APPLICATION of SENSITIVITY ANALYSIS

PARAMETER CHANGES
in
NONLINEAR HYDRAULIC CONTROL SYSTEMS

Said Farahat

A Thesis

in

The Departement

of

Mechanical Engineering

Presented in Partial Fulfillment of the Requirements
for the Degree of Master of Engineering at
Concordia University
Montreal, Quebec, Canada

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ABSTRACT

Application of Sensitivity Analysis
to
Parameter Changes
in
Nonlinear Hydraulic Control Systems

Said Farahat

In this thesis, the sensitivity analysis is applied to a novel electrohydraulic servovalve which is a nonlinear system. This system sensitivity study differs from previous studies by considering the dynamic behaviour and nonlinearity of the system performance. Four different sensitivity analysis methods are compared to each other by studying the sensitivity of the actuator piston velocity of above servovalve with respect to eighteen parameters. All the methods show that the area of the actuator piston is the most sensitive parameter and the static friction is the insensitive parameter.

By using the best method among the above mentioned methods, the sensitivity of the state variables of the sample system (other than the velocity of the actuator piston) have been studied. It is shown that the sensitivity of the actuator piston velocity and the opening area of the servovalve in one hand and pressures of the either sides of the actuator piston

in the other hand are almost similar. Also these pressures are very sensitive to the orifice opening, although two other state variables almost are not sensitive to this parameter.

Having studied the system sensitivity with nominal value of the parameters, the most five sensitive parameters (K_a, M, A_p, C_d, V_{in}) have been chosen for locating insensitive system. For this purpose one of these parameters has been changed at a time and the system sensitivities behaviour with respect to different parameters have been evaluated. Study of these behaviours shows that most of them are almost linear except for the system sensitivity with respect to the parameter Mass (total mass in motion) according to the parameter A_p (area of the actuator piston) changes. This behaviour increases dramatically by decreasing the parameter A_p, and it seems it will become unstable by decreasing A_p more than 50%.

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NOMENCLATURE

A_o	- orifice opening of metering valve (m^2)
A_p	- actuator piston area (m^2)
a_v	- servovalve opening (m^2)
C_d	= flow discharge coefficient
C_ℓ	- leakage coefficient
f	- system state model function
F_f	- coulomb friction (N)
F_d	- dynamic coulomb friction (N)
F_s	- static coulomb friction (N)
F_{sum}	- total applied force on piston (N)
i	- servovalve current (mA)
I_E	- Entirety-Index
I_{dE}	- Sensitivity of Entirety-Index (Double Sensitivity Index)
K_a	- servovalve amplifier gain (mA/V)
K_f	- velocity feedback gain (V/m/s)
K_v	- servovalve area constant (m^2/m)
K_x	- servovalve torque motor constant (m/mA)
M	- total mass in motion (Kg)
$P_{1,2}$	- pressures in actuator chambers (N/m^2)
P_s	- supply pressure (N/m^2)
P_r	- relief valve setting pressure (N/m^2) (back pressure)

$q_{1,2}$	=	flows rate in/out of the actuator (m^3/s)
q_{ℓ}	=	leakage flow (m^3/s)
q_{ss}	=	steady state flow (m^3/s)
t	=	time (s)
t_d	=	delay of direction control valve (s)
T	=	period of square wave input signal (s)
$\bar{u}$	=	input vector
u_p	=	actuator piston velocity (m/s)
$v_{1,2}$	=	volumes of oil under compression (m^3)
$v_{s,r}$	=	supply/return pipes and chamber oil volumes (m^3)
V_0	=	piston reference velocity (m/s)
V_f	=	velocity feedback voltage (Volt)
V_i	=	input signal voltage (Volt)
$\bar{x}$	=	state vector
y	=	load displacement (m)
α	=	system parameter
β	=	bulk modulus of fluid (N/m^2)
γ	=	double sensitivity function
λ	=	sensitivity function
Φ	=	objective function
ρ	=	density of the oil (Kg/m^3)
τ_v	=	servovalve time constant (s)

To my Parents

Who have encouraged me during the whole period of my education

and

To the people of the

Islamic Republic of IRAN

CHAPTER 1

Introduction

1.1 Historical Review

1.1.1 Control Systems

Before World War II, the design of control systems was primarily an art. During and after the war, considerable effort was expended on the design of closed-loop feedback control systems, and negative feedback was used to improve performance and accuracy. The first theoretical tools used were based upon the work of Bode and Nyquist. In particular, concepts such as frequency response, bandwidth, gain (in decibels), and phase margin were used to design servomechanisms in the frequency domain in a more or less trial-and error fashion. This was, in a sense, the beginning of modern automatic-control engineering.

The theory of servomechanisms developed rapidly from the end of the war to the beginning of the fifties. Time-domain criteria, such as rise time, settling time, and peak overshoot" ratio, were commonly used, and the introduction of the "root-locus" method by Evans in 1948 provided both a bridge between the time- and frequency-domain methods and a significant new design tool. During this period, the primary concern of the control engineer was the design of linear servomechanisms. Slight nonlinearities in the plant and in the power-amplifying elements could be tolerated since the use of negative feedback made the system response relatively insensitive to variations and disturbances.

The competitive era of rapid technological change and aerospace exploration which began around mid-century generated stringent accuracy and cost requirements as well as an interest in nonlinear control systems, particularly relay (bistable) control systems. This is not surprising, since the relay is an exceedingly simple and rugged power amplifier. Two approaches, namely, the "describing-function" and "phase-space methods", were used to meet the new design challenge. The describing-function method enabled the engineer to examine the stability of a closed-loop nonlinear system from a frequency-domain point of view, while the phase-space method enabled the engineer to design nonlinear control systems in the time domain.

1.1.2 Sensitivity

Sensitivity considerations have long been of concern in connection with dynamic systems. The study of the influence of the coefficients of differential equation on its solution started with the origins of differential equation. However, for a long period of time, those considerations were merely of mathematical interest [22].

This situation has changed basically with the development of the highly-sophisticated methods of modern control theory and their applications by engineers. Historically, sensitivity considerations have provided a fundamental motivation for the use of feedback and are largely responsible for its development into what is called modern control theory, implying the principles of optimization and adaptation.

Therefore, it is quite natural that the basic concepts in this area were already given in the fundamental literature on feedback control systems forty years ago. Bode [5] was the first to establish the significance of sensitivity in the design of the feedback control systems. He has introduced a proper sensitivity definition on the basis of the frequency domain.

In its subsequent development it seemed that automatic control theory should include the study of sensitivity as an essential component. However, with few exceptions, the sensitivity problem was not even discussed in the academic texts on automatic control in the following decade. It was mainly the problem of accuracy in network-analyzers and analog computers that gave new impulses to the theory of sensitivity during the fifties [43]. Many basic methods were also worked out in connection with the design of electronic networks [6,9]. Toward the end of this period the ideas of Bode were rediscovered in control engineering with the appearance of adaptive systems, more precisely, as a reaction to their appearance. Horowitz [27] has developed the methods of frequency domain to a high extent and has applied them with great success to design of low sensitivity conventional feedback control systems (see also Horowitz [26]).

Beginning in the period 1958-1960, the number of publications devoted to sensitivity considerations in the time domain rose considerably due to the development of state space methods in control engineering and the availability of the digital computer. This also gave rise to a new interest in the general sensitivity problem in automatic control systems with an overwhelming number of papers [32,45] and even some book publications [12,62,63]. In particular, the essential contributions of Kokotović and co-workers [33,34], Perkins and Cruz [15,47-50] and Kreindler [35-37] should be mentioned in this connection.

In 1963 Dorato [19] called attention to the problem of parameter sensitivity of the performance index of optimal control systems. In the sequel, many papers were published clarifying certain unexpected problems emerging from this definition of sensitivity [46,55,58,67,68]. Excellent reviews of the significant publications of that period are given in Kokotović and Rutman [32] and Nuguyen Thuong Ngo [45].

1.1.3 Optimality

Optimization--finding the best way to do things--is obviously of interest in the practical world of production, trade, and politics, where small changes in efficiency can spell the difference between success or disaster for any enterprise, be it neighborhood store, mammoth industrial complex, or governing political party. Today as always many important decisions are made simply by describing the system under study as precisely and quantitatively as possible, selecting some measure of system effectiveness, and then seeking the state of the system which gives the most desirable value of this criterion. Since description and understanding of systems is the traditional task of the engineers, economists, and other applied scientists, hence optimal control is one of the most active research areas of modern technology. It is applied in a variety of fields, such as aerospace, chemical industry, nuclear reactors, transportation and many others. A considerable number of textbooks and monographs covering this subject has appeared during the past ten years. One of the major practical problems in this field is the numerical solution of optimal control problems. Numerous techniques for solving this type of problems have been developed recently.

Over a span of almost two centuries, the only mathematical methods known for handling optimization problems were the classical differential and variational calculus. With the rise of "operation research" since the Second World War, there has been renewed interest in optimization methods for dealing with problems not solvable by classical methods. During the years following war a powerful techniques for solving finite dimensional optimization problems was developed; namely, mathematical programming. Initially it was applied mainly in operations research problems. Only during the past nine years some effort was done in applying mathematical programing techniques in numerical solutions of optimal control problems. It was demonstrated in many practical cases that a numerical solution, utilizing mathematical programing, was obtained where other methods failed. The material covering the work in applying mathematical programing to optimal control problems is scattered in various journals, theses and reports.

During the past ten years a great deal of research activity has been witnessed in the field of optimal control. As a result, numerous papers and a number of books [5,6,10,11,13,14] have been published on the subject of optimization techniques. The quantitative design of control systems is no longer a trial-and error effort but rather a precise science involving applied mathematics and high-speed computers. During the early stage of development, control-system studies were characterized by such tools as stability analysis, frequency response, root locus, phase plane, and describing function. These methods, though widely known and of practical use, can be applied only to non-stringent design problems involving single-variable systems, time-invariant systems, and systems with unconstrained variables.

With the advent of the space age, the control engineer is faced with the challenge of designing a great variety of systems having stringent requirements. Common interests have also been found in the design of social systems, nuclear reactor systems, and transportation systems, which require a considerable degree of sophistication in control theory and technology. Some of the more rigid conditions and requirements which characterize these modern systems are multiple input-output, constraints on state and input variables, stochastic systems, unknown or time-varying parameters, large-scale systems, and time-delay systems.

Minimum-time control laws (in terms of switch curves and surfaces) were obtained for a variety of second- and third-order systems in the early fifties. Proofs of optimality were more or less heuristic and geometric in nature. However, the idea of determining an optimum system with respect to a specific performance measure, the response time, was very appealing; in addition, the precise formulation of the problem attracted the interest of the mathematician.

The time-optimal control problem was extensively studied by mathematicians in United States and the Soviet Union. In the period from 1953 to 1957, Bellman, Gamkrelidze, Krasovskii, and LaSalle developed the basic theory of minimum-time problems and presented results concerning the existence, uniqueness, and general properties of the time-optimal control. The recognition that control problems were essentially problems in the calculus of variations soon followed.

1.2 Survey of Previous Works

1.2.1 Sensitivity

The sensitivity of a dynamic system to variations of its parameters is one of the basic aspects in the of dynamic systems. The question of parameter sensitivity particularly arises in the fields of engineering where mathematical models are used for the purposes of analysis and synthesis [22]. In order to be able to give a unique formulation of the mathematical problem, the mathematical model is usually assumed to be known exactly. This assumption is strictly speaking, unrealistic since there is always a certain discrepancy between the actual system and its mathematical model. This is due to the following reasons :

(1) A real system cannot be identified exactly because of the restricted accuracy of the measuring devices.

(2) A theoretical concept cannot be implemented exactly because of manufacturing tolerances.

(3) The behaviour of any real system changes with time in an often unpredictable way caused by environmental, material property, or operational influences.

(4) Mathematical models are often simplified or idealized intentionally in order to simplify the mathematical problem or to make it solvable at all.

For these reasons the results of mathematical syntheses need not necessarily be practicable. They may even be very poor, e.g., if there are considerable parameter deviations between the real system and the mathematical model and the solution is very sensitive to the parameters. Therefore, it should be part of the solution to a practical problem to know the parameter sensitivity prior to its implementation or to reduce the sensitivity systematically if this turns out to be necessary.

This is of particular importance if optimization procedures are involved, since it is in the nature of optimization to extremize a certain performance index for the special set of parameters. Furthermore, there are many other problems where sensitivity considerations are either useful or mandatory. Some examples are the application of gradient methods, adaptive and self-learning systems, the design of insensitive and suboptimal control system, the determination of allowed tolerance in the design of networks, the calculation of optimal input signals for parameter identification, analog and digital simulation of dynamic systems, and so forth.

The essential ideas of sensitivity hitherto published can be traced back to a few principles and basic concepts of a general theory, called sensitivity theory [22]. This theory can be seen as parallel to the signal theory already well developed for dynamic systems. Thus, sensitivity theory can be interpreted as a section of a general system theory, taking into account parameter variation as inputs instead of signals. It is the major objective of this section to take the initiative in setting up and introducing such a general sensitivity theory, which can also be applied to fields other than technical ones such as economics or social sciences.

As in the case of signal theory, it is useful to subdivide sensitivity theory into two categories: sensitivity analysis and synthesis. Sensitivity analysis provides the basic methods to study the sensitivity of a system to parameter variations. On the other hand, sensitivity synthesis is defined as the design of dynamic system, especially feedback systems, with due regard to sensitivity specifications, say, to obtain minimal or (in some cases) maximal sensitivity to parameter variations.

In order to outline the sensitivity analysis in more detail, recall that, in general, the dynamics of a system can be represented by a single block (Fig. 1.1), which will, for short,

Fig. 1.1 General representation of a dynamic system.

be called the system. From a mathematical point of view what we call a system is the explicitly or implicitly given relationship between the input signal u(t) and the output signal x(t). In general, u(t) and x(t) can be vectors. The character of this relationship is commonly called the structure of the system. For example, the structure of the system may be characterized by [22]:

- -the order of differential or difference equation,
- -linearity or nonlinearity,
- -the order of the numerator and denominator of a rational transfer function,
- -the rationality or irrationality of transfer function.

The quantitative properties of the system are characterized by the system parameters. Typical parameters are :

-initial conditions,
-time-invariant or time-variant coefficients,
-natural frequencies, pulse frequencies,
-sampling periods, sampling instants,
-pulse width or magnitude,
-dead times (or time delays).

Dynamic processes in a system, say, the change of the state or the output variable with time, can be caused (Fig. 1.2) by :

(1) the influence of input signals,

(2) the change of parameters.

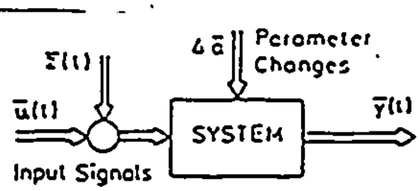

Fig. 1.2 Quantities affecting the dynamics of a system.

While studying the influence of input signals, the dynamic of the system are usually considered only as a function of the input signal, assuming that the relationship is qualitatively and quantitatively unchanged. This is the subject matter of conventional system theory.

While studying the influence of parameters, the dynamics of the system are considered as a function of changes in the parameters (or of the structure of the system, because the change of system parameters can also change the system structure). The dependence of the system dynamics on the parameters is called sensitivity. Strictly, parameter sensitivity can be defined as follows [22]:

Definition : parameter sensitivity is the effect of parameter changes on the dynamics of a system, the time response, the state, the transfer function, or any other quantity characterizing the system dynamics.

With regard to the mathematical treatment of the sensitivity problem, it is useful to distinguish between two types of parameter deviations:

(1) errors and tolerances of the underlying mathematical model (those parameter changes that are time-invariant), and also varying (quasi-constant) parameters; and

(2) changes in the parameters with time.

Parameter changes of the first category can be caused by :

-tolerances of manufacturing (When realizing a system)
-measurement errors (when identifying a system),
-approximations up the mathematical model,
-seasoning of elements (erosion, abrasion, wear, etc.).

Parameter changes of the second category can be caused by :

-seasoning of elements (erosion, abrasion, wear, etc.),
-changes in environmental conditions (temperature, humidity, gravitation, etc.)
-changes in operation conditions (load changes, change of inertia by fuel consumption, influence ofnonlinearities, etc.)

Either of the two categories of parameter changes requires its own methods of treatment.

Parameter changes of both categories appear in any engineering system. Thus, sensitivity analysis can be regarded, along with signal analysis, as a necessary tool in the treatment of engineering systems.

1.2.2 Optimization theory

When an individual is confronted with a problem, he must progress through an alternating sequence of evaluations and decisions. Greber [25] lists six cardinal steps on which evaluations and decisions are made in the solution of engineering problems, namely,

1. Recognition of need.
2. Formulation of the problem.
3. Resolving the problem into concepts that suggest a solution.
4. Finding elements for the solution.
5. Synthesizing the solution.
6. Simplifying and optimising the solution.

The order in which these steps are followed can differ considerably from one problem to another. Insight gained at any given step may be employed to modify conclusions of other steps : We should visualize a set of feedback paths which allow transition from any step to any preceding step in accordance with the dictates of a given problem. For example, the step of "synthesizing the solution" or the "formulation of the problem" may be modified or augmented by considerations associated with "simplifying and optimising the solutions."

Problems are generally associated with physical things: without a thorough understanding of the physical principles upon which a given problem solution depends, the application of optimization principles is of dubious value. There is no substitute for knowledge of physical principles and devices, nor is there any substitute for an inventive idea. The ideal role that optimization plays--in the solution of problems is evidenced in the following statement: After constraints that must be satisfied by the problem solution are defined, either directly or indirectly, all significant forms of solution which satisfy the constraints should be conceived; and from the generally infinite number of such solutions, the one or ones which are best under some criteria of goodness should be extracted by using optimization principles. As with most ideals of optimization are not easily achieved: the identification of all significant forms of solution to a given problem can be accomplished in special cases only, and limitations on time available to produce the solution to a given problem are always present. Thus, the good designer or manager does the best that he can, all factors considered.

Problems which involve the operation or the design of systems are generally of the type to which optimization principles can be beneficially applied. Moreover, problems of analysis can be viewed as optimization problems, albeit trivial ones: for example, if a linear circuit is given with specified voltage and current sources and specified initial conditions, the problem of finding the current distribution in the circuit as a function of time admits to a unique solution, and we could say in such cases that the unique solution is the optimal solution.

Whenever we use "best" or "optimum" to describe a system, the immediate question to be asked is, "Best with respect to what criteria and subject to what limitations?" Given a specific measure of performance and a specific set of constraints, we can designate a system as optimum (with respect to the performance measure and the constraints) if it "performs" as well as, if not better than, any other system which satisfies the constraints. The term suboptimum is used to describe any system which is not optimum (with respect to the given performance measure and constraints). Specific uses of the term suboptimum vary. It can be used in reference to systems which are not optimum because of parameter variations, or in reference to systems which are not optimum because they are designed to satisfy additional constraints, or in reference to any system which ~~is~~ to be compared to a reference optimum one.

Great advances have been made in optimization theory since 1940. For example, almost all of the material in this section has been developed since that time. In the words of Athans [3], "At the present time, the field of optimization has reached a certain state of maturity, and it is regarded as one of the areas of most fervent research." The one factor that has influenced this rapid growth of optimization theory more than any other has been the parallel development of computer equipment with which optimization theory can be applied to broad classes of problems. In the remainder of this section, we examine the general nature of problems that are treated and those aspects of optimization that prevail throughout different literatures.

performance measure

To design or plan something so that it is best in some sense, is to use optimization. The sense in which the something (e.g., a system) is best, or is to be best, is a very pertinent factor. Then term performance measure is used here to denote that which is to be maximized or minimized (to be extremized). Other terms are used in this regard, e.g., objective function, performance index, performance criterion, cost function, return function, and figure of merit.

Constraints

Any relationship that must be satisfied is a constraint. Constraints are classified either as equality constraints or as inequality constraints. Arguments of constraint relationships are related in some well-defined fashion to arguments of corresponding performance measures. Thus, if a particular performance measure depends on parameters and functions to be selected for the optimum, the associated constraints depend, either directly or indirectly, on at least some of the same parameters and functions. Constraints limit the set of solutions from which an optimal solution is to be found.

Constraint also arise from the operating environment of physical systems; for example, a physical system must operate satisfactorily over some specified range of temperatures and must be able to withstand some degree of vibrational stress. An important aspect of optimization is the sensitivity of system performance with respect to environmental changes or uncertainties in the parameters and factors that characterize the system. Thus, we may wish to include certain constraints in a design for the sole purpose of obtaining an assured degree of insensitivity in the system.

Optimization problems

It is easy to categorize optimization problems according to mathematical characteristics, as is done in this section, but it should be clearly understood that any problem associated with a physical system generally fits into one of several classes, depending on the assumptions and approximations that are made in mathematical characterizations of the system and its associated performance measure. We often form a given system model so that a convenient type of analysis is particularly appropriate; we should always go back to the actual system (or to a more realistic model) to check results obtained. Moreover, even if the mathematical structure of a problem is precisely defined, the solution of the problem is generally approachable by use of several different optimization techniques, each of which has relative advantages. As graphically illustrated by Mulligan [44], therefore, we should not tie a given problem form too rigidly to a single optimization technique. Similarly, we should not artificially limit a given approach to solution to a particular problem type.

Approaches to solution

A classical approach to solution of optimization problems is the following [51]:

(1) find necessary conditions that the optimum must satisfy by using differential properties of certain optimal solutions;
(2) solve the equations that constitute the necessary conditions to obtain candidates for the optimum;
(3) test the candidates for the optimum by using necessary and sufficient condition tests.

Optimization procedures that parallel the preceding approach are generally refereed to as indirect methods of solution; indirect only in the sense that optimal solutions are determined primarily on the basis of differential properties of the functions or functional involved. In contrast, direct methods of solution require use of the performance measure and the constraint equations of a given problem, and systematic recursive methods are employed to obtain an optimal solution. It is not always possible, nor is it necessary, to clearly distinguish between direct and indirect methods; a comprehensive optimization procedure may beneficially employ both.

Geometrical interpretations of problems often afford insight into methods of solution. Solutions and solution techniques may be considered in terms of a multidimensional euclidean space (E^n or E_n are sometimes used to denote an n dimensional euclidean space). The current trial point and pertinent information concerning the current and preceding trial points represent the state of the solution in a multidimensional space, and this state information in conjunction with the equations that govern the solution scheme is used to determine the next trial point- the sequential search techniques are of this type. Geometrically, most constraint relationships define allowable regions in euclidean space. The performance measure and the constraints associated with linear programming theory are particularly well suited for geometrical interpretations: the region in which a general linear programming solution is lie is a convex hyper-polyhedron (a hyper-polyhedron is a polyhedron in a euclidean space of more than three dimensions). It is shown [51] that an optimal solution to general linear programming problem is always associated with one (or more) of the a vertices of a convex hyper-polyhedron.

Geometrical insight is also of value in the solution of some problems. We may view the column matrix $\bar{x}(t)$ as a state vector in an n-dimensional euclidean space called state space, and the control matrix $\bar{u}(t)$ as a vector in an r-dimensional euclidean space. Each trajectory in control space gives rise to a trajectory in state space. With automatic feedback control, the opposite is partially true: State information at any given instant of time influences the control applied at that instant of time or time at a slightly later instant. These geometrical interpretations facilitate the development of necessary conditions for the optimum.

Constraint relationships are taken into account in optimization procedures in many ways, but one way that prevails through major classes of problems is that of performance weighting which is closely associated with Lagrange multipliers on the one hand and with penalty coefficients on the other. Reasons that we might wish to weight several factors of interest in a performance measure is that performance measure is not necessarily a single entity. As a very limited example of the relationship of this weighting process to the use of Lagrang multipliers and penalty coefficients, consider the minimization of the performance measure Φ_a :

$$\Phi_a = \phi_o(\alpha_1,\alpha_2) + h\ \phi_1(\alpha_1,\alpha_2) \tag{1.1}$$

Where h is a positive weighting factor, and both $\phi_o \equiv \phi_o(\alpha_1,\alpha_2)$ and $\phi_1 \equiv \phi_1(\alpha_1,\alpha_2)$ are bounded real-valued functions. It is assumed that a minimum of both ϕ_o and ϕ_1 is desired, but because of their mutual dependence on α_1 and α_2 a trade-off must be effected. If h approaches zero, the minimum of Φ_a approaches the minimum of ϕ_o: but if h is allowed to be arbitrarily large, the minimum of Φ_a/h approaches the minimum of ϕ_1.

As a modification of the above conditions, suppose that ϕ_1 is required to be a constant c_1. But suppose that we proceed to minimize Φ_a of equation 1.1, with h not specified in advance, and find the minimum $\Phi_a^*(h)$ of Φ_a in terms of h, and also find the corresponding values $\alpha_1^*(h)$ and $\alpha_2^*(h)$ of c_1 and α_2. If h can be evaluated so that $\phi_1[\alpha_1^*(h),\alpha_2^*(h)]$ equals c_1, the desired result is obtained (a rigorous development of this fact is given in [51]). In this case, h is called a Lagrange multiplier, after the famous mathematician Joseph Louis Lagrange (1736-1813) who introduced this approach.

Alternatively, suppose ϕ_1 is required to equal c_1, as before, but suppose that Φ_p is minimized, rather than Φ_a, where Φ_p is a penalized performance measure and is expressed by :

$$\Phi_p = \phi_0(\alpha_1,\alpha_2) + h[\phi_1(\alpha_1,\alpha_2)-c_1]^2 \qquad (1.2)$$

If h is assigned a very large value, values of α_1 and α_2 that give $\phi_1 \neq c_1$ generally result in an inordinately large value of Φ_p: that is, the performance measure is harshly penalized when the constraint is violated (much), and therefore those values of α_1 and α_2 which yield the minimum of Φ_p also yield (within a controllable degree of accuracy) the constrained minimum of $\phi_0(\alpha_1,\alpha_2)$.

One of the advantages of the so-called indirect methods is that closed form solution are obtainable for certain forms of problems. But for sufficiently complex problems, all feasible approaches to solution, including the indirect approaches, require the use of high-speed, general-purpose computers during some phase of the solution. The direct methods are specifically suited to computer approaches to solution, and, as with any numerical solution scheme, scaling of variables and the appropriate introduction of new variables (for example, a linear transformation of variables) may significantly influence the accuracy of the solution and the time required to obtain the solution.

The dynamic programming approach to solution is based on Bellmans principle of optimality. This principle applies to systems for which a concept of state can be inferred and ordered sequence of stages of solution exists; at each stage of the solution, a decision is made which affects the present and subsequent stages of solution. For such problems, the principle of optimality is embodied in the following statement: whatever the initial state and initial decisions are, the decisions applied at remaining stages of solution must be optimum, with respect to the state resulting from the initial decision, if the overall decision process is to be potentially optimum.

Finally, for special classes of problems, inequality relationships can be used to deduce optimal solution. This is especially true of those problems associated with linear dynamic systems for which performance is measured in terms of an appropriate norm on abstract Hilbert or Banach spaces.

Linear Programming

The linear in "linear programming" indicates that only linear equations are involved. The programming in "linear programming" indicates that various variables are to be programmed -programmed in the sense of being scheduled or selected- to optimize a linear performance measure. In the usual linear programming problem [12] the performance measure is a specified linear algebraic equation, and other linear algebraic equations act as constraints on the optimization process. The simplex technique [51] is an efficient method for solving linear programming problems which are of a general form. Certain modification of the simplex technique are particularly well-suited for digital computer solution, and routines for these are commonly available at digital computer centers.

The original form of the simplex algorithm was developed by George B. Dantzig in 1947 and was formally published in 1951 [18]. Many variations of the original technique have been developed since, but it is significant that the original simplex algorithm is still the best procedure for the solution of the general linear programming problem when manual computations are used [2].

Dual Problems

Another approach to solution of optimization problems involves the concept of dual problems. For each problem of linear programming, for example, there exists a well-defined dual problem. the solutions of the dual and primal (original) problems are related in a definite way, and the is easier to solve in given case should be solved to obtain solution to both problems.

If a problem has dual, the problem is posed in terms of the same types of variables as the original (primal) problem, but with the roles of certain variables being interchanged. Thus, for electrical duals, current is the dual of voltage and vice versa; for mechanical duals, force is the dual of velocity and vice versa. The solution of a dual problem is related in some well-defined way to the primal problem; and in certain cases the solution of the dual problem is easier to obtain. It is for this reason, primarily, that dual problems are considered.

The existence of duals for linear programming problems was first proposed by Von Neumann in 1947 [65]. Much of the initial work on duality properties can be traced to Gale, Kuhn, and Tucker [23]. Of the known dual problems of linear programming, one of fundamental importance is the symmetric dual problem from which other useful dual problems can be developed.

Search Techniques and Nonlinear Programming

The primary problem considered for the methods which are introduced in this section is that of finding the extrema of a performance measure which is a nonlinear real-valued function of n parameters. The function may be given analytically or it may be determined experimentally; noise and experimental error may or may not be associated with the function; the function may or may not exhibit discontinuities; and constraint equations may exist which limit the arguments of the performance measure. In the latter case, the problem is called the nonlinear programming problem, in analogy with the naming of linear programing.

Search techniques are called direct methods of solving problems of the type posed in the preceding paragraph. This is in contrast to indirect methods, ones based solely on differential properties that certain classes of functions exhibit at points of extrema. The set of search techniques may be subdivided in many ways: discrete search versus continuous search; non-sequential search versus sequential search; local search versus global search; search with quadratic convergence versus search without quadratic convergence; and so forth. Specific methods which fall within any one of these categories have merit for certain problems. Given a particular optimization problem, a searcher's prime concern is to utilize a search technique which not only solves the problem, but also solves it efficiently; we might say that a searcher seeks the search technique which is "optimum" for his optimization problem.

The efficiency of a given search technique is affected by certain global and local properties of functions. For functions of one variable , Newton-Raphson [51,66] and Cubic-Convergent [51] searches are more appropriately used on analytically given functions while Quadratic-Convergent [53,69], Fibonacci [31,51] and Golden Section [51] searches may be more useful when data are obtained experimentally. The importance of efficient one-dimensional search is heightened by the fact that many n-dimensional search techniques incorporate a sequence of one-dimensional search in n-dimensional space [51].

Non-sequential search methods (e.g., Random [7] and Factorial [51] searches) are generally inefficient, but are useful under important special conditions. Univariate (one variable at a time) search [51] and relaxation search [59] techniques are often convenient when data are obtained experimentally, but these techniques do not provide rapid convergence to the optimum of most analytically given functions. Basic gradient methods [16] are perhaps the best-known search techniques: continuous gradient search [51] can be programmed on general-purpose analog computers; best-step steepest ascent [38] is a stepping-stone to the more efficient acceleration-step search [21] and Newton search [51] is a generalization of the one-dimensional Newton-Raphson search technique. The method of parallel tangents [21,52] and conjugate-direction search techniques [4,53,69] are based in part on the gradient concept, and exhibit the desirable property of quadratic convergence. That the Gradient and Conjugate methods have proved to be highly efficient is shown by an overall comparison at [51].

The first application of gradient search was given by Cauchy, 1847: he outlined a procedure for solving a set of simultaneous algebraic equations by using search techniques [51]. This procedure is incorporated (Chapter 3 of this thesis) in the penalty-function method of solving the general nonlinear programing problem. Once a performance measure is augmented by a penalty function, any of the known search methods may be used in obtaining the optimum.

1.3 Scope of the Research Work

The Scope of this thesis is to apply the sensitivity analysis to a sample nonlinear system which is a novel

electrohydraulic servovalve. This system sensitivity study differs from previous studies by considering the dynamic behaviour and nonlinearity of the system performance.

The thesis divided into seven chapters. in the first chapter, historical development of sensitivity and optimality and their applications are briefly described. Also in this chapter, the previous works that have been done on different aspects of the sensitivity and optimality analysis are reviewed.

In the second chapter, theory of the sensitivity functions, according to the definition which has been used all over this thesis, has been introduced. Then four different methods for sensitivity analysis have been developed. These methods are Vilenius Method, Revised Vilenius Method, Individual Characteristics Method and Entirety-Index Method respectively. All these methods, but first one, has been introduced in this thesis for the first time.

In the third chapter after introducing the objective function for optimization methods the theory of the basic Gradient methods is explained in general, and Steepest Ascent [Descent] in particular. These methods are considered both as continuous and as discrete methods. Then double sensitivity functions ($\bar{y}_{jk}$) are developed. Using these functions the double sensitivity matrices which will be used for locating insensitive system, can be built up. At the end of this chapter by using the sensitivity functions, the sensitivity of the performance index is analyzed.

In the fourth chapter at the beginning, mathematical model of the sample system is analyzed. Then the simulation of the system state variables are developed. And it is shown how to produce the proper initial values for this simulation, by using relation between the steady state value of the different state variables. At the end of the chapter, the stability of the system considering the time delay of the direction control valve is analyzed. In addition the dependence of the saturation values of the system's state variables on the fluid flow saturation is discussed.

In the fifth chapter, at the beginning, the necessary matrix ($\partial\bar{f}/\partial\bar{X}$) and vectors ($\partial\bar{f}/\partial\alpha_j$) for sensitivity analysis have been derived. Then using these matrix and vectors four different sensitivity analysis methods are applied to actuator piston velocity of the sample system. Using the last method, the sensitivity of the state variables of the sample system, other than the velocity of the actuator piston, are studied.

At the end of this chapter the effects of the time delay of the direction control valve are discussed. And it is shown how the effects of this special parameter can be studied by graphical methods by plotting the performance of the system with and without time delay.

In the sixth chapter a combinational optimization method which could overcome the difficulties of the system, is introduced. Then the five most important parameters (K_a, Mass, A_p, C_d and V_{in}) are chosen to study the behaviour of the system sensitivity. For this purpose each of these parameters are changed in the range of ±50% at a time.

After studying these sensitivity behaviour plots, different combination of these state variables sensitivity are considered. According to the study of all these sensitivity behaviours, at the end of this chapter a general design procedure is discussed.

The seventh chapter gives the highlights of the research work and some suggestions for the future work.

CHAPTER 2

Sensitivity Analysis

2.1 Introduction

Dynamic systems can be characterized in several ways: in the time domain, in the frequency domain, or in terms of a performance index. There is evidently an adequate number of ways to define the sensitivity function of a dynamic system. The definition that is actually used depends on the form of the mathematical model as well as on the purpose of consideration. For example, if the system is represented by a transfer function, the sensitivity will be defined on the basis of the parameter-induced change of the transfer function; whereas in case of a state space representation, the natural basis of the sensitivity definition will be the parameter-induced change of the trajectory.

Thus, the sensitivity functions can be classified into the following three categories [22]:

(1) sensitivity functions in the time domain,

(2) sensitivity functions in the frequency or z-domain,

(3) performance-index sensitivity.

In Reference [22] the most important representatives of each category are introduced and discussed. Besides these sensitivity functions there are so-called sensitivity measures that are defined on the entirety of the sensitivity functions and, therefore, allow for a global characterization of the sensitivity by a single number. These entirety measures are used as an index in the method of section 2.6.

The oldest definition of a sensitivity function was given by Bode [5]. This definition is based on the transfer function and was restricted to infinitesimal parameter deviations. In the sequel, Horowitz [27] gave a different interpretation of Bode's sensitivity function and also used it with great success for the design of control systems in the frequency domain [26,28]. Perkins and Cruz [48] extended Bode's sensitivity function in different directions, also establishing its significance for time domain considerations.

In connection with simulations on network analyzers and analog computers, the output sensitivity functions were introduced in the fifties mainly by Bykhovskiy [10] and Miller and Murray [43]. In the early sixties this definition was extended to the state space, resulting in the so-called trajectory sensitivity function [15,54,62]. The discussion of the merit of the time domain sensitivity functions has not yet come to an end [26]. However, there is no doubt that they play an important role in the comparison of open- and closed-loop systems as well as in the design of optimal controls. In 1963 Dorato [19] introduced the so-called performance-index sensitivity.

Besides the sensitivity functions mentioned above there are various special sensitivity definitions, such as the sensitivity of the overshoot in the time or frequency domain, the eigenvalue (pole or zero) sensitivity, and so on. Definitions such as these may be very helpful in the characterization of the sensitivity of a system in a certain aspect such as its relative stability.

2.2 Basic Theory

Nonlinearities in the system models of electro-hydraulic control servos complicate the application of the sensitivity analysis. The basis for the first order sensitivity models that can be applied to electro-hydraulic position control servos can be introduced as follow [64] :

$$\dot{\bar{x}} = \bar{f}(\bar{x}, \bar{u}, \bar{\alpha}) \qquad (2.1)$$

where $\bar{x}$ is the n-dimensional state vector

$\bar{u}$ is the r-dimensional input vector

$\bar{\alpha}$ is the p-dimensional parameter vector

It is assumed that unique solutions of (2.1) exist for all initial conditions and for all values of $\bar{\alpha}$. Furthermore, it is assumed that $\bar{f}$ is continuously twice differentiable with respect to $\bar{x}$ and $\bar{\alpha}$.

Denote the nominal solution of equation (2.1) :

$$\bar{x}_n(t) = \bar{\phi}(t, \bar{\alpha}_n) \qquad (2.2)$$

where $\bar{\alpha}_n$ is the nominal value[1] of $\bar{\alpha}$.

Denote the vector sensitivity functions :

$$\bar{\lambda}^j = \left(\frac{\partial \bar{x}}{\partial \alpha_j}\right)_n \qquad j = 1 \ldots p \qquad (2.3)$$

Assuming that $\bar{u}$ is independent of $\bar{\alpha}$ and differentiating equation (2.1) partially with respect to α_j we obtain the sensitivity equations in the form :

$$\dot{\bar{\lambda}}^j = \left(\frac{\partial \bar{f}}{\partial \bar{x}}\right)_n \bar{\lambda}^j + \left(\frac{\partial \bar{f}}{\partial \alpha_j}\right)_n \qquad j = 1 \ldots p \qquad (2.4)$$

1- Subscript n referring to nominal values where $(\partial\bar{f}/\partial\bar{x})_n$ is the Jacobian matrix evaluated on the nominal solution.

The initial conditions for (2.3) are :

$$\bar{\lambda}_0^j = \left(\frac{\partial\bar{x}_0}{\partial\alpha_j}\right)_n \qquad j = 1 \ldots p \qquad (2.5)$$

where $\bar{x}_0 = \bar{\phi}(t_0, \bar{\alpha}_n)$ is the initial condition of (2.1).

The sensitivity equations of (2.4) are linear differential equations with time-varying coefficients. There will be $n(p+1)$ equations (n state variable equations and nxp sensitivity equations) to be solved to produce the system states and the sensitivity functions. These equations can be solved using a computer simulation. The block diagram of the procedure in Fig. 2.1 is the first order sensitivity model of the system.

In the system models of electro-hydraulic control servos the function $\bar{f}$ is continuous everywhere. On the other hand, in the corner of some nonlinearity its first derivative is discontinuous. Between these discontinuity points $\bar{f}$ is continuously ~~twice~~ differentiable with respect to $\bar{x}$ and $\bar{\alpha}$. So in the intermediate areas the sensitivity equations can be defined in the form of (2.4). In solving the vector sensitivity functions one has to change the form of the state

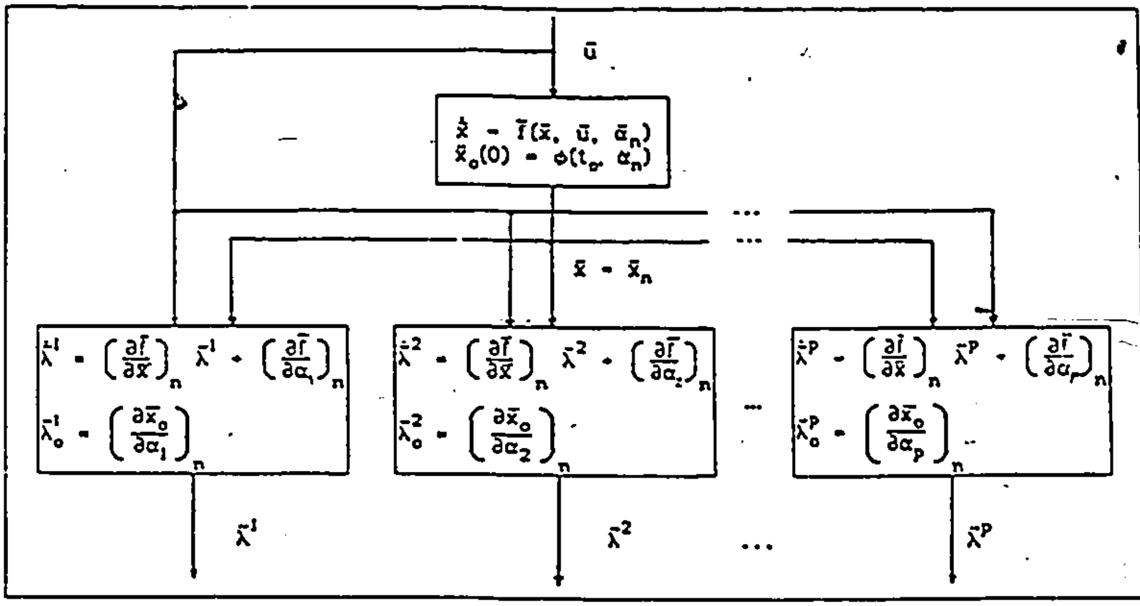

Fig. 2.1 Sensitivity model of a system with continuously twice differentiable state function $\bar{f}$ with respect to $\bar{x}$ and $\bar{\alpha}$. (Reproduced from Ref. [64])

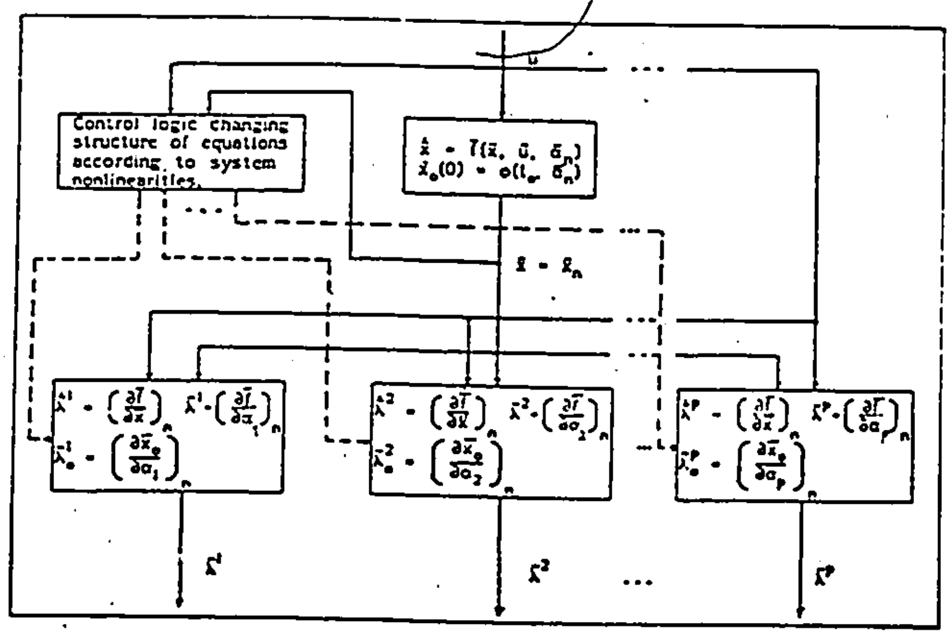

Fig. 2.2 Sensitivity model of a system with special type nonlinearities. Between the corner points of these nonlinearities the state function $\bar{f}$ is assumed to be continuously twice differentiable with respect to $\bar{x}$ and $\bar{\alpha}$. (Reproduced from Ref. [64])

function and sensitivity equations as one moves from one area to another. On the other hand, in the new area the initial conditions of the altered equations are replaced by the final conditions of the previous area. In the computer simulation this is done simply by the control logic, which recognises the area in which we operate during the solution, and in moving from one area to another the structure of the state function and sensitivity equations is changed automatically to represent the conditions in the new area. So the initial conditions of the equations in the new area are automatically given the values of the final conditions in the previous area. The sensitivity model of this case is shown in Fig. 2.2. In this model the parameter influence on the discontinuity of the first derivative is not taken into considerations.

In addition to the sensitivity functions, the complete differential variation $\delta \bar{x}$ of the nominal solution (2.2), which is :

$$\delta \bar{x}(t) = \bar{\phi}(t, \bar{\alpha}_n) - \bar{\phi}(t, \bar{\alpha}) \tag{2.6}$$

has to be known, and is due to the parameter variation :

$$\delta \bar{\alpha} = \bar{\alpha} - \bar{\alpha}_n \tag{2.7}$$

Using Taylor's theorem, equation (2.6) may be written :

$$\delta\bar{x} = \left(\frac{\partial\bar{x}}{\partial\bar{\alpha}}\right)_n \delta\bar{\alpha} + \text{higher order terms} \qquad (2.8)$$

where $(\partial\bar{x}/\partial\bar{\alpha})_n$ is the $n \times p$ matrix of the sensitivity functions. The vector sensitivity functions are the columns of the sensitivity matrix. Once the vector sensitivity functions have been known, according to equation (2.8), the first order approximation of the variation δx can be calculated.

2.3 Vilenius Method

This method was introduced by Professor M.J. Vilenius [64] and has been applied to an electro-hydraulic position control servo. The main idea in this method is that, once one knows the size of the parameter variation $\delta\alpha$, one is able to calculate the size of the variation of the nominal step response of x_i by only taking into account the first order terms in equation (2.8) as follows :

$$\delta x_i = \sum_{j=1}^{p} \lambda_i^j \delta\alpha_j \qquad i=1,n \qquad (2.9)$$

With equation (2.9) we are able to calculate the size of any influence of parameter variation on the step response of x_i at every time instant. To simplify the comparison between the different parameters equation 2.9 can be scaled with the steady state value x_{is} and consider only the maximum values of $\delta x_i / x_{is}$ and also change one parameter at a time. Thus the equation for comparisons will be as follows :

$$\left.\frac{\delta x_i}{x_{is}}\right|_{maxj} = \frac{\lambda_i^j|_{max}\delta\alpha_j}{x_{is}} \qquad (2.10)$$

By means of simulation studies it has been found [64] that the first order sensitivity model is still very accurate when the variations in the parameter vector α are 10 percent. By comparison, Daniels, Lee and Pal [17] noticed that first order sensitivity functions give satisfactory results up to 20 percent parameter variations. So, if one is looking at the influences of 1 percent parameter changes, he can be sure that the first order sensitivity model gives results accurate enough for comparisons. Giving 1 percent change for the parameters ($\delta\alpha_j = 0.01\ \alpha_{jn}$) and taking the maximum values $\lambda_i^j|_{max}$ according[a] to simulation programs, the maximum variations $\delta\alpha_i/x_{is}|_{maxj}$ can be computed by equation 2.10.

2.4 Revised Vilenius Method

This method is the same as the first method except that instead of computing :

$$\frac{\delta x_i}{x_{is}}$$

where x_{is} is the steady state step size, one should calculate :

$$\frac{\delta x_i(t)}{x_i(t)}$$

instantaneously, and then capture the maximum value. This gives a better index of comparison. Note that :

$$\delta \dot{x}_i = \sum_{j=1}^{p} \lambda_i^j \delta\alpha_j$$

then,

$$\frac{\delta x_i(t)}{x_i(t)} = \sum_{j=1}^{p} \boxed{\frac{\lambda_i^j(t)}{x_i(t)}} \delta\alpha_j \qquad (2.11)$$

Consequently, it is sufficient to compute

$$\frac{\lambda_1^j(t)}{x_1(t)} \qquad j = 1 \text{ to } p$$

$$\frac{\lambda_2^j(t)}{x_2(t)} \qquad j = 1 \text{ to } p$$

$$\vdots$$

and then capture the maximum in each case. The only problem which remains yet, is the calculation of $\delta x_i(t)/x_i(t)$ when $x_i(t) \approx 0.0$. To overcome this problem we can consider only the case where $x_i > \xi x_{is}$, where ξ can be $0 < \xi < 1.0$.

2.5 Individual Characteristics Method

As it was mentioned in the first two methods, they choose only λ_i^j at one instant of time, which is maximum in one or the other way. This λ_i^j not only does not have information about other instants of time , but also its information at that special point of time is a combination of different characteristics of the system performance changes (e.g. amplitude, frequency, ... changes).

To make these problems more clear, they are presented in Fig. 2.3. It is assumed that curves 'a' and 'b', are system performances before and after change of parameter α_j, respectively. Amplitude differences between the two curves at different points of times show the $\lambda_i^j\delta\alpha_j$ values. As it can be seen at point 'C' this value ($\lambda_i^j\delta\alpha_j$) is equal to zero. But it does not mean that the parameter change has no effect on the system performance, rather it means that different characteristics changes of the system performance neutralize effects of each other at that point of time. Hence it will be better to study individual characteristics of the system performance separately.

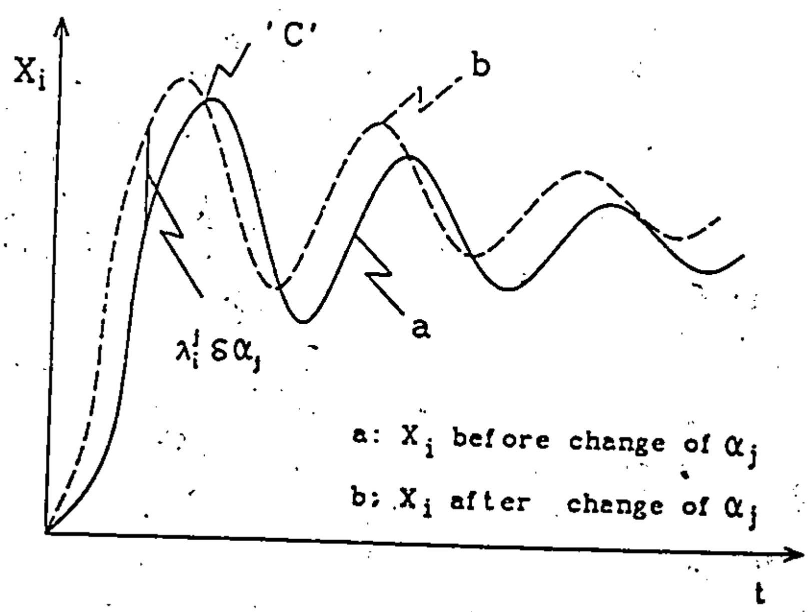

Fig. 2.3. $\lambda_i^j\delta\alpha_j$, the difference of two performances amplitudes.

2.5.1 Graphical Analysis

In the Individual Characteristics Method, it is tried to analyze different characteristics changes of state variables individually. For this purpose, it is tried to extract these characteristics changes from differences between state variable 'a' and 'b' in Fig. 2.3. One way of extracting the individual characteristics is the graphical method, where the curve 'b' can be derived directly by changing parameter α_j by a known amount. Then normalizing the effects of all state variable characteristics changes except one, by normalizing the curves with respect to those characteristics.

Some of characteristics such as overshoot or decrement depend on amplitude change of state variables. Hence, first of all, the real amplitude change will be extracted. The real amplitude change means the amplitude change between two corresponding points on curves (such as the first maxima), which can be located by normalizing steady state and frequency changes. To normalize state variables with respect to the steady state value, we can simply divide state variable during all periods of times by the steady state value of that state variable. Then the steady state value of all state variables will be equal to unity and the value of the state variables amplitudes at the points of time other than the steady state time will increase or decrease with respect to the original steady state values of that state variable.

Because of the frequency change of state variable the corresponding points on two different state variables ('a' and 'b') does not occur at the same time. Hence, it will make it difficult to recognize two corresponding points on curves 'a' and 'b'. Therefore, by normalizing the frequency of two different state variables, it will be tried to obtain each pair of corresponding points on 'a' and 'b' (such as peak points) in one vertical line which represent one instant of time. Normalizing the frequency of the state variables, means to change a pair of curves with different frequencies (Fig. 2.4 a) to another pair of curves with the same frequency (Fig. 2.4 b), by expanding and shifting one of the curves in order to bring together the crossing points of curves with common steady state line.

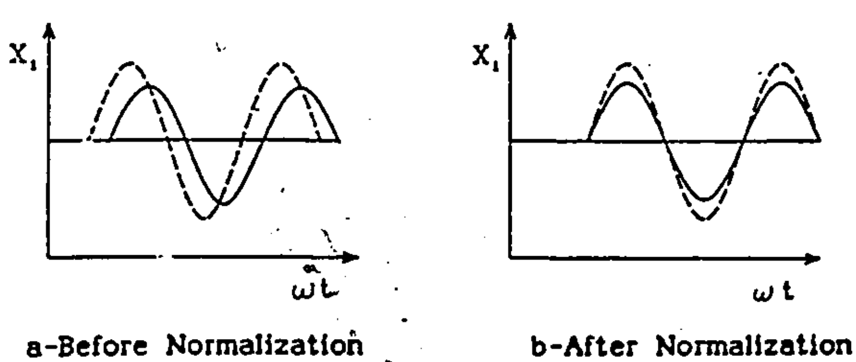

Fig. 2.4. Frequency Normalization

Applying this frequency normalization to nonlinear system state variables (such as the applied system in Chapter 4), shows that it is not able to put all the corresponding points of two different state variables on the same vertical line. That means, for example, the corresponding maximum and minimum points on two different state variables after frequency normalization occur at different normalized time yet. Hence difficulty of locating corresponding points with respect to each other still exists.

Another problem in this graphical method occurs when λ_i^j is calculated using the difference between state variables 'a' and 'b'. This will cause a large error on λ_i^j. It can be shown that this error is large, because the difference between the two state variable according to one percent change of parameter is usually less than one percent of the output value at that instant of time. If one can expect one percent error of numerical calculations of state variable then this error surpasses the difference between state variables, consequently surpasses the individual characteristic values.

But this problem can be overcome by calculating λ_i^j analytically using Eq. (2.4) and getting new state variable 'b' by adding $\lambda_i^j \delta\alpha_j$ to the original state variable 'a' at each instant of time, instead of calculating it (state variable 'b') using Eq. (2.4) again, considering parameter change $\delta\alpha_j$.

2.5.2 Analytical Expressions

In an analytical method some mathematical expressions have to be determined. These expressions give the individual characteristics change of the state variable of the system. Then it is possible to calculate these characteristics (e.g. T_r, time rise) changes by one run of simulation program during the calculation of the sensitivity functions ($\bar{\lambda}^j$).

Actually it is impossible to derive simple expressions similar to the sensitivity functions expression (2.4) for the sensitivity of performance characteristics (such as $\partial T_r/\partial\alpha_j$). Because these kinds of performance characteristics can only be derived from special points such as maxima or crossing points of state variable curve with steady state line. These special points on new state variable 'b' (as it was mentioned before) depend on new performance characteristics. There is no direct relation between these special points on state variable 'b' and their corresponding points on the original state variable 'a'. Therefore a special analytical method will be investigated.

First the performance of interested characteristics are introduced :

X_{ss} = Steady state value

T_r = Rise Time

P_o = Percentage Overshoot

F_f = Frequency

D_c = Decrement

where , according to Fig. 2.5, they are defined as below :

$X_{ss} = x_{1s}$

$T_r = t_1$

$P_o = 100a/x_{1s}$

$F_r = 2\pi/(t_4-t_1)$

$D_c = b/a$

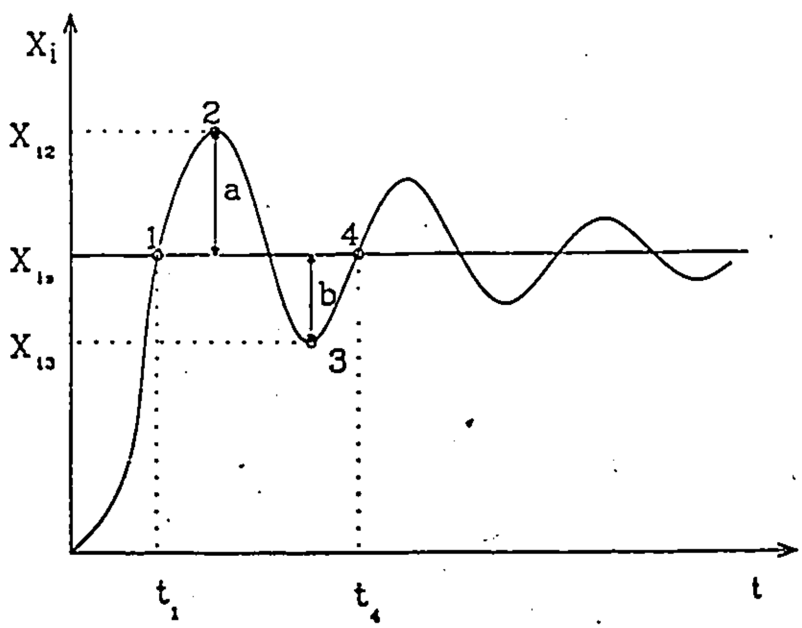

Fig. 2.5 Individual Characteristics of Performance

D_c (decrement) is introduced instead of T_s (settling Time) because $\partial T_s/\partial \alpha_j$ is not continuous over the time. This can be illustrated by using Fig. 2.6 where by changing α_j in such a direction which decreases the amplitude of performance, the settling time value jumps from T_{s1} to T_{s2}.

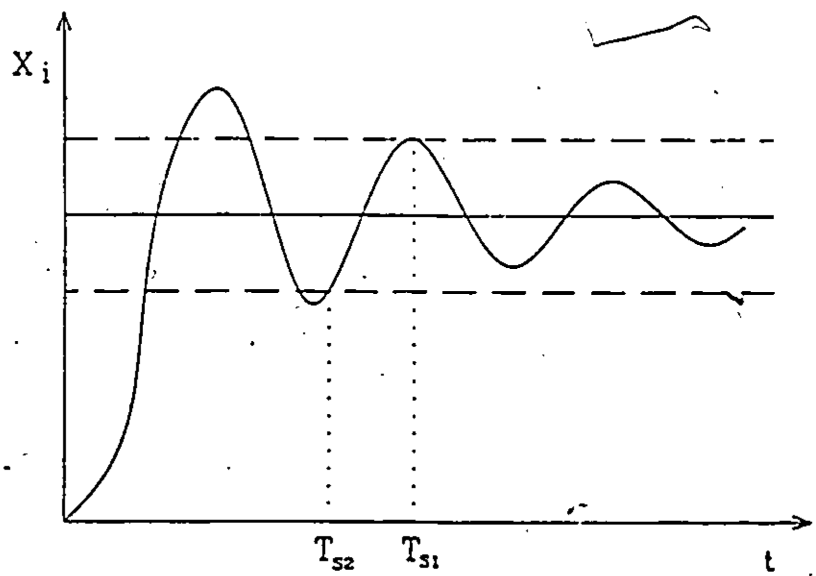

Fig. 2.6. Discontinuity of Settling Time Sensitivity ($\partial T_s/\partial \alpha_j$)

Now to calculate these five characteristics of the system performance, first of all we need to calculate the $X_{ss} = x_{is}$ and by using the coordinates of four points 1 to 4 on the curve, we can calculate other characteristics (T_r, P_o,...).

To calculate X_{ss} the equations which represent the relation between state variables of the system during steady state can be derived, by modifying the mathematical model of that system. These equations for our sample system will be derived in section 4.3.

Having X_{ss}, to calculate t_1, t_4, x_{12}, x_{13} some expressions have to be developed. For x_{12} and x_{13} :

$$\frac{dx_i}{dt} = 0.0 \qquad \text{at } t = t_2 \text{ or } t_3$$

because points 2 and 3 are local maximum and minimum points. And,

$$x_i^j = x_i^o + \lambda_i^j \delta\alpha_j$$

where,

$x_i^j = x_i$ after change of α_j

$x_i^o = x_i$ before change of α_j

then,

$$\frac{dx_i^j}{dt} = \frac{dx_i^o}{dt} + \frac{d\lambda_i^j}{dt}\delta\alpha_j = \dot{x}_i^o + \dot{\lambda}_i^j\delta\alpha_j \qquad (2.12)$$

where x_i^o is x_i before changing any parameter and x_i^j is x_i after changing parameter j.

then,

$$\dot{x}_i^o(t) + \lambda_i^{j}(t)\delta\alpha_j = 0.0 \qquad j=1,...p \quad \& \quad t = t_2 \text{ or } t_3 \qquad (2.13)$$

From (2.13), t_2 and t_3 can be calculated, consequently x_{12} and x_{13} can be determined too.

To calculate t_1 and t_4, it is possible only by looking for a point with $x_1 = x_{1s}$. x_{1s} can be calculated in the beginning of the simulation program by means of steady state equations for all the state variables with respect to different parameters.

Actually in numerical methods we do not have exactly the coordinates of these points (points 1 to 4). So one has to calculate their coordinates by linear interpretation, using coordinates of two points one just before and one just after them.

2.6 Entirety-Index Method

As it was noticed, the different methods for the sensitivity analysis (methods 1-3) have various advantages and disadvantages. The first method (Vilenius Method, capturing maximum sensitivity with respect to steady state value of state variable), and the second method (Revised one, considering sensitivity functions instantaneously and capturing the maximum sensitivity function value with respect to state variable at same instant of time), do not have all the properties of sensitivity functions. On the contrary, Individual Characteristics Method gives most of the system performance properties sensitivity (such as time rise, overshoot,...), but it makes it difficult to use directly these results for some special purposes such as system optimization, and insensitivity. Then a simple method which has all or most of the system performance characteristics sensitivity has to be introduced. Actually what we mean by simple and informative method is an index (Entirety-Index; I_E) which makes the numerical calculation as simple as possible; and improvement of those system performance characteristics sensitivity will decrease this index (for purpose of locating insensitive system).

For this purpose instead of capturing maximum value of sensitivities, such as the first and the second methods, we can integrate $\lambda_i^j \delta\alpha_j$, absolute or square value, over some period of time $(0:T_1)$. If one wants to include the effect of the steady state value difference of the state variable in this integration, one should continue the integration somewhat through the steady state portion, e.g. $T_1 = T_s$, where T_s is settling time. The integral of $|\lambda_i^j \delta\alpha_j|$ simply represent the area between two state variable of the system (Fig. 2.7). So absolute value of $\lambda_i^j \delta\alpha_j$ is considered in this method. This integral can be calculated numerically by considering two different cases as shown in Fig. 2.8.

Refer to Fig. 2.8 b in this case the sensitivity function becomes zero and there are two different sign of sensitivity function in either side of this point $(\lambda^j = 0)$. The ΔI_E equation in this case shows that the area of two triangles are calculated separately and then the result is the sum of these areas. If the crossing point is called x_c and corresponding time t_c, ΔI_E for this case can be derive as :

$$\Delta I_E = \text{Area of triangle } x_i^j x_i^o x_c + \text{Area of triangle } x_i^{j'} x_i^{o'} x_c$$

$$\Delta I_E = \left[|\lambda_i^j \delta\alpha_j|(t_c - t_i) + |\lambda^j_{i+1} \delta\alpha_j|(t_{i+1} - t_c) \right] / 2 \qquad (2.14)$$

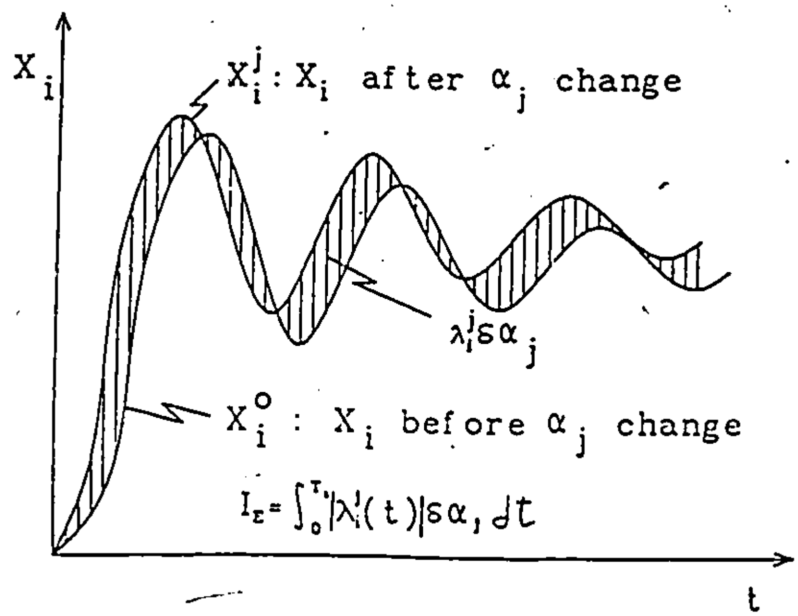

Fig. 2.7 Entirety-Index of state variable

and since the two triangles are similar then :

$$\frac{t_c - t_i}{t_{i+1} - t_c} = \frac{|\lambda_i^j \delta\alpha_j|}{|\lambda_{i+1}^j \delta\alpha_j|} = \frac{|\lambda_i^j|}{|\lambda_{i+1}^j|}$$

Because $\lambda_i^j / \lambda_{i+1}^j < 0$ then :

$$\frac{t_c - t_i}{t_{i+1} - t_c} = \frac{-\lambda_i^j}{\lambda_{i+1}^j}$$

This can be rewritten as :

$$t_c - t_i = \frac{-\lambda_i^j}{\lambda_{i+1}^j - \lambda_i^j}(t_{i+1} - t_i)$$

and,
$$t_{i+1}-t_c = \frac{\lambda^j_{i+1}}{\lambda^j_{i+1}-\lambda^j_i}(t_{i+1}-t_i)$$

By substituting these equations in Equ. (2.14):

$$\Delta I_E=\left|(\lambda^{j\,2}_i+\lambda^{j\,2}_{i+1})\delta\alpha_j/(\lambda^j_i-\lambda^j_{i+1})\right|(t_{i+1}-t_i)/2 \qquad (2.15)$$

In the next chapter (Chap. 3), it will be shown that how this index can be used for purpose of locating optimal or insensitive system. Also for using this Entirety Index in optimization methods (Chapter 6), we have to normalize it. That means to divide $I_E(i,j)$ by $X_{is} \times T_{is}$.

where X_{is} = steady state value of state variable X_i

T_{is} = settling time of state variable X_i

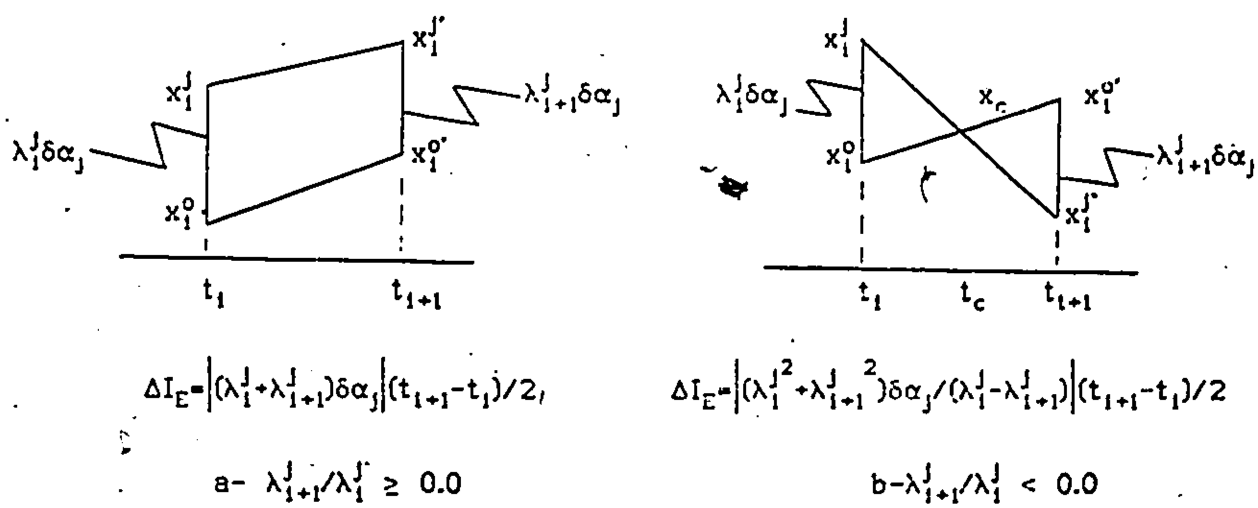

Fig. 2.8. Steps of Entirety-Index Integral

2.7 Summary

In this chapter theory of the sensitivity functions, according to the definition which is used all over this thesis, has been introduced. Then four different methods for sensitivity analysis have been developed. These methods are Vilenius Method, Revised Vilenius Method, Individual Characteristics Method and Entirety-Index Method respectively. All these methods, but first one, has been introduced in this thesis for the first time.

CHAPTER 3

Optimality Analysis

3.1 Introduction

The use of optimization techniques is of fundamental importance in system design; it is necessitated by the practical fact that the system design which is the best in some specified senses is the one which sells, all other things being equal. Granted, in certain instances the optimal design is obvious, e.g., the greater the value of a certain parameter α, the better the system performance, but this is a trivial case. In many instances, constraints are imposed on the parameters of the system, and the treatment of these constraints requires the application of more sophisticated optimization techniques.

It is interesting to note that nature itself generally takes an optimal course. For example, in a classic work by James Clerk Maxwell [41], it is noted that the current distribution in a resistor-source network is the one which, in addition to satisfying Kirchhoff's laws, results in minimum of energy dissipation.

To apply optimization methods to a system, one needs an objective function. So we consider some performance measure Φ as an objective function :

$$\Phi = \phi(\alpha_1, \alpha_2, \ldots, \alpha_n)$$

where $\phi \equiv \phi(\alpha_1, \alpha_2, \ldots, \alpha_n)$ is a known real-valued function of the arguments $\alpha_1, \alpha_2, \ldots, \alpha_n$. The set $\{\alpha_1, \alpha_2, \ldots \alpha_n\}$ of real arguments of ϕ is denoted by $\bar{\alpha}$ [1]. For operational purposes, it is often convenient to view $\bar{\alpha}$ as an $n \times 1$ column matrix with entries α_i. Unless otherwise restricted, $\bar{\alpha}$ may assume any value in n-dimensional Euclidean space E^n. When n is equal to 2, for example, $\bar{\alpha}$ assumes values in the ordinary two-dimensional space of analytic geometry.

One of the principal problems of optimal design is to determine the particular values of α (values of the entries of $\bar{\alpha}$) which result in the attainment of local maxima and local minima of the performance measure Φ in a subset R_x of n-dimensional Euclidean space [S1].

1- α notation has been used as an argument of function to fit our sensitivity models definition.

3.2 Gradient Methods

As it was mentioned in Chapter 1 there are different kinds of optimization methods for different kinds of systems. In the following paragraphs the theory of basic Gradient methods is explained in general, and the Steepest Ascent [Descent] in particular.

3.2.1 Common Features

Common to all gradient search techniques is the use of the gradient $\nabla\phi = \bar{g} = [\partial\phi/\partial\alpha_1 \ \partial\phi/\partial\alpha_2 \ \dots \ \partial\phi/\partial\alpha_n]^T$. In the case that $\phi(\bar{\alpha})$ is determined experimentally or numerically, a discrete approximation to the gradient is used; and also it is possible to use statistical procedures to estimate the gradient $\bar{g}$ from experimentally obtained data [51].

All gradient methods are governed, at least in part, by the following equation :

$$\bar{\alpha}^{k+1} = \bar{\alpha}^k - a\, H\, \bar{g}|_{\bar{\alpha}=\bar{\alpha}^k} \qquad (3.1)$$

in which $\bar{\alpha}^k$ is the "old" value of $\bar{\alpha}$, $\bar{\alpha}^{k+1}$ is the "new" value of $\bar{\alpha}$, $\bar{g} = \nabla\phi$ is the gradient of ϕ in column-vector form, H is an $n \times n$ matrix, and a is a real number. Gradient methods differ in the way which H and a are selected at $\bar{\alpha} = \bar{\alpha}^k$.

For example, if H is taken to be the matrix which contains all zero entries, except for a single 1 in the ith row on the main diagonal, and if a is taken to be $1/(\partial^2\phi/\partial\alpha_i^2)$, then equation 3.1 reduces to :

$$\alpha_i^{k+1} = \alpha_i^k - \left.\frac{\partial\phi/\partial\alpha_i}{\partial^2\phi/\partial\alpha_i^2}\right|_{\bar{\alpha}=\bar{\alpha}^k}$$

and,

$$\alpha_j^{k+1} = \alpha_j^k \qquad \text{for } j \neq i$$

and the result is Southwell's relaxation search [51].

Because the gradient of $\phi(\bar{\alpha})$ is generally affected by a change in scale of any given α_i, it is not surprising that the rate of convergence of basic gradient methods depends on the scale used [16]. Another drawback of basic gradient methods is that they are relatively inefficient when ridges or ravines are salient. Extensions of the basic gradient methods alleviate these faults to a great extent, as is evidenced in [16].

3.2.2 Continuous Steepest Ascent [Descent]

Cauchy first introduced the concept of the steepest descent in 1847 to be used for solving simultaneous equations. The key idea supporting continuous steepest ascent is that a maximum is sought by always proceeding in the direction which yields the greatest rate of increase of $\phi(\bar{\alpha})$. It is as a blindfolded men who strives to reach the top of a hill by always climbing the steepest slope.

To use the method, the following question must be resolved. Given an initial starting point $\bar{\alpha}^o = [\alpha_1^o \; \alpha_2^o \; \ldots \; \alpha_n^o]^T$, in what direction in the n-dimensional Euclidean space of the α_i's from $\bar{\alpha} = \bar{\alpha}^o$ does $\phi(\bar{\alpha})$ tend to increase the most? It is shown in the next few paragraphs that the gradient direction $g(\bar{\alpha}^o)$ yields the greatest incremental increase of $\phi(\bar{\alpha})$ for a fixed incremental distance moved from $\bar{\alpha} = \bar{\alpha}^o$. The derivation follows that of Kelley [30].

Let the fixed incremental distance moved from $\bar{\alpha} = \bar{\alpha}^o$ be denoted by ϵ. By Pythagoras' theorem,

$$\epsilon^2 = (\delta\alpha_1^o)^2 + (\delta\alpha_2^o)^2 + \ldots + (\delta\alpha_n^o)^2 \qquad (3.2)$$

Let $\bar{\alpha}_i$ denote the value of $\bar{\alpha}$ obtained by an incremental move :

$$\begin{aligned}\bar{\alpha}^1 &= [\alpha_1^o + \delta\alpha_1^o \quad \alpha_2^o + \delta\alpha_2^o \quad \dots \quad \alpha_n^o + \delta\alpha_n^o]^T \\ &= \bar{\alpha}^o + \delta\bar{\alpha}^o \end{aligned} \tag{3.3}$$

In seeking a maximum of $\phi(\bar{\alpha})$ by steepest ascent search, the object is to maximize $\phi(\bar{\alpha}^o + \delta\bar{\alpha}^o)$ by appropriate selection of the $\delta\alpha_i^o$'s. Recall, however, that the $\delta\alpha_i^o$'s are constrained by equation 3.2. Thus, the Lagrange multiplier technique [51] is applicable : the augmented function Φ_a is defined by :

$$\phi_a = \phi(\bar{\alpha}^o + \delta\bar{\alpha}^o) + h\sum_{j=1}^{n} (\delta\alpha_j^o)^2 \tag{3.4}$$

and the necessary condition for a maximum of ϕ_a is :

$$\frac{\partial\phi_a}{\partial\delta\alpha_j^o} = 0 \tag{3.5}$$

or,

$$\left.\frac{\partial\phi}{\partial\alpha_j}\right|_{\bar{\alpha}=\bar{\alpha}^o+\delta\bar{\alpha}^o} + 2h\delta\alpha_j^o = 0 \tag{3.6}$$

for all j. Hence,

$$\delta\alpha_j^o = \frac{-1}{2h} \left.\frac{\partial\phi(\bar{\alpha})}{\partial\alpha_j}\right|_{\bar{\alpha}=\bar{\alpha}^o+\delta\bar{\alpha}^o} \tag{3.7}$$

which in matrix form is :

$$\delta\bar{\alpha}^o = \frac{-1}{2h}\nabla\phi(\bar{\alpha}^o + \delta\bar{\alpha}^o) = \frac{-1}{2h}\bar{g}(\bar{\alpha}^o + \delta\bar{\alpha}^o) \tag{3.8}$$

The Lagrange multiplier h is evaluated by using the result given by 3.7 in equation 3.2 :

$$\epsilon^2 = \frac{1}{4h^2}\sum_{j=1}^{n}\left(\frac{\partial\phi}{\partial\alpha_j}\right)^2\Bigg|_{\bar{\alpha}=\bar{\alpha}^o+\delta\bar{\alpha}^o} = \frac{1}{4h^2}\bar{g}(\bar{\alpha}^o + \delta\bar{\alpha}^o)^T\bar{g}(\bar{\alpha}^o + \delta\bar{\alpha}^o)$$

from which it follows that :

$$\frac{-1}{2h} = [\bar{g}(\bar{\alpha}^o + \delta\bar{\alpha}^o)^T\bar{g}(\bar{\alpha}^o + \delta\bar{\alpha}^o)]^{-1/2}\epsilon$$

This result is used to eliminate -1/(2h) from equation (3.8) :

$$\delta\bar{\alpha}^o = \epsilon\bar{g}(\bar{\alpha}^o + \delta\bar{\alpha}^o)/[\bar{g}(\bar{\alpha}^o + \delta\bar{\alpha}^o)^T\bar{g}(\bar{\alpha}^o + \delta\bar{\alpha}^o)]^{1/2} \tag{3.9}$$

If the incremental distance ϵ is sufficiently small, then so also is each $\delta\alpha_i$; and $\bar{g}(\bar{\alpha}^o + \delta\bar{\alpha}^o)$ can be simply replaced by $\bar{g}(\bar{\alpha}^o)$. Also note that the denominator of the right-hand member of 3.9 is a positive number, except at stationary points in which it equals to zero. It is convenient to lump this term with ϵ, i.e., to form a new increment :

$$\Delta\tau \triangleq \epsilon/[\bar{g}(\bar{\alpha}^o)^T\bar{g}(\bar{\alpha}^o)]^{1/2}$$

and rather than view $\Delta\tau$ as a function of $\bar{\alpha}^o$, $\Delta\tau$ is held fixed by assigning an appropriate value of the increment ϵ for each value of $\bar{\alpha}$. In other words $\epsilon \triangleq \Delta\tau[\bar{g}(\bar{\alpha}^o)^T\bar{g}(\bar{\alpha}^o)]^{1/2}$ where $\Delta\tau$ is constant. The net result is that :

$$\delta\alpha_i\big|_{\bar{\alpha}=\bar{\alpha}^o} = \Delta\tau[\partial\phi/\partial\alpha_i]_{\bar{\alpha}=\bar{\alpha}^o} \qquad i=1,2,\ldots,n \qquad (3.10)$$

or in column-vector form :

$$\delta\bar{\alpha}^o = \Delta\tau\bar{g}(\bar{\alpha}^o) \qquad (3.11)$$

for steepest ascent search.

Up to this point, $\Delta\tau$ and ϵ have been assumed to be small increments, small enough so that $\nabla\phi(\bar{\alpha}^o + \delta\bar{\alpha}^o) \cong \nabla\phi(\bar{\alpha}^o)$. If $\Delta\tau$, and therefore $\delta\alpha_i$, is allowed to approach 0+, equations 3.10 and 3.11 will reduce to a set of first-order differential equation :

$$\frac{d\alpha_i}{d\tau} = \frac{\partial\phi}{\partial\alpha_i} \qquad i=1,2,\ldots,n \qquad (3.12)$$

It is conceivable that one might unknowingly start the solution of 3.12 with initial conditions at a relative minimum or at a saddle point of $\phi(\bar{\alpha})$. In such a case, all $\partial\phi/\partial\alpha_i$ are initially zero, and theoretically the search would not begin. In practice, however, noise in the system is sufficient to deviate the solution from either a minimum or a saddle point; and once away, the solution diverges from these unstable equilibrium points. In fact, Zellnick et al. [70] found it difficult to determine the character of functions in the vicinity of saddle points because their search techniques lead them abruptly away.

3.2.3 Discrete Steepest Ascent [Descent]

A discrete version of steepest ascent search is obtained from equation 3.11, namely,

$$\bar{\alpha}^{k+1} - \bar{\alpha}^{k} = \Delta\tau\bar{g}|_{\bar{\alpha}=\bar{\alpha}^{k}} \qquad (3.13)$$

which is rearranged to correspond in form to gradient search in general, equation 3.1, as follows :

$$\bar{\alpha}^{k+1} = \bar{\alpha}^{k} + \Delta\tau I\bar{g}|_{\bar{\alpha}=\bar{\alpha}^{k}} \qquad (3.14)$$

In this special case, the matrix H of equation 3.1 equals the identity matrix I, the nonzero entries of which are "ones" on the major diagonal, and a' of equation 3.1 equals $-\Delta\tau$

The step size $\Delta\tau$ remains to be determined. Note that by replacing $\bar{\alpha}$ in $\phi(\bar{\alpha})$ with $\bar{\alpha}^k + \Delta\tau\bar{g}(\bar{\alpha}^k)$, the function $\phi[\bar{\alpha}^k + \Delta\tau\bar{g}(\bar{\alpha}^k)]$ is a function of the parameter $\Delta\tau$ only, i.e., the function is in a parametric form. There are an unlimited number of ways in which $\Delta\tau$ can be selected, all of which correspond to some form of one-dimensional search (Section 6-3, [51]). Lapidus et al. [38] compare six of these, each but slightly different, by applying them to a common problem.

Because of the computations involved in evaluating the gradient of $\phi(\bar{\alpha})$ at a given point, it is usually advantageous to make the most of each gradient computation before making another; that is, to search in the direction of the gradient until $\partial\phi[\bar{\alpha}^k + \Delta\tau\bar{g}(\bar{\alpha}^k)]/\partial\Delta\tau = 0$ for some $\Delta\tau$. This approach is referred to in the literature [8] as the method of optimum steepest ascent, an unfortunate designation in that quite often improvements in efficiency can be made by incorporating additional features, as is done in Section 6-7 of [51]. In this work, therefore, the phrase "best-step steepest ascent" is used in place of "optimum steepest ascent."

Figure 3.1 depicts a search conducted by best-step steepest ascent when $\tilde{\alpha} = [\alpha_1, \alpha_2]$. Note that if a different scale is employed for the α_1 coordinate, as in Figure 3.2, the number of iterations is changed considerably. Hence, the desirability of proper scaling is clearly evident.

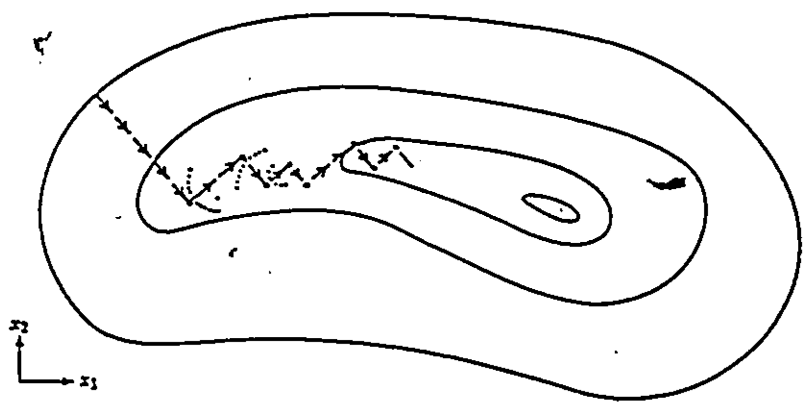

Fig. 3.1. Example of best-step steepest ascent search.

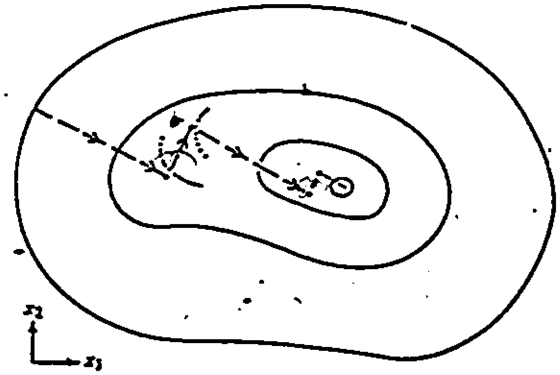

Fig. 3.2. Example of best-step steepest ascent search with proper scaling.

3.3 Double Sensitivity

In Chapter 2, different methods and indices for sensitivity analysis of system are defined. In order to reach the least sensitive system we have to know how a sensitivity index changes with respect to each parameter change. Therefore the sensitivity of sensitivity functions which are called Double Sensitivity Functions ($\bar{y}_{jk}$) are required. They will be described with more details. In Chapter 2 the sensitivity functions are introduced as :

$$\bar{\lambda}^j = \left(\frac{\partial \bar{x}}{\partial \alpha_j}\right)_n \qquad j = 1, \dots p$$

and,

$$\dot{\bar{x}} = \bar{f}(\bar{x},\bar{u},\bar{\alpha})$$

then,

$$\dot{\bar{\lambda}}^j = \left(\frac{\partial \bar{f}}{\partial \bar{x}}\right)_n \bar{\lambda}^j + \left(\frac{\partial \bar{f}}{\partial \alpha_j}\right)_n \qquad j = 1, \dots p$$

and the double sensitivity function can be introduced as

$$\bar{y}_{jk} = \frac{\partial \bar{\lambda}^j}{\partial \alpha_k} = \frac{\partial^2 \bar{x}}{\partial \alpha_j \partial \alpha_k} \qquad (3.15)$$

then provided that $\bar{\alpha}$ is independent of t,

$$\dot{\bar{y}}_{jk} = \frac{\partial \dot{\bar{\lambda}}^j}{\partial \alpha_k}$$

or,

$$\dot{\bar{y}}_{jk} = \frac{\partial}{\partial\alpha_k}\left\{\left(\frac{\partial\bar{f}}{\partial\bar{x}}\right)\lambda^j + \left(\frac{\partial\bar{f}}{\partial\alpha_j}\right)\right\}$$

$$\dot{\bar{y}}_{jk} = \left(\frac{\partial^2\bar{f}}{\partial\bar{x}^2}\frac{\partial\bar{x}}{\partial\alpha_k} + \frac{\partial^2\bar{f}}{\partial\bar{x}\partial\alpha_k}\right)\bar{\lambda}^j + \left(\frac{\partial\bar{f}}{\partial\bar{x}}\right)\frac{\partial^2\bar{x}}{\partial\alpha_j\partial\alpha_k} + \frac{\partial^2\bar{f}}{\partial\bar{x}\partial\alpha_j}\frac{\partial\bar{x}}{\partial\alpha_k} + \frac{\partial^2\bar{f}}{\partial\alpha_j\partial\alpha_k}$$

substituting the following expressions in the above equations :

$$\begin{cases} \bar{\lambda}^k = \dfrac{\partial\bar{x}}{\partial\alpha_k} \\ \bar{y}_{jk} = \dfrac{\partial^2\bar{x}}{\partial\alpha_j\partial\alpha_k} \end{cases}$$

gives :

$$\dot{\bar{y}}_{jk} = \left(\frac{\partial^2\bar{f}}{\partial\bar{x}^2}\bar{\lambda}^k\right)\bar{\lambda}^j + \frac{\partial^2\bar{f}}{\partial\bar{x}\partial\alpha_k}\bar{\lambda}^j + \frac{\partial\bar{f}}{\partial\bar{x}}\bar{y}_{jk} + \frac{\partial^2\bar{f}}{\partial\bar{x}\partial\alpha_j}\bar{\lambda}^k + \frac{\partial^2\bar{f}}{\partial\alpha_j\partial\alpha_k} \quad (3.16)$$

Dimension of all the elements of Eq. 3.16 has been ginen in App. II. To use these double sensitivity functions in order to reach the least sensitive system, Double Sensitivity Index will be introduced in the remaining of this section.

Fig 3.3 shows $\lambda_i^j\delta\alpha_j$ as a function of time, before ('a') and after ('b') parameter α_k change. The norizental axis which

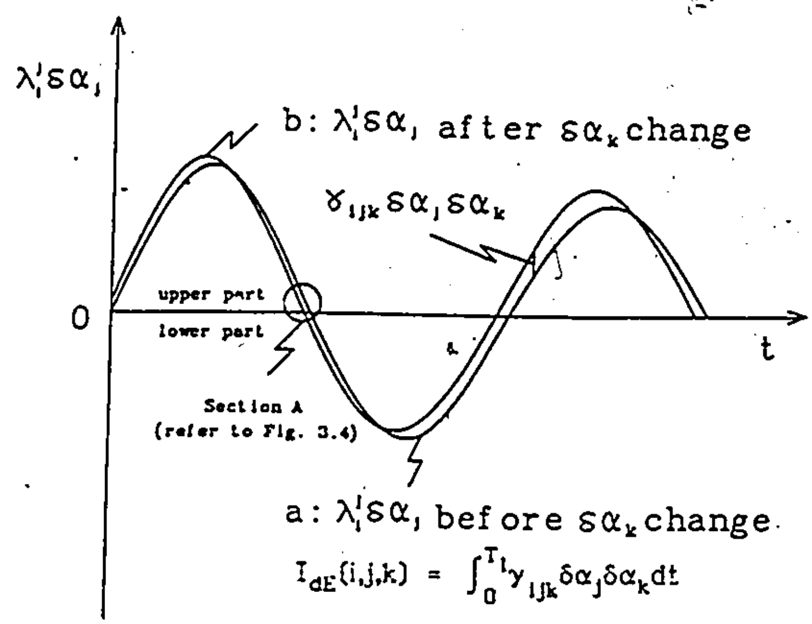

Fig 3.3 Double Sensitivity Index

represents $\lambda_i^j\delta\alpha_j$ equal to zero, can be assumed as old state variable of system before changing any parameter. Hence the area between curve 'a' and this axis will be Entirety-Index which has introduced in Section 2.6. Changing parameter α_k affects on this area or Entirety-Index. This change of area will be called double sensitivity index which can be calculated using $\bar{\gamma}_{jk}$ functions (App. II).

Because of using the absolute value (not square value) in the integration of the sensitivity Index (I_E), it is easier to calculate the double sensitivity index (I_{dE}) from $\bar{\gamma}_{jk}$. But, because the effect of γ_{ijk} on the sensitivity index at each instant of time depends on the sign of γ_{ijk} and λ_i^j, we cannot simply integrate the γ_{ijk} during all the time period. Then the integration of positive λ's and negative λ's should be calculated separately. Still, we have another problem when the two sensitivity functions (before and after α_k change) cross each other ($\gamma_{ijk}=0$) or one of them becomes zero between two adjacent steps. This problem can be solved by dividing different cases of the problem in different categories. There are sixteen different cases which are considered in the following paragraphs.

Fig 3.4 enlarged one of the calculation steps, and illustrated different terms which have been used in various

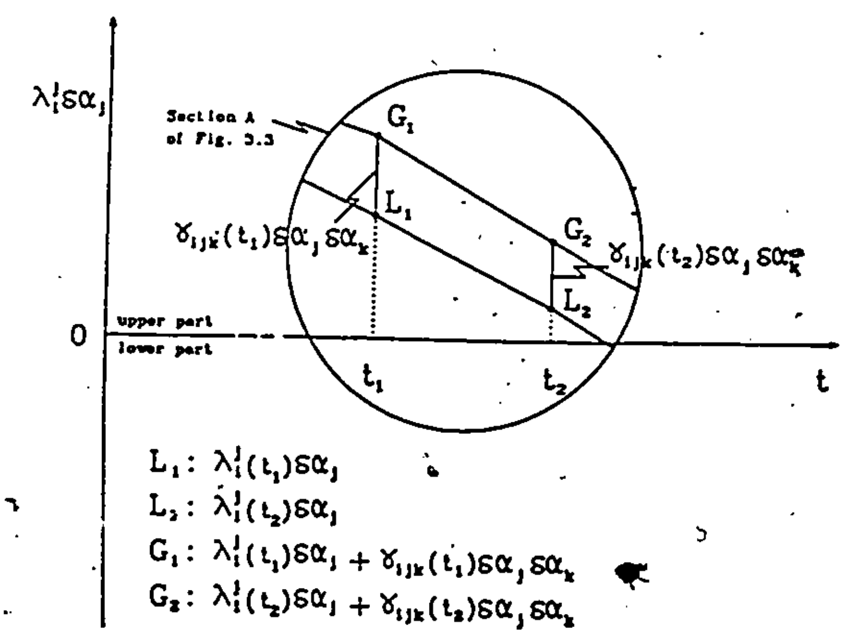

Fig. 3.4 Numerical calculation of double sensitivity index.

categories explanations. Each category has been labeled by a code which is called I_B. This code is a four digit binary integer $(b_1 b_2 b_3 b_4)$. Each bit of this code represents the position of one of the calculation area corner (rectangular $L_1 G_1 G_2 L_2$, Fig. 3.4). These relations have been shown in Table 3.1. b_i equals to one means ith corner is in upper part and b_i equals to zero means ith corner is in lower part.

i	corner	b_i	Mathematical Description
1	G_2	0	$\left[\lambda_i^j(t_2)\delta\alpha_j+\gamma_{ijk}(t_2)\delta\alpha_j\delta\alpha_k\right]<0$
		1	$\left[\lambda_i^j(t_2)\delta\alpha_j+\gamma_{ijk}(t_2)\delta\alpha_j\delta\alpha_k\right]\geq 0$
2	G_1	0	$\left[\lambda_i^j(t_2)\delta\alpha_j+\gamma_{ijk}(t_2)\delta\alpha_j d\alpha_k\right]<0$
		1	$\left[\lambda_i^j(t_2)\delta\alpha_j+\gamma_{ijk}(t_2)\delta\alpha_j\delta\alpha_k\right]\geq 0$
3	L_2	0	$\left[\lambda_i^j(t_2)\delta\alpha_j\right]<0$
		1	$\left[\lambda_i^j(t_2)\delta\alpha_j\right]\geq 0$
4	L_1	0	$\left[\lambda_i^j(t_1)\delta\alpha_j\right]<0$
		1	$\left[\lambda_i^j(t_1)\delta\alpha_j\right]\geq 0$

Table 3.1 I_B code bits (b_i) description.

Fig. 3.5 shows one sample of different possibilities of each categories. Also Fig. 3.6 gives expressions for calculation of each step in different categories.

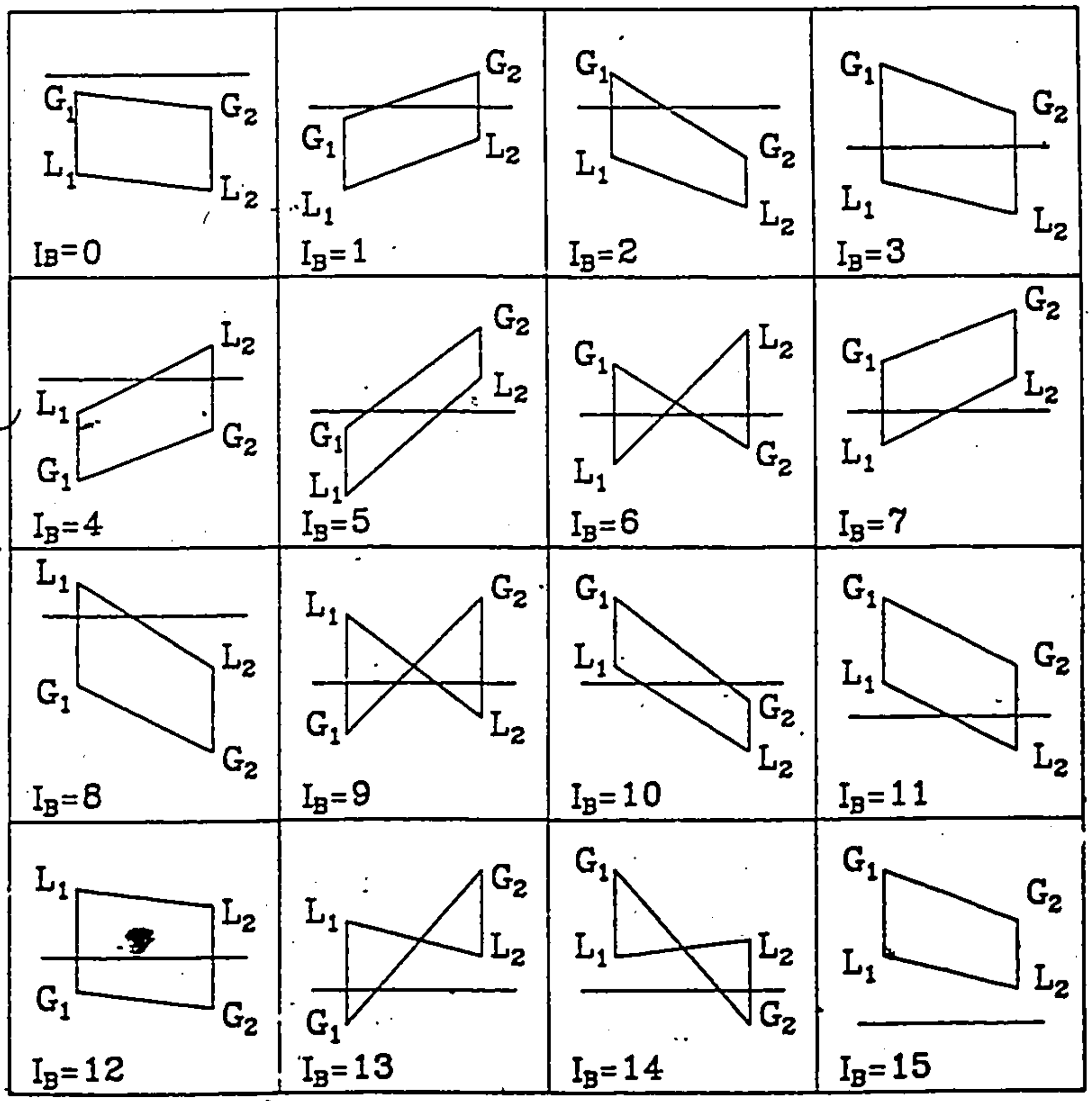

Fig. 3.5 One sample from each category.

I_B	ΔI_{dE}
0	$\left[\gamma_{ijk}(t_1)+\gamma_{ijk}(t_2)\right]\delta\alpha_j\delta\alpha_k\Delta t$
1	$-\left[\left[\gamma_{ijk}(t_1)+\gamma_{ijk}(t_2)\right]\delta\alpha_j\delta\alpha_k + 2\dfrac{\left[\lambda_i^j(t_2)\delta\alpha_j+\gamma_{ijk}(t_2)\delta\alpha_j\delta\alpha_k\right]^2}{\left[\lambda_i^j(t_1)\delta\alpha_j+\gamma_{ijk}(t_1)\delta\alpha_j\delta\alpha_k\right]-\left[\lambda_i^j(t_2)\delta\alpha_j+\gamma_{ijk}(t_2)\delta\alpha_j\delta\alpha_k\right]}\right]\Delta t$
2	$\left[\left[\gamma_{ijk}(t_1)+\gamma_{ijk}(t_2)\right]\delta\alpha_j\delta\alpha_k - 2\dfrac{\left[\lambda_i^j(t_2)\delta\alpha_j\right]^2}{\lambda_i^j(t_1)\delta\alpha_j - \lambda_i^j(t_2)\delta\alpha_j}\right]\Delta t$
3	$I_{S1}\left[\left[\gamma_{ijk}(t_1)+\gamma_{ijk}(t_2)\right]\delta\alpha_j\delta\alpha_k + 2\left[\dfrac{\left[\lambda_i^j(t_2)\delta\alpha_j+\gamma_{ijk}(t_2)\delta\alpha_j\delta\alpha_k\right]^2}{\left[\lambda_i^j(t_1)\delta\alpha_j+\gamma_{ijk}(t_1)\delta\alpha_j\delta\alpha_k\right]-\left[\lambda_i^j(t_2)\delta\alpha_j+\gamma_{ijk}(t_2)\delta\alpha_j\delta\alpha_k\right]} - \dfrac{\left[\lambda_i^j(t_2)\delta\alpha_j\right]^2}{\lambda_i^j(t_1)\delta\alpha_j - \lambda_i^j(t_2)\delta\alpha_j}\right]\right]\Delta t$ where I_{S1} is the sign of $\left[\lambda_i^j(t_1)\delta\alpha_j+\gamma_{ijk}(t_1)\delta\alpha_j\delta\alpha_k\right]$
4	$\left[\left[\gamma_{ijk}(t_1)+\gamma_{ijk}(t_2)\right]\delta\alpha_j\delta\alpha_k + 2\dfrac{\left[\lambda_i^j(t_1)\delta\alpha_j+\gamma_{ijk}(t_1)\delta\alpha_j\delta\alpha_k\right]^2}{\left[\lambda_i^j(t_2)\delta\alpha_j+\gamma_{ijk}(t_2)\delta\alpha_j\delta\alpha_k\right]-\left[\lambda_i^j(t_1)\delta\alpha_j+\gamma_{ijk}(t_1)\delta\alpha_j\delta\alpha_k\right]}\right]\Delta t$
5	$\left[\left[\gamma_{ijk}(t_1)+\gamma_{ijk}(t_2)\right]\delta\alpha_j\delta\alpha_k - 2\left[\left(\lambda_i^j(t_2)\delta\alpha_j+\gamma_{ijk}(t_2)\delta\alpha_j\delta\alpha_k\right) - \left(\lambda_i^j(t_1)\delta\alpha_j+\gamma_{ijk}(t_1)\delta\alpha_j\delta\alpha_k\right)\right]\right]\Delta t$
6	$I_{S2}\left[\left[\gamma_{ijk}(t_1)+\gamma_{ijk}(t_2)\right]\delta\alpha_j\delta\alpha_k + 2\left[\dfrac{\left[\lambda_i^j(t_1)\delta\alpha_j+\gamma_{ijk}(t_1)\delta\alpha_j\delta\alpha_k\right]^2}{\left[\lambda_i^j(t_2)\delta\alpha_j+\gamma_{ijk}(t_2)\delta\alpha_j\delta\alpha_k\right]-\left[\lambda_i^j(t_1)\delta\alpha_j+\gamma_{ijk}(t_1)\delta\alpha_j\delta\alpha_k\right]} - \dfrac{\left[\lambda_i^j(t_2)\delta\alpha_j\right]^2}{\lambda_i^j(t_1)\delta\alpha_j - \lambda_i^j(t_2)\delta\alpha_j}\right]\right]\Delta t$ where I_{S2} is the sign of $\left[\lambda_i^j(t_2)\delta\alpha_j+\gamma_{ijk}(t_2)\delta\alpha_j\delta\alpha_k\right]$
7	$-\left[\left[\gamma_{ijk}(t_1)+\gamma_{ijk}(t_2)\right]\delta\alpha_j\delta\alpha_k + 2\dfrac{\left[\lambda_i^j(t_1)\delta\alpha_j\right]^2}{\lambda_i^j(t_1)\delta\alpha_j - \lambda_i^j(t_2)\delta\alpha_j}\right]\Delta t$
8	$\left[\left[\gamma_{ijk}(t_1)+\gamma_{ijk}(t_2)\right]\delta\alpha_j\delta\alpha_k - 2\dfrac{\left[\lambda_i^j(t_1)\delta\alpha_j\right]^2}{\lambda_i^j(t_2)\delta\alpha_j - \lambda_i^j(t_1)\delta\alpha_j}\right]\Delta t$

Fig. 3.6 Expressions for numerical calculation in each category.

I_B	ΔI_{dE}
9	$I_{S2}\left[\left[\gamma_{ijk}(t_1)+\gamma_{ijk}(t_2)\right]\delta\alpha_j\delta\alpha_k + 2\left[\dfrac{\left[\lambda_i^j(t_1)\delta\alpha_j+\gamma_{ijk}(t_1)\delta\alpha_j\delta\alpha_k\right]^2}{\left[\lambda_i^j(t_2)\delta\alpha_j+\gamma_{ijk}(t_2)\delta\alpha_j\delta\alpha_k\right]-\left[\lambda_i^j(t_1)\delta\alpha_j+\gamma_{ijk}(t_1)\delta\alpha_j\delta\alpha_k\right]} - \dfrac{\left[\lambda_i^j(t_2)\delta\alpha_j\right]^2}{\lambda_i^j(t_1)\delta\alpha_j - \lambda_i^j(t_2)\delta_{a_j}}\right]\right]\Delta t$ where I_{S2} is the sign of $\left[\lambda_i^j(t_2)\delta\alpha_j+\gamma_{ijk}(t_2)\delta\alpha_j\delta\alpha_k\right]$
10	$\left[\left[\gamma_{ijk}(t_1)+\gamma_{ijk}(t_2)\right]\delta\alpha_j\delta\alpha_k + 2\left[\lambda_i^j(t_2)\delta\alpha_j + \lambda_i^j(t_1)\delta\alpha_j\right]\right]\Delta t$
11	$-\left[\left[\gamma_{ijk}(t_1)+\gamma_{ijk}(t_2)\right]\delta\alpha_j\delta\alpha_k - 2\dfrac{\left[\lambda_i^j(t_1)\delta\alpha_j+\gamma_{ijk}(t_1)\delta\alpha_j\delta\alpha_k\right]^2}{\left[\lambda_i^j(t_1)\delta\alpha_j+\gamma_{ijk}(t_1)\delta\alpha_j\delta\alpha_k\right]-\left[\lambda_i^j(t_2)\delta\alpha_j+\gamma_{ijk}(t_2)\delta\alpha_j\delta\alpha_k\right]}\right]\Delta t$
12	$I_{S1}\left[\left[\gamma_{ijk}(t_1)+\gamma_{ijk}(t_2)\right]\delta\alpha_j\delta\alpha_k + 2\left[\dfrac{\left[\lambda_i^j(t_2)\delta\alpha_j+\gamma_{ijk}(t_2)\delta\alpha_j\delta\alpha_k\right]^2}{\left[\lambda_i^j(t_1)\delta\alpha_j+\gamma_{ijk}(t_1)\delta\alpha_j\delta\alpha_k\right]-\left[\lambda_i^j(t_2)\delta\alpha_j+\gamma_{ijk}(t_2)\delta\alpha_j\delta\alpha_k\right]} - \dfrac{\left[\lambda_i^j(t_2)\delta\alpha_j\right]^2}{\lambda_i^j(t_1)\delta\alpha_j - \lambda_i^j(t_2)\delta_{a_j}}\right]\right]\Delta t$ where I_{S1} is the sign of $\left[\lambda_i^j(t_1)\delta\alpha_j+\gamma_{ijk}(t_1)\delta\alpha_j\delta\alpha_k\right]$
13	$-\left[\left[\gamma_{ijk}(t_1)+\gamma_{ijk}(t_2)\right]\delta\alpha_j\delta\alpha_k + 2\dfrac{\left[\lambda_i^j(t_2)\delta\alpha_j\right]^2}{\lambda_i^j(t_2)\delta\alpha_j - \lambda_i^j(t_1)\delta\alpha_j}\right]\Delta t$
14	$-\left[\left[\gamma_{jik}(\text{[illegible]})+\gamma_{jik}(t_2)\right]\delta\alpha_j\delta\alpha_k + 2\dfrac{\left[\lambda_i^j(t_2)\delta\alpha_j+\gamma_{ijk}(t_2)\delta\alpha_j\delta\alpha_k\right]^2}{\left[\lambda_i^j(t_1)\delta\alpha_j+\gamma_{ijk}(t_1)\delta\alpha_j\delta\alpha_k\right]-\left[\lambda_i^j(t_2)\delta\alpha_j+\gamma_{ijk}(t_2)\delta\alpha_j\delta\alpha_k\right]}\right]\Delta t$
15	$\left[\gamma_{jik}(t_1)+\gamma_{jik}(t_2)\right]\delta\alpha_j\delta\alpha_k\Delta t$

Fig. 3.6 Continue.

APPLICATION of SENSITIVITY ANALYSIS

PARAMETER CHANGES
in
NONLINEAR HYDRAULIC CONTROL SYSTEMS

Said Farahat

A Thesis

in

The Departement

of

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C O N C O R D I A   U N I V E R S I T Y

Graduate Studies

This is to certify that the thesis prepared

by: Said Farahat

Entitled: Application of Sensitivity Analysis to Parameter Changes in Nonlinear Hydraulic Control Systems

and submitted in partial fulfillment of the requirements for the degree

of Master of Engineering (Mechanical)

complies with the regulations of this University and meets the accepted standards with respect to originality and quality.

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ABSTRACT

Application of Sensitivity Analysis
to
Parameter Changes
in
Nonlinear Hydraulic Control Systems

Said Farahat

In this thesis, the sensitivity analysis is applied to a novel electrohydraulic servovalve which is a nonlinear system. This system sensitivity study differs from previous studies by considering the dynamic behaviour and nonlinearity of the system performance. Four different sensitivity analysis methods are compared to each other by studying the sensitivity of the actuator piston velocity of above servovalve with respect to eighteen parameters. All the methods show that the area of the actuator piston is the most sensitive parameter and the static friction is the insensitive parameter.

By using the best method among the above mentioned methods, the sensitivity of the state variables of the sample system (other than the velocity of the actuator piston) have been studied. It is shown that the sensitivity of the actuator piston velocity and the opening area of the servovalve in one hand and pressures of the either sides of the actuator piston

in the other hand are almost similar. Also these pressures are very sensitive to the orifice opening, although two other state variables almost are not sensitive to this parameter.

Having studied the system sensitivity with nominal value of the parameters, the most five sensitive parameters (K_a, M, A_p, C_d, V_{in}) have been chosen for locating insensitive system. For this purpose one of these parameters has been changed at a time and the system sensitivities behaviour with respect to different parameters have been evaluated. Study of these behaviours shows that most of them are almost linear except for the system sensitivity with respect to the parameter Mass (total mass in motion) according to the parameter A_p (area of the actuator piston) changes. This behaviour increases dramatically by decreasing the parameter A_p, and it seems it will become unstable by decreasing A_p more than 50%.

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