Integrated Production-Inventory Models in Steel Mills Operating in a Fuzzy Environment

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ABSTRACT

Integrated Production-Inventory Models in Steel Mills Operating in a Fuzzy Environment

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Despite the paramount importance of the steel rolling industry and its vital contributions to a nation's economic growth and pace of development, production planning in this industry has not received as much attention as opposed to other industries. The work presented in this thesis tackles the master production scheduling (MPS) problem encountered frequently in steel rolling mills producing reinforced steel bars of different grades and dimensions. At first, the production planning problem is dealt with under static demand conditions and is formulated as a mixed integer bilinear program (MIBLP) where the objective of this deterministic model is to provide insights into the combined effect of several interrelated factors such as batch production, scrap rate, complex setup time structure, overtime, backlogging and product substitution, on the planning decisions.

Typically, MIBLPs are not readily solvable using off-the-shelf optimization packages necessitating the development of specifically tailored solution algorithms that can efficiently handle this class of models. The classical linearization approaches are first discussed and employed to the model at hand, and then a hybrid linearization-Benders decomposition technique is developed in order to separate the

complicating variables from the non-complicating ones. As a third alternative, a modified Branch-and-Bound (B&B) algorithm is proposed where the branching, bounding and fathoming criteria differ from those of classical B&B algorithms previously established in the literature. Numerical experiments have shown that the proposed B&B algorithm outperforms the other two approaches for larger problem instances with savings in computational time amounting to 48%.

The second part of this thesis extends the previous analysis to allow for the incorporation of internal as well as external sources of uncertainty associated with end customers' demand and production capacity in the planning decisions. In such situations, the implementation of the model on a rolling horizon basis is a common business practice but it requires the repetitive solution of the model at the beginning of each time period. As such, viable approximations that result in a tractable number of binary and/or integer variables and generate only exact schedules are developed. Computational experiments suggest that a fair compromise between the quality of the solutions and substantial computational time savings is achieved via the employment of these approximate models.

The dynamic nature of the operating environment can also be captured using the concept of fuzzy set theory (FST). The use of FST allows for the incorporation of the decision maker's subjective judgment in the context of mathematical models through flexible mathematical programming (FMP) approach and possibilistic programming (PP) approach. In this work, both of these approaches are combined where the volatility in demand is reflected by a flexible constraint expressed by a fuzzy set having a triangular membership function, and the production capacity is expressed as

a triangular fuzzy number. Numerical analysis illustrates the economical benefits obtained from using the fuzzy approach as compared to its deterministic counterpart.

Dedicated to

… My family …

For their endless love and support, and having always believed in me no matter what

… My fiancée …

For giving my life a whole different meaning and making it so wonderful

I cannot wait to meet with you all after this time that we had to spend apart

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I won’t forget to thank the person who enlightens my life day after day, my fiancée. I could not have asked for a better soul mate than her and I cannot wait to share every moment of my life with her.

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Rami Afif As'ad

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LIST OF ACRONYMS

PPC	-	Production planning and control
DM	-	Decisions maker
FP	-	Finished product (steel bars)
RM	-	Raw material (steel billets)
SG	-	Steel grade
MPS	-	Master production schedule
BOM	-	Bill of material
DLSP	-	Dynamic lot-sizing problem
CLSP	-	Capacitated lot-sizing problem
USILSP	-	Uncapacitated single item lot-sizing problem
CSILSP	-	Capacitated single item lot-sizing problem
CMILSP	-	Capacitated multi item lot-sizing problem
NP	-	Non polynomial
DP	-	Dynamic programming
MTS	-	Make-to-stock
MTO	-	Make-to-order
LP	-	Linear program
MILP	-	Mixed integer linear program
MINLP	-	Mixed integer nonlinear program
MIBLP	-	Mixed integer bilinear program
FMIBLP	-	Fuzzy mixed integer bilinear program
FMP	-	Flexible mathematical programming
PP	-	Possibilistic programming
QP	-	Quadratic program
FST	-	Fuzzy set theory
MF	-	Membership function
TMF	-	Triangular membership function
TFN	-	Triangular fuzzy number

RLT	-	Reformulation-linearization technique
B&B	-	Branch-and-bound
BD	-	Benders decomposition
CV	-	Complicating variable
NCV	-	Non-complicating variable
MP	-	Master problem
SP	-	Sub-problem

LIST OF SYMBOLS

Indices:

i : Index of available RM (billets) dimensions (cross sectional area and length), $i = 1,...,I$

j : Index of FP (rebars) dimensions (cross-sectional area), $j = 1,.....,J$

k : Index of steel grades, $k = 1$ for grade 60 and $k = 2$ for grade 40, with grade 60 being better.

t : Index of time periods (days), $t = 1,.....,T$, where T is the planning horizon.

Input Parameters:

CR_{it}^{k} : Cost of purchasing one unit of RM *i* of SG *k* in time period *t*.

OR_{it}^{k} : Fixed cost of ordering RM *i* of SG *k* in time period *t* (independent of the order quantity).

IR_{it}^{k} : Cost of holding one unit of raw material *i* of SG *k* in stock for one time period (from t to $t+1$).

PC_{ijt} : Cost of producing one unit of FP *j* from RM *i* in time period *t*.

PO_{t} : Overtime production cost per hour in period *t*.

SC_{t} : Production line setup cost per hour in period *t*.

IF_{jt}^{k} : Cost of holding one unit of FP *j* of SG *k* in stock for one time period (from t to $t+1$).

BC_{jt}^{k} : Cost of backlogging one unit of FP *j* of SG *k* for one time period (from t to $t+1$).

SP_{jt}^{k} : Selling price of one unit of FP *j* of SG *k* in period *t*.

M_{it}^{k} : Upper limit on the supply capacity of RM *i* of SG *k* in period *t*.

ρ_{ij} : Yield resulting from producing FP *j* using RM *i* (independent of the steel grade), $0 \le \rho_{ij} < 1$ ($\rho_{ij} = 0$ if RM *i* is not to be used in the production of FP *j*).

α_{ij} : Rate of producing FP *j* from RM *i*.

D_{jt}^{k} : Anticipated demand for FP *j* of steel grade *k* in time period *t*.

A_t : Available regular production time in period *t* (in hours).

A_{ot} : Maximum allowable overtime production hours in period *t*.

b_t : Fixed production batch size (60 tons).

ST_{ij} : Minor setup time for a batch of FP *j* produced from RM *i*.

$\tilde{\tau}_{ij}$: Fuzzy production time per ton of FP *j* produced from RM *i*

$\tilde{A}_t$: Fuzzy total available production time in period *t*.

θ : Minimum acceptable possibility level.

Decision variables:

Q_{it}^{k} : Quantity of RM *i* of SG *k* purchased in period *t*.

G_{it}^{k} : $\begin{cases} 1 & \text{if RM } i \text{ of SG } k \text{ is purchased in period } t \\ 0 & \text{Otherwise} \end{cases}$

Sb_{ijt}^{k} : $\begin{cases} 1 & \text{if a setup for FP } j \text{ made from RM } i \text{ both having SG } k \text{ is carried out at the beginning of period } t \text{ (major setup).} \\ 0 & \text{Otherwise} \end{cases}$

Sd_{ijt}^{k} : Number of minor setups for FP *j* produced from RM *i* both having SG *k* conducted during period *t* (between batches).

S_{ijt}^{k} : Total setup time in period *t* for FP *j* produced from RM *i* both having SG *k*.

X_{ijt}^{k} : Quantity of FP *j* of SG *k* produced from RM *i* during period *t*.

O_t : Overtime production capacity used in period *t* (in hours).

W_{jt}^{k} : Quantity of FP *j* of SG *k* used to satisfy the demand for the corresponding product (i.e. same dimensions) with SG 40 in time period *t*.

I_{it}^{k} : Inventory level for RM *i* of SG *k* at the end of period *t*.

I_{jt}^{k} : Inventory level for FP *j* of SG *k* at the end of period *t*.

B_{jt}^{k} : Backlogging level for FP *j* of SG *k* at the end of period *t*.

Chapter 1

Introduction

1.1 Background and Motivation

The intensive competition in today's operating environment has made it increasingly important for industrial enterprises to continuously seek the best practices towards managing their operations and, eventually, differentiate themselves from their competitors. In particular, the optimization of production and inventory related decisions provides a vital step towards a better fulfillment of customers' needs at a minimum cost. Such decisions determine the required machining capacity, workforce levels, space utilization among other factors, which all combined have a direct impact on the financial health of an organization.

Steel manufacturing, for instance, represents one of the backbone industries greatly affecting a nation's economic growth and pace of development. Needless to say, a substantial portion of today's indispensable products that are used on a daily basis and serve multiple purposes have steel ingredient in them, in one form or another, ranging from sophisticated high-tech products such as cars and airplanes, to much simpler products such as kitchen utensils. Steel mills, which are the focus of this research, produce a variety of the most essential products including steel wires, pipes, bars, rods and sheets. In North America only, more than 100 million tons of steel are produced annually with an estimated value of over 50 billion dollars (Denton *et al.* 2003).

In general, the iron and steel industry is characterized by being both capital and energy intensive and, as such, the importance of effective production planning in such industry is by no means less than that in any other industry. For a rolling mill producing between 300,000 and one million tons of steel annually, the capital investment is measured in tens of millions of dollars (Denton *et al.* 2003). Upon realizing the major investments associated with the construction and operations of steel plants, the main concern of steel manufacturers has been the adoption of the latest technology advances in the production process as well as finding better ways to manage the rapid increase in product variety. However, in spite of the significance of steel industry, planning and scheduling problems in iron and steel production have not drawn as wide attention of the production and operations management research community as many other industries such as metal cutting and electronics industry (Tang *et al.* 2001). The work presented in this thesis targets this deficiency and bridges the gap between theory and practice through dealing with a realistic case study encountered in steel rolling mills. From a theoretical standpoint, the production planning problem addressed in this thesis falls under the broad class of the well known dynamic lot-sizing problem (DLSP). However, several practical extensions are incorporated in order to account for the technological constraints associated with the manufacturing process. Hence, it is important to step back and establish the theoretical background behind the basic DLSP and shed some light on some of its distinguishing characteristics.

1.2 A closer look at the dynamic lot-sizing problem

Typically, this class of problems refers to medium term planning decisions in which the production mix and quantities are usually planned ahead on a weekly or a monthly basis. The planning horizon is divided into a number of discrete time intervals of equal length, each having its own demand, hence the term dynamic. In a manufacturing firm context, a lot refers to the items produced consecutively, one after the other, on the same machine, production line, or facility since the last setup. The lot size is simply the number of items contained in a particular lot. Hence, lot-sizing is the activity to obtain simultaneously in which period, and in which number (i.e. lot size) different items should be produced such that the production plan is feasible. In general, making the right decisions in lot-sizing will affect directly the system performance and its productivity, which are important for a manufacturing firm's ability to compete in the market (Karimi *et al.* 2003).

However, a clear distinction has to be made between two different types of the LSPs. First, the continuous time scale, constant demand and infinite time scale lot sizing problems. The credit of initiating the work related to this problem goes to Harris (1913) where he introduced the widely used nowadays economic order quantity (EOQ) model. The basic economic manufacturing quantity (EMQ) model and the economic lot scheduling problem (ELSP) also fall under this category. The other class deals with the discrete time scale, dynamic demand and finite time horizon problems, usually referred to as the dynamic lot sizing problem (DLSP). Typically, the latter case involves the development of a mathematical model that seeks the minimum setup cost, production cost and inventory holding cost. In reality, these costs differ from one item to another depending on the complexity and size of the item. Other types of costs include backorder, lost sales, outsourcing and rework cost depending on the context of the problem. The simplest form of the DLSP, which is the uncapacitated single item lot-sizing problem (USILSP), has been easily formulated in the literature as a mixed integer linear program (MILP) as follows:

Minimize $$Z = \sum_{t=1}^{T} \left(S_t Y_t + C_t X_t + h_t I_t \right) \quad (1.1)$$

Subject to

$$I_t = I_{t-1} + X_t - d_t \qquad t = 1, \ldots.., T \quad (1.2)$$

$$X_t \le M_t Y_t \qquad t = 1, \ldots.., T \quad (1.3)$$

$$Y_t \in \{0,1\} \qquad t = 1, \ldots.., T \quad (1.4)$$

$$X_t, I_t \ge 0 \qquad t = 1, \ldots.., T \quad (1.5)$$

Where:

S_t : Setup cost in period t

C_t : Variable unit production cost in period t

h_t : Unit inventory holding cost in period t

d_t : Demand in period t

$$Y_t = \begin{cases} 1 & \text{if } X_t > 0 \\ 0 & \text{Otherwise} \end{cases}$$

X_t : Production quantity in period t

I_t : Inventory at the end of period t

$$M_t = \sum_{k=t}^{T} d_k$$

The objective is to minimize the total cost composed of setup, production and inventory holding costs. Constraints (1.2) represent the inventory balance equations, and constraints (1.3) are the fixed charge constraints which establish the relation between the binary variable Y_t and the continuous variable X_t, and also set an upper bound on the production quantity per period.

1.3 Characteristics of the dynamic lot-sizing models

The DLSP covers a wide spectrum of production planning problems encountered in several industrial applications. Depending on the features of each, the DLSPs range from simple ones, which can be solved to optimality with exact algorithms such as the USILSP, to much more complicated ones (NP-complete) for which no optimal solution exists and a heuristic solution is adopted. Figure 1.1 provides a classification of the DLSPs based on the characteristics distinguishing them from one another. The figure combines and extends the work of Karimi *et al.* (2003) and Haase (1994).

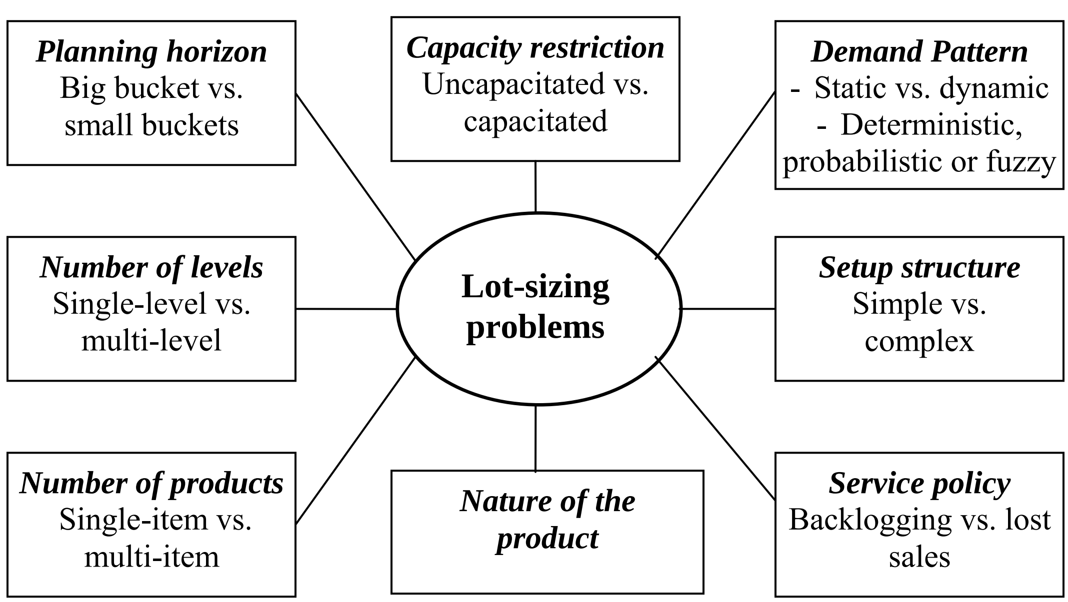

Figure 1.1: Characteristics of the dynamic lot-sizing problem

1.3.1 Planning Horizon

The planning horizon for the DLSP is usually finite and it might be quite short that only one item can be produced in that period, or long enough to accommodate the production of several items within the same period. In the latter case, the problem is called big bucket where this differs from the small bucket, the first case, in the fact that it also considers the sequence of the production lots, which gives rise to the well known lot sizing and scheduling problem. In the literature, lot sizing and scheduling decisions are usually treated independently for simplifying the overall decision making problem (Ozdamar *et al.* 1998).

1.3.2 Number of levels

Under this characteristic, the problems are classified as single-level or multi-level. When there does not exist a parent-component relationship between the items, or when the product is simple in the sense that the end product is directly produced from the raw material with no intermediate subassemblies, a single-level problem arises. Examples include metal casting and forging operations. However, when there exists a parent-component relationship among the items such that the demand at one level depends on that for its parent's level, we have a multi-level problem. Three types of multi-level problems can be further distinguished based on the product structure. These are: serial, assembly and general as shown in Figure 1.2 below. Obviously, the single level problems are a lot easier to solve as compared to the multi-level models.

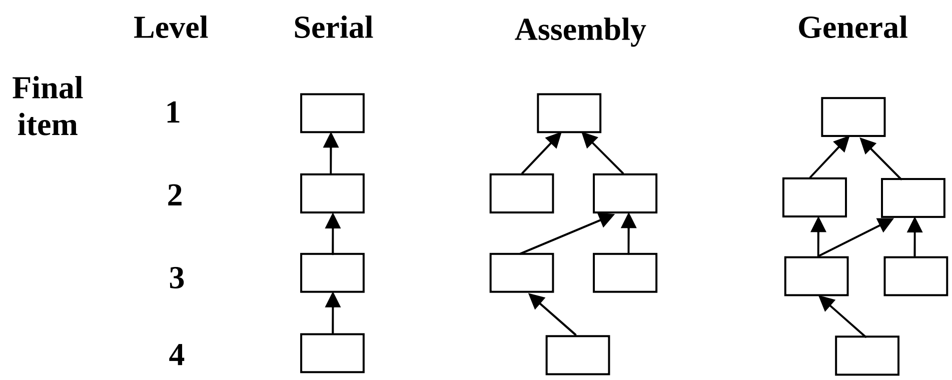

Figure 1.2: Various product structures

1.3.3 Number of products

The number of end items accounted for in a lot-sizing model greatly affects the complexity of the problem. If there is only one end item (final product) to plan for, then a single-item problem arises. On the contrary, if planning is carried out for multiple end items, the problem becomes more involved and it is a multi-item planning problem.

1.3.4 Resource constraints

In practice, a resource might refer to a machine capacity, storage space, available budget or manpower. Once abundant amount of resources exist, the problem is coined as "uncapacitated". When there exist a restriction on one of the resources, e.g., production capacity, which resembles most practical real life systems, the problem is "capacitated" which adds another dimension of complexity to the model.

1.3.5 Nature of the product

The nature of the end items also affects the production strategy, as perishable items shall not be produced way ahead of the time they are actually needed since they might become obsolete while still in stock. The diary products set a good example for such items. Conversely, automobiles, for instance, represent those items which deteriorate over time reducing their value once sold after being in stock for a long time. Non-deteriorating items, those maintaining their value regardless of the storage period, are easier to deal with as it might be more economical to produce them ahead of time in periods of excess capacity.

1.3.6 Demand Pattern

The firm might be facing a static demand over time, or one that changes from a time period to another (dynamic demand). Viewing it from a different perspective, if the demand is known with a high degree of certainty, then it is called “deterministic”. However, obtaining an exact value of the market demand might not be easily achievable or even not at all in case of a highly volatile demand, which necessitates the use of probability distributions to represent the “stochastic demand”. Another pattern that is commonly overlooked by researchers is dealing with the demand as a fuzzy quantity. That is, instead of assigning it a single crisp value, the demand is represented by a set that spans over a certain range, with a likelihood value specifying the degree of compatibility of each element with the set. The range and the compatibility (membership value) for each element is decided upon through experience and human subjective judgment, as explained in Section 1.4.

1.3.7 Setup structure

Typically, whenever the manufacturing process is switched from producing one product to another, a setup activity is incurred, which entails a setup time as well as an associated setup cost. A clear exception is the labor based operations where machinery is not involved such as some assembly processes. While a simple setup is the one independent of the products sequence, a setup activity that depends on the sequence of the products gives rise to a more complicated problem, which can be modeled as the famous traveling salesman problem (TSP).

1.3.8 Service policy

The inventory levels maintained by the firm depend on its policy when it comes to demand fulfillment. Backlogging occurs when the demand of the current period cannot be satisfied on time and is thus satisfied in future periods. On the other end, a demand that is not satisfied momentarily in the same period with no chance of fulfilling it afterwards is lost and hence the title “lost sales”. The options of outsourcing or working overtime could also be utilized to handle the excess demand.

At this juncture, it is important to establish the equivalence between the master production scheduling (MPS) problem and a specific class of DLSP. Both MPS and the big-bucket, multiple-item, single-level DLSP establish the production quantities for the final product in each period. The inventory management literature usually coins the term DLSP to refer to this class of problems while the production planning researchers mostly coin the MPS term. In this thesis, the production planning problem is tackled at the MPS level and hence the two terms are used interchangeably.

1.4 Fuzzy set theory as applied to the lot-sizing problem

In reality, firms operate in a rapidly changing and constantly evolving environment, where several external factors such as market, technology, etc., play an important role. As a result, the certainty assumption imposed in most lot-sizing models is often unsatisfied as the estimation of model parameters is based on the prediction of future events. Whenever there is a high level of ambiguity involved, the Fuzzy set theory (FST) approach stands out as the favorable one. In the second and third parts of this thesis, we make use of the FST to account for the uncertainties associated with customer demand and production capacity. The suitability of choosing the fuzzy tool to represent these quantities is briefly discussed below.

1.4.1 Fuzzy demand

When dealing with a make-to-stock (MTS) environment, the decisions made in a typical lot-sizing problem are of medium range planning horizon, which have a typical time frame ranging from a week to several months. Since the main goal of a lot-sizing model is to optimally decide on the production quantities and timings for a certain number of time periods in the future, lots of uncertainties are involved due to the continuously evolving nature of the environment in which a firm operates which makes the forecasts for future customer demand less reliable. With the multi-item case, the uncertainty is even more obvious in the sense that it might be associated with the volume or the mix. This uncertainty that is associated with future demand can be specified based on experts' opinion and managerial subjective judgment. Possible representation for an uncertain demand using fuzzy sets is as follows:

(a) Demand is around d_m, but definitely not less than d_l and not greater than d_u. This is represented by a triangular fuzzy number.

(b) Demand definitely falls in the interval $\left[d_l, d_u\right]$ with a high degree of possibility to fall in the smaller interval $\left[d_{lm}, d_{um}\right]$. This is called a trapezoidal fuzzy number.

(c) Demand is much larger than d_l, represented by a linear membership function.

(d) Demand is much smaller than d_u, represented by a linear membership function.

The above natural language expressions can be represented as fuzzy sets with the possibility distributions shown in Figure 1.3.

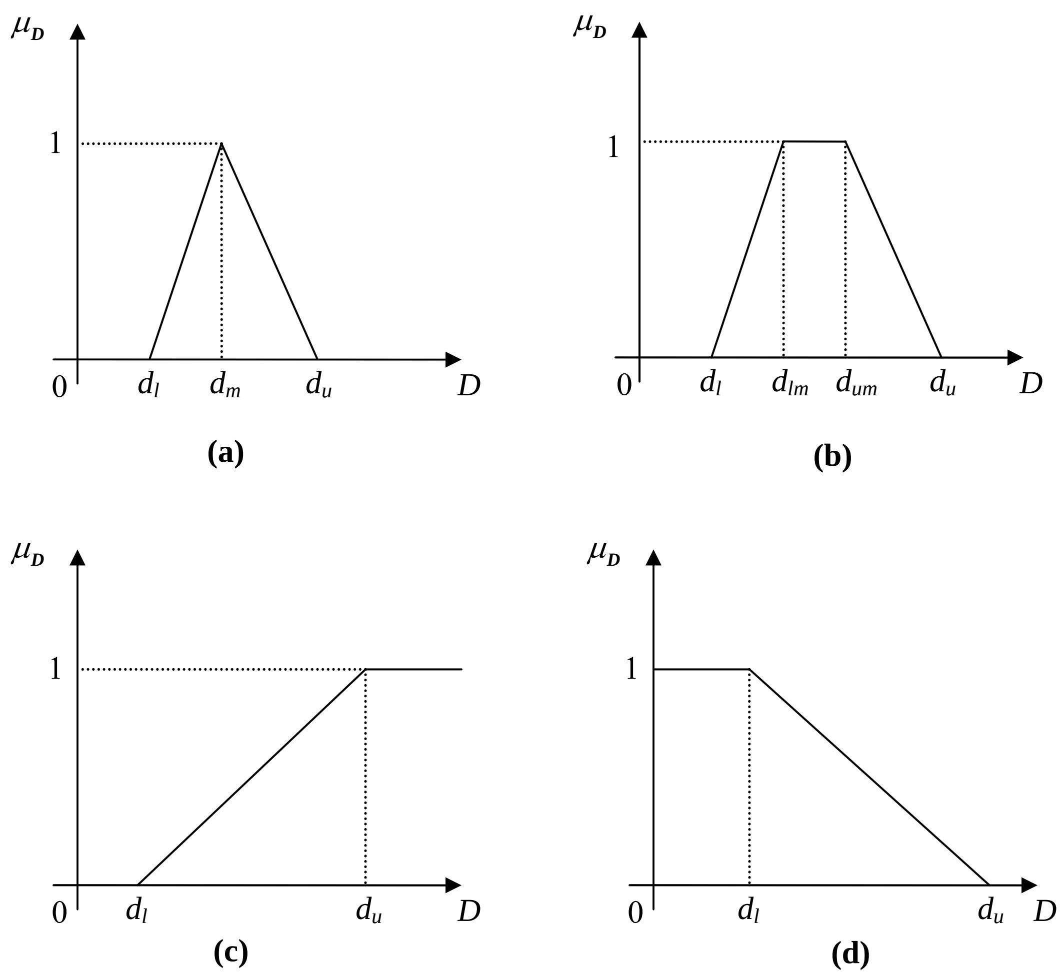

Figure 1.3: Alternative fuzzy sets to represent fuzzy demand

A membership function of fuzzy customer demand can be derived either from subjective manager belief or from its probability distribution if it exists (Dubois and Prade 1994). However, it is important to note that possibility distribution differs from the probability distribution both in principle and in practice. Petrovic *et al.* (1999) demonstrate the suitability of using fuzzy sets to describe customer demand through an example:

"Consider customer demand as in Figure 1.3 (b). Suppose that circumstances have brought into existence a strong belief that customer demand can be $2 + d_u$ *with possibility 1. In such a case, it is easy to modify the existing possibility distribution by simply adding a new possible value of demand with no other changes of the distribution. Let us notice that such an intervention, having a probability distribution, is not straightforward at all"*.

In some practical situations, the demand pattern can be adequately modeled with deterministic or probabilistic values. However, in other situations, the validity of such representation is highly questionable. Examples include:

1. A company entering a new market sector and aiming towards building a chain of new customers.

2. The product is newly introduced to the market, and the company has no clue of what the future demand pattern would look like.

3. In the absence of reliable historical data of the demand that are representative of the future demand, or when these data are not a trusted source any more due to a change in the operating environment or other factors (the global financial crisis that took place in 2008-2009 sets a good example for this case).

As the first to introduce the application of fuzzy set theory to the DLSP, Lee *et al.* (1990) identified two advantages of using fuzzy numbers and membership functions to model demand:

1. Fuzzy set theory allows both the uncertain demand and the subjective judgment of the decision maker to be incorporated into the lot-sizing decisions.
2. Fuzzy part period balancing, which is a heuristic used to solve the USILSP, provides a richer source of data for the decision maker to use in terms of the membership value associated with the lot sizes and costs.

1.4.2 Fuzzy capacity

Production processes typically operate at a finite rate which, in most practical situations, limits the firm's capability to supply a wider range of products to various markets. In most of finite capacity lot-sizing models available in the literature, the capacity is assumed to have a crisp value known in advance. There are several factors associated with the capacity level. Namely, machines, workforce, shop floor space requirements, and availability of raw material, or work-in-process once needed, and budget availability. During the operation of the production processes, unforeseen events associated with one of the previous factors might be encountered.

Obviously, the major elements of a production process are machines, equipments and tools. As a machine is composed of several mechanical and electrical components, there is always a possibility of a machine breakdown or a failure of parts happening. Even with a preventive maintenance (PM) policy in place, a PM activity is unlikely to restore the machine back to the "as good as new" condition unless a complete overhaul takes place. Furthermore, in case of an unplanned process failure, the duration of a corrective maintenance (CM) action aimed at restoring the process to the operational status depends on the parts to be fixed or replaced and on the availability of spare parts, as needed. Another incident frequently encountered is the production of faulty or defective items. Such items consume partial capacity, during production, with no actual contribution to the output delivered to the customer. In addition, there is a setup activity associated with switching production from one item to another, and the duration of this setup may vary between workers depending on experience. A setup delay can happen or, conversely, a setup might be accomplished faster than usual due to a skilled worker performing it.

Even when machine's availability turns out to be precisely as expected, the workers responsible for operating those machines add another dimension of uncertainty to the capacity. Workers absenteeism or on-job injuries due to a hazardous working environment, which is the case for steel mills, could cause the machine to be idle for some time unless a substitute worker is readily available. With no obstacles concerning machines, labor or space, the production process might still starve due to late deliveries of work-in-process from the previous production process, or a late arrival of a shipment of raw material from an external supplier.

The need to deal with capacity as a vague and imprecise quantity rather than assigning it a single crisp value stems from the uncertainties associated with the above mentioned factors affecting capacity. Unplanned machine breakdown, faulty production, workers injury, space limitation and electricity outage, all result in a production time capacity being less than what is planned for. On the other hand, working extra shifts or overtime, and workers operating more efficiently contribute to a capacity larger than what was thought of. Hence, the assumption of constant capacity is clearly an oversimplified version of the situations encountered in practice, where the above factors are completely ignored. A better approach would be to represent the capacity with a fuzzy set, triangular or trapezoidal as shown in parts (a) and (b) of Figure 1.3 respectively. The most likely value(s) would represent the expected available capacity based on experience, intuition and subjective human judgment. The left-most and the right-most values represent the most pessimistic and the most optimistic capacity levels, respectively.

1.5 Research Objectives

Effective production planning at steel rolling mills bares a great importance and plays a major role towards reducing the high costs associated with constructing and operating steel plants. Moreover, there are some distinguishing features of the steel rolling process that sets it apart from the other industries and that needs to be taken into consideration in order for the developed production plans to be implementable. These features include complex setup time structure, batch manufacturing, scrap and production rates that depend on the input and output material, overtime, allowed backlogging and one-way substitutability of the end-items.

Our objective in the first phase of this research is to develop an optimized master production schedule (MPS) that considers the above interrelated factors under static demand conditions. Studying these factors under static conditions shall provide greater insights on the combined effect they have on the production planning decisions and the tradeoffs amongst them. As the problem involves multiple inputs and multiple outputs (i.e., both are available in different sizes and grades as explained later in Chapter 3), the objective is not only to optimize the production and inventory of the end-items, but also to establish the input-output combinations and accordingly the raw material (i.e., input) procurement and inventory policy. It is also the objective of this phase to define and develop a unified framework for the existing solution methodologies capable of handling the proposed mathematical model. We seek to study several alternative solution algorithms with varying performance capabilities in terms of efficiency and quality of solutions obtained.

The second phase of this research is geared towards optimizing the production and inventory related decisions while taking into account the dynamic nature of the operating environment. This dynamicity is mainly attributed to highly changing customers' preferences coupled with their heightened expectations of shortened delivery lead times. Under these conditions of demand volatility, the development of an optimized MPS turns out to be a challenging task. Hence, to achieve the stated objective, we need to develop efficient mathematical models that capture the frequent changes in the problem parameters and quickly react to these changes on a rolling horizon basis.

The rigidity requirement of classical mathematical programming techniques is overcome through the use of FST which allows for uncertainties in demand to be taken into account. The second face of this research also aims at establishing the benefits obtained via adopting fuzzy mathematical programming techniques as opposed to the use of its crisp counterpart. We emphasize on establishing the missing distinction in the literature between the two approaches typically adopted to handle fuzziness within mathematical programming models, namely flexible mathematical programming (FMP) and possibilistic programming (PP). In the context of FMP, aggregation operators are used to combine the fuzzy sets defining the objective function and the constraints in order to obtain the fuzzy decision set. This research shall utilize two different operators, one of which is compensatory while the other is not, in an attempt to evaluate the benefits obtained via each operator in terms of savings in the total costs incurred.

In the last part of this research, the goal is to incorporate both internal as well as external sources of uncertainty associated with production capacity and customers' demand into the planning decisions. Hence, we need to investigate how various sources of uncertainty can be represented differently and incorporated simultaneously within the same mathematical model through the combined utilization of FMP and PP approaches. Since the decision maker specifies the minimum acceptable possibility level, it is also the objective of this analysis to study the effect of varying the possibility levels on the planning decisions, and the total cost incurred. .

1.6 Research Methodology

An outline of the adopted research methodology to tackle the production planning problem at hand is given in Figure 1.4. After conducting several visits to the plant and identifying a realistic problem encountered frequently in steel rolling mills, a literature review is carried out and the relevant theoretical basis for such problem is established. Then, the necessary data is acquired and the problem is formally defined along with the stipulated assumptions and the distinguishing characteristic of this industry. The technique of mathematical programming is employed in order to optimize the operations at the steel mill under static conditions through the construction of a mathematical model that incorporates the technological constraints associated with the manufacturing process. The solution to the proposed model is obtained through several solution algorithms; namely the classical linearization approach, a hybrid linearization-Benders decomposition approach and a modified branch-and-bound algorithm, where all these algorithms were coded in AMPL programming language and solved using CPLEX 11.0 solver. To account for demand uncertainties, the original model is first applied on a rolling horizon basis where the decisions concerning the most immediate time period only are implemented before rolling the horizon forward and updating the problem parameters. This requires the repetitive solution of the exact model and hence viable approximations that target the complicating aspects of the exact model are developed. The alternative approach to deal with demand uncertainty is the use of FST where the material balance constraints are treated as flexible/soft constraints. The resulting fuzzy model is non-symmetric which calls for the fuzzification of the objective function first. The linearization approach coupled with exterior penalty function methods (EPFM) is adopted in order to identify the interval of allowance on the decision maker's (DM) aspiration level. The aggregation of the fuzzy sets representing the DM aspiration level as well as the constraints is accomplished using two different aggregation operators which results in two variants of the approximate auxiliary models. In the last phase, both uncertain demand and production capacity are accounted for in a fuzzy model where the concepts of FMP and PP are jointly employed. In addition, we utilize the weighted average method in order to deal with triangular fuzzy numbers and the fuzzy ranking method in order to deal with constraints involving fuzzy quantities on both sides. To

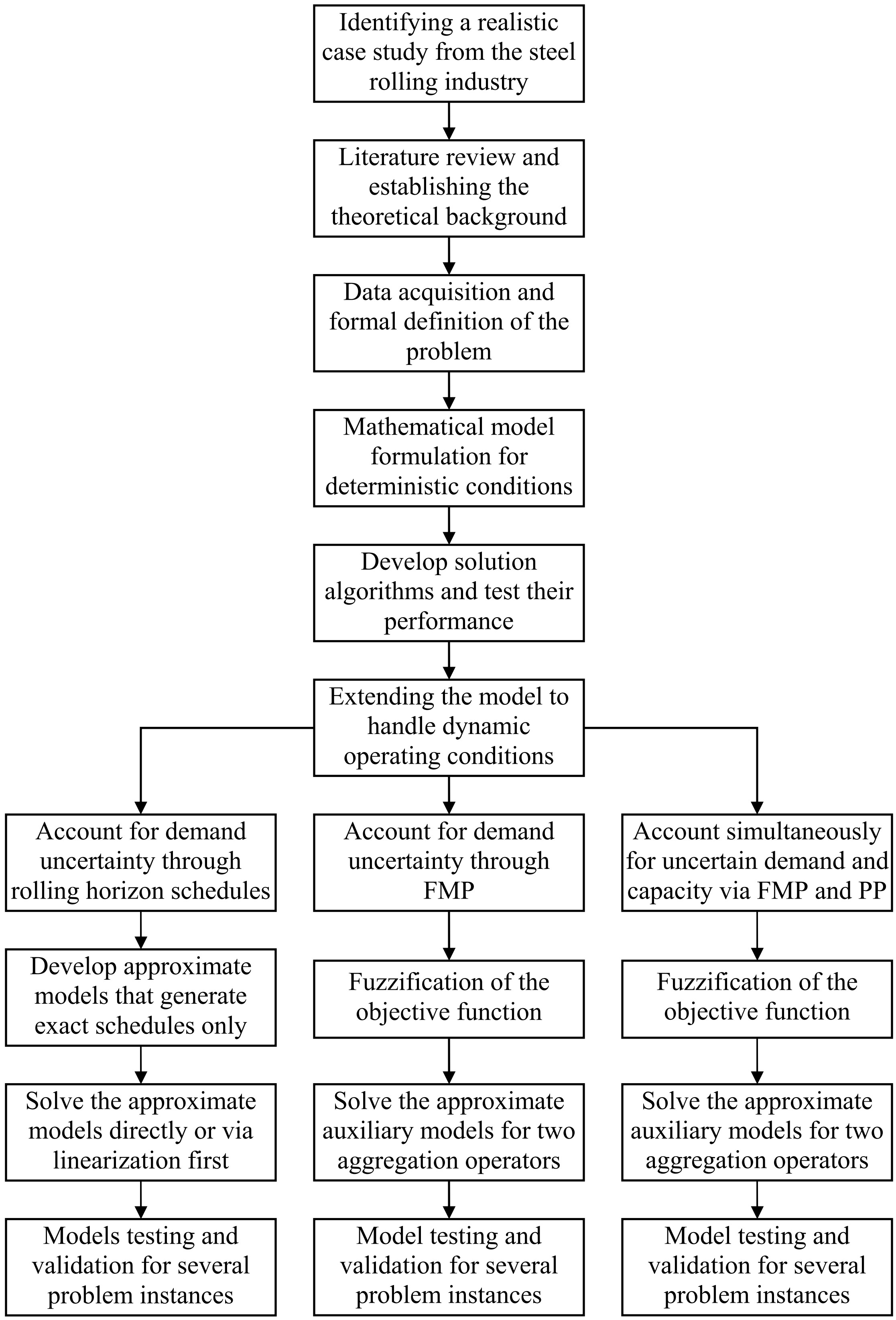

Figure 1.4: Research methodology

serve testing and validation purposes for the developed models, several problem instances with different degrees of complexity are prepared. The values of the input parameters are generated from realistic data ranges so that the practicality of the developed models and solution algorithms is ensured.

1.7 Thesis outline

The remainder of this thesis is organized as follows. Chapter 2 reviews the literature for the general DLSP, in its basic and extended forms, and the more relevant production planning practices in the steel rolling industry. In Chapter 3, a formal definition of the problem is given along with a mathematical formulation that addresses the various aspects of the operating environment and the manufacturing process. This initial model assumes the availability of highly reliable demand forecasts and relatively accurate capacity estimates. Hence, it seeks to provide some insights into the impact of several interrelated factors on the planning decisions under stable operating conditions. As the developed model is a mixed integer bilinear program (MIBLP), the theoretical background for those solution methodologies that can be directly applied or specifically modified to solve this class of models is provided in the appendix. The application/customization of these algorithms to the model at hand is detailed in Chapter 4 and also their performance is benchmarked against one another for several problem instances.

The second part of the research takes the problem one step closer to reality through incorporating uncertainties associated with end customer demand in the planning process. Chapter 5 addresses these uncertainties through the inclusion of both the forecasted demand and the confirmed customers' orders and then applying the resulting model on a rolling horizon basis, where production in the most immediate time period is established based on the confirmed orders. Due to the substantial computation time required to solve the exact model at the beginning of each period, approximate models that result in a tractable number of binary and/or integer variables are developed and tested. In Chapter 6, the alternative approach of utilizing FST, which allows for the incorporation of the decision maker's preference modeling, is presented. A brief background is first established followed by a clear distinction between the FMP and PP, which represent the commonly used concepts to handle existent fuzziness in mathematical programs. The "min operator" and the "convex combination of the min and max operators" are used to aggregate the fuzzy sets representing the objective function and the constraints. The resulting auxiliary models are solved under different settings of the problem parameters in an attempt to quantify the benefits obtained form using the fuzzy models instead of the crisp ones.

In the last part of this research, Chapter 7 addresses jointly the uncertainties associated with the demand and the production capacity in the same model. The importance of this model is to illustrate how different uncertainty sources can be expressed through combining the concepts of FMP and PP in one mathematical model. The utilization of the weighted average method and the fuzzy ranking technique shall be of particular interest to future researchers. A summary of the research work, concluding remarks and suggestions for future research directions are stated in Chapter 8.

Chapter 2

Literature Review

2.1 Introduction

The production planning problem addressed in this thesis can be viewed as an instance of the well know dynamic lot-sizing problem (DLSP) with several extensions that are needed to account for the actual practice. As shall be explained in Chapter 3, the steel rolling process has a continuous flowshop layout and, as far as this work is concerned, can thus be dealt with as a single stage Lot-sizing problem (LSP). Undoubtedly, this class of LSPs is simpler to handle as opposed to the multi-stage version, since the latter involves deciding upon the production or purchasing lot sizes for several items constituting a product's bill-of-material (BOM). Hence, the work reviewed in this chapter focuses on the single stage DLS models as these are more relevant to the production planning problem at hand. The literature review presented in this chapter is divided into two main sections. The first discusses the work related to the LSP in the general context, in its basic and extended forms, while the second considers the more related work addressing production planning problems as applied to the steel industry. As we introduce some of the concepts and solution methodologies utilized in this research, the relevant literature will be reviewed in the respective chapters.

2.2 Single Stage Dynamic Lot-Sizing Problem

Due to its wide spread applications in various industries, the DLSP has received a great deal of interest from industrial practitioners as well as academic researchers. Since the introduction of this problem through the famous work of Wagner-Whitin (1958) and Manne (1958), there have been numerous amounts of research addressing the DLSP in different sittings and under various assumptions. The state-of-the-art advances in the DLSP could be found in De Bodt *et al.* (1984), Bahl *et al.* (1987), Maes and Van Wassenhove (1988), Wolsey (1995), Drexl and Kimms (1997), Karimi *et al.* (2003), Brahimi *et al.* (2006), Jans and Degraeve (2007, 2008) and Quadt and Kuhn (2008). However, as reported by Karimi *et al.* (2003): "There has been little literature regarding problems such as capacitated lot sizing problems (CLSP) with backlogging or with setup times and setup carry-over". The review of this section is divided into subsections, each considering the work related to an issue that is addressed in the thesis.

2.2.1 Basic lot-sizing models

Maes and Van Wassenhove (1986a) points out that capacitated lot sizing models are powerful and very flexible but slow (or impossible) to solve if the problem instance is very large. In terms of complexity theory, the capacitated single item lot-sizing problem (CSILSP) is NP-hard in general. It is even NP-hard for very special cases (Bitran and Yanasse 1982). However, Chen *et al.* (1994) proved through a pseudo-polynomial dynamic programming algorithm that the CSILSP is not NP-hard in the strong sense.

Since it was first introduced, the DLSP has seen many contributions, some of which were from a modeling perspective through providing tighter formulations of the problem, while others developed solution algorithms that outperform the already existing ones in either the solution time or the quality of the solutions obtained or even both. This section intends to highlight the advances made in the basic DLSP (i.e. with no extensions).

Wagner and Whitin (1958) and Manne (1958) started a whole new research direction in their seminal papers. Wagner-Whitin (WW) proved that there exists an optimal solution to the uncapacitated SILSP in which production never takes place in a period while having inventory left from the previous period. This property implies that production in one period, if any, should satisfy the demand for an integral number of consecutive periods. Based on this property, they developed a dynamic programming (DP) algorithm of $O(m^2)$, with m being the number of time periods. The complexity of the DP algorithm for the uncapacitated SILSP was independently reduced from $O(T^2)$ for the case of WW to $O(T \log T)$ by several authors including Federgruen and Tzur (1991), Wagelmans *et al.* (1992) and Aggarwal and Park (1993). Manne (1958) proposed an innovative formulation for the capacitated version of the problem with setup times incorporated. He explicitly models all the possible schedules with different setup sequences. Evans (1985) proposed a shortest path formulation for the USILSP based on a graph representation of the problem. As the reformulations provide improved lower bounds on the optimal solution value, their LP solution can also be used to construct heuristic solutions. Belvaux and Wolsey (2001) explain how many extensions of the basic lot sizing models can be modeled in order to obtain better formulations. A recent survey of the modeling techniques as applied to the LSPs is given by Jans and Degraeve (2008).

From a solution methodology perspective, there exists several heuristics that are dedicated for solving the USILSP. Those heuristics are usually easier to implement than the WW algorithm. Examples include economic order quantity based on average demand, least period cost (Silver and Meal 1973), least unit cost, part-period balancing among many others. A discussion of such heuristics can be found in Silver *et al.* (1998). Moreover, Bitran *et al.* (1984), Axsäter (1985) and Coleman (1992) studied the performance of some of these heuristics and developed worst-case bounds under several demand classes. A dynamic programming based algorithm for the USILSP was developed By Kirca (1990). First, the algorithm generates the set of all feasible cumulative production levels that may occur in an optimal solution, and then a DP procedure is carried out over this set. Diaby (1993) developed an efficient post-optimization procedure that re-computes the optimal schedule starting from the WW solution for the case where some set ups are imposed or prohibited. Chung *et al.* (1994) constructed a hybrid algorithm combining both DP and branch and bound towards solving the same problem.

On the other hand, the CMILSP has received remarkable attention as it is a more challenging problem to solve. Several exact solution methods have been proposed. Barany *et al.* (1984) and Pochet and Wolsey (1991) used valid inequalities (strong cutting planes) to solve the problem. Eppen and Martin (1987) established a shortest path formulation of the DP recursion presented in WW, where this network formulation sets a tighter lower bound for the problem. Constantino (1998) also derived families of strong valid inequalities and showed that they are sufficient to completely describe the convex hulls of the sets of feasible solutions. Belvaux and Wolsey (2000) developed an efficient branch-and-cut based software that includes preprocessing and cutting planes for a variety of lot sizing models.

Apart from the exact methods, there are some heuristics that have been specifically tailored for solving the CMILSP. Dixon and Silver (1981) extended the Silver-Meal heuristic to the capacitated multi item case. The criterion is to select that item for which a one period increase in the supply results in the largest decrease in average cost per unit time per unit of capacity absorbed (Jans and Degraeve 2007). Dogmaraci *et al.* (1981) developed a forward sweep algorithm along with a greedy search starting from the lot-for-lot solution. Karni and Roll (1982) use the WW schedules as a starting point and try to achieve feasibility while optimizing cost through shifting production. Maes and Van Wassenhove (1986c) implemented several cost criteria in their ABC heuristic to determine whether or not to include next period's demand, and several other rules to determine the order of the items. Finally, Kirka and Kökten (1994) developed an efficient item-by-item heuristic where items are selected one at a time, and then a single item problem is solved with adapted capacities and extra bounds on production and inventory to ensure the feasibility of the overall problem.

Another approach towards solving the CMILSP is the use of Lagrangian relaxation, e.g., Thizy and Van Wasenhove (1985), Trigerio *et al.* (1989) and Diaby *et al.* (1992). The traditional practice in this approach is to get rid of the complicating capacity constraint through a dualized term in the objective function coupled with a specific set of positive multipliers. The resulting problem is a lot easier to solve and may be decomposed into separate single item uncapacitated subproblems for each item.

Polynomial approximation technique has also been applied towards solving the LSPs. Bitran and Matsuo (1986) proposed a pseudo-polynomial approximation algorithm for the CMILSP based on Manne's (1958) formulation. Gavish and Johnson (1990) developed a fully polynomial approximation scheme for the capacitated single item lot scheduling problem. Furthermore, Van Hoesel and Wagelmans (2001) presented a fully polynomial algorithm for the CSILSP, which produces solutions with a relative deviation from the optimality that is bounded by a constant.

Meta-heuristics such as Tabu search (e.g. Hindi 1996) and Genetic algorithms (e.g. Gutierrez *et al.* 2001) have also been specifically developed to solve the CMILSP. It is interesting to note that no implementation of meta-heuristics for solving the CSILSP can be found, as concluded by Brahimi *et al.* (2006) in their review paper. On the other hand, Cattrysse *et al.* (1990) discussed the set-partitioning formulation of the CMILSP and used heuristics to convert the possibly fractional solution from the column generation step to a feasible integer one. In a different heuristic, Hindi (1995) implemented the branch and bound method as a solution strategy for the CMILSP. Chen and Thizy (1990) gave a comprehensive analysis of relaxation methods for the problem and showed it to be strongly NP-hard. A comparison of the performance of several solution heuristics can be found in Maes and Van Wassenhove (1986b).

2.2.2 Lot sizing models with Extensions

The numerous extensions of the basic lot sizing problem discussed in the literature demonstrate its flexibility to model a variety of industrial problems. Each of the following subsections presents the advances made towards modeling as well as solving an extended version of the basic LSP.

2.2.2.1 Lot sizing models with allowed stockouts

In practice, the capacity is typically finite and bounded by several factor such as machines, workers, availability of raw material and storage areas among many others. In such situations, a manufacturer might not be capable of completely fulfilling a certain period's demand on time due to insufficient capacity. There are two strategies to deal with the remaining portion of demand or the "unmet demand". First, it might be lost in the sense that a competitor will be the one satisfying this portion of demand. This explains the situation where we have "lost sales", a phenomenon that usually takes place in the retailing industry. There is a certain attached cost resulting from revenue loss, or penalty cost due to loss of customer goodwill. Secondly, the unmet demand can be satisfied at a later period of time. If the whole demand is satisfied late, this is referred to as "complete backlogging" (the words backlogging and backordering can be used interchangeably). In the case of late fulfillment of only a portion of the demand, “partial backlogging” takes place. From a mathematical perspective, permitting backlogging means that inventory levels can be negative. In the steel industry, most customers are long term customers, and the manufacturer might make use of his power alongside customer’s loyalty to backlog a portion of the demand at a certain additional cost, called “backlogging cost”.

Both cases have been addressed in the literature, with the lost sales situation being dealt with to a lesser extent. Sandbothe and Thompson (1990) proposed a necessary condition for an optimal solution and obtained an $O(T^3)$ algorithm when production capacity is constant, and an $O(2^T)$ algorithm for the case of time-varying production capacity. In a 1993 paper, they extended their work through imposing restrictions on both production and inventory capacities, and obtained an algorithm with the $O(2^T)$ time complexity. Aksen *et al.* (2003) introduced a profit maximization model for the USILSP with lost sales, where costs and selling prices were assumed to be time-variant. They showed that losing demand in spite of a nonzero inventory at hand can sometimes be more profitable if costs or prices vary. Liu *et al.* (2004) developed a strongly polynomial algorithm for the lost sales case with bounded inventory, non-increasing setup cost, and time varying production, inventory holding and lost sales costs. Liu and Tu (2008) studied the CSILSP with limited inventory capacity and time-varying functions of demands and costs.

One of the earliest works to consider backlogging is due to Zangwill (1966a), where he proposed a deterministic single-item multi-period production and inventory model having concave production cost and piecewise concave inventory costs. The inventory can be backlogged to a maximum of α periods, where α is called the backlog limit. Zangwill (1966b) extended his previous work to the multi-product multi-facility case with facilities being in series or in parallel, under the same cost structure. Love (1973) was the first to present an $O(T^3)$ DP algorithm with constant inventory capacity, concave production and holding costs. Swoveland (1975) developed a solution algorithm for the case of a piecewise concave production and holding costs or backlogging costs. Moreover, Gupta and Brennan (1992) introduced an easy and robust alternative to the WW backorder algorithm, proposed by Webster (1989). Federgruen and Tzur (1993) proposed a simple $O(n \log n)$ solution algorithm for the CSILSP with backlogging. Miller and Yang (1994) employed lagrangian decomposition and lagrangian relaxation to exploit the underlying network structure of the CMILSP with backlogging. The alternative plant location and shortest path reformulations for the ULSP with backlogging were presented by Pocuhet and Wolsey (1988). Recently, Chu and Chu (2007) developed a polynomial algorithm for the CSILSP with bounded inventory and backlogging or outsourcing.

2.2.2.2 Lot sizing models incorporating setup times and/or overtime

In most practical situations, a setup is incurred whenever the manufacturing process switches between two different products. This setup consumes partial capacity and hence it needs to be explicitly accounted for in the mathematical model. The Silver-Meal lot sizing heuristic for single item problems was extended to the case of regular and overtime production capacities by Dixon *et al.* (1983). Also, Özdamar and Bozyel (2000) extended the latter work to the case of CMILSP with overtime decisions, and presented several meta-heuristics to solve the problem. Starting from an initial lot-for-lot approach, Trigeiro (1989) developed a heuristic algorithm for the CLSP with setup times that is also based on the Silver-Meal heuristic. Trigeiro *et al.* (1989) and Hindi *et al.* (2003) considered the CMILSP with setup times and obtained a lower bound on the value of the objective function by Lagrangian relaxation with subgradient optimization. The polyhedral structure and valid inequalities of the single period relaxation for the CMILSP with setup times are also presented in Miller (2003a,b). Jans and Degraeve (2004) start with the network formulation to come up with improved lower bounds for the problem. In a recent paper, Absi and Kedad-Sidhoum (2008) addressed a more generalized version of the CMILSP with setup times in which the demand can be totally or partially lost.

2.2.2.3 Lot sizing models with product substitution

Some products, such as integrated circuits and steel bars, are produced in different grades with varying performance characteristics. In such a situation, the manufacturer may occasionally choose to downgrade a product instead of backordering the demand for a similar product with the lower grade. The term “downgrading” has been previously established in the literature, and it refers to instances where class j product is used to satisfy the excess demand for that of class i, where $i \geq j$ (i.e., product j has a better quality). For example, Bitran and Dasu (1992) presented the case of Semiconductor industry and called the demand substitution structure where a higher quality chip satisfies the demand for the lower one “downgrading”. For the same industry, Bassok *et al.* (1999) addressed this type of substitution structure and called it “downward substitution”. In general, the issue of demand substitution has been considered in a variety of contexts for traditional production planning and the available literature could be broadly classified into three streams of work, as pointed out by Rajaram and Tang (2001).

However, most papers concentrate on the substitution problem in the context of single period models (Li *et al.* 2007). Balakrishnan and Geunes (2000) considered the material requirement planning problem with substitutions in a multi-period horizon and derived a DP algorithm that obtains the production and substitution quantities in each period. Li *et al.* (2006, 2007) dealt with the DLSP in the context of a hybrid manufacturing/remanufacturing system with product substitution. In their analysis, a new product is offered in place of a remanufactured one when there is a remanufactured product shortage. They developed a DP algorithm for the uncapaciatetd case, and a genetic algorithm for the capacitated one.

2.2.2.4 Fuzzy Lot-Sizing models

Most of the studies on Lot-sizing models assume that all problem parameters are known with a high level of confidence. However, this is not always the case in reality since many uncertainties are involved. Although accounting for such uncertainties poses as an important issue, few publications have dealt with the DLSP in a fuzzy sense. Lee *et al.* (1990) incorporated fuzzy demand into the part-period balancing heuristic. They also (1991) compared the performance of three lot sizing algorithms when demand is fuzzy. Fung *et al.* (2003) solved the more general aggregate production planning problem with fuzzy demand and fuzzy capacity. At last, Pai (2003) made use of the fuzzy set theory to solve the CLSP with fuzzy capacity.

2.3 General production planning literature in steel plants

In spite of the significance of steel industry, planning and scheduling problems in iron and steel production have not drawn as wide attention of the production and operations management researchers as many other industries such as metal cutting and electronics industry (Tang *et al.* 2001). As pointed out by Dutta and Fourer (2004), very little work has been done in the area of inventory control, manufacturing control and multi-period linear programming modeling in the steel industry. Since this thesis deals with optimizing the product mix in a steel mill as explained in more details in Chapter 3, the review focuses on the related work with a glimpse of other issues discussed in the literature.

The first attempt towards formulating the production process at a steel plant as a linear program was made by Fabian (1958) where he developed an integrated linear programming model for iron making, steel making and rolling operations. This work was later extended by Lawrence and Flowerdew (1963) where an economic model for production planning was developed. The authors stressed on the system approach through proposing a qualitative framework for the whole steel plant rather than individual processes. For the blending problem, Fabian (1967) proposed a cost minimization model that can simultaneously determine: optimal raw material purchasing policies, optimal raw materials inventory levels, least cost blend of raw materials, optimal furnace scheduling, long-range production plan and optimal maintenance plan. Westerberg *et al.* (1977) presented the case of a Swedish steel mill that uses up to 15 different types of scrap and alloys which are melted together to produce stainless steel finished products. The problem was formulated as a traditional blending model with the objective of minimizing cost subject to weight and metallurgical composition restrictions.

The steel industry has also been investigated from an investment planning point of view. Kendrick *et al.* (1984) presented three mathematical models for investments analysis in the steel industry, two of which are static models formulated as linear programs and the third is a dynamic one formulated as a mixed-integer program. Anandaligam (1987) made use of stochastic programming to plan for investments in environments where demand projections and technological coefficients are not known with certainty.

The Development of a database for generic mathematical programs was achieved by Fourer (1997). The model could be utilized by any steel plant to fit its operations simply by supplying its own data. Based on Fourer's work, Hung (1991) studied the importance of inventories and the linkage between the time periods in a plate mill. The complicated steel production process was represented by a network composed of facility nodes, material nodes and material flow arcs, and a profit maximization linear program was formulated based on the network representation.

With the increasing steel products variety, it becomes increasingly important for steel manufacturers to adopt new strategies towards improving the service level and reducing the service time. Sharma and Sinha (1991) discussed the various issues affecting the choice of an optimum product mix in a steel plant, and described an optimization model for determining such mix. The network approach was used by Sashidhar and Achray (1991) to deal with the problem of production planning at a steel mill with the objective of maximizing capacity utilization. In their work, production is planned according to customers' priorities, where different customers are assigned different priorities. Denton et al. (2003) developed a decision support system that allows inventory planners to analyze various scenarios, and identify which slabs shall be produced for make-to-stock (MTS). In a recent paper, Kerkkanen (2007) established a new inventory policy in order to enable a comparison between a make-to-order (MTO) system and a hybrid MTO/MTS system. He presented the case of a small MTO steel mill that is considering whether it would be profitable and feasible to make some semi-finished products to stock. Based on the input-output concept, Li and Shang (2001) developed a production planning model for a large steel corporation in China. The hierarchical production planning problem in a make-to-order (MTO) steel fabrication plant was addressed by Neureuther *et al.* (2004). They presented linear programming models towards obtaining the aggregate production plan and then disaggregating it into MPS for the end-items.

Unlike general production scheduling in other industries, iron and steel production scheduling problems have to meet specific requirements related to material continuity, processing times at various production stages, transportation and waiting times between operations. Scheduling production at different stages of steel manufacturing has been studied in the literature. Examples include Vonderembse and Haessler (1982), Jacobs *et al.* (1988), Boukas *et al.* (1990), Diaz *et al.* (1991), Assaf *et al.* (1997), Box and Herbe (1988) and Tang and Liu (2007). As pointed out by Cowling and Rezig (2000), Scheduling of the continuous caster is governed by the production push of liquid steel, which must be cast as it arrives, whereas hot strip mill scheduling is governed principally by the production pull of customer orders for coils of steel. The short term scheduling problem for each process considered in isolation is itself difficult, containing a wealth of NP-hard packing, sequencing and scheduling problems (Garey and Johnson 1979).

Several attempts have been made towards scheduling operations at steel mills and continuous casting machines (CCM) separately. The steel mill case was investigated by Redwine and Wismer (1974), Arizono *et al.* (1991) and Lopez *et al.* (1998). On the other hand, optimizing the operations at a CCM is of great importance as a CCM can be used to eliminate a number of processing steps associated with the traditional ingot/bloom based production process. Vasko and Friedel (1982) proposed a DP formulation which maximizes the case bloom tonnage that can be processed through a finishing mill. Lally *et al.* (1987) constructed a mixed-integer linear programming model to the problem of caster scheduling, where steel is being cast on a continuous basis. However, the model did not incorporate all the complexities of a real continuous caster. Tang *et al.* (2000) presented a more sophisticated nonlinear model based on actual production situations considering both punctual delivery and continuity of the production operation. The model is in line with the idea of just-in-time (JIT) production and provides a way to overcome machine conflicts. Zanoni and Zavanella (2005) established a linear programming model which gives the optimal production sequence of the billets, ordered by the customers, while taking into account the limitations in warehouse space availability. The study focuses on the optimization of the production schedule of a CCM, through determining the quality and dimensions of the billets to be produced such that the total cost of holding, production, and penalty (due to late deliveries) is minimized.

Strategic planning in the steel industry plays an important role in determining a company's ability to survive in today's competitive market. As noted by Denton *et al.* (2003), the steel industry has received a great deal of attention in the field of strategy and operations research, because of the heavy pressure from worldwide competition. The earliest work along this line of research was conducted by Bielefeld *et al.* (1986) and Sinha *et al.* (1995). Chen and Wang (1997) developed a strategic linear programming model for a steel plant from a supply chain perspective. Thus, the model seeks the optimal production plan, raw material supply and finished product distribution. However, the optimal solution of the problem is found with a reduced number of variables and heuristics are not presented for a more realistic solution (Zanoni and Zavanella 2005). Singer and Donoso (2006) showed how a mathematical programming model that can assist in strategic decision-making by forecasting the results of possible actions. The model relies on Activity Based Costing (ABC) for calculating unit product cost, and on dynamic Activity Based Management (ABM) for assessing the feasibility of production plans. A decision support system for strategic and operational planning for process industries was developed by Dutta and Fourer (2004).

However, except for the work of Fourer (1997), Hung (1991), Dutta and Fourer (2004) and Singer and Donoso (2006), most other articles presented a one shot model (single period) ignoring the dynamicity nature of the problem in the real life context. The time varying demand, purchasing and selling prices necessitate treating the production planning problem in a steel plant as a multi-period model. Depending on the specific problem context, the use of the model and the planning horizon, a period could be an hour, a day, a month or even a year. This research presents multi-period, multi-input and multi-output production planning models in which the dynamic nature of the operating environment is taken into account. This represents one of the potential areas for future research explicitly stated in Dutta and Fourer (2004): "Simultaneous optimization of product-mix, inventory, and transportation problems over multiple periods".

In the next chapter, we provide a detailed discussion of the manufacturing process at the steel mill under consideration along with the distinguishing features of this industry. The production planning problem of interest is then stated and the mathematical model for deterministic demand conditions is also developed.

Chapter 3

PART-I: Problem Definition and Mathematical Modeling under Deterministic Conditions

3.1 Introduction

In reality, the high cost figures associated with the construction and operation of steel plants necessitate a periodic revision of the production technologies employed as well as continuously seeking the best managerial practices to handle the rapid increase in product variety. The steel industry is considered to be crucial for many countries' economic competitiveness especially in today's environment which is characterized by lowered barriers to market entry and constantly changing customers' preferences. Despite the great significance of the steel rolling industry in particular and the ample uses of its end products in every nation's daily life, it seems that researchers have generally overlooked the importance of developing optimized production plans that take into account the practical aspects distinguishing this industry from all other industries.

This chapter starts by introducing the manufacturing process at a typical medium-sized steel mill producing round shaped steel bars, which stands as an essential raw material for various construction projects. Since a well defined problem represents a key milestone towards obtaining its solution, the distinguishing features and the technological constraints associated with manufacturing process are clearly identified. The production planning problem under deterministic demand conditions is then formulated using the well-established mathematical programming techniques. The objective of the model is to study the combined effect of several interrelated factors that jointly affect the operations and the production related decisions at the steel mill under consideration. The properties of the presented mathematical model are discussed along with the complicating aspects pertaining to the necessity of developing efficient solution algorithms for such model.

3.2 The manufacturing process

The problem at hand concerns the production of reinforced round steel bars (rebars) from an externally supplied raw material, which is steel billets having a square cross-sectional area. The steel billets (bars) are purchased (produced) in two different steel grades (grade 40 and 60) and have several dimensions that are the same for both grades (Tables 3.1 and 3.2). Due to technical considerations regarding yield and scrap rate, there exist some restrictions on the possible billet-rebar combinations. For instance, a billet of dimensions $100mm \times 100mm \times 6m$ (index $i = 6$ in Table 3.1) is not to be used as an input material in the manufacturing of a $32mm$ diameter steel bar. The two grades differ mainly in the chemical composition, metallurgical structure and carbon content, which eventually lead to varying mechanical properties and performance. In particular, grade 60 has higher values for yield strength and ultimate tensile strength ($YS = 421\ N/mm^2, UTS = 621\ N/mm^2$) as compared to those for grade 40 ($YS = 300\ N/mm^2, UTS = 500\ N/mm^2$). As such, grade 60 steel is considered to be of better quality and is thus sold (or procured) at a higher price. Clearly, a steel bar of a certain grade can only be produced from a billet of that particular grade.

Table 3.1: Raw material dimensions

Index (i)	Width (mm)	Height (mm)	Length (m)
1	130	130	12
2	130	130	8
3	130	130	6
4	125	125	8
5	125	125	6
6	100	010	6

Table 3.2: Finished product dimensions

Index (j)	Diameter (mm)	Index (j)	Diameter (mm)
1	32	**6**	18
2	28	**7**	16
3	25	**8**	14
4	22	**9**	12
5	20	**10**	10

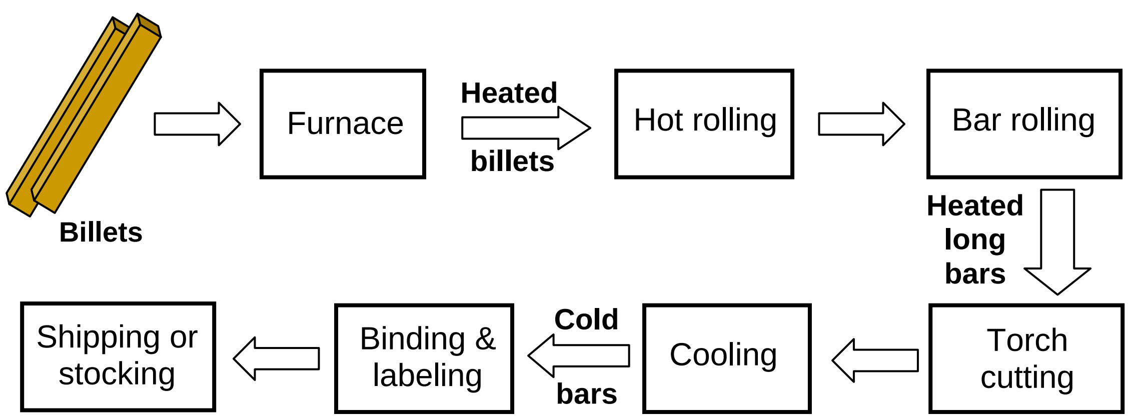

Figure 3.1: The manufacturing process in the steel bar rolling industry

Figure 3.1 above shows a graphical display of the different manufacturing operations involved in the production of the steel bars. The production process starts by placing a batch of billets, with uniform dimensions, into the furnace where they are heated up to 1200°C. The furnace has a fixed capacity of 60 tons/hour and the number of billets placed at once in the furnace depends on the size (dimension) of those billets. Clearly, the smaller the size, the more billets can be accommodated within the furnace. The heated billets are then taken out of the furnace to the rolling mill where the hot rolling operation takes place followed by the bar rolling operation. After rolling the billets into long bars of the desired cross-sectional area and a standard length, the bars are pushed on the cooling bed to allow the product to cool down. The next step is to bind the bundle of products and label the bundle. At last, the bars are either stored in the warehouse or shipped directly to the customer. In fact, since the various products follow the same routing (i.e., same sequence of operations), the bar rolling industry resembles a flowshop production environment.

The processing times depend on which rebar is being produced from which billet (i.e. on the dimensions of both). This is because a billet may be required to pass through several rolling stands depending on the desired diameter of the rebar to be produced. For instance, it takes longer rolling time to produce a steel bar of diameter 10 *mm* from a certain billet, than to produce a bar of diameter 32 *mm* using the same billet. In addition, the higher the number of rolling passes that a billet has to go through, the more return scrap is expected. In any case, the rolling capacity is clearly bounded by the capacity of the furnace (60 tons/hour) which feeds the rolling operation. However, billets of bigger dimensions are cheaper to buy, and bars of smaller diameters are sold to the customer at a more expensive price. This may justify the production of a small diameter rebar from a bigger dimension billet in spite of excess processing time and higher chances of scrap produced. The operations at the steel mill are characterized by the following features:

(1) Setup time: One of the most distinguishing and complicating features of steel mills operation is the setup time structure. The setup activities include those associated with the furnace, such as placing the new batch of billets inside and adjusting the settings, as well as those activities carried out on the rest of the production line, such as rolls and stands changing, guides and grooves changing, runner way and time billet changing, and speed reference adjustments. Hence, when the setup is carried out between batches, the setup time depends on the raw material (as the number of billets placed in the furnace depends on the size) and on the finished product (as the rollers, grooves and speed have to be adjusted according to the rebar to be produced). Whenever the term “minor setup” is quoted in this thesis, it refers to this type of setup (i.e., the between batches setup).

On the other hand, when the setup is carried out at the beginning of the day, it is time dependent rather than product dependent. To achieve a longer service life for the refractory, which is an insulation material on the inside of the furnace, an upper limit on the rate of furnace temperature increase per hour is imposed. Moreover, the refractory also sets a minimum allowable value for the furnace temperature at all times indicating that some burners have to be left functioning even after production is complete. Hence, at the beginning of each business day, the furnace temperature has to be elevated gradually to 1200°C before starting production. This temperature elevation time depends mainly on the idle time since the production of the last batch in the previous day. For example, if the production line runs for 20 hours per day, this means the furnace is stopped for 4 hours, and hence it will take almost 2 hours to heat it back to 1200°C at the beginning of the next day. On the other hand, a 10 working hours per day means longer stoppage time for the furnace, which would take 5 hours to heat the furnace back to the same temperature. Clearly, a solid mathematical relationship between the idle time at the end of period t and the setup time at the beginning of period $t+1$ needs to be established prior to developing a mathematical model for the production planning problem at hand. In the remainder of this thesis, a setup conducted at the beginning of the day is referred to as "major setup". It should be noted that such setup time structure is frequently encountered in other industries such as metal rolling (other than steel) and plastics manufacturing in which the product undergoes a heat treatment phase during the production process.

(2) Product substitution: In practice, the steel mill has the option to:

(a) Fulfill the unmet portion of the demand for an out-of-stock lower grade rebar (i.e., grade 40) with a same sized rebar of the higher grade (i.e., grade 60) in the same time period as to meet the promised delivery schedule. This substitution scenario yields:

- Increased customer expectation for future shipments.

- Lost profit (due to selling a higher quality product at the price of the lower quality one).

(b) Backlog and match the order with the delivery at a later period in time. In this case,

- A backlogging cost is incurred.

- Might eventually lead to the loss of customer goodwill (once this option is adopted repeatedly with the same customer).

(3) Overtime: As the major setup time depends on the working hours, it would make sense for the company to consider the option of working overtime hours especially in periods of excess demand. Although it costs more to produce on overtime basis, it might be economical to do so as this avoids the backlogging cost and reduces the furnace setup time at the beginning of the next business day.

(4) Yield: The number of steel bars produced from the same billet is a function of the dimensions of both the billet and the rebar to be produced. For instance, a billet of dimension index ($i = 1$) would give 17 round bars of 16 *mm* diameter and 60 *m* long each. Alternatively, a billet that is 8 *m* long and of the same cross sectional area ($i = 2$) would give 11 bars of 16 *mm* diameter and 60 *m* long each. Moreover, the manufacturing process in the steel industry, like all other industries, is not a perfect one in the sense that it produces a portion of defective items. However, the resulting nonconforming items can be sold as scrap steel to other manufacturers. The percentage of scrap produced depends on the billet-rebar combination.

(5) Time dependent raw material purchasing cost and finished product selling prices: In reality, the market prices of the reinforced steel bars as well as the billets have been subjected to drastic variations in the last decade or so. Hence, such variation in prices has to be taken into consideration to better reflect the reality. In fact, to serve a broader range of planning purposes, the proposed mathematical formulation (Section 3.4) assumes that all cost parameters are time dependent.

3.3 Problem description

In this section, a realistic, multi-input, multi-output and multi-time period production planning problem encountered at a medium-sized steel mill is considered for analysis. For an industry characterized by significant operating costs and high energy consumption, the optimization of production and inventory related decisions is of paramount importance. Typically, product differentiation in the rolling operation increases as the steel billets proceed on their journey through the rolling stands towards the finished steel bars. As such, the billet-rebar combination not only determines to a great extent the number of rolling passes that the product has to go through, and hence the energy consumption, but also the quantity of both types of materials to be kept in stock, hence the procurement, inventory holding and opportunity costs.

The problem at hand is a short term planning problem in which production is planned at the master production schedule (MPS) level. The goal is to determine the daily/weekly production lot sizes for the various end items (rebars) such that customers' demands over the planning horizon are fulfilled at a minimal total cost. In particular, there are four types of decisions that the decision maker seeks to optimize: (1) which products to produce in each period; (2) how much of each product shall be produced; (3) the allocation of the products to satisfy the customers' demand (since demand substitution is allowed); and (4) the raw-material finished-product combination (i.e. which raw material shall be used to produce which product). Having established the problem statement, we next stipulate the assumptions under which the mathematical model in the next section is developed:

(1) Batch production: In accordance with the economies of scale and to utilize the available capacity to its fullest, the production takes place in batches of size 60 tons each (i.e., the furnace capacity).

(2) Batch uniformity: This applies to both the billets placed at once in the furnace and to the steel bars produced from the same batch. The usage of mixed sized billets or the production of mixed sized bars is not allowed from within the same batch.

(3) Deterministic demand: The steel industry is characterized by having customers that are in most cases long term loyal customers (Chen and Wang 1997, Kerkkänen 2007). Although, the model developed in this chapter addresses the problem under deterministic demand conditions, this assumption is to be relaxed in subsequent chapters to take into account situations involving highly volatile demand.

(4) One way substitutability: This term was coined by Rajaram and Tang (2001) which, in the context of our problem, refers to the case where a grade 60 steel bar of a certain diameter could be used to fulfil a portion of the unmet demand for a similar steel bar of grade 40, given that such substitution is suitable for the intended engineering application, but not the other way around. This "downgrading" is motivated by a variety of reasons, for example, to prevent customer dissatisfaction, to reduce setup costs, or to reduce inventory costs (Bitran and Dasu 1992).

3.4 Mathematical formulation of the problem

For many decades, the tools of operations research have been successfully employed towards the modeling and solution of real world systems leading to a significant increase in the productivity of various countries' economies. This section employs the techniques of mathematical programming in order to optimize the operations at the steel mill under the previously stipulated assumptions. We first start by establishing the relationship between the idle time at the end of period $t-1$ and the major setup time in period t since such a relation is used as input to the mathematical model. As a matter of fact, the dependent variable (y), which denotes the major setup time in this case, is directly proportional to the independent variable (x) denoting the idle time, as can be seen in Figure 3.2.

The plot suggests that the relation is approximately linear where the general equation for the best fitted line using simple regression analysis is given by:

$$y = ax + b \tag{3.1}$$

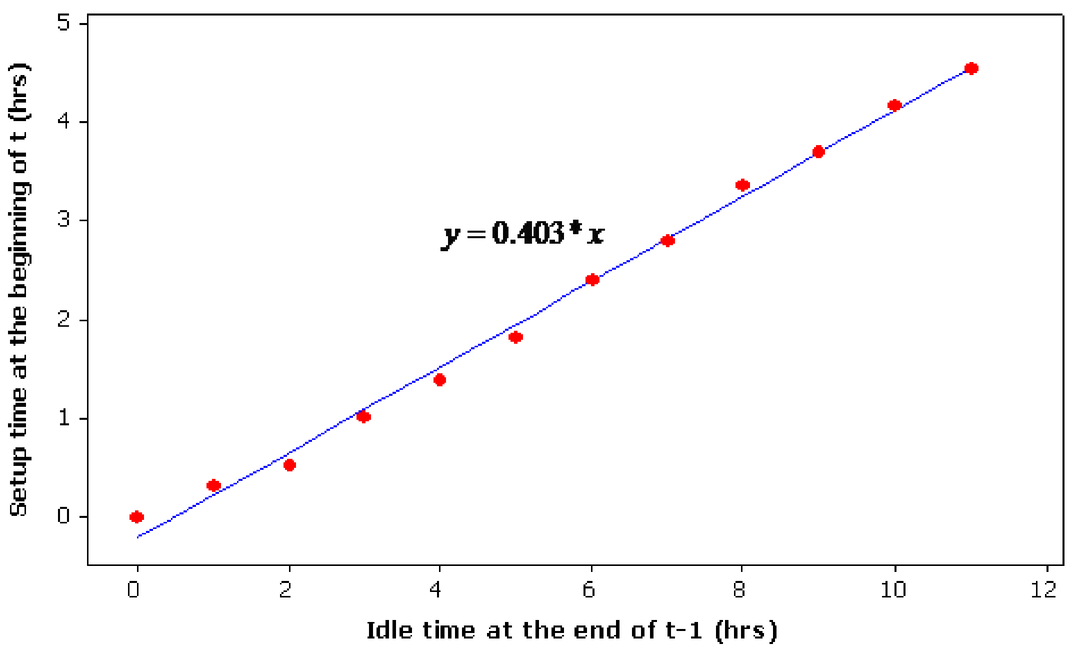

Figure 3.2: The linear relation as obtained from regression analysis

Clearly, a is the slop of the fitted line and b is the $y-$intercept. Since the unlikely event of a continuous production in period $t-1$ (i.e., a zero idle time) indicates a zero major setup time in period t, the value of b is set to zero. Regression analysis utilizes the method of least squared errors (LSE) in order to obtain the optimized value of the slope, which yields a value for $a = 0.403$ using the MINITAB software after setting $b = 0$.

We next present the mathematical formulation for the production planning problem at hand. This operational model establishes the raw material purchasing quantities, regular time and overtime based production quantities, inventory levels, backorder and substitution quantities for each product in each time period such that the total cost over the planning horizon is minimized. In developing the mathematical model, quantities of materials, whether billets or rebars, are measured in tons and the production rate is measured in tons per hour. An index, whether i for raw material (RM) or j for finished product (FP), refers to the same dimension in both steel grades (SG). This notation greatly assists in formulating the problem as a mathematical model.

The model seeks to minimize the total costs incurred in addition to the penalties resulting from downward substitution. The total cost is composed of RM ordering, RM purchasing, RM inventory holding, setup, regular time production, overtime production, FP holding and backorder costs. The last term in the objective function is an additional penalty term which ensures that demand substitution is further discouraged since in this case the higher grade steel is sold at the price of the lower quality one entailing a lost profit to the company. The objective function along with the set of constraints involved is set out as follows:

$$Min \quad Z = \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{2}\sum_{t=1}^{T}\left(SC_t S_{ijt}^k + PC_{ijt} X_{ijt}^k\right) + \sum_{j=1}^{J}\sum_{k=1}^{2}\sum_{t=1}^{T}\left(IF_{jt}^k I_{jt}^k + BC_{jt}^k B_{jt}^k\right) + \sum_{t=1}^{T} PO_t O_t$$

$$+ \sum_{i=1}^{I}\sum_{k=1}^{2}\sum_{t=1}^{T}\left(OR_{it}^k G_{it}^k + CR_{it}^k Q_{it}^k + IR_{it}^k I_{it}^k\right) + \sum_{j=1}^{J}\sum_{t=1}^{T} W_{jt}^1\left(SP_{jt}^1 - SP_{jt}^2\right) \qquad (3.2)$$

Subject to

$$Q_{it}^k \le M_{it}^k G_{it}^k \qquad \forall i,t,k \qquad (3.3)$$

$$\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{2} Sb_{ijt}^k = 1 \qquad \forall t \qquad (3.4)$$

$$S_{ijt}^k = 0.4 \times Sb_{ijt}^k \left(24 - \left(\sum_{i'=1}^{I}\sum_{j'=1}^{J}\sum_{k'=1}^{2}\left(\frac{X_{i'j',t-1}^{k'}}{\alpha_{i'j'}} + S_{i'j',t-1}^{k'}\right)\right)\right) + ST_{ij} \times Sd_{ijt}^k \qquad \forall i,j,t,k \qquad (3.5)$$

$$\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{2}\left(\frac{X_{ijt}^k}{\alpha_{ij}} + S_{ijt}^k\right) \le A_t + O_t \qquad \forall t \qquad (3.6)$$

$$O_t \le A_{ot} \qquad \forall t \qquad (3.7)$$

$$X_{ijt}^k = b_t \times \left(Sb_{ijt}^k + Sd_{ijt}^k\right) \times \rho_{ij} \qquad \forall i,j,t,k \qquad (3.8)$$

$$I_{it}^k = I_{i,t-1}^k + Q_{it}^k - b_t \times \sum_{j=1}^{J}\left[Sb_{ijt}^k + Sd_{ijt}^k\right] \qquad \forall i,t,k \qquad (3.9)$$

$$I_{j,t}^1 - B_{j,t}^1 = I_{j,t-1}^1 - B_{j,t-1}^1 + \sum_{i=1}^{I} X_{ijt}^1 - W_{jt}^1 - D_{jt}^1 \qquad \forall j,t \qquad (3.10)$$

$$I_{j,t}^2 - B_{j,t}^2 = I_{j,t-1}^2 - B_{j,t-1}^2 + \sum_{i=1}^{I} X_{ijt}^2 + W_{jt}^1 - D_{jt}^2 \qquad \forall j,t \qquad (3.11)$$

$$I_{j0}^k = B_{j0}^k = B_{jT}^k = 0 \qquad \forall j,k \qquad (3.12)$$

$$Sd_{ijt}^k, Q_{it}^k, X_{ijt}^k, O_t, W_{jt}^k, I_{it}^k, I_{jt}^k, B_{jt}^k \ge 0 \qquad \forall i,j,t,k \qquad (3.13)$$

$$G_{it}^k, Sb_{ijt}^k \in \{0,1\} \qquad \forall i,j,t,k \qquad (3.14)$$

$$Sd_{ijt}^k \in N \qquad \forall i,j,t,k \qquad (3.15)$$

- Constraint set (3.3) ensures that the quantity of each RM *i* purchased in time period *t*, if any, is limited by the supplier capacity in that period. This 'fixed charge' constraint establishes the relation between the binary and continuous purchasing-related variables.
- Only one major setup for a certain RM, FP and SG combination can take place during a time period, which is stated in constraint set (3.4).
- Equation (3.5) specifies the total setup time for a certain (i, j, k) combination in any period, which is simply the sum of both major and minor setup times. The total production and setup times are subtracted from the available 24 hours a day to give the idle time. The result is then multiplied by 0.4, which is the coefficient obtained earlier using regression analysis, and by the binary major setup variable to indicate that only the chosen (i, j, k) combination shall contribute to the total setup time.
- Constraint set (3.6) states that the total production and setup times for all products should not exceed the available uptime, whether regular production time or overtime, in any time period.
- Naturally, the maximum number of overtime hours worked per day is bounded by a certain allowable value, as specified by constraint (3.7).
- Since the steel bars rolling industry is a batch process, the production quantity of a certain FP from a particular RM, X_{ijt}^{k}, is a function of the number of batches produced, batch size and the yield, which is stated in constraint (3.8). In fact, Constraint set (3.8) coupled with equation (3.5) ensure the overlap of the longest minor setup with the major setup. That is, the first batch produced at the

beginning of the day is for that RM-FP combination that requires the longest minor setup time.

- Constraint set (3.9) defines the inventory balance equation for a particular RM *i* at the end of time period *t* as the sum of the previous period's stock balance and the quantity procured in the current period minus the quantity consumed in the production of the various end items in the current period.
- Constraint sets (3.10) and (3.11) provide the finished products inventory balance equations for both steel grades. Depending on its economic feasibility, these constraints allow for the demand of a grade 40 steel bar to be partially fulfilled by a similar sized grade 60 rebar. The inventory balance equations for both grades are better understood using Figure 3.3 below.

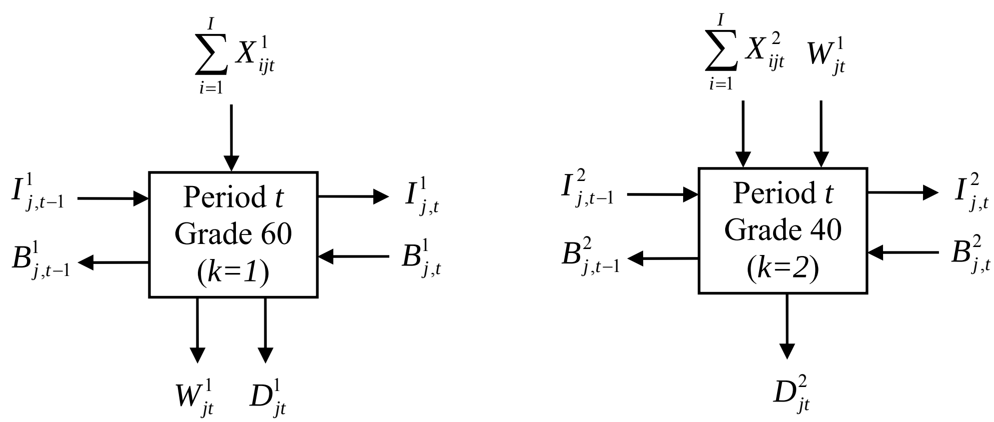

Figure 3.3: Inventory balance for both grades in a particular time period

- Without loss of generality, the initial inventory and backorder levels are set to zero as shown in constraint (3.12) which also ensures that the demand for all the finished products throughout the planning horizon is met through setting the ending backorder levels to zero.

- Constraints (3.13) – (3.15) represent the non-negativity, binary and integrality restrictions on the respective decision variables.

It should also be noted that, in any time period, the consumption rate of a specific raw material to produce the various finished products should not exceed the purchased quantity of that raw material in that same period in addition to the previously available stock. That is

$$b_t \times \sum_{j=1}^{J} \left[Sb_{ijt}^{k} + Sd_{ijt}^{k} \right] \leq Q_{it}^{k} + I_{i,t-1}^{k} \qquad \forall\, i,t,k \tag{3.16}$$

However, constraint set (3.16) is implied by constraint set (3.9) along with the non-negativity restriction on the variables I_{it}^{k}.

3.5 Properties of the Mathematical Model

The model developed above is, in a sense, a typical dynamic capacitated multi-item lot-sizing problem (CMILSP) formulation with various practical extensions incorporated into the model. A much simpler version, which is the capacitated single-item lot-sizing problem (CSILSP), is NP-hard in general. It is even NP-hard for very special cases (Bitran and Yanasse 1982). As pointed out by Maes and Van Wassenhove (1986), capacitated lot-sizing models are powerful and very flexible but slow (or impossible) to solve if the problem instance is very large. Prior to establishing the solution methodology for the mathematical model, the properties of the model have to be explored in order to gain more insights towards solving the model. These properties are:

- **Non-separable:** Although the two steel grades share the same combination of dimensions for both RM and FP, the problem is not separable into two smaller sub-problems each dealing with one steel grade at a time. This is the case since it is not known in advance how much capacity should be allocated to each steel grade, and this would also eliminate the possibility for demand substitution.
- **Bilinear program:** Clearly, the model at hand is a mixed integer nonlinear program (MINLP) as it involves a mixture of continuous and integer variables, and the non-linearity arises due to the existence of a nonlinear constraint (Equation 3.5) that also appears in the objective function (once the variable S_{ijt}^{k} is substituted for by its expression). In particular, the model falls under a special category of nonlinear programs called mixed integer bilinear programs (MIBLP) in which the binary variable Sb_{ijt}^{k} is multiplied by the continuous variables $X_{i'j',t-1}^{k'}$ and $S_{i'j',t-1}^{k'}$, one at a time, as can be seen in equation (3.5). The bilinearity property these models possess is due to the fact that, for fixed Sb_{ijt}^{k} values, the original model reduces to a mixed integer linear program (MILP), and for fixed $X_{i'j',t-1}^{k'}$ and $S_{i'j',t-1}^{k'}$ values, it again reduces to a MILP in the space of the other decision variables. Such models have also been addressed in the literature under the title "mixed 0-1 quadratic programs" (e.g., Adams et al. 2004, Adams and Forrester 2007).
- **Non-convexity:** In the context of nonlinear programming, the simplest mathematical models to obtain a solution for are the convex programs, which involve the minimization of a convex function subject to a convex set. In such a

case, classical optimization techniques can be used to obtain the optimal solution, where the local optimum coincides with the global one. Unfortunately, the bilinearity existent in the model above causes it to be non-convex as we shall prove in what follows. In order to prove non-convexity of the model, it suffices to show that either the objective function is non-convex or that the feasible region, as defined by the set of constraints, is not a convex set. Since a linear function is both convex and concave, we target constraint (3.5) and show that it is not convex which entails that the feasible region is not a convex set. For a function to be non-convex, its Hessian matrix must not be positive definite or positive semidefinite. First, let us rearrange the terms in constraint (3.5) and rewrite it as follows:

$$S_{ijt}^{k} - 9.6 \times Sb_{ijt}^{k} + 0.4 \times Sb_{ijt}^{k} \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{k=1}^{2} \left(\frac{X_{ij,t-1}^{k}}{\alpha_{ij}} + S_{ij,t-1}^{k} \right) + ST_{ij} \times Sd_{ijt}^{k} = 0 \qquad (3.17)$$

Let the left hand side (LHS) above be denoted as $g(S_{ijt}^{k}, Sb_{ijt}^{k}, X_{ij,t-1}^{k}, S_{ij,t-1}^{k}, Sd_{ijt}^{k})$ and assume, for the time being, that this function is twice differentiable (i.e., relax the integrality and binary restriction on the respective decision variables). Then, the vector of the first order partial derivatives (i.e., the gradient) is given by:

$$\nabla g \begin{pmatrix} S_{ijt}^{k} \\ Sb_{ijt}^{k} \\ X_{ij,t-1}^{k} \\ S_{ij,t-1}^{k} \\ Sd_{ijt}^{k} \end{pmatrix} = \begin{pmatrix} 1 \\ -9.6 + 0.4 \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{k=1}^{2} \left(\frac{X_{ij,t-1}^{k}}{\alpha_{ij}} + S_{ij,t-1}^{k} \right) \\ \frac{0.4}{\alpha_{ij}} \times Sb_{ijt}^{k} \\ 0.4 \times Sb_{ijt}^{k} \\ ST_{ij} \end{pmatrix} \qquad (3.18)$$

The symmetric Hessian matrix, which is comprised of the second order partial derivatives, is obtained from Equation (3.18) as follows:

$$H\begin{pmatrix} S_{ijt}^{k} \\ Sb_{ijt}^{k} \\ X_{ij,t-1}^{k} \\ S_{ij,t-1}^{k} \\ Sd_{ijt}^{k} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \dfrac{0.4}{\alpha_{ij}} & 0.4 & 0 \\ 0 & \dfrac{0.4}{\alpha_{ij}} & 0 & 0 & 0 \\ 0 & 0.4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{3.19}$$

The following theorem is needed in order to draw conclusions concerning function convexity based on the obtained Hessian matrix.

† **Theorem 3.1:** (Bazaraa *et al.* 1993, page 96)

Let $\boldsymbol{H}$ be a symmetric $n \times n$ matrix with elements h_{ij}, if $h_{ii} = 0$ for any $i \in \{1,...,n\}$, then we must have $h_{ij} = h_{ji} = 0$ for all $j = 1,...,n$ as well, or else $\boldsymbol{H}$ is not positive semidefinite. †

By looking back at the Hessian matrix, notice that although $h_{22} = 0$ for example, $h_{23} = h_{32} = 0.4/\alpha_{ij} \neq 0$ and $h_{24} = h_{42} = 0.4 \neq 0$. As such, the conditions of the theorem do not hold and the above Hessian matrix is not positive semidefinite. In fact, upon coding the above model in AMPL programming language (Fourer *et al.* 2003) and attempting to solve it using CPLEX 11.0 solver, the obtained output message has reaffirmed the non-convexity of the model.

In brief, the presented mathematical model is a non-separable, non-convex, mixed integer bilinear program. These complicating features of the mathematical model explain the incapability of the commercial off-the-shelf optimization packages (e.g., AMPL/CPLEX and LINGO) to directly obtain the solution to this class of models even for small problem instances. Hence, there is a need for a specifically tailored solution algorithm that can efficiently handle such class of mathematical models as shall be presented in the next chapter.

3.6 Summary

This chapter has discussed the production planning problem encountered on a frequent basis at the maser production scheduling level in steel rolling mills. The problem was formulated as a MIBLP in which the technological constraints associated with the manufacturing process have been taken into account. The model studies the combined effects of several interrelated factors characterizing this type of industry such as complex setup time structure, batch production, scrap rate, overtime, backlogging and product substitution, on the planning decisions. The objective is to establish the daily/weekly production lot sizes such that the assumed static customer demand is fulfilled at a minimal total cost. Upon studying the properties of the presented mathematical model, it turned out that such model is not readily solvable using the commercial optimization packages. Our efforts in the next chapter are directed towards developing efficient solution algorithms that have the ability to solve similar models within a reasonable amount of computational time.

Chapter 4

Application of the Solution Methodologies to the Proposed Mathematical Model

4.1 Introduction

This chapter seeks to attain the solution to the production planning model through the implementation and/or customization of the solution techniques discussed in the appendix. In general, solving the equivalent MILP resulting from the linearization approaches directly using the MIP solver or obtaining the solution to the bilinear model via branch-and-bound (B&B) algorithms yield an optimal solution unless the solver is stopped after a pre-specified length of run time where the solution is considered satisfactory in this case. Benders decomposition (BD) approach, on the other hand, does not necessarily render an optimal solution depending on the value set for the accepted tolerance. We start by presenting how a generic linearization approach applies to the model at hand followed by the development of a modified B&B algorithm and lastly a hybrid linearization-BD approach. In this chapter, the solutions obtained to the model at hand via these solution techniques are the optimal ones. The performance of three solution algorithms is tested under the same set of input parameters for several problem instances of varying complexities.

4.2 Linearization techniques

Since the mathematical background for the linearization approaches has previously been established, we directly employ the technique of Glover (1975) to the MIBLP model presented in Chapter 3. Note that the RLT of Adams and Sherali (1990) is not applicable to the current model as it deals with constraints that are either a function of the binary variable or the continuous one, but not both, which is not the case here. Recall that in this model, the bilinearity appears due to multiplying the binary variable Sb_{ijt}^{k} by the continuous variables $X_{i'j',t-1}^{k'}$ and $S_{i'j',t-1}^{k'}$ as can be seen in equation (3.5) and in the objective function (in the variable S_{ijt}^{k}). Following Glover's linearization scheme, Equation (3.5) is replaced by the following sets of constraints in the linearized version of the model:

$$S_{ijt}^{k} = 9.6 \times Sb_{ijt}^{k} - 0.4 \times y_{ijt}^{k} + ST_{ij} \times Sd_{ijt}^{k} \qquad \forall i, j, t, k \tag{4.1}$$

$$L_{ijt}^{k} Sb_{ijt}^{k} \leq y_{ijt}^{k} \leq U_{ijt}^{k} Sb_{ijt}^{k} \qquad \forall i, j, t, k \tag{4.2}$$

$$\sum_{i'j'k'} \left(\frac{X_{i'j',t-1}^{k'}}{\alpha_{i'j'}} + S_{i'j',t-1}^{k'} \right) - U_{ijt}^{k}\left(1 - Sb_{ijt}^{k}\right) \leq y_{ijt}^{k} \leq \sum_{i'j'k'} \left(\frac{X_{i'j',t-1}^{k'}}{\alpha_{i'j'}} + S_{i'j',t-1}^{k'} \right) - L_{ijt}^{k}\left(1 - Sb_{ijt}^{k}\right)$$

$$\forall i, j, t, k \tag{4.3}$$

where L_{ijt}^{k} and U_{ijt}^{k} represent the lower and upper bounds on the term $\sum_{i'j'k'} \left(\frac{X_{i'j',t-1}^{k'}}{\alpha_{i'j'}} + S_{i'j',t-1}^{k'} \right)$ respectively. Since this term physically denotes the total production and setup time in period $t-1$, the value of the lower bound, L_{ijt}^{k}, is simply set to zero and that for the upper bound is $U_{ijt}^{k} = A_t + A_{ot}$, which is the total available regular and overtime hours of production capacity.

Constraints (4.1) – (4.3) are necessary in order to establish the equivalence $y_{ijt}^{k} = Sb_{ijt}^{k} \sum_{i'j'k'} \left(\frac{X_{i'j',t-1}^{k'}}{\alpha_{i'j'}} + S_{i'j',t-1}^{k'} \right)$ for all i, j, t and k. Once the binary variables Sb_{ijt}^{k} assume a value of zero, constraint set (4.2) is binding while constraint set (4.3) is the binding one whenever Sb_{ijt}^{k} assume a value of one. Since the newly added variables, y_{ijt}^{k}, appear in the objective function with a negative sign, the model will seek the maximum possible value for this variable. This indicates that the left hand side inequalities of constraints (4.2) and (4.3) are redundant and can be dropped out of the model.

Clearly, the linearization approaches, in general, suffer from a common drawback that affects their performance. They involve a radical increase in the number of problem variables and constraints and, as such, the gains to be derived from dealing with linear functions are quite likely to be nullified by the increased problem size (Glover 1975). This increase in the problem size has a major impact on the CPU computation time especially for large scale models since the added variables and constraints are accounted for at all branching nodes in the MIP solver. In particular, the linearized version of the model at hand involves the addition of $I \times J \times T \times K$ variables and $2 \times I \times J \times T \times K$ constraints. The performance of the linearization approach for several problem instances of different sizes is reported in Section 4.5.

4.3 Modified Branch-and-Bound algorithm

The production planning problem at hand, like many other real world problems, is a combinatorial optimization problem that has a very large, but finite, number of feasible solutions. However, solving such discrete optimization problems to optimality through explicit enumeration is normally impossible or is an immense job requiring substantial amount of computational time. For the linearization approach discussed earlier, the shortcomings associated with this approach contribute to a much needed alternative and a more efficient solution procedure.

In this section, a modified version of the long established B&B algorithm is developed in which different branching, bounding and fathoming strategies are employed. The basic idea here is to get rid of bilinearity through proper substitution of the complicating binary variables, Sb_{ijt}^{k}, while simultaneously obtaining the bound at each node via such substitution. Once the values of these variables are set to either zero or one, the resulting reduced size MILP becomes a lot easier to solve. Fortunately, constraint set (3.4) states that among all possible RM, FP and SG combinations, there exists only one possible combination for which a major setup could take place in period t. Hence, setting $Sb_{i'j't}^{k'} = 1$ for a certain value of i', j' and k' entails that $Sb_{ijt}^{k} = 0$ for either one of $i \neq i'$, $j \neq j'$, $k \neq k'$ and for the same t. This reduces the number of possibilities (i.e. branches to be explored) to $I \times J \times K$ for each t. Moreover, the possible number of $Sb_{ijt}^{k} = 1$ combinations at optimality is now $(IJK)^{T}$ as a direct result of constraint (3.4). However, fully exploring this many possibilities is a tedious and a time consuming task especially for large scale models.

Therefore, a clever enumeration algorithm for such possibilities is much needed. The proposed B&B based solution algorithm is stated formally as follows:

<u>Step 1:</u> Set $Sb_{ijt}^{k} = 0$ for $\forall\, i,\ j, k, t$ throughout the model, ignore Equation (3.4) and then solve.

The resulting MILP problem is a relaxed version of the original problem and its solution provides a lower bound on the optimal value for the original problem since the major setup cost is set to zero. However, its solution is not feasible to the original problem as it violates equation (3.4).

<u>Step 2:</u> Set $t = 1$ and substitute $Sb_{ij1}^{k} = 1$ for each branch emanating from the original node, with $I \times J \times K$ possible branches, while keeping the substitution $Sb_{ijt}^{k} = 0$ for $\forall i, j, k$ and $t > 1$.

The resulting solution to each of these subproblems gives a lower bound on the optimal objective function value of the original problem. Among the resulting $I \times J \times K$ subproblems, branch from the one with the lowest value of the objective function, as this is the most promising node (ties are broken arbitrarily).

<u>Step 3:</u> Repeat step 2 for $t = 2,3,...,T$ with each value of t corresponding to one level in the tree. Again, there would be $I \times J \times K$ possible branches from each node.

Clearly, the lower bound on the optimal objective function value increases as we go down the tree since larger portions of the major setup cost are being accounted for.

<u>Step 4:</u> The solution with the minimum objective function value obtained at the lowest level of the tree (corresponding to $t = T$) is called the incumbent, which is used as an upper bound during the search of the unexplored branches of the tree. The search continues with other branches and the value of this incumbent is compared

with the bounds obtained at each node in order to make the fathoming decisions. If the value of the objective function calculated at a particular node is larger than the incumbent, that branch is fathomed. Otherwise, the value of the incumbent is updated whenever a lower value incumbent is attained, and then the new incumbent is used to make the fathoming decisions.

It should be noted that at each node of the tree, the MILP model is solved without equation (3.4) but it is this constraint that drove the branching scheme in the first place. The tree exploring strategy is depth-first since this allows a faster recovery of an incumbent, which can be used to make the fathoming decisions. The width of the tree depends on the values of I, J and K, and the number of different levels is equal to $T+1$. The obtained MILP models at each node are directly solved using AMPL/CPLEX 11.0 solver.

To serve illustrative purposes, Figure 4.1 depicts a graphical representation for the progress of the proposed algorithm as applied to a small problem instance ($I = K = 2$, $J = 3$ and $T = 4$). Implementing the algorithm yields a tree with 5 levels (since $T+1=5$) and 12 nodes (since $I \times J \times K = 12$) at each level starting from level 2. The nodes shown with a solid line are the ones yielding the minimum objective function value (i.e. most promising) among all other nodes in the same level. The branches in the solid line represent the path connecting these nodes which leads to the incumbent at the bottom of the tree. Hence, the values of the major setup variables corresponding to the incumbent in Figure 4.1 are $Sb_{221}^{2} = Sb_{232}^{1} = Sb_{113}^{2} = Sb_{224}^{2} = 1$. The incumbent value can now be used to make the fathoming decisions for the unexplored branches of the tree, which could eventually result in an optimal solution different from the current incumbent (indicating that a lower value incumbent has been found).

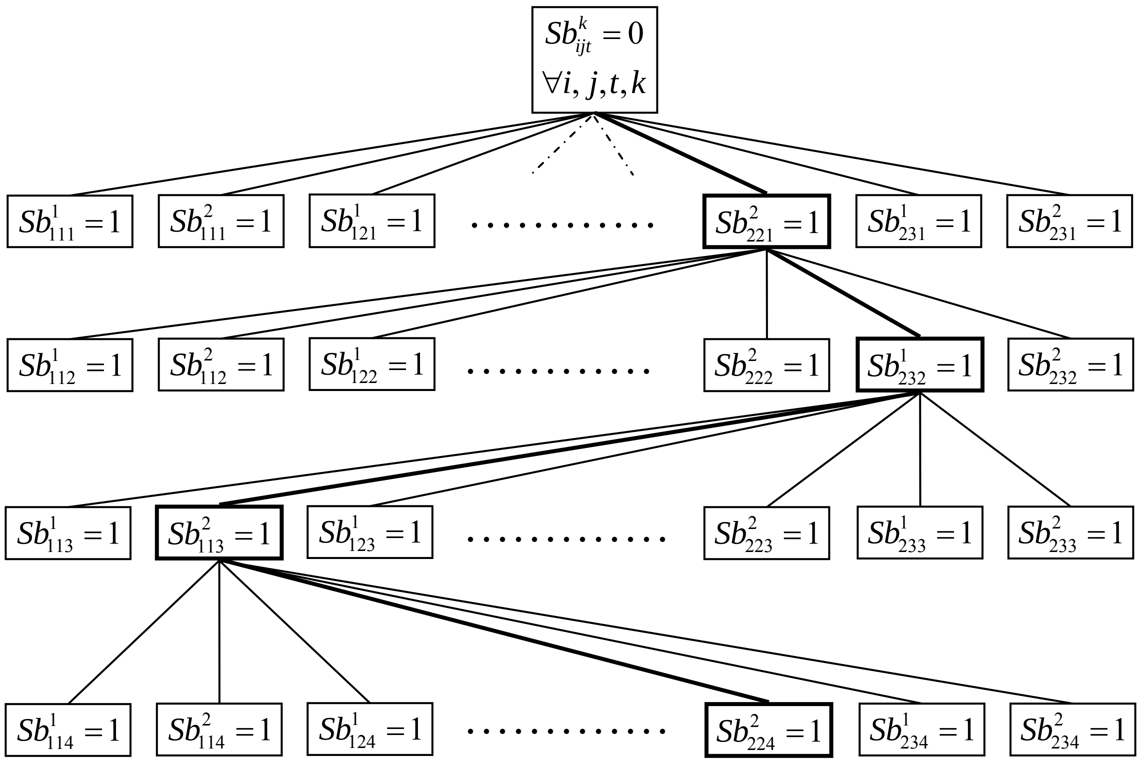

Figure 4.1: Partial tree resulting from applying the proposed B&B algorithm for a small problem instance

At this juncture, a clear distinction has to be made between the classical B&B algorithm and the one proposed in this work. The classical B&B algorithm utilizes LP-relaxation of the binary variables Sb_{ijt}^{k} in order to obtain the bound at the initial node. Such a relaxation does not resolve either bilinearity or non-convexity embedded in the model and hence the resulting model is still not directly solvable using optimization software. As a matter of fact, the relaxed version of the model is now "pure bilinear" (two continuous variables are multiplied by one another) instead of being "mixed-integer bilinear", and there exists a reformulation-linearization technique established in the literature for this class of bilinear models (Sherali and Alameddine 1992). Table 4.1 provides more insights into the differences between the classical B&B algorithm and the modified one.

Table 4.1: A comparison between the classical B&B algorithm and the modified B&B algorithm proposed in this thesis

	Classical B&B	**Modified B&B**
Branching	- Two branches emanate from each node corresponding to one of the binary variables being assigned a value of either zero or one (Except at an incumbent or at an integer solution where no more branching takes place). - Hence, the number of nodes at any level of the tree is twice that of the higher level.	- Utilizes Constraint (3.4) to obtain $I \times J \times K$ branches from each node, where each branch corresponds to one of the binary variables being equal to one. - Each level in the tree corresponds to a single t value. - This branching scheme results in a tree with $I \times J \times K$ nodes at each level of the tree.
Bounding	Relaxes the binarity restriction on all binary variables (except the branching variable) and allows those variables to assume any value between zero and one (i.e. LP relaxation).	Assumes the values of all binary variables (other than the branching variable) are set to zero (those in the same level are set to zero due to constraint (3.4) and others to obtain the bound and reduce the original problem to a MILP).
Incumbent	Corresponds to an integer solution obtained at any node (this might occur at any level of the tree)	Obtained only at the lowest level of the tree, since that is when Constraint (3.4) is satisfied

It is important to point out that there exist several practical problems to which the proposed B&B based algorithm is applicable. Examples include, but not limited to: (1) Small-bucket dynamic lot-sizing problems where a single product may be produced in each time period, (2) Portfolio management problems where a new investment opportunity is decided upon at the beginning of each year, and (3) Vendor selection problem where a product may be solely supplied by a single source for a specific time period. Section 4.5 presents a numerical example for a small problem instance as well as the computational experiments for various problem sizes.

4.4 Hybrid Linearization-Benders decomposition approach

The solution algorithm presented in this section is a two phase methodology that first applies the linearization approach in order to obtain the linearized model and then applies Benders decomposition (BD) to this MILP model instead of solving it directly via the MIP solver. The motivation to use BD is its ability to handle the complicating variables separately which is likely to save in the computational time needed. More importantly, we seek to provide an application of BD to the solution of mathematical models involving complicating constraints as well (constraint 3.5 in this case). On page 244 of their book, Conejo *et al.* (2006) point out that considering nonlinear constraints as complicating constraints and treating them through linearization procedures do not lead generally to a decomposed problem. Therefore, we present an instance of a mathematical model to which linearization techniques are applied first and then the linearized model is handled with Benders decomposition approach.

Following the BD methodology, the complicating variables in the model at hand are the binary variables Sb_{ijt}^{k} and G_{it}^{k}, as well as the integer variables Sd_{ijt}^{k}. Since the continuous variables S_{ijt}^{k} and X_{ijt}^{k} are functions of the Sb_{ijt}^{k} and Sd_{ijt}^{k} variables as can be seen in equations (3.5) and (3.8) respectively, their values are optimized via solving the master problem which is given by:

Master problem:

$$
\begin{aligned}
Min \quad Z_{MP} &= \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{2}\sum_{t=1}^{T}\left(SC_t S_{ijt}^{k} + PC_{ijt} X_{ijt}^{k}\right) \\
&+ \sum_{i=1}^{I}\sum_{k=1}^{2}\sum_{t=1}^{T}\left(OR_{it}^{k} G_{it}^{k} + CR_{it}^{k} Q_{it}^{k} + IR_{it}^{k} I_{it}^{k}\right) + \alpha
\end{aligned} \tag{4.4}
$$

Subject to

$$\alpha \geq Z_{SP}^{(v)} + \sum_{ijtk} \mu_{ijt}^{k(v)}\left(S_{ijt}^{k} - S_{ijt}^{k(v)}\right) + \sum_{ijtk} \lambda_{ijt}^{k(v)}\left(X_{ijt}^{k} - X_{ijt}^{k(v)}\right) \qquad v = 1,\ldots,V \tag{4.5}$$

$$Q_{it}^{k} \leq M_{it}^{k} G_{it}^{k} \qquad \forall i,t,k \tag{4.6}$$

$$\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{2} Sb_{ijt}^{k} = 1 \qquad \forall t \tag{4.7}$$

$$S_{ijt}^{k} = 9.6 \times Sb_{ijt}^{k} - 0.4 \times y_{ijt}^{k} + ST_{ij} \times Sd_{ijt}^{k} \qquad \forall i, j, t, k \tag{4.8}$$

$$L_{ijt}^{k} Sb_{ijt}^{k} \leq y_{ijt}^{k} \leq U_{ijt}^{k} Sb_{ijt}^{k} \qquad \forall i, j, t, k \tag{4.9}$$

$$\sum_{i'j'k'}\left(\frac{X_{i'j',t-1}^{k'}}{\alpha_{i'j'}} + S_{i'j',t-1}^{k'}\right) - U_{ijt}^{k}\left(1 - Sb_{ijt}^{k}\right) \leq y_{ijt}^{k} \leq \sum_{i'j'k'}\left(\frac{X_{i'j',t-1}^{k'}}{\alpha_{i'j'}} + S_{i'j',t-1}^{k'}\right) - L_{ijt}^{k}\left(1 - Sb_{ijt}^{k}\right)$$

$$\forall i, j, t, k \tag{4.10}$$

$$X_{ijt}^{k} = b_t \times \left(Sb_{ijt}^{k} + Sd_{ijt}^{k}\right) \times \rho_{ij} \qquad \forall i, j, t, k \tag{4.11}$$

$$I_{it}^{k} = I_{i,t-1}^{k} + Q_{it}^{k} - b_t \times \sum_{j=1}^{J}\left[Sb_{ijt}^{k} + Sd_{ijt}^{k}\right] \qquad \forall i, t, k \tag{4.12}$$

$$Sd_{ijt}^{k}, Q_{it}^{k}, X_{ijt}^{k}, I_{it}^{k}, \alpha \geq 0 \qquad \forall i, j, t, k \tag{4.13}$$

$$G_{it}^{k}, Sb_{ijt}^{k} \in \{0,1\} \qquad \forall i, j, t, k \tag{4.14}$$

$$Sd_{ijt}^{k} \in N \qquad \forall i, j, t, k \tag{4.15}$$

Notice that the variable α assumes only non-negative values (Constraint 4.13) since it represents an underestimate for the sub-problem's cost function. Once the values of the complicating variable are established, they are substituted into the sub-problem in order to optimize the production capacity, finished product inventory and backlogging, and the substitution related decisions. The sub-problem at iteration v is formulated below:

<u>Sub-problem:</u>

$$Min \quad Z_{SP}^{(v)} = \sum_{t=1}^{T} PO_t O_t + \sum_{j=1}^{J}\sum_{k=1}^{2}\sum_{t=1}^{T}\left(IF_{jt}^{k} I_{jt}^{k} + BC_{jt}^{k} B_{jt}^{k} + M_{jt}^{k} R_{jt}^{k} \right) + \sum_{j=1}^{J}\sum_{t=1}^{T} W_{jt}^{1}\left(SP_{jt}^{1} - SP_{jt}^{2} \right) \tag{4.16}$$

Subject to

$$\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{2}\left(\frac{X_{ijt}^{k}}{\alpha_{ij}} + S_{ijt}^{k} \right) \le A_t + O_t \qquad \forall t \tag{4.17}$$

$$O_t \le A_{ot} \qquad \forall t \tag{4.18}$$

$$I_{j,t-1}^{1} - B_{j,t-1}^{1} - I_{j,t}^{1} + B_{j,t}^{1} + \sum_{i=1}^{I} X_{ijt}^{1} - W_{jt}^{1} + R_{jt}^{1} = D_{jt}^{1} \qquad \forall j,t \tag{4.19}$$

$$I_{j,t}^{2} - B_{j,t}^{2} = I_{j,t-1}^{2} - B_{j,t-1}^{2} + \sum_{i=1}^{I} X_{ijt}^{2} - W_{jt}^{2} \qquad \forall j,t \tag{4.20}$$

$$W_{jt}^{1} + W_{jt}^{2} + R_{jt}^{2} = D_{jt}^{2} \qquad \forall j,t \tag{4.21}$$

$$I_{j0}^{k} = B_{j0}^{k} = B_{jT}^{k} = 0 \qquad \forall j,k \tag{4.22}$$

$$O_t, W_{jt}^{k}, I_{jt}^{k}, B_{jt}^{k}, R_{jt}^{k} \ge 0 \qquad \forall j,t,k \tag{4.23}$$

$$S_{ijt}^{k} = S_{ijt}^{k(v)} \quad : \quad \mu_{ijt}^{k(v)} \qquad \forall i,j,t,k \tag{4.24}$$

$$X_{ijt}^{k} = X_{ijt}^{k(v)} \quad : \quad \lambda_{ijt}^{k(v)} \qquad \forall i,j,t,k \tag{4.25}$$

The nonnegative variables R_{jt}^{k} are added to the inventory balance constraints (4.19) and (4.21) to avoid the infeasibility of the sub-problem through balancing out the shortages in the finished products. The penalty factor M_{jt}^{k} associated with these variables should be large enough in order to ensure no shortages at final solution (i.e., $R_{jt}^{k} = 0$ for all j, t, k). The flowchart in Figure 4.2 illustrates how the BD algorithm works.

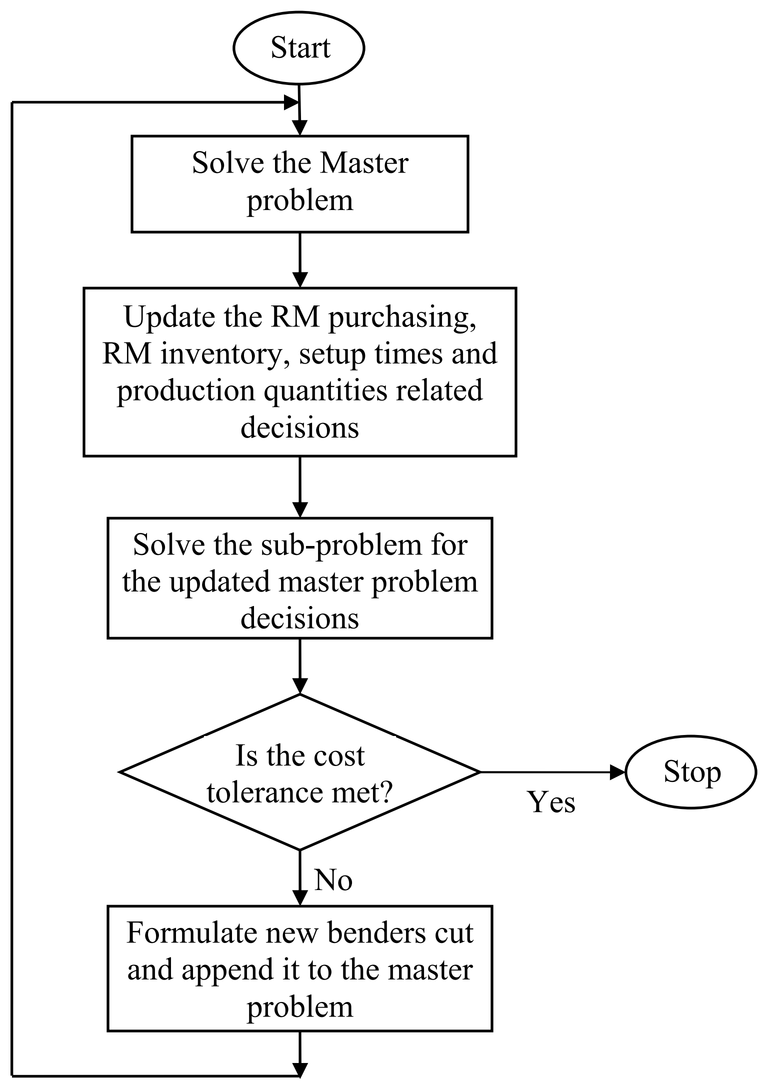

Figure 4.2: A flowchart for the decomposition structure

The solution to the sub-problem provides marginal information on the goodness of the decisions made at the master problem which is reflected in the obtained values for the associated dual variables. After each iteration v, those dual variables are fed back into the master problem, through the optimality cut (constraint 4.5), which helps refine the decisions made at the previous iteration. Note that only the variables S_{ijt}^{k} and X_{ijt}^{k} need to be included in the optimality cut since they are the only variables from the master problem that appear in the sub-problem and can affect the sub-problem's objective function value. The addition of any other variable (e.g., Sb_{ijt}^{k} and Sd_{ijt}^{k}) to the optimality cut will result in a zero value for their associated dual variables in all the iterations, and consequently, this would not have any impact on the solution or the convergence rate. As mentioned in the appendix, the algorithm terminates when the cost tolerance $Z_{SP}^{(v)} - \alpha^{(v)} < \varepsilon$ is met.

4.5 Computational analysis

The purpose of this section is to provide more insights into the mechanism of the proposed B&B algorithm through a simple numerical example. Also, the performance of the three solution algorithms is benchmarked against one another for several problem instances of varying complexities.

4.5.1 A numerical example

To serve for illustrative purposes, a relatively small problem instance where I=K=2, $J = 3$ and $T = 4$ is solved in this subsection through applying the B&B algorithm. Table 4.2 shows problem parameters involving only the time index, which are available regular time and overtime capacity, batch size, the per hour overtime cost and setup cost. Since higher values for the index *j* indicate smaller rebar diameters (see Table 3.2), the per unit production cost from the same billet (i.e. same *i*) increases for increasing values of *j* as seen in Table 4.3. The finished product related parameters, including inventory holding cost, backordering cost, selling prices and demand are shown in Tables 4.4 and 4.5. On the other hand, Table 4.6 provides the values for the raw material related parameters, including procurement cost, ordering cost, inventory holding cost and maximum supply capacity. Finally, Table 4.7 gives the yields, production rates and minor setup times for each raw material-finished product combination.

Table 4.2: Problem parameters involving only time index

Time period	A_t	A_{ot}	b_t	PO_t	SC_t
***t* = 1**	16	4	60	300	400
***t* = 2**	16	4	60	300	400
***t* = 3**	16	4	60	300	400
***t* = 4**	16	4	60	300	400

Table 4.3: Production cost

PC_{ijt}	***i* = 1**			***i* = 2**		
	***j* = 1**	***j* = 2**	***j* = 3**	***j* = 1**	***j* = 2**	***j* = 3**
***t* = 1**	25	30	35	20	26	30
***t* = 2**	25	30	35	20	26	30
***t* = 3**	25	30	35	20	26	30
***t* = 4**	25	30	35	20	26	30

Table 4.4: Inventory holding and backordering costs

Time Period	IF_{jt}^{k}						BC_{jt}^{k}					
	***j* = 1**		***j* = 2**		***j* = 3**		***j* = 1**		***j* = 2**		***j* = 3**	
	***k* = 1**	***k* = 2**	***k* = 1**	***k* = 2**	***k* = 1**	***k* = 2**	***k* = 1**	***k* = 2**	***k* = 1**	***k* = 2**	***k* = 1**	***k* = 2**
***t* = 1**	32	30	35	33	39	37	25	25	28	28	34	34
***t* = 2**	33	31	36	34	41	39	26	26	30	30	35	35
***t* = 3**	30	28	33	31	40	38	27	27	32	32	37	37
***t* = 4**	32	30	35	33	41	39	30	30	34	34	38	38

Table 4.5: Demand and selling prices for end items

Time Period	SP_{jt}^{k}						D_{jt}^{k}					
	j = 1		***j = 2***		***j = 3***		***j = 1***		***j = 2***		***j = 3***	
	k = 1	***k = 2***	***k = 1***	***k = 2***	***k = 1***	***k = 2***	***k = 1***	***k = 2***	***k = 1***	***k = 2***	***k = 1***	***k = 2***
t = 1	1800	1600	1830	1630	1900	1700	60	130	100	90	60	100
t = 2	1800	1600	1830	1630	1900	1700	70	100	120	55	110	75
t = 3	1840	1640	1860	1660	1940	1740	90	85	80	110	95	40
t = 4	1820	1620	1860	1660	1930	1730	140	70	90	150	65	50

Table 4.6: Raw material related costs and supplying limits

Time Period	CR_{it}^{k}				OR_{it}^{k}				IR_{it}^{k}				M_{it}^{k}			
	i = 1		***i = 2***		***i = 1***		***i = 2***		***i = 1***		***i =2***		***i = 1***		***i = 2***	
	k = 1	***k = 2***	***k = 1***	***k = 2***	***k = 1***	***k = 2***	***k = 1***	***k = 2***	***k = 1***	***k = 2***	***k = 1***	***k = 2***	***k = 1***	***k = 2***	***K = 1***	***k = 2***
t = 1	650	550	700	600	2000	2000	2200	2200	22	20	25	23	300	300	300	300
t = 2	660	560	710	610	2050	2050	2250	2250	23	21	26	24	300	300	300	300
t = 3	640	540	700	600	2100	2100	2300	2300	22	20	25	23	300	300	300	300
t = 4	670	570	720	620	2100	2100	2300	2300	23	21	26	24	300	300	300	300

Table 4.7: Yields, production rates and minor setup times

Raw material	ρ_{ij}			α_{ij}			ST_{ij}		
	j = 1	***j = 2***	***j = 3***	***j = 1***	***j = 2***	***j = 3***	***j = 1***	***j = 2***	***j = 3***
i = 1	0.94	0.90	0.87	56	54	52	0.25	0.40	0.50
i = 2	0.96	0.93	0.91	58	56	54	0.40	0.50	0.75

Implementing the proposed solution algorithm yields a tree with 5 levels (since $T+1=5$) and 12 nodes (since $I \times J \times K = 12$) at each level starting from level 2 (a partial tree is seen in Figure 4.3). At this stage of tree exploration, several branches can already be fathomed as they yield an objective function value that is higher than the current incumbent. In particular, starting from level 2 in the tree, the branches emanating from the three nodes where $Sb_{211}^{1} = Sb_{221}^{1} = Sb_{221}^{2} = 1$ can be fathomed. Several other nodes at the lower levels of the tree shall be fathomed as well. The remaining branches (i.e., those emanating from nodes with lower objective function value than that of the incumbent) need to be explored, and the current incumbent is to

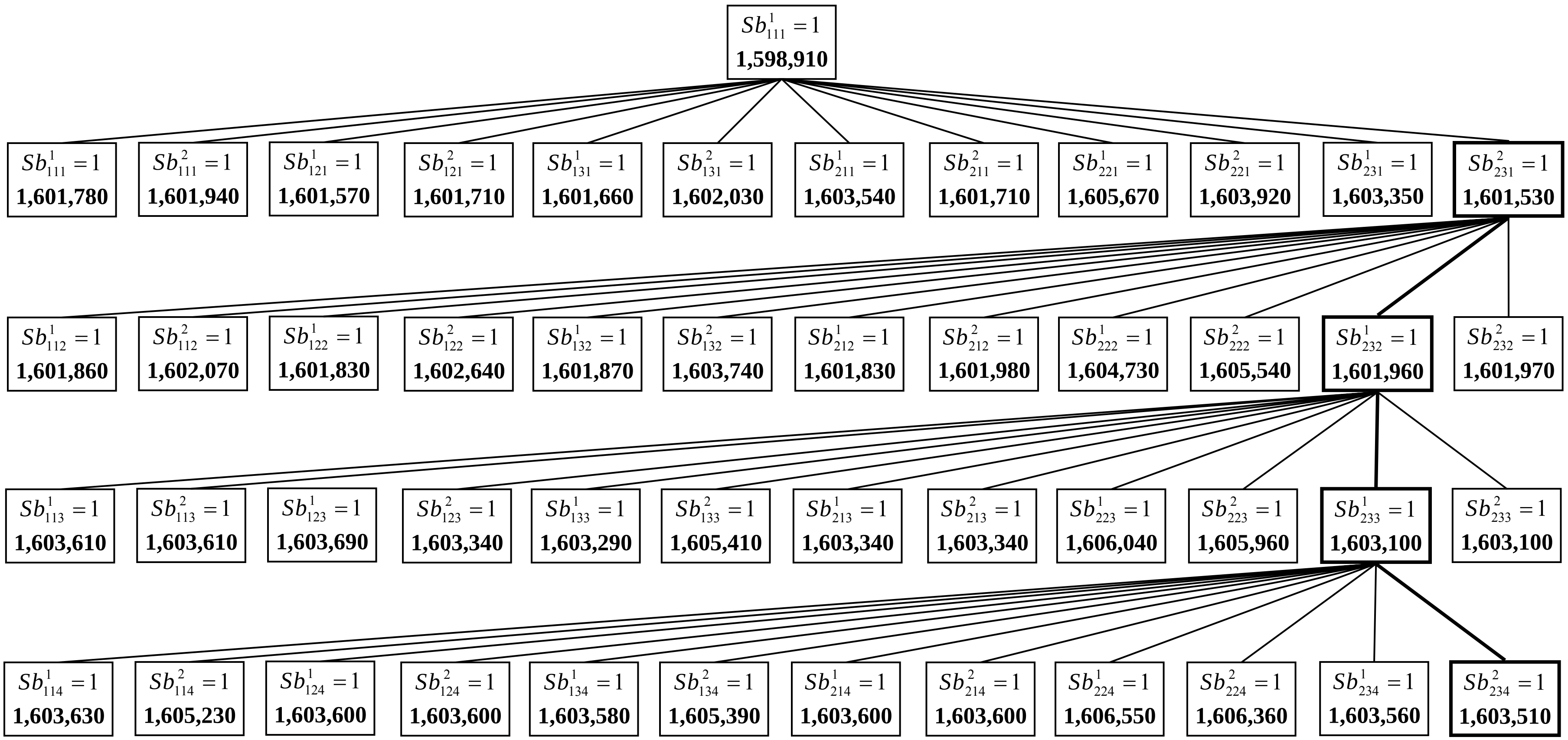

Figure 4.3: Partial tree resulting from applying the proposed B&B

be updated accordingly whenever a lower value is obtained (step 4 of the algorithm). As it turns out, the current incumbent value dominates all other branches, with the exception of only one branch resulting in the same objective function value (i.e., alternative optima). Since the planning horizon is 4 days, we would have four major setups corresponding to one at the beginning of each day. The optimal values of these major setup variables are $Sb_{231}^{2} = Sb_{232}^{1} = Sb_{233}^{1} = Sb_{234}^{2} = 1$, which correspond to the first incumbent obtained (or alternatively, $Sb_{231}^{2} = Sb_{232}^{1} = Sb_{233}^{2} = Sb_{234}^{2} = 1$).

Table 4.8: Finished product inventory, backorder and substitution quantities

I_{jt}^{k}	j = 1		j = 2		j = 3	
	k = 1	k = 2	k = 1	k = 2	k = 1	k = 2
t = 1	0	0	0	26	0	9.2
t = 2	1	0	23	0	0	0
t = 3	26.2	12.4	0	0	13.6	0
t = 4	0.2	0	15	0	0.8	0
B_{jt}^{k}	j = 1		j = 2		j = 3	
	k = 1	k = 2	k = 1	k = 2	k = 1	k = 2
t = 1	3.6	14.8	0	0	7.8	0
t = 2	0	17.8	0	10	0.6	19.2
t = 3	0	0	3	12	0	4.6
t = 4	0	0	0	0	0	0
W_{jt}^{k}	j = 1		j = 2		j = 3	
	k = 1	k = 2	k = 1	k = 2	k = 1	k = 2
t = 1	0	130	8	82	0	100
t = 2	39.4	60.6	19	36	46.6	28.4
t = 3	0	85	0	110	0	40
t = 4	0	70	0	150	0	50

Table 4.9: Optimal raw material purchasing and inventory policy

Time period	Q_{it}^{k}				G_{it}^{k}				I_{it}^{k}			
	i = 1		i = 2		i = 1		i = 2		i = 1		i = 2	
	k = 1	k = 2	k = 1	k = 2	k = 1	k = 2	k = 1	k = 2	k = 1	k = 2	k = 1	k = 2
t = 1	240	120	0	300	1	1	0	1	0	0	0	60
t = 2	240	0	240	0	1	0	1	0	0	0	0	0
t = 3	300	300	300	300	1	1	1	1	240	180	60	120
t = 4	0	0	0	0	0	0	0	0	0	0	0	0

The resulting values for the rest of the decision variables are shown in Tables 4.8-4.10 and the optimal objective function value is $Z^* = 1,603,510$. The resulting values of the demand substitution variables for grade 60 steel (W_{jt}^1) shown in Table 4.8 indicate that, under the given parameters' values, it is economically advantageous for the company to consider the demand substitution option, since $W_{12}^1 = 39.4$, $W_{21}^1 = 8$, $W_{22}^1 = 19$ and $W_{32}^1 = 46.6$.

Table 4.10: Optimal setup and production related decisions

Sd_{ijt}^k	***i = 1***						***i = 2***					
	j = 1		***j = 2***		***j = 3***		***j = 1***		***j = 2***		***j = 3***	
	k = 1	***k = 2***	***k = 1***	***k = 2***	***k = 1***	***k = 2***	***k = 1***	***k = 2***	***k = 1***	***k = 2***	***k = 1***	***k = 2***
t = 1	1	0	2	2	1	0	0	2	0	0	0	1
t = 2	1	0	3	0	0	0	1	1	0	0	2	0
t = 3	0	0	1	2	0	0	2	2	0	0	1	1
t = 4	1	0	2	3	1	0	1	1	0	0	0	0
S_{ijt}^k	***i = 1***						***i = 2***					
	j = 1		***j = 2***		***j = 3***		***j = 1***		***j = 2***		***j = 3***	
	k = 1	***k = 2***	***k = 1***	***k = 2***	***k = 1***	***k = 2***	***k = 1***	***k = 2***	***k = 1***	***k = 2***	***k = 1***	***k = 2***
t = 1	0.25	0	0.8	0.8	0.5	0	0	0.8	0	0	0	4.75
t = 2	0.25	0	1.2	0	0	0	0.4	0.4	0	0	3.932	0
t = 3	0	0	0.4	0.8	0	0	0.8	0.8	0	0	4.266	0.75
t = 4	0.25	0	0.8	1.2	0.5	0	0.4	0.4	0	0	0	2.471
X_{ijt}^k	***i = 1***						***i = 2***					
	j = 1		***j = 2***		***j = 3***		***j = 1***		***j = 2***		***j = 3***	
	k = 1	***k = 2***	***k = 1***	***k = 2***	***k = 1***	***k = 2***	***k = 1***	***k = 2***	***k = 1***	***k = 2***	***k = 1***	***k = 2***
t = 1	56.4	0	108	108	52.2	0	0	115.2	0	0	0	109.2
t = 2	56.4	0	162	0	0	0	57.6	57.6	0	0	163.8	0
t = 3	0	0	54	108	0	0	115.2	115.2	0	0	109.2	54.6
t = 4	56.4	0	108	162	52.2	0	57.6	57.6	0	0	0	54.6

4.5.2 Further computational experiments

At this juncture, a comparison between the performance of the classical linearization approach (Glover 1975), the proposed B&B algorithm and the hybrid linearization-BD (L-BD) algorithm is due. For a realistic problem size (I=T=5,

$J = 7$ and $K = 2$), the linearization approach adds 350 variables and 700 constraints to the original model. On the other hand, the B&B based algorithm involves the solution of a reduced size MILP at each node (350 less binary variables and 355 less constraints as compared to the original problem). Regardless of the problem size, the L-BD algorithm adds one additional constraint to the master problem representing the optimality cut after each iteration.

All three algorithms were coded using AMPL programming language (Fourer *et al.* 2003) and solved using CPLEX 11.0 solver, where the solver option is set to solve integer problems using the built-in branch and cut algorithm. For the sake of comparison, ten different problem instances are tested, and each problem instance is solved to optimality, under the same set of input parameters for all algorithms. The numerical experiments, in this chapter and throughout this thesis, are implemented on a single CPU with 4-2.2 GHz AMD Opteron 64-bit processors and 16 GB RAM. The values for the different input parameters are generated within certain range of intervals, as shown in Table 4.11.

Table 4.11: Selected range of values for input parameters in the test problems

Input parameter	Range of values	Input parameter	Range of Values
CR_{it}^{k}	(550 , 850)	BC_{jt}^{k}	(25 , 40)
OR_{it}^{k}	(2000, 2800)	SP_{jt}^{k}	(1500 , 2000)
IR_{it}^{k}	(15 , 30)	M_{it}^{k}	(150 , 300)
PC_{ijt}	(10 , 45)	ρ_{ij}	(0.82 , 0.98)
PO_{t}	(150 , 450)	α_{ij}	(49 , 58)
SC_{t}	(300 , 1000)	D_{jt}^{k}	(0 , 100)
IF_{jt}^{k}	(30 , 50)	ST_{ij}	(0.25 , 1.0)

The obtained results for the ten problem instances are reported in Table 4.12. It should be noted that the generation of the input parameters within different ranges of intervals will have an influence on the time it takes all algorithms to render an optimal solution. As can be seen in Table 4.12, the linearization approach and the L-BD attain the optimal solution in less time as compared to the B&B algorithm for small problem instances (P1 and P2). However, as the problem size increases, the B&B algorithm tends to outperform both of these approaches, although a larger problem size yields a bigger tree with, most likely, more nodes to be explored. While the B&B algorithm performs complete enumeration for small problem instances, tight bound and fathoming work effectively in larger problems. The time reduction resulting from the implementation of the B&B algorithm for larger problem instances may be attributed to two reasons. First, the binary variables Sb_{ijt}^{k} are considered as parameters at each node of the B&B algorithm while they remain as decisions variables in the other two approaches. Second, the constraints added through the linearization approach (4.1-4.3) are more involved as they all contain either the binary variable Sb_{ijt}^{k} or the integer variable Sd_{ijt}^{k} or even both, which requires longer solution time from the IP solver.

Although the L-BD algorithm separates the complicating variables before solving the resulting problems in the MIP solver, no savings in solution time were obtained as compared to the linearization approach which directly solves the MILP model using the MIP solver. This clearly indicates that it is not necessary for the BD algorithm to yield savings in computational time once applied to problems involving complicating constraints. The last column in Table 4.12 shows the savings in the solution time obtained from the use of the B&B algorithm as compared to the classical linearization approach, which could amount to 48% as seen in P4.

Table 4.12: Numerical comparison for the performance of the three solution approaches

PR. CODE	Problem size (I×J×T×K)	Linearization			Modified Branch & Bound			Linearization - Benders Decomposition			% solution time savings
		No. of variables	No. of constraints	Solution time (sec)	No. of variables[1]	No. of constraints[2]	Solution time (sec)	No. of variables	No. of constraints	Solution time (sec)	
P1	1×2×2×2	74	140	0.42	58	106	8.59	83	149	3.55	--
P2	1×3×3×2	159	297	0.71	123	222	106.32	178	316	84.14	--
P3	2×3×3×2	267	507	1,836	195	360	1,501	286	526	2,709	18
P4	2×3×4×2	358	678	17,961	262	482	9,305	383	703	31,186	48
P5	3×4×4×2	644	1,232	94,027	452	844	68,113	677	1265	128,763	28
P6	4×4×4×2	828	1,592	125,691	572	1,076	88,476	861	1625	153,631	30
P7	4×5×4×2	1,010	1,946	169,829	690	1,302	106,177	1051	1987	*	37
P8	5×5×4×2	1,234	2,386	*	834	1,582	121,830	1275	2427	*	N/A
P9	5×6×4×2	1,456	2,820	*	976	1,856	134,252	1505	2869	*	N/A
P10	5×7×5×2	2,101	4,071	*	1,401	2,666	174,514	2172	4142	*	N/A

[1]Number of variables in the MILP solved at each node.
[2]Number of constraints in the MILP solved at each node.
[*]Code execution was interrupted after 50 hours of run time with no results obtained.
N/A: The basis for carrying out the comparison is not available

4.6 Summary

This chapter has presented three exact solution algorithms for the production planning MIBLP model presented in Chapter 3. The first solution methodology is based on the classical linearization approach adopted for obtaining an equivalent larger sized MILP through the addition of auxiliary variables and constraints and then solving the resulting model directly using the MIP solver. The second solution methodology, however, modifies the long-established B&B algorithm and utilizes the special problem structure to minimize the number of branches and obtain the bound at each node. The third algorithm utilizes the Benders decomposition technique to decompose the MILP model resulting from the linearization approach into two easier problems and then solving these problems using the MIP solver.

The computational experiments have illustrated the ability of the B&B algorithm to solve realistic problem sizes involving 5 different billets, 7 different rebars, 2 steel grades with a planning horizon of one week (5 working days). In essence, this algorithm provides a more efficient alternative for solving bilinear models in which the number of possible combinations for the values of the complicating binary variables is limited. It stands as a first implementation of a B&B based algorithm towards solving MIBLP models since a review of the literature reveals no such implementation except for the pure bilinear case.

Chapter 5

PART-II: Rolling Horizon Approximations for Production Planning with Demand Volatility

5.1 Introduction

The analysis presented so far has dealt with the production planning problem at steel mills considering static or deterministic demand conditions. Although steel manufacturers mostly deal with long term loyal customers (Chen and Wang 1997, Kerkkänen 2007), the end customers' demand may differ from a predetermined forecasted value especially in periods of high demand. Needless to say, such demand variations have a major impact on the production and inventory related decisions and the incorporation of these uncertainties into the planning process is of paramount importance. As pointed out by Mula *et al.* (2006), models for production planning which do not recognize the uncertainty can be expected to generate inferior planning decisions as compared to models that explicitly account for uncertainty.

This chapter addresses the dynamic nature of the operating environment through implementing the developed production plan on a rolling horizon basis. As such, the model developed in Chapter 3 is adjusted in order to allow for the incorporation of demand forecasts and confirmed customers' orders, where both quantities are updated every period as new information becomes available. For reasons of efficiency and practicality, rolling horizon decision making is a common business practice in a dynamic environment (Chand *et al.* 2002). However, rolling horizon schedules suggest solving the mathematical model at hand repeatedly at the beginning of each period, a practice that requires substantial amount of computational efforts especially for large size problems. Hence, it is the objective of this chapter to introduce approximate models that, once implemented on a rolling horizon basis, yield reduced problem dimensionality with significant savings in computational time while still providing practical proxies for the exact model.

5.2 Decision making under uncertainty

In reality, industrial firms operate in a constantly changing environment that causes production planning related problems, such as the one at hand, to be dynamic in nature. Essentially, the instability associated with such problems is caused by external factors (e.g. supplier's late delivery and customer's demand volatility) as well as internal ones (e.g. changing capacity and higher scrap ratio produced). Hence, in most practical situations, the underlying assumptions of standard mathematical programming concerning certainty is often unsatisfied as the estimation of model parameters is usually reached at through anticipating future events. The exact value of such parameters will become known only after the solution has been chosen and implemented (Hillier and Lieberman 2005).

Generally speaking, when it comes to mathematical modeling, there are several ways to account for the uncertainties associated with the problem parameters of interest, as seen in Figure 5.1. The first and customary practice is to assume that all parameters are known a priori and seek the solution to the mathematical model under the assumed values. Having the optimal solution at hand, a post optimality analysis is carried out in order to generate a series of improving approximations to the ideal course of action as well as identify the sensitive parameters, those whose values cannot be altered without changing the optimal solution. However, Hillier and Lieberman (2005) note that if some of the problem parameters have large variance, this approach of dealing with uncertainty is insufficient.

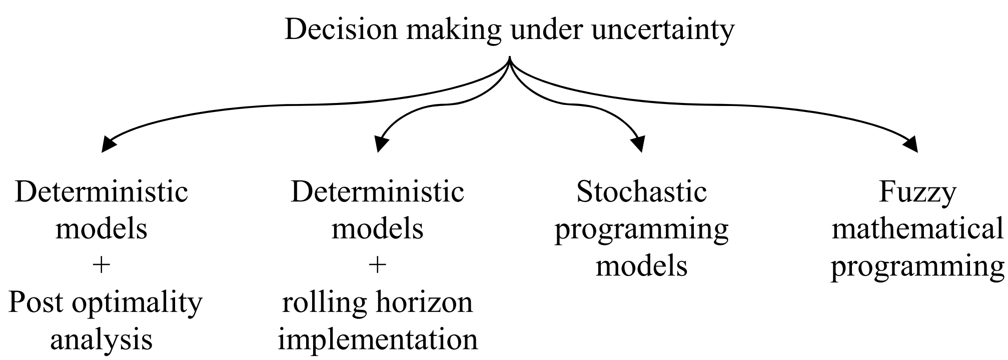

Figure 5.1: Approaches to incorporate uncertainty in mathematical models

For multi-period planning models, the dynamic implementation of the mathematical model on a rolling horizon basis is a more viable alternative especially when there is a high degree of uncertainty involved. This approach calls for periodically updating the model parameters as the horizon rolls forward and new information becomes available, and then re-solving the model with the updated values. For complex and large scale models, decision makers would likely prefer a procedure that facilitates the attainment of quick solutions rather than the repetitive solution of the model which might require extensive computational efforts.

The last two approaches, namely stochastic programming and fuzzy mathematical programming are more appropriate for situations involving a high degree of uncertainty. The former approach accounts explicitly for the uncertainty through treating some or all problem parameters as random variables that are modeled by discrete or continuous probability distributions. The underlying assumption here is the existence of reliable past data that can be used to derive the probability distributions based on some statistical techniques.

On the other hand, fuzzy mathematical programming stands out as the most convenient alternative in the absence of historical data or when these data is no longer reliable. This approach allows the decision makers to incorporate their intuition and subjective managerial judgments into the mathematical model through the use of fuzzy set theory (FST). In addition, qualitative expressions can now be modeled using the concept of possibility distribution (e.g. Dubois and Prade 1994). Broadly speaking, fuzzy mathematical programming encompasses flexible mathematical programming and possibilistic programming, where the decision of which technique to adopt depends on the nature of the existent fuzziness in the model.

This chapter tackles the uncertainties associated mainly with end customers' demand through rolling horizon schedules but with an *added twist*. We leave out the application of the alternative fuzzy approach to subsequent chapters.

5.3 The general rolling horizon practice

The implementation of production plans on a rolling horizon basis is a widely accepted practice as it allows for the dynamic nature associated with practical problems to be accounted for. According to Clark (2005), a schedule that is optimal for forecast demand over a given horizon will almost certainly be sub-optimal when implemented for the actually occurring demand. Essentially, for any rolling horizon schedule, there is a number of design factors that are particularly important. These are: forecast error, lot-sizing rule, length of replanning interval or frequency of replanning, and the choice of forecast window length. However, it is not our focus in this research to analyze these factors since their impact, both solely and combined, on the planning process has been amply investigated in the literature (e.g. Sethi and Sorger 1991, Lin and Krajewski 1992, Venkataraman 1996, Venkataraman and Nathan 1999, and Venkataraman and D'itri 2001). For instance, Lin and Krajewski (1992) concluded that the choice of lot-size rule under condition of demand uncertainty may not be as important as other aspects of MPS system design. Baker (1977) carried out an early experimental study concerning the effectiveness of rolling horizon decisions and suggested, with exceptions however, that such schedules are quite efficient. The rolling horizon outcomes can be very different from the static ones (Clark and Clark 2000). Drexel and Kimms (1997) note that little research has been carried out into the capacitated lot-sizing problem on a rolling horizon basis. Sahin *et al.* (2008) pointed out that although rolling schedules are commonly applied in the industry, they still yield heuristic long-term solutions even if optimal production schedules are determined at every replanning iteration.

In general, At the beginning of the first period, the typical practice under a rolling horizon policy calls for establishing the MPS for a certain number of future time periods, known as the planning horizon, based on the currently available relevant information (e.g. demand forecasts, available capacity, inventory and backlog records, etc.). However, only the current period's decisions actually become firm and are implemented. At the beginning of the second period, the horizon is rolled forward and the MPS is updated as more reliable data about the future becomes available. Again, only the second period's decisions are actually implemented and the process continues. In principle, rolling schedules provide the decisions to be carried out over a number of time periods where only the most immediate decisions are implemented before the multi-period model is re-run. It should be noted, however, that the update process (i.e. model re-running) does not necessarily take place every time period, in which case the number of periods for which the decisions are actually implemented is referred to as the replanning frequency. As pointed out by Venkataraman and D'itri (2001), rolling horizon schedules are considered to be more efficient as the methodology restricts implementation to the immediate period for which demand information is least subject to error.

5.4 Exact mathematical modeling

This section modifies the original mathematical model presented in Chapter 3 to consider demand volatility through the incorporation of demand forecasts as well as confirmed customers' orders in the planning decisions. Clearly, steel bars represent a fundamental and a much needed merchandise that constitute an essential material for mega and micro construction projects alike. This gives rise to a common phenomenon encountered frequently in the steel rolling industry, which is the so called “rush orders”. In particular, the steel mill under consideration constantly faces high levels of demand volatility in the form of last minute changes in confirmed customers’ orders. Such demand instability is mainly attributed to (1) An alteration in the previously agreed upon delivery dates as requested by the long-term loyal customers due to a construction project being ahead or behind schedule, and/or (2) Newly arriving orders usually placed by customers requiring smaller amounts in which case, from a customer’s perspective, such orders need not be placed early ahead of time. Hence, this dynamic nature of the master production scheduling problem at hand is better captured via the use of rolling horizon schedules.

To develop an implementable MPS, the production quantities have to be adjusted for inventory, customer orders, demand forecasts, and production capacity. Let FD_{jt}^{k} denote the forecasted demand for FP j of steel grade k in time period t, and CO_{jt}^{k} denote confirmed customers’ orders for FP j of steel grade k to be delivered in period t. The modified MIBLP is then given as follows:

$$Min \quad Z=\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{2}\sum_{t=1}^{T}\left(SC_t S_{ijt}^{k}+PC_{ijt}X_{ijt}^{k}\right)+\sum_{j=1}^{J}\sum_{k=1}^{2}\sum_{t=1}^{T}\left(IF_{jt}^{k}I_{jt}^{k}+BC_{jt}^{k}B_{jt}^{k}\right)+\sum_{t=1}^{T}PO_t O_t$$
$$+\sum_{i=1}^{I}\sum_{k=1}^{2}\sum_{t=1}^{T}\left(OR_{it}^{k}G_{it}^{k}+CR_{it}^{k}Q_{it}^{k}+IR_{it}^{k}I_{it}^{k}\right)+\sum_{j=1}^{J}\sum_{t=1}^{T}W_{jt}^{1}\left(SP_{jt}^{1}-SP_{jt}^{2}\right) \qquad (5.1)$$

Subject to

$$Q_{it}^{k} \le M_{it}^{k}G_{it}^{k} \qquad \forall\, i,t,k \qquad (5.2)$$

$$\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{2} Sb_{ijt}^{k} = 1 \qquad \forall t \tag{5.3}$$

$$S_{ijt}^{k} = 0.4 \times Sb_{ijt}^{k}\left(24 - \left(\sum_{i'=1}^{I}\sum_{j'=1}^{J}\sum_{k'=1}^{2}\left(\frac{X_{i'j',t-1}^{k'}}{\alpha_{i'j'}} + S_{i'j',t-1}^{k'}\right)\right)\right) + ST_{ij} \times Sd_{ijt}^{k} \qquad \forall i, j, t, k \tag{5.4}$$

$$\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{2}\left(\frac{X_{ijt}^{k}}{\alpha_{ij}} + S_{ijt}^{k}\right) \le A_t + O_t \qquad \forall t \tag{5.5}$$

$$O_t \le A_{ot} \qquad \forall t \tag{5.6}$$

$$X_{ijt}^{k} = b_t \times \left(Sb_{ijt}^{k} + Sd_{ijt}^{k}\right) \times \rho_{ij} \qquad \forall i, j, t, k \tag{5.7}$$

$$I_{it}^{k} = I_{i,t-1}^{k} + Q_{it}^{k} - b_t \times \sum_{j=1}^{J}\left[Sb_{ijt}^{k} + Sd_{ijt}^{k}\right] \qquad \forall i, t, k \tag{5.8}$$

$$I_{j,t}^{1} - B_{j,t}^{1} = I_{j,t-1}^{1} - B_{j,t-1}^{1} + \sum_{i=1}^{I} X_{ijt}^{1} - W_{jt}^{1} - Max\left\{FD_{jt}^{1}, CO_{jt}^{1}\right\} \qquad \forall j, t \tag{5.9}$$

$$I_{j,t}^{2} - B_{j,t}^{2} = I_{j,t-1}^{2} - B_{j,t-1}^{2} + \sum_{i=1}^{I} X_{ijt}^{2} - W_{jt}^{2} + W_{jt}^{1} - Max\left\{FD_{jt}^{2}, CO_{jt}^{2}\right\} \qquad \forall j, t \tag{5.10}$$

$$I_{j0}^{k} = B_{j0}^{k} = B_{jT}^{k} = 0 \qquad \forall j, k \tag{5.11}$$

$$Sd_{ijt}^{k}, Q_{it}^{k}, X_{ijt}^{k}, O_t, W_{jt}^{k}, I_{it}^{k}, I_{jt}^{k}, B_{jt}^{k} \ge 0 \qquad \forall i, j, t, k \tag{5.12}$$

$$G_{it}^{k}, Sb_{ijt}^{k} \in \{0,1\} \qquad \forall i, j, t, k \tag{5.13}$$

$$Sd_{ijt}^{k} \in N \qquad \forall i, j, t, k \tag{5.14}$$

The inventory balance constraints in the model are adjusted for the maximum value of customers' orders and demand forecasts as seen in Equations (5.9) and (5.10). In the remainder of this chapter, the above model is referred to as the exact model, or the full scale model. The next section presents approximate models which provide a quick remedy to the challenging and time consuming task associated with the repetitive solution of the exact model to optimality as new values for problem parameters become available.

5.5 Approximate rolling horizon models

Due to the uncertainties involved, solving the exact model at the beginning of the first period and freezing the resulting MPS for the entire planning horizon (i.e., implementing the MPS all the way) will most certainly produce poor quality solutions. Alternatively, the decision maker is better off implementing only the first period's decisions and then rolling the horizon forward before implementing the second period's decisions in light of the newly available data, and so on. Under the proposed scenario, the decisions associated with the most immediate time period are made based on the confirmed customers' orders. On the other hand, for the unimplemented portion of the cycle, production is planned according to the maximum of forecasted demand and confirmed orders, where both of these quantities are updated periodically as the horizon rolls forward. Such decision making criterion for a planning horizon of $T = 4$ is depicted in Figure 5.2, where $FD_{jt}^{k}(t-1)$ and $CO_{jt}^{k}(t-1)$ denote demand forecasts and confirmed orders at the end of period $t-1$, respectively. The dashed line box in the figure indicates the implemented master production schedule. In this policy, changes in the actual rolled bars selling prices and billets purchasing prices are also accounted for as the horizon rolls forward.

Following the proposed rolling horizon scheme, the exact model is re-solved at the beginning of each period and the production decisions are updated accordingly. However, due to complexities associated with solving the exact model to optimality, the process of frequently re-solving such model consumes an impractical amount of computational time. As such, three families of approximate models, which only generate exact schedules for the immediate time periods, are developed. The basic idea behind such approximation is that the key complicating aspects of the exact model, which prevent the attainment of quick solutions such as major setup time and minimum batch size restriction, are relaxed only for future time periods.

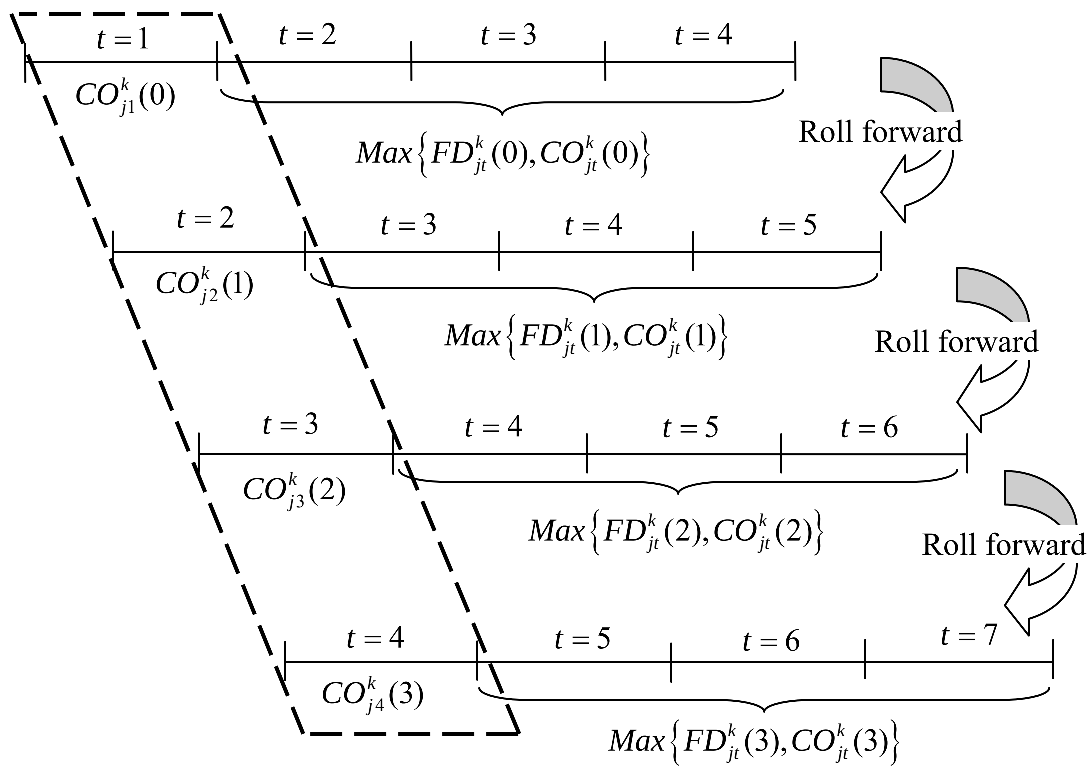

Figure 5.2: The employed rolling horizon strategy

As pointed out by Clark and Clark (2000), plant schedulers far prefer models that make decisions quickly since many scenarios, usually concerning demand forecast and machine availability, often have to be evaluated with what-if analysis before a final decision is made. In addition, it is difficult to obtain precise estimates for the backlogging cost since it is highly dependent on the market conditions and the importance of the customer. As such, the values used for such cost are usually based on subjective judgment and vague information. Hence, instead of solving the exact model optimally which often requires substantial computational efforts and achieves only limited added value, the decision maker would rather make use of smaller alternative models that can provide faster responses without big losses in the quality of the decisions made.

In this section, we seek to resolve the excess computational time, associated with obtaining the optimal solution to the exact model, via proper relaxation of the complicating variables and/or constraints, resulting in a reduced problem size dimensionality and great savings in computational efforts. Only those approximate models that generate exact schedules for the most immediate time period are considered. Hence, the relaxations are only applied to the unimplemented time periods of the planning horizon. Three varieties of alternative models are considered, where each model targets a certain complicating aspect of the exact model. The quality of the schedules obtained from applying these models is benchmarked against one another and against those of the exact model, where the last comparison is only possible for small problems that are optimally solvable in feasible computing time. Since the analysis is carried out on a rolling schedule basis which requires the repetitive solution of the model, our work shall provide the decision maker with practical and more efficient alternatives towards the quick attainment of good quality solutions (i.e. within the proximity of those obtained through solving the exact model). Although the generated solutions might be sub-optimal, they still provide substantial improvement on intuitive or indeed rule-generated production schedules.

In general, for a rolling horizon policy that calls for implementing the decisions concerning, say, the first τ periods, Clark and Clark (2000) were the first to address questions like: Is it necessary to have 0/1 solution values for the binary setup variables in the last $T-\tau$ periods (i.e., the unimplemented portion of the planning horizon)? Why not reduce the number of 0/1 variables by representing the setups for the last $T-\tau$ periods as relaxed continuous variables between 0 and 1? They also argue that even more computing time is economized by eliminating the binary setup variables from the last $T-\tau$ periods. This removal is compensated for by increasing the values of the unit production times to take the setup times into account. Their argument is backed up by an established fact in the literature (e.g., Trigeiro *et al.* 1989) which demonstrates the noticeable increase in models complexity once the setup times are explicitly accounted for.

Looking back at the exact model at hand, it is observed that the binary variables concerning the major setup time, Sb_{ijt}^{k}, induce bilinearity and hence prevent the direct attainment of quick solutions except via classical linearization approaches that have doubtable efficiency. This fact motivates the relaxation of the binary restriction (constraint 5.13) on these Sb_{ijt}^{k} variables only for the unimplemented periods in which those variables may now assume a continuous value between 0 and 1. In our case, such a relaxation would greatly reduce the number of 0/1 Sb_{ijt}^{k} variables (from $I \times J \times T \times K$ to $I \times J \times K$) and is expected to save on the solution time. This relaxation scheme was employed by Clark and Clark (2000) and was proven to provide quick and good quality solutions.

However, for our model, this approach converts the "mixed integer bilinear" program into a "pure bilinear" one since now two continuous variables are multiplied by one another. The resulting model is still non-convex and bilinear but it is solvable using a specific reformulation-linearization approach established in the literature for this class of bilinear models (Sherali and Alameddine 1992). Alternatively, assuming the most immediate period is $t = \tau$, we retain the binary variables Sb_{ijt}^{k} for $t = \tau$ and drop them out of the model for periods $t = \tau + 1, ..., \tau + T - 1$. The removal of such variables is compensated for by a reduction in the regular time available capacity, A_t, in periods $t = \tau + 1, ..., \tau + T - 1$ that is equal to half the major setup time in the most immediate period $t = \tau$. That is, the adjusted capacity A_t^a is now given by:

$$A_t^a = A_\tau - \frac{1}{2}\left[9.6 - 0.4\left(\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{2}\left(\frac{X_{ij,\tau-1}^{k}}{\alpha_{ij}} + S_{ij,\tau-1}^{k}\right)\right)\right] \quad \text{for} \quad t = \tau + 1, ..., T + \tau - 1 \quad (5.15)$$

The elimination of the major setup time in future time periods contributes to an extra production capacity that is actually not available, and hence, a capacity adjustment like the one proposed above is needed for these periods. Although the choice of he multiplicative term (0.5) might seem subjective, the justification behind it comes from a practical perspective. In reality, the production quantities are established on a weekly basis at the beginning of each week and, typically, the major setup time for the first period (i.e., $\tau = 1$ in this case) is longer than that in subsequent periods due to shorter working hours in the weekend. Rolling the horizon one period ahead, the ending inventory levels for RM and FP, the demand forecasts and confirmed orders, the selling and purchasing prices are updated after each period. The resulting model is a MILP that is directly solvable using off-the-shelf optimization packages such as AMPL/CPLEX. This model will be referred to as model ARH in the remainder of this chapter.

The second approximate model, called BRH, addresses another source of complexity in the exact model arising from the integrality restriction on the variables Sd_{ijt}^{k} (constraint 5.14). Recall that these variables denote the number of minor setups, or equivalently the number of batches produced after the initial batch, where the furnace capacity restricts the size of each batch to 60 tons each, which better complies with the economies of scale principles. As established earlier, lot-sizing rules are one of the design factors when it comes to developing MPS on a rolling horizon basis. As such, this model eliminates the restriction of minimum batch size production through relaxing the integrality restriction on the Sd_{ijt}^{k} variables (i.e., allowing the production of partial batches) for the unimplemented time periods only (i.e. $t = \tau + 1, ..., \tau + T - 1$). This allows for a better understanding of how lot-sizing rules impact the total cost in rolling horizon schedules. For the Sd_{ijt}^{k} variables to assume continuous values, they need to be redefined as follows:

Sd_{ijt}^{k} : Quantity of batches of FP *j* produced from RM *i* both having steel grade *k* conducted during period *t* (i.e. after producing the first batch).

To serve for better approximation purposes, the setup time expression (Equation 5.4) for periods $t = \tau + 1, ..., \tau + T - 1$ would still be calculated based on integral number of batches. In reality, a minor setup takes place between each batch produced and the time for this setup is independent of the batch size. Therefore, the updated expression for the setup time in this case is given by:

$$S_{ijt}^{k} = 0.4 \times Sb_{ijt}^{k}\left(24 - \left(\sum_{i'j'k'}^{I}\left(\frac{X_{i'j',t-1}^{k'}}{\alpha_{i'j'}} + S_{i'j',t-1}^{k'}\right)\right)\right) + ST_{ij} \times Sd_{ijt}^{k} \qquad \forall\, i, j, k, t = \tau \quad (5.16)$$

$$S_{ijt}^{k} = 0.4 \times Sb_{ijt}^{k}\left(24 - \left(\sum_{i'j'k'}^{I}\left(\frac{X_{i'j',t-1}^{k'}}{\alpha_{i'j'}} + S_{i'j',t-1}^{k'}\right)\right)\right) + ST_{ij} \times \left\lceil Sd_{ijt}^{k} \right\rceil$$

$$\forall i, j, k, t = \tau + 1, ..., \tau + T - 1 \qquad (5.17)$$

where $\left\lceil Sd_{ijt}^{k} \right\rceil$ denotes the smallest integer greater than or equal to Sd_{ijt}^{k}. Note that the resulting model is still a MIBLP since the binary variables Sb_{ijt}^{k} were retained for all time periods in this model. As such, adopting the classical linearization scheme of Glover (1975), the following sets of constraints overcome the existent bilinearity and replace the above constraints of (5.16) and (5.17):

$$S_{ijt}^{k} = 9.6 \times Sb_{ijt}^{k} - 0.4 \times y_{ijt}^{k} + ST_{ij} \times Sd_{ijt}^{k} \qquad \forall\, i, j, k, t = \tau \qquad (5.18)$$

$$S_{ijt}^{k} = 9.6 \times Sb_{ijt}^{k} - 0.4 \times y_{ijt}^{k} + ST_{ij} \times \left\lceil Sd_{ijt}^{k} \right\rceil \qquad \forall i, j, k, t = \tau + 1, ..., \tau + T - 1 \quad (5.19)$$

$$L_{ijt}^{k} Sb_{ijt}^{k} \le y_{ijt}^{k} \le U_{ijt}^{k} Sb_{ijt}^{k} \qquad \forall\, i, j, t, k \qquad (5.20)$$

$$\sum_{i'j'k'}\left(\frac{X_{i'j',t-1}^{k'}}{\alpha_{i'j'}} + S_{i'j',t-1}^{k'}\right) - U_{ijt}^{k}\left(1 - Sb_{ijt}^{k}\right) \le y_{ijt}^{k} \le \sum_{i'j'k'}\left(\frac{X_{i'j',t-1}^{k'}}{\alpha_{i'j'}} + S_{i'j',t-1}^{k'}\right) - L_{ijt}^{k}\left(1 - Sb_{ijt}^{k}\right)$$

$$\forall\, i, j, t, k \qquad (5.21)$$

Clearly, the resulting linearized version of the model is now readily solvable using AMPL/CPLEX 11.0.

In the above analysis, model ARH tackles the binary restriction on the Sb_{ijt}^{k} variables while model BRH tackles the integrality restriction on the Sd_{ijt}^{k} variables without touching the Sb_{ijt}^{k} variables. This quantifies the quality of the solutions as well as the savings in computational time obtained using each approximation criterion separately. Towards achieving even further savings in the computational time, the approximation schemes adopted in models ARH and BRH are combined. Hence, this hybrid model, called CRH, contains no Sb_{ijt}^{k} variables in the periods $t = \tau + 1, ..., \tau + T - 1$ and assumes that the Sd_{ijt}^{k} variables are continuous during those periods as well. However, for the most immediate period $t = \tau$, all of the full scale model constraints are retained so that only exact schedules are generated.

5.6 Computational experiments

The purpose of this section is to provide insights into the performance of the proposed models in terms of the quality of solutions obtained (i.e., closeness of the objective function value obtained via approximations to that of the exact model), savings in computational time and reduction in problem dimensionality. To ensure a fair comparison, ten problem instances of different sizes are tested and each model is solved under the same set of input parameters. All models were coded using AMPL programming language (Fourer *et al.* 2003) and solved using CPLEX 11.0 solver, where the solver option is set to solve integer problems using the built-in branch and cut algorithm. The values for the different input parameters are generated within certain range of intervals, as shown in Table 5.1.

Table 5.1: Selected range of values for input parameters in the test problems

Input parameter	Range of values	Input parameter	Range of values
CR_{it}^{k}	(500 , 800)	BC_{jt}^{k}	(20 , 35)
OR_{it}^{k}	(2000 , 2200)	SP_{jt}^{k}	(1300 , 1700)
IR_{it}^{k}	(10 , 20)	M_{it}^{k}	(150 , 300)
PC_{ijt}	(15 , 35)	ρ_{ij}	(0.80 , 0.95)
PO_{t}	(300 , 500)	α_{ij}	(50 , 58)
SC_{t}	(600 , 800)	FD_{jt}^{k} , CO_{jt}^{k}	(0 , 100)
IF_{jt}^{k}	(15 , 30)	ST_{ij}	(0.25 , 1.0)

It should be noted that all models are applied dynamically on a rolling horizon basis in which only the first period's outcome contributes to the total cost function. In other words, the reported results are for the implemented plans and, as such, the comparison is only made between exact schedules generated through the different models. The objective is to study the effect of the relaxations made to future (i.e., unimplemented) time periods on the current period's decisions. Under such a rolling horizon policy, the results of the current period establish the next periods' major setup time (if any), and the initial levels of RM inventory, FP inventory and backlog record for the next run. In addition, before rerunning the model, any encountered changes in the values of the problem parameters, mainly demand forecasts and customers' orders are updated in the model. The obtained results for the exact model, for those instances solvable within feasible computational time, along with the three approximate ones are reported in Tables 5.2-5.5 below. Note that the solution time indicates the total time it takes to solve the model T times, and the total cost is the one resulting from the implemented plans. The reported number of variables and constraints for the exact model as well as model BRH are for the linearized version of those models, while these numbers are for the reduced size MILP in case of models ARH and CRH.

Table 5.2: Obtained numerical results for the exact model

Problem Instance	Values of Indices				No. of Variables	No. of Constraints	Solution time (sec)	Total Cost
	I	J	T	K				
1	1	2	2	2	74	140	0.95	778,368
2	1	3	3	2	159	297	2.48	1,112,105
3	2	3	3	2	267	507	8,289	1,151,720
4	2	3	4	2	358	678	33,694	1,542,301
5	3	4	4	2	644	1,232	*	*
6	4	4	4	2	828	1,592	*	*
7	4	5	4	2	1,010	1,946	*	*
8	5	5	4	2	1,234	2,386	*	*
9	5	6	4	2	1,456	2,820	*	*
10	5	7	5	2	2,101	4,071	*	*

*Code execution was interrupted after 12 hours of run time with no results obtained.

Table 5.3: Obtained numerical results for model ARH

Problem Instance	Values of Indices				No. of Variables	No. of Constraints	Solution time (sec)	Total Cost
	I	J	T	K				
1	1	2	2	2	62	111	0.94	778,368
2	1	3	3	2	129	229	3.73	1,077,312
3	2	3	3	2	207	373	9.04	1,127,154
4	2	3	4	2	274	495	57.27	1,520,070
5	3	4	4	2	476	869	600.41	1,588,562
6	4	4	4	2	604	1,109	7,855	1,445,971
7	4	5	4	2	730	1,343	9,249	1,723,936
8	5	5	4	2	884	1,633	16,154	1,687,157
9	5	6	4	2	1,036	1,917	29,373	1,721,446
10	5	7	5	2	1,471	2,737	41,069	2,279,638

Out of the four solvable test problems for the exact model, better proxies are achieved via model ARH (instances 1, 3 and 4) while models BRH and CRH provide for a better approximation as for the second problem instance. In particular, the worst deviation, in terms of objective function value, among all models is not more than 3.65% under tight capacity conditions.

Table 5.4: Obtained numerical results for model BRH

Problem Instance	Values of Indices				No. of Variables	No. of Constraints	Solution time (sec)	Total Cost
	I	**J**	**T**	**K**				
1	1	2	2	2	78	143	0.82	776,051
2	1	3	3	2	165	295	1.40	1,095,979
3	2	3	3	2	279	505	2.73	1,109,694
4	2	3	4	2	370	663	11.83	1,504,809
5	3	4	4	2	668	1,205	108.33	1,567,408
6	4	4	4	2	860	1,557	2,915	1,419,194
7	4	5	4	2	1,050	1,903	2,602	1,723,807
8	5	5	4	2	1,284	2,333	6,274	1,687,766
9	5	6	4	2	1,516	2,757	15,007	1,706,723
10	5	7	5	2	2,171	3,927	27,838	2,279,037

If we are to benchmark the values of the approximate models' total cost against one another for all test problems, there exists a maximum of 3.44% deviation for problem instance number 6. However, for situations where there is abundant capacity available (i.e. total demand $<<$ capacity), one would expect all models to produce very similar, if not exactly the same, production schedules. This is due to the fact that, under low demand scenario, the model actually has less number of non-zero decision variables to optimize since backlogged quantities, overtime hours and substitution quantities are all set to zero.

Table 5.5: Obtained numerical results for model CRH

Problem Instance	Values of Indices				No. of Variables	No. of Constraints	Solution time (sec)	Total Cost
	I	**J**	**T**	**K**				
1	1	2	2	2	66	115	0.80	764,689
2	1	3	3	2	135	229	1.22	1,095,979
3	2	3	3	2	219	373	1.26	1,116,810
4	2	3	4	2	286	483	1.77	1,514,967
5	3	4	4	2	500	845	1.81	1,549,973
6	4	4	4	2	636	1,077	1.81	1,396,213
7	4	5	4	2	770	1,303	1.83	1,723,856
8	5	5	4	2	934	1,583	2.08	1,686,208
9	5	6	4	2	1,096	1,857	2.30	1,674,332
10	5	7	5	2	1,541	2,597	4.09	2,278,262

From a solution time perspective, major benefits are obtained through making use of the approximate models instead of the exact one without, as just illustrated, compromising the quality of such solutions. By comparing the times for models ARH and BRH, we notice that more computational time savings are attained from applying model BRH (up to 81.96% reduction for instance No. 7). This shows that the minimum batch size restriction associated with the Sd_{ijt}^{k} variables (i.e., integrality restriction) contributes a lot more to the complexity of the exact model, hence greatly limiting its applicability, as compared to the binary restriction on the variables Sb_{ijt}^{k}. It is obvious that since model CRH combines the relaxations employed in the other two approximate models, it would result in substantial savings in computational time. For example, it took the exact model 33,694 seconds (9.36 hours) to solve problem instance No. 4 while model CRH solved it in less than two seconds with a difference in the total cost of only 1.77%. For a larger problem size (I=T=5, $J=7$ and $K=2$) the solutions to the exact model are not attainable within 12 hours of computational time, while such problem is solved via model CRH within 4 seconds only.

If the models were to be implemented on a static horizon basis, one knows that model CRH provides a lower bound for both models ARH and BRH which in turn would yield a lower bound on the objective function value of the exact model. However, rolling horizon outcomes are usually different from static horizon ones and, as such, relaxing some of the constraints for the unimplemented time periods does not necessarily guarantee a lower objective function value for the more restricted model once both models are applied on a rolling horizon basis and the implemented plans are compared. As a matter of fact, model ARH results in a lower total cost for problem instance No. 2 as compared to that obtained from model CRH. In addition, in 4 out of the 10 test problems, model BRH yield a total cost that is lower than that of model CRH.

5.7 Summary

This chapter has dealt with the master production scheduling problem in steel rolling mills operating in an environment of dynamic demand. The problem was formulated as a mixed integer bilinear program, in which both forecasted demand and confirmed customers' orders are incorporated. To better capture the existent volatility in demand, the developed model was implemented on a rolling horizon basis where the values for the forecasts and orders, among other problem parameters, were updated every period in light of the newly available information obtained form the market. As the developed production schedule is only implemented for the most immediate time period, production and inventory related decisions for this period were based on the actual orders received.

Since solving the exact model to optimality turns out to be a challenging task, the alternative approach of dealing with approximate models is adopted. Three variants of such models were developed where each of these models tackles the sources of complexity associated with the exact model, either one at a time or both at once, resulting in smaller models with a tractable number of binary and/or integer variables. Based on numerical experiments for several test problems, it is observed that such approximate models are particularly efficient and thus have a great potential in production planning problems where optimal solutions to full scale mathematical models are not quickly attainable. Hence, a fair compromise between the quality of the solutions and substantial computational time savings is achieved via the employment of these approximate models.

Chapter 6

Modeling Demand Uncertainty in Production Planning through Fuzzy Set Theory Approach

6.1 Introduction

Fuzzy set theory (FST) is a theory of graded concepts (a matter of degree), but not a theory of chance (Lai and Hwang 1992a). It allows for the development of more robust and flexible models that better capture the human aspects involved in real world complex systems. In general, FST represents an attractive tool to aid research in production management when the dynamics of the production environment limit the specification of model objectives, constraints and the precise measurement of model parameters (Guiffrida and Nagi 1998). Unlike the rolling horizon approach adopted in the previous chapter, the utilization of FST alleviates the necessity for repeatedly solving the model at the beginning of each time period. In general, there exist several advantages to the utilization of FST to handle such type of uncertainties: (1) It generally allows for the transformation of qualitative estimates that are expressed linguistically based on human perception into exact quantitative values, expressed in terms of fuzzy sets.

(2) It provides for a broader alternative to account for uncertainty as compared to the classical approach that utilizes probability distributions. When there is a lack of evidence or lack of certainty in evidence, the standard probabilistic reasoning methods are not appropriate (Petrovic and Petrovic 1999).

(3) From a mathematical programming perspective, fuzzy models overcome the rigid requirements characterizing standard deterministic models. This greatly facilitates the decision making process in most real life applications where the determination of the exact model parameters poses a crucial practical challenge.

(4) Obtaining an initial compromise solution from the fuzzy model allows the decision maker to specify which further information should be obtained to improve the solution. This procedure obviously offers the possibility to limit the acquisition and processing of information to the relevant components and therefore information costs will be distinctly reduced (Rommelfanger 1996).

(5) The fuzzy models provide decision makers with some flexibility to incorporate their own priority into the model through the selection of aspiration levels (in the case of a fuzzy objective function) and the tolerance interval. As such, alternative decision plans corresponding to different degrees of satisfaction are easily attainable.

In this chapter, the vagueness and impreciseness associated with the estimation of future demand figures is captured through the use of FST, originally introduced in the seminal work of Zadeh (1965). In addition, we clearly establish the difference between the two approaches used to handle fuzzy quantities in mathematical models, namely flexible mathematical programming (FMP) and possibilistic programming (PP), since such distinction has never been made solid in the literature. We then adopt the FMP approach to handle uncertainties in demand through expressing the demand constraint as a flexible one. Our objective is to emphasize on the superiority of this approach for modeling production planning problems and illustrate the economic benefits obtained through the use of fuzzy models as compared to traditional deterministic models. To facilitate understanding the analysis presented in this chapter, some basic FST related background needs to be established first.

6.2 A brief introduction to fuzzy set theory

In classical set theory, the relation between an object x and the ordinary set of objects A is defined by the characteristic function or the indicator function as follows:

$$\Upsilon_A(x) = \begin{cases} 1 & if \quad x \in A \\ 0 & if \quad x \notin A \end{cases} \tag{6.1}$$

Hence, a member either belongs or does not belong to a particular set. Fuzzy set theory, however, simulates the actual practice and allows for a member to belong to different sets with various degrees of membership. The following formal definitions are based on the work of Rommelfanger (1996) and Demirli (2005).

Definition 6.1: Let X denote the universal set (also referred to as the universe of discourse), then the set of ordered pairs:

$$A = \left\{ \left(x, \mu_A(x) \right) \mid x \in X \right\}, \quad \text{where} \quad \mu_A : X \to [0,1] \tag{6.2}$$

is called a fuzzy set in X. The function $\mu_A(x)$ is called the membership function which establishes the grade of membership of element x in A.

Definition 6.2: Let A be a fuzzy set in X. The support of A, denoted by *supp(A)*, is the crisp set of X whose elements have nonzero membership grades in A:

$$supp(A) = \{x \in X \mid \mu_A(x) > 0\} \tag{6.3}$$

The core of A, denoted by *core(A)*, is the crisp set of X whose elements have a membership grade in A equals to one:

$$core(A) = \{x \in X \mid \mu_A(x) = 1.0\}$$

The height of A, denoted by *h(A)*, is the largest membership grade obtained by any element in that set:

$$h(A) = \sup_{x \in X} \mu_A(x) \tag{6.4}$$

Definition 6.3: A fuzzy set A is called normal when $h(A) = 1$; it is called subnormal when $h(A) < 1$.

Definition 6.4: A fuzzy set A in a convex set X is called convex if:

$$\mu_A\left(\lambda x_1 + (1-\lambda)x_2\right) \geq Min\left(\mu_A(x_1), \mu_A(x_2)\right), \quad x_1, x_2 \in X, \quad \lambda \in [0,1] \tag{6.5}$$

Definition 6.5: A convex normalized fuzzy set $A = \{(x, \mu_A(x)) \mid x \in R\}$ on the real line R such that:

(I) there exist exactly one $x_0 \in R$ with the membership degree $\mu_A(x_0) = 1,$ and

(II) $\mu_A(x)$ is piecewise continuous in R,

is called a fuzzy number.

Definition 6.6: (Fuzzy Decision making of Bellman and Zadeh (1970)

Let X be the set of alternatives, which defines the set to all possible solutions to a decision problem. The fuzzy goal G and a fuzzy constraint C are fuzzy sets on X and characterized by their respective membership functions $\mu_G : X \rightarrow [0,1]$ and $\mu_C : X \rightarrow [0,1]$. Then, G and C combine to form a decision, D, which is a fuzzy set resulting from the intersection of G and C (i.e., $D = G \cap C$). The fuzzy decision set is characterized by its membership function:

$$\mu_D(x) = \mu_G(x) \wedge \mu_C(x) = Min(\mu_G(x), \mu_C(x)) \tag{6.6}$$

and the corresponding maximizing decisions can then be defined as:

$$Max\mu_D(x) = MaxMin(\mu_G(x), \mu_C(x)) \tag{6.7}$$

It should be noted that definition 7.6 requires the assumption of symmetry to be satisfied in order to reach decisions that satisfy both the goal "and" the constraint.

6.3 Flexible mathematical programming (FMP) vs. possibilistic programming (PP)

There are two approaches that researchers use in order to deal with fuzzy mathematical models where the suitability of the chosen approach depends on the nature of the existent fuzziness in the model. The two approaches are flexible mathematical programming (FMP), in which the rigidity restriction on the comparison operator (i.e., equality or inequality operators) is relaxed resulting in a soft or flexible set of constraints, and possibilistic programming (PP), in which the imprecise coefficients are expressed as fuzzy numbers having a specified possibility distribution. We distinguish between these two approaches in the general context of fuzzy mathematical programming before applying such distinction to the model at hand. It is of great importance to understand this distinction while modeling fuzziness/imprecision in mathematical programming problems.

Flexibility is modeled by fuzzy sets and may reflect the fact that constraints or goals are linguistically formulated; their satisfaction is a matter of tolerance and degrees or fuzziness (Mula *et al.* 2006). FMP utilizes subjective preference-based membership functions where these functions are constructed by eliciting the preference information from the decision makers. In this scenario, the membership function is equivalent to the utility function when the membership functions of FMP are based on a preference concept like the utility theory (Lai and Hwang 1992a). Hence, the grade of a membership function indicates a subjective degree of the decisions maker's satisfaction within given tolerances. Clearly, the characteristic function in FMP is a membership function. Applications of FMP approach can be found in Pendharkar (1997), Miller *et al.* (1997), Itoh *et al.* (2003) and Mula *et al.* (2006).

On the other hand, possibility theory is adopted in situations where the model parameters are imprecise due to uncertainty in the data or lack of knowledge (i.e. epistemic uncertainty). It is noted that the possibility measure of an event might be interpreted as the possibility degree of its occurrence under a possibility distribution; an analogous to - yet different from - probability distribution (Lai and Hwang 1992a). In essence, an important difference between the probability theory and the possibility theory is that the sum of probabilities for all possible outcomes should be equal to one, while the sum of possibilities for all possible events under possibility theory need not be equal to one. Hence, the possibility distribution is constructed by considering the possible occurrence of events where the grade of possibility indicates the subjective or objective degree of a particular event occurring. Clearly, the characteristic function in PP is a possibility distribution. PP has also been widely applied to various problems as can be seen in Liang (2007), Demirli and Yimer (2008), Liang (2008), and Yimer and Demirli (2009).

In general, possibility theory would be adapted to uncertain situations in which the demand is fuzzy due to lack of information, and is represented by a fuzzy number, but the demand constraint still holds as a strict (in)equality. As such, once the demand has been defuzzified, the resulting crisp constraint holds as a rigid requirement and the decision maker has no choice but to seek a solution that fully satisfies such constraint. On the other hand, flexible mathematical programming deals with situations involving either fuzzy or crisp demand but the corresponding constraint is rather flexible. Therefore, the existent flexibility within the model allows for “approximate” fulfillment of the demand, chosen subjectively according to the decision maker’s preference, which indicates that the comparison operator is being treated as a fuzzy relationship.

Turning back to the problem at hand, the model developed in this chapter determines the daily/weekly production lot sizes and serves for short-term planning purposes. However, the fuzzy nature of the demand in the form of last minute orders, called “rush orders”, still raises a major concern to the decision maker. For an essential material such as steel bars, the realized demand in a particular time period could be greater or less than the confirmed orders established at the beginning of the planning horizon. While the former case is mainly attributed to a collection of small magnitude orders usually placed by short term customers, the latter is due to an alteration on a short notice of a previous order typically placed by long term customers. Depending on the magnitude of such orders, they could cause major disruptions to the production system which induces a high associated cost. Hence, it seems only logical to allow some degree of flexibility into the mathematical model so that it possesses the ability to whether satisfy all orders or only some orders while completely ignoring some other ones, whichever option deems more profitable. By treating the demand constraint as flexible rather than a rigid one, the emphasis now is on to which extent the demand shall be met instead of enforcing a strict policy of being 100% responsive to a fluctuating customer demand. This resembles the actual situation where the decision maker has the choice to accept, reject or partially fulfill an order as soon as it arrives. A good practical example is the famous Japanese automaker Company “Nissan” which faces a highly fluctuating demand and deals with it through ignoring some of the orders since such fluctuations are costly to meet. In the fuzzy model presented in Section 6.5, we mimic the decision maker’s ability to reject or accept rush orders, whether partially or fully, through representing the flexible demand constraints by fuzzy sets having triangular membership functions (TMFs). The allowable deviations from the most likely value are expressed as percentages of the confirmed orders.

6.4 Fuzzy mathematical programming

The uncertainties associated with the operating environment of the steel mill under consideration necessitate the use of FST to explicitly account for the dynamic nature of the problem. In general, whenever FST is employed in the context of mathematical programming, the decision maker is faced with important factors that need to be carefully addressed towards a better representation of the problem at hand:

(1) *Sources of uncertainty*: Depending on the problem, it might be difficult to obtain precise estimates for certain problem parameters such as cost coefficients, supply and demand quantities, machine capacities and supplier lead time. In essence, one has to establish what exactly is fuzzy about the problem at hand before anything else.

(2) *Representation of fuzziness*: As discussed earlier, accommodating for fuzzy quantities in the context of mathematical programming is accomplished through the use of flexible programming or possibilistic programming depending on the nature of the existent fuzziness associated with the parameters of concern.

(3) *Form of membership functions*: This involves defining the support of the membership function as well as the form of the function over the identified interval. There exist several propositions for modeling the membership function including monotonic linearly increasing or decreasing, piecewise linear, triangular or trapezoidal, concave shape or s-shape (Rommelfanger 1996).

(4) *Aggregation operators (aggregators)*: In analogy to the deterministic mathematical programming case, a fuzzy decision set is reached at through aggregating the membership functions of the fuzzy objective function and constraints. Such aggregation is achieved through several operators that have been well established in the literature (e.g., Bellman and Zadeh 1970, Werners 1988 and Zimmerman and Zysno 1980).

While the approach of flexible mathematical programming (FMP), which seems to be more efficient computationally (Zimmermann 2001), has been successfully applied by several authors, the vast majority have adopted linear membership functions throughout their work since such form is computationally efficient and entails a reduced dimensionality for the resulting equivalent crisp model (e.g., Pendharkar 1997, *Miller et al.* 1997, Itoh *et al.* 2003). However, for a better resemblance of reality, we believe that the flexible demand constraint is better represented by a fuzzy set having a triangular membership function (TMF).

In the context of FMP, there are two classes of models: symmetric and non-symmetric models. The symmetric models are based on the definition of a fuzzy decision originally proposed by Bellman and Zadeh (1970) in which they assume that objective(s) and constraint(s) are an ill-structured situation and can be represented by fuzzy sets. These models involve less computational effort and are thus easier to handle since a decision can be readily stated as the confluence of the fuzzy objective(s) and fuzzy constraint(s). However, they suffer from two shortcomings. First, the interval of allowance (i.e., support) of the fuzzy set representing the objective function should be given initially. As such, this interval is usually decided upon blindly based on a purely subjective judgment obtained from the decision maker. Secondly, requiring the initial determination of such interval pays little or no attention to the decision maker's preference on the degree of flexibility of the respective fuzzy constraints. As pointed out by Lai and Hwang (1992a), in real world problems, it is unrealistic to initially ask the decisions maker to give this interval without providing any information about it. For these models, Zimmerman (1976) was the first to confirm the existence of an equivalent ordinary linear program through the application of the "min" operator and linear membership functions.

The other class, which deals with non-symmetric models, is more involved computationally since the problem now involves the determination of an extremum of a crisp function over a fuzzy domain. Based on the concept of a maximizing set, two approaches can be used to handle non-symmetric models (Zimmermann 1985):

(1) The determination of the fuzzy set decision.

(2) The determination of a crisp maximizing decision by aggregating the objective function, after appropriate transformations, with the constraints.

In what follows, we adopt the notation of Zimmermann (1985, 2001) in the derivation of a generic fuzzy model using the second approach for the case of TMFs for the fuzzy demand constraints.

Consider the linear system:

$$\begin{aligned} &\textit{Minimize} \quad z = c^T x \\ &\text{Subject to} \quad \left.\begin{aligned} Ax &\le b' \\ Dx &\cong b \\ x &\ge 0 \end{aligned}\right\} \tilde{R} \end{aligned} \tag{6.8}$$

with $c, x \in \Re^n$, $b' \in \Re^{m_1}$, $b \in \Re^{m_2}$, $A \in \Re^{m_1 \times n}$, $D \in \Re^{m_2 \times n}$

Model (6.8) is non-symmetric and includes a single crisp objective that the decision maker seeks to minimize subject to a combination of m_1 crisp constraints (some of which might assume a "$\ge$" or "$=$" form) and m_2 fuzzy constraints, where the notion "$\cong$" represents the fuzzified version of "=" and reads "essentially equal to". Both sets of constraints define the fuzzy feasible region $\tilde{R}$. A realistic example of model (6.8) is a production planning problem involving a set of resources (modeled above as crisp constraints) and an uncertain demand value (modeled as fuzzy material balance constraints). Typically, a low demand value yields lower production, setup and inventory holding and backorder costs than a higher value for the demand.

Let the membership functions of the triangular fuzzy sets representing the m_2 fuzzy constraints be defined as follows:

$$\mu_i(x) = \begin{cases} \dfrac{D_i x - \beta b_i}{b_i(1-\beta)} & if \quad \beta b_i \le D_i x < b_i \\ 1 - \dfrac{D_i x - b_i}{b_i(\alpha - 1)} & if \quad b_i \le D_i x < \alpha b_i \\ 0 & otherwise \end{cases} \tag{6.9}$$

Where $D_i x$ and b_i denote the left and right hand sides of the i^{th} fuzzy constraint respectively, $i = 1,....,m_2$, and $\beta < 1 < \alpha$. The constants α and β may assume a subscript i each indicating different levels of allowable deviations from the most likely value b_i for the i^{th} fuzzy constraint.

To achieve symmetry between the objective function and the constraints, we represent the objective function by a fuzzy set having a monotonically decreasing linear membership function (Figure 6.1b), where the tolerance interval for such set is determined via solving the following two mathematical programs:

$$\begin{aligned} &Minimize && z = c^T x \\ &\text{Subject to} && Ax \le b' \\ & && Dx = \beta b \\ & && x \ge 0 \end{aligned} \tag{6.10}$$

yielding a value $z_l = (c^T x)_{opt}$, and

$$\begin{aligned} &Minimize && z = c^T x \\ &\text{Subject to} && Ax \le b' \\ & && Dx = \alpha b \\ & && x \ge 0 \end{aligned} \tag{6.11}$$

yielding a value $z_u = (c^T x)_{opt}$

Hence, the membership function of the objective function is given by:

$$\mu_z(x) = \begin{cases} 1 & if \quad c^T x < z_l \\ \dfrac{z_u - c^T x}{z_u - z_l} & if \quad z_l \le c^T x < z_u \\ 0 & if \quad z_u \le c^T x \end{cases} \tag{6.12}$$

Plots of the membership functions for the fuzzy sets representing the constraints as well as the objective function are shown in Figure 6.1. The next step is to define the two different aggregators used to obtain the maximizing decision set from the fuzzy sets of the constraints and the objective function.

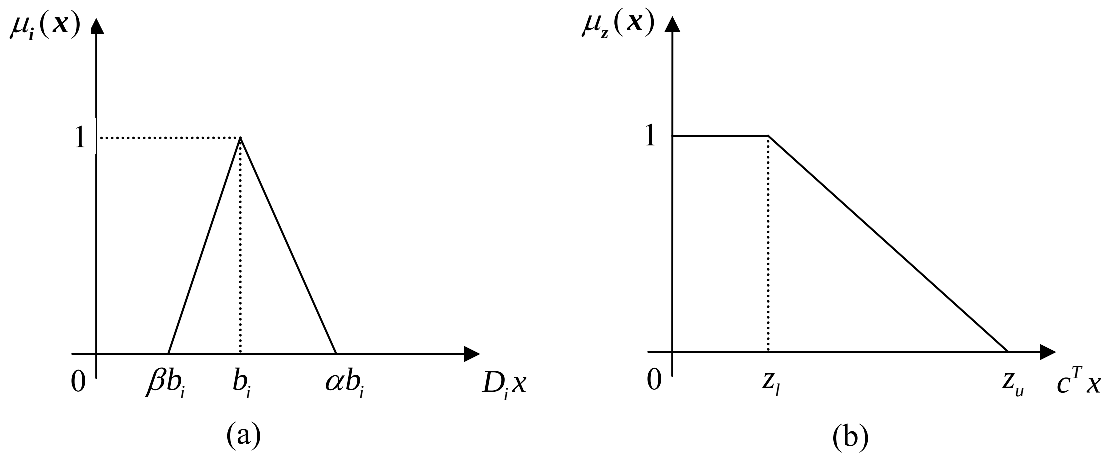

Figure 6.1: Fuzzy sets for the constraints (a) and the objective function (b)

6.4.1 The logical "min" operator

Introduced by Bellman and Zadeh (1970) as a fuzzy conjunction operator, the "min" operator forms the fuzzy decision set $\tilde{D}$ through taking the intersection of the fuzzy goal and fuzzy constraints (see Definition 6.6). Following this approach, the aggregating "min" operator characterizes the membership function of $\tilde{D}$ as follows:

$$\mu_{\tilde{D}}(x) = \min_{i=1}^{m_2} \{\mu_i(x), \mu_z(x)\} \tag{6.13}$$

Since the decision maker is interested in a crisp solution rather than a fuzzy set, the maximizing optimal decision is obtained via solving the following problem:

$$\max_{x\geq 0} \mu_{\tilde{D}}(x) = \max_{x\geq 0} \min_{i=1}^{m_2} \{\mu_i(x), \mu_z(x)\} \tag{6.14}$$

Introducing one new variable $\lambda \in [0,1]$, which essentially corresponds to $\mu_{\tilde{D}}(x)$ in (6.13), and carrying out some manipulations, the approximate auxiliary formulation for model (6.8) is given by:

$$\begin{aligned}
&\textit{Maximize} \quad && \lambda \\
&\text{Subject to} && Ax \leq b' \\
& && \lambda(z_u - z_l) + c^T x \leq z_u \\
& && \lambda b_i(\beta - 1) + D_i x \geq \beta b_i \qquad i = 1,\dots,m_2 \\
& && \lambda b_i(\alpha - 1) + D_i x \leq \alpha b_i \qquad i = 1,\dots,m_2 \\
& && x, \lambda \geq 0 \\
& && \lambda \leq 1
\end{aligned} \tag{6.15}$$

Hence, the maximizing crisp solution is obtained through solving model (6.15), which has one more variable and $m_2 + 1$ additional constraints. It should be noted that although the logical "min" operator has proven to be computationally efficient, it is not compensatory in the sense that it provides no tradeoff between the degrees of membership of the fuzzy sets at hand. As explained by Miller *et al.* (1997), there may not be any change in the degree of membership of the resulting aggregated fuzzy set even if the degree of membership in some of the fuzzy sets intersected is increased. To overcome such limitation, an alternative approach is presented below.

6.4.2 The "convex combination of min/max operators"

To allow for a certain amount of compensation between the degrees of membership of the fuzzy sets involved, Werners (1988) proposed the "convex combination of min operator and max operator". Having established the lowed and upper values for the objective function tolerance interval (z_l and z_u) as illustrated earlier, the fuzzy decision set is then defined as:

$$\mu_{\tilde{D}}(x) = \gamma \min_{i=1}^{m_2} \{\mu_i(x), \mu_z(x)\} + (1-\gamma) \max_{i=1}^{m_2} \{\mu_i(x), \mu_z(x)\} \tag{6.16}$$

A value of 0.6 for the parameter γ, which stands for the degree of compensation, has proven to be an effective one in most circumstances (Zimmermann 2001). Introducing two new auxiliary variables $\lambda_1, \lambda_2 \in [0,1]$ where:

$$\lambda_1 = \min_{i=1}^{m_2} \{\mu_i(x), \mu_z(x)\} \quad \text{and} \quad \lambda_2 = \max_{i=1}^{m_2} \{\mu_i(x), \mu_z(x)\} \tag{6.17}$$

The variables λ_1 and λ_2 essentially represent the degree of satisfaction of the least satisfied and the most satisfied constraint, respectively. Hence, the approximate auxiliary formulation for model (6.8) using the "convex combination of min/max" operator is given by:

$$\begin{aligned}
&\textit{Maximize} && \gamma\lambda_1 + (1-\gamma)\lambda_2 \\
&\text{Subject to} && Ax \leq b' \\
& && \lambda_1(z_u - z_l) + c^T x \leq z_u \\
& && \lambda_2(z_u - z_l) + c^T x \leq z_u + M\pi_0 \\
& && \lambda_1 b_i(\beta - 1) + D_i x \geq \beta b_i && i = 1, \ldots, m_2 \\
& && \lambda_2 b_i(\beta - 1) + D_i x \geq \beta b_i - M\pi_i && i = 1, \ldots, m_2 \\
& && \lambda_1 b_i(\alpha - 1) + D_i x \leq \alpha b_i && i = 1, \ldots, m_2
\end{aligned} \tag{6.18}$$

$$\lambda_2 b_i (\alpha - 1) + D_i x \le \alpha b_i + M\,\pi_i \qquad i = 1,....,m_2$$

$$\sum_{i=0}^{m_2} \pi_i \le m_2$$

$$x, \lambda_1, \lambda_2 \ge 0$$

$$\pi_i \in \{0,1\}$$

$$\lambda_1, \lambda_2 \le 1$$

Where M is a very large positive number. The constraint $\sum_{i=0}^{m_2} \pi_i \le m_2$ ensures that at least one π_i is set to a zero value indicating that the corresponding constraint on λ_2 is satisfied. Notice the increase in model (6.18) dimensionality as compared to that of models (6.8) and (6.15) above, due to the introduction of auxiliary variables and constraints.

6.5 The fuzzy production planning model

The fuzzy model presented in this section is similar to the original model initially introduced in Chapter 3 except that the MPS is now constructed based on confirmed customers orders which appear in the flexible demand balance constraints. The mathematical model is presented below.

$$Min \quad Z = \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{2}\sum_{t=1}^{T}\left(SC_t S_{ijt}^k + PC_{ijt} X_{ijt}^k\right) + \sum_{j=1}^{J}\sum_{k=1}^{2}\sum_{t=1}^{T}\left(IF_{jt}^k I_{jt}^k + BC_{jt}^k B_{jt}^k\right) + \sum_{t=1}^{T} PO_t O_t$$
$$+ \sum_{i=1}^{I}\sum_{k=1}^{2}\sum_{t=1}^{T}\left(OR_{it}^k G_{it}^k + CR_{it}^k Q_{it}^k + IR_{it}^k I_{it}^k\right) + \sum_{j=1}^{J}\sum_{t=1}^{T} W_{jt}^1\left(SP_{jt}^1 - SP_{jt}^2\right) \qquad (6.19)$$

Sibject to

$$Q_{it}^k \le M_{it}^k G_{it}^k \qquad \forall\, i,t,k \qquad (6.20)$$

$$\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{2} Sb_{ijt}^{k} = 1 \qquad \forall t \tag{6.21}$$

$$S_{ijt}^{k} = 0.4 \times Sb_{ijt}^{k}\left(24 - \left(\sum_{i'=1}^{I}\sum_{j'=1}^{J}\sum_{k'=1}^{2}\left(\frac{X_{i'j',t-1}^{k'}}{\alpha_{i'j'}} + S_{i'j',t-1}^{k'}\right)\right)\right) + ST_{ij} \times Sd_{ijt}^{k} \qquad \forall i, j, t, k \tag{6.22}$$

$$\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{2}\left(\frac{X_{ijt}^{k}}{\alpha_{ij}} + S_{ijt}^{k}\right) \leq A_t + O_t \qquad \forall t \tag{6.23}$$

$$O_t \leq A_{ot} \qquad \forall t \tag{6.24}$$

$$X_{ijt}^{k} = b_t \times \left(Sb_{ijt}^{k} + Sd_{ijt}^{k}\right) \times \rho_{ij} \qquad \forall i, j, t, k \tag{6.25}$$

$$I_{it}^{k} = I_{i,t-1}^{k} + Q_{it}^{k} - b_t \times \sum_{j=1}^{J}\left[Sb_{ijt}^{k} + Sd_{ijt}^{k}\right] \qquad \forall i, t, k \tag{6.26}$$

$$I_{j,t-1}^{1} - B_{j,t-1}^{1} - I_{j,t}^{1} + B_{j,t}^{1} + \sum_{i=1}^{I} X_{ijt}^{1} - W_{jt}^{1} \cong CO_{jt}^{1} \qquad \forall j, t \tag{6.27}$$

$$I_{j,t}^{2} - B_{j,t}^{2} = I_{j,t-1}^{2} - B_{j,t-1}^{2} + \sum_{i=1}^{I} X_{ijt}^{2} - W_{jt}^{2} \qquad \forall j, t \tag{6.28}$$

$$W_{jt}^{1} + W_{jt}^{2} \cong CO_{jt}^{2} \qquad \forall j, t \tag{6.29}$$

$$I_{j0}^{k} = B_{j0}^{k} = B_{jT}^{k} = 0 \qquad \forall j, k \tag{6.30}$$

$$Sd_{ijt}^{k}, Q_{it}^{k}, X_{ijt}^{k}, O_t, W_{jt}^{k}, I_{it}^{k}, I_{jt}^{k}, B_{jt}^{k} \geq 0 \qquad \forall i, j, t, k \tag{6.31}$$

$$G_{it}^{k}, Sb_{ijt}^{k} \in \{0,1\} \qquad \forall i, j, t, k \tag{6.32}$$

$$Sd_{ijt}^{k} \in N \qquad \forall i, j, t, k \tag{6.33}$$

The above mathematical model is fuzzy mixed integer bilinear program (FMIBLP), where the fuzziness existent in the model arises due to constraints (6.27) and (6.29) which are treated as flexible constraints and are represented by fuzzy sets having triangular membership functions. Clearly, conventional mathematical approaches lack the ability to directly handle such class of models due to the fuzzy nature of these models. As such, a quick remedy to the existent fuzziness is to adopt the procedure outlined in Section 6.4 with a minor modification that incorporates bilinear rather than linear programs. The upper and lower tolerance limits on the objective function value (i.e., z_u and z_l) are obtained via fixing the customers orders at their three possible values, "$\beta * CO_{jt}^k$, CO_{jt}^k, $\alpha * CO_{jt}^k$" for all j, t, k and then solving the corresponding three crisp models (the solution algorithm is outlined in the next section). Let z_1, z_2 and z_3 be the obtained values for the objective function of the three models, then $z_l = min\ (z_1, z_2, z_3)$ and $z_u = max\ (z_1, z_2, z_3)$, which define the membership function for the objective function as shown in Equation (6.12).

In most situations and for realistic cost figures, solving the model with the smallest possible value for the demand, $\beta * CO_{jt}^k$, and the highest possible value, $\alpha * CO_{jt}^k$, yields the values for z_l and z_u respectively. This is the case since a high demand value in period t is likely to reduce the major setup cost in period $t+1$ but it also entails an increase in most other cost components such as RM purchasing, RM and FP inventory holding, production and minor setup costs. Having the membership functions for the objective function and the constraints at hand, the above production planning fuzzy model is transformed into an equivalent and larger sized crisp model, depending on the aggregation operator being used as detailed below.

6.6 Auxiliary models description

We present the approximate auxiliary models resulting from the application of the two different aggregation operators described in Section 6.4. The adoption of these two particular operators stems from the fact that most other operators, especially those that are compensatory in nature, produce crisp models that are no longer linear.

Following the procedure outlined in Section 6.4.1, the application of the "logical min" operator results in the following approximate auxiliary model:

(Crisp model with "min" operator – "Crisp-Min")

Maximize λ

S.t

Constraints (6.20 – 6.26) & (6.28) & (6.30-6.33)

$$\lambda(z_u - z_l) + Z \leq z_u$$

$$\lambda * CO_{jt}^{1}(\beta - 1) + I_{j,t-1}^{1} - B_{j,t-1}^{1} - I_{j,t}^{1} + B_{j,t}^{1} + \sum_{i=1}^{I} X_{ijt}^{1} - W_{jt}^{1} \geq \beta * CO_{jt}^{1} \qquad \forall j,t$$

$$\lambda * CO_{jt}^{1}(\alpha - 1) + I_{j,t-1}^{1} - B_{j,t-1}^{1} - I_{j,t}^{1} + B_{j,t}^{1} + \sum_{i=1}^{I} X_{ijt}^{1} - W_{jt}^{1} \leq \alpha * CO_{jt}^{1} \qquad \forall j,t$$

$$\lambda * CO_{jt}^{2}(\beta - 1) + W_{jt}^{1} + W_{jt}^{2} \geq \beta * CO_{jt}^{2} \qquad \forall j,t$$

$$\lambda * CO_{jt}^{2}(\alpha - 1) + W_{jt}^{1} + W_{jt}^{2} \leq \alpha * CO_{jt}^{2} \qquad \forall j,t$$

$$0 \leq \lambda \leq 1$$

Where Z is the objective function of the original fuzzy model given by Equation (6.19). Contrary to the fuzzy model, the objective of Model "*Crisp-Min*" is to maximize the aspiration level of the decision maker. The original objective (6.19) is now represented by a constraint in the fuzzy model and each of the fuzzy material balance constraints (6.27 and 6.29) is now represented by two sets of constraints. However, the non-fuzzy sets of constraints remain unaltered. Applying the "convex combination of the min/max" operator as explained in sub-section 6.4.2, the resulting approximate auxiliary model is as follows:

(Crisp model with "convex combination of Min/Max" operator – "Crisp-Comb")

Maximize $\gamma\lambda_1 + (1-\gamma)\lambda_2$

S.t

Constraints (6.20 – 6.26) & (6.28) & (6.30-6.33)

$$\lambda_1(z_u - z_l) + Z \le z_u$$

$$\lambda_2(z_u - z_l) + Z \le z_u + M\pi_1$$

$$\lambda_1 * CO_{jt}^1(\beta - 1) + I_{j,t-1}^1 - B_{j,t-1}^1 - I_{j,t}^1 + B_{j,t}^1 + \sum_{i=1}^{I} X_{ijt}^1 - W_{jt}^1 \ge \beta * CO_{jt}^1 \qquad \forall j,t$$

$$\lambda_2 * CO_{jt}^1(\beta - 1) + I_{j,t-1}^1 - B_{j,t-1}^1 - I_{j,t}^1 + B_{j,t}^1 + \sum_{i=1}^{I} X_{ijt}^1 - W_{jt}^1 \ge \beta * CO_{jt}^1 - M\pi_2 \qquad \forall j,t$$

$$\lambda_1 * CO_{jt}^1(\alpha - 1) + I_{j,t-1}^1 - B_{j,t-1}^1 - I_{j,t}^1 + B_{j,t}^1 + \sum_{i=1}^{I} X_{ijt}^1 - W_{jt}^1 \le \alpha * CO_{jt}^1 \qquad \forall j,t$$

$$\lambda_2 * CO_{jt}^1(\alpha - 1) + I_{j,t-1}^1 - B_{j,t-1}^1 - I_{j,t}^1 + B_{j,t}^1 + \sum_{i=1}^{I} X_{ijt}^1 - W_{jt}^1 \le \alpha * CO_{jt}^1 + M\pi_2 \qquad \forall j,t$$

$$\lambda_1 * CO_{jt}^2(\beta - 1) + W_{jt}^1 + W_{jt}^2 \ge \beta * CO_{jt}^2 \qquad \forall j,t$$

$$\lambda_2 * CO_{jt}^2(\beta - 1) + W_{jt}^1 + W_{jt}^2 \ge \beta * CO_{jt}^2 - M\pi_3 \qquad \forall j,t$$

$$\lambda_1 * CO_{jt}^2(\alpha - 1) + W_{jt}^1 + W_{jt}^2 \le \alpha * CO_{jt}^2 \qquad \forall j,t$$

$$\lambda_2 * CO_{jt}^2(\alpha - 1) + W_{jt}^1 + W_{jt}^2 \le \alpha * CO_{jt}^2 + M\pi_3 \qquad \forall j,t$$

$$\pi_1 + \pi_2 + \pi_3 \le 2$$

$$\pi_1, \pi_2, \pi_3 \in \{0,1\}$$

$$0 \le \lambda_1, \lambda_2 \le 1$$

It is the objective of model *"Crisp-Comb"* to achieve a balance between the "max" operator (i.e., degree of satisfaction of the most satisfied constraint) and the "min" operator (i.e., degree of satisfaction of the least satisfied constraint). While the original objective function (6.19) is now represented by two constraints, each of the fuzzy constraints (6.27 and 6.29) is represented by four sets of constraints: two for the "min" operator and the other two for the "max" operator. Moreover, three binary variables have been added to the model: one for the objective function and the other two correspond to each set of the fuzzy constraints.

6.7 Solution Algorithm

The three variants of the original fuzzy model, which are obtained via fixing the demand at its three respective values (pessimistic, most likely, optimistic), as well as the auxiliary models resulting from the application of the two different aggregation operators (i.e., "*Crisp-Min*" and "*Crisp-Comb*") are all mixed integer bilinear programs (MIBLP). In Chapter 4, we discussed three different "exact" solution algorithms that generate the optimal solution for this class of models, with varying computational efficiency. In this section, however, we propose a "non-exact" solution methodology that is comprised of two phases. In phase 1, the linearization scheme of Glover (1975) is employed (once more) in order to yield an equivalent linear model. In phase 2, an exterior penalty function based approach that targets the complicating constraints is applied to the linearized model resulting from phase 1. The justification behind the use of an algorithm that is likely to generate "near optimal" production plans under the dynamic operating conditions is provided later.

Recall that the generic linearization approach of Glover (1975) results in the following sets of constraints to replace constraint (6.22):

$$S_{ijt}^{k} = 9.6 \times Sb_{ijt}^{k} - 0.4 \times y_{ijt}^{k} + ST_{ij} \times Sd_{ijt}^{k} \qquad \forall i, j, t, k \tag{6.34}$$

$$L_{ijt}^{k} Sb_{ijt}^{k} \leq y_{ijt}^{k} \leq U_{ijt}^{k} Sb_{ijt}^{k} \qquad \forall i, j, t, k \qquad (6.35)$$

$$\sum_{i'j'k'} \left(\frac{X_{i'j',t-1}^{k'}}{\alpha_{i'j'}} + S_{i'j',t-1}^{k'} \right) - U_{ijt}^{k} \left(1 - Sb_{ijt}^{k}\right) \leq y_{ijt}^{k} \leq \sum_{i'j'k'} \left(\frac{X_{i'j',t-1}^{k'}}{\alpha_{i'j'}} + S_{i'j',t-1}^{k'} \right) - L_{ijt}^{k} \left(1 - Sb_{ijt}^{k}\right)$$
$$\forall i, j, t, k \qquad (6.36)$$

Although the linearized model resolves the bilinearity issue, such solution might not be attainable within acceptable time limit especially as the problem size increases. This is mainly due to the complicating constraint (6.33) which limits the production to full batches only (i.e., number of batches produced assume only integer values). The impact of this constraint on the computational complexity was quantified in the approximate rolling horizon models of Chapter 5. In general, Steel mills would typically prefer to produce full batches as this implies a more efficient utilization of the available capacity and better complies with the economies of scale. However, in reality, production does not always take place in full batches particularly when the Company is facing low demand or when there is a limited storage space available or a shortage in the raw material supplies. Moreover, since it is difficult in practice to obtain a precise estimate for the cost figures, especially when it comes to backlogging cost for instance, the use of a model that provides valid and efficient proxies for the total cost function is highly desirable in these situations. The classical approach within the context of flexible mathematical programming to deal with impreciseness in cost parameters (e.g. Zimmerman 1976) is to represent the objective function with a monotonically decreasing linear fuzzy set where the interval of allowance for such set (i.e., z_l and z_u) is decided upon based on a purely subjective judgment obtained from the decision maker. Our approach, however, provides viable estimates, if not the exact value, for the case where production takes place in integral batches. More importantly, it also takes into account the flexibility in customers' orders that the decision maker is willing to undertake during the planning process which naturally contributes to the total cost to be incurred. Hence, restricting the production to full batches achieves only limited added value at the expense of substantial computational efforts. Wang (1997) points out that, in general, the data are imprecise in a fuzzy environment, thus, it is meaningless to calculate an exact solution. According to Hoffman and Padberg (2007), in today's changing and competitive industrial environment the difference between using a "quickly derived solution" and using sophisticated mathematical models to find an "optimal solution" can determine whether or not a company survives.

It should be noted, however, that relaxing the integrality restriction on the number of batches produced implies an additional cost that shall be augmented into the objective function. The augmentation of the deviations from integer batches, we refer to the sum of such deviations as "integrality gap", is accounted for through a penalizing factor that is increased iteratively in an attempt to reduce these deviations and eventually reach a solution that assumes integer values or within some allowable gap ε. This procedure falls under the category of exterior penalty function methods (EPFM) which represent an indirect optimization approach. We provide below a glimpse of these well-established methods where more details can be usually found in integer and nonlinear optimization textbooks such as Bazaraa *et al.* (1993) and Griva *et al.* (2009).

In its original context, EPFM augments all the constraints into the objective function to transform the original problem into an unconstrained optimization problem. A sequential unconstrained minimization technique (SUMT) is then employed in order to approach the feasible region of the original problem from the outside, hence the term exterior, until the solution of the unconstrained problem is made (nearly) equivalent to that of the original one through increasing the value of the penalty factor in each iteration. This sequential increase in the penalty parameter avoids the ill-conditioning difficulties associated with the Hessian matrix as the value of such parameter approaches infinity (more on ill-defining condition can be found in Sherali *et al.* (2001), Griva *et al.* (2009) and Bazaraa *et al.* (1993). In general, penalty function methods are usually more convenient than barrier methods for problems with equality constraints (Griva *et al.* 2009).

Prior to augmenting the integrality restriction to the objective function, it has to be first reformulated as an equality constraint. Hence, constraint (6.33) can be equivalently expressed using either one of the following two forms:

$$\lceil Sd_{ijt}^{k} \rceil - Sd_{ijt}^{k} = 0 \qquad \text{or} \qquad Sd_{ijt}^{k} - \lfloor Sd_{ijt}^{k} \rfloor = 0$$

where $\lceil Sd_{ijt}^{k} \rceil$ and $\lfloor Sd_{ijt}^{k} \rfloor$ represent the smallest integer greater than or equal to Sd_{ijt}^{k} and the largest integer less than or equal to Sd_{ijt}^{k}, respectively. Although both equations serve the same purpose mathematically, the first is more appropriate in our analysis as it shows by how much the production is short of a complete batch. In reality, this difference has the physical interpretation of "capacity underutilization" as it shows the amount that could have been produced using the same setup. Hence, the capacity underutilization cost (CUUC) is given by:

$$CUUC = \mu_l \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{k=1}^{K} \sum_{t=1}^{T} \left(\left\lceil Sd_{ijt}^{k} \right\rceil - Sd_{ijt}^{k} \right) \tag{6.37}$$

The penalty parameter, μ_l, bares the meaning of an underutilization cost per batch and l is the iteration counter. Deviations from integrality for any RM-FP-SG combination have the same relative importance and are therefore penalized equally at a particular iteration. Otherwise, subscripts i, j and k can be included in the parameter μ_l in order to assign different weights to deviations from integrality for different combinations. The auxiliary function involving the penalty term is thus given by $Z_p = Z + CUUC$. Clearly, if the deviations in equation (6.37) add up to zero, an optimal solution has been found where all the Sd_{ijt}^{k} variables assume integer values.

On another note, the setup time calculation is still based on an integral number of batches since in reality a setup takes place regardless of the batch size produced. As such, the setup time constraint (6.34) is now reformulated as:

$$S_{ijt}^{k} = 9.6 \times Sb_{ijt}^{k} - 0.4 \times y_{ijt}^{k} + ST_{ij} \times \left\lceil Sd_{ijt}^{k} \right\rceil \qquad \forall i, j, t, k \tag{6.38}$$

Hence, the three crisp variants of the original fuzzy model, obtained through fixing the demand at its three respective values, as well as the approximate auxiliary crisp models obtained after aggregation are now solved using the penalized objective function Z_p instead of Z while dropping the integrality restriction (6.33) out of the model. Clearly, such changes are also made to the linearized version of these models. At each iteration, the resulting MILP model is solved directly using AMPL/CPLEX 11.0 solver. The iterative increase in the penalty parameter coupled with a setup time, and accordingly a setup cost, that is calculated based on integral batches, shall force the model to seek the production of full batches as much as possible. We point out here that the above application of EPFM to the models at hand is similar in principle to the idea of Lagrangian relaxation except that in the latter case the penalty terms (denoted as Lagrange multipliers) are optimized, rather than decided upon subjectively and increased iteratively, via sub-gradient optimization techniques for example. The solution algorithm can now be formally stated as follows:

Initialization step:

- Obtain the linearized version of the model, replace Z with Z_p and Sd_{ijt}^{k} with $\lceil Sd_{ijt}^{k} \rceil$ in constraint (6.34), drop constraint (6.33) out of the model.
- Define a small tolerance value $\varepsilon \geq 0$ as the deviations related termination scalar, and $\tau \geq 0$ as the time related termination criterion.
- Choose an initial solution (not necessarily feasible), say η_1, a penalty parameter $\mu_1 > 0$ and a scalar $\delta > 1$.
- Let the iteration counter $v = 1$ and go to step 1.

Step 1:

- Starting with η_v, solve the minimization problem at hand directly using AMPL/CPLEX 11.0 solver.
- Let η_{v+1} be the optimal solution and go to step 2.

Step 2:

If $\sum_{ijkt}\left(\left\lceil Sd_{ijt}^{k}\right\rceil - Sd_{ijt}^{k}\right) \leq \varepsilon$ or if solution time $= \tau$, stop; otherwise, let $\mu_{v+1} = \delta\mu_{v}$ and $v = v+1$ and go back to step 1.

The value of the allowable computational time to attain a solution is decided upon by the decision maker. This time related termination criterion is added to the algorithm since for very high values of the penalty parameter μ, the capacity underutilization cost now bares more weight than the rest of the cost components and the model will continuously seek the minimum possible value for it. As the value of μ approaches infinity, no savings in computational time is achieved as compared to the original model which assumes integer values for the variables Sd_{ijt}^{k}. Hence, the objective is to find a penalty value that provides for a good compromise between the deviation from integral batches and the required solution time.

6.8 Computational Experiments

To provide more insights into the performance of the developed mathematical models as well as that of the above solution methodology, ten problem instances of different sizes are tested. For each of these ten problems, the two auxiliary models (i.e., "*Crisp-Min*" and "*Crisp-Comb*") are solved for the same set of input parameters in order to ensure a fair comparison. While those values for the rest of input parameters are retained once solving the three variants of the fuzzy model, demand changes its value corresponding to either one of the three possible values (pessimistic, most likely, optimistic). As before, the numerical experiments are implemented on a single CPU with 4-2.2 GHz AMD Opteron 64-bit processors and 16 GB RAM. All models were coded using AMPL programming language and solved using CPLEX 11.0 solver, where the solver option is set to solve integer problems using the built-in branch and cut algorithm. The values for the different input parameters are generated within certain range of intervals, as shown in Table 6.1.

Table 6.1: Selected range of values for input parameters in the test problems

Input parameter	Range of values	Input parameter	Range of Values
CR_{it}^{k}	(500 , 700)	BC_{jt}^{k}	(20 , 30)
OR_{it}^{k}	(2000 , 2200)	SP_{jt}^{k}	(1300 , 1700)
IR_{it}^{k}	(10 , 20)	M_{it}^{k}	(150 , 300)
PC_{ijt}	(15 , 25)	ρ_{ij}	(0.80 , 0.95)
PO_{t}	(300 , 400)	α_{ij}	(50 , 58)
SC_{t}	(600 , 800)	CO_{jt}^{k}	(0 , 100)
IF_{jt}^{k}	(15 , 30)	ST_{ij}	(0.25 , 1.0)

The iterative procedure of the solution algorithm is illustrated in Table 6.2 for a small problem instance ($I = 1,\ J = T = K = 2$) where the demand is set to its most likely value. It took the algorithm five iterations to achieve a zero integrality gap, at which point the augmented cost matches the original one. As expected, this gap gets smaller as more weight is assigned to such deviations through increasing the value of the penalty parameter, where $\delta = 10$. The complete results for all problem instances are shown in Table 6.3, where the three columns under the various demand scenarios show the Z_p values obtained for the penalty parameter value indicated. Upon solving the three variants of the fuzzy model corresponding to the three demand values, the maximum value for the integrality gap among these models for each problem instance is reported in the "max gap" column. Although the value of $\alpha - 1$ is set equal to that of $1-\beta$ indicating a symmetric triangular membership functions for the various products demands, the obtained values for the augmented objective function are not symmetric which can be attributed in part to the bilinearity existent in the model. While the first six problems meet the first stoppage criterion in which the integrality gap is less than a small tolerance value ($\varepsilon = 1$), a running time of 12 hours for the last four problems is the deciding factor. In all test problems, fixing the demand at its most pessimistic and optimistic values yield the values for z_l and z_u, respectively.

Table 6.2: Summary of the results for a small problem instance

Iteration counter (v)	Penalty (μ_v)	Integrality Gap	Capacity underutilization cost (CUUT)	Z	Z_p
1	10	3.3077	33	457,306	457,339
2	100	2.3077	231	457,360	457,591
3	1,000	1.3077	1,308	457,662	458,969
4	10,000	1.3077	13,077	457,662	470,739
5	100,000	0	0	509,187	509,187

Table 6.3: The obtained results for the ten problem instances

Problem instance	Problem size (I×J×T×K)	Pessimistic demand	Most likely demand	Optimistic demand	Penalty (μ)	Max. gap	α	β
1	(1×2×2×2)	395,534	509,187	594,937	100,000	0	1.2	0.8
2	(1×3×3×2)	792,958	912,498	1,086,102	100,000	0	1.2	0.8
3	(2×3×3×2)	747,795	877,137	1,021,215	100,000	0	1.2	0.8
4	(2×3×4×2)	1,191,695	1,360,073	1,534,557	100,000	0	1.2	0.8
5	(3×4×4×2)	1,370,708	1,546,901	1,711,847	100,000	0.0823	1.2	0.8
6	(4×4×4×2)	1,256,410	1,425,603	1,606,243	10,000	0.9075	1.15	0.85
7	(4×5×4×2)	1,412,311	1,562,135	1,750,538	10,000	1.5659*	1.15	0.85
8	(5×5×4×2)	1,440,165	1,602,495	1,791,857	10,000	1.8368*	1.15	0.85
9	(5×6×4×2)	1,483,096	1,637,422	1,808,136	10,000	2.6960*	1.15	0.85
10	(5×7×5×2)	2,079,562	2,255,339	2,417,524	10,000	4.7902*	1.15	0.85

***** Time related stoppage criterion (Code execution was interrupted after 12 hours of run time)

Having obtained the lower and upper limits on the decision maker's aspiration level, the two approximate auxiliary models to the original fuzzy model can now be constructed. The results of these crisp models, for the same values of the penalty parameter used earlier, are shown in Table 6.4.

Table 6.4: Results of the two crisp models

Problem instance	Penalty (μ)	*"Crisp-Min"* Model		*"Crisp-Comb"* Model				
		λ	Z_{pc1}	λ_1	λ_2	$\gamma\lambda_1+(1-\gamma)\lambda_2$	Z_{pc2}	(π_1, π_2, π_3)
1	100,000	0.7135	452,663	0.7079	1.0	0.8247	453,780	(1,0,1)
2	100,000	0.5898	913,206	0.5539	1.0	0.7323	923,729	(1,0,1)
3	100,000	0.4702	892,653	0.4361	1.0	0.6617	901,976	(1,1,0)
4	100,000	0.5162	1,357,572	0.4711	1.0	0.6827	1,373,035	(1,1,0)
5	100,000	0.6667	1,484,410	0.5920	1.0	0.7552	1,509,893	(1,1,0)
6	10,000	0.7053	1,359,506	0.6423	1.0	0.7854	1,381,545	(1,1,0)
7	10,000	0.7567	1,494,602	0.6989	1.0	0.8193	1,514,151	(1,1,0)
8	10,000	0.6483	1,563,855	0.6005	1.0	0.7603	1,580,666	(1,1,0)
9	10,000	0.6336	1,602,191	0.5896	1.0	0.7538	1,616,492	(1,1,0)
10	10,000	0.6024	2,213,936	0.5612	1.0	0.7367	2,227,859	(1,1,0)

From an aspiration level perspective, a comparison between both models reveals that Model "*Crisp-Comb*", which uses the "convex combination of min/max" operator, achieves a higher value than that achieved by Model "*Crisp-Min*", which uses the "min" operator. Although the value of λ_1 in Model "*Crisp-Comb*" is always slightly less than that of λ in Model "*Crisp-Min*", it is actually λ_2 that pushes the overall objective function of Model "*Crisp-Comb*" to outperform that of Model "*Crisp-Min*". In all problem instances, λ_2 assumes a value of one indicating a 100%

satisfaction for the most satisfied constraint. However, this constraint is not always the same as indicated by the values of the binary variables π_1, π_2, π_3 shown in the right most column of Table 6.4. However, in terms of the augmented objective function values (Z_{pc1} and Z_{pc2}), the total cost of Model "*Crisp-Min*" is slightly less than that of Model "*Crisp-Comb*".

The advantage of using the fuzzy set theory approach to represent uncertainties in demand becomes obvious once the resulting auxiliary models are compared with the crisp counterpart of the original fuzzy model. That is, an initial non-fuzzy formulation of the problem would treat the fuzzy constraints as strict equalities with the demand being set to its most likely value. Comparing Models' "*Crisp-Min*" and "*Crisp-Comb*" total cost with that resulting from the most likely demand given in Table 6.3 shows cost savings in most of the problem instances. In particular, Model "*Crisp-Min*" has outperformed the original non-fuzzy model in 8 out of the 10 problem instances with a slight cost increase for the remaining two problems. Model "*Crisp-Comb*", on the other hand, results in cost savings in 7 out of the 10 problem instances. A notable increase in the cost savings, as a result of using the fuzzy approach, is attained as the problem size increases.

6.9 Summary

This chapter has addressed the problem of developing master production schedule taken into account the uncertainties associated with the end customer demand through the use of fuzzy set theory (FST). In reality, the decision maker has the ability to accept or reject orders; a fact that is better accounted for in a mathematical model through flexible demand constraints having triangular membership functions. Depending on the degree of flexibility that the planner is willing to incorporate, the limits for the aspiration level are determined and a linear membership function is formed to represent such level. Due to the difficulty of precisely estimating some of the cost parameters, “near optimal” solutions were obtained through applying the classical approach of exterior penalty function methods to a linearized version of the original model. Such approach results in substantial computational time savings while still providing valid proxies for the original objective function value.

Besides its major benefit of avoiding the rigid requirements associated with crisp mathematical programming, the adopted fuzzy approach has proved to result in cost savings in most situations as compared to the classical deterministic approach. In general, the performance of the fuzzy models is affected by the choice of the membership functions and the aggregation operators in the sense that different forms of functions and operators produce different outcomes. In an attempt to provide a comparative study between different aggregation operators, we employed the long established classical “min” operator and the “convex combination of the min/max” operator in order to obtain the fuzzy decision set.

Chapter 7

PART-III: Accounting for Uncertain Demand and Capacity Using Fuzzy Set Theory Approach

7.1 Introduction

The previous chapter has illustrated the usefulness of the fuzzy set theory (FST) approach to handle external sources of uncertainties associated with the end customer demand. In this chapter, we take the previous analysis one step further and demonstrate the capability of FST to simultaneously handle internal as well as external sources of uncertainty into the production planning process. In particular, situations involving imprecise production capacity, such as steel rolling mills, are tackled through fuzzy mathematical programming techniques. With the latest advances in technology and the sophisticated manufacturing processes used in several industries, planning production quantities based on a crisply defined future production capacity has become unrealistic as many unforeseen events that directly affect such capacity might occur.

It is the objective of this chapter to illustrate how uncertainty in demand and imprecise available production capacity and production time per unit can be handled together through combining the techniques of flexible mathematical programming

(FMP) with that of possibilistic programming (PP) within the same mathematical model. Under the PP approach, we provide a clear distinction between the appropriate techniques adopted to handle constraints involving fuzzy coefficients only on one side or on both sides of the constraints. In addition, the effect of the decision maker's minimum acceptable possibility level on the final aspiration levels attained is quantified in the numerical analysis section.

7.2 Possibilistic programming with imprecise technological coefficients

Possibilistic programming (PP) is concerned with mathematical programs involving imprecise coefficients that are restricted by possibilistic distributions. In this section, we consider linear programming problems with imprecise technological coefficients which can be stated as (Lai and Hwang 1992b):

$$Min \quad Z = cx$$

$$S.t. \quad x \in X = \left\{ x \mid \tilde{A}x \le b, x \ge 0 \right\} \tag{7.1}$$

In the model above, c and b are assumed to be defined crisply while $\tilde{A}$ is assumed imprecise. This study adopts the form of triangualr possibility distribution to represnet the imprecise coefficients. Generally speaking, a triangular fuzzy number $\tilde{A}$ is fully characterized using three prominent data values:

(1) The most likely value, A^m, that highlights the most belonging memebr to the set of possible values (possibility degree = 1 once normailized).

(2) The smallest possible value, A^s, that least belongs to the set of possible values on the lower end of the possibility distribution (possibility degree = 0 once normailized).

(3) The highest possible value, A^h, that least belongs to the set of possible values on the upper end of the possibility distribution (possibility degree = 0 once normailized).

In the litarature, the “smallest possible” and “highest possible” values are usually referred to as the “most pessimistic” and the “most optimistic” values, respectively. Since the “most optimistic” and “most pessimistic” values may switch locations depending on the physical meaning of the quantity being represnted by a triangular fuzzy number, we prefer the more generic expressions for these prominent points as illustrated above.

The problem that arises when attempting to solve model (7.1) is how to conduct the comparison between the fuzzy consumed resources ($\tilde{A}x$) and the available crisp resources (b). To resolve such fuzzy constraints, we present two approaches that seek to obtain crisp representative numbers for the fuzzy consumed resources $\tilde{A}x$. Tanaka *et al.* (1984) suggested using the weighted average of the upper and lower limits and then substituting this average back into the original model. Following this approach, the constraint $\tilde{A}x \le b$ is now replaced by the auxiliary constraint:

$$wA^s x + \left(1 - w\right) A^h x \le b \qquad (7.2)$$

where $0 < w < 1$. The alternative approach, proposed by Lai and Hwang (1992b), incorporates the three prominent values to obtain the following auxiliary constraint instead:

$$w_1 A^s x + w_2 A^m x + w_3 A^h x \le b \qquad (7.3)$$

where $w_1 + w_2 + w_3 = 1$. Although the decision maker would typically assign the weights to the three possible values, Lai and Hwang (1992b) obtain the most likely solution via the substitution $w_1 = \frac{1}{6}$, $w_2 = \frac{4}{6}$ and $w_3 = \frac{1}{6}$. According to them, the most likely values are often the most important ones and accordingly should have more weights. The two boundary values, however, provide boundary solutions and should bare less weight. If the minimal acceptable possibility θ is given, the auxiliary crisp constraint now becomes:

$$\frac{1}{6}\left[A_\theta^s x + 4A_\theta^m x + A_\theta^h x \right] \le b \tag{7.4}$$

The inclusion of the most likely value in the second approach entails that different solutions will be obtained for different possibilistic distributions, as shown in Figure 7.1 (Lai and Hwang 1992a). Considering the minimal acceptable possibility is θ, equation (7.2) will give the same answer while equation (7.3) would give a higher crisp value for Ax than for $A'x$.

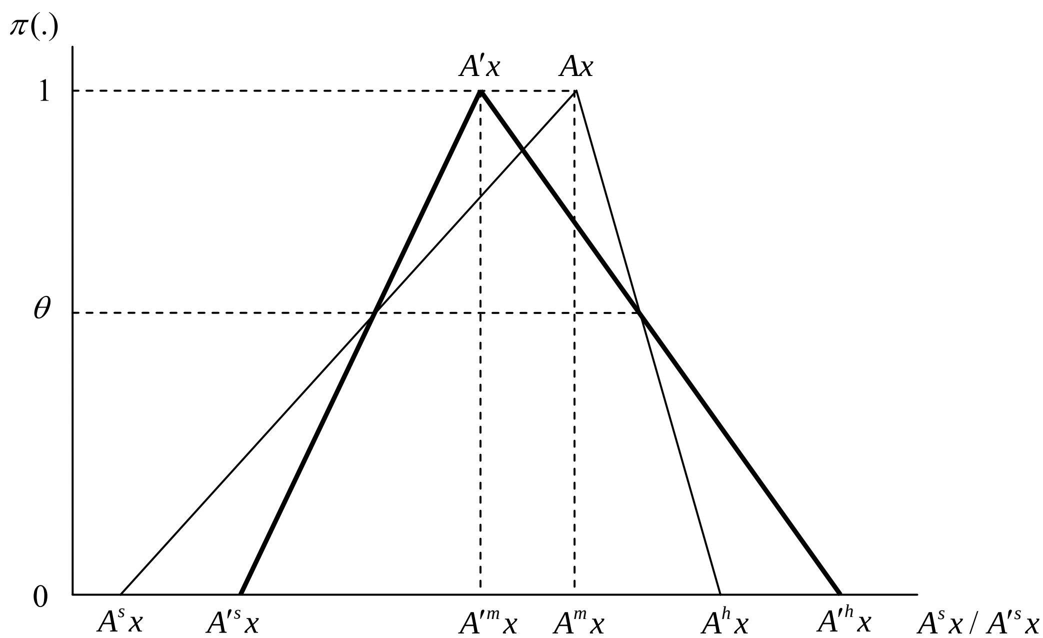

Figure 7.1: Ax is greater than $A'x$ for the accepted possibility level of θ

7.3 Possibilistic programming with imprecise resources and technological coefficients

In the context of possibilistic programming (PP), another problem that frequently arises in practice is the existences of fuzzy numbers representing imprecise quantities on both sides of the constraints. For instance, the constraint $\tilde{A}x \le \tilde{b}$ involves fuzzy parameters (i.e., fuzzy consumption rate of the resources) as well as fuzzy resources available ($\tilde{b}$). Therefore, a comparison needs to be made between two fuzzy numbers which is achieved through the fuzzy ranking approaches. These approaches represent one of the important topics in PP and are used to solve models of the form:

$$
\begin{aligned}
&Min \quad Z = cx \\
&S.t. \quad \tilde{A}x \le \tilde{b} \\
&\qquad\quad x \ge 0
\end{aligned}
\tag{7.5}
$$

In this section, we will discuss the fuzzy ranking approach proposed by Ramik and Rimanek (1985). Reconsider model (7.5) as follows:

$$
\begin{aligned}
&Min \quad Z = cx \\
&S.t. \quad \tilde{a}_{i1}x_1 + \tilde{a}_{i2}x_2 + ... + \tilde{a}_{in}x_n \le \tilde{b}_i \,, \quad i = 1,...,m \\
&\qquad\quad x \ge 0
\end{aligned}
\tag{7.6}
$$

where in this particular case, $\tilde{a}_{i1}$, $\tilde{a}_{i2}$, …, $\tilde{a}_{in}$, $\tilde{b}_i$, for all i are trapezoidal fuzzy numbers. Consider the two fuzzy numbers $\tilde{a}_{ij} = (m_{ij}, n_{ij}, \alpha_{ij}, \beta_{ij})$ and $\tilde{b}_i = (p_i, q_i, \tau_i, \delta_i)$ as shown in Figure 7.2 (Lai and Hwang 1992a). For $\tilde{a}_{ij}$, the left most and right most values with possibility degree of 1 are m_{ij} and n_{ij} respectively, while α_{ij} and β_{ij}

represent the respective left and right spreads. A similar argument applies to the fuzzy number $\tilde{b}_i$.

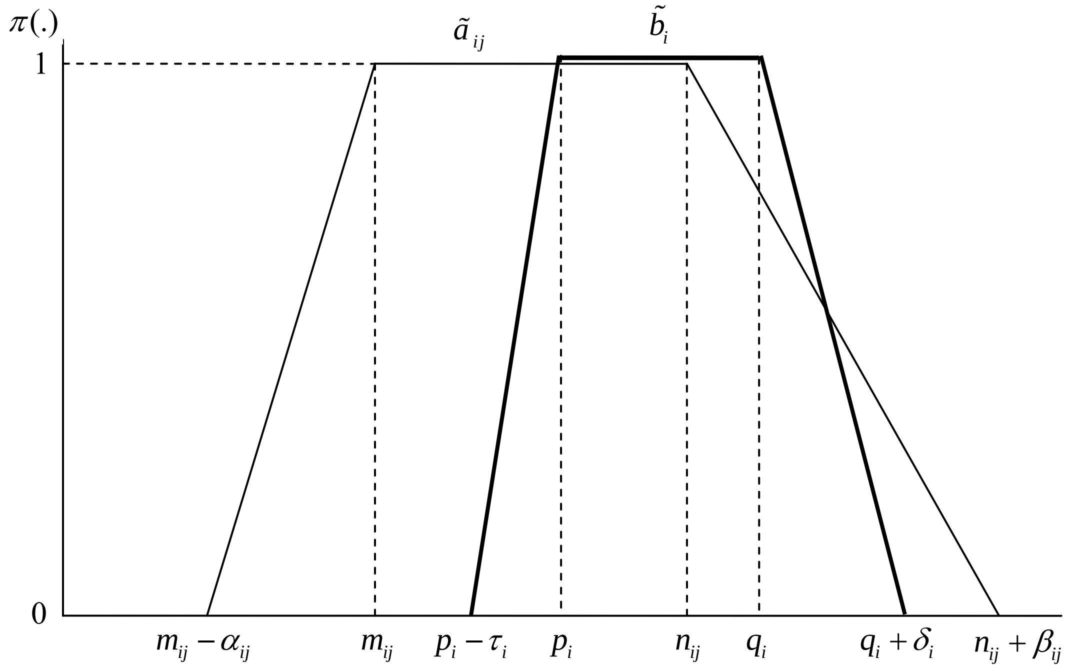

Figure 7.2: The representation of fuzzy inequality of two fuzzy numbers

Using the set-inclusion concept, Ramik and Rimanek (1985) asserted that:

$\tilde{a}_{ij} \le \tilde{b}_i$ if and only if $m_{ij} \le p_i$, $n_{ij} \le q_i$, $m_{ij}-\alpha_{ij} \le p_i-\tau_i$ and $n_{ij}+\beta_{ij} \le q_i+\delta_i$ (7.7)

Excluding the case of $x_1 = x_2 = ... = x_n = 0$, we obtain the following equivalent crisp linear programming problem:

$$
\begin{aligned}
&Min \quad Z = cx \\
&S.t. \\
&\qquad \sum_j m_{ij}x_j \le p_i \qquad \forall i \\
&\qquad \sum_j n_{ij}x_j \le q_i \qquad \forall i \\
&\qquad \sum_j \left(m_{ij}-\alpha_{ij}\right)x_j \le p_i-\tau_i \qquad \forall i
\end{aligned} \tag{7.8}
$$

$$\sum_j \left(n_{ij} + \beta_{ij}\right)x_j \leq q_i + \delta_i \qquad \forall i$$

$$x \geq 0$$

The solution to model (7.8) is also considered to be the solution to model (7.6). We also point out that other fuzzy ranking approaches exist such as that of Tanaka *et al.* (1984) for triangular fuzzy numbers.

7.4 The fuzzy production planning model

In the context of fuzzy mathematical programming, two very different issues can be addressed: fuzzy or flexible constraints for fuzziness, and fuzzy coefficients for lack of knowledge or epistemic uncertainty (Peidro *et al.* 2009). As illustrated in the previous chapter, several authors have addressed problems in which either type of fuzziness arises. In fact, it was not till recently when only few authors have simultaneously considered problems involving both types of fuzziness (e.g., Mula *et al.* 2007 and Peidro *et al.* 2009).

Typically, steel mills, as well as other process industries, operate in an environment in which internal and external sources of uncertainty affect its mode of operation and consequently the production planning related decisions. Such uncertainties are associated with the available production capacity, which in most practical situations is hard to specify precisely, and the anticipated customers' demand, which is frequently prone to errors due to changing customers' preferences (as established earlier). The fuzzy mixed integer bilinear programming (FMIBLP) model presented in this section jointly contemplates the possible lack of knowledge associated with the production capacity and existing fuzziness in the anticipated demand. Hence, the model involves flexible demand constraints as well as hard constraints comprising fuzzy quantities on either one side or both sides of the constraints. In fact, the presented fuzzy model deviates from those in the literature in combining the approaches of flexible mathematical programming (FMP) and possibilistic programming (PP) where also the weighted average method (Lai and Hwang 1992b) and the fuzzy ranking approach (Ramik and Rimanek 1985) are concurrently employed in order to handle fuzzy quantities within the mathematical model.

In reality, it is difficult to establish a precise value for the production capacity, which is mainly comprised of available workforce level and machine uptime. There are a lot of unforeseen events that contribute to unstable production capacity including machine breakdowns, longer or shorter than expected repair time and/or setup time, variable scrap rate resulting from faulty production as well as worker injuries and/or absenteeism. Hence, the available production time is better represented by a fuzzy number having a triangular possibility distribution where the optimistic values of this distribution denote the possibility of working overtime hours. Moreover, the nature of the bar rolling process, the uniformity of raw material supplied and the quality of the production line setup might occasionally cause a steel bar to drift out of its intended path or get stuck on its way through the different rolling stands causing a complete stoppage to the whole production line in either case. Thus, it seems more practical to also assume the production rate (or equivalently, the production time per ton) to be rather a fuzzy number having a triangular possibility distribution.

From a demand perspective, the “rush orders” phenomenon that the steel mill suffers from is also taken into account during the planning process through flexible demand constraints. The fuzzy mathematical model presented here jointly considers both types of uncertainties. As done in Chapter 6, the flexible demand constraints are also represented by fuzzy sets having triangular membership functions. In addition, we assume that the available capacity and the production time per ton are fuzzy numbers having triangular possibility distribution. Such distribution allows for both positive and negative deviations from a “most likely” value which better reflects on reality. The fuzzy model is formally stated as follows:

$$Min \quad Z = \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{2}\sum_{t=1}^{T}\left(SC_t S_{ijt}^k + PC_{ijt} X_{ijt}^k\right) + \sum_{j=1}^{J}\sum_{k=1}^{2}\sum_{t=1}^{T}\left(IF_{jt}^k I_{jt}^k + BC_{jt}^k B_{jt}^k\right)$$
$$+\sum_{i=1}^{I}\sum_{k=1}^{2}\sum_{t=1}^{T}\left(OR_{it}^k G_{it}^k + CR_{it}^k Q_{it}^k + IR_{it}^k I_{it}^k\right) + \sum_{j=1}^{J}\sum_{t=1}^{T} W_{jt}^1\left(SP_{jt}^1 - SP_{jt}^2\right) \quad (7.9)$$

S.t

$$Q_{it}^k \le M_{it}^k G_{it}^k \qquad \forall i,t,k \qquad (7.10)$$

$$\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{2} Sb_{ijt}^k = 1 \qquad \forall t \qquad (7.11)$$

$$S_{ijt}^k = 0.4 \times Sb_{ijt}^k\left(24 - \left(\sum_{i'=1}^{I}\sum_{j'=1}^{J}\sum_{k'=1}^{2}\left(\tilde{\tau}_{i'j'} X_{i'j',t-1}^{k'} + S_{i'j',t-1}^{k'}\right)\right)\right) + ST_{ij} \times Sd_{ijt}^k \quad \forall i,j,t,k \quad (7.12)$$

$$\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{2}\left(\tilde{\tau}_{ij} X_{ijt}^k + S_{ijt}^k\right) \le \tilde{A}_t \qquad \forall t \qquad (7.13)$$

$$X_{ijt}^k = b_t \times \left(Sb_{ijt}^k + Sd_{ijt}^k\right) \times \rho_{ij} \qquad \forall i,j,t,k \qquad (7.14)$$

$$I_{it}^k = I_{i,t-1}^k + Q_{it}^k - b_t \times \sum_{j=1}^{J}\left[Sb_{ijt}^k + Sd_{ijt}^k\right] \qquad \forall i,t,k \qquad (7.15)$$

$$I^{1}_{j,t-1} - B^{1}_{j,t-1} - I^{1}_{j,t} + B^{1}_{j,t} + \sum_{i=1}^{I} X^{1}_{ijt} - W^{1}_{jt} \cong CO^{1}_{jt} \qquad \forall j,t \qquad (7.16)$$

$$I^{2}_{j,t} - B^{2}_{j,t} = I^{2}_{j,t-1} - B^{2}_{j,t-1} + \sum_{i=1}^{I} X^{2}_{ijt} - W^{2}_{jt} \qquad \forall j,t \qquad (7.17)$$

$$W^{1}_{jt} + W^{2}_{jt} \cong CO^{2}_{jt} \qquad \forall j,t \qquad (7.18)$$

$$I^{k}_{j0} = B^{k}_{j0} = B^{k}_{jT} = 0 \qquad \forall j,k \qquad (7.19)$$

$$Sd^{k}_{ijt}, Q^{k}_{it}, X^{k}_{ijt}, W^{k}_{jt}, I^{k}_{it}, I^{k}_{jt}, B^{k}_{jt} \geq 0 \qquad \forall i,j,t,k \qquad (7.20)$$

$$G^{k}_{it}, Sb^{k}_{ijt} \in \{0,1\} \qquad \forall i,j,t,k \qquad (7.21)$$

$$Sd^{k}_{ijt} \in N \qquad \forall i,j,t,k \qquad (7.22)$$

Recall that the notion "$\cong$" in constraints (7.16) and (7.18) represents the fuzzified version of "=" and reads "essentially equal to". On the other hand, the notions "$\tilde{\tau}_{ij}$" and "$\tilde{A}_t$" indicate traingular fuzzy numbers characterizing the imprecise production time per ton and total available production time.

The fuzzy model at hand includes constraints (7.12), which have only imprecise technological coefficients (i.e., a fuzzy production time per ton) resulting in a fuzzy quantity only on one side of the constraint, and constraints (7.13) which involve both imprecise technological coefficients as well as imprecise availability of resources inducing a comparison between two fuzzy quantities existing on both sides of the constraint. As illustrated earlier, the first case can be handled using the weighted average method (WAM), originally proposed by Lai and Hwang (1992b) and successfully applied afterwards to several problems including aggregate production planning (e.g., Liang 2007) and supply chain management (Liang 2008). Following this method, the fuzzy number $\tilde{\tau}_{ij}$ is defuzzified and converted to an equivalent crisp value given the decision's maker minimum acceptable possibility level, and a subjectively chosen set of weights w_1, w_2, w_3. Clearly, the resulting crisp value depends on the initial possibility distribution for fuzzy number, the minimum acceptable possibility level and the weights assigned, as can be seen in Figure 7.3. In general, based on the similarity of triangles, the three prominent values corresponding to the possibility level θ are obtained as follows:

$$\tau_{ij,\theta}^{s} = \tau_{ij}^{s} + \theta\left(\tau_{ij}^{m} - \tau_{ij}^{s}\right) \qquad \forall i, j, \theta \tag{7.23}$$

$$\tau_{ij,\theta}^{m} = \tau_{ij}^{m} \qquad \forall i, j, \theta \tag{7.24}$$

$$\tau_{ij,\theta}^{h} = \tau_{ij}^{m} + \left(1-\theta\right)\left(\tau_{ij}^{h} - \tau_{ij}^{m}\right) \qquad \forall i, j, \theta \tag{7.25}$$

Notice that the value $\tau_{ij,\theta}^{m}$ is always equal to the initial most likely value τ_{ij}^{m} and thus remains unchanged regardless of chosen acceptable possibility level θ.

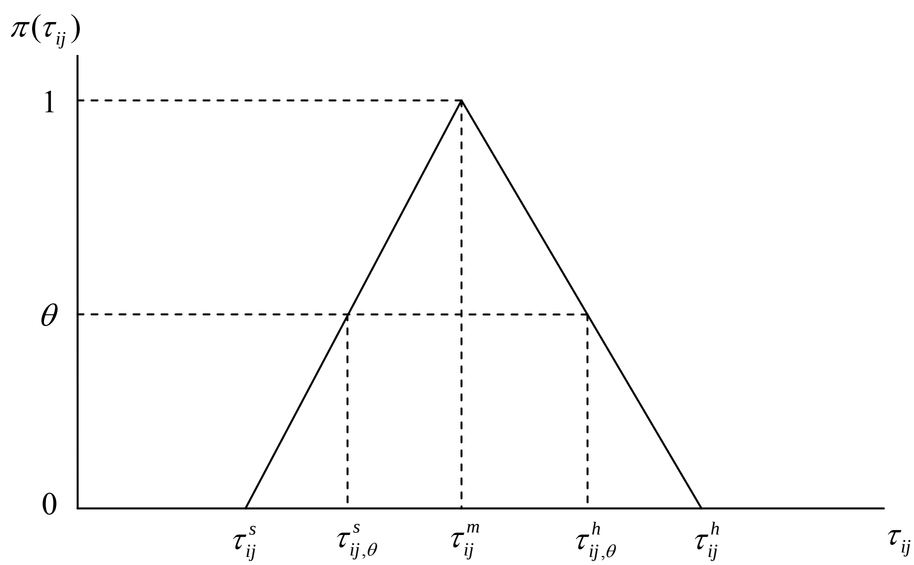

Figure 7.3: Triangular possibility distribution for the production time per ton τ_{ij}

Having these three values at hand, the auxiliary crisp equivalence of equation (7.12) is obtained as:

$$S_{ijt}^{k} = 0.4 \times Sb_{ijt}^{k}\left(24 - \left(\sum_{i'=1}^{I}\sum_{j'=1}^{J}\sum_{k'=1}^{2}\left(\left(\frac{1}{6}\tau_{i'j',\theta}^{s} + \frac{4}{6}\tau_{i'j',\theta}^{m} + \frac{1}{6}\tau_{i'j',\theta}^{h}\right)X_{i'j',t-1}^{k'} + S_{i'j',t-1}^{k'}\right)\right)\right) + ST_{ij} \times Sd_{ijt}^{k} \tag{7.26}$$

In this study, we apply the concept of the most likely values proposed in the original paper (Lai and Hwang 1992b), where $w_1 = w_3 = 1/6$ and $w_2 = 4/6$.

The second scenario, which involves fuzzy quantities on both sides of constraint (7.13) calls for the employment of fuzzy ranking techniques (e.g. Ramik and Rimanek 1985). Following this approach, constraint set (7.13) is replaced by the following three sets of constraints:

$$\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{2}\left(\tau_{ij,\theta}^{s} X_{ijt}^{k} + S_{ijt}^{k}\right) \le A_{t,\theta}^{s} \qquad \forall t \tag{7.27}$$

$$\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{2}\left(\tau_{ij,\theta}^{m} X_{ijt}^{k} + S_{ijt}^{k}\right) \le A_{t,\theta}^{m} \qquad \forall t \tag{7.28}$$

$$\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{2}\left(\tau_{ij,\theta}^{h} X_{ijt}^{k} + S_{ijt}^{k}\right) \le A_{t,\theta}^{h} \qquad \forall t \tag{7.29}$$

As the resulting auxiliary model still involves flexible constraints, this model is solved under the three possible demand values to establish the upper and lower limits for the linear fuzzy set representing the objective function as presented in Chapter 6. Also, the models are solved in a similar fashion where the exterior penalty function method (EPFM) is employed coupled with sequential minimization techniques (SMT) to minimize the deviations from integral batches. The classical "Min" operator and the "convex combination of Min/Max" operator are both used to obtain the approximate auxiliary models.

7.5 Computational Experiments

In this section, the proposed fuzzy production planning model is solved for ten problem instances of various complexities. Since the obtained auxiliary models depend on the selected value for the minimal acceptable possibility level θ, our objective is to first establish or quantify the effect of varying the subjectively chosen parameter θ on the total cost obtained for the same set of input parameters in all the ten problems. Indeed, Lai and Hwang (1992b) suggest providing the decision maker with solution tables for different values of θ. Hence, after generating the problem instances from the ranges shown in Table 7.1, the three variants of the auxiliary flexible mathematical model (obtained from fixing the demand at its three possible values) are solved for $\theta = 0.25$, $\theta = 0.5$ and $\theta = 0.75$. As before, the model is coded in AMPL and solved using CPLEX 11.0 solver where the experiments are run on a single CPU with 4-2.2 GHz AMD Opteron 64-bit processors and 16 GB RAM.

Table 7.1: Selected range of values for input parameters in the test problems

Input parameter	Range of values	Input parameter	Range of Values
CR_{it}^{k}	(500 to 700)	BC_{jt}^{k}	(20 to 30)
OR_{it}^{k}	(2000 to 2200)	SP_{jt}^{k}	(1300 to 1700)
IR_{it}^{k}	(10 to 20)	M_{it}^{k}	(150 to 300)
PC_{ijt}	(15 to 25)	ρ_{ij}	(0.80 to 0.95)
$\tilde{A}_t$	(14, 16, 18) to (16, 18 ,20)	$\tilde{\tau}_{ij}$	(1/58, 1/54, 1/49) to (1/54, 1/48, 1/43)
SC_t	(600 , 800)	CO_{jt}^{k}	(0 to 80)
IF_{jt}^{k}	(15 to 30)	ST_{ij}	(0.25 to 1.0)

The results for the three values for the acceptable possibility level are shown in Tables 7.2-7.4 below. To help visualize the obtained results, Figures 7.4-7.6 illustrate the change in the value of the total cost for each of the problems under the three demand scenarios. As can be seen, there is no significant difference in the cost obtained as a result of changing the acceptable possibility level. This minor difference is seen in the figures showing a “seemingly like” one curve which is actually three curves that plot “more or less” on top of each other. More importantly, there is no consistent pattern to the behavior of the cost figures due to changing θ values. For instance, in problem 9, the minimum cost figures for the pessimistic, most likely and optimistic demand are obtained when $\theta = 0.5$, $\theta = 0.25$ and $\theta = 0.75$, respectively. In problem 10, setting $\theta = 0.25$ results in the highest cost under all demand scenarios although this last consistency seems to be just a coincidence as suggested by the results obtained for the rest of the problems. This analysis suggests that changing the value of θ does not significantly affect the solutions obtained. Hence, we carry out the remaining tests on the resulting equivalent crisp models (after using the aggregation operators) for only one θ value, $\theta = 0.5$.

Since the linear fuzzy set characterizing the decision maker’s aspiration level is now established, the next step is to aggregate this set with the feasible solutions set defined by the constraints in order to obtain the fuzzy decision set using the previously established “Min” operator, and “Convex Combination of Min/Max” operator. We denote the first crisp model as “Crisp-Min2” and the second crisp model “Crisp-Comb2” to distinguish them from those obtained in the previous chapter. The results for the ten problem instances for both models are shown in Table 7.5.

Table 7.2: The obtained results for the ten problem instances for $\theta = 0.25$

Problem instance	Problem size (I×J×T×K)	pessimistic demand	Most likely demand	Optimistic demand	Penalty (μ)	Max. gap	α	β
1	(1×2×2×2)	380,383	489,386	577,717	100,000	0	1.2	0.8
2	(1×3×3×2)	769,570	853,446	1,013,994	100,000	0	1.15	0.85
3	(2×3×3×2)	738,739	841,402	965,628	100,000	0	1.15	0.85
4	(2×3×4×2)	1,053,541	1,184,086	1,306,895	100,000	0.0081	1.15	0.85
5	(3×4×4×2)	1,184,422	1,268,363	1,356,954	100,000	0.3024	1.1	0.9
6	(4×4×4×2)	1,125,123	1,239,076	1,334,282	10,000	0.7962	1.1	0.9
7	(4×5×4×2)	1,290,874	1,402,208	1,500,961	10,000	1.9028*	1.1	0.9
8	(5×5×4×2)	1,326,954	1,431,513	1,550,492	10,000	2.3878*	1.1	0.9
9	(5×6×4×2)	1,461,830	1,588,559	1,708,474	10,000	2.6863*	1.1	0.9
10	(5×7×5×2)	2,017,305	2,115,941	2,237,036	10,000	4.3577*	1.1	0.9

***** Time related stoppage criterion (Code execution was interrupted after 12 hours of run time)

Table 7.3: The obtained results for the ten problem instances for $\theta = 0.5$

Problem instance	Problem size (I×J×T×K)	pessimistic demand	Most likely demand	Optimistic demand	Penalty (μ)	Max. gap	α	β
1	(1×2×2×2)	380,383	489,386	572,107	100,000	0	1.2	0.8
2	(1×3×3×2)	769,570	846,952	1,013,920	100,000	0	1.15	0.85
3	(2×3×3×2)	738,225	840,976	965,306	100,000	0	1.15	0.85
4	(2×3×4×2)	1,053,541	1,184,655	1,305,361	100,000	0	1.15	0.85
5	(3×4×4×2)	1,182,391	1,268,363	1,356,954	100,000	0.2280	1.1	0.9
6	(4×4×4×2)	1,125,181	1,239,003	1,334,557	10,000	0.7435	1.1	0.9
7	(4×5×4×2)	1,295,916	1,411,518	1,501,294	10,000	1.9382*	1.1	0.9
8	(5×5×4×2)	1,317,144	1,432,356	1,557,540	10,000	2.5520*	1.1	0.9
9	(5×6×4×2)	1,460,381	1,590,823	1,707,538	10,000	2.8467*	1.1	0.9
10	(5×7×5×2)	2,009,941	2,111,660	2,229,487	10,000	4.0779*	1.1	0.9

***** Time related stoppage criterion (Code execution was interrupted after 12 hours of run time)

Table 7.4: The obtained results for the ten problem instances for $\theta = 0.75$

Problem instance	Problem size (I×J×T×K)	pessimistic demand	Most likely demand	Optimistic demand	Penalty (μ)	Max. gap	α	β
1	(1×2×2×2)	380,383	489,386	572,107	100,000	0	1.2	0.8
2	(1×3×3×2)	769,570	846,731	1,013,994	100,000	0	1.15	0.85
3	(2×3×3×2)	738,872	841,655	965,717	100,000	0	1.15	0.85
4	(2×3×4×2)	1,053,541	1,183,541	1,303,976	100,000	0	1.15	0.85
5	(3×4×4×2)	1,180,617	1,268,547	1,356,954	100,000	0.1907	1.1	0.9
6	(4×4×4×2)	1,125,928	1,239,421	1,334,285	10,000	0.6013	1.1	0.9
7	(4×5×4×2)	1,292,386	1,409,180	1,509,396	10,000	1.8559*	1.1	0.9
8	(5×5×4×2)	1,317,728	1,430,793	1,555,141	10,000	2.5121*	1.1	0.9
9	(5×6×4×2)	1,468,541	1,595,085	1,698,357	10,000	2.7642*	1.1	0.9
10	(5×7×5×2)	2,010,782	2,106,754	2,221,203	10,000	4.6310*	1.1	0.9

* Time related stoppage criterion (Code execution was interrupted after 12 hours of run time)

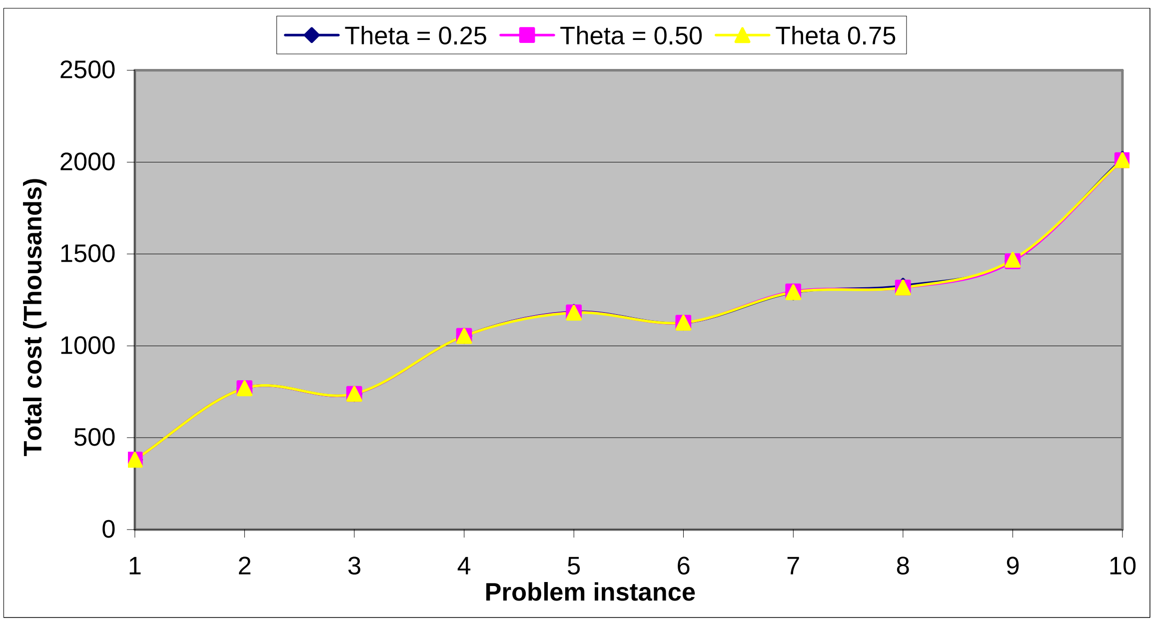

Figure 7.4: Total cost for various θ values under pessimistic demand scenario

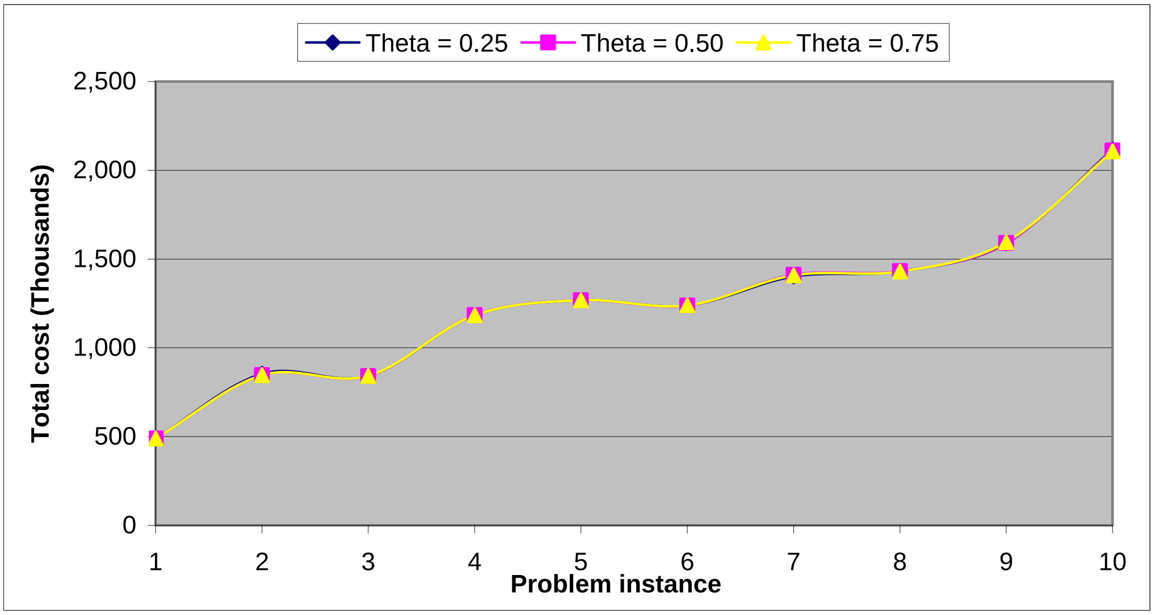

Figure 7.5: Total cost for various θ values under the most likely demand scenario

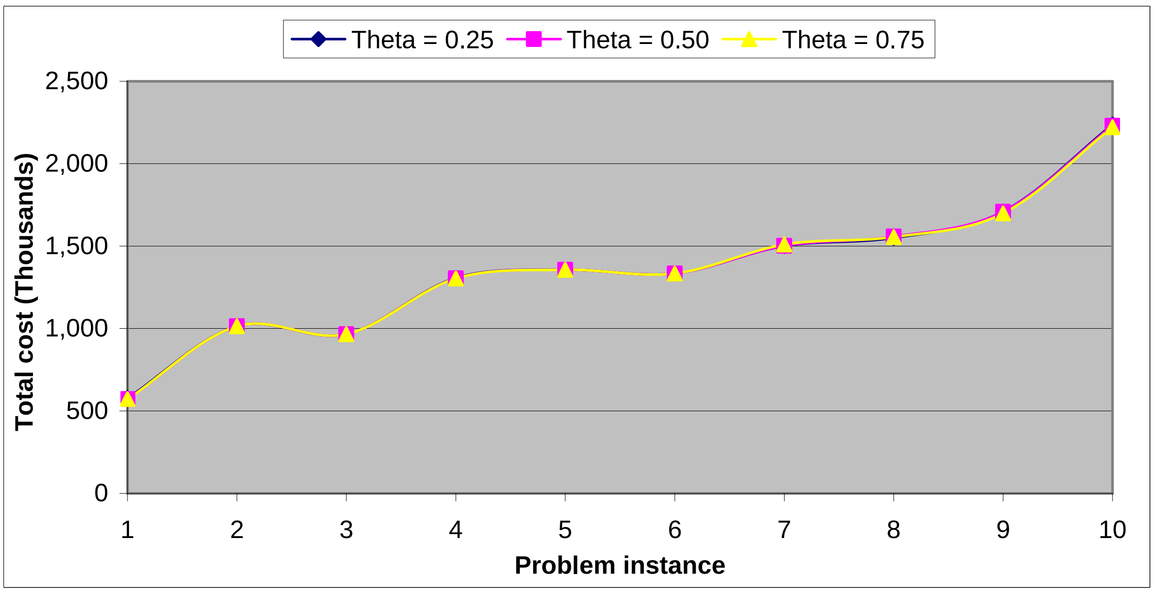

Figure 7.6: Total cost for various θ values under optimistic demand scenario

Comparing the values for the aspiration level obtained using both operators, we notice that Model *"Crisp-Min2"* yields a lower value than that of Model *"Crisp-Comb2"*. The resulting λ_2 value, which expresses the degree of satisfaction of the most satisfied constraints, is responsible for increasing the aspiration level once the combination of λ_1 and λ_2 is calculated. However, it should be noted that λ_2 achieves unity in 9 out of the 10 test problems indicating that, in the remaining problem, 100% satisfaction for the most satisfied constraint could not be achieved. Note that this constraint is not always the same as indicated by the values of the binary variables π_1, π_2, π_3 shown in the right most column of Table 7.5. However, in terms of the augmented objective function values (Z_{pc1} and Z_{pc2}), the total cost of Model "*Crisp-Min2*" is always slightly less than that of Model "*Crisp-Comb2*".

Table 7.5: Results of the two crisp models

Problem instance	Penalty (μ)	"*Crisp-Min2*" Model		"*Crisp-Comb2*" Model				
		λ	Z_{pc1}	λ_1	λ_2	$\gamma\lambda_1+(1-\gamma)\lambda_2$	Z_{pc2}	(π_1, π_2, π_3)
1	100,000	0.6138	454,426	0.6098	1	0.7659	455,194	(1,0,1)
2	100,000	0.5399	881,995	0.5354	1	0.7213	883,095	(1,0,1)
3	100,000	0.5501	840,389	0.5391	1	0.7235	842,886	(1,1,0)
4	100,000	0.6369	1,144,976	0.6343	1	0.7806	1,145,632	(1,1,0)
5	100,000	0.6067	1,251,047	0.5785	1	0.7471	1,255,969	(1,0,1)
6	10,000	0.6239	1,203,927	0.6046	1	0.7628	1,207,968	(1,1,0)
7	10,000	0.6803	1,361,575	0.6727	0.9747	0.7935	1,363,136	(1,1,0)
8	10,000	0.7441	1,378,661	0.7187	1	0.8312	1,384,767	(1,1,0)
9	10,000	0.7572	1,520,390	0.7254	1	0.8352	1,528,250	(1,1,0)
10	10,000	0.7680	2,060,876	0.7403	1	0.8441	2,066,957	(1,1,0)

At this juncture, a comparison between the performances of the resulting auxiliary models using both operators with the crisp counterpart of the original fuzzy model is due. As defined in the previous chapter, the crisp counterpart refers to initial deterministic formulation of the production planning problem with rigid demand constraint where the value of the demand is set at the most likely value. Upon comparing Models' "*Crisp-Min2*" and "*Crisp-Comb2*" total cost with that resulting from the most likely demand given in Table 7.3 shows a cost savings in most of the problem instances. Except for the second problem instance, Model "*Crisp-Min2*" has outperformed the original non-fuzzy model for all other test problems. On the other hand, Model "*Crisp-Comb2*" results in cost savings in 8 out of the 10 problem instances. This illustrates the economical benefits obtained from adopting the FST to explicitly account for uncertainties in the context of mathematical models.

7.6 Summary

This chapter has simultaneously addressed both external uncertainties associated with customers' demand as well as internal uncertainties characterizing the availability of the manufacturing process. A fuzzy mathematical model that jointly accounts for both uncertainties is presented in which the techniques of flexible mathematical programming (FMP) and possibilistic programming (PP) were adopted. To handle fuzzy coefficients existent in the model, average weighted method with the most likely solutions and fuzzy ranking approach were utilized. The results obtained illustrate the benefits obtained from utilizing FST to model uncertainties as compared to the deterministic approach. The combination of FMP and PP principles to treat production planning related problems has not received much attention from researchers and shall thus be a valuable addition to the available literature. This allows for handling fuzzy quantities of various natures within the same mathematical model.

Chapter 8

Conclusions and Future Research Directions

8.1 Summary and Conclusions

The problem of developing production plans that efficiently utilize the available resources in process industries, such as steel rolling industry, is of particular interest to researchers as well as industrial practitioners. Steel manufacturers nowadays face a stiff competition which brings out the need for consistent efforts towards reducing costs and improving the customer service level. In particular, production and inventory related decisions in steel rolling mills, which are characterized by being both capital and energy intensive, bare a great importance as they have a direct impact on the financial health of the organization. In this research, the master production scheduling (MPS) problem in steel rolling mills under static and dynamic operating conditions has been investigated. As this industry has some distinguishing characteristics associated mainly with the manufacturing process, an implementable production schedule has been developed where these features were taken into account. At first, the MPS was developed under static demand conditions where the effect of several interrelated factors on the planning decisions was evaluated. Mathematical programming techniques have been employed in order to determine the optimized production schedules and raw material purchasing quantities while satisfying the end customer demand.

Since uncertainty is an inherent part of the production environment in which steel mills operate, two different approaches were adopted to incorporate the uncertainty in demand into the planning decisions. The first approach to consider demand uncertainty involved establishing approximate models that are implemented on a rolling horizon basis where the problem parameters are updated at the beginning of each period. These approximate models generate only exact schedules and comprise a tractable number of binary and/or integer variables resulting in substantial savings in the computational time needed to solve them. The second approach to address demand uncertainty involved the use of FST in order to express the demand constraints as flexible constraints in the mathematical model. The utilization of this approach has resulted in economical benefits as compared to the use of the deterministic models.

Towards developing a production schedule that considers uncertainties in demand as well as production capacity, the approaches of flexible mathematical programming (FMP) and possibilistic programming (PP) have been utilized. The combined use of these approaches allows uncertainties of different nature to be incorporated into the same mathematical model. To handle fuzzy quantities involved in the model, we utilized both the weighted average method and the fuzzy ranking approach. Throughout this thesis, the (near) optimal solutions to the developed mathematical models were obtained via various solution algorithms that have different computational capabilities.

8.2 Thesis contributions

The research presented in this thesis has been classified in three parts where each part tackles a certain aspect of the production planning problem. The first part, which treats the problem under static demand conditions, has made the following contributions:

- **Literature classification:** The production planning problem at hand represents an instance of the well known dynamic lot-sizing problem (DLSP). Hence, before attempting to tackle the problem, the related literature concerning the general DLSP has been classified based on their distinguishing characteristics. In particular, the contributions made by researchers to the capacitated multi-item DLSP from a modeling and solution algorithms perspectives have been highlighted. We also pointed out the equivalence between the MPS problem, as coined in the production planning literature, and a special class of the DLS problem, as coined in the inventory management literature.
- **Mathematical modeling:** The optimized master production schedule is obtained through the development of a mixed integer bilinear program (MIBLP) that addresses the problem under static demand conditions. The novelty to this model is that it jointly accounts for the distinguishing features and technological constraints associated with the manufacturing process of the bar rolling industry including complex setup time structure, raw-material finished-product dependent scrap and production rates, batch manufacturing, overtime, backlogging and product substitution. The objective of the model is to provide insights into the combined effect of these interrelated factors. Since this deterministic model

establishes the production and inventory related decisions based solely on customer demand, it is most useful in periods where the company faces low demand volatility.

- **Solution Algorithms:** The main contribution in this perspective is a modified-branch-and-bound (B&B) algorithm that exploits the special structure of the mathematical model in order to minimize the number of branches to be explored. Instead of relaxing the binary restriction and obtaining the bound through continuous relaxations, our algorithm makes proper substitutions of the binary variables which resolves the existent bilinearity in the model and allows for the attainment of the bound at each node. An alternative algorithm is the hybrid linearization-Benders decomposition (BD) approach where the bilinear model is linearized first and then the resulting MILP is solved using BD. Although numerical experiments have illustrated that it is more efficient to directly solve the linearized model using MIP solver such as CPLEX 11.0, the hybrid algorithm provides an alternative solution methodology to models involving complicating constraints. The typical approaches to handle these models in the literature are Dantzig-Wolfe decomposition and Lagrange decomposition. The modified B&B algorithm have shown better performance capabilities than the linearization approach for larger problem sizes with a savings in computational time of up to 48%.

In the second part of this research, demand volatility, which takes the form of "rush orders" due to changing customers' preferences and heightened expectations of shortened delivery lead time, is addressed. Two different approaches are employed in order to account for demand uncertainties in the planning decisions which results in the following contributions:

- **Viable rolling horizon approximations:** Developing the production schedule on a rolling horizon basis, where the schedule is updated periodically as new information becomes available, requires rerunning the exact model at the beginning of each time period, and hence consumes a great deal of computational time. As a remedy to this issue, we proposed several viable approximations that generate exact schedules and relax the complicating constraints only during the unimplemented periods of the planning horizon. These complicating constraints, such as integrality restriction on the number of batches produced, prevent the attainment of quick solutions, hence the approximations allow for quantifying the effect of the complicating constraints on the computational time needed and on the total cost obtained. Computational experiments have shown that the developed approximate models provide valid proxies to the exact model and result in substantial savings in computational time.

- **Flexible mathematical model:** Most previous applications of the flexible mathematical programming (FMP) approach have assumed that the fuzzy set representing the objective function is known a priori and is represented by a linear function and the fuzzy set representing the flexible constraint is also linear. Our analysis departs from these assumptions and deals with non-symmetric models in which the fuzzy set representing the flexible constraints is represented by a triangular membership function. As such, instead of establishing the interval of allowance on the decision maker's aspiration level (i.e., objective function) based solely on hunches and pure subjective judgment, the amount of flexibility determined by the fuzzy set of the flexible constraints is used to estimate this interval. Since such an estimate need not be accurate, a non-exact solution algorithm based on the exterior penalty function method (EPFM) was implemented.

The last part of this research is directed towards simultaneously treating the existing uncertainties in customer demand and available production capacity, resulting in the following contribution:

- **Integration of FMP and PP:** In practice, accounting for uncertainties through the use of fuzzy set theory (FST) into mathematical models might take several forms depending on the nature of the existent fuzziness in the problem at hand. Not only we have combined the approaches of FMP and PP into the same mathematical model, but we also presented a situation that calls for the joint implementation of the weighted average method and fuzzy ranking approaches in order to handle fuzzy coefficients appearing on one side or both sides of some constraints. This contribution opens the door to promising research avenues since it allows for handling situations involving various sources of uncertainties. Using these techniques, all uncertainties can now be incorporated into one fuzzy model and then defuzzified into an equivalent auxiliary crisp model whose solution can be found using classical optimization tools.

8.3 Recommendations for future research

Modeling and optimization of process industries in general has been and continues to be one of the active research topics requiring further investigation. The work that has been conducted along this line of research besides the work presented in this thesis suggest the following future research directions:

- Developing mathematical models that jointly optimize production and transportation related decisions. In the steel industry for instance, most of the raw material is bulky which greatly limits the choices for the appropriate transportation modes. This in turn might have an effect on the availability of raw materials in the timings and amounts needed.
- Developing multi-objective production-inventory models where, in addition to cost minimization, other performance measures such as machine utilization and scrap percentage are also optimized. This allows for the attainment of a compromise solution that satisfies all of the performance measures to the best possible extent.
- Implementing heuristic search algorithms to solve mixed integer bilinear programs (MIBLP) similar to the ones presented in this research. These meta-heuristics, such as Genetic algorithms, simulated annealing, and tabu search, have proven successful in obtaining "good quality" solutions to various combinatorial optimization problems within reasonable amount of computational time. Hence, the implementation of these heuristics search methods is an area to be tested furthermore.

- Typically, the existing approaches adopted to handle fuzzy models involve first "defuzzifying" the model and then solving the resulting auxiliary crisp model without directly attempting to handle the fuzzy model in a more efficient way. As noted by Lai and Hwang (1992a), research on developing fuzzy algorithms and coding existing fuzzy approaches in computer packages is urgently needed.
- In practice, firms might face randomness in addition to imprecision/fuzziness where both phenomena need to be accounted for in the planning process. In the literature, very little research has been done in the area of probabilistic fuzzy models and their solution methodologies. This represents an interesting area which requires further investigation.
- The presented analysis can also be extended to account for the more general situation where the rolling mill has more than a single production line and production rates and throughput differ from one production line to the other. In this scenario, a scheduling problem arises where the assignment of jobs to machines (i.e., production lines) is determined such that a performance measure, such as total flow time or makespan, is minimized.

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Appendix

Theoretical Background for Solution Algorithms to Mixed Integer Bilinear Programs

A.1 Introduction

The material presented in this appendix is devoted to studying a special class of mathematical programs referred to as mixed-integer bilinear programming problems (MIBLP) or Mixed 0-1 Quadratic programs. This type of problems arises in various practical situations such as production-distribution planning (Vaish 1974), location-allocation modeling (Sherali and Adams 1984), supply chain reconfiguration and supplier selection (Osman and Demirli 2010) among many others. For instance, while the continuous variables in these models may establish the quantities shipped between designated origin-destination points, the binary variables may signify the decisions whether or not to construct intermediate service or processing facilities on a transshipment network. In general, MIBLP problems may be viewed as a generalization of the fixed-charge location or flow problem (Salkin 1975).

The difficulties associated with directly solving MIBLP problems using optimization packages bring out the need for specifically tailored solution algorithms that can efficiently handle such models and exploit the special structure they possess. Fortunately, these models have been investigated in the literature and solution algorithms have been proposed. Throughout this appendix, three solution algorithms that can be applied, either directly or after some modification, to the solution of MIBLPs are presented in their general context. The application, or the customization, of these algorithms to the model developed in Chapter 3 was detailed in Chapter 4 of this thesis.

A.2 Linearization techniques

The earliest work in this line of research is due to Peterson (1971) where he proposed a linearization mechanism that handles bilinear models involving the multiplication of a continuous variable by a binary one. Glover (1975) extended this work to the more general case where the binary variable is multiplied by a linear function in the binary and/or continuous variables. This class of linearization approaches is readily applicable to bilinear models like the one presented in Chapter 3. As pointed out by Adams and Sherali (1993), one may attempt to solve MIBLP directly via the generalized Benders algorithm, or one may choose to linearize MIBLP using the techniques of Glover (1975) or Peterson (1971) and solve the resulting MILP problem. However, the success obtained via these methods is highly dependent on the specific problem (Adams and Sherali 1990).

The other class of linearization strategies is the so called reformulation-linearization technique (RLT) which requires manipulating the original model before linearizing it. The lead researchers in this class are Profs. Adams and Sherali as they were the first to initialize it and also provide for more efficient strategies later on, as can be seen in Adams and Sherali (1986, 1990) and Sherali and Adams (1990, 1994).

The difference is that RLT provides a hierarchy of formulations that promote tighter linear programming relaxations to the original model. Essentially, an integer linear formulation whose continuous, or linear programming, relaxation closely approximates the convex hull of feasible integer solutions in the vicinity of the optimum is computationally advantageous (Adams and Sherali 1990). The disadvantage of RLT is that those tighter approximations are obtained while paying little regard to problem size.

The general form of a mixed 0-1 quadratic program, denoted as (P1) below, may be stated as follows (Adams and Forrester 2007, Adams *et al.* 2004):

$$\text{(P1):} \quad \textit{Minimize} \quad l\,(\boldsymbol{x},\boldsymbol{y}) + \sum_{j=1}^{n} g_j\,(\boldsymbol{x},\boldsymbol{y})\,x_j \qquad \text{(A.1)}$$

$$\text{Subject to} \quad \sum_{j=1}^{k} h_j\,(\boldsymbol{x},\boldsymbol{y})\,x_j \le \alpha \qquad \text{(A.2)}$$

$$(\boldsymbol{x},\boldsymbol{y}) \in \boldsymbol{X} \equiv \left\{(\boldsymbol{x},\boldsymbol{y}) \in \boldsymbol{S} :\ \boldsymbol{x} \in \{0,1\},\ \boldsymbol{y} \ge 0\right\} \qquad \text{(A.3)}$$

Here, $\boldsymbol{S}$ represents a polyhedral set in the n binary variables $\boldsymbol{x}$ and m continuous variables $\boldsymbol{y}$. Also, $l\,(\boldsymbol{x},\boldsymbol{y})$, $g_j\,(\boldsymbol{x},\boldsymbol{y})$ and $h_j\,(\boldsymbol{x},\boldsymbol{y})$ for all j are linear functions in the $\boldsymbol{x}$ and $\boldsymbol{y}$ variables and α is a scalar quantity. It is assumed, without loss of generality, for each j that $g_j\,(\boldsymbol{x},\boldsymbol{y})$ and $h_j\,(\boldsymbol{x},\boldsymbol{y})$ are not functions of the variable x_j, since $x_j^2 = x_j$ and that it does not contain a term of degree 0.

Typically, linearization approaches target the existent bilinearity in the model and seek to provide an equivalent MILP formulation to the original model. In order to achieve linearity, auxiliary variables and constraints are employed, with the newly defined variables replacing predesignated nonlinear expressions, and with the additional constraints enforcing that the new variables equal their nonlinear counterparts at all binary realizations of the 0-1 variables (Adams *et al.* 2004). Note that although the bilinear model may be equivalently represented by two different linear programming formulations, their size and continuous relaxations may drastically differ depending on how the auxiliary variables and constraints are defined. What these linearization approaches strive to achieve is a tradeoff between the size of the resulting formulation and its strength (i.e., tightness of the bounds obtained). Generally speaking, formulations whose continuous relaxations provide tight approximations of the convex hull of solutions to the original nonlinear problem outperform the weaker representations (Adams *et al.* 2004). Hence, a linearization technique that provides a tight linear programming relaxation, while keeping the problem computationally tractable, is highly desirable (Adams and Sherali 1990).

As pointed out earlier, the linearization scheme of Glover (1975) can be directly applied to the bilinear model of Chapter 3 with straightforward modifications. The justification for choosing this scheme in this work is that it promotes very concise mixed 0-1 linear representation of mixed 0-1 quadratic programs (Adams *et al.* 2004). For a problem involving n binary variables, Glover's technique achieves linearity through the introduction of n unrestricted continuous variables and $4n$ linear inequality constraints. Employing this technique to the above general problem (P1) gives the following model, denoted by (LP1):

(LP1): $$\text{Minimize} \quad l\,(\boldsymbol{x}, \boldsymbol{y}) + \sum_{j=1}^{n} z_j \tag{A.4}$$

Subject to

$$L_j x_j \leq z_j \leq U_j x_j \qquad \forall j = 1, \ldots, n \tag{A.5}$$

$$g_j(\boldsymbol{x}, \boldsymbol{y}) - U_j(1 - x_j) \leq z_j \leq g_j(\boldsymbol{x}, \boldsymbol{y}) - L_j(1 - x_j) \qquad \forall j = 1, \ldots, n \tag{A.6}$$

$$\sum_{j=1}^{k} \xi_j \leq \alpha \tag{A.7}$$

$$\tilde{L}_j x_j \leq \xi_j \leq \tilde{U}_j x_j \qquad \forall j = 1, \ldots, k \tag{A.8}$$

$$h_j(\boldsymbol{x}, \boldsymbol{y}) - \tilde{U}_j(1 - x_j) \leq \xi_j \leq h_j(\boldsymbol{x}, \boldsymbol{y}) - \tilde{L}_j(1 - x_j) \qquad \forall j = 1, \ldots, k \tag{A.9}$$

$$(\boldsymbol{x}, \boldsymbol{y}) \in \boldsymbol{X}$$

For each j, the four inequalities (A.5) and (A.6) associated with z_j enforce the equivalence $z_j = g_j(\boldsymbol{x}, \boldsymbol{y})\, x_j$ for binary x_j. Given any $(\boldsymbol{x}, \boldsymbol{y}) \in \boldsymbol{X}$, if some $x_j = 0$, then (A.5) ensures that $z_j = 0$ with (A.6) being redundant. If some $x_j = 1$, then (A.6) ensures that $z_j = g_j(\boldsymbol{x}, \boldsymbol{y})$ with (A.5) being redundant. Also, L_j and U_j represent the respective lower and upper bounds on the linear functions $g_j(\boldsymbol{x}, \boldsymbol{y})$ over $(\boldsymbol{x}, \boldsymbol{y}) \in \boldsymbol{X}$, where these bounds are obtained as follows:

$$L_j = min\left\{g_j(\boldsymbol{x}, \boldsymbol{y}) : (\boldsymbol{x}, \boldsymbol{y}) \in \boldsymbol{X}^{\boldsymbol{R}}\right\} \tag{A.10}$$

$$U_j = max\left\{g_j(\boldsymbol{x}, \boldsymbol{y}) : (\boldsymbol{x}, \boldsymbol{y}) \in \boldsymbol{X}^{\boldsymbol{R}}\right\} \tag{A.11}$$

where $\boldsymbol{X}^{\boldsymbol{R}}$ denotes any relaxation of $\boldsymbol{X}$ in the variables $(\boldsymbol{x}, \boldsymbol{y})$. A similar argument applies to the newly added variables ξ_j to establish the equivalence $\xi_j = h_j(\boldsymbol{x}, \boldsymbol{y})\, x_j$ through inequalities (A.8) and (A.9) at both binary realizations of the variables x_j, where the bounds are established in a similar fashion using (A.10) and (A.11) with $h_j(\boldsymbol{x}, \boldsymbol{y})$ instead of $g_j(\boldsymbol{x}, \boldsymbol{y})$.

We next present a proposition that helps reduce the size of the resulting linearizaed model.

† **Proposition A.1:** (Adams and Forrester 2007)

Since the variables z_j have nonnegative objective coefficients in (LP1), the right inequalities of (A.5) and (A.6) can be omitted without changing the optimal objective values. These same inequalities can also be omitted in the presence of constraints of the form $\sum_{j=1}^{n} a_j z_j \leq \kappa$ with a_j and κ nonnegative scalars. Similarly, if the variables z_j were to appear in the objective of (LP1) with non-positive coefficients, then the left inequalities of (A.5) and (A.6) could be omitted, even in the presence of constraints of the form $\sum_{j=1}^{n} a_j z_j \geq \kappa$ with a_j and κ nonnegative scalars. †

Since the ξ_j variables do not appear in the objective, the proposition allows for the removal of the $2k$ right inequalities in (A.8) and (A.9).

A.3 Branch-and-Bound algorithms

Branch-and-bound (B&B) algorithms are most widely used solution search methods for solving large scale NP-hard problems. In essence, The B&B approach can be thought of as a simple divide-and-conquer strategy that attains the solution to an optimization problem through an implicit enumerative scheme. The utility of the method drives from the fact that, in general, only a small fraction of the possible solutions need actually be enumerated, the remaining solutions being eliminated from consideration through the application of bounds that establish that such solutions cannot be optimal (Mitten 1970). Glover and Magee (1996) point out that B&B method can often be tailored to exploit special problem structures, thereby allowing these structures to be handled with greater efficiency and reduced computer memory.

From an application perspective, the first implementation of a B&B algorithm dates back to Land and Doig (1960) for the linear case and to Dakin (1965) for the nonlinear case. Since then, the algorithm has seen various improvements and was used to efficiently solve problems of different nature including mixed integer nonlinear programs (e.g., Gupta and Ravindran 1985, Brochers and Mitchell 1994, and Leyffer 2001). In particular, B&B methods have long been used to solve a wide range of the well known problems in the area of operations research including quadratic assignment problem (Bazaraa and Kirca 1983, Hahn *et al.* 1998), fixed charge transportation problem (Kennington and Unger 1976), knapsack problem (Kolesar 1967), and facility location problem (Akinc and Khumawala 1977) to name a few.

To illustrate how the general algorithm works, consider the following generic mathematical program, denoted as (P2):

$$\text{(P2)} \quad Min \quad Z = f\left(x, y\right) \tag{A.12}$$

$$S.t. \quad g_j\left(x, y\right) \le 0 \qquad j \in J \tag{A.13}$$

$$x \in X \;,\; y \in Y \tag{A.14}$$

Where x and y are vectors of the continuous and discrete variables, respectively, $f(.)$ and $g(.)$ are convex differentiable functions, J is the index set of inequalities. Commonly, the set X is defined as $X = \left\{ x \mid x \in R^n, Dx \le d, x^L \le x \le x^U \right\}$

and the set Y is defined as $Y = \left\{ y \mid Z^m, Ay \le a \right\}$, which in most cases is restricted to binary values, $y \in \left\{0,1\right\}^m$. As noted by Grossmann (2002), in most applications of interest the objective and constraint functions $f(.)$, $g(.)$ are linear in y (e.g., fixed cost charges and mixed logic constraints): $f(x,y) = c^T y + r(x)$, $g(x,y) = By + h(x)$. If the functions $r(x)$ and $h(x)$ are linear, then problem (P2) is a mixed integer linear program (MILP) while it is a mixed integer nonlinear program (MINLP) if either function is not linear. In the second scenario, the nonlinearity is caused by higher orders of x and not due to a multiplicative term of the form xy. This emphasizes on the fact that B&B algorithms have been successfully used to solve MILP and MINLP but no track for a MIBLP application was found in the literature. However, Quesada and Grossmann (1995) developed a B&B algorithm for "pure" bilinear models in which both set of variables x and y assume continuous values, and as such, this method is not applicable to the model at hand.

The classical B&B algorithms, or the simplex-based methods, start by solving the continuous relaxation of the original problem (P2) at the root node (Figure A.1) in the hope that the obtained solution will be feasible integer and, hence, solves (P2) as well. This relaxation yield a LP in case (P2) is a MILP and a NLP in case (P2) is originally a MINLP. The basic idea of a relaxation strategy is to identify a relaxed problem that is significantly easier to solve than the original but that is still strong enough to have a fair chance of providing an optimal solution to the original, or at least a good *bound* (Glover and Magee 1996). Once non-integer solutions are obtained, one of the integer variables assuming a fractional value at optimality is picked, say y_j, and two new subproblems (S1 and S2) are constructed through fixing the variable at either zero or one. The search process continues by *branching* from the nodes requiring further exploration with a new branching variable y_i, where $i \neq j$, and so on until all subproblems have been eliminated from consideration. Along the search process, the integer solution yielding the best (i.e., lowest) *bound* on the objective function of the original problem is recorded and it is called the "incumbent" and denoted by Z^*. *Fathoming* is a key part of the search method to reduce the search space, but it requires one to decide whether a better solution might exist further down the space tree (Yimer 2006). A subproblem can be fathomed in one of three cases:

a. The subproblem has no feasible solution.
b. The value of the objective function of the solution to the subproblem, say Z_s, is higher than that of the incumbent, i.e., $Z_s > Z^*$.
c. The solution to the subproblem is an integer solution (if $Z_s < Z^*$ in this case, then an improved value for the incumbent solution is at hand and Z^* is rest to Z_s).

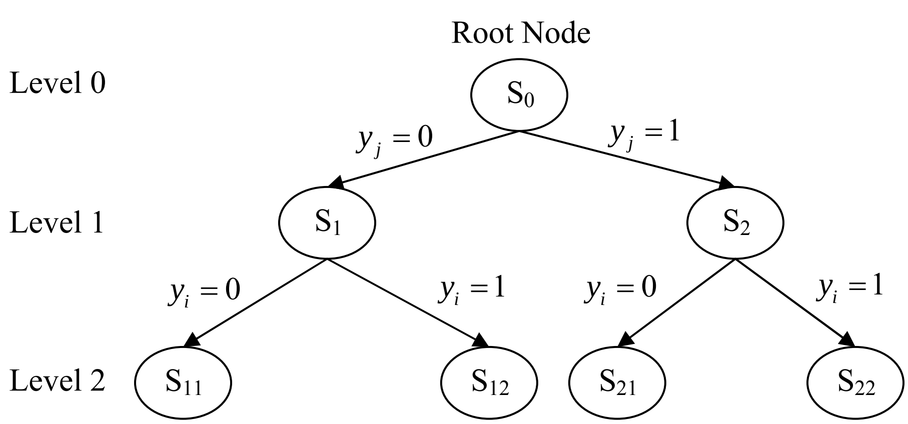

Figure A.1: A portray of the branch-and-bound algorithm

Clearly, a *bound* obtained at a particular node is always better or at worse the same as that obtained at the descendent nodes of the lower level. Once there are no more nodes to explore, the incumbent solution found so far is recorded as the optimal solution to the original problem and the algorithm is terminated.

A.4 Benders decomposition algorithm

This technique was originally proposed by Benders (1962) for the solution of MILP and was later extended by Geoffrion (1972) to handle MINLP through the use of nonlinear duality theory. Since its introduction, Benders decomposition has gained wide popularity among researchers and has been successfully applied to several problems such as supply chain network design and reconfiguration (Osman and Demirli 2010), multi-commodity distribution planning (Cakir 2009, Geoffrion and Graves 1974), power flow and transmission optimization problems (Binato *et al.* 2001, Alguacil and Conejo 2000) among many others. A review of the applications of Benders decomposition to the fixed charge network design problem is given by Costa (2005).

The applicability of Benders decomposition (BD) arises mainly in mathematical models involving complicating variables, where in most practical situations these variables assume either integer or binary values. Basically, the idea is to decompose the model through dealing with the complicating variables (CVs) separately into the "master problem" (MP) and the non-complicating variables (NCVs) into the "sub-problem" (SP). The MP is composed of the objective function terms involving the CVs, the constraints on these variables as well as the added cuts obtained from the solution of the SP. The values of the NCVs are optimized via solving the SP which fixes the values of the CVs to those obtained via solving the MP. The dual multipliers associated with fixed values for the CVs are obtained from the SP and are used to construct an optimality cut that is added iteratively to the MP. After each iteration, a convergence check is carried out on the upper and lower bounds obtained by solving the two problems. Let us consider the following MILP involving both types of variables (Conejo *et al.* 2006):

(P3) *Min* $$Z = \sum_{i=1}^{n} c_i x_i + \sum_{j=1}^{m} d_j y_j \qquad \text{(A.15)}$$

S.t.

$$\sum_{i=1}^{n} a_{li} x_i + \sum_{j=1}^{m} e_{lj} y_j \le b_l \qquad l = 1,\ldots,q \qquad \text{(A.16)}$$

$$x_i^{down} \le x_i \le x_i^{up}, \quad x_i \in N \qquad i = 1,\ldots,n \qquad \text{(A.17)}$$

$$y_j^{down} \le y_j \le y_j^{up}, \quad y_j \in R \qquad j = 1,\ldots,m \qquad \text{(A.18)}$$

In the above model, x_i is considered to be the complicating variable, which can also be binary instead, and y_j is the non-complicating variable. Note that BD is also applicable to linear programs where the complicating variables in this case would be those preventing the solution to the problem by blocks. The BD algorithm calls for decomposing (P3) into the following two problems:

<u>*Master problem (MP):*</u>

$$Min \quad Z_{MP} = \sum_{i=1}^{n} c_i x_i^{(v)} + \alpha \qquad \text{(A.19)}$$

S.t.

$$\alpha \ge \sum_{j=1}^{m} d_j y_j^{(k)} + \lambda_i^{(k)} \left(x_i - x_i^{(k)} \right) \qquad k = 1,\ldots,v-1 \qquad \text{(A.20)}$$

$$x_i^{down} \leq x_i \leq x_i^{up}, \qquad x_i \in N \qquad\qquad i = 1, \ldots, n \tag{A.21}$$

$$\alpha \geq \alpha^{down} \tag{A.22}$$

<u>*Sub-problem (SP):*</u>

$$Min \quad Z_{SP} = \sum_{j=1}^{m} d_j y_j \tag{A.23}$$

S.t.

$$\sum_{j=1}^{m} e_{lj} y_j \leq b_l - \sum_{i=1}^{n} a_{li} x_i \qquad\qquad l = 1, \ldots, q \tag{A.24}$$

$$y_j^{down} \leq y_j \leq y_j^{up}, \qquad y_j \in R \qquad\qquad j = 1, \ldots, m \tag{A.25}$$

$$x_i = x_i^{(v)} : \quad \lambda_i \qquad\qquad i = 1, \ldots, n \tag{A.26}$$

At each iteration, a feasible solution to the SP implies an additional optimality cut of the form (A.20) that is appended to the MP. However, for the case of binary variables, Codato and Fischetti (2004) proposed the addition of a combinatorial feasibility cut to the master problem after each iteration where no feasible solution to the SP exists. The role of this cut is to force the MP to generate 0-1 values that differ from those causing infeasibility in the previous iteration by at least one. For binary x_i variables, the feasibility cut has the general form:

$$\sum_{i:x_i^{(k)}=0}^{n} x_i + \sum_{i:x_i^{(k)}=1}^{n} \left(1 - x_i\right) \geq 1 \qquad\qquad k = 1, \ldots, v-1 \tag{A.27}$$

As we proceed iteratively with the algorithm, the values of the variable α keep on increasing, or possibly stay the same, due to the added optimality cuts after each feasible iteration. However, the objective function of the SP might increase or decrease depending on the values of the CVs obtained from the MP. The algorithm generally seeks to close the gap between the upper and lower bounds obtained as follows:

$$z_{up}^{(v)} = \sum_{i=1}^{n} c_i x_i^{(v)} + \sum_{j=1}^{m} d_j y_j^{(v)} \tag{A.28}$$

$$z_{down}^{(v)} = \sum_{i=1}^{n} c_i x_i^{(v)} + \alpha^{(v)} \tag{A.29}$$

An optimal solution is found when $z_{up}^{(v)} - z_{down}^{(v)} < \varepsilon$, where ε is a small tolerance value. As the variable α provides an underestimate to the SP objective function value, the actual bounds given by equations (A.28) and (A.29) need not be computed and an optimal solution is at hand when the condition $\sum_{j=1}^{m} d_j y_j^{(v)} - \alpha^{(v)} < \varepsilon$ is satisfied. Due to its iterative nature and similar cut generation format, sometimes Benders decomposition is considered as the dual procedure of the Dantzig-Wolfe decomposition (Cakir 2009).

A.5 Summary

This appendix has presented the theoretical background for three solution algorithms that can be used to solve the general class of MIBLP models. The first solution methodology is based on the classical linearization approach adopted for obtaining an equivalent larger sized MILP through the addition of auxiliary variables and constraints. The second solution methodology is the long-established B&B algorithm which exploits the special problem structure to attain the optimal solution through implicit enumeration of a small portion of the feasible solutions. The third approach is Benders decomposition, which is mostly used in the solution of mathematical models involving complicating variables. The next chapter illustrates how such algorithms can be customized to solve the production planning problem at hand.