation shows that:

$$T_x \ \alpha \ x^{1/2}$$

Thus that

$$\frac{\delta T_x}{\delta x} \ \alpha \ x^{-1/2}$$

$$\delta^2 T_x / \delta x^2 \ \alpha \ x^{-3/2}$$

Moreover, the order of magnitude analysis of the terms of equation (4.3) shows that $\rho_o C_{po} U_o \frac{\delta T_x}{\delta x}$ is of the order of $10^{-2}\ x^{-1/2}$ while $\frac{K \delta^2 T_x}{\delta x^2}$ is of the order of $10^{-1} x^{-3/2}$ with x >10,000m, the order of magnitude analysis shows that $K \frac{\delta^2 T_x}{\delta x^2}$ is negligible. Therefore (4.3) reduces to"

$$\rho_o C_{po} U_x \frac{\delta T_x}{\delta x} \simeq H_1 \text{ constant} \quad \ldots\ldots (4.4)$$

and $U_x \propto 1/\delta T_x/\delta x$

or

$U_x \propto x^{1/2}$

Finally,

$$\frac{U_{x_1}}{U_{x_2}} \simeq \sqrt{\frac{x_1}{x_2}} \quad \ldots\ldots (4.5)$$

where subscript 1, 2 refers to two different locations within the neutral corridor. Defining U_S as the surface wind velocity at the city center, where $x_1 = L$, and U_B the reference surface wind velocity where $x_2 = d$, one can obtain:

$$U_s = U_B \sqrt{\frac{L}{d}} \quad \ldots\ldots (4.6)$$

According to the present geographical set-up L = 13260m and d = 10160m.

It may be noted that this relation applies to points being on the same streamline. When, in a region where there are variations in the terrain and mixing height as it occurs in our case, the wind flow pattern is rather complicated. However, on account of the accuracy of the method no correction is carried out for the estimation of the vertical wind velocity profile at sampling Station 1 from that at the Botanical Garden.

4.3 DETERMINATION OF THE VERTICAL WIND VELOCITY-PROFILE AT BOTANICAL GARDEN

Wind velocity measurements were conducted at two different heights above ground (z_0 = 6.1m and z_1 = 59.7m). These measurements were taken during a period of time extending from 7 p.m. to 9 p.m. on 16th February 1975.

The corresponding average wind velocities U_0 and U_1 were 4.02 m/s and 7.20 m/s respectively.

Using equation (3.6)

$$U = U_0 \left(\frac{z}{z_0}\right)^m$$

and the measured wind velocities above, the value of m is computed (m=0.255). Thus, the vertical wind velocity profile is expressed as

$$\therefore \quad U_B = 2.54\, z^{0.255} \quad \ldots\ldots\ldots\ldots\ldots \ (4.7)$$

From this, the wind velocities at heights $z_3 = 10$m and $z_4 = 14$m are evaluated respectively as:

$$U_3 = 4.57 \text{ m/s}$$

$$U_4 = 4.98 \text{ m/s}$$

4.4 DETERMINATION OF THE VERTICAL WIND VELOCITY PROFILE AT SAMPLING STATION 1

Using relation (3.6) the wind velocities at height $z_3 = 10$m and $z_4 = 14$m are evaluated as follows:

Applying the relation (4.6) :

$$U_S = U_B \sqrt{\frac{L}{d}} \quad (L = 13.260\text{m and } d = 10160\text{m})$$

to each velocity of the set:

$$\left(\begin{array}{l} z_3 = 10\text{m} \\ U_{B_3} = 4.57 \text{ m/s} \end{array} \right. \qquad U_{S_3} = 5.14 \text{ m/s}$$

$$\left(\begin{array}{l} z_4 = 14\text{m} \\ U_{B_4} = 4.98 \text{ m/s} \end{array} \right. \qquad U_{S_4} = 5.67 \text{ m/s} \Big)$$

Assuming a power law vertical wind velocity profile at sampling Station 1

$$U_S = cz^n$$

Substituting the above two computed set of values for z and U_S, c and n are evaluated as 2.61 and 0.29 respectively.

Thus the vertical wind velocity profile at sampling Station 1 is estimated as:

$$U_S = 2.61\ z^{0.29} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (4.8)$$

The vertical gradient of the vertical wind velocity profile is estimated as:

$$\frac{dU_s}{dz} = 0.76\ z^{-0.71} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (4.9)$$

And the flow in a unit width corridor of height z is given by:

$$\int_0^z U_S dz = 2.02z^{1.29} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (4.10)$$

It may be noted that since the shear energy losses are neglected, the wind velocity at sampling Station 1 is overestimated. It would be thus, of interest, to check this wind velocity against reality.

CHAPTER V

DISCUSSION OF RESULTS

Before discussing the results in detail, a brief review of the three model set ups chosen for analysis is presented in Table I. The main parameters and the various physical box arrangements which are already explained in Chapter III are also included in Table I. It may be noted that the governing type of diffusion, unless otherwise mentioned, is obtained by comparing the calculated values of the diffusion parameters due to mechanical and thermal effects.

SET-UP I (Table II)

Set up I is based on the Ragland set up for each of the three boxes and the Hsien To Liu (21) recommendations concerning the selection of the box height Δz ($8m < \Delta z < 15$ m). Here an arbitrary common box height of 10m has been chosen. Box 1 is the ground based box where the diffusion, which is of mechanical origin, occurs at the top of the box only. The source strength is evaluated by:

a) Evenly distributing the SO_2 emission rate due to both industrial activities and heating of buildings (7.4 mg/s).

b) Evenly distributing the SO_2 emission rate due to heating of buildings only (4.31 mg/s).

The known concentration C_1 and the corresponding unknown background concentration B_1 are referred to as the ground level ambient air concentrations.

In box 2, diffusion at the bottom is of mechanical origin while diffusion at the top is of thermal origin.

In box 3, diffusions at the bottom and at the top are of thermal origin.

SET UP II

As explained in Chapter III, the second and third set ups are chosen to minimize the influence of the street canyon effect on concentration calculation by reducing the height of the box and positioning the first box at the mean roof level (7 to 10m). Under this situation, mechanical diffusion occurs both at the bottom and top of box 1. On account of the small height of box 1, the source strength is evaluated assuming:

a) that all emission sources are located in box 1. (7.4 mg/s and 4.3 mg/s).

b) that only half of the emission sources (heating of buildings only) are located in box 1 (2.15mg/s).

The heights of the boxes 2 and 3 are chosen in accordance with Raglands suggestion (46) that "the first four boxes be 4m high, while the remaining boxes be selected to fill up the height up to the mixing height.

SET UP III

In set ups II and III the boxes 1 and 2 are identical. Set up III is introduced to explore the effect of uniform box height on background concentrations. Accordingly, a box height of 3m is chosen for all the boxes under this set up.

RESULTS

Based on the previous discussion, background concentrations as well as the four input parameters are calculated for each box of the three set ups. The method of calculations and the corresponding results are summarized in Tables II to IV. Here each table refers to a particular box in a defined set up arrangement. The modified general form of the equation of conservation of mass as explained in Chapter II, is used. The applicable form of this equation for individual cases for each box in a particular model set up is given at the top of each table for reference. The corresponding values of Δx, Δz, Q and the SO_2 concentration in the box C_i based on measurements are also included in each table. The diffusion parameter e_i (either of mechanical or thermal origin as the case may be) is calculated using the applicable formulae is also included in the table. It may be noted that the value of the diffusion parameter out of a given box is taken as the diffusion into the top box. Further, for case where the first box is not ground based (e.g. set up II and III) $e_0 \neq 0$. Advection a_i is calculated using equation (4.3). Substituting the above quantities in equation (2.6), the unknown parameter, the background concentration B_i, is evaluated for each case in the last column.

SET UP I

In box 1, relatively low background concentrations are obtained even when using the lower source strength (based on heating of buildings only, $Q\Delta x = 4.3$ mg/m² - s). Here the background concentration B_1 (Table II-1) reaches only 80% of the concentration in box C_1. In boxes 2 and 3, the calculated background concentrations B_2 and B_3 are slightly smaller (98% and 97%) than the concentration in boxes C_2 (Table II-2) and C_3

(Table II-3) respectively.

SET UP II (Tables III)

In box 1, the calculated background concentration B_1 is still much lower than concentration in box C_1, even with the lowest value of Q ($Q\Delta x$ =2.15 mg/s). In box 2 the calculated background concentration B_2 amounts to 92% of the concentration in box C_2. This low value of the background concentration is due to the larger SO_2 mass flow diffused at the top of the box, which, in turn, is due to the larger value of Δz for box 3. Further, it may be noted that diffusion at the top of box 2 is assumed to be of thermal origin (lower value than mechanical diffusion). However, even this lower value of e_2 does not compensate the effect of Δz on C_3. In box 3, the background concentration B_3 is relatively smaller than the concentration in box 3 (B_3 = 99.5% of C_3). This is due to the equalization of the diffusive mass flow at the top and at the bottom of box 3. This equalization may be attributed to the relatively low diffusion at the top of box 3 (e_3). The low value of diffusion e_3 is due to the fact that the diffusivity at the top of box 3 is distributed over a box of larger height (Δz = 7m).

SET UP III (Table IV)

Box 1 is identical to the corresponding box in set up II. In box 2 and 3 both calculated background concentration B_2 and B_3 reach 95% of the corresponding concentrations inside the box 2 and 3 (C_2 and C_3).

For each set up the vertical background concentration profile is developed in Table V by:

a) Applying a linear correlation between the background concentrations and corresponding heights at the middle of the box (fig.8).

b) Calculating the linear relationship between background concentrations in box 2 and 3, and height at the middle of the corresponding box (i.e. excluding first box). [Fig.9]. The vertical concentration profile "C" is also plotted on each of these two graphs for reference. Theoretical, and experimental results [Summers, Shirr (47)] have shown that the SO_2 pollutants within an urban adiabatic mixing layer is well mixed both in the vertical and horizontal direction. Thus, the concentrations at two nearby locations (approximately 500m apart), within the urban area having uniform emission source strength must be approximately equal. Since the area of investigation is similar to the above mentioned situation, the difference in concentration between C_i and B_i should be small.

However, it may be noted that the concentration C_1 is much different from B_1 in Fig. 8. This may be attributed to:

1. The crude approximation used in calculating the diffusion in the lowest box 1. Here, diffusion is assumed to be uniformly distributed over the height of the box, in spite of the presence of the buildings.
2. The large value of the source strength with respect to the surface concentration C_1.
3. The absence of provision for SO_2 scavenging by buildings.

In order to eliminate these three sources of error in the estimation of the background concentrations B_1, the vertical background concentration profiles are re-evaluated neglecting the first box 1 in each set up.

Fig. 9 shows the modified representation of the vertical concentrations (B_i and C_i). Here, one can see that Set Up III gives a better representation of the vertical background concentration profile. This is attributed to the following reasons:

a) choice of smaller box size.

b) choice of identical box height.

It may be noted that even when the box height for Set Up II is smaller (7m) than that of Set Up I (10m), the latter set up gives a much better respresentation. This is due to the non uniform height of the boxes in Set Up II.

Thus, the use of the present method to determine the vertical SO_2 concentration is limited to the heights above the mean roof level choosing smaller uniform box sizes.

The above results are based on the assumptions that SO_2 does not engage in chemical reactions in the atmosphere under investigation. In order to obtain the actual SO_2 background concentrations, the effect of chemical reaction on the loss of SO_2 per box has to be evaluated.

Besides, it may be noted that the estimated aerodynamic surface roughness (Z_0 = 1.5m) agrees well with previous measurements in similar urban situations (48). However, on account of the absence of wind velocity measurements, the estimated vertical wind velocity profile cannot be checked against reality. Nevertheless, since in the energy equation (4.2), the shear energies are neglected, the wind velocity is overestimated. Accuracy of the method can be improved by measurements of environmental variables such as aerodynamic surface roughness Z_0, concentration C_i, etc.

Finally, a model set up having small and constant box height (3m) located above the mean roof level provides a good representation of the estimated vertical background concentration profile.

CHAPTER VI

SUMMARY & CONCLUSIONS

A mathematical box model is used to estimate the ambient air background concentrations 500m upwind of the Montreal urban community sampling station 1. The estimation is based on measured SO_2 ambient air concentrations at sampling station 1. All other numerical values are derived from local environmental data files.

Initially, a method to evaluate the surface wind velocities at sampling station I in a neutral atmosphere is developed. The vertical wind velocity profile is represented by a power law. The evaluated exponent of the power law compares well with that measured in other North American cities (49). However, on account of the lack of measurements the degree of accuracy of the method is not checked.

As a second step, three box model set ups are developed to:

a) Account for the street canyon effect.

b) Evaluate the respective effect of uniform box height and sudden change in box height on the estimation of background concentrations. A vertical background concentration profile is developed for each set up. All developed concentration profiles are compared with the vertical concentration profile in boxes.

Based on the present investigations the following conclusions can be drawn:

1. Estimation of environmental variable such as the characteristic aerodynamic surface roughness length (Z_0 = 1.5m) agrees well with previous measurements.
2. Lower background concentrations are obtained below the mean roof level (7m) even using lower values of the source strength. (3.4 mg-s over an area of 500m^2). Thus, the present method of modelling cannot be used below the mean roof level.
3. Smaller box height provide a better accuracy of estimation.
4. The boxes should be of constant height.
5. Comparing the three different model set ups, Set Up III (having small and constant box height (3m) located above the mean roof level) provides a better representation of the estimated vertical concentration profile.
6. Despite the fact that no provisions are made for SO_2 scavenging by chemical reactions, the model is valid at least over a short distance.

Finally, it may be noted that in classical estimation of urban SO_2 concentration using box models, it is assumed that the air entering the city is clean. The calculations of background concentrations are evaluated for each box. However, on account of the actual suburban housing development, the air entering the densely built up area (here called the city) may not be clean. Thus, the used assumption of the zero value of background concentration may not be valid any more.

FURTHER REMARKS

The results of the present investigation provide useful information to future investigators of the Montreal surface air quality. As a further extension of the present study, one could:

a) Improve the estimation of environmental variables such as the aerodynamic roughness Z_0, vertical wind velocity profile, vertical concentration profile.

b) Estimate background concentrations for other wind directions and/or stability conditions.

c) Provide for SO_2 reaction processes.

TABLE 1

BOX MODELLING - UNKNOWN BACKGROUND CONCENTRATIONS, B_1, B_2, B_3

ASSUMPTIONS	SET UP I			SET UP II			SET UP III		
CONCERNING	Box 1	Box 2	Box 3	Box 1	Box 2	Box 3	Box 1	Box 2	Box 3
ARRANGEMENT OF BOXES	Ground Based	Above Box 1	Above Box 2	Based at EL.7m	Above Box 1	Above Box 2	Based at EL.7m	Above Box 1	Above Box 2
Box Height	10m	10m	10m	3m	3m	7m	3m	3m	3m
INPUT PARAMETERS DIFFUSION									
At bottom origin & direction	- -	Mechanical $-(e_1)$ IN	Thermal $(-e_2)$ IN	Mechanical $-(e_0)$ OUT	Mechanical $-(e_1)$ IN	Thermal $-(e_2)$IN	Mechanical $-(e_0)$ OUT	Mechanical $-(e_1)$ IN	Mechanical $-(e_2)$ IN
At top origin & direction	Mechanical (e_1) (OUT)	Thermal (e_2) OUT	Thermal (e_3) OUT	Mechanical (e_1) OUT	Thermal* (e_2) OUT	Thermal* (e_3)OUT	Mechanical (e_1) OUT	Mechanical (e_2) OUT	Thermal (e_3) OUT
ADVECTION									
$a = 2.02z^{1.29}$ IN	$B_1\{a\}_0^{10}$	$B_2\{a\}_{10}^{20}$	$B_3\{a\}_{20}^{30}$	$B_1\{a\}_7^{10}$	$B_2\{a\}_{10}^{13}$	$B_3\{a\}_{13}^{20}$	$B_1\{a\}_7^{10}$	$B_2\{a\}_{10}^{13}$	$B_3\{a\}_{13}^{16}$
OUT	$C_1\{a\}_0^{10}$	$C_2\{a\}_{10}^{20}$	$C_3\{a\}_{20}^{30}$	$C_1\{a\}_7^{10}$	$C_2\{a\}_{10}^{13}$	$C_3\{a\}_{13}^{20}$	$C_1\{a\}_7^{10}$	$C_2\{a\}_{10}^{13}$	$C_3\{a\}_{13}^{16}$
SOURCE STRENGTH								-	-
$Q\Delta x$	7.4 mg/s	-	-	7.4 mg/s	-	-	7.4 mg/s	-	-
Heating of Blg on.	4.31 mg/s	-	-	4.31 mg/s	-	-	4.31 mg/s	-	-
Half Exhausts in box concentration	-	-	-	2.15 mg/s	-	-	2.15 mg/s	-	-
IN BOX	For z = 5m	For z =15m	For z =25m	For z =8.5m	For z =11.5m	For z =16.5m	For z = 8.5m	For z =11.5m	For z=14.5
Above Box 3			C_4 at z = 35m		(*Assumed)	C_4 at z=23.5m			C_4 at z=17.5m
Below Box 1					Origin)	C_0 at z=3.5m			C_0 at z=3.5m

TABLE H-1 - SET UP I - Box 1

$$a_1 B_1 = a_1 C_1 + e_1(C_1 - C_2) + Q\Delta x \quad (2.3)$$

$\Delta x = 500\text{m}$ $\Delta z = 10\text{m}$

$C_1 = 1.527\ \text{mg/m}^3$

$C_2 = 1.553\ \text{mg/m}^3$ From equation (3.13)

$C_3 = 1.579\ \text{mg/m}^3$ Appendix V

Q from Appendix IV $C_4 = 1.604\ \text{mg/m}^3$

$U = 2.61z^{0.29}$ (4.8) $a_1 = 2.02z^{0.29}\Big/_{0}^{10}$ (4.10) $du/_{dz} = 0.76^{-0.71}$ (4.9)

$U_*/_U = 0.4/\text{Ln}\,\frac{z + Z_0}{Z_0}$ (3.4) $K_z = 0.4\ U_*Z$ (3.3) $e_1 = K_z\ \frac{\Delta x}{\Delta z}$ (3.2)

With Z_o = 1.5m from Appendix III

Z top m	$U_*/_U$	U at top m/s	U_* at top m/s	K_z top m^2/s	e_1 m^3/s	$C_1 - C_2$ mg/m^3	$e_1(C_1-C_2)$ mg/s	a_1 m^3/s	a_1C_1 mg/s	$Q\Delta x$ mg/s	a_1B_1 mg/s	B_1 mg/m^3
10	.196	5.09	1.0	4	200	-0.026	-5.2	39.39	60.15	7.4 (a)	43.65	1.18
										3.41 (b)	50.64	1.29

TABLE II-2 SET UP I - Box 2

$a_2B_2 = a_2C_2 + e_2(C_2-C_3) - e_1(C_1-C_2)$ (2.3)

$\Delta x = 500\text{m}$

$\Delta z = 10\text{m}$

$C_1 = 1.527\ \text{mg/m}^3$

$C_2 = 1.553\ \text{mg/m}^3$ From equation 3.13

$C_3 = 1.579\ \text{mg/m}^3$ & Appendix V

$C_4 = 1.604\ \text{mg/m}^3$

$U = 2.61z^{0.29}$ (4.8)

$a_2 = 2.02z^{1.29}\Big|_{10}^{20}$ (4.10)

$du/_{dz} = 0.76z^{-0.71}$ (4.9)

$U_* = 0.4z\ du/_{dz}$ (3.5)

$e_2 = K_z \dfrac{\Delta x}{\Delta z} = 50K_z$ (3.2)

$K = 0.4zU_*$ (3.3)

THERMAL DIFFUSION

z top	$du/_{dz}$ top	U_* top	K_z top	e_2 top	$C_2 - C_3$	$e_2(C_2-C_3)$	$e_1(C_1-C_2)$	a_2	a_2C_2	a_2B_2	B_2
m	1/s	m/s	m²/s	m³/s	mg/m³	mg/s	mg/s	mg/m³	mg/s	mg/s	mg/m³
20	0.0906	0.725	5.8	290	-0.025	-7.25	-5.2*	56.92	88.40	86.35	1.517

MECHANICAL DIFFUSION

z(top)	$U_*/_U$ top	U at top	U_* at top	K_z top	e_1
m	–	m/s	m/s	m²/s	m³/s
20	.08	6.22	0.496	3.97	198.5

*FROM PREVIOUS TABLE

TABLE II-3 - SET UP I - Box 3

$a_3B_3 = a_3C_3 + e_3(C_3-C_4) - e_2(C_2-C_3)$ (2.5)

$\Delta x = 500m$

$\Delta z = 10m$

$C_1 = 1.527\ mg/m^3$
$C_2 = 1.553\ mg/m^3$
$C_3 = 1.579\ mg/m^3$
$C_4 = 1.604\ mg/m^3$

(3.13)
Appendix V

$U = 2.61z^{0.29}$ (4.8)

$du/_{dz} = 0.76^{-0.71}$ (4.9)

$K_z = 0.4z\ U_*$ (3.3)

$a_3 = 2.02z^{1.29}\Big|_{20}^{30}$ (4.10)

$U_* = 0.4z\ du/_{dz}$ (3.5)

$e_3 = K_z \dfrac{\Delta x}{\Delta z} = 50K_z$ (3.2)

z top	du/dz top	U_* top	K_z top	e_3	C_3-C_4	$e_3(C_3-C_4)$	$e_2(C_2-C_3)$	a_3	a_3C_3	a_3B_3	B_3
m	1/s	m/s	m^2/s	m^3/s	mg/m^3	mg/s	mg/s	m^3/s	mg/s	mg/s	mg/m^3
30	0.068	0.816	9.792	498.6	-0.025	-12.24	-7.25	105.6	166.70	161.71	1.531

TABLE III-1 - SET UP II - Box 1
IV-1 - SET UP III - Box 1

$a_1B_1 = a_1C_1 + e_1(C_1-C_2) - Q\Delta x - e_0(C_0-C_1)$ (2.3)

$\Delta x = 500m$ $\Delta z = 3m$

$C_0 = 1.523$ mg/m^3
$C_1 = 1.536$ mg/m^3
$C_2 = 1.544$ mg/m^3
$C_3 = 1.557$ mg/m^3
$C_4 = 1.575$ mg/m^3

From equation (3.13)
and Appendix V

$U = 2.61z^{0.29}$ (4.8) $a_1 = 2.02z^{0.29}\Big|_7^{10}$ (4.10) $du/dz = 0.76z^{-0.71}$ (4.9)

$U_*/U = 0.4/\mathrm{Ln}\,\frac{3 + 1.5}{1.5}$ (3.4) $K_z = 0.4\,U_*z$ (3.3) $e_1 = K_z\,\frac{\Delta x}{\Delta z} = \frac{500}{3}\,K_z = 166.7K_z$ (3.2)

z top m	$U_*/_U$ top -	U top m/s	U_* top m/s	K_z top m^2/s	e_1 top m^3/s	$C_1 - C_2$ mg/m^3	$e_1(C_1-C_2)$ mg/s	a_1 m^3/s	a_1C_1 mg/s
10	.196	5.09	1.0	4	666.7	-0.008	-5.33	14.52	22.1

z bottom m	$U_*/_U$ bott. -	U bott. m/s	U_* bott. m/s	K_z bott. m^2/s	e_0 bott. m^3/s	(C_0-C_1) mg/m^3	$e_0(C_1-C_0)$ mg/s	$Q\Delta x$ mg/s	a_1B_1 mg/s	B_1 mg/m^3
7	.231	4.59	1.06	2.968	494.8	-0.013	6.43	7.4	12.10	.833
								4.31	18.9	1.30
								2.15	21.05	1.45

TABLE III - 2 SET UP II - Box 2

$a_2B_2 = a_2C_2 + e_2(C_2-C_3)-e_1(C_1-C_2)$ (2.4)

$\Delta x = 500m$

$\Delta z = 3m$

$C_0=1.523\ mg/m^3$

$C_1=1.536\ mg/m^3$

$C_2=1.544\ mg/m^3$

$C_3=1.557\ mg/m^3$

$C_4=1.575\ mg/m^3$

From equation (3.13) and Appendix V

$U = 2.61z^{0.29}$ (4.8) $a_1 = 2.02z^{1.29}\Big|_{10}^{13}$ (4.10) $du/_{dz} = 0.76z^{-0.71}$ (4.9)

AT TOP: $U_* = 0.4z\ du/_{dz}$ (3.5) $K_z = 0.4U_*z$ (3.3) $e_2 = K_z \frac{\Delta x}{\Delta z} = 166.7\ K_z$ (3.3)

AT BOTTOM: SEE PREVIOUS TABLE FOR BOX 1

z top m	$du/_{dz}$ top 1/s	U_* top m/s	K_z top m^2/s	e_2 top m^3/s	C_2-C_3 mg/m^3	$e_2(C_2-C_3)$ mg/s	a_2 m^3/s	a_2C_2 mg/s	$e_1(C_1-C_2)$ mg/s	a_2B_2 mg/s	B_2 mg/m^3
13	.123	.640	3.33	554.7	-0.013	-7.21	15.86	24.49	-5.33*	22.61	1.426

*FROM PREVIOUS TABLE

TABLE III-3 SET UP II - Box 3

$a_3B_3 = a_3C_3 + e_3(C_3 - C_4) - e_2(C_2 - C_3)$ (2.5)

$\Delta x = 500m$

$\Delta z = 7m$

$C_0 = 1.523\ mg/m^3$
$C_1 = 1.536\ mg/m^3$
$C_2 = 1.544\ mg/m^3$
$C_3 = 1.557\ mg/m^3$
$C_4 = 1.575\ mg/m^3$

From equation (3.13)
and Appendix V

$U = 2.61z^{0.29}$ (4.8) $a_3 = 2.02z^{1.29}\Big|_{13}^{20}$ (4.10) $du/_{dz} = 0.76z^{-0.71}$ (4.9)

AT TOP $U_* = 0.4z\ du/_{dz}$ (3.5) $K_z = 0.4U_*z$ (3.3) $e_3 = K_z \dfrac{\Delta x}{\Delta z} = 71.43K_z$ (3.2)

AT BOTTOM SEE PREVIOUS TABLE FOR BOX 2

z top m	$du/_{dz}$ top 1/s	U_* top m/s	K_z top m^2/s	e_3 top m^3/s	C_3-C_4 mg/m^3	$e_3(C_3-C_4)$ mg/s	$e_2(C_2-C_3)$ mg/s	a_3 m^3/s	a_3C_3 mg/s	a_3B_3 mg/s	B_3 mg/m^3
20	0.090	0.725	5.798	414.1	-0.018	-7.454	-7.21	41.06	63.93	63.69	1.55

TABLE IV-2 - SET UP III - Box 2

$a_2B_2 = a_2C_2 - e_1(C_1 - C_2) + e_2(C_2 - C_3)$ (2.4)

$U = 2.61z^{0.29}$ (4.8) , $du/_{dz} = 0.76z^{-0.71}$ (4.9) $a_2 = 2.02z^{1.29}\Big|_{10}^{13}$ (4.10) - $e_2 = K_z \frac{\Delta x}{\Delta z}$

$U_*/_U = 0.4 \text{Ln} \frac{z + z_o}{z_o}$ (3.4) $K_z = 0.4U_*z$ (3.3)

$C_0 = 1.523$

$C_1 = 1.536$ From equation (3.13)

$C_2 = 1.544$ and Appendix v

$C_3 = 1.552$

$C_4 = 1.560$

z top m	$U_*/$ $\underline{U}$ top	U top m/s	U_* at top m/s	K_z top m^2/s	e_2 m^3/s	C_2-C_3 mg/m^3	$e_2(C_2-C_3)$ mg/s	a_2 m^3/s	a_2C_2 mg/m^3
13	o.176	5.49	.968	5.03	554.3	-.0008	-4.435	15.87	24.50

$e_1(C_1-C_2)$ mg/s
5.334

a_2B_2 mg/s	B_2 mg/m^3
23.12	1.47

TABLE IV-3 - SET UP III - Box 3

$a_3B_3 = a_3C_3 - e_2(C_2 - C_3) + e_3(C_3 - C_4)$ (2.5)

$U = 2.61z^{0.29}$ (4.8) $du/_{dz} = 0.76z^{-0.71}$ (4.9) $a_3 = 2.02z^{1.29}\Big|_{13}^{16}$ (4.10)

$U_* = 0.4z\, du/_{dz}$ (3.5) $K_z = 0.4U_* z$ (3.3)

$C_0 = 1.523$
$C_1 = 1.536$
$C_2 = 1.544$
$C_3 = 1.552$
$C_4 = 1.560$

From equation (3.13)
and Appendix V

z top m	$du/_{dz}$ top 1/s	U_* at top m/s	K_z at top m^2/s	e_3 m^3/s	C_3-C_4 mg/m^3	$e_3(C_3-C_4)$ mg/s	a_3 m^3/s	a_3C_3 mg/s
16	0.106	0.933	5.97	995.2	-0.008	-7.9616	16.92	26.34

FROM LAST TABLE:

$e_2(C_2-C_3)$ mg/s	a_3B_3 mg/s	B_3 mg/m^3
6.711	25.09	1.478

TABLE V

BACKGROUND CONCENTRATION PROFILES

1) BEST LINEAR FIT OF BACKGROUND CONCENTRATIONS

$$B = a + bz$$

WITH: $$a = \frac{(\Sigma B)\Sigma z^2 - \Sigma z \Sigma Bz}{3\Sigma z^2 - (\Sigma z)^2}$$

$$b = \frac{3\Sigma zB - \Sigma z \Sigma B}{3\Sigma z^2 - (\Sigma z)^2}$$

CORRELATION: $$r = \frac{3\Sigma zB - \Sigma z \Sigma B}{\sqrt{3\Sigma z^2 - (\Sigma z)^2}\sqrt{3\Sigma B^2 - (\Sigma B)^2}}$$

SET UP I

	z	B	Z^2	Bz	B^2
Box 1	5	1.29	25	6.45	1.66
Box 2	15	1.517	225	22.76	2.301
Box 3	25	1.531	625	38.275	2.344
	45	4.338	875	67.485	6.305
	Σz	ΣB	Σz^2	ΣBz	ΣB^2

CALCULATION EXAMPLE

$$a = \frac{4.338 \times 875 - 45 \times 67.435}{3 \times 875 - 45^2} = 1.291$$

$$b = \frac{3 \times 67.485 - 45 \times 4.338}{3 \times 875 - 45^2} = 0.0103$$

$$B = 0.0103z + 1.291$$

CORRELATION: $$r = \frac{3 \times 67.485 - 45 \times 4.338}{\sqrt{3 \times 875 - 45^2}\sqrt{3 \times 6.305 - (4.378)^2}} = .88$$

TABLE V.(CONT'D)

SET UP II

	z	B	Z^2	Bz	B^2
Box 1	8.5	1.45	72.25	12.325	2.1025
Box 2	11.5	1.426	133.25	16.40	2.033
Box 3	16.5	1.55	272.25	25.58	2.403
	36.5	4.426	477.75	54.305	6.5385
	Σz	ΣB	Σz^2	ΣBz	ΣB^2

a = 1:19 b = 0.0135

$$B = 0.0135z + 1.31$$

CORRELATION: r = 0.60

TABLE V (CONT'D)

SET UP III

	z	B	Z^2	Bz	B^2
Box 1	8.5	1.45	72.25	12.325	2.1003
Box 2	11.5	1.47	132.25	16.905	2.1609
Box 3	14.5	1.478	210.25	21.43	2.184
	34.5	4.398	414.75	50.66	6.4474
	Σz	ΣB	Σz^2	ΣBz	ΣB^2

$a = 1.248 \qquad b = 0.0168$

$$B = 0.004z + 1.413$$

CORRELATION: $r = 0.66$

2) BACKGROUND CONCENTRATION PROFILES IGNORING BOX I

$$B = a + bz$$

$$a = \frac{B_2 z_3 - B_3 z_2}{z_3 - z_2} \qquad b = \frac{B_2 - B_3}{z_2 - z_3}$$

SET UP I -(CALCULATION EXAMPLE)

$$a = \frac{1.517 \times 25 - 1.531 \times 15}{25 - 15} = 1.496 \qquad b = \frac{1.517 - 1.531}{15 - 25} = 0.0014$$

$$B = 0.0014z + 1.496$$

TABLE V (CONT'D)

SET UP II

$a = 1.141$

$b = 0.0248$

$B = 0.0248z + 1.141$

SET UP III

$a = 1.459$

$b = 0.00267$

$B = 0.00267 + 1.459$

TABLE V (CONT'D)

BACKGROUND CONCENTRATION PROFILES

SET UP I

BOX I INCLUDED: $B = 0.0156z + 1.189$

Correlation: $r = 0.88$

BOX I EXCLUDED: $B = 0.0014z + 1.496$

SET UP II

BOX I INCLUDED: $B = 0.0215z + 1.19$

Correlation: $r = 0.60$

BOX I EXCLUDED: $B = 0.00248z + 1.141$

SET UP III

BOX I INCLUDED: $B = 0.0168z + 1.248$

Correlation: $r = 0.66$

BOX I EXCLUDED: $B = 0.00267z + 1.459$

(Selected background concentration profile)

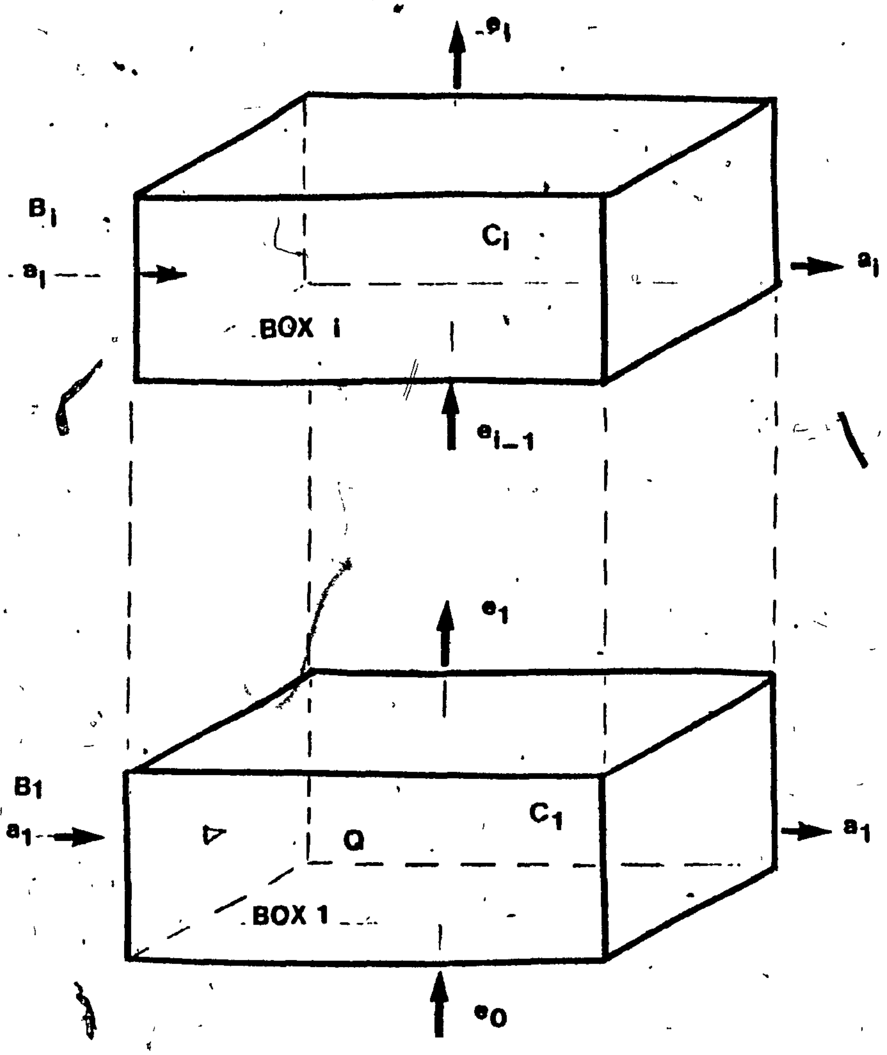

FIG I MODEL INPUT PARAMETERS

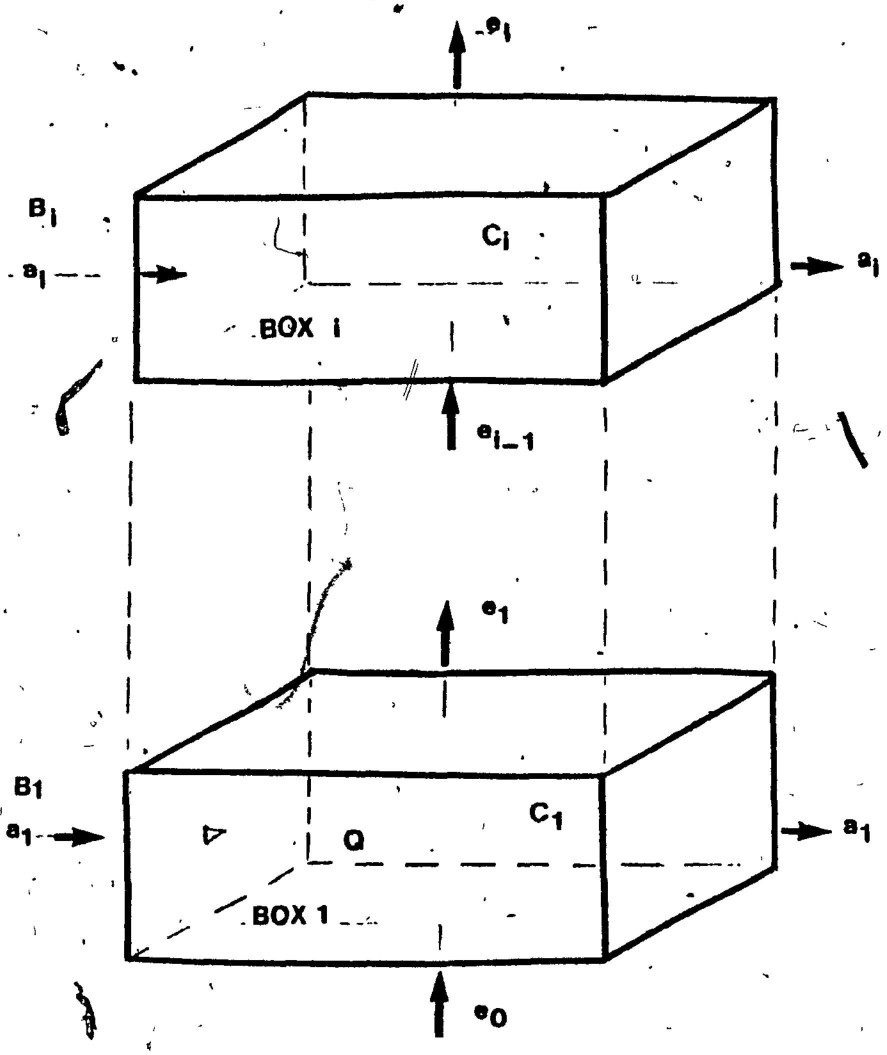

FIG I MODEL INPUT PARAMETERS

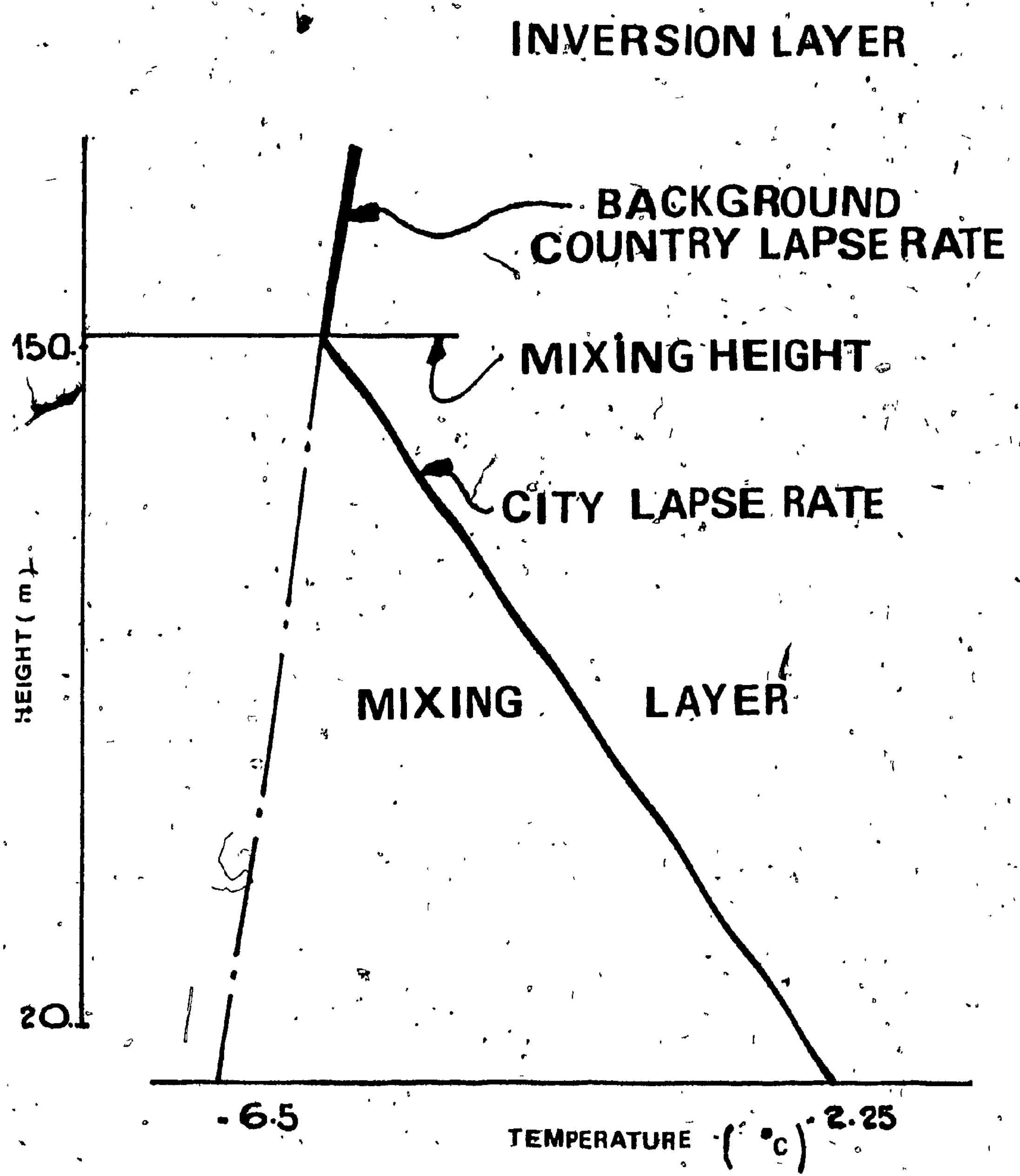

FIG 2 _ CITY VERTICAL TEMPERATURE PROFILES

(FROM LOCAL MEASURENTS—SEE APPENDIX II)

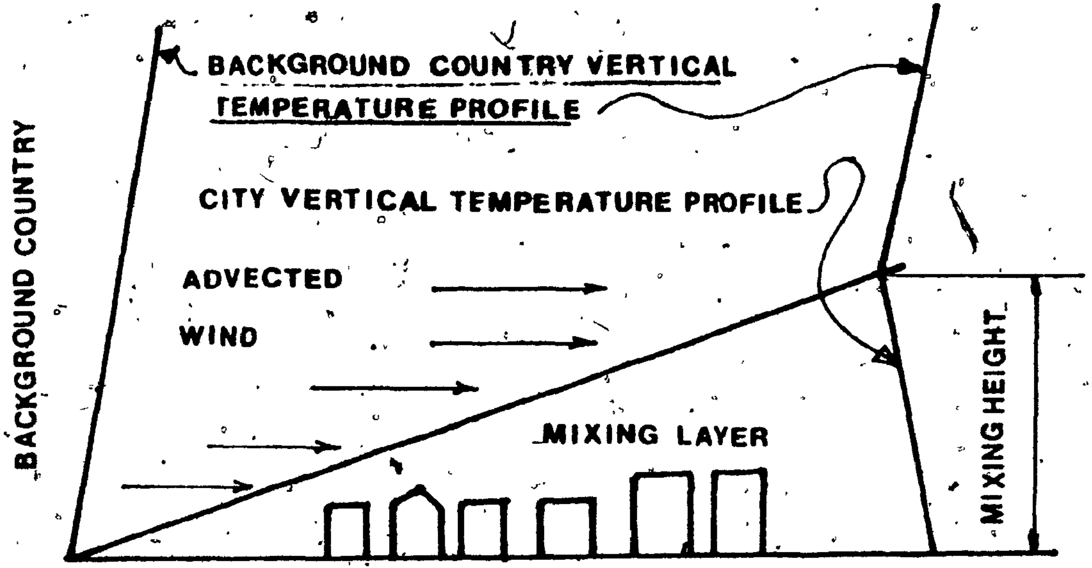

FIG 3 - BUILD UP OF AN URBAN MIXING LAYER

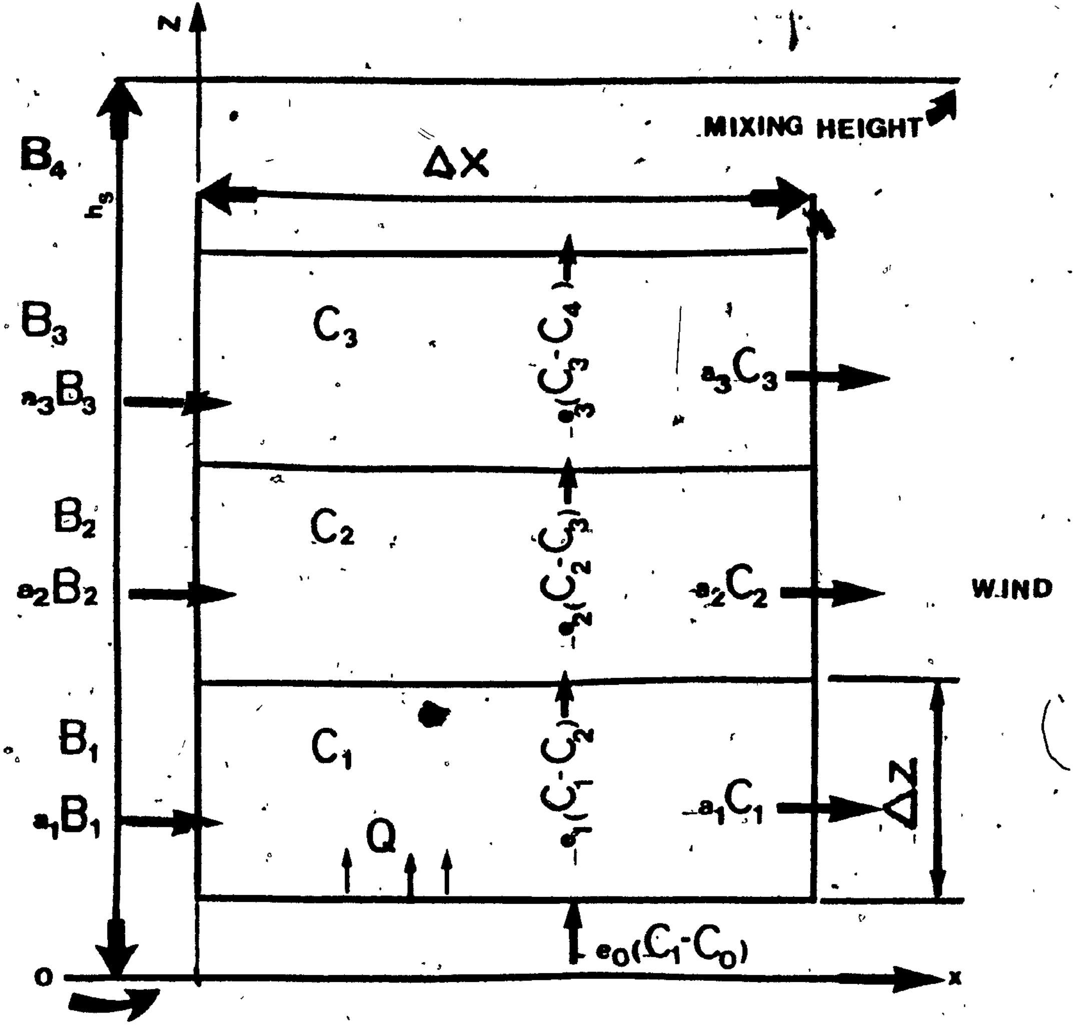

FIG 4 SET UP OF BOXES CROSS SECTION

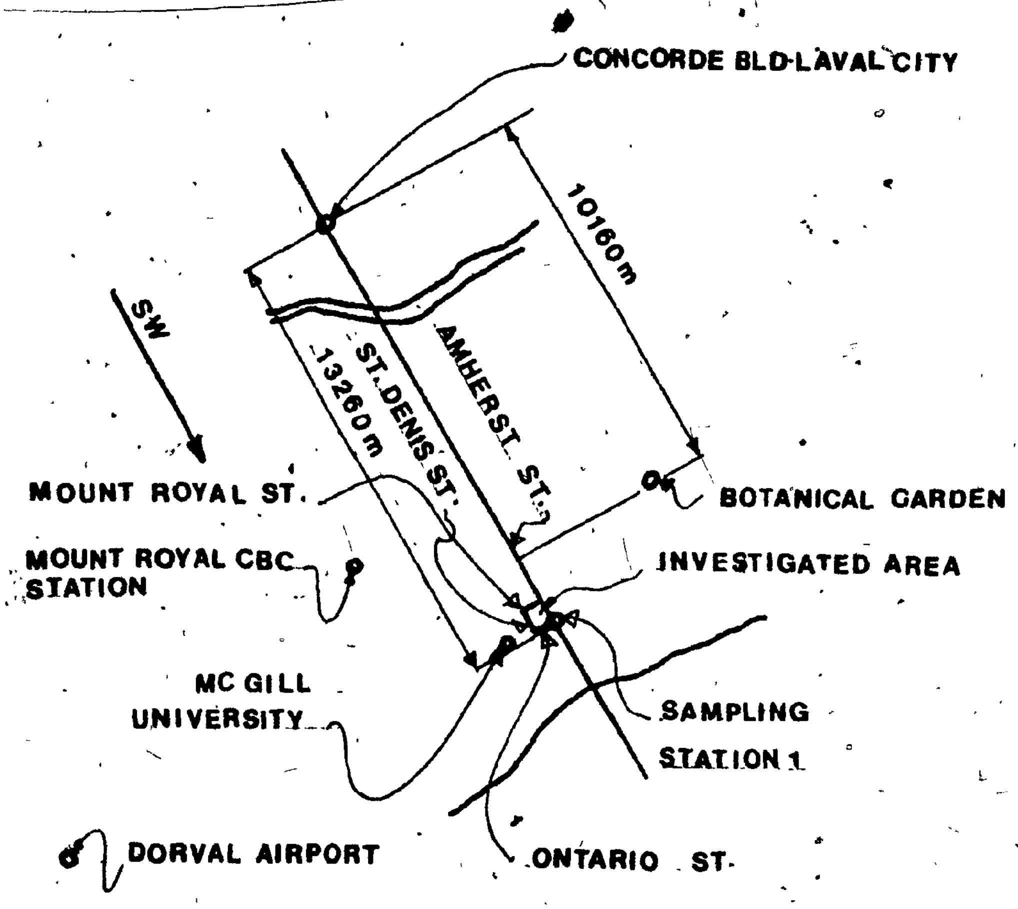

FIG 5 - INVESTIGATED AREA (NTS)

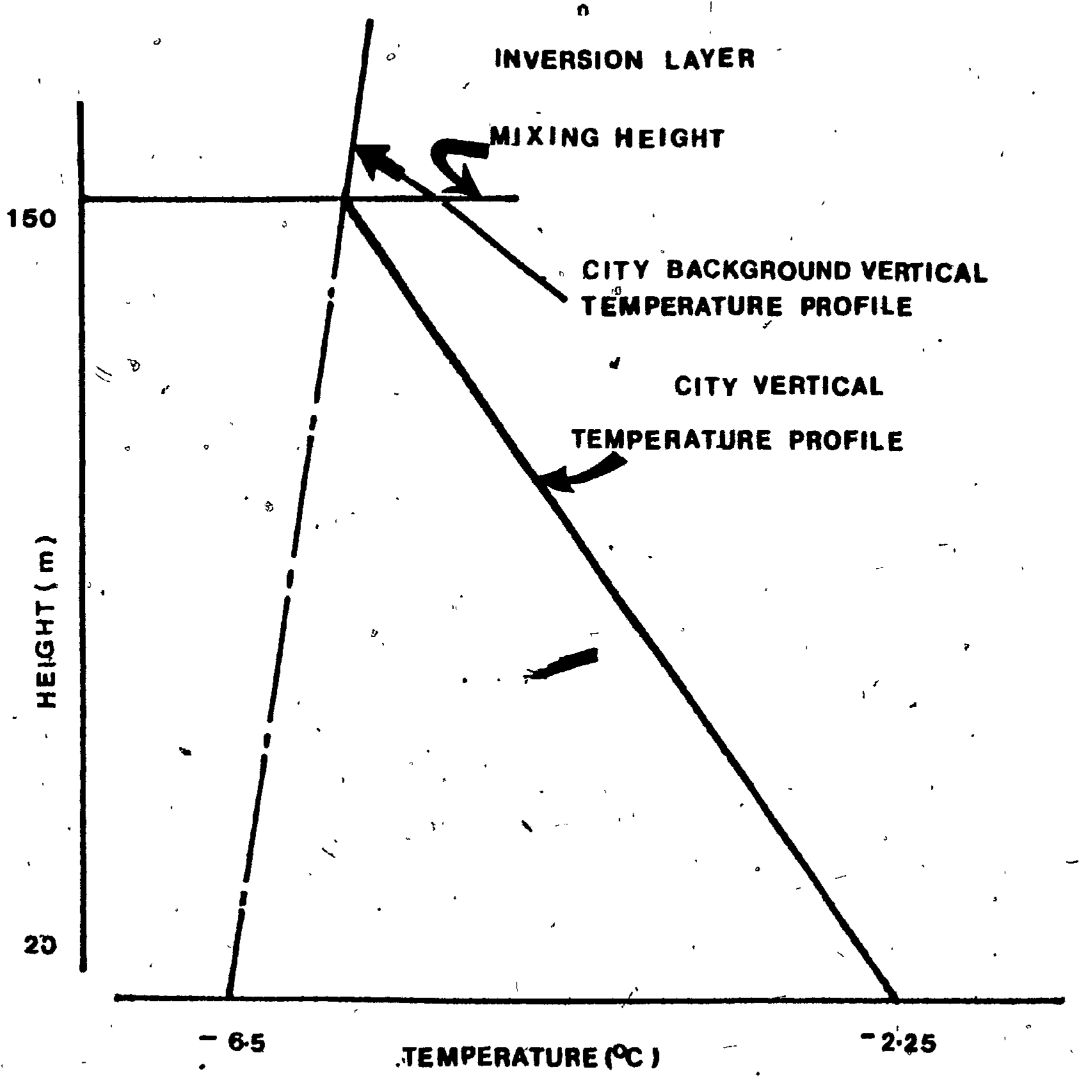

FIG. 6—DETERMINATION OF THE MIXING HEIGHT AT THE BOTANICAL GARDEN

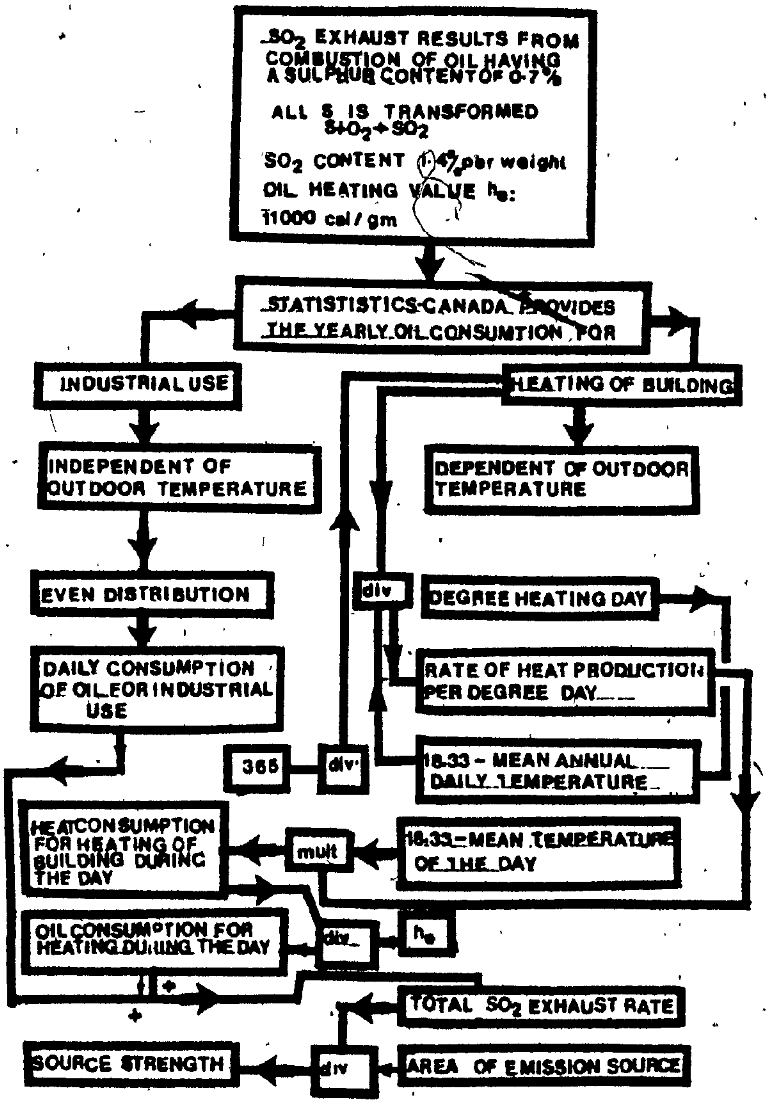

FIG 7 BLOCK DIAGRAM SHOWING THE ESTIMATION OF THE SOURCE STRENGTH

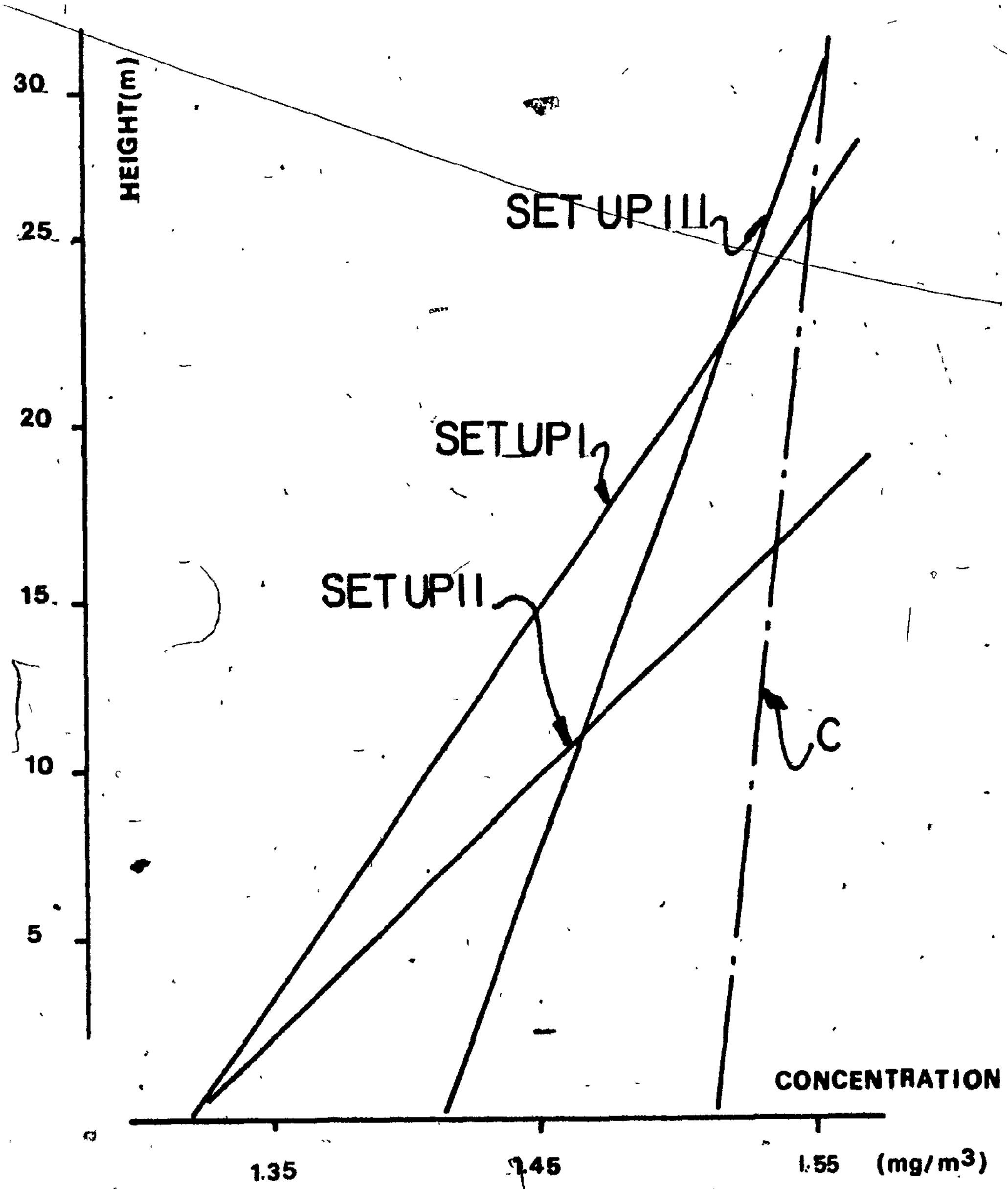

FIG. 8. BACKGROUND CONCENTRATION PROFILE INCLUDING BOX I

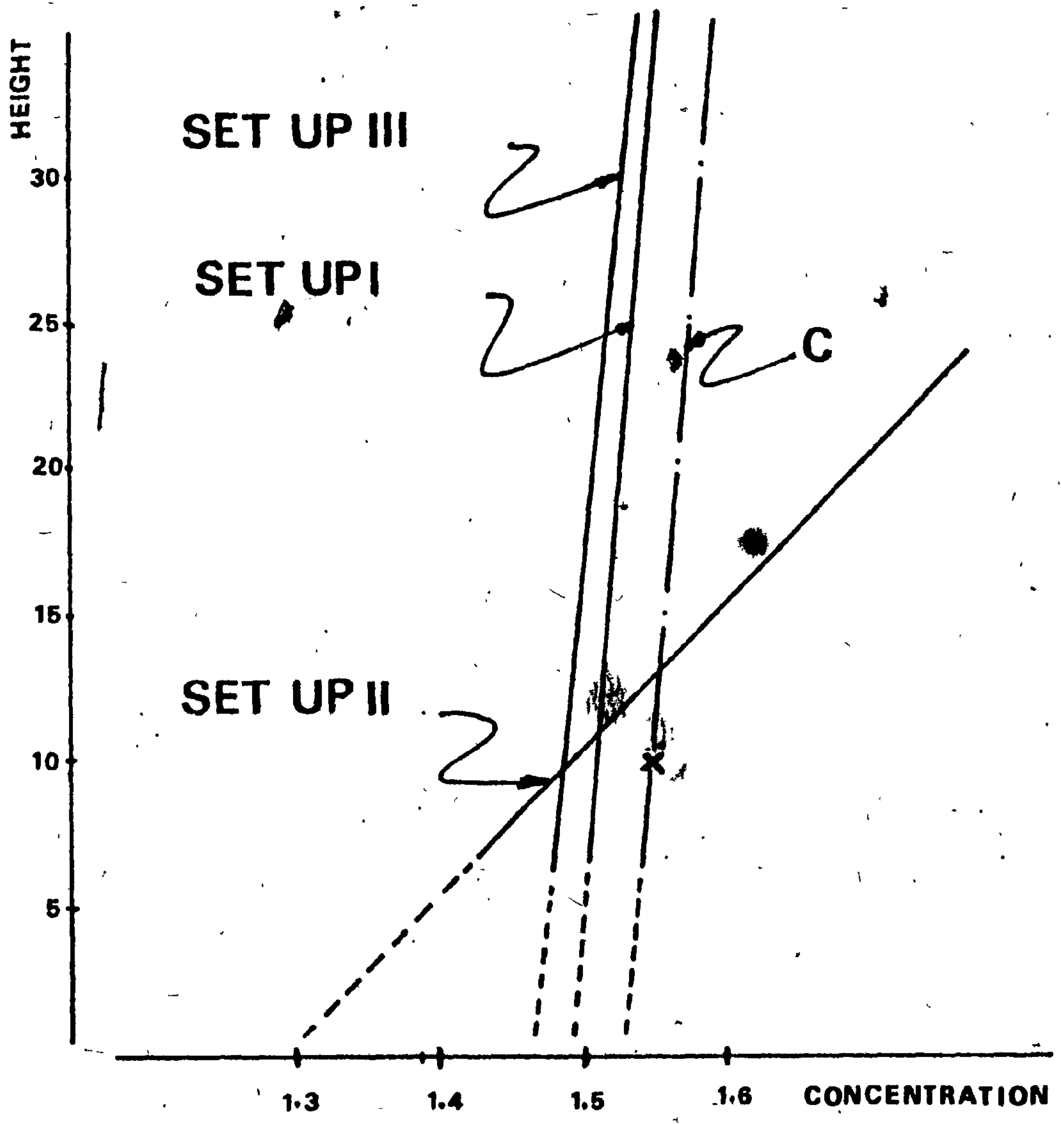

FIG 9 – BACKGROUND CONCENTRATION PROFILE
IGNORING BOX 1

LIST OF REFERENCES

1. Martin D.O., An urban diffusion model for estimating long term average values of air quality. Journal of Air Pollution Control Association, 21 pp.16.19.

2. Turner D.B., Workbook of atmospheric dispersion estimates. Public Health Service Publication 999-A p.26, 1970.

3. Eschenroeder A.Q. and Martinez J.R., Concepts and applications of photochemical smog models. Adv. Chem. 113.1d (1972).

4. Shir C.C. and Shieh L.J., A generalized urban air pollution model and its application to the study of SO_2 distribution in the St. Louis metropolitan area, IBM Research Laboratory Report RJ 1227, San Jose, California.

5. Reiquam H., An atmospheric transport and accumulation model for airsheds. Atmospheric Environment p.223 (1970).

6. Ragland K.W., Multiple box model for dispersion of air pollutants from area sources. Atmospheric Environment. Pergamon Press 1973, paragraph 2 p.1020.

7. IBID, paragraph 1 p.1019.

8. Shenfeld L., A lecture on meteorological aspects of air pollution, and dispersion estimates. p.2.

9. IBID, p.4.

10. IBID, p.4

11. Seinfeld J.H., Air pollution physical and chemical fundamentals McGraw-Hill 1975 paragraph 6.2, p.294.

12. Eskinazi S., Fluid Mechanics and Thermodynamics of our environment. Academic Press 1975, paragraph 4.5 p.90.

13. Seinfeld J.H. Air pollution physical and chemical fundamentals, McGraw-Hill 1975, paragraph 3.2.1, p.117.

14. IBID, paragraph 3.2.3, p.120.

LIST OF REFERENCES (CONT'D)

15. Eskinazi, S., Fluid Mechanics and Thermodynamics of our environment, Academic Press 1975 paragraph 3.2 p.65.

16. Summers, P.W., An urban heat island model. Its role in air pollution problems with application to Montreal. First Canadian Conference in Micrometeorology, April 1965 pp 1-5.

17. Ragland, K.W., Multiple box models for dispersion of air pollutants from Area sources. Atmospheric environment. Pergamon Press 1973 Vol.7 p.1030.

18. IBID, paragraph 2 p.1020.

19. Oke, T.R. and East C., The urban boundary layer in Montreal. Urban boundary layer meteorology, 1971, p.695.

20. Steven D. Reynolds & al. Mathematical modelling of photochemical air pollution I formulation of the model. Atmospheric environment, Pergamon Press 1973, paragraph 4.3 p.1054.

21. Hsien Ta Liu & Allied, Studies of particulate plume diffusion over Laboratory wind-generated waterwaves Atmospheric environment, Pergamon Press 1973, paragraph 3.4 p.880.

22. Johnson, W.B. Ludwig, F.L., Moon, A.E., and Maccuso, R.L., A practical multipurpose urban diffusion model for carbon monoxide. National air pollution control administration. Standford Research Institute, Meulopark, California, 1970.

23. Summers, P.W., An urban heat island model. Its role in air pollution problems with application to Montreal. First Canadian Conference in micrometeorology. Toronto April 1965 p.6.

24. IBID, p.6

25. Seinfeld, J.H., Air pollution physical and chemical fundamentals, McGraw-Hill 1975 paragraph 3.7.1 p.112.

26. Summers, P.W., An urban heat island model. Its role in air pollution problems with application to Montreal. First Canadian Conference in micrometeorology. April 1965 p.20.

LIST OF REFERENCES (CONT'D)

27. Eskinazi, S., Fluid mechanics and thermodynamics of our environment. Academic Press 1975, paragraph 8.4, p.241.

28. IBID, paragraph 9.5 p.229.

29. Ragland, K.W., Multiple box model for dispersion of air pollutants from area sources. Atmospheric Environment. Pergamon Press 1973. Paragraph 3, pp 1022 and 1025.

30. IBID, paragraph 3 p. 1023.

31. IBID, IBID.

32. Seinfeld, J.H., Air pollution physical and chemical fundamentals, McGraw-Hill 1975, paragraph 6.1. p.262.

33. Ragland, K.W., Multiple box model for dispersion of air pollutants from area sources. Atmospheric environment. Pergamon Press 1973. Paragraph 2 p.1020.

34. Summers, P.W., An urban heat island model, its role in air pollution problems with applications to Montreal. First Canadian Conference in micrometeorology, Toronto 1965. Paragraph 3, p.6.

35. IBID, pp 11-17.

36. Statistics Canada 1974, Refined petroleum products catalogue CS45-208.

37. IBID, 1971, Census of Canada, Housing Heating Equipment & Fuel, Cat. 93-526.

38. Carrier Air Conditioning Company - Handbook of airconditioning system design, McGraw-Hill 1965 p.3.

39. IBID p.10

LIST OF REFERENCES (CONT'D)

40. Urban Community of Montreal. Economic Development Council.

41. Oke T.R. & East C., The urban boundary Layer in Montreal. Urban boundary layer meteorology, 1975 p.691.

42. Johnson W.B., Ludwig F.L., Moon A.E., and Maccuss R.L., A practical multipurpose urban diffusion model for carbon monoxide. National air pollution control and administration. Standford Research Institute, Meulopark California 1970.

43. Wendell L.L., A preliminary examination of mesoscale wind fields and transport determined from a network of wind towers. NOAA technical memo ERLM ARTM 25, Silver Spring, Md.

44. Seinfeld J.H., Air pollution physical and chemical fundamentals. McGraw-Hill 1975 paragraph 5.1, p.226.

45. Summers P.W., An urban heat island model. Its role in air pollution to Montreal. First Canadian Conference in micrometeorology, Toronto 1965 p.6.

46. Ragland K.W., Multiple box model for dispersion of air pollutants from area sources. Atmospheric environment. Pergamon Press 1973. Paragraph 4 p.1026.

47. Shirr C.C. and Shieh L.J., A generalized urban air pollution model and its application to the study of the SO_2 distributions in the St. Louis metropolitan area, IBM Research Laboratory Report 1227, San Jose, California.

48. Cook N.J., Simulating the lower third of the urban adiabatic layer in a wind tunnel. Atmospheric environment, Pergammon Press 1973, paragraph 2 p.692.

49. IBID, paragraph 3 p.693.

APPENDIX I
REFERENCE ELEVATIONS

The following listing indicates elevation above ground and mid-sea level (M.S.L.) of sites involved in that report.

LOCATION	MEASURING STATION	ELEVATION (m)	
		ABOVE GROUND	ABOVE M.S.L.
METEOROLOGICAL TOWER in Botanical Garden (Provincial Environment)	HIGH	59.7	114.7
	LOW	6.1	61.1
C.B.C. TOWER on Mt. Royal operated by the Meteorological dept. of McGill Univ.	HIGH No Measurements		313.9
	LOW	10	257.5
DORVAL AIRPORT		10	41.1
McGill OBSERVATORY		10	57
M.U.C. SAMPLING STATION 1 (Corner Amherst & Ontario St.) Ground		10	32.6
Reference Elevation		Local ground level	

APPENDIX II

BASIC DATA FOR EVALUATION OF VERTICAL TEMPERATURE AND WIND PROFILES

II A TEMPERATURE

	MEASUREMENTS				CALCULATIONS
	LOCATIONS	TEMP. °C	TEMP. (°C) DIFFERENCE	DIFFERENCE IN HEIGHT (m)	LAPSE RATE °C/m
BACKGROUND	C.B.C. Mt.R	-	1.015	56.4	-0.0018
COUNTRY	Dorval	-6.5	-	-	-
BOTANICAL GARDEN	Meteorological Tower in Bot. Garden	-	-0.56	53.6	+0.00104
CITY CENTER	McGill	-1.6	-	-	-

Surface temperature at the Meteorological Tower of the Botanical Garden

(Equation 15 of Reference 26)

$$T_b = T_m - \Delta T(1 - \sqrt{\frac{d}{L}})$$

with: $d = 10160\text{m}$
$L = 13450\text{m}$
$\Delta T = -5^{o}\text{C}$
$T_b \simeq -2.25^{o}\text{C}$

APPENDIX II (CONT'D)

BASIC DATA FOR EVALUATION OF VERTICAL TEMPERATURE AND WIND PROFILES

II B WIND

	MEASUREMENTS		
	LOCATION	WIND VELOCITY m/s	REMARKS
BACKGROUND	C.B.C. Mt.Royal	10.2	Questionable measurements
COUNTRY	Dorval	2.4	—
BOTANICAL	High	7.24	
GARDEN	Low	4.02	

APPENDIX II (CONT'D)

Determination of the mixing height at City Center

According to Summers, the mixing height h_s at City Center will be deduced from that at the location of the reference wind profile by (equation 11 of reference 42):

$$h_s = h_B\sqrt{\frac{L}{d}}$$

h_B, which is the mixing height at the meteorological tower of the Botanical Garden is estimated from fig.2 at 154m with

$$\sqrt{\frac{L}{d}} = 1.14, \text{ one have}$$

$$h_s \simeq 176\text{m}$$

Therefore, the total height of the Boxes at City Center shall not exceed 176m.

APPENDIX II (CONT'D)

EVALUATION OF THE ADIABATIC LAPSE RATE

Relative humidity measured at McGill 50%

Surface temperature at McGill -1.6°C

Saturation pressure at that temperature 5.4mb

The vapor pressure that exists on the earth:

$$pw_o = 0.5 \times 5.4 = 2.7\text{mb}$$

Atmospheric pressure at McGill 985 mb

with C_p air = 1,015 J/g-°K

and C_p water vapor = 3.98 J/g-°K

Humidity ratio at the surface of the earth:

$$\omega = 0.6215 \frac{\times 2.7}{985-2.7} = .0017\text{g/g of dry air}$$

ADIABATIC LAPSE RATE FROM (3.1)

$$\Gamma_d = \frac{-9.81}{1{,}015(1-0.0017)+3.98\times0.0017} \approx -0.00962^{\circ}\text{C/m}$$

APPENDIX III

EVALUATION OF THE AERODYNAMIC SURFACE ROUGHNESS

According to Lettau, the aerodynamical roughness at surface level is given by:

$$Z_o = \frac{\overline{H}a}{2\overline{A}}$$

where:

$\overline{H}$ is the effective height of the roughness elements. (m)

a is the silhouette area seen by the wind. (m^2)

$\overline{A}$ is the lot area (total area of the region divided by the number of elements). (m^2)

Modelling Assumptions:

Based on local observations, it is assumed that the roughness elements are consisting of buildings of 7.0m high. The buildings are distributed in 34 square blocks consisting in 24m of building frontage and 5.m wide streets and yard which are discontinued by Cherrier Street. The investigated area is 1000m wide, 25m deep (34 blocks).

Effective height of the roughness elements: $\overline{H} = 7.0m$

Silhouette area seen by the wind because the frontage area is at an angle of 8^0 with the N.W. wind:

The silhouette area seen by the wind is:

$$a = .5 \times 7 \times 34 \times \frac{1}{3} \times \cos 8^0 = 353\ m^2$$

Lot Area:

In a 1000m square area the lot area is:

$$\overline{A} = 1000 \times 25/34 = 780m^2$$

Finally:

$$Z_o = \frac{7 \times 353}{2 \times 780} \approx 1.5m$$

APPENDIX IV

EVALUATION OF THE SOURCE STRENGTH

Sulphur dioxide is mainly exhausted by oil fired machinery used for heating of buildings and industrial activities. At a constant sulphur content, total exhausted mass flow is proportional to the total rate of oil consumption.

1) UNIFORMLY DISTRIBUTED TOTAL SOURCE STRENGTH

Oil consumption rate for heating purposes:

Rate of heat production per degree day

$$h_d = \frac{W_o \times h_e}{365 \times (18.33 - T_m)} \qquad (3.9)$$

where:

$W_o = 4.62\ 10^{12}$ gm/year (Ref.36 & 37)

$h_e = 11000$ cal/gm

$T_m = 5.96^{o}C$

$h_d = 1.13\ 10^{13}$ cal/day

Amount of heat for heating of building H_h

$$H_h = h_d\ (18.33 - T_d) \qquad (3.8)$$

where:

$T_d = -1.6^{o}C$

$H_h = 2.25\ 10^{14}$ cal/day

Daily oil consumption for heating of building: (3.10)

$$W_h = 2.23\ 10^{14}/11000 = 2.05\ 10^{10}\ \text{gm/day}$$

APPENDIX IV (CONT'D)

W_i = yearly consumption/$_{365}$

$= 5.25 10^{12}$ gm/y $/_{365} = 1.43 10^{10}$ gm/day

Total daily oil consumption:

$W_T = W_i + W_h = 2.05 10^{10} + 1.43 10^{10} = 3.58 10^{10}$ gm/day

SO_2 emission rate:

$$E = W_T \times 1.4\% \times \frac{1}{24 \times 3600} \qquad (3.12)$$

$= 4.31\ 10^4$ g/s

Evenly distributed source strength:

$Q = E/_{area}$ of greater Montreal

from (40) $3.66\ 10^8 m^2$

$Q = 14.7\ \mu g/m^2$-s

Source strength in a box of source area Δx = 500m

$Q\Delta x = 14.7 \times 500/1000 = 7.35$ mg/s

2) UNFORMLY DISTRIBUTED SOURCE STRENGTH DUE TO HEATING OF BUILDING ONLY.

In that case $W_T = W_h = 2.05\ 10^{10}$ gm/day

$E = 3.88$ gm/s

$Q = 8.62\ \mu g/m^2 - s$

and $Q\Delta x = 8.62 \times 500/1000 = 4.31$ mg/s

APPENDIX V

EVALUATION OF THE VERTICAL CONCENTRATION PROFILE

The set of coordinates taken from measurement taken by T. Oke and C. East (41) is:

$$C_1 = 1.59 \text{ mg/m}^3 \quad \text{at } z_1 = 25\text{m}$$

$$C_2 = 1.90 \text{ mg/m}^3 \quad \text{at } z_2 = 145\text{m}$$

The linear relation between these two coordinates is given by:

$$\frac{C - C_2}{z - z_2} = \frac{C_2 - C_1}{z_2 - z_1}$$

OR:

$$\frac{C - 1.90}{z - 145} = \frac{1.90 - 1.59}{145 - 25} = 0.00258$$

And finally:

$$C - 1.90 = 0.00258z - 0.386$$

$$C = 0.00258z + 1.514 \quad (\text{mg/m}^3)$$

END

13 02 80

FIN

[illegible]