On the Concept of Force: A Comment on Lopes Coelho

Calvin S. Kalman,
Department of Physics Concordia University
Montreal, QC, Canada H4B 1R6
and
Department of Educational and Counseling Psychology, McGill University
Montreal, QC, Canada, H3A 2T5
Phone (514) 848-2424 x3284
Fax (514) 848-2828
Email Calvin.Kalman@Concordia.ca

Abstract: This article presents a supplement to Coelho's excellent article concerning the definition of force by first defining mass and then momentum. Replacing force with the concept of a field is also briefly noted.

Coelho (2010) has given an excellent presentation of "the concepts and criticism of force in the works of Newton, Euler, d'Alembert, Lagrange, Lazare Carnot, Saint-Venant, Reech, Kirchhoff, Mach, Hertzand Poincare … an overview of definitions of force in contemporary textbooks … an answer to the question is given: how to understand force within the framework of the laws of motion." In this article, I present a supplementary view on understanding force within the laws of motion. This is followed by a brief note on replacing force with the concept of a field.

> As the equation referred to as Newton's second axiom (F = ma) is composed of three variables, the definition of mass will have to be given by force and acceleration. As force is defined by the same equation, it follows that it depends on what mass and acceleration are. As, however, what mass might be depends on force, we remain not knowing what both are. This kind of definition was criticized by Mach in 1868, [Coelho (p.105)]

Arons (1990, p. 51) points out that there are two ways of properly approaching the first law. Firstly, "in Mach's sequence (Mach, 1983) inertial mass is defined first." Force is then defined through Newton's second law. In the second method, which Arons calls "Newtonian", force is defined first and then mass is defined through Newton's second law. Arons has discussed the "Newtonian" method in detail. Following Weinstock (1961) Arons also briefly mentions following Mach's sequence in terms of accelerating bodies. Coelho asks "Let us consider if the defining of force could be improved" (p. 105). Let us see how this could be done. Suppose that we define the mass first. Then in my opinion, it is better to start with bodies moving with different velocities on an airtable. This device consists of a horizontal table over which jets of air move at high velocity. Fairly heavy discs or pucks placed on the table are supported by the air currents so that there is very little resistance to the motion of the pucks on the table. Consider two such pucks of different weights set in motion at two different velocities, v_1,v_2 for a head on collision. After colliding the pucks rebound with velocities v'_1,v'_2. In repeating this experiment many times with different initial velocities v_1,v_2 and carefully measuring the velocity of rebound v'_1,v'_2 we discover that in every case

$$\frac{\Delta v_1}{\Delta v_2} = -c_{12} \qquad (1)$$

where c_{12} is a constant, that is it is independent of the initial velocities and the rebound velocities v'_1,v'_2. Indeed c_{12} depends only upon the particular bodies set in motion. Now since v_1 and v_2 are in opposite directions, it would seem likely and indeed it does happen that $\frac{\Delta v_1}{\Delta v_2}$ is always negative so that c_{12} is always positive.

In this experiment the state of motion of each puck alters since the velocities v_1,v_2 of the puck change. The positive constant c_{12} of eq. 1 has something to do with the relative difficulty of changing the states of motion of the two pucks. Let us call the measure of the resistance of a body to changes in its state of motion the
[inertial] mass of the body. Now let us select one particular body to have unit mass. Then we define the [inertial] mass m_2 of any other body to be the value of the constant c_{12} obtained in a collision on an airtable with the body of unit mass:

$$\Delta v_1 = = -m_2\Delta v_2 \qquad (2)$$

To make the concept of inertial mass clear we can present to students the example of a toy car and a regular car traveling at the same velocity. It is harder to stop the regular car than the toy car. We can state that it is harder to stop the regular car than the toy car because the regular car has a much larger mass than the toy car. We can then state that in addition to mass, another factor must be taken into account in calculating the resistance of a body to a change in its state of motion; the faster a body is moving the harder it is to stop. The complete measure of resistance to change in the motion of a body is the momentum, p:

momentum $p = mv$ (3)

The state of motion of a body is characterized by the momentum mv of a body. The greater the value of the magnitude of the momentum mv of a body, the harder it is to stop the body in a given elapsed time. Indeed Newton characterized the “quantity of motion” of a body by “mv”.

Precisely what then is force? According to the first law, bodies remain in their state of motion unless acted upon by an external force. As presented then force must produce a change in the state of motion of a body. Now since the state of motion is characterized by the momentum of the body, $p = mv$, $\frac{d\mathbf{p}}{dt}$ the instantaneous rate of change of the momentum, must represent the change in the state of motion produced by the action of a force. This is indeed the form in which Newton presented the second law: the sum of forces acting on a body is given by $\boxed{\Sigma \mathbf{F} \; \alpha \; \frac{d\mathbf{p}}{dt}}$.

It is convenient to choose the units of force so that the proportionality constant equals 1. Then if the mass is constant, we have the usual form of

Newton's second law, **F**=m**a**.

Physics today, replaces the concept of force with the concept of the field. The gravitational attraction between two bodies of respective [gravitational] masses m_1 and m_2 is given by Newton's Universal law of gravity:

$$\mathbf{F} = G\,\frac{m_1\,m_2}{r^2}\,\hat{\mathbf{r}} \tag{4}$$

In eq. 4, mass is seen as the source of a gravitational attraction between any two bodies in the universe. Then in the equation

$$G\,\frac{m_1\,m_2}{r^2}\,\hat{\mathbf{r}} = \mathbf{F} = m_1\mathbf{g}(\mathbf{r})\;, \tag{5}$$

mg can be thought of not as Newton's law F=ma applied to the specific case of gravity, but rather as the specific prescription of how gravity affects a body of mass m_1 at a specific distance r from a second body of mass m_2. The affect of gravity on the body of mass m_1 is caused by the presence of the body of mass m_2. In this case

$$\mathbf{g}(\mathbf{r}) = \frac{\mathbf{F}}{m_1} = G\,\frac{m_2}{r^2}\,\hat{\mathbf{r}}\;, \tag{6}$$

represents the presence of gravity emanating from the body of mass m_2. The body of mass m_1 located at the point r then does not experience a force at a distance caused by the body of mass m_2, but instead interacts with the gravitational field $g(\hat{\mathbf{r}})$. A pictorial representation of the field can be obtained by drawing "lines of force" entering each body. The result is figure 1. The direction of a line of force is the direction of the force produced by the action of the gravitational field on a small "test body".

Arons AB (1990) A guide to introductory physics teaching. Wiley, New York.

Coelho RL (2010) On the Concept of Force: How Understanding its History can Improve Physics Teaching. Sci Edu 19:91–113

Mach E (1893), The Science of Mechanics. Open Court Publishing Company, Chicago.

Weinstock R (1961) Laws of Classical Motion. What's F? What's m? What's a?, Am J Phys 29(10): 698-702

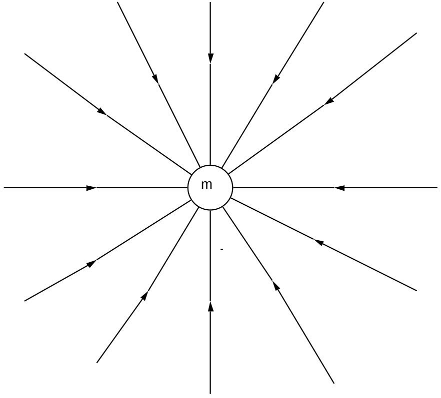

Fig. 1. Lines of force entering a body of mass m