PRODUCTION OF CHARGINOS AND NEUTRALINOS FOR THE REACTION

$e^+e^- \rightarrow Z' \rightarrow \tilde{\chi}_i^+\tilde{\chi}_j^-, \tilde{\chi}_i^0\tilde{\chi}_j^0$

IN $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$

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Abstract

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All left-right models contain a second Z^0, Z'. If the Z' is massive enough it will decay into charginos $\tilde{\chi}_i^{\pm}$ and neutralinos $\tilde{\chi}_i^0$. The particular model being considered is an extension of the supersymmetric standard model (SSM) which has been modified to include both left and right symmetry groups: $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. In this thesis the cross sections have been calculated for the production of charginos and neutralinos in the reaction $e^+e^- \rightarrow Z' \rightarrow \tilde{\chi}_i^+\tilde{\chi}_j^-$ or $\tilde{\chi}_i^0\tilde{\chi}_j^0$. Evaluating the Z'-reaction at the center-of-mass energy up to 2 TeV for different sets of parameters.

Choosing masses of $M_{Z'} = 400$ GeV and $M_{\tilde{\chi}_2^0} = 63\,GeV$, $M_{\tilde{\chi}_2^\pm} = 54\,GeV$, with $g_R \cong g_L$, $\Gamma_{Z'} \cong 12\,GeV$, $M_R = 300\,GeV$, $M_L = 50\,GeV$ and $\mu = -10\,GeV$ the cross sections were found to be: $\sigma(e^+e^- \rightarrow Z' \rightarrow \tilde{\chi}_2^+\tilde{\chi}_2^-) \cong 0.21\,nb$ and $\sigma(e^+e^- \rightarrow Z' \rightarrow \tilde{\chi}_2^0\tilde{\chi}_2^0) \cong 77\,pb$.

Choosing masses of $M_{Z'} = 400$ GeV and $M_{\tilde{\chi}_2^0} = 171.7\,GeV$, $M_{\tilde{\chi}_2^\pm} = 80\,GeV$, with $g_R \cong g_L$, $\Gamma_{Z'} \cong 12\,GeV$, $M_R = 1000\,GeV$, $M_L = 250\,GeV$ and $\mu = -80\,GeV$ the cross sections were found to be: $\sigma(e^+e^- \rightarrow Z' \rightarrow \tilde{\chi}_2^+\tilde{\chi}_2^-) \cong 0.11\,nb$ and $\sigma(e^+e^- \rightarrow Z' \rightarrow \tilde{\chi}_2^0\tilde{\chi}_2^0) \cong 15\,pb$.

Finally, Choosing $M_{\tilde{\chi}_i} \neq M_{\tilde{\chi}_j}$ masses of $M_{Z'} = 400$ GeV and $M_{\tilde{\chi}_1^0} = 158\,GeV$, $M_{\tilde{\chi}_2^0} = 171.7\,GeV$, $M_{\tilde{\chi}_2^\pm} = 80\,GeV$, $M_{\tilde{\chi}_1^\pm} = 45.3\,GeV$, with $g_R \cong g_L$, $\Gamma_{Z'} \cong 12\,GeV$, $M_R = 1000\,GeV$, $M_L = 250\,GeV$ and $\mu = -80\,GeV$ the

cross sections were found to be: $\sigma\left(e^+e^- \to Z' \to \tilde{\chi}_2^+\tilde{\chi}_1^-\right) \cong 0.32\,nb$ and $\sigma\left(e^+e^- \to Z' \to \tilde{\chi}_2^0\tilde{\chi}_1^0\right) \cong 48\,pb$.

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Introduction

The last twenty years have seen a remarkable revamping of the pre-existing concepts of particle physics. A highly inhomogeneous system of describing particle interactions via phenomenology has given way to what we now refer to as the standard model, a theory which describes in detail, all of the available data from the world of high energy physics.

The standard model (SM) actually consists of two separate parts; the Glashow-Weinberg-Salam (GWS) theory, (based on $SU(2)_L \times U(1)_1$ gauge symmetry), which may be described as a quantum gauge field theory which unifies electromagnetic and weak interactions; and quantum chromodynamics (based on $SU(3)_C$ symmetry), the modern field theory of the strong interactions. Both are gauge theories, and it is because of this, that they are thought to represent the low energy limit of a single comprehensive gauge field theory which would be responsible for the unification of all forces known to exist in nature.

Some of the problems associated with the standard model indicate that there remain fundamental topics of concern which have yet to be properly addressed. A fundamental theory should be wholly predictive, and yet , the standard model is subject to the perils of arbitrary parameterization. Masses of the particles and the coupling strength of their interactions must be inserted "by hand".

In fact, in the standard model the coupling strengths do not even appear to be related in any logical manner.

Grand unified theories (GUTS), predict that at a high energy scale, located somewhere above a unification scale M_X, all phenomena satisfy a single symmetry G. Spontaneous breaking of the symmetry of the group G would then be responsible for generating the different couplings g_s, g, g', which arise at low energies. There exists at present, very little evidence to support these ideas.

One of the fundamental difficulties with these theories has become known as the "gauge hierarchy problem". The standard model incorporates a scalar sector, which remains, as yet, untested. The Higgs particles associated with this part of the model have particularly appealing qualities, one of which is the ability to possess a non-vanishing vacuum expectation value (VEV) without breaking Lorentz invariance. Perturbation theory, however, yields quadratic divergences in the masses, pushing them to the order of the Planck mass unless the perturbation series were to cancel to 26 decimal places. This is the essence of the gauge hierarchy problem (GHP).

One very elegant way of dealing with the GHP and simultaneously incorporate gravity into the unified models is to introduce a boson-fermion symmetry or supersymmetry. Supersymmetry resolves the GHP by including a bosonic/fermionic partner for every respective fermion/boson in the nonsupersymmetric theory. Unbroken supersymmetry has the bosonic-fermionic partners possessing equal masses and equal coupling strengths. Thus, in this theory, the mass of the Higgs Boson is not renormalized, since boson and fermion loops give contributions of equal and opposite signs to the perturbation expansion. In broken supersymmetry, the Higgs mass is renormalized, but provided that the mass of the "partner" states is small enough, the Higgs mass will remain small.

Another of the problems associated with the standard model is the "skew" nature, or inherent left-handed symmetry. The parity associated with the weak interactions leaves one questioning the reason for which nature has chosen to exhibit only left-handed particle states. In recent years, models have been proposed which try to explain the reasons for this handedness, associating left-handed particles with a lower energy scale, and proposing the existence of right-handed equivalents at, higher energy scales.

The model which is used in this thesis addresses the problems listed above by incorporating into a single theory, the concepts of left-right symmetry and supersymmetry. The subject of interest in this work involves the proposed production of neutralino and chargino pairs in electron-positron collisions. These paticle states , which arise via supersymmetry, are of interest since they may give rise to very clean experimental signatures. The production of these states in right-handed Z^0-boson decay, and their scattering cross-section calculations are the focus of this thesis.

Chapter 1

The Supersymmetric Standard Model (SSM)

1.1 Introduction

Before outlining the supersymmetric standard model, it is instructive to first describe the standard model in order that its shortcomings might be presented as motivation for extending the model. The standard theory of electroweak interactions (the Glashow-Weinberg-Salam Model), is a unified quantum gauge field theory based on the gauge group $SU(2)_L \times U(1)_Y$, which combines the electromagnetic interactions with the weak interactions[1]. The combination with quantum chromodynamics, a theory of strong interactions[2] based on $SU(3)$ group symmetry, seems to yield a highly satisfactory description of the fundamental interactions of quarks and leptons. The Electro-Weak theory is well tested and predictive. Quantum chromodynamics by contrast, has not been as thoroughly tested.

While this partially unified theory of particle interactions has been shown to be quite successful in predicting the existence of such phenomena as neutral currents and the existence of the W^+, W^- and Z^0 intermediate vector bosons, there remain several untested areas; for example, the "top-quark", a fermion state of the theory is predicted to exist in the 20-200 GeV energy range[3] and has almost been located experimentally.

Similarly, the Higgs sector[4] of the theory (containing spin-0 particles) has yet to be experimentally confirmed. It is, however, an essential component of the theory, as it is responsible for generating the masses of the fermion and gauge boson masses of the theory, via spontaneous symmetry breaking. In the case of particle physics, perturbation theory is used to calculate fluctuations around a minimum energy. Expanding around an unstable point yields a divergent series. Expanding about a stable vacuum, on the other hand, yields a converging series, and a mass for the Higgs particle. Higgs particles also possess very interesting and useful properties including their ability to have a non-vanishing vacuum expectation value (VEV), while maintaining Lorentz invariance. Perturbation theory yields quadratic divergences for the masses of the Higgs particles. One approach to eliminating such divergences in the Higgs sector is to invoke supersymmetry, as will be explained below.

1.2 Supersymmetry and its Application to Particle Physics

1.2.1 Motivation for Supersymmetry

The current understanding of field theories which can be used to describe fundamental physical phenomena qualifies a "particle" as one of two possible types of quantized field, the boson, or the fermion. It is the bosons (in particular, vector fields of spin-1), which are responsible for mediating forces, while fermions (spin-1/2) are the "stuff" of which matter (quarks and leptons) is made.

One of the reasons supersymmetry has been the focus of so much attention over the last two decades, is that its application to particle physics allows many of the inherent problems of existing theories to be dealt with naturally. Supersymmetry combines bosons and fermion into multiplets. This removes the distinction between matter and interaction. Each fermion is given an equivalent bosonic partner and visa versa. Thus, both the particle and the "superpartner" can be considered to be carriers of force, but fermions, are subject to the constraints of the Pauli exclusion principle and as a result, cannot contribute to coherent potentials. Thus it is only phenomenology which distinguishes matter from forces; i.e., since no two identical fermions may occupy the same point in space-time they manifest themselves as physical "particles", while the classical fields which arise from the superposition of bosons, yield an impression of the presence of "forces".

Initially it was thought that symmetries which transform "forces" into "matter" would be at odds with field theory. Coleman and Mandula[5] showed, in 1967, that internal symmetries cannot be unified with space-time symmetries within the context of a relativistically-invariant field theory. This "no-go theorem" states that charge operators which have eigenvalues which represent internal quantum numbers, must commute with energy, momentum and angular momentum operators, thus implying translational and rotational invariance. The generators of the Lorentz transformations are in fact, the only symmetry generators which translate under both translations and rotations. Thus, eigenstates which yield different eigenvalues of mass and spin operators cannot be related by generators of internal symmetries, i.e., particles whose masses or spins are not identical cannot be contained within an irreducible multiplet of symmetry groups.

Fortunately, the theorem applies only to symmetry transformations which adhere to real Lie algebras. The generators of Lie groups of symmetry transformations are charge operators which obey well defined commutation relations. If symmetry operations which have generators which obey "anticommutation" relations are included, it becomes possible to include particles with different spins in the same multiplet. In 1973, Wess and Zumino[6] proposed a renormalizable model involving the interaction of two spin-0 particles with a spin-1/2 particle. These particles were related by a symmetry transformation, thus they could be placed in the same multiplet.

The true importance of supersymmetry lies within its ability to address two fundamentally important issues. First, there exists the possiblility of resolving the gauge hierarchy problem. Secondly, and perhaps more importantly, the possibility of unifying all known forces resides in the nature of supersymmetry. It turns out that "local" supersymmetry implies "supergravity" and thus provides a natural framework in which to attempt to link all of the fundamental interactions.

1.2.2 The Algebra of Supersymmetry

It has been shown that supersymmetry can be used to transform bosons into fermions and visa versa. In this section, a brief explaination of the symmetry which is responsible for this possibility shall be provided. The generator Q, of a transformation which turns a half-integral spin entity into an integral spin entity must be fermionic in character. Thus we may designate it as Q_α, a left-handed Weyl spinor which transforms as a (1/2,0) representation under the set of Lorentz transformations. Its Hermetian adjoint, which transforms as (0,1/2) should be denoted $\overline{Q}_\beta$. It can be shown that the anticommutator of any operator with its adjoint is non-zero. We therefore have:

$$\{Q_\alpha, \overline{Q}_\beta\} \neq 0 \tag{1.1}$$

$\{Q_\alpha, \overline{Q}_\beta\}$ transforms as (1/2,1/2) under the Lorentz transformations. There exists only one operator (the energy-momentum tensor P_μ) which transforms in this manner. Then, Q_α and P_μ must be related entities. Thus, it may be shown that the supersymmetry algebra[7] is defined by:

$$[Q_\alpha, P_\mu] = 0 \tag{1.2}$$

$$\{Q_\alpha, \overline{Q}_\beta\} = 2\sigma^\mu_{\alpha\beta} P_\mu \tag{1.3}$$

where the σ^μ are the Pauli matrices and the constant 2 is invoked for the sake of convenience.

Choosing $\mu = 0$ in eq.(1.2) shows that the Hamiltonian $H = P_0$ commutes with the generator Q. Thus, all non-zero energy states are paired by the action of Q. As Q is fermionic in nature, this pairing condition requires that all supersymmetric multiplets contain one degree of bosonic freedom for every degree of fermionic freedom. These "partners" must then, have equal masses.

Eq.(1.3) relates the Hamiltonian to the supercharges, and using $\sigma^\mu_{\alpha\beta}\sigma^{\alpha\beta}_\nu = 2g^\mu_\nu$, the important result

$$H = P_0 = \tfrac{1}{4}\left(\overline{Q}_1 Q_1 + Q_1 \overline{Q} + \overline{Q}_2 Q_2 + Q_2 \overline{Q}_2\right) \tag{1.4}$$

may be derived, showing that the energy must be ≥ 0.

If we consider a globally supersymmetric theory (not broken spontaneously) in which we define $|0\rangle$ as the vacuum state, then we must have $Q_a|0\rangle=0$. Then, since the anticommutator of Q is the Hamiltonian, we find that the energy of the vacuum is

$$E_{vac} \equiv \langle 0|H|0\rangle . \tag{1.5}$$

Spontaneously breaking the symmetry requires that Q acting on the vacuum be non-zero: $Q_a|0\rangle \neq 0$, leaving a non-zero vacuum energy.

1.2.3 Implications of Supersymmetry

First, we address the question of parity in supersymmetric theories. A quantum number designated R-parity[8] must be assigned to all particles in order to maintain a coherent theory. It may be defined:

$$R_p = (-1)^{3B+L+2S} = \begin{cases} +1\text{: ordinary particle} \\ -1 \text{ supersymmetric particle} \end{cases} \tag{1.6}$$

where B is the Baryon number

L is the Lepton number

S is the spin .

We see that the conservation of baryon and lepton numbers yields conservation of R-parity. Replacement of a normal particle with a supersymmetric particle leads to violation of angular momentum conservation by a value of $\pm\frac{1}{2}$.

R-parity carries with it some important consequences which are relevant to the production of chargino and neutralino states:

1) Normal particles always give rise to PAIRS of supersymmetric particles.

2) Supersymmetric particle decay will always yield supersymmetric products.

3) The lightest supersymmetric particle (LSP) will be stable.

An outline of the spectrum of supersymmetric particles is given in Ref[9]. As the degrees of bosonic and fermionic freedom must always be equal, every particle must have a superpartner. Thus, every quark will have a corresponding "squark", each lepton a "slepton", etc. ... . Similarly, the gauge bosons will have fermions for partners: "photinos" to photons, "gluinos" to gluons, "winos" to W's, "zinos" to Z's and "higgsinos" to Higgs'. The various particles, superpartners and their spins are listed in Table I.

The couplings associated with the supersymmetric theories are in fact the same as those found in the standard model. The interaction portion of the Lagrangian corresponding to a fermion transforms under supersymmetry according to [10]:

$$g\bar{f}_L \gamma_\mu f_L A^\mu \rightarrow \sqrt{2}\, g \bar{f}_L \tilde{A}_L \tilde{f}_l \; + \text{H.C.} \tag{1.7}$$

where $\tilde{A}_L$ is the corresponding gaugino, $\tilde{f}_L$ the "squark" or "slepton". Thus, the strengths of the couplings remain unaffected by the supersymmetry transformations.

Unbroken supersymmetry would have particles and their superpartners degenerate in mass, but as these "theoretical" particles have not been observed in nature, we require "spontaneously broken supersymmetry". If the symmetry is brcken at a scale $\leq$ 1 TeV, mass degeneracy is removed while couplings remain unaffected.

An interesting result arising from supersymmetry theory is that winos and charged higgsinos carry the same value of conserved quantum numbers. As a result, they may intermix to produce particles appropriately deemed "charginos". Similarly, neutral gauginos mix with neutral higgsinos to yield the "neutralinos". The interactions of these quantum fields are the essential focus of this thesis.

1.3 The Gauge Hierarchy Problem (GHP)

One of the most appealing qualities of supersymmetric models is that they can be used to tackle the gauge hierarchy problem. The nature of the problem shall be addressed here, along with an explanation of how SUSY models work to resolve it.

It has been mentioned that the standard model provides a highly accurate description of currently visible phenomenology, but fails in its inability to explain many fundamental isssues, among them, values of constants including charge and coupling strengths. It is expected that the solution to these questions lies within the framework of a more fundamental theory than the standard model. Based on grand unified theories (GUTS), it is predicted that the unification of all forces can only be accomplished at energies which lie between 10^{14} GeV and 10^{19} GeV (the Planck scale - the point at which gravity becomes important). Some argue that no new physics should exist between the energy scales currently being investigated (10^2 GeV) and the GUTS scale, but this view leads to certain fundamental issues; in particular, those which involve linking widely separated scales in a natural manner: the Higgs' vacuum expectation value (≅250 GeV) and the GUT scale.

The GHP exists only if certain assumptions are made:

1) The standard model may successfully describe physics which occurs up to energies $\cong M$ much higher than the 250 GeV weak scale.

2) At the scale M, the standard model breaks down, and a new physics occurs - possibly GUTs, gravity, etc. ... .

3) The physics of low energies is not highly affected by the values of fundamental parameters of high enegy scales.

While the first two assumptions are self explanatory, the third requires some qualification: The standard model is a quantum field theory and as such, it has certain constraints placed upon it. The path integral which defines such a theory is only defined in so much as its short-distance content may be regularized, i.e. it requires a "cutoff" distance in order not to diverge. Thus, a prescription which specifies a cutoff momentum, (as well as a coupling constant g and a mass parameter μ (both generally momentum dependent), must be employed. This leaves us with a defined theory (for momenta less than k).

Renormalization theory allows us to gather quantum effects involving momenta larger than k, and place them in the values of the constants g and μ - transforming them into functions of k. The parameters g and μ ,then, depend only on physics which occurs at length scales less than k^{1} in magnitude.

In order to illustrate the difficulties introduced by the GHP, it is instructive to consider a hypothetical unified field theory. If we demand that this theory contain no other low energy fields than those required by the standard model, then, for a cutoff k less than M, the low energy physics should be that of the standard model. This should include the gauge couplings $g_3(k), g_2(k), g_1(k)$, the Yukawa coupling , $g(k)$, quartic scalar couplings $\lambda(k)$ and the mass parameter $\mu^2(k)$.

$\mu^2(k)$ is the negative mass squared of the Higgs fiels. The associated potential is

$$V(\phi) = \lambda\phi^4 - \mu^2\phi^2 \tag{1.8}$$

These parameters are non-trivially dependent on the construction of the particular unified field theory at distances shorter than M^{-1}.

The standard model dictates that all particle masses are directly proportional to the VEV of the Higgs, $\langle\phi\rangle$;

$$\text{quark and leptor masses: } m_{q,l} \cong g_Y\langle\phi\rangle \tag{1.9}$$

$$\text{Z and W masses: } m_{Z,W} \cong g_2\langle\phi\rangle \tag{1.10}$$

$$\text{Higgs mass: } m_H \cong \sqrt{\lambda}\langle\phi\rangle \tag{1.11}$$

The VEV of the Higgs is directly calculable from $\lambda(k)$, $\mu(k)$, $g_1(k)$, and $g(k)$. Minimizing the potential $V(\phi)$ (ignoring quantum fluctuations), it is found that:

$$\langle\phi\rangle = \mu(k)/2\sqrt{\lambda(k)} \quad . \tag{1.12}$$

If, however, quantum fluctuations become important, i.e. when the cutoff is $\cong M$, k is no longer a reasonable momentum limit. Instead, a low energy cutoff, k', should be used - 1 TeV as an example. As k'<<< k, an approximation of k'=0 is acceptable, leaving:

$$\langle\phi\rangle \cong \mu(0)/2\sqrt{\lambda(0)} \quad . \tag{1.13}$$

Therefore, a reasonable approximation of the Higgs' VEV can be determined via the "renormalized" mass $\mu(0)$ and coupling $\lambda(0)$ calculations.

The source of the GHP may be illustrated by calculating $\mu^2(0)$. The individual contributions generated in the $\mu(0)$ series are all cutoff dependent, and proportional to k^2 ,

$$\therefore \ \mu^2(0) = \mu^2(k) + k^2\left(C_1\lambda_1 + C_2 g^2 + \cdots\right). \tag{1.14}$$

It is expected that the parameters fit the unified field theory when k is at its maximum, i.e. order of M, such that

$$\mu^2(0) = \mu^2(M) + M^2\left(C_1\lambda(M) + \cdots\right) \quad . \tag{1.15}$$

Since $\mu(0)$ should be $\cong\langle\phi\rangle\sqrt{\lambda(0)}$, then with a typical value of $\sqrt{\lambda}=1$, $\mu(0)$ is of the order 10^2 GeV.

For a scale $M \cong 10^{15}$ GeV, eq.(1.15) would be written:

$$\mu^2(0)/M^2 = 10^{-26} = \mu^2(M)/M^2 + (C_1\lambda + \cdots) \qquad (1.16)$$

What this means is that in order for the known particle spectrum to be what it is, $\mu^2(M)/M^2$ must cancel out the series to 26 decimal places. $\mu^2(M)/M^2$, however , is only relevant to small distance physics, and so the liklihood of such a cancellation occuring is inconceivable within the bounds of probability theory. There is then, no justifiable explaination for the existence of two scales separated by an energy difference of 10^{13} GeV, as this sort of division would be deemed "unnatural". This is the "technical GHP", the more commonly addressed GHP being the philosophical question as to why there should exist two such varied scales of energy.

Invoking supersymmetry allows us to do away with the GHP by means of natural cancellations for every contribution of the series. Every bosonic partner of a fermionic particle (and similarly fermionic partners for bosons) couples with the same strength and possess the same mass as the original particle state. Contributions to the Higgs mass are then equally bosonic and fermionic. Bosonic contributions carry an opposite sign from fermionic contributions and thus, the divergences which arise in the standard model are cancelled out in supersymmetric theories.

The coefficients $C_1, C_2, \ldots$ of the series vanish, leaving a divergenceless series.

It must be noted, however, that superpartners of known particle states would have been detected by this point if in fact such states possessed degenerate masses. To date, no such states have been detected. It is necessary then, that supersymmetry is broken, leaving superpartners with different masses than their standard model counterparts. This being the case, divergences re-emerge in the Higgs mass series. These "new" divergences are much more manageable than those arising from the original series. This yields a much more appealing situation, as the problem of naturalness or grand unified theories does not resurface.

1.4 Broken Supersymmetry

It is well understood that if nature is intrinsically supersymmetric, then supersymmetry must be broken if the existing particle spectrum is to be explained. Mechanisms of supersymmetry breaking have been thoroughly investigated. It has been shown that only two such mechanisms exist; **1)** the O'Raifeartaigh[11] (or F-type) mechanism, and **2)** the Fayet-Illiopoulos[12] (or D-type) mechanism. These techniques however, impose strict constraints on the theory. This makes for tedious and undesirable calculation.

The alternative is to break supersymmetry "by hand", which is accomplished by adding explicit mass terms to the

Lagrangian of the theory. This requires care, since only certain types of terms will leave the theory "renormalizable". Giardello and Grisaru[13] have presented a comprehensive list of mass terms which will not introduce quadratic divergences. They are deemed "soft-breaking" terms:

$$\tilde{M}_1 \operatorname{Re} A^2 + \tilde{M}_2 \operatorname{Im} A^2 + C\left(A^3 + h.c.\right) + \tilde{M}_3\left(\lambda^a \lambda^a + \overline{\lambda}^a \overline{\lambda}^a\right) + \tilde{M}_4\left(\lambda' \lambda' + \overline{\lambda}' \overline{\lambda}'\right) \quad (1.17)$$

- A^2 and A^3 are group invariant combinations of the scalar fields A_i (eg. $A^3 = d_{ijk} A_i A_j A_k$, etc. ...).
- λ is the fermionic partner of the gauge boson.
- $\tilde{M}_1$ is the parameter which distinguishes the mass of the complex scalar field A_i from its fermionic partner ψ_i.
- If A_i is expressed in terms of two real spin zero fields, then $\tilde{M}_2$ splits the masses of these fields.
- The coupling constant C corresponds to a new (non-supersymmetric) scalar interaction term.
- $\tilde{M}_3$ and $\tilde{M}_4$ are Majorana mass terms for the gauginos corresponding to the groups G (non-abelian group) and $U(1)$.

1.5 The Higgs Sector in Supersymmetric Theories

The standard model uses the Higgs field $,\phi,$ to generate masses for the quarks, leptons and intermediate vector bosons. Masses can be given to both "up" and "down" quarks because ϕ forms an $SU(2)$ doublet with its complex conjugate. This, then allows for Yukawa couplings which take the form:

$$g_D Q_L \phi D_R + g_u Q_L \phi^* U_R \tag{1.18}$$

Supersymmetry has such couplings developing from cubic terms in the superpotential W, which are formed from chiral supermultiplets of the same chirality. Now, if ϕ is defined as a left-handed chiral supermultiplet, ϕ^* will be right-handed. Q_L, D_R and U_R will be left-handed fields. Thus, $Q_L \phi Q_R$ couplings are permitted, but $Q_l \phi U_R$ couplings are not. This difficulty may be solved by introducing a second Higgs doublet, $\tilde{\phi}$, which transforms like ϕ^* under the electroweak group, but which is left-handed. Therefore eq. (1.18) may be replaced by the terms

$$g_D Q_L \phi D_R + g_u Q_L \tilde{\phi} U_R \tag{1.19}$$

in the superpotential W.

1.6 Supersymmetric Lagrangians

Haber and Kane[14], have provided an in depth discussion regarding the construction of interaction Lagrangians. Here we outline some of the major points presented in Ref.[14].

Supersymmetric gauge theories are constructed to combine gauge bosons V_μ^a with their (two component) gaugino fermionic partners λ^a in the adjoint representation of the gauge group G, along with matter fields consisting of complex scalar fields A_i with two component fermions ψ_i which transform under some representation R of G. There are then, three different types of interactions to consider. They are:

1) <u>Self-interaction of the gauge multiplets</u>: These terms consist of three- and four-gauge boson vertices and involve the covariant derivative $D_\mu = \partial_\mu + \frac{1}{2} g\left(\tau^a \cdot W_\mu^a\right)$ (where the τ^a are the usual Pauli matrices). In addition to boson-boson interactions, gauginos also interact with the gauge field via the term

$$ig f_{abc} \lambda^a \sigma^\mu \overline{\lambda}^b V_\mu^c , \tag{1.20}$$

where the f_{abc} are the structure constants of G.

2) <u>Interactions of the gauge and matter multiplets</u>:

The interaction terms are:

$$-gT^a_{ij}V^a_\mu\left(\overline{\psi}_i\overline{\sigma}^\mu\psi_j + iA^*_i\overleftrightarrow{\partial}_\mu A_j\right) + ig\sqrt{2}T^a_{ij}\left(\lambda^a\psi_jA^*_i - \overline{\lambda}^a\overline{\psi}_iA_j\right) + g^2\left(T^aT^b\right)_{ij}V^a_\mu V^{\mu b}A^*_iA_j\,, \tag{1.21}$$

where $T^a = \tau^a/2$ is the Hermitian group generator in the representation R.

3) <u>Self-Interaction of the matter multiplet</u>:

It is essential to introduce some notation at this point. The superpotential W is some cubic gauge-invariant function of the scalar matter fields A_i (independent of A^*_i). We define the auxiliary functions:

$$F_i = \partial W/\partial A_i\,, \tag{1.22}$$

$$D^a = gA^*_iT^a_{ij}A_j\,. \tag{1.23}$$

This allows us to write the ordinary scalar potential V as:

$$V = \tfrac{1}{2}D^aD^a + F^*_iF_i\,. \tag{1.24}$$

The Yukawa interactions are given by:

$$-\tfrac{1}{2}\left[\left(\partial^2 W / \partial A_i \partial A_j\right)\psi_i\overline{\psi}_j + \left(\partial^2 W / \partial A_i \partial A_j\right)^* \psi_i\overline{\psi}_j\right] \qquad (1.25)$$

These formulae may be generalized to groups G which contain a U(1) factor. Accordingly, the above interactions would be modified in the following way:

i) No interaction between the U(1) gaugino λ' and the U(1) gauge field V_μ': Set $f_{abc} = 0$.

ii) U(1) gauge multiplet: the term $gT_{ij}^a V_\mu^a$ is replaced in eq. (1.20) by $\frac{1}{2}g' y_i \delta_{ij} V'_\mu$', (no sum over i), where y_i is the U(1) quantum number of the matter multiplet (A_i, ψ_i).

Chapter 2

The Left-Right Supersymmetric Model

2.1 Introduction

It has been shown that the standard model, based on $SU(3)_C \times SU(2)_L \times U(1)_Y$,[15] symmetry has served as a highly effective means of describing low-energy phenomena. The model, however, cannot be a complete description of the "nature" of physics, since it fails to address certain fundamental questions. Having introduced the minimal supersymmetric standard model in the previous chapter, it is time to introduce the motivations behind extending such models to incorporate left-right symmetry.

2.2 Motivations for the Model

The original motivation for the extension of the minimal supersymmetric standard model to include left-right symmetry was to investigate possible mechanisms for parity violation in weak interactions. Left-right- (LR) symmetric models provide a framework in which weak interactions obey all space-time symmetries, along with the strong, electromagnetic and gravitational interactions. In such models, the asymmetry observed at normal energies results from the non-invariance of the vacuum under parity symmetry[16]. Left-right models based on $SU(2)_L \times SU(2)_R \times U(1)$

symmetry arre especially appealing since they reproduce features of $SU(2)_L \times U(1)_Y$ at low energies.

Beyond the obvious appeal of parity explanations, there are other important reasons for considering left-right models. Most importantly, there is the question of the neutrino mass. If the neutrino does in fact have a mass, then the left-right models provide a natural framework in which to work. Then, there is the question of possible substructures to quarks and leptons. If forces acting at this level are similar to QCD, then there are arguments which suggest that $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, not $SU(2)_L \times U(1)_Y$ is the symmetry of weak interactions. The B-L (Baryon minus Lepton) quantum number is the only quantum number of the standard model which has not been gauged. As such, it implies that there exists a deeper symmetry structure, so that the replacement of $U(1)_Y$ (to which we can attach no physical significance) with $U(1)_{B-L}$, gives physical meaning to all generators of the theory.

CP violation[17] has been a hotbed for investigations which point toward left-right models. The CP-violations which arise in the Cabbibo-Kobayashi-Maskawa (CKM) parameterization of generation mixing for three generations are dependent upon only one parameter, called δ_{KM} (the KM phase). There exists no explanation for the milliweak strength of the CP violation. The suppression of V+A currents[18] can be used as an explanation of CP-violation strength in left right models where only two generations are present.

2.3 Description of the Model

2.3.1 Notation

"Component fields" are used throughout the description of the model. Fields and superpartners are denoted by the same symbol, the latter being distinguishable from the former by a tilde placed over the symbol. The exception to the rule is the case of gauge bosons and gauginos which are represented by different symbols. Two-component spinor notation is used and all conventions are described in Appendix A.

Taking leptons as an example, we have:

$$L \equiv \begin{pmatrix} \nu \\ e \end{pmatrix}_L \text{, and its supersymmetric partner } \tilde{L}_L \equiv \begin{pmatrix} \tilde{\nu} \\ \tilde{e} \end{pmatrix}_L \quad . \quad (2.1)$$

In this case, the field introduce via supersymmetry is bosonic. Higgs fields (Δ_L), on the other hand, possess fermionic superpartners $\tilde{\Delta}_L$.

2.3.2 The Fields

A) Matter Fields:

In order to simplify the description, only a single generation of quarks and leptons is considered. $SU(2)_{L,R}$ doublets of two-component spinors are classed into leptons and quarks:

$$L_{L,R} \equiv \begin{pmatrix} \nu \\ e \end{pmatrix}_{L,R} \quad ; \quad Q_{L,R} \equiv \begin{pmatrix} u \\ d \end{pmatrix}_{L,R} \tag{2.2}$$

Similarly, the supersymmetric partners are defined:

$$\tilde{L}_{L,R} \equiv \begin{pmatrix} \tilde{\nu} \\ \tilde{e} \end{pmatrix}_{L,R} \quad ; \quad \tilde{Q}_{L,R} \equiv \begin{pmatrix} \tilde{u} \\ \tilde{d} \end{pmatrix}_{L,R} \quad . \tag{2.3}$$

B) Gauge Fields:

$SU(2)_{L,R}$ gauge fields are triplets of vector bosons; $W^{\mu}_{L,R}$. The fermion superpartners are two-component gauginos; $\lambda_{L,R}$. $U(1)_{B-L}$ gauge fields are singlet vector bosons, V^{μ}, with superpartner gauginos, λ_V. The $SU(2)_{L,R}$ gauge couplings are respectively g_L and g_R. Similarly, the coupling $U(1)_{B-L}$ gauge coupling is g_V.

C) Higgs Fields:

There are many possibilities for the Higgs sector of the theory. The particular choice of interest involves two bi-doublets:

$$\Phi_d(\tfrac{1}{2},\tfrac{1}{2},0) \ , \ \Phi_u(\tfrac{1}{2},\tfrac{1}{2},0) \ , \qquad (2.4)$$

and four triplets

$$\Delta_R(0,1,2) \ , \ \Delta_l(1,0,2) \ , \qquad (2.5)$$

$$\delta_r(0,1,-2) \ , \ \delta_l(1,0,-2) \ . \qquad (2.6)$$

This is twice the number of fields required by non-supersymmetric theories. Φ_u is responsible for giving masses to the quarks of $SU(2)_{L,R}$ doublets, Φ_d , the lower. These have fermionic partners $\tilde{\Phi}_d$ and $\tilde{\Phi}_u$.

The members of the $\Delta_{L,R}$ triplets possess charges 0, +1 and +2 respectively. The superpartner $\tilde{\Delta}_{L,R}$ has the same charges as the quantum numbers are equal. As a result, triangle anomalies should be present in the fermionic sector of the theory. In order to counteract this effect, it is necessary to introduce two Higgs triplets $\delta_{L,R}$ and their superpartners , $\tilde{\delta}_{L,R}$ whose members possess $U(1)_{B-L}$ charges which give rise to electric charges of 0, -1 and -2. The "triangle-loop" contributions of the $\tilde{\delta}_{L,R}$ carry opposite signs to those of the $\tilde{\Delta}_{L,R}$ resulting in an overall cancellation. These $\delta_{L,R}$ Higgs and $\tilde{\delta}_{L,R}$ higgsino fields do not aquire VEV's and are not involved in the process of spontaneous symmetry breaking. In addition, there fields couple to very few fields in the theory. The entire LRSM Lagrangian is given in Ref[9].

2.4 Symmetry Breaking

This model has been constructed to contain four distinct symmetries before symmetry breaking, three gauge symmetries and one discreet parity symmetry, i.e. $g_l = g_R$. The breaking of these symmetries is accomplished in three stages:

$$SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times P \xrightarrow[M_P]{} SU(2)_L \times SU(2)_R \times U(1)_{B-L} \quad ,$$

$$SU(2)_L \times SU(2)_R \times U(1)_{B-L} \xrightarrow[M_{W_R}]{} SU(2)_L \times U(1)_Y \quad ,$$

$$SU(2)_L \times U(1)_Y \xrightarrow[M_{W_L}]{} U(1)_{em} \quad . \tag{2.7}$$

Parity alone is broken at the first stage at the M_P (Planck mass) scale[19]; here there is no gauge boson produced. The g_L and g_R couplings are then, no longer equal. In the second stage $\langle \Delta_R \rangle \neq 0$ is responsible for breaking $SU(2)_R \times U(1)_{B-L}$ to $U(1)_{B-L}$. It is possible to choose the Higgs multiplets in such a way as to break parity symmetry and $SU(2)_R$ symmetry at the same scale,i.e., $M_P = M_{W_R}$. The final breaking is brought about by $\langle \Phi \rangle \neq 0$ and $\langle \Delta_L \rangle \neq 0$.

In order to ensure that $U(1)_{em}$ remains unbroken,here, as in the standard model, only neutral Higgs fields are allowed to possess nonzero VEV's. Their values are given below:

$$\langle \Delta_L \rangle = \begin{bmatrix} 0 & 0 \\ \upsilon_l & 0 \end{bmatrix} \quad , \quad \langle \Delta_R \rangle = \begin{bmatrix} 0 & 0 \\ \upsilon_R & 0 \end{bmatrix} \quad , \quad \langle \Phi \rangle = \begin{bmatrix} \kappa & 0 \\ 0 & \kappa' e^{i\alpha} \end{bmatrix} \tag{2.8}$$

$\langle \Phi \rangle$ causes W_L and W_R to mix with a CP-violating phase $e^{i\alpha}$. In this paper the VEV's of the Higgs field have been taken to be:

$$\langle \Delta_R \rangle = \begin{bmatrix} 0 & 0 \\ \upsilon_R & \end{bmatrix}, \ \langle \Delta_L \rangle = \langle \delta_{L,R} \rangle \equiv 0, \ \langle \Phi_u \rangle = \begin{bmatrix} \kappa_u & 0 \\ 0 & 0 \end{bmatrix}, \ \langle \Phi_d \rangle = \begin{bmatrix} 0 & 0 \\ 0 & \kappa_d \end{bmatrix}. \tag{2.9}$$

Here, the assignments $\upsilon_L = 0$ and $\kappa' = 0$ have been made for the following reasons; $\upsilon_R >> \max(\kappa, \kappa') >> \upsilon_L$ is required strictly by arguments regarding phenomenological hierarchy[20]. $\kappa' = 0$ is necessary in order for flavour changing neutral currents to cancel as required.

Chapter 3

Vector Boson Mass-Eigenstates

3.1 Introduction

Weak interaction eigenstates aquire mass via the process of spontaneous symmetry breaking. These vector bosons have an associated mass matrix which must be diagonalized in order to yield "physical" mass eigenstates. This chapter focuses on the evaluation of these matrices, as their expressions are required in the determination of the gauge boson - chargino/neutralino cross sections.

The generation of vector boson masses in the LR-SUSY model can be separated into two stages. This is possible since the values of the VEV's chosen for the Higgs fields allows us to distinguish left and right breaking scales ($\upsilon_R >> \kappa_u, \kappa_d$ and $\kappa_u' = \kappa_d' = \upsilon_L = 0$). The first stage has $\langle \Delta_R \rangle$ generating masses for $W_R^{\pm}, W_R^0$ and V_μ. Subsequently, the mixing of the two neutral states W^0 and V_μ give rise to the physical fields Z_R and B_μ.

The second breaking takes place at a much lower energy scale. It involves $\Phi_{u,d}$, a field which couples to both left- and right-handed fields. This is responsible for mixing W_L and W_R, but to such a small degree that it warrants the treatment of right-handed fields as "effectively decoupled" at this energy. Thus, it is only $W_L^{\pm}, W_L^0$ and B which aquire masses at this level. The neutral fields W_l^0 and B mix and give rise to the Z_L^0 and A_μ bosons which are familiar from the standard model.

3.2 Right-Handed Vector Bosons

The Lagrangian term responsible for the first stage of boson mass generation is

$$+Tr\left|\left(-i\tfrac{g_R}{2}\,\tau\cdot W_\mu^R - ig_{1'}V_\mu\right)\tau\cdot\Delta_R\right|^2 \tag{3.1}$$

Substitution of the VEV $\langle\Delta_R\rangle$ in the place of Δ_R yields the physical fields

$$Z_R = \frac{g_R W_R^0 - 2g_{1'}V}{\left(g_R^2 + 4g_{1'}^2\right)^{1/2}} \tag{3.2}$$

$$B = \frac{g_R V + 2g_{1'}V}{\left(g_R^2 + 4g_{1'}^2\right)^{1/2}} \tag{3.3}$$

with repective masses of

$$M_{Z_R} = \tfrac{1}{\sqrt{2}}V_R\left(g_R^2 + 4g_{1'}^2\right)^{1/2}, \tag{3.4}$$

and

$$M_B = 0. \tag{3.5}$$

B_μ is the massless gauge boson of the symmetry group $U(1)_Y$. $W_R^{\pm}$ and Z_R are very massive states and therefore decouple from the low-energy theory. B_μ is, then, the only field which continues through to the next stage of symmetry breaking.

3.3 Left-Handed Vector Bosons

The relevant Lagrangian terms to be considered in the second stage of symmetry breaking are:

$$Tr\left|\partial_\mu \Phi_u - \frac{ig_L}{2}\tau\cdot W_\mu^L \Phi_u + \Phi_u \frac{ig_R}{2}\tau\cdot W_\mu^R\right|^2 + Tr\left|\partial_\mu \Phi_d - \frac{ig_L}{2}\tau\cdot W_\mu^L \Phi_d + \Phi_d \frac{ig_R}{2}\tau\cdot W_\mu^R\right|^2 \tag{3.6}$$

Since $W_R^\pm$ and Z_R have effectively decoupled, the charged bosons emerging at this stage are to a good approximation, $W_L^\pm$, whose masses are (in this approximation):

$$M_{W_L} \cong \tfrac{1}{\sqrt{2}} g_L \left(\kappa_u^2 + \kappa_d^2\right)^{1/2} . \tag{3.7}$$

The neutral bosons must be expressed in terms of the fields B and Z_R:

$$W_R^0 = \frac{g_R Z_R + 2 g_V B}{\left(g_R^2 + 4 g_V^2\right)^{1/2}} \tag{3.8}$$

We may define the gauge coupling constant g' of $U(1)_Y$, as

$$g' \equiv \frac{g_R g_V}{(g_R^2 + 4 g_V^2)^{1/2}} \quad , \tag{3.10}$$

in order to express the neutral mass eigenstates as

$$Z_L = \frac{g_l W_l^0 - 2g' B}{\left(g_L^2 + 4g'^2\right)^{1/2}} \quad , \tag{3.11}$$

of mass

$$M_{Z_L} = \left[\left(\kappa_u^2 + \kappa_d^2\right)\left(g_L^2 + 4g'^2\right)\right]^{1/2} \tag{3.12}$$

along with the massless photon given by

$$A_\mu = \frac{2g' W_L^0 + g_L B}{\left(g_l^2 + 4g'^2\right)^{1/2}} \quad . \tag{3.13}$$

Chapter 4

Charginos (Charged Gauginos and Higgsinos)

4.1 Introduction

Gauginos[21], Higgs bosons and higgsinos possess certain identical quantum numbers which make it possible for these fields to mix. As in the case of vector boson fields just discussed, after the breaking of gauge symmetries, supersymmetric particles may combine to yield "physical" states. In the case of charged particles, these are deemed "charginos". The identification of such species is a more involved process than in the case of vector boson mass eigenstates, owing to the presence of superymmetry breaking terms in the interaction Lagrangian. This chapter shall be concerned with the elucidation of such states.

4.2 The Mixing of Gauginos and Higgsinos

Gauginos mix with higgsinos via the "$\lambda\psi A$" coupling of eq.(1.21). The non-zero VEV's of the Higgs fields generate fermionic mass terms. These massive fermions combine to produce new "physical" fields. Soft supersymmetry-breaking terms are responsible for contributing to these fermion masses.

The terms of the Lagrangian that are relevant to the mixing may be called L_{GH}:

$$
\begin{aligned}
L_{GH} = {} & i\sqrt{2}\,Tr\left\{(\tau\cdot\Delta_L)^{+}(g_L\tau\cdot\lambda_L + 2g_V\lambda_V)\tau\cdot\tilde{\Delta}_L\right\} + h.c \\
& + i\sqrt{2}\,Tr\left\{(\tau\cdot\Delta_R)\;(g_R\tau\cdot\lambda_R + 2g_V\lambda_V)\tau\cdot\tilde{\Delta}_R\right\} + h.c. \\
& + \tfrac{1}{\sqrt{2}}\,Tr\left\{\Phi_d(g_L\tau\cdot\lambda_L + g_R\tau\cdot\lambda_R)\tilde{\Phi}_d\right\} + h.c. \\
& + \tfrac{1}{\sqrt{2}}\,Tr\left\{\Phi_u(g_L\tau\cdot\lambda_L + g_R\tau\cdot\lambda_R)\tilde{\Phi}_u\right\} + h.c. \\
& + m_L\left(\lambda_L^a\lambda_L^a + \overline{\lambda}_L^a\overline{\lambda}_L^a\right) + m_R\left(\lambda_R^a\lambda_R^a + \overline{\lambda}_R^a\overline{\lambda}_R^a\right) \\
& + m_V\left(\lambda_V\lambda_V + \overline{\lambda}_V\overline{\lambda}_V\right) + Tr\left(\mu_1\left[\tau_1\tilde{\phi}_u\tau_1\right]^T\tilde{\Phi}_d\right) \\
& + Tr\left\{\mu_2(\tau\cdot\tilde{\Delta}_L)(\tau\cdot\tilde{\delta}_L)\right\} + Tr\left\{\mu_2(\tau\cdot\tilde{\Delta}_R)(\tau\cdot\tilde{\delta}_R)\right\}
\end{aligned}
\tag{4.1}
$$

4.3 Chargino Mixing

Terms in eq.(4.1) involving charged fields give rise to chargino mixing. Substituting the VEV's of the Higgs fields in eq.(2.9) into eq.(4.2) yields the chargino-mixing Lagrangian L_{CM}:

$$
\begin{aligned}
L_{CM} = {} & \left\{i\lambda_R\left(\sqrt{2}g_R v_R\tilde{\Delta}_R^{+} + g_R\kappa_d\tilde{\phi}_d^{+}\right) + i\lambda_L^{-}g_L\kappa_d\tilde{\phi}_d^{+} + i\lambda_R^{+}g_R\kappa_u\tilde{\phi}_u^{-} + i\lambda_L^{+}g_L\kappa_u\tilde{\phi}_u^{-} + m_L\lambda_L^{+}\lambda_L^{-}\right. \\
& \left. + m_R\lambda_R^{+}\lambda_R + \mu_1\tilde{\phi}_u^{+}\tilde{\phi}_d^{-} + \mu_1\tilde{\phi}_u^{-}\tilde{\phi}_d^{+}\right\} + h.c.
\end{aligned}
\tag{4.2}
$$

where the simplification $\mu_2 = \mu_3 = 0$ has been made.

As in the case of vector bosons, each stage of symmetry breaking may be considered individually. The mass term of the fermionic partner of W_R^{+} is responsible for the first stage:

$$L_{\omega} = -\sqrt{2} g_R \upsilon_R \overline{\tilde{W}_R^{+}} \tilde{W}_R^{+} + h.c \tag{4.3}$$

where,

$$\tilde{W}_R^{+} = \begin{pmatrix} \tilde{\Delta}_R^{+} \\ i\bar{\lambda}_R \end{pmatrix}. \tag{4.4}$$

$\tilde{W}_R^{+}$ is a four-component Dirac spinor. Since Majorana fermions cannot carry conserved additive quantum numbers, all charged fermions must combine into four-component Dirac spinors.

From eq.(4.3) it can be seen that the mass of the $\tilde{W}_R^{+}$ is $\sqrt{2} g_R \upsilon_R$, and equal to the W_R^{+} , mass. $\tilde{W}_R$, however, does not develop an equivalent mass indicating that supersymmetry must have been broken. As in vector boson theory, the particles produced at the first stage of symmetry breaking are very massive. Such massive states will decouple from the low-energy theory.

The second stage involves the remaining terms of eq.(4.2). The Lagrangian terms are eq.(4.2) - eq.(4.3), which may be written as the matrix equation:

$$L_C = -\tfrac{1}{2}\begin{pmatrix} \psi^{+} & \psi^{-} \end{pmatrix} \begin{pmatrix} 0 & X^T \\ X & 0 \end{pmatrix} \begin{pmatrix} \psi^{+} \\ \psi^{-} \end{pmatrix} + h\,c \tag{4.5}$$

Defining

$$\psi^- \equiv \left(-i\lambda_L^- \quad -i\lambda_R^- \quad \tilde{\phi}_u^- \quad \tilde{\phi}_d^-\right) \qquad (4.6)$$

$$\psi^+ \equiv \left(-i\lambda_L^+ \quad -i\lambda_R^+ \quad \tilde{\phi}_u^+ \quad \tilde{\phi}_d^+\right) \qquad (4.7)$$

$$X = \begin{pmatrix} m_l & 0 & 0 & g_L\kappa_d \\ 0 & m_R & 0 & g_R\kappa_d \\ g_L\kappa_u & g_R\kappa_u & 0 & -\mu_1 \\ 0 & 0 & -\mu_1 & 0 \end{pmatrix} \qquad (4.8)$$

The mass eigenstates may be defined as

$$\chi_i^- = V_{ij}\psi_j^- \quad , \quad \chi_i^+ = U_{ij}\psi_j^+ \quad , \quad i,j = 1,2,3,4 \qquad (4.9)$$

where V and U are unitary matrices chosen to satisfy

$$U^* \times V^{-1} = M_D \qquad (4.10)$$

where M_D is a diagonal matrix with non-negative entries. Using these definitions we may write eq.(4.5)

$$-\left(\chi_i^-(M_D)_{ij}\chi_j^+ + h.c.\right) \qquad (4.11)$$

and using eq.(A19), then this may be written in four-component notation:

$$-\sum_{i=1}^{4} M_i \bar{\tilde{\chi}}_i \tilde{\chi}_i \quad , \qquad (4.12)$$

where the $\tilde{\chi}_i$ are charged four-component Dirac spinors:

$$\tilde{\chi}_i = \begin{pmatrix} \chi_i^+ \\ \bar{\chi}_i^- \end{pmatrix} . \qquad (4.13)$$

Expressions for the matrices U and V must be solved for, and subsequently, the masses M_i. M_D is constrained to have only non-negative entries. The eigenvalues of X may, however, be either positive or negative. Positive square roots of the eigenvalue problem for $X^{\dagger}X$ will then be the diagonal entries of M_D. From eq.(4.11) we have:

$$M_D^2 = VXX^{+}V^{-1} = U^{*}XX^{+}\left(U^{*}\right)^{-1} \qquad (4.14)$$

Thus, the eigenvectors corresponding to the eigenvalues of $X^{\dagger}X$ will be the diagonal matrices U^{*} and V. The solutions to this equation have been presented in Ref.[22]. All calculations will be expressed in terms of the matrices U^{*} and V.

Chapter 5

Neutralinos (Neutral Gauginos and Higgsinos)

5.1 Introduction

The neutral states which result from the mixing of massive gauginos and higgsinos can be elucidated in much the same manner as the chargino states of the previous chapter. There exist, however, certain differences which must be noted; these are:

1) One extra gauge field $\lambda_{I'}$ is involved

2) Since neutralinos do not carry charge, they may be represented as Majorana spinors.

The neutralinos of this theory are produced at two stages of symmetry breaking, as was the case for the gauge bosons and chargino states. The "heavy" neutralinos are produced at the first stage of symmetry breaking, "light" neutralinos in the second. Since the amount of mixing between these states is minimal, it is acceptable to make the approximation that mass eigenstates may be calculated at each scale independently.

5.2 The Neutralino Mass Lagrangian

The terms of relevance are contained in eq.(4.1). The neutral terms arising from this equation are:

$$\begin{aligned} L_{NM} = & \Big\{ -i\lambda_R^0 \sqrt{2} g_R \upsilon_R \tilde{\Delta}_R^0 + i\lambda_V^0 2\sqrt{2} g_V \upsilon_R \tilde{\Delta}_R^0 + i\lambda_R^0 \tfrac{1}{\sqrt{2}} g_R \kappa_u \tilde{\phi}_{1u}^0 \\ & - i\lambda_L^0 \tfrac{1}{\sqrt{2}} g_L \kappa_u \tilde{\phi}_{1u}^0 - i\lambda_R^0 \tfrac{1}{\sqrt{2}} g_R \kappa_d \tilde{\phi}_{2d}^0 + i\lambda_L^0 \tfrac{1}{\sqrt{2}} g_L \kappa_d \tilde{\phi}_{2d}^0 \\ & + m_L \lambda_L^0 \lambda_L^0 + m_R \lambda_R^0 \lambda_R^0 + m_V \lambda_V^0 \lambda_V^0 + 2\mu_1 \tilde{\phi}_{1u}^0 \tilde{\phi}_{2d}^0 \Big\} + h.c. \end{aligned} \qquad (5.1)$$

5.3 "Heavy" Neutralinos

The mass eigenstates which develop in the first stage of symmetry breaking derive from terms in eq(5.1). These "heavy" states involve terms which contain υ_R (we may define the Lagrangian as L_{NMH}):

$$L_{NMH} = \Big\{ -i\lambda_R^0 \sqrt{2} g_R \upsilon_R \tilde{\Delta}_R^0 + i\lambda_V^0 2\sqrt{2} g_V \upsilon_R \tilde{\Delta}_R^0 \Big\} + h.c. \qquad (5.2)$$

We define:

$$(\xi^0)^T \equiv \begin{pmatrix} -i\lambda_R^0 & -i\lambda_V^0 & \tilde{\Delta}_R^0 \end{pmatrix} \qquad (5.3)$$

Using this, eq.(5.2) may be rewritten as

$$L_{NMH} = -\tfrac{1}{\sqrt{2}} \upsilon_R (\xi^0)^T Y \xi^0 + h.c. \qquad (5.3)$$

with

$$Y = \begin{pmatrix} 0 & 0 & -g_R \\ 0 & 0 & 2g_V \\ -g_R & 2g_V & 0 \end{pmatrix} \quad . \tag{5.4}$$

The mass eigenstates may then be defined as:

$$\chi_i^0 = N_{ij}\xi_j^0 \qquad (i, j = 1,2,3) \tag{5.5}$$

where N is a unitary matrix which satisfies

$$N^{*}YN^{-1} = N_D \quad . \tag{5.6}$$

N_D is a diagonal matrix possessing only non-negative elements and may be found in the same manner as in the previous section, by squaring the matrix equation. Solutions for these matrices can be found in Ref.[22].

Using eqs.(5.5) and (A19), eq.(5.3) may be expressed in terms of four component neutral Majorana spinors. We may define:

$$\tilde{\chi}_i^0 = \begin{pmatrix} \chi_i^0 \\ \bar{\chi}_i^0 \end{pmatrix} \tag{5.7}$$

and then the mass term will be

$$-\tfrac{1}{\sqrt{2}} \sum_i M_i \bar{\tilde{\chi}}_i^0 \tilde{\chi}_i^0 \tag{5.8}$$

where M_i are the diagonal elements of N_D.

Now the matrix,

$$Y'Y = \begin{pmatrix} g_R^2 & -2g_R g_{Y'} & 0 \\ -2g_R g_{Y'} & 4g_{Y'}^2 & 0 \\ 0 & 0 & g_R^2 + 4g_{Y'}^2 \end{pmatrix} \tag{5.9}$$

which has eigenvalues 0, $(g_R^2 + 4g_{Y'}^2)$, and $(g_R^2 + 4g_{Y'}^2)$,the diagonal entries of N_D^2. The diagonalizing matrix is therefore:

$$N = \begin{pmatrix} \frac{g_R}{\sqrt{2}(g_R^2 + 4g_{Y'}^2)^{1/2}} & \frac{-2g_{Y'}}{\sqrt{2}(g_R^2 + 4g_{Y'}^2)^{1/2}} & -\frac{1}{\sqrt{2}} \\ \frac{g_R}{\sqrt{2}(g_R^2 + 4g_{Y'}^2)^{1/2}} & \frac{-2g_{Y'}}{\sqrt{2}(g_R^2 + 4g_{Y'}^2)^{1/2}} & +\frac{1}{\sqrt{2}} \\ \frac{2g_{Y'}}{(g_R^2 + 4g_{Y'}^2)^{1/2}} & \frac{g_R}{(g_R^2 + 4g_{Y'}^2)^{1/2}} & 0 \end{pmatrix} \tag{5.10}$$

Eqs.(5.5), and (5.7) are then used to determine the expression fro the "physical" neutralinos which arise from the irst symmetry breaking:

$$\tilde{\chi}_{Z1} = \begin{pmatrix} \frac{-i(g_R\lambda_R^0 - 2g_{Y'}\lambda_{Y'})}{\sqrt{2}(g_R^2 + 4g_{Y'}^2)^{1/2}} - \frac{\tilde{\Delta}_R^0}{\sqrt{2}} \\ \frac{+i(g_R\bar{\lambda}_R^0 - 2g_{Y'}\bar{\lambda}_{Y'})}{\sqrt{2}(g_R^2 + 4g_{Y'}^2)^{1/2}} - \frac{\bar{\tilde{\Delta}}_R^0}{\sqrt{2}} \end{pmatrix}, \tag{5.11}$$

with a mass

$$\tfrac{1}{\sqrt{2}}\,\upsilon_R\left(g_R^2+4g_V^2\right)^{1/2} \quad ; \tag{5.12}$$

$$\tilde{\chi}_{Z2}=\begin{pmatrix}\dfrac{-i\left(g_R\lambda_R^0-2g_V\lambda_V\right)}{\sqrt{2}\left(g_R^2+4g_V^2\right)^{1/2}}+\dfrac{\tilde{\Delta}_R^0}{\sqrt{2}}\\ \dfrac{+i\left(g_R\overline{\lambda}_R^0-2g_V\overline{\lambda}_V\right)}{\sqrt{2}\left(g_R^2+4g_V^2\right)^{1/2}}+\dfrac{\overline{\tilde{\Delta}}_R^0}{\sqrt{2}}\end{pmatrix}, \tag{5.13}$$

also with a mass of

$$\tfrac{1}{\sqrt{2}}\,\upsilon_R\left(g_R^2+4g_V^2\right)^{1/2} \quad ,$$

and

$$\tilde{\chi}_B=\begin{pmatrix}\dfrac{-i\left(g_R\lambda_V^0+2g_V\lambda_R^0\right)}{\left(g_R^2+4g_V^2\right)^{1/2}}\\ \dfrac{+i\left(g_R\overline{\lambda}_V^0-2g_V\overline{\lambda}_R^0\right)}{\left(g_R^2+4g_V^2\right)^{1/2}}\end{pmatrix}. \tag{5.14}$$

with zero mass. This is the superpartner of B_μ.

The two massive Majorana fermions may be expressed as a single Dirac spinor:

$$\tilde{\zeta}^0=\begin{pmatrix}\tilde{\Delta}_R^0\\ \dfrac{i\left(g_R\overline{\lambda}_R^0-2g_V\overline{\lambda}_V\right)}{\left(g_R^2+4g_V^2\right)^{1/2}}\end{pmatrix}, \tag{5.15}$$

the superpartner of Z_R.

The massive $\tilde{\zeta}^0$ decouples at this energy, leaving $\tilde{\chi}_B$ to continue through to the second stage of symmetry breaking.

5.4 "Light" Neutralinos

At the low energy scale, the particles which are of interest are no longer those which were expressed in eq.(5.1). This is due to the reshuffling of fields at high energy. It is therefore necessary to reexpress the fields λ_R^0 and λ_{V}^0 in terms of λ_Z^0 and λ_B^0. From the definitions:

$$\lambda_Z^0 \equiv \frac{g_R \lambda_R^0 - 2g_V \lambda_V^0}{\left(g_R^2 + 4g_V^2\right)^{1/2}} \tag{5.16}$$

$$\lambda_B^0 \equiv \frac{g_R \lambda_V^0 - 2g_V \lambda_R^0}{\left(g_R^2 + 4g_V^2\right)^{1/2}} \tag{5.17}$$

we get

$$\lambda_R^0 \equiv \frac{g_R \lambda_V^0 - 2g_V \lambda_B^0}{\left(g_R^2 + 4g_V^2\right)^{1/2}} \tag{5.18}$$

and

$$\lambda_V^0 \equiv \frac{g_R \lambda_B^0 - 2g_V \lambda_V^0}{\left(g_R^2 + 4g_V^2\right)^{1/2}} \quad . \tag{5.19}$$

Substitution of eq.(5.18) and (5.19) in eq.(5.1) yields the light neutralino mass Lagrangian L_{NMI}, which may be simplified by discarding contributions of the fields which decouple at high energy, eg. $\tilde{\Delta}_R^0$ and λ_Z^0.

$$L_{NMl} = \left\{ -i\lambda_I^0 \tfrac{1}{\sqrt{2}} g_I \kappa_u \tilde{\phi}_{1u}^0 + \frac{i\lambda_B^0 \sqrt{2} g_V g_R \kappa_u \tilde{\phi}_{1u}^0}{\left(g_R^2 + 4g_V^2\right)^{1/2}} + i\lambda_L^0 \tfrac{1}{\sqrt{2}} g_L \kappa_d \tilde{\phi}_{2d}^0 \right.$$
$$\left. - \frac{i\lambda_B^0 \sqrt{2} g_V g_R \kappa_d \tilde{\phi}_{2d}^0}{\left(g_R^2 + 4g_V^2\right)^{1/2}} + m_I \lambda_L^0 \lambda_L^0 + \frac{4m_R g_V^2 \lambda_B^0}{g_R^2 + 4g_V^2} + \frac{m_V g_R^2 \lambda_B^0 \lambda_B^0}{g_R^2 + 4g_V^2} + 2\mu_1 \tilde{\phi}_u^0 \tilde{\phi}_d^0 \right\} + h.c. \quad . \tag{5.20}$$

The mass eigenstates may then be determined by the same procedure as in the L_{NML} case: First defining

$$\left(\Omega^0\right)^T \equiv \left(-i\lambda_I^0 \quad -i\lambda_B^0 \quad \tilde{\phi}_{1u}^0 \quad \tilde{\phi}_{2d}^0\right) , \tag{5.21}$$

then eq.(5.20) may be written as:

$$L_{NMl} = -\tfrac{1}{2}\left(\Omega^0\right)^T Z \Omega^0 + h.c. \tag{5.22}$$

where

$$Z = \begin{pmatrix} m_I & 0 & -\frac{1}{\sqrt{2}} g_L \kappa_u & \frac{1}{\sqrt{2}} g_L \kappa_d \\ 0 & \dfrac{m_V g_R^2 + 4m_R g_V^2}{g_R^2 + 4g_V^2} & \dfrac{\sqrt{2} g_V g_R \kappa_u}{\left(g_R^2 + 4g_V^2\right)^{1/2}} & \dfrac{-\sqrt{2} g_V g_R \kappa_u}{\left(g_R^2 + 4g_V^2\right)^{1/2}} \\ -\frac{1}{\sqrt{2}} g_I \kappa_u & \dfrac{\sqrt{2} g_V g_R \kappa_d}{\left(g_R^2 + 4g_V^2\right)^{1/2}} & 0 & -2\mu_1 \\ \frac{1}{\sqrt{2}} g_I \kappa_d & \dfrac{-\sqrt{2} g_V g_R \kappa_d}{\left(g_R^2 + 4g_V^2\right)^{1/2}} & -2\mu_1 & 0 \end{pmatrix} \tag{5.23}$$

The physical states are then defined by:

$$\chi_i^0 = B_{ij}\Omega_j^0 \quad , \quad (i, j = 1,2,3,4) \tag{5.24}$$

The χ_i^0 are four-component Majorana spinors whose masses are the positive square roots of $Z_D^2 = B^* Z B^{-1}$, where the matrices B^* and B^{-1} diagonalize Z. These matrices have been solved for and have been presented in Ref.[22].

Chapter 6

Chargino/Neutralino-Gauge Field Interactions

6.1 Introduction

Since the introduction of supersymmetry, experimentalists have been extremely busy trying to identify physical states associated with this "new" physics. This chapter is concerned with some of the fundamental interactions of supersymmetric particles with members of the known particle spectrum.

6.2 The Chargino/Neutralino-Gauge Field Lagrangian

The first step in developing cross section calculations for chargino and neutralino production in electron-positron collisions is to determine the contributions to the chargino/neutralino-gauge field Lagrangian L_{CNZ}. It should be noted that some of these fields will not contribute directly to these interactions, but only after mixing. Here we shall be concerned solely with one generation of the physical fields.

The Lagrangian[9] terms are:

L_{CNZ} =

$$+Tr\left[\bar{\tilde{\Phi}}_u \bar{\sigma}_\mu \left(-\tfrac{1}{2} g_L \tau \cdot W_\mu^L - \tfrac{1}{2} g_R \tau \cdot W_\mu^R\right) \tilde{\Phi}_u\right] + h.c.$$
$$+Tr\left[\bar{\tilde{\Phi}}_d \bar{\sigma}_\mu \left(-\tfrac{1}{2} g_L \tau \cdot W_\mu^L - \tfrac{1}{2} g_R \tau \cdot W_\mu^R\right) \tilde{\Phi}_d\right] + h.c.$$
$$+\tfrac{1}{2} g_L \bar{\lambda}_L \bar{\sigma}_\mu T_a G_\mu^a \lambda_L + \tfrac{1}{2} g_R \bar{\lambda}_R \bar{\sigma}_\mu T_a G_\mu^a \lambda_R$$

(6.1)

The expansion of each term with its Hermitian conjugate yields explicit two-spinor expressions for the Lagrangian. Here we develop these contributions, one at a time, beginning with the left-hand Lagrangian.

The expression

$$+Tr\left[\bar{\tilde{\Phi}}_u \bar{\sigma}_\mu \left(-\tfrac{1}{2} g_L \tau \cdot W_\mu^L - \tfrac{1}{2} g_R \tau \cdot W_\mu^R\right) \tilde{\Phi}_u\right] + h.c. \tag{6.2}$$

involves the $\tilde{\Phi}_u \tau \cdot \tilde{\Delta}_L$ term which has been determined to have the form

$$\bar{\tilde{\Phi}} = \begin{pmatrix} \bar{\tilde{\phi}}_{1u}^0 & \bar{\tilde{\phi}}_{2u} \\ \bar{\tilde{\phi}}_{1u}^{\cdot} & \bar{\tilde{\phi}}_{2u}^0 \end{pmatrix} \tag{6.3}$$

using an extension of the Gell-Mann-Nishijima formula.

The expanded form of eq.(6.2) is, then:

$$-iTr\left[\begin{pmatrix}\bar{\tilde{\phi}}^0_{1u} & \bar{\tilde{\phi}}_{2u}\\ \bar{\tilde{\phi}}^+_{1u} & \bar{\tilde{\phi}}^0_{2u}\end{pmatrix}\begin{pmatrix}\bar{\sigma}_\mu & 0\\ 0 & \bar{\sigma}_\mu\end{pmatrix}\begin{pmatrix}\frac{1}{2}g_L W^0_{\mu L} & \frac{1}{\sqrt{2}}g_L W^+_{\mu L}\\ \frac{1}{\sqrt{2}}g_L W_{\mu L} & -\frac{1}{2}g_L W^0_{\mu L}\end{pmatrix}\begin{pmatrix}\tilde{\phi}^0_{1u} & \tilde{\phi}^+_{1u}\\ \tilde{\phi}^-_{2u} & \tilde{\phi}^0_{2u}\end{pmatrix}\right] + \text{h.c.}$$

(6.4)

which yields the terms

$$-\tfrac{1}{2}g_L\Big[\ W^0_{\mu L}\big(\ \bar{\tilde{\phi}}^0_{1u}\bar{\sigma}_\mu\tilde{\phi}^0_{1u}+\bar{\tilde{\phi}}^0_{1u}\bar{\sigma}_\mu\tilde{\phi}^0_{1u}-\bar{\tilde{\phi}}^+_{2u}\bar{\sigma}_\mu\tilde{\phi}^+_{2u}+\bar{\tilde{\phi}}^-_{2u}\bar{\sigma}_\mu\tilde{\phi}^-_{2u}$$
$$+\bar{\tilde{\phi}}^+_{1u}\bar{\sigma}_\mu\tilde{\phi}^+_{1u}+\bar{\tilde{\phi}}_{1u}\bar{\sigma}_\mu\tilde{\phi}_{1u}-\bar{\tilde{\phi}}^0_{2u}\bar{\sigma}_\mu\tilde{\phi}^0_{2u}-\bar{\tilde{\phi}}^0_{2u}\bar{\sigma}_\mu\tilde{\phi}^0_{2u}\ \big)\Big]$$

(6.5)

the Hermitian conjugate term having been evaluated in the same manner, and neglecting those in W^+ and W^-.

Similarly the term

$$+Tr\left[\bar{\tilde{\Phi}}_d\bar{\sigma}_\mu\left(-\tfrac{1}{2}g_l\,\tau\cdot W^L_\mu-\tfrac{1}{2}g_R\,\tau\cdot W^R_\mu\right)\tilde{\Phi}_d\right]+h.c. \qquad (6.6)$$

gives rise to similar contributions.

Next, the left-hand gauge field contribution is considered;

$$\tfrac{1}{2}g_L\bar{\lambda}_L\bar{\sigma}_\mu T_a G^a_\mu\lambda_L\ , \qquad (6.7)$$

where the T_a are $\frac{1}{2}$ times the regular Pauli matrices τ_a. The expanded form yields the two spinor terms:

$$+ig_L W^0_{\mu l}\left(\bar{\lambda}^+_L\bar{\sigma}_\mu\lambda^+_l-\bar{\lambda}^-_L\bar{\sigma}_\mu\lambda^-_L\right)\ . \qquad (6.8)$$

where terms in W^+ and W^- have been ignored.

The total two-spinor interaction Lagrangian is:

$$
\begin{aligned}
L_{CNZ} &= \\
i\{\ & g_L W^0_{\mu L}\Big[\ \left(\bar{\lambda}^+_L \bar{\sigma}_\mu \lambda^+_L - \bar{\lambda}^-_L \bar{\sigma}_\mu \lambda^-_L\right) + \\
& -\tfrac{1}{2}\Big[\ \big(\ \bar{\tilde{\phi}}^0_{1u}\bar{\sigma}_\mu\tilde{\phi}^0_{1u} + \bar{\tilde{\phi}}^0_{1u}\bar{\sigma}_\mu\tilde{\phi}^0_{1u} - \bar{\tilde{\phi}}^+_{2u}\bar{\sigma}_\mu\tilde{\phi}^+_{2u} - \bar{\tilde{\phi}}^-_{2u}\bar{\sigma}_\mu\tilde{\phi}^-_{2u} \\
& +\bar{\tilde{\phi}}^+_{1u}\bar{\sigma}_\mu\tilde{\phi}^+_{1u} + \bar{\tilde{\phi}}^-_{1u}\bar{\sigma}_\mu\tilde{\phi}^-_{1u} - \bar{\tilde{\phi}}^0_{2u}\bar{\sigma}_\mu\tilde{\phi}^0_{2u} - \bar{\tilde{\phi}}^0_{2u}\bar{\sigma}_\mu\tilde{\phi}^0_{2u}\ \big)\Big] \\
& +\bar{\tilde{\phi}}^+_{1d}\bar{\sigma}_\mu\tilde{\phi}^+_{1d} + \bar{\tilde{\phi}}^-_{1d}\bar{\sigma}_\mu\tilde{\phi}^-_{1d} - \bar{\tilde{\phi}}^0_{2d}\bar{\sigma}_\mu\tilde{\phi}^0_{2d} - \bar{\tilde{\phi}}^0_{2d}\bar{\sigma}_\mu\tilde{\phi}^0_{2d}\ \big)\Big] \qquad \Big] \\
& -\tfrac{1}{2}\Big[\ \big(\ \bar{\tilde{\phi}}^0_{1d}\bar{\sigma}_\mu\tilde{\phi}^0_{1d} + \bar{\tilde{\phi}}^0_{1d}\bar{\sigma}_\mu\tilde{\phi}^0_{1d} - \bar{\tilde{\phi}}^+_{2d}\bar{\sigma}_\mu\tilde{\phi}^+_{2d} + \bar{\tilde{\phi}}^-_{2d}\bar{\sigma}_\mu\tilde{\phi}_{2d}
\end{aligned}
$$

(6.11)

At this stage it is instructive to convert to four-spinor notation as this makes the identification of physical states explicit. Therefore it is necessary to define the weak interaction eigenstates in terms of Dirac and Majorana four-spinors:

$$
\tilde{F}_{1u} \equiv \begin{pmatrix} \tilde{\phi}^+_{1u} \\ \bar{\tilde{\phi}}^-_{1u} \end{pmatrix} \ ; \ \tilde{F}_{2u} \equiv \begin{pmatrix} \tilde{\phi}^+_{2u} \\ \bar{\tilde{\phi}}^-_{2u} \end{pmatrix} \ ; \ \tilde{W}_L \equiv \begin{pmatrix} \lambda^+_L \\ \bar{\lambda}_l \end{pmatrix}
$$
$$
\tilde{F}_{1d} \equiv \begin{pmatrix} \tilde{\phi}^+_{1d} \\ \bar{\tilde{\phi}}^-_{1d} \end{pmatrix} \ ; \ \tilde{F}_{2d} \equiv \begin{pmatrix} \tilde{\phi}^+_{2d} \\ \bar{\tilde{\phi}}^-_{2d} \end{pmatrix}
$$

(6.12)

and

$$
\tilde{F}^0_{1u} \equiv \begin{pmatrix} \tilde{\phi}^0_{1u} \\ \bar{\tilde{\phi}}^0_{1u} \end{pmatrix} \ ; \ \tilde{F}^0_{2u} \equiv \begin{pmatrix} \tilde{\phi}^0_{2u} \\ \bar{\tilde{\phi}}^0_{2u} \end{pmatrix} \ ,
$$
$$
\tilde{F}^0_{1d} \equiv \begin{pmatrix} \tilde{\phi}^0_{1d} \\ \bar{\tilde{\phi}}^0_{1d} \end{pmatrix} \ , \ \tilde{F}^0_{2d} \equiv \begin{pmatrix} \tilde{\phi}^0_{2d} \\ \bar{\tilde{\phi}}^0_{2d} \end{pmatrix}
$$

(6.13)

Using eq.(6.12), (6.13), and (A18) we may convert the following:

$$\begin{aligned}
\left(\bar{\lambda}^{+}_{L}\bar{\sigma}_{\mu}\lambda^{+}_{L}-\bar{\lambda}^{-}_{L}\bar{\sigma}_{\mu}\lambda^{-}_{L}\right) &= \bar{\tilde{W}}_{L}\gamma^{\mu}P_{L}\tilde{W}_{L}+\bar{\tilde{W}}_{L}\gamma^{\mu}P_{R}\tilde{W}_{L} \\
\left(\bar{\tilde{\phi}}^{0}_{2u}\bar{\sigma}_{\mu}\bar{\tilde{\phi}}^{0}_{2u}+\bar{\tilde{\phi}}^{0}_{2u}\bar{\sigma}_{\mu}\bar{\tilde{\phi}}^{0}_{2u}\right) &= \bar{\tilde{F}}^{0}_{2u}\gamma^{\mu}P_{L}\tilde{F}^{0}_{2u}-\bar{\tilde{F}}^{0}_{2u}\gamma^{\mu}P_{R}\tilde{F}^{0}_{2u} \\
\left(\bar{\tilde{\phi}}^{+}_{2u}\bar{\sigma}_{\mu}\bar{\tilde{\phi}}^{+}_{2u}+\bar{\tilde{\phi}}^{-}_{2u}\bar{\sigma}_{\mu}\bar{\tilde{\phi}}^{-}_{2u}\right) &= \bar{\tilde{F}}_{2u}\gamma^{\mu}P_{L}\tilde{F}_{2u}-\bar{\tilde{F}}_{2u}\gamma^{\mu}P_{R}\tilde{F}_{2u} \\
\left(\bar{\tilde{\phi}}^{0}_{1u}\bar{\sigma}_{\mu}\bar{\tilde{\phi}}^{0}_{1u}+\bar{\tilde{\phi}}^{0}_{1u}\bar{\sigma}_{\mu}\bar{\tilde{\phi}}^{0}_{1u}\right) &= \bar{\tilde{F}}^{0}_{1u}\gamma^{\mu}P_{L}\tilde{F}^{0}_{1u}-\bar{\tilde{F}}^{0}_{1u}\gamma^{\mu}P_{R}\tilde{F}^{0}_{1u} \\
\left(\bar{\tilde{\phi}}^{+}_{1u}\bar{\sigma}_{\mu}\bar{\tilde{\phi}}^{+}_{1u}+\bar{\tilde{\phi}}^{-}_{1u}\bar{\sigma}_{\mu}\bar{\tilde{\phi}}^{-}_{1u}\right) &= \bar{\tilde{F}}_{1u}\gamma^{\mu}P_{L}\tilde{F}_{1u}-\bar{\tilde{F}}_{1u}\gamma^{\mu}P_{R}\tilde{F}_{1u}
\end{aligned} \tag{6.14}$$

In addition, we must express the fields $W^{0}_{\mu L}$ and V_{μ} in terms of the physical fields Z_{L} and A_{μ}, which arise after the breaking of symmetry. These fields were discussed in chapter 3 and have the forms:

$$Z_{L}=\frac{g_{L}W^{0}_{L}-2g'B}{\left(g_{L}^{2}+4g'^{2}\right)^{1/2}} \tag{6.15}$$

and

$$A_{\mu}=\frac{2g'W^{0}_{L}+g_{L}B}{\left(g_{L}^{2}+4g'^{2}\right)^{1/2}} \quad , \tag{6.16}$$

where the coupling constant g' has been defined in terms of the coupling constants g_{R} and $g_{l'}$:

$$g'\equiv\frac{g_{l'}g_{R}}{\left(g_{R}^{2}+4g_{l'}^{2}\right)^{1/2}} \tag{6.17}$$

rearranging eq.(6.15) and (6.16) we are left with the expression:

$$W_l^0 = \frac{g_L Z_L + 2g' A_\mu}{\left(g_L^2 + 4g'^2\right)^{1/2}} \tag{6.18}$$

We define

$$\cos\theta_W \equiv \frac{g_l}{\left(g_L^2 + 4g'^2\right)^{1/2}} \quad , \quad \sin\theta_W \equiv \frac{2g'}{\left(g_l^2 + 4g'^2\right)^{1/2}} . \tag{6.19}$$

Using eq.(A28) we have expressed the fields in terms of their left-handed and right-handed projection operators. We may separate the interaction Lagrangian into individual chargino and neutralino Lagrangians. Combining all of the above expressions involving charged states yields the four-spinor chargino Lagrangian. In the choice of VEV's of this model, the states involving $\tilde{F}_{1d}$ and $\tilde{F}_{2u}$ remain unmixed and constitute eigenstates of the theory. The expression for the chargino interaction Lagrangian is:

$$L_C =$$

$$-i\left\{\left(\frac{g_L\cos\theta_W}{\cos\theta_W}Z_L^\mu + eA_\mu + \frac{g_R g'}{g_Y}Z_R\right)\times\right.$$
$$\left(-\bar{\tilde{F}}_{1u}\gamma^\mu P_L\tilde{F}_{1u} + \bar{\tilde{F}}_{1u}\gamma^\mu P_R\tilde{F}_{1u} + \bar{\tilde{F}}_{2u}\gamma^\mu P_L\tilde{F}_{2u} - \bar{\tilde{F}}_{2u}\gamma^\mu P_R\tilde{F}_{2u}\right.$$
$$\left.-\bar{\tilde{F}}_{1d}\gamma^\mu P_L\tilde{F}_{1d} + \bar{\tilde{F}}_{1d}\gamma^\mu P_R\tilde{F}_{1d} + \bar{\tilde{F}}_{2d}\gamma^\mu P_L\tilde{F}_{2d} - \bar{\tilde{F}}_{2d}\gamma^\mu P_R\tilde{F}_{2d}\right)$$
$$+eA_\mu\left(\bar{\tilde{W}}_L\gamma^\mu P_L\tilde{W}_L + \bar{\tilde{W}}_L\gamma^\mu P_R\tilde{W}_L + \bar{\tilde{W}}_R\gamma^\mu P_L\tilde{W}_R + \bar{\tilde{W}}_R\gamma^\mu P_R\tilde{W}_R\right)$$
$$+Z_L\left(g_L\cos\theta_W\left[\bar{\tilde{W}}_L\gamma^\mu P_L\tilde{W}_L + \bar{\tilde{W}}_L\gamma^\mu P_R\tilde{W}_L\right] - g'\sin\theta_W\left[\bar{\tilde{W}}_R\gamma^\mu P_L\tilde{W}_R + \bar{\tilde{W}}_R\gamma^\mu P_R\tilde{W}_R\right]\right)$$
$$\left.+Z_R\left(\frac{g_R g'}{g_Y}\left(\bar{\tilde{W}}_R\gamma^\mu P_L\tilde{W}_R + \bar{\tilde{W}}_R\gamma^\mu P_R\tilde{W}_R\right)\right)\right\}$$

(6.20)

Performing the same steps with the uncharged higgsino and gaugino components yields the analagous neutralino interaction Lagrangian:

$$L_N =$$
$$-i\left\{\left(\frac{g_L\cos2\theta_W}{\cos\theta_W}Z_L^\mu + \frac{g_R g'}{g_Y}Z_R\right)\times\right.$$
$$\frac{1}{2}\left(-\bar{\tilde{F}}^0_{1u}\gamma^\mu P_L\tilde{F}^0_{1u} + \bar{\tilde{F}}^0_{1u}\gamma^\mu P_R\tilde{F}^0_{1u} + \bar{\tilde{F}}^0_{2u}\gamma^\mu P_L\tilde{F}^0_{2u} - \bar{\tilde{F}}^0_{2u}\gamma^\mu P_R\tilde{F}^0_{2u}\right.$$
$$\left.\left.-\bar{\tilde{F}}^0_{1d}\gamma^\mu P_L\tilde{F}^0_{1d} + \bar{\tilde{F}}^0_{1d}\gamma^\mu P_R\tilde{F}^0_{1d} + \bar{\tilde{F}}^0_{2d}\gamma^\mu P_L\tilde{F}^0_{2d} - \bar{\tilde{F}}^0_{2d}\gamma^\mu P_R\tilde{F}^0_{2d}\right)\right\}$$

(6.21)

At this stage the $\tilde{W}_L$, $\tilde{W}_R$, $\tilde{F}_u$ and $\tilde{F}_d$ fields must be expressed in terms the chargino and neutralino mass eigenstates. The interacting neutralino states are defined in eq.(5.21). The corresponding chargino eigenstates are defined in eqs.(4.6) and (4.7).

It is instructive to consider the low energy theory, as its results are familiar from MSSM theory. Thus, we may reexpress the low energy chargino interaction Lagrangian using eqs.(4.6), (4.7) and (4.9):

$$\chi_1^+ = -iV_{11}\lambda_L^+ - iV_{12}\lambda_R^+ + V_{13}\tilde{\phi}_u^+ + V_{14}\tilde{\phi}_d^+ \qquad (6.22)$$

and

$$\chi_1^- = -iU_{11}\lambda_L^- - iU_{12}\lambda_R^- + U_{13}\tilde{\phi}_u + U_{14}\tilde{\phi}_d \quad , \qquad (6.23)$$

with equivalent expressions for $\chi_2^+, \chi_3^+, \chi_4^+$.

The right-hand projection of $\tilde{W}_L$ can be expressed

$$P_R\tilde{W}_l = \begin{pmatrix} 0 \\ -i\bar{\lambda}_L^- \end{pmatrix} = \begin{pmatrix} 0 \\ -i\left(U_{11}^*U_{11} + U_{21}^*U_{21} + U_{31}^*U_{31} + U_{41}^*U_{41}\right)\bar{\lambda}_l \end{pmatrix} \quad , \qquad (6.24)$$

since, by the properties of unitary matrices, the bracketted term is equal to one. Using eq.(6.23), and equivalent expressions for $\tilde{\chi}_2^-, \tilde{\chi}_3^-, \tilde{\chi}_4^-$, the right-hand side of eq.(6.24) can be reexpressed as:

$$P_R\left(U_{11}\tilde{\chi}_1 + U_{21}\tilde{\chi}_2 + U_{31}\tilde{\chi}_3 + U_{41}\tilde{\chi}_4\right) \quad , \qquad (6.25)$$

giving;

$$P_R\tilde{W}_1 = P_R(U_{11}\tilde{\chi}_1 + U_{21}\tilde{\chi}_2 + U_{31}\tilde{\chi}_3 + U_{41}\tilde{\chi}_4) \quad . \tag{6.26}$$

In the same manner, expressions for all of the Lagrangian contributions can be developed:

$$\overline{\tilde{W}}_L\gamma^\mu P_L = \left(V_{11}\overline{\tilde{\chi}}_1 + V_{12}\overline{\tilde{\chi}}_2 + V_{13}\overline{\tilde{\chi}}_3 + V_{14}\overline{\tilde{\chi}}_4\right)\gamma^\mu P_L$$

$$P_L\tilde{W}_L = P_L\left(V_{11}^*\tilde{\chi}_1 + V_{21}^*\tilde{\chi}_2 + V_{31}^*\tilde{\chi}_3 + V_{41}^*\tilde{\chi}_4\right)$$

$$\overline{\tilde{W}}_l\gamma^\mu P_R = \left(U_{11}^*\overline{\tilde{\chi}}_1 + U_{12}^*\overline{\tilde{\chi}}_2 + U_{13}^*\overline{\tilde{\chi}}_3 + U_{14}^*\overline{\tilde{\chi}}_4\right)\gamma^\mu P_R$$

$$P_R\tilde{W}_R = P_R\left(U_{12}\tilde{\chi}_1 + U_{22}\tilde{\chi}_2 + U_{32}\tilde{\chi}_3 + U_{42}\tilde{\chi}_4\right)$$

$$\overline{\tilde{W}}_R\gamma^\mu P_L = \left(V_{21}\overline{\tilde{\chi}}_1 + V_{22}\overline{\tilde{\chi}}_2 + V_{23}\overline{\tilde{\chi}}_3 + V_{24}\overline{\tilde{\chi}}_4\right)\gamma^\mu P_L$$

$$P_L\tilde{W}_R = P_L\left(V_{12}^*\tilde{\chi}_1 + V_{22}^*\tilde{\chi}_2 + V_{32}^*\tilde{\chi}_3 + V_{42}^*\tilde{\chi}_4\right)$$

$$\overline{\tilde{W}}_R\gamma^\mu P_R = \left(U_{21}^*\overline{\tilde{\chi}}_1 + U_{22}^*\overline{\tilde{\chi}}_2 + U_{23}^*\overline{\tilde{\chi}}_3 + U_{24}^*\overline{\tilde{\chi}}_4\right)\gamma^\mu P_R$$

$$\overline{\tilde{F}}_{1u}\gamma^\mu P_R = \left(U_{31}^*\overline{\tilde{\chi}}_1 + U_{32}^*\overline{\tilde{\chi}}_2 + U_{33}^*\overline{\tilde{\chi}}_3 + U_{34}^*\overline{\tilde{\chi}}_4\right)\gamma^\mu P_R$$

$$P_L\tilde{F}_{1u} = P_L\left(V_{13}^*\tilde{\chi}_1 + V_{23}^*\tilde{\chi}_2 + V_{33}^*\tilde{\chi}_3 + V_{43}^*\tilde{\chi}_4\right)$$

$$\overline{\tilde{F}}_{1u}\gamma^\mu P_L = \left(V_{31}\overline{\tilde{\chi}}_1 + V_{32}\overline{\tilde{\chi}}_2 + V_{33}\overline{\tilde{\chi}}_3 + V_{34}\overline{\tilde{\chi}}_4\right)\gamma^\mu P_L$$

$$P_R\tilde{F}_{1u} = P_R\left(U_{31}\tilde{\chi}_1 + U_{32}\tilde{\chi}_2 + U_{33}\tilde{\chi}_3 + U_{34}\tilde{\chi}_4\right)$$

$$\bar{\tilde{F}}_{2d}\gamma^{\mu}P_{R}=\left(U^{*}_{41}\bar{\tilde{\chi}}_{1}+U^{*}_{42}\bar{\tilde{\chi}}_{2}+U^{*}_{43}\bar{\tilde{\chi}}_{3}+U^{*}_{44}\bar{\tilde{\chi}}_{4}\right)\gamma^{\mu}P_{R}$$

$$P_{L}\tilde{F}_{2d}=P_{L}\left(V^{*}_{14}\tilde{\chi}_{1}+V^{*}_{24}\tilde{\chi}_{2}+V^{*}_{34}\tilde{\chi}_{3}+V^{*}_{44}\tilde{\chi}_{4}\right)$$

$$\bar{\tilde{F}}_{2d}\gamma^{\mu}P_{L}=\left(V_{41}\bar{\tilde{\chi}}_{1}+V_{42}\bar{\tilde{\chi}}_{2}+V_{43}\bar{\tilde{\chi}}_{3}+V_{44}\bar{\tilde{\chi}}_{4}\right)\gamma^{\mu}P_{L}$$

$$P_{R}\tilde{F}_{2d}=P_{R}\left(U_{14}\tilde{\chi}_{1}+U_{24}\tilde{\chi}_{2}+U_{34}\tilde{\chi}_{3}+U_{44}\tilde{\chi}_{4}\right)$$

$$\bar{\tilde{F}}^{0}_{1u}\gamma^{\mu}P_{L}=\left(B_{31}\bar{\tilde{\chi}}^{0}_{1}+B_{31}\bar{\tilde{\chi}}^{0}_{2}+B_{33}\bar{\tilde{\chi}}^{0}_{3}+B_{34}\bar{\tilde{\chi}}^{0}_{4}\right)\gamma^{\mu}P_{L}$$

$$P_{L}\tilde{F}^{0}_{1u}=P_{L}\left(B^{*}_{13}\tilde{\chi}^{0}_{1}+B^{*}_{23}\tilde{\chi}^{0}_{2}+B^{*}_{33}\tilde{\chi}^{0}_{3}+B^{*}_{43}\tilde{\chi}^{0}_{4}\right)$$

$$\bar{\tilde{F}}^{0}_{1u}\gamma^{\mu}P_{R}=\left(B^{*}_{31}\bar{\tilde{\chi}}^{0}_{1}+B^{*}_{32}\bar{\tilde{\chi}}^{0}_{2}+B^{*}_{33}\bar{\tilde{\chi}}^{0}_{3}+B^{*}_{34}\bar{\tilde{\chi}}^{0}_{4}\right)\gamma^{\mu}P_{R}$$

$$P_{R}\tilde{F}^{0}_{1u}=P_{R}\left(B_{13}\tilde{\chi}^{0}_{1}+B_{23}\tilde{\chi}^{0}_{2}+B_{33}\tilde{\chi}^{0}_{3}+B_{43}\tilde{\chi}^{0}_{4}\right)$$

$$\bar{\tilde{F}}^{0}_{2d}\gamma^{\mu}P_{L}=\left(B_{41}\bar{\tilde{\chi}}^{0}_{1}+B_{42}\bar{\tilde{\chi}}^{0}_{2}+B_{43}\bar{\tilde{\chi}}^{0}_{3}+B_{44}\bar{\tilde{\chi}}^{0}_{4}\right)\gamma^{\mu}P_{L}$$

$$P_{L}\tilde{F}^{0}_{2d}=P_{L}\left(B^{*}_{14}\tilde{\chi}^{0}_{1}+B^{*}_{24}\tilde{\chi}^{0}_{2}+B^{*}_{34}\tilde{\chi}^{0}_{3}+B^{*}_{44}\tilde{\chi}^{0}_{4}\right)$$

$$\bar{\tilde{F}}^{0}_{2d}\gamma^{\mu}P_{R}=\left(B^{*}_{41}\bar{\tilde{\chi}}^{0}_{1}+B^{*}_{42}\bar{\tilde{\chi}}^{0}_{2}+B^{*}_{43}\bar{\tilde{\chi}}^{0}_{3}+B^{*}_{44}\bar{\tilde{\chi}}^{0}_{4}\right)\gamma^{\mu}P_{R}$$

$$P_{R}\tilde{F}^{0}_{2d}=P_{R}\left(B_{14}\tilde{\chi}^{0}_{1}+B_{24}\tilde{\chi}^{0}_{2}+B_{34}\tilde{\chi}^{0}_{3}+B_{44}\tilde{\chi}^{0}_{4}\right)$$

(6.27)

The total chargino-gauge boson interaction Lagrangian may therefore be expressed:

$L_C =$

$$
\begin{aligned}
Z_L^{\mu}\Bigg\{ & \frac{g_L \cos 2\theta_W}{\cos\theta_W}\bar{\tilde{\chi}}_i^{+}\gamma^{\mu}\Big[\left(V_{i3}V_{j3}^{*}+V_{i4}V_{j4}^{*}\right)P_L+\left(U_{i3}U_{j3}^{*}+U_{i4}U_{j4}^{*}\right)P_R\Big]\tilde{\chi}_j^{+} \\
& + g_L \cos\theta_W Z_L^{\mu}\gamma_{\mu}\bar{\tilde{\chi}}_i^{+}\left(V_{i1}V_{j1}^{*}P_L+U_{i1}U_{j1}^{*}P_R\right)\tilde{\chi}_j^{+} \\
& - g'\sin\theta_W Z_L^{\mu}\gamma_{\mu}\bar{\tilde{\chi}}_i^{+}\left(V_{i2}V_{j2}^{*}P_L+U_{i2}U_{j2}^{*}P_R\right)\tilde{\chi}_j^{+} \Bigg\} \\
+Z_R^{\mu}\Bigg\{ & \frac{g_R g'}{g_Y}\bar{\tilde{\chi}}_i^{+}\gamma^{\mu}\Big[\left(V_{i2}V_{j2}^{*}+V_{i3}V_{j3}^{*}+V_{i4}V_{j4}^{*}\right)P_L \\
& +\left(U_{i2}U_{j2}^{*}+U_{i3}U_{j3}^{*}+U_{i4}U_{j4}^{*}\right)P_R\Big]\tilde{\chi}_j^{+} \Bigg\} \\
+A^{\mu}\Bigg\{ & e\delta_{ij}\bar{\tilde{\chi}}_i^{+}\gamma^{\mu}\Big[\left(V_{i1}V_{j1}^{*}+V_{i2}V_{j2}^{*}+V_{i3}V_{j3}^{*}+V_{i4}V_{j4}^{*}\right)P_L \\
& +\left(U_{i1}U_{j1}^{*}+U_{i2}U_{j2}^{*}U_{i3}U_{j3}^{*}+U_{i4}U_{j4}^{*}\right)P_R\Big]\tilde{\chi}_j^{+} \Bigg\}
\end{aligned}
$$

(6.28)

Similarly, the neutralino-gauge boson interaction Lagrangian may be expressed:

$L_N =$

$$
\begin{aligned}
Z_I^{\mu}\Bigg\{ & \frac{g_I \cos 2\theta_W}{2\cos\theta_W}\bar{\tilde{\chi}}_i^{0}\gamma^{\mu}\Big[\left(B_{i3}B_{j3}^{*}+B_{i4}B_{j4}^{*}\right)P_L+\left(B_{i3}B_{j3}^{*}+B_{i4}B_{j4}^{*}\right)P_R\Big]\tilde{\chi}_j^{0} \\
+Z_R^{\mu}\Bigg\{ & \frac{g_R g'}{2g_1}\bar{\tilde{\chi}}_i^{0}\gamma^{\mu}\Big[\left(B_{i3}B_{j3}^{*}+B_{i4}B_{j4}^{*}\right)P_I+\left(B_{i3}B_{j3}^{*}+B_{i4}B_{j4}^{*}\right)P_R\Big]\tilde{\chi}_j^{0} \Bigg\}
\end{aligned}
$$

(6.29)

We may simplify the above expressions by expressing them in terms of the matrices defined below:

$$O'^{L}_{ij} = \cos^2\theta_W V_{i1}V^*_{j1} - \frac{g'}{2g_L}\sin 2\theta_W V_{i2}V^*_{j2} + \cos 2\theta_W \left(V_{i3}V^*_{j3} + V_{i4}V^*_{j4}\right)$$
$$O'^{R}_{ij} = \cos^2\theta_W U_{i1}U^*_{j1} - \frac{g'}{2g_L}\sin 2\theta_W U_{i2}U^*_{j2} + \cos 2\theta_W \left(U_{i3}U^*_{j3} + U_{i4}U^*_{j4}\right)$$

(6.30)

$$Q^{L}_{ij} = V_{i2}V^*_{j2} + V_{i3}V^*_{j3} + V_{i4}V^*_{j4}$$
$$Q^{R}_{ij} = U_{i2}U^*_{j2} + U_{i3}U^*_{j3} + U_{i4}U^*_{j4}$$

(6.31)

and

$$O''^{L}_{ij} = -\tfrac{1}{2}B_{i3}B^*_{i3} + \tfrac{1}{2}B_{i4}B^*_{i4}$$
$$O''^{R}_{ij} = -O''^{L*}_{ij}$$

(6.32)

The left-hand chargino Lagrangian may therefore be expressed

$$L^{L}_{C} = \left(\frac{g_L}{\cos\theta_W}\right) Z^{\mu}_{L}\left[\bar{\tilde{\chi}}^{+}_{i}\gamma_{\mu}\left(O'^{L}_{ij}P_L + O'^{R}_{ij}P_R\right)\tilde{\chi}^{+}_{i}\right] \qquad (6.33)$$

The left-hand neutralino Lagrangian is

$$L^{L}_{N} = \tfrac{1}{2}Z^{\mu}_{L}\left[\bar{\tilde{\chi}}^{0}_{i}\gamma_{\mu}\left(O''^{L}_{ij}P_L + O''^{R}_{ij}P_R\right)\tilde{\chi}^{0}_{i}\right] \qquad (6.34)$$

The respective right-hand Lagrangians are

$$L^{R}_{C} = \left(\frac{\cos 2\theta_W}{\cos\theta_W}\right) g_R Z^{\mu}_{R}\left[\bar{\tilde{\chi}}^{+}_{i}\gamma_{\mu}\left(Q^{L}_{ij}P_L + Q^{R}_{ij}P_R\right)\tilde{\chi}^{+}_{i}\right] \qquad (6.35)$$

and

$$L_N^R = \frac{1}{2} Z_R^\mu \left[\bar{\tilde{\chi}}_i^0 \gamma_\mu \left(O''^L_{ij} P_L + O''^R_{ij} P_R \right) \tilde{\chi}_i^0 \right] \tag{6.36}$$

Chapter 7

Chargino and Neutralino Production in e^+e^- Collisions

7.1 Introduction

Experimental identification of supersymmetric particles has become a major focus of the new generation of high-energy accelerators. Here we calculate cross sections for the production of charginos and neutralinos in e^+e^--collisions.

7.2 Cross Section Calculations for the Process $e^+e^- \to Z^0_{L,R} \to \tilde{\chi}^+_i \tilde{\chi}^-_j, \tilde{\chi}^0_i \tilde{\chi}^0_j$

Chargino and neutralino production in high-energy collisions has already been studied in depth for the Minimal Supersymmetric Standard Model. For a review of this material one may refer to Ref.[14]. This work is concerned with the lowest-order process for the decay of the Z^0_l- and Z'-bosons into charginos and neutralinos in e^+e^- collisions for the left-right supersymmetric model. The process of interest is:

$$e^+e^- \to Z, Z' \to \tilde{\chi}^+_i \tilde{\chi}^-_j, \tilde{\chi}^0_i \tilde{\chi}^0_j$$

We define the four-momenta:

Incoming e^+ : q_1 ; Incoming e^- : q_2 ; Outgoing $\tilde{\chi}^+$: p_1 ; Outgoing $\tilde{\chi}^-$: p_2

Thus, the conserved four-momenta are:

$$k^2 = (q_1 + q_2)^2 = (p_1 + p_2)^2 . \tag{7.1}$$

For the sake of convenience, the calculation will be performed for the Z' interaction, the left-handed equivalent requiring only minor substitutions. The neutralino cross section is calculated in the same manner.

The Z'-propagator is defined:

$$D_{Z'}(k^2) \equiv \frac{-i}{k^2 - M_{Z'}^2 + i\varepsilon}\left[g_{\mu\nu} + \frac{(\xi - 1)k_\mu k_\nu}{(k^2 - \xi M_{Z'}^2 + i\varepsilon)}\right] \tag{7.2}$$

where $\varepsilon \to \Gamma_{Z'} M_{Z'}$ at the singular point $k^2 = M_{Z'}^2$, and

$\xi = 1$ in the 't Hooft-Feynman gauge,

$\xi = 0$ in the Landau gauge,

$\xi = \infty$ in the unitary gauge .

Since charginos and neutralinos are physical states, this calculation is constrained to be performed in the Landau gauge.

The vertices are defined:

$$Z'-\tilde{\chi}_i^+\tilde{\chi}_j^-\text{-vertex:} \quad -i\frac{g_R g'}{g_{1'}}\gamma^\mu\left(Q_{ij}^L P_L + Q_{ij}^R P_R\right) \;, \tag{7.3}$$

where,

$$P_R = \frac{(1+\gamma_5)}{2} \text{ and } P_L = \frac{(1-\gamma_5)}{2} \;. \tag{7.4}$$

$$e^+e^- - Z'\text{- vertex:} \; -i\frac{g_R}{\cos\theta_W}\gamma^\nu\left(c_V^e - c_A^e\gamma_5\right) \;, \tag{7.5}$$

where the c_V^e and c_A^e are the vector and axial-vector couplings of the Left-Right Supersymmetric model:

$$c_V = -16 + 15\sin^2\theta_W \quad \text{and} \quad c_A = -3\sin^2\theta_W \tag{7.6}$$

7.3 Invariant Amplitude $\mathcal{M}$ for the $Z'-\tilde{\chi}_i^+\tilde{\chi}_j$ Interaction

The invariant amplitude $\mathcal{M}$ for the $e^+e^- \to Z' \to \tilde{\chi}_i^+\tilde{\chi}_j$ interaction is written:

$$\mathcal{M} = \left\{ \left[i\frac{g_R}{\cos\theta_W}\bar{\tilde{\chi}}_j(q_2,s_2)\gamma^\mu\left(Q_{ij}^L P_L + Q_{ij}^R P_R\right)\tilde{\chi}_i(q_1,s_1) \right] \times \right.$$
$$\left. \left[\frac{-i}{(k^2 - M_{Z'}^2 + i\varepsilon)}\left(g_{\mu\nu} - \frac{k_\mu k_\nu / M_{Z'}^2}{(k^2 - M_{Z'}^2 + i\varepsilon)}\right) \right]\left[i\frac{g_R}{\cos\theta_W}\bar{u}(p_2,r_2)\gamma^\nu\left(c_V^e - c_A^e\gamma_5\right)u(q_1,s_1) \right] \right\} \tag{7.7}$$

where the terms in $k_\mu k_\nu \to 0$ by virtue of the fact that

$$k_\mu = p_{2\mu} - p_{1\mu} \tag{7.8}$$

and the Dirac equation gives

$$p_{2\mu}\bar{u}(p_2,r_2)=0 \quad \text{and} \quad p_{1\mu}\bar{u}(p_1,r_1)=0 \tag{7.9}$$

We shall call $\mathcal{M}_1$

$$i\left(\frac{g_R^2 g'}{g_V \cos\ \theta_W}\right)\left|D_{Z'}(k^2)\right|\bar{\tilde{\chi}}_j(q_2,s_2)\gamma^\mu Q_{ij}^L P_L\tilde{\chi}_i(q_1,s_1)\bar{u}(p_2,r_2)\gamma^\nu(c_V^e-c_A^e\gamma_5)\bar{u}(p_1,r) \tag{7.10}$$

and $\mathcal{M}_2$

$$i\left(\frac{g_R^2 g'}{g_V \cos\ \theta_W}\right)\left|D_{Z'}(k^2)\right|\bar{\tilde{\chi}}_j(q_2,s_2)\gamma^\mu Q_{ij}^R P_R\tilde{\chi}_i(q_1,s_1)\bar{u}(p_2,r_2)\gamma^\nu(c_V^e-c_A^e\gamma_5)\bar{u}(p_1,r) \tag{7.11}$$

The average square of the invariant amplitude; $|\mathcal{M}|^2=\mathcal{M}\mathcal{M}^+$, is then defined:

$$|\mathcal{M}|^2_{ave}=\tfrac{1}{4}\sum_{spins}|\mathcal{M}_1+\mathcal{M}_2|^2=\tfrac{1}{4}\sum_{spins}\left[\ |\mathcal{M}_1|^2+|\mathcal{M}_1\mathcal{M}_2^*|+|\mathcal{M}_2\mathcal{M}_1^*|+|\mathcal{M}_2|^2\ \right] \tag{7.12}$$

where we average over initial spins and sum over final spins. Applying succesive Fierz transformations of the form

$$\left[\bar{u}(f)\gamma^a u(l)\right]\left[\bar{\upsilon}(\bar{l})\gamma_a\upsilon(\bar{f})\right]=-\left[\bar{u}(f)\gamma^a\upsilon(\bar{f})\right]\left[\bar{\upsilon}(\bar{l})\gamma_a u(l)\right] \tag{7.13}$$

and evaluating spin sums according to

$$\sum_{r_1}u_1(p_1,r_1)\bar{u}_1(p_1,r_1)=\not{p}_1+M_{e^-}$$

$$\sum_{r_2}u_2(p_2,r_2)\bar{u}_2(p_2,r_2)=\not{p}_2-M_{e^-}$$

$$\sum_{s_1}\tilde{\chi}_1(q_1,s_1)\bar{\tilde{\chi}}_1(q_1,s_1)=\not{q}_1+M_{\chi^-}$$

$$\sum_{s_2} \tilde{\chi}_1(q_2,s_2)\bar{\tilde{\chi}}_2(q_2,s_2) = \not{q}_2 - M_{\chi}$$

(7.14)

where it should be noted that at this energy, the electron and positron masses may be approximated to equal zero, thus

$$\sum_{r_1} u_1(p_1,r_1)\bar{u}_1(p_1,r_1) \cong \not{p}_1 \tag{7.15}$$

and

$$\sum_{r_2} u_2(p_2,r_2)\bar{u}_2(p_2,r_2) \cong \not{p}_2 \tag{7.16}$$

then we may evaluate each of the contributing terms, $|\mathcal{M}_1|^2, |\mathcal{M}_1\mathcal{M}_2^*|^2, |\mathcal{M}_2\mathcal{M}_1^*|^2, |\mathcal{M}_2|^2$ individually.

As an example, we evaluate $|\mathcal{M}_1|^2$:

$$-\left(\frac{g_R^2 g'}{g_{V'}\cos\theta_{W'}}\right)^2 |Q_{ij}^L|^2 |D_{Z'}(k^2)|^2 \times$$
$$\left\{ \left[\bar{\tilde{\chi}}_j(q_2,s_2)\gamma^\mu(1-\gamma_5)\tilde{\chi}_i(q_1,s_1)\bar{\tilde{\chi}}_i(q_1,s_1)\gamma^\nu(1-\gamma_5)\tilde{\chi}_2(q_2,s_2) \right] \times \right.$$
$$\left. \left[\bar{u}(p_2,r_2)\gamma_\mu(c_V^e - c_A^e\gamma_5)u(p_1,r_1)\bar{u}(p_1,r_1)\gamma_\nu(c_V^e - c_A^e\gamma_5)\bar{u}(p_2,r_2) \right] \right\}$$

(7.17)

Completing the spin summations and evaluating the products with the help of the trace theorems

$$Tr\left[\gamma_\mu\gamma_\nu\right] = 4g_{\mu\nu}$$

$$Tr\left[\gamma_\alpha\gamma_\mu\gamma_\beta\gamma_\nu\right] = 4\left(g_{\alpha\mu}g_{\beta\nu} - g_{\alpha\beta}g_{\mu\nu} + g_{\alpha\nu}g_{\mu\beta}\right)$$

$$Tr\left[\gamma_5\gamma_\alpha\gamma_\mu\gamma_\beta\gamma_\nu\right] = 4i\varepsilon_{\alpha\mu\beta\nu}$$

$$Tr\left[odd\ \#\ \gamma's\right] = 0$$

$$Tr\left[\gamma_5\gamma_\alpha\gamma_\beta\right] = Tr\left[\gamma_5\gamma_\alpha\gamma_\mu\gamma_\beta\right] = 0$$

(7.18)

and defining

$$c_R \equiv c_V - c_A \text{ and } c_L \equiv c_V + c_A \tag{7.19}$$

we are left with an expression for $|\mathcal{M}_1|^2$ of

$$32\left(\frac{g_R^4 g'^2}{g_V^2 \cos^2\theta_W}\right)\left|Q_{ij}^l\right|^2\left|D_{Z'}(k^2)\right|^2 \times$$
$$\left\{ (c_R^2 + c_L^2)\left[(q_2\cdot p_2)(q_1\cdot p_1) + (q_2\cdot p_1)(q_1\cdot p_2)\right]\right.$$
$$\left. + (c_R^2 - c_L^2)\left[(q_2\cdot p_2)(q_1\cdot p_1) - (q_2\cdot p_1)(q_1\cdot p_2)\right] \right\}$$

(7.20)

In a similar manner the expressions for $\left|\mathcal{M}_1\mathcal{M}_2^*\right|^2$, $\left|\mathcal{M}_2\mathcal{M}_1^*\right|^2$, and $\left|\mathcal{M}_2\right|^2$ are determined to be:

$$\left|\mathcal{M}_{u_1}\mathcal{M}_{u_2}^*\right|^2 = 32\left(\frac{g_R^4 g'^2}{g_{Y'}^2\cos^2\theta_W}\right)\left|Q_{ij}^L Q_{ij}^{R*}\right|\left|D_{Z'}(k^2)\right|^2 * \left\{ \left(c_R^2 + c_L^2\right)\left(p_2\cdot p_1\right) \right\} \quad (7.21)$$

$$\left|\mathcal{M}_{u_2}\mathcal{M}_{u_1}^*\right|^2 = 32\left(\frac{g_R^4 g'^2}{g_{Y'}^2\cos^2\theta_W}\right)\left|Q_{ij}^{L*} Q_{ij}^{R}\right|\left|D_{Z'}(k^2)\right|^2 * \left\{ \left(c_R^2 + c_L^2\right)\left(p_2\cdot p_1\right) \right\} \quad (7.22)$$

$$\left|\mathcal{M}_{t_2}\right|^2 = 32\left(\frac{g_R^4 g'^2}{g_{Y'}^2\cos^2\theta_W}\right)\left|Q_{ij}^R\right|^2\left|D_{Z'}(k^2)\right|^2 \times \left\{ \left(c_R^2 + c_L^2\right)\left[\left(q_2\cdot p_2\right)\left(q_1\cdot p_1\right) + \left(q_2\cdot p_1\right)\left(q_1\cdot p_2\right)\right] - \left(c_R^2 - c_L^2\right)\left[\left(q_2\cdot p_2\right)\left(q_1\cdot p_1\right) - \left(q_2\cdot p_1\right)\left(q_1\cdot p_2\right)\right] \right\} \quad (7.23)$$

Now it is conventional to express these quantities in terms of related Mandelstam variables defined (for incoming particles of momenta p_A and p_B and outgoing particles of momenta p_C and p_D):

$$\begin{aligned} s &= \left(p_A + p_B\right)^2 \\ t &= \left(p_A - p_C\right)^2 \\ u &= \left(p_A - p_D\right)^2 \end{aligned} \quad (7.24)$$

In this particular case we have

$$k^2 = s = \left(q_1 + q_2\right)^2 = \left(p_1 + p_2\right)^2 \quad (7.25)$$

which allow us to express the momenta products (neglecting the electron and positron masses):

$$(p_1 \cdot p_2) = \tfrac{1}{2} s$$

$$(q_1 \cdot q_2) = \tfrac{1}{2}\left(s - M_{\tilde{\chi}^+}^2 - M_{\tilde{\chi}^-}^2\right)$$

$$(p_2 \cdot q_2) = \tfrac{1}{2}\left(M_{\tilde{\chi}^-}^2 - t\right)$$

$$(p_1 \cdot q_2) = \tfrac{1}{2}\left(M_{\tilde{\chi}^-}^2 - u\right)$$

$$(p_2 \cdot q_1) = \tfrac{1}{2}\left(M_{\tilde{\chi}^+}^2 - u\right)$$

(7.26)

This process was evaluated in the center-of-mass frame of reference. We have

$$u = \tfrac{1}{2}\left(M_{\tilde{\chi}_i^+}^2 + M_{\tilde{\chi}_j^-}^2 - s - \sqrt{\left[s - \left(M_{\tilde{\chi}_i^+} + M_{\tilde{\chi}_j^-}\right)^2\right]\left[s - \left(M_{\tilde{\chi}_i^+} - M_{\tilde{\chi}_j^-}\right)^2\right]} \cos\theta_{CM} \right)$$

(7.27)

$$t = \tfrac{1}{2}\left(M_{\tilde{\chi}_i^+}^2 + M_{\tilde{\chi}_j^-}^2 - s + \sqrt{\left[s - \left(M_{\tilde{\chi}_i^+} + M_{\tilde{\chi}_j^-}\right)^2\right]\left[s - \left(M_{\tilde{\chi}_i^+} - M_{\tilde{\chi}_j^-}\right)^2\right]} \cos\theta_{CM} \right)$$

(7.28)

Taking all terms into consideration we may express $|\mathcal{M}|^2_{av}$ for the process $e^+e^- \rightarrow Z' \rightarrow \tilde{\chi}_i^+ \tilde{\chi}_j^-$:

$$\left(\frac{32 g_R^4 g'^2}{g_{V'}^2 \cos^2\theta_W}\right)\frac{\left(c_R^2 + c_L^2\right)}{\left(s - M_{Z'}^2 + i\varepsilon\right)^2}\Bigg\{\left(\left|Q_{ij}^L\right|^2 + \left|Q_{ij}^R\right|^2\right)\frac{1}{4}\Big[\; 2M_{\tilde{\chi}_j^-}^2 M_{\tilde{\chi}_i^+}^2\left(M_{\tilde{\chi}_j^-}^2 + M_{\tilde{\chi}_i^+}^2 - s\right)\left(M_{\tilde{\chi}}^2 + M_{\tilde{\chi}_i^+}^2\right) + s^2$$

$$+\left[s^2 - 2s\left(M_{\tilde{\chi}_j^-}^2 + M_{\tilde{\chi}_i^+}^2\right) + \left(M_{\tilde{\chi}_j^-}^2 + M_{\tilde{\chi}_i^+}^2\right)^2\right]\cos^2\theta_{CM}\;\Big] + \left(O_{ij}^L O_{ij}^{R*} + O_{ij}^R O_{ij}^{L*}\right)\left[\; sM_{\tilde{\chi}} M_{\tilde{\chi}_i^+} \;\right]\Bigg\}$$

(7.29)

7.4 Differential Cross Sections and Total Cross Sections

The differential cross section for two body scattering $1+2 \rightarrow 3+4$ of non-identical particals is given by:

$$\frac{d\sigma}{d\Omega_{CM}} = \frac{|q'|}{|q|}\frac{1}{64\pi^2 s}\left|M(s,t)\right|^2 \qquad (7.30)$$

where in the case of mixed handedness, only the s-channel contributes. For our process we have:

$$4q^2 = \frac{\lambda\left(s, M_{e^+}^2, M_{e^-}^2\right)}{s} = \frac{\left[s - \left(M_{e^+} + M_{e^-}\right)^2\right]\left[s - \left(M_{e^+} - M_{e^-}\right)^2\right]}{s}, \qquad (7.31)$$

$$4q'^2 = \frac{\lambda\left(s, M_{\tilde{\chi}^+}^2, M_{\tilde{\chi}^-}^2\right)}{s} = \frac{\left[s - \left(M_{\tilde{\chi}^+} + M_{\tilde{\chi}}\right)^2\right]\left[s - \left(M_{\tilde{\chi}^+} - M_{\tilde{\chi}}\right)^2\right]}{s}, \qquad (7.32)$$

so that in the high-energy approximation $M_{e^+}, M_e \rightarrow 0$, then

$$\sqrt{\frac{\lambda\left(s, M_{\tilde{\chi}^{+}}^{2}, M_{\tilde{\chi}}^{2}\right)}{\lambda\left(s, M_{e^{+}}^{2}, M_{e}^{2}\right)}} \cong \left[\ 1-\frac{2}{s}\left(M_{\tilde{\chi}^{+}}^{2}+M_{\tilde{\chi}^{-}}^{2}\right)+\frac{1}{s^{2}}\left(M_{\tilde{\chi}^{+}}^{2}-M_{\tilde{\chi}^{-}}^{2}\right)^{2}\ \right]^{1/2} \qquad (7.33)$$

The partially integrated differential cross section for the process $e^{+}e^{-} \rightarrow Z' \rightarrow \tilde{\chi}_{i}^{+}\tilde{\chi}_{j}^{-}$ is given by

$$\frac{d\sigma}{d(\cos\theta_{CM})} = \frac{1}{256\pi}\left(\frac{g_{R}^{4}g'^{2}}{g_{Y}^{2}\cos^{2}\theta_{W}}\right)\frac{\left(c_{R}^{2}+c_{L}^{2}\right)}{s\left(s-M_{Z'}^{2}+i\varepsilon\right)^{2}}\left[\ 1-\frac{2}{s}\left(M_{\tilde{\chi}_{i}^{+}}^{2}+M_{\tilde{\chi}_{j}^{-}}^{2}\right)+\frac{1}{s^{2}}\left(M_{\tilde{\chi}^{+}}^{2}-M_{\tilde{\chi}^{-}}^{2}\right)^{2}\ \right]^{1/2} \times$$

$$\Bigg\{ \left(\left|Q_{ij}^{L}\right|^{2}+\left|Q_{ij}^{R}\right|^{2}\right)\Bigg(\left[\ s^{2}-\left(M_{\tilde{\chi}_{i}^{+}}^{2}-M_{\tilde{\chi}_{j}^{-}}^{2}\right)^{2}\ \right]$$

$$+\left[s^{2}-2s\left(M_{\tilde{\chi}_{i}^{+}}^{2}+M_{\tilde{\chi}_{j}^{-}}^{2}\right)+\left(M_{\tilde{\chi}_{i}^{+}}^{2}-M_{\tilde{\chi}_{j}^{-}}^{2}\right)^{2}\right]\cos^{2}\theta_{CM}\ \Bigg)$$

$$+\left(Q_{ij}^{L}Q_{ij}^{R*}+Q_{ij}^{R}Q_{ij}^{L*}\right)\left[4sM_{\tilde{\chi}_{i}^{+}}M_{\tilde{\chi}_{j}^{-}}\right]\Bigg\}$$

(7.34)

Integrating over θ_{CM} gives the total cross section

$$\sigma_{TOT} = \frac{1}{128\pi}\left(\frac{g_{R}^{4}g'^{2}}{g_{Y}^{2}\cos^{2}\theta_{W}}\right)\frac{\left(c_{R}^{2}+c_{L}^{2}\right)}{s\left(s-M_{Z'}^{2}+i\varepsilon\right)^{2}}\left[\ 1-\frac{2}{s}\left(M_{\tilde{\chi}_{i}^{+}}^{2}+M_{\tilde{\chi}_{j}^{-}}^{2}\right)+\frac{1}{s^{2}}\left(M_{\tilde{\chi}^{+}}^{2}-M_{\tilde{\chi}^{-}}^{2}\right)^{2}\ \right]^{1/2} \times$$

$$\Bigg\{ \left(\left|Q_{ij}^{L}\right|^{2}+\left|Q_{ij}^{R}\right|^{2}\right)\frac{1}{3}\left[s^{2}-s\left(M_{\tilde{\chi}_{i}^{+}}^{2}+M_{\tilde{\chi}_{j}^{-}}^{2}\right)+\left(M_{\tilde{\chi}_{i}^{+}}^{2}-M_{\tilde{\chi}_{j}^{-}}^{2}\right)^{2}\right]$$

$$+\left(Q_{ij}^{L}Q_{ij}^{R*}+Q_{ij}^{R}Q_{ij}^{L*}\right)\left[4sM_{\tilde{\chi}_{i}^{+}}M_{\tilde{\chi}_{j}^{-}}\right]\Bigg\}$$

(7.35)

The corresponding differential cross section and total cross section for neutralino production are respectively

$$\frac{d\sigma}{d(\cos\theta_{CM})}=\frac{1}{256\pi}\left(\frac{g_R^4 g'^2}{g_{V}^2\cos^2\theta_W}\right)\frac{\left(c_R^2+c_L^2\right)}{s\left(s-M_{Z'}^2+i\varepsilon\right)^2}\left[\ 1-\frac{2}{s}\left(M_{\tilde{\chi}_i^0}^2+M_{\tilde{\chi}_j^0}^2\right)+\frac{1}{s^2}\left(M_{\tilde{\chi}_i^0}^2-M_{\tilde{\chi}_j^0}^2\right)^2\ \right]^{1/2}\times$$
$$\left\{\ \left(\left|O''^{L}_{ij}\right|^2+\left|O''^{R}_{ij}\right|^2\right)\left(\ \left[\ s^2-\left(M_{\tilde{\chi}_i^0}^2-M_{\tilde{\chi}_j^0}^2\right)^2\ \right]\right.\right.$$
$$\left.+\left[s^2-2s\left(M_{\tilde{\chi}_i^0}^2+M_{\tilde{\chi}_j^0}^2\right)+\left(M_{\tilde{\chi}_i^0}^2-M_{\tilde{\chi}_j^0}^2\right)^2\right]\cos^2\theta_{CM}\ \right)$$
$$\left.+\left(O''^{L}_{ij}O''^{R*}_{ij}+O''^{R}_{ij}O''^{L*}_{ij}\right)\left[4sM_{\tilde{\chi}_i}M_{\tilde{\chi}_j}\right]\ \right\}$$

(7.36)

and

$$\sigma_{TOT}=\frac{1}{128\pi}\left(\frac{g_R^4 g'^2}{g_{V}^2\cos^2\theta_W}\right)\frac{\left(c_R^2+c_L^2\right)}{s\left(s-M_{Z'}^2+i\varepsilon\right)^2}\left[\ 1-\frac{2}{s}\left(M_{\tilde{\chi}_i^0}^2+M_{\tilde{\chi}_j^0}^2\right)+\frac{1}{s^2}\left(M_{\tilde{\chi}_i^0}^2-M_{\tilde{\chi}_j^0}^2\right)^2\ \right]^{1/2}\times$$
$$\left\{\ \left(\left|O''^{L}_{ij}\right|^2+\left|O''^{R}_{ij}\right|^2\right)\frac{1}{3}\left[s^2-s\left(M_{\tilde{\chi}_i^0}^2+M_{\tilde{\chi}_j^0}^2\right)+\left(M_{\tilde{\chi}_i^0}^2-M_{\tilde{\chi}_j^0}^2\right)^2\right]\right.$$
$$\left.+\left(O''^{L}_{ij}O''^{R*}_{ij}+O''^{R}_{ij}O''^{L*}_{ij}\right)\left[4sM_{\tilde{\chi}_i}M_{\tilde{\chi}_j}\right]\ \right\}$$

(7.37)

Cross sections for the left-handed interactions have also been calculated. The results are:

For the process $e^+e^- \rightarrow Z_L^0 \rightarrow \tilde{\chi}_i^+ \tilde{\chi}_j^-$:

$$\frac{d\sigma}{d(\cos\theta_{CM})} = \frac{1}{256\pi}\left(\frac{g_I^4}{\cos^4\theta_W}\right)\frac{\left(c_R^2+c_L^2\right)}{s\left(s-M_{Z_L}^2+i\varepsilon\right)^2}\left[\ 1-\frac{2}{s}\left(M_{\tilde{\chi}_i^+}^2+M_{\tilde{\chi}_j^-}^2\right)+\frac{1}{s^2}\left(M_{\tilde{\chi}^+}^2-M_{\tilde{\chi}^-}^2\right)^2\ \right]^{1/2}\times$$
$$\left\{ \left(\left|O'^L_{ij}\right|^2+\left|O'^R_{ij}\right|^2\right)\left(\left[\ s^2-\left(M_{\tilde{\chi}_i^+}^2-M_{\tilde{\chi}_j^-}^2\right)^2\ \right]\right.\right.$$
$$\left.+\left[s^2-2s\left(M_{\tilde{\chi}_i^+}^2+M_{\tilde{\chi}_j}^2\right)+\left(M_{\tilde{\chi}_i^+}^2-M_{\tilde{\chi}_j^-}^2\right)^2\right]\cos^2\theta_{CM}\ \right)$$
$$\left.+\left(O'^L_{ij}O'^{R*}_{ij}+O'^R_{ij}O'^{L*}_{ij}\right)\left[4sM_{\tilde{\chi}_i^+}M_{\tilde{\chi}_j^-}\right]\ \right\}$$

(7.38)

and

$$\sigma_{TOT} = \frac{1}{128\pi}\left(\frac{g_I^4}{\cos^4\theta_W}\right)\frac{\left(c_R^2+c_I^2\right)}{s\left(s-M_{Z'}^2+i\varepsilon\right)^2}\left[\ 1-\frac{2}{s}\left(M_{\tilde{\chi}_i^+}^2+M_{\tilde{\chi}_j^-}^2\right)+\frac{1}{s^2}\left(M_{\tilde{\chi}^+}^2-M_{\tilde{\chi}^-}^2\right)^2\ \right]^{1/2}\times$$
$$\left\{ \left(\left|O'^L_{ij}\right|^2+\left|O'^R_{ij}\right|^2\right)\frac{1}{3}\left[s^2-s\left(M_{\tilde{\chi}_i^+}^2+M_{\tilde{\chi}_j^-}^2\right)+\left(M_{\tilde{\chi}_i^+}^2-M_{\tilde{\chi}_j^-}^2\right)^2\right]\right.$$
$$\left.+\left(O'^L_{ij}O'^{R*}_{ij}+O'^R_{ij}O'^{L*}_{ij}\right)\left[4sM_{\tilde{\chi}_i^+}M_{\tilde{\chi}_j}\right]\ \right\}$$

(7.39)

For the process $e^+e^- \to Z_L^0 \to \tilde{\chi}_i^0 \tilde{\chi}_j^0$:

$$\frac{d\sigma}{d(\cos\theta_{CM})} = \frac{1}{256\pi}\left(\frac{g_I^4}{\cos^4\theta_W}\right)\frac{\left(c_R^2 + c_L^2\right)}{s\left(s - M_{Z_L}^2 + i\varepsilon\right)^2}\left[\ 1 - \frac{2}{s}\left(M_{\tilde{\chi}_i^0}^2 + M_{\tilde{\chi}_j^0}^2\right) + \frac{1}{s^2}\left(M_{\tilde{\chi}_i^0}^2 - M_{\tilde{\chi}_j^0}^2\right)^2\ \right]^{1/2} \times$$

$$\left\{ \left(\left|O''^L_{ij}\right|^2 + \left|O''^R_{ij}\right|^2\right)\left(\left[\ s^2 - \left(M_{\tilde{\chi}_i^0}^2 - M_{\tilde{\chi}_j^0}^2\right)^2\ \right] \right.\right.$$

$$\left. + \left[s^2 - 2s\left(M_{\tilde{\chi}_i^0}^2 + M_{\tilde{\chi}_j^0}^2\right) + \left(M_{\tilde{\chi}_i^0}^2 - M_{\tilde{\chi}_j^0}^2\right)^2\right]\cos^2\theta_{CM}\ \right)$$

$$\left. + \left(O''^L_{ij} O''^{R*}_{ij} + O''^R_{ij} O''^{L*}_{ij}\right)\left[4sM_{\tilde{\chi}_i} M_{\tilde{\chi}_j}\right] \right\}$$

(7.40)

and

$$\sigma_{TOT} = \frac{1}{128\pi}\left(\frac{g_I^4}{\cos^4\theta_W}\right)\frac{\left(c_R^2 + c_L^2\right)}{s\left(s - M_{Z_L}^2 + i\varepsilon\right)^2}\left[\ 1 - \frac{2}{s}\left(M_{\tilde{\chi}_i^0}^2 + M_{\tilde{\chi}_j^0}^2\right) + \frac{1}{s^2}\left(M_{\tilde{\chi}_i^0}^2 - M_{\tilde{\chi}_j^0}^2\right)^2\ \right]^{1/2} \times$$

$$\left\{ \left(\left|O''^L_{ij}\right|^2 + \left|O''^R_{ij}\right|^2\right)\frac{1}{3}\left[s^2 - s\left(M_{\tilde{\chi}_i^0}^2 + M_{\tilde{\chi}_j^0}^2\right) + \left(M_{\tilde{\chi}_i^0}^2 - M_{\tilde{\chi}_j^0}^2\right)^2\right]\right.$$

$$\left. + \left(O''^L_{ij} O''^{R*}_{ij} + O''^R_{ij} O''^{L*}_{ij}\right)\left[4sM_{\tilde{\chi}_i} M_{\tilde{\chi}_j}\right] \right\}$$

(7.41)

Analytical expressions and numerical solutions for chargino and neutralino masses in the fully left-right supersymmetric model (LRSM) have been determined by Frank, Kalman and Saif[22]. Two charginos $\tilde{\chi}_i^{\pm}$ and three neutralinos $\tilde{\chi}_i^0$ are generated from the breaking of the LRSM to the MSSM. Four additional charginos and four neutralinos are generated from the breaking of the MSSM. The masses of the charginos in the LRSM, are dependent upon a variety of parameters. They are: gaugino mass parameters M_L and M_R associated with the gauge groups $SU(2)_L$ and $SU(2)_R$ respectively; the higgsino mass parameter μ; and $\tan\theta_\kappa = \kappa_u / \kappa_d$, where κ_u and κ_d are the VEV's of φ_u and ϕ_d respectively. Masses have been calculated using several different combinations of choices for the above-mentioned parameters[23].

For any choice of M_l and M_R, the chargino masses can be larger than M_W and M_Z. The numerical solutions for the neutralinos all provide for small regions of $|\mu|$ where the LSP $\tilde{\chi}_1^0$ is very light. For large values of μ, the masses of the neutralinos are heavier than M_{W_L}.

Since no pairs of unidentifiable charged particles are observed from Z_l decay[24], $\tilde{\chi}_1^{\pm} > 45.2$ GeV. It is expected that higher energies than those currently available, will be necessary for the detection of these particular supersymmetric particles. Since decays arising from electron-positron collisions tend to give rise to very clean signatures it is expected that energies around the Z' should be the ideal place to see the $\tilde{\chi}_1^{\pm}$.

Conclusion

This thesis is concerned with the identification of chargino and neutralino states using the fully left-right supersymmetric model (LRSM). The process of interest is $e^+e^- \to Z' \to \tilde{\chi}_j^+\tilde{\chi}_i^-, \tilde{\chi}_j^0\tilde{\chi}_i^0$. The total cross-sections are given in eqs.(7.34)-(7.41). The future CLIC at CERN[25] should provide center of mass energies on the order of 1 to 2 TeV. Evaluating at $\sqrt{s}$= 2 TeV , and taking a reasonable assumption for $M_{Z'}$ (400 GeV)[26], chargino and neutralino mass-dependent values have been calculated. Frank, Kalman and Saif[27] have determined possible values for the chargino and neutralino masses for different combinations of parameters. The width of the Z' was calculated from the general form[28]

$$\Gamma\left(Z' \to \tilde{f}_{L,R}\bar{\tilde{f}}_{L,R}\right) = \frac{M_{Z'}}{12\pi}\left(1-4\eta_{\tilde{f}}\right)^{3/2}\frac{1}{4}C_f\left(\upsilon_f{}' \mp a_f{}'\right)^2 \tag{7.42}$$

where $\upsilon_f{}'$ and $a_f{}'$ are the vector and axial couplings of the generic fermions $\tilde{f}$, and $\eta_{\tilde{f}} \equiv \left(M_a/M_b\right)^2$, with $a=\tilde{f}$ and C_f is the colour factor. The form which applies in this this case is also used by Frank, Kalman, and Saif[29] to predict the Z' decay rate for $M_{Z'} = 0.5\,TeV$. At this energy the value was found to be $\Gamma_{Z'} \approx 18\,GeV$. The same calculation for $M_{Z'} = 0.4\,TeV$ yields a value of $\Gamma_{Z'} \approx 12\,GeV$. This value was used in the determination of numerical values for the cross sections for different choices of parameters and masses. These choices were made in accordance with values provided in Ref.[28].

Since the chargino and neutralino masses have been chosen in the range > 50 GeV, one would expect their widths to be on the order of a fraction of a GeV. Unless the charginos or neutralinos in question turn out to be the LSP (stable), the decay of these particles should occur well within the detector. With such widths, we would expect to observe not the charginos and neutralinos themselves, but their decay products.

In the case of charginos, the expected decays are the following[30].

$$\tilde{\chi}_i^{+} \to l^{+} + \nu + \tilde{\chi}_k^0 \tag{7.43}$$

$$\tilde{\chi}_i^{+} \to l^{+} + \tilde{\nu} \tag{7.44}$$

$$\tilde{\chi}_i^{+} \to q + \bar{q}' + \tilde{\chi}_k^0 \tag{7.45}$$

$$\tilde{\chi}_i^{+} \to q + \bar{q}' + \tilde{g}, \quad \tilde{g} \to q + \bar{q}' + \tilde{\chi}_k^0 \tag{7.46}$$

where $\tilde{\chi}_k^0$ is the lightest neutralino mass eigenstate, $\tilde{\nu}$ is the sneutrino, and $\tilde{g}$ is the gluino.

In the case of the neutralinos, the decay products are[31]

$$\tilde{\chi}_i^0 \to l^{+} + l^{-} + \tilde{\chi}_k^0 \tag{7.47}$$

$$\tilde{\chi}_i^0 \to \nu + \tilde{\nu} + \tilde{\chi}_k^0 \tag{7.48}$$

$$\tilde{\chi}_i^0 \to q + \tilde{q} + \tilde{\chi}_k^0 \tag{7.49}$$

$$\tilde{\chi}_i^0 \to q + \tilde{q} + \tilde{g}, \quad \tilde{g} \to q + \tilde{q} + \tilde{\chi}_k^0 \tag{7.50}$$

Using $M_{\chi_i^{+}}$ = 54.6 GeV , $M_{\tilde{\chi}_i^0}$ = 63.2 GeV , $\tan\theta_K = 1\,6$, $M_L = 50\,GeV$, $M_R = 300\,GeV$, $\mu = -10\,GeV$ and $g_l \approx g_R$, the cross sections were determined to be: $\sigma(e^{+}e^{-} \to Z' \to \tilde{\chi}_2^{+}\tilde{\chi}_2^{-})\approx$ 0.21 nb, and $\sigma(e^{+}e^{-} \to Z' \to \tilde{\chi}_2^0\tilde{\chi}_2^0)\approx$ 77 pb .

Using $M_{\chi_i^{+}}$ = 80 GeV , $M_{\tilde{\chi}_i^0}$ = 171.7 GeV , $\tan\theta_K = 1.6$, $M_L = 250\,GeV$, $M_R = 1000\,GeV$, $\mu = -80\,GeV$ and $g_l \approx g_R$, the cross sections were determined to be: $\sigma(e^+e^- \rightarrow Z' \rightarrow \tilde{\chi}_2^+\tilde{\chi}_2^-) \approx$ 0.11 nb, and $\sigma(e^+e^- \rightarrow Z' \rightarrow \tilde{\chi}_2^0\tilde{\chi}_2^0) \approx$ 15 pb .

Finally, choosing $M_{\tilde{\chi}_i} \neq M_{\tilde{\chi}_j}$, the following cross sections were determined. Using $M_{\tilde{\chi}_2^+}=$ 80 GeV, $M_{\tilde{\chi}_1^+}=$ 45.3 GeV , $M_{\tilde{\chi}_2^0}=$ 171.7 GeV, and $M_{\tilde{\chi}_1^0}=$158 GeV , $\tan\theta_\Lambda = 1.6$, $M_l = 250\,GeV$, $M_R = 1000\,GeV$, $\mu = -80\,GeV$ and $g_L \approx g_R$, the cross sections were determined to be: $\sigma(e^+e^- \rightarrow Z' \rightarrow \tilde{\chi}_2^+\tilde{\chi}_1^-) \approx$ 0.32 nb, and $\sigma(e^+e^- \rightarrow Z' \rightarrow \tilde{\chi}_2^0\tilde{\chi}_1^0) \approx$ 48 pb .
Graphs showing cross-sections as a function of center-of-mass energy are shown in Figs. I to VI.

Appendix A

Spinor Notation and Conventions

These are the conventions used by Haber and Kane[14]. They are essentially the same as those used by Wess and Bagger, but use a different metric convention.

The metric is given as

$$g_{\mu\nu} = \mathrm{diag}(1,-1,-1,-1) \tag{A1}$$

The momentum four-vector is $p^{\mu} = (E, \mathbf{p})$. The Pauli matrices are:

$$\sigma^{\mu} = (1, \vec{\sigma}) \quad , \quad \overline{\sigma}^{\mu} = (1, -\vec{\sigma}) \tag{A2}$$

The two-component spinor ξ_{α} transforms under a matrix M of $SL(2,C)$. In a similar fashion, the spinors $\overline{\xi}_{\dot{\alpha}}$, ξ^{α}, and $\overline{\xi}^{\dot{\alpha}}$ transform under M^{*}, M^{-1}, and $(M^{-1})^{*}$ respectively. We may therefore define:

$$\overline{\xi}_{\dot{\alpha}} \equiv \xi^{*}_{\alpha}$$

The dirac equation in two-component notation is:

$$\left(\overline{\sigma}_{\mu} p^{\mu}\right)^{\dot{\alpha}\beta} \xi_{\beta} = m\overline{\eta}^{\dot{\alpha}} \quad , \quad \left(\sigma_{\mu} p^{\mu}\right)_{\alpha\dot{\beta}} \overline{\eta}^{\dot{\beta}} = m\xi_{\alpha} \tag{A3}$$

This allows the introduction of four-component notation. Usually, a four-component spinor is introduced which satisfies

$$\left(\gamma_\mu p^\mu - m\right)\Psi = 0 \tag{A4}$$

It follows that

$$\Psi = \begin{pmatrix} \xi_\alpha \\ \overline{\eta}^\alpha \end{pmatrix} \quad , \quad \gamma_\mu = \begin{pmatrix} 0 & \sigma_{\mu\alpha\beta} \\ \overline{\sigma}_\mu^{\alpha\beta} & 0 \end{pmatrix} \tag{A5}$$

$$\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \tag{A6}$$

$$\sigma^{\mu\nu} = \tfrac{i}{2}\left[\gamma^\mu, \gamma^\nu\right] = 2i\begin{pmatrix} \sigma_\alpha^{\mu\nu\beta} & 0 \\ 0 & \overline{\sigma}_\beta^{\mu\nu\alpha} \end{pmatrix} \tag{A7}$$

where

$$\sigma_\alpha^{\mu\nu\beta} = \tfrac{1}{4}\left(\sigma_{\alpha a}^\mu \overline{\sigma}^{\nu a\beta} - \sigma_{\alpha a}^\nu \overline{\sigma}^{\mu a\beta}\right) . \tag{A8}$$

$$\overline{\sigma}_\beta^{\mu\nu\alpha} = \tfrac{1}{4}\left(\overline{\sigma}^{\mu a\alpha}\sigma_{a\beta}^\nu - \overline{\sigma}^{\nu a\alpha}\sigma_{a\beta}^\mu\right) \tag{A9}$$

This is called the chiral representation of the γ-matrices. We define the left- and right-hand projection operators by

$$p_L \equiv \tfrac{1}{2}\left(1 - \gamma_5\right) \quad , \quad p_R \equiv \tfrac{1}{2}\left(1 + \gamma_5\right) \tag{A10}$$

Then, using the notation $\psi_{L,R} \equiv P_{L,R}$, it is seen that

$$\Psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} \tag{A11}$$

It should be noted that most books define $\sigma_\mu = (1, \vec{\sigma})$ and $\bar{\sigma}_\mu = (1, -\vec{\sigma})$ as opposed to eq.(A2). This would lead to an interchange of ψ_L and ψ_R in eq.(A11).

As usual one defines $\overline{\Psi} = \Psi^\dagger \gamma^0$. The charge conjugation operator C allows the definition of the charge conjugated spinor:

$$\Psi^c = C\overline{\Psi}^T \tag{A12}$$

In the chiral representation, $C = -i\gamma^2\gamma^0$. In two-component notation, one defines an antisymmetric tensor $\left(\varepsilon^{\alpha\beta} = -\varepsilon^{\beta\alpha}\right)$:

$$\varepsilon^{\alpha\beta} = -\varepsilon_{\alpha\beta} = i\sigma^2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \tag{A13}$$

The $\varepsilon_{\alpha\beta}$ can raise andlower spinor indices

$$\xi^\alpha = \varepsilon^{\alpha\beta}\xi_\beta \quad , \quad \xi_\beta = \varepsilon^{\beta\alpha}\xi^\alpha \tag{A14}$$

Identical relations hold when replacing undotted with dotted spinor indices in eqs.(A13) and (A14).

$$C = -i\gamma^2\gamma^0 = \begin{pmatrix} \varepsilon_{\beta\alpha} & 0 \\ 0 & \varepsilon^{\dot\beta\dot\alpha} \end{pmatrix} \tag{A15}$$

and

$$\overline{\Psi}^T = \begin{pmatrix} \eta^\alpha \\ \bar{\xi}_{\dot\alpha} \end{pmatrix} \tag{A16}$$

It follows that

$$\Psi^c = \begin{pmatrix} \eta^\alpha \\ \bar{\xi}_{\alpha} \end{pmatrix} \tag{A17}$$

A four-component Majorana spinor has the property that $\eta = \xi$ which implies that $\psi^c \Psi^c = \Psi$. That is,

$$\Psi_M = \begin{pmatrix} \xi_\alpha \\ \bar{\xi}^{\dot\alpha} \end{pmatrix} = \begin{pmatrix} \psi_L \\ i\sigma^2 \psi_L^* \end{pmatrix} \tag{A18}$$

The translation of two-component formalism into four-component fromalism is accomplished using the following set of equations:

$$\overline{\Psi}_1 \Psi_2 = \eta_1 \xi_2 + \overline{\eta}_2 \bar{\xi}_1 \tag{A19}$$

$$\overline{\Psi}_1 \gamma_5 \Psi_2 = -\eta_1 \xi_2 + \overline{\eta}_2 \bar{\xi}_1 \tag{A20}$$

$$\overline{\Psi}_1 \gamma^\mu \Psi_2 = \bar{\xi}_1 \overline{\sigma}^\mu \xi_2 - \overline{\eta}_2 \overline{\sigma}^\mu \eta_1 \tag{A21}$$

$$\overline{\Psi}_1\gamma^\mu\gamma_5\Psi_2 = -\overline{\xi}_1\overline{\sigma}^\mu\xi_2 - \overline{\eta}_2\overline{\sigma}^\mu\eta_1 \tag{A22}$$

$$-\tfrac{1}{2}\overline{\Psi}_1\sigma^{\mu\nu}\Psi_2 = \eta_1\sigma^{\mu\nu}\xi_2 + \overline{\eta}_2\sigma^{\mu\nu}\overline{\xi}_1 \tag{A23}$$

where the subscripts 1 and 2 label two different four-component spinors and their associated two-component spinors. In eqs.(A19)-(A23), the following have been used:

$$\eta\xi = \eta^\alpha\xi_\alpha = \xi\eta \tag{A24}$$

$$\overline{\eta}\overline{\xi} = \overline{\eta}_\alpha\overline{\xi}^\alpha = \overline{\xi}\overline{\eta} \tag{A25}$$

$$\overline{\eta}_2\overline{\sigma}^\mu\eta_1 = \overline{\eta}_{2\alpha}\overline{\sigma}^{\mu\alpha\beta}\eta_{1\beta} = \eta_1\overline{\sigma}^\mu\overline{\eta}_2 \tag{A26}$$

$$\overline{\eta}\overline{\sigma}^{\mu\nu}\overline{\xi} = \overline{\eta}_\alpha\overline{\sigma}_\beta^{\alpha\mu\nu}\overline{\xi}^\beta = -\overline{\xi}\overline{\sigma}^{\mu\nu}\overline{\eta} \tag{A27a}$$

$$\eta\sigma^{\mu\nu}\xi = \eta^\alpha\sigma_\alpha^{\beta\mu\nu}\xi_\beta = -\xi\sigma^{\mu\nu}\eta \tag{A27b}$$

In eqs.(A24)-(A27), the first equality is one of definition. The second equality follows from eqs.(A13),(A14) and the fact that spinors anticommute. The definition of $\overline{\eta}\overline{\xi}$ has been chosen so that $(\eta\xi)^{\dagger} = \overline{\eta}\overline{\xi}$. Using eqs.(A19)-(A23), all Lagrangians written in two-component form can be converted to four-component form, to satisfy the present conventions. In this respect it is particuarly useful to rewrite eqs.(A19)-(A23) as follows:

$$\overline{\Psi}_1 P_L \Psi_2 = \eta_1 \xi_2 \,.$$

$$\overline{\Psi}_1 P_R \Psi_2 = \overline{\eta}_1 \overline{\xi}_2 \,.$$

$$\overline{\Psi}_1 \gamma^\mu P_L \Psi_2 = \overline{\xi}_1 \overline{\sigma}^\mu \xi_2$$

$$\overline{\Psi}_1 \gamma^\mu P_R \Psi_2 = -\overline{\eta}_1 \overline{\sigma}^\mu \eta_2$$

(A28)

where $P_{L,R}$ are the projection operators defined in eq.(A10). From this, one can build up four-component Dirac and Majorana spinors and interactions from a Lagrangian expressed in two-component notation.

To conclude, below are some useful identities which follow from eqs.(A18)-(A23). If Ψ_1 and Ψ_2 are anticommuting four-component Majorana spinors, then:

$$\overline{\Psi}_1 \Psi_2 = \overline{\Psi}_2 \Psi_1 \tag{A29}$$

$$\overline{\Psi}_1 \gamma_5 \Psi_2 = \overline{\Psi}_2 \gamma_5 \Psi_1 \tag{A30}$$

$$\overline{\Psi}_1 \gamma_\mu \Psi_2 = -\overline{\Psi}_2 \gamma_\mu \Psi_1 \tag{A31}$$

$$\overline{\Psi}_1 \gamma_\mu \gamma_5 \Psi_2 = \overline{\Psi}_2 \gamma_\mu \gamma_5 \Psi_1 \tag{A32}$$

$$\overline{\Psi}_1 \sigma_{\mu\nu} \Psi_2 = -\overline{\Psi}_2 \sigma_{\mu\nu} \Psi_1 \tag{A33}$$

One useful consequence of the above is:

$$\overline{\Psi}_1 \gamma^\mu P_L \Psi_2 = -\overline{\Psi}_2 \gamma^\mu P_R \Psi_1 \tag{A34}$$

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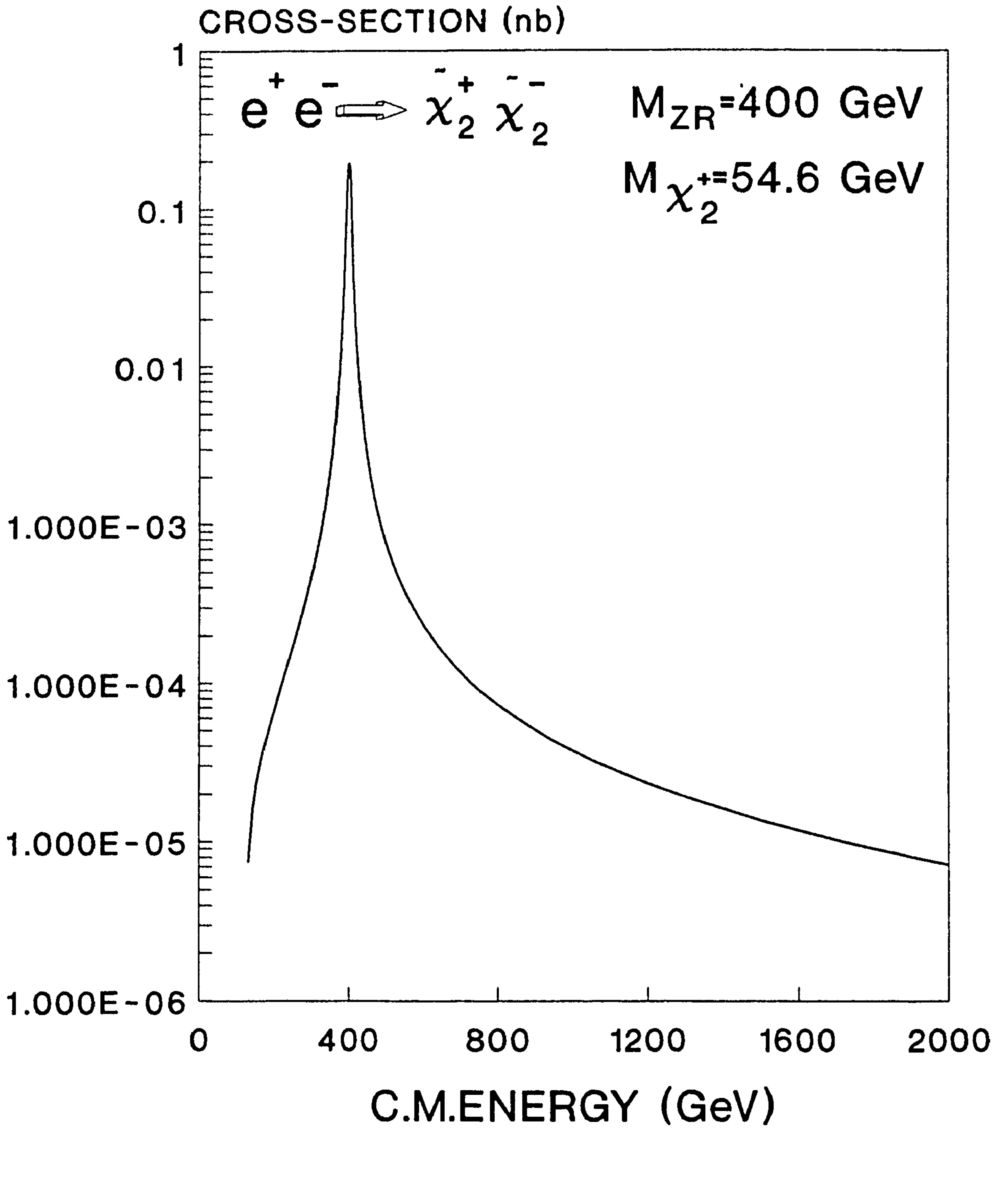
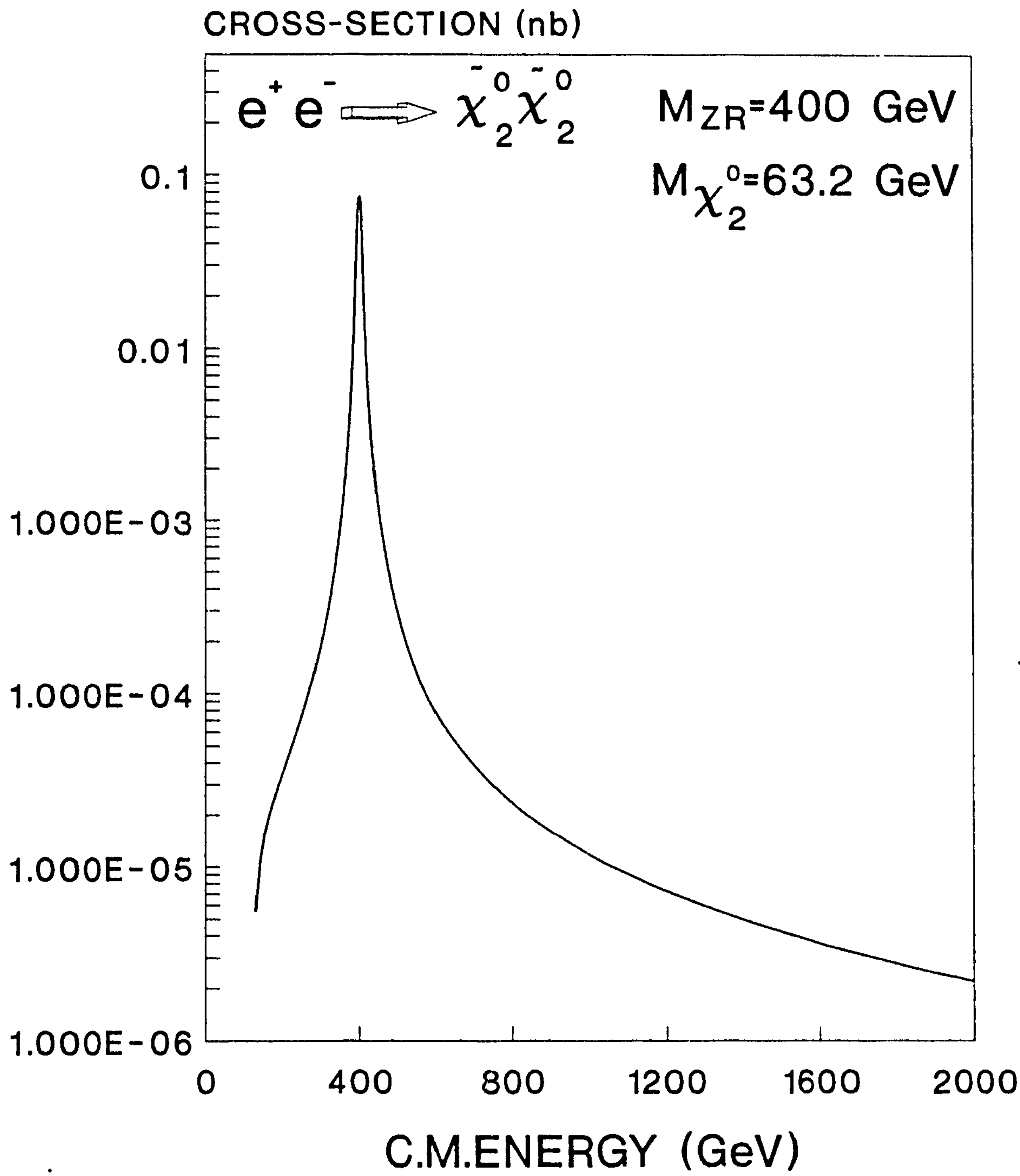
Table I. The particle content for the L-R Supersymmetric Model.

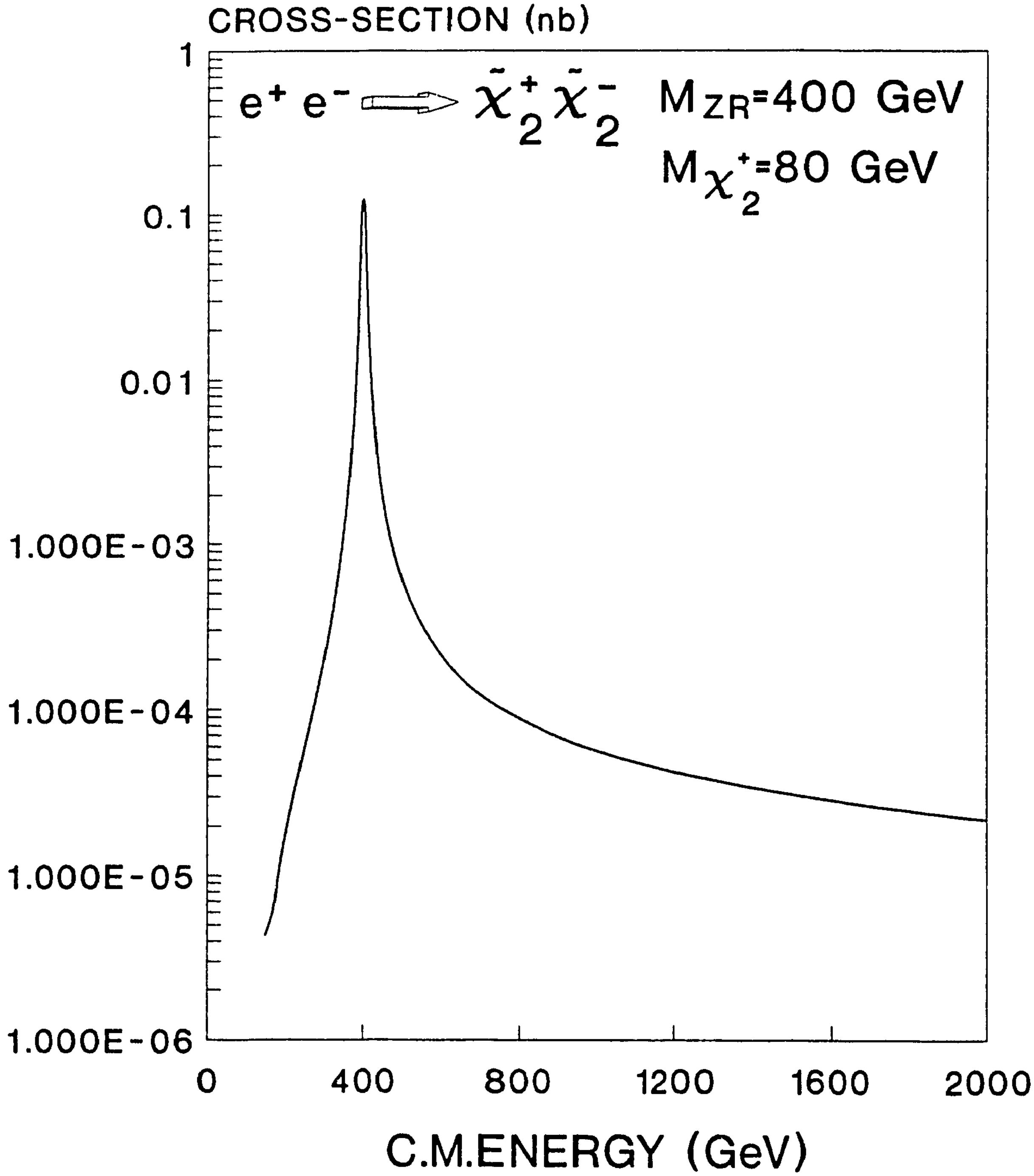
Matter field: Fermions	Matter field: Scalars	$SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ Quantum numbers[a]		
$Q_{L,R} = \begin{pmatrix} u \\ d \end{pmatrix}_{L,R}$		1/2(0)	0(1/2)	1/3
	$\tilde{Q}_{L,R} = \begin{pmatrix} \tilde{u} \\ \tilde{d} \end{pmatrix}_{L,R}$	1/2(0)	0(1/2)	1/3
$L_{L,R} = \begin{pmatrix} \upsilon \\ e \end{pmatrix}_{L,R}$		1/2(0)	0(1/2)	−1
	$\tilde{L}_{L,R} = \begin{pmatrix} \tilde{\upsilon} \\ \tilde{e} \end{pmatrix}_{L,R}$	1/2(0)	0(1/2)	−1
$\tilde{\phi}_{u,d} = \begin{pmatrix} \tilde{\phi}_1^0 & \tilde{\phi}_1^+ \\ \tilde{\phi}_2^- & \tilde{\phi}_2^0 \end{pmatrix}_{u,d}$		1/2	1/2	0
	$\phi_{u,d} = \begin{pmatrix} \phi_1^0 & \phi_1^+ \\ \phi_2^- & \phi_2^0 \end{pmatrix}_{u,d}$	1/2	1/2	0
$\tilde{\Delta}_{L,R} = \begin{pmatrix} \tilde{\Delta}^+/\sqrt{2} & \tilde{\Delta}^{++} \\ \tilde{\Delta}^0 & -\tilde{\Delta}^+/\sqrt{2} \end{pmatrix}$		1(0)	0(1)	2
	$\Delta_{L,R} = \begin{pmatrix} \Delta^+/\sqrt{2} & \Delta^{++} \\ \Delta^0 & -\Delta^+/\sqrt{2} \end{pmatrix}$	1(0)	0(1)	2
$\tilde{\delta}_{L,R} = \begin{pmatrix} \tilde{\delta}^+/\sqrt{2} & \tilde{\delta}^{++} \\ \tilde{\delta}^0 & -\tilde{\delta}^+/\sqrt{2} \end{pmatrix}$		1(0)	0(1)	−2
	$\delta_{L,R} = \begin{pmatrix} \delta^+/\sqrt{2} & \delta^{++} \\ \delta^0 & -\delta^+/\sqrt{2} \end{pmatrix}$	1(0)	0(1)	−2
Gauge Bosons	Gauge Fermions			
$W_{L,R} = \begin{pmatrix} W^+ & W^0 & W^- \end{pmatrix}_{L,R}$		1(0)	0(1)	0
V		0	0	0
	$\lambda_{L,R} = \begin{pmatrix} \lambda^+ & \lambda^0 & \lambda^- \end{pmatrix}_{L,R}$	1(0)	0(1)	0
λ_V		0	0	0

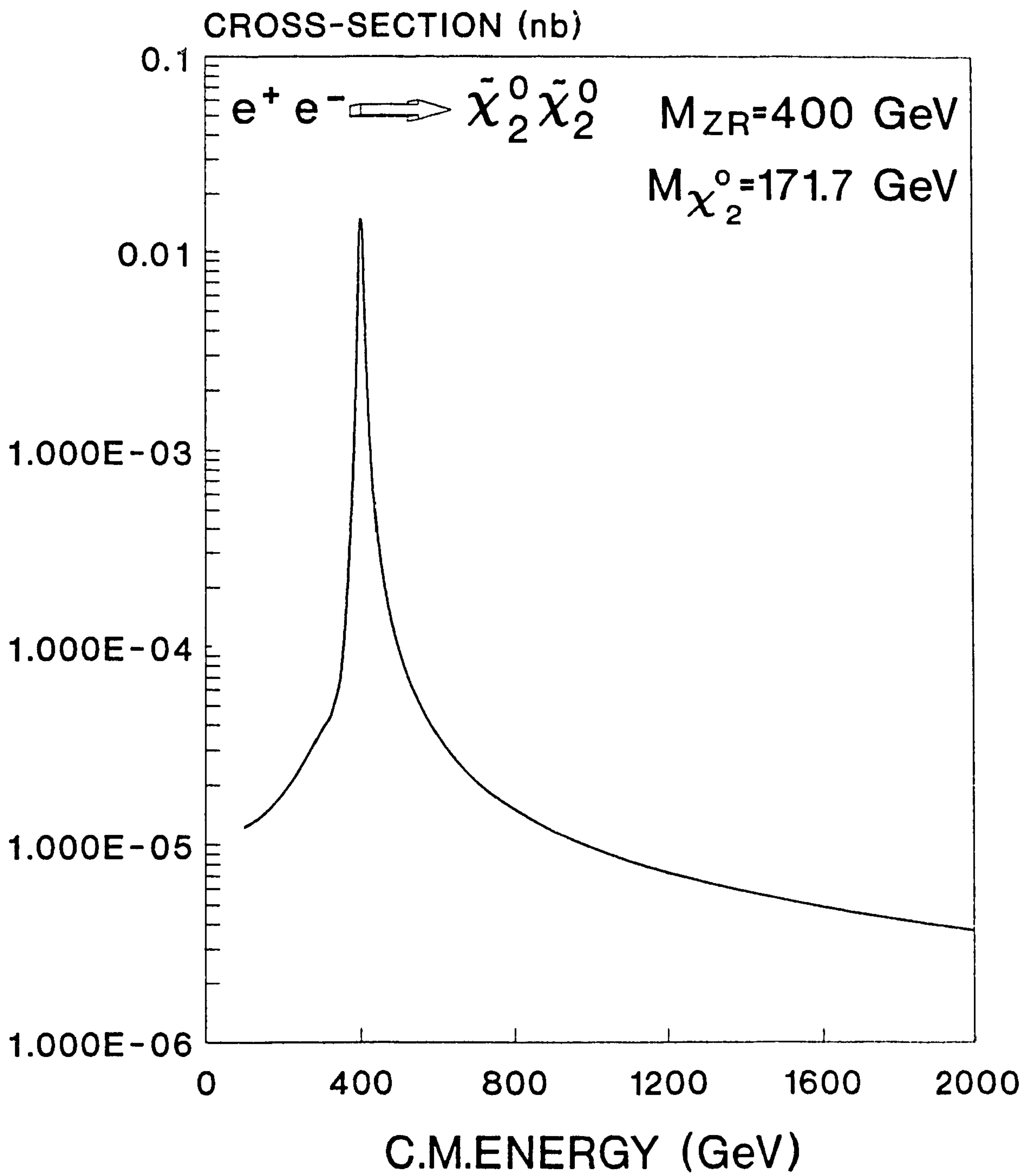
[a]The numbers inside the brackets are the quantum numbers corresponding to the right handed particles.


CROSS-SECTION (nb)
1
0.1
0.01
1.000E-03
1.000E-04
1.000E-05
1.000E-06
$e^+e^- \Rightarrow \tilde{\chi}_2^+ \tilde{\chi}_2^-$
M_{ZR}=400 GeV
$M_{\chi_2^{+-}}$=54.6 GeV
0
400
800
1200
1600
2000
C.M.ENERGY (GeV)

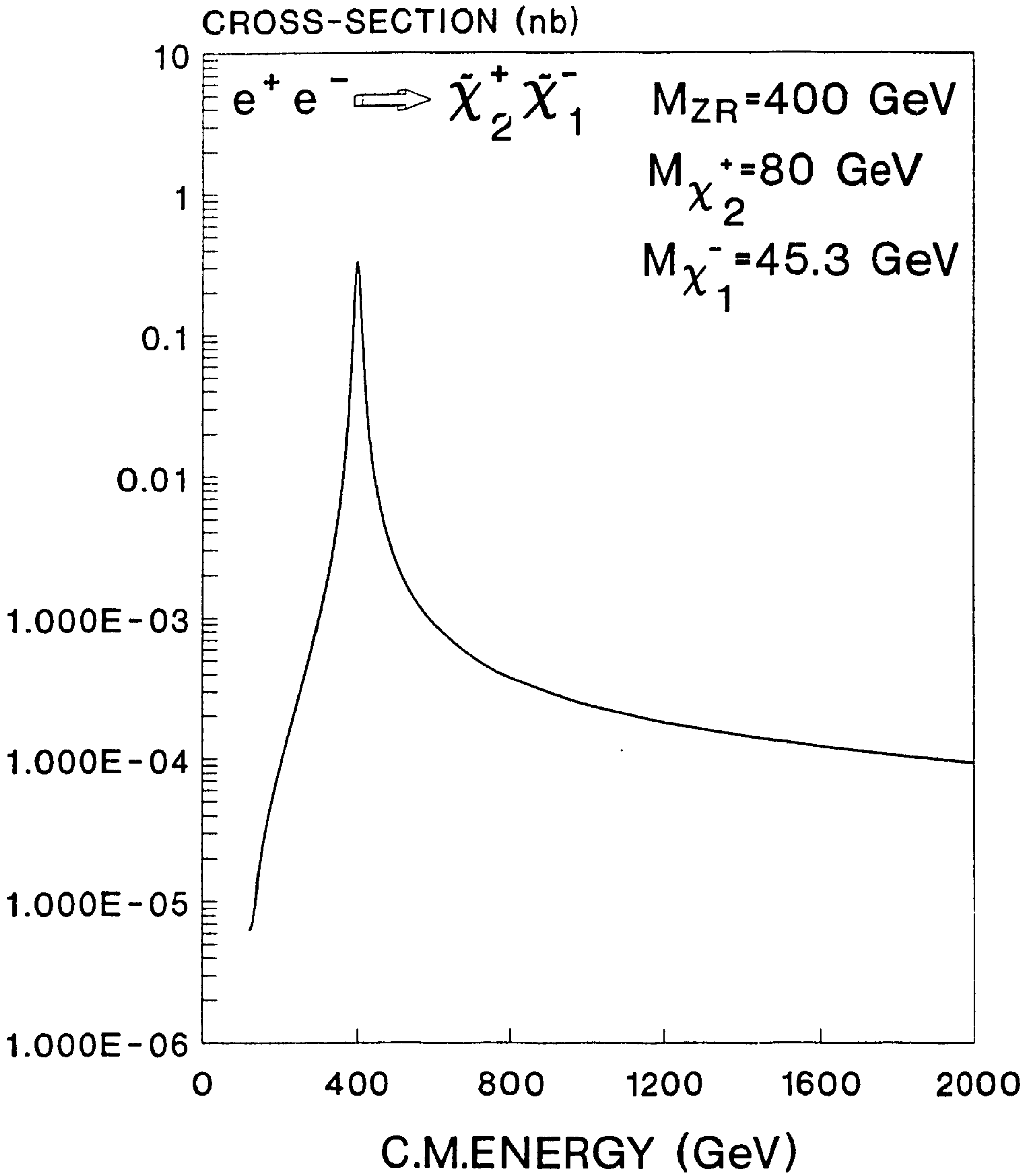


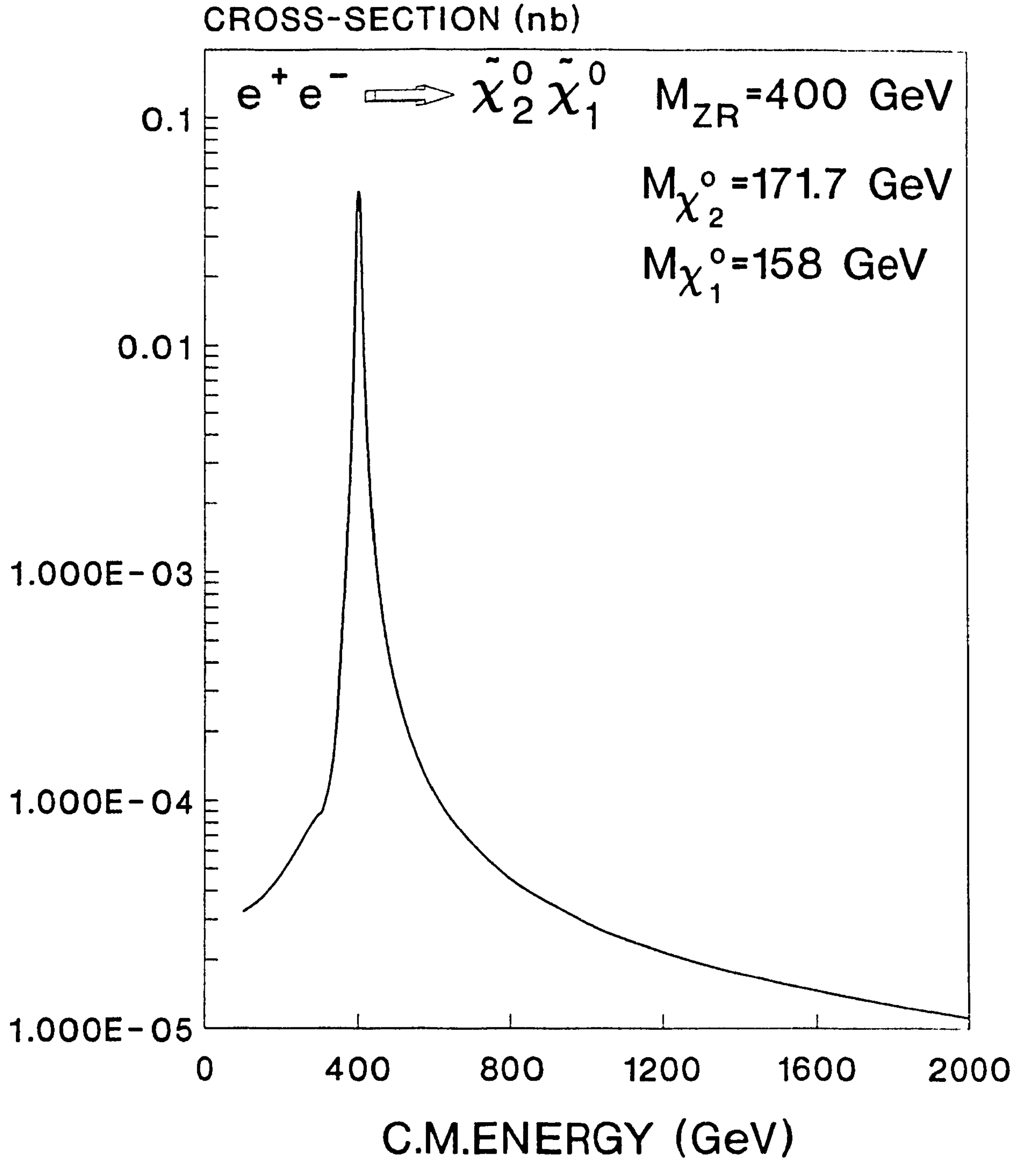
CROSS-SECTION (nb)
e+ e- ⟹ χ̃°2 χ̃°2
MZR=400 GeV
Mχ2°=63.2 GeV
0.1
0.01
1.000E-03
1.000E-04
1.000E-05
1.000E-06
0
400
800
1200
1600
2000
C.M.ENERGY (GeV)


CROSS-SECTION (nb)
1
0.1
0.01
1.000E-03
1.000E-04
1.000E-05
1.000E-06
$e^+ e^- \Longrightarrow \tilde{\chi}_2^+ \tilde{\chi}_2^-$
M_{ZR}=400 GeV
$M_{\chi_2^+}$=80 GeV
0
400
800
1200
1600
2000
C.M.ENERGY (GeV)



CROSS-SECTION (nb)
0.1
0.01
1.000E-03
1.000E-04
1.000E-05
1.000E-06
$e^+e^- \Rightarrow \tilde{\chi}^0_2\tilde{\chi}^0_2$
M_{ZR}=400 GeV
$M_{\chi^o_2}$=171.7 GeV
0
400
800
1200
1600
2000
C.M.ENERGY (GeV)


CROSS-SECTION (nb)
$e^+e^- \Rightarrow \tilde{\chi}_2^+ \tilde{\chi}_1^-$
M_{ZR}=400 GeV
$M_{\chi_2^+}$=80 GeV
$M_{\chi_1^-}$=45.3 GeV
10
1
0.1
0.01
1.000E-03
1.000E-04
1.000E-05
1.000E-06
0
400
800
1200
1600
2000
C.M.ENERGY (GeV)



CROSS-SECTION (nb)
$e^+e^- \Longrightarrow \tilde{\chi}_2^0 \tilde{\chi}_1^0$
M_{ZR}=400 GeV
$M_{\chi_2^o}$=171.7 GeV
$M_{\chi_1^o}$=158 GeV
0.1
0.01
1.000E-03
1.000E-04
1.000E-05
0
400
800
1200
1600
2000
C.M.ENERGY (GeV)