ε'/ε in A Pseudo Manifestly Left-Right Hand Symmetric Model

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ABSTRACT

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CP-violation in the neutral kaon system is investigated in some detail. ε'/ε is calculated to lowest order for the low energy case using Feynman diagrams in the Standard Model as well as in a Pseudo-manifestly Left-Right Hand Symmetric Model.

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CONTENTS

INTRODUCTION

In particle physics one studies three interactions:strong, electromagnetic and weak. The theory of these interactions is understood on the basis of symmetries. But experimental data has shown that some important symmetries, for instance, charge conjugation and parity symmetry (CP), are violated in processes involving weak interaction. As a result, CP-violation becomes an important feature of weak processes. But the origin of CP-violation remains mysterious. It is , therefore, of basic importance theoretically to solve this puzzle and many phenomenological models have attempted to do so. In view of the great success of the Standard Model in describing weak interactions, the various extensions of the Standard Model get considerable attention. An attractive one is the Left-Right Symmetric Model, candidate for explaining parity violation, which is a natural extension of Standard Model and hence, contains all advantages of the latter. In this thesis we shall discuss CP-violation in the Left-Right Model.

Apart from smallness, a remarkable feature of CP-violation is that it has been till now seen only in neutral kaon system. CP-violation can be described by two parameters ε' and ε which are expressed relative to observables (η_{00}, η_{+-}) as well to model parameters (such as δ, α_1, β_1 and so on). Thus ε'/ε is important to select out correct models.

The aim of this thesis is to calculate the CP-violation parameters ε and ε'/ε for the kaon system. The work is organized as follows:

Some background material on the Standard Model and Left-Right Symmetric Model have been provided in *chapter 1* and *2* respectively. In *chapter 3* we discuss CP-violation in the kaon system. The CP-violation parameters ε'/ε are calculated to the lowest order for the low energy case in the Standard Model in *chapter 4* and in the Pseudo-manifest Left-Right Symmetric Model in *chapter 5*, respectively. The notation and conventions used throughout the thesis are given in the Appendix.

Chapter 1

The Standard Model

The Standard Model has two distinct parts, the standard electroweak model and QCD. They are all non-Abelian gauge field theories. The latter deals with the color SU(3) interaction between quarks and gluons, which is believed to responsible for the strong interaction. The former unifies the weak and electromagnetic interactions using the SU(2)·U(1) gauge group. Major experimental support for the model came with the observation of weak neutral currents in 1973, followed by the discovery of the weak gauge bosons themselves ($W^{\pm}$ and Z) in 1983. The model has proved to be very successful phenomenologically and is in detailed agreement with all observed electroweak phenomena.

In this chapter we give a brief survey of some of the basic aspects of the Standard Model. More complete treatments may be found in references [1-8].

1.1 Quantum Electrodynamics (QED)

1.1.1 The Lagrangian

In Nature there exist four fundamental interactions. They are widely believed to be described by gauge field theories which are of a particular kind of field theories based on the gauge principle. The gauge principle is the requirement that the theory be invariant under local gauge transformations.

A prototypical example of gauge field theory is the electrodynamics of a charged particle. Starting with Maxwell's equations for the electromagnetic fields and the Dirac equation, such a theory is constructed. The theory is constrained to obey Lorentz invariance, invariance under space inversion and time reversal, and renormalizability(see subsection 1.2.2), as well as invariance under the following Abelian local gauge transformations:

$$\Psi \longrightarrow \exp(-ie\vartheta(x))\Psi, A^{\mu} \longrightarrow A^{\mu}+\frac{1}{e}\partial^{\mu}\vartheta(x), \quad (1.1)$$

with ϑ depending on x. Then a Lagrangian (density) which describes Quantum Electrodynamics has the following form:

$$L=-(1/4)F^{\mu\nu}F_{\mu\nu}+\bar{\psi}[i\gamma^{\mu}(\partial_{\mu}+ieA_{\mu})-m]\psi$$
$$\equiv-1/4F^{\mu\nu}F_{\mu\nu}+\bar{\psi}(i\gamma^{\mu}D_{\mu}-m), \qquad (1.2)$$

where ψ is the charged fermion field corresponding to Fermi-Dirac particles with charge e (e > 0) and bare mass m, $F_{\mu\nu}$ is the electromagnetic field tensor

$$F_{\mu\nu} = \partial_{\mu}A_{\nu}-\partial_{\nu}A_{\mu}, \qquad (1.3)$$

D_{μ} is the covariant derivative

$$D_{\mu} = \partial_{\mu} + iQeA_{\mu}, \qquad (1.4)$$

and A_{μ} is a vector gauge field describing a massless field quantum which couples to the Fermi-Dirac particle.

Defining an electromagnetic current by

$$j^{\mu} \equiv \bar{\psi}\ \gamma^{\mu}\ \psi, \qquad (1.5)$$

it is easily shown that j^{μ} is a conserved current. That is

$$\partial^{\mu} j_{\mu} = 0 \tag{1.6}$$

And hence the electric charge

$$e \equiv \int j^0 \, d^3x; \tag{1.7}$$

is conserved. In terms of j^{μ}, the interaction term in eq. (1.2) may be written as

$$L_I = - e \, j^{\mu} A_{\mu}. \tag{1.8}$$

The other two terms in eq. (1.2) are, respectively,

$$L_R = -1/4 \, F^{\mu\nu} F_{\mu\nu}, \tag{1.9}$$

which describes the radiation itself, and

$$L_F = \bar{\psi}(i\gamma^{\mu}\partial_{\mu} - m)\psi, \tag{1.10}$$

which describes the fermion field.
Noting that L_F is invariant under global but not local U(1) gauge transformations, we see that both A_{μ} and L_I are the natural consequences of the requirement of the gauge principle.

We have to quantize the gauge field theory to obtain a consistent quantum theory of the gauge field. Quantization is not a unique procedure and a variety of quantization methods exist which lead to the same physical prediction. There are three well-known ways of quantization which are equivalent to each other:

(1) Canonical operator formalism[9];

(2) Functional-integral formalism[10];

(3) Stochastic formalism[11].

In the traditional canonical operator formalism, one regards fields as operators and sets up canonical commutation relations for them. All the Green functions which characterize the quantum theory of fields may be calculated as vacuum expectation values of the product of the field operators. In the Feynman functional-integral formalism, the fields are c-numbers and the Lagrangian is of the classical form. The Green functions are obtained by integrating the product of the fields over all of their possible functional forms with a suitable weight. In the stochastic formalism, one notes the similarity between the functional-integral expressions of Green functions in Euclidian space and statistical averaging, and regards the field as a stochastic variable. The Green functions are then given by the statistical average of the product of the fields in equilibrium.

When we quantize the gauge theory we shall be in trouble. For example, if we try to quantize the theory in canonical formalism for the L_R of (1.9), then we shall obtain a vanishing canonical momentum. The difficulty arises from the freedom of gauge transformations. One way of getting rid of the difficulty is to eliminate this freedom by fixing the gauge. For instance in the Lorentz gauge (or covariant gauge), we set the covariant constraint

$$\partial^{\mu} A_{\mu}^{a} = 0. \tag{1.11}$$

The following noncovariant gauges are also frequently used: Coulomb (radiation) gauge $\partial_i A_i^a = 0$, axial gauge $A_3^a = 0$ and temporal gauge $A_0^a = 0$. Note that the physical predictions stemming from the Lagrangian are gauge-independent.

After fixing the gauge the Lagrangian will in general be modified. For example, in the Lorentz gauge the Lagrangian of eq. (1.2) is modified to

$$L = L_R + L_F + L_I - \frac{1}{2\alpha} (\partial^{\mu} A_{\mu})^2. \qquad (1.12)$$

where α is a constant called the gauge parameter.

As mentioned above, physical predictions are independent of the values of α. One often fixes α, for example, $\alpha = 1$ (Feynman gauge) and $\alpha = 0$ (Landau gauge).

1.2 Quantum Chromodynamics (QCD)

1.2.1 The Lagrangian

The Lagrangian for QED is based on the Abelian local gauge group $U(1)_Q$, where the electric charge Q is the group generator. In an analogous way, but with the U(1) gauge group replaced by the non-Abelian color group SU(3), we can develop the theory of QCD which deals with the color interactions of quarks and gluons. Under the SU(3) group of color transformations, three colored quarks transform as the fundamental(3) representation and eight massless gauge bosons calledgluons transform as the adjoint (8) representation of the group. The generators T^a, a = 1,2,...,8 satisfy

$$[T^a,T^b] = if^{abc}T^c, \qquad (1.13)$$

where the structure constant f^{abc} is a totally antisymmetric tensor, and the summation on repeated indices is understood. Thus the gauge transformation reads

$$\psi'_i = U_{ij}\psi_j, \quad U = exp(-iT^a\vartheta^a), \qquad (1.14)$$

where ψ is a quark field. and the subscripts i,j are color indices that assume the values of 1,2,3, ϑ^a are phase angles which depend on x. The covariant derivative in the fundamental representation is defined by

$$(D_\mu)_{ij} = \delta_{ij}\ \partial_\mu - ig\ T^a_{ij}\ A^a_\mu, \qquad (1.15)$$

and the gluon field-strength tensor is defined by

$$F^a_{\mu\nu} = \partial_\mu\ A^a_\nu - \partial_\nu\ A^a_\mu + g\ f^{abc}\ A^b_\mu\ A^c_\nu, \qquad (1.16)$$

where g is the strong coupling constant.

Thus,the classical Lagrangian of QCD is given by

$$L = -1/4\ F^a_{\mu\nu}F^{a\mu\nu} + \Sigma^f_{k=1}\ \bar{\psi}^i_k(i\gamma^\mu\ D^{ij}_\mu - m_k\delta^{ij})\ \psi^j_k\ , \qquad (1.17)$$

where the summation on k runs over all quark flavors. This Lagrangian is invariant under the gauge transformation (eq. 1.14), provided that the eight gluon field potentials A^a_μ transform according to

$$A^a_\mu \longrightarrow A^a_\mu + f^{abc}\ \vartheta^b A^c_\mu - \frac{1}{g}\ \partial_\mu\vartheta^a. \qquad (1.18)$$

The generators T^a are frequently represented by the Gell-Mann matrices λ^a, which are traceless Hermitian 3×3 matrices satisfying the Lie algebra

$$[\lambda_a, \lambda_b] = 2i\ f_{abc}\lambda_c. \tag{1.19}$$

A remarkable feature of the Lagrangian of eq. (1.7) is that as a consequence of the non-Abelian nature of the theory, it contains both trilinear and quadrilinear self-couplings of the gluon fields. This self-coupling is the main source of asymptotic freedom in QCD see eq. (1.24) and is also the most crucial difference between QCD and QED.

The Feynman rules for tree-level graphs can be deduced from the Lagrangian found in eq. (1.17).

As mentioned before, the gluon fields can be expressed in a variety of gauges. Graphs involving gluon loops , and in particular helicity 0 contributions, introduce some new problems, which lead to a potential violation of unitarity. Unitarity can be restored by introducing into the theory negative metric "particles" (ghosts) which are scalars, but which obey Fermi statistics. For a covariant gluon gauge the contribution of the ghost must be added to every gauge-field loop graph in order to obtain the correct result. But in a physical or axial gauge, which has a more complicated gluon propagator but only helicity 1 contributions, these ghosts do not appear. Such problems do not arise in QED.

Including the ghost fields χ^a, an effective quantum Lagrangian (for one species of quarks) is

$$L = L_G + L_{GF} + L_{FP} + L_F, \tag{1.20}$$

$$L_{G} = -1/4\ F^{a}_{\mu\nu}F^{a\mu\nu}, \tag{1.21}$$

$$L_{GF} = -\frac{1}{2\alpha}\ (\partial^{\mu}A^{a}_{\mu})^{2}, \tag{1.22}$$

$$L_{FP} = (\partial_{\mu}\chi^{a*})\ D^{ab}_{\mu}\ \chi^{b}, \tag{1.23}$$

$$L_{F} = \bar{\psi}^{i}(i\gamma^{\mu}\ D^{ij}_{\mu} - m\delta^{ij})\ \psi^{j}, \tag{1.24}$$

where D^{ab}_{μ} is the covariant derivative in the adjoint representation, defined by

$$D^{ab}_{\mu} = \delta^{ab}\ \partial_{\mu} - g\ f^{abc}\ A^{c}_{\mu}. \tag{1.25}$$

The suffixes stand for "gauge", "gauge fixing", "ghost(FP)" and "fermion" terms respectively. The above Lagrangian of eq.(1.20) forms the basis of QCD (for one species of quarks).

The general set of Feynman rules for QCD can be conveniently derived from this Lagrangian. One finds that, although the rules depend on the particular choice of gauge, all physical results are gauge-invariant. So, the ghost fields, though they must be encountered at intermediate stages, play no role in the final results[12].

1.2.2 Renormalization

A major obstacle to the application of quantum field theories is that naively they predict that all physical observable quantities such as charge, mass, etc., are infinite. Physically, these infinities are consequences of naive definitions of charge, mass, etc.; Mathematically, they arise from divergent loop diagram integrations. The solution to the problem is reparametrization or renormalization. A renormalization prescription consists, in general, of the three following parts:

(*i*) Regularization

Regularization is a procedure which makes the divergent integrals to be mathematically manageable, and has no physical consequences. There are a variety of regularization schemes. The most common one is dimensional regularization, in which all the loop integrals are evaluated in $(4-\varepsilon)$ dimensions and the results are continued analytically to $\varepsilon \longrightarrow 0$, whereupon the divergences can be readily identified. For example, for one-loop quark self-energy in an arbitrary covariant gauge α, using dimensional regularization we obtain the following results

$$\Sigma(p) = -\alpha(g_0^2/12\pi^2)p^{\nu}\gamma_{\nu}[\tfrac{2}{\varepsilon} -\gamma+1+\ln(4\pi)-\ln(-p^2/\mu^2)+O(\varepsilon),$$

$$= p^{\nu}\gamma_{\nu}\Sigma'(p)+O(\varepsilon), \tag{1.26}$$

where for simplicity we have taken quark mass $m = 0$. Here $\gamma = 0.57721\ldots$is the Euler constant and μ is an arbitrary mass scale such that

$$g^2 = g_0^2\,\mu^{\varepsilon}, \tag{1.27}$$

where g_0 is a dimensionless gauge coupling constant. The new gauge coupling constant g is no longer dimensionless.

To implement dimensional regularization the following definitions are needed:

a) The *d*-dimensional space-time has the metric

$$g^{\mu\nu} = (+,-,\ldots,-).$$

b) The Dirac matrix satisfies the anticommutation relations

$$\{\gamma^{\mu},\gamma^{\nu}\} = 2g^{\mu\nu}. \qquad (1.28)$$

c) Tr[1] = 4 in the space of the γ-matrices.

d) The integral measure is $\int d^{d}k/(2\pi)^{2}$.

e) γ_5 is an object which satisfies $\{\gamma_5,\gamma^{\mu}\} = 0$. (1.29)

Since in dimensional regularization nothing has been violated except that space-time is not 4-dimensional, all the physical requirements are preserved. Hence, this regularized theory is Lorentz invariant, gauge invariant, unitary, etc.[13,14]. In this sense dimensional regularization is the most suitable renormalization for gauge theories.

(*ii*) Renormalization Scheme

The prescription for subtracting divergences in Green functions is called the renormalization scheme. Two different renormalization schemes are always connected by a finite renormalization.

Let us consider the quark propagator $S_{ij}(p)$ with m = 0,

$$S_{ij}(p) = -\delta_{ij}/p^{\mu}\gamma_{\mu}[1+\sigma(p^2)], \qquad (1.30)$$

where

$$\sigma(p^2) = \Sigma'(p) + O(g_0^4), \qquad (1.31)$$

with terms of order ε set equal to zero, and use of eq.(1.26) has been made to write eq. (1.30).

We renormalize $S_{ij}(p)$ by a multiplicative factor Z_2,

$$S_{Rij}(p) = Z_2^{-1} S_{ij}(p). \tag{1.32}$$

we expand Z_2 inpowers of g_0 and write

$$Z_2 = 1 - z_2 + O(g_0^4), \tag{1.33}$$

with z_2 the term of order g_0^2 which is assumed to be divergent. Substituting eq. (1.33) for Z_2 in eq. (1.32) and using eq. (1.30) we obtain to order g_0^2,

$$S_{Rij}(p) = -\delta_{ij}/p^{\mu}\gamma_{\mu}[1+\sigma(p^2)-z_2]. \tag{1.34}$$

Since S_{Rij} is the renormalized propagator, it should be free of divergences and hence $\sigma(p^2)-z_2$ has to be finite. Thus the divergence in $\sigma(p^2)$ should be cancelled by that of z_2. This requirement determines the constant z_2 up to a finite addive constant. In order to fix this arbitrary finite constant in z_2, we need an additional requirement which sets up a renormalization scheme. There are a variety of renormalization schemes depending on the choice of the above additional requirement. For example,

a) On-shell subtraction: Z_2 is determined on the mass shell of quarks, i.e., by the condition

$$S_{Rij}(p) \sim \delta_{ij}/(m-p^{\mu}\gamma_{\mu}) \text{ for } p^{\mu}\gamma_{\mu} \sim m. \tag{1.35}$$

b) Off-shell subtraction (or MOM): we require that

$$S_{Rij}(p) \sim \delta_{ij}/p^{\mu}\gamma_{\mu} \text{ for } p^2 \sim -\lambda^2 < 0. \tag{1.36}$$

c) Minimal subtraction (MS):

$$z_2 = -\alpha(g_0^2/12\pi^2)\frac{2}{\varepsilon}. \tag{1.37}$$

Note here that this scheme may be converted to the MOM scheme by setting $\lambda^2 = 4\pi e^{1-\gamma}\mu^2$.

d) Modified minimal subtraction ($\overline{MS}$):we require that

$$z_2 = \Sigma'(p) - 1. \tag{1.38}$$

The above four different renormalization schemes provide different forms for the renormalized propagator. In general the form of the Green functions varies from scheme to scheme.

Whether an interaction is renormalizable or not may be determined by power-counting (see, for example, [7]). For a renormalizable interaction, we have to redefine the coupling g, mass m, and all fields (wave- functions) A_μ^a, χ_1^a, χ_2^a and ψ, for example, by

$$A_\mu^a = \sqrt{Z_3}\, A_{r\mu}^a, \quad \chi_{1,2}^a = \sqrt{Z_3'}\, \chi_{1,2,r}^a, \quad \psi = \sqrt{Z_2}\psi, \tag{1.39}$$

$$g = Z_g\, g_r, \quad \alpha = Z_3\, \alpha_r, \quad m = Z_m m_r, \tag{1.40}$$

and then remove divergences order by order through subtracting counter terms which contain divergent pieces. Here, in eqs. (1.39) and (1.40), the subscript r denotes a renormalized quantity.

QCD theory is renormalizable, which can be rigorously proved (see, for example, [7]).

Starting with eq.(1.29) after applying eqs. (1.39) and (1.40), we obtain the renormalized Lagrangian for QCD as follows

$$L = L_r + L_c, \qquad (1.41)$$

where L_r is precisely equal to the Lagrangian of eq.(1.20) if the quantities A^a_μ, $\chi^a_{1,2}$, ψ, g, α and m are replaced by the corresponding renormalized ones respectively. The couter-term Lagrangian L_c is given by

$$\begin{aligned}
L_c = &-(Z_3-1)\tfrac{1}{4}(\partial_\mu A^a_{r\nu}-\partial_\nu A^a_{r\mu})(\partial^\mu A^{a\nu}_r-\partial^\nu A^{a\mu}_r) \\
&+(Z_3'-1)i(\partial^\mu\chi^a_{1r})(\partial_\mu\chi^a_{2r})+(Z_2-1)\bar{\psi}^i_r(i\gamma^\mu\partial_\mu-m_r)\psi^i_r \\
&-(Z_2Z_m-1)m_r\bar{\psi}^i_r\psi^i_r \\
&-(Z_gZ_3^{3/2}-1)\tfrac{1}{2}g_r f^{abc}(\partial_\mu A^a_{r\nu}-\partial_\nu A^a_{r\mu})A^{b\mu}_r A^{c\nu}_r \\
&-(Z_g^2Z_3^2-1)\tfrac{1}{4}g_r^2 f^{abe}f^{cde}A^a_{r\mu}A^b_{r\nu}A^{c\mu}_r A^{d\nu}_r \\
&-(Z_gZ_3'Z_3^{1/2}-1)\, i\, g_r f^{abc}(\partial^\mu\chi^a_{1r})\chi^b_{2r}A^c_{r\mu} \\
&+(Z_gZ_2Z_3^{1/2}-1)\, g_r\bar{\psi}^i_r T^a_{ij}\gamma^\mu\psi^j_r A^a_{r\mu}.
\end{aligned} \qquad (1.42)$$

Here,the renormalization constants,Z_3,Z_3',Z_2,Z_m and Z_g should be determined by adjusting L_c so as to cancel overall divergences appearing in higher-order Feynman amplitudes. Of course a suitable renormalization scheme is to be chosen to carry out this process.

The renormalized Feynman rules for QCD may be derived from eq.(1.41).

1.2.3.Renormalization Group Equation (RGE)

In subtracting the divergences we inevitably introduce an arbitrary mass scale μ. Therefore, renormalization is

μ-dependent. For example, the truncated connected Green function $\Gamma^{(n)}(p_i;m,g)$, which corresponds to the Feynman amplitude, can be written

$$\Gamma_B^{(n)}(p_i;m_B,g_B,\lambda) = [Z_\phi(\frac{\lambda}{\mu},\frac{m}{\mu},g)]^{-n/2}\,\Gamma^{(n)}(p_i;m,g), \quad (1.43)$$

where λ is some parameter such that the result is finite as $\lambda \longrightarrow \infty$. The subscript ϕ shows corresponds to the ϕ^4 theory which is used here for simplicity, B denotes "bare", and p_i represents the set of n particle momenta. The arguments of Z_ϕ have been written as ratios, since Z_ϕ is dimensionless.

If μ change to μ', then $\Gamma^{(n)}$ change to $\Gamma^{(n)\prime}$,

$$\Gamma^{(n)\prime} = z(\mu',\mu)\Gamma^{(n)}, \quad (1.44)$$

where $z(\mu',\mu) = [Z_\phi(\mu)/Z_\phi(\mu')]^{-n/2}$, (1.45)

which is finite since the divergent part in $Z(\mu')$ is cancelled out by that of $Z(\mu)$ owing to its multiplicative nature. Eq.(1.45) defines a set of finite renormalizations $\{z(\mu',\mu)\}$ for varying renormalization scales μ' and μ. This set is, in fact an Abelian group, called the renormalization group. Physics is invariant under the renormalization group. This renormalization symmetry is the basis of RGE which is a differential equation expressing the response of Green functions and parameters (e.g.,coupling constants and masses) to the change of the scale μ. In the present case, the RGE can be obtained by differentiating eq. (1.43) with respect to μ,

$$(\mu\frac{\partial}{\partial\mu} + \beta\frac{\partial}{\partial g} - n\gamma + \mu\frac{dm}{d\mu}\frac{\partial}{\partial m})\Gamma^{(n)} = 0, \quad (1.46)$$

where

$$\beta = \mu\frac{dg}{d\mu}, \quad \gamma = \frac{\mu}{2} Z_{\phi}^{-1}(\frac{d}{d\mu}Z_{\phi}) = \frac{\mu}{2} \frac{d}{d\mu}(\ln Z_{\phi}). \qquad (1.47)$$

The quantities β and γ are, in fact, finite as $\lambda \longrightarrow \infty$ and, since they are dimensionless, do not depend on μ after taking this limit; so that $\beta = \beta(g)$ and $\gamma = \gamma(g)$.

Introducing a scaling factor for the momenta, σ, so that by varying σ the momenta can be taken to arbitrarily large values, for the massless case (since QCD is massless) we obtain the following solution for eq.(1.46)

$$\Gamma^{(n)}(\sigma\ p_i;g,\mu) = \sigma^{4-n}F\Gamma^{(n)}(p_i;\bar{g}(g,t),\mu), \qquad (1.48)$$

where $t = \ln\sigma$, $\bar{g}$ is the value of g at $t = 0$ (or $\sigma = 1$),

$$F = exp\{-n\int_0^t \gamma[\bar{g}(g,t')]\ dt'\}. \qquad (1.49)$$

Eqn. (1.48) is a powerful result; it relates the value of $\Gamma^{(n)}$, evaluated at a momentum scale σp_i, but with the latter computed using the effective coupling constant $\bar{g}$.

In QCD the situation is somewhat more complicated, novertheless, features similar to those of the ϕ^4 theory emerge. A similar RGE is found, but it has two γ-functions, γ_F and γ_A, for the quarks and gluons in the fundamental (F) and adjoint(A) representations respectively.

1.2.4. Coupling Constant

Using the RGE we can obtain the coupling constaint. For example, for the effective $q\bar{q}g$ vertex coupling $\bar{g}$, we have

$$\bar{g}^2(Q^2) = \bar{g}^2(\mu^2)/[1+2A\ \bar{g}^2(\mu^2)\ \ln(Q^2/\mu^2)], \qquad (1.50)$$

where A is calculable, $Q^2 = -q^2$ with q denoting the gluon momentum.

The evolution of the coupling constaint depends critically on the form of β. At the one-loop level, calculations give the following[15]:

$$\text{QED: } \beta = +(e^3/12\pi^2) + O(e^5) \qquad (1.51)$$

$$\phi^4\text{: } \beta = (3g^2/16\pi^2) + O(g^3), \qquad (1.52)$$

$$\text{QCD: } \beta = -(g^3/16\pi^2)[11-2n_F/3] + O(g^5). \qquad (1.53)$$

For QED, the calculation of β is that of the vacuum polarization term for the photon propagator. In QCD there are additional propagator contributions from gluon (and ghost) loops. Here, n_F is the number of quark flavors.

The main difference between QCD and QED (or a ϕ^4 theory) is that $\beta(\bar{g})$ is negative (provided $n_f < 17$), and so $\bar{g}$ decreases with increasing t, leading to asymptotic ($t\longrightarrow\infty$) freedom. In this limit we obtain (perturbatively) the zeroth order amplitude and, at finite t, there are corrections to this "free particle" amplitude of the form indicated by eq. (1.48). The evolution of $\bar{g}$ with some large scale $Q^2(=\sigma^2\mu^2$, with $g = \bar{g}$ for $\sigma = e^t = 1)$ is given by eq.(1.50). With the aid of eq.(1.53), we find

$$A = \beta_0/32\pi^2, \quad \beta_0 = (33-2n_f)/3. \qquad (1.54)$$

It is customary to quote the QCD coupling constant in terms of $\alpha_s = \bar{g}^2/4\pi$. From (1.50) and (1.54) we obtain the leading order expression

$$\alpha_s(Q^2) = 12\pi/[(33-2n_F)\ln(Q^2/\Lambda^2)], \qquad (1.55)$$

where the scale Λ is defined by $\Lambda = \mu \, exp\, \{-4\pi/[\beta_0\alpha_s(\mu^2)]\}$, whose value is determined from experiment. In addition, Λ is not the same in the different renormalization schemes, and roughly $\Lambda_{MOM}/\Lambda_{\overline{MS}}/\Lambda_{MS} \approx 5.5/2.7/1.0$. Eq.(1.55) is valid for $Q^2 \gg \Lambda^2$.

1.3. Standard Electroweak Theory (GWS Model)

1.3.1. The SU(2)×U(1) invariant Lagrangian

The standard theory of electroweak interactions is based on the gauge group SU(2)×U(1) and is known as the Glashow-Weinberg-Salam (GWS) model. Glashow originally unified the weak and electromagnetic interactions using this gauge group[16], and Weinberg and Salam showed how the weak gauge bosons could acquire their mass without destroying the renormalizability of the theory[17,18].

The group SU(2)×U(1) has four vector fields, three associated with the adjoint representation of SU(2), which we denote W^i_μ with $i = 1,2\ 3$, and one with U(1) denoted by B_μ. The covariant derivative is defined as

$$D_\mu = \partial_\mu + ig\, T_i\, W^i_\mu + ig' \frac{Y}{2} B_\mu, \qquad (1.56)$$

where g, g' and $T_i, Y/2$ are the couplings and generators of the SU(2) and U(1), respectively. The T^i satisfy the SU(2) algebra

$$[T_i, T_j] = i\varepsilon_{ijk} T_k, \tag{1.57}$$

where ε_{ijk} is the antisymmetric permutation tensor. $Y/2$ satisfies

$$Q = T_3 + Y/2, \tag{1.58}$$

where Q is the charge operator, and the factor 1/2 is purely conventional. T_i and Y are referred to as the weak isospin and weak hyper-charge generators, respectively, and their eigenvalues for fermion fields are listed in Table 1.

Table 1 *Weak isospin,hypercharge and electric charge values of left- and right-hand leptons and quarks*

L	T_3	Q	Y	R	T_3	Q	Y
ν_L	1/2	0	-1	ν_R	0	0	0
e_L	-1/2	-1	-1	e_R	0	-1	-2
u_L	1/2	2/3	1/3	u_R	0	2/3	4/3
d'_L	-1/2	-1/3	1/3	d'_R	0	-1/3	-2/3
c_L	1/2	2/3	1/3	c_R	0	2/3	4/3
s'_L	-1/2	-1/3	1/3	s'_R	0	-1/3	-2/3
t_L	1/2	2/3	1/3	t_R	0	2/3	4/3
b'_L	-1/2	-1/3	1/3	b'_R	0	-1/3	-2/3

Parity violation is incorporated by assigning the left- and right-handed components of the fermions to different group representations. All the left-handed fermions are taken to transform as doublets under SU(2), while the right-handed fermions are singlets. For example, the first generation of leptons and quarks belong to the SU(2) multiplets

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}, \quad e_R^-, \begin{pmatrix} u \\ d' \end{pmatrix}, \quad u_R, \quad d'_R, \tag{1.59}$$

and so the SU(2) generator act as follows:

$$T_i\psi_L = \tfrac{1}{2}\tau_i\psi_L, \qquad T_i\psi_R = 0, \tag{1.60}$$

where the τ_i are the 2×2 Pauli matrices,

$$\psi_R = \tfrac{1}{2}(1+\gamma^5)\psi, \qquad \psi_L = \tfrac{1}{2}(1-\gamma^5)\psi, \tag{1.61}$$

The d and s quarks occur in the multiplets in the "rotated" form

$$d' = d\cos\vartheta_c + s\sin\vartheta_c,$$

$$s' = -d\sin\vartheta_c + s\cos\vartheta_c, \tag{1.62}$$

where ϑ_c is the Cabibbo angle (see section 1.3.4). The data give $\sin\vartheta_c \simeq 0.22$. Similar assignments are made for the other generations of fermions like ν_μ, μ^-, c, s'.....

The group structure permits an arbitrary hypercharge assignment for each left-handed doublet and each right-handed singlet, and so we have chosen Y to give the correct electric charges according to eq.(1.58). Evidently, charge quantization must be put in by hand in this SU(2)×U(1) theory. The local phase transformations corresponding to the weak hypercharge (U_1) and weak isospin (U_2) are

$$U_1 = exp[-ig'\tfrac{Y}{2}\vartheta(x)], \quad U_2 = exp[-ig\tfrac{\tau}{2}\cdot\alpha(x)], \tag{1.63}$$

where U_1 and U_2 correspond to the U(1) group and SU(2) group, respectively. The combined SU(2)×U(1) transformation is

$$U = U_2 U_1 = exp(-i[g\frac{\tau}{2}\cdot\alpha(x)+g'\frac{Y}{2}\ I\ \vartheta(x)]). \qquad (1.64)$$

With the inclusion of the gauge boson kinetic energy terms, the SU(2)×U(1) invariant Lagrangian takes the form

$$L_1 = \sum_f[\bar{f}_L\gamma^\mu(iD_\mu)f_L+\bar{f}_R\gamma^\mu(i\partial_\mu-g'\frac{Y}{2}B_\mu)f_R]$$

$$-\frac{1}{4}W^i_{\mu\nu}W^{\mu\nu}_i-\frac{1}{4}B_{\mu\nu}B^{\mu\nu}, \qquad (1.65)$$

where the sum is over all left- and right-handed fermion fields (f_L and f_R,respectively). The corresponding gauge transformations are

$$W'^\mu = W^\mu + g\vec{\alpha}\times W^\mu + \partial^\mu\vec{\alpha}, \qquad (1.66)$$

$$B'^\mu = B^\mu + \partial^\mu\vartheta. \qquad (1.67)$$

The field strength tensors of the SU(2) and U(1) gauge fields are given by

$$W^i_{\mu\nu} = \partial_\mu W^i_\nu - \partial_\nu W^i_\mu - g\ \varepsilon_{ijk}\ W^j_\mu\ W^k_\nu, \qquad (1.68)$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu. \qquad (1.69)$$

1.3.2. The Higgs Mechanism

The Lagrangian (1.65) describes massless fermions. This is not a realistic theory. Therefore, we must modify it so that gauge-bosons and fermions become massive and the photon remains massless without destroying the renormalizability of the theory. To do this, we introduce elementary scalar (Higgs) fields ϕ. We have to add to L_1 the following two Lagrangians for the scalar fields. The first term is

$$L_2 = L_D + L_H, \tag{1.70}$$

where $$L_D = (D_\mu\phi)^\dagger(D_\mu\phi), \tag{1.71}$$

which is produced by gauge-invariantly coupling ϕ to the gauge bosons through the covariant derivative D_μ; and

$$L_H = -V(\phi) \quad \text{with } V(\phi) = -\mu^2(\phi^\dagger\phi)+\lambda(\phi^\dagger\phi)^2, \tag{1.72}$$

Note that L_H contains a self-interaction between the Higgs fields. Here λ must be positive for $V(\phi)$ to be bounded from below. The second term is

$$L_3 = - G_Y[(\bar{\psi}_L\phi)\ \psi_R + \bar{\psi}_R\ (\phi^\dagger)\ \psi_L] \tag{1.73}$$

which couples ϕ to the fermions through so-called "Yukawa" couplings.

To keep L_3 gauge invariant, ϕ must belong to a SU(2)×U(1) multiplet. The most economical choice is to arrange ϕ in an isospin doublet with Y=1 and T=1/2:

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \text{ where } \begin{matrix} \phi^+ = \frac{1}{\sqrt{2}}\ (\phi_1 + i\phi_2) \\ \phi^0 = \frac{1}{\sqrt{2}}\ (\phi_3 + i\phi_4) \end{matrix} \tag{1.74}$$

with ϕ_i real, while the Hermitian conjugate doublet $\phi^\dagger$ describes the antiparticles ϕ^- and $\bar{\phi}^0$. The charge assignments of the components of ϕ follow from eq.(1.73). We require the coefficient of $\phi^\dagger\phi$ to be positive. Indeed, with μ^2 and λ positive the Higgs potential $V(\phi)$ is at its minimum when $\phi^\dagger\phi = \mu^2/2\lambda$.

We choose the minimum that has the vacuum expectation values

$$<0|\phi_i|0> = 0, \quad i = 1,2,4,$$

$$<0|\phi_3|0> = v = \sqrt{\mu^2/\lambda}. \qquad (1.75)$$

The particle quanta of the theory correspond to quantum fluctuations of $\phi_3(x)$ about the value $\phi_3 = v$, rather than to $\phi_3(x)$ itself, that is, to

$$H(x) = \phi_3(x) - v. \qquad (1.76)$$

It is therefore desirable to re-express the Lagrangian in terms of H rather than ϕ_3. We then find that L_D and L_3 contain boson and fermion mass terms of the form

$$(gv)^2 W_\mu W^\mu \quad \text{and} \quad (G_Y v)\bar{\psi}\psi. \qquad (1.77)$$

By choosing the nonvanishing expectation value to be that of the natural field ϕ_3, we ensure that the vacuum is invariant under $U(1)_{em}$ of QED, and that the photon remain massless. Then eq.(1.74) gives

$$<\phi> = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}. \qquad (1.78)$$

Because $\tau_1<\phi> \neq 0$, $Y<\phi> \neq 0$, (1.79)

both SU(2) and $U(1)_Y$ are broken, but

$$Q<\phi> = (\tau_3/2 + Y/2)<\phi> = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ v \end{pmatrix} = 0, \qquad (1.80)$$

and hence the vacuum remains invariant under $U(1)_{em}$ gauge transformations. We therefore expect three massive gauge bosons and one massless gauge boson.

To obtain all the interactions and masses generated by the Higgs mechanism, we need only substitute

$$\phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v+H(x) \end{pmatrix}. \tag{1.81}$$

(as in (1.76)) into the Lagrangian for the Higgs sector, which is the sum of L_2 and L_3. We then find that of the four scalar fields $\phi_i(x)$ of (1.74), the only one that remains is $H(x)$. The other three fields are spurious and we can remove all trace of them from the Lagrangian. To see this, we write $\phi(x)$ in terms of $H(x)$ and three new fields $\vartheta_i(x)$, with $i = 1,2,3$, defined by

$$\phi(x) = \frac{1}{\sqrt{2}} exp[i\tau\cdot\vartheta(x)v] \begin{pmatrix} 0 \\ v+H(x) \end{pmatrix}. \tag{1.82}$$

The ϑ_i and H fully parametrize all possible deviations from the vacuum. Given this form, we can use the gauge freedom to set $\vartheta_i = 0$. This choice is known as the "unitary" gauge. (This choice is equivalent to the condition found in eqs.(1.75) & (1.76)). However, we cannot have just lost three degrees of freedom as a result of spontaneously breaking the symmetry and translating the field variables. What has happened is that in generating masses for the three weak bosons we have increased their polarization degrees of freedom from 2 to 3. They can now have longitudinal polarization, too. The phase ϑ_i of three of the Higgs fileds have been surrendered to make the gauge fields massive. The gauge bosons $W^{\pm}$, Z(see next subsection) become massive by "eating" three of the "would-be-massless" Goldstone bosons, $\phi^{\pm}$ and $i\frac{1}{\sqrt{2}}(\phi^0-\bar{\phi}^0)$, out of the four in the original compex doublet.

1.3.3. The Gauge Boson Masses, Mixing, and Couplings

The masses of the gauge boson can be found by substituting (1.78) into (1.71). The relevent term in L_D is

$$|(ig\ [\tau_i/2]\ W^i_\mu + i\ [g'/2]\ YB_\mu)\langle\phi\rangle|^2$$

$$= \tfrac{1}{8}\, v^2(gW^3_\mu - g'B_\mu)^2 + 0(g'W^3_\mu + gB_\mu)^2 + (\tfrac{1}{2}\, vg)^2 W^+_\mu W^{-\mu}, \qquad (1.83)$$

where $$W^\pm \equiv \frac{1}{\sqrt{2}}\,(W^1 \mp iW^2). \qquad (1.84)$$

The mass matrix of the neutral fields is off-diagonal in the (W^3,B) basis. As expected, one of the mass eigenvalues is zero, and we have displayed this in eq. (1.83) with the orthogonal combination of fields to that in the first term. The normalized neutral mass eigenstates are thus

$$Z_\mu = [gW^3_\mu - g'B_\mu]/[g^2+g'^2]^{1/2}$$

$$= W^3_\mu \cos\vartheta_W - B_\mu \sin\vartheta_W,$$

$$A_\mu = [g'W^3_\mu + gB_\mu]/[g^2+g'^2]^{1/2}$$

$$= W^3_\mu \sin\vartheta_W + B_\mu \cos\vartheta_W, \qquad (1.85)$$

where ϑ_W is the Weinberg or weak mixing angle, defined by

$$\cos\vartheta_W = g/(g^2+g'^2)^{1/2},\ \sin\vartheta_W = g'/(g^2+g'^2)^{1/2}. \qquad (1.86)$$

Thus we see that

$$M_W = \tfrac{1}{2}\, vg,\ M_Z = \tfrac{1}{2}\, v(g^2+g'^2)^{1/2},\ M_\gamma = 0, \qquad (1.87)$$

and so $$M_W/M_Z = \cos\vartheta_W. \qquad (1.88)$$

The inequality $M_W \neq M_Z$ is due to the mixing between the W^3_μ and B_μ fields.

We can rewrite the fermion-gauge boson electroweak interaction terms in L_1 in terms of the physical fields $W^{\pm}$, Z, and A in the form

$$L_{ew} = -\frac{1}{\sqrt{2}}g(J^{\mu}_{cc}W^{+}_{\mu} + J^{\mu\dagger}_{c}W^{-}_{\mu}) - [gg'/(g^2+g'^2)^{1/2}]J^{\mu}_{em}A_{\mu}$$

$$-(g^2+g'^2)^{1/2}J^{\mu}_{nc}Z_{\mu}. \qquad (1.89)$$

Defining the SU(2) and U(1) currents J_i and J_Y by

$$J^{\mu}_{i} \equiv \bar{\psi}_L\gamma^{\mu}\ \tfrac{1}{2}\tau_i\psi_L, \quad J^{\mu}_{Y} = \bar{\psi}\ \gamma^{\mu}Y\ \psi, \qquad (1.90)$$

respectively, we can identify the physical currents as linear combinations of J_i and J_Y, Thus

$$J^{\mu}_{cc} = \tfrac{1}{2}(J^{\mu}_{1}+iJ^{\mu}_{2}) \qquad (1.91)$$

is seen to be the weak charge-current which couples to the W^{+} boson. The coupling g is therefore related to the Fermi coupling G_F by

$$g^2 = 4\sqrt{2}\ M^2_W\ G_F. \qquad (1.92)$$

Hence, we can determine the vacuum expectation value of the Higgs field,

$$v = 2\ M_W/g = (\sqrt{2}G_F)^{-1/2} = 246\ \text{Gev}, \qquad (1.93)$$

using $G_F = 1.16637(2)\times10^{-5}\ \text{Gev}^{-2}$. (1.94)

The current in the second term of eq.(1.89) is

$$J^{\mu}_{em} = J^{\mu}_{3} + \tfrac{1}{2}J^{\mu}_{Y} = \bar{\psi}\ \gamma^{\mu}\ Q\ \psi, \qquad (1.95)$$

and so by construction is just the usual electromagnetic current. Hence, the electromagnetic charge is

$$e = gg'/(g^2+g'^2)^{1/2} = g\sin\vartheta_W = g'\cos\vartheta_W. \qquad (1.96)$$

Finally, we identify the weak neutral-current coupling to the Z boson in L_{ew} as

$$J^\mu_{nc} = J^\mu_3 - \sin^2\vartheta_W\, J^\mu_{em} = \bar{\psi}\, \gamma^\mu\, [\tfrac{1}{2}(1-\gamma^5)T_3 - \sin^2\vartheta_W\, Q]\psi$$

$$\equiv \bar{\psi}\, \gamma^\mu\, \tfrac{1}{2}(C_V - C_A\gamma^5)\psi, \qquad (1.97)$$

where $C^f_V \equiv T^f_3 - 2Q_f\sin^2\vartheta_W$, $C^f_A \equiv T^f_3$, (1.98)

the values of which are listed for the various fermions in Table 2. Note that, unlike J_{cc}, J_{nc} couples to both right- and left-handed fermions.

Table 2. Vector and axial-vector couplings of leptons and quarks to the weak neutral current

f	Q	C^f_A	C^f_V
ν	0	1/2	1/2
e	-1	-1/2	$-1/2+2\sin^2\vartheta_W \simeq -0.04$
q(u-type)	2/3	1/2	$1/2-(4/3)\sin^2\vartheta_W \simeq 0.19$
q(d-type)	-1/3	-1/2	$-1/2+(2/3)\sin^2\vartheta_W \simeq -0.35$

(using $\sin\vartheta_W \simeq 0.23$)

It is customary to introduce the parameter

$$\rho \equiv M_W^2/(M_Z^2 \cos^2\vartheta_W), \qquad (1.99)$$

which specifies the relative strength of the neutral- and charge-current weak interactions. The GWS model with a single Higgs doublet has $\rho=1$, which is in excellent agreement with experiment.

In the minimal model, we have

$$M_W = \frac{1}{2} vg = \frac{1}{2} ve(\sin\vartheta_W)^{-1} \simeq 80 \text{ Gev},$$

$$M_Z = M_W/\cos\vartheta_W \simeq 90 \text{ Gev}, \tag{1.100}$$

using $\sin\vartheta_W \simeq 0.23$. These predictions are in excellent agreement with the masses of the $W^{\pm}$ and Z bosons that were subsequently discovered.

As for the Higgs particle, we have

$$M_H^2 = 2\lambda v^2 = 2\mu^2. \tag{1.101}$$

But the mass of the Higgs is not predicted, since neither μ^2 nor λ is determined, only their ratio v^2. On the other hand, the Higgs couplings to other bosons is completely determined. For instance, we can find

$$g(H\, W^+\, W^-) = gM_W \quad \text{and} \quad g(H\, H\, W^+\, W^-) = g^2/4, \tag{1.102}$$

when we substitute (1.81) into L_D.

1.3.4. Fermion Masses, Mixing, and Couplings

The Higgs-fermion couplings give masses to the fermi ns. For example, substituting (1.81) for the electron doublet in L_3, we obtain

$$L_Y^e = -\frac{1}{\sqrt{2}} G_e (v+H)(\bar{e}_L e_R + \bar{e}_R e_L)$$

$$\equiv -m_e(\bar{e}e) - \frac{1}{v} m_e(\bar{e}eH), \tag{1.103}$$

revealing that the electron's mass and coupling are

$$m_e = \frac{1}{\sqrt{2}} G_e v, \text{ and } g(\bar{e}eH) = \frac{1}{v} m_e = gm_e / 2M_W. \tag{1.104}$$

Since G_e is arbitrary, the m_e not predicted, but its Higgs coupling is specified and, being proportional to m_e / M_W, is very small.

The quark masses (and couplings) are generated in analogous manner. However, eq.(1.81) gives a mass only to the lower member of the fermion doublet, and to generate a mass for the upper member we must construct from ϕ a new Higgs doublet with a neutral upper member, viz.,

$$\phi_c = i\tau_3\phi^* = \begin{pmatrix} \bar{\phi}^0 \\ -\phi^- \end{pmatrix} \xrightarrow[\text{breaking}]{} \frac{1}{\sqrt{2}}\begin{pmatrix} v+H(X) \\ 0 \end{pmatrix}. \qquad (1.105)$$

Owing to the special properties of SU(2), ϕ_c transforms identically to ϕ, but has opposite weak hypercharge, $Y(\phi_c) = 1$. The most general SU(2)×U(1) invariant Yukawa terms for the (u,d) quark doublet are then

$$L_Y^u = - G_d(\bar{u},\bar{d})_L \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} d_R - G_u(\bar{u},\bar{d})_L \begin{pmatrix} \phi^{-0} \\ -\phi^- \end{pmatrix} u_R + h.c.,$$

which, on substitution of (1.81) and (1.105), reveals that the mass and $q\bar{q}H$ coupling terms are

$$L_Y^u = - (m_d\bar{d}d + m_u\bar{u}u)(1 + \frac{H}{v}), \qquad (1.106)$$

where $m_q = \frac{1}{\sqrt{2}}G_q v$, and $h.c.$ denotes hermitian conjugate.

We must distinguish carefully between the gauge interaction states, which we have denoted by d′, s′, etc., and the mass eigenstates of a given flavor d, s, etc. (for which we will here use the notation $d=d_1$, $s=d_2$, and so on), because there is mixing between quark fields of the same charge. Suppose there are N quark doublets (u_i,d_i) with

$i=1,2,\ldots,N$ corresponding to the different generations of quarks. The Yukawa terms can then take the form

$$L_Y = -(\bar{d}'_{iL} M^d_{ij}\ d'_{jR} + \bar{u}'_{iL} M^u_{ij}\ u'_{jR})(1 + \frac{H}{v}) + h.c., \qquad (1.107)$$

where in the gauge eigenstate basis, q', the quark mass matrices M^d and M^u need not be diagonal, or indeed symmetric or hermitian. However,each matrix M can be made real and diagonal by making suitable unitary transformations, U_L and U_R, of the left- and right-handed components of the appropriate quark fields, respectively. Thus for the u type quarks we take

$$u'_L = U^u_L\ u_L, \qquad u'_R = U^u_R\ u_R, \qquad (1.108)$$

and similarly for the d type quarks, such that

$$U^\dagger_L\ M\ U_R = m, \qquad (1.109)$$

where m is a diagonal matrix that has positive mass eigenvalues. For M^u these eigenvalues are $m_u, m_c, \ldots,$and similarly on diagonalizing M^d we get m_d, m_s, ...

It is apparent from (1.107) that the $Hq\bar{q}$ couplings are diagonalized simultaneously with the quark masses, and that they are

$$g(Hq\bar{q}) = m_q / v = gm_q / 2M_W. \qquad (1.110)$$

Moreover, it is evident that the neutral currents, which were flavor-diagonal in the interaction basis (q_i), remain flavor-diagonal in the mass basis (q_i), since for, example,

$$\bar{u}'_L \gamma^\mu u'_L = \bar{u}_L\ U^u_L{}^\dagger \gamma^\mu U^u_L u_L = \bar{u}_L \gamma^\mu u_L. \qquad (1.111)$$

So, in the minimal model (with a single Higgs doublet and with the quarks arranged in doublets) there are no flavor-changing currents like d⟶s.

However, there exists mixing in the charged-current in the mass basis:

$$J^{u}_{cc} = \bar{u}'_L \gamma^{\mu} d'_L = \bar{u}_L \gamma^{\mu} (U^{u\dagger}_L U^{d}_L) d_L$$

$$= (\bar{u},\bar{c},\ldots)_L \gamma^{\mu} V (d,s,\ldots)^{T}_L, \qquad (1.112)$$

where T denotes transpose, $V \equiv U^{u\dagger}_L U^{d}_L$ is a unitary N×N matrix that can be determined by observing flavor-changing weak processes.

A unitary matrix V has N^2 real, independent elements, but of these (2N-1) phases can be absorbed into the definition of the 2N quark fields q_L of different flavor and so are unmeasurable. The matrix V therefore contains

$$N^2-(2N-1) = (N-1)^2 \qquad (1.113)$$

observables. Now an orthogonal N×N matrix has only N(N-1)/2 real parameters and V must also contain

$$(N-1)^2-N(N-1)/2 = (N-1)(N-2)/2 \qquad (1.114)$$

phase factors. So, it is not possible, for the case of N > 2 to make V real by redefining the quark phases. The $(N-1)^2$ parameters of V have to be determined empirically. For N=2 we have only one parameter, the Cabibbo angle[19]. Hence

$$\mathcal{L}_{cc} = -\frac{g}{\sqrt{2}} W^{+}_{\mu} (\bar{u},\bar{c},)_L \gamma^{\mu} \begin{pmatrix} \cos\vartheta_c & \sin\vartheta_c \\ -\sin\vartheta_c & \cos\vartheta_c \end{pmatrix} . \qquad (1.115)$$

Since the coupling constants are all real, the lagrangian is CP conserving. For N=3

$$L_{cc} = -\frac{g}{\sqrt{2}} W^{+}_{\mu} (\bar{u},\bar{c},\bar{t})_L \gamma^{\mu} \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} + h.c$$

(1.116)

The matrix V has four parameters; three rotation angles ϑ_i (i=1,2,3), and one phase δ. This matrix can be written in the form (KM)[20]

$$V = \begin{pmatrix} c_1 & s_1 c_3 & s_1 s_3 \\ -s_1 c_2 & c_1 c_2 c_3 - s_2 s_3 exp(i\delta) & c_1 c_2 s_3 + s_2 c_3 exp(i\delta) \\ s_1 s_2 & -c_1 s_2 c_3 - c_2 s_3 exp(i\delta) & -c_1 s_2 s_3 + c_2 c_3 exp(i\delta) \end{pmatrix},$$

(1.117)

where $c_i = \cos\vartheta_i$ and $s_i = \sin\vartheta_i$. CP invariance implies $V=V^*$, thus δ is the phase that measures CP violation.Hence CP violation can be described by the standard model with three generations. Note that there is no CP violation in either the electromagnetic or weak neutral currents.

The KM matrix (1.117) can be obtained by the following product of three rotational and one phase matrices[21];

$$V= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_2 & s_2 \\ 0 & -s_2 & c_2 \end{pmatrix} \begin{pmatrix} c_1 & s_1 & 0 \\ -s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\delta} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_3 & s_3 \\ 0 & -s_3 & c_3 \end{pmatrix}$$

(1.118)

Other parameterizations of V exist[22-24].

The Lagrangian of the Higgs coupling to a lepton pair for three generations of leptons can be formulated in a similar way to that for quarks allowing, in general, for transitions between generations. The resulting consequences include the possibility of neutrino oscillations. We assume here that the weak and physical bases of the leptons are the same (this freedom exists provided the neutrino are massless), so that

$$L_{LH} = -(1 + \frac{H}{V})(\Sigma_l \; m_l \; \bar{l} \; l), \qquad (1.119)$$

where $l = e, \mu, \tau, \nu_e, \nu_\mu, \nu_\tau$. Here we assumed the existence of a right-handed ν_R, unlike the GWS model. Note that each mass term in (1.119) is arbitrary, and thus represents an additional parameter in the standard model.

The standard model contains a considerable number of parameters: g, g', m_H, λ, 2N quark masses and $(N-1)^2$ mixing angles and phases for N quark generations and a similar number for N lepton generations if the neutrino masses are taken to be non-zero. Thus we have, in total, $2(N^2+1)+4 = 2N^2+6$ parameters. For N=3, this means that we have a total of 24 free parameters (or 17 if we assume massless neutrinos). Of these only two (g and g') are not associated with the Higgs field. Hence the introduction of a fundamental scalar solves the mass generation problem only at the expense of introducing many arbitrary parameters.

1.3.5. The Final Lagrangian

The complete Lagrangian is

$$L = -\frac{1}{4} W_{\mu\nu}W^{\mu\nu} - \frac{1}{4} B_{\mu\nu}B^{\mu\nu} + \bar{L}\gamma^{\mu} iD_{\mu}L + \bar{R}\gamma^{\mu}(i\partial_{\mu} - g'\frac{Y}{2} B_{\mu})R$$

$$+ |iD_{\mu}\phi|^2 - V(\phi) - (G_1\bar{L}\phi R + G_2\bar{L}\phi_c R + h.c.). \qquad (1.120)$$

The standard GWS model is renormalizable, so that meaningful perturbative calculations can be carried out. The important proof that the theory is renormalizable (using the general R_{ξ} rather than the unitary gauge) was given by 't Hooft[25]. More details on this topic can be found in, for example Talor[26].

Chapter 2.

Left-Right Hand Symmetric Model and CP-Violation

2.1. The Chief Limitations of The Standard Model

Although the standard model of the particle theory is a great success (indeed there no confirmed experimental results that contradict it), it contains undoubtedly some theoretical deficencies. Its chief limitations are listed briefly as follows:

1). It does not include gravity.

2). The gauge group $SU(3)_c \times SU(2) \times U(1)_y$ is a product of three disconnected sets of gauge transformations, and hence has three independent coupling constants, α_s, g and g′. It offers a glimpse of unification in the breakdown of $SU(2) \times U(1)_Y \longrightarrow U(1)_Q$, but this does not take us very far, or in another words, it is not really unified theory.

3). It is strange that one of these factor groups, SU(2), distinguishes between left and right handed states. Thus both the P- and CP-violation are put in by hand, and hence no deep insight is gained into the origin of CP- violation.

4). The Higgs mechanism, which is crucial to the success of the standard model, requires an inelegant and arbitrary addition to the Lagrangian.

5). The theory offers no explanation for family replication. The old question of "who ordered the muon", has changed into why are there three (or more) families, but it still remains unanswered.

6). Related to the above (5) is the origin of the parameters in the mass matrix. The number of free parameters in the standard model totals 19 (including Λ_{QCD} and ϑ, for more details on the latter, see, for example, [15, 27]) for three families of fermions. There will be even more parameters if the neutrinos have finite masses and mixing. So many parameters make a theorist uncomfortable. Although the question of how many free parameters one should "expect" in the theory of everything belongs more to philosophy than to physics, the object of physics is to explain as much as possible with as little arbitrary input as possible. Hence, any model that might explain or relate some of the above parameters is worth considering.

It is reasonable to conclude therefore that the standard model can not be the final theory and we must look beyond it for answers to these questions. Several avenues have been explored in this search for physics beyond the standard model: Technicolor, Supersymmetry, Compositeness, Grand-unification and Left-Right Symmetry. Each approach addresses a particular aspect of this search and is not necessarily in conflict with the others. In what follows our attention will be concentrated on the left-right symmetric theories[28-32], which has been the focus of a great deal of theoretical as well as experimental activity in recent years.

2.2. Why Left-Right Symmetry?

The original motivation for the introduction of left-right symmetric models[30,31] based on an $SU(2)_L \times SU(2)_R \times U(1)$ gauge group was to provide an understanding of the origin of parity violation in weak interactions. According to this point of view, the weak interaction Lagrangian prior to breaking of gauge symmetries respects all spacetime symmetries, as do the other forces of nature - the strong, electromagnetic and gravitational interactions. The observed parity violation at low energies is then attributed to non-invariance of the vacuum[30,31] under parity. The most interesting feature of this scenario is that it reproduces all the features of $SU(2)_L \times U(1)$ models at low energies and as we move up in energy, new effects associated with the parity invariance of the original Lagrangian are supposed to appear.

There exist several other considerations having to do with the weak interaction that find their place naturally in a left- right symmetrical model rather than the stadard model. Foremost among them is the neutrino mass. We do not know whether the neutrino has a mass. Laboratory experiments involving tritium decay, which indicate end point behavior of the decay spectrum characteristic of a non-vanishing neutrino mass are controversial. There exist astrophysical considerations[33,34] having to do with missing mass of the universe, galactic clusters and galaxy formation etc., which are easily understood if the neutrino has a non-vanishing mass in the electron volt range. The most natural framework to understand a non-zero neutrino mass is the left-right symmetric models.

Secondly, if weak interaction symmetries are to arise out of a more fundamental substructure of quarks and leptons, and if the forces at the substructure level are assumed to be similar to those operating in nuclear physics, i.e. QCD, then, convincing arguments exist[35], which imply that $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ rather than $SU(2)_L \times U(1)$ is the weak interaction symmetry group.

Another deficiency of the standard model is the lack of any physical meaning of the $U(1)_Y$ generator, which in the left-right symmetric models becomes the (B - L) quantum number[36]. All the weak interaction symmetry generators then have a physical meaning. As if suggesting a deeper symmetry structure in the $SU(2)_L \times U(1)_Y$ model, the only anomaly free quantum number left un-gauged by the Standard Model is B - L and once B - L is included as a gauge generator, the weak gauge group becomes $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ and the electric charge is given by[36]

$$Q = I_{3L} + I_{3R} + \frac{B - L}{2} \tag{2.1}$$

where B and L are the baryon and lepton numbers respectively. Finally, we have to consider the status of CP-violation in gauge theories. It is interesting to note that in the standard model, three generations are required in order to have nontrivial CP-violation and CP-violation is parameterized by only one phase, δ_{KM}, the Kobayashi-Maskawa phase. But the model provides no hint as to why the observed CP-violation has milli-weak strength. The left-right models provide a more appealing alternative[30], where the smallness of CP-violation is related to the suppression of V+A currents, i.e.,

$$\eta_{+-} \simeq [M_L^{(W)} / M_R^{(W)}]^2 \sin\delta. \qquad (2.2)$$

If both parity and CP-violations owe their origin to spontaneous breakdown of gauge symmetries, eq.(2.2) can then be proved[37,38], for three generations and becomes valid regardless of the contribution of the Higgs sector.

2.3. The Standard Left-Right Symmetric Model (L-R Model)

Spontaneous P and CP violation together with the minimal number of two scalar doublets lead directly to the standard L-R model[39,40] with gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$.

A brief summary of CP-violation in this model is given below. For more details, see, for example, references [28-31, 41, 42].

The Standard L-R model is based on the gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ with quarks and leptons being assigned to irreducible representations of the gauge group as follows:

The quantum number assignments for Fermions in $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ are as follows:

$$Q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L \qquad (\tfrac{1}{2}, 0, \tfrac{1}{3}).$$

$$Q_R = \begin{pmatrix} u \\ d \end{pmatrix}_R \qquad (0, \tfrac{1}{2}, \tfrac{1}{3}).$$

$$\psi_L = \begin{pmatrix} \nu \\ e^- \end{pmatrix}_L \qquad (\tfrac{1}{2}, 0, -1). \tag{2.3}$$

$$\psi_R = \begin{pmatrix} \nu \\ e^- \end{pmatrix}_R \qquad (0, \tfrac{1}{2}, -1).$$

where the family index on the Q and ψ has been omitted.

The gauge invariant Lagrangian for Q and ψ has the form

$$\begin{aligned} L = {} & \frac{i}{2} g_L \{\bar{Q}_L \gamma_\mu \vec{\tau} Q_L + \bar{\psi}_L \gamma_\mu \vec{\tau} \psi_L\} \vec{W}_L , \\ & + \frac{i}{2} g_R \{\bar{Q}_R \gamma_\mu \vec{\tau} Q_R + \bar{\psi}_R \gamma_\mu \vec{\tau} \psi_R\} \vec{W}_R , \\ & + (\frac{i}{6} g' \bar{Q} \gamma_\mu Q - \frac{i}{2} g' \bar{\psi} \gamma_\mu \psi) B_\mu , \end{aligned} \tag{2.4}$$

where $\vec{W}_L$, $\vec{W}_R$ and B are the gauge bosons corresponding to the groups $SU(2)_L$, $SU(2)_R$ and $U(1)_{B-L}$ respectively; $\vec{\tau}$ are the Pauli matrices.

We now require that the theory be invariant under parity operation, P under which fields transform as follows:

$$\psi_L \leftrightarrow \psi_R \quad \text{and} \quad Q_L \leftrightarrow Q_R \ , \ \vec{W}_L \leftrightarrow \vec{W}_R. \tag{2.5}$$

This requires that $g_L = g_R$ reducing the number of arbitrary gauge coupling constants to two as in the standard model. The electric charge formula (eqn., 2.1) then implies that,

$$\frac{1}{e^2} = \frac{2}{g^2} + \frac{1}{g'^2} \tag{2.6}$$

where $g \equiv g_L = g_R$. Therefore, as in the standard model, we can parameterize g and g′ in terms of two parameters: the electric charge of the electron, e and the Weinberg angle, $\sin\vartheta_W$, with

$$\sin^2\vartheta_W = e^2 / g^2. \tag{2.7}$$

From eqns.(2.6) and (2.7) it follows that

$$g'^2 = e^2 / \cos 2\vartheta_W. \tag{2.8}$$

In order to break the gauge symmetry down to $U(1)_{em}$ and to maintain left-right symmetry, we must choose Higgs multiplets, which are left-right symmetric. There are various ways to achieve this goal. In the early days of the development of the left-right symmetry, the breaking of the gauge symmetry[28-32,41], was implemented by choosing the Higgs multiplets:

$$\chi_L(\tfrac{1}{2},0,1) \oplus \chi_R(0,\tfrac{1}{2},1) \text{ and } \phi \equiv (\tfrac{1}{2},\tfrac{1}{2},0) \tag{2.9}$$

In 1980, it was shown[43,44] that, in order to understand the smallness of the neutrino mass, it is preferable to introduce the following set of Higgs multiplets:

$$\Delta = (1,0,2);\ \Delta = (0,1,2) \text{ and } \phi\ (\tfrac{1}{2},\tfrac{1}{2},0). \tag{2.10}$$

In what follows, we will work with the second set of Higgs multiplets. The conclusions concerning CP-violation will be independent of this choice.

The gauge symmetry breaking proceeds in two stages. In the first stage, the electrically neutral component of the Δ_R-multiplet denoted by Δ_R^0 acquires a vev:

$$\langle\Delta_R^0\rangle = v_R \tag{2.11}$$

and breaks the gauge symmetry down to $SU(2)_L \times U(1)_Y$ where $Y/2 = I_{3R} + (B-L)/2$. The parity symmetry breaks down at this stage. The second stage of the breaking is caused by the electrically neutral components in ϕ. Writing ϕ explicitly as

$$\phi = \begin{pmatrix} \phi_1^0 & \phi_1^+ \\ \phi_2^- & \phi_2^0 \end{pmatrix}, \tag{2.12}$$

we choose the vacuum expectation values of ϕ as:

$$\langle\phi\rangle = \begin{pmatrix} \kappa & 0 \\ 0 & \kappa' \end{pmatrix} \tag{2.13}$$

with κ, $\kappa' \ll v_R$. This choice of $\langle\phi\rangle$ breaks the gauge symmetry down to $U(1)_{em}$. Here we have chosen κ and κ' to be real for simplicity.

At the first stage, the charged right handed gauge bosons $W_R^\pm$ and a neutral gauge boson Z_2 acquire masses proportional to v_R and become much heavier than the usual left-handed $W_L^\pm$ and Z_1-bosons which pick up masses proportional to κ and κ' only at the second stage. In general there is mixing between the different gauge bosons. W_{3L}, W_{3R}, and B mixing is described by a 3×3 mass matrix. The 2×2 charged gauge boson matrix is given by

$$\begin{array}{c c} & \begin{array}{cc} W_L^+ & W_R^+ \end{array} \\ \begin{array}{c} W_L^- \\ W_R^- \end{array} & \begin{pmatrix} \frac{g^2}{2}(\kappa^2+\kappa'^2+2v_L^2) & g^2\kappa\kappa' \\ g^2\kappa\kappa' & \frac{g^2}{2}(\kappa^2+\kappa'^2+2v_R^2) \end{pmatrix}, \end{array} \qquad (2.14)$$

where v_L denotes the vev of Δ_L^0 which is much smaller than κ,κ'. The eigenstates of this matrix are

$$W_1 = W_L\cos\zeta + W_R\sin\zeta,$$

$$W_2 = -W_L\sin\zeta + W_R\cos\zeta, \qquad (2.15)$$

where $\tan\zeta \approx \kappa\kappa' / v_R^2$. (2.16)

We will assume in what follows that $\kappa' \ll \kappa$, so that the mixing parameter ζ is very small (i.e. $W_1 \simeq W_L$ and $W_2 \simeq W_R$ to a good approximation) and the masses of $W_{1,2}$ are given by

$$m^2(W_1) \simeq \frac{g^2}{2}(\kappa^2+\kappa'^2) \; ; \quad m^2(W_2) \simeq \frac{g^2}{2}(\kappa^2+\kappa'^2+2v_R^2) \; . \qquad (2.17)$$

The charged current weak interactions can be written as (suppressing the generation indices)

$$L^{cc} = \frac{g}{\sqrt{2}}(\bar{u}_L\gamma_\mu d_L + \bar{\nu}_L\gamma_\mu e_L)W_L^+ + \frac{g}{\sqrt{2}}(\bar{u}_R\gamma_\mu d_R + \bar{\nu}_R\gamma_\mu e_R)W_R^+$$

$$+ g^2\kappa\kappa' W_L^- W_R^+ + h.c. \qquad (2.18)$$

It is clear that for $m(W_R^+) \gg m(W_L^+)$, the charged current weak interactions will appear to be almost maximally parity violating at low energies. Any deviation from the pure left-handed (or V-A) strcture of charged weak current will constitute evidence for the right-handed currents and therefore a left-right symmetric structure of weak interactions.

The mass matrix for the neutral weak bosons W_{3L}, W_{3R} and B is given by

$$
\begin{array}{c} \\ W_{3L} \\ W_{3R} \\ B \end{array}
\begin{array}{c} \begin{array}{ccc} W_{3L} & W_{3R} & B \end{array} \\
\begin{bmatrix}
\frac{g^2}{2}(k^2+k'^2+4v_L^2) & -\frac{g^2}{2}(k^2+k'^2) & -2gg'v_L^2 \\
-\frac{g^2}{2}(k^2+k'^2) & \frac{g^2}{2}(k^2+k'^2+4v_R^2) & -2gg'v_R^2 \\
-2gg'v_L^2 & & 3g'^2(v_L^2+v_R^2)
\end{bmatrix}
\end{array}
\tag{2.19}
$$

In the approximation of $v_L \ll \kappa, \kappa' \ll v_R$, the eigenstates of this matrix are given by:

$$
\begin{aligned}
A &= \sin\vartheta_W(W_{3L} + W_{3R}) + \sqrt{\cos 2\vartheta_W}\, B, \\
Z_1 &= \cos\vartheta_W\, W_{3L} - \sin\vartheta_W \tan\vartheta_W\, W_{3R} - \tan\vartheta_W \sqrt{\cos 2\vartheta_W}\, B, \\
Z_2 &= [\sqrt{\cos 2\vartheta_W}\,/\cos\vartheta_W]\, W_{3R} - \tan\vartheta_W\, B,
\end{aligned}
\tag{2.20}
$$

with masses:

$$
\begin{aligned}
&m_A = 0,\ m^2(Z_1) \simeq (g^2 / 2\cos\vartheta_W)(\kappa^2 + \kappa'^2), \text{ and} \\
&m^2(Z_2) \simeq 2(g^2 + g'^2)v_R^2.
\end{aligned}
\tag{2.21}
$$

The field A_μ corresponds to the photon. Note that $m(W_L)$ and $m(Z_1)$ satisfy the mass relation $m(W_L) \simeq m(Z_1)\cos\vartheta_W$ of the standard model. The interactions of W_L and Z_1 also reduce to those of the standard model in the limit of $m(W_R)$ and $M(Z_2) \longrightarrow \infty$. The interaction of Z_1 and Z_2 is given as follows:

$$L_{WK} = \frac{g}{\cos\vartheta_W} Z_{1\mu}[J_{1\mu} - c_W \eta(\sin^2\vartheta_W \, J_{L\mu} + \cos^2\vartheta_W \, J_{R\mu})]$$

$$+ \frac{g}{\cos\vartheta_W \sqrt{\cos 2\vartheta_W}} Z_{2\mu}(\sin^2\vartheta_W \, J_{L\mu} + \cos^2\vartheta_W \, J_{R\mu}),$$

(2.22)

where $J_{L,R\ \mu} = \sum_i \bar{f}_i \gamma_\mu [I_{3L,R} - Q\sin^2\vartheta_W] f_i$. (2.23)

I_3 denotes weak isospin, $\eta = [m(W_L)/m(W_R)]^2$ and c_W is a parameter depending on the Weinberg angle and is of order 1/4. i is the generation index.

We are now ready to discuss the bounds on the masses of the W_R and Z_2 bosons as well as the left-right mixing parameter ζ. The most model independent limit is on the mass of the Z_2-boson. This is obtained by analyzing neutrino neutral current data, where one searches for deviations from the predictions of GWS model. The present experimental accuracy in the neutral current data implies that[45]

$M(Z_2) \geq 275$ Gev. (2.24)

To obtain limits on $m(W_R)$ and ζ, an obvious thing to do is to look for deviations from the predictions of V-A theory for muon decay. The most stringent limits at this moment come from the measurement of the ξ-parameter in μ-decay at TRIUMF[46] using 100 % stopped polarized muons and the limits are:

$$m(W_R) \geq 400 \text{ Gev for arbitrary } \zeta$$

$$\zeta \leq 0.041 \text{ for } m(W_W) \longrightarrow \infty. \qquad (2.25)$$

In general these two bounds are correlated and one gets elliptical regions in the $m(W_R)$-ζ plane which give the allowed and forbidden values for the above parameters.

For values of the right-handed Majorana neutrino mass close to but larger than the mass of the muon, the above analysis does not shed light on the strength of the right-handed current interactions and one must look at weak processes involving only hadrons. There are two methods of approach. In one procedure $K \longrightarrow 3\pi$ decays are analyzed[47] to search for deviations from current algebra results for $K \longrightarrow 3\pi$ decay parameters such as the slope in the Dalitz plot in the presence of right-handed currents. It is found that, making plausible assumptions about hadronic matrix elements, one can obtain[47]

$$m(W_R) \geq 325 \text{ Gev and } \zeta \leq 4\times10^{-3}. \qquad (2.26)$$

In obtaining this bound as well as the bounds from the μ-decay, it is assumed that the quark (and lepton) mixing angles in the left- and right-handed charged currents are equal. It must be also be emphasized that the bounds in eq. (2.26) are more model dependent than the ones in eq.(2.21)

In the second approach, Beall and others[48] observed that a much more improved bound on $m(W_R)$ can be obtained by analyzing the contributions of the right-handed currents to

K_1-K_2 mass difference. For the case of equal left- and right-handed mixing angles, the following result is obtained:

$$[m(W_L) \;/\; m(W_R)]^2 < \frac{1}{430} \text{ or } m(W_R) \geq 1.6 \text{ Tev.} \qquad (2.27)$$

2.4. Fermion Mass Matrices and CP-Violation

In gauge theories, the primordial gauge interactions are always CP- conserving. From discussions of the standard model, we know that CP-violation can arise either from intrinsically complex coupling parameters involving Higgs fields (such as Yukawa or ϕ^4-type couplings) or from complex vacuum expectation values of the Higgs fields in the theory. The former case is generally labled as hard CP-violation. The term Soft CP-violation is not only used for the latter case but also used to describe situations where a dimensional coupling is complex in a gauge theory. Accordingly, CP-violation in L-R models can also be devided into two categories: 1.Hard CP-violation and 2.Soft CP-violation.

The first step in discussion of CP-violation is to obtain the mass matrices for quarks and leptons: M_u,M_d, for up and down quarks respectively and M_e,M_ν for charged lepton and neutrino sectors respectively. The mass matrices are a combination of Yukawa couplings and nontrivial phases in vev's of Higgs field. The M_u and M_d are then diagonalized by choosing a new basis for quarks:

$$U_L M_u V_R^{\dagger} = D_u \text{ and } V_L M_d V_R^{\dagger} = D_d. \qquad (2.28)$$

substituting these in the charged current weak interactions given in eqn.(2.18), we obtain

$$L^{cc} = \frac{g}{\sqrt{2}}(\bar{p}_L \gamma_\mu U N_L + \bar{\nu}_L \gamma_\mu K_L E_L) W_L^+ + \frac{g}{\sqrt{2}}(\bar{p}_R \gamma_\mu V N_R + \bar{\nu}_R \gamma_\mu K_R E_R) W_R^+$$

$$+ h.c. \qquad (2.29)$$

where

$$P = (u,c,t,\ldots),\ N = (d,s,b,\ldots),\ \nu = (\nu_e,\nu_\mu,\nu_\tau,\ldots),$$

$$E = (e,\mu,\tau,\ldots),\ U = U_L V_L^{\dagger},\ V = U_L V_L^{\dagger}. \qquad (2.30)$$

The forms of $K_{L,R}$ are more complicated and depend on whether the neutrinos are Dirac or Majorana particles. Note that if $M_{u,d}$ are complex then U and V are unitary matrices with complex phases in them. This leads to CP-violation in weak interactions. The question of under what conditions these complex phases can arise and when they are genuine depends on the properties of the Yukawa couplings under parity and CP operations

2.4.1. Constraits of Parity Invariance on Yukawa Couplings

The Yukawa couplings, h_{ab} and $\tilde{h}_{ab}$, for quarks can be written for the case of a single Higgs field of type $\phi(2,2,0)$ as follows:

$$L_Y = \sum_{a,b} [h_{ab}\bar{Q}_{La}\phi\, Q_{Rb} + \tilde{h}_{ab}\bar{Q}_{La}\tilde{\phi}\, Q_{Rb}] + h.c. \tag{2.31}$$

where $\tilde{\phi} = \tau_2\, \phi^*\, \tau_2$ (2.32)

and a and b label the generation indices. To study constraints of parity invariance[49], let us define the parity transformation of quarks and Higgs fields as follows:

$$P:\ Q_L \leftrightarrow Q_R \text{ and } \phi \longrightarrow A\phi^\dagger + B\tilde{\phi}^\dagger. \tag{2.33}$$

Substituting eq.(2.33) in eq.(2.32), we find that in order for the theory to be parity invariant, we must have

$$hA + \tilde{h}B^* = h^\dagger \text{ and } hB + \tilde{h}A^* = \tilde{h}^\dagger. \tag{2.34}$$

Applying these transformations to the kinetic energy term for ϕ , we find that its parity invariance implies

$$|A|^2 + |B|^2 = 1 \text{ and } AB^* = 0. \tag{2.35}$$

In other words, either (a) $A = e^{i\alpha}$, $B = 0$, or (b) $A = 0$, $B = e^{i\alpha}$. In case (a), we have,

$$h = e^{-i\alpha}h^\dagger \text{ and } \tilde{h} = e^{i\alpha}\tilde{h}^\dagger \tag{2.36}$$

and in case (b), we find[32],

$$h = e^{-i\alpha}\tilde{h}^\dagger \text{ and } \tilde{h} = e^{i\alpha}h^\dagger \tag{2.37}$$

To see what this implies for up and down quark mass matrices we assume that ϕ has the following vev's:

$$\langle\phi\rangle = \begin{pmatrix} \kappa & 0 \\ 0 & \kappa' \end{pmatrix} \text{ and } \langle\tilde{\phi}\rangle = \begin{pmatrix} \kappa'^* & 0 \\ 0 & \kappa^* \end{pmatrix}, \tag{2.38}$$

where κ and κ' are complex. From eq.(2.31), we then find

$$M_u = h\kappa + \tilde{h}\kappa'^* \text{ and } M_d = h\kappa' + \tilde{h}\kappa^*. \tag{2.39}$$

In case (b), eq.(2.37) implies that,

$$M_u = e^{i\alpha}\, \tilde{h}^\dagger \kappa + \tilde{h}\kappa'^* \text{ and } M_d = e^{-i\alpha}\tilde{h}^\dagger \kappa' + \tilde{h}\kappa^*.$$

This implies that

$$M_u^\dagger = e^{i\alpha} M_d. \qquad (2.40)$$

This is unacceptable since it implies that up and down quark masses are equal. Thus, for the minimal model with one set of ϕ and $\tilde{\phi}$ Higgs multiplet, the parity transformation case (b) is unacceptable. Of course, if there are more than one ϕ, this can be acceptable and in general leads to unequal mixing angles in the left- and right-handed currents. In what follows, we will consider the parity transformation properties corresponding to case (a) and ensuing constraints eq.(2.36) on the Yukawa couplings. It is now worth pointing out that by redefining $\phi \longrightarrow e^{i\alpha/2}\phi$, eq.(2.36) simplifies to:

$$h = h^\dagger \text{ and } \tilde{h} = \tilde{h}^\dagger. \qquad (2.41)$$

Due to this possibility of redefining the phase the definition of parity transformation for all values of α is identical. Note, however, that the phase of ϕ (and $\tilde{\phi}$) is no longer arbitrary.

An important consequence of eq.(2.41) is that, if κ and κ' are real, the up and down quark mass matrices are hermitian. This implies that, in eq. (2.28),

$$U_L = U_R \text{ and } V_L = V_R \text{ and } U = V, \qquad (2.42)$$

which leads to the physically important result that, if $\langle\phi\rangle$ is real, parity invariance implies the equality of the quark mixing angles in the left- and right-handed sector.

2.4.2. Constraints of CP-Invariance on the Yukawa Couplings

Let us define the CP-transformation[50] properties of the quark(Q) and Higgs field ϕ which leaves the gauge interaction invariant:

$$Q_L(\vec{x},x_0) \longrightarrow U_{cp}\ \gamma_0\ C\ \bar{Q}_L^T\ (-\vec{x},x_0),$$

$$Q_R(\vec{x},x_0) \longrightarrow V_{cp}\ \gamma_0\ C\ \bar{Q}_R^T\ (-\vec{x},x_0),$$

$$\vec{W}^{\mu}_{L,R}(\vec{x},x_0)\cdot\vec{\tau} \longrightarrow (-1)^{\delta_{\mu 0}}\ \vec{W}^{\mu}_{L,R}(-\vec{x},x_0)\cdot\vec{\tau}^T,$$

$$B_\mu(\vec{x},x_0) \longrightarrow (-1)^{\delta_{\mu 0}}\ B_\mu(-\vec{x},x_0). \tag{2.43}$$

For simplicity we will choose U_{cp} and V_{cp} to be diagonal unitary matrices. For the ϕ transformation we find that, (dropping space time dependence)

$$\begin{pmatrix} \phi \\ \tilde{\phi} \end{pmatrix} \longrightarrow H\begin{pmatrix} \phi^* \\ \tilde{\phi}^* \end{pmatrix} \tag{2.44}$$

with

$$\text{(a).}\quad H = \begin{pmatrix} e^{i\beta} & 0 \\ 0 & e^{-i\beta} \end{pmatrix} \tag{2.45a}$$

$$\text{or (b).}\quad H = \begin{pmatrix} 0 & e^{i\beta} \\ e^{-i\beta} & 0 \end{pmatrix} \tag{2.45b}$$

Invariance of the Yukawa coupling in eq.(2.31) under the CP-transformation leads to the following constraints on h and $\tilde{h}$:

$$\text{Case (a):}\quad e^{i\beta}V_{cp}h^T U^*_{cp} = h^\dagger,\ e^{-i\beta}V_{cp}\tilde{h}^T U^*_{cp} = \tilde{h}^\dagger. \tag{2.46}$$

$$\text{Case (b)}: e^{i\beta} V_{cp} h^T U^*_{cp} = \tilde{h}^\dagger,\ e^{-i\beta} V_{cp} \tilde{h}^T U^*_{cp} = h^\dagger. \qquad (2.47)$$

In case (b), eq.(2.47) implies that

$$e^{-i\beta}\, U_{cp}\, M^*_u\, V^*_{cp} = M_d. \qquad (2.48)$$

Eq.(2.48) is unacceptable since this implies that $m_u m_c m_t \dots = m_d m_s m_b \dots$ which is in conflict with experiments for values of $m_t \geq 30$—40 Gev. We will, therefore, focus on case (a) only. In this case, we see that if we choose
$U_{cp} = 1$ and $V_{cp} = e^{-i\beta}$, we find

$$h^T = h \text{ and } \tilde{h}^T = e^{2i\beta}\, \tilde{h}. \qquad (2.49)$$

In particular for $\beta = \pi / 2$, we find h is real and symmetric whereas $\tilde{h}$ is imaginary and antisymmetric. This case is of phenomenological interest[51].

In general, eq. (2.49) implies that the matrix $\tilde{h}$ has the following form:

$$\begin{bmatrix} 0 & ae^{-i\beta} & be^{-i\beta} & \cdots \\ ae^{i\beta} & 0 & ce^{-i\beta} & \cdot \\ be^{i\beta} & ce^{i\beta} & 0 & \ddots \\ \vdots & \vdots & \vdots & \end{bmatrix} \qquad (2.50)$$

where a,b,c,...are real. To study the implications of this case for CP-violation, we need to know the phase of the vev's χ and χ'. It can be proved that in the minimal model the vev's acquire phase such that the mass matrices become real leading to a completely CP-conserving theory[42].

It is further worth pointing out that $\beta = 0$ is a special case, which differs from the $\beta \neq 0$ case in the that the number of arbitrary parameters increases by N_g and the model loses all predictivity.

For the case of arbitrary U_{cp} and V_{cp}, CP-invariance implies an infinite sequence of constraints on h and $\tilde{h}$ as follows:

$$\mathrm{Tr}\,[h^{T2P_1}\,\tilde{h}^{T2P_2}\,h^{T2P_3}\,\tilde{h}^{T2P_4}] = \mathrm{Tr}\,[h^{2P_1}\,\tilde{h}^{2P_2}\,h^{2P_3}\,\tilde{h}^{2P_4}]. \tag{2.51}$$

Violation of any of these constraints would imply the existence of genuine CP-violation in the Yukawa couplings. The nontriviality of this result is that, even for complex h and $\tilde{h}$, the theory may be CP-conserving if eq.(2.51) is satisfied. The number of independent equations of type (2.51) is identical with the number of independent CP phases in the model[52].

It must, however, be emphasized that the conditions in eq.(2.51) guarantee CP-conservation only prior to spontaneous breakdown of symmetry. Subsequent to spontaneous breakdown, genuine CP-violation can appear if the $<\phi>$ breaks the CP-transformation law in eq.(2.44), i.e., if it has a form other than the following:

$$<\phi> = e^{i\beta/2}\begin{pmatrix} \kappa & 0 \\ 0 & \kappa' \end{pmatrix} \tag{2.52}$$

(κ, κ' are real).

2.5. Counting Genuine CP-Phases in L-R Models

An important prelude to the phenomenological study of CP-violation in a gauge model is to identify the number of genuine CP- phases in the weak currents. The general strategy is to first diagonalize complex mass matrices to obtain the unitary mixing matrices that appear in weak currents such as the ones in eq. (2.30) and then, to see how many of the phases in them can be absorbed into quark field redefinitions while at the same time keeping quark masses real.

To carry out this procedure, we need to know the structure of the mass matrices[49,53] in the left-right symmetric theories. We, therefore, start from the Yukawa coupling given in eq. (2.31) and distinguish the two cases: (1) intrinsic and (2) spontaneous CP-violation. In the first case (refered to earlier as hard CP-violation), we will simply impose the left-right symmetry constraints on the Yukawa couplings and no constraints of CP-invariance will be assumed. Then, in case (1), we must have

$$h = h^{\dagger} \text{ and } \tilde{h} = \tilde{h}^{\dagger}. \tag{2.53}$$

On the other hand, in the case of spontaneous CP-violation, we will assume the Lagrangian to be both CP and P invariant and specialize to the case, where $U_{cp} = V_{cp} = 1$ and $\beta = 0$ in eq. (2.43) and (2.44) respectively. We, then, have for case (2), in addition to eq. (2.53)

$$h = h^{T} \text{ and } \tilde{h} = \tilde{h}^{T}. \tag{2.54}$$

i.e. all elements of the matrices h and $\tilde{h}$ are real.

To obtain the mass matrices, we need the vev's of the field ϕ. Depending on whether $<\phi>$ is real or complex, case (1) leads to two kinds of mass matrices discussed below in case (1) and case (3) respectively.

Case (1): Manifest Left-Right Symmetry

$<\phi> = \begin{pmatrix} \kappa & 0 \\ 0 & \kappa' \end{pmatrix}$, κ and κ' are real. The up and down quark mass matrices are hermitian, i.e.,

$$M_u = h\kappa + \tilde{h}\kappa' = M_u^\dagger, \; M_d = h\kappa' + \tilde{h}\kappa = M_d^\dagger. \tag{2.55}$$

This implies that,

$$U \, M_u \, U^\dagger = D_u \text{ and } V \, M_d \, V^\dagger = D_d. \tag{2.56}$$

This, in turn, leads to the result that the unitary mixing matrices that appear in the left- and right-handed charged weak currents are the same. Thus, in this case, known in the literature as manifest left-right symmetry, both the left- and right-handed mixing angles as well as the phases are equal (i.e. $\vartheta_L^i = \vartheta_R^i$ and $\delta_L^i = \delta_R^i$).

The counting of non-trivial CP-phases in this case is the same as in the standard model and one has for the number of nontrivial CP-phases, N_p, the result:

$$N_p = (N_g - 1)(N_g - 2) \, / \, 2. \tag{2.57}$$

Case (2): Pseudo-Manifest Left-Right Symmetry

In the case of spontaneous CP-violation, to obtain CP-violation, we would require $<\phi>$ to be complex, i.e.

$$<\phi> = e^{i\alpha} \begin{pmatrix} \kappa & 0 \\ 0 & \kappa' \end{pmatrix}. \tag{2.58}$$

(The vacuum expectation value can be written in this form by an appropriate gauge SU(2) rotation). We then get,

$$M_u = h\kappa e^{i\alpha} + \tilde{h}\kappa' e^{-i\alpha} \text{ and } M_d = h\kappa' e^{i\alpha} + \tilde{h}\kappa e^{-i\alpha}. \tag{2.59}$$

$M_{u,d}$ are therefore complex and symmetric matrices. They can in general be diagonalized as follows[49]:

$$U_L\ M_u\ U_L^{\dagger} J_u = D_u \text{ and } V_L\ M_d\ V_L^{T} J_d = D_d. \tag{2.60}$$

This implies that

$$|P_R> = J_u^* U^* |P_R^0> \text{ and } |N_R> = J_d^* V^* |N_R^0> \tag{2.61}$$

where $|P^0>$ and $|N^0>$ represent column vectors that denote the up-quark and down-quark weak eigenstates respectively. Substituting this in the weak current (2.29) we get the following relation between the left- and right-handed mixing angles: $V = J_u\ U^*\ J_d^*$. (2.62)

We can now count the number of non-trivial CP-phases N_P by adding to the standard model result, the new phases that parameterize J_u and J_d^* (remembering that $J_{u,d}$ are diagonal unitary matrices), i.e.

$$N_P = N_{STD} + N_{Ju} + N_{Jd}. \tag{2.63}$$

For N_g generations, $N_{STD} = (N_g - 1)(N_g - 2) / 2$ (2.64)

and

$$N_{Ju} + N_{Jd} = 2N_g - 1. \tag{2.65}$$

An important implication of this case is that while there are new phases from right handed sector, the mixing angles between the left- and right-handed sectors are the same. This is important because for CP-conserving processes the only new parameters one has to deal with are $m(W_R)$ and the W_L-W_R mixing ζ.

Case (3):Non-Manifest Left-Right Symmetry

In this case the mass matrix is neither hermitian nor symmetric so that in general $U_L \neq U_R$ and $V_L \neq V_R$ leading to different mixing angles as well as phases for the left- and right-handed sector. The number of non-trivial CP-phases are:

$$N_P = N_{STD} + N_R = (N_g - 1)(N_g - 2) / 2 + N_g(N_g + 1) / 2. \tag{2.66}$$

Theoretically, this situation arises[32] from Case (1), if $<\phi>$ is complex or if there are more than one ϕ with parity transformation taking $\phi \longrightarrow \tilde{\phi}^{\dagger}$.

Chapter 3.

CP-Violation in the Neutral Kaon System

3.1 Introduction

Since the beginning of physics, symmetry considerations have provided an extremely powerful and useful tool in the effort to understand nature. Gradually they have become the backbone of the theoretical formulation of physical laws.

There are four groups of symmetries, or broken symmetries, that are found to be of importance in physics:

1. Permutation symmetry:Bose-Einstein and Fermi-Dirac statistics.

2. Continuous space-time symmetries, such as translation, rotation, acceleration, etc.

3. Discrete symmetries, such as parity, time reversal, particle-anti-particle conjugation, etc.

4. Unitary symmetries, which include

U_1-symmetries such as those related to conservation of charge, baryon number, lepton number, etc.

SU_2(isospin)-symmetry,

SU_3(color)-symmetry,

and

SU_n(flavor)-symmetry.

Among these, the first two groups, together with some of the U_1-symmetries and perhaps the SU_3(color)-symmetry in the last group, are believed to be exact. All the rest seem to be broken. We are interested here in three discrete symmetries which relate a given state or process to one other state or process; these are:

P: Parity, or the inversion of all spatial coordinates,

T: Time-reversal, the replacement of t by $-t$,

C: Particle-antiparticle conjugation, the replacement of all particles by their antiparticles. In the first instance, these symmetries were discovered as symmetries of established physical laws. Thus Wigner[54] discovered that P and T are symmetries of the Schrodinger equation applied to atomic and molecular systems. Similarly, C was discovered as a symmetry of quantum electrodynamics[55].

Nuclear physics required the introduction of two interactions: the weak interaction to describe beta-decay, and the strong interaction for nuclear forces. Because these interactions had no classical analogs, they had to be invented *ab initio*. Then, it became natural to assume that the same symmetries held for the new interactions. However, the consequences of making these assumptions were not analyzed in any systematic fashion for a long time. An important point made by Luders in 1954[56] was that if P-symmetry is assumed, then the consequences of C-invariance are identical to those of T invariance. A way of stating this result is that, although it is easy to construct theories that violate C, P or T symmetry, every relativistic local quantum field theory is invariant under the combined symmetry operation CPT.

It was pointed out by Lee and Yang in 1956[57] that no experiments involving weak interactions tested the parity symmetry P. This led to the discovery in 1957[58] that parity symmetry was violated maximully in nuclear beta-decay as well as in pion and muon decay. The resulting V-A theory that became established after these discoveries involved a maximal violation of both C and P but retained invariance under the CP and the T transformations. This possibility was first emphasized by Landau and others[59] after the Lee-Yang paper but before the experiments.

Even before P violation was discovered, Lee et al.[60] looked at possible ways of testing the C and T symmetries subject to the overall CPT invariance. They found that K^0 decay was of particular interest. Their analysis is in terms of C violation, but the quantities they discuss(α and the charge asymmetry r) are equally measures of CP-violation. (To say this in another way, the C-violating observables discussed do not involve any P violation so they are also CP -violating.) Once it was realized that C was violated in the weak interactions, it was important to test the CP symmetry within the K^0 system.

If CP invariance is assumed, the K^0 states with definite masses and lifetimes are K_1 and K_2 which are even and odd under CP. It then follows that only K_1 can decay into two pions while K_2 would decay into three pions. This prediction, originally made by Gell-Mann and Pais[61], was verified with the observation of two types of decay, a short-lived K_s going to $\pi^+\pi^-$ or $\pi^0\pi^0$ and a long-lived K_L going to three pions. In 1964, Christenson et al.[62] discovered that K_L had a small branching ratio into $\pi^+\pi^-$. This decay, indicated at the same time in an experiment by Abashian et al[63], strongly suggested that CP invariance was not an exact symmetry. On the other hand, because the CPT theorem has been shown to hold in the neutral K system[64] a violation of time-reversal invariance is demonstrated in these experiments. In other words, CP violation is telling us that not only is there is a fundamental asymmetry between matter and antimatter, but also at some tiny level, interactions will show an asymmetry under the reversal of time.

Contrary to the violation of parity and charge conjugation which were maximal, the observed CP violation was "small". The immediate questions were then:Is CP violated in any other process? What is the origin of CP violation? Why is it small? Consequently, a very active field of experimental as well as theoretical research, devoted to the study of CP violation, come into being. It is interesting to note that soon after the discovery of CP violation many possible "explanations" were put forward, such as

the superweak interaction (see 3.4),

electromagnetic CP violation (see 3.4),

and so on. Some propositions were much more drastic such as the abolition of the superposition principle or the existence of a "shadow world".

Looking at the present literature, one will immediately realize that the CP scenarios of today have very little in common with those of two decades ago. The reason is not only because experiments have succeeded in ruling out most of the early models of CP violation but, much more importantly, because of the revolutionary developments in particle physics. The concept of local gauge symmetry has, by now, become a deep rooted foundation on which all the modern theories stand. In particular, the Standard Model is indispensable in our present understanding of physics in general and thus also CP violation. The Electroweak Model works so well that one is compelled to explore the problem of CP violation in its framework or "beyond".

Many speculations have been offered since 1964 on the possible origins of CP violation in K_L^0 decay. Despite the high precision of the experimental results, few models have been ruled out. One difficulty in testing models is that CP violation has been seen only in the neutral kaon system. Future measurements of the neutral charmed meson system, and especially the neutral bottom quark meson, may shed light on the matter, but the experimental problems are formidable.

3.2 CP violation in K^0 decay

The valence quark composition of the K^+ and K^- mesons is $u\bar{s}$ and $\bar{u}s$ respectively. It is thus natural to assign the valence quark composition $d\bar{s}$ and $\bar{d}s$ to their corresponding neutral counterparts, K^0 and $\bar{K}^0$. That is, the neutral kaons created in a strong, electromagnetic or weak process have, at the moment of creation (proper time $t=0$), definite strangeness. The K^0 and $\bar{K}^0$ have S=1 and -1 respectively. Since strangeness is conserved in the strong and electromagnetic interactions, and since the kaon is the lightest strange particle, the K^0 and $\bar{K}^0$ can only decay weakly and both lead to a $u\bar{u}d\bar{d}$ final state in the lowest order decay processes.

According to the GWS model, the W only couples to left-handed fermions or right-handed antifermions. The operation of charge conjugation C turns a left-handed quark into a left-handed antiquark, which has no coupling to the W. Subsequent operation of the parity transformation P gives a right-handed antiquark (P changes $\mathbf{r} \longrightarrow -\mathbf{r}$, $\mathbf{p} \longrightarrow -\mathbf{p}$, but $\sigma=\mathbf{r}\times\mathbf{p} \longrightarrow \mathbf{r}\times\mathbf{p}$), which can couple to the W. Thus weak decays are expected to be eigenstates of CP. Now, the K^0 and $\bar{K}^0$ particles are not eigenstates of CP, but are transformed into one another by this operation. Taking the convention that

$$CP|K^0\rangle=|\bar{K}^0\rangle, \quad CP|\bar{K}^0\rangle=|K^0\rangle, \tag{3.1}$$

we can construct the following linear combinations of K^0 and

$\bar{K}^0$, which are CP eigenstates

$$|K_1^0\rangle=\frac{1}{\sqrt{2}}[|K^0\rangle+|\bar{K}^0\rangle], \quad CP=+1 \tag{3.2a}$$

$$|K_2^0\rangle=\frac{1}{\sqrt{2}}[|K^0\rangle-|\bar{K}^0\rangle], \quad CP=-1 \tag{3.2b}$$

Alternatively, we can express the strangeness eigenstates in terms of the states of definite CP

$$|K^0\rangle=|\bar{d}s\rangle=\frac{1}{\sqrt{2}}[|K_1^0\rangle+|K_2^0\rangle] \quad , \tag{3.3a}$$

$$|\bar{K}^0\rangle=|s\bar{d}\rangle=\frac{1}{\sqrt{2}}[|K_1^0\rangle-|K_2^0\rangle], \tag{3.3b}$$

The possible hadronic final states for neutral kaon decay are 2π and 3π. These states have CP eigenvalues +1 and -1 respectively. This can be seen as follows. In the pion the quark and antiquark have zero orbital angular momentum l and also zero net spin. The "intrinsic" parity of the pion is $P_\pi=(-1)^{l+1}= -1$. The 2π system is invariant under C and has zero orbital angular momentum, hence $CP(2\pi) = (P_\pi)^2 = 1$. In the 3π system, the pions are in states of zero relative orbital angular momentum. Hence, $CP(3\pi)=(P_\pi)^3=-1$. Thus, the final states are eigenstates of CP, but the initial states K^0 and $\bar{K}^0$ are not. For CP-conserving decays, it is the K_1^0 and K_2^0 components which will decay and with which we can associate specific lifetimes.

The phase space factors for the 2π and 3π decay modes of the kaon are substantially different. The $K\longrightarrow 3\pi$ phase space integral is much less than that for $K\longrightarrow 2\pi$. Hence, it is expected that the K_2^0 component has a substantially longer lifetime than that of K_1^0. Experimentally, *short* (K_S) and *long* (K_L) lived components are indeed observed, with lifetimes $\tau_s=0.892\times10^{-10}$s and $\tau_L=5.18\times10^{-8}$s respectively.

If CP was conserved, then, after sufficiently long time, neutral decays to two pions would no longer occur. However, in 1964 it was observed [62] that $K_L^0\longrightarrow\pi^+\pi^-$ indeed occured, and with a rate, about 2×10^{-3}. Thus, Cronin, Fitch and co-workers had discovered that CP was violated in this decay. This important observation has been confirmed in this and other decay modes of the K^0 system. However, no evidence for CP violation in other hadron decays has so far been found.

Let us consider the time evolution of a neutral kaon system, allowing for the possibility of CP violation. At time t we have, in general,

$$|\psi(t)>=a(t)\,|K^0>+\bar{a}(t)\,|\bar{K}^0>. \tag{3.4}$$

The time evolution is given, for this coupled two-channel system, by

$$i\frac{d}{dt}|\psi(t)> = H|\psi(t)> = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} |\psi(t)>, \tag{3.5}$$

where the Hamiltonian H,in analogy with the single-component case,can be expressed as

$$H = M - i\,\Gamma/2 = \begin{bmatrix} M_{11} - i\Gamma_{11}/2 & M_{12} - i\Gamma_{12}/2 \\ M_{21} - i\Gamma_{21}/2 & M_{22} - i\Gamma_{22}/2 \end{bmatrix}, \qquad (3.6)$$

where M and Γ are called the *mass* and *decay matrices* respectively. Since they represent observable quantities, these matrices are Hermitian;hence $M_{12}= M^{*}_{21}$ and $\Gamma_{12}= \Gamma^{*}_{21}$. However, H is not Hermitian, otherwise the K^0 would not decay. The CP-conserving strong and electromagnetic interactions do not cause $K^0 \longrightarrow \bar{K}^0$ transitions, nor K^0 decay, and hence contribute only to the diagonal elements of M. Thus, if H represents the weak Hamiltonian, then the mass matrix can be written

$$M_{12}=m(K^0)+<K^0|H|K^0>+\sum_n |<K^0|H|n>|^2/[m(K^0)-E_n],$$

$$M_{22}=m(\bar{K}^0)+<\bar{K}^0|H|\bar{K}^0>+\sum_n |<\bar{K}^0|H|n>|^2/[m(\bar{K}^0)-E_n], \qquad (3.7)$$

$$M_{11}=M^{*}_{21}=\sum_n <K^0|H|n><n|H|\bar{K}^0>/[m(K^0)-E_n],$$

where the sum extends to the n possible zero-strangeness intermediate states of energy E_n. CPT invariance implies that $m(k^0)=m(\bar{K}^0)$, and that $H_{11}=H_{22}$ (i.e., $M_{11}=M_{22}$, $\Gamma_{11}=\Gamma_{22}$). In the discussion below it is assumed that the strong and electromagnetic interactions conserve CPT. The term $<K^0|H|\bar{K}^0>$ has been omitted from M_{12}, since direct $|\Delta S|=2$ transitions are not allowed in the standard model.

The decay matrix Γ can be expressed as a sum of the contributions corresponding to distinct S=0 physical states.

Conservation of probability implies that

$$\frac{d}{dt}\langle\psi|\psi\rangle = -2\pi \sum_F \rho_F |\langle F|H|\psi\rangle| \quad , \tag{3.8}$$

where F is the final state, and ρ_F is the density of final states. From eqs. (3.5) and (3.6) we can write

$$\frac{d}{dt}\langle\psi|\psi\rangle = \langle\dot{\psi}|\psi\rangle+\langle\psi|\dot{\psi}\rangle = -\langle\psi|\Gamma/2-iM|\psi\rangle-\langle\psi|\Gamma/2+iM|\psi\rangle$$

$$= -\langle\psi|\Gamma|\psi\rangle. \tag{3.9}$$

Hence, from eqs. (3.8) and (3.9), we have

$$\langle\psi|\Gamma|\psi\rangle = 2\pi \sum_F \rho_F|\langle F|H|\psi\rangle|^2, \quad \Gamma_{11}= 2\pi \sum_F \rho_F|\langle F|H|K^0\rangle|^2$$

$$\Gamma_{22} = 2\pi \sum_F \rho_F|\langle F|H|\bar{K}^0\rangle|^2, \ \Gamma_{12}=\Gamma^*_{21}= 2\pi \sum_F \rho_F\langle F|H|K^0\rangle^*\langle F|H|\bar{K}^0\rangle. \tag{3.10}$$

If H is invariant under CPT, then

$$\langle F|H|K^0\rangle=\langle F|(CPT)^{-1}H(CPT)|K^0\rangle$$

$$=\langle\bar{K}^0|H|\bar{F}'\rangle=\langle\bar{F}'|H|\bar{K}^0\rangle^*, \tag{3.11}$$

where $\bar{F}'$ is the charge conjugate of final state F, but with the spins reversed. CPT invariance of H thus implies that $\Gamma_{11}=\Gamma_{22}$, since the sum in (3.10) is over all possible final states F. If T invariance holds for H, then the off-diagonal elements are real and equal, and so $M_{12}=M_{21}=M^*_{12}$ and $\Gamma_{12}=\Gamma_{21}=\Gamma^*_{12}$. If CPT and CP (or T) invariance both hold, then $M_{11}=M_{22}$, $\Gamma_{11}=\Gamma_{22}$, $M_{12}=M_{21}$ and $\Gamma_{12}=\Gamma_{21}$.

An alternative way to derive these relationships for the mass and decay matrices is to write

$$M-i\Gamma/2 = a_0 I_2 + a_1\sigma_1 + a_2\sigma_2 + a_3\sigma_3 , \qquad (3.12)$$

where σ_i are the Pauli matrices. For the phase convention (3.1), the operator for CP is CP=σ_1. The operator for time reversal is T = K, the complex conjugation operator. Thus, if R = CPT, then CPT invariance implies that $RHR^{-1} = H^\dagger$, where the form of the right-hand side follows from the anti-unitary nature of K. Application of this to (3.12), and consideration of the equivalent operations for T and CP, gives $a_3 = 0$ (CPT), $a_2 = 0$ (T) and $a_2 = a_3 = 0$ (CP).

In general we can express K_S and K_L as follows(cf.(3.2a))

$$|K_S\rangle = (p|K^0\rangle + q|\bar{K}^0\rangle)/[|p|^2+|q|^2]^{1/2},$$

$$|K_L\rangle = (r|K^0\rangle + s|\bar{K}^0\rangle)/[|r|^2+|s|^2]^{1/2}. \qquad (3.13)$$

The mass and widths of the physical (decaying) states correspond to the eigenvalues of the matrix H, given by (3.6) or (3.12), namely

$$\begin{bmatrix} a_0+a_3 & a_1-ia_2 \\ a_1+ia_2 & a_0-a_3 \end{bmatrix}\begin{bmatrix} p \\ q \end{bmatrix} = \lambda_S\begin{bmatrix} p \\ q \end{bmatrix}, \qquad (3.14)$$

with similar equations for λ_L (with r and s replacing p and q). The eigenvalues of (3.14) are $\lambda = a_0 \pm a$, so that $\lambda_S = a_0 + a$ and $\lambda_L = a_0 - a$, where $a = (a_1^2+a_2^2+a_3^2)^{1/2}$. Using these results, together with (3.6), we obtain

$$a_0 = (M_{11}+ M_{22})/2 - i(\Gamma_{11}+ \Gamma_{22})/4$$

$$= (M_S+ M_L)/2 - i(\Gamma_S+ \Gamma_L)/4,$$

$$a_1 = \mathrm{Re}\, M_{12} - i\, \mathrm{Re}\, \Gamma_{12}/ 2,$$

$$a_2 = - \mathrm{Im}\, M_{12} + i\, \mathrm{Im}\, \Gamma_{12}/ 2,$$

$$a_3 = (M_{11} - M_{22})/ 2 - i(\Gamma_{11} - \Gamma_{22})/ 4,$$

$$a = (M_S - M_L)/ 2 - i(\Gamma_S-\Gamma_L)/ 4 \simeq a_1, \qquad (3.15)$$

where the latter approximation follows from the fact that a_2 and a_3 are small.

The eigenvectors p, q, r and s in (3.13) can now be found from (3.15) and its equivalent for K_L. This gives

$$p/q = (a_1-ia_2)/(a-a_3) \simeq 1-ia_2/a_1+a_3/a_1,$$

$$r/s = -(a_1-ia_2)/(a_1+a_3) \simeq -(1-ia_2/a_1-a_3/a_1).$$

Defining $\varepsilon = -ia_2/(2a_1)$ and $\Delta = a_3/(2a_1)$(i.e. both small), we obtain, to a good approximation,

$$|K_S> = [(1+\varepsilon+\Delta)|K^0>+(1-\varepsilon-\Delta)|\bar{K}^0>]/ 2^{1/2},$$

$$|K_L> = [(1+\varepsilon-\Delta)|K^0>-(1-\varepsilon+\Delta)|\bar{K}^0>]/ 2^{1/2}, \qquad (3.16)$$

where the parameters ε and Δ are, from (3.15),

$$\varepsilon = [-\mathrm{Im}\, M_{12}+i\mathrm{Im}\, \Gamma_{12}/ 2]/[(\Gamma_S-\Gamma_L)/ 2+i(M_S-M_L),$$

$$\Delta = [i(M_{11}-M_{22})+(\Gamma_{11}-\Gamma_{22})/ 2]/ 2[(\Gamma_S-\Gamma_L)/ 2+i(M_S-M_L)].$$

(3.17)

Note that if CPT is conserved, then $a_3 = 0$, and hence $\Delta = 0$; so that (3.16) is then specified by the single parameter ε. Thus, a non-zero value of Δ (i.e.non-diagonal mass matrix) would indicate that CPT was violated. T invariance gives

$a_2 = 0$, and $\varepsilon = 0$. If CP is conserved, then (3.16) takes the form of (3.2). The overlap of the eigenstates for K_L and K_S is given by

$$\langle K_L | K_S \rangle = 2(\mathrm{Re}\ \varepsilon + i\ \mathrm{Im}\ \Delta). \tag{3.18}$$

3.3 Isospin Decomposition

Because of the definite CP eigenvalues of the final state pions, the decay modes $K_L \longrightarrow \pi^+\pi^-$ and $K_L \longrightarrow \pi^0\pi^0$ are CP violating. The quantities which can be extracted experimentally are

$$\eta_{+-} = \langle \pi^+\pi^- | H | K_L \rangle \ / \ \langle \pi^+\pi^- | H | K_S \rangle = |\eta_{+-}| \exp(i\phi_{+-}), \tag{3.19}$$

$$\eta_{00} = \langle \pi^0\pi^0 | H | K_L \rangle \ / \ \langle \pi^+\pi^- | H | K_S \rangle = |\eta_{00}| \exp(i\phi_{00}). \tag{3.20}$$

From Bose symmetry, the $\pi\pi$ state must have $I = 0$ or 2. The $\pi^+\pi^-$ and $\pi^0\pi^0$ states are related to the states of definite isospin by the usual Clebsch-Gordan coefficients

$$\langle \pi^+\pi^- | = (1/3)^{1/2} \langle I = 2 | + (2/3)^{1/2} \langle I = 0 |, \tag{3.21}$$

$$\langle \pi^0\pi^0 | = (2/3)^{1/2} \langle I = 2 | - (1/3)^{1/2} \langle I = 0 |. \tag{3.22}$$

The decay system is specified by four weak decay amplitudes, namely

$$\langle I | H | K^0 \rangle = A_I \exp(i\delta_I), \quad \langle I | H | \bar{K}^0 \rangle = \bar{A}_I \exp(i\delta_I), \tag{3.23}$$

where $I = 0$ or 2. The phase shifts δ_0 and δ_2 arise from final state interactions amongst the pions. If CPT invariance holds, then $\bar{A}_I = A_I^*$ and, since the overall phase is unobservable, we can choose A_0 to be real. If CPT is violated then $\bar{A}_0 \neq A_0$, but one can still choose A_0 and $\bar{A}_0$ to have the same phase with $\bar{A}_0/A_0$ real. A measure of CPT violation for I = 0 is given by

$$\lambda_0 = (\bar{A}_0 - A_0)/(\bar{A}_0 + A_0). \tag{3.24}$$

The corresponding parameter λ_2 is complex. Note that the amplitude A_2 violates the $|\Delta I| = 1/2$ rule, and is thus only about 4% of A_0. If we let $z = (A_2+\bar{A}_2)/(A_0+\bar{A}_0)$, then using eq. (3.19), together with eqs. (3.16), (3.21) and (3.23), we obtain

$$\eta_{+-} \simeq \varepsilon_0 + \varepsilon' \tag{3.25}$$

where

$$\varepsilon_0 = \varepsilon - \Delta - \lambda_0,$$

$$\varepsilon' \simeq 2^{-1/2}[(A_2-\bar{A}_2)-z(A_0-\bar{A}_0)](A_0+\bar{A}_0)^{-1}\exp[i(\delta_2-\delta_0)]. \tag{3.26}$$

Similarly, for the $\pi^0\pi^0$ decay mode

$$\eta_{00} = \varepsilon_0 - 2\varepsilon'. \tag{3.27}$$

For the case of CPT invariance, $A_0 = \bar{A}_0$ and $\Delta = \lambda_0 = 0$, so that

$$\varepsilon_0 = \varepsilon, \ \varepsilon' \simeq i\ 2^{-1/2}\ A_0^{-1}\ \mathrm{Im}\ A_2 \exp[i(\delta_2-\delta_0)]. \tag{3.28}$$

Note that η_{+-} and η_{00} can be non-zero, even if there is no K_S^0-K_L^0 mixing ($\varepsilon = 0$). Such direct CP violation would give a non-zero value of ε'.

The quantity ε, called the CP impurity parameter, measures the departure of the mass eigenstates K_L^0 and K_S^0 from being CP eigenstates. We can get the usually used form of it from eq.(3.17). Let

$$\Delta m \equiv m(K_S)-m(K_L) \simeq -2 \text{ Re } M_{12}, \quad (3.29)$$

$$\Delta\Gamma \equiv \Gamma_S-\Gamma_L \simeq 2 \text{ Re } \Gamma_{12}. \quad (3.30)$$

where the above approximations are obtained from the fact the phases of M_{12} and Γ_{12} are small because of the smallness of CP violation. From eq. (3.10) and the $|\Delta I| = 1/2$ rule, our phase convention ($\bar{A}_0/ A_0 = 1$) implies that

$$\Gamma_{12} \simeq \Gamma_{21}, \text{ real.} \quad (3.31)$$

Noting that due to the experimental relationship[5]

$$\Delta M \simeq -\Delta\Gamma / 2, \quad (3.32)$$

eq. (3.17) becomes

$$\varepsilon \simeq 2^{-1/2} \exp[i\pi/4] \text{ Im } M_{12} / \Delta M. \quad (3.33)$$

The parameter ε', characterizes the "direct" CP violation giving rise to different phases for the weak amplitudes for A_0 and A_2 respectively will pick up a phase factor.

The reason is as follows. If we start with the conventional choice of quark field phases and the K-M matrix (where the weak couplings among light quarks are real), A_0 is not real since the effective hamiltonian for $\Delta S = 1$ weak decay contains CP violating terms. These arise from "penguin" diagrams involving virtual c and t quarks generated when strong interaction corrections to the weak interaction hamiltonian are taken into account. In this case, A_0 picks up a small imaginary part. We can restore the standard phase convention where A_0 is real simply by redefining the phase of the K^0 and $\bar{K}^0$ states:

$$|K^0\rangle \longrightarrow e^{-i\xi}\ |K^0\rangle \text{ and } |\bar{K}^0\rangle \longrightarrow e^{i\xi}\ |\bar{K}^0\rangle, \qquad (3.34)$$

so that

$$(A_0)_{quark} \longrightarrow e^{-i\xi}(A_0)_{quark} = A_0. \qquad (3.35)$$

At the same time the previously (in the quark basis) real amplitude A_2 picks up a phase $exp[-i\xi]$ and is complex in the basis where A_0 is real. Thus from (3.28), we obtain

$$|\varepsilon'| = 2^{-1/2}\ |\xi|\ |A_2 / A_0|. \qquad (3.36)$$

Similarly, in the quark bases((3.34)),

$$\text{Im } \langle K^0|M|\bar{K}^0\rangle \longrightarrow \text{Im } (e^{2i\xi}\ \langle K^0|M|\bar{K}^0\rangle)$$

$$\approx \text{Im } \langle K^0|M|\bar{K}^0\rangle + 2\xi \text{ Re } \langle K^0|M|\bar{K}^0\rangle \qquad (3.37)$$

gives

$$\text{Im } M_{12} / \Delta M \longrightarrow \text{Im } M_{12} / \Delta M + \xi, \qquad (3.38)$$

using eq.(3.29). Thus, from eq.(3.33), we obtain

$$|\varepsilon| = 2^{-3/2}\ |\varepsilon_m + 2\xi|, \qquad (3.39)$$

where

$$\varepsilon_m \equiv e^{i\pi/4} \text{ Im } M_{12} / (2^{1/2}\Delta M), \qquad (3.40)$$

and

$$\xi \equiv \text{Im } A_0 / \text{Re } A_0. \qquad (3.41)$$

Eqs.(3.39) and (3.36) will be bases of our calculations of ε and ε′ later.

3.4 Modes of CP-Violation

Since CP violation has so far only been observed in the neutral kaon system, it has proven difficult to rule out entirely most proposed models of CP violation. The relative strength of the CP violating interaction in any model can usually be adjusted to give the correct magnitude of CP violation in K^0 decays. Some of the various suggested origins of CP violation are as follows (for a detailed review see reference [65]):

i). CP violation in electromagnetic interactions.

If the origin of the CP violation is in the electromagnetic interaction, then the required violation is large(~ 0.1). However, the experimental limits on C, P and T violations in electromagnetic interactions are better than 10^{-12}. A non-zero neutron electric dipole moment (d_n), can only arise if both P and T are violated. If the electromagnetic interaction is T-violating and P-conserving, then the normal T-conserving and P-violating weak interaction is needed to generate d_n. Very roughly, one might expect $d_n \sim 10^{-20}$ e cm , or, with a more sophisticated treatment, $d_n \sim 10^{-23}$ e cm. The current experimental limit is $d_n < 6\times10^{-25}$ e cm[1].

ii). Millistrong interaction.

A small ($\sim 10^{-3}$) violation of CP in the strong interaction is needed, with the $K_L^0 \longrightarrow 2\pi$ occurring via a second order transition in this CP-violating interaction. Since P is conserved to better than 10^{-5} in strong interactions, there must be C (and T) violation in strong interactions of about 10^{-3}. An electric dipole moment for the neutron of $d_n \sim 10^{-21}$ e cm is expected.

iii). Milliweak interactions.

In this model there is, in addition to the CP-conserving weak interaction, a small ($\sim 10^{-3}$) CP-violating piece, giving first-order CP-violating effects. No direct evidence has been found in any weak process for T violation; however, it is difficult, in general, to turn these limits into rigorous limits on a possible milliweak interaction. A value of $d_n \sim 10^{-23}$ e cm is expected in this model.

iv). Superweak interaction.

In this model[66] a $\Delta S = 2$ interaction is postulated, which causes $K^0 \longleftrightarrow \bar{K}^0$, and hence $K_L^0 \longleftrightarrow K_S^0$, transitions. These transitions are first order in the superweak interaction, which has some coupling strength F, $F \sim 10^{-8}\ G_F$. The CP violation arises entirely from the mass matrix, and not from the ordinary weak interactions which give rise to the $K \longrightarrow 2\pi$ transitions. Hence $\bar{A}_2 = A_2$ so that $\varepsilon' = 0$ and, further, a_2 is real. Thus the model predicts that

$$\eta_{+-} = \eta_{00} = \varepsilon,\ \phi_{+-} = \phi_{00} = \tan^{-1}(2\Delta m/\Gamma_S). \qquad (3.37)$$

However the non-zero value of ε', suggested by the recent data, would imply that the superweak model is not the only source of CP violation. Note that the superweak theory implies that any effects outside the K^0 system are very small, for example, $d_n \leq 10^{-29}$ e cm.

v). Standard model.

In the six-quark scheme depicted by the KM matrix, the phase δ represents CP violation. The neutron dipole moment can be predicted in this model. Estimates give[67] $d_n \lesssim 10^{-27}$ e cm; however, these are sensitive to the input parameters (e.g. m_t), and many estimates give $d_n < 10^{-30}$ e.cm.

The discussion of ε and ε' in this model will be left to the next chapter.

The above general formalism treating the CP violation in the neutral kaon system can be found, for example, in any of references [1, 4-6].

Chapter 4.

ε'/ε In The Mini Standard Model

4.1. A Brief Review

As mentioned in *Chapter 3*, the two sets of CP-violation parameters $\{\eta_{+-}, \eta_{00}\}$ and $\{\varepsilon, \varepsilon'\}$ are related to each other by the following relationships

$$\eta_{+-} \approx \varepsilon+\varepsilon' \quad \text{and} \quad \eta_{00} \approx \varepsilon-2\varepsilon'. \qquad (3.25)\&(3.27)$$

Therefore, they are approximately equivalent to each other. The former are experimental observables; the latter have the advantage of separating direct CP-violation from impure CP-violation. In addition, their values depend on the model used, i.e., they are also, in general, functions of model parameters. Thus, ε and ε' are important in selecting the correct model. In theoretical calculations, they are determined by the box and penguin diagrams (that will be seen later).

In the Standard Model, the imaginary parts of both box and penguin diagrams are proportional to $\sin\delta$, the non-trivial phase in the KM matrix. Thus the standard model accommodates CP violation but does not predict it, since $\sin\delta$ is arbitrary. Moreover, if the mass matrix CP violation (ε) is of Standard Model origin, i.e.is due to a KM matrix with a non-zero phase, then (barring "accidental" cancellations), ε'/ε is non-zero, i.e.,there is direct CP violation as well. The task at hand in determining ε'/ε consists of *a*) overcoming calculational difficulties and *b*) determining (or at least constraining) $\sin\delta$ from other experimental and theoretical inputs.

The evolution of the theoretical outlook and expectations for ε'/ε can roughly be divided into two periods: the phenomenological era (1973-1983) and the era of the Electroweak penguin (1989-present). The followings is a brief survey in chronological order. For more detailed review, see [67].

4.1.1. Phenomenological Era

In 1973 Kobayashi and Maskawa[20] showed that four quarks (and minimal gauge bosons, Higgses, etc.) were insufficient to produce CP-violation, i.e., that extra fields are needed. They suggested, among other scenarios, a six quark model. They made no prediction for ε'/ε.

Following the discovery of the charm quark (or rather the J/ψ)[68], in 1976 Weinberg[69] proposes a model with extra Higgses that could account for CP violation (see 1981) solely in the Higgs sector, with no phases in the quark mass matrix. This was particularly interesting before six quarks were known to exist, since if there are six quarks there is no particular reason for the mass matrix phase to be zero.

In the same year, Ellis, Gaillard and Nanopoulos[70] have penguins (not by name,) but estimate them to have magnitude similar to the W loop diagram that results if one neglects the gluon in the penguin diagram. The resultant $1/M_W^2$ suppression led them to estimate $|\varepsilon'/\varepsilon| \leq 1/450$. In September Penguins get named; bottom (or rather the Upsilon) gets discovered[71].

Three years later, Gilman and Wise published two papers. In the first paper[72] the ratio of the imaginary and real parts of the penguin amplitude are calculated to the lowest order. The calculation depends on $\vartheta_2 = \tan^{-1}(V_{td}/V_{cd})$ [these days expected to be in the range 1-5°], μ (a light hadron mass scale) and m_t. The phase $\sin\delta$ is fixed by using the measured value of ε. They estimated $|\varepsilon'/\varepsilon| \approx 1/13$ to 1/100. The first number comes from taking m_t = 15 Gev, μ = 0.2 Gev, and $\vartheta_2 = 15°$. $|\varepsilon'/\varepsilon|$ decreases as m_t, m_c/μ and ϑ_2 increase. In the second Gilman and Wise[73] paper the ratio of the imaginary and real parts of the penguin amplitude is calculated by doing an all orders leading logarithm calculation using successively W boson very heavy; top quark very heavy; bottom quark very heavy; charm quark very heavy. The parameters are ϑ_2 and m_t again, and the QCD scale parameter Λ in eq.(1.55). In addition α_s evaluated at the scale of light hadrons was varied between 0.75 and 1.25. With Λ^2 = 0.1 Gev^2, $|\varepsilon'/\varepsilon| \approx 1/50$ to 1/150 for m_t = 15-30 Gev, $\vartheta_2 = 15°$; with Λ^2 = 0.01Gev^2, $|\varepsilon'/\varepsilon| \approx 1 / 200$ to 1/350 for m_t = 15-30 Gev, $\vartheta_2 = 15°$.

In 1981, Deshpande[74] and Sanda[75] rule out Weinberg's CP-violation model by calculating the penguin to get $|\varepsilon'/\varepsilon|$

$\approx$ 0.045 (this number might be modified by the inclusion of non-gluonic penguin diagrams).

After this period of establishing how to calculate a penguin diagram, a somewhat more phenomenological era was entered, from about 1983 to 1987. During this time period changes in the theoretical estimation of $|\varepsilon'/\varepsilon|$ came more from the incorporation of bounds or improved values for input parameters in the calculation than any changes in the way the penguins were calculated.

In 1983, Gilman and Hagelin[76] used bounds from $K_L \longrightarrow \mu\mu$, as well as the experimentally measured value of ε, to come up with the bound

$$|\varepsilon'/\varepsilon| \gtrsim 2\times10^{-3}\ (0.33/B_K) \times \text{penguin uncertainties.} \tag{4.1}$$

The same year Gilman and Hagelin[77], and Buras et al[78]., used measurements of the b lifetime along with bounds on $\Gamma(b\longrightarrow u)/\Gamma(b\longrightarrow c)$ to get bounds such as

$$|\varepsilon'/\varepsilon| \gtrsim 0.005 \text{ to } 0.01. \tag{4.2}$$

These bounds, however, are only good for m_t in the then expected range 30 to 50 Gev, and drop sharply for large m_t.

We conclude the discussion of this phenomenological era by showing the situation in 1987. With gluonic penguins recalculated for large m_t and $B\bar{B}$ mixing constraints taken into account, the value

$$10^{-3} \lesssim |\varepsilon'/\varepsilon| \lesssim 7\times10^{-3} \tag{4.3}$$

was considered[79] representative.

4.1.2. The Era of the Electroweak Penguin

The main differences between 1981 and 1991 is as follows. First, there now exist more / different information on input parameters. In particular m_t is expected to be high and we have more information from $\Gamma(b\rightarrow)/\Gamma(b\rightarrow c)$ limits, and from the measurement of $B\bar{B}$ mixing. As a result, the Standard Model expectations for ε'/ε have moved from a range of 1/50 to 1/200 to a few $\times 10^{-3}$, probably $\gtrsim 1\times 10^{-3}$.
Second, as a consequence of m_t being large, photon, and most importantly, Z^0 penguins, are not negligible. Also the Z^0 contribution tends towards canceling the gluonic contribution. While the photon and Z^0 contributions are α/α_s suppressed, they are A_0/A_2 enhanced, since they can contribute to the $\Delta I = 3/2$ amplitude as well as to the $\Delta I = 1/2$ amplitude, unlike the gluonic contribution.

The Standard Model expectations for ε'/ε then move to a range of -0.3 to 2×10^{-3} depending on m_t. This is particularly notable in that ε'/ε being identically zero is not excluded in the Standard Model, contrary to the beliefs held for many years now.

In 1989, Flynn and Randall[80] calculated the effects of the photon and Z^0 penguins. The photon penguin *increases* ε'/ε, and for this reason was generally more or less ignored in past calculations, since it tends to cancel the effects due to isospin breaking corrections from π^0 mixing with η and η', which are estimated to decrease ε'/ε by about 25 to 45%. But the dominant effects is the decrease due to the Z^0

contribution, for m_t greater than about 100 Gev.

A similar calculation was done shortly afterwards by Buchalla et al.[81]. Their results are in good agreement with those of Flynn and Randall, and we present some of their phenomenological results here.

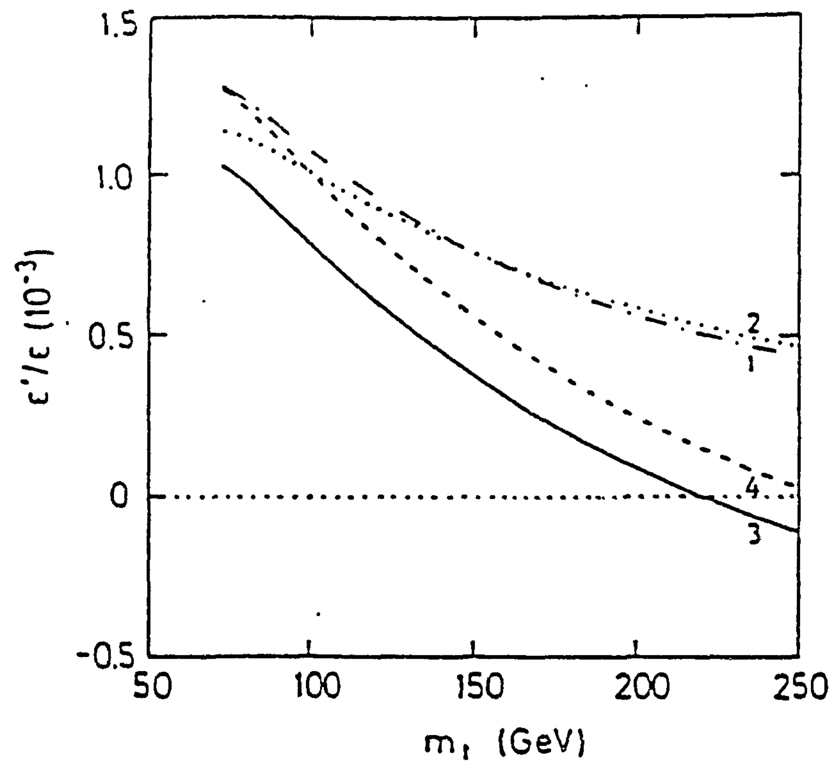


Fig.4.1 Penguin dependence of ε'/ε versus m_t, from Ref. [81], see text for details.

Figure 4.1 summarizes the effect of adding a Z^0 penguin on ε'/ε. A central set of values is used: $B_K = 0.75$, $\bar{R} = \Gamma(b \longrightarrow ue^- \bar{\nu}) / \Gamma(b \longrightarrow ce^- \bar{\nu}) = 0.02$, $s_{23} \approx V_{cb} = 0.05$, $\Lambda = 0.2$ Gev, and $m_s = 175$ Mev. Curve 1) is the pure QCD case, the inclusion of gluonic penguins alone, or setting $\alpha_{QED} = 0$. Curve 2) shows the result of including the $\pi^0\eta\eta'$ effects and the photonic penguins, without the Z^0 penguins, showing that these diagrams do indeed cancel to good approximation. Curve 3) is the full analysis of Ref.81, including Z^0 penguins, W box diagrams, and using the 1/N approach to estimate matrix elements. Curve 4), for comparison, is the same calculation using the vacuum insertion approximation to estimate mass matrix elements.

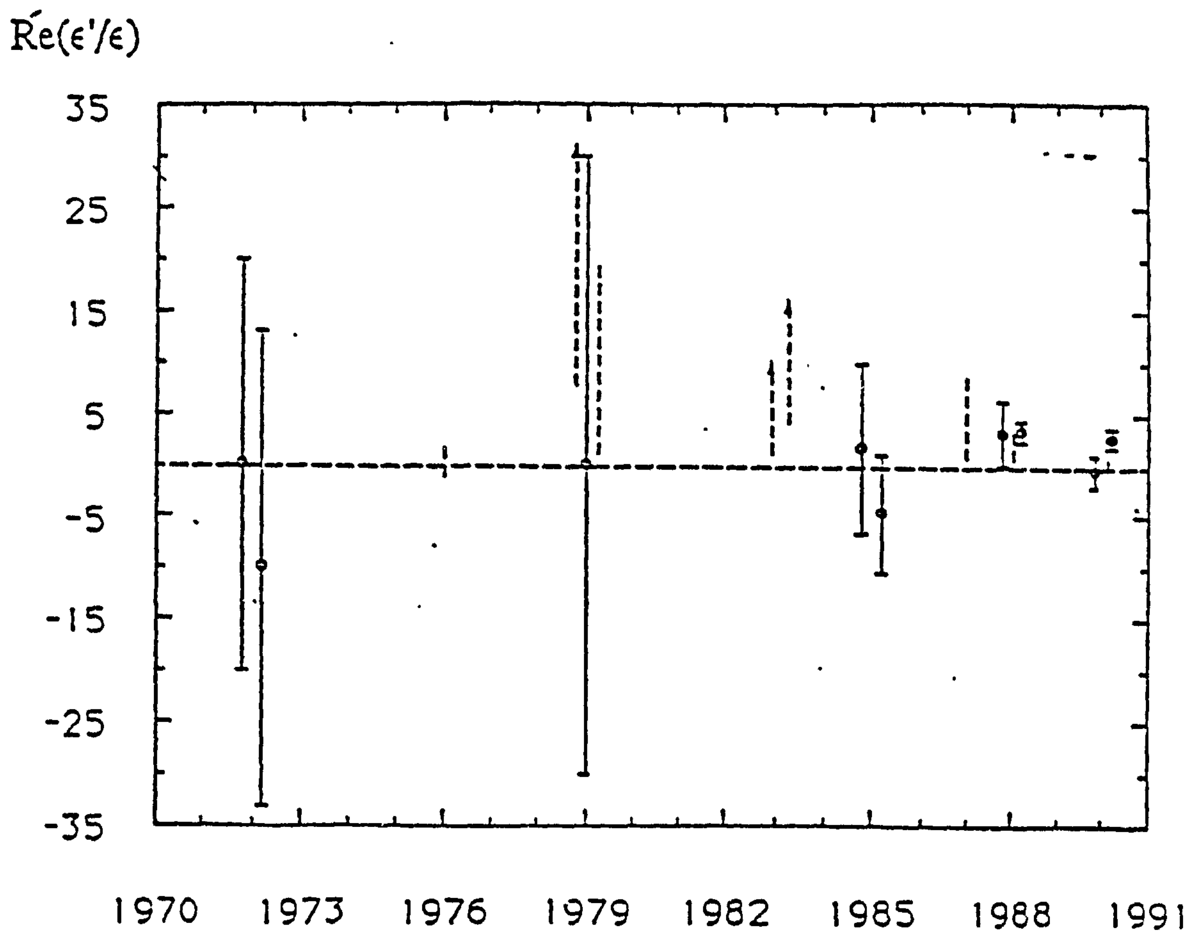


Fig.4.2 Experimental measurements of $Re(\varepsilon'/\varepsilon)$ (in units of 10^{-3}) from the last two decades (solid), along with evolution of the theoretical expectation for ε'/ε (dashed).

Figure 4.2 consists of the evolutions of both the experimental measurement and the theoretical expectation for ε'/ε over the last two decades.

The experimental results shown, in chronological order, are: 1972, Banner et al.,[82] Brookhaven/Princeton; 1972, Holder et al.,[83] CERN; 1979, Christenson et al.,[84] Brookhaven/NYU; 1985, Black et al.,[85] Brookhaven/Yale; 1985, Bernstein et al.[86], Fermilab/U.Chicago/Saclay (E731); 1988, Woods et al.,[87] E731; 1988, Burkhardt et al.,[88] CERN(NA31); 1990, Patterson et al.,[89] E731; 1990, latest NA31 result.[67] Note that not all measurements from the same experiment are independent, e.g., the latest NA31 number quoted above includes the 1988 number averaged in.

The latest measurements (both from 1990) are those of E731 and NA31

$$-0.4\pm1.4\pm0.6\times10^{-3} \text{ (E731)}; \; 2.7\pm0.9\times10^{-3} \text{ (NA31)}. \qquad (4.4)$$

From fig.4.2 we see a trend of convergence, and that ε'/ε is once more expected to be smaller than previous expectations. While the theoretical picture of ε'/ε has clearly undergone much evolution over the past two decades, there are still many missing pieces. The values of some of these (e.g., m_t, $\bar{R}$, and $B\bar{B}$ mixing) will hopefully be clarified by experiments in the near future, especially the next generation of ε'/ε measurements.

As an exercise and a preparation we calculate ε'/ε to the lowest order under the condition of $m_t \ll M_W$ in the Standard Model.

4.2. ε'/ε in the Standard Model

4.2.1. The Box Diagrams and ε

The $K^0 \leftrightarrow \bar{K}^0$ transition matrix M_{12} is given by the following box graphs:

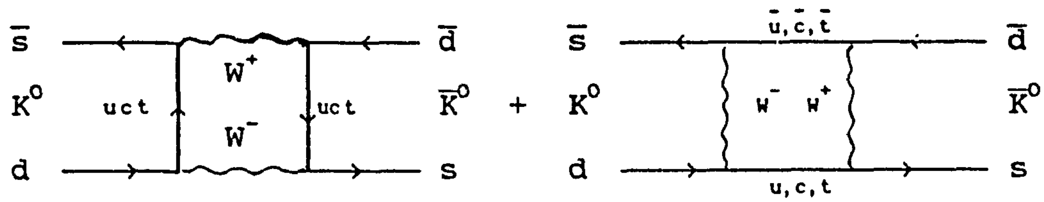


Fig.4.3 the box-graphs that contributes to ε_m

If we neglect all the external momenta associated with the external quark lines in comparison with the W-mass, then the usual Feynman rules give

$$M_{12} = 2\int \frac{d^4k}{(2\pi)^4} \left(\frac{-ig}{\sqrt{2}}\right)^4 \sum_{i,j=u,c,t} \bar{s}\,\gamma_\mu\, LV_{is}V_{id} \frac{i(k_\alpha\gamma^\alpha+m_i)}{k^2-m_i^2}\gamma_\nu Ld\cdot$$

$$\bar{s}\,\gamma_\vartheta\, LV_{js}V_{jd} \frac{i(k_\beta\gamma^\beta+m_j)}{k^2-m_j^2}\gamma_\phi Ld \frac{-ig^{\mu\phi}}{k^2-m_W^2}\frac{-ig^{\vartheta\nu}}{k^2-m_W^2}$$

$$= + \frac{g^4}{2}\int \frac{d^4k}{(2\pi)^4}\left(\frac{1}{k^2-m_W^2}\right)^2 \bar{s}\gamma_\mu k_\alpha\gamma^\alpha\gamma_\nu Ld\ \bar{s}\gamma^\nu k_\beta\gamma^\beta\gamma^\mu Ld$$

$$\sum_{i,j} \frac{V_{is}V_{id}V_{js}V_{jd}}{(k^2-m_i^2)(k^2-m_j^2)},$$

where V_{ij} is a matrix element of the KM matrix (see eq.(1.117)). In obtaining the above last line, use nas been made of the following relationships:

$L^2 = L$, $R\,L = 0$, $\{\gamma_\mu, \gamma^5\} = 0$, with $L \equiv \frac{1-\gamma^5}{2}$, and $R \equiv \frac{1+\gamma^5}{2}$.

From the KM matrix we have

$$\sum_{i,j} \frac{V_{is}V_{id}V_{js}V_{jd}}{(k^2-m_i^2)(k^2-m_j^2)} = \Big(\sum_i \frac{V_{is}V_{id}}{(k^2-m_i^2)}\Big)^2$$

$$= \Big(\frac{c_1 s_1 c_3}{k^2-m_u^2} + \frac{-s_1 c_2(c_1 c_2 c_3 - s_2 s_3 e^{i\delta})}{k^2-m_c^2} + \frac{-s_1 c_2(c_1 s_2 c_3 + c_2 s_3 e^{i\delta})}{k^2-m_c^2}\Big)^2$$

$$\equiv F(k^2; m_i^2). \tag{4.5}$$

Noting that all external momenta have been neglected, we have

$$M_{12} = -\frac{g^4}{2}\; \bar{s}\gamma_\mu \gamma^\alpha \gamma_\nu L\, d\; \bar{s}\gamma^\nu \gamma^\beta \gamma^\mu L\, d\; I_{\alpha\beta}, \tag{4.6}$$

$$\text{where} \quad I_{\alpha\beta} \equiv \int \frac{d^4 k}{(2\pi)^4} \frac{k_\alpha k_\beta}{(k^2-m_W^2)^2}\, F(k^2; m_i^2). \tag{4.7}$$

The integral (4.6) is safely convergent even in the absence of the W propagators, so we assume $m_i^2 \ll m_W^2$ and set the W propagators to $\approx 1/m_W^2$. We will also assume $m_{c,t}^2 \gg m_u^2$ and set $m_u \approx 0$. The pole term in the integral, in the above approximation which is $F(k^2; m^2)$, can be written as

$$F \simeq \Big(\frac{s_1 c_1 c_3}{k^2}\Big[\frac{c_2^2\, m_c^2}{k^2-m_c^2} + \frac{s_2^2\, m_t^2}{k^2-m_t^2}\Big] + \frac{s_1 s_2 c_2 s_3 e^{i\delta}(m_t^2-m_c^2)}{(k^2-m_c^2)(k^2-m_t^2)}\Big)^2, \tag{4.8}$$

On the other hand, $I_{\alpha\beta}$ has the form of $g_{\alpha\beta} I$. Therefore, the coefficient multiplying I in eq.(4.6) has the form

$$-\frac{g^4}{2}\ \bar{s}\gamma_\mu\gamma_\alpha\gamma_\nu L\, d\ \bar{s}\gamma^\nu\gamma^\alpha\gamma^\mu L\, d.$$

It can be simplified to

$$-2g^4\ \bar{s}\gamma_\mu L\, d\ \bar{s}\gamma^\mu L\, d, \qquad (4.9)$$

using the Dirac algebra

$$\gamma^\mu\gamma^\alpha\gamma^\nu = g^{\mu\alpha}\gamma^\nu + g^{\alpha\nu}\gamma^\mu - g^{\mu\nu}\gamma^\alpha - i\varepsilon^{\mu\alpha\nu\beta}\gamma_5\gamma^\beta$$

with $\varepsilon^{\mu\alpha\nu\beta}\gamma_{\nu\alpha\mu}\gamma\gamma = -6i\gamma_5\gamma^\beta$.

Finally, we obtain

$$M_{12} = \langle\bar{K}^0|M_{12}|K^0\rangle$$

$$= -2g^4\ \langle\bar{K}^0|\bar{s}\gamma_\mu Ld|0\rangle\langle 0|\bar{s}\gamma^\mu Ld|K^0\rangle\ I$$

$$\simeq -\frac{1}{4}\ g^4 f_K^2 m_K\ I, \qquad (4.10)$$

where f_K is the kaon decay constant and m_K the kaon mass. In the second line we inserted the vacuum intermediate states, and in the third line the term $f_K^2 m_K^2\ /\ 2m_K$ is identified with the $|K\rangle \longrightarrow |0\rangle$ current.

In the approximation of eq.(4.8) we have

$$\mathrm{Im}M_{12} = A[2\int d^4k(\frac{s_1 s_2 c_2 s_3 \sin\delta\,(m_t^2-m_c^2)}{(k^2-m_c^2)\,(k^2-m_t^2)})\ (\frac{s_1 c_1 c_2^2 c_3 m_c^2}{k^2-m_c^2} +$$

$$+ \frac{s_1 c_1 s_2^2 c_3 m_t^2}{k^2-m_t^2} + \frac{k^2 s_1 s_2 c_2 s_3 \cos\delta\,(m_t^2-m_c^2)}{(k^2-m_c^2)(k^2-m_t^2)} \}], \tag{4.11}$$

$$-\Delta m \simeq 2\mathrm{Re}M_{12} = A[2\int d^4k \{ \frac{s_1^2 c_1^2 c_3^2}{k^2} [\frac{c_2^2 m_c^2}{k^2-m_c^2} + \frac{s_2^2 m_c^2}{k^2-m_t^2}]^2 +$$

$$+ \frac{2s_1^2 c_1 s_2 c_2 s_3 c_3 \cos\delta\,(m_t^2-m_c^2)}{(k^2-m_c^2)(k^2-m_t^2)} [\frac{c_2^2 m_c^2}{k^2-m_c^2} + \frac{s_2^2 m_c^2}{k^2-m_t^2}] +$$

$$+ \frac{k^2 s_1^2 s_2^2 c_2^2 s_3^2 \cos 2\delta\,(m_t^2-m_c^2)^2}{(k^2-m_c^2)(k^2-m_t^2)} \}]. \tag{4.12}$$

Using Feynman parameters and the Feynman integral formula,we find after some calculation

$$\left|\frac{\mathrm{Im}M_{12}}{\Delta M}\right| \simeq 2s_1^2 c_1 s_2 c_2 s_3 c_3 \sin\delta\,(m_t^2-m_c^2)\{ c_2^2 m_c^2 [\frac{m_t^2}{(m_t^2-m_c^2)^2}\ln\frac{m_t^2}{m_c^2} - \frac{1}{m_t^2-m_c^2}] + s_2^2 m_t^2 [\frac{m_c^2}{(m_t^2-m_c^2)^2}\ln\frac{m_c^2}{m_t^2} \quad \frac{1}{m_t^2-m_c^2}]\}$$

$$\times \frac{1}{s_1^2 c_1^2 c_3^2 [c_2^4 m_c^2 + s_2^4 m_t^2 + \frac{2s_2^2 c_2^2 m_t^2 m_c^2}{m_t^2-m_c^2} \ln\frac{m_c^2}{m_t^2}}; \tag{4.13}$$

where we have dropped from both the numerator and denominator terms which are relatively small due to $s_3^2 \ll c_3^2$. Since $\cos^2\vartheta_c \simeq 0.9481 \pm 0.004$ by comparing nuclear β-decay with μ-decay, while $\sin\vartheta_c \simeq 0.230 \pm 0.003$ from semileptonic decays of baryons, we find $\cos^2\vartheta_c + \sin^2\vartheta_c \simeq 1.001 \pm 0.004$ experimentally. Allowing one standard deviation, this implies that in the KM model (see eqn.(1.117)) there must be a small "leakage" of the weak coupling of the u quark to the b quark : $s_1^2 s_3^2 < 0.003$, since $s_1^2 \simeq \sin^2\vartheta_c \simeq 0.05$, and therefore, $s_3^2 < 0.06$.

Let $\eta \equiv m_c^2/m_t^2$, and defining $\eta' \coloneqq \eta/(1-\eta)$, then eq.(4.13) becomes

$$|\mathrm{Im}\ M_{12}/\Delta M| = 2s_2c_2s_3\sin\delta\ F(\eta), \tag{4.14}$$

where

$$F(\eta) = \frac{s_2^2(1+\eta'\ln\eta)-c_2^2(\eta+\eta'\ln\eta)}{c_1c_3(c_2^4\eta+s_2^4-2s_2^2c_2^2\eta'\ln\eta)}. \tag{4,15}$$

The parameters in the eq.(4.14) are rather poorly known, but the predicted value of ε ($|\varepsilon|\simeq \mathrm{Im}M_{12}/\Delta M$) is about the correct magnitude.

However, there are some problems in calculating the box-graph. The first one is how to make the $\bar{s}$ and d quarks into a K^0, and the s and $\bar{d}$ quarks into a $\bar{K}^0$. To solve this problem we used the vacuum insertion approximation (eqn.4.10). By convention the parameter B_K is defined to gauge differences between this value of (4.10) and other theoretical models. Langacker has shown that theoretical calculations of B_K have large variations[90]. Another problem is that there are contributions to ΔM other than the box diagram. These are so-called "long distance" effects which arise out of the virtual transitions $K_L \leftrightarrow \pi^0(\eta)$, $K_S \leftrightarrow \pi\pi$[91]. The "long distance" effects are large and have not been accurately calculated. However, the imaginary part of the box diagram is not expected to be affected.

The calculations of this subsection are for 2nd-order processes. As mentioned before there are also possible 1st-order diagrams, for example, the so-called penguin diagrams, which contribute to ε′. We shall consider it in next subsection.

4.2.2. The Penguin Diagrams and ε′

In the quark basis of (3.34) we have

$$|\varepsilon'| \approx 2^{-1/2}|\xi|\cdot|A_2/A_0|. \qquad (3.36)$$

Thus the calculations of ε′ or the ratio $|\varepsilon'/\varepsilon|$ attributed to that of the phase of ε′, i.e., ξ, and ξ can be determined by the so-called penguin diagrams of Fig.4.4.(a)

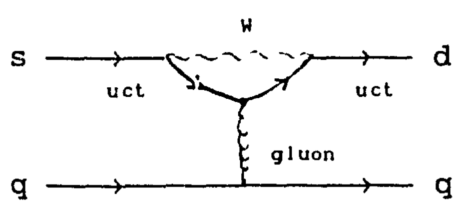


Fig.4.4 (a). Penguin diagrams that contribute for ΔS=1 processes.

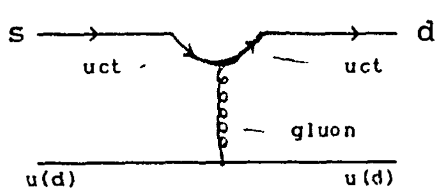


Fig.4.4(b).

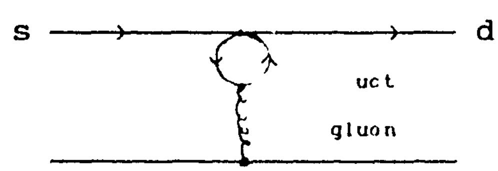


Fig.4.4(c).

In the case of low energy, $M_W^2 \gg k_W^2$, where k_W is the momentum transferred by the W particle, we can then reduce the graph in fig.4.4(a) to the four-fermion point interaction (see fig.4.4(b)). The gap in the four-fermion vertex in fig.4.4(b) separates two white V-A currents found in parentheses in the expression (for $|\Delta S|=1$)

$$\frac{G_F}{\sqrt{2}} V_{fd} V_{fs} (\bar{d}^{\alpha i} (O_\mu)^\beta_\alpha \, \delta^k_i \, f_{\beta k}) (\bar{f}^{\gamma l} (O^\mu)^\delta_\gamma \, \delta^m_l \, s_{\delta m}), \qquad (4.16)$$

where $O_\mu \equiv \gamma_\mu(1-\gamma_5)$, V_{fd}, V_{fs} are elements of KM matrix, f=u,c,t, Greek letters denote Dirac indices (0,1,2,3), and Latin letters denote color indices (1,2,3). The calculation can be simplified by the Fierz transformation: the operators $\bar{f}$ and f are now in one bracket, and $\bar{d}$ and s in the other one (see fig.4.4(c)). By using the relations

$$(O_\mu)^\beta_\alpha (O^\mu)^\delta_\gamma = -(O_\mu)^\delta_\alpha (O^\mu)^\beta_\gamma ,$$

$$\delta^k_i \delta^m_l = \tfrac{1}{3}\delta^m_i \delta^k_l + \tfrac{1}{2}\lambda^m_i \lambda^k_l ,$$

and taking into account that fermion operators anticommute, we obtain

$$\bar{d}O_\mu f \bar{f} O^\mu s = \tfrac{1}{3}\bar{f}O_\mu f \bar{d} O^\mu s + \tfrac{1}{2}\bar{f}O_\mu \lambda^a f \bar{d} O^\mu \lambda^a s . \qquad (4.17)$$

The vertex (emission of a gluon) is $g\bar{f}\frac{1}{2}\gamma_\nu f$. We have to find the trace over Dirac and color indices, hence, the first term of eq. (4.17) gives zero. For the loop in second term, by making the approximation $m_W \gg m_{c,t} \gg m_u$, we have, ignoring m_f:

$$-\mathrm{Tr}\ \frac{\lambda^a}{2}\frac{\lambda^b}{2}\int\frac{d^4k}{(2\pi)^4}\ \mathrm{Tr}\ \gamma_\mu \frac{1}{(k+q)_\alpha\gamma^\alpha}\ \gamma_\nu\ \frac{1}{k_\beta\gamma^\beta}$$

$$\equiv 1\frac{\delta^{ab}}{2}(q^2 g_{\mu\nu} - q_\mu q_\nu)I_0, \tag{4.18}$$

The loop in question is exactly the same (with the exception of additional color matrices) as that found in QED in the case of vacuum polarization calculations. Up to a dimensionless factor I_0, the form of the integral is determined by the transversality condition, which demands, in particular, that the quadratic divergence be absent.

In order to find I_0, multiply the left- and right-hand parts by $g^{\mu\nu}$. This gives

$$3q^2 I_0 = -\int\frac{d^4k}{(2\pi)^4}\ \mathrm{Tr}\ \gamma_\mu \frac{1}{(k+q)_\alpha\gamma^\alpha}\ \gamma^\mu\ \frac{1}{k_\beta\gamma^\beta}$$

$$= \int\frac{d^4k}{(2\pi)^4}\ \mathrm{Tr}\ \gamma_\mu \frac{1}{k_\alpha\gamma^\alpha}\ q_\lambda\gamma^\lambda\ \frac{1}{k_\rho\gamma^\rho}\ q_\sigma\gamma^\sigma \frac{1}{k_\tau\gamma^\tau}\ \gamma^\mu\ \frac{1}{k_\beta\gamma^\beta}$$

$$= 2\int\frac{d^4k}{(2\pi)^4}\ \mathrm{Tr}\ \frac{1}{k_\alpha\gamma^\alpha}\ q_\lambda\gamma^\lambda\ \frac{1}{k_\rho\gamma^\rho}\ q_\sigma\gamma^\sigma\ \frac{1}{k^2}$$

$$= \frac{8}{(2\pi)^4}\int\frac{d^4k}{k^6}\ [2(k\cdot q)^2 - k^2q^2]$$

$$= \frac{-4q^2}{(2\pi)^4}\int\frac{d^4k}{k^4} = -\frac{q^2}{4\pi^2}\int\frac{dk^2}{k^2}. \tag{4.19}$$

Finally, $$I_0 = -\frac{1}{12\pi^2}\ ln\frac{m_f^2}{\mu^2}\ , \tag{4.20}$$

where the upper integration limit is chosen to be m_f^2 for f-quark loop. (If the gluon momentum transfer q^2 is much larger than all the quark masses entering in the quark loop,

then the penguin graphs vanish, since their contribution is proportional to $V_{ud}V_{us} + V_{cd}V_{cs} + V_{td}V_{ts} = 0$.)
However, for problems involving strange particle decay $q^2 \ll m_c^2, m_t^2$ and the cancellation is not complete. This is essentially the crucial observation of Shiftman et.al.[91] The lower integration limit is chosen to be μ^2 because the free quark propagator must be modified at $k^2 < \mu^2$ due to confinement. As a result the contribution becomes

$$\frac{G_F}{\sqrt{2}} V_{fd}V_{fs}\frac{g^2}{12\pi^2}\, ln\frac{m_f^2}{\mu^2}\, [\bar{d}\gamma_\mu(1-\gamma^5)\lambda^a s]\frac{\delta^{ab}}{2}[\bar{u}\gamma^\mu\frac{\lambda^b}{2}u + \bar{d}\gamma^\mu\frac{\lambda^b}{2}d]$$

$$= \frac{G_F}{\sqrt{2}}\frac{\alpha_s}{12\pi} V_{fd}V_{fs}\, ln\frac{m_f^2}{\mu^2}\, [\bar{d}\gamma_\mu(1-\gamma^5)\lambda^a s][\bar{u}\gamma^\mu\lambda^a u + \bar{d}\gamma^\mu\lambda^a d].$$

(4.21)

Neglecting m_u, and noting that s_1 and s_3 are treated as small quantities[92], we have

$$H^{penguin}_{effective} = \frac{G_F}{\sqrt{2}}\frac{\alpha_s}{12\pi}[s_1c_2^2\, ln\frac{m_c^2}{\mu^2} + is_1s_2c_2s_3\, \sin\delta\, ln\frac{m_c^2}{\mu^2} +$$

$$+ s_1s_2^2\, ln\frac{m_t^2}{\mu^2} - is_1s_2c_2s_3\, \sin\delta\, ln\frac{m_t^2}{\mu^2}]\times$$

$$\times[\bar{d}\gamma_\mu(1-\gamma^5)\lambda^a s][\bar{u}\gamma^\mu\lambda^a u + \bar{d}\gamma^\mu\lambda^a d + \ldots] + h.c. \quad (4.22)$$

Gilman and Wise[92] were the first to propose to parameterize the result by the fractional contribution, f (not to be too far from 1), of the real part of the H_e^P to $K \longrightarrow 2\pi(I=0)$,

$$\xi = \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0} = f\,\frac{\mathrm{Im}H_e^P}{\mathrm{Re}H_e^P} = f s_2 c_2 s_3 \sin\delta\, \frac{ln\eta}{c_2^2 ln(m_c^2/\mu^2) + s_2^2 ln(m_t^2/\mu^2)}$$

$$= f s_2 c_2 s_3 \sin\delta\, \frac{ln\eta}{ln(m_c^2/\mu^2) - s_2^2 ln\eta}. \qquad (4.23)$$

The disadvantage of this method is that neither the denominator of eq.(4.23) nor f is reliably known. Thus the predictive power is lessened because of lack of knowledge.

Guberina and Peccei[93] first advocated another approach proceeding through direct calculation of $\mathrm{Im}A_0$ in a quark model by means of the operator product expansion and renormalization group techniques, and then taking the $\mathrm{Re}A_0$ from experiment, their results are

$$\xi = -0.11 s_2 s_3 \sin\delta\, \frac{<\pi^+\pi^-|O_5|K^0>}{0.43\ \mathrm{GeV}^3},$$

where O_5 is the penguin operator. Their calculation is quite complicated. They produced a smaller ξ than Gilman and Wise, and probably more realistic.

The above calculation is a typical one representing the treatment of ε'/ε in the first period as mentioned before. The key point is the assumption of "$m_t \ll m_W$".

Chapter 5

ε'/ε in Left-Right Symmetric Models

5.1. Introduction

As mentioned in Chapter 2, CP-violation is put in by hand in the Standard Model. It is with the goal of understanding the deeper meaning of CP-violation that the left-right symmetric models of CP-violation were first proposed[29]. Therefore, it is important to calculate ε'/ε in left-right models and then to compare the result with experiments. In what follows we shall calculate ε'/ε for the pseudo-manifest case in the left-right model. For convenience, the definitions of ε' and ε are as follows:

$$|\varepsilon'| \simeq 2^{-1/2}|\xi|\cdot|A_2/A_0|. \qquad (3.36)$$

$$|\varepsilon| = 2^{-3/2}\,|\varepsilon_m+2\xi|, \qquad (3.39)$$

where

$$\varepsilon_m \equiv e^{i\pi/4}\,\mathrm{Im}\,M_{12}\,/\,(2^{1/2}\Delta M), \qquad (3.40)$$

and

$$\xi \equiv \mathrm{Im}\,A_0\,/\,\mathrm{Re}\,A_0. \qquad (3.41)$$

The pseudo-manifest, Left-Right Model has six CP-violation phases (see eqn.2.65). For this case, left- and right-handed quark mixing angles are same. We can take[42]

$$U^L = V = \begin{pmatrix} c_1 & s_1c_3 & s_1s_3 \\ -s_1c_2 & c_1c_2c_3 - s_2s_3 exp(i\delta) & c_1c_2s_3 + s_2c_3 exp(i\delta) \\ s_1s_2 & -c_1s_2c_3 - c_2s_3 exp(i\delta) & -c_1s_2s_3 + c_2c_3 exp(i\delta) \end{pmatrix},$$

where V is the KM matrix, and

$$U^R = \begin{pmatrix} e^{i\alpha_1} & & \\ & e^{i\alpha_2} & \\ & & e^{-i(\alpha_1+\alpha_2)} \end{pmatrix} V^* \begin{pmatrix} e^{i\beta_1} & & \\ & e^{i\beta_2} & \\ & & e^{-i(\beta_1+\beta_2)} \end{pmatrix} e^{i\gamma}$$

$$= e^{i\gamma} \begin{pmatrix} c_1 e^{i(\alpha_1+\beta_1)} & -s_1c_3 e^{i(\alpha_1+\beta_2)} \\ s_1c_1 e^{i(\alpha_2+\beta_1)} & (c_1c_2c_3 - s_2s_3e^{-i\delta})e^{i(\alpha_2+\beta_2)} \\ s_1s_2 e^{-i(\alpha_1+\alpha_2-\beta_1)} & (c_1s_2c_3+c_2s_3e^{-i\delta})e^{-i(\alpha_1+\alpha_2-\beta_2)} \end{pmatrix.}$$

$$\begin{matrix} -s_1c_3 e^{i(\alpha_1-\beta_1-\beta_2)} \\ (c_1c_2s_3+s_2s_3e^{-i\delta})e^{i(\alpha_2-\beta_1-\beta_2)} \\ (c_1s_2s_3-c_2c_3e^{-i\delta})e^{-i(\alpha_1+\alpha_2+\beta_1+\beta_2)} \end{matrix}\Bigg). \quad (5.1)$$

In order to simplify writing, let

$$a_i^k \equiv U_{is}^{k*} U_{id}^k, \quad \lambda_i \equiv U_{is}^{R*} U_{id}^L, \quad \lambda_i' \equiv U_{is}^{L*} U_{id}^R, \quad (5.2)$$

with k = L,R, i = c,t.

Noting that s_1 and s_3 are small quantities and neglecting relative small quantities, we then have

$$a_c^L \simeq s_1c_2^2 + is_1s_2c_2s_3 \sin\delta, \quad a_t^L \simeq s_1s_2^2 - is_1s_2c_2s_3 \sin\delta,$$

$$a_c^R = a_c^{L*} e^{i(\beta_1 - \beta_2)} ; \qquad a_t^R = a_t^{L*} e^{i(\beta_1 - \beta_2)} ;$$

$$\lambda_c = a_c^{L*} e^{-i(\alpha_2 + \beta_2 + \gamma)} ; \qquad \lambda_c' = a_c^L \, e^{i(\alpha_2 + \beta_1 + \gamma)} ;$$

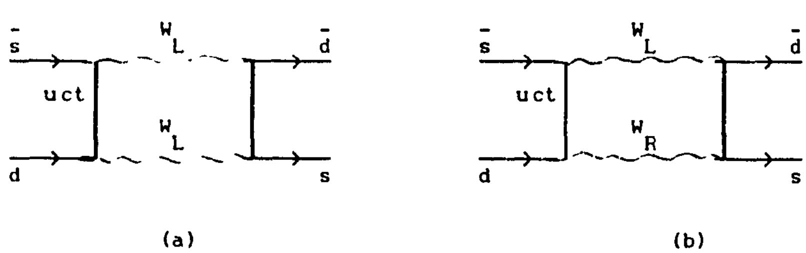
$$\lambda_t = a_t^{L*} e^{i(\alpha_1 + \alpha_2 - \beta_2 - \gamma)} ; \qquad \lambda_t' = a_t^L \, e^{-i(\alpha_1 + \alpha_2 - \beta_1 - \gamma)} .$$

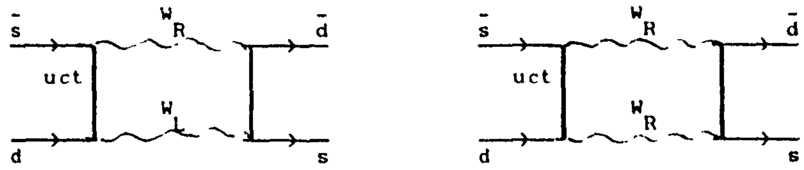
... (5.3)

In the calculations of *Chapter 4* we assumed $m_R \gg m_L \gg m_t \gg m_c \gg m_u \simeq 0$. But, as mentioned before, m_t may be heavy. Indeed, the recent results[94] of the CDF group at the Tevatron and the UA2 group suggest that $m_t > 60$ GeV. On the other hand, the minimal Higgs model[95] for $m_t \gg m_b$ gives $m_t \lesssim$ 200 GeV,[1] and we have at present, in spite of rather loose, experimental lower bounds on the mass of W_R, namely, $m_R \geq$ 300 GeV[90]. So, it is reasonable to base our calculation on the following assumptions:

$$m_R \gtrsim m_t , \; m_t \lessgtr m_L , \; m_R, m_L, m_t \gg m_c \gg m_u \simeq 0. \qquad (5.4)$$

5.2. The Leading Contributions of Box Graphs: ε_m





(b′)　　　　　　　　(c)

Fig.5.1, S-channel diagrams of the box-graphs that contribute to the mass matrix of K_L-K_S system.

The main contributions to ε_m are coming from the box diagrams in Fig.5.1 and the corresponding t-channel diagrams. The leading correction to the result obtained in the standard model arises from the diagrams in Fig.5.1(b) and (b′) and their t-channel counterparts which give the same contribution. Let the free quark amplitude be $A_{LR} = L_{ef}$ (effective Lagrangian) = H_{ef}(effective Hamiltonian). Thus, in the limit where the external quark momenta are assumed to be negligible compared to the loop momenta, we obtain, in the 't Hooft-Feynman gauge (color indices are already omitted)

$$A_{LR} = 4 \left(\frac{g}{\sqrt{2}}\right)^4 \int \frac{d^4q}{(2\pi)^4} \sum_{(i,j=c,t)} \lambda_i \lambda'_j \; \bar{s}\gamma_\mu L \frac{q_\alpha \gamma^\alpha + m_j}{q^2 - m_j^2} \gamma_\nu R d \cdot$$
$$\bar{s}\gamma_\rho L \frac{q_\beta \gamma^\beta + m_i}{q^2 - m_i^2} \gamma_\vartheta L d \; \frac{g^{\mu\vartheta} g^{\nu\rho}}{(q^2 - M_L^2)(q^2 - M_R^2)},$$
$$\equiv 2 \left(\frac{g}{\sqrt{2}}\right)^4 \hat{O}_{LR} \sum_{i,j} m_i m_j \lambda_i \lambda_j I_{ii}, \qquad (5.5)$$

where the factor 4 before the integral indicates that the contributions from counterparts have been included, and

$$L \equiv \frac{1+\gamma^5}{2}, \quad R \equiv \frac{1-\gamma^5}{2},$$

$$\hat{O}_{LR} \equiv (\bar{s}\gamma_\mu\gamma_\nu Rd)(\bar{s}\gamma^\mu\gamma^\nu Ld), \tag{5.6}$$

$$I_{ij} \equiv 4 \int \frac{d^4q}{(2\pi)^4} \frac{1}{(q^2-m_i^2)(q^2-m_j^2)(q^2-M_L^2)(q^2-M_R^2)}$$

$$= \frac{i}{4\pi^2 M_R^4} \left(\frac{\beta ln\beta}{(1-\beta)(\varepsilon_i-\beta)(\varepsilon_j-\beta)} + \frac{\varepsilon_i ln\varepsilon_i}{(1-\varepsilon_i)(\beta-\varepsilon_i)(\varepsilon_j-\varepsilon_i)} + \right.$$

$$\left. + \frac{\varepsilon_j ln\varepsilon_j}{(1-\varepsilon_j)(\beta-\varepsilon_j)(\varepsilon_i-\varepsilon_j)} \right), \tag{5.7}$$

where $\beta \equiv M_L^2 / M_R^2$, $\varepsilon_i \equiv m_i^2 / M_R^2$. (5.8)

Introducing new parameters $\eta_i \equiv m_i^2 / M_L^2$,noting that $\varepsilon_i \equiv \beta\eta_i$ we can rewrite (5.7) as follows

$$I_{ij} = \frac{i\beta}{4\pi^2 M_L^4} \left(\frac{ln\beta}{(1-\beta)(\eta_i-1)(\eta_j-1)} + \frac{\eta_i(ln\beta + ln\eta_i)}{(1-\beta\eta_i)(1-\eta_i)(\eta_j-\eta_i)} + \right.$$

$$\left. + \frac{\eta_j(ln\beta + ln\eta_j)}{(1-\beta\eta_j)(1-\eta_j)(\eta_i-\eta_j)} \right). \tag{5.7$'$}$$

Noting that I_{ij} is invariant under $i \leftrightarrow j$, furthermore,that $\beta \ll 1$ (see eq.2,27),and that $\eta_c \ll 1$, $\eta_t \gg \eta_c$, we have

$$I_{tc} = I_{ct} \simeq \frac{i\beta}{4\pi^2 M_L^4} \left(\frac{\eta_c}{\eta_t} ln\beta\eta_c + \frac{ln\eta_t + \beta\eta_t ln\beta}{(\eta_t-1)(1-\beta\eta_t)} \right),$$

$$I_{cc} \simeq \frac{-i\beta}{4\pi^2 M_L^4} \left(\ln\eta_c + 1 \right),$$

$$I_{tt} \simeq \frac{i\beta}{4\pi^2 M_L^4} \cdot \frac{1}{1-\eta_t} \left(\frac{\ln\beta}{1-\eta_t} - \frac{1+\ln\beta\eta_t}{1-\beta\eta_t} - \frac{(1-2\beta\eta_t)\eta_t \ln\beta\eta_t}{(1-\eta_t)(1-\beta\eta_t)^2} \right). \tag{5.9}$$

On the other hand, noting that

$$\gamma^\mu\gamma^\nu = g^{\mu\nu} - i\sigma^{\mu\nu}, \text{ and } \gamma^5\sigma^{\mu\nu} = \sigma^{\mu\nu}\gamma^5 = \frac{i}{2}\,\varepsilon^{\alpha\beta\mu\nu}\,\sigma_{\alpha\beta}\ ,$$

we can prove that

$$\hat{O}_{LR} \equiv (\bar{s}\gamma_\mu\gamma_\nu Rd)(\bar{s}\gamma^\mu\gamma^\nu Ld) = 4\ O_{LR}\ , \tag{5.10}$$

with $O_{LR} \equiv (\bar{s}Ld)(\bar{s}Rd)$. In addition to $\hat{O}_{LR}$ we need to evaluate the following two operators which we encounter in calculations of A_{LL} and A_{RR}:

$$\hat{O}_{LL} \equiv (\bar{s}\gamma_\mu\gamma_\nu\gamma_\alpha Ld)(\bar{s}\gamma^\alpha\gamma^\nu\gamma^\mu Ld), \text{and}$$

$$\hat{O}_{RR} \equiv (\bar{s}\gamma_\mu\gamma_\nu\gamma_\alpha Rd)(\bar{s}\gamma^\alpha\gamma^\nu\gamma^\mu Rd).$$

Similarly we can prove that

$$\hat{O}_{LL} = 4\ O_{LL}\ , \text{ and } \hat{O}_{RR} = 4\ O_{LL}\ , \tag{5.11}$$

$$\text{with } O_{LL} \equiv (\bar{s}\gamma_\mu Ld)(\bar{s}\gamma^\mu Ld). \tag{5.12}$$

The equations (5.11) hold, because we cannot distinguish between $\hat{O}_{LL}$ and $\hat{O}_{RR}$ in the case of pure left-handed or pure right-handed current. Moreover, we have to evaluate the matrix elements $M_{LL} \equiv \langle\bar{K}^0|O_{LL}|K^0\rangle$ and $M_{LR} \equiv \langle\bar{K}^0|O_{LR}|K^0\rangle$, which can be done with the aid of vacuum saturation as follows[48,70,99]:

$$M_{LL} \simeq M_{LL}^{vac} = \frac{2}{3} f_k^2 \, m_k^2 \, / \, 2m_k \, , \tag{5.13}$$

where f_k is the decay constant of kaon.

Using the divergence equation:

$$\bar{\psi}_1 \gamma_5 \psi_2 = -i \partial_\mu (\bar{\psi}_1 \gamma^\mu \gamma_5 \psi_2) / (m_1 + m_2) \, ,$$

we obtain

$$M_{LR} \simeq M_{LR}^{vac} = \frac{1}{2} [\, m_k^2 \, / \, (m_s + m_d)^2 + \frac{1}{6} \,] \, f_k^2 \, m_k^2 \, / \, 2m_k$$

$$\simeq 7.7 M_{LL}^{vac} \, , \tag{5.14}$$

where we have used the current-mass

m_s = 150 MeV and m_d = 7 MeV.

Substituting the above results into eqn.(5.5), we obtain

$$A_{LR} \simeq \frac{-(8 \times 7.7) \, G_F^2 \, M_L^2 \, O_{LL}}{\pi^2} \, \beta \, e^{i(\beta_1 - \beta_2)} \, \{ \, |a_c^L|^2 \, \eta_c (1 + ln\eta_c) -$$

$$- |a_t^L|^2 \frac{\eta_t}{1-\eta_t} \, [\, \frac{ln\beta}{1-\eta_t} - \frac{1 + ln\beta\eta_t}{1-\beta\eta_t} - \frac{(1-2\beta\eta_t) \, \eta_t \, ln\eta_t}{(1-\eta_t)(1-\beta\eta_t)^2} \,] \, -$$

$$- (a_c^L \, a_t^{L*} \, e^{i(\alpha_1 + 2\alpha_2)} + c.c.) \sqrt{\eta_c \eta_t} \, [\, \frac{\eta_c}{\eta_t} \, ln\beta\eta_c \, -$$

$$- \frac{ln\eta_t + \beta\eta_t \, ln\beta}{(1-\eta_t)(1-\beta\eta_t)} \,] \, \} . \tag{5.15}$$

Similarly, we can get

$$A_{LL} \simeq \frac{-G_F^2 M_L^2 O_{LL}}{\pi^2} \left((a_c^L)^2 (1 + 2\eta_c \ln\eta_c) + 2 a_c^L a_t^L \frac{1}{\eta_t} \times \right.$$

$$\times \left[\eta_c^2 \ln\frac{1}{\eta_c} - \frac{1}{2} + \frac{1}{(\eta_t - 1)^2} (\eta_t^2 \ln\eta_t - \frac{\eta_t^2 - 1}{2})\right] +$$

$$\left. + \frac{(a_t^L)^2}{(\eta_t - 1)^3} (\eta_t^2 - 2\eta_t \ln\eta_t - 1) \right), \qquad (5.16)$$

$$A_{RR} \simeq \frac{-G_F^2 M_L^2 O_{LL}}{\pi^2} \beta \left((a_c^R)^2 (1 + 2\beta\eta_c \ln\beta\eta_c) + \frac{a_c^R a_t^R}{\beta\eta_t} \times \right.$$

$$\times \left[-2\beta^2 \eta_c^2 \ln(\beta\eta_c) - 1 + \frac{1}{(1-\beta\eta_t)^2} (2\beta^2 \eta_t^2 \ln(\beta\eta_t) - \right.$$

$$\left. - \beta^2 \eta_t^2 + 1)\right] - \left. \frac{(a_t^R)^2}{(1-\beta\eta_t)^3} [\beta^2 \eta_t^2 - 2\beta\eta_t \ln(\beta\eta_t) - 1] \right).$$

$$(5.17)$$

Thus total amplitude is

$$A_t = A_{LL} + A_{RR} + A_{LR} . \qquad (5.18)$$

This sum omits two classes of box graphs whose contributions are small. First are the graphs, allowed under the assumption of mixing between W_L and W_R. These are proportional to $\tan^2\zeta < (0.06)^2$ (because $W_L = W_1\cos\zeta - W_2\sin\zeta$, $W_R = W_1\sin\zeta + W_2\cos\zeta$). Because of our choice of gauge , there is also the set of graphs wherein one or both of the W's are replaced by unphysical scalars. These graphs, however, are suppressed by the mass-dependent scalar couplings[96].

In what follows we shall calculate the corrections to the Standard Model. Noting that $ImA_{LL}\ /\ \Delta m$ should be the result in Standard Model, thus we have

$$\varepsilon_m = 2\,\frac{ImM_{12}}{\Delta m}$$

$$= 2\left[\frac{Im\langle\bar{K}^0|A_{LL}|K^0\rangle}{\Delta m} + \frac{Im\langle\bar{K}^0|A_{RR}|K^0\rangle}{\Delta m} + \frac{Im\langle\bar{K}^0|A_{LR}|K^0\rangle}{\Delta m}\right]$$

$$\equiv \varepsilon_m^{STD}\,(1+\rho+\rho'), \qquad (5.19)$$

$$\text{with } \rho \equiv \frac{Im\langle\bar{K}^0|A_{RR}|K^0\rangle}{Im\langle\bar{K}^0|A_{LL}|K^0\rangle}, \text{ and } \rho' \equiv \frac{Im\langle\bar{K}^0|A_{LR}|K^0\rangle}{Im\langle\bar{K}^0|A_{LL}|K^0\rangle}. \qquad (5.20)$$

Omitting the relatively small quantities, we have

$$\rho \simeq \frac{\beta\ c_1c_3\ sin2(\beta_1-\beta_2)\ \{2c_2^2\beta\eta_c ln(\beta\eta_c) + (1-\beta\eta_t)^{-2}(1-c_2^2\beta\eta_t)^2 + 2s_2^2\ \beta\eta_t\ ln(\beta\eta_t)\ (1-\beta\eta_t)^{-3}(1-c_2^2\beta\eta_t)\}}{2s_2c_2s_3\ sin\delta\ \{c_2^2\eta_c\ ln\eta_c + (1-\eta_t)^{-2}(c_2^2\eta_t^2 - \eta_t - s_2^2) - (1-\eta_t)^{-3}\ (1-c_2^2\eta_t + s_2^2)\}}, \qquad (5.21)$$

$$\rho' \simeq \frac{(8\times 7.7)\beta\ sin(\beta_1-\beta_2)\ c_1c_3\{\ s_1^2c_1c_3\ (\ c_2^4\eta_c\ [1+ln\eta_c] - s_2^4\eta_t(1-\eta_t)^{-2}(1-\beta\eta_t)^{-2}[-1 + (1-\beta ln\beta)\eta_t - \beta(1-ln\beta)\eta_t^2 -}{2s_2c_2s_3\ sin\delta\ \{c_2^2\eta_c\ ln\eta_c + (1-\eta_t)^{-2}(c_2^2\eta_t^2 - \eta_t - s_2^2) - (1-\eta_t)^{-3}\ (1-c_2^2\eta_t + s_2^2)\}}$$

$$\times \frac{- (1 - \beta\eta_t - \beta\eta_t^2) ln\eta_t]) - 2[s_1^2 c_1 s_2^2 c_3 \; cos(\alpha_1 + 2\alpha_2) -}{} \times$$

$$\times \frac{- s_3 \; sin\delta \; sin(\alpha_1 + 2\alpha_2)] \sqrt{\eta_c/\eta_t} \; (1 - \eta_t)^{-1}(1 - \beta\eta_t)^{-1} \cdot}{} \times$$

$$\times \frac{[\beta(\eta_c ln\eta_c - ln\beta)\eta_t^2 + (ln\beta + ln\eta_c)(1 - \eta_t)\eta_c - \eta_t ln\eta_t])}{}. \tag{5.22}$$

It is apparently that ρ and ρ' depend on the mass of the top quark . In what follows we shall consider several extreme cases:

i).$m_t \simeq M_L$, i.e. $\eta_t \simeq 1$. In this case, we have

$$\rho \simeq \frac{\beta \; c_1 c_3 \; sin2(\beta_1 - \beta_2) \; \{[1 + 2c_2^2 \beta\eta_c ln(\beta\eta_c)] + [2\beta(s_2^2 ln\beta -}{2s_2 c_2^3 s_3 \; sin\delta \; \eta_c(1 + ln\eta_c)} \times$$

$$\times \frac{- c_2^2)]\eta_t - [2s_2^2 c_2^2 \; \beta^2 \; ln\beta]\eta_t^2\}}{}, \tag{5.23}$$

$$\rho' \simeq \frac{(8\times 7.7)\beta \; sin(\beta_1 - \beta_2) \; c_1 c_3 \{ \; s_1^2 c_1 c_3 \; (\; c_2^4 \eta_c \; [1 + ln\eta_c] -}{2s_2 c_2^3 s_3 \; sin\delta \; \eta_c(1 + ln\eta_c)} \times$$

$$\times \frac{- s_2^4[- \frac{11}{16} + \frac{15}{8}(1 - \beta ln\beta)\eta_t - \frac{1}{8}\beta(11 - 35 ln\beta)\eta_t^2]) - \sqrt{\eta_c \eta_t} \; \cdot}{} \times$$

$$\times \frac{[s_1^2 c_1 s_2^2 c_3 \; cos(\alpha_1 + 2\alpha_2) - s_3 \; sin\delta \; sin(\alpha_1 + 2\alpha_2)][6\beta^2(\eta_c}{} \times$$

$$\times \frac{ln\eta_c - ln\beta)\eta_t^2 - 2\beta[4\eta_c ln\eta_c - 2ln\beta + 1]\eta_t + [\eta_c ln(\beta\eta_t) + 1]\}}{}.$$

(5.24)

Noting that $\eta_c \sim 0.00035$, $\beta \sim 0.0023$, thus $O(|ln\beta|) \sim \sim O(|ln\eta_c|) \sim O(1)$, and that $s_2^2 \ll c_2^2$, we can further simplify eqns.(5.23) and (5.24) as follows:

$$\rho \simeq \frac{\beta\ c_1 c_3\ sin2(\beta_1-\beta_2)\ \{1 - 2c_2^2\beta\eta_t - 2s_2^2c_2^2\beta^2 ln\beta\ \eta_t^2\ \}}{2s_2c_2^3s_3\ sin\delta\ \eta_c(1 + ln\eta_c)},$$

(5.23)'

$$\rho' \simeq \frac{(8\times7.7)\beta\ sin(\beta_1-\beta_2)\ c_1c_3\{\ s_1^2c_1c_3\ (\ c_2^4\eta_c\ [1 + ln\eta_c] -}{2s_2c_2^3s_3\ sin\delta\ \eta_c(1 + ln\eta_c)}\times$$

$$\times\frac{- s_2^4[- \frac{11}{16} + \frac{15}{8}\ \eta_t - \frac{1}{8}\beta(11 - 35ln\beta)\eta_t^2]) + 2\sqrt{\eta_c\eta_t}\ \cdot}{}\times$$

$$\times\frac{[s_1^2c_1s_2^2c_3\ cos(\alpha_1 + 2\alpha_2) - s_3\ sin\delta\ sin(\alpha_1 + 2\alpha_2)][3\beta^2\ ln\beta}{}\times$$

$$\times\frac{\eta_t^2 + \beta\ (1 - 2ln\beta)\ \eta_t - 2]\}}{}. \qquad (5.24)'$$

It is easy seen that the dependence of ρ and ρ' on the top quark mass is based on a power series and is not logarithmic.

ii). $M_R \gg m_t \gg M_L$, i.e. $\eta_t \gg 1$ but $\beta\eta_t \ll 1$.

$$\rho \simeq \frac{\beta\ c_1 c_3\ sin2(\beta_1-\beta_2)\ (1 - 2c_2^2\beta\eta_t)}{2s_2 c_2^3 s_3\ sin\delta}, \tag{5.25}$$

$$\rho' \simeq \frac{(8\times7.7)\beta\ sin(\beta_1-\beta_2)\ c_1c_3(\ s_1^2c_1c_3\ (\ c_2^4\eta_c\ [1 + ln\eta_c] -}{2s_2c_2^3s_3\ sin\delta\ \ \eta_c(1 + ln\eta_c)}\times$$

$$\times\underline{- s_2^4[1 - \beta\eta_t\ (1 - ln\beta - ln\eta_t)]) - 2\beta\sqrt{\eta_c\eta_t}\ \cdot}\times$$

$$\times\underline{[s_1^2c_1s_2^2c_3\ cos(\alpha_1 + 2\alpha_2) - s_3\ sin\delta\ sin(\alpha_1 + 2\alpha_2)]}. \tag{5.26}$$

iii). $m_t \simeq M_R$, i.e., $\beta\eta_t \simeq 1$.

$$\rho \simeq \frac{\beta\ c_1c_3\ sin2(\beta_1-\beta_2)[c_2^2 + 2\beta\eta_c\ ln(\beta\eta_c)]}{2s_2c_2s_3\ sin\delta}, \tag{5.27}$$

$$\rho' \simeq \frac{(8\times7.7)\beta\ sin(\beta_1-\beta_2)\ c_1c_3(\ s_1^2c_1c_3\ (\ c_2^4\eta_c\ [1 + ln\eta_c] +}{2s_2c_2s_3\ sin\delta}\times$$

$$\times\underline{+ s_2^4[- \frac{15}{8}\beta\ \eta_t + \frac{1}{8}\beta(11 + 3ln\beta) + (\frac{59}{8} - 8ln\beta)\beta^2\eta_t^2]) -}\times$$

$$\times\underline{-[s_1^2c_1s_2^2c_3\ cos(\alpha_1 + 2\alpha_2) - s_3\ sin\delta\ sin(\alpha_1 + 2\alpha_2)]}\times$$

$$\frac{\beta^2 \sqrt{\eta_c \eta_t}\ ln\beta\ (-3\beta\eta_t^2 + 2\eta_t - 1) \}}{} . \qquad (5.28)$$

From eq. (5.16), we have

$$\varepsilon_m^{STD} = \frac{\text{Im}M_{12}}{\text{Re}M_{12}} \simeq \frac{2s_2 s_3\ sin\delta\ \{\ c_2^2(1 + 2\eta_c ln\eta_c) + [2c_2^2(1 - ln\eta_t)\cdot}{} \times$$

$$\frac{\eta_t^3 - 2(c_2^2 - 2s_2^2 ln\eta_t)\eta_t^2 + 2c_2^2\eta_t + s_2^2]/[2\eta_t(\eta_t - 1)^3]\ \}}{} . \qquad (5.29)$$

5.3. The Contribution of Penguin Graphs: ε′ or ξ

When the mass of the top quark is heavy, as mentioned before, γ-penguin and Z^0-penguin have to be taken into account. Thus ξ will be determined by the following penguin graphs[97,98]:

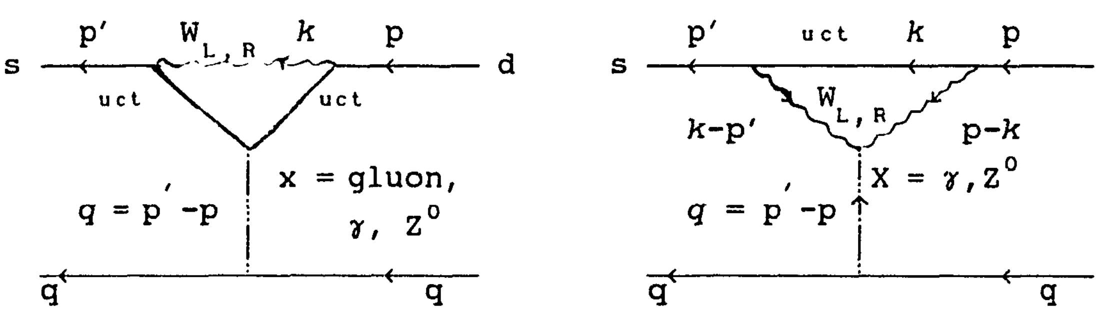


Fig.5.2 (a) (b)

The loop integral is, in general, divergent. After renormalization the divergent part will be absorbed into the renormalization parameters and the finite difference of the loop integral will remain. The effective hamiltonian corresponding to the process will be determined only by the finite difference. Therefore we introduce symbol *Fin* I that denotes the finite difference of the loop integral after renormalization.

As in section 4.2, we shall work in the Feynman gauge and neglect the unphysical field as well as the mixing between left- and right-handed currents. Because the right-handed Z particle Z_2 is very heavy, the contribution coming from Z_2-penguin graphs is assumed to be negligible compared to that from the left-handed Z (that is Z^0) penguin graphs. In addition, we shall also omit all external momenta and u quark contribution.

At low energy case, for any 4-momentum $p \equiv (p_0, \vec{p})$ we have $p_0^2 \gg \vec{p}^2$. Noting that $m_s^2 \gg m_d^2$, we obtain

$$q^2 = (p' - p)^2 \simeq m_s^2 , \tag{5.30}$$

$$\frac{1}{q^2 - M_Z^2} \simeq - \frac{1}{M_Z^2} , \quad (M_Z^2 \gg q^2) \tag{5.31}$$

where q^2 is the momentum transferred by the propagator x(=gluon, or photon, or Z^0-boson).

Let $x_i \equiv m_i^2 / M_L^2$, i = c,t. (5.32)

For simplicity, we shall write $F(x_i)$ in the form $F_j(x)$ or

$F^j(x)$. In what follows we shall calculate the lowest order radiative correction.

For figure (a), in the case of $x = \gamma$ and left-handed current, the loop integral is

$$I_L = -\frac{1}{2} eg^2 \sum_j a_j^L \int \frac{d^4k}{(2\pi)^4} \frac{\gamma_\mu L(k_\rho\gamma^\rho + m_j)\gamma_\alpha(k_\lambda\gamma^\lambda + m_j)\gamma^\mu L}{(k^2 - m_j^2)^2(k^2 - M_L^2)}$$

$$= 2eg^2\sum_j a_j^L \int \frac{d^4k}{(2\pi)^4} \int_0^1 dz \frac{z[(\gamma^\mu\gamma_\alpha\gamma^\nu)k_\mu k_\nu + m_j^2\,\gamma_\alpha]\,L}{[k^2 + (M_L^2 - m_j^2)z - M_L^2]^3}$$

$$\equiv 2eg^2\sum_j a_j^L\, I_\alpha^j(x).$$

With the aid of dimensional regularization, we can find the finite part of $I_\alpha^j(x)$ (cut off at the scale M_L^2).

$$Fin\ I_\alpha^j(x) = \frac{i\pi^2}{2(2\pi)^4}\gamma_\alpha\, L \int_0^1 dz\{z\ln[(x-1)z+1] - \frac{xz}{(x-1)z+1}\}$$

$$\equiv \frac{i\pi^2}{2(2\pi)^4}\gamma_\alpha\, L\, A_j(x),$$

where $A_j(x) = \frac{3-5x}{4(x-1)} + \frac{x^2}{2(x-1)^2}\ln x.$ (5.33)

Thus we obtain

$$Fin\ I_L = \frac{ieg^2}{16\pi^2}\gamma_\alpha\, L \sum_j a_j^L\, A_j(x),$$

$$H_\gamma^a(L) = \frac{\sqrt{2}\alpha}{m_s^2\pi} G_F M_L^2\, \bar{s}\gamma_\alpha L d\, \bar{f}\gamma^\alpha f \sum_j a_j^L\, A_j(x), \quad (5.34)$$

$$H_\gamma^a(R) = \frac{\sqrt{2}\alpha}{m_s^2\pi} G_F M_L^2 \bar{s}\gamma_\alpha L d \; \bar{f}\gamma^\alpha f \sum_j a_j^R A_j(\beta x), \tag{5.35}$$

In view of the parity invariance in the L-R model, we have replaced $\bar{s}\gamma_\alpha Rd$ with $\bar{s}\gamma_\alpha Ld$ in eq. (5.33) (Noting that they are all pure left- or right-handed current). Here A(x) is a dimensionless function.

Similarly , and taking the trace over color space,

$\mathrm{Tr}\, \frac{\lambda^a}{2}\frac{\lambda^b}{2} = \frac{1}{2}\delta_{ab}$, we obtain for gluon-penguins

$$H_g^a(L) = \frac{\alpha_s G_F M_L^2}{\sqrt{2} m_s^2 \pi} \bar{s}\gamma_\alpha L d \; \bar{f}\gamma^\alpha f \sum_j a_j^L A_j(x), \tag{5.36}$$

$$H_g^a(R) = \frac{\alpha_s G_F M_L^2}{\sqrt{2} m_s^2 \pi} \bar{s}\gamma_\alpha L d \; \bar{f}\gamma^\alpha f \sum_j a_j^L A_j(\beta x), \tag{5.37}$$

For the Z^0-penguin, the loop integration in the case of the left-handed current is

$$I_L = \frac{-eg^2}{2\sin\vartheta_W \cos\vartheta_W} \sum_j a_j^L \int \frac{d^4k}{(2\pi)^4} \frac{\gamma_\mu L(k_\rho\gamma^\rho + m_j)\gamma_\alpha (C_L L + C_R R)}{(k^2 - m_j^2)^2 (k^2 - M_L^2)} \times$$

$$\times (k_\lambda \gamma^\lambda + m_j)\gamma^\mu L$$

$$= \frac{2eg^2}{\sin\vartheta_W \cos\vartheta_W} \sum_j a_j^L \int_0^1 dz \int \frac{d^4k}{(2\pi)^4} \frac{z[C_L \gamma^\mu \gamma_\alpha \gamma^\nu k_\mu k_\nu + C_R m_j^2 \gamma_\alpha]\; L}{[k^2 + (M_L^2 - m_j^2)z - M_L^2]^3}$$

$$= \frac{2eg^2}{\sin\vartheta_W \cos\vartheta_W} \sum_j a_j^L I_\alpha^j(x).$$

$$Fin\ I^{j}_{\alpha}(x) = \frac{i\pi^2}{2(2\pi)^4}\gamma_\alpha L \int_0^1 dz\{C_L z ln[(x-1)z+1] - \frac{C_R xz}{(x-1)z+1}\}$$

$$= \frac{i\gamma_\alpha L}{32\pi^2} B_j(x),$$

$$\text{with } B_j(x) = \frac{(3-x)C_L - 4xC_R}{4(x-1)} + \frac{x(C_L x - 1)}{2(x-1)^2} ln x, \qquad (5.38)$$

$$\text{where } C_L = \frac{1}{2} - \frac{2}{3} sin^2\vartheta_W \text{ and } C_R = -\frac{2}{3}sin^2\vartheta_W. \qquad (5.39)$$

Thus we obtain

$$H^a_z(L) = -\frac{2G_F^2M_L^2}{\pi^2 cos^2\vartheta_W}\frac{M_L^2}{M_Z^2}\bar{s}\gamma_\alpha Ld\ \bar{f}\gamma^\alpha(C_L L+C_R R)f \sum_j a^L_j B_j(x)$$

$$= -\frac{2G_F^2M_L^2C_L}{\pi^2 cos^2\vartheta_W}\frac{M_L^2}{M_Z^2}\bar{s}\gamma_\alpha Ld\ \bar{f}\gamma^\alpha f \sum_j a^L_j B_j(x). \qquad (5.40)$$

In the last step we used parity conservation, and omitted the L in the $\bar{f}\gamma^\alpha Lf$. Similarly, we have

$$H^a_z(R) = -\frac{2G_F^2M_L^2C_R}{\pi^2 cos^2\vartheta_W}\frac{M_L^2}{M_Z^2}\bar{s}\gamma_\alpha Ld\ \bar{f}\gamma^\alpha f \sum_j a^R_j B_j(\beta x), \qquad (5.41)$$

$$\text{with } C_j(\beta x) = \frac{(3-\beta x)C_R - 4\beta xC_L}{4(\beta x-1)} + \frac{\beta x[(\beta x-2)C_R+2C_L]}{2(\beta x-1)^2} ln\beta x. \qquad (5.42)$$

In what follows we consider figure (b). For the γ-penguin, in the case of left-handed current, Feynman rules give

$$I_L = \frac{ieg^2}{2} \sum_j a_j^L \int \frac{d^4k}{(2\pi)^4} \frac{\gamma_\mu L(k_\rho \gamma^\rho + m_j)\gamma_\nu \; L\,[g^{\mu\nu}(2k-p'-p)^\alpha +}{(k^2-m_j^2)[(p-k)^2 - M_L^2][(p'-k)^2 - M_L^2]}$$

$$\times \frac{+ g^{\nu\alpha}(p'-k)^\mu + g^{\alpha\mu}(p-k)^\nu}{}$$

$$= \frac{ieg^2}{2} \sum_j a_j^L \int \frac{d^4k}{(2\pi)^4} \frac{-2k_\lambda \gamma^\lambda (2k-p'-p)^\alpha + (p'-k)_\lambda \gamma^\lambda k_\rho \gamma^\rho \gamma^\alpha +}{(k^2-m_j^2)[(p-k)^2 - M_L^2][(p'-k)^2 - M_L^2]}$$

$$\times \frac{+ \gamma^\alpha \; k_\rho \gamma^\rho \; (p-k)\gamma_\lambda^\lambda}{} \; L$$

$$= \frac{ieg^2}{2} \sum_j a_j^L \, I_\alpha^j \; .$$

Omitting p′ and p in the denominator and using Feynman parameterization, we obtain

$$I_\alpha^j = -4 \int_0^1 dz \int \frac{d^4k}{(2\pi)^4} \frac{z\{[2g^{\mu\nu}\gamma^\alpha + \gamma^\mu \gamma^\alpha \gamma^\nu]k_\mu k_\nu + \quad\}L}{[k^2 - (M_L^2-m_j^2)z - m_j^2]^3},$$

where the dots denote the term linear in k, which vanishes when integrated over k. Also we used the relation, $2p^\alpha = p_\lambda \gamma^\lambda \; \gamma^\alpha + \gamma^\alpha p_\lambda \gamma^\lambda$. Thus

$$Fin \; I_\alpha^j = \frac{i3\gamma_\alpha L}{8\pi^2} \int_0^1 dz \; (z \; ln[x-(x-1)z])$$

$$= \frac{i\gamma_\alpha L}{8\pi^2} D_j(x),$$

$$\text{with } D_j(x) = \frac{3(1-3x)}{4(x-1)} + \frac{3x^2}{2(x-1)^2} \ln x. \tag{5.43}$$

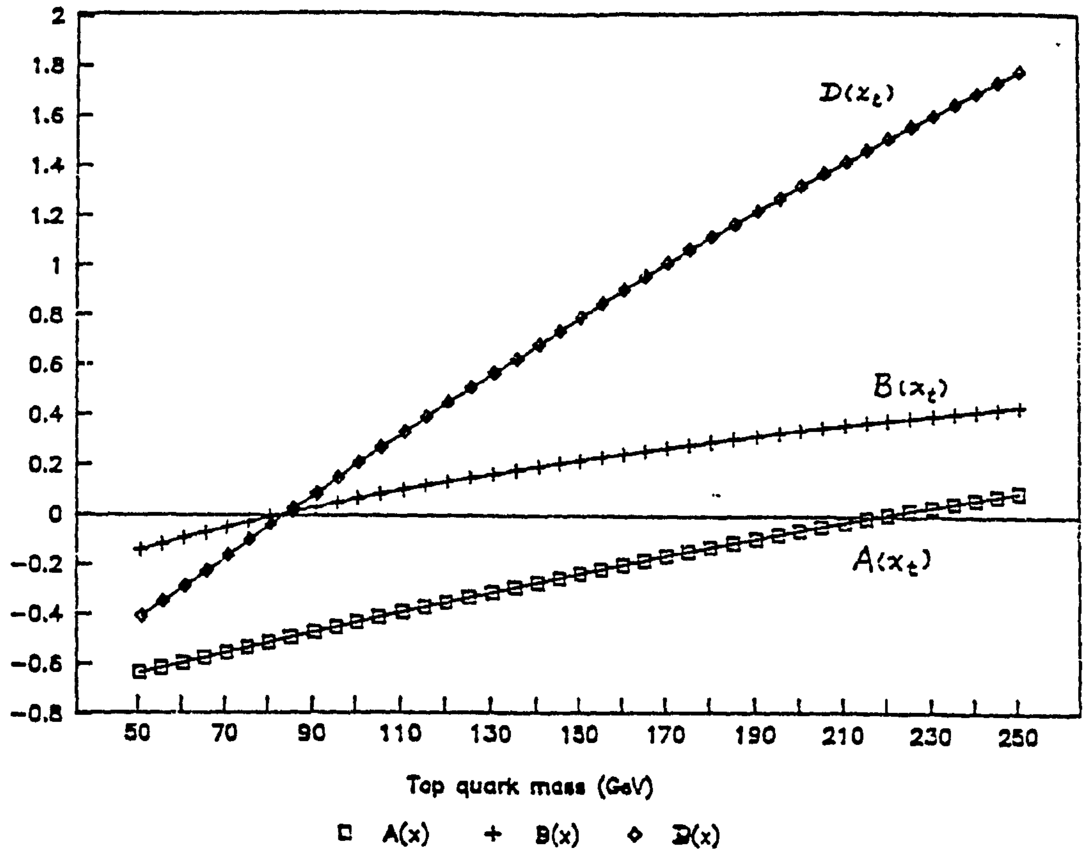


Fig.5.3, $A(X_t)$, $B(X_t)$ and $D(X_t)$ as functions of m_t for M_W = 80 GeV.

Finally, we obtain

$$H^b_\gamma(L) = \frac{\sqrt{2}\alpha}{m_s^2 \pi} G_F M_L^2 \, \bar{s}\gamma_\alpha L d \, \bar{f}\gamma^\alpha f \sum_j a_j^L D_j(x), \tag{5.44}$$

$$H^b_\gamma(R) = \frac{\sqrt{2}\alpha}{m_s^2 \pi} G_F M_L^2 \, \bar{s}\gamma_\alpha L d \, \bar{f}\gamma^\alpha f \sum_j a_j^R D_j(\beta x), \tag{5.45}$$

$$H^b_Z(L) = - \frac{2G_F^2 M_L^2 C_L}{\pi^2} \frac{M_L^2}{M_Z^2} \bar{s}\gamma_\alpha L d \, \bar{f}\gamma^\alpha f \sum_j a_j^L D_j(x). \tag{5.46}$$

$$H_Z^b(R) = -\frac{2G_F^2M_L^2C_R}{\pi^2}\frac{M_L^2}{M_Z^2}\bar{s}\gamma_\alpha Ld\ \bar{f}\gamma^\alpha f\ \sum_j a_j^R\ D_j(\beta x). \qquad (5.47)$$

$$\text{Let } y \equiv \pi^{-1}G_FM_L^2\ \bar{s}\gamma_\alpha Ld\ \bar{f}\gamma^\alpha f. \qquad (5.48)$$

Thus the contributions from the above penguins are as follows:

$$H_g = H_g^L + H_g^R = \frac{\alpha_s}{\sqrt{2}m_s^2}\ y\ \sum_j [a_j^L\ A_j(x) + (\ L \longrightarrow R, x \longrightarrow \beta x\ )].$$

$$H_\gamma = H_\gamma^a(L) + H_\gamma^a(R) + H_\gamma^b(L) + H_\gamma^b(R)$$

$$= \frac{\sqrt{2}\alpha}{m_s^2}\ y\ \sum_j \{a_j^L\ [A_j(x) + D_j(x)] + (\ L \longrightarrow R, x \longrightarrow \beta x\ )\}.$$

Noting that $M_L^2\ /\ M_Z^2 = cos^2\vartheta_W$, we have

$$H_Z = H_Z^a(L) + H_Z^a(R) + H_Z^b(L) + H_Z^b(R)$$

$$= -\frac{2G_F}{\pi}\ y\ \sum_j\ \{a_j^L\ C_L[B_j(x)+cos^2\vartheta_W\ D_j(x)]+(L \longrightarrow R, x \longrightarrow \beta x)\}.$$

$$H_{tot} = H_g + H_\gamma + H_Z$$

$$= y\ \sum_j\ \{a_j^L[\frac{\alpha_s+2\alpha}{\sqrt{2}m_s^2}\ A_j(x) - C_L\frac{2G_F}{\pi}\ B_j(x) + (\frac{\sqrt{2}\alpha}{m_s^2} - C_L\frac{2G_F}{\pi}$$

$$\times cos^2\vartheta_W)\ D_j(x)] + (L \longrightarrow R, x \longrightarrow \beta x)\}.$$

Let $a \equiv \dfrac{\alpha_s + 2\alpha}{\sqrt{2} m_s^2}$, $\quad b \equiv C_L \dfrac{2G_F}{\pi}$, $\quad c \equiv C_R \dfrac{2G_F}{\pi}$,

$$d \equiv \frac{\sqrt{2}\alpha}{m_s^2} - C_L \frac{2G_F}{\pi} \cos^2\vartheta_W , \quad e \equiv \frac{\sqrt{2}\alpha}{m_s^2} - C_R \frac{2G_F}{\pi} \cos^2\vartheta_W . \qquad \text{(5.49)}$$

All these coefficients are constant. Let

$$E_j(x) \equiv aA_j(x) - bB_j(x) + dD_j(x),$$

$$F_j(x) \equiv aA_j(\beta x) - cB_j(\beta x) + eD_j(\beta x).$$

Noting that $0 \ll x_c \ll 1$, $0 \ll \beta \ll 1$, we have

$x \ln x \xrightarrow{x \longrightarrow 0+} 0$, and $\beta x \ln \beta x \xrightarrow{\beta x \longrightarrow 0+} 0$. Thus

$$E_c(x) \simeq - \tfrac{3}{4} (a - bC_L + d) = \textit{constant}, \text{ denoted by } E_c ;$$

$$F_j(x) \simeq - \tfrac{3}{4} (a - cC_R + e) = \textit{constant}, \text{ denoted by } F_c .$$

We can , therefore, write

$$H_{tot} = y \, (E_c a_c^L + F_c a_c^R + E_t(x) \, a_t^L + F_t(\beta x) \, a_t^R),$$

This depends only on the top quark mass m_t. Separating the real part from the imaginary part, we obtain

$$\text{Im } H_{tot} \simeq y \, s_1 \{ s_2 c_2 s_3 \sin\delta [E_c - E_t(x)] + c_1 \sin(\beta_1 - \beta_2) \cdot$$

$$[c_2^2 F_c - s_2^2 F_t(\beta x)] \}.$$

$$\text{Re } H_{tot} \simeq y \, s_1 c_1 c_3 \{ c_2^2 [E_c + F_c \cos(\beta_1 - \beta_2)] + s_2^2 [E_t(x) +$$

$$+ \cos(\beta_1-\beta_2) F_t(\beta x)] \} .$$

Because we considered only pure left- or right-handed currents, we can, like Gilman and Wise[92], assume that the result can be parameterized by the fractional contribution, f (which is not far from unity[91]), of the real part of the H_{tot} to $K \longrightarrow 2\pi(I=0)$,

$$\xi = \frac{\mathrm{Im}\ A_0}{\mathrm{Re}\ A_0} = f \frac{\mathrm{Im}\ H_{tot}}{\mathrm{Re}\ H_{tot}}$$

$$= f \frac{s_2 c_2 s_3 \sin\delta [E_c - E(x_t)] + c_1 \sin(\beta_1-\beta_2) [c_2^2 F_c - s_2^2 F(\beta x_t)]}{c_1 c_3 \{ [c_2^2 E_c + s_2^2 E(x_t)] + \cos(\beta_1-\beta_2) [c_2^2 F_c + s_2^2 F(\beta x_t)] \}} . \qquad (5.50)$$

In what follows we shall consider two extreme cases:

i). $m_t \simeq M_L$, we have $x_t \simeq 1$. In this case, the first term of $E(x_t) \longrightarrow \infty$, the second term of $E(x_t)$ has a finite limit; and $F(\beta x_t)$ has a finite limit too. Thus

$$\xi \simeq -f\ c_2\ s_3 \sin\delta\ /\ c_1 s_2\ c_3 . \qquad (5.51)$$

ii). $m_t \simeq M_R$, i.e., $\beta x_t \simeq 1$.

If we exchange $E(x_t)$ with $F(\beta x_t)$ in (*i*), then the conclusion of (*i*) doesn't change. Therefore

$$\xi \simeq -f \tan(\beta_1 - \beta_2)\ /\ c_3 . \qquad (5.52)$$

Conclusion

In chapter *3* we introduced a general formalism treating CP-violation in the neutral kaon system and derived the general expressions of ε' and ε. In chapter *4* we calculated ε'/ε under the assumption of $m_t \ll m_L$ and re-gained the results obtained by Ellis, Gilman and Wise, which is typical during the period of 1979–1989. Inchapter 5 without any limitation on the top quark mass (m_t), we calculated ε'/ε to the lowest order at low energy case in the Pseudo-manifestly Left-Right Symmetric Model. Our results have shown that

i). The ε_m or ξ (equivalently, ε' or ε) dependence on the top quark mass is as a power-series and not logarithmically.

ii). The ε_m or ξ doesn't depend on the CP-violation phase angle γ. The reason is that we assumed that the mixing between left- and right-handed currents was small and so could be neglected In the case of a pure left-handed current, $e^{i\gamma}$ is absent; and in the case of a pure right-handed current $e^{i\gamma}$ and $e^{-i\gamma}$ are present in pairs and cancel each other.

iii). Our results contain too many parameters: f, δ, α_1, α_2, β_1, and β_2, not known for the left-right model. Therefore, it is difficult to compare our results with experimental data directly.

Some modifications of the work done here are as follows.

i) Corrections should be made to all orders. This can be accomplished using the operator product expansion and renormalization group techniques. Such a calculation is very complicated. For higher order corrections we still expect that the mixing between left- and right-handed currents could be omitted because of the smallness of $tan^2\zeta$.

ii) δ, α_1, α_2, β_1, and β_2 should be determined independently of ε'/ε as is done found in the Standard Model (see secton **4.1**).

References

1. Peter, R., "Electroweak Interactions", Cambridge Univ.Press, 1990.
2. Halzen, F. et al., "Quarks and Leptons", John Wiley and Sons, 1984.
3. Ryder, L.H., "Quantum Field Theory", Cambridge Univ. Press, 1985.
4. Collins, P.D.B. et al., "Particle Physics and Cosmology", John Wiley and Sons, 1989.
5. Lee, T.D., "Particle Physics and Introduction to Field Theory", Harwood Academic Publishers, 1988.
6. Commins,et al., "weak Interactions of Leptons and Quarks", Cambridge Univ. Press, 1983.
7. Muta, T., "Foundations of Quantum Chromodynamics", World Scientific, 1987.
8. Pokorski, S., "Gauge Field Theories", Cambridge Univ. Press, 1987.
9. W. Heisenberg and W. Pauli, Z. Phys. **56**(1929)1; 59(1930)168
10. R. P. Feynman, Rev. Mod. Phys. **20**(1948)367.
11. G. Parisi and Y.Wu, Sci. Sinica **24**(1981)483.
12. Fujikawa, et al., Phys. Rev. **D6**(1972)2923.
13. G.'t Hooft et al, Nucl.Phys.**B44**(1972)189.
14. E.R.Speer, J.Math.Phys.**15**(1974)1.
15. T.-P. Cheng et al, "Gauge Theory of Elementary Particle Physics", Clarendon, Oxford.
16. S.L.Glashow, Nucl.Phys.**22**(1961)579.

17. S.Weinberg, Phys.Rev.Lett.**19**(1967)1264.

18. A.Salam, In *Elementary Particle Theory,* ed.Svartholm, W., Almquist and Wiskell, Stockholm.

19. N.Cabibbo, Phys.Rev.Lett.**10**(1963)531.

20. M.Kobayashi, Prog.Theor.Phys.**49**(1973)652.

21. C.Jarlskog, "New Phenomena in Lepton-Hadron Physics", ed.D.E.C.Fries & J.Wess, Plenum, New York & London.

22. L.Maiani, *Prog.1977 Int.Symp.on Lepton and Photon Interactions at High Energies,* DESY, Hamberg.

23. L.Wolfenstein, Phys.Rev.Lett.**51**(1983)1945.

24. Harari H.& Nir Y., Phys.Lett.**B**195(1987)586.

25. 't Hooft G., Nucl.Phys.**B**33(1971)173; **B**35(1971)167.

26. Taylor J.C., "Gauge Theories of Weak Interactions", Cambridge Univ.Press, Cambridge, UK, 1978.

27. Coleman S., "Aspects of Symmetry", Cambridge Univ.Press, Cambridge, 1985.

28. J.C.Pati et al, Phys.Rev.**D**10(1974)275.

29. R.N.Mohapatra et al, Phys.Rev.**D**11(1975)566.

30. R.N.Mohapatra et al, Phys.Rev.**D**11(1975)2558.

31. G.Senjanovic et al, Phys.Rev.**D**12(1975)1502.

32. R.N.Mohapatra, *New Frontiers in High Energy Physics,* ed.B.Kursunoglu et al, Plenum Press, 1978.

33. R.Cowsic et al, Phys.Rev.Lett.**29**(1972)669.

34. S.S.Gershtein et al, JETP Letters, **4**(1966)120.

35. H.Harari et al, Phys.Lett.**B**98(1981)269.,
O.W.Greenberg et al, Phys.Lett.**B**99(1981)339,
R.Barbieri et al, Phys.Lett.**B**105(1981)369,

R.Casalbuoni et al, Phys.Lett.**B103**(1981)113,
R.N.Mohapatra, Proceedings of Neutrino Mass Miniconference, Telemark, AIP publication.

36.R.N.Mohapatra et al, Phys.Lett.**B91**(1980)222,
A.Davidsm, Phys.Rev.**D20**(1979)776.

37.D.Chang, Nucl.Phys.**B214**(1983)435.

38.G.Branco, Nucl.Phys.**B221**(1983)317.

39.H.Albrecht et al, Phys.Lett.**B192**(1987)245.

40.M.Artuso et al, Phys.Rev.Lett.**62**(1989)2233.

41.R.N.Mohapatra, *Left-Right Symmetric Models of Weak Interactions,* in "Quarks, Leptons and Beyond", ed.H.Fritzsch, et al., Plenum Press, New York, 1985.

42.R.N.Mohapatra, *CP-Violation and Left-Right Symmetry,* in "CP Violation", ed C.Jarlskog, World Scientific Publishing Co.Pte.Ltd.1989.

43.R.N.Mohapatra et al, Phys.Rev.Lett.**44**(1980)912.

44.R.N.Mohapatra et al, Phys.Rev.**D23**(1981)165.

45.L.Durkin et al, Phys.Lett.**B166**(1986)436.

46.D.P.Stocker et al, Phys.Rev.Lett.**54**(1985)1887,
J.Carr et al, Phys.Rev.Lett.**51**(1983)627.

47.J.Donoghue et al, Phys.Lett.**B113**(1982)383.

48.G.Beall et al, Phys.Rev.Lett.**48**(1982)848.

49.R.N.Mohapatra et al, Phys.Rev.**D17**(1978)2642.

50.G.Ecker et al, Nucl.Phys.**B247**(1984)70.

51.M.Gronau et al, Phys.Lett.**B168**(1986)248.

52.G.Branco and M.Rebelo, CERN Preprint TH-4388/86.

53.R.N.Mohapatra et al, Phys.Rev.**D17**(1978)1876.

54. E.P.Wigner, Z.Phys.43(1927)624;Gött.Nachr.(1932)546 or
Group Theory, Academic Press, 1959, chapter 26.

55. H.A.Kramers, Proc.Acad.Sci.Amsterdam 40(1937)814.

56. G.Luders, Annals of Physics 2(1957)1.

57. T.D.Lee et al, Phys.Rev.104(1956)254.

58. C.S.Wu et al, Phys.Rev.105(1957)1413.

59. L.Landau, Nucl.Phys.3(1957)127,
T.D.Lee et al, Phys.Rev.105(1957)1671,
A.Salam, Nuovo Cimento 5(1957)1671,
R.Gatto, Phys.Rev.106(1957)168.

60. T.D.Lee et al, Phys.Rev.106(1957)340.

61. M.Gell-Mann et al, Phys.Rev.97(1955)1387.

62. J.H.Christenson et al, Phys.Rev.Lett.13(1964)138.

63. A.Abashian et al, Phys.Rev.Lett.13(1964)243.

64. J.W.Cronin, Rev.Mod.Phys.53(1981)373.

65. Wolfenstein, Ann.Rev.Nucl.Sci.36(1986)137.

66. Wolfenstein, Phys.Rev.Lett.13(1964)562.

67. J.F.Paula, "ε'/ε:Revew and Recent Progress", LBL-30619.

68. J.J.Aubert et al., Phys.Rev.Lett.33(1974)1404;
J.E.Augustin et al., Phys.Rev.Lett.33(1974)1406.

69. S.Weinberg, Phys.Rev.Lett.37(1976)657.

70. J.Ellis et al., Nucl.Phys.B109(1976)213

71. S.W.Herb et al., Phys.Rev.Lett.39(1977)252;
K.Ureno et al., Phys.Rev.Lett.42(1979)486

72. F.J.Gilman et al., Phys.Lett.83B(1979)83.

73. F.J.Gilman et al., Phys.Rev.D20(1979)2392.

74. N.G.Deshpande, Phys.Rev.D23(1981)2647.

75.A.J.Sanda, Phys.Rev.**D23**(1981)2654.

76.F.J.Gilman et al., Phys.Lett.**126B**(1983)111.

77.F.J.Gilman et al., Phys.Lett.**133B**(1983)443.

78.A.J.Buras et al, Nucl.Phys.**B238**(1984)529.

79.L.Wolfenstein, in *Proceedings of the Second Lake Louise Winter Institute on New Frontiers in Particle Physics, Selected Topics in Electroweak Interactions* (Chateau Lake Louise, Canada, 15-21 February 1987).

80.J.M.Flynn and L.Randall, Phys.Lett.**224B**(1989)221.

81.G.Buchalla, A.J.Buras, and M.K.Harlander, Nucl.Phys. **B337**(1990)313.

82.M.Banner et al., Phys.Rev.Lett.**28**(1972)1597.

83.M.Holder et al., Phys.Lett.**40B**(1972)141.

84.J.H.Christenson et al., Phys.Rev.Lett.**43**(1979)1209.

85.J.K.Black et al., Phys.Rev.Lett.**54**(1985)1628.

86.R.H.Bernstein et al., Phys.Rev.Lett.**54**(1985)1631.

87.M.Woods et al., Phys.Rev.Lett.**60**(1988)1695.

88.H.Burkhardt et al., Phys.Lett.**B206**(1988)169.

89.J.R.Patterson et al., Phys.Rev.Lett.**64**(1990)1491.

90.P.Langacker, in *Proceeding of the 1985 Int.Symposium on Lepton and Photon Interactions at High Energies*,ed.Konum, M.,and Takahashi,K., Kyoto Univ.(1986).

91.M.A.Shifman et al, JETP Lett.**22**(1975)55;
Nucl.Phys.**B120**(1977)316; JETP(Sov.Phys.), **45**(1977)670.

92.F.Gilman & Wise, Phys.Lett.**93B**(1980)129.

93.B.Guberina &R.D.Peccei, Nucl.Phys.**B163**(1980)289.

94.D.Baden, *Talk at the* XXIV*th Rencontres de Moriond* "

Electroweak Interaction and Unified Theories", 1989, and P.S.Wells, ibid.

95. Veltman M. Nucl.Phys.**B123**(1977)89.

96. G.Beall et al., Phys.Rev.Lett.**48**(1982)848;
T.Inami et al., Prog.Theor.Phys.**65**(1981)297.

97. G.Ecker and M.Wise, Nucl.Phys.**B258**(1985)328-360.

98. G.Buchalla et al., "The Anatomy of $\varepsilon' / \varepsilon$ in the Standard Model", Nucl.Phys.**B337**(1990)313-362.

99. M.K.Gaillard et al., Phys.Rev.**D10**(1974)897.

APPENDIX

A.1 Units

The convention $h/(2\pi) = c = 1$ is used unless these symbols explicitly appear in the formula. Some useful constants are

$$h/(2\pi) = 6.5822\times10^{-22}\ \text{MeV}$$

$$= 1.9733\times10^{-11}\ \text{MeV cm}$$

$$= 197.33\ \text{MeV fermi}$$

$$\alpha = e^2/(4\pi) = 1/137.036$$

A.2 Metric And γ-Matrices

The metric in Minkowski space $\{x^\mu : \mu = 1,2,3\ \}$ is given by $g^{\mu\nu}$ with

$g^{00} = +1,\ g^{ii} = -1$, otherwise $= 0$, here $i = 1,2,3$.

γ-matrices, γ^μ, satisfy the anticommutation relation

$$\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2g^{\mu\nu}.$$

γ^5 is defined as $\gamma^5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3$ and satisfies

$$\gamma^5\gamma^\mu + \gamma^\mu\gamma^5 = 0.$$

In the Dirac representation, the explicit 4×4 form of γ-matrices reads

$$\gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},\ \vec{\gamma} = \begin{pmatrix} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{pmatrix},\ \gamma^5 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},$$

where 1 and 0 are the 2×2 unit and zero matrices respectively and $\vec{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$ with σ_i (Pauli matrices) given by

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

A.3 Parameters

The parameters in the minimal standard model are related by the following equations:

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2}, \quad g\, sin\vartheta_W = g'\, cos\vartheta_W = e, \quad M_W = \frac{1}{2}\, g\, v,$$

$$M_Z = \frac{1}{2}\, v \sqrt{(g^2 + g'^2)}, \quad M_W = M_Z\, cos\vartheta_W\ .$$