CHARGINO-NEUTRALINO PRODUCTION IN $p\bar{p}$-COLLISIONS FOR THE LEFT-RIGHT SUPERSYMMETRIC MODEL

Harith N. Saif

A Thesis

in

The Department

of

Physics

Presented in Partial Fulfillment of the Requirements

for the Degree of Doctor of Philosophy at

Concordia University

Montréal, Québec, Canada

December 1992

ISBN 0-315-84666-6

Abstract

An extension of the supersymmetric standard model to the supersymmetric $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ model is considered. The gauge group contains bi-doublet and triplet Higgs fields. In this work we have considered chargino (charged gauginos mixed with higgsinos) and neutralino (neutral gauginos mixed with higgsinos) eigenstates for the left-right supersymmetric (L-R SUSY) model. After mixing, in the minimal supersymmetric model (MSSM), there are two charginos and four neutralinos. In the L-R supersymmetric model, in addition to four charginos there are three neutralinos generated from the first symmetry breaking, and four neutralinos generates from the second symmetry breaking. The mixings are in general model dependent. In the minimal SUSY and L-R SUSY models, these mixings can be parameterized in terms of a few parameters. We find analytical expressions and numerical solutions for the mass eigenstates with some restrictions on the L-R parameters.

We also investigate the possibility of detecting chargino and neutralino production in $p\bar{p}$-collisions at CDF, namely $p\bar{p} \to W^-_{L,R} + X \to \tilde{\chi}^-_j \tilde{\chi}^0_i + X$. The expressions for these *inclusive cross-sections* are solved numerically, using parton model distributions. We have used $\tilde{M}_{\chi^-}$=45 GeV, $\tan\beta$=1 and a lower bound on the lightest supersymmetric particle (LSP) mass, $\tilde{\chi}$=14 GeV. Finally we take $M_{W_R}\geq$300 GeV for

$g \simeq g_L \simeq g_R$. In the $p\bar{p}$-center of mass frame we find the cross sections are: $\sigma_L(p\bar{p} \to W_L^- + X \to \tilde{\chi}_j^- \tilde{\chi}_i^0 + X) \simeq 0.11$ nb, and $\sigma_R(p\bar{p} \to W_R^- + X \to \tilde{\chi}_j^- \tilde{\chi}_i^0 + X) \simeq 4.6$ pb, and thus $\sigma_L \simeq 24\sigma_R$ at $\sqrt{s}=1.8$ TeV. Cross sections are also given for larger values of the center of mass energy $\sqrt{s}$ up to those available at the SSC. The results are compared with the prompt-lepton background of the $W_{L,R}^-$ decays from $p\bar{p} \to W_{L,R}^- + X \to \ell^- \nu_{\ell_{L,R}} + X$. Both decays for $W_{L,R}^-$-bosons show Jacobian peaks for $p_T = \sqrt{\hat{s}}/2 \simeq M_{WL}/2 \simeq 40$ GeV ($p_T \simeq 150$ GeV for M_{WR}) at $\theta = 90^\circ$.

Furthermore the chargino signature unlike the prompt-lepton background is symmetric under the Jacobian peak. We also exhibit the dependence of the angular distribution of the chargino on the c.m angle θ for $p_T \simeq 40$ GeV, 150 GeV at $\sqrt{s}=1.8$ TeV.

Acknowledgements

I specially wish to thank my supervisor Dr. C.S. Kalman, for his guidance and help during the preparation of the thesis. I also wish to acknowledge Dr. M. Frank, for her helpful discussions, suggestions and support. I wish to express my gratitude to the Physics Department at Concordia University for direct financial support.

Using the facilities of VAX2 in the department of Computer Science are greatly appreciated.

I wish to express my gratitude to the Iraqi Government, especially the Scientific Research Council/Baghdad for a four year Scholarship. Finally, I would like to mention my family for their encouragement and support.

Contents

List of Tables

Lists of Figures

Figure | Page

Introduction

The standard model (SM) of particle physics is a partially unified quantum gauge field theory for the electromagnetic and weak interactions, which exhibits a broken $SU(2)_L \times U(1)_Y$ gauge symmetry (Glashow-Weinberg-Salam model), together with the $SU(3)_C$ symmetry (quantum chromodynamics) for the strong interaction. It seems to give a completely satisfactory account of the interactions of the "*fundamental*" particles, which are the quarks and the leptons.

However, the standard model can not be regarded as a satisfactory or final answer to the problems with which it deals. The model suffers from the need for a large number of wholly arbitrary parameters, including the masses of the particles and the coupling strengths of their interactions. Once the strong interaction is combined with the electromagnetic gauge theory, the gauge group is enlarged to $SU(3)_C \times SU(2)_L \times U(1)_Y$, but there is no prediction of the strong coupling relative to the electromagnetic coupling.

This latter problem is resolved by embedding the large group in one simple group, the so-called "*grand unified group*". The essential idea, based on the gauge group SU(5), was proposed by Georgi and Glashow (1974). SU(5) predicts that, at high energy scale, above some unification scale M_X, all phenomena satisfy the symmetry G. The different couplings g_s, g, g', observed at low energies would then arise after the symmetry group G is spontaneously broken, first at mass scale M_X($\approx 10^{15}$ GeV) then followed by electroweak breaking at scale of M_W. In the grand unified theories, one must assume that a cancellation occurs between terms in the series representing the Higgs mass. The terms are each of order 10^{15} GeV. This yields a residue of order 10^2 GeV, which is the size of the Higgs required for the weak bosons to remain light. These are two aspects of the so-called "*gauge hierarchy problem*". At present, there is very little evidence in favor of a grand unified theory. In the SU(5) grand unified theory, there is no evidence for any of the new interactions associated with the new vector bosons, although great experimental effort has gone into looking for the predicted proton decay events.

An elegant way of solving this hierarchy problem is to introduce a boson-fermion symmetry (*supersymmetry*). In the supersymmetric standard model (SSM) the mass of each scalar particle must be equal to that of its fermion partners. This implies the Higgs mass is no longer quadratically divergent since the boson and fermion loop corrections have opposite signs and cancel each other. Naturalness in the hierarchy problem requires that the masses of the super-partners of the ordinary particles should be in the range of $\approx$1 TeV. These ideas will be discussed in chapter I in our thesis.

One of the unsolved problems, which the standard model was not successful in resolving, is the origin of parity violation in low-energy physics. An interesting approach, within the framework of gauge theories, the idea of "*left-right symmetry*" models was proposed in 1973-1974. This idea has found its realization in the $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ gauge group. An important feature of this model is that, at low energies, it reproduces all the features of the $SU(2)_L \times U(1)_Y$ model. As we move up in energies new physics should appear, such as a second neutral Z^0-boson and right-handed boson, $W_R^{\pm}$. In the left-right symmetric model (LRSM), one can expand the gauge boson and Higgs boson sectors, and combine the L-R symmetric structure with a supersymmetric structure. These models are constructed in chapter II.

If there is new physics beyond the standard model, several questions naturally arise. First, at what energy scale will the new physics enter and, second, how will the new physics be identified? There are strong theoretical arguments that new physics will occur in the range 100 GeV-1 TeV. This is a very exciting possibility since the lower end of this energy range is already being examined for the first time by CERN $p\bar{p}$ Colliders, the e^+e^- Collider at CERN (LEP), and the Collider Detector at Fermilab (CDF). At the end of the decade we expect to have pp collisions at super-high energies ($\approx$40 TeV c.m) for the Superconducting Super Collider (SSC) and the Large Hadronic Collider (LHC) at CERN ($\approx$17 TeV c.m).

An aspect of searching for the superpartners to the normal particles which has received a lot of attention in hadronic collisions is *chargino* and *neutralino* pair production. These are perhaps the most promising of the supersymmetry partners for detection and study, because they may give the cleanest experimental signatures.

The limit on their masses is based on the missing momentum signal associated with chargino and neutralino decay into an undetected stable neutralino. Their production via the decay of left- and right-handed $W^{\pm}$-bosons in the $p\bar{p}$ collisions is the main subject of this thesis. This is found in chapters III, IV, and V.

Chapter I

Supersymmetric Standard Model (SSM)

I.1. Introduction.

The standard theory of electroweak interactions (or the Glashow-Weinberg-Salam model), based on the gauge group $SU(2)_L \times U(1)_Y$, is a partially unified quantum gauge field theory for the electromagnetic and weak interactions.[1] This model together with the SU(3) symmetric quantum chromodynamics for the strong interactions[2] seems to give a completely satisfactory account of the fundamental particle interactions of quarks and leptons.

There is no doubt that the standard model, based on the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$, has been very successful in making predictions confirmed by experimental discoveries such as the neutral current and the $W^{\pm}$ and Z^0 vector bosons that mediate the weak interactions at the CERN $p\bar{p}$-collider.[3] Of course the standard model is not completely tested; there are indeed many predictions which are no checked at all. These include for instance: the observation of one fermion, the so-called *"top-quark"*. According to the model the top-quark should exit in the energy region 20-200 GeV.[4]

Another prediction is the existence of the Higgs sector (scalar particles with spin-0). This sector[5] remains rather mysterious, and the predicted particle has not been found. The Higgs sector, however, is very important for the standard model since the masses of the gauge bosons $W^{\pm}$, Z^0 and the fermions are generated by parameters in this sector after the breakdown of the gauge symmetries.

The Higgs particles have several nice properties. They are the only particles to possess nonvanishing vacuum expectation values (VEV's) without breaking Lorentz invariance. But the Higgs masses, are subject to quadratic divergences in perturbation theory which push them to orders of the Planck mass. An approach to this problem is to use a higher symmetry to eliminate the quadratic divergence in the Higgs sector. An example is the supersymmetry theories as we will indicate in the next sections.

I.2. Supersymmetric Theories of Particles.

I.2.1. Generalities.

Supersymmetry is a symmetry that transforms bosons into fermions and vice versa. The generator Q of these transformations should therefore have fermionic character. Since the supersymmetry generators carry one half unit of spin they obey anticommutation relations. Supersymmetry transformations are generated by a 4-component Majorana spin-1/2 symmetry operator Q, which satisfies the algebra:[6-8]

$$[Q, P^{\mu}] = 0 \tag{I.1}$$

$$\{Q, \bar{Q}\} = -2\gamma_{\mu} P^{\mu} \tag{I.2}$$

where the energy momentum operator is $P^{\mu}=i\partial^{\mu}$. From this algebra one can immediately read off two important consequences: From eq.(I.1) for $\mu=0$ we see that Q commutes with the Hamiltonian $H=P^{0}$, which means that spinor charges are conserved.[9,10] Equation (I.2), the anticommutation relation, indicates that two successive SUSY transformations involve the structure of space time.

The supersymmetric algebra, in its simplest form (N=1), has a self-conjugate spin-1/2 Majorana generator Q with the property that it changes the total angular momentum J by half a unit and turns boson fields into fermion fields and vice versa.[10] There could exist N different operators Q^{i} (i=1,...,N) of this kind, but only N=1 SUSY allows fermions in chiral representations. A local gauge symmetry involves spin-1 gauge fields and a local symmetry for the Poincaré group involves spin-2 gravitons.

In the spirit that all fundamental symmetries are local gauge symmetries, it is also possible to make supersymmetry local. Since the anticommutation relations of the supersymmetry involve the Poincaré group generator P^{μ}, local supersymmetry involves both the supersymmetric partners of the graviton and the gauge bosons—the gravitino of spin-3/2 and the gaugino of spin-1/2. In global supersymmetry models the gravitino is not present.

In supersymmetry, particles are assigned a quantum number known as R-parity defined by[11]

$$R_p=(-1)^{3B+L+2S}=\begin{cases} +1: \text{ for ordinary particles} \\ -1: \text{ for supersymmetric particles} \end{cases}$$

where B is the baryon number, L is the lepton number, and S is the spin. The latter formula illustrate that the conservation of baryon and lepton numbers (or simply their difference) necessarily implies R-parity conservation. If we try to replace a normal particle with its superpartner we would violate angular momentum conservation by ±1/2. Hence any amplitude must contain an even number of superpartners. If the R-parity is conserved, it has important phenomenological consequences:

- supersymmetric partners will be produced in pairs starting from normal particles,
- the decay of supersymmetric partners will contain a supersymmetric partner,
- the lightest supersymmetric partner (LSP) will be stable. It is thought that the neutralino $\tilde{\chi}^0$ is likely to be the LSP, though other candidates are possible.[12]

In supersymmetric theories the number of fermion and boson degrees of freedom must match. For instance, for each quark chirality state, such as q_L, there is a boson supersymmetric partner in this case the "*squark*", $\tilde{q}_L$. In the supersymmetric theories, it is necessary to have two doublets of Higgs fields in order to give masses to all the quarks and leptons and there are corresponding spin-1/2 "*higgsino*" partners:[10,13]

$$\begin{pmatrix} H_1^0 \\ H_1^- \end{pmatrix} \begin{pmatrix} H_2^0 \\ H_2^- \end{pmatrix} ; \begin{pmatrix} \tilde{H}_1^0 \\ \tilde{H}_1^- \end{pmatrix} \begin{pmatrix} \tilde{H}_2^0 \\ \tilde{H}_2^- \end{pmatrix} \tag{I.3}$$

The particles *winos, zinos, photinos* and *gluinos*, are the spin-1/2 superpartners of the $W^{\pm}$, Z^0, γ and gluons respectively. All the superpartners of leptons and quarks, which have spin-0, often called are "*sleptons*" and "*squarks*". The particle content of the supersymmetric standard model is shown in Table I.1.

The charged weak gauginos or winos and the charged higgisinos will mix among themselves, since they carry the same values of conserved quantum numbers, forming the so-called "*charginos*".[12] Additionally and for the same reason, the neutral electro-weak gauginos or zinos mix with the neutral higgisinos and form the so-called "*neutralinos*". Together the charginos and neutralinos sparticles are the main subject of this thesis.

Table I.1: *Spectrum of SUSY particles.*

Ordinary Particles	Spin	Spartners		Spin
$q_{L,R}$	1/2	$\tilde{q}_{L,R}$		0,0
$\ell_{L,R}$	1/2	$\tilde{\ell}_{L,R}$		0,0
γ	1	$\tilde{\gamma}$	gauginos	1/2
g	1	$\tilde{g}$	gauginos	1/2
W	1	$\tilde{W}$	gauginos	1/2
Z^0	1	$\tilde{Z}^0$	gauginos	1/2
H	0	$\tilde{H}$	Higgsinos	1/2

Supersymmetry introduces no new coupling. Sleptons and electroweak gauginos interact with the same coupling strength g as the leptons and gauge bosons, and squarks and gluinos with the coupling strength g_s as the quarks and gluons. This is because in the SUSY extension of the standard model, the interaction of a fermion with a gauge boson transforms under supersymmetry as follows:[10]

$$g\bar{f}_L\gamma_\mu f_L A^\mu \rightarrow \sqrt{2}\, g\bar{f}_L\tilde{A}_L\tilde{f}_L + \text{H.C.} \tag{I.4}$$

where $\tilde{A}$ is the corresponding gaugino and $\tilde{f}_L$ is the corresponding squark or slepton. If supersymmetry were unbroken, the supersymmetric partners would have the same masses as their corresponding particles. Supersymmetry must be broken, because the observed particles do not exhibit boson-fermion degeneracy. However, if we assume the symmetry is broken spontaneously at a scale $\lesssim 1$ TeV, the mass degeneracy is removed but the predicted relations among couplings remain exact.

I.2.2. The gauge hierarchy problem (GHP).

Although the standard model is completely successful in accounting for low-energy particle physics (i.e., describe physics at energy scale of order ≈ 100 GeV), it leaves unexplained why the gauge group of strong and electroweak interactions is $SU(3)_C \times SU(2)_L \times U(1)_Y$, with different gauge couplings and why the fermion quantum numbers have their particular values. In 1974 an elegant unification scheme was proposed[14] according to which the standard model at energies $\approx M_W$ is the remnant of a bigger SU(5) symmetry which is spontaneously broken at some unifying scale M_X. Such a theory[14] is characterized by only one gauge coupling constant g_{GUT} at energies $\approx M_X$ where breaking of SU(5) down to $SU(3)_C \times SU(2)_L \times U(1)_Y$ occurs.

The essential idea of grand unification is to try to embed the standard model in some simple gauge group G, and at high energies, above some unification scale M_X, all phenomena satisfy the symmetry G[14]. The different couplings g_s, g, g' observed at low energies would then arise because the unified group G is spontaneously broken, first at mass scale M_X then followed by the electroweak breaking at the scale of M_W. In grand unified theories (GUT's), there are two types of Higgs particles which cause the symmetry breakdown: the usual Higgs with $M_H=O(M_W)$, and heavy Higgs Φ with $M_\Phi=O(M_X)$.

In the standard model there are 12 gauge bosons (the eight gluons, $W^\pm$, Z^0, and γ). In a grand unified theory there are additionally gauge bosons, X, which link the quarks and leptons that lie within the same multiplet of the group G. These X bosons, which mediate new interactions, violate the conservation of baryon number, B, and lepton number, L. For instance, the most spectacular feature of this GUT's is the prediction[14-16] of proton decay with a lifetime $\approx\alpha_G^{-2}M_X^4/m_p^5\approx2\times10^{29}$ years (α_G is the GUT coupling). Such a decay occurs only through lepton and baryon number violating interactions, which are mediated by a virtual superheavy gauge boson, X, with mass $M_X\approx10^{15}$ GeV. Since the experimental limits on the proton lifetime are of order $\tau_p\geq10^{31}$ year,[15-17] M_X is at least of order $10^{16}\sqrt{\alpha_G}$ GeV. Such interactions seem to be a basic ingredient, along with CP-violation, for the explanation of the observed baryon asymmetry of the Universe.[18]

Most of the unification models proposed so far contain two characteristic mass scales, M_W and M_X; M_W is the scale where breaking of the elecroweak $SU(2)_L \times U(1)_Y \rightarrow U_{EM}(1)$ takes place and M_X is the scale where unification of all forces (but gravity) occurs. There is a vast separation of ≥13 orders of magnitude of these two scales. Not only that but this vast separation is not stable under renomalizations. These are the two aspects of the so-called "*gauge hierarchy problem*".[19,20] The first problem is: these models provide us with no information as to why $M_W/M_X \leq O(10^{-13})$ and hence cannot be complete as physical theories. The second aspect is the "*naturalness*" problem, which means that the radiative corrections involving superheavy gauge and scalar particles to the low-energy scalars (the normal Higgs) of the theory do not cancel but contribute through the graph of Fig. I.1 to give corrections:[19]

$$\delta M_H^2 = M_W^2 \,[O_1(g^2,\lambda) + O_2(g^2,\lambda) M_X^2/M_W^2], \qquad (I.5)$$

where g^2 is a typical gauge coupling and λ is a quartic coupling appearing in the Higgs potential. Obviously, δM_H^2 cannot be $O(M_W^2)$ unless $O_2(g^2,\lambda)=O(10^{-26})$. This means that, in order to keep the light Higgs H, while the Φ is heavy, we must ensure cancellations of the divergences to an accuracy:

$$(M_H/M_\Phi)^2 \approx (M_W/M_X)^2 \approx 10^{-26}, \qquad (I.6)$$

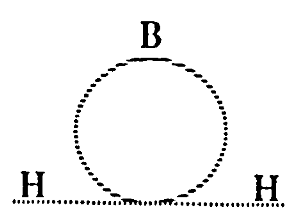

Fig. I.1 *Quadratically divergent contribution to the mass of a scalar particle.*

for $M_X \simeq 10^{15}$ GeV, which must be put in "*by hand*". Such a fine tuning of the parameters to 26 decimal places is really hard to conceive, and what is worse is that this has to readjusted to each order of perturbation theory.[19] This destabilization does not affect the low-mass ($\simeq M_W$) gauge particles of the theory.

An elegant way of solving this hierarchy problem is to introduce a boson-fermion symmetry (*supersymmetry*). If boson-fermion pairs have identical couplings, their contributions to loop diagrams, such as Fig. I.1, are of opposite sign and cancel each other. Naturalness in the hierarchy problem requires the mass of the Higgs to be not much larger than $\simeq 1$ TeV.

I.2.3. Motivations for supersymmetry.

We have seen that the origin of the *gauge hierarchy problem* is the difficulty of including fundamental scalar Higgs fields in the standard model: consider the Higgs potential,[9]

$$V(\phi) = -\mu^2|\phi|^2 + \lambda|\phi|^4. \qquad (I.7)$$

The Higgs mass is then given by: $M_H = (2\mu^2)^{1/2} = (2\lambda)^{1/2}\upsilon$, where υ is the vacuum expectation value of the Higgs field and given by,

$$\upsilon = (\mu^2/\lambda)^{1/2} \simeq 250 \text{ GeV},$$

where λ is the same as in eq.(I.5). Although λ is unknown, we require $\lambda<1$ so that $M_H<1$ TeV. Experimental bounds[21] on

M_H come from LEP; $M_H \geq 40$ GeV. The parameter μ^2 in eq.(I.7) receives a contribution due to the graph of Fig. I.1 which is quadratically divergent. Hence the masses of such scalars, M_H, are subject to quadratically divergent renormalization corrections,

$$\delta M_H^2 \approx \lambda \Lambda^2 \tag{I.8}$$

where Λ is a physical cutoff (of dimension of a mass) in the TeV region.

We have said in the previous section that there is no natural way to sustain a light Higgs of mass $O(M_W)$ together with a heavy Higgs of mass $O(M_X)$. There will be radiative corrections to the light Higgs mass of $O(M_X)$ that automatically destroy the hierarchy. Only an "*unnatural*" fine tuning of parameters, order by order in perturbation theory,[10] could keep a light Higgs of mass $O(M_W)$.

In exact supersymmety the mass of each scalar particle must be equal to that of its fermion partners. This implies the absence of quadratic divergences[9,10] since corrections to the scalar mass, M_H, is cancelled by the contributions from the supersymmetric partners of the Higgs boson, as shown in Fig. I.2, i.e.,

$$\delta M_H^2 \approx \lambda (m_B^2 - m_F^2) \approx 0 \tag{I.9}$$

Although experimental support for SUSY is lacking, many people believe that it is only a matter of time before evidence of SUSY, albeit in some broken form, will be found. We need a finite Higgs mass of $O(M_W)$ to produce the observed electroweak symmetry breaking, so we want to break supersymmetry "*gently*" so that the masses of the boson-fermion multiplet are split and eq.(I.9) becomes:

$$\delta M_H^2 = \lambda (m_B^2 - m_F^2) \tag{I.10}$$

So if eq.(I.10) is to give $\delta M_H^2 \leq M_W^2$, supersymmetric partners of the ordinary particles must be found with masses $\lesssim$1 TeV.

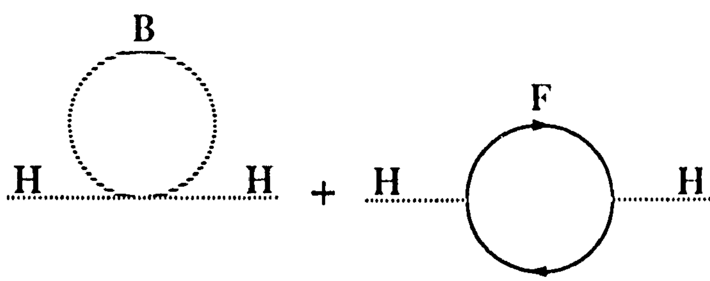

Fig. I.2 *Quadratic divergence contribution to the mass of a scalar particle (dashed line) that cancels the contribution of fig. I.1 in a supersymmetric model. The solid line represents the fermionic partner of the scalar particle.*

I.3. The Minimal Supersymmetric Standard Model (MSSM).

The Minimal Supersymmetric Standard Model[22] (MSSM) is the supersymmetric extension of the standard model (SM) with the minimum number of new particles and new interactions. In this model the Higgs boson receives corrections that are limited by the extent of supersymmetry breaking. The minimal supersymmetric model is the only structure in which the problems of naturalness and hierarchy are resolved, while retaining the Higgs bosons as truly elementary spin-0 particles.

In the standard model only one Higgs doublet, together with its complex conjugate, is required to give mass to the quarks and leptons. In the supersymmetric theory, at least two Higgs doublets are needed to give mass to both up and down quarks.[23-25] Furthermore, two Higgs doublets, are necessary for cancellation of anomalies.[26]

Once electroweak symmetry is broken, the particles with the same spin, electric charge and color (gauginos and higgsinos) mix to form mass eigenstates (charginos and neutralinos). The mixing between the partners of the left- and right-handed fermion, $\tilde{f}_L$ and $\tilde{f}_R$, is proportional to the corresponding fermion mass. The gluinos, $\tilde{g}$, are the only spin-1/2 color octet fermions and so cannot mix, and then, their interactions are fixed by QCD. Gauginos and higgsinos are spin-1/2 weakly interacting particles and so they mix once $SU(2)_L \times U(1)_Y$ is broken. After mixing, in the MSSM, there exist four neutralinos, $\tilde{\chi}^0_1$, $\tilde{\chi}^0_2$, $\tilde{\chi}^0_3$, $\tilde{\chi}^0_4$ and two charginos $\tilde{\chi}^\pm_1$, $\tilde{\chi}^\pm_2$ (labelled in order of increasing mass).[22,27] In our model, the L-R supersymmetric model, there are seven neutralinos: three generated by the first stage of breaking, after acquiring VEV υ_R, and four generated by the second stage of breaking, after acquiring

VEV's κ_u and κ_d. These particles will be discussed in detail in chapters III and IV. The mixings are in general model dependent. In the minimal susy model and L-R susy model, however, they can be parameterized in terms of a few parameters.

I.4. The Scale of Supersymmetry Breaking.

The particles observed in nature show no sign whatsoever of a degeneracy between fermions and bosons. Since nature is not supersymmetric, SUSY must be spontaneously broken. Consider eq.(I.2); it has immediate consequences since it relates the supercharges to the Hamiltonian and we have,[9]

$$H = P^0 \simeq QQ^{\dagger}, \tag{I.11}$$

where Q is the SUSY generator. Thus H is semipositive definite ($H \geq 0$). This implies that in supersymmetric theories the vacuum is well defined. If supersymmetry is unbroken this implies $Q|0\rangle=0$, where $|0\rangle$ is a vacuum state. We then conclude for the vacuum energy:

$$E_{vac} = \langle 0|H|0\rangle = 0. \tag{I.12}$$

If, on other hand, supersymmetry is spontaneously broken we have:[9]

$$Q|0\rangle = |\psi_G\rangle \neq 0; \; E_{vac} \neq 0, \tag{I.13}$$

where $Q|0>$ is a fermionic state, denoted by $|\psi_G>$. This means that, if supersymmetry is spontaneously broken there will be a massless fermion the so-called "*Goldstone fermion or Goldstino*"† created out of the vacuum.[28,29]

In the case of spontaneously broken global supersymmetry: the Goldstone fermion is the lightest supersymmetric particle (since it is massless), which would couple to every particle and its superpartner. This gives rise to a number of interesting production and decay mechanisms, which could be relevant to experiment if the Goldstino coupling were large and this requires the scale of supersymmetry breaking to be less than a TeV. However, in SUSY models involving supergravity, and on spontaneous breaking of the symmetry, neither the Goldstino nor the gravitino remain massless. The gravitino absorbs the Goldstino and becomes massive;[9,10]

$$(m=0,\ \text{spin-}3/2) + (m=0,\ \text{spin-}1/2) \rightarrow (m\neq 0,\ \text{spin-}3/2).$$

The Goldstino becomes the missing helicity $\pm 1/2$ components of the massive gravitino. The gravitino in most recent models is not the lightest supersymmetric particle. The supersymmetric particle most likely to be the lightest and

---

† This situation is analogous to the case of ordinary spontaneously broken symmetries in which massless Goldstone bosons are created out of the vacuum. In the standard model, the gauge bosons, $W^{\pm}$, Z^{0} become massive by "eating" three of the massless Goldstone bosons.

stable is a neutralino (this is discussed in chapter IV).

Supersymmetry must be broken spontaneously at some mass scale $M_s(\approx 10^{10}$ GeV), which implies $M_s \gg M_W$. If M_s is sufficiently lar,e, gravity is no longer neglected. We then have the exciting possibility that it is gravitational effects that are responsible for SUSY breaking.

One wants to extract the low-energy physics of these models, that is, those effects which are not suppressed by the gravitational coupling itself. These are most simply obtained by considering[30] local supersymmetry breaking. In the unbroken case, the graviton and gravitino are degenerate and massless. If SUSY is broken, there will be mass splitting: the graviton will stay massless and the gravitino will acquire a mass:

$$m_{3/2} = \left(\frac{8\pi}{3}\right)^{1/2} \frac{M_s^2}{M_P} \approx M_W, \tag{I.14}$$

where M_s is the supersymmetry breaking scale $\approx 10^{10}$ GeV, and M_P is the Planck mass scale $\approx 10^{19}$GeV. The result of this procedure is an effective "*low energy*" theory, occuring at a mass scale $M_s \gg M_W$, where its Lagrangian is just, the supersymmetric Lagrangian of the ordinary and super-matter fields plus supersymmetry breaking terms[12,31]

$$\mathcal{L} = \mathcal{L}_{susy} + \mathcal{L}_{breaking}. \tag{I.15}$$

Here

$$\mathcal{L}_{susy} = \mathcal{L}_{susy}\,[SU(3) \times SU(2) \times U(1);\, f] \tag{I.16a}$$

$$f = f_Y + \mu \hat{H}_1 \hat{H}_2 ; \text{ where} \tag{I.16b}$$

$$f_Y = \lambda_R^{ij} \hat{R}_i \hat{L}_j \hat{H}_1 + \lambda_D^{ij} \hat{D}_i \hat{Q}_j \hat{H}_1 + \lambda_U^{ij} \hat{U}_i \hat{Q}_j \hat{H}_2$$

$$\mathcal{L}_{\text{breaking}} = -m_{3/2} \sum_i |\phi_i|^2 - M \sum_a \lambda_a \lambda_a + [A m_{3/2} f_y +$$

$$+ (A - 1) m_{3/2} \, \mu \hat{H}_1 \hat{H}_2 + \text{H.C.}] \tag{I.16c}$$

where λ_R^{ij}, λ_D^{ij}, λ_U^{ij} are lepton and quark Yukawa couplings, $\hat{H}_1$ and $\hat{H}_2$ are the Higgs superfields, $\hat{Q}$ and $\hat{L}$ are the SU(2) weak-doublet quark and lepton superfields, respectively, $\hat{U}$ and $\hat{D}$ are SU(2) singlet quark superfields and $\hat{R}$ is an SU(2) weak-singlet charged lepton superfield. Because of the gauge invariance the $\hat{H}_1 \hat{Q} \, \hat{U}$ coupling is prohibited; thus, no up-quark mass can be generated if $\hat{H}_2$ is omitted, ϕ_i and λ_a are scalar and gaugino fields respectively. The parameters μ, $m_{3/2}$, M, A, can all be taken real and have dimensions of mass. The appearance of a mass scale μ in eq.(I.16b) is essential for the breaking of supersymmetry[12,31]. When $\mu \neq 0$ supersymmetry is broken and the two Higgs doublets acquire vacuum expectation values:

$$\langle \hat{H}_1 \rangle = \begin{pmatrix} v_1 \\ 0 \end{pmatrix}, \qquad \langle \hat{H}_2 \rangle = \begin{pmatrix} 0 \\ v_2 \end{pmatrix}. \tag{I.17}$$

As a conclusion, we have seen, one expects the SUSY breaking scale to be O(≈1 TeV), so the difference in mass between a partner and its superpartner is likely to be of this order, and it is therefore not too surprising that no sparticles have been found yet. An order of magnitude increase in energy should, however, show evidence for SUSY.

References

[1] S.L. Glashow, *Nucl. Phys.* 22 (1961) 579; S. Weinberg, *Phys. Rev. Lett.* **19** (1967) 1264; A. Salam, in *Elementary Particle Theory* (ed., N. Svartholm), Almqvist and Wiksells, Stockholm 1968).

[2] For reviews, see H.D. Politzer, *Phys. Rep.* **14** (1974) 129; W. Marciano and H. Pagels, *Phys. Rep.* **36** (1978) 137.

[3] UA1 Collaboration, G. Arnison et al., *Phys. Rep. Lett.* **122B** (1983) 103; **126B** (1983) 398; **129B** (1983) 273; UA2 Collaboration, G. Banner et al., *Phys. Lett.* **122B** (1983) 476; P. Bagnaia at al., *Phys. Lett.* **129B** (1983) 130.

[4] M. Veltman, *Acta Phys. Pol.* **B8** (1977) 475.

[5] P.W. Higgs, *Phys. Rev. Lett.* **12** (1964) 132.

[6] Yu A. Gol'fand, and E.P. Likhtman, *J.E.T.P. Lett.* **13** (1971) 323.

[7] D.V. Volkov and V.P. Akulov, *Phys. Lett.* **46B** (1973) 109.

[8] J. Wess and B. Zumino, *Nucl. Phys.* **B70** (1974) 39.

[9] H.P. Nilles, *Phys. Rep.* **110** (1984) 1; Max Planck Inst. Preprint MPI-Ph/91-106 (Dec. 1991).

[10] V. D. Barger and R.J.N. Phillips, *Collider Pysics*, (Addison-Wesley, California, 1987) ch.14; P.D. Collins, A.D. Martin, E.J. Squires, *Particle Physics and Cosmology*, (Wiley-Interscience Pub., New York, 1989) ch.10.

[11] P. Fayet, *Unification of the Fundamental Particle Interactions*, Proc. Europhysics Study Conf., Erice 1987 eds. S.Ferrara, J. Ellis and P. Van Nieuwenhuizen (Plenum, New York, 1987).

[12] For review, see for instance: H.E. Haber and G.L. Kane, *Phys. Rep.* **117** (1985) 75.

[13] J.F. Gunion, H.E. Haber, G. Kane, and S. Dawson, *The Higgs Hunter's Guide*, (Addison-Wesley Pub., California, 1990) ch.4.

[14] H. Georgi and S.L. Glashow, *Phys. Rev. Lett.* **32** (1974) 438.

[15] H. Georgi, H.R. Quinn and S. Weinberg, *Phys. Rev. Lett.* **33** (1974) 451.

[16] A.J. Buras, J. Ellis, M.K. Gaillard and D.V. Nanopoulos, *Nucl. Phys.* **B135** (1978) 66; See also M.S. Chanowitz, J.Ellis and M.K. Gaillard, *Nucl. Phys.* **B128** (1977) 506.

[17] For a review on proton decay, see K. Enqvist and D.V. Nanopoulos, *Prog. Part. Nucl. Phys.* **16** (1986) 1.

[18] A.D. Sakharov, *Pis'ma Zh. Eksp. Teor. Fiz.* **5** (1967) 32; M. Yoshimura, *Phys. Rev. Lett.* **41** (1978) 381; *Phys. Lett.* **B88** (1979) 294; A.Yu. Ignatiev, N.V. Krasnikov, V.A. Kuzmin and A.N. Tavkhelidze, *Phys. Lett.* **B76** (1978) 436; S. Dimopoulos and L. Susskind, *Phys. Rev.* **D18** (1978) 4500; *Phys. Lett.* **B81** (1979) 416; J. Ellis, M.K. Gaillard and D.V. Nanopoulos, *Phys. Lett.* **B80** (1979) 360; A.D. Sakharov, *Pis'ma Zh. Eksp. Teor. Fiz.* **76** (1979) 1172. D. Toussaint, S.B. Treiman, F. Wilczek and A. Zee, *Phys. Rev.* **D19** (1979) 1036; S. Weinberg, *Phys. Rev. Lett.* **42** (1979) 850; D.V. Nanopoulos and S. Weinberg, *Phys. Rev.* **D20** (1979) 2484.

[19] A.B. Lahanas and D.V. Nanopoulos, *Phys. Rep.* **145** (1987) 1.

[20] See refs.[9,10] and: P. West, *Introduction to Supersymmetry and Supergravity*, (World Scientific, Singapore, 1990) ch.19.

[21] ALEPH Collaboration, D. Decamp *et al.*, *Phys. Lett.* **B246** (1990) 306; OPAL Collaboration, M. Akrawy *et al.*, *Phys. Lett.* **B236** (1990) 224; DELPHI Collaboration, P. Abreu *et al.*, *Nucl. Phys.* **B342** (1990) 1; L3 Collaboration, B. Adeva *et al.*, *Phys. Lett.* **B248** (1990) 203.

[22] M. Peskin, Proc. 1981 Int. Symp. on *Lepton and Photon Interaction at High Energies*, Bonn (1981).

[23] H. Baer *et al.*, in *Research Directions of the Decade*, Proceedings of Workshop, Snowmass, Colorado, 1990.

[24] P. Fayet, *Nucl. Phys.* **B90** (1975) 104.

[25] E. Witten, *Nucl. Phys.* **B231** (984) 419; S. Dimopoulos and H. Georgi, *Nucl. Phys.* **B193** (1983) 150; N. Sakai, *Z. Phys.* **C11** (1981) 153.

[26] K. Inoue, A. Komatsu and S. Takeshita, *Prog. Theor. Phys.* **67** (1982) 927; **71** (1984) 413.

[27] D.J. Gross and R. Jackiw, *Phys. Rev.* **D6** (1972) 477; C. Bouchiat and J. Illiopoulos and Ph. Meyer, *Phys. Lett.* **38B** (1972) 519; H. Georgi and S. Glashow, *Phys. Rev.* **D6** (1972) 429; L. Alvarez-Gaume and E. Witten, *Nucl. Phys.* **B234** (1983) 269.

[28] D. Freedman, P. van Nieuwenhuizen and S. Ferrara; *Phys. Rev.* **D13** (1976) 3214; S. Deser and B. Zumino, *Phys.*

Lett. **B62** (1976) 335.

[29] J. Goldstone, *Nuovo Cim.*, **19** (1961) 154; see also J. Goldstone, A. Salam and S. Weinberg, *Phys. Rev.* **127**, (1962) 965.

[30] R. Barbieri, S. Ferrara and C. Savoy, *Phys. Lett.* **B119** (1982) 343; A. Chamseddine, R. Arnowitt and P. Nath, *Phys. Rev. Lett.* **49** (1982) 970.

[31] R. Barbieri, *Riv. Nuovo Cim.* **11** (1988) 1; V. Bertin, Ph.D. Thesis, University De Paris-Sud Centre D'Orsay, Paris, 1991.

Chapter II

Left-Right Supersymmetric Model, Lagrangians and Feynman Rules

II.1. Introduction.

Although the standard model, based upon the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$,[1] has been very successful in describing the low-energy phenomena, it leaves a lot of unanswered questions. One of these questions involves understanding the origin of parity violation at the low-energy scale. An interesting approach is to assume that the interaction Lagrangian is intrinsically left-right symmetric (LRS). Within the framework of gauge theories this idea has found its realization in the $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ models.[2] At low energy, this model reproduces all the features of the $SU(2)_L \times U(1)_Y$ model, and as the energy increases new effects associated with the parity invariance of the Lagrangian are supposed to appear. For instance, we have two charged gauge bosons $W_L^{\pm}$ correspond to $SU(2)_L$ and are the same as $W^{\pm}$ of the standard model $SU(2)_L \times U(1)_Y$, while $W_R^{\pm}$ corresponding to $SU(2)_R$ are new. The $U(1)_{B-L}$ factor is also different from the standard model $U(1)_Y$; the ordinary quarks and leptons couple to B-L.

In sec. II.2, we describe the $SU(2)_L \times U(1)_Y$ gauge symmetry. In sec. II.3, we have extended the standard model

$SU(2)_L \times U(1)_Y$ into a fully left-right supersymmetric model $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. We have shown in section II.4 some constraints on the right-handed gauge boson mass, W_R, mass and the mixing angle. In the last section, we restrict the Lagrangian and Feynman rules to the $W^-_{L,R}$-gauge boson interactions with the chargino and neutralino.

II.2. $SU(2)_L \times U(1)_Y$ Gauge Symmetry.

Spontaneous breaking of gauge symmetries is the crucial ingredient in the model of unified weak and electromagnetic interactions constructed by Glashow, Weinberg and Salam.[1] The Lagrangian of the theory contains terms corresponding to massless gauge bosons and also massless leptons, which are invariant under the symmetry group. In the electroweak $SU(2)_L \times U(1)_Y$ model, breaking the symmetry is accomplished by a complex Higgs doublet, three components of which become the longitudinal polarization states of the gauge bosons. The remaining component manifests itself as a neutral scalar Higgs particle. By choosing the nonvanishing vacuum expectation value to be that of the neutral field, we ensure the vacuum is invariant under $U(1)_{EM}$ of QED, and that the photon remains massless.

In the $SU(2)_L \times U(1)_Y$ model, the right-handed fermions are assigned to transform under $U(1)_Y$ only; no right-handed neutrino is introduced. Left-handed fermions transform under both $SU(2)_L$ and $U(1)_Y$. For instance, in first generation of leptons and quarks, the left-handed fermions isospin doublets ψ_L and the right-handed fermions isospin singlets ψ_R are[3];

$$\psi_L = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \quad \psi_R = e^-_R, \tag{II.1}$$

and

$$\psi_L = \begin{pmatrix} u \\ d \end{pmatrix}_L, \quad \psi_R = u_R, \text{ or } d_R. \tag{II.2}$$

The generators of the two groups satisfy,

$$Q = T_3 + Y/2 \tag{II.3}$$

in analogy with the Gell-Mann-Nishijima formula for strong interaction quantum numbers. Here Q is the charge operator, generates the group $U(1)_{EM}$ and the operators T_3 and Y generate the symmetry groups $SU(2)_L$ and $U(1)_Y$ respectively. The weak quantum numbers for the first generation of leptons and quarks are listed in Table II.1. The massless gauge fields in this model are an isotriplet W_μ for $SU(2)_L$ and a singlet B_μ for $U(1)_Y$. The Lagrangian is,[3]

Table II.1: *Weak isospin and hypercharge quantum numbers of the first generation of leptons and quarks.*

Lepton	T	T_3	Q	Y	Quark	T	T_3	Q	Y
ν_e	1/2	1/2	0	-1	u_L	1/2	1/2	2/3	1/3
e^-_L	1/2	-1/2	-1	-1	d_L	1/2	-1/2	-1/3	1/3
					u_R	0	0	2/3	4/3
e^-_R	0	0	-1	-2	d_R	0	0	-1/3	-2/3

$$\mathcal{L} = -\tfrac{1}{4}\mathbf{W}^{\mu\nu}\cdot\mathbf{W}_{\mu\nu} - \tfrac{1}{4}B^{\mu\nu}B_{\mu\nu} + i\bar{\psi}\gamma^{\mu}D_{\mu}\psi \qquad \text{(II.4)}$$

with a separate fermion term for each field ψ_L and ψ_R. The first and the second terms are the kinetic energy and self-coupling of the $\mathbf{W}_\mu$ and $B_{\mu\nu}$ fields, where the field tensors $\mathbf{W}_{\mu\nu}$ and $B_{\mu\nu}$ are defined as

$$\mathbf{W}_{\mu\nu} = \partial_\mu \mathbf{W}_\nu - \partial_\nu \mathbf{W}_\mu - g\mathbf{W}_\mu \times \mathbf{W}_\nu; \text{ and } B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu.$$

The covariant derivative is,

$$D_\mu = \partial_\mu + ig\frac{\vec{\tau}}{2}\cdot\mathbf{W}_\mu + ig'\frac{Y}{2}B_\mu$$

where $\vec{\tau}$ are Pauli matrices $(T=\vec{\tau}/2)$. We introduce the Higgs fields, ϕ, which couple gauge-invariantly to the gauge bosons through the covariant derivative,

$$\partial_\mu\phi\partial^\mu\phi^\dagger \equiv |\partial_\mu\phi|^2 \rightarrow \left|\left(\partial_\mu + igT\cdot\mathbf{W}_\mu + ig'\frac{Y}{2}B_\mu\right)\phi\right|^2 \qquad \text{(II.5)}$$

and to the fermion through so-called *"Yukawa"* couplings of the form

$$-G_Y[(\bar{\psi}_L\phi)\psi_R + \bar{\psi}_R(\phi^\dagger\psi_L)]. \qquad \text{(II.6)}$$

We require the Higgs field, ϕ, to be an SU(2) doublet if eq.(II.6) is gauge-invariant. We take the Higgs doublet to be

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \text{ with } \phi^+ = (\phi_1 + i\phi_2)/\sqrt{2} \;;\; \phi^0 = (\phi_3 + i\phi_4)/\sqrt{2}. \tag{II.7}$$

with ϕ_i (i=1,..,4) real. Eq.(II.6) is only $SU(2)_L \times U(1)_Y$ gauge-invariant if ϕ is an isospin doublet (T_3=1/2) with weak hypercharge Y=1 (see, Table II.1):

In addition to the Lagrangian in eq.(II.4), we have to add an $SU(2)_L \times U(1)_Y$ gauge invariant Lagrangian for the scalar fields,[3]

$$\mathcal{L}_\Phi = |D_\mu \Phi|^2 - V(|\Phi|^2), \tag{II.8}$$

The general form of the scalar potential V is,

$$V(\Phi) = -\mu^2(\Phi^\dagger\Phi) + \lambda(\Phi^\dagger\Phi)^2. \tag{II.9}$$

where $\mu^2>0$ and $\lambda>0$. Suppose that the vacuum expectation value for $\Phi(x)$ is

$$<\Phi> = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \upsilon \end{pmatrix} \tag{II.10}$$

The gauge boson masses are then identified by substituting the vacuum expectation value Φ, in the Lagrangian eq.(II.5). The relevant term is,

$$\left|\left(ig\frac{\vec{\tau}}{2}.W_\mu + i\frac{g'}{2}B_\mu\right)\Phi\right|^2 = (\upsilon g/2)^2 W^+_\mu W^{-\mu}$$

$$+ \frac{1}{8}\upsilon^2\left[g^2(W^3_\mu)^2 - 2gg'W^3_\mu B^\mu + g'^2 B^2_\mu\right] \tag{II.11}$$

The first term gives mass of the charged boson, $M^2_W W^+W^-$, with

$$M_W = \frac{1}{2}\upsilon g \tag{II.12}$$

The second term is off-diagonal in the W^3_μ and B_μ basis, which gives:

$$\frac{1}{8}\upsilon^2\left[g^2(W^3_\mu)^2 - 2gg'W^3_\mu B^\mu + g'^2 B^2_\mu\right] = \frac{1}{8}\upsilon^2\left[g\ W^3_\mu - g'B_\mu\right]^2$$

$$+ 0\left[g'W^3_\mu + gB_\mu\right]^2 \tag{II.13}$$

Now, the physical fields Z_μ and A_μ obtained by diagonalizing the W^3_μ and B_μ basis correspond to

$$\frac{1}{2}M^2_Z Z^2_\mu + \frac{1}{2}M^2_A A^2_\mu,$$

Normalizing the fields, we get

$$Z_\mu = \frac{gW_\mu - g'B_\mu}{(g^2 + g'^2)^{1/2}}; \quad \text{with } M_Z = \frac{1}{2}\upsilon(g^2 + g'^2)^{1/2} \tag{II.14}$$

$$A_\mu = \frac{g' W_\mu^3 + g B_\mu}{(g^2 + g'^2)^{1/2}}; \qquad \text{with } M_A = 0. \tag{II.15}$$

For the electromagnetic interaction to be unified with the weak interaction in this model, the electromagnetic term $ieQA_\mu$ must be contained in the neutral term $i(gW_\mu^3 T_3 + g'\frac{Y}{2}B_\mu)$ of the covariant derivative. Therefore, the W_μ^3 and B_μ fields must be linear combinations of A_μ and the neutral field Z_μ. This can be written as

$$\begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix}, \tag{II.16}$$

where θ_W is the electroweak mixing angle. Hence,

$$ig\, W_\mu^3 T_3 + ig'\frac{Y}{2}B_\mu = iA_\mu \left[g\sin\theta_W T_3 + g'\cos\theta_W \,\frac{Y}{2} \right]$$

$$+ iZ_\mu \left[g\cos\theta_W T_3 - g'\sin\theta_W \,\frac{Y}{2} \right]. \tag{II.17}$$

For the coefficient of A_μ to be equal $ieQ = ie(T_3 + Y/2)$, we need: $g = e/\sin\theta_W$; $g' = e/\cos\theta_W$; and hence

$$\frac{1}{g^2} + \frac{1}{g'^2} = \frac{1}{e^2} \tag{II.18}$$

From eqs.(II.10) and (II.12) we have,

$$M_W/M_Z = \cos\theta_W \tag{II.19}$$

The inequality $M_Z \neq M_W$ is due to the mixing between the W^3_μ and B_μ fields.

II.3. Left-Right Symmetric Model (LRSM).

The left-right symmetric model[2] is based on the gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. A central reason for considering this model rather than the standard model is that the scale of parity violation and the strength of new interactions involving the right handed currents. In this framework, the weak interaction respects all space-time symmetries, as do the other interactions at sufficiently high energies. Second, if the weak interaction symmetries are to arise out of a more fundamental substructure of quarks and leptons then $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ arises as the natural weak interaction symmetry rather than $SU(2)_L \times U(1)_Y$. Another important reason for considering this model is that if the neutrino has a mass, then this kind of model[4] becomes the most natural framework in which to work. Another reason is that the L-R symmetric model can give rise to CP-violation for only two generations, and can account for its strength by relating it to the suppression of V+A currents. Another deficiency of the standard model is the lack of any physical meaning of the U(1) generator, which in the left-right symmetric model becomes B-L.[4] Once B-L is included as a gauge generator, the weak gauge group becomes $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, and the electric charge is given by:[4]

$$Q = T_{3L} + T_{3R} + \frac{B-L}{2} \qquad \text{(II.20)}$$

where B is the baryon number, L is the lepton number, and $T_{3L,R}$ is the third component of the left-right isospin, related to the weak-gauge groups $SU(2)_L$ and $SU(2)_R$ respectively.

In analogy with the standard model, we find from eq.(II.20) that the hypercharge $Y=2T_{3R} + (B-L)$. We define the quantum numbers: $(T_{3L}, T_{3R}, B-L)$ for the following fermion doublets;[5,6] for the quark fields,

$$Q_{L,R} = \begin{pmatrix} u \\ d \end{pmatrix}_{L,R}, \text{ with quantum numbers } (1/2,0,1/3) \text{ and}$$

$$(0,1/2,1/3) \text{ respectively.} \qquad \text{(II.20)}$$

For the leptons we have,

$$L_{L,R} = \begin{pmatrix} \nu \\ e \end{pmatrix}_{L,R}, \text{ with quantum numbers } (1/2,0,-1) \text{ and}$$

$$(0,1/2,-1) \text{ respectively.} \qquad \text{(II.22)}$$

The gauge invariant Lagrangian for Q and L leads to the following gauge interactions with fermions:[6]

$$\mathcal{L}_{WK} = \frac{i}{2} g_L \left[\bar{Q}_L \gamma_\mu \vec{\tau} Q_L + \bar{L}_L \gamma_\mu L_L \right] W_L + \frac{i}{2} g_R \left[\bar{Q}_R \gamma_\mu \vec{\tau} Q_R \right.$$

$$\left. + \bar{L}_R \gamma_\mu \vec{\tau} L_R \right] W_R + \frac{i}{2} g' \left[\frac{1}{3} \bar{Q} \gamma_\mu Q - \bar{L} \gamma_\mu L \right] B_\mu, \qquad \text{(II.23)}$$

where W_L, W_R, B_μ are the gauge bosons corresponding to the gauge groups $SU(2)_L$, $SU(2)_R$ and $U(1)_{B-L}$ respectively; g_L, g_R and g' are the corresponding gauge coupling constants. If one now requires the model to be invariant under the parity operation, P, and the fields transform as follows:

$$L_L \leftrightarrow L_R;\ Q_L \leftrightarrow Q_R;\ \text{and}\ W_L \leftrightarrow W_R,$$

then $g_L \approx g_R \approx g$ reducing the number of the gauge coupling constants to two as in the standard model. This implies that,

$$\frac{1}{e^2} = \frac{2}{g^2} + \frac{1}{g'^2} \tag{II.24}$$

Therefore, as in the standard model, one can parameterize g and g' in terms of the electric charge e, and the Weinberg angle, θ_W with

$$\sin^2\theta_W = e^2/g^2. \tag{II.25}$$

From eqs.(II.24) and (II.25) it follows that

$$\cos^2 2\theta_W = e^2/g'^2. \tag{II.26}$$

In order to maintain left-right symmetry, we choose Higgs multiplets in a way that the minimal set required to break the symmetry down to the $U(1)_{EM}$ are[7]

$$\phi(1/2,1/2,0), \tag{II.27}$$

and

$$\Delta_L = (1,0,2);\ \Delta_R=(0,1,2). \tag{II.28}$$

In component form they can be expressed as

$$\begin{pmatrix} \phi_1^0 & \phi_1^+ \\ \phi_2^- & \phi_2^0 \end{pmatrix}, \tag{II.29}$$

and

$$\begin{pmatrix} \Delta^+/\sqrt{2} & \Delta^{++} \\ \Delta^0 & -\Delta^+/\sqrt{2} \end{pmatrix}_{L,R} \tag{II.30}$$

The gauge fields contain two triplets groups W_L^μ and W_R^μ corresponding to the gauge groups $SU(2)_L$ and $SU(2)_R$ respectively, and one singlet, B_μ belong to the gauge group $U(1)_{B-L}$.

II.4. Left-Right Supersymmetric Model.

The L-R supersymmetric version,[8] is an extension of the supersymmetric model $SU(2)_L \times U(1)_{I3R} \times U(1)_{B-L}$ found in Ref. [9]. In the $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ model, the triplet vector boson $(W^\pm,W^0)_{L,R}$ and their superpartners $(\lambda^\pm,\lambda^0)_{L,R}$ are assigned to the gauge groups $SU(2)_{L,R}$; the singlet gauge boson V_μ and its superpartner λ_V is assigned to the gauge group $U(1)_{B-L}$; g_L, g_R and g_V are the gauge coupling constants corresponding to the groups $SU(2)_L$, $SU(2)_R$ and $U(1)_{B-L}$ respectively. The Higgs fields of this model are:

(i) bi-doublet:

$$\phi_u(1/2,1/2,0),\ \phi_d(1/2,1/2,0), \tag{II.31}$$

and

(ii) four-triplets:

$$\left.\begin{array}{l} \Delta_L(1,0,2) \text{ and } \Delta_R(0,1,2), \\ \delta_L(1,0,-2) \text{ and } \delta_R(0,1,-2). \end{array}\right\} \tag{II.32}$$

Supersymmetry is responsible for doubling the number of the Higgs fields; ϕ_u and ϕ_d are required in order to give masses to both the up and down quarks, and δ_L and δ_R, with B-L quantum number -2, are introduced to cancel anomalies in the fermionic sector that would otherwise happen. The new Higgs fields δ_L and δ_R do not acquire VEV's and play no role in spontaneous symmetry breaking. The normal particles of the model, their superpartners, and their quantum numbers are given in Table II.

According to the existence of the discrete parity symmetry, P, the model has only two gauge coupling constants before symmetry breaking, $g_L{\simeq}g_R{\simeq}g$ and g_V. Symmetry breaking occurs in three stages:[10]

$$SU(2)_L\times SU(2)_R\times U(1)_{B-L}\times P \xrightarrow[M_P]{} SU(2)_L \times SU(2)_R\times U(1)_{B-L},$$

$$SU(2)_L \times SU(2)_R \times U(1)_{B-L} \xrightarrow[M_{WR}]{<\Delta_R>\neq 0} SU(2)_L \times U(1)_Y,$$

Table II: *The particle content for the L-R supersymmetric model with their quantum numbers (the numbers inside the brackets belong to the R-handed particles).*

Field Matter: Fermions	Field Matter: Scalars	$SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ Quantum numbers		
$Q_{L,R}=\begin{pmatrix} u \\ d \end{pmatrix}_{L,R}$		1/2(0)	0(1/2)	1/3
	$\tilde{Q}_{L,R}=\begin{pmatrix} \tilde{u} \\ \tilde{d} \end{pmatrix}_{L,R}$	1/2(0)	0(1/2)	1/3
$L_{L,R}=\begin{pmatrix} \nu \\ e \end{pmatrix}_{L,R}$		1/2(0)	0(1/2)	-1
	$\tilde{L}_{L,R}=\begin{pmatrix} \tilde{\nu} \\ \tilde{e} \end{pmatrix}_{L,R}$	1/2(0)	0(1/2)	-1
$\tilde{\phi}_{u,d}=\begin{pmatrix} \tilde{\phi}_1^o & \tilde{\phi}_1^+ \\ \tilde{\phi}_2^- & \tilde{\phi}_2^o \end{pmatrix}_{u,d}$		1/2	1/2	0
	$\phi_{u,d}=\begin{pmatrix} \phi_1^o & \phi_1^+ \\ \phi_2^- & \phi_2^o \end{pmatrix}_{u,d}$	1/2	1/2	0
$\tilde{\Delta}_{L,R}=\begin{pmatrix} \frac{\tilde{\Delta}^+}{\sqrt{2}} & \tilde{\Delta}^{++} \\ \tilde{\Delta}^o & \frac{-\tilde{\Delta}^+}{\sqrt{2}} \end{pmatrix}_{L,R}$		1(0)	0(1)	2
	$\Delta_{L,R}=\begin{pmatrix} \frac{\Delta^+}{\sqrt{2}} & \Delta^{++} \\ \Delta^o & \frac{-\Delta^+}{\sqrt{2}} \end{pmatrix}_{L,R}$	1(0)	0(1)	2
$\tilde{\delta}_{L,R}=\begin{pmatrix} \frac{\tilde{\delta}^-}{\sqrt{2}} & \tilde{\delta}^o \\ \tilde{\delta}^{--} & \frac{-\tilde{\delta}^-}{\sqrt{2}} \end{pmatrix}_{L,R}$		1(0)	0(1)	-2
	$\delta_{L,R}=\begin{pmatrix} \frac{\delta^-}{\sqrt{2}} & \delta^o \\ \delta^{--} & \frac{-\delta^-}{\sqrt{2}} \end{pmatrix}_{L,R}$	1(0)	0(1)	-2

$$SU(2)_L \times U(1)_Y \xrightarrow[M_{WL}]{<\phi_{u,d}>\neq 0} U(1)_{EM}. \qquad (II.33)$$

At the first stage only the parity symmetry is broken at a mass scale $M \leq M_P$ (Planck mass scale)[2,10] and the weak gauge symmetry remain unbroken. The parity breaking at the first stage results in $g_L \neq g_R$ leaving the W_L and W_R massless. The second stage breaks the gauge symmetry $SU(2)_R \times U(1)_{B-L}$ to $U(1)_Y$ of the standard model at mass scale $M_{WR} << M_P$, and $<\Delta_R> \neq 0$ (M_{WR} is the mass of the right handed gauge boson). It is possible to choose the Higgs multiplets in a way that the parity and $SU(2)_R$ are broken at the same scale, i.e., $M_P = M_{WR}$. The final stage is brought by $<\phi>_{u,d} \neq 0$ and $<\Delta_L> \neq 0$. As in the standard model, in order to ensure that $U(1)_{EM}$ remains unbroken, only the neutral Higgs fields are allowed to have non-zero vacuum expectation values VEV's. The values are:[2]

$$<\Delta_L> = \begin{pmatrix} 0 & 0 \\ \upsilon_L & 0 \end{pmatrix}, \quad <\Delta_R> = \begin{pmatrix} 0 & 0 \\ \upsilon_R & 0 \end{pmatrix}, \quad <\phi> = \begin{pmatrix} \kappa & 0 \\ 0 & \kappa' e^{i\omega} \end{pmatrix}$$

(II.34)

$<\phi>$ causes the mixing of W_L and W_R with a CP-violating phase $e^{i\omega}$. In the present paper the VEV's of the Higgs fields are taken as:

$$<\Delta_L> = 0; \quad <\Delta_R> = \begin{pmatrix} 0 & 0 \\ \upsilon_R & 0 \end{pmatrix}, \quad <\phi_u> = \begin{pmatrix} \kappa_u & 0 \\ 0 & 0 \end{pmatrix}, \quad <\phi_d> = \begin{pmatrix} 0 & 0 \\ 0 & \kappa_d \end{pmatrix}$$

(II.35)

where $\upsilon_L = \kappa' = 0$ have been taken, because of the following reasons: firstly as a stringent case of the phenomenologically required hierarchy[8,10] $\upsilon_R \gg \max(\kappa,\kappa') \gg \upsilon_L$, and secondly as due to the required cancellation of flavor-changing neutral currents. The Higgs fields acquire non-zero VEV's [eq.(II.35)] to break both parity and $SU(2)_R \times U(1)_{B-L}$ $(g_L \simeq g_R)$ as[11]

$$SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times P \xrightarrow[\langle\Delta_L\rangle=0]{\langle\Delta_R\rangle\neq 0} SU(2)_L \times U(1)_Y$$

where $\langle\phi_{u,d}\rangle$ breaks $SU(2)_L \times U(1)_Y \xrightarrow{\langle\phi_{u,d}\rangle\neq 0} U(1)_{EM}$.

(II.36)

In the first stage of breaking, the charged right-handed gauge bosons, $W_R^{\pm}$, and neutral right-handed gauge boson, Z_R^0, acquire masses proportional to υ_R and become much heavier than the usual left-handed $W_L^{\pm}$ and the Z^0-bosons which pick up masses proportional to κ_u and κ_d only at the second stage. In general the gauge group eigenstates, $W_{L,R}$, mix with each other to form mass eigenstates $W_{1,2}$ of mass $M_{W1,2}$,

$$\begin{pmatrix} W_1 \\ W_2 \end{pmatrix} = \begin{pmatrix} \cos\eta & e^{i\omega}\sin\eta \\ -e^{i\omega}\sin\eta & \cos\eta \end{pmatrix} \begin{pmatrix} W_L \\ W_R \end{pmatrix}, \qquad \text{(II.37)}$$

where η is the mixing angle; $\tan\eta \simeq \kappa\kappa'/\upsilon_R^2$; and ω is a CP-violating phase. The mass eigenvalues are:[12,13]

$$M^2_{W1,2} = \frac{1}{2}\left[M^2_{WL} + M^2_{WR} \mp [(M^2_{WR} - M^2_{WL})^2 + 4|M^2_{WLR}|^2]^{1/2}\right]$$

(II.38)

where

$$\tan 2\eta = \frac{\mp 2M^2_{WL,R}}{M^2_R - M^2_L}, \tag{II.39}$$

and $M_{WL,R}$ is mass mixing of W_L-W_R system. Since, we have $\kappa'=\upsilon_L=0$, $M_{WL,R}=0$, $\eta \to 0$. $M_{W1} \simeq M_{WR}$ and $M_{W2} \simeq M_{WR}$ is a good approximation in the absence of mixing.

II.5. Constraints on the Right-Handed Gauge boson Mass, M_{WR}.

Many studies have been done on $SU(2)_L \cdot SU(2)_R \cdot U(1)_{B-L}$ model, and many restrictions have been presented on M_{WR} and η.[13,14] Almost all of these restrictions involve extra assumptions, especially on the Higgs structure and the Yukawa couplings of the models. The two most widely adopted assumptions are:

(i) Manifest L-R symmetry; an unrealistic assumption that CP-violation is generated by complex Yukawa couplings, but that the vacuum expectation values (VEV) of the Higgs fields which generate the fermion masses are real.[15]

(ii) Pseudo-manifest L-R symmetry; the assumption that both P and CP-violation arise from spontaneously breaking, which means the Yukawa couplings are real.[16]

Such assumptions may cause serious difficulties when embedded in grand unified theories (GUT) or when their cosmological implications are considered, especially the baryon asymmetry and domain wall problems.[17] Some authors have assumed that the discrete L-R symmetry P is not a good symmetry at low energy (TeV) scale, which means the symmetry is broken at a much higher scale than the $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ breaking scale. This implies that the gauge couplings g_L and g_R are not equal.[18] We consider the $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ gauge structure to be more fundamental, so it is worthwhile to examine some phenomenological limits without assuming manifest or pseudo-manifest L-R symmetry. In particular, we investigate the existing constraints and limits on M_{WR} and η. The most important are:

- The K_L-K_S mass difference Δm_K can yield a very stringent bound[19,20] M_{WR}>1.4–2.5 TeV, depending on certain theoretical assumptions. However for certain values of the quark matrix elements for the right-handed currents, there is no useful constraint on M_{WR}.[21] The weakest limit from Δm_K yields $M_{WR} \geq 300$ GeV for $g_R \simeq g_L$, independent of the properties of the right-handed neutrinos. $B_d\bar{B}_d$ mixing also yields a stringent constraint on M_{WR}, but there are no significant constraints from $B_s\bar{B}_s$ and $D\bar{D}$ mixing.[22]

- In L-R symmetry models, unlike the standard model, we have a right-handed neutrino, which is inevitably massive. There are extremely severe constraints based upon observations from astrophysics and cosmology.[23] For instance, if the $M_{\nu R}$ is lighter than 1 MeV and was produced in enough numbers in the early Universe, it would have contributed significantly to the expansion rate of the Universe, thus affecting the ratio n/p leading to the production of too much helium. The dominant production mechanism is $e^+e^- \to \nu_{eR}\bar{\nu}_{eR}$ via Z′ exchange. The limit depends on both the M_{WR} and Z′ mass in a complicated model dependent way, typically yielding a lower bound of around 1 TeV (at $g_L \simeq g_R$).

- There are stringent constraints from the energetics of Supernova 1987A.[24] If $M_{\nu R} \leq 10$ MeV then it could be produced in the core of the supernova via the charged current process $e^-_R p \to \nu_{eR} n$. For a certain range of the strength of this charged-current interaction, either of two things can happen. *(i)* ν_R has a mean free path larger than the core radius so that it escapes. This can cause the integrated ν_R luminosity to saturate the total energy that can be released in the neutron-star formation, thereby putting an upper bound on the strength of the charged-current interactions. *(ii)* On the other hand, if ν_R has a mean free path smaller than the core radius, it gets trapped and its subsequent thermalization can reduce the luminosity. This gives a very strong limit; $M_{WR} \geq 23$ TeV.

- For Dirac neutrinos ($M_{\nu R} \simeq 10-50$ MeV), constraints from Δm_K, $B_d\bar{B}_d$ mixing, b-decay and muon decay result in limiting M_{WR} to the range between 500-700 GeV. For the case of Majorana neutrinos, the combination of Δm_K, $B_d\bar{B}_d$ mixing, b-decay and the contribution of right-handed currents to neutrinoless double β-decay[25] can also give correlated bounds between the mass of the right-handed neutrino $M_{\nu R}$ and M_{WR}. As a typical bound, if $M_{\nu R}$ is $\simeq 50$ GeV then $M_{WR} \geq 1.5$ TeV.

There would be a good chance of detecting W_R at the SSC, which should be sensitive up to 8-9 TeV.[16]

There are some stringent limits on the left-right mixing angle η:

- One can look for deviations from the predictions of V-A theory for muon decay. However, since right-handed leptonic charged currents involve the right-handed neutrino field, to carry out this analysis, one needs the mass of the right-handed neutrino. The most stringent limits come from the measurement of the η-parameter in μ-decay at TRIUMF[26] are: $M_{WR} \geq 400$ GeV for arbitrary ζ and $\zeta \to 0.041$ for $M_{WR} \to \infty$.
- Based on the hadronic decay $K \to 3\pi$ one can obtain:[27]

$$M_{WR} \geq 325 \text{ GeV} \quad \text{for} \quad \eta \leq 4\text{x}10^{-3}.$$

In obtaining this bound as well as the bounds from the μ-decay, it is assumed that the quark and lepton mixing angles in the left- and right-handed charged currents are equal. This is however more model dependent than the bound based on μ-decay.

- The most stringent bounds on M_{WR} and η from the combination of the SN 1987A observations would imply,[24]

$$M_{WR} \geq 23 \text{ TeV and } \eta < 10^{-5} \text{ for } m_{\nu R} \leq 10 \text{ MeV}.$$

Such a small value of $m_{\nu R}$ for these values of M_{WR} and η is ruled out by the present laboratory limits from μ-decay which indicate that,[26]

$$M_{WR} \geq 514 \text{ GeV for } \eta = 0.$$

In this work, we consider the absence of W_L-W_R mixing ($\eta \to 0$), and also the absence of unusual contributions to the processes such as K-$\bar{K}$ mixing.[16,28] With reasonable assumptions $M_{WR} \geq 300$ GeV for $g_L \simeq g_R \simeq g$.

II.6. Lagrangians and Feynman Rules for the Left-Right Supersymmetric Model.

II.6.1. General structure of the model.

The model here is found in ref. [8]. The interaction Lagrangian of this model is constructed in concordance with the rules given in Refs. [29] and [30]. Supersymmetric gauge theories consist of gauge bosons V_μ^a ard their gaugino fermionic partners λ^a in the adjoint representation of the gauge group G, and matter multiplets $(A_i \psi_i)$ in some chosen representations of G. λ^a and ψ_i are two component fermions. In the strictly supersymmetric case one has the following terms:

1. *Kinetic terms.*

2. *Self-interaction of the gauge multiplets:* These terms contain three and four gauge-boson vertices where the covariant derivative is given by $D_\mu = \partial_\mu + igT^a W_\mu^a$; where T^a are the SU(2) group generators, $T^a = \tau^a/2$, where τ^a denote the Pauli matrices (a=1,2,3) and W_μ^a are the gauge fields. The gauginos interact with gauge fields through the following term:

$$ig f_{abc} \lambda^a \sigma^\mu \bar{\lambda}^b V_\mu^c \qquad \text{(II.40)}$$

where f_{abc} are the structure constants of G.

3. *Interactions terms of the gauge field with matter mult-*

iplets: These terms are,

$$\left.\begin{array}{l} -\,gT^a_{ij}V^a_\mu(\bar{\psi}_i\bar{\sigma}^\mu\psi_j + iA^*_i\overleftrightarrow{\partial}^\mu A_j), \\ ig\sqrt{2}T^a_{ij}(\lambda^a\psi_j A^*_i - \bar{\lambda}^a\bar{\psi}_i A_j), \\ g^2(T^aT^b)_{ij}V^a_\mu\ V^{\mu b}A^*_i A_j \end{array}\right\} \tag{II.41}$$

4. *Self-interactions of the matter multiplets:* The superpotential W is some cubic gauge-invariant function of the scalar matter fields A_i (independent on A^*_i). Define the auxiliary functions,

$$F_i = \partial W/\partial A_i, \quad D^a = gA^*_i T^a_{ij} A_j \tag{II.42}$$

Then, the scalar supersymmetric potential can be written as,

$$V = \tfrac{1}{2}D^aD^a + F_iF^*_i \tag{II.43}$$

Yukawa interactions are given by

$$-\,\tfrac{1}{2}[(\partial^2 W/\partial A_i\partial A_j) + \text{H.C.}] \tag{II.44}$$

For the U(1) factor there is no gaugino-gaugino-gauge interaction (i.e., set $f_{abc}=0$) and the product $gT^a_{ij}V^a_\mu$ is replaced by $\frac{1}{2}gy_i\delta_{ij}V_\mu$, where y_i is the U(1) quantum number of matter multiplet (A_i,ψ_i). In general the U(1) D field may be shifted by the so called Fayet-Illiopoulos term:[31]

$$D' = \frac{1}{2} g y_i A_i^* A_i + \xi \tag{II.45}$$

The constant ξ is different from zero and in most realistic models this constant is absent. Thus, the scalar potential can be written now as

$$V = \frac{1}{2}[D^a D^a + D'^2] + F_i F_i^* \tag{II.46}$$

To construct models which keep the nice property of such supersymmetric theories—lack of quadratic divergences—and which simultaneously are experimentally acceptable, it is necessary to add to the above Lagrangian explicit *soft supersymmetry breaking terms.* All admissible expressions are of the form:[32]

$$\tilde{M}_1 \mathrm{Re}A^2 + \tilde{M}_2 \mathrm{Im}A^2 + y(A^3 + \mathrm{H.C.}) + \tilde{M}_3(\lambda^a\lambda^a + \bar{\lambda}^a\bar{\lambda}^a)$$

$$+ \tilde{M}_4(\lambda'\lambda' + \bar{\lambda}'\bar{\lambda}') \tag{II.47}$$

where A^2 and A^3 denote symbolically all possible gauge invariant combinations of the scalar fields A_i (e.g., $A^3 \equiv d_{ijk} A_i A_j A_k$, etc.). These terms split the masses of scalars and fermions present in the supersymmetry multiplets and introduce new, nonsupersymmetric trilinear scalar couplings. The coupling constant y corresponds to a new (nonsupersymmetric) scalar interaction term, $\tilde{M}_3$ and $\tilde{M}_4$ are Majorana mass terms for the gauginos corresponding to the groups G and U(1) respectively, and λ is the superpartner, the gaugino, of the gauge boson. In our present work, we only consider the Lagrangian for the interactions of charginos and neutralinos with the left- and right-handed gauge bosons.

II.6.2. The scalar supersymmetric potential.

Before we write the gaugino and higgsino mixing Lagrangian in the following sections, we have to compute first the scalar field potential, V, in the L-R SUSY model. This scalar potential which arises from the superpotential can be computed as the sum of "D" and "F" terms (Refs. 29 and 30) and the soft-breaking potential, V_{soft}. Thus, the scalar potential is written using eq.(II.45), by using the identity

$$\tau^a_{ij}\tau^a_{kl} = 2\delta_{il}\delta_{jk} - \delta_{ij}\delta_{kl}, \qquad \text{(II.48)}$$

The result for V is

$$V = \tfrac{1}{2}|D|^2 + |F|^2 + V_{soft}, \qquad \text{(II.49)}$$

where the D terms [eqs.(II.42) and (II.45)] are given by

$$|D|^2 = \tfrac{1}{2}g_L \sum_L \Big|\sum_\psi \psi^\dagger G_L \psi\Big|^2 + \tfrac{1}{2}g_R \sum_R \Big|\sum_\psi \psi^\dagger G_R \psi\Big|^2 + \tfrac{1}{2}g_V \Big|\sum_\psi \psi^\dagger V \psi\Big|^2, \qquad \text{(II.50)}$$

where $\psi = \tilde{Q}_{L,R}$, $\tilde{L}_{L,R}$, $\Phi_{u,d}$, $\Delta_{L,R}$, $\delta_{L,R}$; and $G_{L,R}$, and V are the generators of the gauge groups.

$$|F|^2 = |\varepsilon_u^Q \tilde{Q}_L \tilde{Q}_R + \varepsilon_u^L \tilde{L}_L \tilde{L}_R|^2 + |\varepsilon_d^Q \tilde{Q}_L \tilde{Q}_R + \varepsilon_d^L \tilde{L}_L \tilde{L}_R|^2$$

$$+ |\varepsilon_u^Q \Phi_u \tilde{Q}_R + \varepsilon_d^Q \Phi_d \tilde{Q}_R|^2 + |\varepsilon_u^Q \Phi_u \tilde{Q}_L + \varepsilon_d^Q \Phi_d \tilde{Q}_L|^2$$

$$+ |\varepsilon_u^L \Phi_u \tilde{L}_R + \varepsilon_d^L \Phi_d \tilde{L}_R + 2\varepsilon_{LR} (\tau.\Delta_L) \tilde{L}_L|^2$$

$$+ |\varepsilon_u^L \Phi_u \tilde{L}_L + \varepsilon_d^L \Phi_d \tilde{L}_L + 2\varepsilon_{LR} (\tau.\Delta_R) \tilde{L}_R|^2 + \text{H.C.}, \qquad \text{(II.51)}$$

where $\varepsilon^Q_{u,d}$, $\varepsilon^L_{u,d}$, $\varepsilon_{L,R}$ are the lepton and quark Yukawa couplings.

$$V_{soft} = m_S \Big[\varepsilon_u^Q \tilde{Q}_L^\dagger \Phi_u \tilde{Q}_R + \varepsilon_d^Q \tilde{Q}_L^\dagger \Phi_d \tilde{Q}_R + \varepsilon_u^L \tilde{L}_L^\dagger \Phi_u \tilde{L}_R + \varepsilon_d^L \tilde{L}_L^\dagger \Phi_d \tilde{L}_R$$

$$+ \varepsilon_{LR} [\tilde{L}_L^T \tau_1 (\tau.\Delta_L) \tilde{L}_L + \tilde{L}_R^T \tau_1 (\tau.\Delta_R) \tilde{L}_R] + \mu_1 (\tau_1 \Phi_u \tau_1)^T \Phi_d$$

$$+ \mu_2 (\tau.\Delta_L)(\tau.\delta_L) + \mu_3 (\tau.\Delta_R)(\tau.\delta_R) \Big] + m_{QL}^2 \tilde{Q}_L^\dagger \tilde{Q}_L + m_{QR}^2 \tilde{Q}_R^\dagger \tilde{Q}_R$$

$$+ m_{LL}^2 \tilde{L}_L^\dagger \tilde{L}_L + m_{LR}^2 \tilde{L}_L^\dagger \tilde{L}_R. \qquad \text{(II.52)}$$

where μ_i (i=1,2,3) are the higgsino mass parameters; the parameters m_S, m_{QL}, m_{QR}, m_{LL} and m_{LR} have dimensions of mass.

It is necessary to introduce the *soft SUSY-breaking terms* (without them even using two Higgs doublets it is impossible to break spontaneously the gauge symmetry). Equation (II.47) gives the mass terms for gauginos:

$$M_L(\lambda_L^a\lambda_L^a + H.C.) + M_R(\lambda_R^a\lambda_R^a + H.C.) + M_V(\lambda_V\lambda_V + H.C.) \tag{II.53}$$

where M_L, M_R and M_V are the gaugino mass parameters associated with the gauge groups $SU(2)_L$, $SU(2)_R$ and $U(1)_{B-L}$ respectively.

II.6.3. The charged gaugino-higgsino mixing Lagrangian.

In this Lagrangian, we consider the term $(\lambda\psi A)$ in eq.(II.41), the scalar potential in eq.(II.49), and the soft SUSY breaking mass terms for the charged gauginos (the superpartners of the charged gauge bosons) given in eq.(II.53). As a results of this coupling, fermion masses are generated by the VEV's of the Higgs fields given in eq.(II.35). Substituting eq.(II.35) in eqs.(II.41) and (II.49), we get:

$$\mathcal{L}_{CH} = \Big[i\lambda_R^-(\frac{1}{\sqrt{2}}g_R\upsilon_R\tilde{\Delta}_R^+ + g_R\kappa_d\tilde{\phi}_d^+) + i\lambda_L^- g_L\kappa_d\tilde{\phi}_d^+ + i\lambda_R^+ g_R\kappa_u\tilde{\phi}_d^-$$
$$+ i\lambda_L^+ g_L\kappa_u\tilde{\phi}_u^- + M_L\lambda_L^+\lambda_L^- + M_R\lambda_R^+\lambda_R^- + \mu\tilde{\phi}_u^+\tilde{\phi}_d^- + \mu\tilde{\phi}_u^-\tilde{\phi}_d^+\Big] + H.C. \tag{II.54}$$

where we assume for simplicity that $\mu_1=\mu$ and $\mu_2=\mu_3=0$. From eq.(II.54) one can write the mass term as,

$$\mathcal{L}_{mass} = \frac{i}{\sqrt{2}} g_R \upsilon_R \lambda_R^- \tilde{\Delta}_R^+ + \text{H.C.} \tag{II.55}$$

which can be written in four-component notation as:

$$\mathcal{L}_{mass} = -\tilde{M}_{WR} \bar{\tilde{W}}_R \tilde{W}_R, \tag{II.56}$$

where $\tilde{W}_R = \begin{pmatrix} \tilde{\Delta}_R^+ \\ i\bar{\lambda}_R^- \end{pmatrix}$, is a four-component Dirac spinor. At this stage susy is unbroken, and the right-handed gaugino mass, $\tilde{M}_{WR} = \frac{1}{\sqrt{2}} g_R \upsilon_R$ is the same as that of the gauge boson mass, M_{WR}.

II.6.4. The neutral gaugino-higgsino mixing Lagrangian.

In analogy with the previous section, we consider the term $(\lambda\psi A)$ in eq.(II.41), the scalar potential in eq.(II.48), and the soft SUSY breaking mass terms for the neutral gauginos (the superpartners of the neutral gauge bosons) given in eq.(II.52). As a results of this coupling, fermion masses are generated by the neutral Higgs fields. We get:

$$\begin{aligned}\mathcal{L}_{NH} = \Big[&- \frac{i}{\sqrt{2}} g_R \upsilon_R \tilde{\Delta}_R^0 \lambda_R^- + \frac{2i}{\sqrt{2}} g_V \upsilon_R \tilde{\Delta}_R^0 \lambda_V^0 + \frac{i}{\sqrt{2}} g_R \kappa_u \tilde{\phi}_u^0 \lambda_R^0 \\ &- \frac{1}{\sqrt{2}} g_L \kappa_u \tilde{\phi}_u^0 \lambda_L^0 - \frac{i}{\sqrt{2}} g_R \kappa_d \tilde{\phi}_d^0 \lambda_R^0 + \frac{i}{\sqrt{2}} g_L \kappa_d \tilde{\phi}_d^0 \lambda_L^0 + \\ &+ M_L \lambda_L^0 \lambda_L^0 + M_R \lambda_R^0 \lambda_R^0 + M_V \lambda_V^0 \lambda_V^0 + 2\mu \tilde{\phi}_u^0 \tilde{\phi}_d^0 \Big] + \text{H.C.}\end{aligned} \tag{II.57}$$

II.6.5. The gauge boson masses.

In section II.2 of this chapter we have explained briefly the gauge symmetry Lagrangian and extracting the masses of the gauge bosons. In this section we have used the Lagrangian and symmetry breaking, to obtain the masses for the left- and right-handed gauge bosons. In the first stage of symmetry breaking, the Higgs field acquires VEV $<\Delta_R>$. This process generates masses fo the right-handed gauge bosons, $W_R^{\pm}$ and W_R^0. In the second stage the VEV's $<\phi>_{u,d}$ of the Higgs field generate masses for left-handed gauge bosons, $W_L^{\pm}$ and W_L^0.

The right-handed gauge boson masses are obtained by substituting the VEV for the Higgs field Δ_R, in eq.(II.5). The relevant term is:

$$\left|\left(ig_R \frac{\vec{\tau}}{2}.W_\mu^R + ig_V V_\mu\right)\Delta_R\right|^2 =$$

$$= \frac{1}{4}\left|\left[\begin{pmatrix} g_R W^0_{\mu R} & g_R(W^1_{\mu R} - iW^2_{\mu R}) \\ g_R(W^1_{\mu R} + iW^2_{\mu R}) & -g_R W^0_{R\mu} \end{pmatrix} + 2g_V V_\mu\right]\begin{pmatrix} 0 & 0 \\ \upsilon_R & 0 \end{pmatrix}\right|^2 \tag{II.58}$$

$$= \left(\frac{1}{\sqrt{2}}\upsilon_R g_R\right)^2 W^{+}_{\mu R} W^{-\mu}_{R} + \frac{1}{4}\upsilon_R^2 (W^0_{\mu R},\ V_\mu)\begin{pmatrix} g_R^2 & -2g_V g_R \\ -2g_V g_R & 4g_V^2 \end{pmatrix}\begin{pmatrix} W_R^{0\mu} \\ V^\mu \end{pmatrix} \tag{II.59}$$

where $W^{\pm}_{\mu R} = (W^1_{\mu R} \mp iW^2_{\mu R})/\sqrt{2}$. Comparing the first term in eq.(II.59) with the mass term expected for the right-handed gauge boson, $M^2_{WR} W^{+}_{\mu R} W^{-\mu}_{R}$, we have

$$M_{WR} = g_R \upsilon_R / \sqrt{2} \qquad \text{(II.60)}$$

The remaining term is off-diagonal in the $W^0_{\mu R}$ and V_μ basis, and can be written:

$$\tfrac{1}{4}\upsilon_R^2[g_R^2 W^{02}_{\mu R} - 4g_R g_V W^0_{\mu R} V_\mu + 4g_V^2 V_\mu^2] = \tfrac{1}{4}\upsilon_R^2[g_R W^0_{\mu R} - 2g_V V_\mu]^2$$

$$+ O[2g_V W^0_{\mu R} + g_R V_\mu]^2. \qquad \text{(II.61)}$$

The matrix in eq.(II.59) has two eigenvalues: $(g_R^2 + 4g_V^2)$ and 0. Equation (II.61) is a linear combination of the fields $W^0_{\mu R}$, V_μ are given by the new physical fields $Z_{\mu R}$ and B_μ occur in the diagonalized mass matrix in eq.(II.59), so that eq.(II.61) must be identified with

$$\tfrac{1}{2}M_Z^2 Z^2_{\mu R} + \tfrac{1}{2}M_B^2 B^2_\mu.$$

Therefore, on normalizing the fields, we have

$$Z_{\mu R} = \frac{g_R W^0_{\mu R} - 2g_V V_\mu}{(g_R^2 + 4g_V^2)^{1/2}}; \qquad \text{with } M_{ZR} = \frac{1}{\sqrt{2}}\upsilon_R (g_R^2 + 4g_V^2)^{1/2} \qquad \text{(II.62)}$$

and

$$B_\mu = \cdot \frac{2g_V W^0_{\mu R} + g_R V_\mu}{(g_R^2 + 4g_V^2)^{1/2}}; \qquad \text{with } M_B = 0. \qquad \text{(II.63)}$$

The massless eigenstate B_μ is the gauge boson of the gauge group $U(1)_Y$, which survives the breaking of $SU(2)_R \times U(1)_{B-L}$. The massive eigenstates $W^\pm_{\mu R}$ and $Z_{\mu R}$ are decoupled from the low-energy theory, leaving only B_μ to go through the second stage of symmetry breaking.

In the second stage of symmetry breaking, the left-handed gauge boson masses are obtained by substituting the VEV's $\langle\phi\rangle_{u,d}$, eq.(II.35), for the Higgs fields in eq.(II.5). The relevant terms are

$$\tfrac{1}{4}|(ig_L\vec{\tau}.W^L_\mu + ig_R\vec{\tau}.W^R_\mu)\phi_u|^2 + \tfrac{1}{4}|(ig_L\vec{\tau}.W^L_\mu + ig_R\vec{\tau}.W^R_\mu)\phi_d|^2. \tag{II.64}$$

Since the right-handed gauge bosons decouple from the first part, we neglect the terms containing the charged gauge bosons $W^\pm_{\mu R}$ and $Z_{\mu R}$. We retain the neutral boson $W^0_{\mu R}$, which can be written in terms of the fields $Z_{\mu R}$, B_μ as:

$$W^0_R = \frac{g_R Z_{\mu R} + 2g_V B_\mu}{(g_R^2 + 4g_V^2)^{1/2}} \tag{II.65}$$

Inserting (II.65) in (II.64), we find

$$\begin{aligned} &= [\tfrac{1}{2}g_L^2(\kappa_u^2 + \kappa_d^2)]W^+_{\mu L}W^{-\mu}_L \\ &+ \tfrac{1}{4}(\kappa_u^2 + \kappa_d^2)(W^0_{\mu L}, B_\mu)\begin{pmatrix} g_L^2 & -2g_L g' \\ -2g_L g' & 4g'^2 \end{pmatrix}\begin{pmatrix} W^{0\mu}_L \\ B^\mu \end{pmatrix}, \end{aligned} \tag{II.66}$$

where we have used: $W^\pm_{\mu L} = (W^1_{\mu L} \mp W^2_{\mu L})/\sqrt{2}$, and g' is the coupling constant of the gauge group $U(1)_Y$, which is given by

$$g' = \frac{g_R g_V}{(g_R^2 + 4g'^2)^{1/2}} \tag{II.67}$$

Comparing the first term of eq.(II.66) with the mass term, $M_{WL}^2 W_{\mu L}^{+} W_L^{-\mu}$;

$$M_{WL} = \frac{1}{\sqrt{2}} g_L (\kappa_u^2 + \kappa_d^2)^{1/2} \tag{II.68}$$

The remaining term is off-diagonal in the $W_{\mu L}^0$ and B_μ which can be written as

$$\frac{1}{4}(\kappa_u^2 + \kappa_d^2)[g_L^2 (W_{\mu L}^0)^2 - 2g_L g' W_{\mu L}^0 B_\mu + g'^2 B_\mu^2]$$

$$= \frac{1}{4}(\kappa_u^2 + \kappa_d^2)[g_L W_{\mu L}^0 - 2g' B_\mu]^2 + 0[2g' W_{\mu L}^0 + g_L B_\mu]^2 \tag{II.69}$$

The matrix in eq.(II.66) has two eigenvalues, $g_L^2 + 4g'^2$ and 0. Similarly, the physical fields $Z_{\mu L}$ and A_μ arise from diagonalizing the mass matrix, so that eq.(II.69) must be identified with;

$$\frac{1}{2} M_Z^2 Z_{\mu L}^2 + \frac{1}{2} M_A^2 A_\mu^2 .$$

Therefore, on normalizing the fields, we have,

$$Z_{\mu L} = \frac{g_L W_{\mu L}^0 - 2g' B_\mu}{(g_L^2 + 4g'^2)^{1/2}}$$

with mass $M_{ZL} = \frac{1}{\sqrt{2}}(\kappa_u^2 + \kappa_d^2)^{1/2}(g_L^2 + 4g'^2)^{1/2}$, (II.70)

and the photon field A_μ given by

$$A_\mu = \frac{2g_L W^0_{\mu L} + g' B_\mu}{(g_L^2 + 4g'^2)^{1/2}} \qquad \text{with} \quad M_A = 0. \tag{II.71}$$

II.6.6. Chargino-neutralino interactions with the gauge bosons.

Based on eqs.(II.40) and (II.41), one can write the Lagrangian in two component notation;

$$\mathcal{L}_{WCN} = \frac{i}{2} g_L W_L^{-\mu} (\bar{\lambda}_L^{+} \bar{\sigma}_\mu \lambda_L^0 - \bar{\lambda}_L^{0} \bar{\sigma}_\mu \lambda_L^-)$$

$$+ \frac{ig_L}{\sqrt{2}} \Big[W_L^{-\mu} \, (\bar{\tilde{\phi}}_u^{+} \bar{\sigma}_\mu \tilde{\phi}_d^- - \bar{\tilde{\phi}}_u^{-} \bar{\sigma}_\mu \tilde{\phi}_d^+) + W_L^{-\mu} (\bar{\tilde{\phi}}_d^{+} \bar{\sigma}_\mu \tilde{\phi}_u^- - \bar{\tilde{\phi}}_d^{-} \bar{\sigma}_\mu \tilde{\phi}_u^+)$$

$$+ W_L^{-\mu} (\bar{\tilde{\Delta}}_L^{+} \bar{\sigma}_\mu \tilde{\Delta}_L^- - \bar{\tilde{\Delta}}_L^{-} \bar{\sigma}_\mu \tilde{\Delta}_L^+) + W_L^{-\mu} (\bar{\tilde{\Delta}}_L^{++} \bar{\sigma}_\mu \, \tilde{\Delta}_L^{--} - \bar{\tilde{\Delta}}_L^{--} \bar{\sigma}_\mu \tilde{\Delta}_L^{++}) \Big]$$

$$+ \text{R.H.} + \text{H.C.} \tag{II.72}$$

where $\bar{\sigma}^\mu$ are Pauli matrices (see, appendix A). The four-component Majorana spinors are given by

$$\tilde{W}^0_{L,R} = \begin{pmatrix} -i\lambda^0_{L,R} \\ i\bar{\lambda}^0_{L,R} \end{pmatrix}, \quad \tilde{W}_{L,R} = \begin{pmatrix} -i\lambda^+_{L,R} \\ i\bar{\lambda}^-_{L,R} \end{pmatrix}, \quad \tilde{\Phi}_{u,d} = \begin{pmatrix} \tilde{\phi}^+_{u,d} \\ \bar{\tilde{\phi}}^-_{u,d} \end{pmatrix},$$

$$\tilde{H}^1_{L,R} = \begin{pmatrix} \tilde{\Delta}^+_{L,R} \\ \bar{\tilde{\Delta}}^-_{L,R} \end{pmatrix}, \text{ and } \tilde{H}^2_{L,R} = \begin{pmatrix} \tilde{\Delta}^{++}_{L,R} \\ \bar{\tilde{\Delta}}^{--}_{L,R} \end{pmatrix}. \tag{II.73}$$

Inserting (II.73) into (II.72) we get

$$\mathcal{L}_{WCN}= \frac{i}{2} g_L W_L^{-\mu}\bar{\tilde{W}}_L^0 \gamma_\mu \tilde{W}_L + \frac{ig_L}{\sqrt{2}} W_L^{-\mu}\Big[\bar{\tilde{\Phi}}_u \gamma_\mu \tilde{\Phi}_d$$

$$+ \bar{\tilde{\Phi}}_d \gamma_\mu \tilde{\Phi}_u + \bar{\tilde{H}}_L^1 \gamma_\mu \tilde{H}_L^1 + \bar{\tilde{H}}_L^2 \gamma_\mu \tilde{H}_L^2\Big] + R.H. + H.C. \qquad \text{(II.74)}$$

For the lightest supersymmetric particle (LSP), $\tilde{\chi}$, we can write the Feynman rules for the $W^-_{L,R}-\tilde{\chi}^+_j-\tilde{\chi}$ interactions. The left-right gaugino fields are given in terms of the LSP $\tilde{\chi}$ and zino $\tilde{Z}$ as follows;[8,10]

$$\left.\begin{aligned}\tilde{W}_L^0 &= \cos\theta_W \tilde{Z}_L + \sin\theta_W \tilde{\chi}\\ \tilde{W}_R^0 &= \frac{\sqrt{\cos 2\theta_W}}{\cos\theta_W}\tilde{Z}_R + \sin\theta_W \tan\theta_W \tilde{Z}_L + \sin\theta_W \tilde{\chi}\end{aligned}\right\} \qquad \text{(II.75)}$$

Inserting eq.(II.75) into the $(W^{-\mu}-\tilde{W}^0-\tilde{W})_{L,R}$ interaction term and making use of the chargino eigenstates,

$$P_R \tilde{W}_L = P_R(U_{11}\tilde{\chi}_1 + U_{21}\tilde{\chi}_2 + U_{31}\tilde{\chi}_3 + U_{41}\tilde{\chi}_4),$$

$$\bar{\tilde{W}}_L P_L = (U^*_{11}\bar{\tilde{\chi}}_1 + U^*_{21}\bar{\tilde{\chi}}_2 + U^*_{31}\bar{\tilde{\chi}}_3 + U^*_{41}\bar{\tilde{\chi}}_4) P_L,$$

$$\bar{\tilde{W}}^c_L P_L = (V^*_{11}\bar{\tilde{\chi}}^c_1 + V^*_{21}\bar{\tilde{\chi}}^c_2 + V^*_{31}\bar{\tilde{\chi}}^c_3 + V^*_{41}\bar{\tilde{\chi}}^c_4) P_L,$$

$$P_R \bar{\tilde{W}}^c_L = P_R(V_{11}\bar{\tilde{\chi}}^c_1 + V_{21}\bar{\tilde{\chi}}^c_2 + V_{31}\bar{\tilde{\chi}}^c_3 + V_{41}\bar{\tilde{\chi}}^c_4). \qquad \text{(II.76)}$$

where $\tilde{\chi}^c_j$ (j=1,..,4) are the charge conjugate states,

$P_{L,R}=(1 \mp \gamma_5)/2$, and V_{j1}, U_{j1} are matrices diagonalizing the chargino mass matrix. Then, the resulting interaction is (we have neglected the terms containing $\bar{\tilde{Z}}_L$ and $\bar{\tilde{Z}}_R$),

$$\mathcal{L}_{WN(LSP)} = \frac{1}{2} g_L \sin\theta_W W_L^{-\mu} \bar{\tilde{\chi}} \gamma_\mu (P_R U_{11} + P_L V_{11}^*) \tilde{\chi}_1^+$$

$$+ \frac{1}{2} g_L \sin\theta_W W_L^{-\mu} \bar{\tilde{\chi}} \gamma_\mu (P_R U_{21} + P_L V_{21}^*) \tilde{\chi}_2^+$$

$$+ \frac{1}{2} g_L \sin\theta_W W_L^{-\mu} \bar{\tilde{\chi}} \gamma_\mu (P_R U_{31} + P_L V_{31}^*) \tilde{\chi}_3^+$$

$$+ \frac{1}{2} g_L \sin\theta_W W_L^{-\mu} \bar{\tilde{\chi}} \gamma_\mu (P_R U_{41} + P_L V_{41}^*) \tilde{\chi}_4^+$$

$$+ \text{R.H.} + \text{H.C.} \tag{II.77}$$

The corresponding Feynman rules are shown in Fig. II.1*a*. To get the neutralino physical states $\tilde{\chi}_i^0$, from eq.(II.74) the $W_{L,R}^- - \tilde{\chi}_j^+ - \tilde{\chi}_i^0$ interaction term becomes (inserting $P_L + P_R = 1$),

$$\mathcal{L}_{WCN} = \frac{-i g_L}{2} W_L^{-\mu} \bar{\tilde{\chi}}_i^0 \gamma_\mu [O_{ij}^L P_L + O_{ij}^R P_R] \tilde{\chi}_j^+ + \text{R.H.} + \text{H.C.} \tag{II.78}$$

where $O_{ij}^L = -N_{i4} V_{j2}^* + N_{i2} V_{j1}^*$; $O_{ij}^R = -N_{i3}^* U_{j2} + N_{i2}^* U_{j1}$

(II.79)

and N_{ij} are unitary matrices which diagonalize the neutral fermion states. The Feynman rules for the $W_{L,R}^-$-chargino-neutralino interactions are shown in Fig. II.1*b*.

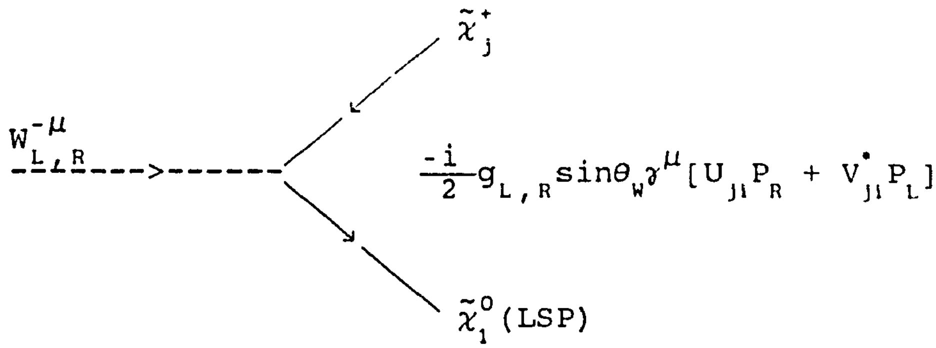

Fig. II.1 *Feynman rules for the interaction of the* $W^-_{L,R}$ *with the chargino* $\tilde{\chi}^+_j$ *and lightest supersymmetric particle* $\tilde{\chi}^0_1$.

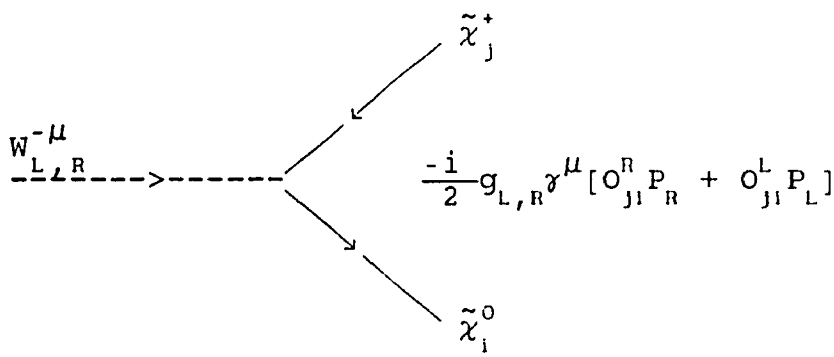

Fig. II.2 *Feynman rules for the interaction of the* $W^-_{L,R}$ *with the chargino* $\tilde{\chi}^+_j$ *and neutralino* $\tilde{\chi}^0_i$.

References

[1] S. Bludman, *II Nuovo Cimento* 9 (1958) 433; S.L. Glashow, *Nucl. Phys.* **22** (1961) 579; A. Salam and J.C. Ward, *Phys. Lett.* **13** (1964) 168; S. Weinberg, *Phys. Rev. Lett.* **19** (1967) 1264; A. Salam, in *Elementary Particle Theory* (ed. N. Svartholm), Almquist and Forlag, Stockholm, 1968; S. Glashow, J. Illiopoulos and L. Maiani, *Phys. Rev.* **D2** (1970) 1285; M. Kobayashi and M. Maskawa, *Prog. Theor. Phys.* **49** (1973) 652.

[2] J.C. Pati and A. Salam, *phys.Rev.* **D10** (1974) 275; R.N. Mohapatra and J.C. Pati, *Phys.Rev.* **D11** (1975) 566, 2558; G. Senjanovic and R.N. Mohapatra, *Phys. Rev.* **D12** (1975) 1502.

[3] R.J.N. Phillips, *Collider Physics*, Addison-Wesley Pub., California, 1987, ch.2.

[4] R.N. Mohapatra and R.E. Marshak, *Phys. Lett.* **91B** (1980) 222; A. Davidson, *Phys. Rev.* **D20** (1979) 776.

[5] J.F. Gunion, H.E. Haber, G. Kane, and S. Dawson, *The Higgs Hunter's Guide*, Addison-Wesley Pub., California, 1990, ch.6.

[6] R.N. Mohapatra and G. Senjanovic, *Phys. Rev. Lett.* **44**, (1980) 912; R.N. Mohapatra, in *Advanced Series on Directions in High Energy Physics*, Vol.3, ed. C. Jarlskog, Word Scientific Pub., 1989; R.N. Mohapatra and R.E. Marshak, *Phys. Rev. Lett.* **44**, (1980) 1316.

[7] Ref.[6] and R.N. Mohapatra and G. Senjanovic, *Phys. Rev.* **D23** (1981) 165.

[8] R.M. Francis, M. Frank and C.S. Kalman, *Phys. Rev.* **D43** (1991) 2369.

[9] M. Frank and C.S. Kalman, *Phys. Rev.* **D38** (1988) 1469.

[10] R.N. Mohapatra and G. Senjanovic, *Phys. Rev. Lett.* 46 (1981) 1315; and *Phys. Rev.* **D24** (1981) 704; D. Chang, R.N. Mohapatra, and M.K. Parida, *Phys. Rev.* D30 (1984) 1052, and *Phys. Rev. Lett.* **52** (1984) 1072.

[11] J.F. Gunion et al., *Phys. Rev.* **D4**, (1990) 1546.

[12] P. Herczeg, Phys. Rev. D34 (1986) 3449.

[13] For review, see for example R.N. Mohapatra, in *New Frontiers in High Energy Physics*, New York, 1978, ed. B. Kursunoglu et al.; R.N. Mohapatra, *Unification and Supersymmetry*, The Frontiers of Quark-Lepton Physics, p.118-138, Springer, New York, 1986.

[14] See for example Ref. [13] and P. Langacker, in Proceedings of the 1985 International Symposium on *Lepton and Photon Interactions at High Energies*, eds. M. Konuma and K. Takahashi p.186, Nisha, Kyoto, 1986.

[15] M.A.B. Bég et al., *Phys. Rev. Lett.* **38** (1977) 1252; G. Senjanovic, *Nucl. Phys.* **B153** (1979) 334.

[16] See, Ref. [13] and H. Harari and M. Leurer, *Nucl. Phys.* **B233** (1984) 221; P. Langacker and S. Uma Sanker, *Phys. Rev.* **D40** (1989) 1569.

[17] See for instance P. Langacker, *Comm. Nucl. and Part. Phys.* **15** (1985) 41.

[18] D. Chang et al., *Phys. Rev.* D31 (1985) 1718; J.M. Gibson and R.E. Marshak, *ibid*, P.1705; Y. Tosa et al.,

Phys. Rev. **D28** (1983) 1731.

[19] G. Beall, M. Bander and A. Soni, *Phys. Rev. Lett.* **48** (1982) 848.

[20] G. Ecker and W. Grimus, *Nucl. Phys.* **B258** (1985) 328.

[21] The limits from Δm_K for the quark matrix elements of the right-handed currents were first considered by F.I. Olness and M.E. Ebel, *Phys. Rev.* **D30** (1984) 1034. See P. Basak et al., *Z. Phys. C-Particles and Fields* **20** (1983) 305.

[22] G. Altareli and P. Franzini, *Z. Phys. C-Particles and Fields* **37** (1988) 271; see also a review of $B_d\bar{B}_d$ mixing for P. Franzini, CERN-TH-5083.

[23] D. Seckel, *Phys. Rev. Lett.* **60** (1988) 1793; R. Barbieri and R.N. Mohapatra, *Phys. Rev.* **D39** (1989) 1229.

[24] G. Steigman, K.A. Olive, and D. Schramm, *Phys. Rev. Lett.* **43** (1979) 239; *Nucl. Phys.* **B180** (1981) 497.

[25] W.C.Haxton and G.J. Stephenson, *Prog. Nucl. Part. Phys.* **12** (1984) 409; R.N. Mohapatra, *Phys. Rev.* **D34** (1986) 909; M. Doi et al., *Prog. Th. Phys.* (Supp) **83** (1985) 1.

[26] J. Carr, et al., *Phys. Rev. Lett.* **51** (1983) 627; D.P. Stoker et al., *ibid* **54** (1985) 1887.

[27] J.F. Donoghue and B.R. Holstein, *Phys. Lett.* **B113** (1982) 382.

[28] J.L. Rosner and E. Takasugi, *Phys. Rev.* **D42** (1992) 241.

[29] H.E. Haber and G.L. Kane, *Phys. Rep.* **117** (1985) 75; J.F. Gunion and H.E. Haber, *NUcl. Phys.* **B272** (1986) 1.

[30] J. Rosiek, *Phys. Rev.* **D41** (1990) 3464.

[31] P. Fayet and J. Illiopoulos, *Phys.Lett.* **51B** (1974) 461.

[32] L. Girardello and M.T. Grisaru, *Nucl.Phys.* **B194** (1982) 65.

Chapter III

Chargino Mass Eigenstates

III.1. Introduction.

The charginos are mass eigenstates, $\tilde{\chi}^{\pm}_{i}$ (i=1,2) and thus linear combinations of charged gauginos and higgsinos. In the MSSM the two gauge bosons $W^{\pm}$, and the charged Higgs bosons H^{+}_{1} and H^{-}_{2} from the two weak doublets needed in a minimal supersymmetry theory, have supersymmetric partner $\tilde{W}^{\pm}$, $\tilde{H}^{+}_{1}$ and $\tilde{H}^{-}_{2}$. In the left-right supersymmetric model we have four charginos, $\chi^{\pm}_{j}$ (i=1,..,4), corresponding to the two gauginos $\lambda^{\pm}_{L}$ and $\lambda^{\pm}_{R}$, the superpartners of the gauge bosons $W^{\pm}_{L}$ and $W^{\pm}_{R}$, and the two higgsinos, $\tilde{\phi}^{\pm}_{u,d}$. In section III.2, we discuss the chargino masses and their mixing for the minimal supersymmetric standard model. The L-R supersymmetric model will be discussed in section III.3.

III.2. Chargino Masses for the Minimal Supersymmetric Standard Model (MSSM).

III.2.1. Chargino mixing.

The electroweak gauginos and higgsinos are all spin-1/2 weakly interacting charginc particles and so mix once SU(2) $\times$ U(1) is broken. After mixing, there exist two charginos $\tilde{\chi}^{\pm}_{1}$, $\tilde{\chi}^{\pm}_{2}$. The mixing is in general model-dependent. In the minimal supersymmetric model, the corresponding mixing mass matrices can be parametrised in terms of three unknown parameters. They are choosed to be:[1,2]

1. A supersymmetry higgsino mass parameter μ, which mixes the higgsino fields H_1 and H_2 that give masses to the up and down quarks respectively.
2. The ratio $\tan\beta=\upsilon_2/\upsilon_1$, where υ_1 and υ_2 are the vacuum expectation values of the Higgs fields which couple to d- and u-type quarks respectively.
3. The supersymmetry breaking $SU(2)_L$ gaugino mass parameter M.

In the neutralino sector there is also a fourth parameter, the $U(1)_Y$ gaugino mass parameter M′ which is related to the gaugino mass M when the usual assumption of the grand unified theory (GUT) is applied. Additionally this assumption implies a relation between M and the $SU(3)_C$ gaugino mass parameter which is the gluino mass $\tilde{M}_g$. At the electroweak energy scale one expects,

$$M'= \frac{5}{3}(g'^2/g^2)M \tag{III.1}$$

and

$$M= (g^2/g_s^2)\tilde{M}_g \tag{III.2}$$

where g', g, g_s are the gauge couplings of the $U(1)_Y$, $SU(2)_L$ and $SU(3)_C$ gauge group respectively. From eqs.(III.1) and (III.2) we get $M'\simeq 0.5M$ and $M\simeq 0.3\tilde{M}_g$. There is a slight theoretical prejudice as to the value of $\tan\beta$. This is basically due to the relation:[1]

$$\lambda_t = \frac{gm_t}{\sqrt{2}\, M_W \sin\beta} \qquad \text{(III.3)}$$

where λ_t is the top-quark Yukawa coupling, m_t the top-quark mass, and g is the weak coupling constant. Thus, the large top-quark mass can be explained by the maximum value of $\sin\beta$. The general tendency of the model then is to have values of $\tan\beta>1$.

As mentioned before, the minimal supersymmetric model contains two Higgs doublets. Of the 8 degrees of freedom, 3 are absorbed in giving mass to the gauge bosons $W^{\pm}$ and Z^0. This leaves a total of five physical Higgs bosons:[3,4] two charged Higgs bosons $H^{\pm}$, with mass $M_{H^{\pm}}$; two neutral Higgs scalars h^0(light) and H^0(heavy), with masses M_{h^0}, M_{H^0} and one neutral pseudoscalar Higgs A^0, with mass M_{A^0}. All couplings and masses of the Higgs sector are determined by two parameters; $\tan\beta=\upsilon_2/\upsilon_1$ and $M_{H^{\pm}}$. The pseudoscalar and scalar-Higgs boson masses are given in terms of these two parameters by:[3,4]

$$M^2_{A^0} = M^2_{H^{\pm}} + M^2_W \,, \qquad \text{(III.4)}$$

and

$$M^2_{H^0,h^0} = \frac{1}{2}\left[(M^2_{A^0} + M^2_Z) \pm \sqrt{(M^2_{A^0} + M^2_Z)^2 - 4M^2_Z M^2_{A^0}\cos^2 2\beta}\ \right]. \qquad \text{(III.5)}$$

These relations imply the following constraints:

(i) $M_{H^0} > M_Z$, *(ii)* $M_{h^0} \leq M_{A^0}$,

(iii) $M_{h^0} \le M_Z |\cos(2\beta)| \le M_Z$, and $M_{H^\pm} \ge M_W$.

Furthermore, the phase of the Higgs fields can be chosen as $0 \le \beta \le \pi/2$. Typically, we expect $\tan\beta > 1$; the nearer $\tan\beta$ is to 1 the lighter the h^0.

The charginos can be written as four-component Dirac fermions which arise due to the mixing of winos, $\tilde{W}^-$, $\tilde{W}^+$, and the charged higgsinos, $\tilde{H}_1^-$ and $\tilde{H}_2^+$. (For chargino mixings in two component notation, see Appendix B). Because there are actually two independent mixings, $(\tilde{W}^-, \tilde{H}_1^-)$ and $(\tilde{W}^+, \tilde{H}_2^+)$, one needs two unitary mixing matrices to diagonalize the mass matrix. We define the mass term in the Lagrangian as[6]

$$\mathcal{L}_m = (\psi^-)^T X \psi^+ + \text{H.C.} \tag{III.6}$$

where the Dirac fermions are,

$$\psi_j^- = (\tilde{W}^-, \tilde{H}_1^-), \quad \psi_j^+ = (\tilde{W}^+, \tilde{H}_2^+) \tag{III.7}$$

and

$$X = \begin{pmatrix} M & M_W\sqrt{2}\,\sin\beta \\ M_W\sqrt{2}\,\cos\beta & \mu \end{pmatrix}, \tag{III.8}$$

where M, μ, and $\tan\beta$ are defined previously, and

$$M_W = \frac{1}{2}\, g(v_1^2 + v_2^2)^{1/2}, \tag{III.9}$$

where v_i are the vacuum expectation values defined by[6]

$$\langle H_1 \rangle = \frac{\upsilon}{\sqrt{2}}\cos\beta \equiv \upsilon_1; \text{ and } \langle H_2 \rangle = \frac{\upsilon}{\sqrt{2}}\sin\beta \equiv \upsilon_2 \qquad \text{(III.10)}$$

Let the mass-eigenstates be defined by

$$\chi_i^+ = V_{ij}\psi_j^+ , \qquad \chi_i^- = U_{ij}\psi_j^- , \qquad (i,j=1,2), \qquad \text{(III.11)}$$

where U and V are unitary matrices chosen such that

$$M_D = U^* X\ V^{-1}, \qquad \text{(III.12)}$$

where M_D is the diagonal mass matrix, which is required to contain real and non-negative entries. It is useful to consider the eigenvalues of $X^\dagger X$. The positive square roots of the eigenvalues of $X^\dagger X$ will be the diagonal elements M_D. Thus, eq.(III.12) becomes:

$$M_D^2 = V\ X^\dagger X V^{-1} = U^* X X^\dagger (U^*)^{-1}. \qquad \text{(III.13)}$$

The diagonalizing matrices U^* and V can easily be obtained by computing the eigenvectors corresponding to the eigenvalues of $X^\dagger X$ and $XX^\dagger$ respectively.

III.2.2. Solutions for the chargino mass eigenstates for the MSSM: Asymptotic results.

Before presenting some numerical results from eq.(III.12) [or eq.(III.13)]. We present simple analytical expressions, in the limit of large M and μ. In order to do this, we need to diagonalize the mass matrix M_D, by finding the eigenvectors of the mixing matrices U, V. We first assume that $|M\mu| \gg \frac{1}{2} M_W^2 \sin 2\beta$. Then, we find:

$$U \simeq \begin{pmatrix} & \dfrac{M_W\sqrt{2}(Mc_\beta + \mu s_\beta)}{M^2 - \mu^2} \\ \dfrac{-M_W\sqrt{2}(Mc_\beta + \mu s_\beta)}{M^2 - \mu^2} & 1 \end{pmatrix}, \qquad \text{(III.14)}$$

and

$$V \simeq \begin{pmatrix} 1 & \dfrac{M_W\sqrt{2}(Ms_\beta + \mu c_\beta)}{M^2 - \mu^2} \\ \dfrac{-M_W\sqrt{2}(Ms_\beta + \mu c_\beta)}{M^2 - \mu^2} & 1 \end{pmatrix}. \qquad \text{(III.15)}$$

where $s_\beta = \sin\beta$, and $c_\beta = \cos\beta$. The corresponding chargino masses are [we use eq.(III.12)]:

$$\tilde{M}_{\chi_1^+} \simeq M + M_W^2 \left(\frac{M + \mu \sin 2\beta}{M^2 - \mu^2} \right), \qquad \text{(III.16)}$$

$$\tilde{M}_{\chi_2^+} \simeq |\mu| + M_W^2 \left(\frac{\mu + M \sin\beta}{\mu^2 - M^2} \right). \qquad \text{(III.17)}$$

These masses, eqs.(III.16) and (III.17), are in agreement with Ref. [7].

III.3. Chargino Masses for the Left-Right Supersymmetric Model.

III.3.1 Chargino mixing.

Let us recall eq.(II.54) for the chargino-higgsino mixing Lagrangian, and exclude the mass term, which has been already discussed:

$$\mathcal{L}_{ch} = \Big[i\lambda^-_R g_R \kappa_L \tilde{\phi}^+_d + i\lambda^-_L g_L \kappa_d \tilde{\phi}^+_d + i\lambda^+_R g_R \kappa_u \tilde{\phi}^-_d$$

$$+ i\lambda^+_L g_L \kappa_u \tilde{\phi}^-_u + M_L \lambda^+_L \lambda^-_L + M_R \lambda^+_R \lambda^-_R + \mu \tilde{\phi}^+_u \tilde{\phi}^-_d + \mu \tilde{\phi}^-_u \tilde{\phi}^+_d \Big] + \text{H.C.} \tag{III.18}$$

All the notations are already defined in the previous chaper. Equation (III.16) can be written in two component otation in matrix form as

$$\mathcal{L}_{ch} = -\tfrac{1}{2} (\psi^+ \psi^-) \begin{pmatrix} 0 & M^{CT} \\ M^C & 0 \end{pmatrix} \begin{pmatrix} \psi^+ \\ \psi^- \end{pmatrix} + \text{H.C.} \tag{III.19}$$

where

$$\psi^+ = (-i\lambda^+_L,\ -i\lambda^+_R,\ \tilde{\phi}^+_u,\ \tilde{\phi}^+_d); \tag{III.20}$$

$$\psi^- = (-i\lambda^-_L,\ -i\lambda^-_R,\ \tilde{\phi}^-_u,\ \tilde{\phi}^-_d); \quad \text{and} \tag{III.21}$$

$$M^C = \begin{pmatrix} M_L & 0 & 0 & g_L\kappa_d \\ 0 & M_R & 0 & g_R\kappa_d \\ g_L\kappa_u & g_R\kappa_u & 0 & -\mu \\ 0 & 0 & -\mu & 0 \end{pmatrix}. \tag{I˙I.22}$$

In the L-R supersymmetric model we have four charginos $\tilde{\chi}_i^{\pm}$ (i=1,...,4) instead of two. This is because we have two Higgs doublet mixed with the $W^{\pm}_{L,R}$. Before we present an analytical expression of the chargino masses for some limiting cases, it is more convenient to discuss the chargino matrix (III.22) in terms of the four parameters M_L, M_R, μ, and $\tan\theta_\kappa=\kappa_u/\kappa_d$, where κ_u, κ_d are the vacuum expectation values [eq.(II.31)] of the Higgs fields which couple to the u-quark and d-quark respectively. Substituting the values of κ_u and κ_d in $M_{WL}(\equiv M_W)$ [eq.(II.54)], we get

$$\left.\begin{aligned} g\kappa_u &= \sqrt{2}\, M_W \sin\theta_\kappa \\ g\kappa_d &= \sqrt{2}\, M_W \cos\theta_\kappa \end{aligned}\right\} \qquad \text{(III.23)}$$

Inserting eq.(III.23) into the chargino mixing matrix, eq.(III.22), we get

$$M^C = \begin{pmatrix} M_L & 0 & 0 & \sqrt{2}\, M_W \cos\theta_\kappa \\ 0 & M_R & 0 & \sqrt{2}\, M_W \cos\theta_\kappa \\ \sqrt{2}\, M_W \sin\theta_\kappa & \sqrt{2}\, M_W \sin\theta_\kappa & 0 & -\mu \\ 0 & 0 & -\mu & 0 \end{pmatrix}$$

(III.24)

As in the MSSM, we need two unitary matrices, U and V, to diagonalize the elements of the matrix M_D;

$$M_D = U^* M^C V^{-1}. \qquad \text{(III.25)}$$

The eigenvalues of the 4×4 matrix, M^c, can be either positive or negative whereas we require M_D to contain only non-negative entries. Therefore, it is important to consider the eigenvalues of the matrix $M^{c\dagger}M^c$(or $M^cM^{c\dagger}$). In analogy with the supersymmetric standard model, then we have from eq.(III.22),

$$M_D^2 = V(M^{c\dagger}M^c)V^{-1} = U^*(M^cM^{c\dagger})(U^*)^{-1}. \qquad \text{(III.26)}$$

The eigenvectors of the matrices U^* and V can be found by computing the eigenvalues of $M^{c\dagger}M^c$ and $M^cM^{c\dagger}$ respectively.

III.3.2. Solutions for the chargino mass eigenstates for the L-R supersymmetric Model: Asymptotic results.

The matrix (III.24) gives quartic solution, by solving the *characteristic equation*,[8]

$$[\mathbf{A} - \lambda_i^2 I]\mathbf{x}_i = 0, \qquad \text{(III.27)}$$

where $\mathbf{A}=M^{c\dagger}M^c$, $M^cM^{c\dagger}$ $(M^{c\dagger}M^c \neq M^cM^{c\dagger})$, λ_i^2 and $\mathbf{x}_i$ $(i=1,\ldots,4)$ are set of eigenvalues and eigenvectors of the matrix $\mathbf{A}$ respectively and I is a unitary matrix. We have diagonalized the mass matrix M_D^2 in eq.(III.26) by finding the eigenvalues and the eigenvectors of the *characteristic equation* (III.27).

Using the *Maple*[9] programming language greatly simplifies the calculations. The eigenvalues of the matrices $M^{c\dagger}M^c$

and $M^c M^{c\dagger}$ are rather lengthy and the general forms of U and V are very complicated. Therefore, we shall give only the final steps of the solution. In analogy with the supersymmetric standard model, we assume, in the limit of large M_R, M_L, μ, that

$$|M_R\mu| \gg M_W^2\sin^2\theta_\kappa, \quad |M_L\mu| \gg M_W^2\sin^2\theta_\kappa,$$

and similarly for $\sin^2\theta_\kappa \leftrightarrow \cos^2\theta_\kappa$. We find that the chargino masses are:

$$\tilde{M}_{\chi_1^\pm} \simeq M_L + \frac{D + M_L^2(M_L^2 A + B) - 2M_L^6(\mu^2 + 2M_W^2) - M_L^6 M_R^2}{2M_L(M_R^2 - M_L^2)(M_L^4 - \mu^4)} \tag{III.28}$$

$$\tilde{M}_{\chi_2^\pm} \simeq M_R - \frac{M_R^2 M_L^2 B + M_L^2 D + M_R^4\,[M_L^2 A - M_L^4(\mu^2 + 2M_W^2) - M_L^4 M_R^2]}{2M_L^2 M_R(M_R^2 - M_L^2)(M_L^4 - \mu^4)}$$

$$\frac{- M_L^2 M_R^6(\mu^2 + 2M_W^2)}{2M_L^2 M_R(M_R^2 - M_L^2)(M_L^4 - \mu^4)}, \tag{III.29}$$

$$\tilde{M}_{\chi_3^\pm} \simeq \mu + \frac{M_L^2\mu^6 + \mu^2 M_L^2 M_R^2(\mu^2 + 2M_W^2) - D + 4\mu^4 M_W^2 + \mu^8 - \mu^2 B}{4\mu^3(M_L^2 - \mu^2)(M_R^2 - \mu^2)}$$

$$\frac{- \mu^4 A - 2\mu^4 M_W^2 M_R^2}{4\mu^3(M_L^2 - \mu^2)(M_R^2 - \mu^2)}, \tag{III.30}$$

and

$$\tilde{M}_{\chi_4^{\pm}} \simeq \mu + \frac{M_L^2\mu^6 + \mu^2 M_L^2 M_R^2(\mu^2 + 2M_W^2) + D + 4\mu^4 M_W^2 + 3\mu^8}{4\mu^3(M_L^2 + \mu^2)(M_R^2 + \mu^2)}$$

$$\frac{-\mu^2 B + \mu^4 A + 2\mu^4 M_W^2 M_R^2 + 2M_R^2\mu^6}{4\mu^3(M_L^2 + \mu^2)(M_R^2 + \mu^2)}, \qquad \text{(III.31)}$$

where

$$A \equiv cd - a^2 + f^2 - ab + (e + \mu^2)(c + d) + \mu^2 e,$$

$$B \equiv -(e + \mu^2)(cd + f^2) - e\mu^2(c + d) + a^2(\mu^2 + c)$$

$$+ ab(\mu^2 + d), \text{ and}$$

$$D \equiv \mu^2 e(f^2 + cd) - a\mu^2(ac + bd) \qquad \text{(III.32)}$$

and

$$a = \sqrt{2}\, M_W(M_R\cos\theta_K - \mu\sin\theta_K),\ b = \sqrt{2}\, M_W(M_L\cos\theta_K - \mu\sin\theta_K),$$

$$c = M_R^2 + 2M_W^2\sin^2\theta_K,\ d = M_L^2 + 2M_W^2\sin^2\theta_K,$$

$$e = \mu^2 + 4M_W^2\cos^2\theta_K, \text{ and } f = 2M_W^2\sin^2\theta_K. \qquad \text{(III.33)}$$

By computing the eigenvectors, we find the two mixing matrices V, and U^*.

$$V_{ij} = \begin{pmatrix} 1 & V_{12} & 0 & V_{14} \\ V_{21} & 1 & 0 & V_{24} \\ V_{31} & V_{32} & 1 & 1 \\ V_{41} & V_{42} & 1 & -1 \end{pmatrix} \qquad \text{(III.34)}$$

where the matrix elements V_{ij} are

$$V_{21} = \frac{fa - b(d - M_R^2 - 2M_R\beta)}{fb - a(c - M_R^2 - 2M_R\beta)}, \qquad \text{(III.35)}$$

$$V_{31}= \frac{-a^2+ (e - \mu^2 - 2\mu\gamma)(d - \mu^2- 2\mu\gamma)}{fa - b(d - \mu^2- 2\mu\gamma)}, \tag{III.36}$$

$$V_{41}= \frac{a^2- (e + \mu^2 - 2\mu\gamma)(d + \mu^2- 2\mu\gamma)}{fa - b(d + \mu^2- 2\mu\gamma)}, \tag{III.37}$$

$$V_{12}= \frac{fb - a(c - M_L^2 - 2M_L\alpha)}{fa - b(d - M_L^2 - 2M_L\alpha)}, \tag{III.38}$$

$$V_{32}= \frac{-ab + f(e - \mu^2 - 2\mu\gamma)}{af - b(d - \mu^2- 2\mu\gamma)}, \tag{III.39}$$

$$V_{42}= \frac{-ab + f(e + \mu^2 - 2\mu\sigma)}{af - b(d + \mu^2- 2\mu\sigma)}, \tag{III.40}$$

$$V_{14}= \frac{f^2- (c - M_L^2- 2M_L\alpha)(d - M_L^2 - 2M_L\alpha)}{fa - b(d - M_L^2 - 2M_L\alpha)}, \tag{III.41}$$

and

$$V_{24}= \frac{-f^2+ (c - M_R^2- 2M_R\beta)(a - M_R^2 - 2M_R\beta)}{fb - a(c - M_R^2 - 2M_R\beta)}. \tag{III.42}$$

where the parameters α, β, γ, and σ are given by:

$$\alpha= \frac{D + M_L^2(M_L^2 A + B) - 2M_L^6(\mu^2+ 2M_W^2) - M_L^6 M_R^2}{2M_L(M_R^2 - M_L^2)(M_L^4 - \mu^4)}, \tag{III.43}$$

$$\beta = \frac{M_R^2 M_L^2 B + M_L^2 D + M_R^4 \,[M_L^2 A - M_L^4(\mu^2 + 2M_W^2) - M_L^4 M_R^2]}{2M_L^2 M_R (M_R^2 - M_L^2)(M_L^4 - \mu^4)}$$

$$\frac{-M_L^2 M_R^6(\mu^2 + 2M_W^2)}{2M_L^2 M_R (M_R^2 - M_L^2)('I_L^4 - \mu^4)}, \qquad \text{(III.44)}$$

$$\gamma = \frac{M_L^2\mu^6 + \mu^2 M_L^2 M_R^2(\mu^2 + 2M_W^2) - D + 4\mu^4 M_W^2 + \mu^8 - \mu^2 B}{4\mu^3(M_L^2 - \mu^2)(M_R^2 - \mu^2)}$$

$$\frac{-\mu^4 A - 2\mu^4 M_W^2 M_R^2}{4\mu^3(M_L^2 - \mu^2)(M_R^2 - \mu^2)}, \qquad \text{(III.45)}$$

and

$$\sigma = \frac{M_L^2\mu^6 + \mu^2 M_L^2 M_R^2(\mu^2 + 2M_W^2) + D + 4\mu^4 M_W^2 + 3\mu^8}{4\mu^3(M_L^2 + \mu^2)(M_R^2 + \mu^2)}$$

$$\frac{-\mu^2 B + \mu^4 A + 2\mu^4 M_W^2 M_R^2 + 2M_R^2\mu^6}{4\mu^3(M^2 + \mu^2)(M^2 + \mu^2)}. \qquad \text{(III.46)}$$

The mixing matrix U^* is,

$$U^* = U = \begin{pmatrix} 1 & U_{12} & U_{13} & 0 \\ U_{21} & 1 & U_{23} & 0 \\ U_{31} & U_{32} & -1 & 1 \\ U_{41} & U_{42} & 1 & 1 \end{pmatrix}, \qquad \text{(III.47)}$$

where the matrix element are given by

$$U_{21} = \frac{mh - i(k - M_R^2 - 2M_R\beta)}{mi - h(j - M_R^2 - 2M_R\beta)}, \qquad \text{(III.48)}$$

$$U_{31} = \frac{h^2 - (k - \mu^2 - 2\mu\gamma)(l - \mu^2 - 2\mu\gamma)}{mh - i(k - \mu^2 - 2\mu\gamma)}, \qquad \text{(III.49)}$$

$$U_{41} = \frac{-h^2 - (k + \mu^2 - 2\mu\gamma)(l + \mu^2 - 2\mu\gamma)}{hm - i(k + \mu^2 - 2\mu\gamma)}, \qquad \text{(III.50)}$$

$$U_{12} = \frac{im - h(j - M_L^2 - 2M_L\alpha)}{hm - i(k - M_L^2 - 2M_L\alpha)}, \qquad \text{(III.51)}$$

$$U_{32} = \frac{-hi + m(l - \mu^2 - 2\mu\gamma)}{hm - i(k - \mu^2 - 2\mu\gamma)}, \qquad \text{(III.52)}$$

$$U_{42} = \frac{hi + m(l + \mu^2 - 2\mu\sigma)}{hm - i(k + \mu^2 - 2\mu\sigma)}, \qquad \text{(III.53)}$$

$$U_{13} = \frac{-m^2 + (j - M_L^2 - 2M_L\alpha)(k - M_L^2 - 2M_L\alpha)}{hm - i(k - M_L^2 - 2M_L\alpha)}, \qquad \text{(III.54)}$$

and

$$U_{23} = \frac{-m^2 + (j - M_R^2 - 2M_R\beta)(k - M_R^2 - 2M_R\beta)}{im - h(j - M_R^2 - 2M_R\beta)}. \qquad \text{(III.55)}$$

where

$$h = \sqrt{2}\, M_W(M_R \sin\theta_K - \mu\cos\theta_K), \quad i = \sqrt{2}\, M_W(M_L \sin\theta_K - \mu\cos\theta_K),$$
$$j = M_R^2 + 2M_W^2\cos^2\theta_K, \quad k = M_L^2 + 2M_W^2\cos^2\theta_K,$$
$$l = \mu^2 + 4M_W^2\sin^2\theta_K, \text{ and } m = 2M_W^2\cos^2\theta_K. \qquad \text{(III.56)}$$

III.4. Numerical Results.

We obtained in the previous sections, analytical solutions for chargino masses, eqs.(III.28)-(III.31) based on some approximations. In this section we solve for the masses numerically and exactly without any approximations using eq.(III.26). We have shown the relation between the chargino masses and the higgsino mass parameter, μ, in the range -1000 to 1000 GeV.

Case I: In the MSSM.

For our numerical results we take M=50, 250 GeV, corresponding to gluino masses of roughly 168 and 840 GeV respectively, as obtained from eq.(III.2). We also examine $\tan\beta=1.6$ and 4 (recent searches at LEP imply that $\tan\beta\geq1.6$, see Ref. [10]). Figures III.1*a* and III.1*b* show the chargino masses as a function of μ for M=50 at $\tan\beta=1.6$ and 4. The very small dependence of the chargino masses on $\tan\beta$ is illustrated for M=250 GeV in Figs. III.2*a* and III.2*b*.

Case II: In the L-R supersymmetric model.

We have assumed that the right-handed gaugino mass parameter M_R has two values 300 GeV and 1 TeV. (This is for illustrative purposes as M_R could be much heavier than that).[11] We take the same values of M_L as we used in examining the MSSM (50 and 250 GeV ($M=M_L$)). We also examine two values of $\tan\theta_\kappa=\kappa_u/\kappa_d$, 1.6 and 4. Figures III.3*a* and III.3*b* show the chargino masses versus μ for $M_L=50$ and

250 GeV and two values of M_R=300 GeV and 1 TeV at the fixed value $\tan\theta_K$=1.6. Figures III.4*a* and III.4*b* show similar relations at $\tan\theta_K$=4. For both cases all these figures show some important general features of the mass spectra:

1. In the MSSM for all choices of M there are regions in the vicinity of small μ where $\tilde{\chi}_1^{\pm}$ are very light (as shown in Figs. III.1*a* and III.1*b*). Generally, the $\tilde{\chi}_1^0$ is the LSP, but there is always a region of small positive μ for which $\tilde{\chi}_1^{\pm}$ is the LSP. However, the experiment at LEP tells us that the lightest chargino $\tilde{\chi}_1^{\pm}$ is heavier than about 45 GeV.[12]

2. In the MSSM, when M=250 GeV, we find (Figs. III.2*a* and III.2*b*) that at large $|\mu|$, $\tilde{\chi}_1^{\pm}$ is heavier than the left-handed gauge boson masses.

3. In the L-R supersymmetric model, for instance at a value of M_R=300 GeV, we have used the same values of M_L (=M)=50 GeV and $\tan\theta_K$=1.6 and 4 as in the MSSM. There are very tiny regions in the vicinity of small μ where $\tilde{\chi}_1^{\pm}$ ($\leq$20 GeV) are very light (Fig. III.3*b*). Thus $\tilde{\chi}_1^{\pm}$ could be the LSP.

4. When M_R=300 GeV and M_L=50 GeV, we find that at large values of μ the mass of charginos $\tilde{\chi}_k^{\pm}$ (k=2,3,4), except $\tilde{\chi}_1^{\pm}$, are heavier than the left-handed gauge boson, M_{WL}, M_Z (Figs. III.3*a* and III.3*b*). At M_R=1 TeV and M_L=250 GeV, all the chargino masses are much heavier than M_{WL}, M_Z. Also we find in the L-R supersymmetric model that $\tilde{M}_{\chi_4^{\pm}}$>1 TeV (Figs. III.4*a* and III.4*b*). In comparison; in the MSSM, as we mentioned in chapter I, that none of the supersymmetric particles are expected to be heavier than about 1 TeV.

Therefore, we can conclude from this section that the L-R supersymmetric model shows a very different spectrum of the charginos than found in the MSSM.

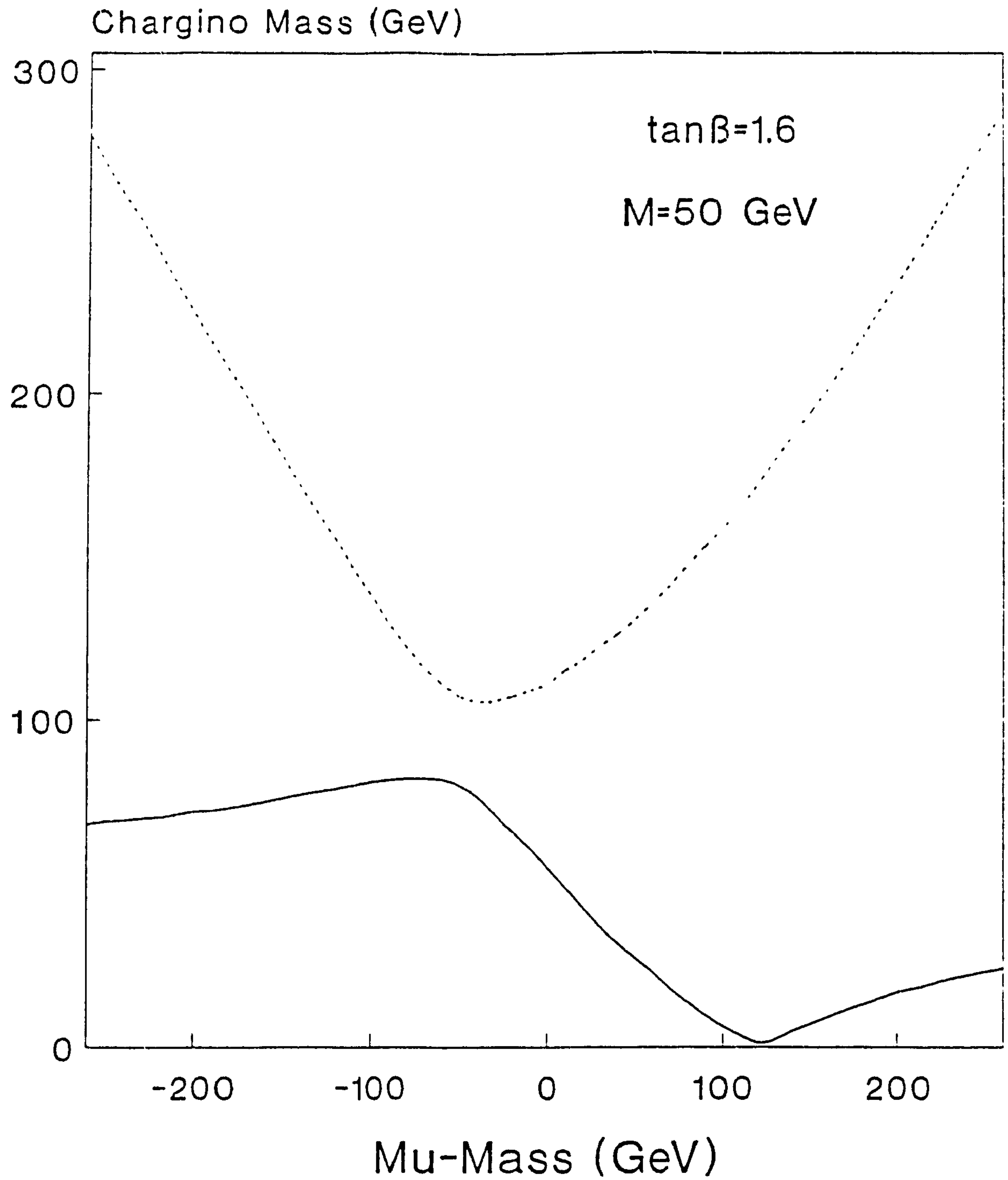

Fig. III.1a: *Masses of charginos, in the MSSM, as a function of μ. We take $\tan\beta=1.6$ and M=50 GeV. The curves are: heavy solid, $\tilde{M}_{\chi_1^\pm}$; and dashed, $\tilde{M}_{\chi_2^\pm}$.*

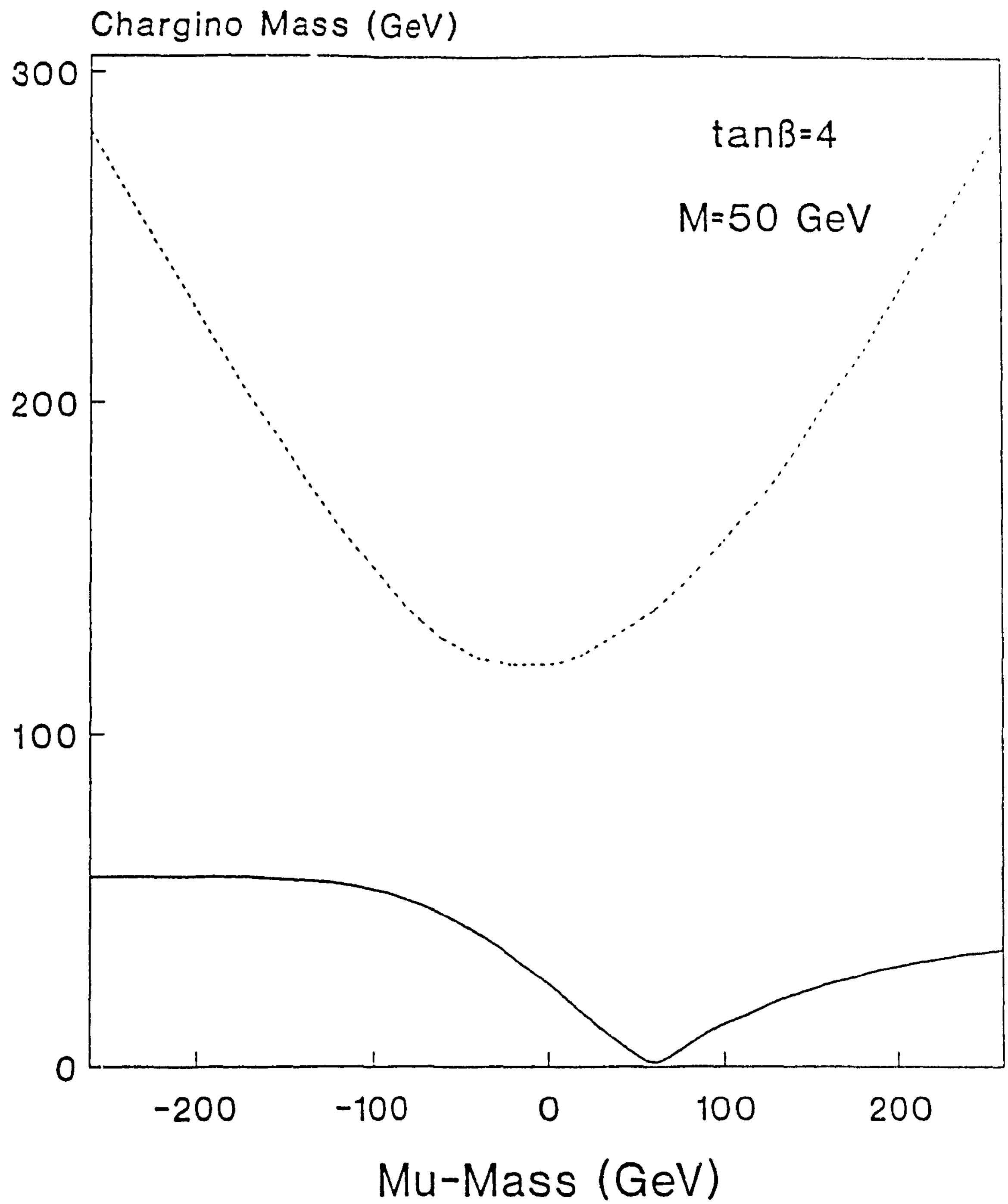

Fig. III.1b: *Masses of charginos, in the MSSM, as a function of* μ. *We take* $\tan\beta=4$ *and* $M=50$ *GeV. The curves are: heavy solid,* $\widetilde{M}_{\chi_1^\pm}$; *and dashed,* $\widetilde{M}_{\chi_2^\pm}$.

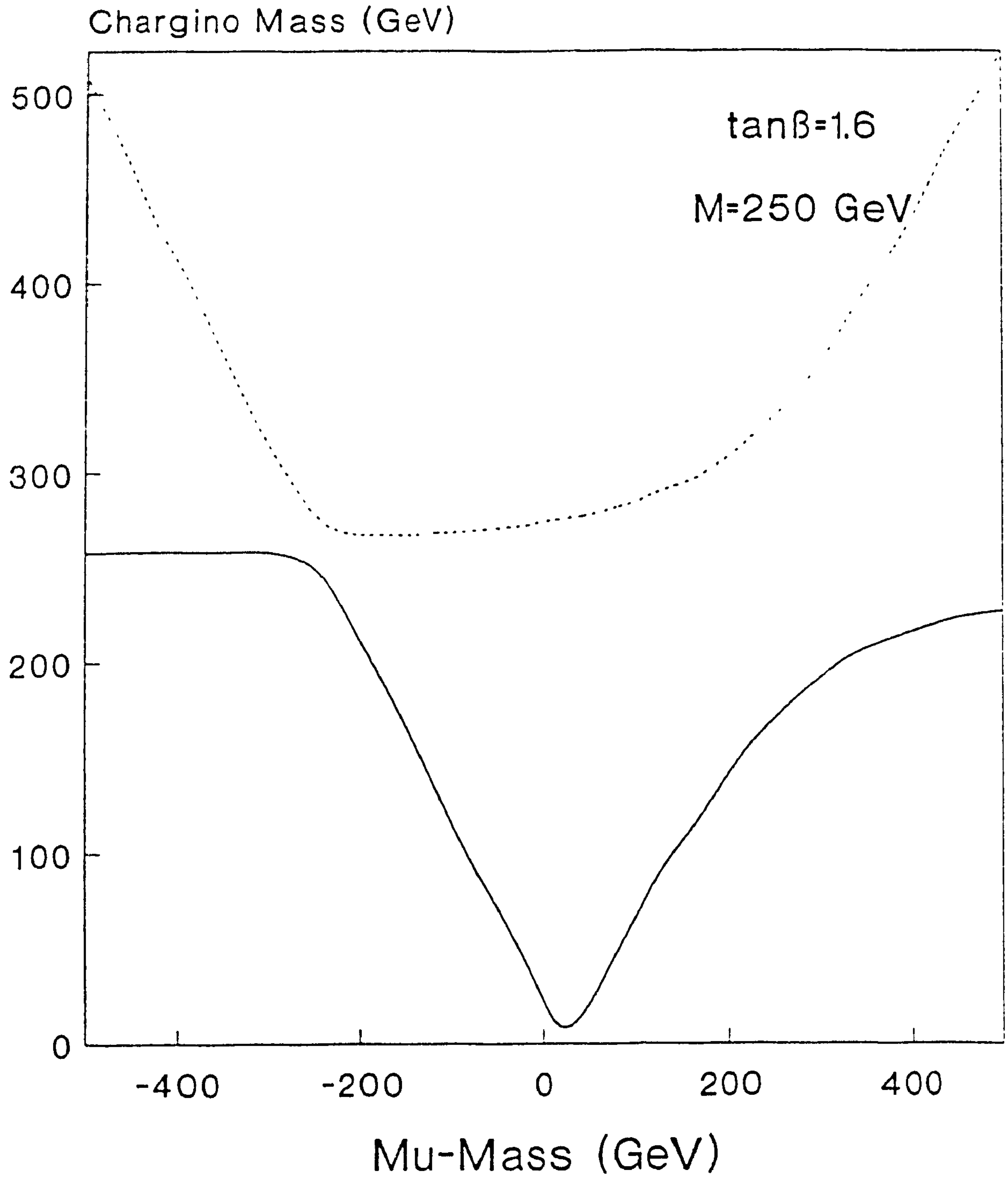

Fig. III.2a: *Masses of charginos, in the MSSM, as a function of* μ. *We take* $\tan\beta=1.6$ *and* $M=250$ *GeV. The curves are: heavy solid,* $\widetilde{M}_{\chi_1^\pm}$; *and dashed,* $\widetilde{M}_{\chi_2^\pm}$.

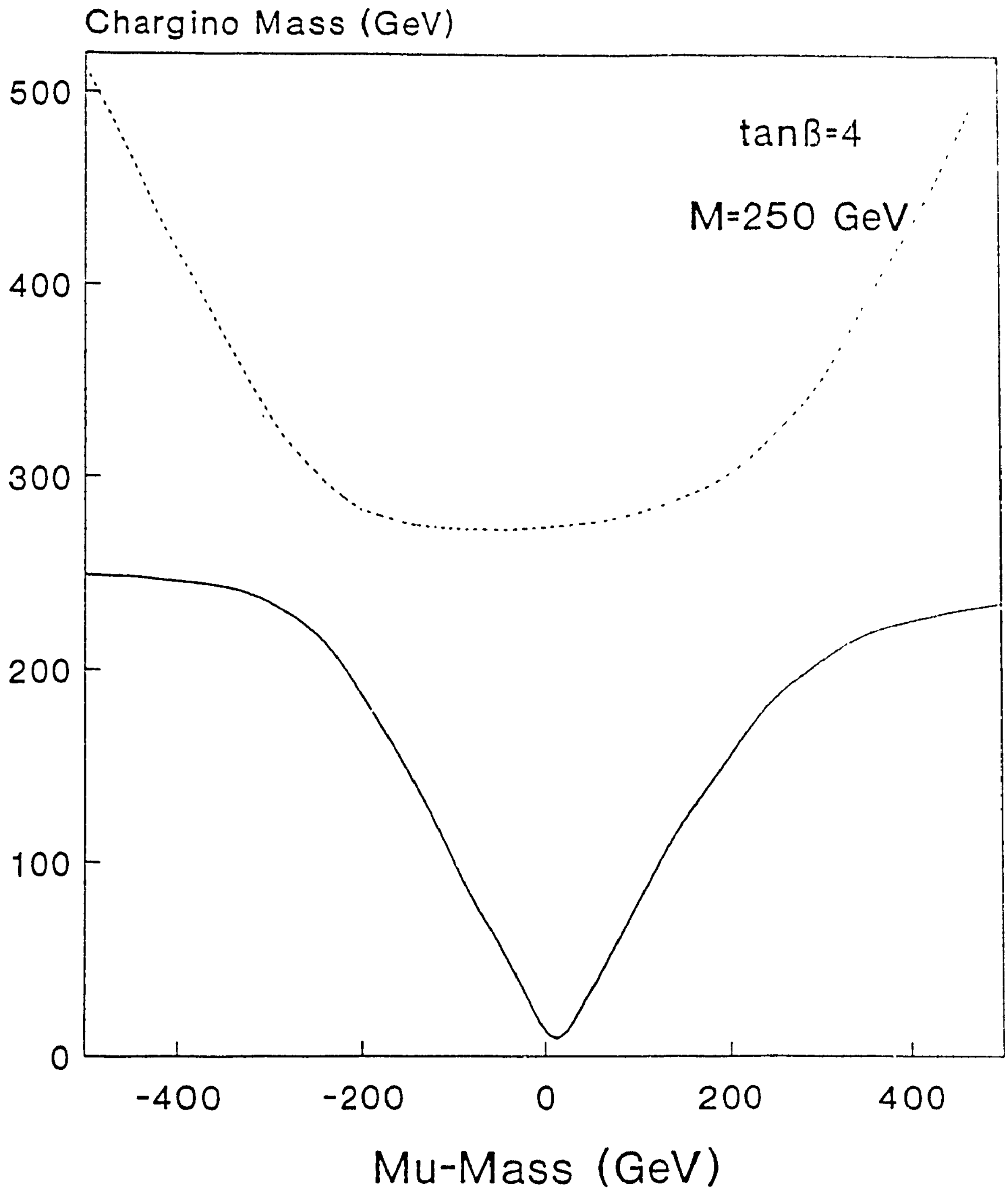

Fig. III.2b: *Masses of charginos, in the MSSM, as a function of μ. We take $\tan\beta=4$ and $M=250$ GeV. The curves are: heavy solid, $\tilde{M}_{\chi_1^\pm}$; and dashed, $\tilde{M}_{\chi_2^\pm}$.*

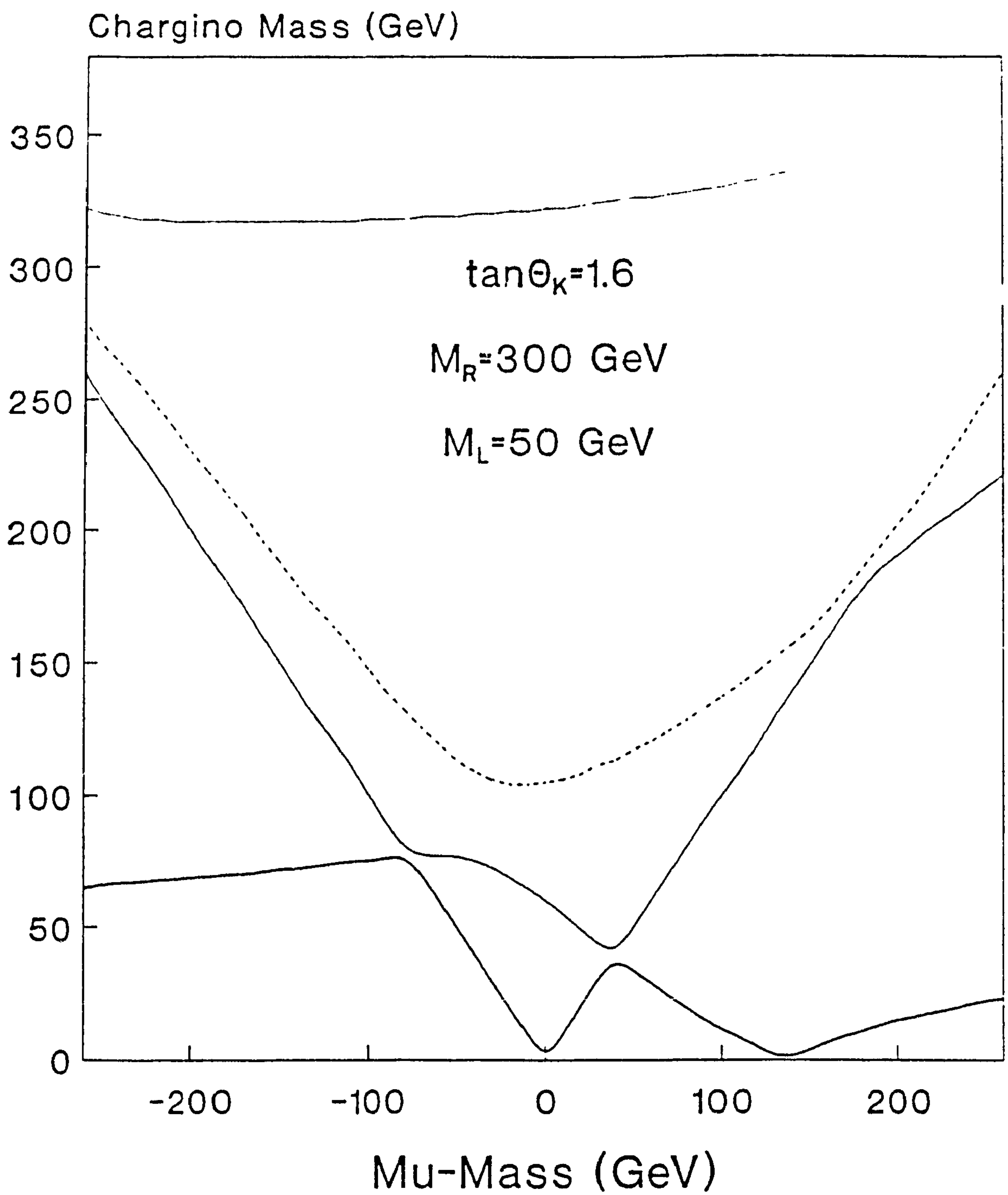

Fig. III.3a: *Masses of charginos, in the L-R supersymmetric model, as a function of μ. We take $\tan\theta_K=1.6$, $M_L=50$ GeV and $M_R=300$ GeV. The curves are: heavy solid, $\tilde{M}_{\chi_1^\pm}$; light solid, $\tilde{M}_{\chi_2^\pm}$; dashed $\tilde{M}_{\chi_3^\pm}$; and dotted, $\tilde{M}_{\chi_4^\pm}$.*

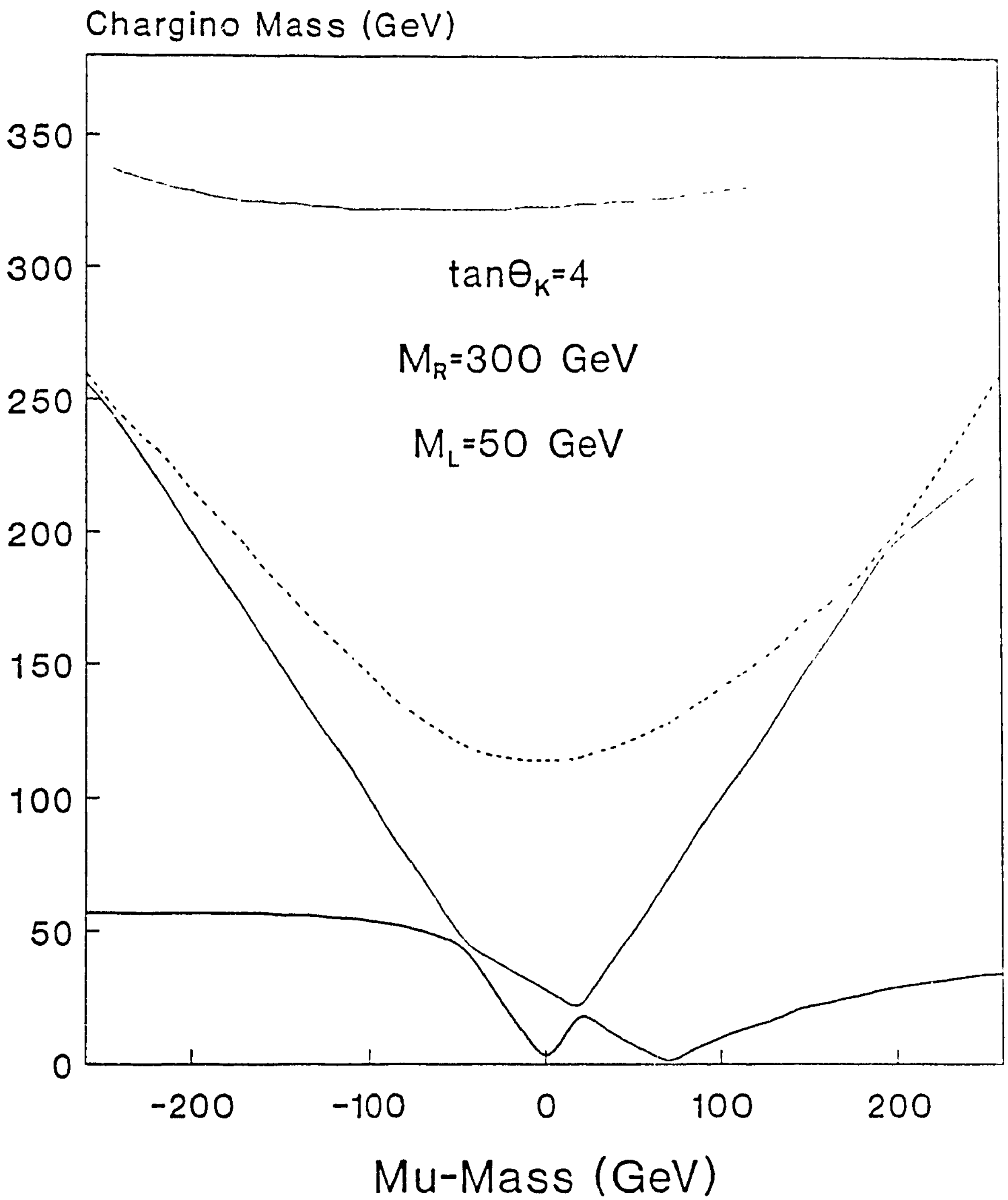

Fig. III.3b: *Masses of charginos, in the L-R supersymmetric model, as a function of μ. We take $\tan\theta_K=4$, $M_L=50$ GeV and $M_R=300$ GeV. The curves are: heavy solid, $\tilde{M}_{\chi_1^\pm}$; light solid, $\tilde{M}_{\chi_2^\pm}$; dashed $\tilde{M}_{\chi_3^\pm}$; and dotted, $\tilde{M}_{\chi_4^\pm}$.*

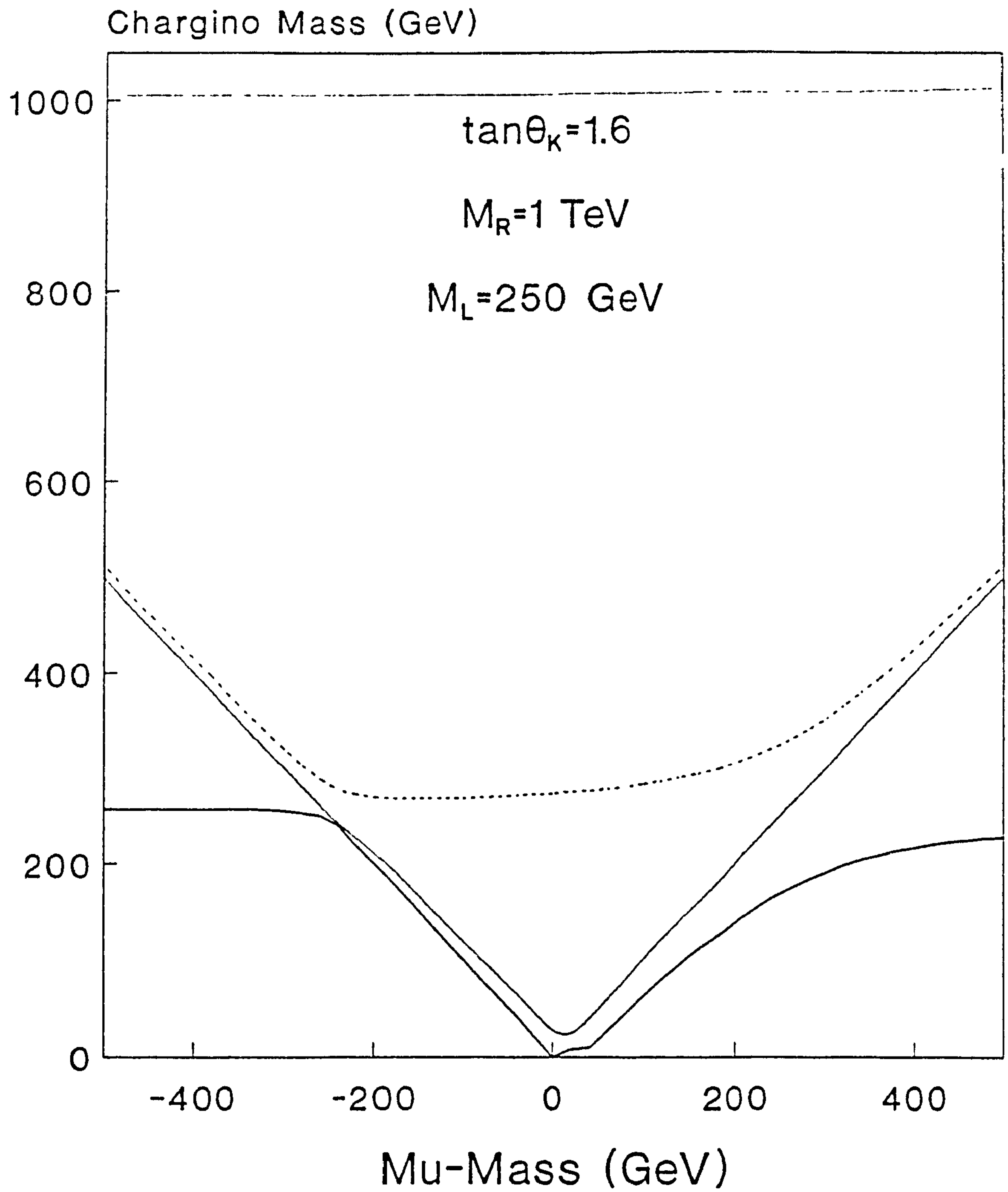

Fig. III.4a: *Masses of charginos, in the L-R supersymmetric model, as a function of μ. We take $\tan\theta_K$=1.6, M_L=250 GeV and M_R=1 TeV. The curves are: heavy solid, $\tilde{M}_{\chi_1^\pm}$; light solid, $\tilde{M}_{\chi_2^\pm}$; dashed $\tilde{M}_{\chi_3^\pm}$; and dotted, $\tilde{M}_{\chi_4^\pm}$.*

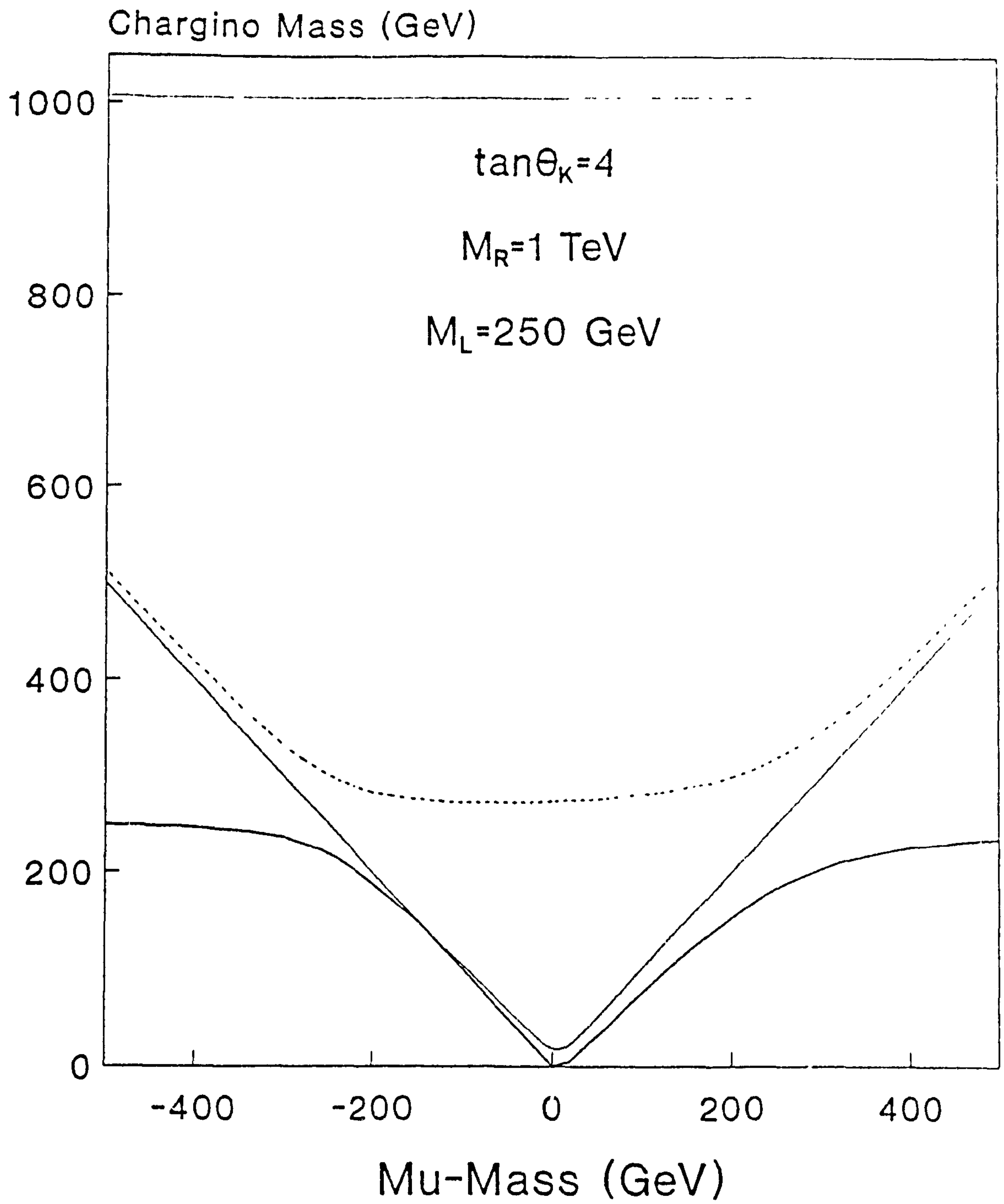

Fig. III.4b: *Masses of charginos, in the L-R supersymmetric model, as a function of* μ. *We take* $\tan\theta_K=4$, $M_L=250$ *GeV and* $M_R=1$ *TeV. The curves are: heavy solid,* $\tilde{M}_{\chi_1^\pm}$; *light solid,* $\tilde{M}_{\chi_2^\pm}$; *dashed* $\tilde{M}_{\chi_3^\pm}$; *and dotted,* $\tilde{M}_{\chi_4^\pm}$.

References

[1] H. Baer *et al.*, in *Research Directions of the Decade*, Proceedings of Workshop, Snowmass, Calorado, 1990; C. Ahn *et al.*, SLAC-Report no.329, UC-34D, 1988.

[2] L. Roszkowski, *Phys. Lett.* B252 (1990) 471.

[3] J. Gunion and H. Haber, *Nucl. Phys.* **B**272 (1986) 1; *Nucl. Phys.* **B**278 (1986) 449.

[4] J.F. Gunion, *et al.*, *Phys. Rev.* **D38** (1988) 3444; J.F. Gunion, G. Gamberini, and S.F. Novaes, *Phys. Rev.* **D38** (1988) 3481.

[5] B. Grinstein, J. Polchinki and M.B. Wise, *Phys. Lett.* **B**130, (1983) 285.

[6] R. Barbieri, *Riv. Nuovo Cim.* **11** (1988) 1; H.E. Haber and G.L. Kane, *Phys. Rep.* **117** (1985) 75.

[7] J.F. Gunion and H.E. Haber, *Phys. Rev.* D37 (1988) 2515.

[8] L.W. Johnson and R.D. Riess, *Introduction to Linear Algebra*, Addison-Wesley Pub., 1981.

[9] Waterloo *Maple* Corp.; see *Maple V Language Reference Manual* (Springer-Verlag, New York, 1991).

[10] Ref. [2] and L. Roszkowski, *Phys. Lett.* **B**262 (1991) 59.

[11] P. Langacker and S. Uma Sanker, *Phys. Rev.* **D40** (1989) 1569; J.L. Rosner and E. Takasugi, *Phys. Rev.* **D42** (1990) 241.

[12] For review, see e.g., G. Altarelli, CERN preprint CERN-TH-5834/90 (Aug. 1990); E. Fernandez, CERN preprint CERN CERN-PPE/90-151 (Oct. 1990); F. Dydak, CERN preprint CERN-PPE/91-14 (Jan. 1991).

Chapter IV

Neutralino Mass Eigenstates

IV.1. Introduction.

The spin-1/2 partners of gauge bosons and Higgs bosons are perhaps the most promising of the supersymmetry partners for detection and study, because they are likely to have the cleanest experimental signatures. The properties of the neutralino particles depend on the mixing of the weak eigenstates with each other to produce the mass eigenstates. The mass eigenstates correspond to the physical particles that could be detected experimentally.

Neutralinos can be produced at any high luminosity collider, e^+e^- or $p\bar{p}$, and indications in most models are that they are light. Thus limitations on the values of their masses is a definitive constraint on the parameters supersymmetry models.

The minimal set of neutralino particles arises as the spin-1/2 supersymmetry partners of W^0, B^0, H_1^0, H_2^0: the neutral wino $\tilde{W}^0$, the bino $\tilde{B}^0$, and the two neutral higgsino $\tilde{H}_1^0$, $\tilde{H}_2^0$ respectively. [($W^\pm$,W^0) is in an SU(2) triplet , B^0 is an SU(2) singlet, and H_1^0 and H_2^0 form an SU(2) doublet. One could equally well consider $\tilde{Z}^0$, $\tilde{\gamma}$, the partners of Z^0, γ, instead of $\tilde{W}^0$, $\tilde{B}^0$].[1-3]

IV.2. Neutralino Masses for the MSSM.

IV.2.1 Neutralino mixing.

In the MSSM, the discussion of the neutralino mass eigenstates is similar to the chargino mass eigenstates, but the main complication here is that there are four neutral states . We define the four-neutralinos in the four-component fermion fields:[1-3]

$$\psi_j^o = (\tilde{W}^0,\ \tilde{B}^0,\ \tilde{H}_1^0,\ \tilde{H}_2^0)\,. \qquad \text{(IV.1)}$$

The neutralino mass matrix in the Lagrangian is given by:

$$\mathcal{L}_m = -\ \frac{1}{2}\ (\psi^0)^T Y \psi^0 + \text{H.C.}, \qquad \text{(IV.2)}$$

where Y is in general a complex symmetric matrix given by†

$$Y = \begin{pmatrix} M' & 0 & -M_Z\sin\theta_W\cos\beta & M_Z\sin\theta_W\sin\beta \\ 0 & M & M_Z\cos\theta_W\cos\beta & -M_Z\cos\theta_W\sin\beta \\ -M_Z\sin\theta_W\cos\beta & M_Z\cos\theta_W\cos\beta & 0 & -\mu \\ M_Z\sin\theta_W\sin\beta & -M_Z\cos\theta_W\sin\beta & -\mu & 0 \end{pmatrix}, \qquad \text{(IV.3)}$$

where $M_Z = \frac{1}{2}\ [(g^2 + g'^2)(\upsilon_1^2 + \upsilon_2^2)]^{1/2}$, and θ_W is the conventional Weinberg angle; all other terms above have been pre-

---

† Y is symmetric because of the Majorana nature of the neutralino particles. As a result, only one diagonalizing matrix N, eq.(IV.5), is required in this case.

viously defined. We define two-component mass eigenstates using[1]

$$\chi_i^0 = N_{ij}\psi_j^0 , \qquad i,j=1,\ldots,4, \tag{IV.4}$$

where N is a unitary matrix satisfying

$$N_D = N^* Y N^{-1}, \tag{IV.5}$$

where N_D is the diagonal neutralino mass matrix. To determine N, it is easiest to square eq.(IV.5) obtaining

$$N_D^2 = N Y^\dagger Y N^{-1}. \tag{IV.6}$$

In analogy with the chargino mass matrix, eq.(III.9), N can be obtained by finding the eigenvectors corresponding to the eigenvalues of $Y^\dagger Y$. One can choose N such that the elements of the diagonal matrix N_D are real and non-negative. The general form which diagonalize the matrix (IV.3) is quite lengthy and the Majorana mass eigenstate fields $\tilde{\chi}_i^0$ are in general complicated combinations of $\tilde{W}^0$, $\tilde{B}^0$, $\tilde{H}_1^0$, $\tilde{H}_2^0$ [1-3]:

$$\tilde{\chi}_i^0 = N_{i1}\tilde{W}^0 + N_{i2}\tilde{B}^0 + N_{i3}\tilde{H}_1^0 + N_{i4}\tilde{H}_2^0, \tag{IV.7}$$

where N_{ij} are the matrix elements given by eq.(IV.4).

IV.2.2 Solutions for the neutralino mass eigenstates for the MSSM: Asymptotic solutions.

We solve eq.(IV.6) analytically by diagonalizing the matrix N_D, in a manner similar to that of the chargino masses, using the *"maple"* programming facilities. In order to simplify the solutions, we first assume, in the limit of large M, M', or large μ, that $|M \pm \mu|$, $|M' \pm \mu| \gg M_Z$. The final results of the neutralino masses are:[4]

$$\tilde{M}_{\chi_1^0} \simeq M' + \frac{M_Z^2 (M' + \mu \sin 2\beta) \sin^2\theta_W}{M^2 - \mu^2}, \tag{IV.17}$$

$$\tilde{M}_{\chi_2^0} \simeq M + \frac{M_Z (M + \mu \sin 2\beta) \cos^2\theta_W}{M^2 - \mu^2}, \tag{IV.18}$$

$$\tilde{M}_{\chi_3^0} \simeq |\mu| - \frac{M_Z^2 (1 - 2\sin\beta)(\mu + M\sin^2\theta_W + M'\cos^2\theta_W)}{2(M' + \mu)(M + \mu)}, \tag{IV.19}$$

$$\tilde{M}_{\chi_4^0} \simeq |\mu| + \frac{M_Z^2 (1 + 2\sin\beta)(\mu - M\sin^2\theta_W - M'\cos^2\theta_W)}{2(M' - \mu)(M - \mu)}. \tag{IV.20}$$

The neutralino mixing matrix is obtained by computing the eigenvectors corresponding to the eigenvalues, eqs.(IV.17)-(IV.20), using the *characteristic equation* of the matrix, which is given in eq.(III.26). We find the mixing matrix, N is

$$N=\begin{pmatrix} 1 & n_{21} & \frac{-M_Z s_W (M' c_\beta + \mu s_\beta)}{M'^2 - \mu^2} & \frac{M_Z s_W (M' s_\beta + \mu c_\beta)}{M'^2 - \mu^2} \\ n_{12} & 1 & \frac{M_Z c_W (M c_\beta + \mu s_\beta)}{M^2 - \mu^2} & \frac{-M_Z c_W (M s_\beta + \mu c_\beta)}{M^2 - \mu^2} \\ \frac{-M_Z s_W (s_\beta - c_\beta)}{\sqrt{2}(\mu + M')} & \frac{M_Z c_W (s_\beta - c_\beta)}{\sqrt{2}(\mu + M)} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{M_Z s_W (s_\beta + c_\beta)}{\sqrt{2}(\mu - M')} & \frac{-M_Z c_W (s_\beta + c_\beta)}{\sqrt{2}(\mu - M)} & \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \end{pmatrix}, \tag{IV.21}$$

where

$$n_{12} \simeq \frac{-M_Z^2 \sin 2\theta_W (M' + \mu \sin 2\beta)}{2(M' - M)(M'^2 - \mu^2)}, \tag{IV.22}$$

and

$$n_{21} \simeq \frac{M_Z^2 \sin 2\theta_W (M + \mu \sin 2\beta)}{2(M' - M)(M^2 - \mu^2)}. \tag{IV.23}$$

The masses and the mixing matrix N [eqs.(IV.17)-(IV.23)] are in agreement with the previous results (see, Ref. [4]).

IV.3. Neutralino Masses for L-R Supersymmetric Model.

IV.3.1 Neutralino Mixing.

As was the case for the charginos in the previous chapter, the heavy particles decouple from the Lagrangian. The neutralino particles are produced in two stages of symmetry breaking. The first stage, involving the VEV υ_R of the neutral Higgs Δ_R is responsible for giving mass to the heavy neutralino. The second stage, involving VEV's, κ_u, κ_d

of the Higgs ϕ_u, ϕ_d, is responsible for giving masses to the light neutralinos. The amount of mixing between the heavy and light neutralino eigenstates is small so that enough to make a reasonable approximation, one can calculate the neutralino mass eigenstates for both stages as independent cases.

I. Neutralino mass eigenstates are generated by the first stage of symmetry breaking:

We recall the Lagrangian, eq.(II.57), for the neutralino masses from chapter three and we write the part which involve the first stage of symmetry breaking. These terms contain υ_R;

$$\mathcal{L}_{HN} = -\frac{i}{\sqrt{2}}\lambda_R^0 g_R \upsilon_R \tilde{\Delta}_R^0 + \frac{2i}{\sqrt{2}}\lambda_V^0 g_V \upsilon_R \tilde{\Delta}_R^0 + \text{H.C.} \qquad \text{(IV.24)}$$

We write the two-component fermion fields:

$$\eta_m^0 = (-i\lambda_R^0, -i\lambda_V^0, \tilde{\Delta}_R^0), \qquad m=1,2,3 \qquad \text{(IV.25)}$$

Thus, eq.(VI.24) takes the form

$$\mathcal{L}_{HN} = -\frac{\upsilon_R}{2\sqrt{2}}(\eta^0)^T Z \eta^0 + \text{H.C.} \qquad \text{(IV.26)}$$

where

$$Z = \begin{pmatrix} 0 & 0 & -g_R \\ 0 & 0 & 2g_V \\ -g_R & 2g_V & 0 \end{pmatrix}. \qquad \text{(IV.27)}$$

We define two-component mass eigenstates using

$$\chi^0_m = A_{mn}\eta_n \,, \qquad m,n = 1,2,3, \qquad \text{(IV.28)}$$

where A_{mn} is a unitary matrix satisfying

$$A_D = A^* Z A^{-1}, \qquad \text{(IV.29)}$$

where A_D is the diagonal neutralino mass matrix. To determine A, it is necessary to square eq.(IV.29) yielding

$$A_D^2 = A Z^\dagger Z A^{-1}. \qquad \text{(IV.30)}$$

Since the matrix Z is symmetric, only one mixing matrix is required to diagonalize A_D. The eigenvalues of the product $Z^\dagger Z$ are,

$$\lambda_1 = (g_R^2 + 4g_V^2),\ \lambda_2 = (g_R^2 + 4g_V^2) \text{ and } \lambda_3 = 0, \qquad \text{(IV.31)}$$

which are used to find the eigenvectors. Normalizing the eigenvectors yields the mixing matrix

$$A = \frac{1}{g_1}\begin{pmatrix} g_R & g_R & 2g_V \\ -2g_V & -2g_V & g_R \\ g_R & g_R & 0 \end{pmatrix}, \qquad \text{(IV.32)}$$

where $g_1 = (g_R^2 + 4g_V^2)^{1/2}$.

The four-component (Majorana fermions) mass eigenstates are the neutralinos which are defined in terms of the two-component χ_m^0 fields by

$$\tilde{\chi}_m^0 = \begin{pmatrix} \chi_m^0 \\ \bar{\chi}_m^0 \end{pmatrix}. \qquad \text{(IV.32)}$$

Using eqs.(IV.28) and (IV.32), we find that the physical eigenstates, $\tilde{\chi}_m^0$ arising from the first stage of the symmetry breaking are;

$$\tilde{\chi}_1^0 = \frac{1}{g_1} \begin{pmatrix} -i(g_R\lambda_R^0 - 2g_V\lambda_V^0) + g_R\tilde{\Delta}_R^0 \\ i(g_R\bar{\lambda}_R^0 - 2g_V\bar{\lambda}_V^0) + g_R\bar{\tilde{\Delta}}_R^0 \end{pmatrix} \qquad \text{(IV.33)}$$

with mass $\frac{1}{2\sqrt{2}}\upsilon_R g_1$

$$\tilde{\chi}_2^0 = \frac{1}{g_1} \begin{pmatrix} -i(g_R\lambda_R^0 - 2g_V\lambda_V^0) + g_R\tilde{\Delta}_R^0 \\ i(g_R\bar{\lambda}_R^0 - 2g_V\bar{\lambda}_V^0) + g_R\bar{\tilde{\Delta}}_R^0 \end{pmatrix} \qquad \text{(IV.34)}$$

with mass $\frac{1}{2\sqrt{2}}\upsilon_R g_1$ and

$$\tilde{\chi}_3^0 = \frac{1}{g_1} \begin{pmatrix} -i(g_R\lambda_V^0 + 2g_V\lambda_R^0) \\ i(g_R\bar{\lambda}_V^0 + 2g_V\bar{\lambda}_R^0) \end{pmatrix}, \qquad \text{(IV.35)}$$

with zero mass.

Thus, the neutralino spectrum at the first stage of symmetry breaking contains: two Majorana fermions, degenerate in mass, and one massless fermion.

II. Neutralino mass eigenstates are generated by the second stage of symmetry breaking:

At the low energy scale, the fields, λ_R^0 and λ_V^0, found in eq.(II.57) have to be rewritten in terms of λ_{ZR}^0 (the superpartner of $Z_{\mu R}$ given in eq.(II.61)) and λ_B^0 defined as:

$$\lambda_{ZR}^0 \equiv (g_R\lambda_R^0 - 2g_V\lambda_V^0)/g_1 \tag{IV.36}$$

$$\lambda_B^0 \equiv (g_R\lambda_V^0 + 2g_V\lambda_R^0)/g_1 . \tag{IV.37}$$

These give

$$\lambda_R^0 \equiv (g_R\lambda_{ZR}^0 + 2g_V\lambda_B^0)/g_1 \tag{VI.38}$$

and

$$\lambda_V^0 \equiv (g_R\lambda_B^0 - 2g_V\lambda_{ZR}^0)/g_1 . \tag{IV.39}$$

Substituting eqs.(IV.38) and (IV.39) in eq.(II.57), and removing the contributions of the fields which have decoupled (such as, $\tilde{\Delta}_R^0$ and λ_{ZR}^0), then the Lagrangian for light neutralinos becomes

$$\begin{aligned}\mathcal{L}_{LN} = &- \frac{i}{\sqrt{2}} g_L \kappa_u \tilde{\phi}_u^0 \lambda_L^0 + i\sqrt{2}\,\frac{g_R g_V}{g_1} \kappa_u \tilde{\phi}_u^0 \lambda_B^0 \\ &- i\sqrt{2}\,\frac{g_R g_V}{g_1} \kappa_d \tilde{\phi}_d^0 \lambda_B^0 + \frac{i}{\sqrt{2}} g_L \kappa_d \tilde{\phi}_d^0 + M_L \lambda_L^0 \lambda_L^0 \\ &+ [(4M_R g_V^2 + M_V g_R^2)/g_1^2]\lambda_B^0\lambda_B^0 + 2\mu\tilde{\phi}_u^0\tilde{\phi}_d^0 + \text{H.C.}\end{aligned} \tag{IV.40}$$

In the case with four light-neutralinos, we define the two component fermion fields

$$\xi^0_l = (-i\lambda^0_L,\ -i\lambda^0_B,\ \tilde{\phi}^0_u,\ \tilde{\phi}^0_d). \tag{IV.41}$$

Then the mass term in the Lagrangian, eq.(IV.39), can be written as

$$\mathcal{L}_{LN} = -\ \tfrac{1}{2}(\xi^0)^T M^n \xi^0 + \text{H.C.} \tag{VI.42}$$

where M^n is in general a complex *symmetric* matrix given by

$$M^n = \begin{pmatrix} M_L & 0 & -\frac{1}{\sqrt{2}}\, g_L\kappa_u & \frac{1}{\sqrt{2}}\, g_L\kappa_d \\ 0 & \dfrac{4M_R g_V^2 + M_V g_R^2}{g_1^2} & \dfrac{\sqrt{2}\, g_V g_R \kappa_u}{g_1} & \dfrac{-\sqrt{2}\, g_V g_R \kappa_d}{g_1} \\ -\frac{1}{\sqrt{2}}\, g_L\kappa_u & \dfrac{\sqrt{2}\, g_V g_R \kappa_u}{g_1} & 0 & -2\mu \\ \frac{1}{\sqrt{2}}\, g_L\kappa_d & \dfrac{-\sqrt{2}\, g_V g_R \kappa_d}{g_1} & -2\mu & 0 \end{pmatrix}. \tag{IV.43}$$

As usual, we define two-component mass eigenstates using

$$\chi^0_k = B_{kl}\xi^0_l\ , \qquad k,l=1,\ldots,4 \tag{IV.44}$$

where B_{kl} is a unitary matrices satisfying:

$$M_D = B^* M^n B^{-1}, \tag{IV.45}$$

where M_D is the diagonal neutralino mass matrix, and

$$M_D^2 = B M^{n\dagger} M^n B^{-1}. \tag{IV.46}$$

Similarly B can be obtained by finding the eigenvectors corresponding to the eigenvalues of $M^{n\dagger}M^n$. The four-component mass-eigenstates are the Majorana neutralinos which are defined in terms of the two-component $\tilde{\chi}_k^0$ fields. From eqs.(VI.41) and (VI.44) we find:

$$\tilde{\chi}_k^0 = \begin{pmatrix} \chi_k^0 \\ \bar{\chi}_k^0 \end{pmatrix} \qquad (k=1,\ldots,4). \tag{IV.47}$$

We diagonalize the mass matrix M_D in the limit of large $M_{L,R}$, or large $|\mu|$. We assume that,

$$\left.\begin{array}{l} (i)\ |M_{L,R} \pm \mu| \gg M_Z, \\ (ii)\ M_R > M_V,\ [4M_R g_V^2 + M_V g_R^2]/g_1^2 \simeq 4M_R g_V^2/g_1^2. \end{array}\right\} \tag{IV.48}$$

Before continuing, note that:

- In order to compare with the MSSM, it is more convenient to write the masses in terms of the parameters M_R, M_L, μ, and $\tan\theta_\kappa$ rather than the parameters $g \simeq g_L \simeq g_R$, g_V, g_1, κ_u and κ_d. From eq.(II.69) the mass of the left-handed Z^0-boson is

$$M_Z = \frac{1}{\sqrt{2}}[(g^2 + 4g'^2)(\kappa_u^2 + \kappa_d^2)]^{1/2}. \qquad \text{(IV.49)}$$

■ From eq.(II.66), the gauge coupling of the subgroup $U(1)_Y$ is

$$g' \simeq \frac{gg_V}{(g^2 + 4g_V^2)^{1/2}} \simeq \frac{gg_V}{g_1}; \quad \text{Thus}$$

$$\tan\theta_W \simeq \frac{g'}{g} = \frac{g_V}{g_1}, \qquad \text{(IV.50)}$$

where θ_W is the Weinberg angle.

■ In the L-R supersymmetric model, we define the ratio of the vacuum expectation values as

$$\tan\theta_K = \kappa_u/\kappa_d. \qquad \text{(IV.51)}$$

Then we write the following relations (substituting eqs.(IV.50) and (IV.51) in the Z_L^0-boson mass, eq.(II.70)):

$$g\kappa_u \simeq \frac{\sqrt{2}\, M_Z \sin\theta_K}{(1 + 4\tan^2\theta_W)^{1/2}}, \qquad \text{(IV.52}a\text{)}$$

$$g\kappa_d \simeq \frac{\sqrt{2}\, M_Z \cos\theta_K}{(1 + 4\tan^2\theta_W)^{1/2}}. \qquad \text{(IV.52}b\text{)}$$

Now, substituting eq.(IV.52*a*,*b*) in eq. (IV.43), we write the final results of the neutralino masses:

$$\tilde{M}_{\chi_1^o} \simeq M_L + \frac{M_Z^2[M_L + 2\mu\sin 2\theta_K]}{(M_L^2 - 4\mu^2)(1 + 4\tan^2\theta_K)}, \qquad \text{(IV.53)}$$

$$\tilde{M}_{\chi_2^o} \simeq 4M_R\tan^2\theta_W + \frac{2M_Z^2\tan^2\theta_W[2M_R\tan^2\theta_W + \mu\sin 2\theta_K]}{(4M_R^2\tan^4\theta_W - \mu^2)(1 + 4\tan^2\theta_W)}, \qquad \text{(VI.54)}$$

$$\tilde{M}_{\chi_3^o} \simeq |2\mu| - \frac{M_Z^2(1 - \sin 2\theta_K)[2\tan^2\theta_W(2\mu + M_L + M_R) + \mu]}{2(M_L + 2\mu)(2M_R\tan^2\theta_W + \mu)(1 + 4\tan^2\theta_W)}, \qquad \text{(VI.55)}$$

$$\tilde{M}_{\chi_4^o} \simeq |2\mu| + \frac{M_Z^2(1 + \sin 2\theta_K)[2\tan^2\theta_W(2\mu - M_L - M_R) + \mu]}{2(M_L - 2\mu)(2M_R\tan^2\theta_W - \mu)(1 + 4\tan^2\theta_W)}. \qquad \text{(IV.56)}$$

In order to find the general form of the neutralino mixing matrix, B_{kl}, we have to find the corresponding eigenvectors of the masses given in eqs.(IV.53)-(IV.56). From eq.(III.26), we find the mixing matrix

$$
B_{kl}=\begin{pmatrix}
1 & b_{12} & \dfrac{-M_Z(M_L c_K+2\mu s_K)}{(M_L^2-4\mu^2)t} & \dfrac{M_Z(M_L s_K+2\mu c_K)}{(M_L^2-4\mu^2)t} \\[2ex]
b_{21} & 1 & \dfrac{M_Z^2 t_W(2t_W^2 s_K+\mu c_K)}{(4M_R^2 t_W^4-\mu^2)t} & \dfrac{-M_Z^2 t_W(2t_W^2 c_K+\mu s_K)}{(4M_R^2 t_W^4-\mu^2)t} \\[2ex]
\dfrac{M_Z(s_K-c_K)}{\sqrt{2}(2\mu+M_L)t} & \dfrac{-M_Z t_W(s_K-c_K)}{\sqrt{2}(\mu+2M_R t_W^2)t} & \dfrac{1}{\sqrt{2}} & \dfrac{1}{\sqrt{2}} \\[2ex]
\dfrac{-M_Z(s_K+c_K)}{\sqrt{2}(2\mu-M_L)t} & \dfrac{M_Z t_W(s_K+c_K)}{\sqrt{2}(\mu-2M_R t_W^2)t} & \dfrac{1}{\sqrt{2}} & \dfrac{-1}{\sqrt{2}}
\end{pmatrix}
\tag{IV.57}
$$

where we use the shorthand notations $s_K=\sin\theta_K$, $c_K=\cos\theta_K$, $t_W=\tan\theta_W$, $t=(1+4\tan^2\theta_W)^{1/2}$, and

$$
b_{12}\simeq\frac{2M_Z^2 t_W(M_L+2\mu\sin 2\theta_K)}{(4M_R t_W^2-M_L)(M_L^2-4\mu^2)t^2}, \tag{IV.58}
$$

$$
b_{21}\simeq\frac{M_Z^2 t_W(2t_W M_R+\mu\sin 2\theta_K)}{(M_L-4M_R t_W^2)(4M_R^2 t_W^4-\mu^2)t^2}. \tag{IV.59}
$$

We use the mixing matrix found in eq.(IV.57) to find the mass eigenstates for the light-neutralinos resulting from the second stage of symmetry breaking

$$\tilde{\chi}_1^0 = \begin{pmatrix} -i(\lambda_L^0 + b_{21}\lambda_B^0) + (b_{31}\tilde{\phi}_u^0 + b_{41}\tilde{\phi}_d^0) \\ i(b_{12}\bar{\lambda}_L^0 + \bar{\lambda}_B^0) + (b_{32}\bar{\tilde{\phi}}_u^0 + b_{42}\bar{\tilde{\phi}}_d^0) \end{pmatrix}, \tag{IV.60}$$

$$\tilde{\chi}_2^0 = \begin{pmatrix} -i(b_{12}\lambda_L^0 + \lambda_B^0) + (b_{32}\tilde{\phi}_u^0 + b_{42}\tilde{\phi}_d^0) \\ i(b_{21}\bar{\lambda}_L^0 + \bar{\lambda}_B^0) + (b_{23}\bar{\tilde{\phi}}_u^0 + b_{24}\bar{\tilde{\phi}}_d^0) \end{pmatrix}, \tag{IV.61}$$

$$\tilde{\chi}_3^0 = \begin{pmatrix} -i(b_{13}\lambda_L^0 + b_{23}\lambda_B^0) + (\tilde{\phi}_u^0 + \tilde{\phi}_d^0)/\sqrt{2} \\ i(b_{31}\bar{\lambda}_L^0 + b_{32}\lambda_B^0) + (\bar{\tilde{\phi}}_u^0 + \bar{\tilde{\phi}}_d)/\sqrt{2} \end{pmatrix}, \tag{IV.62}$$

$$\tilde{\chi}_4^0 = \begin{pmatrix} -i(b_{14}\lambda_L^0 + b_{24}\lambda_B^0) + (\tilde{\phi}_u^0 - \tilde{\phi}_d^0)/\sqrt{2} \\ i(b_{41}\bar{\lambda}_L^0 + b_{42}\lambda_B^0) + (\bar{\tilde{\phi}}_u^0 - \bar{\tilde{\phi}}_d)/\sqrt{2} \end{pmatrix}. \tag{IV.63}$$

where b_{kl} are the elements of the mixing matrix B_{kl} given by (IV.57). Thus the neutralino spectrum at this mass scale contains four Majorana fermions, $\tilde{\chi}_k^0$, given by eqs.(IV.60)-(IV.63).

IV.3.2. Special solutions of the neutralino mass eigenstates: A consistency check.

In the previous sections, we have solved the neutralino mass-eigenstates using the diagonalization methods. We have seen that, there are four light-neutralino masses in the neutralino spectrum. In this special method, we are verifying our results, which are given in eqs.(IV.53)-(IV.56). We have used the first order perturbation theory in our solutions, neglecting the high orders. We assume that this can be written as

$$\left.\begin{aligned} \tilde{M}_{\chi_1^o} &= \lambda_1 = \rho/g_1^2 + \alpha, & \tilde{M}_{\chi_2^o} &= \lambda_2 = M_L + \beta, \\ \tilde{M}_{\chi_3^o} &= \lambda_3 = |2\mu| + \gamma, & \tilde{M}_{\chi_4^o} &= \lambda_4 = |2\mu| + \sigma. \end{aligned}\right\} \qquad \text{(IV.64)}$$

where $\rho = (4M_R g_V^2 + M_V g_R^2)$, and $\tilde{M}_{\chi_k^o}$ or λ_k $(k=1,\ldots,4)$ are the diagonal elements of M_D. The neutralino masses are chosen, eq.(IV.64), in such a way that the parameters α, β, γ and σ are small enough in order to satisfy the following conditions:

(i) The parameters α, β, γ, σ are first order perturbating terms $O[M_Z^2/(M \pm \mu)^2]_{L,R}$, $O[M_Z^2/(M_L \pm \mu)(M_R \pm \mu)]$ and neglecting the high orders, like $O[M_Z^2/(M_{L,R} \pm \mu)^2]^2$,...etc.

(ii) Using the matrix M^n, eq.(IV.43), and the eigenvalues λ_k to solve the determinate of the eq.(III.27). We find

$$\begin{aligned} &\lambda^4 - \lambda^3(M_L + \rho/g_1^2) + \lambda^2\Big[M_L\rho/g_1^2 - 4\mu^2 \\ &- (g^2/2g_1^2)(4g_V^2 + g_1^2)(\kappa_u^2 + \kappa_d^2)\Big] + \lambda\Big[4\mu^2(M_L + \rho/g_1^2) \\ &+ (g^2/2g_1^2)(4M_L g_V^2 + \rho)(\kappa_u^2 + \kappa_d^2) - 2\mu(g^2/g_1^2)(4g_V^2 + g_1^2)\kappa_u\kappa_d\Big] \\ &+ \Big[-4M_L\mu^2\rho/g_1^2 + 2\mu(g^2/g_1^2)(4M_L g_V^2 + \rho)\kappa_u\kappa_d\Big] = 0 \qquad \text{(IV.65)} \end{aligned}$$

(iii) From eq.(VI.64), we find

$$(\lambda - M_L - \alpha)(\lambda - \rho/g_1^2 - \beta)(\lambda - 2\mu - \gamma)(\lambda + 2\mu - \sigma) = 0. \tag{IV.66}$$

This becomes.

$$\lambda^4 - \lambda^3(M_L + \rho/g_1^2 + \alpha + \beta + \gamma + \sigma) + \lambda^2\Big[(\rho/g_1^2 + M_L)$$

$$(\gamma + \sigma) + \rho/g_1^2(\alpha + M_L) + \beta M_L - 2\mu(\gamma - \sigma) - 4\mu^2\Big]$$

$$+ \lambda\Big[2\mu(\rho/g_1^2 + M_L)(\gamma - \sigma) + 4\mu^2(\alpha + \beta) + 4\mu^2(\rho/g_1^2 + M_L)$$

$$- M_L(\rho/g_1^2)(\gamma + \sigma)\Big] + \Big[-2M_L\mu(\rho/g_1^2)(\gamma - \sigma)$$

$$- 4\mu^2(M_L\rho/g_1^2 + \alpha\,\rho/g_1^2 + \beta\mu^2)\Big] = 0. \tag{IV.67}$$

Comparing eq.(IV.65) with eq.(IV.67), we get four equations

$$\alpha + \beta + \gamma + \sigma = 0. \tag{IV.68a}$$

$$- M_L\alpha - 4(\rho/g_1^2)\beta - 2\mu(\gamma - \sigma) = - (g^2/2g_1^2)(4g_V^2 + g_1^2)$$

$$(\kappa_u^2 + \kappa_d^2). \tag{IV.68b}$$

$$2\mu(\rho/g_1^2 + M_L)(\gamma - \sigma) - (4\mu^2 + M_L\rho/g_1^2)(\gamma + \sigma) =$$

$$- 2\mu(g^2/g_1^2)(4g_V^2 + g_1^2)\kappa_u\kappa_d + (g^2/2g_1^2)(4M_L g_V^2 + \rho)$$

$$(\kappa_u^2 + \kappa_d^2). \tag{IV.68c}$$

$$4\mu\,[(\rho/g_1^2)\alpha + M_L\beta] + 2M_L(\rho/g_1^2)(\gamma - \sigma) =$$

$$-(2g^2/g_1^2)(4M_L g_v^2 + \rho)\kappa_u\kappa_d \qquad \text{(IV.68d)}$$

We have solved eqs.(IV.68a-d), and we find the parameters α, β, γ, and σ are:

$$\alpha = \frac{(g^2/2)[M_L(\kappa_u^2 + \kappa_d^2) + 4\mu\kappa_u\kappa_d]}{[M_L^2 - 4\mu^2]}, \qquad \text{(IV.69)}$$

$$\beta = \frac{2g_v^2(g^2/g_1^2)[(\rho/g_1^2)(\kappa_u^2 + \kappa_d^2) + \mu\kappa_u\kappa_d]}{[(\rho/g_1)^2 - 4\mu^2]}, \qquad \text{(IV.70)}$$

$$\gamma = \frac{-g^2[4(g_v^2/g_1^2)(2\mu + M_L) + (2\mu + \rho/g_1^2)](\kappa_u - \kappa_d)^2}{4(M_L + 2\mu)(\rho/g_1^2 + 2\mu)}, \qquad \text{(IV.71)}$$

$$\sigma = \frac{g^2[4(g_v^2/g_1^2)(2\mu - M_L) + (2\mu - \rho/g_1^2)](\kappa_u + \kappa_d)^2}{4(M_L - 2\mu)(\rho/g_1^2 - 2\mu)}. \qquad \text{(IV.72)}$$

Substituting eqs.(IV.69)-(IV.72) in eq.(IV.64) and using $\rho/g_1^2 \simeq 4M_R g_v^2/g_1^2$, eq.(IV.48), and the relations given in eq.(IV.52), we get:

$$\tilde{M}_{\chi_1^0} \simeq M_L + \frac{M_Z^2[M_L + 2\mu\sin 2\theta_\kappa]}{(M_L^2 - 4\mu^2)(1 + 4\tan^2\theta_W)}, \qquad \text{(IV.73)}$$

$$\tilde{M}_{\chi_2^0} \simeq 4M_R\tan^2\theta_W + \frac{2M_Z^2\tan^2\theta_W[2M_R\tan^2\theta_W + \mu\sin 2\theta_K]}{(4M_R^2\tan^4\theta_W - \mu^2)(1 + 4\tan^2\theta_W)}, \tag{IV.74}$$

$$\tilde{M}_{\chi_3^0} \simeq |2\mu| - \frac{M_Z^2(1 - \sin 2\theta_K)[2\tan^2\theta_W(2\mu + M_L + M_R) + \mu]}{2(M_L + 2\mu)(2M_R\tan^2\theta_W + \mu)(1 + 4\tan^2\theta_W)}, \tag{IV.75}$$

$$\tilde{M}_{\chi_4^0} \simeq |2\mu| + \frac{M_Z^2(1 + \sin 2\theta_K)[2\tan^2\theta_W(2\mu - M_L - M_R) + \mu]}{2(M_L - 2\mu)(2M_R\tan^2\theta_W - \mu)(1 + 4\tan^2\theta_W)}, \tag{IV.76}$$

Thus, eqs.(IV.73)-(IV.76) are similar to eqs.(IV.53)-(IV.56). Repeating the same calculations used in the minimal supersymmetric model, we also get similar equations to eqs.(IV.17)-(IV.20).

IV.4. Numerical Results.

In this section we present some numerical results similar to use found for the chargino masses in the previous chapter. We assume that $M_R > M_V$. Thus the approximation that the parameter $\rho \simeq 4M_R g_V^2/g_1^2$ is still valid in this analysis. We use M_Z=91.15 GeV, the recent measurement made at CERN.[14] From eqs. (IV.6) and (IV.46), we find the eigenvalues of both matrices Y and M^n. The positive square roots of the eigenvalues of the matrices $Y^\dagger Y$ and $M^{n\dagger}M^n$ will be the diagonal entries of Y and M^n respectively. One can also use eqs.(IV.5) and (IV.45), without taking the square roots of the eigenvalues. As in the case of the charginos, we have shown the relation between the neutralino masses and the μ parameter in the range between -1000 to 1000 GeV.

case (I): In the MSSM.

We take M=50 and 250 GeV, which based on eq.(III.2) corresponds to gluino masses of roughly 168 and 840 GeV respectively. We also consider $\tan\beta$=1.6 and 4. Figures IV.1*a* and IV.1*b*, show the neutralino masses as a function of μ for M=50 at $\tan\beta$=1.6 and 4. The very small dependence of the $\tilde{\chi}_k^0$ masses on $\tan\beta$ is illustrated for M=250 GeV in Figs. IV.2*a* and IV.2*b*.

Case II: In the L-R supersymmetric model.

Figures VI.3*a*,*b* and VI.4*a*,*b* show similar relations for neutralino masses versus μ. As in the MSSM, we take similar values of M_L(=M): 50 and 250 GeV; and M_R=300 GeV and 1 TeV. We also examine two values of $\tan\theta_K=\kappa_u/\kappa_d$; 1.6 and 4. The important general features of the mass spectra are listed below:

1. In the MSSM, for all choices of M there are regions in the vicinity of small μ where the $\tilde{\chi}_1^0$ is very light. For instance, we see a very tiny region for $\tilde{\chi}_1^0$ (≈13 GeV), for M=250 GeV at $\tan\beta$=1.6 as shown in Fig. IV.2*a*. Generally, in the MSSM, $\tilde{\chi}_1^0$ is the LSP. Now, the lightest neutralino has been only mildly constrained to be heavier than about 13 GeV[6] by combining the results of the ALEPH direct search for neutralinos with the UA2 lower bound on the gluino mass.

(This limit is improved to about 20 GeV by the new, preliminary data from CDF).

In the previous chapter, in the charginos case, we have noted that there is always a region of small positive μ (much smaller for the L-R supersymmetric model) for which $\tilde{\chi}_1^{\pm}$ is the LSP, but once the constraints from the experiment are applied, the condition $\tilde{M}_{\chi_1^{\pm}} > \tilde{M}_{\chi_1^0}$ does not restrict the allowed μ-M parameter space any further.[6] Also, LEP tells us that the lightest chargino $\tilde{\chi}_1^{\pm}$ is heavier than about 45 GeV.[7-9]

2. In the MSSM, when M≥200 GeV, we find (see, Figs. IV.2*a* and IV.2*b*) that at large $|\mu|$ not only $\tilde{\chi}_2^0$, $\tilde{\chi}_3^0$, and $\tilde{\chi}_4^0$ are heavier than the left-handed gauge boson masses, but so is the LSP.

3. In L-R supersymmetric model, there are always very tiny regions of $|\mu|$ region where the LSP, $\tilde{\chi}_1^0$, is very light. For example, in Fig. IV.3*a*, for M_L=50 GeV and M_R=300 GeV, we find a region where the mass can be as small as 17 GeV for $\tan\theta_K$=1.6. In Fig. IV.3*b*, we find a region where the mass can be as light as 5 GeV for $\tan\theta_K$=4.

4. When M_R=300 GeV and M_L=50 GeV, we find that for large values of μ, the mass of the neutralinos $\tilde{\chi}_k^0$ (k=1,...,4) are heavier than left-handed gauge boson, and the masses are even larger for M_R=1 TeV and M_L=250 GeV and can become even heavier than the right-handed W-boson as shown in Figs. IV.4*a* and IV.4*b*. Also we find in the L-R supersymmetric model, for large values of μ, the mass of fourth neutralino, $\tilde{\chi}_4^0$, is larger than 1 TeV.

We conclude this section by noting that the L-R supersymmetric model could add new limits on the supersymmetric particles. For instance the LSP, $\tilde{\chi}_1^0$, mass could be much lighter than expected in MSSM. Secondly is that, not all the supersymmetric particles are in the range ≤ 1 TeV. This implies limits not only on M_R but also on the other parameters.

VI.5. Constraints From Search on Charginos and Neutralinos.

We have shown from the previous sections that the masses and mixings of the gaugino and higgsino sector of the MSSM are determined by the parameters (M or $\tilde{M}_g$, μ, and $\tan\beta$), when the usual assumption of grand unification is applied. In the case of the L-R supersymmeric model the masses and mixing of the gaugino and higgsino sector are determined by the parameters M_R, M_L, μ, M_V and $\tan\theta_K$. The LEP experiments can place constraints on these parameters for the MSSM, in a variety of ways.[10] Charginos, if light enough, should be copiously pair-produced at LEP in Z^0 decay. Based upon experiments at L3,[7] ALEPH,[8] and OPAL [9] at LEP the lower limit on the chargino mass is about 45 GeV. The search for the Neutralino is obviously much more difficult. No absolute lower limit on the neutralino mass is known, although ALEPH,[11] excluded large parts of the parameter space μ-M. CDF[12] and UA2[13] collaborations have searched for the gluino (the superpartner of the gluon) and the squark (the superpartner of the quark), and presented the following bounds on their masses

$$\tilde{M}_q > 74 \text{ GeV}, \quad \tilde{M}_g > 73 \text{ GeV} \quad \text{(CDF)}$$

and

$$\tilde{M}_q > 74 \text{ GeV}, \quad \tilde{M}_g > 79 \text{ GeV} \quad \text{(UA2)}$$

The assumption of grand-unification allows us to use the gluino mass, eq.(III.2) to obtain:

$$M>22 \text{ GeV (CDF)}, \; M>23.5 \text{ GeV (UA2)}.$$

On the other hand, it is likely[14] that CDF will improve the gluino mass bound to $\tilde{M}_g \geq 140$ GeV (or $M \geq 41.6$ GeV), given the much larger data sample now collected at the Tevatron. (So far, the experimental limits on these particles for the L-R supersymmetric model have not been extrapolated).

One should remember that the above bounds on the squark and the gluino mass have been obtained assuming that only the direct decays $\tilde{q} \rightarrow q\tilde{\chi}_1$ and $\tilde{g} \rightarrow q\bar{q}\tilde{\chi}_1$ are possible. The lightest neutralino $\tilde{\chi}_1^0$, which is assumed to be the LSP, escapes the detector. However, Once[15] $\tilde{q}$ and $\tilde{g}$ become heavy enough so that their decays into charginos and neutralinos other than the LSP are kinematically accessible, these often dominate the direct decays to the LSP.

It has been pointed out by several authors[16] that cascade processes through charginos and heavier neutralinos, as well as the effect of $\tilde{M}_{\chi_1^0} \neq 0$, can significantly reduce these bounds. To be more specific, it is shown in Ref. [17] that the CDF bound of 73 GeV can be reduced by 3-30 GeV, whereas the squark bound is reduced by up to 10 GeV, depending on the values of μ and $\tan\beta$. For values of $\tan\beta$ not much above

1 there still remains a tiny region of small $|\mu|$ and M, where the LSP is almost a pure photino. It is also pointed out[17] that in the region of small $|\mu|$ and $M \gg |\mu|$, where the LSP is almost a pure higgsino, the process $\tilde{g} \to q\bar{q}\tilde{\chi}_1^0$ is greatly suppressed by the $\tilde{q}q\tilde{\chi}_1^0$ coupling, and no bound on $\tilde{M}_g$ can be given.

The combined bounds (from CDF and UA2) provide a lower limit on the mass of $\tilde{\chi}_1^0$ of about 13 GeV (as shown in Fig. VI.2*a* for M=250 GeV and $\tan\beta$=1.6 GeV), although for smaller values of $\tan\beta$ there are still tiny regions of small $|\mu|$ for which the LSP can be as light as 10 GeV.[5] Future searches for the chargino and neutralino will be able to exclude the LSP mass below about 14 GeV for $\tan\beta \geq 1$, and about 20 GeV for $\tan\beta \geq \sqrt{2}$. Recent Higgs searches at LEP imply that $\tan\beta > 1.6$.[5] An upper bound can also be placed on the mass of the LSP. Since none of the supersymmetric particles is expected to be heavier than about 1 TeV and directly from the relation (III.2) we find the rough bound:[18]

$\tilde{M}_g \leq 1$ TeV; $M \leq 300$ GeV and $\tilde{M}_{\chi_1^0} \leq 150$ GeV.

For completeness, we have shown in our results, that in the L-R supersymmetric model, new limits could be applied on the LSP's mass (≤ 5 GeV), which could be lighter than expected in the MSSM, Figs. IV.4*b*. Where the lower bound $\simeq 13$ GeV. Depending on the M_R assumptions, we also find that some of the SUSY particles could be heavier than 1 TeV. Certainly the future hadron colliders will tell us more about the right-handed bosons and supersymmetry.

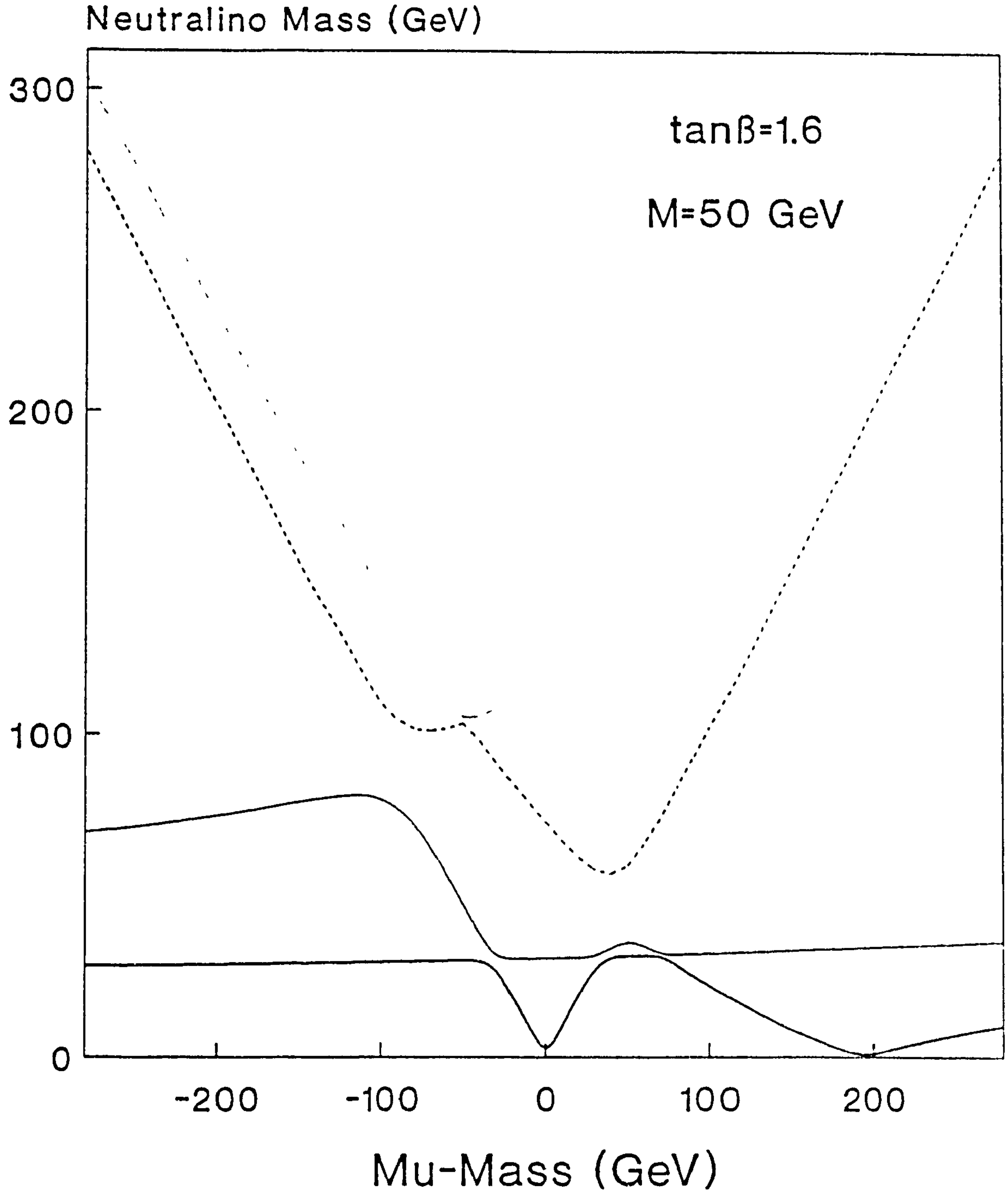

Fig. IV.1a: *Masses of neutralinos, in the MSSM, as a function of μ. We take $\tan\beta=1.6$ and $M=50$ GeV. The curves are: heavy solid, $\tilde{M}_{\chi_1^0}$; light solid, $\tilde{M}_{\chi_2^0}$; dashed $\tilde{M}_{\chi_3^0}$; and dotted, $\tilde{M}_{\chi_4^0}$.*

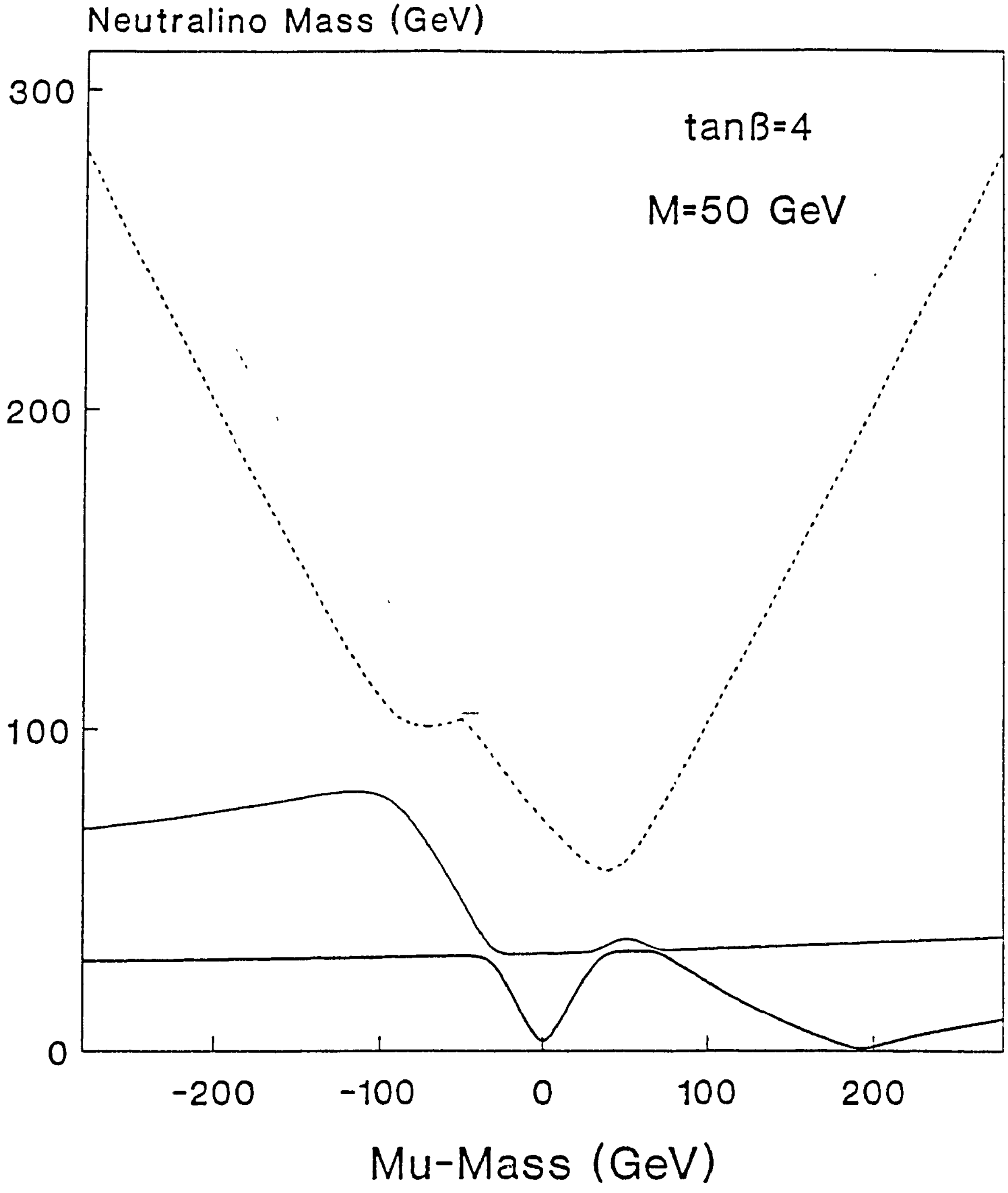

Fig. IV.1b: *Masses of neutralinos, in the MSSM, as a function of μ. We take $\tan\beta=4$ and $M=50$ GeV. The curves are: heavy solid, $\tilde{M}_{\chi_1^0}$; light solid, $\tilde{M}_{\chi_2^0}$; dashed $\tilde{M}_{\chi_3^0}$; and dotted, $\tilde{M}_{\chi_4^0}$.*

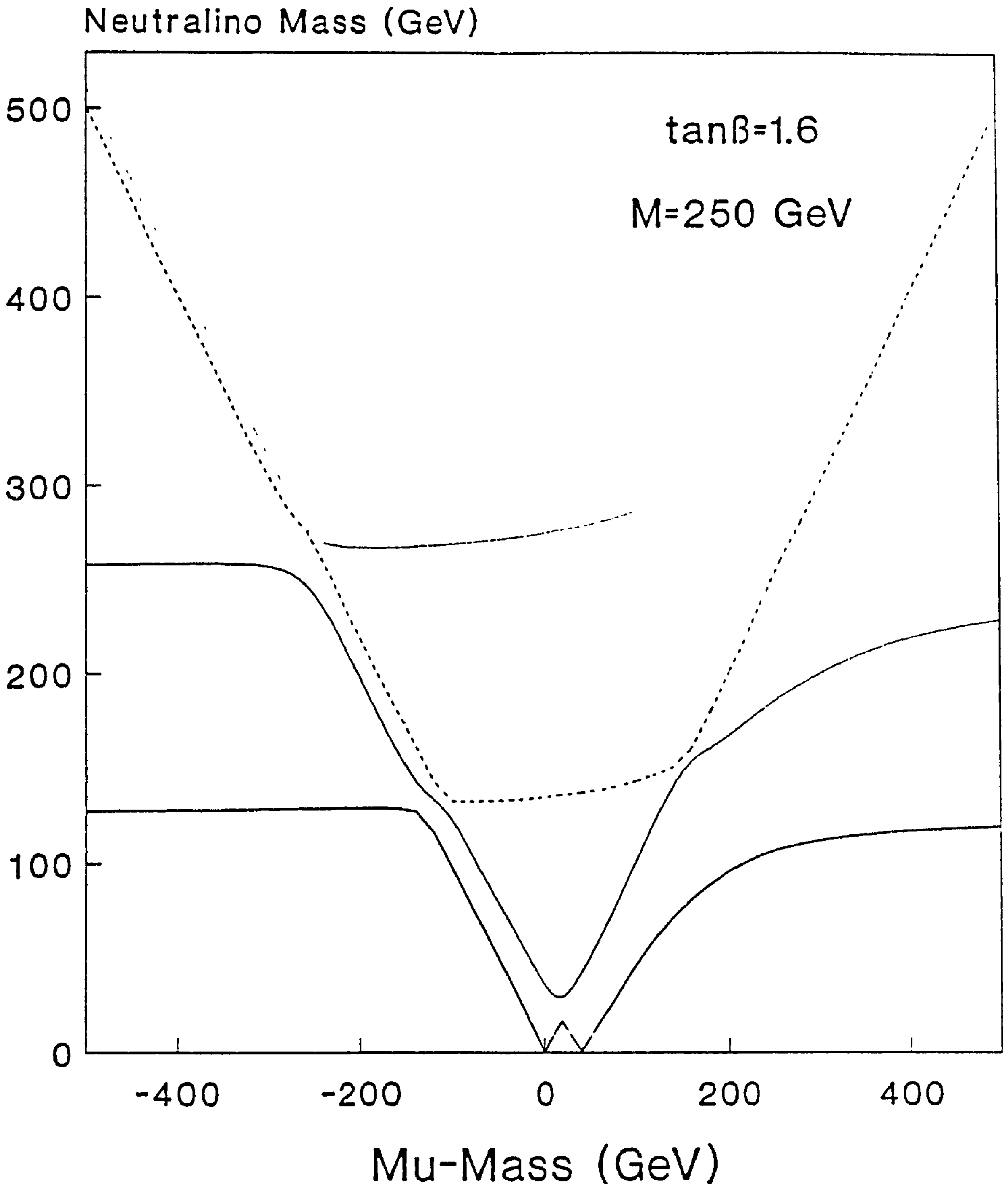

Fig. IV.2a: *Masses of neutralinos, in the MSSM, as a function of μ. We take $\tan\beta=1.6$ and $M=250$ GeV. The curves are: heavy solid, $\tilde{M}_{\chi_1^0}$; light solid, $\tilde{M}_{\chi_2^0}$; dashed $\tilde{M}_{\chi_3^0}$; and dotted, $\tilde{M}_{\chi_4^0}$.*

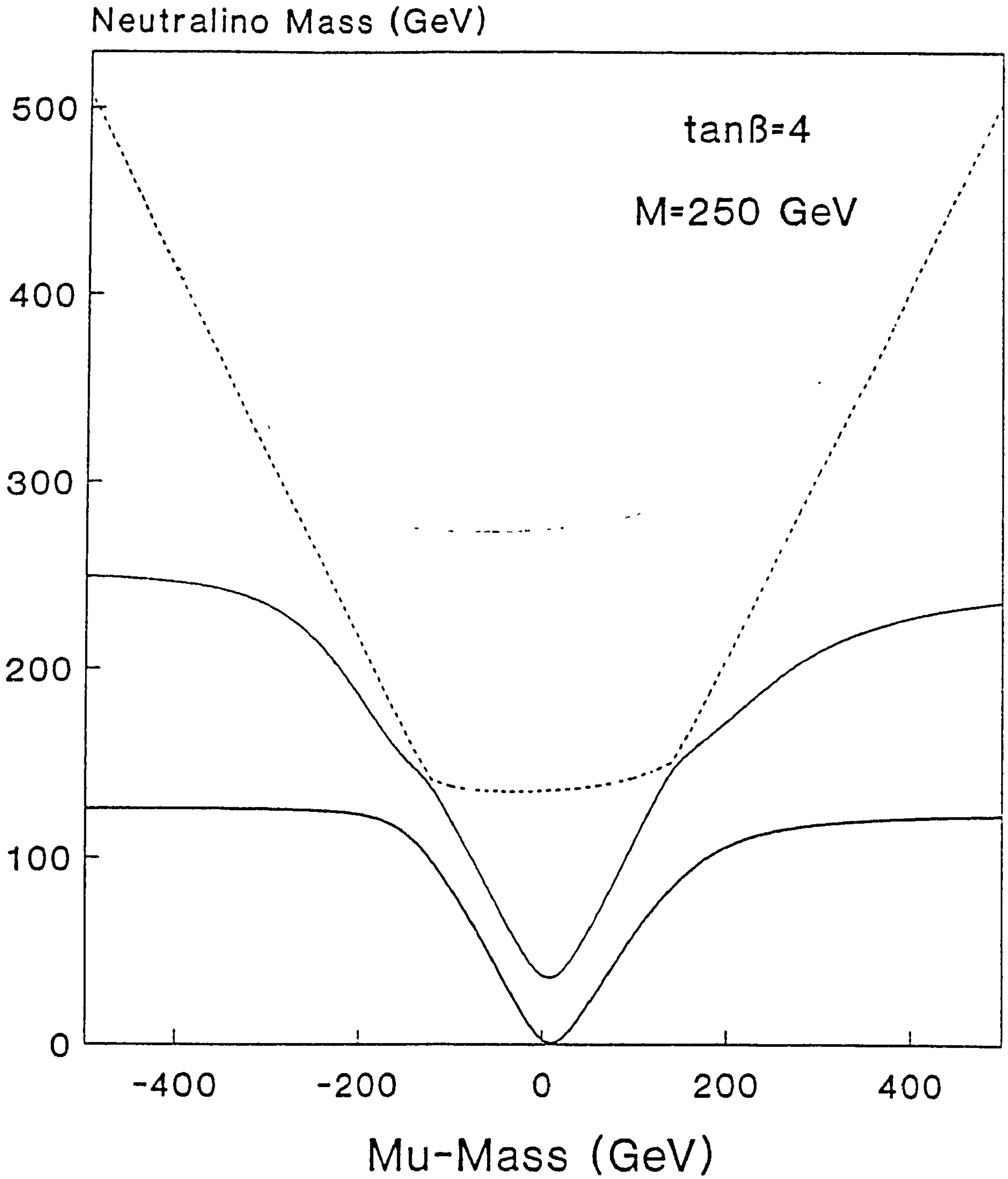

Fig. IV.2b: *Masses of neutralinos, in the MSSM, as a function of μ. We take $\tan\beta=4$ and M=250 GeV. The curves are: heavy solid, $\tilde{M}_{\chi_1^0}$; light solid, $\tilde{M}_{\chi_2^0}$; dashed $\tilde{M}_{\chi_3^0}$; and dotted, $\tilde{M}_{\chi_4^0}$.*

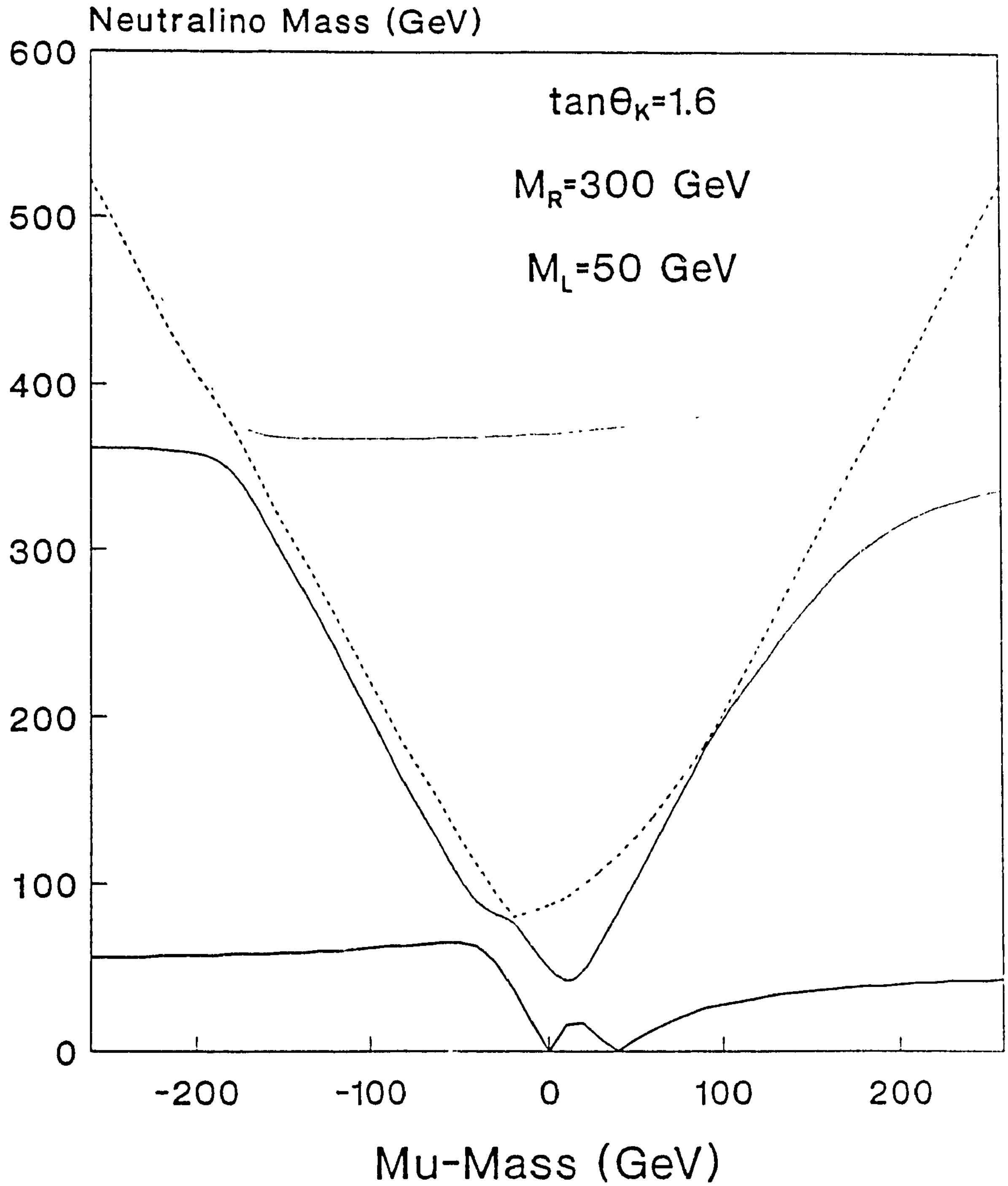

Fig. IV.3a: *Masses of neutralino, in the L-R supersymmetric model, as a function of μ. We take $\tan\theta_\kappa$=1.6, M_L=50 GeV and M_R=300 GeV. The curves are: heavy solid, $\tilde{M}_{\chi^0_1}$; light solid, $\tilde{M}_{\chi^0_2}$; dashed $\tilde{M}_{\chi^0_3}$; and dotted, $\tilde{M}_{\chi^0_4}$.*

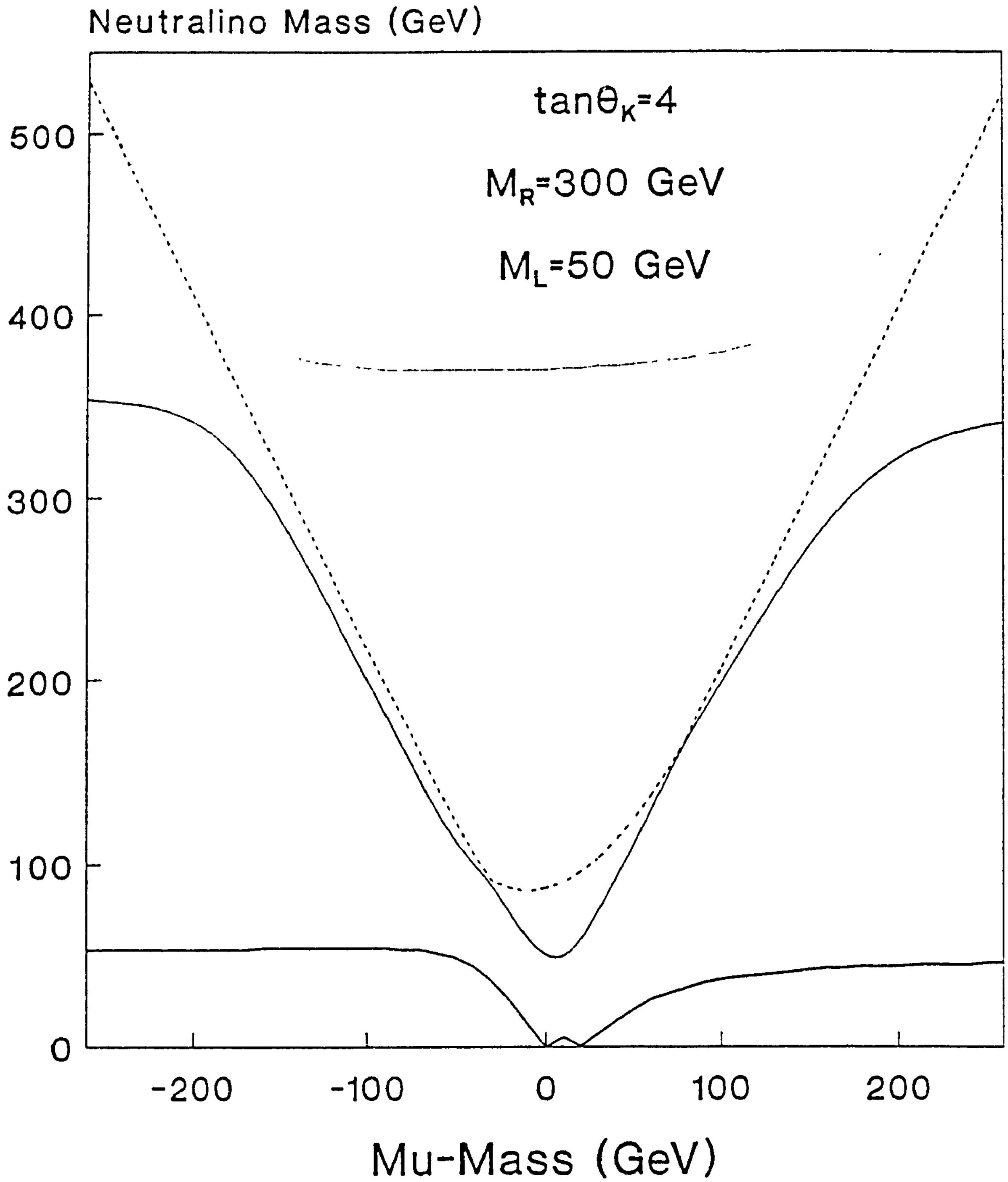

Fig. IV.3b: *Masses of neutralino, in the L-R supersymmetric model, as a function of* μ. *We take* $\tan\theta_K=4$, $M_L=50$ *GeV and* $M_R=300$ *GeV. The curves are: heavy solid,* $\tilde{M}_{\chi_1^0}$; *light solid,* $\tilde{M}_{\chi_2^0}$; *dashed* $\tilde{M}_{\chi_3^0}$; *and dotted,* $\tilde{M}_{\chi_4^0}$.

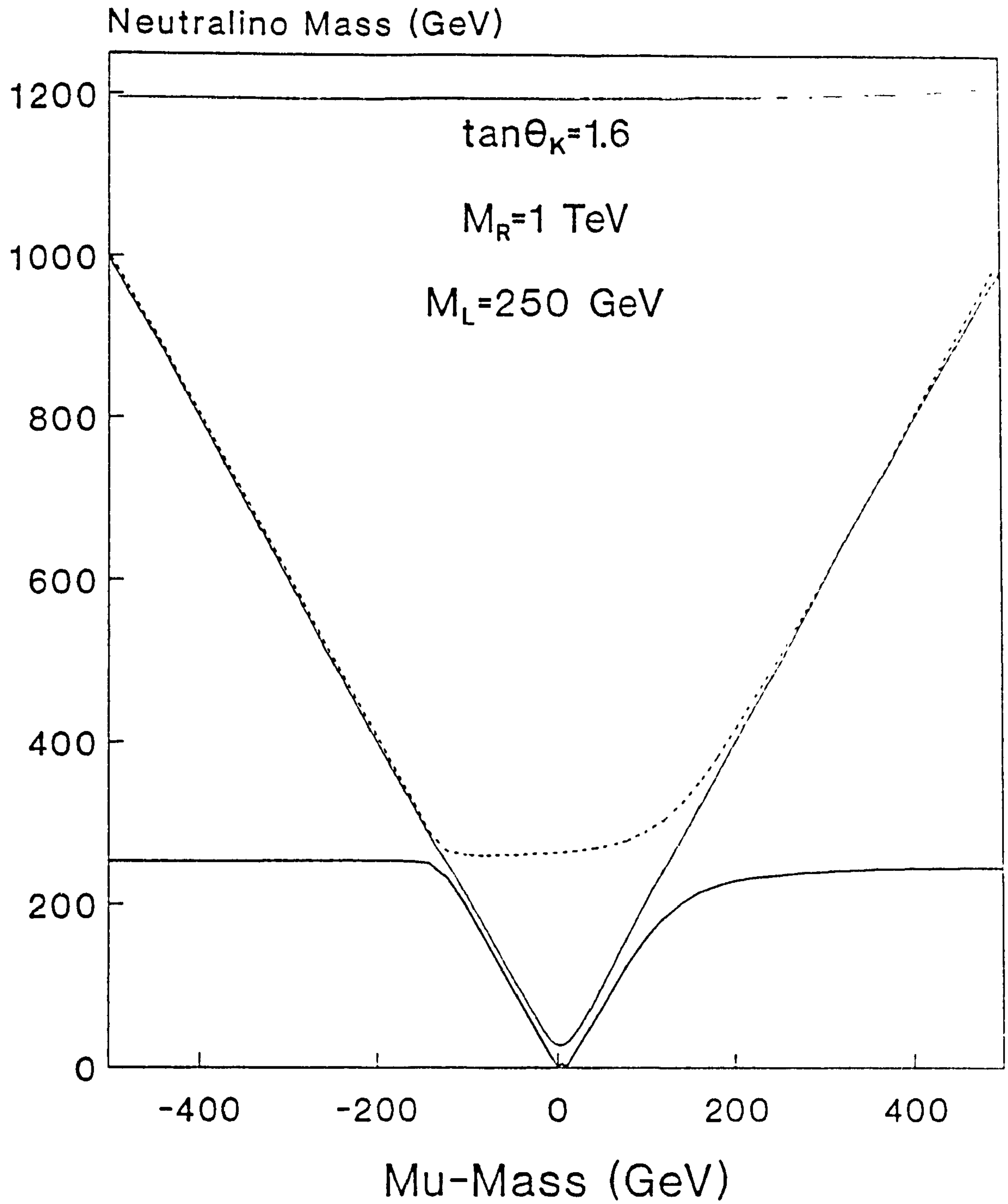

Fig. IV.4a: *Masses of neutralino, in the L-R supersymmetric model, as a function of μ. We take $\tan\theta_K=1.6$, $M_L=250$ GeV and $M_R=1$ TeV. The curves are: heavy solid, $\tilde{M}_{\chi_1^0}$; light solid, $\tilde{M}_{\chi_2^0}$; dashed $\tilde{M}_{\chi_3^0}$; and dotted, $\tilde{M}_{\chi_4^0}$.*

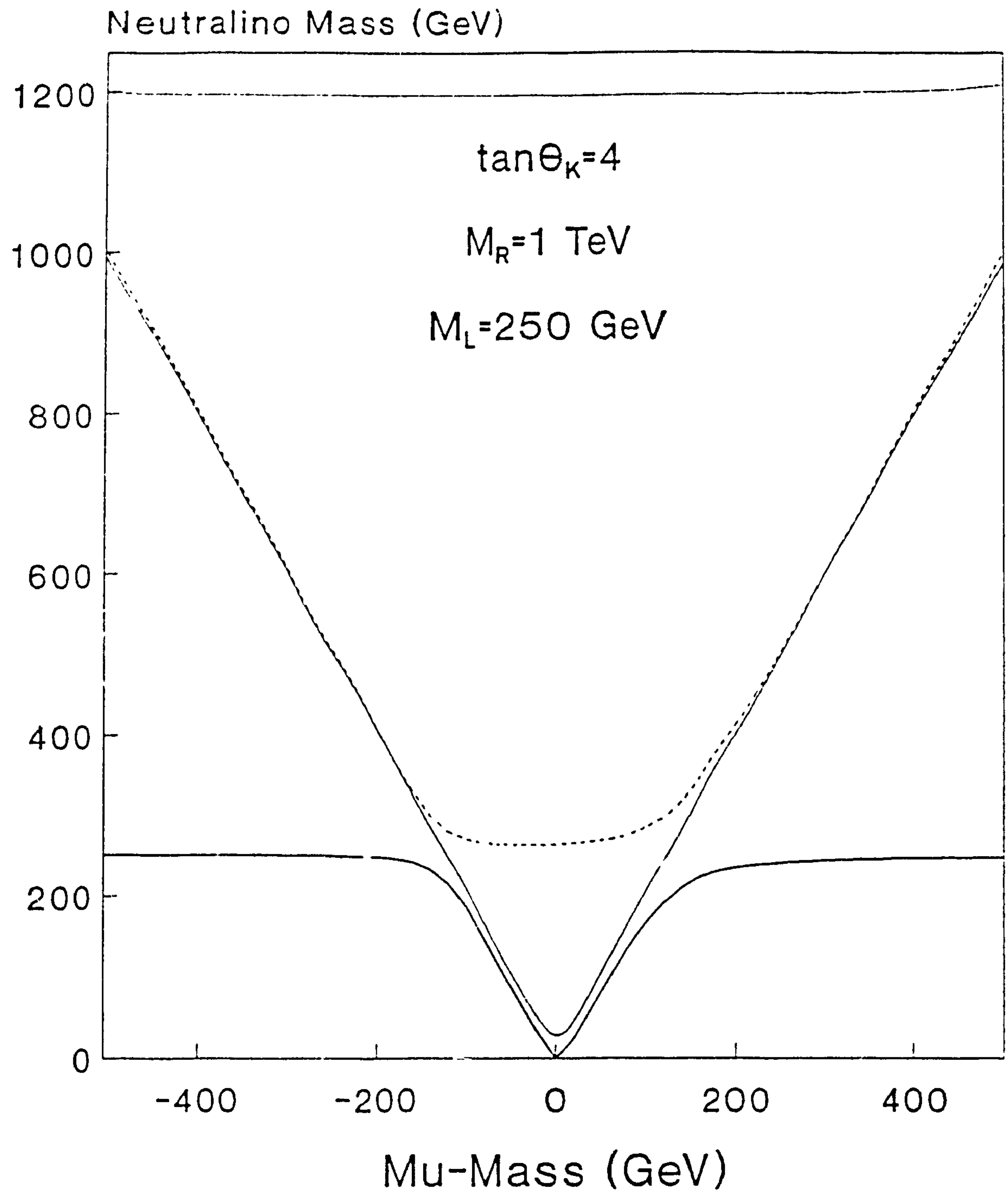

Fig. IV.4b: *Masses of neutralino, in the L-R supersymmetric model, as a function of μ. We take $\tan\theta_K=4$, $M_L=250$ GeV and $M_R=1$ TeV. The curves are: heavy solid, $\tilde{M}_{\chi_1^0}$; light solid, $\tilde{M}_{\chi_2^0}$; dashed $\tilde{M}_{\chi_3^0}$; and dotted, $\tilde{M}_{\chi_4^0}$.*

References

[1] See the review article; H.E. Haber and G.L. Kane, *Phys. Rep.* **110** (1984) 1.

[2] J. Gunion and H. Haber, *Nucl. Phys.* **B272** (1986) 1; *Nucl. Phys.* **B278** (1986) 449.

[3] B. Grinstein, J. Polchinski and M.B. Wise, *Phys. Lett.* **B130** (1983) 285; J. Ellis, J.S. Hagelin, D.V. Nanopoulos and M. Srednicki, *Phys. Lett.* **B127** (1983) 233; J. Ellis *et al.*, *Nucl. Phys.* **B238** (1984) 453.

[4] J.F. Gunion and H.E. Haber, *Phys. Rev.* **D37** (1988) 2515.

[5] ALEPH collaboration, S.L. Wu, DELPHI collaboration, A. Firestone, L3 collaboration, J. Branson, and OPAL collaboration, W. Gary, talks given at the Annual Rice University Meeting of the Division of *Particles and Fields* of the APS, Houston, Texas, 1990 (unpublished).

[6] L. Roszkowski, *Phys. Lett.* **B262** (1991) 59.

[7] L3 collaboration, B. Adeva *et al.*, *Phys. Lett.* **B233** (1989) 530.

[8] ALEPH collaboration, D. Decamp *et al.*, *Phys. Lett.* **B236** (1990) 86.

[9] OPAL collaboration, M.Z. Akrawy *et al.*, *Phys. Lett.* **B240** (1990) 261.

[10] H. Baer, M. Drees and Tata, *Phys. Rev.* **D41** (1990) 3414; L. Krauss, *Phys. Rev. Lett.* **64** (1990)) 999.

[11] ALEPH collaboration, D. Decamp *et al.*, LAL preprint LAL 90-26 (May 1990); J.F. Grivaz, talk first Intern. Symp. on *Particles, Strings, Cosmology* (PASCOS-90) (Boston,

March 1990).

[12] CDF collaboration, F. Abe *et al.*, *Phys. Rev. Lett.* **62** (1989) 1825.

[13] UA2 collaboration, J. Alitti *et al.*, *Phys. Lett.* **B235** (1990) 363.

[14] CDF collaboration, S. Geer, talk first Intern. Symp. on *Particles, Strings, Cosmology* (PASCOS-90) (Boston, March 1990).

[15] H. Baer *et al.*, *Phys. Lett.* **161B** (1985) 175; G. Gamberini, Z. *Phys.* **C30** (1986) 605.

[16] H. Baer *et al.*, *Phys. Lett.* **B161** (1985) 175; G. Gamberini, Z. *Phys.* **C30** (1986) 605; R.M. Barnett, H. Haber and G. Kane, *Nucl. Phys.* **B267** (1986) 625; H. Baer, D. Karatas and X. Tata, *Phys. Lett.* **B183** (1987) 220; R. Barbieri, G. Gamberini, G. Gudice and G. Ridolfi, *Nucl. Phys.* **B301** (1988) 15.

[17] H. Baer, X. Tata and J. Woodside, *Phys. Rev. Lett.* **63** (1989) 352.

[18] L. Roszkowski, *Phys. Lett.* **B252** (1990) 471; H. Baer, X. Tata, J. Woodside, *Phys. Rev.* **D44** (1991) 207.

Chapter V

Chargino and Neutralino Production in $p\bar{p}$-Collisions

V.1. Introduction.

If the supersymmetric particles have masses that are larger than $M_Z/2$, then it may be that their presence can only be established at the very high energy hadronic colliders. Thus, it is very important to study the production of new particles at the hadronic colliders to compare the cross sections for the production of supersymmetric particles with standard model processes.

A reaction which has received a lot of attention in hadronic collisions is gluino pair production. We have already quoted the bound on the gluino mass,[1,2] based on the missing momentum p_T signal associated with the gluino decay into an undetected stable neutralino $\tilde{g} \to q\bar{q}\chi_1^0$. If one assumes that $\tilde{\chi}_1^0$ is considerably lighter than the gluino, the sensitivity to this signal depends only on the gluino mass.

The extraction of new physics from the background will be a formidable task at hadronic colliders. Of special interest, which is our purpose in this chapter, is the production of charginos and neutralinos in hadronic colliders. These particles occur in the final state decay products of the $W^{\pm}$ and Z^0 bosons,[1,2] specifically with:

$p\bar{p} \to W^{\pm} + X(\text{hadrons}) \to \tilde{\chi}_j^{\pm}\chi_i^0 + X(\text{hadrons})$ with $\tilde{\chi}_i^0 \to \tilde{\chi}_\ell^0$ + coplanar jets or monojets or coplanar lepton pairs (where i>k) with missing momentum. In general SUSY particles will decay through multi-step cascades which terminate in the LSP. Within the MSSM, all the cascade decays and sparticle masses are completely determined by fixing the parameters set (M or $\tilde{M}_g$, $\tilde{M}_q$, μ, $\tan\beta$, M_{H^+}, and M_t).[3]

Eventually, at a high luminosity supercollider this process, in question, will provide the most definitive test of whether SUSY is present on the weak scale, because it has a clear signature and a cross section that is large enough to see even for quite heavy neutralinos.

In this chapter we consider the lowest-order cross section of the process $\bar{p}p \to W^-_{L,R} + X \to \tilde{\chi}^-_j \tilde{\chi}^0_i + X$ and $W^-_{L,R}$-boson decay at the Fermilab Tevatron $p\bar{p}$-Collider (CDF) of 1.8 TeV center of mass energy. Parton model distributions which are used in our calculations, are discussed in the next section. The results are compared with the conventional $W^-_{L,R} \to \ell^- \nu_{\ell L,R}$ direct decay as a background signature in the lepton transverse momentum p_T region. We also study the chargino signature through the Jacobian peak in the neighborhood of $p_T \simeq (M_W/2)_{L,R}$. Since the charged fermion, $\tilde{\chi}^-_j$, or the neutral fermion, $\tilde{\chi}^0_i$, decays further into $\tilde{\chi}^0_1$ and a pair of charged and neutral conventional leptons or two collimated jets. Since neutralinos escape undetected, one should look for the chargino signature by examining events which include lepton or collimated jets.

V.2. Cross Sections and Decay Rates.

V.2.1. Cross-sections.

Chargino and neutralino production in physics colliders have been already studied in detail for the standard model. As a review see for instance Refs. [4-8]. In our work we consider the lowest-order decay of the $W^-_{L,R}$-boson into charginos and neutralinos in $p\bar{p}$-collisions for the left-right supersymmetric model. The process in question is:

$$p\bar{p} \rightarrow W^-_{L,R} + X \rightarrow \tilde{\chi}^-_j \tilde{\chi}^0_i + X, \tag{V.1}$$

This process gives rise to the experimental signature associated with the direct $W^-_{L,R} \rightarrow \ell^- \bar{\nu}_{\ell L,R}$ decay in the process:

$$p\bar{p} \rightarrow W^-_{L,R} + X \rightarrow \ell^- \bar{\nu}_{\ell L,R} + X, \tag{V.2}$$

The process (V.1) could give rise to the experimental background signature associated with $W^-_{L,R}$-decay, process (V.2), namely the presence of high transverse momentum of the lepton ($\ell^-=e^-,\mu^-,\tau^-$). The possibility of detecting $W^-_{L,R} \rightarrow \tilde{\chi}^-_j \tilde{\chi}^0_i$ decay against a dominant background due to the conventional decay $W^-_{L,R} \rightarrow \ell^- \bar{\nu}_{\ell L,R}$ depends on the observation that the lepton distribution exhibits forward-backward asymmetry. This is because of the typical $(1 \mp \cos\hat{\theta})^2$ distribution of lepton pairs with V-A coupling. The decay of $W^-_{L,R}$ to charginos and neutralinos is on the other hand symmetric. We begin with the subprocess

$$\bar{q}(p_1) + q'(p_2) \to \tilde{\chi}^-_j(k_1) + \tilde{\chi}^0_i(k_2). \tag{V.3}$$

Neglecting the quark masses, in the $\bar{q}q'$-center of mass energy system, we define the spin averaged, color averaged matrix element squared (we have used the Lagrangian from the second chapter).

$$|\mathcal{M}|^2_{ave} = \frac{g^4}{6} |V_{q,q'}|^2 |D_W(\hat{s})|^2_{L,R} \Big((|O^L_{ij}|^2 + |O^R_{ij}|^2) \times$$

$$\Big[(p_1\cdot k_1)(p_2\cdot k_2) + (p_2\cdot k_1)(p_1\cdot k_2)\Big] + (O^L_{ij}O^{R*}_{ij} + O^{L*}_{ij}O^R_{ij}) \times$$

$$\frac{1}{4} \tilde{M}_{\chi^-}\tilde{M}_{\chi^0} (p_1\cdot p_2)\Big) \tag{V.4}$$

where $|V_{q,q'}|^2$ is the quark mixing matrix†, and

$$D_W(\hat{s})_{L,R} = [(\hat{s} - M^2_W) + iM_W\Gamma_W]^{-1}_{(L,R)} \tag{V.5}$$

is the $W^-_{L,R}$-propagator. The matrix elements O^L_{12} and O^R_{12} are: $O^L_{12} = \sin\theta_W V^*_{21} = \sin\theta_W \sin\phi_+$, and $O^R_{12} = \sin\theta_W U_{21} = \sin\theta_W \sin\phi_-$, where the relevant elements of the mixing matrices U and V are given by,

$$U_{21} = O_- \ , \ V_{21} = O_+ \ , \ \text{and } O_\pm = \begin{pmatrix} \cos\phi_\pm & \sin\phi_\pm \\ -\sin\phi_\pm & \cos\phi_\pm \end{pmatrix} \tag{V.6}$$

---

† The dominant contributions are: $|V_{\bar{u}d}| \simeq |V_{\bar{c}s}| \simeq |V_{\bar{t}b}| \simeq 1$, and all other elements due to quark mixing matrix or the CKM matrix are small. Thus, summing over all quark generations $N_G \simeq 3$.

The decay width $\Gamma_{WL,R}$ of the W^--boson for $g \simeq g_L \simeq g_R$ is [9]

$$\Gamma_{WL} = 9\Gamma^0(W_L^- \to e^-\bar{\nu}_e) = 9\left[\frac{g^2 M_{WL}}{48\,\pi}\right], \tag{V.7a}$$

and

$$\Gamma_{WR} = 12\Gamma^0(W_R^- \to e^-\bar{\nu}_{eR}) = 12\left[\frac{g^2 M_{WR}}{48\,\pi}\right]. \tag{V.7b}$$

We multiply eq.(V.7b) by the factor (12), since $M_{WR} > M_t$ and the decay $W_R^- \to \bar{t} + b$ is kinematically allowed. Thus, for $M_{WR} \geq 300$ GeV, we get $\Gamma_{WR} \geq 5\Gamma_{WL}$. We define the following kinematic invariants in the $\bar{q}q'$-center of mass energy system:

$$\left.\begin{aligned}
\hat{s} &\simeq 2p_1.p_2 = \tilde{M}^2_{\chi^0} + \tilde{M}^2_{\chi^-} - 2k_1.k_2 \\
\hat{t} &\simeq \tilde{M}^2_{\chi^-} - 2p_1.k_1 \simeq \tilde{M}^2_{\chi^0} - 2p_2.k_2 \\
\hat{u} &\simeq \tilde{M}^2_{\chi^-} - 2p_2.k_1 \simeq \tilde{M}^2_{\chi^0} - 2p_1.k_2
\end{aligned}\right\} \tag{V.8}$$

with

$$\hat{s} + \hat{t} + \hat{u} \simeq \sum_i M_i^2 \simeq \tilde{M}^2_{\chi^-} + \tilde{M}^2_{\chi^0} \tag{V.9}$$

Substituting eqs.(V.8) and (V.9) in eq.(V.4) we get:

$$\begin{aligned}
|\mathcal{M}|^2_{ave} = \frac{g^4}{24}\,|V_{q,q'}|^2\,|D_W(\hat{s})|^2_{L,R}\Bigg\{&(|O^L_{ij}|^2 + |O^R_{ij}|^2)\,\times \\
&\left[(\tilde{M}^2_{\chi^-} - \hat{t})(\tilde{M}^2_{\chi^0} - \hat{t}) - (\tilde{M}^2_{\chi^-} - \hat{u})(\tilde{M}_{\chi^0} - \hat{u})\right] \\
&+ \frac{\hat{s}}{2}\,\tilde{M}_{\chi^-}\tilde{M}_{\chi^0}\,(O^L_{ij}O^{R*}_{ij} + O^{L*}_{ij}O^R_{ij})\Bigg\}.
\end{aligned} \tag{V.10}$$

It is more convenient to write our results in terms of new variables x, y and z:

$$x= \tilde{M}_{\chi}-\tilde{M}_{\chi^{o}}/\hat{s}, \qquad y= \tilde{M}^2_{\chi^{o}}/\hat{s}, \qquad z= \tilde{M}^2_{\chi^-}/\hat{s}. \tag{V.11}$$

Thus, eq.(V.10) can be written as,

$$|\mathcal{M}|^2_{ave}= \frac{\hat{s}^2 g^4}{48} \; |V_{q,q'}|^2 |D_W(\hat{s})|^2_{L,R} \Bigg((|O^L_{ij}|^2 + |O^R_{ij}|^2) \times$$

$$\left[1 - (y-z)^2 + \lambda^{1/2}(1,y,z)\cos\hat{\theta}\right] +$$

$$+ x(O^L_{ij} O^{R*}_{ij} + O^{L*}_{ij} O^R_{ij})\Bigg). \tag{V.12}$$

Here the kinematic invariants $\hat{t}$, $\hat{u}$, can be written in terms of eq.(V.11) as,

$$\left.\begin{aligned} \hat{t} &\simeq - \tfrac{1}{2}\hat{s}\; [(1-y-z) - \lambda^{1/2}(1,y,z)\cos\hat{\theta}] \\ \hat{u} &\simeq - \tfrac{1}{2}\hat{s}\; [(1-y-z) + \lambda^{1/2}(1,y,z)\cos\hat{\theta}] \end{aligned}\right\}, \tag{V.13}$$

where the triangle function λ is given by,

$$\lambda(1,y,z)=1+y^2+z^2-2y-2z-2yz. \tag{V.14}$$

The angle $\hat{\theta}$ specifies the chargino or lepton direction with respect to the incident p-direction in the W^--rest frame. The differential cross section of the subprocess (V.3) is given by:[10]

$$\frac{d\sigma(\tilde{\chi}^-_j\tilde{\chi}^o_i)}{d(\cos\hat{\theta})} = \left(\frac{|\mathcal{M}|^2_{ave}}{32\pi\hat{s}}\right) \lambda^{1/2}(1,y,z). \tag{V.15}$$

Substituting the value of $|\mathcal{M}|^2_{ave}$ in eq.(V.15) we get:

$$\frac{d\sigma(\tilde{\chi}^-_j\tilde{\chi}^o_i)}{d(\cos\hat{\theta})} = \frac{\hat{s}g^4}{1536\pi}|V_{q,q'}|^2|D_W(\hat{s})|^2_{L,R} \times$$

$$\left\{(|O^L_{ij}|^2 + |O^R_{ij}|^2)\left[1 - (y-z)^2 + \lambda^{1/2}(1,y,z)\cos\hat{\theta}\right]\right.$$

$$\left. + x(O^L_{ij}O^{R*}_{ij} + O^{L*}_{ij}O^R_{ij})\right\}. \tag{V.16}$$

Equation (V.16) concerns a subprocess of the total cross-section (V.1). In order to find the total cross-section, we consider the standard parton model:

V.2.2. Parton model.

Most of the collisions are called "*soft*", which means that the colliding protons might stay together and just scatter elastically. No energy is transferred to the target. The outgoing particles are in groups and follow a path not far from the beam direction. When the energy is transferred to the target it (i.e., the target) often breaks into several particles. Then the scattering can occur at large angles, giving some collision products with large transverse momenta relative to the beam direction. Such inelastic collisions are also called "*hard*" collisions.[11] The simplest inelastic cross-section to measure is the so-called "*inclusive*" cross-section. For these cross-sections, which correspond to processes such as (V.1) and (V.2), only the final chargino (or lepton) is observed.

In the parton model, the incoming proton is composed of partons i carrying fractional longitudinal momentum represented by x_i $(0 \le x_i \le 1)$. The various scattered and spectator partons are assumed to fragment to final states with probability one. A charged constituent quark coming from the proton carries a momentum $x_1 P$, and mass $m = x_1 M$ (where P is the proton momentum in the $p\bar{p}$-centre of mass frame and M is the mass of the proton(antiproton)). Similarly an antiquark coming from the antiproton carries a momentum $-x_2 P$. The corresponding Feynman diagram for the $p\bar{p}$-collisions is given by Fig. V.1.

Suppose that $f(x_1)$ is the *probability distribution* for finding a parton f with momentum fraction x_1, and $\bar{f}(x_2)$ is the corresponding probability distribution for finding an antiparton with momentum fraction x_2. Then one can obtain the probability distribution for proton-antiproton collisions, by multiplying the subprocess eq.(V.15) with the *probabilities* for finding a quark of type α, with momentum fraction x_1, and an antiquark of the same type with momentum fraction x_2, namely

$$\left(f_\alpha(x_1)\,dx_1\right) \times \left(\bar{f}_\alpha(x_2)\,dx_2\right)$$

There is of course another contribution for which the antiquark has fraction x_1 and antiquark x_2,

$$\left(\bar{f}_\alpha(x_1)\,dx_1\right) \times \left(f_\alpha(x_2)\,dx_2\right)$$

Thus, the inclusive differential cross section for $p\bar{p}$-collisions is:[9,11]

$$\frac{d\sigma}{d\xi\, d(\cos\hat{\theta})} = K\int \frac{d\hat{s}}{s} \sum_{q,\bar{q}} \left[f_{\bar{q}}(x_1) f_q(x_2) + f_q(x_1) f_{\bar{q}}(x_2)\right] \cdot$$
$$\left(\frac{d\hat{\sigma}}{d(\cos\hat{\theta})}(\bar{q}q' \to \tilde{\chi}_j^- \tilde{\chi}_i^0)\right) \qquad (V.17)$$

where K is the Dr·ll-Yan correction factor: this factor,

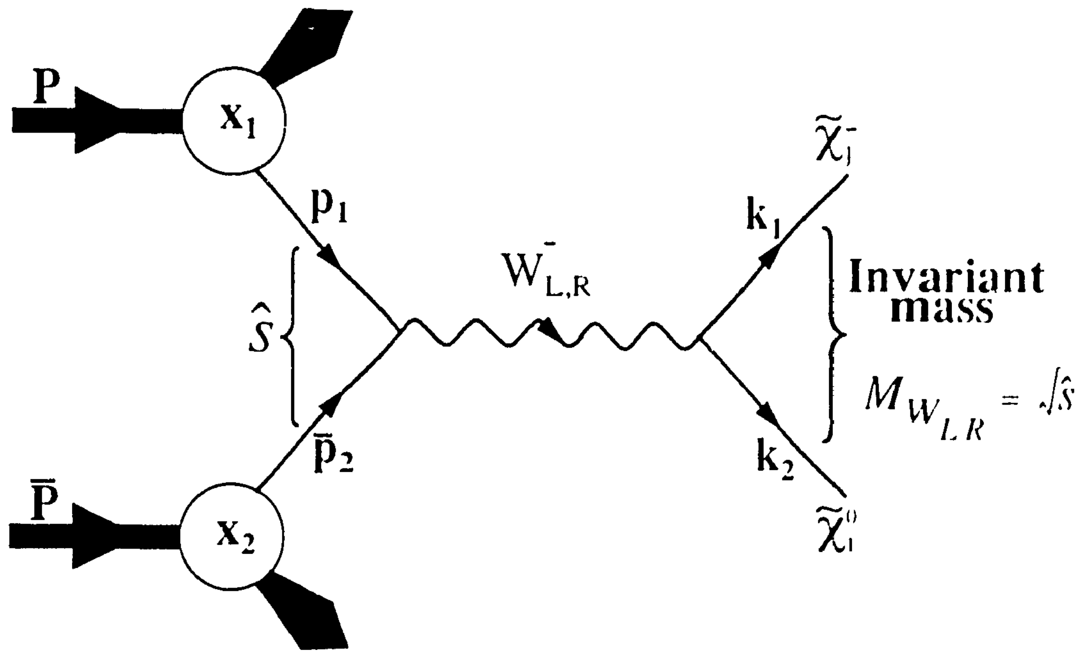

Fig. V.1 *Parton Model description of chargino $\tilde{\chi}_j^-$ and neutralino $\tilde{\chi}_i^0$ production in $p\bar{p}$-collisions through $W^-_{L,R}$-bosons.*

commonly referred to as K-*factor* is given by[12]

$$K = 1 + \frac{8\pi}{9}\,\alpha_s(M_W^2), \tag{V.18}$$

where α_s is understood to mean $\alpha_s(M_W^2)$ thinking of α_s as a function evaluated in terms of the renormalization group. The functions $f(x_1)$ and $f(x_2)$ are also related to the Drell-Yan cross section for producing a $W^-_{L,R}$-boson from annihilation of a quark-antiquark pair with fractional longitudinal momenta x_1, x_2. If there are N_f numbers of quark flavors active at the W^--mass scale,[13,14]

$$\sum_{q,\bar{q}} f_{\bar{q}}(x_1) f_q(x_2) = u(x_1)d(x_2) + \left(\frac{N_f}{2} - 1\right)s(x_1)s(x_2) \tag{V.19}$$

$$\sum_{q,\bar{q}} f_q(x_1) f_{\bar{q}}(x_2) = \frac{N_f}{2}\, s(x_1)s(x_2) \tag{V.20}$$

where the quark structure functions $u(x_1)$ and $d(x_2)$ include both valence and sea contributions. The latter contributions include the strange quark structure function $s(x)$. In our calculations we have used the Duke and Owens parameterization for the quark structure functions found in Ref. [15]. The longitudinal scaling variables x_1 and x_2 are related to s and ξ, where ξ is the rapidity of the W^--boson in the $\bar{q}q'$-centre of mass frame and defined by

$$\xi \equiv x_1 - x_2 \text{ and } x_{1,2} = (M_W/\sqrt{s})\,e^{\pm\xi} \tag{V.21}$$

Here it is assumed that $\hat{s}=M_W^2$ is the appropriate scale of the quark distributions. We integrate eq.(V.17) over $\hat{\theta}$, and replace $|D_W(\hat{s})|^2_{L,R}$ in eq.(V.12) with the narrow-width approximation $\pi[\delta(\hat{s} - M_W^2)/(M_W\Gamma_W)]_{L,R}$. The integral over $d\hat{s}$ takes out the delta function. The total cross section for W^--production is obtained by integration over the full range of the W^--rapidity limits $-\ln\sqrt{s}/M_{WL,R} \le \xi \le \ln\sqrt{s}/M_{WL,R}$. Comparing with process (V.2), the differential cross-section at the parton level is given by,[10-12]

$$\frac{d\sigma}{d\xi d(\cos\hat{\theta})} = K\int \frac{d\hat{s}}{s} \sum_{q,\bar{q}} \left[f_{\bar{q}}(x_1) f_q(x_2) + f_q(x_1) f_{\bar{q}}(x_2)\right] \cdot \left(\frac{d\hat{\sigma}}{d(\cos\hat{\theta})} (\bar{q}q' \to \ell^- \bar{\nu}_{\ell L,R})\right), \qquad (V.22)$$

where

$$\frac{d\hat{\sigma}}{d(\cos\hat{\theta})} (\bar{q}q' \to \ell^- \bar{\nu}_{\ell L,R}) = \frac{g^4}{8\pi} |V_{q,q'}|^2 |D_W(\hat{s})|^2_{L,R} \times \hat{s}(1 - \cos\hat{\theta})^2. \qquad (V.23)$$

The expression for $\hat{\theta}$ can be transformed from the $\bar{q}q'$-frame to the $p\bar{p}$-frame:

$$(1 - \cos\hat{\theta}) = (2p_T/x_1\sqrt{s})[(1 - \cos\theta)/\sin\theta], \qquad (V.24)$$

$$(1 + \cos\hat{\theta}) = (2p_T/x_2\sqrt{s})[(1 + \cos\theta)/\sin\theta], \qquad (V.25)$$

where θ is the angle at which the chargino or lepton is produced in $p\bar{p}$-centre of mass frame.

V.2.3. Decays of the $W^-_{L,R}$-boson.

It is important to give an estimate of the branching ratios of the chargino and neutralino in the decay of the $W^-_{L,R}$-boson and compare it with that of the conventional process $W^-_{L,R} \to \ell^- \bar{\nu}_{\ell L,R}$. For our process, i.e.,

$$\bar{\ }_{L,R}(P) \to \tilde{\chi}^-_j(k) + \tilde{\chi}^0_i(k') \tag{V.26}$$

where P, k, k' are the four-momenta of the vector boson, chargino and neutralino respectively. We find the matrix element squared of the $W^-_{L,R}$-boson decay at $g \approx g_L \approx g_R$ is,

$$|\mathcal{M}|^2_{ave} = \frac{g^2}{6}\left((|O^L_{ij}|^2 + |O^R_{ij}|^2) \left[(k.k') + \frac{2(P.k)(P.k')}{M^2_{WL,R}} \right] \times \right.$$

$$\left. 3\tilde{M}_{\chi}\text{-}\tilde{M}_{\chi^0} \; (O^L_{ij}O^{R*}_{ij} + O^{L*}_{ij}O^R_{ij}) \right), \tag{V.27}$$

where $P = k + k'$. In eq.(V.27) we average over W^--boson polarization and sum over the fermion spins. For the above decay, we find the decay width in the rest frame is (evaluation of the phase space integration is given in the appendix C)

$$\Gamma(W^-_{L,R} \to \tilde{\chi}^-_j \tilde{\chi}^0_i) = \frac{\alpha^2}{3} \Gamma^0_{WL,R} \Bigg((|O^L_{ij}|^2 + |O^R_{ij}|^2) \cdot$$

$$\left[2 - x - y - (x - y)^2\right] + 6z(O^L_{ij} O^{R*}_{ij} + O^{L*}_{ij} O^R_{ij}) \Bigg) \cdot$$

$$\left[(1 + y - x)^2 - 4y\right]^{1/2}, \qquad \text{(V.28)}$$

where the variables x, y and z are defined by

$$x = \tilde{M}^2_{\chi^0}/M^2_{WL,R};\quad y = \tilde{M}^2_{\chi^-}/M^2_{WL,R};\quad z = \tilde{M}_{\chi^-} \tilde{M}_{\chi^0}/M^2_{WL,R};$$

and $\Gamma^0_{WL,R}$ is given by eqs.(V.7*a*,*b*). The contribution of the chargino and neutralino signature to the experimental background signature is found to be in the ratios:

$$R \equiv \frac{\Gamma(W^-_{L,R} \to \tilde{\chi}^-_j \tilde{\chi}^0_i)}{\Gamma(W^-_{L,R} \to \ell^- \bar{\nu}_{eL,R})} = \frac{\alpha^2}{3} \Bigg((|O^L_{ij}|^2 + |O^R_{ij}|^2) \left[2 - x - y - (x - y)^2\right]$$

$$+ 6z(O^L_{ij} O^{*R}_{ij} + O^{L*}_{ij} O^R_{ij}) \Bigg) \left[(1 + y - x)^2 - 4y\right]^{1/2} \qquad \text{(V.29)}$$

The present experimental lower bounds on the masses obtained at LEP[16-18] are 45 GeV for $\tilde{M}_{\chi^-}$ and for the LSP 14 GeV.[19,20] Furthermore using recent measurements of the W-boson mass obtained by the CDF[21] at the Tevatron: M_{WL}=80 GeV, with Γ_{WL}=2.2 GeV and $\sin^2\theta_W$=0.23. This implies that

$$R(W_L^- \to \tilde{\chi}_j^- \tilde{\chi}_i^0) \simeq 0.32, \quad \text{for } \Gamma(W_L^- \to e^- \bar{\nu}_e) \simeq 0.22 \text{ GeV} \qquad \text{(V.30)}$$

We expect normally to be able to see W_R decays to a charged lepton and a right-handed neutrino.[22,23] We find the decay width (neglecting the lepton masses, and setting $g \simeq g_L \simeq g_R$) is

$$\Gamma(W_R^- \to \ell^- \bar{\nu}_{\ell R}) = \frac{g^2}{48\pi} M_{WR} \left(1 - \frac{m_{\nu R}^2}{2M_{WR}^2} - \frac{m_{\nu R}^4}{2M_{WR}^4}\right)\left(1 - \frac{m_{\nu R}^2}{M_{WR}^2}\right).$$

(V.31)

If the right-handed neutrino is light ($m_{\nu R} \leq 10$ MeV) and if it has a charged current coupling, then using eq.(V.31) with $M_{WR} \geq 300$ GeV we find that:

$$R(W_R^- \to \tilde{\chi}_j^- \tilde{\chi}_i^0) \simeq 0.12; \quad \text{for } \Gamma(W_R^- \to e^- \bar{\nu}_{eR}) \simeq 0.84 \text{ GeV} \qquad \text{(V.33)}$$

The possibility of detecting $W_{L,R}^- \to \tilde{\chi}_j^- \tilde{\chi}_i^0$ decay against a dominant background due to the conventional decay $W_{L,R}^- \to \ell^- \bar{\nu}_{\ell L,R}$ depends on the observation that the lepton distribution exhibits forward-backward asymmetry. This is because of the typical $(1 \mp \cos\hat{\theta})^2$ distribution of lepton pairs with V-A coupling. The decay of $W_{L,R}^-$ to charginos is on the other hand symmetric.

V.3. Numerical Results.

The inclusive cross-section for the parton model in eq.(V.17) is solved by numerical integration at $\sqrt{s}=1.8$ TeV, the $p\bar{p}$-collider energy at Fermilab (CDF). We assume

M_{WL}=80 GeV, Γ_{WL}=2.2 GeV, $x_W \simeq 0.23$ and $\tan\beta$=1, and based upon the weakest limit from Δm_K and astrophysics we take $M_{WR} \geq 300$ GeV. The decay at this value is: $\Gamma_{WR} \simeq 8.25$ GeV at $g \simeq g_L \simeq g_R$. Equation (V.17) is normalized by including the QCD-motivated correction factor K≃1.22 at CDF (K≃1.31 at CERN Collider energies), which is given in Ref. [14]. The cross section for $W^-_{L,R} \rightarrow \tilde{\chi}^-_j \tilde{\chi}^0_i$ decays ($\tan\beta \geq 1$) are shown in Fig. V.2 and $W^-_{L,R} \rightarrow \ell^- \bar{\nu}_{\ell L,R}$ decays are shown in Fig. V.3. From these figures, process $p\bar{p} \rightarrow W^-_{L,R} + X \rightarrow \tilde{\chi}^-_j \tilde{\chi}^0_i + X$, note that the cross sections at 1.8 TeV are:

$$\sigma_L (p\bar{p} \rightarrow W^-_L + X \rightarrow \tilde{\chi}^-_j \tilde{\chi}^0_i + X) \simeq 0.11 \text{ nb},$$

$$\sigma_R (p\bar{p} \rightarrow W^-_R + X \rightarrow \tilde{\chi}^-_j \tilde{\chi}^0_i + X) \simeq 4.6 \text{ pb, and}$$

thus $\sigma_L \geq 24\ \sigma_R$ assuming the couplings g_L, g_R for the sub-groups $SU(2)_L$ and $SU(2)_R$ respectively, are equal. Using the same numerical integration procedure for the process:

$$p\bar{p} \rightarrow W^-_{L,R} + X \rightarrow \ell^- \bar{\nu}_{\ell L,R} + X, \text{ at } \sqrt{s}=0.63 \text{ TeV we get;}$$

$$\sigma_L (p\bar{p} \rightarrow W^-_L + X \rightarrow \ell^- \bar{\nu}_{\ell L} + X) \simeq 0.58 \text{ nb.}$$

which is in agreement with previous results.[24]

At $\sqrt{s}$=1.8 TeV we find:

$$\sigma_L (p\bar{p} \rightarrow W^-_L + X \rightarrow \ell^- \bar{\nu}_{\ell L} + X) \simeq 1 \text{ nb, and}$$

$$\sigma_R (p\bar{p} \rightarrow W^-_R + X \rightarrow \ell^- \bar{\nu}_{\ell R} + X) \simeq 37.7 \text{ pb.}$$

In Fig. V.4 we show predictions for the chargino transverse momentum from $W^-_{L,R} \rightarrow \tilde{\chi}^-_j \tilde{\chi}^0_i$ decay at $\sqrt{s}$=1.8 TeV at different values of centre of mass angle θ at which the chargino is produced relative to the $\bar{p}$-direction. Similar distributions for the direct decay of $W_{L,R} \rightarrow \ell^- \bar{\nu}_{\ell L,R}$ are shown in Fig. V.5. Both decays show Jacobian peaks for $p_T = \sqrt{\hat{s}}/2 \simeq M_{WL}/2 \simeq 40$ GeV at

$\theta=90^{\circ}$. Similarly for the right-handed gauge boson, M_{WR}, the curves peak at $p_T \geq 150$ GeV. For angle $\theta=10^{\circ}$ there are no peaks. Furthermore the chargino signature unlike the prompt-lepton background is symmetric under the Jacobian peak. In Figs. V.6 and V.7, we exhibit the dependence of the angular distribution of the chargino or lepton on the c.m. angle θ for $p_T \simeq M_{WL,R}/2$ (i.e., $\hat{\theta} \to \pi/2$). Note that at this value, the asymmetry almost disappears. In conclusion, we have shown that the chargino and neutralino production processes could give rise to a distinctive signal in the lepton distributions from $W^-_{L,R}$-boson production in $p\bar{p}$-collisions.

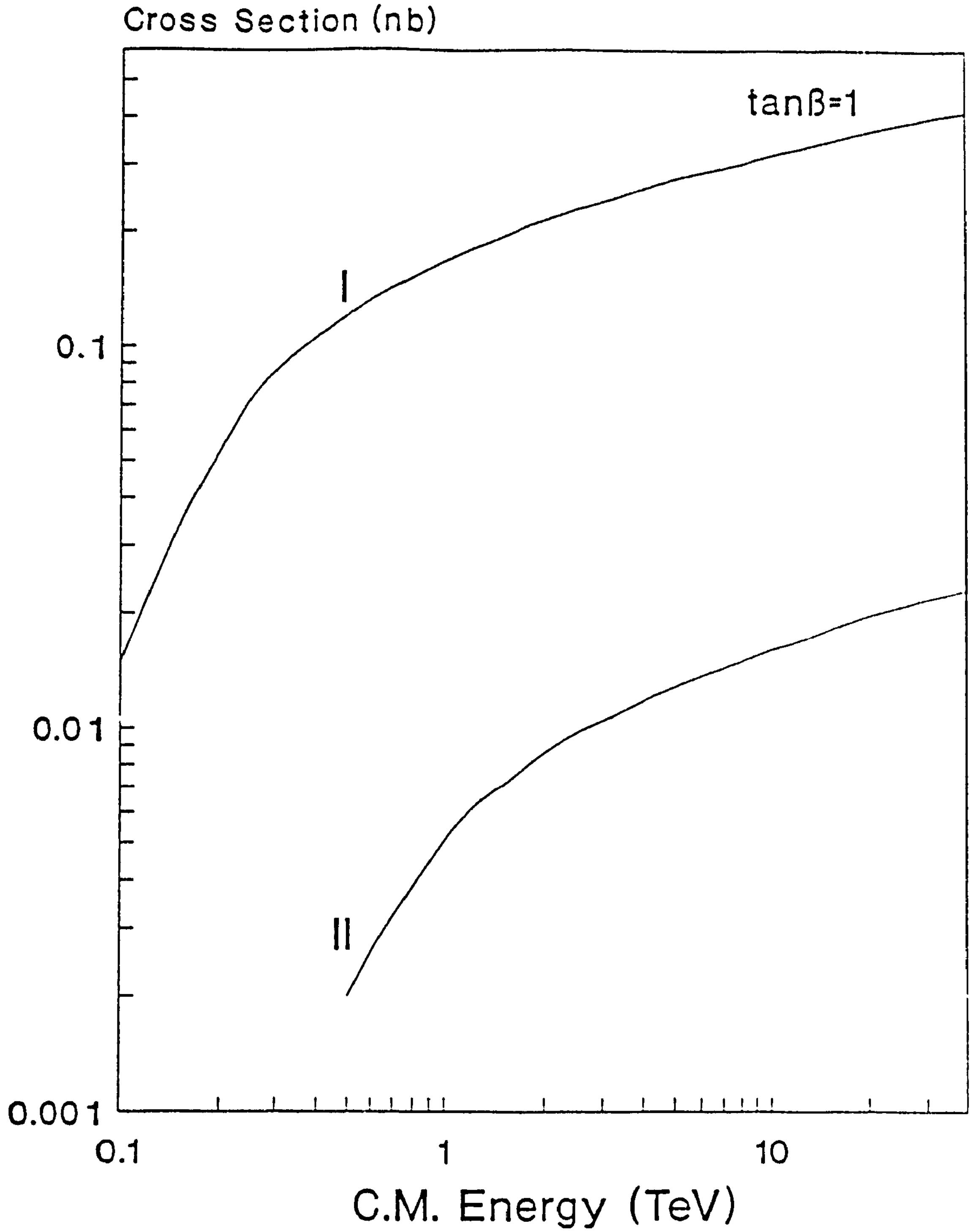

Fig. V.2: *Cross-sections of* $p\bar{p} \rightarrow W^-_{L,R} + X \rightarrow \tilde{\chi}^-_j \tilde{\chi}^0_1 + X$, *versus the centre of mass energy* $\sqrt{s}$. *We take* $\tan\beta=1$, $\tilde{M}_{\chi^0}=14$ *GeV, and* $\tilde{M}_{\chi^-}=45$ *GeV. The curves are:*
(I) for $\sigma_L(p\bar{p} \rightarrow W^-_L + X \rightarrow \tilde{\chi}^-_j \tilde{\chi}^0_1 + X)$ *and*
(II) for $\sigma_R(p\bar{p} \rightarrow W^-_R + X \rightarrow \tilde{\chi}^-_j \tilde{\chi}^0_1 + X)$.

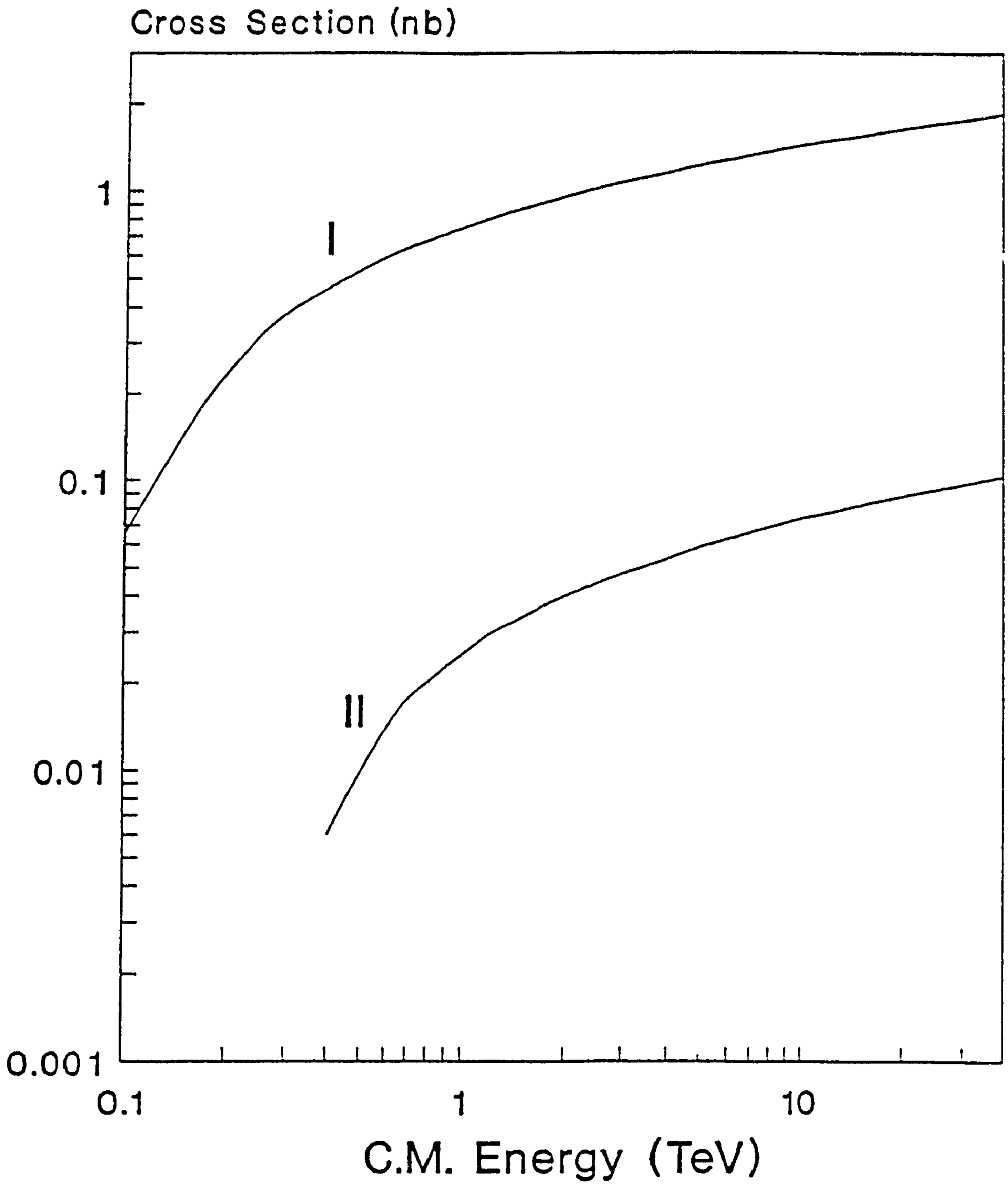

Fig. V.3: *Cross-sections of* $p\bar{p} \to W^-_{L,R} + X \to \ell^- \bar{\nu}_{\ell L,R} + X$, *versus the center of mass energy* $\sqrt{s}$.
The curves are:
(I) for $\sigma_L(p\bar{p} \to W^-_L + X \to \ell^- \bar{\nu}_{\ell L} + X)$ *and*
(II) for $\sigma_R(p\bar{p} \to W^-_R + X \to \ell^- \bar{\nu}_{\ell R} + X)$.

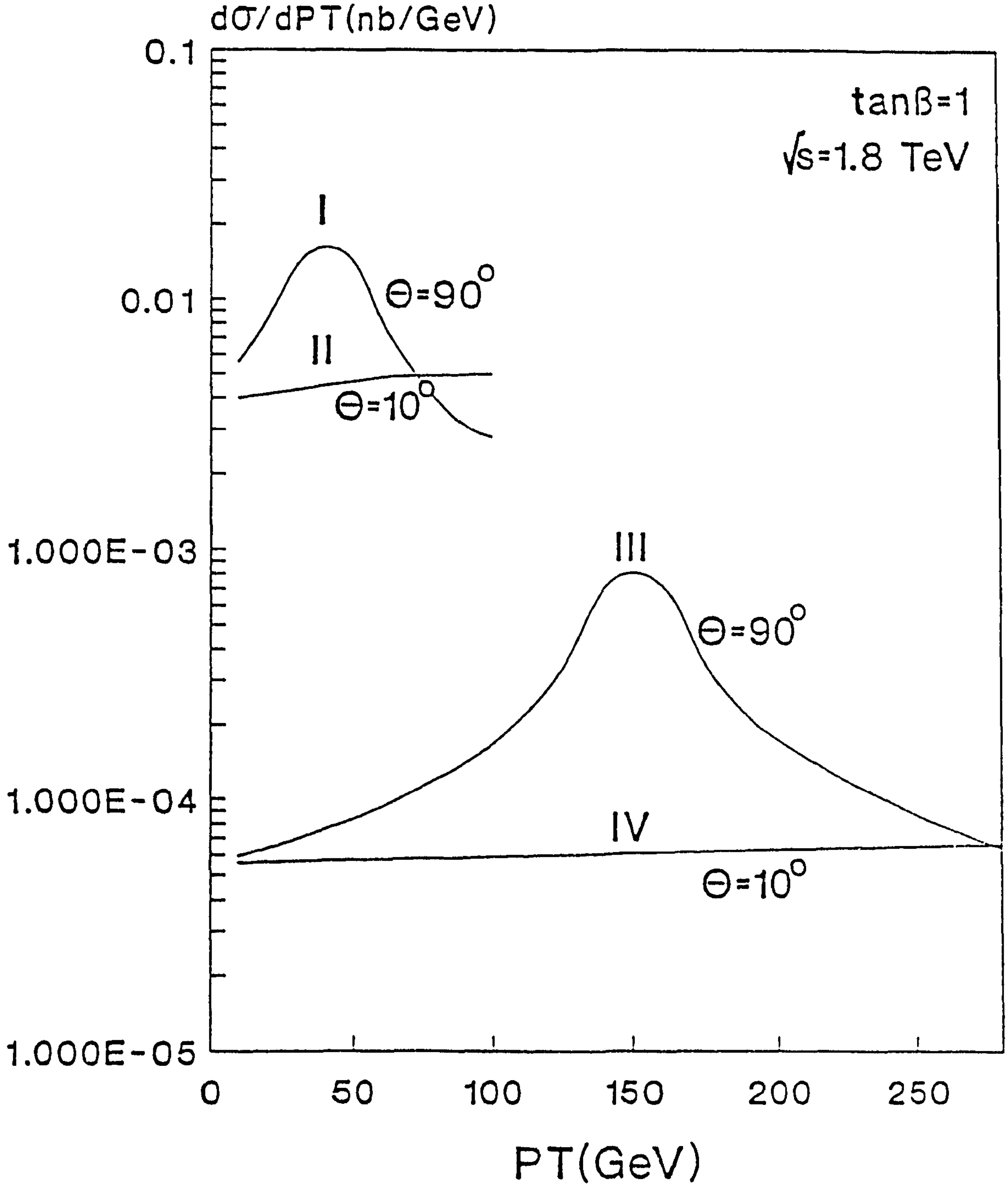

Fig. V.4: *The transverse momentum distribution of the chargino at $\sqrt{s}$=1.8 TeV for $\theta=90^\circ$, 10°. We take $\tan\beta=1$, $\tilde{M}_{\chi^0}$=14 GeV and $\tilde{M}_{\chi^-}$=45 GeV. The curves are:*
(I) and (II) $p\bar{p} \to W_L^- + X \to \tilde{\chi}_j^- \tilde{\chi}_1^0 + X$,
(II) and (II) $p\bar{p} \to W_R^- + X \to \tilde{\chi}_j^- \tilde{\chi}_1^0 + X$.

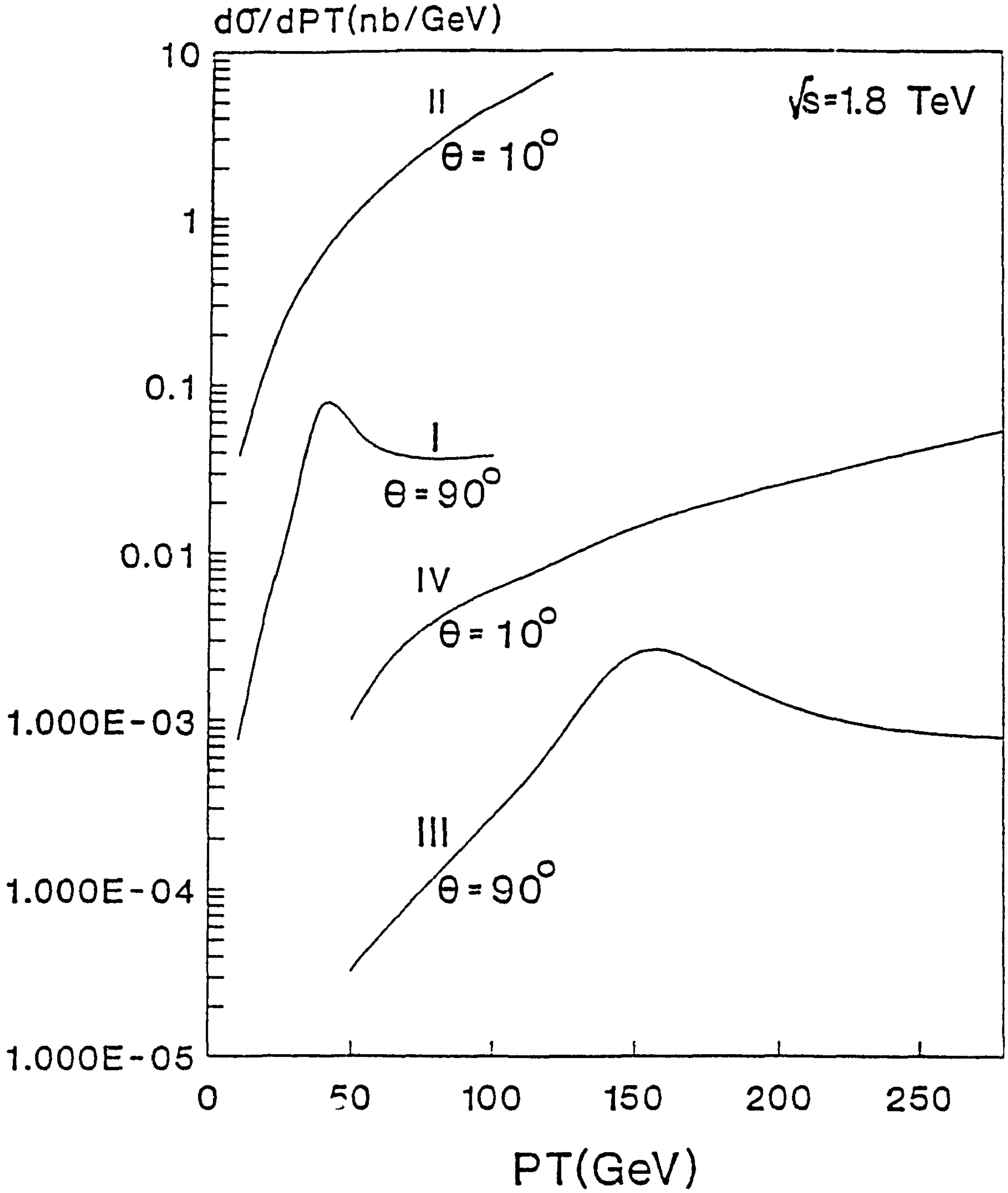

Fig. V.5: *The transverse momentum distribution of the lepton at $\sqrt{s}$=1.8 TeV for θ=90°, 10°. The curves are: (I) and (II) for* $p\bar{p} \to W_L^- + X \to \ell^-\bar{\nu}_{\ell L} + X$, *(III) and (IV) for* $p\bar{p} \to W_R^- + X \to \ell^-\bar{\nu}_{\ell R} + X$.

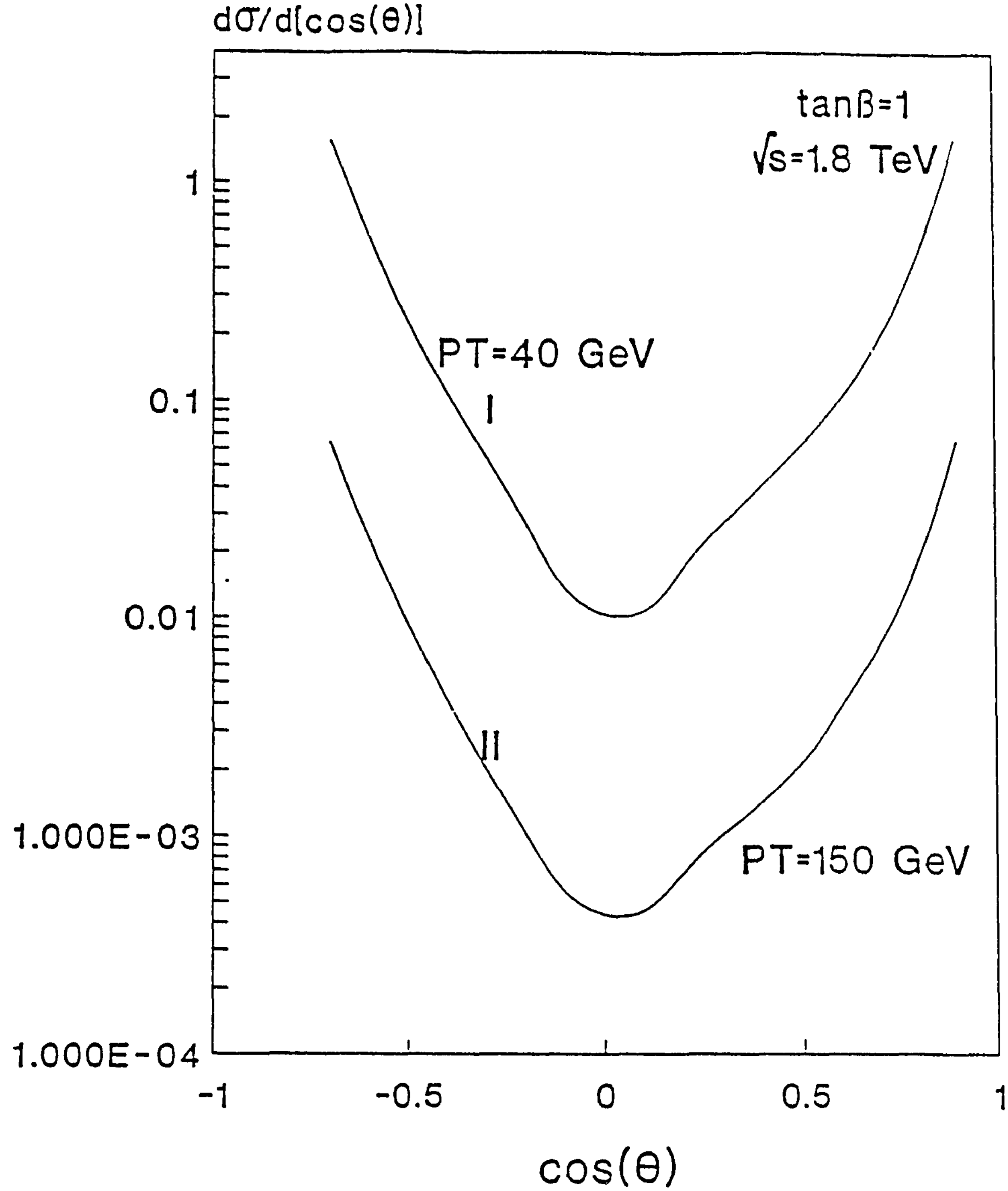

Fig. V.6: *The angular distributions of the emitted* $\tilde{\chi}_j^-$ *at* $\sqrt{s}$*=1.8 TeV. We take* $\tan\beta=1$, $\tilde{M}_{\chi^0}=14$ *GeV and* $\tilde{M}_{\chi^-}=45$ *GeV. The curves are:*
(I) for $p\bar{p} \rightarrow W_L^- + X \rightarrow \tilde{\chi}_j^- \tilde{\chi}_1^0 + X$ $(p_T \simeq 40$ *GeV) and*
(II) for $p\bar{p} \rightarrow W_R^- + X \rightarrow \tilde{\chi}_j^- \tilde{\chi}_1^0 + X$ $(p_T \simeq 150$ *GeV).*

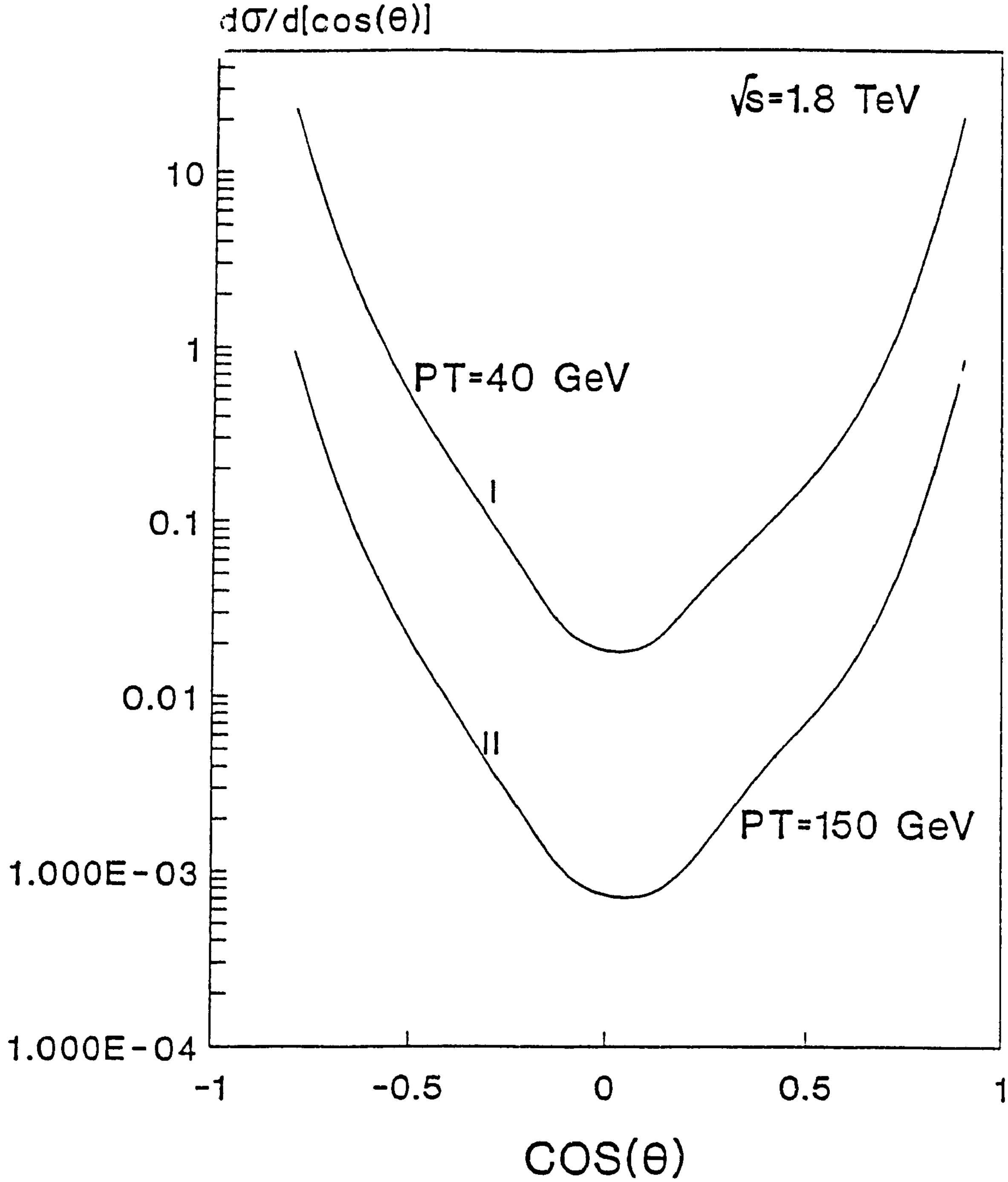

Fig. V.7: *The angular distributions of the emitted ℓ^- at $\sqrt{s}$=1.8 TeV. The curves are:*
(I) for $p\bar{p} \to W_L^- + X \to \ell^-\bar{\nu}_{\ell L} + X$ ($p_T \simeq 40$ GeV) and
(II) for $p\bar{p} \to W_R^- + X \to \ell^-\bar{\nu}_{\ell R} + X$ ($p_T \simeq 150$ GeV).

References

[1] CDF collaboration, F. Abe *et al.*, *Phys. Rev. Lett.* 62 (1989) 1825.

[2] UA2 collaboration, J. Alitti *et al.*, *Phys. Lett.* B235 (1990) 363.

[3] H. Baer *et al.*, in *Research Directions of the Decade*, Proceedings of Workshop, Snowmass, Colorado, 1990; H. Baer, X.Tata, and J.Woodside, *Phys. Rev.* D44, 1991, 207.

[4] H.E. Haber and G.L. Kane, *Phys. Rep.* **117** (1985) 75; R. Barbieri, *Riv. Nuovo Cim.* 11 (1988) 1.

[5] R.M. Barnett, K.S. Lackner, and H.E. Haber, *Phys. Rev. Lett.* **51** (1983) 176; R.M. Barnett, H.E. Haber, and K.S. Lackner, *Phys. Rev.* **D29** (1984) 1381; R.M. Barnett and H. E. Haber, *Phys. Rev.* **D31** (1985) 85.

[6] B. Grinstein, J. Polchinski, and M.B. Wise, *Phys. Lett.* **B130** (1983) 2515;J. Ellis, J.S. Hagelin, D.V. Nanopoulos and M. Srednicki, *Phys. Lett.* **B127** (1983) 233; A.H. Chamseddine, P. Nath, P. Arnowitt, *Phys. Lett.* **B129** (1983) 445.

[7] P. Salati and J.C. Wallet, *Phys. Lett.* **122B** (1983) 397; K. Grassie and P.N. Pandita, *Phys. Rev.* **D30** (1984) 22; A. Bartl, H. Fraas, W. Majerotto, Z. *Phys.* **C30**, (1986) 441; *Nucl. Phys.* **B278** (1986) 1; H. Baer and K. Hagiwara, and X. Tata, *Phys. Rev.* **D35** (1987) 1598. M. Drees, C.S. Kim, X. Tata, *Phys. Rev.* **D37** (1988) 784.

[8] J. Ellis and G. Ridolfi and F. Zwirner, *Phys. Lett.* 237B (1990) 423.

[9] V.D. Barger and R.J.N. Phillips, *Collider Physics*, ch.8, Addison-Wesley Pub., California, 1987.

[10] B.De Wit and J.Smith, *Field Theory in Particle Physics*, ch.3, vol.1, North-Holland Phys. Pub., Amsterdam, 1986.

[11] For review, see for example; W. Marciano and H. Pagels, *Phys. Rep.* **36C** (1978) 137; A.H. Mueller, *Phys. Rep.* **73C**, (1981) 237; G. Altarelli, *Phys. Rep.* **81** (1982) 1; J.F. Owens, *Rev. Mod. Phys.* **59** (1987) 465.

[12] See for instance, Ref. [9] sec. 7.11; D.W. Duke and R.G. Roberts, *Phys. Rep.* **120**, (1985) 275; H. Bear and M.H. Reno, *Phys. Rev.* **D43** (1991) 2892.

[13] F. Halzen, A.D. Martin and D.M. Scott, *Phys. ReV.* D25 (1982) 754; R. Barbieri, N. Cabibbo, L. Maiani and S. Petrarca, *Phys. Lett.* **B127** (1983) 458.

[14] S. Gottlieb and T.J. Weiler, *PHys. Rev.* **D29** (1984) 2005; *ibid* **D32** (1985) 1119.

[15] For the quark-structrure functions parametrization, see for example: Ref. [9], ch.5; and J.F. Owens and E. Reya, *Phys. Rev.* **D17** (1978) 3003; D.W. Duke and J.F. Owens, *Phys. Rev.* **D30** (1984) 49.

[16] L3 collaboration, B. Adeva et al., *Phys. Lett.* **B233** (1989) 530.

[17] ALEPH collaboration, D. Decamp et al., *Phys. Lett.* **B236** (1990) 86.

[18] OPAL collaboration, M.Z. Akrawy et al., *Phys. Lett.* **B240** (1990) 261.

[19] ALEPH collaboration, D. Decamp et al., LAL Preprint

LAL 90-26(1990); J.F. Grivaz, First International Symp. on, *Particles, Strings, and Cosmology*, Boston, 1990 (PASCOS-90).

[20] L. Roszkowski, *Phys. Lett.* **B252** (1990) 471.

[21] For recent measurements of M_W see, F. Abe et al., *Phys. Rev.* **D43** (1991) 2070.

[22] W. Keung and G. Senjanovic, *Phys. Rev. Lett.* **50** (1983) 1427.

[23] J.L. Rosner and E. Takasugi, *Phys. Rev.* **D42** (1990) 241.

[24] V. Barger et al., *Phys. Rev.* **D29** (1984) 2020.

Chapter VI

Conclusions and Prospects

In this thesis, we consider the L-R Supersymmetric Standard Model, the extension of the Minimal Supersymmetric Model (MSSM), based on the gauge groups $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ with doublet and triplet Higgs fields. In the L-R model, and in the MSSM, gauginos and higgsinos are spin-1/2 weakly interacting particles and so they mix once $SU(2)_L \times U(1)_Y$ is broken. After mixing, in the L-R supersymmetric model, there are four charginos $\tilde{\chi}^{\pm}_j$ (j=1,...,4) and seven neutralinos $\tilde{\chi}_i$ (i=1,...,7). The first three neutralinos are generated by the first stage of breaking after the right-handed Higgs acquires a vacuum expectation values, $\langle\Delta_R\rangle$ (However, one of them remains massless). The other four neutralinos are generated by the second stage of breaking after the doublet Higgs acquire VEV's, $\langle\phi\rangle_{u,d}$.

We examined in detail analytical and numerical solutions for the masses for some particular values of the left and right parameters (M_V, M_L, M_R, and $\tan\theta_K$). The results in the MSSM are in agreement with previous results. In the L-R supersymmetric model, the mass spectra for charginos and neutralinos is very different from that found in the MSSM.

We also consider ways of finding evidence for the left-right supersymmetric model in $p\bar{p} \to W^-_{L,R} + X \to \tilde{\chi}^0_i\tilde{\chi}^-_j + X$. The total *inclusive* cross-sections are solved by numerical integration using parton model distributions for $\sqrt{s}$=1.8 TeV. We use tanβ=1, and based upon the weakest limit from Δm_K and astrophysics M_{WR} of about 300 GeV, we find

$$\sigma_L(\bar{p}p \rightarrow W_L^- + X \rightarrow \tilde{\chi}_j^- \tilde{\chi}_i^0 + X) \simeq 0.11 \text{ nb, and}$$

$$\sigma_R(\bar{p}p \rightarrow W_R^- + X \rightarrow \tilde{\chi}_j^- \tilde{\chi}_i^0 + X) \simeq 4.6 \text{ pb,}$$

Thus $\sigma_L \geq 24\sigma_R$, assuming the couplings g_L, g_R for the subgroups $SU(2)_L$ and $SU(2)_R$ respectively, are equal.

Comparing our results with the lepton background signal for the process $p\bar{p} \rightarrow W_{L,R}^- + X \rightarrow \ell^- \bar{\nu}_{\ell L,R} + X$, we find at $\sqrt{s}$=1.8 TeV,

$$\sigma_L(p\bar{p} \rightarrow W_L^- + X \rightarrow \ell^- \bar{\nu}_{\ell L} + X) \simeq 1 \text{ nb, and}$$

$$\sigma_R(p\bar{p} \rightarrow W_R^- + X \rightarrow \ell^- \bar{\nu}_{\ell R} + X) \simeq 37.7 \text{ pb.}$$

Both decays show Jacobian peaks for $p_T = \sqrt{\hat{s}}/2$ $M_{WL}/2 \sim 40$ GeV ($p_T \simeq 150$ GeV for M_{WR}) at $\theta=90^\circ$, whereas the decays have no peaks for angle $\theta=10^\circ$. Furthermore the chargino signature unlike the prompt—lepton background is symmetric under the Jacobian peak.

Further investigations that should be considered are increasi..g the c.m. energy from CDF (~1.8 TeV) to SSC (40 GeV) and consider the pair production of $\tilde{\chi}_i^0 \tilde{\chi}_j^0$ and $\tilde{\chi}_i^- \tilde{\chi}_j^+$ via neutral current interactions ($Z_{L,R}^0$-exchange) in $p\bar{p}$- and pp-collisions. To this end, one can use the analytical expressions (asymptotic results) for chargino and neutralino mass eigenstates to consider the properties of the Higgs particles in the L-R SUSY through the decays: $\tilde{\chi}_i^+ \rightarrow \tilde{\chi}_j^0 + W^+$; $\tilde{\chi}_i^0 \rightarrow \tilde{\chi}_j^0 + Z^0$; $\tilde{\chi}_i^\pm \rightarrow \tilde{\chi}_j^0 + \phi^\pm$; $\tilde{\chi}_i^0 \rightarrow \tilde{\chi}_j^0 + \phi^0$ (i>j).

Appendix A

Conventions

Most of the conventions in this thesis are taken from Ref. [1], and the rest are taken from Ref. [2]. We write our metric to be:

$$g_{\mu\nu} = \mathrm{diag}(1,-1,-1,-1). \tag{A.1}$$

The momentum four vector is $p^{\mu} = (E, p)$.

$$\sigma^{\mu} = (1,\vec{\sigma}) \ ; \ \bar{\sigma}^{\mu} = (1,-\vec{\sigma}) \tag{A.2}$$

denotes the Pauli matrices. Spinors can be written in two- or four-component notation. If the two-component spinor ξ_{α} transform under a matrix M, the spinors $\bar{\xi}_{\alpha}$, ξ^{α}, and $\bar{\xi}^{\dot{\alpha}}$ transform under M^{*}, M^{-1} and $(M^{-1})^{*}$ respectively. The Dirac equation in two-component notation is:

$$(\bar{\sigma}_{\mu}p^{\mu})^{\dot{\alpha}\beta}\ \xi_{\beta} = m\bar{\eta}^{\dot{\alpha}}, \quad (\sigma_{\mu}p^{\mu})_{\alpha\dot{\beta}}\ \bar{\eta}^{\dot{\beta}} = m\xi_{\alpha} \tag{A.3}$$

This allows us to introduce four-component notation. One introduces a four-component spinor which satisfies

$$(\gamma_{\mu}p^{\mu} - m)\psi = 0. \tag{A.4}$$

It follows that

$$\psi = \begin{pmatrix} \xi_\alpha \\ \bar{\eta}^{\dot{\alpha}} \end{pmatrix}, \quad \gamma_\mu = \begin{pmatrix} 0 & \sigma_{\mu\nu\dot{\beta}} \\ {}_\mu\bar{\sigma}^{\dot{\alpha}\beta} & 0 \end{pmatrix}, \tag{A.5}$$

$$\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \tag{A.6}$$

$$\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu] = 2i\begin{pmatrix} \sigma^{\mu\nu\beta}_{\alpha} & 0 \\ 0 & \bar{\sigma}^{\mu\nu\dot{\alpha}}_{\dot{\beta}} \end{pmatrix}, \tag{A.7}$$

where

$$\sigma^{\mu\nu\beta}_{\alpha} = \frac{1}{4}(\sigma^\mu_{\alpha\dot{\alpha}}\,\bar{\sigma}^{\nu\dot{\alpha}\beta} - \sigma^\nu_{\alpha\dot{\alpha}}\,\bar{\sigma}^{\mu\dot{\alpha}\beta}), \tag{A.8}$$

$$\bar{\sigma}^{\mu\nu\dot{\alpha}}_{\dot{\beta}} = \frac{1}{4}(\bar{\sigma}^{\mu\dot{\alpha}\alpha}\sigma^\nu_{\alpha\dot{\beta}} - \bar{\sigma}^{\nu\dot{\alpha}\alpha}\sigma^\mu_{\alpha\dot{\beta}}). \tag{A.9}$$

This is called the chiral representation of the γ-matrices. We define the four-component spinors:

$$\psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}, \tag{A.10}$$

where $\psi_{L,R} = P_{L,R}\psi$, and the left- and right-handed projection operators are given by

$$P_{L,R} = \frac{1}{2}(1 \mp \gamma_5). \tag{A.11}$$

The charge conjugated spinor can be written in terms of the charge conjugation operator C as:

$$\psi^C = C\bar{\psi}^T \ ; \ C = -i\gamma^2\gamma^0. \tag{A.12}$$

In two-component notation, one defines an antisymmetric tensor ($\varepsilon^{\alpha\beta} = -\varepsilon^{\beta\alpha}$):

$$\varepsilon^{\alpha\beta} = -\varepsilon^{\beta\alpha} = i\sigma^2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \tag{A.13}$$

The $\varepsilon_{\alpha\beta}$ tensor can be used to raise and lower spinor indices:

$$\xi^\alpha = \varepsilon^{\alpha\beta}\xi_\beta, \quad \xi_\beta = \varepsilon_{\beta\alpha}\xi^\alpha. \tag{A.14}$$

Then, in chiral representation we have:

$$C = -i\gamma^2\gamma^0 = \begin{pmatrix} \varepsilon_{\beta\alpha} & 0 \\ 0 & \varepsilon^{\dot{\beta}\dot{\alpha}} \end{pmatrix}, \tag{A.15}$$

and

$$\bar{\psi}^T = \begin{pmatrix} \eta^\alpha \\ \bar{\xi}_{\dot{\alpha}} \end{pmatrix}, \quad \psi^c = \begin{pmatrix} \eta_\beta \\ \bar{\xi}^{\dot{\beta}} \end{pmatrix}. \tag{A.16}$$

A four-component Majorana spinor has the property that $\eta = \xi$, which implies that $\psi^c = \psi$;

$$\psi_M = \begin{pmatrix} \xi_\alpha \\ \bar{\xi}^{\dot{\alpha}} \end{pmatrix}. \tag{A.17}$$

The following equations are translated from two-components into four-components:

$$\bar{\psi}_1\psi_2 = \eta_1\xi_2 + \bar{\eta}_2\xi_1, \quad \bar{\psi}_1\gamma_5\psi_2 = -\eta_1\xi_2 + \bar{\eta}_2\xi_1,$$

$$\bar{\psi}_1\gamma^{\mu}\psi_2 = \bar{\xi}_1\bar{\sigma}^{\mu}\xi_2 - \bar{\eta}_2\bar{\sigma}^{\mu}\eta_1, \quad \bar{\psi}_1\gamma^{\mu}\gamma_5\psi_2 = -\bar{\xi}_1\bar{\sigma}^{\mu}\xi_2 - \bar{\eta}_2\bar{\sigma}^{\mu}\eta_1,$$

$$\frac{1}{2}\, i\bar{\psi}_1\sigma^{\mu\nu}\psi_2 = \eta_1\sigma^{\mu\nu}\xi_2 - \bar{\eta}_2\sigma^{\mu\nu}\bar{\xi}_1. \tag{A.18}$$

where the subscripts 1 and 2 label two different four-component spinors. In eqs.(A.18), we have used:

$$\eta\xi = \xi\eta, \quad \bar{\eta}\bar{\xi} = \bar{\xi}\bar{\eta}, \quad \bar{\eta}_2\bar{\sigma}^{\mu}\eta_1 = -\eta_1\bar{\sigma}^{\mu}\bar{\eta}_2,$$

$$\bar{\eta}\,\bar{\sigma}^{\mu\nu}\bar{\xi} = -\bar{\xi}\bar{\sigma}^{\mu\nu}\bar{\eta}, \quad \eta\sigma^{\mu\nu}\xi = -\xi\sigma^{\mu\nu}\eta, \tag{A.19}$$

and

$$\bar{\psi}_1 P_L\psi_2 = \eta_1\xi_2, \quad \bar{\psi}_1 P_R\psi_2 = \bar{\eta}_2\bar{\xi}_1, \quad \bar{\psi}_1\gamma^{\mu}P_L\psi_2 = \bar{\xi}_1\bar{\sigma}^{\mu}\xi_2,$$

$$\bar{\psi}_1\gamma^{\mu}P_R\psi_2 = -\bar{\eta}_2\bar{\sigma}^{\mu}\eta_1. \tag{A.20}$$

Finally, we write some useful relationsfor four-component Majorana spinors using eqs.(A.15) and (A.16);

$$\bar{\psi}_1\psi_2 = \bar{\psi}_2\psi_1, \quad \bar{\psi}_1\gamma_5\psi_2 = \bar{\psi}_2\gamma_5\psi_1, \quad \bar{\psi}_1\gamma_{\mu}\psi_2 = -\bar{\psi}_2\gamma_{\mu}\psi_1,$$

$$\bar{\psi}_1\gamma_{\mu}\gamma_5\psi_2 = \bar{\psi}_2\gamma_{\mu}\gamma_5\psi_1, \quad \bar{\psi}_1\sigma_{\mu\nu}\psi_2 = -\bar{\psi}_2\sigma_{\mu\nu}\psi_1, \tag{A.21}$$

and

$$\bar{\psi}_1\gamma_{\mu}P_L\psi_2 = -\bar{\psi}_2\gamma_{\mu}P_R\psi_1. \tag{A.22}$$

Appendix B

Chargino and Neutralino Mixing in Two Component Notation

B.1. Chargino mixing.[1,3]

The mixing of the charginos, $\tilde{\chi}^{\pm}_{i}$ (i=1,2) has been discussed in chapter three in the terms of four-component Dirac fermions. In this section the mixing process is put in terms of two-component notation. We define two unitary mixing matrices:

$$\begin{aligned} \psi^{+}_{j} &= (-i\lambda^{+}, \psi^{+}_{H2}), \\ \psi^{-}_{j} &= (-i\lambda^{-}, \psi^{-}_{H1}), \end{aligned} \qquad j=1,2 \tag{B.1}$$

with $\lambda^{\pm}=\frac{1}{\sqrt{2}}(\lambda^{1} \mp i\lambda^{2})$. The mass term in the Lagrangian is:

$$\mathcal{L}_{m} = -\frac{1}{2}(\psi^{+}\psi^{-}) \begin{pmatrix} 0 & X^{T} \\ X & 0 \end{pmatrix} \begin{pmatrix} \psi^{+} \\ \psi^{-} \end{pmatrix}, \tag{B.2}$$

where

$$X= \begin{pmatrix} M & M_{W}\sqrt{2}\ \sin\beta \\ M_{W}\sqrt{2}\ \cos\beta & \mu \end{pmatrix}. \tag{B.3}$$

Here M is the gaugino mass parameter associated with the gauge group SU(2) of the standard model, μ is a supersymmetric Higgs mass parameter, $\tan\beta=\upsilon_{2}/\upsilon_{1}$, and

$M_W = \frac{1}{2}g(\upsilon_1^2 + \upsilon_2^2)^{1/2}$. Finally υ_i are the vacuum expectation values of the Higgs doublets;

$$< H_1 > = \begin{pmatrix} \upsilon_1 \\ 0 \end{pmatrix}, \quad <H_2>= \begin{pmatrix} 0 \\ \upsilon_2 \end{pmatrix}. \tag{B.4}$$

Let the mass eigenstates be defined as

$$\chi_i^+ = V_{ij}\psi_j^+ , \qquad \chi_i^- = U_{ij}\psi_j^- , \ (i,j=1,2) \tag{B.5}$$

where U and V are unitary matrices chosen such that

$$M_D = U^* X V^{-1}, \tag{B.6}$$

where M_D is the diagonal mass matrix with real and non-negative entries. Eq.(B.2) can be written as,

$$- [\chi_i (M_D)_{ij} \chi_j^+ + \text{H.C.}] \tag{B.7}$$

Using eq.(A.16), we may write eq.(B.7) in four-component as,

$$- [\tilde{M}_+ \bar{\tilde{\chi}}_1 \tilde{\chi}_1 + \tilde{M}_- \bar{\tilde{\chi}}_2 \tilde{\chi}_2], \tag{B.8}$$

where $\tilde{\chi}_1$ and $\tilde{\chi}_2$ are the charged four-component Dirac spinor

$$\tilde{\chi}_1 = \begin{pmatrix} \chi_1^+ \\ \bar{\chi}_1^- \end{pmatrix}, \quad \tilde{\chi}_2 = \begin{pmatrix} \chi_2^+ \\ \bar{\chi}_2^- \end{pmatrix}. \tag{B.9}$$

It is useful to consider the eigenvalue problem ɔr $X^{\dagger}X$. The positive square roots of the eigenvalues of $X^{\dagger}X$ will diagonalize M_D elements. Thus, eq. (B.6) becomes:

$$M_D^2 = V\, X^{\dagger}XV^{-1} = U^{*}XX^{\dagger}(U^{*})^{-1}. \tag{B.10}$$

The exact solution for the chargino masses for the 2×2 matrix can be found. From (B.9) we find the masses,

$$\tilde{M}^2_{\chi 1,\chi 2} = \frac{1}{2}\left\{M^2 + \mu^2 + 2M_W^2 \mp \left[(M^2 - \mu^2)^2 + 4M_W^4\cos 2\beta + \right.\right.$$

$$\left.\left. + 4M_W^2(M^2 + \mu^2 + 2M\mu\sin 2\beta)\right]\right\} \tag{B.11}$$

$$U = O_- \;, \quad V = \begin{cases} O_+ & ,\ \det X \geq 0 \\ \sigma_3 O_+ & ,\ \det X < 0 \end{cases} \;, \quad O_\pm = \begin{pmatrix} \cos\phi_\pm & \sin\phi_\pm \\ -\sin\phi_\pm & \cos\phi_\pm \end{pmatrix}. \tag{B.12}$$

where the Pauli matrix σ_3 is inserted when det $X<0$, so that the masses $M^2_{\chi 1,\chi 2}$ which appear as diagonal elements of M_D are both positive. In the supersymmetric limit $\mu=0$ and $\upsilon_1=\upsilon_2$ ($\beta=45^{\circ}$). Then $\phi_+=\phi_-=\phi$ and the physical eigenstates are the Dirac spinors

$$\tilde{\chi}_1 = \begin{pmatrix} -i\lambda^{+}\cos\phi + \psi^{1}_{H2}\sin\phi \\ i\bar{\lambda}^{-}\cos\phi + \bar{\psi}^{2}_{H1}\sin\phi \end{pmatrix} \text{ with mass} = \tilde{M}_{\chi 1}, \text{ and} \tag{B.13}$$

$$\tilde{\chi}_2 = \begin{pmatrix} -i\lambda^+\sin\phi - \psi^1_{H2}\cos\phi \\ -i\bar{\lambda}^-\sin\phi + \bar{\psi}^2_{H1}\cos\phi \end{pmatrix} \text{ with mass} = \tilde{M}_{\lambda 2}. \tag{B.14}$$

In the limit $M=\mu=0$ ($\phi=45^\circ$) we have two degenerate states with mass M_W:

$$\tilde{\omega}^-_1 = \frac{1}{\sqrt{2}}(\tilde{\chi}^C_2 + \tilde{\chi}^C_1), \quad \tilde{\omega}^+_2 = \frac{1}{\sqrt{2}}(\tilde{\chi}_2 - \tilde{\chi}_1), \tag{B.15}$$

where C is the charge conjugation operator given in eq.(A.15), and $\tilde{\omega}^-_1$, $\tilde{\omega}^+_2$ are four-component Dirac fermions given by:

$$\tilde{\omega}^-_1 = \begin{pmatrix} \psi^2_{H1} \\ i\bar{\lambda}^+ \end{pmatrix}, \quad \tilde{\omega}^-_1 = \begin{pmatrix} \psi^1_{H2} \\ i\bar{\lambda}^- \end{pmatrix}. \tag{B.16}$$

B.2. Neutralino mixing.[1,3]

In the case with four neutralinos, $\tilde{\chi}^0_i$ ($i=1,\ldots,4$), we define the two-component fermion fields

$$\psi^0_j = (-i\lambda', -i\lambda^3, \psi^0_{H1}, \psi^0_{H2}), \tag{B.17}$$

where λ^3 is the neutral wino and λ' is the bino. These fields can also be expressed in terms of two-component notation given by linear combination of the photino and zino:

$$\lambda_Z = \lambda^3\cos\theta_W - \lambda'\sin\theta_W, \tag{B.18a}$$

$$\lambda_\gamma = \lambda^3\sin\theta_W + \lambda'\cos\theta_W. \tag{B.18b}$$

So, in place of eq.(B.14), it will be usful to define

$$\psi'^{o}_{j}=(-\ i\lambda_{\gamma},-i\lambda_{Z},\psi^{0}_{H1},\psi^{0}_{H2}). \tag{B.19}$$

The mass term in the Lagrangian is given by

$$\mathcal{L}_{m}= -\ \frac{1}{2}(\psi^{0})^{T}Y\psi^{0} + \text{H.C.}, \tag{B.20}$$

where Y is in general a complex *symmetric matrix* given by

$$Y= \begin{pmatrix} M' & 0 & -M_{Z}\sin\theta_{W}\cos\beta & M_{Z}\sin\theta_{W}\sin\beta \\ 0 & M & M_{Z}\cos\theta_{W}\cos\beta & -M_{Z}\cos\theta_{W}\sin\beta \\ -M_{Z}\sin\theta_{W}\cos\beta & M_{Z}\cos\theta_{W}\cos\beta & 0 & -\mu \\ M_{Z}\sin\theta_{W}\sin\beta & -M_{Z}\cos\theta_{W}\sin\beta & -\mu & 0 \end{pmatrix}, \tag{B.21}$$

where $M_{Z}= \frac{1}{2}[(g^{2}\ +\ g'^{2})(\upsilon_{1}^{2}\ +\ \upsilon_{2}^{2})]^{1/2}$ and θ_{W} is the conventional Wienberg angle. We define two-component mass eigenstates

$$\chi^{0}_{i}= N_{ij}\psi^{0}_{j}\ , \qquad i,j=1,\ldots,4. \tag{B.22}$$

where N is a unitary matrices satisfying

$$N_{D}= N^{*}YN^{-1}, \tag{B.23}$$

where N_{D} is the diagonal neutralino mass matrix. To determine N, it is easiest to square eq.(B.20) obtaining

$$N_D^2 = NY^\dagger Y N^{-1}. \tag{B.24}$$

One can choose N such that the elements of the diagonal matrix N_D are real and non-negative. The four-component mass eigenstates are the neutralinos which are defined in terms of the two-component $\tilde{\chi}_i^0$ fields by

$$\tilde{\chi}_i^0 = \begin{pmatrix} \chi_i^0 \\ \bar{\chi}_i^0 \end{pmatrix}, \; (i = 1, \ldots, 4). \tag{B.25}$$

Note that the $\tilde{\chi}_i^0$ are Majorana fermions. The mass term becomes

$$\frac{1}{2} \sum_i \tilde{M}_i \bar{\tilde{\chi}}_i^0 \tilde{\chi}_i^0, \tag{B.26}$$

where $\tilde{M}_i$ are the diagonal elements of N_D. The special case $M=M'$ and $\mu=0$ corresponds to a massive photino with mass $\tilde{M}$ which decouples from the other fermions.

Appendix C

Integrated Cross-Section for Chargino and Neutralino Production

C.1. Parton model distributions.

C.1.1. Cross-section for subprocess.[4,5]

Consider the first order production of the chargino and neutralino in $W^-_{L,R}$-decay for the $\bar{q}q'$-collisions. The process in terms of the momenta labels is:

$$\bar{q}(p_1) + q'(p_2) \to W^-_{L,R} \to \tilde{\chi}^-_j(k_1) + \tilde{\chi}^0_i(k_2). \tag{C.1}$$

The amplitude of this interaction at $g \approx g_L \approx g_R$ is:

$$\mathcal{M} = \frac{ig}{4}\left(V_{q,q'}\right)\bar{\upsilon}(k_2)\gamma_\mu\left[O^L_{ij}(1-\gamma_5) + O^R_{ij}(1+\gamma_5)\right]\upsilon(k_1)$$
$$\times\left(\frac{-ig^{\mu\nu}}{q^2 - M_W{}^2 + i\Gamma_W M_W}\right)_{L,R}\bar{u}(p_2)\,\frac{-ig}{2\sqrt{2}}\,\gamma_\nu(1-\gamma_5)u(p_1) \tag{C.2}$$

where $\bar{\upsilon}(k_2)$, $\upsilon(k_1)$, $\bar{u}(p_2)$, and $u(p_1)$ are respectively, the Dirac spinors of the neutralino, chargino, antiquark, and quark, $V_{q,q'}$ is the quark mixing matrix, and

$$O^L_{ij} = -N_{i4}V^*_{j2} + N_{i2}V^*_{j1}, \quad O^R_{ij} = -N^*_{i3}U_{j2} + N_{i2}U_{j1}.$$

where N_{ij} are unitary matrices which diagonalize the neutral fermion states. The vertex of W_R^--boson-lepton-lepton interactions can be found from the Lagrangian given in chapter II:

$$-\frac{g_R}{\sqrt{2}}\,[\bar{\nu}\gamma^{\mu}P_R e\; W_{\mu}^{-} + \bar{e}\gamma^{\mu}P_R\nu\; W_{\mu}^{+}] \tag{C.3}$$

where we have made use of eq.(A.15). In the Feynman gauge, the $W_{L,R}^-$-boson propagator is given by

$$\left(\frac{-ig^{\mu\nu}}{q^2 - M^2 + i\Gamma_W M_W}\right)_{L,R}, \tag{C.4}$$

where q is the four-vector momentum of the gauge boson. The vertex W_R^--boson-chargino-neutralino has been already discussed in chapter II. Consider the Mandelstam variables $\hat{s}$, $\hat{t}$, and $\hat{u}$ defined by

$$\left.\begin{aligned} q^2 = \hat{s} &= (p_1 + p_2)^2 = (k_1 + k_2)^2, \\ \hat{t} &= (p_1 - k_1)^2 = (p_2 - k_2)^2, \\ \hat{u} &= (p_1 - k_2)^2 = (p_2 - k_1)^2. \end{aligned}\right\}. \tag{C.5}$$

Note that

$$\left.\begin{aligned} \hat{s} &\simeq 2p_1.p_2 = \tilde{M}_{\chi^0}^2 + \tilde{M}_{\chi^-}^2 - 2k_1.k_2 \\ \hat{t} &\simeq \tilde{M}_{\chi^-}^2 - 2p_1.k_1 \simeq \tilde{M}_{\chi^0}^2 - 2p_2.k_2 \\ \hat{u} &\simeq \tilde{M}_{\chi^-}^2 - 2p_2.k_1 \simeq \tilde{M}_{\chi^0}^2 - 2p_1.k_2 \end{aligned}\right\}. \tag{C.6}$$

We average over the spins of the incoming particles and sum over the spins of the particles of the final state, i.e.,

$$|\mathcal{M}|_{ave} = \frac{1}{4} \sum_{spins} |\mathcal{M}|^2 \tag{C.7}$$

Before evaluating the traces of the squared matrix element, $|\mathcal{M}|^2$, we note the following:

$$\left.\begin{aligned}
&\sum_{s1} u(p_1, s_1)\bar{u}(p_1, s_1) \simeq p_1 \cdot \gamma, \\
&\sum_{s2} u(p_2, s_2)\bar{u}(p_2, s_2) \simeq p_2 \cdot \gamma, \\
&\sum_{r1} \upsilon(k_1, r_1)\bar{\upsilon}(k_1, r_1) = (k_1 \cdot \gamma - \tilde{M}_{\chi^-}), \\
\text{and} \\
&\sum_{r2} \upsilon(k_2, r_2)\bar{\upsilon}(k_2, r_2) = (k_2 \cdot \gamma + \tilde{M}_{\chi^0}).
\end{aligned}\right\} \tag{C.8}$$

where s_1, s_2, r_1, and r_2 are the spin of the quark, antiquark, chargino, and neutralino respectively. As a convention, we assume that $\tilde{\chi}^+_j$ is the *particle* and $\tilde{\chi}^-_j$ is the *antiparticle*. Thus, based upon eqs.(C.2), (C.6) we get

$$|\mathcal{M}|^2_{ave} = \frac{g^4}{6} |V_{q,q'}|^2 |D_W(\hat{s})|^2_{L,R} \Bigg((|O^L_{ij}|^2 + |O^R_{ij}|^2) \times$$

$$\Big[(p_1 \cdot k_1)(p_2 \cdot k_2) + (p_2 \cdot k_1)(p_1 \cdot k_2)\Big] + (O^L_{ij} O^{R*}_{ij} + O^{L*}_{ij} O^R_{ij})$$

$$\times \frac{1}{4} \tilde{M}_{\chi^-} \tilde{M}_{\chi^0} (p_1 \cdot p_2) \Bigg), \tag{C.9}$$

where

$$D_W(\hat{s})_{L,R} = [(\hat{s} - M_W^2) + iM_W\Gamma_W]^{-1}_{(L,R)} \qquad (C.10)$$

Using the relations in (C.4), eq.(C.9) becomes:

$$|\mathcal{M}|^2_{ave} = \frac{g^4}{24} |V_{q,q'}|^2 |D_W(\hat{s})|^2_{L,R} \Bigg\{ (|O^L_{ij}|^2 + |O^R_{ij}|^2) \times$$

$$\left[(\tilde{M}^2_{\chi^-} - \hat{t})(M^2_{\chi^0} - \hat{t}) - (\tilde{M}^2_{\chi^-} - \hat{u})(\tilde{M}^2_{\chi^0} - \hat{u}) \right]$$

$$+ \frac{\hat{s}}{2} \tilde{M}_{\chi^-} \tilde{M}_{\chi^0} (O^L_{ij} O^{R*}_{ij} + O^{L*}_{ij} O^R_{ij}) \Bigg\}. \qquad (C.11)$$

The differential cross-section for the subprocess can be written as:

$$d\hat{\sigma} = \frac{1}{2\lambda^{1/2}(\hat{s}, m^2_{\bar{q}}, m^2_{q'})} (2\pi)^4 \delta^4(p_1 + p_2 - k_1 - k_2)$$

$$\times |\mathcal{M}(p_1, p_2, k_1, k_2)|^2_{ave} \frac{d^3k_1}{(2\pi)^3 2\omega_1} \frac{d^3k_2}{(2\pi)^3 2\omega_2}. \qquad (C.12)$$

It is convenient to introduce the triangle function

$$\lambda(x,y,z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz. \qquad (C.13)$$

Eq.(C.12) is evaluated in the center of mass frame, where the four-vector momenta of the given paticles are $p_1^\mu = (E_1, \mathbf{p}_1)$, $p_2^\mu = (E_2, -\mathbf{p}_2)$, $k_1^\mu = (\omega_1, \mathbf{k}_1)$, and $k_2^\mu = (\omega_2, -\mathbf{k}_2)$, and $\mathbf{p}_1 = -\mathbf{p}_2 = \mathbf{p}$; $\mathbf{k}_1 = -\mathbf{k}_2 = \mathbf{k}$. The d^3k_2 integration takes care of $\delta^3(\mathbf{p}_1 + \mathbf{p}_2 - \mathbf{k}_1 - \mathbf{k}_2)$ yielding $\mathbf{k}_2 = \mathbf{p}_1 + \mathbf{p}_2 - \mathbf{k}_1$, and integration with respect to k_1 eliminates $\delta(E_1 + E_2 - \omega_1 - \omega_2)$. Thus the differential cros-section becomes

$$\frac{d\hat{\sigma}}{d(\cos\hat{\theta})} = \frac{1}{32\pi\hat{s}} \left(\frac{\lambda(\hat{s}, \tilde{M}^2_{\chi^-}, \tilde{M}^2_{\chi^0})}{\lambda(\hat{s}, m^2_{\bar{q}}, m^2_{q'})} \right)^{1/2} |\mathcal{M}|^2_{ave}. \qquad (C.14)$$

It is most convenient to introduce new variables y and z:

$$y = \tilde{M}^2_{\chi^0}/\hat{s}, \quad z = \tilde{M}^2_{\chi^-}/\hat{s}, \qquad (C.15)$$

Thus, eq.(C.15) becomes:

$$\frac{d\hat{\sigma}}{d(\cos\hat{\theta})} = \frac{1}{32\pi s} \lambda^{1/2}(1,y,z)\,|\mathcal{M}|^2_{ave}, \qquad (C.16)$$

where we neglect the quark and antiquark masses in the triangle function $\lambda(\hat{s}, m^2_{\bar{q}}, m^2_{q'})$.

C.1.2. Evaluation of the parton model cross-section. [6-8]

Chargino and Neutralino production in $p\bar{p}$-collisions is based on quark-antiquark annihilation into a vector boson or photon, which subsequently decays into charginos and neutralinos . In our process, the W^--boson is the mediating particle and carries four-momentum $Q_\mu = (Q, Q_0)$ with

$$Q = (x_1 - x_2)P, \quad Q_0 = (x_1 + x_2)E, \qquad 0 \le x_{1,2} \le 1$$

where E is the proton(antiproton) energy in the centre-of-mass frame, and $s=4E^2$. Neglecting the proton(antiproton) masses so that $|P|=E=\frac{1}{2}\sqrt{s}$, it follows that

$$\hat{s}=Q^2= x_1x_2s, \quad y= \frac{1}{2}\ln(x_1/x_2), \tag{C.17}$$

where y is the rapidity of the W^--boson in the centre of mass frame. The momentum fractions x_1, x_2 are related to the variables y, $\hat{s}$. The transformations from x_1, x_2 to y, $\hat{s}$ are

$$x_{1,2}= \frac{M_{WL,R}}{\sqrt{\hat{s}}}e^{\pm y}, \tag{C.18}$$

and

$$\frac{d^2\sigma}{dx_1dx_2}= s\frac{d^2\sigma}{dyd\hat{s}}, \tag{C.19}$$

The differential cross-section for $p\bar{p}$-scattering can be written as:

$$\frac{d^2\sigma}{dx_1dx_2} = KC_{q,q'}\sum_{q,q'} \left[f_{q'}(x_1)f_{\bar{q}}(x_2) + f_{\bar{q}}(x_1)f_{q'}(x_2)\right] \cdot \hat{\sigma}, \tag{C.20}$$

where $\hat{\sigma}$ is the total cross section for the subprocess $\bar{q}q' \to \tilde{\chi}^-_j\tilde{\chi}^0_1$, given by eq.(C.16). The functions $f_{q'}(x_1)$ and $f_{\bar{q}}(x_2)$ $(q \leftrightarrow q')$ are the quark(antiquark) distributions inside the proton and antiproton, respectively. In eq.(C.20), we indicate the summation over the quark(antiquark) flavours that lead to the annihilation reaction into the W^--boson. Colour average factors[†] are accounted for by $C_{\bar{q},q'}$ (=1/9), and the K-factor is the first order QCD correction. Substituting eqs.(C.17)-(C.19) in (C.20) we get the so-called Drell-Yan formula (at y=0)

$$\frac{d^3\sigma}{dyd\hat{s}d(\cos\hat{\theta})} = K\, C_{\bar{q},q'} \sum_{q,q'} \left[f_{q'}(x_1) f_{\bar{q}}(x_2) + f_{\bar{q}}(x_1) f_{q'}(x_2) \right]$$
$$\times\ (x_1 x_2/\hat{s}) \left(\frac{d\hat{\sigma}}{d(\cos\hat{\theta})} \right). \qquad \text{(C.21)}$$

In the $\bar{q}q' \to \tilde{\chi}_j^- \tilde{\chi}_i^0$ subprocess centre of mass frame, the transverse momenta $\hat{p}_T$ of $\tilde{\chi}_j^-$ and $\tilde{\chi}_i^0$ have the same magnitude:

$$\hat{p}_T = \frac{1}{2}\sqrt{\hat{s}}\ \sin\hat{\theta} \qquad \text{(C.22)}$$

To change variables in the differential cross section from $d(\cos\hat{\theta})$ to $d\hat{p}$, we use the Jacobian of eq.(C.22) and we find:

$$\frac{d\hat{\sigma}}{d\hat{p}_T} = \left(\frac{2\sin\hat{\theta}}{\hat{s}^{1/2}\cos\hat{\theta}} \right) \frac{d\hat{\sigma}}{d(\cos\hat{\theta})} \qquad \text{(C.23)}$$

where

$$\cos\hat{\theta} = \frac{2}{\hat{s}^{1/2}} \left[\hat{s}/4 - \hat{p}_T^2 \right]^{1/2}$$

---

† Since the results for the annihilation of a quark-antiquark of a given colour does not depend on the colour itself, summing over colours gives a factor 3, so that combining the factors gives $3\times C_{\bar{q},q'} = \frac{1}{3}$.

The divergence at $\hat{\theta}=\pi/2$ which is the upper endpoint $\hat{p}_T=\frac{1}{2}\sqrt{\hat{s}}\simeq M_{WL,R}/2$ of the $\hat{p}_T$ distribution stems from the Jacobian factor and is known as a *Jacobian peak*. The integration over $\hat{s}$ removes the singularity and leaves a Jacobian peak of finite height near $\hat{p}_T=M_{WL,R}/2$. Thus, using eqs.(C.21) and (C.23), we can write the differential cross-section:

$$\frac{d\sigma}{dp_T}= KC_{\bar{q},q'}\iiint \sum_{q,q'}\left[f_{q'}(x_1)f_{\bar{q}}(x_2) + f_{\bar{q}}(x_1)f_{q'}(x_2)\right]$$

$$\times \left(\frac{2\sin\hat{\theta}}{\hat{s}^{1/2}\cos\hat{\theta}}\right)\left(\frac{d\hat{\sigma}}{d(\cos\hat{\theta})}\right)\left(\frac{x_1x_2}{\hat{s}}\right) d\hat{s}dyd(\cos\hat{\theta}), \qquad \text{(C.24)}$$

where $x_{1,2}$ are given in terms of y by eq.(C.18).

C.1.3. $W^-_{L,R}$-Decay.

The amplitude for $W^-_{L,R}(P) \rightarrow \tilde{\chi}^-_j(-k) + \tilde{\chi}^0_i(k')$ decay mode is given by (at $g\simeq g_L\simeq g_R$)

$$\mathcal{M}= i\frac{g}{4}\varepsilon_\mu(P)\bar{u}(k')\gamma^\mu\left[O^L_{ij}(1 - \gamma_5) + O^R_{ij}(1 + \gamma_5)\right]v(-k),$$

(C.25)

where $\varepsilon_\mu(P)$ is the gauge boson polarization vector, and $\bar{u}(k')$ and $v(-k)$ are the spinors of the chargino and neutralino respectively. Averaging $|\mathcal{M}|^2$ over the W^--polarizations and summing over the fermion spins, we get

$$|\mathcal{M}|^2_{ave} = \frac{1}{3}\sum_{spins}|\mathcal{M}|^2 \tag{C.26}$$

$$= \frac{2g^2}{3}\left\{\left[(k.k') + \frac{2(P.k)(P.k')}{M_W^2}\right](|O^L_{ij}|^2 + |O^R_{ij}|^2) \right.$$

$$\left. + 3\tilde{M}_{\chi^0}\tilde{M}_{\chi^-}(O^L_{ij}O^{R*}_{ij} + O^{L*}_{ij}O^R_{ij})\right\} \tag{C.27}$$

Using the following kinematic invariants:

$$P = k + k', \quad M_W^2 = \tilde{M}^2_{\chi^-} + \tilde{M}^2_{\chi^0} + 2k.k', \tag{C.28}$$

the differential decay rate in the W^--boson rest frame is

$$d\Gamma(W^-_{L,R} \to \tilde{\chi}^-_j\chi^0_i) = \frac{1}{2M_W}|\mathcal{M}|^2_{ave}d(PS), \tag{C.29}$$

where d(PS) is the invariant differential phase space, which can be written as

$$d(PS) = (2\pi)^4\delta^4(P - k - k')\frac{d^3k}{2\omega(2\pi)^3}\frac{d^3k'}{2\omega'(2\pi)^3}. \tag{C.30}$$

The integration over the phase space is

$$\int d(PS) = \frac{1}{8\pi}\lambda^{1/2}(1, \tilde{M}^2_{\chi^-}/M_W^2, \tilde{M}^2_{\chi^0}/M_W^2). \tag{C.31}$$

Substituting eq.(C.29) in eq.(C.31), we find the total decay width

$$\Gamma(W^-_{L,R} \to \tilde{\chi}^-_j \chi^0_i) = \frac{\alpha^2}{3} \Gamma^0(W^-_{L,R} \to \ell^- \bar{\nu}_\ell) \Big((|O^L_{ij}|^2 + |O^R_{ij}|^2)$$

$$\times \left[2 - x - y - (x - y)^2\right] + 6z(O^L_{ij} O^{R*}_{ij} + O^{L*}_{ij} O^R_{ij}) \Big)$$

$$\times \left[(1 + y - x)^2 - 4y\right]^{1/2}, \qquad \text{(C.32)}$$

where

$$x = \tilde{M}^2_{\chi^0}/M^2_{WL,R}, \quad y = \tilde{M}^2_{\chi^-}/M^2_{WL,R}, \quad z = \tilde{M}_{\chi^0}\tilde{M}_{\chi^-}/M^2_{WL,R}.$$

This result can be compared with

$$\Gamma(W^-_L \to \ell^- \nu_\ell) = 3\Gamma(W^-_L \to e^- \nu_e) = 9\frac{g^2 M_{WL}}{48\pi},$$

$$\Gamma(W^-_R \to \ell^- \nu_{\ell R}) = 3\Gamma(W^-_R \to e^- \nu_{eR}) = 12\frac{g^2 M_{WR}}{48\pi}.$$

References

[1] H.E. Haber and G.L. Kane, *Phys. Rep.* **117** (1985) 75.

[2] P. West, *Introduction to Supersymmetry and Supergravity*, 2nd ed., World Scientific Pub., Singapore, 1990.

[3] J. Gunion and H. Haber, *Nucl. Phys.* B272 (1986) 1; *Nucl. Phys.* B278 (1986) 449.

[4] R.M. Barnett and H. E. Haber, *Phys. Rev.* **D31** (1985) 85.

[5] B. De Wit and J. Smith, *Field Theory in Particle Physics* ch.3, vol.1, North-Holland Phys. Pub., Amsterdam 1986.

[6] Ref. [2] ch.6.

[7] V.D. Barger and R.J.N. Phillips, *Collider Physics*, ch.8, Addison-Wesley, California, 1987.

[8] For review, see for example; W. Marciano and H. Pagels, *Phys. Rep.* 36C (1978) 137; A.H. Mueller, *Phys. Rep.* 73C, (1981) 237; G. Altarelli, *Phys. Rep.* 81 (1982) 1; J.F. Owens, *Rev. Mod. Phys.* 59 (1987) 465.

Appendix D

PHYSICAL REVIEW D VOLUME 44, NUMBER 4 15 AUGUST 1991

Graviton-electron interactions

H. N. Saif
Physics Department, Concordia University, 1455 de Maisonneuve Boulevard West, Montreal, Quebec, Canada H3G 1M8

(Received 22 February 1991; revised manuscript received 24 April 1991)

First-order cross sections for the processes of graviton-Compton scattering, $ge^- \rightarrow \gamma e^-$, bremsstrahlung, and pair production by gravitons in the Coulomb field, are calculated. The calculations, which are linear in the gravitational coupling κ, are obtained in the extreme relativistic limit.

I. INTRODUCTION

Several processes in which gravitons, the quanta of the gravitational field, interact with other elementary particles are studied in the literature. Since the gravitational coupling strength $\kappa = \sqrt{8\pi G}$ is extremely weak, where G is the Newtonian gravitational constant ($G = 6.7 \times 10^{-39}\,\mathrm{GeV}^{-2}$), only processes linear in κ are usually considered. Although no real attempt is made to estimate the overall background of gravitational radiation in the Universe, which would clearly be of considerable astrophysical significance, linear processes can be distinctly singled out that contribute to increase the gravitational radiation in the Universe. Some such processes have a direct astrophysical interest; for instance, photoproduction and bremsstrahlung generate sizable amounts of gravitational radiation which can be comparable in magnitude to those of classical processes.

Previous work was done by Weber and Hinds [1], Weinberg [2], Carmeli [3], Boccaletti and Occhionero [4], Boccaletti [5], and Papini [6]. They have shown that in astrophysical applications the gravitational radiation power in quantum processes could be as high as in the classical ones. Papini and Valluri [7] considered the process of photoproduction of gravitons in static magnetic and Coulomb fields in the first- and second-order perturbation theory and applied the results for studying the gravitational radiation from some astrophysical objects. Although the experimental implications of quantum gravity are normally far beyond the range of contemporary experimental physics, some interesting astrophysical objects have recently been observed that emit extremely-high-energy electromagnetic radiation which could be produced by processes involving gravitons. Indeed it is likely that objects such as Cygnus X-3 or neutron stars radiate a significant fraction of their energy in the form of very-high-energy gravitons. Also, there is strong indirect evidence that the rate at which the rotation periods of some massive binary-star systems are slowing down is consistent with the expectation of energy loss due to the emission of gravitational radiation [3].

Part of the motivation for considering linear processes for quantum gravity is due to the question of renormalizability, which now appears as a major obstacle to constructing a complete quantum theory of gravity. For nongravitational radiation, renormalizable quantum field theories exist which seem to describe nature adequately. However, when one considers gravity because the gravitational coupling constant κ of Einstein's theory of gravity is dimensional, the corresponding quantum field theory is nonrenormalizable. Although in the case of pure gravity the one-loop divergences can be eliminated by field renormalization [9–11], physical divergences remain for the more realistic situation of combined matter interactions. It is implicitly hoped that the first-order terms of perturbation theory are valid for the processes considered.

The main purpose of this paper is to give an estimate of the cross section at the tree level (order of κ^2), for the processes $ge^- \rightarrow \gamma e^-$, bremsstrahlung and pair production by gravitons in a Coulomb field. The motivation is to find some process which would enable extremely high-energy gravitons to be detected. In Secs. II–IV, we calculate the cross sections for these processes. Section V contains the conclusions. We use units $\hbar = c = 1$.

II. GRAVITON-COMPTON SCATTERING

The first-order contribution to the reaction $ge^- \rightarrow \gamma e^-$ is described by the two diagrams of Fig. 1, where k, k', ω, ω' are the four-momenta of the initial graviton, the final photon, the initial electron, and the final electron, respectively. The quantities $\epsilon_{\mu\nu}$, ϵ_β^* are the polarization tensor and vector of the initial graviton and the final photon, respectively. The intermediate four-vector momenta are $q' = (k - p) = (k' - p')$, $q = (p - k') = (p' - k)$. The corresponding Feynman rule for the electron-electron-graviton vertex is $\kappa[(p - q')_\mu \gamma_\nu - \gamma_\mu (p - q')_\nu]$ ($p \rightarrow p'$ and $q \rightarrow q'$ in the second diagram), which has already been discussed in the literature [3,10–12] (more details are given in Appendix A). The matrix element for both diagrams is given by

$$\mathcal{M}_{fi} = -e^{-}\sqrt{8\pi G}\,\epsilon_{\mu\nu}\epsilon_\beta^*\bar{u}(p',s') \times \left\{ [(p+q')_\mu\gamma_\nu + \gamma_\mu(p+q')_\nu]\frac{q'+m}{q'^2-m^2}\gamma_\beta + \gamma_\beta\frac{q+m}{q^2-m^2}[(p'+q)_\mu\gamma_\nu + \gamma_\mu(p'+q)_\nu] \right\} u(p,s) . \quad (2.1)$$

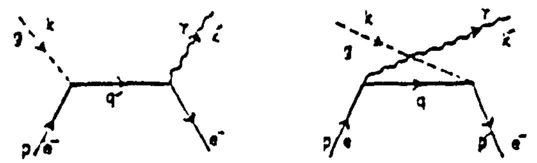

FIG. 1. Graviton-Compton scattering $ge^- \to \gamma e^-$.

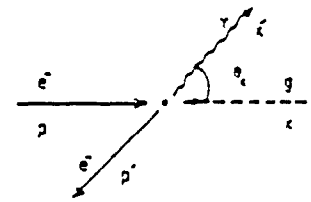

FIG. 2. g-e collision c.m. frame.

where $u(p,s)$, $u(p',s')$ are the Dirac spinors for the initial and final electron and $\not q = q_\mu \gamma^\mu = (q\cdot\gamma)$. Disregarding polarization effects, we average the cross section over the initial spins of the graviton and electron, and sum over the polarizations of the final electron and photon. Neglecting the electron masses and squaring Eq. (2.1) one obtains

$$|\mathcal{M}_{fi}|^2 = 8\pi G e^2 \frac{1}{4} \sum_{\text{polar}} \varepsilon_{\mu\nu} \varepsilon^*_{\alpha\beta} \varepsilon_\sigma \varepsilon^*_\rho (\bar u' T_{\mu\nu\rho} u)(\bar u \bar T_{\sigma\alpha\beta} u') \tag{2.2}$$

$$= -2\pi G e^2 \mathrm{Tr}[\tfrac{1}{2} g_{\sigma\rho} (g_{\mu\alpha} g_{\nu\beta} + g_{\mu\beta} g_{\nu\alpha} - g_{\mu\nu} g_{\alpha\beta}) \times (p'\cdot\gamma T_{\mu\nu\rho} p\cdot\gamma \bar T_{\alpha\beta\sigma})] , \tag{2.3}$$

where we have used the polarization sums of the photon and graviton [3,13,14]:

$$\sum_{\lambda'} \varepsilon_\sigma(k',\lambda') \varepsilon_\rho(k',\lambda') = -g_{\sigma\rho} , \tag{2.4}$$

$$\sum_{\lambda} \varepsilon_{\mu\nu}(k,\lambda) \varepsilon^*_{\alpha\beta}(k,\lambda) = \tfrac{1}{2}(g_{\mu\alpha} g_{\nu\beta} + g_{\mu\beta} g_{\nu\alpha} - g_{\mu\nu} g_{\alpha\beta}) , \tag{2.5}$$

with

$$T_{\mu\nu\rho} = [(2p+k)_\mu \gamma_\nu + \gamma_\mu (2p+k)_\nu] \frac{\not p + \not k}{(p+k)^2} \gamma_\rho + \gamma_\rho \frac{\not p - \not k}{(p'-k)^2} [(2p'-k)_\mu \gamma_\nu + \gamma_\mu (2p'-k)_\nu] \tag{2.6}$$

and

$$\bar T_{\sigma\alpha\beta} = \gamma_\sigma \frac{\not p - \not k}{(p-k)^2} [(2p-k)_\alpha \gamma_\beta + \gamma_\alpha (2p+k)_\beta] + [(2p'-k)_\alpha \gamma_\beta + \gamma_\alpha (2p'-k)_\beta] \frac{\not p - \not k}{(p'-k)^2} \gamma_\sigma . \tag{2.7}$$

We define the kinematical invariants $s = q'^2 = (k+p)^2 = (k'+p')^2$, $t = (k-k')^2 = (p-p')^2$, $u = q^2 = (p-k')^2 = (p'-k)^2$, and $s+u+t \cong 0$. Equation (2.3) is evaluated in the center-of-mass frame, where the four-vector momentum of the given particles are (Fig. 2) $k^\mu = (\omega, \mathbf{k})$, $p^\mu = (E, -\mathbf{k})$, $k'^\mu = (\omega', \mathbf{k}')$, and $p'^\mu = (E', -\mathbf{k}')$. After very lengthy calculations of traces in Eq. (2.3), we get the differential cross section

$$d\sigma_{\text{Comp}} = \frac{4G\alpha}{s} \left[t + \frac{1}{su}(2t^3 + 3t^2 s + 3t^2 u + ts^2 + 2tsu + tu^2) \right] d\Omega' , \tag{2.8}$$

where α is the fine-structure constant and Ω' is the emitted photon solid angle. Setting $s = 4k^2$, $t = -2k^2(1-\cos\theta_{\text{c.m.}})$, $u = -2k^2(1+\cos\theta_{\text{c.m.}})$ and integrating over the angle $\theta_{\text{c.m.}}$ (where $\theta_{\text{c.m.}}$ is the c.m. angle) we get $\sigma_{\text{comp}} = 8\pi\alpha G = 0.5\times 10^{-66}\ \text{cm}^2$, which no longer depend on the energy of the colliding particles. The radiation of the final particles becomes strongest in the directions of the initial momenta. This result is in agreement with the results of Vladimirov [15] for the collision $e^+e^- \to \gamma g$. Actually this is expected since these two processes are related by crossing symmetry. The calculation by Papini and Valluri [7] of one vertex γ-g interaction also shows that the results are energy independent.

III. GRAVITON BREMSSTRAHLUNG IN THE COULOMB FIELD

We consider graviton emission in a collision between an electron and a nucleus ($e^- - Z \to e^- - g - Z$). The momentum $k' = p' - p - k$ is the four-vector momentum transfer to the nucleus. Since the recoil of the nucleus is neglected, the time component $k'_0 = 0$. According to Fig. 3 the matrix element $\mathcal{M}_{fi}$ has the form

$$\mathcal{M}_{fi} = -e\sqrt{8\pi G}\, \Phi(|\mathbf{k}'|) \varepsilon^*_{\alpha\beta}(k,\lambda) \bar u(p',s') \times \left\{ [(p'+q')_\alpha \gamma_\beta + \gamma_\alpha (p'+q')_\beta] \frac{\not q'}{q'^2} \gamma_0 - \gamma_0 \frac{\not q}{q^2} \times [(p+q)_\alpha \gamma_\beta + \gamma_\alpha (p+q)_\beta] \right\} u(p,s) , \tag{3.1}$$

where the intermediate four-momenta are $q' = p - k' = k + p'$, $q = p - k = p' - k'$, and $\Phi(|\mathbf{k}'|)$ is the scalar potential of the external field: for a Coulomb field

$$\Phi(|\mathbf{k}'|) = -4\pi Z e^- / |\mathbf{k}'|^2 , \tag{3.2}$$

After averaging the cross section over the initial spin of

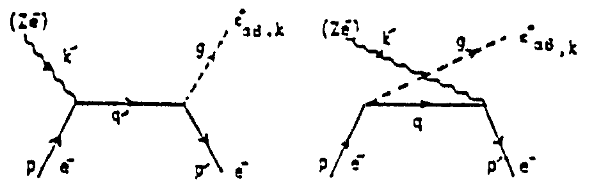

FIG. 3. Graviton bremsstrahlung in the Coulomb field.

the electron and summing over the polarizations of the final electron and graviton, by squaring (3.1) one obtains

$$|M_{fi}|^2=8\pi G\frac{Z^2e^4}{|\mathbf{k}'|^4}\frac{1}{2}\sum_{\text{polar}}\varepsilon_{\mu\nu}\varepsilon^*_{\alpha\beta}(\bar{u}'T_{\mu\nu}u)(\bar{u}\bar{T}_{\alpha\beta}u')\ . \tag{3.3}$$

Inserting (2.5) into (3.3), we get

$$|M_{fi}|^2=4\pi G\frac{Z^2e^4}{|\mathbf{k}'|^4}\mathrm{Tr}[\tfrac{1}{2}(g_{\mu\alpha}g_{\nu\beta}+g_{\mu\beta}g_{\nu\alpha}-g_{\mu\nu}g_{\alpha\beta})\times(p'\cdot\gamma T_{\mu\nu}p\cdot\gamma\bar{T}_{\alpha\beta})]\ , \tag{3.4}$$

where

$$T_{\mu\nu}=[(2p'+k)_\mu\gamma_\nu+\gamma_\mu(2p'+k)_\nu]\frac{\not p'-\not k}{(p'-k)^2}\gamma_0+\gamma_0\frac{\not p-\not k}{(p-k)^2}[(2p-k)_\mu\gamma_\nu+\gamma_\mu(2p-k)_\nu] \tag{3.5}$$

and

$$\bar{T}_{\mu\nu}=\gamma_0T^\dagger_{\alpha\beta}\gamma_0$$
$$=\gamma_0\frac{\not p'+\not k}{(p'+k)^2}[(2p'+k)_\alpha\gamma_\beta-\gamma_\alpha(2p'+k)_\beta]+[(2p-k)_\alpha\gamma_\beta+\gamma_\alpha(2p-k)_\beta]\frac{\not p-\not k}{(p-k)^2}\gamma_0\ . \tag{3.6}$$

The differential cross section for bremsstrahlung is given by [16]

$$d\sigma_{\text{brems}}=\frac{1}{(2\pi)^5}|M_{fi}|^2\frac{E'\omega}{E}\,d\omega\,d\Omega_k\,d\Omega'\ , \tag{3.7}$$

where $d\Omega_k\,d\Omega'$ are the solid angles of the graviton emission and the final electron. Inserting the value of $|M_{fi}|^2$ in Eq. (3.7), we obtain the following expression for bremsstrahlung cross section:

$$\begin{aligned}d\sigma_{\text{brems}}=\;&\frac{32Z^2\alpha^2G}{|\mathbf{k}'|^4}\frac{E'\omega}{E}\,d\omega\,d(\cos\theta)d(\cos\theta')\\&\times\Bigg\{(p\cdot k)-2(p_0k_0)+\frac{1}{2(p'\cdot k)(p\cdot k)}\\&\quad\times[3(p'\cdot p)^2(p\cdot k)+2(p'\cdot p)^3-3(p'\cdot p)^2(p'\cdot k)-4(p'\cdot p)(p'_0p_0)(p\cdot k)-4(p'_0p_0)(p'\cdot p)^2\\&\quad+4(p'_0p_0)(p'\cdot k)(p'\cdot p)-2(p'\cdot p)(p'\cdot k)(p\cdot k)+(p'\cdot p)(p'\cdot k)^2+(p'_0k_0)(p'\cdot p)(p\cdot k)+2(p'_0k_0)(p'\cdot p)^2\\&\quad-(p'_0k_0)(p'\cdot p)(p'\cdot k)-(p'_0p'_0)(p\cdot k)^2-2(p'_0p'_0)(p'\cdot p)(p\cdot k)+(p'_0p'_0)(p'\cdot k)(p\cdot k)-2(p'_0p_0)(p'\cdot k)(p\cdot k)\\&\quad-(p'_0p_0)(p'\cdot k)^2+(p'\cdot p)(p\cdot k)^2-(p'_0p_0)(p\cdot k)^2+(p_0p_0)(p'\cdot k)(p\cdot k)+2(p_0p_0)(p'\cdot k)(p'\cdot p)\\&\quad-(p_0p_0)(p'\cdot k)^2-(p_0k_0)(p'\cdot p)(p\cdot k)-2(p_0k_0)(p'\cdot p)^2+(p_0k_0)(p'\cdot p)(p'\cdot k)]-(p'\cdot k)-2(p'_0k_0)\Bigg\}\ ,\end{aligned} \tag{3.3}$$

where k_0,p_0,p'_0 are the energy components of the graviton, the initial, and the final electron, respectively; θ,θ' the angles between $\mathbf{k}$ and $\mathbf{p},\mathbf{p}'$, respectively. It is convenient to write, at extremely high energy $E\gg m$ the relations

$$p'\cdot p\cong p'\cdot k-p\cdot k-\mathbf{k}'^2/2\ ,\quad\text{where } p'^2=p^2=m^2,\ \ k^2=0\ , \tag{3.9}$$

$$p\cdot k=E\omega(1-\cos\theta)=\omega\delta,\quad\text{where } \delta=E(1-\cos\theta)\ , \tag{3.10}$$

and

$$p'\cdot k=E'\omega(1-\cos\theta')=\omega\delta'\ ,\quad\text{where } \delta'=E'(1-\cos\theta')\ , \tag{3.11}$$

substituting (3.9)–(3.11) in (3.3). The integration of Eq. (3.8) over the angles θ,θ' is rather lengthy (see Refs. [16–19] for photon bremsstrahlung). We shall give only the final result (a few steps are described in Appendix B):

$$\begin{aligned}d\sigma_{\text{brems}}=\;&32Z^2\alpha^2G(E'/E)(d\omega/\omega)\\&\times\{\tfrac{1}{2}-\beta L(1-3E'/2E-E'^2/E^2)+L(-\tfrac{11}{4}-3E/16E'-5E'/16E)-\beta(\tfrac{3}{4}-9E'/3E-2E/3E')\\&\quad-\beta'(\tfrac{1}{4}-29E/16E'-9E'/16E)-\beta\beta'(1-5E'/4E)\}\ ,\end{aligned} \tag{3.12}$$

where $\beta=\ln[(E-p)/(E-p)]\cong2\ln(2E/m)$; $\beta'=\ln[(E'-p')/(E'-p')]\cong2\ln(2E/m)$, and $L=\ln[(EE'-pp'$

$-m^2)/(EE'-pp'-m^2)] \cong 2\ln(2EE'/m\omega)$ as defined by Bethe and Heitler [18]. The presence of the logarithm of a large quantity [the ratio $(2EE'/m\omega) >> 1$ even if $\omega \cong E$] should be noted, the logarithmic terms become the principal ones in (3.12). Finally, we shall give the limiting formula for the region near the end of the spectrum, when the extreme-relativistic electron radiates almost all its energy $\omega \cong E >> E' >> m$, then $L = \beta'$ one can easily find

$$d\sigma_{\text{brems}} = 32Z^2\alpha^2 G\, d\omega/E(2\beta/3 - 13\beta'/8) \;. \tag{3.13}$$

Equation (3.13) covers all the range of ω values for extremely relativistic initial electron.

IV. PAIR PRODUCTION BY A GRAVITON IN THE COULOMB FIELD

The process of pair production by a graviton in the field of a nucleus ($g + Z \to e^- + e^+ + Z$) is very closely related to the process of bremsstrahlung in the previous section. Figure (4) shows the corresponding diagrams, where $E, p \to -E, -p; \omega, k \to -\omega, -k$; and $\varepsilon^*_{\alpha\beta} \to \varepsilon_{\mu\nu}$. The four-vector momentum transfer to the nucleus can be written as $k' = p' + p - k$, and the energy transition is $\omega = E + E'$. The matrix element M_{fi} is, therefore,

$$M_{fi} = -e\sqrt{8\pi G}\,\Phi(|\mathbf{k}'|)\varepsilon_{\mu\nu}\bar{u}(p',s')$$
$$\times \left[[(p'+q')_\mu \gamma_\nu + \gamma_\mu (p'+q')_\nu] \frac{\not{q}'}{q'^2}\gamma_0 \frac{\not{q}}{q^2}[(-p+q)_\mu\gamma_\nu + \gamma_\mu(-p+q)_\nu] \right] \bar{v}(-p,s) \;, \tag{4.1}$$

where the intermediate four-momenta are $q' = -p + k' = -k + p'$, $q = -p + k = p' - k'$, and $\bar{v}(-p,s)$ is the Dirac spinor of the emerging positron. The differential cross section for the pair production is

$$d\sigma_{\text{pair}} = [1/(2\pi)^5]|M_{fi}|^2 (EE'/\omega)dE\, d\Omega'\, d\Omega \;; \tag{4.2}$$

we have multiplied (3.7) by $(E^2/\omega^2)\,dE/d\omega$ and replaced by $d\Omega_k$ by $d\Omega$, the solid angle of the emerging positron (see Ref. 16, Sec. 91]). By squaring (4.1) one obtains

$$|M_{fi}|^2 = 8\pi G \frac{Z^2 e^4}{|\mathbf{k}'|^4} \frac{1}{2} \sum_{\text{polar}} \varepsilon_{\mu\nu}\varepsilon^*_{\alpha\beta}(\bar{u}' T_{\mu\nu} v)(\bar{v}\bar{T}_{\alpha\beta} u') \tag{4.3}$$

$$= 4\pi G \frac{Z^2 e^4}{|\mathbf{k}'|^4} \mathrm{Tr}[\tfrac{1}{2}(g_{\mu\alpha}g_{\nu\beta} + g_{\mu\beta}g_{\nu\alpha} - g_{\mu\nu}g_{\alpha\beta})(p'\cdot\gamma\, T_{\mu\nu}\, p\cdot\gamma\, \bar{T}_{\alpha\beta})] \;, \tag{4.4}$$

where

$$T_{\mu\nu} = [(2p'-k)_\mu\gamma_\nu + \gamma_\mu(2p'-k)_\nu] \frac{-\not{k}+\not{p}'}{(-k+p')^2}\gamma_0 + \gamma_0 \frac{-\not{p}+\not{k}}{(-p+k)^2}[(-p+k)_\mu\gamma_\nu + \gamma_\mu(-p+k)_\nu] \tag{4.5}$$

and

$$\bar{T}_{\alpha\beta} = \gamma_0 T^\dagger_{\alpha\beta}\gamma_0$$
$$= \gamma_0 \frac{-\not{k}+\not{p}'}{(-k+p')^2}[(2p'-k)_\alpha\gamma_\beta + \gamma_\alpha(2p'-k)_\beta] + [(-p+k)_\alpha\gamma_\beta + \gamma_\alpha(-p+k)_\beta]\frac{-\not{p}+\not{k}}{(-p+k)^2}\gamma_0 \,. \tag{4.6}$$

By means of (4.4) and (4.2) the cross section can be now written as

$$d\sigma_{\text{pair}} = -\frac{32Z^2\alpha^2 G}{|\mathbf{k}'|^4}\frac{EE'}{\omega}\, dE\, d(\cos\theta)\, d(\cos\theta')$$
$$\times \Bigg[(p\cdot k) - 2(p_0 k_0) - \frac{1}{2(p'\cdot k)(p\cdot k)}$$
$$\times [3(p'\cdot p)^2(p\cdot k) - 2(p'\cdot p)^3 + 3(p'\cdot p)^2(p'\cdot k) - 4(p'\cdot p)(p\cdot k)(p_0' p_0)$$
$$+ 4(p'\cdot p)^2(p_0' p_0) - 4(p'\cdot p)(p'\cdot k)(p_0' p_0) - 2(p'\cdot p)(p'\cdot k)(p\cdot k)$$
$$-(p'\cdot p)(p'\cdot k)^2 + (p_0' k_0)(p'\cdot p)(p\cdot k) - 2(p'\cdot p)^2(p_0' k_0) + (p_0' k_0)(p'\cdot p)(p'\cdot k)$$
$$-(p_0' p_0')(p\cdot k)^2 - 2(p_0' p_0')(p'\cdot p)(p\cdot k) - (p_0' p_0')(p'\cdot k)(p\cdot k)$$
$$+ 2(p_0' p_0)(p'\cdot k)(p\cdot k) - (p_0' p_0)(p'\cdot k)^2 - (p'\cdot p)(p\cdot k)^2 + (p_0' p_0)(p\cdot k)^2$$
$$-(p_0 p_0)(p'\cdot k)(p\cdot k) - 2(p_0 p_0)(p'\cdot k)(p'\cdot p) - (p_0 p_0)(p'\cdot k)^2 - (p'\cdot p)(p\cdot k)(p_0 k_0)$$
$$-2(p'\cdot p)^2(p_0 k_0) - (p'\cdot p)(p'\cdot k)(p_0 k_0)] + (p'\cdot k) - 2(p_0' k_0) \Bigg] \tag{4.7}$$

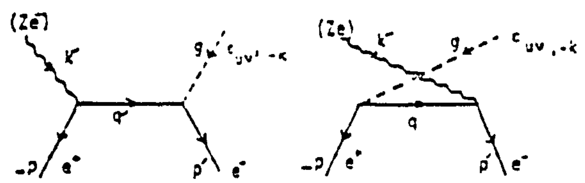

FIG. 4. Pair production by a graviton in the Coulomb field.

The electron and the positron are emitted at angles θ, θ' relative to the direction of the incident graviton. The integration over the angles θ, θ' is completely analogous to the bremsstrahlung case [Eq. (3.8)]. We assume the electron and positron share equally the energy of the graviton $\omega = E + E' \simeq 2E$, then $\beta = \beta' \simeq 2\ln(2E/m)$ and $L \simeq 2\ln(E/m)$. We find the differential cross section is

$$d\sigma_{\text{pair}} = Z^2\alpha^2 G(dE/E)(7\beta^2/2 + 201\beta/5 - 31/5) \ . \quad (4.8)$$

Integration of Eq. (4.8) over E from $m \to \omega$ gives the total cross section for pair production by a graviton having a given energy $\omega \gg m$:

$$\begin{aligned}\sigma_{T\,\text{pair}} = Z^2\alpha^2 G\{&\tfrac{7}{3}\ln^3(\omega/m) + \ln^2(\omega/m)\\ &\times[7\ln(2/m) + \tfrac{201}{10}] + \ln(\omega/m)\\ &\times[7\ln^2(2/m) + \tfrac{201}{5}\ln(2/m)] - \tfrac{31}{5}\} \ .\end{aligned} \quad (4.9)$$

As in bremsstrahlung, the logarithmic terms in pair production become the principal one in (4.8) and (4.9). Taking $\omega = 10^4$ TeV, $Z = 1$, then $\sigma_{T\,\text{pair}} \simeq 10^{-66}$ cm^2. Near threshold, i.e., $\omega \to 2m$, Eq. (4.9) can be reduced to $\sigma_{\text{Th}} = (370)Z^2\alpha^2 G \simeq 0.05 \times 10^{-66}$ cm^2.

V. CONCLUSIONS

We have calculated the first-order cross sections for the processes graviton-Compton scattering $(ge^- \to \gamma e^-)$, bremsstrahlung, and pair production by a graviton in the Coulomb field at extremely high energies. In the process $ge^- \to \gamma e^-$ the final result is energy independent, which is in agreement with the results obtained in Refs. [7] and [15]. We have treated the graviton bremsstrahlung and pair production in a manner similar to photon bremsstrahlung and pair production [16–18]. Since the bremsstrahlung and pair production depend on the logarithmic terms, one might hope, at least from the astrophysical point of view, that these processes are significant.

ACKNOWLEDGMENTS

The author would like to thank Professor P. J. S. Watson at Carleton University for valuable discussions. The author also wishes to thank Professor C. S. Kalman for reading the manuscript and making helpful comments.

APPENDIX A

Lagrangian and the Feynman rule

The Lagrangian density for a spin-$\frac{1}{2}$ fermion in a gravitational field is given by the sum of the Dirac and Einstein Lagrangian densities [10]

$$\mathcal{L}(\bar{e}, \psi) = -\tfrac{1}{2}\bar{e}\kappa^{-2}R(\bar{g}) - \bar{e}\bar{\psi}\gamma^a e_a^\mu D_\mu\psi, \quad \kappa^2 = 8\pi G \ , \quad (A1)$$

where $\bar{g}$ is expressed in terms of $\bar{e}$ by the relation

$$e_\mu^a e_\nu^b \eta_{ab} = g_{\mu\nu} \ . \quad (A2)$$

The matrix e_a^μ (the inverse of e_μ^a) is a set of vierbein fields or tetrad fields (which is defined as the matrix square root of the metric tensor $g_{\mu\nu}$), and e is given by

$$e \equiv \det(e_\mu^a) = [\det(e_a^\mu)]^{-1} = [-\det(g_{\mu\nu})]^{1/2} \ . \quad (A3)$$

η_{ab} is the Minkowski metric tensor; R is the curvature scalar; $\bar{\psi}$ and ψ are fermion fields, which can be introduced into general relativity [20] by describing them with respect to local Lorentz frames; they are defined to be world scalars and transform as ordinary spinors under local Lorentz transformations of the vierbein frames (Lorentz spinors). The covariant derivatives D_μ can be introduced as a covariant world vector and a Lorentz spinor,

$$D_\mu = \partial_\mu + \tfrac{1}{2}\sigma^{ab}\omega_{\mu ab} \ , \quad (A4)$$

where $\sigma^{ab} = \frac{1}{4}(\gamma^a\gamma^b - \gamma^b\gamma^a)$ and

$$\omega_{\mu ab} = [e_a^\nu(\partial_\mu e_{b\nu} - \partial_\nu e_{b\mu}) + \tfrac{1}{2}e_a^\rho e_b^\sigma(\partial_\sigma e_{c\rho} - \partial_\rho e_{c\sigma})e_\mu^c]_{(ab)} \ .$$

The last symbol denotes antisymmetrization in (ab), $\omega_{\mu ab}$ is a covariant vector; under local Lorentz transformations it is not a tensor, but acquires an inhomogeneous term which is needed to make $D_\mu\psi$ a Lorentz spinor. The fields $\bar{e}, \psi$ can be written as a s[illegible] of background and quantum fields:

$$\bar{e}_\mu^a = e_\mu^a - \kappa c_\mu^a, \quad \psi \to \kappa^{-1}\xi - \psi \ . \quad (A5)$$

The factor κ has been inserted to give the quantum fields canonical dimension (units $\hbar = c = 1$ are used). The quantum field c_μ^a can be considered just like other matter fields, such as photons and fermions, and ξ is a fermion field. We expand $\mathcal{L}(e - \kappa c, \kappa^{-1}\xi - \psi)$ in quantum fields (c, ψ) around the background fields (e, ξ). For the first variational derivative (neglecting the other Lagrangians) one has [10]

$$\begin{aligned}\mathcal{L}_1(e, c; \xi, \psi) = \kappa^{-1}e\{&c_\mu^a[2G_a^\mu - T_a^\mu(e, \xi)]\\ &- \bar{\xi}\gamma^\mu D_\mu\psi - \bar{\psi}\gamma^\mu D_\mu\xi\} \ ,\end{aligned} \quad (A6)$$

where $G_a^\mu = e_{a\nu}G^{\nu\mu}$ and $G^{\mu\nu}$ is the symmetric Einstein tensor $R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R$ ($R^{\mu\nu}$ is the Ricci tensor). $\mathcal{L}_1$ vanishes if and only if the classical field equations are satisfied by the background field, namely,

$$G_{\mu\nu} = -\tfrac{1}{2}T_{\mu\nu}, \quad \gamma^\mu D_\mu\xi = (D_\mu\bar{\xi})\gamma^\mu = 0 \ . \quad (A7)$$

The Einstein equations consist of a symmetric part

$G_{\mu\nu}=-\frac{1}{2}(T_{\mu\nu}-T_{\nu\mu})$ and an antisymmetric part $T_{\mu\nu}-T_{\nu\mu}=0$. The fermion stress tensor $T^{\mu}_{a}=-\delta\mathcal{L}^{D}/\delta e^{a}_{\mu}$ is

$$T^{a}_{\mu}=-\bar{\zeta}\gamma^{a}D_{\mu}\zeta-\delta^{a}_{\mu}\bar{\zeta}\gamma^{\nu}D_{\nu}\zeta-\frac{1}{2}D_{\lambda}(\bar{\zeta}\gamma^{\lambda}\sigma^{\mu a}\zeta)$$
$$-\frac{1}{2}D_{a}[\bar{\zeta}(\gamma^{\mu}\sigma^{\lambda a}+\gamma^{\lambda}\sigma^{\mu a})\zeta] \quad \text{(A8)}$$

and is *a priori* nonsymmetric, but it becomes symmetric, conserved, and traceless as a consequence of the Dirac equation $\gamma^{\mu}D_{\mu}\zeta=0$ [for convenience, we have neglected the fermion mass in Eq. (A1)], which reduces it to the usual expression $T_{\mu\nu}=\bar{\zeta}(\gamma_{\mu}\bar{D}_{\nu}+\gamma_{\nu}\bar{D}_{\mu})\zeta$. Thus, the corresponding Feynman rule for a spin-$\frac{1}{2}$ fermion-graviton vertex is $\kappa(\gamma_{\mu}P_{\nu}+\gamma_{\nu}P_{\mu})$, $P=p+q$ (where p and q are the momentum components of the initial and final fermion states).

APPENDIX B

Integrated cross section

The integrals to be evaluated have the general form

$$I_{m,n}=\int d(\cos\theta')k'^{-2m}\delta'^{-n}\ , \quad \text{(B1)}$$

$$m=-1,0,1,2,\quad n=-2,-1,0,1\ .$$

where $\delta'=E'(1-\cos\theta')$, if one writes $k'^{2}=(\mathrm{p}-\mathrm{p}'-\mathrm{k})^{2}=(T^{2}+p'^{2})(1-\mathrm{p}'\cdot\mathrm{c})$, with $\mathrm{c}=2\mathrm{T}/(T^{2}+p^{2})$, $\mathrm{T}=p-k$, and $\delta'=E'(1-\mathrm{p}'\cdot\mathrm{d})$, with $\mathrm{d}=\mathrm{k}/\omega E'$, then the integrals reduce to the form

$$I_{m,n}=(p'^{2}+T^{2})^{-m}E'^{-n}$$
$$\times\int d(\cos\theta')(1-\mathrm{p}'\cdot\mathrm{c})^{-m}(1-\mathrm{p}'\cdot\mathrm{d})^{-n}\ . \quad \text{(B2)}$$

We find the following cases: $I_{1,0}$, $I_{2,0}$, $I_{0,1}$, $I_{0,2}$, $I_{1,1}$, $I_{1,-1}$, $I_{-1,1}$, $I_{2,1}$, $I_{2,-1}$, $I_{2,-2}$. The integrals for $m=0$ or $n=0$ may be easily evaluated by choosing c or d as the polar axis for the integration over θ', then

$$I_{1,0}=(p'^{2}+T^{2})^{-1}\int_{-1}^{1}d(\cos\theta'_{p',c})(1-\mathrm{p}'\cdot\mathrm{c})^{-1}$$
$$=(p'c)^{-1}(p'^{2}+T^{2})^{-1}\ln[(1+p'c)/(1-p'c)]$$
$$=(p'T)^{-1}\beta_{T}\ ,$$

where $\beta_{T}=\ln[(T+p')/(T-p')]$, at $E\gg m$ $\beta_{T}\simeq(L+\beta')/2$, with

$$L=\ln\frac{EE'+pp'-m^{2}}{EE'-pp'-m^{2}}\simeq 2\ln\frac{2EE'}{m\omega}\ .$$

and

$$\beta'=\ln[(E'+p')/(E'-p')]\simeq 2\ln(2E'/m)\ ,$$

$$I_{2,0}=(p'^{2}+T^{2})^{-2}\int_{-1}^{1}d(\cos\theta'_{p',c})(1-\mathrm{p}'\cdot\mathrm{c})^{-2}$$
$$=2(p'^{2}+T^{2})^{-2}(1-p'^{2}c^{2})^{-1}=2(T^{2}-p'^{2})^{-2}\ ,$$

$$I_{0,1}=(E'p'b)^{-1}\ln[(1+p'd)/(1-p'd)]=\beta'/p'\ ,$$

$$I_{0,2}=2E'^{-2}(1-p'^{2}b^{2})^{-1}=2(E'^{2}-p'^{2})^{-1}\ .$$

The integrals such as $I_{1,-1}$, $I_{-1,1}$, $I_{2,-1}$, and $I_{2,-2}$ can be expressed in terms of the others by choosing c as the polar axis for the integration over θ'. For example,

$$I_{1,-1}=(p'^{2}+T^{2})^{-1}E'\int_{-1}^{1}d(\cos\theta'_{p',c})(1-\mathrm{p}'\cdot\mathrm{c})^{-1}(1-\mathrm{p}'\cdot\mathrm{d})$$
$$=(p'^{2}+T^{2})^{-1}E'\int_{-1}^{1}d(\cos\theta'_{p',c})(1-\mathrm{p}'\cdot\mathrm{d})^{-1}[1-(\mathrm{p}'\cdot\mathrm{c})(\mathrm{c}\cdot\mathrm{d})/c^{2}]$$
$$=(p'^{2}+T^{2})^{-1}E'I_{1,0}[1-\mathrm{c}\cdot\mathrm{d}/c^{2}]\ ;$$

using $\mathrm{c}=2\mathrm{T}/(p'^{2}+T^{2})$; $\mathrm{d}=\mathrm{k}/\omega E'$, and $2\mathrm{k}\cdot\mathrm{T}=p^{2}-T^{2}-\omega^{2}$, one finds

$$I_{1,-1}=\frac{E'}{2Tp'}\beta_{T}[1-(p^{2}-T^{2}-\omega^{2})(T^{2}+p'^{2})/4\omega ET^{2}]\ ,$$

$$I_{-1,1}=(p'^{2}+T^{2})E'^{-1}I_{0,1}[1-\mathrm{c}\cdot\mathrm{d}/d^{2}]$$
$$=\frac{p'^{2}+T^{2}}{p'}\beta'-\frac{p^{2}-T^{2}-\omega^{2}}{\omega}\ .$$

The integrals $I_{1,1}$, $I_{2,1}$ may be easily evaluated by using the Feynman integral

$$(\alpha\gamma)^{-1}=\int_{-1}^{1}\frac{dx}{[\alpha x-\gamma(1-x)]^{2}}\ , \quad \text{(B3)}$$

and those find by differentiating (B3) with respect to α and γ. For example

$$I_{1,1}=(p'^{2}+T^{2})^{-1}E'^{-1}\int d(\cos\theta'_{p',c})(1-\mathrm{p}'\cdot\mathrm{c})^{-1}(1-\mathrm{p}'\cdot\mathrm{d})^{-1}$$
$$=(p'^{2}+T^{2})^{-1}E'^{-1}\int_{0}^{1}dx\int_{-1}^{1}d(\cos\theta'_{p',h})(1-\mathrm{p}'\cdot\mathrm{h})^{-2}\ ,$$

where $\mathrm{h}=\mathrm{c}x+\mathrm{d}(1-x)$. If h is now as the polar axis, the integration over θ' may be performed, giving $I_{1,1}=2(p'^{2}+T^{2})^{-1}E'^{-1}\int_{0}^{1}\xi^{-1}dx$, where ξ is given as a quadratic in x:

$$\xi=1-p'^{2}h^{2}=1-p'^{2}c^{2}-2xp'^{2}(\mathrm{c}\cdot\mathrm{d}-b^{2})-x^{2}p'^{2}(\mathrm{c}-\mathrm{d})^{2}\ ,$$

then one can obtain $I_{11}=\omega(p'p)^{-1}(T^2-p'^2)^{-1}L$. By a similar method one also gets

$$
\begin{aligned}
I_{21} &= 4(p'^2-T^2)^{-2}E'^{-1}\int_0^1 x\xi^{-2}\,dx \\
&= \frac{-32p'^4p^2}{\omega^2E'^4}\left[\frac{p^2-T^2-\omega^2}{(p'^2-T^2)^3}-\frac{4\omega E'T^2}{(p'^2-T^2)^4}\right]-\frac{\omega^2}{(p'p^3)(T^2-p'^2)}[E'(p^2-T^2-\omega^2)-\omega(p'^2-T^2)]L
\end{aligned}
$$

[1] J. Weber and G. Hinds, Phys. Rev. **128**, 2414 (1962).
[2] S. Weinberg, Phys. Rev. **140**, B512 (1965).
[3] M. Carmeli, Phys. Rev. **158**, 1248 (1966).
[4] D. Boccaletti and F. Occhionero, Nuovo Cimento Lett. **2**, 549 (1971).
[5] D. Boccaletti, Nuovo Cimento Lett. **4**, 929 (1972).
[6] G. Papini, Can. J. Phys. **52**, 880 (1974).
[7] G. Papini and S. R. Valluri, Can. J. Phys. **53**, 2306 (1975); **53**, 2312 (1975); **53**, 2315 (1975); **54**, 76 (1976); **56**, 801 (1978); Phys. Rep. **33C**, 51 (1977); Astron. Astrophys. **208**, 345 (1989).
[8] P. D. B. Collins, A. D. Martin, and E. J. Squires, *Particle Physics and Cosmology* (Wiley, New York, 1989), Chap. 11.
[9] See, for instance, G. 't Hooft and M. Veltman, Ann. Inst. Henri Poincare **20A**, 69 (1972); B. S. DeWitt, Phys. Rev. **162**, 1239 (1967).
[10] S. Deser and P. van Nieuwenhuizen, Phys. Rev. D **10**, 411 (1974).
[11] S. Deser, in *Quantum Gravity*, Proceedings of the Oxford Symposium, edited by C. J. Isham, R. Penrose, and D. W. Sciama (Oxford University Press, Oxford, 1975).
[12] H. Pagels, Phys. Rev. **144**, B1250 (1966).
[13] M. D. Scadron, *Advanced Quantum Theory* (Springer, New York, 1979), Chap. 14.
[14] S. Weinberg, Phys. Rev. **135**, B1049 (1964).
[15] Y. S. Vladimirov, Zh. Eksp. Teor. Fiz. **45**, 251 (1963) [Sov. Phys. JETP **18**, 176 (1964)].
[16] V. B. Berestetskii, E. M. Lifshitz, and L. P. Pitaevskii, *Quantum Electrodynamics*, 2nd ed. (Pergamon, Oxford, 1982), Sec. 93.
[17] R. L. Gluckstern and M. H. Hull, Phys. Rev. **90**, 1030 (1953).
[18] H. A. Bethe and W. Heitler, Proc. R. Soc. London **A146**, 83 (1934).
[19] J. M. Jauch and F. Rohrlich, *The Theory of Photons and Electrons*, 2nd ed. (Springer, New York, 1976), Chap. 15.
[20] H. Weyl, Z. Phys. **56**, 330 (1929).