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INTRODUCING A DEPENDENCE STRUCTURE TO THE OCCURRENCES IN STUDYING PRECISE LARGE DEVIATIONS FOR THE TOTAL CLAIM AMOUNT

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Introducing a Dependence Structure to the Occurrences in Studying Precise Large Deviations for the Total Claim Amount

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Abstract

In this paper we study precise large deviations for a compound sum of claims, in which the claims arrive in groups and the claim numbers in the groups may follow a certain negative dependence structure. We try to build a platform both for the classical large deviation theory and for the modern stochastic ordering theory.

Keywords: Consistent variation; Matuszewska index; Negative cumulative dependence; Precise large deviations; Random sums; Stop-loss order.

1 Introduction

Inspired by the recent works of Cline and Hsing (1991), Klüppelberg and Mikosch (1997), Tang et al. (2001), and Ng et al. (2004), in the present paper we are interested in precise large deviations for the random sum

$$S_t = \sum_{k=1}^{N_t} X_k, \qquad t \geq 0. \tag{1.1}$$

Here $\{X_k, k = 1, 2, \ldots\}$ is a sequence of independent, identically distributed (i.i.d.), and nonnegative heavy-tailed random variables, representing the sizes of successive claims, with

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common distribution function $F = 1 - \overline{F}$ and finite mean $\mu > 0$; N_t, $t \geq 0$, is a nonnegative, nondecreasing, and integer-valued process, representing the number of claims by time t, with a mean function $\lambda_t = \mathrm{E}[N_t] < \infty$ for each $t \geq 0$ and $\lambda_t \to \infty$ as $t \to \infty$; and, as usual, $\sum_{k \in \emptyset}(\cdot) = 0$ by convention. The process $\{N_t, t \geq 0\}$ and the sequence $\{X_k, k = 1, 2, \ldots\}$ are assumed to be mutually independent. Our goal is to establish a precise large deviation result that for any fixed $\gamma > 0$, the relation

$$\Pr(S_t - \mu\lambda_t > x) \sim \lambda_t \overline{F}(x), \qquad t \to \infty, \tag{1.2}$$

holds uniformly for $x \geq \gamma\lambda_t$. Hereafter, all limit relationships are for $t \to \infty$ unless stated otherwise. The uniformity of relation (1.2) means

$$\lim_{t\to\infty} \sup_{x \geq \gamma\lambda_t} \left| \frac{\Pr(S_t - \mu\lambda_t > x)}{\lambda_t \overline{F}(x)} - 1 \right| = 0.$$

This is crucial for our purpose.

As an application of (1.2), we consider the calculation of the stop-loss premium of the random sum S_t with large retention d, say $d = d(t) \geq (\mu + \gamma)\lambda_t$. Write $x_+ = \max\{x, 0\}$. As t increases, applying the uniformity of the asymptotic relation (1.2) we have

$$\begin{aligned}
\mathrm{E}\left[(S_t - d)_+\right] &= \int_d^\infty \Pr(S_t > x)\,\mathrm{d}x \\
&= \int_d^\infty \Pr(S_t - \mu\lambda_t > x - \mu\lambda_t)\,\mathrm{d}x \\
&\sim \lambda_t \int_d^\infty \overline{F}(x - \mu\lambda_t)\mathrm{d}x \\
&= \lambda_t \mathrm{E}\left[(X_1 + \mu\lambda_t - d)_+\right].
\end{aligned}$$

Clearly, the calculation of the right-hand side of the above is much simpler than the calculation of the stop-loss premium of S_t itself. For further applications of precise large deviations to insurance and finance, we refer the reader to Klüppelberg and Mikosch (1997), Mikosch and Nagaev (1998), and Embrechts et al. (1997, Chapter 8), among many others.

In this paper we shall consider the following special case of the random sum (1.1), in which the claim arrivals follow a compound renewal counting process:

1. the arrival times $0 = \sigma_0 < \sigma_1 < \sigma_2 < \ldots$ constitute a renewal counting process

$$\tau_t = \sum_{k=1}^{\infty} 1_{(\sigma_k \leq t)}, \qquad t \geq 0, \tag{1.3}$$

where 1_A denotes the indicator function of a set A and the interarrival times $\sigma_k - \sigma_{k-1}$, $k = 1, 2, \ldots$, are i.i.d. with $\mathrm{E}[\sigma_1] < \infty$;

2. at each arrival time σ_k, a group of Z_k claims arrives, and Z_k, $k = 1, 2, \ldots$, constitute a sequence of nonnegative, integer-valued, and identically distributed random variables, that may be independent, but can also follow a certain dependence structure;

3. the number of claims by time t is therefore a process

$$N_t = \sum_{k=1}^{\infty} Z_k 1_{(\sigma_k \le t)} = \sum_{k=1}^{\tau_t} Z_k, \qquad t \ge 0; \tag{1.4}$$

4. the sequences $\{X_k, k = 1, 2, \ldots\}$ and $\{Z_k, k = 1, 2, \ldots\}$ and the process $\{\tau_t, t \ge 0\}$ are mutually independent.

Clearly, if each Z_k is degenerate at 1, the model above reduces to the ordinary renewal model. Generally speaking, however, it describes a nonstandard risk model since the random sum (1.1) is equal to

$$S_t = \sum_{k=1}^{Z_1} X_k + \sum_{k=Z_1+1}^{Z_1+Z_2} X_k + \cdots + \sum_{k=Z_1+\cdots+Z_{\tau_t-1}+1}^{Z_1+\cdots+Z_{\tau_t}} X_k = A_1 + A_2 + \cdots + A_{\tau_t},$$

where $\{A_n, n = 1, 2, \ldots\}$, though independent of $\{\tau_t, t \ge 0\}$, is no longer a sequence of i.i.d. random variables.

A reference related to the present model is Denuit et al. (2002), who understood the random variable Z_k above as the occurrence of the kth claim, hence as a Bernoulli variate I_k, $k = 1, 2, \ldots$, and who assumed that the sequence $\{I_k, k = 1, 2, \ldots\}$ follows a certain dependence structure. In this way, the random sum (1.1) is equal to

$$S_t = \sum_{k=1}^{\tau_t} X_k I_k.$$

See also Ng et al. (2004, Section 5.1).

The remaining part of the paper is organized as follows. Section 2 recalls some preliminaries about heavy-tailed distributions; Section 3 establishes the precise large deviations for a standard case where the claim numbers in groups, Z_k, $k = 1, 2, \ldots$, are independent; and, after introducing a kind of negative dependence structure, Section 4 further extends the result to a nonstandard case where the claims numbers Z_k, $k = 1, 2, \ldots$, follow this dependence structure.

2 Distributions of consistent variation

In this paper, we will assume that the common claim size distribution F is heavy tailed. More precisely, we assume that F has a consistent variation, denoted by $F \in \mathcal{C}$. By definition, a distribution function F belongs to the class $\mathcal{C}$ if and only if $\overline{F}(x) > 0$ for all real numbers x and, moreover,

$$\lim_{y \nearrow 1} \limsup_{x\to\infty} \frac{\overline{F}(xy)}{\overline{F}(x)} = 1. \tag{2.1}$$

Discussions and applications of this class can be found, for example, in Cline (1994), Schlegel (1998), Jelenković and Lazar (1999, Section 4.3), Ng et al. (2004), and Tang (2004).

Specifically, the class $\mathcal{C}$ covers the famous class $\mathcal{R}$, which consists of all distribution functions with regularly varying tails in the sense that there is some $\alpha > 0$ such that the relation

$$\lim_{x\to\infty} \frac{\overline{F}(xy)}{\overline{F}(x)} = y^{-\alpha} \tag{2.2}$$

holds for any $y > 0$. We denote by $F \in \mathcal{R}_{-\alpha}$ the regularity property in (2.2).

A simple example to illustrate that the inclusion $\mathcal{R} \subset \mathcal{C}$ is strict is the distribution function of the random variable

$$X = (1+U)2^N,$$

where U and N are independent random variables with U uniformly distributed on $(0,1)$ and N geometrically distributed satisfying $\Pr(N=n) = (1-p)p^n$ for $0<p<1$ and $n = 0, 1, \cdots$; see Kaas et al. (2004) and Cai and Tang (2004).

For a distribution function F and any $y > 0$, as done recently by Tang and Tsitsiashvili (2003), we set

$$\overline{F}_*(y) = \liminf_{x\to\infty} \frac{\overline{F}(xy)}{\overline{F}(x)}$$

and then define

$$J_F^+ = -\lim_{y\to\infty} \frac{\log \overline{F}_*(y)}{\log y}. \tag{2.3}$$

We call J_F^+ the upper Matuszewska index of the distribution function F. For more details of the Matuszewska indices, see Bingham et al. (1987, Chapter 2.1), Cline and Samorodnitsky (1994), as well as Tang and Tsitsiashvili (2003). Clearly, if $F \in \mathcal{C}$ then $J_F^+ < \infty$ and if $F \in \mathcal{R}_{-\alpha}$ then $J_F^+ = \alpha$.

Tang and Tsitsiashvili (2003, Lemma 3.5) proved the following result:

Lemma 2.1. *For a distribution function F with upper Matuszewska index $J_F^+ < \infty$, it holds for any $p > J_F^+$ that $x^{-p} = o(\overline{F}(x))$.*

By this lemma it is easy to see that if $F(x)1_{(0\le x<\infty)}$ has a finite mean then $J_F^+ \geq 1$.

3 A standard case with independent occurrences

3.1 The first main result

Ng et al. (2004) investigated the random sum (1.1) and obtained the following general result:

Proposition 3.1. *Consider the random sum (1.1). If $F \in \mathcal{C}$ and N_t satisfies*

$$\mathrm{E}\left[N_t^p 1_{(N_t > \eta\lambda_t)}\right] = O\left(\lambda_t\right) \tag{3.1}$$

for some $p > J_F^+$ and all $\eta > 1$, then for any fixed $\gamma > 0$, the precise large deviation result (1.2) holds uniformly for $x \geq \gamma\lambda_t$.

Now we state the first main result of this paper.

Theorem 3.1. *Consider the compound model introduced in Section 1. In addition to the assumptions made there, we assume that the claim numbers Z_k, $k = 1, 2, \ldots$, are independent. If the claim size distribution $F \in \mathcal{C}$ and*

$$\mathrm{E}\left[Z_1^p\right] < \infty \tag{3.2}$$

for some $p > J_F^+$, then for any fixed $\gamma > 0$, the precise large deviation result (1.2) holds uniformly for $x \geq \gamma\lambda_t$.

As can easily be seen, Theorem 3.1 above improves Theorems 2.3 and 2.4 of Tang et al. (2001) in several directions.

3.2 Two lemmas

Before giving the proof of Theorem 3.1, we need some preliminaries. First we show, in the spirit of Fuk and Nagaev (1971) (see also Nagaev (1976) for additional erratum and extension), a general inequality for the tail probability of sums of i.i.d. random variables.

Lemma 3.1. *Let $\{Z_k, k = 1, 2, \ldots\}$ be a sequence of i.i.d. nonnegative random variables with $\mathrm{E}\left[Z_1^p\right] < \infty$ for some $p \geq 1$. Then for any $\gamma > \mathrm{E}\left[Z_1\right]$, there is some $C > 0$ irrespective to x and m such that for all $m = 1, 2, \ldots$ and $x \geq \gamma m$,*

$$\Pr\left(\sum_{k=1}^{m} Z_k > x\right) \leq Cmx^{-p}. \tag{3.3}$$

Proof. For the case $p = 1$, (3.3) is a trivial consequence of Chebyshev's inequality. Then, we assume $p > 1$ and define $\tilde{p} = \min\{p, 2\}$. With an arbitrarily fixed constant $v > 0$, by Theorem 2 of Fuk and Nagaev (1971) we obtain

$$\Pr\left(\sum_{k=1}^{m} Z_k > x\right) \leq m \Pr\left(Z_1 > x/v\right) + P_v(x) \tag{3.4}$$

with

$$P_v(x) = \exp\left\{ v - \frac{x - m\mathrm{E}\left[Z_1 1_{(Z_1 \le x/v)}\right]}{x/v} \log\left(\frac{x(x/v)^{\tilde{p}-1}}{m\mathrm{E}\left[Z_1^{\tilde{p}} 1_{(Z_1 \le x/v)}\right]} + 1 \right) \right\}.$$

Since $x \geq \gamma m$ and $\gamma > \mathrm{E}[Z_1]$, some simple calculation leads to

$$P_v(x) \le \mathrm{e}^v \left(\frac{\gamma}{v^{\tilde{p}-1}\mathrm{E}\left[Z_1^{\tilde{p}}\right]} \right)^{-v(1-\mathrm{E}[Z_1]/\gamma)} x^{-(\tilde{p}-1)v(1-\mathrm{E}[Z_1]/\gamma)}.$$

It follows that for all large v, say $v \ge v_0 > 0$,

$$P_v(x) = o\left(x^{-p}\right).$$

For the first term on the right-hand side of (3.4), by Chebyshev's inequality, it holds that

$$m \Pr\left(Z_1 > x/v\right) \le (x/v)^{-p} m\mathrm{E}\left[Z_1^p\right].$$

This proves that inequality (3.3) holds for some constant $C > 0$. □

For any fixed $t > 0$, the random variable τ_t defined by (1.3) has certain finite exponential moments, hence has finite moments of all orders; see Stein (1946). We reformulate Lemma 3.5 of Tang et al. (2001) below.

Lemma 3.2. *Let $\{\tau_t, t \ge 0\}$ be a renewal counting process defined by (1.3). Then for any $p > 0$ and $\eta > 1$,*

$$\sum_{m > \eta\mathrm{E}[\tau_t]} m^p \Pr\left(\tau_t \ge m\right) = o(1).$$

3.3 Proof of Theorem 3.1

In view of Proposition 3.1, it suffices to prove that N_t satisfies assumption (3.1). For this purpose we recall an elementary inequality that for any real numbers a_1, a_2, ..., any $m = 1, 2, \ldots$ and any $r \ge 0$,

$$\left| \sum_{k=1}^{m} a_k \right|^r \le \max\{m^{r-1}, 1\} \sum_{k=1}^{m} |a_k|^r. \tag{3.5}$$

By this inequality and relation (1.4), one easily checks that for the number p given in (3.2) and for each $t > 0$,

$$\mathrm{E}\left[N_t^p\right] \le \mathrm{E}\left[\tau_t^p\right] \mathrm{E}\left[Z_1^p\right] < \infty. \tag{3.6}$$

Denote by Δ the forward difference operator and by $\lfloor a \rfloor$ the largest integer that is not larger than a. In view of (3.6), for any $\eta > 1$ and $p_1 \in (J_F^+, p)$, we can use summation by parts to obtain

$$\begin{aligned}
\mathrm{E}\left[N_t^{p_1} 1_{(N_t>\eta\lambda_t)}\right] &= \sum_{n>\eta\lambda_t} n^{p_1} \Pr(N_t = n) \\
&= -\sum_{n>\eta\lambda_t} n^{p_1} \Delta \Pr(N_t \geq n) \\
&= \sum_{n>\eta\lambda_t} \Pr(N_t \geq n) \Delta n^{p_1} + (\lfloor \eta\lambda_t \rfloor + 1)^{p_1} \Pr(N_t \geq \lfloor \eta\lambda_t \rfloor + 1) \\
&= I_1(t) + I_2(t). \qquad (3.7)
\end{aligned}$$

Recalling relation (1.4), we deal with $I_1(t)$ as

$$\begin{aligned}
I_1(t) &\sim p_1 \sum_{n>\eta\lambda_t} n^{p_1-1} \Pr(N_t \geq n) \\
&= p_1 \sum_{n>\eta\lambda_t} n^{p_1-1} \left(\sum_{0\leq m\leq \sqrt{\eta}\mathrm{E}[\tau_t]} + \sum_{m>\sqrt{\eta}\mathrm{E}[\tau_t]} \right) \Pr\left(\sum_{k=1}^{m} Z_k \geq n, \tau_t = m \right) \\
&= I_{11}(t) + I_{12}(t).
\end{aligned}$$

Note that

$$\lambda_t = \mathrm{E}[N_t] = \mathrm{E}[Z_1]\,\mathrm{E}[\tau_t]. \qquad (3.8)$$

By Lemma 3.1, we know that for some constant $C_1 > 0$,

$$\begin{aligned}
I_{11}(t) &\leq p_1 \sum_{n>\eta\lambda_t} n^{p_1-1} \Pr\left(\sum_{1\leq k\leq \sqrt{\eta}\mathrm{E}[\tau_t]} Z_k \geq n \right) \\
&\leq C_1 \mathrm{E}[\tau_t] \sum_{n>\eta\lambda_t} n^{p_1-1} n^{-p} = o(\mathrm{E}[\tau_t]) = o(\lambda_t).
\end{aligned}$$

Successively applying Chebyshev's inequality and (3.5), we obtain that

$$\begin{aligned}
I_{12}(t) &= p_1 \sum_{n>\eta\lambda_t} n^{p_1-1} \sum_{m>\sqrt{\eta}\mathrm{E}[\tau_t]} \Pr\left(\sum_{k=1}^{m} Z_k \geq n \right) \Pr(\tau_t = m) \\
&\leq p_1 \sum_{n>\eta\lambda_t} n^{p_1-1-p} \sum_{m>\sqrt{\eta}\mathrm{E}[\tau_t]} \mathrm{E}\left[\sum_{k=1}^{m} Z_k \right]^p \Pr(\tau_t = m) \\
&\leq p_1 \mathrm{E}[Z_1^p] \sum_{n>\eta\lambda_t} n^{p_1-1-p} \sum_{m>\sqrt{\eta}\mathrm{E}[\tau_t]} m^p \Pr(\tau_t = m).
\end{aligned}$$

Hence by Lemma 3.2, $I_{12}(t) = o(1)$. This proves that

$$I_1(t) = o(\lambda_t). \qquad (3.9)$$

Now we turn to $I_2(t)$. Analogously, for some $C_2 > 0$,

$$\begin{aligned} I_2(t) &\le C_2\lambda_t^{p_1}\left(\sum_{0\le m\le\sqrt{\eta}\mathrm{E}[\tau_t]} + \sum_{m>\sqrt{\eta}\mathrm{E}[\tau_t]}\right)\Pr\left(\sum_{k=1}^{m} Z_k \ge \eta\lambda_t\right)\Pr\left(\tau_t = m\right) \\ &= I_{21}(t) + I_{22}(t). \end{aligned}$$

Again by Lemma 3.1 and relation (3.8), we have for some $C_3 > 0$,

$$I_{21}(t) \le C_3\lambda_t^{p_1-p}\sum_{0\le m\le\sqrt{\eta}\mathrm{E}[\tau_t]} m\Pr\left(\tau_t = m\right) = o(\mathrm{E}\left[\tau_t\right]) = o(\lambda_t).$$

Since τ_t has finite moments of all orders, by Chebyshev's inequality and relation (3.8), one easily sees that

$$I_{22}(t) \le C_2\lambda_t^{p_1}\Pr\left(\tau_t > \sqrt{\eta}\mathrm{E}\left[\tau_t\right]\right) = o(1).$$

This proves that

$$I_2(t) = o\left(\lambda_t\right). \tag{3.10}$$

Plugging (3.9) and (3.10) into (3.7) we finally obtain that

$$\mathrm{E}\left[N_t^{p_1}1_{(N_t>\eta\lambda_t)}\right] = o(\lambda_t).$$

Hence, N_t satisfies assumption (3.1) with $p_1 > J_F^+$ replacing p. This ends the proof of Theorem 3.1. □

4 A nonstandard case with dependent occurrences

4.1 An equivalent statement of assumption (3.1)

As we have seen in Section 3, the proof of Theorem 3.1 heavily depends on the independence assumptions made. In the following result, we rewrite the left-hand side of (3.1) as the expectation of a nondecreasing and convex function of N_t. This enables us to check some nonstandard cases by using the well-developed stochastic ordering theory.

Lemma 4.1. *Let $\{N_t, t \ge 0\}$ be a nonnegative process with mean function $\lambda_t = \mathrm{E}\left[N_t\right]$, which satisfies $\lambda_t < \infty$ for any $t \ge 0$ and $\lambda_t \to \infty$. Then for any fixed $p > 0$, the following two assertions are equivalent:*

A. for any $\eta > 1$,

$$\mathrm{E}\left[N_t^{p}1_{(N_t>\eta\lambda_t)}\right] = O\left(\lambda_t\right); \tag{4.1}$$

B. for any $\eta > 1$,

$$\mathrm{E}\left[(N_t - \eta\lambda_t)_+^{p}\right] = O\left(\lambda_t\right). \tag{4.2}$$

Proof. The proof of the implication $A \Longrightarrow B$ is trivial since

$$\mathrm{E}\left[(N_t - \eta\lambda_t)_+^p\right] \le \mathrm{E}\left[N_t^p 1_{(N_t > \eta\lambda_t)}\right].$$

To verify the other implication $B \Longrightarrow A$, let $\eta > 1$ be arbitrarily fixed. We derive that

$$\begin{aligned}
\mathrm{E}\left[(N_t - \sqrt{\eta}\lambda_t)_+^p\right] &= \mathrm{E}\left[(N_t - \sqrt{\eta}\lambda_t)^p \left(1_{(N_t > \eta\lambda_t)} + 1_{\left(\eta\lambda_t \ge N_t > \sqrt{\eta}\lambda_t\right)}\right)\right] \\
&\ge \mathrm{E}\left[(N_t - \sqrt{\eta}\lambda_t)^p \, 1_{(N_t > \eta\lambda_t)}\right] \\
&\ge (1 - \sqrt{\eta}/\eta)^p \, \mathrm{E}\left[N_t^p 1_{(N_t > \eta\lambda_t)}\right].
\end{aligned}$$

Since by condition B the left-hand side of the above is $O(\lambda_t)$, we immediately obtain relation (4.1) with $\eta > 1$ being arbitrarily given. This ends the proof of Lemma 4.1. □

4.2 Negative cumulative dependence and convex order

Recently, Denuit et al. (2001) extended the notion of bivariate positive quadrant dependence (PQD) to arbitrary dimension by introducing the notion of positive cumulative dependence (PCD). The analysis there indicates that PCD can well keep the intuitive meaning of PQD.

In a similar fashion, we introduce here the notion of negative cumulative dependence (NCD) as follows. Let $\{Z_1, Z_2, \ldots, Z_m\}$ be a sequence of random variables. For $\mathcal{I} \subset \{1, 2, \ldots, m\}$, we denote by $S_{\mathcal{I}}$ the sum of the random variables Z_k whose indices are in the set $\mathcal{I}$. We say that the family of random variables $\{Z_1, Z_2, \ldots, Z_m\}$ is NCD if for any $\mathcal{I} \subset \{1, 2, \ldots, m\}$ and $k \in \{1, 2, \ldots, m\} - \mathcal{I}$, the inequality

$$\Pr(S_{\mathcal{I}} > x_1, Z_k > x_2) \le \Pr(S_{\mathcal{I}} > x_1) \Pr(Z_k > x_2)$$

holds for all real numbers x_1 and x_2. We say that an infinite family of random variables $\{Z_k, k = 1, 2, \ldots\}$ is NCD if each of its finite subfamilies is NCD.

Given two random variables Y_1 and Y_2, we say Y_1 precedes Y_2 in the stop-loss order, written as $Y_1 \le_{sl} Y_2$, if the inequality

$$\mathrm{E}\left[\phi(Y_1)\right] \le \mathrm{E}\left[\phi(Y_2)\right] \tag{4.3}$$

holds for all nondecreasing and convex functions ϕ for which the expectations exist. It is worth mentioning that $Y_1 \le_{sl} Y_2$ and $\mathrm{E}[Y_1] = \mathrm{E}[Y_2]$ if and only if inequality (4.3) holds for all convex functions ϕ for which the expectations exist. Reviews on the stochastic ordering can be found in Dhaene et al. (2002a,b).

As usual, we write by $\{Z_k^{\perp}, k = 1, 2, \ldots\}$ the independent version of the sequence $\{Z_k, k = 1, 2, \ldots\}$, that is, the random variables $Z_k^{\perp}$, $k = 1, 2, \ldots$, are mutually independent and for each $k \in \{1, 2, \ldots\}$ the random variables $Z_k^{\perp}$ and Z_k have the same marginal distribution. We have the following result:

Lemma 4.2. *Let $\{Z_k, k = 1, 2, \ldots\}$ be a sequence of NCD random variables and let $\{Z_k^{\perp}, k = 1, 2, \ldots\}$ be its independent version. Then the inequality*

$$\sum_{k=1}^{m} Z_k \leq_{sl} \sum_{k=1}^{m} Z_k^{\perp}$$

holds for each $m = 1, 2, \ldots$.

Proof. The proof for $m = 2$ can be given in a similar way as the proof of Theorem 2 of Dhaene and Goovaerts (1996). The remainder of the proof can be given by proceeding along the same lines as in the proof of Theorem 3.1 of Denuit et al. (2001), only changing the directions of some inequalities. □

4.3 The second main result and its proof

Now we are ready to state the second main result of this paper.

Theorem 4.1. *Consider the compound model introduced in Section 1. In addition to the assumptions made there, we assume that the claim numbers Z_k, $k = 1, 2, \ldots$, are NCD. If $F \in \mathcal{C}$ and (3.2) holds for some $p > J_F^+$, then for any fixed $\gamma > 0$, the precise large deviation result (1.2) holds uniformly for $x \geq \gamma\lambda_t$.*

Proof. As done in the proof of Theorem 3.1, it suffices to prove that N_t satisfies assumption (3.1). By Lemma 4.1, this amounts to proving that (4.2) holds for some $p > J_F^+$ and all $\eta > 1$. Recall (1.4) and Lemma 4.2. We have

$$\begin{aligned} \mathrm{E}\left[(N_t - \eta\lambda_t)_+\right]^p &= \sum_{m=1}^{\infty} \mathrm{E}\left[\left(\sum_{k=1}^{m} Z_k - \eta\lambda_t\right)_+^p\right] \Pr(\tau_t = m) \\ &\leq \sum_{m=1}^{\infty} \mathrm{E}\left[\left(\sum_{k=1}^{m} Z_k^{\perp} - \eta\lambda_t\right)_+^p\right] \Pr(\tau_t = m) \\ &= \mathrm{E}\left[\left(N_t^{\perp} - \eta\lambda_t\right)_+^p\right], \end{aligned} \tag{4.4}$$

where $N_t^{\perp} = \sum_{k=1}^{\tau_t} Z_k^{\perp}$ with $\{Z_k^{\perp}, k = 1, 2, \ldots\}$ and $\{\tau_t, t \geq 0\}$ independent. The proof of Theorem 3.1 has shown that the relation

$$\mathrm{E}\left[\left(N_t^{\perp}\right)^p 1_{(N_t > \eta\lambda_t)}\right] = O(\lambda_t)$$

holds for some $p > J_F^+$ and all $\eta > 1$. Thus, applying Lemma 4.1 once again, the relation

$$\mathrm{E}\left[\left(N_t^{\perp} - \eta\lambda_t\right)_+^p\right] = O(\lambda_t)$$

holds for some $p > J_F^+$ and all $\eta > 1$. By (4.4) we conclude that, as desired, the relation

$$\mathrm{E}\left[(N_t - \eta\lambda_t)_+^p\right] = O\left(\lambda_t\right)$$

holds for some $p > J_F^+$ and all $\eta > 1$. This ends the proof of Theorem 4.1. □

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