A STUDY OF SUBSONIC FLOW LOSSES

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ABSTRACT

Shapiro's pressure loss calculation method, applied to flows in constant-area passages, is compared to Benedict's variable-area method. Both methods are applicable to steady, one-dimensional, adiabatic compressible flows. Although the two methods are different in their respective derivations and mathematical formulations, they prove to be numerically identical for the constant-area case.

Based on the approaches of Shapiro and Benedict, an alternative method is developed for estimating pressure losses in variable-area flow passages. The developed method is a more general and direct approach to flow loss calculation than either of the two reviewed methods.

ACKNOWLEDGEMENTS

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TABLE OF CONTENTS

page

Chapter 3

COMPARISON OF SHAPIRO'S AND BENEDICT'S METHODS

Chapter 4

ALTERNATIVE LOSS CALCULATION METHOD

LIST OF TABLES

LIST OF FIGURES

NOMENCLATURE

A	Cross-Sectional Area
A_w	Wetted Surface Area
C_p	Specific Heat at Constant Pressure
D	Hydraulic Diameter
F	Frictional Head
f	Friction Factor
f_F	Fanning Friction Factor
$\overline{f_F}$	Mean Fanning Friction Factor
f_n	Denotes Function
g	Acceleration of Gravity
K	Loss Coefficient
L_{max}	Maximum Passage Length for Continuous Flow
M	Mach Number
P	Non-Dimensionalized Pressure Factor
p	Absolute Pressure
Q	External Heat Transfer per LB.
R	Pressure Ratio
Re	Renoylds Number
Rg	Gas Constant
T	Absolute Temperature
u	Specific Internal Energy
v	Specific Volume

V	Velocity
w	Mass Rate of Flow
$\bar{W}$	External Work per LB
x	Distance in the Direction of Flow
z	Potential Head
δ	Inexact Differential
α	Flow Number
γ	Isentropic Exponent
Γ	Generalized Compressible Flow Function
ρ	Density
t_w	Wall Shearing Stress

SUBSCRIPTS

o	Denotes Stagnation
1	Denotes Initial Condition
2	Denotes Final Condition
s	Denotes Static
t	Denotes Total
w	Denotes Wall

SUPERSCRIPTS

*	Denotes Critical State or Maximum Value

INTRODUCTION

One of the most common problems in engineering concerns the determination of pressure losses in flow passages, due mainly to viscous wall friction. The one-dimensional approach to this problem includes estimating a loss coefficient for the section of the passage under consideration. This loss coefficient is generally a function of the friction factor f which is defined in the well known Moody diagram*. The friction factor, in turn, depends on the relative roughness of the flow passages and the equivalent flow Reynolds number. After the loss coefficient is obtained one can follow any of the several available procedures to estimate the total or/and static pressure drops.

Two of the better known pressure loss calculation methods, those of Shapiro [1]** and Benedict [2], are discussed in the main body of this study. The presentation emphasizes the main differences and similarities in the derivation of the two approaches. An attempt is made to evolve an alternative procedure which combines both methods and adapts them to the particular needs of aerodynamics flow loss estimation.

All three above mentioned methods consider compressible subsonic flows with friction. The following are the underlying assumptions common to the three flow analyses.

(i) The flow under consideration is steady and one-dimensional, i.e. all fluid properties are uniform over any cross-section of the duct.

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* see Appendix A-2
** denotes Reference

(ii) No external work is done by the flow during the flow process.

(iii) Differences in elevation produce negligible changes compared with frictional effects.

(iv) The analysis is restricted to perfect gases, i.e. a perfect gas obeys the following two equations:

$$pv = RgT$$

$$\gamma = \text{constant}$$

where

p denotes pressure

v denotes specific volume

Rg denotes gas constant

T denotes temperature

γ denotes specific heat ratio

(v) Within any cross-section of the flow passage the various fluid properties are related isentropically. Flow between any two calculation planes, e.g. inlet and exit of duct, is adiabatic with friction.

Chapter 1 of the present study gives a brief description of Shapiro's flow loss calculation method for constant-area flow passages. Shapiro's loss coefficient and pressure correlations are expressed in terms of the inlet and critical (=1.0) Mach numbers.

The method of Benedict, outlined in Chapter 2, is allegedly valid for both constant and variable-area duct flows. The two principal features of the method are:

a) The loss coefficient, expressed in terms of parameters

R_1, R_2 and the 'total flow number' α_{t1} at inlet. Parameters R_1, R_2 and α_{t1} are defined as

R_1 = ratio of static to total pressure at inlet of flow passage

R_2 = ratio of static pressure at exit to total pressure at inlet of flow passage

$$\text{'total flow number'}, \ \alpha_{t1} = \frac{W/\Delta t}{Ap_{t1}}\left(\frac{p_t v_t}{g}\right)^{1/2} \equiv \frac{w\sqrt{(Rg/g)T_t}}{Ap_{t1}}$$

The derivation of the loss coefficient relationship is based on the assumption that the flow passage is of constant area.

b) The Generalized Compressible Flow Function, Γ, which may be defined as the ratio of the 'total flow number' at any arbitrary state to the critical 'total flow number'. The variable-area aspect is treated as an isentropic change of the Γ-function.

The combination of the two mentioned concepts forms the workings of Benedict's method.

Chapter 3 presents a discussion of the differences as well as the similarities of the two mentioned methods.

The Alternative pressure loss calculation approach, described in Chapter 4, follows Shapiro's line of analysis up to the point of establishing the final working equations. The resultant working equations are then expressed in terms of the inlet and exit Mach numbers. The variable-area case is handled by utilizing the expressions for total and static flow numbers at two arbitrary duct cross-sections. The 'static flow number' α_s may be defined similarly to the 'total flow number' α_t mentioned earlier as

$$\text{static flow number}, \alpha_s = \frac{w \sqrt{(Rg/g)\ T_t}}{A\ p_s}$$

By expressing α_s and α_t in terms of Mach number one can write that

$$\frac{p_{t2}}{p_{t1}} = \frac{A_1\ \alpha_{t1}}{A_2\ \alpha_{t2}} \equiv \text{fn}\ (M_1,\ M_2,\ A_1/A_2)$$

$$\frac{p_{s2}}{p_{s1}} = \frac{A_1\ \alpha_{s1}}{A_2\ \alpha_{s2}} \equiv \text{fn}\ (M_1,\ M_2,\ A_1/A_2)$$

where

subscripts 1 and 2 indicate inlet and exit of flow passage respectively

p_{t2}/p_{t1} is the total pressure ratio for flow passage

p_{s2}/p_{s1} is the static pressure ratio for flow passage

A_1/A_2 is the area ratio for flow passage

Following the Conclusions of this study, Appendixes A, B and C present details and/or auxiliary information pertaining to Shapiro's, Benedict's and the Alternative methods respectively.

Chapter 1 THE METHOD OF SHAPIRO

The constant-area flow loss calculation method of Shapiro is described in Chapter 6 of Reference 1. The entire analysis is carried out in differential form employing the infinitesimal control volume flow model of Fig. 1-1.

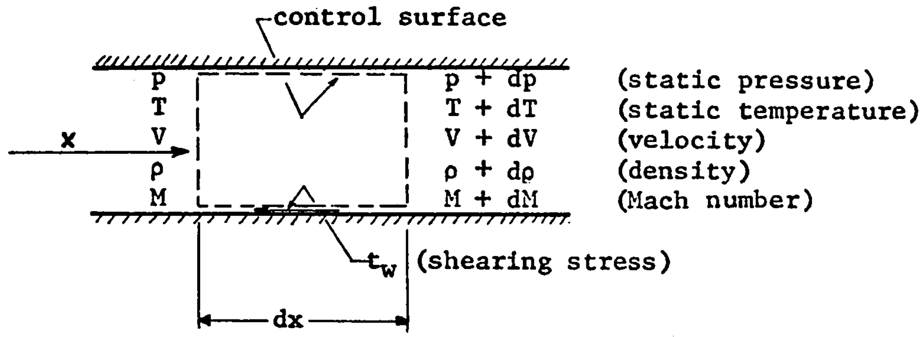


FIG. 1-1 INFINITESIMAL CONTROL VOLUME

1.1 Basic Equations and Definitions

The following is a summary of the basic equations used by Shapiro in his derivation of the calculation method.

Perfect Gas Law Equation

$$p = \rho R_g T \tag{1-1}$$

Definition of Mach number

$$M^2 = V^2/\gamma R g T \tag{1-2}$$

Steady Flow Energy Equation

$$C p d T + d\left(\frac{V^2}{2}\right) = 0 \tag{1-3}$$

Continuity Equation

$$w/A = \rho V \tag{1-4}$$

Momentum Equation

$$-A dp - t_w dA_w = w dV \tag{1-5}$$

where A is the cross-sectional area of passage and dAw is the wetted wall area over which the shearing stress t_w acts.

The friction factor* and hydraulic diameter are defined respectively as follows:

$$f_F = \frac{\text{wall shearing stress}}{\text{dynamic head}}$$

$$= \frac{t_w}{pV^2/2} \tag{1-6}$$

f_F is also called the Fanning friction factor

$$D = \frac{4 \times (\text{cross-sectional area})}{\text{wetted perimeter}}$$

$$= \frac{4A}{dAw/dx} = 4\frac{A}{dAw}dx \tag{1-7}$$

1.2 Main Differential Equations

Applying logarithmic differentiation to relations (1-1), (1-2) and (1-4), and by modifying equation (1-3), Shapiro arrives at the following convenient expressions:

$$\frac{dp}{p} = \frac{d\rho}{\rho} + \frac{dT}{T} \tag{1-8}$$

$$\frac{dM^2}{M^2} = \frac{dV^2}{V^2} - \frac{dT}{T} \tag{1-9}$$

$$\frac{d\rho}{\rho} + \frac{1}{2}\frac{dV^2}{V^2} = 0 \tag{1-10}$$

$$\frac{dT}{T} + \frac{\gamma-1}{2}M^2\frac{dV^2}{V^2} = 0 \tag{1-11}$$

A subsequent substitution of equations (1-1), (1-2), (1-4), (1-6) and (1-7) into the Momentum expression (1-5) results in

$$\frac{dp}{p} + \frac{\gamma M^2}{2}4f_F\frac{dx}{D} + \frac{M^2\gamma}{2}\frac{dV}{V^2} = 0 \tag{1-12}$$

* see Appendix A-1

Another useful relationship is obtained by logarithmically differentiating both sides of the isentropic expression $p_o/p = (1 + \frac{\gamma-1}{2}M^2)^{\frac{\gamma}{\gamma-1}}$. This gives

$$\frac{dp_o}{p_o} = \frac{dp}{p} + \frac{\gamma M^2/2}{1 + \frac{\gamma-1}{2} M^2} \frac{dM^2}{M^2} \tag{1-13}$$

Relationships (1-8) to (1-13) make up the basic differential equations of the method.

Next, the loss coefficient $4f_F\frac{dx}{D}$ is selected as the independent variable. Employing the algebraic procedure of elimination, Shapiro manipulates equations (1-8) to (1-13) to obtain the following essential differential equations:

$$\frac{dM^2}{M^2} = \frac{(1 + \frac{\gamma-1}{2} M^2).\gamma M^2}{1 - M^2} 4f_F\frac{dx}{D} \tag{1-14}$$

$$\frac{dp}{p} = -\frac{\gamma M^2[1 + (\gamma-1)M^2]}{2(1-M^2)} 4f_F\frac{dx}{D} \tag{1-15}$$

$$\frac{dp_o}{p_o} = \frac{-\gamma M^2}{2} 4f_F\frac{dx}{D} \tag{1-16}$$

Equations (1-14) to (1-16) form the backbone of Shapiro's constant-area flow method. They express the effect of friction upon flow Mach number and pressure. A cursory analysis of these expressions reveals that in an adiabatic flow with friction (for $M < 1$) both static and stagnation pressures decrease, whereas the Mach number and, hence, velocity increase in the direction of flow.

1.3 Working Formulas

In order to develop the necessary working expressions for the method, Shapiro chooses M^2 as the independent variable. Equations (1-14) to (1-16) then take the following forms:

$$4f_F\frac{dx}{D} = \frac{1 - M^2}{\gamma M^4(1 + \frac{\gamma-1}{2}M^2)} dM^2 \tag{1-17}$$

$$-\frac{dp}{p} = \frac{1 + (\gamma-1)M^2}{2M^2(1 + \frac{\gamma-1}{2}M^2)} dM^2 \tag{1-18}$$

$$-\frac{dp_o}{p_o} = \frac{1 - M^2}{2(1 + \frac{\gamma-1}{2}M^2)} dM^2 \tag{1-19}$$

Subsequent integration of these equations between the limits specified below,

Variable	integration limits
M^2	M^2 and 1
x	0 and L_{max}
p	p and p*
p_o	p_o and p_o*

yields

$$4\bar{f}_F\frac{Lmax}{D} = \frac{1-M^2}{\gamma M^2} + \frac{\gamma+1}{2\gamma} \ln\left[\frac{(\gamma+1)M^2}{2(1 + \frac{\gamma-1}{2}M^2)}\right] \tag{1-20}$$

where

$$\bar{f}_F = \frac{1}{Lmax}\int_o^{Lmax} f_F dx$$

L_{max} is the constant-area duct length at the end of which the flow is choked

$$\frac{p}{p^*} = \frac{1}{M}\left[\frac{\gamma + 1}{2(1 + \frac{\gamma-1}{2}M^2)}\right]^{1/2} \tag{1-21}$$

$$\frac{p_o}{p_o^*} = \frac{1}{M}\left[\frac{2(1 + \frac{\gamma-1}{2}M^2)}{\gamma + 1}\right]^{\frac{\gamma+1}{2(\gamma-1)}} \tag{1-22}$$

The parameters marked with an asterisk in the above relationships represent values at the critical state. The above equations are applicable for ducts of any length irrespective whether the flow reaches a critical condition at the exit of the duct or not. In the latter case the problem is treated by considering an imaginary duct extension where the flow is choked at the exit.

Expressions (1-20) to (1-22) are the working formulas of Shapiro's constant-area flow method. They are represented in tabular and graphical forms in Table I-1 and Fig. 1-2 respectively.

For practical applications the change in loss coefficient and the pressure ratios between planes 1 and 2 of a flow channel may be obtained thus:

$$4\overline{f}_F\frac{L}{D} = \left(4\overline{f}_F\frac{L_{max}}{D}\right)_{M_1} - \left(4\overline{f}_F\frac{L_{max}}{D}\right)_{M_2} \tag{1-23}$$

$$\frac{p_2}{p_1} = \frac{(p/p^*)_{M_2}}{(p/p^*)_{M_1}} \tag{1-24}$$

$$\frac{p_{02}}{p_{01}} = \frac{(p_0/p_0^*)_{M_2}}{(p_0/p_0^*)_{M_1}} \tag{1-25}$$

Appendix A-2 presents a worked example to illustrate the use of Fig. 1-2 and equations (1-23) to (1-25).

TABLE I-1

DATA FOR SHAPIRO'S METHOD

Perfect Gas, $\gamma = 1.4$

M	p/p^*	p_0/p_0^*	$4f_F L_{max}/D$
0.00	∞	∞	∞
.05	21.903	11.5914	280.02
.10	10.9435	5.8218	66.922
.15	7.2866	3.9103	27.932
.20	5.4555	2.9635	14.533
.25	4.3546	2.4027	8.4834
.30	3.6190	2.0351	5.2992
.35	3.0922	1.7780	3.4525
.40	2.6958	1.5901	2.3085
.45	2.3865	1.4486	1.5664
.50	2.1381	1.3399	1.06908
.55	1.9341	1.2549	.72803
.60	1.7634	1.1882	.49081
.65	1.6183	1.1356	.32460
.70	1.4934	1.09436	.20814
.75	1.3848	1.06242	.12728
.80	1.2892	1.03823	.07229
.85	1.2047	1.02067	.03632
.90	1.12913	1.00887	.014513
.95	1.06129	1.00215	.003280
1.00	1.00000	1.00000	0

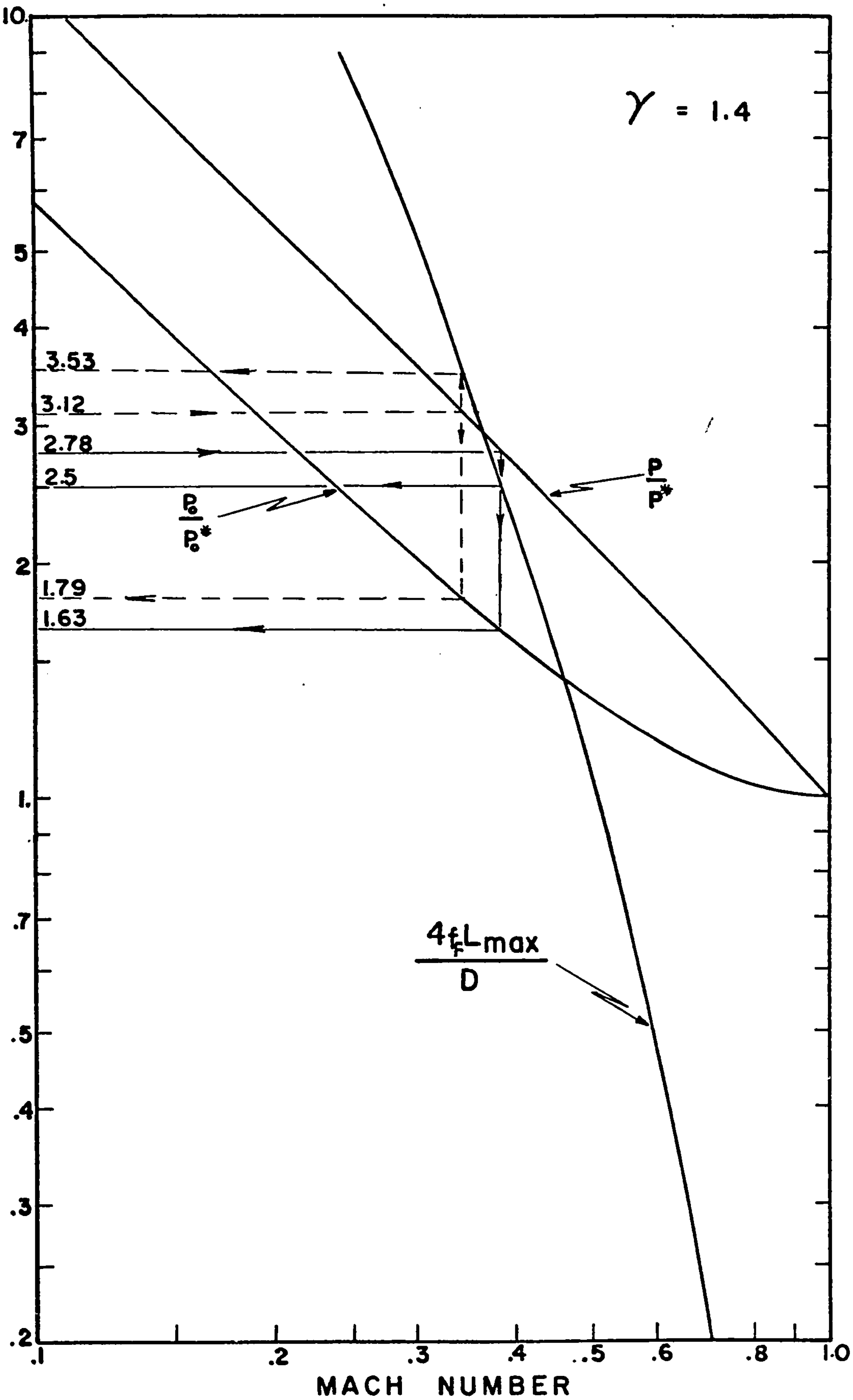


FIG. 1–2 SHAPIRO'S METHOD CURVES

Chapter 2 THE METHOD OF BENEDICT

The method of pressure loss calculation as developed by Benedict is outlined in References 2 and 3. The part of the method presented below is limited to compressible fluid flows in the subsonic range.

2.1 Basic Equations

The following is a list of basic equations used to derive the method:

General Energy equation [4]

$$\delta Q + \delta \overline{W} = du + pdv + vdp + \frac{VdV}{g} + dz \qquad (2\text{-}1)$$

First Law of Thermodynamics, corollary [4],

$$\delta Q + \delta F = du + pdv \qquad (2\text{-}2)$$

where δF is frictional head loss

Darcy - Weisbach Equation for frictional head loss

$$\delta F = f\frac{dx}{D}\frac{V^2}{2g} \qquad (2\text{-}3)$$

where V is the average velocity of flow. This expression is used here as a definition of the friction factor $f(=4f_F)$*

Continuity Equation

$$W/\Delta t = AV/v \qquad (2\text{-}4)$$

where $W/\Delta t$ is the mass rate of flow

Isentropic Gas Law relationship

$$pv^{\gamma} = c_c = \text{constant} \qquad (2\text{-}5)$$

Perfect Gas Law Equation

$$p/_{\rho} = RgT \qquad (2\text{-}6)$$

* see Appendix A-1

2.2 Loss Coefficient Relationship

The loss coefficient is defined by Benedict as

$$K = \int f \frac{dx}{D}$$

The derivation of K relation to other flow parameters is carried out by considering a flow passage of uniform cross-section.

First, equations (2-1) and (2-2) are combined to give

$$\delta F = -vdp - \frac{vdv}{g} \tag{2-7}$$

Subsequently, relationship (2-5) is introduced into (2-7) and the resultant expression integrated between the static and stagnation states at a point ($\delta F = 0$), producing

$$V^2 = \frac{2g\gamma}{\gamma-1}(p_t v_t - pv) \tag{2-8}$$

A further combination of the preceding equation with the continuity relation (2-4) results in

$$V = \frac{2(gp_t v_t)^{1/2}}{\left[\frac{R^2}{\alpha^2} + \frac{2(\gamma-1)}{\gamma}\right]^{1/2} + \frac{R}{\alpha}} \tag{2-9}$$

where

$R = p/p_{t1}$, a variable, a function of static pressure only (for a given inlet total pressure p_{t1}).

$\alpha_{t1} = \frac{W/\Delta t}{Ap_{t1}} \frac{(p_t v_t)^{1/2}}{g}$, 'total flow number' at inlet, a constant for given area and p_{t1}

$p_t v_t = RgT_t$, a constant for the adiabatic case

Equation (2-9) concludes the first step in the derivation. In the next step, the joint manipulation of expressions (2-3), (2-4) and (2-6) gives

$$dK = \frac{\delta F}{V^2/2g} = -\frac{2gAp_{t1}}{W/\Delta t} \cdot \frac{dR}{V} - \frac{2dV}{V}$$

Substituting expression (2-9) into the above relationship and integrating [7] the resultant equation, one obtains an expression for the loss coefficient in the following form:

$$K = \frac{1}{2}\frac{R}{\alpha_{t1}}\left[\frac{R^2}{\alpha_{t1}^2}+\frac{2(\gamma-1)}{\gamma}\right]^{1/2}\Big|_2^1 + \frac{1}{2}\frac{R}{\alpha_{t1}}\Big|_2^1 -$$

$$\frac{\gamma+1}{\gamma}\ln\left\{\left[\frac{R^2}{\alpha_{t1}^2}+\frac{2(\gamma-1)}{\gamma}\right]^{1/2}+\frac{R}{\alpha_{t1}}\right\}\Big|_2^1 \qquad (2\text{-}10)$$

In the above expression the friction factor f is assumed to be either independent of L or to have an average value over the duct length L.

2.3 Generalized Compressible Flow Function [3]

The Generalized Compressible Flow Function is the chief innovation of Benedict's method. To derive its characteristic relationship, equations (2-4), (2-5) and (2-8) are rearranged to the following forms:

$$\rho VA = \text{constant} \qquad (2\text{-}11)$$

$$\rho = \rho_t\left[\frac{p}{p_t}\right]^{1/\gamma} \qquad (2\text{-}12)$$

$$V = \left[\frac{2\gamma}{\gamma-1}\left(p_t/\rho_t - p/\rho\right)\right]^{1/2} \qquad (2\text{-}13)$$

Furthermore, introducing relations (2-12), (2-13) and (2-6) into (2-11), Benedict obtains a new expression for continuity:

$$(RgT_t)^{-1/2} p_t A \left[\frac{p}{p_t}\right]^{1/\gamma}\left[1-(p/p_t)^{\frac{\gamma-1}{\gamma}}\right]^{1/2} = \text{constant}$$

which after subsequent generalization becomes

$$(T_{t2}/T_{t1})^{1/2}(p_{t1}/p_{t2})(A_1/A_2)\{(p_1/p_{t1})^{1/\gamma}[1-(p_1/p_{t1})^{\frac{\gamma-1}{\gamma}}]^{1/2}\} =$$

$$(p_2/p_{t2})^{1/\gamma}[1-(p_2/p_{t2})^{\frac{\gamma-1}{\gamma}}]^{1/2} \qquad (2\text{-}14)$$

Considering now expression

$$P = (p/p_t)^{1/\gamma} \, [1-(p/p_t)^{\frac{\gamma-1}{\gamma}} \,]^{1/2}$$

one can determine from equation (2-14) that maximum P occurs when

$$\frac{p}{p_t} = [\frac{2}{\gamma+1}]^{\gamma/(\gamma-1)} \qquad (\gamma \neq 1)$$

which is the familiar relation of p/p_t at the critical state, p^*/p_t^*. At the critical condition

$$P^* = (2/(\gamma+1))^{1/(\gamma-1)} \, [(\gamma-1)/(\gamma+1) \,]^{1/2}$$

Henceforth, Benedict proceeds to define a Generalized Compressible Flow Function as

$$\Gamma = P/P^*$$

$$= \frac{(p/p_t)^{1/\gamma} \, [1-(p/p_t)^{\frac{\gamma-1}{\gamma}} \,]^{1/2}}{[(2/(\gamma+1)]^{1/(\gamma+1)}[(\gamma-1)/(\gamma+1)]^{1/2}} \qquad (2\text{-}15)$$

where Γ varies between 0 and 1 for any flow process. A transformed continuity equation, (2-14), may hence be stated as

$$\Gamma_2 = \Gamma_1[(T_{t2}/T_{t1})^{1/2}(p_{t1}/p_{t2})(A_1/A_2)] \qquad (2\text{-}16)$$

An alternative definition of Γ [5] may be obtained by defining the total flow number as

$$\alpha_t = \frac{(W/\Delta t)}{p_t A}(RgT_t/g)^{1/2}$$

$$= \{[2\gamma/(\gamma-1)](p/p_t)^{2/\gamma}[1-(p/p_t)^{\frac{\gamma-1}{\gamma}} \,]\}^{1/2} \qquad (2\text{-}17)$$

The critical total flow number is then

$$\alpha_t^* = [\, 2\gamma/(\gamma-1)[2/(\gamma+1)]^{\frac{2}{\gamma-1}} (\gamma-1)/(\gamma+1)]^{1/2}$$

Hence,

$$\Gamma = \alpha_t/\alpha_t^*$$

$$= \frac{(p/p_t)^{1/\gamma} [1-(p/p_t)^{\frac{\gamma-1}{\gamma}}]^{1/2}}{[(2/(\gamma+1)]^{1/(\gamma-1)}[(\gamma-1)/(\gamma+1)]^{1/2}} \qquad (2\text{-}18)$$

The preceding equation and expression (2-15) are identical.

2.4 Pressure Loss Calculation Procedure

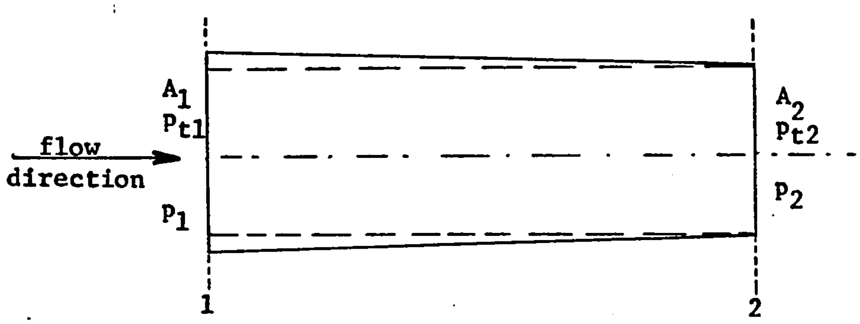


FIG. 2-1 FLOW MODEL

Referring to Fig. 2-1, the entire total pressure loss calculation procedure may be divided into four steps:

i) In the first step Benedict [2,3] handles the effect of area variation by representing it as an isentropic change of state. The change is from a given initial state, corresponding to initial area, to a new initial state, corresponding to final area. From equation (2-16) it follows that

$$\Gamma_1' = \Gamma_1 (A_1/A_2)$$

where Γ_1' corresponds to the new initial state. From the area-modified initial state calculation then proceeds via a constant-area flow process. The 'modified duct' is shown in broken lines in Fig. 2-1.

ii) Utilizing equation (2-17) to express the total flow number α_{t1} in terms of R_1,

$$\alpha_{t1} = \{[2\gamma/(\gamma-1)]R_1^{2/\gamma}[1-R_1^{\frac{\gamma-1}{\gamma}}]\}^{1/2} \tag{2-19}$$

Equations (2-10) and (2-19) are then solved* [2] simultaneously to yield α_{t2} as a function of the initial conditions, R_2 and K.

iii) Having found α_{t2} one can thus obtain Γ_2 from

$$\Gamma_2 = \alpha_{t2} / \alpha_t^*$$

iv) Henceforth, from equation (2-16) it follows that

$$\Gamma_2 = \Gamma_1' \, (p_{t1}/p_{t2})$$

or

$$\Delta p_t = p_{t1}(\Gamma_2-\Gamma_1')/\Gamma_2$$

The tabulated data and graph pertaining to Benedict's method are presented in Table II-1 and Fig. 2-2 respectively. A worked example on the use of the Table and graph is included in Appendix B-2.

*See Appendix B-1 for a possible solution

TABLE II-1

DATA FOR BENEDICT'S METHOD

GENERALIZED FANNO FLOW TABLE

(γ = 1.4, α* = .68473)

EXIT STATIC PRESSURE TO INLET TOTAL PRESSURE (P_2/P_{t1}), AND INLET TOTAL PRESSURE TO EXIT TOTAL PRESSURE (P_{t1}/P_{t2}) VERSUS LOSS COEFFICIENT (K OR fL/D)

Press Ratio P/P_t	Mach No. M	Temp Ratio T/T_t	Flow No α	Flow Factor Γ	.1	.2	.4	.6	1	2	4	6	10	20	K*
.999	.038	1.000	.0447	.06528	.99890 1.00010	.99880 1.00020	.99860 1.00040	.99840 1.00060	.99800	.99700	.99499	.99297	.98893	.97876	493.5268
.99	.120	.997	.1407	.20543	.98898 1.00101	.98796 1.00202	.98592 1.00405	.98388 1.00609	.97978 1.01022	.96944 1.02077	.94843 1.04290	.92691 1.06556	.88225 1.11923	.75857 1.29596	45.48177
.98	.170	.994	.1978	.28894	.97793 1.00203	.97585 1.00408	.97169 1.00821	.96750 1.01233	.95908 1.02031	.93765 1.04324	.89313 1.09280	.84606 1.15043	.74204 1.30127	.32715 2.57883	21.07852
.97	.209	.991	.2410	.35193	.96684 1.00307	.96365 1.00618	.95728 1.01247	.95085 1.01890	.93784 1.03213	.90439 1.05772	.83293 1.15221	.75352 1.26216	.55333 1.64972		13.09545
.96	.242	.988	.2767	.40412	.95570 1.00413	.95138 1.00832	.94258 1.01586	.93368 1.02554	.91539 1.04393	.86936 1.09461	.76589 1.22532	.64123 1.42584			9.17812
.95	.272	.985	.3076	.44929	.94452 1.00521	.93901 1.01051	.92787 1.02138	.91656 1.03263	.89344 1.05638	.83216 1.12449	.68841 1.31988	.48060 1.72254			6.87217
.94	.299	.982	.3351	.48938	.93330 1.00631	.92654 1.01274	.91283 1.02603	.89885 1.03940	.87038 1.06958	.79218 1.15807	.59227 1.45313				5.36456
.93	.324	.979	.3599	.52555	.92202 1.00742	.91395 1.01503	.89753 1.03083	.88072 1.04746	.84577 1.08363	.74853 1.19652	.44508 1.68330				4.30904
.92	.347	.976	.3825	.55858	.91069 1.00856	.90124 1.01737	.88195 1.03578	.86208 1.05527	.82034 1.09666	.69973 1.24162					3.53348
.91	.370	.973	.4033	.58898	.89929 1.00972	.88840 1.01976	.86606 1.04091	.84288 1.06355	.79354 1.11465	.64307 1.29643					2.94285
.90	.391	.970	.4226	.61715	.88783 1.01090	.87541 1.02221	.84980 1.04622	.82301 1.07234	.76504 1.13244	.57223 1.36726					2.48050
.89	.411	.967	.4405	.64338	.87629 1.01210	.86225 1.02473	.83313 1.05176	.80234 1.08153	.73438 1.15175	.46172 1.47278					2.11056
.88	.431	.964	.4573	.66790	.86458 1.01333	.84693 1.02731	.81598 1.05752	.78076 1.09126	.70081 1.17323						1.80929
.87	.451	.961	.4731	.69088	.85298 1.01458	.83539 1.02997	.79829 1.06354	.75804 1.10162	.66313 1.19757						1.56034
.86	.469	.958	.4879	.71249	.84118 1.01586	.82163 1.03270	.77996 1.06984	.73390 1.11275	.61897 1.22604						1.35213
.85	.487	.955	.5018	.73284	.82927 1.01717	.80761 1.03552	.76085 1.07648	.70792 1.12480	.55273 1.26095						1.17618
.84	.505	.951	.5149	.75203	.81725 1.01851	.79329 1.03843	.74080 1.08350	.67948 1.13601	.46630 1.30964						1.02621
.83	.523	.948	.5274	.77016	.80509 1.01988	.77863 1.04144	.71957 1.09096	.64750 1.15273							.89741
.82	.540	.945	.5391	.78729	.79278 1.02128	.76354 1.04458	.69583 1.09897	.60983 1.16961							.78610
.81	.557	.942	.5502	.80348	.78026 1.02274	.74800 1.04783	.67205 1.10765	.56111 1.18981							.68937
.80	.574	.938	.5607	.81880	.76757 1.02423	.73188 1.05123	.64437 1.11718	.46509 1.21742							.60491
.79	.590	.935	.5706	.83330	.75464 1.02576	.71507 1.05479	.61199 1.12796								.53086
.78	.607	.931	.5800	.84701	.74145 1.02734	.69738 1.05853	.57087 1.14051								.46572
.77	.623	.928	.5888	.85997	.72793 1.02897	.67855 1.06251	.49854 1.15680								.40825
.76	.639	.925	.5972	.87222	.71403 1.03066	.65819 1.06676									.35743
.75	.654	.921	.6052	.88378	.69965 1.03242	.63562 1.07137									.31239
.74	.670	.918	.6126	.89469	.68468 1.03425	.60951 1.07648									.27243
.73	.686	.914	.6137	.90497	.66856 1.03617	.57574 1.08230									.23692
.72	.701	.910	.6263	.91464	.65222 1.03820	.52125 1.08954									.20536
.71	.717	.907	.6325	.92371	.63409 1.04035										.17731
.70	.732	.903	.6383	.93222	.61361 1.04275										.15237
.69	.748	.899	.6438	.94016	.58939 1.04536										.13023
.68	.763	.896	.6488	.94756	.55581 1.04842										.11059

CONTINUATION OF ISENTROPIC ENVELOPE

Press Ratio P/P_t	Mach No M	Temp Ratio T/T_t	Flow No α	Flow Factor Γ_{isen}
.66	.794	.886	.6579	.96079
.64	.825	.880	.6656	.97199
.62	.855	.872	.6719	.98123
.60	.886	.864	.6769	.98859
.58	.918	.856	.6807	.99409
.56	.949	.847	.6832	.99778
.54	.981	.838	.6845	.99970
.52828	1.00	.83333	.68473	1.00
.52	1.013	.830	.6846	.99985
.50	1.046	.820	.6835	.99825
.48	1.080	.811	.6812	.99490
.46	1.114	.801	.6777	.98979
.44	1.150	.791	.6730	.98291
.42	1.186	.780	.6671	.97424
.40	1.223	.770	.6599	.96375
.38	1.262	.758	.6514	.95139
.36	1.302	.747	.6417	.93712
.34	1.344	.735	.6306	.92088
.32	1.387	.722	.6180	.90258

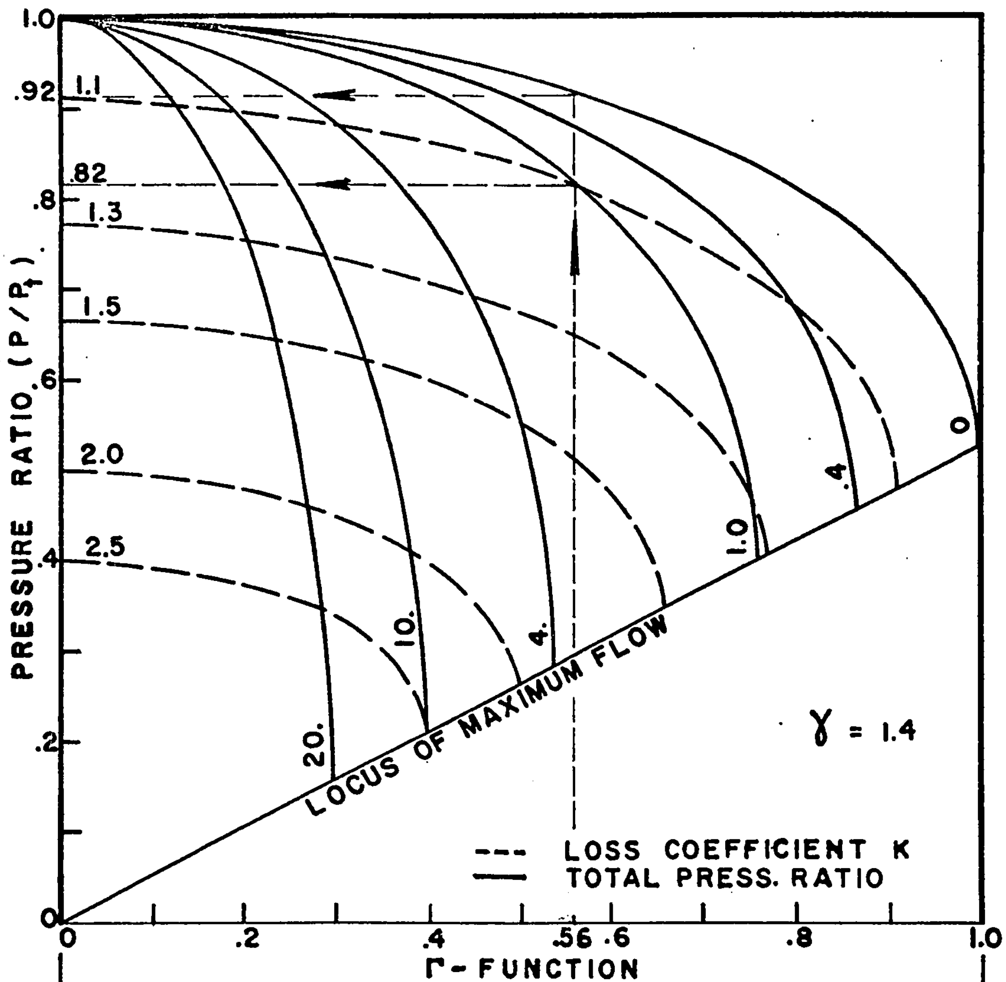
NOTE ON USE OF TABLE:

UNDER LOSS COEFFICIENT IS TABULATED P_2/P_{t1} AS UPPER FIGURE AND P_{t1}/P_{t2} AS LOWER FIGURE.

EXAMPLE:
AT P_1/P_{t1} = .80, M_1 = .574, T_1/T_t = .938, α_1 = .5607, Γ_1 = .81880, K* = .60491,
AND AT K = .4 P_2/P_{t1} = .64437
WHILE P_{t1}/P_{t2} = 1.11718

NOTE: 1. K* is the value of K evaluated between the passage entry plane 1 (see Fig.2-1) and the critical condition.

2. The values of α are the same as α_t in the text.



NOTE: 1. the envelope (K = 0) represents the isentropic flow solutions;

2. the above graph is for a constant-area case;

3. 'locus of maximum flow' line separates the subsonic and supersonic regimes.

FIG. 2-2 BENEDICT'S METHOD GRAPH

Chapter 3 COMPARISON OF SHAPIRO'S AND BENEDICT'S METHOD'S

3.1 Correlation of Loss Coefficients

i) Modification of Benedict's expression for K

Considering equation (2-10)

$$K = \frac{1}{2}\frac{R}{\alpha_{t1}}\left[\frac{R^2}{\alpha_{t1}^2}+\frac{2(\gamma-1)}{\gamma}\right]^{1/2}\Big|_2^1 + \frac{1}{2}\frac{R}{\alpha_{t1}}\Big|_2^1 -$$

$$\frac{\gamma+1}{\gamma}\ln\left\{\left[\frac{R^2}{\alpha_{t1}^2}+\frac{2(\gamma-1)}{\gamma}\right]^{1/2}+\frac{R}{\alpha_{t1}}\right\}\Big|_2^1 \qquad (2\text{-}10)$$

one recalls that

$$\alpha_{t1} = \frac{W/\Delta t}{Ap_{t1}}\left[\frac{p_t v_t}{g}\right]^{1/2} \equiv \frac{w\sqrt{RgT_t/g}}{Ap_{t1}}$$

$$R = p/p_{t1}$$

Hence, one can write

$$R/\alpha_{t1} = (1/\alpha_{t1})\, R = [\, Ap_{t1}/w\sqrt{RgT_t/g}\,]\; p/p_{t1}$$

or

$$R/\alpha_{t1} = 1/w\sqrt{RgT_t/g}\;/Ap_s = 1/\alpha_s \qquad (3\text{-}1)$$

Subsequent substitution of (3-1) into equation (2-10) results in K as a function of one variable, the static flow number α_s :

$$K = \frac{1}{2\alpha_s}\left[\frac{1}{\alpha_s^2}+\frac{2(\gamma-1)}{\gamma}\right]^{1/2}\Big|_2^1 + \frac{1}{2\alpha_s^2}\Big|_2^1 -$$

$$\frac{\gamma+1}{\gamma}\ln\left\{\frac{1/\alpha_{s1}+[1/\alpha_{s1}^2+2(\gamma-1)/\gamma]^{1/2}}{1/\alpha_{s2}+[1/\alpha_{s2}^2+2(\gamma-1)/\gamma]^{1/2}}\right\} \qquad (3\text{-}2)$$

ii) Modification of Shapiro's Expression for Loss Coefficient

Considering equation (1-14)

$$\frac{dM^2}{M^2} = \gamma\frac{M^2[1+M^2(\gamma-1)/2]}{1-M^2}\, 4f_F\,\frac{dx}{D} \tag{1-14}$$

denote .

$$dK = 4f_F\,\frac{dx}{D} \qquad \left(\text{ since } K = \int f\frac{dx}{D} = \int 4f_F\,\frac{dx}{D}\right)$$

$$m = M^2$$

Then equation (1-14) becomes

$$\frac{dm}{m} = \frac{\gamma m\,[1+ m(\gamma-1)/2]}{1 - m}\,dK$$

or after some rearrangement,

$$dK = \frac{1 - m}{\gamma m^2\,[1+ m(\gamma-1)/2]}\,dm$$

Integrating now both sides of the above equation results in

$$K = \left| -\frac{1}{\gamma M^2} + \frac{\gamma+1}{\gamma}\ln\{[\,\frac{1+M^2(\gamma-1)/2}{M^2}\,]^{1/2}\}\right|_{M_1^2}^{M_2^2} \tag{3-3}$$

From the isentropic relationship for α_s one can write that

$$\alpha_s^2 = \gamma M^2(1+ M^2[\gamma-1]/2)$$

which after some algebraic transformation gives

$$M^2 = \sqrt{(\gamma-1)^{-2} + 2\alpha_s^2/[\gamma(\gamma-1)]} - 1/(\gamma-1) \tag{3-4}$$

A subsequent combination of equations (3-3) and (3-4) produces

$$K = \left| \bar{a} + (\gamma+1)/(2\gamma)\ln(\bar{b}) \right|_{\alpha_{s1}}^{\alpha_{s2}}$$

where

$$\bar{a} = \frac{1}{\gamma} \frac{1}{\frac{1}{(\gamma-1)} - [\frac{1}{(\gamma-1)^2} + \frac{2\alpha_s^2}{\gamma(\gamma-1)}]^{1/2}}$$

$$\bar{b} = \frac{1 + \frac{\gamma-1}{2}[(\frac{1}{(\gamma-1)^2} + \frac{2\alpha_s^2}{(\gamma-1)})^{1/2} - \frac{1}{\gamma-1}]}{[\frac{1}{(\gamma-1)^2} + \frac{2\alpha_s^2}{\gamma(\gamma-1)}]^{1/2} - \frac{1}{\gamma-1}}$$

Part $\bar{a}$ is then multiplied and divided by the term $\{ \frac{1}{\gamma-1} + [\frac{1}{(\gamma-1)^2} + \frac{2\alpha_s^2}{\gamma(\gamma-1)}]^{1/2} \}$
The transformed $\bar{a}$ becomes

$$\bar{a} = -\frac{1}{2\alpha_s}[\frac{1}{\alpha_s^2} + \frac{2(\gamma-1)}{\gamma}]^{1/2} + \frac{1}{2\alpha_s^2} \qquad (3\text{-}6)$$

Similarly, multiplying and dividing $\bar{b}$ by the term $\{[\frac{1}{(\gamma-1)^2} + \frac{2\alpha_s^2}{\gamma(\gamma-1)}]^{1/2} + \frac{1}{\gamma-1}\}$

gives, after some algebraic rearrangement and simplification,

$$\bar{b} = \frac{\gamma}{4}\{[\frac{1}{\alpha_s^2} + \frac{2(\gamma-1)}{\gamma}]^{1/2} + \frac{1}{\alpha_s}\}^2 \qquad (3\text{-}7)$$

Since $K = \bar{a} + \frac{\gamma+1}{2\gamma}\ln(\bar{b})$, a combination of equations (3-6) and (3-7) results in the following new expression for K.

$$K = \frac{1}{2\alpha_s}[\frac{1}{\alpha_s^2} + \frac{2(\gamma-1)}{\gamma}]^{1/2} \Big|_2^1 + \frac{1}{2\alpha_s^2} \Big|_2^1 -$$

$$\frac{\gamma+1}{\gamma}\ln\{\frac{1/\alpha_{s1} + [1/\alpha_{s1}^2 + 2(\gamma-1)/\gamma]^{1/2}}{1/\alpha_{s2} + [1/\alpha_{s2}^2 + 2(\gamma-1)/\gamma]^{1/2}}\} \qquad (3\text{-}8)$$

A comparison of relations (3-2) and (3-8) reveals them to be identical, proving that both Shapiro's and Benedict's loss coefficients are the same.

3.2 Other Differences and Similarities

Considering Shapiro's expressions (1-23) and (1-25) for total pressure one can deduce that

$$\frac{p_{02}}{p_{01}} = \frac{M_1}{M_2}\left[\frac{1 + (\gamma-1)M_2^2/2}{1 + (\gamma-1)M_1^2/2}\right]^{\frac{\gamma+1}{2(\gamma-1)}} \tag{3-9}$$

As for Benedict's case, a combination of equations (2-16) and (2-18), when $T_{t2} = T_{t1}$ and $A_1 = A_2$, gives

$$p_{t2}/p_{t1} = \alpha_{t1}/\alpha_{t2}$$

Utilizing the well known isentropic expression for α_t

$$\alpha_t = \frac{w\sqrt{T_t Rg/g}}{A p_t}$$

$$= \frac{\sqrt{\gamma}M}{[1 + (\gamma-1)M^2/2]^{(\gamma+1)/(\gamma-1)/2}}$$

one can express p_{t2}/p_{t1} as

$$p_{t2}/p_{t1} = \frac{M_1}{M_2}\left[\frac{1 + (\gamma-1)M_2^2/2}{1 + (\gamma-1)M_1^2/2}\right]^{\frac{\gamma+1}{2(\gamma-1)}} \tag{3-10}$$

Since equations (3-9) and (3-10) are identical, one concludes that the total pressure ratios for both methods are the same. Similar reasoning can be applied to the static pressures.

The preceding conclusions prove that both Shapiro's and Benedict's pressure loss calculation procedures are quantitatively identical for the constant-area flow case.

Considering now the qualitative merits of the two methods it may be worthwhile to note the following.

a) From overall considerations, Shapiro's approach appears to have more unity, as a whole, and a greater generality. This is mainly due to the use of Mach number as the common parameter. This fact also makes Shapiro's approach simpler to derive and understand, as opposed to Benedict's.

The Γ-function in Benedict's approach does not play the same unifying or other role as does the Mach number in Shapiro's case. Since values of the total flow number α_t are generally available from aerodynamics and other tables, the Γ-function concept may be considered as redundant for the present type of flow analysis.

b) A comparison of Tables I-1 and II-1 reveals Benedict's tabulated parameters to be more applicable for solving duct pressure loss problems. This is especially true for 'non-choked' passage flows where Benedict's parameters p_t/p_{t1}, p_2/p_{t1}, p_{t1}/p_{t2} and K are more to the point than Shapiro's p/p^*, p_o/p_o^* and $4f_F Lmax/D$. The worked examples in Appendixes A and B give ample evidence to this effect.

c) If both methods are considered for analysis by computer, it may be conjectured that Shapiro's method would be easier and more straightforward to use for that purpose. Referring to section 2.3 of this write-up, step ii) in Benedict's method would probably involve some iteration which is an added complication to the method.

d) An added advantage of Benedict's method over Shapiro's is its adaptability for approximate variable-area flow loss calculation. The Alternative method described in Chapter 4 demonstrates how Shapiro's approach can be utilized for that purpose.

Chapter 4 ALTERNATIVE LOSS CALCULATION METHOD

4.1 Definition of Loss Coefficient

Based on equations (1-20) and (3-2), two auxiliary expressions for the loss coefficient K are derived as follows.

a) K Relationship in Terms of M_1 and M_2

Consider equation (3-3), which if evaluated between the indicated limits gives

$$K = \frac{1}{\gamma} \frac{M_2^2 - M_1^2}{M_1^2 M_2^2} - \frac{\gamma+1}{2\gamma} \ln\left\{ \frac{M_2^2(1+[\gamma-1]M_1^2/2)}{M_1^2(1+[\gamma-1]M_2^2/2)} \right\} \qquad (4-1)$$

Expression (4-1) is a more general version of Shapiro's loss coefficient. Putting M_2 to 1 in equation (4-1) results in Shapiro's form of the loss coefficient, (1-20).

b) Approximated K Coefficient

Evaluating equation (3-2) between the stated limits one obtains

$$K = \frac{X_1}{2\alpha_{s1}} - \frac{X_2}{2\alpha_{s2}} - \frac{(\gamma+1)}{\gamma} \ln\left\{ \frac{X_1}{X_2} \right\}$$

where

$$X_1 = [1/\alpha_{s1}^2 + 2(\gamma-1)/\gamma]^{1/2} + 1/\alpha_{s1}$$

$$X_2 = [1/\alpha_{s2}^2 + 2(\gamma-1)/\gamma]^{1/2} + 1/\alpha_{s2}$$

By further algebraic manipulation the ratio $\frac{X_1}{X_2}$ can be simplified to

$$\frac{X_1}{X_2} = \frac{\frac{1}{\alpha_{s2}} - [\frac{1}{\alpha_{s2}^2} + \frac{2(\gamma-1)}{\gamma}]^{1/2}}{\frac{1}{\alpha_{s1}} - [\frac{1}{\alpha_{s1}^2} + \frac{2(\gamma-1)}{\gamma}]^{1/2}}$$

Henceforth, consider the series expansion of the term $(1/\alpha_s^2 + 2[\gamma-1]/\gamma)^{1/2}$.

$$[1/\alpha_s^2 + 2(\gamma-1)/\gamma]^{1/2} \equiv \alpha_s^{-1} + \alpha_s(\gamma-1)/\gamma - \alpha_s^3(\gamma-1)/(2\gamma) + \cdots$$

Since for subsonic flows $\alpha_s < 1$, the preceding series is a rapidly converging one. Hence, substituting the first two terms of the above series expansion into equation (4-2) results in the following approximation of K coefficient.

$$K_\alpha = 1/\alpha_{s1}^2 - 1/\alpha_{s2}^2 - \frac{\gamma+1}{\gamma} \ln[\alpha_{s2}/\alpha_{s1}] \qquad (4\text{-}3)^*$$

4.2 Derivation of Pressure Correlations

The development of useful pressure correlations presented in this subsection follows mainly along the lines of Shapiro's approach. All pressure correlations are derived as functions of Mach number, at the inlet and exit of a passage.

* See Appendix C-2 for error estimation

a) Pressure Ratio Correlations

From equations (1-22) and (1-25), the total pressure ratio can be expressed as

$$p_{t2}/p_{t1} = \frac{M_1}{M_2}\left[\frac{1 + (\gamma-1)M_2^2/2}{1 + (\gamma-1)M_1^2/2}\right]^{\frac{\gamma+1}{2(\gamma-1)}} \tag{3-10}$$

Similarly, by combining expressions (2-21) and (2-24) one obtains

$$p_{s2}/p_{s1} = \frac{M_1}{M_2}\left[\frac{1+M_1^2(\gamma-1)/2}{1+M_2^2(\gamma-1)/2}\right]^{1/2} \tag{4-4}$$

Furthermore, the term $(1 + \frac{\gamma-1}{2}M^2)^{1/2}$ can be represented as a series expansion

$$[1+M^2(\gamma-1)/2]^{1/2} \equiv 1 + (\gamma-1)[M/2]^2 - (\gamma-1)[M/2]^4 + \cdots -$$

Where, for our case of $M < 1$, the series converges rapidly. Substituting then the first two terms of the above series expansion into equation (4-4) results in the following approximation:

$$(p_{s2}/p_{s1})_1 = \left\{\frac{1+\frac{\gamma+1}{4}M_2^2}{1+\frac{\gamma+1}{4}M_1^2}\right\}\frac{M_1}{M_2} \tag{4-5}$$

b) Pressure Difference Correlations

Referring to equations (3-10) and (4-4), one can define the pressure difference relations as

$$\Delta p_t/p_{t1} = 1 - p_{t2}/p_{t1} \tag{4-6}$$

and

$$\Delta p_s/p_{s1} = 1 - p_{s2}/p_{s1} \tag{4-7}$$

Dividing both sides of equation (4-6) by the corresponding sides of equations (4-7) results in

$$\frac{\Delta p_t}{\Delta p_s}\frac{p_{s1}}{p_{t1}} = \frac{p_{t2}/p_{t1} - 1}{p_{s2}/p_{s1} - 1} = \frac{(Y_2/Y_1)^{(\gamma+1)/[2(\gamma-1)]} M_1/M_2 - 1}{(Y_1/Y_2)^{1/2}\, M_1/M_2 - 1}$$

where

$$Y_1 = (1 + M_1^2[\gamma-1]/2)$$
$$Y_2 = (1 + M_2^2[\gamma-1]/2)$$

If, again, both sides of the preceding equation are divided by the corresponding sides of the generally known isentropic expression

$$p_{s1}/p_{t1} = [1 + M_1^2(\gamma-1)/2]^{-\gamma/(\gamma-1)}$$

one obtains

$$\Delta p_t/\Delta p_s = \frac{Y_2^{\phi} - Y_1^{\phi} M_2/M_1}{Y_2^{1/2} - Y_1^{1/2} M_2/M_1}(Y_2 Y_1)^{1/2} \tag{4-8}$$

where

$$\phi = (\gamma+1)/[2(\gamma-1)]$$

Expression (4-8) can be also presented in an approximated simpler form. This is achieved by substituting for the terms Y^{ϕ} and $Y^{1/2}$ the first two terms of their series expansions:

$$Y^{\phi} = [1+M^2(\gamma-1)/2]^{\phi} \equiv 1+M^2(\gamma+1)/4 + M^4(\gamma+1)(\gamma+3)/32 + \cdots$$
$$Y^{1/2} = [1+M^2(\gamma-1)/2]^{1/2} \equiv 1+M^2(\gamma-1)/4 - M^4(\gamma-1)/32 + \cdots$$

The resulting approximation becomes

$$\left[\frac{\Delta p_t}{\Delta p_s}\right]_1 = \frac{(1-[\gamma+1]M_1M_2/4)(1+[\gamma-1]M_1^2/4)(1+[\gamma-1]M_2^2/4)}{1+(\gamma-1)(M_1^2+M_1M_2+M_2^2)/4} \qquad (4\text{-}9)$$

At this point it should be reiterated that all the preceding pressure correlations of this section were derived on the assumption of an adiabatic ($T_{t2} = T_{t1}$), constant-area flow passage. An adaptation, following Benedict, of the above correlations for a variable-area case is presented below.

Utilizing the well known expression for the total flow number α_t,

$$\alpha_t = \frac{w\sqrt{T_t Rg/g}}{Ap_t}$$

$$= \frac{\sqrt{\gamma}M}{[1+(\gamma-1)M^2/2]^{(\gamma+1)/(\gamma-1)/2}}$$

one obtains

$$\alpha_{t1}/\alpha_{t2} = p_{t2}A_2/p_{t1}A_1$$

$$= \frac{M_1}{M_2}\left[\frac{1+(\gamma-1)M_2^2/2}{1+(\gamma-1)M_1^2/2}\right]^{\frac{\gamma+1}{2(\gamma-1)}}$$

and thus

$$p_{t2}/p_{t1} = \frac{M_1}{M_2}\left[\frac{1+(\gamma-1)M_2^2/2}{1+(\gamma-1)M_1^2/2}\right]^{\frac{\gamma+1}{2(\gamma-1)}}\frac{A_1}{A_2} \qquad (4\text{-}10)$$

Since

$$\alpha_s = \frac{w\sqrt{T_t Rg/g}}{Ap_s} \equiv \{\gamma M^2[1+(\gamma-1)M^2/2]\}^{1/2}$$

one can state that

$$\alpha_{s1}/\alpha_{s2} = p_{s2}A_2/p_{s1}A_1 = \frac{M_1}{M_2}\left[\frac{1+M_1^2(\gamma-1)/2}{1+M_2^2(\gamma-1)/2}\right]^{1/2}$$

Hence,

$$p_{s2}/p_{s1} = \frac{M_1}{M_2}\left[\frac{1+M_1^2(\gamma-1)/2}{1+M_2^2(\gamma-1)/2}\right]^{1/2}\frac{A_1}{A_2} \qquad (4\text{-}11)$$

For the constant-area case relations (4-10) and (4-11) become respectively identical to expressions (3-10) and (4-4).

Other useful pressure relationships, $\Delta p_t/p_{t1}$, $\Delta p_s/p_{s1}$ and $\Delta p_t/\Delta p_s$, for the variable-area case can be easily derived, if necessary, employing similar techniques as were used for the case of constant-area flow.

4.3 Working Graphs

To complete the presentation of the Alternative pressure loss calculation method, working graphs are presented in Figures 4-1 to 4-4 at the end of this section. These are plots of equations (4-1), (3-10), (4-4), (4-6), (4-7) and (4-8).

If the graphs are to be used for variable-area passages the following must apply:

1.	Instead of $\frac{p_{t2}}{p_{t1}}$	use	$\frac{p_{t2}}{p_{t1}}\frac{A_2}{A_1}$
2.	Instead of $\frac{p_{s2}}{p_{s1}}$	use	$\frac{p_{s2}}{p_{s1}}\frac{A_2}{A_1}$
3.	Instead of $\frac{\Delta p_t}{p_{t1}}$	use	$1 - \frac{p_{t2}A_2}{p_{t1}A_1}$
4.	Instead of $\frac{\Delta p_s}{p_{s1}}$	use	$1 - \frac{p_{s2}A_2}{p_{s1}A_1}$
5.	Instead of $\frac{\Delta p_t}{\Delta p_s}$	use	$\frac{(1-p_{t2}A_2/p_{t1}A_1)}{(1-p_{s2}A_2/p_{s1}A_1)}\frac{p_{t1}}{p_{s1}}$

From the preceding table it appears that for a variable-area case the use of Fig. 4-4 is too involved for efficient calculation.

The use of the working graphs is illustrated in Appendix C-1 by means of two examples. In passing, it must be stated that the plots in Figures 4-1 to 4-4 are only a demonstration of the workings of the Alternative method. In order for the method to become of general use, more detailed graphs and tabulated data may be required. These additional data can be obtained by means of the computer program used in providing the data for Figures 4-1 to 4-4. The listing of the program is included in Appendix C-3.

Expressions (4-3), (4-5) and (4-9) comprise the set of simplified approximations to be used for hand calculations if necessary. Of these, $(p_{s2}/p_{s1})_{\prime}$ is correct to within 0.5% of exact value, (4-4), for Mach numbers up to 1.0. The $(\Delta p_t/\Delta p_s)_{\prime}$ is correct to within 1.0% of exact value, (4-8), for values of M_1 and M_2 less than 0.4 and 0.6 respectively.

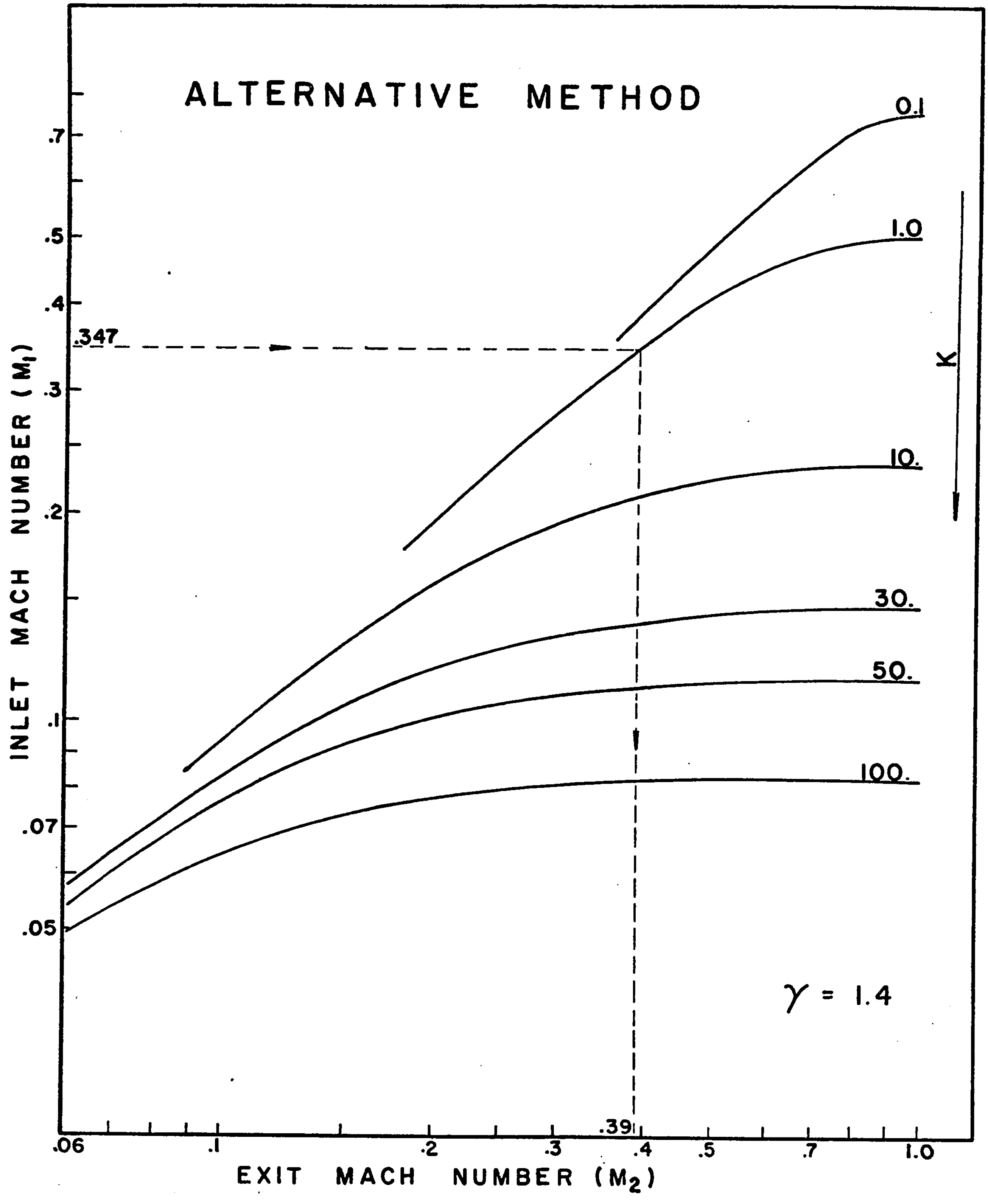


FIG. 4-1 LOSS COEFFICIENT VARIATION

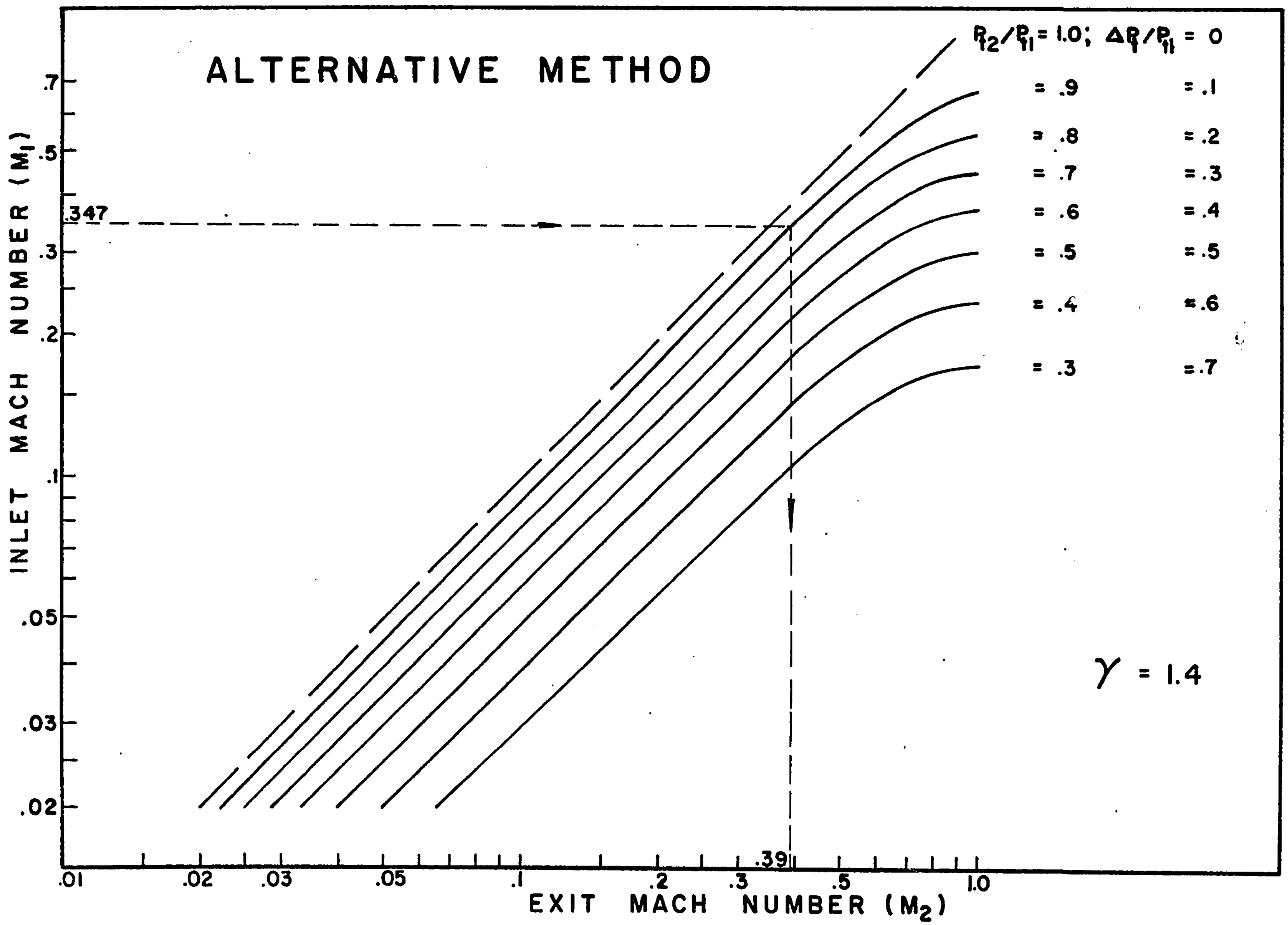


FIG. 4–2 TOTAL PRESSURE RATIO VARIATION

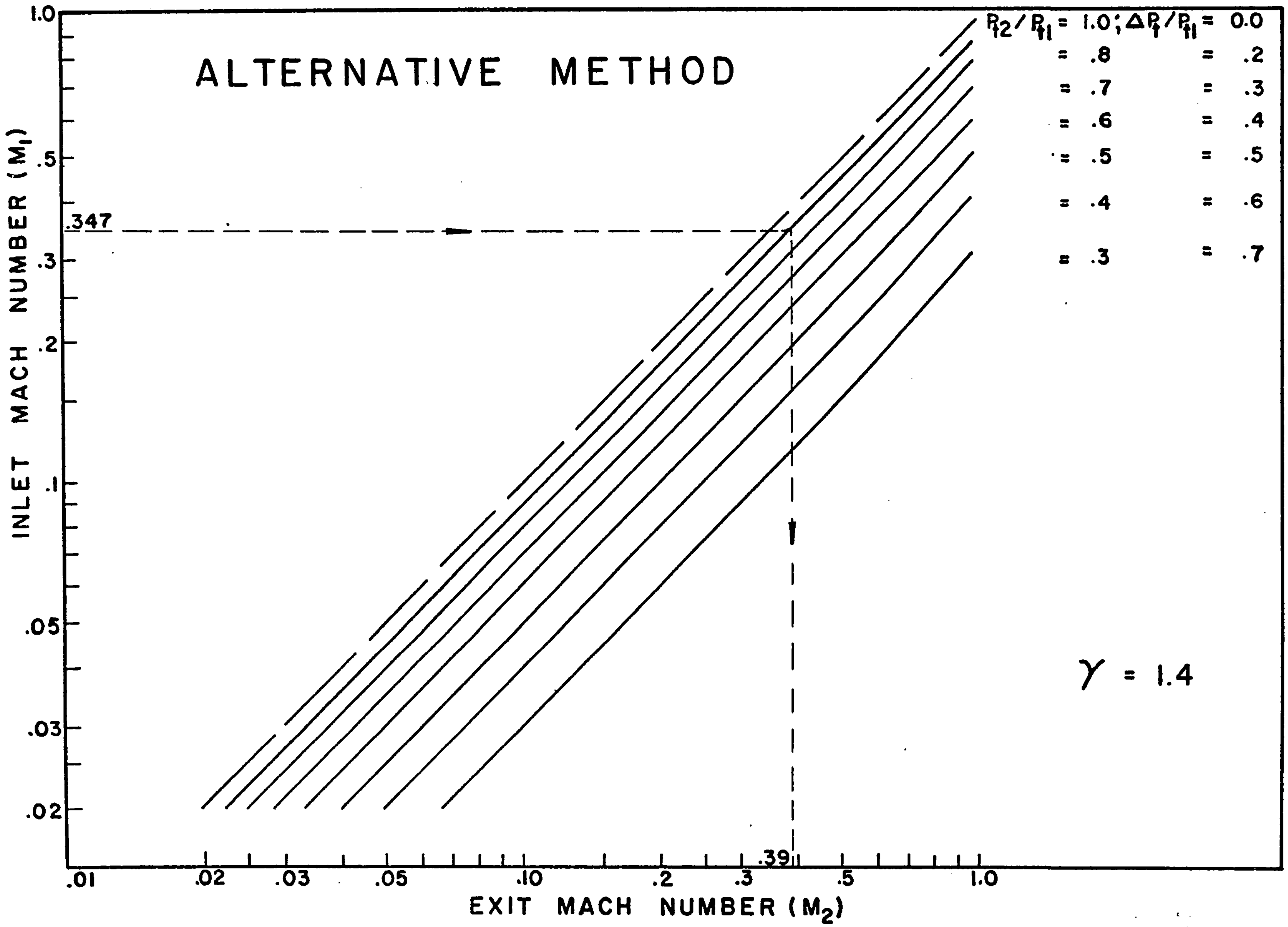


FIG. 4–3 STATIC PRESSURE RATIO VARIATION

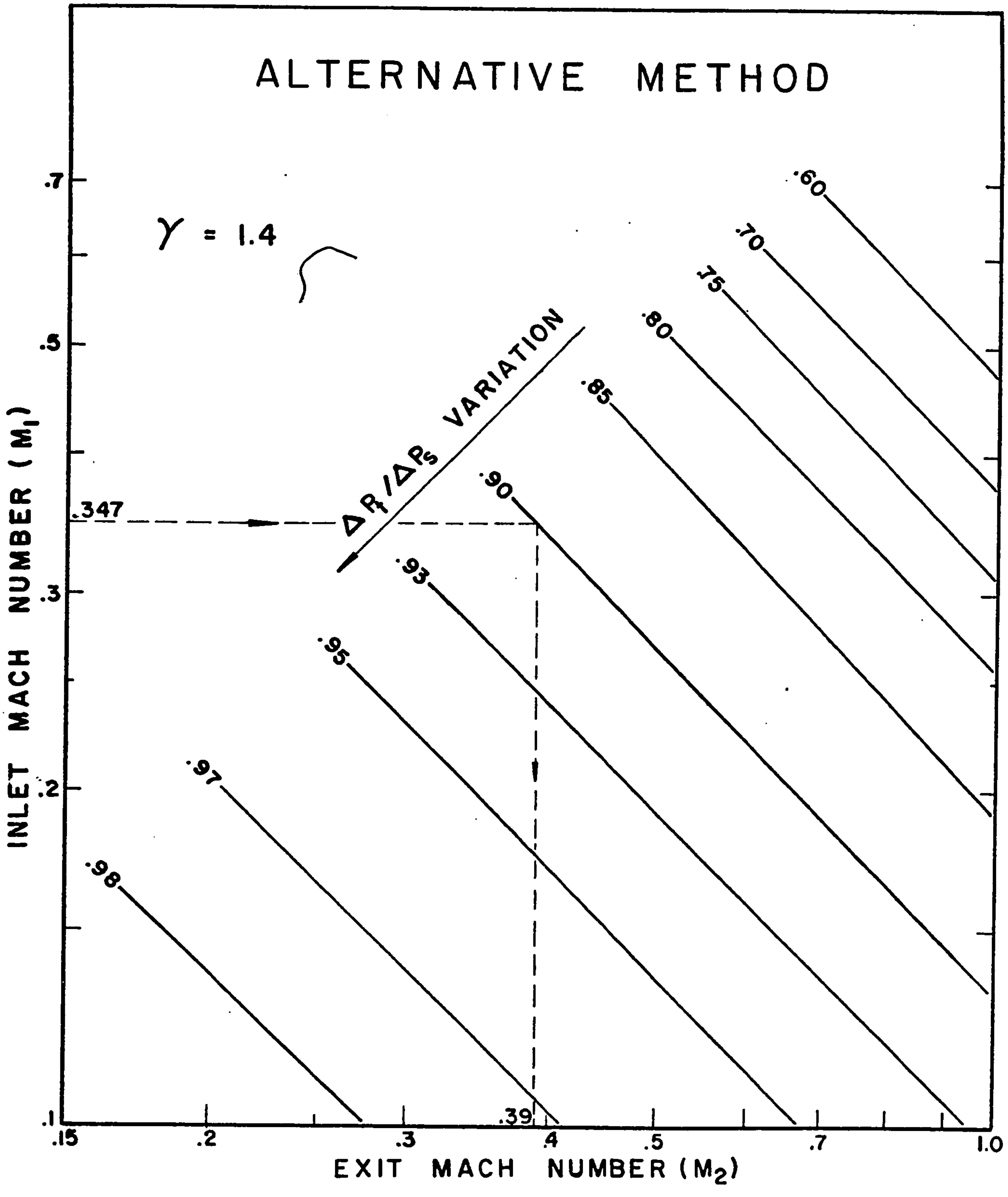


FIG. 4-4
PRESSURE DIFFERENCE RATIO VARIATION

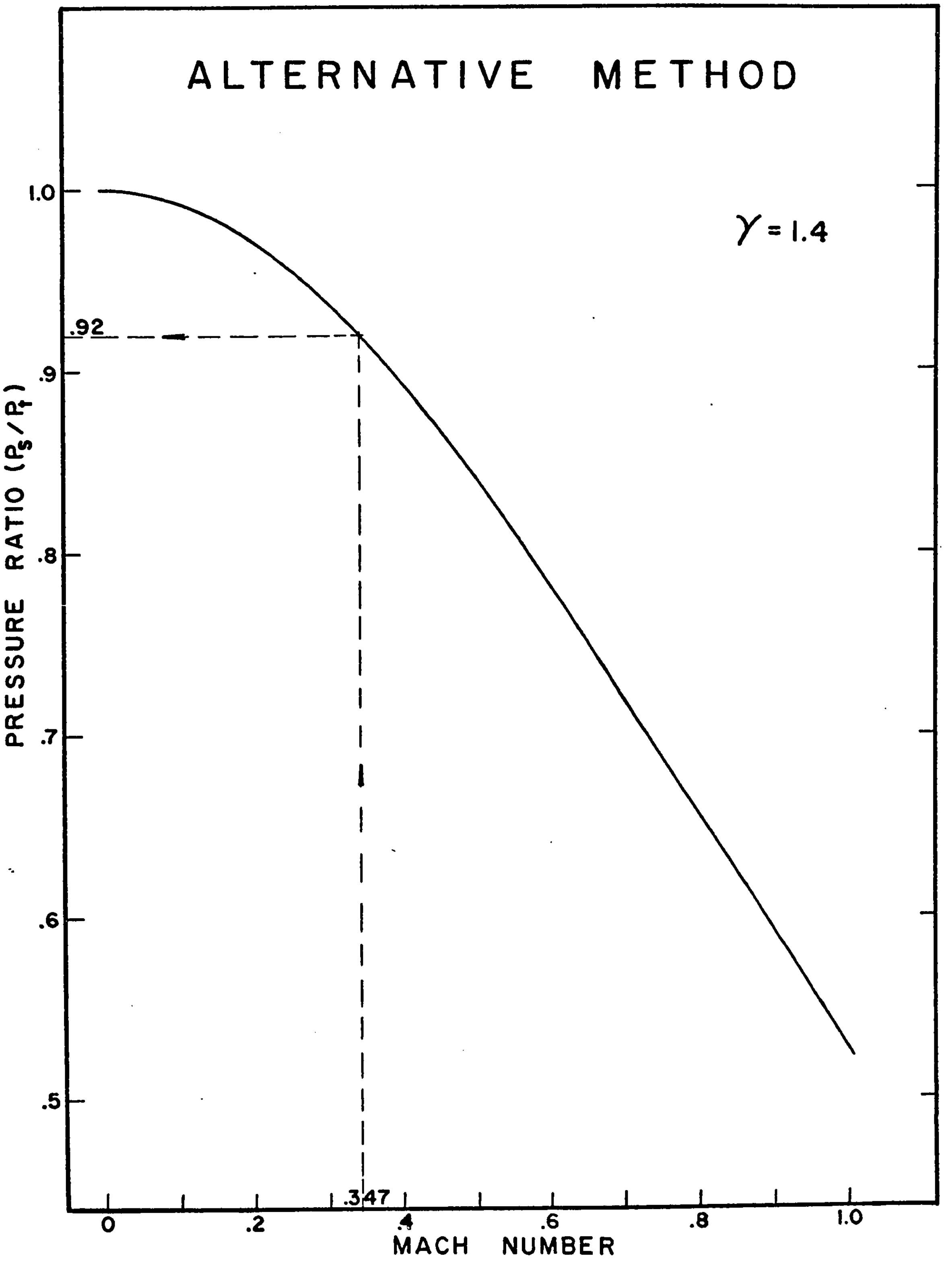


FIG. 4-5 PRESSURE RATIO VS MACH NUMBER

CONCLUSIONS

The foregoing study consisted primarily of reviewing two known pressure loss calculation methods. A third alternative method, utilizing the advantages of the two other methods, was developed and presented as part of this study. The main features of the three methods are as follows.

1. Numerical accuracy. Since the expressions for the loss coefficients of the three methods are identical, numerical calculations by any of the three procedures must be the same for constant-area flow. This is further substantiated by the three solutions of the same worked example included separately in Appendixes A, B and C.

2. Generality of Application. As in the case of Shapiro's approach, the use of Mach number for the common independent variable gives the Alternative method a greater unity and generality than that of Benedict for constant-area flow. On the other hand, the Alternative approach, by virtue of its added capability of handling variable-area cases, has a wider range of application than Shapiro's method. Further, for the case of variable area flow, the greater availability from various technical sources of tabulated α_t data, as opposed to Γ-function data, may give the Alternative approach an added advantage over Benedict's method.

3. Directness of applications. Comparing the three solutions of the same example one concludes that Shapiro's solution is the most involved one of the three. Indeed, if one takes Shapiro's method in order to calculate the generally needed pressure loss Δp_s or Δp_t, it is

necessary to go first through determining the flow parameters at the critical state and only then the final conditions can be found. All this tends to obscure and lengthen the flow loss calculation procedure. Both Benedict's and the Alternative methods are more direct and explicit in this respect.

4. Method versatility. Shapiro's approach is easier to adapt for computer analysis than Benedict's method. This holds also for the Alternative approach. One other feature, peculiar only to the Alternative method, is the derived set of simplified approximations to be used for hand calculations, whenever neither proper graphs nor tables are available. The approximate expressions include the loss coefficient $K\alpha$, the pressure ratio $(p_{s2}/p_{s1})_1$, and $(\Delta p_t/\Delta p_s)_1$.

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A P P E N D I X 'A'

SHAPIRO'S METHOD

APPENDIX A-1

Friction Factor f

The values for the friction factor may be obtained from the generally known Moody diagram included in this Appendix. The Moody diagram defines f values for three different flow regimes: the laminar flow, fully developed turbulent flow, and flow in the transitional zone.

The friction factor definition for the laminar zone is based on the Hagen-Poiseuille equation for pipe flow:

$$f = \frac{64}{Re}$$

where Re is the flow Reynolds number referred to the hydraulic diameter of the flow channel. The transition zone friction factor is defined on the basis of Colebrooks' equation [6]:

$$\frac{1}{f} = -\ 0.86\ \ln \left(\frac{\epsilon/D}{3.7} + \frac{2.51}{Re\sqrt{f}}\right)$$

where

ϵ/D is the relative roughness

Re is the Reynolds number, as above

In general it may be stated that for

laminar zone	$f = fn(Re)$ only
transition zone	$f = fn(Re_1, \epsilon/D)$
turbulent zone	$f = fn(\epsilon/D)$ only

The Fanning friction factor, f_F, is defined as

$$f_F = f/4$$

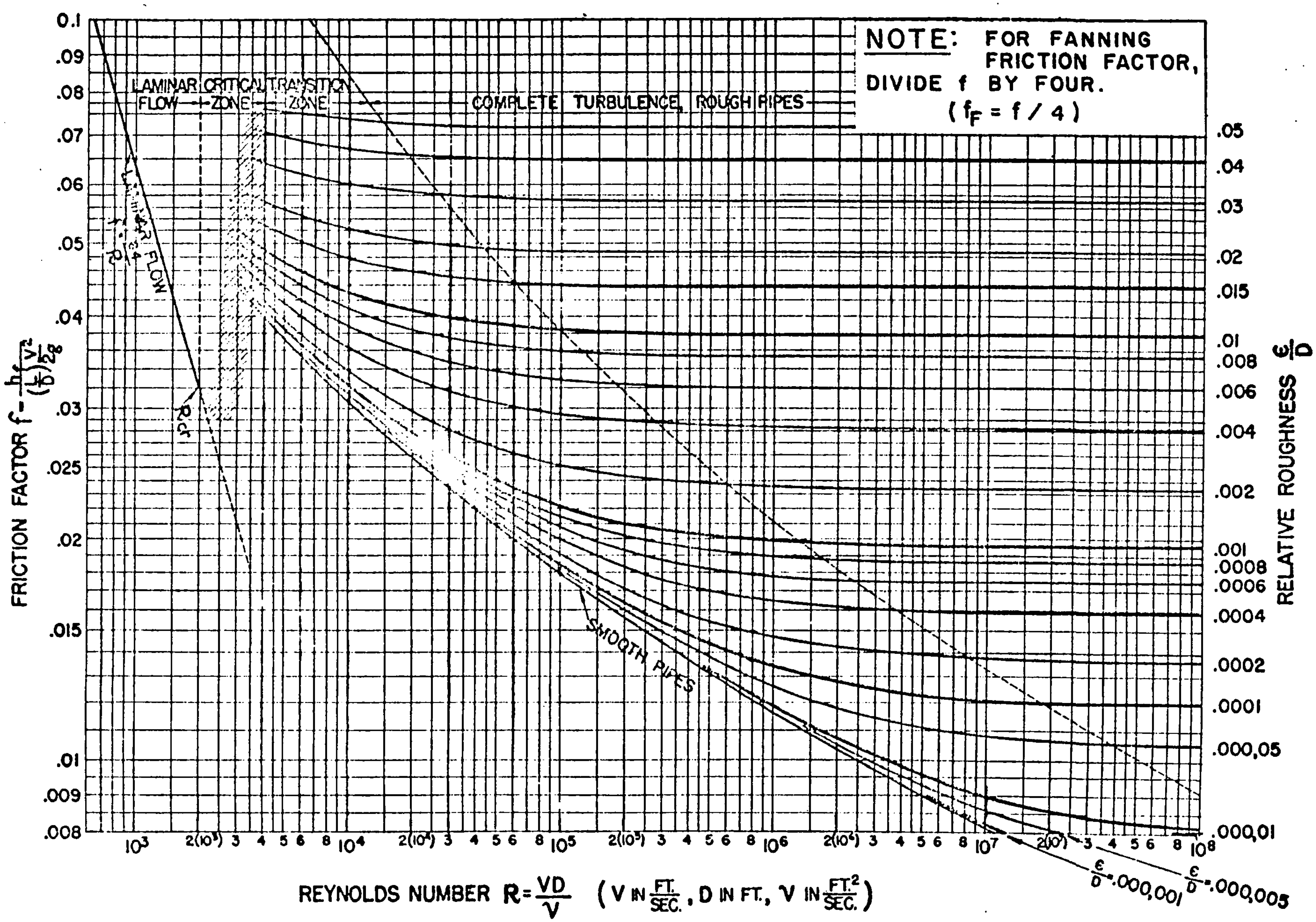

FIG. A-1 MOODY DIAGRAM

Worked example

In a constant cross-sectional passage through which flows air (γ=1.4) the following is known:

$$M_1 = .347$$

$$p_1 = 18.4 \text{ psia}$$

$$p_2 = 16.4 \text{ psia}$$

$$p_{s1}/p_{t1} = 0.92$$

Find the pressure loss and the loss coefficient of the passage flow.

Solution:

Knowing p_1/p_{o1}, p_{t1} = 20.0 psia.

From Fig. 2-2 one then determines

$$p_{o1}/p_o{*} = 1.79$$

$$p_1/p{*} = 3.12$$

$$(4f_F\ Lmax/D)_1 = 3.53$$

which enables to calculate $p_2/p*$

$$p_2/p{*} = \frac{p_2}{p_1} \cdot p_1/p{*}$$

$$= \frac{16.4}{18.4}(3.12) = 2.78$$

Subsequently, from Fig. 2-2, for $p_2/p* = 2.78$, it follows that

$$M_2 = .39$$

$$p_{o2}/p_o{*} = 1.63$$

$$(4f_F\ Lmax/D)_2 = 2.5$$

Hence, one can determine p_{o2}/p_{o1}

$$p_{o2}/p_{o1} = \frac{p_{o2}}{p_o^*} / \frac{p_{o1}}{p_o^*}$$

$$= 1.63/1.79$$

$$= 0.91$$

Thus:

$$\Delta p_o = (1 - p_{o2}/p_{o1})\, p_{o1}$$

$$= (.09).20$$

$$= 1.8 \text{ psi}$$

$$\Delta p = 18.4 - 16.4 = 2.0 \text{ psi}$$

$$4f_F L/D = \left(4f_F \frac{L_{max}}{D}\right)_1 - \left(4f_F \frac{L_{max}}{D}\right)_2$$

$$= 1.03$$

A P P E N D I X 'B'

BENEDICT'S METHOD

Pressure loss calculation, step (ii)

One of the possible ways to express α_{t2} as a function of R_2 may be as follows. First, evaluate equation (2-10) so that K expression obtains the form

$$\frac{\gamma+1}{\gamma}\ln\left\{\left[\frac{R_2^2}{\alpha_{t1}^2}+\frac{2(\gamma-1)}{\gamma}\right]^{1/2}+\frac{R_2}{\alpha_{t1}}\right\}-\frac{1}{2}\frac{R_2}{\alpha_{t1}}-\frac{1}{2}\frac{R_2}{\alpha_{t1}}\left[\frac{R_2^2}{\alpha_{t1}^2}+\frac{2(\gamma-1)}{\gamma}\right]^{1/2}-K+C_K=0$$

where

$$C_K=\frac{1}{2}\frac{R_1}{\alpha_{t1}}\left[\frac{R_1^2}{\alpha_{t1}^2}+\frac{2(\gamma-1)}{\gamma}\right]^{1/2}+\frac{1}{2}\frac{R_1}{\alpha_{t1}}-\frac{\gamma+1}{\gamma}\ln\left\{\left[\frac{R_1^2}{\alpha_{t1}^2}+\frac{2(\gamma-1)}{\gamma}\right]^{1/2}+\frac{R_1}{\alpha_{t1}}\right\}$$

Considering γ, K, C_K to be known, one can solve for R_2/α_{t1} by using any of the known numerical method procedures.

In order to proceed further it is useful to redefine R_2/α_{t1} as

$$\frac{R_2}{\alpha_{t1}}=\frac{1}{\alpha_{s2}}=\frac{1}{\frac{W/\Delta t}{Ap_2}\left[\frac{p_t v_t}{g}\right]^{1/2}}$$

where

α_{s2} - is the 'static flow number' which can be expressed as a function of Mach number as

$$\alpha_{s2}=\sqrt{\gamma}M\,(1+M_2^2[\gamma-1]/2)^{1/2} \qquad \text{(B-1)}$$

Solving then (B-1) for M_2 gives

$$M_2=\left(\sqrt{(\gamma-1)^{-2}+2\alpha_{s2}^2/[\gamma(\gamma-1)]}-1/(\gamma-1)\right)^{1/2} \qquad \text{(B-2)}$$

On the other hand, the total flow number α_{t2} at station 2 may be also defined in terms of M_2 as

$$\alpha_{t2} = \frac{\sqrt{\gamma} M_2}{[1 + (\gamma-1)M_2^2/2]^{(\gamma+1)/(\gamma-1)/2}} \tag{B-3}$$

A further combination of equations (B-2) and (B-3) produces

$$\alpha_{t2} = \frac{[\gamma(\gamma-1)(\lambda-1)]^{1/2}}{[\ (\lambda+1)/2\ ]^{\phi}}$$

where

$$\lambda = [1 + 2\alpha_{t1}^2(\gamma-1)/\gamma R_2^2]^{1/2}$$

$$\phi = (\gamma+1)/[2(\gamma-1)]$$

APPENDIX B-2

Worked example

Consider the adiabatic flow of air (γ=1.4, Rg=53.3 $\frac{ft.}{°R}$) from the inlet total pressure of 20 psia and an inlet total temperature of 573°R. Find the static and total pressure drops through a constant-area duct (25 sq. in.) if the flow rate is 5 lb/sec and the loss coefficient is 1.

Solution (see Fig. 2-2):

At

$$\alpha_{t1} = \frac{W/\Delta t}{A p_{t1}} \left(\frac{R g T_t}{g}\right)^{1/2}$$

$$= 0.3825$$

obtain from Table II-1

$p_1/p_{t1} = 0.92$	$p_1 = 18.4$ psia
$p_2/p_{t1} = 0.82034$	$p_2 = 16.4$ psia
$p_{t1}/p_{t2} = 1.09866$	$p_{t2} = 18.2$ psia

Thus

$$p_1 - p_2 = 2.0 \text{ psi}$$

$$p_{t1} - p_{t2} = 1.8 \text{ psi}$$

Considering now Fig. 2-2,

At Γ

$$\Gamma = \alpha/\alpha^*$$

$$= .5586$$

obtain

$$p_1/p_{t1} = 0.92$$

$$p_2/p_{t1} = 0.82$$

$$p_{t1}/p_{t2} = 1.099$$

These values give substantially the same results as the tabular method.

A P P E N D I X 'C'

ALTERNATIVE METHOD

Worked examples

Example 1:

In a constant cross-sectional passage through which flows air (γ=1.4) the following is known:

$M_1 = 0.347$

$p_{s1} = 18.4$ psia

$p_{s2} = 16.4$ psia

Find the total pressure loss.

Solution:

From the static pressures:

$p_{s2}/p_{s1} = 0.891$

Hence, from Fig. 4-3 one obtains

$M_2 = 0.39$

Going then to Fig. 4-4 gives

$\Delta p_t/\Delta p_s \approx 0.9$ psi

Hence,

$$\Delta p_t = (0.9)\ (18.4 - 16.4)$$
$$= 1.8 \text{ psi}$$

Example 2:

Change the given conditions in Example 1 as follows

$M_1 = .347$

$p_{t1} = 20$ psia

$K = 1.0$

$A_2/A_1 = .96$

Find the total and static pressure losses.

Solution:

From Fig. 4-1 one obtains

$$M_2 = 0.39$$

then from Fig. 4-2

$$\frac{P_{t2}}{P_{t1}} \frac{A_2}{A_1} = 0.91$$

Hence,

$$\Delta P_t = P_{t1} - \frac{P_{t2}A_2}{P_{t1}A_1} \cdot P_{t1}/(A_2/A_1)$$

$$= 20(1 - 0.91/0.96)$$

$$= 1.02 \text{ psi}$$

Going now to Fig. 4-5:

$$\frac{P_{s1}}{P_{t1}} = .92$$

Hence,

$$P_{s1} = 20 \; . \; (0.92) = 18.4 \text{ psia}$$

Then from Fig. 4-3:

$$\frac{P_{s2}}{P_{s1}} \cdot \frac{A_2}{A_1} \simeq 0.891$$

or

$$\Delta P_s = 18.4 \; . \; (1 - 0.891/.96)$$

$$= 1.325 \text{ psi}$$

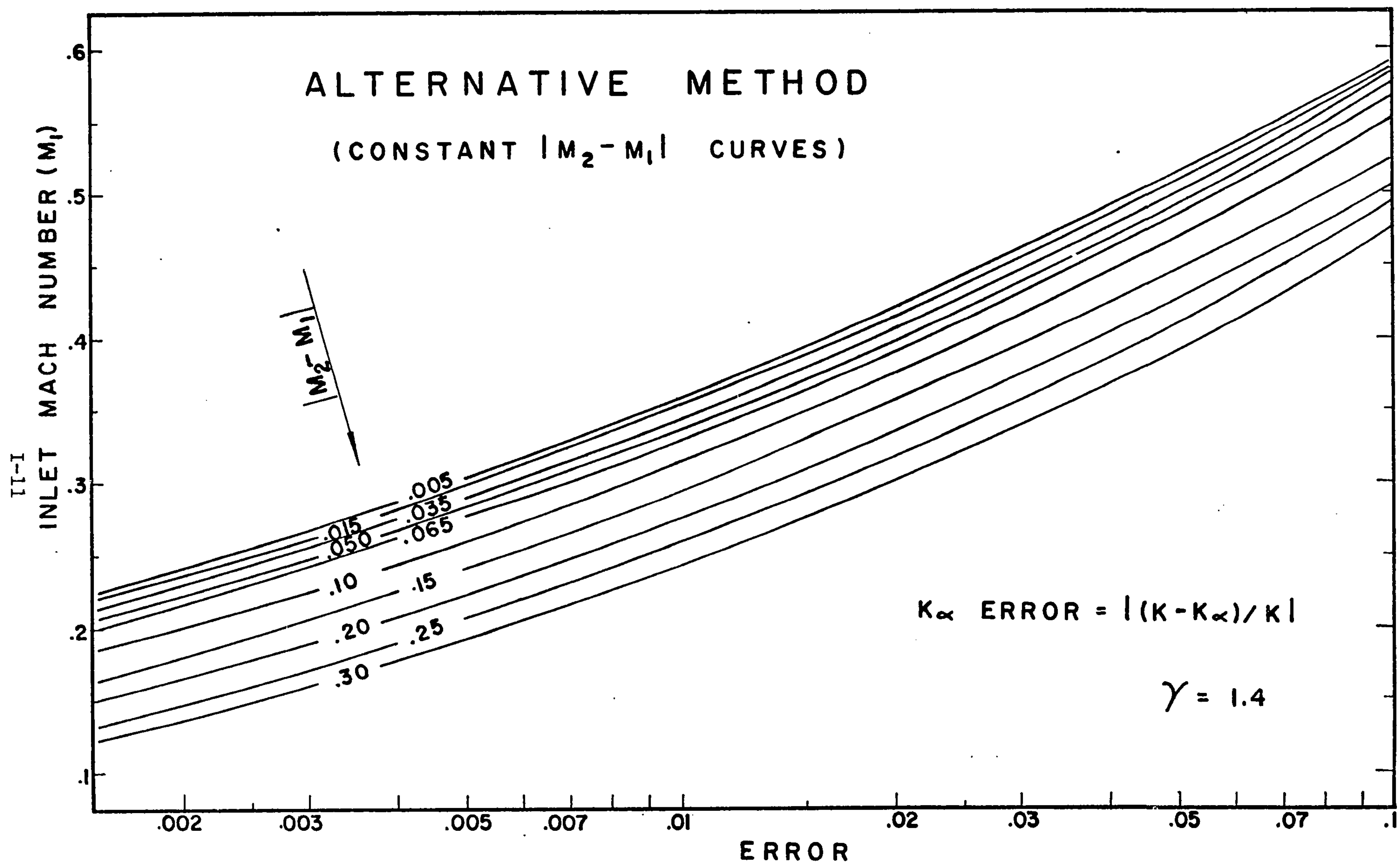

FIG. C-1 K_α ERROR VS INLET MACH NUMBER

APPENDIX C-3

COMPUTER PROGRAM CODE

YM1	denotes	M_1	INPUT
*DM	"	$\Delta M = M_2 - M_1$	INPUT
NYM1	"	No. of values of M_1	INPUT
*NDM	"	No. of values of ΔM	INPUT
XK	denotes	K	
*XK2	"	K_1	
*XKB1	"	$\overline{K}$	
XKA	"	K_α	
PTR	denotes	p_{t2}/p_{t1}	
DPTR	"	$p_{t2}/p_{t1} - 1$	
PSR	"	p_{s2}/p_{s1}	
DPSR	"	$p_{s2}/p_{s1} - 1$	
PSR1	"	$(p_{s2}/p_{s1})_1$	
DPR	"	$\Delta p_t/\Delta p_s$	
DPR1	"	$(\Delta p_t/\Delta p_s)_1$	
*XKINC	"	K' or K"	
EKA	denotes error of	K_α	
*EK2	"	K_1	
*EKB1	"	$\overline{K}$	
EPSR	denotes error of	$(p_{s2}/p_{s1})_1$	
EDPR1	"	$(\Delta p_t/\Delta p_s)_1$	
*EKINC	denotes error of	XKINC	

* denotes parameters not pertaining to this study.

```
      PROGRAM WIERBA
C     FLOW LOSSES
      DIMENSION YM1(10),DM(15),XK(10,15),XKA(10,15),XK1(10,15),XK2(10,15
     1),XK3(10,15),XKB(10,15),DPR3(10,15),DPR2(10,15),XKINC(10,15)
      DIMENSION EKA(10,15),EK1(10,15),EK2(10,15),EK3(10,15),EKB(10,15)
      DIMENSION PTR(10,15),PSR(10,15),DPR(10,15),DPR1(10,15),DPTR(10,15)
     1,DPSR(10,15),XK31(10,15),XKB1(10,15),EK31(10,15),EKB1(10,15),PTR1(
     210,15),PSR1(10,15),EPTR(10,15),EPSR(10,15),EDPR1(10,15),EDPR2(10,1
     35),EDPR3(10,15),EDPR4(10,15),DPR4(10,15),EKINC(10,15)
      WRITE(61,99)
   99 FORMAT(12H FLOW LOSSES//////)
    1 READ(60,100)LAST RUN
      IF (LAST RUN.EQ.1R$) 2,10
    2 RETURN
  100 FORMAT(49H                                                ,30XR1)
   10 READ(60,21)NYM1,NDM,G
      WRITE(61,21)NYM1,NDM,G
      READ(60,20) (YM1(I),I=1,NYM1)
      READ(60,20)(DM(J),J=1,NDM)
      WRITE(61,20)(YM1(I),I=1,NYM1)
      WRITE(61,20) (DM(J),J=1,NDM)
   21 FORMAT(2I4,F10.3)
   20 FORMAT( 8F10.3)
      WRITE(61,22)
   22 FORMAT(/////)
      G1=(G-1.)*.5
      G2=(G+1.)*.5/G
      G3=(G+1.)*.5/(G-1.)
      G4=(G+1.)/4.
      DO 30 I=1,NYM1
      DO 31 J=1,NDM
      A=2.*DM(J)/(G*YM1(I)**2*(YM1(I)+2.*DM(J)))
      A1=1.+G1*YM1(I)**2
      B=YM1(I)+2.*DM(J)*A1
      C=YM1(I)*(1.+G1*(YM1(I)**2+2.*DM(J)*YM1(I)))
      XK1(I,J)=A-G2*ALOG (B/C)
      XK2(I,J)=A-G2*ALOG (1.+2.*DM(J)/(YM1(I)+G1*YM1(I)**3))
      XK3(I,J)=A-G2*ALOG (1.+2.*DM(J)/YM1(I))
      XK31(I,J)=(2.*DM(J)/(G*(YM1(I)+DM(J))))*(1./YM1(I)**2-G4*2.)
      YMB=YM1(I)+DM(J)*.5
      D=YMB+DM(J)+G1*YMB**3
      E=YMB-DM(J)+G1*YMB**3
      XKB(I,J)=2.*DM(J)/(G*YMB**3)-G2*ALOG (D/E)
```

```
      XKB1(I,J)=2.*DM(J)*(1./YMB**2-G4*4./(1.+G1*YMB**2)/2.)/G/YMB
      YM2=YM1(I)+DM(J)
      A2=1.+G1*YM2**2
      XK(I,J)=(YM2**2-YM1(I)**2)/(G*YM1(I)**2*YM2**2)-G2*ALOG (YM2**2*A1
     1/(YM1(I)**2*A2))
      AS1=G*YM1(I)**2*A1
      AS2=G*YM2**2*A2
      XKA(I,J)=1./AS1-1./AS2-G2*ALOG (AS2/AS1)
      EK1(I,J)=(XK(I,J)-XK1(I,J))/XK(I,J)
      EK2(I,J)=(XK(I,J)-XK2(I,J))/XK(I,J)
      EK3(I,J)=(XK(I,J)-XK3(I,J))/XK(I,J)
      EKB(I,J)=(XK(I,J)-XKB(I,J))/XK(I,J)
      EKA(I,J)=(XK(I,J)-XKB(I,J))/XK(I,J)
      EK31(I,J)=(XK(I,J)-XK31(I,J))/XK(I,J)
      EKB1(I,J)=(XK(I,J)-XKB1(I,J))/XK(I,J)
      PTR(I,J)=(A2/A1)**G3*(YM1(I)/YM2)
      PTR1(I,J)=(1.+G4*YM2**2)*YM1(I)/((1.+G4*YM1(I))*YM2)
      DPTR(I,J)=PTR(I,J)-1.
      PSR(I,J)=(A1/A2)**.5*(YM1(I)/YM2)
      PSR1(I,J)=(1.+G1*.5*YM1(I)**2)*YM1(I)/((1.+G1*.5*YM2**2)*YM2)
      DPSR(I,J)=PSR(I,J)-1.
      DPR(I,J)=((A2**G3-YM2/YM1(I)*A1**G3)*A2**.5)/((A1**.5-YM2/YM1(I)*
     1A2**.5)/A1**.5)
      F1=1.+(G-1.)*YM1(I)**2/4.
      F2=1.+(G-1.)*YM2**2/4.
      DPR1(I,J)=(1.-G4*YM1(I)*YM2)*F2/((1.+G1/2.*(YM1(I)**2+YM1(I)*YM2+Y
     1M2**2))/F1)
      DPR2(I,J)=(1.+G1*YMB**2)**(G/(G-1.))
      DPR3(I,J)=1.+G*YMB**2*.5
      DPR4(I,J)=1.+G*YMB**2*.5+G*YMB**2/8.
      XKINC(I,J)=-DPR(I,J)*DPSR(I,J)*2./(G*YM1(I)**2)
      EKINC(I,J)=(XK(I,J)-XKINC(I,J))/XK(I,J)
      EPTR(I,J)=(PTR(I,J)-PTR1(I,J))/PTR(I,J)
      EPSR(I,J)=(PSR(I,J)-PSR1(I,J))/PSR(I,J)
      EDPR1(I,J)=(DPR(I,J)-DPR1(I,J))/DPR(I,J)
      EDPR2(I,J)=(DPR(I,J)-DPR2(I,J))/DPR(I,J)
      EDPR3(I,J)=(DPR(I,J)-DPR3(I,J))/DPR(I,J)
      EDPR4(I,J)=(DPR(I,J)-DPR4(I,J))/DPR(I,J)
   31 CONTINUE
   30 CONTINUE
      WRITE (61,22)
      DO 40 I=1,NYM1
      DO 40 J=1,NDM
   40 WRITE(61,53)YM1(I),DM(J),XK(I,J),XK1(I,J),XK2(I,J),XK3(I,J),XKA(I,
     1J),XKB(I,J),XKINC(I,J)
```

```
      WRITE(61,22)
      DO 41 I=1,NYM1
      DO 41 J=1,NDM
   41 WRITE(61,52)YM1(I),DM(J),FK1(I,J),EK2(I,J),EK3(I,J),FKA(I,J),FKB(I
     1,J),EK31(I,J),FKB1(I,J)
   52 FORMAT(2F10.3,7F10.5)
      WRITE(61,22)
      DO42 I=1,NYM1
      DO42 J=1,NDM
   42 WRITE(61,53)YM1(I),DM(J),PTR(I,J),PTR1(I,J),DPR(I,J),DPR1(I,J),DPR
     12(I,J),DPR3(I,J),DPR4(I,J)
   53 FORMAT(2F10.3,7F10.4)
      WRITE(61,22)
      DO 43 I=1,NYM1
      DO 43 J=1,NDM
   43 WRITE(61,53)YM1(I),DM(J),XK(I,J),XK31(I,J),XKB1(I,J),DPTR(I,J),DPS
     1R(I,J),PSR(I,J),PSR1(I,J)
      WRITE(61,22)
      DO 44 I=1,NYM1
      DO 44 J=1,NDM
   44 WRITE(61,52)YM1(I),DM(J),EPTR(I,J),EPSR(I,J),EDPR1(I,J),EDPR2(I,J)
     1,EDPR3(I,J),EDPR4(I,J),EKINC(I,J)
            GO TO   1
      END
```

FORTRAN DIAGNOSTIC RESULTS FOR WIERRA