Three Essays on Linear Asset Pricing Models

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ABSTRACT

Three Essays on Linear Asset Pricing Models

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This disertation includes three essays on linear asset pricing models.

The first chapter is concerned with the effects of including a low-variance factor which leads to a small signal-to-noise ratio in an asset pricing model. We rely on local asymptotics and define the low-variance as local-to-zero by being inversely related to the sample size. When a low-variance factor is present, the commonly applied Fama-Macbeth two-pass regression procedure yields misleading results. Local asymptotic analysis and simulation evidence indicate that the beta of the low-variance factor, risk premiums corresponding to all factors and the magnitude of associated variances are all unreliably estimated. Moreover, t- and F- statistics are unable to detect whether risk premiums are significantly different from zero. Additional simulation results also reveal that Kleibergen's statistic has some ability to detect the usefulness of different factors.

In the second chapter, I investigate the finite sample properties of the two-pass regression, the t-statistic, statistics proposed by Kleibergen (2009) and the specification tests when the first-pass regression slope coefficients — betas — are large, small and zero. In particular, I explore the effect of the number of assets on the properties of the statistics. The results reveal that most of the statistics tend to reach a conclusion that the factor should be included in the model or the model is correct more often that it should, especially when betas are small and the number of assets is large. The diagnosis of the results shows that the source of the problem lies in the large bias

of the estimated risk premiums and the poor estimation of the variance-covariance matrix of the error terms in the first-pass regression.

The third chapter explores an economic explanation of commodity prices by considering the macro-economic exposure of commodity returns. Through estimating the stochastic discount factor representation of the linear asset pricing model, I find that investors are compensated for exchange-rate risk. The result is robust to different estimation methods, to different data sets and over longer periods of time.

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Introduction

Asset pricing and portfolio management are the main building blocks of modern finance theory. Understanding how an asset is priced and how optimal portfolios are constructed has important practical implications for both policy makers and practitioners. Asset pricing theories [e.g. the capital asset pricing model (CAPM), the intertemporal CAPM and arbitrage pricing theory (APT)] typically suggest there is a linear relation between expected returns and sensativity to some systematic factors (e.g. macroeconomic or financial variables). However, until now, there is no consensus on which factors should be included in an asset pricing model. Among the possible reasons for dissatisfaction with the model are the estimation and inference procedures used to evaluate it.

There are two representations of a linear asset pricing model: the beta representation and the stochastic discount factor (SDF) representation. Under the beta representation, expected returns are expressed as a linear function of systematic risks (betas) and risk premiums, corresponding to some macroeconomic or financial factors. This reflects the fundamental principle that investors need to be compensated for bearing more risks. The SDF representation characterizes the law of one price which means that the expectation of discounted returns equals one. A linear asset pricing model implies that the SDF is a linear function of the factors.

A common estimation methodology of the beta representation is called the Fama-Macbeth (FM) two-pass regression proposed by Fama and Macbeth (1973). In the first pass, the betas associated with the factors are estimated. In the second pass, the corresponding risk premiums are estimated. One interesting question asked in the asset pricing literature is whether the risk premiums are different from zero, or equivalently, whether the factors are priced. Since the properties of the estimated risk premiums depend on whether an asset pricing model is correctly specified, a model specification test is also important.

In the empirical asset pricing literature, the covariance of the returns and a factor included in a linear asset pricing model may be small. This implies that either the betas are small or zero [e.g. Lustig and Verdelhan (2007)] or the relative variations of the factors to the returns are very small (e.g. the default premium, consumption and labor growth rate, etc).

The small covariance of the returns and factors may distort the properties of both the finite sample and large sample properties of the estimated risk premiums and the statistics used to test whether a factor is priced or whether the model is correctly specified. Kan and Zhang (1999) are the first to address this problem. In their paper, they investigate both the finite sample and large sample properties of the student t-test when the factor included in the model has zero betas with the returns and the model is misspecified. They show that the t-test tends to favor the conclusion that the factor should be included in the model when actually it should not. Kleibergen (2009) extends Kan and Zhang (1999) by including the cases when betas are small. He finds that the asymptotic distributions of the estimated risk premiums follow non-standard distributions when betas are small or zero under both correctly specified and misspecified models. Kleibergen (2009) then constructs some new statistics (FAR, FM-LM and GLS-LM) and argues that these new tests do not depend on the magnitude of the betas.

In the first chapter, I further extend Kan and Zhang (1999) and Kleibergen (2009) to the case when the variance of the factors is small. I rely on local asymptotics and define the low-variance as local-to-zero by being inversely related to the sample size. The large sample distributions of the estimated parameters in the two-pass regression are derived assuming the error terms in the first pass are independent and identically distributed both over time and across assets. I expect the results under heteroskedasticity and autocorrelation to be similar to those derived in this chapter. The results show that the estimated betas associated with high-variance factors in the first pass have the same asymptotic distributions as those when there is no low-variance factor in the model. All estimated parameters in the second pass are inconsistent and have non-standard asymptotic distributions. Thus, the standard t- and F-statistics, which rely on consistent and normally distributed estimates, become invalid in this case. Furthermore, the asymptotic bias of these parameters depends on the true values of the betas, the risk premium associated with the low-variance factor, the localizing constant (which controls the variance of the factor) and the distribution of estimated beta corresponding to the low-variance factor.

Next, I investigate the finite sample properties of the estimated parameters by simulation. I provide results for a two-factor model with one low-variance factor in both a simple simulation and a more realistic simulation. My results show that the finite-sample behavior of the estimated parameters is close to their large-sample behavior. The smaller the variance of the low-variance factor for a fixed sample size, the larger the bias for the estimated second-pass parameters and the more severe size distortion for the t- and the F-statistics. I also compare the behavior of F-statistic and Kleibergen's (2009) factor Anderson-Rubin (FAR) statistic for testing whether the estimated coefficients in the second-pass regression jointly equal their true values. I find that the performance of the F-statistic is very similar to that of the t-statistic. However, the FAR statistic has a small size distortion.

In the second chapter, I conduct a simulation study to compare the finite sample behavior of the FM two-pass procedure, the t-statistic, the FM-LM, GLS-LM and FAR statistics and three model specification tests when betas are large, small and zero. The case when betas are zero and the model is misspecified is also included. In particular, the effect of the number of assets on the properties of estimators and the statistics is investigated. I also provide a diagnosis of the size distortion of the FAR, GLS-LM statistics and two model specification tests. In order to screen other factors which can affect the properties of the FM procedure and the associated statistics, I fix the signal-to-noise ratio to be one. In addition, I assume the betas are generated from a uniform distribution in order for the mean and variance of the betas to be the same for different Ns.

The finite sample behavior of the FM procedure and the associated statistics has not received enough attention. When the betas are large, Shanken and Zhou (2007) investigate through simulation the finite sample properties of the FM procedure, the t-statistic and the modified model specification test by Shanken (1985). The results show that the finite sample properties are close to their large sample properties. Kan and Zhang (1999) provide both analytical and simulation results on the t-statistic for the case when betas are zero and the model is misspecified. They reveal that the t-statistic has a large size distortion. Chen and Kan (2005) analyze the finite sample properties of the estimated risk premiums revealing that the unconditional mean of the estimated risk premiums is a complicated function of the betas, dispersion of the factors, dispersion of the error terms, T and N. However, how these variables by themselves affect the estimated risk premiums has not been explored. Kleibergen (2009) also provides simulation results on the estimated risk premiums when betas are zero and the model is either correctly specified or misspecified. He further compares the size and power of the Wald statistic and the statistics he proposed when betas are small at $N = 25$ and $T = 143$. However, he assumes the product of betas and the sample means of the factors are independent of the factors in a small sample.

Our results show that when betas are small, the estimated risk premiums are seriously biased. The bias seems to converge as the number of assets increases. The FM-LM test has almost the correct size when the model is correctly-specified regardless of the number of assets, the sample size and magnitude of betas. None of the tests behave well when the model is misspecified. Both the GLS-LM and the FAR statistics are affected by the number of assets. However, the performance of the GLS-LM statistic is much better than that of the FAR when the number of assets is large. The inaccurate estimation of the variance covariance matrix of the error terms is one of the main reasons for the poor performance of the FAR and GLS-LM when betas are small and zero. The t-statistic has large size distortions when betas are small in a small sample. The problem gets worse when the number of assets is large. It is mainly due to the large bias of the estimated risk premiums as N rises.

The specification test with Shanken's correction has a very good size property given the assets are independent both across assets and over time. The behavior of the specification tests without Shanken (1985)'s correction is similar to the FAR statistics. After correcting the variance covariance matrix of the error term, the size distortion is reduced. However, there is still a size distortion when the number of assets is large.

In the third chapter, I investigate how returns of long-only commodity portfolios are determined in equilibrium. Or in other words, what kind of risks US investors are compensated for through buying long-only commodity portfolios.

Historical evidence shows that long-only commodity portolios have average excess returns which are similar to those of stocks [e.g. Bodie and Rosansky (1980), Erb and Harvey (2006)]. Researchers tried to investigate whether asset pricing models constructed for equities are applicable to commodities [e.g. Dusak (1973), Bodie and Rosansky (1980), Jagannathan (1985), Bessembinder (1992) and Roache (2008)]. However, the results are mixed.

Based on the theory of storage and the theory of normal backwardation and in a small open economy, I construct a 3-factor model including the stock market factor, the real interest rate and the exchange rate. I find that exchange rate risk is priced. The results are invariant to returns constructed by Standard and Poor (The SP-GSCI mono-indices) and those constructed by the US commodity research bureau. They are also robust to different estimation methods and under both correctly specified and misspecified models.

CHAPTER 1

Inference in Asset Pricing Models with a Low-Variance Factor

1.1. Introduction

Asset pricing and portfolio management are the main building blocks of modern finance theory. Understanding how an asset is priced and how optimal portfolios are constructed has important practical implications for both policy makers and practitioners. Asset pricing theories typically model expected returns as a linear function of systematic risks and risk premiums corresponding to some macroeconomic or financial factors. An interesting aspect that emerges from the descriptive analysis of factor models is that the variability of some macroeconomic factors (e.g. the default premium, consumption and labor growth rate, etc) is very small compared to the variability of asset returns. We refer to the small relative variability of the factors as low signal-to-noise ratio (SNR). In Figure 1 we present two examples of relative return variability of the Fama and French 10 portfolio formed on momentum and the growth rate of per capita US labor income from July 1963 to December 1990. As it can be observed even the return with a minimum variance is much more volatile than the labor factor.

Figure 1.1. The relative variation of the returns of Fama and French 10 portfolio formed on momentum and the growth rate of per capita US labor income from July 1963 to December 1990

sample/low beta/writings/disertation/LMJ3P200.wmf

Relative variation of the returns with minimum variance and the labor

sample/low beta/writings/disertation/LMJ3P201.wmf

Relative variation of the returns with maximum variance and the labor

The commonly applied methodologies for estimating the models is the Fama and Macbeth procedure, proposed by Fama and Macbeth (1973). In the first pass, asset returns are regressed on the factors to obtain the coefficients, betas. In the second pass, the estimated betas become the regressors, and the corresponding coefficients, the risk premiums, are calculated.

It has been documented that the large sample inference on the risk premiums is valid given a correctly specified model. Over the last twenty years, researchers in financial economics have been actively exploring properties of estimation and testing asset pricing models under model misspecification, error-in-variable (EIV) and possibly irrelevant factors. For example, the FM procedure treats the estimated betas as the true betas in the second pass regression, which causes an EIV problem. Shanken (1992) analyzes the asymptotic properties of the estimated risk premiums by taking account of the EIV problem under conditional homoskedastic error terms. He argues that the usual FM standard errors are incorrect and proposes the EIV-adjusted standard errors. Later, Jagannathan and Wang (1998) extend the properties to the case of weakly stationary and ergodic errors. From a model misspecification perspective, Kan & Zhang (1999) investigate the properties of the two-pass procedure when a factor is independent of the asset returns. They argue that the t-statistic tends to over-reject the null and lead to the conclusion that the factor is useful. Kleibergen (2009) generalizes Kan and Zhang (1999) and argues that the t-statistic is unreliable when the true betas associated with the factors are either zero or close to zero. He also proposes some new test statistics, including the factor Anderson-Rubin statistic (FAR).

There is little discussion about the effect on estimation and test statistics under a low SNR. In a framework of predicting asset returns using an explanatory variable, Torous and Valkanov (2000) argue that the low SNR renders unreliable estimation, inference and forecasting. Gospodinov (2009) shows that the low relative variation of the forward premium to the exchange rate returns creates a large bias and variability of the estimated slope parameters in a differenced forward premium regression, and leads to size distortions of the t statistic. Chen and Kan (2005) point out that in a one-factor linear asset pricing model, the magnitude of the finite sample percentage biases of the estimated zero-beta rate and the risk premium by GLS, is an inverse function of the relative variances of the true betas and that of the estimated betas.

To the best of our knowledge, this is the first paper which investigates the properties of the two-pass regression when a low-variance factor, which leads to a small SNR, is present in an asset pricing model. We analyze the properties of the parameters corresponding to both the low-variance factor and the other factors. Our first contribution is that we derive the large sample distribution of the two-pass estimator when a model includes a low-variance factor using local asymptotic analysis. In order to account for the low variability of the factor, we parameterize its variance as local-to-zero by being inversely related to the sample size. This also implies that the information used to estimate the parameters associated with the low-variance factor remains low when the sample size increases. The standard asymptotics assumes the information increases as the sample size increases. Thus the estimates become less and less volatile through time. In order to control other determinants, e.g. heteroskedasticity and autocorrelation, which possibly influence the estimators, and concentrate on the effect of the variance, we derive the asymptotic distributions of Fama-Macbeth two-pass estimator assuming the error term is independent and identically distributed (iid) across time and across assets. The distribution under heteroskedasticity and autocorrelation can be derived in a similar fashion as in Shanken (1992) or Jagnnathan and Wang (1998). We expect that the results are similar to those under the stronger assumptions.

The finite sample properties of this estimator are then analyzed via simulation. We also compare the performances of the t-statistic, F-statistic and Kleibergen's FAR statistic. The reason why we investigate the FAR statistic is because Kleibergen (2009) also studies the problem that the returns and the factors have small or zero covariances. The difference is that Kleibergen investigates the properties of the estimator when the betas are small and zero, while, we are concerned about the small variance of the factor.

Our theoretical results show that the asymptotic distributions of the estimated betas associated with high-variance factors are the same as those when there is no low-variance factor in the model. However, the distribution of the estimated beta associated with the low-variance factor is inconsistent and converges to a normally distributed random variable. The asymptotic variance increases as the SNR or the variance of the low-variance factor decreases. This implies that if the variance of a factor in an asset pricing model is very small, only the beta associated with that factor is not consistently estimable.

Further, the estimated risk premiums and zero-beta rate in the second-pass regression are inconsistent and their asymptotic distributions are non-standard and depend on the magnitude of the risk premium corresponding to the low-variance factor. This implies that even though the asset pricing model is correct, the expected return calculated from the model is not reliable. In the asset pricing literature, many researchers are interested in testing whether a particular factor is or a group of factors are priced. Our results show that the conclusion might be erroneous if the model includes a low-variance factor and is estimated by the FM procedure. This is because the standard inference procedures, such as t-statistic and F-statistic, which are used for these purposes and which require asymptotic normal distribution of estimators, are no longer valid.

The source of this problem is that the small variance of a factor leads to a large volatility of the estimated beta associated with the low-variance factor. In the limit, the dispersion of the estimated beta does not shrink to zero. Therefore, the inconsistency of the estimated beta leads to the bias and non-standard distributions of the estimated risk premiums. It thus leads to invalid t- and F-statistics.

By simulation, we demonstrate that, when the sample size is fixed, the asymptotic biases of all parameters in the second-pass regression increases as the variance of the low-variance factor decreases. The estimated parameters are not informative at all when its variance is very small. Furthermore, the estimated zero-beta rate and the risk premiums corresponding to the high-variance factors, possibly have larger asymptotic biases than those associated with the low-variance factor. In addition, the t- and F-statistics have large size distortions. They tend to over-reject the true values more often than they should, especially when the varaince is very low. Compared to the t-statistic and F-statistic, Kleibergen's FAR statistic performs more adequately. The empirical size is close to its nominal size.

The rest of the chapter is organized as follows. Section 1.2 reviews the models, the FM two-pass methodology, the t-statistic, the F-statistic and Kleibergen's FAR statistic. Section 1.3 derives the asymptotic properties of the risk premium estimates when the variance of an included factor in an asset pricing model is low. In section 1.4, we show the simulation results. A summary of the conclusions is presented in Section 1.5.

1.2. The estimating and testing procedures

In this section, we first review the expected return-beta representation of a linear factor model and then present the popular Fama-Macbeth two-pass estimation and testing procedures.

The notation we use in this chapter is the following:

For example, let $R = \begin{bmatrix} R_{11} & R_{12} & ... & R_{1T} \\ R_{21} & R_{22} & ... & R_{2T} \\ & & ... & \\ R_{N1} & R_{N2} & ... & R_{NT} \end{bmatrix}$ denote a N by T matrix of returns on N test assets over T periods.

Whereupon, R_i for $i = 1, ..., N$ is a column vector formed by the transpose of its row i; R_t for $t = 1, ..., T$ is a column vector representing the t^{th} column of R.

Also, let $F = \begin{bmatrix} f_{11} & f_{12} & ... & f_{1K} \\ f_{21} & f_{22} & ... & f_{2K} \\ & & ... & \\ f_{T1} & f_{T2} & ... & f_{TK} \end{bmatrix}$ be a $T \times K$ matrix of risk factors where f_t for $t = 1, ..., T$ is a $K \times 1$ column vector transposed from the t^{th} row of F.

Finally, let $B = \begin{bmatrix} \beta_{11} & \beta_{12} & ... & \beta_{1K} \\ \beta_{21} & \beta_{22} & ... & \beta_{2K} \\ & & ... & \\ \beta_{N1} & \beta_{N2} & ... & \beta_{NK} \end{bmatrix}$ be a N by K matrix of beta coefficients associated with the risk factors, where β_i for $i = 1, ..., N$ is a column vector from the transpose of row i of B.

The notations of the population time series mean and sample time series average of a matrix are $E((\bullet)_t)$ and $\overline{(\bullet)}$. For example, $E(R_t)$ denotes the population time serie mean of R and $\overline{R} = \frac{1}{T}\sum_{t=1}^{T} R_t$. $E(f_t)$ means the population time series mean of F and $\overline{f} = \frac{1}{T}\sum_{t=1}^{T} f_t$.

The population time series mean and sample time series average for asset i is denoted as $E((\bullet)_t^i)$ and $\overline{(\bullet)}^i$. For example, the sample time series average of returns and the error term in a regression is $\overline{R}^i$ and $\overline{\epsilon}^i$.

$\widehat{(\bullet)}$ represents the estimated value of $(\bullet)$, $(\bullet)_*$ indicates the demeaned value of $(\bullet)$ and $M_{(\bullet)} = I - (\bullet)((\bullet)'(\bullet))^{-1}(\bullet)'$ is a symmetric and idempotent matrix used to project a vector onto the space orthogonal to $(\bullet)$. ι_N and ι_T are N by 1 and T by 1 vectors of ones, respectively, and 0_N denots an N by 1 vector of zeros. All the covariance matrices are assumed to be positive definite and all the matrices except the projection matrices are assumed to have full rank. "$\xrightarrow{d}$" means convergence in distribution. "$\xrightarrow{p}$" indicates convergence in probability.

1.2.1. The model

We define R_t as the returns from N assets at time t, f_t as K systematic risk factors at time t of which a linear combination is able to explain the returns at time t, $B = [\beta_1, \beta_2 ... \beta_N]'$ as a N by K matrix and β_i for $i = 1, ..., N$ as a K by 1 vector. Therefore, the following relation holds

$$R_t = \alpha + Bf_t + \epsilon_t \text{ for } t = 1, ..., T, \tag{1.1}$$

where α is a N by 1 vector, $B = cov(R_t, f_t)var(f_t)^{-1}$ and ϵ_t is a N by 1 vector.

Let $E(R_t)$ denote the expectation of R_t. The asset pricing theories suggest that

$$E(R_t) = \lambda_0 \iota_N + B\lambda_F, \tag{1.2}$$

where λ_0 is a scalar, called the zero-beta rate denoting the expected return when all betas are zero; $\lambda_F = [\lambda_1, \lambda_2, ..., \lambda_K]'$ is a K by 1 vector of risk premiums or the prices of the risks corresponding to the factors; and ι_N is a N by 1 vector of ones.

The asset pricing model is designed to answer the question: why do different assets have different expected returns? This representation implies that it is because different assets are exposed to different systematic risks. In equilibrium, investors need to be compensated with higher returns for holding riskier assets.

1.2.2. Fama-Macbeth Two-pass procedure

The most commonly used methodology in estimating linear asset pricing models is the FM procedure, which includes a time series regression in the first pass and a cross-sectional regression in the second pass. The following assumptions are imposed in this section:

Assumption 1: *Assume ϵ_t is indepedent and identically distributed with $E(\epsilon_t|F) = 0_N$ and $Var(\epsilon_t|F) = \Sigma$, a N by N positive definite matrix.*

Assumption 2: *The factors $F = [f_1, ..., f_K]$ are a T by K full column rank matrix. Let $\overline{f} = \frac{1}{T}\sum_{t=1}^{T} f_t$, $\mu_F = E(f_t)$ and $\Sigma_F = Var(f_t)$. f_t for $t = 1, ..., T$ is weakly stationary, ergodic and asymptotically normally distributed with $\sqrt{T}(\overline{f} - \mu_F) \xrightarrow{d} N(0, \Sigma_F)$.*

Let R_i be a T by 1 vector of returns of asset i for T periods and F be a T by K matrix of K factors for T periods. β_i can be estimated from the following time series regression:

$$R_i = \alpha_i \iota_T + F\beta_i + \epsilon_i, \text{ for } i = 1, ..., N. \tag{1.3}$$

Therefore,

$$\widehat{\beta}_i = (F_*' F_*)^{-1} F_*' R_i, \tag{1.4}$$

where $F_* = F - \iota_T \overline{f}'$ and ι_T is a T by 1 vector of ones.

In order to estimate the zero-beta rate and risk premiums, Fama and Macbeth suggest calculating their values at each time t first from a cross-sectional regression.

Define λ_{0t} as the zero-beta rate at time t, $\lambda_{Ft} = [\lambda_{1t}, \lambda_{2t}, ..., \lambda_{kt}]'$ as the risk premiums at time t and $\lambda_t = [\lambda_{0t}\ \lambda'_{Ft}]'$. If B is known, the cross-sectional regression at each time t is

$$R_t = \lambda_{0t}\iota_N + B\lambda_{Ft} + u_t, \text{ for } t = 1, ..., T, \tag{1.5}$$

where $u_t = B(f_t - \mu_F) + \epsilon_t$ and $\sqrt{T}\overline{u} \xrightarrow{d} N(0, B\Sigma_F B' + \Sigma)$ under Assumptions 1 and 2.

Since B is unknown, the estimated beta $\widehat{B}$ is employed instead of B in empirical analysis.

Let $\widehat{X} = [\iota_N\ \widehat{B}]$ and $\widehat{\lambda}_t = [\widehat{\lambda}_{0t}\ \widehat{\lambda}'_{Ft}]'$. Therefore,

$$\widehat{\lambda}_t = (\widehat{X}'\widehat{X})^{-1}\widehat{X}'R_t. \tag{1.6}$$

$\{\widehat{\lambda}_t\}_{t=1}^T$ can be considered as drawn from the same distribution with mean λ and covariance V. Therefore, the estimated parameters $\widehat{\lambda} = [\widehat{\lambda}_0, \widehat{\lambda}'_F]'$ can be obtained as

$$\widehat{\lambda} = \frac{1}{T}\sum_{t=1}^{T}\widehat{\lambda}_t. \tag{1.7}$$

Let $\lambda = [\lambda_0, \lambda'_F]'$. The estimated variance covariance matrix of the term $\sqrt{T}(\widehat{\lambda}-\lambda)$, can be calculated as

$$\widehat{V} = \frac{1}{T}\sum_{t=1}^{T}(\widehat{\lambda}_t - \widehat{\lambda})(\widehat{\lambda}_t - \widehat{\lambda})'. \tag{1.8}$$

The idea of this procedure is to treat $\{\lambda_t\}_{t=1}^T$ as a random sample with mean λ and covariance V.

The t-statistic can be used to test whether the risk premiums equal their true values. Under $H_0 : \lambda_k = \lambda_{k,0}$, for $k = 1, ..., K$, the t-statistic is

$$t = \frac{\widehat{\lambda}_k - \lambda_{k,0}}{\widehat{s}_k/\sqrt{T}}, \text{ for } k = 1, ..., K, \tag{1.9}$$

where $\widehat{s}_k = [\widehat{V}^{\frac{1}{2}}]_{kk}$, which is the $[k, k]_{th}$ element of the standard error of the estimated covariance matrix for the risk premiums. The critical value is obtained from a t distribution with degree of freedom $T - 1$ or from a standard normal distribution.

The F-statistic can be applied to test the joint hypothesis that the risk premiums equal their true values. For example, under $H_0 : \lambda_F = \lambda_{F,0}$

$$F = \frac{(\widehat{\lambda}_F - \lambda_{F,0})'\widehat{V}^{-1}(\widehat{\lambda}_F - \lambda_{F,0})}{k} \xrightarrow{d} F(k, N - k). \tag{1.10}$$

The drawback of the Fama and MacBeth procedure is that it treats $\widehat{B}$ as the true B. It thus ignores the estimation error in B. Black, Jensen and Scholes (1972) create a method of grouping the stocks into portfolios to mitigate the error-in-variable (EIV) problem. However, the grouping methods may neglect useful information in the data. Shanken (1992) analyzes the asymptotic properties of the two-pass regression methodology by taking account of the EIV problem and proposes the EIV-adjusted variance.

Under Assumptions 1 and 2, $\widehat{\beta}_i$ is unbiased, $\sqrt{T}$-consistent and follows an asymptotic normal distribution as

$$\sqrt{T}(\widehat{\beta}_i - \beta_i) \xrightarrow{d} N(0, \sigma_i^2 \Sigma_F^{-1}), \tag{1.11}$$

where $\sigma_i^2 I_T$ is the variance of ϵ_i and Σ_F^{-1} is the inverse of the variance covariance matrix of the factors.

As suggested by Shanken (1992), the asymptotic covariance matrix of the risk premiums is given by

$$\widetilde{V} = (1+c)\Omega + \Sigma_F^*, \tag{1.12}$$

where $c = \lambda_F' \Sigma_F^{-1} \lambda_F$, $\Omega = A\Sigma A'$, $A = (X'X)^{-1}X'$ and $\Sigma_F^* = \begin{bmatrix} 0 & 0 \\ 0 & \Sigma_F \end{bmatrix}$.

After some derivations, we can find that $\widetilde{V} = V + c\Omega$, where $V = \Omega + \Sigma_F^*$. The term $\sqrt{T}(\widehat{\lambda} - \lambda)$ can be represented as $\sqrt{T}(\widehat{\lambda} - \lambda) = (\widehat{X}'\widehat{X})^{-1}\widehat{X}'\sqrt{T}(\overline{R} - E(R_t)) - (\widehat{X}'\widehat{X})^{-1}\widehat{X}'\sqrt{T}(\widehat{B} - B)\lambda_F$. Therefore, V is the asymptotic covariance matrix of the first part and $c\Omega$ is the asymptotic covariance matrix of the second part. [1]

1.2.3. FAR statistic

Kleibergen (2009) argues that when the elements of B are small or zero, $\widehat{\lambda}_F$ is not $\sqrt{T}$ consistent and normally distributed. Instead, it converges to a random vector when the model is correct and diverges when the model is misspecified. The t-statistic is not applicable under these circumstances. Kleibergen derived the statistic based on GMM and the instrumental variable statistics. This statistic remains unaltered asymptotically when the factors are nearly useless or the betas are low.

Assume the asset-pricing conditions [equation (1.2)] hold and the returns and factors are generated from the following regressions:

$$R_t = \lambda_0 \iota_N + B(f_t - \mu_F + \lambda_F) + \epsilon_t \tag{1.13}$$

$$f_t = \mu_F + v_t, \tag{1.14}$$

where the covariance of ϵ_t and v_t is zero.

[1] This result can be inferred from Shanken (1992) or Jagannathan and Wang (1998).

Let $\overline{f_t} = f_t - \overline{f}, \varepsilon_t = \epsilon_t + B\overline{v}$ and $\overline{v} = \frac{1}{T}\sum_{t=1}^{T} v_t$. After some transformation, equation (1.13) can also be written as

$$R_t = \lambda_0 \iota_N + B(\overline{f_t} + \lambda_F) + \varepsilon_t. \tag{1.15}$$

The asymptotic distributions of the statistics are derived under the following assumptions:

Assumption 3: *As the number of time series observations T becomes large,*

$$\frac{1}{\sqrt{T}}\sum_{t=1}^{T}\left(\begin{pmatrix} 1 \\ f_t \end{pmatrix} \otimes (R_t - \lambda_0\iota_N - B(\overline{f_t} + \lambda_F))\right) \xrightarrow{d} \begin{pmatrix} \varphi_R \\ \varphi_B \end{pmatrix}$$

with $(\varphi_R' \, \varphi_B')'\tilde{\ }N(0, \Psi)$, $\Psi = Q \otimes \Sigma$, *and* $Q = \begin{pmatrix} Q_{11} & Q_{1F} \\ Q_{F1} & Q_{FF} \end{pmatrix} = \begin{pmatrix} 1 & \mu_F' \\ \mu_F & \Sigma_F + \mu_F\mu_F' \end{pmatrix}$.

Let $\mathbb{R}_t$ be the excess returns through subtracting the $1st$ to $(n-1)^{th}$ returns by the n^{th} returns and $\mathbb{C}$ be the excess beta. Thus, λ_0 is removed from the model. Therefore, the following conditions hold

$$E(\mathbb{R}_t) = \mathbb{C}\lambda_F$$
$$cov(\mathbb{R}_t, f_t) = \mathbb{C}var(f_t)$$
$$E(f_t) = \mu_F.$$

When an asset pricing model is correctly specified, under $H_0 : \lambda_F = \lambda_{F,0}$, $\mathbb{C}$ can be estimated as $\widehat{\mathbb{C}} = \sum_{t=1}^{T}\mathbb{R}_t(\overline{f_t} + \lambda_{F,0})[\sum_{j=1}^{T}(\overline{f_j} + \lambda_{F,0})(\overline{f_j} + \lambda_{F,0})']^{-1}$.

Lemma 2 in Kleibergen (2009): *Under $H_0 : \lambda_F = \lambda_{F,0}$, and Assumption 3,*

$$\sqrt{T}\begin{pmatrix} \overline{\mathbb{R}} - \widehat{\mathbb{C}}\lambda_{F,0} \\ vec(\widehat{\mathbb{C}} - \mathbb{C}) \end{pmatrix} \xrightarrow{d} \begin{pmatrix} \Psi_{\mathbb{R}} \\ \Psi_{\mathbb{C}} \end{pmatrix},$$

where $\overline{\mathbb{R}}$ is the time series mean of $\mathbb{R}$. $\Psi_{\mathbb{R}}$ and $\Psi_{\mathbb{C}}$ are independent and asymptotically normally distributed random variables

$\Psi_{\mathbb{R}} \sim N(0, (1 - \lambda'_{F,0}(V_{FF} + \lambda_{F,0}\lambda'_{F,0})\lambda_{F,0}) \otimes \Sigma_{N-1}),$

$\Psi_{\mathbb{C}} \sim N(0, (V_{FF} + \lambda_{F,0}\lambda'_{F,0}) \otimes \Sigma_{N-1}),$

and $\Sigma_{N-1} = \Sigma_{11} - \iota_{N-1}\varpi_{1n} - \varpi_{n1}\iota'_{N-1} + \iota_{N-1}\varpi_{nn}\iota'_{N-1}$ *for* $\Sigma = \begin{pmatrix} \Sigma_{11} & \varpi_{1n} \\ \varpi_{n1} & \varpi_{nn} \end{pmatrix}$.

Therefore, under $H_0 : \lambda_F = \lambda_{F,0}$, the FAR statistic is

$$\text{(1.16)}\quad \text{FAR}(\lambda_{F,0}) = \frac{T}{1 - \lambda'_{F,0}\widehat{Q}(\lambda_F)^{-1}_{FF}\lambda_{F,0}}(\overline{\mathbb{R}} - \widehat{\mathbb{C}}\lambda_{F,0})'\widetilde{\Sigma}^{-1}(\overline{\mathbb{R}} - \widehat{\mathbb{C}}\lambda_{F,0}) \xrightarrow{d} \chi^2(N-1),$$

where $\widehat{Q}(\lambda_{F,0}) = \frac{1}{T}\sum_{t=1}^{T}(\overline{f_t} + \lambda_{F,0})(\overline{f_t} + \lambda_{F,0})'$ and

$\widetilde{\Sigma} = \frac{1}{T-k}\sum_{t=1}^{T}(\overline{\mathbb{R}} - \widehat{\mathbb{C}}(\overline{f_t} + \lambda_{F,0}))(\overline{\mathbb{R}} - \widehat{\mathbb{C}}(\overline{f_t} + \lambda_{F,0}))'.$

This test is similar to the Anderson-Rubin (1949) statistic in the instrumental variables regression model and is proportional to the square of the Hansan-Jagannathan (1997) distance when it is evaluated at $\lambda_{F,0}$.

1.3. Properties of the estimators with a low-variance factor

When the variances of all the factors included in a linear asset pricing model are large, the Fama-Macbeth two-pass estimator has an asymptotic normal distribution and the t-statistic performs adequately. In this section, we show that if there is a factor with a low variance which leads to a low SNR in a linear asset pricing model, the betas associated with the low-variance factor are not trustworthy. All estimated risk premiums and the zero-beta rate are unreliable. In particular, the estimated beta associated with the low-variance factor in the first-pass regression is inconsistent. Its asymptotic variance increases as the SNR decreases. The other estimated betas have the same distributions as there is no low-variance factor in the model. The estimated parameters in the second-pass regression are inconsistent. They are not normally distributed as usual. Instead, they have non-standard distributions.

For illustrative purposes, we derive the asymptotic distribution of the $\widehat{B}$ and the $\widehat{\lambda}$ in the second pass when the model includes only one low-variance factor.

Let $F = [Z \ f_K]$ and $B = [\Gamma \ b]$, where $Z = [f_1 ... f_{K-1}]$ are $K-1$ factors with large variances, f_K is a factor with a small variance, $\Gamma = [\gamma_1, \gamma_2, ..., \gamma_N]'$ and γ_i for $i = 1, ..., N$ is a $K-1$ by 1 vector. Therefore, the specification of the first-step regression can be written as[2]

$$R_i = \beta_{0i}\iota_T + Z\gamma_i + f_K b_i + \epsilon_i. \tag{1.17}$$

If the model is correctly specified, the cross-sectional relation of the returns and the betas is the following:

$$E(R_t) = \lambda_0\iota_N + \Gamma\lambda_1 + b\lambda_K. \tag{1.18}$$

It can also be written in a compact format, which is

$$E(R_t) = X\lambda, \tag{1.19}$$

where $X = [\iota_N \ \Gamma \ b]$ and $\lambda = [\lambda_0 \ \lambda_1' \ \lambda_K]'$.

Let $M_{f_{K*}} = I - f_{K*}(f_{K*}'f_{K*})^{-1}f_{K*}'$ and $M_{z*} = I - Z_*(Z_*'Z_*)^{-1}Z_*'$. Then, the OLS estimators of the risk parameters are given by

$$\widehat{\gamma}_i = (Z_*'M_{f_{K*}}Z_*)^{-1}Z_*'M_{f_{K*}}R_{i*} \tag{1.20}$$

[2] A more general case is that $E(R) = \lambda_0\iota_N + y\theta + \Gamma\lambda_1 + b\lambda_2$, where y is some variable representing firm characteristics which some researchers in empirical analysis use to test the validity of an asset pricing model. If the risk parameters are sufficient to explain the cross-sectional variation of asset returns, in other words, the cross-sectional relation of the expected returns and the risk parameters are correct, θ is zero. The student t-statistic is used to test whether θ is zero. An insignificant t-statistic indicates that the proposed asset pricing model is valid. Jagannathan and Wang (1998) provide an econometric analysis of the two-step regression when the second step includes a variable representing firm characteristic. For example, Fama and French (1992) use the variable — firm size — to test whether it is able to explain the variation of the cross section of returns besides the market risk parameter.

$$\widehat{b}_i = (f'_{K*} M_{Z_*} f_{K*})^{-1} f'_{K*} M_{Z_*} R_{i*}. \tag{1.21}$$

Assumption 4: *The error term ϵ_{it} in the first-step regression for each asset i at each time t is independent and identically distributed across time and across assets, conditional on F, with $E(\epsilon_{it}|F) = 0$ and $Var(\epsilon_{it}|F) = \sigma^2$.*

The advantage of the asymptotic analysis over the finite sample analysis is that we do not need to assume the error terms follow specific distributions. In order to focus on the effect of the small SNR on the estimating and testing asset pricing models, we do not explicitely allow for heteroskedasticity and autocorrelation in our analysis in this part. However, the properties of $\widehat{\lambda}$ under cross-sectional heteroskedasticity and autocorrelation can be derived by following a similar procedure in deriving the properties of the OLS estimator in models with iid errors.

Assumption 5: *The SNR is $\tau = \frac{\sigma_{f_K}}{\sigma} = \frac{a}{\sqrt{T}}$, where σ_{f_K} is the standard deviation of the factor f_K and $a > 0$ is some fixed constant.*

Assumption 6: *The factors Z are generated by a stationary and ergodic process with $p\lim_{T\to\infty} \frac{Z'_* Z_*}{T} = \Sigma_Z$ a positive definite matrix* and *$p\lim_{T\to\infty} f'_{K*} f_{K*} = a^2\sigma^2$.*

In Assumption 5, we parameterize the SNR as local-to-zero. The parameterization provides a convenient tool for analyzing the large sample properties of the estimator, when the variance of a factor in a linear asset pricing model is very small. This implies that the variance of the factor is a small fraction of the variance of the noise component $\sigma^2_{f_K} = \frac{a^2\sigma^2}{T}$. The normalization factor $T^{1/2}$ is chosen to match the rate of convergence of the estimated σ_{f_K} from the regression $f_{Kt} = \mu_{f_K} + \sigma_{f_K} \sum_{j=0}^{\infty} \theta_j \nu_{t-j}$. [3]. The SNR parameter a controls the departures of the σ_{fK} from zero.

[3] Gospodinov (2009) applies the local-to-zero parameterization in explaining the forward premium puzzle. Torous and Valkanov (2000) implement a similar method in analyzing the properties of predictive regressions when the signal-to-noise ratio is low.

The following theorem characterizes the properties of the estimated betas under the previous three assumptions.

Theorem 1. Under Assumptions 4, 5 and 6, we have

(i) $\sqrt{T}(\widehat{\gamma}_i - \gamma_i) \xrightarrow{d} N(0, (\Sigma_z - \varpi_{zf_K}\varpi'_{zf_K})^{-1}\sigma^2)$,

where $\varpi_{zf_K} = [\rho_{f_1 f_K}\sigma_{f_1} \ \rho_{f_2 f_K}\sigma_{f_2} \ ... \ \rho_{f_{K-1} f_K}\sigma_{f_{K-1}}]'$ *and* ρ *is the correlation coefficient.*

(ii) $\widehat{b}_i - b_i \xrightarrow{d} N(0, \frac{1}{a^2(1-\varpi'_{zf_K}\Sigma_z^{-1}\varpi_{zf_K})})$.

Proof. See Appendix. □

Theorem 1 shows that under Assumptions 4, 5 and 6, only the estimated betas associated with the low-variance factors are affected by the low variance. In particular, Condition (i) demonstrates that the estimated betas associated with high-variance factors are consistent and converge to the same normal distribution at rate $\sqrt{T}$, as those when there is no factor with a low variance in the regression. This implies that the properties of the risk parameters, B, associated with the large-variance factors are still standard. Condition (ii) shows that the estimated risk parameter corresponding to the factor with a low variance is no longer consistent. While, it converges to a normally distributed random variable, with the asymptotic mean being the true b. The variance of the $\widehat{b}$ does not decrease as T increases, and $\widehat{b}$ is not consistently estimable. Furthermore, the dispersion of the estimates increases as a decreases, since the asymptotic variance of this parameter is inversely related to a.

In summary, this theorem shows that the estimated betas with high-variance factors through the Fama-Macbeth procedure are still valid. However, the estimated parameter associated with a low-variance factor is not consistently estimable and highly volatile as the SNR is close to zero.

Given the results in Theorem 1, we can derive the asymptotic distribution of the estimated parameters in the second pass. From equation (1.17), (1.18) and (1.19), we can obtain

$$\overline{R} = \widehat{X}\lambda + B(\overline{f} - \mu_F) + \overline{\epsilon} - (\widehat{\Gamma} - \Gamma)\lambda_1 - (\widehat{b} - b)\lambda_K. \tag{1.22}$$

Let $\widehat{A} = (\widehat{X}'\widehat{X})^{-1}\widehat{X}'$. Then, the estimated parameters in the second pass can be expressed as follows:

$$\widehat{\lambda} = \lambda + \widehat{A}B(\underset{(1)}{\overline{f}} - \mu_F) + \underset{(2)}{\widehat{A}\overline{\epsilon}} - \widehat{A}(\underset{(3)}{\widehat{\Gamma} - \Gamma})\lambda_1 - \widehat{A}(\underset{(4)}{\widehat{b} - b})\lambda_K, \tag{1.23}$$

where $\widehat{X} = [\iota_N, \widehat{\Gamma}, \widehat{b}]$. Since $\widehat{b} - b = O_p(1)$ from Theorem 1, all parts except the last part of $\widehat{\lambda}$ in equation (1.23) converge in probability to zero. Therefore, the large sample distribution of $\widehat{\lambda}$ depends only on $\lambda - \widehat{A}(\widehat{b} - b)\lambda_K$. We can further decompose part (4) in equation (1.23) into three parts $\widehat{\eta}_0$, $\widehat{\eta}_1$, $\widehat{\eta}_K$, corresponding respectively to $\widehat{\lambda}_0$, $\widehat{\lambda}_1$ and $\widehat{\lambda}_K$. Let $\widehat{D} = [\iota_N, \widehat{\Gamma}]$, $M_{\widehat{D}} = I - \widehat{D}(\widehat{D}'\widehat{D})^{-1}\widehat{D}'$ and $M_{\widehat{b}} = I - \widehat{b}(\widehat{b}'\widehat{b})^{-1}\widehat{b}'$. Then

$$\widehat{\eta}_0 = (\iota_N' M_{\widehat{b}}\iota_N \widehat{\Gamma}' M_{\widehat{b}}\widehat{\Gamma} - \iota_N M_{\widehat{b}}\widehat{\Gamma}\widehat{\Gamma}' M_{\widehat{b}}\iota_N)^{-1}(\widehat{\Gamma}' M_{\widehat{b}}\widehat{\Gamma}\iota_N M_{\widehat{b}} b - \iota_N M_{\widehat{b}}\widehat{\Gamma}\widehat{\Gamma}' M_{\widehat{b}} b)\lambda_K \tag{1.24}$$

$$\widehat{\eta}_1 = (\iota_N M_{\widehat{b}}\iota_N \widehat{\Gamma}' M_{\widehat{b}}\widehat{\Gamma} - \iota_N M_{\widehat{b}}\widehat{\Gamma}\widehat{\Gamma}' M_{\widehat{b}}\iota_N)^{-1}(\iota_N M_{\widehat{b}}\iota_N \widehat{\Gamma}' M_{\widehat{b}} b - \widehat{\Gamma}' M_{\widehat{b}}\iota_N \widehat{\Gamma}' M_{\widehat{b}}\iota_N b)\lambda_K \tag{1.25}$$

$$\widehat{\eta}_K = \lambda_K - (\widehat{b}' M_{\widehat{D}}\widehat{b})^{-1}\widehat{b}' M_{\widehat{D}} b\lambda_K. \tag{1.26}$$

Given the asymptotic distributions of the estimated betas, the limiting representation of $[\widehat{\lambda}_0, \widehat{\lambda}_1', \widehat{\lambda}_K]'$ has the form

$$D_0(z_b) = \lambda_0 - C_1\lambda_K \tag{1.27}$$

$$D_1(z_b) = \lambda_1 - C_2\lambda_K \tag{1.28}$$

$$D_2(z_b) = \lambda_K - \lambda_K + C_3\lambda_K. \tag{1.29}$$

where $z_b \,\tilde{}\, N(ab, \frac{1}{(1-\varpi_{zf_K}'\Sigma_z^{-1}\varpi_{zf_K})}I)$,

$M_{z_b} = I - z_b(z_b'z_b)^{-1}z_b$ and $M_D = I - D(D'D)^{-1}D'$,

$C_1 = (\iota_N' M_{z_b}\iota_N\Gamma' M_{z_b}\Gamma - \iota_N M_{z_b}\Gamma\Gamma' M_{z_b}\iota_N)^{-1}(\Gamma' M_{z_b}\Gamma\iota_N M_{z_b}b - \iota_N M_{z_b}\Gamma\Gamma' M_{z_b}b)$,

$C_2 = (\iota_N M_{z_b}\iota_N\Gamma' M_{z_b}\Gamma - \iota_N M_{z_b}\Gamma\Gamma' M_{z_b}\iota_N)^{-1}(\iota_N M_{z_b}\iota_N\Gamma' M_{z_b}b - \Gamma' M_{z_b}\iota_N\Gamma' M_{z_b}\iota_N b)$,

$C_3 = a(z_b' M_D z_b)^{-1}z_b' M_D b$.

The following theorem establishes the asymptotic properties of the estimated zero-beta rate and the risk premiums in the second pass under Assumptions 4, 5 and 6.

Theorem 2. Given Assumptions 4, 5 and 6, $(\widehat{\lambda} - \lambda) = O_p(1)$ and each component is asymptotically non-degenerate.

Proof. See Appendix. □

Theorem 2 means that all estimated risk premiums and the estimated zero-beta rate are inconsistent and do not converge to their true values as the sample size increases. Compared with $\widehat{b}$, $\widehat{\lambda}$ converges to a non-standardly distributed random vector. Therefore, their properties are very difficult to analyze. This implies that standard inference on the estimated risk premiums and the zero-beta rate can be highly unreliable. Furthermore, it is noteworthy that a low-variance factor not only renders its own estimated risk premium to be inconsistent, but it also contaminates the properties of all the other parameters appearing in the second pass regression.[4] Theorem 2 also implies that the t-statistic and F-statistic may be inappropriate to test whether a particular factor or a group of factors is priced or not when the linear asset pricing model includes a low-variance factor.[5]

Even though the asymptotic distribution of $\widehat{\lambda}$ is non-standard, the inspection of the biases and standard errors reveals some useful information. From equation (1.27), (1.28), and (1.29), we observe that the asymptotic biases of $[\widehat{\lambda}_0, \widehat{\lambda}_1', \widehat{\lambda}_K]'$ are equal to $E(D(z_b)) - \lambda = [E(D_0(z_b)) - \lambda_0, E(D_1(z_b)') - \lambda_1', E(D_2(z_b)) - \lambda_K]$. These expressions suggest that the magnitude of the biases depends on the value of λ_K given a certain realization of z_b. The larger the λ_K, the larger the absolute value of the asymptotic bias. Since the distribution of z_b is a function of a, which controls the the SNR, we expect the bias of $\widehat{\lambda}_K$ also to depend on a.

We expect that the Fama-Macbeth standard errors and even the EIV-adjusted standard errors are not applicable in a large sample when the variance of a factor is small. From the previous analysis we know that the limiting distribution of $\widehat{\lambda}$ does not depend on the distribution of either of the factors, the error terms ϵ or the distribution of the betas associated with high-variance factors. It only depends on the distribution of the estimated beta with the low-variance factor and the true values of the risk premiums. Therefore, the usual Fama-Macbeth variance $\widehat{V}$ is not informative at all because it only accounts for the covariance of the first two parts in equation (1.23). Though Shanken's adjusted variance accounts for the variance of the last two

[4]If the second pass regression includes the firm characteristic factor y, the estimated parameter associated with y also becomes inconsistent and has an asymptotic non-standard distribution. For researchers who are interested in whether y is able explain the cross-sectional variation of the returns, their decisions based on the t-statistic may not be reliable.

[5]The t-statistic has been shown to be invalid in testing linear asset pricing models with a useless factor. Kan & Zhang (1999) argue that the t-statistic is not able to distinguish a useful factor and a useless factor. Kleibergen (2009) demonstrates that the t-statistic is not appropriate when the betas are low and the number of assets are large.

parts, it is not applicable either. The reason is that it is based on a consistent and asymptotically normally distributed $\widehat{b}$. In practice, even when the variance of the factor is not extremely small, these results give us an idea that as the variance of the factor becomes smaller, the distribution of the estimated risk premiums and that of the zero-beta rate is driven more by the distribution of the estimated beta associated with the low-variance factor and its true risk premium, than those with high variance factors. Furthermore, if λ_K is big, the absolute values of the last part of $D_0(z_b)$ and $D_1(z_b)$ are large. Therefore, the asymptotic variances for the market factor and the zero-beta rate are large. However, the number of assets has an opposite effect on the variance of $\widehat{\lambda}$. The reason for this is that $\widehat{\eta}_0$, $\widehat{\eta}_1$ and $\widehat{\eta}_K - 1$ can be regarded as OLS estimators of a regression of b on $\widehat{X}$ multiplied by λ_K. As we know, as the sample size increases, the variance of an OLS estimator decreases. Therefore, as N increases, the variance of $\widehat{\lambda}$ decreases.

Therefore, the t-statistic and the F-statistic under the Fama-Macbeth standard errors and the EIV-adjusted standard errors may be highly misleading. As can be inferred from Theorem 1, if the variance of the factor is very small, the estimated risk parameters are very imprecise, even when the sample size is large. The t- and F-statistics using the Fama-Macbeth estimated variance, which do not account for the estimation of the risk parameters can lead to a large size distortion due to the high imprecision of the estimated betas. These statistics under Shanken's correction are possibly less size distorted since this correction accounts for the estimation error of the betas. However, since the estimated risk parameters associated with the low variance factor are inconsistent, the benefits of Shanken's correction are limited. In comparison, the limiting distributions of Kleibergen's statistics are not affected by the magnitude of betas and are expected to provide a better approximation in this case.

1.4. Simulation

To evaluate the finite sample properties of the Fama-Macbeth two-step procedure, the t- and F-statistics and Kleibergen's FAR statistic when the variance of a factor is low, we provide a Monte Carlo simulation. First, we simulate a two-factor model using a similar setup as that in our analytical part. After that, we generalize the simulation to a more realistic two-factor model. The data we use in this section are based on the Fama-French 10 portfolios formed on momentum, the Fama-French 25 portfolios formed on size and book-to-market, the value-weighted returns on all NYSE and AMEX stocks (the market factor) and percentage change of US per capita labor income. All these data are from July 1963 to December 1990.

1.4.1. Simulation I

Let R_{it} be the return of the i^{th} asset at time t. The data generating process is:

$$R_{it} = \lambda_0 + \beta_i'(f_t - \mu_F + \lambda_F) + \epsilon_{it} \text{ for } t = 1, ..., T \text{ and } i = 1, ..., N$$

$$f_t = \mu_F + v_t,$$

where $v_t \tilde{} N(0, \Sigma_F)$, $\beta_i = [\gamma_i, b_i]'$, $f_t = [f_{1t}, f_{Kt}]$. The parameters γ_i are set to be the estimated betas corresponding to the market factor from a regression of asset returns on the market factor and the labor factor. b_i is generated as five times the estimated beta associated with the labor factor from the regression. Before multiplying by 5, we modify the magnitude of beta to be 0.2 if its absolute value is less than 0.2.[6] By adopting this transformation, we avoid the low beta problem. The error term ϵ_{it} is drawn from a normal distribution with mean zero and variance $\sigma^2 = 10$, which is close to the average of the diagonal of the covariance matrix of the error term in the regression with actual data. The factors f_t are generated from a multivariate normal

[6] When N=10, no modification is needed for the betas. When N=25, there are only few betas less than 0.2.

distribution with $\mu_F = [0.447, 0.571]'$[7], where 0.571 is the mean of the labor factor. ${\sigma_f}_1^2 = 29.8$ is the variance of the excess value-weighted return, including all NYSE, AMEX, and NASDAQ stocks from July 1926 to March 2008. ${\sigma_f}_K^2$ is set to $\frac{a^2\sigma^2}{T}$ as in Assumption 6. $\rho = -0.1$ is the correlation covariance of the market factor and the labor factor. The zero-beta rate is $\lambda_0 = 0.1$ and the true risk premiums are $\lambda_F = \mu_F$.

In order to explore the effect of the magnitude of the SNR on the properties of the estimators, we vary the sample size by letting $T = 160, 330, 640$ and $a = 0.3, 1, 2$. We also vary the number of assets with $N = 10, 25$. In the end, we calculate the empirical size of the t-, F- and FAR statistics. The nominal sizes are set to be 5% for all the statistics. The results are based on 5,000 Monte Carlo replications. Note that the beta is fixed for all scenarios under a fixed N, but it is different when N is different.

1.4.1.1. Results. Table 1.1 shows the SNR and the variance of the labor factor in each scenario. The true variance of the labor factor is 0.118 when $T = 330$. Therefore, the case in which $T = 330$ and $a = 2$, is calibrated to match the true data. The largest variance of the low-variance factor, we considered in this chapter, is 0.250. It arises in the case when $T = 160$ and $a = 2$. The lowest variance is 0.001 when $T = 640$ and $a = 0.3$. Because we assume the variance of the error term is 10, the largest SNR in our case is 0.158 and the lowest one is 0.012. When $T = 330$ and $a = 2$, the SNR is 0.110.

In Table 1.2, we summarize the results of the mean bias, median bias and variance of the average of the estimated betas.[8] The true average of the betas are $[1.113, 4.164]$ for $N = 10$ and $[1.133, 2.528]$ for $N = 25$. From this table, we can observe that the

[7] 0.6667 is the historical monthly market risk premium since the historical annulized market risk premium is 8%. We use 0.4467 here in order to be able to compare the results from the more realistic simulation setup later.

[8] In this table, we do not report the statistics of each of the N estimated betas but only those of the average of the N estimated betas. However, in our opinion, the distribution of each of the N betas may affect the properties of the estimated parameters in the second pass.

mean bias and median bias of the average market beta and the labor beta are very small, especially for the market beta. That is to say the mean and median for both the market factor and the labor factor are very close to their true values. Further, the variance of the market beta is tiny. It also decreases as T increases and is nearly constant as a changes. In comparison, the variance of the labor beta is much larger. It remains almost constant as T increases but becomes larger as a decreases. These observations are compatible with Theorem 1. The reason, why the variance is smaller for $N = 25$ than for $N = 10$, is because we report the variance of the average estimated betas and $\overline{\widehat{B}} = \frac{1}{N}\sum_{i=1}^{N}\widehat{\beta}_i \xrightarrow{d} N(\frac{1}{N}\sum_{i=1}^{N}\beta_i, \frac{1}{N}var(\widehat{\beta}_i))$. In addition, the properties of the estimated betas related to the sample size also imply that dependence on part (4) in equation (1.23) becomes stronger as the sample size increases.

Table 1.3 presents the percentage biases of the estimated risk premiums and the zero-beta rate. This shows that the biases increase as a decreases. From our analysis in the previous section, we know that the asymptotic biases of the estimated parameters only depend on the distribution of z_b. Note that a only appears in the mean of the distribution. Therefore, as a decreases, the mean of z_b decreases. This implies that in equations (1.27) and (1.28), when the mean of z_b decreases, the magnitude of $E(D_0(z_b)) - \lambda_0$ and $E(D_1(z_b)') - \lambda_1'$ increases. Since the asymptotic bias of the labor factor equals $E(D_2(z_b)) - \lambda_K = E(a(z_b' M_D z_b)^{-1} z_b' M_D b) - \lambda_K$, the increase of the magnitude of the bias may due to the joint effect of the decrease of a and the decrease of the mean of z_b and a.

In addition, the magnitude of the percentage biases for the market factor and the zero-beta rate is much larger than those for the labor factor. In other words, if an asset pricing model includes a low-variance factor and the beta associated with the low-variance factor is large, not only the estimate for the low-variance factor is poorly behaved, the estimates for the high-variance factors are even more strongly affected.

The simulation results show that $|E(C_2)| > |E(C_1)| > |1 - E(C_3)|$. Furthermore, the percentage bias of the estimated parameters depends also on the true value of the lambdas. The smaller the $\frac{\lambda_K}{\lambda_1}$ and $\frac{\lambda_K}{\lambda_0}$, the smaller the percentage biases of the market factor and the zero-beta rate. It is also worth noticing that under our setup, the market factor has an upward bias; the zero-beta rate has a downward bias; and the labor factor is biased toward zero. It is in agreement with our conjecture that the $E(C_1\lambda_K)$ is positive and $E(C_2\lambda_K)$ is negative and $E(C_3\lambda_K)$ is greater than zero but less than one. For $N = 25$, the properties of the estimated parameters are similar to those when $N = 10$.

Table 1.4 provides the sample standard deviations, Fama-Macbeth standard errors and Shanken's EIV-adjusted standard errors in each scenario. We observe that the difference between the Fama-Macbeth standard errors and the other two types of standard errors is pretty large. This further explains that the last two terms in equation (1.23) play an important role in determining the properties of $\widehat{\lambda}$. Furthermore, the Fama-Macbeth standard errors for all the estimated parameters decrease as the variance of the factor decreases, regardless of the reduction of a or the growth of T. The sample standard errors and Shanken's EIV-adjusted standard errors tend to increase and then decrease when a gets smaller. Moreover, all standard deviations of the estimated parameters tend to decrease as T increases. This implies that the estimated coefficients become more and more concentrated on some particular values, as the sample size increases. However, based on our previous analysis, these values may not be the same as the true values of the risk premiums and the zero-beta rate. In addition, the fact that the standard errors decrease when N increases may partly be due to more information included in estimation of $\widehat{\lambda}$.

In Table 1.5, we present the means, medians and confidence intervals of the estimated parameters in the second pass. As we have metioned before, the true values of the parameters in the second pass are $\lambda = [0.1, 0.447, 0.571]$. By presenting these results, we would like to emphasize the poor behavior of the estimated risk premiums and the zero-beta rate. For all estimated parameters, the mean and median differ from their true values, especially for the market factor and the zero-beta rate. The difference increases as the variance of the labor factor decreases. At the same time, the confidence intervals shift away from the true values, as a decreases. In the end, when the variance of the labor factor is very small, the confidence intervals fail to include the true values. This indicates that $\widehat{\lambda}$ are not informative about the true values of the risk premiums and zero beta rate when the variance of the factor is very small. It also conforms to our expectations that the estimated premiums center on values which are different from their true values.

Table 1.6 summarizes the empirical size of the t-statistic with Fama-Macbeth variance and t-statistic with Shanken's adjusted variance. For all parameters, both the t-statistic and the EIV-adujsted t increase as a decreases and as the sample size increases. The reason for this is that the bias increases faster when a decreases. For example, when $T = 330$ and a decreases from 2 to 1, the percentage changes of the biases for all estimated parameters are around 300%. Meanwhile, it is only around 30% for the EIV-adjusted standard errors. Further, when the variance of the labor factor is smaller than or equal to 0.063, even the EIV-adjusted t-statistic has a size distortion which is more than double of its true size. When $a = 0.3$, $T = 640$ and the variance of the labor factor is 0.001, the empirical size of both t-statistic is very close to one. It is also noteworthy to mention that the empirical size of the t-statistic for the labor factor tends to be larger than those for the other two variables, even though the other two variables have large percentage biases. This indicates that it is more likely for researchers to find the a low variance factor be priced even though it is not. In addition, the over-rejections of the t-statistic also tend to be larger when $N = 25$ compared to the case when $N = 10$.[9]

In Table 1.7, we compare the results of the F-statistic based on the Fama-Macbeth variance, the F based on the EIV-adjusted variance and Kleibergen's FAR statistic. All of these statistics jointly test the null that the risk premiums equal to their true values under each scenario. Our results show that the performances of both F-statistic are similar to those of the t-statistic. The empirical sizes are much larger than their true sizes. Meanwhile, the FAR statistic performs more adequately. However, the FAR tends to under-reject the null when the variance of the factor is small.

1.4.2. More Realistic Simulations

In this part, we investigate the properties of the Fama-Macbeth two-pass regression under more realistic simulation setups. We consider the scenarios of $T = 160, 330$ and $N = 10, 25$. We employ a two-factor model. The setup is similar to the previous simulation setup. However, there are a few modifications. Here, we use the true market factor and the labor factor instead of generating the factors by matching their mean and variances. When $T = 160$, the variance of the labor factor is modified by multiplying the variance of labor for $T = 330$ by $\frac{\sqrt{330}}{\sqrt{160}}$. This modification allows us to compare the results with the previous section. The error terms are simulated by boostrapping the error terms from the regression of the 10 portfolios on the factors and of the 25 portfolios on the factors. By implementing the bootstrap method, we avoid assuming any specific distributions for the returns. In addition, the betas are the estimated betas without multiplying by 5.

[9] Even though, in our opinion, it is not appropriate to compare the magnitude of the bias and variance of the estimated parameters for different number of assets, we think the comparison of the t-statistic across assets make sense.

In Table 1.8, we present the mean biases, median biases and sample variances for the estimated betas; means, medians and confidence intervals for the estimated lambdas; biases and standard errors of the estimated lambdas. The true beta is $[1.113, 0.833]$ when $N = 10$ and $[1.133, 0.506]$ when $N = 25$. The true variance of the labor factor is 0.118 when $T = 330$ and 0.244 when $T = 160$. These scenarios are pretty similar to those when $a = 2$ in the previous subsection. However, here a is not 2 anymore since the variance of the error term ranges from 5.695 to 26.985 for $N = 10$ and from 1.518 to 23.826 for $N = 25$. Furthermore, the betas for the labor factor is much smaller here than those in the previous simulation.

In general, the results are very similar under this setup to those under the previous setup. We only point out some interesting differences in the magnitude of the various statistics. First, the variances of the average of betas for the labor factor are larger than those in the case when $a = 2$ but smaller than those when $a = 1$ in the previous subsection. Compared to the cases of $T = 160$ and $T = 330$ when $a = 2$ in the previous setup, the percentage biases of the estimated lambdas here are larger, especially for the labor factor. One possible reason is that the variances for the estimated betas are larger than those in the simple simulation part. It thus results in a small SNR or a small a given a fixed T. As we have discussed in the previous subsection, the smaller the a, the larger the bias. The increase of the percentage bias of the estimated labor premium may also be due to the fact that b is smaller than that in the previous cases. The smaller b leads to a larger magnitude of the term $E(D_2(z_b)) - \lambda_K$. At the same time, the standard errors of the estimated market risk premium and the estimated zero-beta rate are smaller than those reported in the previous section. Therefore, the confidence intervals for these two parameters are tighter than those under the previous setup. This means that as the sample size increases and the variance of the factor decreases, the estimated market risk premium and the zero-beta rate concentrate on values which are different from their true values.

Table 1.9 shows the Fama-Macbeth t- and F-statistics, the EIV-adjusted t- and F-statistics and the Kleibergen's FAR statistic. Compared to the results when $a = 2$, all the t- and F-statistics have much larger size distortions, except for the zero-beta rate and the market factor when $T = 160$ and $N = 10$. In particular, the size distortion for the labor factor is very large. Therefore, in practice, when the SNR is very low, researchers should be aware of the problem of the over-rejection of the t-statistic for all the parameters included in the second pass regression. As compared to the F-statistic, the FAR statistic has a size close to the nominal size and it is recommended that this statistic be used in applied work when factors with low variance are included in the model.

1.5. Conclusion

In this chapter, we provide analytical and simulation results on the properties of the Fama-Macbeth two-pass procedure in estimating and testing an asset pricing model when the model includes a low-variance risk factor. We derive the large sample distribution of the estimated parameters in the two-pass regression assuming the error term in the first pass is independent and identically distributed both across time and across assets. Our results show that the estimated betas associated with high-variance factors in the first pass have asymptotic distributions that are the same as those when there is no low-variance factor in the model. All estimated parameters in the second pass are inconsistent and have non-standard asymptotic distributions. Thus, the standard t- and F-statistics, which rely on consistent and normally distributed estimates, become invalid in this case. Furthermore, the asymptotic bias of these parameters depends on the true values of the betas, the risk premium associated with the low-variance factor, the SNR parameter and the distribution of the estimated beta corresponding to the low-variance factor.

Furthermore, we investigate the finite sample properties of the estimated parameters using simulation. We provide results of a two-factor model with one low-variance factor in both a simple simulation and a more realistic simulation. Our results show that the finite-sample behavior of the estimated parameters is close to the large-sample approximations derived in the theoretical part. The smaller the variance of the low-variance factor for a fixed sample size, the larger the bias for the estimated second-pass parameters and the more severe the size distortions for the t- and the F-statistics. We also compare the behavior of F-statistic and Kleibergen's (2009) factor Anderson-Rubin (FAR) statistic for jointly testing whether the estimated coefficients in the second-pass regression equal their true values. We find that the performance of the F-statistic is very similar to that of the t-statistic and is characterized by large size distortions. By contrast, the FAR statistic has a small size distortion and appears to be well behaved in the presence of a low-variance factor.

1.6. Appendix

Proof of Theorem 1. By eliminating β_0 in 1.17, we obtain

$$R_{i*} = Z_*\gamma_i + f_{K*}b_i + \epsilon_{i*}. \tag{A.1}$$

Multiplying both sides of A.1 by $\mathrm{M}_{f_{K*}}$, we get

$$M_{f_{K*}}R_{i*} = M_{f_{K*}}Z_*\gamma_i + M_{f_{K*}}\epsilon_{i*}.$$

Therefore, $\widehat{\gamma}_i$ can be obtained as

$$\widehat{\gamma}_i = (Z_*'M_{f_{K*}}Z_*)^{-1}Z_*'M_{f_{K*}}\epsilon_{i*} + \gamma_i = (Z_*'M_{f_{K*}}Z_*)^{-1}Z_*'M_{f_{K*}}\epsilon_i + \gamma \tag{A.2}$$

or

$$\sqrt{T}(\widehat{\gamma}_i - \gamma_i) = (\frac{Z_*'M_{f_{K*}}Z_*}{T})^{-1}(\frac{Z_*'M_{f_{K*}}\epsilon_i}{\sqrt{T}}). \tag{A.3}$$

Note that $\underset{T\to\infty}{p\lim}\frac{Z_*'M_{f_{K*}}Z_*}{T} = \underset{T\to\infty}{p\lim}(\frac{Z_*'Z_*}{T} - \frac{Z_*'f_{K*}(f_{K*}'f_{K*})^{-1}f_{K*}'Z_*}{T}) = \Sigma_Z - \varpi_{Zf_K}\varpi_{Zf_K}'$

using $\underset{T\to\infty}{p\lim}\frac{Z_*'Z_*}{T} = \Sigma_Z$, $\underset{T\to\infty}{p\lim}(f_{K*}'f_{K*})^{-1} = (a^2\sigma^2)^{-1}$

and assuming that $\underset{T\to\infty}{p\lim}\frac{Z_*'f_{K*}}{\sqrt{T}} = \sqrt{T}cov(Z,f_K) = \varpi_{Zf_K}\sigma a$ exists.

Also, by the Central Limit Theorem,

$$\frac{Z_*'M_{f_{K*}}\epsilon_i}{\sqrt{T}} \xrightarrow{d} N(0, \underset{T\to\infty}{p\lim}\frac{Z_*'M_{f_{K*}}Z_*}{T}\sigma^2).$$

Therefore,

$$\sqrt{T}(\widehat{\gamma}_i - \gamma_i) \xrightarrow{d} N(0, (\Sigma_Z - \varpi_{Zf_K}\varpi_{Zf_K}')^{-1}\sigma^2),$$

assuming $\underset{T\to\infty}{p\lim}(\frac{Z_*' M_{f_{K*}} Z_*}{T})^{-1} = (\Sigma_Z - \varpi_{Zf_K}\varpi_{Zf_K}')^{-1}$ exists.

By multiplying both sides of A.1 by M_{Z_*}, we get

$$M_{Z_*}R_{i*} = M_{Z_*}f_{K*}b_i + M_{Z_*}\epsilon_{i*}.$$

Therefore

$$\widehat{b} = (f_{K*}' M_{Z_*} f_{K*})^{-1} f_{K*}' M_{Z_*}\epsilon_{i*} + b = (f_{K*}' M_{Z_*} f_{K*})_i^{-1} f_{K*}' M_{Z_*}\epsilon_i + b. \tag{A.4}$$

Note that,

$$\underset{T\to\infty}{p\lim} f_{K*}' M_{Z_*} f_{K*} = \underset{T\to\infty}{p\lim}(f_{K*}' f_{K*} - \frac{f_{K*}' Z_*}{\sqrt{T}}(\frac{Z_*' Z_*}{T})^{-1}\frac{Z_*' f_{K*}}{\sqrt{T}}) = \sigma^2 a^2 (1 - \varpi_{Zf_K}' \Sigma_Z^{-1}\varpi_{Zf_K})M,$$

assuming $\underset{T\to\infty}{p\lim}(f_{K*}' M_{Z_*} f_{K*})^{-1} = \frac{1}{\sigma^2 a^2 (1 - \varpi_{Zf_K}' \Sigma_Z^{-1}\varpi_{Zf_K})}$ exists,

and

$$f_{K*}' M_{Z_*}\epsilon_i \xrightarrow{d} N(0, \underset{T\to\infty}{p\lim} f_{K*}' M_{Z_*} f_{K*}\sigma^2) = N(0, \sigma^2 a^2 (1 - \varpi_{Zf_K}' \Sigma_Z^{-1}\varpi_{Zf_K}).$$

Therefore,

$$\widehat{b}_i - b_i = (f_{K*}' M_{Z_*} f_{K*})_i^{-1} f_{K*}' M_{Z_*}\epsilon_i \xrightarrow{d} N(0, \frac{1}{a^2 (1 - \varpi_{Zf_K}' \Sigma_Z^{-1}\varpi_{Zf_K})}).$$

$\square$

Proof of Theorem 2. From equation (2.3), the time series average of asset i is

$$\overline{R}^i = \beta_0 + \overline{f}'\beta_i + \overline{\epsilon}^i, \tag{A.5}$$

and the expectation of R_t for asset i is

$$E(R_t^i) = \beta_0 + \mu_F'\beta_i \tag{A.6}$$

From (A.5) and (A.6), by getting rid of β_0, we obtain

$$\overline{R}^i = E(R_t^i) + (\overline{f} - \mu_F)'\beta_i + \overline{\epsilon}^i \tag{A.7}$$

or

$$\overline{R}^i = \widehat{X}_i\lambda + \upsilon_i \tag{A.8}$$

The estimator of λ in equation (1.23) is obtained from this equation.

We can decompose part (4) in equation (1.23) as estimated parameters $\widehat{\eta}_{-K}$ and $\widehat{\eta}_K$ corresponding to $[\widehat{\lambda}_0, \widehat{\lambda}_1']'$ and $\widehat{\lambda}_K$

using the regression

$$(\widehat{b} - b)\lambda_K = \widehat{D}\eta_{-K} + \widehat{b}\eta_K + \upsilon$$

Following similar arguments as in the deviation of $[\widehat{\Gamma}, \widehat{b}]$, we obtain

$$\widehat{\eta}_{-K} = (\widehat{D}'M_{\widehat{b}}\widehat{D})^{-1}\widehat{D}'M_{\widehat{b}}(\widehat{b} - b)\lambda_K$$

and

$$\widehat{\eta}_K = (\widehat{b}'M_{\widehat{D}}\widehat{b})^{-1}\widehat{b}'M_{\widehat{D}}(\widehat{b} - b)\lambda_K$$

We have that

(1) $\xrightarrow{p} 0$ for $\overline{f} \xrightarrow{p} \mu_F$ and $(\widehat{X}'\widehat{X})^{-1}\widehat{X}'B = O_p(1)$

(3) $\xrightarrow{p} 0$ for $\overline{\epsilon} \xrightarrow{p} 0$ and $(\widehat{X}'\widehat{X})^{-1}\widehat{X}' = O_p(1)$

(4) $\xrightarrow{p} 0$ for $\widehat{\Gamma} \xrightarrow{p} \Gamma$ and $(\widehat{X}'\widehat{X})^{-1}\widehat{X}' = O_p(1)$.

Also

$$\widehat{\eta}_{-K} = [\widehat{D}'\widehat{D} - \widehat{D}'\widehat{b}(\widehat{b}'\widehat{b})^{-1}\widehat{b}'\widehat{D}]^{-1}[\widehat{D}'(\widehat{b} - b) - \widehat{D}'\widehat{b}(\widehat{b}'\widehat{b})^{-1}\widehat{b}'(\widehat{b} - b)]\lambda_K$$

$$= [\widehat{D}'\widehat{D} - \widehat{D}'\widehat{b}(\widehat{b}'\widehat{b})^{-1}\widehat{b}'\widehat{D}]^{-1}[\widehat{D}'\widehat{b}(\widehat{b}'\widehat{b})^{-1}\widehat{b}'b - \widehat{D}'b]\lambda_K = O_p(1)$$

$$\widehat{\eta}_K = [\widehat{b}'\widehat{b} - \widehat{b}'\widehat{D}(\widehat{D}'\widehat{D})^{-1}\widehat{D}'\widehat{b}]^{-1}[\widehat{b}'(\widehat{b} - b) - \widehat{b}'\widehat{D}(\widehat{D}'\widehat{D})^{-1}\widehat{D}'(\widehat{b} - b)]\lambda_K$$

$$= 1 + [\widehat{b}'\widehat{b} - \widehat{b}'\widehat{D}(\widehat{D}'\widehat{D})^{-1}\widehat{D}'\widehat{b}]^{-1}[\widehat{b}'\widehat{D}(\widehat{D}'\widehat{D})^{-1}\widehat{D}'b - \widehat{b}'b]\lambda_K = O_p(1)$$

for $\widehat{D} \xrightarrow{p} D$ and $\widehat{b} \xrightarrow{d} N(b, \frac{1}{a^2(1-\varpi'_{Zf_K}\Sigma_Z^{-1}\varpi_{Zf_K})})$.

Therefore,

$\widehat{\lambda} = O_p(1)$. $\square$

Table 1.1. Signal-to-noise ratios and variances of the labor factor

N	var_F2			SNR		
T	$a = 0.3$	$a = 1$	$a = 2$	$a = 0.3$	$a = 1$	$a = 2$
160	0.006	0.063	0.250	0.024	0.079	0.158
330	0.003	0.030	0.121	0.017	0.055	0.110
640	0.001	0.016	0.063	0.012	0.040	0.079

Notes: Var_F2 denotes the variance of the labor factor in the model. SNR represents the SNR. "a" is the localizing constant.

Table 1.2. Mean bias, median bias and variances of the average of the estimated betas

		$a = 0.3$		$a = 1$		$a = 2$	
T		market	labor	market	labor	market	labor
				$N = 10$			
160	mbias	0.000	0.009	-0.000	-0.003	-0.000	-0.004
	medbias	0.000	-0.000	0.000	-0.002	-0.000	-0.001
	var	0.000	1.145	0.000	0.102	0.000	0.026
330	mbias	0.000	0.015	-0.000	0.000	0.000	-0.001
	medbias	0.000	0.015	0.000	0.003	0.000	-0.004
	var	0.000	1.103	0.000	0.100	0.000	0.026
640	mbias	0.000	-0.017	0.000	0.002	0.000	-0.000
	medbias	0.000	0.006	-0.000	0.002	-0.000	-0.002
	var	0.000	1.107	0.000	0.103	0.000	0.026
				$N = 25$			
160	mbias	-0.000	-0.001	-0.000	0.003	-0.000	0.000
	medbias	-0.000	0.004	-0.000	0.005	-0.000	0.001
	var	0.000	0.458	0.000	0.040	0.000	0.010
330	mbias	-0.000	-0.006	-0.000	0.001	-0.000	-0.000
	medbias	0.000	-0.009	-0.000	0.002	-0.000	-0.000
	var	0.000	0.445	0.000	0.040	0.000	0.010
640	mbias	-0.000	-0.003	-0.000	0.002	0.000	-0.003
	medbias	-0.000	-0.001	0.000	0.001	0.000	-0.003
	var	0.000	0.464	0.000	0.039	0.000	0.011

Notes: The results in this table are for the cross-sectional average of the estimated betas. mbias represents the mean bias. medbias denotes the median bias. var means the variance.

Table 1.3. Percentage biases of the parameters in the second pass

N		T	zero-beta	market	labor
10	$a = 0.3$	160	-107.798	25.599	-0.821
		330	-128.636	30.008	-0.868
		640	-138.649	32.143	-0.888
	$a = 1$	160	-30.748	7.373	-0.243
		330	-46.308	10.807	-0.312
		640	-55.492	12.876	-0.359
	$a = 2$	160	-5.708	1.410	-0.055
		330	-11.532	2.733	-0.083
		640	-15.837	3.694	-0.105
25	$a = 0.3$	160	-47.830	11.413	-0.692
		330	-51.355	12.148	-0.698
		640	-53.577	12.591	-0.706
	$a = 1$	160	-9.205	2.242	-0.150
		330	-10.788	2.589	-0.156
		640	-11.854	2.801	-0.163
	$a = 2$	160	-1.597	0.413	-0.037
		330	-2.485	0.611	-0.041
		640	-2.975	0.721	-0.043

Notes: In this table, we summarize the percentage bias of the second-pass estimates.

Table 1.4. Sample standard deviations, Fama-Macbeth standard errors and Shanken's EIV-adjusted standard errors

	$a = 0.3$			$a = 1$			$a = 2$		
	zbeta	market	labor	zbeta	market	labor	zbeta	market	labor
				$N = 10, T = 160$					
SVAR	4.107	3.964	0.101	3.740	3.733	0.121	2.655	2.665	0.098
FM	1.168	1.190	0.027	1.555	1.614	0.055	1.639	1.706	0.069
EIV	3.765	3.588	0.087	3.620	3.642	0.122	2.620	2.671	0.099
				$N = 10, T = 330$					
SVAR	3.569	3.442	0.087	3.753	3.746	0.116	2.679	2.694	0.088
FM	0.877	0.884	0.019	1.258	1.293	0.041	1.383	1.424	0.049
EIV	3.180	3.025	0.069	3.640	3.648	0.116	2.690	2.722	0.090
				$N = 10, T = 640$					
SVAR	3.169	3.073	0.080	3.751	3.748	0.113	2.666	2.686	0.085
FM	0.656	0.658	0.014	0.992	1.015	0.031	1.100	1.129	0.037
EIV	2.655	2.523	0.057	3.550	3.554	0.110	2.671	2.697	0.087
				$N = 25, T = 160$					
SVAR	1.421	1.372	0.053	1.103	1.098	0.051	0.761	0.809	0.053
FM	0.428	0.580	0.015	0.473	0.619	0.029	0.480	0.625	0.046
EIV	1.237	1.206	0.039	1.067	1.094	0.053	0.729	0.813	0.053
				$N = 25, T = 330$					
SVAR	1.326	1.259	0.049	1.061	1.039	0.046	0.669	0.683	0.035
FM	0.308	0.411	0.010	0.345	0.442	0.018	0.350	0.447	0.025
EIV	1.163	1.100	0.036	1.041	1.025	0.047	0.664	0.696	0.036
				$N = 25, T = 640$					
SVAR	1.264	1.184	0.048	1.038	0.993	0.043	0.627	0.624	0.030
FM	0.224	0.297	0.007	0.251	0.320	0.012	0.257	0.325	0.015
EIV	1.113	1.034	0.034	1.013	0.978	0.045	0.626	0.630	0.030

Notes: zbeta denotes the zero-beta rate. SVAR means the sample variance. It is the variance of all the estimated parameters.

Table 1.5. Means, medians and confidence intervals of the estimated parameters in the second pass

		$a = 0.3$			$a = 1$			$a = 2$		
T		zbeta	market	labor	zbeta	market	labor	zbeta	market	labor
					$N = 10$					
160	mu	-10.680	11.882	0.102	-2.975	3.740	0.433	-0.471	1.076	0.540
	med	-10.730	11.916	0.102	-2.964	3.747	0.431	-0.455	1.099	0.538
	CI	-19.091	3.685	-0.101	-10.43	-3.625	0.199	-5.681	-4.253	0.348
		-2.459	19.872	0.301	4.376	11.155	0.671	4.901	6.455	0.741
330	mu	-12.764	13.851	0.075	-4.531	5.274	0.393	-1.053	1.668	0.524
	med	-12.795	13.903	0.078	-4.602	5.372	0.387	-1.083	1.726	0.522
	CI	-20.070	7.078	-0.110	-11.902	-2.510	0.181	-6.322	-3.727	0.358
		-5.721	20.928	0.242	3.236	12.472	0.639	4.280	6.969	0.704
640	mu	-13.765	14.805	0.064	-5.449	6.198	0.366	-1.484	2.097	0.511
	med	-13.891	14.910	0.067	-5.559	6.337	0.361	-1.591	2.236	0.508
	CI	-19.837	8.467	-0.107	-12.534	-1.583	0.159	-6.596	-3.507	0.354
		-7.307	20.787	0.213	2.391	13.355	0.605	4.051	7.258	0.693
					$N = 25$					
160	mu	-4.683	5.545	0.176	-0.821	1.448	0.485	-0.060	0.631	0.550
	med	-4.717	5.575	0.177	-0.856	1.472	0.482	-0.065	0.641	0.549
	CI	-7.379	2.822	0.068	-2.889	-0.808	0.394	-1.525	-0.974	0.451
		-1.815	8.136	0.278	1.461	3.553	0.595	1.451	2.174	0.655
330	mu	-5.036	5.873	0.172	-0.979	1.603	0.482	-0.149	0.720	0.548
	med	-5.105	5.935	0.172	-0.993	1.634	0.480	-0.147	0.731	0.547
	CI	-7.461	3.234	0.072	-3.062	-0.496	0.400	-1.483	-0.616	0.480
		-2.202	8.196	0.270	1.159	3.573	0.580	1.154	2.078	0.619
640	mu	-5.258	6.071	0.171	-1.085	1.698	0.478	-0.197	0.769	0.547
	med	-5.356	6.156	0.172	-1.125	1.742	0.475	-0.193	0.768	0.546
	CI	-7.516	3.537	0.075	-3.009	-0.353	0.404	-1.409	-0.467	0.490
		-2.537	8.141	0.261	1.086	3.514	0.569	1.063	1.956	0.609

Notes: zbeta denotes the zero-beta rate. mu represents the mean. med means the median. CI represents the conference interval. The true parameters are [0.1, 0.447, 0.571].

Table 1.6. Empirical size of the t-statistics

		$a = 0.3$			$a = 1$			$a = 2$		
T		zbeta	market	labor	zbeta	market	labor	zbeta	market	labor
					$N = 10$					
160	t	0.976	0.981	1.000	0.571	0.572	0.646	0.223	0.203	0.178
	EIV	0.827	0.866	0.961	0.166	0.189	0.281	0.033	0.031	0.055
330	t	0.995	0.998	0.999	0.771	0.775	0.831	0.355	0.349	0.345
	EIV	0.936	0.951	0.987	0.336	0.355	0.442	0.067	0.070	0.093
640	t	0.998	0.999	1.000	0.863	0.871	0.908	0.520	0.515	0.527
	EIV	0.964	0.971	0.993	0.454	0.472	0.555	0.114	0.120	0.152
					$N = 25$					
160	t	0.994	0.995	1.000	0.561	0.455	0.728	0.227	0.145	0.116
	EIV	0.910	0.943	0.999	0.173	0.175	0.422	0.069	0.049	0.071
330	t	0.997	0.999	1.000	0.694	0.636	0.885	0.335	0.231	0.260
	EIV	0.945	0.968	1.000	0.218	0.243	0.505	0.075	0.064	0.113
640	t	0.999	0.999	1.000	0.800	0.774	0.936	0.464	0.373	0.481
	EIV	0.959	0.980	1.000	0.265	0.301	0.563	0.089	0.090	0.151

Notes: zbeta means the zero-beta rate. t denotes the t-statistic with the Fama-Macbeth variance. EIV represents the t-statistic with Shanken's EIV-adjusted variance.

Table 1.7. F-statistic and FAR

N		10			25		
T		$a=0.3$	$a=1$	$a=2$	$a=0.3$	$a=1$	$a=2$
160	F	1.000	0.621	0.108	1.000	0.707	0.109
	EIV	0.940	0.119	0.010	1.000	0.309	0.041
	FAR	0.030	0.036	0.042	0.027	0.035	0.079
330	F	1.000	0.837	0.272	1.000	0.900	0.271
	EIV	0.974	0.251	0.020	0.999	0.404	0.068
	FAR	0.031	0.032	0.036	0.032	0.033	0.044
640	F	1.000	0.944	0.521	1.000	0.953	0.504
	EIV	0.986	0.392	0.058	1.000	0.448	0.095
	FAR	0.025	0.033	0.034	0.033	0.041	0.042

Notes: F denotes the F-statistic with the Fama-Macbeth variance. EIV represents the F-statistic with Shanken's EIV-adjusted variance. FAR is Kleibergen's FAR statistic.

Table 1.8. Mean biases, median biases and sample variances for the estimated betas means, medians and confidence intervals for the estimated lambdas;

	N	10			25		
T		zbeta	market	labor	zbeta	market	labor
160	mbias	-	0.000	0.005	-	0.001	0.001
	medbias	-	0.000	0.003	-	0.001	0.001
	var	-	0.001	0.069	-	0.000	0.025
	pbias	-8.215	2.264	-0.781	-5.870	1.462	-0.732
	SVAR	1.714	1.612	0.219	0.679	0.615	0.108
	FM	1.431	1.386	0.181	0.588	0.633	0.100
	EIV	1.764	1.692	0.222	0.658	0.688	0.110
	mulam	-0.722	1.458	0.125	-0.487	1.100	0.153
	medlam	-0.680	1.424	0.127	-0.479	1.094	0.155
	CIlam	-4.243	-1.651	-0.313	-1.830	-0.062	-0.051
		2.601	4.673	0.551	0.776	2.333	0.373
330	mbias	-	0.000	0.004	-	0.000	0.002
	medbias	-	0.000	0.003	-	0.000	0.627
	var	-	0.000	0.067	-	0.000	0.025
	pbias	-15.675	4.479	-0.828	-8.352	2.615	-0.744
	SVAR	1.528	1.435	0.167	0.549	0.500	0.083
	FM	1.185	1.136	0.127	0.445	0.472	0.067
	EIV	1.563	1.483	0.166	0.529	0.538	0.079
	mulam	-1.468	2.447	0.098	-0.735	1.615	0.146
	medlam	-1.434	2.421	0.103	-0.727	1.609	0.145
	CIlam	-4.595	-0.317	-0.240	-1.836	0.630	-0.019
		1.479	5.366	0.421	0.322	2.619	0.312

Notes: zbeta denotes the zero-beta rate. mbias, medbias and var represent mean bias, median bias and the variance of the average of the estimated betas respectively. pbias, SVAR, FM and EIV denote the percentage bias, the sample variance, the Fama-Macbeth variance and Shaken's EIV-adjusted variance of the second-pass estimates. mulam, medlam and CIlam represent the mean, the median and the confience interval of the second-pass estimates. The true values of the second-pass estimates are [0.1, 0.447, 0.571].

Table 1.9. T-statistic, F-statistic and FAR

	N	10			25		
T		zbeta	market	labor	zbeta	market	labor
160	t	0.131	0.151	0.688	0.207	0.172	0.966
	EIV	0.026	0.036	0.599	0.149	0.124	0.939
	F	-	0.218		-	0.574	
	F_EIV	-	0.127		-	0.495	
	FAR	-	0.058		-	0.064	
330	t	0.315	0.470	0.902	0.479	0.695	0.998
	EIV	0.132	0.280	0.819	0.358	0.605	0.991
	F	-	0.837		-	0.997	
	F_EIV	-	0.665		-	0.981	
	FAR	-	0.048		-	0.056	

Notes: zbeta means the zero-beta rate. t denotes the t-statistic with the Fama-Macbeth variance. EIV represents the t-statistic with Shanken's EIV-adjusted variance. F denotes the F-statistic with the Fama-Macbeth variance. F_EIV represents the F-statistic with Shanken's EIV-adjusted variance. FAR is Kleibergen's FAR statistic.

CHAPTER 2

Finite-Sample Properties of Alternative Tests on Beta-Pricing Models in the Cases of Large, Small and Zero Betas

2.1. Introduction

In empirical analysis of the asset pricing models, the correlation between a non-traded factor and asset returns is typically small. For example, the true betas might be very small or even zero [e.g. Lustig and Verdelhan (2007)]. The small or zero betas might distort both the asymptotic and finite sample distributions of the risk premiums. Kan and Zhang (1999) point out that when betas associated with a factor are zero (or the factor is useless for all assets) and the model is misspecified, the t-statistic, for testing whether the factor should be included in the model or not has a size distortion. Asymptotically, the t-statistic tends to infinity with probability one. These findings imply that a useless factor has a large possibility of being considered as useful based on inferences drawn from the t-statistic. Kleibergen (2009) extends the results from Kan and Zhang (1999). He derives the asymptotic distributions of the estimated risk premiums in the cases of low betas and zero betas, under both a correctly specified model and a misspecified model. However, his results are built on the assumption that the covariance of the error terms and the regressors are zero in a large sample. The results indicate that the estimated risk premiums converge to a random variable instead of being $\sqrt{T}$-consistent as is the case for correctly specified models and diverge under misspecified models. Thus, the t-statistic does not follow student-t distributions under this setup. In other words, results based on the t-statistic in these cases are unreliable. Kleibergen (2009) constructs some new statistics [the Fama-Macbeth Lagrange Multiplier test (FM-LM), the Generalized Least Square Lagrange Multiplier test (GLS-LM) and Factor Anderson-Rubin test (FAR)] and shows that these statistics are asymptotically invariant to the magnitude of the betas.

The asymptotic properties of the FM procedure and these statistics have already been analyzed in the literature. However, their finite sample properties and their performance when the number of assets is large have not received enough attention. Shanken and Zhou (2007) analyze by simulation the small sample properties of the FM procedure and the t-statistic when betas are large. They observe that the estimated risk premiums do not have a large bias and the t-test has a correct size in a small sample. The bias is larger when the number of assets is larger. Chen and Kan (2005) analyze the finite sample properties of the estimated risk premiums revealing that the unconditional mean of the estimated risk premium is a complicated function of the betas, dispersion of the factors, dispersion of the error terms, number of time series observations and number of test assets. However, how these variables by themselves affect the estimated risk premiums has not been explored. Kleibergen (2009) also provides simulation results on the estimated risk premiums when betas are zero and the model is either correctly specified and misspecified. He further compares the size and power of the Wald statistic and the statistics he proposed when betas are small with $N = 25$ and $T = 143$. However, he assumes that the product of the betas and the sample means of the factors are independent of the factors in a small sample.

In constructing an asset pricing model, it is also interesting to know whether the model is correctly specified or not. One way to test such a hypothesis against a general alternative is to check whether the pricing errors (the error term in the second-pass regression) are significantly different from zero. This can be obtained by checking whether the square of the weighted pricing errors follows a central χ^2 distribution. The weights are the inverse of the covariance of the pricing errors[1]. Shanken (1985) modifies the specification test based on GLS estimators and argues that the modified version has better finite-sample behavior. An appealing feature of this statistic is that it is similar to the HJ-distance[2] with the weighting matrix as the covariance of the pricing errors in the stochastic discount factor (SDF) representation[3] of asset pricing models. The pricing errors in the SDF representation are close to the pricing errors in the two-pass regression. Zhou and Shanken (2007) investigate the finite sample properties of Shanken's modified specification tests when betas are large. We extend their results to cases when betas are small or zero.

In this chapter, we mainly focus on the finite sample properties of the estimated risk premiums, the FM procedure, the t-statistic, Kleibergen's statistics and the specification tests when betas are large, small and zero. Further, we investigate the performance of these statistics when the number of assets is large. In addition, we provide diagnostics of the poor behavior of some statistics.

Our results show that the FM-LM statistic has nearly the correct size regardless of the magnitude of the betas, the sample size and the number of assets, when the model is correctly specified. None of the tests behaves well when the model is misspecified. Both the GLS-LM and the FAR statistics are affected by the number of assets. However, the performance of the GLS-LM statistic is much better than that of the FAR when the number of assets is large, under the assumption that the returns are independent across assets and over time. Inaccurate estimation of the variance-covariance matrix of the error terms is one of the main reasons for the poor performance of the FAR and GLS-LM when betas are small and zero. The t-statistic

[1]Cochrane (2005) provides a detailed analysis of this statistic.
[2]See Hansen and Jagannathan (1997).
[3]We will introduce the SDF representation of the asset pricing models in Section 2.

has large size distortions when betas are small in a small sample. The problem gets worse when the number of assets is large. This is mainly due to the large bias of the estimated risk premiums as N increases.

The specification test with Shanken's correction has a very good size property when the assets are indepedent both across assets and over time. The behavior of the specification tests without Shanken (1985) correction is similar to the FAR statistic. After correcting the variance covariance matrix of the error term, the size distortion is reduced. However, there is still a size distortion when the number of assets is large.

The rest of the chapter is organized as follows. Section 2.2 reviews the models, pricing errors, the FM two-pass methodology, the t-statistic, Kleibergen's statistics and the specification tests. Section 2.3 introduces the simulation setup. Section 2.4 presents the simulation results and discusses the size properties of the statistics. A summary of the main findings is provided in Section 2.5.

2.2. Estimation and Testing Procedures

The expected return-beta representation of a linear factor model is presented in this section, along with the popular Fama-Macbeth two-pass procedure, t-statistic, specification tests and Kleibergen's statistics.

The notation we use in this chapter is the following:

For example, let $R = \begin{bmatrix} R_{11} & R_{12} & ... & R_{1T} \\ R_{21} & R_{22} & ... & R_{2T} \\ & & ... & \\ R_{N1} & R_{N2} & ... & R_{NT} \end{bmatrix}$ denote a N by T matrix of returns on N test assets over T periods.

Whereupon, R_i for $i = 1, ..., N$ is a column vector formed by the transpose of its row i; R_t for $t = 1, ..., T$ is a column vector representing the t^{th} column of R.

Also, let $F = \begin{bmatrix} f_{11} & f_{12} & ... & f_{1K} \\ f_{21} & f_{22} & ... & f_{2K} \\ & & ... & \\ f_{T1} & f_{T2} & ... & f_{TK} \end{bmatrix}$ be a $T \times K$ matrix of risk factors where f_t for $t = 1, ..., T$ is a $K \times 1$ column vector transposed from the t^{th} row of F.

Finally, let $B = \begin{bmatrix} \beta_{11} & \beta_{12} & ... & \beta_{1K} \\ \beta_{21} & \beta_{22} & ... & \beta_{2K} \\ & & ... & \\ \beta_{N1} & \beta_{N2} & ... & \beta_{NK} \end{bmatrix}$ be a N by K matrix of beta coefficients associated with the risk factors, where β_i for $i = 1, ..., N$ is a column vector from the transpose of row i of B.

The notations of the population time series mean and sample time series average of a matrix are $E((\bullet)_t)$ and $\overline{(\bullet)}$. For example, $E(R_t)$ denotes the population time serie mean of R and $\overline{R} = \frac{1}{T}\sum_{t=1}^{T} R_t$. $E(f_t)$ means the population time series mean of F and $\overline{f} = \frac{1}{T}\sum_{t=1}^{T} f_t$.

$\widehat{(\bullet)}$ represents the estimated value of $(\bullet)$, $(\bullet)_*$ indicates the demeaned value of $(\bullet)$ and $M_{(\bullet)} = I - (\bullet)((\bullet)'(\bullet))^{-1}(\bullet)'$ is a symmetric and idempotent matrix used to project a vector onto the space orthogonal to $(\bullet)$. ι_N and ι_T are N by 1 vector and T by 1 vector of ones, respectively, and 0_N denots an N by 1 vector of zeros. All the covariances matrices are assumed to be positive definite and all the matrices are assumed to have full rank. "$\xrightarrow{d}$" means convergence in distribution. "$\xrightarrow{p}$" indicates convergence in probability.

2.2.1. The models and the pricing errors

There are two ways to represent a linear asset pricing model. One is the beta-representation that we adopt in this chapter. The other is the stochastic discount factor (SDF) representation. It is more general than the beta-representation due to its ability to incoporate both linear and nonlinear asset pricing models. We will introduce it briefly in this section in order to form a link between the pricing errors in these two formulations.

We define R_t as the returns from N assets at time t, f_t as K systematic risk factors at time t of which a linear combination is able to explain the returns at time t, $B = [\beta_1, \beta_2 ... \beta_N]'$ as a N by K matrix and β_i for $i = 1, ..., N$ as a K by 1 vector. The following relation is assumed to hold

$$R_t = \alpha + Bf_t + \epsilon_t \text{ for } t = 1, ..., T \tag{2.1}$$

where α is a N by 1 vector, $B = cov(R_t, f_t)var(f_t)^{-1}$ and ϵ_t is a N by 1 vector.

Let $E(R_t)$ denotes expectation of R_t. Asset pricing theories suggest that

$$E(R_t) = \lambda_0 \iota_N + B\lambda_F. \tag{2.2}$$

where λ_0 is a scalar, called the zero-beta rate denoting the expected return when all betas are zero; $\lambda_F = [\lambda_1, \lambda_2, ..., \lambda_K]'$ is a K by 1 vector of risk premiums or the prices of the risks corresponding to the factors; and ι_N is a N by 1 vector of ones.

The asset pricing model is designed to answer the question: why do different assets have different expected returns? This representation implies that it is because different assets possess different systematic risks. In equilibrium, investors need to be compensated with higher returns for holding riskier assets.

Let $e = E(R_t) - \lambda_0 \iota_N - B\lambda_F$ be the pricing errors. If an asset pricing model is correct, $e = 0_N$. Otherwise, $e \neq 0_N$. This resembles the pricing errors in the SDF formulation. The SDF representation postulates that the price of an asset is determined by its future discounted payoffs. This implies that the discounted returns for all assets equal to 1 (Law of one price). Let m be the SDF. The SDF model can be written as $1_N = E(mR_t)$. For a linear asset pricing model, m is a linear function of the factors f_t, e.g. $m = a + b'f_t$, where a and b are unknown parameters. Thus, the pricing error under the SDF formula is $e_{SDF} = E(mR_t) - 1_N$.

The SDF m can take more general forms than $m = a + b'f_t$. Since the risk-free rate is also an asset, it also satisfies the equation $1 = E(mR_{f_t})$. The SDF formula can also be written as $0_N = E(mR_t^e)$ where $R_t^e = R_t - R_{ft}$. Therefore, m can be normalized to be $m = 1 + d(f_t - \mu_F)$. In this case, $e_{SDF} = E(mR_t) - 0_N$.

It can be shown that the beta representation implies the SDF representation. And a SDF m, which is a linear function of f_t, leads to a beta model. Cochrane (2005) have detailed proofs under both $m = a+b'f_t$ assuming $\mu_F = 0$ and $m = 1+d'(f_t - \mu_F)$ and assuming $E(R_t^e) = B\lambda_F$. The former case can be easily generalized without assuming $\mu_F = 0$. It can also be proved that $e_{SDF} = e$ in the latter case and e_{SDF} equals e up to a scale.

2.2.2. Fama-Macbeth Two-pass Procedure

The most commonly used methodology in estimating linear asset pricing models is the FM procedure, which includes a time series regression in the first pass and a cross-sectional regression in the second pass. The following assumptions are imposed in this section:

Assumption 1: *Assume ϵ_t is indepedent and identically distributed with $E(\epsilon_t|F) = 0_N$ and $Var(\epsilon_t|F) = \Sigma$ a positive definite matrix.*

Assumption 2: *The factors $F = [f_1, ..., f_K]$ is a T by K full column rank matrix. Let $\overline{f} = \frac{1}{T}\sum_{t=1}^{T} f_t$, $\mu_F = E(f_t)$ and $\Sigma_F = Var(f_t)$. f_t for $t = 1, ..., T$ is weakly stationary, ergodic and asymptotically normally distributed with $\sqrt{T}(\overline{f} - \mu_F) \xrightarrow{d} N(0, \Sigma_F)$.*

Let R_i be a T by 1 vector of returns of asset i for T periods and F be a T by K matrix of K factors for T periods. β_i can be estimated from the following time series regression:

$$R_i = \alpha_i \iota_T + F\beta_i + \epsilon_i, \text{ for } i = 1, ..., N \tag{2.3}$$

as

$$\widehat{\beta}_i = (F_*' F_*)^{-1} F_*' R_i. \tag{2.4}$$

where $F_* = F - \iota_T \overline{f}'$ and ι_T is a T by 1 vector of ones.

In order to estimate the zero-beta rate and risk premiums, Fama and Macbeth suggest to calculate their values at each t from a cross-sectional regression. Define λ_{0t} to be the zero-beta rate at time t, $\lambda_{Ft} = [\lambda_{1t}, \lambda_{2t}, ..., \lambda_{kt}]'$ be the risk premiums at time t and $\lambda_t = [\lambda_{0t}\ \lambda_{Ft}']'$. If B is known, the cross-sectional regression at each time t is

$$R_t = \lambda_{0t}\iota_N + B\lambda_{Ft} + u_t, \text{ for } t = 1, ..., T, \tag{2.5}$$

where $u_t = B(f_t - \mu_F) + \epsilon_t$ and $\sqrt{T}\overline{u} \xrightarrow{d} N(0, B\Sigma_F B' + \Sigma)$ under Assumptions 1 and 2.

Since B is unknown, $\widehat{B}$ is employed instead of B in empirical analysis.

Let $\widehat{X} = [\iota_N\ \widehat{B}]$ and $\widehat{\lambda}_t = [\widehat{\lambda}_{0t}\ \widehat{\lambda}_{Ft}']'$. Then,

$$\widehat{\lambda}_t = (\widehat{X}'\widehat{X})^{-1}\widehat{X}' R_t. \tag{2.6}$$

$\{\widehat{\lambda}_t\}_{t=1}^T$ can be considered as drawn from the same distribution with mean λ and covariance V. Therefore, the estimated parameters $\widehat{\lambda} = [\widehat{\lambda}_0, \widehat{\lambda}_F']'$ are given by

$$\widehat{\lambda} = \frac{1}{T}\sum_{t=1}^{T}\widehat{\lambda}_t. \tag{2.7}$$

Let $\lambda = [\lambda_0, \lambda_F']'$. The estimated variance covariance matrix of $\sqrt{T}(\widehat{\lambda} - \lambda)$, can be calculated as

$$\widehat{V} = \frac{1}{T}\sum_{t=1}^{T}(\widehat{\lambda}_t - \widehat{\lambda})(\widehat{\lambda}_t - \widehat{\lambda})'. \tag{2.8}$$

The t-statistic can be used to test whether whether the risk premiums equal their true values. Under $H_0 : \lambda_k = \lambda_{k,0}$, for $k = 1, ..., K$, the t-statistic is

$$t = \frac{\widehat{\lambda}_k - \lambda_{k,0}}{\widehat{s}_k/\sqrt{T}}, \text{ for } k = 1, ..., K, \tag{2.9}$$

where $\widehat{s}_k = [\widehat{V}^{\frac{1}{2}}]_{kk}$, which is the $[k,k]_{th}$ element of the standard error of the estimated covariance matrix for the risk premiums. The critical value is obtained from a t distribution with $T-1$ degrees of freedom or from a standard normal distribution.

It is also interesting to test whether a particular model is correctly specified or whether the pricing errors are jointly zero. The common practice in the empirical asset pricing literature is to add firm-specific factors in the second-pass regression and test whether the coefficients associated with these factors are significantly different from zero. However, there are some alternative econometric methods which can also accomplish this task.

From equations (2.1) and (2.2), it can be found that

$$\overline{R} = X\widehat{\lambda} + \widehat{e},$$

where $\overline{R} = \frac{1}{T}\sum_{t=1}^{T} R_t$, $X = [\iota_N \; B]$, $\widehat{e} = M_X(B(\overline{f}-\mu_F)+\overline{\epsilon})$, $M_X = I_N - X(X'X)^{-1}X'$, I_N is a N by N identity matrix and $\overline{\epsilon} = \frac{1}{T}\sum_{t=1}^{T}\epsilon_t$. Under Assumptions 1 and 2, it can be proved[4] that $\widehat{e}$ has the following distribution:

$$\sqrt{T}\widehat{e} \xrightarrow{d} N(0, \Sigma_{\widehat{e}}) \tag{2.10}$$

where $\Sigma_{\widehat{e}} = M_X' \Sigma M_X$.

Therefore,

$$Q_s = T\widehat{e}'\Sigma_{\widehat{e}}^{-1}\widehat{e} \xrightarrow{d} \chi^2_{N-1} \tag{2.11}$$

Since B and Σ are unknown, their estimated counterparts are used in practice. Thus, $\widehat{e} = \overline{R} - \widehat{X}\widehat{\lambda}$, $\widehat{\Sigma}_{\widehat{e}} = M_{\widehat{X}}'\widehat{\Sigma}M_{\widehat{X}}$ where $M_{\widehat{X}} = I_N - \widehat{X}(\widehat{X}'\widehat{X})^{-1}\widehat{X}'$.

The drawback of the Fama and MacBeth procedure is that it treats $\widehat{B}$ as the true B. It thus ignores the estimation error in $\widehat{B}$. Black, Jensen and Scholes (1972) propose a method of grouping the stocks into portfolios to mitigate the error-in-variable (EIV) problem. However, the grouping methods may neglect useful information in the data. Shanken (1992) analyzes the asymptotic properties of the two-pass regression methodology by taking account of the EIV problem and proposes the EIV-adjusted variance.

Under assumptions 1 and 2, $\widehat{\beta}_i$ is unbiased, $\sqrt{T}$-consistent and follows an asymptotic normal distribution

$$\sqrt{T}(\widehat{\beta}_i - \beta_i) \xrightarrow{d} N(0, \sigma_i^2\Sigma_F^{-1}), \tag{2.12}$$

where $\sigma_i^2 I_T$ is the variance of ϵ_i and Σ_F^{-1} is the inverse of the variance covariance matrix of the factors.

[4] See Shanken (1992) and Cochrane (2005).

As suggested by Shanken (1992), the asymptotic covariance matrix of the risk premiums is

$$\widetilde{V} = (1+c)\Omega + \Sigma_F^*, \tag{2.13}$$

where $c = \lambda_F'\Sigma_F^{-1}\lambda_F$, $\Omega = A\Sigma A'$, $A = (X'X)^{-1}X'$ and $\Sigma_F^* = \begin{bmatrix} 0 & 0 \\ 0 & \Sigma_F \end{bmatrix}$.

After some derivation, it can be shown that $\widetilde{V} = V + c\Omega$, where $V = \Omega + \Sigma_F^*$. Furthermore, the term $\sqrt{T}(\widehat{\lambda} - \lambda)$ can be represented as $\sqrt{T}(\widehat{\lambda} - \lambda) = (\widehat{X}'\widehat{X})^{-1}\widehat{X}'\sqrt{T}(\overline{R} - E(R_t)) - (\widehat{X}'\widehat{X})^{-1}\widehat{X}'\sqrt{T}(\widehat{B} - B)\lambda_F$. Therefore, V is the asymptotic covariance matrix of the first part and $c\Omega$ is the asymptotic covariance matrix of the second part. [5]

In this case, $\widehat{\Sigma}_{\widehat{e}} = M_{\widehat{X}}'\widehat{\Sigma}M_{\widehat{X}}(1+\widehat{c})$. We denote the modified specification test by Q_{sc}.

However, both Q_s and Q_{sc} have χ^2 distributions when the sample size is large. Shanken (1985) proposes a similar test which reflects the small-sample properties of the $\widehat{\Sigma}$. The test has the form

$$Q_c = T\widetilde{e}'\widehat{\Sigma}^{-1}\widetilde{e}/(1+\widehat{c}),$$

where $\widetilde{e} = \overline{R} - \widehat{X}\widetilde{\lambda}$ and $\widetilde{\lambda} = (\widehat{X}'\widehat{\Sigma}^{-1}\widehat{X})^{-1}\widehat{X}'\widehat{\Sigma}^{-1}\overline{R}$. Then, it follows that $Q_F = (T-N+1)Q_c/T(N-K-1)$ has an approximate F-distribution with degrees of freedom $N-K-1$ and $T-N+1$.

2.2.2.1. Kleibergen's statistics. Kleibergen (2009) argues that when the elements of B are small or zero, $\widehat{\lambda}_F$ are not $\sqrt{T}$ consistent and normally distributed. Instead, they converge to random variables when the model is correct and diverge when the model is misspecified. The t-statistic is not valid under these circumstances. He proposes some new tests – the FM-LM, GLS-LM, FAR statistics – based on GMM

[5] This result can be inferred from Shanken (1992) or Jagannathan and Wang (1998).

and the instrumental variable statistics in weakly identified models. He argues that these statistics are valid asymptotically regardless of the magnitude of the betas.

Assume the asset-pricing conditions [equation (2.2)] hold and the returns and factors are generated from the following regressions:

$$R_t = \lambda_0 \iota_N + B(f_t - \mu_F + \lambda_F) + \epsilon_t \tag{2.14}$$

$$f_t = \mu_F + v_t, \tag{2.15}$$

where covariance of ϵ_t and v_t is zero.

Let $\overline{f_t} = f_t - \overline{f}, \varepsilon_t = \epsilon_t + B\overline{v}$ and $\overline{v} = \frac{1}{T}\sum_{t=1}^{T} v_t$. After some transformation, equation (2.14) can also be written as

$$R_t = \lambda_0 \iota_N + B(\overline{f_t} + \lambda_F) + \varepsilon_t. \tag{2.16}$$

The asymptotic distributions of the statistics are derived under the following assumption:

Assumption 3: *Assume that as the number of time series observations T becomes large,*

$$\frac{1}{\sqrt{T}}\sum_{t=1}^{T}\left(\begin{pmatrix} 1 \\ f_t \end{pmatrix} \otimes (R_t - \lambda_0 \iota_N - B(\overline{f_t} + \lambda_F))\right) \xrightarrow{d} \begin{pmatrix} \varphi_R \\ \varphi_B \end{pmatrix}$$

with $(\varphi_R' \, \varphi_B')' \sim N(0, \Psi)$, $\Psi = Q \otimes \Sigma$, *and* $Q = \begin{pmatrix} Q_{11} & Q_{1F} \\ Q_{F1} & Q_{FF} \end{pmatrix} = \begin{pmatrix} 1 & \mu_F' \\ \mu_F & \Sigma_F + \mu_F \mu_F' \end{pmatrix}$.

Let $\mathbb{R}_t$ be the excess returns through subtracting the $1st$ to $(n-1)^{th}$ returns by the n^{th} returns and $\mathbb{C}$ be the excess beta. Thus, λ_0 is removed from the model. Therefore, the following conditions hold

$$E(\mathbb{R}_t) = \mathbb{C}\lambda_F$$

$$cov(\mathbb{R}_t, f_t) = \mathbb{C}var(f_t)$$
$$E(f_t) = \mu_F.$$

When an asset pricing model is correctly specified, under $H_0 : \lambda_F = \lambda_{F,0}$, $\mathbb{C}$ can be estimated as $\widehat{\mathbb{C}} = \sum_{t=1}^{T} \mathbb{R}_t(\overline{f_t} + \lambda_{F,0})[\sum_{j=1}^{T}(\overline{f_j} + \lambda_{F,0})(\overline{f_j} + \lambda_{F,0})']^{-1}$.

Lemma 2 in Kleibergen (2009): *Under $H_0 : \lambda_F = \lambda_{F,0}$, and Assumption 3,*

$$\sqrt{T}\begin{pmatrix} \overline{\mathbb{R}} - \widehat{\mathbb{C}}\lambda_{F,0} \\ vec(\widehat{\mathbb{C}} - \mathbb{C}) \end{pmatrix} \xrightarrow{d} \begin{pmatrix} \Psi_{\mathbb{R}} \\ \Psi_{\mathbb{C}} \end{pmatrix},$$

where $\overline{\mathbb{R}}$ is the time series mean of $\mathbb{R}$. $\Psi_{\mathbb{R}}$ and $\Psi_{\mathbb{C}}$ are independent and asymptotically normally distributed random variable

$\Psi_{\mathbb{R}} \sim N(0, (1 - \lambda'_{F,0}(V_{FF} + \lambda_{F,0}\lambda'_{F,0})\lambda_{F,0}) \otimes \Sigma_{N-1})$
and $\Psi_{\mathbb{C}} \sim N(0, (V_{FF} + \lambda_{F,0}\lambda'_{F,0}) \otimes \Sigma_{N-1})$
where $\Sigma_{N-1} = \Sigma_{11} - \iota_{N-1}\varpi_{1n} - \varpi_{n1}\iota'_{N-1} + \iota_{N-1}\varpi_{nn}\iota'_{N-1}$ *for* $\Sigma = \begin{pmatrix} \Sigma_{11} & \varpi_{1n} \\ \varpi_{n1} & \varpi_{nn} \end{pmatrix}$.

Below we introduce several statistics that are constructed based on the result in Lemma 2.

2.2.2.2. Fama-Macbeth LM statistic. The LM statistic is constructed using the information included in the constrained model. This test builds on the estimated pricing errors and the estimated betas. Under $H_0 : \lambda_F = \lambda_{F,0}$ and Assumption 3 and conditional on $\widehat{\mathbb{C}}$, $(\overline{\mathbb{R}} - \widehat{\mathbb{C}}\lambda_{F,0})'\widehat{\mathbb{C}} \rightarrow_d N(0, (1 - \lambda'_{F,0}(V_{FF} + \lambda_{F,0}\lambda'_{F,0})\lambda_{F,0}) \otimes \widehat{\mathbb{C}}'\Sigma_{N-1}\widehat{\mathbb{C}})$ and $\widetilde{\Sigma}_{N-1} \xrightarrow{p} \Sigma_{N-1}$. Note that $(\overline{\mathbb{R}} - \widehat{\mathbb{C}}\lambda_{F,0})$ and $\widehat{\mathbb{C}}$ are independent in large samples. Therefore, letting $P = \frac{T}{1 - \lambda'_{F,0}\widehat{Q}(\lambda_F)^{-1}_{FF}\lambda_{F,0}}$,

$$FM - LM(\lambda_{F,0}) = P(\overline{\mathbb{R}} - \widehat{\mathbb{C}}\lambda_{F,0})'\widehat{\mathbb{C}}(\widehat{\mathbb{C}}'\widetilde{\Sigma}_{N-1}\widehat{\mathbb{C}})^{-1}\widehat{\mathbb{C}}'(\overline{\mathbb{R}} - \widehat{\mathbb{C}}\lambda_{F,0}) \xrightarrow{d} \chi^2(k), \tag{2.17}$$

where $\widehat{Q}(\lambda_F) = \frac{1}{T}\sum_{t=1}^{T}(\overline{f_t} + \lambda_F)(\overline{f_t} + \lambda_F)'$ and

$$\widetilde{\Sigma}_{N-1} = \frac{1}{T-k}\sum_{t=1}^{T}(\overline{\mathbb{R}} - \widehat{\mathbb{C}}(\overline{f_t} + \lambda_{F,0}))(\overline{\mathbb{R}} - \widehat{\mathbb{C}}(\overline{f_t} + \lambda_{F,0}))'.$$

The drawback of this test is that it is not invariant to transformations of the asset returns.

2.2.2.3. GLS-LM statistic. In order to resolve the problem of the FM-LM test, Kleibergen (2009) proposes the GLS-LM statistic. The only difference between these two statistics is that the estimated pricing errors are standardized by $\widetilde{\Sigma}_{N-1}^{-1}$ with $\widetilde{\Sigma}_{N-1}^{-1} \xrightarrow{p} \Sigma_{N-1}^{-1}$. Therefore,

(2.18)

$$GLS - LM(\lambda_{F,0}) = P(\overline{\mathbb{R}} - \widehat{\mathbb{C}}\lambda_{F,0})'\widetilde{\Sigma}_{N-1}^{-1}\widehat{\mathbb{C}}(\widehat{\mathbb{C}}'\widetilde{\Sigma}_{N-1}^{-1}\widehat{\mathbb{C}})^{-1}\widehat{\mathbb{C}}'\widetilde{\Sigma}_{N-1}^{-1}(\overline{\mathbb{R}} - \widehat{\mathbb{C}}\lambda_{F,0}) \xrightarrow{d} \chi^2(k).$$

However, when the number of returns can be very large, for example 100 in Jagannathen and Wang (1996), the variance covariance matrix $\widetilde{\Sigma}_{N-1}$ can have a very large dimension. The inverse of the matrix can be hard to calculate.

2.2.2.4. The FAR statistic. The FAR test is two times the difference between the unrestricted and restricted criterion functions and is given by

$$FAR(\lambda_{F,0}) = P(\overline{\mathbb{R}} - \widehat{\mathbb{C}}\lambda_{F,0})'\widetilde{\Sigma}_{N-1}^{-1}(\overline{\mathbb{R}} - \widehat{\mathbb{C}}\lambda_{F,0}) \xrightarrow{d} \chi^2(N-1). \tag{2.19}$$

It is similar to the Anderson-Rubin (1949) statistic in the instrumental variables regression model and it is proportional to the square of the unconstrained Hansen-Jagannathan (1997) distance when it is evaluated at $\lambda_{F,0}$.

2.3. Simulation Setup

In this section, we investigate by simulation the finite sample properties of the FM estimation procedure, the corresponding t-statistic, Kleibergen's statistics and the specification tests discussed in the previous section. We also explore the performance of these statistics when the number of assets is large. The data we use in this section includes the Fama-French 6, 25 and 100 portfolios formed on size and book-to-market and the excess market return proxied by the difference between the value-weighted returns on all NYSE, AMEX, and NASDAQ stocks and the one-month T-bill rate. All these data are obtained from Kenneth French's data library[6].

We start with a simple case where the model has only one factor and the returns are independent and identically distributed. The factor is simulated by matching the mean $\mu_F = 0.649$ and variance $\sigma_F^2 = 29.841$ of the excess market factor. The returns are generated from (2.14). ϵ_t is drawn independently from a normal distribution with mean zero and covariance matrix given by the variance of the factor multiplyed by an N by N identity matrix. Thus, we are able to maintain the SNR (defined as the ratio of the standard deviation of the factor and that of the error term) to be 1. In the first chapter of this dissertation, we show that when the signal-to-noise ratio is small, the estimated risk premiums have a large bias and the t-statistic has a substantial size distortion. The standard t-test tends to conclude that the factor priced, (or equivalently, a significant risk premium) more often than it should. The smaller the signal-to-noise ratio, the worse the size distortion. The zero-beta rate is set to $\lambda_0 = 0.3$ which is close to the sample risk-free rate starting from June 1927 to present. In order to detect the effect of different betas on the properties of the statistics, we consider four scenarios. In the first scenario, $\beta_i \sim U(1,2)$ for $i = 1, ..., N$, which is close to the estimated betas corresponding to a regression of the FM portfolios on the market factor. In the second scenario, $\beta_i \sim U(0.01, 0.1)$ for $i = 1, ..., N$. In the first two scenarios, $\lambda_F = 3.89$. In the third scenario, all βs are zero. In the fourth scenario, the returns are generated by matching the mean and variance of the returns in the

[6]We are very grateful to Kenneth French for making the data available.

first scenario[7]. This is a case similar to that considered in Kan and Zhang (1999). Since the factors are useless in the last two cases, it is reasonable to set $\lambda_F = 0$.

The sample sizes are $T = 320, 640$ which are often used in the empirical asset pricing literature. In order to explore the effects of the number of assets on the properties of the FM procedure and various statistics, we vary the number of assets by letting $N = 6, 25, 100$. We compare the empirical sizes of the various statistics and the nominal sizes which are set at 1%, 5% and 10%. All reported results are based on $J = 5,000$ Monte Carlo replications.

2.4. Results

2.4.1. Parameter Estimates

In table 2.1, we present the percentage biases of the estimated risk premiums when betas are large. The biases of the estimated risk premiums are very small (the magnitude is less than 4%) and are negative in all cases. As N increases, the bias tends to increase a little bit. For example, when $N = 6$ and $T = 320$, the bias is -2.4%. It then grows to -3.7% when $N = 100$. The bias tends to decrease a bit as T rises. For example, it diminishes from -3.7% to -2.1% when $N = 100$ and T goes from 320 to 640. These results are consistent with Shanken and Zhou (2007). In their paper, they investigate the cases when $N = 25$ and 48 and $T = 60$ to 960.

Table 2.2 provides the percentage biases of the estimated risk premiums when betas are small.[8] The biases are still negative. However, the magnitude of the estimated risk premiums is more than 20 times larger than those when betas are large. For example, the bias is -83.4% in the low beta case, while it is -2.5% in the high beta case when $N = 25$ and $T = 320$. Even when the sample size rises from 320 to 640,

[7] We tried different values of the zero-beta rate. It does not seem to have a large effect on the properties of the estimation and testing results.

[8] When $N = 6$, the sample mean of the bias is not stable. For example, we have tried several experiments letting $J = 100000$ when $T = 320$, the mean of the biases fluctuates from -0.72 to 0.95.

the bias only falls to -74.6%. It implies that the standard large sample distribution might not be able to approximate the finite sample distribution of the estimated risk premium well. When N increases, the biases seem to converge to a number around -82%.

Table 2.3 shows the biases of the estimated risk premiums when betas are zero. Similar to the case when betas are large, all biases are small. However, the sign of the bias is not always negative. For example, when $T = 320$ and $N = 6$, the bias is -0.02. When $T = 320$ and $N = 100$, it is 0.01.[9]

Table 2.4 presents the biases of the estimated risk premiums when betas are zero and the model is misspecified. Overall, the biases are larger than those in the case when betas are zero and the model is correctly specifed. The sign can be either positive or negative. When T increases, the magnitude of the biases tends to rise. For example, when $N = 6$ and T grows from 320 to 640, the magnitude of the bias increases from 0.131 to 0.256. When N rises, the magnitude of the biases seem to decrease. For example, when $T = 320$ and N increases from 6 to 100, the magnitude of the bias decreases in absolute value from -0.131 to 0.

2.4.2. Tests on Risk Premiums

Table 2.5 shows the empirical sizes of the various tests of the hypothesis that the risk premium equals its true value at 1%, 5% and 10% significance levels when betas are large. All statistics have nearly correct sizes except for the FAR statistic and the t-statistic which exhibit some size distortions. For example, when $T = 320$ and $N = 25$ at the 5% level, the empirical sizes are 6.6% for the FM t-test (FM-t), 5.1% for the t-test with Shanken's correction (t-SK), 8.3% for FAR, 5.8% for FM-LM and 6.2% for GLS-LM. The t-SK has better size properties than the FM-t test since it

[9] We have also tried $J = 10000$, the magnitude of the biases is still very small. However, the results are not exactly the same as we report in Table 2.3.

corrects the EIV problem in the estimated betas. The GLS-LM statistic performs a little bit worse than the FM-LM statistic.

The sample size and number of assets do not seem to have strong effects on all the tests except for the FAR test. The FAR statistic performs poorly when N is large and/or T is small. For example, when $N = 100$ and $T = 320$ at the 5% level, FAR has an empirical size of 51.1%. It decreases to 20.4% when T equals 640. Thus sample size does seem to have a strong effect on correcting the size of FAR.

In Table 2.6, we provide the actual sizes of various statistics when betas are small. Compared to the cases when betas are large, both the t-statistic and t-test with Shanken's correction have very poor size properties under small betas. When the number of assets is large, the size distortions deteriorate. For example, when $T = 320$ and $N = 6$ at 5% level, the actual sizes for FM-t and t-SK are 28.7% and 25.9%, respectively. When $N = 100$, the sizes rise to 100% and 99.9%, respectively. Increasing the sample size does not seem to improve the situation very much. For $N = 6$ when $T = 640$ at 5%, the sizes decline to 25% for FM-t and 23% for t-SK. For $N = 100$, the results are almost the same for $T = 320$ and $T = 640$. It confirms that the finite sample distribution of the estimated risk premiums are quite different from the standard asymptotic approximations when the betas are small.

Kleibergen's statistics perform better than the t-statistic. The empirical size of the FM-LM statistic is around the true size. The performance of the GLS-LM is very close to FM-LM, especially when N is small. The performance of FAR is very close to that when betas are large. For example, when $N = 100$ and $T = 320$, the sizes of FAR are 28.9% versus 28.9% at 1% level, 51.1% versus 53% at 5% level, 62.9% versus 65.1% at 10% level. All these simulation results show that in small samples, Kleibergen's statistics are not significantly affected by the magnitude of the betas.

In Table 2.7, we present the empirical sizes of the various statistics when betas are zero. In other words, the factor is useless in explaning the time-series variation of the returns. Therefore, the risk premium associated with the factor is supposed to be zero. The results show that similar to the cases when betas are small, the FM-LM statistic has the best size properties among all the other tests. The empirical sizes are close to the nominal sizes regardless of the number of assets and the sample size. However, GLS-LM does not behave as well as in the previous cases, especially when N is large and T is small. This test is more likely to conclude that the factor is useful when it is actually useless. For example, when $N = 100$ and $T = 320$ at 1%, 5% and 10%, the empirical sizes of GLS-LM are 6.5%, 16.1% and 24.5%, respectively. The t-test behaves very well when N is small. For example, when $N = 25$ and $T = 320$ at the 5% level, the size is 4.5%. However, when N is large, it tends to under-reject the null. Unlike the cases when betas are large and small, the t-SK has worse size properties than the FM-t when the factor is useless. It under-rejects the null all the time for various numbers of assets and sample sizes considered. The FAR has similar properties as in the previous two cases. The only difference is that the size distortion becomes even worse when N is large. For example, when $N = 100$ and $T = 320$ at 5% level, the sizes are 79.9% for zero beta, 53% for small beta and 51.1% for large beta.

Table 2.8 provides the results of the various statistics when the factor is a useless factor and the model is misspecified. It is a case similar to that considered in Kan and Zhang (1999). The results show that all tests have very large size distortions.

2.4.3. Specification Tests

Table 2.9 shows the performance of the specification tests when betas are high. Q_s is based on the FM procedure. Q_{sc} is the statistic with Shanken's correction. Q_s has severe size distortions for the sample sizes considered. The size distortions become larger when the number of assets increases. For example, for Q_s at 5% significance level and $T = 320$, the rejection rates are 19.3% when $N = 6$, 53.9% when $N = 25$ and 99.7% when $N = 100$. The size properties of Q_{sc} are much better than that of Q_s in all cases. However, they are also over-sized when N is large. For example, when $T = 320$ and the true size is 5%, the estimated sizes of Q_{sc} are 6% for $N = 6$, 10.7% for $N = 25$ and 81.7% for $N = 100$. The sample size does not seem to have a large effect on the size properties of the Q_s, and Q_{sc} when N is small. However, given 100 assets, the size of Q_{sc} decreases relatively quickly. It reduces from 81.7% to 37.1%[10]. The specification test Q_F proposed by Shanken (1985) seems to outperform all the other tests, especially when N is large. Its size is close to the true size regardless of the number of assets and the sample size. For example, when $T = 320$ at 5% significance level, the estimated sizes are 5.5% for $N = 6$, 5.7% for $N = 25$ and 6.7% for $N = 100$. When $N = 100$ and T increases from 320 to 640, the empirical size decreases from 6.7% to 5.4%. Our results are consistent with Shanken and Zhou (2007) who find that the Q_F test performs well regardless of the sample size and number of assets when betas are large.

Table 2.10 presents the specification tests when betas are small. Compared to the cases when betas are large, Q_s has smaller size distortions. However, there is still a big chance to over-reject the null and conclude that the risk premium is significantly different from zero. The size distortions of the Q_{sc} and Q_F seem to be larger than those when betas are large, especially when N and T are large. For example, when $N = 100$, $T = 640$ at 5% level, Q_s falls from 98.1% to 65.3%; Q_{sc} rises from 37.1% to 56.1%; and Q_F grows from 5.4% to 13.9%. Q_F does show some size distortions when betas are small and N is large. When $N = 100$ and $T = 320$, the sizes of Q_F

[10]When the sample size is 1280, the estimated size further lowers to 16.82%.

are almost twice as large as the true sizes. It is also interesting to notice that unlike the case when betas are large, the size of Q_F deteriorates with the sample size. For example, it rises from 10.3% to 13.9% when $N = 100$ and T increases from 320 to 640. This might indicate that the distribution of Q_F with small betas is different from the F distribution.

In Table 2.11, we provide the results when betas are zero. Compared to the previous two cases, both Q_s and Q_{sc} have smaller sizes when betas are zero. For example, when $N = 25$ and $T = 320$ at the 5% level, the sizes of Q_s and Q_{sc} are 10.3% and 8.3% when betas are zero, while they are 53.9% and 10.7% when betas are large and 16.6% and 13.2% when betas are small. However, when N is large and T is small, there is still a serious size distortion. For example, when $N = 100$ and $T = 320$, the sizes are 63%, 79.5% and 86.4% at 1%, 5% and 10% levels, respectively. Similar to the case when betas are small, Q_F tends to increase when N rises. However, in this case Q_F under-rejects the null. For example, when $N = 6$ and $T = 320$ at 5% level, the size of Q_F is 2.8%. When $N = 100$, it increases to 4.5%.

2.4.4. Source of the Size Distortions

In this part, we first show that the size distortions of the t-statistic are mainly due to the biases of the estimated risk premiums and the result that the estimated risk premiums may not follow a normal distribution. Then, we show that the size distortion of Kleibergen's statistics and the specification tests are mainly a result of the poor estimation of the variance covariance of the estimated risk premiums.

The possible reasons for the size distortion of the t-statistic are the bias, the poor estimation of the variance of the estimated risk premiums and the dependence of the numerator and denominator of the t-statistic. It may also be due to the non-normal distribution of the estimated risk premiums in a finite sample especially when betas are small.

Table 2.12 presents the empirical sizes of the t-statistic using the bias-corrected risk premiums when betas are large and small. Compared to Table 2.5 and Table 2.6, the size distortion is smaller. When betas are small and N is large, for example, $N = 100$ and $T = 320$, the empirical size of the t test without and with Shanken's correction decrease from 1 and 0.999 to 0.031 and 0.026. However, they are a bit below the nominal size 5%. It is also interesting to notice that when betas are small, Shanken's correction seems to over-estimate the variance of the estimated risk premiums. It might also imply that the estimated risk premiums are not normally distributed when betas are small.

From section 2.2, we can derive that $\widehat{\lambda} = \lambda + (\widehat{X}'\widehat{X})^{-1}\widehat{X}'(\overline{R} - E(R_t)) - (\widehat{X}'\widehat{X})^{-1}\widehat{X}'(\widehat{B} - B)\lambda_F$. It can further be shown that $\widehat{\lambda} = \lambda + (\widehat{X}'\widehat{X})^{-1}\widehat{X}'\overline{\epsilon} + (\widehat{X}'\widehat{X})^{-1}\widehat{X}'(\overline{f} - \mu_F) - (\widehat{X}'\widehat{X})^{-1}\widehat{X}'(\widehat{B} - B)\lambda_F$ where $\widehat{X} = [\iota_N \ \widehat{B}]$. Note that $\widehat{B} = B + \frac{1}{\sqrt{T}}\psi_B$, where ψ_B follows a normal distribution. When B is large, $\frac{1}{\sqrt{T}}\psi_B$ can be ignored since as T increases, $\frac{1}{\sqrt{T}}\psi_B$ converges to zero based on standard asymptotics. However, when B is very small or even zero, $\frac{1}{\sqrt{T}}\psi_B$ shouldn't be ignored especially in finite samples. Therefore, the large sample distribution of $\widehat{\lambda}$ using standard asymptotics may not provide a good approximation to its finite sample distribution. It also implies that the finite sample distribution of $\widehat{\lambda}$ might be different from a normal distribution.

There are several reasons for the size distortion of Kleibergen's statistics and the specification tests Q_s and Q_{sc} in finite samples. For example, the finite-sample distribution of the pricing errors may be different from their large sample distribution. The $(\overline{\mathbb{R}} - \widehat{\mathbb{C}}\lambda_{F,0})$ and $\widehat{\mathbb{C}}$ may not be indepedent in finite samples. We show that the poor estimation of the variance covariance matrix in finite samples is one of the main reasons for the size distortion when betas are small and zero.

Table 2.13 reports the empirical sizes of the Kleibergen's statistics for different magnitudes of betas when the estimated variance covariance matrix is substituted by its true counterpart. As we can see, the empirical size of all the statistics are very close to the true size 5%, regardless of the sample size and the number of assets. For example, the empirical size of the FAR test was 52.1% when $T = 320$ and $N = 100$ and now it is 5.2%.

Table 2.14 presents the emprical sizes of the specification tests Q_s and Q_{sc} with the true variance covariance matrix of ϵ_t. The size distortion reduces a lot compared to the results based on the estimated variance covariance matrix. For example, when $N = 100$ and $T = 320$, the empirical sizes of the Q_s and Q_{sc} are 91.4% and 88.6% at 5% significance level with the estimated variance covariance matrix. They are reduced to 18.4% and 14.1%. But even after correcting the variance covariance matrix, the Q_s and Q_{sc} still seem to be affected by the number of assets when betas are small. The size distortion deteriorates when N rises. For example, when N increases from 6 to 100, and $T = 640$, Q_s increases from 12.8% to 25.7%.

2.5. Conclusion

In this paper, we provide some simulation results on the finite sample properties of the Fama-Macbeth two-pass procedure, the t statistics and Kleibergen's statistics (FM-LM, GLS-LM and FAR) and the specification tests in estimating and testing an asset pricing model when the magnitude of the betas is large, small and zero. In particular, the effect of the number of assets on the estimation procedure and test statistics are presented. We further provide a diagnosis on the poor size properties of some statistics.

Our results are based on a one-factor model assuming error terms in the first-pass regression are iid over time and across assets and follow normal distributions.

The factor is also generated from an iid normal distribution. Four experiments are conducted: a correctly specified model with high betas, small betas and zero betas and a misspecified model with zero betas. The number of assets includes 6, 25 and 100. The sample sizes are 320 and 640.

Our results show that when betas are small, the estimated risk premiums are seriously biased. The bias seems to converge as the number of assets increases. The FM-LM test has almost correct sizes when the model is correctly-specified regardless of the number of assets, the sample size and magnitude of betas. No tests behave well when the model is misspecified. Both the GLS-LM and the FAR statistics are affected by the number of assets. However, the performance of the GLS-LM statistic is much better than that of the FAR when the number of assets is large. The inaccurate estimation of the variance covariance matrix of the error terms is one of the main reasons for the poor performance of the FAR and GLS-LM when betas are small and zero. The two t-statistics have large size distortions when betas are small in a small sample. The problem gets worse when the number of assets is large. This is mainly due to the large bias of the estimated risk premiums as N rises.

The specification test with Shanken (1985)'s correction has a better size property, compared to the other two specification tests. The behavior of the specification tests without Shanken's correction is similar to the FAR statistics. After correcting the variance covariance matrix of the error term, the size distortion reduces. However, there is still a size distortion when the number of assets is large.

We recommend the FM-LM statistic and the specification test with Shanken's correction in empirical analysis due to their better size properties in finite samples regardless of the magnitude of the betas.

Table 2.1. Percentage biases of the estimated risk premium when betas are large

T	N	6	25	100
320	λ_F	-0.024	-0.025	-0.037
640	λ_F	-0.010	-0.017	-0.021

Note: beta ˜ $U(1,2)$, the true risk premium is $\lambda_F = 3.89$.

Table 2.2. Percentage biases of the estimated risk premium when betas are small

T	N	6	25	100
320	λ_F	-0.728	-0.834	-0.820
640	λ_F	-0.607	-0.746	-0.744

Note: beta ~ $U(0.01, 0.1)$,the true risk premium is $\lambda_F = 3.89$.

Table 2.3. Biases of the estimated risk premium when betas are zero

T	N	6	25	100
320	λ_F	-0.020	0.003	0.010
640	λ_F	-0.042	-0.008	0.010

Note: the true risk premium is $\lambda_F = 0$

Table 2.4. Biases of the estimated risk premium when betas are zero and the model is misspecified

T	N	6	25	100
320	λ_F	-0.131	-0.065	-0.000
640	λ_F	-0.256	0.084	-0.095

Note: the true risk premium is $\lambda_F = 0$

Table 2.5. Empirical sizes of the tests on whether the true risk premium is significantly different from zero when betas are large

	N	6			25			100		
T		0.01	0.05	0.10	0.01	0.05	0.10	0.01	0.05	0.10
	FM-t	0.029	0.090	0.157	0.015	0.066	0.122	0.015	0.070	0.133
	t-SK	0.015	0.056	0.102	0.010	0.051	0.101	0.012	0.064	0.122
320	FAR	0.015	0.058	0.114	0.020	0.083	0.140	0.289	0.511	0.629
	FM-LM	0.015	0.060	0.107	0.013	0.058	0.114	0.011	0.056	0.120
	GLS-LM	0.015	0.060	0.110	0.015	0.062	0.122	0.011	0.053	0.104
	FM-t	0.026	0.096	0.164	0.022	0.081	0.140	0.014	0.066	0.122
	t-SK	0.010	0.053	0.107	0.014	0.060	0.113	0.012	0.058	0.111
640	FAR	0.012	0.053	0.113	0.018	0.069	0.135	0.070	0.204	0.310
	FM-LM	0.012	0.055	0.110	0.014	0.063	0.125	0.010	0.059	0.107
	GLS-LM	0.014	0.056	0.115	0.020	0.074	0.133	0.013	0.064	0.121

Note: beta ~ $U(1,2)$. "FM-t" denotes the FM t test. "t-SK" means the FM t test under Shanken's EIV adjustment.

Table 2.6. Empirical size of the tests on whether the true risk premium is significantly different from zero when betas are small

	N	6			25			100		
T		0.01	0.05	0.10	0.01	0.05	0.10	0.01	0.05	0.10
320	FM-t	0.131	0.287	0.382	0.662	0.841	0.904	0.998	1.000	1.000
	t-SK	0.100	0.259	0.360	0.647	0.825	0.887	0.997	0.999	1.000
	FAR	0.013	0.055	0.108	0.022	0.085	0.142	0.289	0.530	0.651
	FM-LM	0.009	0.053	0.107	0.009	0.046	0.095	0.010	0.049	0.104
	GLS-LM	0.011	0.053	0.108	0.013	0.058	0.118	0.026	0.096	0.160
640	FM-t	0.118	0.250	0.346	0.621	0.813	0.882	0.997	1.000	1.000
	t-SK	0.097	0.230	0.326	0.602	0.788	0.862	0.995	1.000	1.000
	FAR	0.013	0.057	0.106	0.013	0.064	0.127	0.072	0.207	0.321
	FM-LM	0.008	0.045	0.092	0.011	0.050	0.095	0.011	0.050	0.096
	GLS-LM	0.009	0.046	0.093	0.010	0.058	0.112	0.016	0.069	0.132

Note: beta ˜ $U(0.01, 0.1)$. "FM-t" denotes the FM t test. "t-SK" means the FM t test under Shanken's EIV adjustment.

Table 2.7. Empirical sizes of the tests on whether the true risk premium is significantly different from zero when the betas are zero

	N	6			25			100		
T		0.01	0.05	0.10	0.01	0.05	0.10	0.01	0.05	0.10
320	FM-t	0.010	0.050	0.100	0.011	0.045	0.091	0.003	0.024	0.058
	t-SK	0.000	0.002	0.015	0.003	0.027	0.071	0.002	0.021	0.056
	FAR	0.012	0.057	0.108	0.029	0.107	0.183	0.633	0.799	0.868
	FM-LM	0.012	0.057	0.106	0.010	0.053	0.103	0.009	0.050	0.101
	GLS-LM	0.011	0.055	0.108	0.019	0.071	0.135	0.065	0.161	0.245
640	FM-t	0.010	0.046	0.097	0.008	0.041	0.091	0.006	0.033	0.075
	t-SK	0.000	0.002	0.017	0.001	0.024	0.071	0.005	0.030	0.070
	FAR	0.010	0.052	0.103	0.020	0.078	0.141	0.177	0.358	0.478
	FM-LM	0.010	0.051	0.104	0.010	0.057	0.113	0.010	0.048	0.096
	GLS-LM	0.012	0.051	0.102	0.012	0.057	0.110	0.027	0.095	0.163

Note: "FM-t" denotes the FM t test. "t-SK" means the FM t test under Shanken's EIV adjustment.

Table 2.8. Empirical sizes of the tests on whether the true risk premium is significantly different from zero when betas are zero

	N	6			25			100		
T		0.010	0.050	0.100	0.01	0.05	0.10	0.01	0.05	0.10
320	FM-t	0.666	0.747	0.790	0.586	0.685	0.737	0.786	0.837	0.859
	t-SK	0.071	0.296	0.474	0.503	0.652	0.722	0.777	0.832	0.857
	FAR	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	FM-LM	0.911	0.935	0.947	0.650	0.781	0.831	0.313	0.425	0.499
	GLS-LM	0.581	0.680	0.734	0.291	0.429	0.510	0.258	0.383	0.460
640	FM-t	0.652	0.737	0.781	0.666	0.745	0.784	0.847	0.882	0.901
	t-SK	0.056	0.271	0.443	0.588	0.720	0.771	0.841	0.880	0.899
	FAR	0.100	0.100	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	FM-LM	0.853	0.897	0.915	0.406	0.548	0.631	0.835	0.888	0.912
	GLS-LM	0.513	0.633	0.691	0.383	0.512	0.579	0.273	0.408	0.485

Note: "FM-t" denotes the FM t test. "t-SK" means the FM t test under Shanken's EIV adjustment.

Table 2.9. Model specification tests when betas are large

	N	6			25			100		
T		1%	5%	10%	1%	5%	10%	1%	5%	10%
320	Q_s	0.076	0.193	0.286	0.329	0.539	0.652	0.993	0.997	0.999
	Q_{sc}	0.014	0.060	0.118	0.033	0.107	0.182	0.647	0.817	0.880
	Q_F	0.013	0.055	0.110	0.012	0.057	0.105	0.013	0.067	0.128
640	Q_s	0.075	0.186	0.274	0.285	0.496	0.624	0.944	0.981	0.990
	Q_{sc}	0.012	0.055	0.107	0.020	0.081	0.147	0.180	0.371	0.498
	Q_F	0.011	0.053	0.105	0.012	0.055	0.113	0.012	0.054	0.112

Note: beta ~ $U(1,2)$. Q_s represents the specification test; Q_{sc} denotes the specification test after Shanken's EIV adjustment; Q_F is the specification test modified by Shanken (1985).

Table 2.10. Model specification tests when betas are small

	N	6			25			100		
T		1%	5%	10%	1%	5%	10%	1%	5%	10%
	Q_s	0.021	0.089	0.161	0.060	0.166	0.255	0.798	0.914	0.949
320	Q_{sc}	0.011	0.056	0.106	0.042	0.132	0.210	0.754	0.886	0.930
	Q_F	0.009	0.049	0.100	0.017	0.066	0.127	0.025	0.103	0.187
	Q_s	0.023	0.096	0.172	0.057	0.159	0.253	0.439	0.653	0.757
640	Q_{sc}	0.014	0.063	0.114	0.039	0.118	0.195	0.337	0.561	0.666
	Q_F	0.013	0.062	0.109	0.024	0.088	0.152	0.037	0.139	0.236

Note: beta ˜ $U(0.01, 0.1)$. Q_s represents the specification test; Q_{sc} denotes the specification test after Shanken's EIV adjustment; Q_F is the specification test modified by Shanken (1985).

Table 2.11. Model specification tests when betas are zero

	N	6			25			100		
T		1%	5%	10%	1%	5%	10%	1%	5%	10%
320	Q_s	0.011	0.053	0.105	0.027	0.103	0.177	0.630	0.795	0.864
	Q_{sc}	0.006	0.033	0.067	0.020	0.083	0.144	0.607	0.781	0.851
	Q_F	0.005	0.028	0.060	0.006	0.036	0.081	0.008	0.045	0.087
640	Q_s	0.010	0.053	0.102	0.018	0.076	0.143	0.175	0.356	0.480
	Q_{sc}	0.004	0.028	0.064	0.014	0.057	0.114	0.159	0.334	0.457
	Q_F	0.004	0.027	0.059	0.009	0.038	0.078	0.010	0.043	0.087

Note: Q_s represents the specification test; Q_{sc} denotes the specification test after Shanken's EIV adjustment; Q_F is the specification test modified by Shanken (1985).

Table 2.12. Empirical sizes of the bias corrected t tests

		high			low		
T	N	6	25	100	6	25	100
320	FM-t	0.090	0.055	0.047	0.060	0.046	0.031
	t-SK	0.047	0.042	0.044	0.011	0.032	0.026
640	FM-t	0.091	0.072	0.049	0.065	0.054	0.046
	t-SK	0.049	0.053	0.042	0.020	0.041	0.039

Note: "FM-t" denotes the FM t test. "t-SK" means the FM t test under Shanken's EIV adjustment. The significance level is 5%.

Table 2.13. Kleibergen's statistics with true error variance

	N	high			low			zero		
T		6	25	100	6	25	100	6	25	100
320	FAR	0.090	0.058	0.117	0.049	0.051	0.055	0.052	0.043	0.052
	FM-LM	0.104	0.000	0.000	0.048	0.049	0.065	0.047	0.051	0.045
	GLS-LM	0.119	0.190	0.438	0.053	0.051	0.049	0.045	0.050	0.046
640	FAR	0.068	0.062	0.099	0.057	0.048	0.048	0.047	0.051	0.054
	FM-LM	0.081	0.000	0.000	0.045	0.051	0.055	0.049	0.054	0.054
	GLS-LM	0.091	0.228	0.407	0.047	0.053	0.053	0.050	0.056	0.053

Note: The significance level is 5%.

Table 2.14. Model specification tests with true error variance

	N	high			low			useless		
T		6	25	100	6	25	100	6	25	100
	Q_s	0.175	0.435	0.863	0.059	0.093	0.184	0.051	0.045	0.052
320	Q_{sc}	0.053	0.054	0.066	0.037	0.070	0.141	0.029	0.033	0.046
	Q_s	0.178	0.446	0.862	0.077	0.128	0.257	0.053	0.051	0.053
640	Q_{sc}	0.049	0.047	0.060	0.046	0.090	0.165	0.033	0.037	0.045

Note: Q_s represents the specification test; Q_{sc} denotes the specification test after Shanken's EIV adjustment. The significance level is 5%.

CHAPTER 3

Macroeconomic Factors and the Cross-Section of Commodity Returns

3.1. Introduction

The surge in commodity futures prices in recent years has engendered renewed interest in commodities from investors, policy makers and financial economists. Since the third quarter of 2007, the nominal prices of most commodities have reached record highs. Now, commodities are considered an alternative asset class due to their excellent ability to offer diversified benefits compared with other assets.[1] Understanding how commodity prices are determined or why some commodities pay higher average returns than others, can help investors to grasp trading opportunities and guide public and private project decisions.

Research has revealed that historical average returns of long-only portfolios of commodity futures are similar to the average equity returns. For example, Bodie and Rosansky (1980) form an equally-weighted portfolio of 23 commodities from December 1949 to December 1976. They find that the portfolio has an excess annual return of 9.77% which is close to that of common stocks at 9.42%. Erb and Harvey (2006) obtain a comparable result using the Goldman Sacks Commodity Index (GSCI) from December 1969 to May 2004. The annualized compound returns of the GSCI and S&P 500 are 12.24% and 11.20%, respectively. A similar result emerges from the empirical analysis in this chapter with the GSCI from January 1986 to July 2008.[2] Commodities are also very heterogeneous. For example, from January 1986 to July

[1] For example, Greer (2000).

[2] See Table 1.

2008, the historical annualized excess return of crude oil is 16.311% with a standard deviation of 33.553, while the average return of cotton is 0.768% with a standard deviation of 24.614.[3]

The historical evidence indicates that investors are able to obtain excess returns by investing in commodity futures. However, some studies [Arnott and Bernstein (2002), Erb and Harvey (2006)] point out that the performance of the future returns should not be inferred from the past performance of returns. Forward-looking returns should be based on an understanding of the fundamental determinants of assets returns. Further, compared to stocks and bonds, which are determined by future cash flows of companies, commodities are affected by demand and supply. This implies that macroeconomic variables may play an important role in the price determination of commodities. Therefore this chapter investigates the macroeconomic or fundamental risks, faced by those US investors who employ a buy and hold strategy, from investing in commodity futures.

The results reveal that investors are compensated on average for taking on exchange rate risk. If a commodity futures contract offers a low return when the US dollar appreciates (or other currencies(y) depreciate(s)), investors in this contract, expect to obtain an excess return in equilibrium. This is reasonable since, when the dollar increases, for some commodities world demand decreases and foreign exporters will be willing to export more commodities to the US. Thus, both the spot prices and futures prices will decrease. Such commodities therefore offer low returns. US investors who hold contracts in these commodities require compensation for the exchange rate risk.

[3] See Table 2.

There have been several studies investigating how commodity futures prices are determined in equilibrium. Most of the research is based on the hypothesis that commodity futures are assets. Consequently, asset pricing models constructed for equities should also apply to commodity futures. However, the results emerging from the existing studies are mixed until now. Dusak (1973) is the first to fit commodity futures returns into a capital asset pricing model. She examines three commodity futures contracts in the agricultural sector from 1952 to 1967. But she does not find that the commodity futures are exposed to the stock market factor. Bodie and Rosansky (1980) analyze the quarterly returns of 23 commodities from 1950 to 1976 and find a significant negative market risk premium. Jagannathan (1985) tests the consumption-based Intertemporal Capital Asset Pricing Model (ICAPM) using corn, wheat and soybeans from January 1960 to December 1978. This specification is not supported by the data. Bessembinder (1992) tests whether futures and equities have uniformity in their risk and return relationships. Using both equity returns and 22 futures returns from January 1967 to December 1989, he finds that the integration of the equities and futures markets can not be rejected. The data include financial futures, agriculture futures, foreign currency futures and mineral futures. More recently, Roache (2008) tests Merton (1973)'s ICAPM by using the Fama-Macbeth procedure with time varying betas. He uses the percentage price change of 17 nearest-to-maturity commodity futures contracts constructed by the Commodity Research Bureau (CRB) from January 1973 to February 2008. He finds that investors are compensated for bearing interest rate risk.

This chapter differs from the previous literature in several respects. First, the potential factors included in the model are chosen based on theoretical arguments. Second, these factors are widely believed to possess the ability to determine commodity prices and have been found to be priced by equity returns. Third, the S&P GSCI returns used are known to be capable of reflecting the returns earned by investors through holding commodity futures well. Fourth, the estimation method applied is more conservative than the other available methods. Finally, the results are robust to different estimation methods, different data sets and longer time series.

There are several methodologies used to estimate and test a linear asset pricing model: for example, the Fama-Macbeth procedure, the Generalized Method of Moments Stochastic Discount Factor (GMM-SDF) method, etc. However, when macroeconomic variables are included in the model, many complications arise in the application of these methods. These complications include inference problems arising from weak identification, low variability of the factors and misspecified models. In order to alleviate these problems, a conservative approach is employed by adopting a recently proposed demeaned GMM-SDF method. This method has been argued to be invariant to affine transformations of the factors, and to have better power in rejecting misspecified models when the factors have low correlation with returns [see, for example, Cochrane (2005), Burnside (2007) and Kan and Robotti (2008)]. Most importantly, in determining whether a factor is priced, possible model misspecification is explicitly taken into account. This point is often ignored in the existing research.

The rest of the chapter is organized as follows. Section 3.2 explains the determination of commodity returns. Section 3.3 reviews the models and the demeaned GMM-SDF methodology. Section 3.4 describes the properties of the data. Section 3.5 explains the empirical evidence and discusses the results. Section 3.6 discusses the conclusions.

3.2. The determination of commodity returns

Unlike stock prices, which depend on the future cash flow of companies, commodities are affected by demand and supply. Gospodinov and Ng (2010) derive the determination of the expected spread of commodity prices. The model used in this chapter follows their arguments. Let $F_{t,T}$ denote time t price of a futures contract which matures at time T. S_t is the underlying commodity's spot price at time t. $i_{t,T}$ is the nominal interest rate from time t to T. The theory of storage shows that the futures price is determined by two components: 1) the gain forgone from buying the commodity instead of investing in riskless assets, $S_t(1 + i_{t,T})$; 2) the marginal convenience yield (net of storage cost) by holding a unit of commodity. Convenience yield is the benefit from holding commodities. It varies with the supply and demand of the commodities. For example, the convenience yield of a particular commodity tends to be small when there are large supplies of the commodity. Gorton, Hayashi and Rouwenhorst (2008) show that the convenience yield follows a decreasing, nonlinear relationship with inventories. Bollinger and Kind (2010) extract the convenience yield from the Schwartz (1997) three-factor futures pricing model using the Kalman filter. They find risk premiums embeded in the convenience yield. The relation can be formulated as:

$$F_{t,T} = S_t(1 + i_{t,T}) - CY_{t,T}. \tag{3.1}$$

The theory of storage implies that the nominal interest rate and the convenience yield carry information about the determination of the futures price.

An alternative view of the determination of the futures prices is the theory of normal backwardation. This theory reveals that a futures price is a biased predictor of the expected future spot price. Keynes (1930) proposes that producers short contracts in the futures market to hedge risks due to possible spot price decreases. Thus, they transfer these risks to long-side investors of the contracts. Long-side investors should be compensated with a risk premium. This indicates that the risk premium, or the components of the risk premium (e.g. possibly some financial or macroeconomic variables), affect futures prices. Define $\Psi_{t,T}$ as the risk premium. Then,

$$F_{t,T} = E_t(S_T) - \Psi_{t,T}. \tag{3.2}$$

From the previous two equations, the expected spread of spot prices can be derived, yielding

$$E_t(S_T - S_t) = S_t i_{t,T} - CY_{t,T} + \Psi_{t,T}.$$

Dividing throughout by S_t and let $R_{T-t} = \frac{S_T - S_t}{S_t}$, $cy_{t,T} = \frac{CY_{t,T}}{S_t}$ and $\varphi_{t,T} = \frac{\Psi_{t,T}}{S_t}$ gives

$$E_t(R_{t,T}) = i_{t,T} - cy_{t,T} + \varphi_{t,T}. \tag{3.3}$$

Since $i_{t,T} = i^r_{t,T} + EI_t$, where EI_t is the expected inflation at time t, the expected return can also be represented as

$$E_t(R_{t,T}) = i^r_{t,T} + EI_t - cy_{t,T} + \varphi_{t,T}. \tag{3.4}$$

Equation (3.4) implies that the expected spot return from time t to T is comprised of four components: the real interest rate, expected inflation, the convenience yield and the risk premium. From the theory of normal backwardation, $\varphi_{t,T}$ includes information on the futures price and the spot price, implying that it is not mutually exclusive with the first three terms in the right hand side of the equation (3.4).

3.3. Estimation and testing procedures

In this section, we first review the SDF representation of a linear factor model. Then, we describe the demeaned GMM-SDF procedure. In the end, the statistics for testing the slope parameters of the SDF model are presented.

3.3.1. The stochastic discount factor representation

The SDF representation of an asset pricing model explains directly how an asset is priced. The price of an asset equals its expected discounted payoffs. Since the price of the return of an asset is one, we obtain the following pricing formula

$$E(m_t R_t) = \iota_N, \tag{3.5}$$

where m_t is the SDF, R_t is a vector of gross returns on N test assets and ι_N is a N by 1 vector of ones. The SDF form is a fundamental ingredient of all asset pricing models. It encompasses, for example, linear asset pricing models and consumption-based asset pricing models. It is also applicable to all assets, including stocks, bonds, commodities, and so on. Different asset pricing models imply different SDFs. For a linear asset pricing model, $m_t = a + b' f_t$, where a and b are coefficients; f_t is a K by 1 vector, representing K financial or macroeconomic factors at time t, from which the return R_t is generated.

In the empirical asset pricing literature, excess returns are more often used than gross returns. Since the risk free rate also satisfies equation (3.5), it can be written as

$$E(m_t R_t^e) = 0_N, \tag{3.6}$$

where $R_t^e = R_t - R_{ft}$ is the excess return and 0_N is a vector of zeros.

The left hand side of the equation (3.6) can be called the pricing error. We denote it as $e = E(m_t R_t^e)$, that is the pricing error is zero when the SDF is in the set of correct SDFs.

The idea of the linear asset pricing model is simple. However, until now, there is no consensus about which model provides the best approximation despite the large number of studies in the literature. Evidently what factors should be contained in the model is a rather difficult question.

3.3.2. Estimation

3.3.2.1. The demeaned GMM-SDF method. The GMM-SDF method is used in the literature to estimate the SDF parameters and testing the specification of linear asset pricing models. Unfortunately, the SDF parameters in $m_t = a + b' f_t$ are not identifiable. For example, if $a = 0$ and $b = 0$, the pricing errors are zero. The mean of the SDF can not be identified either. For example, $E(2 * m_t R_t^e) = E(m_t R_t^e) = 0$. The common practice is to estimate a normalized version of the SDF, namely, $m_t = 1 - b^{n\prime}(f_t - E(f_t))$ or $m_t = 1 - b^{n\prime} f_t$. The former is called the demeaned GMM-SDF method. The latter is named the traditional GMM-SDF method.

Both the demeaned GMM-SDF method and the traditional GMM-SDF method are reviewed in Cochrane (2005). The demeaned procedure can be viewed as a regression of expected excess returns on the covariance of the returns and the factors. In comparison, the traditional one can be considered a regression of expected returns on the second moments of the returns and the factors. Kan & Robotti (2008) point out that the demeaned procedure is invariant to affine transformations of the factors. However, the traditional GMM is not. Thus, the traditional HJ-distance will possibly provide incorrect model rankings if the mean of the SDF is not restricted to be the same. For example, by modifying the mean of the factor, the GMM procedure could potentially favor a very poor model[4]. Kan and Robotti (2008) also provide a modified HJ-distance for the demeaned method. The demeaned SDF is

$$m_t = 1 - b^{d\prime}(f_t - \mu_F), \tag{3.7}$$

where $\mu_F = E(F)$.

The moment conditions of the GMM procedure are $E(g_t) = \begin{bmatrix} E(f_t - \mu_F) \\ E\{R_t^e[1 - (f_t - \mu_F)' b^d]\} \end{bmatrix}$.

The parameters $\theta = [\mu_F' \; b^{d\prime}]'$ can be estimated by minimizing a quadratic function

$$Q_n = g_T(\theta)' A_T g_T(\theta),$$

where $g_T(\theta) = \begin{bmatrix} g_{1T}(\theta) \\ g_{2T}(\theta) \end{bmatrix} = \frac{1}{T}\sum_{t=1}^{T} g_t(\theta) = \begin{bmatrix} \frac{1}{T}\sum_{t=1}^{T}(f_t - \mu_F) \\ \frac{1}{T}\sum_{t=1}^{T}\{R_t^e[1 - (f_t - \mu_F)' b^d]\} \end{bmatrix}$ is the sample counterpart of the previously metioned moment conditions, $g_t(\theta) = \begin{bmatrix} g_{1t}(\theta) \\ g_{2t}(\theta) \end{bmatrix} = \begin{bmatrix} f_t - \mu_F \\ R_t^e[1 - (f_t - \mu_F)' b^d] \end{bmatrix}$, A_T is the prespecified weighting matrix represented as $A_T = \begin{bmatrix} I_K & 0_{KN} \\ 0_{KK} & \widehat{V}_{RF}' W_2 \end{bmatrix}$, and $\widehat{V}_{RF} = \frac{1}{T}\sum_{t=1}^{T}(R_t^e - \widehat{\mu}_R)(f_t - \widehat{\mu}_F)'$, $\widehat{\mu}_R = \frac{1}{T}\sum_{t=1}^{T} R_t^e$. Let 0_{KN} and 0_{KK} denote K by N and K by K matrices of zeros respectively, I_K be a K by K identity matrix, W_2 be the weighting matrix in the quadratic form $Q_{2n} = g_{2T}(b^d)' W_2 g_{2T}(b^d)$. In this chapter, we let $W_2 = \widehat{V}_{RR}^{-1} = [\frac{1}{T}\sum_{t=1}^{T}(R_t^e - \widehat{\mu}_R)(R_t^e - \widehat{\mu}_R)']^{-1}$, which is the variance of the returns. This permits the calculation of the modified HJ-distance measure which will be reviewed later. Therefore,

$$\widehat{\theta} = \arg\min Q(\theta).$$

[4]Details can be found on page 818-819 in Kan and Robotti (2008).

By solving the minimization problem, we obtain

$$\widehat{\mu}_F = \frac{1}{T}\sum_{t=1}^{T} f_t$$

and

$$\widehat{b^d} = (\widehat{V}'_{RF}\widehat{V}_{RR}^{-1}\widehat{V}_{RF})^{-1}\widehat{V}'_{RF}\widehat{V}_{RR}^{-1}\widehat{\mu}_R. \tag{3.8}$$

The variance of $\widehat{\theta} = [\widehat{\mu}'_F\ \widehat{b^d}']'$ can be estimated by

$$\widehat{\Omega} = \frac{1}{T}(A_T D_T^d)^{-1} A_T \widehat{V^d} A_T (D_T^{d\prime} A'_T)^{-1}, \tag{3.9}$$

where $D_T^d = \begin{bmatrix} -I_K & 0_{KK} \\ \widehat{\mu}_R \widehat{b^a}' & -\widehat{V}_{RF} \end{bmatrix}$.

In order to account for possible serial correlation in the moment conditions, we employ the Newey and West (1987) estimator which is consistent and ensures that $\widehat{V^d}$ is positive definite. It can be calculated as

$$\widehat{V^d} = \widehat{v^d}(0) + \sum_{j=1}^{m}(1 - \frac{j}{m+1})(\widehat{v^d}(j) + \widehat{v^d}(j)'), \tag{3.10}$$

where $\widehat{v^d}(j) = \frac{1}{T}\sum_{t=1}^{T} g_t(\widehat{\theta}) g_{t+j}(\widehat{\theta})$. Since the sample size is only 271, m is set to 2.

When a model is misspecified, the variance of $\widehat{\theta}$ can be derived by the delta method:

$$\widehat{\Omega}_m = \frac{1}{T}\sum_{j=-\infty}^{\infty}\frac{1}{T}\sum_{t=1}^{T} q_t(\widehat{\theta}) q'_t(\widehat{\theta}), \tag{3.11}$$

where $q_t(\widehat{\theta}) = \widehat{H}\widehat{V}'_{RF}\widehat{V}_{RR}^{-1}(R_t^e - \widehat{\mu}_R)(1 - \widehat{y}_t) + \widehat{H}[(f_t - \widehat{\mu}_F) - \widehat{V}'_{RF}\widehat{V}_{RR}^{-1}(R_t^e - \widehat{\mu}_R)]\widehat{a}_t + \widehat{b^d}$ with $\widehat{H} = (\widehat{V}'_{RF}\widehat{V}_{RR}^{-1}\widehat{V}_{RF})^{-1}$, $\widehat{y}_t = 1 - b^{d\prime}(f_t - \widehat{\mu}_F)$, $\widehat{a}_t = \widehat{e}'_m \widehat{V}_{RR}^{-1}(R_t^e - \widehat{\mu}_R)$ and $\widehat{e}_m = \widehat{\mu}_R - \widehat{V}_{RF}\widehat{b^d}$.

Kan & Robotti (2008) argue that the traditional HJ-distance is not appropriate here since it measures the distance between the proposed SDF to the set of all correct SDFs. In this case, the mean of the SDF has been restricted to be 1. A more suitable HJ-distance will be the distance of the proposed SDF to the set of correct SDFs with a unit mean.

The modified sample HJ-distance can be calculated as the following:

$$HJ(\widehat{\theta}) = \sqrt{g_{2T}'(\widehat{b^d})\widehat{V}_{RR}g_{2T}(\widehat{b^d})}, \tag{3.12}$$

where $g_{2T}(\widehat{b^d}) = \frac{1}{T}\sum_{t=1}^{T}(R_t^e(1-(f_t-\widehat{\mu}_F)'\widehat{b^d}))$.

The modified HJ-distance is asymptotically distributed as

$$T[HJ(\widehat{\theta})]^2 \xrightarrow{d} \sum_{i=1}^{N-K}\lambda_i^d v_i \text{ as } T\to\infty, \tag{3.13}$$

where $\{\lambda_i^d\}_{i=1}^{N-K}$ are $(N-K)$ nonzero eigenvalues of $L^d = X(V_{RR})^{1/2}V_{22}(V_{RR})^{1/2}X$ with $X = [I_N - (V_{RR})^{1/2}V_{RF}(V_{RF}'V_{RR}V_{RF})^{-1}V_{RF}'(V_{RR})^{1/2}]$, $(V_{RR})^{1/2}$ is the upper-triangular matrix from the Cholesky decomposition of $V_{RR} = E((R_t^e-\mu_R)(R_t^e-\mu_R))$ and $\mu_R = E(R_t^e)$ and $\{v_i\}_{i=1}^{N-K}$ are $N-K$ independent $\chi^2(1)$ distributed random variables.

Let $\widehat{u} = \sum_{i=1}^{N-K}\widehat{\lambda}_i^d v_i$, where $\widehat{\lambda}_i$ is the sample counterpart of λ_i^d. The empirical p-value of the modified HJ-distance can be calculated by simulating different samples of $\{\widehat{u}_j\}_{j=1}^{J}$ and comparing them with the estimated $T[HJ(\widehat{\theta})]^2$. The empirical p-value is defined as

$$p_{HJ} = \frac{1}{J}\sum_{j=1}^{J}I(\widehat{u}_j \geq T[HJ(\widehat{\theta})]^2), \tag{3.14}$$

where J being the number of simulations, $I(\cdot)$ is an indicator function with $I(\cdot) = 1$ if $\widehat{u}_j \geq T[HJ(\widehat{\theta})]^2$, zero otherwise.[5]

3.4. Data Description

3.4.1. Commodity futures returns

The futures data used are components of S&P GSCI Excess Returns obtained from Morningstar, Inc. All the commodities included in the GSCI are chosen due to its liquidity. The returns are constructed from the prices of nearest-to-maturity and second nearest-to-marturity futures contracts. The contracts are rolled to the next nearest-to-maturity contracts on the fifth to ninth business days in the month before the maturity month.[6] Ideally, all the 24 components should be included. However, in order to have a long time series, only 14 commodities which can be dated back to January 1986 are selected. Platinum is also included. The return series of platinum is constructed the same way as the components of the GSCI Excess Return. So there are 15 commodities starting from January 1986 to July 2008, or 271 data points. The 15 commodities belong to five sectors: 7 in agriculture (cocoa, coffee, corn, cotton, soybean, sugar, wheat), 2 in livestock (lean hogs, live cattle), 2 in industrial metals (copper, platinum), 2 in precious metals (gold, silver) and 2 in the energy sector (crude oil and heating oil). The monthly returns are compounded on a daily basis.

3.4.2. The factors

From equation (3.4), expected spot returns are determined by the real interest rate, the factors affecting convenience yield and components of risk premiums. The theory of storage and the theory of normal backwardation suggest that the real interest rate

[5]The calculation of the empirical p-value of the modified HJ-distance is similar to that of the traditional HJ-distance described in Jagannathan and Wang (1996).

[6]The description of the data can be found in the following link:
http://www.standardandpoors.com/

and components of the risk premiums comprise information on the determination of futures prices. Erb and Harvey (2006) point out that futures returns (or the change of the futures prices) are a combination of the spot returns and roll returns. Therefore, there are reasons to believe that the real interest rate includes information about futures returns. Further, the commodities are dollar denominated. The variation of the exchange rate is widely believed to affect the supply and demand of commodities. Therefore, exchange rate growth should be an important factor in the determination of futures returns. In addition, we include stock market returns. There is no consensus on the variables influencing the convenience yield and the components of the risk premium. The stock market returns can be used as a proxy for all the other information aside from the real interest rate and exchange rate growth. The factors are described and our choices are elaborated upon in the following paragraphs.

The market factor is obtained from Kenneth R. French's website. It is the value-weighted return on all NYSE, AMEX, and NASDAQ stocks in excess of the one-month T-bill rate. Market indices are included based on the following considerations. According to Chen, Roll, and Ross (1986), macroeconomic factors can not be expected to possess all the information available to the market in a month. It is thus necessary to include market indices since the stock prices are affected instantly by public information. Furthermore, the theory of ICAPM demonstrates that market indices represent stock market performance, while, macroeconomic factors represent investment opportunities. Investors make investment decisions based on these two criteria. Ideally, the market factor should include all the assets available to investors. It should thus include commodity futures. However, since the commodity futures are only a small fraction in the market portfolios, it is reasonable to include stock returns alone.

The real interest rate is measured by subtracting expected inflation from the nominal interest rate. The nominal interest rate is the three month T-bill rate obtained from Federal Reserve Bank of St.Louis. Frankel (2006) proposes an overshooting theory to argue that commodity prices have an inverse relation with the real interest rate. Specifically, high interest rates decrease the demand for storable commodities and increase their supply. As the interest rate decreases, investors move their money from equities into commodities. Therefore, commodities are expected to inversely related to interest rate movements.

The real interest rate can be considered a proxy for the state of investment opportunities [e.g. Merton (1973) and Cox, Ingersoll and Ross (1985)]. It is the real interest rate which affects consumers intertemporal consumption and investment decisions.

The exchange rate is the US effective exchange rate on major currencies. It is constructed by the Federal Reserve with the object of inducing market pressure on the US dollar. Commodities have a global market. Their price depends on global supply and demand. Many commodities are denominated in US dollars. Therefore, investors face a risk of exchange rate fluctuations. For example, both Mundell (2002) and Frankel (2008) have documented a link between the commodity price cycle and the dollar cycle.

The exchange rate variable is priced in international asset pricing models. The idea is that world security markets can be viewed as one market. Investors from different countries face different exchange rates. Investors demand higher expected returns to be compensated for the exchange rate risk. The discussion of exchange rate risk is included in Solnik (1974), Adler and Dumas (1983), among others.

3.4.3. Summary Statistics

3.4.3.1. Properties of the returns and the factors. In Table 3.1, commodity returns are compared with other asset returns. Stock returns are represented by the return on the SP 500 index. The bond is the Ibbotson US long term coporate bond.

The compound annualized average is 8.047, which is similar to that of the stock returns 7.206. Each is larger than the average of bond returns, 3.917. Commodities have a standard deviation 18.848, which is 4 percentage points more than that for stocks and 11.6 percentage points more than that for bonds. Commodities have the lowest Sharpe ratio due to their high volatility.

Table 3.2 summarizes the statistical properties of individual commodity returns. Most commodities have positive returns in this period. Energy and industrial metals have larger average returns than other commodities. Compared to the sample period in Erb and Harvey (2006), gold and silver perform much better in the sample period. The returns increase from -4.81 and -5.30 to 1.037 and 2.712 for gold and silver respectively. Agriculture and energy tend to have large standard deviations. The risk premiums are significant for industrial metals and energy. All commodities have positive skewness, except lean hogs and live cattle. The large excess kurtosis indicates that many commodity futures returns have fatter tails than a normal distribution. The first-order autocorrelations of commodity returns are very small, indicating that these returns are not persistent.

Since it is typically believed that some commodities act as a hedge against inflation, some unconditional correlations between these asset returns and inflation are considered. Table 3.3 shows the correlation of excess returns and inflation in two different sample periods. One is from January 1986 to July 2008, the other from June 2004 to July 2008. Commodity returns have negative correlations with both stocks and bonds and a positive correlation with inflation. The magnitude of the correlations is pretty small in the whole sample period. The correlations become larger in the more recent sample period. For example, the correlation of the returns of stocks and commodities is -0.116 for the more recent sample period compared to -0.070 for the whole sample period.

Table 3.4 presents the properties of the factors. The market factor has a monthly average of 0.573 and nearly zero autocorrelation. It is the most volatile among the three factors. The monthly average of the exchange rate growth is negative, with a first-order autocorrelation 0.336. The real interest rate has an annualized average of 1.464 and is very persistent with an autocorrelation of 0.963.

Table 3.5 summarizes the correlations of the factors. All these factors have very small correlations with each other.

Table 3.6 shows the estimated betas. Most commodities tend to have negative betas associated with real interest rate. This is consistent with Frankel (2006) who argues that a low real interest rate leads to high commodity prices. Gold and silver have significantly negative relations with the real interest rate at the 5% significance level. When the value of US dollar decreases, the price of commodities tends to decrease. Gold and industrial metals show significant relations with the exchange rate at the 5% significance level. Gold is the only commodity which has a significant beta with market returns at the 5% significance level. This is consistent with the argument that commodity futures do not have close relations with stocks.

3.4.3.2. Commodity Returns over the business cycles. Gorton and Rouwenhorst (2006) argue that the negative correlation of commodity porfolio returns and equity returns is due to the different behavior of the commodity index over the business cycles. Their results are based on analyzing an equally weighted commodity index from July 1959 to December 2004. They show that the commodity index earns higher returns in the late expansion and early recession than those in their counter-stages of the business cycles, on average.

In this part, we examine the performance of the GSCI excess index returns and some individual commodity returns from the five sectors over the different stages of business cycles on average. We also investigate their performance in the latest business cycle, which is the cycle from March 2001 to December 2007. The business cycles are identified by the National Bureau of Economic Research. The commodity returns are dated back to its earliest initiation time[7]. The data on the GSCI index, the agriculture commodities and live cattle involve 6 expansions and 5 recessions.

Table 3.7 presents the averages of commodity returns at different stages of the business cycles. The GSCI index behaves in a similar pattern as that reported by Gorton and Rouwenhorst (2006). Most representative individual commodities have positive returns during expansions. In particular, the return of the crude oil is 1.507, which is the highest among all commodities. In the early expansion, the index return is very small and all individual commodities show negative returns except for live cattle and crude oil. The crude oil is very unique in the sense that it has much higher returns in the early expansion than in the late expansion. It is also interesting to notice that gold and silver have negative returns during all stages of the expansion.

Commodities do not seem to perform well in recessions on average. The index and most of the commodities demonstrate negative returns. Even though crude oil has a positive return, the magnitude is not large. Most individual commodities show negative returns in early stages of recession on average. However, the index return is positive. It is most probably due to the crude oil. It performs very well in early expansion, with a return 7.182. Nevertheless, its return in the latest recession is -6.294, far smaller than the other commodities. Gold and silver seem to be a very good hedge against the late recessions on average. They have the best performances among all the commodities. However, their returns in the early stage of the recessions are the smallest among all commodites. It is also interesting to notice that even though the index shows a higher return on average in the early recession than in the late

[7]The initiation date can be found at the bottom of the table.

recession, most individual commodities deliver smaller returns than those in the late recession.

Table 3.8 presents the results for the latest business cycle, from March 2001 to December 2007. The GSCI provides a higher return in the early expansion than in the late expansion, which is different from its average over multiple business cycles. Most of the commodities show positive returns during all stages of expansion. Copper has an exceptional performance with average returns above 2% at all stages. Further, the behavior of the gold and silver are quite different from their averages. Both of them have substantial positive returns.

In the latest recession, from March 2001 to November 2001, the GSCI index performs much worse than its historical average, with a return 2.36% lower than the average. Soybean and gold have positive returns. Crude oil performs much worse than its average. Soybean has an exceptional return in the early recession, which is 5.33%. The index shows a smaller return in the early recession than that in the late recession, which is consistent with the case in Table 6. But the four commodities in the agricultural and live stock sectors perform better in the early recession than in the late recession in this case.

3.4.3.3. Predictability of Commodity Returns. There is a strand of literature analyzing the prediction of commodity returns using common predictors similar to those predicting equity returns. Since commodity returns are also assets, the prediction of commodity returns should have the same explanation as equities. One is due to market inefficiency, which means prices deviate irrationally from fundamentals. The other is due to the changing of equilibrium expected returns through time. If some variables are able to predict commodity returns, it may imply that the asset pricing model holds from time to time conditional on the information proxied by these variables.

In this part, the results of the predictive ability of three variables are presented: the yield spread (YS), the dividend yield (DY) and the short rate (SR). The yield spread and the dividend yield have been argued to be proxies for the business cycle [see, for example, Fama and French (1989)]. Bessembinder and Chan (1992) analyze 12 futures markets from January 1975 to December 1989 and document that T-bill yield, dividend yield and yield spread have the ability to predict commodity returns. More recently Hong and Yogo (2009) construct an equally-weighted portfolio using 34 fully collaterized commodity futures and find that the short rate and yield spread possess the ability to predict commodity returns in the sample period from January 1965 to December 2008.

The dividend yield is constructed using the value-weighted return including dividends and excluding dividends as described in Fama and French (1988). It is the summation of the 12-month dividends over the price at the beginning of the period. The yield spread is the difference between Moody's seasoned Aaa corporate bond yield and the 1-month T-bill rate following Fama and French (1989). The short rate is the 1-month T-bill rate obtained from Kenneth R. French's website. Table 3.9 presents the properties of the three information variables from December 1985 to June 2008. The first-order autocorrelation of the dividend yield is 0.986, which is the largest among all three variables. The yield spread is the least persistent, with the autocorrelation 0.9051. Furthermore, the correlation of the yield spread and the dividend yield is very small (-0.025). In comparison, the correlations of the short rate with the dividend yield and the yield spread are much larger.

In Table 3.10, we report the regression of the commodity returns on the lagged information variables as in Hong and Yogo (2009). The variables are detrended by subtracting the previous 12-month moving average to guard against spurious regression bias as suggested by Ferson, Sarkissian, and Simin (2003). The results show that these variables do not have statistically significant power to predict the GSCI index in the sample period. For individual commodities, we observe that the dividend yield is significant for copper and wheat at the 5% significance level. The yield spread and the short rate exhibit some predictive power only for copper at the 5% significance level and for crude oil at the 10% significance level.[8] The results are somehow consistent with the results in Hong and Yogo (2009). They do not find that these variables possess significant power in predicting commodity index returns in the sample period from 1987 to 2008.

3.5. Empirical Evidence

3.5.1. Estimation Results

The results investigating the validity of CAPM are presented in panel A of Table 3.11. The demeaned method shows that the t-value for the market factor is 1.506 assuming a correctly specified model and 1.263 under a potentially misspecified model. This result is consistent with Dusak (1973) and Bodie and Rosansky (1980). The HJ-distance measure is not statistically different from zero at the 10% significance level. This can be attributed to the very volatile nature of commodity returns.

The results from the three-factor model are shown in panel B of Table 3.11. The demeaned method provides similar results under a correctly specified model and under a misspecified model. This is expected since the HJ-distance is 0.214 with a p-value of 0.817. The pricing error is very small in this case. The t-values of the exchange rate growth are -2.893 under a correctly specified model and -2.694 under a misspecified model. Therefore, it can be concluded that exchange rate growth is priced but the real interest rate is not. This indicates that investors are not compensated for the real

[8] We also experimented with combinations of two information variables out of these three in a conditional asset pricing model with the market, real interest rate and exchange rate as the state variables. We did not find any significant results.

interest rate risk. This result is different from Roache (2008), who shows that the real interest rate is priced for the period from January 1973 to February 2008. However, based on the theory of storage and the theory of backwardation, the real interest rate does affect commodity returns. Furthermore, from Table 3.5, we observe that its correlation with the market factor and the exchange rate is very small. This implies that it also brings extra information in explaining commodity returns. Therefore, the 3-factor model is favoured.

In order to compare different models visually, the scatter graphs of the estimated expected returns versus realized average returns under different model specifications are plotted. The lines in the graphs are 45-degree lines: the closer the scatter points are to the line, the better the fit of the model. Figure 1 shows the plots of different model specifications. From top to bottom, the models are CAPM with stock market returns as the only factor and the three-factor model. The points in the first graph spread out widely. The estimated expected returns of many commodities are very different from the corresponding realized expected returns. This indicates that the CAPM has a very poor fit. The second graph shows that, after including the exchange rate and the real interest rate in the model, the estimated expected returns become much closer to the corresponding realized returns. This reinforces the belief that the three-factor model is much better than the CAPM.

3.5.2. Robustness Check

For a robustness check, we first apply the traditional GMM to estimate the above models. The advantage of the traditional GMM-SDF to the demeaned method is that the traditional method is a one-step method, while the demeaned method is a two-stage procedure. In Table 3.12, the results for the 3-factor model estimated by the traditional method are reported. The results are quite similar to the those estimated by the demeaned method in terms of both sign and statistical significance. The exchange rate is more significant this time with a t-value -2.982.

We also tried the estimation using the Fama-Macbeth method to estimate the risk premiums directly. Table 3.13 shows that the exchange rate is also priced by using the method with and without Shanken's correction [see Shanken (1992)].

In order to check whether the results hold in a long period, returns are constructed using commodity data obtained from US Commodity Research Bureau (CRB). Since CRB only has price data instead of return data, returns are constructed by adding the percentage net basis to the percentage change of the commodity price. The percentage net basis is defined as the ratio of the net convenience yield – marginal convenience yield minus per-unit storage cost – to the previous period's price. The marginal convenience yield is obtained from the theory of storage. By using the same 15 commodities from April 1983 to July 2008, we find that the exchange rate is also priced. Table 3.14 shows that the t-value is -3.724 assuming the model is correct and -3.314 under a potentially misspecified model.

The high persistence of the real interest rate process poses some empirical challenges to the stationarity assumption that allows the inclusion of real interest rate in the model in levels. Also, some experimentation with different real interest rate processes are undertaken: for example, the real interest rate constructed by subtracting the previous 12-month moving average and a process that subtracts the previous 3-year moving average. In all these cases, the exchange rate is found to be significant at the 5% significance level.

3.5.3. Possible Economic Explanations

The result using the GSCI excess commodity futures from January 1986 to July 2008 indicates that, after controlling for real interest rate movements and the market returns, the risk premium of the exchange rate is significant and has a negative sign. This means that commodities whose returns are negatively correlated with the exchange rate should be expected to have higher returns, in equilibrium. For example, as the US dollar appreciates or other currencies depreciate relative to the US dollar, prices of some dollar-denominated commodities drop. The reason can be that the world demand for these commodities decreases. It can also be due to the increased supply of commodities. Foreign exporters, especially the suppliers of storable goods, are willing to provide more commodities because the appreciation of the dollar offers them higher returns. This is because the supply of storable commodities is able to increase in the short term, given enough inventories. Therefore, the return to US investors on these commodities decreases. They are expected to receive a reward for bearing the risk in equilibrium.

3.6. Conclusion and Discussion

This study contributes to the empirical literature that explores the determination of commodity futures prices in equilibrium. It also sheds some light on the current debate on the profitability of investing in long-only portfolios of commodity futures and the predictability of common futures returns.

In this study, macroeconomic determinants of the commodity returns in both a static asset pricing model framework and by allowing the risk premiums to vary over time, has been investigated. The macroeconomic factors included are the stock market factor, the real interest rate and the exchange rate. These factors have been argued to be the main determinants of commodity prices. They have also been found to be priced by equity returns. This study also considers the predictability of the commodity futures returns using three variables: the short rate, the dividend yield and the yield spread. The results show that these variables tend to have very weak predictive power only for copper and wheat among all the 15 commodities.

The model is estimated in its SDF representation using the demeaned GMM-SDF method. This method is invariant to linear transformations of the factors. It also has some power to detect useless factors. We report both results under the correctly specified model and under model misspecification. Since the GMM method we are using is not an efficient GMM (because it uses a prespecified weighting matrix), the standard errors obtained might be larger than those with the efficient GMM. This could be one reason for statistical insignificance for the interest rate. On the other hand, this method allows us to make objective comparisons across different SDF and model specifications using a common weighting matrix.

The main result of the chapter is that exchange rate risk is priced. Specifically, the commodities which are negatively related to the exchange rate provide higher expected returns in equilibrium. A possible explanation could lie in the willingness of commodity suppliers to provide more commodities and/or that the world demand shrinks when the dollar value increases, or both. Thus, the prices of some commodities fall. US investors are worse off and they ask for compensation for the exchange rate risk they are bearing. The results are robust across different estimation methods and different data sets and over a longer time period.

Figure 3.1. Estimated expected returns versus realized expected returns
$sample/lowbeta/writings/disertation/LMJ3P202.wmf$
$sample/lowbeta/writings/disertation/LMJ3P203.wmf$

Note: The lines in the graphs are the 45-degree lines. The closer the scatter points to the line, the better the fit of the model. The top graph is the plot of the CAPM. The bottom graph is the plot of the 3-factor model including the market factor, the real interest rate factor and the exchange rate factor.

Table 3.1. Comparison of the commodity futures return and other asset returns

	average	std	Sharpe ratio
GSCI	8.047	18.848	0.427
stock	7.206	14.825	0.486
bond	3.917	7.249	0.540

Note: the results in the table are compound annualized average returns, standard deviation and Sharpe ratio calculated from monthly returns from January 1986 to July 2008. The "GSCI" is the Goldman Sacks Commodity Index. The "stock" represents the SP 500 index. The "bond" is the Ibbotson US long term coporate bond. All returns are excess returns.

Table 3.2. Properties of commodity futures returns

	average	std	Sharpe ratio	t-stat	skewness	kurtosis	AR(1)
cocoa	-4.976	28.567	-0.174	-0.847	0.814	4.680	-0.092
coffee	-3.578	39.219	-0.091	-0.441	1.001	5.553	-0.006
corn	-4.487	24.700	-0.182	-0.882	0.976	9.198	0.002
cotton	0.768	24.614	0.031	0.148	0.513	3.989	-0.037
soybean	4.868	22.346	0.218	1.013	0.119	4.361	-0.078
sugar	7.753	33.056	0.235	1.077	0.619	5.038	0.062
wheat	-1.971	23.063	-0.085	-0.410	0.263	3.218	0.010
lean hogs	-0.868	24.673	-0.035	-0.168	-0.044	3.881	-0.070
live cattle	3.980	13.568	0.293	1.369	-0.523	5.987	0.010
copper	17.921	26.578	0.674	2.968	0.852	5.856	0.101
gold	1.037	13.707	0.076	0.358	0.518	3.814	-0.017
silver	2.712	24.195	0.112	0.526	0.405	4.110	-0.097
platinum	9.980	20.761	0.481	2.186	0.709	6.808	-0.048
crude oil	16.311	33.553	0.486	2.154	0.125	7.160	0.111
heating oil	13.695	33.704	0.406	1.819	0.532	4.313	0.030

Note: This table summarizes the compound annualized average, standard deviation, Sharpe ratio, t-statistic, skewness, kurtosis and the first-order autocorrelation of the individual commodity returns. The data series are monthly excess returns from January 1986 to July 2008.

Table 3.3. Correlation of stocks, bonds, commodity futures and inflation

	GSCI	stock	bond
January 1986-July 2008			
stock	-0.070	-	-
bond	-0.052	0.168	-
inf	0.074	-0.167	-0.168
June 2004-July 2008			
stock	-0.116	-	-
bond	-0.087	0.006	-
inf	0.134	-0.279	-0.378

Note: The results are correlations of the excess returns and inflation in two different sample periods. One is from January 1986 to July 2008. The other one is from June 2004 to July 2008. The "GSCI" is the Goldman Sacks Commodity Index. The "stock" represents the SP 500 index. The "bond" is the Ibborton US long term coporate bond. The "inf" represents the inflation.

Table 3.4. Properties of factors

	market	rir	ex
average	0.573	1.464	-0.198
std	4.332	1.720	1.665
AR(1)	0.047	0.963	0.336

Note: The table summarizes the average, standard deviation and first-order autocorrelation of the factors. The "market" is the market factor, which is the value-weighted return on all NYSE, AMEX, and NASDAQ stocks in excess of the one-month T-bill rate. The "rir" represents the real interest rate measured by subtracting the expected inflation (constructed by the survey research center of University of Michigan) from the three month T-bill rate. The "ex" represents the US effective exchange rate on major currencies. The data series are from January 1986 to July 2008.

Table 3.5. Correlation of the factors

	market	rir	ex
market	1	-	-
rir	0.019	1	-
ex	-0.047	0.069	1

Note: The results are the correlations of the four factors. The "market" is the market factor, which is the value-weighted return on all NYSE, AMEX, and NASDAQ stocks in excess of the one-month T-bill rate. The "rir" represents the real interest rate measured by subtracting the expected inflation (constructed by the survey research center of University of Michigan) from the three month T-bill rate. The "ex" represents the US effective exchange rate on major currencies. The data series are from January 1986 to July 2008.

Table 3.6. Estimated betas

	market	rir	ex	R^2
cocoa	-0.183*	-0.605*	-0.293	0.030
coffee	0.135	-0.282	0.295	0.006
corn	0.110	-0.109	0.345	0.011
cotton	0.081	0.438	0.050	0.014
soybean	0.053	-0.483*	0.106	0.018
sugar	-0.018	0.190	-0.031	0.001
wheat	0.102	-0.198	0.102	0.007
lean hogs	0.024	0.416*	-0.042	0.010
live cattle	0.071	0.099	-0.032	0.008
copper	0.132	0.143	-0.828**	0.039
gold	-0.142***	-0.277**	-0.483***	0.081
silver	0.087	-0.570***	-0.106	0.024
platinum	0.120	-0.175	-0.528***	0.034
crude oil	-0.167	0.339	-0.547	0.016
heating oil	-0.125	0.155	-0.666*	0.016

Note: The results in this table are the estimated betas obtained from regressions of the individual excess commodity returns on the four factors. * , **, and *** denote significance at 10%, 5% and 1% levels, respectively. The standard errors are the robust standard errors. The "market" is the market factor, which is the value-weighted return on all NYSE, AMEX, and NASDAQ stocks in excess of the one-month T-bill rate. The "rir" represents the real interest rate measured by subtracting the expected inflation (constructed by the survey research center of University of Michigan) from the three month T-bill rate. The "ex" represents the US effective exchange rate on major currencies. The data series are from January 1986 to July 2008.

Table 3.7. Commodity futures returns at different stages of business cycles

	expansion	EE	LE	recession	ER	LR
GSCI	0.602	0.090	1.114	-0.328	0.544	-1.201
corn	0.000	-0.553	0.554	-0.463	-0.601	-0.325
soybean	0.820	-0.021	1.662	-0.198	0.479	-0.876
wheat	0.348	-0.577	1.272	-1.274	-2.320	-0.227
live cattle	0.468	0.223	0.713	-0.023	-0.210	0.164
copper	0.655	-0.270	1.580	-1.377	-2.606	-0.149
gold	-0.573	-0.928	-0.219	-0.699	-3.394	1.996
silver	-0.272	-0.464	-0.080	-3.554	-8.848	1.740
crudeoil	1.507	2.126	0.887	0.444	7.182	-6.294

Note: The table presents the monthly averages of excess commodity returns at different stages of the business cycles. The "EE", "LE", "ER" and "LR" represent early expansion, late expansion, early recession and late recession, respectively. The "GSCI" is the Goldman Sacks Commodity Index.

Table 3.8. Commodity futures returns at the most recent business cycle from March 2001 to December 2007

	expansion	EE	LE	recession	ER	LR
GSCI	1.961	2.158	1.763	-2.691	-1.221	-4.162
corn	-0.244	-1.087	0.599	-1.178	0.143	-2.499
soybean	1.652	1.777	1.528	0.756	5.331	-3.818
wheat	0.521	-0.708	1.750	-0.367	0.326	-1.059
live cattle	0.348	0.913	-0.218	-1.719	-0.237	-3.201
copper	2.616	2.156	3.077	-1.095	-3.139	0.950
gold	1.342	1.338	1.346	0.601	0.642	0.561
silver	1.788	1.871	1.704	-0.705	-0.822	-0.588
crudeoil	2.160	3.639	0.681	-3.954	-0.534	-7.374

Note: The table presents the monthly averages of excess commodity returns in the latest business cycle, which is the cycle from March 2001 to December 2007. The "EE", "LE", "ER" and "LR" represent early expansion, late expansion, early recession and late recession, respectively. The "GSCI" is the Goldman Sacks Commodity Index.

Table 3.9. Descriptive statistics for the information variables

variable	average	std deviation	AR(1)	correlation with	
-	-	-	-	DY	YS
DY	2.489	0.928	0.986	-	-
YS	0.244	0.116	0.905	-0.025	-
SR	0.373	0.160	0.944	0.573	-0.668

Note: The table shows the average, standard deviation, first-order autocorrelation and correlations of the lagged information variables as in Hong and Yogo (2009). The variables are detrended by subtracting the previous 12-month moving average to guard against spurious regression bias as suggested by Ferson, Sarkissian, and Simin (2003). The "YS", "DY" and "SR" denote the yield spread, the dividend yield and the short rate. The sample period is from January 1965 to December 2008.

Table 3.10. Descriptive statistics for the information variables

	YS	*t*-stat	DY	*t*-stat	SR	*t*-stat	R^2	F-test	p-value
GSCI	8.064	1.03	2.044	1.33	8.810	1.13	0.010	1.28	0.282
cocoa	9.127	0.67	4.524	2.03	-2.373**	-0.18	0.018	1.52	0.208
coffee	32.025	1.38	0.653	0.20	34.427	1.49	0.012	0.76	0.516
corn	-6.826	-0.68	2.317	1.22	-7.595	-0.72	0.007	0.70	0.551
cotton	-15.531	-1.38	0.480	0.25	-8.060	-0.66	0.010	1.02	0.382
soybean	4.250	0.46	3.290	1.82*	-3.981	-0.42	0.016	1.16	0.325
sugar	4.232	0.20	3.303	1.34	11.656	0.55	0.011	1.04	0.374
wheat	-15.785	-1.29	4.472	2.83**	-9.688	-0.85	0.038	4.75	0.003**
lean hogs	-12.607	-0.98	-2.999	-1.60	-6.821	-0.53	0.011	1.02	0.382
live cattle	0.639	0.09	-0.940	-0.79	4.335	0.60	0.006	0.48	0.698
copper	40.374	2.53**	4.433	2.19**	50.977	3.13***	0.073	4.77	0.003**
gold	-6.650	-0.95	0.834	0.80	-4.211	-0.61	0.008	0.84	0.475
silver	-16.237	-1.50	0.441	0.24	-13.569	-1.28	0.007	0.94	0.420
platinum	-16.203	-1.50	1.074	0.58	-10.819	-0.99	0.014	1.17	0.320
crude oil	25.433	1.95*	3.021	1.00	28.740	1.94*	0.015	1.91	0.128
heating oil	20.706	1.11	2.805	0.97	19.468	1.12	0.009	0.69	0.559

Note: The table shows the results from a regression of the excess commodity returns on the lagged information variables as in Hong and Yogo (2009). The variables are detrended by subtracting the previous 12-month moving average to guard against spurious regression bias as suggested by Ferson, Sarkissian, and Simin (2003). The "YS", "DY" and "SR" denote the yield spread, the dividend yield and the short rate. The sample period is from January 1965 to December 2008.

Table 3.11. Estimation results using demeaned SDF method

panel A				
risk permiums	market	rir	ex	HJ-dist
estimates	0.075	-	-	0.295
t	1.506	-	-	-
t_mis	1.263	-	-	-
p_value	-	-	-	0.116
panel B				
estimates	0.126	0.026	-0.419	0.214
t	1.905	0.172	-2.893	-
t_mis	1.859	0.145	-2.694	-
p_value	-	-	-	0.817

Note: The table shows the estimation results from the CAPM and our four-factor asset pricing model. They are estimated by the demeaned GMM/SDF method. The "t_mis" represents the t value under a misspecified model. The "HJ-distance" is the HJ-distance statistic. The "market" is the market factor, which is the value-weighted return on all NYSE, AMEX, and NASDAQ stocks in excess of the one-month T-bill rate. The "rir" represents the real interest rate measured by subtracting the expected inflation (constructed by the survey research center of University of Michigan) from the three month T-bill rate. The "ex" represents the US effective exchange rate on major currencies. The data series are from January 1986 to July 2008.

Table 3.12. Estimation results using traditional SDF method

risk permiums	market	rir	ex	HJ-dist
estimates	0.084	0.136	-0.323	0.159
t	1.752	1.336	-2.982	-
p_value	-	-	-	0.827

Note: The table shows the estimation results from the 3-factor asset pricing model. It estimated by the traditional GMM/SDF method. The "HJ-distance" is the HJ-distance statistic. The "market" is the market factor, which is the value-weighted return on all NYSE, AMEX, and NASDAQ stocks in excess of the one-month T-bill rate. The "rir" represents the real interest rate measured by subtracting the expected inflation (constructed by the survey research center of University of Michigan) from the three month T-bill rate. The "ex" represents the US effective exchange rate on major currencies. The data series are from January 1986 to July 2008.

Table 3.13. Estimation results using the Fama-Macbeth two-pass regression

risk permiums	market	rir	ex
estimates	1.152	0.463	-1.348
t	0.899	1.245	-3.102
t-SK	0.714	0.918	-2.356

Note: The table shows the estimation results from the 3-factor asset pricing model. It estimated by the Fama-Macbeth method. The "market" is the market factor, which is the value-weighted return on all NYSE, AMEX, and NASDAQ stocks in excess of the one-month T-bill rate. The "rir" represents the real interest rate measured by subtracting the expected inflation (constructed by the survey research center of University of Michigan) from the three month T-bill rate. The "ex" represents the US effective exchange rate on major currencies. The data series are from January 1986 to July 2008.

Table 3.14. Estimation results using demeaned SDF method with CRB data

risk permiums	market	rir	ex	HJ-dist
estimates	0.207	-0.032	-0.711	0.325
t	2.410	-0.265	-3.724	-
t_mis	2.318	-0.222	-3.314	0.669

Note: The table shows the estimation results from the 3-factor asset pricing model. It estimated by the demeaned GMM/SDF method. The returns are constructed using the prices from CRB. The "market" is the market factor, which is the value-weighted return on all NYSE, AMEX, and NASDAQ stocks in excess of the one-month T-bill rate. The "rir" represents the real interest rate measured by subtracting the expected inflation (constructed by the survey research center of University of Michigan) from the three month T-bill rate. The "ex" represents the US effective exchange rate on major currencies. The data series are from April 1983 to July 2008.

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