

**THE PREDICTION OF FLOW PATTERNS, LIQUID HOLDUP AND PRESSURE
LOSSES OCCURRING DURING CONTINUOUS TWO-PHASE FLOW IN
HORIZONTAL PIPELINES**

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ABSTRACT

THE PREDICTION OF FLOW PATTERNS, LIQUID HOLDUP AND PRESSURE LOSSES OCCURRING DURING CONTINUOUS TWO-PHASE FLOW IN HORIZONTAL PIPELINES

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The need for a general method of predicting the energy or pressure losses that occur when two phases, one gaseous and the other liquid, flow simultaneously through a horizontal conduit has been stressed for several decades.

The purpose of this report is to investigate the correlation with which the pressure losses occurring in continuous two-phase gas-liquid flow in horizontal flowline can be calculated and thereby to enable the optimum design of such flowlines.

A general discussion of the two-phase flow in horizontal pipelines is given in Chapter I. Here the design activities and earlier investigations of Horizontal Multiphase Flow are discussed.

The first correlation is useful for the prediction of pressure losses in "Liquid Holdup", and the second correlation is an "energy loss factor" which, when used with the two-phase power balance permits pressure losses to be predicted. The third correlation is a flow pattern map with which two-phase flow patterns may be predicted.

A step by step procedure for using the correlations is given in Chapter V to predict the pressure loss in horizontal two-phase flow pipe.

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NOMENCLATURE

<u>Symbol</u>	<u>Quantity</u>	<u>Units</u>
A	Internal cross section of pipe	ft ²
B _o	Formation-volume factor for oil	-
B _g	Formation-volume factor for gas	-
B _w	Formation-volume factor for water	-
C	Coefficient for liquid viscosity number	-
D _B	Base pipe diameter = 0.08333 ft.	ft
D _G	Equivalent gas phase diameter	ft
D _L	Equivalent liquid phase diameter	ft
f	Darcy-Weisbach friction factor	-
F _T	Friction factor calculated using superficial velocities	-
f _R	Froude number	-
g	Acceleration of gravity	ft/sec ²
g _c	Conversion constant equal to 32.174	lb _m ft/lb _f sec ²
GLR	Gas/Liquid ratio	SCF/bbl
GR	Ratio of gas mass rate to total mass rate	-
h	Elevation	ft
H _L	Liquid holdup	-
H _L '	Liquid holdup factor assuming no slip	-
k	Mass ratio of gas to liquid	-
L	Equivalent length of pipe and fittings	ft
LR	Ratio of liquid mass rate to total mass rate	-

<u>Symbol</u>	<u>Quantity</u>	<u>Units</u>
M_T	Mass per time	lb _m /sec
N_D	Pipe diameter number = $D \sqrt{\rho_L g / \sigma}$	-
N_{GV}	Gas velocity number = $v_{SG} \sqrt{\rho_L / g \sigma}$	-
N_L	Liquid viscosity number = $\mu_L \sqrt{g / \rho_L \sigma}$	-
N_{LB}	Base water viscosity number	-
N_{LV}	Liquid velocity number = $v_{SL} \sqrt{\rho_L / g \sigma}$	-
P	Pressure	psi
\bar{P}	Average pressure for decrement	psia
P_a	Atmospheric pressure	psia
P_B	Base pressure for gas measurements	psia
(P/P_B)	Pressure number	-
q_L	Liquid production rate	bbt/day
q_G	Gas production rate	SCF/day
q_T	Total production rate	ft ³ /day
$(R_e)_L$	Reynolds number for liquid phase	-
$(R_e)_G$	Reynolds number for gas phase	-
$(R_e)_{TP}$	Reynolds number for two-phase mixture	-
R_s	Solution gas/oil ratio	SCF/bbl
$(SG)_L$	Specific gravity of liquid phase	-
$(SG)_G$	Specific gravity of gas phase	-

<u>Symbol</u>	<u>Quantity</u>	<u>Units</u>
$(SG)_m$	- Specific gravity of two-phase mixture	-
T	- Temperature	$^{\circ}R$
\bar{T}	- Average temperature for increment	$^{\circ}R$
T_b	- Base temperature for gas measurements	$^{\circ}R$
v	- Velocity	ft/sec
v_m	- Velocity of gas-liquid mixture at flowing conditions	ft/sec
v_{SG}	- Superficial velocity of gas at flowing conditions based on total area of conduit	ft/sec
\bar{v}_{SG}	- Average superficial velocity of gas at flowing conditions	ft/sec
v_{SL}	- Superficial velocity of liquid at flowing conditions based on total areas of conduit	ft/sec
\bar{v}_{SL}	- Average superficial velocity of liquid at flowing conditions	ft/sec
V	- Specific volume of fluid at flowing conditions	ft^3/lb_m
V_e	- Volume of pipe element	ft^3
V_m	- Specific volume of gas-liquid mixture at flowing conditions	ft^3/lb_m
\bar{V}_m	- Integrated average specific volume of gas-liquid mixture at flowing conditions	ft^3/lb_m
W_f	- Irreversible energy losses	$lb_f ft/lb_f$
W_e	- External work done by the flowing fluid	$lb_f ft/lb_f$
W_e	- Weber number	-
WOR	- Water-oil-ratio	-

SymbolQuantityUnits

z

Compressibility factor for gas

Greek Symbols α

Arbitrary constant

 β

Arbitrary constant

 Δ

Difference

 ϵ

Absolute roughness

ft

 μ

Viscosity

cp

 μ_G

Viscosity of gas phase

cp

 μ_L

Viscosity of liquid phase

cp

 μ_m

Viscosity of gas-liquid mixture

cp

 ρ

Density

 lb_m/ft^3 ρ_G

Density of gas phase at flowing conditions

 lb_m/ft^3 $\bar{\rho}_G$

Integrated average density of gas phase at flowing conditions

 lb_m/ft^3 ρ_L

Density of liquid phase at flowing conditions

 lb_m/ft^3 $\bar{\rho}_L$

Integrated average density of liquid phase at flowing conditions

 lb_m/ft^3 ρ_m

Density of gas-liquid mixture at flowing conditions

 lb_m/ft^3 $\bar{\rho}_m$

Integrated average density of gas-liquid mixtures at flowing conditions

 lb_m/ft^3 σ

Surface tension of liquid-air interface

dynes/cm

 ψ

Arbitrary function

CHAPTER I INTRODUCTION

Due to the frequent occurrence of multiphase, gas-liquid flow in pipelines and the desire to accurately calculate the pressure losses that occur in these pipelines, this type of flow is of considerable interest to the petroleum, chemical, and nuclear industries. In the petroleum industry, gas-liquid mixtures have been transported over relatively long distances (100 mi) in a common line due to the advent of centralized gathering and separation systems.

The problem of designing such two-phase flowlines came sharply into focus since the beginning of offshore oil field development. The corrosive nature of ocean water on pipes prohibits the installation of separation systems far offshore and then transporting the gas phase and the liquid phase to land based storage facilities in separate pipelines. In addition, construction and maintenance costs of this excess equipment are too severe. Therefore, long multiphase flowlines must be designed, and one of the main problems involved is the prediction of the pressure losses that occur when a gas-liquid mixture flows in a closed conduit.

When oil and gas are produced from any given natural reservoir, the energy stored in the reservoir is consumed in three distinct flow processes. The three flow processes are depicted in Figure 1 which was taken from an extensive study by Hagedorn [1]. First, the fluids must flow through the porous medium to the wellbores. A potential gradient

CONTINUOUS TWO PHASE FLOW PROBLEM

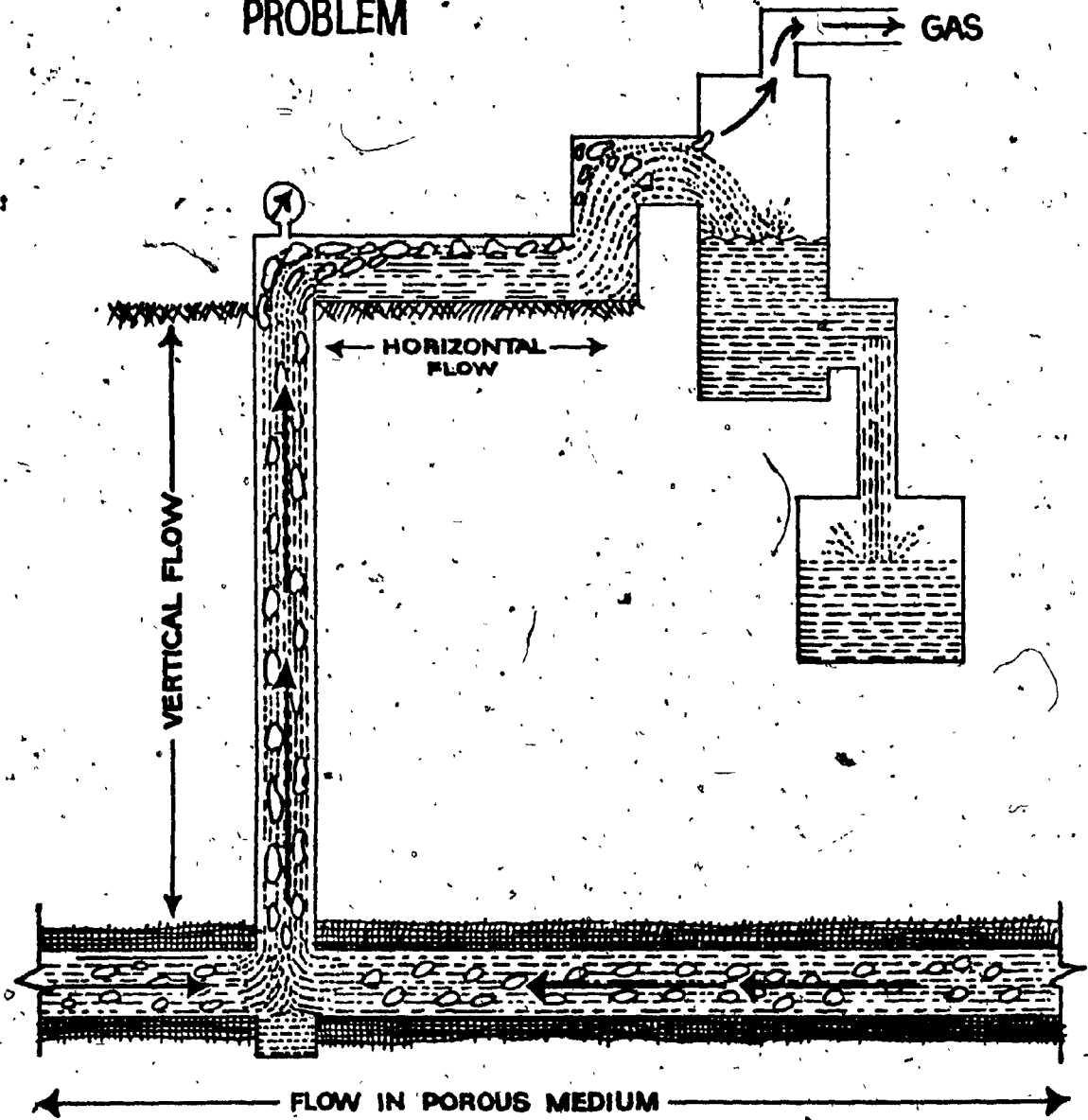


FIG.1

THREE TWO-PHASE FLOW PROCESSES.

AFTER HARDEN [1]

must exist for this flow to originate. Secondly, energy is consumed in large quantities as the fluids flow up the wellbores. Lastly, either reservoir energy or input mechanical energy is consumed as the fluids traverse flowlines from the wellheads to the separation.

It must be stressed that two-phase flow is not limited to oil flowlines. This phenomenon commonly exists in chemical plants, refineries, atomic reactors, evaporator tubes, and many other processes encountered in the chemical process industries. Many different processes give rise to numerous types of gas-liquid flow. A great number of these types of two-phase flows are discussed thoroughly by Gouse [2, 3, 4, 5].

The main objective of all multiphase flow studies has been to develop a technique with which the static pressure drop can be calculated. Static pressure losses in gas-liquid flow are quite different from those encountered in single phase flow. This is due to the fact that in most cases an interface exists and the gas slips past the liquid. The interface may be smooth or have varying degrees of roughness depending on the flow pattern. Therefore, a transfer of energy from the gaseous phase to the liquid phase may take place while energy is lost from the wetting phase at the pipe wall. Such an energy transfer may be in the form of heat exchange or in the form of acceleration and wall friction. Since each phase must flow through a smaller area than it would if it flowed alone, amazingly high pressure losses occur when compared to single phase flow.

Most of the previous investigators of two-phase flow phenomena have chosen to separate their experimental data into several groups of observed flow patterns or regimes. Separate energy loss correlations have then been developed for each flow regime. This appears to be a logi-

cal approach to correlate widely scattered data. The problem arises, however, in determining which particular flow pattern exists for a certain set of flow conditions and in selecting the correct energy loss correlation for that pattern. The problem is complicated further by the existence of several correlations by many authors for any one particular flow regime. The number of reported flow mechanisms for gas-liquid mixtures varies from four to ten depending on the method by which the regimes are separated.

Several previous investigators have measured a quantity defined as "liquid holdup." Liquid holdup is simply that fraction of a unit volume of pipe that is occupied by flowing liquid. A knowledge of the variation of liquid holdup along the length enables one to calculate the average linear velocities of each phase and consequently the difference between the phase velocities may be calculated. The difference between the velocity of the two phases is known as the "slip velocity." Due to the slippage of the gas over the liquid, energy is transferred across the interface between the phases within the system. Therefore, it is quite clear that liquid holdup is an important parameter in multiphase flow situations.

Many techniques have been used to measure liquid holdup and likewise many methods have been proposed to correlate liquid holdup with flow parameters and the physical properties of the flowing fluids. These methods of measurement and liquid holdup correlations are discussed in detail in a later chapter.

The subject of multiphase flow has stirred the minds of men for some time and the literature on the subject is voluminous.

A chronological review of the previous gas-liquid flow studies that are applicable to this study follows in the next section.

The purpose of this report is to develop a best available correlation with which the pressure losses occurring in continuous two-phase gas-liquid flow in horizontal flowline can be calculated and thereby to enable one to obtain the optimum design of such flowline.

EARLIER INVESTIGATIONS OF MULTIPHASE FLOW

A great deal of research has been conducted in the general area of multiphase, confined flow. Interest in this field has increased greatly since the year 1830 when the first known work in this area was published. Figure 2, taken from a literature survey by Gouse [2], shows the number of publications for all gas-liquid flow studies excluding atomization, cavitation, and condensation. In general, multiphase flow is a subject of considerable interest since early 1960. In the review that follows, only the studies that deal directly with steady-state, horizontal multiphase flow are discussed, since a complete review of all multiphase flow literature would fill several volumes.

The evolution of a fundamental knowledge of the gas-liquid flow problem is by no means at an advanced stage at the present time. However, some progress of limited applicability has been made in the last quarter of a century in the area of empirical correlations. Several two-phase flow relationships have been developed using somewhat different approaches.

Previous investigators have followed one of five broadly classified approaches listed by Dukler, et al. [6] as follows:

1. Empirical correlations alone.
2. Empirical correlations with dimensional analysis.
3. Empirical correlations utilizing similarity analysis and model theory.

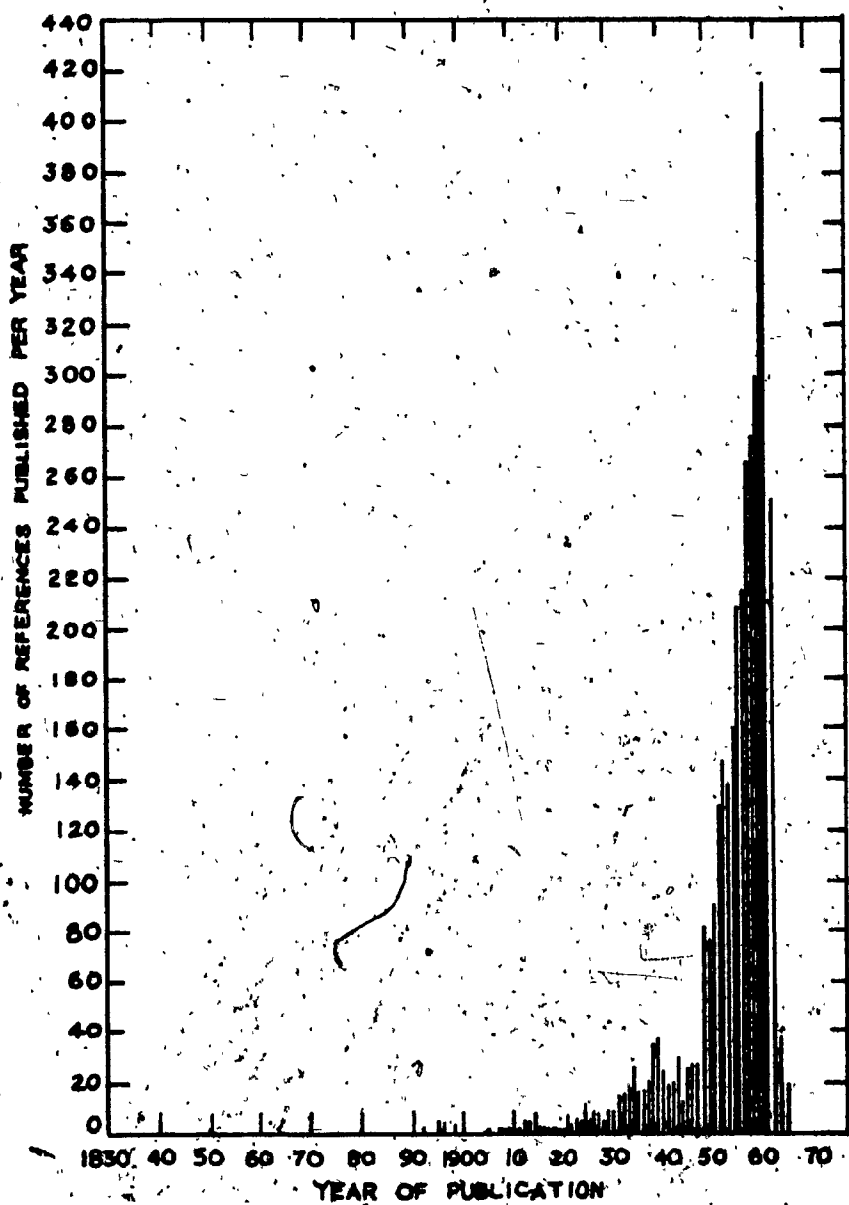


Fig. 2. Number of Publications on Two-Phase Gas-Liquid Flow Phenomena Appearing Per Year Versus the Year of Publication Excluding Atomization, Cavitation, and Condensation (Courtesy Rose) [3]

- 4. Mathematical analysis and theoretical development of equations using simplified physical models.
- 5. Semi-empirical correlations developed from the conservation of energy and momentum equations using experimental data to evaluate the transport terms.

The first practical correlation evolved in 1949 from the study conducted at the University of California by Lockhart and Martinelli and others [7, 8, 9, 10]. Several different fluids were used in this study in short test pipes whose diameters varied from 2.54 centimeters to .1488 centimeters or capillary size. After an extensive mathematical development, they resorted to experimental data to evaluate certain constants in order to develop the pressure loss correlations.

Lockhart and Martinelli proposed four pressure loss correlations for four different flow mechanisms. The flow mechanisms consisted of (1) laminar flow in both phases (2) turbulent flow in both phases (3) and (4) one phase laminar and the other turbulent. Three correlating parameters χ , ϕ_L , and ϕ_G , were defined as follows:

$$\chi = \sqrt{\left(\frac{\Delta P}{\Delta L}\right)_L / \left(\frac{\Delta P}{\Delta L}\right)_G} \dots\dots\dots 1-1a$$

$$\phi_L = \sqrt{\left(\frac{\Delta P}{\Delta L}\right)_{T.P.} / \left(\frac{\Delta P}{\Delta L}\right)_L} \dots\dots\dots 1-1b$$

$$\phi_G = \sqrt{\left(\frac{\Delta P}{\Delta L}\right)_{T.P.} / \left(\frac{\Delta P}{\Delta L}\right)_G} \dots\dots\dots 1-1c$$

where

$$\left(\frac{\Delta P}{\Delta L}\right)_L$$

is the pressure gradient that would exist if the liquid were flowing alone in the conduit.

$$\left(\frac{\Delta P}{\Delta L}\right)_G$$

is the pressure gradient that would exist if the gas were flowing alone in the conduit.

$$\left(\frac{\Delta P}{\Delta L}\right)_{T.P.}$$

is the actual two-phase pressure gradient.

These investigators found that ϕ_L and ϕ_G were both correlative with X and flow mechanism. They further found that liquid holdup could be correlated with X . Therefore, a total of nine correlations are presented in their pioneering study. The detail of these correlations can be found in the original reference [7].

Several authors [11, 12] refer to the work of Lockhart and Martinelli as the classical approach to the problem and some [13] state that these correlations have received universal acceptance. However, other investigators such as Hoogendorn [14], Isbin [15], Ciafalonì [16], and others have shown that the Lockhart and Martinelli correlations are by no means a panacea to the problem.

A possible explanation of the shortcomings of these correlations consists of three parts. First, Lockhart and Martinelli conducted experiments on gas-liquid flow at near atmospheric pressure. Secondly, the data were taken in small diameter tubes in the laboratory. These two limiting conditions on the data could explain why the correlations do not always work when applied to conditions of high pressures and large diameter pipes.

Lastly, in using the correlations, one has extreme difficulty in determining which flow mechanism exists; therefore, if an error is made at this critical stage, the wrong pressure loss correlation is used. It should be pointed out that the lines separating the flow mechanisms are actually broad zones of overlapping flow mechanisms. Ciufaloni shows that large errors are encountered when using the correlations near these transition regions. Gazley [17] have shown errors of +44% to -50% when the correlations were tested with their experimental data.

The year 1949 was also the year that Bergelin et al. [18] published part one of their multiphase flow study. These authors showed that the Lockart and Martinelli correlations were not completely general or adequate. A general method of predicting two-phase pressure drop was not proposed, but it was suggested that a friction factor chart using gas properties and a gas friction factor might be useful.

The gas phase pressure loss was defined by Bergelin, et al. [18]

as

$$-\left(\frac{dP}{dX}\right)_{G.P.} = \frac{f'_G W_G^2 (P_G + C_I)}{2 g_c P_G S_G} \dots \dots \dots 1-2$$

where

f'_G = gas phase friction factor

W_G = mass flow rate of gas

P_G = pipe wall perimeter wetted by the gas phase

C_I = length of interfacial chord

S_G = area open to gas flow

X = distance travel

The friction factor, f'_G was found to correlate with a Reynolds modulus which is defined as

$$R_{eg} = \frac{D'_G W_G}{\mu_G S_G}$$

where D'_G = an equivalent gas phase diameter.

Experiments were carried out in such a manner that the interfacial position or height was measured. This indicates that stratified flow was prevalent.

Bergelin et al. pointed out a very important factor, namely that the length of the test section greatly affected the flow pattern. They showed that waves form in a 16-foot (4.88 meters) length at lower air rates than in a 10-foot (3.048 meter) length. The question of the reliability of short tube multi-phase flow data immediately arises. Possibly, entrance effects cause short tube data to be unreliable. This observation was confirmed by the study of Eaton, B.A. [19] in which flow patterns were observed to change as the fluids traversed a 518.16 meter length.

During 1952 Poettmann and Carpenter [20] published the result of their study on vertical two-phase flow in oil well tubing. Poettmann and Carpenter developed their correlation on the basis of an energy balance and a two-phase energy loss factor chart. This approach has proved to be sound and the correlation has been used quite successfully for many years. Hagedorn [1] used a similar approach in his study of vertical multiphase flow in long tubes to develop what appears to be a very good correlation for predicting vertical two-phase pressure gradients.

Schneider [21] in 1953 developed another two-phase friction

factor correlation and in 1954, Schneider, White, and Huntington [22] published the results of all this work.

The Schneider friction factor, $F_{T.P.}$, was correlated by —

$$F_{T.P.} = \psi \left[\frac{W_L U_L}{W_G U_G} \right] \dots \dots \dots 1-4$$

where

ψ = constant

W_L = mass of liquid flow

U_L = velocity of liquid flow

and a two-phase friction factor chart was prepared.

In 1954 Alves [23] published the results of his multi-phase flow study. No general correlation was proposed in this work, as Alves specifically stated that his study and solution were for a particular chemical engineering process. Baker [24] began to work on the multiphase flow problem about this same time. Baker has since published several articles on the subject, and his method of solution will be discussed later in this section.

Chenoweth and Martin [12] in 1955 ran a series of 264 tests in large pipes and at mean pressures up to 100 psia to check the Lockhart and Martinelli correlation under these conditions. The results of this study indicate that the Lockhart and Martinelli correlations became progressively worse as the mean pressure and pipe diameters are increased. Chenoweth and Martin proceeded to develop what they called an improved correlation along the same lines as the ones by Lockhart and Martinelli. The improved correlations are limited to turbulent flow and the improvement appears to be limited to their data. Baker [24] used their correla-

tion on field data and showed that large errors are encountered.

White and Huntington [25] conducted an extensive search for the answers to the mysteries of multiphase flow. The study was conducted in the 1950's in the laboratory and published in 1955. The data was obtained via a clear plastic tube at or near one atmosphere mean pressure. Color motion pictures were made of the flow regimes and some six different patterns were defined and mapped. Pressure losses were correlated by means of an energy loss factor and a mass flow ratio. Different correlations were proposed for the different flow patterns. White and Huntington state specifically in their summary that the flow pattern map should not be used for conditions where the mean pressure is greater than one atmosphere. Therefore, for higher pressure conditions, it is impossible to be sure of the flow regime that exists and which pressure loss correlation should be applied.

In 1956, Bertuzzi, Tek and Poettmann [26] proposed a somewhat different approach to the problem by introducing a new Reynolds function with which the energy loss factor could be correlated. These authors developed an energy balance for two-phase horizontal flow and, using this equation with experimental data, calculated the over-all energy losses due to irreversibilities. They found that the loss factor could be correlated by a Reynolds function as follows:

$$\text{Reynolds function} = \psi_1 (R_G^a \cdot R_L^b) \dots \dots \dots 1-5$$

where

R_G = Reynolds number of the gas assuming it flows alone

R_L = Reynolds number of the liquid assuming it flows alone

$$a = \frac{K}{K + 1}$$

$$b = 1/e^{0.1K}$$

$$K = \frac{W_g}{W_L} \frac{\text{mass rate of gas}}{\text{mass rate of fluid}}$$

$$\psi_1 = \text{constant}$$

The energy loss factor chart prepared from this correlation consists of several lines with a certain range of K values associated with each line. Each line has a transition region and overlap similar to that of the Moody diagram [27] for single phase flow.

The correlation of Bertuzzi, Tek, and Poettmann has the advantages of being relatively free of flow pattern effect and is simply to use. The Reynolds function will reduce to the single phase Reynolds number as K approaches infinity (for all gas flow) or as K approaches zero (for all liquid flow). However, it should be pointed out that the acceleration term was set equal to zero in the energy balance; therefore, acceleration pressure losses were not properly accounted for. Lubinski [20] has pointed out that in some cases acceleration can take on a real significance. Baker [24] shows that this correlation can give results that are in extreme error. The reasons for the failure can be found in detail in the original reference.

Investigators were active in 1957 collecting experimental data in separate efforts to solve the problem. Among these investigators were Hoogendorn [14], Baker [24], Huntington [22], Reid [28], Yocum [29], and others.

Yocum presented a paper in 1957 in which the frictional losses occurring in long oil field flowlines were correlated against the square of the Froude number $(U^2/gz)^2$. The assumptions and development of this correlation can be found in the original paper. However, this type of friction factor model has not received much acceptance in other multiphase flow literature.

A solution to the horizontal two-phase flow problem that has received a considerable amount of acceptance in the oil industry is that which Baker [30] proposed in 1958. Baker incorporated the Lockhart and Martinelli correlations [7] into his solution. His main contribution was made in postulating an equation for each different flow pattern. Baker also proposed a chart with which the flow patterns could be predicted. The merits of this flow pattern map will be discussed in a later chapter.

Chisholm and Laird [31] studied the problem using the basic approach of Lockhart and Martinelli [7]. However, they extended the correlation to include pipes of varying degrees of wall roughness.

In the final correlation,

$$\left[\frac{(\Delta P / \Delta L)_{T.P.}}{(\Delta P / \Delta L)_L} \right]^{-1} \text{ was found to vary } \dots \dots \dots 1-6a$$

with

$$\lambda = \left(\frac{G_L}{G_G} \right)^{0.875} \left(\frac{\mu_L}{\mu_G} \right)^{0.125} \left(\frac{\rho_G}{\rho_L} \right)^{0.5} \dots \dots \dots 1-6b$$

according to the following equation:

$$\left[\frac{(\Delta P / \Delta L)_{T.P.}}{(\Delta P / \Delta L)_L} \right]^{-1} = C/X^M \dots \dots \dots 1-7$$

where C and M were constants determined by the flow rates and tube rough-

ness. The correlation can also be expressed as follows:

$$\frac{\Delta P_{T.P.}}{\Delta P_L} = 1 + C \left(\frac{G_G}{G_L} \right)^{0.875M} \left(\frac{\mu_G}{\mu_L} \right)^{1.25M} \left(\frac{\rho_L}{\rho_G} \right)^{0.5M} \dots \dots \dots 1-8$$

The authors have excluded data whose values of X are less than 0.40. Baker, in his discussion of the Chisholm and Laird correlation, points out that most gas condensate two-phase flow lines fall within the excluded range. Hence, another correlation is added to the list of empirical relationships with limited applicability.

Next in the series of reported horizontal multiphase pressure loss correlations is that of Hoogendorn [14] which was developed from data taken in 24, 50, 91 and 140mm diameter pipes. Hoogendorn varied the roughness of the 50mm pipe and, in addition, he determined the liquid holdup for all tests by a new capacitance technique [4, 14]. Flow patterns were observed and a pattern was developed. Hoogendorn reported that ΔP_{accel} amounted to as much as 15% of $\Delta P_{T.P.}$ in the smaller pipes.

The data were used in the familiar Lockhart and Martinelli correlation, but the results showed that the correlation did not fit the data when large pipes and high pressures were utilized. Also, the correlation proved to be inaccurate for some flow patterns under any conditions.

Hoogendorn proceeded to develop several friction factor relationships and pressure loss correlations corresponding to the several different flow patterns. These equations and charts can be found in the original reference. These correlations have not been widely used.

Bankoff [32] in 1960, published a two-phase pressure loss correlation and a holdup correlation that have been checked extensively by Dukler [6] with the data in the data bank at the University of Houston.

These correlations were found to be unsatisfactory in most cases. Hughmark [33, 34] improved Bankoff's holdup correlation in 1962, and Dukler found this correlation to be the best holdup correlation to date.

Ros [15] published the results of an extensive study on vertical two-phase flow in 1961. A dimensional analysis was presented that utilized the liquid density, interfacial tension, and gravity as repeating variables. Hagedorn [1] in 1963-64, utilized this dimensional analysis very successfully to correlate vertical two-phase flow. This analysis, as well as the holdup correlation of Hughmark [34] will be discussed in more detail in the chapter on holdup.

The next in the series of horizontal two-phase fluid flow studies that has been published is that of Dukler et al. [35]. As has been previously mentioned, these investigators have accumulated all the published data on horizontal two-phase flow and formed what they call the data bank. The data consist of both short tube laboratory data and long tube oil field data. Over 20,000 experimental measurements have been taken, half of which have been taken since 1959. The data have been culled and approximately 2,600 data points remain as consistent data. Dukler, et al., report that some of the data contain very large errors. These authors have discarded a great deal of short tube data where end effects were known to be large.

The culled data have been used in what Dukler et al. believe to be the five most reliable two-phase pressure loss correlations. The correlations tested were those of Lockhart and Martinelli [27], Baker [30], Bankoff [32], Chenoweth and Martin [12] and Yagi [39]. These correlations are five of about twenty-five published correlations, but the others have received much less acclaim. Results of the comparison indi-

cate that the Lockhart and Martinelli and Baker correlation are still the most reliable of the five.

Three holdup correlations were tested with the culled holdup data. The Hagedorn [14], the Lockhart and Martinelli [7], and the Hughmark [34], correlations were tested with the results indicating that the Hughmark correlation is the most reliable. Hughmark's holdup correlation is an extension of the original work of Bankoff [32], which will be discussed in a later chapter.

In Part B of the article by Dukler *et al.* [35], the approach through similarity analysis is introduced and developed. The parameters for two-phase flow corresponding to the Euler and Reynolds numbers for single phase flow are developed. These parameters are given below.

$$N_{ReT.P.} = L \bar{v} M \left[\frac{\rho_L \frac{\lambda^2}{H_L} + \rho_G \frac{1-\lambda^2}{H_G} C_1}{\mu_L \lambda + \mu_G (1-\lambda) G_2} \right] \dots \dots \dots 1-9a$$

and

$$N_{EuT.P.} = 2f_{T.P.} = \left[\frac{dP/dX}{\frac{\bar{v} M^2}{g C_L}} \right] \left[\frac{1}{\rho_L \frac{\lambda^2}{H_L} + \rho_G \frac{(1-\lambda)^2}{H_G} C_2} \right] \dots \dots \dots 1-9b$$

where

$$\lambda = q_L/q_L + q_G$$

$$H_L = \text{liquid holdup}$$

$$H_G = 1-H_L = \text{Gas holdup}$$

$$N_{ReT.P.} = \text{two-phase Reynolds number}$$

$$N_{EuT.P.} = \text{two-phase Euler number which equals twice the friction factor}$$

From dynamic similarity, the authors define the two-phase density and viscosity as follows:

$$\rho_{T.P.} = \rho_L \frac{\lambda^2}{H_L} + G \frac{(1-\lambda)^2}{H_G} C_1 \dots\dots\dots 1-10a$$

$$\mu_{T.P.} = \mu_L \lambda + \mu_G (1-\lambda) C_2 \dots\dots\dots 1-10b$$

Various assumptions were made to evaluate the C's, and this was done by considering some special cases of two-phase flow. One special case is for homogeneous flow with no slip between the phases, in which case $C_1 = C_2 = 1$.

All of these correlations must utilize liquid holdup and this study does not include a reliable holdup correlation.

Banoczy [17] published the study on multiphase flow in which he modified the Lockhart and Martinelli correlations [7]. This correlation appears to be only slightly better than the original and has not been verified with any other data.

The latest work on horizontal two-phase fluid flow has been published is that of Mandhane, J.M., Gregory, G.A., and Aziz, K.A. [36]. These investigators have accumulated approximately 10,600 observations on horizontal two-phase flow and formed what they call the U.C. Multiphase-Flow Data Bank [37], and developed a new flow pattern map; this map is in over-all best agreement with the observations in the data bank. However, this flow pattern has not yet received much acceptance in other multiphase flow literature.

Presently, many investigations on horizontal two-phase fluid flow are under way at the University of Calgary, Alberta, and University

of Texas, Texas, USA. The conclusion reached after a complete study of the preceding literature on horizontal multiphase flow is that a general two-phase pressure loss correlation is not available and that the hold-up correlations in the literature are not reliable. None of the preceding correlations is extremely good over a wide range of conditions. It is therefore very difficult to say which correlation is best.

CHAPTER II
LIQUID HOLDUP CORRELATION

2.1 REASONS FOR MEASURING LIQUID HOLDUP

In a previous section, "liquid holdup" was defined as that portion of a unit volume of pipe that is occupied by flowing liquid. The complement of liquid holdup is sometimes called the "void fraction." The term "void fraction" is a misnomer since it designates that portion of a unit volume of pipe occupied by flowing gas. Nevertheless, it is a common term in two-phase flow literature. Void fraction will be called "gas holdup" hereafter and is related to liquid holdup by the following equation:

$$H_G = 1 - H_L \dots \dots \dots (2.1)$$

One may wonder why it is important to measure holdup since it appears at first glance that one should be able to calculate this quantity when the flow rates, pressure, temperature, and fluid properties are known. This is possible when there is no slip velocity between the two phases. This "no slip" condition does not exist under actual flowing conditions, but is approached in some cases. The evidence that there exists a relative velocity between the gas and liquid phase is shown in Fig. 3, taken from a literature survey, Eaton [19]. Values for liquid holdup were calculated (assuming no slip velocity exists) by —

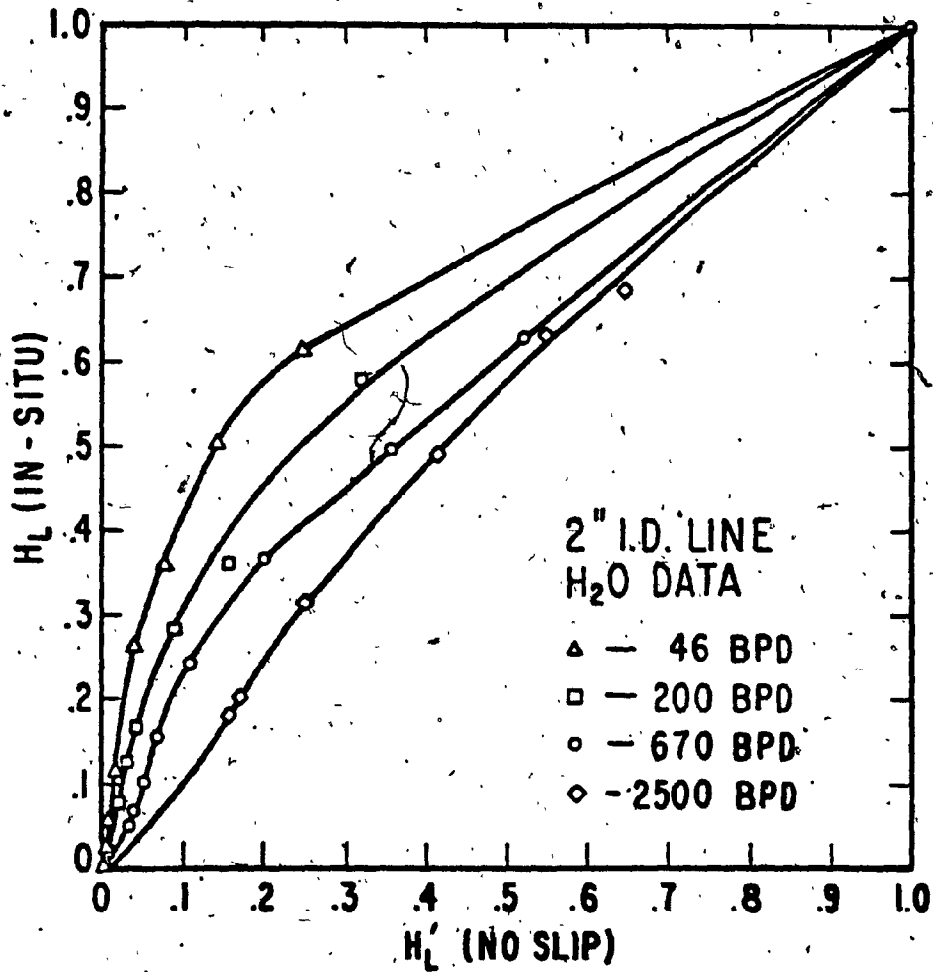


FIG. 3 COMPARISON OF MEASURED FLOWING LIQUID HOLDUP WITH CALCULATED "NO-SLIP" LIQUID HOLDUP (AFTER EATON) [19]

$$H_L' \text{ (no slip)} = \frac{q_L}{q_L + q_G} \dots \dots \dots (2.2)$$

where the q 's are volumetric flow rates considered at the pressures and temperatures at which the actual liquid holdup values were measured. The "no slip" values and the actual experimental values of liquid holdup are compared in Fig. 3. It was noticed that in every case the actual H_L is greater than the calculated H_L' (no slip). The actual values of H_L are greater than the "no slip" values due to the fact that the gas slips past the liquid. This slippage actually causes the liquid phase to move and, in most cases, to accelerate.

A knowledge of liquid holdup also enables one to predict the two-phase flowing mixture density. The density of a flowing gas-liquid mixture is given by

$$\rho_M = \rho_L H_L + \rho_G (1 - H_L) \dots \dots \dots (2.3)$$

In a similar manner, the viscosity of a gas-liquid mixture may be defined by

$$\mu_M = \mu_L H_L + \mu_G (1 - H_L) \dots \dots \dots (2.4)$$

or, as was done by Hagedorn [1].

$$\mu_M = \mu_L^{H_L} \cdot \mu_G^{(1 - H_L)} \dots \dots \dots (2.5)$$

A knowledge of liquid holdup does not shed any light on the real fluid distribution in a pipeline. Two flow regimes of widely differing geometry can have the same value for liquid holdup. The real significance of a knowledge of holdup is that it enables one to calculate the area through which each phase is flowing at any point in a pipeline. The

studies of [4, 6, 19, 30] show that liquid holdup and the cross sectional area of each phase change all along the flowline for any given set of conditions.

The obvious conclusion is that a correlation of liquid holdup against gas-liquid flow rates, fluid properties, and geometry of the system would be extremely useful for the calculation of real average velocities of each phase and the evaluation of the transport terms in any flow situation.

In order to obtain a useful holdup correlation, experimental data had to be obtained. There are several methods by which experimental holdup values may be measured.

2.2 METHOD OF MEASUREMENT OF LIQUID HOLDUP

There are several ways in which liquid holdup can be measured, and these techniques fall generally into two classes:

I. Indirect holdup measurement

- a) By the concentration of dissolved radioisotopes in the liquid phase
- b) Acoustic velocity in a two-phase mixture
- c) Dielectric constant of a two-phase mixture
- d) Electric resistivity of a two-phase mixture
- e) Thermal conductivity of a two-phase mixture
- f) Capacitance of a two-phase mixture
- g) Ability of a mixture to absorb nuclear particles

- 1) Alpha particles
- 2) Beta particles
- 3) Neutrons

II. Direct holdup measurement

- a) Direct measurement of each phase by use of quick-closing valves
- b) Photography
- c) Sampling probes.

Several other techniques have been used to measure holdup and Gouse [4] has discussed their merits as well as some of the techniques listed above.

Selecting a holdup calculation method is not the same type of problem as selecting a method for computing the frictional pressure gradient as discussed in Chapter I. Of the limited number of holdup correlations available, as discussed in Chapter I, Dukler [6, 35], analyzed over a wide range of values the best holdup correlations published by using his data bank. His analysis in terms of average error and standard deviation are presented in Table 1. Of the correlations Dukler tested Hugmark's [33], Martinelli's [8], Hoogendoorn's [14] and discussed Eaton's [19].

These correlations have limited applicability. Eaton's [19] holdup correlation ranges from approximately 0.01 to 0.7, Martinelli's from approximately 0.05 to 0.84 and Hugmark's appears to be valid from approximately 0.1 to 1.00. Dukler et al, in their studies, show that both Hugmark and Martinelli's fall below R_e (in place liquid holdup) = 0.2. In this range, Eaton's [19] correlation is recommended.

Table 1 shows Hugmark's correlation to be best in the middle range and Hoogendoorn's for R_e close to 1.00. Dukler et al recommended Hugmark's correlation generally and used it to test his results with other equations.

Hugmark's [33] correlations have received a considerable amount

TABLE I
COMPARISON OF HOLDUP CORRELATIONS

Range of Measured R_e	Hoogendoorn			Hugmark			Martineil			
	\bar{d}	σ	\bar{y}	\bar{d}	σ	\bar{y}	\bar{d}	σ	\bar{y}	n
0.80 - 0.99	-1.8	10.0	5.0*	-3.0	10.0	6.0	2.0	13.0	10.0	193
0.60 - 0.79	-0.05	11.0	10.0	-6.1	13.0	12.5	0.9	9.3	10.0*	111
0.40 - 0.59	2.9	18.0	17.5*	-11.0	20.0	20.0	8.5	20.0	17.5	124
0.20 - 0.39	0.5	33.0	27.5	-11.0	22.0	22.5*	23.0	33.0	35.0	145
0.10 - 0.19	18.0	74.0	25.0	20.0	44.0	27.5*	51.0	72.0	57.5	74
0.06 - 0.099	69.0	72.0	57.5	69.0	41.0	32.5*	57.0	71.0	42.5	34
0.01 - 0.059	295	208	-	303	262	-	276	377	-	40

* denotes correlation that seemed to predict best for each horizontal row.

where R_e = in place liquid holdup
 \bar{d} = any given value
 \bar{y} = mean value of d
 n = total number of d values
 d, σ, \bar{y} are parameters expressed in percentages.

of acceptance in the oil industry. The detail of these correlations can be found in the original references. However, the calculation scheme of Martinelli's is simple, while the correlations of Martinelli and Eaton are explicit in holdup, Hughmark's is not so easily resolved. According to Hughmark [33] R_e is given by:

$$W_t/W_o = 1 - (\rho_l/\rho_g)[1 - K/(1 - R_1)] \quad \dots \dots \dots (2.6)$$

Solving for R_1 :

$$R_1 = 1 - K \left[1 + \left(\frac{\rho_g}{\rho_l} \right) \left(\frac{W_t}{W_g} - 1 \right) \right] \quad \dots \dots \dots (2.7)$$

where

K is a parameter that is a function of $\delta = (R_e)^{1/6} (Fr)^{1/8} / \lambda^{1/4}$.

If the Reynolds and Froud numbers are

$$\left. \begin{aligned} R_e &= DG_t / [R_1 \mu_l (1 - R_1) \mu_g] \\ Fr &= V_{ns}^2 / g_c D \end{aligned} \right\} \dots \dots \dots (2.8)$$

if we let:

$$C_1 = 0.642 V_{ns}^{0.50} G_t^{0.1667} D^{0.0427} / V_{ST}^{0.25}, \quad \dots \dots \dots (2.9)$$

then, for Hughmark's definitions, becomes:

$$\delta = C_1 / [R_1 (\mu_l - \mu_g) + \mu_g]^{0.1667} \quad \dots \dots \dots (2.10)$$

Hughmark presents a chart of K vs δ for a graphical solution of liquid holdup.

2.3 EFFECT OF PRESSURE AND TEMPERATURE ON FLUID PROPERTIES

The previously described liquid holdup function contains several fluid properties that are functions of both temperature and pressure of the system. The functional relationships are not known exactly in many cases.

Interfacial or surface tension appeared several times in the holdup function. This quantity was corrected for temperature changes and solution gas effects by the charts shown by Katz, et al. [38]. Baker [30] also presents some experimental data concerning the behavior of this variable. The effect of changes in surface tension appeared to be rather slight, but these changes were accounted for in each case.

Chew and Connally [13] developed a correlation whereby the gas-free oil viscosity may be corrected for solution gas. They found that the viscosity of gas-saturated crude oil was related to the gas-free viscosity, or dead oil viscosity, at any given temperature by

$$\mu_{OS} = A(\mu_{OD})^b \dots \dots \dots (2.11)$$

where

μ_{OS} = viscosity of gas saturated crude oil

μ_{OD} = dead oil viscosity.

Dead oil viscosity is given by Figure 4 and the intercept A and slope b are given by Figure 5. Then equation (2.11) may be evaluated for μ_{OS} .

Figure 6 is a chart for a graphical solution of equation (2.11).

Solution gas must always be evaluated and the familiar Standing correlation may be used for this purpose. This correlation may be found in the textbook by Katz, et al. [38].

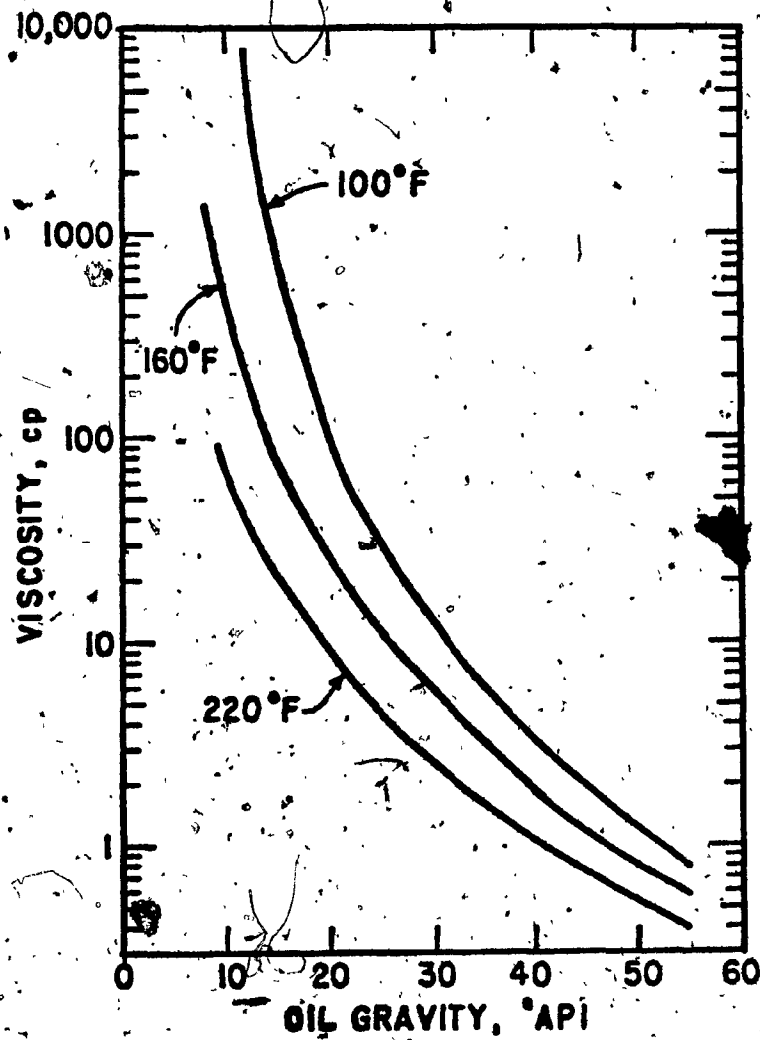


FIG. 4 VISCOSITY OF GAS-FREE
CRUDE OILS AT ATMOSPHERIC PRESSURE.

(After Frick²⁴)

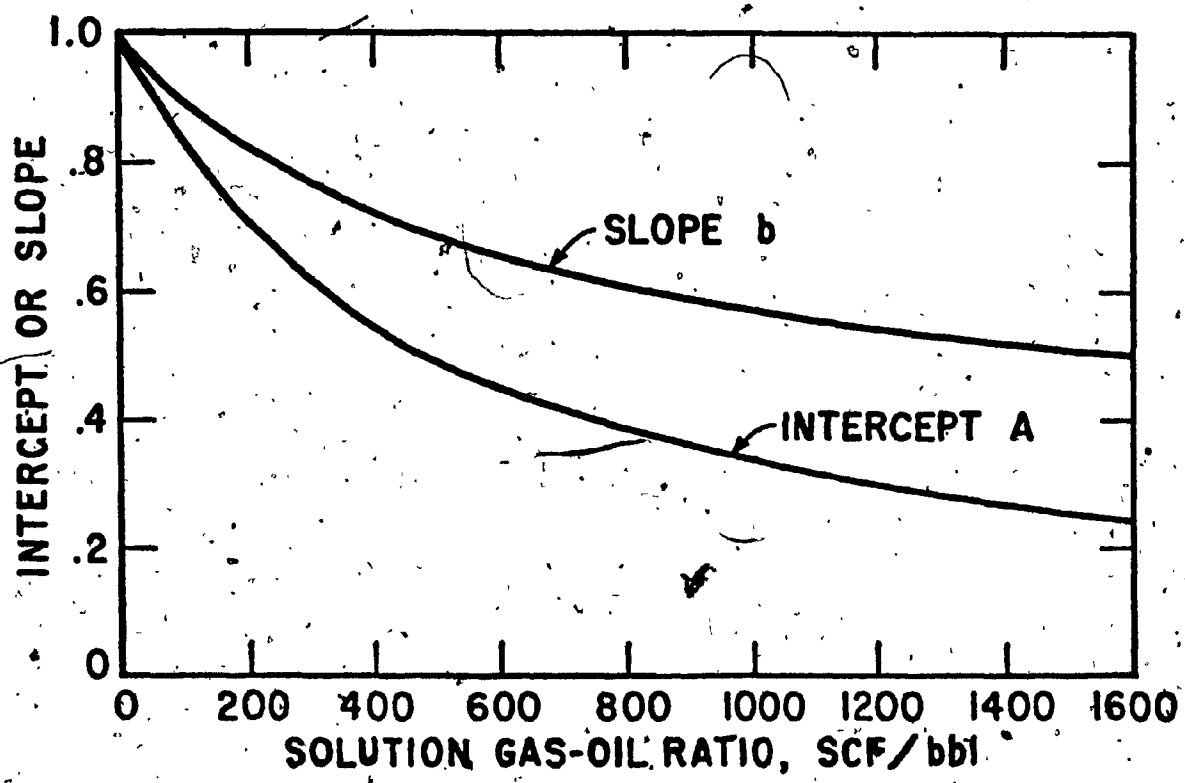


FIG. 5 . A AND b FACTORS FOR USE IN CHEW AND CONNALLY CORRELATION. (After Chew and Connally [13])

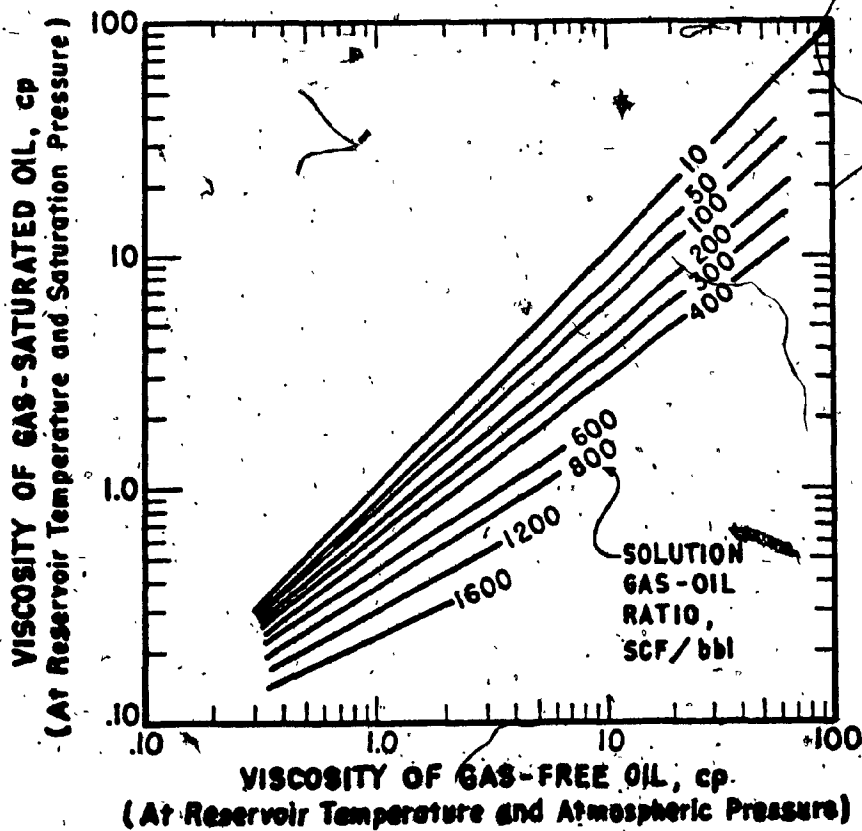


FIG. 6. VISCOSITY OF GAS-SATURATED CRUDE OILS AT RESERVOIR TEMPERATURE AND PRESSURE. (After Chew and Connally [13])

CHAPTER III

DEVELOPMENT OF THE GENERAL PRESSURE LOSS CORRELATION

3.1 THE ENERGY BALANCE

Any fluid that flows from one arbitrary point in a given system to another point in the system has associated with each unit mass of fluid a certain quantity of energy. The energy is of many forms such as kinetic energy, potential energy, and others. As each unit mass of fluid enters point one in the system, it carries a certain amount of total energy with it. When the same unit mass of fluid passes point two in the system, it has associated with it that same total energy minus a quantity of energy lost in the system. The different forms of energy may be interchanged or a conversion of one form into another may take place during the flow process, but the total energy must be conserved. It is obvious that a quantity of energy is always lost or converted, since a pressure drop or potential drop always accompanies a real flow process in a constant diameter conduit. Therefore, if these losses can be accounted for, the total energy at two points in any system can be balanced. Hence, the energy balance can be the basis for any fluid flow situation whether it be single or multiphase flow, provided all the energy terms can be evaluated.

The basic energy balance equation in symbolic differential form based on one pound of flowing fluid is —

$$144 VdP + g/g_c dh + \frac{vdv}{g_c} dW_f + g/g_c dW_e = 0 \dots\dots\dots 3.1$$

Equation (3.1) assumes only steady-state flow which simply means a flow that is independent of time. The first term in the equation represents the change in pressure-volume energy while the second term is the symbol for a change in potential energy due to a change in elevation. The third term is the kinetic energy change term. The irreversible losses or changes are represented by the fourth term and the fifth term is the energy change due to external shaft work done by the fluid as it flows between the two arbitrary points.

The second and last terms of equation (3.1) may be eliminated for horizontal flow in which no external work is done by the fluid. Horizontal flow simply means that there is no change in elevation between points 1 and 2. Therefore,

$$h = \text{constant} \dots\dots\dots 3.2$$

$$dh = 0$$

and

$$W_e = 0 \dots\dots\dots 3.3$$

$$dW_e = 0$$

since no external work is involved in this case.

The energy balance for one pound of flowing fluid is then given by

$$144 VdP + \frac{vdv}{g_c} + g/g_c dW_f = 0 \dots\dots\dots 3.4$$

If we assume a gas-liquid mixture flowed between point X₁, where the static pressure was P₁, and point X₂, where the static pressure was P₂. Let M_L and M_G represent the pounds mass per unit time of flowing liquid and flowing gas respectively. Then, an energy balance may be written

for the respective quantities of each phase. For the liquid phase, the balance is

$$144 M_L v_L dP + \frac{M_L v_L dv_L}{g_c} + g/g_c M_L dW_{fL} = 0 \quad \dots \dots \dots 3.5$$

and for the gas phase

$$144 M_G v_G dP + \frac{M_G v_G dv_G}{g_c} + g/g_c M_G dW_{fG} = 0 \quad \dots \dots \dots 3.6$$

Equations (3.5) and (3.6) may be added to obtain a total energy balance for both phases

$$144 [M_L v_L + M_G v_G] dP + \frac{1}{g_c} [M_L v_L dv_L + M_G v_G dv_G] + g/g_c [M_L dW_{fL} + M_G dW_{fG}] = 0 \quad \dots \dots \dots 3.7$$

One should keep in mind that the two-phase mixture flows from X_1 to X_2 where the pressures are P_1 and P_2 , respectively. Then, equation (3.7) may be integrated between these limits.

$$144 \int_{P_1}^{P_2} [M_L v_L + M_G v_G] dP + \frac{M_L}{g_c} \int_{v_{L1}}^{v_{L2}} v_L dv_L + \frac{M_G}{g_c} \int_{v_{G1}}^{v_{G2}} v_G dv_G + g/g_c \int [M_L dW_{fL} + M_G dW_{fG}] = 0 \quad \dots \dots \dots 3.8$$

The last integral in equation (3.8) represents the sum of all the irreversible energy losses. It is impossible to evaluate each term separately because the interchange of energy across the gas-liquid interface cannot be ascertained. Then, it becomes convenient to combine all of these energy losses into one term as —

$$M_T dW_{fT} = M_L dW_{fL} + M_G dW_{fG} \dots \dots \dots 3.9$$

where

$$M_T = M_L + M_G \dots \dots \dots 3.10$$

One may choose to employ the Darcy-Weisbach definition of the changes in frictional losses for single phase flow for use in multiphase flow situations. This is given by —

$$M_T \frac{dW_{fT}}{dx} = \frac{f M_T \bar{v}_M^2}{2gD} \dots \dots \dots 3.11$$

Combining equations (3.11) and (3.9) and (3.8) results in

$$144 \int_{P_1}^{P_2} [M_L V_L + M_G V_G] dP + \frac{M_L}{g_c} \int_{v_{L1}}^{v_{L2}} v_L dv_L + \frac{M_G}{g_c} \int_{v_{G1}}^{v_{G2}} v_G dv_G + \int_{X_1}^{X_2} \frac{f M_T \bar{v}_M^2}{2g_c D} dx = 0 \dots \dots \dots 3.12$$

The integration may be performed directly on the last three terms of equation (3.12) to obtain —

$$144 \left[M_L \int_{P_1}^{P_2} V_L dP + M_G \int_{P_1}^{P_2} V_G dP \right] + \frac{M_L}{g_c} \frac{\Delta v_L^2}{2} + \frac{M_G}{g_c} \frac{\Delta v_G^2}{2} + \frac{f M_T \bar{v}_M^2}{2g_c D} \Delta X = 0 \dots \dots \dots 3.13$$

Equation (3.13) still contains two integrals that must be evaluated, but the functional relationship of V with P may not be known exactly. The functional relationship can be approximated quite accurately with a linear

relationship, provided the pressure decrements are fairly small. The average integrated volumes of each phase, \bar{V}_L and \bar{V}_G , between pressure limits P_1 and P_2 are given by

$$\bar{V}_L = \frac{\int_{P_2}^{P_1} V_L dP}{P_1 - P_2} = - \frac{\int_{P_1}^{P_2} V_L dP}{P_1 - P_2} \dots \dots \dots 3.14$$

and

$$\bar{V}_G = \frac{\int_{P_2}^{P_1} V_G dP}{P_1 - P_2} = - \frac{\int_{P_1}^{P_2} V_G dP}{P_1 - P_2} \dots \dots \dots 3.15$$

The values of the integrals are approximated for small pressure decrements by —

$$\int_{P_1}^{P_2} V_L dP = - \bar{V}_L (P_1 - P_2) = - \bar{V}_L \Delta P \dots \dots \dots 3.16$$

and

$$\int_{P_1}^{P_2} V_G dP = - \bar{V}_G (P_1 - P_2) = - \bar{V}_G \Delta P \dots \dots \dots 3.17$$

All of the integrated average quantities are guaranteed to exist in the pressure range P_1 to P_2 by the theorem of the mean for integrals.

Substitution of equations (3.16) and (3.17) back into equation (3.13) gives the result —

$$-144[M_L \bar{V}_L + M_G \bar{V}_G] \Delta P + \frac{M_L \Delta v_L^2 + M_G \Delta v_G^2}{2g_c} + \frac{f M_T \bar{v}_M^2}{2g_c D} \Delta X = 0 \quad \dots \dots \dots 3.18$$

Note that since M_L , M_G , and M_T all have units of mass per time, the equation (3.18) is an "energy-per-time" balance or, more precisely, a power balance between points X_1 and X_2 . The power balance can be carried one step further by noting that \bar{V}_L and \bar{V}_G , the integrated average specific volumes, are related to the integrated average liquid and gas densities, $\bar{\rho}_L$ and $\bar{\rho}_G$, respectively by

$$\bar{\rho}_L = \frac{1}{\bar{V}_L}, \quad \bar{\rho}_G = \frac{1}{\bar{V}_G} \quad \dots \dots \dots 3.19$$

Employing the volume-density relationships for substitution into equation (3.18) results in —

$$-144 \left[\frac{M_L}{\bar{\rho}_L} + \frac{M_G}{\bar{\rho}_G} \right] \Delta P + \frac{M_L \Delta v_L^2 + M_G \Delta v_G^2}{2g_c} + \frac{f M_T \bar{v}_M^2}{2g_c D} \Delta X = 0 \quad \dots \dots \dots 3.20$$

The first term of equation 3.20 represents the change in pressure-volume energy per unit time between pressures P_1 and P_2 . Kinetic energy changes are represented in the second term while all irreversible energy changes are combined into the last term. All of the terms have the units of power which are foot-pounds per second. The coefficient f in the last term of equation (3.20) represents an "energy loss factor" and is dimensionless.

The energy loss factor will in effect make equation (3.20) balance to zero regardless of the manner in which the irreversible energy losses occur.

A knowledge of liquid holdup is absolutely necessary in order to evaluate the transport terms that make up the second term of equation (3.20). The finite differences in the actual average linear velocities of each phase are represented by Δv_L and Δv_G , respectively, between points X_1 and X_2 . Of course, these changes may be negligible in some cases. In fact, most investigators have chosen to neglect acceleration pressure drop entirely.

Hoogendorn [14], however, showed that the acceleration pressure drop could amount to as much as fifteen percent of the total static pressure drop. Therefore, these terms should be evaluated even though they will be negligible in a great majority of the cases. To do so, one must be able to predict liquid holdup, since the real velocities are dependent upon H_L . Superficial velocities of each phase may be obtained by assuming that each phase flows alone in the conduit. Then,

$$\begin{aligned}\bar{v}_{SL} &= q_L/A_p \\ \bar{v}_{SG} &= q_G/A_p \\ \bar{v}_M &= \frac{q_L + q_G}{A_p} = \bar{v}_{SL} + \bar{v}_{SG} \dots \dots \dots 3.21\end{aligned}$$

The actual average velocities are related to the superficial velocities by —

$$\bar{v}_L = \frac{q_L}{A_L} = \frac{q_L}{A_P H_L} = \frac{\bar{v}_{SL}}{H_L}$$

$$\bar{v}_G = \frac{q_G}{A_G} = \frac{q_G}{A_P (1 - H_L)} = \frac{\bar{v}_{SG}}{(1 - H_L)} \dots \dots \dots 3.22$$

The liquid holdup correlation presented in Chapter II will be used hereafter to evaluate actual average velocities.

Equation (3.20) still contains two unknowns, namely ΔP and f , for any given set of flow rates, pipe size, pipe length, and flowline temperature. Hence, an energy loss correlation is necessary whereby f is a function of known flow, fluid, and pipe properties. In order to obtain such a correlation, experimental data had to be utilized.

The energy loss factors were found by solving equation (3.20) for the energy loss factor f , as shown below —

$$f = \frac{2g_c D}{M_T \bar{v}_M^2 (\Delta X)} \left[144 \Delta P \left(\frac{M_L}{\rho_L} + \frac{M_G}{\rho_G} \right) - \frac{(M_L \Delta v_L^2 + M_G \Delta v_G^2)}{2g_c} \right] \dots \dots \dots 3.23$$

Every term to the right of the equal sign can be evaluated with experimental data and the energy loss factor can be calculated.

3.2 PRESSURE LOSS CORRELATION

Several of the previous investigators [26, 14, 25, 40] of multiphase flow have attempted to correlate a so-called "friction factor" with flow parameters and fluid properties. These friction factor models were discussed in Chapter I and have also been discussed by Gouse [5]. It should be pointed out that the term "friction factor" is, in reality, an energy loss factor containing all the unknown losses, including fric-

tional losses. Therefore, the term "frictional factor" which is a misnomer, will be replaced by a more general term called the "Energy loss factor". These factors provide a convenient means of balancing the total energy at any two points in a system. Therefore, an energy loss factor correlation is extremely useful in predicting pressure losses in multi-phase flowlines. The merits of this energy loss factor correlation will be discussed in the next section.

3.3 PRESSURE DROP CALCULATION METHODS

In general, pressure drop receives contribution from three effects: friction, acceleration and elevation. Thus —

$$\left(\frac{\partial P}{\partial Z}\right) = \left(\frac{\partial P}{\partial Z}\right)_f + \left(\frac{\partial P}{\partial Z}\right)_{ac} + \left(\frac{\partial P}{\partial Z}\right)_{cd} \dots \dots \dots 3.24$$

where

$$\frac{\partial P}{\partial Z} = \text{Total pressure gradient.}$$

For horizontal flow the last term is of course zero. Thus —

$$\left(\frac{\partial P}{\partial Z}\right) = \left(\frac{\partial P}{\partial Z}\right)_f + \left(\frac{\partial P}{\partial Z}\right)_{ac} \dots \dots \dots 3.25$$

of the number of pressure drop correlations available, as discussed in Chapter I. We are going to present the three (3) Methods of Calculation:

- I Lockhart-Martinelli Correlation
- II Dukler Correlation
 - a. No slip or homogeneous flow (Case I)
 - b. Constant slip (Case II)
- III Baker Correlation.

I. Lockhart-Martinelli Correlation

Of all purely empirical equations, Lockhart-Martinelli's takes the simplest approach. It considers the ratio of the frictional pressure drops that each phase would experience if it occupied the entire conduit alone, $(\partial P/\partial Z)_l$, $(\partial P/\partial Z)_g$. The square root of the ratio of these two terms is the correlating parameter X , Lockhart-Martinelli parameter:

$$X = [(\partial P/\partial Z)_l / (\partial P/\partial Z)_g]^{1/2} = (f_l G_l^2 \rho_g / f_g G_g^2 \rho_l)^{1/2} \dots \dots \dots 3.26$$

As mentioned previously, pressure drop is higher for two-phase flow than for single phase. Lockhart and Martinelli denoted the ratio on the frictional pressure drop for two-phase flow to the corresponding pressure drop calculated for one phase:

$$\phi_l^2 = (\partial P/\partial Z)_{tp} / (\partial P/\partial Z)_l, \phi_g^2 = (\partial P/\partial Z)_{tp} / (\partial P/\partial Z)_g \dots \dots \dots 3.27$$

If the pressure drop $(\partial P/\partial Z)_l$ is known and the factor ϕ_l^2 can be found, then the two-phase frictional pressure-drop can be computed from:

$$\begin{aligned} r_f = (\partial P/\partial Z)_{tp} &= (\partial P/\partial Z)_l \phi_l^2 = (\partial P/\partial Z)_g \phi_g^2 \\ &= f_l G_l^2 \phi_l^2 / 2g_c \rho_l D = f_g G_g^2 \phi_g^2 / 2g_c \rho_g D \dots \dots \dots 3.28 \end{aligned}$$

This simple concept, coupled with a very large amount of experimentation to determine ϕ_l and ϕ_g as functions of X resulted in the first really reliable estimate for the two-phase frictional pressure drop. Table 2 tabulates ϕ_l and ϕ_g for four different flow mechanisms: (1) laminar flow in both phases (2) turbulent flow in both phases (3) and (4) one phase laminar and the other turbulent. Table 2 tabulates ϕ_l and ϕ_g for each mechanism.

The mechanisms are determined by calculating the superficial Reynolds number for each phase, where $R_{es} = \frac{6.31W}{dU}$. If the Reynolds number is greater than 2,000, the flow is turbulent; if less than 2,000 the flow is viscous. Thus, if the gas-phase Reynolds number were 100,000 and the liquid phase number were 500, the flow would be described as turbulent gas flow, viscous liquid flow; the corresponding ϕ_1 would be $(\phi_1)_{vt}$.

Follow these steps to use Martinelli's correlation:

1. Compute the parameter X by estimating the liquid and mass flowrates G_g and G_l , and then compute the pressure drops for friction only, $(\partial P/\partial Z)_l$ and $(\partial P/\partial Z)_g$, with the usual single-phase method.
2. Calculate the superficial-phase Reynolds numbers and determine which of the four types of flow exist in the conduit. Then read the values of the parameter ϕ from Table 2.
3. Using Eq. (3.28) calculate the two-phase pressure drop, τ_p .

Table 2
VALUES OF MARTINELLI FUNCTIONS WITH INDEPENDENT VARIABLE X

X	All Mechanisms R_1	Turbulent-Turbulent $\phi_{l,tt}$	Viscous-Turbulent $\phi_{l,vt}$	Turbulent-Viscous $\phi_{l,tv}$	Viscous-Viscous $\phi_{l,vv}$
0.01		(1.28)	(120.0)	(112.0)	(105.0)
0.02		(68.4)	(64.0)	(59.0)	(53.5)
0.04		38.5	(34.0)	(31.0)	(28.0)
0.07	(0.04)	24.4	20.7	(19.3)	(17.0)
0.10	0.05	18.5	15.2	(14.5)	(12.4)
0.2	0.05	11.2	-8.90	(8.70)	(7.00)
0.4	0.14	7.05	5.62	(5.50)	4.25
0.7	0.19	5.04	4.07	(4.07)	3.08
1.0	0.23	4.20	3.48	(3.48)	2.61
2.0	0.31	3.10	2.62	(2.62)	2.06
4.0	0.40	2.38	2.05	(2.15)	1.76
7.0	0.48	1.96	1.73	(1.83)	1.60
10.0	0.53	1.75	1.59	(1.66)	1.50
20.0	0.66	1.48	(1.40)	(1.44)	1.36
40.0	0.76	1.29	(1.25)	(1.25)	1.25
70.0	0.84	1.17	(1.17)	(1.17)	(1.17)
100.0	(0.90)	1.11	(1.11)	(1.11)	(1.11)

where R_1 = Inplace liquid holdup
 R_g = Inplace gas holdup
 ϕ_l = Liquid phase
 ϕ_g = Gas phase
 X = Lockhart-Martinelli parameters.

II. Dukler's Correlation

Dukler's two methods, Case I and Case II, are important because both yield more reliable estimates.

a. No slip or Homogeneous Flow

In homogeneous flow, a two-phase fluid is considered as a single-phase fluid whose properties are the volumetric averages of the two-phase fluid. The key concept is that of flowing-volume holdup λ , defined either as the ratio of the liquid volumetric flowrate to the total volumetric flow, or as the ratio of the liquid superficial velocity to the total superficial velocity:

$$\lambda = Q_L / (Q_L + Q_G) = V_{SL} + V_{SG} \dots \dots \dots 3.29$$

The properties of density and viscosity attributed to this single-phase fluid are discussed in Chapter I. The sum of the superficial gas velocity and the superficial liquid velocity is assumed to be the velocity of the homogeneous fluid:

$$V_{ns} = V_{SL} + V_{SG} \dots \dots \dots 3.30$$

Dukler, A.E., et al [35], similarity analysis revealed that the Euler number for homogeneous flow is:

$$Eu_{ns} = (\partial P / \partial Z)_{ns} \rho_{ns} D / G_t^2 \dots \dots \dots 3.31$$

Since $Eu_{ns} = f_{ns} / 2$, rearrangement gives:

$$(\partial P / \partial Z)_{ns} = (f_{ns} G_t^2) / (2g_c \rho_{ns} D) \dots \dots \dots 3.32$$

All these terms are easily found except for f_{ns} . But remember that we are now treating the fluid as a pseudo single-phase, and that many empirical relations exist for smooth-tube friction factors for single phase. We

can therefore use the homogeneous Reynolds number:

$$Re_{ns} = DG_t/\mu_{ns} \dots\dots\dots 3.33$$

and a convenient friction-factor equation, to calculate the friction-factor f_{ns} . This factor may even be corrected for conduit roughness, just as in single-phase flow, so that a friction factor greater than or equal to the corresponding smooth-tube friction factor for any given Reynolds number always results.

The friction factor, f_{ns} , may be evaluated from Re_{ns} and the following smooth-tube friction factor equation:

$$f_{ns} = \left(2 \log \left[\frac{(Re_{ns})}{4.5223 \log (Re_{ns}) - 3.8215} \right] \right)^{-2} \dots\dots\dots 3.34$$

It is very simple to calculate the pressure differential for Dukler's homogeneous or no-slip flow. Besides being more accurate than the Lockhart-Martinelli equation, it is much simpler to use, requires little more effort than a single-phase pressure-drop calculation, and does not require consideration of flow regimes. Conventional pressure-drop charts may even be used by converting the gas-liquid flow to some common basis such as gpm. or thousand cu. ft./day, weighting the density on a flowing-volume basis, and correcting for weighted kinematic viscosity, if necessary.

Remember that in using no-slip flow to estimate two-phase pressure drop, no standard correction factor can be applied to approximate the actual two-phase pressure drop that might be experienced in a line.

Caution: The homogeneous pressure drop must always be less than the actual two-phase pressure drop.

Thus, no-slip flow provides the lowest pressure drop that is

feasible in a real situation for two-phase flow, giving the engineer a limiting boundary for estimating pipe diameter. A correlation that gives an estimate for pressure drop lower than the homogeneous estimate--for a given range of operating conditions--should be assigned little or no credence.

b. Constant Slip

Dykler's constant-slip method is the most accurate presently available over a wide range. It has not attained the popularity in industry that its validity deserves, probably because of the computational complexity of the Equation for $(\partial P/\partial Z)$. The acceleration term is especially complicated. The detail of this study can be found in the original reference [35].

The simplify relations, taking advantage of the incremental algorithm and the mechanical energy balance —

$$\beta = (\rho_l/\rho_{ns})(\lambda^2/R_L) + (\rho_g/\rho_{ns})[(L-\lambda)^2/R_g] \dots \dots \dots 3.35$$

The Case II Euler number is:

$$Eu_{tp} = [(\partial P/\partial Z)g_c \rho_{ns} D]/G_t^2 \beta \dots \dots \dots 3.36$$

Hence, remembering that $f_{tp} = Eu_{tp}/2$:

$$(\partial P/\partial Z)_f = \beta f_{tp} G_t^2 / 2g_c \rho_{ns} D \dots \dots \dots 3.37$$

The Case II Reynolds number is:

$$Re_{tp} = \left(\frac{\rho_l \lambda^2}{R_L} + \frac{\rho_g (1-\lambda)^2}{(1-R_L)} \right) DV_{ns} / \mu_{ns} = \beta DG_t / \mu_{ns} = \beta Re_{ns} \dots \dots \dots 3.38$$

Again, the problem is finding the two-phase friction factor, f_{tp} . No ready-made empirical relations exist for the two-phase friction factor as defined

by Dukler. With characteristic engineering insight, however, he calculated a friction factor, f_0 , by substituting the Case II Reynolds number Re_{tp} into a friction-factor equation such as Eq. 3.34. He computed the normalized friction factor f_{tp}/f_0 , where f_{tp} was determined from the previously mentioned data banks at the same conditions as f_0 . He found that f_{tp}/f_0 could be correlated by a function of the flowing volume holdup λ , defined as $\alpha(\lambda)$. The method may be developed in the following manner:

$$f_0 = \left(2 \log \left[\frac{(Re_{tp})}{4.5223 \log (Re_{tp}) - 3.8215} \right] \right)^{-2} \dots \dots \dots 3.39$$

$$\alpha(\lambda) = f_{tp}/f_0 = 1.0 - [\ln \lambda / \xi] \dots \dots \dots 3.40$$

where $\xi = 1.281 + 0.478 \ln \lambda + 9.444 (\ln \lambda)^2 + 0.094 (\ln \lambda)^3 + 0.00843 (\ln \lambda)^4$.

Then $\alpha(\lambda)$ is plotted in Fig. and

$$f_{tp} = f_0 \alpha(\lambda) \dots \dots \dots 3.41$$

and

$$\tau_f = (\partial P / \partial Z)_{tp} = \alpha(\lambda) f_0 8 G_{tz} / 2 g_c \rho_n s D \dots \dots \dots 3.42$$

II. Baker Correlation

Baker [24] proposed a method in 1960, which has received considerable amount of acceptance in the oil industry. Baker incorporated the Lockhart and Martinelli, et al, correlations [7] into his solution.

Baker postulates an equation for each different flow pattern and a chart with which the flow patterns could be predicted. The merit of this flow pattern map will be discussed in "Flow Region Correlation", chapter.

One can establish a particular flow region from the Baker para-

meters Bx and By, where

$$By = 2.16 \cdot W \sqrt{A \rho_L \rho_V} \dots \dots \dots 3.43$$

$$Bx = 531 \left(\frac{W_L}{W_V} \right) \left(\frac{\rho_L \rho_V}{\rho_L^{2/3}} \right) \left(\frac{U_L^{1/3}}{\sigma} \right) \dots \dots \dots 3.44$$

By - depends on the vapor phase flow rate

Bx - depends on the weight flow ratio and the physical properties of the liquid and vapour phases.

Using the Lockhart and Martinelli parameters, Baker proposed the following correlations for each different flow pattern.

(a) For froth or bubble flow:

$$d_{Gtt} = \frac{14.2 \times 0.75}{L^{.1}}$$

Both phases must be flowing turbulently for this equation to hold. This is symbolized by the tt subscripts.

(b) For plug flow:

$$d_{Gtt} = \frac{27.315 \times .855}{L^{.17}}$$

(c) For stratified flow:

$$d_{Gtt} = 15400 \times L^{.8}$$

(d) For wave flow:

Given by a figure in Baker's paper. [24].

(e) For slug flow:

$$d_{Gtt} = \frac{1190 \times .515}{L^{.5}}$$

* Types of flow are defined in the flow regime chapter.

(f) For annular flow:

$$\beta_{Gtt} = (4.8 - 0.3125D) (X^{.343} - .921D)$$

Once the flow pattern is determined, then β_{Gtt} is found from one of the preceding equations. Then from the definitions of β_G , β_L , and X the two-phase pressure loss is given by

$$\Delta P_{T.P.} = \Delta P_G \beta_{Gtt}^2$$

where

ΔP_G = the pressure loss that would occur if the gas flowed alone in the pipe.

3.4 COMPARISON

A.E. Dukler and his co-workers in 1964 published the results of their comprehensive study on two-phase flow [19]. In the first paper, Dukler discusses the development of a data bank of some 2600 culled data points out that were consistent, accurate and representative of two-phase flow systems. The data bank was then used to compare the accuracy of existing correlations.

At the time of Dukler's work some 25 correlations for horizontal flow pressure loss had been published. Dukler's compared the five (5) most popular methods and two of his [9, 10, 11, 12, 19, 24], against his culled data points. The results are summarized in Table 3.

The lower the value of the statistical parameters ψ and σ , then better the correlation; the asterisk denotes the most accurate correlation for each condition. The results indicate that Case II is the superior method of calculating $\partial P/\partial Z$. But Case I is also consistently

Table 3
COMPARISON OF CORRELATIONS USING DUKLER'S DATA BANK

Co. No.	In. CP.	Correlations										Dukler's Case I		Dukler's Case II		Number of Data Points				
		\bar{u}	σ	ϕ	\bar{u}	σ	ϕ	\bar{u}	σ	ϕ	\bar{u}	σ	ϕ	\bar{u}	σ		ϕ			
1	1	64.2	40.0	45.0	-8.5	17.8	15.0	-6.6	10.1	10.0*	40.9	29.1	30.0	-9.4	17.9	17.5	-25.2	18.2	13.5	224
	3	77.4	335	87.5	11.2	55.6	30.0	3.8	29.1	20.0*	163	123	-	-1.0	29.7	20.0	8.6	24.8	12.0*	230
	20	30.7	89.5	40.0	42.5	94.2	65.0	-5.6	24.7	20.0*	481	192	-	0.0	46.7	25.0	6.7	24.4	18.6*	156
2	1	-13.6	60.3	65.0	-2.7	24.8	20.0*	9.2	37.7	25.0	162	228	-	-11.2	13.8	12.5	2.4	16.4	15.5*	320
	3	19.3	79.0	82.5	8.4	45.3	45.0	-4.7	22.9	25.0*	82.3	74.5	80.0	-1.4	39.4	25.0	1.6	19.7	16.0*	393
	20	73.0	199	90.0	95.4	268	-	13.2	52.9	30.0*	271	329	-	-19.1	22.6	22.5	10.3	27.2	20.0*	401
3 1/2	1	11.5	79.2	82.5	15.0	40.2	30.0*	31.0	50.2	47.5	27.6	104	97.5	-18.5	29.2	22.5	-0.3	25.8	26.2*	109
	3	7.1	60.1	72.5	27.8	62.0	45.0	16.3	39.3	22.5*	84.5	86.1	-	-0.9	34.6	25.0*	9.3	24.9	25.0	67
	20	31.8	60.6	47.5	51.0	91.8	57.5	-0.4	26.2	22.5*	147	83.4	-	7.3	33.5	25.0	10.6	24.5	18.6	111
5 1/2	1	-79.5	11.6	10.0	51.2	23.7	30.0	38.3	12.2	12.5*	93.3	22.3	22.5	16.4	20.0	17.5*	50.6	16.8	19.3	24
	3	-0.5	44.6	45.0	20.0	57.5	45.0	11.6	41.5	37.5*	106	80.8	-	2.0	31.7	25.0	11.2	19.2	16.0*	131
	20	9.6	47.8	50.0	37.1	79.4	47.5	-7.0	24.8	25.0*	120	69.0	-	7.2	29.8	20.0	7.3	22.7	14.0*	122

where D = nominal tube size
 u_s = nominal liquid viscosity
 \bar{u} , σ and ϕ = parameters expressed as percentages
 *Asterisk denotes correlation that seemed to predict best for each horizontal row.

better than Lockhart Martinelli's method.

The key comparison parameter. His standard deviation σ , exceeds 35% for five groups of Martinelli data. Two groups for Case I and no groups for Case II. The Martinelli correlation performed best for low viscosity liquid in small tube sizes, conditions under which most of the Martinelli data were obtained. - The other 4 correlation results were not acceptable.

CHAPTER IV

FLOW REGIME CORRELATION

4.1 FLOW REGIME

Nearly all the investigators of horizontal two-phase flow have presented different pressure loss correlations for the several different flow patterns that occur during such flow processes. Flow pattern has been treated as an independent variable in practically all of the existing correlations. Yocum [29] and Bertuzzi, et al. [26] are exceptions to the general rule, since they show no flow pattern effect. Several methods of predicting flow patterns have been presented in the form of "flow pattern maps." These maps consist of plots of two independent flow parameters, such as mass flow rates of each phase, with each pattern falling in a certain area of the plot.

Various flow pattern maps which have appeared in the literature are briefly discussed below in chronological order.

Bergelin and Gazley [41] suggested one of the first flow pattern maps. Their diagram, based on the air-water system in a 1 in. pipe, uses the liquid and gas mass flow rates, M_L and M_G , as the coordinates. Johnson and Abou-Sabe [42] proposed a flow pattern map which is very similar to that of Bergelin and Gazley and is based on air-water data in 0.87 in. pipe.

Alves [23] suggested a map based on data for air-water and air-

oil mixtures in a 1-in. pipe utilizing the superficial liquid and gas velocities, V_{SL} and V_{SG} , as the coordinates. He was able to represent both of the systems on a single map. Alves also proposed a visual classification of basic flow patterns for horizontal pipes. The merits of this flow pattern will be discussed in a later chapter.

Baker [24] proposed a flow pattern map based on the data of Jenkins [43], Gazley [44], Alves [23], and Kosterin [45]. Most of their data are for the air-water system. The detail of Baker flow pattern will be discussed in a later chapter.

White and Huntington [25] proposed a flow pattern map based on their data obtained in 1-, 1 1/2-, and 2-in. pipes with gas-oil, air-oil, and air-water systems. They used liquid and gas mass velocities, L and G , as the coordinates.

Hoogendoorn [25] used the mixture velocity, V_M , and the input gas volume fraction, C_G , as coordinates as first proposed by Kosterin [45] in a flow pattern map which is based on several air-oil and air-water systems. Hoogendoorn observed modest effects due to pipe diameter and liquid properties at liquid viscosity less than 50 cP.

Govier and Omer [46] presented a map based on their data for air-water system in a 1.026-in. pipe. The liquid and gas mass velocities, L and G , were used as the coordinates.

Scott [47] modified Baker's [24] diagram by making use of the more recent data of Hoogendoorn [14], and Govier and Omer [46]. The modified diagram does not have clearcut transition boundaries but instead shows relatively wide bands depicting regions of transition from one flow pattern to another.

Eaton et al. [19] obtained extensive data on natural gas, water, natural gas-crude oil, and natural gas-distillate systems in 5.08 (cm) and 10.16 (cm) pipes. Knowles [48] using the data of this Eaton study, utilized several of the existing correlations including that of Baker in an attempt to find a correlation suitable for a wide range of flow conditions. Knowles found that none of the existing flow pattern maps are adequate for the prediction of the flow patterns that occurred during the Eaton experiments. These comparisons are shown in his report [48].

A new pattern correlation was presented by Knowles and is included here, Knowles defined two dimensionless groups by means of dimensional analysis. These groups are: 1) a two-phase Reynolds function and 2) a two-phase Weber function.

The Reynolds function is given by

$$(R_e)_{T.P.} = \frac{M_{T.P.}^2}{D\mu_{T.P.}} \dots \dots \dots (4.1)$$

where

$$\mu_{T.P.} = \mu_L H_L + \mu_G (1-H_L)$$

The Weber function is given by

$$(W_e)_{T.P.} = \frac{\rho_L v_L^2 D H_L^{1/2}}{\sigma} + \frac{\bar{\rho}_G v_S^2 D (1-H_L)^{1/2}}{\sigma} \dots \dots \dots (4.2)$$

where

$$v_S = v_G - v_L \dots \dots \dots (4.3)$$

= slip velocity

These two dimensionless groups appear to separate the different flow pattern data quite well. Figure 7 shows the lines separating the flow regimes. Very few data points show any overlap, as can be seen in Knowles original work. The two dimensionless groups given previously may

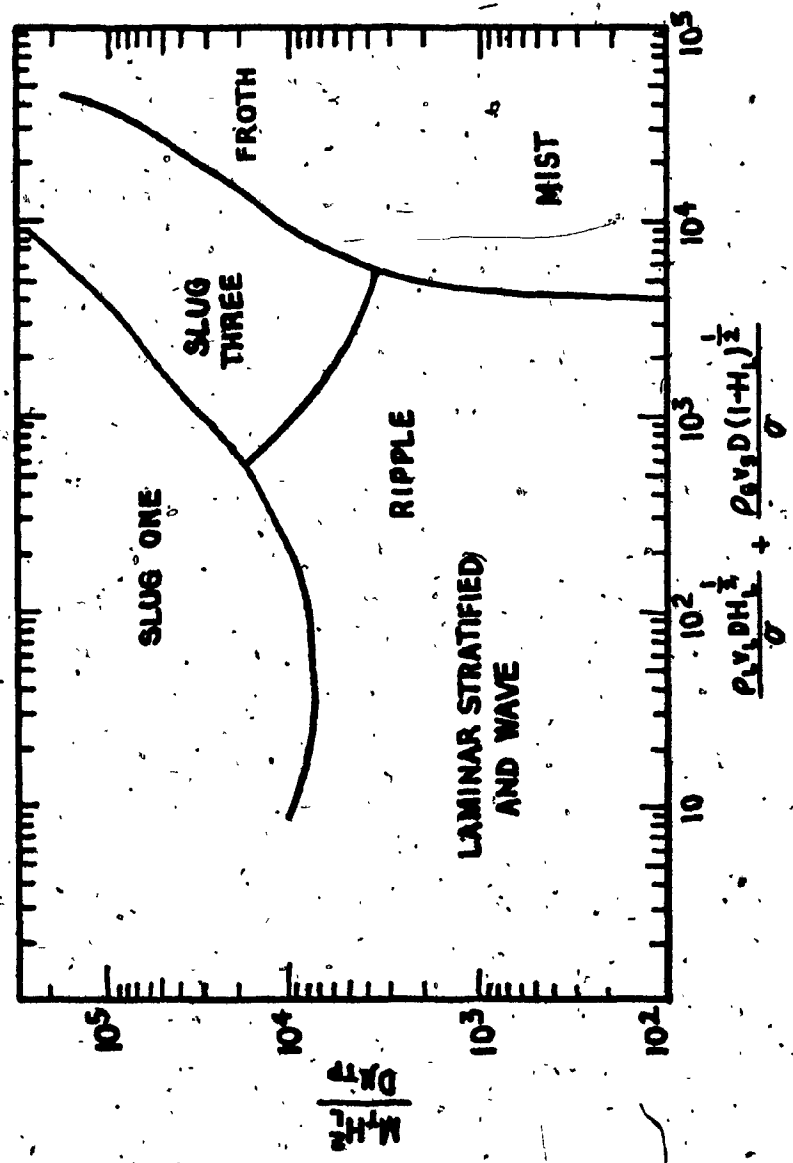


FIG. 7 FLOW PATTERN CORRELATION (AFTER KNOWLES(18))

be evaluated with data and Figure 7 may be used to predict the flow regimes.

Note that the in situ liquid volume fraction H_L , must be known in order to use the Eaton et al. map. Moreover, their definitions of flow patterns are somewhat different from those commonly found in the literature which results in apparent subdivisions of the usually defined region.

Al-Sheikh et al. [49] have taken an entirely different approach to develop a method to predict flow pattern. Their correlation is based on the AGA-API Two-Phase Flow Data Bank referred to by Dukler et al. [6]. They use a total of 4475 data points and produce a complex correlation which requires a set of twelve figures on ten different coordinate systems. They did not attempt to define lines of separation between different flow patterns but have tried to enclose all of the data belonging to a particular pattern into a flow pattern. Observations are completely reliable. A sequential procedure required to predict the flow pattern. Because the boundaries of their regions are highly irregular, this method is not readily suited for a computer oriented study.

Govier and Aziz [50] have presented a revised version of the Govier and Omer [46] flow pattern map. The revision is based on, in addition to the Govier and Omer data, the data of Baker [24] and Hoogendoorn [14], and others. The coordinate system for this revised diagram is also different from that originally used by Govier and Omer in that the superficial liquid and gas velocities, V_{SL} and V_{SG} , are used, as originally suggested by Alves [23].

Govier and Aziz also suggest that with suitable modification to the coordinates, the revised Govier and Omer map can be used with other

than the air-water system. Specifically, these authors recommend that fluid property parameters, defined as

$$X = \left(\frac{\rho_G}{0.0808} \right)^{1/3} Y \dots \dots \dots (4.4)$$

$$Y = \left[\left(\frac{\rho_L}{62.4} \right) \left(\frac{72.4}{\sigma} \right) \right]^{1/4} \dots \dots \dots (4.5)$$

be used to multiply the actual superficial fluid velocities as follows:

$$\tilde{V}_{SG} = X V_{SG} \dots \dots \dots (4.6)$$

$$\tilde{V}_{SL} = Y V_{SL} \dots \dots \dots (4.7)$$

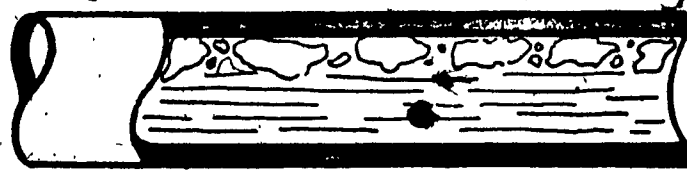
In eqns. (4.4) and (4.5), ρ_L and ρ_G are expressed in (lb./ft³), and σ has units (dyne cm). V_{SL} and V_{SG} are then used in the normal way with the revised Govier and Omer map. This is referred to as the revised Govier and Omer map with physical property parameters. The quantities V_{SL} and V_{SG} thus represent "effective" superficial velocities for all systems except air-water; for the air-water system, they are the actual superficial velocities.

4.2 FLOW PATTERNS

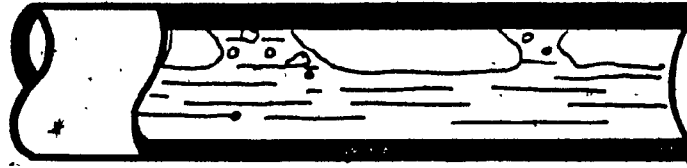
The following is a brief description of Alves [23] visual classification of basic flow patterns for horizontal pipes, a sketch of which appears in Fig. 8.

(a) Spray or Dispersed Flow - When nearly all of the liquid is entrained as spray by the gas the flow is dispersed. This type of flow can be expected when the vapour content is more than roughly 30% of the total

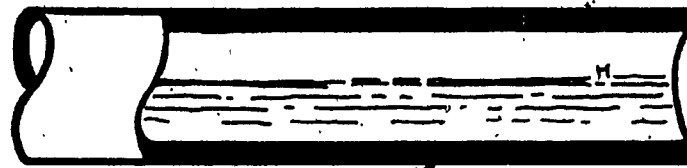
FLOW PATTERNS



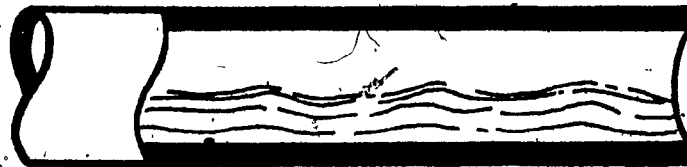
Bubble



Plug



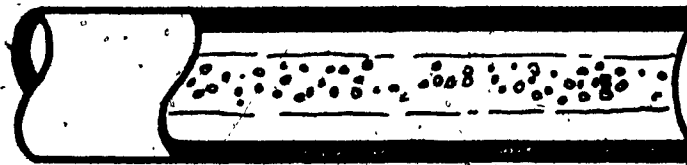
Stratified



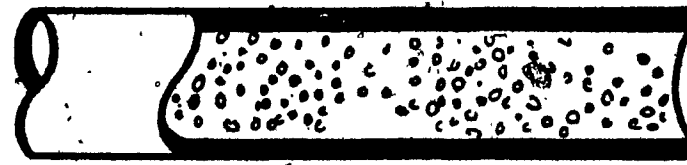
Wavy



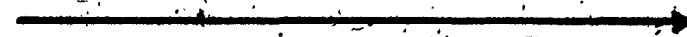
Slug



Annular



Spray



Flow

HORIZONTAL FLOW PATTERNS
FIG. 8 (AFTER ALVES) [3]

weight flow rate. Some overhead condenser and reboiler return lines have dispersed flow.

(b) Annular Flow - Annular flow develops when liquid forms a film or ring around the inside wall of the pipe, and gas flow at a higher velocity as a central core.

(c) Stratified Flow - In this liquid flows along the bottom of the pipe which the gas flows above it. The gas liquid interface remains smooth and the fraction occupied by each phase remains constant.

(d) Wave Flow - Wave flow is similar to the stratified flow except that gas flow at high velocities and disturbs the interface so that waves are formed which travel in the direction of flow.

(e) Plug Flow - In this flow there are alternating plugs of gas and liquid which move along the upper section of the pipe. The lower section is liquid.

(f) Bubble Flow - In this, gas flows in the form of bubbles along the upper surface of the pipe. The velocity of these bubbles and the velocity of this liquid are approximately equal. If the bubbles become dispersed through the liquid, this is known as froth flow.

This type of flow can be expected when the vapour content is less than about 30% of the total weight flow rate.

(g) Slug Flow - Slug flow develops when waves of liquid are picked up periodically in the form of frothy slugs that move at a greater velocity than the average liquid velocity. Typically, this phenomenon may occur in a pocketed line between an overhead condenser at grade and an elevated

reflux drum. At low velocities, liquid collects at the low point and periodically can become a slug in the line. With a sufficiently high velocity, the liquid phase can be carried through without developing slug flow.

4.3 EVALUATION OF EXISTING FLOW PATTERN MAPS

Of the limited number of diagrams available, as discussed in Chapter I and Chapter IV, were written into the computer program in terms of the original coordinate systems used by G.W. Gregory et al. [50]. Two parameters, defined below, were calculated to facilitate comparisons.

$$\alpha_1 = \left(\frac{\text{Number of points correctly predicted to lie in flow regime 1}}{\text{Number of points which were observed in flow regime 1}} \right) \times 100 \dots (4.8)$$

not to a particular flow regime.

$$\alpha_2 = \left(\frac{\text{Total number of observations correctly predicted to lie in their respective flow regimes}}{\text{Total number of observations}} \right) \times 100 \dots (4.9)$$

Results of the comparison based on the air-water data are shown in Table 4. It should be noted that in Table 4 the bubble and elongated (sometimes referred to as plug) flow regimes are grouped together as simply bubble flow. Also no distinction is drawn between annular and annular-mist flow. The region designated in this study as dispersed bubble is sometimes also referred to in the literature as froth flow.

The results of the comparison of flow pattern maps for all 5,935 observations are shown in Table 5.

It is interesting to note that while the fluid property corrections defined by eqns. (4.4) and (4.5) were expected to improve the reifa-

TABLE 4
COMPARISON OF FLOW PATTERN MAPS USING AIR-WATER DATA

<u>Flow Pattern</u>	<u>Number of Observations</u>	<u>Values of α</u>		
		<u>Baker</u>	<u>Hoogendoorn</u>	<u>Revised Govier-Omer</u>
Bubble	12	58.4	91.6	58.4
Stratified	229	72.1	89.1	52.8
Wave	162	9.9	64.2	43.2
Slug	453	49.0	57.8	73.7
Annular-mist	320	97.5	94.1	95.0
Dispersed bubble	2	0.0	0.0	0.0
TOTAL --	<u>1178</u>			
β		61.3	74.9	71.0

TABLE 5

COMPARISON OF FLOW PATTERN MAPS USING ALL AVAILABLE DATA

<u>Flow Pattern</u>	<u>No. of observations</u>	<u>Values of α</u>			<u>Revised Govier-Omer with physical property parameters</u>
		<u>Baker</u>	<u>Hoogendoorn</u>	<u>Revised Govier-Omer</u>	
Bubble	758	20.4	64.6	32.4	23.3
Stratified	397	54.7	80.9	51.6	46.3
Wave	783	16.0	41.5	17.5	16.5
Slug	2020	24.0	58.7	63.4	58.6
Annular-mist	1746	84.7	90.5	91.0	93.2
Dispersed bubble	231	2.2	0.0	0.0	0.0
TOTAL --	5935				
β		41.5	65.7	58.3	55.6

bility of the revised Govier and Omer map, in fact the reverse is true for most of the flow regimes. It is also interesting to note the total failure of all of these maps to predict the dispersed bubble flow regime. In fact, there is some evidence that the data (at least the observation regarding flow pattern) are partly at fault. When all 5,935 points are plotted on V_{SL} , V_{SG} coordinates, a substantial number of the observations designated as "dispersed bubble" are seen to lie in the region of high gas rate and low liquid rates where one would normally expect annular-mist flow to predominate. In any case, this flow pattern designation represents less than 4% of the total data and its importance will not be overstated here.

All of the maps do very well with respect to the annular-mist flow regime, whereas without exception they tend to predict wave flow very poorly. For all maps, there is evidence that the boundary between dispersed bubble and slug flow is located at a liquid flow rate that is too low. From the calculated values of β , it is apparent that the map presented by Hoogenboom [14] is substantially more reliable than that of Baker [24]. But map presented by Baker [24] has been used extensively, both in literature and in industrial design calculations.

CHAPTER V
GENERAL METHOD OF APPLICATION

5.1 FLOW PATTERN PREDICTION

As was previously mentioned, there have been many flow regime maps developed for prediction purposes. Baker [24] presented the correlation that is most widely used in the petroleum industry. Baker [24] plotted B_x versus B_y parameter as a function of the modified ratio of liquid to vapour phase mass velocity, where B_x and B_y are fluid property correction factors and are defined as:

$$B_y = 2.16 W_v/A \sqrt{\rho_L \rho_v} \dots \dots \dots (5.1)$$

$$B_x = 531 \left(\frac{W_L}{W_v} \right) \left(\frac{\sqrt{\rho_L \rho_v}}{\rho_L^{2/3}} \right) \left(\frac{\mu_L^{1/3}}{\sigma_L} \right) \dots \dots \dots (5.2)$$

where B_y depends on the vapor phase flow rate, liquid and vapor densities and pipe size. The practical significance of pipe size is that by changing pipe diameters, the type of flow might also be changed, which also changes friction losses in the pipe.

B_x depends on the weight flow ratio and the physical properties of the liquid and vapor phases. For W_L/W_v we can substitute the ratio of % liquid to % vapour. Once B_x is calculated it does not change with alternative pipe diameters. The position of the B_x line changes only if

the liquid vapor proportion changes and to a much lesser extent if the physical properties of the flowing liquid and vapour change. This can occur in long pipelines when relatively high friction losses reduce the pressure. Consequently, the vapor content of the mixture increases with a corresponding decrease in vapour density. The B_x line will shift somewhat to the left.

In Baker parameters Fig. No. 9, the intersection of B_x and B_y determines the flow region for the calculated liquid vapour ratio and the physical properties of the liquid and vapour. Increasing vapour content, the point of intersection moves up and to the left.

The border of the various flow patterns are shown as lines in the Fig. 9. In reality, these borders are not lines but zones of transition between flow patterns.

5.2 PRESSURE DROP PREDICTION

Two general approaches towards the prediction of pressure drops have been used. One approach attempts to correlate pressure drop in terms of dimensionless groups and the other parameters as is done in single-phase flow. The other analyzes the hydrodynamics of individual flow patterns. The former approach suffers in that the mechanisms differ over the range of flow condition while the latter requires an approximate knowledge of the flow pattern which was explained earlier.

The total pressure drop in a two-phase flow system is often expressed by the equation, which was explained earlier.

$$\Delta P_{\text{total}} = \Delta P_{\text{two phase flow}} + \Delta P_{\text{elevation change}} + \Delta P_{\text{acceleration change}}$$

7

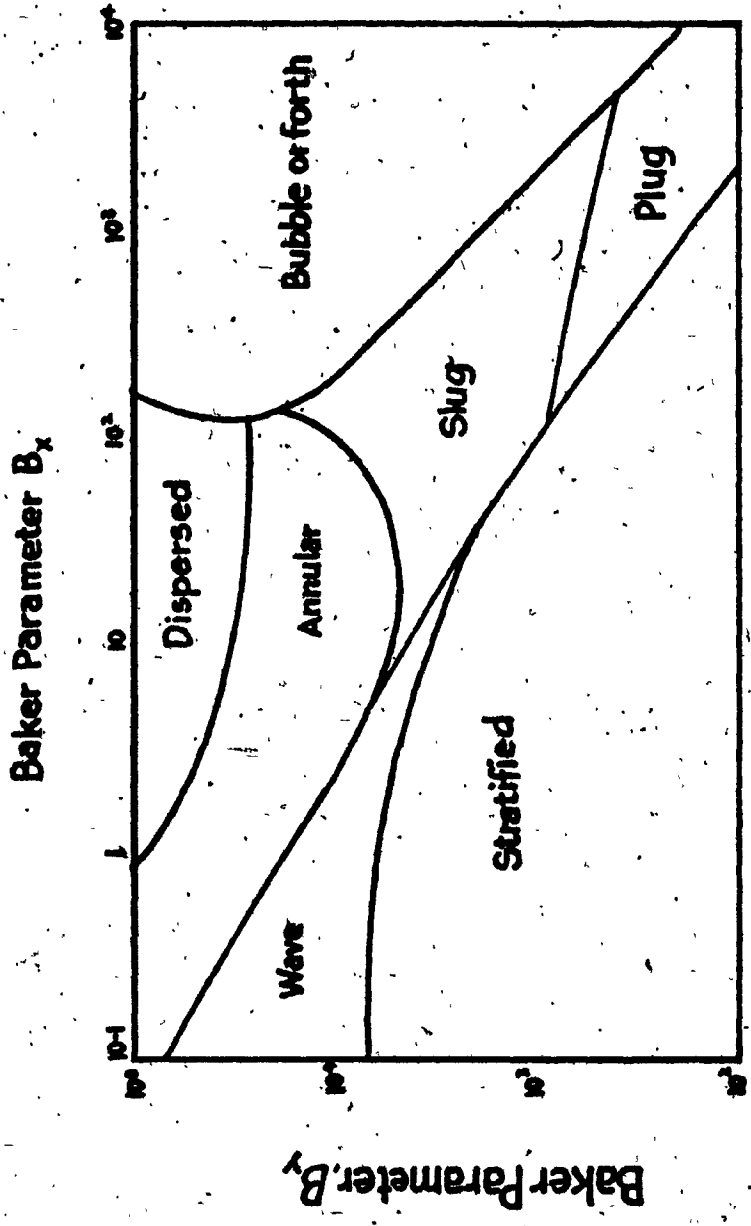


Fig.9 Baker parameters determine type of two-phase-flow.
(After Baker) [24]

The most important pressure drop considerations are $\Delta P_{\text{two phase flow}}$ and $\Delta P_{\text{elevation}}$ difference. The pressure drop due to an acceleration change is usually neglected. If the pipeline is horizontal, the elevation correction term is zero. If the pipeline is vertical, the correction term can be calculated using an average mixture density in the standard manner. The calculation of unit losses for two phase flow (vapour liquid mixtures) are based on the method of Lockhart and Martinelli [7]. Only essential relationships are repeated here and 100 ft pipeline is taken as basis of calculation. The General Equation

$$\Delta P_{100}(\text{two phase}) = \Delta P_{100}(\text{vapour}) \phi^2 \dots \dots \dots (5.3)$$

we calculate the pressure drop of the vapour phase by assuming that only vapour flows in the pipeline.

The calculated vapour-phase unit loss is then corrected with the correlations as shown in Chapter III (Section 3.3). The form of the correlation is

$$\phi = a x^b \dots \dots \dots (5.4)$$

where a includes the vapour phase flowrate and the pipe cross-section, b is constant and x is the Lockhart Martinelli two-phase flow modulus, as explained in Chapter III.

$$x^2 = \Delta P_{100}(\text{liquid}) / \Delta P_{100}(\text{vapour}) \dots \dots \dots (5.5)$$

$\Delta P_{100}(\text{liquid})$ is calculated by assuming only liquid flows in the pipe and $\Delta P_{100}(\text{vapour})$ by assuming only vapour flows in the same size pipe.

By using DARCY [51] relation in equation 5.5 the two-phase flow modulus becomes

$$x^2 = (W_L/W_V)^2 (\rho_V/\rho_L)(f_L/f_V) \dots \dots \dots (5.6)$$

where f_L and f_V are the liquid and vapour phase friction factor.

The modulus can be attained directly by calculating the liquid and vapour phase, Reynolds numbers and using Moody friction factors [27] (Fig. 10).

Reynolds numbers are calculated separately for the vapour and liquid phases by using the same pipe diameter, and corresponding flow rates and viscosities.

$$N_{RC} = 6.31 W/d\mu \dots \dots \dots (5.7)$$

A convenient form of Darcy equation for unit pressure-loss correlation in pipelines for liquid or vapor

$$\Delta P_{100} = 0.000336 (fw^2)/d^5\rho \dots \dots \dots (5.8)$$

We use the same diameter for the liquid and vapour phase calculations.

A generalized form suggested by Blazius [52] for the Lockhart-Martinelli [7] relation expresses the friction factors for turbulent flow as:

$$f_L = 0.046(N_{Re})_L^{-0.2} \dots \dots \dots (5.9)$$

$$f_V = 0.046/(N_{Re})_V^{-0.2} \dots \dots \dots (5.10)$$

Using equations (5.7), (5.9), and (5.10) to simplify the Equation (5.6), the two-phase flow modulus becomes -

$$X^2 = (W_L/W_V)^{1.8} (\rho_V/\rho_L (\mu_L/\mu_V))^{0.2} \dots \dots \dots (5.11)$$

The Blazius Friction factor does not take into account the relative roughness of the pipe wall and the consequent relative reduction of the friction factor as the pipe diameter increases. The Moody friction factor [27] will give somewhat smaller values for f_L/f_w and consequently smaller

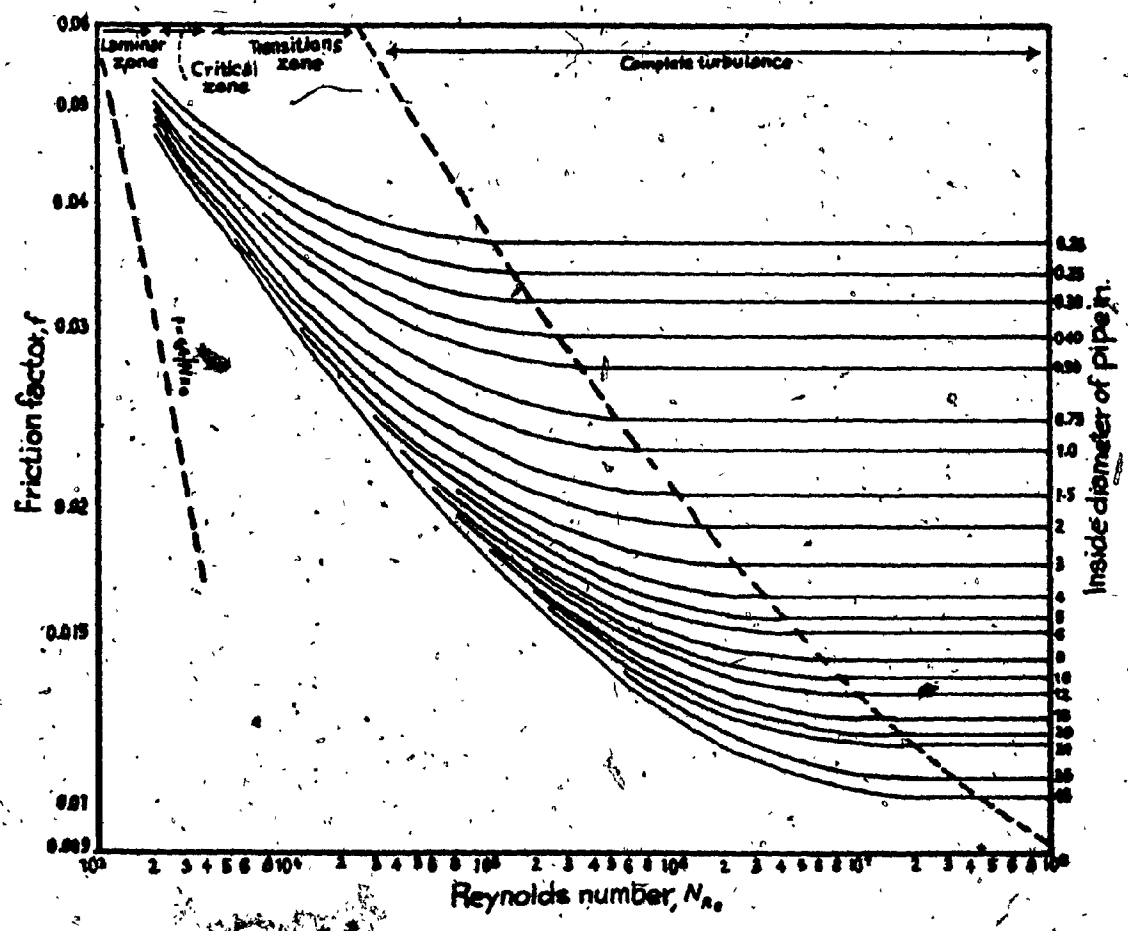


Fig 10 Moody Friction Factors.
(After Moody) [27]

unit losses than the Blasius form of f_L/f_v , depending on pipe size and liquid vapour proportion. Overall friction loss in the pipeline line or process piping between two points will be

$$\Delta P(\text{two phase}) = P_{100}(\text{two-phase})(L/100). \dots \dots (5.12)$$

where L is the equivalent length of pipe and fittings.

5.3 EXAMPLE PROBLEM

This example problem demonstrates the use of General Flow Pattern Prediction and the General pressure drop correlation. Find the unit loss in a 12-inch schedule 40 pipe for the following flow data. [53].

	<u>Liquid</u>	<u>Vapour</u>
Flow "W" lb/hr.	300,000	350,000
Molecular weight M	78.8	75.7
Density lb/ft ³	33.5	2.00
Viscosity ρ , cp.	0.1	0.01
Surface tension dyne/cm.	5.7	-

Solution

From piping design data book.

Schedule 40 5.5 pipe

$$O = 12 \text{ in.}$$

$$d = 11.938 \text{ in.}$$

$$A = 0.7773 \text{ ft}^2$$

We first calculate the flow regime coordinates (Baker Parameters) [24].

$$\begin{aligned} B_x &= 531 \left(\frac{W_L}{W_V} \right) \left(\frac{\sqrt{\rho_L \sigma}}{\rho_L^{2/3}} \right) \left(\frac{\mu_L^{1/3}}{\sigma} \right) \\ &= 531 \left(\frac{300,000}{350,000} \right) \left(\frac{\sqrt{33.5 \times 2}}{33.5^{2/3}} \right) \left(\frac{0.1^{1/3}}{5.7} \right) \\ &= 29.3 \end{aligned}$$

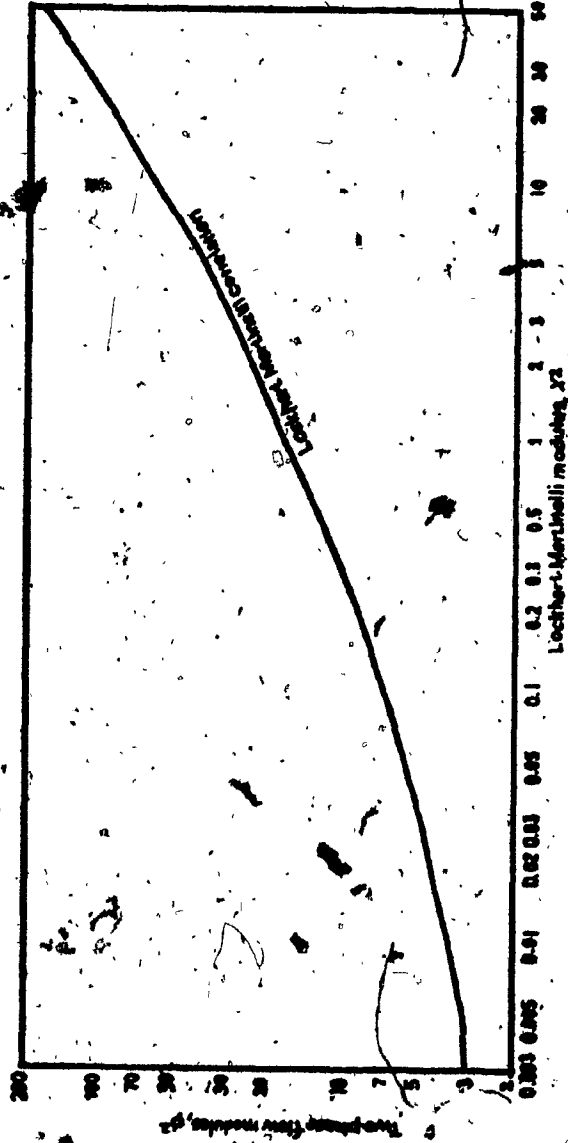


Fig 11 Lockhart-Martinelli Correlation to establish two-Phase flow modulus
(After Lockhart-Martinelli) [7]

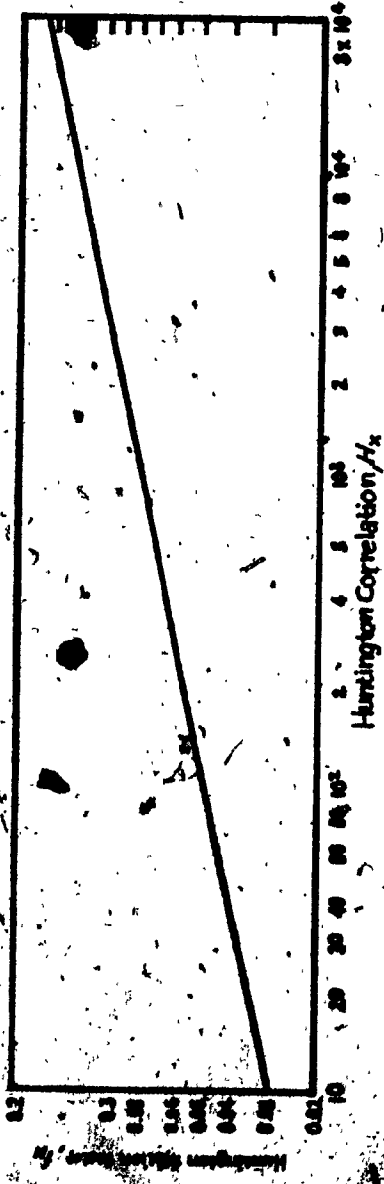


Fig 12 Huntington Correlation
(After Huntington) [22]

$$B_y = 2.16 W_v / A \sqrt{\rho_L \rho_v}$$

$$= \frac{2.16(350,000)}{0.7773 \sqrt{33.52(2)}} = 118,000$$

With these values of B_x and B_y we enter Fig. 9 of "Flow Pattern Regions" and find the intersect in the dispersed flow region.

Next we calculate the Modulus X^2 from Equation (5.11).

$$X^2 = (W_L / W_v)^{1.8} (\rho_v / \rho_L) (\mu_L / \mu_v)^{0.2}$$

$$X^2 = \left(\frac{300,000}{350,000} \right)^{1.8} \left(\frac{2}{33.5} \right) \left(\frac{0.1}{0.01} \right)^{0.2} = 0.072$$

$$X^2 = 0.072$$

with the value $X^2 = 0.072$ we enter Fig. 11 "Lockhart Martinelli Correlation"

$$X^2 = 0.072$$

$$\phi^2 = 6.3$$

Next we calculate the Reynolds number to find the friction factor (Vapour Phase) from Equation 5.5.

$$(N_{Re})_v = 6.31 W / d \mu$$

$$d = 11.938 \text{ in.}$$

$$= \frac{6.31 \times 350,000}{11.938(0.01)}$$

$$= 18.5 \times (10)^6$$

with this Reynolds number we enter Fig. 10 "Friction Factors in New Steel pipe"

$$f_v = 0.013$$

Next we calculate the Vapour Phase unit loss by using Equation (5.8).

$$\Delta P_{100}(\text{vapour}) = \frac{0.00036(0.13)(350,000)^2}{242,470(2)}$$

$$= 1.1 \text{ psi/100 ft}$$

From Equation (5.3).

$$\Delta P_{100}(\text{two phase}) = P_{100}(\text{vapour})\phi^2$$

$$\Delta P_{100}(\text{dispersed}) = 1.1 \times 6.3 = 6.93 \text{ psi/100 ft}$$

Dispersed force unit loss: 6.93 psi/100 ft.

EXAMPLE 2

Find the unit loss in a 8-in. schedule 40 pipe for the following flow data. [53].

	<u>Liquid</u>	<u>Vapour</u>
Flow W, lb/hr	6,150	21,500
Density ρ , lb/ft ³	52	1.92
Viscosity, μ , cp.	0.1	0.01
Surface tension, σ dyne/cm.	6.25	-

Pipe Data

$$d = 6.065 \text{ in}$$

$$d^5 = 8,206 \text{ in}^5$$

$$A = 0.2 \text{ ft}^2$$

$$B_x = \left(\frac{6,150}{21,500} \right) \left(\frac{52(1.92)}{52^{2/3}} \right) \left(\frac{0.1^{1/3}}{6.25} \right) = 8$$

$$B_y = \frac{2.16(21,500)}{0.2 \sqrt{52(1.92)}} = 23,200$$

With these coordinates we find that the point of intersection in Fig. 9 falls in the annular flow region.

Two phase flow modulus x^2 .

$$x^2 = \left(\frac{6.150}{21,500} \right)^{1.8} \left(\frac{1.92}{52} \right) \left(\frac{0.1}{0.01} \right)^{0.2} = 0.00619$$

From Chapter III, section 3.3, we find that the two-phase flow correlation for annular flow is

$$\phi = ax^b$$

where

$$a = 4.8 - (0.3125 \times 6.065) = 2.9$$

$$b = 0.343 - (0.021 \times 6.065) = 0.216$$

$$x^2 = 0.00619$$

$$x = 0.0787$$

$$\phi = ax^b$$

$$= 2.9(0.0787)^{0.216} = 1.67$$

$$\phi^2 = 2.80$$

Reynolds Number

$$N_{Rev} = \frac{6.31(21,500)}{6.065(0.01)} = 2.24 \times 10^6$$

with this Reynolds number we enter to the Fig. 10.

$$f_v = 0.015$$

vapour phase unit loss by Equation 5.8.

$$\Delta P_{100}(\text{vapour}) = \frac{.000336(0.015)(21,500)^2}{8.206(1.92)}$$

By substituting this value in the equation 5.3.

$$\Delta P_{100}(\text{Annular}) = .15 \times 2.8 = 0.42 \text{ psi/100 ft.}$$

Note: The annular flow relation in Table 1 correlates well with pipe size up to 10 in. Above this negative values are obtained so for pipe size larger than 10 in. use I.D. = 10 in. = d.

Stratified flow is similar to the wave flow except the interface has waves moving in the direction of flow. The best method for calculating the unit loss for these types of flow is the Schneider-White-Huntington correlation [21].

$$H_x = (W_L/W_v) / (\mu_L/\mu_v) \dots \dots \dots (5.13)$$

After computing H_x we find the fraction factor f_H from Fig.12 and then use it in the equation of Darcy relation to determine the unit loss.

$$\Delta P_{100}(\text{wa}) = 0.00036 f_H (W_v)^2 / d^5 \rho_v \dots \dots \dots (5.14)$$

Plug flow develops when the interface is high in the pipe cross section. The waves congregate and touch the high point of the pipe wall. Then, plugs of liquid alternating with pockets of gas in the liquid alternating with pockets of gas in the liquid close to the upper wall move along the horizontal line.

Crowly-flow lines have such types of flow.

Bubble Flow - Calculating the unit loss procedure is similar to that of dispersed flow. The correlation for double flow is shown in Chapter III, Section 3.3.

Slug Flow - Unit Loss

Unit losses for slug flow in process piping are generally not calculated. In borderline cases, unit losses for slug flow are smaller than those for bubble or annular flow.

Slug flow causes pressure fluctuations in piping, which can upset process conditions and cause inconsistent instrument sensing. Slug flow can be avoided in process piping by:

- Reducing line sizes to the minimum permitted by available pressure differentials.
- Designing for parallel pipe runs that will increase flow capacity without increasing overall friction loss.
- Using valved auxiliary pipe runs to regulate alternative flowrates and thus avoid slug flows.
- Using a low-point effluent drain or bypass.
- Arranging the pipe configurations to protect against slug flow. For example, in a pocketed line where liquid can collect, slug flow might develop.

Slug flow will not occur in a gravity-flow line, flow through the branch, at a low point can provide sufficient turbulence for more-effective liquid carryover.

A diameter adjustment, coupled with gas injections, can also alter a slug-flow pattern to bubble or dispersed flow.

CHAPTER VI

CONCLUSIONS

6.1 CONCLUSIONS

The present literature review of two-phase flow in long horizontal circular conduits has led to conclusions that are important from the author's viewpoint.

A great deal of experimental work on this problem has been done by people in the chemical, petroleum, and nuclear industries as well as in many Universities.

Most of this research in the chemical and nuclear industries has been concerned with specific processes and does not approach the problem in a general manner. However, several attempts have been made to develop generalized correlations that can be used for process design as well as two-phase flow pipeline design. None of these attempts has been successful and a great amount of disagreement exists among the authors of multiphase flow literature. This is probably due to the fact that none of the existing correlations adequately account for all of the variables involved in the problem.

The correlations presented in this study may be used to accurately predict the pressure loss for horizontal multiphase flow pipelines. Interpolation between the lines of the energy loss factor correlation allows application of these correlation to all pipelines whose inside diameters are from 2 to 17 inches.

ACCURACY AND LIMITATIONS OF METHOD DESCRIBED

The application of correlations for two-phase flow to process piping design and to pipeline design is arbitrary.

Experiments are usually done with small diameter and straight horizontal pipe. Under laboratory conditions flow patterns are kept constant and flow conditions consistent. However, most process piping in most cases has changing flow pattern in various segments of the line because of the three dimensional pipe configuration in which one finds horizontal and vertical runs, elevations manifolds pipe components reducers and other restrictions.

Sizable deviations can be expected in prediction of friction loss compared to actually measured values. However, if actual pipeline closely resemble the experimental conditions, deviations are small. A. E. Dukler, et. al. [6] compared the values obtained by the described method using his data bank with a number of other suggested models and found the deviation within the permissible level. Table 3, page 49 gives the general comparison of these values.

The author used this correlation to design 60 ft. long and 80 in. dia. pipe of two phase monoethanolamine (MEA) and natural gas from scrubbing plant to gasoline plant*.

*The calculated pressure drop was 8.75 psi and observed pressure drop was 7.00 psi and the % error was 20%.

This correlation is widely used in the petroleum industry in process piping.

BIBLIOGRAPHY

1. HAGEDORN, A.R. "Experimental Study of Pressure Gradients Occurring During Continuous Two-Phase Flow in Small Diameter Vertical Conduits," ASME (1964).
2. GOUSE, S.W. "An Introduction to Two-Phase Flow," M.I.T., D.S.R. Report 8734-3 (June, 1964).
3. GOUSE, S.W. "Two-Phase Gas-Liquid Flow Oscillations: Preliminary Survey," M.I.T., D.S.R. Report 8734-5 (July, 1964).
4. GOUSE, S.W. "Void Fraction Measurement", M.I.T., D.S.R. Report 8734-2 (April, 1964):
5. GOUSE, S.W. "Heat Transfer and Fluid Flow Inside a Horizontal Tube Evaporator--Phase I," M.I.T. D.S.R. Report 9649-1 (June, 1964).
6. DUKLER, A.E., et al. "Frictional Pressure Drop in Two-Phase Flow: A Comparison of Existing Correlations for Pressure Loss and Holdup," A.I.Ch.E. Journal (January, 1964).
7. LOCKHART, R.W., and MARTINELLI, R.C. "Proposed Correlation of Data for Isothermal Two-Phase Two Component Flow in Pipe Lines," Chem. Engr. Progr. (1949) 45, 39.
8. MARTINELLI, R.C., et al. "Isothermal Pressure Drop for Two-Phase, Two-Component Flow in a Horizontal Pipe," Trans., ASME (1944) 66, 39-151.
9. MARTINELLI, R.C., and NELSON, D.B. "Prediction of Pressure Drop During Forced-Circulation Boiling of Water," Trans., ASME (August, 1948), 70, 695.
10. MARTINELLI, R.C. et. al. "Two-Phase, Two-Component Flow in the Viscous Region," Trans., A.I.Ch.E. (1946) 42, 681-705.
11. AGRAWAL, S.S., GREGORY, G.A., and GOVIER, G.W., "An Analysis of Horizontal Stratified Two Phase Flow in Pipes," Can. Jour. Ch.E., 51, 280 (1973).

12. CHENOWETH, J.M., and MARTIN, M.W. "Turbulent Two-Phase Flow," Petroleum Refiner (October, 1955).
13. CHEW, Ju-Nam, and CONNELLY, C.A., Jr. "A Viscosity Correlation for Gas-Saturated Crude Oils," Trans. AIME (1959).
14. HOOGENDORN, C.J. "Gas-Liquid Flow in Horizontal Pipes," Chem. Engr. Sci. (1959).
15. ISBIN, H.S., SHER, N.C., and EDDY, K.C. "Void Fractions in Two-Phase Steam-Water Flow," A.I.Ch.E. Journal (1957).
16. CIAFALONI, F. "Investigation of Two-Phase Horizontal Flow Evaluation Methods with a Procedure to Optimize Well Production Rates by Machine Computation," M.S. Thesis, University of Texas (1963).
17. BAROCZY, C.J. "A Systematic Correlation for Two-Phase Pressure Drop." A.I.Ch.E. Preprint 37 (Aug. 1965).
18. BERGELIN, et. al. "Co-Current Gas-Liquid Flow," (I) "Flow in Horizontal Tubes," Heat Flow and Fluid Mechanics Institute, Berkeley, California (1949) (available from ASME).
19. EATON, B.A., ANDREWS, D.E., KNOWLES, C.R., SILVERBERG, I.H., and BROWN, K.E., The Prediction of Flow Patterns, Liquid Hold-up and Pressure Losses Occurring During Continuous Two-Phase Flow in Horizontal Pipelines, J. Petroleum Technology, 19, 815 (1967).
20. POETTMANN, F.H., and CARPENTER, P.G. "The Multiphase Flow of Gas, Oil, and Water Through Vertical Flow Strings with Application to the Design of Gas-Lift Installations," Drilling and Production Practice (1952).
21. SCHNEIDER, F.N., WHITE, P.P., and HUNTINGTON, R.L. "Horizontal Two-Phase Oil and Gas Flow," Pipe Line Industry (October, 1954).
22. HUNTINGTON, R.L., BRIGHAM, W.E. and HOLSTEIN, E.D. "How Uphill and Downhill Flow Affect Pressure Drop in Pipelines Through Hilly Country," Oil and Gas Journal (November 11, 1957).
23. ALVES, G.E. "Concurrent Liquid-Gas Flow in a Pipeline Contractor". Chem. Eng. Progr. (July, 1960).
24. BAKER, Ouid. "Design of Pipelines for the Simultaneous Flow of Oil and Gas". Oil and Gas Journal (1954).

25. WHITE, P.D., and HUNTINGTON, R.L. "Horizontal Co-Current Two-Phase Flow of Fluid in Pipe Lines," Petroleum Engineer (August, 1955) 27, 40.
26. BERTUZZI, A.F., TEK, M.R., and POETTMAN, F.H. "Simultaneous Flow of Liquid and Gas Through Horizontal Pipe," Trans., AIME (1956) 207, 17.
27. MOODY, L.F. "Friction Factors in Pipe Flow," Trans., ASME (1944) 66, 671.
28. REID, R.C., and REYNOLDS, A.B., et al. "Two-Phase Pressure Drops in Large Diameter Pipes," A.I.Ch.E. Journal (September, 1957), 3, 321.
29. YOCUM, B.T. "Two-Phase Flow in Well Flowlines", AIME Paper No. 924-G(1957).
30. BAKER, Ovid. "Multiphase Flow in Pipe Lines". Oil and Gas Journal (November 10, 1958).
31. CHISHOLM, D., and LAIRD, A.D.K. "Two-Phase Flow in Rough Tubes", Trans., ASME (1958).
32. BANKOFF, S.G. "Variable Density Single Fluid Model for Two-Phase Flow". Trans. A.S.M.E. (1960), 265-272.
33. HUGHMARK, G.A. "Holdup in Gas-Liquid Flow," Chem. Engr. (April, 1962).
34. HUGHMARK, G.A., and PRESSBURG, B.S. "Holdup and Pressure Drop with Gas-Liquid Flow in a Vertical Pipe," A.I.Ch.E. Journal (December, 1961).
35. DUKLER, A.E., et al. "Frictional Pressure Drop in Two-Phase Flow: B. An Approach Through Similarity Analysis," A.I.Ch.E. Journal (January, 1964).
36. MANDHANE, J.M., GREGORY, G.A., and AZIZ, K. "A Flow Pattern Map for Gas-Liquid Flow in Horizontal Pipes," The International Journal of Multiphase Flow, 1, 537 (1974).
37. MANDHANE, J.M., GREGORY, G.A., and AZIZ, K., User's Manual. University of Calgary Multiphase Pipe Flow Data Bank, University of Calgary (1974).

38. KATZ, D.L., et al. Handbook of Natural Gas Engineering, New York-Toronto-London: McGraw Hill Book Co., Inc., 1959.
39. YAGI, S. Chemical Engineering, 1954, 18, 2.
40. BAKER, O., BRAINERO, H.W., GOLDVEN, C.L., FLANIGAN, O., AND WELCHEN, J.K. "Gas-Liquid Flow in Pipelines: Design Manual". American Gas Association, Arlington, Va. 22209, October 1970.
41. BERGELIN, O.P. and GAZLEY, C., Proc. Heat Transfer and Fluid Mechanics Inst., May 1949, p. 5.
42. JOHNSON, H.A., and ABOU-SABE, A.H. Trans. A.S.M.E., 74 (1952) 977.
43. JENKINS, R., M.S. Thesis, University of Delaware, 1947.
44. GAZLEY, C., Ph.D. Thesis, University of Delaware, 1949.
45. KOSTERIN, S.I., AKAD, Isv., Nauk, S.S.S.R., Otd. Tekh. Nauk., No. 12, 1949, pp. 1824.
46. GOVIER, G.W. and OMER, M.M. Can. J. Chem. Eng., 40 (1962) 93.
47. SCOTT, D.S., in Advances in Chemical Engineering, Vol. 4, Academic Press, New York, 1963, p. 200.
48. KNOWLES, C.R. "The Effect of Flow Patterns on Pressure Loss in Multiphase, Horizontal Flow. Chemical Engineering (1965).
49. AL-SHEIKH, J.N., SAUNDERS, D.E. and BRÖDKEY, R.S., Can. J. Chem. Eng., 48 (1970), 21.
50. GOVIER, G.W., and AZIZ, K. The Flow of Complex Mixtures in Pipes, Van-Nostrand-Reinhold Co., New York (1972).
51. STREETER, V.L. Fluid Mechanics, New York: McGraw-Hill Book Co., Inc., (1958).
52. BRIGHAM, W.E., HOLSTEIN, E.D., and HUNTINGTON, R.L. "How Uphill and Downhill Flow Affect Pressure Drop in Two-Phase Pipe Lines", Oil and Gas Journal (1957).
53. KERN, R., HOFFMAN-LA ROCHE, "Piping Design for Two-Phase Flow", Chemical Engineering (1975).